Evaluation of ASHRAE Dilution Models to Estimate Dilution from Rooftop Exhausts

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ABSTRACT

Re-entrainment of building exhausts may lead to poor indoor air quality, potential health hazards, worker complaints, and lower productivity. To minimize re-entrainment, the American Society of Heating, Refrigerating, and Air Conditioning Engineers (ASHRAE) recommends minimum dilution models $D_r$ and $D_s$ to estimate worst-case dilutions at fresh-air intakes. The $D_r$ and $D_s$ models predict plume center-line (worst-case) dilution at roof level, assuming that the plume has a Gaussian concentration profile in both the vertical and lateral directions. The $D_r$ model considers the effect of plume rise; however, the $D_s$ model assumes negligible plume rise and is primarily recommended for wall vents and capped stacks. This paper evaluates the ASHRAE (2003, 2007) dilution models using data from wind tunnel and field experiments carried out with typical low-rise and high-rise buildings. Some comparisons with the 2011 dilution models are also presented. The effectiveness of the dilution models in modeling the downwash effect of a rooftop structure (RTS) on plume dispersion is also evaluated. Comparisons between estimated and measured concentration data obtained from present and previous studies indicate that the ASHRAE model estimates are sensitive to building geometry, stack height, exhaust speed, sampler location, and the downwash effect of RTS. Depending on the interpretation and user experience, the models may significantly underpredict or overpredict the dilution level at fresh-air intakes. For a typical low-rise building, the ASHRAE 2003 $D_r$ model overpredicted dilutions by a factor of 10 when the upwind RTS was within 15 m (49.2 ft) of the stack and by a factor of 2 with no RTS present. On the other hand, the ASHRAE 2007 $D_r$ model was conservative for nearly all cases evaluated with dilution estimates 10 to 100 times lower than the measured values. The ASHRAE 2003 and 2007 $D_r$ models underpredicted dilutions by a factor of 10 to 100 irrespective of building height and whether or not an RTS was present. The 2011 dilution models are generally more conservative than the 2007 model. For both $D_r$ and $D_s$ models, the predictions improved with increase in distance from the stack. Some basic knowledge of building aerodynamics is helpful in the application of the ASHRAE dilution models, which otherwise should be used with caution.

INTRODUCTION

One of the major causes of poor indoor air quality in buildings is due to exhaust reingestion at fresh-air intakes. This may lead to potential health hazards and lower productivity for people working in these buildings. Universities, hospitals, and industrial laboratories, as well as manufacturing facilities, are particularly vulnerable to this phenomenon since they emit a wide range of toxic and odorous chemicals. Consequently, numerous incidents of poor indoor air quality have been documented (Meroney 1999).

The most fundamental and effective way to minimize the problem of exhaust reingestion is the suitable design and placement of exhaust stacks and air intakes in buildings. This can be accomplished through the use of numerical dispersion models. The most widely used analytical models for estimating dispersion of building roof exhausts are the minimum dilution models recommended by ASHRAE. These models are largely derived from works of Halitsky (1963) and Wilson and colleagues (1979, 1985, 1994, and 1998). ASHRAE also provides a Geometric Stack Design method to estimate minimum stack height to avoid plume entrainment using the flow recirculation zones of a building.
and is based on water flume experiments of Wilson et al. (1979).

Few studies have evaluated the ASHRAE dilution models and most of them have investigated pre-2000 versions (e.g., Schuyler and Turner 1989; Petersen and Wilson 1989; Lowrey and Jacko 1996; Wilson and Lamb 1994; Lazure et al. 1998; and Stathopoulos et al. 2002). The ASHRAE 2003 $D_r$ model was evaluated using field measurements by Stathopoulos et al. (2003). The dispersion experiments were conducted on the roof of a typical low-rise building for stack heights 1 and 3 m (3.3 and 9.8 ft). Concentration measurements were obtained for various wind directions and exhaust speeds. The authors showed that the ASHRAE 2003 $D_r$ model could lead to unconservative dilution estimates at distances less than 20 m (65.6 ft) from the stack. Saathoff et al. (2009) evaluated ASHRAE 2003 and 2007 $D_r$ and $D_s$ dilution models using primarily wind tunnel experiments. The influence of building height and downwash effect of an RTS on roof-level dilutions was also evaluated. The authors showed that the ASHRAE 2003 $D_r$ model does not take into consideration the downwash effect of RTS effectively and significantly overpredicted dilutions for the low-rise building. On the other hand, the ASHRAE 2007 $D_r$ model predicted reasonable dilution values. The ASHRAE 2003/2007 $D_s$ model underpredicted dilutions for all cases tested. However, the observations were limited to a stack height of 4 m (13.1 ft) and one exhaust momentum ratio ($M = 2$).

The present study aims at evaluating the ASHRAE 2003 and ASHRAE 2007 $D_r$ and $D_s$ minimum dilution models using data obtained from wind tunnel experiments with isolated buildings and field experiments with buildings located in urban environment for a wide range of stack heights and exhaust speeds. The effectiveness of the models for cases where the stack is downwind of a rooftop structure is investigated as well. A brief description of the ASHRAE Geometric Stack Design method is also provided. Dilution estimates obtained with the ASHRAE dilution models are compared with the experimental results from the present study and from previous studies.

**EXPERIMENTAL METHODOLOGY**

Wind tunnel experiments were carried out in the boundary layer wind tunnel at Concordia University. The wind tunnel working section is 1.8 by 1.8 m (5.9 by 5.9 ft) and the length is 12.2 m (40.0 ft). Complete details on the wind tunnel may be found in Stathopoulos (1984). Experiments were carried out using an isolated building of variable height. The building models were constructed at a scale of 1:200 and had a square plan with along-wind length (L) and cross-wind width (W) of 50 m (164.0 ft). The full-scale building heights (H) were 15 m (49.2 ft) and 60 m (197.0 ft). The stack was located at a distance of 0.4L from the building leading edge. A wide rooftop structure representing a typical penthouse with height (h) of 4 m (13.1 ft), cross-wind width (w) of 30 m (98.4 ft) and along-wind length (l) of 8 m (26.2 ft) was located upstream of the stack with $x_s = 0.5h$, where $x_s$ is the distance from the structure to the stack. Figure 1 shows the positions of the stack, rooftop structure, and the sampling locations.

An urban boundary layer for neutral conditions (i.e., no thermal stratification) with power law exponent ($\alpha$) of 0.31 was simulated for the present study. Sulfur hexafluoride ($\text{SF}_6$) was used as the tracer gas. Since Sulfur hexafluoride is

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**Figure 1** Schematic view of building, RTS, and sampling locations a) elevation, b) plan.
heavier than air, a mixture of SF₆ and nitrogen was used as the exhaust gas to eliminate buoyancy effects. The gas was released from an adjustable brass stack with a full-scale diameter \( d_e \) of 0.6 m (2 ft) with \( h_s = 0.25h, 0.75h, 1.25h, \) and \( 1.75h \), which corresponds to full-scale heights of 1, 3.5, and 7 m (3.3, 9.8, 16.4, and 23.0 ft), respectively. Concentration measurements were obtained along the plume centerline for two wind directions, \( \theta = 0^\circ \) to \( 45^\circ \). A key parameter for simulating plume dispersion at a model scale is the exhaust momentum ratio, \( M \left( \frac{\rho_e}{\rho_a} \right)^{0.5} \frac{w_e}{U_H} \), i.e., the ratio of the square root of the momentum fluxes, where \( \rho_e \) is the density of the exhaust and \( \rho_a \) is the ambient air density. Since the measurements were carried out with nonbuoyant exhausts, \( \rho_e = \rho_a \) and \( M = \frac{w_e}{U_H} \). The \( M \) values ranged from 1 to 5, representing moderately strong (> 5 m/s) to moderately weak (1–2 m/s) wind speeds. Further details on atmospheric boundary layer and exhaust simulation may be found in Gupta et al. (2006).

**ASHRAE MINIMUM DILUTION MODELS**

**ASHRAE Geometric Stack Design Method**

ASHRAE (2003, 2007) provides a Geometric Stack Design method (AGM) for estimating minimum stack height to avoid plume entrainment in the flow recirculation zones of a building. The AGM is based on a water flume study conducted by Wilson (1979). Even though it is not a quantitative dispersion model, it allows designers to determine the minimum required stack height by avoiding plume entrainment. The AGM is applicable to isolated rectangular buildings. Note that AGM is not recommended if the exhausts are highly toxic in nature.

The AGM requires the dimensions of the building recirculation zones. Figure 2a shows the recirculation zones for a typical low-rise building. These are expressed in terms of the scaling length, \( R \), which is defined as:

\[
R = B_s^{0.067} B_L^{0.33}
\]

where \( B_s \) is the smaller of building height or crosswind width and \( B_L \) is the larger of these dimensions. Note that in Equation 1, \( B_L \leq 8B_s \). When the long dimension exceeds the short dimension by more than 8 times, the long dimension has no added effect. The dimensions of flow recirculation zones that form on the building and rooftop structures are:

\[
H_c = 0.22R
\]

\[
X_c = 0.5R
\]

\[
L_c = 0.9R
\]

\[
L_r = 1.0R
\]

where \( H_c \) is the maximum height of the roof recirculation zone, \( X_c \) is the distance from the leading edge to \( H_c, L_c \) is the length of the roof recirculation zone, and \( L_r \) is the length of the building wake zone.

As shown in Figure 2a, the flow above the roof has three regions: \( Z_1, Z_2, \) and \( Z_3 \). The design method assumes that the boundary of the high turbulence region \( (Z_3) \) is defined by a line with a slope of 10:1 extending from the top of the leading

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**Figure 2a**  Geometric stack design method (Wilson 1979).
edge separation bubble. The location of the plume relative to the recirculation zones is determined by taking into account plume rise due to exhaust momentum and assuming a conical plume with a slope of 5:1.

Figure 2b from ASHRAE (2007) shows the application of AGM for an exhaust stack located near the leading edge of a typical low-rise building. The plume height is determined from the following expression:

$$ h_p = h_s + h_r - h_d $$

where $h_p$ is the plume rise and $h_d$ is the reduction in plume height due to stack tip downwash. The plume rise, which is assumed to occur instantaneously, is calculated using the formula of Briggs (1984):

$$ h_r = 3\beta d_e \left( \frac{w_e}{U_H} \right) $$

where $\beta$ is known as the capping factor. For uncapped stacks, $\beta = 1$; for capped stacks, $\beta = 0$. To account for the stack tip downwash caused by low exit velocities or high wind speeds ($w_e / U_H < 3.0$), Wilson et al. (1998) recommended the following formula for the stack wake downwash adjustment:

$$ h_d = d_e \left( 3.0 - \beta \frac{w_e}{U_H} \right) $$

where $d_e$ is the internal diameter of the stack.

Note that for high exhaust velocities or low wind speeds ($w_e / U_H > 3.0$) there is no stack tip downwash ($h_d = 0$).

The height $h_{\text{small}}$ shown in Figure 2b is known as the capped stack height, which is the stack height required to escape the tallest recirculation zone ($h_{\text{top}}$) without any contribution from plume rise. The stack height from AGM can be determined by applying the plume rise and stack downwash correction to $h_{\text{small}}$, using the following formula:

$$ h_{\text{s(AGM)}} = h_{\text{small}} - h_r + h_d $$

**ASHRAE $D_r$ Minimum Dilution Model**

ASHRAE (2003) recommends a modified Gaussian plume dispersion model for emissions from rooftop stacks, based on water flume simulations of Wilson et al. (1998). The model predicts plume center-line (worst-case) dilution at roof level, $D_r$, assuming that the plume has a Gaussian concentration profile in both the vertical and lateral directions. Roof-level dilution at a receptor distance $x$ from the stack is given as:

$$ D_r = 4 \frac{U_H \sigma_y d_e \sigma_z}{w_e d_e d_e} \exp \left[ \frac{h_p^2}{2\sigma_z^2} \right] $$

where $\sigma_y$ and $\sigma_z$ are the plume spreads in the crosswind and vertical directions. The plume height is calculated from Equation 6. The equations for $\sigma_y$ and $\sigma_z$ are those used in the Industrial Source Complex Screening Tool (ISCST) dispersion model, which was developed by the EPA (1995). The sigma values are adjusted from a 60-minute averaging time to a 2-minute averaging time using the 0.2 power law applied to both vertical and crosswind spreads. The normalized crosswind and vertical spreads are given by the following equations:

$$ \frac{\sigma_y}{d_e} = (0.071) \left( \frac{t_{\text{avg}}}{2.0} \right)^{0.2} \frac{x}{d_e} + \frac{\sigma_o}{d_e} $$

$$ \frac{\sigma_z}{d_e} = 0.071 \frac{x}{d_e} + \frac{\sigma_o}{d_e} $$

![Figure 2b](image-url)  
*Flow recirculation regions and exhaust-to-intake stretched-string distances (ASHRAE 2007).*
where \( t_{avg} \) is the concentration averaging time in minutes, and \( \sigma_{o} \) is the initial source size that accounts for stack diameter and for dilution due to jet entrainment during plume rise. The ratio of \( \sigma_{o} \) to \( d_{e} \) is given by:

\[
\frac{\sigma_{o}}{d_{e}} = \left( 0.125 \beta \left[ \frac{w_{e}}{U_{H}} \right] + 0.911 \beta \left[ \frac{w_{e}}{U_{H}} \right] ^{2} + 0.250 \right)^{0.5} \tag{13}
\]

ASHRAE (2003) recommends the following criteria when calculating the effective plume height, \( h_{p} \), for various building/stack configurations. If the calculated \( h_{p} \) is greater than \( h_{\text{small}} \), the physical stack height should be used when calculating \( h_{p} \). However, if \( h_{p} \) is smaller than \( h_{\text{small}} \) and greater than \( h_{\text{top}} \), the physical stack height should be reduced by \( h_{\text{top}} \). On the other hand, if \( h_{p} \) is less than both \( h_{\text{small}} \) and \( h_{\text{top}} \), the physical stack height should be set at 0 when calculating \( h_{p} \).

This plume height criteria make the ASHRAE 2003 \( D_{r} \) model difficult to apply. To simplify the calculations, the \( D_{r} \) model was revised by ASHRAE in 2007. Equation 10 was replaced by:

\[
D_{r} = 4 \frac{U_{H} \sigma_{z} \sigma_{x}}{w_{e} d_{e} \sigma_{x} d_{e}} \exp \left[ \frac{\zeta^{2}}{2 \sigma_{z}^{2}} \right] \tag{14}
\]

where \( h_{p} \) is replaced with a new parameter known as the vertical separation factor, \( \zeta \). The value of \( \zeta \) is estimated from the following:

\[
\zeta = \begin{cases} 
    h_{p} - h_{\text{top}} & \text{if } h_{p} > h_{\text{top}} \\
    0 & \text{if } h_{p} < h_{\text{top}}
\end{cases} \tag{15}
\]

Note that \( h_{\text{small}} \) has been eliminated in the 2007 \( D_{r} \) model.

ASHRAE 2011 minimum dilution models have been changed compared to the 2003 and 2007 models. New formulations for estimating plume rise (\( h_{p} \)), plume spread parameters (\( \sigma_{x} \) and \( \sigma_{z} \)) and dilution for shorter time periods have been suggested. Plume rise (\( h_{p} \)) is estimated as:

\[
h_{p} = \min \{ \beta h_{x}, \beta h_{f} \} \tag{16}
\]

Where \( \beta \) is the stack capping factor (see Equation 7), and \( h_{x} \) and \( h_{f} \) are estimated as:

\[
h_{x} = \frac{3}{4} \left[ \frac{V_{e}^{2} \sigma_{x}^{2} d_{e}^{3} X}{\beta^{2} U_{e}^{2}} \right] \tag{17}
\]

\[
h_{f} = 0.9 \left( \left[ \frac{V_{e}^{2} \sigma_{x}^{2} d_{e}^{3}}{4} \right] \left[ \frac{U_{H}}{U_{*}} \right] \right)^{0.5} \tag{18}
\]

where \( U_{e} \) is the friction velocity (m/s), and \( \beta_{j} \) is the jet entrainment coefficient calculated by:

\[
\beta_{j} = \frac{1}{3} \left[ \frac{U_{H}}{U_{e}} \right] \tag{19}
\]

The logarithmic wind profile equation is:

\[
\frac{U_{H}}{U_{*}} = 2.5 \ln (H/Z_{o}) \tag{20}
\]

where \( Z_{o} \) is the surface roughness length (m). It may be noted that the plume rise as per ASHRAE 2007 (Equation 8) were functions of the exhaust velocity ratio \( (V_{e} / U_{H}) \) and stack diameter \( (d_{e}) \) whilst the 2011 version also incorporates the effects of wind profile and stack-receptor distance \( (X) \).

The revised plume spread parameters (\( \sigma_{x} \) and \( \sigma_{z} \)) are calculated using the formulations of Cimoreli et al. (2005):

\[
\sigma_{x} = (i_{x}^{2} + \sigma_{0}^{2})^{0.5} \tag{21}
\]

\[
\sigma_{z} = (i_{y}^{2} + \sigma_{0}^{2})^{0.5} \tag{22}
\]

where \( i_{x}, i_{y}, \) and \( i_{z} \) are the turbulence intensities in \( x, y, \) and \( z \) directions:

\[
i_{x} = (0.24 + 0.096 \log_{10} [Z_{o}] + 0.016 [\log_{10} Z_{o}]^{2}) (\ln [30/Z_{o}] / \ln [Z/Z_{o}]) \tag{23}
\]

\[
i_{y} = 0.75 i_{x} \tag{24}
\]

\[
i_{z} = 0.5 i_{x} \tag{25}
\]

In the 2011 model \( \sigma_{o} \) is set equal to 0.35 \( d_{e} \) (m), and \( Z \) is the height of the plume above the rooftop (m). The dilution is calculated using Equation 14, which is equivalent to 10–15 minutes field-averaging time.

**ASHRAE \( D_{s} \) Minimum Dilution Model**

Another dilution model recommended by ASHRAE is the \( D_{s} \) model. This is same for both ASHRAE 2003 and ASHRAE 2007 and is primarily recommended for wall vents, capped stacks, or for cases in which the plume height is below the maximum height of recirculation zones. For the ASHRAE 2011 model the equation for \( D_{s} \) remains the same; however, plume spreads are calculated using Equations 21 and 22.

The \( D_{s} \) model is similar in form to the \( D_{r} \) model, except that the plume height is set equal to zero and \( x \) is replaced by the stretched-string distance \( S \), which is the distance between the nearest edge of the exhaust to the nearest edge of the intake. The dilution at a receptor from the exhaust at distance \( S \) is given as:

\[
D_{s} = 4 \frac{w_{e} \sigma_{x} \sigma_{z}}{S d_{e} d_{e}} \tag{26}
\]
Experimental Results

Normalized Dilutions

The dilutions obtained in the wind tunnel experiments were converted to normalized dilution $D_N$, as suggested by Wilson et al. (1998). The usefulness of $D_N$ derives from the fact that it remains relatively constant for a variety of full-scale atmospheric conditions and varying ratio of exhaust velocity to wind speed. In the present study, the wind speed at the building height was kept constant and exhaust speed was varied to obtain different values of exhaust momentum ratio, $M$. However, the results are applicable for different wind speeds since plume rise only depends on the exhaust momentum ratio.

Normalized dilution was derived assuming the roof-level concentrations follow a Gaussian concentration profile (Wilson et al. 1998). The present study is only concerned with roof-level concentrations, and in particular, concentrations on the plume centerline, where maximum concentrations occur. The plume center-line concentration at roof or ground level with the standard Gaussian equation is given by the following expression (Turner 1994):

$$C = \frac{C_r Q}{\pi U_H \sigma_y \sigma_z} \exp \left( \frac{-\left(h_s + \Delta h\right)^2}{2\sigma_z^2} \right)$$

(27)

where $Q$ is the flow rate of the exhaust, $C$ is the receptor concentration, $C_r$ is the exhaust concentration and $\Delta h$ is the effective plume rise ($h_r - h_d$). Rearranging Equation 27 and substituting minimum dilution $D_{\text{min}} = C_r / C$ gives:

$$\frac{D_{\text{min}} Q}{U_H} = \frac{\pi \sigma_y \sigma_z}{4} \exp \left( \frac{-\left(h_s + \Delta h\right)^2}{2\sigma_z^2} \right)$$

(28)

The left hand side of Equation 28 will be nondimensional, if divided by the square of any length scale. An appropriate length scale could be any of the building dimensions or length scales obtained from the ASHRAE Geometric Method. In the present study the height of the building was used for nondimensionalizing Equation 28. Thus, the normalized dilution $D_N$ is calculated by the following equation:

$$D_N = \frac{D_{\text{min}} Q}{U_H H^2} = \frac{\pi \sigma_y \sigma_z}{H^2} \exp \left( \frac{\left[h_s + \Delta h\right]^2}{2\sigma_z^2} \right)$$

(29)

It should be noted that the use of building height as a nondimensionalizing parameter is strictly arbitrary and does not indicate that dilutions are inversely proportional to the square of building height. Any other building characteristic length can also be used. Similarly, for comparison with wind tunnel data, the ASHRAE $D_r$ and $D_s$ model estimates have been normalized using the following equations:

$$D_{N,r} = \frac{D_r Q}{U_H H^2}$$

(30)

$$D_{N,s} = \frac{D_s Q}{U_H H^2}$$

(31)

Critical Cases Used for Evaluating ASHRAE Models

From a stack design point of view, only cases with worst-case dilutions (maximum concentrations) are important for comparison. Figure 3 shows the variation of $D_N$ measured in the wind tunnel with distance from the stack for the low-rise and high-rise building with and without the RTS for $\theta = 0^\circ$ and $\theta = 45^\circ$. Results are presented for $h_s = 0.75h$ (3 m [9.8 ft]) and $M = 2$. Note that, for consistency, distance from the stack ($x$) is normalized with the height of the RTS ($h$), even for the cases where RTS was not present.

Figure 3a shows that for the low-rise building with no RTS, the lowest rooftop dilutions occurred for $\theta = 0^\circ$. For this case, $D_N$ values are approximately 10 times lower than those obtained for $\theta = 45^\circ$. High $D_N$ values for $\theta = 45^\circ$ occur due to the absence of the separation bubble that forms at the roof-leading edge for $0^\circ \leq \theta \leq 30^\circ$. For $30^\circ < \theta < 60^\circ$, delta wing vortices are formed around the building upwind edges. Similar results have been reported by Schulman and Scire (1991) for a typical low-rise building with no RTS. However, for the building with the RTS, $\theta = 45^\circ$ generated the lowest dilutions, consistent with findings from Saathoff et al. (2009). It can be seen that $D_N$ values dropped sharply with the addition of the RTS. This occurs due to a reduction in plume height as a result of downwash as indicated by Saathoff et al. (2002) and Gupta et al. (2005). A recirculation cavity is formed downwind of the RTS, which entrains much of the plume causing low $D_N$ values at roof level.

For the high-rise building (Figure 3b) the lowest rooftop dilutions occurred for $\theta = 0^\circ$ irrespective of whether or not the RTS was present. Due to the formation of a large rooftop recirculation zone, the effect of the RTS on $D_N$ values is not that significant. For a moderately tall building, the RTS and stack are generally engulfed inside the separation bubble. Consequently, the entire plume is entrained within the bubble causing low dilutions at roof level. Similar observations were made for other $h_s$ and $M$ values tested for both low-rise and high-rise buildings.

Since ASHRAE minimum dilution models are evaluated in the present study, worst-case scenarios are important. Thus, concentration measurements have been analyzed for the following cases:

- $\theta = 0^\circ$ high-rise with and without RTS
- $\theta = 0^\circ$ low-rise without RTS
- $\theta = 45^\circ$ low-rise with RTS
Building Recirculation Zones

The estimated dimensions of the recirculation zones for the test buildings and the RTS using AGM (Equations 1 to 5) are summarized in Table 1 and shown in Figure 4. The dimensions of capped stack height ($h_{small}$) for the three building configurations are also shown. Note that for the low-rise building with the RTS, the highest point of recirculation $h_{top}$ increased to 5.7 m (18.7 ft) from 4.9 m (16.1 ft) with no RTS. This occurred due to a separation bubble formed on the roof of the RTS.

The plume height ($h_p$) and vertical separation factor ($\xi$) are key inputs in ASHRAE dilution models and depend on the dimensions of building recirculation zones. As an example, $h_p$ and $\xi$ for $h_s = 3$ m (9.8 ft) are shown in Table 2 for $M = 1$ and 3. The values of $h_p$ and $\xi$ for the three building configurations were calculated using Equations 6 and 15, respectively.

For the low-rise building with no RTS, $h_{top} = 4.9$ m (16.1 ft) and $h_{small} = 6$ m (19.7 ft) as shown in Figure 4. For $M = 1$, $h_p = 3.6$ m (11.8 ft), thus, $h_p < h_{top}$ and $h_p < h_{small}$. As per ASHRAE (2003, 2007) recommendations, the effective value of $h_p$ and $\xi$ is 0. Thus, for this case, the $D_s$ model is applicable. For $M = 3$, $h_p = 8.4$ m (27.6 ft) and $h_p > h_{small}$. Since $h_p > h_{small}$, $h_p = 8.4$ m (27.6 ft) and $h_p > h_{top}$, $\xi = h_p - h_{top} = 3.5$ m (11.5 ft). For this case, the $D_r$ model is applicable for both 2003 and 2007 versions. The above applies also for the low-rise building with the RTS.

In the case of the high-rise building, for both $M = 1$ and $M = 3$, $h_p < h_{top}$ and $h_p < h_{small}$. Consequently, the effective value of $h_p$ and $\xi$ is 0. Thus, the $D_s$ model is applicable.

Table 1. Dimensions of Building Recirculation Zones as Specified by Wilson (1979)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Low-Rise Building height — 15 m (49.2 ft)</th>
<th>High-Rise Building height — 60 m (197 ft)</th>
<th>RTS height — 4 m (13.1 ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum height of recirculation region ($H_c$)</td>
<td>4.9 m (16.1 ft)</td>
<td>11.7 m (38.4 ft)</td>
<td>1.7 m (5.6 ft)</td>
</tr>
<tr>
<td>Distance from leading edge to $H_c$ ($X_c$)</td>
<td>11.2 m (36.7 ft)</td>
<td>26.6 m (87.3 ft)</td>
<td>3.9 m (12.8 ft)</td>
</tr>
<tr>
<td>Along-wind length of recirculation zone ($L_c$)</td>
<td>20.1 m (65.9 ft)</td>
<td>47.8 m (156.8 ft)</td>
<td>7.0 m (23.0 ft)</td>
</tr>
<tr>
<td>Length of wake recirculation region ($L_r$)</td>
<td>22.3 m (73.1 ft)</td>
<td>44.6 m (146.3 ft)</td>
<td>7.8 m (25.6 ft)</td>
</tr>
</tbody>
</table>
Figure 4  Dimensions of separation bubble, $h_{top}$ and $h_{small}$ for the low-rise and high-rise building with and without the RTS based on ASHRAE Geometric Method.
Model Evaluation for High-Rise Building

In addition to the wind tunnel data obtained in the present study, field data from Stathopoulos et al. (2002) have been used to evaluate the model for a moderately tall building. Four field tests were conducted on the rooftop of a nearly cubical building with \( H = 62 \) m (203.3 ft) located in an urban environment. The tests were conducted for a short stack with \( h_s = 0.5 \) m (1.6 ft) and \( d_s = 0.7 \) m (2.3 ft), located on the southwest side of the roof (see Figure 5a). During each test, ten 15-minute samples were collected at 15 different sampling locations on the building roof. Further details of the experimental procedures can be found through Stathopoulos et al. (2002).

Field data obtained on the roof of the high-rise building for Tests 1 and 3 are used for model evaluation. For these tests, the wind direction was nearly normal to the building leading face, making them suitable for evaluation. The \( M \) value varied from 2.3 to 3. For the lowest value of \( M = 2.3 \), from Equation 6, \( h_p = 4.3 \) m (14.1 ft). For this case, \( h_{\text{top}} = 13 \) m (42.6 ft). Since \( h_p < h_{\text{top}} \), plume rise is insufficient to allow the plume to escape the recirculation zone and thus, the \( D_s \) model is applicable.

Figure 5a shows the field data obtained on the roof of the high-rise building for Tests 1 and 3. The normalized dilution values estimated with the ASHRAE (2003, 2007, and 2011) \( D_s \) models are also presented. A significant level of scatter is apparent in the field data. This occurs due to fluctuations in wind direction, wind speed, and upstream building effects. Note that sampling locations are a bit sparse and not all sampling locations were on the plume centerline. However, for model evaluation, the lower bound to the field data may be used to represent the worst-case scenario. Normalized \( D_N \) values obtained along the plume centerline in the present study for the high-rise building for \( M = 2, h_s = 1 \) m (3.3 ft) and \( \theta = 0^\circ \) are also presented.

The predicted dilutions close to the stack are nearly one tenth of the measured data, although the level of conservatism decreases for samplers located farther from the stack. Since plume height is less than the height of the recirculation bubble, the emissions are expected to be entrained within recirculation zone. The contribution of plume rise is negligible in the ASHRAE (2003, 2007 and 2011) \( D_s \) models and therefore it significantly underpredicted the dilutions. Some additional comparisons of the \( D_s \) model estimates with the wind tunnel data for the high-rise building are shown in Figure 5b. The results are presented for \( h_s = 0.75h \) and \( M = 1 \) to 5. For all \( M \) values, the \( D_s \) model underestimated the dilutions at nearly all receptors by at least a factor of 10.

Model Evaluation for Low-Rise Building without RTS

Figure 6 shows along-wind \( D_N \) profiles of wind tunnel measurements from the present study and ASHRAE \( D_r \) model estimates. Also shown are wind tunnel data from Schulman and Scire (1991) and water flume measurements from Wilson et al. (1998). The building dimensions and experimental parameters for these studies are shown in Table 3. Note that the concentration data obtained from Schulman and Scire (1991) have been converted to \( D_N \) values using Equation 29. For all cases shown, the wind direction was normal to the building leading face, which is the critical direction for buildings with no RTS.

Some variations between dilution values obtained from various studies are noted. This is expected since there are some differences in the experimental parameters, such as terrain roughness, stack location with respect to the building leading edge, stack height, building dimensions, and model scale. The \( D_N \) values from Schulman and Scire (1991) are towards the higher side for \( M = 5 \) (Figure 7b) compared with the other studies. This was probably due to higher plume height compared to the other studies (see Table 3). However, the \( D_N \) profiles obtained from the previous wind tunnel/water flume studies are generally similar to present study data.

The ASHRAE 2003 \( D_r \) model overpredicted dilution values slightly. Since the 2003 \( D_r \) model is based on the results of Wilson et al. (1998), dilutions obtained with this model are also similar to Wilson’s data. However, the ASHRAE 2007 \( D_r \) model underpredicted the dilutions by approximately a factor of 10. This is mainly attributed to the reduction applied to the plume height (see Equation 15). The ASHRAE 2011 model is even more conservative than the 2007 model.

### Table 2. Estimated Values of \( h_p \) and \( \zeta \) for \( h_s = 3 \) m (9.8 ft)

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.6 m (11.8 ft)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>8.4 m (27.6 ft)</td>
<td>8.4 m (27.6 ft)</td>
<td>3.5 m (11.5 ft)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 5a  Model evaluation with field concentration data from Stathopoulos et al. (2003) for a typical high-rise building for $h_s = 0.5 \, \text{m} (1.6 \, \text{ft}), \theta = 210^\circ - 220^\circ$ and $M \sim 2-3$. Present study data corresponds to $M = 2$ and $h_s = 1 \, \text{m} (3.3 \, \text{ft})$.

Figure 5b  Predicted (solid lines) vs. measured in wind tunnel (symbols) $D_N$ values for the high-rise building with no RTS for $\theta = 0^\circ$, $h_s = 0.75h$, $M = 1$ to 5.
Figure 6  Model validation with concentration data from previous studies for the low-rise building with no RTS for $h_s = 3\ m$ (9.8 ft), and $\theta = 0^\circ$: a) $M = 3$ and b) $M = 5$.

Table 3.  Experimental Parameters (Full-Scale) Used by the Present and Previous Studies

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Model scale</td>
<td>1:200</td>
<td>1:100</td>
<td>1:240</td>
</tr>
<tr>
<td>Upstream terrain</td>
<td>urban</td>
<td>suburban</td>
<td>suburban</td>
</tr>
<tr>
<td>Power law exponent ($\alpha$)</td>
<td>0.32</td>
<td>0.20</td>
<td>0.26</td>
</tr>
<tr>
<td>Upstream roughness ($z_o$)</td>
<td>1.1 m (3.6 ft)</td>
<td>0.3 m (1.0 ft)</td>
<td>0.38 m (1.2 ft)</td>
</tr>
<tr>
<td>Stack diameter ($d_e$)</td>
<td>0.60 m (2.0 ft)</td>
<td>0.75 m (2.5 ft)</td>
<td>0.61 m (2.0 ft)</td>
</tr>
<tr>
<td>Wind direction ($\theta$)</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>Stack height ($h_s$)</td>
<td>3 m (9.8 ft)</td>
<td>1.5 m (4.9 ft)</td>
<td>2.1 m (6.9 ft)</td>
</tr>
<tr>
<td>Exhaust momentum ($M$)</td>
<td>3, 5</td>
<td>3, 5</td>
<td>3, 5</td>
</tr>
<tr>
<td>Stack location (from leading edge)</td>
<td>0.4 L</td>
<td>0.5 L</td>
<td>0.3 L</td>
</tr>
<tr>
<td>Plume height ($h_p$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M = 3$</td>
<td>8.4 m (27.6 ft)</td>
<td>8.3 m (25.7 ft)</td>
<td>7.6 m (24.9 ft)</td>
</tr>
<tr>
<td>$M = 5$</td>
<td>11 m (36.1 ft)</td>
<td>12.8 m (42.0 ft)</td>
<td>11.3 m (37.1 ft)</td>
</tr>
<tr>
<td>Building height (H)</td>
<td>15 m (49.2 ft)</td>
<td>15 m (49.2 ft)</td>
<td>12 m (39.3 ft)</td>
</tr>
</tbody>
</table>
Additional results for the low-rise building for $\theta = 0^\circ$ are shown in Figure 7, which also shows $D_N$ values obtained with the normalized ASHRAE 2003 and ASHRAE 2007 $D_r$ models. Results are shown for $h_s = 0.75h$ and $M = 1$ to 5. As discussed previously, for the low-rise building with no RTS, the $D_s$ model is applicable for $M = 1$ and the $D_r$ model is applicable for $M \geq 2$. It is worth noting that the measured $D_N$ values for $M \geq 2$ always decrease as $x / h$ increases. However, the ASHRAE 2003 $D_r$ model (Figure 7a) predicts that $D_N$ increases with distance up to $x < 2.5h$ and then decreases for $x \geq 2.5h$. The maximum $D_N$ values measured in the wind tunnel tests for each $M$ value always occurred near the stack. In general, the ASHRAE 2003 $D_r$ estimates compare well for $M = 5$ but the model overpredicts dilutions for $M = 2$ and $M = 3$. The estimates are at least two times higher than the measured values at samplers located within a distance of $x < 2h$.

The $D_N$ profiles shown in Figure 7b indicate that the ASHRAE 2007 $D_r$ model predicts significantly lower dilutions than the measured values for all $M$ values and at nearly all sampling locations. In contrast to the measured data, the estimated values did not vary much with distance from the stack. The model underpredicted by a factor of 10 to 100 for samplers located at $x < 4h$ and nearly 5 times at $x > 4h$. This is attributed to the reduction applied to the plume height.

The $D_N$ profiles obtained with the ASHRAE 2007 $D_r$ model are significantly different from those obtained with the ASHRAE 2003 version. The large difference between dilutions obtained with 2003 and 2007 $D_r$ models is attributed to the way plume height is calculated in each version. The $D_r$ model limits the contribution of plume height near the stack. Close to the stack, the ratio $h_p^2 / 2\sigma_z^2$ in the 2003 $D_r$ model and $\xi^2 / 2\sigma_z^2$ in the 2007 $D_r$ model become very large, causing the exponential terms in Equations 10 and 14 to overpredict roof-level dilutions. In order to limit the overprediction in dilution near the stack, the ratios $h_p^2 / 2\sigma_z^2$ and $\xi^2 / 2\sigma_z^2$ are not allowed to exceed values of 5 and 7, respectively. Consequently, $h_p$ and $\xi$ are limited to values of 3.16$\sigma_z$ and 3.74$\sigma_z$ for the 2003 and 2007 $D_r$ models, respectively. Hence, for tall stacks and high exhaust speeds, the 2003 model predicts lower $D_N$ values close to the stack than the 2007 model. Increasing the $\xi$ value from 3.16$\sigma_z$ to 3.74$\sigma_z$ in the 2007 $D_r$ model improved the dilution estimations close to the stack. However, based on the value of $h_{top}$, the model applies a reduction to $h_p$ (see Equation 15), which causes a further significant reduction in dilution.

The $D_r$ model estimates are also shown in Figure 7 and correspond to $M = 1$. Note that the $D_r$ model trend is opposite to the trend shown for the measured values. Since the $D_r$ model assumes that the plume is released at the building roof level with virtually no plume rise, the estimates are significantly conservative. In general, the estimated values are 10 to 100 times lower than the measured values.

**Model Evaluation for a Low-Rise Building with an RTS**

Saathoff et al. (2002) conducted field tests to evaluate the dispersion of exhaust from a rooftop stack in an urban...
environment. An induction type fan was used in this study. Measurements were obtained on the roof of a 3-storey laboratory building at Concordia University in Montreal, Quebec, Canada. This study was chosen to evaluate the capability of the ASHRAE dilution models in dealing with the downwash effect of the RTS. Tracer gas (SF$_6$) was emitted from a 3.1 m (10.2 ft) tall stack with a diameter of 0.9 m (3.0 ft). The stack was located approximately 1 m (3.3 ft) downstream of the RTS in consideration, which had a height of 2.2 m (7.2 ft) and crosswind width of 35 m (114.8 ft) as shown in Figure 8. The samplers were placed on the roof and penthouse of the building. Three 150-minute tests were conducted when the stack was downwind of the RTS. During each field test, ten 15-minute samples were collected at each measurement location and the wind speed and wind direction were recorded with a sonic anemometer located 3 m (9.8 ft) above the RTS. Note that the exit velocity profile inside an induction fan is not uniform, which was used in the field tests as well. Although wind tunnel measurements were carried out with a uniform exit velocity profile, the concentration data compared well with field data. Further details are provided in Saathoff et al. (2002).

Figure 8 shows the hourly mean dilution values from Saathoff et al. (2002) for Test 2, which had an average $M$ value of 3. The ASHRAE $D_r$ model estimates are also shown. The ASHRAE 2003 model overpredicted dilutions by nearly 10 times for $x < 20$ m (65.6 ft), which indicates the ineffectiveness of the 2003 $D_r$ model in considering the downwash effect of an RTS. On the other hand, the ASHRAE 2007 $D_r$ model estimates are reasonable and within a factor of 2 of the measured dilutions for $x < 20$ m (65.6 ft). However, the 2007 model appears to significantly underpredict dilutions at distances greater than 20 m (65.6 ft) from the stack. The reduction applied to the plume in the 2007 $D_r$ model appears to be reasonable in this case. With the RTS, the 2011 model was also more conservative than the 2007 model, especially within 15 m from the stack beyond which the estimates were similar to the 2007 model.

Some additional cases tested in the wind tunnel for the low-rise building with the RTS are presented in Figure 9. Note that results for the RTS case are presented for the oblique wind case. This is the worst-case scenario for the low-rise building with the RTS. The ASHRAE 2003 $D_r$ model estimates are presented in Figure 9a. Since the ASHRAE 2003 $D_r$ model does not take into account the downwash effect of an RTS on plume rise, the estimates remain the same as those for the building with no RTS. However, as shown previously in Figure 3, an RTS can significantly decrease dilutions (i.e., increase concentrations) at the roof level. As a result, the $D_r$ model significantly overestimated the dilutions for the building with the RTS. Estimated $D_N$ values were higher at least 100 times for $x < 4h$ and 5 to 10 times for $x \geq 4h$ compared to measured values.

The ASHRAE 2007 $D_r$ model estimates for the building with the RTS are shown in Figure 9b. The level of conservatism for the building with the RTS is less than that for the building with no RTS and it decreases with increase in $M$. The reduction applied to the plume height appears to be reasonable for $M = 5$. However, the estimated dilutions were
still conservative by a factor of 5 to 10 at samplers located within \( x < 2h \).

The ASHRAE 2003 and 2007 \( D_r \) model estimates for the RTS case are also shown in Figure 9 and correspond to \( M = 1 \). Similar to the case where no RTS was present, the results clearly indicate a significant level of conservatism in \( D_r \) model estimates at all sampling locations. Since dilutions decreased significantly for the low-rise building with the RTS, the level of conservatism also decreased to 5 to 10 times compared with 10 to 100 times for the building with no RTS.

### Generalizing ASHRAE Models Performance

To generalize the performance of ASHRAE dispersion models for different values of \( h_s \) and \( M \), the parameter \( \varphi \) is introduced, which is defined as:

\[
\varphi = \frac{\text{Dilution estimated with ASHRAE models}}{\text{Dilution measured in the wind tunnel}}
\]

Depending on the magnitude of \( \varphi \), the results are categorized as follows. If \( \varphi \) is around 1, the model estimates are acceptable; if \( \varphi \) is greater or smaller than 1, the model may be too little unconservative or too conservative, respectively.

The ratios \( \varphi_r \) and \( \varphi_s \) were calculated for all values of \( h_s \) and \( M \) values tested, where subscripts \( r \) and \( s \) represent the values obtained with ASHRAE \( D_r \) and \( D_s \) models, respectively. For example, Figures 10a and 10b show the variation of \( \varphi_r \) with \( x / h \) in the case of the low-rise building without the RTS. The ratio \( \varphi_r \) is shown for \( h_s = 0.75h \) and \( M = 2, 3, \) and 5 for which the \( D_r \) model was applied. Note that \( \varphi_r \) values obtained with the 2003 and 2007 \( D_r \) models generally follow a similar trend for different \( M \) values. The trend is indicated by a solid line, which is the average of \( \varphi_r \) values for \( M = 2, 3, \) and 5. For all building configurations presented, similar trends for \( \varphi_r \) vs. \( x / h \) were found for other values of \( h_s \) (0.75h, 1.25h, and 1.75h). Therefore, one trend can be used to represent the ASHRAE model behavior for all \( M \) values tested for a particular stack height.

### ASHRAE 2003 \( D_r \) Dilution Model

Figure 11a shows the variation of \( \varphi_r \) with \( x / h \) for the ASHRAE 2003 \( D_r \) model for the low-rise building with and without the RTS. In general, for the building with no RTS, the \( D_r \) model estimates were generally twice as large as measured (\( \varphi_r \approx 2 \)) for \( h_s \leq 1.25h \), except at samplers close to the stack (\( x < h \)). The maximum difference between \( D_r \) model estimates and wind-tunnel results was found for the tall stack (\( h_s = 1.75h \)), where \( \varphi_r \) values were significantly low (\( \varphi << 1 \)) near the stack. As indicated previously, \( h_s^2 / 2\sigma_z^2 \leq 5 \) in the 2003 \( D_r \) model. Consequently, for \( h_s = 1.75h \), plume height is limited at many sampling locations. Consequently, the model is significantly conservative close to the stack (\( \varphi_r \approx 0.01 \) to 0.1). For the building with the RTS, \( \varphi_r >> 1 \) for all \( h_s \), except for \( h_s = 1.75h \) as shown in Figure 12b.
Figure 10 Variation of $\varphi_r$ values with distance from the stack for the low-rise building with no RTS for $\theta = 0^\circ$, $h_s = 0.75h$, $M = 2$, 3 and 5; a) ASHRAE 2003 $D_t$ model and b) ASHRAE 2007 $D_t$ model.

Figure 11a ASHRAE 2003 $D_t$ model performance for $h_s = 0.25h$ to 1.75h where $h = 4$ m (13.1 ft): a) low-rise building with no RTS and b) low-rise building with the RTS. Note that each trend represents averaged values for $M = 1, 2, 3$, and 5.
Thus, the 2003 \(D_r\) model significantly overpredicted dilutions for the low-rise building, especially when the RTS was present. In this case, the model did not take into account the downwash effect of the RTS effectively.

**ASHRAE 2007 \(D_r\) Dilution Model**

Figure 11b shows the variation of \(\varphi_r\) with \(x/h\) for the ASHRAE 2007 \(D_r\) model for the low-rise building with and without the RTS. As indicated by low values of \(\varphi\) (\(\varphi_r \approx 0.01 - 0.1\) for \(x < 4h\)), the \(D_r\) model significantly underpredicted dilutions for the building with no RTS. For the building with the RTS, the model estimates for \(h_s \leq 1.25h\) were about one-fourth of measured dilutions \(\varphi_r \approx 0.25\). However, for the tall stack \((h_s = 1.75h)\), \(\varphi_r \approx 0.1\) at nearly all sampling locations. Thus, for a typical low-rise building with no RTS, the 2007 \(D_r\) model is conservative. However, for the building with the RTS, the model estimates were reasonable for short to moderately tall stacks \((h_s \leq 1.25h)\) due to the reduction applied to the plume height. With the taller stack \((h_s = 1.75h)\), the model estimates were conservative by a factor of nearly 10 at all sampling locations.

**ASHRAE (2003, 2007) \(D_s\) Dilution Model**

The ASHRAE (2003, 2007) \(D_s\) model results for the high-rise building, and the low-rise building with and without RTS are presented in Figures 12a to 12c, respectively. Note that for the low-rise building, each trend represents average values of \(\varphi\) for \(M = 1\) and \(M = 2\). For the high-rise building, data for \(M = 1, 2, 3,\) and 5 have been averaged to produce the trend lines. As indicated by low values of \(\varphi\), the \(D_s\) model significantly underpredicts dilutions for the cases shown. The \(\varphi_s\) values varied from 0.01 to 0.1 for the low-rise building with no RTS and 0.1 to 0.5 for the low-rise building with the RTS. The \(\varphi_s\) values for the high-rise building were similar to the low-rise building with no RTS. Since the \(D_s\) model does not take into account the effect of plume height, the conservatism increases with an increase in stack height.

**CONCLUSIONS**

The ASHRAE 2003, 2007, and 2011 minimum dilution models were evaluated using data obtained from wind tunnel and field experiments. Concentration measurements obtained on the plume centerline for typical low-rise and high-rise buildings were compared with estimates from the ASHRAE models for different stack heights and exhaust speeds. The effectiveness of the ASHRAE models in addressing the downwash effect of an RTS on plume was also evaluated. The main conclusions from the present study are:

1. The ASHRAE 2003 \(D_r\) model is generally unconservative for short stacks \((h_s \leq 5m, 16.4\ ft)\) and conservative for tall stacks \((h_s > 5m, 16.4\ ft)\). For stacks downwind of an RTS, the model may be significantly unconservative due to downwash effects.
2. The ASHRAE 2007/2011 \(D_r\) model provided conservative dilution estimates, especially when an RTS was present.
3. The ASHRAE 2003/2007) and ASHRAE 2011 \(D_s\) model is expected to be conservative for all cases.
4. In general, the accuracy of the ASHRAE (2003, 2007, and 2011) dilution models increases with distance from the stack.

5. The ASHRAE 2007 $D_r$ model is simpler to apply than the ASHRAE 2003 and 2011 $D_r$ models. In addition, the downwash effect of RTS is addressed more effectively in the 2007 model. Thus, the ASHRAE 2007 $D_r$ model is recommended.

Building exhaust design depends on a large number of parameters such as exhaust types, design criteria, and building type. The designer must determine what level of conservatism is required for a specific emission source. For example, Petersen and Wilson (1989) noted that for odors, peak concentrations may be more important than the average values typically estimated by numerical models. Peak concentrations could be two to three times higher than the average values. Although ASHRAE dilution models are useful for preliminary analysis, they should be used with caution and by an experienced engineer with some basic knowledge of flow around buildings.

REFERENCES


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*Figure 12* ASHRAE (2003, 2007) $D_r$ model performance: a) low-rise building with no RTS, b) low-rise building with RTS, and c) high-rise building. Note that for the low-rise building, each trend represents averaged values for $M = 1$ and 2 for $h_s = 0.25h$ and $0.75h$ where $h = 4$ m (13.1 ft). For the high-rise building each trend represents averaged values for $M = 1, 2, 3,$ and 5 and for $h_s = 0.25h$ to $1.75h$. 

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The diagrams show the performance of the ASHRAE (2003, 2007) $D_r$ model from different perspectives:

- **Figure 12a**: Graph of $D_r$ versus distance from the stack for a low-rise building with no rooftop structures (RTS).
- **Figure 12b**: Graph of $D_r$ versus distance from the stack for a low-rise building with RTS.
- **Figure 12c**: Graph of $D_r$ versus distance from the stack for a high-rise building.


