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Ramsey-optimal Tax Reforms and Real Exchange Rate Dynamics

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**Abstract**

We solve the Ramsey-optimal tax plan for a small open economy with an endogenously-determined real exchange rate. The open economy constrains the government's setting of the capital income tax rate since physical capital cannot be dominated in rate of return by foreign assets. However, the endogenous real exchange rate loosens this constraint relative to a one good open economy model in which the real exchange rate is necessarily fixed. We find that, the dynamics of the two good small open economy model more closely resemble those of a closed economy model than a one good small open economy model.

*Keywords:* Optimal fiscal policy, Tax reforms, Welfare

*JEL Codes:* E32, E52, F41.

## 1. Introduction

Much is known in the macroeconomics literature about Ramsey-optimal taxation in closed environments as evidenced by the extensive literature that traces back to Lucas and Stokey (1983), Chamley (1986) and Judd (1985).[1] A key finding in the literature is that capital income should initially be taxed at very high rates, but in the long run not at all. The intuition for this finding is as in Ramsey (1927): the government should tax most heavily those factors that are in inelastic supply.

Far less is known about Ramsey-optimal taxation in the context of open economy macroeconomic models. In Correia (1996) and the subsequent literature on Ramsey taxation in open economy models, there is a homogeneous good, thereby fixing the real exchange rate at one. In such an environment, the government is constrained in its initial setting of the capital income tax rate. In particular, a no-arbitrage condition forces the after-tax return to capital to equal the exogenous world real interest rate. As shown in Correia (1996), the government finds it best not to increase the capital income tax rate, even in the very short term.

Our paper is the first to address Ramsey optimal taxation with distinct domestic and foreign goods, and so an endogenous real exchange rate. While the Ramsey planner is still constrained by the no-arbitrage condition described above, real exchange rate movements mean that the relevant world return is no longer fixed. The solutions of the model show that the dynamics of the two good small open economy model fairly closely resemble those of a closed economy model, not the one good small open economy model. As in the closed economy model, the Ramsey

[1] By "Ramsey-optimal taxation" we mean work that solves for at least a set of tax rates as well as the path for government debt. The path for government debt is often implicit. This definition excludes those papers that hold fixed the level of government debt.

tax plan in the two good model calls for very high capital income tax rates in the short run. Satisfying the no arbitrage condition requires an initial real exchange rate depreciation (when the capital income tax rate is high), followed by an appreciation. Due to the initially high capital income taxation, government debt falls sharply in the closed and two good small open economy models; in the one good open economy model, there is little change in government debt.

Arguably, the two good model, with its endogenous determination of the real exchange rate, better describes the environment faced by governments than either the one good open economy model or the closed economy model. The differences in the capital income tax rate dynamics across the two small open economy models point to the importance of real exchange rate determination.

One may well wonder why the government does not simply apply very high tax rates early in its implementation of the Ramsey tax program in order to become a net creditor (drive its debt very negative), financing its expenditures on public goods from its interest revenue. That is, why does the government not choose a value for its debt so that it replicates the Pareto optimal outcome with no taxes? The answer is that achieving such a level of government debt is, evidently, too costly. Indeed, the existing literature does *not* predict zero labor income tax rates precisely because the Lagrange multiplier on the implementability constraint binds. This point is made more formally in Section 3.

Following the Ramsey literature, the government is restricted to linear tax schedules, although it is free to tax labor and capital income at different rates. The government is able to fully and credibly commit to the Ramsey optimal taxation program. Adopting a residence-based taxation scheme, income from foreign bonds are implicitly taxed by the foreign government, not the domestic government. In the model, the representative household values government provision of public goods. A practical advantage of analyzing a *small* open economy is that strategic interactions between governments can be ignored.

The bulk of Section 2 develops the two good small open economy model; towards the end of this section is a brief description of how the two good model can be reduced to either a closed economy model, or a one good small open economy model. Fiscal policy – the Ramsey problem – is presented in Section 3. The model is calibrated in Section 4. The key results, in the form of time paths for macroeconomic variables, are discussed in Section 5. The implications of alternative settings for preference parameters and the trade parameters are presented in Section 6. Some final remarks are made in Section 7.

## 2. A Two Good, Small Open Economy Model

In the two good model, private consumption is a composite of domestic and foreign goods. Attention is focused on the home or domestic economy; an asterisk is used to denote values of foreign, or rest of the world, variables.

*Households*

The typical domestic household starts period $t$ with three assets: $k_{t-1}$ units of domestic capital, $d_{t-1}$ units of domestic government debt, and $b_{t-1}$ units of internationally traded bonds. Capital income is taxed at the rate $\tau_t^k$. The gross return to a unit of capital is, then, $R_t^k \equiv 1-\delta+(1-\tau_t^k)r_t$ where $r_t$ is the real rental rate for capital and $\delta$ the depreciation rate of capital. The gross return to a unit of government debt is $R_t^d$. Finally, international bonds pay off in terms of foreign output; this return, $R^b$, is assumed to be constant. Conceptually, the international bonds are state contingent. However, since the analysis focuses on perfect foresight equilibria, state contingent notation is suppressed in the interest of a cleaner presentation. The fact that the international bonds are state contingent means that the implicit assumption that only domestic households own domestic capital and domestic government debt is without loss of generality.

The representative domestic household receives utility from a private composite consumption good, $c_t$, a government or public good, $g_t$, and disutility from working, $h_t$. The household's problem is:

$$\max_{\{c_t,h_t,k_t,d_t,b_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U\ (c_t, g_t, h_t) \tag{1}$$

subject to the sequence of budget constraints,

$$p_t c_t + k_t + \frac{d_t}{R_t^d} + e_t \frac{b_t}{R^b} \leq \left(1-\tau_t^h\right) w_t h_t + R_t^k k_{t-1} + d_{t-1} + e_t b_{t-1} + \tau. \tag{2}$$

The last two terms on the right-hand side of (2) are the proceeds of previous period purchases of domestic and international bonds. Since the international bond is denominated in terms of foreign output, the proceeds from this bond are converted into units of domestic output via the real exchange rate, $e_t$, which is expressed in terms of the number of units of domestic output per unit of foreign output. The first two terms on the right-hand side are payments to labor and capital: in addition to the capital income components discussed above, $w_t$ is the real wage which is taxed at the rate $\tau_t^h$. Finally, $\tau$ is a lump-sum transfer.

In addition to bond purchases (the last two terms on the left-hand side of (2)), the domestic household purchases private consumption goods and capital. The price of a unit of consumption in terms of domestic output is $p_t$; its derivation is described shortly.

The private consumption good is a composite of domestic, $c_{ht}$, and foreign, $c_{ft}$, goods,

$$c_t = \left[ \varphi^{\frac{1}{\mu}} c_{ht}^{\frac{\mu-1}{\mu}} + (1-\varphi)^{\frac{1}{\mu}} c_{ft}^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}} \quad (3)$$

where $\mu > 0$ is the elasticity of substitution between domestic and foreign goods; $\varphi = 1-(1-n)\gamma$; $n$ is the relative size of the domestic economy; and $\gamma$ is a measure of trade openness. Solving the relevant cost minimization problem yields the relative price for aggregated private domestic consumption,

$$p_t = \left[ \varphi + (1-\varphi) e_t^{1-\mu} \right]^{\frac{1}{1-\mu}}, \quad (4)$$

as well as demands for domestic and foreign goods,

$$c_{ht} = \varphi \left[ \varphi + (1-\varphi) e_t^{1-\mu} \right]^{\frac{\mu}{1-\mu}} c_t \quad (5)$$

$$c_{ft} = (1-\varphi) \left[ \varphi e_t^{\mu-1} + 1 - \varphi \right]^{\frac{\mu}{1-\mu}} c_t. \quad (6)$$

The corresponding expressions for foreign households are:

$$c_{ht}^* = \varphi^* \left[ \varphi^* + (1-\varphi^*) e_t^{1-\mu} \right]^{\frac{\mu}{1-\mu}} c^* \quad (7)$$

$$c_{ft}^* = (1-\varphi^*) \left[ \varphi^* e_t^{\mu-1} + 1 - \varphi \right]^{\frac{\mu}{1-\mu}} c^* \quad (8)$$

where $\varphi^* = n\gamma$.

*Firms*

The representative firm has access to a neoclassical production function, $F$. The firm rents capital and hires labor on competitive factor markets to maximize period-by-period profits,

$$F(k_{t-1}, h_t) - w_t h_t - r_t k_{t-1}.$$

*Government*

The problem of a benevolent government planner is analyzed in Section 3. For the purpose of analyzing the competitive equilibrium, it is sufficient to note that the government finances the stream of public goods, $g_t$, and lump-sum transfers, $\tau$, by either issuing debt, $d_t$, or levying taxes to satisfy its budget constraint

$$\frac{d_t}{R_t^d} - d_{t-1} = g_t + \tau - \tau_t^h w_t h_t - \tau_t^k r_t k_{t-1} \tag{9}$$

as well as the usual transversality condition concerning its debt. In (9), it is understood that the quantities are expressed per capita.

*Competitive Equilibrium*

The definition of a competitive equilibrium is standard: Given prices and government actions, households solve their utility maximization problems, firms solve their profit maximization problems, the government satisfies its budget constraint and 'no Ponzi scheme' condition, and markets clear.

As in De Paoli (2009), the small open economy is the limit case (as $n \to 0$) of a two country model. The domestic goods market clearing condition is

$$y_t = (1-\gamma) p_t^{\mu} c_t + \gamma e_t^{\mu} c^* + k_t - (1-\delta) k_{t-1} + g_t. \tag{10}$$

In addition, the balance of payments must be satisfied:

$$\gamma e_t^{\mu-1} c^* - \gamma \left[ (1-\gamma) e_t^{\mu-1} + \gamma \right]^{\frac{\mu}{1-\mu}} c_t + b_{t-1} - \frac{b_t}{R^b} = 0. \tag{11}$$

The first term is exports, denominated in foreign output while the second term is imports similarly expressed in units of foreign output. The remaining terms are the redemption of international bonds, and purchases of new international bonds. The specific expressions for imports and exports come from the demands (6) and (7), expressed in domestic per capita terms, for the limit as $n$ approaches zero.

*Model Variants*

The Ramsey optimal taxation literature has considered two other classes of models that can be viewed as special cases of the two good, small open economy model. The bulk of the Ramsey optimal taxation literature has focused on closed economies. In this case, set foreign bond holdings to zero ($b_t = 0$). There are no foreign goods and so no consumption aggregator, and the relative price of the consumption good is unity ($p_t = 1$).

The open economy models analyzed in the literature have one good. In this case, since domestic and foreign goods are perfect substitutes, the real exchange rate ($e_t$) is one as is the relative price of consumption ($p_t$). There is no need to introduce the consumption aggregator, (3). The goods market clearing condition now reads

$$y_t = c_t + nx_t + k_t - (1-\delta)k_{t-1} + g_t \tag{12}$$

where $nx_t$ is net exports. In turn, net exports are related to international bonds via

$$\frac{b_t}{R^b} - b_{t-1} = nx_t.$$

## 3. The Ramsey Taxation Problem

The problem of the benevolent Ramsey planner is to choose a sequence for tax rates so as to maximize lifetime utility of the representative household given that the resulting allocation constitutes a competitive equilibrium. The usual approach employed in the Ramsey optimal taxation literature is to develop an implementability condition that eliminates all prices and tax rates; the planner then chooses an allocation directly subject to this implementability condition. Issues related to the open economy setting are addressed below.

To flesh out the derivation of the implementability condition, write the household's problem as choosing $\{c_t, h_t, k_t, d_t, b_t\}_{t=0}^{\infty}$ to maximize

$$\mathcal{L} = \sum_{t=0}^{\infty} \left\{ \beta^t U(c_t, g_t, h_t) + \lambda_t \left[ \left(1-\tau_t^h\right) w_t h_t + R_t^k k_{t-1} + d_{t-1} + e_t b_{t-1} + \tau - p_t c_t - k_t - \frac{d_t}{R_t^d} - e_t \frac{b_t}{R^b} \right] \right\}. \quad (13)$$

The associated first-order conditions are:

$$\beta^t U_c(c_t, g_t, h_t) = \lambda_t p_t \quad (14)$$

$$\beta^t U_h(c_t, g_t, h_t) + \lambda_t (1-\tau_t^h) w_t = 0 \quad (15)$$

$$\lambda_t = \lambda_{t+1} R_{t+1}^k \quad (16)$$

$$\frac{\lambda_t}{R_t^d} = \lambda_{t+1} \quad (17)$$

$$e_t \frac{\lambda_t}{R^b} = e_{t+1} \lambda_{t+1}. \quad (18)$$

Substituting (14) to (18) into

$$\sum_{t=0}^{\infty} \lambda_t \left[ \left(1-\tau_t^h\right) w_t h_t + R_t^k k_{t-1} + d_{t-1} + e_t b_{t-1} + \tau - p_t c_t - k_t - \frac{d_t}{R_t^d} - e_t \frac{b_t}{R^b} \right], \quad (19)$$

and recognizing that each term in the sum must equal zero, delivers the implementability condition,

$$\sum_{t=0}^{\infty} \beta^t \left[ U_c(c_t, g_t, h_t)(c_t - \tau) + U_h(c_t, g_t, h_t) h_t \right] = \frac{U_c(c_0, g_0, h_0)}{p_0} \left\{ R_0^k k_{-1} + d_{-1} + e_0 b_{-1} \right\} \quad (20)$$

At this stage, we write the Ramsey problem as: maximize (1) subject to the implementability condition, (20), the pricing equation, (4), feasibility, (10), the balance of payments equation, (11), and *in addition* the set of no-arbitrage conditions implicit in e(16) to (18):

$$R_{t+1}^{k} = R_{t}^{d} = \frac{e_{t+1}R^{b}}{e_{t}}. \tag{21}$$

The easiest way to impose the set of return arbitrage conditions in (21) is via an international risk-sharing condition which we now develop. Notice that (14) and (18) imply the following Euler equation:

$$\frac{e_{t}U_{c}(c_{t}, g_{t}, h_{t})}{p_{t}} = \beta R^{b} \frac{e_{t+1}U_{c}(c_{t+1}, g_{t+1}, h_{t+1})}{p_{t+1}}. \tag{22}$$

The corresponding Euler equation for foreign households is

$$\frac{U_{c}^{*}(c_{t}^{*}, g_{t}^{*}, h_{t}^{*})}{p_{t}^{*}} = \beta R^{b} \frac{U_{c}^{*}(c_{t+1}^{*}, g_{t+1}^{*}, h_{t+1}^{*})}{p_{t+1}^{*}}; \tag{23}$$

the real exchange rate does not appear in (23) since international bonds are denominated in units of foreign output. As in Chari et al. (2000), solve (22) and (23) for the common bond return, $R^{b}$, then iterate backwards:

$$\frac{e_{t}U_{c}(c_{t}, g_{t}, h_{t}) / p_{t}}{U_{c}^{*}(c_{t}^{*}, g_{t}^{*}, h_{t}^{*}) / p_{t}^{*}} = \frac{e_{-1}U_{c}(c_{-1}, g_{-1}, h_{-1}) / p_{-1}}{U_{c}^{*}(c_{-1}^{*}, g_{-1}^{*}, h_{-1}^{*}) / p_{-1}^{*}}. \tag{24}$$

The right-hand side of (24) is given by history and so constant. An implication of (24) is that the real exchange rate is determined by the ratios of prices and marginal utilities of private consumption between domestic and foreign households, and the arbitrary factor of proportionality given by history. Since the rest of the world is assumed to be in a steady state, this condition can be written

$$\frac{e_{t}U_{c}(c_{t}, g_{t}, h_{t})}{p_{t}} = \vartheta. \tag{25}$$

This is the relevant risk-sharing condition for the Ramsey problem and necessarily implies that the no-arbitrage conditions (21) are satisfied.

Rather than dealing with the balance of payments equation in (11), it is more convenient to work with the international solvency condition that is obtained by iterating forward on (11):

$$b_{-1}+\sum_{t=0}^{\infty}\frac{\gamma e_t^{\mu-1}c^{*}-\gamma\left[(1-\gamma)e_t^{\mu-1}+\gamma\right]^{\frac{\mu}{1-\mu}}c_t}{(R^b)^t}. \tag{26}$$

This international solvency condition states that the present value of net exports must equal net foreign indebtedness. Notice that the steady state version of the bond accumulation Euler equation (22) implies that $\beta R^b=1$; in other words, domestic households are as patient as foreign households. This fact will be used below to simplify the Ramsey planner's problem.

The Ramsey taxation problem can, now, formally be written:

$$\begin{aligned}\mathcal{L} = \max_{\{c_t,g_t,h_t,k_t,e_t,p_t\}_{t=0}^{\infty}} &\sum_{t=0}^{\infty}\beta^t\{U\ (c_t,g_t,h_t)\\
&+\phi_t\left[F(k_{t-1},h_t)+(1-\delta)k_{t-1}-(1-\gamma)p_t^{\mu}c_t-\gamma e_t^{\mu}c^{*}-k_t-g_t\right]\\
&+\xi_t\left[\frac{e_tU_c(c_t,g_t,h_t)}{p_t}-\vartheta\right]\\
&+\zeta_t\left[\left[1-\gamma+\gamma e_t^{1-\mu}\right]^{\frac{1}{1-\mu}}-p_t\right]\\
&+\Lambda\left[U_c(c_t,g_t,h_t)(c_t-\tau)+U_h(c_t,g_t,h_t)h_t\right]\\
&+\Omega\left[\gamma e_t^{\mu-1}c^{*}-\gamma[(1-\gamma)e_t^{\mu-1}+\gamma]^{\frac{\mu}{1-\mu}}c_t\right]\}\\
&-\Lambda\frac{U_c(c_0,g_0,h_0)}{p_0}\left\{\left[(1-\tau_0^k)F_k(k_{-1},h_0)+1-\delta\right]k_{-1}+d_{-1}+e_0b_{-1}\right\}\\
&+\Omega b_{-1}.\end{aligned} \tag{27}$$

Purists may object that leaving in the prices $p_t$ and $e_t$ means that we have not properly written down the Ramsey problem. In principle, the price equation and international risk-sharing condition, (4) and (24), could be used to solve out for $p_t$ and $e_t$ in terms of quantities.[2] Doing so

[2] Without specifying a functional form for utility, $p_t$ and $e_t$ would necessarily be implicit functions of the quantities.

leads to messy expressions in the Ramsey problem that would have to be differentiated to obtain the associated first-order conditions. In any event, $p_t$ and $e_t$ can be thought of as "book keeping devices": $p_t$ stands in for the marginal rate of transformation between private home and foreign goods, while $e_t$ corresponds to the price adjusted marginal rate of substitution between consumption of home and foreign households.

It is extremely convenient that neither government debt nor international bonds appear in the Ramsey problem (27) since it is well known that the dynamics of the equations governing these variables are inherently unstable. To solve for these bond sequences, solve backwards from the final steady state; in this way, small numerical errors will not accumulate.

In computing solutions to Ramsey problems, one typically starts with some guess for $\Lambda$, the multiplier associated with the implementability condition, then solve for the allocation. Given the resulting allocation, check whether the implementability condition is satisfied. If not, make an appropriate adjustment to the value of the multiplier, and re-solve for the allocation. To this, the open economy adds $\Omega$, the multiplier on the international solvency condition, and a subsequent check on whether this condition is satisfied. Conditional on the guesses for the multipliers on the implementability and international solvency conditions, the model is solved using an extended path algorithm (Fair and Taylor, 1983), specifying an initial steady state and a 'no change' terminal conditions (for details, see Auray et al., 2016).

Of the Ramsey planner's first-order conditions, the one of intrinsic interest is that with respect to capital:

$$\phi_t = \beta\phi_{t+1}\left[F_k(k_t, h_{t+1}) + 1 - \delta\right]. \tag{28}$$

Compare this equation with the corresponding household Euler equation, obtained by combining (14) and (16), along with the equilibrium expression for the gross return to capital:

$$\frac{U_c(c_t, g_t, h_t)}{p_t} = \beta \frac{U_c(c_{t+1}, g_{t+1}, h_{t+1})}{p_{t+1}} \left[ (1-\tau_{t+1}^k) F_k(k_t, h_{t+1}) + 1 - \delta \right]. \tag{29}$$

Mutual consistency of these two equations delivers the classic Chamley-Judd prescription that in the long run, when the economy converges to its eventual final steady state, capital income should not be taxed.

## 4. Calibration

*Functional Forms*

The utility function is of the constant relative risk aversion variety

$$U(c,g,h) = \frac{\left(C(c,g)(1-h)^{\chi}\right)^{1-\sigma}}{1-\sigma},$$

where $C(c,g)$ is an aggregator over private and public consumption goods, given by

$$C(c,g) = \left[ (1-\kappa) c^{\frac{\psi-1}{\psi}} + \kappa g^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}.$$

It is understood that the consumption aggregator is Cobb-Douglas when $\psi = 1$, and the utility function is logarithmic when $\sigma = 1$.

Production is Cobb-Douglas:

$$y = F(k,h) = k^{\alpha} h^{1-\alpha}.$$

*Parameterization*

Some of the parameters are set exogenously. A model period corresponds to one year. The world real interest rate is, then, set to the conventional 4%: $R^b = 1.04$. The analysis of the tax reforms is somewhat more straightforward when utility is additively separable. Thus, the coefficient of relative risk aversion, $\sigma$, is set to one which implies logarithmic preferences, and the consumption aggregator is Cobb-Douglas ($\psi = 1$). The trade openness parameter, $\gamma$, is set to $0.3$ on the basis that the world share of imports is $30\%$; and the elasticity of substitution between home and foreign goods, $\mu$, is set to $1.5$ as in Backus et al. (1992). Section 6 explores the implications of alternative values for $\gamma$ and $\mu$.

For a number of parameters, there is a direct link between their values and calibration targets. Thus, $\alpha = 0.3$ and $\delta = 0.075$ based on evidence on capital's share of income and the depreciation rate for capital presented in Gomme and Rupert (2007). The discount factor must be consistent with the steady state version of the international bond Euler equations, (22): $\beta = 1/R^b$. Factor income tax rates are $\tau^n = 28.59\%$ and $\tau^k = 37.10\%$ based on average effective tax rates for the U.S. over the period 2005–07; see Auray et al. (2017) for details. The real exchange rate is normalized to $e = 1$. From (4), it then follows that $p = 1$ in steady state.

Two parameters remain to be calibrated: $\kappa$, the preference weight on public goods, and $\chi$, the preference weight on leisure. Given the parameters discussed above, a steady state delivers values for private and public consumption ($c$ and $g$), hours worked ($n$), the capital stock ($k$) and foreign composite consumption ($c^*$) that satisfy steady state versions of the labor-leisure Euler equation,

$$(1-\tau_t^h)F_2(k_{t-1},h_t)\frac{U_c(c_t,g_t,h_t)}{p_t} + U_h(c_t,g_t,h_t) = 0, \tag{30}$$

the capital accumulation Euler equation (29), feasibility (10) as well as: zero net exports, and a government share of output of $19.55\%$. The parameters $\kappa$ and $\chi$ are, then, set such that average hours worked is 30% of the time endowment (a value consistent with U.S. time use surveys; see Gomme and Rupert (2007)), and equality of the marginal utility of private goods with the marginal utility of public goods. The resulting parameter values are: $\kappa = 0.2323$ and $\chi = 1.33$.

That the real exchange rate is normalized to equal one while net exports are set to zero implies that the steady states of the open economy models coincide with that of the closed economy model. Requiring that the intratemporal marginal rate of substitution between private and public goods is one is motivated by the fact that this setting is consistent with what would be chosen by a benevolent social planner given that the marginal rate of transformation between these two goods is one. Consequently, in solving the Ramsey problem, there is no obvious means to improve the representative household's lifetime utility by simply reallocating private and public goods so that their marginal rate of substitution equals their marginal rate of transformation.

Finally, the lump-sum transfer, $\tau$, is chosen so that the government debt-output ratio is equal to one – a value close to that currently observed for the U.S. and a number of EU countries.

## 5. Tax Reforms

At time $0$, the government announces its policy in the form of time paths for public spending and income tax rates: $\{g_t, \tau_t^h, \tau_{t+1}^k\}_{t=0}^{\infty}$. As is common in the Ramsey taxation literature, the government is not free to choose the initial capital income tax rate, $\tau_0^k$, otherwise it would set this tax rate sufficiently high to drive its debt negative enough to finance all of its current and future expenditures from the interest income. The short run dynamics of these tax reforms are presented in Fig. 1 while the long run steady states are summarized in Table 1.

**Figure 1:** Dynamic Paths For the Closed and Small Open Economy Models

To start, consider government policy, starting with the closed economy model, then the two good open economy model. The most dramatic effects are with respect to the capital income tax rate and government debt. In the closed economy model, the capital income tax rate rises from $37.1\%$ to $260\%$ in period $1$, after which it immediately falls to around $0\%$.[3] In the first period, the government actually *subsidizes* labor income ($\tau_0^h = -24\%$), presumably to boost hours worked and so output. This tax rate then rises to around its eventual long run value of $32\%$, roughly $3.7$ percentage points above its initial level. While public goods fall in the short term, the new steady state sees an $11\%$ increase. The initial rise in the capital income tax rate drives down government debt to such an extent that the government becomes a net creditor (government debt is negative). The combination of somewhat higher government spending, positive revenue from government debt, and an expansion of the labor income tax base eventually leads to a modest $3.65$ percentage point rise in the labor income tax rate.

**Table 1:** Initial and Terminal Steady States

| | Initial | Closed | One Good Small Open Economy | Two Good Small Open Economy |
|---|---|---|---|---|
| $\tau^h$ | 0.2859 | 0.3224 | 0.4114 | 0.3920 |
| $\tau^k$ | 0.3710 | 0.0000 | 0.0000 | 0.0000 |
| $y$ | 0.3709 | 0.4608 | 0.4582 | 0.4500 |
| $c/y$ | 0.6814 | 0.6300 | 0.5516 | 0.5778 |

[3] The path for the capital income tax rate is broadly consistent with theoretical results for the Ramsey taxation literature which describe what happens in the short term (very high capital income tax rates) and in the very long term (zero capital income taxation), but is largely silent on what happens in the medium term.

| | | | | |
|---|---|---|---|---|
| $k/y$ | 1.6409 | 2.6087 | 2.6087 | 2.6087 |
| $h$ | 0.3000 | 0.3055 | 0.3038 | 0.2984 |
| $g/y$ | 0.1955 | 0.1744 | 0.1931 | 0.2136 |
| $d/y$ | 1.0000 | –0.2884 | 0.8366 | –0.0788 |
| $b/y$ | 0.0000 | | –1.5511 | –0.1504 |
| $nx/y$ | 0.0000 | | 0.0597 | 0.0058 |
| $e$ | 1.0000 | | 1.0000 | 1.0410 |
| $\omega$ | | 5.5320 | 5.2334 | 4.3614 |

For the two good small open economy model, the dynamics of the government policy variables are broadly similar to those in the closed economy model. While the initial hike in the capital income tax rate is impressive $(\tau_1^k = 216\%)$, it falls short of the $260\%$ tax rate in the closed economy model. The fall in government debt is correspondingly somewhat smaller as seen in Fig. 1(i), although in both models the government ends up a net creditor. Relative to the closed economy, the two main factors in the open economy leading to a higher labor income tax rate are: less revenue from government assets, and greater public spending (see Fig. (c)). In the long run, the labor income tax rate is $39.2\%$, $10.3$ percentage points higher than its initial value.

Fiscal policy in the one good open economy model differs starkly from the other two model economies. In this case, the Ramsey government never raises the capital income tax rate. To the contrary, as in Correia (1996), it immediately drops this tax rate to its long run value of zero. This result can be explained via the no-arbitrage condition between the returns to capital and the international bond. For the one good open economy model, the equilibrium version of this condition reads

$$(1-\tau_{t+1}^{k})F_1(k_t,h_{t+1})+1-\delta=R^b. \tag{31}$$

An increase in the future capital income tax rate lowers the return to capital. Maintaining equality between the return to capital and the fixed world real interest rate leads to capital flight. Instead, the Ramsey planner lowers the future capital income tax rate, and capital floods in. Such an increase in the capital stock raises real wages. It would seem that the increase in labor income tax revenue more than makes up for the lost capital income tax revenue which seems plausible since the labor income tax base is more than twice the size of the capital income tax base (their income shares were calibrated to $70\,\%$ and $30\,\%$, respectively).

To understand the differences in the paths of the capital income tax rate in the two open economy environments, compare the no-arbitrage condition for the one good model with its two good model counterpart:

$$(1-\tau_{t+1}^{k})F_1(k_t,h_{t+1})+1-\delta=\frac{e_{t+1}}{e_t}R^b. \tag{32}$$

The essential difference is the two exchange rate terms in (32). From Fig. 1(j), the planner initially depreciates the real exchange rate ($e_0$ rises above its initial steady state value), then sharply appreciates it. The timing of the real exchange rate movements serve to reduce the effective return to international bonds (the right-hand side of (32)). As a result, in the two good environment, the planner can markedly raise the capital income tax rate with only a modest effect on investment.

Returning to fiscal policy in the one good open economy model, government debt changes little, principally because the planner never chooses to raise the capital income tax rate. The long run increase in public goods along with little change in government debt necessitate a higher long run increase in the labor income tax rate: $12.55$ percentage points, the largest increase in the three economic environments considered here.

The dynamics of macroeconomic variables can be attributed largely to the path of the capital income tax rate. In the closed economy, the initial spike in this tax rate depresses investment, allowing private consumption to rise. Remember that the planner must respect the competitive equilibrium conditions. Germane to this discussion is the household's capital accumulation Euler equation, (29). The immediate effect of the capital income tax dropping to zero, is a sharp rise in the return to capital. In response, the household increases its investment at the expense of its consumption. At the same time, the planner lowers public consumption in order to free up output for investment. The resulting decrease in private consumption raises the intertemporal marginal rate of substitution which limits the short term increase in capital. As a result, capital rises gradually to its new long run value as depicted in Fig. 1(d).

It is well known that open economy macroeconomic models disconnect the intertemporal marginal rate of substitution from the intertemporal marginal rate of transformation. This point is well illustrated in the one good open economy model: the domestic economy can sharply increase its capital stock without sacrificing consumption. In fact, in the one good model, the international risk-sharing condition (25) along with separable preferences implies no change in private consumption. Similar considerations are in play in the two good open economy model, although in this case the planner chooses a more gradual buildup of the capital stock than is seen in the one good model. The initial real exchange rate depreciation raises the relative price of aggregate private consumption, which serves to moderate the increase in private consumption in the initial period. The subsequent appreciation leads to a fall in the relative price of consumption (the path of this price follows that of the real exchange rate). As discussed in Benigno and De Paoli (2010), this real exchange rate appreciation tends to raise the path for consumption and lower that of hours.

As discussed earlier, in the closed and two good open economy models, the initial spike in the capital income tax rate leads to a one period increase in private consumption. On its own, such an increase in consumption would tend to reduce hours worked, an effect operating through the labor-leisure Euler equation (30). To forestall this drop in hours, and associated fall in output, the planner temporarily reduces the labor income tax rate. In fact, as discussed earlier, in the closed economy, the planner actually subsidizes wage income. Such considerations are absent from the one good open economy model, and in this case the labor income tax immediately rises, resulting in a temporary decline in hours worked as shown in Fig. 1(e).

In the one good open economy model, the sharp rise in the capital stock depicted in Fig. 1(d) is financed chiefly through negative net exports (Fig. 1(l)) leading to a negative net foreign asset position as shown in Fig. 1(k). The home economy subsequently pays for this infusion of capital by running positive trade balances. In contrast, in the two good setting, the initial real exchange rate depreciation makes home goods cheaper relative to foreign goods; consequently, on impact exports rise while imports fall leading to a modest increase in net exports, and an initial rise in net foreign assets. The subsequent real exchange rate appreciation and buildup of capital lead to negative net exports and net foreign assets go negative. In the long run, the home country is a net foreign debtor and so runs trade balance surpluses to service this debt.

To evaluate the efficacy of the tax reforms, compute the welfare benefit as the percentage of private consumption that can be taken from households (holding fixed both public consumption and hours worked) that leaves them with the same utility as the original steady state. That is, the welfare benefit is measured by the value of $\omega$ that satisfies

$$\sum_{t=0}^{\infty}\beta^t U((1-\omega)c_t, g_t, h_t) = \frac{U(c_0, g_0, h_t)}{1-\beta}. \tag{33}$$

For the closed economy model, the welfare benefit of the tax reform is $5.5\%$ of consumption; for the two good small open economy model, $4.4\%$; and for the one good small open economy model, $5.2\%$. It may seem odd that both open economy models predict a *lower* welfare benefit of the Ramsey tax reform. After all, the aforementioned disconnect between the intertemporal marginal rate of substitution and marginal rate of transformation suggests that the open economies afford better utility smoothing than the closed economy. The explanation is that in the open economy settings, international risk sharing implies that some of the benefits of the tax reform are enjoyed by foreign households.

*Assessment*

The sharp contrast in the results across these economic models highlight the underlying economic mechanisms. The fixed real exchange rate in the one good small open economy model severely limits the government's ability to raise capital income tax revenue, an effect operating through a return arbitrage condition equating the return on international bonds with the return to capital. This consideration is entirely absent from the closed economy model, and the government chooses a very high capital income tax rate for one year (one model period). While the aforementioned arbitrage condition is present in the two good small open economy model, its effects are tempered by movements in the real exchange rate which allow the effective return on international bonds to respond to the tax reforms.

The scant literature on Ramsey-optimal tax reforms in open economies has focused on the one good small open economy model; see, for example, Correia (1996). As shown in this section, fixing the real exchange rate is far from an innocuous assumption.

The results in this section lead to two conclusions. First, considering the international dimension of tax reforms is important: The arbitrage condition in the small open economy models constrains the government's choice for the capital income tax rate in the short term. Second, the real exchange rate dynamics that arise from the two good small open economy model – but not in the one good model – moderates the effects of this arbitrage constraint. As a result, the benchmark two good small open economy model's dynamics more closely resemble those of the closed economy model than the one good small open economy model.

## 6. Alternative Parameter Settings

Most of the parameters in the calibration are well pinned down. Chief among those that are not: the coefficient of relative risk aversion, the elasticity of substitution between private and public goods, the degree of trade openness, and the elasticity of substitution between home and foreign goods. Implications for alternative values for these parameters are considered in this section.

*Preference Parameters*

The benchmark calibration sets the coefficient of relative risk aversion, $\sigma$, and the elasticity of substitution between private and public consumption goods, $\psi$, equal to one. The advantage of these parameter values is that together they imply that utility is additively separable between private consumption, public consumption, and leisure. This separability implies that the cross partial derivatives of the utility function are zero which made it easier to work through the results in Section 5.

To start, consider the effects of a plausibly higher setting for risk aversion: $\sigma = 2$. As shown in Fig. 2, the time paths for the government policy variables are qualitatively very similar to the benchmark setting with logarithmic preferences. The paths of private consumption and hours worked are somewhat smoother reflecting the fact that higher risk aversion also implies a lower intertemporal elasticity of substitution. Fluctuations in the real exchange rate and net exports are somewhat larger. Indeed, with higher risk aversion, net exports are more negative (relative to the benchmark calibration) for years 1 to 10 of the tax reform period. As a result, the home country becomes a larger net foreign debtor. The welfare benefit of the tax reform program falls from $4.4$ % for the benchmark calibration to $3.4\%$.

**Figure 2:** Dynamic Paths For Alternative Preference Parameter Values

Less substitutability between private and public consumption ($\psi = 1/2$) moderates the movements in private and public consumption, although qualitatively the dynamics of these variables are quite similar to the benchmark calibration. To afford the higher path for private consumption, households work more than in the benchmark. The one period rise in the capital income tax rate is larger: $227\%$ compared to $216\%$. The paths for the real exchange rate, net exports and net foreign assets are quite similar to the benchmark case. The welfare benefit of the Ramsey tax reform rises from $4.4\%$ to $4.7\%$.

*Trade Parameters*

The trade openness parameter, $\gamma$, was set to $0.3$ on the basis that the world import share is around $30\%$. The U.S. is less open than the world as a whole: its import share in the early 2000s is closer to $15\%$. Here, $\gamma$ is set to $0.05$ which corresponds to the U.S. import share circa 1970.

**Figure 3:** Dynamic Paths for Alternative Trade Parameters

As shown in Fig. 3, a less open economy naturally has time paths that look much more like a closed economy than the benchmark model. The one period spike in the capital income tax rate, $235\%$, lies between that of the closed economy ($260\%$) and the benchmark model ($216\%$). Due to the higher capital income tax rate, government debt falls more when the economy is less open. As in the closed economy model, labor income is initially subsidized, albeit at a lower rate ($\tau_0^h = -1.5\%$ rather than $-24\%$). At the start of the tax reform, there is a stronger depreciation of the real exchange rate ($24\%$ compared to $19\%$), followed by an attenuated appreciation relative to the initial real exchange rate ($22\%$ versus $21\%$). The exchange rate dynamics can be understood through the arbitrage condition equating the effective returns on international bonds and capital (see (32)). The larger increase in the capital income tax rate in period $1$ leads to a similarly larger change in the effective return to capital. Effective return equality then requires a more substantial appreciation in the real exchange rate between dates $0$ and $1$. Further, given that the economy is far less open, the effects of these real exchange rate movements on the domestic macroeconomy are less onerous. The welfare benefit of the tax reform is virtually the same as the benchmark model: $4.4\%$ of consumption.

**Figure 4:** Decomposition of Composite Home Consumption

Finally, the elasticity between domestic and foreign goods in the private consumption aggregator, (3), is $\mu = 1.5$ in the benchmark model, a value used by Backus et al. (1992) and much of the subsequent international finance literature with distinct home and foreign goods. While there is much recent controversy in the empirical literature concerning this elasticity, De Paoli (2009) shows that with complete international financial markets (and so complete risk sharing), what matters is whether home and foreign goods are substitutes in utility $(\mu > 1)$ or complements ($\mu < 1$). With this in mind, set this elasticity to $0.8$. Fig. 3 shows that this setting for $\mu$ has little effect on the model's time paths *except* for net exports and net foreign assets. In the benchmark calibration, the dynamics of net exports follow that of the real exchange rate. The long run real exchange rate depreciation is associated with positive net exports, and a negative net foreign asset position. These dynamics are reversed when the trade elasticity, $\mu$, is less than one. Fig. 4 digs deeper into these dynamics, presenting the paths of domestic consumption of the home- and foreign-produced goods. When these goods are substitutes in utility, as they are in the benchmark calibration, domestic households can relatively easily substitute between these two goods. As a result, the initial depreciation of the real exchange rate – which makes foreign goods relatively more expensive – leads to a substitution from foreign- to domestically-produced goods. In contrast, when the two goods are complements in utility, the two goods tend to be consumed together. Consequently, when $\mu < 1$, domestic consumption of the home and foreign goods both increase. These results are an illustration of the Marshall-Lerner condition: the balance of trade improves following a real exchange rate depreciation if the sum of the absolute values of the import and export elasticities is larger than one. Given the similarity of the time paths of private and public consumption as well as hours worked, it is not surprising that the welfare benefit of the tax reform is $4.4\%$ as it is in the benchmark calibration.

*Summary*

Relative to the benchmark small open economy model, the alternative preference parameter values considered in this section lead to only modest changes in fiscal policy and so in the time paths of macroeconomic variables. The largest differences are associated with the behavior of net exports and net foreign assets when the trade elasticity between home and foreign goods, $\mu$, is less than one. The welfare benefit of the Ramsey tax plan remains substantial for the alternative parameter values considered.

## 7. Conclusion

This paper looked at Ramsey-optimal taxation in three economic environments: a closed economy, a one good small open economy, and a two good small open economy. All three deliver the traditional result that capital income should not be taxed in the long run; see Chamley (1986); Judd (1985) and the subsequent literature. However, the short run dynamics of these models differ considerably. In the closed economy, the government initially taxes capital income very heavily, driving down government debt. Contrast these dynamics with those of the one good open economy: immediately drop the tax rate on capital income to zero. Opening the economy to trade imposes an additional constraint on the benevolent government planner in the Ramsey optimal taxation problem: In choosing the tax rate on capital income, the rate of return on capital, net of taxes, cannot be dominated by the return on international assets. The one good open economy environment precludes real exchange rate movements. Lowering the capital income tax rate to zero raises the return to capital. Consequently, the economy experiences a large inflow of capital (negative net exports) in order to restore equality of the return to capital with the fixed return to international assets, without sacrificing domestic private consumption. These dynamics are similar to those reported in Correia (1996).

The results in this paper show that the differences in the short run dynamics of the capital income tax rate are an artifact of the fixed exchange rate inherent to the one good open economy model. When the home and foreign countries produce differentiated goods, the real exchange rate is endogenously determined. In such a setting, the relevant return to foreign assets is no longer fixed, but rather includes a term reflecting real exchange rate movements. The Ramsey planner manipulates the real exchange rate – a sharp depreciation followed by an even sharper appreciation – which lowers the effective return to foreign assets, giving the planner room to raise the capital income tax rate. As a result, the capital income tax dynamics in the two good open economy environment look quite similar to those of the closed economy.

Two lessons can be drawn. First, opening the economy to trade is important because it imposes on the Ramsey planner an additional constraint: capital cannot be dominated in rate of return by foreign assets. Second, real exchange rate dynamics are important. The fixed real exchange rate inherent to the one good open economy model probably does not reflect the environment faced by policymakers. Certainly, the results from the one good model are not representative of open economy models more generally.

**Appendix A. The Ramsey Allocation is also a Competitive Equilibrium**

This appendix shows that the allocation chosen by the Ramsey planner,

$$\{c_t, g_t, h_t, k_t, e_t, p_t\}_{t=0}^{\infty} \tag{A.1}$$

can be supported as a competitive equilibrium.

To start, factor prices are obtained from the first-order conditions to the firm's problem:

$$r_t = F_1(k_{t-1}, h_t), \quad w_t = F_2(k_{t-1}, h_t). \tag{A.2}$$

Next, combine (14) and (15) as

$$U_h(c_t, g_t, h_t) + (1-\tau_t^h) F_2(k_{t-1}, h_t) \frac{U_c(c_t, g_t, h_t)}{p_t} \tag{A.3}$$

which gives the labor income tax rate. Combine (14) and (16) to obtain

$$\frac{U_c(c_t, g_t, h_t)}{p_t} = \beta \frac{U_c(c_{t+1}, g_{t+1}, h_{t+1})}{p_{t+1}} \left[1 - \delta + (1-\tau_{t+1}^k) F_1(k_t, h_{t+1})\right] \tag{A.4}$$

which gives the capital income tax rate.

Given factor prices in (A.2), the Ramsey allocation is consistent with firm profit maximization since (A.2) are the firm's first-order conditions. We need to show that the Ramsey allocation is consistent with household optimization. The implementability condition (20) in the Ramsey problem implies that the household budget constraint (2) holds with equality. Setting the tax rates to satisfy (A.3) and (A.4) means that the household first-order conditions (14) and (16) are satisfied. For household optimization, it remains to show that the first-order conditions (17) and (18) hold. (14) and (17) imply

$$\frac{e_t U_c(c_t, g_t, h_t)}{p_t R^b} = \beta \frac{e_{t+1} U_c(c_{t+1}, g_{t+1}, h_{t+1})}{p_{t+1} R^b}. \tag{A.5}$$

If the international risk-sharing condition is satisfied (as it is in the Ramsey planner's problem), then

$$\frac{e_t U_c(c_t, g_t, h_t)}{p_t} = \vartheta, \tag{A.6}$$

and so (A.5) is necessarily satisfied given $R^b = \beta$ (as it is in our calibration). It is, then, straightforward to show that (17) and (18) are satisfied. We have, then, shown that the Ramsey allocation is consistent with household optimizing behavior.

All that remains to show is that the government budget constraint is satisfied for the Ramsey allocation. Recall that the household budget constraint (2) is satisfied by virtue of the planner satisfying the implementability condition. Use (A.2) to substitute for factor prices in (2):

$$p_t c_t + k_t + \frac{d_t}{R_t^d} + e_t \frac{b_t}{R^b} = (1-\tau_t^h) F_2(k_{t-1}, h_t) h_t + (1-\tau_t^k) F_1(k_{t-1}, h_t) k_{t-1} + (1-\delta) k_{t-1} + d_{t-1} + e_t b_{t-1} + \tau. \tag{A.7}$$

Rearrange the terms in (A.7) as

$$\begin{aligned} p_t c_t + k_t + \frac{d_t}{R_t^d} + e_t \frac{b_t}{R^b} &= F_2(k_{t-1}, h_t) h_t + F_1(k_{t-1}, h_t) k_{t-1} \\ &\quad - \tau_t^h F_2(k_{t-1}, h_t) h_t - \tau_t^k F_1(k_{t-1}, h_t) k_{t-1} + (1-\delta) k_{t-1} + d_{t-1} + e_t b_{t-1} + \tau. \end{aligned} \tag{A.8}$$

Since the production function is homogeneous of degree one, Euler's theorem for homogeneous functions implies that $F_2(k_{t-1}, h_t) h_t + F_1(k_{t-1}, h_t) k_{t-1} = F(k_{t-1}, h_t)$. Use this fact and the domestic goods market clearing condition (10) to obtain

$$p_t c_t + \frac{d_t}{R_t^d} + e_t \left[ \frac{b_t}{R^b} - b_{t-1} \right] = (1-\gamma) p_t^{\mu} c_t + \gamma e_t^{\mu} c^* + g_t - \tau_t^h F_2(k_{t-1}, h_t) h_t - \tau_t^k F_1(k_{t-1}, h_t) k_{t-1} + d_{t-1} + \tau. \tag{A.9}$$

Next, since the international solvency condition in the Ramsey problem, (26), was developed from the balance of payments equation, (11), it follows that (11) is satisfied for the Ramsey allocation. Rearrange the terms in (11) as

$$e_t \left[ \frac{b_t}{R^b} - b_{t-1} \right] = \gamma e_t^{\mu-1} c^* - \gamma \left[ (1-\gamma) e_t^{\mu-1} + \gamma \right]^{\frac{\mu}{1-\mu}} c_t \tag{A.10}$$

and substitute into (A.9):

$$p_t c_t + \frac{d_t}{R_t^d} + \gamma e_t^{\mu-1} c^* - \gamma\left[(1-\gamma)e_t^{\mu-1} + \gamma\right]^{\frac{\mu}{1-\mu}} c_t = (1-\gamma)p_t^{\mu} c_t + \gamma e_t^{\mu} c^* + g_t$$
$$- \tau_t^h F_2(k_{t-1}, h_t)h_t - \tau_t^k F_1(k_{t-1}, h_t)k_{t-1} + d_{t-1} + \tau. \tag{A.11}$$

In equilibrium the consumption bundler earns zero profits; thus, the bundler's revenue equals its expenses:

$$p_t c_t = c_{ht} + e_t c_{ft}. \tag{A.12}$$

For the small open economy, $\varphi = 1-\gamma$ and so (4) to (6) read

$$p_t = \left[1-\gamma+\gamma e_t^{1-\mu}\right]^{\frac{1}{1-\mu}}, \tag{A.13}$$

$$c_{ht} = (1-\gamma)\left[1-\gamma+\gamma e_t^{1-\mu}\right]^{\frac{\mu}{1-\mu}} c_t, \tag{A.14}$$

$$c_{ft} = \gamma\left[(1-\gamma)e_t^{\mu-1} + \gamma\right]^{\frac{\mu}{1-\mu}} c_t. \tag{A.15}$$

Substituting (A.12) to (A.15) into (A.11) and canceling terms leaves the government budget constraint (9).

## Appendix B. Complete Descriptions of the Closed and One Good Open Economy Models

*Appendix B.1. The Closed Economy Model*

The representative household's problem is:

$$\max_{\{c_t, h_t, k_t, d_t\}} \sum_{t=0}^{\infty} \beta^t U\ (c_t, g_t, h_t) \tag{B.1}$$

subject to

$$c_t + k_t + \frac{d_t}{R_t^d} \leq \left(1-\tau_t^h\right) w_t h_t + \left[(1-\tau_t^k) r_t + 1 - \delta\right] k_{t-1} + d_{t-1} + \tau. \tag{B.2}$$

Firms maximize period-by-period profits:

$$\max_{k_{t-1},h_t} F(k_{t-1},h_t) - r_t k_{t-1} - w_t h_t. \tag{B.3}$$

The government budget constraint is (9).

The Ramsey problem is:

$$\begin{aligned}\mathcal{L} = \max_{\{c_t,g_t,h_t,k_t\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t \{U\ (c_t,g_t,h_t) \\ & + \phi_t \left[F(k_{t-1},h_t) + (1-\delta)k_{t-1} - c_t - k_t - g_t\right] \\ & + \Lambda\left[U_c(c_t,g_t,h_t)(c_t-\tau) + U_h(c_t,g_t,h_t)h_t\right]\} \\ -\Lambda U_c(c_0,g_0,h_0) & \left\{\left[(1-\tau_0^k)F_k(k_{t-1},h_t) + 1 - \delta\right]k_{-1} + d_{-1}\right\}.\end{aligned} \tag{B.4}$$

*Appendix B.2 The One Good Small Open Economy Model*

The representative domestic household solves:

$$\max_{\{c_t,h_t,k_t,d_t,b_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U\ (c_t,g_t,h_t) \tag{B.5}$$

subject to:

$$c_t + k_t + \frac{d_t}{R_t^d} + \frac{b_t}{R^b} \le \left(1-\tau_t^h\right) w_t h_t + R_t^k k_{t-1} + d_{t-1} + b_{t-1} + \tau. \tag{B.6}$$

The typical firm's problem and government budget constraint are as above.

The associated Ramsey problem:

$$\begin{aligned}\mathcal{L} = \max_{\{c_t,g_t,h_t,k_t\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t \{U\ (c_t,g_t,h_t) \\ & + \phi_t \left[F(k_{t-1},h_t) + (1-\delta)k_{t-1} - c_t - nx_t - k_t - g_t\right] \\ & + \xi_t \left[U_c(c_t,g_t,h_t) - \vartheta\right] \\ & + \Lambda\left[U_c(c_t,g_t,h_t)(c_t-\tau) + U_h(c_t,g_t,h_t)h_t\right] \\ & + \Omega n x_t \\ -\Lambda \frac{U_c(c_0,g_0,h_0)}{p_t} & \left\{\left[(1-\tau_0^k)F_k(k_{t-1},h_t) + 1 - \delta\right]k_{-1} + d_{-1} + b_{-1}\right\} \\ +\Omega b_{-1}. &\end{aligned} \tag{B.7}$$

**Appendix C. First-order Conditions to the Ramsey Problem: The Two Good Small Open Economy Model**

Defining

$$W(c_t, g_t, h_t) \equiv U(c_t, g_t, h_t) + \Lambda\left[U_c(c_t, g_t, h_t)(c_t - \tau) + U_h(c_t, g_t, h_t)h_t\right], \tag{C.1}$$

the Ramsey problem can be written:

$$\begin{aligned}\mathcal{L} = \max_{\{c_t, g_t, h_t, k_t, e_t, p_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \{ & W(c_t, g_t, h_t) \\ & + \phi_t\left[F(k_{t-1}, h_t) + (1-\delta)k_{t-1} - (1-\gamma)p_t^{\mu} c_t - \gamma e_t^{\mu} c^* - k_t - g_t\right] \\ & + \xi_t\left[\frac{e_t U_c(c_t, g_t, h_t)}{p_t} - \vartheta\right] \\ & + \zeta_t\left[1 - \gamma + \gamma e_t^{1-\mu} - p_t^{1-\mu}\right] \\ & + \Omega\left[\gamma e_t^{\mu-1} c^* - \gamma[(1-\gamma)e_t^{\mu-1} + \gamma]^{\frac{\mu}{1-\mu}} c_t\right]\} \\ & - \Lambda \frac{U_c(c_0, g_0, h_0)}{p_0}\left\{\left[(1-\tau_0^k)F_k(k_{-1}, h_0) + 1 - \delta\right]k_{-1} + d_{-1} + e_0 b_{-1}\right\} \\ & + \Omega b_{-1}.\end{aligned} \tag{C.2}$$

The $t = 0$ first-order conditions are:

$$\begin{aligned} 0 = \; & W_c(c_0, g_0, h_0) - \phi_0(1-\gamma)p_0^{\mu} + \xi_0 \frac{e_0 U_{cc}(c_0, g_0, h_0)}{p_0} - \Omega\gamma\left[e_0^{\mu-1}(1-\gamma) + \gamma\right]^{\frac{\mu}{1-\mu}} \\ & - \Lambda \frac{U_{cc}(c_0, g_0, h_0)}{p_0}\left\{\left[(1-\tau_0^k)F_k(k_{-1}, h_0) + 1 - \delta\right]k_{-1} + d_{-1} + e_0 b_{-1}\right\}\end{aligned} \tag{C.3}$$

$$\begin{aligned} 0 = \; & W_g(c_0, g_0, h_0) - \phi_0 + \xi_0 \frac{e_0 U_{cg}(c_0, g_0, h_0)}{p_0} \\ & - \Lambda \frac{U_{cg}(c_0, g_0, h_0)}{p_0}\left\{\left[(1-\tau_0^k)F_k(k_{-1}, h_0) + 1 - \delta\right]k_{-1} + d_{-1} + e_0 b_{-1}\right\}\end{aligned} \tag{C.4}$$

$$0 = W_h(c_0, g_0, h_0) + \phi_0 F_h(k_{-1}, h_0) + \xi_0 \frac{e_0 U_{ch}(c_0, g_0, h_0)}{p_0} - \Lambda \frac{U_{ch}(c_0, g_0, h_0)}{p_0}\left\{\left[(1-\tau_0^k)F_k(k_{-1}, h_0) + 1 - \delta\right]k_{-1} + d_{-1} + e_0 b_{-1}\right\} - \Lambda \frac{U_c(c_0, g_0, h_0)}{p_0}(1-\tau_0^k)F_{kh}(k_{-1}, h_0)k_{-1} \tag{C.5}$$

$$0 = \beta\phi_1\left[1 + F_k(k_0, h_1) - \delta\right] - \phi_0 \tag{C.6}$$

$$0 = \phi_0\mu(1-\gamma)p_0^{\mu-1}c_0 + \xi_0 \frac{e_0 U_c(c_0, g_0, h_0)}{p_0^2} + \zeta_0(1-\mu)p_0^{-\mu} + \Lambda \frac{U_c(c_0, g_0, h_0)}{p_0^2}\left\{\left[(1-\tau_0^k)F_k(k_{-1}, h_0) + 1 - \delta\right]k_{-1} + d_{-1} + e_0 b_{-1}\right\} \tag{C.7}$$

$$0 = -\phi_0\gamma\mu e_0^{\mu-1}c^* + \xi_0 \frac{U_c(c_0, g_0, h_0)}{p_0} + \zeta_0\gamma(1-\mu)e_0^{-\mu} + \Omega\gamma(\mu-1)e_0^{\mu-2}c^* + \Omega\gamma(1-\gamma)\mu\left[(1-\gamma)e_0^{\mu-1} + \gamma\right]^{\frac{\mu}{1-\mu}} e_0^{\mu-2}c_0 - \Lambda \frac{U_c(c_0, g_0, h_0)}{p_0}b_{-1} \tag{C.8}$$

For $t \geq 1$:

$$0 = W_c(c_t, g_t, h_t) - \phi_t(1-\gamma)p_t^{\mu} + \xi_t \frac{e_t U_{cc}(c_t, g_t, h_t)}{p_t} - \Omega\gamma\left[e_t^{\mu-1}(1-\gamma) + \gamma\right]^{\frac{\mu}{1-\mu}} \tag{C.9}$$

$$0 = W_g(c_t, g_t, h_t) - \phi_t + \xi_t \frac{e_t U_{cg}(c_t, g_t, h_t)}{p_t} \tag{C.10}$$

$$0 = W_h(c_t, g_t, h_t) + \phi_t F_h(k_{t-1}, h_t) + \xi_t \frac{e_t U_{ch}(c_t, g_t, h_t)}{p_t} \tag{C.11}$$

$$0 = \beta\phi_{t+1}\left[1 + F_k(k_t, h_{t+1}) - \delta\right] - \phi_t \tag{C.12}$$

$$0 = \phi_t\mu(1-\gamma)p_t^{\mu-1}c_t + \xi_t \frac{e_t U_c(c_t, g_t, h_t)}{p_t^2} + \zeta_t(1-\mu)p_t^{-\mu} \tag{C.13}$$

$$
\begin{aligned}
0 = \quad & -\phi_t \gamma \mu e_t^{\mu-1} c^* + \xi_t \frac{U_c(c_t, g_t, h_t)}{p_t} + \zeta_t \gamma (1-\mu) e_t^{-\mu} \\
& + \Omega \gamma (\mu - 1) e_t^{\mu-2} c^* + \Omega \gamma (1-\gamma) \mu \left[ (1-\gamma) e_t^{\mu-1} + \gamma \right]^{\frac{\mu}{1-\mu}} e_t^{\mu-2} c_t
\end{aligned} \tag{C.14}
$$

In addition, (4), (10) and (A.6) must be satisfied.

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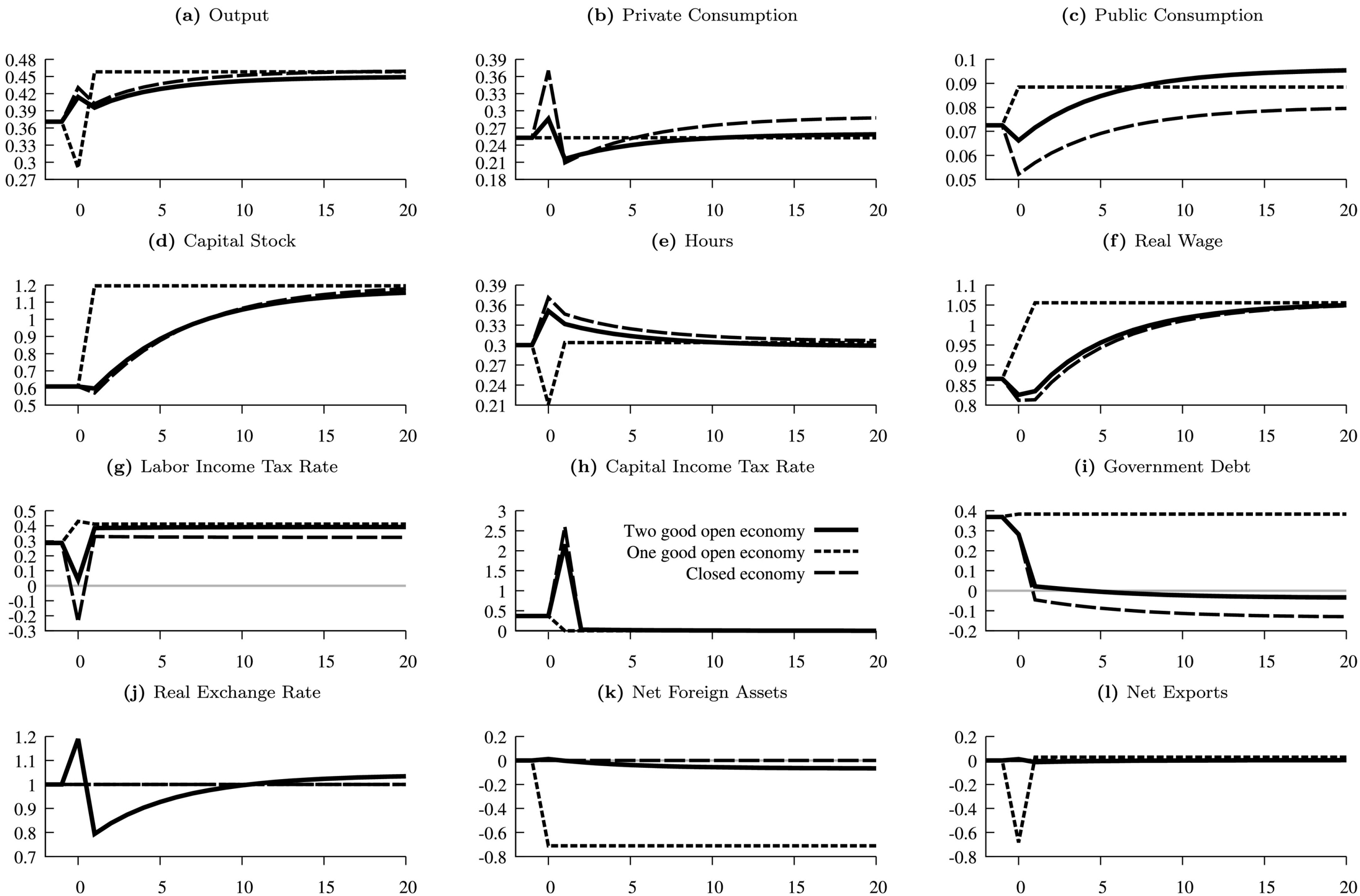
(a) Output
(b) Private Consumption
(c) Public Consumption
(d) Capital Stock
(e) Hours
(f) Real Wage
(g) Labor Income Tax Rate
(h) Capital Income Tax Rate
(i) Government Debt
(j) Real Exchange Rate
(k) Net Foreign Assets
(l) Net Exports
Two good open economy
One good open economy
Closed economy
0
5
10
15
20

Figure 1

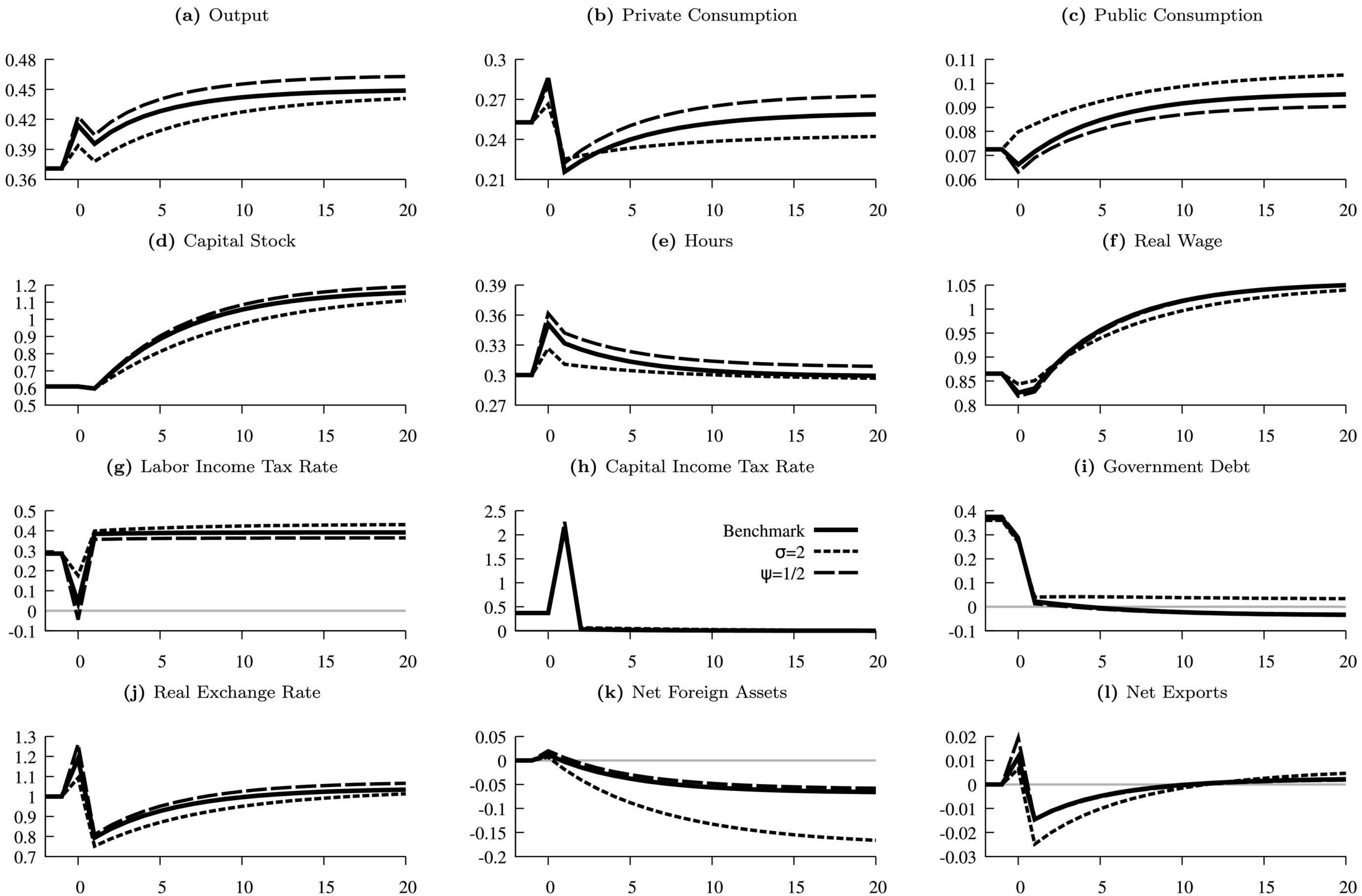
(a) Output
(b) Private Consumption
(c) Public Consumption
(d) Capital Stock
(e) Hours
(f) Real Wage
(g) Labor Income Tax Rate
(h) Capital Income Tax Rate
(i) Government Debt
(j) Real Exchange Rate
(k) Net Foreign Assets
(l) Net Exports
Benchmark
σ=2
ψ=1/2

Figure 2

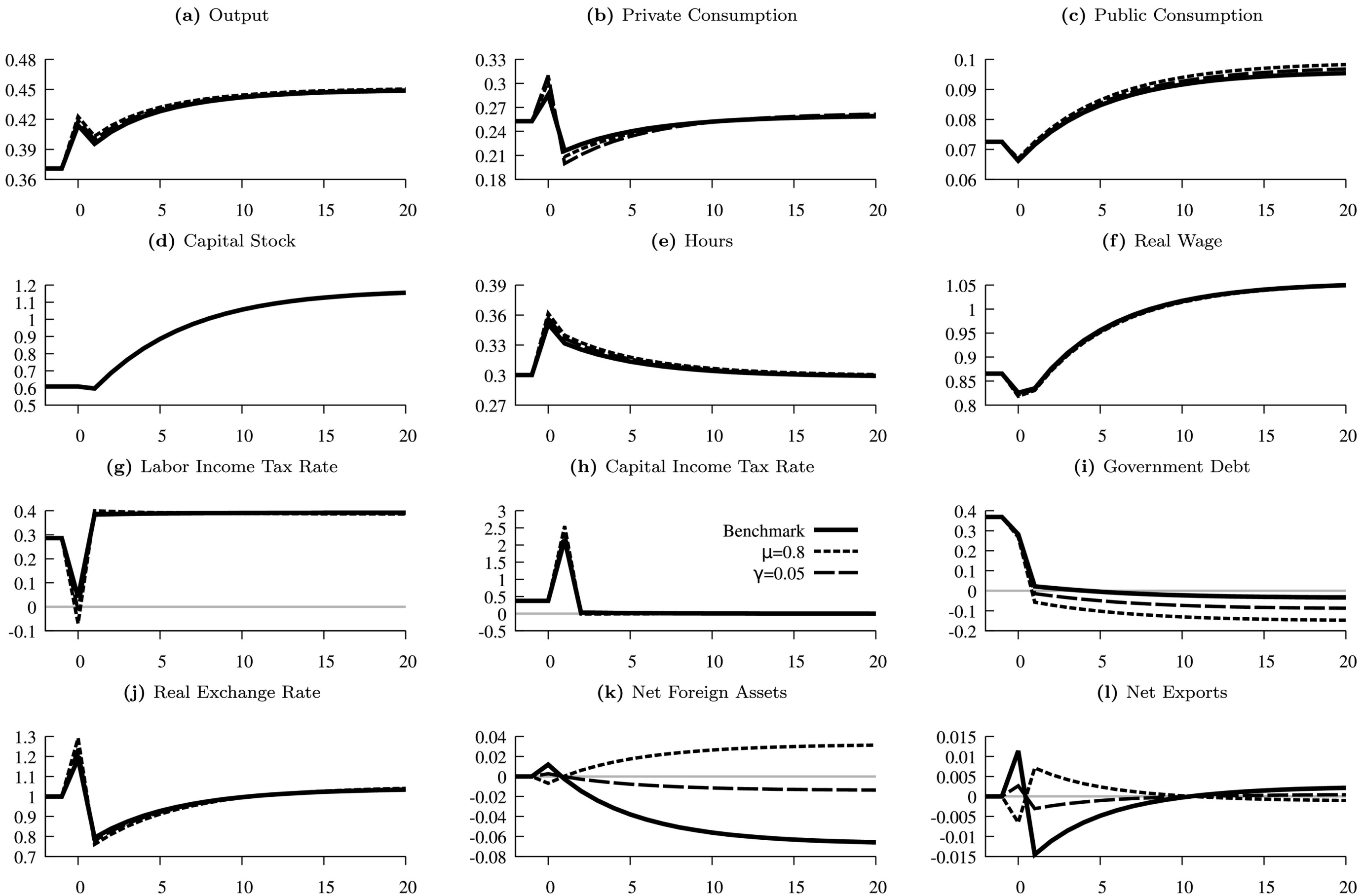
(a) Output
(b) Private Consumption
(c) Public Consumption
(d) Capital Stock
(e) Hours
(f) Real Wage
(g) Labor Income Tax Rate
(h) Capital Income Tax Rate
(i) Government Debt
(j) Real Exchange Rate
(k) Net Foreign Assets
(l) Net Exports
Benchmark
μ=0.8
γ=0.05
0
5
10
15
20

Figure 3

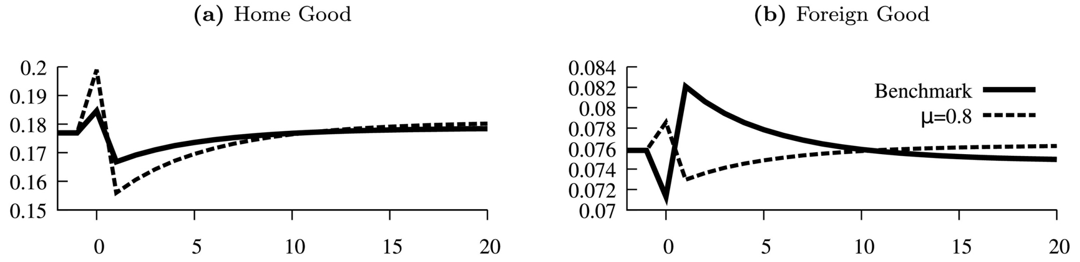
(a) Home Good
0.2
0.19
0.18
0.17
0.16
0.15
0
5
10
15
20
(b) Foreign Good
0.084
0.082
0.08
0.078
0.076
0.074
0.072
0.07
Benchmark
μ=0.8
0
5
10
15
20

Figure 4