A Mixed Integer Programming Model to Minimize Fuel Consumption in Freight Train Operation

Zenan Zhang

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_____Chair
Dr. Masoumeh Kazemi Zanjani

_____Examiner
Dr. Onur Kuzgunkaya

_____Examiner
Dr. Zhi Chen

_____Supervisor
Dr. Mingyuan Chen

Approved by _____
      Chair of Department or Graduate Program Director

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Abstract

Fuel consumption is among the most important criteria in train operations. Considerable efforts have been made to identify optimal speed for enhanced fuel economy. The train speed control is achieved through switching the power notches, which sets constant level of fuel supply to the engine in a typical diesel-electric locomotive. A global optimal speed, however, cannot be considered appropriate due to the localized variations in the track gradient and curvature, apart from the variations in the load. An efficient train operation also requires the train completes its journey within a given travel time. This study aims at determining optimal train speeds to minimize the total fuel consumption in completing the journey, while considering the local variations in the track geometry and other properties. A nonlinear mixed integer programming model is formulated to solve the considered problem using an off-the-shelf optimization software package. A multiphase-steps improved method is proposed for solving the considered problem more effectively. Numerical examples are presented to illustrate the developed model and solution method.

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Chapter 1 Introduction

In this chapter, we introduce the motivation of this study and briefly introduce the considered problem.

1.1 Motivation

In recent years, energy efficiency in transportation has attracted notable attention due to increasing fuel price and awareness of environmental protection. Transportation nowadays is essential both for trade and business and for connecting people and communities. Recently, Canadian government published a Transportation 2030 Strategic Plan for ensuring a better, smarter, cleaner and safer transportation system throughout the country. Rail transportation sector is one of the three main parts in the transportation system. Up to now, there are 41,711 route- kilometers (km) of track with 19 intermodal terminals in Canada (*Transportation in Canada 2011*, 2012). In each year the rail transport industry generates approximate $10 billion in value, 95% of which comes from rail freight operations and approximately 5% from commuter, intercity and tourist passenger rail services in major urban centers, corridors and regions (*Transportation in Canada 2018*, 2019). To establish an efficient, safe and competitive rail system is the main objective of the public, railway operators, governments and other stake holders in Canada. Reducing fuel consumption also reduces greenhouse emission in the development of sustainable rail transportation in Canada and North America.

Reducing fuel consumption is the main research direction for efficient rail system operations. Researchers have built different mathematical models to minimize fuel consumption of train.

Analytical solutions, heuristics and other methods are applied to obtain efficient train control and to minimize fuel consumption. Therefore, a detailed train model and the method for minimizing fuel consumption have an important role.

1.2 Objectives and Contributions

The main objective of this thesis is to propose a strategy for minimizing fuel consumption of train operations in completing a journey between two adjacent stations. Based on the previous research, a new strategy for train drivers to operate trains using less fuel consumption by selecting optimal notches is considered. A nonlinear programming model is formulated to minimize fuel consumption in train operation. We propose a heuristic solution method of a multiple-steps to search for near optimal solutions to reduce computational time. Computational experiments using realistic example problems show very encouraging results.

1.3 Thesis Structure

Literature review of optimal fuel consumption in train operation is presented in Chapter 2. Based on the previous research, a detailed nonlinear discretized optimization model that determines the notch position in each segment and a heuristic solution method are introduced in Chapter 3. Chapter 4 includes the computational results in different conditions and constraints. At the end of this chapter, to prove the feasibility of heuristic solution method, the comparison results of heuristic solution method and using original equal segment model are presented. Conclusion and future research are introduced in Chapter 5.

Chapter 2 Literature Review

In this Chapter, we introduce the literature review in the field of minimizing fuel consumption in train operations.

2.1 Introduction

With development of economy and urbanization, transportation has a significant role in both passenger transport and freight. Rail transportation is always regarded as a low cost and high quantity transport method. To reduce energy cost of rail transportation has been the focus issue for researchers and practitioners in rail operations and management. Yang, Li, Ning and Tang (2016) states that *"Timetable optimization and energy-efficient driving, as two mainly used train operation methods in relation to the tractive energy consumption"*. This thesis focuses on the method using energy-efficient driving to minimize fuel consumption of train. In this chapter, we review the related literature from the perspective of two different model types: continuous and discrete models.

2.2 Continuous Models

A continuous model refers to a mathematical model where the state variables change continuously over time. From the related literature, authors applied Lagrange transfer and other mathematical methods to obtain optimal or near-optimal solutions. Compared with discrete models, solving continuous models may take more computational times and obtain more accurate solutions.

2.2.1 Continuous Models with Analytical Solutions

Many researchers have developed mathematical optimization formula to solve various train optimization control problems. Researchers developed such models to find optimal speed profiles of the train movement using mathematical analysis. The development of solution methods may require complicated analytical procedures to obtain accurate solutions.

Howlett (1990) considered an optimal train control problem for a train travelling on a level track between two stations within certain time period. The author developed an optimization model to minimize fuel consumption satisfying Pontryagin principle. The optimization model was based on that proposed in Milroy (1980) and other related development in the literature. The model considered 4 different phases of operation: acceleration, constant speed, coasting and braking. The optimal solution presents fuel consumption in each phase and accurate conversion points between the 4 phases within the given distance and time of the journey. The model was solved to achieve the minimum journey cost. The results show that certain parameters of the model can determine the complete journey within certain limit of the train speed.

Cheng, Cheng, Song and Zhao (2000) proposed a mathematical model based on that in Howlett (1996) for minimizing energy consumption in train control problem. Adjusting the values of two parameters in the equations is essential to control the switching points between successive phases of train operations. In their research the overall trip was divided into several segments and applied certain control switches in each segment to solve the considered problem.

Computer simulation was used to verify the developed results. To verify the feasibility of the developed algorithm, the authors compared two different methods with one based on Newton iterations and the other using a genetic penalty function. From the experimental results, it is observed that alternating power phase and coast phase was the best strategy for energy saving for the considered train operations.

Liu and Golovitcher (2003) discussed an optimal train operation process to minimize energy consumption. Based on previous researches work, the authors used a continuous traction force model considering constant efficiency. A Langragian based solution method was used to solve the developed differential equation with different optimal control phases represented by three model parameters. The input information stored in an on-board database is utilized in the calculation process for fast computation. A complete solution using the maximum principle in Ioffe and Tikhomirov (1974) for solving such problem is presented. From the results of the example problems considering electric and diesel locomotives, the authors found a set of optimal controls, control switching graphs and optimal complementary conditions in solving these problems.

Chang and Morlok (2005) considered the relationship between vehicle speed profiles and fuel consumption in rail transportation under clear boundary conditions. A more detailed and direct model was proposed based on several similar models in earlier studies. In this model, fuel consumption is considered to have a positive correlation with propulsion work. Inherent resistance of the train is considered as the only variable to determine the vehicle speed and the

model can be solved by a quadratic velocity formula. The authors conducted comprehensive analysis for different train speed trajectories to obtain accurate speed curves. They include level tangent paths, level tangent paths with acceleration and deceleration, level tangent combined with level curved section, level tangent section combined with upgrade tangent section and level tangent section combined with downgrade tangent section. The results show that the constant speed is the optimal speed profile for fuel consumption of a land transportation vehicle such as a train under different guideway characteristics.

Bai, Mao, Zhou, Ding and Dong (2009) discussed the influence of different coasting distances in train trajectory for energy saving. In the mathematical model developed by the authors, freight train is considered as a mass belt and the total energy cost consists of resistance work, kinetic energy loss, gravitational potential energy difference and energy consumed by running the locomotive. A relationship between speed uniformity and work of resistance is presented to show the train with a constant speed in overcoming the least resistance. To reduce kinetic energy loss caused by braking, different coasting distances in different running schemes are applied to reduce the distance of deceleration braking. A 30.4km train track in China was selected in the case study. The result shows selecting a 3km coasting distance ahead of a lower speed restriction rail section and a 5km coasting distance ahead of the braking for stop can lead to 9% energy saving and only increase 0.5% of running time.

Albrecht, Howlett and Pudney (2016) investigated the cost-time curves for individual trains to generate energy-efficient timetables in complex rail networks. A mathematical model was

developed with energy density per unit mass function and two auxiliary functions to determine optimal driving or braking speed. A two-stage method was developed to find a precise cost-time curve. The first stage is to find optimal transit times for each individual journey segment, and the second stage is to synchronize the collective movement of trains optimally by adjusting the arrival and departure times. Using two example problems with several instances, the authors presented precise cost-time curves and analyzed their impact on optimal energy cost.

2.2.2 Continuous Models with Heuristic Solutions

Heuristic algorithms have been widely applied to solve difficult optimization problems. In addition, meta-heuristic algorithms based on genetic algorithms (GA) and simulated annealing (SA) have also been developed to find optimal or near-optimal solutions of such problems.

Chang and Sim (1997) considered an optimal train movement problem and developed genetic algorithms (GA) based solution method to solve it. Solving the considered problem is to obtain optimal train coast control in each interstation run. The object function or the fitness value include quantified energy consumption, train punctuality and passenger comfort. The GA-based controller is to determine the global optimal solution of the considered problem.

Açkbas and Söylemez (2008) developed a method to determine optimal train coasting points to reduce energy consumption in based on a multi-line and multi-train model. To find optimal train coasting points, a fitness function with of weighted target travel time and energy consumption was proposed. The solution method based on artificial neural networks (ANN)

and genetic algorithms (GA) was used to solve model with the fitness function. The ANN was trained using simulation to estimate energy consumption and travel time for various sets of coasting points. GA was applied to calculate different target values and weights of parameters in solving the multi-objective optimization problems. A 5km section of metro line in Turkey was used to test the model and the solution method, the results show that the ANN can be trained with acceptable error margins to minimize the fuel consumption for the considered railway system.

2.2.3 Continuous Models Based on Other Methods

Wong and Ho (2004) considered the problem of locating single or multiple coasting points in an inter-station track on train fuel consumption. They developed a mathematical model with the object function to determine the distance from the chosen coasting point to the real coasting point when train achieves the desired run-time. Both direct and heuristic search methods were proposed to solve the considered problem. In direct method, Golden Section search, Fibonacci search and Gradient search were tested and compared to obtain the best result in minimum computational time. Genetic algorithm was selected as the most efficient solution method for solving this problem. From simulation results, both direct and heuristic methods provide acceptable solutions with reasonable average number of iterations. Direct methods have less average number of iterations while heuristic methods can obtain better solutions in solving the problem.

To achieve maximum energy saving in train operation, Li, Li and Liu (2009) introduced a

solution algorithm based on optimal principle of adaptive control. A nonlinear constrained mathematical model with boundary conditions was developed. The output of the model obtains a deviation value under ideal conditions. The value adjusts the real-time handle position to approach the ideal operating characteristics in actual operation. Microcontroller Unit (MCU) was used to obtain the deviation value. Considering three different routes of a possible train journey, the authors presented a flow chart to select the best suitable strategy. An actual case with train route from Jinzhou to Shenyang using an ND5390 locomotive were used to ensure the feasibility of the model solution. The result shows that around 0.2ton of fuel consumption can be reduced if the developed control strategy is followed.

In Miyatake and Ko (2010), the authors discussed the problem of train speed profiles and energy storage devices control in railway systems for minimizing total energy consumption. The model considers 8 characteristics influencing optimal train operations. Three different methods, dynamic programming (DP), gradient method, and sequential quadratic programming (SQP) were used to solve the developed model separately. Simulation results show clearly that the importance of speed profiles and optimal state of charge profiles of energy storage for optimal train operations.

Su and Tan (2011) solved an optimal train operation control problem for minimizing the fuel consumption of the diesel locomotive by adjusting the driver's operation. Three modes of operation are considered: traction, coasting and braking in formulating the mathematical model. An idling fuel consumption status was considered, and 7 traction handle positions of locomotive determines fuel consumption. The authors calculated the difference in kinetic

energy between coasting and braking. The results based on a DF7G diesel locomotive show that lower speed and increasing coasting period time can save more energy and decreasing the speed from 80KM/H to 70KM/H or to 60KM/H through coasting before braking can also lead to better fuel efficiency.

2.3 Discrete Models

Discrete system modelling is to use discrete mathematical models to formulate certain train operations. The input variables of train operations such as train speed and traction are obtained in discrete time points. Compared with continuous system modelling, discrete system models normally take less computational time to reach optimal or near optimal solutions.

2.3.1 Discrete Models with Optimal Solutions

Cheng and Howlett (1992) developed an extended model based on that proposed in Benjamin, Long, Milroy and Pudney (1986) for providing train drivers with fuel-efficient driving strategies. In developing the model, it is assumed that each throttle setting determines a constant rate of fuel supply and the rate of fuel supply is directly proportional to the power generated by the train engine. A solution method for minimizing fuel consumption in train operation control was proposed. The authors considered critical velocities and used the method to determine the maximum energy savings in a given sequence of throttle settings. Lagrangean function and Kuhn-Tucker conditions are applied to determine the motion equation of the locomotive. The optimal Lagrange solution determines three critical values of the velocity and ensures the feasibility of the driving strategy. The result shows a strategy with a speedholding

phase is the minimum cost strategy in comparing with other situations.

Howlett, Milroy and Pudney (1994) explored optimal control strategy in solving a real-time railway vehicle control problem for both suburban rail trains and long-haul trains. The goal is to minimize total fuel consumption. In the model, train is regarded as a point mass and the traction system consists of train dynamics and traction losses. Train energy is lost to frictional resistance and braking, and remaining energy is stored as kinetic energy and potential energy. The authors considered engine, resistance and gradient decelerations as the main factors to determine the speed of train. The developed model is similar to that in Howlett (1990), satisfying Pontryagin Principle, Lagrange multipliers were used to determine the change point between two different phases. It is found that the speed limit only occurs at the end of the control interval or at the speed change point on the level track. The results show that significant fuel consumption can be reduced, and time accuracy can be improved if the math model and solution method are implemented.

Howlett (1996) continued his research on a train travelling between two stations with non-constant gradient. A train operation model to minimize fuel consumption considering typical diesel-electric locomotives was proposed. In the model, the train is controlled through a finite sequence of traction phases. The final phase, the braking phase, can be on a level track or a non-level track. The rate of fuel supply is determined by discrete throttle settings controlled by train operators. Considering certain motion equations and locomotive performance characteristics, the track gradient is represented as a piecewise-constant gradient. Based on the

developed model, the authors stated that alternating the coasting phases and the maximum power phase and extending the time of speedholding phase as much as possible can result in reducing fuel consumption.

Kim and Chien (2010) considered train operation control tactics under various track alignment using computer simulation for train movement. They calculated energy consumption and estimate travel time. A modified time-driven train performance simulation model was proposed based on work in Howard, Linda and Wong (1983) and other related literature. The model consists of three modules: train traction, track alignment, and train control. The track alignment module converts the value calculated by train traction module and to pass to train control module to determine the correct form of motion. Numeral examples were presented to demonstrate the applicability of the model to estimate travel time and energy consumption. The results indicate that if implemented, the control strategy may lead to a 66.7% reduction of energy consumption with slight increase of travel time.

Ye and Liu (2017) investigated an optimal train control problem based on closed-form expressions. They proposed a mathematical model to formulate the optimal train control problem and solved it as a nonlinear programming problem. The train is considered as a point-mass with running resistance following the Davis formula (Davis, 1926). Speed limit and gradient are represented by piecewise constants in each subsection of the track. Two methods were proposed to solve the model. The first method assumes a continuous train movement sequence on each subsection of the track. The second method applies a constant force on the

train on each subsection of the track. The two methods were tested using a numerical example based on a line section of Beijing Subway.

2.3.2 Discrete Models with Heuristic Solutions

Fu, Gao and Li (2009) investigated a train operation control problem for energy saving considering disturbances condition from trains in other lines. The authors proposed a model with nonlinear constraints based on mechanics and Newton's Laws. In formulating their model, they considered that train was not allowed to pass the station in certain time period if there are other trains occupying the track at the station. Considering the computer requirement and performance simplicity, the authors developed a solution method based on genetic algorithm (GA) to solve the nonlinear optimization problem. A table listing all control modes including two different schemes was presented to be applied for a variety of tracks. The authors used simulation to generate the speed-position curve of the train under disturbance and compared it with that for the train without such disturbances. The simulation results show that energy savings can be achieved if the proposed control strategy is implemented.

Kim and Chien (2011) considered optimal train operation for minimum energy consumption based on the train performance simulation (TPS) model developed in Kim and Chien (2010). The authors extended the existing model by adding a function to reflect the coupling forces resulted from the constant damping and constant spring force with effect of track curvature along the horizontal direction of train travel. They also added a new constraint reflecting the comfort level of the passengers and assumed that the train is equipped with a regenerative

braking system. A solution method based on simulated annealing (SA) search was developed to determine the best starting point of coasting and corresponding coasting speed with limited time and distance. In addition, a sensitivity analysis was conducted to investigate the impact of various values of model parameters. The result shows that as the maximum allowable time increases, the best coasting point moves backward and the corresponding speed increases.

Zhang, Wang, Tang and Wang (2011) considered an optimal train operation control problem and proposed a heuristic method to solve the problem in minimizing fuel consumption. The power of the considered train is controlled by certain throttle settings and the force used to brake or track is constant in each throttle setting in formulating the model. An adjustable slope between two maximum balancing speeds was used.

2.3.3 Discrete Models Based on Other Methods

Effati and Roohparvar (2006) considered a large-scale constrained optimization problem for train trajectory planning aiming to minimize fuel costs. A straight segment of track covering hills and valleys was used as a simplified trajectory in formulating the model. The control variables are the acceleration generated by train engine and deceleration braking. Measure theory method and iterative dynamic programming (IDP) were used to solve the problem. In using the measure theory method, the authors followed the optimal control theory created in Rubio (1993) and built a nearly-optimal piecewise constant control. The authors used iterative dynamic programming to determine the minimum fuel consumption. Computational results show that the graphs of trajectory and velocity functions and piecewise control functions.

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Gu, Meng and Ma (2014) studied the energy saving problem for controlling two successive trains controlled by a moving block signaling system (MBS). A nonlinear programing model is formulated to model the train operation optimization problem. The decision variables are to decide switching speed between each running phase. Running state and constraints for the leading train are discussed separately for moving and stopping states. A simulation study is conducted using with actual train system data in two successive stations in Beijing, China to test the proposed control strategy.

Gu, Tang, Cao and Song (2014) developed a solution procedure combining analytical and numerical methods to solve an energy-efficient train operation control problem in urban railway systems. Five possible driving modes were considered based on the work in Liu and Golovitcher (2003). Pontryagin maximum principle was applied for solving the objective function of the formulated model. The solution procedure is based on the sequential quadratic programming (SQP) method and can be used to solve small to medium sized problems. The model and solution method were tested using an example problem based on a real train line in Beijing area with encouraging testing results.

Saadat, Esfahanian and Saket (2016) studied an energy-efficient operation control problem with diesel-electric locomotives. A more detailed train model considering many subsystems of a locomotive was proposed. The speed of the considered train is controlled by the diesel engine throttle or braking against force of adhesion and friction. The authors proposed a fuzzy look-ahead control strategy by using ahead path data to solve the considered problem. Fuzzy look-

ahead controller consists of speed controller and fuzzy look-ahead controller. The position of the throttle and braking are controlled by speed controller. The model and solution were texted using a numerical example based on a real application. Computer simulation results show that the optimal control strategy may reduce fuel consumption by about 7.5% with 0.8% increase of travel time.

Saadat, Esfahanian and Saket (2017) proposed a solution method based on fuzzy look-ahead control aiming to reduce fuel consumption of diesel-electric locomotives and hybrid power train. The new hybrid power locomotives considered in this research use lithium-ion battery packs instead of the diesel-electric engines in locomotives. A specific consumption graph was developed to show the relationship between speed and torque of the diesel engines based on the mathematical model developed in Saadat et al. (2016). The fuzzy look-ahead control strategy based on battery statues as well as the ahead gradient and speed is used in real time to reduce fuel consumption. The authors conducted simulation studies with parameter values adopted from GM SD40-2 locomotives. The results show that hybrid locomotives with fuzzy look-ahead controller can achieve better fuel efficiency and shorter travel time.

2.4 Summary

From the literature review, we notice that research in modeling train operation and optimization problems is extensive. Most of the reviewed papers show researchers focus on finding the best switch points of train speed profile using different methods. In this thesis we develop a nonlinear mathematical model with discrete distance variables to determine the locations of the

switch points of power notches in running a freight train with diesel-electric locomotive. A new heuristic solution method is developed to solve the proposed model with details given in the next chapter.

Chapter 3 Mathematical Programming Model for Train Control

In this chapter, we present the problem description and the detailed mathematical programming model. A new heuristic solution method is developed to search for optimal or near-optimal solutions of the developed model with reduced computational time in the second part of this chapter.

3.1 Introduction

We consider freight train operation control problem with the train pulled by centralized traction power from a diesel-electric locomotive. A nonlinear mathematical programming model is formulated to describe the train operations. The objective function of the developed model is to minimize fuel consumption by using different train notch selections within given travel time and speed limit. The nonlinear mixed integer programming model is formulated to solve the problem considering local variations in track geometry and other properties. We divide the train journey into three phases: starting phase, running phase and braking phase. In formulating the model, we consider that the train journey is separated with several equidistant smaller segments. The point connecting 2 consecutive segments corresponds to the time when the power notch can be changed from the position in the segment prior to the point to new position in the segment after the connecting point.

3.2 Problem Description

According to the International Energy Agency (IEA), rail is the most efficient and the lowest emitting modes of transport. To minimize fuel consumption in train operations is an important

issue in the field of rail transit. Modern trains are normally divided into electric draw power and diesel draw power by different motive power. For diesel draw power train, the energy transmission may be either diesel-electric, diesel-mechanical or diesel-hydraulic with diesel-electric being the dominant. Especially in North America, diesel-electric trains are the most common mode of rail transportation for both passenger and freight trains. Typically, the diesel-electric locomotives have 9 different notch positions from 0 to 8. Train drivers control the speed of trains by selecting different notch positions. Different notch positions determine different levels of the locomotive power. Higher notch position gives a higher power from the diesel engine so that the train can move with a higher speed.

In this thesis we focus on the energy-efficient operation for diesel-electric trains. Typical diesel-electric trains consist of Diesel Multiple Units (DMUs) and locomotive traction power centralized trains. The two kinds of diesel-electric trains have different power distributions while their operation systems are similar. In such systems, several power notches are controlled by train driver to control the train speed. Different notches lead to different levels of fuel consumption and different running effects. One of the effective approaches to reduce fuel consumption is to develop an optimal locomotive control strategy to assist the train operators in selecting different notches. Such a strategy leads to more consistent and less fuel consumption in operating a train.

From the literatures in this area of research, the solution of the problem typically starts from that the train journey is divided into 4 phases: acceleration, constant speed, coasting and

braking. In this thesis, we consider that the train is in one of the three states of power, coast (idle notch) or brake controlled by train operator to complete a journey. The control strategy begins with a starting phase followed by a running phase with coast and power alternating. The journey ends with a brake phase of a semi-final coasting phase and a final braking phase.

The fuel consumption rate of the diesel engine of the locomotive is determined by the notch positions. The energy powered by the diesel engine is transmitted to the traction motor to bring the forward traction of the train and the power of other on-board equipment such as blowers, radiator fans, and air compressors. Higher notches determine higher output power and transmit more electrical energy. In running the train, notch may advance more than one position at a time except that in the starting phase. Notch is normally adjusted to the first position in order to make the starting phase more smoothly when diesel engine is fired to start the train.

Train operators control the train by shifting the power notches of the engine. Hence fuel consumption rate and engine power level are changed at the switching points along the track. An optimal control strategy, if established, is to determine the notch switching points to minimize fuel consumption with respect to all constraints and train operation conditions. To determine the points that the notch may be changed during a train journey, we divide the entire length of the journey to a number of smaller and equal size segments in formulating the optimization model.

3.3 Assumptions

Based on the description of the problem, a discretized optimization model is formulated to

determine the notch level for train operation to minimize fuel consumption. The following assumptions are considered in formulating the mathematical model.

1. The train is modeled as a rigid body (Yi, 2018) as the shape change of the train is considered negligible in terms of train movement and fuel consumption.

2. The notch position can only be changed at the connecting points of two adjacent segments.

3. The power position can only be switched to the first notch when the train starts from its idle state so that the train will move smoothly in the starting phase.

4. The train does not stop at any middle point of its journey between the starting and ending points.

5. Brake state will only be applied in the last segment of the journey.

6. The speed of the train must not exceed the allowable maximum speed specified for each segment (Ye and Liu, 2017).

7. The speed of the train in during the time of passing a segment approximated by the average speed at the two ending points of the segment.

8. Initial speed and final speed of the train journey are zero.

3.4 Notations

We introduce the notations and symbols used in the discrete optimization model as follow.

Index sets

$k = \{0,1, \dots, K\}$ The number of notches

$j = \{1,2, \dots, J\}$ The number of segments

$p = \{1,2,..,P_j\}$ The number of circular curves in j^{th} segment

Parameters

m Locomotive mass

M Total cars mass

η_a Transformation ratio of the diesel engine

λ_h Surrounding air temperature coefficient

λ_p Track altitude coefficient

g Gravitational constant

L_t Train length in meters

L_c Circular curve length in meters

R_r The curve radius in meters

A Curve resistance coefficient

T Total time limit of the train journey

S_t Distance of train journey

V_{sf} Final speed of the starting phase

S_{fb} Distance of the final brake phase

V_{jmax} Speed limit for the train passing the jth segment

Decision Variables

$z_{ik} = \begin{cases} 1, & \text{if the } k\text{-th notch is used in segment } j \\ 0, & \text{otherwise} \end{cases}$

Other Variables

e_{kj} The k^t fuel consumption rate in the jth segment

$\bar{a}_s$ Approximate acceleration in the starting phase

$\bar{V}_s$ Average speed in the starting phase

$\bar{\iota}_j$ Average track gradient in the jth segment

$\bar{F}_{cj}$ Average curve resistance in the jth segment

$\bar{a}_j$ Approximate acceleration in the jth segment

$\bar{V}_j$ Average speed in the jth segment

V_{j-1} Initial speed in the jth segment

V_j Final speed of jth segment

V_{fc} Final speed in the semi-final coast

t_b Time in the braking phase

t_{fc} Time in the semi-final coast

t_{fb} Time in the final brake

Y_n Notch changed number in running phase in the nth step

J_n Segments number in the nth step

E_n Fuel consumption in the nth step

3.5 Discrete Optimization Model

In this section, we first perform force analysis on the considered train operations followed by

model development.

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3.5.1 Force Analysis

In analyzing forces affecting train movement, we follow the basic analysis in Yi (2018) to express the traction force as power defined by the rate of doing work or the rate of using energy. The relationship between power, traction force and train speed is given by

$$P = \frac{F_T \times V}{3600} \qquad (3.1)$$

where P is the diesel engine power in kilowatt, F_T is the tractive force powered by diesel engine in newton, V is the instant train velocity or speed in kilometer per hour.

The power of train is determined by the notch positions. As a result, the formula can be expressed by

$$F_T = \frac{3600 \times N_k \times \eta_a}{V} \qquad (3.2)$$

where N_k is the locomotive power at k^{th} notch in kilowatt. $k = \{0, 1, \dots, K\}$. η_a is the diesel engine transformation ratio since the power produced by the diesel engine is not all available for traction with loss to on-board equipment and other losses. Typically, η_a ranges from 0.8 to 0.85.

The effective power of a diesel engine is related to the amount of air entering the cylinder (Yi, 2018). In our model, we introduce two correction coefficients considering air temperature and altitude influence. As a result, the actual tractive force is given by

$$F_{AT} = F_T \times \lambda_h \times \lambda_p \qquad (3.3)$$

where λ_h is the coefficient of surrounding air temperature and λ_p is the track altitude coefficient.

When a train is running on the railway track, there is running resistance acting on the train. We consider that the total running resistance equals to the sum of locomotive running resistance and cars running resistance.

$$R = R_L + R_C \qquad (3.4)$$

where R is the total running resistance, R_L is the running resistance of locomotive and R_C is the running resistance of cars. Based on the Davis formula (Davis, 1926), the running resistance of locomotive and cars can be expressed by

$$R_L = m \times g \times (a_1 + b_1 V + c_1 V^2) \qquad (3.5)$$

$$R_C = M \times g \times (a_2 + b_2 V + c_2 V^2) \qquad (3.6)$$

where m is the locomotive mass, M is the cars mass, g is the gravitational constant, a_1, b_1 and c_1 are the coefficients corresponding to the type of locomotive and a_2, b_2 and c_2 are the coefficients corresponding to the characteristics of cars.

Railway tracks may not be placed along a straight line on a horizontal surface. In the modeling, we consider external forces caused by track gradient and curve resistance (Liu and Golovitcher, 2003) acting on the train. the external force caused by track gradient can be expressed by

$$F_g = (m + M) \times g \times \sin \alpha \qquad (3.7)$$

where α is the angle between track and horizontal. Because the angle α is generally very small, we can let $\sin \alpha \approx \tan \alpha$. The external force caused by track gradient can then be expressed by

$$F_g = (m + M) \times g \times \tan \alpha = \frac{(m+M) \times g \times i}{1000} \qquad (3.8)$$

where i is the gradient (‰) of the track. When slope is upgrade, i is positive and when slope is downgrade, i is negative.

The external force caused by curve resistance is related to the instant value of the curve radius, speed of the train and other characters of the track. In practice, it may be difficult to determine the accurate value of this force. In developing our model, we follow that in Yi (2018) and use the following equation:

$$f_c = \frac{A}{R_r} \times g \tag{3.9}$$

where f_c is the unit curve resistance, A is the curve resistance coefficient related to designed speed of train and R_r is the curve radius. To calculate the total curve resistance force on the entire train, if the circular curve length is equal or longer than the train length, we use:

$$F_c = f_c \times (M + m) = \frac{A \times (M+m) \times g}{R_r} \qquad (L_c \geq L_t) \tag{3.10}$$

where F_c is the total curve resistance, L_c is the length of circular curve and L_t is the length of train. If the circular curve length is shorter than the train length, we use:

$$F_c = f_c \times (M + m) \times \frac{L_c}{L_t} = \frac{A \times (M+m) \times L_c \times g}{R_r \times L_t} \qquad (L_c < L_t) \tag{3.11}$$

Resultant Force Analysis

Based on Newton's Second Law and work-energy principle, train motion formula can be obtained. Since the train is considered as a rigid body, the train motion not only contains translational motion of train, but also contains the rotation of wheels. For train translational motion, the kinetic energy is $\frac{1}{2}mv^2$. For rotation of wheels, the kinetic energy is $\frac{\sum N\omega_R^2}{2}$ (Yi, 2018). The total kinetic energy in the train system is given by

$$KE = \frac{1}{2}(m+M)V^2 + \frac{\sum N\omega_R^2}{2} = \frac{(m+M)V^2}{2}\left(1 + \sum\frac{N}{(m+M)R_a^2}\right) \tag{3.12}$$

where KE is the total kinetic energy, N is the moment of inertia of rotating parts, ω_R is the angular speed of wheels and R_a is the average rotation radius of rotation parts. We let $\gamma = \sum\frac{N}{(m+M)R_a^2}$. Then γ is the mass coefficient of rotation parts. The value of γ is 0.06 (Yi, 2018) for a typical train. As a result, the kinetic energy increment can be expressed by

$$dKE = (m+M)(1+\gamma)VdV \tag{3.13}$$

Since the kinetic energy increment will be calculated by

$$dKE = C \times V \times dt \tag{3.14}$$

where C is the resultant force of train. As a result, the train acceleration or the derivative of train velocity over time can be expressed as:

$$\frac{dV}{dt} = \begin{cases} \frac{F_{AT}-R-F_g-F_c}{(1+\gamma)(m+M)}, & \text{train is in strating or running phase} \\ \frac{-F_B-R-F_g-F_c}{(1+\gamma)(m+M)}, & \text{train is in braking phase} \end{cases} \tag{3.15}$$

where F_B is the brake force that opposes to the direction of train motion. F_B is assumed to only apply in braking phase and is related to instant speed and friction coefficient of track. When the train is in starting or running phase and $L_c \geq L_t$, the equation (3.15) can be expanded to become:

$$\frac{dV}{dt} = \frac{(3600\lambda_h\lambda_p\eta_a)\frac{N_k}{V} - (mgc_1+Mgc_2)V^2 - (mgb_1+Mgb_2)V - \frac{(m+M)g}{1000}i - \left(mga_1+Mga_2+\frac{A(M+m)}{R_r}w\right)}{(1+\gamma)(m+M)} \tag{3.16}$$

where w is $\{1,0\}$ set, when there is a circular curve on the track, w equals to 1. Otherwise w equals to 0. When train is in starting or running phase and $L_c < L_t$, we have:

$$\frac{dV}{dt} = \frac{(3600\lambda_h\lambda_p\eta_a)\frac{N_k}{V} - (mgc_1+Mgc_2)V^2 - (mgb_1+Mgb_2)V - \frac{(m+M)g}{1000}i - \left(mga_1+Mga_2+\frac{A\times(M+m)\times L_c}{R_r\times L_t}w\right)}{(1+\gamma)(m+M)} \tag{3.17}$$

When train is in braking phase and $L_c \geq L_t$, we have:

$$\frac{dV}{dt} = \frac{-F_B(V) - (mgc_1+Mgc_2)V^2 - (mgb_1+Mgb_2)V - \frac{(m+M)g}{1000}i - \left(mga_1+Mga_2+\frac{A(M+m)}{R_r}w\right)}{(1+\gamma)(m+M)} \tag{3.18}$$

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When train is in braking phase and $L_c < L_t$, we have:

$$\frac{dV}{dt} = \frac{-F_B(V)-(mgc_1+Mgc_2)V^2-(mgb_1+Mgb_2)V-\frac{(m+M)g}{1000}i-\left(mga_1+Mga_2+\frac{A\times(M+m)\times L_c}{R_r\times L_t}w\right)}{(1+\gamma)(m+M)} \quad (3.19)$$

3.5.2 Train Motion Analysis

From the force analysis presented in the previous subsection, train motion equation is a differential equation with independent variables of train speed and instant track gradient. Analytical solutions of the developed differential equation can be difficult to obtain by integration while numerical solutions can be determined without much difficulty. Hence in this research, we divide the train journey into several equidistance smaller segments. In each segment, the total force can be considered as constant and the train motion can be considered in constant acceleration or deceleration movement. We use average speed, track gradient and curve resistance to approximate instant speed, track gradient and curve resistance respectively. The resultant force in each segment then can be considered constant if the length of the segment is sufficiently small.

Train Starting Phase

We divide train movement into three phases. The first phase is the starting phase. This phase starts the first segment. In the starting phase, the fuel consumed to start the train is not considered as part of the total fuel consumption. We assume that the notches can only be switched into the first position when train starts so that the operation in the starting phase will be smoothly. The average acceleration in the starting phase can be expressed by

$$\bar{a}_s = A_1\frac{N_1}{\bar{V}_s} - A_2\bar{V}_s^2 - A_3\bar{V}_s - A_4 \quad (3.20)$$

where $A_1 = \frac{3600\lambda_h\lambda_p\eta_a}{(1+\gamma)(m+M)}$, $A_2 = \frac{mgc_1+Mgc_2}{(1+\gamma)(m+M)}$, $A_3 = \frac{mgb_1+Mgb_2}{(1+\gamma)(m+M)}$, and $A_4 = \frac{\frac{(m+M)g}{1000}\bar{\iota}_j+(mga_1+Mga_2+\bar{F}_{cj})}{(1+\gamma)(m+M)}$ where $\bar{\iota}_j$ is the average gradient in j^{th} segment and $\bar{F}_{cj}$ is the average curve resistance force in j^{th} segment. $j = \{1,2,\dots,J\}$ where J is the number of total segments. And $\bar{F}_{cj} = \frac{\sum_{p=1}^{P}L_c}{s}\sum_{p=1}^{P}F_{cp}$ where F_{cp} is the pth curve resistance in the jth segment, $p = \{1,2,\dots,P_j\}$ where P_j is the total number of circular curves in the jth segment.

The distance that the train will travel in the starting phase is calculated by

$$S_s = \frac{V_{sf}^2 - V_0^2}{2\bar{a}_s} \tag{3.21}$$

where s_s is the distance of starting phase, V_0 is the initial speed of train journey, V_{sf} is the final speed of starting phase and is generally between 20km/h and 30km/h. The time length of the starting phase is given by

$$t_s = \frac{s_s}{\bar{V}_s} \tag{3.22}$$

where $\bar{V}_s = \frac{V_{sf}+V_0}{2}$ and it is the average speed in the starting phase.

Train Running Phase

The second phase is running phase. The train acceleration in this phase can be expressed by

$$\bar{a}_j = A_1 \frac{N_k}{\bar{V}_j} - A_2 \bar{V}_j^2 - A_3 \bar{V}_j - A_4 \tag{3.23}$$

where $\bar{a}_j$ is the average acceleration in j^{th} segment and $\bar{V}_j$ is the average speed in j^{th} segment. The time of running phase is given by

$$t_r = \sum_{j=1}^{n-1} t_{rj}$$

$$t_{rj} = \frac{s-s_s}{\bar{V}_j} \quad (j = 1) \tag{3.24}$$

$$t_{rj} = \frac{s}{\bar{V}_j} \quad (j = 2,3,\dots,J\text{-}1)$$

where t_r is the total time for running phase, t_{rj} is the time of each segment for running phase.

s is the distance of each segment. The initial speed and final speed in each segment can be calculated by

$$
\begin{cases} s - s_s = \dfrac{V_j{}^2 - V_{sf}^2}{2\bar{a}_j} \\ \bar{V}_j = \dfrac{V_j + V_{sf}}{2} \end{cases} \qquad (j = 1)
$$

$$
\begin{cases} s = \dfrac{V_j^2 - V_{j-1}^2}{2\bar{a}_j} \\ \bar{V}_j = \dfrac{V_j + V_{j-1}}{2} \end{cases} \qquad (j = 2,3,\ldots,J\text{-}1) \qquad\qquad (3.25)
$$

where V_{j-1} is the initial speed of jth segment and V_j is the final speed of jth segment.

Train Braking Phase

The last phase is the braking phase. This phase consists of two sub-phases: semi-final coasting phase and the final braking phase. In the semi-final coasting phase, no traction force is applied to the train with the notch at N_0. When brake is applied, the train will enter the final braking phase. In developing our model, we assume that brake force is a constant, and the value of the force can be obtained from experimental data. In addition, we assume that the distance covered in the final braking phase is a constant and is longer than 1.5km. As a result, the average train acceleration and average train speed in semi-final coasting phase are

$$
\begin{cases} \bar{a}_{fc} = -A_2\bar{V}_{fc}{}^2 - A_3\bar{V}_{fc} - A_4 \\ s - s_{fb} = \dfrac{V_{fc}^2 - V_j^2}{2\bar{a}_{fc}} \\ \bar{V}_{fc} = \dfrac{V_{fc} + V_j}{2} \end{cases} \qquad (j = J - 1) \qquad\qquad (3.26)
$$

where $\bar{a}_{fc}$ is the average train acceleration in semi-final coasting phase, $\bar{V}_{fc}$ is the average speed in semi-final coasting phase, V_{fc} is the final speed of semi-final coasting phase and s_{fb} is the distance of final braking phase. The time length of the braking phase is calculated by

$$\begin{cases} t_b = t_{fc} + t_{fb} \\ t_{fc} = \frac{s - s_{fb}}{\bar{V}_{fc}} \\ t_{fb} = \frac{2s_{fb}}{V_{fc} + V_j} \end{cases} \qquad (j = J) \qquad (3.27)$$

where t_b is the time length of braking phase, t_{fc} is the time length of semi-final coasting phase and t_{fb} is the time length of final braking phase.

3.5.3 The Main Mathematical Model

The main purpose of solving the considered problem is to minimize fuel consumption by properly shifting train notches during the journey. In the proposed model, the objective function is to minimize the fuel consumption by changing notch positions. We use e_k in kilogram per hour to represent the train fuel consumption rate in k^{th} notch. The value of e_k is determined by the notch positions. As a result, the objective function of the model is

$$E = E_s + E_r + E_b = e_1 \times t_s + \sum_{k=1}^{K} \sum_{j=1}^{J-1} e_{kj} \times t_{rj} \times z_{jk} + e_0 \times t_{fc} \qquad (3.28)$$

where E is the total fuel of train journey, E_s is the total fuel consumption of starting phase, E_r is the total fuel consumption of running phase, E_b is the total fuel consumption of braking phase and e_{kj} is the fuel consumption rate in the jth segment. z_{jk} is the binary variable determining the notch using in the jth segment.

The complete model to minimize the above function under considered constraint functions is given below:

$$\text{Min } E = E_s + E_r + E_b = e_1 \times t_s + \sum_{k=1}^{K} \sum_{j=1}^{J-1} e_{kj} \times t_{rj} \times z_{jk} + e_0 \times t_{fc} \qquad (3.29)$$

$$\text{s.t } V_j \leq \min(V_{jmax}, V_{j+1max}) \qquad (3.30)$$

$$t_s + t_r + t_b \leq T \qquad (3.31)$$

$$t_s, t_r, t_{rj}, t_b, t_{fc}, t_{fb} \geq 0 \tag{3.32}$$

$$V_0 = V_n = 0 \tag{3.33}$$

$$V_j, \bar{V}_j, \bar{V}_s, V_{sf}, V_{fc}, \bar{V}_{fc} \geq 0 \tag{3.34}$$

$$s = \frac{S_t}{n} \tag{3.35}$$

$$V_{J-1} \geq \sqrt{2 \times \bar{a}_{fc} \times s} \tag{3.36}$$

$$\sum_{k=1}^{K} Z_{jk} = 1 \tag{3.37}$$

where V_{jmax} is the maximum allowed speed in the jth segment, T is the total allowed time, S_t is the distance of train journey. Constraint Eq.(3.30) gives the speed limit in each segment, total travel time limit is imposed by Eq.(3.31). Eq.(3.32) and Eq.(3.34) define the nature of the variables. Eq.(3.33) defines the initial speed and final speed of train journey. Eq.(3.35) gives the relationship between the distance of each segment and the distance of train journey. Eq.(3.36) ensures train having sufficient speed to get to the station during braking phase. Eq.(3.37) ensures only one notch position can be applied in each segment.

Solving the above presented nonlinear optimization model is NP-hard. In the next subsection, we present a heuristic solution method to solve the problem to find near- optimal solutions within reasonable computational time for the purpose of practical applications.

3.6 Heuristic Solution Method

In this section, we introduce a heuristic solution method to solve the optimization model developed in the previous subsection.

The solution of the proposed discrete optimization model has binary-integer variables. If the train journey distance has large number of segments with smaller segment size, the solution of the model can be closer to the real optimal solution. To do so, the computational time will be longer since each additional segment corresponds to 8 additional notch decisions. The proposed heuristic solution method attempts to take a trade-off between solution accuracy and computational time. It will start with smaller number of segments of relatively large size hence the problem will have fewer number of binary variables. Solution of such problem can be found in less computational time. The heuristic solution method will continue by adding smaller segments around the points where notch changes take place from the solution in the previous iteration. It will continue until the objective function value converges. Details of the proposed solution method are given below.

Step 1

Divide the total distance of the train journey into several relatively large segments and solve the train control optimization model presented in Eqs.(3.29) to (3.36).

Step 2

Based on the results from the Step 1, if the notch changes take place at all points connecting each segment, double the number segments by reducing the size of each segment by half. Go to Step 1 and resolve the optimization model. If the solution from Step 1 shows that the notch changes take place at certain but not all the points, then smaller segments will be added around the points where notch changes are required from the solution found in Step 1. No segments

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will be added around other points where notch changes are not required from the solution found in Step 1. With the total journey distance divided by all current segments, go to Step 1 and solve the train control optimization problem.

Step 3

The iterative solution method will terminate if the objective function of the proposed model converges, i.e., the difference between the objective function values of the previous and the current solution is less than a certain given value. Alternatively, the search process can terminate if the size of the new segment to add is less than a given threshold. The other stopping criterion can be that the total number of iterations should not exceed a pre-set given number. The flow chart of the heuristic solution method is shown in Figure 3.1.

3.7 Summary

In this chapter, we analysis the forces of freight train operations during the journey of certain distance on a railway track with slopes and curvature. A mixed integer nonlinear programming model is proposed to solve the considered train control problem to minimize fuel consumption. We also present a heuristic solution method to find close-to-optimal solutions with less computational time. In the next chapter, we will present several numerical example problems to demonstrate the developed model and solution method.
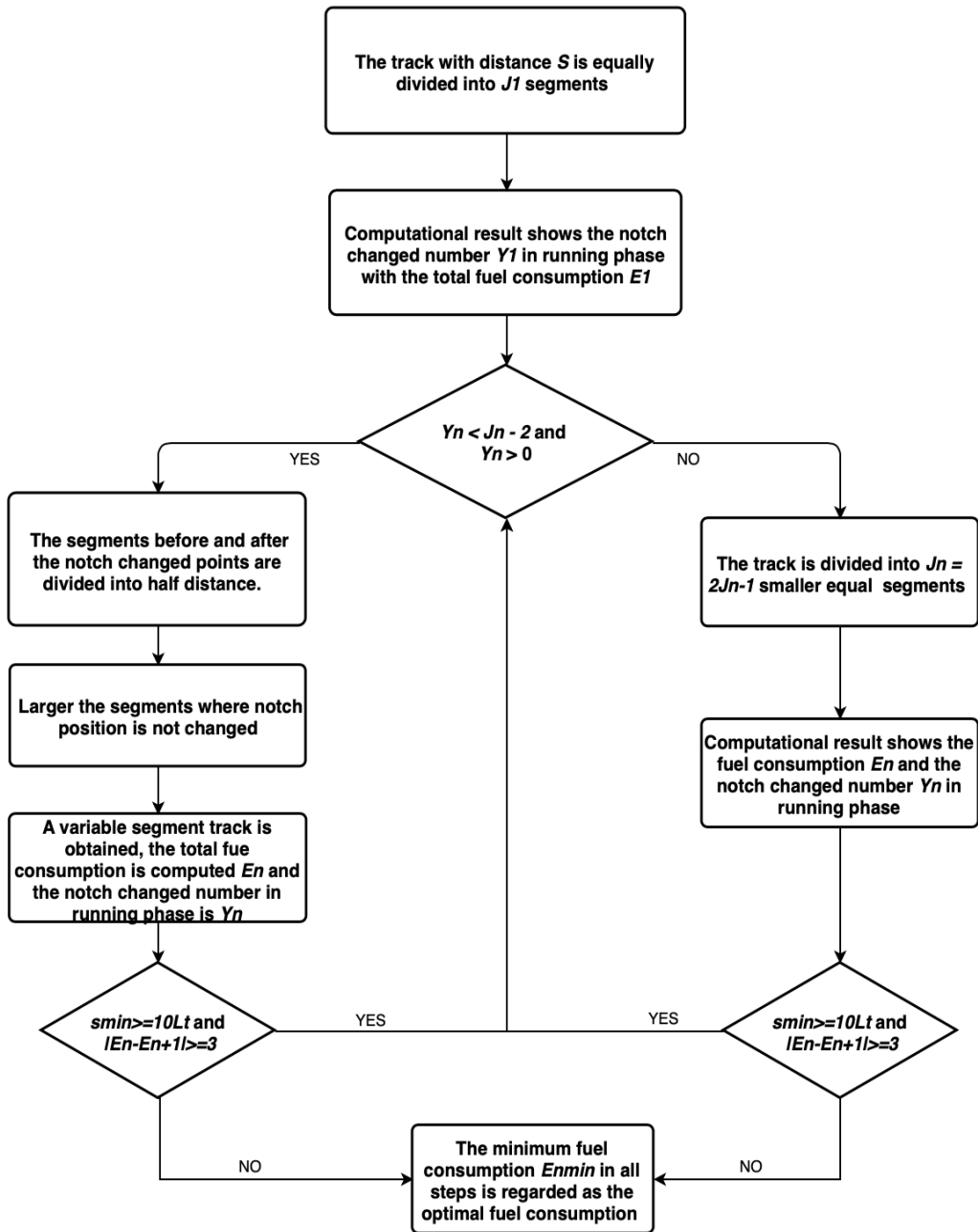
Figure 3.1 Flow Chart of Heuristic Solution Method

Chapter 4 Numerical Analysis and Results

In this chapter, numerical examples are presented. These problems and several instances are solved following the heuristic method discussed in Chapter 3. Computational results are analyzed in demonstrating the effectiveness and efficiency of the developed heuristic solution method. The solution method is coded in LINGO 18 and Matlab A2016R. All example problems were solved on a PC computer with Intel i7 CPU. The Matlab and LINGO codes are presented in Appendix A, Appendix B and Appendix C, at the end of this thesis.

4.1 Data Sources and Assumptions

Data of Train

In testing the developed model and solution method, data based on the diesel-electric locomotive SD40-2 are used, since this locomotive type is widely in use in North America according to *EMD Locomotives* (2006). We consider that all train cars are 60 foot's-standard boxcars and mounted with rolling bearings. The SD40-2 locomotive is 68 feet and 10 inches (20.98m) long and has 9 notch levels (0 to 8) with a braking switch. Different notch levels correspond to different fuel consumption rates and power levels. Data related to the locomotive power and energy consumption are shown in Table 4.1.

Table 4.1 Locomotive Fuel Consumption and Power Levels

Notch k	0	1	2	3	4	5	6	7	8
e_k (liter/h)	25	35	75	140	220	320	430	540	650
N_k (kw)	0	120	285	600	775	990	1230	1910	2240

The considered freight car each is 67 feet and 11 inches (20.70m) long with weight of 80 tons. The considered train has 15 freights in the considered example problem. We calculate the resistance force for the locomotive and each freight car based on the same Davis formula. The resistance force coefficients of the locomotive and the freight cars are normally obtained based on field and laboratory experiments. Such data are not available for the considered locomotive type while similar data can be found for locomotive type DF4 widely used in China's rail freight transportation. The length of DF4 locomotive is 20.50m. since the differences between the exterior shapes of DF4 and SD40-2 are insignificant, the available resistance force coefficients of DF4 are used in testing the developed model. These data are summarized in Table 4.2 and Table 4.3.

Table 4.2 Other Parameters of Train

m	M	η_a	L_t	A
167 ton	1200 ton	0.82	327.88 m	600

Table 4.3 Locomotive and Train Cars Resistance Coefficients

Resistance Coefficient	a_i	b_i	c_i
Cars ($i = 2$)	0.92	0.0048	0.000125
Locomotive ($i = 1$)	2.28	0.0293	0.000178

Rail Track Data

In the testing example problems, we consider 2 cases where the distances of the train journey are 60km and 120km, respectively, at a horizontal level at 500m altitude, and hence the value

of track altitude coefficient is 1.0. The end speed of the starting phase is assumed to be 20km/h and the distance of final braking phase to be 2km. Other data such as surrounding air temperature coefficient, track gradient, length of circular curve in each segment, curve radius and speed limits for the journey distances of 60km and 120km trip are shown in Table 4.4 and Table 4.5, respectively.

Table 4.4 Parameters of 60km Track

Distance (km)	V_{max} (km/h)	i (‰)	L_c (km)	R_r (km)	λ_h
0-10	100	0	0	0	0.982
10-20	90	0	0.8	5.75	0.982
20-30	80	2	0	0	0.982
30-40	100	-2	0	0	0.952
40-50	100	0	0	0	0.952
50-60	100	0	0	0	0.952

Table 4.5 Parameters of 120km Track

Distance (km)	V_{max} (km/h)	i (‰)	L_c (km)	R_r (km)	λ_h
0-10	100	0	0	0	0.982
10-20	90	0	0.8	5.75	0.982
20-30	80	2	0	0	0.982
30-40	100	-2	0	0	0.952
40-50	100	0	0	0	0.952
50-60	100	0	0	0	0.952
60-70	90	0	1.0	4.80	0.952
70-80	90	0	0.9	3.48	0.921
80-90	90	1	0	0	0.921
90-100	100	0	0	0	0.921
100-110	100	-2	1.2	4.80	0.921
110-120	100	0	0	0	0.921

4.2 Computational Results

In this section, several computational results are presented in different situations. Firstly, we compare the fuel consumption of the train in different segments. After that, the result of notch changed point and fuel consumption are given in different time limits. Finally, we compute the fuel consumption and computational time by using heuristic solution method and compare the results with that in track divided into equal segments.

4.2.1 Minimum Fuel Consumption in Different Segments

We present the relationship of different segments and fuel consumption by dividing the distance in different segments. The segment number is divided from less to more. Due to computational efficiency requirement, the number of segments should not be unnecessarily large. For a 60km track, the segment number is from 5 to 24 and for a 120km track, the segment number is from 10 to 48. The time limit for 60km track is 1.05 hour and for 120km is 1.9 hour, respectively. From the results, we obtain the optimal selection of notch and fuel consumption in different segments based on the mathematical model formulating in Chapter 3. The results of fuel consumption and computational time in different segments of 120km track and 60km track are given in Table 4.6 and Table 4.7 respectively. The other results of 60km track and 120km track are shown in Appendix D, at the end of this thesis.

Table 4.6 Fuel Consumption and Computational time of 120km Track

Segments	Fuel Consumption (liter)	Computational Time (second)
10	399.3	49.6
11	373.4	29.6
12	366.9	26.0
13	354.6	77.1
14	348.8	61.2
15	355.8	64.9
16	359.8	283.8
17	343.6	17.9
18	339.2	43.9
19	340.2	63.5
20	339.6	147.6
24	338.5	99.8
30	338.0	191.2
40	333.7	897.6
48	335.9	5414.5

Table 4.7 Fuel Consumption and Computational Time of 60km Track

Segments	Fuel Consumption (liter)	Computational Time (Second)
5	217.8	1.4
6	207.8	3.9
7	192.0	2.6
8	191.1	5.1
9	178.2	3.6
10	177.7	16.4
12	174.4	8.2
15	172.8	8.4
20	172.8	117.3
24	171.5	53.6

Based on the Table 4.6 and 4.7, we can obtain that with the number of divided segments increase, the computational time tends to increase longer. The scatter diagram for the relationship between fuel consumption and different segments of 60km track and 120km track

is shown in Figure 4.1 and Figure 4.2 respectively.
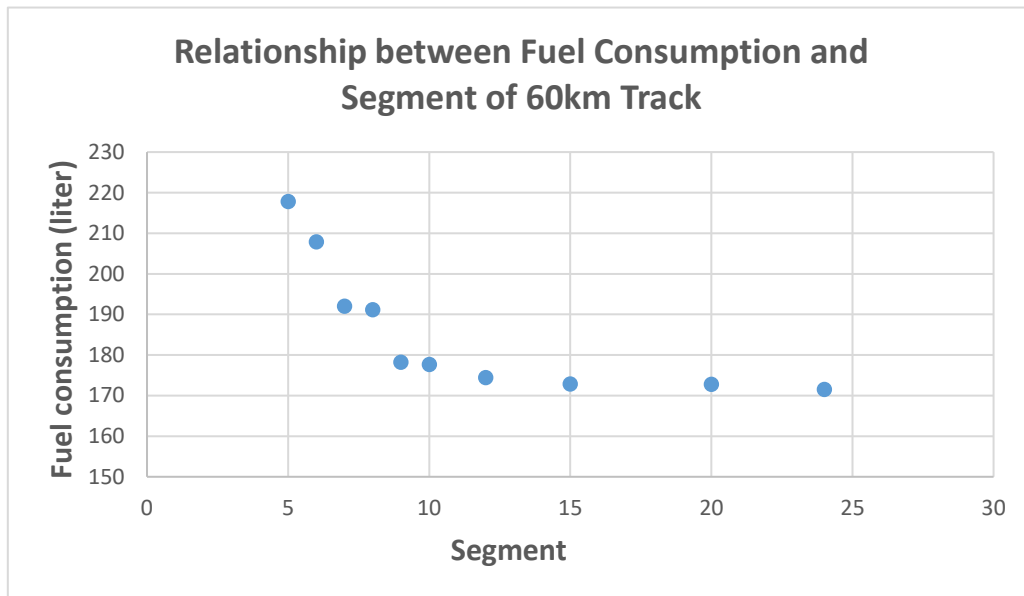
Figure 4.1 Relationship between Fuel Consumption and Segment of 60km Track
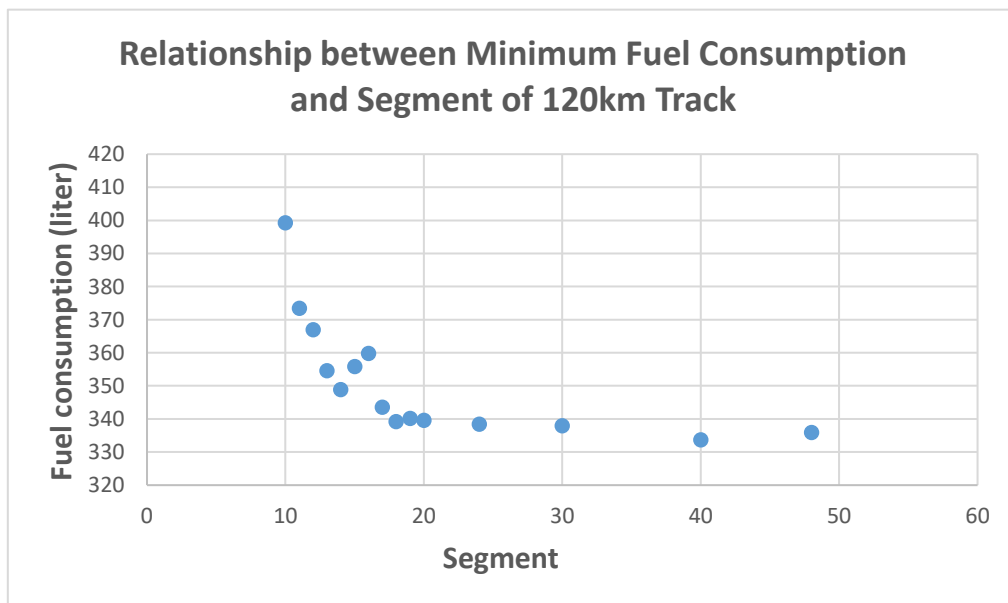
Figure 4.2 Relationship between Fuel Consumption and Segment of 120km Track

From the scatter diagram, we observed that, firstly, as the divided segment increases, the fuel

consumption deceases fast. The number of divided segments is from 5 to 15 in 60km track, and

the number of divided segments is from 10 to 18 in 120km track. After that, when the number of divided segments is enough, the fuel consumption decreases slowly and trends to a minimum value. In our tests, the minimum fuel consumption is 171.5 liters when track is divided into 24 segments in 60km track example, and for 120km track, the minimum fuel consumption is 333.7 liters when track is divided into 40 segments. From the tendency shown in the scatter diagram, we can obtain that the minimum fuel consumption is approximate 170 liters in 60km track example and approximate 330 liters in 120km track example.

4.2.2 Minimum Fuel Consumption in Different Time Limits

Based on the results from last section, we can obtain that the fuel consumption by dividing track into several equal size segments. However, it is hard and takes lots of time to find the best segment for minimizing fuel consumption. Searching for optimal solutions will take much longer computational time when more segments are used since more binary variables are required for solving the same problem. As a result, we take 6-segment for 60km track and 12-segment for 120km track as examples to find the relationship of different time limits and notch changed number. Here we present the relationship in running phase since notch is at the same position in starting phase and braking phase. The results are shown in Table 4.8 and Table 4.9 respectively.

Table 4.8 Results of 60km Track in Different Time Limits

Time (hour)	Fuel Consumption (liter)	Notch Position (running phase)	Notch Changed Number (running phase)
1.0	234.8	6,3,6,2,7	4
1.05	207.8	6,3,3,3,7	2
1.1	192.7	3,4,4,3,7	3
1.15	181.8	4,4,4,4,4	0
1.2	166	4,3,3,3,5	2
1.25	152.9	3,4,3,3,4	3

Table 4.9 Results of 120km Track in Different Time Limits

Time (hour)	Fuel Consumption (liter)	Notch Position (running phase)	Notch Changed Number (running phase)
1.9	366.9	4,5,3,3,7,3,3,3,7,3,3	6
2.0	338.3	4,4,4,3,3,3,3,3,7,3,3	3
2.1	310.6	3,3,3,3,3,3,3,3,7,3,3	2
2.2	293.5	3,4,3,3,3,3,4,3,3,3,3	4
2.3	272.9	3,3,3,3,2,3,3,3,3,3,3	2
2.4	264.2	3,3,3,2,3,3,3,3,3,3,2	3

From Table 4.8 and 4.9, it is clear found that the optimal fuel consumption goes down as the time limit goes up. As for the relationship of notch changed number and time limit, we can observe that when the total travel time is relatively relaxed, notch would not be changed in every changed point during the whole trip. In 60km track example, notch needs to change 4 times in running phase when time limit is 1 hour, while notch only needs to change 2 times when time limit is 1.3 hours. As for 120km track example, notch needs to change 9 times in running phase when time limit is 1.7 hours, while notch only needs to change 2 times when time limit is 2.3 hours. As a result, notch position may not change in every segment during the whole trip, larger number of smaller segments may not necessarily lead to significantly better results but very likely require much longer computational time.

To improve solution quality and avoid unnecessary computational burden, in this work, we propose a heuristic solution method to reduce computational time. The detail procedure of the heuristic solution method is given in Chapter 3. Since the amount of computation mainly depends on the number of variables, especially binary variables, a less computational time can be expected.

4.2.3 Fuel Consumption with Heuristic Solution Method

To illustrate the above-mentioned improved approach, we consider a 6-segments 60km track and a 12-segments 120km track as examples. The limit time for 60km track is from 1 to 1.25 hours and the limit time for 120km is from 1.9 to 2.4 hours. We consider that the track is divided into relatively large equal segments in the first run of the model. The distance of each segment in 60km track and 120km track are the same in 10km. And we follow the three steps of heuristic solution method shown in Chapter 3. The determined conditions to terminate the iterative solution method is that the absolute value of difference between the objective function values of the previous and the current solution is less than 3 liters, or the new minimum segment is less than 10 times train long. To prove the feasibility of heuristic solution method, we compare the fuel consumption and computational time with the results in 24 segments of 60km track and 40 segments in 120km track that is the optimal segments selection in our tests. The results of the comparison of 60km track and 120km track are shown in Table 4.10 and Table 4.11 respectively. The corresponding relationship of speed and distance and the other data results are shown in Table E.1, Table E.2, Table E.3 and Table E.4 in Appendix E respectively.

Table 4.10 Compared Results of 60km Track

Distance of Trip	60km					
Time Limit (Hour)	1	1.05	1.1	1.15	1.2	1.25
Fuel Consumption in 24 equal segments (Liter)	183.1	171.5	161.9	153.1	146.7	140.6
Computational Time in 24 equal segments (Second)	99.4	53.6	77.5	32.1	54.7	55.2
Fuel Consumption by Heuristic Solution Method (Liter)	184.6	177	167.4	152.7	149.0	145.1
Computational Time by Heuristic Solution Method (Second)	30.2	47.6	25.8	41.9	24.3	26.0

Table 4.11 Compared Results of 120km Track

Distance of Trip	120km					
Time Limit (Hour)	1.9	2.0	2.1	2.2	2.3	2.4
Fuel Consumption in 40 equal segments (Liter)	333.7	312.8	294.7	280.5	268.0	258.8
Computational Time in 40 equal segments (Second)	897.6	243.7	121.6	98.7	163.0	45.7
Fuel Consumption by Heuristic Method (Liter)	337.4	323.5	310.6	280.4	270.2	263.5
Computational Time by Heuristic Method (Second)	364.5	355.5	32.5	48.7	36.6	29.9

From the data shown in Table 4.10 and Table 4.11, we can obtain that with the time limit increases, the fuel consumption both in heuristic solution method and using equal segments decreases. And comparison result of fuel consumption in 24 equal segments and heuristic solution method of 60km track is shown in Figure 4.3. The comparison result of computational time in 40 segments and heuristic solution method of 60km track is shown in Figure 4.4. As

for the comparison result of fuel consumption in 40 equal segments and heuristic solution method of 120km track is shown in Figure 4.5. The comparison result of computational time in 40 equal segments and heuristic solution method of 120km track is shown in Figure 4.6.
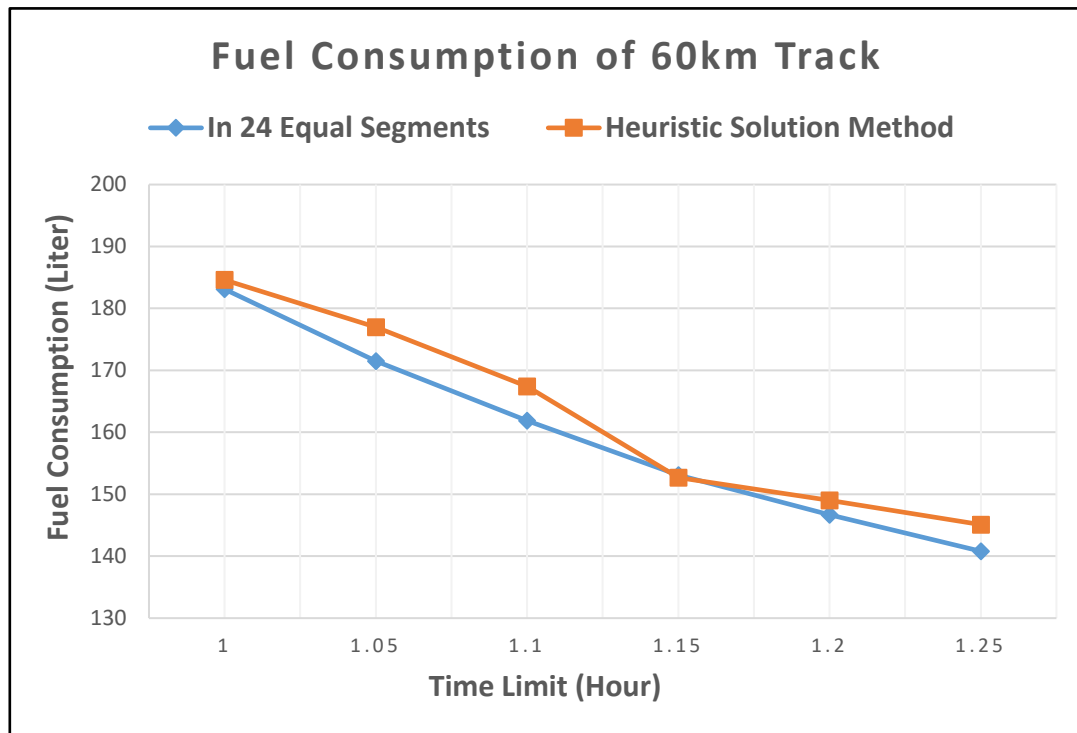
Figure 4.3 Fuel Consumption Comparison of 60km Track
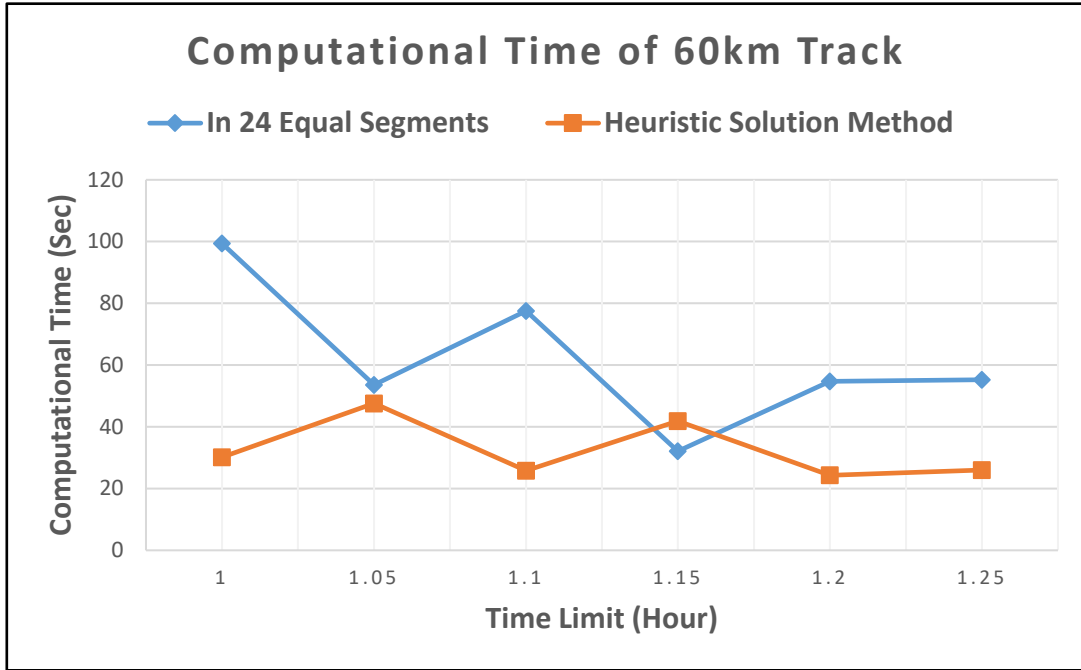
Figure 4.4 Computational Time Comparison of 60km Track
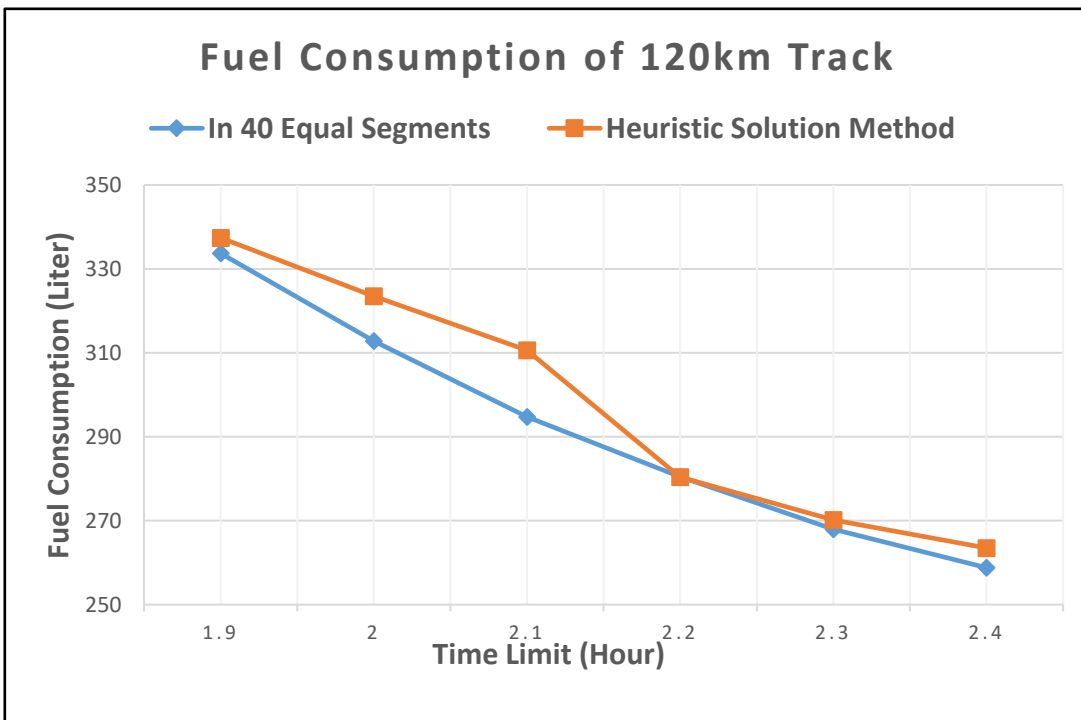
Figure 4.5 Fuel Consumption Comparison of 120km Track
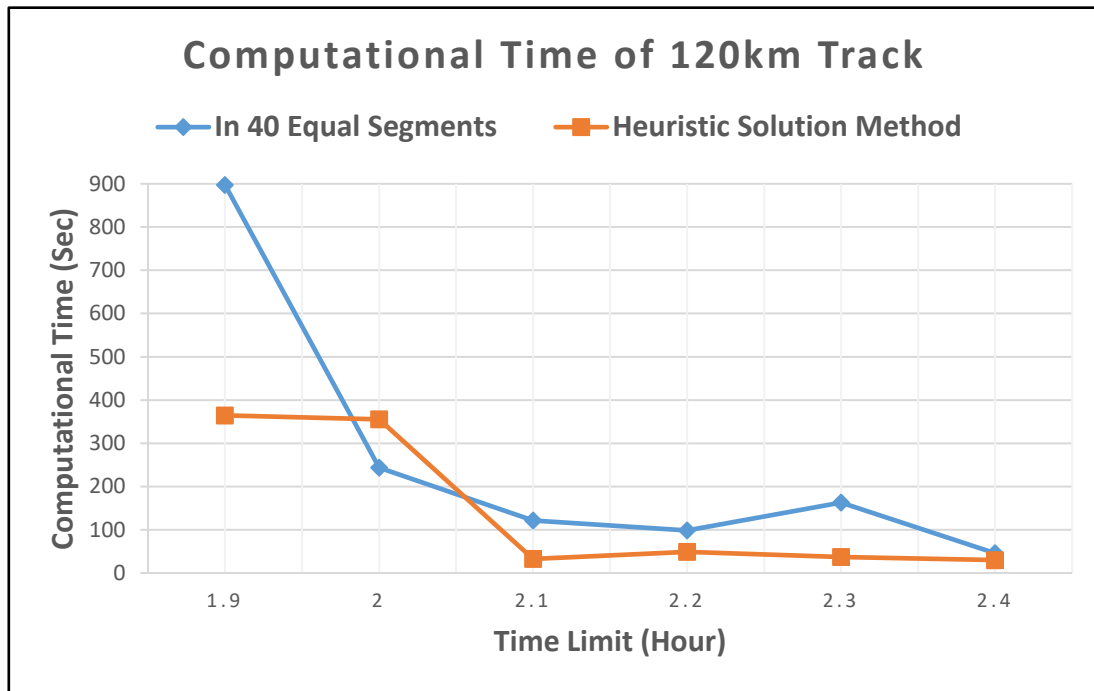
Figure 4.6 Computational Time Comparison of 120km Track

From the results shown in the figures, the fuel consumption goes down with the time limit goes up. The fuel consumption computed by heuristic solution method and the minimum value computed in equal segments in our tests is very close. The average difference of fuel consumption is 3.2 liters in 60km track and 6.2 liters in 120km track. The maximum difference of fuel consumption is 5.5 liters in 60km track and 15.9 liters in 120km track respectively. But as for the computational time, using heuristic solution method is obviously less than that computed in equal size segments. In 60km track example, the computational time of 24 equal segments is less than that by heuristic solution method only in the time limit of 1.15 hours. In 120km track example, the computational time of 40 equal segments is less than that by heuristic solution method only in the time limit of 2.0 hours. The average saved time for 60km track is 29.5 sec and for 120km track is 117.1 sec. The improved efficient of computational time is 47.5% and 44.7% respectively. As a result, the heuristic solution method is an efficient method

to find a near-optimal fuel consumption with less computational time.

4.3 Summary

In this chapter, we used Lingo and Matlab to solve the fuel consumption optimization problem in different variables and conditions. Firstly, we compute the mathematical model formulated in last chapter with different equal size segment. After that, different time limits are considered to compare fuel consumption and notch changed number. Based on the previous data, in order to achieve similar result with less computational time, we propose a multi-steps approach to improve the solution iteratively. 60km track and 120km track examples were used to test the formulated model and the solution methodology. Preliminary computational experiments show very encouraging results.

Chapter 5 Summary and Future Research

In this chapter, we first present a summary of this thesis research followed by suggestions for future research in this topic area.

5.1 Summary

In this thesis, the problems regarding fuel-efficient freight train operations on a limited length of rail track and various control strategies are studied. A nonlinear mix integer programming model for freight train operation and control in completing a journey between 2 train stops is developed. The objective of solving the considered problem is to minimize fuel consumption of the train in completing the journey between two adjacent stations. The purpose of this research is to develop a computerized solution procedure to assist train operators in switching locomotive control notches in completing the required journey with best fuel efficiency. It requires that the considered locomotive control problems be solved without extended computational time on widely available computing platforms such as a PC computer. With this consideration, we developed a heuristic solution method to solve the NP-hard nonlinear programming problem with reduced computational time so that the applications of the model and solution method will be feasible in practice. The details of the developed mathematical model and the heuristic solution method with numerical examples are presented in Chapters 3 and 4 in this thesis.

In developing the mathematical model, we considered the train as a rigid body and analyzed the different force acting on the train. The total trip is divided into several equal size segments

and in each segment, we regard the motion of the train with constant acceleration or constant deceleration. The instant train speed, instant track gradient and instant track curve resistance at any point in a track segment are approximated by average speed, average gradient and average curve resistance respectively, at the starting and ending points of the segment in the computation.

To solve the mathematical model to optimality, a multiple-step heuristic solution method is proposed to reduce the number of variables in the model for faster computation in solving real size practical problems. The multiple-step method first solves the problem with number of equal size segments. In the following steps of the solution procedure, it generates more segments of reduced sizes around the points where notch changes take place from the solution found in the first step. It continues until the objective function values converge or the segments size is reduced below a given threshold value. Several numerical example problems and different instances were used to test the developed model and solution method extensively with results showing the advantages of the development made in this thesis. These are the main contributions made in this thesis research.

5.2 Future Research

The mathematical model developed in this research is based on discretization of the distance of the considered freight train journey. The optimal solution from solving this model will always be an approximation comparing to those found in solving a corresponding model with continuous distance variable. The solution method may also be improved if alternative search

methods are used in generating the segments of the train journey. In future research work in this area, a continuous variable model may be developed so that the notch switching will not be limited to certain points during the train journey. In addition, new model and solution methods can be developed allowing multiple stops during the train journey. The current model development is limited to that the train takes a "non-stop" journey. It requires to apply the current solution method several times if it used to solve multiple-stop problems.

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Appendix A

Starting phase computation in Matlab R2016a:

```
N=[0 120 250 570 860 1200 1600 1910 2240]; %the power of diesel engine
e=[25 35 75 140 220 320 430 540 650]; %the fuel consumption rate in each segment
i=[0 0 2 -2 0 0 0 0 1 0 -2 0]; %the gradient in each segment
v0=0; %initial speed
v1=20; %final speed in staring phase
S=120; %total distance
J=12;   %total segment
s=S/J; %the distance in each segment
m=167; %the mass of locomotive
w=15; %the number of car
M=80*w; %the total mass of cars
Lt=(20.98+ 20.46*w)/1000; %the length of train
Lc=[0 0.8 0 0 0 0 1 0.9 0 0 1.2 0]; %the distance of curve in each segment
R=[0 5.75 0 0 0 0 4.8 3.48 0 0 4.8 0]; %the radius of curve in each segment
gamma=0.06;
c1=0.00178;
b1=0.293;
a1=22.8;
c2=0.00125;
b2=0.048;
a2=9.2;
for j=1:12
    if Lc(1,j)==0
        Fc(1,j)=0;
    elseif Lc(1,j)>=Lt
        Fc(1,j)=600*(m+M)*9.8/(R(1,j)*1000);
    else
            Fc(1,j)=600*(m+M)*Lc(1,j)*9.8/(R(1,j)*Lt*1000);
    end
end
Fac=Lc.*Fc/s; %the force of curve
as=(55.2614*0.982*(N(1,2)/(v0+v1))-(0.0007*((v0+v1)*(v0+v1))+0.2319*(v0+v1)+36.0984)-
(0.0036*((v0+v1)*(v0+v1))+0.273*(v0+v1)+104.666))/(1+gamma); %the acceleration in starting phase
ss=(v1^2-v0^2)/(2*as); %the distance of starting phase
ts=ss/10; %the time of starting phase
Es=e(1,2)*ts; %the fuel consumption in starting phase
sr1=s-ss; % remaining distance in first segment
```

Appendix B

Code of track in equal segments in Lingo 18.0:

60km track (24 segments in 1.05 hours as example):

```
SETS:
nodes / 1..24/: v,vmax; !number of nodes;
segments /1..23/: d, e, a, w, t, fac, tem; !number of segments;
notches /1..9/: r, p; !number of notches;
nodes_notches(segments,notches): z; !number of binary variables;

ENDSETS

DATA:
vmax = 100 100 100 100 90 90 90 90 80 80 80 80 80 100 100 100 100 100 100 100 100 100 100 100; !the
max allowed speed in each segment;
ef=0; !the gradient in last segment;
facf=0; !the curve resistance in last segment;
tem= 0.982 0.982 0.982 0.982 0.982 0.982 0.982 0.982 0.982 0.982 0.982 0.982 0.952 0.952 0.952 0.952
0.952 0.952 0.952 0.952 0.952 0.952 0.952 ; !the coefficient of surrounding air temperature in each
segment ;
p = 0 120 285 600 775 990 1230 1910 2240; !the power of diesel engine;
r = 25 35 75 140 220 320 430 540 650; !the fuel consumption rate in each segment;
Tmax = 0.9275; !time limit;
d=1.2747 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5; !the distance
of each segment;
e= 0 0 0 0 0 0 0 0 2 2 2 2 -2 -2 -2 -2 0 0 0 0 0 0 0; !the gradient in each segment;
fac= 0 0 0 0 112 112 112 112 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; !the curve resistance in each segment;


@ole('locomotive.xlsx','speed')=v; !output the speed in each node in EXCEL;

@ole('locomotive.xlsx','t')=t; !output the time in each segment in EXCEL;

@ole('locomotive.xlsx','accerate')=a; !output the acceleration in each segment in EXCEL;

@ole('locomotive.xlsx','z')=z; !output the binary variables in each segment;


ENDDATA

min = @sum (segments(i):@sum(notches(k): t(i)*z(i,k)*r(k)) )+25*tfc+4.2886; !minimum value of fuel
consumption;

! subject to;
@sum (segments(i): t(i)) <= Tmax-tfc-tfb ; !allowed time in running phase;
```

@for(segments(i): t(i)*(v(i)+v(i+1)) = 2*d(i)); !the relationship of speed, time and distance in each segment;

@for (nodes (i): v(i) <= Vmax(i)); !the speed in each segment does not exceed than the maximum allowed speed;

@for (segments(i): v(i+1) = v(i)+a(i)*t(i)); !the relationship of speed, acceleration and time in each segment;

@for (segments (i): a(i) *(1+0.06)= 55.2614* tem(i)*@sum (notches(k): p(k)*z(i,k))/(v(i+1)+v(i))
-(0.0007*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.2319*(v(i)+v(i+1))+36.0984)
-(0.0036*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.273*(v(i)+v(i+1))+104.666)
-127*e(i)-0.0097*fac(i)); !the expression of resultant force;

@for (segments (i): @sum (notches (k): z(i,k)) = 1); !define binary variable;

@for(nodes_notches : @bin(z));

@for(segments : @Free(a)); !define acceleration;

(vfc*vfc-v(24)*v(24))=1*(-(0.0007*(v(24)+vfc)*(v(24)+vfc)+0.2319*(v(24)+vfc)+36.0984)
-(0.0036*((v(24)+vfc)*(v(24)+vfc))+0.273*(v(24)+vfc)+104.666)-127*ef-0.0097*facf); !the expression of speed in last segment;

tfc=1/(vfc+v(24)); !the time of semi-final coast in braking phase;

tfb=4/vfc; !the time of final brake in braking phase;

v(1) = 20.0; !define the final speed of starting phase;

v(24)*v(24) >=2*((v(24)-vfc)/tfc)*2.5; !ensure enough speed to arrive at station;

END

120km track (24 segments in 1.9 hours as example):

SETS:
nodes / 1..24/: v,vmax; !number of nodes;
segments /1..23/: d, e, a, w, t, fac, tem; !number of segments;
notches /1..9/: r, p; !number of notches;
nodes_notches(segments,notches): z; !number of binary variables;

ENDSETS

DATA:
vmax = 90 100 90 90 80 80 80 100 100 100 100 100 90 90 90 90 90 90 90 100 100 100 100 100; !the max allowed speed in each segment;
ef=0; !the gradient in last segment;
facf=0; !the curve resistance in last segment;
tem= 0.982 0.982 0.982 0.982 0.982 0.982 0.952 0.952 0.952 0.952 0.952 0.952 0.952 0.952 0.921 0.921 0.921 0.921 0.921 0.921 0.921 0.921 0.921;   !the coefficient of surrounding air temperature in each segment ;
r = 25 35 75 140 220 320 430 540 650; !the fuel consumption rate in each segment;
p = 0 120 285 600 775 990 1230 1910 2240; !the power of diesel engine;
Tmax = 1.7775; !time limit;

d=3.7747 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5; !the distance of each segment;

e= 0 0 0 0 2 2 -2 -2 0 0 0 0 0 0 0 1 1 0 0 -2 -2 0; !the gradient in each segment;

fac= 0 0 112 112 0 0 0 0 0 0 0 0 167 167 208 208 0 0 0 0 201 201 0; !the curve resistance in each segment;

@ole('locomotive.xlsx','speed')=v; !output the speed in each node in EXCEL;

@ole('locomotive.xlsx','t')=t; !output the time in each segment in EXCEL;

@ole('locomotive.xlsx','accerate')=a; !output the acceleration in each segment in EXCEL;

@ole('locomotive.xlsx','z')=z; !output the binary variables in each segment;

ENDDATA

min = @sum (segments(i):@sum(notches(k): t(i)*z(i,k)*r(k)))+25*tfc+4.2886; !minimum value of fuel consumption;

!subject to;
@sum (segments(i): t(i)) <= Tmax-tfc-tfb ; !allowed time in running phase;
@for(segments(i): t(i)*(v(i)+v(i+1)) = 2*d(i)); !the relationship of speed, time and distance in each segment;
@for (nodes (i): v(i) <= Vmax(i)); !the speed in each segment does not exceed than the maximum allowed speed;
@for (segments(i): v(i+1) = v(i)+a(i)*t(i)); !the relationship of speed, acceleration and time in each segment;
@for (segments (i): a(i) *(1+0.06)= 55.2614* tem(i)*@sum (notches(k): p(k)*z(i,k))/(v(i+1)+v(i))
-(0.0007*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.2319*(v(i)+v(i+1))+36.0984)
-(0.0036*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.273*(v(i)+v(i+1))+104.666)
-127*e(i)-0.0097*fac(i)); !the expression of resultant force;
@for (segments (i): @sum (notches (k): z(i,k)) = 1); !define binary variable;
@for(nodes_notches : @bin(z));
@for(segments : @Free(a)); !define acceleration;
(vfc*vfc-v(24)*v(24))=6*(-(0.0007*(v(24)+vfc)*(v(24)+vfc)+0.2319*(v(24)+vfc)+36.0984)
-(0.0036*((v(24)+vfc)*(v(24)+vfc))+0.273*(v(24)+vfc)+104.666)-127*ef-0.0097*facf); !the expression of speed in last segment;
tfc=6/(vfc+v(24));   !the time of semi-final coast in braking phase;
tfb=4/vfc; !the time of final brake in braking phase;
v(1) = 20.0; !define the final speed of starting phase;
v(24)*v(24) >=2*((v(24)-vfc)/tfc)*5; !ensure enough speed to arrive at station;

END

Appendix C

Variable size segment model code based on heuristic solution method in Lingo 18.0:

60km track (6 segments in 1.05 hours as example):

The first step:

SETS:
nodes / 1..6/: v,vmax; !number of nodes;
segments /1..5/: d, e, a, w, t, fac, tem; !number of segments;
notches /1..9/: r, p; !number of notches;
nodes_notches(segments,notches): z; !number of binary variables;

ENDSETS

DATA:
vmax = 100 90 80 80 100 100; !the max allowed speed in each segment;
ef=0; !the gradient in last segment;
facf=0; !the curve resistance in last segment;
tem= 0.982 0.982 0.982 0.952 0.952; !the coefficient of surrounding air temperature in each segment ;
p = 0 120 285 600 775 990 1230 1910 2240; !the power of diesel engine;
r = 25 35 75 140 220 320 430 540 650; !the fuel consumption rate in each segment;
tmax=0.9275; !time limit;
d=8.7747 10 10 10 10; !the distance of each segment;
e= 0 0 2 -2 0; !the gradient in each segment;
fac= 0 112 0 0 0; !the curve resistance in each segment;

@ole('locomotive.xlsx','speed')=v; !output the speed in each node in EXCEL;

@ole('locomotive.xlsx','t')=t; !output the time in each segment in EXCEL;

@ole('locomotive.xlsx','accerate')=a; !output the acceleration in each segment in EXCEL;

@ole('locomotive.xlsx','z')=z; !output the binary variables in each segment;

ENDDATA

min = @sum (segments(i):@sum(notches(k): t(i)*z(i,k)*r(k)))+25*tfc+4.2886; !minimum value of fuel consumption;

!Subject To;
@sum (segments(i): t(i)) <= Tmax-tfc-tfb ; !allowed time in running phase;
@for(segments(i): t(i)*(v(i)+v(i+1)) = 2*d(i)); !the relationship of speed, time and distance in each segment;

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@for (nodes (i): v(i) <= Vmax(i));   !the speed in each segment does not exceed than the maximum allowed speed;

@for (segments(i): v(i+1) = v(i)+a(i)*t(i)); !the relationship of speed, acceleration and time in each segment;

@for (segments (i): a(i) *(1+0.06)= 55.2614* tem(i)*@sum (notches(k): p(k)*z(i,k))/(v(i+1)+v(i))
-(0.0007*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.2319*(v(i)+v(i+1))+36.0984)
-(0.0036*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.273*(v(i)+v(i+1))+104.666)
-127*e(i)-0.0097*fac(i)); !the expression of resultant force;

@for (segments (i): @sum (notches (k): z(i,k)) = 1); !define binary variable;

@for(nodes_notches : @bin(z));

@for(segments : @Free(a)); !define acceleration;

(vfc*vfc-v(6)*v(6))=16*(-(0.0007*(v(6)+vfc)*(v(6)+vfc)+0.2319*(v(6)+vfc)+36.0984)
-(0.0036*((v(6)+vfc)*(v(6)+vfc))+0.273*(v(6)+vfc)+104.666)-127*ef-0.0097*facf); !the expression of speed in last segment;

tfc=16/(vfc+v(6)); !the time of semi-final coast in braking phase;

tfb=4/vfc; !the time of final brake in braking phase;

v(1) = 20.0; !define the final speed of starting phase;

v(6)*v(6) >=2*((v(6)-vfc)/tfc)*10; !ensure enough speed to arrive at station;

END

The second step:

SETS:
nodes / 1..10/: v,vmax; !number of nodes;
segments /1..9/: d, e, a, w, t, fac, tem; !number of segments;
notches /1..9/: r, p; !number of notches;
nodes_notches(segments,notches): z;   !number of binary variables;

ENDSETS

DATA:
vmax = 100 100 90 90 80 80 100 100 100 100; !the max allowed speed in each segment;
ef=0; !the gradient in last segment;
facf=0; !the curve resistance in last segment;
tem= 0.982 0.982 0.982 0.982 0.982 0.952 0.952 0.952 0.952;   !the coefficient of surrounding air temperature in each segment ;
p = 0 120 285 600 775 990 1230 1910 2240; !the power of diesel engine;
r = 25 35 75 140 220 320 430 540 650; !the fuel consumption rate in each segment;
tmax=0.9275;   !time limit;
d=3.7747 5 5 5 10 5 5 5 5; !the distance of each segment;
e= 0 0 0 0 2 -2 -2 0 0; !the gradient in each segment;
fac= 0 0 112 112 0 0 0 0 0; !the curve resistance in each segment;

@ole('locomotive.xlsx','speed')=v; !output the speed in each node in EXCEL;

@ole('locomotive.xlsx','t')=t; !output the time in each segment in EXCEL;

@ole('locomotive.xlsx','accerate')=a; !output the acceleration in each segment in EXCEL;

@ole('locomotive.xlsx','z')=z; !output the binary variables in each segment;

ENDDATA

min = @sum (segments(i):@sum(notches(k): t(i)*z(i,k)*r(k)))+25*tfc+4.2886; !minimum value of fuel consumption;

!Subject To;
@sum (segments(i): t(i)) <= Tmax-tfc-tfb ; !allowed time in running phase;
@for(segments(i): t(i)*(v(i)+v(i+1)) = 2*d(i)); !the relationship of speed, time and distance in each segment;
@for (nodes (i): v(i) <= Vmax(i)); !the speed in each segment does not exceed than the maximum allowed speed;
@for (segments(i): v(i+1) = v(i)+a(i)*t(i)); !the relationship of speed, acceleration and time in each segment;
@for (segments (i): a(i) *(1+0.06)= 55.2614* tem(i)*@sum (notches(k): p(k)*z(i,k))/(v(i+1)+v(i))
-(0.0007*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.2319*(v(i)+v(i+1))+36.0984)
-(0.0036*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.273*(v(i)+v(i+1))+104.666)
-127*e(i)-0.0097*fac(i)); !the expression of resultant force;
@for (segments (i): @sum (notches (k): z(i,k)) = 1); !define binary variable;
@for(nodes_notches : @bin(z));
@for(segments : @Free(a)); !define acceleration;
(vfc*vfc-v(10)*v(10))=16*(-(0.0007*(v(10)+vfc)*(v(10)+vfc)+0.2319*(v(10)+vfc)+36.0984)
-(0.0036*((v(10)+vfc)*(v(10)+vfc))+0.273*(v(10)+vfc)+104.666)-127*ef-0.0097*facf); !the expression of speed in last segment;
tfc=16/(vfc+v(10)); !the time of semi-final coast in braking phase;
tfb=4/vfc; !the time of final brake in braking phase;
v(1) = 20.0; !define the final speed of starting phase;
v(10)*v(10) >=2*((v(10)-vfc)/tfc)*10; !ensure enough speed to arrive at station;

END

The third step:

SETS:
nodes / 1..16/: v,vmax; !number of nodes;
segments /1..15/: d, e, a, w, t, fac, tem; !number of segments;
notches /1..9/: r, p; !number of notches;

nodes_notches(segments,notches): z; !number of binary variables;

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ENDSETS

DATA:
vmax = 100 100 100 100 90 90 90 90 80 80 100 100 100 100 100 100; !the max allowed speed in each segment;
ef=0; !the gradient in last segment;
facf=0; !the curve resistance in last segment;
tem= 0.982 0.982 0.982 0.982 0.982 0.982 0.982 0.982 0.982 0.952 0.952 0.952 0.952 0.952 0.952;   !the coefficient of surrounding air temperature in each segment ;
p = 0 120 285 600 775 990 1230 1910 2240; !the power of diesel engine;
r = 25 35 75 140 220 320 430 540 650; !the fuel consumption rate in each segment;
tmax=0.9275; !time limit;
d=1.2747 2.5 2.5 2.5 2.5 2.5 2.5 2.5 15 2.5 2.5 2.5 2.5 2.5 2.5; !the distance of each segment;
e= 0 0 0 0 0 0 0 0 0.6667 -2 -2 0 0 0 0; !the gradient in each segment;
fac= 0 0 0 0 112 112 112 112 0 0 0 0 0 0 0; !the curve resistance in each segment;

@ole('locomotive.xlsx','speed')=v; !output the speed in each node in EXCEL;

@ole('locomotive.xlsx','t')=t; !output the time in each segment in EXCEL;

@ole('locomotive.xlsx','accerate')=a; !output the acceleration in each segment in EXCEL;

@ole('locomotive.xlsx','z')=z; !output the binary variables in each segment;

ENDDATA

min = @sum (segments(i):@sum(notches(k): t(i)*z(i,k)*r(k)))+25*tfc+4.2886; !minimum value of fuel consumption;

!Subject To;
@sum (segments(i): t(i)) <= Tmax-tfc-tfb ; !allowed time in running phase;
@for(segments(i): t(i)*(v(i)+v(i+1)) = 2*d(i)); !the relationship of speed, time and distance in each segment;
@for (nodes (i): v(i) <= Vmax(i)); !the speed in each segment does not exceed than the maximum allowed speed;
@for (segments(i): v(i+1) = v(i)+a(i)*t(i)); !the relationship of speed, acceleration and time in each segment;
@for (segments (i): a(i) *(1+0.06)= 55.2614* tem(i)*@sum (notches(k): p(k)*z(i,k))/(v(i+1)+v(i))
-(0.0007*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.2319*(v(i)+v(i+1))+36.0984)
-(0.0036*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.273*(v(i)+v(i+1))+104.666)
-127*e(i)-0.0097*fac(i)); !the expression of resultant force;
@for (segments (i): @sum (notches (k): z(i,k)) = 1); !define binary variable;
@for(nodes_notches : @bin(z));
@for(segments : @Free(a)); !define acceleration;
(vfc*vfc-v(16)*v(16))=16*(-(0.0007*(v(16)+vfc)*(v(16)+vfc)+0.2319*(v(16)+vfc)+36.0984)

-(0.0036*((v(16)+vfc)*(v(16)+vfc))+0.273*(v(16)+vfc)+104.666)-127*ef-0.0097*facf); !the expression of speed in last segment;

tfc=16/(vfc+v(16)); !the time of semi-final coast in braking phase;

tfb=4/vfc; !the time of final brake in braking phase;

v(1) = 20.0; !define the final speed of starting phase;

v(16)*v(16) >=2*((v(16)-vfc)/tfc)*10; !ensure enough speed to arrive at station;

END

120km track (12 segments in 1.9 hours as example):

The first step:

SETS:
nodes / 1..12/: v,vmax; !number of nodes;
segments /1..11/: d, e, a, w, t, fac, tem; !number of segments;
notches /1..9/: r, p; !number of notches;
nodes_notches(segments,notches): z; !number of binary variables;

ENDSETS

DATA:
vmax = 100 90 80 80 100 100 90 90 90 90 100 100;   !the max allowed speed in each segment;
ef=0;   !the gradient in last segment;
facf=0; !the curve resistance in last segment;
tem= 0.982 0.982 0.982 0.952 0.952 0.952 0.952 0.921 0.921 0.921 0.921;   !the coefficient of surrounding air temperature in each segment ;
p = 0 120 285 600 775 990 1230 1910 2240; !the power of diesel engine;
r = 25 35 75 140 220 320 430 540 650; !the fuel consumption rate in each segment;
tmax=1.7775; !time limit;
d=8.7747 10 10 10 10 10 10 10 10 10 10; !the distance of each segment;
e= 0 0 2 -2 0 0 0 0 1 0 -2; !the gradient in each segment;
fac= 0 112 0 0 0 0 167 208 0 0 201; !the curve resistance in each segment;

@ole('locomotive.xlsx','speed')=v; !output the speed in each node in EXCEL;

@ole('locomotive.xlsx','t')=t; !output the time in each segment in EXCEL;

@ole('locomotive.xlsx','accerate')=a; !output the acceleration in each segment in EXCEL;

@ole('locomotive.xlsx','z')=z; !output the binary variables in each segment;

ENDDATA

min = @sum (segments(i):@sum(notches(k): t(i)*z(i,k)*r(k)))+25*tfc+4.2886; !minimum value of fuel

consumption;

!Subject To;
@sum (segments(i): t(i)) <= Tmax-tfc-tfb ; !allowed time in running phase;
@for(segments(i): t(i)*(v(i)+v(i+1)) = 2*d(i)); !the relationship of speed, time and distance in each segment;
@for (nodes (i): v(i) <= Vmax(i)); !the speed in each segment does not exceed than the maximum allowed speed;
@for (segments(i): v(i+1) = v(i)+a(i)*t(i)); !the relationship of speed, acceleration and time in each segment;
@for (segments (i): a(i) *(1+0.06)= 55.2614* tem(i)*@sum (notches(k): p(k)*z(i,k))/(v(i+1)+v(i))
-(0.0007*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.2319*(v(i)+v(i+1))+36.0984)
-(0.0036*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.273*(v(i)+v(i+1))+104.666)
-127*e(i)-0.0097*fac(i)); !the expression of resultant force;
@for (segments (i): @sum (notches (k): z(i,k)) = 1); !define binary variable;
@for(nodes_notches : @bin(z));
@for(segments : @Free(a)); !define acceleration;
(vfc*vfc-v(12)*v(12))=16*(-(0.0007*(v(12)+vfc)*(v(12)+vfc)+0.2319*(v(12)+vfc)+36.0984)
-(0.0036*((v(12)+vfc)*(v(12)+vfc))+0.273*(v(12)+vfc)+104.666)-127*ef-0.0097*facf); !the expression of speed in last segment;
tfc=16/(vfc+v(12)); !the time of semi-final coast in braking phase;
tfb=4/vfc;   !the time of final brake in braking phase;
v(1) = 20.0; !define the final speed of starting phase;
v(12)*v(12) >=2*((v(12)-vfc)/tfc)*10; !ensure enough speed to arrive at station;

END

The second step:

SETS:
nodes / 1..21/: v,vmax; !number of nodes;
segments /1..20/: d, e, a, w, t, fac, tem; !number of segments;
notches /1..9/: r, p; !number of notches;
nodes_notches(segments,notches): z; !number of binary variables;

ENDSETS

DATA:
vmax = 100 100 90 90 80 80 80 100 100 100 100 100 90 90 90 90 90 90 100 100 100; !the max allowed speed in each segment;
ef=0; !the gradient in last segment;
facf=0; !the curve resistance in last segment;
tem= 0.982 0.982 0.982 0.982 0.982 0.982 0.952 0.952 0.952 0.952 0.952 0.952 0.952 0.921 0.921 0.921

0.921 0.921 0.921 0.921;   !the coefficient of surrounding air temperature in each segment ;

p = 0 120 285 600 775 990 1230 1910 2240; !the power of diesel engine;

r = 25 35 75 140 220 320 430 540 650; !the fuel consumption rate in each segment;

tmax=1.7775; !time limit;

d=3.7747 5 5 5 5 5 5 5 5 5 5 5 10 5 5 5 5 5 5 10; !the distance of each segment;

e= 0 0 0 0 2 2 -2 -2 0 0 0 0 0 0 1 1 0 0 -2; !the gradient in each segment;

fac= 0 0 112 112 0 0 0 0 0 0 0 0 167 208 208 0 0 0 0 201; !the curve resistance in each segment;

@ole('locomotive.xlsx','speed')=v; !output the speed in each node in EXCEL;

@ole('locomotive.xlsx','t')=t; !output the time in each segment in EXCEL;

@ole('locomotive.xlsx','accerate')=a; !output the acceleration in each segment in EXCEL;

@ole('locomotive.xlsx','z')=z; !output the binary variables in each segment;

ENDDATA

min = @sum (segments(i):@sum(notches(k): t(i)*z(i,k)*r(k)))+25*tfc+4.2886; !minimum value of fuel consumption;

!Subject To;

@sum (segments(i): t(i)) <= Tmax-tfc-tfb ; !allowed time in running phase;

@for(segments(i): t(i)*(v(i)+v(i+1)) = 2*d(i)); !the relationship of speed, time and distance in each segment;

@for (nodes (i): v(i) <= Vmax(i)); !the speed in each segment does not exceed than the maximum allowed speed;

@for (segments(i): v(i+1) = v(i)+a(i)*t(i)); !the expression of resultant force;

@for (segments (i): a(i) *(1+0.06)= 55.2614* tem(i)*@sum (notches(k): p(k)*z(i,k))/(v(i+1)+v(i))

-(0.0007*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.2319*(v(i)+v(i+1))+36.0984)

-(0.0036*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.273*(v(i)+v(i+1))+104.666)

-127*e(i)-0.0097*fac(i)); !the expression of resultant force;

@for (segments (i): @sum (notches (k): z(i,k)) = 1); !define binary variable;

@for(nodes_notches : @bin(z));

@for(segments : @Free(a)); !define acceleration;

(vfc*vfc-v(21)*v(21))=16*(-(0.0007*(v(21)+vfc)*(v(21)+vfc)+0.2319*(v(21)+vfc)+36.0984)

-(0.0036*((v(21)+vfc)*(v(21)+vfc))+0.273*(v(21)+vfc)+104.666)-127*ef-0.0097*facf); !the expression of speed in last segment;

tfc=16/(vfc+v(21)); !the time of semi-final coast in braking phase;

tfb=4/vfc; !the time of final brake in braking phase;

v(1) = 20.0;   !define the final speed of starting phase;

v(21)*v(21) >=2*((v(21)-vfc)/tfc)*10; !ensure enough speed to arrive at station;

END

The third step:

SETS:

nodes / 1..27/: v,vmax; !number of nodes;

segments /1..26/: d, e, a, w, t, fac, tem; !number of segments;

notches /1..9/: r, p; !number of notches;

nodes_notches(segments,notches): z; !number of binary variables;

ENDSETS

DATA:

vmax = 100 100 100 100 90 90 90 80 80 80 80 80 100 100 100 100 100 100 90 90 90 90 90 90 90 90 100; !the max allowed speed in each segment;

ef=0; !the gradient in last segment;

facf=0;　　!the curve resistance in last segment;

tem= 0.982 0.982 0.982 0.982 0.982 0.982 0.982 0.982 0.982 0.982 0.982　 0.952 0.952 0.952 0.952 0.952 0.952 0.952 0.952　 0.921 0.921 0.921 0.921 0.921 0.921 0.921;　 !the coefficient of surrounding air temperature in each segment ;

p = 0 120 285 600 775 990 1230 1910 2240; !the power of diesel engine;

r = 25 35 75 140 220 320 430 540 650; !the fuel consumption rate in each segment;

tmax=1.7775; !time limit;

d=1.2747 2.5 2.5 2.5 5 2.5 2.5 2.5 2.5 2.5 2.5 15 2.5 2.5 2.5 2.5 2.5 2.5 15 2.5 2.5 2.5 2.5 2.5 2.5 20; !the distance of each segment;

e= 0 0 0 0 0 0 2 2 2 2 -1.3333 0 0 0 0 0 0 0 0 1 1 1 1 -1; !the gradient in each segment;

fac= 0 0 0 0 112 112 112 0 0 0 0 0 0 0 0 0 0 180.6667 208 208 0 0 0 0 100.5; !the curve resistance in each segment;

@ole('locomotive.xlsx','speed')=v; !output the speed in each node in EXCEL;

@ole('locomotive.xlsx','t')=t; !output the time in each segment in EXCEL;

@ole('locomotive.xlsx','accerate')=a; !output the acceleration in each segment in EXCEL;

@ole('locomotive.xlsx','z')=z; !output the binary variables in each segment;

ENDDATA

min = @sum (segments(i):@sum(notches(k): t(i)*z(i,k)*r(k)))+25*tfc+4.2886; !minimum value of fuel consumption;

!Subject To;

@sum (segments(i): t(i)) <= Tmax-tfc-tfb ; !allowed time in running phase;

@for(segments(i): t(i)*(v(i)+v(i+1)) = 2*d(i)); !the relationship of speed, time and distance in each segment;

@for (nodes (i): v(i) <= Vmax(i)); !the speed in each segment does not exceed than the maximum allowed speed;

@for (segments(i): v(i+1) = v(i)+a(i)*t(i)); !the relationship of speed, acceleration and time in each segment;

@for (segments (i): a(i) *(1+0.06)= 55.2614* tem(i)*@sum (notches(k): p(k)*z(i,k))/(v(i+1)+v(i))
-(0.0007*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.2319*(v(i)+v(i+1))+36.0984)
-(0.0036*((v(i)+v(i+1))*(v(i)+v(i+1)))+0.273*(v(i)+v(i+1))+104.666)
-127*e(i)-0.0097*fac(i)); !the expression of resultant force;
@for (segments (i): @sum (notches (k): z(i,k)) = 1); !define binary variable;
@for(nodes_notches : @bin(z));
@for(segments : @Free(a)); !define acceleration;
(vfc*vfc-v(27)*v(27))=16*(-(0.0007*(v(27)+vfc)*(v(27)+vfc)+0.2319*(v(27)+vfc)+36.0984)
-(0.0036*((v(27)+vfc)*(v(27)+vfc))+0.273*(v(27)+vfc)+104.666)-127*ef-0.0097*facf); !the expression of
speed in last segment;
tfc=16/(vfc+v(27)); !the time of semi-final coast in braking phase;
tfb=4/vfc; !the time of final brake in braking phase;
v(1) = 20.0; !define the final speed of starting phase;
v(27)*v(27) >=2*((v(27)-vfc)/tfc)*10; !ensure enough speed to arrive at station;

END

Appendix D

The detail data of minimum fuel consumption in different segments:

60km track in 1.05hours:

Table D.1 The Relationship of Speed and Distance by Equal Segment in 60km Track

5 Segments		6 Segments		7 Segments		8 Segments	
Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)
0	0.0	0	0.0	0	0.0	0	0.0
1.23	20.0	1.23	20.0	1.23	20.0	1.23	20.0
12	88.7	10	84.0	8.6	65.2	7.5	76.6
24	76.6	20	70.4	17.2	68.6	15	68.7
36	59.2	30	36.3	25.8	64.0	22.5	64.4
48	87.4	40	79.9	34.4	78.7	30	74.0
58	44.6	50	99.7	43	91.2	37.5	90.4
60	0.0	58	66.7	51.6	80.3	45	77.1
		60	0.0	58	52.2	52.5	68.5
				60	0.0	58	43.8
						60	0.0

9 Segments		10 Segments		12 Segments		15 Segments	
Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)
0	0.0	0	0.0	0	0.0	0	0.0
1.23	20.0	1.23	20.0	1.23	20.0	1.23	20.0
6.7	88.0	6	84.6	5	78.8	4	76.2
c13.4	77.0	12	64.3	10	72.6	8	71.7
20.1	69.6	18	62.6	15	68.1	12	68.2
26.8	73.4	24	61.8	20	65.2	16	65.7
33.5	67.5	30	79.6	25	76.9	20	64.1
40.2	88.4	36	57.7	30	56.1	24	63.0
46.9	65.6	42	72.8	35	72.4	28	72.1
53.6	62.6	48	80.2	40	81.1	32	52.3
58	41.1	54	72.0	45	73.8	36	71.0
60	0.0	58	54.7	50	68.6	40	81.3
		60	0.0	55	65.2	44	75.3
				58	51.9	48	70.7
				60	0.0	52	67.2
						56	64.7
						58	56.0
						60	0.0

20 Segments		24 Segments	
Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)
0	0.0	0	0.0
1.23	20.0	1.23	20.0
3	66.3	2.5	56.3
6	64.9	5	74.7
9	63.8	7.5	72.0
12	63.0	10	69.7
15	62.4	12.5	67.8
18	62.0	15	66.2
21	79.4	17.5	65.0
24	75.3	20	64.0
27	64.2	22.5	71.7
30	74.2	25	76.1
33	63.2	27.5	65.4
36	53.9	30	55.6
39	64.4	32.5	65.4
42	70.9	35	72.0
45	75.6	37.5	76.9
48	79.0	40	80.7
51	74.8	42.5	76.9
54	71.3	45	73.6
57	61.2	47.5	70.9
58	56.8	50	68.5
60	0.0	52.5	66.7
		55	60.8
		57.5	52.6
		58	50.3
		60	0.0

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Table D.2 The Other Data by Equal Segment in 60km Track

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
5 Segments	0	1.23	163.20	0.12	1
	1.23	12	346.70	0.20	6
	12	24	-83.30	0.15	4
	24	36	-98.70	0.18	5
	36	48	172.40	0.16	4
	48	58	-285.30	0.15	0
	58	60	-495.60	0.09	0
6 Segments	0	1.23	163.20	0.12	1
	1.23	10	379.27	0.17	6
	10	20	-105.09	0.13	3
	20	30	-181.67	0.19	3
	30	40	252.92	0.17	3
	40	50	178.52	0.11	7
	50	58	-330.49	0.10	0
	58	60	-1111.67	0.06	0
7 Segments	0	1.23	163.20	0.12	1
	1.23	8.6	262.54	0.17	4
	8.6	17.2	26.06	0.13	4
	17.2	25.8	-35.55	0.13	3
	25.8	34.4	122.40	0.12	7
	34.4	43	124.00	0.10	3
	43	51.6	-109.43	0.10	4
	51.6	58	-280.66	0.10	0
	58	60	-652.50	0.08	0
8 Segments	0	1.23	163.20	0.12	1
	1.23	7.5	435.39	0.13	6
	7.5	15	-76.20	0.10	3
	15	22.5	-38.55	0.11	3
	22.5	30	89.09	0.11	7
	30	37.5	179.81	0.09	3
	37.5	45	-148.70	0.09	3
	45	52.5	-83.62	0.10	3
	52.5	58	-246.93	0.10	0
	58	60	-486.67	0.09	0

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
9 Segments	0	1.23	163.20	0.12	1
	1.23	6.7	674.21	0.10	7
	6.7	13.4	-135.57	0.08	3
	13.4	20.1	-81.74	0.09	3
	20.1	26.8	40.83	0.09	7
	26.8	33.5	-62.31	0.09	3
	33.5	40.2	244.80	0.09	3
	40.2	46.9	-263.69	0.09	1
	46.9	53.6	-28.34	0.10	3
	53.6	58	-239.34	0.09	0
	58	60	-411.00	0.10	0
10 Segments	0	1.23	163.20	0.12	1
	1.23	6	707.74	0.09	7
	6	12	-252.35	0.08	1
	12	18	-17.34	0.09	3
	18	24	-8.78	0.10	3
	24	30	210.47	0.08	7
	30	36	-250.84	0.09	3
	36	42	163.80	0.09	3
	42	48	93.87	0.08	3
	48	54	-103.44	0.08	3
	54	58	-288.21	0.06	0
	58	60	-781.43	0.07	0
12 Segments	0	1.23	163.20	0.12	1
	1.23	5	769.99	0.08	7
	5	10	-94.93	0.07	3
	10	15	-62.66	0.07	3
	15	20	-38.88	0.08	3
	20	25	166.33	0.07	7
	25	30	-276.51	0.08	3
	30	35	210.23	0.08	3
	35	40	132.11	0.07	3
	40	45	-111.67	0.06	3
	45	50	-74.06	0.07	3
	50	55	-46.63	0.07	3
	55	58	-265.28	0.05	0
	58	60	-648.75	0.08	0

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
	0	1.23	163.20	0.12	1
	1.23	4	975.35	0.06	8
	4	8	-84.35	0.05	3
	8	12	-60.53	0.06	3
	12	16	-41.39	0.06	3
	16	20	-26.51	0.06	3
	20	24	-17.31	0.06	3
15 segments	24	28	153.02	0.06	7
	28	32	-307.42	0.06	3
	32	36	287.84	0.06	3
	36	40	195.74	0.05	3
	40	44	-116.67	0.05	3
	44	48	-84.78	0.05	3
	48	52	-59.60	0.06	3
	52	56	-40.61	0.06	3
	56	58	-291.53	0.03	0
	58	60	-800.00	0.07	0
	0	1.23	163.20	0.12	1
	1.23	3	1125.02	0.04	8
	3	6	-30.83	0.05	3
	6	9	-22.58	0.05	3
	9	12	-16.38	0.05	3
	12	15	-12.52	0.05	3
	15	18	-8.96	0.05	3
	18	21	409.43	0.04	7
	21	24	-105.77	0.04	3
	24	27	-257.63	0.04	3
20 segments	27	30	230.46	0.04	7
	30	33	-251.03	0.04	3
	33	36	-181.77	0.05	3
	36	39	205.59	0.05	3
	39	42	148.34	0.04	3
	42	45	113.31	0.04	3
	45	48	88.93	0.04	3
	48	51	-108.44	0.04	3
	51	54	-85.12	0.04	3
	54	57	-222.24	0.05	1
	57	58	-221.78	0.02	0
	58	60	-811.43	0.07	0

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
	0	1.23	163.20	0.12	1
	1.23	2.5	1088.11	0.03	7
	2.5	5	481.32	0.04	7
	5	7.5	-80.47	0.03	3
	7.5	10	-64.76	0.04	3
	10	12.5	-52.42	0.04	3
	12.5	15	-41.24	0.04	3
	15	17.5	-32.12	0.04	3
	17.5	20	-24.81	0.04	3
	20	22.5	208.41	0.04	7
	22.5	25	129.96	0.03	7
24 segments	25	27.5	-303.92	0.04	3
	27.5	30	-235.63	0.04	3
	30	32.5	235.84	0.04	3
	32.5	35	181.61	0.04	3
	35	37.5	146.00	0.03	3
	37.5	40	119.97	0.03	3
	40	42.5	-119.68	0.03	3
	42.5	45	-98.60	0.03	3
	45	47.5	-80.19	0.03	3
	47.5	50	-64.40	0.04	3
	50	52.5	-51.08	0.04	3
	52.5	55	-148.50	0.04	2
	55	57.5	-186.53	0.04	1
	57.5	58	-230.87	0.01	0
	58	60	-628.75	0.08	0

120km track in 1.9hours:

Table D.3 The Relationship of Speed and Distance by Equal Segment in 120km Track

10 Segments		11 Segments		12 Segments		13 Segments	
Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)
0	0.0	0	0.0	0	0.0	0	0.0
1.23	20.0	1.23	20.0	1.23	20.0	1.23	20.0
12	88.7	10.91	86.3	10	68.3	9.23	98.4
24	76.6	21.82	76.3	20	76.7	18.46	71.7
36	59.2	32.73	48.7	30	39.2	27.69	36.3
48	81.5	43.64	80.5	40	79.9	36.92	79.8
60	66.7	54.55	67.2	50	99.8	46.15	98.8
72	68.7	65.46	68.7	60	79.4	55.38	80.1
84	68.1	76.37	61.4	70	67.3	64.61	68.5
96	57.0	87.28	90.0	80	61.3	73.84	62.9
108	92.5	98.19	71.5	90	88.1	83.07	59.9
118	50.4	109.1	84.8	100	71.6	92.3	86.3
120	0.0	118	46.7	110	85.9	101.53	54.9
		120	0.0	118	51.9	110.76	81.4
				120	0.0	118	50.5
						120	0.0

14 Segments		15 Segments	
Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)
0	0.0	0	0.0
1.23	20.0	1.23	20.0
8.57	96.0	8	63.9
17.14	79.6	16	62.0
25.71	69.5	24	61.4
34.28	79.2	32	76.7
42.85	79.9	40	91.0
51.42	69.1	48	80.8
59.99	63.5	56	70.2
68.56	61.1	64	64.4
77.13	59.4	72	61.5
85.7	59.0	80	59.6
94.27	84.6	88	89.4
102.84	71.3	96	75.2
111.41	74.6	104	66.4
118	45.8	112	86.0
120	0.0	118	60.9
		120	0.0

16 Segments		17 Segments		18 Segments		19 Segments	
Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)
0	0.0	0	0.0	0	0.0	0	0.0
1.23	20.0	1.23	20.0	1.23	20.0	1.23	20.0
7.5	62.6	7.06	89.8	6.67	88.0	6.32	86.2
15	61.7	14.12	77.7	13.34	77.0	12.64	76.3
22.5	61.2	21.18	69.7	20.01	69.6	18.96	69.4
30	73.7	28.24	74.7	26.68	73.4	25.28	65.4
37.5	79.4	35.3	67.5	33.35	67.5	31.6	71.2
45	69.8	42.36	88.0	40.02	88.4	37.92	90.5
52.5	93.7	49.42	76.1	46.69	76.9	44.24	79.0
60	79.4	56.48	68.3	53.36	69.2	50.56	70.9
67.5	69.2	63.54	63.9	60.03	64.5	56.88	65.7
75	63.5	70.6	61.5	66.7	61.9	63.2	62.8
82.5	60.5	77.66	59.7	73.37	60.9	69.52	61.2
90	81.8	84.72	59.2	80.04	88.9	75.84	59.9
97.5	70.9	91.78	80.4	86.71	63.5	82.16	59.1
105	64.4	98.84	70.4	93.38	61.0	88.48	78.0
112.5	85.7	105.9	64.4	100.05	59.8	94.8	69.7
118	62.7	112.96	74.7	106.72	77.4	101.12	64.5
120	0.0	118	52.8	113.39	69.0	107.44	86.9
		120	0.0	118	48.4	113.76	69.3
				120	0.0	118	50.3
						120	0.0

20 Segments		24 Segments		30 Segments	
Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)
0	0.0	0	0.0	0	0.0
1.23	20.0	1.23	20.0	1.23	20.0
6	90.1	5	78.8	4	71.7
12	69.5	10	72.6	8	68.3
18	65.5	15	68.1	12	85.6
24	63.3	20	65.2	16	79.0
30	79.9	25	76.9	20	73.8
36	58.0	30	56.1	24	69.8
42	72.9	35	72.4	28	74.8
48	80.2	40	81.1	32	54.7
54	72.0	45	73.8	36	72.1
60	66.7	50	68.6	40	82.0
66	63.4	55	65.2	44	75.9
72	61.6	60	63.0	48	71.1
78	60.1	65	61.6	52	67.5
84	59.3	70	85.5	56	65.0
90	82.0	75	76.8	60	63.2
96	66.0	80	70.2	64	62.0
102	76.0	85	58.1	68	61.2
108	81.6	90	78.5	72	82.1
114	72.6	95	71.6	76	75.6
118	55.3	100	66.7	80	70.5
120	0.0	105	77.3	84	79.4
		110	78.6	88	66.2
		115	61.6	92	63.7
		118	48.0	96	61.9
		120	0.0	100	60.8
				104	60.0
				108	74.4
				112	76.4
				116	62.9
				118	54.0
				120	0.0

40 Segments		40 Segments		48 Segments		48 Segments	
Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)
0	0.0	63	65.3	0	0.0	62.5	62.4
1.23	20.0	66	63.8	1.23	20.0	65	76.8
3	62.5	69	62.7	2.5	59.7	67.5	73.5
6	62.1	72	79.0	5	76.2	70	70.7
9	79.4	75	74.5	7.5	73.2	72.5	68.1
12	75.4	78	70.7	10	70.7	75	66.0
15	72.0	81	67.7	12.5	68.6	77.5	64.3
18	69.2	84	65.3	15	66.9	80	77.2
21	67.1	87	59.4	17.5	65.5	82.5	69.9
24	65.4	90	73.8	20	64.5	85	63.4
27	74.8	93	66.4	22.5	71.9	87.5	73.5
30	63.8	96	60.2	25	61.5	90	66.6
33	74.0	99	67.8	27.5	70.4	92.5	64.8
36	63.0	102	73.0	30	60.1	95	63.4
39	70.1	105	76.8	32.5	68.3	97.5	62.3
42	74.9	108	79.7	35	74.1	100	61.5
45	78.5	111	75.2	37.5	78.6	102.5	68.9
48	81.3	114	65.0	40	82.0	105	74.3
51	76.7	117	55.0	42.5	78.1	107.5	78.5
54	72.9	118	50.4	45	74.6	110	81.7
57	69.7	120	0.0	47.5	71.7	112.5	74.7
60	67.3			50	69.2	115	66.2
				52.5	67.2	117.5	57.8
				55	65.6	118	55.6
				57.5	64.3	120	0.0
				60	63.3		

Table D.4 The Other Data by Equal Segment in 120km Track

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
10 Segments	0	1.23	163.20	0.12	1
	1.23	12	346.68	0.20	6
	12	24	-83.33	0.15	4
	24	36	-98.73	0.18	5
	36	48	131.17	0.17	3
	48	60	-91.68	0.16	3
	60	72	11.20	0.18	4
	72	84	-3.37	0.18	4
	84	96	-58.05	0.19	4
	96	108	221.54	0.16	5
	108	118	-300.88	0.14	0
	118	120	-630.00	0.08	0
11 Segments	0	1.23	163.20	0.12	1
	1.23	10.91	363.45	0.18	6
	10.91	21.82	-74.22	0.13	4
	21.82	32.73	-157.95	0.17	4
	32.73	43.64	188.06	0.17	3
	43.64	54.55	-89.93	0.15	3
	54.55	65.46	9.29	0.16	4
	65.46	76.37	-43.58	0.17	3
	76.37	87.28	198.36	0.14	7
	87.28	98.19	-136.94	0.14	3
	98.19	109.1	95.89	0.14	3
	109.1	118	-272.46	0.14	0
	118	120	-518.89	0.09	0
12 Segments	0	1.23	163.20	0.12	1
	1.23	10	242.93	0.20	4
	10	20	61.26	0.14	5
	20	30	-217.40	0.17	3
	30	40	242.34	0.17	3
	40	50	178.23	0.11	7
	50	60	-182.19	0.11	3
	60	70	-88.69	0.14	3
	70	80	-39.03	0.16	3
	80	90	200.00	0.13	7
	90	100	-131.67	0.13	3
	100	110	112.52	0.13	3
	110	118	-282.93	0.12	0
	118	120	-648.75	0.08	0

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
13 Segments	0	1.23	163.20	0.12	1
	1.23	9.23	579.79	0.14	7
	9.23	18.46	-246.41	0.11	2
	18.46	27.69	-206.62	0.17	3
	27.69	36.92	273.30	0.16	3
	36.92	46.15	183.72	0.10	7
	46.15	55.38	-181.23	0.10	3
	55.38	64.61	-92.79	0.12	3
	64.61	73.84	-40.45	0.14	3
	73.84	83.07	-19.74	0.15	3
	83.07	92.3	208.70	0.13	7
	92.3	101.53	-239.96	0.13	1
	101.53	110.76	196.07	0.14	3
	110.76	118	-281.09	0.11	0
	118	120	-631.25	0.08	0
14 Segments	0	1.23	163.20	0.12	1
	1.23	8.57	600.21	0.13	7
	8.57	17.14	-167.90	0.10	3
	17.14	25.71	-88.06	0.11	3
	25.71	34.28	84.47	0.12	7
	34.28	42.85	5.74	0.11	1
	42.85	51.42	-93.73	0.12	3
	51.42	59.99	-42.73	0.13	3
	59.99	68.56	-17.96	0.14	3
	68.56	77.13	-12.01	0.14	3
	77.13	85.7	-2.57	0.14	3
	85.7	94.27	214.24	0.12	7
	94.27	102.84	-120.75	0.11	3
	102.84	111.41	28.25	0.12	1
	111.41	118	-261.92	0.11	0
	118	120	-508.89	0.09	0

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
15 Segments	0	1.23	163.20	0.12	1
	1.23	8	271.69	0.16	4
	8	16	-14.47	0.13	3
	16	24	-4.71	0.13	3
	24	32	132.11	0.12	7
	32	40	150.10	0.10	3
	40	48	-110.49	0.09	4
	48	56	-99.81	0.11	3
	56	64	-48.95	0.12	3
	64	72	-22.35	0.13	3
	72	80	-14.50	0.13	3
	80	88	277.98	0.11	8
	88	96	-146.28	0.10	3
	96	104	-78.19	0.11	3
	104	112	186.69	0.10	3
	112	118	-313.69	0.08	0
	118	120	-870.00	0.07	0
16 Segments	0	1.23	163.20	0.12	1
	1.23	7.5	280.47	0.15	4
	7.5	15	-7.57	0.12	3
	15	22.5	-4.03	0.12	3
	22.5	30	112.33	0.11	7
	30	37.5	57.74	0.10	1
	37.5	45	-94.57	0.10	3
	45	52.5	260.33	0.09	7
	52.5	60	-164.91	0.09	3
	60	67.5	-101.41	0.10	3
	67.5	75	-50.35	0.11	3
	75	82.5	-24.58	0.12	3
	82.5	90	201.82	0.11	7
	90	97.5	-111.35	0.10	3
	97.5	105	-58.41	0.11	3
	105	112.5	213.36	0.10	3
	112.5	118	-328.67	0.07	0
	118	120	-1045.00	0.06	0

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
17 Segments	0	1.23	163.20	0.12	1
	1.23	7.06	656.68	0.11	7
	7.06	14.12	-143.05	0.08	3
	14.12	21.18	-84.19	0.10	3
	21.18	28.24	51.65	0.10	7
	28.24	35.3	-73.07	0.10	3
	35.3	42.36	226.14	0.09	3
	42.36	49.42	-138.63	0.09	3
	49.42	56.48	-79.74	0.10	3
	56.48	63.54	-41.20	0.11	3
	63.54	70.6	-20.93	0.11	3
	70.6	77.66	-15.26	0.12	3
	77.66	84.72	-4.57	0.12	3
	84.72	91.78	209.20	0.10	7
	91.78	98.84	-105.84	0.09	3
	98.84	105.9	-57.30	0.10	3
	105.9	112.96	100.70	0.10	1
	112.96	118	-273.35	0.08	0
	118	120	-660.00	0.08	0
18 Segments	0	1.23	163.20	0.12	1
	1.23	6.67	674.21	0.10	7
	6.67	13.34	-135.57	0.08	3
	13.34	20.01	-81.74	0.09	3
	20.01	26.68	40.83	0.09	7
	26.68	33.35	-62.31	0.09	3
	33.35	40.02	244.80	0.09	3
	40.02	46.69	-142.38	0.08	3
	46.69	53.36	-85.16	0.09	3
	53.36	60.03	-46.31	0.10	3
	60.03	66.7	-24.79	0.11	3
	66.7	73.37	-9.93	0.11	3
	73.37	80.04	314.60	0.09	7
	80.04	86.71	-290.14	0.09	3
	86.71	93.38	-23.41	0.11	3
	93.38	100.05	-10.85	0.11	3
	100.05	106.72	181.87	0.10	2
	106.72	113.39	-92.81	0.09	3
	113.39	118	-257.43	0.08	0
	118	120	-605.00	0.08	0

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
19 Segments	0	1.23	163.20	0.12	1
	1.23	6.32	691.21	0.10	7
	6.32	12.64	-128.31	0.08	3
	12.64	18.96	-79.04	0.09	3
	18.96	25.28	-42.87	0.09	3
	25.28	31.6	62.85	0.09	7
	31.6	37.92	247.55	0.08	3
	37.92	44.24	-154.58	0.07	3
	44.24	50.56	-96.69	0.08	3
	50.56	56.88	-55.59	0.09	3
	56.88	63.2	-29.49	0.10	3
	63.2	69.52	-16.49	0.10	3
	69.52	75.84	-11.97	0.10	3
	75.84	82.16	-7.81	0.11	3
	82.16	88.48	205.11	0.09	7
	88.48	94.8	-96.61	0.09	3
	94.8	101.12	-55.24	0.09	3
	101.12	107.44	268.71	0.08	3
	107.44	113.76	-218.55	0.08	2
	113.76	118	-270.85	0.07	0
	118	120	-628.75	0.08	0
20 Segments	0	1.23	163.20	0.12	1
	1.23	6	807.54	0.09	8
	6	12	-273.56	0.08	1
	12	18	-44.45	0.09	3
	18	24	-23.97	0.09	3
	24	30	198.62	0.08	7
	30	36	-252.60	0.09	3
	36	42	162.68	0.09	3
	42	48	93.41	0.08	3
	48	54	-103.72	0.08	3

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
20 Segments	54	60	-62.15	0.09	3
	60	66	-35.56	0.09	3
	66	72	-18.61	0.10	3
	72	78	-15.50	0.10	3
	78	84	-7.61	0.10	3
	84	90	267.93	0.08	7
	90	96	-197.54	0.08	3
	96	102	118.58	0.08	3
	102	108	72.34	0.08	3
	108	114	-115.59	0.08	3
	114	118	-287.53	0.06	0
	118	120	-790.00	0.07	0
24 Segments	0	1.23	163.20	0.12	1
	1.23	5	769.99	0.08	7
	5	10	-94.93	0.07	3
	10	15	-62.66	0.07	3
	15	20	-38.88	0.08	3
	20	25	166.33	0.07	7
	25	30	-276.51	0.08	3
	30	35	210.23	0.08	3
	35	40	132.11	0.07	3
	40	45	-111.67	0.06	3
	45	50	-74.06	0.07	3
	50	55	-46.63	0.07	3
	55	60	-28.05	0.08	3
	60	65	-17.48	0.08	3
	65	70	352.29	0.07	7
	70	75	-141.45	0.06	3
	75	80	-96.55	0.07	3
	80	85	-156.04	0.08	3
	85	90	278.33	0.07	7
	90	95	-103.55	0.07	3
	95	100	-67.73	0.07	3
	100	105	152.56	0.07	3
	105	110	20.10	0.06	2
	110	115	-238.06	0.07	1
	115	118	-271.28	0.05	0
	118	120	-600.00	0.08	0

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
	0	1.23	163.20	0.12	1
	1.23	4	855.27	0.06	7
	4	8	-59.77	0.06	3
	8	12	331.90	0.05	7
	12	16	-135.65	0.05	3
	16	20	-99.17	0.05	3
	20	24	-71.01	0.06	3
	24	28	89.95	0.06	7
	28	32	-325.00	0.06	3
	32	36	275.74	0.06	3
	36	40	190.17	0.05	3
	40	44	-120.52	0.05	3
	44	48	-87.89	0.05	3
	48	52	-62.00	0.06	3
30 Segments	52	56	-42.38	0.06	3
	56	60	-28.19	0.06	3
	60	64	-18.34	0.06	3
	64	68	-13.30	0.06	3
	68	72	375.25	0.06	7
	72	76	-128.58	0.05	3
	76	80	-94.04	0.05	3
	80	84	167.10	0.05	7
	84	88	-239.94	0.05	3
	88	92	-41.15	0.06	3
	92	96	-27.16	0.06	3
	96	100	-17.53	0.07	3
	100	104	-11.12	0.07	3
	104	108	241.34	0.06	3
	108	112	38.47	0.05	1
	112	116	-236.51	0.06	1
	116	118	-295.38	0.03	0
	118	120	-771.43	0.07	0

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
	0	1.23	163.20	0.12	1
	1.23	3	986.24	0.04	7
	3	6	-8.29	0.05	3
	6	9	409.72	0.04	7
	9	12	-105.25	0.04	3
	12	15	-83.32	0.04	3
	15	18	-64.20	0.04	3
	18	21	-48.63	0.04	3
	21	24	-36.27	0.05	3
	24	27	218.25	0.04	7
	27	30	-254.62	0.04	3
	30	33	234.83	0.04	7
	33	36	-249.78	0.04	3
	36	39	155.35	0.05	3
40 Segments	39	42	117.95	0.04	3
	42	45	92.27	0.04	3
	45	48	73.43	0.04	3
	48	51	-120.87	0.04	3
	51	54	-95.71	0.04	3
	54	57	-74.40	0.04	3
	57	60	-56.79	0.04	3
	60	63	-43.72	0.05	3
	63	66	-32.31	0.05	3
	66	69	-23.56	0.05	3
	69	72	386.23	0.04	7
	72	75	-115.42	0.04	3
	75	78	-90.80	0.04	3
	78	81	-70.03	0.04	3
	81	84	-52.99	0.05	3
	84	87	-124.01	0.05	3
	87	90	320.72	0.05	7

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
40 Segments	90	93	-173.61	0.04	3
	93	96	-130.31	0.05	3
	96	99	162.90	0.05	3
	99	102	122.26	0.04	3
	102	105	95.04	0.04	3
	105	108	75.32	0.04	3
	108	111	-117.91	0.04	3
	111	114	-238.09	0.04	1
	114	117	-200.21	0.05	1
	117	118	-227.60	0.02	0
	118	120	-630.00	0.08	0
48 Segments	0	1.23	163.20	0.12	1
	1.23	2.5	1241.97	0.03	8
	2.5	5	447.14	0.04	7
	5	7.5	-88.81	0.03	3
	7.5	10	-71.88	0.03	3
	10	12.5	-58.40	0.04	3
	12.5	15	-46.17	0.04	3
	15	17.5	-36.12	0.04	3
	17.5	20	-28.01	0.04	3
	20	22.5	204.02	0.04	7
	22.5	25	-278.08	0.04	3
	25	27.5	235.15	0.04	7
	27.5	30	-268.55	0.04	3
	30	32.5	210.43	0.04	3
	32.5	35	165.60	0.04	3
	35	37.5	134.52	0.03	3
	37.5	40	111.21	0.03	3
	40	42.5	-126.93	0.03	3
	42.5	45	-104.99	0.03	3
	45	47.5	-85.74	0.03	3
	47.5	50	-69.13	0.04	3
	50	52.5	-55.05	0.04	3
	52.5	55	-43.33	0.04	3
	55	57.5	-33.75	0.04	3
	57.5	60	-26.05	0.04	3
	60	62.5	-21.29	0.04	3
	62.5	65	401.50	0.04	7
	65	67.5	-99.57	0.03	3
	67.5	70	-80.94	0.03	3
	70	72.5	-71.55	0.04	3

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
48 Segments	72.5	75	-56.85	0.04	3
	75	77.5	-44.61	0.04	3
	77.5	80	364.80	0.04	7
	80	82.5	-214.59	0.03	3
	82.5	85	-172.20	0.04	3
	85	87.5	275.71	0.04	7
	87.5	90	-193.39	0.04	3
	90	92.5	-46.35	0.04	3
	92.5	95	-36.09	0.04	3
	95	97.5	-27.82	0.04	3
	97.5	100	-21.26	0.04	3
	100	102.5	194.77	0.04	3
	102.5	105	154.54	0.03	3
	105	107.5	126.05	0.03	3
	107.5	110	104.46	0.03	3
	110	112.5	-219.11	0.03	2
	112.5	115	-239.62	0.04	1
	115	117.5	-207.87	0.04	1
	117.5	118	-248.65	0.01	0
	118	120	-794.29	0.07	0

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Appendix E

The final optimal results of 60km track by heuristic solution method:

Table E.1 The Relationship of Speed and Distance by Heuristic Solution Method in 60km Track

1.0 Hour		1.05 Hour		1.1 Hour		1.15 Hour	
Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)
0	0.0	0	0.0	0	0.0	0	0.0
1.23	20.0	1.23	20.0	1.23	20.0	1.23	20.0
2.5	56.3	2.5	56.3	2.5	56.3	2.5	56.3
5	74.7	5	57.6	5	74.7	5	57.6
7.5	72.0	7.5	75.3	7.5	72.0	7.5	58.6
10	82.6	10	72.4	10	69.7	10	59.2
20	69.7	12.5	70.0	20	63.8	12.5	59.7
22.5	78.0	15	68.1	22.5	71.6	15	60.0
25	67.1	17.5	66.5	25	61.2	17.5	60.2
27.5	73.4	20	79.5	27.5	70.3	20	76.4
30	62.9	35	53.3	30	60.0	22.5	65.6
32.5	70.2	37.5	63.9	32.5	68.2	25	55.9
35	75.5	40	83.9	35	74.1	35	59.9
55	69.5	42.5	79.7	50	72.6	37.5	68.1
58	56.5	45	76.0	58	36.4	40	74.0
60	0.0	47.5	72.9	60	0.0	42.5	71.2
		50	82.7			45	68.8
		58	48.4			47.5	66.9
		60	0.0			50	58.5
						55	59.2
						58	45.4
						60	0.0

1.2 Hour		1.25 Hour	
Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)
0	0.0	0	0.0
1.23	20.0	1.23	20.0
2.5	56.3	10	60.4
5	57.6	12.5	52.2
7.5	58.6	15	54.8
10	50.5	17.5	56.5
40	64.3	20	57.7
42.5	63.3	22.5	68.7
45	62.5	25	58.6
47.5	61.9	27.5	49.9
50	76.6	30	43.3
58	41.3	50	79.3
60	0.0	58	44.5
		60	0.0

Table E.2 The Other Data by Heuristic Solution Method in 60km Track

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
	0	1.23	163.20	0.12	1
	1.23	2.5	1088.11	0.03	7
	2.5	5	481.32	0.04	7
	5	7.5	-80.47	0.03	3
	7.5	10	329.26	0.03	7
	10	20	-98.66	0.13	3
1.0 Hour	20	22.5	245.67	0.03	8
	22.5	25	-315.19	0.03	3
	25	27.5	176.52	0.04	7
	27.5	30	-287.17	0.04	3
	30	32.5	195.45	0.04	3
	32.5	35	155.59	0.03	3
	35	55	-21.94	0.28	3
	55	58	-259.69	0.05	0
	58	60	-807.14	0.07	0

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
	0	1.23	163.20	0.12	1
	1.23	2.5	1088.11	0.03	7
	2.5	5	29.71	0.04	3
	5	7.5	468.06	0.04	7
	7.5	10	-83.59	0.03	3
	10	12.5	-68.34	0.04	3
	12.5	15	-54.46	0.04	3
	15	17.5	-42.91	0.04	3
1.05 Hour	17.5	20	380.28	0.03	7
	20	35	-116.01	0.23	3
	35	37.5	249.44	0.04	3
	37.5	40	589.39	0.03	7
	40	42.5	-137.04	0.03	3
	42.5	45	-113.95	0.03	3
	45	47.5	-93.57	0.03	3
	47.5	50	304.57	0.03	7
	50	58	-301.85	0.12	0
	58	60	-455.00	0.08	0
	0	1.23	163.20	0.12	1
	1.23	2.5	1088.11	0.03	7
	2.5	5	481.32	0.04	7
	5	7.5	-80.47	0.03	3
	7.5	10	-64.76	0.04	3
	10	20	-39.42	0.15	3
	20	22.5	211.32	0.04	7
1.1 Hour	22.5	25	-275.78	0.04	3
	25	27.5	238.73	0.04	7
	27.5	30	-267.50	0.04	3
	30	32.5	211.26	0.04	3
	32.5	35	166.14	0.04	3
	35	50	-7.09	0.20	3
	50	58	0.00	0.15	0
	58	60	0.00	0.11	0

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
1.15 Hour	0	1.23	163.20	0.12	1
	1.23	2.5	1088.11	0.03	7
	2.5	5	29.71	0.04	3
	5	7.5	21.41	0.04	3
	7.5	10	15.60	0.04	3
	10	12.5	10.56	0.04	3
	12.5	15	7.79	0.04	3
	15	17.5	5.76	0.04	3
	17.5	20	441.15	0.04	7
	20	22.5	-305.54	0.04	3
	22.5	25	-237.28	0.04	3
	25	35	23.20	0.17	3
	35	37.5	212.00	0.04	3
	37.5	40	166.63	0.04	3
	40	42.5	-82.33	0.03	3
	42.5	45	-66.22	0.04	3
	45	47.5	-52.60	0.04	3
	47.5	50	-208.84	0.04	1
	50	55	7.79	0.08	3
	55	58	-240.88	0.06	0
	58	60	-515.32	0.09	0
1.2 Hour	0	1.23	163.20	0.12	1
	1.23	2.5	1088.11	0.03	7
	2.5	5	29.71	0.04	3
	5	7.5	21.41	0.04	3
	7.5	10	-176.60	0.05	1
	10	40	26.45	0.52	3
	40	42.5	-26.12	0.04	3
	42.5	45	-20.01	0.04	3
	45	47.5	-15.23	0.04	3
	47.5	50	408.12	0.04	7
	50	58	-260.12	0.14	0
	58	60	-426.65	0.10	0

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	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
1.25 Hour	0	1.23	163.20	0.12	1
	1.23	10	184.90	0.22	3
	10	12.5	-184.25	0.04	1
	12.5	15	55.89	0.05	3
	15	17.5	38.74	0.04	3
	17.5	20	27.54	0.04	3
	20	22.5	276.09	0.04	7
	22.5	25	-257.23	0.04	3
	25	27.5	-188.44	0.05	3
	27.5	30	-122.31	0.05	3
	30	50	110.44	0.33	3
	50	58	-267.89	0.13	0
	58	60	-494.44	0.09	0

The final optimal results of 120km track by heuristic solution method:

Table E.3 The Relationship of Speed and Distance by Heuristic Solution Method in 120km Track

1.9 Hour		2.0 Hour		2.1 Hour		2.2 Hour	
Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)
0	0.0	0	0.0	0	0.0	0	0.0
1.23	20.0	1.23	20.0	1.23	20.0	1.23	20.0
2.5	56.3	10	60.4	10	60.4	2.5	56.3
5	74.7	20	60.8	20	60.8	5	57.6
7.5	72.0	22.5	70.1	30	33.6	7.5	58.6
10	69.7	25	75.1	40	79.9	10	59.2
15	66.2	27.5	78.3	50	67.7	70	60.3
17.5	79.3	30	67.4	60	62.4	72.5	59.8
20	75.9	40	85.1	70	60.5	75	59.5
22.5	65.2	55	65.4	80	59.0	77.5	59.3
25	72.3	70	60.4	90	88.0	80	59.1
27.5	76.5	72.5	75.2	100	71.5	95	47.8
30	65.7	75	71.9	110	85.8	110	78.5
45	80.8	77.5	69.1	118	51.9	118	43.5
47.5	77.0	80	66.8	120	0.0	120	0.0
50	73.7	82.5	75.4				
52.5	70.9	85	80.6				
55	68.6	87.5	73.0				
57.5	66.7	90	66.1				
60	79.1	92.5	64.5				
75	63.2	95	63.1				
77.5	76.6	110	79.4				
80	73.1	118	44.7				
82.5	66.3	120	0.0				
85	60.3						
87.5	71.9						
90	65.2						
110	76.6						
118	41.3						
120	0.0						

2.3 Hour		2.4 Hour	
Distance (km)	Speed (km/h)	Distance (km)	Speed (km/h)
0	0.0	0	0.0
1.23	20.0	1.23	20.0
15	66.9	20	71.4
30	39.0	25	51.7
32.5	56.6	30	39.4
35	61.4	35	57.0
37.5	65.1	40	72.9
40	71.8	45	56.1
42.5	69.3	50	58.0
45	67.3	90	59.0
47.5	65.6	95	50.7
50	64.3	100	46.5
52.5	63.3	105	53.8
55	62.5	110	70.9
57.5	61.9	118	34.2
60	61.4	120	0.0
85	53.9		
110	71.6		
118	35.0		
120	0.0		

Table E.4 The Other Data by Heuristic Solution Method in 120km Track

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
	0	1.23	163.20	0.12	1
	1.23	2.5	1088.11	0.03	7
	2.5	5	481.32	0.04	7
	5	7.5	-80.47	0.03	3
	7.5	10	-64.76	0.04	3
	10	15	-47.33	0.07	3
	15	17.5	382.83	0.03	7
	17.5	20	-107.60	0.03	3
	20	22.5	-302.47	0.04	3
	22.5	25	196.78	0.04	7
	25	27.5	124.08	0.03	7
	27.5	30	-306.15	0.04	3
	30	45	73.45	0.20	3
1.9 Hour	45	47.5	-120.00	0.03	3
	47.5	50	-98.88	0.03	3
	50	52.5	-80.44	0.03	3
	52.5	55	-64.60	0.04	3
	55	57.5	-51.26	0.04	3
	57.5	60	361.88	0.03	7
	60	75	-75.20	0.21	3
	75	77.5	374.91	0.04	7
	77.5	80	-104.73	0.03	3
	80	82.5	-191.27	0.04	3
	82.5	85	-150.47	0.04	3
	85	87.5	306.68	0.04	7
	87.5	90	-184.21	0.04	3
	90	110	40.20	0.28	3
	110	118	-259.96	0.14	0
	118	120	-425.80	0.10	0

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
2.0 Hour	0	1.23	163.20	0.12	1
	1.23	10	184.90	0.22	3
	10	20	2.47	0.17	3
	20	22.5	243.09	0.04	7
	22.5	25	146.44	0.03	7
	25	27.5	96.64	0.03	7
	27.5	30	-316.94	0.03	3
	30	40	135.15	0.13	3
	40	55	-98.67	0.20	3
	55	70	-21.28	0.24	3
	70	72.5	402.52	0.04	7
	72.5	75	-96.82	0.03	3
	75	77.5	-78.41	0.04	3
	77.5	80	-62.65	0.04	3
	80	82.5	242.49	0.04	7
	82.5	85	162.53	0.03	7
	85	87.5	-233.76	0.03	3
	87.5	90	-190.23	0.04	3
	90	92.5	-43.57	0.04	3
	92.5	95	-33.83	0.04	3
	95	110	77.44	0.21	3
	110	118	-269.71	0.13	0
	118	120	-498.72	0.09	0
2.1 Hour	0	1.23	163.20	0.12	1
	1.23	10	184.90	0.22	3
	10	20	2.47	0.17	3
	20	30	-128.17	0.21	3
	30	40	262.74	0.18	3
	40	50	-89.86	0.14	3
	50	60	-34.68	0.15	3
	60	70	-11.81	0.16	3
	70	80	-8.59	0.17	3
	80	90	213.12	0.14	7
	90	100	-131.55	0.13	3
	100	110	112.58	0.13	3
	110	118	-282.53	0.12	0
	118	120	-674.58	0.08	0

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
2.2 Hour	0	1.23	163.20	0.12	1
	1.23	2.5	1088.11	0.03	7
	2.5	5	29.71	0.04	3
	5	7.5	21.41	0.04	3
	7.5	10	15.60	0.04	3
	10	70	1.06	1.00	3
	70	72.5	-10.68	0.04	3
	72.5	75	-7.99	0.04	3
	75	77.5	-5.95	0.04	3
	77.5	80	-4.43	0.04	3
	80	95	-40.25	0.28	3
	95	110	129.16	0.24	3
	110	118	-268.72	0.13	0
	118	120	-473.18	0.09	0
2.3 Hour	0	1.23	163.20	0.12	1
	1.23	15	148.12	0.32	3
	15	30	-98.75	0.28	3
	30	32.5	336.09	0.05	3
	32.5	35	114.14	0.04	2
	35	37.5	93.52	0.04	2
	37.5	40	183.20	0.04	3
	40	42.5	-69.70	0.04	3
	42.5	45	-55.53	0.04	3
	45	47.5	-43.73	0.04	3
	47.5	50	-34.07	0.04	3
	50	52.5	-26.30	0.04	3
	52.5	55	-20.15	0.04	3
	55	57.5	-15.34	0.04	3
	57.5	60	-11.61	0.04	3
	60	85	-17.24	0.43	3
	85	110	44.26	0.40	3
	110	118	-243.62	0.15	0
	118	120	-306.65	0.11	0

	Start Point(km)	End Point (km)	Acceleration (km/h^2)	Time (Hour)	Notch Position
	0	1.23	163.20	0.12	1
	1.23	20	125.21	0.41	3
	20	25	-243.03	0.08	3
	25	30	-112.10	0.11	3
	30	35	169.98	0.10	2
	35	40	205.84	0.08	3
2.4 Hour	40	45	-215.60	0.08	1
	45	50	20.92	0.09	3
	50	90	1.46	0.68	3
	90	95	-90.93	0.09	3
	95	100	-40.76	0.10	3
	100	105	73.78	0.10	1
	105	110	213.26	0.08	3
	110	118	-244.89	0.15	0
	118	120	-292.58	0.12	0