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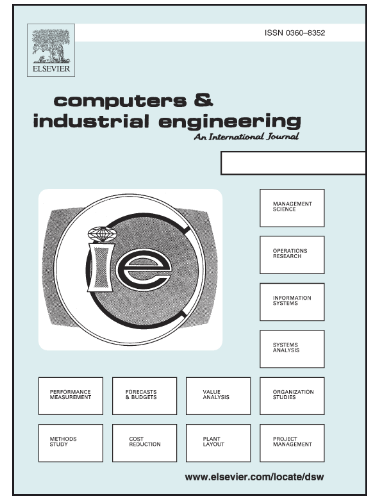

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ABSTRACT

Aircraft en-route flight planning is one of the major challenges for Air Traffic Control operations. Poor planning results in undesirable congestion in the air-traffic network, causing major economic losses for both airline companies and the public. Furthermore, heavy congestion generates flight safety risks due to increased possibility of mid-air conflict. To address these problems, this paper introduces a non-time segmented en-route flight plan formulation with rerouting options for aircrafts in a 3-dimensional (3D) capacitated airspace. Novelty of the proposed mathematical model is the non-time segmented formulation that captures exact arrival and departure times to/from each air-sector. The proposed formulation also incorporates sector capacity changes due to changing weather conditions during planning horizon. Moreover, the speed dependent fuel consumption rate is introduced as a factor in the zone-based air traffic flow management problem. In order to handle the problem sizes similar to those in real-world cases, we proposed a sequential solution heuristics. The performance of the sequential solution method is demonstrated through various test cases.

Keywords: Air traffic flow management; non-time segmented formulation; conflict-free flight; re-routing; fuel consumption; dynamic air-sector capacity

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1. Introduction

Since 1980s, congestion in United States (US) air-traffic network has become an increasing problem with serious economic and social consequences (Octavio Richetta, 1995). Past three decades, the demand for air transportation around the world has precipitated a significant growth in passenger volume, as well as the number of airline companies providing service. According to International Air Transport Association (IATA) forecasts, the demand for air transportation will double over the next 20 years (IATA, 2016). Other studies further suggest that the demand for air-traffic will continue increasing (Matos and Powell, 2003 and Boeing, 2011). The resulting traffic conditions will cause significant bottlenecks in most parts of the world. As a result of this augmenting demand for the air transportation, the Air Traffic Flow Management (ATFM) has been an increasingly challenging task for both academia and industry (Bertsimas *et al.*, 2008). Therefore, the development of new methodologies and strategies for managing air-traffic flow in densely populated airspaces and airports is vital for the future of the air transportation industry. Among various alternatives, the development of modern tools for capacity management in which the flow rate of air traffic is matched with the available capacity of air-segments and airports in the Air Traffic Control (ATC) network requires special attention from both academia and industry. The capacity management in aerospace is known as the Flow Management Problem (FMP), which is particularly important when weather conditions significantly decrease the capacity of the ATC system (Richitta, 1995).

The mathematical model proposed in this paper aims to improve the efficiency of ATC by incorporating the aircrafts' exact arrival and departure times and ensures the avoidance of mid-air collision. While the traditional concerns, the minimization of delays and earliness represent the main body of the objective function, the cost of fuel consumption as a function of average aircraft-speed at each air-sector has successfully been introduced in the formulation. In this paper, the entire airspace is divided into capacitated air segments. The en-route flight plans for all aircrafts are determined in such way that the capacities of air-segments are upheld and mid-air conflicts are avoided. Unlike most other research work on the same topic, the described mathematical model avoids time-segmented modeling approach.

The remainder of this paper is organized as follows. First, a brief literature review in the realm of the air traffic management is provided. Next, the model and its distinctive features are discussed. In section 4, proposed solution strategies are presented and performance gains have been demonstrated. Finally, conclusions and future work are summarized in section 5.

2. Literature Review

The ATFM problem under several different extensions has been well studied in the literature. Hence, in this section we only cover the literature that is most relevant to the work presented in the paper. For a more general discussion on the air-traffic management problem, readers are referred to the works of Navazio and Romanin-Jacur (1998) and de Neufville et al. (2013).

Earlier works in the area of ATFM problem mostly focus on single airport traffic-congestion problems which study the impact of ground and airborne delays on airport congestion (Bertsimas, 2008). Earlier works on ground delay strategies were introduced by Odoni (1987) and Richetta (1995). More recently, the importance of mid-air congestion has been increasingly discussed in the literature (Bertsimas *et al.*, 2008). Current air navigation systems utilize air-sectors to guide airplanes from their origins to their destinations. Air sectors are capacitated geometries where each sector has a unique capacity that can be safely handled at any given time by the Air Traffic Controllers (ATCOs). Hence the objective is to control the flow of aircraft through air sectors in such way that capacity constraints are not violated. One of the earlier attempts to study the en-route capacity management problem as part of ATFM was the work of Helme (1992). In this work, Helme proposed a multi-commodity minimum-cost flow model on a time-space network that utilizes the airborne and ground delays for effectively managing the air traffic flow in a given network under temporarily reduced capacities. More recently, Bertsimas, Lulli and Odoni (2008) proposed an integer programing model for ATFM problems that combines all phases of a flight for achieving a safe, efficient and on-time traffic flow. Later, Bertsimas et *al.* (2011) applied some of these modeling techniques to handle large-scale real-life traffic conditions.

The concept of free flight has further complicated the air traffic management problem in that individual aircrafts are given the autonomy to make real-time rerouting decisions. It was widely accepted that the free flight concept would better manage the increased demand for air transportation by dividing the responsibility for real-time decision making between pilots and ATCOs. Hence, more economic benefits for airline companies and customers can be attained (Hilburn *et al*., 1997, Galdino *et al*., 2007). Both free flight concept and increasing demand for air transportation led researchers to study ATFM problem with rerouting options. In the current business model, all commercial traffic provides their projected en-route flight plan to ATC authorities for approval. Once the aircraft is airborne, the navigation is controlled by ATCOs. Under severe weather and/or traffic conditions, deviation from the original flight-plans may be desirable to better satisfy business and customer objectives.

Consequently, researchers have started tackling congestion in the entire airspace rather than at a single airport. Helm (1992) introduced one of the earliest rerouting formulations. Bertsimas and Stock Patterson (2000) and later Dell'Olmo and Lulli (2002) developed mathematical models to allow aircraft to divert from their original flight-plans in response to changing traffic and weather conditions during flight. Other notable works on rerouting problem are Leal de Matos, Chen and Ormerod (2001), Ma, Cui and Cheng (2004) and Bertsimas, Lulli and Odoni (2011)

One of the major drawbacks of the current literature is the formulation of ATFM problem using time segmentation where the duration of planning horizon is segmented in smaller time intervals (periods) and decisions are made at the beginning of each period. Depending on the period length, the time segmented formulation leads to major errors in determining the exact location of aircraft. Hence, the airspace capacity may easily be violated and the risk of mid-air conflicts is increased. In this paper, the ATFM problem is modeled using a non-time segmented formulation, which provides the exact arrival and departure times of aircraft at each sector. Furthermore, the proposed formulation guarantees that the capacity of each sector is not violated during the entire planning horizon. While the model is a powerful tool for handling scheduling phase for all aircrafts, it complies very well with the objectives of the free flight concept as well.

3. Formulation of air traffic flow problem in capacitated air sectors

We formulated the ATFM problem as a MILP model in a constrained airspace. Aircraft leaves an origin airport to reach the destination airport through travelling a number of transitional air sectors (See Figure 1 for illustration). Air sectors are 3D geometries defined by air transportation authorities for air traffic controllers to manage the traffic more efficiently and safely (Pasquini, Pozzi, 2005). In the proposed mathematical model, the airspace is modeled as a graph $G(V,E)$ where $v \in V$ is an air sectors connected to neighboring air-sectors through an edge $e \in E\left(e = \{v_i, v_j\}\right)$. In order to manage the traffic at each air-sector effectively and safely, an air-sector capacity C_v is introduced (at any given time, the number of aircrafts at the air-sector cannot exceed the given capacity). In the model, the travelling time at a sector $\left(t_v^f\right)$ is determined from the aircraft's average speed (ϑ_v^f) at the sector. The fuel consumption rate $FUEL_v^f$ of an aircraft is formulated as a function of aircraft's average speed at the air-sector and imbedded in the MILP model as a cost function. The relationship between the fuel consumption and the aircraft-speed is established from the available industry data (Clarke et al., 2008). Finally, the minimization of total flight-cost incurred from delays, earliness and fuel consumption is formulated as an objective function. In Figure 1, a sample airspace configuration is illustrated.

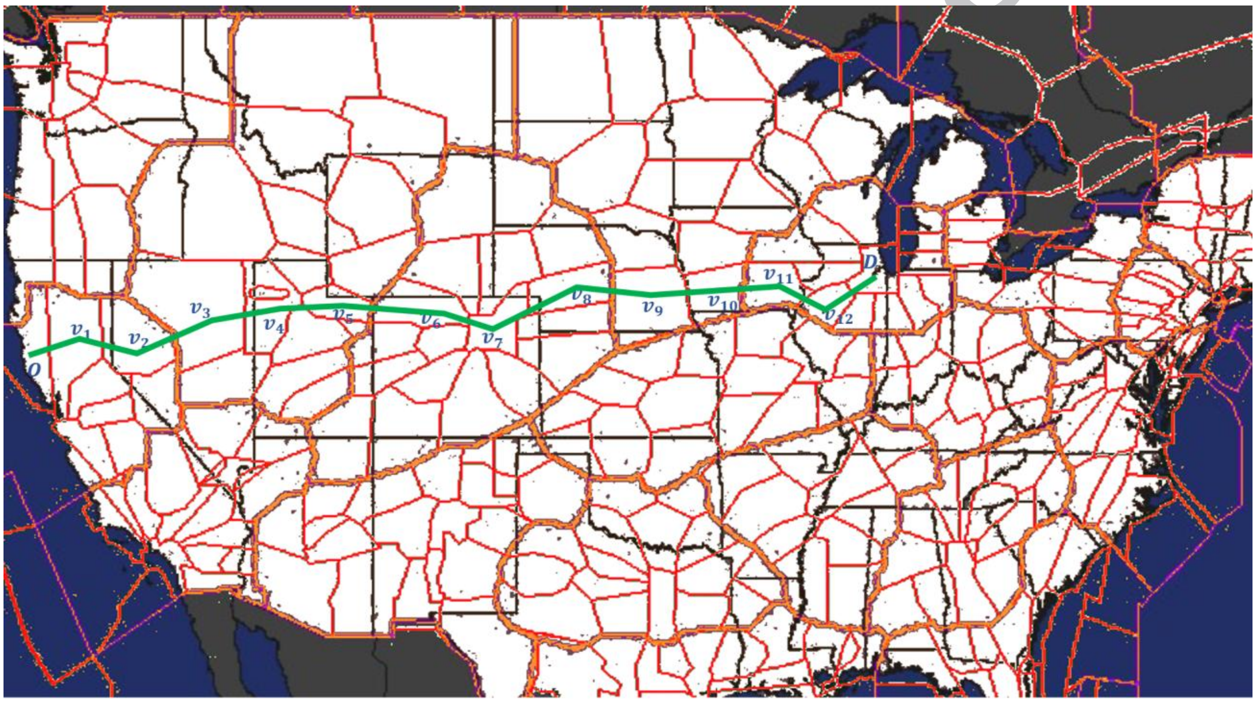


Figure 1: High-altitude air-sector map of United States: An aircraft travelling from San Francisco to Chicago may be required to travel through 12 different air-sectors

3.1 Assumptions

The following assumptions are made in order to model the air traffic management problem discussed in this paper:

- The journey of an aircraft is only considered from an origin to a destination airport. The proposed model does not consider the assignments of an aircraft on different flights during the planning horizon.

- All aircrafts are introduced to the problem from an origin airport and sink at a destination airport.
- An aircraft may only visit an air-sector once during its journey
- The average aircraft speed at each sector is bounded by ϑ_v^{min} and ϑ_v^{max}
- The capacity of air sectors including airports may change during the planning horizon due to weather conditions (CAP_v^t).

3.2 Model formulation

3.2.1 Objective function and constraints

The primary objective of the mathematical model is to determine a flight plan (R_f) for all aircraft $f \in F$ as a set of visited air-sectors $\left(y_v^f = 1 \ \forall v \in V\right)$ from an origin ORG^f to a destination $DEST^f$. In addition to the list of visited links, the formulation extracts arrival time at visited air sector $\left(a_v^f\right)$, departure time from the air-sector $\left(d_v^f\right)$ and the average speed of the aircraft while inside of an air sector $\left(\vartheta_v^f\right)$. Consequently, costs that are incurred from early and late arrivals $\left(T_{LATE}^f, T_{EARLY}^f\right)$, and fuel consumption cost as a function of average speed are determined.

A. Objective function

$$MIN \sum_{f \in F}\left(C_{LATE}^f T_{LATE}^f + C_{EARLY}^f T_{EARLY}^f\right) + C_{FUEL} \sum_{f \in F} \sum_{v \in V : y_v^f = 1} D_v FCR_v^f \qquad (1)$$

where C_{LATE}^f, C_{EARLY}^f and C_{FUEL} are the unit costs for late arrival, early arrival and gallon of jet-fuel respectively. In the objective function, FCR_v^f is per Nautical Mile fuel consumption rate and

D_v is the distance travelled in the air-sector. The calculation fuel consumption rate $\left(FCR_v^f\right)$ which is a function of average aircraft speed is discussed in Equation 22 later in the text.

B. Routing constraints

In the model, we introduce a binary variable $x_{vv'}^f$, to trace the aircraft's journey from air-sector v to the adjacent air-sector v' $\left(if\ x_{vv'}^1 = 1\ then\ y_v^f = y_{v'}^f = 1; \text{and air} - \text{sector } v \text{ is visited first}\right)$. Notation $V(v)$ describes the set of all adjacent air-sectors of sector v.

$$\sum_{v' \in V(ORG^f)} x_{ORG^f v'}^f = \sum_{v' \in V(DEST^f)} x_{v' DEST^f}^f = 1 \qquad \forall f \in F \tag{2}$$

$$\sum_{v' \in V(ORG^f)} x_{v' ORG^f}^f = \sum_{v' \in V(DEST^f)} x_{DEST^f v'}^f = 0 \qquad \forall f \in F \tag{3}$$

$$\sum_{v' \in V(v)} x_{v'v}^f = \sum_{v' \in V(v)} x_{vv'}^f \leq 1 \qquad \forall f \in F;\ \forall v \in V \backslash \{ORG^f, DEST^f\} \tag{4}$$

$$y_v^f = \sum_{v' \in V(v)} x_{v'v}^f \qquad \forall f \in F;\ \forall v \in V \backslash ORG^f \tag{5}$$

Constraint (2) ensures that all aircrafts leave from their origin airports and reach their destinations. The given formulation in Constraint (3) guarantees that aircrafts would not return to their origin before the associated flight is completed and there is no departure from the destination under the same flight. Finally in Constraint (4), it is ensured that an aircraft entering to an air sector is forced to depart that sector if it is not an origin or a destination sector. The binary decision variable y_v^f is utilized to monitor if an aircraft enters the particular air-sector in Constraint (5).

C. Speed and timing constraints

Flight duration at a given air-sector depends on the speed $\left(\vartheta_v^f\right)$ and the distance travelled. Given that air-sector length D_v is known, the flight duration in the sector is $t_v^f = {D_v}/{\vartheta_v^f}$, which is a nonlinear expression as ϑ_v^f is a decision variable. In order to linearize the flight duration, speed is defined as "time to travel a unit distance", $\tau_v^f = {1}/{\vartheta_v^f}$. Consequently, $t_v^f = \tau_v^f D_v$ which is a linear expression. Consequently, the following set of constraints are introduced to determine the arrival time to a Node (a_v^f), the departure time from a Node (d_v^f), and the amount of lateness (T_{LATE}^f) and earliness (T_{EARLY}^f) for all Nodes on the path of an aircraft.

Constraints (6-11) are subject to $\forall f \in F$.

$$d_v^f = a_v^f + \tau_v^f D_v = a_v^f + t_v^f \leq y_v^f M \qquad \forall v \in V \tag{6}$$

$$\vartheta_v^{MIN} y_v^f \leq \tau_v^f \leq \vartheta_v^{MAX} y_v^f \qquad \forall v \in V \tag{7}$$

$$a_v^f = d_{v'}^f + \varphi_{v'v}^f - \omega_{v'v}^f \qquad \forall v \in V \backslash ORG^f, \forall v' \in V(v) \backslash DEST^f \tag{8}$$

$$\varphi_{v'v}^f + \omega_{v'v}^f \leq M\left(1 - x_{v'v}^f\right) \qquad \forall v \in V \backslash ORG^f, \forall v' \in V(v) \backslash DEST^f \tag{9}$$

$$a_{ORG^f}^f \geq T_{DEP}^f \tag{10}$$

$$d_{DEST^f}^f - T_{ARV}^f = T_{LATE}^f - T_{EARLY}^f \tag{11}$$

In Constraint (6), based on the arrival time $\left(a_v^f\right)$ and the aircraft's flight time $\left(t_v^f = \tau_v^f D_v\right)$, the departure time from the air-sector calculated. We also ensure that $a_v^f = d_v^f = 0$ if $y_v^f = 0$. In order to ensure the feasibility of Constraint (6) at all time, the value of τ_v^f is constrained by y_v^f

using the Constraint (7) between a minimum (ϑ_v^{MIN}) and maximum (ϑ_v^{MAX}) allowable speed. In our model, the arrival time at a new air-sector is equal to the departure time from the previous sector. The Constraints (8 and 9) controls the transition between two consecutive air-sectors without allowing any delays ($a_v^f = d_{v'}^f$, given that $x_{v'v}^f = 1$). Decision variables $\varphi_{v'v}^f$ *and* $\omega_{v'v}^f$ are introduced to capture the difference between a_v^f and $d_{v'}^f$. Constraint (9) ensures that for cases where the binary decision variable $x_{v'v}^f = 1$, then the difference between a_v^f and $d_{v'}^f$ is guaranteed to be zero, Hence, $a_v^f = d_{v'}^f$. Constraint (10) ensures that aircraft leaves the airport earliest at its scheduled departure time $\left(T_{DEP}^f\right)$. Finally, in constraint (11), the tardiness $\left(T_{LATE}^f\right)$ or the earliness $\left(T_{EARLY}^f\right)$ are determined based on the scheduled arrival time $\left(T_{ARV}^f\right)$.

D. Capacity constraint:

Constraints (12-17) are subject to: $\forall f, f' \in F;\ \forall v \in V$

$$a_v^{f'} \le a_v^f + M\left(1 - \alpha_v^{ff'}\right) + M\left(2 - y_v^f - y_v^{f'}\right) \tag{12}$$

$$a_v^f \le a_v^{f'} + M\alpha_v^{ff'} + M\left(2 - y_v^f - y_v^{f'}\right) \tag{13}$$

$$a_v^f \le d_v^{f'} + M\left(1 - \beta_v^{ff'}\right) + M\left(2 - y_v^f - y_v^{f'}\right) \tag{14}$$

$$d_v^{f'} \le a_v^f + M\beta_v^{ff'} + M\left(2 - y_v^f - y_v^{f'}\right) \tag{15}$$

$$\alpha_v^{ff'} + \alpha_v^{f'f} \ge y_v^f + y_v^{f'} - 1 \tag{16}$$

$$\beta_v^{ff'} + \beta_v^{f'f} \ge y_v^f + y_v^{f'} - 1 \tag{17}$$

At the time of a new arrival to an air-sector, in order to monitor the existing traffic conditions, three sets of decision variables are introduced. Constraints (12) and (13) determine if aircraft f arrives to sector v after aircraft f' $\left(\alpha_v^{ff'} = \begin{cases} 1, & a_v^{f'} \leq a_v^f \\ 0, & otherwise \end{cases}\right)$. Similarly, Constraints (14) and (15) determines if aircraft f arrives at air-sector v before aircraft f' leaves the sector $\left(\beta_v^{ff'} = \begin{cases} 1, & a_v^{f} \leq d_v^{f'} \\ 0, & otherwise \end{cases}\right)$. Finally, Constraints (16) and (17) are used to associate binary decision variables $\alpha_v^{ff'}$ and $\beta_v^{ff'}$ with the decision variable y_v^f. If both aircrafts f and f' transfer through the same air-sector, than conditions $\alpha_v^{ff'} + \alpha_v^{f'f} = 1$ and $\beta_v^{ff'} + \beta_v^{f'f} = 1$ should be sustained. Constraints 12-17 are only enforced when both aircrafts use the same airspace. For conditions $y_v^f + y_v^{f'} \leq 1$, decision variables $\alpha_v^{ff'}$ and $\beta_v^{ff'}$ may take an arbitrary value in {0 or 1}. Yet, this does not impact the flight planning decisions. Due to the limited capacity in the network, optimization model enforces either $\alpha_v^{ff'} = 0$ and/or $\beta_v^{ff'} = 0$ for cases where $y_v^f + y_v^{f'} \leq 1$ so that the available capacity at the airspace can be allocated for other aircrafts.

In the next set of constraints, we utilize the binary decision variables $\alpha_v^{ff'}$ and $\beta_v^{ff'}$ to count the number of aircrafts present at the air-sector at the time of aircraft f's arrival. Constraints (18-20) are subject to $\forall f, f' \in F; \forall v \in V$

$$\alpha_v^{ff'} + \beta_v^{ff'} - 1 \leq \lambda_s^{ff'} \tag{18}$$

$$\lambda_v^{ff'} \leq \alpha_v^{ff'} \tag{19}$$

$$\lambda_v^{ff'} \leq \beta_v^{ff'} \tag{20}$$

$$\sum_{f' \in F : f \neq f'} \lambda_v^{ff'} \leq CAP_v - 1 \qquad \forall f \in F \,\&\, \forall v \in V \tag{21}$$

The binary decision variable $\lambda_{\text{v}}^{ff'} = 1$ indicates that f' is already at the air sector v at the time aircraft f enters it $\left(\lambda_{\text{v}}^{ff'} = \begin{cases} 1, & a_v^{f'} \leq a_v^{f} \leq d_v^{f'} \\ 0, & otherwise \end{cases}\right)$. Constraints (18-20) are designed to link $\alpha_v^{ff'}$ and $\beta_v^{ff'}$ with the decision variable $\lambda_v^{ff'}$. Constraint (21) guarantees that capacity constraints will not be violated for any flight.

E. Fuel consumption cost as a function of speed

It is a well-known fact that fuel consumption rates vary depending on the speed for most transportation vehicles. Industry data suggests that fuel consumption rate for all aircraft gradually reduces as the aircraft speed is increased until an optimum speed is reached; thereafter, the fuel consumption rate increases. The study conducted by Clarke et al. (2008) clearly demonstrates the existence of such relationships for several commercial aircrafts (see Figure 2 for illustration).

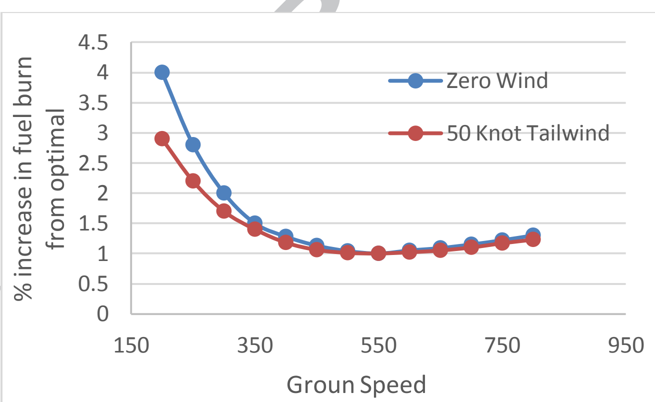
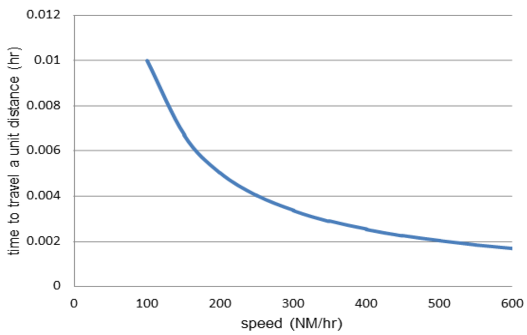
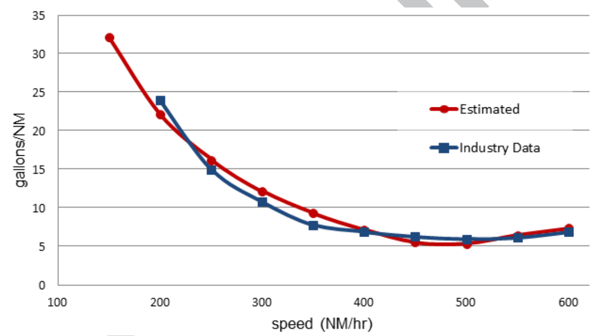


Figure 2: Fuel Consumption vs. Aircraft Speed-Industry Data (Clark et al., 2008)

In this paper, the speed is formulated as "time to travel a unit distance" (τ_v^f). When plotted against the ground speed, a pattern (Figure 3) similar to the fuel consumption rate patern given in Figure 2 is observed. Consequently, the expression for the fuel consumption rate (FCR_v^f) as a function of τ_v^f is derived in Equation 22.



(a) Time to travel a unit distance at a given speed



(b) Estimation of industry data as a function of τ_v^f

Figure 3: Estimation of industry data for fuel consumption rate change with speed.

$$FCR_v^f = \begin{cases} FCR^{f^*}\left(k_1 y_v^f + k_2\left(\tau_v^f - \dfrac{y_v^f}{\vartheta^{f^*}}\right)\right), & \tau_v^f \leq \dfrac{1}{\vartheta^{f^*}} \\ FCR^{f^*}\left(k_1 y_v^f + k_2\left(\dfrac{y_v^f}{\vartheta^{f^*}} - \tau_v^f\right)\right), & otherwise \end{cases} \tag{22}$$

where ϑ^{f^*} is the optimum speed that leads to minimum fuel consumption and FCR^{f^*} is the per Nautical Mile (NM) fuel consumption rate at the optimum speed. Coefficients k_1 and k_2 are shape parameters used to smooth the curve. $FCR_v^f - \tau_v^f$ relationship depicted in Figure (3-b) is obtained for $FCR^{f^*} = 5$ gallons, $k_1 = 0.8$ and $k_2 = 1000$. The given speed-fuel consumption relationship can be adjusted for different types of the aircraft by controlling the shape parameters. The fuel consumption rate (FCR_v^f) obtained in Equation (22) is utilized in the objective function to minimize the total fuel consumption during flight while considering the safety and delay factors.

3.2.2 Extended formulation for sequential solution

In order to solve the ATFM problem for larger instances within a reasonable time window, a heuristic, namely sequential planning, has been proposed. The sequential planning method determines flight plans for aircraft in a sequential order according to their departure times. First, the aforementioned ATFM formulation is solved for a set of aircrafts (aircrafts that are scheduled to leave their origins earlier, $f^+ \in F^+$ where F^+ includes N^+ aircrafts) and a flight plan $R^{f^+} = \left\{ y_v^{f^+}, a_v^{f^+}, t_v^{f^+};\ \forall v \in V\colon y_v^{f^+} = 1 \right\}$ are obtained. Next, the mathematical model is solved for a new set of aircrafts, $f^- \in F^-$ given that the flight plans for the previous set, $R^{f^+}, f^+ \in F^+$ are known and included in the problem as inputs. Once the problem is solved for aircrafts in second aircraft-set ($f^- \in F^-$), the list for already-solved aircrafts is updated $F^+ = F^+ \cup F^-$ and the new set is introduced until a flight plan for $\forall f \in F$ is determined. In order to solve the sequential planning method, in addition to the capacity constraints (12-21), following set of constraints that determines if the newly introduced aircrafts ($f^- \in F^-$) and previously solved aircrafts ($f^+ \in F^+$) present at the same air-sector at the same time ($\lambda_v^{f^+ f^-} = 1$).

All constraints from number (23) to number (32) are subject to: $\forall f^- \in F^-; \forall f^+ \in F^+; \forall v \in V$

$$a_v^{f^-} \leq a_v^{f^+} + M\left(1 - \alpha_v^{f^+ f^-}\right) + M\left(y_v^{f^+} + y_v^{f^-} - 1\right) \tag{23}$$

$$a_v^{f^+} \leq a_v^{f^-} + M\alpha_v^{f^+ f^-} + M\left(y_v^{f^+} + y_v^{f^-} - 1\right) \tag{24}$$

$$a_v^{f^+} \le d_v^{f^-} + M\left(1 - \beta_v^{f^+f^-}\right) + M\left(y_v^{f^+} + y_v^{f^-} - 1\right) \tag{25}$$

$$d_v^{f^-} \le a_v^{f^+} + M\beta_v^{f^+f^-} + M\left(y_v^{f^+} + y_v^{f^-} - 1\right) \tag{26}$$

$$\alpha_v^{f^+f^-} + \alpha_v^{f^-f^+} \ge y_v^{f^+} + y_v^{f^-} - 1 \tag{27}$$

$$\beta_v^{f^+f^-} + \beta_v^{f^-f^+} \ge y_v^{f^+} + y_v^{f^-} - 1 \tag{28}$$

$$\alpha_v^{f^+f^-} + \beta_v^{f^+f^-} - 1 \le \lambda_v^{f^+f^-} \tag{29}$$

$$\lambda_v^{f^+f^-} \le \alpha_v^{f^+f^-} \tag{30}$$

$$\lambda_v^{f^+f^-} \le \beta_v^{f^+f^-} \tag{31}$$

$$\sum_{f' \in F^- : f^- \neq f'} \lambda_v^{f^-f'} + \sum_{f^+ \in F^+} \lambda_v^{f^-f^+} \le CAP_v - 1 \qquad \forall f^- \in F^-, \forall v \in V \tag{32}$$

Constraints from (12) to (20) are rewritten for the newly introduced aircraft $(f^- \in F^-)$ vs. aircrafts from previously solved set $(f^+ \in F^+)$ in Constraints (23-31) for determining if two aircraft spend any time at the same air-sector $\left(\lambda_{\mathrm{v}}^{f^-f^+} = 1\right)$. The presence of two aircraft from the set F^- at the same air-sector at the same time is determined through constraints (12-20). Finally, in constraint 32, it is ensured that no capacity violation is experienced.

3.2.3 Impact of Dynamic Capacity on ATFM

As discussed earlier, the capacity of an air-sector (CAP_v) may change over a period $T = \{t_1, t_2 \cdots, t_n\}$, depending on various factors such as weather conditions, military activities, etc. as illustrated in Figure 4. In this section, a dynamic capacity (CAP_v^t) control policy is modeled as part of the previously introduced MILP model.

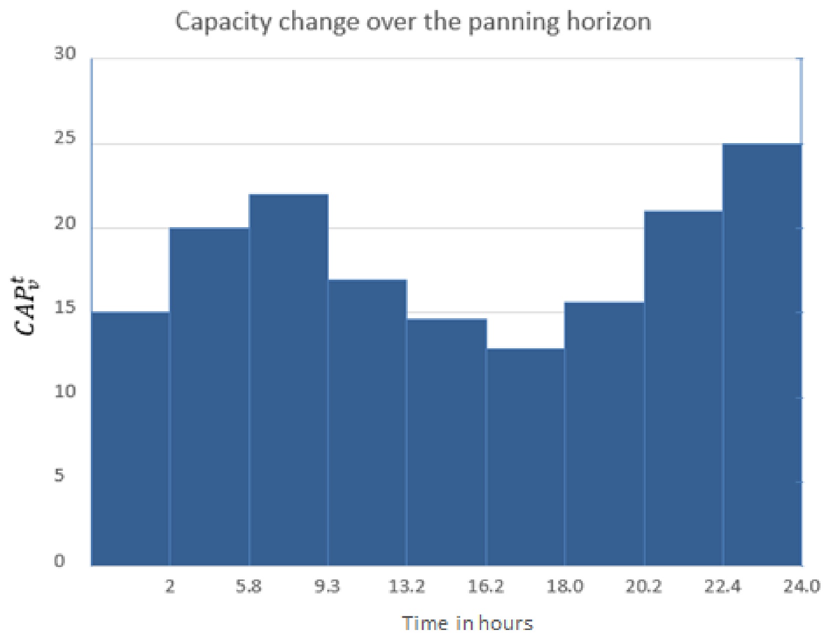


Figure 4: Capacity change over the time

Constraints (33-35) below are subject to: $\forall f \in F; \forall v \in V$.

$$a_v^f \leq t_i + \varepsilon - M\Psi_{iv}^f \qquad \forall t \in T \qquad (33)$$

$$t_{i+1} \leq a_v^f + \varepsilon - M\Omega_{iv}^f \qquad \forall t \in T \qquad (34)$$

$$\Psi_{iv}^f + \Omega_{iv}^f \geq 2z_{iv}^f \qquad \forall t \in T \qquad (35)$$

$$\sum_{i \in T} z_{iv}^f = y_v^f \qquad (36)$$

Constraint (33) checks the condition if an aircraft enters the air-sector after the time t_i: $\Psi_{iv}^f = \begin{cases} 1, & t_i \leq a_v^f \\ 0, & otherwise \end{cases}$ and Constraint (34) checks if the same aircraft enters the air-sector before time t_{i+1}: $\Omega_{iv}^f = \begin{cases} 1, & a_v^f \leq t_{i+1} \\ 0, & otherwise \end{cases}$; where ε is a small positive number, used to model constraints (33 and 34) with a "less than or equal to" condition instead of "less than" condition. Consequently, Constraints (35) along with the Constraint (36) introduces a new binary variable $\left(z_{iv}^f\right)$ to determine the index of the period that aircraft enters to the air-sector. In Constraint (36), it is ensured that all aircrafts with $y_v^f = 1$ are assigned to an appropriate capacity index z_{iv}^f. Finally, the Constraint (32) is modified to incorporate dynamic capacity changes in the model as:

$$\sum_{f' \in F^- : f^- \neq f'} \lambda_v^{f^- f'} + \sum_{f^+ \in F^+} \lambda_v^{f^- f^+} \leq \sum_{i \in T} z_{iv}^f \, CAP_v^i - 1 \tag{37}$$

4. Experimental Results and Discussions

In order to test the capabilities of the mathematical model, a set of sample cases have been generated. In all cases, the objective is to determine a set of conflict free en-route flight plans for all aircraft with a minimum operating cost (minimized delays and earliness, and the fuel consumption). Figure 5 illustrates a sample network with 50 air-sectors. In Table 1, a sample flight data is provided. For all flights, origin and destination airports and their scheduled arrival and departure times are randomly generated. All sectors have an original capacity of 4 aircraft.

Designed scenarios were tested on various synthetically generated traffic conditions. Corresponding mathematical models were solved in IBM ILOG CPLEX Optimization Studio 12.5.1.0, using Optimization Programming Language (OPL) on a personnel computer with 64 bit operating system, 3.40 GHz Intel Core i7-2600 CPU and 16.0 GB RAM.

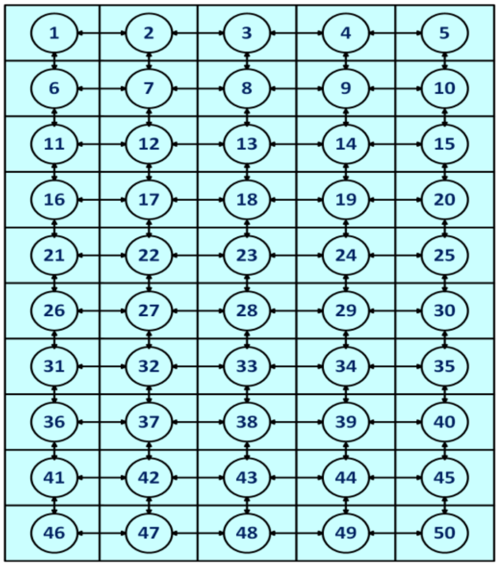


Figure 5: Sample Network diagram and network relations (50 air-sectors)

Table 1: Sample input data include origin and destination and scheduled arrival and departure times

Flight ID	Origin Node	Dest Node	Sch. Dept. Time	Sch. Ariv. Time
1	5	27	0	150
2	14	42	14	164
3	24	5	17	167
4	37	4	24	174
5	22	15	40	190
6	28	4	40	190
7	45	17	43	193
8	11	29	57	207
9	50	16	62	212
10	38	4	68	218
11	15	4	80	230
12	12	33	96	246
13	35	8	113	263
14	30	10	114	264
15	10	20	132	282

4.1 Solution Strategy 1: Global Optimality for the Planning Horizon

The main objective of the ATFM problem is to determine flight plans for all aircraft optimally at the beginning of a planning horizon. In other words, all aircraft are introduced to the system at the same time with their scheduled departure and arrival times, origin and destination airports as inputs. Consequently, the developed MILP formulation is solved for all aircrafts. Due to the nature of all scheduling and sequencing problems, the computation time increases exponentially as the number of aircrafts increases (see Table 2 for experimental results), which makes the approach unattractive for real-time implementation. Despite computational challenges, the proposed method is more accurate as it captures the arrival and departure times at sectors in continuous time domain. All other known methods formulate the time as discrete intervals during planning periods; therefore, the arrival and departure times of an aircraft at a sector is decided at the beginning of each period, which may lead to significant errors (Agustin, 2010, Potts, 2009). In the past, the time-space diagrams have been used by researchers to illustrate the schedule of aircrafts, trains or crews (Clausen, 2010). In our case, we utilize time-space diagrams to illustrate flight plans and assurance of capacity constraints. As shown in the time-space diagram (Figure 6), the proposed formulation determines arrival and departure times at air sectors in a continuous format and complies perfectly with the air-sector capacity constraints. Upper right corner of the figure 6, details of the highlighted section is depicted. In the highlighted section, lines that represent the aircraft, separated from each other by 0.5 point so the number of aircraft can be observed more clearly. The goal is not to have more than 4 lines at the same time between two black-solid lines. It is clear from the figure that, capacity constraint is respected at all times.

Table 2: Summary of Results for a network of 50 air-sectors and 170 links

Number of aircrafts	Execution time (seconds)	Objective function: Minimize Total Delays/Earliness	Execution Time per aircraft (seconds)
10	3.11	885.00	0.31
15	6.77	975.00	0.45
20	88.11	1,184.00	4.41
25	248.00	1,437.00	9.92
30	391.72	1,673.00	13.06
35	33,507	2,004.00	957.34

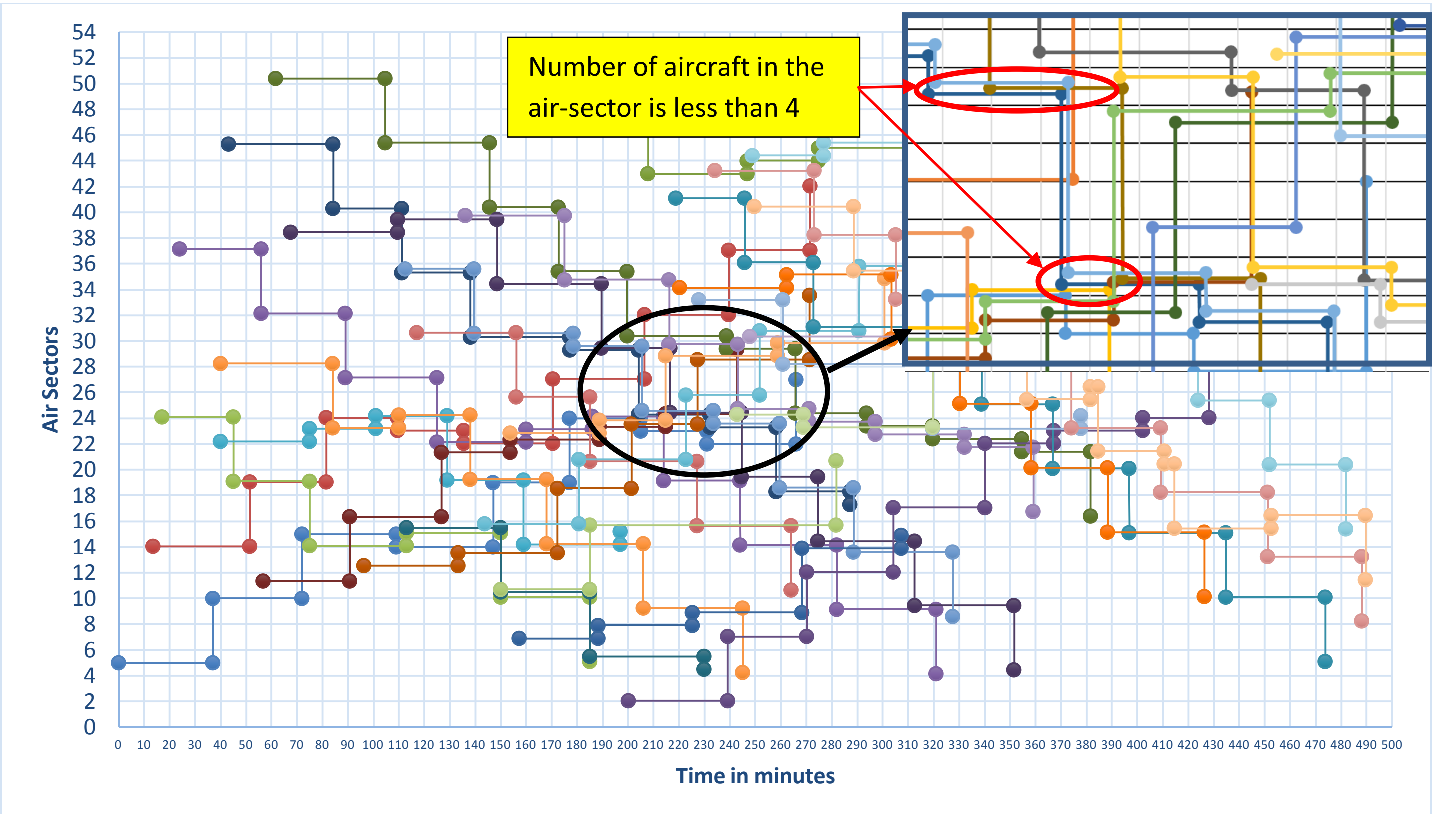


Figure 6-Time-Space Diagram for 30 aircraft

4.2 Solution Strategy 2: Sequential Solution

Due to the business objectives of airline companies and airports, aircrafts leave their origin airports in a sequential order according to their scheduled departure times. Consequently, there is only a small chance for two aircraft leaving from the same airport to be in a conflict situation. In this section, we take advantage of the sequential departure rule and solve the test cases according to the formulation described in section 3.2.2. Aircrafts are introduced to the model in batches according to their departure times. At the beginning of the planning horizon, it is assumed that the airspace is empty; hence the performance measures (flight-time) for earlier flights are found to be better. As time advances and the traffic conditions reach a steady state, the performance measures reflect realistic flight conditions.

In order to test the proposed sequential solution approach, a problem set with 355 flights was randomly generated. In average, every 2 minutes, a new aircraft is introduced to the system. Aircrafts stayed in the airspace for average of 400 minutes. A steady state conditions were observed after 225 aircrafts were introduced to the airspace (average number of aircrafts in the airspace at the steady-state was 232). As expected, the sequential solution method converge to a solution much faster. Each batch can be solved at an average 164 seconds and a solution for all 355 aircrafts was obtained in 194 minutes. However, majority of this time was due to post-processing between batches. The actual processing time at the solver was recorded 3.8 seconds per batch and 269 seconds for all 355 aircrafts. Once steady state is reached, the processing time to solve each batch is steady as well (see Figure 7 for illustration).

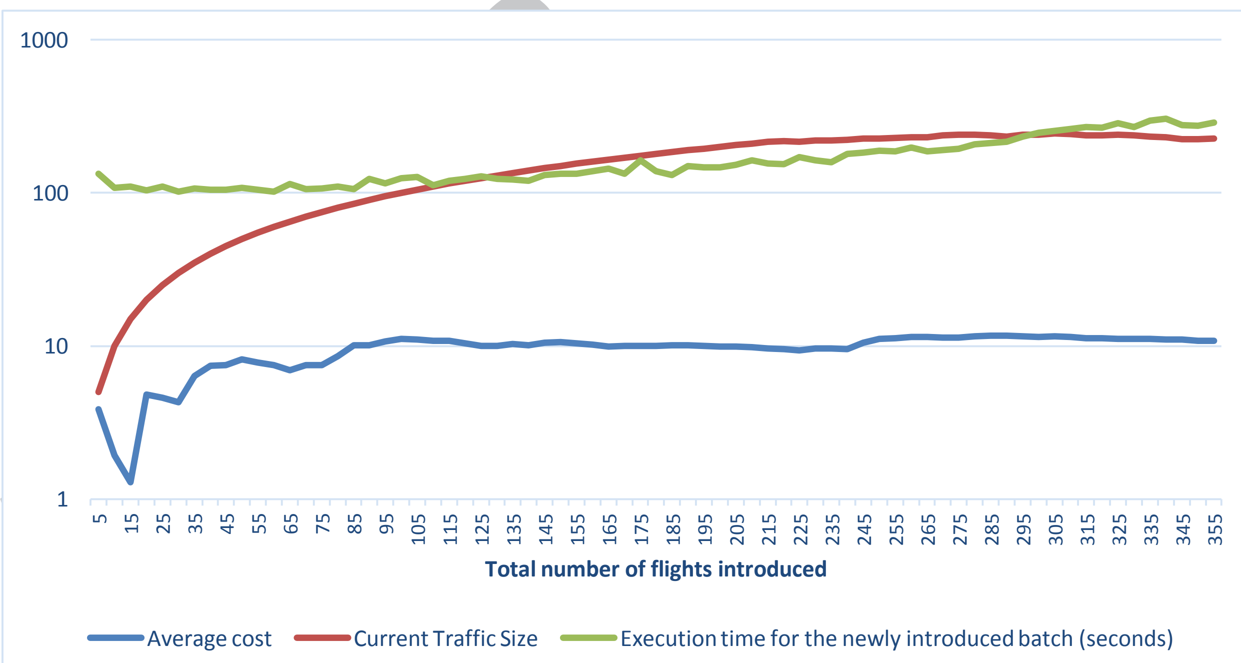


Figure 7: Results of sequential solution for 355 aircrafts on a logarithmic scale

5. Conclusions and Future Work

This paper introduces an air traffic flow management formulation using continuous time decision variables. The en-route flight plans from origin to destination airports are determined as a sequence of visited air sectors and the arrival and departure times to these sectors for all aircrafts with an objective to minimize the flight cost incurred from the delays and fuel consumption. The proposed MILP model successfully introduces the speed dependent fuel consumption calculation in the formulation. Moreover, the dynamic air-sector capacity change due to various factors such as weather conditions has been explicitly modeled. Finally, the performance of the model is tested on a various traffic conditions using different solution strategies. The proposed sequential solution strategy is capable of tackling real-size traffic conditions. While sequential solution methods seem to favor earlier flights, at the steady-state traffic conditions such behavior is not observed. Hence it can be concluded that the proposed model can successfully help decision makers to determine en-route flight plans for all aircraft safely and more economically.

In a future work, more advanced solution strategies such as column generation can be utilized to solve the proposed mathematical model. The mathematical model discussed in the paper provides further opportunities for the operations research community to improve the computation time of the ATFM problem with continuous time variables.

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HIGHLIGHTS

- Air traffic flow management problem with continuous time variables.
- Use of time segmentation where decisions are made in a periodical order is omitted
- Travelling time is formulated as a function of speed using linear expressions
- A linear approximation for the fuel consumption as a function of speed is introduced
- Results for a number of case studies are discussed