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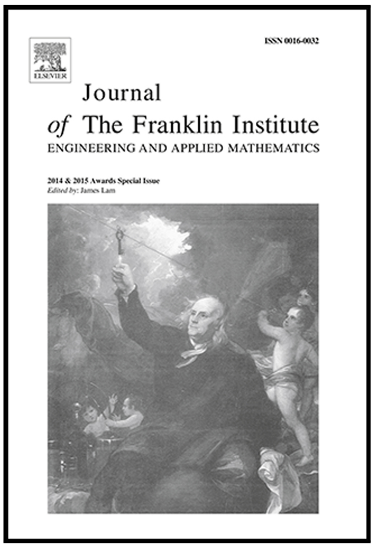

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Highlights

- Formation control of mobile agents based on the cyclic pursuit idea is studied.
- The existing results have limitations to control the swarm centroid and angular rate.
- Under the proposed approach, the swarm centroid and angular rate are controllable.
- This issue increases the applications of the proposed strategy in the real world.

A Cyclic Pursuit Framework for Networked Mobile Agents Based on Vector Field Approach

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Abstract

This paper proposes a pursuit formation control scheme for a network of double-integrator mobile agents based on a vector field approach. In a leaderless architecture, each agent pursues another one via a cyclic topology to achieve a regular polygon formation. On the other hand, the agents are exposed to a rotational vector field such that they rotate around the vector field centroid, while they keep the regular polygon formation. The main problem of existing approaches in the literature for cyclic pursuit of double-integrator multiagent systems is that under those approaches, the swarm angular velocity and centroid are not controllable based on missions and agents capabilities. However, by employing the proposed vector field approach in this paper, while keeping a regular polygon formation, the swarm angular velocity and centroid can be determined arbitrary. The obtained results can be extended to achieve elliptical formations with cyclic pursuit as well. Simulation results for a team of eight mobile agents verify the accuracy of the proposed control scheme.

Keywords: Cyclic pursuit, double-integrators, formation control, mobile agents, swarm, vector field.

1. Introduction

Autonomous coordination of mobile agents has been an interesting area of research in robotics and control engineering over the past decade. A multiagent system (MAS) offers a lot of advantages such as reliability, flexibility, and robustness, and has lots of applications in surveillance, search and rescue missions, maintenance, monitoring missions, and so on [1, 2]. One of challenging problems in this area of research is to maintain relative positions among agents in a desired formation. Based on the type of communication topology associated with a MAS, formation control approaches can be categorized in two classes, namely, *acyclic* and *cyclic*. In acyclic approaches, agents are connected via a directed topology, and no loops exist in the associated communication topology. In an acyclic approach, an agent is considered as a leader and other agents are followers which track the leader. Since there are no loops in the associated communication topology,

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stability analysis of this structure is reduced to two agents behaviors (an agent following another agent) which simplifies stability analysis and implementation of these approaches. However, all the decisions are made by the leader, and no feedbacks are sent from the followers to the leader [3, 4]. To increase the degree of precision in leader tracking and formation keeping, the leader can be considered virtually with a known trajectory [5, 6]. On the other hand, in a cyclic approach, agents are connected via a topology such as a ring topology such that no independent leader exists in the MAS. In other words, each agent is a follower of another agent, and compared with acyclic approaches, this issue increases the degree of autonomy in the MAS [7, 8, 9].

Cyclic pursuit is one of swarming behaviors based on cyclic communication topologies which for first time was mathematically studied in [10] and [11]. This behavior is inspired by biological organisms such as beetles, ants, and dogs to evade predators or search foods. Indeed, by their rotational motion around a centroid, the probability to find predators or foods is increased [7, 8]. This maneuver recently has been employed for regular polygon formation control of mobile agents. In this condition, perfect coverage would be feasible by employing a limited number of agents. For instance, in [12], regular polygon formations of first-order kinematics based on cyclic topologies around a centroid was studied in which the centroid was obtained from an agreement problem. Based on that idea, regular polygon formations of unicycle kinematics with cyclic pursuit was studied in [7] and [8] in which each agent pursued another one in a regular polygon formation, while all the agents rotated on a circle. Since the agents were considered with constant forward speeds, in the case of identical speeds, they rotated after achieving a regular polygon formation. That approach was extended to first-order kinematics with unit speeds under bidirected time-varying networks in [13]. Similar approaches were introduced in [14] and [15] in which each agent speed was proportional to the distance from the leading agent. In [16], a vision based methodology for cyclic pursuit of first-order kinematics in regular polygon formations was proposed. In [17] and [18], the application of cyclic pursuit for the target-capturing problem in first-order kinematics was studied. Pursuit strategies for non-holonomic first-order agents with different constant speeds were introduced in [19] and [20]. In [21] and [22], the cyclic pursuit idea was employed for rendezvous of first-order kinematics, and in [23, 24, 25, 26], circular motion of non-holonomic first-order agents was studied.

In practice, a real robot cannot be modeled precisely by first-order kinematics, because the motion of a real robot should be controlled by using engines or motors which provide forces and torques; therefore, the behavior of a real robot should be described by a second-order model [27, 28, 29, 30]. Hence, pursuit strategies for formation control of double-integrator mobile agents were developed as well. For instance, in [31], pursuit formation control of double-integrator mobile agents with application to spacecraft formation flying was studied. Under those approaches, achieving regular polygon formations of double-integrator mobile agents was guaranteed, and it was shown that the swarm angular velocity only depended on the number of agents. By using the feedback linearization technique, that approach was employed for formation control of mobile robots in [32], and in [33], by using the trajectory information of a moving target, those results were extended to moving target tracking while keeping a regular polygon formation around the target. The main problem of the above-mentioned approaches was that under those approaches, the swarm angular velocity was not controllable which limited the applications of those approaches. In other words, under those approaches, the swarm angular velocity could not be controlled based on missions and agents capabilities. In [34], a hierarchical strategy for pursuit formation control of double-integrators mobile agents was proposed. However, under that approach, the swarm centroid was not controllable, and hence it could not be applicable for missions such as monitoring a specific environment or guarding a specific target.

This paper is devoted to cyclic pursuit control of double-integrator MASs in regular polygon formations. Under the proposed control scheme, the agents are exposed to a clockwise/counter-clockwise rotational vector field around a centroid. In this condition, based on a cyclic (ring) communication topology, the agents keep relative angles around the centroid such that a regular polygon formation with cyclic pursuit is achieved. Accordingly, the performance of the MAS under the combination of the vector field and the pursuit formation control strategy is mathematically analyzed. Despite existing similar approaches in the literature, by tuning the vector field parameters, it is feasible to control the swarm angular velocity and centroid in arbitrary values. The obtained results are extendable to elliptical formations with cyclic pursuit as well. In summary, compared with existing similar results in the literature, the main advantages of the proposed approach in this paper are as follows:

- Considering double integrator dynamics which makes the results more practical than the existing approaches in the literature based on first-order kinematics and unicycles with constant forward speeds.
- The ability to control the swarm centroid and angular velocity arbitrary.

It should be noted that in [33, 35, 36, 37, 38, 39, 40, 41], the problem of cooperative control of MASs under cyclic topologies is studied. However, those approaches mainly differ from the proposed approach in this paper since no regular polygon formation keeping is studied.

The rest of the paper is organized as follows. Preliminaries are provided in the next section. The problem statement is given in Section 3. Section 4 introduces the rotational vector field. The pursuit formation control scheme is presented in Section 5, and the obtained results are extended to elliptical formations in Section 6. Finally, Section 7 provides simulation results, and conclusions reside in Section 8.

2. Preliminaries

In this section, notations and the concept of circulant matrices are provided.

2.1. Notations

$\mathbb{R}$ expresses the set of real numbers, and $\mathbb{R}_+$ presents the set of positive real numbers. $\mathbb{C}$ denotes the set of complex numbers. $\mathbf{0}_{n\times m}$ is an $n \times m$ matrix with zero entries. $\mathbf{1}_n$ is an $n \times 1$ vector with all entries equal to 1. $\otimes$ denotes the Kronecker product. x^* denotes the transpose conjugate of x. x_0 expresses the initial value of x. min means minimum. $\Re(\cdot)$ denotes the real part of a complex number. $\mathrm{sgn}(\cdot)$ expresses the sign function. $\det(\cdot)$ denotes the determinant of a matrix. $\mathrm{diag}(x_1, x_2, \ldots, x_n)$ shows a diagonal matrix composed of $x_1, x_2, \ldots, x_n$, and $\mathrm{R}(\cdot)$ presents a rotation matrix which can be stated as follows:

$$\mathrm{R}(\cdot) = \begin{bmatrix} \cos(\cdot) & \sin(\cdot) \\ -\sin(\cdot) & \cos(\cdot) \end{bmatrix}$$

where for $\theta_1, \theta_2 \in \mathbb{R}$, we have $\mathrm{R}(\theta_1 + \theta_2) = \mathrm{R}(\theta_1)\mathrm{R}(\theta_2)$, and $\mathrm{R}(\theta_1)^{-1} = \mathrm{R}(-\theta_1)$.

2.2. Circulant Matrices

Consider $c_1, c_2, \dots, c_n \in \mathbb{R}$. A circulant matrix composed of $c_1, c_2, \dots, c_n$ can be defined as follows:

$$\mathrm{circ}(c_1, c_2, \dots, c_n) = \begin{bmatrix} c_1 & c_2 & \dots & c_n \\ c_n & c_1 & \dots & c_{n-1} \\ \vdots & \vdots & \ddots & \vdots \\ c_2 & c_3 & \dots & c_1 \end{bmatrix}. \tag{1}$$

In other words, the matrix defined in (1) can be specified by $c_1, c_2, \dots, c_n$ which appear in the first row of the matrix, respectively, and each row of the matrix is formed by its previous row that is right shifted with wrapping around. Moreover, the following important lemma holds for a circulant matrix.

Lemma 1. *[42] By defining $\omega = e^{\frac{2\pi}{n}\mathrm{j}}$ where $\mathrm{j} = \sqrt{-1}$, the circulant matrix defined in (1) can be stated as follows:*

$$\mathrm{circ}(c_1, c_2, \dots, c_n) = \mathbf{F}_n^* \mathrm{diag}(d_1, d_2, \dots, d_n)\mathbf{F}_n$$

in which $\mathbf{F}_n^*\mathbf{F}_n = \mathbf{I}_n$ where

$$\mathbf{F}_n^* = \frac{1}{\sqrt{n}} \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \omega & \omega^2 & \dots & \omega^{n-1} \\ 1 & \omega^2 & \omega^4 & \dots & \omega^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{n-1} & \omega^{2(n-1)} & \dots & \omega^{(n-1)(n-1)} \end{bmatrix}. \tag{2}$$

In this condition,

$$\begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} = \sqrt{n}\mathbf{F}_n^* \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}. \tag{3}$$

In the next section, the mentioned cyclic pursuit problem is stated in more details.

3. Problem Statement

Consider a team of n double-integrator mobile agents which the ith one is described as follows:

$$\ddot{\mathbf{p}}_i = \mathbf{u}_i \tag{4}$$

where $\mathbf{p}_i \in \mathbb{R}^2$ denotes the agent position with respect to a fixed coordinate frame and $\mathbf{u}_i \in \mathbb{R}^2$ is the control input. The objective is to propose a control strategy for the MAS such that the ith agent pursues the $(i+1)$th one (with wrapping around) with a relative angle $\vartheta \in \mathbb{R}$ around a centroid $\mathbf{c} \in \mathbb{R}^2$ and with an arbitrary desired angular velocity $\varpi \in \mathbb{R}$. This centroid can be a

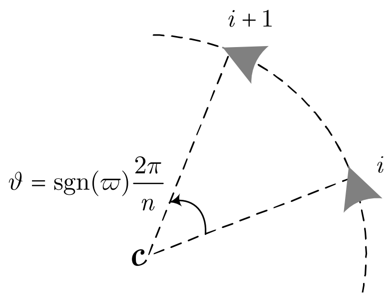

Figure 1: Cyclic pursuit configuration where the direction of motion of each agent is depicted by an arrow.

specific position in an environment to be covered or a target to be guarded. To achieve this goal, by considering Fig. 1, the desired trajectory of the ith agent can be stated as follows:

$$\mathbf{p}_{di} = \mathbf{c} + \mathrm{R}(\vartheta)(\mathbf{p}_{i+1} - \mathbf{c}) \tag{5}$$

where $\mathbf{p}_{di}$ is the desired trajectory of the ith agent in the formation. If we suppose that the ith agent is following the $(i+1)$th one, a regular polygon of the agents can be achieved if

$$\vartheta = \mathrm{sgn}(\varpi)\frac{2\pi}{n}.$$

We suppose that each agent can measure its position and velocity in the fixed coordinate frame.

Remark 1. It is worth mentioning that although the formation centroid is known for the swarm, this global information is necessary for missions such as monitoring a specific environment or guarding a specific target. These missions are basically different from those considered in [7, 12, 34] in which the agents reached agreement on a centroid. In those papers, regular polygon formations were achieved based on formulations such that there was no control on the swarm centroid. Hence, they were not applicable for missions such as monitoring a specific environment or guarding a specific target.

Following the desired trajectory defined in (5), if all the agents are exposed to a rotational vector field around $\mathbf{c}$, a regular polygon formation of the mobile agents with cyclic pursuit is achieved. To achieve this goal, first let us propose a rotational vector field around the centroid $\mathbf{c}$ under which each agent rotates with angular velocity ϖ.

4. Rotational Vector Field

Using vector fields is one of popular techniques for motion planning and coordination of mobile agents which was introduced in [43, 44, 45] for manipulators and mobile robots obstacle avoidance. According to this technique, any point of a plane or space is exposed to a virtual velocity/force vector based on its relative position with respect to the origin of the vector field. Hence, by choosing the vector field functions properly, it is feasible to lead a mobile agent toward a desired point/path for path planning or target seeking.

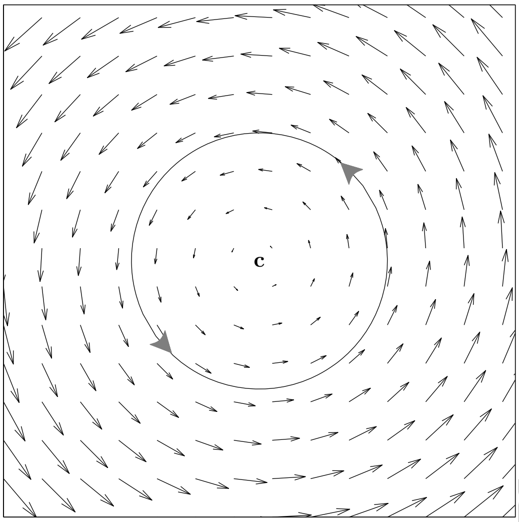

Figure 2: Trajectory of a mobile agent exposed to a counter-clockwise vector field around the centroid $\mathbf{c}$ when ϖ is positive. The vector field will be clockwise if ϖ is negative.

In [46], a first-order rotational vector field (based on position measurement) around rectangular obstacles was introduced such that it provided repulsive vectors for obstacle avoidance. However, to guarantee rotational motion of double-integrators, it is required to extend it to a second-order vector field (based on position and velocity measurement). The following theorem develops a second-order vector field for rotational motion of double-integrator mobile agents which is shown in Fig. 2.

Theorem 1. *Consider the MAS described in (4). The following control law provides a rotational vector field under which the ith agent rotates around a centroid* $\mathbf{c}$ *with angular velocity* ϖ*:*

$$\mathbf{u}_i = (\lambda\varpi\mathbf{T} - \varpi^2\mathbf{I}_2)\bar{\mathbf{p}}_i - \lambda\dot{\mathbf{p}}_i \tag{6}$$

where $\bar{\mathbf{p}}_i = \mathbf{p}_i - \mathbf{c}$, $\lambda \in \mathbb{R}_+$, *and*

$$\mathbf{T} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

Proof. By substituting (6) into (4), one can get

$$\ddot{\mathbf{p}}_i = (\lambda\varpi\mathbf{T} - \varpi^2\mathbf{I}_2)\bar{\mathbf{p}}_i - \lambda\dot{\mathbf{p}}_i. \tag{7}$$

Then, since $\dot{\mathbf{c}} = \ddot{\mathbf{c}} = \mathbf{0}_{2\times1}$, the Laplace transformation of (7) yields

$$\bar{\mathbf{p}}_i^s = (s^2\mathbf{I}_2 + \lambda\mathbf{I}_2 s - \lambda\varpi\mathbf{T} + \varpi^2\mathbf{I}_2)^{-1}((\lambda + s)\bar{\mathbf{p}}_{i0} + \dot{\mathbf{p}}_{i0})$$

where $\bar{\mathbf{p}}_i^s$ indicates the Laplace transform of $\bar{\mathbf{p}}_i$. The poles of the system describe its response which can be obtained from the roots of the following characteristic equation:

$$\det(s^2\mathbf{I}_2 + \lambda\mathbf{I}_2 s - \lambda\varpi\mathbf{T} + \varpi^2\mathbf{I}_2) = 0$$

implying that

$$\det\left(\begin{bmatrix} s^2+\lambda s+\varpi^2 & \lambda\varpi \\ -\lambda\varpi & s^2+\lambda s+\varpi^2 \end{bmatrix}\right)=0. \tag{8}$$

Therefore, the poles of the system can be obtained from solution of (8) which results in

$$\begin{aligned} s_{1,2} &= \pm \mathrm{j}\varpi, \\ s_{3,4} &= -\lambda \pm \mathrm{j}\varpi. \end{aligned}$$

Since $\lambda > 0$, s_3 and s_4 are stable and they will not appear in the steady state response of the agent. In this condition, as $t \to \infty$, the agent response can be represented as follows:

$$\bar{\mathbf{p}}_i^s = \frac{\mathbf{a}_i}{s-\mathrm{j}\varpi} + \frac{\mathbf{a}_i^*}{s+\mathrm{j}\varpi}$$

where $\mathbf{a}_i \in \mathbb{C}$. Therefore, there exists $a_{1i}, a_{2i}, \varphi_{1i}, \varphi_{2i} \in \mathbb{R}$ such that as $t \to \infty$,

$$\bar{\mathbf{p}}_i = \begin{bmatrix} a_{1i}e^{\phi_{1i}} + a_{1i}e^{-\phi_{1i}} & a_{2i}e^{\phi_{2i}} + a_{2i}e^{-\phi_{2i}} \end{bmatrix}^\top \tag{9}$$

where

$$\begin{aligned} \phi_{1i} &= \mathrm{j}(\varphi_{1i} + \varpi t), \\ \phi_{2i} &= \mathrm{j}(\varphi_{2i} + \varpi t). \end{aligned}$$

By substituting (9) into (7), one can get

$$\begin{aligned} a_{1i} &= a_{2i}, \\ \varphi_{2i} &= \varphi_{1i} - \frac{\pi}{2} \end{aligned}$$

which imply rotational motion of the agent on a circle around the centroid $\mathbf{c}$ with angular velocity ϖ, and this completes the proof. ■

Example 1. Consider two mobile agents with initial positions [6 4] and [2 5] and initial velocities $[2\ -1]$ and $[-2\ -4]$, respectively. Let $\mathbf{c} = [5\ 5]^\top$, $\lambda = 1$, and their desired angular velocity be -1 and 1, respectively. Under the vector field defined in (6), the trajectories of the agents are depicted in Fig. 3 which confirm the agents rotational motion around $\mathbf{c}$.

Based on the proposed vector field, the next section proposes a formation control strategy to achieve regular polygon formations of double-integrator mobile agents with cyclic pursuit around $\mathbf{c}$.

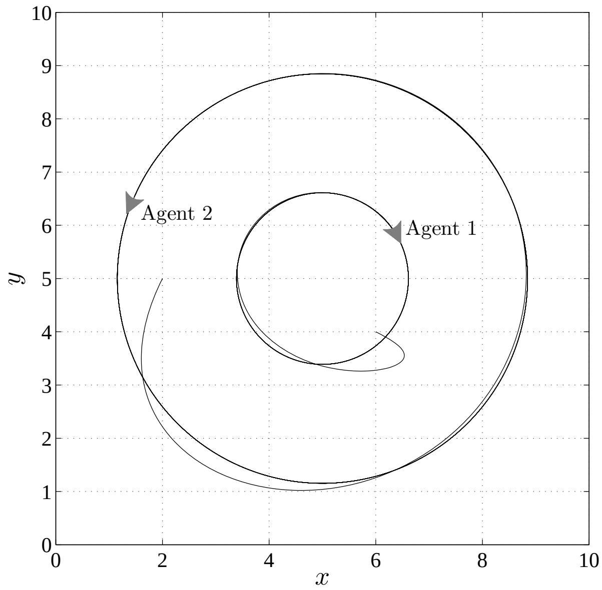

Figure 3: Trajectories of the agents in Example 1.

5. Regular Formations of Mobile Agents

Suppose that the ith mobile agent is exposed to a rotational vector field, while by receiving the position and velocity information, $(\mathbf{p}_{i+1}, \dot{\mathbf{p}}_{i+1})$, from the $(i+1)$th agent, it follows the desired trajectory defined in (5). To achieve this goal, we will design a control law as an additive term in (7) such that a regular polygon formation of mobile agents with cyclic pursuit around $\mathbf{c}$ is obtained. Therefore, the ith agent dynamics can be considered as follows:

$$\ddot{\mathbf{p}}_i = \mathbf{u}_{ri} + \mathbf{u}_{ci} \tag{10}$$

where $\mathbf{u}_{ri}$ provides the rotational vector field defined in (6), and $\mathbf{u}_{ci}$ is the formation control strategy. In this condition, by substituting (6) into (10), one can get

$$\ddot{\mathbf{p}}_i = (\lambda\varpi\mathbf{T} - \varpi^2\mathbf{I}_2)\bar{\mathbf{p}}_i - \lambda\dot{\mathbf{p}}_i + \mathbf{u}_{ci} \tag{11}$$

which since $\dot{\mathbf{c}} = \ddot{\mathbf{c}} = \mathbf{0}_{2\times1}$, can be represented in state space form as follows:

$$\begin{bmatrix} \ddot{\bar{\mathbf{p}}}_i \\ \dot{\bar{\mathbf{p}}}_i \end{bmatrix} = \mathbf{M}\begin{bmatrix} \dot{\bar{\mathbf{p}}}_i \\ \bar{\mathbf{p}}_i \end{bmatrix} + \mathbf{B}\mathbf{u}_{ci} \tag{12}$$

where

$$\mathbf{M} = \begin{bmatrix} -\lambda\mathbf{I}_2 & \lambda\varpi\mathbf{T} - \varpi^2\mathbf{I}_2 \\ \mathbf{I}_2 & \mathbf{0}_{2\times2} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{0}_{2\times2} \end{bmatrix}.$$

Therefore, the objective is to design $\mathbf{u}_{ci}$ for the ith agent to follow the desired trajectory defined in (5), while it satisfies (7). In other words, under the proposed vector field, while satisfying (7), as $t \to \infty$, $\mathbf{u}_{ci}$ should guarantee

$$\bar{\mathbf{p}}_i = \mathrm{R}(\vartheta)\bar{\mathbf{p}}_{i+1}$$

which implies a regular polygon formation. The following theorem proposes a proper control law for $\mathbf{u}_{ci}$. The main idea of the proposed control law is based on the consensus theory such that the agents reach a regular polygon formation in the rotational vector field. Indeed, since no leader exists in the network, a consensus-like formulation is employed.

Theorem 2. *Consider the double-integrator MAS defined in (4) exposed to a rotational vector field defined in (6). Considering the control law presented in (10)-(12), under a cyclic communication topology, a regular polygon formation is achieved by designing* $\mathbf{u}_{ci}$ *as*

$$\mathbf{u}_{ci} = \mathrm{R}(-i\vartheta)\mathbf{K}\begin{bmatrix}\dot{\xi}_i - \dot{\xi}_{i+1}\\ \xi_i - \xi_{i+1}\end{bmatrix} \tag{13}$$

in which $\xi_i = \mathrm{R}(i\vartheta)\bar{\mathbf{p}}_i$*, and*

$$\mathbf{K} = -\varepsilon\Big(1 - \cos\frac{2\pi}{n}\Big)^{-1}\mathbf{B}^\top\mathbf{P} \tag{14}$$

where $\varepsilon \geq 1$ *which is an arbitrary parameter, and* $\mathbf{P}$ *is a symmetric positive definite matrix obtained from solution of the following Riccati equation:*

$$\mathbf{M}^\top\mathbf{P} + \mathbf{P}\mathbf{M} + \mathbf{I}_4 - \mathbf{P}\mathbf{B}\mathbf{B}^\top\mathbf{P} = \mathbf{0}_{4\times4}. \tag{15}$$

Proof. By multiplying (12) by $\mathbf{I}_2 \otimes \mathrm{R}(i\vartheta)$, one can get

$$\begin{bmatrix}\ddot{\xi}_i\\ \dot{\xi}_i\end{bmatrix} = \begin{bmatrix}-\lambda\mathrm{R}(i\vartheta) & \lambda\varpi\mathrm{R}(i\vartheta)\mathbf{T} - \varpi^2\mathrm{R}(i\vartheta)\\ \mathrm{R}(i\vartheta) & \mathbf{0}_{2\times2}\end{bmatrix}\begin{bmatrix}\dot{\bar{\mathbf{p}}}_i\\ \bar{\mathbf{p}}_i\end{bmatrix} + \begin{bmatrix}\mathrm{R}(i\vartheta)\\ \mathbf{0}_{2\times2}\end{bmatrix}\mathbf{u}_{ci}$$

where since $\mathrm{R}(i\vartheta)\mathbf{T} = \mathbf{T}\mathrm{R}(i\vartheta)$, it can be expressed as

$$\begin{bmatrix}\ddot{\xi}_i\\ \dot{\xi}_i\end{bmatrix} = \mathbf{M}\begin{bmatrix}\dot{\xi}_i\\ \xi_i\end{bmatrix} + \mathbf{B}\mathrm{R}(i\vartheta)\mathbf{u}_{ci}. \tag{16}$$

Therefore, substituting (13) into (16) leads to

$$\begin{bmatrix}\ddot{\xi}_i\\ \dot{\xi}_i\end{bmatrix} = \mathbf{M}\begin{bmatrix}\dot{\xi}_i\\ \xi_i\end{bmatrix} + \mathbf{B}\mathbf{K}\begin{bmatrix}\dot{\xi}_i - \dot{\xi}_{i+1}\\ \xi_i - \xi_{i+1}\end{bmatrix}$$

which can be stated for the whole MAS as follows:

$$\dot{\Xi} = (\mathbf{I}_n \otimes \mathbf{M} + \mathbf{L} \otimes \mathbf{B}\mathbf{K})\Xi \tag{17}$$

where

$$\Xi = \begin{bmatrix}\dot{\xi}_1^\top & \xi_1^\top & \dot{\xi}_2^\top & \xi_2^\top & \ldots & \dot{\xi}_n^\top & \xi_n^\top\end{bmatrix}^\top,$$

and $\mathbf{L} \in \mathbb{R}^{n\times n}$ specifies the MAS communication topology which has a circulant form as

$$\mathbf{L} = \mathrm{circ}(1, -1, 0, \ldots, 0).$$

On the other hand, since $\mathbf{L}$ is a circulant matrix, by invoking Lemma 1, it can be restated in the following form:

$$\mathbf{L} = \mathbf{F}_n^* \mathbf{J} \mathbf{F}_n$$

where by considering (2) and (3) and by letting $\nu = n - 1$, it can be said that $\mathbf{J}$ has a diagonal form as

$$\mathbf{J} = \text{diag}(0, 1 - \omega, 1 - \omega^2, \ldots, 1 - \omega^\nu). \tag{18}$$

Therefore, (17) can be redefined as follows:

$$\dot{\Xi} = (\mathbf{I}_n \otimes \mathbf{M} + \mathbf{F}_n^* \mathbf{J} \mathbf{F}_n \otimes \mathbf{BK})\Xi. \tag{19}$$

If we define $\Theta = (\mathbf{F}_n \otimes \mathbf{I}_4)\Xi$, (19) yields

$$\dot{\Theta} = (\mathbf{I}_n \otimes \mathbf{M} + \mathbf{J} \otimes \mathbf{BK})\Theta. \tag{20}$$

Now, by considering (18), (20) can be restated as

$$\dot{\Theta} = \begin{bmatrix} \mathbf{M} & \mathbf{0}_{4\times4\nu} \\ \mathbf{0}_{4\nu\times4} & \mathbf{I}_\nu \otimes \mathbf{M} + \acute{\mathbf{J}} \otimes \mathbf{BK} \end{bmatrix} \Theta$$

where

$$\acute{\mathbf{J}} = \text{diag}(1 - \omega, 1 - \omega^2, \ldots, 1 - \omega^\nu) \tag{21}$$

which implies that

$$\Theta = \begin{bmatrix} e^{\mathbf{M}t} & \mathbf{0}_{4\times4\nu} \\ \mathbf{0}_{4\nu\times4} & e^{\mathbf{I}_\nu \otimes \mathbf{M}t + \acute{\mathbf{J}} \otimes \mathbf{BK}t} \end{bmatrix} \Theta_0. \tag{22}$$

Now, let us analyze $e^{\mathbf{I}_\nu \otimes \mathbf{M}t + \acute{\mathbf{J}} \otimes \mathbf{BK}t}$. By considering (14) and (21), we have

$$\mathbf{I}_\nu \otimes \mathbf{M} + \acute{\mathbf{J}} \otimes \mathbf{BK} = \text{diag}(\Delta_1, \Delta_2, \ldots, \Delta_\nu) \tag{23}$$

where

$$\Delta_j = \mathbf{M} - \varepsilon\Big(1 - \cos\frac{2\pi}{n}\Big)^{-1}(1 - \omega^j)\mathbf{BB}^\top\mathbf{P}.$$

On the other hand, $1 - \omega^j$, $j \in \{1, 2, \ldots, \nu\}$, are depicted in Fig. 4. Therefore, by considering Fig. 4, it can be observed that

$$\min_{j=1,2,\ldots,\nu} \Re(1 - \omega^j) = 1 - \cos\frac{2\pi}{n}. \tag{24}$$

In this condition, since $\varepsilon \geq 1$, from (24), one can conclude that

$$\Re\bigg(\varepsilon\Big(1 - \cos\frac{2\pi}{n}\Big)^{-1}(1 - \omega^j)\bigg) \geq 1, j \in \{1, 2, \ldots, \nu\}. \tag{25}$$

It is easy to see that the pair $(\mathbf{M}, \mathbf{B})$ is controllable and the Riccati equation given in (15) has a solution. In this condition, since $\mathbf{P}$ is obtained from the Riccati equation of the pair $(\mathbf{M}, \mathbf{B})$, $\mathbf{M} - \mathbf{BB}^\top\mathbf{P}$ is Hurwitz. As a result, for any gain $\Re(\kappa) \geq 1$, $\mathbf{M} - \kappa\mathbf{BB}^\top\mathbf{P}$ is Hurwitz [47]; hence,

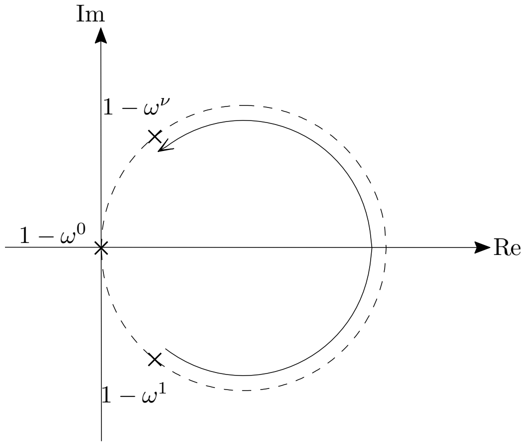

Figure 4: The eigenvalues of $\mathbf{L}$.

from (25), it follows that Δ_j, $j \in \{1, 2, \ldots, \nu\}$, are Hurwitz which by considering (23), they result in

$$\lim_{t\to\infty} e^{\mathbf{I}_\nu\otimes\mathbf{M}t+\hat{\mathbf{J}}\otimes\mathbf{BK}t} = \mathbf{0}_{4\nu\times 4\nu}. \tag{26}$$

Therefore, from (26), as $t \to \infty$, (22) leads to

$$\Theta = \begin{bmatrix} e^{\mathbf{M}t} & \mathbf{0}_{4\times 4\nu} \\ \mathbf{0}_{4\nu\times 4} & \mathbf{0}_{4\nu\times 4\nu} \end{bmatrix} \Theta_0.$$

Now, since $\Theta = (\mathbf{F}_n \otimes \mathbf{I}_4)\Xi$, as $t \to \infty$, we have

$$\Xi = (\mathbf{F}_n^* \otimes \mathbf{I}_4) \begin{bmatrix} e^{\mathbf{M}t} & \mathbf{0}_{4\times 4\nu} \\ \mathbf{0}_{4\nu\times 4} & \mathbf{0}_{4\nu\times 4\nu} \end{bmatrix} (\mathbf{F}_n \otimes \mathbf{I}_4)\Xi_0. \tag{27}$$

In this condition, by substituting $\mathbf{F}_n$ from (2) into (27), one can observe that

$$\Xi = \left(\frac{\mathbf{1}_n\mathbf{1}_n^\top}{n} \otimes e^{\mathbf{M}t}\right)\Xi_0$$

which implies that

$$\begin{bmatrix} \dot{\xi}_i \\ \xi_i \end{bmatrix} = e^{\mathbf{M}t}\text{ave}\left(\begin{bmatrix} \dot{\xi}_0 \\ \xi_0 \end{bmatrix}\right), i \in \{1, 2, \ldots .n\},$$

and therefore, $\xi_i = \xi_{i+1}$ and $\dot{\xi}_i = \dot{\xi}_{i+1}$. Hence, as $t \to \infty$, the following results can be obtained:

i) Since $\xi_i = \xi_{i+1}$ and $\xi_i = \mathrm{R}(i\vartheta)\bar{\mathbf{p}}_i$, one can get

$$\mathrm{R}(i\vartheta)\bar{\mathbf{p}}_i = \mathrm{R}((i+1)\vartheta)\bar{\mathbf{p}}_{i+1}.$$

In other words,

$$\bar{\mathbf{p}}_i = \mathrm{R}(\vartheta)\bar{\mathbf{p}}_{i+1}$$

which confirms that the desired trajectory defined in (5) is achieved and a regular polygon formation is obtained.

ii) Since $\xi_i = \xi_{i+1}$ and $\dot{\xi}_i = \dot{\xi}_{i+1}$; then, $\mathbf{u}_{ci} = \mathbf{0}_{2\times1}$. Thus, from (11), it can be said that

$$\ddot{\mathbf{p}}_i = (\lambda\varpi\mathbf{T} - \varpi^2\mathbf{I}_2)\bar{\mathbf{p}}_i - \lambda\dot{\mathbf{p}}_i$$

which by invoking Theorem 1, it follows that while achieving a regular polygon formation, the agents rotate around the centroid $\mathbf{c}$ with angular velocity ϖ.

Therefore, the proof is completed. ■

Remark 2. It is worth noting that the proposed vector field $\mathbf{u}_{ri}$ was designed separately independent of the formation control strategy $\mathbf{u}_{ci}$. However, since $\mathbf{u}_{ri}$ was a function of the ith agent states, in the proof of Theorem 2, it was necessary to investigate the performance of the MAS under the combined controllers $\mathbf{u}_{ri} + \mathbf{u}_{ci}$.

Remark 3. It should be noted that the main contributions of the paper are in the area of formation control based on "cyclic topologies". However, the proposed approach is extendable to more general communication topologies as well. Note that the base of the proof of Theorem 2 is designing $\mathbf{K}$ such that $\mathbf{I}_\nu \otimes \mathbf{M} + \acute{\mathbf{J}} \otimes \mathbf{BK} = \mathrm{diag}(\Delta_1, \Delta_2, \ldots, \Delta_\nu)$ is Hurwitz stable, because as $\mathbf{I}_\nu \otimes \mathbf{M} + \acute{\mathbf{J}} \otimes \mathbf{BK}$ is Hurwitz stable, based on the arguments given after (26) in the proof of Theorem 2, the agents converge to a regular polygon formation with cyclic pursuit. To achieve this goal, the smallest real part of the nonzero eigenvalues of $\mathbf{L}$ (which is equal to $1 - \cos(2\pi/n)$ for a cyclic topology) is used in $\mathbf{K}$ such that

$$\mathbf{K} = -\varepsilon\Big(1 - \cos\frac{2\pi}{n}\Big)^{-1}\mathbf{B}^\top\mathbf{P}.$$

Because in this condition, we have

$$\Delta_j = \mathbf{M} - \varepsilon\Big(1 - \cos\frac{2\pi}{n}\Big)^{-1}(1 - \omega^j)\mathbf{B}\mathbf{B}^\top\mathbf{P},$$

and since

$$\Re\Big(\varepsilon\Big(1 - \cos\frac{2\pi}{n}\Big)^{-1}(1 - \omega^j)\Big) \geq 1, j \in \{1, 2, \ldots, \nu\},$$

from [47], the Hurwitz stability of $\mathbf{I}_\nu \otimes \mathbf{M} + \acute{\mathbf{J}} \otimes \mathbf{BK}$ can be concluded. Now, in the case of more general communication topologies, by redefining $\mathbf{K}$ as

$$\mathbf{K} = -\varepsilon\Upsilon^{-1}\mathbf{B}^\top\mathbf{P},$$

a regular polygon formation is achieved by redesigning $\mathbf{u}_{ci}$ as

$$\mathbf{u}_{ci} = \mathrm{R}(-i\vartheta)\mathbf{K}\sum_{j=1}^{n} a_{ij}\begin{bmatrix}\dot{\xi}_i - \dot{\xi}_{i+1}\\ \xi_i - \xi_{i+1}\end{bmatrix}$$

where Υ is the smallest real part of the nonzero eigenvalues of the general $\mathbf{L}$, $a_{ij} = 1$ if the ith agent follows the jth one, and $a_{ij} = 0$ otherwise. In this condition, based on arguments the same as the proof of Theorem 2, we can say that $\mathbf{I}_\nu \otimes \mathbf{M} + \acute{\mathbf{J}} \otimes \mathbf{BK}$ is Hurwitz stable which results in convergence of the agents to a regular polygon formation with cyclic pursuit (see the arguments given after (25) in the proof of Theorem 2).

In the proposed approach, it was supposed that $\vartheta = \mathrm{sgn}(\varpi)\frac{2\pi}{n}$, i.e., each agent pursues its front agent. However, a similar procedure can prove Theorem 2 when $\vartheta = -\mathrm{sgn}(\varpi)\frac{2\pi}{n}$.

By introducing an elliptical rotational vector field, in the next section, the obtained results are extended to a case when the agents are moving on an ellipse.

6. Extension to Elliptical Formations

Consider an ellipse represented by the equation below:

$$\alpha^2(x - x_c)^2 + \beta^2(y - y_c)^2 = 1$$

where $\alpha, \beta \in \mathbb{R}_+$, and $\begin{bmatrix} x_c & y_c \end{bmatrix}$ denotes the ellipse centroid. This ellipse is the squashed form of a circle about $\begin{bmatrix} x_c & y_c \end{bmatrix}$ with the transformation matrix below:

$$\mathbf{T}_s = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}.$$

In other words, the circle is a special case of the ellipse when $\alpha = \beta$. Moreover, the ellipse can be rotated through an angle $\phi \in [0, 2\pi)$ around $\begin{bmatrix} x_c & y_c \end{bmatrix}$ by using the rotation matrix $\mathrm{R}(\phi)$. As depicted in Fig. 5, ellipses can be considered parallel to the mentioned ellipse on which the vector field rotates around the centroid $\mathbf{c} = \begin{bmatrix} x_c & y_c \end{bmatrix}^\top$. In this condition, to provide this elliptical vector field, $\mathbf{u}_{ri}$ defined in (6) can be modified via the similarity transformation matrix $\mathbf{S} = \mathrm{R}(\phi)\mathbf{T}_s$ as

$$\mathbf{u}^m_{ri} = \mathbf{S}^{-1}(\lambda\varpi\mathbf{T} - \varpi^2\mathbf{I}_2)\mathbf{S}\bar{\mathbf{p}}_i - \mathbf{S}^{-1}\lambda\mathbf{S}\dot{\mathbf{p}}_i$$

which can be simplified as follows:

$$\mathbf{u}^m_{ri} = (\lambda\varpi\mathbf{S}^{-1}\mathbf{TS} - \varpi^2\mathbf{I}_2)\bar{\mathbf{p}}_i - \lambda\dot{\mathbf{p}}_i \tag{28}$$

where $\mathbf{u}^m_{ri}$ is the modified control law to provide the mentioned rotational elliptical vector field.

Moreover, to achieve a formation of agents exposed to the introduced elliptical vector field, first let us restate $\mathbf{u}_{ci}$ defined in (13) as follows:

$$\mathbf{u}_{ci} = \mathrm{R}(-i\vartheta)\mathbf{K}\left(\begin{bmatrix} \mathrm{R}(i\vartheta)\dot{\mathbf{p}}_i \\ \mathrm{R}(i\vartheta)\bar{\mathbf{p}}_i \end{bmatrix} - \begin{bmatrix} \mathrm{R}(i\vartheta + \vartheta)\dot{\mathbf{p}}_{i+1} \\ \mathrm{R}(i\vartheta + \vartheta)\bar{\mathbf{p}}_{i+1} \end{bmatrix}\right).$$

Therefore, by applying the similarity transformation matrix $\mathbf{S}$, the modified form of $\mathbf{u}_{ci}$ can be stated as follows:

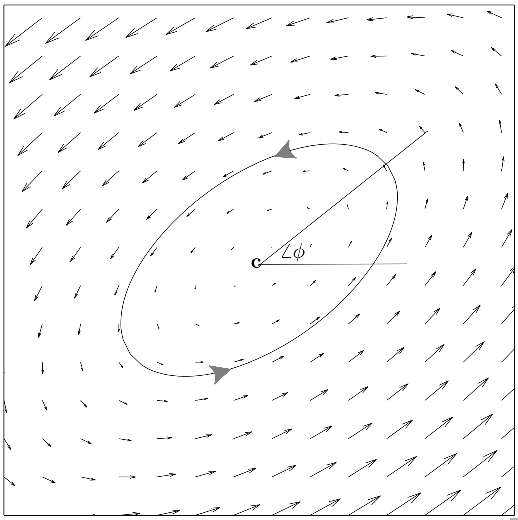

Figure 5: Trajectory of a mobile agent exposed to a counter-clockwise elliptical vector field around the centroid **c**.

$$\mathbf{u}_{ci}^{m} = \mathbf{S}^{-1}\mathrm{R}(-i\vartheta)\mathbf{K}\left(\begin{bmatrix}\mathrm{R}(i\vartheta)\mathbf{S}\dot{\mathbf{p}}_i \\ \mathrm{R}(i\vartheta)\mathbf{S}\bar{\mathbf{p}}_i\end{bmatrix} - \begin{bmatrix}\mathrm{R}(i\vartheta+\vartheta)\mathbf{S}\dot{\mathbf{p}}_{i+1} \\ \mathrm{R}(i\vartheta+\vartheta)\mathbf{S}\bar{\mathbf{p}}_{i+1}\end{bmatrix}\right) \tag{29}$$

where $\mathbf{u}_{ci}^{m}$ is the modified form of $\mathbf{u}_{ci}$ to obtain an elliptical polygon formation of the mobile agents.

The proposed control approach is examined in the following section.

7. Simulation Results

Let us consider a MAS containing eight mobile agents whose initial states are given in Table 1. According to the control law proposed in Theorem 2, it is necessary for each agent to receive

Table 1: Initial Conditions of the Agents

Agent	Position	Velocity	Agent	Position	Velocity
1	[14 10]	[2 0]	5	[4 0]	[2 1]
2	[20 13]	[3 1]	6	[5 8]	[−1 1]
3	[20 5]	[−1 −1]	7	[6 12]	[3 3]
4	[10 2]	[−1 2]	8	[2 16]	[4 7]

the position and velocity information of another agent under a cyclic directed communication topology. By letting $\varpi = -0.2$ and $\lambda = 1$, the Riccati equation given in (15) yields

$$\mathbf{P} = \begin{bmatrix} 0.9878 & 0.0000 & 0.9756 & 0.0994 \\ 0.0000 & 0.9878 & -0.0994 & 0.9756 \\ 0.9756 & -0.0994 & 1.9788 & 0.0000 \\ 0.0994 & 0.9756 & 0.0000 & 1.9788 \end{bmatrix}.$$

Now, by considering $\varepsilon = 5$ and $\mathbf{c} = \begin{bmatrix} 10 & 10 \end{bmatrix}^\top$, the trajectories of the mobile agents are depicted in Fig. 6-a. Based on the desired trajectory defined in (5), the formation error vector of the ith agent can be defined as follows:

$$\mathbf{e}_i = \mathbf{p}_i - \mathrm{R}(\vartheta)\mathbf{p}_{i+1}.$$

Therefore, based on this definition, the norms of the formation error vectors are depicted in Fig. 6-b confirming that the formation error vectors converged to zero and a regular polygon formation is obtained. Moreover, Fig. 6-c demonstrates the angular velocities of the agents around the centroid $\mathbf{c}$ and verifies that the angular velocities are converged to -0.2. By considering all the above-mentioned issues, the agents are rotating around a *desired centroid* with a *controlled angular velocity*, while an octagonal formation is achieved.

The octagonal formation can be squashed to an elliptical octagonal formation by employing the modified control laws defined in (28) and (29). Let $\alpha = 1, \beta = 2$, and $\phi = \pi/8$. In this condition, Fig. 7 demonstrates the elliptical octagonal formation of the mobile agents with cyclic pursuit around $\mathbf{c}$.

8. Conclusion and Future Work

Pursuit formation control of double-integrator mobile agents based on a vector field approach was studied in this paper. First a rotational vector field for rotational motion of mobile agents was proposed. Then, sufficient conditions were proposed such that the agents reached a regular polygon formation with cyclic pursuit under the introduced rotational vector field. Despite existing approaches in the literature for cyclic pursuit of double-integrator mobile agents, it was feasible to control the pursuit angular velocity and centroid in arbitrary values. Extension of the results when only relative states are measurable and experimental study of the proposed formation control scheme are other challenging problems to be investigated as future work.

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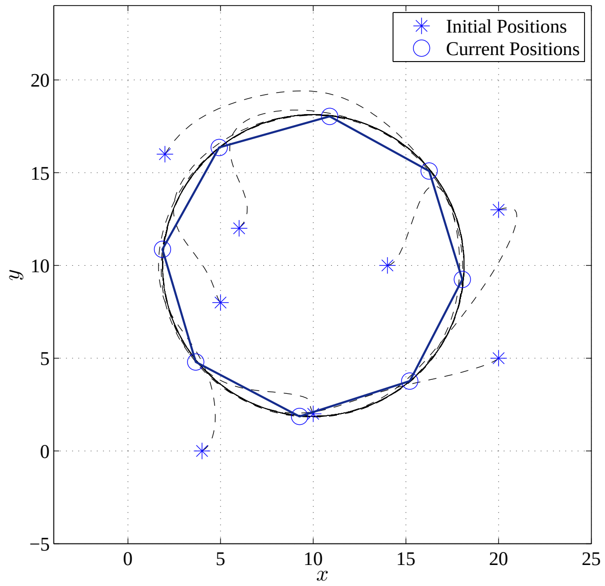
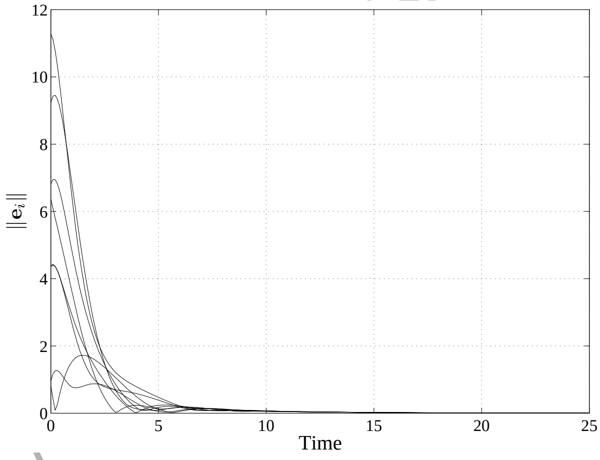
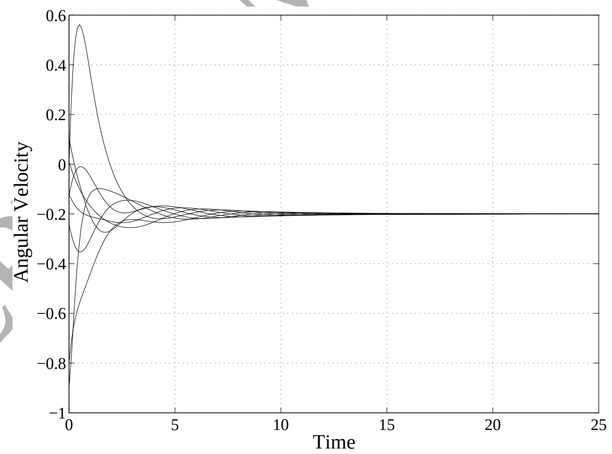


(a) Trajectories of the agents.



(b) Norms of the formation errors.



(c) Agents angular velocities around the centroid.

Figure 6: Octagonal formation of the eight mobile agents with cyclic pursuit around the centroid.

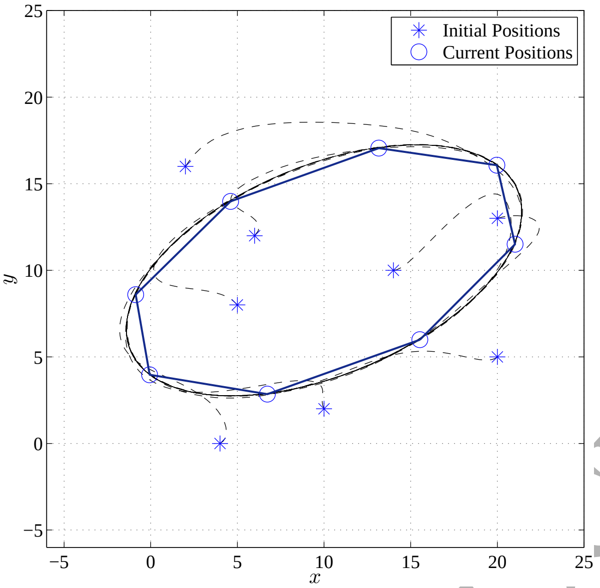


Figure 7: Elliptical octagonal formation of the eight mobile agents with cyclic pursuit around the centroid.

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