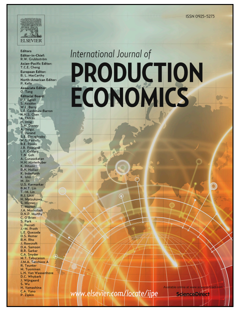

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Cost-sharing mechanism for product quality improvement in a supply chain under competition

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Abstract

For the growing competition in the modern sophisticated business environment, there is an increasing trend to launch new products or to improve the quality of the end products in order to attract more consumers. But the rising costs or uncertainties for this innovation require firms to collaborate with each other. In this paper, we study how a retailer and each of two competing manufacturers can be benefited by collaborative product quality improvement strategies in a supply chain. In our model, two competing manufacturers invest in the quality improvement of their respective product and a common retailer sells those higher quality substitutable products to the end consumers. To incentivize the manufacturers in product quality improvement initiatives, we address cost-sharing mechanisms between the retailer level and the manufacturer level. To focus on the importance of collaborative product quality improvement, we study the quality improvement initiatives both in collaborative and non-collaborative scenarios. Through game-theoretic framework, we address mainly three different contract scenarios: (1) WP contract in which both manufacturers accept wholesale price contract, (2) CSC contract in which both manufacturers accept cost-sharing contract, and (3) WC contract in which only one manufacturer accepts cost-sharing contract and the other accepts wholesale price contract. Depending on the mechanism how the retailer decides its cost-sharing proportion with the manufacturer, we further develop and analyze two models of cost-sharing for each of CSC and WC contracts. Our results show that both cost-sharing contracts result in higher quality improvement levels and higher profits in the supply chain as compared to WP contract. In addition, the retailer can enhance his profit level by reducing the differentiation between retail prices (intense price competition) of two products. On the contrary, both manufacturers can sufficiently increase their profit levels by raising the differentiation between two products with respect to quality (lower quality competition). We further investigate those scenarios in which the retailer would be interested in sharing more quality investment costs with the manufacturers.

Keywords: supply chain management; quality management; competition; coordination; game theory

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1. Introduction

In recent years, product quality is getting greater attention by the consumers. Moreover, in some industries, after the price of the product, the quality has become the second most important factor influencing consumers' purchasing decisions. Hence, many industries are now adopting product quality improvement as a powerful competitive tool (He et al., 2016) in fulfilling consumers' expectation level. A failure to meet this expectation may be a key reason behind the loss of goodwill toward a company. For instance, in 2011, the brand's ranking of Ford fell 10 places in Consumer Report's annual auto reliability survey due to deterioration of quality and subsequently, 'quality' became 'job 1' at Ford (Durbin and Krisher, 2011). As a result of year-long fixation on quality, Ford ranked at 4th position according to latest edition of J.D. Power's Initial Quality Study beating BMW, Hyundai even Toyota, also. This is the best position for Ford in the 31-year history of the report (Rosevear, 2017).

However, there are two major dimensions through which quality of the product is measured: quality dimensions of products and quality dimensions of services (Bergman and Klefsjö, 2010). In our present study, our interest is on the first type of the quality of products. In practice, product quality concept includes many further sub-dimensions of products like reliability, maintenance, maintainability, environmental impact, safety and durability of the product etc. Hence, it is very important for an industry to design appropriate quality improvement strategies and to address suitable coordinating mechanisms which result in greater product quality and larger profits in the supply chain. From this perspective, the retail giant Wal-Mart can be seen as a quality oriented company. Before purchasing the products from its suppliers and before ensuring the compatibility of the products, Wal-Mart always goes through the manuals of the suppliers providing detailed guidelines and standards of the products in order to meet Wal-Mart's quality standard and policies (Crosby, 1979).

In order to meet or even exceed their quality requirement applied to products, Wal-Mart works in close partnership with their suppliers (Essays, 2013).

On the other hand, product quality is important to those consumers who are ready to pay a high price to get a high-quality product in return. But the higher quality of the products sometimes leads to extra cost burden to the manufacturers due to higher production costs, this results in turn, high product price possibility. Hence, demand and profits are influenced by these quality improvement practices (Banker et al., 1998; Baiman et al., 2000) in competitive environment. Hence, it is very important to understand for the retailer, how to motivate the manufacturers to product quality improvement initiatives. Moreover, how to balance between quality improvement strategy and pricing strategy in presence of competition is a very interesting research topic.

Hence, our primary objective is to study different pricing and quality improvement strategies of the channel members under both non-collaborative and collaborative quality improvement scenarios in presence of competition. In our study, consumers are sensitive not only to the price but also to the quality of the product. Similar to the definition of quality in Banker et al. (1998), in this paper, by the term "quality" we mean both design and conformance quality characteristics of interest to the consumers. Most of the existing literature on quality management has been focused either in the direction of coordinating the supply chain (Reynier and Tapiero, 1995; Baiman et al., 2000; Chao et al., 2009; Wang et al., 2015; He et al., 2016 etc.) or in the direction of competing market segment (Moorthy, 1988; Banker et al., 1998; Chambers et al., 2006; Giri et al.,2015 etc.), but little attention has been paid to the joint effect of these two research directions (Chen et al. 2015). Hence, in our present study, our aim is to contribute the existing literature by integrating these two research directions. Hence, the important research questions in our present study are: (1) how to design suitable coordinating mechanisms between each manufacturer and the retailer to make spontaneous participation of the manufacturers into quality improvement initiatives?; (2) what is the impact of these contracts on the equilibrium decisions of the supply chain?; (3) is intense price competition or intense quality competition between two products beneficial to the channel members? and (4) which supply chain strategy is best-off in producing the highest quality product or earning the highest profit?

In this model, we consider product quality improvement initiatives among the manufacturers. In order to light on how to diverse this quality improvement efforts among the channel members, we analyze two-competing-manufacturers and one-retailer supply chain model under game theoretic framework. We assume two products compete with respect to price and quality, both. We capture these competitions by the product substitution in terms of both price and quality. On the other hand, to focus the impact of the collaborative product quality improvement on the profits of the supply chain, we consider this quality improvement in both non-collaborative and collaborative scenarios. We study the non-collaborative scenario under well known wholesale price (WP) contract in which the manufacturers are only responsible for the quality improvement of the products whereas there is no contribution of the retailer. We then convert this non-collaborative quality improvement scenario to a collaborative one by addressing cost-sharing (CS) mechanism (Chao et al., 2009; Yan et al. 2015; Ghosh et al., 2015; He et al., 2016 etc.) between the common retailer and each of the manufacturers. Under this mechanism, the retailer shares the quality investment cost with the manufacturers. Depending on whether both manufacturers accept the CS contract, we study the CS contract under two scenarios: CSC contract (in which both manufacturers accept the CS contract) and WC contract (in which only one manufacturer accepts the CS contract. We further consider each of the above two CSC and WC contracts in two different scenarios: Simple cost-sharing contract, in which the retailer determines his cost-sharing fraction by optimizing his profit and Cost-sharing through bargaining contract, in which both the retailer and each of the manufacturers bargain on the cost-sharing fraction.We name simple cost-sharing contact as SCS and WCS in case of CSC contract and WC contract, respectively. Similarly, Cost-sharing through bargaining contract is named as CSB and WCB under CSC and WC contracts, respectively.

In terms of collaborative product quality improvement approach, our analysis shows that both CS contracts (CSC and WC) can always coordinate the supply chain in the sense that both CS contracts are able to improve product quality improvement levels, profits of all channel members as well as total supply chain profit as compared to WP contract. Moreover, our results show that for higher quality improvement cost the retailer reduces his share in quality investment cost with the manufacturers due to reduction in consumers demand

and hence to maintain his profitability. More interestingly we further find that CSB contract is always the best choice to the manufacturers, consumers and the total supply chain but it is not the best choice to the retailer. SCS is the best choice for the retailer as compared to other contract scenarios from profitability's viewpoint.

In terms of competition between two products, we find that price competition between two substitutable products is always profitable to consider than that with respect to quality. Moreover, the retailer would be interested to share more quality investment cost with the manufacturers when price competition intensity is high and quality competition intensity is low between two products. It reduces the quality improvement cost burden of the manufacturers and consequently, it increases the quality improvement levels of the products.

The rest of the paper is organized in the following way. The next section provides a review of the related literature. In Section 3, we develop our model with appropriate descriptions. In Section 4, we derive the equilibrium solutions of our supply chain model in different coordinating contracts. In section 5, we discuss the impact of coordinating contracts and competition on the equilibrium decisions. Moreover, in this section, a comparison study is carried out under different scenarios. Finally, we conclude our paper with managerial explanations and with further research directions in Section 6. All proofs are depicted in the Appendix.

2. Literature Review

The contribution of this paper to the existing literature is that we identify how a retailer collaborates with a manufacturer in product quality improvement initiatives in presence of competition. Therefore, this paper contributes to several streams of research which are broadly classified into two streams: (1) quality management with different coordinating contracts, and (2) quality management in presence of competition. We address the contributions and the limitations of the existing literature in the following way:

Since last two decades, there is a growing literature on quality management and coordinating contracts in the supply chain. Following are some researches in quality management that investigate how to coordinate the supply chain to improve the performance of the entire supply chain. The initial work was done by Reyniers and Tapiero (1995) who studied the impact of coordinating contracts on the supplier's quality level and a buyer's inspection policy in both non-cooperative and cooperative settings. They further highlighted the importance of strategic and contractual issues in quality management. Considering a "risk-neutral settings", Baiman et al. (2000) analyzed the relationship among product quality, the cost of quality, and the information that can be contracted on. Lim (2001) extended Reyniers and Tapiero (1995)'s work by considering a supply chain with incomplete information. Then, Singer et al. (2003) intended to explain the strategic behavior regarding quality within a supplier-retailer partnership in a disposable product industry. By addressing mutually beneficial transfer contract they were able to improve product quality as well as profits within the supply chain. Balachandran and Radhakrishnan (2005) developed a model by considering both single moral-hazard and double moral-hazard cases in quality investment effort. By addressing warranty/penalty contracts they further investigated their model under three different scenarios: whether penalty (i) is based on internal information, (ii) is based on external failure , (iii) satisfies the fairness criterion. Zhu et al. (2007) considered a single-supplier-single-buyer supply chain in which the buyer designed the product and owned the brand whereas the supplier produced the product for that buyer. They further investigated how to coordinate the channel members regarding quality improvement efforts. In make-to-order (MTO) environment, Xiao et al. (2011) developed a model to investigate how to coordinate one-supplier-one-retailer supply chain with a quality assurance policy via a revenue-sharing contract. El Ouardighi (2014) investigated the impact of revenue sharing contract on the design quality of a particular finishes product by considering one-manufacturer-one-supplier supply chain model. Wang et al. (2015) developed a supply chain model in which an upstream supplier invests in innovation and a downstream manufacturer sells those improved products to consumers. They addressed three widely used contracts and concluded that the revenue-sharing contract coordinates the supply chain whereas the other two contracts (the wholesale price contract and the quality-dependent wholesale price contract) may or may not.

But all the references, mentioned above, did not consider the contract of sharing the cost of quality management effort among the channel members. The initial work with a cost-sharing contract in quality management literature was carried out by Chao et al. (2009).

Apart from this, they proposed two more (revenue-sharing, and effort-sharing) collaborative arrangements between a manufacturer and a supplier to induce quality improvement efforts. At the same time, Bhaskaran and Krishnan (2009) also considered cost-sharing contract together with other two (revenue sharing and innovative effort sharing) contracts to collaborate in the development and launching a new product within a supply chain. However, the type of innovation defined in their study is different from us in the sense that in our study we focus on the improvement of the existing product quality within the supply chain. Yan (2015) in his recent work, considered a joint pricing and product quality decision problem in a one-manufacturer-one-retailer supply chain and compared the performance of three different contract formats (two-part tariff contract, revenue-sharing contract, and effort cost-sharing contract) for this decentralized system. Ghosh et al (2015) also considered cost-sharing contract for a one-manufacturer-one-retailer supply chain model from greening level's perspective. Recently, He et al. (2016) developed a single-manufacturer-single-supplier supply chain model with reference effects in supplier quality management. By considering the total cost-sharing contract they were able to improve the performance of the supply chain in the presence of transfer payment. However, in our present study, we add this stream by identifying the impact of both non-cooperative (ie., WP, SCS, WCS contract scenarios) and cooperative (SCB, WCB contract scenarios) behavior (games) among the channel members on the pricing and quality decisions in the supply chain.

On the other hand, the second stream of literature captures the research on quality management in a competitive environment. In all the above studies, the effect of quality maintenance and coordination mechanisms are investigated for the single manufacturer and single retailer supply chain. Neither of the above references includes the effect of price and quality competition or cooperation. But depending on different choices of consumer, a retailer can buy different-quality products at different prices. Since the last two decades, there has been an increasing trend in many industries where competition is shifting from price and product quantity to product quality and service in the specific market segment (He et al., 2016 etc). Early research which included attributes like product quality and service can be found in economics literature such as Spence (1975) and Dixit (1979). Moorthy (1988) considered competition in a duopoly market through both price and quality. Banker et al. (1998) then considered supply chain models of oligopolistic competition to investigate whether equilibrium levels of quality increased in competition intensity. In a similar way to Moorthy (1988), Chambers et al. (2006) considered the impact of variable production costs on competitive behavior in a duopoly where manufacturers compete on both quality and price in a two-stage game. Xie et al. (2011) considered quality improvement in a given segment of the market, shared by two supplier-manufacturer supply chains which offer a given product at the same price but compete on quality. Giri et al. (2015) developed a supply chain model where multiple oligopolistic manufacturers compete for both quality and selling price of a product with a deterministic demand pattern.

All the above references on the quality management literature, researches had been carried out either in the direction of coordinating the supply chain or in competing-market scenarios. Very little attention has been paid to the joint effect of these two streams. In our present study, our goal is to enrich this gap of the joint effects of both coordination and competition in quality management literature. In this study, our aim is to investigate the impact of demand variability of the two products on the optimal decisions as well as to investigate how the channel members collaborate with each other in product quality improvement initiatives. Recently, Chen et al. (2015) considered a cooperative quality improvement strategy with both price and quality competition. But, our model is different from Chen et al. (2015)'s model in the sense that instead of downstream competition in our model, we consider upstream competition at the horizontal level. Moreover, different from them we have considered both non-cooperative as well as cooperative game frameworks at the same study. Chen et al. (2015) focused on the cooperative quality investment issues on the quality decisions of the supply chain. While in the present study, we aim to investigate the impact of price competition, quality competition as well as cooperative quality improvement strategies on both pricing and quality decisions. Through our analysis, we find whether immense quality differentiation (lower quality competition) between two products is profitable to the manufacturers or huge price differentiation (lower price competition) between those products is beneficial to the retailer.

Our modeling approach in studying the coordinating contracts leads to the use of the non-cooperative game as well as cooperative bargaining (Nash, 1950; Nash, 1953) as modeling tools. In recent years, there has been a wide variety of research papers that apply non-cooperative game (Choi, 1991; Choi, 1996 etc) as well as cooperative bargaining (Kohli and Park, 1989; Bhaskaran and Krishnan, 2009) framework to the field of supply chain management. For a detailed survey of the existing literature on the applications of non-cooperative games readers are referred to Cachon and Netessine (2004). On the other hand, for a detailed review of operations management work which applies cooperative bargaining framework, we would like readers to refer to Nagarajan and Sosic (2008).

The papers closest in spirit to ours are Choi (1991) and Chakraborty et al. (2015) with substitutable products, where quality and cooperative bargaining issues are not considered, Giri et al. (2015) with quality competition but coordination and cooperative bargaining are not considered, Chen et al. (2015)'s model where downstream competition is considered without considering bargaining issues and Ghosh et al. (2015) where cooperative bargaining is considered for green products without considering any competition among the channel members.

In brief, the main contribution of our work is that we shed light on how the retailer collaborates with the manufacturers to share their quality improvement efforts in presence of upstream competition (rather than downstream competition). We achieve this by addressing a coordinating contract with both non-cooperative game and cooperative bargaining framework (rather than non-cooperative only) in both scenarios: when both manufacturers accept CS contract (CSC) and when only one manufacturer accepts that contract (WC); and consider sequential decisions of the channel members with the assumption that quality decisions always take place before pricing decisions.

3. The Model

In our study, we consider a supply chain consists of two competing manufacturers $M_i,\ i = 1, 2$ (she) who sell the products through a common retailer R (he). The retailer sells those two competing brands with varying degrees of product substitutability. In our study, two manufacturers compete on both price and quality of the products. Due to technological innovation, nowadays, it is quite easy for consumers to compare the prices as well as the qualities of different brands of similar products. Hence, our primary interest is to investigate quality investment and pricing decisions, simultaneously, in a supply chain which produces goods against specific orders placed by the retailer.

Our basic model is developed under non-collaborative quality investment scenario in which only manufacturers incur the cost of quality improvement of their respective product while both the retailer and the manufacturer benefit from this quality improvement efforts. However, retailer's involvement can have a significant positive impact on profits of both parties (manufacturer and retailer) as well as the supply chain (Zhu et al., 2007). In our study, we capture the competition between two products in terms of both price and quality by the product substitution. Hence, market demand function must adequately reflect the substitutability of those two products. Thus, our basic model uses the following demand function that captures the product substitution in terms of price (Jeuland and Shugan, 1983; Choi, 1991; Choi, 1996; Ingene and Parry, 1995, Chakraborty et al., 2015 etc.) as well as in terms of quality (Banker et al., 1998; Chen et al., 2015, Giri et al., 2015) in the following way

$$d_i = d_i(P_i, P_j, \theta_i, \theta_j) = \alpha_i - \beta_i P_i + \gamma_i P_j + \delta_i \theta_i - \lambda_i \theta_j, \; i = 1, 2 \text{ and } j = 3 - i, \qquad (1)$$

where d_i is demand function of the ith product with retail price P_i and quality improvement level (QIL) θ_i given that the price and the QIL of the competitor brand are P_j and θ_j, respectively. Here $\alpha_i(> 0), i = 1, 2$ is the initial market size of the ith product. We assume that the initial market size α_i of ith product is large enough so that the demand d_i and hence the order quantity Q_i will be non-negative always[1].

β_i and δ_i measure one's own price and quality sensitivity, respectively, whereas $\gamma_i(> 0$ and $\lambda_i(> 0)$ measure the cross price and quality sensitivity coefficients between two brands and $\beta_i, \delta_i, \gamma_i, \lambda_i \in \mathbb{R}^+$. The parametric restriction $\beta_i > \gamma_i$ and $\delta_i > \lambda_i)$ are necessary for the demand function to be well behaved[2]. The differences $(\beta_i - \gamma_i)$ and $(\delta_i - \lambda_i)$ are inversely proportional to the degree of product substitutability (i.e., the competition) between the two products (Choi, 1991).

[1]Since, in our model consumers demand is assumed to be deterministic, demand (d_i)=order quality (Q_i), always in our model.

[2]This indicates that own price of a product (brand) has negative impact whereas own QIL of a product (brand) has a positive impact on its demand function. This assumption is consistent with our notion that lower price and higher quality product give higher demand

It is to be perceived that demand function (1) reflects a "quality" sensitive consumer market where higher quality and lower price of own product give higher demand. Further, if one manufacturer improves the quality level of her own brand then it certainly affects the demand for the competing product with lower quality. This insists the competing manufacturer raise the quality level of her product in order to maintain her market share. If retail prices are set to be equal (i.e., $P_i = P_j$) (Xie et al., 2011), higher quality is always able to attract more consumers in the same market.

All the channel members are assumed to be risk neutral in our study. Quality improvement investment is an upfront investment and is a function of θ_i (QIL) of the form $\eta_i \theta_i^2$, where η_i is the quality improvement investment parameter for ith product. This cost structure consistent with the existing literature (Banker et al., 1998) and increases quadratically with QIL suggesting the diminishing returns on the quality investment (Tsay and Agrawal, 2000). In our present study, we assume that this quality investment does not increase the marginal cost of the product produced by each manufacturer which is a common assumption in the literature (Kim & El ouardighi, 2007; Bhaskaran & Krishnan, 2009; He et al.,2016 etc.).

It is assumed that the manufacturers are always able to produce the required order quantity $Q_i,\ i = 1, 2$ in time for the start of the selling season. The lead times of both products are assumed to be zero (Chao et al., 2009; Bhaskaran and Krishnan, 2009; Chen et al., 2015; He et al., 2016 etc.). In addition, the underlying assumption is that $P_i > w_i > c_i$, where w_i and c_i are the wholesale price and production cost of ith product for $i = 1, 2$. These inequalities assure that the chain will not produce infinite quantities of the product and each firm has a positive profit. Furthermore, if m_i be the retail margin for the ith product then P_i, w_i and m_i are related as $P_i = w_i + m_i$.

Thus, in our basic supply chain model with a typical wholesale price (WP) contract, two competing manufacturers are the decision makers of the QILs ($\theta_i,\ i = 1, 2$) and wholesale prices of their own product (brand). On the other hand, the retailer is the decision maker of the retail prices of both products. It is relatively easy to change the price of any product quickly but in most of the cases, changes in the quality of a product are time-consuming (Olbrich and Jansen 2014). Hence in our study, each manufacturer first decides the QIL of her own product and then based on this QIL, she sets the wholesale price. Thus, QIL will affect her wholesale price[3].

Based on the above assumptions the profits of the retailer and the manufacturers of our basic model under wholesale price (WP) contract are given as

$$\Pi_R = \sum_{i=1}^{2}(P_i - w_i)d_i \text{ for } j = 3-i, \tag{2}$$

$$\Pi_{M_i} = (w_i - c_i)d_i - \eta_i\theta_i^2, \text{ for } i = 1,\ 2, \tag{3}$$

where $d_i(i = 1, 2)$ are defined in (1). Under the common knowledge assumption, we consider both vertical interaction between each manufacturer and the retailer as well as horizontal interaction between two competing manufacturers. We consider a three-stage dynamic game-theoretic framework to decide the decision entities of each channel members. Since manufacturers are the decision makers of quality improvement levels, there is no loss of generality if manufacturers are assumed to have enough power to control the market. Hence, in this study, we consider a Manufacturer-Stackelberg (MS) game to define the vertical interaction between each manufacturer and the retailer (eg:). We define MS game in the following way:

Under MS game, both manufacturers have Stackelberg leadership on pricing decisions in vertical level with manufacturers as the Stackelberg leader and the retailer as the follower. On the other hand, having equal power both manufacturers play Nash game in horizontal level. Hence, quality competition and price competition between two products under MS game take place in the following sequence in time:

1. Both manufacturers first decide quality improvement level (QIL) of their respective brand in simultaneous move.

2. By knowing each other's QILs they decide per unit wholesale price of their respective product, simultaneously, taking into consideration the reaction functions of the retailer.

3. Finally, the retailer chooses the retail margins of both products (m_1, m_2), which imme-

[3]This assumption is intuitive since higher quality improvement of any product means higher price investment and this subsequently compels the manufacturers to increase their unit wholesale price accordingly.

diately implies the retail prices $P_1 = m_1 + w_1$ and $P_2 = m_2 + w_2$ of those products at stage 3. Then the actual market demand is realized and is fulfilled at the decided retail prices and with chosen quality levels.

From the above game sequence, it is evident that both manufacturers first decides the QIL of their own product and their pricing decisions (wholesale price) are based on these quality improvement levels (QILs). The concept of the game sequence is commonly used in the literature (for example, Choi, 1991; Choi, 1996; Lee and Staelin, 1997; Lu et al., 2011 etc.).

Before analyzing the various equilibrium solutions under different game scenarios we first consider an integrated system as a bench mark, in which all the decisions are optimized to maximize the performance of the entire supply chain. Then we characterize the equilibrium solutions under a decentralized supply chain under wholesale price (WP) contract, where the manufacturers incur the complete cost of quality improvement of the products. Then we address a coordinating mechanism to enhance individual profits as well as total supply chain efficiency. In order to encourage the manufacturers to engage in quality improvement initiatives, in our present study, we consider a cost-sharing (CS) contract between each manufacturer and the retailer in two different scenarios: one in which both manufacturers accept cost-sharing contract and second in which only one manufacturer accepts cost-sharing contract. Moreover, for each CS contract, we develop and analyze two models depending on how the retailer determines its cost sharing proportion with the corresponding manufacturer: One where the retailer determines the cost-sharing parameter by optimizing his profit (Simple cost-sharing contract), second where each manufacturer and the common retailer bargain on the cost-sharing fraction (Cost-sharing contract through bargaining). Throughout this article we use subscript to denote supply chain member and superscript to denote coordinating contract. For example, by $\Pi_R^{WP}(\cdot)$ we denote the profit of the retailer under the wholesale price (WP) contract.

3.1. Integrated system

We consider an integrated system where a single decision maker sets QILs and the retail prices of two different brands by maximizing the total profit of the supply chain. If

$\Pi^I(P_1, P_2, \theta_1, \theta_2)$ denotes the profit of the integrated system then the corresponding optimization problem is

$$\max_{P_1,P_2,\theta_1,\theta_2} \Pi^I(P_1, P_2, \theta_1, \theta_2) = \sum_{i=1}^{2} \Big[(P_i - c_i)d_i - \eta_i\theta_i^2\Big] \quad \text{for } j = 3 - i, \tag{4}$$

For the rest of the paper we assume that

A1: $4\beta_i\beta_j - (\gamma_i + \gamma_j)^2 > 0$; **A2:** $\eta_i > max\left\{\frac{2(\beta_i\lambda_j^2+\beta_j\delta_i^2)}{4\beta_i\beta_j-(\gamma_i+\gamma_j)^2}, \frac{\delta_i(\delta_i-\lambda_i)}{(2\beta_i-\gamma_i)}\right\}$ for $j = 3 - i,\ i = 1, 2$; **A3:** $\eta \geq \frac{\delta(\delta-\lambda)(\beta-\gamma)(2\beta-\gamma)+\beta^2(\delta-\lambda)^2}{(\beta-\gamma)(2\beta-\gamma)^2}$; **A4:** $\frac{\gamma\delta}{2} < \beta\lambda < \gamma\delta$ and **A5:** $c_i < \frac{\alpha_i}{\beta_i-\gamma_i}$ for $i = 1, 2$.

These conditions are sufficient for many subsequent analytical results and valid in a wide range of the parametric values. Later in Section 5, we derive such parametric range in which all conditions valid simultaneously.

Proposition 1. *The integrated system has unique optimal solutions for retail prices and quality improvement levels under the parametric restrictions* **A1** *and* **A2** *and are given by* $P_i^{I^*}$ *and* $\theta_i^{I^*}$, $i = 1, 2$ *(see Appendix).*

The joint concavity of the objective function (4) is needed for the existence of the unique optimal solutions. Assumptions **A1** and **A2** are the sufficient conditions for this concavity. In order to brief the manuscript, those calculations are not included into the manuscript. If both products are symmetric in production costs (*i.e.*, $c_1 = c_2 = c$) and if the retailer is symmetric in initial market size of each product (α_i), product's own price and own quality sensitivity parameter (β_i and δ_i); product's cross price and cross quality sensitivity parameter (γ_i and λ_i) and quality improvement investment parameter (η_i) (*i.e.*, $\alpha_1 = \alpha_2 = \alpha,\ \beta_1 = \beta_2 = \beta,\ \gamma_1 = \gamma_2 = \gamma,\ \eta_1 = \eta_2 = \eta,\ \delta_1 = \delta_2 = \delta,\ \lambda_1 = \lambda_2 = \lambda$), then the problem becomes symmetric and it becomes quite tractable. We summarize the optimal solutions under symmetric assumption in the appendix.

4. Game analysis of decentralized system

In this section, we derive the equilibrium solutions under different contracts for MS game scenario. The most important part of this derivation process is the sequence of decision making and the informational assumptions, based on which our model is developed. In all possible scenarios, we derive the equilibrium solutions of our dynamic games using backward induction approach.

4.1. WP contract

Our basic model is developed under wholesale price (WP) contract in which non-cooperative quality improvement initiative is considered between the retailer and each of the manufacturers. Under this contract, the decisions of the manufacturers are their respective quality improvement level (QIL) and wholesale price while the retailer is the decision maker of retail prices of both products. In this contract, we first derive the retailer's reaction functions that represent optimal retail prices for both products (brands) by maximizing the profit function as given by (2) subject to the equation (1)[4]. Thus, we obtain the reaction functions as functions of QILs and wholesale prices. For these given reactions of the retailer, we solve for QILs and wholesale prices of the manufacturers in turn, by maximizing their profits in simultaneous Nash game. To derive the decision variables of each manufacturer we solve backward. Thus, we derive first the wholesale prices as functions of QILs in a simultaneous move and then we derive QILs, simultaneously.

Lemma 1. *(a) In WP contract, for given manufacturers' wholesale prices and QILs decisions, the optimal reaction function of the retailer can be expressed as*

$$\begin{aligned} P_i^*(\theta_1, \theta_2, w_1, w_2) \;=\; & \frac{1}{\Delta_1^{WP}}\Big[\alpha_j(\gamma_i+\gamma_j) + 2\alpha_i\beta_j + w_i\Big(2\beta_i\beta_j - \gamma_i(\gamma_i+\gamma_j)\Big) \\ & + w_j\Big(\beta_j(\gamma_i+\gamma_j) - 2\beta_j\gamma_j\Big) + \theta_i\Big(2\beta_j\delta_i - \lambda_j(\gamma_i+\gamma_j)\Big) \\ & + \theta_j\Big(\delta_j(\gamma_i+\gamma_j) - 2\beta_j\lambda_i\Big)\Big] \text{ for } i,j = 1,2 \text{ and } j \neq i, \end{aligned} \tag{5}$$

where $\Delta_1^{WP} = 4\beta_1\beta_2 - (\gamma_1+\gamma_2)^2 > 0$, *under assumptions* **A1**.
(b) $\frac{\partial P_i^*}{\partial \theta_i} > 0,\ \frac{\partial P_i^*}{\partial w_i} > 0$ *for* $i = 1,2$.

Thus, under the WP contract, the retailer increases the retail price of a product whenever the corresponding manufacturer increases the quality improvement level and the wholesale price of its product. This also corroborate with our existing notion that higher quality implies higher price. Substituting the reaction functions (5) into (3) we derive optimal

[4] Since, it is easily verified that the objective function of the retailer is concave with respect to his decision variables. This ensures the existence of the optimal retail prices which maximize the retailer's profit function.

decisions about wholesale price of the ith manufacturer as

$$
\begin{aligned}
w_i^*(\theta_1,\theta_2) \quad = \quad & \frac{1}{\Delta_2^{WP}}\Big[\theta_i\Big\{H_j(\gamma_i+\gamma_j)+4\beta_jG_i\Big\}+\theta_j\Big\{G_j(\gamma_i+\gamma_j)+4\beta_jH_i\Big\} \\
& +\Big\{E_j(\gamma_i+\gamma_j)+4\beta_jE_i\Big\}\Big] \text{ for } i,j=1,2 \text{ and } j\neq i, \qquad (6)
\end{aligned}
$$

where $\Delta_2^{WP} = (\beta_1\beta_2-\gamma_1\gamma_2)\Big\{16\beta_1\beta_2-(\gamma_1+\gamma_2)^2\Big\} > 0$, under assumptions **A1** and since $\beta_i > \gamma_i$ always for $i = 1, 2$. Subsequently, we get the following lemma:

Lemma 2. $\frac{\partial w_i^*}{\partial \theta_i} > 0$ *for* $i = 1, 2$.

The above lemma implies that each manufacturer increases unit wholesale price of its product whenever he increases the quality improvement level of the corresponding product. This is consistent with our existing intuition that higher quality investment cost impels the manufacturers to claim higher per unit wholesale price from the retailer. Using these above expressions in (3) we derive the equilibrium QIL of ith product as

$$
\begin{aligned}
\theta_i^{WP^*} \quad = \quad & \frac{1}{\Delta_3^{WP}}\Big[G_iG_j\Big\{G_j(\gamma_i+\gamma_j)+4\beta_jH_i\Big\}\Big\{E_i(\gamma_i+\gamma_j)+4\beta_iE_j-\Delta_2^{WP}c_j\Big\} \\
& +G_i\Big\{E_j(\gamma_i+\gamma_j)+4\beta_jE_i-\Delta_2^{WP}c_i\Big\}\Big\{2\eta_j\Delta_1^{WP}\Delta_2^{WP}-G_j\{H_i(\gamma_i+\gamma_j)+4\beta_iG_j\}\Big\}\Big] \\
& \text{for } i,j=1,2 \text{ and } j\neq i. \qquad (7)
\end{aligned}
$$

Expression for Δ_3^{WP} is provided in the appendix. Substituting this QILs in the expressions (5) and (6), we obtain the equilibrium solutions in terms of wholesale prices and retail prices. Finally, replacing these equilibrium prices and quality improvement levels in demand and profits we can easily get the equilibrium quantities and profits in decentralized system. The notations and full results including prices and profits under symmetric assumption are presented in the appendix [5].

Lemma 3. *Under symmetric assumptions and under assumptions* **A2** *and* **A5**
(a) equilibrium wholesale prices, retail prices and quality improvement levels are always

[5] Choi (1991) established the similar results for the integrated system and decentralized channel and those are derived further for the quality sensitive product to motivate our proposed coordination mechanism. When $\delta_1 = \delta_2 = \delta = 0$, and $\lambda_1 = \lambda_2 = 0$ *i.e.*, if consumer sensitivity parameters to quality is assumed to be zero then all the results for decentralized system under WP contract are converted to choi (1991)'s results for Manufacturer-Stackelberg game.

positive,
(b) the contribution margin of each manufacturer is always non-negative.

This fact ensures that for non-negative quality improvement level, retailer's and manufacturers' profits will always be positive for Manufacturer-Stackelberg game.

4.2. Coordination mechanisms: Cost-sharing (CS) contract

In this section, we will consider a cost-sharing (CS) contract between each manufacturer and the common retailer. Coordinating contracts always have a significant impact, as in our basic model under WP contract, the manufacturers only incur the product quality improvement efforts, while both the retailer and the manufacturer can be benefited from this quality improvement initiatives. But due to the higher quality investment costs, manufacturers often struggle to take such initiatives. Hence, in order to incentivize the manufacturers to participate in product quality improvement initiatives, CS contract can play an important role. More importantly, we seek to find if the CS contract is beneficial from the retailer's point of view and why the retailer would be motivated to share the product quality improvement cost with the manufacturers. Further, through this contract, a collaborative effort will be reflected in product quality improvement initiatives.

The main focus is on how to design CS contract between each manufacturer and the retailer so that both the retailer and the manufacturers will be benefited from this contract as compared to WP contract. Under this CS contract, in order to encourage the manufacturers in the quality improvement initiatives, the common retailer offers to the ith manufacturer to share ϕ_i $(0 \leq \phi_i < 1)$ proportion of the total investment associated to quality improvement of the ith product. Now, manufacturers can accept or reject that offer. If the ith manufacturer accepts the offer then retailer shares a ϕ_i proportion of the total quality related investment with the corresponding manufacturer and the ith manufacturer incurs only $(1-\phi_i)$ proportion of that investment. Since, our main interest is to investigate the positive impact of CS contract over WP contract, we consider only those scenarios where at least one of the two manufacturers accepts such contract. We later analyze what motivates these manufacturers to accept this contract. Therefore, depending on whether the manufacturers accept or reject CS contract, two scenarios arise: (i) in which both manufacturers accept

CS contract (CSC) and, (ii) where one manufacturer accepts CS contract while the other continues with WP contract (WC).

4.2.1. CSC contract in which both manufacturers accept cost-sharing contract

Under this contract, the profits of the retailer and the ith manufacturer are given by

$$\Pi_R^{CSC} = \sum_{i=1}^{2}\Big[(P_i - w_i)d_i - \phi_i\eta_i\theta_i^2\Big] \tag{8}$$

$$\Pi_{M_i}^{CSC} = (w_i - c_i)d_i - \eta_i(1-\phi_i)\theta_i^2. \tag{9}$$

Under this contract, decisions of the manufacturers are their respective wholesale price and the QIL while the retailer decides the cost-sharing fraction (CSF)s together with retail prices. Depending on the mechanisms how the retailer determines his CSFs ϕ_i for $i = 1,\ 2$, we further develop and analyze two models of cost-sharing: (i) Simple cost-sharing (SCS) contract where the retailer decides the CSFs by maximizing his profit, and (ii) Cost-sharing through bargaining (CSB) contract where both the retailer and each manufacturer negotiate over these CSFs. Hence, in order to describe the decision structures among the channel members under the coordinated scenario, we consider four-stage game-theoretic structures as shown in Figure 1.

Figure 1:Sequence of decisions under cost-sharing contracts

4.2.1 (a) Simple cost-sharing (SCS) contract

Under this SCS contract, the common retailer is the decision maker of the CSFs and he sets CFSs by optimizing his own profit. The game sequence under this contract is given below:

1. Before the start of the selling season, the retailer offers the SCS contract to ith manufacturer. If the ith manufacturer accepts the contract, then the retailer bears the ϕ_i fraction of the total quality investment cost with that manufacturer and the remaining $(1-\phi_i)$ fraction of this investment is incurred by that manufacturer.

2. As Stackelberg leaders, the manufacturers move first by announcing their own QILs and wholesale prices in turn. At this stage 1, they first decide the QIL of their own product,

simultaneously. Then after knowing these QILs, at stage 2, they decide respective per unit wholesale price in simultaneous move taking CSFs and the retailer's reaction functions into consideration.

3. For these given QILs and wholesale prices, the retailer then sets his retail prices taking CSFs into consideration at stage 3.

4. Finally, at stage 4, the retailer decides the CSFs for the given retail prices, wholesale prices and QILs by maximizing his profit functions.

Similar to WP contract, we derive all equilibrium solutions backward. All notations related to SCS contract are presented in the appendix. Following the above game sequence, we derive retailer's reaction functions on retail prices and manufacturers' decisions on their respective wholesale prices which are same as given in (5) and (6). Substituting (5) and (6) into profit function (9) we derive the equilibrium QIL of ith manufacturer ($i = 1, 2$) as

$$\begin{aligned} \theta_i^{SCS}(\phi_1, \phi_2) \;=\; & \frac{1}{\Delta_3^{SCS}}\Big[G_i G_j\big\{G_j(\gamma_i+\gamma_j)+4\beta_j H_i\big\}\big\{E_i(\gamma_i+\gamma_j)+4\beta_i E_j - c_j\Delta_2^{WP}\big\} \\ & + G_i\big\{E_j(\gamma_i+\gamma_j)+4\beta_j E_i - \Delta_2^{WP}c_i\big\}\big[2\eta_j(1-\phi_j)\Delta_1^{WP}\Delta_2^{WP} \\ & - G_j\big\{H_i(\gamma_i+\gamma_j)+4\beta_i G_j\big\}\big]\Big],\; j = 3-i. \end{aligned} \tag{10}$$

Lemma 4. $\frac{\partial \theta_i^{SCS}(\phi_1,\phi_2)}{\partial \phi_i} > 0$ *for* $i = 1, 2$.

Hence, quality improvement level (QIL) of each product increases when the retailer enhances its cost-sharing fraction (CSF) with the corresponding manufacturer. This is very intuitive since higher CSF reduces the cost burden due to quality improvement on the manufacturers. Subsequently, the manufacturers increase their respective QIL since quality has positive impact on the demand functions.

Derivation of expression (10) is given in the appendix. Substituting this $\theta_i^{SCS}(\phi_1, \phi_2)$ in (5) and (6), the actual values of wholesale prices and the retail prices can be realized for given CSFs ϕ_is. For the given CSFs, the equilibrium solutions under symmetric assumptions with $\phi_1 = \phi_2 = \phi$ are given in the appendix. When $\phi = 0$, then SCS contract converted into WP contract. That is why, when $\phi = 0$ all solutions corresponding to SCS contract coincide with those for WP contract under symmetric assumptions. Hence, under the symmetric assumption, the retailer's decision problem takes the form

$$
\begin{aligned}
& \max_{\phi} \Pi_R^{SCS}(\phi) \quad \text{where} \\
\Pi_R^{SCS}(\phi) &= \frac{2\Big\{4\beta^2\eta^2(1-\phi)^2 - \phi\eta\delta^2(\beta-\gamma)\Big\}\Big\{\alpha - c(\beta-\gamma)\Big\}^2}{(\beta-\gamma)\zeta}, \text{with} \\
\zeta &= \Big[4\eta(1-\phi)(2\beta-\gamma) + \delta\lambda - \delta^2\Big]^2. \qquad (11)
\end{aligned}
$$

Proposition 2. *Under the symmetric assumption, there exists a local optimal solution that maximizes the retailer's profit (11) under SCS contract and the optimal value of ϕ is given by* $\phi^{SCS^*} = \frac{\left[4\eta\left\{2\beta(\gamma\delta-\beta\lambda)+\gamma\delta(\beta-\gamma)\right\}+\delta^2(\beta-\gamma)(\delta-\lambda)\right]}{4\eta\left[2\beta^2(\delta-\lambda)+\delta(\beta-\gamma)(2\beta-\gamma)\right]}$[6].

Substituting the above optimal value of ϕ into the above results corresponding to SCS contract (see Appendix), we get equilibrium solutions under SCS contract and those are presented in the appendix. From this equilibrium solutions, it can be verified that under assumptions **A3** and **A5** the equilibrium wholesale cost and retail price are always positive under SCS contract. It is easy to verify that under symmetric assumption, the parametric restriction **A2** dominates restriction **A3**.

4.2.1 (b) Cost-sharing through bargaining (CSB) contract

In this section, let us consider a cost-sharing (CS) contract where the CSF is determined through bargaining between each manufacturer and the retailer. Our proposed bargaining game is based on the Nash bargaining process as proposed by John Nash (1950, 1953). The first three stages of CSB contract are identical with those of SCS contract. The only difference between the two contracts lies at stage 4. In this contract, the CSF is determined from the Nash bargaining optimization problem for given retail prices, wholesale prices and QILs in the following way:

Let $\Pi_R^{WP^*}$ and $\Pi_{M_i}^{WP^*}$ be the optimal profits for the retailer and the ith manufacturer, respectively, in Manufacturer-Stackelberg (MS) game under WP contract. Let $\Pi_{R_P}^{CSB^*}$ and $\Pi_{M_{i_P}}^{CSB^*}$

[6] Numerically using Built-in software Mathematica 11.0 (1996) we verified that this local optimal solution is actually the global optimal solution of ϕ.

be the Pareto improved profits of the retailer and the ith manufacturer, respectively, under CSB contract. Under WP contract, $\Pi_d^{WP^*} = \Pi_R^{WP^*} + \sum_{i=1}^{2} \Pi_{M_i}^{WP^*}$.

Now, let $\Pi_{d_P}^{CSB^*} = \Pi_{R_P}^{CSB^*} + \sum_{i=1}^{2} \Pi_{M_{i_P}}^{CSB^*}$. Then $\Pi_d^{WP^*}$ and $\Pi_{d_P}^{CSB^*}$ are the equilibrium channel profit under WP contract and under cost-sharing through bargaining (CSB) contract, respectively. If Π_I^* denotes the optimal profit of the integrated system, then from the definition of the Pareto improvement, we have $\Pi_{R_P}^{CSB^*} \geq \Pi_R^{WP^*}$, $\Pi_{M_{i_P}}^{CSB^*} \geq \Pi_{M_i}^{WP^*}$ for $i = 1, 2$. In addition we have $\Pi_d^{WP^*} \leq \Pi_{d_P}^{CSB^*} \leq \Pi_I^*$.

It is to be noted that Nash bargaining game is considered at the final step (i.e., at Stage 4). Hence, we can define the decision set κ of Pareto improvement as

$$\kappa \;=\; \left\{\left(\phi_1^{CSB},\ \phi_2^{CSB}\right) : \Pi_{R_P}^{CSB^*} \geq \Pi_R^{WP^*},\ \Pi_{M_{i_P}}^{CSB^*} \geq \Pi_{M_i}^{WP^*} \text{ for } i = 1, 2\right\}. \tag{12}$$

This κ is assumed to be compact and closed [7]. Then the optimization problem of the Nash ([1950], [1953]) bargaining game takes the form

$$\max_{\left(\phi_1^{CSB},\ \phi_2^{CSB}\right) \in \kappa} \sum_{i=1}^{2} \Pi_{B_i} \ \textit{where}$$
$$\Pi_{B_i} = \left(\Pi_{R_P}^{CSB^*} - \Pi_R^{WP^*}\right)\left(\Pi_{M_{i_P}}^{CSB^*} - \Pi_{M_i}^{WP^*}\right). \tag{13}$$

Due to the complicated form of the objective function, it is difficult to derive ‘closed-form analytical solutions’ of $\phi_1^{CSB^*}$, $\phi_2^{CSB^*}$ under CSB contract. However, we can borrow numerical methods in order to gain the insights of our proposed model into management implications *i.e.*, how well this CSB contract works in practice.

4.2.2. WC contract in which one manufacturer accepts cost-sharing contract and the other manufacturer accepts wholesale price contract

Without loss of generality, suppose manufacturer M_1 accepts WP contract and manufacturer M_2 accepts CS contract offered by the retailer. In this contract scenario, the retailer

[7] Compactness of κ ensures that κ is bounded and closed. Boundedness property of κ is the basic requirement for a solution to exist. Closeness of κ implies that there exists a maximum in κ. On the other hand, convexity is required to ensure the uniqueness property of the maximum solution.

shares, say ϕ_2 proportion of quality improvement cost of Product 2 with M_2 whereas M_1 alone incurs the quality improvement cost of Product 1. Under this contract, expressions for the profits of the retailer and manufacturer M_2 are given by

$$\Pi_R^{WCS} = (P_1 - w_1)d_1 + (P_2 - w_2)d_2 - \phi_2\eta_2\theta_2^2 \tag{14}$$

$$\Pi_{M_1}^{WCS} = (w_1 - c_1)d_1 - \eta_1\theta_1^2 \tag{15}$$

$$\Pi_{M_2}^{WCS} = (w_2 - c_2)d_1 - (1-\phi_2)\eta_1 2\theta_2^2. \tag{16}$$

Similar to CSC contract, depending on the mechanism how the retailer determines its proportion (ϕ_2) of cost sharing with M_2, we consider two models of cost-haring: (i) simple cost-sharing (WCS) and (ii) cost-sharing through bargaining (WCB) in which the retailer and manufacturer M_2 negotiate over ϕ_2.

4.2.2 (a) Simple cost-sharing (WCS) contract

Under this contract, the sequence of the game is similar to SCS contract. Moreover, the retailer's reaction functions on retail prices and manufacturers' pricing decisions are the same as given by (5) and (6). It is to be noted that for given (5) and (6) both manufacturers decide their respective QIL in simultaneous move. Hence, substituting those into profit functions (15) and (16) we derive the equilibrium QIL of ith manufacturer as functions of ϕ_2 and are given by

$$\begin{aligned}
\theta_1^{WCS}(\phi_2) = {} & \frac{1}{\Delta_3^{WCS}}\Big[G_1G_2\big\{G_2(\gamma_1+\gamma_2)+4\beta_2H_1\big\}\big\{E_1(\gamma_1+\gamma_2)+4\beta_1E_2-c_2\Delta_2^{WP}\big\} \\
& +G_1\big\{E_2(\gamma_1+\gamma_2)+4\beta_2E_1-\Delta_2^{WP}c_1\big\}\big[2\eta_2(1-\phi_2)\Delta_1^{WP}\Delta_2^{WP} \\
& -G_2\big\{H_1(\gamma_1+\gamma_2)+4\beta_1G_2\big\}\big]\Big], \qquad (17) \\
\theta_2^{SCS}(\phi_2) = {} & \frac{1}{\Delta_3^{WCS}}\Big[G_1G_2\big\{G_1(\gamma_1+\gamma_2)+4\beta_1H_2\big\}\big\{E_2(\gamma_1+\gamma_2)+4\beta_2E_1-c_1\Delta_2^{WP}\big\} \\
& +G_2\big\{E_1(\gamma_1+\gamma_2)+4\beta_1E_2-\Delta_2^{WP}c_2\big\}\big[2\eta_1\Delta_1^{WP}\Delta_2^{WP} \\
& -G_1\big\{H_2(\gamma_1+\gamma_2)+4\beta_2G_1\big\}\big]\Big], \qquad (18)
\end{aligned}$$

where Δ_3^{WCS} is defined in the appendix.

Lemma 5. $\frac{\partial\theta_1(\phi_2)}{\partial\phi_2} > 0$, $\frac{\partial\theta_2(\phi_2)}{\partial\phi_2} > 0$.

The first part of the lemma implies that CSF ϕ_2 has positive impact on the QILs of both Product 1 and Product 2. In WCS contract, QIL of Product 1 of M_2 increases whenever the retailer enhances its cost-sharing fraction ϕ_2 with M_2. The underlying reason can be explained in that way. Higher CSF ϕ_2 implies higher quality improvement level (QIL) of Product 2 (**Lemma 4**). Since demand function is positively affected by QIL of its own product and negative influenced by that of its competitor's product, M_1 set the QIL of its product in such a way that it increases whenever the retailer increases ϕ_2. Thus, the manufacturer M_1 tries to maintain consumer demand for its product. The explanation for the second part of **Lemma 5** is similar to that of '**Lemma 4**.

4.2.2 (b) Cost-sharing through bargaining (WCB) contract

Under this contract, the retailer and the manufacturer M_2 negotiate over the cost-sharing fraction ϕ_2. This bargaining game is similar as described in CSB. If $\Pi_{d_P}^{WCB^*}$ denotes the equilibrium channel profit under WCB contract, then $\Pi_{d_P}^{WCB^*} = \Pi_{R_P}^{WCB^*} + \Pi_{M_1}^{WCB^*} + \Pi_{M_{2_P}}^{WCB^*}$, where $\Pi_{M_1}^{WCB^*}$ is the equilibrium profit of M_1 where as $\Pi_{M_{2_P}}^{WCB^*}$ is the pareto improved profit of M_2 under WCB contract. Similar to CSB contract, we can define the decision set κ' of pareto improvement as

$$\kappa' \; = \; \Big\{\phi_2^{WCB} : \Pi_{R_P}^{WCSB^*} \geq \Pi_R^{WP^*}, \Pi_{M_{2_P}}^{WCB^*} \geq \Pi_{M_2}^{WP^*}\Big\}. \tag{19}$$

Hence, the optimization problem under Nash bargaining game takes the form

$$\begin{aligned} &\max_{\phi_2^{WCB}\in\kappa'} \Pi_B \; where \\ &\Pi_B = \left(\Pi_{R_P}^{WCB^*} - \Pi_R^{WP^*}\right)\left(\Pi_{M_{2_P}}^{WCB^*} - \Pi_{M_2}^{WP^*}\right). \end{aligned} \tag{20}$$

In the following through computational analysis we discuss the impact of WCB contract as well as how well this contract works in practice.

5. Discussion

In order to discuss the performance of above coordinating contracts on the equilibrium solutions and the impact of price and quality competitions on quality improvement levels as well as on pricing strategies, we carry out a comparison study among different scenarios (Integrated, WP, SCS, CSB, WCS, WCB) through analytical as well as computational investigations. Moreover, we carry out a sensitivity analysis to find the impact of quality investment parameters and asymmetric demand functions on the equilibrium solutions. The important results are depicted in propositions 3-6 and observations 1-7.

For computational approach, model parameters are drawn from: $c_1 = 10,\ c_2 = 10;,\ \alpha_1 = 1000,\ \alpha_2 = 1000,\ \beta_i \in \Big\{45, 47, 49, 51, 53, 55\Big\},\ \gamma_i \in \Big\{30, 32, 34, 36, 38, 40\Big\},\ \delta_i \in \Big\{20, 21, 22, 23, 24, 25\Big\},\ \lambda_i \in \Big\{5, 6, 7, 8, 9, 10\Big\},\ \eta_i \in \Big\{10, 12, 14, 16, 18, 20\Big\}$ for $i = 1,\ 2$, though all results are hold good under more general settings[8]

5.1. Impact of coordinating contract

Is it beneficial to the retailer to offer cost-sharing contract to reduce the burden of the manufacturers in quality improvement efforts? Which coordinating strategy is the best choice to the retailer as well as to the manufacturers? We'll try to find the answers in the subsequent results.

Proposition 3. *Under symmetric assumption, the equilibrium values of QILs and unit wholesale prices in SCS contract are related with those in WP contract as: (i)* $\theta^{SCS^*} > \theta^{WP^*}$*, (ii)* $w^{SCS^*} > w^{WP^*}$ *under assumptions* **A2**, **A4** *and production cost restriction* **A5**.

The above result suggests that in SCS contract the manufacturers are able to produce higher quality product as compared to WP contract. Intuitively we can say that cost-sharing with the retailer under SCS contract now reduces the quality improvement cost burden of the manufacturers and consequently, it enables the manufacturers to produce higher-quality products in SCS contract in comparison to WP contract. Consequently, this higher QIL

[8]It is to be noted that production costs always satisfy the parametric restriction $c_i \leq \frac{\alpha_i}{(\beta_i - \gamma_i)}$ $(i = 1, 2)$ as given by assumption **A5**. Through experiment we find a parametric region in which both cost-sharing contracts can coordinate the supply chain as well as all assumptions **A1** to **A5** sustain and is given by: $\beta_i - \gamma_i \geq 15,\ \delta_i - \lambda_i \geq 12,\ c_i \leq 66.67,\ \eta_i \geq 10$ for $i = 1,\ 2$. Although, the same results may still hold in violation of some or all of these assumptions.

forces the manufacturer to raise her wholesale price in SCS contract as compared to WP contract.

Observation 1: The manufacturers' equilibrium QILs, wholesale prices and the retailers' retail prices have the following relationships under all scenarios

(a) $\theta^{I^*} > \theta^{CSB^*} > \theta_2^{WCB^*} > \theta^{SCS^*} > \theta_2^{WCS^*} > \theta^{WP^*} \approx \theta_1^{WCB^*} \approx \theta_1^{WCS^*}$

(b) $w^{CSB^*} > w_2^{WCB^*} > w^{SCS^*} > w_2^{WCS^*} > w^{WP^*} \approx w_1^{WCB^*} \approx w_1^{WCS^*}$

(c) $P^{CSB^*} > P^{SCS^*} > P_2^{WCB^*} > P_2^{WCS^*} > P_1^{WCB^*} > P_1^{WCS^*} > P^{WP^*}$.

Figure 2: Comparisons of the optimal QILs under different scenarios

From the above result, it is interesting to note that from the quality level's perspective, negotiation during cost-sharing (CSB) contract is more beneficial than non-cooperative scenario under SCS contract (Figure 2). Moreover, as the contract shifts from both manufacturers accept CS contract to both manufacturers accepts WP contract (CSC→WC→WP), the quality improvement level (QIL) decreases. Furthermore, by accepting CS contract, a manufacturer will always be able to enhance QIL of its product as compared to WP scenario. Since, quality investment cost increases quadratically with QIL, higher QIL implies higher quality investment cost. Hence, this higher QIL of the product subsequently forces the respective manufacturer to charge higher wholesale price for that product (Figure 3-a,c,e). On the other hand, under coordinating contract, due to sharing the quality investment costs with the manufacturers and due to higher wholesale prices, the retailer sets higher retail prices for those higher quality products in order to maintain his retail margins from those products (Figure 3-b,d,f). The significant observation from the last part of (c) is that in WC scenario in which only one of the two manufacturers accepts CS contract, the retailer increases the retail price of the other manufacturer's product, although there is no significant change of QIL of the other product. The retailer does so to maintain the price competition between the two products. In Observation 3, we discuss why the retailer does so.

Thus, from consumer's point of view, higher QILs of the products result in the higher prices to purchase the products. This may reduce consumer demand. Hence, the manufacturers and the retailer would be interested in participating in cost-sharing contract only when this contract results in more profits than that of the WP contract. The impact of the cost-sharing contract on profitability is depicted in the following results.

Figure 3: Comparisons of the optimal prices under different scenarios

Observation 2: Equilibrium profits of the two manufacturers, the retailer and the total channel under different possible scenarios relate respectively as follows:

(a) $\Pi_M^{CSB*} > \Pi_M^{SCS*} > \Pi_{M_2}^{WCB*} > \Pi_{M_2}^{WCS*} > \Pi_{M_1}^{WCB*} > \Pi_{M_1}^{WCS*} > \Pi_M^{WP*}$

(b) $\Pi_R^{SCS*} > \Pi_R^{CSB*} > \Pi_R^{WCB*} > \Pi_R^{WCS*} > \Pi_R^{WP*}$.

(c) $\Pi^{I*} > \Pi_{SC}^{CSB*} > \Pi_{SC}^{SCS*} > \Pi_{SC}^{SCS*} > \Pi_{SC}^{WCB*} > \Pi_{|SC}^{WCS*} > \Pi_{SC}^{WP*}$

We can explain the underlying reasons of this result in the following ways: Previous result indicates that retail prices are higher in coordinating scenarios. On the other hand, both cost-sharing contracts (CSC and WC) are able to successfully increase consumer demand in comparison to WP contract (Figure 4 (a,b,c)). This means, despite of higher retail prices, consumer demand enhances in cost-sharing contracts due to the offering of higher-quality products. Subsequently, by accepting the CS contract, the manufacturers get the chance to keep higher profit-margins from those higher-quality products. On the other hand, in spite of sharing the quality improvement costs with the manufacturers and taking the burden of higher wholesale prices, the retailer earns higher profit in CS contracts because of increased quality levels of the products, which subsequently increases the consumer demand and hence there arise an opportunity for the retailer to keep higher profit-margins from those products. This is the reason why the retailer should prefer to offer CS contracts over WP contract. In addition, under CSC contract (in which both manufacturers accept CS contract), the manufacturers are always better-off in CSB contract in which they possess the power to bargain over cost-sharing fractions (CSFs). The underlying reasons follow from the previous result which states that in CSB contract, the manufacturers produce higher-quality products for which they charge higher wholesale prices as compared to SCS and WP contracts. Subsequently, the manufacturers earn the highest profit in CSB contract (Figure 5-a,c,e). Similarly, in WC contract (in which only M_2 accepts CS contract) manufacturer M_2 is better-off in WCB contract as compared to WCS and WP contract. Furthermore, considering both CS contracts (CSC and WC), both manufacturers are more profitable in CSC contract as compared to WC contract.

Figure 4: Comparisons of consumer demands under different scenarios

On the other hand, from the retailer's point of view, comparing two CS scenarios, under CSC contract, the retailer earns a higher profit in SCS contract where he decides the cost-sharing fraction (CSF)s by maximizing his profit (Figure 5-b,d,f). But under WC contract, the retailer earns a higher profit in WCB contract in comparison to WCS contract. But from the channel profit's perspective, CSB and WCB contracts are always beneficial in comparison to the other cost-sharing (SCS and WCS) contracts (Figure 6-a,b,c).

From the above results and discussion, we ultimately find that manufacturers (M_2) can produce better quality products in CSB (WCB) contract as compared to other decentralizes scenarios. Moreover, due to combined effect of retail price and quality level, consumers are also always better-off in CSB contract and in WCB contract (for Product 2) in which both manufacturers and manufacturer M_2 bargain on the CSFs, respectively. Interestingly, we further find that while CSB contract is beneficial to the manufacturers, consumers and the total supply chain, it is not the best choice for the retailer in CSC contract. Ideally, in CSC contract, the retailer would like to participate in SCS contract where he would have the control on the decision of the CSFs.

5.2. Impact of price and quality competition

How would the price competition $(\beta - \gamma)$ and the quality competition $(\delta - \lambda)$ affect the quality improvement decisions and the other consequent strategies of the channel members? Significant outcomes are depicted in the following results which are consistent under all possible scenarios.

Observation 3: Price competition between two products has a positive influence on the quality *improvement levels (QILs) of those products. Further, it has a positive impact on the unit prices, profits of individual channel members as well as on the total channel profit.*

Intuitively we can say that higher price competition (i.e., lower $(\beta - \gamma)$) implies higher product substitutability between two products. Hence, for fixed quality competition, if the products are more competitive with respect to the price, then in order to differentiate their respective product (brand), both manufacturers improve the QIL of their own product (Figure 2-a). These higher QILs force the manufacturers as well as the retailers to increase their unit prices (Figure 3-a,b). On the other hand, Figure 4-a indicates that price competition also has a positive impact on the consumer demand functions. This higher consumer demand enables the manufacturers and the retailer to set higher profit margins for those products. Due to these higher profit margins, profits of all the channel members increase with the increasing price competition, (Figure 5-a,b). Consequently, the total profit of the supply chain also enhances as well (Figure 6-a).

Observation 4: Quality competition between two products has a negative impact on the QILs of both products. Moreover, this competition is negatively related to the unit prices as well as profit functions of the retailer, the manufacturers and the total channel, as well.

Figure 5: Comparisons of retailer's and manufacturers' profits under different scenarios

Figure 4-b shows that quality competition has a negative effect on the consumer demand functions of both products. This force the retailer to reduce his profit margins from both products. On the other hand, due to the reduction in consumer demand, both manufacturers also reduce their profit margin for their own product (Figure 3-c,d). This lower profit margins indirectly force the manufacturers to lower the QILs of their products (Figure 2-b). Hence, more quality competition leads to lower profits for all the channel members (Figure 5-c,d). Consequently, total channel profit also decreases with the increase of this quality competition between two products (Figure 6-b). This claim is consistent with the Xie et al. (2011)'s

results derived for two supply chains which offer products at the same price but compete on quality.

Proposition 4. *If condition* **A4** *is satisfied, the optimal cost-sharing fraction (CSF)* ϕ^{SCS^*} *under SCS contract increases with the increase of price competition level and decreases with quality competition level between the two products.*

This proposition implies that the CSF decision of the retailer is influenced by the intensity of competition (price as well as quality competition) between the two products. We can explain the underlying reasons as: price competition has a positive impact on the QILs of the products (*Observation 3*) and consequently increases consumer demand of the products. Hence, with the increase of this competition, the retailer would raise his CSFs to encourage the manufacturers in the quality improvement initiatives. On the contrary, quality competition has a negative impact on QILs (*Observation 4*) and hence on consumer demand. Thus, quality competition between the products is not desirable to the manufacturers as well as to the retailer. That is why with the increase of price competition level or with the decrease in quality competition level the retailer would be interested in sharing more quality investment cost with the manufacturers.

From the above discussion it is clear that while price competition between two products can induce better quality products as well as better profitability in the supply chain, quality competition between products is not profitable to consider from quality levels and profits' viewpoints. Moreover, while price competition can successfully implement the CS contract mechanisms, quality competition can not.

Figure 6: Comparisons of total channel profits under different scenarios

5.3. Sensitivity analysis of the equilibrium solutions

Since quality improvement level of a product depends generally on the quality improvement cost, it is interesting to know what is the impact of quality investment parameters ($\eta_i,\ i = 1, 2$) on the optimal decisions of our supply chain. Moreover, how does this parameter affect the coordinating contracts addressed in this study? On the other hand, since consumers are sensitive to both price and quality of the products, what would be the optimal strategies of all the channel members if the retailer faces asymmetric demand functions for those competing products? We are thus interested to find the impact of the quality investment parameters and asymmetric demand functions on the equilibrium decisions.

5.3.1. Impact of quality investment parameter

Proposition 5. *Higher quality investment parameter always results in lower quality improvement levels (QILs) for both products.*

In our study, quality improvement cost $\eta_i\theta_i^2$ increases linearly with the quality investment parameter η_i and quadratically with QIL θ_i. Hence, for a higher η_i, the manufacturers decrease the QIL of their own product in order to reduce the quality improvement cost burden. This is the reason why manufacturers grapple with improving the quality of the products for higher quality improvement cost (Figure 2-c). This result is also consistent with the existing notion.

Observation 5: As the quality investment parameter η_i, $i \in \{1, 2\}$, increases, the equilibrium unit prices as well as equilibrium profits of the supply chain decreases under all possible scenarios.

The consequence of Proposition 5 leads to lower consumer demand functions of corresponding products (Figure 4-c), which in turn force the manufacturers and the retailer to reduce their profit margins by reducing their unit prices (Figure 3-e,f). Thus, these lower consumer demand and lower profit margins jointly decrease the profitability of the supply chain (Figure 5-e,f, and Figure 6-c).

The following result depicts the impact of this quality investment parameter on the coordinating contracts.

Proposition 6. *In SCS contract, the optimal cost-sharing fraction (CSF) ϕ^{SCS^*} decreases with the increase of quality investment parameter η under symmetric assumption.*[9]

Since the previous results indicate that higher quality investment parameter has a negative impact on both QILs and consumer demand functions of the products, it is very intuitive that the retailer would hesitate to share more quality improvement cost with the manufacturers

[9] Our numerical experiment shows that this result is also true for the other cost-sharing scenarios (CSB, WCS, WCB) and for asymmetric demand functions as well.

Table 1: Equilibrium results in different scenarios with asymmetric own price sensitiveness: $\beta_1 = 45, \beta_2 = 50$; $\gamma_1 = \gamma_2 = 30$; $\delta_1 = \delta_2 = 20$; $\lambda_1 = \lambda_2 = 5$

Scenario	θ^*	W^*	P^*	ϕ^*	Demand	Π_{M^*}	Π_{R^*}	Π_{D^*}
Integrated	$\theta_1^{I^*} = 28.33$	—	$P_1^{I^*} = 46.69$	—	$d_1 = 647.61$	—	—	28979.6
	$\theta_2^{I^*} = 24.28$	—	$P_2^{I^*} = 43.45$	—	$d_2 = 572.01$	—	—	
WP	$\theta_1^{WP^*} = 7.88$	$w_1^{WP^*} = 25.34$	$P_1^{WP^*} = 45.68$	—	$d_1 = 345.22$	$\Pi_{M_1}^{WP^*} = 4675.08$	13477.4	22292.6
	$\theta_2^{WP^*} = 6.92$	$w_2^{WP^*} = 23.59$	$P_2^{WP^*} = 42.59$	—	$d_2 = 339.83$	$\Pi_{M_2}^{WP^*} = 4140.13$		
SCS	$\theta_1^{SCS^*} = 18.11$	$w_1^{SCS^*} = 27.79$	$P_1^{SCS^*} = 51.29$	$\phi_1^{SCS^*} = 0.524$	$d_1 = 400.27$	$\Pi_{M_1}^{SCS^*} = 5560.08$	14898.20	25362.9
	$\theta_2^{SCS^*} = 15.94$	$w_2^{SCS^*} = 25.62$	$P_2^{SCS^*} = 47.53$	$\phi_2^{SCS^*} = 0.530$	$d_2 = 390.50$	$\Pi_{M_2}^{SCS^*} = 4904.66$		
CSB	$\theta_1^{CSB^*} = 22.99$	$w_1^{CSB^*} = 28.87$	$P_1^{CSB^*} = 53.78$	$\phi_1^{CSB^*} = 0.583$	$d_1 = 424.60$	$\Pi_{M_1}^{CSB^*} = 5946.42$	14880.20	25973.9
	$\theta_2^{CSB^*} = 19.99$	$w_2^{CSB^*} = 26.53$	$P_2^{CSB^*} = 49.74$	$\phi_2^{CSB^*} = 0.587$	$d_2 = 413.21$	$\Pi_{M_2}^{CSB^*} = 5147.23$		
WCS	$\theta_1^{WCS^*} = 7.40$	$w_1^{WCS^*} = 25.34$	$P_1^{WCS^*} = 46.63$		$d_1 = 345.10$	$\Pi_{M_1}^{WCS^*} = 4745.34$	13953.1	23403.3
	$\theta_2^{WCS^*} = 15.45$	$W_2^{WCS^*} = 25.32$	$P_2^{WCS^*} = 45.32$	$\phi_2^{WCS^*} = 0.512$	$d_2 = 383.03$	$\Pi_{M_2}^{WCS^*} = 4704.83$		
WCB	$\theta_1^{WCB^*} = 7.72$	$w_1^{WCB^*} = 25.45$	$P_1^{WCB^*} = 47.19$		$d_1 = 347.62$	$\Pi_{M_1}^{WCB^*} = 4773.86$	13986.7	23644.9
	$\theta_2^{WCB^*} = 18.51$	$W_2^{WCB^*} = 25.95$	$P_2^{WCB^*} = 46.97$	$\phi_2^{WCB^*} = 0.569$	$d_2 = 398.769$	$\Pi_{M_2}^{WCB^*} = 4884.34$		

for higher improvement cost. Thus, to maintain own profitability, the retailer would reduce his share in quality improvement cost.

The above claim is also consistent with Ghosh et al. (2015)'s result for the green supply chain. In addition, we find during this study that CSFs are also affected by the price and quality competition levels between two products (Proposition 4).

5.3.2. Impact of asymmetric demand

Observation 6: (a) Higher own price sensitiveness and higher cross quality sensitiveness of a product (brand) lead to lower quality improvement level (QIL), lower unit wholesale price and lower retail price. Furthermore, it results in lower profit for the corresponding manufacturer.

(b) Higher cross price sensitiveness and higher own quality sensitiveness lead to higher QIL and higher unit prices of the product. Moreover, the corresponding manufacturer earns more than his competitor.

We can explain the underlying reasons of this result as follows: higher own price sensitivity implies lower price competition (lower substitution) between two products, which leads to lower QIL to the corresponding product (Section 4.2). This lower QIL leads directly to lower consumer demand for that product (brand). Consequently, this lower demand forces

Table 2: Equilibrium results in different scenarios with asymmetric cross price sensitiveness: $\gamma_1 = 30, \gamma_2 = 32$; $\beta_1 = \beta_2 = 45$; $\delta_1 = \delta_2 = 20$; $\lambda_1 = \lambda_2 = 5$

Scenario	θ^*	W^*	P^*	ϕ^*	Demand	Π_{M*}	Π_{R*}	Π_{D*}
Integrated	$\theta_1^{I^*} = 38.40$	—	$P_1^{I^*} = 61.26$	—	$d_1 = 611.08$	—	—	44156.1
	$\theta_2^{I^*} = 38.61$	—	$P_2^{I^*} = 61.43$	—	$d_2 = 776.36$	—	—	
WP	$\theta_1^{WP^*} = 8.24$	$W_1^{WP^*} = 26.01$	$P_1^{WP^*} = 53.10$	—	$d_1 = 360.46$	$\Pi_{M_1}^{WP^*} = 5153.49$	20357	31542.7
	$\theta_2^{WP^*} = 9.15$	$W_2^{WP^*} = 27.53$	$P_2^{WP^*} = 54.36$	—	$d_2 = 394.74$	$\Pi_{M_2}^{WP^*} = 6032.13$		
SCS	$\theta_1^{SCS^*} = 24.67$	$W_1^{SCS^*} = 30.02$	$P_1^{SCS^*} = 63.74$	$\phi_1^{SCS^*} = 0.623$	$d_1 = 450.88$	$\Pi_{M_1}^{SCS^*} = 6730.76$	24126	38821.1
	$\theta_2^{SCS^*} = 24.84$	$W_2^{SCS^*} = 31.67$	$P_2^{SCS^*} = 64.99$	$\phi_2^{SCS^*} = 0.576$	$d_2 = 488.10$	$\Pi_{M_2}^{SCS^*} = 7964.35$		
CSB	$\theta_1^{CSB^*} = 32.90$	$W_1^{CSB^*} = 32.03$	$P_1^{CSB^*} = 69.03$	$\phi_1^{CSB^*} = 0.671$	$d_1 = 496.11$	$\Pi_{M_1}^{CSB^*} = 7372.67$	23955.5	40060.8
	$\theta_2^{CSB^*} = 32.52$	$W_2^{CSB^*} = 33.71$	$P_2^{CSB^*} = 70.24$	$\phi_2^{CSB^*} = 0.629$	$d_2 = 533.922$	$\Pi_{M_2}^{CSB^*} = 8732.54$		
WCS	$\theta_1^{WCS^*} = 7.39$	$w_1^{WCS^*} = 26.05$	$P_1^{WCS^*} = 55.40$		$d_1 = 361.40$	$\Pi_{M_1}^{WCS^*} = 5254.19$	21619.0	34181.2
	$\theta_2^{WCS^*} = 23.45$	$W_2^{WCS^*} = 30.81$	$P_2^{WCS^*} = 60.80$	$\phi_2^{WCS^*} = 0.554$	$d_2 = 468.78$	$\Pi_{M_2}^{WCS^*} = 7308.06$		
WCB	$\theta_1^{WCB^*} = 7.98$	$w_1^{WCB^*} = 26.27$	$P_1^{WCB^*} = 56.62$		$d_1 = 366.34$	$\Pi_{M_1}^{WCB^*} = 5321.81$	21695.9	34722.7
	$\theta_2^{WCB^*} = 28.42$	$W_2^{WCB^*} = 31.99$	$P_2^{WCB^*} = 63.23$	$\phi_2^{WCB^*} = 0.606$	$d_2 = 495.18$	$\Pi_{M_2}^{WCS^*} = 7705.03$		

the corresponding manufacturer and the retailer to reduce their profit margins from that product. The joint effect of lower consumer demand and lower profit margin ultimately result in lower profit to the manufacturer with the higher price sensitive product (Table 1 and Table 2).

On the other hand, higher cross price sensitivity leads to higher price competition (i.e., higher substitution) between the two products (brands). This results in higher QIL of that product (Section 5.2). Consequently, higher QIL and higher cross price sensitivity leads to more consumers to shift their choice to this product from the competing one. This higher demand allows the manufacturer and the retailer to raise the unit prices of that product. Hence, a manufacturer earns a higher profit with this higher cross price sensitiveness (Table 3 and Table 4).

Observation 7: Asymmetric own price sensitiveness and cross quality sensitiveness have a negative impact on profits. However, asymmetric cross price sensitiveness and own quality sensitiveness have a positive influence on the profits of the supply chain.

We can explain the underlying reasons in the following way: we observe that higher price sensitivity of a product results in slightly lower QIL for the other product as compared to the symmetrical scenario (Table 1 and Table 5). This leads to a reduction in demand for the other product as well. Consequently, it reduces the unit wholesale price of the other

Table 3: Equilibrium results in different scenarios with asymmetric own quality sensitiveness: $\delta_1 = 20, \gamma_2 = 24$; $\beta_1 = \beta_2 = 45$; $\gamma_1 = \gamma_2 = 30$; $\lambda_1 = \lambda_2 = 5$

Scenario	θ^*	W^*	P^*	ϕ^*	Demand	Π_{M*}	Π_{R*}	Π_{D*}
Integrated	$\theta_1^{I^*} = 39.28$	—	$P_1^{I^*} = 63.82$	—	$d_1 = 676.94$	—	—	47598.0
	$\theta_2^{I^*} = 56.35$	—	$P_2^{I^*} = 68.17$	—	$d_2 = 1002.99$	—	—	
WP	$\theta_1^{WP^*} = 8.52$	$W_1^{WP^*} = 26.53$	$P_1^{WP^*} = 51.84$	—	$d_1 = 371.92$	$\Pi_{M_1}^{WP^*} = 5421.2$	19422.9	30458
	$\theta_2^{WP^*} = 10.93$	$W_2^{WP^*} = 27.39$	$P_2^{WP^*} = 52.97$	—	$d_2 = 391.37$	$\Pi_{M_2}^{WP^*} = 5613.93$		
SCS	$\theta_1^{SCS^*} = 23.66$	$W_1^{SCS^*} = 30.84$	$P_1^{SCS^*} = 63.96$	$\phi_1^{SCS^*} = 0.592$	$d_1 = 468.83$	$\Pi_{M_1}^{SCS^*} = 7483.37$	24099.7	39883.4
	$\theta_2^{SCS^*} = 33.28$	$W_2^{SCS^*} = 33.95$	$P_2^{SCS^*} = 68.01$	$\phi_2^{SCS^*} = 0.584$	$d_2 = 538.87$	$\Pi_{M_2}^{SCS^*} = 8300.32$		
CSB	$\theta_1^{CSB^*} = 32.47$	$W_1^{CSB^*} = 33.26$	$P_1^{CSB^*} = 70.45$	$\phi_1^{CSB^*} = 0.642$	$d_1 = 523.27$	$\Pi_{M_1}^{CSB^*} = 8393$	23928.8	41710.2
	$\theta_2^{CSB^*} = 43.93$	$W_2^{CSB^*} = 37.11$	$P_2^{CSB^*} = 75.46$	$\phi_2^{CSB^*} = 0.630$	$d_2 = 609.94$	$\Pi_{M_2}^{CSB^*} = 9388.45$		
WCS	$\theta_1^{WCS^*} = 7.84$	$w_1^{WCS^*} = 27.15$	$P_1^{WCS^*} = 56.48$		$d_1 = 385.89$	$\Pi_{M_1}^{WCS^*} = 6003.14$	21877.1	35610.9
	$\theta_2^{WCS^*} = 31.64$	$W_2^{WCS^*} = 33.16$	$P_2^{WCS^*} = 64.30$	$\phi_2^{WCS^*} = 0.566$	$d_2 = 521.19$	$\Pi_{M_2}^{WCS^*} = 7730.6$		
WCB	$\theta_1^{WCB^*} = 8.83$	$w_1^{WCB^*} = 27.66$	$P_1^{WCB^*} = 58.75$		$d_1 = 397.413$	$\Pi_{M_1}^{WCB^*} = 6239.51$	21909.6	36601.1
	$\theta_2^{WCB^*} = 39.26$	$W_2^{WCB^*} = 35.31$	$P_2^{WCB^*} = 68.69$	$\phi_2^{WCB^*} = 0.613$	$d_2 = 569.492$	$\Pi_{M_2}^{WCS^*} = 8451.98$		

product. On the other hand, higher price sensitivity leads to the lower retail price of the corresponding product (*Observation 6*). Since higher price competition (lower price difference) is beneficial to the retailer (Section 5.2), the retailer reduces slightly the retail price of the other product to maintain the competition level. The combined effect of lower profit margins and lower consumer demand results in lower profits of all the channel members and the total profit, as well. The underlying reasons for the remaining result can be explained similarly.

6. Conclusions

This paper analyzes the product quality improvement strategies in both cooperative and non-cooperative scenarios. By considering a non-collaborative scenario with WP contract and collaborative scenario with cost-sharing (SCS, CSB, WCS and WCB) contracts, we investigate whether collaborative quality improvement initiative is beneficial to all channel members. In addition, by considering both price and quality competition between the two products (brands) we identify how the manufacturers set the quality levels of their own product and how the retailer takes the pricing decisions (retail price) regarding those two competitive products. The significant insights of our results can be summarized as follows:

- Intense price competition always leads to higher quality product and higher profits to all channel members.
- Quality competition is not desirable between the products as it has a negative impact on the QILs as well as on the profits in the supply chain.[10]
- For higher quality investment cost, the manufacturers decrease the QIL of their own product in order to reduce quality improvement cost burden.
- The retailer would be interested in sharing more quality investment cost with the manufacturers when they raise the differentiation between two products with respect to quality (lower quality competition).
- The retailer would reduce his share (i.e., the cost-sharing fraction (CSF)) whenever the quality improvement cost increases, as it decreases consumer demand.
- More quality sensitiveness of the consumers induces the manufacturers to produce a higher quality product.

In addition, we observe the following results associated with the coordinating contracts:

- Both cost-sharing (CSC and WC) contracts result in higher QILs and higher profits in the supply chain as compared to WP contract. But it is more profitable from QILs and profits' viewpoint if both manufacturers accept the cost-sharing contract (CSC).
- Highest quality product is always produced in the Integrated system. But considering decentralized system, CSB contract produces better quality product as compared to the other decentralized (SCS, WCS, WCB and WP) scenarios.
- Due to the combined effects of retail price and QIL, consumers are always better-off in CSB contract in which the manufacturers bargain on the CSFs. This leads to highest demand in CSB contract as compared to other contracts (SCS, WCS, WCB and WP).
- By accepting cost-sharing (CS) contract, a manufacturer will always be able to increase more quality improvement level (QIL) of its product in comparison to WP contract.

[10] This result is coherent with the claim of Xie et al (2011) for two competing supply chains.

Moreover, CSC contract (in which both manufacturers accept CS contract) scenario is the best option for both manufacturers as compared to WC contract (in which only one manufacturer accepts CS contract).

- The manufacturers are always best-off in CSB contract as compared to all other scenarios. This contract is also profitable from total channel profit's viewpoint.

- Comparing all scenarios, from the profit level's perspective, the retailer would always like to participate in SCS contract in which he would have control over the CSFs. But in WC contract in which only one manufacturer accepts CS contract, the retailer is profitable if the retailer bargains over CSF with the corresponding manufacturer.

The above findings are also important from the managerial point of view. From the manufacturers' viewpoint, both cost-sharing (CS) contracts (collaborative quality improvement scenario) are always profitable to consider in comparing to WP contract (non-collaborative quality improvement scenario). But negotiation over CSFs (CSB, WCB) always benefits the manufacturers. Hence, as a Stackelberg leader, the manufacturers would always like to go with CSB or WCB contract offered by the retailer. Again, if quality improvement cost is high, the manufacturers would decrease the QIL of their own product and thus reduce their quality investment cost burden. Moreover, since, intense price competition has a positive impact on the QILs and hence on the consumer demand functions, it encourages the manufacturers to set higher profit margins from those products.

On the other hand, both CS contracts are also beneficial to the retailer from a profitability point of view in comparison to the WP contract. But the retailer is benefited more in SCS contract than other CS contracts (CSB, WCS, WCB). Hence, the retailer would always prefer to go with SCS contract under CSC contracts in which both manufacturers accepts CS contract and with WCB contract in which the retailer does negotiation over CSF with the corresponding manufacturer. Again, since, lower quality improvement cost can raise the QILs of the products and therefore the consumer demand, the retailer would be interested in sharing more quality investment cost with the manufacturers only when this investment cost is low. Further, since the high price competition is profitable to the retailer, he would like to set the retail prices for both products very close to each other.

Table 4: Equilibrium results in different scenarios with asymmetric cross quality sensitiveness: $\lambda_1 = 5, \lambda_2 = 7$; $\beta_1 = \beta_2 = 45$; $\gamma_1 = \gamma_2 = 30$; $\delta_1 = \delta_2 = 20$

Scenario	θ^*	W^*	P^*	ϕ^*	Demand	Π_{M^*}	Π_{R^*}	Π_{D^*}
Integrated	$\theta_1^{I^*} = 27.13$	—	$P_1^{I^*} = 51.97$	—	$d_1 = 616.51$	—	—	35861.6
	$\theta_2^{I^*} = 31.92$	—	$P_2^{I^*} = 52.41$	—	$d_2 = 649.20$	—	—	
WP	$\theta_1^{WP^*} = 8.01$	$W_1^{WP^*} = 26.11$	$P_1^{WP^*} = 50.22$	—	$d_1 = 362.39$	$\Pi_{M_1}^{WP^*} = 5196.22$	17424.2	27717.1
	$\theta_2^{WP^*} = 8.26$	$W_2^{WP^*} = 26.03$	$P_2^{WP^*} = 50.11$	—	$d_2 = 360.61$	$\Pi_{M_2}^{WP^*} = 5096.62$		
SCS	$\theta_1^{SCS^*} = 18.59$	$W_1^{SCS^*} = 28.714$	$P_1^{SCS^*} = 56.91$	$\phi_1^{SCS^*} = 0.519$	$d_1 = 421.01$	$\Pi_{M_1}^{SCS^*} = 6217.56$	19573.5	31934
	$\theta_2^{SCS^*} = 21.13$	$W_2^{SCS^*} = 28.93$	$P_2^{SCS^*} = 57.20$	$\phi_2^{SCS^*} = 0.570$	$d_2 = 425.95$	$\Pi_{M_2}^{SCS^*} = 6142.91$		
CSB	$\theta_1^{CSB^*} = 23.47$	$W_1^{CSB^*} = 29.91$	$P_1^{CSB^*} = 60.01$	$\phi_1^{CSB^*} = 0.576$	$d_1 = 448.09$	$\Pi_{M_1}^{CSB^*} = 6586.87$	19490.3	32587.4
	$\theta_2^{CSB^*} = 27.16$	$W_2^{CSB^*} = 30.29$	$P_2^{CSB^*} = 60.57$	$\phi_2^{CSB^*} = 0.626$	$d_2 = 456.62$	$\Pi_{M_2}^{CSB^*} = 6510.21$		
WCS	$\theta_1^{WCS^*} = 7.75$	$w_1^{WCS^*} = 26.31$	$P_1^{WCS^*} = 52.33$		$d_1 = 366.98$	$\Pi_{M_1}^{WCS^*} = 5385.42$	18552.1	30040.5
	$\theta_2^{WCS^*} = 20.78$	$W_2^{WCS^*} = 28.89$	$P_2^{WCS^*} = 55.69$	$\phi_2^{WCS^*} = 0.553$	$d_2 = 425.15$	$\Pi_{M_2}^{WCS^*} = 6182.94$		
WCB	$\theta_1^{WCB^*} = 8.26$	$w_1^{WCB^*} = 26.52$	$P_1^{WCB^*} = 53.38$		$d_1 = 371.76$	$\Pi_{M_1}^{WCB^*} = 5460.11$	20117.8	30456.2
	$\theta_2^{WCB^*} = 25.53$	$W_2^{WCB^*} = 29.98$	$P_2^{WCB^*} = 57.87$	$\phi_2^{WCB^*} = 0.608$	$d_2 = 449.6$	$\Pi_{M_2}^{WCS^*} = 6433.57$		

From the above managerial implications, it is clear that to be more successful, the manufacturers should raise the differentiation between the two products with respect to QILs and coordinate with the retailer through CSB contract. On the other hand, the retailer should increase price competition intensity by reducing the price differentiation between the products and deal with strategically less powerful manufacturers so that the retailer would be able to convince them to participate in SCS contract which is more profitable for the retailer as compared to CSB contract.

Our present study can be extended in different ways. Consideration of power channel structure at a horizontal level between two competing manufacturers would be an interesting extension of our study. Depending on the leadership on quality setting stage and price setting stage, this power channel structure may create different scenarios in the supply chain. It would be interesting to see how our present results be influenced by these power channel structures.

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Table 5: Equilibrium results in different scenarios for symmetrical demand with: $\beta_i = 45, \gamma_i = 30, \delta_i = 20, \lambda_i = 5$ for $i \in \{1, 2\}$

Scenario	θ^*	w^*	P^*	ϕ^*	Demand	Π_{M^*}	Π_{R^*}	Π_{D^*}
Integrated	34.0	—	55.33	—	680	—	—	38533.3
WP	8.39	26.26	50.66	—	365.92	5247.79	17852.9	28348.5
SCS	21.93	29.65	59.13	0.571	442.13	6626.09	20565.5	33817.7
CSB	28.33	31.25	63.12	0.625	478.12	7149.67	20445.8	34745.2
WCS	$\theta_1^{WCS^*} = 7.79$	$w_1^{WCS^*} = 26.38$	$P_1^{WCS^*} = 52.63$		$d_1 = 368.76$	$\Pi_{M_1}^{WCS^*} = 5437.63$	18896.4	30603.2
	$\theta_2^{WCS^*} = 21.05$	$W_2^{WCS^*} = 29.15$	$P_2^{WCS^*} = 56.22$	$\phi_2^{WCS^*} = 0.552$	$d_2 = 430.92$	$\Pi_{M_2}^{WCS^*} = 6269.22$		
WCB	$\theta_1^{WCB^*} = 8.30$	$w_1^{WCB^*} = 26.60$	$P_1^{WCB^*} = 53.67$		$d_1 = 373.57$	$\Pi_{M_1}^{WCB^*} = 5513.13$	18938.8	31053.8
	$\theta_2^{WCB^*} = 25.62$	$W_2^{WCB^*} = 30.21$	$P_2^{WCB^*} = 58.36$	$\phi_2^{WCB^*} = 0.605$	$d_2 = 454.76$	$\Pi_{M_2}^{WCS^*} = 6601.95$		

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APPENDIX

Notations and symbols

$$
\begin{aligned}
E_i &= 2\alpha_i\beta_i\beta_j - \alpha_i\gamma_j(\gamma_i+\gamma_j) - \alpha_j\beta_i(\gamma_j-\gamma_i) + 2\beta_i c_i(\beta_i\beta_j - \gamma_i\gamma_j) \\
F &= \beta_1\beta_2 - \gamma_1\gamma_2 \\
G_i &= 2\beta_i\beta_j\delta_i + \beta_i\lambda_j(\gamma_j-\gamma_i) - \gamma_j\delta_i(\gamma_i+\gamma_j) \\
H_i &= \lambda_i\gamma_j(\gamma_i+\gamma_j) - \beta_i\delta_j(\gamma_j-\gamma_i) - 2\lambda_i\beta_i\beta_j \text{ for } i,j=1,2,\ j\neq i \\
\Delta_1^{WP} &= 4\beta_1\beta_2 - (\gamma_1+\gamma_2)^2,\ \Delta_2^{WP} = F\Big\{16\beta_1\beta_2 - (\gamma_1+\gamma_2)^2\Big\} \\
\Delta_3^{WP} &= \Big[2\eta_1\Delta_1^{WP}\Delta_2^{MS} - G_1\Big\{H_2(\gamma_1+\gamma_2)+4\beta_2G_1\Big\}\Big]\Big[2\eta_2\Delta_1^{WP}\Delta_2^{MS} - G_2\Big\{H_1(\gamma_1+\gamma_2)+4\beta_1G_2\Big\}\Big] \\
&\quad -\Big[G_1\Big\{G_2(\gamma_1+\gamma_2)+4\beta_2H_1\Big\}\Big]\Big[G_2\Big\{G_1(\gamma_1+\gamma_2)+4\beta_1H_2\Big\}\Big] \\
\Delta_3^{SCS} &= \Big[2\eta_1(1-\phi_1)\Delta_1^{WP}\Delta_2^{WP} - G_1\{H_2(\gamma_1+\gamma_2)+4\beta_2G_1\}\Big]\Big[2\eta_2(1-\phi_2)\Delta_1^{WP}\Delta_2^{WP} \\
&\quad -G_2\{H_1(\gamma_1+\gamma_2)+4\beta_1G_2\}\Big] - \Big[G_1\{G_2(\gamma_1+\gamma_2)+4\beta_2H_1\}\Big] \\
&\quad \times\Big[G_2\{G_1(\gamma_1+\gamma_2)+4\beta_1H_2\}\Big] \\
\Delta_3^{WCS} &= \Big[2\eta_1\Delta_1^{WP}\Delta_2^{WP} - G_1\{H_2(\gamma_1+\gamma_2)+4\beta_2G_1\}\Big]\Big[2\eta_2(1-\phi_2)\Delta_1^{WP}\Delta_2^{WP} \\
&\quad -G_2\{H_1(\gamma_1+\gamma_2)+4\beta_1G_2\}\Big] - \Big[G_1\{G_2(\gamma_1+\gamma_2)+4\beta_2H_1\}\Big] \\
&\quad \times\Big[G_2\{G_1(\gamma_1+\gamma_2)+4\beta_1H_2\}\Big]
\end{aligned}
$$

Equilibrium retail prices and QILs for Integrated system

$$
\begin{aligned}
P_i^{I^*} &= \frac{1}{\Delta_1^I}\Big[\Big\{(\gamma_i+\gamma_j)-B\Big\}\Big\{\alpha_j - c_i(\gamma_i - B) + c_j\Big(\beta_j - A_j\Big)\Big\} + \Big(2\beta_j - A_j\Big) \\
&\quad \times\Big\{\alpha_i - c_j(\gamma_j - B) + c_i\Big(\beta_i - A_i\Big)\Big\}\Big], \\
\theta_i^{I^*} &= \frac{\Big\{\delta_i\Big(P_i^{I*}-c_i\Big) - \lambda_j\Big(P_j^{I*}-c_j\Big)\Big\}}{2\eta_i}, \text{where } \Delta_1^I = \prod_{i=1}^{2}\Big(2\beta_i - A_i\Big) - \Big\{(\gamma_1+\gamma_1)-B\Big\}^2, \\
&\quad A_i = \frac{\delta_i^2}{2\eta_i} + \frac{\lambda_i^2}{2\eta_j},\ B = \sum_{i,j=1}^{2}\Big(\frac{\delta_i\lambda_j}{2\eta_i}\Big) \text{ for } i,j=1,2,\ j\neq i.
\end{aligned}
$$

Equilibrium solutions under symmetric assumptions for Integrated system

$$
\begin{aligned}
\theta^{I^*} &= \frac{(\delta-\lambda)\Big\{\alpha - c(\beta-\gamma)\Big\}}{2\eta\Big\{2\beta - \frac{1}{2\eta}(\delta^2+\lambda^2) + \frac{\delta\lambda}{\eta} - 2\gamma\Big\}} \\
P^{I^*} &= c + \frac{\Big\{\alpha - c(\beta-\gamma)\Big\}}{\Big\{2\beta - \frac{1}{2\eta}(\delta^2+\lambda^2) + \frac{\delta\lambda}{\eta} - 2\gamma\Big\}} \\
d^{I^*} &= \frac{(\beta-\gamma)\Big\{\alpha - c(\beta-\gamma)\Big\}}{\Big\{2\beta - \frac{1}{2\eta}(\delta^2+\lambda^2) + \frac{\delta\lambda}{\eta} - 2\gamma\Big\}} \\
\Pi^{I^*} &= \frac{\Big\{4\eta(\beta-\gamma) - (\delta-\lambda)^2\Big\}\Big\{\alpha - c(\beta-\gamma)\Big\}^2}{2\eta\Big\{2\beta - \frac{1}{2\eta}(\delta^2+\lambda^2) + \frac{\delta\lambda}{\eta} - 2\gamma\Big\}^2}
\end{aligned}
$$

Equilibrium solutions under symmetric assumptions for WP contract

$$\theta^{WP^*} = \frac{\delta\Big\{\alpha - c(\beta-\gamma)\Big\}}{\Big(8\beta\eta - 4\eta\gamma + \delta\lambda - \delta^2\Big)}$$

$$P^{WP^*} = \frac{2\alpha\eta(3\beta-2\gamma) + c(\beta-\gamma)(\delta\lambda + 2\beta\eta - \delta^2)}{(\beta-\gamma)\Big(8\beta\eta - 4\eta\gamma + \delta\lambda - \delta^2\Big)}$$

$$w^{WP^*} = \frac{4\alpha\eta + c\Big\{4\beta\eta + \delta\lambda - \delta^2\Big\}}{\Big(8\beta\eta - 4\eta\gamma + \delta\lambda - \delta^2\Big)}$$

$$m^{WP^*} = \frac{2\beta\eta\Big\{\alpha - c(\beta-\gamma)\Big\}}{(\beta-\gamma)\Big(8\beta\eta - 4\eta\gamma + \delta\lambda - \delta^2\Big)}$$

$$d^{WP^*} = \frac{2\beta\eta(1-\phi)\Big\{\alpha - c(\beta-\gamma)\Big\}}{\Big(8\beta\eta - 4\eta\gamma + \delta\lambda - \delta^2\Big)}$$

$$\Pi_M^{WP^*} = \frac{\eta(8\beta\eta - \delta^2)\Big\{\alpha - c(\beta-\gamma)\Big\}^2}{\Big(8\beta\eta - 4\eta\gamma + \delta\lambda - \delta^2\Big)^2}$$

$$\Pi_R^{WP^*} = \frac{8\beta^2\eta^2\Big\{\alpha - c(\beta-\gamma)\Big\}^2}{(\beta-\gamma)\Big(8\beta\eta - 4\eta\gamma + \delta\lambda - \delta^2\Big)^2}$$

$$\Pi_{SC}^{WP^*} = \frac{2\eta\Big\{4\beta\eta(3\beta-\gamma) - \delta^2(\beta-\gamma)\Big\}\Big\{\alpha - c(\beta-\gamma)\Big\}^2}{(\beta-\gamma)\Big(8\beta\eta - 4\eta\gamma + \delta\lambda - \delta^2\Big)^2}$$

For given CSF ϕ equilibrium solutions under symmetric assumptions for SCS contract

$$
\begin{aligned}
\theta^{SCS}(\phi) &= \frac{\delta\Big\{\alpha - c(\beta-\gamma)\Big\}}{\Big[4\eta(1-\phi)(2\beta-\gamma)-\delta^2+\delta\lambda\Big]} \\
P^{SCS}(\phi) &= \frac{2\alpha\eta(1-\phi)(3\beta-2\gamma)+c(\beta-\gamma)\Big\{2\beta\eta(1-\phi)-\delta^2+\delta\lambda\Big\}}{(\beta-\gamma)\Big[4\eta(1-\phi)(2\beta-\gamma)-\delta^2+\delta\lambda\Big]} \\
w^{SCS}(\phi) &= \frac{4\alpha\eta(1-\phi)+c\Big\{4\beta\eta(1-\phi)-\delta^2+\delta\lambda\Big\}}{\Big[4\eta(1-\phi)(2\beta-\gamma)-\delta^2+\delta\lambda\Big]} \\
m^{SCS}(\phi) &= \frac{2\beta\eta(1-\phi)\Big\{\alpha - c(\beta-\gamma)\Big\}}{(\beta-\gamma)\Big[4\eta(1-\phi)(2\beta-\gamma)-\delta^2+\delta\lambda\Big]} \\
d^{SCS}(\phi) &= \frac{2\beta\eta(1-\phi)\Big\{\alpha - c(\beta-\gamma)\Big\}}{\Big[4\eta(1-\phi)(2\beta-\gamma)-\delta^2+\delta\lambda\Big]} \\
\Pi_M^{SCS}(\phi) &= \frac{\eta(1-\phi)\Big(8\beta\eta(1-\phi)-\delta^2\Big)\Big\{\alpha - c(\beta-\gamma)\Big\}^2}{\Big[4\eta(1-\phi)(2\beta-\gamma)-\delta^2+\delta\lambda\Big]^2} \\
\Pi_R^{SCS}(\phi) &= \frac{2\Big\{4\beta^2\eta^2(1-\phi)^2-\phi\eta\delta^2(\beta-\gamma)\Big\}\Big\{\alpha - c(\beta-\gamma)\Big\}^2}{(\beta-\gamma)\Big[4\eta(1-\phi)(2\beta-\gamma)-\delta^2+\delta\lambda\Big]^2} \\
\Pi_{SC}^{SCS}(\phi) &= \frac{2\eta\Big\{4\beta\eta(1-\phi)^2(3\beta-2\gamma)-\delta^2(\beta-\gamma)\Big\}\Big\{\alpha - c(\beta-\gamma)\Big\}^2}{(\beta-\gamma)\Big[4\eta(1-\phi)(2\beta-\gamma)-\delta^2+\delta\lambda\Big]^2}
\end{aligned}
$$

Equilibrium solutions under symmetric assumptions for SCS contract

$$\phi^{SCS^*} = \frac{4\eta\Big\{2\beta(\gamma\delta-\beta\lambda)+\gamma\delta(\beta-\gamma)\Big\}+\delta^2(\beta-\gamma)(\delta-\lambda)}{4\eta\Big[2\beta^2(\delta-\lambda)+\delta(\beta-\gamma)(2\beta-\gamma)\Big]}$$

$$\theta^{SCS^*} = \frac{\Big\{\alpha-c(\beta-\gamma)\Big\}\Big\{2\beta^2(\delta-\lambda)+\delta(\beta-\gamma)(2\beta-\gamma)\Big\}}{2\Big[4\eta(\beta-\gamma)(2\beta-\gamma)^2-\delta(\beta-\gamma)(\delta-\lambda)(2\beta-\gamma)-\beta^2(\delta-\lambda)^2\Big]}$$

$$P^{SCS^*} = \frac{\alpha(3\beta-2\gamma)\Big\{8\eta(2\beta-\gamma)-\delta(\delta-\lambda)\Big\}+c\kappa_1}{4\Big[4\eta(\beta-\gamma)(2\beta-\gamma)^2-\delta(\beta-\gamma)(\delta-\lambda)(2\beta-\gamma)-\beta^2(\delta-\lambda)^2\Big]}$$

where $\kappa_1 = \Big\{8\beta\eta(\beta-\gamma)(2\beta-\gamma)-\delta(\beta-\gamma)(\delta-\lambda)(5\beta-2\gamma)-4\beta^2(\delta-\lambda)^2\Big\}$

$$w^{SCS^*} = \frac{\alpha(\beta-\gamma)\Big\{8\eta(2\beta-\gamma)-\delta(\delta-\lambda)\Big\}+c\kappa_2}{2\Big[4\eta(\beta-\gamma)(2\beta-\gamma)^2-\delta(\beta-\gamma)(\delta-\lambda)(2\beta-\gamma)-\beta^2(\delta-\lambda)^2\Big]}$$

where $\kappa_2 = \Big\{8\beta\eta(\beta-\gamma)(2\beta-\gamma)-\delta(\beta-\gamma)(\delta-\lambda)(3\beta-\gamma)-2\beta^2(\delta-\lambda)^2\Big\}$

$$m^{SCS^*} = \frac{\beta\Big\{\alpha-c(\beta-\gamma)\Big\}\Big\{8\eta(2\beta-\gamma)-\delta(\delta-\lambda)\Big\}}{4\Big[4\eta(\beta-\gamma)(2\beta-\gamma)^2-\delta(\beta-\gamma)(\delta-\lambda)(2\beta-\gamma)-\beta^2(\delta-\lambda)^2\Big]}$$

$$d^{SCS^*} = \frac{\beta(\beta-\gamma)\Big\{\alpha-c(\beta-\gamma)\Big\}\Big\{8\eta(2\beta-\gamma)-\delta(\delta-\lambda)\Big\}}{4\Big[4\eta(\beta-\gamma)(2\beta-\gamma)^2-\delta(\beta-\gamma)(\delta-\lambda)(2\beta-\gamma)-\beta^2(\delta-\lambda)^2\Big]}$$

$$\Pi_M^{SCS^*} = \frac{(\beta-\gamma)(2\beta-\gamma)\Big\{\alpha-c(\beta-\gamma)\Big\}^2\kappa_3}{16\Big[4\eta(\beta-\gamma)(2\beta-\gamma)^2-\delta(\beta-\gamma)(\delta-\lambda)(2\beta-\gamma)-\beta^2(\delta-\lambda)^2\Big]^2}$$

where $\kappa_3 = 128\beta\eta^2(\beta-\gamma)(2\beta-\gamma)-8\eta\delta\Big\{2\beta(\delta-\lambda)(3\beta-2\gamma)+\delta(2\beta-\gamma)(\beta-\gamma)\Big\}$
$+\delta^2(\delta-\lambda)\Big\{\beta(\delta-\lambda)+(\gamma\delta-\beta\lambda)\Big\}$

$$\Pi_R^{SCS^*} = \frac{\Big\{\alpha-c\beta-\gamma)\Big\}^2\kappa_4}{8\Big[4\eta(\beta-\gamma)(2\beta-\gamma)^2-\delta(\beta-\gamma)(\delta-\lambda)(2\beta-\gamma)-\beta^2(\delta-\lambda)^2\Big]^2}$$

where $\kappa_4 = 64\beta^2\eta^2(\beta-\gamma)(2\beta-\gamma)^2-4\eta\Big[4\beta^2(\delta-\lambda)\Big\{\delta(\beta-\gamma)(2\beta-\gamma)+\beta^2(\delta-\lambda)\Big\}$
$+\delta^2(\beta-\gamma)^2(2\beta-\gamma)^2\Big]-\delta^2(\beta-\gamma)(\delta-\lambda)\Big\{\delta(\beta-\gamma)(2\beta-\gamma)+\beta^2(\beta-\lambda)\Big\}$

$$\Pi_{SC}^{SCS^*} = \frac{\Big\{\alpha-c\beta-\gamma)\Big\}^2\kappa_5}{8\Big[4\eta(\beta-\gamma)(2\beta-\gamma)^2-\delta(\beta-\gamma)(\delta-\lambda)(2\beta-\gamma)-\beta^2(\delta-\lambda)^2\Big]^2}$$

where $\kappa_4 = 64\beta\eta^2(\beta-\gamma)(2\beta-\gamma)^2(3\beta-\gamma)-4\eta\Big\{4\beta\delta(\beta-\gamma)(2\beta-\gamma)(4\beta-\gamma)(\delta-\lambda)$
$+4\beta^4(\delta-\lambda)^2+\delta^2(\beta-\gamma)^2(2\beta-\gamma)^2\Big\}+\beta\delta^2(\beta-\gamma)(\delta-\lambda)^2(3\beta-\gamma)$

Proof of Proposition 1

To establish the existence and uniqueness of the optimal solutions of the integrated system it is sufficient to show that the objective function is strictly concave with respect to its decision variables. Since, profit functions are continuous and twice differentiable, to establish the strictly concavity of the objective function as given by (4) with respect to its decision variables $P_1, P_2, \theta_1, \theta_2$, it is sufficient to show that objective function is negative definite. Thus, we have to show that principal minors of the hessian matrix H of the objective function are alternatively, (-)ve, (+)ve and (-)ve, in order i.e., $D_1^I(P_1, P_2, \theta_1, \theta_2) < 0$, $D_2^I(P_1, P_2, \theta_1, \theta_2) > 0$, $D_3^I(P_1, P_2, \theta_1, \theta_2) < 0$ and $D_4^I(P_1, P_2, \theta_1, \theta_2) > 0$, respectively where $D_r^I(P_1, P_2, \theta_1, \theta_2)$ denotes the principal minor of the hessian H of rth order, $r = 1, 2, 3, 4$.Now

$$
\begin{aligned}
D_1^I &= -2\beta_1 < 0,\\
D_2^I &= 4\beta_1\beta_2 - (\gamma_1+\gamma_2)^2 > 0, \text{ under } \mathbf{A1},\\
D_3^I &= -2\eta_1\Big\{4\beta_1\beta_2 - (\gamma_1+\gamma_2)^2\Big\} - 2\Big\{\lambda_2\delta_1(\gamma_1+\gamma_2) - (\beta_1\lambda_2^2 + \beta_2\delta_1^2)\Big\}\\
&< 0, \text{ under assumptions } \mathbf{A1 \text{ and } A2} \text{ and}\\
D_4^I &= \sum_{i=1, j=3-i}^{2} 4\eta_i\Big\{\lambda_i\delta_j(\gamma_i+\gamma_j) - (\beta_j\lambda_i^2 + \beta_i\delta_i^2)\Big\} + (\delta_1\delta_2 - \lambda_1\lambda_2)^2\\
&\quad +4\eta_1\eta_2\Big\{4\beta_1\beta_2 - (\gamma_1+\gamma_2)^2\Big\} \text{ under } \mathbf{A2}\\
&> 0.
\end{aligned}
$$

This ensures that the objective function of the integrated system is strictly concave. This completes the proof of the proposition. ■

Proof of Lemma 1

(a) For given quality improvement level (QIL)s and wholesale prices, using equation (1) and the retail margins $m_i = P_i - w_i,\ i \in \{1, 2\}$, the first order optimality condition of profit function (2) gives

$$\frac{\partial \Pi_R}{\partial P_1} = -2\beta_1 P_1 + (\gamma_1+\gamma_2)P_2 + \delta_1\theta_1 - \lambda_1\theta_2 + \beta_1 w_1 - \gamma_2 w_2 + \alpha_1 = 0, \qquad (B.1)$$

$$\frac{\partial \Pi_R}{\partial P_2} = (\gamma_1+\gamma_2)P_1 - 2\beta_2 P_2 - \lambda_2\theta_1 + \delta_2\theta_2 - \gamma_1 w_1 + \beta_2 w_2 + \alpha_2 = 0. \qquad (B.2)$$

Solving $(B.1)$ and $(B.2)$ simultaneously and after some simplification we get the expressions for $P_i^*(\theta_1, \theta_2, w_1, w_2)$ as given in equation (5).

(b) Differentiating the retailer's reaction function (5) partially with respect to w_i and θ_i we have respectively $\frac{\partial P_i^*(\theta_i,\theta_j,w_i,w_j)}{\partial w_i} = \frac{2\beta_i\beta_j - \gamma_i(\gamma_i+\gamma_j)}{\Delta_1^{WP}}$ and $\frac{\partial P_i^*(\theta_i,\theta_j,w_i,w_j)}{\partial \theta_i} = \frac{2\beta_j\delta_i - \lambda_j(\gamma_i+\gamma_j)}{\Delta_1^{WP}}$. Since,$\Delta_1^{WP} = 4\beta_1\beta_2 - (\gamma_1 + \gamma_2)^2 > 0$ by assumption **A1**, to establish the positivity of $\frac{\partial P_i^*(\theta_i,\theta_j,w_i,w_j)}{\partial w_i}$ and $\frac{\partial P_i^*(\theta_i,\theta_j,w_i,w_j)}{\partial \theta_i}$, we just have to prove the positivity of the expressions $2\beta_i\beta_j - \gamma_i(\gamma_i + \gamma_j)$ and $2\beta_j\delta_i - \lambda_j(\gamma_i + \gamma_j)$. Since $\beta_i,\ \beta_j > \gamma_i$ and $\delta_i,\ \delta_j > \lambda_i$ for $i, j = 1, 2$ and $j \neq i$ always, we have $2\beta_i\beta_j - \gamma_i(\gamma_i + \gamma_j) > 0$ and $2\beta_j\delta_i - \lambda_j(\gamma_i + \gamma_j) > 0$.

This completes the proof. ■

Derivation of Equations (6) and (7)

Substituting the retailers' reaction function (5) in equation (3) we get the ith manufacturer's profit function as a function of QIL and wholesale price. Thus, the profit of the ith manufacturer can be given as

$$\Pi_{M_i}(w_i, w_j, \theta_i, \theta_j) = (w_i - c_i)d_i^*(w_i, w_j, \theta_i, \theta_j) - \eta_i\theta_i^2, \tag{B.3}$$

where $d_i^*(w_i, w_j, \theta_i, \theta_j) = \alpha_i - \beta_i P_i^*(w_i, w_j, \theta_i, \theta_j) + \gamma_i P_j^*(w_i, w_j, \theta_i, \theta_j) + \delta_i\theta_i - \lambda_i\theta_j$ for $i, j = 1, 2$ and $j \neq i$.

Differentiating $(B.3)$ with respect to w_1 and w_2 and equating to zero we get

$$\frac{\partial \Pi_{M_1}(w_1, w_2, \theta_1, \theta_2)}{\partial w_1} = -4\beta_1 F w_1 + (\gamma_1 + \gamma_2)F w_2 + \theta_1 G_1 + \theta_2 H_1 + E_1 = 0, \tag{B.4}$$

$$\frac{\partial \Pi_{M_2}(w_1, w_2, \theta_1, \theta_2)}{\partial w_2} = (\gamma_1 + \gamma_2)F w_1 - 4\beta_2 F w_2 + \theta_1 H_2 + \theta_2 G_2 + E_2 = 0, \tag{B.5}$$

where $F,\ E_i,\ G_i,\ H_i,\ i \in \{1, 2\}$ are defined in **Notations and Symbols** of this appendix. Solving $(B.4)$ and $(B.5)$ simultaneously we get the expressions for $w_i(\theta_1, \theta_2)$ as functions of $\theta_1,\ \theta_2$ as are given in equation (6).

In the next step, substituting (6) in $(B.3)$ and differentiating with respect to θ_1, θ_2 and equating to zero in turn we get a system of linear equations of θ_1 and θ_2 as

$$\begin{aligned}
&\theta_1\Big[G_1\Big\{H_2(\gamma_1+\gamma_2)+4\beta_2 G_1\Big\}-2\eta_1\Delta_1^{WP}\Delta_2^{WP}\Big]+\theta_2\Big[G_1\Big\{G_2(\gamma_1+\gamma_2)+4\beta_2 H_1\Big\}\Big]\\
&+G_1\Big[E_2(\gamma_1+\gamma_2)+4\beta_2 E_1-c_1\Delta_2^{WP}\Big]=0, \qquad (B.6)\\
&\theta_1\Big[G_2\Big\{G_1(\gamma_1+\gamma_2)+4\beta_1 H_2\Big\}\Big]+\theta_2\Big[G_2\Big\{H_1(\gamma_1+\gamma_2)+4\beta_1 G_2\Big\}-2\eta_2\Delta_1^{WP}\Delta_2^{WP}\Big]\\
&+G_2\Big[E_1(\gamma_1+\gamma_2)+4\beta_1 E_2-c_2\Delta_2^{WP}\Big]=0, \qquad (B.7)
\end{aligned}$$

Solving $(B.6)$ and $(B.7)$ simultaneously we get the expressions for equilibrium quality improvement level (QIL)s under WP contract as are given in (7). ■

Proof of Lemma 2

Differentiating the expression (6) partially with respect to θ_i we have $\frac{\partial w_i^*(\theta_i\theta_j)}{\partial\theta_i}=\frac{1}{\Delta_2^{WP}}\Big[H_j(\gamma_i+\gamma_j)+4\beta_j G_i\Big]$ for $i.j=1,2$ and $j\neq i$.
We have $\Delta_2^{WP}=(\beta_1\beta_2-\gamma_1\gamma_2)\Big[16\beta_1\beta_2-(\gamma_1+\gamma_2)^2\Big]>0$ by assumption **A1**. Hence, to prove the positivity of $\frac{\partial w_i^*(\theta_i\theta_j)}{\partial\theta_i}$ it is sufficient to prove the positivity of $H_j(\gamma_i+\gamma_j)+4\beta_j G_i$ for $i,j=1,2$ and $j\neq i$. We have after some simplification

$$\begin{aligned}
H_j(\gamma_i+\gamma_j)+4\beta_j G_i &= \Big[4\beta_j\delta_j-\lambda_j(\gamma_i+\gamma_j)\Big]\Big[2\beta_i\beta_j-\gamma_i(\gamma_i+\gamma_j)\Big]\\
&\quad+(\gamma_i-\gamma_j)\Big[4\beta_j\lambda_i-\delta_i(\gamma_i+\gamma+j)\Big]\\
&> 0,\ \text{since}\ \Big\{\beta_i,\beta_j\Big\}>\gamma_i\text{and}\Big\{\delta_i,\delta_j\Big\}>\lambda_i\ \text{for}\ i,j=1,2\ \text{and}\ j\neq i.
\end{aligned}$$

This completes the proof. ■

Proof of Lemma 3

(a) Under symmetric assumption as defined in Section *3.1*, from the expression (7), the equilibrium quality improvement level of each manufacturer under WP contract can be given as

$$\theta^{WP^*} = \frac{\delta\Big\{\alpha - c(\beta-\gamma)\Big\}}{8\beta\eta - 4\eta\gamma + \delta\lambda - \delta^2} \quad (B.8)$$
$$> 0, \text{ always under assumptions } \mathbf{A3} \text{ and } \mathbf{A5}.$$

Substituting (B.8) in equation (6), the equilibrium wholesale price of each manufacturer under symmetric assumption can be given as

$$w^{WP^*} = \frac{4\alpha\eta + c\Big\{4\beta\eta + \delta\lambda - \delta^2\Big\}}{8\beta\eta - 4\eta\gamma + \delta\lambda - \delta^2} \quad (B.9)$$
$$> 0, \text{ always under assumptions } \mathbf{A2} \text{ and } \mathbf{A3}.$$

Finally, substituting $(B.8)$ and $(B.9)$ in (5) we get equilibrium retail price in WP contract under symmetric assumption and is given by

$$P^{WP^*} = \frac{2\alpha\eta(3\beta - 2\gamma) + c(\beta-\gamma)(2\beta\eta + \delta\lambda - \delta^2)}{(\beta-\gamma)(8\beta\eta - 4\eta\gamma + \delta\lambda - \delta^2)}$$
$$> 0, \text{ under assumption } \mathbf{A3} \text{ and since } \beta > \gamma \text{ always.}$$

(b) In WP contract the contribution margin of each manufacturer under symmetric assumption is

$$m^{WP^*} = w^{WP^*} - c = \frac{2\beta\eta\Big\{\alpha - c(\beta-\gamma)\Big\}}{(\beta-\gamma)(8\beta\eta - 4\eta\gamma + \delta\lambda - \delta^2)}$$
$$> 0, \text{ under assumptions } \mathbf{A3}, \mathbf{A5} \text{ and since } \beta > \gamma \text{ always.}$$

This completes the proof of the lemma. ■

Derivation of Equation (10)

Since, cost-sharing fraction (CSF)s are decided at the last stage of the manufacturer-Stackelberg game, the retailer's reaction functions on retail prices and manufacturers' pricing decisions are same as WP contract and are given by expressions (5) and (6). Hence, substituting (5) and (6) into profit function (9), differentiating with respect to QILs $\{\theta_1, \theta_2\}$ and equate it to zero we get

$$\theta_1\Big[G_1\Big\{H_2(\gamma_1+\gamma_2)+4\beta_2G_1\Big\}-2\eta_1(1-\phi_1)\Delta_1^{MS}\Delta_2^{MS}\Big\}+\theta_2\Big[G_1\Big\{G_2(\gamma_1+\gamma_2)+4\beta_2H_1\Big\}\Big]$$
$$+G_1\Big[E_2(\gamma_1+\gamma_2)+4\beta_2E_1-c_1\Delta_2^{MS}\Big]=0, \quad (B.10)$$
$$\theta_1\Big[G_2\Big\{G_1(\gamma_1+\gamma_2)+4\beta_1H_2\Big\}\Big]+\theta_2\Big[G_2\Big\{H_1(\gamma_1+\gamma_2)+4\beta_1G_2\Big\}-2\eta_2(1-\phi_2)\Delta_1^{MS}\Delta_2^{MS}\Big]$$
$$+G_2\Big[E_1(\gamma_1+\gamma_2)+4\beta_1E_2-c_2\Delta_2^{MS}\Big]=0. \quad (B.11)$$

Solving $(B.10)$ and $(B.11)$ simultaneously we get the equilibrium QILs under SCS contract as are given in (10), Δ_3^{SCS} is the same as defined in **Notations and Symbols** of this appendix. ■

Proof of Lemma 4

Differentiating the equation (10) partially with respect to ϕ_i we have

$$\begin{aligned}\frac{\partial\theta_i^{SCS}(\phi_i,\phi_j)}{\partial\phi_i} &= \frac{1}{(\Delta_3^{SCS})^2}2\eta_i\Delta_1^{WP}\Delta_2^{WP}\Big[2\eta_j(1-\phi_j)\Delta_1^{WP}\Delta_2^{WP}-G_j\Big\{H_i(\gamma_i+\gamma_j)+4\beta_iG_j\Big\}\Big]\\ &\quad\times\Big[G_iG_j\Big\{G_j(\gamma_i+\gamma_j)+4\beta_jH_i\Big\}\Big\{E_i(\gamma_i+\gamma_j)+4\beta_iE_j-c_j\Delta_2^{WP}\Big\}\\ &\quad+G_i\Big\{E_j(\gamma_i+\gamma_j)+4\beta_jE_i-c_i\Delta_2^{WP}\Big\}\Big\{2\eta_j(1-\phi_j)\Delta_1^{WP}\Delta_2^{WP}\\ &\quad-G_j\Big\{H_i(\gamma_i+\gamma_j)+4\beta_iG_j\Big\}\Big]\Big]\\ &> 0, \text{ under assumptions } \mathbf{A1}, \mathbf{A2} \text{ and } \mathbf{A5}.\end{aligned}$$

This completes the proof of the lemma. ■

Proof of Proposition 2

Under symmetric assumption and for given CSF ϕ, the objective function of the retailer under SCS contract is

$$\Pi_R^{SCS}(\phi)=\Pi_R^{SCS}=\frac{2\Big\{\alpha-c(\beta-\gamma)\Big\}^2\Big\{4\beta^2\eta^2(1-\phi)^2-\phi\eta\delta^2(\beta-\gamma)\Big\}}{(\beta-\gamma)\Big[4\eta(1-\phi)(2\beta-\gamma)-\delta^2+\delta\lambda\Big]^2}. \quad (B.12)$$

This is a function of ϕ only. Hence, to establish the proposition,it is sufficient to prove that Π_R^{SCS} is strictly concave with respect to ϕ. Now differentiating $(B.12)$ with respect to ϕ

twice and putting $\phi = \phi_{Opt}^{SCS^*}$ we get

$$\left[\frac{d^2\Pi_R^{SCS}}{d\phi^2}\right] = -\frac{8\eta^2\delta\Big\{\alpha-(\beta-\gamma)\Big\}^2\Big[2\beta^2(\delta-\lambda)+\delta(\beta-\gamma)(2\beta-\gamma)\Big]}{(\beta-\gamma)\Big[4\eta(1-\phi_{Opt}^{SCS^*})(2\beta-\gamma)-\delta^2+\delta\lambda\Big]^2} < 0.$$

This implies that at $\phi = \phi^{SCS^*}$, Π_R^{SCS} is strictly concave.Hence, there exists an optimal solution of Π_R^{SCS}, which is a local solution. Thus, $\phi = \phi^{SCS^*}$ is the required local optimal solution. This completes the proof. ■

Proof of Lemma 5

The second part of the proof is similar to **Proof of Lemma 4**. For the first part, differentiating (17) partially with respect to ϕ_2 and after some simplification we get

$$\begin{aligned}
\frac{\partial\theta_1(\phi_2)}{\partial\phi_2} &= \frac{2\eta_2\Delta_1^{WP}\Delta_2^{WP}}{(\Delta_3^{WCS})^2}\xi, \text{ where} \\
\xi &= \Big[G_1\Big\{E_2(\gamma_1+\gamma_2)+4\beta_2E_1-\Delta_2^{WP}c_1\Big\}\Big]\Big[G_1\Big\{G_2(\gamma_1+\gamma_2)+4\beta_2H_1\Big\}\Big] \\
&\times\Big[G_2\Big\{G_1(\Gamma_1+\gamma_2)+4\beta_1G_2\Big\}\Big]+\Big[G_2\Big\{E_1(\gamma_1+\gamma_2)+4\beta_1E_2-\Delta_2^{WP}c_2\Big\}\Big] \\
&\times\Big[G_1\Big\{G_2(\gamma_1+\gamma_2)+4\beta_2H_1\Big\}\Big]\Big[2\eta_1\Delta_1^{WP}\Delta_2^{WP}-G_1\Big\{H_2(\gamma_1+\gamma_2)+4\beta_2G_1\Big\}\Big] \\
&> 0, \text{ under assumptions } \mathbf{A1}, \mathbf{A2} \text{ and } \mathbf{A5}.
\end{aligned}$$

This completes the proof of the lemma. ■

Proof of Proposition 3

To prove the results we use the expressions as given in **Equilibrium solutions under symmetric assumptions** of this appendix under different contracts.

(i) $\theta^{SCS^*} > \theta^{WP^*}$: To establish this result it is sufficient to show that $Num\_\theta^{SCS^*} > Num\_\theta^{WP^*}$ and $Den\_\theta^{WP^*} > Den\_\theta^{SCS^*}$ where $Num\_\theta^{SCS^*}$ and $Num\_\theta^{WP^*}$ are the numerators of the equilibrium θ^{SCS^*} and θ^{WP^*},respectively whereas $Den\_\theta^{SCS^*}$ and $Den\_\theta^{WP^*}$ are their respective denominators. The equilibrium QILs in WP and SCS contracts under symmetric assumption are

$$
\begin{aligned}
\theta^{WP^*} &= \frac{\delta\Big\{\alpha - c(\beta-\gamma)\Big\}}{\Big(8\beta\eta - 4\eta\gamma + \delta\lambda - \delta^2\Big)}, \\
\theta^{SCS^*} &= \frac{\Big\{\alpha - c(\beta-\gamma)\Big\}\Big\{2\beta^2(\delta-\lambda) + \delta(\beta-\gamma)(2\beta-\gamma)\Big\}}{2\Big[4\eta(\beta-\gamma)(2\beta-\gamma)^2 - \delta(\beta-\gamma)(\delta-\lambda)(2\beta-\gamma) - \beta^2(\delta-\lambda)^2\Big]} \\
&= \frac{Num\_\theta^{SCS^*}}{Den\_\theta^{SCS^*}}.
\end{aligned}
$$

We can rewrite θ^{WP^*} in the following way:

$$
\begin{aligned}
\theta^{WP^*} &= \frac{2\delta(\beta-\gamma)(2\beta-\gamma)\Big\{\alpha - c(\beta-\gamma)\Big\}}{2\Big[8\beta\eta(\beta-\gamma)(2\beta-\gamma) - 4\eta\gamma(\beta-\gamma)(2\beta-\gamma) - \delta(\delta-\lambda)(\beta-\gamma)(2\beta-\gamma)\Big]} \\
&= \frac{Num\_\theta^{WP^*}}{Den\_\theta^{WP^*}}.
\end{aligned}
$$

We get

$$
\begin{aligned}
Num\_\theta^{SCS^*} - Num\_\theta^{WP^*} &= \Big\{\alpha - c(\beta-\gamma)\Big\}\Big\{2\beta(\gamma\delta - \beta\lambda) + \gamma\delta(\beta-\gamma)\Big\} \\
&> 0, \text{ under assumption } \mathbf{A4} \text{ and production cost restriction (8)}, \\
\Rightarrow Num\_\theta^{SCS^*} &> Num\_\theta^{WP^*}. \qquad (B.13)
\end{aligned}
$$

On the other hand,

$$
\begin{aligned}
Den\_\theta^{WP^*} - Den\_\theta^{SCS^*} &= 4\eta(\beta-\gamma)(2\beta-\gamma)^2 - 4\eta(\beta-\gamma)(2\beta-\gamma)^2 + \beta^2(\delta-\lambda)^2 \\
&= \beta^2(\delta-\lambda)^2 > 0, \\
\Rightarrow Den\_\theta^{WP^*} &> Den\_\theta^{SCS^*}. \qquad (B.14)
\end{aligned}
$$

Combining $(B.13)$ and $(B.14)$ we have we have $\theta^{SCS^*} > \theta^{WP^*}$ under assumption **A4** and production cost restriction **A5**. This is the required result.

(ii) $w^{SCS^*} > w^{WP^*}$: To establish the result we follow the similar approach as discussed in case of Proposition 3(i). In this case, to prove the result it is sufficient to show that

$Num\_w^{SCS^*} > Num\_w^{WP^*}$ and $Den\_w^{WP^*} > Den\_w^{SCS^*}$ where similar to previous proof, $Num\_w^{SCS^*}$ and $Num\_w^{WP^*}$ are the numerators and $Den\_w^{SCS^*}$ and $Den\_w^{WP^*}$ are the denominators of w^{SCS^*} and w^{WP^*}, respectively. Under symmetric assumption, the expression for w^{WP^*} can be rewritten as

$$\begin{aligned} w^{WP^*} &= \frac{\frac{3}{2}(\beta-\gamma)(2\beta-\gamma)\Big\{4\alpha\eta + c\Big(4\beta\eta+\delta\lambda-\delta^2\Big)\Big\}}{\frac{3}{2}(\beta-\gamma)(2\beta-\gamma)\Big(8\beta\eta-4\eta\gamma+\delta\lambda-\delta^2\Big)}. \\ &= \frac{Num\_w^{WP^*}}{Den\_w^{WP^*}} \end{aligned}$$

We have

$$\begin{aligned} w^{SCS^*} &= \frac{\alpha(\beta-\gamma)\Big\{8\eta(2\beta-\gamma)-\delta(\delta-\lambda)\Big\}+c\kappa_2}{2\Big[4\eta(\beta-\gamma)(2\beta-\gamma)^2-\delta(\beta-\gamma)(\delta-\lambda)(2\beta-\gamma)-\beta^2(\delta-\lambda)^2\Big]} \\ &= \frac{Num\_w^{SCS^*}}{Den\_w^{SCS^*}}, \text{ where} \\ \kappa_2 &= \Big\{8\beta\eta(\beta-\gamma)(2\beta-\gamma)-\delta(\beta-\gamma)(\delta-\lambda)(3\beta-\gamma)-2\beta^2(\delta-\lambda)^2. \end{aligned}$$

Now,we have

$$\begin{aligned} Num\_w^{SCS^*}-Num\_w^{WP^*} &= \alpha(\beta-\gamma)\Big\{2\eta(2\beta-\gamma)-\delta(\delta-\lambda)\Big\}+c\Big\{2\beta\eta(\beta-\gamma)(2\beta-\gamma) \\ &\quad -(\beta-\gamma)(\delta-\lambda)\frac{\delta\gamma}{2}-2\beta^2(\delta-\lambda)^2\Big\} \\ &> \alpha(\beta-\gamma)\Big\{2\eta(2\beta-\gamma)-\delta(\delta-\lambda)\Big\} \\ &\quad +\frac{c}{2}\Big\{(\beta-\gamma)(\delta-\lambda)\delta(4\beta-\gamma)+\frac{4\beta^2(\delta-\lambda)^2(2\beta+\gamma)}{(2\beta-\gamma)}\Big\} \text{ by } \mathbf{A2} \\ &> 0, \text{ by } \mathbf{A2}, \\ \Rightarrow Num\_w^{SCS^*} &> Num\_w^{WP^*}, \text{ by assumption } \mathbf{A2}. \qquad (B.15) \end{aligned}$$

Further, we have

$$\begin{aligned} Den\_w^{WP^*}-Den\_w^{SCS^*} &= 4\eta(\beta-\gamma)(2\beta-\gamma)^2+\frac{\delta}{2}(\beta-\gamma)(2\beta-\gamma)(\delta-\lambda)+2\beta^2(\delta-\lambda)^2 \\ &> 0, \\ \Rightarrow Den\_w^{WP^*} &> Den\_w^{SCS^*}. \qquad (B.16) \end{aligned}$$

Combining $(B.15)$ and $(B.16)$ we get the required result $w^{SCS^*} > w^{WP^*}$ under assumption **A2**. This completes the proof of the proposition. ■

Proof of Proposition 4

In our study, we represent the price competition level and the quality competition level between two products by the the parametric differences $(\beta - \gamma)$ and $(\delta - \lambda)$, respectively. Competition levels increase with the decrease of these values. Hence to establish the result we have to just show that ϕ^{SCS^*} decreases with $(\beta - \gamma)$ and increases with $(\delta - \lambda)$. Taking partial differentiation with respect to $(\beta - \gamma)$ we have

$$\begin{aligned}\frac{\partial \phi^{SCS^*}}{\partial(\beta-\gamma)} &= -\frac{1}{\eta\zeta^2}\Big[\Big\{4\eta(2\beta\lambda-\gamma\delta)-\delta^2(\delta-\lambda)\Big\}\Big[2\beta^2(\delta-\lambda)+\delta(\beta-\gamma)(2\beta-\gamma)\Big] \\ &\quad +4\eta\delta(2\beta-\gamma)\Big[2\beta(\gamma\delta-\beta\lambda)+\gamma\delta(\beta-\gamma)\Big] \\ &\quad +\delta^3(\beta-\gamma)(\delta-\lambda)(2\beta-\gamma)\Big] \\ &< 0, \text{ under assumption } \mathbf{A4} \text{ and if assumption } \mathbf{A2} \text{ holds under symmetric assumption,} \\ &\quad \text{where } \zeta = 4\Big[2\beta^2(\delta-\lambda)+\delta(\beta-\gamma)(2\beta-\gamma)\Big].\end{aligned}$$

Similarly, taking partial differentiation with respect to $(\delta - \lambda)$

$$\frac{\partial \phi^{SCS^*}}{\partial(\delta-\lambda)} = \frac{(\beta-\gamma)}{\eta\zeta^2}\Big[\delta^3(\beta-\gamma)(2\beta-\gamma)+8\beta\eta\Big\{2\beta^2\lambda+\gamma\delta(\beta-\gamma)\Big\}\Big] > 0.$$

This completes the proof of the proposition. ■

Proof of Proposition 5

We prove the result for WP contract and SCS contract under symmetric assumptions. Our numerical investigation assures that this result is also true for CSB contract and for asymmetric demand assumptions. Taking partial differentiation of θ^{WP^*} and θ^{SCS^*} with respect to η we get in turn

$$\begin{aligned}
\frac{\partial \theta^{WP^*}}{\partial \eta} &= -\frac{4\delta(2\beta-\gamma)\Big\{\alpha - c(\beta-\gamma)\Big\}}{(8\beta\eta - 4\eta\gamma + \delta\lambda - \delta^2)^2} \\
\frac{\partial \theta^{SCS^*}}{\partial \eta} &= -\frac{2(\beta-\gamma)(2\beta-\gamma)^2\Big\{\alpha - c(\beta-\gamma)\Big\}\Big\{2\beta^2(\delta-\lambda) + \delta(\beta-\gamma)(2\beta-\gamma)\Big\}}{\Big[4\eta(\beta-\gamma)(2\beta-\gamma)^2 - \delta(\beta-\gamma)(\delta-\lambda)(2\beta-\gamma) - \beta^2(\delta-\lambda)^2\Big]^2}.
\end{aligned}$$

From the above results we have $\frac{\partial \theta^{WP^*}}{\partial \eta} < 0$ and $\frac{\partial \theta^{SCS^*}}{\partial \eta} < 0$ under condition **A2**[11]. This completes the proof of the proposition. ■

Proof of Proposition 6

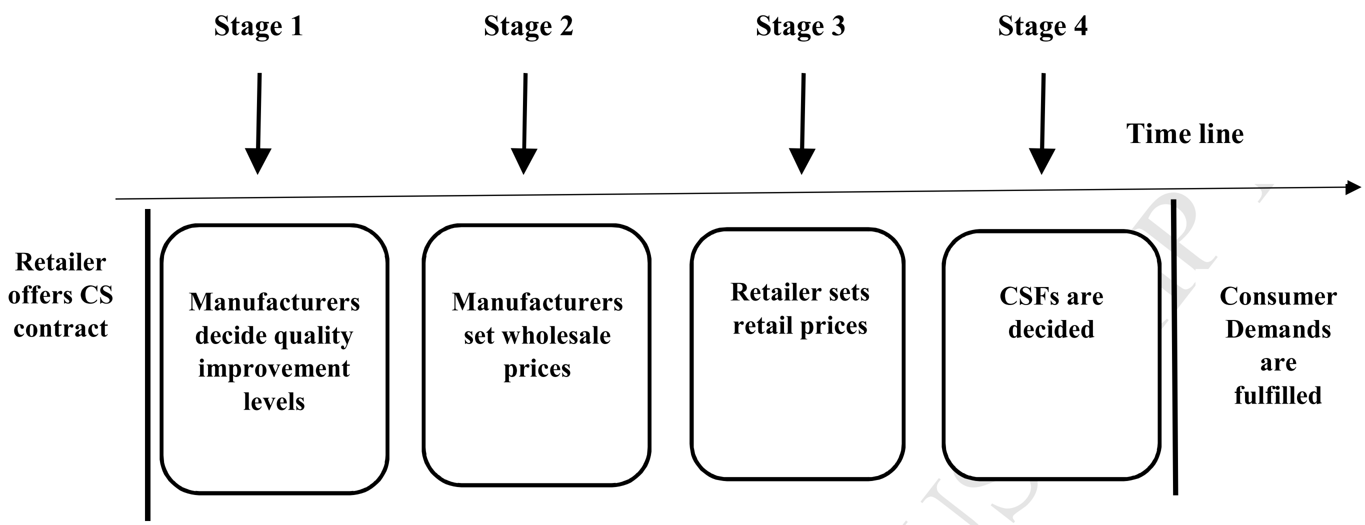
The proof is similar to that of Proposition 4. To prove the result we have to just show that decreases with the increase of η. Taking partial differentiation of ϕ^{SCS^*} with respectη we get

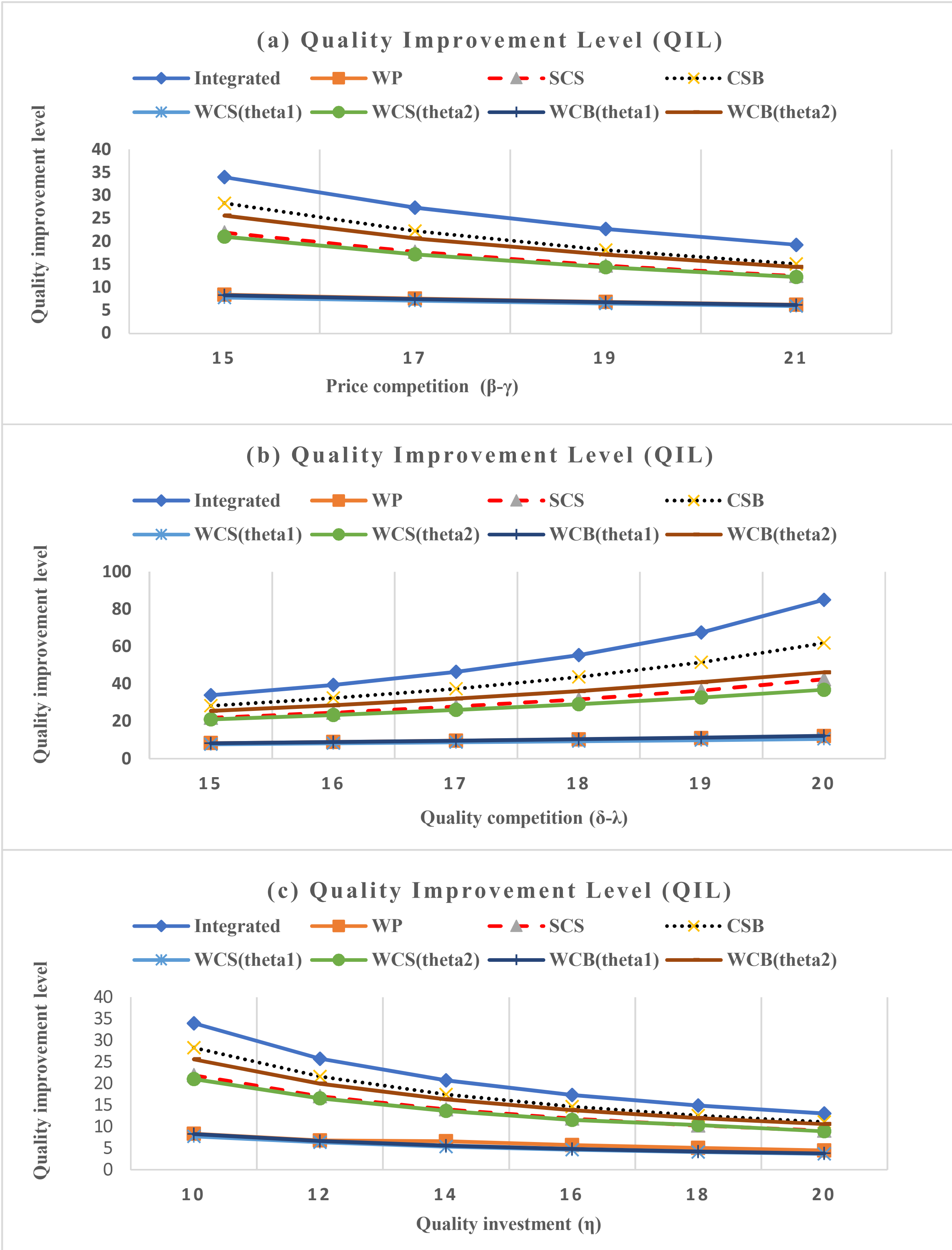
$$\frac{\partial \phi^{SCS^*}}{\partial \eta} = -\frac{\delta^2(\beta-\gamma)(\delta-\lambda)}{\eta^2\zeta}, \text{ where}$$

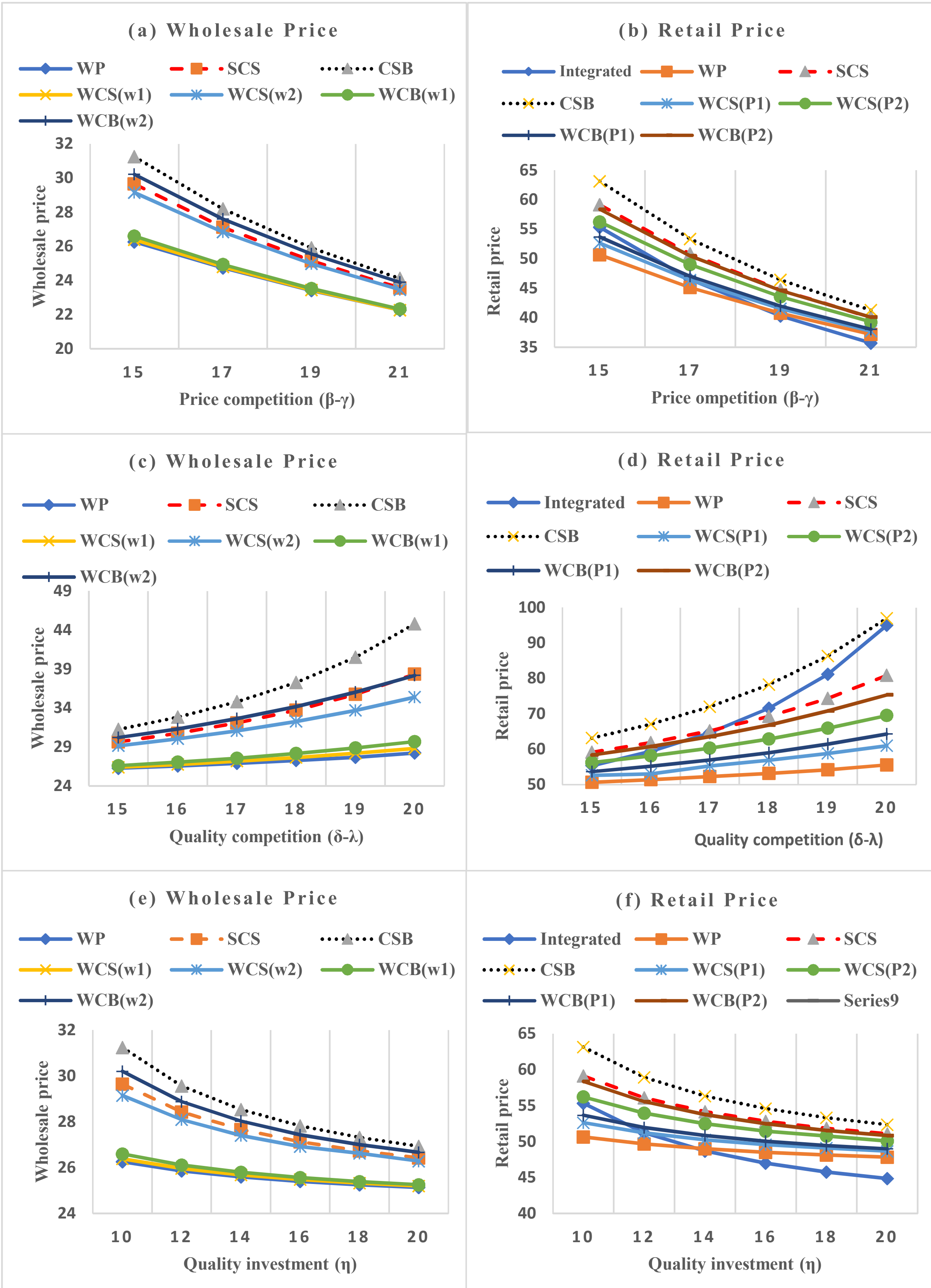
ζ is the same as defined in **Proof of Proposition 4**.

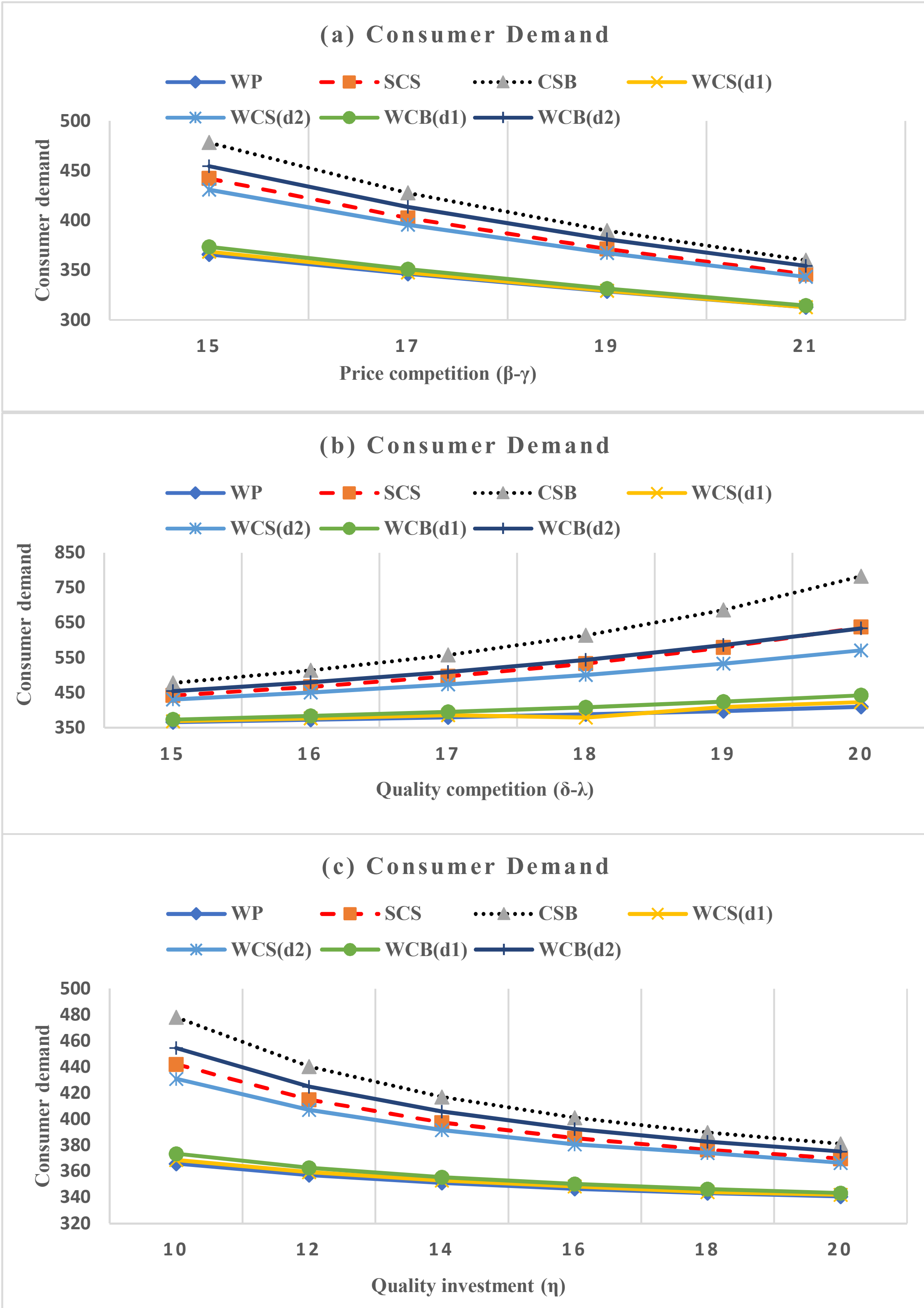
This completes the proof. ■

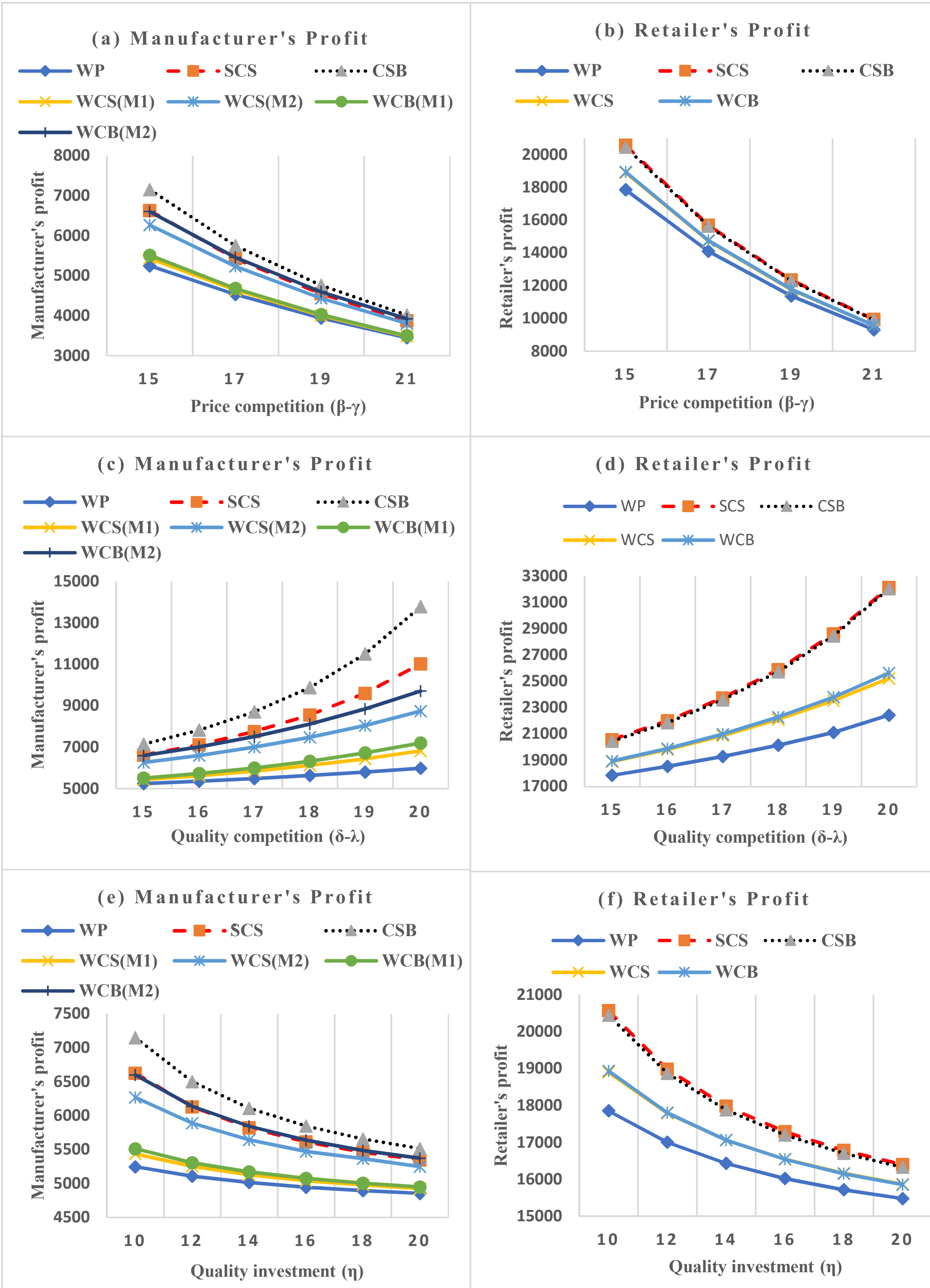
[11] It is to be noted that under symmetric assumptions, the condition **A2** dominates the condition **A3** always.

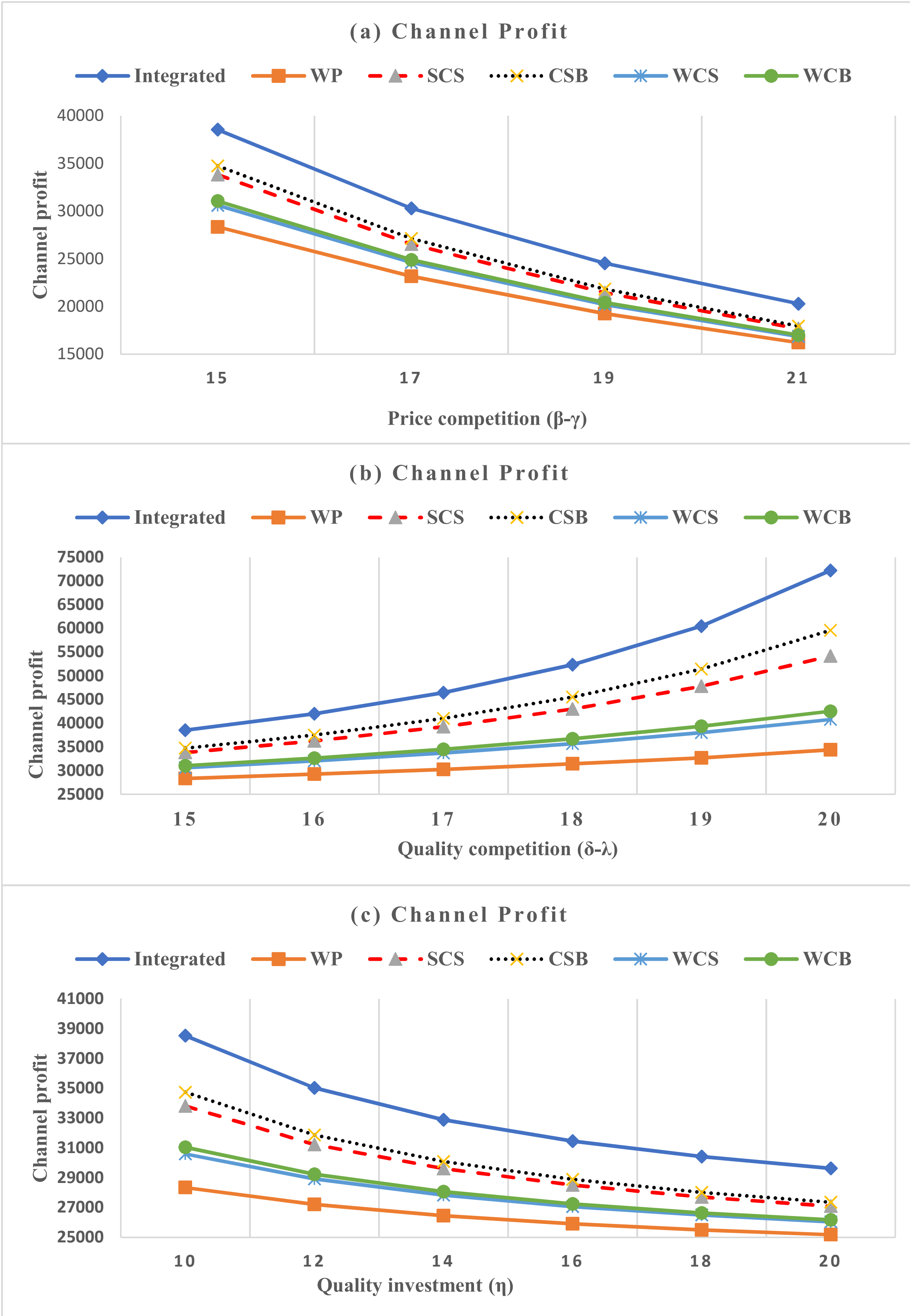

Stage 1
Stage 2
Stage 3
Stage 4
Time line
Retailer offers CS contract
Manufacturers decide quality improvement levels
Manufacturers set wholesale prices
Retailer sets retail prices
CSFs are decided
Consumer Demands are fulfilled


(a) Quality Improvement Level (QIL)
Integrated
WP
SCS
CSB
WCS(theta1)
WCS(theta2)
WCB(theta1)
WCB(theta2)
Quality improvement level
40
35
30
25
20
15
10
5
0
15
17
19
21
Price competition (β-γ)
(b) Quality Improvement Level (QIL)
Integrated
WP
SCS
CSB
WCS(theta1)
WCS(theta2)
WCB(theta1)
WCB(theta2)
Quality improvement level
100
80
60
40
20
0
15
16
17
18
19
20
Quality competition (δ-λ)
(c) Quality Improvement Level (QIL)
Integrated
WP
SCS
CSB
WCS(theta1)
WCS(theta2)
WCB(theta1)
WCB(theta2)
Quality improvement level
40
35
30
25
20
15
10
5
0
10
12
14
16
18
20
Quality investment (η)


(a) Wholesale Price
WP
SCS
CSB
WCS(w1)
WCS(w2)
WCB(w1)
WCB(w2)
Wholesale price
32
30
28
26
24
22
20
15
17
19
21
Price competition (β-γ)
(b) Retail Price
Integrated
WP
SCS
CSB
WCS(P1)
WCS(P2)
WCB(P1)
WCB(P2)
Retail price
65
60
55
50
45
40
35
15
17
19
21
Price ompetition (β-γ)
(c) Wholesale Price
WP
SCS
CSB
WCS(w1)
WCS(w2)
WCB(w1)
WCB(w2)
Wholesale price
49
44
39
34
29
24
15
16
17
18
19
20
Quality competition (δ-λ)
(d) Retail Price
Integrated
WP
SCS
CSB
WCS(P1)
WCS(P2)
WCB(P1)
WCB(P2)
Retail price
100
90
80
70
60
50
15
16
17
18
19
20
Quality competition (δ-λ)
(e) Wholesale Price
WP
SCS
CSB
WCS(w1)
WCS(w2)
WCB(w1)
WCB(w2)
Wholesale price
32
30
28
26
24
10
12
14
16
18
20
Quality investment (η)
(f) Retail Price
Integrated
WP
SCS
CSB
WCS(P1)
WCS(P2)
WCB(P1)
WCB(P2)
Series9
Retail price
65
60
55
50
45
40
10
12
14
16
18
20
Quality investment (η)


(a) Consumer Demand
WP
SCS
CSB
WCS(d1)
WCS(d2)
WCB(d1)
WCB(d2)
Consumer demand
500
450
400
350
300
15
17
19
21
Price competition (β-γ)
(b) Consumer Demand
WP
SCS
CSB
WCS(d1)
WCS(d2)
WCB(d1)
WCB(d2)
Consumer demand
850
750
650
550
450
350
15
16
17
18
19
20
Quality competition (δ-λ)
(c) Consumer Demand
WP
SCS
CSB
WCS(d1)
WCS(d2)
WCB(d1)
WCB(d2)
Consumer demand
500
480
460
440
420
400
380
360
340
320
10
12
14
16
18
20
Quality investment (η)


(a) Manufacturer's Profit
WP
SCS
CSB
WCS(M1)
WCS(M2)
WCB(M1)
WCB(M2)
Manufacturer's profit
8000
7000
6000
5000
4000
3000
15
17
19
21
Price competition (β-γ)
(b) Retailer's Profit
WP
SCS
CSB
WCS
WCB
Retailer's profit
20000
18000
16000
14000
12000
10000
8000
15
17
19
21
Price competition (β-γ)
(c) Manufacturer's Profit
WP
SCS
CSB
WCS(M1)
WCS(M2)
WCB(M1)
WCB(M2)
Manufacturer's profit
15000
13000
11000
9000
7000
5000
15
16
17
18
19
20
Quality competition (δ-λ)
(d) Retailer's Profit
WP
SCS
CSB
WCS
WCB
Retailer's profit
33000
31000
29000
27000
25000
23000
21000
19000
17000
15
16
17
18
19
20
Quality competition (δ-λ)
(e) Manufacturer's Profit
WP
SCS
CSB
WCS(M1)
WCS(M2)
WCB(M1)
WCB(M2)
Manufacturer's profit
7500
7000
6500
6000
5500
5000
4500
10
12
14
16
18
20
Quality investment (η)
(f) Retailer's Profit
WP
SCS
CSB
WCS
WCB
Retailer's profit
21000
20000
19000
18000
17000
16000
15000
10
12
14
16
18
20
Quality investment (η)


(a) Channel Profit
Integrated
WP
SCS
CSB
WCS
WCB
Channel profit
40000
35000
30000
25000
20000
15000
15
17
19
21
Price competition (β-γ)
(b) Channel Profit
Integrated
WP
SCS
CSB
WCS
WCB
Channel profit
75000
70000
65000
60000
55000
50000
45000
40000
35000
30000
25000
15
16
17
18
19
20
Quality competition (δ-λ)
(c) Channel Profit
Integrated
WP
SCS
CSB
WCS
WCB
Channel profit
41000
39000
37000
35000
33000
31000
29000
27000
25000
10
12
14
16
18
20
Quality investment (η)