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An Application of a Cost Minimization Model in Determining Safety Stock Level and Location

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Abstract- In recent decades, the lean methodology and the development of its principles and concepts have widely been applied in supply chain management. One of the most important strategies of being lean is having efficient inventory within the whole supply chain. Managing inventory efficiently requires appropriate management of safety stock in order to compensate the weakness of the supply chain for product availability.A nonlinear cost minimizationsafety stock model with the objective of minimizing the total logistics cost is developed in this paper. This modelis also applied to a real-world case company which is a manufacturer.The model results in optimum levels and locations of safety stock within the company's supply chain in order to minimize total logistics costs.

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Abstract- In recent decades, the lean methodology and the development of its principles and concepts have widely been applied in supply chain management. One of the most important strategies of being lean is having efficient inventory within the whole supply chain. Managing inventory efficiently requires appropriate management of safety stock in order to compensate the weakness of the supply chain for product availability.A nonlinear cost minimizationsafety stock model with the objective of minimizing the total logistics cost is developed in this paper. This modelis also applied to a real-world case company which is a manufacturer.The model results in optimum levels and locations of safety stock within the company's supply chain in order to minimize total logistics costs.

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I. Introduction

Intoday's competitive environment, applying the lean paradigm has been extended to the field of supply chain management.Taylor (1999), Adamides et al. (2008), Kainuma&Tawara (2006), Lamming (1996), Crino et al. (2007), Wu & Wee (2009) researched on lean supply chain. Naylor (1999), Qi et al. (2007), Mason-Jones et al. (2000) compared lean paradigm with other methodologies in supply chain management. Contributors of a supply chain, no matter to which industry they belong, aim to follow a lean philosophy to make their business processes more and more efficient in order to survive on the market. Manufacturers are one of these contributors and inventory plays a paramount role in their efforts to become lean.Chun Wu (2003), Wu (2009), McCullen&Towill (2001) studied on the application of lean manufacturing. There are different inventory drivers such as level of supply chain collaboration and visibility, forecast accuracy, order pattern, and safety stock policy, among others. Therefore, proper management of inventory and consequently safety stock as one of its drivers has become critical objective towards achieving leanness. In this paper, we propose a safety stock cost minimization model in a manufacturing case company thatis attemptingto become lean by managingthe inventory across its supply chain efficiently, and towards this goal, efficient levels and locations of safety stock becomes more and more significantas a prerequisitecondition.

An optimization model of safety stock can be built on different objectives. Minimizingcost, maximizingservice level, and aggregate considerations are examples of such objectives (Silver, 1998). Optimal determination approaches based on cost and service level objectives are more appropriate for practical applications (Inderfurth, 1991). One of the vital goals of the enterprise is to maximize earnings under certain investment conditions (Long et al., 2009). On the other hand, as reducing costs of materials, equipment, and labor is difficult at best in today's competitive market, enterprises are more interested in targeting logistics costs in this regard (Long et al., 2009). In this paper, minimization of logistics costs is selected as the basis of the determination of optimum safety stock.Logistics costs are mainly related to procurement and supply, manufacturing process, and after sales service. Thus, holding and shortage costs are selected as representations of logistics costs in the optimization model. Indeed, product availability is a critical measure for the performance of logistics and supply chain (Coyle et al., 2009). Any obstacles at any node and level of supply chain can result in unavailability of products to their customers. There are different issues that cause disruptions and unavailability of products in the supply chain, as for example variability, whether in demand or lead time; quality issues; or internal and external issues such as low delivery performances, improper scheduling, inadequate product capacity, poor maintenance, among others. Figure. 1 is a schematic of a supply chain with its nodes such as different tiers of suppliers, producer, assembly, distributors, and customer. Any actions taken by any member of the chain can affect the profitability of the others. Therefore, companies have great interest in having better coordination among the contributors of their supply chain (Silver, 1998). Safety stock is essential to compensate for the weakness of the supply chain for part availability and this factor has been considered in the selected optimization model.

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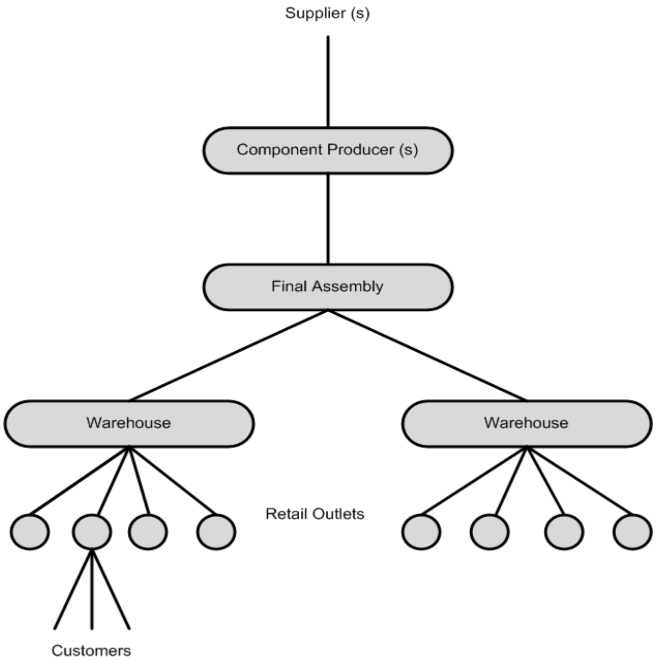

Figure 1 : A schematic of a supply chain

In this paper, we apply a safety stock cost minimization model in a case company which is a manufacturer. In the next section, we provide a review of the literature. In Section 3, we describe the case company. In Section 4, we introduce the model, followed by the model formulation in Section 5. Results are then presented in the next section, followed by validation. We then provide a discussion of the results and their implication, and then conclude with some suggestions for avenues of future research.

II. Literature Review

According to the literature, there are different approaches and methods for determining safety stock under different situations. Some different methods for computing safety stock in the Just In Time (JIT) environments are presented by Natarajan &Goyal (1994). These methods deal with objectives related to service level, expected number of stock outs, tradeoff between stocking out and carrying extra buffer, minimization of total cost comprises of set-up, holding, and shortage costs. Efficient level of inventory in creases the inventory turnover in companies. Reducing the level of inventory helps to increase the turns according to its definition. One of the approaches towards reduction of inventory especially in Just In Time (JIT) environments is reducing lot sizes. On the other hand, smaller lot sizes will lead to uncertainties and consequently stock outs (Natarajan &Goyal, 1994). Therefore, safety stock is really needed to protect against these kinds of uncertainties.

Minner (1997) uses dynamic programming algorithms to find the optimal combinations of coverage times with the target of minimizing the average holding costs in serial, divergent, and convergent inventory systems. In this paper, it is assumed that customer demand is normally distributed and correlations between demands are permitted. One of the outcomes of this paper is that concentrating safety stocks at the first and final stages would be optimal for a serial system with a high enough service level.

A linear programming model with the objective of establishing a trade-off among plan changes, carrying, and shortage costs under resource constraints for a multi-item production system is presented by Kanyalkar & Adil (2009). Plan changes cost is related to the instabilities occur under rolling schedule. These instabilities in the chain affect costs such as setup and expediting costs and they also affect material plans like shortage or excess of components (Kanyalkar & Adil, 2009).

Jung et al. (2008) present a linear programming formulation which includes the control variables of safety stock with the purpose of minimization of the total supply chain's inventory while meeting the target of the service level. This model incorporates the nonlinear performance functions, the interdependence between the service level at upstream and downstream stages of supply chain and also the safety capacity constraint. A section also provided for linearization of the nonlinear functions of the model. Some of the assumptions applied in this model are normally distributed demand, zero lead time at the warehouse, and constant production capacity. In addition, it is assumed that raw material and transportation means in any size are always available.

A dynamic model of the safety stock by assuming a Vendor Managed Inventory (VMI) system is presented by Yuan Li &Jian Li (2009). Under VMI system, the uncertainties related to the efficiency of the supplier disappear and the model considers only the variability sourced by demand.

Patel & Rodrigues (2010) present the dynamics of the model of optimizing safety stock for small-scale aluminum utensil manufacturing industry. This model takes into account factors of demand, production rate, delay, and waste time. Indeed, this paper concentrates on the bullwhip effect in a manufacturing supply chain and tries to reduce it by increasing safety stock.

Zhao et al. (2001) use a simulation approach to evaluate alternative methods of determining the level of safety stock based on historical forecasting errors in multilevel MRP systems. In addition, the relation between the safety stock multiplier and different system performance measures such as total cost, service level, and schedule instability in different methods analyzed and results also provided.

Badinelli (1986) is about combining stock out cost and holding costs functions towards determining the optimal safety tock. It also presents a technique for estimating the stock out with a decision maker's disvalue function.

An approximation model for safety stock in a two echelon distribution system is provided by Desmet et al. (2010). This model tries to incorporate the variance of the retailers and the central warehouse in the replenishment lead time. It also takes into account the variance of the service time of orders at the warehouse as it has significant effect on the system's lead time variance.

Inderfurth (1991)represents a safety stock optimization model in multi-stage problems with divergent structure and provides a dynamic programming algorithm for solving that. The analysis for the impact of the correlation of demands on safety stock allocation has also provided in this paper. This model does not include inter-stage shortage costs by assuming of having a certain capacity of slack resources for operating flexibility.

Inderfurth (1995) is the continuation of hisprevious work in 1991.He extended his study to a case that demand is not only cross-product but also cross-time correlated. Cross-time correlation of demand yields a tendency to keep safety stock at the end-item level, while cross product correlation provides a tendency for holding buffer more in upstream stages. One of the results of this study is that increasing the correlation in both products and time makes the safety stock policy to be more expensive. This research also shows that not taking into account demand correlation may result in incorrect sizing and positioning of safety stock in multi-stage manufacturing systems. Neglecting this may also lead to missed cost reduction opportunities.

A nonlinear integer optimization model with the objective of minimization of the total setup and inventory holding costs by considering service level constraint has been provided by Carlson & Yano (1986). The only variability that is incorporated into the model isrelated to demand. In addition, it is assumed that there are no capacity constraints. The model suggests having safety stock at those stages with high setup or disruption costs.

An optimization model with the purpose of minimizing the total holding and shortage costs is presented by Aleotti Maia & Qassim (1998). Then, an analytical solution provided for finding the preferable case by comparing inventory and opportunity costs. It is concluded that holding inventory at the intermediate levels is not economical if it is solely using for reduction of the frequency of stock out. The model from this paper is expanded for this study and applied in a real-world case company. The reason for this selection is that the objective of this model is the same as the objective of the case company which is minimization of the cost. Determination of the optimal level and location of safety stock in a supply chain with different stages and stochastic environment is a very complex task; therefore, most of the models and approaches provided in this regard have applied certain assumptions in their own cases to make it simpler. Some of these approaches are applicable for only a specific inventory system, some of them limit the distribution of demand, and some of them exclude the suppliers' variability. In this paper, we present a general model with the objective of logistics costs minimization by considering both internal and external variability and taking into account of part availability factor which is very important in the chain.

III. CASE STUDY

The company under study, which we will hereinafter refer to as ABC for the purpose of confidentiality, is a manufacturer in the aerospace industry. The company is characterized by high demand variability and long lead time, among others. ABC is a multi-stage manufacturer. Tiers of suppliers, procurement, manufacturing, final assembly, and customers (internal and external) are different nodes of the ABC's supply chain. The downstream nodes are the upstream nodes' customers, and the replenishmentlead time of customer nodes is the order waiting time provided by their upstream nodes. In addition, ABC has a generally structured multi-stage system and there is no restriction with respect to the number of predecessors and successors of any node. Such multi-stage systems focus considerable attention on setting and positioning safety stock. ABC has two different manufacturing plants (MFs). The procurement department of the company is responsible for procuring the raw materials or semi- finished parts through suppliers to manufacturing plants or even supplying parts from one manufacturing plant to another (inter plants transfers). Indeed, the word "supplier" in the model could be the representative of the external supplier or internal manufacturing entity. It should be noted that procurement's location can be different from manufacturing ones. Finished parts from manufacturing entities have two internal customers that pull their outputs; they are Assembly (ASSY) and Aftermarket(AFM). These two latter entities are the last stages of the internal chain of the company just before the end customer. There are also some external supplied finished parts required for Assembly and Aftermarket that the procurement department is again in charge of supplying them. The Assembly entity has different finished product families with their own specifications.Therefore, if availability of parts (right parts at right time) can be assured for the internal customers, on-time delivery performance to the end customer will be assured as well. This availability should be guaranteed through safety stock, but the optimum safety stock level and location should also minimize logistics costs.

IV. MODEL DESCRIPTION

The optimization model is presented through different possible value streams of each finished product family of the company and developed using lingo optimization software to result in the optimum level of safety stock with its optimum location in the stream. Value stream is the aggregation of all actions needed to bring a specific product through problem-solving task, information management task, and physical transformation task (Womack and Jones, 2003). Value stream mapping as a tool of lean is a method to depict material and information flow throughout whole the chain for both value added and non-value added processes.Value stream is used to give the visibility of the whole supply chain form end to end for each specific part. By applying the model through different value streams, it will not only result in the optimal level of the safety stock but also in the optimal location of it in the supply chain (raw material safety stock, semi-finished part safety stock, or finished part safety stock). Each of the possible value streams of the case company can have different combinations of the chain's contributors before the end customer. In order to limit the number of stages and for simplification, only the last two stages of those value streams that have more than two nodes before the internal customer stage are selected. Therefore, all the previous stages and their connections are being excluded and their performances are being captured only through the input of the latest second stage. The other reason for this limitation is the difficulty in defining the shortage costs in upstream stages of the chain due to lack of visibility and control. Furthermore, the objective of the model is cost minimization, and the upstream stages' contributions towards cost are significantly less than the downstream stages, thus this simplifying assumption should have a negligible effect on overall results. Although, there is a sample (Value Stream 4) presented in "Computational Results" section that goes beyond this limitation just to show the applicability of the model for the whole chain from end to end point.

Shortage cost, overage cost, and delivery performances (percentage of product availability) are the inputs of the model. Different combinations of raw material (semi-finished part) and finished part are considered as indices in the model based on the selected value streams.

V. MODEL FORMULATION

For all value streams, the notations of the model are as follows:

a. Sets and Indices

i Raw material/ semi-finished part

p Finished part

u Customer (ASSY, AFM)

b. Variables

K_i Delivery performance of procurement to manufacturing

K_p Delivery performance of manufacturing orprocurement to customers

c. Parameters[1]

P_i Supplierdelivery performance to procurement (If supplier is a manufacturing plant, thenP_i wouldbe manufacturing performance for semi-finishedpart)

P_p Manufacturing performance for finished part (Ratiobetween on time manufactured and plannedmanufacture of finished part)

C_S Cost of shortage

C_O Cost of overage

x_i Raw material/semi-finished part safety stock

x_p Finished part safety stock

q_i Raw material/semi-finished part quantity ordered

q_p Finished part quantity ordered

q^* On-time delivered quantity of raw material/ semi-finished part or finished part

Figures.2 to 4present variables and parameters in possible value streams for procuring a part to the customer in the case company.

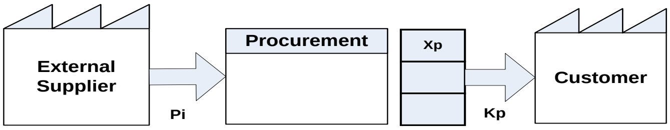

Figure 2 : Variables and parameters in value stream

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[1] It should be noted that index of "p" is used for only those finished parts that are manufactured in ABC. Indeed, for those finished parts that are supplied through suppliers, index of "i" is used.

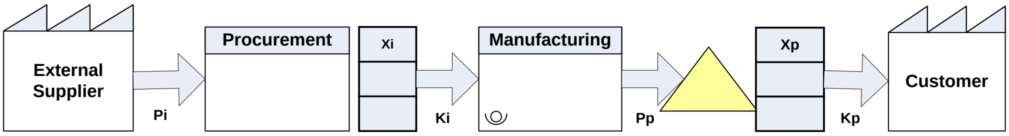

Figure 3 : Variables and parameters in value stream

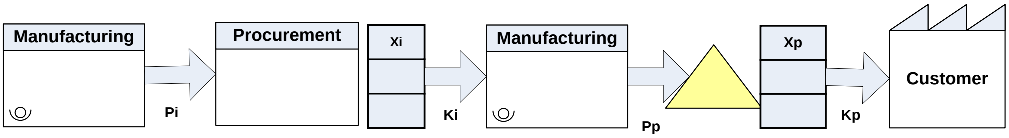

Figure 4 : Variables and parameters in value stream

K_i is the summation of the availability percentage of raw material/semi-finished part for manufacturing through procurement based on the absolute suppliers' performances (P_i) and the availability percentage of procurement's safety stock for that part (x_i/q_i). Indeed, procurement can deliver whatever quantities they received on time through suppliers plus their safety stock to the manufacturing. K_p is the summation of the availability percentage of the finished part which is dependent on the manufacturing performance (P_p) and also their previous stages' performances (K_i) and the availability percentage of manufacturing's safety stock for that part (x_p/q_p). Likewise, manufacturing can deliver what everquantities of finished parts they can produce on time which is also dependent on the deliveries of their previous stages in the chain plus their own safety stock quantities to their customers (ASSY and AFM).

The related formulas of K_i and K_p are as (1) and (2):

$$K_i = P_i + x_i/q_i \tag{1}$$

$$K_p = P_p \times K_i + x_p/q_p \tag{2}$$

In the cases that the finished part is directly procured through the external supplier for the customers, K_p formula will be equal to (1).

P_i and P_p are calculated as average numbers based on historical data from the last year. A report called the First Filled Rate (FFR) is used for calculation of these parameters. This report is used top resent the availability of the right part at the time that is required. The FFR result takes into account the total on hand stock in its calculation which does include safety stock as well. It should be noted that P_i and P_p should be the absolute delivery performance of supplier and manufacturing without the contribution of the safety stock that may be used during last year. Therefore, the safety stock has been excluded from the FFR report for this purpose. In addition, when there are two stages in the selected value stream, the FFR report also includes the contribution of the last second stage's performance in its results for calculating the last stage's performance which is manufacturing. Therefore, this must also be excluded. Indeed, P_p is the manufacturing performance without taking into account the stock out of raw materials (Aleotti Maia & Qassim, 1998). Hence, to calculate the required absolute value of P_p from FFR, three other parameters should be defined. First one is K'_p which is the exact number extracted through FFR, the other one is P'_p which is the FFR's result excluding safety stock contribution. And the third one is K'_i which is the historical previous stage's delivery performance; by dividing this by P'_p the absolute manufacturing performance is measured ($P_p=P'_p/K'_i$). Indeed, there is no direct report for tracking absolute manufacturing performance in the case company. Table1 is a snapshot of a sample FFR and presents the formulas used to eliminate the safety stock from its calculation. As shown through the table, in the 12th week of 2010, the FFR report gives 100% ($K'_p=100\%$) as the delivery performance of manufacturing to its customer because it takes into account the 300 pieces of safety stock for meeting the past and current requirements; however, safety stock must be excluded through this calculation and P'_p becomes 18%. The next step for calculating the absolute manufacturing performance would be the elimination of the effect of the previous stage's performance (K'_i).

About the calculation of P_i in FFR, it should be noted that if the supplier delivers a part on time with the right quality, but defects occurs during transportation from procurement to manufacturing or customer, although the delivery performance of the supplier is 100%, P_i will be 0% since the part is not available for use. Therefore, P_i can also be called "part availability" instead of supplier delivery performance.

It is worth mentioning here that ABC has three different strategies for managing its inventory. It applies a two-bin kanban system for the parts with low costs. The company is moving towards excellence and applying a pull system for managing the inventory of those parts that have high cost with high volume; but this system is not applicable for all parts due to the complexity and lack of required conditions such as having suppliers with delivery performance of higher than 80% and with a supermarket of finished goods, having parts with a robust process and steady volume, among others. Therefore, its inventory strategy for the rest of the parts with high cost and low volume is MRP system. Based on this, a safety stock strategy is really required for this latter category of parts.For calculating q_i and q_p, we need to understand the *risk period*. Risk period consists of a review period and replenishment lead time (Tempelmeier, 2006). The review period is the basis on which the company updates its data. As a result, if a company reviews its data once a week, its review period would be one week. Of course this review period has an effect on the duration that the company should wait to receive its order through the supplier.In the case company of this paper the data are updated daily; therefore, there is no need fordefining the review period.Consequently for parts managed by the MRP system, quantities within the replenishment lead time have found as the most appropriate definition for q_i and $q_p$to result in the proper level of safety stock for the company through the model.In essence, if changes happen in demand within this period (replenishment lead time), we cannot count on the suppliers' support 100% of the time. Safety stock is required for coverage of this variability. The first step for their calculation would be identifying the planned order quantity of each specific part (raw/semi or finished part) per week according to its planning parameters which it itself is related to ordering policies. Some of the examples of planning parameters in this regard are Lot for Lot, Weekly Batch, 2 Weeks Batch, and Fixed Order Quantity, among others. The second step would be the calculation of the average weekly forecast demand of that specific part for the next year. After that, the division of the planned order quantity and average weekly demand would result in the replenishment lead time in weeks. When changes happen in the supply chain such as changes in the demand or capacity ration, entrance of new competitors, introduction of a new product, or retirement of a matured one, the safety stock required for the supply chain must be re-evaluated (Jung et al., 2008).ABC has decided to run the model and update it every quarter, therefore, the weekly demand of the next quarter would be merged based on the calculated replenishment lead time. And finally, the maximum quantity of this combination will be selected as q_i/q_p in order to allow the safety stock strategy to support the *worst case*.

Table 1 : First fill rate report sample

Part Code	Entity	Calendar Week	Stock	Required Past	Required Current	% Met Global (K'p)	Theoretical Safety Stock	Safety Stock On-Hand	q*	P'p
AF1	MF	11.2010	2100	500	500	100	0	0	500	100%
AF1	MF	12.2010	1100	700	560	100	300	300	100	17.85%

* Shaded sections are used to make the FFR report applicable.

* Theoretical safety stock based on historical data.

* Safety Stock On-Hand = Max (0, Min (Stock - Required Past, Theoretical Safety Stock))

* q* = Max(0 ,Min (Stock - Required Past - Safety Stock On Hand, Required Current))

* P'_p = (q*/Required Current)×100

One of the advantages of this method of calculating q_i and q_p is making the market variability involved by taking into account of the forecast demand. It should be mentioned that the planned order quantity for a manufacturing part should always be calculated through its demand only in the plant in which it is being manufactured because the part will be replenished based on the ordering policy in that plant. On the other hand, in the case that a raw material has more than one customer (MF and AFM), calculation of q_i required by manufacturing through weekly demand seen in procurement (entity that receives part through supplier)is not correct because procurement sees the demand of both customers mix. Therefore, the respective q_i must be calculated through the part's parameters (planned order and weekly demand) all in the manufacturing plant that it is going to be used.

Shortage costs (costs of safety stock violation) have different definitions for raw materials (semi-finished parts) and finished parts as they are located in different stages within the chain and their shortages have different effects on the system. The shortage cost of the raw material (semi-finished part) is the summation of the expediting cost on the supplier, expediting cost on transportation, and overtime of the manufacturing section. On the other hand, shortage of the finished part which is required by Assembly, causes disruptions and stock not pulled for all the other parts related to that finished part and also its finished product in different locations of the supply chain. In addition, shortage of the finished part causes the finished assembled product to be held up unreleased. Therefore, the shortage cost is defined as follows:

C_{sp} = (Standard cost of the finished assembled product* average days of holding finished assembled product due to the shortage of the specific finished part during last year*0.1)/365

Coefficient of 10% in the above formula is the annual interest rate that company could receive by putting this amount of money in the bank, although the company has this as inventory buckets instead of cash right now.

The cost of shortage of the finished part required by Aftermarket is defined as the profit that the company will lose by not having the part ready to deliver ontime to the customer, which is the direct cost. Besides that, there are many intangible effects of this shortage that are called indirect costs and are difficult to gauge accurately (Graves et al., 1993). One of them is loss of customers' goodwill that may turn them to other competitors in the future. On the other hand, at the time of shortage of a specific part, the Aftermarket department may rent out another more expensive part instead of the required one to the customer until it arrives. Therefore, the shortage cost of these parts is defined as four times of the standard cost (Stnd.Cost) of the finished part.

The cost of overage is defined as the interest that the company is losing by holding inventory instead of having it in cash. Hence, it is the multiplication of standard cost of the part and the annual interest rate (10%).

As can be seen through the formulas and definitions, a period of one year has been selected for historical data collection. As the factors (such as shortage cost and delivery performances) that are gathered within this time frame are critical to make an appropriate decision about the level and location of safety stock, one year has been selected in order to have a sufficient window view.

Some samples of value streams associated with their models' formulas are presented below.

Value stream 1 shown in Figure.5consists of one raw material/semi-finished part used to make one finished part which has two customers, ASSY and AFM. The corresponding objective function and constraints are presented by (3).

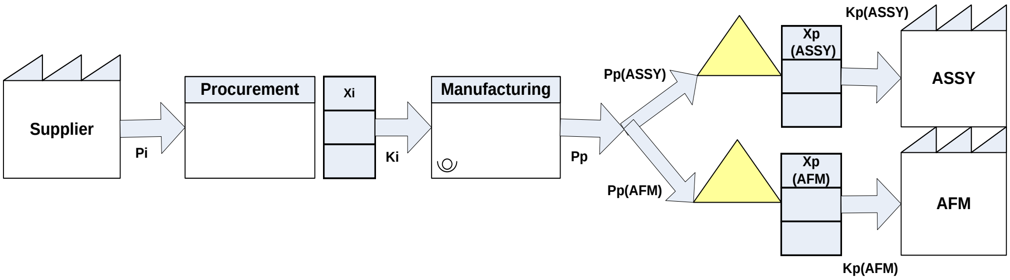

Figure 5 : Value stream 1

$$
\begin{aligned}
&MinC = C_{si}q_i(1-P_i)+C_{oi}q_i(K_i-P_i)+\sum_{u=1}^{2} C_{spu}q_{pu}(1-K_{pu}) \\
&+\sum_{u=1}^{2} C_{opu}q_{pu}(K_{pu}-(P_{pu}\times K_i)) \\
&SubjectTo: \\
&K_i \le 1 \\
&K_i \ge P_i \\
&K_{pu} \le 1, \qquad u=1,2 \\
&K_{pu} \ge P_{pu}\times K_i, \qquad u=1,2
\end{aligned}
\tag{3}
$$

If for this case, there were two different kinds of finished parts but again in demand with both customers, then there should be a summation on both indices of finished part (p) and customer (u) in the objective function:

$$
\begin{aligned}
MinC &= C_{si}q_i(1-P_i)+C_{oi}q_i(K_i-P_i) \\
&+ \sum_{p=1}^{2}\sum_{u=1}^{2} C_{spu}q_{pu}(1-K_{pu}) \\
&+ \sum_{p=1}^{2}\sum_{u=1}^{2} C_{opu}q_{pu}(K_{pu}-(P_{pu}\times K_i))
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
&SubjectTo: \\
&K_i \le 1 \\
&K_i \ge P_i \\
&K_{pu} \le 1, \qquad u,p=1,2 \\
&K_{pu} \ge P_{pu}\times K_i, \qquad u,p=1,2
\end{aligned}
$$

In value stream 2 which is shown in Figure. 6, two raw materials/semi-finished parts are used to make one finished part which has two customers, ASSY and AFM. The corresponding model is also presented by (5).

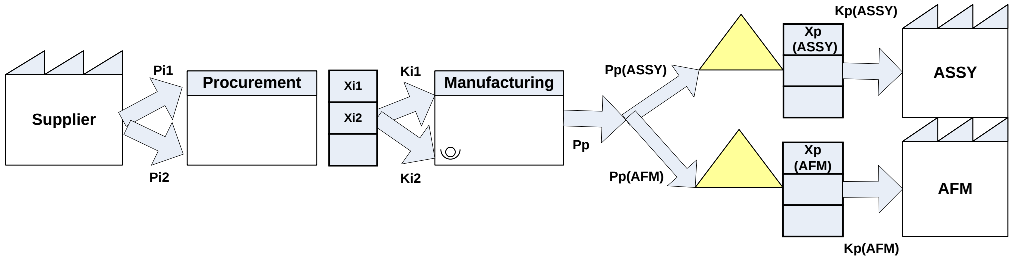

Figure 6 : Value stream 2

$$
\begin{aligned}
MinC &= \sum_{i=1}^{2} C_{si}q_i(1-P_i)+\sum_{i=1}^{2} C_{oi}q_i(K_i-P_i) \\
&+ \sum_{u=1}^{2} C_{spu}q_{pu}(1-K_{pu}) \\
&+ \sum_{u=1}^{2} C_{opu}q_{pu}(K_{pu}-(P_{pu}\prod_{i=1}^{2} K_i))
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
&SubjectTo: \\
&K_i \le 1, \qquad i=1,2 \\
&K_i \ge P_i, \qquad i=1,2 \\
&K_{pu} \le 1, \qquad u=1,2 \\
&K_{pu} \ge P_{pu}\prod_{i=1}^{2} K_i, \qquad u=1,2
\end{aligned}
$$

As before, if there were two different finished parts for the same situation, the model would be changed as (6):

$$
\begin{aligned}
MinC = & \sum_{i=1}^{2} C_{si} q_i (1-P_i) + \sum_{i=1}^{2} C_{oi} q_i (K_i - P_i) \\
& + \sum_{p=1}^{2} \sum_{u=1}^{2} C_{spu} q_{pu} (1-K_{pu}) \\
& + \sum_{p=1}^{2} \sum_{u=1}^{2} C_{opu} q_{pu} (K_{pu} - (P_{pu} \prod_{i=1}^{2} k_i))
\end{aligned}
$$

$$
\begin{aligned}
& SubjectTo: \\
& K_i \le 1, \qquad i = 1,2 \\
& K_i \ge P_i, \qquad i = 1,2 \\
& K_{pu} \le 1, \qquad p,u = 1,2 \\
& K_{pu} \ge P_{pu} \prod_{i=1}^{2} K_i, \qquad p,u = 1,2
\end{aligned}
\tag{6}
$$

As can be seen through the constraints of the model, the company's objective is to have 100% delivery performances. Therefore, the upper boundaries of both stages are assigned to 1 in order to not to allow the model to impose a shortage to the system. Of course, these upper bounds could be less than 1 based on the service level goals in different cases.

By this definition of the model, costs factors would be the indicators for the location of the safety stock and its level would be identified based on the boundaries of the delivery performances.

This optimization model will be linear if there is only one raw material/semi-finished part and optimum point with minimum cost will happen only in one of the four boundaries. Based on this, we assume the optimization model as (7) with only one customer for finished part:

$$
\begin{aligned}
MinC = & C_{si} q_i (1-P_i) + C_{oi} q_i (K_i - P_i) + \\
& C_{spu} q_{pu} (1-K_{pu}) + C_{opu} q_{pu} (K_{pu} - (P_{pu} \times K_i))
\end{aligned}
$$

$$
\begin{aligned}
& SubjectTo: \\
& K_i \le 1 \\
& K_i \ge P_i \\
& K_{pu} \le 1 \\
& K_{pu} \ge P_{pu} \times K_i
\end{aligned}
\tag{7}
$$

Varying the location of the safety stock based on the optimum point in two sample cases of the linear model in (7) are shown with the following feasible regions in Figures.7and8. In addition, Table 2 presents the comparison between the costs in each of the cases and also the recommended location of the model for the safety stock. In this comparison, it is assumed that q_i and q_p are equal.

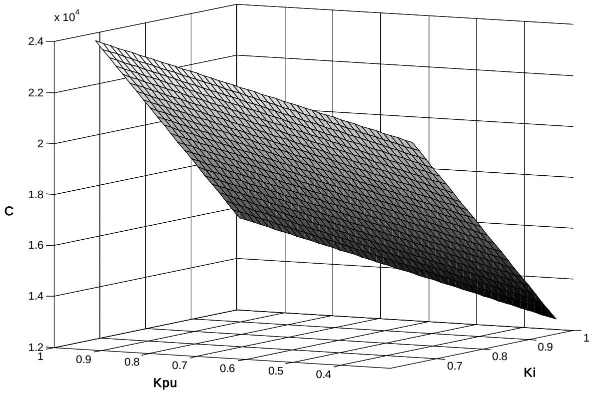

Figure 7 : Location of safety stock-Case 1

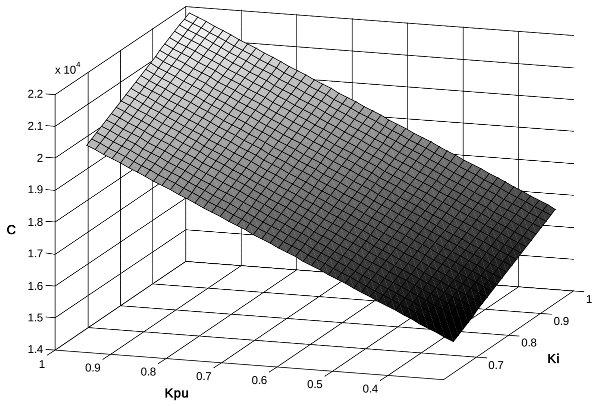

Figure 8 : Location of safety stock-Case 2

Table 2 : Costs Comparison and safety stock locations

Case	Costs Comparison	Safety Stock for Raw Material	Safety Stock for Finished Part
1	Cop>Csp>Csi>Cos	Yes	No
2	Cop>Coi>Csp>Csi	No	No

In order to make the results of the model more effective for the company, one of the most problematic finished product families of the Assembly was selected, and value streams of its finished parts that are going to be assembled were reviewed with the model. As each of the selected final product families could have 100 different value streams in the case company, it was decided to apply the optimization model only for those value streams that end with finished parts that were consistently in shortage report during last year in order to limit samples. Value streams of these pacer parts vary. Some of them could have only the supplier stage before the assembly and some others could be very long. As discussed before, these long value streams were limited by taking into account only parts of level 1 and 2 of its finished product's bill of materials (BOM).

VI. Computational Results

Results of the model applied to some value stream samples of one finished product family in the company are presented in Table 3. This table includes input factors to the model such as delivery performances (P_i,P_p), parts quantities (q_i, q_p), costs (C_s, C_o) along with parameters required to calculate them (K'_i, P'_p, K'_p, standard cost) for each value stream. This table also presents the old and new safety stock levels and total costs (for those cases that all required data were available)to compare previous situation with new one. All historical data presented in this table, as mentioned before in "Model Formulation" section,are based on last year records. In addition, recommendations of the model based on the analysis of the real cases are explained. Lingo 11.0 was used to solve the non-linear optimization model. It should be mentioned that due to confidentiality, masked data are used in this paper.

Table 3 : Computational Results

Value Stream	Part Code	Entity	K'i	Pi	qi	P'p	Pp	qp	K'p	Stnd Cost	Cs	Co	Old xi	New xi	Old xp	New xp	Total Old Cost	Total New Cost
VS1	B	MF	0.65	0.57	1400					$40	$2	$4	0 & 500	602			$497,732	$15,116
VS1	AB	ASSY				0.40	0.62	1100	0.53	$120	$500	$12			1 & 8	429		
VS1	AB	AFM				0.20	0.30	900	0.46	$120	$480	$12			300	630		
VS2	C	MF	0.22	0.22	5					$2,000	$25	$200	0	0			$28,257	$10,757
VS2	D	MF	0.24	0.24	7					$8,000	$30	$800	0	0				
VS2	ACD	ASSY				0	0	7	0	$15,000	$4,000	$1,500			1&2	7		
VS3	E	MF	0.55	0.31	200					$250	$10	$25	&160&34	138			$75,200	$3,450
VS3	AE	ASSY				0.57	1	170	0.57	$400	$1,000	$40			0	0		
VS4	F	MF	0.58	0.37	25					$500	$150	$50	5&9	16			$4,457.5	$913
VS4	AF	ASSY				0.59	1	12	0.58	$1,000	$450	$100			0	0		
VS4	AF	AFM				0.48	0.82	7	1	$1,000	$4,000	$100			24	2		
VS5	G	MF	0.30	0.30	12					$3,000	$45	$300	0	0			$150,378	$6,378
VS5	AG	ASSY				0	0	10	0	$6,000	$15,000	$600			0	10		
VS5	AG	AFM				0	0	0	0	$6,000	$24,000	$600			0	0		
VS6	H	MF	0.15	0.15	10					$4,000	$80	$400	0	9				$3,400
VS6	AH	ASSY				0.25	1	6		$10,000	$800	$1,000			1	0		
VS6	AH	AFM				0.38	1	5		$10,000	$40,000	$1,000			1	0		
VS7	I	MF	0.18	0.18	8					$3,500	$36	$350	0	7				$13,246
VS7	AI	ASSY				0.05	0.27	6		$25,000	$8,000	$2,500			1	5		
VS8	M	MF	0	0	12					$8,000	$15	$800	0	0				$19,980
VS8	AM	ASSY				0.09	0.09	11		$18,000	$6,000	$1,800			3&0&1	11		
VS9	T	MF	0.70	0.59	25					$2,000	$15	$20	6	10			$1,249	$468.96
VS9	L	MF				0.30	0.43	12	0.50	$300	$25	$30			4&3	7		
VS9	N	MF	0.70	0.53	12					$90	$2	$9	14&0&5	6				
VS9	S	MF	0.95	0.95	10					$160	$8	$16	0	1				
VS9	ALNS	ASSY				0.59	1	5	0.85	$3,500	$500	$350			6&3	0		

a) *Value Stream 1*

Shortage costs of ASSY and AFM (customers) are the first two highest costs; therefore, the model has targeted them at first and recommended that the delivery performances in those entities be increased to 100% by keeping safety stock for the finished parts. ASSY and AFM can count on receiving their required demand on time for 0.61% and 0.30% respectively; thus, they need to compensate the 0.39% and 0.70 % of *unavailability* of partsby asking manufacturing to keep safety stock.

Then, the third and fourth highest costs are the overage costs of the same entities. Hence, the model suggests keeping some level of safety stock in the raw material (semi-finished part) level as well to lower the level of finished parts' safety stocks. It is shown that procurement can count on on-time delivery performance of supplier(s) for 0.57% and they have to reimburse the remaining 0.43% by having safety stock. As in this case, safety stock has been increased in both levels of supplier and manufacturing, of course before applying the recommendations, the capacity of both should be checked in order to be aligned with the new level of demand and input respectively.

b) *Value Stream 2*

According to the priority of the costs, shortage should be removed for the Assembly entity by keeping safety stock for its required finished part. In this case, the manufacturing performance is zero; therefore, having safety stock for the rawmaterials' level in case of improving the input ration to this entity will not make any changes. Consequently, there is no choice but to pay for the holding cost for the finished part, although this holding cost is the second highest cost. On the other hand, as soon as manufacturing performance increases even slightly, the level of safety stock required for the finished part will decrease by recommending holding some safety stock for raw materials.

c) *Value Stream 3*

Again the highest cost is the shortage cost of the finished part and an action required to reduce this cost by making K_p(delivery performance) 100%. As the manufacturing performance is 100% ($P_p=1$) and based on the formula of $K_p = P_p \times K_i$, the only way to make $K_p$equal to 1 is by making K_i equal to 1. Therefore, having safety stock for raw material is recommended by the model for this purpose. In sum, in this case, the manufacturing entity has produced whatever they received from procurement; therefore, to improve their delivery performance, the input amount should be improved. Of course, for this kind of change, the capacity of manufacturing should be checked in order to be aligned with its input.

d) *Value Stream 4*

In this case, the highest cost is related to the shortage of finished part required for Aftermarket; hence, safety stock should be kept for this customer. Then, the biggest loss would happen if the company cannot deliver the required demand of ASSY; As manufacturing's performance in response to Assembly's demand is 100% and it can produce whatever it receives from procurement, delivery performance to ASSY will be improved only by increasing input of the raw material to manufacturing. To make a decision about the value of K_i, the model will hit the third highest cost which is the raw material's shortage cost. The selected value for K_i will also affect the level of required safety stock for Aftermarket.

e) *Value Stream 5*

Apparently, it is understood that there is no need for safety stock for Aftermarket as its demand for the next quarter is zero. But, it should be noted that as the manufacturing performance for this customer is zero, safety stock should be considered as soon as demand occurs. On the other hand, for the purpose of cost reduction, delivery performance to Assembly should become 100%. As the manufacturing performance in response to this customer is also zero, the full quantity of the finished part within the replenishment lead time should be kept as safety stock. By improving manufacturing's performance up to 50%, the level of safety stock required to be kept in finished part will be lowered but still there would not be any recommendation for keeping safety stock for raw material. But, as soon as manufacturing's performance increases by more than 50%, the model will suggest starting keeping safety stock in the raw material stage as well and balancing it to minimize the total cost.

f) *Value Stream 6*

Based on the investigation done for this case, it is known that raw material has quality problems most of the times. With this background, the result of the model does make sense: to keep safety stock in that level of the chain.

g) *Value Stream 7*

The model suggests balancing the level of safety stock by keeping it in both raw material and finished part levels and ensuring the on-time delivery to the customer, Assembly.

h) *Value Stream 8*

This value stream includes one raw material and one finished part with only one customer, Assembly, just as in Value Stream 7. As shown previously, safety stock was kept at both levels; but now the model is suggesting keeping safety stock for the finished part only. The reason is that manufacturing performance is almost zero and improving its input will never help to provide on time delivery to Assembly. On the other hand, holding cost of the raw material is really greater than its shortage cost; so, it is not beneficial even for lowering the level of finished part's safety stock.

i) *Value Stream 9*

This sample shows one of the class A finished parts required for Assembly for the selected product family. This finished part has three semi-finished parts (level 2 in finished product's BOM which are L, N, and S in Table 3). "L" is an in-house part and is manufactured in ABC. Furthermore, the manufacturing plant requires raw material (T) to produce this part which is procured through the supplier.Part T is in level 3 in the BOM. Therefore, this sample goes far beyond the limitation of levels 1 and 2, andshows that the model is applicable for all stagesof the value streams as long as the input data of the model are provided.

Manufacturing, receives the two other semi-finished parts (N and S) required forproducing the finished part directly through suppliers. Figures.9and 10present the respective value stream and BOM.

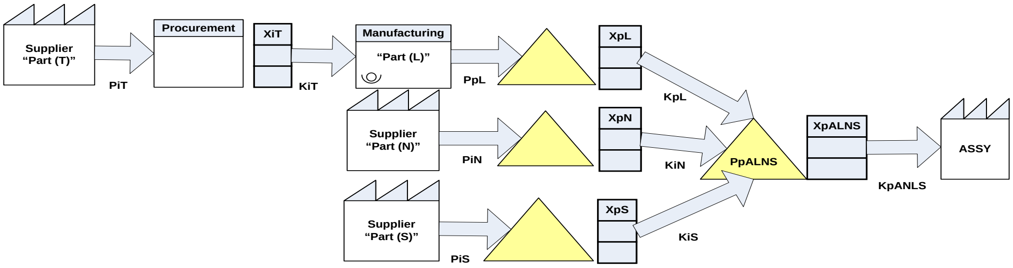

Figure 9 : Value stream

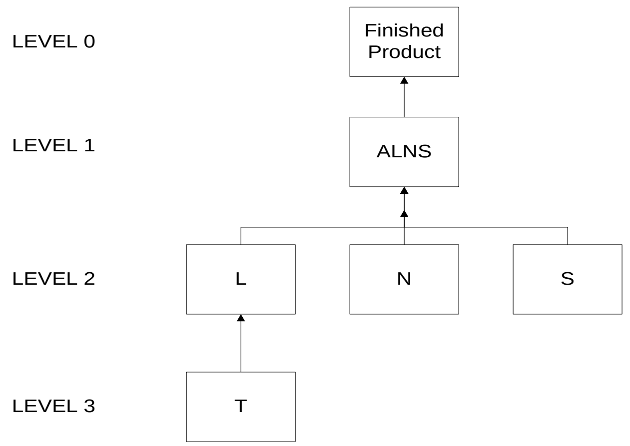

Figure 10 : Bom

Assume that there is a bottleneck in the first value stream in Figure. 9as the manufacturer does not have the capacity for the requested new level of demand. Then the other two value streams can make their delivery performances 100% by keeping safety stock, although the finished product cannot be cleared yet due to the pacer part of the first value stream (if there is no safety stock kept for the finished product). In this situation, there may be some complaints that safety stock must not be kept in the other value streams either since in the end, the company will pay for the holding costs while the finished product cannot be released. The response to this complaint is that if the first value stream comes out of the pacer situation, then another one will become the pacer due to not having safety stock. In essence, bottleneck always moves. Therefore, for this case, it makes sense to keep safety stock only for two of the value streams although the delivery performance of the finished part will not be 100% due to the low performance of the value stream with the bottleneck. On the other hand, by improving the delivery performances even only for two value streams out of three, holding cost of the finished part based on its formula $(C_{op}\times K_p-(P_p\times K_1\times K_2\times K_3))$ will be decreased.

This last value stream (value stream 4), can be a representative case to illustrate the error and especially in this case, the overestimating of safety stock result in the analysis of parts in isolation and not within the chain. If, ALNS was being considered separately and apart of its chain, system may allocate some level of safety stock for that due to the K'_pwhich is 85%. But, when this part is analyzed within its chain, it is understood that the reason for no availability of the finished part is not due to the last stage performance but it is due to the low delivery performances of the semi-finished parts. Therefore, keeping safety stock in the last stage only increases the holding cost of the system.

VII. Validation

In this section, historical data on a raw material part will be used for analysis and compared to the results of the model.

As illustrated in Figure. 11, there were periods in the last 5 months during which the company was in shortage and had negative stock. There was no safety stock assigned to the part during these periods. On the other hand, the stock situation became better starting in week 14 by allocating 600 units of safety stock. Thus the theoretical safety stock was 0 and 600for this part during the last five months. The same analysis in the same period has been done for P_i and K'_i as shown in Figures.12and 13.

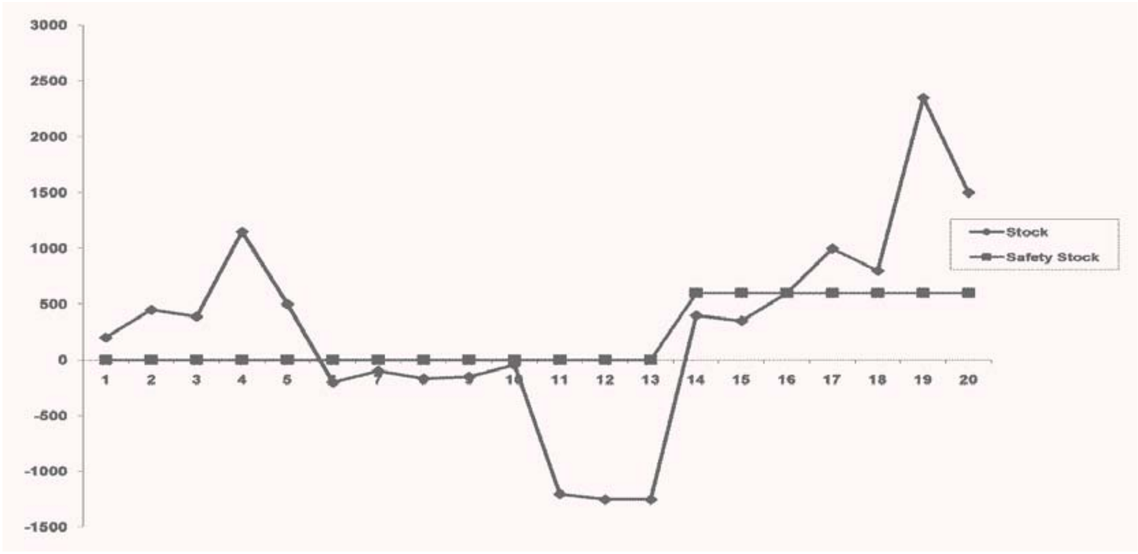

Figure 11 : Past stock situation and safety stock level

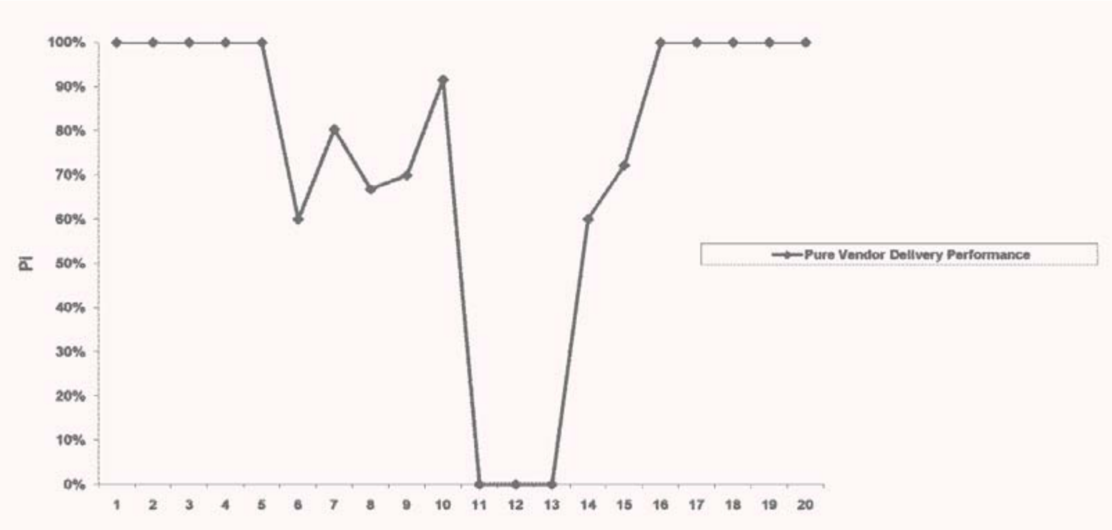

Figure 12 : Absolute part availability percentage without safety stock

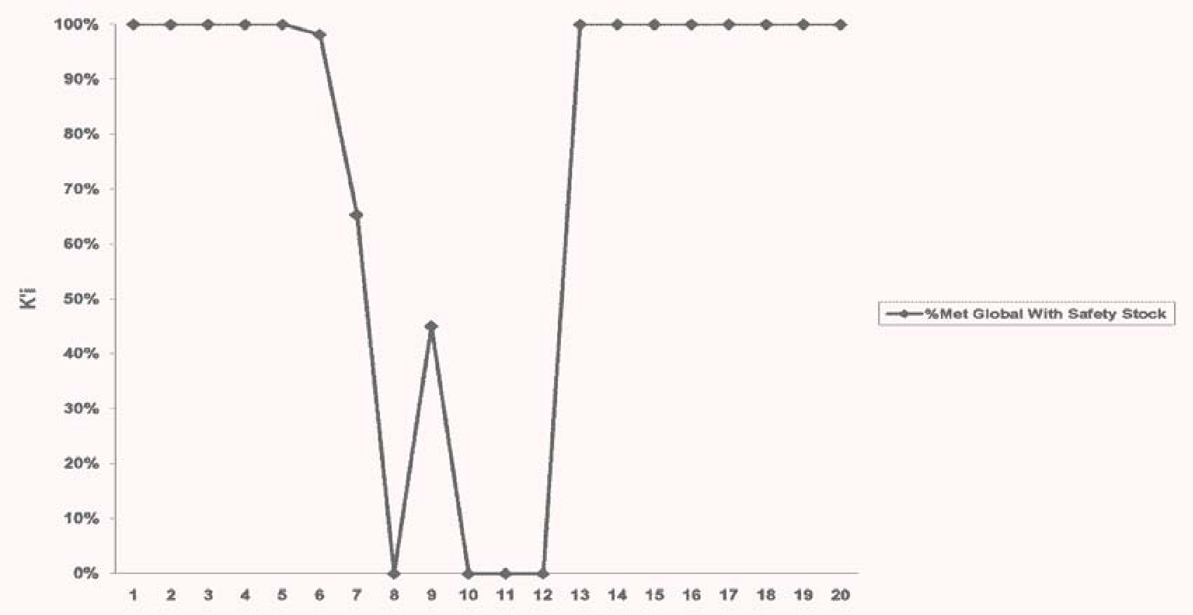

Figure 13 : Procurement delivery performance with safety stock

It can be seen that the weakness of part availability in weeks 13, 14, and 15 had been compensated by safety stock; although this weakness could not be remunerated previously as there was no safety stock. Therefore, it is concluded that by this amount of availability for this part, safety stock is essential to guarantee on-time delivery to manufacturing.

The optimization model was then run for the raw material's value stream. The result of the model was 394 pieces for the raw material's safety stock; but of course this level is based on the next quarter ratio of demand. Indeed, the lower level of safety stock recommended through the model is related to the maximum quantity of this part that will be required in the next three months based on the forecast. And this maximum number is being considered in the model to decide the level of safety stock to guarantee the worst case. On the other hand, it is shown through Figure. 13that by keeping 600 pieces of safety stock, the level of stock is going to be increased and this is not a desired case as holding cost is associated with this increase; therefore, lowering the level of safety stock does make sense.

Figures.14and 15show the historical data of three factors, FFR (%), safety stock fulfill rate (SS FR%), and number of parts with quality issues (QN in pieces) for three different parts. The messages of these charts are provided as well. These messages were aligned with the safety stock model's results obtained for the respective parts.

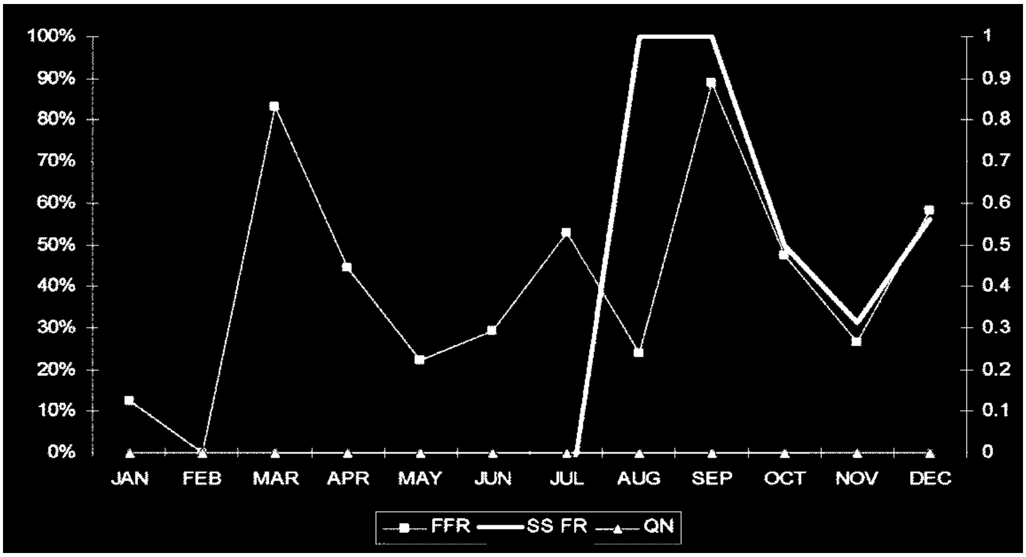

Figure 14 : FFR,SS FR, QN

*There is no quality issue.

*Buffer strategy is required to compensate the low delivery performance.

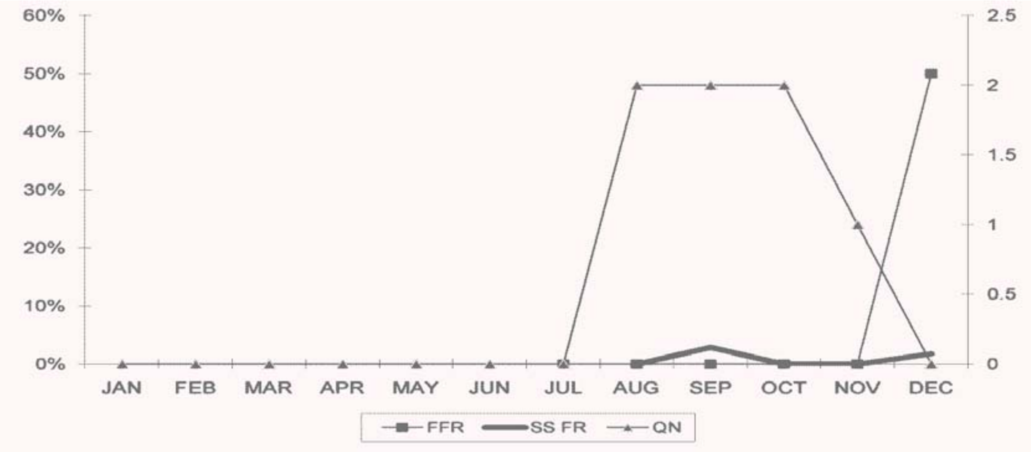

Figure 15 : FFR,SS FR, QN

* Low FFR can be improved by 50% if quality issues solved.

*Additional buffer may be required to increase FFR by 50% and make it 100%.

VIII. Discussion and Implications

The recommendations of the model are according to the current situation of the system. Of course as soon as the company takes action towards improving its system for parts availability within the chain, the results of the model for level and location of required safety stock will be adjusted accordingly.The managerial guidelines that are provided in this section can be used in any kind of manufacturing systems.

Re-sourcing of the suppliers would be a solution for their low delivery performances and quality problems. Increasing the capacity of manufacturing and improving its quality would be a solution for low availability percentage at semi-finished and finished parts level.

On the other hand, in the cases that the company requires keeping some level of safety stock due to the bad performance of vendors (low delivery performance, low quality), it is recommended that a VMI system be applied to have safety stock at the vendors' place.

The existing FFR report in the case company for the Aftermarket entity is based on their forecast demand instead of their firm orders; therefore, the model is not capturing the accurate delivery performance record for them. By deciding the level of safety stock based on the forecast demand, we will put safety stock on top of the safety stock because forecast demand is itself a kind of buffer stock. To solve this problem, it is recommended that ABC design an FFR report specifically for Aftermarket in order to capture the performances in response to only firm orders.

There may be some parts that are dual sourced and there is a quota arrangement between different suppliers, but the FFR report being used in the case company does not include the vendor field in its results. Therefore, it is recommended that the supplier field in the FFR report be considered as well to allow the company to recognize their delivery performances separately and consequently be able to make decisions about re-sourcing more accurately.

One of the other factors other than delivery performance or service level of the suppliers in making decisions in the dual source cases is the *waiting time*for receiving the late parts. Indeed, the company as a customer will select the supplier with the lower waiting time among the ones with the same service level. One way of tracking the waiting time of the supplier is through the calculation of the period within the replenishment lead time in that the company had negative stocks; but it is subject to keeping stock of each supplier separately to be able to relate its negative period to the corresponding supplier. Now consider a case that the supplier of a specific required raw material has the delivery performance of 50%, demand is one piece per week, its replenishment lead time is 10 weeks, and its waiting time is 2 weeks. Assume that the worst case for its q_i for the next quarter is 10 pieces. And again assume that itis the case that the model suggests keeping safety stock for the remaining 50% of the time that the supplier is late, which is equal to 5 pieces. This level of safety stock is equivalent to 5 weeks of demand, although the company will receive its late demand after 2 weeks according to the waiting time of the supplier. Therefore, the company does really need safety stock of 2 weeks instead of 5 weeks. Hence, no matter if it is a dual source case or not, it can be concluded that waiting time is also an important factor for determining the optimum safety stock.

If there is safety stock for the finished assembled product or it is scheduled for build ahead, sizing the required safety stock within the chain should be done by taking into account of these factors as well. One way to get them involved is by converting them to the weeks of demand for each stage and comparing them with the suggested amount of safety stock (like the method suggested for waiting time). But the time lag between the time that we put safety stock for the finished product (or build ahead) and the time that we will have it should also be considered; otherwise, reducing the safety stock within this period by will put the system in a shortage situation.

For some cases where unavailability of a part is solely related to the low delivery performances and not to quality issues, safety lead time can be applied instead of safety stock.

Delivery performances of some parts in their last stage are very low due to different engineering issues such as changing the layout and design consistently. Therefore, recommendation of the model to have safety stock for these parts will make sense only if the cost of reverse engineering of these parts is less than their shortage cost.

If the model suggests increasing the level of safety stock for a specific stage, the company will receive it by the end of the *total lead time* of the chain related to that part. Therefore, if the company adds the extra pieces of safety stock to its demand, it will allow all purchase orders to be expedited although this extra amount is not theactual demand and it is required for safety stock. Hence, the company must inform the suppliers that it needs this portion of demand for their next lead time. On the other hand, it is really important to take into account the lead time of the whole chain, otherwise, it will put them in a shortage situation. As a result, knowing the existence of this time lag makes the selection of the periods for calculating q_i and q_p more accurate. It should be noted that after selecting this appropriate period, standard cost of the parts should also be updated accordingly.

The q_i for those parts that are strategic ones should be validated with the responsible value stream managers. Indeed, quantities of this kind of parts could be really greater than the number which is result in through the mentioned definition for them. There are different indicators that make a part strategic such as the critical parts that are single sourced, or the parts that have limited suppliers or the parts with the resourcing strategy. For example, there could be a single sourced critical part which is received in a batch and based on the experience it is known that if one part of this batch has a quality issue, there is a high possibility that the entire batch needs to be scrapped. Therefore, by having correct level of safety stock for this part, the company can survive and save the supplier's lead time.

IX. CONCLUSIONS

This research extends the work of Aleotti Maia and Qassim (1998). They proposed a nonlinear safety stock optimization model for a system with n suppliers, one manufacturer and one customer with the objective of total inventory cost minimization. In this study we extended the model to be applicable to the whole supply chain of a generally structured multi-stage manufacturing system. Proper required index, parameters, and variables have been introduced and added more flexibility to the model implementation. In addition, the possibility of stock out for all materials at any stage of supply chain (raw material, semi-finished part or finished part) has been taken into account in the model of this study; although it was assumed previously that the material (raw material or semi-finished part) required by manufacturing is always available. This consideration makes the model more realistic. In this research, the safety stock optimization model is provided with the objective function of total logistic costs minimization to result in the optimal level and location of it across the supply chain. The constraints of the model provided for the boundaries of the delivery performances of each stage of the supply chain. Then, we applied the optimization model in a practical real-world problem with different possible value streams. We accurately defined the inputs of the model such as shortage and overage costs and also quantities of the parts. Lingo 11.0 was used to solve the non-linear optimization model.

The weakness of the supply chain must be compensated with safety stock, while it is optimized to meet the desired objective of the business. It has been shown in this paper that in optimizing the safety stock based on a cost minimization objective, not only its level but also its location in the supply chain is important. Indeed, by keeping safety stock in upstream stages, the company will save in holding costs. On the other hand, by keeping safety stock in downstream stages, it will save lead time. Therefore, these two options must be traded off towards optimizing safety stock location for minimizing the total logistics costs. Through this procedure, the company can improve its profitability and also become a superior competitor with its chain.

The first contribution of this paper is developing a nonlinear optimization safety stock model applicable for the whole supply chain. Thus, the applications are not limited to specific stages or levels of the chain. The second contribution of this study is applying the proposed safety stock optimization model to a real-world case company. Through this contribution, it has been shown that analysis of any part in isolation and not within the chain will result in errors (overestimating or underestimating) in safety stock calculation. It also proved that in optimizing the safety stock, not only its level but also its location within the supply chain is really critical.

The optimization model developed in this paper can be adjusted according to the requirements of different value streams of any supply chain. Therefore, it is applicable

to any kind of manufacturing system with the goal of creating flow in their supply chain and reducing logistic costs by applying lean principles.

If a part is procured through more than one supplier, the current model tracks their performance with only one average number representative of all of them. In future work, the model may be extended simultaneously by increasing the accessibility of the other required input data to decide on the level of safety stock foreach of these suppliers separately.

Due to the inaccessibility of the required data, the model is currently limited to the last two stages before the customer in the chain. Again, by enhancing the visibility and control of the upstream stages in the chain, the model can be applied for each specific part from its starting point until the end of the chain. Furthermore, by increasing the accessibility of the data, the cost of shortage of raw material/semi-finished part can be more accurate by adding the re-sequencing cost of manufacturing.

The cost of shortage of the finished part required by Assembly can be more precise by making the average days of shortage weighted based on the frequency of its occurrence (increasing or decreasing trend of shortage).

One of the avenues for future work for this research would be taking into account the factors of waiting time for receiving the late parts, safety stock for the finished assembled product, and build ahead in making the decision for the safety stock.

Sensitivity analysis would be helpful for this model. This kind of analysis will support the system for taking appropriate action towards improving the system. For example, it will help to find out that improving delivery performance even with a slight amount will make a big difference in the level of required safety stock and consequently saving costs for the system.

In order to have a high level view of safety stock kept across the chain, this model can be applied to the aggregate level of stages and entities involved in the chain instead of applying it to the part level. Indeed, q_i and q_p will be the total demand of the downstream stage in a specific period seen by its upstream stage (kits of parts instead of one part). Delivery performances will be delivery performance of each stage to its downstream stage in respond to its whole demand. The parts that were historically pacers with the maximum number of shortages within the total demand of each stage will be selected as the representatives for calculating the shortage and overage costs of the stages for determining the location of safety stock.

X. Appendix

Now, assume a case that there are two different finished parts manufactured in the same plant and they require a common raw material. Model formulation and value stream for this case would be as (8) and Figure. 16:

$$MinC = C_{si}q_i(1-P_i) + C_{oi}q_i(K_i - P_i)$$
$$+ \sum_{p=1}^{2} C_{spu}q_{pu}(1-K_{pu})$$
$$+ \sum_{p=1}^{2} C_{opu}q_{pu}(K_{pu} - (P_{pu} \times K_i)) \tag{8}$$

$$SubjectTo:$$
$$K_i \leq 1$$
$$K_i \geq P_i$$
$$K_{pu} \leq 1, \qquad p = 1,2$$
$$K_{pu} \geq P_{pu} \times K_i, \qquad p = 1,2$$

Procurement sees the summation of demands for both finished parts through manufacturing at once and not separately. Therefore, mathematical proof of (9) is provided to make sure that the used formulation is accurate. Indeed, it is shown that manufacturing plant absorbs the input ration of the raw material based on its performance for each finished part:

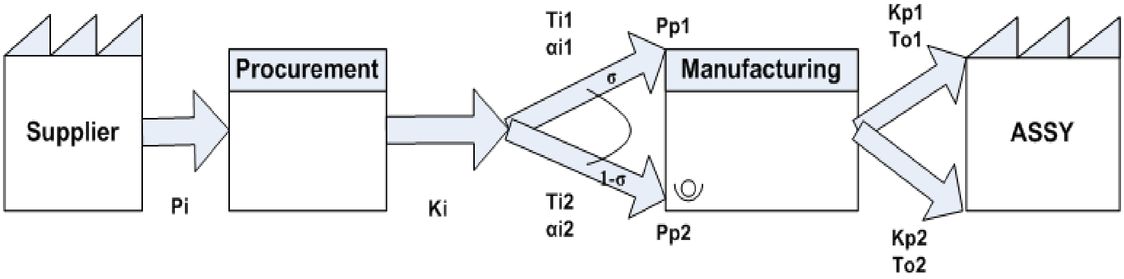

Figure 16 : Value Stream

$T_i = Total\ Input$
$\alpha_i = Input\ On\ Time$

$T_{o_1} = Total\ Output\ 1$
$\alpha_{o_1} = Output\ On\ Time$

$T_{o_2} = Total\ Output\ 2$
$\alpha_{o_2} = Output\ On\ Time$

$$K_i = \frac{\alpha_i}{T_i}, \quad K_{p_1} = \frac{\alpha_{o_1}}{T_{o_1}}, \quad K_{p_2} = \frac{\alpha_{o_2}}{T_{o_2}}$$
$$T_{o_1} + T_{o_2} = T_i \qquad (Input = Output)$$
$$\alpha_{o_1} \geq \alpha_{i_1} \times P_{p_1} \qquad Because\ of\ Safety\ Stock$$
$$\alpha_{o_2} \geq \alpha_{i_2} \times P_{p_2} \qquad Because\ of\ Safety\ Stock$$
$$\alpha_{i_1} = \sigma \times \alpha 1$$
$$\alpha_{i_2} = (1-\sigma) \times \alpha_i$$

$$T_{i_1} = \sigma \times T_i = T_{o_1}$$
$$T_{i_2} = (1-\sigma) \times T_i = T_{o_2}$$

$$K_{p_1} = \frac{\alpha_{o_1}}{T_{o_1}} = \frac{\alpha_{o_1}}{\sigma \times T_i} \geq \frac{\alpha_{i_1}}{\sigma} \times \frac{P_{p_1}}{T_i}$$
$$= \frac{\sigma}{\sigma} \times \frac{\alpha_{i_1}}{T_i} \times P_{p_1} = P_{p_1} \times K_i$$

$$K_{p_1} \geq P_{p_1} \times K_i$$

$$K_{p_2} = \frac{\alpha_{o_2}}{T_{o_2}} = \frac{\alpha_{o_2}}{\sigma \times T_i} \geq \frac{\alpha_{i_2}}{\sigma} \times \frac{P_{p_2}}{T_i}$$
$$= \frac{\sigma}{\sigma} \times \frac{\alpha_{i_2}}{T_i} \times P_{p_2} = P_{p_2} \times K_i \qquad (9)$$

$$K_{p_2} \geq P_{p_2} \times K_i$$

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