Equity Return Forecasting Using Risk-Neutral Option-Implied Moments

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ABSTRACT

Equity Return Forecasting Using Risk-Neutral Option-Implied Moments

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Using OptionMetrics implied volatility surfaces for a sample of S&P 100 constituent stocks as of April 29, 2016, with historical data spanning from January 1996 to April 2016, this research provides additional empirical data regarding the informational content of option implied risk-neutral distributions. Two different methodologies are used to compute option-implied moments for the years 1996-2016: the first is based on Breeden-Litzenberger (1978) and the other on Bakshi, Kapadia and Madan (2003).

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LIST OF ABBREVIATIONS

B-L	Breeden-Litzenberger (1978)
BKM	Bakshi, Kapadia and Madan (2003)
B-S	Black-Scholes
BSIV	Black-Scholes Implied Volatility
CBOE	Chicago Board Options Exchange
CRR	Cox, Ross & Rubinstein Model
GEV	Generalized Extreme Value (distribution family)
ITM	In The Money
LEAPS	Long Term Equity Anticipation Securities
MFIV	Model-Free Implied Volatility
OTM	Out of The Money
RND	Risk-Neutral Density Function
RV	Realized Volatility
S&P	Standard & Poor's
SECID	Security ID
SD or σ	Standard Deviation
SR	Sharpe Ratio
TR	Total Return
WRDS	Wharton Research Data Services

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1. INTRODUCTION

The efficient market hypothesis theory states that asset prices fully reflect all available information. Based on this premise, we can infer market expectations by comparing the price of various financial instruments. A frequently used example is computing inflation expectations by comparing the prices of T-Bills and Treasury Inflation Protected Securities (TIPS). Both securities being identical with the exceptions of TIPS being protected for inflation, we can thus compute the market-expected term structure of inflation. Similarly, we can infer many types of market expectations using the vast amount of ever-increasing financial products created. Every financial product has several market expectations imbedded in their price and it is possible, through mathematical processes, to find what are the expectations attributed to each factor that affect the underlying asset price. By translating prices into concrete market expectation factors, a practitioner that has extensive fundamental knowledge of the topic can more easily determine and explain his reasoning regarding whether markets are pessimist, optimist or in line with what his own predictions would be. This difference between market and the expert's expectations can then be used to help determine whether an asset is undervalued or overvalued.

Among those financial products, derivatives markets provide forward-looking information that can be used by investors, regulators and other economic agents in their decision making. Over the past decades, an extensive literature has been written on topics such as whether option implied volatility provides an accurate forecast of realized volatility and on various methods that can be used to extract implied probability density functions from option prices. Although implied density functions have received much attention, there is fewer research on the application of option-implied information to asset allocation and ability to forecast future stock returns.

Previous papers including among others Bates (1991), Conrad et al. (2009), Rehman and Vilkov (2010) and DeMiguel et al. (2012), suggest option-implied moments might be useful in the prediction of future returns. However, the relationships are not clear as some of these papers report conflicting results. This paper also addresses forecasting future returns using the first four statistical moments of the option implied density function, namely the mean, standard deviation, skewness and kurtosis. It builds on previous literature by comparing results obtained using either a methodology based on Bakshi, Kapadia and Madan (2003) (BKM) or on Breeden-Litzenberger (1978) (B-L) whilst also testing results for a range of time to maturities, two time windows and various levels of interpolation granularity.

Using option prices for a sample of 64 US stocks with American exercise style, this research aims to provide additional empirical data regarding the informational content of option implied risk-neutral distributions. This will be done by extracting and comparing distribution statistics for the years 1996-2016. Results at various levels of interpolated option price data granularity are also compared both for older (prior to June 24th, 2003) and recent data (post June 24th, 2003). This analysis allows to have a better idea of whether there is an increase in result accuracy when using higher granularity and if it justifies the additional computational requirements when compared to using lower granularity. The comparison of interpolated vs non-interpolated data is partly motivated by the fact that exchange rules for strike intervals have changed over the years. While it was common to see very few options contracts traded for a given maturity in the past, in recent years the quantity of option contracts with different strike prices has considerably increased. With new standard listing procedures, strike intervals for equity options are listed in increments of $2.50 for strikes below $25, $5 increments for strikes ranging from $25 to $200 and strikes over $200 are listed in $10 increments. Nevertheless, many stocks are exempt from those standard listing procedures and are listed in $1 increments or even smaller since the $1 Strike pilot program was implemented on June 24th, 2003. With more observable option prices than there were when research in this area first began, it becomes relevant to re-evaluate the need for data interpolation due to the significant computational requirements when applying those additional steps in the calculation of option-implied moments. Observing the extent to which changes in options markets over time affect results obtained is also pertinent.

Primary findings from the thesis include the following: First, realized daily return moments tend not to be well predicted by their respective option-implied moment except for standard deviation which is relatively accurate. Furthermore, firms with higher option-implied volatility & skewness, positive momentum, lower market capitalization, dollar volume, price to cash flows and total debt to EBITDA tend to outperform their opposites. Second, the BKM approach does not appear to be positively impacted by data interpolation whereas the B-L approach benefits from interpolation. However, the interpolation benefits quickly decay when increasing the number of data points. Finally, coefficients for higher moments (skewness & kurtosis) tend to be less stable than mean and standard deviation coefficients through time.

2. LITERATURE REVIEW

The Black-Scholes-Merton model:

Assuming a constant volatility geometric Brownian motion, constant risk-free rates, lognormally distributed asset returns and frictionless markets, the famous Black-Scholes (B-S) formula used to value European options takes as input the time to expiration (T), strike price (K), underlying asset value (S) and the interest rate (r), together with the volatility to output the price. Every variable except the volatility and interest rate are known and either fixed or decreasing at a constant pace (time to maturity). Given interest rates have a relatively small effect on option premiums, it is frequently argued that the implied volatility is the most important input of an option's price. The Black-Scholes implied volatility (IV or BSIV) is the volatility of the underlying, which, when substituted into the B-S formula, gives a theoretical price equal to the market price. It therefore represents the market's view of volatility over the life of the option. When looking at options with the same underlying and time to expiration but different strike prices, we will most likely find that implied volatility follows a skewed shape commonly known as the volatility smile/smirk (depending on the skewness of the smile). Consequently, the assumption of lognormally distributed underlying returns is incorrect since if this assumption was accurate, implied volatility should be the same (flat) for such contracts. While many assumptions made in the B-S formula do not correctly depict reality, investors often choose to simply adjust implied volatility for different strike prices. Thus, one could say that they obtain the right option price using a wrong model. Nevertheless, this model is merely a translation tool between option price and its IV. Using these implied volatility values, it is possible to infer the market expected return probability density over the time to expiration of each group of options. If enough options are traded, these expected return distributions can be available from one week (for weekly options) up to a maximum of 39 months for Long Term Equity Anticipation Securities (LEAPS).

Accuracy of implied volatility:

Before testing strategies that use implied volatility as an input, we need to know the extent to which BSIV is an accurate forecast of realized volatility. On that note, several studies test the ability of option-implied volatility in forecasting the future volatility of the underlying asset.

For instance, Christensen and Prabhala (1998) test the relationship between implied and realized index option volatility for the S&P 100 Index (OEX) between 1983 and 1995. They find that implied volatility predicts future realized volatility better than past volatility. Similarly, Emmanuel and Oscar (2013) study both the S&P 500 and OMX30 from November 2007 to November 2013. Their results also support that implied volatility is an efficient estimator of realized volatility. Muzzioli (2010) also test this relation for the DAX index options market. The results found by his study suggest that both call and put implied volatility forecasts are unbiased and efficient. However, early studies mostly use at-the-money (ATM) implied volatility values for their tests. Alternatively, a study written by Jiang & Tian (2005) empirically test the informational efficiency of the option market using an implied volatility measure that is independent of option pricing models. This implied volatility measure, known as the model-free implied volatility, or MFIV, can be estimated by numerically integrating a cross-section of option prices over a big (approaching infinity) range of strike prices. In other words, as mentioned in Proposition 1 of their research paper, *"The integrated return variance between the current date 0 and a future date T is fully specified by the set of prices of call options expiring on date T"*. The output is a single implied volatility value that is said to encompass all the information available for that specific expiration.

Consistent with this idea, the authors find (with univariate and encompassing regressions) that the MFIV informational content encompasses both BSIV and historical volatility. They also show that both MFIV and BSIV are more accurate predictors when looking at shorter time to expiration values. For example, in Table 4 Panel C of their paper, the regressions that only includes the model-free beta has an adjusted R-squared of 0.75 for 30 days to expiration in contrast to 0.61 for 180 days in Table 6 Panel C. Finally, they also find that both MFIV and BSIV tend to be greater than realized volatility. On that note, using data from January 1996 to February 2003, a research made by Carr & Wu (2009) quantifies the variance risk premium on 35 stocks and 5 indexes. Comparing a synthetized variance swap rate to annualized 30-day realized variance, the authors demonstrate that the market variance risk premiums are significantly negative under both bearish and bullish market conditions. According to the authors, negative variance risk premiums mean that investors are willing to pay a premium to hedge increases in the return variance of the stock market (IV is greater than realized volatility over the long-run, which is consistent with other studies). The variance risk premium is also found to be time-varying and mostly unexplained by Fama-French risk factors (regression unadjusted r-squared values ranging from 0.005 to 0.30 for the different stocks and indexes tested) except for the market expected return and SMB (Small Minus Big) factors, both appearing to be negatively correlated with the variance risk premium.

A common use of implied volatility:

Also known as the investor's fear index, the VIX volatility index published by the Chicago Board Options Exchange (CBOE) is one of the most known and used example of option-implied information. Since 2003, the VIX relies on a model-free methodology that replicates variance swaps using options and is based on a paper published by Demeterfi et al. (1999). The VIX is computed using out-of-the-money and at-the-money call and put options and represents the 30-day option-implied volatility of the S&P500 where the value of the VIX is presented on an annual basis. For example, a VIX value of 20 would suggest that the market expects the next 30 days standard deviation of returns to be approximately equal to $\frac{20}{\sqrt{(365/30)}} = 5.73\%$.

Methodologies used to compute option-implied risk-neutral densities (RNDs):

The AIMR research foundation monograph written by Jackwerth (2004) provides a comprehensive review of the (pre-2004) literature on this topic, covering a wide variety of methodologies and potential applications. Notably, the author mentions that the easiest and most stable methods tend to be within the group of methods for curve-fitting the implied volatility smile. Because observable option prices span over a narrow range of strike prices with relatively large strike price intervals, those methods first try to fit the implied volatility smile with some functions such as polynomials or splines then transform back the IV values to option prices. This create a nearly continuous call (or put) price function.

In the creation of that smooth price function, there are three main aspects which tend to vary, namely the choice of independent variable, the interpolation and extrapolation method.

Observing that implied volatilities tend to be smoother than option prices, Shimko (1993) first proposed a transformation of option prices into implied volatilities. He then used a quadratic interpolation within the IV-Strike space and for probabilities beyond the range of traded strike prices, he simply pasted lognormal tails on the RND, thus the transition between the interpolated and extrapolated sections was not smooth. Although far from perfect, comparing approximation errors of the Shimko extrapolation to a simple truncation method using a parametric Gram-Charlier expansion model, Vial (2013) has shown that Shimko's extrapolation methodology provides more accurate results than a simple truncation. These results coincide with those obtained by Jiang & Tian (2005).

The most commonly used interpolation methods are polynomials and splines. Constructed of piecewise polynomials, splines have the particularity of requiring the interpolated function to pass through every original point and requiring continuity for N-1 order derivatives where N is the order of the spline (i.e. a natural cubic spline first and second derivatives will be continuous whereas the first, second and third derivatives will be continuous for a natural quartic spline.) Forcing the function to pass through all original data points can potentially cause issues. Since differentiation amplifies even the smallest irregularities, errors and frictions can have a considerable effect on the estimated RND and can result in unrealistic interpolation outputs such as negative values and spikes (Figlewski, 2010). Examples of errors and frictions include data errors, infrequent trading, high bid-ask bounces, limited quote precision, etc.

Clews, Parnigirtzoglou and Proudman (2000) used a natural smoothing cubic spline to interpolate the implied volatility curves. By introducing a smoothing parameter with a value ranging from 0 to 1, smoothing splines allow a trade-off between curve smoothness and how closely they fit the observed data points. If this parameter is set to 1, it forces the interpolation to pass through all original points whereas if it is set below 1, it is progressively allowed to deviate from those points. Thus, smoothing splines are particularly useful when used to interpolate noisy data.

A smoothing spline minimizes over all functions s:

$$p \sum_i w_i \big(y_i - s(x_i)\big)^2 + (1-p) \int \left(\frac{d^2 s}{dx^2}\right)^2 dx$$

Where p is the smoothing parameter with value between 0 and 1, w_i are specified weights (if the data is to be weighted), x_i and y_i where i = 1,…, n is the set of observations to interpolate.

Nevertheless, implied moments are less sensitive to measurement errors than densities, meaning that a spline interpolation can still provide very accurate results (Vial, 2013).

Whether using polynomials, natural or smoothing splines, the exponent order level is also of critical importance. High order equations sometime exhibit an oscillatory behavior commonly known as Runge's phenomenon. Spline interpolation is often preferred to polynomial interpolation since it yields similar results at low exponent degree while avoiding this oscillatory behavior at higher degrees. Natural cubic splines also require the second derivative of each polynomial to equal zero at the endpoint knots, thus approaching a linear form at the tails.

Figlewski (2010) suggests the use of fourth-order polynomial to avoid negative values in the derived RND. He argues that negative values that can occur when using third-order splines are due to the discontinuous third derivative of the implied volatility curve translating into a discontinuous first derivative in the RND (after differentiating twice using the B-L 1978 formula).

Several papers including Bates (1991), Campa, Chang and Reider (1998) and Jiang and Tian (2005) use a cubic spline to interpolate IV across strike prices. The interpolated IVs are then used to compute corresponding option prices using the B-S model. Although this methodology uses the B-S model, it does not assume the validity of the model or log-normality of the underlying price process, but rather simply uses it as a tool for mapping option prices.

Some also attempt to extrapolate the volatility smile outside of observable values. The methodologies used greatly differ depending on the research paper. In some case, the authors opt for a truncation, therefore ignoring strike prices beyond the truncation interval, in others they assume that the last known IV on each end is used for extrapolation while in other papers they assume a structure for IV beyond each boundary. Such tail extrapolations may be performed with caution only and one must keep in mind that values outside of the observable range might greatly differ from reality. An example of truncation is done in Jiang & Tian (2005) study of model-free implied volatility. The authors find that the truncation error is relatively small when truncation points are more than 2 standard deviations (σ) from the current futures price. However, in several cases, the range of available strike prices do not even span up to 2σ. In those cases, the use of extrapolation is necessary. In table 1 of their paper, Jiang & Tian (2005) also show using a stochastic volatility and random jump model that the approximation error created from extrapolating by assuming a flat implied volatility function beyond the truncation point is smaller than the truncation error occurring when a simple truncation methodology is used. In addition, they find that the approximation error is greater for options with longer maturities and that approximation error decreases when increasing the range for strike prices from [$0.9S_0$, $1.1S_0$] to [$0.8S_0$, $1.2S_0$] and this effect becomes insignificant after reaching [$0.7S_0$, $1.3S_0$]. Nevertheless, those results may only be applicable to model-free implied volatility and not higher moments such as skewness and kurtosis which are even more sensitive to changes in the tails of the distribution. On that note, Chang, Christoffersen, Jacobs, & Vainberg (2012) tested the accuracy of moments computations by conducting Monte Carlo experiments. The authors look at volatility and skewness approximation error for three different biases, namely discretization of strike price (using interpolation or only discrete strike prices), truncation of the integration domain (range for strike prices) and asymmetry of the integration domain. Results from their experiments suggest that the integration method (cubic spline interpolation and extrapolation with constant implied volatility beyond end-points) used by Jiang & Tian (2005) does a good job at mitigating approximation error. Since implied volatilities are derived using the B-S framework, extrapolating the volatility curve horizontally forces the tails to be lognormal. (Figlewski, 2010; Kostakis, Panigirtzoglou & Skiadopoulos, 2011) Thus, this means that when Shimko (1993) appended log-normal tails to the RND, he was essentially doing a flat IV extrapolation. In contrast, Figlewski (2010) proposed using the Generalized Extreme Value (GEV) to extrapolate the tails. The GEV distribution is built upon the Fisher-Tippett theorem which states that the maximum within samples might converge. The GEV distribution has 3 parameters that require to be set: the first two (μ and σ) respectively control the location and scale of the distribution while the shape parameter (ξ) determines the tail behavior of the distribution. When this parameter takes values smaller, equal or greater than zero, the resulting distributions are named as "Weibull, Gumbel or Fréchet families", respectively. The shape parameter allows to control whether the GEV distribution has smaller, equal or greater tails than the normal distribution. To find which value these parameters should take, the author uses standard optimization procedures with respect to 3 conditions. First, he constraint the total tail probabilities such that the total tail probability for the RND is equal to that of the GEV. Second and third, he selects parameters that will allow the GEV shape density to have the same shape as the RND at the point where the two overlaps and at a more extreme point. While the logic behind the second condition (same value where GEV and RND overlap) is easy to understand, the first condition and the third conditions may not be appropriate in all circumstances. Nevertheless, since extreme returns are rarely observed, the small number of extreme value data makes it more difficult to determine the appropriate shape that should be applied to the tails. This directly impacts, estimated distribution higher moments which tend to be more affected by changes in extrapolation methods in comparison to lower moments (mean and variance) which are not affected in the same magnitude.

Next, once interpolated values have been computed, a function of option prices across strike prices is fitted, thus finding the theoretical value of options for strike prices which are not traded on the market. This methodology then applies the Breeden-Litzenberger (1978) (B-L) result that the implied probabilities can be inferred from the second partial derivative of the European call price function with respect to the strike price: $\frac{\delta^2 C(F_t,K,\tau)}{(\delta K)^2} = e^{-r\tau} f(F_t)$

Where C is the call function, K is the option strike price, r is the risk-free rate, F_t is the value of the underlying future at time t and $f(F_t)$ is the probability density function which describes the possible outcomes for the underlying futures at time t. Put options can also be used.

If the implied volatility smile is relatively smooth, the computed risk-neutral probability distribution will be positive and arbitrage-free. One advantage of the Breeden and Litzenberger procedure is that it is said to be model-free. A variation of this method has been introduced by Malz (1997). By fitting implied volatility across deltas instead of strike price, the author limited the bounds to [-1,0] for put options and [0,1] for call options instead of a range of [0, ∞) when using strike price. However, translating the implied volatility vs delta curve back to implied volatility vs strike price can be a challenge, especially when working with American options which can have several discrete dividend payments until time to expiration.[1]

[1] The derivation of the European-style option with continuous yield dividends formula that can be used to compute strike prices for given delta values is shown in Appendix A. I attempted converting delta values to strike prices using sample data S&P European options and comparing the values obtained to observed market data. Unfortunately, computed strike prices were not accurate for deep in-the-money or deep out-of-the-money options. Furthermore, this formula would need to be adjusted before using it for American options with discrete cash dividend payments. Typically, researchers that used delta interpolation did so using European options.

Non-parametric methods do not assume the data follow a specific distribution and thus work best when there is plenty of data point available whereas parametric models are most efficient in scarce data environment, assuming reasonable parameter assumptions are used. In a paper published by Bakshi, Kapadia and Madan in 2003 (BKM), the authors developed one of the most widely used model-free measure of RND moments. The BKM approach to compute RND moments uses OTM calls and puts to recover the volatility, cubic and quartic contract prices. The formulas apply a weighting structure that is much higher for deep OTM options relative to near-the-money OTM options. This aspect is of crucial importance as it also serves as an argument supporting the validity of the model when used with American options. The authors argue that their model generalizes to American options since OTM options have negligible early exercise premiums and that their model applies greater weights to deep OTM options relative to near-the-money OTM options, knowing that the further out of the money an option is, the lower the probability of early exercise and thus the lower the exercise premium. In addition, several researches including Bliss and Panigirtzoglou (2000) and De Vincent-Humphreys & Puigvert Gutierrez (2010), just to name a few, show that out-of-the-money options tend to be more liquid than in-the-money options. In table 2, p.120 of Bakshi, Kapadia and Madan (2003) the authors provide a comparison of Black-Sholes implied volatilities versus American option implied volatilities (estimated by binomial tree model and using Richardson extrapolation to speed-up the price convergence) for a sample of 10 stocks and the S&P 100 index for the year 1995. Results show that American IV tend to be smaller than B-S IV for OTM puts whereas they tend to be mostly identical for OTM calls. Generally, there is greater difference between American IV and B-S IV for options with greater time to expiration and which are near-the-money rather than deep OTM. However, the authors point out that this difference is within the bid-ask spread and thus considered negligible.

When implementing the BKM approach (just like B-L), the literature is divided when it comes to suggesting a methodology to be used when interpolating and extrapolating option prices across and beyond observable market prices. This step is done so that a greater number of option prices can be used when computing the integral of weighted option prices as seen in formulas for volatility, cubic and quartic contract prices shown in Appendix B.

Using the S&P 500 index for the period of January 1996 to September 2009, Christoffersen et al. (2012) look at 30-day log-return option-implied moments (Volatility, Skewness and Kurtosis) using the BKM approach. Over the studied period, they find that there is a correlation value of 0.997 between the BKM volatility and the VIX index. The estimate of skewness is negative for the entire period, varying between -3 and 0. They also find positive excess kurtosis (kurtosis >3). The overall probability in the tails are thus greater than the normal distribution.

First published in February 2011, the CBOE S&P 500 Skew index is a good example of how the BKM 2003 methodology has been used outside of research papers. The skew index can theoretically be translated to a risk-adjusted probability that the 30-day S&P 500 log-return falls two or three σ below the mean and the VIX can be used as an indicator for the magnitude of σ. (CBOE, 2010)

In Appendix C, I performed and show results of an analysis of ex-post S&P500 30-days and 365-days return when the Skew index is at different levels. Results suggest the skew does not provide an accurate estimation that risk-adjusted probability that the 30-day S&P 500 log-return falls two or three σ below the mean as first suggested by the CBOE skew white paper.

American vs. European options:

So far, both the B-L and the BKM methods are supposed to be computed using European options. However, as mentioned earlier, it is possible to input OTM American options instead since the early exercise premium is small (implied volatility differences are minimal). This allows the estimation of option-implied moments for a much wider class of underlying assets. Tian (2011) tests three alternative methods to extract European option prices from American option prices. Those are the iterative implied binomial tree approach (iIB), the Cox, Ross & Rubinstein binomial tree model (CRR) and the analytical approximation to early exercise premium (AA). While the author favors the iIB approach when using both ITM and OTM options, the CRR still provides accurate results for OTM options and unlike the AA approach, the CRR makes no model assumptions. The implied volatility of an option is found by running the CRR model iteratively with new volatility values until the model and the market price converge. It is also important to note that the CRR model can be adapted to account for discrete dividend payments. As suggested by Black (1975), this can be done by reducing the initial price of the underlying asset by the present value of the expected dividends during the remaining life of the option.

Forecasting future asset returns using option-implied moments:

Several studies investigate whether implied moments can predict realized stock returns or other variables. Bates (1991) is one of the first to study the relation between option prices and expectations of future market crashes. He examines S&P 500 futures option prices during 1985-1987 and concludes that the unusually negative skewness in the option-implied distribution between October 1986 and August 1987 was a sign that the options market expected a crash. An example of application to other variables is a paper from Doran et al. (2008) which investigates the information content of option prices and volatility surrounding analyst forecast revisions. While analyst recommendation changes have been shown to affect ex-post stock prices, the authors look at whether changes in analyst recommendations (for which the date and direction is unknown) happen following changes in implied volatility. In summary, the results suggest information in option market prices and implied volatility leads analyst recommendation changes. In another study, DeMiguel et al. (2012) find that the volatility risk premium and option-implied skewness can be used to improve the out-of-sample performance of portfolios in terms of Sharpe ratio and certainty-equivalent return. On a related note, Conrad et al. (2009) and Rehman and Vilkov (2010) report conflicting results on the relationship between implied skewness (computed using the BKM 2003 methodology) and stock returns. Rehman and Vilkov (2010) find that higher skewness values are associated with higher stock returns whereas Conrad et al. (2009) findings suggest an inverse relationship. In theory, the popularity of lotteries (high chance of small loss, low chance of large gain) suggest that many individuals prefer a positively skewed return distribution to a negatively skewed one. Nevertheless, the two research papers do not use the same data sample/filters and identical methodologies. This suggest that the above relationship is unstable and affected by (minor) changes in data and/or methodology. Finally, another research made by Kostakis, Panigirtzoglou & Skiadopoulos (2011) find that portfolio strategies that use risk-adjusted (using either negative exponential or power utility functions) option-implied distributions for asset allocation have greater Sharpe ratios compared to portfolios that use historical distributions.

Risk-neutral vs. real world probability distributions:

After computing the implied distribution, the impact that risk-aversion have on risk-neutral distributions remains. When interpreting risk-neutral probabilities, we must keep in mind that the actual and risk-neutral distributions will be identical only if the economy wide utility function is risk neutral. An economy wide utility function can be interpreted as a weighted average of all individual investor's utility functions. If an economy was risk neutral, investors would be indifferent between receiving a sure payment or receiving a gamble with identical expected value. However, this does not accurately depict the world we live in. Utility theory typically assumes that individuals are risk-averse. Thus, their utility functions are concave and exhibit diminishing marginal utility. Building upon this, Kahneman & Tversky (1979) developed the prospect theory which states that in addition to being risk-averse in gains, individuals are also loss averse, resulting in a convex (risk-seeking) function in losses and thus an S-shaped utility function. Shefrin and Statman (1985) related the disposition effect (holding an investment that have experienced losses for too long and selling a winning investment too quickly) to the prospect theory.

As previously mentioned, an important limitation of RNDs is the fact that these do not reflect the true distributions and there is a distinct gap between the two (both in location and shape of the distributions). Nevertheless, it is plausible that although option-implied distributions or moments are biased predictors of physical ones, this bias may be small. It is therefore possible that (risk-neutral) option-based forecasts contain information not captured in historical forecasts alone. However, adjusting those risk-neutral distributions may provide information that is more reliable and can be interpreted differently.

There are two main methodologies used by research papers to convert risk-neutral implied moments and distributions to their physical equivalent. The first transforms the Q-distribution using a utility function which is assumed to represent the overall behavior of options market participants. One issue is that market participants may not all have the same utility functions (different wealth level and personal characteristics). Consequently, since trading volume for options is limited, an individual or institution might be responsible for a large portion of that contract's volume (potentially more than 50%). Thus, adjusting the distribution using an economy-wide utility function in a similar fashion for every strike price could be affected by differences in utility functions across options market participants. Although its accuracy is also limited, the second method is simpler as it adjusts the Q-distribution using historical estimates of the risk-premium between the risk-neutral and physical moments. DeMiguel et al. (2012) is an example of paper which assumes the variance risk-premium can be approximated by the proportion of MFIV and realized volatility. Thus, this methodology inherently assumes that risk-premiums are subject to autocorrelation and that the next period volatility risk premium can be well approximated by the previous period (historical) volatility risk premium. In contrast, Carr & Wu (2009) research on variance risk premiums show (in Table 3 of their paper) that nonoverlapping 30-day variance risk-premiums have low autocorrelation estimates and they vary by asset class and through time. This suggests that this month's variance risk-premium can not be used to approximate next month risk premium. To correct for this issue, DeMiguel et al. (2012) opt for using a rolling window instead of a nonoverlapping window. The obtained results closely track realised volatility. Nevertheless, Jackwerth (2004) concludes that the information drawn from the center of risk-neutral distributions with sufficient option prices observed (minimum 5) is quite reliable and can particularly be useful for relative comparison of either one security through time or multiple securities with similar risk-aversion profiles.

Finally, some papers suggest that risk-premiums (which indirectly reflect risk aversion and investor sentiment) are useful for forecasting returns. For example, DeMiguel et al. (2012) find that stocks with a high volatility risk premium tend to outperform those with low volatility risk premium. Recent research investigates topics such as high-order option-implied moments risk-premiums for individual securities and market indexes.

3. DATA

To compute moments of the option-implied distributions, we need to have as many options for a given expiration as we can get. The Volatility Surface file provided by OptionMetrics contains the standardized option prices and interpolated volatility surface for each security. Those are calculated using a kernel smoothing technique as described in Appendix D.

The result is a dataset that provides a smoothed implied-volatility surface for a range of standardized expirations of 30, 60, 91, 122, 152, 182, 273, 365, 547 and 730 calendar days, at deltas ranging from 0.20 to 0.80 for calls and -0.20 to -0.80 for puts. This give a total of 13 implied volatilities which, are then used to calculate the moments of the risk-neutral distribution. The fact that there is data for a constant and reasonable large amount of option contracts for all chosen securities is a major advantage. Thus, the calculations in this analysis are made using the volatility surface file rather than using raw option prices data. Consequently, the implied volatility surface data spanning from January 2nd, 1996 until April 30th, 2016 for each security tested is retrieved from WRDS Ivy DB US OptionMetrics[2]. The securities tested are S&P 100 index constituents as of April 29, 2016 for which data was available for the entire period. S&P 100 index constituent's tickers were obtained from iShares S&P 100 ETF holdings historical data[3]. The choice of S&P 100 as a sample appeared to be appropriate since, to be included in the index, firms must have

[2]The OptionsMetrics Ivy DB US File and Data Reference Manual Version 3.1 Rev.1/17/2017 provides detailed information regarding the files and methodology used by OptionMetrics.
OptionMetrics is one of the most used source of option information for empirical analysis. This database can be used for a wide variety of topics which require historical options data. Some examples include Ordu & Schweizer (2015) which use this data in the context of event studies around M&A activity while a second paper published by the same authors compute put-call parity deviations for a sample of US and Chinese companies with the ultimate objective of determining whether informed traders prefer trading under less restrictive regulatory regimes. Similarly, Cumming, Johanning, Ordu and Schweizer (2017) use standardized options data to compare realized and option-implied volatility around seasoned equity offerings and use observed results to test the performance of straddles around those events.

[3] S&P 100 constituents as of April 29, 2016 were obtained in the "holdings" section of the following website: https://www.ishares.com/us/products/239723/ishares-sp-100-etf

listed options and are among the larger and more stable companies in the S&P 500. In addition, sector balance is considered in the selection of companies for the S&P 100. Using OptionMetrics – Volatility Surface Code Lookup tool, SECID numbers were retrieved for each ticker. Only companies which had data spanning the entire period were kept. This resulted in an ending sample of 64 different companies, a sample size that is large enough, yet still manageable in terms of computational requirements. A list of these companies can be found in Table 1 of the Appendix.

When working with data on option prices and the volatility surface, for several calculations we also need historical close prices for the stocks, dividend distributions and the riskfree rate for the expiration of each option. Those are all retrieved from the OptionMetrics IvyDB US Security_Price, Distribution and Zero_Curve files, respectively. The Security_Price file provides the historical closing prices, holding period daily return and cumulative total return factor. The Distribution file provides information such as historical ex-distribution dates and dollar amounts. Finally, the Zero_Curve file contains historical zero-coupon interest rate curves. The Zero_Curve file is the same for every security. A file named "*Rinterp.mat*" is created from the Zero_Curve file. The *Rinterp.mat* file uses a linear interpolation with clamped ends to interpolate the zero-curve data in such a way that it increases granularity of continuously-compounded zero-coupon interest rates. As a result, zero-coupon interest rates with maturities ranging from 1 to 3659 days are generated. A second interpolation is performed using the last known value (closest previous day available) to generate interest rates values for days which were not already in the original data. This allows to fill gaps which would have caused problems when searching for interest rates values on specific days for which no data was originally available. Consequently, we now have interest rate values for any day of the year (including weekends and holidays) between January 2nd, 1996 and April 29th, 2016. The *Rinterp.mat* file takes the form of a 3659 x 7424 matrix (expiration date x date). A file named "*groupsR.mat*" which contain the 7424 respective dates is also created for logical indexing purpose. Next, a variety of financial ratios are obtained from "Financial Ratios Firm Level by WRDS". The variable "Public_Date" is used as logical indexing to make sure only information that is publicly available is used. This effectively puts a two months delay between market information and financial ratios. In addition, various market data factors are obtained from the OptionMetrics Security_Price files. Those factors include dollar volume, market capitalization and momentum.

Finally, for illustration purposes, Table 1 below provides an overview of sample sizes for each option days to expiration and time window groups.

Table 1: Sample sizes

Days to expiration	**Sample size** **[Jan. 2nd, 1996 to June 23rd, 2003]**	**Sample size** **[June 24th, 2003 to April 30th, 2016]**
30 days	1725	3023
60 days	1718	2950
91 days	1718	2926
122days	1715	2906
152days	1715	2906
182days	1713	2894
273days	1703	2873
365days	1700	2830
547days	1672	2780
730days	1584	2734

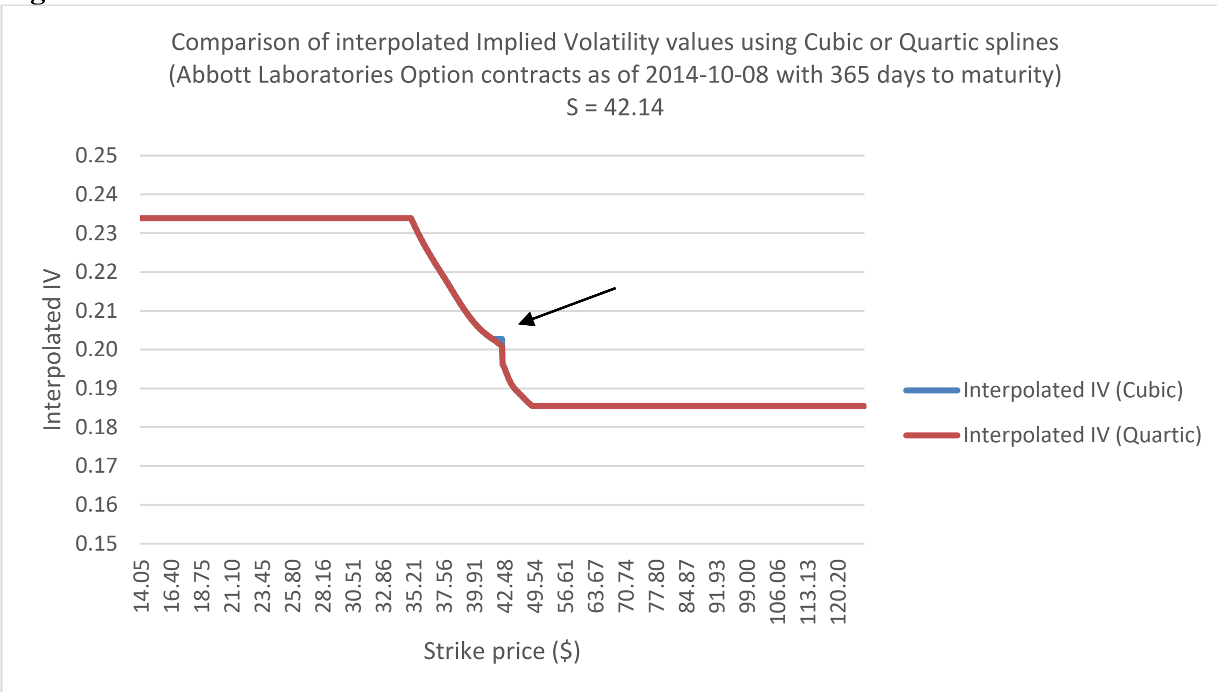
The table above shows the sample size for both time samples in the study and further divided by days to expiration groups.

4. RESEARCH METHODOLOGY

First, the volatility surface data is organized by removing rows with missing values. For example, if a firm has no implied volatility value (impl_volatility was set to -99.99 in the database), this data along with data of the same date and time to expiration for other firms is removed. The same methodology is used to remove other missing values such as impl_volatility = NaN or impl_premium = -99.99. Two additional columns that provide the option's stock prices and risk-free rates are also added to the volatility surface data file. Volatility surface, stock prices and dividends data files for each individual firm are then saved separately and organized by SECID number. After loading data files, for each distinct security, we perform the following steps given that there is a limited amount of data points (strike prices) for each group. First, the Implied Volatility variable is transformed via natural logarithm using the *log(X)* function in MATLAB. The log transformation allows to avoid an issue with cubic splines which are known for overshooting at times and being unstable in certain conditions. This further ensures that no interpolated values are negative. Next, the transformed volatility smile (Strike vs BSIV space) is interpolated using quartic splines and extrapolated using the respective boundary value at the end of the interval over a total of either 100, 300 or 1000 ((502-2) *2) implied volatility values for evenly spaced strike prices points from moneyness range of 1 to 3 for calls and 1/3 to 0.999 for puts. These moneyness ranges thus exclude ITM options.

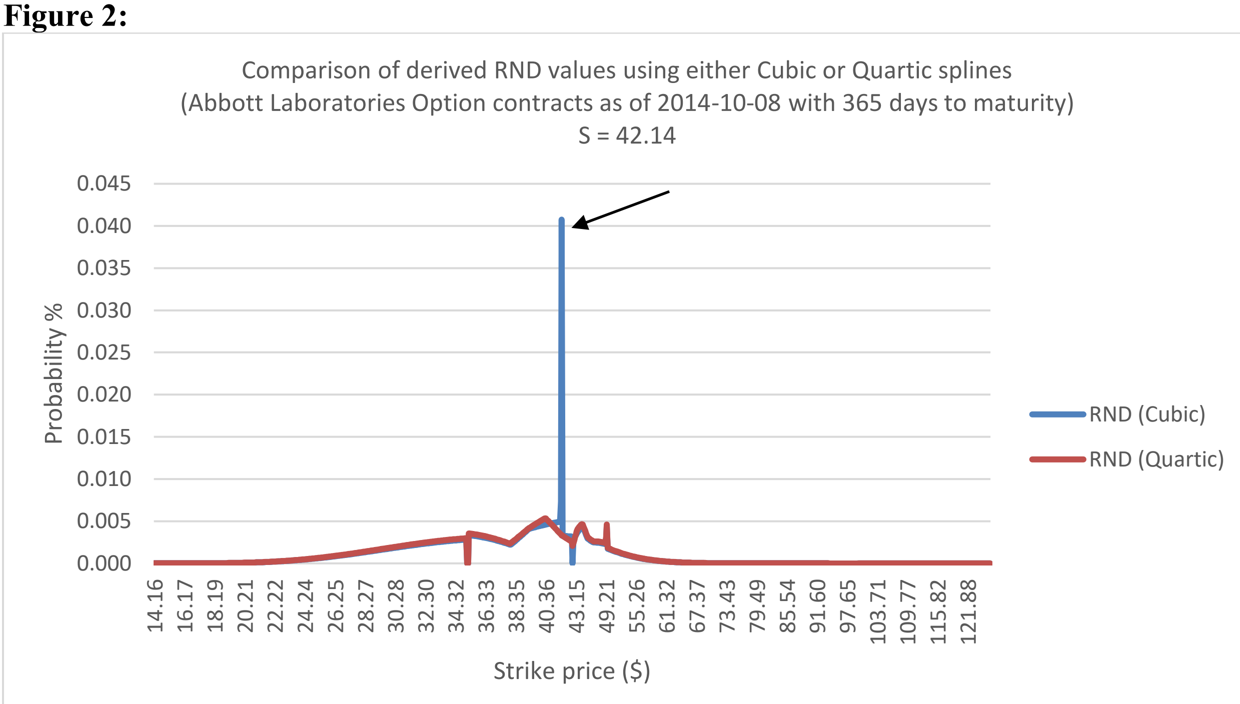
The following figures show the differences between different methodologies tested and the retained methodology (Quartic interpolation with outlier points removal using Hampel).

Figure 1:



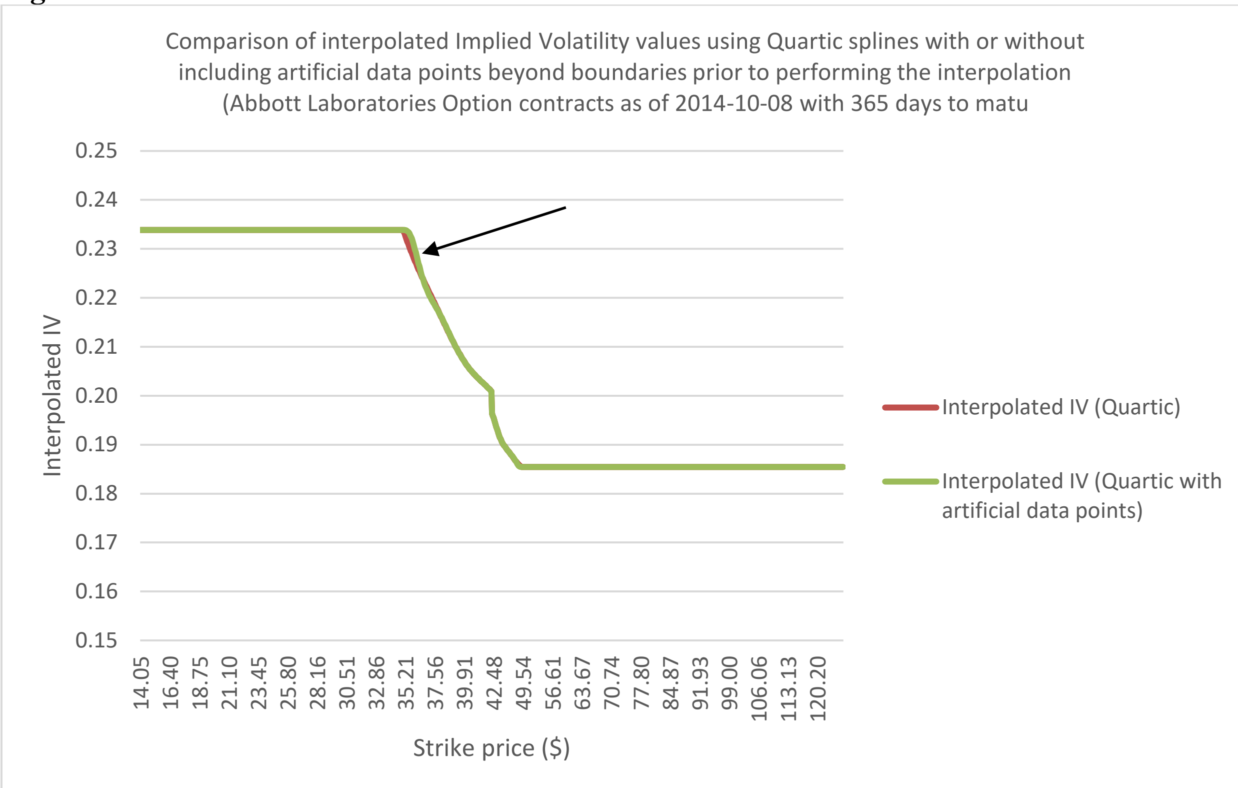
Although barely visible, using a Cubic spline interpolation creates a "plateau" between the strike values of 41.37 and 42.10. This plateau later causes issues when computing the RND since first and second derivative of option prices using those IV values cannot be computed.

Figure 2:



As can be seen in figure 2, the plateau mentioned earlier creates a spike at the second data point following the beginning of the plateau. (spike at 41.43, which is the data point immediately following 41.37.) The quartic spline interpolation does not appear to have this issue. Additionally, at the point K = 42.48, the cubic spline creates a negative interpolation value (which is then changed to 0). This does not occur with the quartic interpolation. An issue remains with both types of interpolation that is at the points K = 35.10 and 35.16, the interpolated values are negative for both types of interpolation used. This is caused by the extrapolation technique used that simply put constant IV values beyond minimum and maximum observable strike prices. This issue may be solved by appending a minimum of N-1 (2+ for cubic, 3+ for quartic) artificial data points beyond boundaries before performing the interpolation, thus allowing for a smoother transition between interpolated and extrapolated values. Figure 3 show the interpolated IV and derived RND with artificial data points beyond boundaries included in the data prior to performing the quartic spline interpolation. Unfortunately, as can be seen in figure 4, although the interpolated IV appears to have smoother transition between interpolated and extrapolated values on the left side, this change worsens the resulting RND by creating further negative values and a spike. Thus, it appears that although imperfect, it is preferable only to include observable data when performing the quartic spline interpolation and to append IV values beyond the minimum and maximum observable values after.

Figure 3:



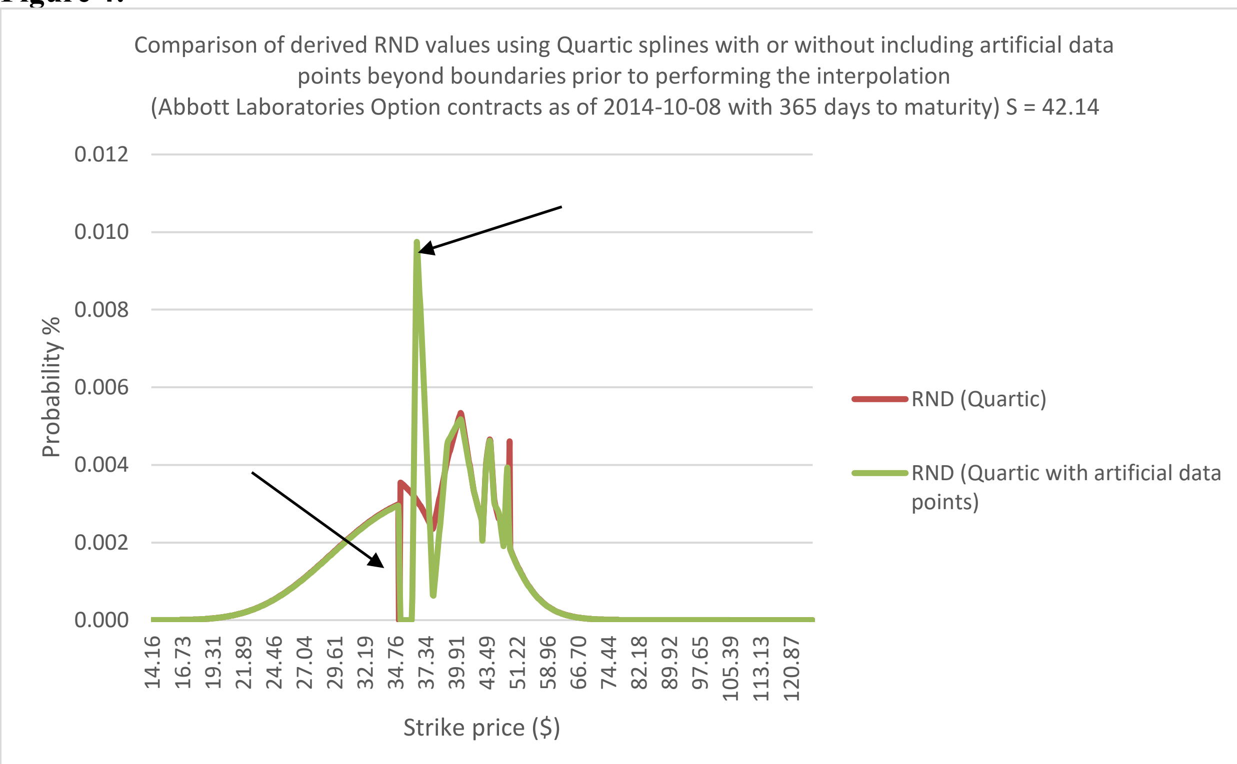
Using the interpolated implied volatility data along with strike price, security prices, time to expiry, interest rate and dividends/distributions values, theoretical call and put option prices for each interpolated value are computed using the Black-Scholes model. This generates series of European option prices based on the original American option price parameters. Using those option prices, option-implied moments are computed for each security using two different methods.

For the first method (B-L), implied moments are estimated by deriving the RND and transforming the stock strike prices to log-returns. Estimating the RND is done using the Breeden-Litzenberger (1978) implied density formula:

$$\frac{\delta^2 C(F_t, K, \tau)}{(\delta K)^2} = e^{-r\tau} f(F_t)$$

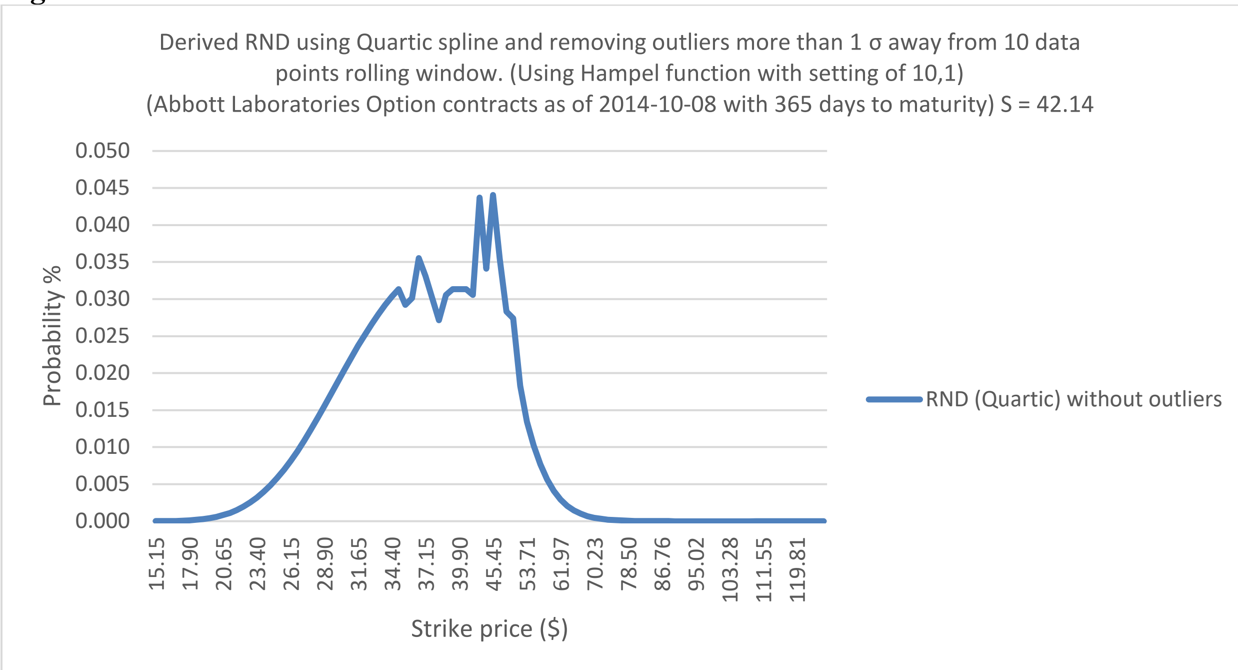
After computing, distributions and log-returns call/put data are concatenated horizontally as follow [Call Put]. The computed distributions are then normalized the so that the sum of the values sum to 1. As seen in figure 4, these values provide probability estimations for a range of strike prices (strike can also be translated into log-return %).

Figure 4:



Finally, outliers are removed using the "Hampel" function in MATLAB. The input arguments used determine outliers by finding valued that are more than one standard deviation away from a 10 data points rolling window. Although still imperfect, the output of this procedure seen in Figure 5 is considerably smoother.

Figure 5:



In the reminder of this paper, I chose to input market-observable data when performing the Quartic spline interpolation, append flat IV tails and smooth the output data by removing outliers. Using this output, I then compute descriptive statistics for each of the implied risk-neutral distributions obtained. Among others, I compute the mean, variance, skewness and kurtosis of the distributions. The following probability distribution descriptive statistics formulas are used:

$$\mu = \sum_{i=1}^{n} x_i \cdot p_i$$

$$\sigma = \sqrt{\sum_{i=1}^{n} p_i \cdot (x_i - \mu)^2}$$

$$Skewness = \sum_{i=1}^{n} p_i \cdot \left(\frac{x_i - \mu}{\sigma}\right)^3$$

$$Kurtosis = \sum_{i=1}^{n} p_i \cdot \left(\frac{x_i - \mu}{\sigma}\right)^4$$

Where x_i represents the strike price, p_i is the estimated probability and n is the number of data points.

For the second method, the model-free moments are computed using the technique proposed by Bakshi, Kapadia, and Madan (2003) which formulas are shown in Appendix B.

Realized daily log-return moments are also computed in addition to the realized total returns. Equipped with the above probability density descriptive statistics, realized daily return descriptive statistics and realized total return, simple regressions are then performed to look at the predictive performance of option-implied moments. Adjusted r-squared, coefficient estimates and p-values for these regressions are organized so that summary statistics (of the 64 firms) for these 3 variables can be computed at each different time to expiration and time windows. Each of the two methods and two-time periods computed first four moments are then regressed linearly against the equivalent realized daily log-return moment and the realized total return.

Finally, multiple linear regressions and quartile portfolios are computed with the inclusion of various fundamental and market factors including: Dollar volume, market capitalization, Price to book (equal to 1/market-to-book), Price to cash flow, ROE, Total debt to EBITDA, Total debt to equity, Momentum (Trailing Twelve Months returns). Quartile portfolios returns effectively divide the 64 sample stocks in 4 equal groups of 16 ranked from the lowest (Q1) to the highest (Q4) factor values. Every day from 1996 to 2016, quartile portfolios are created for each different factor and mean returns for those factor portfolios are computed for that day. The first and fourth quartile 20 year mean returns are then compared and tested for equal (unequal) means using a two-sample t-test.

5. EMPIRICAL RESULTS

The following tables compare the results across two-time periods [January 2nd, 1996 to June 23rd, 2003] and [June 24th, 2003 to April 30th, 2016]. In addition, non-reported tests have been performed to divide by interpolation granularity. Those compare the results across four levels of interpolation granularity [No interpolation, Quartic 52, Quartic 152 and Quartic 502]. In general, the results explained below are observed across all 4 levels of granularity. It also appears that the BKM methodology does not really benefits from interpolating the data whereas the B-L methodology benefits from interpolation.

Option Implied moments vs Realized Daily Log Return % moments:

First, statistical coefficients can vary considerably, sometimes shifting from positive to negative or vice-versa. For example, in table 2 p.31, coefficient values for columns mean, skewness and kurtosis are unstable across time periods whereas coefficients for standard deviation columns are always positive. In general, only implied standard deviation appears to be an accurate predictor (of realized σ) due to a combination of relatively high adj. R^2 values, stable coefficients and statistically significant p-values. Notably, the "post" time window appears to have higher standard deviation adjusted R-squared values for short-term maturity option (<182 days to maturity) whereas adj. R^2 has decreased for long-term maturity options. This result, which is valid for both the B-L (table 2, p.31) and BKM (table 3, p.33) methodologies, suggest that, through time, option implied standard deviation may have become a better predictor of short-term realized standard deviation whereas the long-term options implied standard deviation prediction ability has worsened relative to the previous period. However, there are numerous possible reasons for such a result including the fact that both periods are of unequal length, number and stage of economic cycles, just to name a few. As for other option-implied moments, although p-values are mostly significant, r-squared values are very small. Consequently, option-implied mean, skewness and kurtosis are all relatively poor predictors of their respective moments (realized daily log return % mean, skewness and kurtosis).

Table 2: Option Implied (B-L) vs Realized Daily Log Return % - Quartic 502						
Days to maturity		**Mean (μ)**	**Standard deviation (σ)**	**Skewness**	**Kurtosis**
Adj. R^2	**30**	0.0043	0.5813	0.0024	0.0022
Coef.		-0.0020	0.2324	0.0620	0.0808
P-value		0.1257	0.0000	0.1405	0.2366
Adj. R^2	**60**	0.0067	0.5702	0.0044	0.0052
Coef.		-0.0004	0.1638	0.0627	0.3057
P-value		0.1218	0.0000	0.1145	0.1534
Adj. R^2	**91**	0.0079	0.5408	0.0085	0.0076
Coef.		0.0008	0.1354	0.0656	0.6582
P-value		0.1409	0.0000	0.0670	0.1599
Adj. R^2	**122**	0.0090	0.5139	0.0127	0.0116
Coef.		-0.0002	0.1186	0.0533	1.1075
P-value		0.1504	0.0000	0.0821	0.1079
Adj. R^2	**152**	0.0100	0.4920	0.0167	0.0138
Coef.		-0.0007	0.1079	0.0346	1.2583
P-value		0.1054	0.0000	0.0593	0.0824
Adj. R^2	**182**	0.0119	0.4662	0.0198	0.0176
Coef.		-0.0013	0.0994	0.0074	1.4322
P-value		0.0627	0.0000	0.0548	0.1205
Adj. R^2	**273**	0.0155	0.4305	0.0275	0.0187
Coef.		-0.0008	0.0852	-0.0288	1.1070
P-value		0.0738	0.0000	0.0326	0.0714
Adj. R^2	**365**	0.0192	0.4016	0.0367	0.0218
Coef.		-0.0003	0.0811	-0.0612	0.4930
P-value		0.0607	0.0000	0.0379	0.0738
Adj. R^2	**547**	0.0333	0.3469	0.0547	0.0317
Coef.		-0.0012	0.0743	-0.1365	0.9581
P-value		0.0708	0.0000	0.0606	0.0601
Adj. R^2	**730**	0.0486	0.2966	0.0658	0.0316
Coef.		-0.0015	0.0680	-0.2060	0.7317
P-value		0.0538	0.0000	0.0258	0.1164

The table above shows average Adjusted R-Squared, Coefficient and P-values of all 64 firms regressions done using data for the entire period of January 1996-April 2016 and separated by time to maturity groups. For example, when looking at the column Mean (μ) and row of 30 days to maturity, the values presented is the average of 64 firms linear regressions, with data filtered to keep only options with 30 days to maturity. Each regression looks at the linear relation between y = 30 days Realized daily log return % mean and x = 30 days Option-Implied daily log return % mean (using the Breeden-Litzenberger (1978) methodology). Similarly, for the column Standard deviation (σ), the numbers represent average values for linear regressions between y = 30 days Realized daily log return % standard deviation and x = 30 days Option-Implied daily log return % standard deviation.

Option Implied (B-L) vs Realized Daily Log Return % - Pre/Quartic 52					
Days to maturity		Mean (μ)	Standard deviation (σ)	Skewness	Kurtosis
Adj. R2	**30**	0.0008	0.4907	0.0001	0.0002
Coef.		-0.0058	0.2429	-0.0325	0.0406
P-value		0.0000	0.0000	0.0039	0.0000
Adj. R2	**60**	0.0013	0.5227	0.0000	0.0000
Coef.		-0.0038	0.1662	0.0059	0.0224
P-value		0.0000	0.0000	0.6733	0.2827
Adj. R2	**91**	0.0013	0.5200	0.0002	0.0000
Coef.		-0.0027	0.1376	0.0655	0.0074
P-value		0.0000	0.0000	0.0001	0.8478
Adj. R2	**122**	0.0011	0.5042	0.0002	0.0002
Coef.		-0.0019	0.1206	0.0751	0.2427
P-value		0.0000	0.0000	0.0001	0.0000
Adj. R2	**152**	0.0015	0.4955	0.0000	0.0007
Coef.		-0.0018	0.1108	0.0486	0.5799
P-value		0.0000	0.0000	0.0212	0.0000
Adj. R2	**182**	0.0024	0.4864	0.0000	0.0007
Coef.		-0.0020	0.1041	0.0444	0.6925
P-value		0.0000	0.0000	0.0464	0.0000
Adj. R2	**273**	0.0033	0.4653	0.0001	0.0000
Coef.		-0.0017	0.0945	0.0853	0.2481
P-value		0.0000	0.0000	0.0002	0.0344
Adj. R2	**365**	0.0032	0.4434	0.0002	0.0000
Coef.		-0.0015	0.0924	0.1079	-0.0741
P-value		0.0000	0.0000	0.0000	0.5208
Adj. R2	**547**	0.0038	0.3743	0.0005	0.0000
Coef.		-0.0012	0.0896	0.1603	-0.2711
P-value		0.0000	0.0000	0.0000	0.0751
Adj. R2	**730**	0.0053	0.3078	0.0008	0.0003
Coef.		-0.0013	0.0886	0.2181	-1.1358
P-value		0.0000	0.0000	0.0000	0.0000

Option Implied (B-L) vs Realized Daily Log Return % - Post/Quartic 52					
Days to maturity		Mean (μ)	Standard deviation (σ)	Skewness	Kurtosis
Adj. R2	**30**	0.0034	0.6333	0.0001	0.0003
Coef.		0.0091	0.2937	-0.0338	-0.0519
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**60**	0.0060	0.6148	0.0010	0.0000
Coef.		0.0069	0.1952	-0.1007	-0.0180
P-value		0.0000	0.0000	0.0000	0.0997
Adj. R2	**91**	0.0057	0.5725	0.0004	0.0000
Coef.		0.0048	0.1584	-0.0641	0.0242
P-value		0.0000	0.0000	0.0000	0.1498
Adj. R2	**122**	0.0007	0.5320	0.0000	0.0003
Coef.		0.0014	0.1372	-0.0243	0.1745
P-value		0.0000	0.0000	0.0068	0.0000
Adj. R2	**152**	0.0000	0.4961	0.0002	0.0005
Coef.		-0.0001	0.1234	0.0518	0.3024
P-value		0.1858	0.0000	0.0000	0.0000
Adj. R2	**182**	0.0010	0.4565	0.0010	0.0004
Coef.		-0.0012	0.1119	0.1424	0.3085
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**273**	0.0054	0.3948	0.0056	0.0007
Coef.		-0.0019	0.0921	0.3449	0.5549
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**365**	0.0075	0.3382	0.0084	0.0001
Coef.		-0.0018	0.0850	0.4152	0.2104
P-value		0.0000	0.0000	0.0000	0.0002
Adj. R2	**547**	0.0458	0.2570	0.0063	0.0001
Coef.		-0.0034	0.0740	0.3174	0.0670
P-value		0.0000	0.0000	0.0000	0.0001
Adj. R2	**730**	0.0902	0.2058	0.0024	0.0008
Coef.		-0.0037	0.0677	0.1667	0.4557
P-value		0.0000	0.0000	0.0000	0.0000

The tables above are identical to Table 2 with two differences. They divide the time sample in two and use a 52-interpolation granularity instead of 502. The “Pre” window ranges from January 2nd, 1996 to June 23rd, 2003 while the “Post” time window is from June 24th, 2003 until April 30th, 2016.

Table 3: Option Implied (BKM) vs Realized Daily Log Return % - Quartic 502					
Days to maturity		**Mean (μ)**	**Standard deviation (σ)**	**Skewness**	**Kurtosis**
Adj. R^2	**30**	0.0039	0.5340	0.0027	0.0037
Coef.		-0.0095	0.8597	0.0687	0.1553
P-value		0.1452	0.0000	0.1545	0.1610
Adj. R^2	**60**	0.0060	0.5262	0.0051	0.0107
Coef.		-0.0012	0.4335	0.0645	0.5237
P-value		0.1561	0.0000	0.1180	0.1400
Adj. R^2	**91**	0.0087	0.5024	0.0095	0.0194
Coef.		0.0021	0.2931	0.0520	0.9352
P-value		0.1259	0.0000	0.0905	0.0670
Adj. R^2	**122**	0.0106	0.4784	0.0128	0.0245
Coef.		-0.0008	0.2190	0.0250	1.0783
P-value		0.1294	0.0000	0.0601	0.0353
Adj. R^2	**152**	0.0133	0.4556	0.0159	0.0289
Coef.		-0.0018	0.1752	0.0058	1.1085
P-value		0.0561	0.0000	0.0336	0.0406
Adj. R^2	**182**	0.0167	0.4286	0.0193	0.0332
Coef.		-0.0031	0.1468	-0.0105	1.1618
P-value		0.0452	0.0000	0.0465	0.0376
Adj. R^2	**273**	0.0229	0.3786	0.0266	0.0392
Coef.		-0.0018	0.0950	-0.0359	0.9365
P-value		0.0336	0.0000	0.0236	0.0251
Adj. R^2	**365**	0.0264	0.3373	0.0347	0.0492
Coef.		-0.0009	0.0728	-0.0600	0.8485
P-value		0.0646	0.0000	0.0479	0.0288
Adj. R^2	**547**	0.0431	0.2681	0.0537	0.0644
Coef.		-0.0013	0.0447	-0.0903	0.7797
P-value		0.0512	0.0000	0.0356	0.0561
Adj. R^2	**730**	0.0577	0.2122	0.0713	0.0653
Coef.		-0.0013	0.0288	-0.1345	0.4414
P-value		0.0456	0.0000	0.0713	0.0234

The table above shows average Adjusted R-Squared, Coefficient and P-values of all 64 firms regressions done using data for the entire period of January 1996-April 2016 and separated by time to maturity groups. For example, when looking at the column Mean (μ) and row of 30 days to maturity, the values presented is the average of 64 firms linear regressions, with data filtered to keep only options with 30 days to maturity. Each regression looks at the linear relation between y = 30 days Realized daily log return % mean and x = 30 days Option-Implied daily log return % mean (using the Bakshi, Kapadia and Madan (2003) methodology). Similarly, for the column Standard deviation (σ), the numbers represent average values for linear regressions between y = 30 days Realized daily log return % standard deviation and x = 30 days Option-Implied daily log return % standard deviation.

Option Implied (BKM) vs Realized Daily Log Return % - Pre/Quartic 52					
Days to maturity		Mean (μ)	Standard deviation (σ)	Skewness	Kurtosis
Adj. R2	**30**	0.0023	0.4680	0.0011	0.0006
Coef.		-0.0415	0.6547	0.0720	-0.0850
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**60**	0.0021	0.4885	0.0003	0.0016
Coef.		-0.0145	0.3258	0.0515	-0.2938
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**91**	0.0011	0.4902	0.0000	0.0015
Coef.		-0.0060	0.2215	0.0243	-0.4707
P-value		0.0000	0.0000	0.0193	0.0000
Adj. R2	**122**	0.0006	0.4818	0.0000	0.0010
Coef.		-0.0031	0.1666	-0.0048	-0.5041
P-value		0.0000	0.0000	0.6598	0.0000
Adj. R2	**152**	0.0009	0.4761	0.0000	0.0004
Coef.		-0.0027	0.1348	-0.0223	-0.3786
P-value		0.0000	0.0000	0.0532	0.0000
Adj. R2	**182**	0.0016	0.4709	0.0001	0.0002
Coef.		-0.0028	0.1129	-0.0329	-0.2638
P-value		0.0000	0.0000	0.0064	0.0001
Adj. R2	**273**	0.0027	0.4577	0.0004	0.0001
Coef.		-0.0021	0.0759	-0.0787	-0.2139
P-value		0.0000	0.0000	0.0000	0.0048
Adj. R2	**365**	0.0052	0.4403	0.0004	0.0000
Coef.		-0.0020	0.0585	-0.0773	0.0283
P-value		0.0000	0.0000	0.0000	0.7148
Adj. R2	**547**	0.0120	0.3937	0.0000	0.0000
Coef.		-0.0018	0.0397	-0.0160	0.0335
P-value		0.0000	0.0000	0.1679	0.6565
Adj. R2	**730**	0.0212	0.3380	0.0000	0.0001
Coef.		-0.0016	0.0299	-0.0045	-0.2277
P-value		0.0000	0.0000	0.6822	0.0014

Option Implied (BKM) vs Realized Daily Log Return % - Post/Quartic 52					
Days to maturity		Mean (μ)	Standard deviation (σ)	Skewness	Kurtosis
Adj. R2	**30**	0.0048	0.5424	0.0005	0.0000
Coef.		0.0383	0.5642	0.0515	0.0119
P-value		0.0000	0.0000	0.0000	0.0924
Adj. R2	**60**	0.0039	0.5271	0.0008	0.0002
Coef.		0.0134	0.2942	0.0783	0.0785
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**91**	0.0043	0.5119	0.0011	0.0005
Coef.		0.0081	0.2118	0.1054	0.1809
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**122**	0.0001	0.4980	0.0014	0.0008
Coef.		0.0010	0.1658	0.1151	0.2645
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**152**	0.0010	0.4814	0.0016	0.0010
Coef.		-0.0021	0.1377	0.1227	0.3181
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**182**	0.0061	0.4548	0.0017	0.0012
Coef.		-0.0043	0.1206	0.1264	0.3503
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**273**	0.0157	0.3945	0.0024	0.0006
Coef.		-0.0042	0.0841	0.1347	0.2476
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**365**	0.0187	0.3156	0.0016	0.0004
Coef.		-0.0034	0.0695	0.1044	0.2110
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**547**	0.0515	0.2106	0.0007	0.0021
Coef.		-0.0039	0.0461	0.0568	0.4547
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**730**	0.0787	0.1451	0.0000	0.0053
Coef.		-0.0031	0.0307	0.0087	0.6090
P-value		0.0000	0.0000	0.0718	0.0000

The tables above are identical to Table 3 with two differences. They divide the time sample in two and use a 52-interpolation granularity instead of 502. The “Pre” window ranges from January 2nd, 1996 to June 23rd, 2003 while the “Post” time window is from June 24th, 2003 until April 30th, 2016.

Option Implied moments vs Realized Total Return %:

As can be seen in tables 4 p.35-36 and 5 p.37-38, regardless of the time window and methodology used, option-implied mean coefficients tend to be negatively related to realized total return while option-implied standard deviation columns are mostly positively related to returns. As for option-implied skewness, coefficient signs appear to be unstable. When looking at the entire time window starting in 1996 and ending in 2016, coefficients are positive for both methodologies. However, dividing time periods shows that coefficients for higher moments (skewness & kurtosis) tend to be less stable than mean and standard deviation coefficients through time. This can be observed for both the B-L and BKM methodologies. Option-implied kurtosis coefficient sign is particularly unstable as it goes from negative to positive depending on both the period, methodology used and time to maturity of the options. Overall, the outputs suggest that option-implied mean is negatively related to return and option-implied standard deviation is positively related to realized returns. It would suggest that, on average, companies that are expected to perform poorly and have high expected volatility (as determined by options implied mean and volatility) tend to produce higher long-term returns (potentially due to risk premia). In theory, investors should prefer a return distribution that exhibit higher mean, lower standard deviation, higher skewness and lower kurtosis. These results suggest that adopting a contrarian outlook for the first two moments of the distribution could potentially yield higher returns. In other words, the best time to buy stocks is when implied volatility is high, and expected returns are low which generally happens when pessimism prevails, and the markets are potentially closer to the bottom than to the top. Nonetheless, those relations are quite weak as measured by adj. R^2 and thus, if used alone, would be unlikely to produce positive abnormal returns within a relatively high level of confidence.

Table 4: Option Implied (B-L) vs Realized Total Return % - Quartic 502						
Days to maturity		**Mean (μ)**	**Standard deviation (σ)**	**Skewness**	**Kurtosis**
Adj. R^2	**30**	0.0058	0.0059	0.0024	0.0013
Coef.		-0.2377	0.1342	0.0106	0.0005
P-value		0.1047	0.1025	0.1819	0.2877
Adj. R^2	**60**	0.0077	0.0077	0.0048	0.0031
Coef.		-0.2210	0.1325	0.0215	0.0049
P-value		0.1042	0.0777	0.1203	0.1679
Adj. R^2	**91**	0.0084	0.0087	0.0075	0.0065
Coef.		-0.1795	0.1193	0.0293	0.0200
P-value		0.1005	0.0742	0.1073	0.1042
Adj. R^2	**122**	0.0128	0.0124	0.0111	0.0084
Coef.		-0.2781	0.1839	0.0460	0.0148
P-value		0.1206	0.0892	0.0800	0.1029
Adj. R^2	**152**	0.0160	0.0153	0.0153	0.0114
Coef.		-0.3430	0.2351	0.0772	0.0137
P-value		0.1221	0.0552	0.0499	0.1143
Adj. R^2	**182**	0.0194	0.0194	0.0192	0.0149
Coef.		-0.4461	0.3114	0.1066	0.0137
P-value		0.0907	0.0675	0.0415	0.1029
Adj. R^2	**273**	0.0245	0.0229	0.0275	0.0169
Coef.		-0.5013	0.3687	0.1726	-0.0085
P-value		0.0949	0.0579	0.0459	0.0872
Adj. R^2	**365**	0.0280	0.0255	0.0289	0.0186
Coef.		-0.4432	0.3587	0.1973	0.0415
P-value		0.0359	0.1236	0.0588	0.0874
Adj. R^2	**547**	0.0470	0.0351	0.0358	0.0191
Coef.		-0.8117	0.7123	0.2896	0.0568
P-value		0.0406	0.0631	0.0828	0.0443
Adj. R^2	**730**	0.0634	0.0430	0.0370	0.0220
Coef.		-1.0830	1.1358	0.3365	-0.0048
P-value		0.0516	0.0971	0.0425	0.0807

The table above shows average Adjusted R-Squared, Coefficient and P-values of all 64 firms regressions done using data for the entire period of January 1996-April 2016 and separated by time to maturity groups. For example, when looking at the column Mean (μ) and row of 30 days to maturity, the values presented is the average of 64 firms linear regressions, with data filtered to keep only options with 30 days to maturity. Each regression looks at the linear relation between y = 30 days Realized Total Return % and x = 30 days Option-Implied daily log return % mean (using the Breeden-Litzenberger (1978) methodology). Similarly, for the column Standard deviation (σ), the numbers represent average values for linear regressions between y = 30 days Realized Total Return % and x = 30 days Option-Implied daily log return % standard deviation.

Option Implied (B-L) vs Realized Total Return % - Pre/Quartic 52					
Days to maturity		Mean (μ)	Standard deviation (σ)	Skewness	Kurtosis
Adj. R2	**30**	0.0037	0.0130	0.0038	0.0000
Coef.		-0.2652	0.3822	-0.0344	-0.0009
P-value		0.0000	0.0000	0.0000	0.1433
Adj. R2	**60**	0.0064	0.0142	0.0018	0.0001
Coef.		-0.3644	0.4029	-0.0382	0.0044
P-value		0.0000	0.0000	0.0000	0.0004
Adj. R2	**91**	0.0082	0.0144	0.0001	0.0016
Coef.		-0.4526	0.4475	-0.0145	0.0255
P-value		0.0000	0.0000	0.0009	0.0000
Adj. R2	**122**	0.0089	0.0147	0.0015	0.0034
Coef.		-0.4957	0.4896	0.0656	0.0536
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**152**	0.0106	0.0173	0.0061	0.0050
Coef.		-0.5861	0.5868	0.1644	0.0810
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**182**	0.0138	0.0213	0.0115	0.0044
Coef.		-0.7245	0.7219	0.2625	0.0899
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**273**	0.0166	0.0217	0.0218	0.0011
Coef.		-1.0606	1.0363	0.5002	0.0667
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**365**	0.0147	0.0196	0.0247	0.0000
Coef.		-1.5058	1.5679	0.7888	-0.0042
P-value		0.0000	0.0000	0.0000	0.6238
Adj. R2	**547**	0.0150	0.0233	0.0381	0.0000
Coef.		-2.1578	2.6981	1.4421	-0.0222
P-value		0.0000	0.0000	0.0000	0.0960
Adj. R2	**730**	0.0145	0.0231	0.0365	0.0000
Coef.		-2.9713	4.1826	2.0525	0.0170
P-value		0.0000	0.0000	0.0000	0.4579

Option Implied (B-L) vs Realized Total Return % - Post/Quartic 52					
Days to maturity		Mean (μ)	Standard deviation (σ)	Skewness	Kurtosis
Adj. R2	**30**	0.0000	0.0000	0.0000	0.0001
Coef.		0.0067	-0.0175	0.0002	0.0014
P-value		0.3671	0.0040	0.7834	0.0000
Adj. R2	**60**	0.0000	0.0000	0.0008	0.0001
Coef.		-0.0033	-0.0006	-0.0118	0.0016
P-value		0.6864	0.9136	0.0000	0.0001
Adj. R2	**91**	0.0002	0.0001	0.0010	0.0003
Coef.		-0.0578	0.0197	-0.0166	0.0044
P-value		0.0000	0.0011	0.0000	0.0000
Adj. R2	**122**	0.0061	0.0037	0.0019	0.0000
Coef.		-0.3108	0.1622	-0.0296	0.0015
P-value		0.0000	0.0000	0.0000	0.1041
Adj. R2	**152**	0.0122	0.0077	0.0013	0.0001
Coef.		-0.4498	0.2459	-0.0304	-0.0041
P-value		0.0000	0.0000	0.0000	0.0008
Adj. R2	**182**	0.0197	0.0136	0.0009	0.0003
Coef.		-0.6092	0.3553	-0.0300	-0.0123
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**273**	0.0292	0.0183	0.0001	0.0013
Coef.		-0.7926	0.4760	0.0165	-0.0357
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**365**	0.0252	0.0099	0.0000	0.0000
Coef.		-0.8131	0.4170	0.0121	0.0000
P-value		0.0000	0.0000	0.0033	0.9932
Adj. R2	**547**	0.0674	0.0189	0.0000	0.0002
Coef.		-1.6225	0.7991	-0.0100	-0.0064
P-value		0.0000	0.0000	0.0690	0.0000
Adj. R2	**730**	0.0910	0.0200	0.0000	0.0043
Coef.		-2.2191	1.0847	0.0079	-0.0764
P-value		0.0000	0.0000	0.2281	0.0000

The tables above are identical to Table 4 with two differences. They divide the time sample in two and use a 52-interpolation granularity instead of 502. The "Pre" window ranges from January 2nd, 1996 to June 23rd, 2003 while the "Post" time window is from June 24th, 2003 until April 30th, 2016.

Table 5: Option Implied (BKM) vs Realized Total Return % - Quartic 502					
Days to maturity		**Mean (μ)**	**Standard deviation (σ)**	**Skewness**	**Kurtosis**
Adj. R^2	**30**	0.0055	0.0071	0.0029	0.0030
Coef.		-1.0179	0.6166	0.0100	-0.0113
P-value		0.1225	0.0645	0.1662	0.1431
Adj. R^2	**60**	0.0070	0.0083	0.0057	0.0054
Coef.		-0.5717	0.3658	0.0161	-0.0190
P-value		0.1088	0.0936	0.0911	0.0763
Adj. R^2	**91**	0.0097	0.0096	0.0093	0.0080
Coef.		-0.3001	0.1990	0.0243	-0.0194
P-value		0.1134	0.0994	0.0718	0.1171
Adj. R^2	**122**	0.0158	0.0144	0.0111	0.0116
Coef.		-0.4633	0.2734	0.0312	-0.0337
P-value		0.0946	0.1195	0.0466	0.0806
Adj. R^2	**152**	0.0216	0.0195	0.0153	0.0151
Coef.		-0.5342	0.3487	0.0487	-0.0454
P-value		0.0244	0.0908	0.0400	0.0752
Adj. R^2	**182**	0.0284	0.0272	0.0207	0.0199
Coef.		-0.7299	0.4983	0.0711	-0.0608
P-value		0.0498	0.0850	0.0454	0.0898
Adj. R^2	**273**	0.0367	0.0332	0.0306	0.0284
Coef.		-0.6124	0.4999	0.1076	-0.0911
P-value		0.0622	0.0580	0.0210	0.0829
Adj. R^2	**365**	0.0396	0.0311	0.0375	0.0267
Coef.		-0.3801	0.3736	0.1300	-0.0828
P-value		0.0927	0.0504	0.0243	0.0185
Adj. R^2	**547**	0.0589	0.0413	0.0515	0.0391
Coef.		-0.4664	0.3830	0.1690	-0.1188
P-value		0.0365	0.0741	0.0176	0.0338
Adj. R^2	**730**	0.0727	0.0494	0.0520	0.0501
Coef.		-0.5608	0.4141	0.1713	-0.1632
P-value		0.0475	0.0215	0.0531	0.0359

The table above shows average Adjusted R-Squared, Coefficient and P-values of all 64 firms regressions done using data for the entire period of January 1996-April 2016 and separated by time to maturity groups. For example, when looking at the column Mean (μ) and row of 30 days to maturity, the values presented is the average of 64 firms linear regressions, with data filtered to keep only options with 30 days to maturity. Each regression looks at the linear relation between y = 30 days Realized Total Return % and x = 30 days Option-Implied daily log return % mean (using the Bakshi, Kapadia and Madan (2003) methodology). Similarly, for the column Standard deviation (σ), the numbers represent average values for linear regressions between y = 30 days Realized Total Return % and x = 30 days Option-Implied daily log return % standard deviation.

Option Implied (BKM) vs Realized Total Return % - Pre/Quartic 52					
Days to maturity		Mean (μ)	Standard deviation (σ)	Skewness	Kurtosis
Adj. R2	**30**	0.0095	0.0128	0.0030	0.0051
Coef.		-1.7946	1.0484	0.0196	-0.0172
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**60**	0.0097	0.0139	0.0057	0.0085
Coef.		-1.3438	0.8095	0.0434	-0.0399
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**91**	0.0084	0.0133	0.0083	0.0084
Coef.		-1.1240	0.7140	0.0732	-0.0611
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**122**	0.0080	0.0130	0.0118	0.0079
Coef.		-1.0070	0.6526	0.1054	-0.0759
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**152**	0.0092	0.0152	0.0171	0.0077
Coef.		-1.0466	0.6845	0.1498	-0.0867
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**182**	0.0119	0.0188	0.0217	0.0095
Coef.		-1.1635	0.7482	0.1951	-0.1054
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**273**	0.0158	0.0225	0.0194	0.0096
Coef.		-1.3778	0.8545	0.2580	-0.1285
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**365**	0.0207	0.0270	0.0187	0.0110
Coef.		-1.9411	1.1684	0.3693	-0.1829
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**547**	0.0365	0.0416	0.0359	0.0131
Coef.		-2.6840	1.5591	0.6555	-0.2232
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**730**	0.0444	0.0501	0.0395	0.0128
Coef.		-3.3516	1.9833	0.8534	-0.2511
P-value		0.0000	0.0000	0.0000	0.0000

Option Implied (BKM) vs Realized Total Return % - Post/Quartic 52					
Days to maturity		Mean (μ)	Standard deviation (σ)	Skewness	Kurtosis
Adj. R2	**30**	0.0003	0.0000	0.0000	0.0002
Coef.		-0.1831	0.0214	-0.0005	-0.0017
P-value		0.0000	0.0902	0.3782	0.0000
Adj. R2	**60**	0.0012	0.0002	0.0002	0.0002
Coef.		-0.2924	0.0624	-0.0048	-0.0036
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**91**	0.0018	0.0002	0.0001	0.0003
Coef.		-0.3143	0.0533	-0.0050	-0.0053
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**122**	0.0121	0.0064	0.0000	0.0028
Coef.		-0.7447	0.2680	-0.0023	-0.0186
P-value		0.0000	0.0000	0.0794	0.0000
Adj. R2	**152**	0.0257	0.0172	0.0000	0.0054
Coef.		-1.0134	0.4158	0.0046	-0.0287
P-value		0.0000	0.0000	0.0018	0.0000
Adj. R2	**182**	0.0425	0.0305	0.0006	0.0094
Coef.		-1.3287	0.5754	0.0165	-0.0416
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**273**	0.0590	0.0426	0.0037	0.0170
Coef.		-1.4772	0.6642	0.0495	-0.0633
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**365**	0.0501	0.0266	0.0033	0.0138
Coef.		-1.4189	0.5794	0.0538	-0.0629
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**547**	0.0794	0.0325	0.0045	0.0357
Coef.		-1.8665	0.7206	0.0784	-0.1139
P-value		0.0000	0.0000	0.0000	0.0000
Adj. R2	**730**	0.0969	0.0350	0.0101	0.0573
Coef.		-2.0425	0.7728	0.1316	-0.1472
P-value		0.0000	0.0000	0.0000	0.0000

The tables above are identical to Table 5 with two differences. They divide the time sample in two and use a 52-interpolation granularity instead of 502. The “Pre” window ranges from January 2nd, 1996 to June 23rd, 2003 while the “Post” time window is from June 24th, 2003 until April 30th, 2016.

Table 6: Multiple Regression B-L

Days to maturity	30	60	91	122	152	182	273	365	547	730
Intercept	-0.0472***	-0.0765***	-0.1348***	-0.1980***	-0.2421***	-0.2495***	-0.3591***	-0.2944***	-0.6318***	-0.9025***
	(-14.42)	(-18.40)	(-27.07)	(-31.05)	(-31.97)	(-34.15)	(-28.26)	(-17.58)	(-23.42)	(-20.41)
Option-Implied Mean (B-L)	0.1555***	0.2413***	0.2866***	0.1522***	0.1186***	0.0928***	-0.1011***	-0.1380***	-0.2915***	0.0101
	(10.92)	(15.02)	(16.00)	(7.79)	(5.63)	(4.11)	(-3.91)	(-4.05)	(-7.52)	(0.24)
Option-Implied Standard Deviation (B-L)	0.3061***	0.4135***	0.5029***	0.5020***	0.5543***	0.6064***	0.6434***	0.8683***	1.2998***	2.3550***
	(30.27)	(37.76)	(41.95)	(38.21)	(38.20)	(37.78)	(30.57)	(26.46)	(26.04)	(32.32)
Option-Implied Skewness (B-L)	0.0360***	0.0681***	0.1273***	0.2561***	0.3108***	0.2657***	0.5619***	0.2104***	0.3255***	0.2457***
	(10.65)	(14.02)	(19.46)	(24.89)	(23.48)	(22.00)	(20.05)	(14.87)	(28.39)	(12.47)
Option-Implied Kurtosis (B-L)	0.0086***	0.0160***	0.0326***	0.0501***	0.0616***	0.0617***	0.0913***	0.0572***	0.1234***	0.1958***
	(9.13)	(13.25)	(22.12)	(26.08)	(26.86)	(28.42)	(24.59)	(13.27)	(19.09)	(19.38)
X3*X4 Interaction	-0.0081***	-0.0148***	-0.0302***	-0.0644***	-0.0772***	-0.0590***	-0.1333***	-0.0055*	-0.0203***	-0.0599***
	(-8.28)	(-10.60)	(-15.77)	(-20.93)	(-19.31)	(-16.58)	(-15.51)	(-1.67)	(-19.03)	(-14.45)
Dollar Volume	<0**	<0	<0	<0	<0	<0	<0	-0.0001***	-0.0001***	<0
	(-2.17)	(-0.90)	(-0.10)	(-0.36)	(-0.69)	(0.25)	(-1.15)	(-4.22)	(-5.58)	(-1.38)
Market Capitalization	<0***	<0***	<0***	<0***	<0***	<0***	<0***	<0***	<0***	<0***
	(-15.59)	(-21.52)	(-27.07)	(-30.48)	(-32.56)	(-35.18)	(-35.93)	(-30.44)	(-35.46)	(-34.76)
Price to Book	>0***	>0	<0**	-0.0001***	-0.0002***	-0.0002***	-0.0002***	-0.0001***	<0	-0.0011***
	(3.88)	(1.59)	(-2.06)	(-7.55)	(-9.07)	(-9.74)	(-6.41)	(-3.20)	(-0.03)	(-17.56)
Price to Cash Flow	<0**	-0.0001***	-0.0001***	-0.0001***	-0.0001***	-0.0002***	-0.0008***	-0.0015***	-0.0005***	0.0019***
	(-2.30)	(-7.16)	(-5.27)	(-6.08)	(-10.08)	(-14.77)	(-34.71)	(-43.49)	(-9.60)	(26.00)
ROE	0.0014***	0.0008**	0.0012**	0.0010*	0.0012*	0.0023***	0.0021**	-0.0201***	-0.0932***	-0.9128***
	(5.22)	(2.13)	(2.50)	(1.78)	(1.82)	(3.10)	(2.00)	(-12.58)	(-39.78)	(-131.99)
Total Debt to EBITDA	-0.0003***	-0.0006***	-0.0009***	-0.0012***	-0.0016***	-0.0021***	-0.0038***	-0.0055***	-0.0083***	-0.0231***
	(-9.10)	(-13.20)	(-14.63)	(-15.88)	(-18.86)	(-21.92)	(-27.66)	(-25.85)	(-26.51)	(-52.84)
Total Debt to Equity	<0	>0***	0.0002***	0.0003***	0.0004***	0.0005***	0.0007***	0.0007***	0.0008***	0.0059***
	(-0.50)	(2.70)	(7.19)	(11.85)	(14.31)	(15.74)	(14.39)	(9.91)	(7.41)	(43.21)
RM-RF (SP100-Rf)	0.9726***	0.9694***	0.9647***	0.9601***	0.9685***	0.9695***	0.9601***	0.9914***	1.0768***	1.1716***
	(310.21)	(290.82)	(290.19)	(285.13)	(281.14)	(276.45)	(230.57)	(184.25)	(176.07)	(165.07)
Momentum (TTM)	0.0052***	0.0093***	0.0107***	0.0116***	0.0144***	0.0169***	0.0352***	0.0572***	0.0528***	-0.0016
	(16.08)	(20.28)	(18.67)	(17.10)	(18.47)	(19.13)	(27.72)	(29.61)	(18.50)	(-0.41)
Adj. R-Squared	0.2633	0.2482	0.2535	0.2551	0.2563	0.2580	0.2180	0.1686	0.1742	0.2390
Sample Size	278372	272121	269413	266647	265324	263249	257289	250439	237980	222652

This table shows the relationship between realized N days total returns % (where N represents the number of days to maturity for each respective columns) and different factors using a multiple linear regression. Option-implied moments (first four factors) are computed using the B-L methodology while the X3*X4 interaction is an interaction factor representing the multiplication of option-implied Skewness (B-L) and Option-Implied Kurtosis (B-L). The time sample used is from 1996 to 2016 and includes all 64 firms sample. Sample sizes per time to maturity group are indicated at the bottom. The first row shows coefficients while the second gives the t-values in parentheses.
Significance codes: '***', '**' and '*' respectively denote that the coefficient sign is significant at the 1, 5 and 10% level.

Multiple Regression B-L Coefficient Confidence Intervals (α = 0.01)

Days to maturity	30	60	91	122	152	182	273	365	547	730
Intercept	[-0.06, -0.04]	[-0.09, -0.07]	[-0.15, -0.12]	[-0.21, -0.18]	[-0.26, -0.22]	[-0.27, -0.23]	[-0.39, -0.33]	[-0.34, -0.25]	[-0.7, -0.56]	[-1.02, -0.79]
Option-Implied Mean (B-L)	[0.12, 0.19]	[0.2, 0.28]	[0.24, 0.33]	[0.1, 0.2]	[0.06, 0.17]	[0.03, 0.15]	[-0.17, -0.03]	[-0.23, -0.05]	[-0.39, -0.19]	[-0.1, 0.12]
Option-Implied Standard Deviation (B-L)	[0.28, 0.33]	[0.39, 0.44]	[0.47, 0.53]	[0.47, 0.54]	[0.52, 0.59]	[0.57, 0.65]	[0.59, 0.7]	[0.78, 0.95]	[1.17, 1.43]	[2.17, 2.54]
Option-Implied Skewness (B-L)	[0.03, 0.04]	[0.06, 0.08]	[0.11, 0.14]	[0.23, 0.28]	[0.28, 0.34]	[0.23, 0.3]	[0.49, 0.63]	[0.17, 0.25]	[0.3, 0.36]	[0.19, 0.3]
Option-Implied Kurtosis (B-L)	[0.01, 0.01]	[0.01, 0.02]	[0.03, 0.04]	[0.05, 0.06]	[0.06, 0.07]	[0.06, 0.07]	[0.08, 0.1]	[0.05, 0.07]	[0.11, 0.14]	[0.17, 0.22]
X3*X4 Interaction	[-0.01, -0.01]	[-0.02, -0.01]	[-0.04, -0.03]	[-0.07, -0.06]	[-0.09, -0.07]	[-0.07, -0.05]	[-0.16, -0.11]	[-0.01, 0]	[-0.02, -0.02]	[-0.07, -0.05]
Dollar Volume	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]
Market Capitalization	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]
Price to Book	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]
Price to Cash Flow	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]
ROE	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[-0.02, -0.02]	[-0.1, -0.09]	[-0.93, -0.9]
Total Debt to EBITDA	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[-0.01, 0]	[-0.01, -0.01]	[-0.02, -0.02]
Total Debt to Equity	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.01, 0.01]
RM-RF (SP100-Rf)	[0.96, 0.98]	[0.96, 0.98]	[0.96, 0.97]	[0.95, 0.97]	[0.96, 0.98]	[0.96, 0.98]	[0.95, 0.97]	[0.98, 1.01]	[1.06, 1.09]	[1.15, 1.19]
Momentum (TTM)	[0, 0.01]	[0.01, 0.01]	[0.01, 0.01]	[0.01, 0.01]	[0.01, 0.02]	[0.01, 0.02]	[0.03, 0.04]	[0.05, 0.06]	[0.05, 0.06]	[-0.01, 0.01]

This table is a continuation of table 6 and shows coefficient confidence intervals using an alpha parameter of 0.01. Numbers are rounded to the second decimal. A confidence interval of [0.28, 0.33] for 30 days option-implied standard deviation (B-L) can be interpreted as follows: for a 1 unit increase in option-implied standard deviation (B-L), total realized return would be expected to increase (in average) by an amount ranging between 0.28% and 0.33%. Several variables confidence intervals are [0, 0]. Consequently, although some of these coefficients may be statistically significant, their effect is marginal. (less than 1/100th of a percent.)

Table 7: Multiple Regression BKM

Days to maturity	30	60	91	122	152	182	273	365	547	730
Intercept	0.0211***	0.0435***	0.0557***	0.0816***	0.0667***	0.0491***	0.0743	0.0807***	-0.4264***	-0.2933***
	(13.94)	(15.57)	(13.22)	(13.44)	(9.54)	(6.10)	(5.91)	(3.63)	(-11.46)	(-7.00)
Option-Implied Mean (BKM)	0.3796***	0.4358***	0.5694***	0.5906***	0.4364***	0.1977***	0.3179***	0.4861***	0.1228***	1.7899***
	(4.19)	(6.37)	(9.48)	(11.08)	(8.80)	(4.07)	(6.49)	(8.16)	(1.91)	(25.39)
Option-Implied Standard Deviation (BKM)	0.5359***	0.5522***	0.6133***	0.7042***	0.7497***	0.7438***	0.8850***	1.1663***	1.6777***	2.3550***
	(12.38)	(16.85)	(21.28)	(27.12)	(30.59)	(30.76)	(34.99)	(35.78)	(42.25)	(52.33)
Option-Implied Skewness (BKM)	0.0134***	0.0289***	0.0400*	0.0908***	0.1316***	0.1805***	0.2485***	0.3026***	0.1233***	-0.1263***
	(7.87)	(9.13)	(9.85)	(16.66)	(21.79)	(26.56)	(25.83)	(19.08)	(5.21)	(-5.35)
Option-Implied Kurtosis (BKM)	-0.0023***	-0.0054***	-0.0054***	-0.0098***	-0.0016	0.0091***	0.0133***	0.0180***	0.1686***	0.1340***
	(-7.16)	(-7.85)	(-4.73)	(-5.79)	(-0.82)	(3.97)	(3.70)	(2.85)	(16.24)	(11.60)
X3*X4 Interaction	-0.0020***	-0.0048***	-0.0061***	-0.0179***	-0.0249***	-0.0328***	-0.0464***	-0.0590***	0.0029***	0.0463***
	(-5.95)	(-7.20)	(-6.91)	(-13.68)	(-16.94)	(-19.71)	(-18.78)	(-13.82)	(0.44)	(7.25)
Dollar Volume	<0*	<0	>0	<0	<0	<0	<0	<0**	-0.0001***	0.0001***
	(-2.29)	(-1.58)	(0.04)	(-0.21)	(-0.77)	(-0.24)	(0.37)	(-2.19)	(-4.66)	(2.79)
Market Capitalization	<0***	<0***	<0***	<0***	<0***	<0***	<0***	<0***	<0***	<0***
	(-15.72)	(-21.97)	(-28.50)	(-32.14)	(-34.50)	(-37.43)	(-40.98)	(-35.59)	(-38.47)	(-36.50)
Price to Book	>0***	>0**	<0	-0.0001***	-0.0001***	-0.0002***	-0.0002***	-0.0002***	-0.0003***	-0.0012***
	(4.56)	(2.57)	(-0.73)	(-6.08)	(-7.50)	(-8.78)	(-6.40)	(-5.35)	(-4.87)	(-19.23)
Price to Cash Flow	<0*	-0.0001***	<0***	-0.0001***	-0.0001***	-0.0002***	-0.0008***	-0.0015***	-0.0003***	0.0019***
	(-2.34)	(-6.82)	(-4.79)	(-5.62)	(-9.24)	(-13.42)	(-33.90)	(-42.08)	(-6.58)	(26.81)
ROE	0.0013***	0.0009***	0.0015***	0.0022***	0.0043***	0.0076***	0.0115***	-0.0040***	-0.0572***	-0.8352***
	(4.63)	(2.43)	(3.17)	(3.95)	(6.59)	(10.27)	(10.84)	(-2.47)	(-23.91)	(-116.60)
Total Debt to EBITDA	-0.0003***	-0.0007***	-0.0009***	-0.0012***	-0.0015***	-0.0020***	-0.0036***	-0.0050***	-0.0077***	-0.0225***
	(-9.35)	(-13.80)	(-14.93)	(-15.93)	(-18.42)	(-20.90)	(-25.94)	(-23.71)	(-24.48)	(-51.40)
Total Debt to Equity	<0	<0	0.0001***	0.0002***	0.0003***	0.0004***	0.0005***	0.0006***	0.0008***	0.0054***
	(-1.26)	(1.27)	(5.41)	(9.69)	(11.02)	(12.00)	(11.06)	(8.46)	(7.62)	(39.96)
RM-RF (SP100-Rf)	0.9733***	0.9697***	0.9662***	0.9605***	0.9656***	0.9612***	0.9504***	0.9932***	1.1044***	1.2744***
	(310.08)	(290.62)	(289.30)	(283.70)	(279.03)	(272.96)	(226.66)	(182.00)	(174.14)	(168.69)
Momentum (TTM)	0.0053***	0.0099***	0.0119***	0.0128***	0.0152***	0.0165***	0.0322***	0.0496***	0.0339***	-0.0165***
	(16.48)	(21.80)	(20.84)	(18.80)	(19.58)	(18.57)	(25.22)	(25.62)	(11.86)	(-4.19)
Adj. R-Squared	0.2630	0.2478	0.2515	0.2534	0.2569	0.2605	0.2217	0.1750	0.1881	0.2484
Sample Size	278372	272121	269413	266647	265324	263249	257289	250439	237980	222652

This table shows the relationship between realized N days total returns (where N represents the number of days to maturity for each respective columns) and different factors using a multiple linear regression. Option-implied moments (first four factors) are computed using the BKM methodology while the X3*X4 interaction is an interaction factor representing the multiplication of option-implied Skewness (BKM) and Option-Implied Kurtosis (BKM). The time sample used is from 1996 to 2016 and includes all 64 firms sample. Sample sizes per time to maturity group are indicated at the bottom. The first row shows coefficients while the second gives the t-values in parentheses.
Significance codes: '***', '**' and '*' respectively denote that the coefficient sign is significant at the 1, 5 and 10% level.

Multiple Regression BKM Coefficient Confidence Intervals (α = 0.01)

Days to maturity	30	60	91	122	152	182	273	365	547	730
Intercept	[0.02, 0.02]	[0.04, 0.05]	[0.04, 0.07]	[0.05, 0.08]	[0.05, 0.08]	[0.03, 0.07]	[0.04, 0.11]	[0.02, 0.14]	[-0.52, -0.33]	[-0.4, -0.19]
Option-Implied Mean (BKM)	[0.15, 0.61]	[0.26, 0.61]	[0.41, 0.72]	[0.31, 0.56]	[0.31, 0.56]	[0.07, 0.32]	[0.19, 0.44]	[0.33, 0.64]	[-0.04, 0.29]	[1.61, 1.97]
Option-Implied Standard Deviation (BKM	[0.42, 0.65]	[0.47, 0.64]	[0.54, 0.69]	[0.69, 0.81]	[0.69, 0.81]	[0.68, 0.81]	[0.82, 0.95]	[1.08, 1.25]	[1.58, 1.78]	[2.24, 2.47]
Option-Implied Skewness (BKM)	[0.01, 0.02]	[0.02, 0.04]	[0.03, 0.05]	[0.12, 0.15]	[0.12, 0.15]	[0.16, 0.2]	[0.22, 0.27]	[0.26, 0.34]	[0.06, 0.18]	[-0.19, -0.07]
Option-Implied Kurtosis (BKM)	[0, 0]	[-0.01, 0]	[-0.01, 0]	[-0.01, 0]	[-0.01, 0]	[0, 0.02]	[0, 0.02]	[0, 0.03]	[0.14, 0.2]	[0.1, 0.16]
X3*X4 Interaction	[0, 0]	[-0.01, 0]	[-0.01, 0]	[-0.03, -0.02]	[-0.03, -0.02]	[-0.04, -0.03]	[-0.05, -0.04]	[-0.07, -0.05]	[-0.01, 0.02]	[0.03, 0.06]
Dollar Volume	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]
Market Capitalization	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]
Price to Book	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]
Price to Cash Flow	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]
ROE	[0, 0]	[0, 0]	[0, 0]	[0, 0.01]	[0, 0.01]	[0.01, 0.01]	[0.01, 0.01]	[-0.01, 0]	[-0.06, -0.05]	[-0.85, -0.82]
Total Debt to EBITDA	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[-0.01, 0]	[-0.01, -0.01]	[-0.02, -0.02]
Total Debt to Equity	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0, 0]	[0.01, 0.01]
RM-RF (SP100-Rf)	[0.97, 0.98]	[0.96, 0.98]	[0.96, 0.97]	[0.96, 0.97]	[0.96, 0.97]	[0.95, 0.97]	[0.94, 0.96]	[0.98, 1.01]	[1.09, 1.12]	[1.25, 1.29]
Momentum (TTM)	[0, 0.01]	[0.01, 0.01]	[0.01, 0.01]	[0.01, 0.02]	[0.01, 0.02]	[0.01, 0.02]	[0.03, 0.04]	[0.04, 0.05]	[0.03, 0.04]	[-0.03, -0.01]

This table is a continuation of table 7 and shows coefficient confidence intervals using an alpha parameter of 0.01. Numbers are rounded to the second decimal. A confidence interval of [0.42, 0.61] for 30 days option-implied standard deviation (BKM) can be interpreted as follows: for a 1 unit increase in option-implied standard deviation (BKM), total realized return would be expected to increase (in average) by an amount ranging between 0.42% and 0.61%. Several variables confidence intervals are [0, 0]. Consequently, although some of these coefficients may be statistically significant, their effect is marginal. (less than 1/100th of a percent.)

Table 8: Quartile portfolios mean return differences (Q4 High factor values – Q1 Low factor values)

Days to maturity	30days	60days	91days	122days	152days	182days	273days	365days	547days	730days
Option-Implied Mean (B-L)	-0.0059	-0.0151	-0.0275	-0.0405	-0.0534	-0.0682	-0.1106	-0.1617	-0.2557	-0.3524
Option-Implied Standard Deviation (B-L)	0.0110	0.0203	0.0287	0.0404	0.0529	0.0661	0.1047	0.1543	0.2513	0.3457
Option-Implied Skewness (B-L)	-0.0054	-0.0088	-0.0068	0	0.0146	0.0266	0.0729	0.1302	0.2462	0.3434
Option-Implied Kurtosis (B-L)	0.0024	0.0047	0.0111	0.0121	0.0136	0.0132	0	-0.0319	-0.0869	-0.1171
Skewness * Kurtosis Interaction (B-L)	-0.0041	-0.0071	0	0.0070	0.0177	0.0290	0.0705	0.1174	0.2318	0.3186
Option-Implied Mean (BKM)	-0.0116	-0.0204	-0.0292	-0.0405	-0.0539	-0.0674	-0.1089	-0.1601	-0.2611	-0.3653
Option-Implied Standard Deviation (BKM)	0.0116	0.0203	0.0289	0.0402	0.0536	0.0671	0.1088	0.1600	0.2637	0.3654
Option-Implied Skewness (BKM)	0.0071	0.0143	0.0215	0.0298	0.0409	0.0515	0.0821	0.1193	0.2147	0.3197
Option-Implied Kurtosis (BKM)	-0.0100	-0.0205	-0.0312	-0.0437	-0.0581	-0.0724	-0.1113	-0.1581	-0.2543	-0.3501
Skewness * Kurtosis Interaction (BKM)	0.0084	0.0162	0.0236	0.0323	0.0431	0.0538	0.0886	0.1297	0.2266	0.3329
Dollar Volume	-0.0073	-0.0133	-0.0185	-0.0250	-0.0297	-0.0348	-0.0519	-0.0810	-0.1440	-0.2064
Market Capitalization	-0.0132	-0.0258	-0.0392	-0.0536	-0.0681	-0.0829	-0.1301	-0.1873	-0.3015	-0.4312
Price to Book	0	0	0.0045	0.0052	0.0060	0.0082	0.0139	0.0337	0.0696	0.1329
Price to Cash Flow	0	0	0	-0.0069	-0.0099	-0.0126	-0.0255	-0.0326	0	0
ROE	0	0	-0.0041	-0.0073	-0.0111	-0.0157	-0.0303	-0.0473	-0.0973	-0.1520
Total Debt to EBITDA	-0.0041	-0.0076	-0.0096	-0.0106	-0.0128	-0.0168	-0.0347	-0.0529	-0.0974	-0.1748
Total Debt to Equity	-0.0034	-0.0065	-0.0092	-0.0110	-0.0129	-0.0166	-0.0330	-0.0547	-0.1173	-0.2220
Momentum (TTM)	0	0	0	0	0.0075	0.0121	0.0225	0.0342	0.0261	0

This table shows the difference between the quartile 4 and the quartile 1 (Q4-Q1) mean realized returns for each time to maturity group. For example, the Option-Implied Mean (B-L) 730days value of -0.3524 represents the 20 years mean return difference between a portfolio composed of the companies with the highest (quartile 4) option-implied mean values and those with the lowest option-implied mean. Every day from 1996 to 2016, quartile portfolios are created for each different factor and mean returns for those factor portfolios are computed for that day. The first and fourth quartile 20 year mean returns are then compared and tested for equal (unequal) mean using a two-sample t-test. In the case of Option-Implied Mean (B-L), the alternative hypothesis was chosen, meaning that the 20-year return distribution mean of the two quartile factor portfolios were statistically different at the 5% level. When the outcome was that Q4 mean and Q1 mean are not statistically different from one-another, the value was replaced by 0, in other words, there was no difference in returns between quartile (Q4-Q1) portfolios.

A potential pitfall when performing regressions using large samples is the fact that larger sample sizes have smaller standard error and smaller p-values. Thus, relying solely on coefficient sizes and p-values when performing regressions using large samples could potentially be misleading. As an alternative, the quartiles portfolio methodology is used to compare results.

Table 9: Summary

Factors\Methodology	Table 6 Regression B-L	Table 7 Regression BKM	Table 8 Quartiles (Q4High-Q1Low)	Summary:
Option-Implied Mean (B-L)	Unclear		Negative	Unclear
Option-Implied Standard Deviation (B-L)	Positive		Positive	Higher factor value higher realized returns.
Option-Implied Skewness (B-L)	Positive		Unclear	Unclear.
Option-Implied Kurtosis (B-L)	Positive		Unclear	Unclear
Skewness * Kurtosis Interaction (B-L)	Negative		Positive	Unclear
Option-Implied Mean (BKM)		Positive	Negative	Unclear
Option-Implied Standard Deviation (BKM)		Positive	Positive	Higher factor value higher realized returns.
Option-Implied Skewness (BKM)		Positive	Positive	Higher factor value higher realized returns.
Option-Implied Kurtosis (BKM)		Unclear	Negative	Unclear
Skewness * Kurtosis Interaction (BKM)		Negative	Positive	Unclear
Dollar Volume	Negative	Negative	Negative	Lower factor value higher realized returns.
Market Capitalization	Negative	Negative	Negative	Lower factor value higher realized returns.
Price to Book	Unclear	Unclear	Positive	Unclear
Price to Cash Flow	Negative	Negative	Negative	Lower factor value higher realized returns.
ROE	Unclear	Unclear	Negative	Unclear
Total Debt to EBITDA	Negative	Negative	Negative	Lower factor value higher realized returns.
Total Debt to Equity	Positive	Positive	Negative	Unclear
Momentum (TTM)	Positive	Positive	Positive	Higher factor value higher realized returns.

The table above shows the obtained relationship sign between realized total return and factors using different methodologies. The quartiles methodology compares returns between high and low quartile portfolios. A two-sample t-test is used to see if quartile portfolio returns are significantly different from one-another. The regression B-L and BKM methodologies use multiple regressions to find which factors are statistically significant and determine their coefficients.

It is important to state that the largest contributor to R-squared values shown in table 6 and 7 is by far the RM-Rf (SP100-Rf) factor which represents the S&P 100 market premium over risk-free rates. Thus, while other factors appear significant, their effect is relatively small in absolute term. Table 8 does not include market premium and provides a different methodology which helps determining whether an observed relationship can be confirmed or should rather be contested.

Comparing the multiple regressions (both methodologies) and quartile portfolios results found in table 6, 7 and 8 above, we can observe that option-implied standard deviation is positive and statistically significant for all. This means that, in average, stocks which have higher expected volatility tend to outperform those with lower expected volatility. Similarly, option-implied skewness and momentum (computed as trailing last twelve month returns) are positively related to realized returns most of the time.

On the other hand, dollar volume, market capitalization, price to cash flow and total debt to EBITDA ratios are significantly negatively related to realized returns, suggesting that smaller size companies with high cash flow per share and low debt burden likely outperform large companies that have higher debt to EBITDA ratios and lower cash flow per share. The above-mentioned relations appear to be logical and conform with results obtained from past studies.

However, note that the multiple regressions and quartile portfolio tests are done using the entire time window. Consequently, it is possible that some of these results (particularly option-implied skewness) might not hold within specific sub-periods. We also note that the option-implied skewness * option-implied kurtosis interaction term is positive for both methodologies when using the quartiles portfolio test whereas it is negative when performing the multiple regression. Thus, we cannot conclude with certainty the sign of this interaction term. In theory, risk-averse investors should require a higher expected return for investments that exhibit negative skewness and high kurtosis. As a result, when the interaction value of skewness*kurtosis is large and negative, investors should require higher expected returns. Finally, it is important to mention that some of the above-mentioned coefficient sizes may not only differ in size but also in realizability. For example, looking at table 6 p.40, coefficient intervals for option-implied σ with time to maturity of 182 days values of [0.57, 0.65] can be interpreted as follows: for a 0.1 unit increase in option-implied standard deviation (B-L 182 days), total realized return would be expected to increase (in average) by an amount ranging between 0.057% and 0.065%. The quartile portfolios value 0.0661% for this same variable presents an accurate overview of the size that can realistically be realized in 182 days. On an annual basis (0.0661*(365/182)), alpha generated would be approximately equal to 0.1326%, which is relatively close to the obtained value of 0.1543% when looking at 365 days to expiration groups.

6. CONCLUSIONS

This paper analyzes the information content of option-implied moments. We find that realized daily return moments are not well predicted by their respective option-implied moment except for standard deviation which is relatively accurate. Thus, option-implied standard deviation is a reasonably accurate predictor of future realized daily return standard deviation. In addition, in terms of methodology used, the BKM approach does not appear to be positively impacted by data interpolation whereas the B-L approach benefits from such interpolation. However, the interpolation benefits quickly decay, meaning that balancing the number of interpolated data points (to 52 instead of 152 or 502 for example), can provide a close to optimal prediction ability while greatly reducing the computing requirements. Next, when regressing option-implied moments in addition to various fundamental and market data with realized total returns, we find that firms that have higher option-implied volatility and option-implied skewness, lower market capitalization, dollar volume, price to cash flows and total debt to EBITDA tend to outperform their opposites. Except for option-implied skewness which is found to be unstable across time windows, other results observed are mostly conform with previous literature and have stable coefficient signs. As mentioned earlier, the best time to buy stocks generally is when implied volatility has reached high levels and is starting to decrease due to positive momentum. At that moment, quality mid-cap firms that are financially healthy (Low debt to EBITDA, high cash flow per share) can provide good risk-reward return potential.

7. LIMITATIONS AND FUTURE RESEARCH

One potential pitfall from the research is the fact during sample construction, keeping only firms that remained in the S&P 100 index could have created a survivorship bias. Only firms that maintained the index criterions during the entire time window remain. Consequently, it is possible that some firms stock price declined significantly for some reason, hence generally leading to an increase of that firm's perceived risk. Since only firms which remained in the index are retained, as a by-product, it might also have removed some firms which stock prices never recovered and kept firms that survived through those more difficult times, hence potentially explaining a portion of the excess return obtained from those stocks with higher expected volatility and skewness.

First, future research could be done to address this issue whilst also including smaller capitalization companies that have liquid options market. Similarly, companies from other countries or exchanges could be analyzed.

Finally, other interesting areas for future research could be for instance to test strategies which increases a portfolio allocation towards equity when implied volatility previously reached high levels and began decreasing. In contrast, it would decrease equity allocation and increase fixed income or alternative investments allocation when VIX is increasing while SKEW is at or near high levels.

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9. APPENDICES

Table 10: Company List

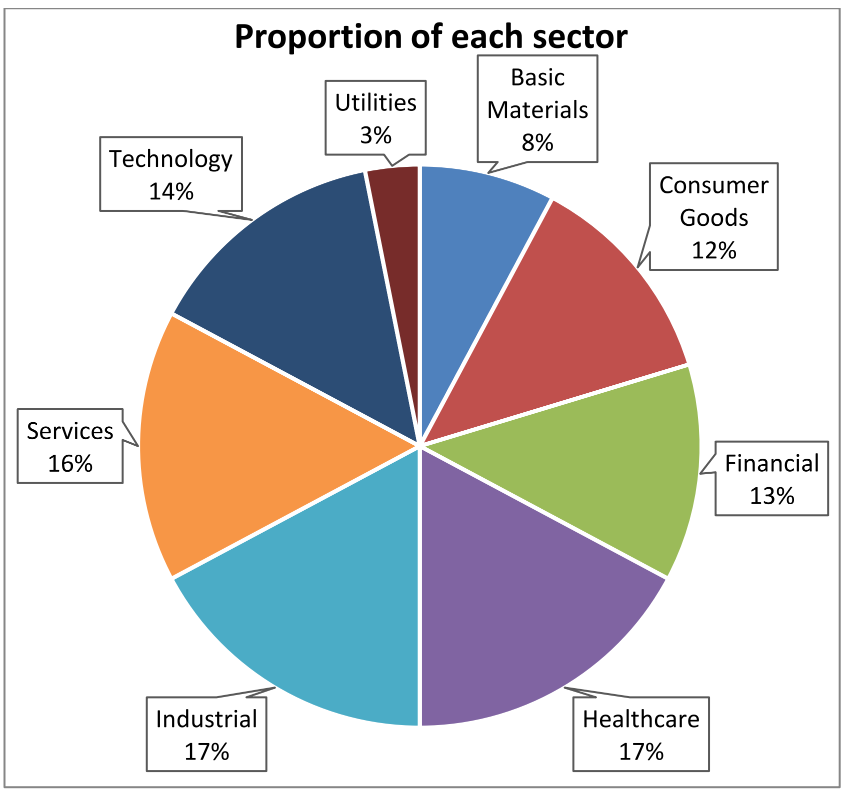
Company name	Ticker	SECID(s)	Industry Group #	Main industry (Sector)
Abbott Laboratories	ABT	100972	521	Healthcare
Allstate Corp	ALL	101273	432	Financial
Time Warner Inc.	TWX	101328	722	Services
American Express Co	AXP	101375	424	Financial
American International Group Inc.	AIG	101397	432	Financial
Amgen Inc.	AMGN	101508	515	Healthcare
Apple Inc.	AAPL	101594	314	Consumer Goods
Bank of America	BAC	101966	410	Financial
Boeing Co	BA	102265	611	Industrial
Bristol-Myers Squibb Co	BMY	102349	510	Healthcare
Capital One Financial Corp	COF	102702	424	Financial
Caterpillar Inc.	CAT	102796	620	Industrial
Celgene Corp	CELG	102822	515	Healthcare
J P Morgan Chase & Co	JPM	102936	410	Financial
Cisco Systems Inc.	CSCO	103042	814	Technology
Coca-Cola Co	KO	103125	348	Consumer Goods
Colgate Palmolive Co	CL	103157	323	Consumer Goods
Comcast Corp	CMCSA	103198	722	Services
Danaher Corp	DHR	103676	622	Industrial
Walt Disney Co	DIS	103879	722	Services
Dow Chemical Co	DOW	103936	110	Basic Materials
E. I du Pont de Nemours & Co	DD	103969	112	Basic Materials
Duke Energy Corp	DUK	103979	913	Utilities
EMC Corp	EMC	104049	813	Technology
Emerson Electric Co	EMR	104286	627	Industrial
Fedex Corp	FDX	104635	773	Services
Ford Motor Co	F	104939	330	Consumer Goods
General Dynamics Corp	GD	105168	611	Industrial
General Electric Co	GE	105169	622	Industrial
Gilead Sciences Inc.	GILD	105243	515	Healthcare
Halliburton Co	HAL	105512	124	Basic Materials
Home Depot Inc.	HD	105759	736	Services
Honeywell Inc.	HON	105785	622	Industrial
Intel Corp	INTC	106203	830	Technology
International Business Machines Corp	IBM	106276	824	Technology
Johnson & Johnson	JNJ	106566	510	Healthcare
Lilly Eli & Co	LLY	106967	510	Healthcare
Lockheed Martin Corp	LMT	107010	611	Industrial
Lowe's Companies Inc.	LOW	107045	736	Services
Mc Donalds Corp	MCD	107318	712	Services
Bank of New York Mellon	BK	107407	422	Financial
Merck & Co Inc.	MRK	107430	510	Healthcare
Microsoft Corp.	MSFT	107525	826	Technology
3M Co	MMM	107616	622	Industrial
Morgan Stanley	MS	107704	420	Financial
Nike Inc.	NKE	108161	321	Consumer Goods
Occidental Petroleum Corp	OXY	108385	121	Basic Materials
Oracle Corp	ORCL	108505	821	Technology
Pepsico Inc.	PEP	108893	348	Consumer Goods
Altria Group Inc.	MO	108965	350	Consumer Goods

Company name	Ticker	SECID(s)	Industry Group #	Main industry (Sector)
Procter & Gamble Co	PG	109224	323	Consumer Goods
Pfizer Inc.	PFE	108948	510	Healthcare
Qualcomm Inc.	QCOM	109348	841	Technology
Raytheon Co	RTN	109497	611	Industrial
AT&T Corp.	T	109775	844	Technology
Schlumberger Ltd	SLB	109956	124	Basic Materials
Southern Co	SO	110337	911	Utilities
Starbucks Corp	SBUX	110472	713	Services
Texas Instruments Inc.	TXN	110972	830	Technology
Union Pacific Corp	UNP	111394	776	Services
United Technologies Corp	UTX	111459	611	Industrial
UnitedHealth Group Inc.	UNH	111469	522	Healthcare
Wal-Mart Stores Inc.	WMT	111860	732	Services
Allergan Inc.	AGN	111907	511	Healthcare

This table presents the list of S&P 100 constituent stocks which were included in the sample studied in this paper.

Figure 6. Sample firm sector proportions

Sector	Total
Basic Materials	5
Consumer Goods	8
Financial	8
Healthcare	11
Industrial	11
Services	10
Technology	9
Utilities	2
Grand Total	**64**



APPENDIX A – Formula to compute the strike price of a continuous dividend yield European option contract given the delta value, stock price, dividends, time to expiration, volatility and risk-free interest rates.

This formula starts from the B-S delta formula and isolates the strike (K) value. Formulas for call and puts are applicable to European options with continuous dividend yield. (Thus, it applies to European option on indexes.) In the case of American option with discrete cash dividends, the formula would need proper adjustments.

Call option delta calculation steps:

$$\Delta = e^{-qt}N(d1)$$

$$\frac{\Delta}{e^{-qt}} = N(d1)$$

$$normsinv\left(\frac{\Delta}{e^{-qt}}\right) = d1$$

$$normsinv\left(\frac{\Delta}{e^{-qt}}\right) = \frac{ln(S/K)+\left(r-q+\sigma^2/2\right)t}{\sigma\sqrt{t}}$$

$$normsinv\left(\frac{\Delta}{e^{-qt}}\right) \times \left(\sigma\sqrt{t}\right) = ln(S/K) + (r - q + \sigma^2/2)t$$

$$normsinv\left(\frac{\Delta}{e^{-qt}}\right) \times \left(\sigma\sqrt{t}\right) - (r - q + \sigma^2/2)t = ln(S/K)$$

$$exp\left(normsinv\left(\frac{\Delta}{e^{-qt}}\right) \times \left(\sigma\sqrt{t}\right) - (r - q + \sigma^2/2)t\right) = S/K$$

Put option delta:

$$\Delta = -e^{-qt}N(-d1)$$

$$d1 = \frac{ln(S/K)+\left(r-q+\sigma^2/2\right)t}{\sigma\sqrt{t}}$$

Formula for European call option:

$$K = \frac{S}{exp\left(normsinv\left(\frac{\Delta}{e^{-qt}}\right)\times\left(\sigma\sqrt{t}\right)-(r-q+\sigma^2/2)t\right)}$$

Formula for European put option:

$$K = \frac{S}{exp\left(-normsinv\left(\frac{\Delta}{-e^{-qt}}\right)\times\left(\sigma\sqrt{t}\right)-(r-q+\sigma^2/2)t\right)}$$

APPENDIX B – Computing Risk-Neutral Implied Moments Using the Bakshi, Kapadia and Madan (2003) Methodology

This appendix shows the formulas used by Bakshi, Kapadia and Madan (2003) and the corresponding MATLAB code used to compute the Risk-Neutral option-implied moments using the BKM methodology. For additional details, please refer to the original paper and/or code. The MATLAB Option Implied Moments toolbox shared on the MATLAB File Exchange by Matthias Held (Updated 30 July 2014).

First, the BKM *theorem 1* states that under all martingale pricing measures, the prices of volatility, cubic and quartic contracts can be recovered from the market prices of OTM European calls and puts. Using those contracts, the mean, volatility, skewness and kurtosis of the risk-neutral return density can be extracted. The authors also assert that their theorem also generalizes to American options. They support this by stating that OTM options have negligible early exercise premiums and that even when early exercise premiums are substantial (for example for OTM options that are close to being ATM), the weight assigned to those options is small.

The notation used for computing these contracts are as follows: S(t) represents the stock price at time t, r is the interest rate, C(t,τ;K) and P(t,τ;K) are prices of call and put options written on the stock with current price S(t), τ is the time to expiration and K is the strike price.

$$Vol^{BKM}(t,\tau) = \sqrt{e^{r\tau}V(t,\tau) - \mu(t,\tau)^2}$$

$$SKEW^{BKM}(t,\tau) = \frac{e^{r\tau}W(t,\tau) - 3\mu(t,\tau)e^{r\tau}V(t,\tau) + 2\mu(t,\tau)^3}{[e^{r\tau}V(t,\tau) - \mu(t,\tau)^2]^{3/2}}$$

$$KURT^{BKM}(t,\tau) = \frac{e^{r\tau}X(t,\tau) - 4\mu(t,\tau)e^{r\tau}W(t,\tau) + 6e^{r\tau}\mu(t,\tau)^2\,V(t,\tau) - 3\mu(t,\tau)^4}{[e^{r\tau}V(t,\tau) - \mu(t,\tau)^2]^2}$$

Where the mean stock return $\mu(t,\tau)$ is:

$$\mu(t,\tau) = e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V(t,\tau) - \frac{e^{r\tau}}{6}W(t,\tau) - \frac{e^{r\tau}}{24}X(t,\tau),$$

the price of the volatility contract $V(t,\tau)$ is:

$$V(t,\tau) = \int_{S(t)}^{\infty} \frac{2\left(1 - ln\left[\frac{K}{S(t)}\right]\right)}{K^2} C(t,\tau;K)dK + \int_{0}^{S(t)} \frac{2\left(1 + ln\left[\frac{S(t)}{K}\right]\right)}{K^2} P(t,\tau;K)dK,$$

the price of the cubic contract $W(t,\tau)$ is:

$$W(t,\tau) = \int_{S(t)}^{\infty} \frac{6\, ln\left[\frac{K}{S(t)}\right] - 3\left(ln\left[\frac{K}{S(t)}\right]\right)^2}{K^2} C(t,\tau;K)dK - \int_{0}^{S(t)} \frac{6\, ln\left[\frac{S(t)}{K}\right] + 3\left(ln\left[\frac{S(t)}{K}\right]\right)^2}{K^2} P(t,\tau;K)dK,$$

and the price of the quartic contract $X(t,\tau)$ is:

$$X(t,\tau) = \int_{S(t)}^{\infty} \frac{12\left(ln\left[\frac{K}{S(t)}\right]\right)^2 - 4\left(ln\left[\frac{K}{S(t)}\right]\right)^3}{K^2} C(t,\tau;K)dK + \int_{0}^{S(t)} \frac{12\left(ln\left[\frac{S(t)}{K}\right]\right)^2 + 4\left(ln\left[\frac{S(t)}{K}\right]\right)^3}{K^2} P(t,\tau;K)dK.$$

The MATLAB function takes the form of:

Output moments = mOption2stat(XC,C,XP,P,S0,DF,N)

Where XC and XP are column vectors of strike prices, C and P are column vectors of Call and Put prices. XC and C must me of the same size and with matching rows. Likewise, for XP and P. S0 is the underlying asset price, DF is the risk-free discount factor (both are scalar values). N is a vector holding the required moments. (i.e. if equal 4, it computes the first four moments)

Using the above formula calls the mOption2stat.m file code and returns the first N standardized moments of an asset's risk neutral log return distribution from traded put and call options traded on that asset, with equal time to expiration.

Appendix C: CBOE SKEW – Comparing Estimated to Realized Risk-Adjusted Probabilities of S&P 500 Log returns Two and Three Standard Deviations below the Mean

In its white paper published in 2010, the CBOE indicates that the SKEW index can be interpreted as a risk-adjusted probability that the one-month S&P 500 log-return falls two or three standard deviations below the mean and that the VIX can be used as an indicator of the magnitude of the standard deviation. Table 2 of their paper shows probabilities which are based on overlaying risk-neutral skewness over a normal distribution, as done in a Gram-Charlier expansion of the normal distribution. Thus, when SKEW = 100, the distribution of the S&P500 log-returns is normal, meaning that the probability of returns +-2σ is 4.6% (2.3% on each side) and +-3σ is 0.3% (0.15% on each side). As skew increases to levels beyond 100, a skewness term is added to the normal distribution, leading to probabilities shown in the replicated table 1 below:

Table A1: Estimated Risk-Adjusted Probabilities of S&P 500 Log Returns Two and Three Standard Deviations below the Mean

Estimated Risk-Adjusted Probabilities of S&P 500 Log Returns		
	S&P 500 30-Day Log Return	
SKEW	2 Std. Dev	3 Std. Dev
100	2.30%	0.15%
105	3.65%	0.45%
110	5.00%	0.74%
115	6.35%	1.04%
120	7.70%	1.33%
125	9.05%	1.63%
130	10.40%	1.92%
135	11.75%	2.22%
140	13.10%	2.51%
145	14.45%	2.81%

Source: CBOE, 2010. “The CBOE Skew Index - SKEW”, SKEW White Paper.

Intrigued by this theory proposition, I performed a quick statistical analysis of historical S&P 500 30-day log returns at various SKEW index level for the period ranging from the 2nd January 1990 to the 31st of July 2017. It was done as follows: First, data for the VIX, SKEW and S&P 500 Total Return was gathered from the CBOE website. Next, daily VIX values were adjusted as follows so that instead of representing annualized implied volatility, it represents monthly implied volatility:

$VIX_{Monthly} = VIX/\sqrt{12}$

Then 30-day realized log returns were divided by $VIX_{Monthly}$ to finally obtain the realized returns in terms of standard deviations ($VIX_{Monthly}$). This information was then divided in skew-value categories such as in Table 1 above. Results obtained are as follow:

Table A2: Realized Risk-Adjusted Probabilities of S&P 500 Log Returns Two and Three Standard Deviations below the Mean

Realized Risk-Adjusted Probabilities of S&P 500 Log Returns			
		S&P 500 30-Day Log Return	
SKEW	**Sample size**	**% <= -2 Std. Dev**	**% <= -3 Std. Dev**
[100-105]	21	0.00%	0.00%
]105-110]	449	1.11%	0.67%
]110-115]	1854	1.24%	0.32%
]115-120]	2094	0.76%	0.05%
]120-125]	1391	0.72%	0.22%
]125-130]	638	0.94%	0.00%
]130-135]	272	1.10%	0.00%
]135-140]	180	0.00%	0.00%
]140-145]	35	0.00%	0.00%
]145+	17	0.00%	0.00%

Finally, the results suggest that the relationship between SKEW index value and realized returns in terms of $VIX_{Monthly}$ is far from values previously estimated by the CBOE in the SKEW white paper. Furthermore, as shown by table 3 and 4 on the following page, small sample sizes for skew values below 105 or above 140 makes it trivial to make any conclusion for those groups. As for groups ranging from 105 to 140, they do not appear to have significant differences. However, testing different time frames (more than 30 days) could provide interesting results.

Table A3:

Frequency (sample size) for different skew values and realized 30 days return in number of STDEV (VIX)										
σ\SKEW	**[100-105]**	**]105-110]**	**]110-115]**	**]115-120]**	**]120-125]**	**]125-130]**	**]130-135]**	**]135-140]**	**]140-145]**	**>145**
]-∞, -4]	0	2	2	0	0	0	0	0	0	0
]-4, -3]	0	1	4	1	3	0	0	0	0	0
]-3, -2]	0	2	17	15	7	6	3	0	0	0
]-2, -1]	0	18	121	113	94	32	15	5	4	1
]-1, 0]	0	134	568	587	381	176	81	80	8	9
]0, 1]	14	247	968	1209	806	377	154	85	21	5
]1, 2]	7	43	174	168	100	47	19	10	2	2
]2, 3]	0	2	0	1	0	0	0	0	0	0
]3, 4]	0	0	0	0	0	0	0	0	0	0
Total	**21**	**449**	**1854**	**2094**	**1391**	**638**	**272**	**180**	**35**	**17**

Table A4:

Frequency (%) for different skew values and realized 30 days return in number of STDEV (VIX)										
σ\SKEW	**[100-105]**	**]105-110]**	**]110-115]**	**]115-120]**	**]120-125]**	**]125-130]**	**]130-135]**	**]135-140]**	**]140-145]**	**>145**
]-∞, -4]	0.00%	0.45%	0.11%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-4, -3]	0.00%	0.22%	0.22%	0.05%	0.22%	0.00%	0.00%	0.00%	0.00%	0.00%
]-3, -2]	0.00%	0.45%	0.92%	0.72%	0.50%	0.94%	1.10%	0.00%	0.00%	0.00%
]-2, -1]	0.00%	4.01%	6.53%	5.40%	6.76%	5.02%	5.51%	2.78%	11.43%	5.88%
]-1, 0]	0.00%	29.84%	30.64%	28.03%	27.39%	27.59%	29.78%	44.44%	22.86%	52.94%
]0, 1]	66.67%	55.01%	52.21%	57.74%	57.94%	59.09%	56.62%	47.22%	60.00%	29.41%
]1, 2]	33.33%	9.58%	9.39%	8.02%	7.19%	7.37%	6.99%	5.56%	5.71%	11.76%
]2, 3]	0.00%	0.45%	0.00%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]3, 4]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Total	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**

Finally, in the tables presented on the following pages, it is worth noting that the 30 and 365 days return frequency for various VIX and SKEW levels appear to follow some pattern. Notably, the worst forward 365 days returns have often occurred when the VIX reached levels between 20 and 30. Furthermore, when the VIX is either below 15 or above 35, it is followed by positive returns close to 90% of the time. Regarding the SKEW, the worst 365 days returns occurred with greater frequency when the indicator reached levels between 110 and 115. In terms of positive returns, one-year realized total returns tend to be highest when reaching points of maximum pessimism (VIX reaching above 35). In overall, it is best to wait for the VIX to reach high levels and wait for it to start declining (which should happen if prices stabilize or increase.) After reaching such levels, seeing both VIX and SKEW declining simultaneously could suggest a potential (365 days time frame) buying opportunity. Naturally, this is complementary to fundamental and technical analysis.

Frequency for different vix values and realized next 30 days total return

Return\VIX	[0,10]	]10,15]	]15,20]	]20,25]	]25,30]	]30,35]	]35,40]	]40,45]	]45,50]	>50
]-100%, -90%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-90%, -80%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-80%, -70%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-70%, -60%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-60%, -50%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-50%, -40%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-40%, -30%]	0.00%	0.00%	0.00%	0.14%	0.15%	0.70%	0.81%	0.00%	0.00%	0.00%
]-30%, -20%]	0.00%	0.00%	0.05%	0.07%	0.15%	1.74%	1.61%	3.80%	0.00%	1.79%
]-20%, -10%]	0.00%	0.00%	1.45%	2.97%	3.20%	1.05%	7.26%	6.33%	15.15%	10.71%
]-10%, 0%]	46.15%	32.66%	35.15%	37.39%	33.69%	23.34%	17.74%	17.72%	42.42%	44.64%
]+0%, +10%]	53.85%	67.34%	62.99%	58.50%	60.37%	67.60%	65.32%	41.77%	18.18%	32.14%
]+10%, +20%]	0.00%	0.00%	0.35%	0.92%	2.44%	5.57%	7.26%	30.38%	18.18%	8.93%
]+20%, +30%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	6.06%	1.79%
]+30%, +40%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+40%, +50%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+50%, +60%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+60%, +70%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+70%, +80%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+80%, +90%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+90%, +100%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Sample size	**26**	**2281**	**1997**	**1412**	**656**	**287**	**124**	**79**	**33**	**56**

Frequency for different vix values and realized next 365 days total return

Return\VIX	[0,10]	]10,15]	]15,20]	]20,25]	]25,30]	]30,35]	]35,40]	]40,45]	]45,50]	>50
]-100%, -90%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-90%, -80%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-80%, -70%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-70%, -60%]	0.00%	0.00%	0.00%	0.07%	0.61%	0.00%	0.00%	0.00%	0.00%	0.00%
]-60%, -50%]	0.00%	0.00%	0.30%	1.49%	1.52%	0.35%	0.00%	0.00%	0.00%	0.00%
]-50%, -40%]	0.00%	0.00%	1.37%	3.33%	5.49%	1.05%	0.00%	0.00%	0.00%	0.00%
]-40%, -30%]	0.00%	0.00%	1.12%	3.26%	0.91%	0.00%	0.00%	0.00%	0.00%	0.00%
]-30%, -20%]	0.00%	0.00%	3.55%	10.00%	5.49%	3.14%	1.61%	0.00%	0.00%	0.00%
]-20%, -10%]	0.00%	0.29%	2.48%	11.70%	12.04%	7.32%	4.84%	5.06%	0.00%	0.00%
]-10%, 0%]	11.11%	6.78%	3.09%	6.74%	6.25%	2.79%	4.03%	0.00%	9.09%	0.00%
]+0%, +10%]	88.89%	34.84%	26.47%	14.18%	13.11%	5.92%	2.42%	0.00%	3.03%	7.14%
]+10%, +20%]	0.00%	40.38%	38.84%	31.99%	29.27%	31.71%	23.39%	3.80%	3.03%	28.57%
]+20%, +30%]	0.00%	14.84%	16.78%	14.61%	24.54%	34.84%	45.97%	55.70%	30.30%	46.43%
]+30%, +40%]	0.00%	2.86%	5.53%	1.91%	0.76%	12.89%	17.74%	22.78%	36.36%	10.71%
]+40%, +50%]	0.00%	0.00%	0.46%	0.71%	0.00%	0.00%	0.00%	12.66%	12.12%	5.36%
]+50%, +60%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	6.06%	1.79%
]+60%, +70%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+70%, +80%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+80%, +90%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+90%, +100%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Sample size	**9**	**2095**	**1972**	**1410**	**656**	**287**	**124**	**79**	**33**	**56**

Frequency for different skew values and realized next 30 days total return

Return%\SKEW	[100-105]	]105-110]	]110-115]	]115-120]	]120-125]	]125-130]	]130-135]	]135-140]	]140-145]	>145
]-100%, -90%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-90%, -80%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-80%, -70%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-70%, -60%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-60%, -50%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-50%, -40%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-40%, -30%]	0.00%	**0.22%**	**0.27%**	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-30%, -20%]	0.00%	**0.45%**	**0.32%**	**0.10%**	**0.22%**	**0.16%**	0.00%	0.00%	0.00%	0.00%
]-20%, -10%]	0.00%	**0.45%**	**2.42%**	**1.57%**	**1.65%**	**1.72%**	**1.83%**	**0.56%**	0.00%	0.00%
]-10%, 0%]	0.00%	**33.85%**	**35.41%**	**32.49%**	**32.93%**	**31.67%**	**34.80%**	**46.67%**	**36.11%**	**58.82%**
]+0%, +10%]	**100.00%**	**61.69%**	**59.31%**	**64.46%**	**64.78%**	**66.15%**	**61.90%**	**52.22%**	**63.89%**	**41.18%**
]+10%, +20%]	0.00%	**3.34%**	**2.15%**	**1.34%**	**0.43%**	**0.31%**	**1.47%**	**0.56%**	0.00%	0.00%
]+20%, +30%]	0.00%	0.00%	**0.11%**	**0.05%**	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+30%, +40%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+40%, +50%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+50%, +60%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+60%, +70%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+70%, +80%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+80%, +90%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+90%, +100%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Sample size	21	449	1858	2096	1394	641	273	180	36	17

Frequency for different skew values and realized next 365 days total return

Return\SKEW	[100-105]	]105-110]	]110-115]	]115-120]	]120-125]	]125-130]	]130-135]	]135-140]	]140-145]	>145
]-100%, -90%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-90%, -80%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-80%, -70%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-70%, -60%]	0.00%	0.00%	**0.27%**	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]-60%, -50%]	0.00%	**0.22%**	**1.45%**	**0.43%**	**0.07%**	0.00%	0.00%	0.00%	0.00%	0.00%
]-50%, -40%]	0.00%	**2.67%**	**3.61%**	**1.63%**	**0.07%**	0.00%	0.00%	0.00%	0.00%	0.00%
]-40%, -30%]	0.00%	**0.45%**	**2.64%**	**1.01%**	**0.15%**	0.00%	0.00%	0.00%	0.00%	0.00%
]-30%, -20%]	0.00%	**2.45%**	**6.35%**	**4.45%**	**2.74%**	0.00%	0.00%	0.00%	0.00%	0.00%
]-20%, -10%]	0.00%	**6.68%**	**9.36%**	**5.03%**	**1.48%**	**0.17%**	0.00%	0.00%	0.00%	0.00%
]-10%, 0%]	0.00%	**2.23%**	**4.25%**	**3.59%**	**7.42%**	**10.34%**	**9.55%**	**7.19%**	**4.17%**	**14.29%**
]+0%, +10%]	**19.05%**	**16.48%**	**18.03%**	**23.65%**	**27.74%**	**24.31%**	**34.09%**	**48.20%**	**41.67%**	**0.00%**
]+10%, +20%]	**52.38%**	**29.62%**	**26.32%**	**33.03%**	**43.92%**	**52.24%**	**50.45%**	**39.57%**	**50.00%**	**42.86%**
]+20%, +30%]	**28.57%**	**23.83%**	**18.57%**	**23.31%**	**15.58%**	**12.24%**	**5.91%**	**5.04%**	**4.17%**	**42.86%**
]+30%, +40%]	0.00%	**13.59%**	**8.02%**	**3.40%**	**0.82%**	**0.69%**	0.00%	0.00%	0.00%	0.00%
]+40%, +50%]	0.00%	**1.78%**	**1.02%**	**0.43%**	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+50%, +60%]	0.00%	0.00%	**0.11%**	**0.05%**	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+60%, +70%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+70%, +80%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+80%, +90%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
]+90%, +100%]	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Sample size	21	449	1858	2089	1348	580	220	139	24	7

Appendix D: OptionMetrics Volatility Surface Methodology

Below is part of the methodology as explained in the OptionMetrics File and Data Reference Manual. For further details, please refer to the .pdf manual itself.

First, in the case of American options, OptionMetrics uses a proprietary pricing algorithm based on the Cox-Ross-Rubinstein (CRR) binomial tree model. Typically, a large number of tree periods (i.e. 1000) are required to obtain good results. The proprietary pricing algorithm has the advantage of achieving convergence towards those results, therefore requiring much lower computational requirement. The implied volatility of an option given its price is found by running the model iteratively with new values until the model price of the option converges to its market price, which is defined as the midpoint between the best closing bid and offer prices.

The Volatility_Surface file contains the interpolated volatility surface for each security on each day, using a methodology based on a kernel smoothing algorithm. This file contains information on standardized options, both calls and puts, with expirations of 30, 60, 91, 122, 152, 182, 273, 365, 547, and 730 calendar days, at deltas of 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, and 0.80 (negative deltas for puts). A standardized option is only included if there exists enough option price data on that date to accurately interpolate the required values.

The standardized option implied volatilities in the Volatility Surface file are calculated using a kernel smoothing technique. The data is first organized by the log of days to expiration and by "call-equivalent delta" (delta for a call, one plus delta for a put). A kernel smoother is then used to generate a smoothed volatility value at each of the specified interpolation grid points.

At each grid point *j* on the volatility surface, the smoothed volatility $\hat{\sigma}_j$ is calculated as a weighted sum of option implied volatilities:

$$\hat{\sigma}_j = \frac{\sum_i V_i \sigma_i \Phi(x_{ij}, y_{ij}, z_{ij})}{\sum_i V_i \, \Phi(x_{ij}, y_{ij}, z_{ij})}$$

where i is indexed over all the options for that day, V_i is the vega of the option, σ_i is the implied volatility, and $\Phi(.)$ is the kernel function:

$$\Phi(x_{ij}, y_{ij}, z_{ij}) = \frac{1}{\sqrt{2\pi}} e^{-\left[\left(x^2/_{2h_1}\right)+\left(y^2/_{2h_2}\right)+\left(z^2/_{2h_3}\right)\right]}$$

The parameters to the kernel function, x_{ij}, y_{ij}, and z_{ij} are measures of the "distance" between the option and the target grid point:

$$x_{ij} = \log(T_i/T_j)$$

$$y_{ij} = \Delta_i - \Delta_j$$

$$z_{ij} = I_{\{CP_i = CP_j\}}$$

where T_i (T_j) is the number of days to expiration of the option (grid point); Δ_i (Δ_j) is the "call-equivalent delta" of the option (grid point); CP_i (CP_j) is the call/put identifier of the option (grid point); and I{.} is an indicator function (=0 if the call/put identifiers are equal, or 1 if they are different).

The kernel "bandwidth" parameters were chosen empirically, and are set as *h1*=0.05, *h2*=0.005, and *h3*=0.001.