Examining Children’s Use of Different Types of Concrete Representations in a Novel Numeration System

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ABSTRACT

Examining Children's Use of Different Types of Concrete Representations

in a Novel Numeration System

By Aryann Blondin

Concrete materials, also called manipulatives, are used in the elementary classroom (e.g., Blondin, Tomaszewski, & Osana, 2017; Moyer, 2001) to illustrate a wide variety of abstract mathematical notions, yet little is known about how the differences in the materials affect the mathematical learning (Belenky & Schalk, 2014). This study examined the differences between concrete proportional and non-proportional models in the context of base-four problems. The aim was to examine the effects of the manipulatives' proportionality on learning and transfer measures, and to see if there were any effects of prior knowledge of numeracy and place value. Following prior knowledge assessment, 52 second-graders were randomly assigned to either the proportional condition (n = 26) or the non-proportional condition (n = 26). Students in the proportional condition used base-four blocks for the instructional intervention and the testing. Students in the non-proportional condition used colored chips. Next, students received an instructional intervention with the manipulatives and completed learning and transfer measures. No condition effects were found. Prior knowledge accounted for a significant portion of the variance on two of the learning measures and one of the transfer measures, but no interaction effects were found. The present study reminds teachers that prior knowledge is important to consider when choosing appropriate tools in the classroom to meet their students' needs. Additionally, the present study reminds researchers that assessing prior knowledge is an important factor when examining the effects of manipulatives. The results of this study contribute to the literature on external knowledge representations.

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Chapter 1: Statement of the Problem

When children learn mathematics, they acquire the ability to externally represent their knowledge to help them communicate and understand complex ideas. These external knowledge representations can be, for example, symbols like numbers or even concrete mathematical tools, such as manipulatives. Children do not always learn on their own how to use these external knowledge representations (Osana & Duponsel, 2016) nor do they intuitively understand the connections between the different types of representations and their underlying conceptual structure (Alfieri, Nokes-Malach, & Schunn, 2014). This is why students need support on how to use external representations to extend their mathematical understanding (Osana, Przednowek, Cooperman, & Adrien, 2017).

Concrete tools are often used in the mathematics classroom (e.g., Blondin, Tomaszewski, & Osana, 2017; Clements, 1999; Moyer, 2001). Manipulatives can be used to support a wide variety of ideas, but they are most often used to represent abstract mathematical notions and support children's understanding of these notions. In the early grades, manipulatives are typically used to concretely illustrate conceptual ideas of our numeration system, such as place value. An example of such manipulatives is base-ten blocks. These blocks illustrate our numeration system by showing how single cubes represent ones, and how ten of these cubes (stuck together) represent a group of ten. Each denomination is conceptually, and physically, ten times bigger than the previous one. While manipulatives such as these are intended to help children understand the mathematical ideas to which they are linked, this objective is not always reached, nor is it always reached in the most optimal way.

Teachers use manipulatives in a variety of ways (Blondin et al., 2017), yet little is known about how different types of concrete materials support students' understanding of the underlying concepts they are intended to represent. Additionally, conclusions from past studies rarely assess the role of participants' prior knowledge on the effects of different manipulative types (e.g., Ginsburg, Jamalian, & Creighan, 2013; Kaminski, Sloutski, & Heckler, 2013). This suggests an important gap in the literature and further study in this area promises to provide important educational insights.

There exists a wide variety of mathematical tools for teachers to use in the classroom, and the sheer number of available tools can present a challenge for teachers. It is not always clear to teachers what type of concrete material is most appropriate for specific students at specific times, particularly given the variance in students' prior knowledge and understanding. Although teachers frequently make decisions in real time about what manipulatives to use in any given situation, there is limited empirical evidence to suggest which type of manipulative is ideal based on students' current understanding (i.e., prior knowledge) and teachers' desired learning objective. The present study will consider the above-mentioned gaps in the literature by testing the effects of different types of manipulatives on students' learning and transfer in a novel numeration system. In addition, students' prior knowledge will be considered to examine any moderating effects.

The study will have theoretical and practical implications. The results of this study will contribute to the literature on external representations and the relationship between the physical features of these representations and their affordances for learning and transfer in mathematics. This will contribute to the literature because previous studies typically focus on either transfer or learning (e.g., Kaminski et al. 2013; McNeil, Uttal, & Sternberg, 2009), but rarely both. For practitioners, the results of this study could provide insight on their teaching practices in terms of having a more nuanced understanding of how to support their student’s learning with concrete materials.

Chapter 2: Literature Review

External Knowledge Representations

Teachers use different tools to illustrate mathematical ideas and concepts. They choose to represent these ideas using what has been called in the literature, “external knowledge representations.” Some examples of external knowledge representations include drawings, written numerals, charts and graphs, and even concrete objects such as manipulatives (Belenky & Schalk, 2014). External knowledge representations can be used either by teachers to explain and illustrate a concept, or by children themselves to show their thinking (Carpenter, Fennema, Franke, Levi, & Empson, 2014).

Manipulatives are a type of external knowledge representation, because they can be viewed as symbols to illustrate ideas. That is, teachers choose specific manipulatives to illustrate specific targeted concepts during instruction. Uttal, Scudder and DeLoache (1997) described mathematical manipulatives as being symbols, because they represent an intended idea (see also Yuan & Uttal, 2017). The main difference between concrete materials like manipulatives, and other less concrete representations, such as charts or graph, is the possibility for physical interaction with the objects (Belenky & Schalk, 2014). Mathematical manipulatives are varied and range from geometry instruments, counting materials, and objects to illustrate base-ten ideas. Base-ten blocks are commonly used to teach the numeration system in the early grades. Teachers use these materials because they physically represent the ideas that are needed to understand the numeration system, such as the idea that a *ten* is ten times larger than a *one*.

For them to understand what the teacher is trying to teach them, children must be able to appropriate the object’s referent. Internal knowledge representations can be characterized as procedural and conceptual knowledge related to targeted areas of knowledge (Goldin, 1998;

Rittle-Johnson, Siegler, & Alibali, 2001). These type of knowledge, namely internal and external knowledge, are often targeted by teachers in the mathematics classroom. In the context of this study, I will define procedural knowledge as a "series of steps, or actions, done to accomplish a goal" or to be able to solve a specific problem (Rittle-Johnson & Schneider, 2015). I will define conceptual knowledge as the knowledge that is not linked to specific problems and procedures, but rather knowledge that can be connected to other types of problems or contexts (Rittle-Johnson & Schneider, 2015). Internal and external knowledge representations are closely related, and different types of external knowledge representations will predict different degrees of conceptual understanding and procedural knowledge for the learners (Belenky & Schalk, 2014).

Many authors describe external knowledge representations using different terminology (e.g., Belenky & Schalk, 2014; Braithwaite & Goldstone, 2013). Despite the different terms, the literature converges on two main types of representations: grounded and idealized. In fact, Belenky and Schalk (2014) explain how representations can be conceptualized on a spectrum, where on one extreme are grounded representations, and on the other end are idealized representations. For a representation to be *grounded*, it needs to be motivated by the context of the question and activates prior knowledge on the topic. On the other hand, for a representation to be considered *idealized*, the representation would not be one that activates prior knowledge; there is nothing inherent about the representation that reflects its meaning, which means that its meaning needs to be taught (Braithwaite & Goldstone, 2013).

Consider the example of representing the quantity 23, in the context of the following problem: "Susan has 12 marbles and her sister gives her 11 more marbles. How many marbles does she have now?". There are many ways to represent the quantity of 23, and some ways will be more grounded than others. If a child takes 12 marbles followed by 11 marbles to represent the total of 23 marbles, we would consider this child's representation as *grounded*, as the representation activates prior knowledge related to the question. However, if a child would write "12 + 11 = 23" in Arabic numerals, one would consider the child's representation to be *idealized* because these symbols must be taught and do not look, in and of themselves, like 12 or 11 or 23 things. The literature would suggest that children respond differently to the degree of groundedness of a representation used during mathematics instruction. Specifically, the type of representation used has been shown to impact the degree of learning and transfer that is observed (Belenky & Schalk, 2014). In the literature review that follows, I will expand on how different types of representations can affect students' learning and transfer.

A point of clarification is needed before beginning the review of the literature. There is considerable overlap between the terms "concrete" and "grounded" in the research on external knowledge representations, but they are not synonymous. It is important to distinguish these two notions and to understand that a concrete representation is not necessarily grounded, and a grounded representation is not necessarily concrete. A concrete external knowledge representation refers to a physical, movable representation, such as manipulatives used in the classroom. In contrast, a grounded representation refers to the internal state of understanding of the person using that representation. For example, to illustrate this important distinction, a physical representation of money, with all its perceptual detail, and a physical representation of bland money (dollar bills with no perceptual details) are both concrete representations (as used in McNeil et al., 2009). However, the representation of the actual money would be considered grounded compared to the bland manipulative because it activates prior knowledge of dollar bills. On the other hand, a diagram of an insect with realistic depictions of its physical features, would be considered a grounded representation but not a concrete one, as the representation is static and cannot be physically manipulated in the same ways as blocks or dollar bills.

Potential Effects of Grounded and Idealized Representations on Learning and Transfer

Because children use different representations in mathematics as they progress through school, and because teachers use them daily in their lessons, representations are important to study and understand. For the purpose of this study, I will look at the effects of different types of external knowledge representations on learning and transfer. Specifically, I will be investigating two different components of learning, namely (a) learning procedures, and (b) learning concepts. I will also investigate the effects of different representations on both near and far transfer. As shall be made clear in the following literature review, research on the effects of grounded and idealized representations on learning and transfer have uncovered contradictory findings.

In relation to learning, the literature would suggest that both grounded and idealized representations have their advantages and disadvantages depending on the outcome measure. For example, McNeil et al. (2009) asked fifth-grade students to solve mathematical problems using concrete objects that represented money. The researchers found that students using grounded representations, that is, concrete objects that represented money with details such as color and decorative features, made more errors than the students using the idealized representations, which were bland dollar bills. Yet, when comparing the types of errors committed by the groups that used the grounded representations and idealized representations, the authors found that the errors committed by the participants using grounded representation were procedural mistakes and not conceptual in nature. This suggests that the grounded representations helped the students learn the targeted concept, but that the activation of the prior knowledge, through the perceptual features on the dollar bills, hindered their accuracy when solving the problems.

Contradicting these findings, Harp and Mayer (1998) reported that grounding science lessons by providing details relevant to the lesson to make it meaningful to students did not help them understand the underlying structure, or concept, targeted by the instruction. Rather, the participants got lost in the aspects of the lesson that were intended to activate their prior knowledge. Thus, in contrast to McNeil et al. (2009) study, Harp and Mayer (1998) found no advantages to using grounded representations in a learning context. In McNeil's et al. (2009) study, the grounded representation and context helped the students learn the underlying concept that was being assessed, whereas in Harp and Mayer's (1998) study, the groundedness of the lesson was not conducive to the students' learning. Harp and Mayer (1998) speculate that the grounded context distracted the students from focusing on the concepts and prevented them from learning. Together, this means that it is still unclear which type of representation is most beneficial for learning purposes in the classroom.

Other studies have focused not on learning, but rather on transfer. When assessing students' transfer to novel concepts, some studies have shown that they benefit from idealized representations (Kaminski, Sloutsky, & Heckler, 2008; Kaminski et al., 2013). In these studies, participants were required to learn a commutative mathematical group of order three (i.e., a mathematical rule based on division with remainders) using either grounded or idealized representations. The participants were engaged in instructional activities that involved either idealized representations (e.g., shapes) or grounded representations (e.g., pictures that depicted real world objects, such as pitchers). The transfer measure required the participants to apply the same commutative mathematical group of order three with a new representation. The representations used on the transfer task were also idealized.

The authors found that the participants in the idealized condition outperformed their counterparts using the grounded representation. This could show that when fewer perceptual details were provided through idealized representations, the participants were able to transfer the concepts to a structurally similar problem. Furthermore, the authors concluded that when concepts are taught with the use of grounded representations, transfer is less likely to happen than through idealized representations because the results seemed to indicate that grounded symbols limit students' ability to generalize their knowledge to different situations (Kaminski et al., 2013). Again, they suggested that students would learn the mathematical rule with any representation, but that grounded representations do not support the transfer.

At the same time, Kaminski, Sloutsky, and Heckler (2013) suggested that these findings have important educational implications for teaching mathematics, namely that idealized representations should be prioritized in the teaching and instruction process. Unfortunately, these interpretations and conclusions are overgeneralized (Jones, 2009). Considering the methodology of the Kaminski et al. (2008; 2013) experiments, the students in the idealized representation group were also given an idealized transfer task. Essentially, the students in the idealized group could have learned the pattern procedurally, but the students in the grounded representation group learned the structure of the problem more conceptually. Students in the grounded group may have learned the necessary knowledge to apply the conceptual structure to the transfer task, but were unable to do so because the underlying structure was not present in the transfer task in the same way they had learned it. This could explain some of the results, which is important to consider bearing in mind the educational claims made by Kaminski et al. (2013) (Jones, 2009).

In a similar study that evaluated the effect of context and representation on transfer, Moreno, Ozogul, and Reisslein (2011) examined the effects of different combinations of representations on students' learning new science concepts related to electrical circuits. In their study, four different combinations of context and representations were studied. In a first phase, the researchers provided students with some instruction that used either a grounded or idealized context to frame the instruction. Following the instructional intervention, the students were asked to solve some problems that incorporated either grounded or idealized representations. The design of the study involved four experimental groups: grounded instruction + problem solving using grounded representations, grounded instruction + problem solving using idealized representations, idealized instruction + problem solving using grounded representations, and idealized instruction + problem solving using idealized representations.

Moreno et al. (2011) found that students who received the grounded instructional intervention and solved the problems with an idealized representation outperformed their peers on a transfer task in the other groups. These findings contradict the recommendation of Harp and Mayer (1998) who concluded that grounded contexts are not advantageous and could confuse or distract students. However, the difference between the two studies relies on what was being assessed. In one case, Harp and Mayer's (1998) study assessed learning, while Moreno's et al. (2011) study assessed the ability to transfer. This could suggest that the effectiveness of the representation depends whether learning or transfer is assessed.

Reflecting on the aforementioned research studies, and concluded by Belenky and Schalk (2014), children benefit the most from grounded representations, free of irrelevant details, when immediate learning (procedural and conceptual) is assessed. While some groundedness is important for children to make sense of the context and problem at hand, there seems to be a limit to the benefits of grounded representations on student performance, particularly when transfer is being assessed. Idealized representations are, perhaps, better for children's deep learning, or structural learning, of the target concepts. However, grounded representations do not hold up as well when they require some near or far transfer, or when children have substantial prior knowledge of the concepts. This is why examining children's prior knowledge of the concept is important.

Prior Knowledge

Very little is known about the effects of prior knowledge on students' learning and transfer with manipulatives and representations. Goldstone and Sakomoto (2003) examined the potential effects of different representations when learning with simulations of complex adaptive system involving ants searching for and managing food supplies. The authors found that participants who had lower prior knowledge of an initial practice simulation performed better on the transfer task, which involved a new simulation that followed the same abstract concept, if they used the idealized representation compared to the grounded representation. No effects of representation type were observed for the participants who had high prior knowledge of the initial simulation. The authors explain the result by saying that the participants in the grounded condition seemed to be adding extra factors and conditions to the rules (e.g., ants enjoyed sharing food, they scared other ants away) before answering the questions on the transfer tasks, whereas the participants in the idealized condition seemed to have understood the abstract principles and concepts that were taught and were able to apply these to analogous situations.

While the participants with lower prior knowledge in the Goldstone and Sakomoto (2003) study benefited more from the idealized representations on a transfer task, Petersen and McNeil (2013) found the opposite in a context where learning was being assessed. In a study with preschoolers, the authors examined the effects of prior knowledge (low and high) and perceptual richness of manipulatives on participants' counting abilities. Petersen and McNeil (2013) found that the children's prior "established" knowledge of what the manipulatives represented (e.g., giraffes, elephants) along with the type of representation (grounded or idealized) influenced students' performance. Petersen and McNeil (2013) assumed prior knowledge and changed the manipulatives in their experiment. The grounded representation helped the students with low prior knowledge of the manipulatives and hindered the students who had high prior knowledge of the manipulatives. The authors advance that a possible reason for this could be because the children's prior knowledge was activated for what the original purpose of the objects and not for the targeted use in the experiment. Regardless of prior knowledge level, performance with the idealized manipulatives was the same.

Other studies were not focused on the differences between grounded and idealized manipulatives, but nevertheless investigated the effect of prior knowledge on children's learning and transfer with manipulatives. Osana and Pitsolantis (2016) examined kindergarteners' ability to learn and transfer base-ten concepts that were taught with manipulatives (base-ten blocks) and investigated possible moderating effects of prior knowledge on numeracy. The authors found that children in both low and high prior knowledge groups were able to learn and use the manipulatives with meaning during the lessons, but the children in the low prior knowledge group had difficulties on transfer tasks. Similarly, in a study on the relation between prior knowledge and manipulative physicality, Lee and Chen (2014) conducted a quasi-experimental study examining the differences between prior knowledge (low/high) and physicality of the manipulatives (virtual vs. physical). The authors found that students with higher prior knowledge outperformed their peers with lower prior knowledge on the learning measures. Additionally, the students with high prior knowledge outperformed their peers in the virtual manipulative condition, compared to the students in the physical manipulative condition.

These studies are important as they suggest that the relation between prior knowledge of a concept and the ability to use different manipulatives affects performance, as well as the ability to transfer targeted concepts to other situations. While there are not many studies that examine the interplay between prior knowledge and representations for learning and transfer, these conclusions suggest that prior knowledge may moderate the effects of the representation type.

Proportional and Non-Proportional representations

Reys, Lindquist, Lambdin, and Smith (2014) explained that children can learn concepts of place value using a proportional or non-proportional representation (Reys et al. used the term "model" and not "representation"). The idea behind the proportional model is that the representation for the quantity 10 is represented by something that is physically 10 times bigger than the representation for the quantity one, and the representation for the quantity 100 is represented by something 10 times bigger than the object representing the quantity 10. Some examples of proportional models that represent the place value system would include base-ten blocks, cubes, and longs. Alternatively, a non-proportional model would be a model where the relationship between the physical sizes of the denominations does not reflect the relative size of the quantities represented. The money system would be an example of a non-proportional model. The size of the coin does not represent the value it holds; a dollar coin is not ten times larger than a dime, for example. Counters (colored chips) and the abacus are other examples of non-proportional models.

Unfortunately, there is no systematic research conducted on the relative effects of proportional and non-proportional models, despite the fact that both are used extensively in mathematics instruction worldwide. I am only aware of one study, conducted by Tracy and Fanelli (2000), that investigated these potential differences. In their quasi-experimental study, the authors evaluated first- and second-graders' ability to learn money concepts with proportional and non-proportional models of money. The authors found that students had difficulties learning the quantitative meaning of the coins in the non-proportional model, which could suggest that there are added difficulties with this type of model. The authors did not speculate on the potential difficulties of this model.

According to Belenky and Schalk's (2014) definition of groundedness, a grounded representation activates some aspect of prior knowledge and there is something inherent in the representation that is actually perceptually meaningful. On the other hand, an idealized representation is more arbitrary and needs to be learned. I argue that this definition is appropriate for the categorization of proportional and non-proportional representations. In this case, the proportional representation would be considered grounded, as it translates more readily to the quantitative referents in the base-ten system; the activation of prior knowledge stems from the physical features of the model. For example, when a child sees that a ten is ten times bigger than a one, and that a hundred is ten times bigger than a ten, the concept of the base-ten conceptual structure may be perceptually more evident. In contrast, a non-proportional model would be considered and idealized representation. The values of the objects in a non-proportional model are arbitrary, and would need to be taught as there is nothing inherent in the color to indicate the objects' quantitative meanings.

As a whole, the literature on external knowledge representations and their effects on learning and transfer reviewed above suggests that different representations have different effects depending on the outcome measure (Kaminski et al., 2013; McNeil et al., 2009; Moreno et al. etc.). The sparse literature investigating the role of prior knowledge also suggests that what children already know and understand affects learning and transfer with manipulatives (Osana &

Pitsolantis, 2016; Petersen & McNeil, 2013). Thus, in the context of studying the effects of representation on learning and transfer, it is important to consider children's prior knowledge of the concepts and materials for instructional purposes.

Present Study

I have described the literature on external knowledge representations, and on how children tend to interpret these different types of representations (e.g., Belenky & Schalk, 2014; Braithwaite & Goldstone, 2013). I have also discussed the potential effects of prior knowledge on understanding and using external knowledge representations (e.g., Goldstone & Sakomoto, 2013; Petersen & McNeil, 2013). Yet, the literature still remains unclear as to the optimal type of concrete material when learning with manipulatives and the role of prior knowledge in this process.

This study will explore differences in children's understanding of grounded and idealized representations in the context of a base-four numeration system. The proposed research is part of a larger project on the affordances of manipulatives and children's learning numeration concepts. In the larger study, Osana, Blondin, Alibali and Donovan (2017) examined three aspects of the manipulatives: (a) perceptual richness, (b) detachability of the manipulatives, and (c) proportionality of the model. The goal of the larger study was to determine which type of representation is most beneficial for learning and transfer in contexts that differed according to the manipulatives' physical features.

In my research, I examined the effects of the manipulatives' physical proportionality. My objective was to examine the relative effects of proportional and non-proportional models on children's learning of the base-four numeration structure, and to test whether prior knowledge moderated these effects. Second grade students were randomly assigned to two different groups (proportional and non-proportional). In the first phase, all the participants were assessed on their prior knowledge of numeracy and place value understanding in base-ten. In the second phase, the participants received an instructional intervention that introduced the students to the base-four numeration system with manipulatives. One group used a proportional model made with attached blocks, and the other group used chips, where the color of the manipulative was associated with a specific quantity. The literature would suggest that these two conditions would perform differently based on the outcome measures of learning and transfer (Harp & Mayer 1998; Kaminski et al., 2013; McNeil et al. 2009).

My research questions were the following:

1. Will manipulative type be a significant predictor of the students' performance on (a) learning procedures, (b) learning concepts, and (c) transfer?

2. Will prior knowledge of place value and numeracy be a significant predictor of students' performance on (a) learning procedures, (b) learning concepts, and (c) transfer?

3. Will there be an interaction between prior knowledge of numeracy and manipulative type on (a) learning procedures, (b) learning concepts, and (c) transfer?

For the first research question, I predicted that condition would account for a significant proportion of the variance in all dependent variables, but in different ways. Specifically, I predicted that the participants in the non-proportional condition (i.e., chips) would score higher than the participants in the proportional condition (i.e., base-four blocks) for the learning procedures measures. For the learning concepts measure, I predicted the opposite: students in the proportional condition would score significantly higher than those in the non-proportional condition. Both predictions aligned with McNeil et al.'s (2009) research, who found that students who used idealized representations (i.e., chips in the present study) scored higher on the accuracy of procedures (i.e., learning procedures measure) and participants using grounded representations (i.e., base-four blocks in the present study) learned the intended mathematical concepts better (i.e. learning concepts measure) than their counterparts. Finally, I predicted that the students in the non-proportional model would score significantly higher on the transfer measures than the students using the proportional model. These predictions aligned with Kaminski et al. (2013) and Moreno et al. (2011) findings where students performed higher with idealized representations on transfer tasks.

For the second research question, I predicted that the prior knowledge would account for a significant predictor for all measures, namely learning procedures, learning concepts, and transfer. Participants with higher prior knowledge scores would outperform the participants with lower prior knowledge scores. I predicted this because of Osana and Pitsolantis (2016) study where children in kindergarten with higher prior knowledge outperformed their peers with lower prior knowledge on learning and transfer measures.

To address the third research question, I predicted a significant interaction between prior knowledge and condition. There is so little consistent literature on the relationship between external knowledge representation and prior knowledge that it was difficult to make a definitive prediction on the specific nature of the interaction. However, I predicted that for the learning measures, participants with higher prior knowledge would score the same across conditions. Participants in the proportional (grounded) condition would outperform their peers in the non-proportional (idealized) condition, if the participants scored lower on the prior knowledge measures. These predictions aligned with Petersen and McNeil's (2009) findings where grounded representations help the participants' performance. On the other hand, I predicted that for the transfer measures, participants with higher prior knowledge would score the same across conditions. Participants in the chips condition would outperform their peers in the blocks condition, if the participants scored lower on the prior knowledge measures. These predictions aligned with Goldstone and Sakomoto's (2003) findings where participants benefitted from idealized representations on the transfer measures.

The results of this study promise to contribute to the literature on external representations and specifically, on the relationship between representations and their affordances for learning and transfer in mathematics. Previous studies typically focus on only one outcome measure of learning or transfer, and few consider the role of prior knowledge. For practitioners, the results of this study could provide insight on their teaching practices to best support their students depending on the desired outcome, whether it would be learning procedures, learning concepts, or transfer purposes.

Chapter 3: Method

As part of a larger study, second-grade students were asked to solve a series of mathematical problems with concrete materials. The objective of the research was to evaluate children's learning and transfer as a function of the type of manipulative they used during an instructional intervention and their prior knowledge of base-ten numeration. Children were given manipulatives to construct orders for a toy company preparing shipments of different sizes. The shipments were packaged in base-four denominations and the children were asked to construct sets of toys in ways that involved regrouping.

Participants and Context

The participants of this study were 56 second-grade students from 12 classrooms in seven different elementary schools in the Montreal area. The schools were located in two different public school districts and two private schools in the suburb of a large metropolitan area in Canada. The Ministère de l'éducation et de l'enseignment supérieur du Québec (MEES) publishes an income index for all public schools in the province. This index represents the proportion of families in each school with an income at or below the poverty line. For the first five public schools that participated in the study, School 1 had an index of 20.49; School 2, School 3, and School 4 had an index of 30.21, 11.68, and 20.76, respectively. School 5, the only school in the second participating school district, had an index of 12.33 (Ministère de l'éducation et de l'enseignement supérieur du Québec, 2017). The data for the two private schools were not available.

Design

The design of the present study was a two-group experimental design. Figure 1 presents the two phases of the study. Across all classrooms, there were 28 students in each condition. The students were randomly assigned to each condition within each classroom. The researcher met with each child individually twice. During the first meeting, two prior knowledge measures were administered. During the second meeting, the researcher delivered an instructional intervention during which the children learned about the toy factory and helped the researcher construct some of the orders. Immediately after the intervention, the researcher administered the Learning Procedures (LP) measure, the Learning Concepts (LC) measure, the Learning Double Regrouping (LR), the Transfer Comparison (TC), and the Transfer New Denomination (TD). All the activities and measures were administered in the same order for every participant.

I randomly assigned the children to conditions that differed with respect to the type of concrete representation (i.e., blocks or chips) provided. In particular, the manipulatives differed with respect to their groundedness: the degree to which they were "instantiated" in some real-world context (Belenky & Schalk, 2014), operationalized in this study as the extent to which they resemble the base-four numeration structure described in the toy factory context.

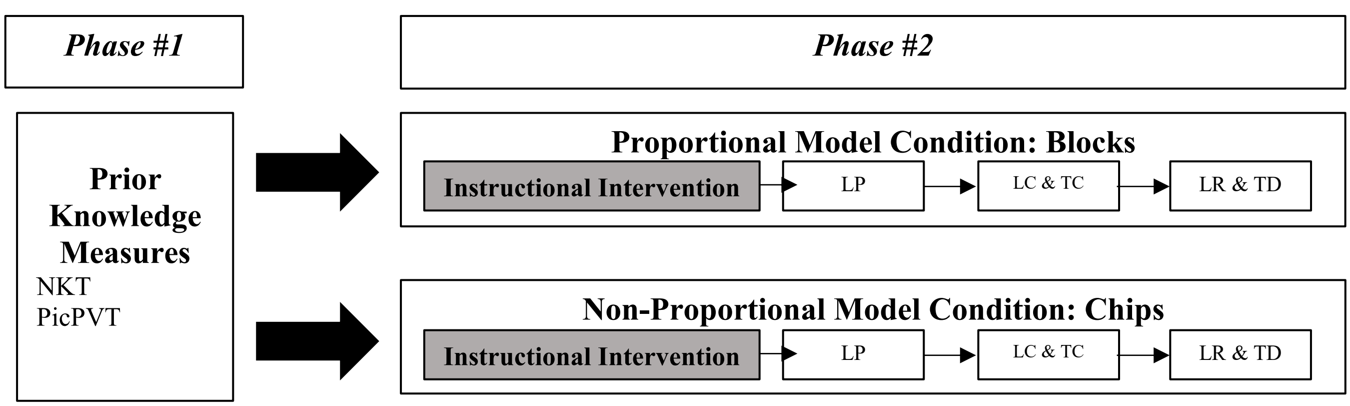

Figure 1. Study design.

In the proportional condition, the researcher provided the children with wooden cubes to represent the teddy bears. The cubes were in the form of units (single blocks), groups of four blocks glued together, and groups of sixteen blocks glued together. In the non-proportional condition, the researcher presented the children with chips of three different colors, with each color representing a different denomination (blue for 1, red for 4, and green for 16). In both the proportional and non-proportional conditions, the researcher showed a large stuffed teddy bear to the children and told them that the manipulatives would be used when preparing the orders because the actual toy was too cumbersome to manoeuvre. This indicated to the children that the objects used to construct the orders are representations of the actual toys in the factory.

After the intervention, the students were assessed on their accuracy in preparing the shipments (Learning Procedures), their learning of the base-four groupings used in the problems (Learning Concepts), extend their knowledge on procedures (Learning Double Regrouping) and their ability to apply the conceptual base-four structure to two transfer tasks (Transfer Comparison and Transfer New Denomination).

Toy Company Shipment Instructional Intervention

The protocol for the instructional intervention is presented in Appendix A. The participants were introduced to FunToyz, a company that manufactures and ships teddy bears. They were asked to help the researcher configure the orders to send out to different fictitious schools. To contextualize the conceptual structure of the base-four system, the participants were introduced to two shipping rules: (a) four toys fit in a bag and four bags fits in a box, and (b) bags and boxes cannot be shipped unless they are full, so as soon as there is enough to fill a box or a bag, it needs to be filled. As the researcher explained these rules, she took one of the manipulatives (either a single, a bag, or a box) and placed it on the correct space on the legend, which is presented in Figure 2. The legend consisted of three distinct pieces of white cardboard that had a small image of the teddy bear, a bag, or a box at the bottom of the cardboard. The legend stayed in front of the child as a reference. For example, during the instruction, when the researcher says, “this is a teddy bear,” she took either one wooden cube or a blue chip, depending on the condition, and placed it on the cardboard that has the image of the teddy bear.

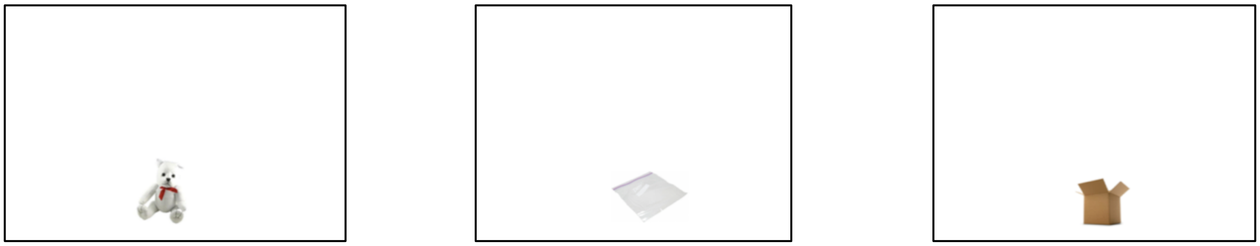

Figure 2. Legends for the teddy bears, bags, and boxes.

The researcher provided five demonstrations on how to prepare the teddy bear shipment packages. The first demonstration was the most complete; it included all the steps necessary for creating a shipment and included both regrouping bears to bags, and bags to boxes. The researcher demonstrated both the rules and all possible regroupings to the children during the first demonstration. This demonstration did not require any participation on the part of the participant. For example, the researcher took 22 teddy bears and prepared them for shipping, talking out loud for the child to hear the regrouping components of the demonstration (i.e., “Do I have four teddy bears? Yes! So, I need to put them in a bag.”). Then, the researcher regrouped four teddy bears, put them in a bag, and repeated the process. The objective of this demonstration was for the child to see how to apply both shipping rules mentioned above. At the end of the first demonstration, the researcher asked the child to answer questions about the rules (e.g., “How many bears fit in a bag?”), and repeated the rules if the child did not answer correctly.

The second demonstration, also done entirely by the researcher, had the objective of showing that one does not always need to regroup; at times the shipment needs no configuration and is already ready to send. The researcher began by saying, “Let’s say I need to ship 1 box, 2 bags, and 2 extra teddy bears. What would my order look like?”. In this demonstration, the researcher again went through the process of verifying if the order is complete (e.g., “Do I have more than 4 teddy bears? No! So, I leave it like that.”) and left the order as is.

The objective of the third demonstration was to re-explain the rule of regrouping bears to bags. The researcher started with four teddy bears and went through the rules. She said, “Do I have four bears? Yes! So, I need to put them in a bag. Do I have four bags? No! So, I leave them like this.”

The fourth demonstration solicited the child’s participation. The objective of this demonstration was for the child to re-explain how to regroup bears to bags. The researcher asked the question: “Let’s say I need to ship 5 teddy bears. What would my order look like? Remember to use the packing rules!” Then, the child, him or herself, prepared the shipment and answered the researcher’s question. The researcher provided corrective feedback to the child, and either gestured to reinforce the rationale for regrouping or modeled the action should the child have difficulty.

The fifth demonstration also solicited the child’s participation. The objective of this demonstration was for the child to re-explain the rule of regrouping bags to boxes. The researcher asked, “Now I need to ship 1 box, 2 bags, 1 extra teddy bear, and 3 more bags. What would my order look like?”. Again, the researcher provided corrective feedback to child using the same procedures as in the fourth demonstration.

Measures

The children were evaluated on two prior knowledge measures: the Number Knowledge Test (NKT) and the Picture Place Value Task (PicPVT). These measures were administered in the first interview. The prior knowledge measures and accompanying protocols can be found in Appendix B. Once the instructional intervention is completed in the second meeting with the participants, the researcher administered the outcome measures, which are presented in Appendix D. The first meeting with the students was not recorded. The scoring sheet used during the administration of the prior knowledge tests during the first interview can be found in Appendix E. The CPV measure was not part of this study. The second meeting with the students was videotaped for subsequent analysis. The participants' responses during the second interview were recorded on a scoring sheet completed by the researcher during the meeting and verified through videotape analysis. The scoring sheet for the second meeting can be found in Appendix F.

Prior knowledge measures. The NKT (Okamoto & Case, 1996) was developed to evaluate children's intuitive knowledge of numbers. The test places children's understanding at one of four different levels (Level 0, 1, 2, and 3), with each level corresponding to a specific age (4, 6, 8, and 10 years old). The NKT is administered individually and participants answer the questions orally. The researcher does not prompt the participant to justify any responses.

Administration continues from Level 0 until the child cannot answer the minimum number of questions correctly for a given level. For Level 0, a minimum of three correct answers are required to continue to the next level, and at each of Levels 1 and 2, five correct answers are required.

There are 5 items at Level 0. On three items, 1 point is assigned for a correct response and 0 points for an incorrect response. The other two items have two parts (a and b), both of which need to be answered correctly for 1 point to be assigned. No points are awarded if one or two incorrect responses are provided on two-part items. A maximum of 5 points can be awarded for Level 0. Level 1 and Level 2 each have 9 items, for a maximum of 9 points. At Level 3, there are 7 items for a maximum of 7 points. The procedure for scoring two-part items is the same for all levels. A total NKT raw score is computed by summing the number of points obtained, with a minimum score of 0 and a maximum score of 30.

The PicPVT is a measure of place value knowledge designed specifically for the present study (Osana & Blondin, 2017). The test was based on a place value measure created by Kamawar et al. (2010) in which participants were asked to assess if the value of the underlined digit in a numeral was the same as a picture of a collection of base-ten blocks. For example, the participant would be shown 6$\underline{2}$8 and would be asked whether the picture (e.g., two single cubes) matches the underlined part of the numeral.

The items on the PicPVT are similar to those on the Kamawar task, but I modified the visual representations used for the quantities to reduce the likelihood of the participants' prior exposure to base-10 blocks, which could confound the results. A sample item is shown in Figure 3. Instead of pictures of base-ten blocks, the representations are hexagons that are placed in groups of ones, tens, or hundreds. Ten single hexagons are grouped together to represent a ten and 100 single hexagons are placed in a group to represent a hundred, but unlike base-ten blocks, the hexagons are not connected when grouped into tens or hundreds. To maintain the physical proportionality across the numeration system, the size of the individual hexagons is the same throughout the task and the spacing between hexagons and their groupings also remain constant.

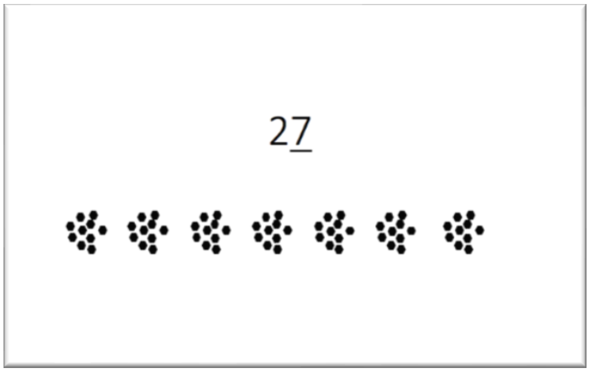

Figure 3. Sample item of the PicPVT for which the correct answer is "no."

To administer the PicPVT, the researcher shows the participant an index card, such as the one shown in Figure 3, and asks, "Does the picture match the underlined part of the number?" The participant answers yes or no. The PicPVT consists of 20 items, 9 of which have correct corresponding representations and 11 have incorrect representations. On 6 items, the units will be underlined (with three correct and three incorrect corresponding hexagonal representations), 7 items will have the tens underlined (with three correct and four incorrect representations), and 7 items will have the hundreds underlined (with three correct and four incorrect representations). There are 7 items with two-digit numbers and 13 items with three-digit numbers. Each correct answer will be assigned 1 point and each incorrect answer 0 points. The total PicPVT score will range from 0 to 20 and all items can be found in Appendix C.

Outcome measures.

Learning Procedures. Immediately after the instructional intervention in the same interview session, the participants were asked to complete the Learning Procedures (LP) measure. This objective of this measures was to test children's ability to procedurally recreate the shipping procedures taught during the instructional intervention. The measure included four items, each of which required the participants to prepare an order using the manipulatives. See Figure 4 for the list of the four items. Two of these tasks assessed their ability to regroup from bears to bags (e.g., Let's say I need to ship 6 teddy bears. What would my order look like?), and two tasks assessed the participants' ability to regroup from bags to boxes (e.g., Now, I need to ship 1 box, 3 bags, 1 extra teddy bear, and 3 more bags. What would my order look like?).

Once the child completed the shipment packages with the manipulatives, the researcher asked a series of *verification questions*, presented in Figure 5. The objective of the verification questions was for the participant to verbalize his or her final answer on each item. The verification questions were asked in order, and the researcher wrote down the participants' answers in the "first attempt" column on the scoring sheet.

Learning for Procedural Accuracy Items:

1. Let's say I need to ship 6 teddy bears. What would my order look like?
2. Now, I need to ship 1 box, 3 bags, 1 extra teddy bear and 3 more bags. What would my order look like?
3. Let's say I need to ship 2 boxes, 2 extra teddy bears, 1 box and 3 more teddy bears. What would my order look like?
4. Now, I need to ship 2 boxes, 3 bags, 2 extra teddy bears and 2 more bags. What would my order look like?

Figure 4. Learning Procedures Items.

Verification Questions:

A. Are there any boxes? *If yes:* Show me where they are? *Let the child answer*. How many are there?

B. Are there any bags? *If yes:* Show me where they are? *Let the child answer*. How many are there?

C. Are there any extras? *If yes:* Show me where they are? *Let the child answer*. How many are there?

Figure 5. Verification Questions.

After the verification questions were asked, and if the participant regrouped according to the shipping rules as demonstrated in the instructional intervention, the researcher moved to the next item. If the participant did not regroup according to the shipping rules, the researcher asked a *check question,* which was: “Check! Did you use the packing rules?” If the child said yes, the researcher moved on to the next item. If the child said no, or proceeded to regroup according to the shipping rules, the researcher asked the *verification questions* once more, this time recording the participants’ answer in the “second attempt” column on the scoring sheet.

Each item on the LP measure was assigned two points if the child created a correct shipment order on the first attempt, one point for a correct order on the second attempt, and zero points if the child did not get the correct order after two attempts. The total LP score will be summed for a maximum of eight points.

Learning Concepts and Transfer Comparison. The Learning Concepts (LC) and Transfer Comparison (TC) measures were administered immediately after the LP. For these two measures, the researcher took a scarf and covered the manipulatives in front of the child. The child was only provided with a paper and marker for these measures.

This section of the testing was comprised of eight items: two recall questions, three LC questions, and three TC questions. The first two items administered were recall questions, and then the next six items alternated between learning concepts and transfer-comparison items. All items can be found in Figure 6. This section of the protocol is designed to assess the participants' knowledge of the size of the first three denominations of the base-four system and regrouping processes in base-four. All items were couched in the context of the toy factory.

<u>Recall Items:</u>

1. How many bears fit in a bag?
2. How many bags fit in a box?

<u>Learning Concepts (LC) and Transfer-Comparison (TC) Items:</u>

3. If you have eight teddy bears, what would the order look like? (LC)
4. Which order has more teddy bears? 2 bags or 1 box? (TC)
5. Let's say I have 9 teddy bears. What would the order look like? (LC)
6. Which order has more teddy bears? 2 boxes or 3 bags? (TC)
7. Let's say I have 2 bags and 4 extra teddy bears. What would the order look like? (LC)
8. Which order has more teddy bears? 3 extra teddy bears, or 1 bag? (TC)

Figure 6. Recall, Learning Concepts and Transfer Comparison Items.

The two recall questions were, "How many bears fit in a bag?" and "How many bags fit in a box?" These first two items tested the participants' memory of the size of each denomination (i.e., 4 bears, 4 bags). For these questions, the researcher corrected the participant if an answer was incorrect. These questions were scored one point if correct, and zero points if incorrect.

The three LC items (items 3, 5, and 7) involved three bears to bags regroupings. These questions were administered and scored in the same way as the learning procedures items. The TC items (items 4, 6, and 8) asked the participant to compare two orders, and decided which one had more teddy bears. The goal of these items was to see if children understood the quantity of teddy bears in bags and boxes. The participants were asked to justify their answers to each TC item. A correct answer was awarded one point and an incorrect answer zero points.

Learning Double Regrouping and Transfer New Denomination. The participants were asked to solve two additional learning items and one transfer item that aimed at evaluating their understanding of grouping patterns in place value numeration systems. The three items are shown in Figure 7. The researcher removed the scarf from the manipulatives and told the participants that they would use the objects for the questions that followed.

Still in the context of the toy factory, the first two problems involved double regrouping and together constituted the Learning Double Regrouping (LR) measure. For these items, the child needed to regroup both the bears and the bags to find the correct answer (e.g., Now, I need to ship 4 bags and 5 teddy bears. What would my order look like?). These items are considered double regrouping because both regroupings, four bears to one bag and four bags to one box, are needed to solve each of these items. For example, for the first item, the child would need to regroup the five teddy bears into one bag and one teddy bear. Then, the child would need to regroup the five bags into one box and one bag. This would result in a final shipment order of one box, one bag, and one extra. These two items were administered and scored in the same way as the learning procedures measure, with a minimum total score of 0 points and a maximum of four points.

The last transfer task, the transfer denomination (TD) task, required the participants to create the additional denomination of 64, which did not appear in any of the shipping problems during the intervention. As such, this question assessed transfer of base-four place value concepts. The item is, “Let’s say I need to ship 5 boxes and 2 bags of teddy bears What would the order look like?” The problem required regrouping 5 boxes into a group of 64, requiring the participant to think of a way of packaging a group of 4 boxes, or 64 teddy bears. The researcher prompted the participant to elaborate his or her answer. Examples of prompts for the child to think about regrouping 4 groups of 16 into one package of 64 include, “What would we do now? What would they go in? Why did you think that?” For the extra denomination question, any response that indicated an understanding of the required regrouping, such as, “Maybe we could put these 4 boxes into an even bigger box,” was awarded 1 point. Responses that did not show regrouping understanding, such as, “I would just send out 5 boxes,” was awarded 0 points.

<u>Learning Regrouping Items:</u>

1. Now I need to ship 4 bags and 5 teddy bears. What would my order look like?
2. Let’s say I need to ship 2 bags and 10 teddy bears. What would my order look like?

<u>Transfer Denomination Item:</u>

3. Let’s say I need to ship 5 boxes and 2 bags of teddy bears. What would the order look like?

Figure 7. Learning Double Regrouping and Transfer New Denomination Items.

Materials

At the start of the instructional intervention, one mid-size stuffed teddy bear (12.5" in height, arm span of 11") was used to show the students, in both conditions, one of the toys manufactured by the toy factory. For the remainder of the instruction and during the administration of some of the measures (i.e., the LP, LR, TD), blocks were used in the proportional condition and chips in the non-proportional condition. The participants in the proportional condition were provided wooden cubes of a single color. Individual ¾" cubes represented the individual teddy bears, bags were represented by 4 cubes glued together in a 2 x 2 configuration, and boxes were represented by 16 cubes glued together in a 2 x 2 x 4 configuration (see Figure 8). I decided to glue the grouped cubes together rather than provide interlocking cubes to prevent the children from trying to separate the cubes when regrouping.

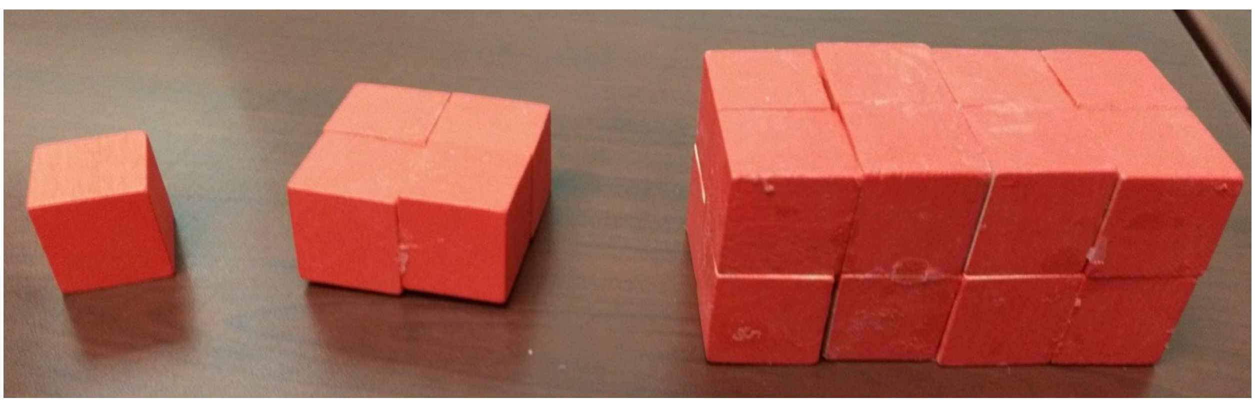

Figure 8. Wooden cubes.

The participants in the non-proportional condition used colored chips. This model used color to determine the value of each chip. In this study, blue chips will be worth 1, red chips will be worth 4, and green chips will be worth 16 (see Figure 9).

Figure 9. Colored chips.

Additionally, during the administration of the LP, LR, TD measures, the researcher provided the participants with a legend (see Figure 2) indicating the value of the manipulatives used for each denomination in each condition (e.g., blue chip = 1, single block = 1).

Chapter 4: Results

The objective of my study was to examine the relative effects of proportional and non-proportional models on children's learning of the base-four numeration structure. I also tested whether prior knowledge predicts how children learn with manipulatives, and whether prior knowledge moderates the effect of manipulative type.

In the following section, I will discuss the findings based on the statistical analyses. The data were grouped by condition across the 11 classrooms. Descriptive statistics and four multiple linear regressions, with Learning Procedures (LP), Learning Concepts (LC), Transfer Comparison (TC), and Learning Double Regrouping (LR) as separate outcome measures, are presented below. A logistic regression with the Transfer New Denomination as an outcome measure is also presented. As one of my objectives was to use prior knowledge scores as predictor variables in my model, I will also elaborate on a rationale for the selection process of the prior knowledge measures.

Participants that did not get a mean score of 1.0 on the Recall measure were removed from the sample; this was the case for four participants (two in each condition), resulting in a final sample size of 26 participants per condition. Because of administrative error on the new denomination measure, six participants had missing data (three in each condition). This resulted in 23 participants per condition in the analysis for this measure.

Prior Knowledge Measures

The prior knowledge measures consisted of the NKT and the PicPVT. My goal was to use both as predictor variables in my multiple regression analyses. Before using the percent scores of the PicPVT, however, I analyzed the data to see how to use this measure.

I initially looked at the scores related to the PicPVT measure in the following way. I separated the items that incorporated a correct representation (correct response was "Yes"; 9 items) from those that had an incorrect representation (correct response was "No"; 11 items). Next, for each of these categories, I separated the items by denomination. This resulted in a 2 x 3 repeated measures design, where the first dimension was correctness ("yes"/"no") and the second dimension was denomination (ones, tens, and hundreds). Crossing these two variables resulted in six different problem types: correct-ones, correct-tens, correct-hundreds, incorrect-ones, incorrect-tens, and incorrect-hundreds.

The mean scores for the correct representation was 0.97 (0.98 for the ones, 0.95 for the tens, and 0.98 for the hundreds). The mean scores for the incorrect representation was 0.75 (0.73 for the ones, 0.75 for the tens, 0.78 for the hundreds). Next, I ran a 2x3 repeated measures ANOVA to see if there were main effects of correctness and denomination and to test for a correctness x denomination interaction. There was a main effect of correctness, $F\,(1,51) = 16.11$, $p < 0.001$, but not of denomination, $F\,(2, 102) = 1.81$, $p = 0.17$. There were no interaction effects. Considering the ceiling effect on the correct representation items and that the mean performance on correct items was significantly higher than the performance on the incorrect items, I decided to drop the correct items from my analyses. This means that only the incorrect items on the PicPVT were included in my analyses.

The raw scores for the NKT were summed and converted to percent scores. Next, I looked at the correlation between the percent NKT score and the percent PicPVT score (including only the incorrect representation items). These two measures were significantly correlated ($r = 0.48$, $p < 0.001$). As the two measures were significantly correlated, I decided to use only the NKT score as my prior knowledge measure in subsequent analyses, because the correlation suggested that the two measures evaluated some similar aspects of prior knowledge. I made this decision bearing in mind that the NKT allowed me to capture participants' prior knowledge of numeracy and place value, whereas the PicPVT evaluated participants' knowledge of place value concepts, only.

Descriptive Statistics for Learning and Transfer Measures

Initially, I scored the participants' responses either on the LP, LC and LR measures as 2, 1, or 0, and 1 or 0 (correct or incorrect) on each item of the TC and TD measures. For the LP, LC and LR, the participants would be scored a 2 if they got the correct answer on the first try, a score of 1 if they got the correct answer after the "check question", and 0 if they got the incorrect answer after both attempts. Descriptive statistics revealed that most items were either scored a 2 or a 0. Specifically, on the LP, only 6% of items were scored a 1, on the LC only 5% and on the LR 3%. For this reason, I decided to revise the scoring rubric. I only considered the participants' first attempt, and scored their response with a score of 1 for correct items, and a 0 as incorrect.

The means and standard deviations for the Learning Procedures (LP), Learning Concepts (LC), Transfer Comparison (TC), Learning Double Regrouping (LR), and Transfer New Denomination (TD) measures by condition can be seen in the Table 1.

Table 1

Means and (Standard Deviations) for Learning and Transfer Measures by Condition

Condition	Learning Measures			Transfer Measures	
	Procedures	Concepts	Regrouping	Comparison	New Denom.
BL	.75 (.33)	.73 (.30)	.73 (.41)	.82 (.36)	.17 (.39)
N	26	26	26	26	23
CH	.85 (.25)	.64 (.40)	.67 (.40)	.60 (.44)	.39 (.50)
N	26	26	26	26	23

Note. All scores reported as proportions. “BL” represents the Base-Four condition. “CH” represents the Chips condition.

The Effects of Concrete Representations and Prior Knowledge on Learning and Transfer

Learning Procedures. A multiple linear regression was conducted with LP as the criterion and the NKT, condition, and an interaction term as the three predictor variables. The multiple regression was conducted hierarchically, where the NKT centered scored predictor and the condition predictor were used in the first block, and the interaction predictor was used in the second block. The model significantly explained the variance in the LP scores, $F\ (2, 49) = 9.64$, $p < 0.001$, $R^2 = 0.28$. The NKT was a significant predictor ($\beta = 0.03$, $t(49) = 4.17$, $p < 0.001$) of LP scores, which means that the scores on the LP increased as prior knowledge increased. Condition was not a significant predictor ($\beta = 0.11$, $t(49) = 1.55$, $p = 0.13$). There were no significant interactive effects between condition and prior knowledge. Statistics reported from the NKT and Condition predictor are from the first block as there was no significant interaction.

Learning Concepts. A multiple linear regression was conducted with LC as the criterion and the NKT, condition, and an interaction term as the three predictor variables. The multiple regression was conducted hierarchically, where the NKT centered scored predictor and the condition predictor were used in the first block, and the interaction predictor was used in the second block. The model significantly explained the variance in the LC scores, F (2, 49) = 11.26, $p < 0.001$, $R^2 = 0.32$. The NKT was a significant predictor ($\beta = 0.04$, t (49) = 4.62, $p < 0.001$) of LC scores, which means that the scores on the LC increased as prior knowledge increased. Condition was not a significant predictor ($\beta = -0.07$, $t(49) = -0.88$, $p = 0.38$). There were no significant interactive effects between condition and prior knowledge. Statistics reported from the NKT and Condition predictor are from the first block as there was no significant interaction.

Transfer Comparison. A multiple linear regression was conducted with TC as the criterion and the NKT, condition, and an interaction term as the three predictor variables. The multiple regression was conducted hierarchically, where the NKT centered scored predictor and the condition predictor were used in the first block, and the interaction predictor was used in the second block. The model significantly explained the variance in the TC scores, F (2, 49) = 6.87, $p = 0.002$, $R^2 = 0.22$. The NKT was a significant predictor ($\beta = 0.03$, t (49) = 3.05, $p = 0.04$) of LCU score, meaning that the scores on the TC increased as prior knowledge increased. Condition was not a significant predictor ($\beta = -0.20$, $t(49) = -1.98$, $p = 0.05$). There were no significant interactive effects between condition and prior knowledge. Statistics reported from the NKT and Condition predictor are from the first block as there was no significant interaction.

Learning Double Regrouping. A multiple linear regression was conducted with LR as the criterion and the NKT, condition, and an interaction term as the three predictor variables. The multiple regression was conducted hierarchically, where the NKT centered scored predictor and the condition predictor were used in the first block, and the interaction predictor was used in the second block. The model could not significantly explain the variance in the LR scores, $F\ (2, 49) = 2.16$, $p = 0.126$, $R^2 = 0.08$.

Transfer New Denomination. A binary logistic regression was performed to evaluate the effects of prior knowledge, condition, and the interaction on the scores of the Transfer New Denomination measure. The logistic regression model was not significant $X^2\ (3) = 3.69$, $p = 0.297$. Nagelkerke R^2 explained 11.1% of the model. The variance in the scores for the new denomination were correctly classified in 73.9% of the cases.

Chapter 5: Discussion

The present study examined the relative effects of proportional and non-proportional models on children's learning of the base-four numeration structure. All participants completed the same prior knowledge measures in a first individual interview, and then received the same instructional intervention and completed the same five outcome measures in a second individual interview. The experimental manipulations entailed the use of different concrete materials during the intervention; one group completed the tasks with base-four blocks, and the other group had coloured chips. The first objective of the study was to determine if proportional and non-proportional concrete materials make a difference in the learning of the base-four numeration structure. The second objective was to test the effects of prior knowledge. Specifically, I tested whether prior knowledge predicts how children learn with manipulatives. The third objective was to see if there was an interaction between prior knowledge and the type of concrete material used when learning about base-four numeration.

In line with my predictions, prior knowledge accounted for a significant proportion of the variance in performance, but this was true only for the learning procedures, learning concepts, and transfer comparison tasks. In other words, children who scored higher on the prior knowledge tasks also scored significantly higher on these three measures. In the context of the double regrouping and new denomination items measures, prior knowledge did not account for a significant portion of the variance.

Previous studies have demonstrated that prior knowledge predicts children's learning in mathematics (e.g., Lee & Chen, 2014; Osana & Pitsolantis, 2016; Petersen & McNeil, 2013). In a recent study, Foster and Osana (2017) examined students' prior knowledge of fraction related concepts as a moderator of students' strategy use on equal sharing problems. The authors designed equal sharing problems that had contexts that ranged from grounded to idealized. For example, problems categorized as grounded involved sharing pizzas, and problems categorized as idealized involved sharing non-words items, such as “gloops” or “bamoes.” They found that students’ prior knowledge of fraction-related concepts indeed predicted their performance on the equal sharing problems. While the context of the Foster and Osana study is slightly different than the present study, the way the authors conceptualized prior knowledge as knowledge of the mathematically related concepts before testing, was similar to how I conceptualized prior knowledge in my study. In addition, their findings, that prior knowledge predicts performance scores supported mine.

Contrary to my predictions, however, condition was not a significant predictor in any of the outcome measures, and there were no interactions between condition and prior knowledge. Recent research by Osana, Blondin, Alibali, and Donovan (2017), however, uncovered different results relating to the effects of proportionality of manipulatives. Osana et al. examined the effects of proportionality of manipulatives with four different conditions. In addition to the chips (non-proportional) and the base-four blocks (proportional) conditions used in this study, the authors also examined differences between two other proportional models: quadriblocks and teddy bear counters. Quadriblocks are individual blocks that can be nested into a box of four and four boxes of four blocks can fit into a larger box of 16. Children in the teddy bear condition worked with perceptually rich (e.g., colorful) teddy bear counters that were grounded in the context of the intervention (FunToyz). Four small bears could be placed in a small bag and four bags could be placed into a box of 16.

Osana et al. (2017) found that students who worked with the non-proportional materials (chips) performed better on learning measures than students who worked with any one of the three proportional materials. I think the differences in my results and those of Osana et al. can be explained by the two additional conditions that were included in their study. In my study, the difference between the two conditions was not large enough to detect any differences. In contrast, when considering the mean scores of all three proportional conditions collapsed in a single "proportional group," as was the case in the Osana et al. (2017) study, proportionality made a difference relative to non-proportional conditions. As such, even though proportionality cannot be considered a significant predictor of performance scores in the present study, it does not mean proportionality in different concrete models should be dismissed in future research and educational contexts. In relation to transfer, Osana et al. (2017) did not find any effects of proportionality on their transfer measures; these findings support my findings as I did not uncover any condition effect on either transfer task.

In the present study, no interactions between prior knowledge and condition were found, possibly in part because condition was not a significant predictor on any of the outcome measures. In contrast, Petersen and McNeil (2013) had found an interaction between prior knowledge of manipulatives and the manipulatives used in their study. Because of the way I tested participants' prior knowledge of related concepts, I argue that my prior knowledge scores resulted in a restricted range (meaning there was not much variance in the NKT scores), which may have lowered the correlation between prior knowledge and the outcome. Petersen and McNeil (2013), on the other hand, selected knowledge levels at opposite ends of the knowledge spectrum (i.e., child has either prior exposure or not). This difference in classification and range of prior knowledge might explain why I did not find any interaction in my study. Additionally, the small sample size left me with low power in my statistical analyses; this could explain my lack of significant findings for condition and interactions. A larger sample size might have allowed me to conclude differently.

The present study will add to the research on teaching with manipulatives and place value in several ways, especially in terms of methodology. My study is one of the first to explicitly study the effects of proportional and non-proportional representations in a learning context using an experimental design. I carefully controlled all aspects of the experiment, including the structure of the intervention across and within both conditions. I also controlled for the use of manipulatives during the experimental manipulation and testing.

Many previous studies were not as methodologically rigorous as the present study. For example, Tracy and Fanelli (2000) examined children's learning of concepts related to money by using different concrete manipulatives, some proportional, others not. Their sampling methods were inadequate as they did not use random assignment to conditions (one whole classroom was used as control, and another class received the instruction), and the number of participants on the different measures varied and were small (e.g., eight participants in one condition and 25 in another one). Although the authors concluded that children had difficulties learning the value of the coins because they were not proportional, the authors did not conduct an experiment to draw such conclusions; rather, their conclusions reflected broad generalizations because their dependent variables were not adequately operationalized.

In addition, McNeil et al.'s (2009) study, where participants used bland or perceptually rich materials based on conditions to solve equations, also had confounds. The authors claimed to have randomly assigned participants to conditions, but the dependent variables were measured through the written work of the students thus preventing the authors from knowing if the participants actually used the manipulatives that were part of the experimental manipulation. In contrast, I individually interviewed the participants in order to make sure that they were using the materials during the intervention and the testing.

Furthermore, previous studies on external knowledge representations operationalized prior knowledge in ways that do not translate well to educational settings. For example, some that looked at prior knowledge in different contexts assumed the participants' level of knowledge. For example, Petersen and McNeil (2013), as discussed previously, had chosen the concrete materials about which the participants would have prior knowledge. The authors classified participants as having prior knowledge of animals and fruits that would be activated when using concrete representations of giraffes and apple counters. On the other hand, the authors assumed that children would have relatively low established knowledge of pompom and bland counter chips.

Other researchers determined participants' prior knowledge based on their responses from the experimental manipulation, which I argue is essentially a learning measure. An example of this would be Goldstone and Sakomoto's (2003) work on simulations and representations, where the authors looked at the effects of different representations used in computer simulations; although the authors conceptualized their measure as prior knowledge, I would argue that it is different from mine and should be conceptualized differently. When using a learning measure as a gauge of prior knowledge, the participants' conceptual, prerequisite knowledge, that they bring to study is not being measured adequately. There is a difference between what participants bring to the study and what is directly provided to them in an intervention.

In sum, the present study is one of the few that explicitly evaluated participants' prior conceptual competence in school mathematics and how that knowledge interacts with tools typically used in the elementary classroom. Thus, my results have considerably more ecological validity than many of the studies in the psychological literature. I demonstrated that prior knowledge allows for higher performance on target outcome measures and this adds to the slim literature on the relationship between prior knowledge and external knowledge representations (e.g., Foster & Osana, 2017; Lee & Chen, 2014; Osana & Pitsolantis, 2016). The findings remind researchers that prior knowledge does matter, and that it is important to test it and account for it, especially in educational contexts.

Although the idea of grounded and idealized representation has been previously studied (e.g., Braithwaite & Goldstone, 2013; Kaminski et al., 2008; McNeil et al., 2009), the present study contributes to the literature by looking at the affordances of grounded and idealized manipulatives in an applied educational mathematical context. Goldstone and Sakomoto's (2003), as well as Kaminski et al.'s (2008) research, was very narrow and not educationally authentic. This did not prevent Kaminski and her colleagues to imply that students are able to learn and transfer better with idealized representations. This statement created considerable turmoil in the field (Jones, 2009). I argue that Kaminski et al.'s (2008) research is not applicable to an educational context, because the experimental testing was decontextualized from authentic educational practice. Learning a commutative mathematical group of order three is not directly relevant to the learning and conceptual understanding of school mathematics. While their claims are partially in line with mine, I do not think that they have the same implications for teaching and learning mathematics.

Conclusions and Implications

There are several limitations to the present study. First, the study was conducted in school settings that were demographically similar. In particular, the participants came mostly from the same neighborhoods and socio-economic settings; the participant pool was homogenous and tended to lean towards higher socio-economic settings as described by the MEESQ (2017) school income index. This means that the results are not necessarily generalizable across the entire second-grade population as not all social groups were adequately represented in the sample.

Second, the interviews were conducted late in the school year. This could have influenced the distribution of the prior knowledge scores as well as the participants' performance on the outcome measures. An identical study with the same participants could have yielded different results if the data had been collected at an earlier time in the school year, when children's place value understanding is still developing.

Although my study is methodologically stronger than previous studies in the area, there are nevertheless some weaknesses related to methodology that should be addressed in future studies. For example, I believe that the learning concept items were not necessarily analogous to the learning procedures items, which was my initial goal. The learning concept items were modified during the piloting phase of the project, when the instruction in the intervention was much less explicit than in the present study. In the pilot, the original learning concepts items were too difficult for the participants, which required us to rewrite them to involve less regrouping. In the present study, however, because the participants' prior knowledge of simple number facts was better developed than that of the pilot participants (e.g., 9 bears is equal to 4 bears plus 4 bears plus 1 bag), they did not need to rely on the concepts of regrouping learned in the intervention to complete the items on the learning concepts measure; they were able to find a way around using the instructed concepts. In addition, future research should include items on bags-to-box regrouping, which was not the case in this protocol (the learning concept items contained only bears to bag regrouping items). Finally, in future studies, the far transfer measure (new denomination) would need to include more than one item. Having additional items for the far transfer measure might have given a better picture of the participants' understanding of the concepts at stake and their ability to transfer these notions to new situations.

In line with the educational contributions of the present study, several future directions would be worthwhile to investigate. Looking at this research from a developmental standpoint, meaning across different ages and different levels of development on the targeted concepts (e.g., Kindergarten through Grade 2), is needed to gain understanding on how developmental mechanisms interact with the use of different types of external knowledge representations. Perhaps examining the same phenomena across a wider age range would allow for more sensitivity to any potential differences between conditions.

Furthermore, there are important educational implications that emerged from this study. The first one is that prior knowledge of numeracy predicts how children learn with manipulatives. As mentioned before, this finding is in line with previous studies that examined the role of prior knowledge (Foster & Osana, 2017; Osana et al., 2017; Osana & Pitsolantis, 2016). This implication might seem like an obvious one, but it is important to remind teaching professionals that what children bring to the classroom does influence their practices and learning of new concepts. Although, there are various ways in which teaching basic numeracy concepts can be achieved, it is important to remember that the children's prior knowledge of these concepts helps them build new knowledge with solid foundations. Teachers cannot dismiss this information when teaching new concepts, nor when teaching with manipulatives and concrete objects.

While the proportionality of concrete materials did not predict participants' performance in the present study, a recent study by Osana et al. (2017) concluded that students did perform better with non-proportional materials on the learning measures. While it has long been thought that proportional models were ideal materials for learners of mathematics, Osana et al. (2017) report otherwise. The effect of proportionality is an important education implication and should be considered by teaching professionals. Additional research on the topic is needed to understand the nuances inherent in students' learning in these contexts.

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Appendix A

Instructional Intervention

Part A: Intervention

Materials:
- Materials for Condition X - Legend for Condition X
Set-up:
Place the Big Teddy Bear on the table. *The manipulative should be placed in front of the child, in the following order: singles, 4s, 16s.* *The legends should be in front of the child, "grouped" with the appropriate manipulatives.*
Intervention
A teddy bear factory named FunToyz makes stuffed teddy bears like this one (show teddy bear*)* and sends them to different schools. FunToyz prepares the orders to send out the teddy bears by using bags and boxes. The company doesn't like to waste, so they only use a bag or a box when it is completely filled. If they can't fill them up, they have extra leftover. The owner of FunToyz wants to know what the orders should look like, so that she can know how many boxes, bags and extras they will need. Because the teddy bears are too big, we're going to pretend these are teddy bears and we will use these *(point to appropriate representation)* to help us know how they should be packed. The packing rules works like this: 1. This is one teddy bear. *RA takes a 1 and puts it on the legend*. 2. When you have 4 teddy bears, you have to put it in a bag to make it full. Let me show you. *RA takes a 4 put it in a bag, and places the bag on the appropriate legend* 3. And then, when you have 4 bags, you have to put it in a box to make it full. Let me show you. *RA a 16, and places the box on the appropriate legend.* 4. Remember, as soon as you have enough to fill a bag or a box, it needs to be filled. <u>Demo 1:</u> So let's say I need to ship all of these teddy bears. *RA takes 22 singles.* What would my order look like? I need to see if I can make bags and boxes. Ok, so do I have more than 4 teddy bears? Yes! So I need to put them in a bag. *RA puts 4 TB in a bag.* Do I have more than 4 teddy bears? Yes! So I need to put them in a bag. *RA puts 4 TB is a bag.* Do I have more than 4 teddy bears? Yes! So I need to put them in a bag. *RA puts 4 TB is a bag.* Do I have more than 4 teddy bears? Yes! So I need to put them in a bag. *RA puts 4 TB is a bag.* Do I have more than 4 teddy bears? Yes! So I need to put them in a bag. *RA puts 4 TB is a bag.*

I don't have 4 teddy bears anymore, so I cannot make another bag. I leave these two extra here.

Now, do I have more than 4 bags? Yes! So I need to put the four bags into a box. *RA puts 4 bags into a box.*
I can't fill another box, because I only have only 1 bag left.

Okay, did I use my packing rules? Let me check. I don't have 4 TB and I don't 4 bags. I'm done. Okay, so my order would be 1 box, 1 bag, and 2 extra teddy bears.

Let's review the packing rules so that you can help me figure out what the orders will look like.

Can you tell me how many teddy bears need to fit into a bag?
If child says 4: Yes, that's right.
If child says other answer: repeat rule #1.
Can you tell me how many bags fit into a box?
If child says 4: Yes, that's right.
If child says other answer: repeat rule #2.
And what are we going to call the bear that don't fit into a bag or a box?
Accept extra, leftover, or synonym.

<u>Demo 2:</u>
Let's do another one.
Let's say I need to ship 1 box, 2 bags and 2 extra teddy bears. What would my order look like? *RA takes appropriate manipulatives.*
Do I have 4 teddy bears? No, so I can't put them in a bag! Do I have 4 bags? No, so I can't put them in a box!
Check! Did I use my packing rules? Do I have more than 4 TB? No. Do I have more than 4 bags? No. Okay so my order would by 1 box, 2 bags, and 2 extra teddy bears. *RA points to appropriate manipulative while saying the order.*

<u>Demo 3:</u>
Are you ready for another one!
Let's say I need to ship 4 teddy bears. What would my order look like?
Do I have 4 teddy bears? Yes! Okay, so I need to put them in a bag. *RA puts the 4 TB in a bag.*
Do I have 4 bags? No.
Check! Did I use my packing rules? Do I have more than 4 TB? No. Do I have more than 4 bags? No. Okay, so my order would be 1 bag!

<u>Demo 4:</u>

Now you do it, it's your turn!
Let's say I need to ship 5 teddy bears. What would my order look like? Remember to use the packing rules! *Let child pick 5 TB and do the order.*
If child is correct: Yes, that's right. As soon as you have 4 teddy bears they need to go in a bag. *RA should gesture between the two legends.*
If child is incorrect: Oops not quite. As soon as you have 4 teddy bears they need to go in a bag. *RA will model action.*

<u>Demo 5:</u>
Okay, now you help me do the next one…
Now I need to ship 1 box, 2 bags, 1 extra teddy bear and 3 more bags. What would my order look like? *Let child work his/her way through the order.*
If child is correct: Yes, that's right. As soon as you have 4 bags they need to go into a box. *RA should gesture between the two legends.*
If child is incorrect: Oops not quite. As soon as you have 4 bags they need to go into a box. *RA will model action.*

Appendix B

Prior Knowledge Measures and Protocols

I. Number Knowledge Test (NKT)

Preliminary

Let's see if you can count from 1 to 10. Go ahead.

Level 0. (4 years old)

Go to Level 1 if 3 or more are correct.

1. Can you count these counters and tell me how many there are?
(place 3 counters in a row in front of the child.)

2a. (Show stacks of Counters, 5 vs. 2, same colour.) Which pile has more?

2b. (Show stacks of Counters, 3 vs. 7, same color.) Which pile has more?

3a. This time, I'm going to ask you which pile has less. (Show stacks of Counters, 2 vs. 6, same color.) Which pile has less?

3b. (Show stacks of Counters, 8 vs. 3, same color.) Which pile has less?

4. I'm going to show you some Counters. (Show a line of 3 Counters of one color [A] and 4 Counters of a different color [B] in a row, as follows: A B A B A B B.)
Count just the (color B) Counters and tell me how many there are.

5. (Clear all the Counters from the previous question. Show a mixed array—not a row—of 8 Counters of one color [A] and 7 Counters of a different color [B].)
Here are some more Counters. Count just the (color A) Counters and tell me how many there are.

Level 1 (6 years old)

Go to Level 2 if 5 or more are correct

1. If you had 4 chocolates and someone gave you 3 more, how many chocolates would you have all together? (Assess strategy used by asking, "How did you figure that out?")

2. What number comes right after 7?

3. What number comes two numbers after 7?

4a. Which is bigger: 5 or 4?

4b. Which is bigger: 7 or 9?

5a. This time, I'm going to ask you about smaller numbers. Which is smaller: 8 or 6?

5b. Which is smaller: 5 or 7?

6a. Which number is closer to 5: 6 or 2? (Show Visual Array 1 after asking the question)

6b. Which number is closer to 7: 4 or 9? (Show Visual Array 2 after asking the question.)

7. How much is 2 + 4? (Children can use fingers for counting. Assess strategy used by asking, "How did you figure that out?")

8. How much is 8 take away 6? (Children can use fingers for counting. Assess strategy used by asking, "How did you figure that out?")

9a. (Show Visual Array 3 of the numerals 8 5 2 6, and ask the child to point to and name each numeral.) When you are counting, which of these numbers do you say first?

9b. When you are counting, which of these numbers do you say last?

Level 2 (8 years old)

Go to Level 3 if 5 or more are correct

1. What number comes 5 numbers after 49? (Assess strategy used by asking, "How did you figure that out?")
2. What number comes 4 numbers before 60? (Assess strategy used by asking, "How did you figure that out?")

3a. Which is bigger: 69 or 71?

3b. Which is bigger: 32 or 28?

4a. This time I'm going to ask you about smaller numbers. Which is smaller: 27 or 32?

4b. Which is smaller: 51 or 39?

5a. Which number is closer to 21: 25 or 18? (Show Visual Array 4 after asking the question.)

5b. Which number is closer to 28: 31 or 24? (Show Visual Array 5 after asking the question.)

6. How many numbers are there between 2 and 6? (Accept either 3 or 4.)
7. How many numbers are there between 7 and 9? (Accept either 1 or 2.)
8. How much is 12 + 54? (Show Visual Array 6. Assess strategy used by asking, "How did you figure that out?")
9. How much is 47 take away 21? (Show Visual Array 7.)

Level 3 (10 years old)

1. What number comes 10 numbers after 99? (Assess strategy used by asking, "How did you figure that out?")
2. What number comes 9 numbers after 99?

3a. Which difference is bigger, the difference between 9 and 6 or the difference between 8 and 3?

3b. Which difference is bigger, the difference between 6 and 2 or the difference between 8 and 5?

4a. Which difference is smaller, the difference between 99 and 92 or the difference between 25 and 11?

4b. Which difference is smaller, the difference between 48 and 36 or the difference between 84 and 73

5. How much is 13 + 39? (Show Visual Array 8. Assess strategy used by asking, "How did you figure that out?")
6. How much is 36 - 18? (Show Visual Array 9. Assess strategy used by asking, "How did you figure that out?")
7. How much is 301 take away 7?

II. Picture Place Value Task (PicPVT)

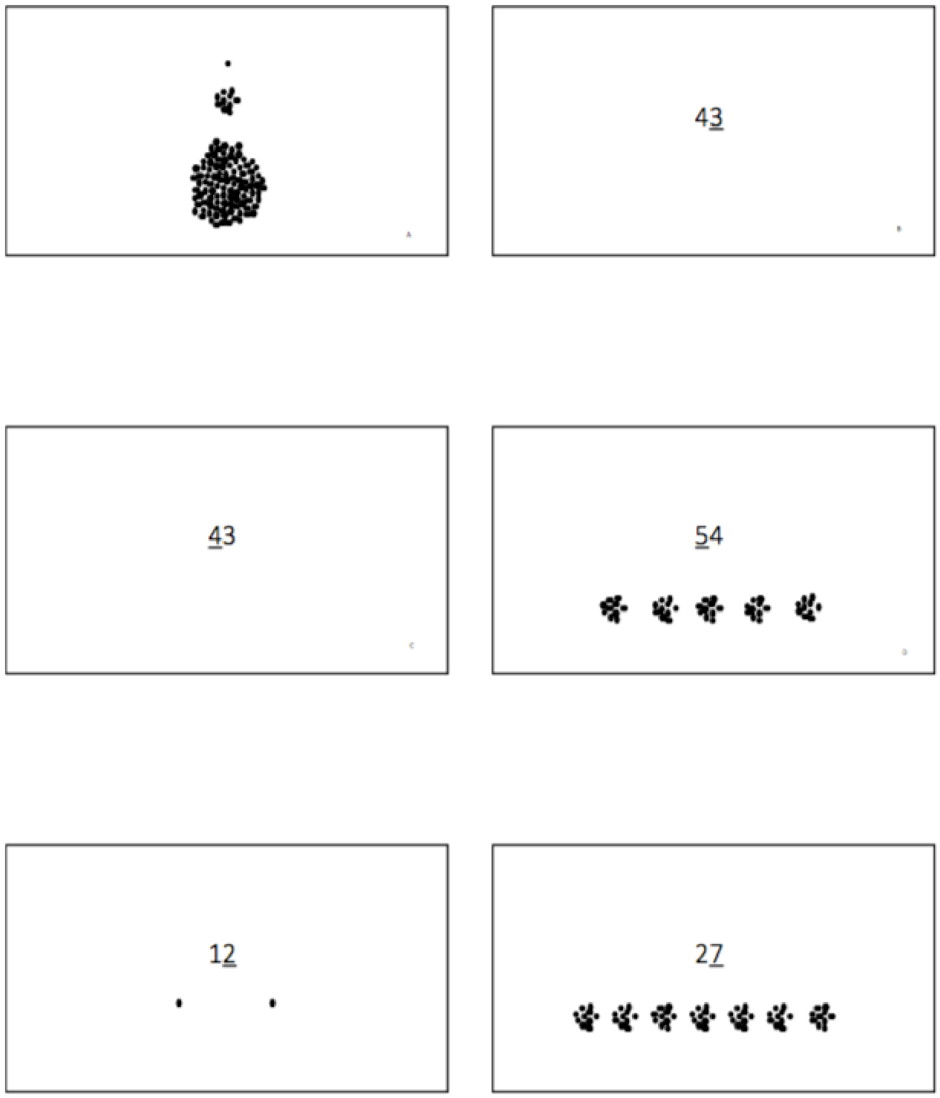
Introduction
This activity uses numbers and shapes. *Show card A* There are the unit for the ones *(point to the unit)* There are the group of ten for the tens (*point to the group of ten*) And there are the group of hundred for the hundreds *(point to the group of 100)* *Show card B* On each card, you will see a number with one part underlined, like this… *(point to the 43)* In this number, the underlined 3 is in the ones place, so it means 3 ones, or just 3. *Show card C* In this one: 43, what place is the underlined 4? What does it mean? *If they say "4 tens" say:* Yes, that's right! *Then give explanation below.* *If they say something else say:* Oops, not quite. Let's look at it closely. *Then give explanation below.* In this number, the underlined 4 is in the tens place, so it means 4 tens, or just 40. Now, I am going to show you a number and a picture. You tell me if the picture matches the underlined part of the number. *Show card D* Does the picture match the underlined part of the number? *Awknowledge the child's answer.* *If they say "Yes" say:* Yes, that's right! *Then give explanation below.* *If they say "No" say:* Oops, not quite. Let's look at it closely. *Then give explanation below.* A 5 in the tens place means 5 tens and the picture shows 5 tens, so the answer is Yes.
The task
Show card 1 Does the picture match the underlined part of the number? *Record child's answer, flip next card, and repeat the question for cards 2-20.*

Items and answer

1. 12 - YES
2. 27 - NO
3. 624 - YES
4. 32 - NO
5. 237 - NO
6. 64 - YES
7. 534 - NO
8. 178 - YES
9. 726 - NO
10. 49 - YES
11. 442 - YES
12. 284 - NO
13. 59 - NO
14. 419 - YES
15. 454 - NO
16. 384 - YES
17. 65 - NO
18. 245 - NO
19. 206 - YES
20. 131 - NO

Appendix C

PicPVT Items



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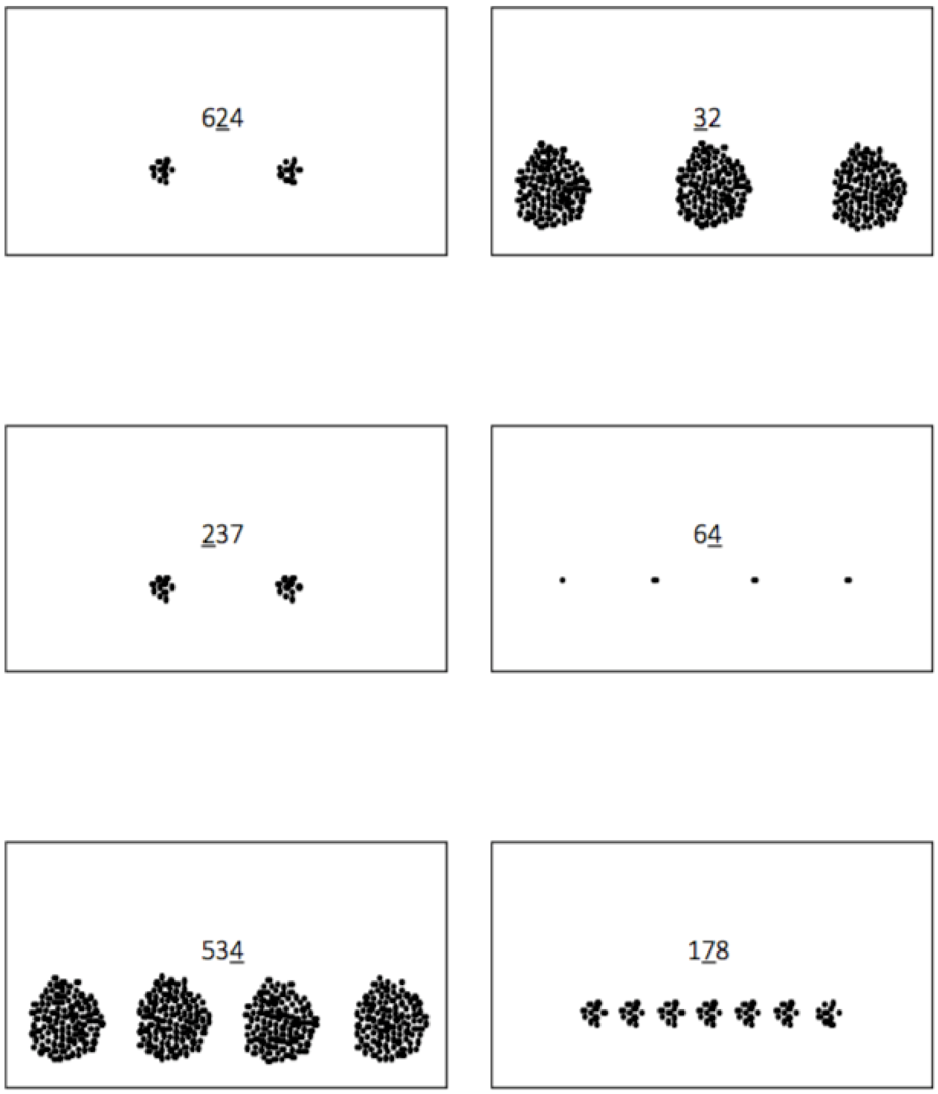

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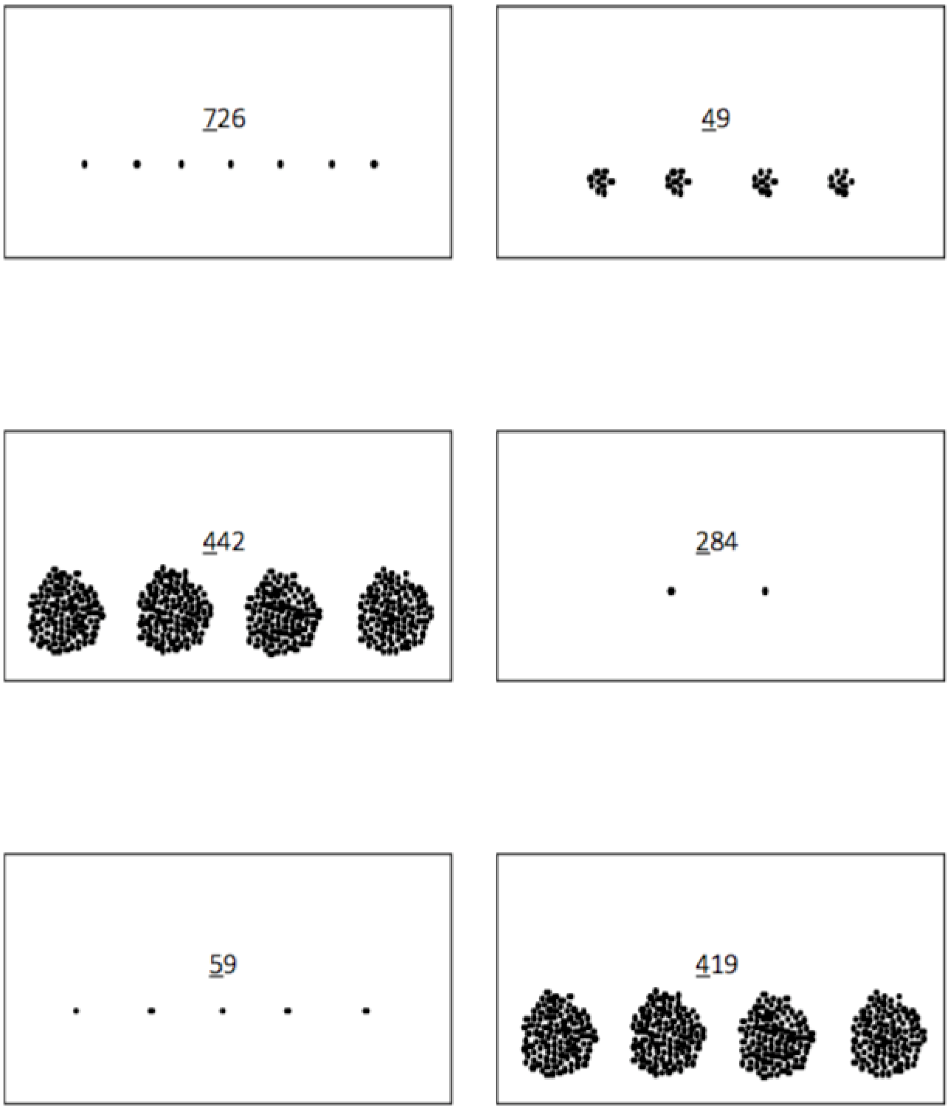

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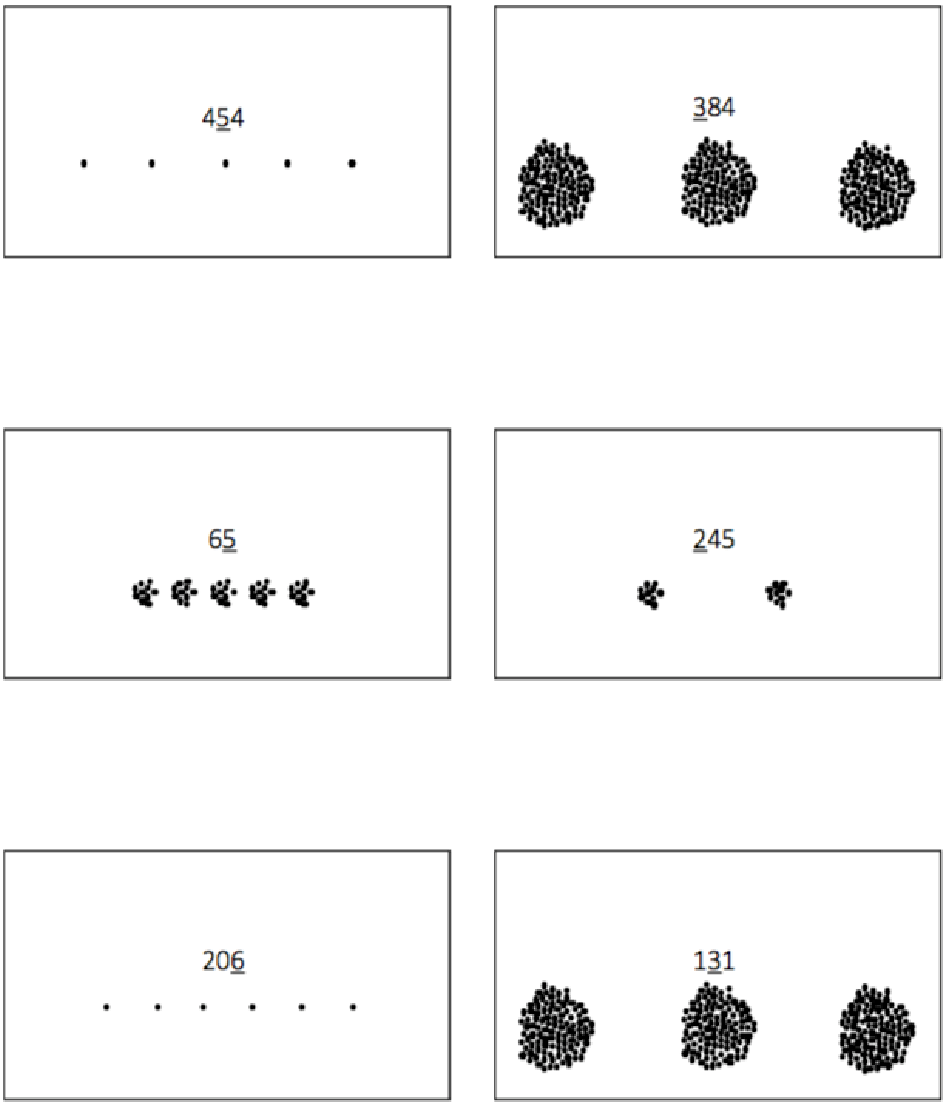

4

Appendix D

Learning and Transfer Measures

Part B: Learning Procedures
Materials:
- Same as Part A
Set-up:
Ask shipment problems to the child. After each problem you will ask the "Verification questions". If the child does not regroup, ask the "Check question". If the child adjusts his/her answer, ask the "Verification questions" again. If not, move on to the next problem. Verification questions: A. Are there any boxes? If yes: Show me where they are? Let the child answer. How many are there? B. Are there any bags? If yes: Show me where they are? Let the child answer. How many are there? C. Are there any extras? If yes: Show me where they are? Let the child answer. How many are there? Check Question: Check! Did you use the packing rules?
Introduction to the task:
Okay, now it's your turn to construct the orders. Make sure to use the packing rules! You can use these *(point to representation)* to help you prepare the order. You can put them here *(point to the space in front of the child)* to show me.
The task:
Read each question twice to the child. 5. Let's say I need to ship 6 teddy bears. What would my order look like? 6. Now, I need to ship 1 box, 3 bags, 1 extra teddy bear and 3 more bags. What would my order look like? 7. Let's say I need to ship 2 boxes, 2 extra teddy bears, 1 box and 3 more teddy bears. What would my order look like? 8. Now, I need to ship 2 boxes, 3 bags, 2 extra teddy bears and 2 more bags. What would my order look like?

Part C: Learning Concepts and Transfer Comparison
Materials:
- White Paper - Marker - Scarf to cover manipulatives
Set-up:
Hide all the manipulatives and the legends from the child's sight. You can use a scarf. *Place the marker and white paper in front of the child.* *Read each question twice to the child.*
Introduction to the task:
I'm going to ask you some questions now. I will read each question twice, but you can ask me to read it again at any time if you need me to. Here you have a marker and paper. You don't have to use the paper and marker if you don't want to, but you can if you need to. NOTE: "[…]" means to let the child answer and then ask the verification/check questions (V/C) on the separate sheet OR "how did you figure that out?" For question 1, and 2 you should tell the child the correct answer if they get the wrong answer.
The task:
Read each question twice to the child. 9. How many bears fit in a bag? 10. How many bags fit in a box? 11. If you have eight teddy bears, what would the order look like? […] V/C 12. Which order has more teddy bears? 2 bags or 1 box? […] How did you figure that out? 13. Let's say I have 9 teddy bears. What would the order look like? […]V/C 14. Which order has more teddy bears? 2 boxes or 3 bags? […] How did you figure that out? 15. Let's say I have 2 bags and 4 extra teddy bears. What would the order look like? […] V/C 16. Which order has more teddy bears? 3 extra teddy bears, or 1 bag? […] How did you figure that out?

Part D: Learning Regrouping and Transfer Denomination

Materials:
- Appropriate representation for Condition - Legend for Condition
Set-up:
The manipulative should be placed in front of the child, in the following order: singles, fours, sixteens. *The legend should be in the child's sight.*
Introduction to the task:
The objective of these questions is to make the child think of the next possible denomination (but don't teach!). You want to question the child and make the child talk about the materials, without asking leading questions (see notes in general comments). I will ask you just three more questions. We are going to use the same packing rules as before to prepare the orders. I will read each question twice, but you can ask me to read it again at any time if you need me to.
The task:
4. Now I need to ship 4 bags and 5 teddy bears. What would my order look like? *[Verification questions/if necessary, Check question]* 5. Let's say I need to ship 2 bags and 10 teddy bears. What would my order look like? *[Verification questions/if necessary, Check question]* 6. Let's say I need to ship 5 boxes and 2 bags of teddy bears. What would the order look like? *Acknowledge child's answer.* [Verification questions only] Oh! Okay, what would we do now? What would they go in? Why did you think that? Or How did you come up with that?

Appendix E

Prior Knowledge Scoring Sheet

PRIOR KNOWLEDGE SCORING SHEET

Part A: Number Knowledge Test

Level 0		Score
1. Count (3)		
2a. More: 5 vs. 2	2b. More: 3 vs. 7	
3a. Less 2 vs. 6	3b. Less: 8 vs. 3	
4. Count B (4)		
5. Count A (8)		
Total: Need 3 to move on		/5

Level 1		Strategy	Score
1. 4 + 3		CU CO R	
2. 7 + 1			
3. 7 + 2			
4a. Bigger: 5 or 4	4b. Bigger: 7 or 9		
5a. Smaller: 8 or 6	5b: Smaller 5 or 7		
6a. Closer to 5: 6 or 2	6b. Closer to 7: 4 or 9		
7. 2 + 4		CU CO R	
8. 8 – 6			
9a. First: 8 5 2 6	Last: 8 5 2 6		
Total: Need 5 to move on			/9

Level 2		Strategy	Score
1. 49 + 5			
2. 60 – 4			
3a. Bigger: 69 or 71	3b. Bigger: 32 or 28		
4a. Smaller: 27 or 32	4b. Smaller: 51 or 39		
5a. Closer to 21: 25 or 18	5b. Closer to 28: 31 or 24		
6. How many number between 2 and 6			
7. How many numbers between 7 and 9			
8. 12 + 54		CO	
9. 47- 21			
Total: Need 5 to move on			/9

Level 3		Strategy	Score
1. 99 + 10			
2. 99 + 9			
3a. Bigger: 9 – 6 or 8 - 3	3b. Bigger: 6 - 2 or 8 - 5		
4a. Smaller: 99 - 92 or 25 - 11	4b. Smaller: 48 – 36 or 84 – 73		
5. 13 + 39			

6. 36 – 18		
7. 301 - 7		
Total:		/7
Total:		/30

Part B: CPV

To get the point, the child would need to say “4 tens” or “40”.

	Score		Score		Score
1) $2\underline{7}$		7) $\underline{1}88$		13) $\underline{5}64$	
2) $1\underline{4}4$		8) $1\underline{9}0$		14) $\underline{3}5$	
3) $6\underline{3}$		9) $\underline{4}16$		15) $\underline{2}2$	
4) $\underline{2}18$		10) $\underline{2}5$		16) $7\underline{1}$	
5) $\underline{5}4$		11) $12\underline{8}$		17) $5\underline{5}$	
6) $12\underline{6}$		12) $\underline{3}21$		18) $\underline{6}12$	

Total correct: /18

Part C: PicPVT

Item #	Child’s Answer	Correct Answer
1. 12	YES: __________ NO: __________	YES
2. 27	YES: __________ NO: __________	NO
3. 624	YES: __________ NO: __________	YES
4. 32	YES: __________ NO: __________	NO
5. 237	YES: __________ NO: __________	NO
6. 64	YES: __________ NO: __________	YES
7. 534	YES: __________ NO: __________	NO
8. 178	YES: __________ NO: __________	YES
9. 726	YES: __________ NO: __________	NO
10. 49	YES: __________ NO: __________	YES
11. 442	YES: __________ NO: __________	YES
12. 284	YES: __________ NO: __________	NO
13. 59	YES: __________ NO: __________	NO
14. 419	YES: __________ NO: __________	YES
15. 454	YES: __________ NO: __________	NO
16. 384	YES: __________ NO: __________	YES
17. 65	YES: __________ NO: __________	NO
18. 245	YES: __________ NO: __________	NO
19. 206	YES: __________ NO: __________	YES
20. 131	YES: __________ NO: __________	NO
TOTAL CORRECT		/20

Total:	/68

Appendix F

Learning and Transfer Scoring Sheet

Learning and Transfer Scoring Sheet

Verification questions:

A. Are there any boxes? If yes: Show me where they are? Let the child answer. How many are there?

B. Are there any bags? If yes: Show me where they are? Let the child answer. How many are there?

C. Are there any extras? If yes: Show me where they are? Let the child answer. How many are there?

Check Question:

Check! Did you use the packing rules?

Part A: Shipment Problems

Problem	Attempt 1	Attempt 2 (if necessary)
1	_____ boxes, _____ bags, ____ extras	_____ boxes, _____ bags, ____ extras
2	_____ boxes, _____ bags, ____ extras	_____ boxes, _____ bags, ____ extras
3	_____ boxes, _____ bags, ____ extras	_____ boxes, _____ bags, ____ extras
4	_____ boxes, _____ bags, ____ extras	_____ boxes, _____ bags, ____ extras

Part B: Learning Questions

Problem	Answer	
1		
2		
3	_____ boxes, _____ bags, ____ extras	_____ boxes, _____ bags, ____ extras
4		
5	_____ boxes, _____ bags, ____ extras	_____ boxes, _____ bags, ____ extras
6		
7	_____ boxes, _____ bags, ____ extras	_____ boxes, _____ bags, ____ extras
8		

Part C: Transfer questions

Problem	Attempt 1	Attempt 2
1	_____ boxes, _____ bags, ____ extras	_____ boxes, _____ bags, ____ extras
2	_____ boxes, _____ bags, ____ extras	_____ boxes, _____ bags, ____ extras
3	_____ boxes, _____ bags, ____ extras	