## Performance of Pile Cap Foundation with Respect to Cap Rigidity

# Soukayna El Hammouli

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By: Soukayna El Hammouli

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Signed by the final Examining Committee:

Dr. Anjan Bhowmick.	Chair
Dr. Biao Li	Examiner
Dr. Mehdi Hojjati	Examiner
Dr. Adel Hanna	Supervisor
Dr. Lan Lin	Supervisor

Approved by Dr. Ashutosh Bagchi Chair of Department or Graduate Program Director

December 2, 2019

Dr. Amir Asif Dean of Faculty

### ABSTRACT

#### Performance of Pile Cap Foundation with Respect to Cap Rigidity

### Soukayna El Hammouli

The thickness of a pile cap is a major parameter in the design of the pile-cap foundation. A thin cap performs as a flexible slab by distributing the load on the piles unevenly, which does not accord with the concept of pile foundations. On the other hand, a thicker and therefore more rigid cap evenly distributes the load on the piles, as well as resisting bending moments and punching shear failure, but may impose additional load on the foundation. In view of these differences, there is a need to determine an optimal cap thickness such that the cap will distribute the load evenly on the piles without imposing excessive loads.

This thesis, therefore, used ABAQUS commercial software to develop a 3-D finite numerical model to simulate pile-cap foundations of 9 and 16 piles under variable pile diameter, pile length, and pile spacing for a range of cap thickness from 0.5m to 3m. The collected data is in the form of the load on individual piles, the load sharing between the pile and the cap, and the vertical displacement of the cap.

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# **TABLE OF CONTENTS**

List of Figures	VII
List of Tables	X

# **CHAPTER 1: INTRODUCTION**

1.1 Background	1
1.2 Objective	3
1.3 Thesis organization	3

# **CHAPTER 2: LITERATURE REVIEW**

2.1 General	5
2.2 Past studies	5
2.2.1 Flexural rigidity	5
2.2.2 Thickness of rigid pile caps	10
2.3 Code procedure for the design of pile caps	12
2.3.1 Canadian design standard A23.3-14	12
2.3.2 American design code ACI318-14	14

# **Chapter 3: NUMERICAL MODELING**

3.1 Description of pile foundation model	5
3.2 Modeling techniques	16
3.2.1 Piles and pile cap1	7
3.2.2 Soil	7
3.2.3 Interactions and contact zones	9
3.2.4 Mesh	20
3.3 Model techniques validation	22

# **CHAPTER 4: ANALYSIS RESULTS**

4.1 Introduction	27
4.2 Effect of pile-cap thickness	27
4.3 Effect of pile spacing	40
4.4 Effect of pile diameter	46
4.5 Effect of pile length	53

# **CHAPTER 5: CONCLUSIONS AND RECOMMENDATIONS**

5.1 Summary and Conclusions	60
5.2 Recommendations for future research	61

# REFERENCES

# LIST OF FIGURES

Figure 2.1 Illustration of small and large pile rafts adapted from Viggiani et al. (2012
Figure 2.2 Curvature versus depth of the cap for a variable number of floors (Cheng, 2005)11
Figure 2.3 Comparison of the depth of a pile cap with different methods (Cheng, 2005)11
Figure 2.4 Regression analysis results as Rabbany et al. (2015)12
Figure 3.1 Elevation view of the foundations: (a) 16-pile foundation, (b) 9-pile foundation15
Figure 3.2 ABAQUS <sup>1</sup> / <sub>4</sub> model: (a) 16-pile foundation, b) 9-pile foundation17
Figure 3.3 ABAQUS <sup>1</sup> / <sub>4</sub> model for the soil in the 16-pile foundation
Figure 3.4 Contact zones defined in ABAQUS20
Figure 3.5 ABAQUS meshing for <sup>1</sup> / <sub>4</sub> 16-pile foundation: (a) partitions assigned, (b) final meshed model
Figure 3.6 Boundary conditions defined in the model
Figure 3.7 Finite element model as Alnuiam et al. (2013)23
Figure 3.8 Comparison of the results of pile cap displacement vs axial loading24
Figure 3.9 Comparison of the results of load sharing percentage vs pile cap displacement25
Figure 4.1 Layout of piles under examination: (a) 16-pile foundation, (b) 9-pile foundation28
Figure 4.2 Pile load vs pile cap thickness for the 16-pile foundation: (a) medium sand soil, (b) dense sand soil
Figure 4.3 Pile load vs pile cap thickness for the 9-pile foundation: (a) medium sand soil, (b) dense sand soil
Figure 4.4 Percentage of load sharing <i>vs</i> pile cap thickness: (a) 16-pile foundation, (b) 9-pile foundation
Figure 4.5 Location of typical points to examine the displacement: (a) 16-pile foundation, (b) 9-pile foundation
Figure 4.6 Vertical displacement in pile cap in 16-pile foundation in medium sand soil

Figure 4.7 Vertical displacement in pile cap in 16-pile foundation in dense sand soil
Figure 4.8 Vertical displacement in pile cap in 9-pile foundation in medium sand soil37
Figure 4.9 Vertical displacement in pile cap in 9-pile foundation in medium sand soil
Figure 4.10 Curvature versus depth of the cap for 9 and 16 pile caps in medium and dense sand
Figure 4.11 Center settlement of the pile cap foundation with variable cap thicknesses for different pile cap foundations
Figure 4.12 Pile load vs pile spacing for the 16-pile foundation: (a) medium sand soil, (b) dense sand soil
Figure 4.13 Pile load vs pile spacing for the 9-pile foundation: (a) medium sand soil, (b) dense sand soil
Figure 4.14 Percentage of load sharing vs pile spacing: (a) 16-pile foundation, (b) 9-pile foundation
Figure 4.15 Vertical displacement in pile cap in 16-pile foundation: (a) medium sand soil, (b) dense sand soil
Figure 4.16 Vertical displacement in pile cap in 9-pile foundation: (a) medium sand soil, (b) dense sand soil
Figure 4.17 Pile load vs pile diameter for the 16-pile foundation: (a) medium sand soil, (b) dense sand soil
Figure 4.18 Pile load vs pile diameter for the 9-pile foundation: (a) medium sand soil, (b) dense sand soil
Figure 4.19 Percentage of load sharing vs pile diameter: (a) 16-pile foundation, (b) 9-pile foundation
Figure 4.20 Vertical displacement in pile cap in 16-pile foundation: (a) medium sand soil, (b) dense sand soil
Figure 4.21 Vertical displacement in pile cap in 9-pile foundation: (a) medium sand soil, (b) dense sand soil
Figure 4.22 Pile load vs pile length for the 16-pile foundation: (a) medium sand soil, (b) dense sand soil

Figure 4.23 Pile load vs pile length for the 9-pile foundation: (a) medium sand soil, (b) dense sand soil
Figure 4.24 Percentage of load sharing vs pile length: (a) 16-pile foundation, (b) 9-pile foundation
Figure 4.25 Vertical displacement in pile cap in 16-pile foundation: (a) medium sand soil, (b) dense sand soil
Figure 4.26 Vertical displacement in pile cap in 9-pile foundation: (a) medium sand soil, (b) dense sand soil

# LIST OF TABLES

Table 3.1 Pile cap parameters considered in the modeling and analysis	.16
Table 3.2 Material properties	.19
Table 3.3 Basic information on models as Alnuiam et al. (2013)	.23
Table 3.4 Information on modeling as Alnuiam et al. (2013)	.24
Table 4.1 Pile load for different pile cap thickness, 16-pile foundation	29
Table 4.2 Pile load for different pile cap thickness, 9-pile foundation	.31
Table 4.3 Pile load for different pile spacing, 16-pile foundation	.41
Table 4.4 Pile load for different pile spacing, 9-pile foundation	.42
Table 4.5 Pile load for different pile diameter, 16-pile foundation	.49
Table 4.6 Pile load for different pile diameter, 9-pile foundation	.49
Table 4.7 Pile load for different pile length, 16-pile foundation	.56
Table 4.8 Pile load for different pile length, 9-pile foundation	.56

## **CHAPTER 1**

## **INTRODUCTION**

### 1.1 Background

Deep foundations are often used when the soil underneath the foundation has the insufficient bearing capacity to support loads imposed. The typical types of deep foundations are pile, pier, caissons, and well-foundation. Among them, the pile foundation is the most used type of deep foundation. The two components of a pile foundation are pile cap and piles, while the pile cap directly connects the superstructure with the substructure. Accordingly, the behavior of the pile cap significantly affects the design of the entire foundation. For example, the loads in piles affect both the design of pile cap and piles while the rigidity of the pile cap on the other affects the load distributed to the piles. Therefore, piles and pile cap are intertwined, i.e., the design of one component will affect the other.

A pile cap can be either rigid or flexible. Theoretically, if the pile cap is rigid, all the piles are equally loaded. However, if the pile cap is flexible, then the piles close to the loading point are heavily loaded compared to others. In some cases, the load in these piles could be 3-4 times of the others. However, in practice, all the piles are designed using the largest demands among the pile group.

Researchers have developed some formulae to determine the rigidity of the pile cap, such as Hain et al. (1978), by considering the effects of the cap flexibility, size and pile group parameters. Randolph (1983), by introducing flexibility matrix method and the average settlement of the cap. Clancy and Randolph (1993), by presenting cap-pile interaction factor and cap and piles stiffness, Viggiani et al. (2012), by including young's modulus and Poisson's ratio for both cap and soil, etc. Some enclose only the parameters related to the pile cap in the formulation, such as pile cap thickness and modulus of the elasticity of the material for the pile cap. Others also introduce parameters associated with piles in the formula, e.g., pile stiffness. Very recently, Rabbany et al. (2018) proposed an equation to estimate pile cap depth based on the external loading. However, it was based on a regression analysis solely on very limited data provided 30 years ago in Reynolds et al. (1988).

In light of lagging research on the investigation of pile cap rigidity, design codes and standards provide requirements for the determination of pile cap thickness. For example, Canadian Concrete Design Standard CSA A23.3-14 specifies two methods for the design of pile caps, which are sectional method and strut-and-tie method (STM). The sectional method is used for the design of "shallow pile caps" since its behavior is very similar to spread footings as shallow foundations. However, if the depth of a pile cap is greater enough, the cap would behave like a deep beam. In this case, STM is also allowed. American Building Code Requirements for Structural Concrete ACI 318-14 defines the requirements for pile cap design in the Section for Deep Foundation. It also allows using STM to design pile caps. In CSA A23.2-14 and ACI 318-14, the depth of pile caps should be determined to satisfy the requirements for both one-way shear and two-way shear.

In the engineering community, STM is commonly used for the design of pile caps since it is well accepted that the beam design method is not valid for elements with discontinuity (i.e., loading and geometric discontinuities), in which the loading in a pile cap supported by multiple piles is discontinued. Numerous studies have been conducted on improving STM, such as Adebar et al. (1990), Adebar and Zhou (1996), Part et al. (2008), Aouza et al. (2009), etc. to debate which either sectional method or strut-and-tie method is more appropriate for the design of pile caps.

### **1.2 Objective**

The results from previous studies have indicated that the pile cap thickness plays a crucial role in the performance of pile foundation, it has a direct impact on the pile cap flexural rigidity, which will, in turn, affect the loads transferred from the superstructure to the substructure. Both CSA A23.3-14 and ACI 318-14 requires that the pile thickness should satisfy the requirements for one-way and two-way shear. However, the design factored shear is determined based on an assumption that the external load is equally taken by all the piles. This assumption may not be valid unless the pile cap is rigid enough to be able to distribute the load evenly. Given this, the objective of this study is to examine the effects of pile cap thickness on the performance of pile foundation in terms of load distribution on piles and deformation of pile cap itself and to conclude if a given pile cap thickness would make the loads distribute more uniformly. Given this, the following tasks were carried out in this study:

- Develop finite element models for pile cap foundations using ABAQUS. One foundation has 16 piles and the other has 9 piles.
- (2) Investigate pile cap thickness on the response of the two foundations and recommend the thickness in which the pile cap is relatively rigid, and the piles carry approximately the same amount of the load.
- (3) Examine the effects of pile spacing, pile diameter, and pile length.

### **1.3 Thesis organization**

This thesis is divided into 5 Chapters,

- Chapter 1 presents the introduction and objectives of the study.
- Chapter 2 presents the literature review on past studies related to the current research.

- Chapter 3 describes the numerical modeling of the pile cap foundation using ABAQUS and the validation of the modeling techniques with the analysis results available in the literature.
- Chapter 4 presents and discusses the analysis results of the parametric study of the pile cap foundation.
- Chapter 5 summarizes the major findings and conclusions from the present study, and recommendations for future studies.

# **CHAPTER 2**

## LITERATURE REVIEW

### 2.1 General

Pile cap or pile raft<sup>1</sup> is a critical element in the load transfer mechanism of the pile foundation since it does not only transfer the loads from the superstructure to the substructure but also it also affects the design of the piles. This chapter provides an overview of the past studies related to the current research work with a focus on the estimate of the rigidity of pile caps and the role of pile cap thickness on the behavior of the pile foundations. Furthermore, the requirements and design procedures stipulated in the code and standard in North-America, e.g., American Code: Building code requirements for structural concrete ACI 318-14 (ACI, 2014) and Canadian standard: Design of concrete structures CSA A23.3-14 (CSA, 2014) are described at the end of this Chapter.

### 2.2 Past studies

#### 2.2.1 Flexural rigidity of pile caps

Hain et al. (1978) analyzed the effectiveness of the pile group in reducing the settlement of the raft by considering the effects of the cap flexibility, size and pile group parameters. They introduced the cap-supporting soil relative stiffness  $K_R$  (Equation 2.1) and the pile-supporting soil stiffness  $K_p$  (Equation 2.2) from in their study.

<sup>&</sup>lt;sup>1</sup> The two terms pile cap and pile raft are exchangeable in this thesis.

$$K_R = \frac{4E_R t_R B_R (1 - v_s^2)}{3\pi E_s L_R^4}$$
(2.1)

$$K_p = \frac{E_p}{E_s} \tag{2.2}$$

Where,

 $E_R$  = Young's modulus of the material for the raft  $E_p$  = Young's modulus of the material for piles  $E_s$  = Young's modulus of the soil  $L_R$ ,  $B_R$ ,  $t_R$  = Length, breadth, and thickness of the raft  $v_s$  = Poisson's ratio of the soil

They reported that  $K_R$  varying from 10 to 0.01 represents very stiff to very flexible rafts while  $K_p$  ranging from 10<sup>5</sup> to 10<sup>2</sup> indicates very stiff to very compressible piles. Furthermore, they concluded that the settlement becomes more effective with the increase of pile stiffness and pile length. In addition, the results from the study suggested that increasing the raft flexibility would increase the differential settlement of piles, reduce the bending moments in the raft.

Randolph (1983) proposed a simple method, namely, the flexibility matrix method, to study the behavior of rafts by considering the interaction between piles and pile raft. The stiffness of rectangular piles rafts can be estimated by Equation 2.3,

$$K_{rs} = \frac{P_p + P_r}{w_{pr}} = \frac{k_p + k_r (1 - 2\alpha_{rp})}{1 - \left(\frac{k_r}{k_p}\right)\alpha_{rp}^2}$$
(2.3)

Where,  $P_p$  = Load carried by piles  $P_r$  = Load carried by raft  $w_{pr}$  = Average settlement of raft  $k_p$  = Stiffness of piles  $k_r$  = Stiffness of raft

 $\alpha_{rp}$  = Interaction factor between piles and raft

Ten years after Randolph (1994) made a modification to Equation 2.3 and proposed Equation 2.4 based on the finding reported in Clancy and Randolph (1993) that the interaction factor  $\alpha_{rp}$  tends to be equal to 0.8 for larger pile foundations.

$$K_{pr} = \frac{1 - 0.6 \left(\frac{k_r}{k_p}\right)}{1 - 0.64 \left(\frac{k_r}{k_p}\right)} k_p$$
(2.4)

Zaman et al. (1993) studied the effect of pile cap thickness and pile inclination on the distribution of displacements using 3D nonlinear finite element analysis. In the modeling, the soil medium was idealized as a generalized plasticity model in where both yielding and failure surfaces were defined by a single mathematical function, and the pile cap and the piles were assumed to be linearly elastic. The flexural rigidity of the pile cap ( $R_P$ ) is expressed as Equation 2.5. They concluded that the pile cap thickness and the pile inclination had a very minor effect on the flexural moment distribution in piles except at the pile head region where the shear force governs the design.

$$R_P = \frac{E_P t^3}{12(1 - v_p^2)} \tag{2.5}$$

Where,

 $E_p$  = Young's modulus of material for piles  $v_p$  = Poisson's ratio of the material for piles t = thickness of pile cap Duan and McBride (1995) investigated the effects of cap stiffness in order to improve the foundation design practice stipulated in the highway bridge foundation design practice of the California department of transportation (Caltrans) (1991). They suggested that a pile cap may be assumed to be rigid if the length to thickness ratio of the overhang is less than or equal to 2.2. where the cantilever is the edges of the overhang of the pile cap. Moreover, they reported that the pile reactions are nonlinearly distributed because of the different elastic spring constants for compression and tension piles and the flexural stiffness of the pile cap.

Ghali (1999) studied the effect of pile cap flexural rigidity and the piles' axial stiffness on the load transfer from column to piles. Ghali concluded that the flexural rigidity of the pile cap significantly affects the deformed shapes of the footing, behavior of the pile cap and the load distribution in piles as well. Most importantly, Ghali made the following recommendation: a pile cap can be considered rigid if the ratio of the distance between the centerline of a column and the centerline of the furthermost corner pile to the thickness of the pile cap is less than 2.4. Otherwise, a detailed finite element analysis is required for the design of the pile cap since a flexible behavior is expected.

Jeong et al. (2007) examined the behavior of interaction among pile cap, piles, and soil. Jeong et al. also conducted a parametric study of the effect of the elastic modulus, the thickness of the pile cap, the length and diameter of piles on the foundation performance as they were the main parameters to affect the cap flexibility. In addition, Jeong et al. concluded that the effect of the cap flexibility is more profound in a pile foundation with large pile diameter and large subgrade soil reaction. It also has significant effects on the forces in piles including the pile head forces, bending moments, and shear forces.

Abbas et al. (2008) evaluated the effect of pile cap system on the behavior of pile group subjected to the axial load, and the effect of pile cap thickness on the shearing forces in piles. They

developed a 2D finite element model for the study where the pile cap was molded as a plate element and the piles were modeled as a spring with a stiffness defined using hyperbolic stress-strain relationship. They reported that smaller pile cap thickness made pile cap more flexible. In addition, they concluded that the shearing forces in piles are affected by the following factors, the location of piles within the group, the pile cap thickness, the state of loading, and the restraint of the pile head.

Viggiani et al. (2012) studied the settlement and load sharing of piled rafts. In their study, two rafts were considered as illustrated in Fig. 2.1. A raft is considered to be a small raft if the ratio of the raft width B to the pile length L is less than 1, otherwise, it is considered as a large raft. The stiffness of small rafts  $K_{rs}$  given by Viggiani et al. is presented in Equation 2.6. Viggiani et al. reported that large rafts are not desirable since the rafts would be too flexible, thus, additional piles are required to avoid excessive settlement. Furthermore, in large rafts, changing the stiffness of rafts would not affect the average and the differential settlement.



Figure 2.1 Illustration of small and large pile rafts adapted from Viggiani et al. (2012).

$$K_{rs} = \frac{4E_r(1 - v_s^2)}{3E_s(1 - v_r^2)} \left(\frac{t_r}{B_r}\right)^3$$
(2.6)

Where,

 $E_r$ ,  $E_s$  = Young's modulus of material for raft and soil respectively

 $v_{r, Vs}$  = Poisson's ratio of the material for raft and soil respectively

 $t_r$ ,  $B_r$  = thickness, and width of raft respectively

#### 2.2.2 Thickness of rigid pile caps

Reynolds et al. (1988) proposed to estimate the thickness of pile cap (h) using Equation 2.7 based on the pile diameter ( $h_p$ ) to satisfy the requirements for anchorage and punching shear. Furthermore, they recommended that the cap thickness should be selected in such a way that bending moments in the column are not considered. However, when two or more piles are placed under one column it is necessary to reinforce the pile-cap for the moments and other forces imposed. Thirty years after, Rabbany et al. (2018) commended that the Reynolds guideline is "very safe but not economical and it's an empirical guideline".

$$h = 2h_p + 100 If h_p \ge 550mm h = \frac{1}{3}(8h_p - 600) If h_p \ge 550mm (2.7)$$

Cheng (2005) investigated the depth of reinforced concrete rigid-pile caps for tall buildings using three-dimensional finite element analysis. He recommended using cap curvature to validate the rigidity of the cap, i.e., a pile cap is considered rigid if a further increase in its depth will not lead to a significant reduction of the cap curvature. A design chart (Fig. 2.2) was provided in Cheng (2005) in which the cap curvature is represented by its out-of-plane deflection divided by the pile spacing. Furthermore, a tangent can be drawn on each graph to obtain the depth for a rigid pile cap.



Figure 2.2 Curvature versus depth of the cap for variable number of floors (Cheng, 2005).

In addition, Cheng compared the depths required for rigid pile caps from his study with those obtained to satisfy the design requirements for flexural shear failure (one-way shear), punching shear failure (two-way shear) and flexural bending failure. The results are presented in Fig. 2.3. As shown in the figure that the depth proposed in Cheng (2005), which is about 1m, works for buildings less than 45 floors. For buildings higher than 45 floors, the pile depth is governed by punching shear.



Figure 2.3 Comparison of the design depth of a pile ap with different methods (Cheng, 2005).

Rabbany et al. (2018) conducted a regression analysis on the pile cap depth from the Strutand-Tie method and the depth formula proposed by Reynolds et al. (1999), which were referred to as Reynolds chart in Fig. 2.4. The R-square was used as a statistical measure to fit the regression line as shown in Fig. 2.4. The minimum depth of the pile cap can be determined using this formula,



Figure 2.4 Regression analysis results in Rabbany et al. (2015).

 $y = 77.6x^{0.5}$  for a given external load, where *x* represents the load in ton and *y* represents the pile depth in mm. This was the first study to connect the pile cap thickness with the external loading.

### 2.3 Code procedures for the design of pile caps

#### 2.3.1 Canadian design standard A23.3-14

CSA A23.3-14 provides following Clauses for design of pile caps including Clause 15.3 (Footings and pile caps supporting circular and regular polygonal columns or pedestals), Clause 15.5 (Shear design of footings and pile caps), Clause 15.6 (Development of reinforcement in footings and pile caps), Clause 15.8.3 (Minimum depth of pile caps), and Clause 15.9 (Transfer of force at base of column, pile cap, wall, or pedestal).

There are methods stipulated in CSA A23.2-14, Part II (design examples and design aids) for the design of pile caps, i.e., the sectional method for shallow pile caps, the Strut-and-Tie (STM) method for deep pile caps. Shallow pile caps are defined where "the distance from the point of zero shear to the face of the column, pedestal, or wall is less than two times the effective shear depth,  $d_v$ , of the footing" (CSA, 2014). Both methods use the same way to determine the pile cap depth. They use the larger depth from steps 1 to 3 for the design of shallow pile caps and larger depth from steps 1 and 2 for the design of deep pile caps as follows:

- Step 1: Determine minimum effective shear depth  $d_v$  based on the requirements for a one-shear check using Clause 11.3.4 and 11.3.6 in CSA A23.3-14 at the critical section defined by Clauses 11.3.2, 15.2.3 and 15.5.3 depending on the pile arrangements.
- Step 2: Determine minimum depth *d* based on the requirements for a two-shear check using Clause 13.3.4 in CSA A23.3-14 at the critical section defined by Clauses 13.3.3, 15.2.3 and 15.5.3.
- Step 3: For shallow pile caps, corner pile shear checks must be conducted as per Clause 3.3.6.3.

In the calculation following the above-mentioned procedure, the design shear is determined by assuming the reaction in each pile due to external loading is the same. Specifically, pile reaction is obtained by using the total load in the column divided by the number of piles in the foundation. The difference between the sectional method and STM is related to the reinforcement design. For the sectional method, the reinforcement is determined based on Clauses 10 (Flexural and axial loads), 15 (Foundations), and 12 (Development and splices of reinforcement). For the STM, the minimum tie reinforcement, minimum pile cap reinforcement, and minimum reinforcement for interfaces should be determined as per Clause 15.9.2. The requirements for anchorage as per Clause 11.4 must also be respected. When using these methods, the following attention must be given:

- STM was developed for pile caps with simple geometries and for concentric loadings. For pile caps with complex geometries and for other loads including bending moments and shear, the sectional method should be applied, and its step-by-step procedure is described in CSA A23.3-14.
- The reinforcement provided by the sectional method would be significantly less than STM, the pile cap could be under reinforced by 20%.

#### 2.3.2 American design code ACI 318-14

The requirements for design of pile caps given in American Code ACI 318-14 include Clause 13.2.7 (Critical sections for shallow foundations and pile caps), Clause 13.2.8 (Development of reinforcement in footings and pile caps), and Clause 13.4.2 (Pile caps) that covers the minimum depth of pile caps and shear design of pile caps (such as one-way shear check, twoway shear check, and determination of factored shear for any section through a pile cap). In general, these requirements are not very different from those in CSA A23.4-14. It is not surprising given the design practice in the two countries are very similar.

Like CSA A23.3-14, ACI 318-14 also allows using STM for the design of pile caps. However, the requirements are much detailed to specify Design strength (Clause 23.3), Strength of struts (Clause 23.4), Strut reinforcement detailing (Clause 23.6), Strength of ties (Clause 23.7), Tie reinforcement detailing (Clause 23.8) and Clause 23.9 (Strength of nodal zones).

## **CHAPTER 3**

## NUMERICAL MODELING

### **3.1 Description of the Numerical Model**

Two foundations, one with 16 piles and the other with 9 piles were considered in the study. Figure 3.1 shows the elevation view of the foundations. The centerline of the piles is aligned in both directions. The pile cap is square with 8 m long which remains unchanged in all the cases for



Figure 3.1 Elevation view of the foundations: (a) 16-pile foundation, (b) 9-pile foundation.

the analysis. Both pile cap and piles are made of concrete. It is necessary to mention that the dimensions presented in Fig. 3.1 are the standard values for the two foundations under examination, such as a diameter of piles of 0.5m, a pile spacing of 2m for the 16-pile foundation and 3m for 9-pile foundation, and a pile length of 8.5m. As described in Chapter 1, this study investigated the following four parameters on the performance of a pile foundation including pile cap thickness, pile spacing, pile diameter, and pile length. The quantities examined for each

parameter are listed in Table 3.1. With respect to the soil mediums, both medium sand and dense sand are assigned to each of the foundations.

Parameter examined	Dimensions
Pile cap thickness	<b>Pile cap thickness:</b> 0.5m, 1.0m, 1.75m, 2.0m, and 3.0m
-	Pile diameter: 0.5m
	Pile spacing: 2.0m (16-pile foundation), 3.0m (9-pile foundation)
	Pile length: 8.5m
Pile spacing	Pile spacing:
	1.25m, 1.5m, 1.75m, 2m, and 2.25 (16-pile foundation)
	1.25m, 1.75m, 2m, 3m, and 3.25m (9-pile foundation)
	Pile cap thickness: 1.75m
	Pile diameter: 0.5m
	Pile length: 8.5m
Pile diameter	<b>Pile diameter:</b> 0.4m, 0.5m, 0.7m, and 1m
	Pile cap thickness: 1.75m
	Pile spacing: 2.0m (16-pile foundation), 3.0m (9-pile foundation)
	Pile length: 8.5m
Pile length	<b>Pile length:</b> 5m, 8.5m, 10m, and 14m
_	Pile cap thickness: 1.75m
	Pile diameter: 0.5m
	Pile spacing: 2.0m (16-pile foundation), 3.0m(9-pile foundation)

Table 3.1 Pile cap parameters considered in the modeling and analysis.

## **3.2 Modeling Techniques**

In this study, the three-dimensional finite element software ABAQUS was used to model the two pile foundations presented in Fig. 3.1. This software has been used many researchers to perform studies on geotechnical engineering (Alkinani et al., 2014; Riyadh et al., 2017; Wang et al., 2018; etc.). The detailed discussion on the techniques for modeling piles, pile cap, soil, boundary conditions, etc. is given in the sections below.

#### 3.2.1 Piles and pile cap

The two elements were model using ABAQUS 3D deformable homogeneous solid element C3D8 (i.e., Continuum, **3-D**, **8**-node). The C3D8 element has been used in several studies on the investigation of pile-pile cap foundations, such as Moayed et al. (2013), Fattah et al. (2013), Ata et al. (2014), etc. In the modeling, full integration instead of reduced integration was assigned to the element. This is due to the fact that the preliminary results indicate that reduced integration provided not correct displacements in the pile cap. As an example, Figure 3.2 shows a quarter of the finite element model for the 16-pile foundation.



Figure 3.2 ABAQUS <sup>1</sup>/<sub>4</sub> model: (a)16-pile foundation, (b) 9-pile foundation.

#### 3.2.2 Soil

Figure 3.3 shows a quarter of the ABAQUS model for the soil in the 16-pile foundation. The soil continuum in Fig. 3.3 is represented by a single layer of sand with a width of 10 m and a length of 17 m. Specifically, the width of the soil is taken as 2.5 times the width of the pile cap (i.e., 4m as only a <sup>1</sup>/<sub>4</sub> of the foundation was modeled) and the length is taken as 2 times the pile length (i.e., 8.5m).

The soil layer was modeled as an elastic-plastic constitutive model following Mohr-Coulomb yield criterion, i.e., yield occurs when the shear stress on any point in a material reaches a value (Equation 3.1) that depends linearly on the normal stress in the same plane.

$$\tau = c - \sigma \tan \phi \tag{3.1}$$

Where,

- $\tau =$  shear stress
- c = cohesion
- $\sigma$  = normal stress
- $\phi$  = angle of shearing resistance



Figure 3.3 ABAQUS <sup>1</sup>/<sub>4</sub> model for the soil in 16-pile foundation.

Other input parameters assigned in ABAQUS to define the soil material including Young's modulus of elasticity (*E*), which was taken from Geotechdata (Geotechdata.info, 2013) for medium and dense sand and it was constant along the depth of the soil, Poisson's ratio (v), friction angle ( $\phi$ ) and dilatancy angle ( $\psi$ ) (Table 3.2).

Parameter	Symbol	Soil		Concrete
		Dense-sand	Medium-sand	
Material density	$P(t/m^3)$	1.63	1.49	2.41
Young's modulus	E (MPa)	65	49	23600
Friction angle	φ (°)	40	30	
Poison's ratio	ν	0.35	0.29	0.21
Angle of dilatancy	ψ (°)	10	0	
Friction coefficient		0.55	0.45	

Table 3.2 Material properties.

#### **3.2.3 Interactions and contact zones**

In this study, three contact zones were defined in ABAQUS to simulate the interaction between pile cap and soil (Zone 1), pile circumference and soil (Zone 2), pile tip and soil (Zone 3) as illustrated in Fig. 3.4.

Using slave and master concept, all the three zones were simulated with surface-to-surface interaction, in which the nodes of soil elements were defined as Slave (red color in Fig. 3.4) while the nodes of pile cap and piles were defined as Master (pink color in Fig. 3.3). Furthermore, the contact zone was assigned by either friction contact (Type I) or frictionless contact (Type II) depending on the expected behavior of the zone. In particular, Type I contact was assigned to Zone 1 and Zone 2 as hard normal behavior and tangential behavior (Friction) with a Penalty method instead of Lagrange method because of its simplicity, good convergence and less

computation time. Type II contact was assigned to Zone 3 using tangential behavior (Frictionless) and hard normal behavior.





Figure 3.4 Contact zones defined in ABAQUS.

#### 3.2.4 Mesh

In ABAQUS, the model is partitioned as an assembly first before meshing is performed. Figure 3.5a illustrates partitions assigned in ABAQUS for the 16-pile foundation. As shown in Fig. 3.5a, the partitions are placed closer to the foundation than the rest of the soil in order to create finer elements around that area. In addition, the partitions were distanced equally around the pile to have uniform meshing for all the piles.

There are two options for meshing that are available in ABAQUS, i.e., structured meshing and sweep meshing. Specifically, the structured meshing is applied for high quality hexahedral or near to perfect shell elements required on solids or surfaces. Kumar (2018) reported that "this technique offers a better mesh control to the user compared to sweep meshing technique. It works by partitioning the complex solids into smaller six or eight-sided parametric solids that can be brick meshed. The nodes at the boundaries are then automatically attached to ensure connectivity". The sweep meshing is used when hexahedral elements are required on solids with minimal geometry editing. Kumar (2018) stated that "the technique automatically identifies a source side and a target side on the geometry, it creates a quadrilateral shell mesh on the source side and sweeps those elements to the target side thereby converting them to hexahedral or bricks". The green region is shown in Fig. 3.5a was meshed using the structured meshing and the yellow regions using the sweep meshing. Figure 3.5b shows the model for the 16-pile foundation after meshing.



Figure 3.5 ABAQUS meshing for <sup>1</sup>/<sub>4</sub> of the 16-pile foundation: (a) partitions assigned, (b) final meshed model.

Figure 3.6 shows the boundary conditions defined in the model. In the coordinate system, the positive direction for the vertical axis Z is downward, the positive direction for horizontal axes X and Y follows the well-known right-hand thumb rule. The restraints for the face nodes, corner nodes and bottom nodes assigned to translation (U) and rotation (UR) about given axes are provided in the figure.



Figure 3.6 Boundary conditions defined in the model.

### 3.3 Model techniques validation

In order to validate the two models developed in this study, the above-described modeling techniques were applied to a pile foundation system considered in Alnuiam et al. (2013) and the results were compared with Alnuiam's. In addition, the data collected from the geotechnical centrifuge testing available in Horikoshi et al. (2003) were also used to verify model techniques explained in Section 3.2.

Alnuiam et al. (2013) used the software Plaxis to create a 3D finite element model to study the performance of pile caps in Toyoura sand. Figure 3.7 shows a quarter model of the pile foundation system examined in which *B* is the width of the pile cap and  $L_p$  is the length of piles. They calibrated the finite element model (i.e., "Prototype" in Table 3.3) by centrifuge testing on an aluminum model (i.e., "Model" in Table 3.3) with parameters and dimensions listed in Table 3.3.



Figure 3.7 Finite element model as Alnuiam et al. (2013).

Parameter	Model	Prototype (n=50)
Diameter (mm)	10	500
Wall thickness (mm)	1	solid
Material	Aluminum	Concrete
Thickness	170 mm	8.5 m
Pile length	71 GPa	41.7 GPa
Modulus of elasticity	40 mm	2.0 m
Raft width (square)	80 mm	4 m
Pile Spacing	40 mm	2 m
Number of piles	4	4

Table 3.3 Basic information on models as in Alnuiam et al. (2013).

In their study, Cone penetration tests were performed in order to get the strength of the sand with the depth of the soil. Based on the test results the behavior of Toyoura sand was modeled

as an elastic-perfectly-plastic Mohr-coulomb constitutive model. Table 3.4 provides the material properties for both soil and concrete in the finite element modeling. It should be noted that Young's modulus is increasing with increasing depth.

Parameter	Soil	Concrete
Constitutive modeling	Mohr-Coulomb	Linear elastic
Unit weight (kN/m <sup>3</sup> )	14.6	23.6
Angle of internal friction	45	-
Modulus of elasticity	4500 kN/m <sup>2</sup>	$23.6 \text{ GN/m}^2$
Poisson's ratio	0.175	0.21
Stiffness increases with depth	Yes	No
Incremental modulus of elasticity	6500	-
Interface reduction factor	0.43	-

Table 3.4 Information on modeling as Alnuiam et al. (2013).

For the purpose of validation, a 3D ABAQUS model was developed for the pile foundation examined by Alnuiam et al. (2013) shown in Fig. 3.7 using the same geometric dimensions of the foundation (Table 3.3) and soil material properties (Table 3.5) following the modeling techniques described in Section 3.2. For ease of discussion, this model is referred to as FEM hereafter. Figure 3.8 presents the results of the displacement in the pile cap *vs* the axial load from FEM



Figure 3.8 Comparison of the results of pile cap displacement vs axial loading.

superimposed with the results given in Horikoshi et al. (2003) (Labelled as HOR in Fig. 3.8) and Alnuiam et al. (2013) (labeled as ALN in Fig. 3.8). It can be seen in the figure that FEM results are very close to HOR and ALN. For example, at the load of 5000 kN, the displacement given by FEM is about 0.032 m while the displacement provided by both HOR and ALN is about 0.029 and 0.031m respectively; at the load of 15000 kN, the displacement given by FEM is about 0.11 m while the displacement provided by HOR and ALN is the same, which is about 0.105 m.

Furthermore, the loads carried by piles from FEM were compared with those provided in Alnuiam et al. (2013) for the case with the pile cap thickness of 2 m, and the ratio of pile spacing to pile diameter of 4 and the results are shown in Fig. 3.9. In the figure, the vertical axis represents the percentage of the loads carried by the group of piles, and the horizontal axis represents the displacement of the pile cap normalized to its total displacement. It can be seen in the figure that



Figure 3.9 Comparison of the results of load sharing percentage vs pile cap displacement.

the two curves follow the same profile, i.e., At initial displacement, the load is carried by piles and this is due to the lack of intimate contact between the pile cap and the soil. However, as the displacement increases, the load carried by piles drops rapidly, e.g., 60% for ALN and 62% for FEM at the displacement of 7%. While, at 80% of the displacement, ALN shows 45% of the load is taken by piles, FEM shows 47% of the load is carried by piles.

The results in Figs. 3.8 and 3.9 have shown that the modeling techniques developed in this study to build up ABAQUS models are able to provide results in good agreement with those from finite element analysis given in Alnuiam et al. (2013) and experimental data reported in Horikoshi (2003). Therefore, it can be concluded that the modeling techniques described in Section 3.2 are acceptable to proceed with detailed analyses to examine the two pile foundations considered in this study.
# **CHAPTER 4**

# **ANALYSIS RESULTS**

## 4.1 General

As described in Chapter 1, the objective of this study is to check the effects of pile cap thickness, pile diameter, pile length, and pile spacing on the behavior of the pile-cap foundation. For this purpose, two foundations, one with 16 piles and the other with 9 piles, were examined; two soil mediums were tested, one is medium sand and the other is dense sand. A working load of 200000 kN was applied in the vertical direction (downward) at the center of the pile cap because it will cause higher deflections and deformations in both cap and piles compared to uniformly distributed vertical loading. The structural response parameters considered in the analysis are divided into two groups, i.e., one for piles and the other for pile cap, to consider

- Pile load carried by each individual pile, and load sharing between piles and pile cap
- Pile cap displacement in the vertical direction at selected points measured at the bottom face of the cap

## 4.2 Effect of pile-cap thickness

### Pile load resistance

Due to the symmetry of the foundation, a quarter of the pile-cap foundation (i.e., shaded area in Fig. 4.1) was examined in this study, in particular, four piles (i.e., Pile 1 to Pile 4 labeled in Fig. 4.1) were selected to investigate the amount of the Load developed in each individual pile under the external loading. The ultimate Load of each pile in the 16-pile foundation and 9-pile foundation for different pile cap thickness varied from 0.5m to 3m is presented in Figs. 4.2 and

4.3, respectively in which Figures 4.2a and 4.3a show the results for the medium sand while Figures 4.2b and 4.3b for dense sand. It should be noted that the Load given by each pile is equivalent to the load carried each pile under the external loading.



Figure 4.1 Layout of piles under examination: (a) 16-pile foundation, (b) 9-pile foundation.

For the 16-pile foundation, the results for the thickness of 0.5m (Fig. 4.2) clearly show that Pile 2 carries the largest amount of the load followed by Piles 1 and 4 while Pile 3 carries the least. This observation is not surprising from a structural point of view as the piles close to the loading point (center piles, e.g., Pile 2) would carry more load while the piles far from the loading (corner piles, e.g., Pile 3) would carry less load. More specifically, for cap thickness of 0.5m, the load resisted by Pile 2 (maximum Load) is 3.7 times that by Pile 3 (minimum Load) for the medium sand and 4.4 times for dense sand (Tables 4.). Furthermore, as illustrated in Fig. 4.2 when the thickness is increased to 1m, this ratio reduces dramatically reaching around 1.6 as listed in Table 4.1 for both soil mediums considered. When the thickness is between 1.5m and 2m this ratio becomes much smaller, i.e., about 1.3 for medium sand soil and 1.2 to1.1for dense sand soil. As presented in Fig.4.2 all 4 piles carry the same load at the thickness of 3m in dense sand soil. It is necessary to mention that once the cap thickness is 1m and above, the load distributed to Piles 1, 3 and 4 is almost the same (Fig. 4.2 and Table 4.1).



Figure 4.2 Pile Load *vs* pile cap thickness for the 16-pile foundation: (a) medium sand soil, (b) dense sand soil.

Soil tune Pile cap		Pile ID						
Son type	thickness (m)	Pile 1	Pile 2	Pile 3	Pile 4			
	0.50	811	1760	472	714			
	1.00	580	814	497	564			
Madium and	1.50	546	670	507	542			
Medium sand	1.75	544	651	513	542			
	2.00	546	644	519	545			
	3.00	567	650	545	569			
	0.50	672	1611	367	588			
Dense sand	1.00	577	774	488	558			
	1.50	543	606	516	536			
	1.75	541	581	526	537			
	2.00	543	569	535	541			
	3.00	568	560	569	567			

Table 4.1 Pile load for different pile cap thickness, 16-pile foundation.

For the 9-pile foundation, it can be seen in Fig. 4.3 for the cap thickness of 0.5m, a large load is developed in Pile 2, which is about 16-18 times in other piles as provided in Table 4.2. Such a result indicates the pile cap is much more flexible in the 9-pile foundation than in a 16-pile foundation in which the ratio was about 4 (Table 4.1). Another reason leading to such a huge

difference in the pile load between Pile 2 and the other three piles is the number of piles associated with the Pile ID (Table 4.2) is different. More specifically, in this foundation, there is only one pile labeled as Pile 2 while there are two piles labeled as Pile 1 and Pile 4, four as Pile 3. As illustrated in Table 4.2 once Pile 2 is removed in the calculation, the ratio of the maximum to the minimum pile load is reduced from 15.9 (medium sand) and 18.4 (dense sand) to 1.9. While for the thickness of 1m, this ratio is about 1.3, and for the thickness of 1.5m, 1.75m, 2m and 3m, the ratio is about 1.2 for medium sand soil, 1.1 to 1.2 for dense sand soil.



Figure 4.3 Pile load *vs* pile cap thickness for the 9-pile foundation: (a) medium sand soil, (b) dense sand soil.

Soil turno	Pile cap	Pile ID					
Son type	thickness (m)	Pile 1	Pile 2	Pile 3	Pile 4		
	0.50	1092	9251	582	1097		
	1.00	750	2182	572	742		
Madium and	1.50	704	1271	580	694		
Medium sand	1.75	701	1129	595	692		
	2.00	704	1043	593	694		
	3.00	732	951	622	721		
	0.50	919	8922	484	929		
Dense sand	1.00	752	2145	578	749		
	1.50	705	1229	599	699		
	1.75	701	1072	617	695		
	2.00	703	976	618	696		
	3.00	731	817	655	724		

Table 4.2 Pile load for different pile cap thickness, 9-pile foundation.

With respect to the effect of the soil medium on the pile load, the results in Figs. 4.2 and 4.3 indicate that dense soil tends to help distribute the load more uniformly across piles than medium sand. This is also supported by the results provided in Tables 4.1 and 4.2 since the ratios of the maximum load resistance to the minimum associated with the dense sand are relatively smaller than those associated with the medium sand.

Theoretically, all the piles would not be able to develop exactly the amount of load. In this study, the external load was considered to be evenly distributed to the piles once the difference of the load resistance in the piles is not greater than 20%. Given this, considering the results for both medium sand soil and dense sand soil, it can be noted that, among a variety of the cap thickness examined, a pile cap with a thickness of 1.75m or above is able to distribute the load uniformly to all the piles except the center pile in the 9-pile foundation. The results for the displacement of the pile cap provided the same conclusion, the detailed discussion is presented in Section 4.2.2.



Figure 4.4 Percentage of load sharing vs pile cap thickness: (a) 16-pile foundation, (b) 9-pile foundation.

Figures 4.4a and 4.4b present the load sharing between piles and pile cap for the 16-pile foundation and 9-pile foundation, respectively. It can be noted that at the cap thickness of 0.5m, most of the load is taken by piles. More specifically, for the 16-pile foundation, piles take about 60-70% of the total load and the pile cap only takes a small portion of the load, which is about 30-40%. For the 9-pile foundation, piles take even more load compared with the 16-pile foundation, i.e., about 75% of the load is taken by the piles while the pile cap takes about 25% of the load. Such results indicate that the pile cap is extremely flexible compared to piles, as a result, it could not be able to develop resistance again external loading. When the pile thickness reaches around 0.75m which is the average between 0.5m and 1m, both pile cap and piles share the same amount of the load, i.e., each takes 50% of the load. When the thickness becomes 1m and above, the load sharing between the pile cap and the piles does not change with the cap thickness. The percentage of the load carried by the pile cap is about 60% for the 16-pile foundation, 70% for the 9-pile foundation. Furthermore, the results in Fig. 4.4 illustrate that the soil medium does not affect the load sharing. It is necessary to mention that the self-weight of the pile cap was considered to determine the load sharing results presented in Fig. 4.4. In this study, it was noted that the self-weight of the cap would affect the load sharing percentage by 6% for the 16-pile foundation, and 4% for the 9-pile foundation.

#### Pile cap deformation

In addition, to examine the load carried by each individual pile, the deformation of the pile cap for different cap thickness was evaluated in this study. Figure 4.5 illustrates the 9 points for the 16-pile foundation, 4 points for the 9-pile foundation selected for this exercise.



Figure 4.5 Location of typical points to examine the displacement: (a) 16-pile foundation, (b) 9-pile foundation.

Figures 4.6 and 4.7 present the displacement at points A, B and C along lines 1, 2, and 3 for the 16-pile foundation for medium sand and dense sand, respectively while Figures 4.8 and 4.9 show the displacement at points A and B along lines 1 and 2 for the 9-pile foundation for medium sand and dense sand, respectively. The dashed line on each plot in the figures represents the average displacement based on the data presented for a given pile cap thickness. As expected, the cap displacement in the foundation with medium sand is larger than that in the foundation with dense sand. This is due to the fact that a loose soil medium provides less support to the pile cap as compared to a dense soil medium. It can be noted that in Figs. 4.6 to 4.9 that there is a great deal of variation among the displacements for the cap thickness of 0.5m especially for the results for the 9-pile foundation (Figs. 4.8 and 4.9).



Figure 4.6 Vertical displacement in pile cap in 16-pile foundation in medium sand soil.







Figure 4.8 Vertical displacement in pile cap in 9-pile foundation in medium sand soil.





On the other hand, the variation is decreasing with the increase of the cap thickness. When the thickness reaches 3m, the displacement becomes uniform that indicates the pile cap behaves like a rigid body. By comparing the discrete displacement with the average value for each cap thickness under investigation ranging from 0.5m to 3m, it is recommended that a pile cap would be considered as rigid if its thickness is not less than 1.75m. Moreover, Figure 4.10 presents the results of the cap curvature vs the cap thickness for the 16 and 9 pile foundations following the approach provided in Cheng (2005) (Figure 2.2, Chapter 2). Chen suggested that a pile cap could be considered rigid if a further increase in its thickness will not lead to a significant reduction of the cap curvature. Given this, a thickness of 1.5m would be considered as a threshold between a flexible cap and a rigid cap.



Figure 4.10 Curvature vs cap thickness for 9- and 16-pile foundation.

In order to examine the effect of the number of piles on the cap rigidity, the number of piles in the cap was varied between zero to 16. Figure 4.11 presents the settlement of the pile caps versus the cap thickness. It can be noted that the settlement decreases by increasing the number of piles in the cap. This can be explained by the fact that the piles act as stiffeners for the cap.



Figure 4.11 Center settlement of the pile cap foundation with variable cap thicknesses for different pile cap foundations.

## 4.3 Effect of pile spacing

Based on the observations discussed above, a thickness of 1.75m is selected for a cap to be rigid and it is being used in the next phase of the study to examine the effects of pile spacing, pile diameter and pile length on the performance of piles and pile cap.

In order to investigate the effect of pile spacing on the pile load resistance and pile cap deformation, for the 16-pile foundation, the pile spacing selected for the investigating is 1.25m, 1.5m, 1.75m, 2m, and 2.25m while for the 9-pile foundation, it is 1.25m, 1.75m, 2.25m, 2.75m, and 3m. Other parameters for this exercise are presented in Tables 3.1 and 3.2, Chapter 3.

#### Pile load resistance

Figures 4.12 and 4.13 show the ultimate load in Piles 1 to 4 for the 16-pile foundation and 9-pile foundation, respectively in which Figures 4.12a and 4.13a illustrate the results for the

foundation in medium sand while Figures 4.12b and 4.13b illustrate the results for the foundation in dense sand.

For the 16-pile foundation, it can be seen in Fig. 4.12 that there is a greater deal of variation on the load among the 5 pile spacings tested in the medium sand soil than in the dense sand soil. Specifically, for the medium sand soil, the ratio of the maximum to the minimum pile load for the five cases considered is around 1.3 while for the dense sand this ratio is much less which is not greater than 1.1 (Table 4.3). Furthermore, the results in Fig. 4.12b clearly demonstrate the load is uniformly distributed to the piles.



Figure 4.12 Pile load *vs* pile spacing for the 16-pile foundation: (a) medium sand soil, (b) dense sand soil.

Table 4.3 Pile	load for different	pile spacing,	16-pile foundation

Soil trmo	Pile spacing		Pile	Batia of the max /min		
Son type	(m)	Pile 1	Pile 2	Pile 3	Pile 4	Ratio of the max./mm.
	1.25	864	1048	797	868	1.32
	1.50	961	1150	881	961	1.31
Medium sand	1.75	907	1110	894	999	1.24
	2.00	918	1084	885	918	1.22
	2.25	834	988	779	815	1.27
	1.25	878	944	886	880	1.08
	1.50	948	989	954	949	1.04
Dense sand	1.75	927	984	970	984	1.06
	2.00	963	991	965	960	1.03
	2.25	912	975	880	888	1.11

For the 9-pile foundation, since the number of piles associated with each pile ID is not the same, as expected the variation in the data shown in Fig. 4.13 is much wider than that in Fig. 4.12. However, it is noted that the difference in the pile load developed among the piles is the same for the five pile spacings considered from 1.25m to 3.25m, which is 1.60 for the medium sand soil and 1.40 for the dense sand soil as listed in Table 4.4. If Pile 2 is eliminated in the calculation, the ratio is reduced to around 1.2 for the medium sand soil, 1.1 for the dense sand soil. According to the threshold defined above, the load in Piles 1, 3, and 4 could be considered as a uniform distribution.



Figure 4.13 Pile load *vs* pile spacing for the 9-pile foundation: (a) medium sand soil, (b) dense sand soil.

Soil trmo	Pile spacing		Pile	Batio of the may (min		
Son type	(m)	Pile 1	Pile 2	Pile 3	Pile 4	Ratio of the max./mm.
	1.25	1246	1650	1032	1229	1.60 (1.21)
	1.75	1226	1659	1029	1198	1.61 (1.19)
Medium sand	2.00	1219	1633	1020	1215	1.60 (1.20)
	3.00	1165	1612	1005	1134	1.60 (1.16)
	3.25	1157	1595	973	1160	1.64 (1.19)
	1.25	1235	1492	1078	1221	1.38 (1.15)
	1.75	1248	1549	1110	1229	1.40 (1.12)
Dense sand	2.00	1250	1558	1112	1248	1.40 (1.12)
	3.00	1258	1608	1116	1237	1.44 (1.13)
	3.25	1261	1614	1099	1264	1.47 (1.15)

Table 4.4 Pile load for different pile spacing, 9-pile foundation.

Based on the ratios of the maximum to the minimum pile load summarized in Tables 4.3 and 4.3, it was noted that ratio at the greatest spacing, i.e., 2.25m for the 16-pile foundation and 3.25m for 9-pile foundation, was larger than those ratios at smaller spacing. This is because piles farther apart make the cap less rigid compared with a cap where piles are closer to each other, which was approved by the results for the cap displacement (see Section 4.2.2).

Figure 4.14 presents the results for the load sharing between piles and pile cap for the five pile spacings considered in the analysis. It can be noted that the load sharing percentage does not



Figure 4.14 Percentage of load sharing *vs* pile spacing: (a) 16-pile foundation, (b) 9-pile foundation

change with the pile spacing. The load carried by the pile cap is about 60% in the case for the 16pile foundation and it is about 70% in the case for the 9-pile foundation, which is consistent with the finding observed in the investigation of the effect of the pile cap thickness on the response of the foundation as discussed in the previous section 4.1.1.

### Pile cap deformation

Figures 4.15 and 4.16 illustrate the displacement of the pile cap for the 16-pile and 9-pile foundation, respectively. The values for the pile spacing selected for presentation are 1.25m (lowest), 1.75m, and 2.25m (highest) for the 16-pile foundation; 1.25m (lowest), 2m, and 3.25m (highest) for the 9-pile foundation. The results in these figures show that the medium sand generates more displacement in the cap than the dense sand. This is because the medium sand provides less support to the cap than the dense sand. It is also noted in the figures that the variation in the displacement becomes wider with the increase of the pile spacing. For example, at the two smaller pile spacing (i.e., 1.25m and 1.75m for the 16-pile foundation, 1.25m, and 2m for the 9-pile foundation), the variation is not very noticeable. However, at the largest pile spacing (i.e., 2.25m for the 16-pile foundation, 3.25m for the 9-pile foundation), the variation is relatively larger, and it indicates that larger pile spacing tends to prevent the external load from being uniformly distributed across piles. This tendency can also be seen in Tables 4.3 and 4.4.



Figure 4.15 Vertical displacement in pile cap in 16-pile foundation: (a) medium sand soil, (b) dense sand soil.



Figure 4.16 Vertical displacement in pile cap in 9-pile foundation: (a) medium sand soil, (b) dense sand soil.

## 4.4 Effect of pile diameter

The diameters selected to examine the effect of pile diameter on the pile load resistance and pile cap deformation are 0.4m, 0.5m, 0.7m, and 1.0m. As mention above in Section 4.1.2, the thickness of the pile cap considered is 1.75, all other parameters used in the analysis are given in Tables 3.1 and 3.2, Chapter 3.

#### Pile load resistance

The load resistance developed in each of the 4 piles (layout is given in Fig. 4.1) associated with different pile diameters is presented in Figs. 4.17 and 4.18 for the case for the 16-pile foundation and the case for the 9-plie foundation, respectively. In each figure, one plot is for the medium sand soil and the other is for the dense sand soil.



Figure 4.17 Pile load *vs* pile diameter for the 16-pile foundation: (a) medium sand soil, (b) dense sand soil.



Figure 4.18 Pile load *vs* pile diameter for the 9-pile foundation: (a) medium sand soil, (b) dense sand soil.

It can be seen in Fig. 4.17, for the case for the 16-pile foundation with medium sand soil (Fig. 4.17a), the load carried by Piles 1, 3, and 4 is quite close while the load carried by Pile 2 is relatively high, which it is about 30% to 40% larger than that by Pile 3 taking the least load among the 4 piles, as shown in Table 4.5. It is also noted that for the 4 diameters considered, pile diameter of 0.5m leads to the smallest difference in the load among the 4 piles, which is also the case for the dense sand soil. Compare with a wider distribution of the load in the piles observed in Fig. 4.17a, the load developed in each pile in the dense sand soil is very close. Although it is seen in the figure that, at the diameter of 1m, the load provided by Pile 2 is relatively higher than that by the other three piles, the difference is not very significant, instead, it is only about 20%. According to the threshold proposed in Section 4.1.2 (i.e., if the load difference in the piles is less than 20%, then the load in each pile is considered approximately the same), the external load would be considered evenly distributed to piles for the four pile diameters considered in this study.

As discussed in Sections 4.1 and 4.2, the results shown in Fig. 4.18 for the 9-pile foundation is not a surprise in which the load provided by Pile 2 is much higher than that provided by Piles 1, 3, and 4 while the load developed in these three piles is very close. More specifically, when Pile 2 is eliminated to determine the difference between the largest and the smallest load in the pile, the ratio is about 15% as given in Table 4.6. Therefore, it can be concluded that Piles 1, 3 and 4 take equal loads.

Soil trmo	Pile diameter		Pile	Potio of the may /min		
Son type	(m)	Pile 1	Pile 2	Pile 3	Pile 4	Ratio of the max./mm.
	0.4	473	520	400	428	1.30
Madium cond	0.5	544	651	513	542	1.27
Medium sand	0.7	759	902	669	774	1.35
	1.0	961	1290	893	993	1.44
	0.4	426	456	418	427	1.09
Dense sand	0.5	541	581	526	537	1.11
	0.7	751	789	693	734	1.14
	1.0	936	1111	927	967	1.20

Table 4.5 Pile load for different pile diameter, 16-pile foundation.

Soil trac	Pile diameter		Pile	Patio of the max /min		
Son type	(m)	Pile 1	Pile 2	Pile 3	Pile 4	Ratio of the max./mm.
	0.4	497	779	468	497	1.66 (1.06)
Madium cand	0.5	701	1067	587	692	1.82 (1.20)
Medium sand	0.7	1013	1638	855	1003	1.92 (1.19)
	1.0	1409	2472	1227	1451	2.01 (1.18)
	0.4	517	794	486	517	1.63 (1.06)
Dense sand	0.5	701	1013	608	695	1.66 (1.15)
	0.7	1007	1615	869	998	1.86 (1.16)
	1.0	1373	2265	1232	1418	1.84 (1.15)

Table 4.6 Pile load for different pile diameter, 9-pile foundation.

It is worth mentioning that the load developed in the piles linearly increases with the increase of the pile diameter as illustrated in Fig. 4.15 and 4.16. This is because the axial stiffness of the pile is linearly proportional to its diameter.

Figures 4.17a and 4.17b present the results for the load sharing between piles and pile cap associated with different pile diameter. It is interesting to observe in the figure that the percentage of the load sharing changes linearly with the increase of the pile diameter. For the 16-pile foundation, at the pile diameter of 0.7m, piles and pile cap share the load equally, i.e., each component takes 50% of the load. At the pile diameter smaller than 0.7m, most of the load is carried by the pile cap, which is consistent with the finding from the investigation into the effect of the pile cap thickness and pile spacing as discussed in the previous Sections 4.1 and 4.2. However, when the diameter becomes larger than 0.7m, the group of piles carries most of the load.



Figure 4.19 Percentage of load sharing *vs* pile diameter: (a) 16-pile foundation, (b) 9-pile foundation.

For example, at the diameter of 1m, 80% of the load is taken by the piles and only 20% of the load is taken by the pile cap. Such a significant difference in the load sharing between the two components might not be appreciated for the design purpose, i.e., the cap would be designed for a little load while piles would be designed for a heavy load. For the 9-pile foundation, diameter of 0.9m is a threshold point, i.e., when the diameter is smaller than 0.9, pile cap carries a great portion of the load which is about 60%-80%; when the diameter is larger than 0.9m (i.e., 1m considered

in the examination), pile cap and group of piles each takes about 50% of the load. Comparing the results between the 16-pile foundation and the 9-pile foundation, it is seen in Fig. 4.19 that the cap in the 9-pile foundation carries about 10%-20% more load than that in the 16-pile foundation. This is due to the significantly reduced number of piles (i.e., 9 vs 16) in the foundation.

### Pile cap deformation

Figures 4.20 and 4.21 show the results of the cap displacement for the 16-pile foundation and 9-pile foundation, respectively. The diameters chosen for the presentation are the smallest diameter of 0.4m and the largest diameter of 1m used in the analysis. It can be noted that the variation in the displacement does vary with the change of the pile diameter for all the cases (16pile foundation and 9-pile foundation, both medium sand soil and dense sand soil). Where the variation of the displacement associated with the diameter of 0.4m is slightly larger than that of the 1m. It can be concluded then that the pile diameter does slightly affect the displacement of the pile cap.



Figure 4.20 Vertical displacement in pile cap in 16-pile foundation: (a) medium sand soil, (b) dense sand soil.



Figure 4.21 Vertical displacement in pile cap in 9-pile foundation: (a) medium sand soil, (b) dense sand soil.

## 4.5 Effect of pile length

The pile length considered in the investigation of its effect on the performance of the foundation is 5m, 8.5m, 10m, and 14m. As used in the analysis for the pile spacing and pile diameter, the thickness of the pile cap is taken as 1.75m in this examination while the values of all other parameters are given in Tables 3.1 and 3.2, Chapter 3. Figures 4.22 and 4.23 present the ultimate Load in each of the four selected piles for the 16-pile and 9-pile foundation, respectively. The layout of the four piles is illustrated in Fig. 4.1.



Figure 4.22 Pile load *vs* pile length for the 16-pile foundation: (a) medium sand soil, (b) dense sand soil.



Figure 4.23 Pile load *vs* pile length for the 9-pile foundation: (a) medium sand soil, (b) dense sand soil.

### Pile load resistance

It can be seen in Fig. 4.22a (for the 16-pile foundation, medium, and dense sand soil) that the load developed in Pile 2 is quite larger compared to that developed in the other three piles while the load in Piles 1, 3, and 4 is very close. More specifically, as shown in Table 2.7, the ratio of the resistance in Pile 2 (maximum load) to that in Pile 3 (minimum load) ranges from about 1.15 for the pile length of 5m to 1.40 for the pile length of 14m. However, for the dense sand soil, the load generated in Pile 2 is not very much different from that in Piles 1, 3 and 4 like observed in the results for the medium sand soil. Instead, the load developed in Pile 2 is about 10% higher than in the other three piles (Table 4.7). Considering such a small difference in the results, the load distribution among the piles can be treated uniformly distributed.

With respect to the results of the 9-pile foundation (Fig.4.23), like the observation discussion in the previous sections and due to the layout of piles in the foundation, there is no doubt that the Load in Pile 2 is significantly larger than in the other three piles. For example, the Load in Pile 2 is about two times in Pile 3 for the pile length of 5m (Table 4.8). However, if Pile 2 is not considered to determine the difference between the load resistance in piles, this ratio is reduced dramatically from 2.11 to 1.28 as shown in Table 4.8. The results in Table 4.8 indicate that the load resistance could be considered uniformly distributed among Piles 1, 3, and 4 for both medium and dense sand soil for all the four values of the pile length under examination, except for the pile length of 5m in medium sand since the ratio of the difference in the load resistance among the four piles is too high to be considered to be uniform.

Furthermore, it can be seen clearly in Figs. 4.22 and 4.23 that the load in Pile 2 (i.e., center pile) increases linearly with the increasing of the pile length. However, this increase grows much faster in the dense sand soil than in the medium sand soil for both the 16-pile and 9-pile foundation. In addition, the results in Tables 4.7 and 4.8 demonstrate that the difference in the load resistance with the increasing of the pile length. It indicates that a larger pile length helps to achieve even load distribution among the piles.

Soil trmo	Pile length		Pile	Datio of the may /min		
Son type	(m)	Pile 1	Pile 2	Pile 3	Pile 4	Ratio of the max./mm.
	5.0	430	540	382	426	1.41
Medium sand	8.5	544	651	513	542	1.27
	10.0	597	674	551	594	1.22
	14.0	698	760	665	694	1.14
	5.0	455	492	447	450	1.10
Dense sand	8.5	541	581	526	537	1.11
	10.0	577	618	562	572	1.10
	14.0	688	738	669	683	1.10

Table 4.7 Pile load for different pile length, 16-pile foundation.

Table 4.8 Pile load for different pile length, 9-pile foundation.

Soil type	Pile length		Pile	Patio of the max /min		
Son type	(m)	Pile 1	Pile 2	Pile 3	Pile 4	Ratio of the max./mm.
	5.0	545	897	426	535	2.11 (1.28)
Modium cond	8.5	701	1067	587	692	1.82 (1.20)
wiedrum sand	10.0	772	1105	641	762	1.72 (1.20)
	14.0	930	1262	796	920	1.59 (1.17)
	5.0	573	876	495	568	1.77 (1.16)
Dense sand	8.5	701	1013	608	695	1.66 (1.15)
	10.0	758	1075	661	753	1.63 (1.15)
	14.0	940	1278	820	936	1.56 (1.15)

The results of the load sharing between piles and pile cap (Fig. 4.24) show that this sharing changes linearly with the increase of the pile length and the soil medium does not affect the load sharing. Given the observations from the results of the pile load resistance, it is expected that the percentage of the load carried by the piles is higher for a larger pile length. For the largest pile length tested, for the 16-pile foundation, piles, and pile cap share the same amount of the load (i.e.50%) while for the 9-pile foundation, piles carry about 35% of the load and pile cap carries 65% of the load. Once the pile length is reduced, the percentage of the load shared by the piles is also reduced. For example, for the 16-pile foundation, for the pile length of 5m, piles take about 30% of the load while the pile cap takes about 70% of the load; for the 9-pile foundation, for the same length, these two percentages are 20% and 80%, respectively.



Figure 4.24 Percentage of load sharing vs pile length: (a) 16-pile foundation, (b) 9-pile foundation.

### Pile cap deformation

Figures 4.25 and 4.26 show the results of the pile cap displacement for the 16-pile foundation and 9-pile foundation, respectively, for the pile length of 5m (Figs. 4.25a and 4.26a) and 14m (Figs. 4.25b and 4.26b). The findings of the results are consistent with those obtained in the evaluation of the effects of pile cap thickness, pile diameter and pile spacing on the response

of the foundation, namely, (i) the vertical displacement of the pile cap is smaller in the dense sand soil than in the medium sand soil, (ii) the variation in the displacement remain the same regardless of the quantity of the parameter under investigation.



Figure 4.25 Vertical displacement in pile cap in 16-pile foundation: (a) medium sand soil, (b) dense sand soil.



Figure 4.26 Vertical displacement in pile cap in 9-pile foundation: (a) medium sand soil, (b) dense sand soil.

However, the results in Figs. 4.25 and 4.26 do show that the displacement changes significantly with the change of the pile length, which was not observed in the results from examining pile cap thickness, pile diameter and pile spacing as discussed in the sections above. In particular, the pile displacement corresponding to the pile length of 14m is about twice that corresponding to the pile length of 5m, for the 16-pile foundation for both soil mediums of medium sand and dense sand. For the 9-pile foundation, this difference reduced slightly to reach 1.5. Such a difference is because the peripheral surface area of 14m-long piles is much larger than that of 5m-long piles. As a result, 14m-long piles provide larger friction Load to the pile cap than 5m-long piles, and accordingly, the pile cap supported by 14m-long piles deforms less than that by 5m long piles.

# **CHAPTER 5**

# **CONCLUSIONS AND RECOMMENDATIONS**

### 5.1 Summary and Conclusions

In pile foundations, the cap is designed to transfer the loads from the superstructure to the piles. The structural design of the cap stipulates that the cap will distribute the load uniformly on the piles. The cap rigidity plays an important role in the loads taken by each individual pile. For a flexible cap, some piles within the group may be overloaded, while others may carry less or be separated from the cap. However, a relatively rigid cap may impose additional load on the piles. Thus, the design of the cap in a pile foundation must be optimized to meet the design requirement without imposing additional load.

In this thesis, a 3-D finite element model was developed to simulate the case of pile/cap/soil system to examine the effects of pile spacing, pile diameter, pile length on the rigidity on the pile cap and accordingly, the load distribution on the piles within the group. The software ABAQUS was used in this analysis. The model was validated by the data available in the literature. Based on the results obtained in this study, the following was concluded:

### With respect to the pile cap thickness:

- 1. The thickness of the pile cap acts as flexible up to a certain thickness, beyond which the cap acts as a rigid slab, where the loads are almost distributed evenly on the piles. The thickness of 1.75 m is recommended as a lower bond for a pile cap to be rigid.
- CSA A23.3-14 specifications for the design of pile caps lead to overestimating the pile cap thickness.

3. The additional load (i.e., load from self-weight) in the cap and piles due to increasing pile cap thickness is about 3% of the total load. Therefore, this extra load should not be a concern in the selection of cap thickness.

### With respect to the pile cap rigidity:

- 1. It increases with increasing pile diameter. This is due to the fact that by increasing the pile diameter, the contact area of piles and the cap increases, which will further reduce the settlement.
- 2. It increases with an increasing number of piles, as piles act as stiffeners to the cap.
- 3. It increases with an increasing pile length.

### With respect to the load sharing:

- 1. Pile spacing does not affect the load sharing between the cap and the group of piles.
- 2. Load sharing increased linearly with increasing pile diameter and pile length.

### **5.3 Recommendations for future research**

- Examine the case of a variety of pile cap geometry in terms of length, width, and thickness for a given layout of the piles to develop a threshold of the relative rigidity between a pile cap and a group of piles such that the loads in piles are well distributed.
- 2. Introduce an optimum percentage for load sharing between a pile cap and a group of piles to achieve the most economical design for both pile cap and piles.
- 3. Investigate the nonlinear behavior of piles.
- 4. Extend the current study to include bending moment and lateral load as external loading.

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