Multi-level production planning with raw-material perishability and inventory bounds

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ABSTRACT

Multi-level production planning with raw-material perishability and inventory bounds

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This thesis focuses on studying one of the most important and fundamental links in supply chain management: production planning. A considerably common assumptions in most of the production planning research literature is that the intermediate items involved in the production process have unlimited *lifespans*, meaning they can be stored and used indefinitely. In real life applications, whether referring to physical exhaustion, loss of functionality, or obsolescence, most items deteriorate over time and cannot be stored infinitely without enforcing specific constraints on a set of crucial production planning decisions. This is specially the case for multi-level production structures. In the thesis, we first introduce the fundamental characteristics in production planning modeling and discuss some of the common elements and assumptions used to model complex production planning problems. We also present an overview of the production planning research evolution. Our attention is then focused on the most relevant modeling approaches for perishability in production planning available in the research literature. We present lot-sizing problems that incorporate raw-material perishability and analyze how these considerations enforce specific constraints on a set of fundamental decisions. Three variants of the two-level lot-sizing problem are studied: with *fixed raw-material shelf-life*, with *raw-material functionality deterioration*, and with *functionality and volume deterioration*. We propose mixed-integer programming formulations for each of these variants and perform computational experiments with sensitivity analyses, showing the added value of explicitly incorporating perishability considerations into production planning problems. Using a Silver-Meal-based rolling-

horizon algorithm, we develop a sequential approach to solve the studied problems and compare the results with our proposed formulations.

We then shift our attention to study the *multi-item, multi-level lot-sizing problem with raw-material perishability and batch ordering*, inspired by an application in advanced composite manufacturing processes. We proposed a mixed-integer programming formulation for the problem and perform computational experiments with sensitivity analyses, demonstrating its potentials for practical applications in planning composite production.

Finally, we address the study of production planning involving *inventory bounds*. This characteristic is shown to be related to the perishable raw-material considerations and constitutes another fundamental aspect of this family of problems. We study the *multi-item uncapacitated lot-sizing problem with inventory bounds*, presenting a new mixed-integer programming formulation for the case of non-speculative (Wagner-Whitin) cost structure using a special set of variables to determine the production intervals for each item. We then reformulate the problem using a variable-splitting technique that allows for a Dantzig-Wolfe decomposition. The Dantzig-Wolfe principle exploits the structure of the problem by decomposing it into two sub-problems: one relating to the production decisions per item and another that relates to the inventory decisions per period. We propose a Column Generation algorithm for solving the Dantzig-Wolfe reformulation. Computational experiments are performed to evaluate the proposed formulations and algorithms on a set of benchmark instances.

This research presents important contributions on a variety of fields related to production planning that had only been partially studied in the literature. It also opens important research paths for the integration of different types of raw-material perishability in multi-level product structures processes, with the study of finished product inventory bounds.

To Johana, Cristina, Carlos Manuel, Catalina, Juan Sebastián, and Valeria:
The core foundations of my inner motivation.

"Avoiding difficult situations or running away from them does not usually take much skill or effort. But doing so prevents you from testing your own limits and from growing. The ability to face difficulties can be crucial for your growth. However, if you are faced with a situation in which the difficulties are simply overwhelming, you should step back for the time being and wait until you have built up enough strength to deal with it skillfully."

- Sayadaw U Tajeniya.

"Of all the footprints, that of the elephant is the deepest and most supreme.
Of all contemplations, that of impermanence is the deepest and most supreme."

- The Buddha (in the Udānavarga).

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All we have is our intentions and motives. We make decisions, we act, and we move forward because of our intentions and motives. From there, the flow of life takes us places and offers us experiences that we could never imagine. We can dream, we can plan for the future, we can visualize ourselves fulfilling our goals, but uncertainty is a fundamental characteristic of our human life.

All we have is our intentions and motives, and they are all we have to offer other people. We make connections and we collaborate based on the quality of our intentions and motives. From there, life brings us to meet and interact with people that, in one way or another, knowing it or not, give us support and guidance on the road.

This PhD has its roots in the deep intention of following a personal passion for *teaching*: where I find everyday meaning and motivation. It has been the most significant and challenging project of my life, and it would not have been possible without the help, complicity, and company of the people with whom I connected.

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Chapter 1

Introduction

In the context of supply chain management, *production planning* constitutes one of the fundamental links to achieve the efficiency and competitiveness desired by any manufacturing organization. In general terms, *production planning* can be defined as the planning of the acquisition of resources and raw material, as well as the planning of the production activities required to transform raw material into finished products, meeting customer demand in the most cost-effective way. Decisions in production planning are fundamentally related to the size and timing of production lots or batches, and the size and timing of raw material acquirement. However, in addition to the basic production planning decisions, other multiple aspects of manufacturing systems must be taken into account to define feasible, realistic and economical production plans. For instance, the availability of resources (machine hours, workforce, materials), production and set-up costs, physical warehouse spaces, special inventory conditions and costs, and other important performance measures [106].

Decisions in production planning often lead to the integration of other operational and strategic decisions, creating more relevant problems such as: inventory management problems [54]; production routing problems [10]; production–scheduling problems [85]; and production–distribution problems [45].

A common assumption in most of the production planning literature is that the finished and intermediate items involved in the production process have unlimited *lifespans*, meaning they can be stored and used indefinitely. However, in practice, most items deteriorate over time, referring not only to physical exhaustion or loss of functionality (usefulness), but also obsolescence. Often, the rate of deterioration is low or can be ignored and there is little need for considering it in the planning process. Nonetheless, in many types of industries, it is common to deal with items that are subject to significant rates of deterioration. These items are referred to as *perishable products*.

Although there are multiple definitions of *perishability* depending on the type of product or system, the concept basically relates to items that cannot be stored infinitely without deterioration or devaluation [29]. Clear cases of this type of products can be found in the food or pharmaceutical industries [46, 125]. For instance, in the yogurt industry [44], perishability is found in every link of the supply chain: from the raw-material (milk) that enters the dairy factories, to the highly perishable intermediate items and, finally, the finished-products which are all stamped with a best-before-date fixing its *shelf-life*. There are also more subjective, but equally relevant cases, as in the competitive technological market, where deterioration does not necessarily refer to the physical condition of a product, but to its marketable (or salable) life [138].

Perishability and deterioration enforce specific constraints on a set of crucial production planning decisions [19], specially in the case of multi-level production structures, where two or more items are produced, and at least one item is required as an input (raw-material, component, part) of another. These intermediate products, either acquired from a supplier or processed internally, can often be inventoried, allowing one to produce and consume them at different moments and rates in time [100]. Most of the data associated with inventories has to be extended in order to track the age and usability status of items with specific time-stamps. Besides the amount of inventory kept in stock, we also need to know when the material has been acquired and to what level it has deteriorated, as well as the impact that such deterioration may have in the production process.

Furthermore, production planning decisions determine the size and timing of *production lots* or *batches*, and therefore, the frequency of set-ups. Meanwhile, set-ups affect lead-times of items waiting in line to be processed, which consequently increases deterioration. To reach acceptable quality levels and/or production yields, a deteriorated material will consume more resource capacity that would otherwise be available for production, and therefore, it will also have an effect on waiting-times. Figure 1.1 shows this interdependent relationship between set-ups, waiting-times, deterioration, and capacity consumption.

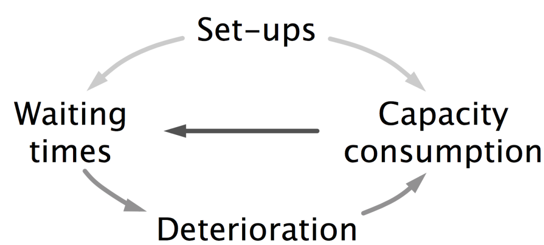

Figure 1.1: Interdependency between set-ups, deterioration, and capacity.

If a perishable item reaches the end of its *shelf-life* and becomes unsuitable for use, it may have to be discarded. Thus, besides the obvious waste of valuable resources and the negative impact it may have on the quality of the finished products, this aspect causes additional costs, as disposed material may need to be transported to a certain disposal site and incur also a treatment cost.

When studying multi-level production planning problems with perishable raw material, it is fundamental to analyze how inventory management can reduce the costly impact these material can have on production and on storage and disposal. A key assumption of our study is that the finished products are non-perishable, or that their shelf-life is long enough so as to reasonably ignore their perishable nature. Therefore, one of the ways in which the optimal solution of these problems tends to be structured is by using raw material for production as soon as possible after a batch has been received, resulting in larger finished product quantities stored in inventory. This solution structure partially reduces the complexity of the perishable raw material inventory management and is able to avoid larger material disposal costs.

Considering this tendency of storing higher finished product inventories, we have opened the study to another fundamental assumption regarding production planning problems: inventory bounds. This consideration arises in various types of production systems where it is common to find that storage levels of products are bounded. These restrictions on the quantities to be stored may be related to physical warehouse space and even to administrative policies, specially for voluminous products, or products requiring special warehouse conditions (i.e., clean rooms, controlled temperatures) [15].

The general research objectives of this thesis can be summarized as follows.

- To investigate the fundamental aspects to consider when integrating raw-material perishability considerations into production planning problems and provide a comprehensive review on the most relevant modeling approaches available in the literature.

- To define a fundamental class of production planning problems involving different types of raw-material perishability considerations and propose mathematical formulations to solve them.

- To perform managerial analyses on the impact of raw-material perishability and deterioration on key production planning decisions.

- To investigate the potential application of multi-level multi-item production planning problems with raw-material perishability in advanced composite manufacturing processes and perform computational experimentation to asses it.

- To study the multi-item uncapacitated lot-sizing problem with inventory bounds and propose mathematical formulations and solution algorithms to solve it, devising the opportunity to integrate it with raw material perishability considerations.

The remainder of this document is organized as follows. In Chapter 2 we present the fundamental characteristics in production planning modeling, introducing the simplest version of the production planning problem. We discuss some of the most common elements and assumptions used to model complex production planning problems. Finally, we present an overview of the production planning research evolution. In Chapter 3 we first bring a review of the different characteristics that can be considered when dealing with perishability, and present a classification framework. We review the most relevant modeling approaches for perishability in production planning. We then present lot-sizing problems that incorporate raw-material perishability and analyze how these considerations enforce specific constraints on a set of fundamental decisions, particularly for multi-level structures. We study three variants of the two-level lot-sizing problem incorporating different types of raw-material perishability: (a) fixed shelf-life, (b) functionality deterioration, and (c) functionality-volume deterioration. We propose mixed-integer programming formulations for each of these variants and perform computational experiments with sensitivity analyses. We analyze the added value of explicitly incorporating perishability considerations into production planning problems. In Chapter 4 we study the *multi-item, multi-level lot-sizing problem with raw-material perishability and batch ordering* inspired by a direct application in advanced composite laminates manufacturing. In particular, we consider an assembly production system in which one item at the lower level (non-perishable final product, representing the advanced composite) facing independent demand is to be produced. Several types of perishable raw-material items at the upstream level are to be procured in batches from suppliers. The upstream level consists of two different inventory levels: a storage location where raw-material batches can be initially stored under special conditions so as to avoid deterioration, and a secondary storage location at the shop floor where raw-material units become available for production after a batch is opened and start deteriorating. We proposed a mixed-integer programming formulation for the problem and perform computational experiments with sensitivity analyses, demonstrating its potentials for practical applications in planning composite production. In Chapter 5 we study the *multi-item uncapacitated lot-sizing problem with inventory bounds*. We present a new mixed-integer programming formulation for the case of non-speculative (Wagner-Whitin) cost structure using a set of variables to determine the production intervals for each item. We then reformulate the problem using a variable-splitting technique that allows for a Dantzig-Wolfe decomposition. The Dantzig-Wolfe principle exploits the structure of the problem by decomposing it into two sub-problems: one relating to the production decisions per item and another that relates to the inventory decisions per period. We propose a column generation algorithm for solving the Dantzig-Wolfe reformulation. Computational experiments are performed to evaluate the proposed formulations and algorithms on a set of benchmark instances. A summary of the thesis follows in Chapter 6.

Bibliographical note.

The content of Chapter 3 is published as "Two-level lot-sizing with raw-material perishability deterioration". *Journal of the Operational Research Society*, 1-16, 2019 [8]. Additionally, the research developments presented in Chapter 3 were presented in the following international conferences, as:

- "Production planning with perishable raw material considerations." In *Optimization Days*, Montreal, Canada, 2014 [2].
- "Production planning with perishable raw material considerations." In *56th CORS Annual Conference, Canadian Operational Research Society*, Ottawa,

Canada, 2014 [3].

- "Production planning with perishable raw material considerations." In *20th IFORS, International Federation of Operational Research Societies*, Barcelona, Spain, 2014 [4].

- "Two-level lot-sizing with raw-material perishability and deterioration: an extended MIP formulation." In *CORS/INFORMS 2015 Joint International Meeting*, Montreal, Canada, 2015 [5].

- "Two-level lot-sizing with raw-material perishability and deterioration: formulations and analysis." In *International Workshop on Lot-Sizing (IWLS)*[1], Montreal, Canada, 2015 [6].

The content of Chapter 4 was submitted for publication as "Multi-level lot-sizing with raw-material perishability, deterioration, and batch ordering: an application of production planning in advanced composite manufacturing" to the *Journal of Computers & Industrial Engineering* in June, 2019.

The content of Chapter 5 was presented in *International Workshop on Lot-Sizing (IWLS)*[1], Glasgow, Scotland, 2017, as "Dantzig-Wolfe reformulations for multi-item lot-sizing problems with inventory bounds" [7].

[1]The International Workshop on Lot-Sizing (IWLS) is on invitation only. A limited number of participants who are active in the field of lot-sizing are invited.

Chapter 2

Production planning problems

In this chapter, we present the fundamental characteristics in production planning modeling, introducing the simplest version of the production planning problem. We discuss some of the most common elements and assumptions used to model complex production planning problems. Finally, we present an overview of the production planning research evolution.

2.1 Production planning fundamentals: the lot-sizing problem

The most fundamental production planning problem is known as the *single-item, single-level, uncapacitated lot-sizing problem* (*ULS*). It corresponds to the planning of a single item production to meet some dynamic demand over a discretized planning horizon, minimizing the sum of production and inventory costs. Although the *ULS* model is the most restrictive in terms of applicability, its importance lies in being the simplest high-level relaxation occurring in most complex production planning models [106]. *ULS* can be described as follows: there is a planning horizon of $T = \{1, \dots, n\}$ periods. The demand for the item in period t is $d_t \geq 0$ for $t = 1, \dots, n$. For each period t, there are unit production costs p_t, unit storage costs h_t for stock remaining at the end of period t, and a fixed set-up (or order placement) cost q_t which is incurred to allow production to take place in period t and is independent of the amount produced.

LS-U is polynomially solvable, since it can be solved using dynamic programming [128]. On the other hand, Barany et al. [26] introduced the (l, S)-*inequalities* for the standard mixed integer programming formulation of the problem, leading to the convex hull of the solutions.

An alternative and useful way to view the *LS-U* is using an adaptation from a well known network optimization problem: the *minimum cost network flow problem* with additional fixed costs for the activation of certain arcs. Figure 2.1 shows an example of this approach for an instance with $n = 3$ periods. The flow in arc $(0, t)$ represents the amount produced in period t, and the flow in arc $(t, t + 1)$ represents the stock at the end of period t.

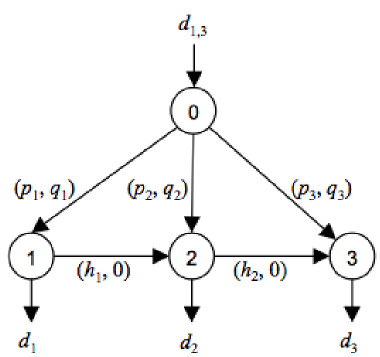

Figure 2.1: *ULS* as a fixed charge network flow.

To mathematically formulate the problem, we define the variables x_t for the amount produced in period t, s_t for the amount of stock at the end of period t, and y_t as a binary set-up variable which must have the value 1 if $x_t > 0$. An mixed integer programming (MIP) formulation for the *ULS* is:

$$\text{minimize} \quad \sum_{t=1}^{n} p_t x_t + \sum_{t=0}^{n} h_t s_t + \sum_{t=1}^{n} q_t y_t \tag{2.1}$$

$$\text{subject to} \quad s_{t-1} + x_t = d_t + s_t \qquad 1 \le t \le n \tag{2.2}$$

$$x_t \le M y_t \qquad 1 \le t \le n \tag{2.3}$$

$$x \in \mathbb{R}^n, s \in \mathbb{R}_+^{n+1}, y \in \{0,1\}^n \tag{2.4}$$

$$s_0 = s_0^*, \;\; s_n = s_n^*, \tag{2.5}$$

where M is a large positive number, expressing an upper bound on the maximum lot size in period t. The objective function (2.1) is the sum of unit production, fixed production, and unit inventory costs. Constraints (2.2) express the demand satisfaction in each period, and are also known as the *flow balance* or *flow conservation* constraints (every feasible solution of *ULS* corresponds to a flow in the network shown in Figure 2.1, where $d_{1,3} = \sum_{i=1}^{3} d_i$ is the total demand). Constraints (2.3) ensure that if there is production in period t (i.e., $x_t > 0$), then the set-up variable in period t is $y_t > 0$ and so necessarily $y_t = 1$. Here, the term M can be replaced by $\sum_{k=t}^{n} d_k$, which is the true upper bound on x_t when there is no ending inventory (i.e., $s_n = 0$). Constraints (2.4) impose the nonnegativity and integrality restrictions on the variables. Constraints (2.5) are optional in the sense that there are model situations in which the initial stock s_0 and the final stock s_n may be decision variables. However, in the majority of cases their values are fixed, and the values s_0^* and s_n^* are part of the available data of the problem.

2.1.1 Modeling features

In addition to the characteristics mentioned so far as part of the simplest and most fundamental production planning problems, there are a number of features that may arise in some applications. They are intended to make the models cover a wide variety of aspects of the supply chain and be more applicable, which makes them more complex and difficult to solve. Some of the most important of these key elements are presented below:

- *Capacity restrictions coming from common resources*: problems with this consideration are commonly known as *Master Production Schedule (MPS)* problems. The main purpose of *MPS* is to plan the production of a set of items, over a short-term planning horizon corresponding at least to the total production cycle. The most fundamental *MPS* model is known as the *multi-item, single level, capacitated lot-sizing model* (*MI-LS-C*), where the production plans of the different items are linked through capacity restrictions from the common resources used.

- *Multi-level product structures*: in a several cases, products interact through multi-level product structures, where an item can be an output and/or an input of some production stage (i.e. sub-assembly), or it may be delivered from an external supplier (raw material). This creates what is known as *precedence constraints* between the supply and the consumption of that product. The *multi-item, multi-level, capacitated lot-sizing model* (*MI-MLS*) is often referred to as the *Material Requirements Planning (MRP)* model. The dependency between items is modeled through a product structure, also called the *Bill of Materials (BOM)*.

- *Backlogging*: the demand satisfaction process may allow demand for finished products to be backlogged. In these cases, it is possible to deliver to a customer later than required. This is a way to balance the lack of available capacity where permitted, and usually involves a penalty for the negative impact on customer satisfaction.

- *Capacity Utilization*: other important elements in production planning models are related to a more precise way of modeling capacity utilization in order to obtain more feasible production plans. For instance, the capacity consumed when a machine starts or finishes a production batch, or when a machine switches from one product to another, may need to be considered. In these cases, we obtain models with set-up times, start-up times, changeover times, or models with sequencing restrictions.

2.2 Evolution of production planning modeling

The emergence of the formal study of production planning dates back to the early 1910s. Ford W. Harris is well known for presenting the *Economic Order Quantity* (EOQ) model in a 1913 paper published in *Factory, The Magazine of Management*. The main purpose of the EOQ model is to determine the order quantity that minimizes the total inventory and ordering costs [62]. As an extention of Harris' work, the *Economic Production Quantity* (EPQ) model was developed, assuming the product orders are available in an incrementally manner [119], while the EOQ model assumes complete and immediate availability. Subsequently, the well known *Statistical Reorder Point* (Q,r) model was introduced with the objective of preventing shortages, introducing the notion of *safety stock* [134]. Harris' original publication, along with his second paper on inventories [63], and the EPQ and Q,r models, laid the foundations for the treatment of stationary demand.

The next major contribution is the introduction of the dynamic version of the economic lot-sizing model, as a generalized version of the EOQ, known as the Wagner-Whitin model [128]. The Wagner-Whitin property provides the optimal lot-sizing policy of having either on-hand inventory or production, but not both in each period of the planning horizon. Wagner and Whitin's work is considered to be the cornerstone for the treatment of time-varying demand.

The following milestone of fundamental importance in this field is the well-known *Material Requirements Planning* (MRP) method. Born in 1964, MRP is the result of the efforts of Joseph Orlicky at one of the pioneer companies in information technology: IBM. The commercial availability of computers in the mid-1950s ushered in a new era of business information processing, with a profound impact in the area of production planning [94]. Orlicky's MRP was the most successful method to take advantage of the new computational capabilities, and it was a major step forward in the standardization and control of production planning systems.

Inspired by some shortcomings in MRP, *Manufacturing Resource Planning* (MRP II) was born in the early 1980s. The development of MRP II is primarily attributed to Oliver Wight [see 132, 133]. MRP II extended the initial MRP systems to cover all aspects of manufacturing processes, including demand planning, sales and operations planning (S&OP), master production schedule (MPS), BOM and inventory control, among others. For a comprehensive review of MRP models, see Baker [23].

During the 1980s and 1990s, the intentions of integrating MRP and MRP II transversally in supply chain and manufacturing facilities, along with the need to introduce new techniques, led to what is known today as *Advanced Planning and Scheduling* (APS) and *Enterprise Resource Planning* (ERP). APS systems provide long, mid and short-term planning of the supply chain, including internal aspects of procurement, production, distribution, and sales [93]. Alternatively, ERP systems not only focus on planning and scheduling of internal resources, they strive to plan and schedule supplier resources as well [34]. In addition, ERP systems include technology aspects, such as friendly graphical user interfaces, relational databases, and computer-aided software engineering tools [9]. For a general review of advantages and critics on ERP systems, see Davenport [41].

For extensive reviews and fundamentals on the lot-sizing problem, we refer the reader to Andriolo et al. [20] and Pochet and Wolsey [106].

2.3 Mathematical programming for production planning

Production planning problems are often modeled using Mixed Integer Programming (MIP), because of problem features such as set-up costs and times, start-up costs and times, machine assignment decisions, and so on. In this section, we review polynomial-time algorithms and polyhedral approaches for special cases in production planning. We also present some of the most relevant MIP techniques used in production planning and approximate solution methodologies.

2.3.1 Polynomial-time algorithms

Dynamic Programming (DP) is the base tool for polynomial-time algorithms in production planning. The Wagner-Whitin (W-W) algorithm is the best example of this fact, where the problems are decomposed into smaller problems that are then solved recursively. In special structured cases, network optimization algorithms are used to find optimal solutions in polynomial-time.

Zangwill [141], published in 1969, is an early paper that studies a basic extension of the *ULS*, described in Section 2.1, and presents exact algorithms to solve it. The author uses the Wagner-Whitin property and single-source flow networks, and proposes a DP algorithm to solve both: an uncapacitated single-item lot-sizing problem with backlogging and an uncapacitated serial multi-level problem. Along the same lines, Florian and Klein [51] discovered properties for the case of constant capacities that allows an $O(n^4)$ DP algorithm to solve these problems. Pochet and Wolsey [105] bring a polyhedral analysis for the special case of *lot-sizing with constant batches* (*LS-CB*), and propose an $O(n^3)$ dynamic programming algorithm to sove the *LS-CB* problem exactly.

To improve the classic W-W algorithm, advocated since 1958 with $O(n^2)$ time, Federgruen and Tzur [48] described an algorithm that solves the general *ULS* in $O(n \log n)$ time. They studied two special cases of the problem, obtaining $O(n)$ times. Subsequently, Wagelmans et al. [127] solved the W-W special case with the possibility of negative costs in $O(n)$ time. Around the same time, the *ULS* problem was studied with different cost structures and with and without backlogging, obtaining the same results as in the previous two studies, adding search techniques to the DP algorithm [11].

Continuing with more recent studies on polynomial algorithms for different forms of lot-sizing problems, Vanderbeck [124] studied the *single-item constant-capacity lot-sizing* problem (*LS-CC*) with start-ups and presented an $O(n^6)$ DP algorithm. *Start-up* times occur in production systems such as the manufacturing of food products or chemicals, where significant clean-ups must take place between different batches of production. Vanderbeck [124] used a column generation approach to solve multi-item problems. In the context of production planning of perishable products, Hsu [66] studied a lot-sizing problem with an age-dependent inventory stock's deterioration rate and carrying cost (perishability and production planning of perishable products will be covered in Chapter 3). The author developed an $O(n^4)$ DP algorithm for the uncapacitated single-item problem with concave costs and no backlogging. Furthermore, Ahuja and Hochbaum [12] studied the linear-cost single-item lot-sizing problem with production, inventory and backlogging capacities, proposing an $O(nlogn)$ network algorithm to solve it.

Lee et al. [75] and Brahimi et al. [30] studied models that involve the so-called *delivery windows* and *production windows*, respectively. The authors proposed polynomial-time algorithms for the single-item cases, and Lagrangian heuristics for multi-item problems. In this same problem area, Wolsey [135] presented a polyhedral analysis of these models, proposing valid inequalities for both, and providing tight extended reformulations and DP algorithms for special cases.

In more recent studies on polynomial-time algorithms, Akbalik and Rapine [13], for example, present two polynomial time algorithms for the constant capacitated lot sizing problem with batch production: the first one is an $O(n^4)$ time algorithm for the case with production capacity being a multiple of the batch size, and the second one an $O(n^6)$ time algorithm for the case with an arbitrary fixed capacity. For their part, Hellion et al. [65] present an $O(n^5)$ algorithm for the single-item capacitated lot-sizing problem with concave production and storage costs, and minimum order quantity, and Hwang et al. [69] present the first polynomial algorithm for the general lot-sizing problem with lost sales and bounded inventory.

2.3.2 Production planning by mixed integer programming

When dealing with MIP models for structured challenging production problems, polyhedral analysis and other theoretical and practical approaches have been one of the favorite and most efficient ways to develop some special purpose solution techniques.

The first set of these techniques seek to strengthen the original problem formulations by adding valid inequelities. One of the milestone studies in this area is the development of the well known (l, S)-*inequalities* [26]. The importance of the (l, S) inequalities is that they define the convex hull of integer solutions for the *ULS* problem, and they can be extended to multi-level lot-sizing problems using echelon reformulations [104].

Subsequently, Pochet [99] addressed the *single-item lot-sizing with constant capacities problem (LS-CC)* and proposed a family of facet-defining inequalities with a heuristic separation algorithm. Leung et al. [76] studied the polyhedral structure of the *LS-CC* and proposed a family of valid inequalities for the multi-item case. The authors showed that the inequalities can be effectively used to develop an cutting plane/branch and bound procedures. Later, Pochet and Wolsey [105] considered the same problem with a variant in which the capacity in each period is an integer multiple of some basic batch size *(lot-sizing with constant batches, LS-CB)*. The authors proposed facet-defining inequalities that can also be applied for the *LS-CC* problem.

Turning to another type of problem, Pochet and Wolsey [102] examined reformulations for the *uncapacitated lot-sizing problem with backlogging*. They considered the effect of using a standard reformulation technique for *fixed charge network flow problems*, and described a family of strong valid inequalities. Küçükyavuz and Pochet [74] described the convex hull of integer solutions to the same problem.

Constantino [38] considered the general *capacitated lot-sizing problem with start-up costs* and derived several families of valid inequalities for the single-item case, which were used in a branch-and-cut procedure. Later, the same author studied the polyhedral characteriscs of the *lot-sizing problem with constant lower bounds* on production [39], where production below some level is not allowed, in order to make full use of resources.

Loparic et al. [80] examined the *ULS* version involving sales instead of fixed demands and safety-stocks. The authors presented an extended formulation for the problem with non-decreasing safety-stocks. For their part, Vyve and Ortega [126] used an extension of the (l, S)-inequalities and described the convex hull of integer solutions for the *uncapacitated lot-sizing problem with fixed charges on stocks*. Furthermore, Atamturk and Küçükyavuz [22] identified facet-defining inequalities for the *lot-sizing problem with fixed inventory costs and inventory bounds*.

Extended reformulations for the initial models is the idea behind the second set of MIP methodologies. An early study by Krarup and Bilde [73] presented the first general extended reformulation for production planning problems: the *facility location (FL) reformulation*. It allows the lot-sizing problem to be seen as a network with *NT* possible "facilities" to open, representing the set-up decisions in each period. For the *ULS* problem, the *LP* relaxation of the *FL* reformulation gives the optimal integral solution. With a similar idea, Rardin and Wolsey [109] defined the *multi-commodity (MC) reformulation* for the *fixed charge netwrok flow* problem. The *MC* reformulation further decomposes the production with respect to which finished-product an item is produced for. This reformulation is stronger than the *FL* one for the *multi-level lot-sizing* problem.

A detailed polyhedral study of different *single-item lot-sizing* problems with W-W costs is provided by Pochet and Wolsey [103]. The authors proposed extended reformulations for the *uncapacitated problem with backlogging* and the *uncapacitated problem with start-up costs*. Miller and Wolsey [88] presented a polyhedral study on the the *multi-item discrete lot-sizing* problem, where only one item can be produced in a time period and production is either 0 or a predefined constant amount for the item. The authors presented extended reformulations for the cases with backlogging and initial inventory variables.

For an extensive review of production planning by mixed integer programming, taking into account the two sets of aforementioned techniques in detail, and depth, besides other complementary techniques, we refer the reader to Pochet and Wolsey [106].

2.3.3 Approximate solution methodologies

When tackling *NP*–hard problems, heuristic methods are commonly used to speed up the process of finding a satisfactory solution. In production planning, complex problems such as *multi-item, multi-level, capacitated lot-sizing* problems have been approached with problem-specific heuristic algorithms. Silver and Meal [111] presented one of the first and classical heuristics for lot-sizing problems. The *Silver-Meal heuristic* determines the average cost per period for producing a sequence of periods and stops production in a period which observes an increase in the average cost. Maes and Van Wassenhove [83] provided a review of early heuristics techniques for lot-sizing problems.

Trigeiro et al. [123], Diaby et al. [43], and Tempelmeier and Derstroff [120], used Lagrangean-based heuristics for different lot-sizing cases. The first one studied the multi-item problem with a single machine shared by all items. The problem is decomposed into uncapacitated single-item problems using Lagrangean relaxation, and thereafter the same are solved by DP. Feasible solutions for the original problem are generated using a smoothing heuristic. Diaby et al. [43] also used a Lagrangean relaxation-based heuristic, this time for very large-scale capacitated lot-sizing problems. The relaxation is made on resources acquirement contraints and the Lagrangean-dual problem is solved using subgradient optimization. The set-up decisions are retained, and transportation problems are solved to determine corresponding optimal production quantities. Moreover, Tempelmeier and Derstroff [120] also decomposed the *capacitad multi-level multi-item lot-sizing* problem into several *uncapacitated single-item lot-sizing* problems with the help of *Lagrangean relaxation.* Lower bounds on the original problem are found from the single-item problems.

Recently, Wu et al. [137] also proposed a Lagrangean relaxation-based heuristic for the *capacitated multi-level lot sizing problem with backorders (LS-CB)*, relaxing the capacity constraints. The relaxation leads to a number of *uncapacitated multi-level, multi-item lot sizing sub-models.* A subgradient optimization procedure is applied to the Lagrangean dual to obtain lower bounds, and a relax-and-fix approach is applied on the LS-CB problem to obtain upper bounds.

Furthermore, objective dividing heuristics that modify the objective coefficients of a model have also been used to solve production planning problems. Katok et al. [72] introduced one of these heuristics for *multi-item lot sizing* problems with *general assembly structures*, multiple constrained resources, and nonzero set-up costs and set-up times. Coefficient modification is applied by allocating set-up times to variable time to find initial solutions. *LP* relaxations of the second stage are used to improve initial solutions. Pochet and Van Vyve [101] proposed a coefficient modification based heuristic to be used within branch-and-cut for a class of *capacitated multi-item multi-level lot-sizig* problems with set-up time. The algorithm is based on modifying the capacities to smooth the relation between the linear and binary variables in the *LP* relaxation. After obtaining integer values for all set-up variables, the heuristic solves only an *LP* problem.

Forward scheme and relax-and-fit heuristics have also been used for solving production planning models. Federgruen and Tzur [49], for example, proposed a time-partitioning heuristic with non-overlapping subproblems for a lot-sizing-distribution network integrated problem where each subproblem is solved to optimality sequentially. For their part, Belvaux and Wolsey [28] studied lot-sizing problems arising both in practice and in the literature, using a branch-and-cut algorithm that employs relax-and-fix heuristics for finding feasible solutions. Stadtler [116] proposed a time-oriented decomposition heuristic to solve the *multi-item multi-level lot-sizing* problem in general product structures with single and multiple constrained resources as well as setup times. Along the same line, Suerie and Stadtler [117] studied the *capacitated lot-sizing problem with linked lot sizes* with a time-oriented decomposition heuristic.

In terms of *local-search-based* heuristics, Gopalakrishnan et al. [55] proposed a tabu-search heuristic for the *LS-C* with set-up carryover. It consists of five basic move types: three for the sequencing decisions and two for the lot-sizing decisions. It allows infeasible solutions to be generated at a penalty, and uses a dynamic tabu list, an adaptive memory, and self-adjusting penalties. Simpson and Erenguc [112] proposed a neighborhood search heuristic that finds an initial feasible solution and improves it using local search. More recently, Guimaraes et al. [60] studied a *single-machine capacitated lot-sizing and scheduling* problem with sequence-dependent setup times and costs. The authors proposed a matheuristic that uses pricing principles within construction and improvement MIP-based heuristics. A partial exploration of distinct neighborhood structures avoids local entrapment and is conducted on a rule-based neighbor selection principle.

For extended reviews of several types of heuristics for different production planning problems, we refer the reader to Jans and Degraeve [70], Goren et al. [56], and Ramezanian et al. [108].

Chapter 3

Raw-Material perishability in production planning

In this chapter, we first bring a review of the different characteristics that can be considered when dealing with perishability, and present a classification framework based on Amorim et al. [19]. Afterwards, we review the most relevant modeling approaches for perishability in production planning. We then present lot-sizing problems that incorporate raw-material perishability and analyze how these considerations enforce specific constraints on a set of fundamental decisions, particularly for multi-level structures. We study three variants of the two-level lot-sizing problem incorporating different types of raw-material perishability: (a) fixed shelf-life, (b) functionality deterioration, and (c) functionality-volume deterioration. We propose mixed-integer programming formulations for each of these variants and perform computational experiments with sensitivity analyses. We analyze the added value of explicitly incorporating perishability considerations into production planning problems. For this, we compare the results of the proposed formulations with those obtained by implementing a sequential approach that adapts a standard two-level lot-sizing solution with a Silver-Meal-based rolling-horizon algorithm.

The content of this chapter has been published as "Two-level lot-sizing with raw-material perishability deterioration", *Journal of the Operational Research Society*, 1-16, 2019 [8]. Additionally, the research developments in this chapter were presented in several international conferences, including the *20th IFORS, International Federation of Operational Research Societies*, Barcelona, Spain, 2014, as "Production planning with perishable raw material considerations" [4], and the *International Workshop on Lot-Sizing (IWLS)*[1], Montreal, Canada, 2015, as "Two-level lot-sizing with raw-material perishability and deterioration: formulations and analysis" [6].

3.1 Perishability in production planning

A general definition of perishability describes it as the decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of an item that results in decreasing usefulness from the original one [131]. As mentioned by Pahl and Voß [96], most authors working in this field use the terms deterioration, perishability, and depreciation interchangeably. Regardlessly, all perishable goods have a fixed maximum lifetime, usually referred to as *shelf-life*. Shelf-life is defined as the maximum length of time during which a product is considered of satisfactory quality and can be stored under specified (or expected) conditions, remaining suitable for use, consumption or for its intended function. It is the length of time that a given item can remain in a salable or functional condition on the shelf of a retailer or in the stock of a manufacturer. Shelf-life is usually considered from the moment the product is produced or acquired.

[1]The International Workshop on Lot-Sizing (IWLS) is on invitation only. A limited number of participants who are active in the field of lot-sizing are invited.

3.1.1 Characteristics of perishability and classification scheme

Different, complementary, and even contradictory classifications have been proposed to deal with perishability and deterioration over the last decades, and there is an obvious overlap in the way perishablility has been characterized [19]. However, we can distinguish three main viewpoints to characterize and classify product perishability: (a) the utility or functionality of the product, (b) the physical state of the product, and (c) the mathematical modeling point of view of perishability.

When the interest is mainly in the utility or functionality of the products [95, 107], and based on the value of the inventory as a function of time, perishability can be classified into: (1) *constant-utility*: items undergo decay but face no appreciable or considerable decrease in value, e.g., prescription drugs; (2) *decreasing-utility*: items lose functional value throughout their shelf-life, e.g., milk, fruits and vegetables; (3) *increasing-utility*: items increase in value, e.g., some wines, cheese, antiques. In the same sense, but with a special interest in how the customer perceives the functional value of items [50], we can make two distinctions: (1) items whose functionality deteriorates over time; (2) items whose functionality does not degrade, but the utility perceived by the customers deteriorates over time, e.g, fashionable clothing and high-technology products.

The emphasis on the physical state of the product can be found in early inventory control papers dealing with perishability. For instance, Ghare and Schrader [53] characterize perishability taking into account the type of deterioration: (1) *direct spoilage*, e.g., vegetables, flowers and fresh food; (2) *physical depletion*, e.g., gasoline and alcohol; (3) *decay and obsolescence*, e.g., newspapers. This perspective refers to the volume (or quantity) loss of product, but not necessarily to the loss of functionality or utility. With a similar interest, Lin et al. [79] take into account age-related perishability characteristics and distinguish between: (1) *age-dependent* on-going deterioration; (2) *age-independent* on-going deterioration, e.g., volatile liquids, radioactive and other chemicals, and grain products.

The third perspective refers to the treatment of perishability from a purely mathematical modeling viewpoint. That is, the interest is not in the origin of the perishable nature of the products, but only in aspects related to the incorporation of perishability in the problem formulation. This is the case of Nahmias [90], who divides perishable products into: (1) with *fixed shelf-life*: cases where the shelf-life is known *a priori* to be a specified length of time; (2) with *random shelf-life*: cases where the product shelf-life is a random variable with a specified probability distribution.

One of the most complete and complementary classification of perishability found in the literature is the one proposed by Amorim et al. [19] shown in Figure 3.1.

		Authority Limits			
		Fixed		Loose	
Physical Product Deterioration	Yes	e.g. Human blood	e.g. Yoghurt	e.g. Cheese, Gasoline	e.g. Fruits, Radioactive materials
	No	Not-realistic	e.g. Daily newspapers	Not-perishable	e.g. Haute-Couture Fashion clothes
		Constant	Decreasing	Constant	Decreasing
		Customer Value			

Figure 3.1: Framework for classifying perishability
Source: Amorim et al. [19]

The framework is a cluster of three classifying dimensions: (1) *physical product deterioration*: reflects if the item is actually suffering physical modifications or not, (2) *authority limits*: represents the external regulations or any other conventions that influence directly the perishability phenomenon, and (3) *customer value*: reflects the customer willingness to pay for a certain good. The *authority limits* dimension is interesting from a mathematical modeling point of view because it may reduce the stochasticity of the perishability phenomenon when a lifetime is fixed and known. The author states that the framework might be applied to all different forms of product perishability, either when it manifests itself through the changing of the physical state or not. It is also flexible enough to be applied to models dealing with any process(es) of the supply chain. The *authority limits* dimension is interesting from a mathematical modelling point of view because the influence of authorities may reduce the stochasticity of the perishability phenomenon when a lifetime is fixed.

Clustering the three dimensions to classify perishability allows having a more integrated understanding of product perishability and provides an important tool for the mathematical programming of problems in this area. An simple example of the applicability of the framework can be seen in a supply chain planning problem of production and distribution of fresh milk, where the product undergoes physical deterioration, its shelf-life is fixed, since there is a Best-Before-Date (BBD) stamp on it, and the perceived customer value decreases over time, since customer will prefer packages with a later BBD comparing with others having an earlier one [19].

Furthermore, another essential distinction when characterizing perishability is the one shown in Figure 3.2 [95]. The leftmost graph represents the course of perishability of products that are considered fully functional during their shelf-life, or whose deterioration before reaching the end of its shelf-life does not need to be considered for practical purposes. We make reference to this case as *perishable items subject to fixed shelf-life*. In contrast, the second and third graphs show the course of perishability of products whose functionality or efficiency level decreases progressively throughout their shelf-life. We make reference to this case as *perishable items subject to functionality deterioration rate*. The second graph shows a gradual discrete treatment of deterioration, and the third graph shows possible courses of continuous deterioration.

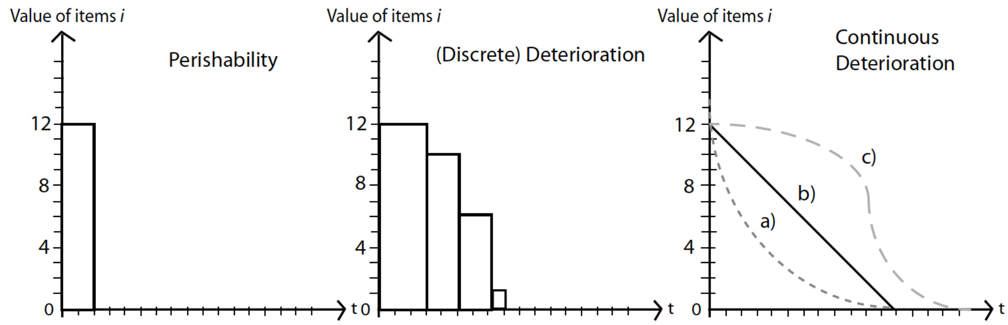

Figure 3.2: Three examples for perishability and deterioration
Source: Pahl and Voß [95]

Although the above classification largely comprises all aspects of perishability considered until now in the literature, there is a feature that, to the best of our knowledge, has not been treated and is the way in which raw-material (and/or intermediate products) perishability and deterioration may affect the production of higher-level items in the product structure in multi-level production systems.

3.1.2 Modeling approaches for perishability

Most of the literature regarding perishable goods is focused on inventory management, pricing and reverse logistics. In general terms, we distinguish two different approaches in which perishability and deterioration are studied in inventory control, scheduling, and production and distribution planning.

The first approach assumes a loss of a portion of inventory, determined by a fixed input parameter (shrinkage factor). Hsu [66] presents an uncapacitated, single-item, lot-sizing problem (LS) using a deterioration rate factor and considering age-dependent inventory costs. The model is generalized to include back-ordering [67] and capacities [130]. Using a similar deteriorating coefficient, Chen and Chen [35] integrate LS and scheduling for a perishable item in maximizing revenue, where demand and production depend on the selling price. Other studies following this approach include Balkhi and Benkherouf [25], Yang and Cheng [139], Skouri and Papachristos [113], and Goyal and Giri [59]. Balkhi [24] considers lot-sizing problems with time-varying demand of deteriorating items as well as the effect of "learning" in production. Lin et al. [79] present a multi-item production-inventory problem with exponential deterioration rates and constant inventory shrinkage factor. Other studies in this area include Tadj et al. [118], Manna and Chaudhuri [84], and Belo-Filho et al. [27]. Li et al. [78] study dynamic pricing and inventory control policies for perishable products with stochastic disturbance.

For a comprehensive review of available literature regarding perishable goods in inventory management, we refer the reader to Nahmias [90], Goyal and Giri [57], Li et al. [77], and Raafat [107]. Amorim et al. [19] and Pahl and Voß [96] present reviews on production-distribution and supply chain planning for deteriorating items.

The second approach attempts to avoid inventory expiration by limiting the number of periods of production to ensure that products do not reach the end of their shelf-life. Entrup et al. [44] develop MIP models following this approach to solve production-scheduling problems of yogurt production with shelf-life-dependent selling price. Their objective is to maximize the contribution margin, and they use a block planning approach, where a block is formed from all product variants based on a same recipe. In the same application area, Amorim et al. [18] propose two multi-objective LS and scheduling MIP models for a pure make-to-order system, and for a hybrid make-to-order/make-to-stock scenario. The authors incorporate the maximization of product freshness as the problem objective function. Pahl and Voß [95] extend this approach without restricting the number of time periods. They allow inventory expiration and penalize it by applying a disposal cost as part of a standard discrete lot-sizing and scheduling problem. Pahl et al. [97] extend this approach to include sequence-dependent setup times and costs.

Other studies relevant to our research are those by Abad [1], who present a constrained non-linear programing model for LS problems of perishable goods with exponential decay, partial back-ordering and lost sales. Teunter and Flapper [121] consider a stochastic Economic Production Quantity model where produced units of a single type product may be non-defective, reworkable-defective, or non-reworkable-defective. In industries such as the food or pharmaceutical industry, reworkable defective products that are perishable, can become technologically obsolete. Another approach involves *traceability* in an economic production quantity problem [129]. Traceability is the ability to trace and follow the product through all stages of production, processing and distribution, which is an important management issue in food industry. Wang et al. [129] develop an operations planning model with production set-up costs, inventory holding costs, raw material costs, product spoilage costs, and recall costs integrating traceability and operational indicators to attain both product quality and minimum impact of product recall. Kallrath (2002) made an overview of some of the most encountered production planning and scheduling problems in the chemical process industry and their specific characteristics. The author took into account and distinguished three classes of production systems: continuous, batch and semi-batch production. Neumann et al. [92] introduce a mixed integer nonlinear programming model for an advanced planning system in the context of batch production for process industries. The model includes constraints referring to perishability of products, where production tasks are assigned to consuming tasks so that no perishable product is kept in stock at any time, i.e. the amount produced by a batch must equal the amount consumed in following tasks without delay.

When considering perishability in well-known *economic lot scheduling problems* (*ELSP*), most literature is limited to adding shelf-life constraints to the original problems [19]. The *ELSP*, a problem shown to be *NP*–hard, is concerned about obtaining a cyclic schedule for several products, for a single resource and under the assumption of a constant demand rate. Chowdhury and Sarker [37], Goyal and Viswanathan [58], and Sarker and Chowdhury [110] studied three different approaches in this field: (1) changing the production rate, (2) changing the cycle time, and (3) changing production rate and cycle time simultaneously with respect to production scheduling and raw material ordering. Furthermore, Soman et al. [115], who provided a review in this topic, stated that, in case of high capacity utilization, the production rate should not be reduced due to quality problems that may arise with this adjustment. On similar lines, Yao and Huang [140] proposed a new *ELSP* model that considers multiple continuously deteriorating items. Lin et al. [79] for their part, studied an *ELSP* with multiple items subject to different exponential deterioration rates. Arbib et al. [21] considered a production scheduling problem for perishable products, under two independent aspects: the relative perishability of products and the feasibility of the completion time. Gawiejnowicz [52] considered two problems of scheduling a set of independent, non-preemptable and proportionally deteriorating jobs on a single machine, where the objective is to minimize the total completion time of jobs subject to a certain machine capacity. Soman et al. [115], and Pahl and Voß [96] present reviews of related literature.

Production time-windows are also used to model perishability constraints [136]. Chiang et al. [36] study a production-distribution problem applied to the newspaper industry. The authors present a simulation-optimization framework and formulate the problem as an extension of the vehicle routing problem with time-windows. Chen et al. [33] study a production-scheduling and vehicle routing problem with time-windows and stochastic demands.

The studies presented above consider product perishability and deterioration in production planning and related problems. However, research on raw-material (and/or intermediate products) perishability and deterioration affecting the production of higher-level items is very limited. Cai et al. [31] and Billaut [29] solve different production scheduling problems with, to certain extent, considerations of raw-material perishability. The former develop a model and an algorithm for the production of seafood related products, and the latter proposes new scheduling problems dealing with perishable raw materials. Billaut [29] describes a *multi-item multi-machine* problem where raw materials are stored into vials and can be used during a limited time after being opened. The problem is to assign a set of jobs that use these raw materials to a performing machine, to determine a starting time for each job, and to determine the opening dates for the vials of each required material. Furthermore, the same author proposes a first MIP formulation for the case of single-item and single-machine, shown to be *NP*–hard, where raw materials are consumed instantaneously, i.e., it is assumed that if the material perishes exactly when a job starts, the consumption is realized by the job first, and the remaining quantity of product is lost.

3.2 Two-Level Lot-sizing with perishable raw material

We consider a production system in which one item (finished product) is to be produced and another item (raw-material), an input of the first, is to be procured from a supplier over a planning horizon with n time periods, $T = \{1, ..., n\}$. This constitutes the simplest version of a two-level product structure. Solving the *two-level lot-sizing problem* (*2LS*) is to determine the production, procurement and inventory plans for the two items to meet the demands of the planning horizon, while minimizing the corresponding costs. As mentioned above, the core aspect of the problems under study is the perishable condition of the raw-material. In particular, we consider three different types of raw-material perishability in solving the *2LS* problem: (a) raw-material with fixed shelf-life (*FS*); (b) raw-material with functionality deterioration (*FD*); and (c) raw-material with functionality and volume deterioration (*FVD*).

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3.2.1 Fixed shelf-life

We first consider the *two-level lot-sizing problem with fixed raw-material shelf-life* (*2LS-FS*). We assume that raw-materials are ordered and received immediately. Associated with each order there are unit batch costs and fixed ordering costs. Received raw-material may be used in production or placed in inventory. However, it can only be kept in stock for a predetermined period of time (shelf-life). If the material reaches the end of its shelf-life and expires, it will be disposed. This causes additional costs depending on when the disposal is made. Raw-material functionality is considered constant during the entire shelf-life period. Production is limited by process capacity and incurs fixed setup costs as well as variable production costs. Demand must be satisfied in every time period. The *2LS-FS* consists of planning the production levels and raw-material ordering for each time period, as well as planning the inventory levels so as to minimize the total production, setup, order-placement, inventory, and raw-material disposal costs.

Applications of *2LS-FS* may arise in the production of plastic films. A plastic film is a thin continuous polymeric material used to separate areas or volumes, to hold items, to act as barriers, or as printable surfaces [64]. Depending on their applications, plastic films can be made from a variety of plastic resins and monomers, which are highly reactive and undergo uncontrolled polymerization. However, they are considered fully functional during their shelf-life. The finished product is not considered to be perishable.

In formulating the *2LS-FS* model, we use the following notation for the input parameters:

d_t $\rightarrow$ demand per period t

a $\rightarrow$ standard unit production time

C_t $\rightarrow$ available process capacity per period t

p_t $\rightarrow$ standard unit production cost per period t

q_t $\rightarrow$ fixed setup cost per period t

h_t $\rightarrow$ unit storage cost per period t

b $\rightarrow$ fixed order batch-size

K_t $\rightarrow$ upper ordering limit per period t

β $\rightarrow$ raw-material shelf-life

r $\rightarrow$ units of raw-material required to produce each finished item

ρ_t $\rightarrow$ fixed cost of placing a raw-material order per period t

ζ_t $\rightarrow$ unit batch cost per period t

γ_t $\rightarrow$ raw-material unit storage cost per period t

ϕ_t $\rightarrow$ raw-material unit disposal cost per period t

In terms of decision variables, we have the following:

Q_t $\rightarrow$ number of batches of raw-material to order in period t

w_{ut} $\rightarrow$ amount of raw-material received in period u used for production in period t

e_t $\rightarrow$ amount of perished raw-material received in period t to be discarded

s_t $\rightarrow$ finished item stock at the end of period t

y_t $\rightarrow$ binary variable equal to 1 if and only if there is a positive production in period t

z_t $\rightarrow$ binary variable equal to 1 if and only if a raw-material order is placed in period t

Variables w_{ut} are defined for $1 \leq u \leq t \leq n$ and $(t-u) < \beta$, given that material received at the beginning of period u can only be used for production during β periods of time (including u). The last period in which material received at period u can be used for production is given by $\Theta_u = \min\{u+\beta-1,\ n\}$. Similarly, let $\Pi_t = \max\{1, t-\beta+1\}$ denote the earliest period in which material can be acquired and still be used in period t. We further assume that even though material received during period u, $\beta < u \leq n$, does not expire during the planning horizon, if not used, will be discarded.

Using these sets of decision variables, the *2LS-FS* can be formulated as follows:

$$\text{minimize} \quad \sum_{t=1}^{n} \left(\sum_{u=\Pi_t}^{t} p_{ut}^{FS} w_{ut} + h_t s_t + q_t y_t + \zeta_t Q_t + \phi_t e_t + \rho_t z_t \right) \tag{3.1}$$

$$\text{subject to} \quad bQ_u = \sum_{t=u}^{\Theta_u} w_{ut} + e_u \qquad u \in T \tag{3.2}$$

$$s_{t-1} + \left(\frac{1}{r}\right) \sum_{u=\Pi_t}^{t} w_{ut} = d_t + s_t \qquad t \in T \tag{3.3}$$

$$\left(\frac{a}{r}\right) \sum_{u=\Pi_t}^{t} w_{ut} \leq C_t y_t \qquad t \in T \tag{3.4}$$

$$Q_t \leq K_t z_t \qquad t \in T \tag{3.5}$$

$$s_t,\ e_t \geq 0 \qquad t \in T \tag{3.6}$$

$$w_{ut} \geq 0 \qquad u, t \in T,\ u \leq t \tag{3.7}$$

$$Q_t \geq 0 \text{ and integer} \qquad t \in T \tag{3.8}$$

$$y_t,\ z_t \in \{0, 1\} \qquad t \in T \tag{3.9}$$

$$s_0 = s_0^*, \tag{3.10}$$

where s_0^* is the number of finished item units available at the beginning of the planing horizon and

$$p_{ut}^{FS} = \begin{cases} \frac{p_t}{r} + \sum_{v=u}^{t-1} \gamma_v & \text{if } u < t, \\ \frac{p_t}{r} & \text{if } u = t. \end{cases}$$

The objective function (3.1) includes production and raw-material inventory holding costs, finished item inventory holding costs, setup costs, raw-material batch costs, raw-material disposal costs, and order-placement costs. Constraints (3.2) state that the amount of raw-material entering the production system at each period u is equal to the amount used for production at subsequent periods, plus the amount that is discarded if not used before the end of its shelf-life. Constraints (3.3) represent finished item inventory balance, whereas constraints (3.4) are capacity limits. Constraints (3.5) are the upper bounds for the amount of raw-material to order at each time period. Constraints (3.6) – (3.7) are non-negativity conditions, and constraints (3.8) – (3.9) are the classical integrality and non-negativity conditions. Constraint (3.10) provides initial finished item units.

Note that we have not used decision variables to explicitly model production levels at every period. The p_{ut}^{FS} coefficient in the objective function includes both finished-item production and raw-material storage costs.

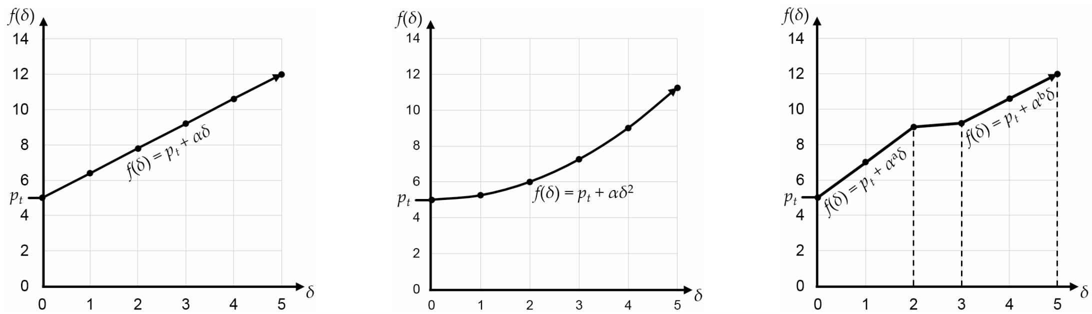
Property 3.1. *When $b = 1$, there exists an optimal solution to 2LS-FS in which $e_u = 0 \ \forall \, u \in T$.*

This property states that if it is possible to order raw-material by units, then one should order the exact amount as needed, i.e., $\sum_{u=1}^{n} Q_u = r \sum_{t=1}^{n} d_t$. We focus all our computational study on the case of raw-material batch ordering ($b > 1$).

3.2.2 Functionality deterioration

The second variant is the *two-level lot-sizing problem with raw-material functionality deterioration* (*2LS-FD*). In this case, raw-material functionality decreases as storage time passes. We refer to functionality as the suitability of the raw-material for being used in the manufacturing process. Deteriorated materials may cause additional production costs and increased resource consumption in achieving the desired product quality and yields. We represent this effect by considering the unit production cost p_t as an arbitrary non-decreasing function $f(\delta)$ where $\delta = (t - u)$, such that $f(0) = p_t$ and $f(\delta) \leq f(\delta + 1)$, for $0 \leq \delta < \beta$.

Figure 3.3 shows three examples of production cost functions with $\beta = 6$ and $p_t = 5$ for all $t \in T$.



(a) Linear function with slope α

(b) Cuadratic function with rate of change $\alpha\delta^2$

(c) Piecewise linear function with slopes α^a and α^b

Figure 3.3: Three production cost functions for an example with $\beta = 6$ and $p_t = 5$

Correspondingly, the updated production and raw-material inventory holding costs per period t considering the use of deteriorated raw-material received in $u \leq t$ is given by:

$$p_{ut}^{FD} = \begin{cases} \frac{f(\delta)}{r} + \sum_{v=u}^{t-1} \gamma_v & \text{if } u < t, \\ \frac{p_t}{r} & \text{if } u = t, \end{cases}$$

Note that the parameter p_{ut}^{FD} is not a decision variable but an input of the problem that depends on u, t, and $f(\delta)$.

As mentioned, the use of deteriorated but otherwise usable raw-material may require longer production time and consume more resource capacity for setting up the production system. For example, in composite manufacturing processes producing polyimide reinforced fiber composites and other products, slightly hardened resin may still be used in production but it usually requires additional cautions that slower operations.

In this regard, unit production time a is replaced as follows:

$$a_{ut}^{FD} = a + \Delta(\delta) \text{ for } 0 \leq u \leq t \leq n, \quad (t-u) < \beta.$$

where $\Delta(\delta)$ is a non-decreasing function with $\Delta(0) = 0$ and $\Delta(\delta) \leq \Delta(\delta + 1)$, for $0 \leq \delta < \beta$.

The *2LS-FD* model can be formulated as follows:

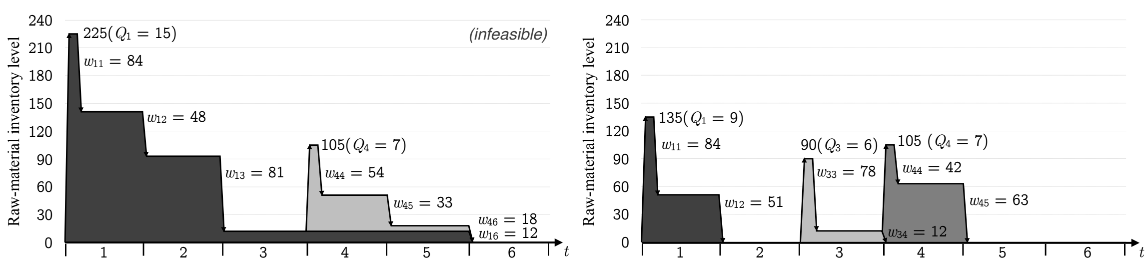
$$\text{minimize} \quad \sum_{t=1}^{n} \left(\sum_{u=\Pi_t}^{t} p_{ut}^{FD} w_{ut} + h_t s_t + q_t y_t + \zeta_t Q_t + \phi_t e_t + \rho_t z_t \right) \tag{3.11}$$

$$\text{subject to} \quad (3.2), (3.3), \ (3.5) - (3.10)$$

$$\left(\frac{a_{ut}^{FD}}{r} \right) \sum_{u=\Pi_t}^{t} w_{ut} \leq C_t y_t \qquad t \in T, \tag{3.12}$$

A production setup here is the realization of all operations required to reconfigure the production process at the end of period t after producing a batch of products. Thus, constraint (3.12), ensures that resource capacity in period t to produce all batches and perform reconfigurations using deteriorated material is not exceeded.

For illustrative purposes, Figure 3.4 shows a comparison of solutions for an instance with $n = 6$, $\beta = 2$, $b = 15$, $r = 3$, and $d = \{28, 16, 27, 18, 11, 10\}$ for the *2LS-FD* problem. Figure 3.3.2 shows raw-material inventory levels considering a durable (non-perishable) raw-material, solved with the standard *2LS* formulation. The *2LS* optimal solution is infeasible for the original *2LS-FD* problem. Figure 3.4b shows the raw-material inventory levels for the actual optimal solution obtained with the *2LS-FD* formulation. As shown in Figure 3.4a, the standard *2LS* solution has



(a) Standard *2LS* optimal solution (b) Optimal *2LS-FD* solution

Figure 3.4: A comparison of solutions for the *2LS-FD*

raw-material orders at $t = 1$ of size $Q_1 = 15$ and at $t = 4$ of size $Q_4 = 7$. Since $b = 15$, the amount of raw-material units which enter the production system at periods 1 and 4 are 225 and 105, respectively. Letting $X_t = \sum_{u=\Pi_t}^{t} w_{ut}$ for $t \in T$, represent the total production at time period t, in terms of production the standard *2LS* leads to the following lot sizes: $X_1 = w_{11}/r = 28$, $X_2 = w_{12}/r = 16$, $X_3 = w_{13}/r = 27$, $X_4 = w_{44}/r = 18$, $X_5 = w_{45}/r = 11$, and $X_6 = (w_{46} + w_{16})/r = 10$. However, this solution is infeasible for the *2LS-FD* variant, since it uses $w_{13} = 81$, $w_{46} = 18$, and $w_{16} = 12$ units of raw-material for production that are in fact lost/perished and have to be disposed, given that $\beta = 2$.

The optimal solution obtained with the *2LS-FD* formulation, as shown in Figure 3.4b, results in $Q_t > 0$ in $t \in \{1, 3, 4\}$, and production in $t \in \{1, 2, 3, 4, 5\}$, which corresponds to lot sizes $X = \{28, 17, 26, 18, 21, 0\}$ and no raw-material lost/perished.

3.2.3 Functionality and volume deterioration

We now propose the *two-level lot-sizing problem with raw-material functionality and volume deterioration* (*2LS-FVD*). Here, the perishability nature of the raw-material not only refers to a functionality loss but, in addition, to a progressive volume loss. This, in fact, generalizes the two previous model variants.

We consider that the amount (volume) of available raw-material decreases as a function of the time it has remained in storage. Thereby, $\nu(\delta)$ where $\delta = (t - u)$, denotes the raw-material volume deterioration function, with $\nu(\delta) \leq \nu(\delta + 1)$, for $0 \leq \delta < \beta$. A new set of decision variables c_{ut} for $1 \leq u \leq t \leq n$ is introduced to represent the amount of raw-material received in time period u and in storage at the end of t. Moreover, the expired raw-material variables e_t now have an additional index to track when the material is received (u) and when it is perished/lost (t): e_{ut} for $1 \leq u \leq t \leq n$.

Applications of this problem can be found in canning processes such as, canning fruits, vegetables, seafood, and meats, among others. The primary objective of food processing is the preservation of highly perishable goods in a stable form to be stored and shipped to distant markets. However, considerable amounts of raw-material may be lost throughout the multiple steps of the production process, which may include preliminary preparation, blanching, and filling (Melrose Chemicals Ltd., 2005).

Although some of the final products are stamped with a best-before-date fixing its shelf-life, their functionality or suitability is sufficiently long so that it is not required to take it into account in the planning horizon. If the planning horizon T refers to a set of days, weeks, or even months of production, a finished-item shelf-life that is in the order of 2 to 6 years does not need to be considered for practical purposes.

The updated production cost per period t (not including raw-material inventory holding costs) considering the use of deteriorated raw-material received in $u \leq t$ is given by:

$$p_{ut}^{FVD} = \begin{cases} \frac{f(\delta)}{r} & \text{if } u < t, \\ \frac{p_t}{r} & \text{if } u = t, \end{cases}$$

where as before, $f(\delta)$ is the production cost function depending on $\delta = t - u$.

The *2LS-FVD* model can be formulated as follows:

$$\text{minimize} \quad \sum_{t=1}^{n} \left(\sum_{u=\Pi_t}^{t} p_{ut}^{FVD} w_{ut} + h_t s_t + q_t y_t + \zeta_t Q_t + \rho_t z_t \right) + \sum_{u=1}^{n} \sum_{t=u}^{\Theta_u} (\gamma_u c_{ut} + \phi_u e_{ut}) \qquad (3.13)$$

$$\text{subject to} \quad (3.3), (3.5) - (3.10), (3.12)$$

$$c_{tt} = (bQ_t - w_{tt})(1 - \nu(0)) \qquad t \in T \qquad (3.14)$$

$$c_{ut} = (c_{u,t-1} - w_{ut})(1 - \nu(\delta)) \qquad u, t \in T, \; 0 < \delta < \beta \qquad (3.15)$$

$$e_{tt} = (bQ_t - w_{tt})(\nu(0)) \qquad t \in T \qquad (3.16)$$

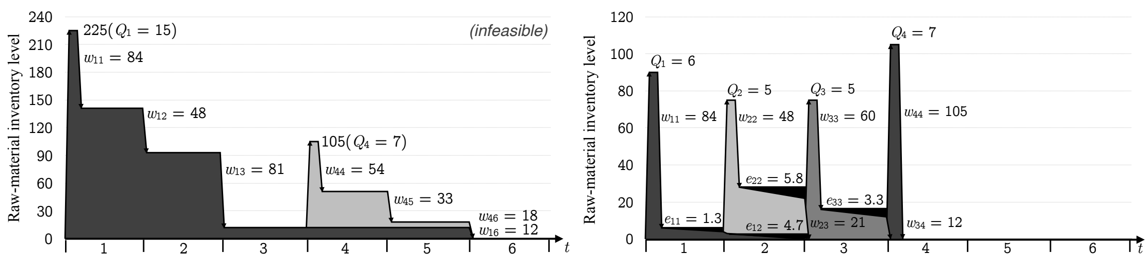
$$e_{ut} = (c_{u,t-1} - w_{ut})(\nu(\delta)) \qquad u, t \in T, \; 0 < \delta < \beta \qquad (3.17)$$

$$c_{ut}, \; e_{ut} \in \mathbb{R}^+ \qquad u, t \in T, \; u \leq t, \qquad (3.18)$$

where constraints (3.14) – (3.15) represent raw-material inventory levels and constraints (3.16) – (3.17) represent raw-material disposal. Since raw-material inventory levels are recursively calculated at each period t depending on the volume deterioration function $\nu(\delta)$, it becomes necessary to remove the raw-material storage costs γ_t from the p_{ut}^{FVD} cost function and apply it directly to the c_{ut} variables in the objective function.

Note that, if $f(\delta) = p_t$ for $0 \leq \delta < \beta$, *2LS-FVD* reduces to a variant with only raw-material volume deterioration. Moreover, if $\nu(\delta) = 0$ for $0 \leq \delta < \beta$, *2LS-FVD* reduces to *2LS-FD*.

Considering the same small instance in Section 3.2.2 with the addition of the volume deterioration rate function $\nu(0) = 0.214$ and $\nu(1) = 1.0$, Figure 3.5 presents a graphical representation of the solution structure and a comparison with the durable (non-perishable) version of the problem.



(a) Standard *2LS* optimal solution (b) Optimal *2LS-FVD* solution

Figure 3.5: A comparison of solutions for the *2LS-FVD*

Figure 3.5a shows the same initial *2LS* solution and Figure 3.5b does so for the actual optimal solution for the *2LS-FVD* version of the problem. The implementation of the *2LS-FVD* MIP formulation results in an optimal solution with $Q_1 = 6$ batches of raw-material received in $t = 1$, which represent 90 units entering the production system. Out of those 90 units, $w_{11} = 84$ are used in period 1, leaving 6 units in inventory. Due to volume deterioration, $e_{11} = 6 \times 0.214 = 1.3$ and since $w_{12} = 0$, $e_{12} = (6 - 1.3) \times 1.0 = 4.7$ units of raw-material perished and are disposed. $Q_2 = 5$ batches are received in $t = 2$, representing 75 units entering the system. $w_{22} = 48$ units are used in $t = 2$, leaving 27 in inventory, and $e_{22} = 27 \times 0.214 = 5.8$ units perished and are disposed. $Q3 = 5$ batches are once more received in $t = 3$, but since 21.2 units are still in storage at the beginning of $t = 3$, $75 + 21.2 = 96.2$ units are available for production. Out of those 96.2 units, $w_{23} + w_{33} = 21 + 60 = 81$ are used, leaving 15.2 in inventory, and $e_{33} = 15.2 \times 0.214 = 3.3$ units of raw-material perished and disposed. Finally, $Q_4 = 7$ raw-material batches are received in $t = 4$, representing 105 units entering the production system. Since 12 units are still in storage at the beginning of $t = 4$, $12 + 105 = 117$ units are available for production, and they are all used with no disposal. This corresponds to the following lot sizes: $X_1 = w_{11}/r = 28$, $X_2 = w_{12}/r = 16$, $X_3 = (w_{23} + w_{33})/r = 27$, $X_4 = (w_{34} + w_{44})/r = 39$, with the following finished-item inventory levels: $s = \{0, 0, 0, 21, 10, 0\}$.

3.3 Computational experiments and analysis

A computational study was conducted in order to gain in-depth understanding of the considered models and to evaluate our MIP formulations. We tested more than 1,500 randomly generated instances for each of the three problem variants. Section 3.3.1 shows how such instances were generated.

For each of our three problem variants, we begin by evaluating the solutions of the standard *2LS*. Results are presented in Section 3.3.2. In Section 3.3.3, we study the added value of the proposed models in comparison with a sequential approach in which production and raw-material related decisions are made independently. In Section 3.3.4, we analyze the way that certain key parameters affect the optimal planning decisions. Section 3.3.5 shows certain computational aspects of the proposed MIP formulations when used with a general purpose solver.

3.3.1 Description of test instances

All problem instances used to perform the computational experiments in section **??** were randomly generated, as follows.

The following parameters were set a priori as a basis for comparison: planning horizon (n), shelf-life (β), batch-size (b).

Problem size (planning horizon): as seen in Table 3.3, in terms of planning horizon, we tested instances with $n \in \{18, 20, 22\}$.

Shelf-life (β): as seen in Table 3.3, for each n value, in terms of shelf-life, we tested instances with values $\beta \in \{2, 4, 6, 8\}$.

Batch-size (b): for each β value, we tested instances with values $b \in \{50, 100, 150, 200, 250\}$.

Other parameter set a priori is $r = 3$ for all problem instances.

The remaining parameters were all generated randomly using a uniform distribution (with $\mathcal{U}\{lower\_limit, upper\_limit\}$ for discrete cases, and $\mathcal{U}(lower\_limit, upper\_limit)$ for continuous cases, as follows:

Demand (d_t): $\mathcal{U}\{150, 300\}$

Unit production time (a): $\mathcal{U}(2.5, 3.5)$

Unit production cost (p_t): $\mathcal{U}(10.0, 13.0)$

Fixed setup cost (q_t): $\mathcal{U}(380.0, 420.0)$

Unit storage cost (h_t): $\mathcal{U}(5.0, 7.0)$

Unit raw-material cost (ζ_t): $\mathcal{U}(1.0, 3.0)$

Upper ordering limit (K_t): the K_t values were generated considering the product of the average demand $\bar{d_t} = 225$ and the bill of material r, divided by the batch size b. And so, assuming $K_{coef} = \frac{\bar{d_t} \times r}{b}$, we used $\mathcal{U}\{K_{coef} \times 4.5, K_{coef} \times 4.75\}$.

With the purpose of integrating variability in a controlled manner, for the remaining parameters, three different value levels were generated each for 1/3 of the instances.

Fixed order-placement cost (ρ_t): A *high level* using $\mathcal{U}(500, 550)$, a *medium level* using $\mathcal{U}(250, 300)$, and a *low level* using $\mathcal{U}(150, 200)$.

Raw-material unit storage cost (γ_t): A *high level* using $\mathcal{U}(8.0, 12.0)$, a *medium level* using $\mathcal{U}(3.0, 7.0)$, and a *low level* using $\mathcal{U}(1.0, 2.0)$.

Finally, the capacitated versions of the problem instances (results presented in Section 3.3.5) used the following:

Available process capacity (C_t): The C_t values were generated considering the upper demand limit 300, as follows: a *high level* using $\mathcal{U}\{300 \times 4.5, 300 \times 4.75\}$, a *medium level* using $\mathcal{U}\{300 \times 4.25, 300 \times 4.5\}$, and a *low level* using $\mathcal{U}\{300 \times 4.0, 300 \times 4.25\}$

3.3.2 A standard two-level lot-sizing model

As described in Section 3.2, the *standard two-level lot-sizing 2LS* problem considers a production system in which one item (finished product) is to be produced and another item (raw-material), an input of the first, is to be procured from a supplier over a planning horizon with n time periods, $T = \{1, ..., n\}$. Solving the *2LS* problem is to determine the production, procurement and inventory plans for the two items to meet the demands of the planning horizon, while minimizing the corresponding costs.

To evaluate the added value of integrating raw-material perishability into classical lot-sizing problems, we initially perform a comparative analysis on the optimal solutions obtained with our MIP formulations and those of the *2LS* model.

For each of our three problem variants, we begin by evaluating the solutions of the *2LS*. If they are feasible for the counterpart problems with raw-material deterioration, these solutions will be compared with the optimal solutions of the proposed MIP formulations. Table 3.1 presents these results for a set of instances with $n = 7$ where the only varying parameters are $\beta = \{2, 3, 4\}$ and $b = \{40, 80, 100, 150, 200, 250\}$. The same instances are used for the computational experiments presented in Section

3.3.3.

Table 3.1: Average *standard 2LS* solution deviations

Variant	β	%inf	%dev	% > 10	max
2LS-FS	2	33.3	8.8	12.6	15.5
	3	16.6	10.2	20.3	18.1
	4	0.0	12.2	21.8	23.6
		16.6	**10.4**	**18.8**	**23.6**
2LS-FD	2	33.3	8.9	12.6	15.5
	3	16.6	10.5	20.3	18.1
	4	0.0	12.3	21.8	23.6
		16.6	**10.6**	**18.8**	**23.6**
2LS-FVD	2	56.2	9.2	33.8	26.3
	3	56.2	11.9	42.9	30.6
	4	56.2	13.0	47.5	34.2
		56.2	**11.4**	**41.4**	**34.2**
Total		**29.8**	**10.8**	**23.9**	**34.2**

The first two columns in Table 3.1 specify the type of problem variant solved and the β values of the instances. The third column shows the percentage of instances for which the standard *2LS* solution is infeasible (%inf) when adapted to solve its counterpart problem variant. The next column shows the average deviation (%dev) of the feasible solutions from the optimal solution of the actual problem considering raw-material perishability. The deviations are computed as $\%dev = [(SOL_{2LS} - OPT)/OPT] \times 100$, where SOL_{2LS} is the objective function value of the feasible solution and OPT the optimal solution value. Finally, the last two columns show the percentage of instances with %dev greater than 10% and the maximum %dev observed, respectively.

As expected, for many problem instances, optimal solutions of the *2LS* model are not feasible for solving their counterpart problems with raw-material perishability, specially for shorter shelf-life instances and for *2LS-FVD*.

We observe that $\%dev$ increases when the β values increase. In total, 29.8% of the *2LS* solutions were infeasible and the average $\%dev$ is 10.8% with the maximum being 34.2%. Averaging the three problem instances, nearly 24% of the instances showed a $\%dev$ greater than 10%.

We further investigate the instances that are infeasible and those that show greater deviations. In general, infeasibility comes from two different but closely related sources. Firstly, the standard *2LS* model may have solutions where production of finished-items is set to use raw-material stored in inventory for periods longer than its shelf-life, i.e. $w_{ut} > 0$ for $(t - u) \geq \beta$. A second source of infeasibility, specifically for *2LS-FVD*, is that the amount of raw-material ordered in any given period is less than required to cover all production for subsequent periods before a new order is placed, i.e. constraints (3.14) and (3.15) are violated.

3.3.3 The value of integrating raw-material perishability into classical lot-sizing

With a clear understanding that the comparison between the standard *2LS* model and our MIP formulations may not seem fair, we make it in order to quantitatively assess the value of integrating perishability and deterioration into classical lot-sizing problems using our proposed models. To continue this assessment, we propose a sequential approach that adapts in a natural and intuitive fashion the initial standard *2LS* solutions to find feasible and possibly improved solutions for the original problem variants. We then compare how our MIP formulations perform compared to this sequential optimization approach.

Considering that the sources of infeasibility are decisions regarding the size and timing of raw-material orders and the use of such material to meet production requirements, it is natural to adapt the solutions by modifying these decisions in a subsequent phase.

With this in mind, the first step of the sequential approach begins by fixing the production-related decisions obtained in the standard *2LS* solution and use them as exogenous decisions for the following step. The second step applies a rolling-horizon algorithm following the basic idea of the Silver-Meal heuristic [111] to solve the remaining sub-problem regarding the raw-material related decisions. Thus, the idea is to order enough raw-material to cover the production of one time period u and then the number of periods to cover is increased (in increments of one period t) until the average cost per period (ACP_{ut}) increases. We begin by defining the terms:

$\hat{x}_t$ → production decisions to fix obtained from standard *2LS* solution for $1 \leq t \leq n$, where $\hat{x}_t = \sum_{u=1}^{t} w_{ut}$.

$\hat{X}_{ut}$ → fixed cumulative production to cover from period u to t.

$\bar{Q}_{ut}$ → order quantity (raw-material batches) in period u to cover fixed production up to t.

$\bar{z}_{ut}$ → binary raw-material for order placement variable.

$\bar{w}_{ut}$ → variables used to modified the original w_{ut} variables within the heuristic to avoid violation of the $(t-u) < \beta$ condition.

ACP_{ut} → average cost per period for an order placed in period u to cover fixed production requirements up to t, where:

$$ACP_{ut} = \frac{\rho_u \bar{z}_u + \zeta_u \bar{Q}_{ut} + \sum_{t=u}^{\Theta_u} p_{ut}^{i} \bar{w}_{ut} + \phi_u \left(b\bar{Q}_{ut} - \sum_{t=u}^{\Theta_u} \bar{w}_{ut} \right)}{(t-u)+1}.$$

Clearly, the original Silver-Meal heuristic cannot be applied to any our problem variants, so an extended version is implemented. For the *2LS-FS* and *2LS-FD* variants, the steps involved in the second step of the sequential approach are shown in Algorithm 1.

For the *2LS-FVD* variant, ACP_{ut} is computed as follows:

$$ACP_{ut} = \frac{\rho_u \bar{z}_u + \zeta_u \bar{Q}_{ut} + \sum_{t=u}^{\Theta_u} p_{ut}^{3} w_{ut} + \sum_{t=u}^{\Theta_u} (\gamma_u c_{ut} + \phi_u e_{ut})}{(t-u)+1},$$

and the steps are shown in Algorithm 2.

Table 3.2 shows the computational results using the sequential approach to solve each of the original problem variants. The first two columns specify the type of

Algorithm 1 Second step procedure of sequential approach for *2LS-FS* and *2LS-FD*

1: Solve standard *2LS*, **return** $\hat{y}_t$, w_{ut} for $u, t \in T$, $u \leq t$
2: $u \leftarrow 1$
3: **while** $t < n$ **do**
4: $\quad t \leftarrow u$
5: $\quad$ **compute** $\hat{X}_{ut} = \dfrac{\sum_{t=u}^{n} w_{ut}}{r}$
6: $\quad$ **let** $\bar{Q}_{ut} = \begin{cases} 0, & \textbf{if } \hat{X}_{ut} = 0 \\ \max\left\{ \left\lceil \dfrac{r\hat{X}_{ut}}{b} \right\rceil, L \right\}, & \textbf{if } \hat{X}_{ut} > 0 \end{cases}$ **and** $\bar{z}_u = \begin{cases} 0, & \textbf{if } \bar{Q}_{ut} = 0 \\ 1, & \textbf{if } \bar{Q}_{ut} > 0 \end{cases}$
7: $\quad$ **compute** $\bar{w}_{ut} = r\hat{x}_t$
8: $\quad$ **compute** ACP_{ut}
9: $\quad$ **if** $ACP_{ut} > ACP_{u,t-1}$ or $t + 1 = \Theta_u$ **then**
10: $\qquad$ go to step 14
11: $\quad$ **else**
12: $\qquad$ $t \leftarrow (t + 1)$ and go to step 5
13: $\quad$ **end if**
14: $\quad$ **Let** $z_u = \bar{z}_{u,t-1}$, $Q_u = \bar{Q}_{u,t-1}$, and $w_{u,t-1} = \bar{w}_{u,t-1}$
15: $\quad$ $u \leftarrow (u + 1)$ and go to step 4
16: **end while**
17: **return** z_u, Q_u, w_{ut} for $u, t \in T$, $0 \leq (t - u) < \beta$

Table 3.2: Average sequential optimization approach results

Variant	β	%opt	%dev	% > 10
2LS-FS	2	39.6	4.5	10.5
	3	52.1	4.2	20.9
	4	62.5	3.9	18.8
		51.4	**4.2**	**16.7**
2LS-FD	2	39.6	5.0	12.5
	3	52.1	4.4	20.9
	4	52.1	4.0	18.8
		47.9	**4.5**	**17.4**
2LS-FVD	2	8.3	8.7	34.4
	3	6.3	12.1	49.0
	4	4.2	14.2	63.6
		6.3	**11.7**	**49.0**
Total		**35.2**	**6.8**	**27.7**

Algorithm 2 Second step procedure of sequential approach for *2LS-FVD*

1: Solve standard *2LS*, **return** $\hat{y}_t$, w_{ut} for $u, t \in T$, $u \leq t$
2: $u \leftarrow 1$
3: **while** $t < n$ **do**
4: $\quad t \leftarrow u$
5: $\quad$ **if** u = t **then**
6: $\quad\quad$ **let** $\bar{Q}_{ut} = \begin{cases} 0, & \textbf{if } \hat{x}_u = 0 \\ \max\left\{\left\lceil \frac{r\hat{x}_u}{b} \right\rceil, L\right\}, & \textbf{if } \hat{x}_u > 0 \end{cases}$ **and** $\bar{z}_u = \begin{cases} 0, & \textbf{if } \bar{Q}_{ut} = 0 \\ 1, & \textbf{if } \bar{Q}_{ut} > 0 \end{cases}$
7: $\quad$ **else**
8: $\quad\quad$ **let** $\bar{Q}_{ut} = \begin{cases} 0, & \textbf{if } \hat{x}_{ut} = 0 \\ \max\left\{\left\lceil \frac{\left[\frac{\bar{c}_{ut}}{(1-\nu(0))} + r\hat{x}_u\right]}{b} \right\rceil, L\right\}, & \textbf{if } \hat{x}_{ut} > 0 \end{cases}$ **and** $\bar{z}_u = \begin{cases} 0, & \textbf{if } \bar{Q}_{ut} = 0 \\ 1, & \textbf{if } \bar{Q}_{ut} > 0 \end{cases}$
9: $\quad$ **end if**
10: $\quad$ **compute** $\bar{w}_{ut}$, $\bar{c}_{ut}$, and $\bar{e}_{ut}$ for all $u \leq t \leq \Theta_u$
11: $\quad$ **compute** ACP_{ut}
12: $\quad$ **if** $ACP_{ut} > ACP_{u,t-1}$ or $t + 1 = \Theta_u$ **then**
13: $\quad\quad$ go to step 17
14: $\quad$ **else**
15: $\quad\quad$ $t \leftarrow (t + 1)$ and go to step 8
16: $\quad$ **end if**
17: $\quad$ **let** $z_u = \bar{z}_{u,t-1}$, $Q_u = \bar{Q}_{u,t-1}$ for $u \leq t \leq \Theta_u$
18: $\quad$ **let** $w_{u,t-1} = \bar{w}_{u,t-1}$, $c_{u,t-1} = \bar{c}_{u,t-1}$, and $e_{u,t-1} = \bar{e}_{u,t-1}$ for $u \leq t \leq \Theta_u$
19: $\quad$ $u \leftarrow (u + 1)$ and go to step 4
20: **end while**
21: **return** $z_u, Q_u, w_{u,t}, c_{ut}, e_{ut}$ for $u, t \in T$, $0 \leq (t - u) < \beta$

Where $\bar{w}_{ut} = r\hat{x}_t$; $\bar{c}_{ut} = (\bar{c}_{u,t-1} - r\hat{x}_t)(1 - \nu(\delta))$ and $\bar{c}_{uu} = r\hat{x}_{u+1}$, and $\bar{e}_{ut} = (\bar{c}_{u,t-1} - r\hat{x}_t)(\nu(\delta))$ and $\bar{e}_{uu} = \left(b\bar{Q}_{ut} - \hat{x}_t\right)(\nu(\delta))$.

problem variant solved and the β values of the instances. The third column shows the percentage of instances for which the sequential approach solution is optimal (%opt). The last two columns show the average deviation (%dev) with respect to the optimal solution (of the non-optimal solutions), and the percentage of instances greater than 10%, respectively.

We can see that in addition to achieving feasibility for all instances, the sequential approach also reached optimal solutions for 51.4%, 47.9%, and 6.3% of the instances for *2LS-FS*, *2LS-FD*, and *2LS-FVD*, respectively. The average deviations (%dev) for the first two problem variants are quite similar, ranging from 3.9% to 5.0%. It is much higher *2LS-FVD*, ranging from 8.7% to 14.2%. Figure 3.6 clusters the three problem variants and graphically shows the sequential approach deviations (%dev) for each batch-size value b. The dotted lines represent the average %dev for each shelf-life β value. It shows a clear trend that the average deviation increases as the batch-size value (b) increases.

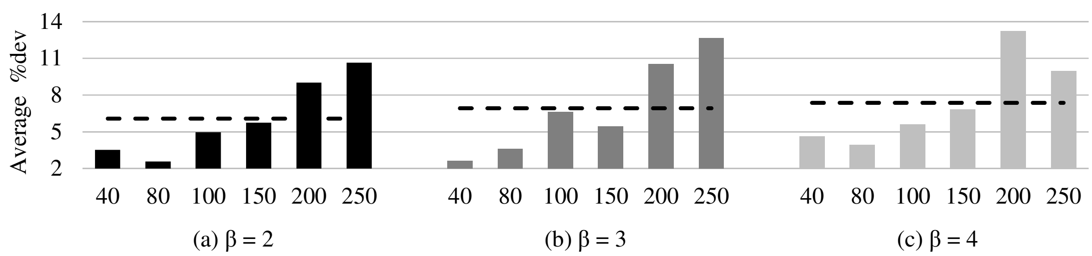
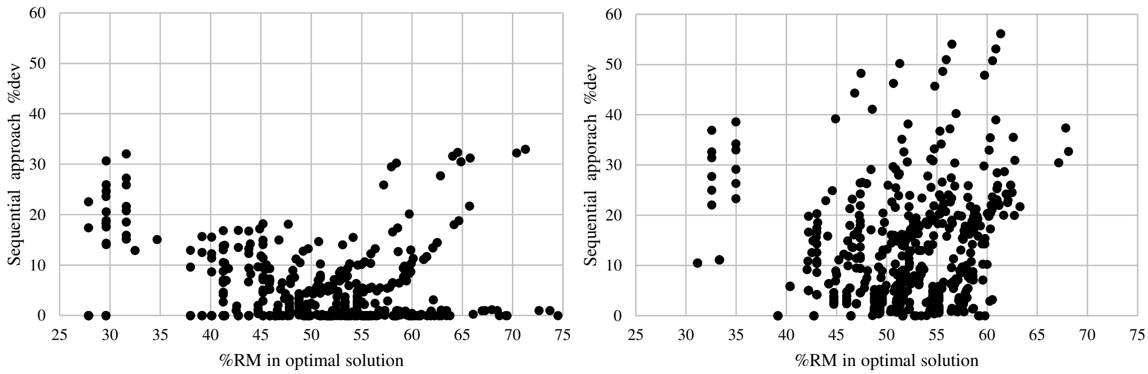


Figure 3.6: Average sequential approach (%dev) by shelf-life (β) and batch size (b)

Since the sequential approach focuses on the modification of raw-material related decisions, it is relevant to see how its deviation changes with respect to the total costs corresponding to these decisions (%RM) from the total value of the optimal solution. Figure 3.7 shows the scatter plots for *2LS-FD* and *2LS-FVD* within the range $25 \leq$ %RM ≤ 75.

Figure 3.7 shows that the deviation values are somewhat scattered. However, we note that for instances with %RM above 65%, the sequential approach found near



(a) *2LS-FD* (b) *2LS-FVD*

Figure 3.7: Average sequential algorithm %dev vs. raw-material costs percentage %RM in optimal solution

optimal solutions for *2LS-FD*. This is not the case for *2LS-FVD*, where higher %RM represent also higher deviations. Similarly, when %RM is below 40%, some instances are optimally solved for *2LS-FD*, whereas the *2LS-FVD* variant shows higher deviations.

The highest deviations for the *2LS-FS* and *2LS-FD* (%dev > 20) were found on instances with $b \geq 100$ in which the sequential approach resulted in solutions with a lower raw-material ordering frequency than the one observed in the optimal solution of the problem. These instances have %RM lower than 35 and greater than > 55. By resulting in a lower raw-material ordering frequency, the fixed order-placement are evidently reduced. However, this reduction is not sufficient compared to the substantial increase in raw-material inventory holding and disposal costs.

On the other hand, the highest deviations for the 2LS-FVD (%dev > 30) were found on instances with $b \geq 80$. In these cases, the discrepancy between the sequential and the integrated solution is that the former makes raw-material orders that are much higher than those required for production. This increases the general raw-material related costs, including inventory and disposal.

3.3.4 Key parameters for optimal planning

The shelf-life parameter β limits the number of periods that the raw-material can remain in storage and be used for production. Along with β, functions $f(\delta)$ and $\nu(\delta)$ to model the loss of material functionality and volume, respectively, constitute the core features of the studied problems. In addition, raw-material order batch-size is another parameter requiring detailed analysis to see how it affects the optimal solutions. For this analysis, we solved a set of problem instances with a planning horizon of $n = 7$ periods, bill of material $r = 3$, lower ordering limit $L = 0$, and various β and b values, keeping all other parameters unchanged. Figure 3.8 shows a comparison of the corresponding raw-material disposal costs $\left(\sum_{u=1}^{n}\sum_{t=u}^{\Theta_u}\left(\gamma_u c_{ut} + \phi_u e_{ut}\right)\right)$ in the optimal solution for each problem variant.

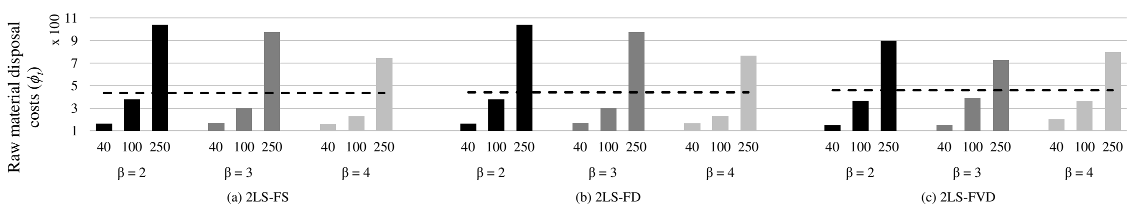


Figure 3.8: Comparison of disposal costs by shelf-life (β) and batch size (b) values

The most important variations in the structure of optimal solutions largely arise from the relation between the order batch-size b, the bill of material r, and the finished-item demand d_t levels. This relation affects the flexibility to manage raw-material inventories and the possibility to avoid disposing units. The greater flexibility is found on instances with ordering batch-size $b = 1$ (see Property 3.1). Results shown in this sections are for instances with $b > 1$. As observed in Figure 3.8, for every problem instance, raw-material disposal increases consistently as the batch-size increases.

For the same β and b values, Figure 3.9 shows the changes in the corresponding order-placement costs ($\sum_{t=1}^{n}\rho_t z_t$) in the optimal solution. These costs represent the frequency with which raw-material order are placed.

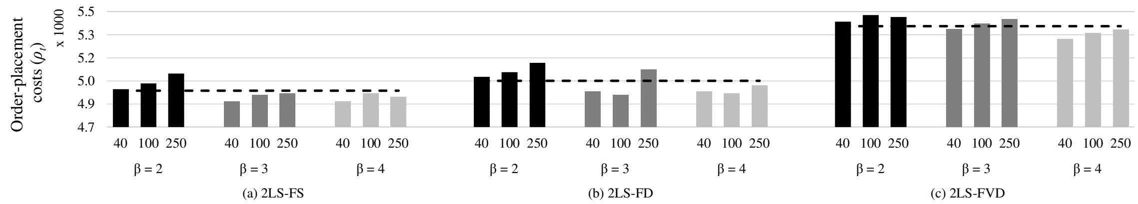


Figure 3.9: Comparison of order-placement costs by shelf-life (β) and batch size (b) values

As observed, longer shelf-lives result in lower average order-placement costs. This is partially attributed to the fact that shorter shelf-lives represent fast functionality and/or volume deterioration, which consequently results in a higher setup frequency. Although not shown in the figure, this fact also results in an increase in the average finished-item inventory holding costs, since production tends to take place in earlier periods to avoid raw-material disposal.

A counterintuitive observation from Figure 3.9 is that higher order-placement costs are found for instances with larger batch-sizes b. This is partially attributed to the fact that larger order batch-sizes result in an increase of raw-material wastage and disposal, as well as higher finished-item inventory costs. This behavior is later observed again in Figure 3.11 for the optimal objective function values.

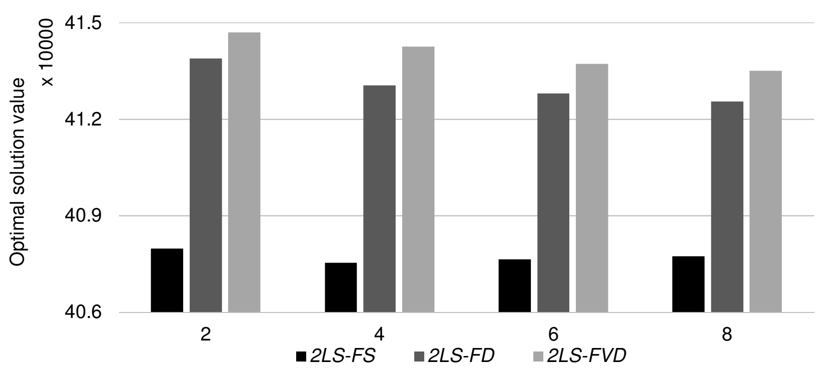


Figure 3.10: Optimal objective function values by shelf-life (β)

To get a more generic view of how shelf-life impacts cost levels, Figure 3.10 shows the changes in the optimal solution values when solving a single problem instance with a planning horizon of $n = 16$ periods and various β values, keeping all other parameters unchanged.

Figure 3.10 shows that the same behavior previously observed is ultimately reflected in the optimal solution values as well.

A clear observation from Figures 3.8 to 3.10 is that the optimal solution values are clearly higher for the *2LS-FVD* problem variant, which is consistent with the significant increase in raw-material disposal and order-placement costs. This is somewhat expected due to the progressive raw-material volume loss in each period, resulting in the need to place orders more frequently. The *2LS-FS* variant does not show substantial changes when β varies.

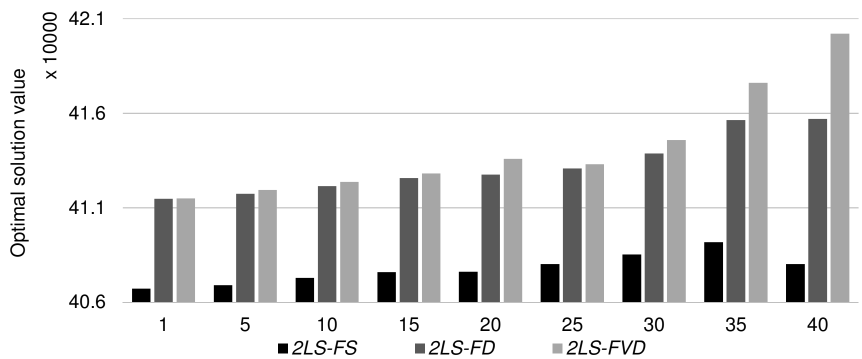


Figure 3.11: Optimal objective function values by batch-size (b)

Finally, Figure 3.11 presents the optimal objective function values for nine different b values. As previously mentioned, for every problem instance, it is clear how these values increase consistently as the batch-size increases.

3.3.5 Computational performance of MIP formulations

For the following computational study, we have implemented our MIP formulations on a set of randomly generated capacitated instances for each problem variant (see Section 3.3.1). All computational experiments were implemented and executed using the Callable Library of IBM CPLEX 12.6.2 on an Intel(R) Xeon(R) CPU E3-1270 v3 processor with 3.50GHz and 24GB of RAM memory and Microsoft Windows 7

Enterprise operating system. A maximum time limit of two hours was used in all experiments.

Table 3.3 presents the computational performance of the test instances for each problem variant. The first three columns show the different variant, n and β values for which results are presented. We show the average performance of 1620 instances for each variant (i.e. 135 for each β value and 540 for each n).

The second couple of columns refer to the number of branch and bound nodes explored by CPLEX showing the minimum value found and the average. The next couple of columns refer to the linear programming gaps (LP Gaps %). And the last two columns refer to the CPU time (in seconds), showing the average and the maximum value observed. Wherever "limit" is registered it means that at least one of the solutions was not solved to optimality within the two hours.

A first observation is that the increase in the shelf-life values corresponds to an increment of the number of branch and bound nodes explored in most of the tested instances. This is partially due to the fact that a lower shelf-life restricts the solution space decreasing the number of possibilities to decide upon regarding the usage of raw-material for production in later periods. Having a longer shelf-life value increases the solution space to explore.

Generally, LP Gaps range from 6.51% to 26.81%. The lowest LP Gaps ($< 10\%$) were observed in *2LS-FS* and *2LS-FD* instances with lower fixed costs of placing raw-material orders ρ_t. Whereas the highest LP Gaps ($> 20\%$) were mostly observed in *2LS-FVD* instances with $b \geq 100$ values. In terms of computational times, the average CPU time ranges between 6.77 and $1,577$ seconds. The highest values (> 60 minutes) were mostly observed in instances with $\beta \geq 4$. Instances that were not solved within the two hour limit have all $\beta \geq 6$.

Table 3.3: Computational performance of MIP formulations

Variant	n	β	Nodes (x1,000) min	Nodes (x1,000) avg	LP Gap (%) min	LP Gap (%) avg	Time (s) avg	Time (s) max	Solved
2LS-FS	18	2	1.69	158.66	7.37	13.56	7.30	66.94	135/135
		4	1.59	740.71	7.31	13.96	35.70	1684.83	135/135
		6	1.12	1,480.00	6.94	14.15	73.64	2131.91	135/135
		8	1.97	3,529.23	7.87	14.21	192.37	limit	134/135
		Total	**1.12**	**1,477.15**	**6.94**	**13.97**	**77.25**	**limit**	**539/540**
	20	2	2.47	663.44	6.51	13.59	30.28	1137.07	135/135
		4	5.80	1,191.96	7.95	13.95	59.54	1077.24	135/135
		6	6.11	4,180.69	7.25	14.30	226.90	limit	134/135
		8	5.43	9,106.74	8.08	14.25	604.65	limit	130/135
		Total	**2.47**	**3,785.71**	**6.51**	**14.02**	**230.34**	**limit**	**534/540**
	22	2	4.76	1,741.72	7.56	13.45	87.56	1685.92	135/135
		4	6.55	5,565.82	7.23	13.95	332.51	limit	133/135
		6	15.93	8,269.32	7.03	13.97	480.59	limit	133/135
		8	21.21	16,195.62	8.40	14.16	1193.36	limit	120/135
		Total	**4.76**	**7,943.12**	**7.03**	**13.88**	**523.50**	**limit**	**521/540**
	Variant Total		**1.12**	**4,401.99**	**6.51**	**13.96**	**277.03**	**limit**	**1,594/1,620**
2LS-FD	18	2	1.28	139.98	7.80	14.08	6.77	60.29	135/135
		4	2.94	661.71	7.32	14.37	36.11	1916.44	135/135
		6	2.15	783.55	7.23	14.46	51.86	918.77	135/135
		8	4.67	1,840.65	8.00	14.46	140.16	limit	134/135
		Total	**1.28**	**856.47**	**7.23**	**14.34**	**58.73**	**limit**	**539/540**
	20	2	0.92	622.76	7.00	14.08	28.87	1054.34	135/135
		4	6.30	1,032.04	7.96	14.38	60.36	902.31	135/135
		6	9.43	3,239.19	7.46	14.64	216.48	2881.92	135/135
		8	6.18	5,500.59	8.33	14.51	452.89	limit	134/135
		Total	**0.92**	**2,598.65**	**7.00**	**14.40**	**189.65**	**limit**	**539/540**
	22	2	3.66	1,715.70	8.16	13.95	88.01	1573.18	135/135
		4	7.19	4,317.31	7.96	14.34	291.21	limit	134/135
		6	6.30	6,087.04	7.50	14.30	491.10	limit	132/135
		8	15.29	11,610.36	8.49	14.41	1135.40	limit	125/135
		Total	**3.66**	**5,932.60**	**7.50**	**14.25**	**501.43**	**limit**	526/540
	Variant Total		**0.92**	**3,129.24**	**7.00**	**14.33**	**249.94**	**limit**	**1,604/1,620**
2LS-FVD	18	2	1.54	232.54	10.80	17.85	13.87	335.50	135/135
		4	3.52	1,107.24	11.04	18.20	114.48	limit	134/135
		6	3.84	1,093.42	11.90	18.30	133.27	2090.75	135/135
		8	3.02	1,015.08	11.97	18.40	142.19	2510.95	135/135
		Total	**1.54**	**862.07**	**10.80**	**18.19**	**100.95**	**limit**	**539/540**
	20	2	1.99	947.69	11.38	17.95	56.41	1132.17	135/135
		4	10.68	2,358.18	11.84	18.22	242.20	6684.31	135/135
		6	4.44	5,677.68	11.75	18.54	765.87	limit	131/135
		8	7.79	4,146.80	11.97	18.37	674.60	limit	130/135
		Total	**1.99**	**3,282.59**	**11.38**	**18.27**	**434.77**	**limit**	**531/540**
	22	2	5.71	3,215.44	11.92	17.82	213.35	6195.72	135/135
		4	9.38	6,323.53	11.61	18.19	708.45	limit	131/135
		6	18.28	5,933.21	11.57	18.33	847.36	limit	132/135
		8	26.13	8,659.96	12.08	18.40	1577.27	limit	119/135
		Total	**5.71**	**6,033.03**	**11.57**	**18.18**	**836.61**	**limit**	**517/540**
	Variant Total		**1.54**	**3,392.56**	**10.80**	**18.21**	**457.44**	**limit**	**1,587/1,620**

3.3.6 Conclusions and future research

From the study presented in Section 3.3, we can infer that there is a significant added value for using our proposed MIP formulations to integrate these considerations into classical lot-sizing models. Clearly, the use of a standard two-level lot-sizing model within a sequential approach is insufficient to solve the problems discussed in this study.

From this research, Chapter investigates the integration of other relevant factors to make our models more robust, such as multi-raw-material items, different product structures, capacity restrictions, time-dependent batch sizes, and other special raw-material inventory-related assumptions. Finally, considering the extensive computational times to solve a portion of medium to large size problem instances, we plan to work in the development of solution algorithms for efficiently solving certain variants of these problems in the future.

Chapter 4

Multi-item, multi-level lot-sizing with raw-material perishability, deterioration, and batch ordering

Advanced composite manufacturing processes use polymers and fibers pre-impregnated with thermoplastic or thermoset resins as raw-materials. These highly sensitive materials are considerably perishable, affecting the production process in several ways and requiring special inventory cares. This perishable condition of raw materials in composite manufacturing constitutes the main motivation for this study.

With production planning in composite manufacturing as an initial motivation and with the possibility of generalizing the problem to other applications, in this chapter we study the *multi-item, multi-level lot-sizing problem with raw-material perishability and batch ordering* (*MI-MLS-FVD*). In particular, we consider an assembly production system in which one item at the lower level (non-perishable final product, representing the advanced composite) facing independent demand is to be produced. Several types of perishable raw-material items (representing various prepregs and adhesives) at the upstream level are to be procured in batches (e.g. boxes, container or packages) from suppliers. The upstream level consists of two different inventory levels: a storage location where raw-material batches can be initially stored (unopened) under special conditions so as to avoid deterioration, and a secondary storage location at the shop floor where raw-material units become available for production after a batch is opened and start deteriorating. We proposed a mixed-integer programming formulation for the problem and perform computational experiments with sensitivity analyses, demonstrating its potentials for practical applications in planning composite production.

The content of this chapter has been submitted for publication as "Multi-level lot-sizing with raw-material perishability, deterioration, and batch ordering: an application of production planning in advanced composite manufacturing" to the *Journal of Computers & Industrial Engineering* in June, 2019.

4.1 Perishability in composite manufacturing

Material performance qualification and product consistency are key requirements in aerospace, defense, marine, automotive, mass transit, and renewable energy sectors. Advanced composite materials exhibit desirable physical and chemical properties that make them widely used in these industries. Automated Tape Laying (ATL) and Automated Fibre Placement (AFP) are currently the two main technologies employed to manufacture advanced composite laminates from unidirectional *prepregs*. ATL is mainly employed to deliver wide prepreg tape onto a surface, while AFP utilises a band of narrow prepreg slices, which are collimated on the head and then delivered together [82].

Prepregs are used in high-performance applications where weight and mechanical properties take precedence over cost. Prepregs are reinforcement materials that have been pre-impregnated with either a thermoplastic or thermoset resin, hence the name *prepreg*. Due to their high sensitivity toward premature aging by cross-linking polymerization, prepreg materials are considerably perishable, affecting the production process in several ways [see 16, 17, 32]. To prevent the cross-linking polymerization reaction that takes place at room temperature, prepregs have to be kept under special freezing conditions. Refrigerated prepregs usually remain usable for three months in most cases, while certain types of prepregs can be good for up to one year, depending upon the particular resin system used. Suppliers usually assign materials a *shelf-life* representing the maximum length of time during which the material can remain outside storage (out-time) in the shop floor being suitable for use. Materials exceeding their shelf-life cannot be relied upon and must be disposed. Once these materials have been used for production they become stable and no longer deteriorate. Thus, the final products are normally considered to be non-perishable.

4.1.1 Prepreg control

Fiber-Reinforced Plastics (FRPs) are composite materials that are made of polymers and fibers. The polymers (prepregs) and the fibers are commonly glass or carbon. FRP have properties of being high-strength and light-weight so they are widely used as an advanced material in automobiles, aircraft and construction. The fibers are bonded to the epoxy resins and can have high economic value [114]. Figure 4.1 shows a general composite material constitution.

Prepreg is the common term for a reinforcing fabric which has been pre-impregnated with a resin system (typically epoxy) that already includes the proper curing agent. As a result, the prepreg is ready to lay into the mold without the addition of any more resin. In order for the laminate to cure, it is necessary to use a combination of pressure and heat.

Thermoset prepregs are produced by saturating a fiber reinforcement with a liquid thermoset resin. Once the resin components are mixed, the cure reactions are initiated

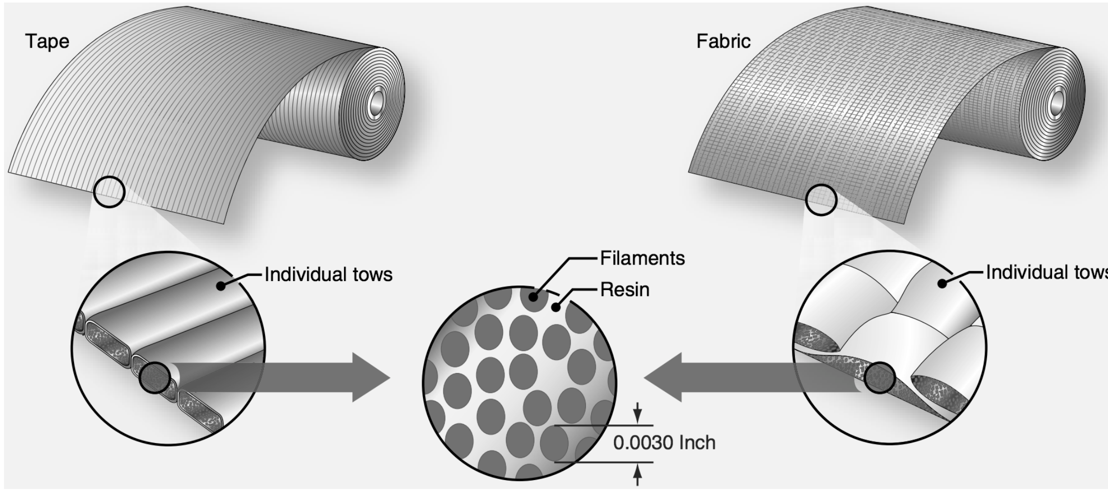


Figure 4.1: Composite material constitution [47]

and the material is no longer stable in the process of transforming from liquid to pliable solid. This is known as the "B-stage". Composite thermoset prepregs and adhesives in the B-stage must be stored in a refrigerator. After the curing process is activated with the application of heat, the material attains again a stabilized stage known as "C-stage". This creates what is called a *perishable condition*, which requires special cares, as the material will advance or age when kept at room temperature. As the resin ages at room temperature, the curing agent slowly reacts with the base resin. Several things happen when the resin advances [32]:

- there is a noticeable loss of tack that can make the plies hard to lay up,
- the prepreg becomes boardy and stiff,
- during cure, there will be less resin flow which can result in thicker than desired parts, and
- in some systems, the resin may not properly cure if taken to extreme "out-time" conditions.

This deterioration and loss of functionality in prepregs and its impact on production is the main motivation for the present study from a production planning perspective.

Prepregs are shipped from suppliers wrapped in plastic in refrigerated trucks or packed in boxes with dry ice. Once received, it is important to immediately store them in refrigerated containers, usually 0°F or lower, where the resin cure reaction is slowed down to residual levels. Freezer life is normally between 3 to 12 months from the date of manufacture, depending upon the particular resin system used. Once a roll is removed from the refrigerator and opened for use, the time of removal must be documented and the material begins its *out-time* at room temperature. During out-time, usually between 10 to 30 days, the material properties are affected by the cure reactions, eventually reaching its *shelf-life* when the material can no longer be used for production and it is scraped or disposed [89].

4.2 Problem Description

We study the *multi-item, multi-level lot-sizing problem with raw-material perishability and batch-ordering* (*MI-MLS-FVD*). Figure 4.2 shows a graphical depiction of the flow of material and the distinction between different levels and types of inventory considered in the *MI-MLS-FVD*.

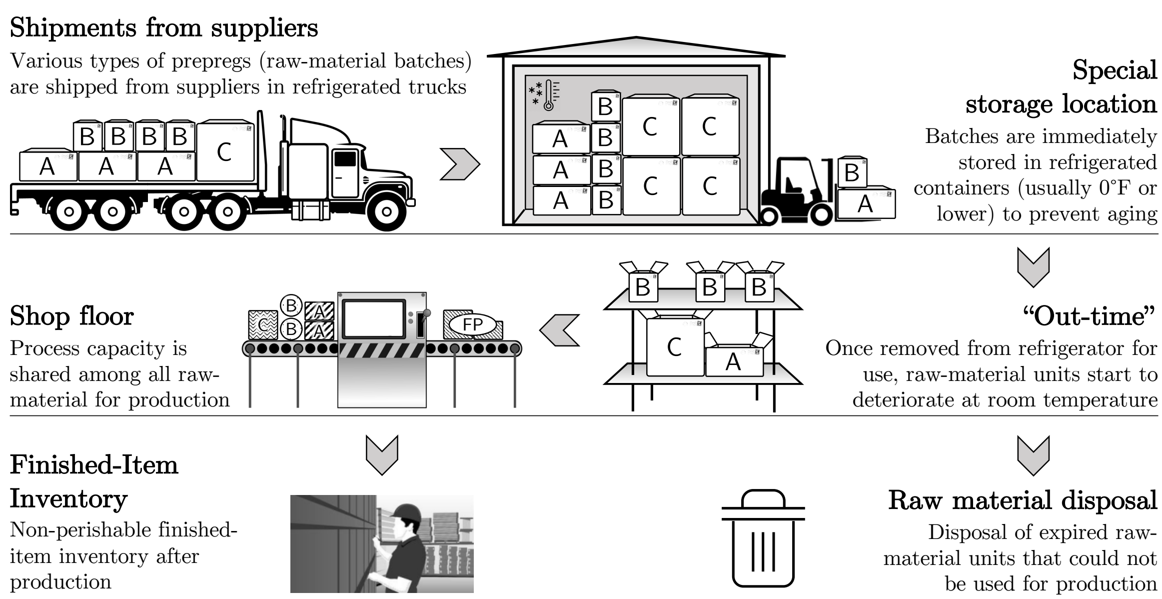


Figure 4.2: Batch-ordering, multi-level inventories, production, and disposal

Several types of perishable raw-material items are procured in batches from suppliers (e.g. prepregs packed in boxes, containers or packages). Once received at the upstream level, two different inventory sub levels are considered: a storage location where raw-material batches can be initially stored (unopened) in refrigerated containers so as to avoid deterioration, and a secondary storage location at the shop floor where raw-material units become available for production after a batch is opened and start deteriorating. As raw-material deteriorates at room temperature during *out-time*, the detriment of its functionality directly impacts production due to increased assembly costs. Additionally, the use of deteriorated material requires longer production time and greater process capacity consumption. Process capacity is considered to be shared among all raw-material items. In addition to the loss of functionality, it is often the case that the perishable nature of the raw-material involves a progressive quantity loss of on hand inventory. The prepregs outside the fridge with premature polymerization causes the material to lose not only its suitability for production, but also a portion of the initial available quantity. Once final product units are assembled they are considered to be non-perishable and can be either used to satisfy demand immediately or placed in inventory to deliver at a later time.

4.3 Model formulation

The main purpose of formulating and solving the *MI-MLS-FVD* is to determine the production, procurement, inventory, and disposal plans over a planning horizon with n time periods. We consider a total of m items including one final product and $m-1$ raw material types required for the assembly. Finished product demand d_t is to be satisfied at the end of every period $t = 1, \ldots, n$. Production incurs a fixed setup cost of q_t^1 and a basic per-unit cost of $\hat{p}_t^1$. Finished-item inventory holding cost at the end of every period is denoted by h_t. Raw-material orders are placed and delivered immediately (no order lead time assumed) in batches of size b^i for every

$i = 2, \ldots, m$, and they entail a fixed ordering cost of q_t^i and a per-batch cost of $\hat{g}_t^i$. Raw-material batches are received at the storage location where they are kept under special conditions to avoid deterioration, incurring a storage cost of f_t^i.

Once a raw-material batch is transferred to the shop floor and opened, it becomes available for production. Raw-material functionality decreases as a function of storage time (out-time) at the shop floor. The use of deteriorated but otherwise usable material causes additional assembly costs and reduces production capacity. As in Section 3.2.2, we represent this by considering the standard per-unit costs $\hat{p}_t^i$ and process capacity consumption $\hat{a}_t^i$ of item $i = 2, \ldots, m$ used for production in period t as general non-decreasing functions $P^i(\delta)$ and $\Delta^i(\delta)$, respectively, where $\delta = (t - k)$ for $1 \leq k \leq t \leq n$ and $0 \leq \delta < \beta^i$. Here, δ is the out-time from time k that a raw-material batch is opened to time t that the material is used for production. β^i represents the shelf-life of raw-material item i. From their definitions, we have $P^i(0) = \hat{p}_t^i$, $P^i(\delta) \leq P^i(\delta + 1)$, $\Delta^i(0) = \hat{a}_t^i$, and $\Delta^i(\delta) \leq \Delta^i(\delta + 1)$. Shelf-life is considered from the time that the raw-material batch is opened and represents the maximum length of time (out-time) during which the item is considered of satisfactory quality and can be stored, remaining suitable for use in whole or partially.

Using the same concept of quantity loss of on hand inventory as in Section 3.2.3, we denote $\nu^i(\delta)$ as the raw material volume deterioration function for item $i = 2, \ldots, m$, where $\delta = (t - u)$, $\nu^i(\delta) \leq \nu^i(\delta + 1)$, for $0 \leq \delta < \beta^i$ and $1 \leq u \leq t \leq n$. We further consider that perished raw material must be discarded, incurring additional disposal costs.

In formulating the *MI-MLS-FVD* model, we use the following notation:

- m $\rightarrow$ number of items (including the final product),
- n $\rightarrow$ number of time periods in planning horizon,
- d_t $\rightarrow$ final product demand per period t, $1 \leq t \leq n$,
- b^i $\rightarrow$ order batch size of item i, with $b^i > 1$, for $2 \leq i \leq m$,

r^i $\rightarrow$ units of item i required to produce a final product (bill of materials), with $r^i > 1$,

β^i $\rightarrow$ shelf-life of item i, for $2 \leq i \leq m$ (we assume no final product perishability, i.e. $\beta^1 \geq n$),

$\hat{a}_t^i$ $\rightarrow$ standard process capacity consumption per unit of item i when used for production at period t, $2 \leq i \leq m$ and $1 \leq t \leq n$,

C_t $\rightarrow$ available process capacity per period t, $1 \leq t \leq n$,

$\hat{p}_t^i$ $\rightarrow$ standard cost per unit of item i used for production at period t, $1 \leq i \leq m$ and $1 \leq t \leq n$,

h_t $\rightarrow$ final product storage cost per period t, $1 \leq t \leq n$,

$\hat{g}_t^i$ $\rightarrow$ per batch cost of item i ordered at period t, $2 \leq i \leq m$ and $1 \leq t \leq n$,

f_t^i $\rightarrow$ storage cost per batch of item i per period t, $2 \leq i \leq m$ and $1 \leq t \leq n$,

γ_t^i $\rightarrow$ storage cost (at shop floor) per unit of item i per period t, $2 \leq i \leq m$ and $1 \leq t \leq n$,

ϕ_t^i $\rightarrow$ disposal cost per unit of item i per period t, $2 \leq i \leq m$ and $1 \leq t \leq n$,

q_t^i $\rightarrow$ fixed set up or ordering cost of item i per period t, $1 \leq i \leq m$ and $1 \leq t \leq n$,

δ $\rightarrow$ out-time duration from time k that a raw-material batch is opened to time t that the material is used for production, $\delta = (t - k)$ for $1 \leq k \leq t \leq n$, and $0 \leq \delta, < \beta^i$,

$P^i(\delta)$ $\rightarrow$ production cost function for item i, $2 \leq i \leq m$,

$\Delta^i(\delta)$ $\rightarrow$ process capacity consumption function for item i, $2 \leq i \leq m$,

$\nu^i(\delta)$ $\rightarrow$ raw material volume deterioration function for item i, $2 \leq i \leq m$.

In terms of decision variables, we have the following:

x_{tl} $\rightarrow$ production of final product at period t to satisfy demand of period l, $1 \leq t \leq l \leq n$,

s_t $\rightarrow$ final product inventory at the end of period t, $1 \leq t \leq n$,

Q^i_{uk} $\rightarrow$ number batches of item i, ordered at period u to open at period k, $2 \leq i \leq m$, $1 \leq u \leq k \leq n$,

w^i_{ukt} $\rightarrow$ amount of item i, from a batch received at period u and opened at period k to use for production at period t, $1 \leq u \leq k \leq t \leq n$ and $0 \leq (t-k) < \beta^i$,

c^i_{ukt} $\rightarrow$ inventory of item i at the end of period t, from a batch received at period u and opened at period k, $2 \leq i \leq m$, $1 \leq u \leq k \leq t \leq n$, and $0 \leq (t-k) < \beta^i$,

e^i_{ukt} $\rightarrow$ amount of perished item i discarded at period t, from a batch received at period u and opened at period k, $2 \leq i \leq m$, $1 \leq u \leq k \leq t \leq n$, and $0 \leq (t-k) < \beta^i$,

y^i_t $\rightarrow$ binary variable equal to 1 if and only if an order of item i is placed ($2 \leq i \leq m$), or there is final product production ($i = 1$), in period t, $1 \leq t \leq n$.

Using these sets of decisions variables, the *MLS-FVD* can be formulated as the following mixed-integer program (MIP):

$$\begin{aligned}\text{minimize} \quad & \sum_{t=1}^{n}\left(\sum_{l=t}^{n}\hat{p}^1_t x_{tl} + h_t s_t\right) + \sum_{i=2}^{m}\sum_{u=1}^{n}\sum_{k=u}^{\Theta^i_u}\sum_{t=k}^{n} p^i_{kt} w^i_{ukt} + \sum_{i=1}^{m}\sum_{t=1}^{n} q^i_t y^i_t \\ & + \sum_{i=2}^{m}\sum_{u=1}^{n}\sum_{k=u}^{n}\left(g^i_{uk} Q^i_{uk} + \sum_{t=k}^{\Theta^i_k}\left(\gamma^i_t c^i_{ukt} + \phi^i_t e^i_{ukt}\right)\right) \quad (4.1)\end{aligned}$$

$$\text{subject to } \sum_{t=1}^{l} x_{tl} = d_l, \qquad 1 \le l \le n \tag{4.2}$$

$$\sum_{u=1}^{t} \sum_{k=\Pi_{ut}^i}^{t} w_{ukt}^i = r^i \sum_{l=t}^{n} x_{tl}, \qquad 2 \le i \le m,\ 1 \le t \le n \tag{4.3}$$

$$c_{ukk}^i = \left(b^i Q_{uk}^i - w_{ukk}^i\right)\left(1 - \nu^i(0)\right), \quad 2 \le i \le m, 1 \le u \le k \le n \tag{4.4}$$

$$c_{ukt}^i = \left(c_{uk,t-1}^i - w_{ukt}^i\right)\left(1 - \nu^i(\delta)\right),$$

$$2 \le i \le m, 1 \le u \le k < t \le n : \delta < \beta^i \tag{4.5}$$

$$e_{ukk}^i = \left(b^i Q_{uk}^i - w_{ukk}^i\right)\left(\nu^i(0)\right), \qquad 2 \le i \le m, 1 \le u \le k \le n \tag{4.6}$$

$$e_{ukt}^i = \left(c_{uk,t-1}^i - w_{ukt}^i\right)\left(\nu^i(\delta)\right),$$

$$2 \le i \le m, 1 \le u \le k < t \le n : \delta < \beta^i \tag{4.7}$$

$$\sum_{i=2}^{m} \sum_{u=1}^{t} \sum_{k=\Pi_{ut}^i}^{t} a_{kt}^i w_{ukt}^i \le C_t y_t^1, \qquad 1 \le t \le n \tag{4.8}$$

$$\sum_{l=t}^{n} x_{tl} \le M_t y_t^1, \qquad 1 \le t \le n \tag{4.9}$$

$$\sum_{k=u}^{n} Q_{uk}^i \le K_u^i y_u^i, \qquad 2 \le i \le m, 1 \le u \le n \tag{4.10}$$

$$s_t = \sum_{l=t+1}^{n} x_{tl}, \qquad 1 \le t \le n \tag{4.11}$$

$$x_{tl},\ s_t \in \mathbb{R}^+, \qquad 1 \le t \le l \le n \tag{4.12}$$

$$Q_{uk}^i \in \mathbb{N}, \qquad 2 \le i \le m, 1 \le u \le k \le n \tag{4.13}$$

$$w_{ukt}^i,\ c_{ukt}^i,\ e_{ukt}^i \in \mathbb{R}^+, \qquad 2 \le i \le m, 1 \le u \le k \le t \le n \tag{4.14}$$

$$y_t^i \in \{0, 1\}, \qquad 1 \le i \le m,\ 1 \le t \le n \tag{4.15}$$

where $p_{kt}^i = \begin{cases} \frac{\hat{p}_t^i}{r^i} & \text{if } k = t, \\ \frac{P^i(\delta)}{r^i} & \text{if } k < t, \end{cases}$ $g_{uk}^i = \begin{cases} \hat{g}_u^i & \text{if } u = k, \\ \hat{g}_u^i + \sum_{t=u}^{k-1} f_t^i & \text{if } u < k, \end{cases}$ and $a_{kt}^i = \begin{cases} \frac{\hat{a}_t^i}{r^i} & \text{if } k = t, \\ \frac{\Delta^i(\delta)}{r^i} & \text{if } k < t. \end{cases}$

Alternatively, we can calculate the raw material batch inventory as $S_k^i = \sum_{k=u+1}^{n} Q_{uk}$ for $2 \le i \le m$ and $1 \le k \le n$.

The objective function (4.1) includes: final product production and inventory holding costs, raw material production related costs, fixed set-up and ordering costs, raw material batch and special inventory holding costs, raw-material (at shop floor) unit inventory holding costs, and disposal costs. Constraints (4.2) make sure that the amounts x_{tl} produced in every period $t = 1, \ldots, l$ are equal to the final product demand d_l at period $l = 1, \ldots, n$. Constraints (4.3) represent the link between the raw material production related variables w^i_{ukt} and the final product production variables x_{tl} for raw material item $i = 2, \ldots, m$ and period $t = 1, \ldots, n$, using the bill of materials parameter r^i. Constraints (4.4) and (4.5) represent raw material inventory levels c^i_{ukt} at the shop floor for raw material item $i = 2, \ldots, m$, and recursively calculate the number of Q^i_{ukt} batches opened at period $k = 1, \ldots, n$, whereas constraints (4.6) and (4.7) compute the raw material disposal units e^i_{ukt}. Constraints (4.8) set the capacity limits for the production process. Constraints (4.9) force the set-up binary variable to be 1 if production takes place ($x_{tl} > 0$) at $t = 1, \ldots, n$. Constraints (4.10) are the ordering binary variables and equal to 1 if a raw material batch order is placed ($Q^i_{uk} > 0$) at $u = 1, \ldots, n$. Constraints (4.11) calculate the final product inventory levels s_t at period $t = 1, \ldots, n$. Finally, constraints (4.12) – (4.15) are the non-negativity and integrality conditions.

4.4 Numerical examples

In this section, we first present a small example instance with detailed data and solution to illustrate the application of the proposed model. Several mid-sized example problems are then presented to further illustrate other aspects of the model such as its performance, solution differences with various data sets and required computational time.

4.4.1 A small problem instance

We first illustrate the problem definition and validate our *MI-MLS-FVD* formulation using the following problem instance. We consider a seven time-period instance ($n = 7$), with one final product and two raw material types required for assembly ($m = 3$). Finished product demand per period is assumed to be $d = (178, 67, 119, 72, 50, 193, 91)$, order batch sizes for each raw material type are $b = (30, 60)$, and their shelf-lives values are $\beta = (3, 2)$. The amount of units of item i required to produce a final product are $r = (4, 2)$.

Capacity consumption, production costs, and raw material volume deterioration functions We assume the same standard capacity consumption per unit for the two raw material types to be $\hat{a} = 2.5$. Since the rate of material deterioration is different for each type, the increased capacity consumption process of items $i = 2, 3$ is represented by two piecewise non-decreasing linear functions $\Delta^2(\delta)$ and $\Delta^3(\delta)$, where $\delta = (t - k)$ for $1 \leq k \leq t \leq n$ is the raw-material out-time and $0 \leq \delta < \beta^i$. In this way, since $\beta = (3, 2)$ for $i = 2, 3$, then $\Delta^2(\delta) = (2.5, 3.333, 4.167)$ for $0 \leq \delta < 3$, and $\Delta^3(\delta) = (2.50, 3.75)$, for $0 \leq \delta < 2$. Similarly, but with a different standard per-unit production cost $\hat{p} = (12, 7)$ for $i = 2, 3$ and the increased assembly costs functions to be $P^2(\delta) = (12, 16.5, 20)$ for $0 \leq \delta < 3$, and $P^3(\delta) = (7, 10.5)$, for $0 \leq \delta < 2$. In terms of raw material volume deterioration, we assume the functions $\nu^2(\delta) = \{0.333, 0.667, 1.0\}$ for $0 \leq \delta < 3$, and $\nu^3(\delta) = \{0.5, 1.0\}$ for $0 \leq \delta < 2$.

Storage, disposal, and fixed set up and ordering costs Table 4.1 shows the the different relevant costs parameters for each item.

Problem Instance Solution Tables 4.2, 4.3, and 4.4 show the *MI-MLS-FVD* optimal solution in detail for the small problem instance. In terms of final item production

Table 4.1: Costs parameters for the small problem instance

Item (i):	1	2	3
$\hat{p}_t^i$, standard per unit production	10	12	7
$\hat{g}_t^i$, per batch ordered	n/a	30	60
f_t^i, per batch storage	n/a	42.96	93
h_t, γ_t^i, per unit storage	5	1.19	0.67
ϕ_t^i, per unit disposal	n/a	4.36	1.13
q_t^i, fixed set up / ordering	3,000	2,764	2,691

and inventory levels, the optimal plan is to produce in periods $t = \{1, 2, 5, 6\}$, resulting in an average inventory level of 90.75 units for the first six periods. As shown in Table 4.2, this represents a total set-up cost of \$12.000, a total final product production cost of \$7.700, and a total final product inventory holding cost of \$2,722.5, adding up to a total of \$22,422.5.

Table 4.2: Optimal values for final product related variables

Time-periods			1	2	3	4	5	6	7	Totals	Associated costs
Section 2.1 *Binary set-up variable at period t*											
y_t^i	$i =$	1	1	1	0	0	1	1	0	**4.0**	**12,000.0**
Section 2.2 *Production of final product at period t to satisfy demand of period l*											
x_{tl}	$t =$	1	178	0	18	0	35	6.5	0	**237.5**	
		2	-	67	101	72	0	0	0	**240.0**	
		5	-	-	-	-	15	0	0	**15.0**	
		6	-	-	-	-	-	186.5	91	**277.5**	
Totals			**178**	**67**	**119**	**72**	**50**	**193**	**91**	**770.0**	**7,700.0**
Section 2.3 *Finished product inventory at the end of period t*											
s_t			59.5	232.5	113.5	41.5	6.5	91	0	**544.5**	**2,722.5**

The procurement plan and special storage decisions for the two raw material types are presented in Table 4.3. It consists of placing and receiving orders in periods $t = \{1, 6\}$ for item $i = 2$ and in periods $t = \{1, 2, 5, 6\}$ for item $i = 3$, respectively. In total, 103 and 26 batches of items 2 and 3 are procured, containing a total of 3,090 ($b^2 = 30$) and 1,560 ($b^3 = 60$) raw material units, respectively. These batch orders are used and inventoried as shown in Sections 3.2 to 3.5 of Table 4.3. Out of the 66 batches of item 2 received in period $u = 1$, 32 batches are opened for production instantaneously (same period they are received), and 34 are kept in stock (unopened). Afterwards, 32 batches are opened in period 2, and the remaining two batches are kept in stock until the end of period 4. They are then opened at period 5. For item 3, batches received at periods 1, 2 and 6 are opened instantaneously. While out of the two batches received at period 5, one is opened and the other is kept in stock until the next period. The total cost with fixed ordering, per-batch ordered, and per batch storage costs associated with these orders is $\$41,353.4$

Table 4.3: Optimal values for raw material batch procurement and inventory variables

Time-periods			1	2	3	4	5	6	7	**Totals**	**Associated costs**
Section 3.1 *Binary ordering variable at period t*											
y_t^i	$i =$	2	1	0	0	0	0	1	0	**2.0**	**5,528.0**
		3	1	1	0	0	1	1	0	**4.0**	**10,764.0**
Section 3.2 *Number of batches of item 2, ordered at period u, to open at period k*											
Q_{uk}^2	$u =$	1	32	32	0	0	2	0	0	**66.0**	
		6	-	-	-	-	-	37	0	**37.0**	
Totals =			**32**	**32**	**0**	**0**	**2**	**37**	**0**	**103.0**	**15,450.0**
Section 3.3 *Number of batches of item 3, ordered at period u, to open at period k*											
Q_{uk}^3	$u =$	1	8	0	0	0	0	0	0	**8.0**	
		2	-	8	0	0	0	0	0	**8.0**	
		5	-	-	-	-	1	1	0	**2.0**	
		6	-	-	-	-	-	8	0	**8.0**	
Totals =			**8**	**8**	**0**	**0**	**1**	**9**	**0**	**26.0**	**7,800.0**
Section 3.4 *Amount of raw material batch inventory of item 2 at the end of period t*											
S_t^2			**34**	**2**	**2**	**2**	**0**	**0**	**0**	**40.0**	**1,718.4**
Section 3.5 *Amount of raw material batch inventory of item 3 at the end of period t*											
S_t^3			**0**	**0**	**0**	**0**	**1**	**0**	**0**	**1.0**	**93.0**

Regarding the activity at the shop floor, Table 4.4 shows how raw material units are used for production, kept in stock (out-time), and disposed.

Section 4.1, 4.3, and 4.5 refer to item $i = 2$. From the 960 units (32 batches $\times$ 30 units/batch) of item $i = 2$ that are opened at period 1, 950 are used for production instantaneously (same period they are opened). As the remaining 10 units start their out-time at the shop floor, they progressively perish and are all disposed. The pace at which they are disposed ($e^2_{11t} = \{3.33, 4.449, 2.221\}$ for $1 \leq t \leq 3$) is determined by the volume deterioration function $\nu^2(\delta)$. All 960, 60, and 1,110 units opened at periods 2, 5, and 6, respectively, are used for production instantaneously.

Sections 4.2, 4.4, and 4.6 in Table 4.4 refer to item $i = 3$. From the 480 units (8 batches $\times$ 60 units/batch) of item $i = 3$ that are opened at period 1, 475 of them are used for production instantaneously. As the remaining 5 units start their out-time, they progressively perish and are all disposed. Out the 60 units opened at period 5, 30 of them are used for production instantaneously, 15 are used at period 6, and the remaining 15 become unusable and are disposed. The pace at which these raw material units are disposed ($e^3_{11t} = \{2.5, 2.5\}$ for $1 \leq t \leq 2$ and $e^3_{555} = 15$) corresponds to the deterioration function $\nu^3(\delta)$.

This activity at the shop floor results in $\$47,792.5$ of raw material cost of production, and \$22.3 and \$66.2 in raw material inventory and disposal costs, respectively.

The total cost of the optimal solution of this example problem is $\$111,656.92$. Finished product related costs account for $\$22,422.4$ (20.1% of total costs), raw material related costs account for $\$41,353.4$ (37% of total costs), and raw material costs due to material perishing and disposal at the shop floor account for $\$47,881$ (42.9% of total costs). Table 4.5 shows a summary of these costs.

4.4.2 Larger size problems and numerical experiments

In addition to the example instance presented above, we use several more data sets to further test the *MI-MLS-FVD* model and to demonstrate its potentials for practical applications in planning composite production. Here, we present optimal solutions based on four data sets with different problem sizes and characteristics. Each data set consists of eight problem instances with 8, 10, 12, and 14 time periods (n) in the planning horizon, respectively. All four data sets consists of problems with $m =$

Table 4.4: Optimal values for raw material usage in the shop floor and disposal variables

Time-periods			1	2	3	4	5	6	7	Totals	Associated costs
Section 4.1 *Amount of item 2, from a batch received at u and opened at k, used for production at t*											
w^2_{1kt}	$k =$	1	950	0	0	-	-	-	-	**950.0**	**11,400.0**
		2	-	960	0	0	-	-	-	**960.0**	**11,520.0**
		5	-	-	-	-	60	0	0	**60.0**	**720.0**
w^2_{6kt}		6	-	-	-	-	-	1,110	0	**1,110.0**	**13,320.0**
Totals =			**950**	**960**	**0**	**0**	**60**	**1,110**	**0**	**3,080.0**	**36,960.0**
Section 4.2 *Amount of item 3, from a batch received at u and opened at k, used for production at t*											
w^3_{1kt}	$k =$	1	475	0	-	-	-	-	-	**475.0**	**3,325.0**
w^3_{2kt}		2	-	480	0	-	-	-	-	**480.0**	**3,360.0**
w^3_{5kt}		5	-	-	-	-	30	15	-	**45.0**	**367.5**
		6	-	-	-	-	-	60	0	**60.0**	**420.0**
w^3_{6kt}		6	-	-	-	-	-	480	0	**480.0**	**3,360.0**
Totals =			**475**	**480**	**0**	**0**	**30**	**555**	**0**	**1,540.0**	**10,832.5**
Section 4.3 *Inventory of item 2 at the end of period t, from a batch received at u and opened at k*											
c^2_{1kt}	$k =$	1	6.67	2.221	0	-	-	-	-	**8.89**	**10.6**
Section 4.4 *Inventory of item 3 at the end of period t, from a batch received at u and opened at k*											
c^3_{1kt}	$k =$	1	2.5	0	-	-	-	-	-	**2.5**	
c^3_{5kt}		5	-	-	-	-	15	0	-	**15.0**	
Totals =			**2.5**	**0**	**0**	**0**	**15**	**0**	**0**	**17.5**	**11.7**
Section 4.5 *Amount of perished item 2 discarded at t, from a batch received at u and opened at k*											
e^2_{1kt}	$k =$	1	3.33	4.449	2.221	-	-	-	-	**10.0**	**43.6**
Section 4.6 *Amount of perished item 3 discarded at t, from a batch received at u and opened at k*											
e^3_{1kt}	$k =$	1	2.5	2.5	-	-	-	-	-	**5.0**	
e^3_{5kt}		5	-	-	-	-	15	0	-	**15.0**	
Totals =			**2.5**	**2.5**	**0**	**0**	**15**	**0**	**0**	**20.0**	**22.6**

Table 4.5: Summary of optimal solution costs

Final product related costs	**22,422.5**	**20.1%**
Fixed Set-up costs	12,000.0	
Final product production costs	7,700.0	
Final product inventory costs	2,722.5	
Raw material batch related costs	**41,353.4**	**37.0%**
Fixed order placement costs	16,292.0	
Raw material batch costs	23,250.0	
Raw material batch special inventory costs	1,811.4	
Raw material (at shop floor) related costs	**47,881.0**	**42.9%**
Raw material production related costs	47,792.5	
Shop floor inventory costs	22.3	
Raw material disposal costs	66.2	
Total costs	**111,656.9**	**100%**

$\{6, 9, 12, 15\}$ items, including the final product. Depending on m, the batch-size (b^i) parameter for $2 \leq i \leq m$ was set a priori, as follows: for instances with $m = 6$, $b = \{20, 40, \ldots, 100\}$; for instances with $m = 9$, $b = \{20, 40, \ldots, 160\}$; for instances with $m = 12$, $b = \{20, 40, \ldots, 220\}$; for instances with $m = 15$, $b = \{20, 40, \ldots, 280\}$.

Depending on the problem sizes, the shelf-life values β^i parameter for each $2 \leq i \leq m$ was randomly generated using a discrete uniform distribution, as follows: for instances with $n = 8$, $\beta \sim \mathcal{U}\{2, 6\}$; for instances with $n = 10$, $\beta \sim \mathcal{U}\{2, 8\}$; for instances with $n = 12$, $\beta \sim \mathcal{U}\{2, 10\}$; for instances with $n = 14$, $\beta \sim \mathcal{U}\{2, 12\}$.

One of the main purposes of these numerical experiments is to compare the structure of the optimal solution depending on n and m, as well as some other key cost parameters. Thus, we introduced more noticeable differences between each pair of instances for set up, inventory holding and disposal costs parameters (q_t^1, h_t, and ϕ_t^i), due to the fact that, based on preliminary experimentation, we found that they are the most fundamental parameters for important changes in the structure of the optimal solutions. Each instance has a "lower cost level" version called "*nxm* A", and a "higher cost level" version called "nxm B" for these three costs. The remaining parameters were randomly generated using continuous and discrete uniform distributions integrating variability in a controlled manner.

The proposed model formulation was implemented using the Callable Library of IBM CPLEX 12.7.0. All tests were carried out on an Intel(R) Xeon(R) CPU E3-1270 v3 processor with 3.50GHz and 24GB of RAM memory and Microsoft Windows 7 Enterprise operating system. Time limit was set to two hours.

Table 4.6 shows computational results for the first two data sets. The first column shows the name of the instance, which also corresponds to its size (nxm) and version ("A", or "B"). The second column shows the computational time, which in the majority of the cases reached the two hours limit. The last three columns show the upper and lower bounds reached, and the optimality gaps. In this table, one can see that the optimality gaps have an increasing tendency as the size of the instance gets larger. The average optimality gap within the two hour limit for instances with $n = 8$ was 2.2%, and for instances with $n = 10$ was 4.8%.

When comparing the instances by the pairs A vs. B (see Table 4.7), we observe that, for instances with fewer items, the lower level cost of A instances tend to have lower optimal solution bounds. When the number of items becomes larger, this tendency is no longer present. The optimality gap, however, except for one case, was always larger for the higher level costs of B instances. The average optimality gaps for A and B instances are similar, less then 3.6%.

Table 4.8 shows the different costs corresponding to the first two sets of instances. Similar to those shown in the small example problem in Section 4.4.1 these costs are divided into the following three categories:

- Final product related costs
  - Set up: fixed set up costs $\sum_{t=1}^{n} q_t^1 y_t^1$,
  - FP Pcc: final product production costs $\sum_{t=1}^{n} \sum_{l=t}^{n} \hat{p}_t^1 x_{tl}$,

Table 4.6: Results for instances with $n = \{8, 10\}$ grouped by size.

nxm	**Time (s)**	**UB**	**LB**	**Gap (%)**
8x6 A	27.2	486,403.1	486,403.1	0.0
8x6 B	22.7	492,086.3	492,086.3	0.0
8x9 A	limit	607,681.2	603,178.1	0.7
8x9 B	limit	651,197.1	645,827.9	0.8
8x12 A	limit	869,025.2	847,438.7	2.5
8x12 B	limit	925,293.6	897,009.8	3.1
8x15 A	limit	1,006,592.3	954,503.5	5.2
8x15 B	limit	907,918.6	860,012.9	5.3
				2.2
10X6 A	1634.7	390,091.6	390,091.6	0.0
10X6 B	449.3	614,301.3	614,301.3	0.0
10X9 A	limit	821,417.8	782,231.9	4.8
10X9 B	limit	658,959.0	635,338.2	3.6
10X12 A	limit	1,231,766.7	1,135,279.7	7.8
10X12 B	limit	1,117,245.4	1,023,071.6	8.4
10X15 A	limit	1,647,093.4	1,540,101.6	6.5
10X15 B	limit	1,451,398.0	1,349,587.1	7.0
				4.8

Table 4.7: Results for instances with $n = \{8, 10\}$ grouped by type A and B.

nxm	**Time (s)**	**UB**	**LB**	**Gap (%)**
8x6 A	27.2	486,403.1	486,403.1	0.0
8x9 A	limit	607,681.2	603,178.1	0.7
8x12 A	limit	869,025.2	847,438.7	2.5
8x15 A	limit	1,006,592.3	954,503.5	5.2
10X6 A	1634.7	390,091.6	390,091.6	0.0
10X9 A	limit	821,417.8	782,231.9	4.8
10X12 A	limit	1,231,766.7	1,135,279.7	7.8
10X15 A	limit	1,647,093.4	1,540,101.6	6.5
				3.4
8x6 B	22.7	492,086.3	492,086.3	0.0
8x9 B	limit	651,197.1	645,827.9	0.8
8x12 B	limit	925,293.6	897,009.8	3.1
8x15 B	limit	907,918.6	860,012.9	5.3
10X6 B	449.3	614,301.3	614,301.3	0.0
10X9 B	limit	658,959.0	635,338.2	3.6
10X12 B	limit	1,117,245.4	1,023,071.6	8.4
10X15 B	limit	1,451,398.0	1,349,587.1	7.0
				3.5

- FP Inv: final product inventory costs $\sum_{t=1}^{n} h_t s_t$,

- Raw material batch related costs

  - Order: fixed ordering costs $\sum_{i=2}^{m} \sum_{t=1}^{n} q_t^i y_t^i$,
  - Batch: raw material per batch costs $\sum_{i=2}^{m} \sum_{t=1}^{n} \sum_{k=t}^{n} \hat{g}_t^i Q_{tk}^i$
  - Batch Inv: raw material special inventory costs $\sum_{i=2}^{m} \sum_{t=1}^{n} \sum_{k=t+1}^{n} f_k^i Q_{tk}^i$

- Raw material (at shop floor) related costs

  - RM Pcc: raw material production related costs $\sum_{i=2}^{m} \sum_{u=1}^{n} \sum_{k=u}^{\Theta_u^i} \sum_{t=k}^{n} p_{kt}^i w_{ukt}^i$, where $p_{kt}^i = \hat{p}_t^i / r^i$ if $k = t$, and $p_{kt}^i = P^i(\delta)/r^i$ if $k < t$.
  - RM Inv: raw material unit inventory (at shop floor) costs $\sum_{i=2}^{m} \sum_{u=1}^{n} \sum_{k=u}^{n} \sum_{t=k}^{\Theta_k^i} \gamma_t^i c_{ukt}^i$,
  - Disposal: raw material disposal costs $\sum_{i=2}^{m} \sum_{u=1}^{n} \sum_{k=u}^{n} \sum_{t=k}^{\Theta_k^i} \phi_t^i e_{ukt}^i$.

Table 4.8: Optimal cost values for instances with $n = \{8, 10\}$

	Final product related costs					**Raw material batch related costs**					**Raw material (at shop floor) related costs**				
nxm	**Set up**	**FP Pcc**	**FP Inv**	**Total**	**%**	**Order**	**Batch**	**Batch Inv**	**Total**	**%**	**RM Pcc**	**RM Inv**	**Disposal**	**Total**	**%**
8x6 A	16,000	17,990	4,241.3	38,231.3	7.9	25,666	93,680	6,504	125,850	25.9	322,038.5	93.8	189.5	322,321.8	66.3
8x6 B	20,000	16,850	8,180.2	45,030.2	9.2	37,611	62,840	6,000	106,451	21.6	337,815.0	816.4	1,973.7	340,605.1	69.2
8x9 A	16,000	16,820	2,998.0	35,818.0	5.9	51,818	90,020	5,892	147,730	24.3	417,180.7	2,151.2	4,801.4	424,133.2	69.8
8x9 B	20,000	17,330	10,670.0	48,000.0	7.4	74,024	118,520	10,120	202,664	31.1	395,180.0	956.5	4,396.6	400,533.1	61.5
8x12 A	16,000	17,110	6,702.0	39,812.0	4.6	70,571	131,560	8,700	210,831	24.3	607,811.4	3,136.1	7,434.7	618,382.2	71.2
8x12 B	20,000	18,150	19,730.0	57,880.0	6.3	98,656	146,320	5,460	250,436	27.1	605,306.2	2,817.7	8,853.8	616,977.6	66.7
8x15 A	16,000	16,860	4,040.7	36,900.7	3.7	93,115	175,480	6,420	275,015	27.3	676,954.1	5,304.9	12,417.6	694,676.6	69.0
8x15 B	20,000	15,140	6,486.3	41,626.3	4.6	129,540	161,200	8,860	299,600	33.0	554,424.0	2,061.5	10,206.8	566,692.3	62.4
	18,000	**17,031.3**	**7,881.1**	**42,912.3**	**6.2**	**72,625.1**	**122,452.5**	**7,244.5**	**202,322.1**	**26.8**	**489,588.7**	**2,167.3**	**6,284.3**	**498,040.2**	**67.0**
10X6 A	20,000	20,710	4,693.1	45,403.1	11.6	40,710	70,860	5,976	117,546	30.1	225,923.4	477.0	742.1	227,142.5	58.2
10X6 B	25,000	22,080	6,730.0	53,810.0	8.8	62,985	130,640	0	193,625	31.5	364,371.9	582.9	1,911.5	366,866.3	59.7
10X9 A	24,000	24,790	2,380.0	51,170.0	6.2	93,846	129,860	7,344	231,050	28.1	533,453.8	2,150.9	3,593.2	539,197.8	65.6
10X9 B	25,000	20,540	7,813.3	53,353.3	8.1	75,854	124,380	5,240	205,474	31.2	388,603.1	2,496.5	9,032.1	400,131.7	60.7
10X12 A	24,000	23,650	4,901.4	52,551.4	4.3	132,502	208,480	10,308	351,290	28.5	812,241.3	5,492.4	10,191.7	827,925.3	67.2
10X12 B	30,000	24,770	17,479.7	72,249.7	6.5	135,150	178,260	11,860	325,270	29.1	699,804.6	4,057.4	15,863.7	719,725.7	64.4
10X15 A	24,000	24,940	4,846.9	53,786.9	3.3	153,684	337,840	8,040	499,564	30.3	1,073,872.5	6,394.2	13,475.8	1,093,742.5	66.4
10X15 B	25,000	22,270	19,232.5	66,502.5	4.6	125,400	242,100	18,040	385,540	26.6	977,814.6	5,373.7	16,167.2	999,355.5	68.9
	24,625	**22,968.8**	**8,509.6**	**56,103.4**	**6.7**	**102,516.4**	**177,802.5**	**8,351.0**	**288,669.9**	**29.4**	**634,510.7**	**3,378.1**	**8,872.1**	**646,760.9**	**63.9**

Table 4.8 shows the raw material associated costs take major portions of the total costs in these instances. On average, for all instances, these costs add up to 93.45% of the total costs. This was to be expected, as we are considering multi-item structures with only one final product and up to 15 types of raw material items. From Table 4.8, we can also observe that, as the size of the instances increases in terms of the number of items m, the final product related costs proportion is reduced. This also corresponds to an increase in the costs associated with raw material batch and raw material handling at the shop floor (last two sets of columns in Table 4.8).

Notably, instances with $m = \{6, 9\}$ averaged 91.9% for raw material related costs, while instances with $m = \{12, 15\}$ average 95.3% for the same cost category. This is because as the instances have a higher number of raw material types considered to produce the one final product, there will be more costs associated with the ordering, storage, handling (at shop floor), and disposal of there items.

When the instance size increases with respect to the planning horizon from $n = 8$ to $n = 10$, the final product related costs proportion also increases, as well as the raw material batch related costs. In this case, only the costs associated with raw material handling at the shop floor decrease.

One of the most important aspects to observe in the cost structure presented in the Table 4.8 and later in Table 4.11 is that, with respect to all other costs, those associated with raw material disposal are relatively low. This is specially informative because it constitutes one of the main values of the proposed model. In previous computational experiments, we adapted the optimal solutions obtained with classic lot sizing models to the studied instances, and the raw material disposal levels were considerably higher.

Table 4.9 shows the computational results for the second data sets. We can see that the tendency for the optimality gaps to increase with the instance sizes is also present. On average, the optimality gap within the two hour limit for instances with $n = 12$ was 7.5%, and for instances with $n = 14$ was 8.7%. When comparing the instances by pairs A vs. B (see Table 4.10), we observe that, for instances with $m = 6$ and $m = 9$, the lower cost level A instances tend to have lower optimal solution bounds. This tendency is no longer present for instances with larger numbers of items. As with the first data sets, except for one case, the optimality gap was always larger for the higher costs level B instances. The lower cost level A instances resulted in an average optimality gap of 6.7%, while that for the B instances is 9.5%.

Table 4.11 shows the different costs corresponding to the last two sets of instances.

Table 4.9: Results for instances with $n = \{12, 14\}$ grouped by size.

nxm	**Time (s)**	**UB**	**LB**	**Gap (%)**
12X6 A	limit	511,633.3	506,448.9	1.0
12X6 B	limit	740,685.9	707,379.3	4.5
12X9 A	limit	1,040,811.2	1,005,228.4	3.4
12X9 B	limit	1,086,433.1	991,328.3	8.8
12X12 A	limit	1,606,875.4	1,504,864.7	6.3
12X12 B	limit	1,249,973.1	1,083,615.3	13.3
12X15 A	limit	1,645,667.2	1,478,286.3	10.2
12X15 B	limit	1,557,413.6	1,367,853.5	12.2
				7.5
14X6 A	limit	722,331.1	701,709.6	2.9
14X6 B	limit	769,199.3	733,757.1	4.6
14X9 A	limit	960,330.1	856,638.5	10.8
14X9 B	limit	1,042,533.5	961,903.6	7.7
14X12 A	limit	1,745,341.0	1,575,429.4	9.7
14X12 B	limit	1,557,248.5	1,349,720.9	13.3
14X15 A	limit	1,888,661.6	1,711,459.1	9.4
14X15 B	limit	1,912,862.7	1,691,936.8	11.5
				8.7

Table 4.10: Results for instances with $n = \{12, 14\}$ grouped by type A and B.

nxm	**Time (s)**	**UB**	**LB**	**Gap (%)**
12X6 A	limit	511,633.3	506,448.9	1.0
12X9 A	limit	1,040,811.2	1,005,228.4	3.4
12X12 A	limit	1,606,875.4	1,504,864.7	6.3
12X15 A	limit	1,645,667.2	1,478,286.3	10.2
14X6 A	limit	722,331.1	701,709.6	2.9
14X9 A	limit	960,330.1	856,638.5	10.8
14X12 A	limit	1,745,341.0	1,575,429.4	9.7
14X15 A	limit	1,888,661.6	1,711,459.1	9.4
				6.7
12X6 B	limit	740,685.9	707,379.3	4.5
12X9 B	limit	1,086,433.1	991,328.3	8.8
12X12 B	limit	1,249,973.1	1,083,615.3	13.3
12X15 B	limit	1,557,413.6	1,367,853.5	12.2
14X6 B	limit	769,199.3	733,757.1	4.6
14X9 B	limit	1,042,533.5	961,903.6	7.7
14X12 B	limit	1,557,248.5	1,349,720.9	13.3
14X15 B	limit	1,912,862.7	1,691,936.8	11.5
				9.5

Table 4.11: Optimal cost values for instances with $n = \{12, 14\}$

	Final product related costs					Raw material batch related costs					Raw material (at shop floor) related costs				
nxm	**Set up**	**FP Pcc**	**FP Inv**	**Total**	**%**	**Order**	**Batch**	**Batch Inv**	**Total**	**%**	**RM Pcc**	**RM Inv**	**Disposal**	**Total**	**%**
12X6 A	24,000	26,740	6,398.0	57,138.0	11.2	39,696	75,040	7,128	121,864	23.8	331,576.0	337.1	718.3	332,631.3	65.0
12X6 B	35,000	29,180	10,111.7	74,291.7	10.0	97,175	134,440	9,000	240,615	32.5	423,394.2	657.2	1,727.8	425,779.2	57.5
12X9 A	24,000	26,580	6,115.9	56,695.9	5.4	79,335	206,020	10,944	296,299	28.5	681,061.1	2,123.0	4,632.2	687,816.3	66.1
12X9 B	35,000	27,140	19,741.6	81,881.6	7.5	134,752	169,000	18,440	322,192	29.7	673,829.3	2,438.5	6,091.8	682,359.5	62.8
12X12 A	28,000	28,820	6,408.6	63,228.6	3.9	132,478	294,140	7,464	434,082	27.0	1,090,378.9	6,355.4	12,830.4	1,109,564.8	69.1
12X12 B	35,000	27,380	20,718.4	83,098.4	6.6	184,418	209,200	11,340	404,958	32.4	729,661.0	7,052.2	25,203.5	761,916.7	61.0
12X15 A	28,000	26,830	4,609.3	59,439.3	3.6	188,355	284,600	18,972	491,927	29.9	1,072,138.9	6,982.2	15,179.8	1,094,300.9	66.5
12X15 B	35,000	27,080	10,172.5	72,252.5	4.6	202,961	243,420	15,800	462,181	29.7	981,242.0	10,088.4	31,649.7	1,022,980.1	65.7
Avg.	**30,500**	**27,468.8**	**10,534.5**	**68,503.3**	**6.6**	**132,396.3**	**201,982.5**	**12,386.0**	**346,764.8**	**29.2**	**747,910.2**	**4,504.2**	**12,254.2**	**764,668.6**	**64.2**
14X6 A	28,000	32,100	9,804.5	69,904.5	9.7	49,227	83,700	14,304	147,231	20.4	504,136.5	339.6	719.6	505,195.7	69.9
14X6 B	35,000	31,570	21,192.0	87,762.0	11.4	60,562	142,520	18,160	221,242	28.8	458,221.2	507.1	1,467.0	460,195.3	59.8
14X9 A	32,000	30,550	6,930.0	69,480.0	7.2	118,494	169,560	26,832	314,886	32.8	568,810.4	2,527.7	4,625.9	575,964.1	60.0
14X9 B	35,000	30,890	16,205.9	82,095.9	7.9	101,063	156,160	9,980	267,203	25.6	677,128.5	3,687.2	12,418.9	693,234.6	66.5
14X12 A	36,000	34,310	6,551.2	76,861.2	4.4	181,291	273,700	6,408	461,399	26.4	1,185,573.8	7,570.0	13,937.0	1,207,080.9	69.2
14X12 B	35,000	27,960	18,759.9	81,719.9	5.2	198,827	285,900	5,020	489,747	31.4	929,463.0	11,111.1	45,207.6	985,781.6	63.3
14X15 A	28,000	30,110	7,138.8	65,248.8	3.5	174,546	362,380	20,532	557,458	29.5	1,230,767.8	12,475.7	22,711.3	1,265,954.8	67.0
14X15 B	35,000	30,170	19,202.9	84,372.9	4.4	223,786	341,580	17,580	582,946	30.5	1,203,371.3	10,209.0	31,963.5	1,245,543.8	65.1
Avg.	**33,000**	**30,957.5**	**13,223.2**	**77,180.7**	**6.7**	**138,474.5**	**226,937.5**	**14,852.0**	**380,264.0**	**28.2**	**844,684.1**	**6,053.4**	**16,631.3**	**867,368.8**	**65.1**

As in Table 4.8, these costs are divided into final product related costs (first set of columns), raw material batch related costs (second set of columns), and raw material (at shop floor) related cost (third set of columns). Similar observations to those from Table 4.8 can be made in Table 4.11 regarding the cost proportions as the number of items increments from $m = 6$ to $m = 15$. Instances with fewer items average 8.8% in terms of the final product related costs proportions, and those with $m = \{12, 15\}$ average 4.5% for the same type of costs. Comparing tables 4.8 and 4.11, one can see that, although the costs are higher in every category for the second set of instances, the proportions are quite similar.

4.5 Conclusions and future research

In the study presented in this chapter, we contribute to the research of production planning by developing and incorporating special constraints for several raw material types that deteriorate and perish depending on the storage and handling conditions. We propose and study a *multi-item, multi-level lot-sizing problem with raw-material perishability and batch ordering* (*MI-MLS-FVD*) inspired by a direct application in advanced composite laminates manufacturing. We consider different inventory levels and conditions: one where raw material batches are kept unopened and under special conditions so as to avoid deterioration, and a second one where raw material batches are opened and used for production and start deteriorating. Following the perishable conditions of *prepregs* used in producing high-performance composite materials, we consider both progressive functionality loss and a volume loss after the raw material batches are opened and begin their *out-time* at room temperature. We present a detailed small example to validate and show the structure of the optimal solution obtained, and performed extensive computational experimentation to evaluate the proposed model formulation and its potentials for practical applications in composite manufacturing production planning and optimization.

From this study, we plan to further explore the structure of the problem and to investigate the impact of material perishability on production efficiency as well as product quality. We also plan to develop integrated production models for process control and optimization in composite manufacturing and other manufacturing systems which may share problem features studied in this work.

Chapter 5

Reformulations for the multi-item lot-sizing problem with inventory bounds

In previous chapters, we have thoroughly studied multi-level lot-sizing problems with perishable raw material considerations. A fundamental aspect to consider in these problems is how inventory management can reduce the costly impact that these materials can have on production and on storage and disposal. A key assumption of our study in previous chapters is that the finished products are non-perishable, or that their shelf-life is long enough so as to reasonably ignore their perishable nature. Therefore, one of the ways in which the optimal solution of these problems tend to be notably structured is by using raw material for production as soon as possible after a batch has been received, and thus, storing large finished product quantities in inventory. This solution structure partially reduces the complexity of the perishable raw material inventory management and is able to avoid considerably larger material disposal costs.

Considering this tendency of storing higher finished product inventories, we have opened the study to another fundamental assumption regarding production planning problems: *inventory bounds* (IB). Whether due to warehouse infrastructure conditions, conditions inherent to the market, or internal administrative policies, there are several industrial application cases where finished products cannot be stored in unlimited quantities from one period to the next. Considering the type of applications that we have studied in Chapters 3 and 4, the approach has been focused on studying problems with a single finished product. One of our future fields of research is the extension of these problems to consider not only multiple types of raw material items, but also the case of few large structural components as finished products, integrating the consideration of IB. As discussed in previously, these problems arise, for example, in advanced composite materials manufacturing processes, which motivate our study [122].

With this in mind, in this chapter we formally study the *multi-item uncapacitated lot-sizing problem with inventory bounds* (*MI-ULS-IB*). We present a new MIP formulation for the case of non-speculative (Wagner-Whitin) cost structures using a special set of variables to determine the production intervals for each item. We then reformulate the problem using a variable-splitting technique that allows for a Dantzig-Wolfe decomposition. The Dantzig-Wolfe principle exploits the structure of the *MI-ULS-IB* by decomposing it into two sub-problems: one relating to the production decisions per item and another that relates to the inventory decisions per period. We propose a column generation algorithm for solving the linear programming relaxation of the Dantzig-Wolfe reformulation. Preliminary computational experiments are performed to evaluate the proposed formulations and algorithms on a set of benchmark instances.

The content of this chapter was presented in the *International Workshop on Lot-Sizing (IWLS)*[1], Glasgow, Scotland, 2017, as "Dantzig-Wolfe reformulations for multi-item lot-sizing problems with inventory bounds" [7]. It constitutes progress and pre-

[1]The International Workshop on Lot-Sizing (IWLS) is on invitation only. A limited number of participants who are active in the field of lot-sizing are invited.

liminary results of our ongoing research project. We then plan to integrate the results and insights of this project with those obtained in our previous research to study lot-sizing problems considering multiple finished products and perishable raw materials with IB. We believe this is a promising research area in the field of production planning that, to the best of our knowledge, has not been studied in the literature so far.

5.1 Multi-item lot-sizing with inventory bounds

In various types of production systems and industries it is common to find that inventory levels of products are bounded. These restrictions on the quantities to be stored may be related to physical warehouse space and even to administrative policies, specially for voluminous products, or products requiring special warehouse conditions (i.e., clean rooms, controlled temperatures) [15]. Storage capacity (inventory bounds, *IB*) considerations are even more relevant for multi-item production structures, where different types of products share storage space. We study the *multi-item uncapacitated lot-sizing problem with inventory bounds* (*MI-ULS-IB*), a problem of special theoretical and practical interest. The *MI-ULS-IB* is very symmetrical, but not equivalent to the multi-item capacitated lot sizing problem (*MI-CLS*), which is widely studied in the literature.

A similar problem to the single-item version of the uncapacitated lot-sizing problem with inventory bounds (*ULS-IB*) was initially studied in 1973 [81]. Then, the author presented a polynomial algorithm for the *LS* with bounded production and inventory considering separable piecewise concave costs. After that, to the best of our knowledge, it was not until 1994 that the problem was again studied, when Pochet and Wolsey [103] presented a polyhedral study on the *ULS-IB* and gave a complete description of the convex hull of solutions for the case of non-speculative (Wagner-

Whitin) cost structure. Most recently, Atamturk and Küçükyavuz [22] performed a polyhedral study and proposed valid inequalities for the *ULS-IB* with linear and fixed inventory costs. Wolsey [135] studied the lot-sizing problem with time windows and nonspecific orders, which is equivalent to the *ULS-IB*, and derived polynomial time dynamic programming algorithms and tight extended formulations for the uncapacitated and constant capacity problems with general costs. Considering bounds on the initial inventory for the discrete version of the *LS*, Di Summa and Wolsey [42] presented extended formulations describing the convex hull of solutions. For the *ULS-IB* that allows backlogging, and considers lost sales, dynamic programming algorithms were presented in [68] and [69], respectively.

General production-distribution planning problems considering *IB* can be found in [98] and [87]. An industrial application example of a multi-item replenishment-storage planning problem with *IB* was presented by Akbalik et al. [14]. Gutierrez et al. [61] presented a variant of the problem with different item weights (or volume), where the bounds are imposed on the total weight of the stock. More formally, the *MI-ULS-IB* was studied by Akbalik et al. [15]. The authors showed that the problem is $\mathcal{NP}$-hard, even for the case of Wagner-Within cost structure. Most recently, Melo and Ribeiro [86] presented a shortest path formulation and a formulation based on the addition of (l, S)-inequalities. The authors also proposed a rounding and relax-and-fix heuristics, which is an MIP based heuristic that has been successfully applied to various $\mathcal{NP}$-hard production planning problems.

5.1.1 Problem description and formulation

The *MI-ULS-IB* can be described as having m different items to be produced over a finite planning horizon of n periods to satisfy the demand d_t^i for each item $i = 1, \ldots, m$ and each period $t = 1, \ldots, n$ (we assume backlogging is not allowed). Let M and T be the set of all items and all periods, respectively. We assume demand is immediately satisfied at the beginning of each period t. Any produced units that are not immediately used to satisfy demand are inventoried in a common storage space. The total amount of inventory in period t is limited by the storage capacity u_t (considering that any item consumes one unit of storage capacity). Producing an item i in any period t incurs a fixed setup cost q_t^i and a variable production cost p_t^i (joint setup costs are not considered). In addition, a holding cost h_t^i is incurred for each unit of item i in stock between period t and $t+1$. We assume no initial and final stocks and nonnegative demands and costs.

Using a classical approach for *LS* problems to formulate the *MI-ULS-IB*, we let variables x_t^i represent the amount of item i produced in period t, and s_t^i the amount of item i in stock at the end of period t. Finally, $y_t^i = 1$ if and only if there is production of item i in period t and $y_t^i = 0$ otherwise. Figure 5.1 shows a graphical representation of the *MI-ULS-IB* using a network where the total amount of product inventory $\sum_{i=1}^m s_t^i$ flowing between each pair of time periods $(t, t+1)$ is bounded by storage capacity u_t.

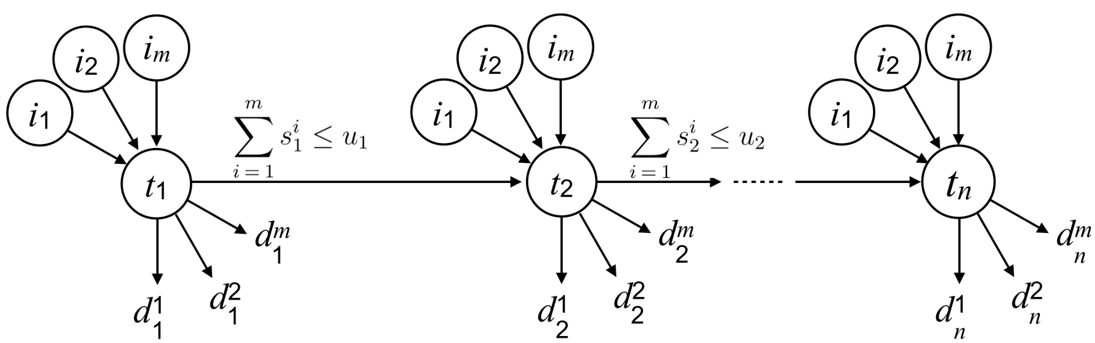


Figure 5.1: Graphical representation of the *MI-ULS-IB*

Similarly to [98] and [87], the *MI-ULS-IB* can be initially formulated as:

$$\textbf{\textit{MI-ULS-IB}} \quad \text{minimize} \quad \sum_{i=1}^{m} \sum_{t=1}^{n} \left(p_t^i x_t^i + h_t^i s_t^i + q_t^i y_t^i\right) \tag{5.1}$$

$$\text{subject to} \quad s^i_{t-1} + x^i_t = d^i_t + s^i_t \qquad i \in M,\ t \in T \tag{5.2}$$

$$x^i_t \leq M y^i_t \qquad i \in M, t \in T, \tag{5.3}$$

$$\sum_{i=1}^{m} s^i_t \leq u_t \qquad t \in T, \tag{5.4}$$

$$x^i_t, s^i_t \geq 0 \qquad i \in M, t \in T \tag{5.5}$$

$$y^i_t \in \{0, 1\} \qquad i \in M, t \in T. \tag{5.6}$$

The objective function (5.1) minimizes the sum of all related costs: variable production, storage, and fixed production costs. Constraint set (5.2) is the inventory balance equation and (5.3) the set-up enforcing equation. Constraint set (5.4) limits the total inventory at a given period t by the inventory bound u_t. Finally, constraint sets (5.5) and (5.6) are non-negativity and integrality constraints, respectively.

5.1.2 Facility location reformulation

To formulate the *MI-ULS-IB* using the facility location approach for *LS* problems [73], let a new set of variables w^i_{lt} represent the amount of item i, measured as a fraction of the demand d^i_t, that is produced in period l to satisfy demand of period t. Accordingly, the facility location MIP reformulation is:

$$\boldsymbol{FLF} \quad \text{minimize} \quad \sum_{i=1}^{m} \sum_{t=1}^{n} \left(\sum_{l=t}^{n} c^i_{tl} w^i_{tl} + q^i_t y^i_t \right) \tag{5.7}$$

$$\text{subject to} \quad \sum_{k=1}^{t} w_{kt}^i = 1 \qquad i \in M,\ t \in T,\ k \leq t \tag{5.8}$$

$$w_{kt}^i \leq y_k^i \qquad i \in M, k \in T, t \in T, \tag{5.9}$$

$$\sum_{i=1}^{m} \sum_{k=1}^{t} \sum_{l=t+1}^{n} d_t^i w_{kl}^i \leq u_t \qquad 1 \leq t \leq n-1, \tag{5.10}$$

$$w_{kt}^i \geq 0 \qquad i \in M, k \in T, t \in T, k \leq t \tag{5.11}$$

$$y_t^i \in \{0,1\} \qquad i \in M, t \in T \tag{5.12}$$

where $c_{tl}^i = d_t^i \left(p_t^i + \sum_{r=t}^{l-1} h_r^i \right)$.

The objective function (5.7) minimizes the sum of all related costs, and also scales the proportional variable w_{lt}^i using the corresponding demand d_t^i. Constraint set (5.8) ensures that for each item i at each period t, demand is met at its entirety using all variables w_{kt}^i from $1 \leq k \leq t$. Constraint set (5.9) is the set-up enforcing equation. Constraint set (5.10) limits the total inventory at a given period t by the inventory bound u_t. Finally, constraints set (5.11) and (5.12) are non-negativity and integrality constraints, respectively.

Alternatively, one can calculate the original production x_t^i and inventory s_t^i variables by adding the following sets of constraints:

$$x_t^i = \sum_{l=1}^{t} d_t^i w_{tl}^i \qquad i \in M, t \in T, \tag{5.13}$$

$$s_{t-1}^i + x_t^i = d_t^i + s_t^i \qquad i \in M, t \in T. \tag{5.14}$$

5.1.3 Cumulative-demand reformulation

We now propose an alternative MIP formulation for the case of non-speculative costs (also known as Wagner-Whitin costs) where producing and storing one unit in a period costs more than producing it later, that is $p_t^i + h_t^i \geq p_{t+1}^i$ for any item i in any period t. This cost structure is very frequent in practical situations and appears in a vast set of the lot-sizing literature. This alternative formulation extends the binary variables y_t^i to determine the production time intervals $[k, t]$, $1 \leq k \leq t \leq n$, for each item and are defined as $y_{kt}^i = 1$ if and only if there is production of item i to cover all demand from period k to t and all other $y_{ut}^i = 0$ for $k \leq u \leq t \leq n$.

Figure 5.2 shows these extended y_{kt}^i variables graphically in a feasible solution for a hypothetical instance with $m = 4$ items and n periods. As the graph shows, a

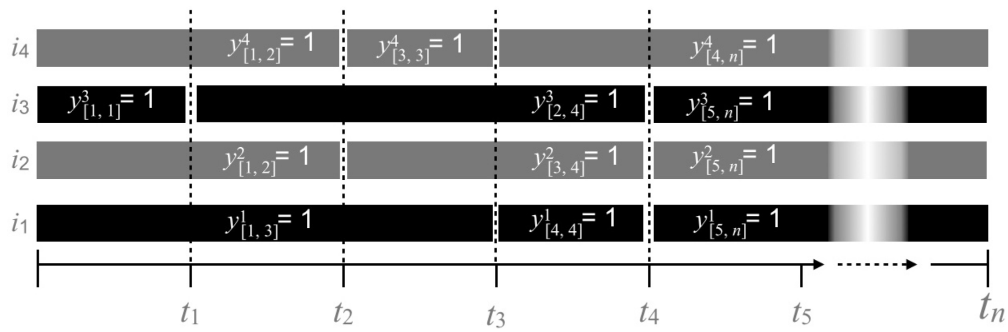


Figure 5.2: Solution for instance with $m = 4$ with extended y_{kt}^i variables

feasible solution is to produce the cumulative demand from periods 1 to 3 for item i_1, to produce just the demand for period 4 in that same period, and then from 5 to n. This is represented by $y_{13}^1 = y_{44}^1 = y_{5n}^1$, and all the rest of variables $y_{kt}^1 = 0$. Similarly, for item i_2, the solution involves producing the cumulative demand from period 1 to 2, 3 to 4, and 5 to n, represented by $y_{12}^2 = y_{34}^2 = y_{5n}^2 = 1$, and all the rest of variables $y_{kt}^2 = 0$. And so too with the rest of the items.

The alternative MIP formulation for the *MI-ULS-IB* is:

$$\textbf{\textit{CDF}} \quad \text{minimize} \quad \sum_{i=1}^{m} \sum_{k=1}^{n} \sum_{t=k}^{n} c_{kt}^i y_{kt}^i \tag{5.15}$$

$$\text{subject to} \quad \sum_{k=1}^{t} \sum_{l=t}^{n} y_{kl}^i = 1 \qquad i \in M,\ t \in T \tag{5.16}$$

$$\sum_{i=1}^{m} \sum_{l=t+1}^{n} \sum_{k=1}^{t} D_{tl}^i y_{kl}^i \leq u_t \qquad t \in T, \tag{5.17}$$

$$y_{kt}^i \in \{0, 1\} \qquad i \in M, k \in T, t \in T, k \leq t, \tag{5.18}$$

where $c^i_{kt} = \begin{cases} q^i_k + p_k d^i_k, & \text{if } k = t, \\ p^i_k \sum_{l=k}^{t} d^i_l + \sum_{l=k}^{t-1} h^i_l d^i_{l+1} & \text{if } k < t, \end{cases}$ and $D^i_{tl} = \sum_{k=t+1}^{l} d^i_k$.

5.2 Variable-splitting reformulation

Based on our *CDF*, we now apply variable-splitting technique (also known as Lagrangean decomposition), originally presented in [91]. More specifically, by duplicating the original variables y^i_{kt} with a new set of binary variables z^i_{kt}. We let $z^i_{kt} = 1$ if and only if there is production of item i at period k to cover all demand from period k to t, and $y^i_{kt} = 1$ if and only if there is inventory of item i to cover all demand from period k to t. Accordingly, we obtain the following reformulation:

$$\textbf{\textit{VPR}} \text{ minimize } \sum_{i=1}^{m}\sum_{k=1}^{n}\sum_{t=k}^{n} q^i_k z^i_{kt} \tag{5.19}$$

$$\text{subject to } \sum_{k=1}^{t}\sum_{l=t}^{n} z^i_{kl} = 1 \qquad i \in M,\ t \in T \tag{5.20}$$

$$\sum_{k=1}^{t} z^i_{kl} = y^i_{tl} \qquad i \in M, 1 \le t \le n-1,\ t+1 \le l \le n \tag{5.21}$$

$$\sum_{i=1}^{m}\sum_{l=t+1}^{n} D^i_{tl} y^i_{tl} \le u_t \qquad t \in T, \tag{5.22}$$

$$z^i_{kl} \in \{0,1\} \qquad i \in M,\ 1 \le k \le l \le n \tag{5.23}$$

$$y^i_{kt} \in \{0,1\} \qquad i \in M, 1 \le t \le n-1,\ t+1 \le l \le n, \tag{5.24}$$

where constraint 5.21 are added as the linking constrains for the two sets of variables z and y. Thus, the *VPR* enables us to decompose the *MI-ULS-IB* into two independent sub-problems. The first sub-problem, associated with variables z^i_{kt}, constitutes an *ULS* for each $i \in M$, making sure that demand d^i_t is satisfied at every period $t \in T$. Each of these problems for $i \in M$ is considered as the most simple dynamic lot-sizing problem. The second sub-problem, which is our "hard" problem, is associated with variables y^i_{kt}, and it constitutes a special case of a *multi-item knapsack problem* (CAP) for $t = 1, \ldots, n-1$, making sure that inventory bounds u_t are satisfied for every $t \in T$.

5.3 Dantzig-Wolfe column generation approach

Based on *VPR*, we now propose a Dantzig-Wolfe column generation (DW-CG) approach to decompose and solve the problem. The DW-CG decomposition principle is a standard way to decompose an integer linear programming model with a large number of variables [40]. In general terms, the idea behind DW-CG is to divide an original linear program, denoted as the *master problem* (*MP*), into two or more inter-related problems: a *restricted master problem* (*RMP*) and one or several *pricing problems* (*PPs*). Given the large number of variables in the *MP*, which in our case refers to *VPR*, the *RMP* contains a small subset of them. The *RMP* is solved to optimality and additional variables are dynamically added at every iteration. Solving the *RMP* is equivalent to solving the original *MP* with several variables fixed to zero, and so the *PPs* are solved to determine whether the current solution is optimal or to identify additional variables to be added to the *RMP*.

5.3.1 The restricted master problem

We let U_i denote the subset of feasible configurations for the *ULS* sub-problem for each $i \in M$, that is the set of production plans satisfying all demands for each item, i.e., $\sum_{k=1}^{t} \sum_{l=t}^{n} \bar{z}^i_{kl} = 1$ for $t \in T$ and $i \in M$. We let C_t denote the subset of feasible configurations for the CAP sub-problem for $t = 1, \ldots, n-1$, that is the set of production plans satisfying all inventory bounds u_t, i.e., $\sum_{i=1}^{m} \sum_{l=t+1}^{n} D^i_{tl} \bar{y}^i_{tl} \leq u_t$ for $t \in T$. Consider also decision variable L^c_i, which is the weight (of fraction) of production plan c for item i, $c \in U_i$; and X^c_t, which is the weight (or fraction) of production plan c for time period t, $c \in C_t$. The cost parameter for each production plan $c \in U_i$ is given by F_i^c for $i \in M$.

Accordingly, the RMP can be stated as follows:

$$\textbf{\textit{RMP}} \ \text{minimize} \quad \sum_{i \in M} \sum_{c \in U_i} F_i^c L_i^c \tag{5.25}$$

$$\text{subject to} \quad \sum_{c \in U_i} L_i^c = 1 \qquad i \in M \tag{5.26}$$

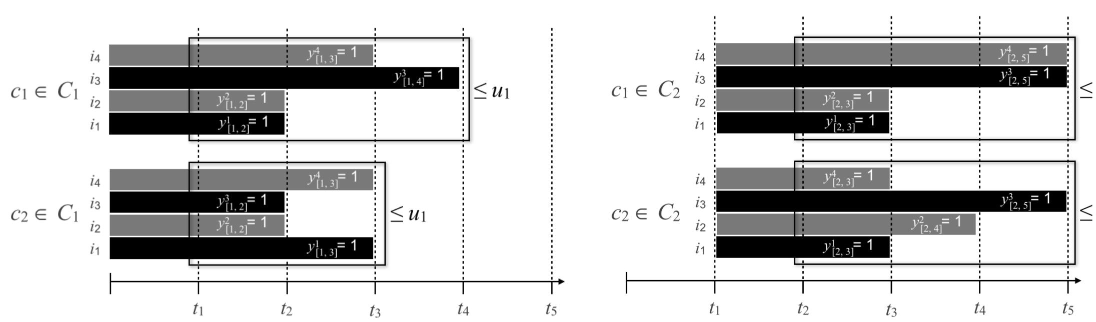
$$\sum_{c \in C_t} X_t^c = 1 \qquad 1 \leq t \leq n-1 \tag{5.27}$$

$$\sum_{c \in U_i : \sum_{k=1}^{t} \bar{z}_{kl}^i = 1} L_i^c = \sum_{c \in C_t : \bar{y}_{tl}^i = 1} X_t^c \qquad i \in M, \quad 1 \leq t \leq n-1,\ t+1 \leq l \leq n \tag{5.28}$$

$$L_i^c \geq 0 \qquad c \in U_i \tag{5.29}$$

$$X_t^c \geq 0 \qquad c \in C_t. \tag{5.30}$$

Figure 5.3 illustrates an example of how the feasible configurations C_t for the CAP are structured using the y_{kt}^i variables.



(a) $c \in C_t$ confirgurations for $t = 1$ (b) $c \in C_t$ confirgurations for $t = 2$

Figure 5.3: Illustrative examples of C_t feasible configurations for the CAP sub-problem

Of the two sub-problems in which we have decomposed the original problem, the CAP sub-problem implies a greater complexity. To improve the performance of the column generation, we further decompose the CAP sub-problem into a smaller set of sub-problems. Instead of solving the overall problem at each iteration, we can divide the problem into blocks of time periods (or time intervals) of the planning horizon T. For this, we let $T(1), \ldots, T(r)$ denote the set of r blocks (or time intervals) in which the planning horizon T is partitioned into. For $b \in [1, \ldots, r]$, each block $T(b)$ corresponds to a time period interval within T, i.e., $T(b) = [l_b, \ldots, u_b] \in T$. We consider a sequential ordering of time periods into block such that $l_1 = 1, u_r = n$, and $l_b = u_{b-1} + 1$ for $b \in [2, \ldots, n]$. The set of decision variables X_b^c denotes the weight of production plan c for $b \in [1, \ldots, r]$. Thus, the *RMP* with block-partitioning can be stated as follows:

$$\textbf{\textit{RMP-Block}} \quad \text{minimize} \quad \sum_{i \in M} \sum_{c \in U_i} F_i^c L_i^c \tag{5.31}$$

$$\text{subject to} \quad \sum_{c \in U_i} L_i^c = 1 \qquad i \in M \tag{5.32}$$

$$\sum_{c \in C_b} X_b^c = 1 \qquad 1 \leq b \leq |r| \tag{5.33}$$

$$\sum_{c \in U_i : \sum_{k=1}^{t} \bar{z}_{kl}^i = 1} L_i^c = \sum_{c \in C_b : \bar{y}_{tl}^i = 1} X_b^c \qquad i \in M,$$

$$1 \leq b \leq |r|, \; l_b \leq t \leq u_b, \; (u_b + 1) \leq l \leq n \tag{5.34}$$

$$L_i^c \geq 0 \qquad c \in U_i \tag{5.35}$$

$$X_b^c \geq 0 \qquad c \in C_b, b \in [1, \ldots, r]. \tag{5.36}$$

Using a small subset of variables from the original sets, the LP relaxation of the *RMP-Block* can be solved using a general-purpose solver. The original master problem (*VPR*) corresponds to the *RMP-Block* in which $L_i^c = X_b^c$ for every $i \in M$ and $b \in [1, \ldots, r]$.

5.3.2 The pricing problems

We let λ_{tl}^i be the dual variables associated with constraints 5.34. Given that the *VPR* contains two sets of decision variables, y and z, there are two pricing problems, one for each set. In each iteration of the column generation algorithm, we solve the pricing problems and add all the variables with negative reduced costs coefficients.

The first pricing problem is associated with the configurations for the *ULS* sub-problem (variables z):

Pricing Problem for $i \in M$

$$\textbf{\textit{ULS}} \text{ minimize } \sum_{k=1}^{n} \sum_{l=k+1}^{n} \left(q_{kl}^i + \sum_{t=k}^{l-1} \lambda_{tl}^i \right) z_{kl}^i + \sum_{k=1}^{n} q_{kk}^i z_{kk}^i \tag{5.37}$$

$$\text{subject to } \sum_{k=1}^{t} \sum_{l=t}^{n} z_{kl}^i = 1 \qquad t \in T \tag{5.38}$$

$$z_{kl}^i \in \{0,1\} \qquad 1 \leq k \leq l \leq T. \tag{5.39}$$

The *ULS* implicitly evaluates the reduced cost of all feasible configurations in U_i. The optimal solution to *ULS* is thus the configuration for the demand d_t^i of each item $i \in M$ being satisfied at every period $t \in T$ having the smallest reduced cost.

Once the *ULS* is solved, the second pricing problem, which is associated with the configurations for the *CAP* sub-problem (variables y), can be stated as:

Pricing Problem (Block) for $1 \leq b \leq |r|$

$$\textbf{\textit{CAP-Block}} \text{ minimize } \sum_{i=1}^{m} \sum_{t=l_b}^{u_b} \sum_{l=t+1}^{n} \lambda_{tl}^i y_{tl}^i \tag{5.40}$$

$$\text{subject to} \quad \sum_{k=l_b}^{t} \sum_{l=t}^{n} z_{kl}^i = 1 \qquad i \in M,\ l_b \le t \le u_b \tag{5.41}$$

$$\sum_{k=l_b}^{t} z_{kl}^i = y_{tl}^i \qquad i \in M,\ l_b \le t \le u_b,\ (t+1) \le l \le n \tag{5.42}$$

$$\sum_{i=1}^{m} \sum_{l=t+1}^{n} D_{tl}^i y_{tl}^i \le u_t \qquad l_b \le t \le u_b \tag{5.43}$$

$$y_{t-1,l}^i \le y_{tl}^i \qquad l_b \le t \le u_b,\ (t+1) \le l \le n \tag{5.44}$$

$$y_{tl}^i \in \{0,1\} \qquad i \in M,\ l_b \le t \le u_b,\ (t+1) \le l \le n \tag{5.45}$$

The *CAP-Block* implicitly evaluates the reduced cost of the feasible configurations in C_t. The solution to *CAP-Block* is thus a configuration for the inventory bounds u_t being satisfied at every period $t \in T$ having negative reduced cost.

Our implementation of the column generation algorithm starts by solving the *ULS* (with no inventory bounds). Using the information from the *ULS* optimal solution, we solve the *CAP-Block* and find feasibility for the *RMP-Block*. Once new columns with negative reduced costs coefficients are found, they are added to *RMP-Block* and the process is repeated with updated sets of columns U_i and C_t.

For our preliminary experiments, the algorithm terminates when the time limit of two hours is reached.

5.4 Preliminary computational experiments

In this section, we present our initial preliminary computational experiments comparing the results obtained with our *CDF* formulation, its linear programming relaxation (*LP*) and our proposed DW-CG algorithm using the *RMP-Block* formulation.

The experiments were implemented using the Callable Library of IBM CPLEX 12.7.0. All tests were carried out on an Intel(R) Xeon(R) CPU E3-1270 v3 processor with 3.50GHz and 24GB of RAM memory and Microsoft Windows 7 Enterprise operating system. For our preliminary experiments, the DW-CG algorithm terminates when the time limit of two hours is reached or when no more columns with negative reduced costs can be found.

In order to assess the performance of each approach, we used an adaptation of the same instances used by Melo and Ribeiro [86]. These instances consider neither production not storage costs. Initially, we are solving smaller versions of these instances with $|M| = 15$ and $|M| = 30$ items and shorter planning horizons of $|T| = 12$ periods.

We are evaluating the quality of the solutions by comparing the lower bounds obtained with our column generation algorithm for different partitions of the planning horizon: $r = \{2, 3, 4\}$.

Both in Table 5.1 and in Table 5.2, the first column gives the name of the instance, based on the following structure: "I_*M*_*N*_*counter*". The second, third, and fourth columns show the value of the optimal solution for each instance obtained with the *CDF*, the linear programming (LP) relaxation bound, and the LP relaxation gap (%). These values were obtained without the time limit of two hours. The last three columns present the lower bounds obtained with our DW-CG algorithm using three different planning horizon partitions, 2, 3, and 4, respectively.

Table 5.1: Results for instances with $|M| = 15$ items and $|T| = 12$ periods

	CDF			***DW-CG*** **(Dev%)**		
Instance	Opt	LP	LP Gap %	$r = 2$	$r = 3$	$r = 4$
I_15_12_01	2,923	2,851.80	2.44	0.29	0.05	1.58
I_15_12_02	2,810	2,750.69	2.11	-	0.64	2.55
I_15_12_03	3,331	3,251.52	2.39	1.46	1.31	1.8
I_15_12_04	3,085	3,035.65	1.60	1.92	0.05	0.94
I_15_12_05	2,655	2,593.59	2.31	1.22	0.61	1.34
I_15_12_06	3,300	3,221.93	2.37	0.47	-	0.97
I_15_12_07	4,466	4,379.62	1.93	0.89	0.05	0.53
I_15_12_08	3,837	3,745.84	2.38	0.32	-	0.93
I_15_12_09	3,633	3,559.72	2.02	0.52	0.62	0.84
I_15_12_10	3,685	3,619.05	1.79	1.52	2.41	2.29
			2.13	**0.96**	**0.72**	**1.38**

Table 5.2: Results for instances with $|M| = 30$ items and $|T| = 12$ periods

	CDF			***DW-CG*** **(Dev%)**		
Instance	Opt	LP	LP Gap %	$r = 2$	$r = 3$	$r = 4$
I_30_12_01	3,115	3,057.34	1.85	2.02	2.04	6.07
I_30_12_02	2,882	2,834.79	1.64	1.71	2.12	4.35
I_30_12_03	3,025	2,950.59	2.46	2.73	4.14	3.53
I_30_12_04	2,652	2,605.26	1.76	1.72	3.08	1.59
I_30_12_05	3,420	3,337.34	2.42	3.33	1.12	3.21
I_30_12_06	2,651	2,590.14	2.3	1.31	2.07	2.49
I_30_12_07	2,572	2,532.46	1.54	2.93	1.99	3.79
I_30_12_08	2,800	2,742.50	2.05	0.05	3.29	2.18
I_30_12_09	2,933	2,861.78	2.43	0.67	3.34	1.38
I_30_12_10	2,806	2,743.64	2.22	0.05	1.33	2.27
			2.07	**1.65**	**2.45**	**3.09**

As seen in table 5.1, for instances with $|M| = 15$ items and $|T| = 12$ periods, our DW-CG algorithm is able to obtain average lower bound gaps in the order of 0.96%, 0.72%, and 1.38% using $r = \{2, 3, 4\}$ blocks, respectively. When using $r = 3$ blocks, the DW-CG algorithm is able to reach lower bound gaps as low as 0.05% within the time limit of two hours. This shows the promising potential of the algorithm when comparing these gaps with the LP Gap average of 2.13%.

For the larger-size instances shown in table 5.2 with $|M| = 30$ items and $|T| = 12$ periods, the DW-CG algorithm is able to average a 1.65% lower bound gap when using $r = 2$ blocks. For these larger-size instances the DW-CG algorithm is also able to reach lower bound gaps in the order of 0.05%.

These results show promising potential of our models and algorithms to extend the problems and instances studied in Chapters 3 and 4 related to composite manufacturing applications [122], to consider the case of few large carbon-fiber based structural components as finished products with IB considerations.

5.5 Conclusions and future research

In this chapter, we have studied the *multi-item uncapacitated lot-sizing problem with inventory bounds* (*MI-ULS-IB*). We presented a new MIP formulation for the case of non-speculative (Wagner-Whitin) cost structure using a set of variables to determine the production intervals for each item. We then reformulate the problem using a variable-splitting technique to a general split-variable model that allows for a Dantzig-Wolfe decomposition. The Dantzig-Wolfe principle exploits the structure of the *MI-ULS-IB* by decomposing it into two sub-problems: one relating to the production decisions per item and another that relates to the inventory decisions per period. We propose a column generation algorithm for solving the Dantzig-Wolfe reformulation and present preliminary computational experiments to evaluate the proposed formulations and algorithms on a set of benchmark instances involving up to 30 items and 12 periods.

From the study presented in Section 5.4, we can infer that our proposed DW-CG algorithm have the potential to be of significant added value for solving *MI-ULS-IB*. However, a stabilization of the DW-CG algorithm is needed in order to be able to solve the full size instances by Melo and Ribeiro [86]. Additional research is also proposed to develop an algorithm to more efficiently solve the *CAP* subproblem and to find upper bounds to solve larger-size instances.

We also plan to further research how the *MI-ULS-IB* can be integrated with the raw-material perishability considerations studied in the previous chapters.

Chapter 6

Summary

This thesis focuses on studying one of the most important and fundamental links in supply chain management, production planning. Although production planning is a field of research in which immense progress has been made since the 1910s, this thesis presents several important contributions in relation to one of the least studied assumptions of the multi-level production planning research: the perishable nature of raw materials. In real life applications, whether referring to physical exhaustion, loss of functionality, or obsolescence, most items deteriorate over time and cannot be stored infinitely without enforcing specific constraints on a set of crucial production planning decisions. This is specially the case for multi-level production structures. Clear cases of this type of products can be found in the food or pharmaceutical industries, but also in industries like the advanced composite manufacturing industry, which originally inspired this thesis. It also addresses the study of production planning involving inventory bounds. This characteristic is shown to be related to the perishable raw-material considerations and constitutes another fundamental aspect of this family of problems.

In Chapter 2, we introduce the fundamental characteristics in production planning modeling and discuss some of the most common elements and assumptions used to model complex production planning problems. An overview of the production planning research evolution is presented.

In Chapter 3 we structure a review of the different characteristics that must be considered when dealing with perishability in production planning, and present a classification framework. We present the most relevant modeling approaches for perishability in production planning available in the research literature. We then present lot-sizing problems that incorporate raw-material perishability and analyze how these considerations enforce specific constraints on a set of fundamental decisions, tackling three variants of the two-level lot-sizing problem incorporating different types of raw-material perishability: *fixed shelf-life*, *functionality deterioration*, and *functionality-volume deterioration*. We propose mixed-integer programming formulations for each of these variants and perform computational experiments with sensitivity analyses, showing the added value of explicitly incorporating perishability considerations into production planning problems. Using a Silver-Meal-based rolling-horizon algorithm, we develop a sequential approach to solve the studied problems and compare the results with our proposed formulations.

With production planning in composite manufacturing as an initial motivation and with the possibility of generalizing the problem to other applications, in Chapter 4 we study the *multi-item, multi-level lot-sizing problem with raw-material perishability and batch ordering*. We proposed a mixed-integer programming formulation for the problem and perform computational experiments with sensitivity analyses, demonstrating its potentials for practical applications in planning advanced composites manufacturing.

In Chapter 5, we formally study the *multi-item uncapacitated lot-sizing problem with inventory bounds*. We present a new mixed-integer programming formulation for the case of non-speculative (Wagner-Whitin) cost structure using a special set of variables to determine the production intervals for each item. We then reformulate the problem using a variable-splitting technique that allows for a Dantzig-Wolfe decomposition. The Dantzig-Wolfe principle exploits the structure of the problem by decomposing it into two sub-problems: one relating to the production decisions per item and another that relates to the inventory decisions per period. We propose a column generation algorithm for solving the Dantzig-Wolfe reformulation. Preliminary computational experiments are performed to evaluate the proposed formulations and algorithms on a set of benchmark instances. The results show the promising potential of our models and algorithm to be extended and applied in composite manufacturing production planning and optimization.

As mentioned above, this thesis constitutes an important contribution within one of the most studied areas of supply chain management, especially for applications in contexts in which multi-level product structures are being restricted by specific characteristics associated with the nature of the raw materials. However, we believe that this field still has ample possibilities for future research. Mainly, the integration of the results and understanding achieved in Chapters 3 and 4 on lot-sizing with perishable raw materials, with the study of models considering inventory bounds, constitutes a very interesting path for future research in the field of production planning. Additionally, another opportunity for future research that arises from this study refers to the development of efficient solution methodologies to solve larger-size instances of the proposed problems that could be found in some practical applications.

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