Government Policy, Trade under Oligopolistic Competition and Pollution

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A THESIS
IN THE DEPARTMENT
OF
ECONOMICS

PRESENTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY (ECONOMICS) AT CONCORDIA UNIVERSITY
MONTREAL, QUEBEC, CANADA

July 2017

CONCORDIA UNIVERSITY
SCHOOL OF GRADUATE STUDIES

This is to certify that the thesis prepared

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Entitled: Government Policy, Trade under Oligopolistic Competition and Pollution

and submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

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Abstract

Government Policy, Trade under Oligopolistic Competition and Pollution

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Concordia University, 2017

This thesis is a study on the strategic interactions between governments and firms under pollution. Governments make environmental policies and put them on their domestic firms. Firms decide their outputs and compete in the international market. Pollution are local or transboundary.

First, we discuss governments' preference when their firms compete Cournot in the international market. Our results suggest that under local pollution governments tend to apply looser environmental policies on their firms which decrease their production costs. However, these polices become more stringent if governments and firms are under transboundary pollution.

Second, we analyze firms' payoffs in Stackelberg competition and find that under this case, both governments and firms obtain less payoffs than what they can get when firms compete Cournot. This is different from the results that a firm can get more profits as the leader in Stackelberg model than its profits in Cournot model if there is no intervention from governments.

Finally, we discover governments' and firms' payoffs by collusion and compare all the possible choices for governments and firms. Our results shows that in the first stage governments choose collude in making environmental policies and in the second stage firms would collude in deciding their outputs.

Acknowledgments

I would like to express all my sincere gratitude to my supervisor, Prof. Effrosyni Diamantoudi. She provided me over the entire course of this thesis by her great guidance, advice and support. My thesis would not have been completed without her strong commitment to excel in her part as my thesis supervisor.

Besides, I would like to thank Prof. Ming Li whose advice are very helpful for me to improve my thesis. I am indebted to the Graduate Program Director, Prof. Szilvia Papai and the Chair of the Department of Economics, Prof. Grec LeBlanc. I would thank to Prof. Licun Xue and Prof. Amy Poteete who are examination committee members for my thesis. I would also like to thank the administrative staff in the Department of Economics, Lise Gosselin, Lucy Gilson, Bonnie Janicki, and Elise M. Melancon, who were always very supportive and helpful during my whole study in the Department.

Finally, I want to express my gratitude to my family who give me much warm and durable support during my whole PH. D. Study.

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Chapter 1
Literature Review & Motivation

As a part of economic development for centuries, trade plays an important role to reduce global poverty by stimulate economic growth, creating jobs, reducing prices, increasing the variety and qualities of goods for consumers, and help countries and firms acquire new technologies. Trade especially international trade significantly increase during the past decades. The development of trade contributes to growth to investment, production, technological progress, and so on.

In 2005, the value of world exports was over $10.3 trillion, one-third of which originated in developing countries and one-third of which was sold in these countries. China is a successful example that developing countries could benefit from international trade. Chinese economy develops fast during the past thirty years and one important reason is its significantly increasing exportation to the rest of the world. International trade also much affect developed countries' economies. Over the past 40 years, international trade has grown from 9.6% to 26% of the U.S. national economy. It implies that more and more U.S. jobs are related to trade and that Americans can buy more low-cost goods from abroad such as Mexico and India. As developing countries (for example China) become richer and more integrated in the global economy, they start to buy more and more American goods. About 45% of U.S. exportation go to developing countries today, compared with 39% ten years ago.

Although the direct effects of international trade on the economy are positive, as measured by Gross Domestic Product, concerns rise with regard to its "non-economic" effects. At the Ministerial meeting of the World Trade Organization (WTO) in Seattle in November 1999, some protestors launched the first of the big anti-globalization demonstrations. They wore turtle costumes to express their concerns that international trade in shrimp was harming sea turtles by ensnaring them in nets. They criticized that a WTO panel had, in the name of free trade, negated the ability of the United States to protect the turtles and undermined the international environment. Subsequently, doubts on international trade and relevant protests became common at international conferences.

The development of international trade stimulates the increase of production of goods. When firms produce more goods, they may generate more pollution from their production activities. Antweiler et al., (2001), Harbaugh et al., (2002), Cole and Elliott (2003), Frankel and Rose (2005) provide a number of empirical studies on the relationship between international trade openness and environmental quality. Economists call pollution as an externality for the economic development. It means that individuals and firms, and sometimes even countries, lack the incentive to control their pollution, because they have strong incentives to keep the competitiveness of their goods in international trade. This often occurs in developing countries which undervalue the environmental cost caused by the pollution. Grossman and Krueger (1993, 1995), and the World Bank (1992) show an inverted U-shaped relationship for a cross section of countries: at relatively low levels of income per capita, economic growth leads to greater environmental damage, until it levels off at an intermediate level of income, after which further growth leads to improvements in the environment. Runge (1987), Grossman and Kreuger (1991) find that as trade grows and incomes raise, demands for a cleaner environment tend to rise correspondingly, and new regulatory constraints induce technological innovations which are more environment-friendly. .

Lucas, et al. (1992), study the toxic intensity implied by the composition of manufacturing output, and state that trade-distorting policies increase pollution in rapidly growing countries. Dean (2002) shows on net a positive effect of liberalization for a certain level of income. Antweiler, Copeland and Taylor (2001) and Copeland and Taylor (2001, 2003, 2004) also find that the net effect of trade liberalization on SO2 concentrations is beneficial. Some scholars state that the negative impacts of trade on the environment are overvalued In face, these impacts vary greatly in degree and by location. Runge (1990), Harold and Runge (1993) show evidence that reducing subsidies and trade distortions in agriculture are often helpful to reduce environmental damages by lowering fertilizer and pesticide use and increasing the efficiency with which water and soil resources are used .

A number of literature (Vogel, 1995; Charnovitz and Weinstein, 2001) show that it is often necessary for governments to intervene international trade by making environmental policies. If no negative effects of trade on environment is found, then the trade effects of the regulation are not at issue. But if such negative effects appear to be present, it opens the way for governments to identify, in which its benefits for the trade are weighed against its harm to the environment (Hudec and Farber, 1992). While firms focus on pursuing their own profits without concerns for the environmental pollution related to their production activities, governments consider both of them to maximize the welfare of the countries. Governments are responsible for making suitable environmental policies to balance their domestic firms' profits and the social environmental damage from the pollution that affect their countries. Barrett (1997, 2003) shows theoretically how multilateral trade sanctions can sometimes successfully enforce a multilateral environmental treaty such as the Montreal Protocol. During past decades, governments impose some standards on environmental externalities which ensures that trade liberalization is ultimately welfare-enhancing. For example, in United States, since 1986 the Environmental Protection Agency (EPA) has required a Toxic Release Inventory (TRI), in which 10,000 U.S. manufacturers report annual releases from their facilities into the air, ground and water of more than 300 toxic chemicals. These chemicals include asbestos, freon, and PCBs, as well as 20 toxic chemical categories such as lead compounds. As this list continues to grow, firms and governments have an increasing basis to apply measures such as TRI releases per product or per dollar of sales. Firms' performance may be different in this issue. Dow Chemical stopped injecting hazardous wastes underground before the TRI began, but its competitor Du Pont chemical failed to do so (Rice, 1993). Such regulatory framework could bring double impacts: it creates a quantitative basis for reducing pollution and increases firms' incentives to move production to foreign countries where such policies are less strict.

However, in contrast to the pollution concerns over the impacts of more liberal trade, some countries most directly involved in trade tend to focus on another main issue, which is trade protectionism disguised as environmental action. In other words, environmental protection can be an excuse for trade protectionism. If governments give in to protectionist arguments and establish trade barriers, growth in trade will become slow and benefit from trade will decrease. Moreover, such barriers may not necessarily end up with a better environment when they cut off payoffs from the trade. Another reason for establishing these barriers is from domestic firms. Competing firms in the international market which may not be particularly friendly to the environment, sometimes seek to advocate or retain barriers to imports in the name of environmental protection, when in fact it is their own profits they are trying to protect. Thus, environmental concerns in such a case is really an excuse for protectionism.

Besides, governments may have incentives to impose weak environmental standards on their domestic firms that compete in the imperfectly competitive international markets. This is another type of trade protectionism that governments neglect or undervalued the environmental damage in their countries. The weak environmental standards make firms profit from the marginal cost of abatement which is less than the marginal damage from pollution. These purposefully designed policies may confer competitive advantage upon the domestic firms against the foreign firms. Grimmett (1991), Reistein (1991), Whalley (1991), Pearce (1992), Anderson and Blackhurst (1992) explore this topic in early 1990s. Barrett (1994) shows that if the domestic industry consists of one firm, the foreign industry is imperfectly competitive, and competition in international markets is Cournot, then the domestic government has an incentive to impose such a weak environmental standard. A number of papers in the literature criticize this sort of weak environmental standard policy. Barrett (1994) demonstrates that environmental policy is inferior to industry policy as an instrument for improving competitiveness. Markusen, Morey and Olewiler (1993) state that if a nation neglects the internal impact in determining an "optimal" emission tax, the welfare losses caused by its unilateral emission tax is greater than expected. Burguet and Sempere (2003) show that decreasing tariff will stimulate governments to make less stringent environmental policies. Diamantoudi and Sartzetakis (2006) discuss the stability of international environmental agreements among governments which make optimal emission standards on their domestic firms' production.

This paper discusses the optimal environmental policy, in which both firm's profits and country's social environmental damage are considered. Government controls the social environmental damage in the country by its environmental policy. Such policy may affect the competitiveness of the domestic firm in the international market since it affects the firm's environmental abatement cost. The firm may have incentive to move to another country where it can profit from a lower environmental abatement cost than it can obtain from its home country because of countries' different environmental policies. McGuire (1982) presents a theoretical analysis of the movement of capital across boundaries resulting from environmental regulation. Low and Yeats (1992) state that environmentally "dirty" industries migrated to lower income countries where environmental standards are weaker. Lucas, Wheeler, and Hettige's (1992) empirical studies test that the OECD countries' environmental policies drive dirty industries to developing countries. Rauscher (1993) also finds that in an open economy, the polluting industry of the home country will migrate to the foreign country with looser environmental policies. However, some scholars hold different opinions. Dean (1992) claims that the link between trade flows and environmental standards is weak or nonexistent. In an earlier paper, Leonard and Duerksen (1980) analyze the investment and trade data to track the relationship between environmental policies and the migration of pollution-intensive industries. Their statistical results reveal that the growth of U.S. investment in developing countries did not exceed the entire overseas investment growth rate.

This paper focuses on how governmental environmental policy and trade interact under different types of pollution and how they affect countries' and firms' payoffs. Unlike Barrett (1994), this paper considers not only local pollution but also transboundary pollution. The paper finds whether the pollution is local or transboundary could be a crucial factor of consideration for government to make its environmental policy. Unlike Hoel and Schneider (1997), this paper treats a firm and its home government as two separate entities. The firm considers only its own profit and neglects the social environmental damage while the government takes both into account. Unlike the paper of Lanoie, Lucchetti, Johnstone and Ambec (2011) which shows evidence that governmental environmental policies will stimulate firms' environmental innovation, we discuss the impacts of governmental environmental policies on firms' competition and profits in this paper.

To be simplified, we analyze the competition between two firms which are located in two different countries. These two firms produce a same product and sell in the international market. We assume that no other producer and no other parties can make influence on the market. It is a case for duopoly and firms can choose different types of competition in the market. They may decide their outputs independently and simultaneously (Cournot model), or move before the other firm (Stackelberg model) or cooperate with each other (Collusion model). Firms' motivation is to maximize their profits, so they need consider their revenues and costs. Each firm's revenue is decided by its output and its' competitor's output. Its cost includes production cost which is fixed and environmental abatement cost which is affected by its government's environmental policy. Governments decide their environmental policies before their firms' productions. Each government maximizes its country's welfare which is its domestic firm's profits minus the relevant social environmental damage in the country. Thus, it needs balance its domestic firm's profits and the environmental cost from the pollution when it make the policy. In this paper, pollution could be purely local or transboundary. We discuss the impact of government environmental policies on firms in different types of competition and under different types of pollution.

Golden (1993) suggests that differences in environmental policies or standards commonly exist across countries, especially between North and South. Mohnen (1988) and Brown, et. al., (1993) discuss important terms of trade effects arising from transboundary externalities. Krutilla (1991), Merrifield (1988), Antle and Just (1992), and Anderson (1992) attempt to integrate externalities theory with the neoclassical theory of international trade. In Chapter 2, we firstly discuss the cases where two firms with same production cost react independently and play Cournot competition in the market. Then, we assume that one firm would decrease its production cost and the competition continues. Then governments would modify their environmental policies and firms would modify their outputs. By comparing the payoffs among different cases, we show how governmental environmental policies affects firms' incentives to decreases their production costs which reflect the technology progress. We also analyze how different types of pollution (local or transboundary) play different roles in this evolution.

In a duopoly competition without intervention from governments, it is not surprising that each firm has incentive to decide its output and produce before its competitor since the first move can bring more profits to the leader firm. However, if we consider the pollution issue and introduce governmental environmental policies, the result might be different. In Chapter 3, we analyze firms' incentives to play as the first mover in the competition. In such Stackelberg competition, payoffs of the leader firm and the follower firm are compared, as well as welfare of their countries. We also compare the profits one firm obtains in the Cournot competition with what it can get as the leader in the Stackelberg competition. Based on results from these comparison, we state that firms prefer compete under Cournot in stead of Stackelberg and their preference are welcome by their governments.

Barrett (1997, 2003) proves that governments have incentives to collude in deciding their environmental policies and making a multilateral environmental treaty such as the Montreal Protocol. In Chapter 4, we analyze the cases for collusion. We assume that two governments could cooperate to make environmental policies together or make them independently and that two firms could cooperate to decide their outputs together or decide them independently. We analyze all the possibilities and compare the payoffs of governments and firms in different cases. It look likes a two-stage game. In the first stage, two governments decide to collude or not in making environmental policies. In the second stage, based on the known environmental policies from their governments, two firms decide to collude or not in deciding their outputs. Then, from the comparison of governments' and firms' payoffs, we find that collusion is the best choice for both governments and firms.

Based on the results we find in Chapters 2, 3 and 4, we make our conclusion in Chapter 5. First, we prove that with governmental environmental policies, firms prefer Cournot competition to Stackelberg competition in the international market. Second, we show that when firms compete Cournot with governmental environmental policies, the circumstance of local pollution stimulates firms to decreases their production costs while the circumstance of transboundary pollution discourages firms to do so. Third, if collusion is a possible choice, governments would make their environmental policies together and firms would also collude in deciding their outputs.

Chapter 2
Impacts of Governmental Environmental Policies on Firms in the Cournot Competition

2.1 Introduction

Cournot competition is a kind of competition in the market in which firms compete on the amount of output whey will produce, which they decide on independently of each other and at the same time. Barrett (1994) claims that under Cournot competition, governments have incentive to make loose environmental polices to increase their domestic firms' competitiveness in the international market. Sanan and Zanaj (2007) discuss firms' environmental innovation under Cournot competition. They state that when firms play a la Cournot, they either both innovate to protect their market share in the output market or they both choose not to innovate.

In this Chapter, we discuss the environmental policies that governments make and apply on their domestic firms which play a la Cournot competition in the inter national market. Such environmental policies could influence firms abatement costs thus influence their competitiveness in the market. Once firms know the environmental policies they have to respect, they would reconsider the levels of their output to pursue maximum profits. Meanwhile, when governments consider the environmental policies, they need care both their domestic firms' profits and the social environmental damage due to the pollution.

The model employed is a two-staged game involving two governments and their two domestic firms. Firms sell all their products in the imperfectly competitive international market. Pollution is local or transboundary. Based on a certain type of pollution, governments move first by making environmental policies for their domestic firms. Firms take these policies as given and compete by choosing their output levels. It is assumed that other countries, whether or not they are consumer countries, have no means of influencing environmental policies in these two producer countries.

2.2 Model

We assume that there exist only two countries A, B. There exists one domestic firm i in the country $i, i \in \{A, B\}$. We denote them firm A and firm B which compete Cournot in the international market. Denote the profits of firm A by π_A and the profits of firm B by π_B. Assuming that firm i's production cost per unit is c_i. Government i decides e_i and apply it on firm i. e_i reflects the technology standard for environmental abatement. As e_i decreases, the technology becomes more advanced which generate less pollution and bring higher environmental abatement cost to the firm. $e_i = 1$ means that the government does not put any technology standard on its firm whose environmental abatement cost is 0. $e_i = 0$ means that the government would require its firm to use the most adavnced technology to totally eliminate the relevant pollution. Thus, the firm would assume the highest abatement cost. To be realistic, we let $1 > e_i > 0$ in our model. Firm i's output is q_i and it generates a certain level of pollution by its production. The pollution causes the social environmental damage only in the country i under local pollution while it causes the equal damage in both countries under transboundary pollution. Since the pollution increases as q_i increases and firm i must meet the requirement of emission standard set by government i, firm i's abatement cost depends on both q_i and e_i. In the model, governments choose the emission standards e_i in the first stage and firms choose the output levels q_i in the second stage. Firms consider only their profits π_i while governments take account into their domestic firms' profits and social environmental damage in their countries. Let W_i be government i's welfare. Government i's welfare equals to its domestic firm i's profits minus the cost of social environmental damage in its country. The environmental damage in the country is caused by the production of the domestic firm. In our model, consumer welfare is not considered. Government i considers its welfare by firm i's profits and the pollution in the country.

2.2.1 Firms' profits

Firms decide their outputs to maximize their profits. Their profits equal to revenues minus costs. Their costs include production cost and environmental abatement cost. They have the same environmental abatement cost parameter d. Based on different cases, their production cost could be same or different.

2.2.1.1 Homogeneous firms in production cost

Two firms are homogeneous in production cost per unit. $c_A = c_B = c$. Their profit functions are as below.

$$\text{Firm } A\text{: } \pi_A = [a - b(q_A + q_B)]q_A - cq_A - d(1 - e_A)q_A \quad (1)$$

$$\text{Firm } B\text{: } \pi_B = [a - b(q_A + q_B)]q_B - cq_B - d(1 - e_B)q_B \quad (2)$$

where a, b, d are positive parameters. Firm i's profit is its total revenue $[a - b(q_A + q_B)]q_i$ minus its cost which is composed of total production cost cq_i and total environmental abatement cost $d(1 - e_i)q_i$.

Two governments consider both domestic firm's profit and social environmental damage in the country. Government i controls firm i's output and pollution by making environmental policy e_i which can affect firm i's environmental abatement cost. Based on the nature of pollution (local or translational), governments' welfare functions are different.

2.2.1.2 Heterogeneous firms in production cost

We consider the cases that two firms are homogeneous in production cost. We assume that firm A's production cost per unit is higher than that of firm B. $c_A = c, c_B = \alpha c$ and $\alpha \in (0, 1)$. Their profit functions are as below.

$$\text{Firm } A\text{: } \pi_A = [a - b(q_A + q_B)]q_A - cq_A - d(1 - e_A)q_A \tag{3}$$

$$\text{Firm } B\text{: } \pi_B = [a - b(q_A + q_B)]q_B - \alpha c q_B - d(1 - e_B)q_B \tag{4}$$

Here firm A's production cost per unit is c and firm B's production cost per unit is αc. As $\alpha \in (0, 1)$, firm B has cost advantage.

2.2.2 Governments' welfare

Governments always make policies before their firms' production. Each country's welfare equals to its domestic firm's profit minus the relevant social environmental damage from the pollution. The pollution could be purely local or transboundary.

2.2.2.1 Governments' welfare with local pollution

Local pollution implies that pollution caused by firm i's production in country i does not influence the other country. Then, governments' welfare functions are:

$$\text{Government } A\text{: } W_A = \pi_A - \frac{t}{2}e_A^2 \tag{5}$$

$$\text{Government } B\text{: } W_B = \pi_B - \frac{t}{2}e_B^2 \tag{6}$$

t is positive parameter. Government i's welfare is its domestic firm's profit π_i minus the social environmental damage $\frac{t}{2}e_i^2$ in the country. In the welfare function, firm i's output q_i and relevant pollution are internalized in the governmental environmental policy e_i.

2.2.2.2 Governments' welfare with transboundary pollution

Transboundary pollution implies that pollution caused by firm i's production in country i not only influences its home country but also influences the other country. In this case, governments' welfare functions are:

Government A: $W_A = \pi_A - \frac{t}{2}(e_A + e_B)^2$ (7)

Government B: $W_B = \pi_B - \frac{t}{2}(e_A + e_B)^2$ (8)

Compared with the case 2.2.1, the only difference here is the social environmental damage which is expressed as $\frac{t}{2}(e_A + e_B)^2$ in stead of $\frac{t}{2}e_i^2$. It shows that government i's policy e_i has a direct impact on the welfare of the other country.

2.2.3 Solution

From the objective functions above, we could obtain the different solutions based on firms' heterogeneities and types of pollution.

2.2.3.1 Homogeneous firms under local pollution and transboundary pollution

For the case which includes two homogeneous firms with local pollution, we have four functions (1), (2), (5), (6).

Assumption 2.1: To make our solutions are interior($1 > e_i^* > 0$, $q_i^* > 0$,for $i \in A, B$), we set restrictions for parameter:

$a - c - d > 0$

$9bt > 4d(a - c)$

First, we solve firms' problems:

$$\max \pi_i = [a - b(q_A + q_B)]q_i - cq_i - d(1 - e_i)q_i \ , \quad \text{for } i \in A, B$$

q_i

$$\text{s.t.} \quad q_i > 0$$
$$e_i \in (0,1)$$

After we get $q_A^*(e_A, e_B)$ and $q_B^*(e_A, e_B)$, we plug them into governments' welfare functions:

$\max W_i = \{a - b[q_A^*(e_A, e_B) + q_B^*(e_A, e_B)]\}q_i^*(e_A, e_B) - cq_i^*(e_A, e_B) - d(1 - e_i)q_i^*(e_A, e_B) - \frac{t}{2}e_i^2$

$$\text{s.t. } e_i \in (0,1), \text{ for } i \in A, B$$

We get e_i^*, then we can get q_i^*,π_i^* and W_i^*.

For two homogeneous firms with transboundary pollution, we have functions (1), (2), (7), (8). The first step is same as the case above. We have $q_A^*(e_A, e_B)$ and $q_B^*(e_A, e_B)$ for firms A, B. The second step is different as governments here should consider not only domestic firm's pollution but also foreign firm's pollution when they decide environmental policies. So we plug $q_A^*(e_A, e_B)$ and $q_B^*(e_A, e_B)$ into (7), (8) to solve:

$\max W_i = \{a - b[q_A^*(e_A, e_B) + q_B^*(e_A, e_B)]\}q_i^*(e_A, e_B) - cq_i^*(e_A, e_B) - d(1 - e_i)q_i^*(e_A, e_B) - \frac{t}{2}(e_A + e_B)^2$

$$\text{s.t. } e_i \in (0,1) \text{ for } i \in A, B$$

Then we get e_i^*, q_i^*,π_i^* and W_i^*.

	Homogeneous firms with local pollution	Homogeneous firms with transnational pollution
e_i^*	$e_A^* = e_B^* = 4d(a-c-d)/(9bt-4d^2)$	$e_A^* = e_B^* = 4d(a-c-d)/(18bt-4d^2)$
q_i^*	$q_A^* = q_B^* = 3t(a-c-d)/(9bt-4d^2)$	$q_A^* = q_B^* = 6t(a-c-d)/(18bt-4d^2)$
π_i^*	$\pi_A^* = \pi_B^* = 9bt^2(a-c-d)^2/(9bt-4d^2)^2$	$\pi_A^* = \pi_B^* = 36bt^2(a-c-d)^2/(18bt-4d^2)^2$
W_i^*	$W_A^* = W_B^* = t(9bt-8d^2)(a-c-d)^2/(9bt-4d^2)^2$	$W_A^* = W_B^* = 4t(9bt-8d^2)(a-c-d)^2/(18bt-4d^2)^2$

Table 2.1 Solution for homogeneous firms under local&transboundary pollution

To easily compare the solutions between two cases above, we put the solutions in the table 2.1. It shows governmental policies, outputs, firms' profits and governments' welfare in the case of homogeneous firms under local&transboundary pollution.

2.2.3.2 Heterogeneous firms under local pollution

If two firms have the different production costs and they interact under local pollution, we have functions (3), (4), (5) and (6) . The analysis is similar to that in 2.3.1. In the first step, we get firms' best response function of their outputs based on the governmental environmental policies. In the second step, we put these best response functions in governments' welfare functions to obtain the optimal emission standards. Then we plug these optimal emission standards into firms' profit functions and governments' welfare functions to obtain the optimal outputs, profits and welfare. The only difference from 2.3.1 is that here firm A and firm B are heterogeneous in production cost.

As $\alpha \in (0,1)$, firm B has the lower production cost than firm A. We need max firm A's and firm B's profit functions and get their best response functions separately.

Assumption 2.2: To make our solutions are interior, we set restrictions for parameter:

$3bt > 4d^2$

$36bdt(a+3c) > 48cd^3 + 27bt(4d^2+3bt)$

$(a-c-d)\left(81b^2t^2 - 108bd^2t + 16d^4 - 16d^3\right) > 0$

$1 > \alpha > 0$

$\alpha > [12d^2(a-c-d) - 9bt(a-2c-d)]/9bct$

$\alpha > [36bdt(a+c+3d) - 48ad^3 - 81b^2t^2]/24cd(3bt-2d^2)$

$\alpha > [16d^3(a+4c-d) - 4d^2(a-5c-d)(9bt-8d^2) - (a-2c-d)(9bt-4d^2)(9bt-12d^2)]/c\left(81b^2t^2 - 80d^4 + 80d^3\right)$

First we solve firms' problems:

For firm A,

$$\max_{q_A} \pi_A = [a - b(q_A+q_B)]q_A - cq_A - d(1-e_A)q_A$$

s.t. $\quad q_A > 0$

$$e_A \in (0,1)$$

And for firm B,

$$\max_{q_B} \pi_B = [a - b(q_A + q_B)]q_B - \alpha c q_B - d(1 - e_B)q_B, \quad \alpha \in (0,1)$$

s.t. $\quad q_B > 0$

$$e_B \in (0,1)$$

We get $q_A^*(e_A, e_B)$ and $q_B^*(e_A, e_B)$. It is not surprising that they are not equal since firms are heterogeneous. Similar as case in 2.3.1,we plug them into governments' welfare functions (5), (6) to get e_i^*, then we plug e_i^* into firms' and governments' objective functions and get q_i^*,π_i^* and W_i^*.

We put the solutions in the table 2.2:

	Heterogeneous firms with local pollution	
	firm A & country A	firm B & country B
e_i^*	$e_A^* = [4d(a - 2c + \alpha c - d)(9bt - 8d^2) - 16d^3(a - 2\alpha c + c - d)]/(9bt - 4d^2)(9bt - 12d^2)$	$e_B^* = [4d(a - 2\alpha c + c - d)(9bt - 8d^2) - 16d^3(a - 2c + \alpha c - d)]/(9bt - 4d^2)(9bt - 12d^2)$
q_i^*	$q_A^* = [(a - 2c + \alpha c - d) + d(2e_A^* - e_B^*)]/3b$	$q_B^* = [(a - 2\alpha c + c - d) + d(2e_B^* - e_A^*)]/3b$
π_i^*	$\pi_A^* = bq_A^{*2}$	$\pi_B^* = bq_B^{*2}$
W_i^*	$W_A^* = \pi_A^* - \frac{te_A^{*2}}{2}$	$W_B^* = \pi_B^* - te_B^{*2}/2$

Table 2.2 Solution for heterogeneous firms under local pollution

Table 2.2 shows governmental policies, outputs, firms' profits and governments' welfare in the case of heterogeneous firms under local pollution.

2.2.3.3 Heterogeneous firms under transboundary pollution

We have functions (3), (4), (7), (8) for two heterogeneous firms with transboundary pollution.

Assumption 2.3: To make our solutions are interior, we set restrictions for parameter:

$a - c - d > 0$

$9bt > 4d^2$

$9bt > 2d(a - c)$

$1 > \alpha > 0$

$\alpha > [9bt(c - 2d) + 4d^2(a - c)]/9bct$

$\alpha > [9bct - 4d^2(a - d)]/c(9bt - 4d^2)$

The first step is similar as that in 2.3.2 and we have firm A's and firm B's best reaction functions. We plug them into governments' welfare functions (7), (8) which reflect the case of transboundary pollution. Then we get e_i^*, q_i^*,π_i^* and W_i^*.

We show the solutions in the table 2.3 below:

	Heterogeneous firms with transnational pollution	
	firm A & country A	firm B & country B
e_i^*	$e_A^* = 4d(a-c-d)/(18bt-4d^2) + 9bct(1-\alpha)/d(18bt-4d^2)$	$e_B^* = 4d(a-\alpha c-d)/(18bt-4d^2) - 9bct(1-\alpha)/d(18bt-4d^2)$
q_i^*	$q_A^* = 3t(2a-c-\alpha c-2d)/(18bt-4d^2)$	$q_B^* = 3t(2a-c-\alpha c-2d)/(18bt-4d^2)$
π_i^*	$\pi_A^* = 9bt^2(2a-c-\alpha c-2d)^2/(18bt-4d^2)^2$	$\pi_B^* = 9bt^2(2a-c-\alpha c-2d)^2/(18bt-4d^2)^2$
W_i^*	$W_A^* = t(9bt-8d^2)(2a-c-\alpha c-2d)^2/(18bt-4d^2)^2$	$W_B^* = t(9bt-8d^2)(2a-c-\alpha c-2d)^2/(18bt-4d^2)^2$

Table 2.3 Solution for heterogeneous firms under transboundary pollution

Table 2.3 shows governmental policies, outputs, firms' profits and governments' welfare in the case of heterogeneous firms under transboundary pollution.

2.3 Results

After comparing the solutions in different cases, we find some results about governments and firms.

Proposition 2.1: For homogeneous firms, $q_i^{*,l} > q_i^{*,t}$ and $\pi_i^{*,l} > \pi_i^{*,t}$, while $e_i^{*,l} > e_i^{*,t}$ and $W_i^{*,l} > W_i^{*,t}$.

The intuitive explanation behind this proposition is that if firms' production cost are equal, governments and firms under local pollution are in a better position than those under transboundary pollution. Under local pollution, pollution from abroad does not affect the home country, so governments would make less stringent environmental policies to help domestic firms to get more profits. Since domestic firms' increasing profits are more than countries' increasing pollution, countries' welfare under local pollution are higher than those under transboundary pollution. Firms' total costs are lower under local pollution because of their lower environmental abatement costs. Thus firms' outputs and profits become more.

We illustrate this result by using a numerical example. We assume $a = 5$, $b = 4$, $c = 2$, $d = 1$ and $t = 0.5$, which satisfy all the restrictions. Then we have solutions:

$e_i^{*,l} = 0.57 > e_i^{*,t} = 0.25, i = \{A, B\}$, which implies governmental environmental policy is less strict in local pollution than in transboundary pollution.

$q_i^{*,l} = 0.214 > q_i^{*,t} = 0.188, i = \{A, B\}$, which implies firms produce more in local pollution than they produce in transboundary pollution.

$\pi_i^{*,l} = 0.184 > \pi_i^{*,t} = 0.141, i = \{A, B\}$, which implies firms get more profits in local pollution than they get in transboundary pollution.

$W_i^{*,l} = 0.102 > W_i^{*,t} = 0.078, i = \{A, B\}$,which implies countries' welfare are higher in local pollution than in transboundary pollution.

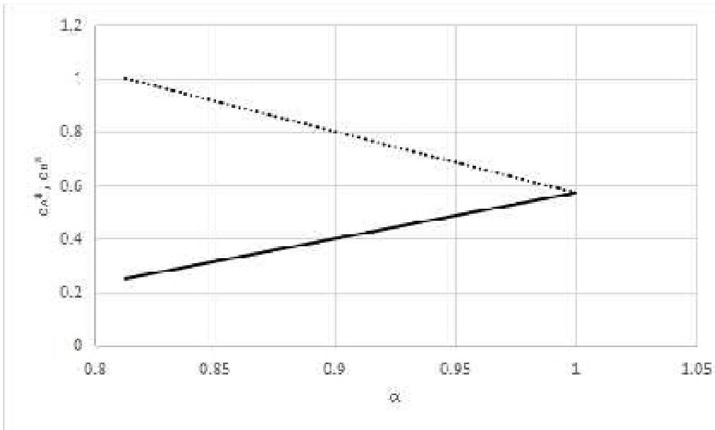
Proposition 2.2: For heterogeneous firms under local pollution, $q_A^{*,l} < q_B^{*,l}$ and $\pi_A^{*,l} < \pi_B^{*,l}$. $e_A^{*,l} < e_B^{*,l}$ and $W_A^{*,l} < W_B^{*,l}$.

Proposition 2.2 implies that under local pollution, the country whose domestic firm has the cost advantage of production has a higher tolerance for the pollution, since its firm's increasing profits are higher than the costs of the increasing pollution. Thus the government would impose a relatively loose environmental policy on its firm which benefits from both the advantage of production and the advantage of abatement. As a result, the firm produces a higher quantity of the products and obtains more profits than its competitor. Its country has higher welfare than its competitor's country.

We use another numerical example with same data to observe governments (figure 1,4) and firms (figure 2,3).

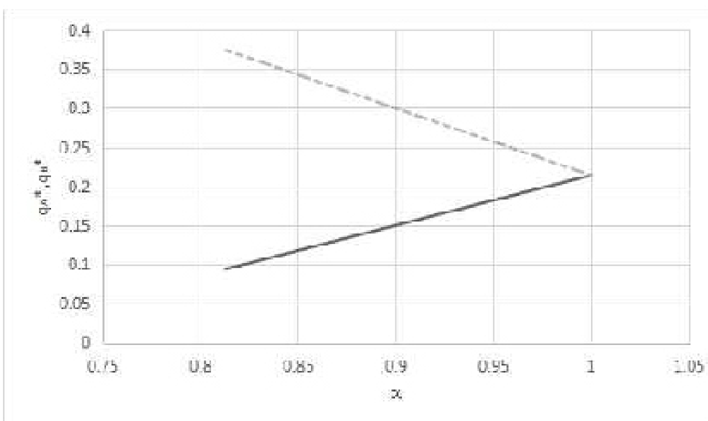
In Figure 1, X-axis is "α" which reflects the heterogeneity of production costs between firms. $\alpha \in (0.813, 1)$ for solutions are interior. The heterogeneity increases as α decreases. Y-axis represents governmental environmental policies. The solid line is for government A and the dotted line is for government B. We observe that $e_A^{*,l}$ is always smaller than $e_B^{*,l}$, which means government B applies a less strict environmental policy than government A. As α decreases, the heterogeneity increases and the gap between $e_A^{*,l}$ and $e_B^{*,l}$ increases. This trend of the gap shows that government A tends to make a more stringent environmental policy while government B tends to make a looser policy. The first policy imposes a higher abatement cost on firm A while the latter policy imposed a lower abatement cost on firm B.

In figure 2, X-axis is "α" and Y-axis represents firms' outputs. The solid line is $q_A^{*,l}$ which is higher than the dotted line which reflects $q_B^{*,l}$. As α decreases, firm B's advantage in production cost increases. Meanwhile its advantage in environmental abatement cost also increases due to a looser environmental policy. Thus the gap between $q_A^{*,l}$ and $q_B^{*,l}$ increases, which means firm B's output becomes more and more than that of firm A.



e_A^*:solid line; e_B^*: dotted line.

Figure 1. Governmental policies under local pollution in heterogeneous case



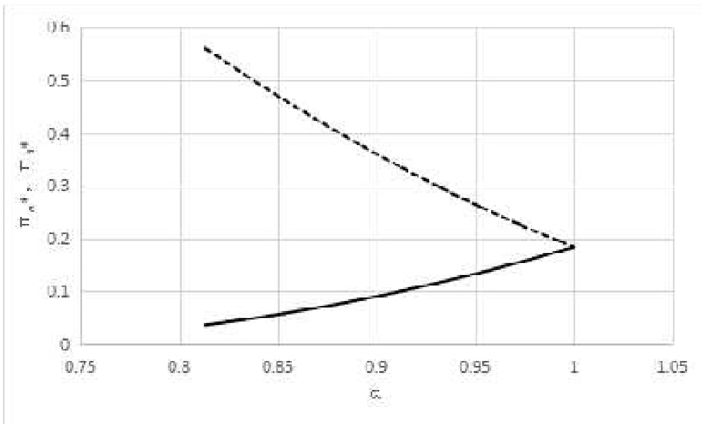
q_A^*:solid line; q_B^*: dotted line.

Figure 2. Firms' outputs under local pollution in heterogeneous case

In figure 3, X-axis is "α" and Y-axis represents firms' profits. Firm A's profit $\pi_A^{*,l}$ (solid curve) is less than firm B's profits $\pi_B^{*,l}$ (dotted curve) and their gap increases as α decreases.

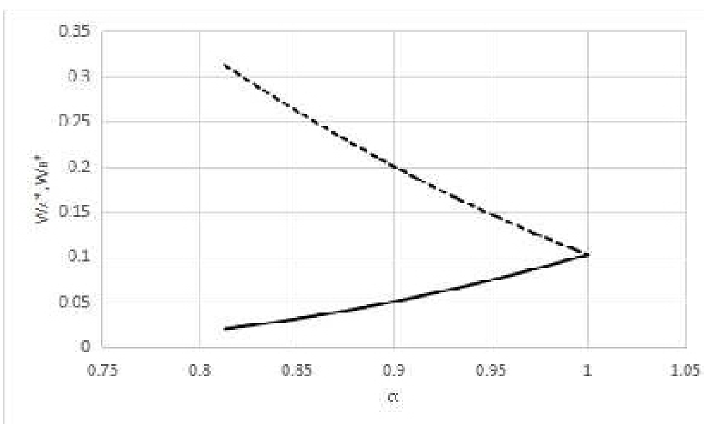
In figure 4, X-axis is "α" and Y-axis represents countries' welfare. Country A's welfare $W_A^{*,l}$ (solid curve) is lower than country B's welfare $W_B^{*,l}$

(dotted curve) and their gap increases as α decreases.



π_A^*:solid curve; π_B^*: dotted curve.

Figure 3. Firms' profits under local pollution in heterogeneous case

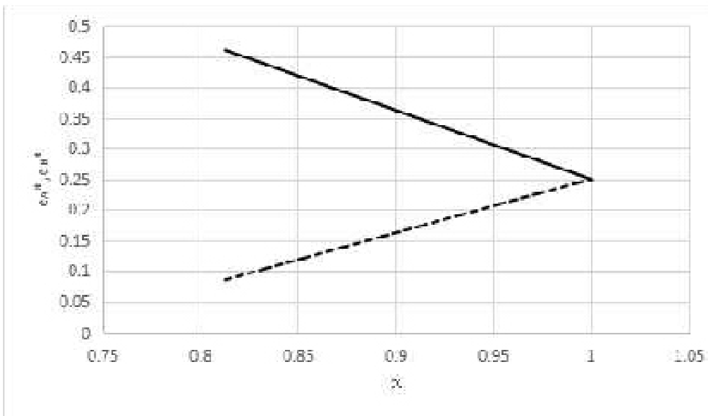


W_A^*:solid curve; W_B^*: dotted curve.

Figure 4. Countries' welfare under local pollution in heterogeneous case

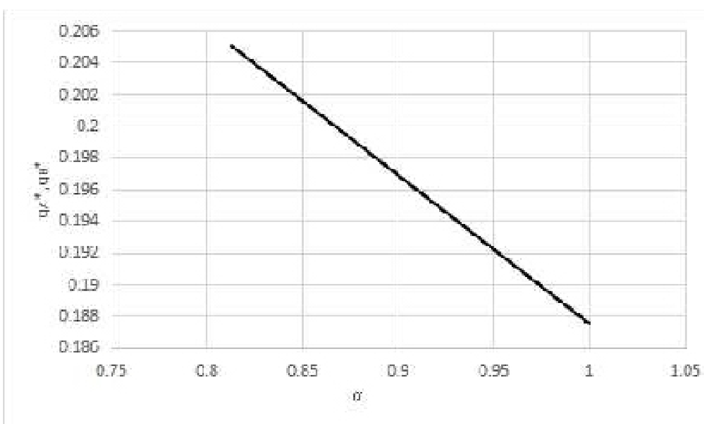
Proposition 2.3: For heterogeneous firms in a transboundary pollution, $q_A^{*,t} = q_B^{*,t}$ and $\pi_A^{*,t} = \pi_B^{*,t}$. $e_A^{*,t} > e_B^{*,t}$ and $W_A^{*,t} = W_B^{*,t}$.

Propositions 2.3 suggests that the firm with lower production cost would suffer from a more strict governmental environmental policy than its competitor under transboundary pollution. Such a policy increases its environmental abatement cost and offsets its advantage in production cost. Thus its output and profit are equal as those of its competitor. The welfare of the two countries are also equal. Figures 5,6,7,8 show how governments and heterogeneous firms are influenced in a transboundary pollution.



e_A^*:solid line; e_B^*: dotted line.

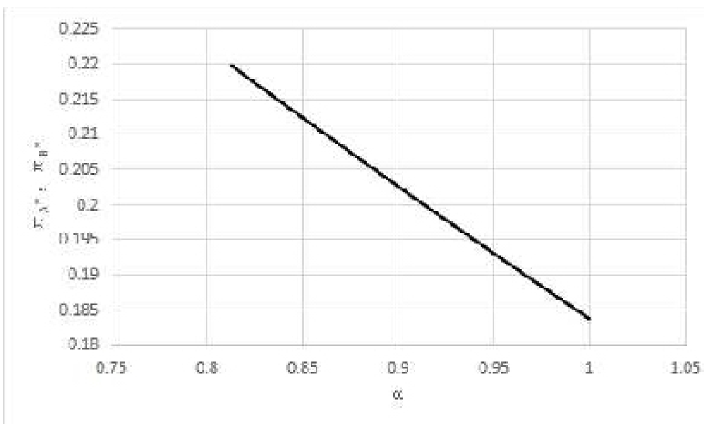
Figure 5. Governmental policies under transboundary pollution in heterogeneous case



q_A^*:solid line; q_B^*: dotted line (coincide).

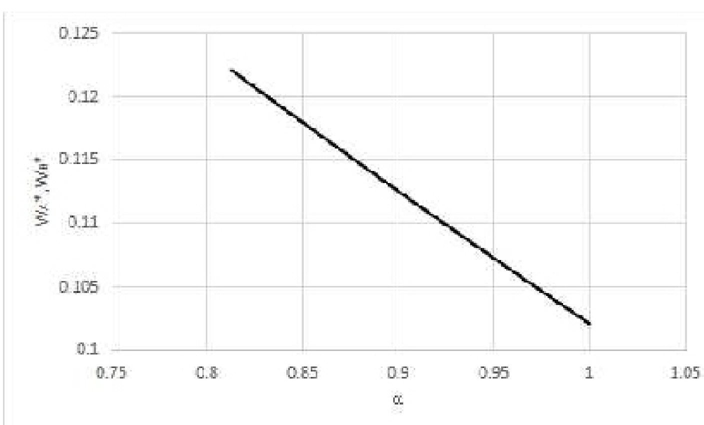
Figure 6. Firms' outputs under transboundary pollution in heterogeneous case

In Figure 5, X-axis is "α" which reflects the heterogeneity of production costs between firms. $\alpha \in (0.813, 1)$ for solutions are interior. The heterogeneity increases as α decreases. Y-axis represents governmental environmental policies. The solid line is for government A and the dotted line is for government B. We observe that $e_A^{*,l}$ is always higher than $e_B^{*,l}$, which means government A applies a less strict environmental policy than government B. This is completely opposite of the result in Figure 1.



π_A^*:solid curve; π_B^*: dotted curve (coincide)

Figure 7. Firms' profits under transboundary pollution in heterogeneous case



W_A^*:solid curve; W_B^*: dotted curve (coincide)

Figure 8. Countries' welfare under transboundary pollution in heteroge-

neous case

In Figure 6, X-axis is "α" and Y-axis represents firms' outputs. The solid line is $q_A^{*,t}$ overlaps the solid line which reflects $q_B^{*,t}$. As α decreases, firm B's production cost decreases. However, as Government B's environmental policy becomes stricter and stricter, firm B's environmental abatement cost increases by a same amount of its decreasing production cost. Consequently, firm B's output remains equal as that of firm A.

In Figure 7, X-axis is "α" and Y-axis represents firms' profits. The solid curve is $\pi_A^{*,t}$ overlaps the dotted curve which reflects $\pi_B^{*,t}$. Since firms' outputs and total costs are same, their profits remain equal as them in homogeneous cases.

In Figure 8, X-axis is "α" and Y-axis represents countries' welfare. Country A's welfare $W_A^{*,t}$ (solid curve) is equal to country B's welfare $W_B^{*,t}$ (dotted curve). The reason is that both their domestic firms' profits and the social environmental damages in their countries are same regardless of the change of "α".

The intuition for Proposition 2.3 is that when pollution is transboundary, both countries face the same loss from pollution. We view such loss as a function of the sum of emissions from each country, then they must face the same marginal loss. Meanwhile the optimal condition says that each country must also be facing the same marginal profit from emission in equilibrium, which, upon examination of the Cournot profits, is a linear function of a country's own equilibrium production quantity, common to both countries. This implies that both countries must produce the same quantity of the good, which in turn implies the environmental policies by the two countries must completely offset the disadvantage in production cost. The equalization results in profits and welfare then follow.

2.4 Conclusion

This paper discusses how governments make environmental policies under different types of pollution. They choose more stringent policies under transboundary pollution than those they choose under local pollution. When firms are homogeneous in production cost, both firms and governments are better off under local pollution.

When firms are heterogeneous in production cost, the results are different. Under local pollution, the government chooses a looser environmental policy if its domestic firm has lower production cost than the foreign firm. The country with the cost advantage benefits more from profits generated by the industry since it is in a better competitive position than the other country, so has a relatively higher tolerance for the resulting pollution. Industry profits are higher in the country with cost advantage both because of lower costs of production and because of lower costs of abatement due to laxer environmental policy. The larger negative effect of pollution is not enough to offset the gain in industry profits. In stark contrast, we find that the cost advantage in production does not matter under transboundary pollution. The country with the cost advantage now sets a more stringent environmental policy and hence higher cost of abatement which completely offsets the cost advantage of production. Thus, firms' profits are equal, as well as countries' welfare. A more stringent environmental policy imposed on the firm with lower production cost implies that such policy under transboundary pollution would discourage he firm' incentive to decrease its production cost, since the increasing benefits from the decrease of its production cost will be shared with the other firm which benefits from a looser environmental policy. In general, we conclude that the governmental environmental policies under local pollution stimulate firms to decrease their production costs while such policies under transboundary pollution discourage firms to make the same efforts.

Chapter 3
Impacts of Governmental Environmental Policies on firms in the Stackelberg Competition

3.1. Introduction

The Stackelberg competition is a kind of competition in the market in which the leader firm moves first and then the follower firms move sequentially. There are some constrains upon the existence of a Stackelberg equilibrium. The leader must know ex ante that he follower observes its action. The follower must have no means of committing to a future non-Stackelberg follower action and the leader must know this. Barrett (1994) discusses the interaction between governmental environmental policy and Stackelberg competition among firms. He states that the government has no incentive to make a weak environmental policy if its domestic firm is Stackelberg leader. Ferreira (2012) finds that governments have incentives to raise environmental taxes under mixed Stackelberg duopoly where two firms compete.

In a traditional Stackelberg competition where two homogeneous firms compete on quantity, the first move often gives the leader firm an advantage in output and profit. Thus both firms have incentives to engage in Stackelberg competition and play as the leader. However, if governmental environmental policies are involved, firms' competition becomes complicated.

It is uncertain that the leader firm could have an advantage in the Stackelberg competition due to two reasons. First, the firm needs consider whether it would suffer a more stringent environmental policy than that it has in Cournot competition. Such policy may offset the advantage it has as the Stackelberg leader. Second, the follower firm may benefit from a looser environmental policy from its government. Thus the first move would bring disadvantage rather than advantage in environmental abatement cost. Consequently, before reacting as the first mover, each firm would calculate the profits it could get and compare it with the profits it could get in the Cournot competition. Certainly, the firm prefer to choose the type of competition which could bring it more profits.

As the model in Chapter 2, the model employed in this chapter is a two-staged game involving two governments and their two domestic firms. Firms sell the same products and compete in the imperfectly competitive international market. Pollution is local or transboundary. Governments move first by making environmental policies for their domestic firms. Firms take these policies as given and compete by choosing their output levels. Each firm may choose to decide its output before its competitor or decide it at the same time with its competitor. Governments know that there are two possibilities for their firms' outputs (outputs under Stackelberg competition and those under Cournot competition), thus they may make corresponding policies based on different outputs.

It is assumed that other countries, whether or not they are consumer countries, have no means of influencing environmental policies in these two producer countries.

3.2. Model under local pollution

We assume that there exist only two countries A, B and let W_A and W_B be the benefits of countries A, B. There exists one domestic firm i in the country $i, i = \{A, B\}$. We denote them firm A which is the leader firm and firm B which is the follower firm in the international market. Denote the profit of firm A by π_A and the profit of firm B by π_B. Assuming that each firm's production cost per unit is $c_.$ Firm i's abatement cost depends on the output q_i and the domestic emission standard e_i as environmental policy $(1 > e_i > 0)$, which is set by government i. In the model, governments choose environmental policies e_i in the first stage and firms choose output levels q_i in the second stage $(q_i > 0)$. Firms consider only their profits π_i while governments take account into their domestic firms' profits and social environmental damage in their countries. Pollution is purely local.

3.2.1 Firms' profits

Firms care only about their own profits without caring the social environmental damage. However, they should consider the environmental policies from their governments which will affect their environmental abatement costs.

Firms' profit functions are as below:

Firm A: $\pi_A = [a - b(q_A + q_B)]q_A - cq_A - d(1 - e_A)q_A$

Firm B: $\pi_B = [a - b(q_A + q_B)]q_B - cq_B - d(1 - e_B)q_B$

$e_i \in (0, 1), q_i > 0$

a, b, c, d are positive parameters. Firm i's profit is its total revenue $[a - b(q_A + q_B)]q_i$ minus its cost which is composed of total production cost cq_i and total environmental abatement cost $d(1 - e_i)q_i$.

3.2.2 Governments' welfare

Each government considers both its domestic firm's profit and the social environmental damage due to the firm's production. Government i controls firm i's output and pollution by making environmental policy e_i which can affect firm i's environmental abatement cost.

Governments' welfare functions are:

Government A: $W_A = \pi_A - (\frac{t}{2})e_A^2$

Government B: $W_B = \pi_B - (\frac{t}{2})e_B^2$

$e_i \in (0, 1)$, $i \in \{A, B\}$.

t is positive parameter which reflects the degree of social environmental damage. Government i's welfare equals to its domestic firm's profit π_i minus the social environmental damage $(\frac{t}{2})e_i^2$ in the country. In the welfare function, firm i's output q_i and relevant pollution are internalized in the governmental environmental policy e_i.

3.2.3 Solution

Before computing the solution, we need some assumption for the model.

Assumption 3.1: To make our solutions are interior($1 > e_i^* > 0$, $q_i^* > 0$,for $i \in A, B$), we set restrictions for parameter:

$a - c - d > 0$

$d^2 \in (0.667bt, 0.889bt)$

$3d(a - c - d)(18d^4 - 25btd^2 + 8b^2t^2) > (6d^4 - 17btd^2 + 8b^2t^2)(8bt - 9d^2)$

First, we solve firm B' problem:

$\max \pi_B = [a - b(q_A + q_B)]q_B - cq_B - d(1 - e_B)q_B$

q_B

s.t. $q_i > 0, i \in \{A, B\}$

$e_i \in (0, 1)$

We get $q_B^*(q_{A,} e_A, e_B)$ and plug it into firm A's profit function and solve firm A's problem:

$$\max_{q_A} \pi_A = [a - b(q_A + q_B)]q_A - cq_A - d(1 - e_A)q_A$$

s.t. $q_i > 0, i \in \{A, B\}$

$e_i \in (0, 1)$

Then we get $q_A^*(e_A, e_B)$ and $q_B^*(e_A, e_B)$. Put them into governments' welfare functions to solve their problems:

max $W_i = \{a - b[q_A^*(e_A, e_B) + q_B^*(e_A, e_B)]\}q_i^*(e_A, e_B) - cq_i^*(e_A, e_B) - d(1 - e_i)q_i^*(e_A, e_B) - \frac{t}{2}e_i^2$

s.t. $e_i \in (0, 1)$, for $i \in A, B$

Finally we get e_i^*, q_i^*,π_i^* and W_i^*. We show them in the table 3.1 below:

	Firm A&Government A	Firm B&Government B
e_i^*	$e_A^* = \frac{2d(a-c-d)(2bt-3d^2)}{(6d^4-17btd^2+8b^2t^2)}$	$e_B^* = \frac{3d(a-c-d)(18d^4-25btd^2+8b^2t^2)}{(6d^4-17btd^2+8b^2t^2)(8bt-9d^2)}$
q_i^*	$q_A^* = 2t\frac{2bt-3d^2}{8b^2t^2-17bd^2t+6d^4}(a-c-d)$	$q_B^* = 2t\frac{bt-2d^2}{8b^2t^2-17bd^2t+6d^4}(a-c-d)$
π_i^*	$\pi_A^* = \frac{2bt^2(a-c-d)^2(2bt-3d^2)^2}{(6d^4-17btd^2+8b^2t^2)^2}$	$\pi_B^* = \frac{4bt^2(bt-2d^2)^2(a-c-d)^2}{(6d^4-17btd^2+8b^2t^2)^2}$
W_i^*	$W_A^* = \frac{2t(bt-d^2)\left(3d^2-2bt\right)^2(a-c-d)^2}{(8b^2t^2-17bd^2t+6d^4)^2}$	$W_B^* = \frac{t(8bt-9d^2)(bt-2d^2)^2(a-c-d)^2}{2(8b^2t^2-17bd^2t+6d^4)^2}$

Table 3.1 Solution for firms in Stackelberg competition under local pollution

Table 3.1 shows governmental policies, outputs, firms' profits and governments' welfare in the case of firms in Stackelberg competition under local pollution.

3.3. Results

Proposition 3.1: In Stackelberg Model, $e_A^* < e_B^*$, $q_A^* < q_B^*$, $\pi_A^* < \pi_B^*$, $W_A^* < W_B^*$ if $d^2 \in (\frac{2}{3}bt, \frac{23+\sqrt{17}}{32}bt)$ and $W_A^* > W_B^*$ if $d^2 \in (\frac{23+\sqrt{17}}{32}bt, \frac{8}{9}bt)$.

Proposition 3.1 shows that government A tends to make a more stringent environmental policy than Government B if it thinks that its domestic firm A will plays Stackelberg as a quantity leader. The reason is that if two firms face a same environmental policy, firm A will produce more than firm B. It implies that country A will suffer from a higher social environmental damage than country B due to a higher output. To avoid this consequence, government A makes a more stringent environmental policy before firm A's activity to raise firm A's environmental abatement cost. Since its cost increases, firm A has to decrease its output and its profit becomes less. Thus, in Stackelberg equilibrium, both outputs and profit of firm A are lower than those of firm B. By limiting firm A's output, government A obtains the optimal benefit for its country. Whether country A's welfare is higher than country B's welfare depends on the values of parameters b, d, t.

Example: let $a = 4, b = 5, c = 2, d = 1, t = 0.25$ which satisfy all the preconditions of the model. We get,

$$e_A^* = \frac{4}{11} < \frac{9}{11} = e_B^*$$

$$q_A^* = \frac{1}{11} < \frac{3}{22} = q_B^*$$

$\pi_A^* = \frac{5}{242} < \frac{45}{484} = \pi_B^*$

$W_A^* = \frac{1}{242} < \frac{9}{968} = W_B^*$ as $d^2 \in (\frac{2}{3}bt, \frac{23+\sqrt{17}}{32}bt)$ holds.

Chapter 2 shows that when two homogeneous firms plays Cournot in stead of Stackelberg in the same model, they get same profits as:

$$\pi_A^{*,C} = \pi_B^{*,C} = \frac{9bt^2(a-c-d)^2}{(9bt-4d^2)^2}$$

and their countries' welfare are:

$$W_A^{*,C} = W_B^{*,C} = \frac{t(9bt-8d^2)(a-c-d)^2}{(9bt-4d^2)^2}$$

While $a - c - d > 0$, $a - c \leq 2d$ and $9bt > 8d^2$ hold.

We can compare these outcomes with firms' profits $\pi_A^{*,S}$, $\pi_B^{*,S}$ and countries' welfare $W_A^{*,S}$, $W_B^{*,S}$ in Stackelberg competition.

Proposition 3.2: $e_B^{*,S} > e_A^{*,C} = e_B^{*,C} > e_A^{*,S}, q_A^{*,C} > q_A^{*,S}$ if $d^2 \in (\frac{2}{3}bt, \frac{19-\sqrt{73}}{12}bt)$, $q_A^{*,C} < q_A^{*,S}$ if $d^2 \in (\frac{19-\sqrt{73}}{12}bt, \frac{8}{9}bt)$, $q_B^{*,S} > q_B^{*,C}$, $\pi_B^{*,S} > \pi_A^{*,C} = \pi_B^{*,C} > \pi_A^{*,S}$, $W_A^{*,C} > W_A^{*,S}$ and $W_B^{*,C} > W_B^{*,S}$.

Proposition 3.2 shows that both firms prefers play Cournot to play as Stackelberg leader, because the profits they get by playing as Stackelberg leader are less than what they can get from Cournot competition. Firms suffer from a more stringent governmental environmental policy if they produce before their competitors in stead of producing simultaneously with them. Thus, even they may have a higher output as Stackelberg leader, they still get less profit than they can get in Cournot competition. Governments obtain higher welfare if their firms play Cournot rather than play Stackelberg.

Governments tend to make relatively stringent environmental policies to discourage firms to play Stackelberg.

Example: we use the same data in the example of Proposition. 1, we get:

$$e_B^{*,S} = \frac{9}{11} = 0.8181 > e_A^{*,C} = e_B^{*,C} = 0.5517 > e_A^{*,S} = \frac{4}{11} = 0.3636$$

$$q_A^{*,C} = 0.1034 > q_A^{*,S} = \frac{1}{11} = 0.0909 \text{ since } d^2 \in (\tfrac{2}{3}bt, \tfrac{19-\sqrt{73}}{12}bt)$$

$$q_B^{*,S} = \frac{3}{22} = 0.1364 > q_B^{*,C} = 0.1034$$

$$\pi_B^{*,S} = 0.0920 > \pi_A^{*,C} = \pi_B^{*,C} = 0.0535 > \pi_A^{*,S} = 0.0207$$

$$W_A^{*,C} = 0.1121 > W_A^{*,S} = 0.0041$$

$$W_B^{*,C} = 0.1121 > W_B^{*,S} = 0.0093$$

3.4 Model under transboundary pollution

Now we discuss a model under transboundary pollution. In this case, firms' profit functions are same as those in the model under local pollution. Firm A is Stackelberg leader and firm B is follower. However, for countries, their welfare are affected by pollution not only from domestic firm but also from foreign firm. It means that the pollution generated in one country would bring the equal social environmental damage to the other country.

The model is as below:

Firm A: $\pi_A = [a - b(q_A + q_B)]q_A - cq_A - d(1 - e_A)q_A$

Firm B: $\pi_B = [a - b(q_A + q_B)]q_B - cq_B - d(1 - e_B)q_B$

Government A: $W_A = \pi_A - \frac{t}{2}(e_A + e_B)^2$

Government B: $W_B = \pi_B - \frac{t}{2}(e_A + e_B)^2$

$$\text{s.t. } e_i \in (0,1), q_i > 0, i = A, B.$$

Assumption 3.2: To make our solutions are interior($1 > e_i^* > 0$, $q_i^* > 0$,for $i \in A, B$), we set restrictions for parameter:

$a - c - d > 0$

$d^2 \in (\frac{5}{12}bt, \frac{6+\sqrt{30}}{3}bt)$

$-4b^2t^2 + 24bd^2t - 6d^4 > d(a - c - d)(6d^2 + bt)$

Similar as under local pollution, we firstly solve firm B' problem:

$$\max_{q_B} \pi_B = [a - b(q_A + q_B)]q_B - cq_B - d(1 - e_B)q_B$$

$$\text{s.t.} \quad q_i > 0, i \in \{A, B\}$$

$$e_i \in (0,1)$$

We get $q_B^*(q_A, e_A, e_B)$ and plug it into firm A's profit function and solve firm A's problem:

$$\max_{q_A} \pi_A = [a - b(q_A + q_B)]q_A - cq_A - d(1 - e_A)q_A$$

$$\text{s.t.} \quad q_i > 0, i \in \{A, B\}$$

$$e_i \in (0, 1)$$

Then we get $q_A^*(e_A, e_B)$ and $q_B^*(e_A, e_B)$. Put them into governments' welfare functions which are different from those under local pollution, then we solve their problems:

$\max W_i = \{a - b[q_A^*(e_A, e_B) + q_B^*(e_A, e_B)]\}q_i^*(e_A, e_B) - cq_i^*(e_A, e_B) - d(1 - e_i)q_i^*(e_A, e_B) - \frac{t}{2}(e_A + e_B)^2$

$$\text{s.t. } e_i \in (0, 1), \text{ for } i \in A, B$$

Finally we get e_i^*, q_i^*,π_i^* and W_i^*. We show them in the table 3.2 below:

	Firm A&Government A	Firm B&Government B
e_i^*	$e_A^* = \frac{d(a-c-d)(12d^2-5bt)}{2(-4b^2t^2+24bd^2t-6d^4)}$	$e_B^* = \frac{d(a-c-d)(6d^2+bt)}{-4b^2t^2+24bd^2t-6d^4}$
q_i^*	$q_A^* = \frac{t(9d^2-2bt)(a-c-d)}{(-4b^2t^2+24bd^2t-6d^4)}$	$q_B^* = \frac{t(8d^2-bt)(a-c-d)}{(-4b^2t^2+24bd^2t-6d^4)}$
π_i^*	$\pi_A^* = \frac{bt^2\left(9d^2-2bt\right)^2(a-c-d)^2}{2(4b^2t^2-24bd^2t+6d^4)^2}$	$\pi_B^* = \frac{bt^2\left(8d^2-bt\right)^2(a-c-d)^2}{(4b^2t^2-24bd^2t+6d^4)^2}$
W_i^*	$W_A^* = \frac{t(a-c-d)^2[4bt(9d^2-2bt)^2-9d^2(8d^2-bt)^2]}{8(4b^2t^2-24bd^2t+6d^4)^2}$	$W_B^* = \frac{t(8bt-9d^2)(8d^2-bt)^2(a-c-d)^2}{8(4b^2t^2-24bd^2t+6d^4)^2}$

Table 3.2 Solution for firms in Stackelberg competition under transboundary pollution

Table 3.2 shows governmental policies, outputs, firms' profits and governments' welfare in the case of firms in Stackelberg competition under transboundary pollution.

3.5 Results in the model under transboundary pollution

Proposition 3.3: $e_A^* < e_B^*, q_A^* > q_B^*$ if $d^2 \in (bt, \frac{6+\sqrt{30}}{3}bt), q_A^* < q_B^*$ if $d^2 \in (\frac{5}{12}bt, bt), \pi_A^* < \pi_B^*, W_A^* < W_B^*$.

Proposition 3.3 shows a similar result as that in Proposition 1. With transboundary pollution, leading firm suffers from a more stringent environmental policy from its government. Thus, it gets less profit than its competitor even when it produce more than its competitor. Its government makes such stringent environmental policy to discourage it to play Stackelberg competition because the government gets lower welfare than the other government in such case.

We use the same date in 3.3 for a numerical example.

Example: $a = 4, b = 5, c = 2, d = 1, t = 0.25$.

$e_A^* = 0.162 < 0.408 = e_B^*$

$q_A^* = 0.092 < 0.095 = q_B^*$

$\pi_A^* = 0.021 < 0.045 = \pi_B^*$

$W_A^* = -0.0196 < 0.0044 = W_B^*$

We can also compare these results with those when firms plays Cournot competition.

Proposition 3.4: $e_A^{*,C} > e_A^{*,S}, e_B^{*,C} < e_B^{*,S}, q_B^{*,C} < q_B^{*,S}, \pi_A^{*,C} > \pi_A^{*,S}, \pi_B^{*,C} < \pi_B^{*,S}, W_A^{*,C} > W_A^{*,S}$

Proposition 3.4 shows that no firm has incentive to play as Stackelberg leader. In stead, both of them tend to play Cournot. If they produce before their competitor, they would suffer a higher environmental policy and get less profit than what they can get from Cournot competition. The government make stringent policy to push its domestic firm to play Cournot since the government can also obtain higher welfare if its firm plays Cournot.

Example: $a = 4, b = 5, c = 2, d = 1, t = 0.25$.

$e_A^{*,C} = 0.216 > 0.162 = e_A^{*,S}$

$e_B^{*,C} = 0.216 < 0.408 = e_B^{*,S}$

$\pi_A^{*,C} = 0.033 > 0.021 = \pi_A^{*,S}$

$\pi_B^{*,C} = 0.033 < 0.045 = \pi_B^{*,S}$

$W_A^{*,C} = 0.023 < -0.0196 = W_A^{*,S}$

3.6 Conclusion

This chapter discusses the impacts of governmental environmental policies on firms in Stackelberg competition. Firm would face a more stringent environmental policy from its government if it plays as Stackelberg leader. Meanwhile, its competitor as the follower could benefit from a looser environmental policy from its government. Governments make different policies for the same reason: balance the domestic firm's profit and the social environmental damage in the country. If one firm moves firstly, its output would significantly increase if there is no intervention from governmental policy. Thus, the pollution from its production would also sharply increase. To avoid the increasing pollution, its government tends to make more stringent environmental policy to discourage its choice to play as the leader. Meanwhile, if one firm moves secondly, its output would significantly decrease if there is no intervention from governmental policy. The pollution from its production would also sharply decrease. To keep the pollution decrease, its government tends to make looser environmental policy to courage its choice to play as the follower.

Therefore, no firm has incentive to play as leader since the leader would get less profit than the follower due to the different governmental environmental policies on them. In stead, they tend to play Cournot from which they obtain more profits and their governments obtain higher welfare than what they could get from Stackelberg competition. These results are same with both local pollution and transboundary pollution.

Chapter 4
Governments' and Firms' collusion under local&transboundary pollution

4.1 Introduction

In Chapter 2 and 3, firms independently decide their outputs and governments independently make their environmental policies. In this Chapter, we discuss the possibilities for collusion. Collusion is an agreement between two or more parties to limit open competition or to obtain a common objective. Within the environmental economics literature, the study of collusion stability has so far been restricted to international environmental agreements (IEAs), where countries collude to reduce emissions together (See Barrett (2003) and Finus (2003)). Governments could choose collude to make same environmental policies while firms could choose to collude to decide their outputs together. Governments and firms can also keep reacting independently as they do in Chapter 2 and 3.

Before making decision on collusion, governments and firms would consider all the choices and their correspondent payoffs. Their objectives are always to maximize their payoffs.

In a typical competition without governments, firms would obtain more profits from their collusion than what they could get in the Cournot competition. However, if governmental environmental policies are involved, the result could be different. As we show in Chapter 3, the firm gets less profits as the leader in the Stackelberg competition than in the Cournot competition. Here the case is similar. If firms collude, their governments may impose more stringent environmental policies on them. Thus, their environmental abatement costs would increase and the collusion might not be a suitable choice for them. Besides, governments may collude too. Governments' decision on their collusion would also affect firms' choice.

As the model in Chapter 2 and 3, the model employed in this chapter is a two-staged game involving two governments and their two domestic firms. Firms sell the same products and compete in the imperfectly competitive international market. Pollution is local or transboundary. Governments move first by making environmental policies for their domestic firms. Firms take these policies as given and compete by choosing their output levels. In the first stage, each government may choose to independently make its environmental policy or make it together with the other government. In the second stage, each firm may choose to independently decide its output or decide it together with its competitor by cooperation. Governments know that there are two possibilities for their firms' outputs (outputs in the Cournot competition and those in firms' cooperation), thus they may choose to cooperate or not based on different outputs. Consequently, we have four possibilities of choices to analyze. We will compare the payoffs of governments and firms in all the four cases and get their preference for choice.

As in Chapter 2 and 3, it is assumed that other countries, whether or not they are consumer countries, have no means of influencing environmental policies in these two producer countries.

4.2 Model under local pollution

There are two countries (governments) A and B where firm A is the only domestic firm in country A and firm B is the only domestic firm in country B. In the first step, Governments make environmental policies to pursue maximum countries' welfare which is the domestic firm's profit minus the pollution in the country. In the second step, firms compete in the market (no other firms) to pursue maximum firms' profit. Government can make the environmental policy independently or cooperate with the other government to make the policy together. Firm can decide its output independently or cooperate with the other firm to decide the output together. As it is a symmetric model, governments always obtain the same welfare and firms always obtain the same profit. Thus, we can think it as a game with two players, government and firm. Since government makes its policy before firm decides its output, it is a sequential game. We assume that any cheating activity is observable which is not considered in the model.

In this sequential game, there are four cases. In case 1, governments make environmental policies independently then firms also play independently (Cournot). In case 2, governments make policies by collusion and firms play Cournot. In case 3, governments make policies independently and firms decide output by collusion. In case 4, both governments and firms make decisions by collusion. Case 1 is discussed in Chapter 2. In this Chapter, we derive the other three cases and discuss them with case 1.

The firms' profit functions and the governments' welfare functions are as below:

Firm A: $\pi_A = [a - b(q_A + q_B)]q_A - cq_A - d(1 - e_A)q_A$

Firm B: $\pi_B = [a - b(q_A + q_B)]q_B - cq_B - d(1 - e_B)q_B$

Government A: $W_A = \pi_A - \frac{t}{2}e_A^2$

Government B: $W_B = \pi_B - \frac{t}{2}e_B^2$

$e_i \in (0, 1)$, $i = A, B$.

Before computing the solution, we need some assumption for the cases under local pollution.

Assumption 4.1: To make our solutions are interior($1 > e_i^* > 0$, $q_i^* > 0, \pi_i^* > 0$ and $W_i^* > 0$ for $i \in A, B$), we set restrictions for parameter:

$a - c - d > 0$

$9bt > 4d(a - c)$

$bt > \frac{8}{9}d^2$

4.2.1 Governments' collusion without firms' collusion

First, we solve firm i's problem:

$$\max_{q_i} \pi_i = [a - b(q_A + q_B)]q_i - cq_i - d(1 - e_i)q_i$$

$$\text{s.t.} \quad q_i > 0, i \in \{A, B\}$$

$$e_i \in (0, 1)$$

Then we get $q_A^*(e_A, e_B)$ and $q_B^*(e_A, e_B)$. Put them into governments' welfare functions to solve their problems:

$\max W = W_A + W_B = \{a - b[q_A^*(e_A, e_B) + q_B^*(e_A, e_B)]\}q_i^*(e_A, e_B) - cq_i^*(e_A, e_B) - d(1 - e_i)q_i^*(e_A, e_B) - \frac{t}{2}e_i^2$

$$\text{s.t. } e_i \in (0, 1), \text{ for } i \in A, B$$

Finally we get e_i^*, q_i^*,π_i^* and W_i^*.

4.2.2 Firms' collusion without governments' collusion

First, firm A and B cooperate. Their total profits function is:

$\pi = \pi_A + \pi_B$

We solve their problem as below:

Max $\quad \pi = \pi_A + \pi_B = [a - b(q_A + q_B)](q_A + q_B) - (c + d)(q_A + q_B) + d(e_A q_A + e_B q_B)$

Then we get $q_A^*(e_A, e_B)$ and $q_B^*(e_A, e_B)$. Put them into government A's and B's welfare functions to solve their problems:

Finally we get e_i^*, q_i^*,π_i^* and W_i^*.

4.2.3 Collusion for both governments and firms

First, firm A and B cooperate. They solve the problem of total profit as in **4.2.2:**

Max $\quad \pi = \pi_A + \pi_B = [a - b(q_A + q_B)](q_A + q_B) - (c + d)(q_A + q_B) + d(e_A q_A + e_B q_B)$

Then we get $q_A^*(e_A, e_B)$ and $q_B^*(e_A, e_B)$. Put them into governments' total welfare function to solve their problems.

Now consider both governments and firms cooperate, two governments solve:

Max $\quad [a - \frac{2(a-c-d)+d(e_A+e_B)}{4}][\frac{2(a-c-d)+de_A+de_B}{4b}] - (c+d)[\frac{2(a-c-d)+de_A+de_B}{4b}] + de_A(\frac{a-c-d+de_A}{4b}) + de_B(\frac{a-c-d+de_B}{4b}) - \frac{t}{2}(e_A^2 + e_B^2)$

e_A, e_B

Finally we get e_i^*, q_i^*,π_i^* and W_i^*.

4.2.4 Comparison among different cases

We show all the solution of different cases in the tables below:

Cases	governmental policy	firm's output
no collusion	$e_{nc}^* = \frac{4d(a-c-d)}{(9bt-4d^2)}$	$q_{nc}^* = \frac{3t(a-c-d)}{(9bt-4d^2)}$
governments collude&firms don't collude	$e_G^* = \frac{2d(a-c-d)}{(9bt-2d^2)}$	$q_G^* = \frac{3t(a-c-d)}{(9bt-2d^2)}$
governments don't collude&firms collude	$e_F^* = \frac{5d(a-c-d)}{(16bt-5d^2)}$	$q_F^* = \frac{4t(a-c-d)}{(16bt-5d^2)}$
governments collude&firms collude	$e_{GF}^* = \frac{d(a-c-d)}{(4bt-d^2)}$	$q_{GF}^* = \frac{t(a-c-d)}{(4bt-d^2)}$

Table 4.1: Government's policy and firm's output under local pollution

Table 4.1 shows different governmental environmental policies and firms' outputs based on different decisions from governments and firms. These policies and outputs would cause different profits for firms and different welfare for governments.

Cases	firm's profit	government's welfare
no collusion	$\pi_{nc}^* = \frac{9bt^2(a-c-d)^2}{(9bt-4d^2)^2}$	$W_{nc}^* = \frac{t(9bt-8d^2)(a-c-d)^2}{(9bt-4d^2)^2}$
governments collude&firms don't collude	$\pi_G^* = \frac{9bt^2(a-c-d)^2}{(9bt-2d^2)^2}$	$W_G^* = \frac{t(a-c-d)^2}{(9bt-2d^2)}$
governments don't collude&firms collude	$\pi_F^* = \frac{32bt^2(a-c-d)^2}{(16bt-5d^2)^2}$	$W_F^* = \frac{t(64bt-25d^2)(a-c-d)^2}{2(16bt-5d^2)^2}$
governments collude&firms collude	$\pi_{GF}^* = \frac{2bt^2(a-c-d)^2}{(4bt-d^2)^2}$	$W_{GF}^* = \frac{t(a-c-d)^2}{(8bt-2d^2)}$

Table 4.2: Government's welfare and firm's profit under local pollution

Table 4.2 shows different profits for firms and different welfare for governments. Both firms and governments would compare their payoffs among different cases and find the best way to maximize their payoffs.

4.3. Results in the model under local pollution

From the comparison among different cases, we find some interesting results related to governments' and firms' payoffs and preference.

Proposition 4.1: $e^*_{nc} > e^*_F > e^*_{GF} > e^*_G$, $q^*_{nc} > q^*_G > q^*_F > q^*_{GF}$.

Proposition 4.1 shows that if governments cooperate, they always make more stringent environmental policies than those they make independently, regardless of firms' behaviors. The main reason is that with collusion government considers not only the pollution in its own country but also that in the other country. Thus governments tend to make more stringent policies to control the whole pollution.

Firms always make higher outputs when they play Cournot than when they cooperate. The reason is that firms' outputs are supposed to be higher by collusion than those under Cournot competition if the costs are same. Higher outputs imply more pollution, thus governments make more stringent environmental policies if they think firms would not cooperate. The objective of such policies is to obtain countries' maximum welfare by balancing firms' profits and relevant pollution. As environmental policies become more stringent, firms' environmental abatement costs become higher. Thus firms have to lower their output by collusion to maintain their optimal profit.

Proposition 4.2: $\pi^*_F > \pi^*_{GF} > \pi^*_G$ and $\pi^*_{nc} > \pi^*_G$, $W^*_{GF} > W^*_F > W^*_G > W^*_{nc}$.

Proposition 4.2 shows that firms prefer to cooperate than compete Cournot since they can obtain higher profit in the first case. This is same as the typical model in which firms' profit are always higher by collusion than those by Cournot. The difference is governments' intervention. Firms face higher costs from more stringent governmental environmental policies if they decide their outputs independently. The reason is that their outputs in the Cournot competition are higher than those in collusion and more outputs mean more pollution which stimulate governments to make more strict environmental policies. The best outcome for governments is that both governments and firms cooperate while the worst case for them is that neither governments nor firms cooperate. Governments' collusion implies that stringent policies make pollution significantly decrease and firms' collusion implies that firms maintain their profit at a certain level. Thus governments can obtain the maximum welfare meanwhile firms can avoid the worst outcome for them.

Proposition 4.2 implies that governments can get their best outcome since they are the player who plays first in this sequential game. Once they choose to cooperate, firms have to choose cooperate too. Firms' best outcome is that neither governments nor firms cooperate. However, as the following player, firms have to wait governments to choose firstly and governments would choose cooperate. Thus firms never have chance to reach their best outcome.

4.4. Model under transboundary pollution

The firms' profit functions are same as those under local pollution. The governments' welfare functions are different from those under local pollution since the pollution in the foreign country would also affect the home country under transboundary pollution. Their functions are as below.

Firm A: $\pi_A = [a - b(q_A + q_B)]q_A - cq_A - d(1 - e_A)q_A$

Firm B: $\pi_B = [a - b(q_A + q_B)]q_B - cq_B - d(1 - e_B)q_B$

Government A: $W_A = \pi_A - \frac{t}{2}(e_A + e_B)^2$

Government B: $W_B = \pi_B - \frac{t}{2}(e_A + e_B)^2$

$e_i \in (0, 1)$, $i = A, B$.

Before computing the solution, we need some assumption for the cases under transboundary pollution.

Assumption 4.2: To make our solutions are interior($1 > e_i^* > 0$, $q_i^* > 0, \pi_i^* > 0$ and $W_i^* > 0$ for $i \in A, B$), we set restrictions for parameter:

$a - c - d > 0$

$9bt > 2d(a - c)$

$bt > \frac{8}{9}d^2$

The computation is similar as those under local pollution. We get the solutions from different cases and show them in the tables below.

Cases	governmental policy	firm's output
no collusion	$e_{nc}^* = \frac{4d(a-c-d)}{(18bt-4d^2)}$	$q_{nc}^* = \frac{6t(a-c-d)}{(18bt-4d^2)}$
governments collude&firms don't collude	$e_G^* = \frac{d(a-c-d)}{(18bt-d^2)}$	$q_G^* = \frac{6t(a-c-d)}{(18bt-d^2)}$
governments don't collude&firms collude	$e_F^* = \frac{5d(a-c-d)}{(32bt-5d^2)}$	$q_F^* = \frac{8t(a-c-d)}{(32bt-5d^2)}$
governments collude&firms collude	$e_{GF}^* = \frac{d(a-c-d)}{(16bt-d^2)}$	$q_{GF}^* = \frac{4t(a-c-d)}{(16bt-d^2)}$

Table 4.3: Government's policy and firm's output under transboundary pollution

Table 4.3 shows governmental environmental policies and firms' outputs based on different decisions from governments and firms under transboundary pollution. These policies and outputs would cause different profits for firms and different welfare for governments.

Cases	firm's profit	government's welfare
no collusion	$\pi_{nc}^* = \frac{36bt^2(a-c-d)^2}{(18bt-4d^2)^2}$	$W_{nc}^* = \frac{4t(9bt-8d^2)(a-c-d)^2}{(18bt-4d^2)^2}$
governments collude&firms don't collude	$\pi_G^* = \frac{36bt^2(a-c-d)^2}{(18bt-d^2)^2}$	$W_G^* = \frac{2t(a-c-d)^2}{(18bt-d^2)}$
governments don't collude&firms collude	$\pi_F^* = \frac{128bt^2(a-c-d)^2}{(32bt-5d^2)^2}$	$W_F^* = \frac{2t(64bt-25d^2)(a-c-d)^2}{(32bt-5d^2)^2}$
governments collude&firms collude	$\pi_{GF}^* = \frac{32bt^2(a-c-d)^2}{(16bt-d^2)^2}$	$W_{GF}^* = \frac{2t(a-c-d)^2}{(16bt-d^2)}$

Table 4.4: Government's welfare and firm's policy under transboundary pollution

Table 4.4 shows different profits for firms and different welfare for governments under transboundary pollution. Both firms and governments would compare their payoffs among different cases and find the best way to maximize their payoffs.

4.5 Results in the model under transboundary pollution

From the comparison among different cases, we find the orders of results from different cases are similar as those under local pollution. There are some difference in firms' profits and countries' welfare, but they would not affect firms' or governments' choices.

Proposition 4.3: $e^*_{nc} > e^*_F > e^*_{GF} > e^*_G$, $q^*_{nc} > q^*_G > q^*_F > q^*_{GF}$.

Proposition 4.3 shows that the orders of governmental environmental policies and firms' outputs among different cases under transboundary pollution are same as those under local pollution. In general, governments tend to make more stringent environmental policies if they choose collude rather than decide independently, since they should consider the pollution in the other country when they collude. Another issue which affects governmental environmental policies is firms' outputs. If firms produce more, it means that the pollution increase correspondingly. In that case, governments tend to make more stringent environmental policies to discourage firms' production and relevant pollution.

Since firms produce more under Cournot competition than what they produce by collusion, they would suffer more stringent environmental policies under Cournot competition if governments collude. Thus, firms would consider to collude and decrease their outputs. Lower outputs do not mean less profits for firms. First, lower outputs raise the price in the market. Second, lower outputs generate less pollution, then governments may make looser environmental policies to decrease firms' environmental abatement costs.

Proposition 4.4: $\pi^*_F > \pi^*_{GF} > \pi^*_G$ and $\pi^*_{nc} > \pi^*_G$; $W^*_{GF} > W^*_G > W^*_{nc}$ and $W^*_{GF} > W^*_F$.

Proposition 4.4 implies that firms prefer collude than react independently, as their profits are higher in the first case. Besides, they don't welcome governments' collusion which would increase their environmental abatement costs and decrease their profits. For example, if governments choose collude in the first stage, firms could produce more if they decide their outputs independently. But if they do so, they can not obtain more profits. In stead, their profits decrease due to more stringent governmental environmental polices. Consequently, governments' collusion may discourage firms to increase their outputs.

Governments also prefer collude than decide independently for higher welfare. The difference is that for firms, they like collusion but they don't like governments collude while for governments, they like not only collusion between themselves but also collusion between firms.

Proposition 4.4 also shows that under transboundary pollution, both governments and firms have same preference as what they have under local pollution. In the first stage, governments choose to collude for making environmental policies. In the second stage, firms choose to collude for deciding their outputs. Proposition 4 implies that the type of pollution (local or transboundary) would not affect governments' and firms' choices.

4.6 Conclusion

This chapter discusses the different payoffs of governments and firms when they make different choices for collusion. We find that governments can obtain maximum welfare if they collude and their domestic firms collude. Although firms can not maximize their profits in this case, they can accept it since they would have the least profits if they reject it. Firms could get more profits if governments don't choose collude. But it is impossible for governments not to choose collude in the first stage. As the second player, firms have to make their decision under governments' collusion in environmental policies.

In the first stage, governments choose collude and make relatively strict environmental policies. In the second stage, firms choose collude to maintain their outputs at a certain level and avoid the most strict environmental policy. Thus, firms' collusion can recover some loss of their profits due to the increasing environmental abatement costs. Finally, by collusion, governments can get their maximum welfare while firms can obtain acceptable profits which are neither the most nor the least.

The results in this chapter advocate both governments and firms to collude. By such collusion, the pollution is controlled at a relatively low level while firms produce less but still can get acceptable profits.

Chapter 5
Conclusion

After discussing governments' welfare and firms' profits in different types of competition (Cournot, Stackelberg and Collusion) and under different types of pollution (pure local or transboundary), we can make some conclusion in this chapter.

First, under the circumstance that both governments and firms make their decisions independently, firms which are homogeneous in costs have no incentives to decide their outputs and produce before their competitors. In stead, they prefer produce at the same time. This is different than what we know that one firm can obtain more profits as a leader in Stackelberg competition than as a player in Cournot competition. The reason is that when governments and environment issues are involved, the firm as a Stackelberg leader would suffer from a more stringent governmental environmental policy than what it has in Cournot competition. Such policy raises its environmental abatement cost and decreases its profits, thus the firm has no incentive to act as the first mover.

Second, once firms choose compete Cournot in the market, the types of pollution would affect their incentives to make efforts on decreasing their production costs. Under local pollution, if one firm's production cost decreases, the firm would benefit from a looser environmental policy from its government and its competitor would suffer from a more stringent governmental environmental policy. Then the firm would obtain more profits than what it can get without governments' intervention. So firms have strong incentives to decrease their production costs under local pollution. However, under transboundary pollution, the decrease of one firm's production cost would make the firm suffer from a more strict environmental policy from its government but its competitor would benefit from a looser governmental environmental policy. Then, the firm's advantage in production costs would be offset by its disadvantage in governmental environmental policies and the firm still get same profits as its competitor which acts as a free rider. Consequently, firms have no incentive to make efforts on decreasing their production costs under transboundary pollution. As the decrease of production cost reflect a technology progress, the result implies that transboundary pollution could be a barrier for technology improvement.

Finally, if governments and firms are free to choose collusion, the types of pollution (local or transboundary) would not affect governments' or firms' choices. Governments certainly choose to cooperate with each other in making their environmental policies. It means that when one government make its policy, it not only consider its domestic firm's profits and the pollution in its country but also consider the foreign firm's profits and the pollution in the other country. Based on these policies, firms would also choose to collude in deciding their total outputs. As a result, collusion is the best choice for both governments and firms. From collusion, governments can obtain their maximum welfare and firms can obtain medium profits.

Discussion in this paper is limited to two countries and two firms. More complicated cases that include more countries and firms remain to be discovered in future research. Besides, the results in this paper are based on the assumption that solutions are interior. The cases for corner solutions is to be discussed in the later study. Moreover, we need more empirical evidence to support the results in this paper.

Bibliography

Anderson, K (1992) "The Standard Welfare Economics of Policies Affecting Trade and the Environment," in The Greening of World Trade Issues, K. Anderson and R. Blackhurst (eds.), Ann Arbor, MI: The University of Michigan Press, pp. 25-48.

Anderson and Blackhurst (1992) *The greening of World trade issues* (Harvester Wheatsheaf, London)

Antweiler, Werner, Brian Copeland and Scott Taylor (2001) "Is Free Trade Good for the Environment?" American Economic Review, 91, no. 4, September, 877-908.

Antle, J. M. and R. E. Just (1992) "Conceptual and Empirical Foundations for Agricultural-Environmental Policy Analysis." Journal of Environmental Quality 21: 307-16.

Barrett (1994) *Strategic environmental policy and international trade*, Journal of Public Economics 54, 325-338.

Barrett, Scott (1997) "The Strategy of Trade Sanctions in International Environmental Agreements," Resource and Energy Economics 19, 345-361.

Barrett, Scott (2003) "Trade Leakage and Trade Linkage," Chapter 12 of Environment and Statecraft: The Strategy of Environmental Treaty-Making, Oxford University Press.

Brown, D. K., A. V. Deardorff and R. M. Stern (1993) "International labor Standards and Trade: A Theoretical Analysis." University of Michigan.

Cole, M.A. and R.J.R. Elliott (2003) Determining the Trade-Environment Composition Effect: The Role of Capital, Labor and Environmental Regulations. Journal of Environmental Economics and Management, 46 (3): 363–383.

Copeland, Brian, and Scott Taylor (2001) "International Trade and the Environment: A Framework for Analysis," NBER Working Paper No. 8540, Oct.

Copeland, Brian, and Scott Taylor (2003) Trade and the Environment: Theory and Evidence (Princeton University Press: Princeton).

Copeland, Brian, and Scott Taylor (2004) "Trade, Growth and the Environment," Journal of Economic Literature 42(1): 7-71. NBER Working Paper No. 9823.

Charnovitz, Steve, and Michael Weinstein (2001) "The Greening of the WTO," Foreign Affairs 80, no. 6, 147-156.

Dean (1992) *Trade and the environment: A survey of the litterature*, International trade and the environment (The World Bank, Washington, DC) 15-28.

Dean, Judy (2002) "Does Trade Liberalization Harm the Environment? A New Test," Canadian Journal of Economics 35, no. 4, November: 819-842.

Ferreira (2012) "Environmental taxes in a public Stackelberg leader duopoly", IEEE 4th International Conference on Nonlinear Science and Complexity (NSC).

Finus (2003) "Stability and design of international environmental agreements: the case of transboundary pollution", The International Yearbook of Environmental and Resource Economics 2003/2004, Edward Elgar, Cheltenham, 2003, pp. 82-158.

Frankel, J. and A. Rose. (2005) In Is Trade Good or Bad for the Environment? Sorting out the Causality. Review of Economics and Statistics. 87 (1): 85–91.

Golden, Tim (1993) "A History of Pollution in Mexico Casts Cloud Over Trade Accord." New York Times. August 16, p. A-1.

Grossman, Gene M., and Alan B. Krueger (1991) "Environmental Impacts of a North American Free trade Agreement." Woodrow Wilson Institute for Public Affairs. Princeton University. October 8.

Grossman, Gene, and Alan Krueger (1993) "Environmental Impacts of a North American Free Trade Agreement," in The U.S.-Mexico Free Trade Agreement, Peter Garber, editor, MIT Press.

Grossman, Gene, and Alan Krueger (1995) "Economic Growth and the Environment," Quarterly Journal of Economics, 110, no. 2, May: 353-77.

Grimmett (1991) *Environmental regulation and the GATT*, Report for congress, Congressional Research Service.

Harbaugh, W., A. Levinson, and D. Wilson (2002) Reexamining Empirical Evidence for an Environmental Kuznets Curve, Review of Economics and Statistics, 84 (3): 541–551.

Harold, Courtney, and C. Ford Runge (1993) "GATT and the Environment: Policy Research Needs," American Journal of Agricultural Economics 75: 789-793.

Hoel and Schneider (1997) *Incentives to participate in an International Environmental Agreement*, Environmental and Resource Economics 9, 153-170.

Hudec, Robert and Daniel Farber (1992) "Distinguishing Environmental Measures from Trade Barriers." University of Minnesota. Workshop on International Economic Policy.

Krutilla, K. (1991) "Environmental Regulation in an Open Economy." Journal of Environmental Economics and Management 20: 127-142.

Leonard and Duerksen (1980) *Environmental Regulations and the Location of Industry: An International Perspective.* Columbia Journal of World Business Summer, 52-68

Low and Yeats (1992) *Do "dirty" industries migrate?*, International trade and the environment (The World Bank, Washington, DC) 89-103.

Lucas, Wheeler, and Hettige's (1992) *Economic development, environmental regulation and the international migration of toxic industrial pollution:1960-1988*, International trade and the environment (The World Bank, Washington, DC) 67-86.

Markusen, Morey and Olewiler (1993) *Environmental Policy when Market Structure and Plant Locations Are Endogenous*, Journal of Environmental Economics and Management 24, 69-86.

McGuire (1982) *Regulation, Factor Rewards, and International Trade*, Journal of Public Economics 17, 335-354.

Merrifield, J. D. (1988) "The Impact of Selected Abatement Strategies on transboundary Pollution, the Terms of Trade, and Factor Rewards: A General Equilibrium Approach." Journal of Environmental Economics and Management 15: 259-284.

Mohnen, V. A. (1988) "The Challenge of Acid Rain." Scientific American 259: 30-38.

Pearce (1992) *Should GATT be reformed for environmental reasons?*, CSERGE Discussion Paper GEC 92-06 (University of East ANglia and University College London).

Rauscher (1991) *Foreign trade and the environment, in H. Siebert, ed., Environmental scarcity*, The international dimension (J.C.B Mohr, Tubingen).

Reistein (1991) *Trade and environment*, Mimeo. (U.S. State Department).

Rice, Faye. (1993) "Who Scores Best on the Environment." Fortune: 114-122.

Runge, C. Ford (1987) "Induced Agricultural Innovation and Environmental Quality: The Case of Groundwater Regulation," Land Economics 63(3).

Runge, C. Ford (1990) "Environmental Risk and the World Economy." The American Prospect 1: 114-118.

Sanin and Zanaj (2007) "Environmental Innovation under Cournot competition" Center for Operations Research and Econometrics, Universite catholique de Louvain.

Vogel, David (1995) Trading Up: Consumer and Environmental Regulation in a Global Economy, Harvard University Press.

Whalley (1991) *The interface between environmental and trade policies*, Economic Journal 101, 100-189.

World Bank (1992) Development and the Environment, World Development Report.

Appendix A: Government policies and firms under Cournot competition

A.1 Computation for solutions

We show the computations for solutions from different cases.

(1) Homogeneous firms with local pollution

First, we solve firms' problems:

$$\max_{q_i} \pi_i = [a - b(q_A + q_B)]q_i - cq_i - d(1 - e_i)q_i \ , \quad \text{for } i \in A, B$$

$$\text{s.t.} \quad q_i > 0$$
$$e_i \in (0, 1)$$

F.O.C. $\frac{\partial \pi_i}{\partial q_i} = 0$

$$\rightarrow q_A^* = \frac{a-c-d(1-e_A)}{2b} - \frac{q_B}{2} \tag{A1}$$

$$q_B^* = \frac{a-c-d(1-e_B)}{2b} - \frac{q_A}{2} \tag{A2}$$

Plugging (A2) into (A1), we get

$$q_A^* = \frac{a-c-d+d(2e_A-e_B)}{3b} \tag{A3}$$

Plugging (A3) into (A2), we get

$$q_B^* = \frac{a-c-d+d(2e_B-e_A)}{3b} \tag{A4}$$

Then we solve governments' problems.

For government A, we solve:

$\max_{e_A} W_A = \pi_A - (\frac{t}{2})e_A^2$

s.t. $e_i \in (0,1)$ for $i \in A, B$

We plug (A3), (A4) into government A's objective function to get:

$\max W_A = \{a - b[\frac{a-c-d+d(2e_A-e_B)}{3b} + \frac{a-c-d+d(2e_B-e_A)}{3b}]\}[\frac{a-c-d+d(2e_A-e_B)}{3b}] - c[\frac{a-c-d+d(2e_A-e_B)}{3b}] - d(1-e_A)[\frac{a-c-d+d(2e_A-e_B)}{3b}] - (\frac{t}{2})e_A^2$

s.t. $e_i \in (0,1)$, for $i \in A, B$

F.O.C. $\frac{\partial W_A}{\partial e_A} = 0$

$\rightarrow \frac{2a}{b} - \frac{4(a-c-d)+2d(e_A+e_B)}{3b} - \frac{2c}{b} + \frac{a-c-d+d(2e_A-e_B)}{b} - \frac{a-c-d+d(2e_A-e_B)}{3b} - \frac{2d}{b}(1 - e_A) - \frac{3te_A}{d} = 0$

$$\rightarrow e_A^* = \frac{1}{-9bt+8d^2}(4d^2e_B - 4ad + 4cd + 4d^2) \quad \text{(A5)}$$

Similarly, we plug (A3), (A4) into government B's objective function to get:

$\max_{e_B} W_B = \{a - b[\frac{a-c-d+d(2e_A-e_B)}{3b} + \frac{a-c-d+d(2e_B-e_A)}{3b}]\}[\frac{a-c-d+d(2e_B-e_A)}{3b}] - c[\frac{a-c-d+d(2e_B-e_A)}{3b}] - d(1-e_B)[\frac{a-c-d+d(2e_B-e_A)}{3b}] - (\frac{t}{2})e_B^2$

s.t. $e_i \in (0,1)$, for $i \in A, B$

F.O.C. $\frac{\partial W_B}{\partial e_B} = 0$

$$\rightarrow e_B^* = \frac{1}{-9bt+8d^2}(4d^2e_A - 4ad + 4cd + 4d^2) \quad \text{(A6)}$$

We plug (A6) into (A5) to get

$$e_A^* = \frac{4d(a-c-d)}{9bt-4d^2} \quad \text{(A7)}$$

Then, plugging (A7) into (A6), we have

$$e_B^* = \frac{4d(a-c-d)}{9bt-4d^2} \tag{A8}$$

So e_A^*, e_B^* in (A7), (A8) are optimal choices for governments A, B. We put them into (A3), (A4) to get firms' optimal choices:

$$q_A^* = q_B^* = \frac{3t(a-c-d)}{9bt-4d^2} \tag{A9}$$

Plugging (A7), (A8), (A9) into (1), (2), (5), (6) in Chapter 2, we obtain firms' optimal profits and governments' optimal welfare:

$$\pi_A^* = \pi_B^* = \frac{9bt^2(a-c-d)^2}{(9bt-4d^2)^2}$$

$$W_A^* = W_B^* = \frac{t(9bt-8d^2)(a-c-d)^2}{(9bt-4d^2)^2}$$

(2) Computations for homogeneous firms with transboundary pollution

For government A, we solve:

$$\max_{e_A} W_A = \pi_A - (\tfrac{t}{2})(e_A + e_B)^2$$

$$\text{s.t.} \quad e_i \in (0,1) \text{ for } i \in A, B$$

We plug (A3), (A4) into government A's objective function to get:

$$\max W_A = \{a - b[\tfrac{a-c-d+d(2e_A-e_B)}{3b} + \tfrac{a-c-d+d(2e_B-e_A)}{3b}]\}[\tfrac{a-c-d+d(2e_A-e_B)}{3b}] - c[\tfrac{a-c-d+d(2e_A-e_B)}{3b}] - d(1-e_A)[\tfrac{a-c-d+d(2e_A-e_B)}{3b}] - (\tfrac{t}{2})(e_A+e_B)^2$$

$$\text{s.t.} \quad e_i \in (0,1), \text{ for } i \in A, B$$

F.O.C. $\frac{\partial W_A}{\partial e_A} = 0$

$$\rightarrow \frac{2a}{b} - \frac{4(a-c-d)+2d(e_A+e_B)}{3b} - \frac{2c}{b} + \frac{a-c-d+d(2e_A-e_B)}{b} - \frac{a-c-d+d(2e_A-e_B)}{3b} - \frac{2d}{b}(1-e_A) - \frac{3t(e_A+e_B)}{d} = 0$$

$$\rightarrow e_A^* = \frac{1}{-9bt+8d^2}\left(4d^2 e_B - 4ad + 4cd + 4d^2 + 9bte_B\right) \quad \text{(A10)}$$

Similarly we solve government B's problem and get

$$e_B^* = \frac{1}{-9bt+8d^2}\left(4d^2 e_A - 4ad + 4cd + 4d^2 + 9bte_A\right) \quad \text{(A11)}$$

Combining (A10) and (A11), we get

$$e_A^* = e_B^* = \frac{4d(a-c-d)}{(18bt-4d^2)} \quad \text{(A12)}$$

Plugging (A12) into (A3), (A4), we obtain

$$q_A^* = q_B^* = \frac{6t(a-c-d)}{(18bt-4d^2)} \quad \text{(A13)}$$

Plugging (A12), (A13) into firms' and governments' objective functions, we get

$$\pi_A^* = \pi_B^* = \frac{36bt^2(a-c-d)^2}{(18bt-4d^2)^2}$$

$$W_A^* = W_B^* = \frac{4t(9bt-8d^2)(a-c-d)^2}{(18bt-4d^2)^2}$$

(3) Heterogeneous firms with local pollution

For firm A,

$$\max_{q_A} \pi_A = [a - b(q_A + q_B)]q_A - cq_A - d(1-e_A)q_A$$

$$\text{s.t.} \quad q_A > 0$$
$$e_A \in (0,1)$$

F.O.C. $\frac{\partial \pi_A}{\partial q_A} = 0$

$$\rightarrow q_A^* = \frac{a-c-d(1-e_A)}{2b} - \frac{q_B}{2} \qquad \text{(A14)}$$

For firm B,

$$\max_{q_B} \pi_B = [a - b(q_A + q_B)]q_B - \alpha c q_B - d(1 - e_B)q_B, \quad \alpha \in (0, 1)$$

$$\text{s.t.} \quad q_B > 0$$
$$e_B \in (0, 1)$$

F.O.C. $\frac{\partial \pi_B}{\partial q_B} = 0$

$$\rightarrow q_B^* = \frac{a-\alpha c-d(1-e_B)}{2b} - \frac{q_B}{2} \qquad \text{(A15)}$$

Combining (A14) and (A15), we get,

$$q_A^* = \frac{[(a-2c+\alpha c-d)+d(2e_A-e_B)]}{3b} \qquad \text{(A16)}$$

$$q_B^* = \frac{[(a-2\alpha c+c-d)+d(2e_B-e_A)]}{3b} \qquad \text{(A17)}$$

Then we solve governments' problems,

For government A, we solve:

$$\max_{e_A} W_A = \pi_A - (\tfrac{t}{2}){e_A}^2$$

$$\text{s.t.} \quad e_A \in (0, 1)$$

We plug (A16), (A17) into government A's objective function to get:

$$\max W_A = \{a - b[\tfrac{(a-2c+\alpha c-d)+d(2e_A-e_B)}{3b} + \tfrac{(a-2\alpha c+c-d)+d(2e_B-e_A)}{3b}]\}\tfrac{[(a-2c+\alpha c-d)+d(2e_A-e_B)]}{3b} - c\tfrac{[(a-2c+\alpha c-d)+d(2e_A-e_B)]}{3b} - d(1-e_A)\tfrac{[(a-2c+\alpha c-d)+d(2e_A-e_B)]}{3b} - (\tfrac{t}{2})e_A^2, \quad \alpha \in (0, 1)$$

s.t. $e_A \in (0,1)$

F.O.C. $\frac{\partial W_A}{\partial e_A} = 0$

$$\rightarrow \frac{2ad}{3b} - \frac{4d[(a-2c+\alpha c-d)+d(2e_A^*-e_B)]}{9b} - \frac{2d[(a-2\alpha c+c-d)+d(2e_B-e_A^*)]}{9b} + \frac{d[(a-2c+\alpha c-d)+d(2e_A^*-e_B)]}{9b} - \frac{2cd}{3b} + \frac{d[(a-2c+\alpha c-d)+d(2e_A^*-e_B)]}{3b} - \frac{2d^2(1-e_A^*)}{3b} - te_A^* = 0 \quad \text{(A18)}$$

For government B, we solve:

$\max W_B = \pi_B - (\frac{t}{2}){e_B}^2$
e_B

s.t. $e_B \in (0,1)$

We plug (A16), (A17) into government B's objective function to get:

$$\max W_B = \{a - b[\frac{(a-2c+\alpha c-d)+d(2e_A-e_B)}{3b} + \frac{(a-2\alpha c+c-d)+d(2e_B-e_A)}{3b}]\}[\frac{(a-2\alpha c+c-d)+d(2e_B-e_A)}{3b}] - \alpha c[\frac{(a-2\alpha c+c-d)+d(2e_B-e_A)}{3b}] - d(1-e_A)[\frac{(a-2\alpha c+c-d)+d(2e_B-e_A)}{3b}] - (\frac{t}{2})e_B^2,\ \alpha \in (0,1)$$

s.t. $e_B \in (0,1)$

F.O.C. $\frac{\partial W_B}{\partial e_B} = 0$

$$\rightarrow \frac{2ad}{3b} - \frac{4d[(a-2\alpha c+c-d)+d(2e_B^*-e_A)]}{9b} - \frac{2d[(a-2c+\alpha c-d)+d(2e_A-e_B^*)]}{9b} + \frac{d[(a-2\alpha c+c-d)+d(2e_B^*-e_A)]}{9b} - \frac{2\alpha cd}{3b} + \frac{d[(a-2\alpha c+c-d)+d(2e_B^*-e_A)]}{3b} - \frac{2d^2(1-e_B^*)}{3b} - te_B^* = 0 \quad \text{(A19)}$$

Combining (A18) and (A19), we get,

$$e_A^* = \frac{[4d(a-2c+\alpha c-d)(9bt-8d^2)-16d^3(a-2\alpha c+c-d)]}{(9bt-4d^2)(9bt-12d^2)} \quad \text{(A20)}$$

$$e_B^* = \frac{[4d(a-2\alpha c+c-d)(9bt-8d^2)-16d^3(a-2c+\alpha c-d)]}{(9bt-4d^2)(9bt-12d^2)} \quad \text{(A21)}$$

Plugging (A20),(A21) into (A16),(A17), we get,

$$q_A^* = \frac{a-2c+\alpha c-d}{3b} + \frac{2d[4d(a-2c+\alpha c-d)(9bt-8d^2)-16d^3(a-2\alpha c+c-d)]}{3b(9bt-4d^2)(9bt-12d^2)} - \frac{d[4d(a-2\alpha c+c-d)(9bt-8d^2)-16d^3(a-2c+\alpha c-d)]}{3b(9bt-4d^2)(9bt-12d^2)} \tag{A22}$$

$$q_B^* = \frac{a-2\alpha c+c-d}{3b} + \frac{2d[4d(a-2\alpha c+c-d)(9bt-8d^2)-16d^3(a-2c+\alpha c-d)]}{3b(9bt-4d^2)(9bt-12d^2)} - \frac{d[4d(a-2c+\alpha c-d)(9bt-8d^2)-16d^3(a-2\alpha c+c-d)]}{3b(9bt-4d^2)(9bt-12d^2)} \tag{A23}$$

Plugging (A20) - (A23) into firms' and governments' objective functions, we get,

$$\pi_A^* = b\{\frac{a-2c+\alpha c-d}{3b} + \frac{2d[4d(a-2c+\alpha c-d)(9bt-8d^2)-16d^3(a-2\alpha c+c-d)]}{3b(9bt-4d^2)(9bt-12d^2)} - \frac{d[4d(a-2\alpha c+c-d)(9bt-8d^2)-16d^3(a-2c+\alpha c-d)]}{3b(9bt-4d^2)(9bt-12d^2)}$$

$$\pi_B^* = b\{\frac{a-2\alpha c+c-d}{3b} + \frac{2d[4d(a-2\alpha c+c-d)(9bt-8d^2)-16d^3(a-2c+\alpha c-d)]}{3b(9bt-4d^2)(9bt-12d^2)} - \frac{d[4d(a-2c+\alpha c-d)(9bt-8d^2)-16d^3(a-2\alpha c+c-d)]}{3b(9bt-4d^2)(9bt-12d^2)}$$

$$W_A^* = b\{\frac{a-2c+\alpha c-d}{3b} + \frac{2d[4d(a-2c+\alpha c-d)(9bt-8d^2)-16d^3(a-2\alpha c+c-d)]}{3b(9bt-4d^2)(9bt-12d^2)} - \frac{d[4d(a-2\alpha c+c-d)(9bt-8d^2)-16d^3(a-2c+\alpha c-d)]}{3b(9bt-4d^2)(9bt-12d^2)}$$
$$\frac{t[4d(a-2c+\alpha c-d)(9bt-8d^2)-16d^3(a-2\alpha c+c-d)]^2}{2(9bt-4d^2)^2(9bt-12d^2)^2}$$

$$W_B^* = b\{\frac{a-2\alpha c+c-d}{3b} + \frac{2d[4d(a-2\alpha c+c-d)(9bt-8d^2)-16d^3(a-2c+\alpha c-d)]}{3b(9bt-4d^2)(9bt-12d^2)} - \frac{d[4d(a-2c+\alpha c-d)(9bt-8d^2)-16d^3(a-2\alpha c+c-d)]}{3b(9bt-4d^2)(9bt-12d^2)}$$
$$\frac{t[4d(a-2\alpha c+c-d)(9bt-8d^2)-16d^3(a-2c+\alpha c-d)]^2}{2(9bt-4d^2)^2(9bt-12d^2)^2}$$

(4) Heterogeneous firms with transboundary pollution

The first step is similar as that in 2.3.2 and we have firm A's and firm B's best response functions (A16), (A17).

We plug (A16), (A17) into (3), (4) in the Chapter 2 to get, For government A,

$\max W_A = \{a - b[\frac{(a-2c+\alpha c-d)+d(2e_A-e_B)}{3b} + \frac{(a-2\alpha c+c-d)+d(2e_B-e_A)}{3b}]\}\frac{[(a-2c+\alpha c-d)+d(2e_A-e_B)]}{3b} - c\frac{[(a-2c+\alpha c-d)+d(2e_A-e_B)]}{3b} - d(1-e_A)\frac{[(a-2c+\alpha c-d)+d(2e_A-e_B)]}{3b} - (\frac{t}{2})(e_A+e_B)^2, \ \alpha \in (0,1)$

s.t. $e_A \in (0,1)$

For government B,

$\max W_B = \{a - b[\frac{(a-2c+\alpha c-d)+d(2e_A-e_B)}{3b} + \frac{(a-2\alpha c+c-d)+d(2e_B-e_A)}{3b}]\}[\frac{(a-2\alpha c+c-d)+d(2e_B-e_A)}{3b}] - \alpha c[\frac{(a-2\alpha c+c-d)+d(2e_B-e_A)}{3b}] - d(1-e_A)[\frac{(a-2\alpha c+c-d)+d(2e_B-e_A)}{3b}] - (\frac{t}{2})(e_A+e_B)^2,$
$\alpha \in (0,1)$

s.t. $e_B \in (0,1)$

We apply First Order Condition to obtain e_A^* and e_B^*.

$$e_A^* = \frac{4d(a-c-d)}{(18bt-4d^2)} + \frac{9bct(1-\alpha)}{d(18bt-4d^2)} \quad \text{(A24)}$$

$$e_B^* = \frac{4d(a-\alpha c-d)}{(18bt-4d^2)} - \frac{9bct(1-\alpha)}{d(18bt-4d^2)} \quad \text{(A25)}$$

Plugging (A24), (A25) into (A16), (A17), we get,

$$q_A^* = \frac{3t(2a-c-\alpha c-2d)}{(18bt-4d^2)} \quad \text{(A26)}$$

$$q_B^* = \frac{3t(2a-c-\alpha c-2d)}{(18bt-4d^2)} \quad \text{(A27)}$$

Plugging (A24) - (A27) into firms' profit functions and governments' welfare functions, we get,

$$\pi_A^* = \pi_B^* = \frac{9bt^2(2a-c-\alpha c-2d)^2}{(18bt-4d^2)^2}$$

$$W_A^* = W_B^* = \frac{t(9bt-8d^2)(2a-c-\alpha c-2d)^2}{(18bt-4d^2)^2}$$

A.2 Proof of Propositions

Here we show the proofs for Propositions 2.1 - 2.3.

(1) Proof of Proposition 2.1:

Since $4d(a-c-d) > 0$, $18bt - 4d^2 > 9bt - 4d^2 > 0$, then

$e_i^{*,l} = 4d(a-c-d)/(9bt-4d^2) > 4d(a-c-d)/(18bt-4d^2) = e_i^{*,t}$

$q_i^{*,l} = 3t(a-c-d)/(9bt-4d^2) = 6t(a-c-d)/(18bt-8d^2) > 6t(a-c-d)/(18bt-4d^2) = q_i^{*,t}$

$\pi_i^{*,l} = 9bt^2(a-c-d)^2/(9bt-4d^2)^2 = 36bt^2(a-c-d)^2/(18bt-8d^2)^2 > 36bt^2(a-c-d)^2/(18bt-4d^2)^2 = \pi_i^{*,t}$

$W_i^{*,l} = t(9bt-8d^2)(a-c-d)^2/(9bt-4d^2)^2 = 4t(9bt-8d^2)(a-c-d)^2/(18bt-8d^2)^2 > 4t(9bt-8d^2)(a-c-d)^2/(9bt-4d^2)^2 = W_i^{*,t}$

(2) Proof of Proposition 2.2:

Since $a - 2\alpha c + c - d > a - 2c + \alpha c - d$, then

$e_A^{*,l} = \frac{[4d(a-2c+\alpha c-d)\left(9bt-8d^2\right)-16d^3(a-2\alpha c+c-d)]}{(9bt-4d^2)(9bt-12d^2)} < \frac{[4d(a-2\alpha c+c-d)\left(9bt-8d^2\right)-16d^3(a-2\alpha c+c-d)]}{(9bt-4d^2)(9bt-12d^2)} < \frac{[4d(a-2\alpha c+c-d)\left(9bt-8d^2\right)-16d^3(a-2c+\alpha c-d)]}{(9bt-4d^2)(9bt-12d^2)} = e_B^{*,l}$

$q_A^{*,l} = [(a-2c+\alpha c-d)+d(2e_A^{*,l}-e_B^{*,l})]/3b < [(a-2\alpha c+c-d)+d(2e_A^{*,l}-e_B^{*,l})]/3b < [(a-2\alpha c+c-d)+d(2e_B^{*,l}-e_A^{*,l})]/3b = q_B^{*,l}$

$\pi_A^{*,l} = b(q_A^{*,l})^2 < b(q_B^{*,l})^2 = \pi_B^{*,l}$ since $q_B^{*,l} > q_A^{*,l} > 0$

$W_B^{*,l} - W_A^{*,l} = ct\,(1-\alpha)\,\frac{9bt-8d^2}{27b^2t^2-48bd^2t+16d^4}\,(2a-c-\alpha c-2d) = ct(1-\alpha)\frac{9bt-8d^2}{(3bt-4d^2)(9bt-4d^2)}(2a-c-\alpha c-2d)$. Preconditions $3bt-4d^2 > 0$, $a-c-d > 0$, $\alpha \in (0,1)$ ensure that $ct(1-\alpha)\frac{9bt-8d^2}{(3bt-4d^2)(9bt-4d^2)}(2a-c-\alpha c-2d) > 0$, so $W_B^{*,l} > W_A^{*,l}$.

(3) Proof of Proposition 2.3:

$e_A^{*,t} - e_B^{*,t} = \frac{18bct(1-\alpha)}{d(18bt-4d^2)} - \frac{4cd(1-\alpha)}{(18bt-4d^2)} = \frac{c(1-\alpha)}{d} > 0$, so $e_A^{*,t} > e_B^{*,t}$.

Firm A's total cost per product is: $c + d(1 - e_A^{*,t})$. Firm B's total cost per product is: $\alpha c + d\{1 - [e_A^{*,t} - \frac{c(1-\alpha)}{d}]\} = \alpha c + d(1 - e_A^{*,t}) + c(1-\alpha) = c + d(1 - e_A^{*,t})$. So firm B's total cost per product equals to that of firm A.

Since firms' total costs per product are equal, they have same outputs and profits.

Both countries' social environmental damages equal to $\frac{t(e_A^{*,t}+e_B^{*,t})^2}{2}$, so $W_A^{*,t} = \pi_A^{*,t} - \frac{t(e_A^{*,t}+e_B^{*,t})^2}{2} = \pi_B^{*,t} - \frac{t(e_A^{*,t}+e_B^{*,t})^2}{2} = W_B^{*,t}$. Countries' welfare are equal.

Appendix B: Government policies and firms under Stackelberg competition

B.1 Computation for solutions

Based on the fact that pollution could be local or transboundary, we compute for different solutions.

(1) Computation under local pollution

As firm A is Stackelberg leader, we firstly solve firm B's problem,

$$\max_{q_B} \pi_B = [a - b(q_A + q_B)]q_B - cq_B - d(1 - e_B)q_B$$

F.O.C. $\quad q_B^* = \frac{a-c-d+de_B}{2b} - \frac{1}{2}q_A \quad$ (B1)

Put (B1) in firm A's profit function to get,

max $[a - b(\frac{1}{2}q_A + \frac{a-c-d+de_B}{2b})]q_A - cq_A - d(1-e_A)q_A$

F.O.C. $$q_A^* = \frac{a-c-d+2de_A-de_B}{2b} \quad \text{(B2)}$$

Put (B2) in (B1) to get,

$$q_B^* = \frac{a-c-d+3de_B-2de_A}{4b} \quad \text{(B3)}$$

Put (B2), (B3) in government A's welfare function to solve,

max $[a - \frac{3(a-c-d)+2de_A+de_B}{4}]\frac{a-c-d+2de_A-de_B}{2b} - c\frac{a-c-d+2de_A-de_B}{2b} - d(1-e_A)\frac{a-c-d+2de_A-de_B}{2b} - \frac{t}{2}e_A^2$

F.O.C. $$e_A^* = \frac{d^2e_B - d(a-c-d)}{2(d^2-bt)} \quad \text{(B4)}$$

Similarly, we put (B2), (B3) in government B's welfare function to get,

$$e_B^* = \frac{6d^2e_A - 3d(a-c-d)}{9d^2-8bt} \quad \text{(B5)}$$

Combining (B4) and (B5), we get,

$$e_A^* = \frac{2d(a-c-d)(2bt-3d^2)}{(6d^4-17btd^2+8b^2t^2)} \quad \text{(B6)}$$

$$e_B^* = \frac{3d(a-c-d)(18d^4-25btd^2+8b^2t^2)}{(6d^4-17btd^2+8b^2t^2)(8bt-9d^2)} \quad \text{(B7)}$$

Now we put (B6), (B7) in (B2), (B3) to get,

$$\begin{aligned} q_A^* &= \frac{a-c-d}{2b} + \frac{2d^2(a-c-d)(2bt-3d^2)}{b(6d^4-17btd^2+8b^2t^2)} - \frac{3d^2(a-c-d)(18d^4-25btd^2+8b^2t^2)}{2b(6d^4-17btd^2+8b^2t^2)(8bt-9d^2)} \\ &= 2t\frac{2bt-3d^2}{8b^2t^2-17bd^2t+6d^4}(a-c-d) \quad \text{(B8)} \end{aligned}$$

$$\begin{aligned} q_B^* &= \frac{a-c-d}{4b} + \frac{9d^2(a-c-d)(18d^4-25btd^2+8b^2t^2)}{4b(6d^4-17btd^2+8b^2t^2)(8bt-9d^2)} - \frac{d^2(a-c-d)(2bt-3d^2)}{b(6d^4-17btd^2+8b^2t^2)} \\ &= 2t\frac{bt-2d^2}{8b^2t^2-17bd^2t+6d^4}(a-c-d) \quad \text{(B9)} \end{aligned}$$

Put (B6), (B7), (B8) and (B9) in firms' profit functions and governments' welfare functions, we have,

$$\pi_A^* = \frac{2bt^2(a-c-d)^2(2bt-3d^2)^2}{(6d^4-17btd^2+8b^2t^2)^2} = \frac{b}{2}(q_A^*)^2$$

$$\pi_B^* = \frac{4bt^2(bt-2d^2)^2(a-c-d)^2}{(6d^4-17btd^2+8b^2t^2)^2} = b(q_B^*)^2$$

$$W_A^* = \frac{2bt^2(a-c-d)^2(2bt-3d^2)^2}{(6d^4-17btd^2+8b^2t^2)^2} - \frac{2td^2(a-c-d)^2(2bt-3d^2)^2}{(6d^4-17btd^2+8b^2t^2)^2} = \frac{2t(bt-d^2)\left(3d^2-2bt\right)^2(a-c-d)^2}{(8b^2t^2-17bd^2t+6d^4)^2}$$

$$W_B^* = \frac{4bt^2(bt-2d^2)^2(a-c-d)^2}{(6d^4-17btd^2+8b^2t^2)^2} - \frac{9td^2(a-c-d)^2(18d^4-25btd^2+8b^2t^2)^2}{2(6d^4-17btd^2+8b^2t^2)^2(8bt-9d^2)^2} = \frac{t(8bt-9d^2)(bt-2d^2)^2(a-c-d)^2}{2(8b^2t^2-17bd^2t+6d^4)^2}$$

(2) Computation under transboundary pollution

First we solve firm B's problem,

$$\max_{q_B} [a-b(q_A+q_B)]q_B - cq_B - d(1-e_B)q_B$$

F.O.C. $\quad q_B^* = \frac{a-c-d+de_B}{2b} - \frac{1}{2}q_A \qquad$ (B10)

Put (B14) in firm A's profit function to get,

$$\max\ [a-b(\tfrac{1}{2}q_A + \tfrac{a-c-d+de_B}{2b})]q_A - cq_A - d(1-e_A)q_A$$

F.O.C. $\quad q_A^* = \frac{a-c-d+2de_A-de_B}{2b} \qquad$ (B11)

Put (B11) in (B10) to get,

$$q_B^* = \frac{a-c-d+3de_B-2de_A}{4b} \qquad \text{(B12)}$$

Put (B11), (B12) in government A's welfare function to solve, max $[a-\frac{3(a-c-d)+2de_A+de_B}{4}]\frac{a-c-d+2de_A-de_B}{2b}-c\frac{a-c-d+2de_A-de_B}{2b}-d(1-e_A)\frac{a-c-d+2de_A-de_B}{2b}-\frac{t}{2}(e_A+e_B)^2$

F.O.C. $\quad e_A^* = \frac{(d^2+bt)e_B-d(a-c-d)}{2(d^2-bt)}$

Similarly, we put (B11), (B12) in government B's welfare function and solve government B's problem to get,

$$e_B^* = \frac{(6d^2+8bt)e_A-3d(a-c-d)}{9d^2-8bt}$$

Combining them and we get,

$$e_A^* = \frac{d(a-c-d)(12d^2-5bt)}{2(-4b^2t^2+24bd^2t-6d^4)}$$

$$e_B^* = \frac{d(a-c-d)(6d^2+bt)}{-4b^2t^2+24bd^2t-6d^4}$$

Put e_A^*, e_B^* in (B11), (B12), we get:

$$q_A^* = \frac{t(9d^2-2bt)(a-c-d)}{(-4b^2t^2+24bd^2t-6d^4)}$$

$$q_B^* = \frac{t(8d^2-bt)(a-c-d)}{(-4b^2t^2+24bd^2t-6d^4)}$$

Then put e_A^*, e_B^*, q_A^*, q_B^* in firms' profit functions and governments' welfare functions, we have:

$$\pi_A^* = \frac{bt^2\left(9d^2-2bt\right)^2(a-c-d)^2}{2(4b^2t^2-24bd^2t+6d^4)^2} = \frac{b}{2}(q_A^*)^2$$

$$\pi_B^* = \frac{bt^2\left(8d^2-bt\right)^2(a-c-d)^2}{(4b^2t^2-24bd^2t+6d^4)^2} = b(q_B^*)^2$$

$$W_A^* = \frac{t(a-c-d)^2[4bt(9d^2-2bt)^2-9d^2(8d^2-bt)^2]}{8(4b^2t^2-24bd^2t+6d^4)^2}$$

$$W_B^* = \frac{t(8bt-9d^2)(8d^2-bt)^2(a-c-d)^2}{8(4b^2t^2-24bd^2t+6d^4)^2}$$

B.2 Proof of Propositions

We show proofs for Propositions 2.1 - 2.4.

(1) Proof of Proposition 2.1

Proof: First, we compare $e_A^* = \frac{2d(a-c-d)(2bt-3d^2)}{(6d^4-17btd^2+8b^2t^2)}$ and $e_B^* = \frac{3d(a-c-d)(18d^4-25btd^2+8b^2t^2)}{(6d^4-17btd^2+8b^2t^2)(8bt-9d^2)}$

Both times $\frac{(6d^4-17btd^2+8b^2t^2)(8bt-9d^2)}{d(a-c-d)}$ which is negative, we have $(4bt-6d^2)(8bt-9d^2)$ and $54d^4-75btd^2+24b^2t^2$.

$54d^4-75btd^2+24b^2t^2 < (4bt-6d^2)(8bt-9d^2) < 0$ since $d^2 \in (\frac{2}{3}bt, \frac{8}{9}bt)$

$$\Rightarrow \frac{2d(a-c-d)(2bt-3d^2)}{(6d^4-17btd^2+8b^2t^2)} < \frac{3d(a-c-d)(18d^4-25btd^2+8b^2t^2)}{(6d^4-17btd^2+8b^2t^2)(8bt-9d^2)} \Rightarrow e_A^* < e_B^*$$

Then, we compare $q_A^* = 2t\frac{2bt-3d^2}{8b^2t^2-17bd^2t+6d^4}(a-c-d)$ and $q_B^* = 2t\frac{bt-2d^2}{8b^2t^2-17bd^2t+6d^4}(a-c-d)$

Both times $\frac{(6d^4-17btd^2+8b^2t^2)}{2t(a-c-d)}$ which is negative, we have $2bt-3d^2$ and $bt-2d^2$. $\Rightarrow bt-2d^2 < 2bt-3d^2 < 0$,since $d^2 \in (\frac{2}{3}bt, \frac{8}{9}bt)$

$\Rightarrow 2t\frac{2bt-3d^2}{8b^2t^2-17bd^2t+6d^4}(a-c-d) < 2t\frac{bt-2d^2}{8b^2t^2-17bd^2t+6d^4}(a-c-d) \Rightarrow q_A^* < q_B^*$

For π_A^* and π_B^*, we have

$\frac{\pi_A^*}{\pi_B^*} = \frac{\frac{b}{2}(q_A^*)^2}{b(q_B^*)^2} = \frac{(q_A^*)^2}{2(q_B^*)^2} = \frac{1}{2}(\frac{2bt-3d^2}{bt-2d^2})^2$. From $bt-2d^2 < 2bt-3d^2 < 0$, we get $(\frac{2bt-3d^2}{bt-2d^2})^2$.

$\Rightarrow \pi_A^* < \frac{1}{2}\pi_B^* < \pi_B^*$

For W_A^* and W_A^*, we have

$$\frac{W_A^*}{W_B^*} = \frac{\frac{2t(bt-d^2)\left(3d^2-2bt\right)^2(a-c-d)^2}{\left(8b^2t^2-17bd^2t+6d^4\right)^2}}{\frac{t(8bt-9d^2)(bt-2d^2)^2(a-c-d)^2}{2\left(8b^2t^2-17bd^2t+6d^4\right)^2}} = \frac{4(bt-d^2)\left(3d^2-2bt\right)^2}{(8bt-9d^2)(bt-2d^2)^2}$$

$$4(bt-d^2)\left(3d^2-2bt\right)^2-(8bt-9d^2)(bt-2d^2)^2 = bt\left(8b^2t^2-23bd^2t+16d^4\right)$$

$bt\left(8b^2t^2-23bd^2t+16d^4\right) > 0$ if $d^2 \in (\frac{23+\sqrt{17}}{32}bt, \frac{8}{9}bt)$, and $bt\left(8b^2t^2-23bd^2t+16d^4\right) < 0$ if $d^2 \in (\frac{2}{3}bt, \frac{23+\sqrt{17}}{32}bt)$

so we get $W_A^* > W_B^*$ if $d^2 \in (\frac{23+\sqrt{17}}{32}bt, \frac{8}{9}bt)$

$$W_A^* < W_B^* \text{ if } d^2 \in (\frac{2}{3}bt, \frac{23+\sqrt{17}}{32}bt)$$

(2) Proof of Proposition 2.2

Proof: $e_A^{*,C} > e_A^{*,S} \Leftrightarrow \frac{4d(a-c-d)}{(9bt-4d^2)} > \frac{2d(a-c-d)(2bt-3d^2)}{(6d^4-17btd^2+8b^2t^2)} \Leftrightarrow \frac{2}{(9bt-4d^2)} > \frac{(3d^2-2bt)}{(-6d^4+17btd^2-8b^2t^2)}$

$$\Leftrightarrow 34btd^2-12d^4-16b^2t^2 > 35btd^2-12d^4-18b^2t^2$$

$$\Leftrightarrow bt(2bt-d^2) > 0 \Leftrightarrow 2bt-d^2 > 0$$

$2bt-d^2 > 0$ holds since $d^2 \in (\frac{2}{3}bt, \frac{8}{9}bt)$,

so $e_A^{*,C} > e_A^{*,S}$.

$e_B^{*,S} > e_B^{*,C} \Leftrightarrow \frac{3d(a-c-d)(18d^4-25btd^2+8b^2t^2)}{(6d^4-17btd^2+8b^2t^2)(8bt-9d^2)} > \frac{4d(a-c-d)}{(9bt-4d^2)}$

$\Leftrightarrow \frac{3(18d^4-25btd^2+8b^2t^2)}{(6d^4-17btd^2+8b^2t^2)(8bt-9d^2)} > \frac{4}{(9bt-4d^2)}$

$\Leftrightarrow 3(9bt-4d^2)(-18d^4+25btd^2-8b^2t^2) > 4(-6d^4+17btd^2-8b^2t^2)(8bt-9d^2)$

$\Leftrightarrow bt\,(40b^2t^2-61bd^2t+18d^4) > 0$

$\Leftrightarrow 40b^2t^2-61bd^2t+18d^4 > 0$

$40b^2t^2-61bd^2t+18d^4 > 0$ if $d^2 > \frac{45}{18}bt$ or $d^2 < \frac{8}{9}bt$. We know that $d^2 \in (\frac{2}{3}bt, \frac{8}{9}bt)$, so $40b^2t^2-61bd^2t+18d^4 > 0$ holds. Thus, $e_B^{*,S} > e_B^{*,C}$.

Suppose $q_A^{*,C} > q_A^{*,S}$ holds, we have

$\frac{3t(a-c-d)}{(9bt-4d^2)} > \frac{2(2bt-3d^2)(a-c-d)}{8b^2t^2-17bd^2t+6d^4} \Leftrightarrow 51btd^2-18d^4-24b^2t^2 > -24d^4-36b^2t^2+70btd^2$

$\Leftrightarrow 6d^4-19btd^2+12b^2t^2 > 0$

$6d^4-19btd^2+12b^2t^2 > 0$ if $d^2 > \frac{19+\sqrt{73}}{12}bt$ or $d^2 < \frac{19-\sqrt{73}}{12}bt$, $6d^4-19btd^2+12b^2t^2 < 0$ if $d^2 \in (\frac{19-\sqrt{73}}{12}bt, \frac{19+\sqrt{73}}{12}bt)$.As $d^2 \in (\frac{2}{3}bt, \frac{8}{9}bt)$, we get:

$q_A^{*,C} > q_A^{*,S}$ if $d^2 \in (\frac{2}{3}bt, \frac{19-\sqrt{73}}{12}bt)$, $q_A^{*,C} < q_A^{*,S}$ if $d^2 \in (\frac{19-\sqrt{73}}{12}bt, \frac{8}{9}bt)$.

$$q_B^{*,S} > q_B^{*,C} \Leftrightarrow \frac{2(bt-2d^2)(a-c-d)}{8b^2t^2-17bd^2t+6d^4} > \frac{3t(a-c-d)}{(9bt-4d^2)}$$

$$\Leftrightarrow (4d^2 - 2bt)(9bt - 4d^2) > (51btd^2 - 18d^4 - 24b^2t^2)$$

$$\Leftrightarrow 2d^4 - 7btd^2 + 6b^2t^2 > 0 \Leftrightarrow d^2(2d^2 - bt) + 6bt(bt - d^2) > 0$$

Since $d^2(2d^2 - bt) + 6bt(bt - d^2) > 0$ holds when $d^2 \in (\frac{2}{3}bt, \frac{8}{9}bt)$, $q_B^{*,S} > q_B^{*,C}$ holds.

$$\pi_B^{*,S} > \pi_B^{*,C} \Leftrightarrow \frac{9bt^2(a-c-d)^2}{(9bt-4d^2)^2} < \frac{4bt^2(bt-2d^2)^2(a-c-d)^2}{(6d^4-17btd^2+8b^2t^2)^2} \Leftrightarrow \frac{9}{(9bt-4d^2)^2} < \frac{4(bt-2d^2)^2}{(6d^4-17btd^2+8b^2t^2)^2}$$

$$\Leftrightarrow 9\left(6d^4 - 17btd^2 + 8b^2t^2\right)^2 < 4(9bt - 4d^2)^2(bt - 2d^2)^2$$

$$\Leftrightarrow (18d^4 - 51btd^2 + 24b^2t^2)^2 < (16d^4 - 44btd^2 + 18b^2t^2)^2$$

$\Leftrightarrow 18d^4 - 51btd^2 + 24b^2t^2 > 16d^4 - 44btd^2 + 18b^2t^2$ since both sides are negative.

Since $18d^4 - 51btd^2 + 24b^2t^2 > 16d^4 - 44btd^2 + 18b^2t^2$ when $d^2 \in (\frac{2}{3}bt, \frac{8}{9}bt)$, $\pi_B^{*,S} > \pi_B^{*,C}$holds.

$\pi_A^{*,C} > \pi_A^{*,S} \Leftrightarrow \frac{9bt^2(a-c-d)^2}{(9bt-4d^2)^2} > \frac{2bt^2(a-c-d)^2(2bt-3d^2)^2}{(6d^4-17btd^2+8b^2t^2)^2} \Leftrightarrow 9(6d^4 - 17btd^2 + 8b^2t^2)^2 > 2(9bt - 4d^2)^2(2bt - 3d^2)^2$

$$\Leftrightarrow (18d^4 - 51btd^2 + 24b^2t^2)^2 > 2(12d^4 - 35btd^2 + 18b^2t^2)^2$$

$$\Longleftrightarrow 18d^4 - 51btd^2 + 24b^2t^2 < \sqrt{2}(12d^4 - 35btd^2 + 18b^2t^2)$$

Since $18d^4 - 51btd^2 + 24b^2t^2 < \sqrt{2}(12d^4 - 35btd^2 + 18b^2t^2)$ holds when $d^2 \in (\frac{2}{3}bt, \frac{8}{9}bt)$,then $\pi_A^{*,C} > \pi_A^{*,S}$ holds.

To prove $W_A^{*,C} > W_A^{*,S}$, we need prove:

$$\frac{t(9bt-8d^2)(a-c-d)^2}{(9bt-4d^2)^2} > \frac{2t(bt-d^2)\left(3d^2-2bt\right)^2(a-c-d)^2}{\left(8b^2t^2-17bd^2t+6d^4\right)^2}$$

$\Leftrightarrow (9bt - 8d^2)\left(8b^2t^2 - 17bd^2t + 6d^4\right)^2 > 2(bt - d^2)(-12d^4 - 18b^2t^2 + 35btd^2)^2$

Since $9bt - 8d^2 > bt - d^2 > 0$, we only need prove:

$$\left(8b^2t^2 - 17bd^2t + 6d^4\right)^2 > 2(-12d^4 - 18b^2t^2 + 35btd^2)^2$$

$\Leftrightarrow 8b^2t^2 - 17bd^2t + 6d^4 < \sqrt{2}(-12d^4 - 18b^2t^2 + 35btd^2)$ since both sides are negative.

$\frac{3}{2}(-12d^4 - 18b^2t^2 + 35btd^2) < \sqrt{2}(-12d^4 - 18b^2t^2 + 35btd^2)$ because $\frac{3}{2} > \sqrt{2} > 0$ and $-12d^4 - 18b^2t^2 + 35btd^2 < 0$

We can also get $8b^2t^2 - 17bd^2t + 6d^4 < \frac{3}{2}(-12d^4 - 18b^2t^2 + 35btd^2)$ since $d^2 \in (\frac{2}{3}bt, \frac{8}{9}bt)$.

So $8b^2t^2 - 17bd^2t + 6d^4 < \sqrt{2}(-12d^4 - 18b^2t^2 + 35btd^2)$ holds. Thus $W_A^{*,C} > W_A^{*,S}$ holds.

To prove $W_B^{*,C} > W_B^{*,S}$, we need prove:

$$\frac{t(9bt-8d^2)(a-c-d)^2}{(9bt-4d^2)^2} > \frac{t(8bt-9d^2)\left(bt-2d^2\right)^2(a-c-d)^2}{2\left(8b^2t^2-17bd^2t+6d^4\right)^2}$$

$\Leftrightarrow 2(9bt-8d^2)\left(8b^2t^2-17bd^2t+6d^4\right)^2 > (8bt-9d^2)\left(bt-2d^2\right)^2(9bt-4d^2)^2$

Since $2(9bt-8d^2) > 8bt-9d^2$, we only need prove:

$\left(8b^2t^2-17bd^2t+6d^4\right)^2 > \left(bt-2d^2\right)^2(9bt-4d^2)^2 = (8d^4-22btd^2+9b^2t^2)^2$

$\Longleftrightarrow 8b^2t^2-17bd^2t+6d^4 < 8d^4-22btd^2+9b^2t^2$ since both sides are negative

$\Longleftrightarrow 14d^4-39btd^2+17b^2t^2 < 0$

Since $14d^4-39btd^2+17b^2t^2 < 0$ holds when $d^2 \in (\frac{2}{3}bt, \frac{8}{9}bt)$, we get $W_B^{*,C} > W_B^{*,S}$.

(3) Proof of Proposition 2.3

Proof: $e_A^* - e_B^* < 0 \Leftrightarrow \frac{d(a-c-d)(12d^2-5bt)}{2(-4b^2t^2+24bd^2t-6d^4)} < \frac{d(a-c-d)(6d^2+bt)}{-4b^2t^2+24bd^2t-6d^4} \Leftrightarrow 12d^2-5bt < 2(6d^2+bt)$

Since $12d^2-5bt < 2(6d^2+bt)$ holds, $e_A^* < e_B^*$ holds.

$q_A^* - q_B^* = \frac{t(9d^2-2bt)(a-c-d)}{(-4b^2t^2+24bd^2t-6d^4)} - \frac{t(8d^2-bt)(a-c-d)}{(-4b^2t^2+24bd^2t-6d^4)} = \frac{t(a-c-d)}{(-4b^2t^2+24bd^2t-6d^4)}(d^2-bt)$. Its sign is uncertain.

$q_A^* > q_B^*$ if $d^2 \in (bt, \frac{6+\sqrt{30}}{3}bt), q_A < q_B$ if $d^2 \in (\frac{5}{12}bt, bt)$.

$\pi_A^* - \pi_B^* = \frac{bt^2\left(9d^2-2bt\right)^2(a-c-d)^2}{2(4b^2t^2-24bd^2t+6d^4)^2} - \frac{bt^2\left(8d^2-bt\right)^2(a-c-d)^2}{(4b^2t^2-24bd^2t+6d^4)^2} = \frac{bt^2(a-c-d)^2}{2(4b^2t^2-24bd^2t+6d^4)^2}[(9d^2-2bt)^2 - 2\left(8d^2-bt\right)^2] < 0$

so $\pi_A^* < \pi_B^*$

Since $\pi_A^* < \pi_B^*$, $W_A^* = \pi_A^* - \frac{t}{2}(e_A^* + e_B^*)^2 < W_B^* = \pi_B^* - \frac{t}{2}(e_A^* + e_B^*)^2$ holds.

(4) Proof of Proposition 2.4

Proof: Preconditions for cournot model with transboundary pollution: $d^2 < \frac{9}{8}bt$, so $d^2 \in (\frac{5}{12}bt, \frac{8}{9}bt)$.

$e_A^{*,C} > e_A^{*,S} \Leftrightarrow \frac{4d(a-c-d)}{(18bt-4d^2)} > \frac{d(a-c-d)(12d^2-5bt)}{2(-4b^2t^2+24bd^2t-6d^4)} \Leftrightarrow 8(-4b^2t^2+24bd^2t-6d^4) > (18bt-4d^2)(12d^2-5bt)$

$\Leftrightarrow 8(-4b^2t^2+24bd^2t-6d^4) > (18bt-4d^2)(12d^2-5bt)$ holds when $d^2 \in (\frac{5}{12}bt, \frac{8}{9}bt)$.

$e_B^{*,C} < e_B^{*,S} \Leftrightarrow \frac{4d(a-c-d)}{(18bt-4d^2)} < \frac{d(a-c-d)(6d^2+bt)}{-4b^2t^2+24bd^2t-6d^4} \Leftrightarrow 4(-4b^2t^2+24bd^2t-6d^4) < (18bt-4d^2)(6d^2+bt)$

$\Leftrightarrow 4(-4b^2t^2+24bd^2t-6d^4) < (18bt-4d^2)(6d^2+bt)$ holds when $d^2 \in (\frac{5}{12}bt, \frac{8}{9}bt)$.

$q_A^{*,C} - q_A^{*,S} = \frac{6t(a-c-d)}{(18bt-4d^2)} - \frac{t(9d^2-2bt)(a-c-d)}{(-4b^2t^2+24bd^2t-6d^4)}$. Its sign is uncertain

$q_B^{*,C} < q_B^{*,S} \Leftrightarrow \frac{6t(a-c-d)}{(18bt-4d^2)} < \frac{t(8d^2-bt)(a-c-d)}{(-4b^2t^2+24bd^2t-6d^4)} \Leftrightarrow 6(-4b^2t^2+24bd^2t-6d^4) < (18bt-4d^2)(8d^2-bt)$

$\Leftrightarrow 6(-4b^2t^2+24bd^2t-6d^4) < (18bt-4d^2)(8d^2-bt)$ holds when $d^2 \in (\frac{5}{12}bt, \frac{8}{9}bt)$.

$\pi_A^{*,C} > \pi_A^{*,S} \Leftrightarrow \frac{36bt^2(a-c-d)^2}{(18bt-4d^2)^2} > \frac{bt^2\left(9d^2-2bt\right)^2(a-c-d)^2}{2(4b^2t^2-24bd^2t+6d^4)^2} \Leftrightarrow 72\left(4b^2t^2-24bd^2t+6d^4\right)^2 > (18bt-4d^2)^2\left(9d^2-2bt\right)^2$

$\Leftrightarrow 72\left(4b^2t^2-24bd^2t+6d^4\right)^2 > (18bt-4d^2)^2\left(9d^2-2bt\right)^2$ holds when $d^2 \in (\frac{5}{12}bt, \frac{8}{9}bt)$.

$\pi_B^{*,C} < \pi_B^{*,S} \Leftrightarrow \frac{36bt^2(a-c-d)^2}{(18bt-4d^2)^2} < \frac{bt^2\left(8d^2-bt\right)^2(a-c-d)^2}{(4b^2t^2-24bd^2t+6d^4)^2} \Leftrightarrow 36\left(4b^2t^2-24bd^2t+6d^4\right)^2 < (18bt-4d^2)^2\left(8d^2-bt\right)^2$

$\Leftrightarrow 36\left(4b^2t^2-24bd^2t+6d^4\right)^2 < (18bt-4d^2)^2\left(8d^2-bt\right)^2$ holds when $d^2 \in (\frac{5}{12}bt, \frac{8}{9}bt)$.

$e_A^{*,C} + e_B^{*,C} = \frac{8d(a-c-d)}{(18bt-4d^2)} < \frac{d(a-c-d)(24d^2-3bt)}{2(-4b^2t^2+24bd^2t-6d^4)} = e_A^{*,S} + e_B^{*,S}$ holds when $d^2 \in (\frac{5}{12}bt, \frac{8}{9}bt)$.

Since $\pi_A^{*,C} > \pi_A^{*,S}$ and $e_A^{*,C} + e_B^{*,C} < e_A^{*,S} + e_B^{*,S}$, $W_A^{*,C} < W_A^{*,S}$.

$W_B^{*,C} - W_B^{*,S} = \frac{4t(9bt-8d^2)(a-c-d)^2}{(18bt-4d^2)^2} - \frac{t(8bt-9d^2)(8d^2-bt)^2(a-c-d)^2}{8(4b^2t^2-24bd^2t+6d^4)^2}$. Its sign is uncertain.

Appendix C

C.1 Computation for solutions under local pollution

(1) Governments collude & Firms don't collude

First, we solve firm i's problem:

$$\max_{q_i} \pi_i = [a - b(q_A + q_B)]q_i - cq_i - d(1 - e_i)q_i$$

$$\text{s.t.} \quad q_i > 0, i \in \{A, B\}$$

$$e_i \in (0, 1)$$

We get:

$$q_A^* = \frac{a-c-d+2de_A-de_B}{3b} \tag{C1}$$

$$q_B^* = \frac{a-c-d+2de_B-de_A}{3b} \tag{C2}$$

Since two governments collude, their total welfare function is:

$$W = [a - \frac{2(a-c-d)+de_A+de_B}{3}]\frac{2(a-c-d)+de_A+de_B}{3b} - (c + d)\frac{2(a-c-d)+de_A+de_B}{3b} + \frac{d(a-c-d)}{3b}(e_A + e_B) + \frac{2d^2}{3b}(e_A^2 + e_B^2 - e_Ae_B) - \frac{t}{2}(e_A^2 + e_B^2)$$

So we put (C1) and (C2) in it and solve the problem:

$$\text{Max } [a - \frac{2(a-c-d)+de_A+de_B}{3}]\frac{2(a-c-d)+de_A+de_B}{3b} - (c + d)\frac{2(a-c-d)+de_A+de_B}{3b} + \frac{d(a-c-d)}{3b}(e_A + e_B) + \frac{2d^2}{3b}(e_A^2 + e_B^2 - e_Ae_B) - \frac{t}{2}(e_A^2 + e_B^2)$$

s.t. $e_i \in (0, 1), i = A, B.$

We get:

$$e_G^* = e_{A,G}^* = e_{B,G}^* = \frac{2d(a-c-d)}{(9bt-2d^2)} \qquad \text{(C3)}$$

Put (C3) into (C1), (C2), we get:

$$q_G^* = q_{A,G}^* = q_{B,G}^* = \frac{3t(a-c-d)}{(9bt-2d^2)} \qquad \text{(C4)}$$

Put (C3), (C4) into governments' and firms' objective functions, we get:

$$\pi_G^* = \pi_{A,G}^* = \pi_{B,G}^* = [a - \frac{6bt(a-c-d)}{(9bt-2d^2)}]\frac{3t(a-c-d)}{(9bt-2d^2)} - (c+d)\frac{3t(a-c-d)}{(9bt-2d^2)} + d\frac{2d(a-c-d)}{(9bt-2d^2)}\frac{3t(a-c-d)}{(9bt-2d^2)}$$

$$= \frac{3t(a-c-d)^2}{(9bt-2d^2)} - \frac{18bt^2(a-c-d)^2}{(9bt-2d^2)^2} + \frac{6td^2(a-c-d)^2}{(9bt-2d^2)^2}$$

$$= \frac{9bt^2(a-c-d)^2}{(9bt-2d^2)^2}$$

$$W_G^* = W_{A,G}^* = W_{B,G}^* = \frac{9bt^2(a-c-d)^2}{(9bt-2d^2)^2} - \frac{2td^2(a-c-d)^2}{(9bt-2d^2)^2}$$

$$= \frac{t(a-c-d)^2}{9bt-2d^2}$$

(2) Governments don't collude & Firms collude

First, firm A and B cooperate. Their total profits function is:

$$\pi = \pi_A + \pi_B$$

$$= [a - b(q_A + q_B)]q_A - cq_A - d(1 - e_A)q_A + [a - b(q_A + q_B)]q_B - cq_B - d(1 - e_B)q_B$$

$$= [a - b(q_A + q_B)](q_A + q_B) - (c + d)(q_A + q_B) + d(e_A q_A + e_B q_B)$$

Then we solve it as:

$$\underset{q_A, q_B}{\text{Max}} \quad [a - b(q_A + q_B)](q_A + q_B) - (c + d)(q_A + q_B) + d(e_A q_A + e_B q_B)$$

$$\text{s.t.} \quad q_i > 0, i \in \{A, B\}$$

$$e_i \in (0, 1)$$

As firms are symmetric, we get:

$$q_A^* = \frac{a-c-d+de_A}{4b} \tag{C5}$$

$$q_B^* = \frac{a-c-d+de_B}{4b} \tag{C6}$$

Then we put (C5), (C6) into governments' welfare functions and solve governments' problems.

For government A,

$W_A = \pi_A - \frac{t}{2}e_A^2 = [a - b\frac{2(a-c-d)+d(e_A+e_B)}{4b}](\frac{a-c-d+de_A}{4b}) - (c+d)(\frac{a-c-d+de_A}{4b}) + de_A(\frac{a-c-d+de_A}{4b}) - \frac{t}{2}e_A^2$

$\underset{e_A}{\text{Max}} \ [a - b\frac{2(a-c-d)+d(e_A+e_B)}{4b}](\frac{a-c-d+de_A}{4b}) - (c+d)(\frac{a-c-d+de_A}{4b}) + de_A(\frac{a-c-d+de_A}{4b}) - \frac{t}{2}e_A^2$

$$\text{s.t.} \quad e_i \in (0, 1), i \in \{A, B\}$$

F.O.C. $(16bt - 6d^2)e_A^* = 5d(a - c - d) - d^2 e_B^*$ (C7)

Similarly, we solve government B's problem to get:

$$(16bt - 6d^2)e_B^* = 5d(a - c - d) - d^2 e_A^* \quad \text{(C8)}$$

Combining (C7) and (C8), we get:

$$e_F^* = e_{A,F}^* = e_{B,F}^* = \frac{5d(a-c-d)}{(16bt-5d^2)} \quad \text{(C9)}$$

Then we put (C9) into to (C5), (C6) to get:

$$q_F^* = q_{A,F}^* = q_{B,F}^* = \frac{4t(a-c-d)}{(16bt-5d^2)} \quad \text{(C10)}$$

Finally we put (C9), (C10) into governments' and firms' objective functions and get:

$$\pi_F^* = \pi_{A,F}^* = \pi_{B,F}^* = [a - \frac{8bt(a-c-d)}{(16bt-5d^2)}]\frac{4t(a-c-d)}{(16bt-5d^2)} - (c+d)\frac{4t(a-c-d)}{(16bt-5d^2)} + \frac{5d^2(a-c-d)}{(16bt-5d^2)}\frac{4t(a-c-d)}{(16bt-5d^2)}$$

$$= \frac{4t(a-c-d)^2}{(16bt-5d^2)} - \frac{32bt^2(a-c-d)^2}{(16bt-5d^2)^2} + \frac{20td^2(a-c-d)^2}{(16bt-5d^2)^2}$$

$$= \frac{32bt^2(a-c-d)^2}{(16bt-5d^2)^2}$$

$$W_F^* = W_{A,F}^* = W_{B,F}^* = \frac{32bt^2(a-c-d)^2}{(16bt-5d^2)^2} - \frac{25td^2(a-c-d)^2}{2(16bt-5d^2)^2}$$

$$= \frac{t(64bt-25d^2)(a-c-d)^2}{2(16bt-5d^2)^2}$$

(3) Governments collude & Firms collude

Now consider government also cooperate, we put (C5), (C6) into the governments' total welfare function and solve:

Max $[a - \frac{2(a-c-d)+d(e_A+e_B)}{4}][\frac{2(a-c-d)+de_A+de_B}{4b}] - (c+d)[\frac{2(a-c-d)+de_A+de_B}{4b}] + de_A(\frac{a-c-d+de_A}{4b}) + de_B(\frac{a-c-d+de_B}{4b}) - \frac{t}{2}(e_A^2 + e_B^2)$

$$\text{s.t.} \quad e_i \in (0,1), i \in \{A, B\}$$

We get:

$$e^*_{GF} = e^*_{A,GF} = e^*_{B,GF} = \frac{d(a-c-d)}{(4bt-d^2)} \tag{C11}$$

We put (C11) into (C5), (C6) to get:

$$q^*_{GF} = q^*_{A,GF} = q^*_{B,GF} = \frac{t(a-c-d)}{(4bt-d^2)} \tag{C12}$$

Finally we put (C11), (C12) into governments' and firms' objective functions and get:

$$\pi^*_{GF} = \pi^*_{A,GF} = \pi^*_{B,GF} = [a - \frac{2bt(a-c-d)}{(4bt-d^2)}]\frac{t(a-c-d)}{(4bt-d^2)} - (c+d)\frac{t(a-c-d)}{(4bt-d^2)} + d\frac{d(a-c-d)}{(4bt-d^2)}\frac{t(a-c-d)}{(4bt-d^2)}$$

$$= \frac{t(a-c-d)^2}{(4bt-d^2)} - \frac{2bt^2(a-c-d)^2}{(4bt-d^2)^2} + \frac{td^2(a-c-d)^2}{(4bt-d^2)^2}$$

$$= \frac{2bt^2(a-c-d)^2}{(4bt-d^2)^2}$$

$$W^*_{GF} = W^*_{A,GF} = W^*_{B,GF} = \frac{2bt^2(a-c-d)^2}{(4bt-d^2)^2} - \frac{td^2(a-c-d)^2}{2(4bt-d^2)^2} = \frac{t(a-c-d)^2}{8bt-2d^2}$$

C.2 Computation for solutions under transboundary pollution

The computation for solutions under transboundary are similar as those under local pollution. The only difference is the government's welfare function.

(1) Governments collude & Firms don't collude

First we solve firms' profits functions and get (C1), (C2).

Since governments collude, we put (C1), (C2) into their total welfare function and solve the problem:

$$\text{Max } [a - \tfrac{2(a-c-d)+de_A+de_B}{3}]\tfrac{2(a-c-d)+de_A+de_B}{3b} - (c+d)\tfrac{2(a-c-d)+de_A+de_B}{3b} + \tfrac{d(a-c-d)}{3b}(e_A+e_B) + \tfrac{2d^2}{3b}(e_A^2+e_B^2-e_Ae_B) - t(e_A+e_B)^2$$
$$\text{s.t.} \quad e_i \in (0,1), i \in \{A,B\}$$

From the First Order Condition, we get:

$$e_G^* = e_{A,G}^* = e_{B,G}^* = \tfrac{d(a-c-d)}{(18bt-d^2)}$$

Similarly as 4.7.1, then we get:

$$q_G^* = q_{A,G}^* = q_{B,G}^* = \tfrac{6t(a-c-d)}{(18bt-d^2)}$$

$$\pi_G^* = \pi_{A,G}^* = \pi_{B,G}^* = [a - \tfrac{12bt(a-c-d)}{(18bt-d^2)}]\tfrac{6t(a-c-d)}{(18bt-d^2)} - (c+d)\tfrac{6t(a-c-d)}{(18bt-d^2)} + d\tfrac{6t(a-c-d)}{(18bt-d^2)}\tfrac{d(a-c-d)}{(18bt-d^2)}$$
$$= \tfrac{6t(a-c-d)^2}{(18bt-d^2)} - \tfrac{72bt^2(a-c-d)^2}{(18bt-d^2)^2} + \tfrac{6td^2(a-c-d)^2}{(18bt-d^2)^2} = \tfrac{36bt^2(a-c-d)^2}{(18bt-d^2)^2}$$

$$W_G^* = W_{A,G}^* = W_{B,G}^* = \tfrac{36bt^2(a-c-d)^2}{(18bt-d^2)^2} - \tfrac{2td^2(a-c-d)^2}{(18bt-d^2)^2} = \tfrac{2t(a-c-d)^2}{(18bt-d^2)}$$

(2) Governments don't collude & Firms collude

First we solve firms' total profits function and get (C5), (C6).

Then we put (C5), (C6) into governments A's and B's individual profits functions and solve their problems.

For government A,

$$\underset{e_A}{\text{Max}}\ [a-b\tfrac{2(a-c-d)+d(e_A+e_B)}{4b}](\tfrac{a-c-d+de_A}{4b})-(c+d)(\tfrac{a-c-d+de_A}{4b})+de_A(\tfrac{a-c-d+de_A}{4b})-\tfrac{t}{2}(e_A+e_B)^2$$

$$\text{s.t.}\quad e_i \in (0,1), i \in \{A,B\}$$

For government B,

$$\underset{e_B}{\text{Max}}\ [a-b\tfrac{2(a-c-d)+d(e_A+e_B)}{4b}](\tfrac{a-c-d+de_B}{4b})-(c+d)(\tfrac{a-c-d+de_B}{4b})+de_B(\tfrac{a-c-d+de_B}{4b})-\tfrac{t}{2}(e_A+e_B)^2$$

$$\text{s.t.}\quad e_i \in (0,1), i \in \{A,B\}$$

From the First Order Condition, we get:

$$e_F^* = e_{A,F}^* = e_{B,F}^* = \tfrac{5d(a-c-d)}{(32bt-5d^2)}$$

Similarly as 4.7.1, then we get:

$$q_F^* = q_{A,F}^* = q_{B,F}^* = \tfrac{8t(a-c-d)}{(32bt-5d^2)}$$

$$\pi_F^* = \pi_{A,F}^* = \pi_{B,F}^* = [a-\tfrac{16bt(a-c-d)}{(32bt-5d^2)}]\tfrac{8t(a-c-d)}{(32bt-5d^2)}-(c+d)\tfrac{8t(a-c-d)}{(32bt-5d^2)}+d\tfrac{5d(a-c-d)}{(32bt-5d^2)}\tfrac{8t(a-c-d)}{(32bt-5d^2)} = \tfrac{8t(a-c-d)^2}{(32bt-5d^2)} - \tfrac{128bt^2(a-c-d)^2}{(32bt-5d^2)^2} + \tfrac{40td^2(a-c-d)^2}{(32bt-5d^2)^2} = \tfrac{128bt^2(a-c-d)^2}{(32bt-5d^2)^2}$$

$$W_F^* = W_{A,F}^* = W_{B,F}^* = \tfrac{128bt^2(a-c-d)^2}{(32bt-5d^2)^2} - \tfrac{50td^2(a-c-d)^2}{(32bt-5d^2)^2} = \tfrac{2t(64bt-25d^2)(a-c-d)^2}{(5d^2-32bt)^2}$$

(3) Governments collude & Firms collude

Now consider government also cooperate, we put (C5), (C6) into the governments' total welfare function and solve:

$$\text{Max } [a-\tfrac{2(a-c-d)+d(e_A+e_B)}{4}][\tfrac{2(a-c-d)+de_A+de_B}{4b}]-(c+d)[\tfrac{2(a-c-d)+de_A+de_B}{4b}]+de_A(\tfrac{a-c-d+de_A}{4b})+de_B(\tfrac{a-c-d+de_B}{4b})-t(e_A+e_B)^2$$

$$\text{s.t.} \quad e_i \in (0,1), i \in \{A,B\}$$

We get:

$$e^*_{GF}=e^*_{A,GF}=e^*_{B,GF}=\tfrac{d(a-c-d)}{(16bt-d^2)}$$

Similarly as 4.7.1, then we get:

$$q^*_{GF}=q^*_{A,GF}=q^*_{B,GF}=\tfrac{4t(a-c-d)}{(16bt-d^2)}$$

$$\pi^*_{GF}=\pi^*_{A,GF}=\pi^*_{B,GF}=[\tfrac{4t(a-c-d)^2}{(16bt-d^2)}-\tfrac{32bt^2(a-c-d)^2}{(16bt-d^2)^2}+\tfrac{4td^2(a-c-d)^2}{(16bt-d^2)^2}=\tfrac{32bt^2(a-c-d)^2}{(16bt-d^2)^2}$$

$$W^*_{GF}=W^*_{A,GF}=W^*_{B,GF}=\tfrac{32bt^2(a-c-d)^2}{(16bt-d^2)^2}-\tfrac{2td^2(a-c-d)^2}{(16bt-d^2)^2}=\tfrac{2t(a-c-d)^2}{16bt-d^2}$$

C.3 Proof of Propositions

(1) Proof of Proposition 4.1

Proof: $\frac{2d(a-c-d)}{(9bt-2d^2)}=\frac{5d(a-c-d)}{(22.5bt-5d^2)}<\frac{5d(a-c-d)}{(16bt-5d^2)} \Rightarrow e^*_G<e^*_F$

$$\tfrac{5d(a-c-d)}{(16bt-5d^2)}>\tfrac{d(a-c-d)}{(4bt-d^2)}=\tfrac{5d(a-c-d)}{(20bt-5d^2)}>\tfrac{5d(a-c-d)}{(22.5bt-5d^2)} \Rightarrow e^*_G<e^*_{GF}<e^*_F$$

$\frac{4d(a-c-d)}{(9bt-4d^2)} = \frac{20d(a-c-d)}{(45bt-20d^2)} > \frac{20d(a-c-d)}{(64bt-20d^2)} \Rightarrow e^*_{nc} > e^*_F$

Thus, $e^*_{nc} > e^*_F > e^*_{GF} > e^*_G$ holds.

$\frac{3t(a-c-d)}{(9bt-4d^2)} > \frac{3t(a-c-d)}{(9bt-2d^2)} \Rightarrow q^*_{nc} > q^*_G$

$\frac{4t(a-c-d)}{(16bt-5d^2)} > \frac{4t(a-c-d)}{(16bt-4d^2)} = \frac{t(a-c-d)}{(4bt-d^2)} \Rightarrow q^*_F > q^*_{GF}$

$q^*_G > q^*_F \Leftrightarrow \frac{3t(a-c-d)}{(9bt-2d^2)} > \frac{4t(a-c-d)}{(16bt-5d^2)} \Leftrightarrow 12bt > 7d^2$

Since $9bt > 8d^2$ holds in the Assumption 1, $12bt > 7d^2$ holds. So $q^*_G > q^*_F$ holds.

Thus $q^*_{nc} > q^*_G > q^*_F > q^*_{GF}$ holds.

(2) Proof of Proposition 4.2

Proof: $\frac{9bt^2(a-c-d)^2}{(9bt-4d^2)^2} > \frac{9bt^2(a-c-d)^2}{(9bt-2d^2)^2} \Leftrightarrow \pi^*_{nc} > \pi^*_G$

$\frac{32bt^2(a-c-d)^2}{(16bt-5d^2)^2} > \frac{2bt^2(a-c-d)^2}{(4bt-d^2)^2} > \frac{9bt^2(a-c-d)^2}{(9bt-2d^2)^2} \Leftrightarrow 36\sqrt{2}bt - 8\sqrt{2}d^2 > 48bt - 12d^2 > 48bt - 15d^2 \Leftrightarrow \pi^*_F > \pi^*_{GF} > \pi^*_G$

We can not compare π^*_{nc} with π^*_F or π^*_{GF}, because $36\sqrt{2}bt - 16\sqrt{2}d^2$ could be smaller than $48bt - 15d^2$ or larger than $48bt - 12d^2$.

Thus we have $\pi^*_F > \pi^*_{GF} > \pi^*_G$ and $\pi^*_{nc} > \pi^*_G$.

$\frac{t(a-c-d)^2}{(8bt-2d^2)} > \frac{t(a-c-d)^2}{(9bt-2d^2)} \Leftrightarrow W^*_{GF} > W^*_G$

$\frac{t(a-c-d)^2}{(9bt-2d^2)} > \frac{t(9bt-8d^2)(a-c-d)^2}{(9bt-4d^2)^2} \Leftrightarrow (9bt-4d^2)^2 > (9bt-2d^2)(9bt-8d^2) \Leftrightarrow 0 > -18btd^2 \Leftrightarrow W^*_G > W^*_{nc}$

$\frac{t(a-c-d)^2}{(8bt-2d^2)} > \frac{t(64bt-25d^2)(a-c-d)^2}{2(16bt-5d^2)^2} \Longleftrightarrow 2(16bt-5d^2)^2 > (8bt-2d^2)(64bt-25d^2) \Longleftrightarrow -320btd^2 > -328btd^2 \Leftrightarrow W^*_{GF} > W^*_F$

$\frac{t(64bt-25d^2)(a-c-d)^2}{2(16bt-5d^2)^2} > \frac{t(a-c-d)^2}{(9bt-2d^2)} \Leftrightarrow (64bt - 25d^2)(9bt - 2d^2) > 2(16bt - 5d^2)^2 \Leftrightarrow 64b^2t^2 > 33btd^2 \Leftrightarrow 64bt > 33d^2$ since $bt > \frac{8}{9}d^2 \Leftrightarrow W_F^* > W_G^*$

Thus, $W_{GF}^* > W_F^* > W_G^* > W_{nc}^*$ holds.

In the first stage, governments could choose collude or not collude. If they don't collude, they would get W_F^* or W_{nc}^*. It is uncertain, since whether $\pi_F^* > \pi_{nc}^*$ is uncertain.

If governments choose collude, they would get W_{GF}^* if firms choose collude and get W_G^* if firms don't collude. We know that firms surely choose collude since $\pi_{GF}^* > \pi_G^*$. So for sure governments would get W_{GF}^* if they choose collude. As $W_{GF}^* > W_F^* > W_G^* > W_{nc}^*$, governments would certainly choose collude and then firms also certainly choose collude.

(3) Proof of Proposition 4.3

Proof: $128bt - 20d^2 > 90bt - 20d^2 \Rightarrow \frac{4d(a-c-d)}{(18bt-4d^2)} > \frac{5d(a-c-d)}{(32bt-5d^2)} \Rightarrow e_{nc}^* > e_F^*$

$\frac{5d(a-c-d)}{(32bt-5d^2)} > \frac{5d(a-c-d)}{(80bt-5d^2)} = \frac{d(a-c-d)}{(16bt-d^2)} > \frac{5d(a-c-d)}{(90bt-5d^2)} = \frac{d(a-c-d)}{(18bt-d^2)} \Rightarrow e_F^* > e_{GF}^* > e_G^*$

Thus, $e_{nc}^* > e_F^* > e_{GF}^* > e_G^*$ holds.

$18bt - d^2 > 18bt - 4d^2 \Rightarrow \frac{6t(a-c-d)}{(18bt-4d^2)} > \frac{6t(a-c-d)}{(18bt-d^2)} \Rightarrow q_{nc}^* > q_G^*$

$bt > \frac{8}{9}d^2 \Rightarrow 96bt - 15d^2 > 72bt - 4d^2 \Rightarrow \frac{6t(a-c-d)}{(18bt-d^2)} > \frac{8t(a-c-d)}{(32bt-5d^2)} \Rightarrow q_G^* > q_F^*$

$\frac{8t(a-c-d)}{(32bt-5d^2)} > \frac{8t(a-c-d)}{(32bt-2d^2)} = \frac{4t(a-c-d)}{(16bt-d^2)} \Rightarrow q_F^* > q_{GF}^*$

Thus, $q_{nc}^* > q_G^* > q_F^* > q_{GF}^*$ holds.

(4) Proof of Proposition 4.4

$$\frac{128bt^2(a-c-d)^2}{(32bt-5d^2)^2} > \frac{128bt^2(a-c-d)^2}{(32bt-2d^2)^2} = \frac{32bt^2(a-c-d)^2}{(16bt-d^2)^2} \Rightarrow \pi_F^* > \pi_{GF}^*$$

$$bt > \frac{8}{9}d^2 \Rightarrow 72\sqrt{2}bt - 4\sqrt{2}d^2 > 96bt - 6d^2 \Rightarrow \frac{32bt^2(a-c-d)^2}{(16bt-d^2)^2} > \frac{36bt^2(a-c-d)^2}{(18bt-d^2)^2} \Rightarrow \pi_{GF}^* > \pi_G^*$$

$$18bt - d^2 > 18bt - 4d^2 \Rightarrow \frac{36bt^2(a-c-d)^2}{(18bt-4d^2)^2} > \frac{36bt^2(a-c-d)^2}{(18bt-d^2)^2} \Rightarrow \pi_{nc}^* > \pi_G^*$$

Thus we have $\pi_F^* > \pi_{GF}^* > \pi_G^*$ and $\pi_{nc}^* > \pi_G^*$.

$$-320btd^2 > -464btd^2 \Rightarrow (32bt-5d^2)^2 > (16bt-d^2)(64bt-25d^2) \Rightarrow \frac{2t(a-c-d)^2}{(16bt-d^2)} > \frac{2t(64bt-25d^2)(a-c-d)^2}{(32bt-5d^2)^2} \Rightarrow W_{GF}^* > W_F^*$$

$$-144btd^2 > -306btd^2 \Rightarrow (18bt-4d^2)^2 > (18bt-16d^2)(18bt-d^2) \Rightarrow \frac{2t(a-c-d)^2}{(18bt-d^2)} > \frac{4t(9bt-8d^2)(a-c-d)^2}{(18bt-4d^2)^2} \Rightarrow W_G^* > W_{nc}^*$$

$$\frac{2t(a-c-d)^2}{(16bt-d^2)} > \frac{2t(a-c-d)^2}{(16bt-d^2)} \Rightarrow W_{GF}^* > W_G^*$$

Thus we have $W_{GF}^* > W_G^* > W_{nc}^*$ and $W_{GF}^* > W_F^*$.

In the first stage, governments could choose collude or not collude. If they don't collude, they would get W_F^* or W_{nc}^*. It is uncertain, since whether $\pi_F^* > \pi_{nc}^*$ is uncertain.

If governments choose collude, they would get W_{GF}^* if firms choose collude and get W_G^* if firms don't collude. We know that firms surely choose collude since $\pi_{GF}^* > \pi_G^*$. So for sure governments would get W_{GF}^* if they choose collude. As $W_{GF}^* > W_G^* > W_{nc}^*$ and $W_{GF}^* > W_F^*$, governments would certainly choose collude and then firms also certainly choose collude.