Costs and Benefits of Telling Children the Quantitative Meaning of Manipulatives

Emmanuelle Adrien

A Thesis

in

The Department

of

Education

Presented in Partial Fulfillment of the Requirements

For the Degree of

Doctor of Philosophy (Education) at

Concordia University

Montreal, Quebec, Canada

February 2020

CONCORDIA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

This is to certify that the thesis prepared

By: Emmanuelle Adrien

Entitled: Costs and Benefits of Telling Children the Quantitative Meaning of Manipulatives

and submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy (Education)

complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

Signed by the final Examining Committee:

______________________________ Chair

Dr. Saul Carliner

______________________________ External Examiner

Dr. Emily R. Fyfe

______________________________ External to Program

Dr. Adam S. Radomsky

______________________________ Examiner

Dr. Sandra Martin-Chang

______________________________ Examiner

Dr. Holly E. Recchia

______________________________ Thesis Supervisor

Dr. Helena P. Osana

Approved by ____

Dr. Sara Kennedy, Chair of Department of Education

___________ 2020 ____

Dr. André G. Roy, Dean of Faculty of Arts and Science

Abstract

Costs and Benefits of Telling Children the Quantitative Meaning of Manipulatives

Emmanuelle Adrien, PhD
Concordia University, 2020

The objective of the present study was to identify the costs and benefits of directly telling students the quantitative referents for manipulatives compared to allowing them to construct meaning for the manipulatives in more open and exploratory learning environments. Sixty-five ($N = 65$) first graders were randomly assigned to one of three conditions that differed in the type of encoding instruction they received: direct instruction (DI), guided exploration (GE), or control. The overarching research question was: How do the ways in which children assign a quantitative referent to a target manipulative (DI vs. GE vs. control) influence their (a) learning, (b) near-transfer abilities, (c) symbolic flexibility and symbolic fluency through far-transfer tasks, and (e) problem-solving accuracy?

Results indicated that direct instruction seemed to be most beneficial for children's learning. In terms of the learning assessment, children from the DI condition benefitted relative to children in the GE condition, in that they needed fewer items and less time before using the target manipulative in the prescribed way. Evidence suggested that children in the DI condition also outperformed their counterparts in the GE condition on a near-transfer task when looking at their initial responses, but when both initial and post-prompt responses were considered, the performance of children in the GE condition was not significantly different from the performance of children in the DI condition. In contrast, students who learned through guided exploration seemed to be more flexible in their use and interpretation of the manipulatives in the context of the far-transfer tasks than those who were told explicitly what the objects represented. The greater flexibility demonstrated by children in the GE condition also conferred an advantage on their accuracy when solving word problems with the manipulatives compared to children in the DI condition.

This study contributes to the existing literature in that it offers a nuanced view of the use of manipulatives in classroom contexts. Results suggest that teachers may wish to tailor their

instructional methods to the learning objectives (e.g., learning, near transfer, far transfer) they have set for their students when using concrete representations with them.

Acknowledgements

I first want to thank my supervisor, Dr. Helena Osana. Thank you for the invaluable guidance and mentorship you have provided me this past decade. Thank you for believing in my abilities and for offering me so many opportunities in research and teaching. Your passion for all things related to teaching and learning mathematics is contagious, and I always left our numerous meetings together feeling excited and motivated. I could not have done all that I have accomplished without you.

I want to thank the faculty members in the Child Studies and Early Childhood Education programs whom have I come to know well since I first started my graduate studies at Concordia and from whom I have received continued support and encouragement. I also want to express my gratitude to Dr. Sandra Martin-Chang and Dr. Holly Recchia for being part of my doctoral committee. Thank you for all your support and input throughout this journey. Thank you to Dr. Emily Fyfe and Dr. Adam Radomsky for agreeing to sit on my Examination Committee and to Dr. Saul Carliner for chairing the defense.

I want to acknowledge the financial support I received from the Social Sciences and Humanities Research Council of Canada for my doctoral research. I also want to express my gratitude to the schools and teachers who welcomed me into their classrooms and to the students who agreed to participate in the study. A big thank you to Alexandra Maduro and Kiera Vaccaro for their help with data collection, to Cassandra Karam for her help with inter-rater reliability, and to Arielle Orsini who collected and coded part of the data. Thank you to all the members (past and present) of the Osana Mathematics Teaching and Learning Lab for your support.

I am fortunate to have an amazing support system made up of family members and friends without whom this accomplishment could not have been possible. Thank you to all the friends I made in graduate school who kept me motivated with work dates and chats about our academic successes and uncertainties. Thank you to the longtime friends who have watched me be a student for so, so long, and who have made sure I keep a balanced lifestyle that includes fun dinners, drinks, and lots of laughter. Thank you to my immediate and extended family for always believing in me and encouraging me. Merci Papa de m'avoir donné le goût de la recherche et pour toute l'aide et l'amour que tu m'as toujours apportés. Merci Noé d'être ma meilleure amie. C'est un grand réconfort de savoir que je peux toujours compter sur toi. Merci Louis pour tout ton support et pour tes encouragements. Thank you Marc for being my rock and my number-one fan, for loving me so much, and for believing in me through the good times and the more difficult ones. Merci Anissa, mon amour chérie, de me rendre si heureuse et de m'avoir aidée à savoir prioriser les choses importantes de la vie. Merci Maman pour tout. Tu m'as toujours encouragée et cru en moi. Tu m'as vu commencer ce long processus et tu me vois le terminer de là-haut. Je sais que tu es fière de moi. Tu me manques et je t'aime.

Contribution of Authors

The work presented in this dissertation and in the references below stems from a study that I designed and carried out in collaboration with my supervisor, Dr. Helena P. Osana. David Uttal provided valuable input and feedback during the elaboration phase of the study. Arielle Orsini worked as a research assistant on the project, and she collected and scored a portion of the data. As the lead author, I was the primary researcher, and I was responsible for collecting participant data, providing training to research assistants, scoring and analyzing the data, and reporting the results.

I presented a part of the results of this dissertation research at the Mathematical Cognition Learning Society (MCLS) conference in June 2019:

Adrien, E., Osana, H. P., Uttal, D., & Orsini, A. (2019, June). *How does type of instruction influence children's use of manipulatives?* [Paper presentation]. Mathematical Cognition and Learning Society conference, Ottawa, Canada.

A new research question emerged from the dissertation data. The results for this new research question are not reported here, but I presented them at the Society for Research in Child Development (SRCD) biennial meeting in March 2019:

Adrien, E., Osana, H. P., Uttal, D., & Orsini, A. (2019, March). *Instructional environment influences how first-graders interact with math manipulatives* [Poster presentation]. Society for Research in Child Development biennial meeting, Baltimore, MD.

Table of Contents

List of Figures

List of Tables

Chapter 1: Statement of the Problem

Mathematics is a discipline in which representations, such as written symbols or concrete models, are ubiquitous. Developing conceptual knowledge of mathematical ideas requires knowing the links between various representations and their referents (Dufour-Janvier et al., 1987; Donovan & Fyfe, 2019; Hiebert, 1984). Teachers have an active role to play in the development of children's understanding of the connections between representations and mathematical notions. The instructional approach through which children will come to make these links might influence how they integrate the underlying concepts and how they use the representations in their mathematical activities (Gravemeijer, 2002).

In addition to using written symbols to represent quantities or other mathematical concepts, physical representations can be used during mathematics instruction. Manipulatives are concrete objects meant to help students represent and understand abstract mathematical ideas, and they are omnipresent in elementary school classrooms (Hiebert, 1992; McNeil & Jarvin, 2007; Moch, 2001; Moyer, 2001). While manipulative-based instruction has been shown to have benefits on different learning outcomes (Marley & Carbonneau, 2014a), the specific conditions under which it is most effective are still not well understood. Current research in this area is moving away from determining *whether* manipulative-based instruction is beneficial and focusing instead on *how* manipulatives should be used in instructional settings (Donovan & Fyfe, 2019; Lazonder & Harmsen, 2016; Wise & O'Neill, 2009).

Several factors of instruction with manipulatives have been studied and include students' developmental level (e.g., DeLoache et al., 1999), students' individual differences in terms of prior knowledge, cognitive factors, students' own interpretations (e.g., Fyfe & Rittle-Johnson, 2016; Osana, Przednowek, et al., 2018), the nature and perceptual features of the manipulatives (e.g., Chao et al., 2000; Petersen & McNeil, 2003), and the structure of the learning environment, including the level of instructional guidance (e.g., Brown et al., 2009; Carbonneau & Marley, 2015; Martin & Schwartz, 2005). Results from existing research are not always conclusive, and the factors listed above might interact to impact the effectiveness of manipulatives-based instruction. More research is therefore necessary to tease apart the contributions of these instructional features and better understand under which exact conditions instruction with manipulatives will be beneficial and for whom.

The current study focuses on the instructional approaches used when teaching with manipulatives. There is evidence that using direct instruction in mathematics instruction leads to positive learning outcomes (see Alfieri et al., 2011) and that when teaching with manipulatives, teachers should make explicit links between the concrete representations and their intended referents (Carbonneau et al., 2013; Carbonneau & Marley, 2015; Osana, Przednowek, et al., 2018). Other researchers, however, believe there are benefits to children constructing their own understandings with scaffolds or guidance from a more knowledgeable other (see Brown et al., 2009). Such benefits include children being better able to abstract the concepts conveyed by the representations, which in turn, might help them transfer knowledge to different contexts and be more flexible in their use and interpretations of the manipulatives (Bonawitz et al., 2011; Brown et al., 2009; Martin & Schwartz, 2005). More research that compares different instructional methods is therefore necessary to better understand how to use manipulatives with children effectively (Carbonneau & Marley, 2015).

The objective of the present study was to identify the costs and benefits of directly telling young elementary school students the quantitative referents for manipulatives compared to allowing them to construct meaning for the manipulatives in more open and exploratory learning environments (Martin & Schwartz, 2005). The present study aimed to extend the literature and examine how the type of instruction young children receive influences how they assign quantitative referents to manipulatives and how they use the manipulatives in learning and transfer tasks. Students were randomly assigned to one of three encoding conditions (direct instruction, guided exploration, control) to develop a specific representation-referent relationship for a target manipulative. One objective of this study was to shed light on the costs and benefits of direct instruction (compared to guided and control) in a manipulative-based instruction context and to draw conclusions on the practices teachers should adopt in the classroom.

Chapter 2: Literature Review

Adults and children alike come across multiple external representations every day (e.g., traffic signs that govern how to conduct oneself on the road; school textbooks that contain pictorial representations, words, and numbers), and they are studied in diverse fields such as linguistics, semiotics, and mathematics (Dufour-Janvier et al., 1987; Mason, 1987). Creating, understanding, and using symbols is believed by some to be the hallmark of human cognition (DeLoache et al., 1999). There are two types of representations: internal representations, which are the mental images that we automatically construct of our reality; and external representations, which have been created to represent a specific reality (DeLoache et al., 1999; Dufour-Janvier et al., 1987). All representations have something in common; they involve two entities: the representing world (or symbol, or signifier) and the represented world (or symbolized, referent, signified; Dufour-Janvier et al., 1987; Gravemeijer, 2002; Kaput, 1987a; Palmer, 1977). Representations, then, stand for or take the place of something else, and they can be of various types, such as concrete materials, spoken symbols, or markings on paper (Behr et al., 1983; DeLoache, 1995; Hiebert, 1988; Hiebert et al., 1997). In the literature, the terms *representations* and *symbols* are often used to refer to the same idea, and I will use them interchangeably. Virtually anything can be a symbol; all that is needed is for someone to intend that some entity stand for something other than itself (Gelman & Ebeling, 1998; Myers & Liben, 2008; Sharon, 2005). Relatedly, anything can be a referent to a symbol, as long as someone specifies a link between it and the entity that it is supposed to represent (DeLoache et al., 1999). Adults have come to take for granted many symbol-referent relations (DeLoache et al., 1998).

The process of symbolizing refers to the passage from the represented world to the representing world (Mason, 1987). When symbolizing or modeling, the original situation needs to be simplified by ignoring the aspects that are less relevant and focusing on those that are more relevant, and a mapping needs to be established between the original situation and the model (Lesh et al., 1987). Kaput (1987b) defined a *symbol scheme* as a collection of characters governed by some more or less explicit rules for their identification and combination. A *symbol system* refers to a symbol scheme, along with its field of reference, and a systematic rule of correspondence existing between the latter two. Whenever someone encounters a set of symbols, he or she will need to read or interpret them and encode the information intended by the symbols.

The uses and advantages of using external representations or symbols are numerous. They can reduce memory load or increase storage capacity, and they can allow their users to encode information in a more manipulable format (Behr et al., 1983), which makes them tools that can be used to communicate. Communication is one of the main goals of symbolic systems (Callaghan & Corbit, 2015). When thinking about symbols that are in the form of marks on paper, one can see that they have many advantages: they are mobile, immutable when transported, reproduced easily, and can be combined and superimposed to allow for an examination of structures and patterns (Latour, 1990).

Representations can also be supports for learning. They can help simplify complex relationships (Behr et al., 1983) and develop conceptual understanding because they can play an intermediary role: when students develop meaning for the tools they use, they can then use these tools to help them develop meaning for other concepts (Hiebert et al., 1997). Representations can also serve as a tool for thinking. Vygotsky (1978) argued that the kinds of tools people use can influence their thoughts in different ways: they can help connect current experiences with past experiences; they can extend mental capabilities by freeing thinking; and they can shape the way people think about problems and influence the problem-solving methods they will develop and use.

Development of Children's Symbolization Abilities

From the time of infancy, children are exposed to a vast range of representations and these early experiences will have an influence on their learning of new representations (Sherin, 2000). Young children have nascent understandings of the relations between representations and their referents. At a very young age, they are able to use symbolic representations to identify and locate objects. DeLoache (see 1987; 1991; 1995) conducted several studies with 2 ½- and 3-year-old children, in which she used a model room (i.e., a small-scale replica) as a symbol for an actual room with furniture. Children were shown the similarities and correspondences between the room and its model. They then watched as the experimenter hid a toy in the model room (e.g., behind the couch). Children were asked to retrieve a hidden toy from the life-size room that could be found in the same location as what they had seen in the model room. It is only at around 3 years of age (not 2 ½) that children understand that the small-scale room is a symbol that stands for the normal-size room and that they can use this information to help them find the hidden toy. DeLoache (1995) called the ability to view an object as its own entity ***and*** as a representation of something else ***dual representation.***

In their pre-school and early school years, children develop symbolic thought; they become increasingly skilled in using different symbol systems to interpret and express meanings (Bialystok, 1992a; Hiebert, 1988). This cognitive development is believed to be driven by children's need to communicate with others (Rochat & Callaghan, 2005). Understanding the symbol-referent relationship for written symbols such as numbers is more challenging than understanding spatial relationships, and it is only at around the age of 4 or 5 that children are able to identify and distinguish written numbers, and attribute functions to them (Martí et al., 2013). Bialystok (1992b) identified three stages for the development of symbolic representations for written letters and numbers. First, children learn how to recite, by rote, each name for the symbols found in each system. In other words, they are able to recite the alphabet and the number sequence. Second, they become able to recognize and correctly name the written notation corresponding to each letter and number and also to produce the written form for these. Lastly, they come to understand the significance (or the referent) for each letter and number. It is not until the age of 6 that most children fully understand the relation between the symbols and the specific meaning of what they represent. Understanding the symbol-referent relation, however, is a necessary but not a sufficient factor for children to be able to use that information to perform an action, such as reading, writing, or performing written computations (Bialystok, 1992b). Children's understanding of symbols might not develop at the same time and pace in all domains; it is hypothesized that their ability to use number notations symbolically occur before their understanding of the symbolic system for language (Bialystok, 2000).

Several studies have been conducted to examine the development of the written notations children produce to represent quantities. Hughes (1986) asked 3- to 6-year-old children to remember how many toy bricks were in different identical tin boxes by writing something down on a piece of paper. He coded children's notations as belonging to one of four categories: *idiosyncratic*, when the notation did not convey information about numerosity; *pictographic*, for drawings that represented both object and quantity; *iconic*, for markings that represented each brick (one-to-one correspondence); and *symbolic*, when conventional symbols were used. He found that as children got older, their notations became more sophisticated: older children tended to use more symbolic notations, whereas only young children used idiosyncratic representations.

Bialystok and Codd (1996) conducted a similar study where they looked at 3- to 5-year-old children's notations for remembering different amounts, but they added a novel task in which children were asked to select a card to represent a given quantity. One card depicted a conventional symbol, another used an iconic representation, and the last one had a picture of one of the objects (e.g., a single fish to represent 8 fish). They found, as had Hughes (1986), that children produced more symbolic notations as they got older. They also found that overall, children were more successful in "reading" notations provided by the experimenter than the ones they had produced; the effect was the greatest for the iconic notations. The authors speculated that children did not understand the principle of symbolic representation. That is, they did not see the iconic markings as having the symbolic meaning of representing a given quantity – they only saw them as graphic displays. As such, Bialystok and Codd concluded that children can be fluent in using numbers to count objects and talk about quantity, but their ability to understand and use written notations for numbers develops later.

Martí et al. (2005) differentiated between two views of notations: one as objects of knowledge (e.g., writing and recognizing numerals), and one as psychological tools for problem solving. Their study dealt with the latter. They wanted to explore the difficulties 5- to 7-year-old children had in the context of two tasks: (a) a production task, in which they had to make a notation that would help them remember qualitative ("what" was in the box) *and* quantitative ("how many" were in the box) information; and (b) an interpretation task, in which children had to use their own notations to reproduce the content that was in the box. The task proved difficult for children, and Martí et al. hypothesized that it was because of a metacognitive restriction: children's mental state in the production task was different from their mental state in the interpretation task. They did not anticipate what information they needed to be able to reproduce the type and number of objects in the box at a later time. Between the ages of 5 and 6, children become able to accurately represent the qualitative aspects, but it is not before the age of 7 or later that they are able to accurately represent the quantitative aspects.

Four principles that could play a role in children's early symbolization abilities have been identified in prior research (see DeLoache & Marzolf, 1992). First, when children can appreciate symbolic relations, it is said that they display symbolic sensitivity, which is a general predisposition or readiness to recognize that one object may stand for another. Second, children must be capable of dual representation, which is the ability to view an object as its own entity and as a representation of something else (DeLoache, 1995). Third, children need to demonstrate representational insight, which means they must know that an object can stand for something it is not. Lastly, when presenting concrete representations to young children, one needs to keep in mind that pictures are easier for them to understand as representations of real-world items than models or objects that stand for abstract concepts.

Representations in Mathematics

The domain of mathematics is centered on symbols. Without symbols, there would be no mathematics as we know it. In its essence, the foundations and methods of mathematics are representational, and no mathematical activity can happen without external material forms of expression (Dufour-Janvier et al., 1987; Kaput, 1987b). Representations (or symbols) can serve many functions. Any material or symbolic object that has the objective to enhance or support mathematical thinking and learning can be thought of as a representational tool (Empson, 1999). There are three broad types of representational tools that can serve as external supports for learning in mathematics: language, concrete representations (e.g., linking cubes, fraction pieces), and symbols (e.g., number systems, algebraic symbols, graphs, diagrams; Empson, 1999; Hiebert et al., 1997).

Mathematical symbols can help learners represent ideas that already have real-world meaning, and these experiences can act as anchors for the symbols (Hiebert et al., 1997). Symbols in mathematics can serve many goals. One of these goals is to support communication; for clear and effective communication to occur, people need to use a common symbol system that refers to shared understandings (Hiebert et al., 1997; Kaput, 1987b). Mathematical symbols can also help with cognitive processing by allowing people to simplify and amplify their mathematical activity (Hiebert et al., 1997). It would be nearly impossible to do mathematics if one were to refer back to the referents at all times (Mason, 1987). The real efficiency and power of using mathematical symbols and algorithms is that it restricts the manipulation to numbers rather than to the quantities themselves. It is much easier and efficient to use symbols than to focus on complex referents and their nuances. The meaning behind the symbols can be accessed if and when it is needed. Established symbol systems can then be used as referents for the development and understanding of more complex concepts (Hiebert, 1988; Kaput, 1987a, 1987b). As the symbolic manipulations happen, one does not need to stop and think about the referents at every single step. This process happens so spontaneously that people often do not realize that they are involved in a symbolizing process (Mason, 1987). Representations are sometimes used to make mathematics more attractive or interesting when they adorn or embellish the pages of textbooks, for example, in the hopes of motivating children or presenting analogies to the real world (Ball, 1983; Dufour-Janvier et al., 1987). For example, there could be an image of kittens on the page of a mathematics textbook next to an addition word problem involving cats. Caution should be used, however, as these features can sometimes prove distracting and can actually hinder learning because children might fail to see the connections between these representations and the concepts they are meant to be learning (Koedinger & Nathan, 2004; McNeil & Jarvin, 2007; McNeil et al., 2009; Petersen & McNeil, 2013).

A myriad of representations exist in mathematics, and many believe that the ability to reason with and among multiple representations exemplifies what it means to be mathematically competent (Lesh, 2000; National Research Council, 2009). Meta-representational competence has been described as the ability to select, construct, and use external representations (diSessa & Sherin, 2000). This idea has been described and studied to a great extent, and the various terms and definitions for meta-representational competence used by different authors each highlight specific aspects (Acevedo Nistal et al., 2009). There are two main constructs that have been considered: (a) the ability to translate meaning from one representation to another (i.e., inter-representational flexibility), and (b) the ability to do transformations within one given representation (intra-representational flexibility; Deliyianni et al., 2016). Other labels have been used for each of these two constructs. For example, inter-representational flexibility has been referred to as representational fluency (Bieda & Nathan, 2009; Ceuppens et al., 2018; Lesh, 1999; Nathan et al., 2002) and representational competence (Shafrir, 1999), whereas intra-representational flexibility has been called adaptivity (Siegler & Lemaire, 1997) and representational flexibility (Lesh et al., 1987). Thomas and Hong (2001) introduced the notion of representational versatility, which is the ability to exhibit both representational fluency and representational flexibility.

As was explained above, the terms flexibility and fluency have been used many different ways in the literature (Acevedo Nistal et al., 2009) and so it is important that I define how I will use these terms in the context of this study. In the present study, I will use the term *representational versatility* to describe an ability that encompasses both *symbolic flexibility* and *symbolic fluency*. I define symbolic flexibility as the ability to assign a new quantitative referent to a known target manipulative and symbolic fluency as the ability to assign a new quantitative referent to a novel manipulative.

Effective Use of Manipulatives in the Classroom

Manipulatives can be described as concrete objects used to help students represent and understand abstract mathematical ideas or to support carrying out mathematical procedures (Hiebert, 1992; Laski et al., 2015; McNeil & Jarvin, 2007; Moyer, 2001) Early theorists such as Piaget, Bruner, and Montessori believed that in order to construct abstract concepts, young children needed to interact with objects in their environment (Burns & Hamm, 2011; McNeil & Jarvin, 2007; Uttal et al., 1997). The assumption underlying these theoretical perspectives is that young children have not yet developed the ability to reason abstractly and will be in a better position to understand certain concepts if first presented using visual representations (as opposed to symbolic representations) with the help of concrete manipulatives (Carbonneau et al., 2013). By making abstract concepts concrete, then, manipulatives can help children solve problems that would otherwise be too difficult (Jacobs & Kusiak, 2006).

Manipulatives have been found to be beneficial for learning in different domains, such as language, science, and mathematics (e.g., Carbonneau et al., 2013; Glenberg, 2008; Sherman & Bisanz, 2009; Zacharia & Olympiou, 2011), and the cognitive abilities that prepare children to use symbols effectively might be similar across domains (Gromko, 1988). Many teachers have come to believe that young children will benefit from touching and moving concrete objects when learning mathematics (Lee & Ginsburg, 2009; Martin & Schwartz, 2005). In fact, one of the objectives listed by the Québec Education Program (Ministère de l'Éducation du Québec, 2001) for elementary education refers to the use of different representations (e.g., objects) to illustrate mathematical concepts. In many jurisdictions in North America, using concrete objects during mathematics instruction to promote student learning is considered an effective pedagogical technique (e.g., Common Core State Standards Initiative, 2010; Moch, 2001). Manipulative-based instruction has been shown to have a positive effect on different learning outcomes, such as recall, application, and transfer (Carbonneau et al., 2013).

One important issue with using manipulatives is that it is possible that students will see the representations as objects themselves, and not as tools that can help them solve mathematical problems (Dufour-Janvier et al., 1987). In other words, they might not have developed dual representation (DeLoache, 1995). When teachers use manipulatives or diagrams to support their students' learning, they need to keep in mind that students need time and guidance to construct and develop meaning for these tools; there is no meaning inherent to the tools themselves (Hiebert et al., 1997; Uttal et al., 2006). Manipulatives on their own do not help students develop conceptual understanding of the mathematics involved (Moyer, 2011; Resnick & Omanson, 1987). Teachers need to realize that manipulatives and other visual models are a means to the mathematics and not the end (Ball, 1992; Petit et al., 2016). Using tools to simply help students generate answers might not be as productive and as beneficial to the development of students' understanding than using them as tools to think with or reflect on different solutions (Empson et al., 2011; Hiebert et al., 1997; Puchner et al. 2008).

To ensure that children are given and use the tools they need to succeed in school, it is important to consider not only the content (i.e., what is being taught), but also the process (i.e., how it is taught; Weisberg et al., 2013). Using manipulatives in the classroom offers no guarantee that students will view them and use them as the teacher intends (i.e., for reflection and thought), so it is critical that teachers think about how they are being introduced, used, and what their relationships are to the underlying concepts they are trying to convey (Ball, 1992; Hiebert et al., 1997). Three aspects related to manipulative-use in the classroom may influence how children develop an understanding of the symbol-referent relationship: (a) the nature and perceptual features of the manipulatives chosen (e.g., Chao et al., 2000; Petersen & McNeil, 2003), (b) the structure of the learning environment and the level of instructional guidance (e.g., Brown et al., 2009; Carbonneau & Marley, 2015; Martin & Schwartz, 2005), and (c) students' individual differences in terms of prior knowledge, cognitive factors, and their own interpretations (e.g., Fyfe & Rittle-Johnson, 2016; Osana, Przednowek, et al., 2018).

Nature of the Manipulatives

Any given manipulative will come with its own set of affordances. Affordances have been defined as "the physical and perceptual properties of an object that direct and constrain a user's actions" (Osana, Blondin, et al., 2018, p. 1; see also Gibson, 1979; Greeno, 1994). It is likely that many different features of concrete objects interact and lead to different learning outcomes. Fyfe and Nathan (2019) proposed a framework to illustrate how representations might differ from one another in terms of their perceptual richness (low to high), their physicality (two- or three-dimensional), the learner's level of familiarity (low to high), and the narrative context in which they are couched (low to high).

One possible drawback of having manipulatives with perceptually-salient surface features is that the features will distract learners from using manipulatives in the intended mathematical manner (McNeil & Jarvin, 2007; McNeil et al., 2009; Petersen & McNeil, 2013). Results from studies focusing on the effect of the salience of manipulatives' perceptual features are not unanimous, however. While some contend that the realistic nature of the manipulatives might help cue real-world knowledge, which can help problem solving (see Martin, 2009; McNeil et al., 2009), others have found that the superficial information for some materials might interfere with learning (Carbonneau & Marley, 2015; Kaminski et al., 2009; Sarama & Clements, 2009).

It is assumed that when students encounter problems in real-world contexts, they will perform better than when they are given problems traditionally given in school (e.g., McNeil et al., 2009). Researchers have found that young Brazilian street vendors, for example, performed better on arithmetic problems that involved real currency than on arithmetic problems that did not (Carraher et al., 1985). Realistic and perceptually-rich manipulatives might activate children's real-world knowledge and help them make sense of abstract concepts (McNeil et al., 2009; McNeil & Jarvin, 2007). Durik and Harackiewicz (2007) found that materials that are highly salient (i.e., enhanced with colors and pictures) might be beneficial for some students because they will make the lessons more interesting. They suggested that visually stimulating and aesthetically pleasing materials might help students who start off with a low level of interest become more involved in the lesson. It is important to note, however, that Durik and Harackiewicz's study was conducted with college students and as such, their findings might not generalize to a younger age group.

Other research examining the effects of manipulative use on student learning has hinted at disadvantages to using highly salient materials in the mathematics classroom. When students are given manipulatives that are perceptually rich, their attention might be drawn to the surface features of the objects and away from the concepts these objects symbolize; in other words, their representations of the perceptual details might compete with their representations of concepts and procedures (Laski et al., 2015; McNeil et al., 2009; McNeil & Jarvin, 2007; Rosen et al., 2018). McNeil et al. (2009) conducted two studies with students in Grades 4 through 6. In their first study, they gave one group of students bills and coins that looked similar to real money that they could use to help them solve word problems involving money. The control group did not receive any bills or coins. The authors found that the control group performed better than the experimental group. This led them to suggest that the perceptually-rich features of the bills and coins hindered the students' performance.

In their second study, McNeil et al. (2009) wanted to see if using "bland" money instead of perceptually-rich money would influence students' performance. They assigned participants to one of three conditions: (a) "perceptually-rich" money, (b) "bland" money (students were given black-and-white bills and coins that did not include extraneous perceptual details), and (c) control (students were not given any bills or coins). The authors found that the students in the "perceptually-rich" condition made more errors than students in the "bland" or control conditions. They also found, however, that the errors made by the students in the "perceptually-rich" condition were less likely to be conceptual errors. This finding suggests that when students were using the realistic money, it activated their real-world knowledge, and they were thus better able to understand the problem conceptually. McNeil et al. contended that if a teacher's goal is to reduce students' errors in terms of accuracy, he or she should steer away from perceptually-rich manipulatives. Perceptually-rich manipulatives can be useful, however, if teachers want to target students' conceptualization of problems.

Carbonneau et al.'s (2013) meta-analysis also yielded trade-off effects of perceptually-rich manipulatives on student learning. When looking at retention of learning, they found that studies that had used perceptually-rich manipulatives had a smaller effect on student performance compared to studies that used bland manipulatives. On the other hand, perceptually-rich manipulatives seemed to enhance students' transfer, perhaps because these manipulatives facilitated a greater conceptual understanding. In one of her studies with scale models, DeLoache (2000) examined how decreasing or increasing the salience of the model influenced young children's ability to see the model as a symbol. She found that when the model was placed behind a window (decreased salience), it made it easier for children to use it as a representation of the life-size room. When children were allowed to play with the model at the beginning of the task (increased salience), however, it made it more difficult for them to understand the "stands for" relation between the model and the normal-size room. Osana, Przednowek, et al. (2018) found similar results in their study of how different ways of encoding plastic chips influenced how second graders learned to use them to represent base-10 denominations (i.e., 1, 10, 100). When children played with the plastic chips during the encoding phase, they had a harder time using the chips as representations of quantities during the instruction compared to the children who had interacted with the chips in a quantitative way during the encoding phase. Perceptually-rich concrete materials, then, may be interesting and engaging for students and help them to better conceptualize problems, but students might consider them only as objects (rather than as symbols), therefore making it difficult for them to understand the dual representation of the manipulatives used in the classroom (Carbonneau & Marley, 2015; DeLoache et al., 1998; McNeil et al., 2009; Sloutsky et al., 2005; Uttal et al., 1997).

Learning Environment and Instructional Guidance

To learn and understand mathematics, one has to make sense of its symbols and understand which conceptual referents they allude to. For people to use symbols in the way they are intended, they need to understand the symbol-referent links. The idea of mapping symbols to concepts, with meaning being a by-product of the relations among representations of the world and mental representations, has its roots in information-processing theory, in which learning is considered an active process (Gravemeijer, 2002; Meira, 1995). To fully appreciate how symbolizing can impact children's learning, however, influences from constructivist and socio-cultural perspectives also need to be taken into account. Teachers have an active role to play in the development of children's connections between representations and mathematics knowledge, and students' learning will be influenced by how they appropriate and internalize the representational tools they use in their mathematical activities (Gravemeijer, 2002).

There is still some debate as to how children should come to understand the symbol-referent links. Many researchers support the idea that teachers should make the links between representations and their referents explicit (see DeLoache et al., 1999; Hiebert, 1984; Osana & Pitsolantis, 2013), whereas others contend that children should have freedom to explore manipulatives on their own (see Martin, 2009; Martin & Schwartz, 2005). These opposing views echo the discussions that take place on instructional approaches more generally. There have been many debates over the past decades as to the most effective teaching approaches, and in the context of mathematics education, these have been called "the math wars" (Fuson, 2009; Schoenfeld, 2004). These debates entail two extreme positions on how children should be taught mathematics to maximize learning outcomes. On one hand, there is the "instructionist" camp, with approaches such as direct instruction, which contends that children learn best through explicit instruction and repetition, and on the other end, there is the "constructivist" camp, with approaches such as discovery learning through which children are said to uncover the key principles underlying their experiences (Roll et al., 2018; Trninic, 2018). Advocates of inquiry learning contend that it can be a powerful method that leads to a deep construction of knowledge, whereas its critics posit that it presents too much of a cognitive load for the learner, thereby reducing the capacity to process and store relevant information into long-term memory (see Alfieri et al., 2011; Hmelo-Silver et al., 2007; Kirschner et al., 2006; Lazonder & Harmsen, 2016).

One of the issues for researchers who have examined the effects of "guidance-type" approaches on learning, however, is that there is a lack of comparability in operational definitions that are used in different studies, and various terms have been used such as cooperative learning, guided inquiry, guided discovery, guided play, questioning, feedback, prompts, and co-construction techniques (Clark, 2009; Clements & Joswick, 2018; Horan & Carr, 2018a; Fyfe & Rittle-Johnson, 2012; Klahr, 2010; Wise & O'Neill, 2009). This issue of inconsistent terminology makes it difficult to compare and generalize across studies. Several definitions of guidance have been proposed in the literature; Lazonder and Harmsen (2016) defined it as "any form of assistance offered before and/or during the inquiry learning process that aims to simplify, provide a view on, elicit, supplant, or prescribe the scientific reasoning skills involved" (p. 687). Other aspects of guidance that have been described are interactions between teacher and students and the presence of scaffolding or supportive and corrective feedback given to students in response to their questions and the difficulties they encounter during their learning attempts (Clark, 2009; Horan & Carr, 2018a). In the current study, three instructional conditions were compared: (a) direct instruction, which refers to a sequence of instruction in which the instructor explicitly tells the child the quantitative referent for a target manipulative (see Kirschner et al., 2006), (b) guided exploration, during which the instruction offers hints and prompts to lead the child to discover the quantitative referent for the target manipulative (similar to what has been called guided discovery; see Mayer, 2004; Fyfe & Rittle-Johnson, 2012), and (c) control, which could be seen as a "pure" discovery condition because it includes no feedback whatsoever from the interviewer.

Mayer (2004) and others (e.g., Carbonneau et al., 2013; Schwartz & Bransford, 1998) suggested that instead of supporting one or the other of the positions advanced as part of this debate, discovery learning should be considered as a range; instructional approaches, then, could be seen as existing along a continuum with direct instruction at one end, "pure" discovery at the other, and guided discovery somewhere in between. Theoretical and empirical support has been found for various points on the continuum (see Hushman & Marley, 2015; Lee & Anderson, 2013; Marley & Carbonneau, 2014b), although "pure" discovery has not been found to be effective, and it has been said that inquiry should be accompanied by teacher guidance (Lazonder & Harmsen, 2016). One meta-analysis (Alfieri et al., 2011) comparing different instructional approaches found that unassisted discovery resulted in poor learning outcomes relative to providing some instructional support. One possible explanation for this is that learning by discovery might require a greater number of mental operations (e.g., selecting relevant information, organizing and integrating new information, metacognition), as well as better executive control of attention, in comparison with learning under a more directive approach.

One argument in favor of a more direct instruction on the link between a manipulative and its referent is that without structure to guide students' learning, students may interact with the objects in ways that do not support learning the intended concept, which will prevent them from constructing knowledge that could transfer to other representations (Brown et al., 2009; Mayer, 2004; Sarama & Clements, 2009; Uttal et al., 2009). Carbonneau and Marley (2015) studied the effectiveness of instruction with manipulatives on preschoolers' learning. They found that manipulative use was most effective when the instruction incorporated high levels of support. Students who received high levels of guidance outperformed those who received low instructional guidance. Osana et al. (2017) also found that instructional guidance led to improved knowledge in their second-grade participants, but this was not the case for all of their measures of place value and regrouping. In some cases, it was when no instructional guidance was offered that knowledge increased, but this was found only when concrete objects were used before written notation.

When structuring the learning environment, many researchers believe that teachers should aim to support learning from manipulatives. It is common practice in the pedagogical methods used in schools for external representations to be proposed to (or imposed on) students, and they are not always encouraged or given the opportunity to construct and explore representations of their own doing (Dufour-Janvier et al., 1987). Teachers may wish to refrain from imposing the use of manipulatives because although they can be helpful for certain students, not all students will need them to reason about how to solve a given problem (Empson et al., 2011). External representations that are imposed on children can lead them to difficulties (e.g., not understanding how to use the tools independently) if the representations are too distant from what the child knows and his or her internal representations, or if the links are not made explicit. If the child is forced to learn how to use a given representation, the use of the specific representation will take precedence over learning the concepts the representation is used for (Dufour-Janvier et al., 1987). Additionally, using direct instruction to teach children how concrete representations should be used before (or instead of) giving them freedom to construct their own meaning might prevent children from using these representations more flexibly or in novel ways in the future (Bonawitz et al., 2011).

Many advantages have been cited for providing opportunities for children to interact with materials in relatively unstructured environments. Martin and Schwartz (2005) found that children who learned fraction concepts in unstructured environments (i.e., they represented fractional quantities with tiles, which required them to think of how to represent parts of wholes) were better able to transfer underlying fraction concepts to new materials compared to children who learned in well-structured environments (i.e., they represented fraction quantities with pie pieces which, by their nature, made part-whole relationships visible). When learners are provided with too much structure, they may become dependent on the external environment and fail to construct meaningful knowledge for themselves (Brown et al., 2009); they may not learn as much or as deeply as they are able (Martin, 2009). Martin and Schwartz contended that individuals need to direct and regulate their own interactions with concrete materials for cognitive development to occur. In their study, they found that when children moved fraction pieces themselves to help them solve a fraction addition problem, they made more conceptual gains compared to a condition in which children saw the pieces already organized in correct groups.

A more balanced approach for teaching that incorporates elements of direct instruction and exploration has been proposed by many researchers (e.g., Fuson, 2009; Honomichl & Chen, 2012; Kuhn, 2007; Lobato et al., 2005). It is possible that when learners already possess some prerequisite knowledge of a concept, unguided instruction will be successful (Kirschner et al., 2006; Schmidt et al., 2007). If teachers act more as facilitators who scaffold student learning rather than sole providers of knowledge, it might lead to positive learning outcomes (see also Hmelo-Silver, 2004).

The findings from the studies described above highlight the need for research on the best ways for teachers to provide instruction when teaching with manipulatives while at the same time allowing their students to regulate and direct their own activity (Martin, 2009). The ways in which different instructional approaches influence different learning outcomes, such as learning and transfer, also need to be studied (Marley & Carbonneau, 2014a).

Individual Differences in Terms of Cognitive Factors

Students' own cognitive abilities might play a role in how they are able to understand the links between mathematical symbols and the concepts they represent. Astle et al. (2013) investigated the contribution of specific cognitive skills (i.e., phonological and visuospatial working memory, and inhibitory control) to 4-year-old children's performance on measures of representational understanding. Children were asked to find stickers in a model room by using information from a drawing of the room (referred to as a *map*). The authors created five versions of the task, in which the correspondence between the sticker and its symbolic representation in the map matched or conflicted in terms of color and/or shape. They found that all three cognitive skills played a role in children's ability to understand representations. The authors stated that working memory was important because children had to hold in their minds the nature of the relation between the symbol and its referent when solving problems with the symbols. More specifically, phonological working memory (i.e., the ability to hold speech-based information in mind and use inner speech to manipulate it; Baddeley, 1990) and visuospatial working memory (i.e., the ability to remember what is being seen as well as where it is in space; Baddeley, 1990) were thought to be important. In other research in the field of mathematics, it has been found that visuospatial working memory can be useful for the encoding, retention, and manipulation of numbers during mental calculations (Holmes & Adams, 2006). In their study, Rasmussen and Bisanz (2005) speculated that preschoolers used a mental model for arithmetic that required visuospatial working memory, and they also found that phonological working memory was involved in children's problem-solving skills in Grade 1.

In their review of theory and research on symbols as tools in the development of inhibitory control skills, Carlson and Beck (2009) reported that symbols play a role in facilitating executive function (including inhibitory control). Inhibitory control (i.e., the ability to stop oneself from doing what is usually done in a particular situation and to instead perform a conflicting action) was also found to be involved in Astle et al.'s (2013) study when children were faced with conflicting representations (e.g., the representation's color did not match the sticker's color). Inhibitory control could potentially be linked to dual representation in that when using a symbol, one needs to inhibit its function as an object in itself and be able to see it for what it represents. Inhibitory control skills and symbolizing skills might, then, be related, and each set of skills might help in the development of the other. In sum, the evidence described here did not directly address the role of cognitive factors on meaning-making when manipulatives and their referents are involved, which is something that deserves greater research attention.

Present Study

There is evidence to support the idea that using direct instruction in mathematics instruction leads to positive learning outcomes (see Alfieri et al., 2011; Lazonder & Harmsen, 2016) and that when teaching with manipulatives, teachers should make explicit links between the concrete representations and their referents for students to use them in the prescribed way (Carbonneau et al., 2013; Carbonneau & Marley, 2015; Fuson et al., 1992; Osana, Przednowek, et al., 2018). Other researchers, however, believe there are benefits to children constructing their own understandings with scaffolds or guidance from a more knowledgeable other (see Brown et al., 2009). Such benefits include children being better able to abstract the concepts conveyed by the representations which might help them transfer their knowledge to different contexts, and they might also be more flexible in their use of the manipulatives (Bonawitz et al., 2011; Brown et al., 2009; Martin & Schwartz, 2005).

Several aspects of instruction with manipulatives have been studied and include students' individual differences in terms of prior knowledge, cognitive factors, and the structure of the learning environment, including the level of instructional guidance (e.g., Brown et al., 2009; Carbonneau & Marley, 2015; Martin & Schwartz, 2005). In the present study, I focused on instructional guidance on manipulative use and its effect on learning and transfer. The specific goal of the present study was to identify the benefits and costs of directly telling students the quantitative referents for manipulatives compared to allowing them to construct meaning for the manipulatives in more open and exploratory learning space. In particular, I was interested in the instructional conditions under which children can develop representational versatility, which I operationalized as being comprised of two related constructs: (a) symbolic flexibility, which I defined as the ability to assign a new quantitative referent to a known target manipulative, and (b) symbolic fluency, which I defined as the ability to assign a new quantitative referent to a new manipulative. Participants were randomly assigned to one of three conditions that differed in the type of encoding instruction they received (i.e., direct instruction, guided exploration, control). The encoding instruction involved wooden polygonal blocks (called "pollies") as the target manipulative; the instructional objective was for participants to come to view each polly as representing the quantity 2. During the session, children used the pollies to represent different quantities. Children's learning and near-transfer abilities were assessed. To do a thorough evaluation of the impact of instructional guidance, it was important to not only investigate immediate learning, but also children's ability to use their knowledge flexibility in transfer activities (Roll et al., 2018), which is why children were also assessed on their far-transfer skills one day after their encoding instruction session.

The overarching research question was: How do the ways in which children assign a quantitative referent to a target manipulative (direct instruction vs. guided exploration vs. control) influence their (a) learning, (b) near-transfer abilities, (c) representational versatility (i.e., symbolic flexibility and symbolic fluency), and (d) their problem-solving accuracy? I hypothesized that for learning and near-transfer tasks, students who learned through direct instruction would benefit relative to students who engaged with the manipulatives in a more exploratory way (Alfieri et al., 2011, Carbonneau & Marley, 2015, Osana et al., 2017). In contrast, those students who were guided in their exploration of the objects' meanings would be more flexible with their use and interpretation than those who were told explicitly what the objects represented (Bonawitz et al., 2011, Martin & Schwartz, 2005); the former group of students might therefore fare better on transfer tasks.

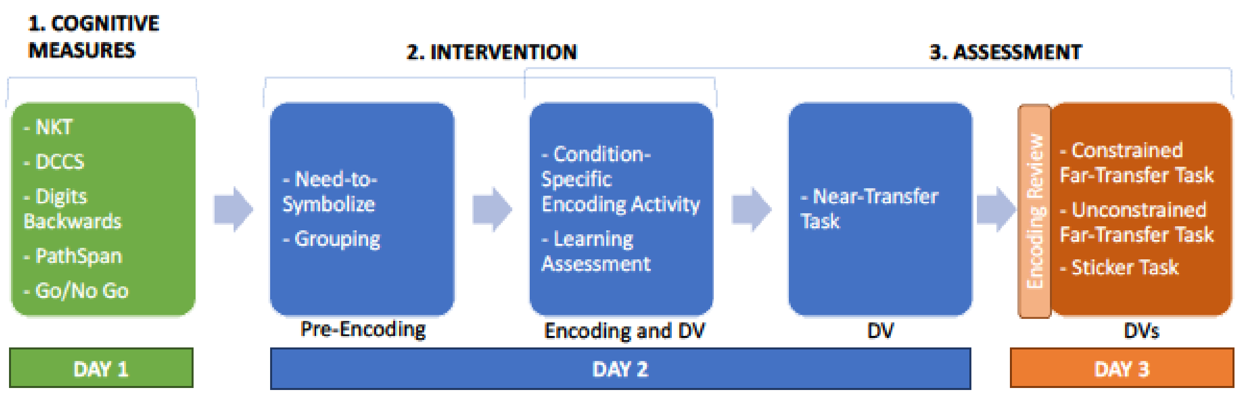
Chapter 3: Method

Design and Procedure

The design of the study is illustrated in Figure 1. Data collection took place in the months of April, May, and June. There were three phases to this study: (a) administration of cognitive measures, (b) intervention, and (c) assessment. Participants took part in three individual meetings, each one lasting approximately 30 minutes. On Day 1, during the administration of cognitive measures phase, they were assessed on their cognitive skills (i.e., number and counting ability, phonological and visuospatial working memory, executive functioning). The next two meetings (on Days 2 and 3, respectively) took place on consecutive days[1], no more than 38 days after Day 1; the mean number of days between Day 1 and Day 2 meetings was 15 (*SD* = 7, min: 6, max: 38). The Day 2 and Day 3 meetings were video recorded.

Figure 1

Design



Note. NKT = Number Knowledge Test; DCCS = Dimensional Change Card Sort.

The intervention phase took place on Day 2 and was designed to manipulate the participants' encoding of the pollies. The intervention consisted of three activities, the first two of which (Pre-Encoding in Figure 1) were meant to prepare them for the encoding activity. They then took part in the encoding activity, the objective of which was for participants to view a target manipulative (called a "polly") as representing a quantity of 2.

[1] There were two students (both from the control condition) who did the Day 3 activities four days after Day 2 activities.

Participants were randomly assigned to one of three encoding conditions: direct instruction (DI), guided exploration (GE), and control. In the DI condition, the interviewer told the children that one polly represented a quantity of 2. In the GE condition, the interviewer constrained the encoding task in a such a way that children "discovered" for themselves, after some prompting, that one polly represented 2. In the control condition, children were free to assign the quantitative referents of their choice to the pollies.

The first part of the assessment phase took place at the end of the meeting on Day 2. As part of the encoding activity, participants represented different quantities with the use of the pollies (e.g., using a referent of 2 for each polly, they would use two pollies to represent a quantity of 4); there was a maximum of eight different quantities that participants could be asked to represent. They received Learning scores based on how many items they attempted before encoding the pollies as 2 and on the length of the encoding session. They then took part in the Near-Transfer task in which they needed to use the pollies to solve multiplication word problems involving quantities that were multiples of 2.

The second part of the assessment phase began on Day 3. Participants first took part in activities designed to remind them of how they had encoded the pollies in the previous day's encoding instruction. They then completed the Constrained Far-Transfer task, in which they were provided with pollies to help them solve multiplication word problems, the Unconstrained Far-Transfer task, in which they were provided with a variety of manipulatives other than pollies to help them solve multiplication word problems, and the Sticker task, in which they had to use the pollies to request a number of stickers of their choice. The objective of the Constrained Far-Transfer task was to assess children's symbol flexibility (i.e., ability to assign a new quantitative referent, other than 1 or 2, to the same target manipulative); the objective of the Unconstrained Far-Transfer task was to assess children's symbolic fluency (i.e., ability to assign a new quantitative referent, other than 1, to a new manipulative); and the objective of the Sticker task was to assess children's symbol flexibility in a context that was personally relevant (i.e., a context that was potentially profitable to them).

Participants

Participants were first-grade students from four different elementary schools in a large urban area in Canada. Within each classroom, participants were randomly assigned to one of the three encoding conditions. Six students came from School A (2 classrooms), 21 students from

School B (2 classrooms), 19 students from School C (2 classrooms), and 19 students from School D (3 classrooms).

The province of Québec publishes an income index and a socio-economic index for all the schools in the province (Ministère de l'Éducation et de l'Enseignement supérieur du Québec, 2019). The income index is the proportion of children from families at or below the poverty line. The socio-economic index is a weighted proportion of (a) the number of children from families with mothers with no post-secondary education (two-thirds of the index) and those coming from families in which both parents were unemployed at the time of the last Canadian census (one-third of the index). Based on these indices, the schools are ranked on a scale of 1 (least disadvantaged) to 10 (most disadvantaged).

The indices reported here are based on information published for the 2017-2018 academic year, which is when the data were collected for the present study. The income index for School A was 26.87 (rank: 9), and the socio-economic index was 13.31 (rank: 8); for School B, the indices were 25.80 (rank: 9) and 14.88 (rank: 9) on the income and socio-economic scales, respectively; for School C, the indices were 25.57 (rank: 9) and 13.01 (rank: 8) on the income and socio-economic scales, respectively; and for School D, the indices were 27.23 (rank: 9) and 12.24 (rank: 8) on the income and socio-economic scales, respectively.

Seventy-two first-grade students (39 girls, 33 boys) constituted the original sample for the study. One student from the DI condition and six students from the GE condition were excluded from the original sample because they failed to encode the polly as having a referent of 2 during the encoding session, so the final sample for this study was 65 students (35 girls, 30 boys): 20 students in the DI condition (10 girls, 10 boys), 25 students in the GE condition (17 girls, 8 boys), and 20 students in the control condition (8 girls, 12 boys). Because the control group did not receive any instruction on a specific referent to use with the pollies, they were not excluded from the study if they failed to assign a referent of 2. The participants' mean age was 7 years, 1 month. The participants' teachers were part of a larger project conducted in partnership with the school board. In conducting this investigation, I complied with the principles of Canada's Tri-Council Policy related to the ethical treatment of the participants. Before data collection began, ethical approval from the university, the school board, and the schools' governing boards were obtained, as well as parental permission (see Appendix A). All children were asked to provide assent (see Appendix B) before participating.

Pre-Encoding Activities

Need-to-Symbolize Activity

All students (regardless of condition) were presented with three different cardboard boxes that were opaque and closed (see Figure 2); the only difference between the boxes was the number of red cubes in each one (i.e., 4, 12, 8; see Figure 3). One at a time, students had to open each box, count the number of cubes in the box, and then close the box. The interviewer provided corrective feedback, if needed, in the case of miscounts. The interviewer then mixed up the boxes and asked students to say how many cubes were in each one without opening them. The objective of this procedure was for students to realize that without an *aide-memoire* (Martí et al., 2005), it was impossible for them to be sure how many cubes were in each box. The goal of the activity was for the students to realize that they needed to find a way to remember the number of cubes in each box. The boxes were put away, but the interviewer explained to the child that they would come back to them and find a way to help them figure out how many cubes were in each box.

Figure 2

Cardboard Box

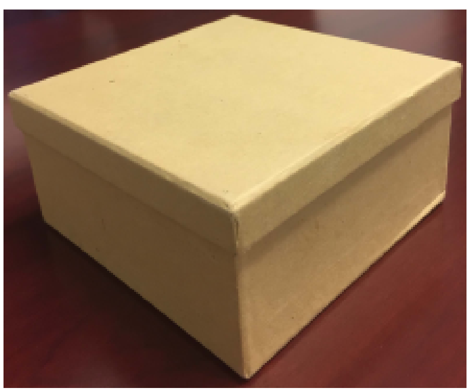
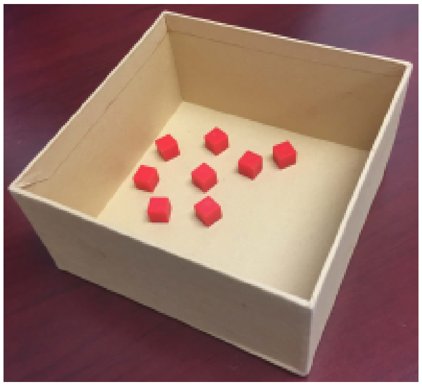


Figure 3

Example of a Quantity of Cubes to be Counted



Grouping Tasks

The next activity in preparation for the encoding activity required all children (regardless of condition) to practice grouping blocks by 2s and skip-counting by 2 with the interviewer. First, the interviewer demonstrated skip-counting by 2 out loud (up to 20) for the child. The child was then asked to skip-count by 2 (with the support of the interviewer, if needed) on his or her own twice more. The interviewer then placed eight blocks (see Figure 4) in front of the child and asked him or her to count them. Then, the interviewer asked the child to arrange the blocks in groups of two (i.e., 4 groups of 2 blocks). The interviewer then modelled how to skip-count by 2 by pointing to each group (i.e., 2-4-6-8). The child was then asked to skip-count by 2 on his or her own while pointing to each group. After this demonstration item, the child was required to group and skip-count two other quantities of blocks (i.e., 6 and 10) on his or her own. The interviewer provided corrective feedback if needed.

Figure 4

Blocks Used During the Grouping Activity

Experimental Manipulations

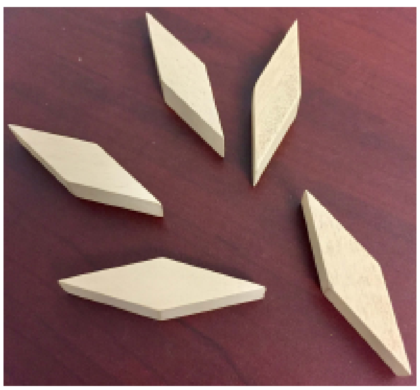
Following the pre-encoding activities, the interviewer brought back the same three boxes that had been used in the Need-to-symbolize activity. The interviewer took out special lids for the boxes (i.e., lids with a clear plastic bin glued on top; see Figure 5) and also introduced the pollies to the children. The pollies were objects that the children were not familiar with, which was important so that prior knowledge would not interfere with their ability to use and see them as symbols of quantity (Petersen & McNeil, 2013). The pollies were simple and bland so that students could focus on deeper mathematical structures and not be distracted by superficial features (Brown et al., 2009). More specifically, the pollies were wooden manipulatives with no markings on them that measured 4.7 cm in length, 1.3 cm in width, and 1.0 cm in height (see Figure 6). The interviewer suggested that the pollies could be placed on the special lid and used as a "secret code" to help remember how many cubes were inside each box. The ways children had to come up with a secret code depended on the instructional condition to which they had been assigned (see Appendix C for the interview protocol).

Figure 5

Opaque Box with Special Lid

Figure 6

Pollies



Conditions

During the encoding activity, children were asked to represent different quantities of cubes with the pollies using a secret code. There was a maximum of eight different quantities that children in the DI and GE conditions were asked to represent (i.e., 4, 12, 8, 14, 6, 16, 10, and 18 – in that order); the first three quantities were the ones in the boxes from the Need-to-Symbolize activity. The interviewer stopped the encoding activity for children in the DI and GE conditions once they had successfully represented two consecutive quantities using a referent of 2 for the pollies without receiving any corrective feedback. Students who were not able to do so were excluded from the study. Children in the control condition were asked to represent the first six quantities. Below, I describe the procedures in each condition.

Direct Instruction Condition

With the covered boxes in view, the interviewer explicitly told students that the secret code was that one polly could be used to represent a quantity of 2 (i.e., two cubes). The interviewer demonstrated this by using the quantity 4 (i.e., four cubes). She placed one polly near a group of two cubes and then modelled how to count with the pollies using a referent of 2 to know how many cubes were represented. The interviewer then put the four cubes back in the box, closed it with the special lid, and placed the pollies in the special lid's plastic box. The child was then asked to use the pollies in the same way to represent the quantities in each of the two remaining boxes that were introduced in the Need-to-symbolize activity (i.e., 12 cubes and 8 cubes, respectively), as well as the other quantities listed above. Corrective feedback was provided if needed, "Remember that each little polly means that there are two blocks."

Guided Exploration Condition

With the covered boxes in view, the interviewer asked students how they could use the pollies as a secret code to represent the number of cubes in the first box (i.e., 4 cubes). The interviewer then provided the child with a small bin filled with 40 pollies, and the child could use the pollies as he or she wished (i.e., use his or her own secret code) to represent the quantity in the box. The interviewer then constrained the task by removing pollies so that there would be half the number of pollies as cubes and asked students again how the pollies could be used to represent the cubes in the same box. The interviewer used prompts and provided feedback until students realized that each polly could be used to represent 2. Some of the prompts included referring to the grouping activity children had previously taken part in (e.g., "Remember how we were counting the blocks before? Could that help you think of a secret code to use with the pollies?"). The interviewer never explicitly told the students they were "right" or "wrong," nor did she ever explain that a referent of 2 should be used for the pollies. The child was then asked to use the pollies to represent the quantities for the two remaining boxes, as well as the other quantities listed above.

Control Condition

The interviewer provided children in this condition with a bin of 40 pollies and asked them how they could use them as a secret code to represent the number of objects in each of the three boxes, as well as the additional three quantities listed above. The interviewer did not offer prompts or provide any feedback, and the students were free to assign any referent they wished to the pollies.

Encoding Review Activity

Children started the third meeting with the interviewer (on Day 3) with an encoding review activity. The objective of the review activity was to remind children of how they had encoded the polly in the previous day's encoding session. The children were asked to, as they had done the day before, use the pollies as a secret code to represent different quantities of cubes (i.e., 8, 4, 10, 12, 6, 14, in that order, for children in the DI and GE conditions; 8, 4, 10, in that order, for children in the control condition) that were in cardboard boxes. There were fewer items for children in the control condition because there was no expectation of them using any specific referent for the pollies. The interviewer provided feedback appropriate to the condition for students in the DI and GE conditions. The encoding review activity ended after students in the DI and GE conditions had successfully represented two consecutive quantities of cubes without any feedback from the interviewer; students who were not able to do so were excluded from the study's remaining tasks (i.e., Constrained and Unconstrained Far-Transfer tasks and Sticker task). The activity ended for students in the control condition after they had represented all three quantities of cubes.

Measures

The measures in this section are described in the order in which they occurred over the course of data collection, from prior knowledge and cognitive measures to the final transfer tasks. These activities run from left to right in Figure 1 and will be described below in turn. All tasks were administered individually.

Prior Knowledge and Cognitive Measures

Children's number and quantity knowledge was measured. Their phonological and visuospatial working memory as well as their executive functioning skills were also assessed, as these cognitive factors have been shown to be related to how children understand concrete or pictorial representations (Astle et al., 2013; Rasmussen & Bisanz, 2005).

Counting Ability

To assess children's counting skills, children were asked to count as high as they could. The interviewer stopped them when they reached 30. This task was meant to serve as a screening task, with children not able to count up to 30 excluded from the study. All participants passed the task.

Number Knowledge Test

The version of the Number Knowledge Test (NKT; McGraw-Hill Education, 2008) used in the present study is based on Okamoto and Case's (1996) NKT. The NKT assesses children's understanding of numbers and quantity. Okamoto and Case found a high reliability coefficient for the NKT based on a sample of 470 students. Other research (Gersten et al., 2005) has indicated that the NKT was correlated with the two mathematics subtests of the Stanford Achievement Test – Ninth Edition (1995).

Students were asked to answer questions that required them to count objects, compare sets of different quantities of objects, show their understanding of how numbers relate to one another, and add and subtract whole numbers. The NKT is organized into four different levels: (a) level 0 (4-year-old level), (b) level 1 (6-year-old level), (c) level 2 (8-year-old level), and (d) level 3 (10-year-old level). If children answered more than half of one level's questions correctly, testing continued to the next level. The administration of the test ended once a child failed to correctly answer more than half of the questions at any one level.

Students received one point for each correct answer. Some items on the NKT have two parts, and to receive one point for these items, both parts had to be answered correctly. The points earned were added to obtain a total raw score for the NKT, which could range from 0 to 30.

Digits Backward

The Digits Backward test is part of the *Memory for digit span assessment* component of the Wechsler Intelligence Scales for Children-Revised (WISC-R; Wechsler, 1974) and was used to assess children's ability to manipulate verbal information while in temporary storage (i.e., phonological working memory). In the task, the child listened to the interviewer read a sequence of single-digit numbers (span ranging from 2 numbers for the easiest items to 8 numbers for the more difficult items). The child then repeated the numbers in reverse order. The interviewer recorded each child's response on a scoring sheet. There were 14 items on the test, organized into seven groups of two items; the span increased by one for each group of items. When a child scored 0 on both items in one group, the test was terminated. Each correct response was worth one point, and points were added to obtain a total score that could range from 0 to 14.

Dimensional Change Card Sort

The Dimensional Change Card Sort (DCCS; Zelazo, 2006) task assesses children's flexible use of rules to govern behavior, which is believed to be a key aspect of executive function. The task comprised three different phases. Before starting the first phase, the interviewer explained and demonstrated the rules of the task. Two sorting trays were placed side by side in front of the child (see Figure 7). Behind the first tray was displayed a laminated image of a blue square, and behind the second tray was displayed a laminated image of a red circle. The interviewer explained to the child that they were going to play a card game called "the color game," and that all the blue cards needed to go in the first tray and all red cards needed to go in the second tray. The interviewer demonstrated this by placing a card with a blue circle in the first tray (face down) and then a card with a red square in the second tray (face down).

In the first phase, the pre-switch phase, the child had to correctly place six cards (given one at a time by the interviewer in random order) in the appropriate tray. Three cards representing a blue circle and three cards representing a red square. The interviewer made sure that no more than two identical pictures were presented in a row. Before giving the first card to the child, the interviewer repeated the rules, "If it's blue, it goes here (*the interviewer pointed to the first tray*), but if it's red, it goes there (*the interviewer pointed to the second tray*)."

Figure 7

Setup for the Dimensional Change Card Sort Task

Immediately after the six trials of the pre-switch phase, the interviewer moved on to the second phase, the post-switch phase, and explained to the child that he or she was going to play a new game, "the shape game." In the shape game, all square cards needed to go in the first tray (i.e., the one with an image of a blue square) and all circle cards needed to go in the second tray (i.e., the one with an image of a red circle). The child then had to correctly place six cards (three blue circle cards and three red square cards, given one at a time by the interviewer in random order) in the appropriate tray. Again, the interviewer made sure that no more than two identical pictures were presented in a row.

Children who correctly placed at least 5 cards out of 6 in the pre-switch phase and at least 5 of 6 in the post-switch phase proceeded to the final phase, the border phase. The interviewer removed all cards for the sorting trays and used a new deck of cards composed of 3 red square cards with no border (same as those used in the previous phases), 3 blue circle cards with no border (same as those used in the previous phases), 3 red square cards with a black border, and 3 blue circle cards with a black border. The interviewer explained the rules of the new game to the child, "If you see a card with a black border, you have to play the color game. If you see a card with no black border, you have to play the shape game." The interviewer demonstrated the procedure to follow with a red square card with no border (rules for the color game were repeated at that time) and with a red square card with a border (rules for the shape game were repeated at that time). On each trial, the interviewer repeated the rules of the new game while handing the child a card at random and making sure that no more than two identical pictures were presented in a row.

Children received a score of 0 if they failed the pre-switch phase (i.e., correctly placed fewer than 5 cards out of 6), a score of 1 if they passed the pre-switch phase but failed the post-switch phase (i.e., correctly placed fewer than 5 cards out of 6), a score of 2 if they passed the pre- and post-switch phases but failed the border phase (i.e., correctly placed fewer than 9 cards out of 12), and a score of 3 if they passed all phases. Scores on the DCCS task thus ranged from 0 to 3.

PathSpan

The PathSpan test (administered via iPad; see sample stimulus in Appendix D) measured children's visuospatial working memory (Hume, 2015). PathSpan is a version of the Corsi Block Tapping Test (Berch et al., 1998), and it has been used with pre-kindergarten and elementary school children (LeFevre et al., 2010). Each child watched as a sequence of lighted buttons were displayed on the iPad screen and then "played the sequence back" by touching the buttons in the same order.

The task began with the interviewer demonstrating and explaining the rules of the task on a practice trial, after which testing began. On the first trial, the sequence length was 2. The sequence length increased by one on each successful trial. The child was given three tries for each sequence length, and the task ended when the child failed all three trials at a particular length. Performance was tracked and scored within the application and then downloaded onto a computer for data analysis. The minimum sequence length the participants had to reproduce was 2, and the maximum possible sequence length was 8. The participants were scored on the maximum sequence length they were able to correctly reproduce. If they failed to reproduce any correct trials, they were given a span length of 0. Scores thus could range from 0 to 8.

Go/No-Go

The Go/No-Go task (administered via iPad; Carleton University Math Lab, 2017; see sample stimulus in Appendix E) assessed children's inhibitory control skills. Children were asked to respond as quickly as possible by touching the iPad screen when they saw a mouse (i.e., go trials) and by not touching anything when they saw a cat (i.e., no-go trials). Children were given practice trials to ensure that they understood the instructions. For each practice trial, feedback was provided on the iPad screen: a smiling face emoji with the word "YES!" appeared if a child touched the screen after seeing a mouse or refrained from touching the screen after seeing a cat, or a thinking face emoji with the words "Try Again" appeared if the child did not touch the screen after seeing a mouse or touched the screen after seeing a cat. The practice portion of the task ended after a child had received a total of four smiling faces as feedback (for three go trials and one no-go trial).

After the practice trials, children were given 40 randomly presented experimental stimuli (30 go trials and 10 no-go trials). Performance was tracked and scored within the application and then downloaded onto a computer for data analysis. The scoring of the task was based on the standardized difference (i.e., *d* score) between the hit rate (correct go responses) and the false alarm rate (incorrect go responses). Higher *d* scores indicated better performance (i.e., better discrimination between hit and false alarm). Scores could range from 0 to 3.77.

Manipulative-Use Measures

All children took part in tasks in which they used different manipulatives. I was interested in the different quantitative referents they were assigning to the manipulatives during the tasks. The tasks are described below in the order they were presented to the children, but the order of the Constrained Far-Transfer and the Unconstrained Far-Transfer tasks was counterbalanced. Half of the sample did the Constrained Far-Transfer task first and the other half did the Unconstrained Far-Transfer task first.

Learning

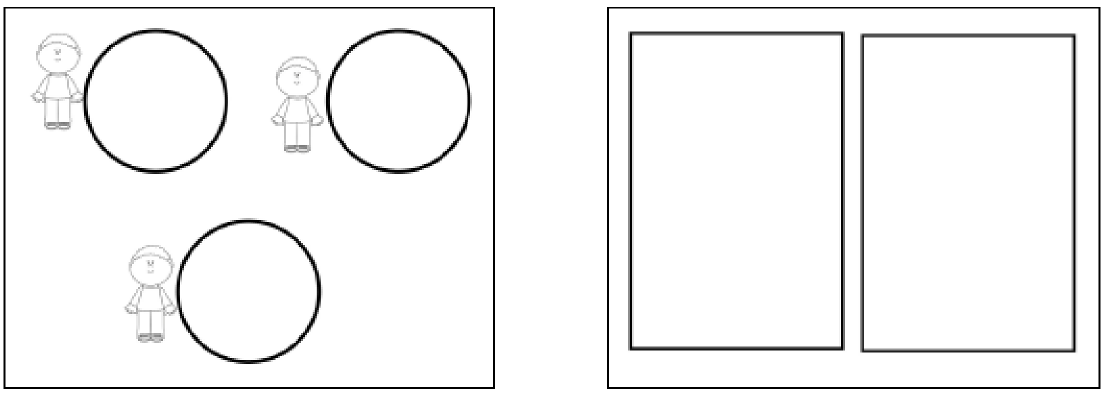
During the encoding activity described above, children were asked to represent different quantities of cubes with the pollies. There was a maximum of eight different quantities that children could be asked to represent (i.e., 4, 12, 8, 14, 6, 16, 10, and 18 – in that order). The encoding activity stopped for children in the DI and GE conditions once they had successfully represented two consecutive quantities using a referent of 2 for the pollies without receiving any corrective feedback from the interviewer. The first quantity (i.e., 4) was a demonstration item for children in the DI condition, and so there was a maximum of seven different quantities they could represent on their own. Children in the control condition were all asked to represent the first six quantities. Children in the DI and GE conditions were given two different learning scores: one for the number of items they had completed and one for the length of time it took for them to successfully use the pollies with a referent of 2 on two consecutive trials (in minutes). The number of items needed could range from 2 to 7 for children in the DI condition and from 2 to 8 for children in the GE condition. These scores were converted to percent by dividing the number of items completed before the task ended by the maximum number of items possible.

Near-Transfer Task

The Near-Transfer task (see interview protocol in Appendix C) was designed to assess whether children could use the pollies with a referent of 2 in a new context (i.e., a word problem context). The interviewer read two multiplication word problems (see Table 1) out loud, one at a time, and asked children to use the pollies to solve each problem. The order of the problems was counterbalanced. The use of manipulatives to solve these types of word problems is appropriate, as it is a common strategy children might naturally use when they first work on solving multiplication problems, typically unfamiliar for children of that age (see Carpenter et al., 1993; Manches & O'Malley, 2016). Both items were multiple groups word problems: one was of the *n* x 2 type and the other of the *n* x 4 type, with *n* being the number of groups described in the problem, and 2 and 4 being the numbers in each group, respectively. Based on observations from pilot data, I provided children with mats (see Figure 8) to help them see the number of groups involved in the problems; children were asked to place the manipulatives in the shapes drawn on the mats.

Figure 8

Example of Mats Used During the Near-Transfer, Constrained and Unconstrained Far-Transfer Tasks



After each problem, the interviewer asked prompts to make sure she understood which referent the child was using for the pollies (e.g., "Can you explain to me how you solved the problem?"). If the child had not used a referent of 2 for the pollies during the initial problem-solving attempt, the interviewer prompted to see if the child could use a different referent for the pollies (e.g., "Is there a way you could use fewer pollies to solve the problem?"). Children received an initial response score and a second response score that took both initial and post-prompt responses into account. The initial response score was computed by summing the number of points on each item (i.e., 1 point if they used a referent of 2 for the pollies and 0 points if they did not) and could have a minimum of 0 and a maximum of 2. The second response score was calculated by summing the number of points on each item, but in this case, 0 points were assigned if the referent of 2 was never used across the two responses and 1 point for a referent of 2 on one or both responses; this second score and could have a minimum of 0 and a maximum of 2.

Table 1

Word Problems Used in the Near-Transfer, Constrained and Unconstrained Far-Transfer Tasks

Task	Problem Label	Word Problem (*grouping*)
Near-Transfer	Problem A	There are three kids in a park. Each kid has 2 balloons. How many balloons do they have altogether? *(3 groups of 2)*
	Problem B	Tad has two bowls. There are 4 meatballs in each bowl. How many meatballs does Tad have altogether? *(2 groups of 4)*
Constrained Far-Transfer	Problem C	Eric puts 3 toy cars in every box. He has 3 boxes. How many toy cars does he have altogether? *(3 groups of 3)*
	Problem D	At the bakery, the baker puts 9 cupcakes in each box. How many cupcakes are in 2 boxes? *(2 groups of 9)*
	Problem E	Louie puts 5 shirts in each drawer of his dresser. There are 3 drawers in his dresser. How many shirts does he have altogether? *(3 groups of 5)*
	Problem F	Robin has 2 packs of gum. There are 15 pieces of gum in each pack. How many pieces of gum does Robin have altogether? *(2 groups of 15)*
Unconstrained Far-Transfer	Problem G	Samantha puts 3 bracelets in each jewelry box. She has 3 jewelry boxes. How many bracelets does she have altogether? *(3 groups of 3)*
	Problem H	A teacher puts 9 crayons in each pencil case. How many crayons are in two pencil cases? *(2 groups of 9)*
	Problem I	Violet wants to give 5 flowers to each one of her 3 friends. How many flowers does she need altogether? *(3 groups of 5)*
	Problem J	There are two aquariums at the pet store. Each aquarium has 15 fishes. How many fishes are there altogether? *(2 groups of 15)*

I coded half of the data and a second coder, a trained graduate student, coded the other half. I established inter-rater reliability by randomly selecting and coding 18% of the data that had been coded by the second coder (i.e., the responses for 11 participants: 4 from the DI condition, 4 from the GE condition, and 3 from the control condition). Percentage of agreement and kappa were calculated. The percentage of agreement was 95% and the kappa coefficient was $\kappa = .93$, $p < .001$. The Near-Transfer task had high levels of internal consistency, for both the initial response items and the post-prompt response items, as determined by Cronbach's alphas of .818 and .867, respectively.

Constrained Far-Transfer Task

The Constrained Far-Transfer Task (see interview protocol in Appendix C) assessed children's symbolic flexibility – that is, their ability to assign a new quantitative referent to the same target manipulative. Participants were provided with a bin containing 40 pollies. The interviewer read a series of four multiple-groups multiplication word problems (see Table 1) out loud and asked children to use the pollies to solve the problems. The word problems differed from the ones used in the Near-Transfer task in that they were not of the n x 2 or n x 4 types. The word problems were of the n x 3, n x 5, n x 9, or n x 15 types, in which there were n groups and 3, 5, 9, and 15 items in each group, respectively. The order of the word problems was counterbalanced. Again, children were provided with mats to help them see the number of groups involved in the problems.

After each problem, the interviewer asked prompts to make sure she understood which referent the child was using for the pollies (e.g., "Can you explain to me how you solved the problem?") and to see if the child could use a different referent for the pollies (e.g., "Can you show me a different way to solve the problem with the pollies?"). The child could therefore make more than one attempt to solve the problem and thus provide more than one answer. Children received a flexibility score for each problem, and that score considered all attempts and answers provided for that problem. They received a score of 0 if they had assigned only a referent of 1 or only a referent of 2 to the pollies at any point while completing the item. They received a score of 1 if they had assigned a referent of 1 *and* 2 to the pollies. Finally, they received a score of 2 if they had assigned a referent other than 1 or 2 to the pollies. I considered that using a referent of 1 only did not demonstrate symbolic flexibility, as it would be the default referent to use based on the literature (Martí et al., 2013; National Research Council, 2009). A referent of 2 only would not show flexibility either because it is the referent that was learned during instruction, and it would not lead to a correct solution to the problems in the Constrained Far-Transfer task. If children used both referents of 1 and 2 in their solutions, it demonstrated a certain level of flexibility because they were able to use two different referents for the same manipulative. Lastly, if children used a referent other than 1 or 2, I considered this a high level of flexibility because they had not encountered these other referents at any point during the instruction.

The scoring used for the Constrained Far-Transfer task could be considered a Likert scale (Boone & Boone, 2012), with a score of 0 indicating responses that were not flexible, a score of 1 indicating responses that were somewhat flexible, and a score of 2 indicating responses that were optimally flexible. Although Likert scales yield ordinal data, many experts in the social sciences analyze Likert scale responses with parametric tests typically used with interval data, especially when the tests or scales have sufficient levels of internal consistency and inter-rater reliability (Sullican & Artino, 2013). Inter-rater reliability for the Constrained Far-Transfer task was conducted with a second coder, a trained undergraduate student, who coded 18% of the data (i.e., the responses for 11 participants: 4 from the DI condition, 4 from the GE condition, and 3 from the control condition), which had been randomly selected. The percentage of agreement was 93% and the kappa coefficient was $\kappa = .86$, $p < .001$. The Constrained Far-Transfer task had a high level of internal consistency, as determined by a Cronbach's alpha of .897. These reliability metrics justify treating the Constrained Far-Transfer task as a measure yielding data on an interval scale. As such, a mean flexibility score for the task was calculated for each student.

Accuracy was also assessed for answers children provided to the Constrained Far-Transfer problems. Because children could make multiple attempts to solve each problem, accuracy was not determined for each participant but rather for each problem-solving attempt across all participants. For each problem-solving attempt, a score of 1 was assigned if the strategy involved the use of the pollies and the final answer obtained was correct. A score of 0 was assigned for wrong answers.

Unconstrained Far-Transfer Task

The Unconstrained Far-Transfer task (see interview protocol in Appendix C) assessed children's symbolic fluency – that is, their ability to assign a new quantitative referent to a new, unfamiliar manipulative. Children's symbolic flexibility was also assessed with this task. The procedure was similar to the one used for the Constrained Far-Transfer task. The interviewer read a series of four n x 3, n x 5, n x 9, and n x 15 multiple-groups multiplication word problems out loud (see Table 1). The order of the word problems was counterbalanced. The children were given a bin with a variety of manipulatives (see Figure 9) and asked to solve the problems using the objects in the bin. Each bin contained 45 blocks of three different colors (15 blocks for each color); 45 square, round, triangular or oval buttons of three different colors (15 buttons for each color); and 45 plastic gems of three different colors (15 gems for each color). There were no pollies in the bin.

Figure 9

Example of Manipulatives Used for the Unconstrained Far-Transfer Task

After each problem, the interviewer asked prompts to make sure she understood which referent the child was using for the manipulatives (e.g., "Can you explain to me how you solved the problem?") and to see if the child could use a different referent for the manipulatives (e.g., "Can you show me a different way to solve the problem with these?"). The child could therefore make more than one attempt to solve the problem and thus provide more than one answer. Children received a fluency score for each problem solved, which considered all attempts and answers provided for that problem. They received a score of 0 if they had assigned only a referent of 1 to the manipulatives while solving the item. They received a score of 1 if they had assigned a referent of 2 to the manipulatives at any point while working on the item. Finally, they received a score of 2 if they had assigned a referent other than 1 or 2 to the manipulatives at any point. I considered that using a referent of 1 only did not demonstrate symbolic fluency, as, again, it would be the default referent to use based on the literature (Martí et al., 2013; National Research Council, 2009). I believed that if children used a referent of 2, they exhibited a certain level of fluency as they would have transferred what they had learned during the instruction to a novel manipulative. Lastly, if children used a referent other than 1 or 2, I considered they demonstrated a high level of fluency, as it would reveal their ability to assign a referent they had not encountered during instruction to a novel manipulative.

As was the case for the Constrained Far-Transfer task, the scoring used for the Unconstrained Far-Transfer task could also be considered a Likert scale (Boone & Boone, 2012), in this case with a score of 0 indicating responses that were not fluent, a score of 1 indicating responses that were somewhat fluent, and a score of 2 indicating responses that were optimally fluent. Inter-rater reliability was conducted with a second coder, a trained undergraduate student, who coded 18% of the data (i.e., the responses for 11 participants: 4 from the DI condition, 4 from the GE condition, and 3 from the control condition), which had been randomly selected. The percentage of agreement was 98% and the kappa coefficient was $\kappa = .95$, $p < .001$. The Unconstrained Far-Transfer task had a high level of internal consistency, as determined by a Cronbach's alpha of .941. These reliability metrics allowed me to treat the data from the Unconstrained Far-Transfer task as interval data (see Sullican & Artino, 2013), justifying the calculation of a mean fluency score for each student.

Accuracy was also assessed for answers children provided to the Unconstrained Far-Transfer problems. Because children could make multiple attempts to solve each problem, the unit of analysis was problem-solving attempt for the whole sample. For each problem-solving attempt, a score of 1 was assigned if the strategy involved the use of the manipulatives and the final answer obtained was correct. A score of 0 was assigned for wrong answers.

Sticker Task

The Sticker task was another measure of children's symbolic flexibility. The objective of the task was to see whether children would decide to assign a "profitable" referent to the pollies to get a reward. The interviewer told children that they would be receiving stickers to thank them for their hard work. The interviewer explained that the number of stickers they would receive would depend on the secret code they would choose for the pollies. They were asked to place pollies on the special lid of an empty box (same type of box used in the Need-to-symbolize and encoding activities; see Figure 5) and to explain their secret code to the interviewer. The number of stickers the children asked for with their representation depended on the number of pollies they placed on the lid and the referent (i.e., the secret code) they assigned to each.

The interviewer then asked the child to close his or her eyes while she placed a sheet of stickers inside the box. All children received the same number of sticker in the end; if they had asked for fewer stickers, the interviewer said that because they did such a good job, they got more than what they had asked for, and if they had asked for more stickers, the interviewer said that this was all the stickers that she had. Children's responses were classified into one of three categories based on the referent they had used for the pollies: (a) referent of 1, (b) referent of 2, or (c) referent other than 1 or 2.

Chapter 4: Results

The results section will report the statistical analyses and provide descriptive examples from the data to illustrate some of the findings. The results are organized into four different sections: (a) prior knowledge and cognitive measures, (b) learning, (c) near-transfer, and (d) far-transfer, which includes representational versatility (i.e., symbolic flexibility and symbolic fluency), manipulative use, and problem-solving accuracy. Before data analysis began, the study's hypotheses, design, sample size, and analysis plan were preregistered at osf.io. The preregistered protocol is currently embargoed, but it can be found in Appendix F.

Of the sample of 65 students, 11 were excluded on at least one of the four tasks. In the report of the analysis for each task below, I will specify the number of excluded participants and the reasons for the exclusions. When considering the full sample of 65 participants, I found that age was not significantly correlated with any of the outcome variables (all ps > .05). There were also no gender or school differences, and there was no effect of order of presentation of the Constrained and Unconstrained Far-Transfer tasks on any outcome variable (all ps > .05).

Prior Knowledge and Cognitive Measures

The means and standards deviations for the prior knowledge and cognitive measures can be found in Table 2. Tables 3, 4, and 5 present the correlations between the prior knowledge and cognitive measures and the outcome measures in each condition, respectively. There were four significant correlations' scores were significantly related to the number of items completed during the encoding session ($p < .001$) for students in the DI condition only. For students in the GE condition, NKT scores were significantly correlated with flexibility scores obtained from the Constrained Far-Transfer task, $p = .02$. Scores on the Digits Backwards task were significantly related to the number of items completed during the encoding session ($p < .001$) for students in the DI condition only. Lastly, PathSpan scores were significantly related to scores obtained on students' initial responses to the Near-Transfer task, but for students in the DI condition only. Because none of the cognitive measures yielded significant correlations in all three conditions, they were not included as covariates in any subsequent analyses.

Table 2

Means and Standards Deviations for the Prior Knowledge and Cognitive Measures

Measure	Condition	*M*	*SD*	*n*
NKT	DI	15.40	4.91	20
	GE	16.48	4.18	25
	Control	15.75	4.45	20
	Total	15.92	4.45	65
DCCS	DI	2.30	.57	20
	GE	2.68	.48	25
	Control	2.55	.51	20
	Total	2.52	.53	65
Digits Backwards	DI	2.90	1.30	20
	GE	3.12	1.33	25
	Control	2.10	1.55	20
	Total	2.74	1.44	65
PathSpan	DI	3.55	1.47	20
	GE	3.68	1.11	25
	Control	3.80	1.44	20
	Total	3.68	1.31	65
Go/No-Go	DI	1.97	.84	20
	GE	2.60	.64	24
	Control	2.05	.79	20
	Total	2.23	.80	64

Note. NKT = Number Knowledge Test; DCCS = Dimensional Change Card Sort

Table 3

Correlations Between Cognitive Measures and Outcome Measures for the DI Condition

Measures	Learning: Items ($n = 20$)	Learning: Time ($n = 20$)	Near-Transfer: Initial ($n = 19$)	Near-Transfer: Post-prompt ($n = 19$)	Constr. Far-Transfer: Flexibility ($n = 19$)	Unconstr. Far-Transfer: Fluency ($n = 19$)
NKT	–.65**	–.21	.38	.28	.40	.28
DCCS	–.58	–.20	–.07	–.13	.42	.18
Digits backwards	–.76**	–.19	.25	.15	.38	.22
PathSpan	–.25	.16	.46*	.36	.32	.05
Go/ No-Go	–.05	.37	–.15	–.01	.03	.15

Note. Constr. = Constrained; Unconstr. = Unconstrained; NKT = Number Knowledge Test; DCCS = Dimensional Change Card Sort

* $p < .05$, ** $p < .001$

Table 4

Correlations Between Cognitive Measures and Outcome Measures for the GE Condition

Measures	Learning: Items ($n = 25$)	Learning: Time ($n = 25$)	Near-Transfer: Initial ($n = 25$)	Near-Transfer: Post-prompt ($n = 25$)	Constr. Far-Transfer: Flexibility ($n = 22$)	Unconstr. Far-Transfer: Fluency ($n = 21$)
NKT	–.18	–.14	.09	.28	.50*	.37
DCCS	–.10	.14	.12	–.25	–.08	–.17
Digits backwards	–.35	–.32	–.15	.03	.01	.22
PathSpan	–.11	.25	–.24	–.15	–.04	.08
Go/ No-Go	–.10[a]	.01[a]	–.08[a]	–.04[a]	–.01[b]	–.30

Note. Constr. = Constrained; Unconstr. = Unconstrained; NKT = Number Knowledge Test; DCCS = Dimensional Change Card Sort

[a]$n = 24$ [b]$n = 21$

*$p < .05$

Table 5

Correlations Between Cognitive Measures and Outcome Measures for the Control Condition

Measures	Learning: Items[a] ($n = 20$)	Learning: Time ($n = 20$)	Near-Transfer: Initial[a] ($n = 20$)	Near-Transfer: Post-prompt[a] ($n = 20$)	Constr. Far-Transfer: Flexibility ($n = 20$)	Unconstr. Far-Transfer: Fluency[a] ($n = 20$)
NKT	–	–.35	–	–	–.09	–
DCCS	–	–.40	–	–	–.25	–
Digits backwards	–	–.35	–	–	.29	–
PathSpan	–	.02	–	–	.20	–
Go/ No-Go	–	.02	–	–	.01	–

Note. Constr. = Constrained; Unconstr. = Unconstrained; NKT = Number Knowledge Test; DCCS = Dimensional Change Card Sort

[a]Correlations could not be computed because the variable was constant.

Five one-way analyses of variance (ANOVA) were performed with each cognitive measure as the dependent variable and condition as the between-groups factor. Results indicated that there was a significant group difference for the Digits Backwards task, $F(2, 62) = 3.18$, $p = .049$, $\eta_p^2 = .09$, $(1 - \beta) = .59$[2]. Post-hoc analyses using a Bonferroni correction did not reveal any specific pairwise differences between the three groups (all *p*s > .05), however. Given that these findings are difficult to interpret, scores on the Digits Backwards task were not included in any of the subsequent analyses.

A significant group difference was also found for the Go/No-Go task, $F(2, 61) = 4.67$, $p = .01$, $\eta_p^2 = .13$, $(1 - \beta) = .77$. Post-hoc analyses using a Bonferroni correction revealed that

[2] $(1 - \beta)$ refers to observed power.

scores for students in the DI condition (M = 1.97, SD = .84) were significantly lower than those of students in the GE condition (M = 2.06, SD = .64), $t(42) = 2.76$, $p = .02$, $d = .86$. Despite these differences, scores on the Go/No-Go task were not included in subsequent analyses because they were not correlated with any of the outcome measures (see Tables 3, 4, and 5). No other group differences were found on any of the other prior knowledge or cognitive measures (all ps > .05).

Learning

Out of the full sample of 65 students, the data of four participants were excluded for the learning assessment, resulting in a sample of 61. The four participants (three from the DI condition and one from the GE condition) did not successfully encode the polly as having a referent of 2 during the encoding session that took place before the learning assessment on the second interview day.

Recall that learning was assessed by examining children's performance during the encoding session in two ways: (a) the number of items needed for children to encode the pollies as having a referent of 2, and (b) the length of the encoding session in minutes, from the start of the session until children had finished correctly solving two items in a row using a referent of 2 for the pollies without feedback. Means and standard deviations on these measures are presented in Table 6. Two independent-samples t-tests were conducted to determine if there were differences in learning between the two instructional conditions (i.e., DI and GE). Data from students in the control condition were not included in this analysis because these students did not receive any type of instruction, and only one participant spontaneously assigned a referent of 2 to the pollies during the encoding session (see below).

The results from the first independent-samples t-test, with mean number of items as the dependent variable, revealed a significant difference between the two groups, $t(39) = -2.34$, p =.02, $d = .75$, $(1 - \beta) = .63$. The results from the second independent-samples t-test, with length of the encoding session as the dependent variable, revealed that students in the GE condition took significantly longer to encode the pollies as having a referent of 2 than students in the DI condition, $t(39) = -2.56$, p =.01, $d = .82$, $(1 - \beta) = .71$.

Table 6

Means and Standard Deviations for the Learning Assessment

	Number of items[a]			Time to learn[b]		
	M	*SD*	*n*	*M*	*SD*	*n*
Direct Instruction	.63	.15	18	8.20	3.53	18
Guided Exploration	.74	.14	23	12.43	6.25	23
Total	.69	.15	41	10.57	5.60	41

Note. All students in the control condition ($n = 20$) completed six items during the encoding session, and the mean session length for these students was 8.67 minutes ($SD = 3.28$)

[a]Reported in percent

[b]In minutes

Students' responses during the encoding session were also examined qualitatively. All students in the DI condition used the pollies as having a referent of 2 during the encoding session. Students in the GE condition were at first given the freedom to decide on their own referent for the pollies and then had to modify that referent in response to the constraints placed on the task by the interviewer. Therefore, students in the GE condition assigned different referents to the pollies during the course of the encoding session before finally assigning a referent of 2 by the end of the session. All of the students in the GE condition started by assigning a referent of 1 to each polly, thereby using the same number of pollies as cubes to represent the given quantities. Four students also experimented with using a referent of 4 for the pollies – some of these students pointed out the four lateral faces or the four edges of the pollies to justify the referent of 4. Most students in the control condition only used 1 as the referent for the pollies. One student, however, used a referent of 2 for some of the items, and another student used the pollies to "draw" the number of cubes he was asked to represent.

Near Transfer

Out of the full sample of 65 students, the data of four participants were excluded for this task, resulting in a sample of 61. Because of administrative error, video data were missing for one participant (DI condition), and three students (all from the DI condition) did not successfully encode the polly as having a referent of 2 during the encoding session that took place before the Near-Transfer task on the second interview day.

Table 7 shows the means and standard deviations for each group on the initial and post-prompt responses. A score of 2 (the highest score possible) would indicate that the student assigned a referent of 2 to the pollies on both items. Because there was no variance on either initial or post-prompt responses for students in the control group, this group was excluded from the analyses for the Near-Transfer task.

Table 7

Means and Standard Deviations for Performance on the Near-Transfer Task

	Near-Transfer (initial response)[a]			Near-Transfer (post-prompt response)[a]		
	M	*SD*	*n*	*M*	*SD*	*n*
Direct Instruction	.88	.99	17	1.35	.86	17
Guided Exploration	.38	.65	24	1.08	.93	24
Control	.00	.00	20	.00	.00	20
Total	.39	.74	61	.80	.93	61

[a]Max: 2, min: 0

To determine group differences in performance on the Near-Transfer task, a 2x2 mixed design ANOVA was conducted with response attempt (initial, post-prompt) as the repeated-measures factor and condition (DI, GE) as the between-groups independent variable. A significant main effect of attempt was found, $F(1, 39) = 23.26$, $p < .001$, $\eta_p^2 = .37$, $(1 - \beta) = 1.00$, indicating that overall the mean score on the post-prompt response was higher than the mean score on the initial response. There was, however, no significant main effect of condition, $F(1, 39) = 2.57$, $p = .117$, $(1 - \beta) = .35$ and no significant interaction between attempt and condition, $F(1, 39) = .95$, $p = .34$, $(1 - \beta) = .16$. My prediction that children in the DI condition would use a referent of 2 more consistently across all problems compared to children in the GE condition was therefore not supported.

Four one-sample *t*-tests were then conducted to determine if the scores on the Near-Transfer task (both the initial and the post-prompt scores) for the DI and GE conditions were significantly different from 0. A value of 0 represents not ever using the polly with a referent of 2. To minimize the risk of a Type I error, an alpha of .0125 (.05/4) was used for each *t*-test. As can be seen from the results presented in Table 8, all four *t*-tests yielded significant results. All initial and post-prompt mean scores on the Near-Transfer task were significantly different from 0 for the DI and GE conditions, meaning that students in these two conditions more often represented the pollies as 2 than any other referent.

Table 8

Results from One-Sample t-*Tests for the Initial and Post-Prompt Performance on the Near-Transfer Task in the DI and GE Conditions*

Variable	Condition	*t*	*df*	*p*	*d*
Initial response	DI	3.67	16	.002	.89
	GE	2.84	24	.009	.58
Post-prompt response	DI	6.47	16	< .001	1.57
	GE	5.72	24	< .001	1.17

Note. DI: $n = 17$; GE: $n = 24$

Despite no significant interaction between attempt and condition, students' response change from their initial to their post-prompt responses were nevertheless examined descriptively. Table 9 shows the distribution of scores for initial and post-prompt responses on the Near-Transfer task for the DI and GE instructional conditions. A score of 0 indicates that the student did not assign a referent of 2 to the pollies on either item. A score of 1 indicates that the student assigned a referent of 2 to the pollies on one item. A score of 2 indicates that the student assigned a referent of 2 to the pollies on both items. Scores for students in the control condition were omitted because they all received a score of 0 both for their initial and post-prompt responses.

Table 9

Frequencies (and Proportions) of Responses Within the DI and GE Conditions on the Near-Transfer Task for Initial and Post-Prompt Scores

	Near-Transfer (initial response)				Near-Transfer (post-prompt response)			
Scores	0	1	2	Total	0	1	2	Total
Direct Instruction	9 (53%)	1 (6%)	7 (41%)	17 (100%)	4 (24%)	3 (18%)	10 (59%)	17 (100%)
Guided Exploration	17 (71%)	5 (21%)	2 (8%)	24 (100%)	9 (38%)	4 (17%)	11 (46%)	24 (100%)

As can be seen in Table 9, there were about five times as many students in the DI condition (41%) whose initial referent for the pollies was 2 on both items (i.e., score of 2) compared to students in the GE condition (8%). Inversely, the proportion of students in the GE condition (71%) whose initial use of the pollies was not with a referent of 2 on both items (i.e., score of 0) was about 1.5 times greater than the proportion of students in the DI condition (53%) who answered similarly.

Students in both the DI and GE conditions were generally better at assigning a referent of 2 for the pollies after being prompted by the interviewer, but students in the GE condition seemed to benefit the most from the prompts. The proportion of students in the GE condition who assigned a referent of 2 to the pollies after being prompted was over five times that for the initial responses (8% on the initial response and 46% on the second). In contrast, the proportion of students in the DI condition who assigned a referent of 2 to the pollies after the prompt was almost 1.5 times that for their initial responses (41% on the initial response and 59% on the second).

A more detailed examination of the change from the initial responses to the post-prompt responses was conducted next. There were nine students in the DI condition who did not assign a referent of 2 to the pollies on either item on their initial responses. Three of these students (33%) were able to do so on one item after being prompted, two of these students (22%) assigned a referent of 2 on both items after being prompted, and four of these students (44%) were still not able to assign a referent of 2 after being prompted. One additional student in the DI condition only assigned a referent of 2 to the pollies on one item before prompting but was able to assign a referent of 2 to both items after prompting. Finally, there were seven students who assigned a referent of 2 on both their initial and post-prompt responses.

There were 17 students in the GE condition who did not assign a referent of 2 to the pollies as an initial response. Three of these students (18%) were able to do so on one item after being prompted, five of these students (29%) were able on both items after being prompted, and nine of these students (53%) were still not able to assign a referent of 2 after being prompted. Five additional students in the GE condition only assigned a referent of 2 to the pollies on one item as their initial response. After prompting, four of these students (80%) were able to do it for both items. Finally, there were two students who assigned a referent of 2 on both their initial and post-prompt responses. Overall, all participants except two (one from the DI condition and one from the control condition) assigned a referent of 1 to the pollies when they failed to assign a referent of 2. When asked to use the pollies to solve the first problem, for example, in which they were asked for the total number of balloons for three children holding two balloons each, children who assigned a referent of 1 to the pollies put down 6 in total. One student did not use a referent for the pollies and instead used the pollies to "draw" the digits representing the quantities in the problem, and another student used an unclear referent (i.e., he was making up stories about the word problems and playing with the pollies).

Far Transfer

Children's far-transfer abilities were examined in different ways, from broader analyses to more fine-grained analyses. First, I assessed children's representational versatility, which entails two different aspects of symbol use: symbolic flexibility and symbolic fluency. Next, I observed how children used the different manipulatives during the Constrained and Unconstrained Far-Transfer tasks; I was interested in the quantitative referents they assigned to these manipulatives. Lastly, children's word-problem-solving accuracy was analyzed.

Representational Versatility

Taken together, students' symbolic flexibility and their symbolic fluency provide information on their representational versatility. Flexibility was determined from students' performance on the Constrained Far-Transfer task: I wanted to know if, when solving the word problems with the pollies, children used only one referent on each item (either 1 or 2) for the pollies, if they used both referents of 1 and 2, or if they used other referents. The Sticker task provided another measure of students' flexibility as it revealed what referent(s) students chose to use for the pollies in a context that was potentially profitable to them. Students' fluency was determined from their performance on the Unconstrained Far-Transfer task: I wanted to know if, when solving the word problems with the manipulatives, children were able to assign a referent other than 1 to the novel objects. Lastly, I also examined the relation between the degree of students' flexibility and fluency.

Flexibility

On the Constrained Far-Transfer task, out of the full sample of 65 students, the data of four participants were excluded ($N = 61$). Two participants in the GE condition did not successfully encode the polly as having a referent of 2 during the review session that took place before the Constrained Far-Transfer task. One participant from the GE condition was absent, and one participant in the DI condition wished to stop the interview.

Table 10 shows students' mean flexibility scores and standard deviations on the Constrained Far-Transfer task in each group. Higher scores represent a higher level of flexibility, with a score of 2 (the maximum possible) indicating that the student assigned a referent other than 1 or 2 to the pollies on all items. A one-way ANOVA was conducted with mean flexibility scores as the dependent variable and condition as the between-groups variable. Results indicate significant group differences, $F(2, 58) = 6.11$, $p = .004$, $\eta_p^2 = .17$, $(1 - \beta) = .87$. Post-hoc analyses using a Bonferroni correction revealed that flexibility scores on the Constrained Far-Transfer task in the GE group were significantly greater than those in the control group, $t(40) = 3.48$, $p = .003$, $d = 1.08$. The scores from the DI group were not significantly different from those in the GE group ($t(39) = 1.38$, $p = .52$), nor from those in the control group ($t(37) = .01$, $p = .15$). In sum, my predictions were partially supported. I had predicted that children in the GE condition would outperform children in the other two conditions, but their performance was only significantly greater than that of children in the control group. In addition, as I had predicted, there was no difference between the DI and control groups.

Table 10

Means and Standard Deviations of Flexibility Scores on the Constrained Far-Transfer Task[a]

	M	*SD*	*n*
Direct Instruction	.36	.54	19
Guided Exploration	.57	.63	22
Control	.04	.17	20
Total	.33	.53	61

Note. Flexibility score was the mean number of points across all four items.

[a]Max: 2, min: 0

On the Sticker task, out of the full sample of 65 students, the data of five participants were excluded ($N = 60$). Two participants (both in the GE condition) did not successfully encode the polly as having a referent of 2 during the review session that took place at the beginning of Day 3. One participant (GE condition) was absent, and two participants (one from the DI condition and one from the GE condition) wished to stop the interview.

The Sticker task was another measure of students' flexibility. Students' chosen referent for the pollies fell into one of three categories: (1) referent of 1, (2) referent of 2, or (3) other or unspecified referent. Only three students were classified in the latter category and so they were excluded from this analysis. A chi-square test of association was conducted between chosen referent (1, 2) and condition (DI, GE, and control). All expected cell frequencies were greater than five. There was a statistically significant association between referent and condition, $\chi^2(2) = 11.43$, $p = .003$. Post-hoc analyses involved pairwise comparisons using z-tests to compare proportions with Bonferroni corrections. The proportion of students in the control condition (18/18 or 100%) who used a referent of 1 was significantly greater than the proportions of students in both the DI (10/19 or 52.6%) and the GE (12/20 or 60.0%) conditions, $p < .05$. The proportions in the latter two groups did not significantly differ from each other.

Fluency

Students' fluency was determined from their performance on the Unconstrained Far-Transfer task. Out of the full sample of 65 students, the data of five participants were excluded for this task ($N = 60$). Two participants (both in the GE condition) did not successfully encode the polly as having a referent of 2 during the review session on Day 3 that took place before the

Unconstrained Far-Transfer task. One participant (GE condition) was absent, and two participants (one from the DI condition and one from the GE condition) wished to stop the interview.

Table 11 shows students' mean fluency scores and standard deviations on the Unconstrained Far-Transfer task for each group. Higher scores represent a higher level of fluency, with a score of 2 (the maximum possible) indicating that the student assigned a referent other than 1 or 2 to the manipulatives on all items. An independent-samples *t*-test was conducted with mean fluency scores as the dependent variable and condition (DI, GE) as the between-groups independent variable. Because there was no variance in performance scores for students in the control group, this group was excluded from the analysis. Results indicate no significant group difference, $t(38) = -1.34$, $p = .19$, $(1 - \beta) = .26$. My prediction that children in the GE condition would outperform children in the DI condition was not supported.

Table 11

Means and Standard Deviations of Fluency Scores on the Unconstrained Far-Transfer Task[a]

	M	*SD*	*n*
Direct Instruction	.25	.54	19
Guided Exploration	.52	.72	21
Control	.00	.00	20
Total	.26	.56	60

Note. Fluency score was the mean number of points across all four items.

[a]Max: 2, min: 0

Two one-sample *t*-tests were then conducted to determine if the scores on the Unconstrained Far-Transfer task for each of the DI and GE conditions were significantly different from 0. A value of 0 would represent only assigning a referent of 1 to the manipulatives for all four items. To minimize the risk of a Type I error, an alpha of .025 (.05/2) was used for each *t*-test. As can be seen from the results presented in Table 12, fluency scores were significantly different from 0 for students in the GE condition, but not for students in the DI condition.

Table 12

Results from One-Sample t*-Tests for Mean Fluency Scores on the Unconstrained Far-Transfer Task for the DI and GE Conditions*

Condition	*n*	*t*	*df*	*p*	*d*
Direct instruction	19	2.02	18	.059	.46
Guided exploration	21	3.32	20	.003	.72

Relation between Flexibility and Fluency

To determine the relation between flexibility and fluency, a Pearson's correlation was conducted. There was a statistically significant, strong positive correlation between flexibility and fluency scores, $r(58) = .78$, $p < .001$. Statistically significant, strong positive correlations between flexibility and fluency scores were also found when looking at the associations within each of the DI and GE conditions, as can be seen in Table 13.

Table 13

Correlations Between Flexibility and Fluency Scores by Condition[a]

Conditions	*r*	*df*	*p*	*n*
DI condition	.75	17	< .001	19
GE condition	.77	19	< .001	21

[a]Correlations could not be computed for the control condition because the fluency variable was constant (i.e., 0).

Manipulative Use

Students were categorized into different profiles based on their manipulative use during the Constrained and Unconstrained Far-Transfer tasks. First, profiles were based the nature of their polly use during the Constrained Far-Transfer task by coding the specific referent(s) they had used across all four items. Next, students' manipulative use during the Unconstrained Far-Transfer task was examined: I wanted to see whether there were group differences in the number of different manipulatives students used to solve each word problem. I also categorized students into different profiles based on the specific referent(s) they used during the task.

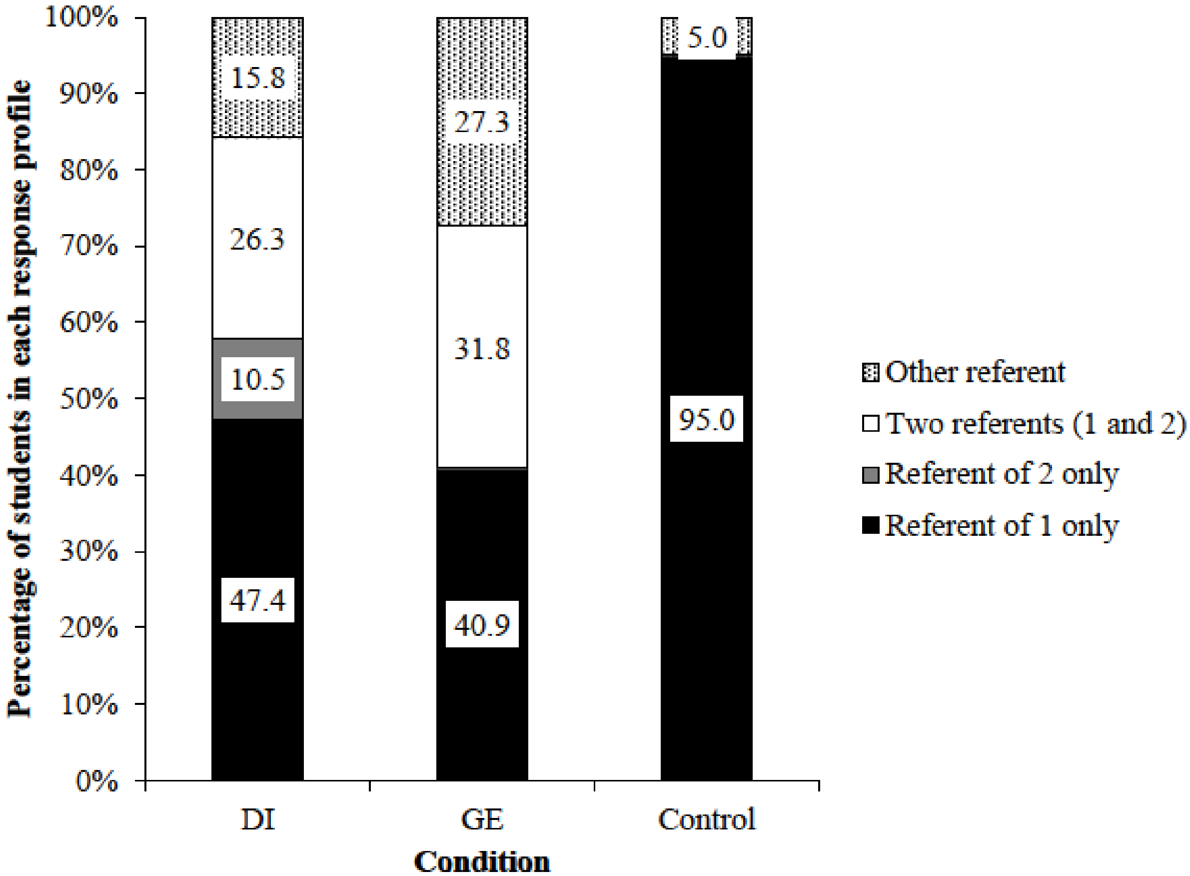
Polly Use During the Constrained Far-Transfer Task

Students were categorized into one of four profiles based on how they answered all four items of the Constrained Far-Transfer task. I was interested in how they viewed the pollies during the task regardless of when their responses were provided (i.e., initial and post-prompt). If students used the pollies only with a referent of 1 on all four items, they were classified in the "Referent of 1 only" profile. If they used the pollies only with a referent of 2 on all four items, they were placed in the "Referent of 2 only" profile. If they had assigned a referent of 1 *and* 2 in at least one of their responses to the four items, they were placed in the "Two referents" profile. Lastly, if they had assigned one or more referents, other than 1 or 2, to the pollies in their responses to at least one of the four items, they were placed in the "Other referent" profile.

Figure 10 shows the percentage of students in each response profile by condition. First, not surprisingly, most of the students in the control condition (19/20 or 95.0%) were in the *Referent of 1 only* profile. There was a higher proportion of students in the *Referent of 1 only* profile in the DI condition (9/19 or 47.4%) compared to the GE condition (9/22 or 40.9%). Second, some students in the DI condition (2/19 or 10.5%) were in the *Referent of 2 only* profile; no student was classified in this profile in the other two conditions. A typical response for students in the DI condition was to attempt to represent the problem with the pollies as 2 and then come to the conclusion that it could not be done as none of the quantities in the problems involved multiples of 2. Third, there was a slightly greater number of students from the GE condition (7/22 or 31.8%) compared to those in the DI condition (5/19 or 26.3%) who were in the *Two referents* profile. These students, at one point or another during the task, viewed the pollies as having a referent of 1 and 2. To illustrate, one student in this profile used one polly as 1 and four pollies, each worth 2, to represent a total quantity of 9. Finally, there were close to twice as many students in the GE condition (6/22 or 27.3%) who were in the *Other referent* profile compared to students in the DI condition (3/19 or 15.8%) and about five times as many when compared to students in the control condition (1/20 or 5.0%). All students in this profile assigned at least one referent, other than 1 or 2, to the pollies on at least one item, and the referents took on a variety of values, namely 3, 4, 5, 9, 15, 19, or 30. For example, to represent a quantity of 15, one child in this category used three pollies, each one having a referent of 5. No student used a referent other than 1 or 2 on all items.

Figure 10

Percentage of Students in Each Response Profile on the Constrained Far-Transfer Task for the DI (n = 19), GE (n = 22), and Control (n = 20) Conditions



To determine whether there was an association between condition and students who used only one or more than one referent, the profiles described above were collapsed into two types of responses: (1) one referent only (i.e., *Referent of 1 only* and *Referent of 2 only* profiles), and (2) two or more referents (i.e., *Two referents* and *Other referent* profiles). A chi-square test of association was conducted between type of responses (one referent only, two or more referents) and condition (DI, GE, and control). All expected cell frequencies were greater than five. There was a statistically significant association between type of response and condition, $\chi^2(2) = 13.73$, $p = .001$. Post-hoc analyses involved pairwise comparisons using z-tests to compare proportions with Bonferroni corrections. The proportion of students in the control condition

(5.0%) who used two or more referents was significantly lower than the proportion of students in both the DI (42.1%) and the GE (59.1%) conditions, $p < .05$, respectively. The proportions in the latter two groups did not differ significantly from each other.

To further understand the different types of responses students gave when they were using two or more referents for the pollies, I next examined the responses of children in the *Two referents* and *Other referent* profiles together. More specifically, I looked at the proportion of problems (out of the total number of problems solved during the task) in which children used two or more referents – that is, 1, 2, or other – on one problem. For example, one child used the pollies with a referent of 1 and 2 in the *same attempt*; to represent the quantity of 3, she used two pollies: one with a referent of 1 and one with a referent of 2. In another example, one child used different referents in *separate attempts* to solve the same problem; to represent the quantity of 5 on one attempt, he used five pollies that each had a referent of 1, and on another attempt for the same problem, he used one polly that had a referent of 5. Children in the GE condition (n = 13) used two or more referents more often than children in the DI condition (n = 8; 78.2% of the time vs. 68.8%, respectively).

Proportion scores were also calculated for the proportion of problems (out of the total number of problems attempted during the task) in which children in the *Other referent* profile used a referent other than 1 or 2. Children in the GE condition appeared to use referents other than 1 or 2 for the pollies more often than students in the DI condition; the mean proportion of students in the GE condition (69.4%, n = 6) was almost one and a half times as high as that of students in the DI condition (50.0%, n = 3).

Manipulative Use During the Unconstrained Far-Transfer Task

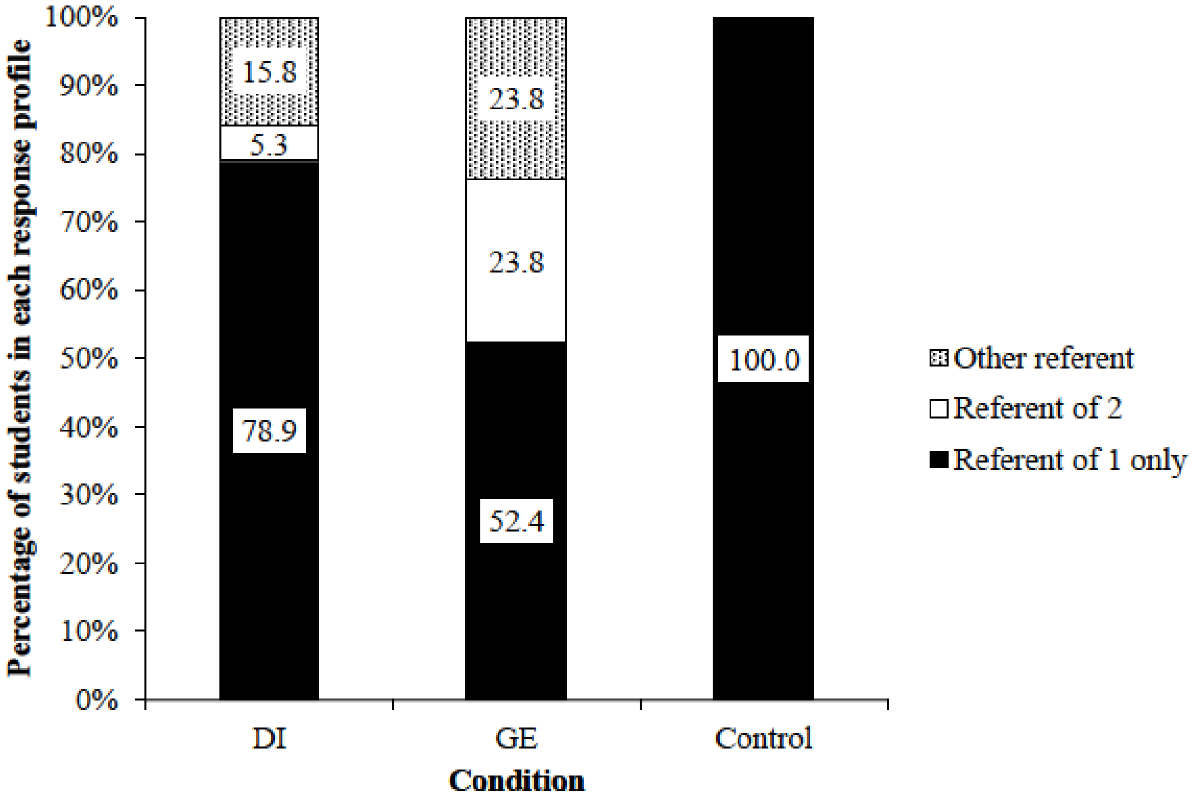
Children's manipulative use during the Unconstrained Far-Transfer task was examined. Children could choose various manipulatives (i.e., blocks, buttons, gems) to solve the word problems. For each of the four word problems, a score was assigned from 1 to 3 to reflect the number of different types of manipulatives they had used. For example, a score of 2 would mean that the child had used two different types of manipulatives (e.g., buttons and gems) to solve the problem. A mean score was then calculated for each child. There were no group differences in the number of different manipulatives children used to solve the word problems, $F(2, 59) = .08$, $p = .92$, $\eta_p^2 = .003$, $(1 - \beta) = .06$. Children (in the whole sample) used an average of 1.63 manipulatives ($SD = .63$).

Students were categorized into one of three profiles based on how they answered all four items of the Unconstrained Far-Transfer task. I was interested in how they viewed the manipulatives during the task regardless of when their responses were provided (i.e., initial and post-prompt). If students only used the same or different manipulatives with a referent of 1 on all four items, they were classified in the "Referent of 1 only" profile. If they had assigned a referent of 2 to either the same or different manipulatives at least once in their responses to the four items, they were classified in the "Referent of 2" profile. Lastly, if they had assigned one or more referents other than 1 or 2 to the same or different manipulatives at least once in their responses to the four items, they were classified in the "Other referent" profile.

Figure 11 shows the percentage of students in each response profile for each one of the three conditions. First, not surprisingly, all of the students in the control condition were in the *Referent of 1 only* profile. There was a higher proportion of students in the *Referent of 1 only* profile in the DI condition (15/19 or 78.9%) compared to the GE condition (11/21 or 52.4%). Next, there were about five times as many students from the GE condition (5/21 or 23.8%) compared to those in the DI condition (1/19 or 5.3%) who were in the *Referent of 2* profile. These students, at one point or another during the task, viewed the manipulatives as having a referent of 2 at least once across the four items. Finally, there were one and a half times as many students in the GE condition (5/21 or 23.8%) who were in the *Other referent* profile compared to students in the DI condition (3/19 or 15.8%). All students in this profile assigned more than one referent to the same or different manipulatives at least once across the four items, and the referents that were different from 1 or 2 took on a variety of values, namely 1.5, 3, 4, 5, 6, 8, 9, 13, or 15. For example, to represent a quantity of 9, one child in this category used two gems, each one with a referent of 4, one button with a referent of 1, and three blocks, each one with a referent of 3.

Figure 11

Percentage of Students in Each Response Profile on the Unconstrained Far-Transfer Task for the DI (n = 19), GE (n = 21), and Control (n = 20) Conditions



Accuracy

An analysis with attempt as the unit of analysis (i.e., all word-problem-solving attempts across all participants) was conducted to determine how instructional condition and type of referent influenced word-problem-solving accuracy for both the Constrained and Unconstrained Far-Transfer tasks, separately. Accuracy was determined by taking the final answer and use of the manipulatives into account. Incorrect final answers because of counting errors were counted as correct. There were some cases where children changed the quantities involved in the world problem (e.g., using 2 toy cars and 3 boxes for Problem C; see Table 1) and attempted to solve their version of the problem; these attempts were removed from the analyses. Because children were invited to provide more than one answer to the word problems, it was not possible to assess accuracy at the student level of analysis, which is why the following analyses were made using attempt as the unit of analysis.

Constrained Far-Transfer Task

I examined children's word-problem-solving accuracy on each problem-solving attempt in the Constrained Far-Transfer task as a function of condition and referent. Taking into account all the problem-solving attempts made by children in all three conditions, there were 299 distinct attempts made on the four items[3]: 82 on problem C, 71 on problem D, 74 on problem E, and 72 on problem F (see Table 1). Students in the DI condition made a total of 100 attempts, students in the GE condition made a total of 116 attempts, and students in the control condition made a total of 83 attempts.

For each attempt, I determined the referent that was used by the student for the pollies: referent of 1 only, referent of 2 only, referents of 1 and 2, and other combinations of referents. I operationalized accuracy as the ability to obtain the correct answer and appropriately represent all the quantities in the problem with the pollies, regardless of referent. Responses from students from the control condition were excluded from the analysis because they used a referent of 1 only in nearly all of their attempts (80/83 or 96.4%).

Table 14 presents the means and standard deviations for performance on the Constrained Far-Transfer task. A 2 x 4 ANOVA was conducted to examine the effects of condition (DI, GE) and referent (referent of 1 only, referent of 2 only, referents of 1 and 2, other combination of referents) on word-problem-solving accuracy. There was a significant main effect of condition, $F(1, 208) = 18.33$, $p < .001$, $\eta_p^2 = .08$, $(1 - \beta) = .99$, a significant main effect of referent type, $F(3, 208) = 36.39$, $p < .001$, $\eta_p^2 = .34$, $(1 - \beta) = 1.00$, and a significant condition by referent interaction (see Figure 12), $F(3, 208) = 5.10$, $p = .002$, $\eta_p^2 = .07$, $(1 - \beta) = .92$. Simple effect analyses revealed that items solved by children in the GE condition were more accurate than items solved by children in the DI condition when referents of 2 only were used, $t(37) = 2.20$, $p = .03$, $d = .85$, and when other referents were used, $t(26) = 4.68$, $p < .001$, $d = 1.71$.

[3] After their initial responses, the interviewer prompted the children to "solve the problem in a different way." Sometimes students offered more than one additional way to solve the problem, and sometimes they were not able to provide an additional response, which is why the total number of attempts does not correspond to the number of participants x the number of items x the number of attempts (i.e., 61 x 4 x 2).

Table 14

Means and Standard Deviations of Accuracy Scores[a] by Referent and Condition for the Constrained Far-Transfer Task

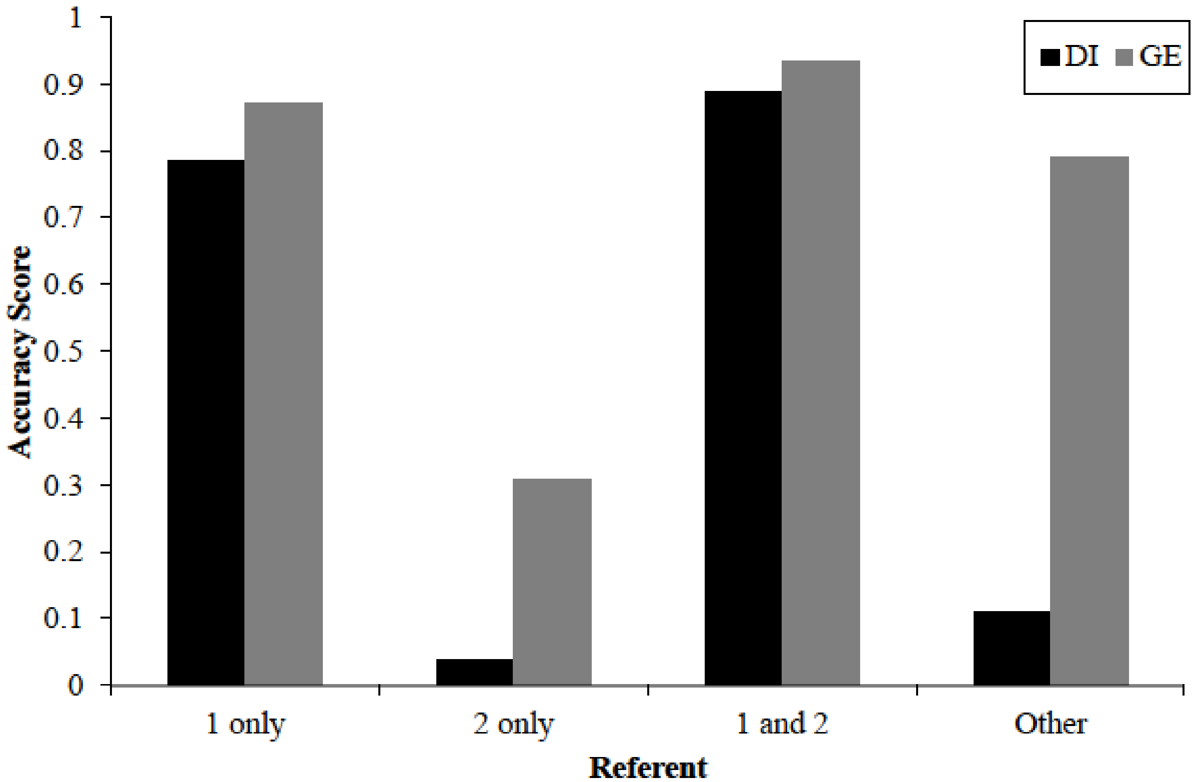
Referent(s)	Condition	*M*	*SD*	*n*
1 only	DI	.79	.41	56
	GE	.89	.34	69
2 only	DI	.04	.20	26
	GE	.31	.48	13
1 and 2	DI	.89	.33	9
	GE	.93	.26	15
Other	DI	.11	.33	9
	GE	.79	.42	19

[a]Max: 1, min: 0

Children showed varying levels of accuracy when using the polly with a referent of 2 only. Consider the problem of nine cupcakes in each of two boxes (Problem D). One child successfully used 9 pollies to represent 18 cupcakes by constructing 9 groups of 2. Another child placed 4 pollies (i.e., 8 cupcakes) in each box and one polly across both boxes. As an example of an unsuccessful attempt when using the pollies as 2, one child in the DI condition did not successfully use them to represent the quantity 9. After putting down four pollies, she said, "If I put one more, it will be 10 and that's too much. I don't know how to do it." Referent use other than 1 or 2 also resulted in varying levels of performance. For example, a child in the DI condition, for the problem involving five shirts in each of three drawers (Problem E), used a referent of 3 to represent the quantity 5, which did not permit the correct answer. In contrast, a child in the GE condition chose to use a referent of 5 for the pollies to represent the quantity 5 and thus was able to successfully solve the problem.

Figure 12

Condition by Referent Interaction for the Constrained Far-Transfer Task



Unconstrained Far-Transfer Task

I examined children's word-problem-solving accuracy on each problem-solving attempt in the Unconstrained Far-Transfer task as a function of condition and referent used. Taking into account all the problem-solving attempts made by children in all three conditions, there were 276 attempts made on the four items[4]: 70 on problem G, 72 on problem H, 67 on problem I, and 67 on problem J (see Table 1). Students in the DI condition made a total of 88 attempts, students in the GE condition made a total of 113 attempts, and students in the control condition made a total of 75 attempts.

[4] After their initial responses, the interviewer prompted the children to "solve the problem in a different way." Sometimes students offered more than one additional way to solve the problem, and sometimes they were not able to provide an additional response, which is why the total number of attempts does not correspond to the number of participants x the number of items x the number of attempts (i.e., 60 x 4 x 2).

For each attempt, I determined the referent used by the student for the manipulatives: referent of 1 only, referent of 2 (only or in combination with a referent of 1), and other referents (referents other than 1 or 2 only). I operationalized accuracy as the ability to obtain the correct answer and appropriately represent all the quantities in the problem with the manipulatives, regardless of referent. Responses from students from the control condition were excluded from this analysis because they used a referent of 1 only for all of their attempts.

Table 15 presents the means and standard deviations for performance on the Unconstrained Far-Transfer task. A 2 x 3 ANOVA was conducted to examine the effects of condition (DI, GE) and referent type (referent of 1 only, referent of 2, other referents) on problem-solving accuracy. There was a significant main effect of referent type only, $F(2, 195) = 7.13$, $p = .001$, $\eta_p^2 = .07$, $(1 - \beta) = .93$. Pairwise comparisons using Bonferroni corrections revealed that when children used a referent of 2 ($M = .65$, $SD = .49$, $n = 26$), they were significantly less accurate than when they used a referent of 1 only ($M = .91$, $SD = .29$, $n = 152$, $t(176) = 3.50$, $p = .002$, $d = 1.76$).

Table 15

Means and Standard Deviations of Accuracy Scores[a] by Referent and Condition for the Unconstrained Far-Transfer Task

Referent(s)	Condition	*M*	*SD*	*n*
1 only	DI	.88	.33	73
	GE	.94	.25	79
2	DI	.67	.50	9
	GE	.65	.49	17
Other	DI	.67	.52	6
	GE	.76	.44	17

[a]Max: 1, min: 0

Chapter 5: Discussion

The present study aimed to identify the costs and benefits of directly telling students the quantitative referents for manipulatives compared to allowing them to construct meaning for the manipulatives in a more exploratory learning environment. First graders were introduced to pollies, which were novel manipulatives, and they were asked to perform different tasks using the pollies in learning, near-transfer, and far-transfer contexts. Participants were randomly assigned to one of three instructional conditions: (a) direct instruction, (b) guided exploration, and (c) control, and the way in which they were introduced to the pollies varied based on condition. The influence of the different instructional conditions on children's learning, near-transfer abilities, representational versatility (i.e., symbolic flexibility and symbolic fluency), and accuracy was examined. The results uncovered learning contexts in which direct instruction was beneficial compared to guided exploration and those in which direct instruction was not as effective compared to guided exploration. These results will be explained in the following sections.

Benefits of Direct Instruction

There is still an ongoing debate in the educational research world as to which instructional approaches are most beneficial for learning (see Lee & Anderson, 2013). The contention that direct instruction leads to superior outcomes when compared to unassisted or minimally-guided discovery methods is well supported in the literature (see Alfieri et al., 2011; Baroody et al., 2015; Klahr & Nigam, 2004; Rosenshine, 2009). Some researchers who have studied instruction with manipulatives in the fields of mathematics and reading have also reported an advantage of explicit instructional methods on students' retention and other problem-solving outcomes (Carbonneau et al., 2013; Carbonneau & Marley, 2015). The present study's finding that children who learned through direct instruction were better than children who learned through guided exploration at reproducing what they had learned during instruction aligns with this line of research; children who received direct instruction learned more quickly how to represent even quantities of cubes with pollies representing the quantity 2. During the instructional session, children who learned through guided exploration were engaged in what some have described as enhanced-discovery tasks, which include techniques such as feedback, scaffolding, guided discovery, and elicited self-explanations (Alfieri et al., 2011). In one of the meta-analyses they report in their article, Alfieri et al. (2011) found that learners who were engaged in such enhanced-discovery tasks seemed to take more time to perform target responses, which is in line with what I found in the present study. Relatedly, I found that children who learned through direct instruction necessitated, on average, fewer items before they saw the pollies as having a referent of 2 compared to the children who learned through guided exploration. This finding is similar to results reported in the second meta-analysis described in Alfieri et al.'s article that compared the effects of explicit-instructional tasks to unassisted discovery learning conditions. The authors found that explicit instruction led to greater performance on measures assessing learning acquisition (which included measurements of learning and of successes and errors during the learning phase).

It appears, then, that when children needed to use the pollies in a specific way, direct instruction was more effective than guided exploration. Instructional conditions that involve a high level of guidance or in which the learning environment is highly structured have been found to be beneficial when the objective is to complete well-known tasks quickly (Martin, 2009). Taking these explanations into consideration, it should not be too surprising that children who learned through guided exploration needed longer (in terms of amount of time and number of items) to encode the target referent. They spent more time interacting with the objects in ways that deviated from the learning objective because they did not have, at least initially, any structure to guide their actions with the pollies (Adrien et al., 2019; Brown et al., 2009). When children learn through direct instruction, in contrast, they are less likely to perform irrelevant actions with the objects because the teacher can focus the learner's attention on the useful aspects and interpretations of what is to be learned (Bonawitz et al., 2011; Sweller et al., 2007). Discovery-based approaches are also said to be demanding in terms of memory and cognitive resources, and so direct instruction might be a less taxing process for learners, allowing them to focus on retention (Klahr, 2009; Rittle-Johnson, 2006; Sweller, 1988). Together, these reasons could explain why children in this study who learned through direct instruction were able to use the pollies in the prescribed way more efficiently than students who learned through guided exploration.

On the Near-Transfer task, children had to use the pollies in the way they had learned during instruction (i.e., with a referent of 2) to solve multiplication word problems. Results from inferential statistical analyses did not reveal any significant differences between children who received direct instruction and those who learned through guided exploration. In their initial and post-prompt responses, children in both conditions used a referent of 2 for the pollies significantly more than other referents. Thus, my prediction that children who learned through direct instruction would outperform those who learned through guided exploration on the Near-Transfer task was not confirmed statistically, although descriptive results were in the expected direction. A descriptive analysis of children's initial responses showed that those who received direct instruction answered in ways that were more aligned with how they had been instructed to use the pollies compared to the initial responses provided by those who learned through guided exploration. Moreover, children in both conditions used the pollies with a referent of 2 more often *after* they had received a prompt from the interviewer (e.g., "Can you show me a different way to solve the problem?") compared to the responses they had given before the prompt. No attempt by condition interaction was found, which again shows that children who learned through direct instruction and those who learned through guided exploration did not differ significantly in their near-transfer abilities.

Results from prior research that had found positive learning outcomes associated with direct instruction (e.g., Alfieri et al., 2011; Carbonneau & Marley, 2015) had led me to predict an advantage for children who learned through direct instruction on learning and near-transfer tasks compared to children who learned through guided exploration. There are other studies, however, that, just like the results from the present study, did not report differences between direct and more exploratory instruction. Hushman and Marley (2015), for example, did not find significant differences on a multiple-choice assessment of cued recall, application, and evaluation of knowledge between direct and guided instructional conditions in their investigation of 9- and 10-year-old children's learning of "control of variable strategies" (CVS; which is used in science). The authors found it challenging to explain these results because other studies on the learning of CVS had used other methods (e.g., interviews) or had used different outcome variables. Hushman and Marley also questioned the validity of some aspects of their multiple-choice assessment.

The instructional conditions used by Schwartz et al. (2011) in their study on adolescents' learning of physics concepts and formulas are similar to the ones used in the current study. They compared the effects of a "tell-and-practice" condition (akin to my direct instruction condition), in which students were told the relevant concepts and formulas before answering practice questions, to those of an "invent-with-contrasting-cases (ICC)" condition, in which children invented their own formulas before being told the concepts and "real" formulas. The ICC condition bears similarities with my guided exploration condition in that in both cases, participants were asked to explore or invent (representations, in the case of the present study, or formulas, in the case of Schwartz et al.'s study) independently before receiving any type of instruction. Schwartz et al. found that although students in the ICC condition developed deeper conceptual understanding that helped them with transfer tasks, both groups showed comparable proficiency at using the formulas they had learned when solving word problems. Both instructional conditions, then, helped children directly apply what they had learned, but only the ICC condition led to positive outcomes on transfer tasks. This finding reflects the results from my study, in that children who received direct instruction and those who learned through guided exploration performed similarly on the Near-Transfer task, which was a task that required them to use the pollies in the same manner as they had been taught (i.e., with a referent of 2).

In a more specific context of mathematics learning with manipulatives, Horan and Carr (2018b) examined the effects of timing and level of guidance on kindergarteners' learning of counting with manipulatives. Children in all four of their guidance instructional conditions used ten boards with which they had to "make" the numbers one through ten by using pennies and nickel strips. The level of guidance children received varied according to the teacher's questions and feedback. In the high guidance condition, the teacher asked questions and provided elaborate feedback with the objective to increase student learning and understanding. In the low guidance condition, the teacher provided procedural instruction (i.e., telling students they had to replace groups of five pennies with a nickel strip) but did not use questioning and offered limited feedback with "yes" or "no" statements. Using the conclusions drawn from Carbonneau et al.'s (2013) meta-analysis, Horan and Carr had hypothesized seeing greater transfer abilities for students who received low instructional guidance, but they failed to find condition differences between the high and low guidance conditions on their transfer task. The authors speculated that they were not able to replicate Carbonneau et al.'s findings perhaps because of the small number of items included in their transfer task or because of the task's limited validity. In addition to these explanations, I propose that the task used by Horan and Carr was a near-transfer task. Carbonneau et al. defined transfer as contexts in which students are asked to extend their knowledge to new situations, such as extending what students learn about addition to multiplication problems. Both the learning and transfer tasks Horan and Carr used in their study involved counting quantities from one to ten. The authors suggested that guidance might not be a moderator for that skill to explain why they had not found any group differences on the learning task. This might perhaps also be the reason why they did not find any group differences for what I would qualify a near-transfer task.

Another way to explain the absence of group differences between the direct and more exploratory instructional conditions for near transfer in both Horan and Carr's (2018b) and my study could be due to what Klahr and Nigam (2004) have called *path-independent transfer*. Path-independent transfer refers to the idea that once children have achieved mastery of a new procedure, the way in which they attained that mastery level will not influence their ability to transfer what they have learned. In the case of the present study, then, it is possible that regardless of how children who learned through direct instruction and those who learned through guided exploration came to view the pollies as representing the quantity 2, once they had assigned that referent to the manipulatives, they were able to transfer and use that knowledge to solve problems in the word-problem context. Thus, even though students in the direct instruction condition performed better during the learning phase and had a higher proportion of correct initial responses in the Near-Transfer task, students in the guided instruction condition "caught up to them" in the context of the Near-Transfer task, particularly when post-prompt responses were considered. More specifically, descriptive results revealed that students who learned through guided exploration seemed to benefit more (compared to students who learned through direct instruction) from the prompts offered during the Near-Transfer task, as seen in the greater proportional change from their initial responses to their post-prompt responses in their use of the pollies with a referent of 2.

Path-independent transfer might also explain Osana, Przednowek, et al.'s (2018) findings from their study of children's use of manipulatives as symbols of quantity. First graders were introduced to chips in different encoding conditions, including one condition in which they saw the chips as board game pieces and another in which they saw the chips as representing specific quantities. After an addition intervention that focused on addition problem-solving procedures (with regrouping) with the chips and written numbers, children in the game pieces condition did as well as children in the quantitative condition on a transfer task in which they had to use the chips quantitatively. Osana, Przednowek, et al. suggested that both conditions might have allowed children to develop the ability to see the chips as representations of something else and that it is why children in these two conditions performed similarly on the transfer task, which could be viewed as another example of path-independent transfer. Relatedly, Osana et al. (2017) found that when second graders received guidance on the link between base-ten blocks and written symbols, the type of guidance (which varied according to representational sequence: blocks-first, written-symbols-first, or iterative) was not related to children's performance on a near-transfer task involving regrouping.

In summary, it seems that direct instruction leads to better outcomes compared to guided exploration for learning assessments. When it comes to near transfer, however, instructional conditions were not found to have an influence on children's performance on the Near-Transfer task when considering both their initial and their post-prompt responses. Even though children who learned through guided exploration took longer and required more items to view the pollies as having a referent of 2, they did learn it in the end, which made them as capable as their counterparts who received direct instruction of solving the word problems in the Near-Transfer task. Successful learning, however, does not automatically translate into successful transfer (Kaminski et al., 2009), and the patterns of results for learning observed in this study were not the same in far-transfer contexts.

Costs of Direct Instruction

My findings indicate that, contrary to what I had found for learning and near transfer, children who received direct instruction did not outperform their counterparts who learned through guided exploration in far-transfer contexts. More specifically, guided exploration was more beneficial for representational versatility (i.e., symbolic flexibility and symbolic fluency) and word-problem-solving accuracy.

Representational Versatility

Children's representational versatility was assessed by examining their symbolic flexibility and their symbolic fluency. Symbolic flexibility was defined as the ability to assign a new quantitative referent to a known manipulative (i.e., a polly), whereas symbolic fluency was defined as the ability to assign a new quantitative referent (i.e., other than 1 or 2) to a novel manipulative. No significant differences were found between the children who learned through guided exploration and those who received direct instruction on the measure of symbolic flexibility. The fact that children who learned through guided exploration did significantly better than children in the control condition and that children who received direct instruction did no different than children in the control condition, however, would indicate that children in the guided exploration condition reached a flexibility threshold that children who learned through direct instruction did not. Thus, my predictions were partially supported: I had predicted that children in the guided exploration condition would outperform children in the other two conditions, but their performance was only significantly greater than that of children in the control group. There was also no difference between the direct instruction and control groups, as predicted. Nevertheless, these results support the idea that certain factors, such instruction, can have an effect on children's representational flexibility (Acevedo Nistal et al., 2009).

In terms of fluency, the results indicated that the students who received guided instruction were more likely to assign a referent other than 1 to the manipulatives, but that students who learned through direct instruction were not. As such, my predictions were partially supported: I had predicted that children in the guided exploration condition would engage with the pollies more flexibly and fluently than the other two conditions, but their performance differed only from the children in the control group and not from those who had received direct instruction. Further, a significant positive correlation between flexibility and fluency was found, which indicates that the higher the flexibility scores, the higher the fluency scores. It therefore appears that symbolic flexibility and symbolic fluency are related constructs, and the mechanisms that favor or hinder each might be similar.

Manipulative Use

A closer investigation of children's manipulative use allowed me to place students into different profiles. This was different from the flexibility and fluency scores they received, because these scores did not give a precise picture of the types of responses the children had provided. In the context of the Constrained Far-Transfer task, one noteworthy finding is that children who received direct instruction were the only ones who gave a referent of 2 only to all four items of the task. There was no such behavior observed in the guided exploration or control conditions. This could mean that for some children who learned through direct instruction, receiving explicit instructions on how to use the pollies prevented them from using the manipulatives in any other way no matter the context. Another interesting finding that emerged from the manipulative use analysis was that a greater proportion of students who learned through guided exploration demonstrated a high level of symbolic flexibility (i.e., by either using referents of 1 and 2 for the pollies or using referents other than 1 or 2) compared to students in the direct instruction and control conditions.

These findings illustrate some of the potential drawbacks of using direct instruction in the context of mathematics instruction that have been raised by other researchers. When children use a symbol system that is new to them, they are initially more rigid and inflexible (DeLoache, 1991), and explicit instruction might exacerbate these types of constrained behaviors. Explicit instruction can lead children to develop knowledge structures and problem-solving behaviors that are overly narrow (De Caro & Rittle-Johnson, 2012). Children who learn through direct instruction might not think of searching for new or alternative ideas because they believe they have been provided with all the information they need (e.g., Bonawitz et al., 2011). Direct instruction might also lead children to fixate on a single procedure and not wonder or care about whether it makes sense or why it works the way it does (Hatano, 2003; Schwartz et al., 2011). For example, Roll et al. (2018) examined the influence of instruction (directive vs. non-directive) in the context of a physics simulation on electric circuits with university students. They found that students in the directive condition used more prescribed and formal strategies and suggested that the detailed guidance they had received might have prevented their self-driven exploration. Similarly, in their study with second, third, and fourth graders, DeCaro and Rittle-Johnson (2012) found that when children were allowed to explore equivalence problems before instruction (compared to receiving instruction and then solving problems), they attempted a larger variety of strategies. If teachers' actions are too prescriptive during instruction, then children might learn to use concrete and abstract symbols in a more procedural manner disconnected from the symbols' meanings; this might have a negative effect on transfer abilities (Belenky & Schalk, 2014; Gravemeijer, 2002).

An analysis of the number and type (i.e., block, button, or gem) of manipulatives children used to solve problems in the Unconstrained Far-Transfer task revealed no significant group differences, but there were differences between the children in the direct instruction and guided exploration conditions in terms of the referents they had assigned to the manipulatives. Interestingly, there was a higher proportion of students in the direct instruction condition who used a referent of 1 only for the novel objects compared to students in the guided exploration condition. That pattern was different in the Constrained Far-Transfer task, where the proportions of children in the direct instruction and guided exploration conditions who used a referent of 1 only for the pollies were similar. These profile differences between the two tasks suggest that children who learned through guided exploration were better able to use the manipulatives as quantitative symbols (with a referent other than 1) when presented with novel manipulatives, whereas children in the direct instruction condition did not transfer that ability to the same extent to a non-polly context.

Accuracy

The analysis of answers to the problems on the Constrained Far-Transfer task generated the most pronounced results. The analysis of problem-solving accuracy by condition and referent type revealed a main effect of condition, indicating that children who learned through guided exploration were generally more accurate when solving the word problems compared to children who received direct instruction, regardless of the referent they used for the pollies. In addition, the referents used moderated this effect: Relative to children who learned through direct instruction, those who learned through guided exploration were more accurate when they used referents of 2 only and when they used referents other than 1 or 2.

That children who learned through guided exploration were able to accurately solve word problems with the pollies using a referent of 2 but that the children who received direct instruction were not as successful is worth exploring. Recall that the problems in the Constrained Far-Transfer task required children to use the pollies to solve multiple-groups multiplication word problems that involved quantities that were not multiples of 2, and so using the pollies with a referent of 2 should not theoretically yield accurate answers. How, then, were children who learned through guided exploration able to arrive at a greater number of correct solutions? They were able to think of creative and flexible ways to use the pollies to solve the problems; for example, for the problem of nine cupcakes in each of two boxes (Problem D), one child placed 4 pollies (i.e., 8 cupcakes) in each box and one polly across both boxes. Contrarily, many children who received direct instruction and who were trying to use a referent of 2 for the pollies appeared "stuck" and were not able to solve the problems with that referent. It is possible, then, that through guided exploration, children developed a type of knowledge, possibly meta-representational awareness, that allowed them to try to make sense of their actions and encouraged them to explore a variety of possibilities to solve the novel problems (diSessa & Sherin, 2000; Hatano, 2003). A close link between solution strategies and representations is believed to exist, where certain representations might encourage or elicit the use of specific strategies (Acevedo Nistal et al., 2009; Tabachnek et al., 1994). In the case of the present study, the specific strategies the children used to represent the word problems might have been related to the specific referents they chose to use for the pollies.

An important take-away from these findings is that children who learned through guided exploration learned the same "content" as children who learned through direct instruction (i.e., that a polly represented the quantity 2), but their superior symbolic flexibility and symbolic fluency abilities allowed them to use that knowledge in a context that differed from instruction, and it also had an influence on their performance. Their greater flexibility and fluency can also explain why children who learned through guided exploration were more accurate than their counterparts who received direct instruction when using referents other than 1 or 2 when solving the problems in the Constrained Far-Transfer task.

Explaining the Results

The results of the far-transfer tasks show an advantage of guided exploration over direct instruction. To explain these findings, I considered the possible mechanisms that were involved and activated during the guided exploration instruction. The guided exploration instruction differed from direct instruction in two ways. First, before receiving any type of instruction or feedback from the interviewer, children in the guided exploration condition were asked to represent quantities in any way they wished; they therefore had the opportunity to explore and use the manipulatives freely. Second, children received instruction that was not explicit. Rather, it was through a series of prompts, questions and careful constraints put in place by the interviewer that they came to view the polly as representing the quantity 2. I propose four different explanations for the findings regarding far transfer: (a) the nature of the guided exploration instruction, (b) children's representational insight, (c) physical actions with the manipulatives, and (d) productive failure. I will explain each in turn.

Nature of Guided Exploration Instruction

The nature of the guided instruction itself might have created opportunities for children to deepen their understanding of the symbolic relations with the manipulatives. During guided exploration, the interviewer asked questions meant to stimulate students' thinking and prompt their understanding of different ways to represent quantities with the manipulatives. Children tried different representations and modified their thinking in response to the interviewer's prompts and questions. These prompts and questions led children to explore the problem, and many benefits of exploration have been described in the literature. When children are exploring, it can help them make sense of their experiences, and they might notice and consider similarities between their experiences and their prior knowledge (Brown et al., 2009; DeCaro & Rittle-Johnson, 2012). In the case of my study, I consider their “prior knowledge” to be their use of the pollies with a referent of 1. During the instruction, in response to the interviewer’s prompts, children might have noticed the similarities between what they had done naturally (i.e., using a referent of 1) and the new referents they were “discovering,” which might have alerted them to important aspects of symbolization. They might also have searched for new information about the type of referent they could use at the point the situation did not allow them to use a referent of 1, which may have encouraged them to revise the schemas they had used up until that point (DeCaro & Rittle-Johnson, 2012). This process might have helped them use their symbolization abilities in a more flexible and fluent way. Problem exploration can facilitate knowledge of problem structure, and this could have helped learners acquire a deeper level of understanding and help them generate and revise possible solutions (DeCaro & Rittle-Johnson, 2012; Schwartz et al., 2009).

In contrast, because of its restrictive nature, direct instruction may inhibit or delay children’s ability to construct deep understanding of concepts and hinder their capacity to transfer knowledge to novel contexts (Brown et al., 2009). For example, Bonawitz et al. (2011) conducted experiments with preschoolers in which they introduced a novel-looking toy that had several non-obvious functions (e.g., making a sound by pulling a tube, turning on a light with a hidden button). Children were assigned to different conditions including the pedagogical condition, in which the experimenter demonstrated some functions for the child, and other conditions in which the experimenter did not. The child was then encouraged to play with the toy. The authors found that the children who were in the pedagogical condition focused on the functions they had been shown, whereas the children in the other conditions explored the toy’s functions more broadly and were thus more likely to discover new information.

Instructional approaches that do not provide a high level of guidance might help learners formulate and understand concepts on their own, and this process might lead to them to develop a deeper understanding of mathematical concepts (Fuson, 2009). This idea of students’ self-construction of knowledge has been deemed beneficial based on constructivist theories (see Cobb et al., 1992; Steffe & Gale, 1995; Von Glaserfeld, 2013). It is believed that when students generate their own explanations, as opposed to hearing a more-knowledgeable other providing the explanation, it could generate greater cognitive engagement, which in turn could lead to learning benefits (Hushman & Marley, 2015; Rittle-Johnson, 2006). In one of their meta-analyses, Alfieri et al. (2011) found that when learners participated in guided discovery and constructed their own explanations, they had better learning outcomes compared to learners who had been given explicit instruction or explanations; they found that this finding was true for all the age groups they studied.

Children's relationship to the pollies themselves might have been different based on the instructional condition. Chase and Abrahamson (2018) found that the fourth- and ninth-grade children learning about algebra concepts who were in a discovery condition had greater learning gains than children in a no-discovery condition. Children in the no-discovery condition learned to use a tool (the virtual program children used to learn algebra concepts) and did not spend time thinking of its underlying structures. Children in the discovery condition, however, used the tool to learn key algebra concepts. In the present study, children who received direct instruction were focused on using the tool (in this case the pollies) as it had been taught (i.e., with a referent of 2), whereas children who learned through guided exploration explored and perhaps thought more deeply about representing quantities in different ways, which might have allowed them to develop a greater flexibility in terms of how to represent quantities specific to the context at hand.

The prompts that the interviewer used during the guided exploration instruction might also have activated children's metacognition, which is the active process of explicitly reflecting on one's cognitive activity (Chi, 2000). These metacognitive abilities could have led children to monitor and evaluate their solutions while they were solving the word problems, and as a result create new knowledge (i.e., assign new referents to the manipulatives) and use it appropriately. There is some research to support the idea that the use of metacognitive prompts during instruction with concrete materials can facilitate procedural fluency, representational flexibility, conceptual understanding, and transfer (Belenky & Nokes, 2009).

Representational Insight

One of the reasons children who learned through guided exploration outperformed children who learned through direct instruction on the far-transfer tasks is because of the benefits they might have gained from seeing multiple representations during the encoding session.

Multiple external representations have been said to help learners gain a deeper understanding of what they are learning (Ainsworth, 1999). During the encoding session, the children in the guided exploration condition saw the pollies as representing the quantity 1 from the outset and as representing the quantity 2, which they "discovered" after the interviewer's prompts and guidance. Some of the children even assigned other referents to the pollies during the course of the encoding session. The students in the direct instruction condition did not have this opportunity.

Symbolic sensitivity is a predisposition to recognizing that one object may stand for something else (DeLoache & Marzolf, 1992). It is believed that symbolic development is cumulative, which means that symbolic sensitivity can increase when children acquire experience using symbols (e.g., DeLoache, 1991, 1995; DeLoache et al. 1999). Marzolf and DeLoache (1994) conducted experiments with young children (2½- and 3-year-olds) and found that experience understanding a symbol-referent relationship in a simple context helped children understand a symbol-referent relationship in a more difficult context, which same-age children without prior symbolization experience did not typically understand. With more symbolic sensitivity, then, a child is more likely to be capable of interpreting other symbols and symbol systems encountered in the future, which will support the development of representational insight, the recognition that something (a symbol) can stand for something other than itself (DeLoache & Marzolf, 1992). In the present study, the children who learned through guided exploration might have demonstrated superior representational insight compared to the children who received direct instruction from having been exposed to two or more different symbol-referent relationships with the pollies. This heightened representational insight might have also helped them assign a completely new referent (i.e., other than 1 or 2) to the pollies when they used them in the context of the Constrained Far-Transfer task. Increased symbolic sensitivity and representational insight might indeed help children expect or even look for other symbol-referent relations (DeLoache & Marzolf, 1992).

The theory described above could also be used to explain the greater symbolic fluency exhibited by children who received the guided exploration instruction on the Unconstrained Far-Transfer task. About half of the children in the guided exploration condition fit in profiles in which they had assigned referent(s) other than 1 only to the new manipulatives compared to about a fifth of the children in the direct instruction condition in the same profile. The greater symbolic sensitivity and representational insight children in the guided exploration condition may have developed during the instruction might have led them to think that a novel entity (in this case the new manipulatives) could be interpreted as representing something other than itself (as representing a quantity other than 1; Acevedo Nistal et al., 2009; DeLoache et al., 1999; Karmiloff-Smith, 1991; Osana, Przednowek, et al., 2018). Furthermore, children in the control condition did not significantly outperform children in the other two conditions on any outcome measure. The control condition was essentially a "pure" discovery condition, and similar findings have been widely reported in prior research (see Abrahamson & Kapur, 2018; Alfieri et al., 2011; Clark, 2009). Together, the findings suggest that children will not spontaneously symbolize in a flexible manner. In the context of this study, the prior experiences that children had with mathematics manipulatives as counters might have influenced their natural inclination to use them with a referent of 1 (National Research Council, 2009).

Physical Interactions

Another possible explanation for the benefits of guided exploration observed in this study is children's physical interaction with the manipulatives. One of the differences between children who learned through direct instruction and those who learned through guided exploration is that during direct instruction, the children did not interact with the manipulatives as much or in the same way as children in the guided exploration condition. In the guided exploration condition, children were required to interact with the objects in ways that would satisfy the constraints imposed by the researcher. Using observational data from the encoding sessions, Adrien et al. (2019) found that children who received guided instruction were more likely to interact with specific parts of the pollies (for example, counting two of the extremities of the pollies "one-two" to justify using a referent of 2), compared to the children in the other two conditions, who interacted with the pollies as a way to keep track of their counts with less attention to the pollies' physical features. Because of the nature of the instruction they received, children who learned through guided exploration explored the affordances of the pollies in a manner that allowed them, in one way or another, to arrive at a referent of 2. The gestures and physical interactions could have had the effect of enacting the symbolic relationships between the pollies and their referents, which could have been abstracted for flexible use in a different context (Nathan, 2008).

The learning that comes from sensorimotor experiences with manipulatives might be more amenable to transfer to new contexts (see also Pouw et al., 2014). In fact, embodied cognition theory proposes that the body and its abilities to perceive and act on the environment contribute to the development of various cognitive processes (Barsalou, 2008; Glenberg, 2008). Embodied cognition proposes that the sensorimotor information that manipulatives provide might help the learner acquire new concepts and deepen existing ones (Pouw et al., 2009). The novel or abstract ideas are grounded in the physical environment, and it is the mapping between the abstraction and the object that may facilitate meaning-making, lead to a deeper understanding, and support the transfer of knowledge to new situations (Alibali & Nathan, 2012; English, 2004; Uttal et al., 2013). Manipulatives, therefore, can be seen as an intermediary between real-world referents and more abstract mathematical ideas or symbols. It seems that when children are first developing their understanding of concepts, they might need to rely more on external or concrete representations to access these concepts, but that over time that need will diminish as they will have internalized external representations into mental images that can help them solve problems (Behr et al., 1983; Fyfe, McNeil, & Son, 2014; Glenberg, 2008; Laski et al., 2015; Petit et al., 2016). Several researchers have explained their findings from the embodied cognition perspective (e.g., Fugate et al., 2019; Nathan, 2008, Nathan et al., 2014; Rosen et al., 2018; Tran et al., 2017), and it is possible that the greater level of physical manipulations with the pollies experienced by children in the guided exploration condition conferred them an advantage on the far-transfer tasks over children who learned through direct instruction.

Martin's (2009; see also Martin & Schwartz, 2005) physically distributed learning (PDL) theory could also explain why guided exploration offered an advantage. Behind PDL is the idea that children's actions and ideas coevolve and influence each other over time. In a study with fraction manipulatives, children made greater conceptual gains when they interacted and moved the manipulatives themselves rather than having pieces already organized into correct groups for them (Martin, 2009). The theory of PDL predicts that an overstructured environment does too much of the "mental work" and reduces students' opportunity to extend what they have learned. In such structured environments, Martin contends, the benefits of working with concrete materials are lost. Relatedly, Osana, Blondin, et al. (2018), in their study examining the physical affordances of manipulatives, found that chips (i.e., non-proportional objects) led to greater learning than a proportional model. They hypothesized that because the chips were not transparent in terms of showing the quantitative relationships between the different values, children were required to "work more" to understand the conceptual rules. The additional efforts the participants deployed to actively interpret the meanings of the manipulatives might have led them to develop a deeper understanding, potentially explaining the greater accuracy observed on the learning task. In the present study, children had to actively discover and interpret the meaning of the pollies, which might have helped them gain a deeper understanding of the act of symbolizing, thereby supporting their far-transfer abilities.

When children take part in constructive and interactive activities, such as the ones in the guided exploration instruction, it can generate new ideas and predictions (Chi, 2009). Of course, the manipulation of concrete objects alone cannot suffice to help children understand symbolic representations of mathematical ideas (Uttal et al., 2009), but it might partly explain why instruction that makes use of these principles is beneficial.

Productive Failure

Another line of research that might explain the results pertains to the benefits of struggle, which Kapur (2014) has referred to as productive failure. By "failure," Kapur meant that children fail to arrive at the correct answer on their own. Kapur conducted a study on mathematics ability with ninth graders and found that students who experienced productive failure outperformed students who learned through direct instruction on measures of conceptual understanding and transfer. Other researchers also underscore the benefits of struggle in learning (e.g., Hiebert & Grouws, 2007). When children struggle, they make efforts to understand and make sense of new concepts, which can help them make connections between facts, ideas, or procedures they have previously learned; this can lead them to re-examine and restructure their knowledge (Hiebert & Grouws, 2007). Children who received the guided exploration instruction at first failed to generate or discover the intended referent on their own, and many continued to struggle even after the interviewer introduced the constraints on the polly use. That failure may have been constructive because it focused their attention on the mechanisms underlying the symbol-referent link, thus promoting transfer. Presumably, different mechanisms were at play during direct instruction. Children were explicitly told what to do and did not have the opportunity to reflect on their own actions or the fact that they were actually symbolizing.

Relatedly, some researchers have studied the impact of sequencing in instruction. When children attempt to solve problems before instruction, findings have suggested that it prepares them for subsequent learning (see Lee & Anderson, 2013). Solving problems before receiving any kind of instruction is effortful, and it is possible that this process helped children attend to the learning activities at a deeper level (DeCaro & Rittle-Johnson, 2012). Schwartz and Martin (2004) conducted a study on ninth graders' learning of statistical concepts. They found that students who took part in invention activities failed to generate conventional solutions when compared to their peers who had only received explicit instruction on the concepts. After the students in the invention group had received instruction, however, they did better than the explicit instruction group in terms of their procedural skills, insight into formulas, and data evaluation abilities. Schwartz and Martin concluded that the invention activities might have prepared students to learn more deeply from instruction. The present study did not deal with sequencing of instruction in the same way, but an argument could be made that in the guided exploration condition, children had to attempt problem solving with little instruction (or only a few prompts from the interviewer) and that they experienced some of the same struggles or difficulties described in research that examines sequencing.

The struggles experienced during guided exploration, then, might have led children to attend to their own actions, which may have directed their thinking toward symbolizing. A related idea is the concept of representational disfluency (Bieda & Nathan, 2009), which can be experienced when students perceive limitations with the representations they are using and are thus motivated to modify them. In Bieda and Nathan's (2009) study on students' understanding of Cartesian graphs, they observed some students transfer to a different representation when their initial representation did not allow a solution. In the present study, the children who were not able to represent the word problems with the pollies in the context of the Constrained Far-Transfer task (because the quantities did not lend themselves to using a pollies with a referent of 2) may have experienced representational disfluency, thus explaining why they changed the referent for the pollies during problem solving.

Contributions to the Literature

The results of the present study contribute in several important ways to the existent literature. I identified three main areas in which I believe my work adds, extends, or nuances the findings and conclusions from prior research. First, because my study focused on the influence of different instructional conditions on children's learning and transfer abilities, it provides empirical evidence that contributes to research relating to the ongoing debates about optimal types of instruction in Science, Technology, Engineering, and Mathematics (STEM) fields in general, and in mathematics in particular. Second, my findings can shed more light on manipulatives-based instructional methods for young children. Lastly, little is known about representational versatility in young children who are learning with concrete objects, and my findings will offer a novel perspective to this area of research.

There is a plethora of research on the most effective pedagogical practices to adopt for STEM teaching and learning, and the recommendations lie on a continuum with two opposing views at each end, which are considered by some to be untenable and extreme (Abrahamson & Kapur, 2018). On one end of the spectrum, there are practices that encourage students to discover all knowledge on their own, and on the other end, there are practices during which the teacher presents explicit information to students. Between these two extremes exist a variety of approaches that are the subject of much research.

Several scholars have proposed a number of different taxonomies for instructional guidance (e.g., Lazonder & Harmsen, 2016). The instruction provided in the present study can fit into the classification scheme proposed by Baroody et al. (2015) in that my direct instruction could be considered "highly guided discovery," because it was well structured, and it provided explicit instructions and practice items. My guided exploration condition could be considered "moderately guided discovery," because it included tasks that were carefully and purposefully presented, and it contained some prompts and non-directive feedback. Lastly, my control condition would be considered "unguided discovery," as the children in that condition did not receive any feedback or support from the interviewer.

Several meta-analyses have been conducted on the influence of pedagogical approaches, but the results are often mixed (e.g., Freeman et al., 2014). For example, Alfieri et al. (2011) found that enhanced discovery methods (e.g., provision of teacher feedback, encouragement of students' self-explanations) generally facilitated deeper learning relative to unassisted discovery or direct teaching. Alfieri et al. also found, however, that explicit instruction led to more favorable learning outcomes when compared to unassisted discovery. My research findings also present trade-off relations between type of instruction and outcome. I found advantages for direct instruction over guided exploration in some contexts (i.e., learning), and I also found that there were some contexts in which direct instruction was not beneficial compared to guided exploration. In line with much of what has been reported in previous research (see Abrahamson & Kapur, 2018; Alfieri et al., 2011; Clark, 2009; Kirschner et al., 2006), I found that children who received no guidance at all did not significantly outperform children in the direct instruction or guided exploration conditions on any of the outcome measures. Another contribution of the current study is that it compared three levels of instruction, whereas some of the prior research examined the effects of one approach compared to a control (e.g., Chase & Abrahamson, 2008; Purpura et al., 2016; Roll et al., 2008). In sum, therefore, the results of the current study presented here do not lend unequivocal support to either direct instruction or guided exploration. Rather, the results offer a nuanced view of the costs and benefits of each approach. It is important to remember, however, that the conclusions stem from a context of instruction with manipulatives, and so the results should be interpreted with this in mind.

As described above, instructional guidance is commonly examined in educational research, but there are considerably fewer investigations on instructional guidance that pertains specifically to manipulatives (Furner & Worrell, 2017). One of the main contributions of this study, then, is that it served to shed more light on the instructional methods that should be used when manipulatives are involved. Again, current research efforts have moved past the examination of whether or not manipulatives are beneficial during instruction (Donovan & Fyfe, 2019; Marley & Carbonneau, 2014a; Wise & O'Neil, 2009); research objectives are now more focused on the conditions under which instruction with manipulatives is most beneficial.

For children to be successful when learning with manipulatives, they must in some way appropriate the connections between the concrete representation and its referent (see Donovan & Fyfe, 2019; Dufour-Janvier et al., 1987; Hiebert, 1984). A productive avenue for future research, therefore, is to focus on the instructional conditions that facilitate children's knowledge of these connections (Gravemeijer, 2002). Carbonneau and Marley (2015) found that when using manipulatives in mathematics instruction with preschoolers, higher levels of guidance were more effective and led to better procedural and conceptual knowledge, and transfer abilities. My results also suggest an advantage for higher levels of guidance, but for learning only (which I would compare to Carbonneau and Marley's procedural knowledge because of the nature of my task). I found that low levels of guidance were associated with better transfer abilities, a finding that deviates from Carbonneau and Marley's results. This discrepancy could be due to the age of the children in both studies; there is some evidence to suggest that younger children benefit more from higher levels of guidance than older children (see Alfieri et al., 2001). Furthermore, unlike the procedures used in my study, there was no lapse of time between the learning activity and the far-transfer task in Carbonneau and Marley's study, which could have, according to the authors, affected children's abilities to consolidate information. In sum, my study contributes to the field by adding another piece of evidence to an area where a consensus has yet to be attained.

Literature in manipulatives-based research has described trade-offs between certain characteristics of manipulatives, such as perceptual richness (e.g., Osana, Blondin, et al., 2018; Petersen & McNeil, 2013) and level of concreteness (e.g., Fyfe, McNeil, & Son, 2014), and learning and transfer. Another contribution of the present study is the finding that there are also trade-offs between instructional approach and mathematical outcomes. In terms of instruction, prior research has typically focused on finding support for one approach or another or on comparing different approaches (e.g., Osana et al., 2017; Osana & Pitsolantis, 2013), but my study examined the conditions under which direct instruction could break down.

Lastly, this study provides insightful information about how instruction can support symbolic flexibility and symbolic fluency in the context of mathematics learning with manipulatives. Little research has focused on contextual factors, including instructional approach, in young children's representational abilities with manipulatives (Acevedo Nistal et al., 2009). The present study, then, can shed light on how representational versatility abilities can have an influence on different outcomes (e.g., problem-solving accuracy) in contexts that have not yet been widely explored in the literature (see Thomas, 2008). Examining children's meta-representational competence in the context of instruction with manipulatives is important and should be considered in future research in that field (diSessa & Sherin, 2000). The results of my study represent a meaningful contribution to the current literature because it addresses the links between instruction, representational abilities, and performance in young children's mathematics.

Conclusion

The present study is not without its weaknesses. One of the tasks (i.e., the Sticker task) that was meant to assess children's symbolic flexibility yielded inconclusive results. Children were asked to use the pollies with a referent of their choice to request a number of stickers, a context that was intended to capitalize on children's inherent interests. The fact that none of the participants assigned a referent that was profitable, and the fact that most participants were "reasonable" in their requests, may indicate that they were too young to consider profiting from the situation. Thus, the task might not have been sensitive to children's symbolic flexibility.

Another methodological weakness of the current study is the small number of items on the outcome measures. Also, in my study, the learning assessment was embedded in the encoding instruction, which was not ideal because it made it difficult to measure learning as a stand-alone construct not influenced by the researcher's instructions. A more suitable learning task would have been conducted outside the instructional context. Lastly, even though patterns of results were in the expected direction, the sample size was not large, and this may explain the lack of power needed to detect significant differences between the direct and guided instruction groups.

Some questions remain unanswered and new questions have arisen from the findings of this study, which means that there are several avenues for future research. The present study focused on examining the costs and benefits of direct instruction for learning and transfer, but other factors could be studied in conjunction with instruction to investigate moderating effects. Age could be one of the factors studied; past developmental research has suggested that younger, less experienced learners might need more explicit guidance than older learners (Carbonneau et al., 2013; Lazonder & Harmsen, 2016). Prior knowledge has also been shown to play a role (see Fyfe & Rittle-Johnson, 2012; Osana & Pitsolantis, 2019; Roll et al., 2018). Although the present study detected no influence of prior mathematical knowledge or other cognitive factors such as working memory, executive function, and inhibitory control skills, other cognitive abilities or different assessments of the same abilities could be examined in a future study.

The development of symbolization, including symbolic sensitivity, has been said to be cumulative (DeLoache 1991; 1995); examining the effects of instruction as a function of symbolization abilities might be particularly insightful. Relatedly, examining in more detail the factors that facilitate children's representational versatility and meta-representational competence would be of value (diSessa & Sherin, 2000; Thomas, 2008). The moderating effects of context-specific and manipulative-specific attributes, such as perceptual richness (low to high), physicality (two- or three-dimensional), learner's level familiarity (low to high), and narrative context in which the representation is couched (low to high), could also be examined (Fyfe & Nathan, 2019).

There are also instruction-related extensions to the current study that could be explored. The effects of frequency and duration of guidance could also have an impact on learning and transfer outcomes (Clark, 2009; Lazonder & Harmsen, 2016), as could delayed transfer tasks to assess the long-term effects of different instructional approaches. In addition, the idea of gradually fading concrete representations and replacing them with abstract ones (i.e., concreteness fading, Donovan & Fyfe, 2019; Fyfe, DeCaro, & Rittle-Johnson, 2014; Fyfe & Nathan, 2019) has generated recent attention. The role of instructional guidance in the transition from concrete to abstract representations is only beginning to gain traction in the research community (see Osana et al., 2017). To gain a better understanding of which aspects of guided exploration are most beneficial would require the explicit comparison of carefully-designed conditions that would differ on the type and amount of feedback children receive (e.g., Fyfe & Rittle-Johnson, 2012, 2016; Lazonder & Harmsen, 2016). Lastly, to maximize applicability to the classroom, replications of current research would take place in classroom settings and include other transfer-related outcomes (e.g., transfer to written representations; Uttal et al., 2003), thereby lending it greater ecological validity (Loehr et al., 2014).

There are several educational implications that stem from this research. First, the findings do not lend support to the use of discovery learning for either learning or transfer (see also Alfieri et al., 2011). What does emerge from the data, however, is that there are benefits to both direct instruction and guided exploration. This does not mean that both approaches need to be combined or used at the same time in the classroom. The choice of selecting one or the other will depend on the teacher's goals and objectives for the students. More specifically, if teachers are aiming for efficient rote learning, then direct instruction might be beneficial. Teaching through guided exploration may take children longer to learn the concepts, but my findings suggest that they can compensate for the additional time by transferring their learning to novel contexts. The idea that educators should consider the alignment between the pedagogical approaches they choose to use and the learning outcomes they desire is important and has been suggested in other research as well (e.g., Belenky & Schalk, 2014; Hiebert & Grouws, 2007; Hushman & Marley, 2015; Osana et al., 2017; Osana, Przednowek, et al., 2018). Indeed, when the children in my study had a high level of representational versatility, they were more accurate when solving word problems. The continued development of these representational abilities should be encouraged in the classroom by providing and linking multiple representations, for example (e.g., Thomas, 2008), as they have been shown in the current study to be related to positive learning outcomes.

References

Abrahamson, D., & Kapur, M. (2018). Reinventing discovery learning: a field-wide research program. *Instructional Science*, *46*(1), 1-10. https://doi.org/10.1007/s11251-017-9444-y

Acevedo Nistal, A., Van Dooren, W., Clarebout, J. E., & Verschaffel, L. (2009). Conceptualising, investigating and stimulating representational flexibility in mathematical problem solving and learning: a critical review. *ZDM Mathematics Education, 41*, 627-636. https://doi.org/10.1007/s11858-009-0189-1

Adrien, E., Osana, H. P., Uttal, D., & Orsini, A. (2019, March). *Instructional environment influences how first-graders interact with math manipulatives* [Poster presentation]. Society for Research in Child Development biennial meeting, Baltimore, MD.

Ainsworth, S. (1999). The functions of multiple representations. *Computers & Education*, *33*(2-3), 131-152. https://doi.org/10.1016/S0360-1315(99)00029-9

Alfieri, L., Brooks, P. J., Aldrich, N. J., & Tenenbaum, H. R. (2011). Does discovery-based instruction enhance learning? *Journal of Educational Psychology*, *103*(1), 1-18. https://doi.org/10.1037/a0021017

Alibali, M. W., & Nathan, M. J. (2012) Embodiment in mathematics teaching and learning: Evidence from learners' and teachers' gestures. *Journal of the Learning Sciences*, *21*(2), 247-286. https://doi.org/10.1080/10508406.2011.611446

Astle, A., Kamawar, D., Vendetti, C., & Podjarny, G. (2013). When this means that: The role of working memory and inhibitory control in children's understanding of representations. *Journal of Experimental Child Psychology*, *116*(2), 169-185. https://doi.org/10.1016/j.jecp.2013.05.003

Baddeley, A. (1990). *Human memory*. Erlbaum.

Ball, D. L. (1992). Magical hopes: Manipulatives and the reform of math education. *American Educator: The Professional Journal of the American Federation of Teachers*, *16*(2).

Ball, D. L. (1993). Halves, pieces, and twoths: Constructing and using representational contexts in teaching fractions. In T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 157-196). Erlbaum.

Baroody, A. J., Purpura, D. J., Eiland, M. D., & Reid, E. E. (2015). The impact of highly and minimally guided discovery instruction on promoting the learning of reasoning strategies for basic add-1 and doubles combinations. *Early Childhood Research Quarterly*, *30*, 93-105. http://dx.doi.org/10.1016/j.ecresq.2014.09.003

Barsalou, L. W. (2008). Grounded cognition. *Annual Review Psychology*, *59*, 617-645. https://doi.org/10.1146/annurev.psych.59.103006.093639

Behr, M. J., Lesh, R., Post, T. R., & Silver, E. A. (1983). Rational number concepts. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 91-126). Academic Press.

Belenky, D. M., & Nokes, T. J. (2009). Examining the role of manipulatives and metacognition on engagement, learning, and transfer. *The Journal of Problem Solving*, *2*(2), 6. https://doi.org/10.7771/1932-6246.1061

Belenky, D. M., & Schalk, L. (2014). The effects of idealized and grounded materials on learning, transfer, and interest: An organizing framework for categorizing external knowledge representations. *Educational Psychology Review*, *26*(1), 27-50. https://doi.org/10.1007/s10648-014-9251-9

Berch, D. B., Krikorian, R., & Huha, E. M. (1998). The Corsi block-tapping task: Methodological and theoretical considerations. *Brain and Cognition*, *38*(3), 317-338. https://doi.org/10.1006/brcg.1998.1039

Bialystok, E. (1992a). The emergence of symbolic thought: Introduction. *Cognitive Development, 7,* 269-272. https://doi.org/10.1016/0885-2014(92)90015-J

Bialystok, E. (1992b). Symbolic representation of letters and numbers. *Cognitive Development, 7,* 301-316. https://doi.org/10.1016/0885-2014(92)90018-M

Bialystok, E. (2000). Symbolic representation across domains in preschool children. *Journal of Experimental Child Psychology*, *76*(3), 173-189. https://doi.org/10.1006/jecp.1999.2548

Bialystok, E., & Codd, J. (1996). Developing representations of quantity. *Canadian Journal of Behavioural Science/Revue canadienne des sciences du comportement*, *28*(4), 281-291. https://doi.org/10.1037/0008-400X.28.4.281

Bieda, K. N., & Nathan, M. J. (2009). Representational disfluency in algebra: Evidence from student gestures and speech. *ZDM Mathematics Education*, *41*(5), 637-650. https://doi.org/10.1007/s10648-014-9259-1

Bonawitz, E., Shafto, P., Gweon, H., Goodman, N. D., Spelke, E., & Schulz, L. (2011). The double-edged sword of pedagogy: Instruction limits spontaneous exploration and discovery. *Cognition*, *120*(3), 322-330. https://doi.org/10.1016/j.cognition.2010.10.001

Boone, H. N., Jr., & Boone, D. A. (2012). Analyzing Likert data. *Journal of Extension*, *50*(2), 1-5.

Brown, M. C., McNeil, N. M., Glenberg, A. M. (2009). Using concreteness in education: Real problems, potential solutions. *Child Development Perspectives*, *3*(3), 160-164. https://doi.org/10.1111/j.1750-8606.2009.00098.x

Burns, B. A., & Hamm, E. M. (2011). A comparison of concrete a virtual manipulative use in third- and fourth-grade mathematics. *School Science and Mathematics, 111*(6), 256-261. https://doi.org/10.1111/j.1949-8594.2011.00086.x

Callaghan, T., & Corbit, J. (2015). The development of symbolic representation. In R. M. Lerner, L. S. Liben, & U. Mueller (Eds.). *Handbook of child psychology and developmental science, Volume 2, Cognitive Processes, 7th edition* (pp. 1-46). Wiley.

Carbonneau, K. J., & Marley, S. C. (2015). Instructional guidance and realism of manipulatives influence preschool children's mathematics learning. *The Journal of Experimental Education*, *83*(4), 495-513. https://doi.org/10.1080/00220973.2014.989306

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology, 105*(2), 380-400. https://doi.org/10.1037/a0031084

Carleton University Math Lab. (2017). Go/No Go (Version 1.1.01) [iPad software]. https://carleton.ca/cacr/math-lab/apps/gono-go-app/

Carlson, S. M., & Beck, D. M. (2009). Symbols as tools in the development of executive function. In A. Winsler, C. Fernyhough, & I. Montero (Eds.), *Private speech, executive functioning, and the development of verbal self-regulation* (pp. 163-175). Cambridge University Press.

Carpenter, T. P., Ansell, E., Franke, M. L., Fennema, E., & Weisbeck, L. (1993). Models of problem solving: A study of kindergarten children's problem-solving processes. *Journal for Research in Mathematics Education*, 428-441. https://doi.org/10.2307/749152

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1985). Mathematics in the street and in schools. *British Journal of Developmental Psychology, 3*, 21-29. https://doi.org/10.1111/j.2044-835X.1985.tb00951.x

Ceuppens, S., Deprez, J., Dehaene, W., & De Cock, M. (2018). Design and validation of a test for representational fluency of 9th grade students in physics and mathematics: The case of linear functions. *Physical Review Physics Education Research*, *14*(2), 1-19. https://doi.org/10.1103/PhysRevPhysEducRes.14.020105

Chao, S. J., Stigler, J. W., & Woodward, J. A. (2000). The effects of physical materials on kindergartners' learning of number concepts. *Cognition and Instruction*, *18*(3), 285-316. https://doi.org/10.1207/S1532690XCI1803_1

Chase, K., & Abrahamson, D. (2018). Searching for buried treasure: uncovering discovery in discovery-based learning. *Instructional Science*, *46*(1), 11-33. https://doi.org/10.1007/s11251-017-9433-1

Chi, M. (2000). Self-explaining expository texts: The dual processes of generating inferences and repairing mental models. In R. Glaser (Ed.), *Advances in instructional psychology*, *Volume 5* (pp. 161-238). Erlbaum.

Clark, R. E. (2009). How much and what type of guidance is optimal for learning from instruction. In Tobias, S., & Duffy, T. M. (Eds.), *Constructivist instruction: Success or failure* (pp. 158-183). Routledge.

Clements, D. H., & Joswick, C. (2018). Broadening the horizons of research on discovery-based learning. *Instructional Science*, *46*(1), 155-167. https://doi.org/10.1007/s11251-018-9449-1

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education*, 2-33. https://doi.org/10.2307/749161

Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

DeCaro, M. S., & Rittle-Johnson, B. (2012). Exploring mathematics problems prepares children to learn from instruction. *Journal of Experimental Child Psychology*, *113*(4), 552-568. http://dx.doi.org/10.1016/j.jecp.2012.06.009

Deliyianni, E., Gagatsis, A., Elia, I., & Panaoura, A. (2016). Representational flexibility and problem-solving ability in fraction and decimal number addition: A structural model. *International Journal of Science and Mathematics Education*, *14*(2), 397-417. https://doi.org/10.1007/s10763-015-9625-6

DeLoache, J. S. (1987). Rapid change in the symbolic functioning of very young children. *Science*, *238*(4833), 1556-1557. https://doi.org/10.1126/science.2446392

DeLoache, J. S. (1991). Symbolic functioning in very young children: Understanding of pictures and models. *Child Development, 62,* 736-752. https://doi.org/10.1111/j.1467-8624.1991.tb01566.x

DeLoache, J. S. (1995). Early understanding and use of symbols: The Model model. *Current Directions in Psychological Science, 4*(4), 109-113. https://doi.org/10.1111/1467-8721.ep10772408

DeLoache, J. S. (2000). Dual representation and young children's use of scale models. *Child Development*, *71*(2), 329-338. https://doi.org/10.1111/1467-8624.00148

DeLoache, J. S., de Mendoza, O. A. P., & Anderson, K. N. (1999). Multiple factors in early symbol use: Instructions, similarity, and age in understanding a symbol-referent relation. *Cognitive Development*, *14*(2), 299-312. https://doi.org/10.1016/S0885-2014(99)00006-4

DeLoache, J. S., & Marzolf, D. P. (1992). When a picture is not worth a thousand words: Young children's understanding of pictures and models. *Cognitive Development*, *7*(3), 317-329. https://doi.org/10.1016/0885-2014(92)90019-N

DeLoache, J. S., Uttal, D. H., & Pierroutsakos, S. L. (1998). The development of early symbolization: Educational implications. *Learning and Instruction, 8*(4), 325-339. https://doi.org/10.1016/S0959-4752(97)00025-X

diSessa A. A., & Sherin, B. L. (2000). Meta-representation: An introduction. *The Journal of Mathematical Behavior*, *19*(4), 385-398. https://doi.org/10.1016/S0732-3123(01)00051-7

Donovan, A. M., & Fyfe, E. (2019, June 21). Connecting concrete objects and abstract symbols promotes children's mathematics learning. https://doi.org/10.31234/osf.io/ye2j6

Dufour-Janvier, B., Bednarz, N., & Belanger, M. (1987). Pedagogical considerations concerning the problem of representation. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 109-122). Erlbaum.

Durik, A. M., & Harackiewicz, J. M. (2007). Different strokes for different folks: How individual interest moderates the effects of situational factors on task interest. *Journal of Educational Psychology, 99*(3), 597-610. https://doi.org/10.1037/0022-0663.99.3.597

Empson, S. B. (1999). Equal sharing and shared meaning: The development of fraction concepts in a first-grade classroom. *Cognition and Instruction, 17*(3), 283-342. https://doi.org/10.1207/S1532690XCI1703_3

Empson, S. B., Levi, L., & Carpenter, T. P. (2011). The algebraic nature of fractions: Developing relational thinking in elementary school. In J. Cai, & E. Knuth (Eds.), *Early algebraization* (pp. 409-428). Springer.

English, L. D. (2004). Mathematical and analogical reasoning. In L. English (Ed.), *Mathematical and analogical reasoning of young learners* (pp. 1–22). Erlbaum.

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences*, *111*(23), 8410-8415. https://doi.org/10.1073/pnas.1319030111

Fugate, J. M., Macrine, S. L., & Cipriano, C. (2019). The role of embodied cognition for transforming learning. *International Journal of School & Educational Psychology*, *7*(4), 274-288. https://doi.org/10.1080/21683603.2018.1443856

Furner, J. M., & Worrell, N. L. (2017). The importance of using manipulatives in teaching math today. *Transformations*, *3*(1), 2.

Fuson, K. C. (2009). Avoiding misinterpretations of Piaget and Vygotsky: Mathematical teaching without learning, learning without teaching, or helpful learning-path teaching? *Cognitive Development*, *24*(4), 343-361. https://doi.org/10.1016/j.cogdev.2009.09.009

Fuson, K. C., Fraivillig, J. L., & Burghardt, B. H. (1992). Relationships children construct among English number words, multiunit base-ten blocks, and written multidigit addition. In J. I. Campbell (Ed.), *Advances in psychology* (Vol. 91, pp. 39-112). North-Holland.

Fyfe, E. R., DeCaro, M. S., & Rittle-Johnson, B. (2014). An alternative time for telling: When conceptual instruction prior to problem solving improves mathematical knowledge. *British Journal of Educational Psychology*, *84*(3), 502-519. https://doi.org/10.1111/bjep.12035

Fyfe, E. R., McNeil, N. M., & Son, J. Y. (2014). Concreteness fading in mathematics and science instruction: A systematic review. *Educational Psychology Review*, *26*(1), 9-25. https://doi.org/10.1007/s10648-014-9249-3

Fyfe, E. R., & Nathan, M. J. (2019). Making "concreteness fading" more concrete as a theory of instruction for promoting transfer. *Educational Review*, *71*(4), 403-422. https://doi.org/10.1080/00131911.2018.1424116

Fyfe, E. R., & Rittle-Johnson, B. (2012). The effects of feedback during exploratory mathematics problem solving: Prior knowledge matters. *Journal of Educational Psychology*, *104*(4), 1094. https://doi.org/10.1037/a0028389

Fyfe, E. R., & Rittle-Johnson, B. (2016). Feedback both helps and hinders learning: The causal role of prior knowledge. *Journal of Educational Psychology*, *108*(1), 82-97. https://doi.org/10.1037/edu0000053

Gelman, S. A., & Ebeling, K. S. (1998). Shape and representational status in children's early naming. *Cognition*, *66*(2), B35-B47. https://doi.org/10.1016/S0010-0277(98)00022-5

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities*, *38*(4), 293-304. https://doi.org/10.1177/00222194050380040301

Gibson, J. J. (1977). The theory of affordances. In R. Shaw & J. Bransford (Eds.), *Perceiving, acting, and knowing: Toward an ecological psychology (pp.* 67-82). Erlbaum.

Glenberg, A. M. (2008). Embodiment for education. In P. Calvo, & T. Gomila (Eds.), *Handbook of cognitive science* (pp. 355-372). Elsevier.

Gravemeijer, K. P. (2002). Preamble: From models to modeling. In K. P. Gravemeijer, R. Lehrer, B. van Oers, & L. Verschaffel (Eds.), *Symbolizing, modeling and tool use in mathematics education* (pp. 7-22). Kluwer Academic.

Greeno, J. G. (1994). Gibson's affordances. *Psychological Review, 101*(2), 336-342. https://doi.org/10.1037/0033-295X.101.2.336

Gromko, J. E. (1998). Young children's symbol use: Common principles and cognitive processes. *Update: Applications of Research in Music Education*, *16*(2), 3-7.

Hatano, G. (2003). Foreword. In A. J. Baroody, & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. xi-xiii). Erlbaum.

Hiebert, J. (1984). Children's mathematics learning: The struggle to link form and understanding. *Elementary School Journal, 84,* 497-513. https://doi.org/10.1086/461380

Hiebert, J. (1988). A theory of developing competence with written mathematical symbols. *Educational Studies in Mathematics, 19,* 333-355. https://doi.org/10.1007/BF00312451

Hiebert, J. (1992). Mathematical, cognitive, and instructional analyses of decimal fractions. In G. Leinhardt, R. Putnam, & R. A. Hattrup (Eds.) *Analysis of arithmetic for mathematics Teaching* (pp. 283–322). Erlbaum.

Hiebert, J., Carpenter, T. R., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., & Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. Heinemann.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). Information Age.

Hmelo-Silver, C. E. (2004). Problem-based learning: What and how do students learn? *Educational Psychology Review*, *16*(3), 235-266. https://doi.org/10.1023/B:EDPR.0000034022.16470.f3

Hmelo-Silver, C. E., Duncan, R. G., & Chinn, C. A. (2007). Scaffolding and achievement in problem-based and inquiry learning: a response to Kirschner, Sweller, and. *Educational Psychologist*, *42*(2), 99-107. https://doi.org/10.1080/00461520701263368

Holmes, J., & Adams, J. W. (2006). Working memory and children's mathematical skills: Implications for mathematical development and mathematics curricula. *Educational Psychology*, *26*(3), 339-366. https://doi.org/10.1080/01443410500341056

Honomichl, R. D., & Chen, Z. (2012). The role of guidance in children's discovery learning. *Wiley Interdisciplinary Reviews: Cognitive Science*, *3*(6), 615-622. https://doi.org/10.1002/wcs.1199

Horan, E., & Carr, M. (2018a). A review of guidance and structure in elementary school mathematics instruction. *Review of Science, Mathematics and ICT Education*, *12*(2), 41-60. https://doi.org/10.26220/rev.2889

Horan, E., & Carr, M. (2018b). How much guidance do students need? An intervention study on kindergarten mathematics with manipulatives. *International Journal of Educational Psychology, 7*(3), 286-316. https://doi.org/10.17583/ijep.2018.3672

Hughes, M. (1986). *Children and number.* Basil Blackwell.

Hume, T. (2015). PathSpan (Version 2.0.1) [iPad software]. https://hume.ca/ix/pathspan

Hushman, C. J., & Marley, S. C. (2015). Guided instruction improves elementary student learning and self-efficacy in science. *The Journal of Educational Research*, *108*(5), 371-381. https://doi.org/10.1080/00220671.2014.899958

Jacobs, V. R., & Kusiak, J. (2006). Got Tools? Exploring Children's Use of Mathematics Tools during Problem Solving. *Teaching Children Mathematics*, *12*(9), 470-477.

Kaminski, J. A., Sloutsky, V. M., Heckler, A. (2009). Transfer of mathematical knowledge: The portability of generic instantiations. *Child Development Perspectives, 3*(3), 151-155. https://doi.org/10.1111/j.1750-8606.2009.00096.x

Kapur, M. (2014). Productive failure in learning math. *Cognitive Science*, *38*(5), 1008-1022. https://doi.org/10.1111/cogs.12107

Kaput, J. J. (1987a). Representation systems and mathematics. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 19-26). Erlbaum.

Kaput, J. J. (1987b). Toward a theory of symbol use in mathematics. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 159-196). Erlbaum.

Karmiloff-Smith, A. (1991). Beyond modularity: Innate constraints and developmental change. In S. Carey, & R. Gelman (Eds.), *The epigenesis of mind: Essays on biology and cognition* (pp. 171-198). Erlbaum.

Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist, 41*, 75–86. https://doi.org/10.1207/s15326985ep4102_1

Klahr, D. (2009). “To every thing there is a season, and a time to every purpose under the heavens”: What about direct instruction? In S. Tobias, & T. M. Duffy (Eds.), *Constructivist instruction* (pp. 291-310). Routledge.

Klahr, D. (2010). Coming up for air: But is it oxygen or phlogiston? A response to Taber’s review of constructivist instruction: Success or failure. *Education Review*, *13*(13), 1-6.

Klahr, D., & Nigam, M. (2004). The equivalence of learning paths in early science instruction: Effects of direct instruction and discovery learning. *Psychological Science*, *15*(10), 661-667. https://doi.org/10.1111/j.0956-7976.2004.00737.x

Koedinger, K. R., & Nathan, M. J. (2004). The real story behind story problems: Effects of representations on quantitative reasoning. *Journal of the Learning Sciences*, *13*(2), 129-164. https://doi.org/10.1207/s15327809jls1302_1

Kuhn, D. (2007). Is direct instruction an answer to the right question? *Educational Psychologist*, *42*(2), 109-113. https://doi.org/10.1080/00461520701263376

Laski, E. V., Jor'dan, J. R., Daoust, C., & Murray, A. K. (2015). What makes mathematics manipulatives effective? Lessons from cognitive science and Montessori education. *SAGE Open*, *5*(2), 1-8. https://doi.org/10.1177/2158244015589588

Latour, B. (1990). Drawing things together. In M. Lynch & S. Woolgar (Eds.), *Representation in scientific practice* (pp. 19-68). MIT Press.

Lazonder, A. W., & Harmsen, R. (2016). Meta-analysis of inquiry-based learning: Effects of guidance. *Review of Educational Research*, *86*(3), 681-718. https://doi.org/10.3102/0034654315627366

Lee, H. S., & Anderson, J. R. (2013). Student learning: What has instruction got to do with it? *Annual Review of Psychology*, *64,* 445-469. https://doi.org/10.1146/annurev-psych-113011-143833

Lee, J. S., & Ginsburg, H. P. (2009). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Australian Journal of Early Childhood, 34*(4), 37-45. https://doi.org/10.1177/183693910903400406

LeFevre, J. A., Fast, L., Skwarchuk, S. L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development*, *81*(6), 1753-1767. https://doi.org/10.1111/j.1467-8624.2010.01508.x

Lesh, R. (1999). The development of representational abilities in middle school mathematics. In I. E. Sigel (Ed.), *Development of mental representation: Theories and application* (pp. 323-350). Erlbaum.

Lesh, R. (2000). What mathematical abilities are most needed for success beyond school in a technology based age of information. In M.O.J. Thomas (Ed.), *Proceedings of TIME*

2000: An international conference on technology in mathematics education (pp. 72– 82). Auckland University of Technology.

Lesh, R., Behr, M., & Post, T. (1987) Rational number relations and proportions. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 41-58). Erlbaum.

Lobato, J., Clarke, D., & Ellis, A. B. (2005). Initiating and eliciting in teaching: A reformulation of telling. *Journal for Research in Mathematics Education, 36*(2), 101-136. https://doi.org/10.2307/30034827

Loehr, A. M., Fyfe, E. R., & Rittle-Johnson, B. (2014). Wait for it... Delaying instruction improves mathematics problem solving: A classroom study. *The Journal of Problem Solving*, *7*(1), 5. http://dx.doi.org/10.7771/1932-6246.1166

Manches, A., & O'Malley, C. (2016). The effects of physical manipulatives on children's numerical strategies. *Cognition and Instruction, 34*(1), 27-50. https://doi.org/10.1080/07370008.2015.1124882

Marley, S. C., & Carbonneau, K. J. (2014a). Future directions for theory and research with instructional manipulatives: Commentary on the special issue papers. *Educational Psychology Review*, *26*(1), 91-100. https://doi.org/10.1007/s10648-014-9259-1

Marley, S. C., & Carbonneau, K. J. (2014b). Theoretical perspectives and empirical evidence relevant to classroom instruction with manipulatives. *Educational Psychology Review*, *26*(1), 1-7. https://doi.org/10.1007/s10648-014-9257-3

Martí, E., Garcia-Mila, M., & Teberosky, A. (2005). Notational strategies for problem solving in 5- to 7-year olds. *European Journal of Developmental Psychology, 2*(4), 364-384. https://doi.org/10.1080/17405620500317607

Martí, E., Scheuer, N., & de la Cruz, M. (2013). Symbolic use of quantitative representations in young children. In B. M. Brizuela & B. E. Gravel (Eds.), *"Show me what you know": Exploring student representations across STEM disciplines* (pp. 7-21). Teachers College Press.

Martin, T. (2009). A theory of physically distributed learning: How external environments and internal states interact in mathematics learning. *Child Development Perspectives, 3*(3), 140-144. https://doi.org/10.1111/j.1750-8606.2009.00094.x

Martin, T., & Schwartz, D. L. (2005). Physically distributed learning: Adapting and reinterpreting physical environments in the development of fraction concepts. *Cognitive Science*, *29*(4), 587-625. https://doi.org/10.1207/s15516709cog0000_15

Marzolf, D. P., & DeLoache, J. S. (1994). Transfer in young children's understanding of spatial representations. *Child Development*, *65*(1), 1-15. https://doi.org/10.1111/j.1467-8624.1994.tb00730.x

Mason, J. (1987). What do symbols represent? In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 73-82). Erlbaum.

Mayer, R. E. (2004). Should there be a three-strikes rule against pure discovery learning? *American Psychologist*, *59*(1), 14. https://doi.org/10.1037/0003-066X.59.1.14

McGraw-Hill Education. (2008). *Number knowledge test*. McGraw-Hill Education.

McNeil, N. M., & Jarvin, L. (2007). When theories don't add up: Disentangling the manipulatives debate. *Theory Into Practice, 46*(4), 309-316. https://doi.org/10.1080/00405840701593899

McNeil, N. M., Uttal, D. H., Jarvin, L., Sternberg, R. J. (2009). Should you show me the money? Concrete objects both hurt and help performance on mathematics problems. *Learning and Instruction, 19*, 171-184. https://doi.org/10.1016/j.learninstruc.2008.03.005

Meira, L. (1995). The microevolution of mathematical representations in children's activity. *Cognition and Instruction, 13*, 269-313. https://doi.org/10.1207/s1532690xci1302_5

Ministère de l'Éducation du Québec. (2001). *Québec Education Program*. http://www.education.gouv.qc.ca/fileadmin/site_web/documents/PFEQ/educprg2001.pdf

Ministère de l'Éducation et de l'Enseignement supérieur du Québec. (2019). *Indices de défavorisation (2017-2018).* http://www.education.gouv.qc.ca/fileadmin/site_web/documents/PSG/statistiques_info_decisionnelle/Indices-defavorisation-2017-2018.xlsx

Moch, P. L. (2001). Manipulatives work! *The Educational Forum, 66*, 81-97. https://doi.org/10.1080/00131720108984802

Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics, 47*(2), 175-197. https://doi.org/10.1023/A:1014596316942

Myers, L. J., & Liben, L. S. (2008). The role of intentionality and iconicity in children's developing comprehension and production of cartographic symbols. *Child Development*, *79*(3), 668-684. https://doi.org/10.1111/j.1467-8624.2008.01150.x

Nathan, M. J. (2008). An embodied cognition perspective on symbols, gesture, and grounding instruction. In M. de Vega, A. Glenberg, & A. Graesser (Eds.), *Symbols and embodiment: Debates on meaning and cognition* (pp. 375-396). Oxford University Press.

Nathan, M. J., Stephens, A. C., Masarik, D. K., Alibali, M. W., & Koedinger, K. R. (2002). In Representational fluency in middle school: A classroom study. In D. S. Mewborn, P. Sztajn, D. Y. White, H. G. Wiegel, R. L. Bryant, & K. Nooney (Eds.), *Proceedings of the 24th annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 462-472). ERIC Clearinghouse for Science, Mathematics and Environmental Education.

Nathan, M. J., Walkington, C., Boncoddo, R., Pier, E., Williams, C. C., & Alibali, M. W. (2014). Actions speak louder with words: The roles of action and pedagogical language for grounding mathematical proof. *Learning and Instruction*, *33*, 182-193. http://dx.doi.org/10.1016/j.learninstruc.2014.07.001

National Research Council. (2009). *Mathematics in early childhood: Learning paths toward excellence and equity*. National Academy Press.

Okamoto, Y., & Case, R. (1996). Exploring the microstructure of children's central conceptual structures in the domain of number. *Monographs of the Society for Research in Child Development, 61*, 27-58. https://doi.org/10.1111/j.1540-5834.1996.tb00536.x

Osana, H. P., Adrien, E., & Duponsel, N. (2017). Effects of Instructional Guidance and Sequencing of Manipulatives and Written Symbols on Second Graders' Numeration Knowledge. *Education Sciences*, *7*(2), 52. https://doi.org/10.3390/educsci7020052

Osana, H. P., Blondin, A., Alibali, M. W., & Donovan, A. M. (2018). *The affordances of physical manipulatives on second-graders' learning of number and place value* [Paper presentation]. American Educational Research Association, New York, NY.

Osana, H. P., & Pitsolantis, N. (2013). Addressing the struggle to link form and understanding in fractions instruction. *British Journal of Educational Psychology, 83*, 29-56. https://doi.org/10.1111/j.2044-8279.2011.02053.x

Osana, H. P., & Pitsolantis, N. (2019). Supporting Meaningful Use of Manipulatives in Kindergarten: The Role of Dual Representation in Early Mathematics. In K. M. Robinson, H. P. Osana, & D. Kotsopoulos (Eds.), *Mathematical learning and cognition in early childhood* (pp. 91-113). Springer.

Osana, H. P., Przednowek, K., Cooperman, A., & Adrien, E. (2018). Encoding Effects on First-Graders' Use of Manipulatives. *The Journal of Experimental Education*, *86*(2), 154-172. https://doi.org/10.1080/00220973.2017.1341862

Palmer, S. (1977). Fundamental aspects of cognitive representation. In: E. Rosch, & B. B. Lloyd (Eds.), *Cognition and categorization* (pp. 259-303). Erlbaum.

Petersen, L. A., & McNeil, N. M. (2013). Effects of perceptually rich manipulatives on preschoolers' counting performance: Established knowledge counts. *Child Development*, *84*(3), 1020-1033. https://doi.org/10.1111/cdev.12028

Petit, M. M., Laird, R. E., Marsden, E. L., & Ebby, C. B. (2016). *A focus on fractions: Bringing research to the classroom, 2nd ed.* Taylor & Francis.

Pouw, W. T., van Gog, T., & Paas, F. (2014). An embedded and embodied cognition review of instructional manipulatives. *Educational Psychology Review*, *26*(1), 51-72. https://doi.org/10.1007/s10648-014-9255-5

Puchner, L., Taylor, A., O'Donnell, B., Fick, K. (2008). Teacher learning and mathematics manipulatives: A collective case study about teacher use of manipulatives in elementary and middle school mathematics lessons. *School Science and Mathematics, 108*(7), 313-325. https://doi.org/10.1111/j.1949-8594.2008.tb17844.x

Purpura, D. J., Baroody, A. J., Eiland, M. D., & Reid, E. E. (2016). Fostering first graders' reasoning strategies with basic sums: the value of guided instruction. *The Elementary School Journal*, *117*(1), 72-100. https://doi.org/10.1086/687809

Rasmussen, C., & Bisanz, J. (2005). Representation and working memory in early arithmetic. *Journal of Experimental Child Psychology*, *91*(2), 137-157. https://doi.org/10.1016/j.jecp.2005.01.004

Resnick, L. B., & Omanson, S. F. (1987). Learning to understand arithmetic. In R. Glaser (Ed.), *Advances in instructional psychology* (Vol. 3, pp. 41-95). Erlbaum.

Rittle-Johnson, B. (2006). Promoting transfer: Effects of self-explanation and direct instruction. *Child Development*, *77*(1), 1-15. https://doi.org/10.1111/j.1467-8624.2006.00852.x

Rochat, P., & Callaghan, T. C. (2005). What drives symbolic development? The case of pictorial comprehension and production. In L. L. Namy (Ed.), *Symbol use and symbolic representation* (pp. 25-46). Erlbaum.

Roll, I., Butler, D., Yee, N., Welsh, A., Perez, S., Briseno, A., Perkins, K., & Bonn, D. (2018). Understanding the impact of guiding inquiry: The relationship between directive support, student attributes, and transfer of knowledge, attitudes, and behaviours in inquiry learning. *Instructional Science, 46*(1), 77-104. https://doi.org/10.1007/s11251-017-9437-x

Rosen, D., Palatnik, A., & Abrahamson, D. (2018). A better story: An embodied-design argument for generic manipulatives. In N. Calder, K. Larkin, & N. Sinclair (Eds.), *Using mobile technologies in the teaching and learning of mathematics* (pp. 189-211). Springer.

Rosenshine, B. (2009). The Empirical Support for Direct Instruction. In S. Tobias, & T. M. Duffy (Eds.), *Constructivist instruction* (pp. 201-220). Routledge.

Sarama, J., & Clements, D. H. (2009). "Concrete" computer manipulatives in mathematics education. *Child Development Perspectives, 3*(3), 145-150. https://doi.org/10.1111/j.1750-8606.2009.00095.x

Schmidt, H. G., Loyens, S. M., Van Gog, T., & Paas, F. (2007). Problem-based learning is compatible with human cognitive architecture: Commentary on Kirschner, Sweller, and Clark (2006). *Educational Psychologist*, *42*(2), 91-97. https://doi.org/10.1080/00461520701263350

Schoenfeld, A. H. (2004). The math wars. *Educational Policy*, *18*(1), 253-286. https://doi.org/10.1177/0895904803260042

Schwartz, D. L., & Bransford, J. D. (1998). A time for telling. *Cognition and Instruction*, *16*(4), 475-5223. https://doi.org/10.1207/s1532690xci1604_4

Schwartz, D. L., Chase, C. C., Oppezzo, M. A., & Chin, D. B. (2011). Practicing versus inventing with contrasting cases: The effects of telling first on learning and transfer. *Journal of Educational Psychology*, *103*(4), 1-17. https://doi.org/10.1037/a0025140

Schwartz, D. L., Lindgren, R., & Lewis, R. (2009). Constructivism in an age of non-constructivist assessments. In S. Tobias, & T. M. Duffy (Eds.), *Constructivist instruction* (pp. 34-61). Routledge.

Schwartz, D. L., & Martin, T. (2004). Inventing to prepare for future learning: The hidden efficiency of encouraging original student production in statistics instruction. *Cognition and Instruction, 22*(2), 129-184. https://doi.org/10.1207/s1532690xci2202_1

Shafrir, U. (1999). Representational competence. In I. E. Sigel (Ed.), *Development of mental representation: Theories and applications* (pp. 371–389). Erlbaum.

Sharon, T. (2005). Made to symbolize: Intentionality and children's early understanding of symbols. *Journal of Cognition and Development*, *6*(2), 163-178. https://doi.org/10.1207/s15327647jcd0602_1

Sherin, B. L. (2000). How students invent representations of motion: A genetic account. *The Journal of Mathematical Behavior*, *19*(4), 399-441. https://doi.org/10.1016/S0732-3123(01)00052-9

Sherman, J., & Bisanz, J. (2009). Equivalence in symbolic and nonsymbolic contexts: Benefits of solving problems with manipulatives. *Journal of Educational Psychology, 101,* 88-100. https://doi.org/10.1037/a0013156

Siegler, R. S., & Lemaire, P. (1997). Older and younger adults' strategy choices in multiplication: Testing predictions of ASCM using the choice/no-choice method. *Journal of Experimental Psychology: General*, *126*(1), 71. https://doi.org/10.1037/0096-3445.126.1.71

Sloutsky, V. M., Kaminsky, J. A., & Heckler, A. F. (2005). The advantage of simple symbols for learning transfer. *Psychometric Bulletin & Review, 12*(3), 508-513. https://doi.org/10.3758/BF03193796

Stanford Achievement Test (9th ed.). (1995). Psychological Corp.

Steffe, L. P., & Gale, J. E. (Eds.). (1995). *Constructivism in education* (p. 159). Erlbaum.

Sullivan, G. M., & Artino, A. R., Jr. (2013). Analyzing and interpreting data from Likert-type scales. *Journal of Graduate Medical Education*, *5*(4), 541-542. http://dx.doi.org/10.4300/JGME-5-4-18

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science*, *12*(2), 257-285. https://doi.org/10.1016/0364-0213(88)90023-7

Sweller, J., Kirschner, P. A., & Clark, R. E. (2007). Why minimally guided teaching techniques do not work: A reply to commentaries. *Educational Psychologist*, *42*(2), 115-121. https://doi.org/10.1080/00461520701263426

Tabachneck, H. J. M., Koedinger, K. R., & Nathan, M. J. (1994). Toward a theoretical account of strategy use and sense-making in mathematics problem solving. In A. Ram, & K. Eiselt (Eds.), *Proceedings of the 16th Annual Conference of the Cognitive Science Society* (pp. 836-841). Erlbaum.

Thomas, M. O. (2008). Conceptual representations and versatile mathematical thinking. In M. Niss (Ed.), *Proceedings of the 10th International Congress on Mathematical Education, 4-11 July 2004* (pp. 1-18). Roskilde University.

Thomas, M. O. J. & Hong, Y. Y. (2001) Representations as conceptual tools: Process and structural perspectives. *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education, 4,* 257–264. https://www.math.auckland.ac.nz/~thomas/index/staff/mt/My%20PDFs%20for%20web%20site/PME01YY.pdf

Tran, C., Smith, B., & Buschkuehl, M. (2017). Support of mathematical thinking through embodied cognition: Nondigital and digital approaches. *Cognitive Research: Principles and Implications*, *2*(1), 16. https://doi.org/10.1186/s41235-017-0053-8

Trninic, D. (2018). Instruction, repetition, discovery: Restoring the historical educational role of practice. *Instructional Science*, *46*(1), 133-153. https://doi.org/10.1007/s11251-017-9443-z

Uttal, D. H., Amaya, M., del Rosario Maita, M., Hand, L. L., Cohen, C. A., O'Doherty, K., & DeLoache, J. S. (2013). It works both ways: Transfer difficulties between manipulatives and written subtraction solutions. *Child Development Research*, *2013*, 1-13. https://doi.org/10.1155/2013/216367

Uttal, D. H., Liu L. L., & DeLoache, J. S. (2006). Concreteness and symbolic development. In L. Balter & C. S. Tamis-LeMonda (Eds.), *Child psychology: A handbook of contemporary issues* (2nd ed.) (pp. 167-184). Psychology Press.

Uttal, D. H., O'Doherty, K., Newland, R., Hand, L. L., & DeLoache, J. (2009). Dual representation and the linking of concrete and symbolic representations. *Child*

Development Perspectives, *3*(3), 156-159. https://doi.org/10.1111/j.1750-8606.2009.00097.x

Uttal, D. H., Scudder, K. V., & DeLoache, J. S. (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of Applied Developmental Psychology, 18*, 37-54. https://doi.org/10.1016/S0193-3973(97)90013-7

Von Glasersfeld, E. (2013). *Radical constructivism*. Routledge.

Vygotsky, L. S. (1978). Tool and symbol use in child development. In M. Cole, V. John-Steiner, S. Scribner, & E. Souberman (Eds.), *Mind in society: The development of higher psychological processes* (pp. 19-30). Harvard University Press.

Wechsler, D. (1974). *Manual for the Wechsler Intelligence Scale for Children-Revised*. Psychological Corporation.

Weisberg, D. S., Hirsh-Pasek, K., & Golinkoff, R. M. (2013). Guided play: Where curricular goals meet a playful pedagogy. *Mind, Brain, and Education*, *7*(2), 104-112. https://doi.org/10.1111/mbe.12015

Wise, A. F., & O'Neill, K. (2009). Beyond more versus less: A reframing of the debate on instructional guidance. In S. Tobias, & T. M. Duffy (Eds.), *Constructivist instruction* (pp. 94-117). Routledge.

Zacharia, Z. C., & Olympiou, G. (2011). Physical versus virtual manipulative experimentation in physics learning. *Learning and Instruction*, *21*(3), 317-331. https://doi.org/10.1016/j.learninstruc.2010.03.001

Zelazo, P. D. (2006). The Dimensional Change Card Sort (DCCS): A method of assessing executive function in children. *Nature Protocols*, *1*(1), 297-301. https://doi.org/10.1038/nprot.2006.46

Appendix A

Parent Consent Form

Université Concordia University

Lester B. Pearson School Board

Commission scolaire Lester-B.-Pearson

DATE

Dear Parent(s) and/or Guardian(s):

The Lester B. Pearson School Board is working in partnership with Concordia University on a math project, which is part of the Board's Trauma Informed Pedagogical Practices and Strategies (TIPPS) initiative. We have been partners in the TIPPS project for the past four years with the main objective of improving children's numeracy skills. This year, we are working to understand the ways in which children in pre-kindergarten, kindergarten, and Grade 1 learn and understand math concepts and symbols. Working with children on their early numeracy skills is critical for their future success in mathematics. To this end, we are inviting your child to participate in our math project.

This year's project is headed by Dr. Helena Osana, Associate Professor of Education and Concordia University Research Chair in Mathematical Cognition and Instruction. Three graduate students have been hired as research assistants on this project. The graduate students are pursuing Master's and Doctorate degrees in Child Studies in the Department of Education at Concordia.

We would like to meet individually on one or two occasions with your child, on separate days, in April, May, or June 2018 to do some counting activities and solve word problems with blocks and other objects. These meetings will be take place in a quiet place on school grounds, such as the school library and will last approximately 30 minutes each. We would like to videotape the meetings, but your child's name and face will not be on the recording.

Please rest assured that we will never share any information about your child's thinking in math with anyone except the research team and teachers who take part in teacher preparation and math professional development in the future. Under no circumstances will we include any information that can identify your child, his or her teacher, the school, or the Board, even should we publish any of the findings.

Your child does not have to be part of this research. It is up to you and your child. Your child can stop being a part of the project at any time. This is your choice.

The consent form for this project is at the end of this letter. If you choose for your child to participate, please fill it out, sign it, and return it to your child's teacher no later than <u>DATE</u>.

Should you have any questions about this project, please feel free to contact Helena Osana at Concordia University (514-848-2424 ex. 2543 or **helena.osana@concordia.ca**). If you have concerns about ethical issues in this research, please contact the Manager, Research Ethics, Concordia University, 514- 848-2424 ex. 7481 or oor.ethics@concordia.ca.
Our findings will help inform teachers on the best ways to use mathematical tools in their lessons to help students understand math concepts. Thank you for your interest in this important project.

Sincerely,

Helena P. Osana, Associate Professor of Education
Concordia University Research Chair in Mathematical Cognition and Instruction
1610 Ste.- Catherine Ouest, FG-6.413
Montreal, QC H3H 2S2
(514) 848-2424 ext. 2543
helena.osana@concordia.ca

INFORMATION AND CONSENT TO PARTICIPATE IN A RESEARCH STUDY

Study Title: Costs and Benefits of Telling Children the Quantitative Meaning of Manipulatives

Researcher: Helena P. Osana
Researcher's Contact Information:

helena.osana@concordia.ca
514-848-2424 ext. 2543
1455 de Maisonneuve Ouest
Montréal, QC
H3G 1M8

Study Investigator: Emmanuelle Adrien
Study Investigator's Contact Information: emmanuelle.adrien@concordia.ca

Source of funding for the study: Social Sciences and Humanities Research Council of Canada (SSHRC)

Your child is being invited to participate in the research study mentioned above. This form provides information about what participating would mean for him or her. Please read it carefully before deciding if you want your child to participate or not. If there is anything you do not understand, or if you want more information, please ask the researcher.

A. PURPOSE

The purpose of the research is to better understand the ways in which children in pre-kindergarten, kindergarten, and Grade 1 learn and understand math concepts and symbols.

B. PROCEDURES

If you decide that your child may participate in this research, he/she will meet individually on one or two occasions with a research assistant. During these individual meetings, your child will do some counting activities and solve word problems with blocks and other objects. These meetings will be conducted in a quiet place on school grounds, such as the school library and will last approximately 30 minutes each.

The research assistant would also like to videotape the conversations they have with your child about math only. This will help us better understand how children think about math concepts. **The focus of**

the recordings will be on your child's hands only. His or her face will never be captured on tape. These video recordings will be used for research and training purposes only and will not be shared with anyone outside the research team.

C. RISKS AND BENEFITS

Potential benefits include being exposed to math ideas and activities that have been shown to help your child think deeply about important concepts in the curriculum. Your child will benefit by thinking and talking about these concepts because they will prepare him or her for the math curriculum taught throughout elementary school and beyond.

Video recordings and other data will be collected for the study. To minimize the risk to participants and to maximize confidentiality, we will ensure that throughout the interview only your child's hands are recorded and not their faces. Additionally, all recordings will be stored in Dr. Osana's locked laboratory on a password-protected device at all times.

During the individual interview, the risks are minimal. It is possible that your child gets bored during the individual interview or gets frustrated because the math questions may be different from the ones he or she experiences in class. If at any point and for whatever reason your child does not want to continue participating, we will stop all activity and bring your child back to the classroom.

D. CONFIDENTIALITY

We will gather the following information as part of this research:

- Videotaped interviews (total 30-60 minutes). Your child will do some counting activities and solve word problems with blocks and other objects during the videotaped interviews.
- Your child's age, gender, and ethnicity.

By allowing your child to participate, you agree to let the researchers have access to the collected information.

We will not allow anyone to access the information, except people directly involved in conducting the research (H. Osana, E. Adrien, D. Uttal (Professor at Northwestern University, USA, collaborator on this project), and [name of research assistants – TBD]), and except as described in this form. Only H. Osana and E. Adrien will have access to your child's name. We will only use the information for the purposes of the research and training described in this form.

To verify that the research is being conducted properly, regulatory authorities might examine the information gathered. By participating, you agree to let these authorities have access to the information.

The information gathered will be kept strictly confidential. This means that the research team (i.e., H. Osana, E. Adrien, and Research Assistants who will be hired to assist in the data collection) will know your child's real identity, but it will not be disclosed.

We will protect the information by keeping it on a password-protected computer in the research lab of H. Osana in the FG Building (FG 6.403) and in a locked filing cabinet in the same office. Only the research assistants on this project will have the password and the key to the filing cabinet. The office is kept locked at all times.

We intend to publish the results of the research, but it will not be possible to identify your child in the published results.

We will destroy the information five years after the last presentation or publication that is generated from this study.

In certain situations we might be legally required to disclose the information your child provides. This includes situations where discoveries, such as child abuse or an imminent threat of serious harm to specific individuals, are uncovered as a result of our interactions. If this kind of situation arises, we will disclose the information as required by law, despite what is written in this form.

E. CONDITIONS OF PARTICIPATION

You do not have to agree for your child to participate in this research. It is purely your decision. If you do agree for your child to participate, he/she can stop at any time. You can also ask that your child's information that was provided not be used, and your choice will be respected. If you decide that you do not want us to use your child's information, you must tell the researcher before June 15, 2018.

We will tell you if we learn of anything that could affect your decision to stay in the research.

There are no negative consequences for not participating, stopping in the middle, or asking us not to use your information.

We will not be able to offer you compensation if you are injured in this research. However, you are not waiving any legal right to compensation by signing this form.

F. PARENTS'/GUARDIANS' DECLARATION

I have read and understood this form. I have had the chance to ask questions and any questions have been answered.

☐ Yes, I agree for my child to participate in this research under the conditions described.

Optional Permission:
☐ I give permission for **non-identifiable** portions of the videos of my child to be shown to educational or scientific audiences (e.g., in a scientific presentation, class, or teacher training).

Child Name: ____

Child's date of birth (Month, Day, Year): __________________________

Parent/Guardian Name: ____________________________________

Signature: ____

Date: ____

If you have questions about the scientific or scholarly aspects of this research, please contact Dr. Helena Osana at Concordia University, the Principal Investigator on the project. Her contact information is on page 1 of this form.

If you have concerns about ethical issues in this research, please contact the Manager, Research Ethics, Concordia University, 514.848.2424 ex. 7481 or oor.ethics@concordia.ca.

☐ I would like a report of the results of this project.

Contact Information for Report: ____

Appendix B

Child Assent Form

MY AGREEMENT

My name is *(RA's name)*. I would like to talk to you about math and learn about the way you do math problems. I would like to play some math games with you and ask you to do some math problems. I would like to videotape you while you do math problems. I will only videotape your hands and not your face. You will meet with me or with one of my friends two times this year.

I won't tell your teacher what you say. I also won't show the videos of your hands to your teacher. I will tell my friend at my school what you said, but I won't tell her your name. I won't show your name to anyone.

You don't have to talk to me if you don't want to. If you change your mind, you can stop talking to me. That would be ok and no one will be mad. You just have to tell me.

Do you understand this? Circle the smiley face if you do and the sad face if you don't.

☺ ☹

Do you want to talk to me about math? Circle the smiley face if you do and the sad face if you don't.

☺ ☹

Child's name: ________________________________

Date: ______________________________________

Appendix C

Interview Protocol

Symbolization Study2 – DAY 1

1. Counting Sequence
2. Number Knowledge Test (NKT)
3. Dimensional Change Card Sort (DCCS)
4. Digits Backward
5. PathSpan
6. Go/No Go
7. Baseline Unconstrained Task

1. Counting Sequence

⇒ SAY: **Can you please count as high as you can?**

Make note of any counting error on NKT scoring sheet (in Preliminary).

Stop the child when he/she reaches 30.

If child can't count in English, ask to count in French. If can't in French – do not continue.

SAY: **Thank you for showing me how you count! Let's do another game now.**

2. NKT

⇒ *Preliminary*: no need to ask if child was able to count to 10 during "Counting Sequence"

3. DCCS

Materials: Trays, cards

⇒ **Pre-switch phase dimension: colour**

- Place trays in front of child (within reaching distance)
- Affix panels behind sorting trays (blue square on the child's left; red circle on the child's right)
- **DEMO:**
- SAY: **Here's a blue square and a red circle. Now we're going to play a card game. This is the color game. In the color game, all the blue ones go here** *(point to the tray on the left),* **and all the red ones go there** *(point to the tray on the right).*
- Take one blue circle and SAY: **See, here's a blue one. So it goes here** *(place it face down in the tray on the left.)* **If it's blue it goes here** *(point),* **but if it's red it goes there** *(point).*
- Take one red square and SAY: **Now here's a red one. Where does this one go?**
  - ➢ If child points to or places in correct tray, SAY: **Very good. You now know how to play the color game.** *If the child had only pointed, SAY:* **Can you help me put this red one down?** *(ensure that the card is face down, turn it over if necessary)*

- If child is incorrect, SAY: **No, this one's red, so it has to go over here in the color game. Can you help me put this red one down?** *(ensure that the card is placed face-down in the appropriate tray).*

- **TRIALS:**
- SAY: **Now it's your turn. So remember, if it's blue it goes here** *(point)*, **but if it's red it goes there** *(point).*
- Pick a card at random, show it to the child, and label it by the relevant dimension only. SAY: **Here's a red/blue one. Where does it go?**
  - Child may place or point (you can place for them); ensure that the card is placed face down in the appropriate tray.
- For each trial, discretely note if the child is correct or incorrect on scoring sheet.
- Whether or not child sorts correctly, SAY: **Let's do another one. / Let's do it again. / How about another one?**
- SAY: **If it's blue it goes here** *(point),* **but if it's red it goes there** *(point).*
- Pick a new card; ensure that the same card doesn't appear more than twice in a row. SAY: **Here's a red/blue one. Where does it go?**
- Do 6 trials in total.
- DO NOT PAUSE BEFORE MOVING ON TO THE POST-SWITCH PHASE.

⇒ **Post-switch phase dimension: shape**

- SAY: **Now we're going to play a new game. We're not going to play the color game anymore. We're going to play the shape game. In the shape game, all the squares go here** *(point to the tray on the left),* **and all the circles go there** *(point to the tray on the right).* **Remember, if it's a square, put it here** *(point),* **but if it's a circle put it there** *(point).* **Okay?**
- Do not remove the cards that were sorted during the pre-switch phase.
- **TRIALS:**
- Pick a card at random *(make sure the same card hasn't been shown on more than two consecutive trials)*, show it to the child, and label it by the relevant dimension only. SAY: **Here's a square/circle. Where does it go?**
  - Child may place or point (you can place for them); ensure that the card is placed face down in the appropriate tray.
- For each trial, discretely note if the child is correct or incorrect on scoring sheet.
- Whether or not child sorts correctly, SAY: **Let's do another one. / Let's do it again. / How about another one?**
- SAY: **If it's a square it goes here** *(point),* **but if it's a circle it goes there** *(point).*
- Pick a new card; ensure that the same card doesn't appear more than twice in a row. SAY: **Here's a square/circle. Where does it go?**
- Do 6 trials in total.
- IF CHILD GOT 5/6 OR 6/6 CORRECT, MOVE ON TO BORDER VERSION.

⇒ **Border version:**

- Remove already-sorted cards from the trays. Keep 4 red squares and 3 blue circles. Set the remaining cards aside.
- Add additional Border test cards (i.e., 4 red squares and 3 blue circles) to the set.
- SAY: **Okay, you played really well. Now I have a more difficult game for you to play. In this game, you sometimes get cards that have a black border around it like this one** *(show a red square with a border).* **If you see cards with a black border, you have to play the color game. In the color game, red ones go here** *(point)* **and blue ones go there** *(point).* **This card's red, so I'm going to put it right there** *(place it face down in the appropriate tray.)* **But if the cards have no black border, like this one** *(show a red square without a border),* **you have to play the shape game. In the shape game, if it's a square, we put it here** *(point),* **but if it's a circle, we put it there** *(point).* **This one's a square, so I'm going to put it right here** *(place it face down in the appropriate tray).* **Okay? Now it's your turn.**
- **TRIALS:**
- SAY: **Remember, if there's a black border, you have to play the color game. But if there's not black border, you have to play the shape game.**
- Pick a card at random, show it to the child, and label it as having a border or not. SAY: **Here's one with/without a black border. Where does it go?**
  - ➢ Child may place or point (you can place for them); ensure that the card is placed face down in the appropriate tray.
- For each trial, discretely note if the child is correct or incorrect on scoring sheet.
- Whether or not child sorts correctly, SAY: **Let's do another one. / Let's do it again. / How about another one?**
- SAY: **Remember, if there's a black border, you have to play the color game. But if there's not black border, you have to play the shape game.**
- Pick a card at random *(ensure the same type of test card – with or without border – is not selected on more than 2 consecutive trials)*, show it to the child, and label it as having a border or not. SAY: **Here's one with/without a black border. Where does it go?**
- Do 12 trials in total.

TABLE 1 | Troubleshooting table.

PROBLEM	SOLUTION
Steps 2–4, 6+7	
Children hesitate	Label the card by the relevant dimension and ask where it goes (e.g., "Here's a rabbit, where does it go?"). If the child still hesitates, say, "Let's do another one," return the skipped card to the pile of to-be-sorted cards, select a new card, label it by the relevant dimension, and ask where it goes.
Children refuse to complete the task	If a child refuses to continue sorting, suggest that he or she may point to the correct box and show you where each card goes. If the child refuses to do this, then terminate the task, as their data will be unusable unless all trials are completed.
Children change response	Allow children to change their responses, scoring only their final response. Do not provide evaluative feedback. Simply say, "Are you sure?" and then proceed to the next trial, saying, "Let's do another one."
Children ask for feedback	Do not provide evaluative or corrective feedback. Simply encourage them to keep playing, saying, "Sort the card," or "Let's do another one," as appropriate.
Children pick up previously sorted cards	Prevent children from picking up previously sorted cards. Tell them, "Those cards have to stay there, but let's do another one."
Children take a break during the task	Discourage children from taking a break until the procedure has been completed, saying, "We're almost done." If children need to take a break during Steps 2, 3, 6 or 7, repeat the interrupted step when they return and then complete the procedure. Only use data from the completed (re-administered) step, not the interrupted one. Children should not take a break during Step 4; this would render the data unusable.

4. Digits Backward

- *Read each digit span **only once** at an even rate of 1 digit per second.*
- SAY: **I am going to say some numbers. Listen carefully and when I stop, I want you to say them backward. If I say 9-2-7, what would you say?**
- Pause for the child to respond.
- If child responds correctly (7-2-9), SAY: **That's right. Let's go on with the rest of the numbers.** Proceed to 1st item.
- If child responds incorrectly, SAY: **No, you would say 7-2-9. I said 9-2-7, so to say it backward you would say 7-2-9. Now try these numbers. Remember, you are to say them backward: 3-6-5.**
- Whether the child is correct or wrong on the 2nd example, the test will proceed.

5. PathSpan

⇒ *Instructions on iPad*

6. Go/No Go

⇒ *Instructions on iPad*

7. Baseline Unconstrained Task

Materials: Container with wooden blocks, scaffolding mats

SAY: **I will read you some word problems. Use these** *(point to container)* **to help you solve the problem.**

1) John has 3 children. He wants to give 2 sandwiches to each one of them. How many sandwiches does he need?

2) Pat has 3 cats, She gives 3 cans of food to each cat. How many cans of food does Pat need for her cats?

STUDY 2 – INTERVIEW PROTOCOL
Day 2

Day 2	Day 3
1) Need-to-symbolize activity	1) Encoding Review
2) Grouping tasks	2) Constrained Transfer*
3) Encoding + Manipulation check	3) Unconstrained Transfer*
4) Near-Transfer	4) Sticker Task
	**counterbalance order*

DAY 2

1) Need-to-Symbolize

Materials: 3 boxes (normal lids) that have different amounts of base-10 "ones" in each one
Items: 4, 12, 8

	Take out the first box. Other boxes are kept out of the child's view.
"I have some blocks in this box. How many blocks are there?"	*Child answers. Provide corrective feedback if needed.*
"Ok, let's look in another box."	*Leave box open, push it aside, and take out a second box.*
Repeat the same procedure for the other 2 boxes.	
"You said there were ***n*** blocks in this box ***(close the lid of the 1st box)***, ***n*** blocks in this box ***(close the lid of the 2nd box)***, and ***n*** blocks in this box ***(close the lid of the 3rd box)***. I will now mix up the boxes."	*Mix them up the boxes out of the view of the child, take one box out at random and ask,*
"Can you tell me how many blocks are in this box?"	*Child will provide an answer. Open the box and check. Repeat the same procedure with the other 3 boxes.*
"It's kind of difficult to know how many blocks are in the boxes when they are closed. I'm going to think about an idea to help us know how many blocks there are while we do another activity."	

2) Grouping tasks

Materials: Square pattern blocks

Items: (8 demo), 6, 10

"Listen to how I am counting. I want you to continue counting in the same way: Two, four, six, …"	*Child continues skip-counting by 2s. Make a note of highest number child says in English.*
"Écoute comment je compte en français. Continue de compter de la même manière: Deux, quatre, six, …"	*Child continues skip-counting by 2s. Make a note of highest number child says in French.*
"Thank you. I want you to know that for all the activities we will do, you can count in English or in French – you decide what is easier for you."	*Continue in the language chosen by the child.*
"Let's count again: Two, four, six, …"	*Count up to 20 with the child; do the sequence 3 times in total.*
"For this game, we have some blocks."	*Empty the bag in front of the child and ask,*
"How many blocks are there?"	*Child counts. If it's not the correct amount, ask the child, "*Can you please count the blocks one more time? *Wait for the child to count. If the child is still incorrect, count the blocks out loud yourself, moving each one aside as you count.*
"I will now make groups of 2 with the blocks. Watch what I do."	*Arrange the blocks by 2s.*
"Look at how I count the blocks now."	*Point to each grouping as you count out loud,*
"Two – four – six…"	
"Can you count the blocks the same way I did?"	*Child will repeat. Provide corrective feedback if needed.*
	Put the blocks back in the bag, and take out the next bag.
"Now you try it on your own. Make groups of 2 with the cubes and count them the same way I just did."	*Child will count by 2s. Provide corrective feedback if needed.*
Repeat same procedure with the last bag.	

3) Encoding + Manipulation check

Materials: Same boxes as in #1, special lids, pollies

Items for encoding: 4, 12, 8 (same boxes as in #1)

Items for manipulation check: 14, 6, 16, 10, 18

"I thought about our problem remembering how many blocks are in each box, and I have an idea. I have these special lids and these pollies that we can use to help us remember how many blocks are in the box."	***Take out a special lid and the bin with little pollies. Take one of the 3 boxes.***
"Let's open this box. I will count how many blocks are inside."	***Take the blocks out and count them out loud.***
"We can put little pollies on the special lids to help us remember how many blocks are in each box. That way, when we close the box, we can look at the little pollies and know how many blocks are inside. The little pollies are like a secret code that will tell us how many blocks are in the box."	

CONDITION #1: Direct instruction

Items for encoding: 4, 12, 8 (same boxes as in #1)

Items for manipulation check: 14, 6, 16, 10, 18

"Do you want to know what the secret code is for the little pollies? For these boxes, our secret code is that each little pollie will mean there are two blocks. Look."	*Group blocks by 2 and place one little pollie near each grouping.*
"Two – four – six – etc."	
"I will now put the blocks back in the box and put the little pollies on the lid. Now I know that there are ***n*** blocks in the blocks because I know the secret code: I know that each little pollie means there are two blocks. I can count the little pollies: two – four – six – etc. This means there are ***n*** blocks!"	*Put box away and take out second box.*
"Now it's your turn. Use the same secret code with the little pollies to show how many blocks are in this box."	*Provide corrective feedback if necessary:* - *Remember the secret code! Each little pollie means there are two blocks.* *Close the lid.*
"Can you tell me how many blocks are inside just by looking at the little pollies?"	*Child will answer. If answer is correct, say* "Yes! You know the secret code!" *If answer is incorrect, say* "You need to remember the secret code. Each little pollie means there are two blocks."
Repeat the same procedure with the last box.	
Manipulation check	
"Now I will put a new number of blocks in the box, and I want you to use the secret code with the little pollies to help you know how many blocks are in the box."	*Put a new amount of blocks in one of the boxes – out of the child's view.* *Child will use the pollies. If correct, say* "Yes! You know the secret code!" *If incorrect, say* "You need to remember the secret code. Each little pollie means there are two blocks. Try again."
Repeat the procedure with new items. Stop the task when child gets two in a row correct.	

CONDITION #2: Guided exploration

Items for encoding: 4, 12, 8 (same boxes as in #1)

Items for manipulation check: 14, 6, 16, 10, 18

	Provide the same number of pollies as blocks in the box.
"How can you use these little pollies as a secret code to help you remember how many blocks are in the box?"	*Child interacts with the blocks and the little pollies. Suggests one way.*
"That's interesting! Now can you think of a different secret code with the pollies to help you remember how many blocks are in the box if you only have this many?"	*Remove little pollies so that their number is half the number of blocks. Child interacts with blocks and little pollies until they come up with little pollie=2.*
Provide prompts if necessary: *1) Remember how we were counting with the little cubes? Can that help you think of a way to find a secret code for the pollies?* *2) Remember how we were grouping the cubes before? Could you do the same thing with these blocks? Can that help you think of a way to find a secret code for the pollies?* *3) Let's put the blocks in the box. Remember, there are n blocks. Now let's put the pollies in the lid. When I see the pollies, I know there are n blocks. How do you think I figured it out? What is my secret code for the pollies?*	
"What is your secret code for the pollies? I don't your code; I want to be able to use the same code as you. How does it help you know how many blocks are in the box?"	
"Now can you use the little pollies to help you remember how many blocks are in this box? Let's open it."	
Repeat the whole procedure with the other two boxes.	
Manipulation check	
"Now I will put a new number of blocks in the box, and I want you to use your secret code with the little pollies to help you know how many blocks are in the box."	*Put a new amount of blocks in one of the boxes – out of the child's view.* *Feedback:* *Level 1*: Cue to remember the encoding: Remember when you were grouping the blocks before and you found a secret code for the pollies? Use the same secret code with the pollies now. *Level 2*: End state: If this many pollies tell us that there are *n* blocks in the box. What do you think the secret code for the pollies is?
Repeat the procedure with new items. Stop the task when child gets two in a row correct.	

<u>CONDITION #3: Control</u>

<u>Items for encoding:</u> 4, 12, 8 (same boxes as in #1)

<u>Items for manipulation check:</u> 14, 6, 16

"How can you use these little pollies as a secret code to help us remember how many blocks are in the box?"	*Child interacts with the blocks and the little pollies. Suggests one way.*
"Can you explain to me how you used the pollies as a secret code to help you say how many blocks are in the box?"	*Child explains*
"That's interesting!"	
"Now can you use your secret code with the little pollies to help us remember how many blocks are in this box?"	*Bring out a new box.* *Repeat the same procedure for the remaining boxes.* *No prompts or feedback should be given.* *Use all 3 encoding items + 3 manipulation check items.*

4) Near-Transfer

Materials: Container with many little pollies, mats.

******Check Drive file to now in which order to read the problems for each child******

"Now I will read you some word problems. Use the little pollies to help you solve each problem." (*It's ok to "unpack" the problem for the kids, i.e., to say it in different words so that they understand the problem. Example: "No, he puts **3** cars **in each box**.")*

A - *(3 groups of 2)* There are three kids in a park. Each kid has 2 balloons. How many balloons do they have altogether?

B - *(2 groups of 4)* Tad has two bowls. There are 4 meatballs in each bowl. How many meatballs does Tad have altogether?

Possible prompts to ask after each problem – *ask as many as needed to achieve each goal*:

Goal #1: Understand what referent the child is assigning to the pollies.

- How did you figure that out?
- Can you explain to me how you solved the problem?
- How would you explain to a friend how you solved the problem with the pollies?
- What are these [point to pollies used; what do they show from the story]? What can they be used for?
- How many [*items from the problem, e.g., toy cars*] are you showing me here [point to the mat]?
- How many *[items from the problem]* is this [show one pollie to the child]?

If child didn't use pollie=2

Goal #2: See if can use a different referent for the pollies.

- Can you show me a different way to solve the problem with the pollies?
- Is there a way you could use fewer pollies to solve the problem? (Or "I don't want to use this many pollies, can you should me how you can use less of them to solve the problem?)
- *If child says something like "he lost 2 cars,"* ask, "Can you show me the same story, the same problem, but with less/fewer pollies?"
- Can you count with the pollies to show me how many [*items from the problem]* there are?
- How many [*items from the problem*] are you showing me here [point to the mat]?
- How many *[items from the problem]* is this [show one pollie to the child]?

STUDY 2 – INTERVIEW PROTOCOL
Day 3

Day 2	**Day 3**
5) Need-to-symbolize activity	5) Encoding Review
6) Grouping tasks	6) Constrained Transfer*
7) Encoding + Manipulation check	7) Unconstrained Transfer*
8) Near-Transfer	8) Sticker task
	**counterbalance order*

DAY 3

1) Encoding Review

<u>Materials:</u> Boxes with special lids, pollies

"Do you remember yesterday (or last time) when you were using a secret code with the little pollies to help us remember how many blocks are in the box? We're going to do the same thing now."	*Take out the box with 8 cubes.*
"How many blocks are there inside?"	*Child counts. Provide corrective feedback if necessary.*

<u>CONDITION #1: Direct instruction</u>

<u>Items:</u> 8, 4, 10, 12, 6, 14

"I want you to use the secret code with the little pollies to help you know how many blocks are in the box."	*Put a new amount of blocks in one of the boxes – out of the child's view.* *Child will use the pollies. If correct, say* "Yes! You know the secret code!" *If incorrect, say* "You need to remember the secret code. Each little pollie means there are two blocks. Try again."
Repeat the procedure with new items. Stop the task when child gets two in a row correct.	

CONDITION #2: Guided exploration

Items: 8, 4, 10, 12, 6, 14

"I want you to use your secret code with the little pollies to help you know how many blocks are in the box."	*Child will use the pollies.*
"What is your secret code for the pollies? I don't your code; I want to be able to use the same code as you. How does it help you know how many blocks are in the box?"	*Feedback:* *Level 1*: Cue to remember the encoding: Remember when you were grouping the blocks last time and you found a secret code for the pollies? Use the same secret code with the pollies now. *Level 2*: End state: If this many pollies tell us that there are *n* blocks in the box. What do you think the secret code for the pollies is?
Repeat the procedure with new items. Stop the task when child gets two in a row correct. If child cannot do two in a row correct, skip directly to the "Sticker Task," do not do the "Constrained Transfer" or the "Unconstrained Transfer" tasks.	

CONDITION #3: Control

Items: 8, 4, 10

"I want you to use your secret code with the little pollies to help you know how many blocks are in the box."	***No prompts or feedback should be given. Do 3 items.***

Check the Drive file to know in which order to read the problems in each task for each student

2) Constrained Transfer

Materials: Container with many little pollies, mats.

"Now I will read you some other word problems. Use the little pollies to help you solve each problem." (*It's ok to "unpack" the problem for the kids, i.e., to say it in different words so that they understand the problem. Example: "No, he puts **3** cars **in each box**.")*

***C** - (3 groups of 3)* Eric puts 3 toy cars in every box. He has 3 boxes. How many toy cars does he have altogether?

***D** - (2 groups of 9)* At the bakery, the baker puts 9 cupcakes in each box. How many cupcakes are in 2 boxes?

***E** - (3 groups of 5)* Louie puts 5 shirts in each drawer of his dresser. There are 3 drawers in his dresser. How many shirts does he have altogether?

***F** - (2 groups of 15)* Robin has 2 packs of gum. There are 15 pieces of gum in each pack. How many pieces of gum does Robin have altogether?

If child gets stuck or is not sure what to do, ask or say, "What are you thinking?" or Explain to me what you're thinking about.

Possible prompts to ask after each problem – *ask as many as needed to achieve each goal*:

Goal #1: Understand what referent the child is assigning to the pollies.

- How did you figure that out?
- Can you explain to me how you solved the problem?
- How would you explain to a friend how you solved the problem with the pollies?
- What are these [point to pollies used; what do they show from the story]? What can they be used for?
- How many [*items from the problem, e.g., toy cars*] are you showing me here [point to the mat]?
- How many *[items from the problem]* is this [show one pollie to the child]?

Goal #2: See if can use a different referent for the pollies.

- Can you show me a different way to solve the problem with the pollies?
- Is there a way you could use fewer pollies to solve the problem? (Or "I don't want to use this many pollies, can you should me how you can use less of them to solve the problem?)
- *If child says something like "he lost 2 cars,"* ask, "Can you show me the same story, the same problem, but with less/fewer pollies?"
- Can you count with the pollies to show me how many *[items from the problem]* there are?
- How many [*items from the problem*] are you showing me here [point to the mat]?
- How many *[items from the problem]* is this [show one pollie to the child]?

3) **Unconstrained transfer task**

Materials: Container with other manipulatives (i.e., jewels, coloured blocks, buttons); scaffolding mats

"I will read some other word problems. Use these *(point to container)* to help you solve the problem." (*It's ok to "unpack" the problem for the kids, i.e., to say it in different words so that they understand the problem. Example: "No, she puts **3** bracelets **in each box**.")*

G - (3 groups of 3) Samantha puts 3 bracelets in each jewellery box. She has 3 jewellery boxes. How many bracelets does she have altogether?

H - (2 groups of 9) A teacher puts 9 crayons in each pencil case. How many crayons are in two pencil cases?

I - (3 groups of 5) Violet wants to give 5 flowers to each one of her 3 friends. How many flowers does she need altogether?

J - (2 groups of 15) There are two aquariums at the pet store. Each aquarium has 15 fishes. How many fishes are there altogether?

If child gets stuck or is not sure what to do, ask or say, "What are you thinking?" or Explain to me what you're thinking about.

Possible prompts to ask after each problem – *ask as many as needed to achieve the goal*:

Goal #1: Understand what referent the child is assigning to the manipulatives.

– How did you figure that out?
– Can you explain to me how you solved the problem?
– How would you explain to a friend how you solved the problem with these [point to manipulatives]?
– What are these [point to manipulatives used; what do they show from the story]? What can they be used for?
– How many [*items from the problem, e.g., toy cars*] are you showing me here [point to the mat]?
– How many *[items from the problem]* is this [point to one manipulative]?

Goal #2: See if can use a different referent for the manipulatives.

– Can you show me a different way to solve the problem with the these [sweeping gesture over manipulative bin]?
– Is there a way you could use fewer of these to solve the problem? (Or "I don't want to use this many, can you should me how you can use less of these to solve the problem?)
– *If child says something like "he lost 2 cars,"* ask, "Can you show me the same story, the same problem, but with less/fewer of these?"
– Can you count with these to show me how many *[items from the problem]* there are?
– How many [*items from the problem*] are you showing me here [point to the mat]?
– How many *[items from the problem]* is this [point to one manipulative]?

4) Sticker Task

"To thank you for your hard work, I will give you some stickers. Put some pollies on the lid, and I will put the right number of stickers for you in the box. You just need to let me know what your secret code is for the pollies."

After the child answers, ask, "How many stickers do you think you will get? How do you know?

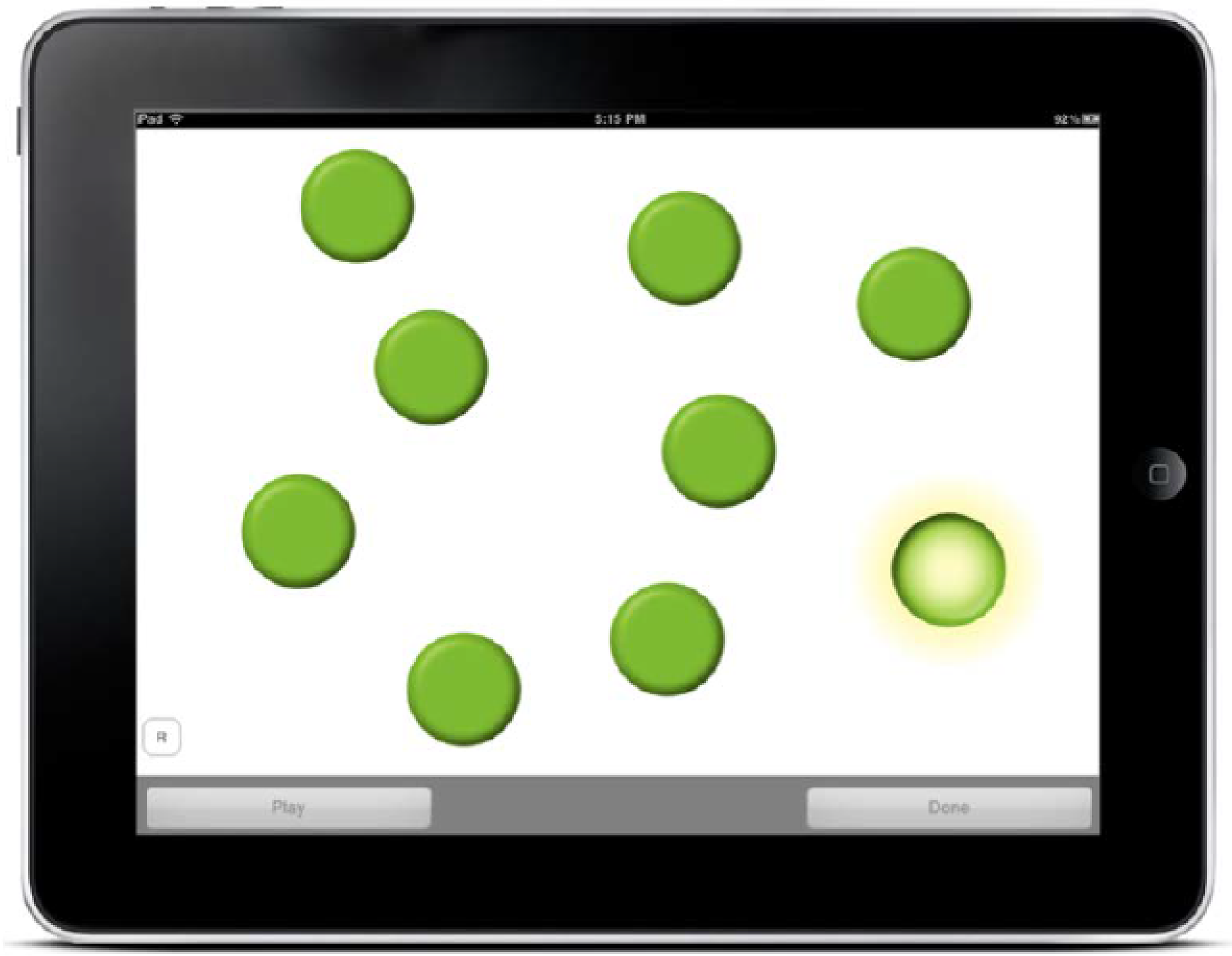
Take the box away and put a full sheet of stickers in it. Put the closed box in front of the child and tell him/her to open it.

Say (depending on the situation), "I really like how you used your secret code to tell me how many stickers you wanted:

- So you can have more than what you asked for!
- But I only have this many, but you can have all of them!

Appendix D

PathSpan



Appendix E

Go/No-Go

Appendix F

Preregistration

osf.io/jbpx9

OSF REGISTRIES

Help Donate

Costs and Benefits of Telling Children the Quantitative Meaning of Manipulatives

Embargoed registration

Overview
Files
Wiki
Components 0
Links 0
Analytics
Comments 0

Preregistration Template from AsPredicted.org

Have any data been collected for this study already?

Yes, at least some data have been collected for this study already

What's the main question being asked or hypothesis being tested in this study?

Study's main objectives: (1) Identifying the benefits and costs of directly telling students the quantitative referents for manipulatives compared to allowing them to construct meaning for the manipulatives in more open and exploratory learning environments. (2) Determining how different instructional conditions (i.e., direct instruction, guided exploration or free exploration) influence how children develop symbolic flexibility (i.e., ability to assign a new quantitative referent to a known target manipulative) and symbolic fluency (i.e., ability to assign a new quantitative referent to a new manipulative).

Research question #1: Are there group differences between students who received direct instruction, took part in guided exploration, and who were in the control group (free exploration) in terms of their performance on the Near-Transfer task, which assesses children's ability to use the target manipulative in the intended way in a context that differs from the encoding instruction context?

Research question #2: Are there group differences between students who received direct instruction, took part in guided exploration, and who were in the control group (free exploration) in terms of their performance on the Constrained Transfer task, which assesses symbol flexibility?

Research question #3: Are there group differences between students who received direct instruction, took part in guided exploration, and who were in the control group (free exploration) in terms of their performance on the Unconstrained Transfer task, which assesses symbolic fluency?

Describe the key dependent variable(s) specifying how they will be measured.

Performance on the Near-Transfer task. This task assesses whether children can use the target manipulative as intended during the encoding instruction in a different context (i.e., solving word problems). Students will be asked to solve two multiplication word problems using the target manipulatives. Scores will be computed by assigning a 1 if the students use the target manipulatives as intended, and a 0 if they do not.

Performance on the Constrained Transfer task. This task assesses children's symbol flexibility (i.e., their ability to assign a new quantitative referent to the same target manipulative). Students will be asked to solve four multiplication word problems using the target manipulatives. Scores will be computed by assigning a 1 if the students use the target manipulatives as new quantities that fit the groupings in the multiplication problems, and a 0 if they do not.

Performance on the Unconstrained Transfer task. This task assesses children's symbolic fluency (i.e., their ability to assign a new quantitative referent to a new, unfamiliar manipulative). Students will be asked to solve four multiplication word problems using manipulatives they choose from a bin containing various manipulatives (excluding the target manipulative). Scores will be computed by assigning a 1 if the students use the new manipulatives as new quantities that fit the groupings in the multiplication problems, and a 0 if they do not.

Contributors
Emmanuelle Adrien

Description
No description

Registration type
Preregistration Template from AsPredicted.org

Date registered
January 17, 2019

Date created
January 17, 2019

Registered from
osf.io/rfm3t

Category
Project

Publication DOI
No publication DOI

Subjects
No subjects

Affiliated institutions
This registration has no affiliated institutions

License
No license

Tags
No tags

Citation
osf.io/jbpx9

How many and which conditions will participants be assigned to?

3 conditions: 1) Direct instruction, 2) Guided exploration, and 3) Free exploration (control)

Specify exactly which analyses you will conduct to examine the main question/hypothesis.

Three separate one-way ANCOVAs with condition as the between-group variable, one for each dependent variable (i.e., performance on the Near-Transfer, Constrained Transfer, and Unconstrained Transfer tasks). In all cases, performance on working memory and inhibitory control tasks will be entered as covariates.

Any secondary analyses?

No.

How many observations will be collected or what will determine sample size? No need to justify decision, but be precise about exactly how the number will be determined.

Data have been collected from 72 participants. Nine participants were excluded because they did not encode the intended quantity for the target manipulatives after the encoding instruction. Data from 63 participants, then, will be used for the analyses. Of these, 19 are in the Direct instruction condition, 24 are in the Guided exploration condition, and 20 are in the Free exploration (control) condition.

Anything else you would like to pre-register? (e.g., data exclusions, variables collected for exploratory purposes, unusual analyses planned?)

DATA COLLECTION:
All the data have been collected for this study, but data pertaining to the research questions listed above have not been coded or analyzed yet. Data collection took place from April 10, 2018 to June 12, 2018.

PREDICTIONS:
Performance on the Near-Transfer task.
– Children in the Direct instruction condition will outperform children in the other two conditions.
– Children in the Guided exploration condition will outperform children in the Free exploration condition.

Performance on the Constrained Transfer task.
– Children in the Guided exploration condition will outperform children in the other two conditions.
– There will be no difference in performance between children in the Direct instruction and Free exploration conditions.

Performance on the Unconstrained Transfer task.
– Children in the Guided exploration condition will outperform children in the other two conditions.
– There will be no difference in performance between children in the Direct instruction and Free exploration conditions.