Concordia University

A History of the Master in the Teaching of Mathematics (MTM) Program 1967 – 2018

written by Anna Sierpinska,

with editorial help of Carol Beddard

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Abstract

The MTM – Master in the Teaching of Mathematics – program has been the oldest graduate program in the Department of Mathematics and Statistics of Concordia University, established in 1967, when Concordia was still the Sir George Williams University. Very quickly, however, the Department developed a Master of Science/Master of Arts in Mathematics program and, later, a PhD program in Mathematics. MTM survived about 50 years, until 2016 when admissions to the program were suspended, or until 2019 when the last MTM student graduated.

In this document, we describe the program, its mission, degree requirements and courses. We present the context of the program's creation in 1967 and the reasons for its suspension 50 years later.

We present also the people – faculty and students – who made the program what it was: a lively space for critical reflection on mathematics education research and practice, always open to trying novel approaches to teaching mathematics.

The tradition of critical reflection and trying different approaches to teaching mathematics has a chance to survive in the department after the suspension of the MTM program; MSc/MA students have the option of doing their research in the area of mathematics education.

The purpose of writing this historical piece is to keep the memory of the MTM program at Concordia alive, and to create an easily accessible reference for graduates of the program when they apply for jobs or further graduate studies.

The document has been written by the last director of the MTM program, Anna Sierpinska, with editorial help of her former student, Carol Beddard, an MTM 2012 graduate.

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The MTM Program at Concordia University

The MTM – Master in the Teaching of Mathematics – program has been the oldest graduate program in the Department of Mathematics and Statistics of Concordia University, established in 1967, when Concordia was still the Sir George Williams University. Very quickly, however, the Department developed a Master of Science/Master of Arts in Mathematics program and, later, a PhD program in Mathematics. MTM survived about 50 years, until 2016 when admissions to the program were suspended, or until 2019 when the last MTM student graduated.

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Description of the program; admission and degree requirements

The mission of the program

The webpage of the Department of Mathematics & Statistics describes the MTM program as follows[1]:

> *The Master in the Teaching of Mathematics (MTM) is an academic program in mathematics education. Its aims are threefold: to improve the professionalism of*

[1] This description of the program is the updated version from 2010-11 and it has not changed since 2012.

secondary school teachers; to prepare college mathematics teachers, and to prepare researchers in mathematics education.

MTM students are:

Exposed to advanced mathematics taught by professional mathematicians

Introduced to current theories, research methods and research results in mathematics education

Encouraged to reflect on and critically evaluate general and specific aspects of mathematical pedagogy at secondary and college levels

Trained in conducting research, presenting their research orally in a professional manner, and writing research reports in mathematics education

All students are encouraged to conduct research, but those who aim mainly at improving their teaching or preparing to teach at the college level are advised to focus on the mathematical content courses, and include three or more advanced mathematics courses within their program. Students who are interested in pursuing their studies at the doctoral level are advised to conduct a more disciplined and rigorous research and write a thesis, under the guidance of a supervisor and committee members. The topic and the research questions are developed together with the student, starting from his or her interests and own questions about the practice or theory of mathematics teaching and learning.[2]

Admission requirements

For admission requirements the webpage refers to the Graduate Calendar which states:

A Bachelor's degree with a minimum GPA of 3.00, an interest in the teaching of pre-university mathematics, as well as an adequate mathematical background including courses equivalent to: a) 6 credits in statistics-probability; b) 6 credits in advanced calculus; c) 6 credits in linear algebra and d) 3 credits in differential equations or algebraic systems. Candidates must be able to demonstrate their capacity for graduate level work in some academic field, not necessarily mathematics. Candidates will normally be interviewed to ensure their suitability for the program. Applicants with a deficiency in their academic background may be required to take up to 12 undergraduate credits in addition to or as a part of the regular graduate program. Promising candidates who lack the requirements for admission may be considered after having completed a qualifying program. Applicants without teaching experience may be admitted to the program provided they satisfy the Graduate Studies Committee of their potential for teaching or for educational research.[3]

[2] Text as appearing on September 13, 2019, at https://www.concordia.ca/artsci/math-stats/programs/graduate/teaching-of-mathematics-mtm.html

[3] https://www.concordia.ca/academics/graduate/calendar/current/fasc/mast-mtm.html

Degree requirements

Regarding the degree requirements, the Calendar further states:

> ***Credits.*** *A fully qualified candidate is required to complete a minimum of 45 credits.*
>
> ***Courses***
> *Students may enter one of the three options below. The choice of the option, the selection of the courses and the thesis or project topic must be approved by the Graduate Program Director. Besides the courses listed in the present section, Master/Magisteriate in the Teaching of Mathematics (MTM) students may take any MAST 600 or higher level course offered in the MSc program, subject to the Graduate Program Director's approval. Students aspiring to become College mathematics teachers upon graduation will be encouraged to take at least three MSc mathematics courses.*
>
> *Thesis Option: MATH 602, 647, 654 and eight additional 3-credit courses.*
>
> *Project Option: MATH 602, 603 and eleven additional 3-credit courses.*
>
> *Course Option: Fifteen 3-credit courses.*[4]

The MTM courses

In the 2019-20 Graduate Calendar, the MTM course descriptions have not changed since 2012:

> *MTM courses fall into six categories:*
>
> 1. *Psychology of Mathematics Education (PME): MATH 630, 649.*
> 2. *Didactics of Mathematics (DM): MATH 624.*
> 3. *Information and Communication Technology (ICT): MATH 633, 634, and 639.*
> 4. *Research in Mathematics Education (RME): MATH 641, 642, 645, and 646.*
> 5. *Mathematics content courses (MC): MATH 601, 613, 616, 618, 621, 622, 625, 626, 627, 637, 640, and 648*
> 6. *Thesis or Extended Project (T/P): Seminar MATH 652; Reading courses MATH 602 and 647; Extended Project MATH 603, and Thesis MATH 654.*
>
> *Each year the Department of Mathematics and Statistics offers a selection of the following courses. Courses are worth 3 credits unless otherwise indicated. [Topics courses all have the same "Note" as MATH 601; we do not repeat it for each subsequent course here, to save space. In the Calendar the Note was repeated.]*

[4] https://www.concordia.ca/academics/graduate/calendar/current/fasc/mast-mtm.html

MATH 601 Topics in Mathematics

Note: The content varies from term to term and from year to year. Students may re-register for this course, provided the course content has changed. Changes in content are indicated by the title of the course.

MATH 602 Readings in Mathematics Education I

This reading course is closely related to the project or thesis. The outcome is a section of the literature review chapter, related to the domain of research that is the focus of the project or thesis.

MATH 603 Extended Project (9 credits)

A student investigates a mathematics education topic, prepares a report, and gives a seminar presentation under the guidance of a faculty member.

MATH 613 Topics in Number Theory

Topics are chosen from the area of Number Theory.

MATH 616 Linear Algebra

This course is an extension of undergraduate courses in linear algebra, covering a selection of topics in advanced linear algebra (e.g. from the theory of general vector spaces, linear and multilinear algebras, matrix theory, etc.)

MATH 618 Topics in the Application of Mathematics

Topics are chosen from the area of the Application of Mathematics.

MATH 621 Geometry

The course offers an insight into Euclidean and Non-Euclidean geometries.

MATH 622 Abstract Algebra

The course looks at objects such as numbers, polynomials, matrices or transformations from an algebraic-structural point of view. The course may aim at proving such “famous impossibilities” as squaring the circle, duplicating the cube, trisecting an angle or solving a polynomial equation of degree 5 or more by radicals.

MATH 624 Topics in Mathematics Education

This course is an overview and critical analysis of theories and technologies of mathematics teaching. Applications of the theories to studying and/or developing teaching situations or tools for specific mathematical topics are examined.

MATH 625 Topology

The course develops elements of the theory of topological spaces and their transformations.

MATH 626 Analysis I

The course is an extension of undergraduate courses in mathematical analysis in the real domain (Analysis I, II; Real Analysis; Measure Theory). Students may substitute this course with any of the MAST 660-669 courses in the MA/MSc program.

MATH 627 Analysis II

The course is an extension of undergraduate courses in mathematical analysis in the complex domain (Complex Analysis I, II). Students may substitute this course with any of the MAST 660-669 courses in the MA/MSc program.

MATH 630 Topics in the Psychology of Mathematics Education

This course studies epistemological, cognitive, affective, social and cultural issues involved in mathematics.

MATH 633 Applications of Technology in Mathematics Curriculum Development

This course is an overview of the impact of information and communication technology on curricula, textbooks and teaching approaches.

MATH 634 Computer Software and Mathematics Instruction

This course is an overview and critical evaluation of computer software designed for use in mathematics instruction.

MATH 637 Statistics and Probability

This course discusses theoretical and applied aspects of statistics and probability. Students may substitute this course with any of the MAST 670-677 courses in the MA/MSc program.

MATH 639 Topics in Technology in Mathematics Education

This course involves the elaboration, experimentation and critical analysis of individual projects of integration of ICT in mathematics education.

MATH 640 Topics in Logic

Topics are chosen from the area of Mathematical Logic.

MATH 641 Survey of Research in Mathematics Education

This course is an overview of recent results in mathematics education research.

MATH 642 Research Methods for Mathematics Education

This course is an overview of qualitative and quantitative methods in mathematics education research.

MATH 645 Topics in Mathematics Education Research

This course is an overview of research literature on a chosen topic or issue in mathematics education.

MATH 646 Research Internship

Students conduct a pilot study or participate in a research project as a research assistant under the supervision of a senior researcher. The outcome is a written report of the study.

MATH 647 Readings in Mathematics Education II

The course is closely related to project or thesis writing. Its outcome is a section of the literature review chapter, focused on the student's particular research question.

MATH 648 Topics in the History of Mathematics

Topics are chosen from the area of the History of Mathematics.

MATH 649 Heuristics and Problem Solving

This course examines cognitive processes, tools and strategies involved in solving mathematical problems.

MATH 652 Seminar in Mathematics Education

This course is primarily a thesis or project preparation seminar but it is open to students in the Course Option as well. The research related to students' research projects is presented and critically evaluated.

MATH 654 Thesis (15 credits)

Students are required to demonstrate their ability to carry out original, independent research. The thesis is researched and written under the direction of a supervisor and thesis committee. Upon completion of the thesis, the student is required to defend his/her thesis before the thesis committee."[5]

[5] https://www.concordia.ca/academics/graduate/calendar/current/fasc/mast-mtm.html

The present (2019-20) status of the MTM Program

At the time of writing this historical piece (2019-20), the MTM Program is "suspended". Admissions have been suspended since January 2016, the last MTM course was offered in the fall of 2018, and the last MTM student graduated in spring 2019. The information on the Mathematics & Statistics department's web page now reads:

> *Teaching of Mathematics (MTM)*
>
> *Note: Admissions have been suspended as of January 2016*
>
> *Candidates interested in Mathematics Education research are advised to apply to study this area within the MSc program in the Department or in the Concordia Individualized Program (INDI). For further information, please contact the Program Director.*[6]

So, the MTM program has been suspended but the area of research in mathematics education at the graduate level has been maintained within the department. The major change that this "merger" of the MTM program with the MSc/MA program entails, is that candidates for studying mathematics education must have an undergraduate degree equivalent to a BSc or a BA in mathematics (with a GPA of at least 3); for admission into the MTM program. Previously, it was enough to have 21 credits (with GPA≥ 3) in undergraduate mathematics (6 credits for Linear Algebra I & II, 6 credits for Multivariate Calculus I & II, 3 credits for Probability, 3 credits for Statistics and 3 credits for either Abstract Algebra or Differential Equations).

Graduates of the MTM program who did not have a full bachelor's degree in mathematics had trouble completing more advanced courses in mathematics and consequently had poor chances to be hired in CEGEPs. It is the policy of some CEGEPs to hire people for mathematics teacher positions who have obtained an MSc in Mathematics or those who have at least passed the mathematical analysis sequence of courses (Analysis I and II, Real Analysis, Measure Theory and Complex Analysis). The MTM degree did not require these courses.

Also, if an MTM student did not already have a teaching certificate, the MTM degree did not allow them to obtain it. In Quebec, only departments of education are authorized to offer programs leading to teaching certificates.

This was one of the reasons why we decided to merge the MTM with the MSc program: to improve the chances of employment of our graduates.

At the beginning of MTM program's existence, its clientele were mostly certified secondary school math teachers, but by the mid-90s, fewer and fewer practicing high school teachers were applying to the MTM program. One of the reasons for this could be changes in the laws governing the remuneration of teachers and professional development[7]. Salaries started to

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[6] https://www.concordia.ca/artsci/math-stats/programs/graduate/teaching-of-mathematics-mtm.html

[7] Education Act can be found at http://legisquebec.gouv.qc.ca/en/ShowDoc/cs/I-13.3

depend less upon the number of educational credits and more on seniority[8] – so it was no longer profitable to take courses at the university in terms of salary increase.

Moreover, under Pauline Marois' leadership as minister of education in Lucien Bouchard's government (1996-2000), an undergraduate degree in a teachable subject and a one year course in pedagogy plus a course in the Quebec educational law became, by law, not enough to get an authorization to teach at a public high school. A full 4-year BEd program with 600 hours of classroom stages at schools was required[9], but it required only 21 credits in undergraduate mathematics. People who had already spent 3-4 years in a BSc or BA program in mathematics, were not enthusiastic about going back to undergraduate studies to get their teaching diploma. They preferred enter a graduate program, either MSc or MTM in order to pursue a career in a CEGEP.[10]

But, with increasing availability of MSc or even PhD mathematicians on the CEGEP job market, it became more and more difficult for an MTM graduate to obtain a mathematics teaching position at a CEGEP. Hence our decision to merge the two programs together gives equal employment prospects to graduates with an interest in all areas of mathematical research, including research in mathematics education.

[8] https://www.payscale.com/research/CA/Job=High_School_Teacher/Salary/3c54ae3e/Montr%C3%A9al-QC
[9] See http://www.education.gouv.qc.ca/en/teachers/teaching-in-quebec/teaching-authorizations/
[10] Presently, some education departments of universities in Quebec offer "professional master's degrees" for people with Bachelor's degrees in "teachable subjects".

The context of the MTM program creation

Listed below are the conditions which accompanied the creation of the MTM program:

1. The process of departmentalization in the SGW University in the 1960s; "by 1962… [a] growing tendency of departments to think in isolation of their own particular needs, and… rivalry between departments for priority" was observed (Clarke, 1977, p. 140). Departments wanted to develop programs (graduate and undergraduate) to suit their particular interests and ambitions. There was a need for a body that would overview and coordinate these various initiatives: "Faculty Council felt that there was a 'need for a planning body re academic programmes, to be looked at from a university rather than a departmental point of view'." (ibid.)

2. Establishment of the "University Committee on Academic Planning" (UCAP) in 1963; any new programme had to be approved by this Committee.[11]

3. By the end of 1963, the development of graduate programs became the main topic of discussion in the UCAP. The Chairman's position was that the creation of graduate programs would be beneficial to the university as a whole in that "the establishment of such programs would inevitably lead to the appointment of specially qualified scholars…" (Chairman's speech quoted in (Clarke, 1977, p. 143)) He also proposed that master's degree programmes should be well established first; doctoral programs were to develop as their "natural extension" (ibid.)

4. Department of Chemistry considered the possibility of offering a master's program leading to an MSc degree in Science Education; this was in response to a request by the Canadian Association of Chemistry Teachers (established in 1958) which claimed that this degree "would serve as a very useful academic qualification for Science teachers in the High School system" (Clarke, 1977, p. 144). But they were later (after the MTM was proposed) to change their mind and propose instead a program "leading to the degree of Master of Science in Chemistry with a teaching option" (ibid.).

5. In years 1963-4, the 5 volumes of the Report of the Commission Parent[12] were published. It recommended the creation of two-year "institutes" or CEGEPs and UCAP became "involved in studying the implications of that report" (Clarke, 1977, p. 146) . One of the implications was that the undergraduate program at the university could be reduced from 4 years to 3 years.

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[11] The UCAP ceased to exist in 1969 and was replaced by the "University Committee on Academic Planning, Priorities and Budget." (Clarke, 1977, p. 148) This committee was too large to get any decisions made and, by 1972, it was declared a failure.

[12] Information on the Report of the Commission Parent can be found at http://www.banq.qc.ca/a_propos_banq/publications/a_rayons_ouverts/aro_94/aro_94_dossier.html There is a photograph of the members of the Commission at https://www.actualites.uqam.ca/2013/le-rapport-parent-un-document-fondateur

Universities could get rid of college level prerequisite courses. The problem was however that "though a number of French CEGEPs had been promptly set up, replacing the classical colleges, the lack of such institutions on the English side meant a long delay in setting up English language CEGEPs. In 1967 there were none on the horizon." (ibid.) SGW maintained its 4-year undergraduate program with some courses classified as "collegial", or belonging to "collegial programs", officially established in 1969 (ibid.) and planned for it to last 5 years, to be closed in 1974.[13] MTM contributed to the creation of English CEGEPs by training teachers of college level mathematics.

6. However, the Commission Parent also postulated (section 132) that SGW, the University of Bishop's College and the University of Sherbrooke be reduced to undergraduate programs only. SGW contested this postulate strongly. Here below is the relevant excerpt from Clarke's account of the coming into being of Concordia University (Clarke, 1977, p. 154).

> *Shortly thereafter, the Principal, Dr. Robert C. Rae, reminded the Board of Governors 'that we had been planning the release of a brochure on four programs of graduate studies at the time at which the Commission Parent report was published. He emphasized that we are concerned with the principle of the right and the need of the University to grant graduate studies rather than with the four proposed programs, and reviewed the reasons why these are needed at the University, particularly to keep alive scholarly activity of professors, to keep on the staff, and to attract the first class professors, and to upgrade and stimulate the work done in the undergraduate faculty by their contacts with advanced students and instructors currently occupied in research at the graduate level. He told the Board that he had sent a letter to the Minister drawing his attention to the problem and soliciting his response. He had not been able at that time to arrange a meeting with the Minister, but the Minister had recently made a statement on the television that the Department of Education had no intention of following the Report to the letter and assuring the public that nothing will be done in haste'.*

Finally, SGW and the other universities singled out by Com. Parent were allowed to keep developing their graduate programs. But SGW decided to proceed more slowly and cautiously, deferring the proposals of two MSc programs (in Physics and Chemistry) for a couple of years. The MTM program was created in 1967 (Clarke, 1977, pp. 156-7). It was created before the MSc/MA program in mathematics (1972). The department of Chemistry, on the other hand, first established a MSc in chemistry (1966) and only later (1973) added the teaching option, which, as mentioned, did not last very long.

7. In the SGW University there was no department of education until 1968. There were only some courses in e.g., philosophy of education, adult education, etc. (Clarke, 1977, p. 126). Starting from the early 60s, the university was receiving "briefs and requests from school boards and teachers' organizations asking the University to do a number of things for teachers

[13] However, in mathematics, college level courses are offered at Concordia to this day and constitute a good source of income for the University.

ranging from the provision of special courses and programs to the setting up of a Faculty of Education which would be an alternative choice, for student teachers, to McGill's Macdonald College, and in direct competition with it. (Clarke, 1977, p. 126).

The Department of Mathematics responded to the appeal by proposing, in 1966, the Master in the Teaching of Mathematics [MTM] program to be run by the department of mathematics; the department of education did not exist at that time. There were plans to establish such a department and in 1966 the University hired a professor, Dr. John Macdonald, and entrusted him with the task of setting it up. The department of education was finally established in 1968, but the beginnings were difficult, with decisions about which programs to develop delayed at different administrative levels for various reasons. Based on information gleaned from Clarke's book, it seems that programs focused on the teaching and learning of particular school subjects (mathematics, science, language, art) were not among the range of possibilities. So, the MTM program remained within the Department of Mathematics.

8. In 1967, the "Provincial Association of Protestant Teachers presented a brief to Sir George Williams [University] with a series of recommendations asking the University to set up a variety of programs, both graduate and undergraduate, for teachers. This brief was extensively studied by the University Academic Planning Committee [UAPC], where three possibilities seemed to emerge: '(1) … set up a full scale operation in the field of education; 2)… pick up items from among recommendations one by one and in stages; 3) institute more graduate programs which would relate more closely to the needs of teachers and eventually set undergraduate programs for the Bachelor of Education Degree' (UAPC minutes, March 7, 1967). (Clarke, 1977, pp. 126-7)

9. The New Math reforms of the 60s created a need for professional development of practicing teachers in logic, abstract algebra, geometry (especially non-Euclidean geometries), topology, etc. This explains the inclusion of courses on these subjects in the MTM curriculum.

The creation of the MTM program

The MTM program was established already in Sir George Williams University. In November 1966, the Department of Mathematics submitted to the Board of Graduate Studies a proposal for the "Master in the Teaching of Mathematics" program. The Board accepted the proposal and recommended its adoption by the University Council. The program was scheduled to be started, and indeed was, in September 1967. The first batch of MTM graduates (11 in total) appeared in the fall 1969 Convocation booklet.

The Proposal

The text of the Proposal was obtained from the University archives. We quote it below:

> *General*
>
> *(a) The Mathematics Department of Sir George Williams University has come to the conclusion that there is an urgent need for a programme of training and re-training of practicing teachers of mathematics. Such a programme would help to increase the number of teachers able to judge and implement necessary changes in the secondary school curriculum[14]. The programme would also train prospective instructors for the new Institutes[15].*
>
> *(b) Existing honours and graduate programmes in mathematics are ill-suited to this purpose. These are designed for the training of research mathematicians and other specialists and consequently tend to be deep but somewhat narrow. The teacher of mathematics requires training in breadth free from inessential technical detail.*

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[14] Allusion to the New Math curriculum reform. Many new textbooks and instructional materials were being published, not all good and some looking like caricatures of advanced mathematics and of sensible mathematical instruction. Characteristic of those reforms were the textbooks and materials written by Georges and Frédérique Papy ("Mathématique Moderne"). Information about the Papys can be found at https://fr.wikipedia.org/wiki/Georges_Papy and also at https://www.eimacs.com/blog/2012/01/georges-papy-mathematics-educator-gifted-math-curriculum/. Hans Freudenthal was vehemently critical of the New Math reforms and his book "Mathematics as an Educational Task", which contains much of his critique, was a must read for anybody interested in mathematics education in the 1960-1990. See a review of this book by David Tall (Tall, 1977). Later there were many caricatures of the New Math circulating in the form of jokes. For example, here is one such joke, distributed, through MAA, in 1999, March 2, and sent by a Samuel S. Kutler, titled "Is mathematics teaching a function of time?":
"Teaching math in 1950: A logger sells a truckload of lumber for $100. His cost of production is 4/5 of the price. What is his profit?
Teaching math in 1960: A logger sells a truckload of lumber for $100. His cost of production is 4/5 of the price or $80. What is his profit?
Teaching math in 1970: A logger exchanges a set 'L' of lumber for a set 'M' of money. The cardinality of set 'M' is 100. Each element is worth $1. Make 100 dots representing the elements of the set 'M'. The set 'C', the cost of production contains 20 fewer points that set 'M'. Represent set 'C' as a subset of the set 'M' and answer the following question: What is the cardinality of the set of profits?"

[15] "Institutes" is the name that was used to refer to the CEGEPs at the time of writing the proposal.

(c) At least 175 American institutions offer Master's degrees specifically designed for teachers of mathematics (including Cornell, Harvard, and Yale) and 25 American universities offer Doctorates in this field.

II. Beginning of programme

The programme is scheduled to begin in September 1967.

III. Curriculum

This programme, both in aim and content follows closely the recommendations of the Mathematical Association of America for the training of teachers of mathematics[i] [see (Hardgrove, 1963), (The Mathematical Association of America, 1960)]. In the opinion of the Department, the student will reach the level of mathematical competence necessary for instructors in the projected Institutes and for leaders in the teaching of high school mathematics. The mathematical prerequisites besides being essential for graduate work also represent the type of background desirable for every teacher of high school mathematics.

(a) Entrance Requirements

Candidates for admission to the programme must have an undergraduate degree, a teacher's certificate, and at least one year's experience in the teaching of secondary school mathematics, as well as an adequate mathematical background up to and including:

(i) A full course in Statistics-Probability

(ii) A full course in Intermediate Analysis

(iii) A full course in Algebra involving a study of fundamental algebraic systems.

Candidates with inadequate background will be required to take the necessary undergraduate courses[16].

[16] By 1990, the requirements of a teaching certificate and experience teaching high school mathematics were dropped.

The struggle to keep MTM alive

Several times throughout its history, the need for the MTM program has been questioned. The first time was in 1973/74, when CEGEP teachers either already had the MTM degree, or did not need it. More information about this time is contained in the interview with Joel Hillel.

Another time was in 1996/97. The movement towards the decrease of governmental spending on education, started in the 1980s, (Ratel & Verrault-Julien, 2006) arrived to its apogee, in Quebec, under the government of Lucien Bouchard, in 1994-2000. Bouchard's goal as prime minister of Quebec, was to bring the finances of the province to "zero deficit". In working towards this goal, Bouchard was greatly helped by Pauline Marois, who became minister of education in his government, after being minister of finance in Jacques Parizeau's government (1992-1995). Universities of Quebec were faced with the consequences of serious cuts to their governmental financing. They had to come up with ways to increase private funding and ways to save money. At Concordia, the then Vice-Rector, Academic, Jack Lightstone, proposed, among other things, to cut academic programs. In a document titled, "Our Immediate Future: Planning Proposals for Adoption in the Winter of 1997" presented to the Senate on February 7th, 1997, he lists the programs that, in his opinion, should be cut (Black, 1997). MTM is named on this list, with the following annotation: "New registrations, FTEs per faculty member and the number of graduates are low. Admissions to the MTM should be suspended".

Joel Hillel, who was Chair of the Department of Mathematics and Statistics at this time, reacted immediately in a letter addressed to Lightstone:

> *Presently, 27 students are enrolled in the MTM program. Quite a few of these students are part-time, but this is in accordance with the definition of the program: MTM is a program for in-service mathematics teachers. It has been classified by the Quebec Ministry of Education as 'un programme de perfectionnement'. In-service teachers can, of course, only be part-time students. We have now 9 practicing teachers in the program. Graduations in the MTM are going at a fast pace: there were 6 graduates in 1996 and we expect 7 this year[17]. New students have been coming: 9 in 1995, 8 in 1996 and January 1997. The program survives on the shoulders on virtually one faculty member who also teaches undergraduate mathematics courses and is the program's director.*
>
> *On the basis of mere numbers, it is incomprehensible why the 'Our immediate future' document recommends that 'Admissions to the MTM program should be suspended'. Of course, if the program is closed, the program director[18] may want to leave which will save the university some salary money (and make it lose some grant money, too).*

[17] In fact, there were 3 graduates in 1997 but 5 more graduated in the spring of 1998.
[18] Anna Sierpinska was the MTM program director at this time.

> *However, she will not have to leave, because she also teaches mathematics and has tenure.*
>
> *Suspending admissions to the MTM is tantamount to a destruction of a well-established graduate program for which Concordia is renowned. (…) (Joel Hillel to Jack Lightstone, February, 1997)*

This proposal of suspending the program came as a surprise to us, because in 1996 we had the approval of the Dean of Arts & Science, Gail Valaskakis, to go ahead with the hiring process of a new faculty member specializing in mathematics education. In January 1997, the process was very advanced, and an offer of employment to a chosen candidate was about to be issued and signed by the Dean. However, it wasn't and we were told that there will be no hiring for the time being.

Joel Hillel managed to save the program by his arguments, and by soliciting support letters from colleagues, mathematics educators from around the world, to be send to the Vice-Rector or to the Dean of Arts & Science. The following colleagues sent letters of support:

Nicolas Balacheff	Colette Laborde
Jean Dionne	Richard Noss
Jean-Luc Dorier	Renan Samurçay
Tommy Dreyfus	Heinz Steinbring

The letters are reproduced in the Appendix, at the end of this document.

The MTM program continued to exist on the shoulders of two faculty specializing in mathematics education. Until 2009 these were Joel Hillel and Anna Sierpinska. After Joel Hillel retired in 2009 until 2018, these were Anna Sierpinska and Nadia Hardy, hired in 2009. Anna Sierpinska retired in 2018. After 2 years of service as chair of the Department, 2014-16, Nadia Hardy became Vice-Provost and could no longer teach any regular courses. There were no full-time faculty to support the program. In January 2016, admissions to the MTM program were suspended. The year 2019 saw the last MTM student, Mandana Mir, graduate.

Faculty associated with MTM – An overview

This section and the next three speak about faculty members who contributed to the MTM program. The first is an overview of the kinds of contributions made by members of the mathematics (and statistics) department at Concordia. The second lists contributions made by visiting professors and postdoctoral fellows. The third contains personal accounts of four department members: Fred Szabo, Bill Byers, Joel Hillel and myself (Anna Sierpinska). The fourth gives some more information on D. A. Wheeler, N. Herscovics, S. H. Erlwanger, and N. Hardy.

Over the years, many department members made contributions to the MTM program, mostly by teaching mathematics courses or designing special courses, keeping in mind the special (and changing) clientele enrolled in the program. A smaller group of faculty members was more involved with the MTM, helping to define and, over time, change the goals of the program, change the course offerings and the content of some courses, act as Program Director, etc. At the beginning, among this latter group were **Norm Smith** and **Martin Harrow**. In Joel Hillel's memory,

> *Norm Smith was critical to the establishment of the program. He was the Chair of the department when the MTM was established and certainly taught linear algebra in the program when I first joined. (Joel Hillel, September 28, 2020, email communication)*

Joel Hillel remembers also that the first MTM program director was Martin Harrow. Martin Harrow, like Norm Smith, was a mathematician (with a PhD in mathematics from McGill) for whom mathematics education was a field of practice, not a domain of scholarly research. The goal of MTM was to improve the knowledge and understanding of mathematics of secondary school and CEGEP teachers, and the way to achieve this was to teach them more mathematics. Mathematicians in the department were busy designing courses in mathematics for MTM students. Hal Proppe remembers that Martin Harrow "occasionally taught differential equations and… advanced calculus adapted to the MTM students"[19]. In the interviews with Fred Szabo and Bill Byers, there will be examples of some of the challenges that this task represented.

By the time **Joel Hillel** became the program's director in 1973, the new cohort of MTM students were demanding more pedagogically oriented courses. The department dealt with this demand in two ways.

One approach was to offer a few intermediate graduate programs, certificate and diploma, for practicing teachers of mathematics, where teachers improved their knowledge of mathematics

[19] E-mail to A. Sierpinska dated 14 October 2020.

for teaching by reflecting on their practice and taking a few mathematics courses. Activities took place in schools where the teachers worked. So, for example, in 1980, there was a Certificate in Mathematics for Teachers – Elementary School Level, and a Certificate in Mathematics for Teachers – Junior High School level (both 15 credits), and there was a Diploma in the Teaching of Mathematics (30 credits), besides the 45-credit Master in the Teaching of Mathematics [20]. The Junior High School Certificate was directed by **Mary** A. **Brian**[21] who was a member of the department of mathematics during 1961-74. **Alberta Boswell** directed the elementary certificate. In the mid-seventies, the Diploma program was directed by **Nicolas Herscovics**[22]. It produced (at least) 14 graduates, the first ones in 1976 and the last ones in 1984. Their names are listed after the MTM graduates, at the end of this document. Joel Hillel remembers also that "the Diploma Program, aside from being a complete package, also served as a kind of 'qualifying year' for the MTM. Students who took the necessary math courses in the Diploma could then be admitted to the MTM without completing the Diploma."[23]

The other way was to seek expertise in what came to be called later on "specialized knowledge of mathematics for teaching" (Ball, Thames, & Phelps, 2008), at the scholarly level, through research and writing, so that it could be formulated and communicated to others, in publications and in those "pedagogically oriented" mathematics courses that the MTM students demanded.

If the development of specialized knowledge of mathematics for teaching was to become one of the goals of a master's program, this required hiring experts in the area. This is not to say that mathematicians in the department didn't possess this knowledge – Norm Smith's often tested students' understanding by posing questions that required going beyond mechanical understanding and Joel Hillel had a talent for inventing such good questions, too – but they may have possessed this knowledge at an intuitive level, and perhaps were not ready to formulate it and communicate to others.

In the interview, Joel Hillel tells the story of how **David Wheeler** – a mathematics educator who certainly had the knowledge and the language to think critically and speak about mathematics teaching and learning – ended in the department. David Wheeler was hired in 1976. His presence in the department seemed to have a stimulating effect on others. There were some members of the department interested in mathematics education already. For example, **Victor Byers**, who joined the department of mathematics of SGWU in 1961, was its chair in 1969-1975,

[20] According to a list of graduates in the fall of 1980 published in Concordia's Thursday Report November 13,1980 at: https://www.concordia.ca/content/dam/concordia/offices/archives/docs/concordia-thursday-report/1980-81/Thursday-Report-1980-November-13.pdf

[21] More information about Mary Brian's activity at Concordia University can be found in a piece written about her in one of Concordia's journals, titled "In Memoriam: Mary A. (McWright) Brian", at http://cjournal.concordia.ca/journalarchives/2005-06/jun_1/007075.shtml.

[22] As remembered by Joel Hillel (e-mail communication on 24.10.2020).

[23] ibid.

and received a PhD in mathematics from McGill in 1969. His first publication in mathematics education appears to be the one co-written with **Nicolas Herscovics**, published in 1977, titled "Understanding school mathematics" (Byers & Herscovics, 1977)[24]. Nicolas Herscovics had been a member of the department since 1965 but did not have a PhD. He decided to become a professional mathematics educator and went on to obtain a PhD in this area.[25] In 1980 **Stanley Erlwanger** was hired. Stanley Erlwanger collaborated well with others. He co-wrote two papers with Victor Byers for *Educational Studies in Mathematics*[26] and four[27] with Josef Brody. **Josef Brody** is a mathematician (hired in 1968, now retired) with a PhD in automata theory who, in the 1980s, developed a passion for the "computational approach" to teaching calculus and has been working on it ever since. I remember him teaching about this approach in one of the MTM courses.

In early 1980s, **Joel Hillel** worked with David Wheeler on problem solving, and then became interested in the use of computers in mathematics education.

As already mentioned before, when **Joel Hillel** became the MTM Program Director (in 1973), there were very few non-mathematical content courses in the MTM program. Hal Proppe remembers that when he joined the department in 1970, MTM had a course on pedagogy taught by **Fred Bedford, "**who was a full professor in our department but served as Assistant Dean in the Faculty of Science until about the time of the merger" (H. Proppe, e-mail, 14 October, 2020). The course was "very popular with the students" (ibid.). Fred Bedford was probably not teaching this course anymore when Joel Hillel became the MTM Program Director, as he (JH) says, "[t]he only non-mathematical content course given in the MTM program that I remember was *History of Mathematical Ideas*, taught by **Paul Rosenbloom** from the Teacher's College of Columbia University."(Joel Hillel, interview, October 17, 2019).

Interests in mathematics education research developed in the department in the late 1970s and 1980s – problem solving, study of processes of understanding and learning mathematics, computational aspects of mathematics and use of computers in mathematics education – resulted in addition of new courses. In particular, the courses:

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[24] The paper refines the famous distinction between relational and instrumental understanding made by Richard Skemp to include intuitive and formal types of understanding in mathematics.

[25] There is more about Nicolas Herscovics in a section devoted to him later on in this document.

[26] S.H. Erlwanger, V. Byers, (1984), Content and Form in Mathematics, *Educational Studies in Mathematics*, 15(3), 259-275

S.H. Erlwanger, V. Byers, (1985), Memory in Mathematical Understanding, *Educational Studies in Mathematics,* 16(3), 259-281

[27] The four papers were published in *International Journal of Mathematical Education in Science and Technology*, as four parts of a larger work, titled "A calculator-based computational approach", in 1987, 1988, 1990 and 1991. Part 1 was on "Linear functions and equations"; part 2 – on "Quadratic functions and equations"; part 3 – on The Differentiation", and part 4 was "On differences and graphs". The first three parts were signed by Erlwanger and Brody; the fourth also by Rosenfeld.

MATH 630 Topics in the Psychology of Mathematics Education

MATH 633 Microcomputers: Applications to the Mathematics Curriculum[28]

MATH 634 Computer Software and Mathematics Instruction

MATH 636 Topics in Computational Mathematics[29]

MATH 639 Topics in Computer Education[30]

MATH 649 Heuristics and Problem Solving

The course on Psychology of Mathematics Education was first taught by David Wheeler (according to Joel Hillel's memory), and, in the 1980s, probably also by Nicolas Herscovics and Stanley Erlwanger. Anna Sierpinska taught it later on. Some of the computer related courses were taught by Fred Szabo. Joel Hillel taught the course on heuristics and problem solving.

(*Back to MTM curriculum changes*) Moreover, there was a belief, fueled by annual conferences of the International Group for the Psychology of Mathematics Education that mathematics education can evolve into a disciplined scientific inquiry with its own specific theories and research methodologies. If MTM was to contribute to this evolution, there was a need to introduce courses on research methodologies for mathematics education, and acquaint the students with research literature. So, courses focused on research were added:

MATH 641 Survey of Research in Mathematics Education

MATH 642 Research Methods for Mathematics Education

MATH 645 Topics in Mathematics Education Research

MATH 646 Research Internship

Added were also two reading courses, associated with writing a thesis (MATH 654 *Thesis*) or a project (MATH 603 *Extended Project*):

MATH 602 Readings in Mathematics Education (Reading Course)

MATH 647 Topics in Mathematics Education (Reading Course)

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[28] Note the now obsolete word "microcomputers" in the name of the course. It was used in 1970s and 80s mainly and later was replaced by "PC" (personal computer). Yet this name of the course survived in the Concordia graduate calendars until 2004. The course was then renamed "Applications of Technology in Mathematics Curriculum Development".

[29] The course MATH 636 Topics in Computational Mathematics was removed from the MTM curriculum in 2012. We believed this course could be replaced by MAST 334 Numerical Analysis cross-listed with MATH 601 Topics in Mathematics.

[30] In 2005, the course MATH 639 Topics in Computer Education was renamed "Topics in Technology in Mathematics Education". It was taught mainly by Fred Szabo, who covered several topics over the years. For example, in the winter 2017, the topic was: Visualization as an instructional tool.

MTM theses were supervised mainly, but not solely, by department members who published in the area of mathematics education. Reviewers could be recruited from among members not necessarily working in this area. Some Extended Projects were supervised by researchers in mathematics education and some by those in different areas of mathematics or statistics. There were many people who were involved in the supervision of MTM students; let me mention a few. I found a record of one MTM thesis supervised by **Victor Byers**[31]. **Harald Proppe** participated in the supervision of a few MTM projects and was frequently solicited to review theses in mathematics education. An example of such participation was when he taught a reading course on complex functions to an MTM student interested in the Riemann hypothesis. The student then wrote a project titled "On the Riemann Zeta function and the consequences of its truth" under the supervision of **Hershy Kisilevsky**. **Fred Szabo**, besides supervising MTM projects, was a reviewer for a few MTM theses and **Syed Twareque Ali** also reviewed several MTM theses.

Galia Dafni and **Alina Stancu** also reviewed theses in mathematics education but these were written by students who were enrolled in MSc or PhD in mathematics programs rather than in MTM. When MTM suspended its admissions in January 2016, several MTM students chose to interrupt their studies in this program and apply to the MSc in mathematics program, transferring some of the credits they had accumulated in MTM to MSc. Galia Dafni was a reviewer of the master's thesis of one of these students and Alina Stancu is supervising the research of another.

Mohammed A. Malik[32] supervised projects on the history of mathematics. **N.T. Srivastava** – on applications of mathematics in statistics and on applications of statistics in social sciences. **Sun Wei** directed a project on the history and current trends in Operations Research.

(*Back to curriculum changes*) The course called "History of Mathematical Ideas" became

MATH 648 Topics in the History of Mathematics

It was taught by mathematicians in the department interested in the history of mathematics. Among these were **Mohammed A. Malik**, **Alexander Shnirelman** and **John Harnad**. These professors were not professional historians of mathematics, but mathematicians with deep knowledge and understanding of vast areas of mathematics and their interconnections, both conceptual and historical. Occasionally, a historian of mathematics from another university was hired to teach this course. This was the case of Alexei Volkov[33]. **Bill Byers** and **Hershy Kisilevsky** designed an undergraduate course MATH 215 *Great ideas in mathematics* and taught it

[31] The student's name was B.P. Kirlin, graduated in Fall 1979, and the thesis was titled "Set theory in high school: an error in timing?"

[32] See his obituary in March 15, 2001 Thursday Report, signed by Joel Hillel, at https://www.concordia.ca/content/dam/concordia/offices/archives/docs/concordia-thursday-report/2000-2001/Thursday-Report-2001-March-1.pdf

[33] More about A. Volkov in the section about "Visiting professors…".

together, in the form of a dialogue. MTM students could take this course for credit as MATH 648 provided they did some extra work at the graduate level for the course.

Mathematics content courses continued to be taught by members of the department with degrees in mathematics but not necessarily working in the area of mathematics education. The names of these courses in the MTM were:

MATH 601 Topics in Mathematics

MATH 610 Computing Systems in Mathematics

MATH 613 Topics in Number Theory

MATH 616 Linear Algebra

MATH 618 Topics in the Application of Mathematics

MATH 621 Geometry

MATH 622 Abstract Algebra

MATH 625 Topology

MATH 626 Analysis I

MATH 627 Analysis II

MATH 636 Topics in Computational Mathematics

MATH 637 Statistics and Probability

MATH 640 Topics in Logic

Only some of these courses were designed as MTM courses, addressed to MTM students: in particular, Geometry, Topology, Topics in Logic, and the MATH 618 course on the topics of Classical Optimization, Physics or Modeling.

Chris Cummins developed a version of the MATH 621 *Geometry* course and taught it many times. When he retired, **Alina Stancu** took over the task and developed another version of the course which she taught once every two years since at least 2010.

Harald Proppe remembers teaching the Topology course for MTM students:

> *I… taught an introductory course in topology which the students enjoyed – basically a classification of compact 2-dimensional manifolds. They were taught the notion of open neighbourhood (in 2 dimensions), orientability, identification topology, etc. The text I used was Chinn and Steenrod, an ideal book with lots of illustrations. I taught it about 3 times in the seventies. (Hal Proppe, e-mail communication, 14 October, 2020)*

Bill Byers taught a topology course for MTM students in more recent times.

Nadia Hardy developed MATH 640 *Topics in Logic* Topic: *Introduction to mathematical logic from Euclid to Gödel*, and taught it three times between 2012 and 2016.

The MATH 618 Application of Mathematics for the topic "The Classical Optimization" was developed by **Richard Hall** who taught it many times. **Syed Twareque Ali** once taught a Physics course to MTM students. **Fred Szabo** taught a course on the topic of mathematical modelling with "Mathematica".

Most mathematical content courses in MTM were cross-listed with MSc courses or with undergraduate courses. This practice (potentially[34]) involved many mathematicians in the department in the education of MTM students. On the "Course Offerings" page of the MTM program[35] these cross-listed courses appeared under the title "Electives" (as opposed to "Core courses" which, in principle, were courses designed especially for MTM students). For example, the Electives for the Fall 2016 were as listed in **Error! Reference source not found.**, and – for the winter 2017, in **Error! Reference source not found.**. In the winter term of 2017, even one of the "core" courses was cross-listed with an MSc mathematics course, and a PhD mathematics course: the MTM course MATH 640 *Topics in Logic* Topic: *Set theory* was cross-listed with MAST 661 *Topics in Analysis* (MSc in mathematics) and MAST 837 *Selected Topics in Analysis*, and was taught by **Alexander Shnirelman**.

[34] There was not necessarily an MTM student in every course posted as cross-listed with an MTM course.
[35] At https://www.concordia.ca/artsci/math-stats/programs/graduate/teaching-of-mathematics-mtm/course-offef-rings.html

COURSE NO.	COURSE NAME	INSTRUCTOR
MATH 601/2, A (MATH 369)	Topics in Mathematics *Topic: "Group Theory"* *Course Outline*	E. Cohen
MATH 601/2, AA (MATH 366)	Topics in Mathematics *Topic: "Complex Analysis I"* *Course Outline*	M. Bertola
MATH 601/2, B (MATH 464)	Topics in Mathematics *Topic: "Real Analysis"* *Course Outline*	R. Raphael
MATH 601/2, C (STAT 349)	Topics in Mathematics *Topic: "Advanced Probability"* *Course Outline*	W. Sun
MATH 601/2, D (STAT 360)	Topics in Mathematics *Topic: "Linear Models"* *Course Outline*	Y.P. Chaubey
MATH 613/2, A (MATH 392)	Topics in Number Theory *Topic: "Elementary Number Theory"* *Course Outline*	H. Kisilevsky
MATH 618/2, B (MATH 479, MAST 680 & MAST 837)	Topics in the Application of Mathematics *Topic: "Convex and Nonlinear Analysis II"* Course Outline	A. Stancu
MATH 626/2, A (MATH 364)	Analysis I Course Outline	P. Gora
MATH 626/2, B (MATH 364)	Analysis I Course Outline	G. Dafni

Figure 1. Elective courses for MTM students posted for the fall term of 2016.

COURSE NO.	COURSE NAME	INSTRUCTOR
MATH 601/4, E (MATH 361)	Topics in Mathematics *Topic: "Operations Research I"* *Course Outline*	N. Rossokhata
MATH 627/4, A (MATH 365)	Analysis II *Course Outline*	A. Stancu
MATH 637/4, A (MAST 333)	Statistics and Probability *Course Outline*	L. Kakinami

Figure 2. Elective courses for MTM students posted for the winter term of 2017.

So, already in two semesters of one academic year, eleven instructors of mathematics content courses were prepared to train MTM students; in 2016-17 they were: **E. Cohen, M. Bertola, W. Sun, Y.P. Chaubey, H. Kisilevsky, A. Stancu, P. Gora, G. Dafni, N. Rossokhata, L. Kakinami** and **A. Shnirelman**. I have not checked, but it is possible that, over the years, every member of the department has taught a course cross-listed with an MTM course in mathematics (or an MTM course). If the course was a graduate MSc or MA course, the instructor did not have to treat MTM students differently than other graduate students. But, as was often the case, if the MTM students were sitting in an undergraduate course, they had to be given some extra work to make the course count as a master's level course. This meant more work for the teacher. Graduate School accepted an undergraduate course to be cross-listed with a graduate course if the teacher was willing to assign some extra graduate level material to the MTM students and assess its mastery by them. In my experience, the instructors of an undergraduate course, approached by an MTM student, never refused to do this extra work.

Moreover, **Alexander Shnirelman** developed a course on Nonstandard Analysis that could be taken by graduate students in the area of mathematics education in the MSc program as one of the topics in the course MAST 653 *Topics in the Foundations of Mathematics*. The course was first given in winter 2019, videotaped and made available online[36].

I (Anna Sierpinska) apologize for not having acknowledged by name every faculty member who has contributed to the MTM program.

[36] The whole course was videotaped by the professor himself and posted on YouTube. The first lecture is at https://www.youtube.com/watch?v=ILDkYszP2lA.

Visiting professors and postdoctoral fellows associated with MTM

Along with professors affiliated with Concordia University, MTM students benefitted from the expertise of numerous researchers in mathematics education who visited the department for a term or two, or stayed longer (if in post-doc position), taught MTM courses, and advised students on their theses and projects. We mention those who visited the department between 1978 and 2018, as remembered by Joel Hillel and Anna Sierpinska, in chronological order.

Alan Bell, UK (1978-79)

We refer the reader to a story of Alan Bell's life and work written after his death on April 5, 2018, by Hugh Burkhardt[37].

David O. Tall[38], UK (1980-81)

At the time of his stay at Concordia, David Tall was known in the community of mathematicians-turned-mathematics educators, such as Joel Hillel, mainly as the creator of "Graphic Calculus"[39], a system of interactive exercises which taught the basic concepts of Calculus, e.g., the derivative, by studying graphs of functions. He was already on his way to become a "psychologist of mathematics education".

Renan Samurçay, France (1984-85)

Renan Samurçay was Gérard Vergnaud's student. She came to Concordia as Joel Hillel's post-doc, collaborating in the LOGO project. At the time of her visit to Concordia, Renan Samurçay was interested in learning mathematics by programing computers to do various mathematical tasks. She published a paper in *For the Learning of Mathematics* 5.1, 1985, titled, "Learning programming: An analysis of looping strategies used by beginning students", (pp. 37-43).

Jean-Luc Gurtner[40], France (1987)

Jean-Luc Gurtner, from Université de Fribourg, was another post-doc with Joel Hillel, collaborating in the LOGO project.

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[37] The story of Alan Bell is at https://www.nottingham.ac.uk/education/documents/news-events/alan-bell.pdf

[38] David Tall's CV can be found at: http://homepages.warwick.ac.uk/staff/David.Tall/pdfs/prof-david-tall-cv.pdf. All his papers are freely available from: http://homepages.warwick.ac.uk/staff/David.Tall/downloads.html

[39] See "Understanding the Calculus" at https://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot1985a-und-calc-mt.pdf.

[40] See J.-L. Gurtner's profile at https://www.researchgate.net/profile/Jean-Luc_Gurtner.

Colette Laborde, France (1991)

Colette Laborde's research developed questions around, on the one hand, students' and teachers' language when solving mathematical problems (the title of her Thèse d'Etat, defended in 1982, was "Langue naturelle et écriture symbolique : deux codes en interaction dans l'enseignement mathématique") which was interesting for Nick Herscovics, and, on the other, the uses of technology in mathematics education (particularly the dynamic geometry system Cabri) which interested Joel Hillel[41]. At the time of her stay in the Department of Mathematics and Statistics at Concordia in the winter term of 1991, she gave a series of seminars to the MTM students. She ended her stay at Concordia at this time with a participation in the CMESG/GCEDM annual conference in Fredericton, New Brunswick, travelling there from Montreal by car with Liora Linchevski, Anna Sierpinska, and David Reid, who was an MTM student at that time. She was well-known to the Canadian mathematics education community. Even before her stay at Concordia, she had three articles published in the "For the Learning of Mathematics" (FLM). One was on the teaching of geometry (Laborde, 1985). Her ability to synthesize large areas of research were highly appreciated by David Wheeler, who asked her to write up her impressions from three ICME congresses, the first in 1984 at Adelaide, the second in 1996 in Sevilla, and the third in 2000 in Tokyo for FLM. After David Wheeler's death, she contributed an article for FLM commemorating him as a person, editor and scholar (Laborde, 2001).

Shlomo Vinner[42], Israel (1990-91)

Shlomo Vinner needs no introduction; he is probably the most cited author in the mathematics education community for the educational phenomena he noticed and the concepts he invented; particularly the distinction between "concept image" and "concept definition" (Tall & Vinner, 1981), and between "analytical" and "pseudo-analytical" thinking (Vinner, 1997).

Liora Linchevski, Israel (1990-91)

Liora Linchevski collaborated with Nick Herscovics on early algebra learning; they published a paper together in *Educational Studies in Mathematics* (Herscovics & Linchevski, 1994).

Maciej Klakla, Poland (1993)

Maciej Klakla was invited to Concordia at the recommendation of Anna Sierpinska because he had proposed a certain conception of understanding of mathematical concepts (Klakla, Klakla, Nawrocki, & Nowecki, 1991), and she was, at this time, working on a book on understanding of mathematics (Sierpinska, 1994). It is through him that she learned about Dyrszlag's model of

[41] Some information about Colette Laborde's more recent research can be gleaned from ResearchGate and from her webpage at http://membres-lig.imag.fr/colette.laborde/index.html.
[42] See Shlomo Vinner's page on ResearchGate at https://www.researchgate.net/profile/Shlomo_Vinner.

understanding mathematical definitions (Dyrszlag, 1978) that were mentioned in her book on understanding, and that Anne Watson found inspiring (Watson & Mason, 1998). In the MTM program, M. Klakla taught the course MATH 621 Geometry, where, among others, he tried to improve the students' spatial visualization abilities.

Ewa Łakoma[43], Poland (1993)

Ewa Łakoma specialized in the problems of teaching and learning probability. She came to Concordia in 1993 for two weeks in October to give a series of workshops on the "local models approach to the teaching of probability", where, rather than apply the general axiomatic theory to solve a problem, participants were encouraged to adapt or construct a model of the particular probabilistic situation in the problem. She gave examples of particularly smart models in the form of graphs for modelling situations with infinite probabilistic spaces. The participants were mainly MTM students, but a few faculty members came to attend, too.

Leslie Lee, Canada (1993)

Leslie Lee taught one course in the MTM program. She specialized in problems of teaching and learning of algebra at pre-university level. At the international level, she is mainly known as co-editor of the monograph on approaches to algebra (Bednarz, Kieran, & Lee, 1996). Her connection to Concordia was through MTM and David Wheeler who supervised her thesis (Lee L. , 1982) and with whom she did research on high school students' learning of algebra, and co-wrote an article, published in *Educational Studies in Mathematics,* where the myth of algebra as "generalized arithmetic" is deconstructed (Lee & Wheeler, 1989): 'Conceiving algebra as "generalised arithmetic" may obscure the many genuine obstacles that the learner has to overcome in moving from fluent performance in arithmetic to fluent performance in algebra while achieving and maintaining a smooth coordination of both modes of action.' She remained a faithful friend of David's, talking to him and comforting him frequently during his battle with terminal cancer illness. She contributed a piece for him in the 21st volume of the journal *For the Learning of Mathematics* that he founded in 1980 (Lee, 2001). In this article, which is, essentially, a transcript of an interview that David Pimm did with her, she tells the story of her admission into the MTM program by David Wheeler (she graduated in the spring of 1983). There is also a photo of Leslie and David standing together and talking at a CMESG conference.

[43] See Ewa Łakoma's page on Research Gate at https://www.researchgate.net/scientific-contributions/78809323_Ewa_Lakoma.

William S. Anglin[44], Canada (1994)

W.S. Anglin taught one course for the MTM students, on philosophy of mathematics. He was writing his book on the subject at this time (Anglin, 1997), and the participants in the course served as the first readers and critics of drafts of the chapters.

Tommy Dreyfus[45], Israel (1997-98)

Like Shlomo Vinner, Tommy Dreyfus needs no introduction, either. He spent his sabbatical year 1997-8 at Concordia at a time where the very existence of the MTM program was threatened (see the section "The struggle to keep MTM alive" above). He helped us to save the program, by writing a letter to the university's administration in its defense. He also taught two MTM courses and collaborated with Joel Hillel and Anna Sierpinska on their SSHRC funded project about teaching linear algebra with technology. This collaboration resulted in several conference presentations and a common paper published in *Recherches en Didactique des Mathématiques* (Sierpinska, Dreyfus, & Hillel, 1999).

Geneviève Boulet, Canada (1997-98)

Geneviève Boulet taught two courses in the MTM program; one in the fall term of 1997:
MATH 630 Topics in the Psychology of Mathematics Education. Topic: On rational numbers,
and the other in the fall of 1998:
MATH 624 Topics in Mathematics Education. Topic: Transition from arithmetic to algebra in high school.

She also directed the thesis of Peter Balyta (Spring 2000), titled, "The effects of using motion detector technology to develop conceptual understanding of functions through dynamic representation in grade 6 students".

Geneviève Boulet's connection to Concordia was through Nicolas Herscovics. She had started her doctoral research under his guidance. When he became too ill to continue, Thomas Kieren stepped in to co-supervise her study with Jacques Bergeron. Her thesis, completed at Université de Montréal in 1993, was entitled "*The construction of the unit fraction concept".*

At the time of teaching MTM courses, Geneviève Boulet was affiliated with Université de Sherbrooke. Since 2001, she has been teaching in Mount Saint Vincent University in Halifax, where she is Associate Professor of Educational Mathematics, in the Faculty of Education.

[44] See https://www.researchgate.net/scientific-contributions/2053996995_W_S_Anglin
[45] Tommy Dreyfus has a well maintained webpage with information about his research and projects at https://english.tau.ac.il/profile/tommyd .

Jana Trgalova, France (1997-1999)

Jana Trgalova was a post-doctoral fellow with Joel Hillel and Anna Sierpinska; she collaborated in research on the teaching and learning of linear algebra in the Cabri-Geometry II environment. She taught 4 courses in the MTM program. The first one was MATH 621 Geometry. For the purposes of this course, she programmed models of non-Euclidean geometries in Cabri; among others – the Poincaré disk model of the plane hyperbolic geometry, within which students could explore "straight lines" and "angles" in the hyperbolic sense and the differences between this geometry and the Euclidean one. She also taught MATH 649 Heuristics and Problem Solving, based on Pólya's writings, MATH 624 Topics in Mathematics Education, on the transition from arithmetic to algebra, and MATH 642 Research Methods for Mathematics Education.

Nada Stehlikova, The Czech Republic (2001)

Nada Stehlikova taught the MATH 621 Geometry course in the MTM program, focusing on different approaches to teaching and theorizing geometry: synthetic, analytical, vectorial…. In research, she was interested in methodologies of discourse analysis in mathematics education. She came in the frame of an agreement on collaboration between the department of didactics of mathematics of the Charles University in Prague and the MTM program. The proposal of the agreement was signed, on the Prague side, by Milan Hejny[46]. In the 1990s, Milan Hejny used to come to Montreal in the summer at the invitation of Joseph Brody to work on approaches to mathematics education. Milan Hejny liked to talk to MTM students about their research – in the summer the MTM students that were around were those writing their theses or projects – and as far as I know, they appreciated the attention he awarded them.

Gerald A. Goldin, U.S.A. (several times in 1990s and 2000-10)

G. A. Goldin came several times to Concordia in 1990 – 2010 to work with Syed Twareque Ali on mathematical physics and to consult the MTM faculty on their research projects. He gave seminars to MTM students. Since he did some research on affect in mathematical problem solving, he was invited, and accepted, to become a collaborator in Anna Sierpinska's SSHRC funded research project titled, "Study of the phenomenon of frustration in mature students re-learning mathematics" (2003-2006: SSHRC Grant # 410-2003-0799).

Asuman Oktaç[47], Mexico, (1999-2000)

Asuman Oktaç did her PhD in group theory in the U.S.A, and then turned to research in mathematics education. She started working in CINVESTAV, Center for Research and Advanced Studies of the National Polytechnic Institute in Mexico city, in its Departamento de Matemática

[46] See biographical article about Milan Hejny at https://www.h-mat.cz/en/prof-milan-hejny
[47] See Asuman Oktaç's list of publications at https://www.researchgate.net/profile/Asuman_Oktac.

Educativa. She came to do her post-doctoral stage with Anna Sierpinska and participated in research on the notion of theoretical thinking and the conditions of success in the learning of linear algebra. A report of this study can be found in (Sierpinska, Nnadozie, & Oktaç, 2002).

Alexei Volkov, Canada (1999-2000)

Alexei Volkov is a historian of mathematics specializing in the history of Chinese mathematics and mathematics education. He taught the Topics in the History of Mathematics course for MTM students and participated in the design of the questionnaire and interviews with students in the 1999-2002 SSHRC funded research project (Grant # 410-99-1416), led by Anna Sierpinska, "Mathematical meaning and students' interpretations of computer representations of linear algebra concepts in specially designed didactic situations", described in (Sierpinska, Nnadozie, & Oktaç, 2002). More information about his more recent research can be gleaned from https://sites.google.com/site/nthuhost/home/professors/professor-alexei-volkov-lang-yuan and from https://www.paris-iea.fr/en/fellows/alexei-volkov-2. On the second website we read that "His recent publications include a co-edited monograph *Computations and Computing Devices in Mathematics Education before the Advent of Electronic Calculators* (Springer, 2018)." The other editor of this publication is Viktor Freiman, now a professor at the Université de Moncton, New Brunswick. Back then, he was a student in Alexei Volkov's class in the MTM program. It is in the MTM program that the two gentlemen met and started this long lasting and fruitful collaboration.

Anna Rybak, Poland, 2004

Anna Rybak was a freshly promoted PhD in mathematics education from the University of Białystok in Poland, specializing in the use of technology in mathematics education. She came to teach an MTM course on this area of theory and practice. She was recommended to Anna Sierpinska by Syed Twareque Ali, a colleague in the Department, a mathematical physicist, who visited the University of Białystok frequently and was a great fan of the Polish soups served in the University's cafeteria. During one such visit, he happened to see, in the corridors of the mathematics department, a large party of people having refreshments and chatting. When he asked what the occasion was, he was told that it was the successful defense of Anna Rybak's PhD thesis in mathematics education. He spoke to her, was quite impressed, and so, when he came back to Montreal, he told Anna Sierpinska about his chance encounter with an interesting mathematics educator from Poland.[48]

[48] S.T. Ali was himself an interesting and much loved member of the department of Mathematics and Statistics at Concordia. See his obituary at https://www.concordia.ca/cunews/artsci/math-stats/2016/01/28/in-memory-of-syed-twareque-ali.html).

Analia Bergé, Canada, 2004-6 and 2018

Analía Bergé first came to Concordia in 2004 and stayed two years as a postdoctoral fellow, on the recommendation of Michèle Artigue, her PhD thesis supervisor. The thesis was an epistemological study of the completeness property of the set of real numbers. For the MTM students, she gave two courses. One, in the fall of 2004, was listed as MATH 645-Topics in Mathematics Education Research, on the topic of epistemology and pedagogy of real numbers as a set. The other – in the fall of 2005, was the course listed as MATH 624 Topics in Mathematics Education, which she chose to focus on the teaching and learning of functions at post-secondary level. She also collaborated with Anna Sierpinska in her SSHRC funded project (2003-2006: SSHRC Grant # 410-2003-0799) titled, "Study of the phenomenon of frustration in mature students re-learning mathematics".

Her second visit to Concordia was from January to December of 2018. By then, she was established as a scholar and professor at the Université du Québec à Rimouski[49]. Her presence in the department this year was extremely important for the MTM program and Anna Sierpinska personally. Anna was planning to retire on June 1, while there were still one or two students left in the MTM program. A mathematics education course was planned for the fall, and no faculty specializing in mathematics education research was active in the department (since Nadia Hardy was then a Vice-Provost and could not teach regular classes). The mathematics education course Analia taught was titled "Topics in Research in Mathematics Education. Topic: Instructional Design in Mathematics Education" and it was already listed as an MSc course: MAST 652[50]. This way, students who took a course on difficulties in mathematics learning with Anna Sierpinska in the fall of 2017, could take the course on design which was conceived as a natural follow up (Sierpinska, 2019). In the summer of 2018, Analia also taught a mathematics course, Analysis II, listed as an elective MTM course, MATH 627 Analysis II.

Christine Knipping, Germany, 2005

Christine Knipping came to Concordia as a post-doctoral fellow at the invitation of Anna Sierpinska. Her doctoral research was on the task of proving in geometry in different high school mathematics classroom cultures, mainly the French and German cultures (Knipping, 2003). At Concordia, she taught an MTM topics course on mathematical reasoning, argumentation and proof. She also collaborated with Anna Sierpinska in the already mentioned SSHRC funded study of frustration in mature students re-learning mathematics. This collaboration resulted in the publication of two articles (Sierpinska, Bobos, & Knipping, 2007;

[49] See https://www.researchgate.net/profile/Analia_Berge.

[50] Its Course Outline can be found at https://www.concordia.ca/content/dam/artsci/math-stats/docs/Outlines%202018-2019/MAST652_2_18.pdf .

2008). Presently (April 2020), Christine Knipping is a professor at the Bremen University in Germany[51].

Araceli Reyes, Mexico, 2008-2009, and 2010

Araceli Reyes, from Instituto Tecnologico Autonomo de Mexico, came to spend the full academic year 2008-09 at Concordia, at the invitation of Joel Hillel. She taught two MTM courses, MATH 645E Topics in Mathematics Education: APOS Theory (in the fall of 2008), and MATH 621 Geometry (in the winter of 2009). At this time, she started supervising an MTM thesis, which investigated college level students' difficulties with "related rates problems" in Calculus from the perspective of APOS theory, and continued supervising it by correspondence (Tziritas, 2011). She had an opportunity to work again face to face with the student, Mathew Tziritas, during her second, brief, stay at Concordia in the summer of 2010. At that time, she taught the MTM course MATH 639D Topics in Technology in Mathematics Education: Use of Computer Algebra Systems in mathematics teaching and learning.

Helena P. Osana, Canada, 2008 and 2012

Helena P. Osana is a faculty member at Concordia University, a professor in the Department of Education. She and Anna Sierpinska collaborated on a few research projects and engaged in interdepartmental teaching exchange to broaden their experience teaching students with different mathematical backgrounds. In the fall of 2008, Helena Osana taught the MTM course MATH 630F, Topics in the Psychology of Mathematics Education: Modes of mathematical thinking, and, in the winter of 2012, the course MATH 642 Research methods for mathematics education. In exchange, in the winter terms of 2013, 2014 and 2015, Anna Sierpinska taught EDUC 387 Teaching Mathematics II, a math methods course for elementary school teachers in the Department of Education, which provided data for several publications and two doctoral theses (Bobos-Kristof, 2015; Pelczer, 2017; Bobos & Sierpinska, 2017).

Edyta Nowinska, Poland & Germany, 2013

Anna Sierpinska "discovered" Edyta Nowinska at the Seventh Congress of the European Society for Research in Mathematics Education, Rzeszów, Poland, 9-13 February, 2011. Edyta Nowinska, originally from Poland, obtained her doctoral degree in mathematics education in Germany, in the Department of Mathematics and Computer Science of the University of Osnabrück. Her thesis, titled, "Kognitionsorientiertes Lehren - Analyse eines Implementationsprojektes zur Einführung des Funktionsbegriffs" was based on a teaching experiment aimed at teaching the formal concept of function. She came to Concordia for the winter 2013 term to teach the course MATH 624K Topics in Mathematics Education: Teaching functions in secondary school. During her stay, she also actively participated in Anna

[51] See http://www.math.uni-bremen.de/didaktik/ma/knipping/index_en.html

Sierpinska's exploratory teaching of a "math methods" course for future elementary teachers in the Department of Education at Concordia. (This experience was described in (Sierpinska, 2016)). Edyta had experience teaching such courses from her work in the Institute of Mathematics Education at the Adam Mickiewicz University in Poznań, Poland, and her expert advice and classroom observations were invaluable for Anna. Presently (April 2020), Edyta Nowinska is a faculty member at the University of Osnabrück[52].

Viktor Freiman, Canada, 2014

Viktor Freiman obtained a doctorate in computer science education from the Moscow State University in Russia in 1990[53]. After he moved to Canada, he obtained a job in a primary school in Montreal, where he taught regular classes and organized mathematical activities for gifted children. Having collected interesting empirical material from these children's solutions, he decided to share it with mathematics education researchers and started coming to meetings and conferences. At one such conference, he met Anna Sierpinska, who told him about the MTM program. He enrolled in it and graduated in 2003, with a thesis on mathematical giftedness and how to identify and foster it (Freiman, 2003). Then he obtained a position at the Université de Moncton[54]. During his sabbatical year in 2014, he was able to come back to Concordia, as a professor this time, to teach the MTM course MATH 649 Heuristics and Problem Solving.

Georgeana Bobos-Kristof, Canada, 2016

Georgeana Bobos-Kristof was another MTM graduate (Fall 2006) who came back to Concordia to teach an MTM course. By then, she had a master's degree in mathematics (also from Concordia; she wrote a thesis under the supervision of Galia Dafni, (Bobos-Kristof, 2007) and a PhD, individualized program[55] (Bobos-Kristof, 2015). She taught MATH 624 Topics in Mathematics Education: Teaching Calculus in College, in the fall of 2016. The course was cross-listed with the MSc course MAST 652 Topics in Research in Mathematics Education Research, because there were already some students in the MSc/MA program in Mathematics specializing in the area of Mathematics Education. Georgeana had experience in research in mathematics education and writing scholarly papers about it, as well as in day-to-day teaching of Calculus in College. She combined a profound understanding of students' ways of thinking in Calculus, with knowledge of the institutional aspects of college level mathematics.

[52] See https://www.mathematik.uni-osnabrueck.de/forschung/ag_kognitive_mathematik/mitglieder/nowinska_edyta.html.
[53] See http://www.policyresearchnetwork.ca/viktor-freiman/.
[54] See https://www.umoncton.ca/prof/umcm-freiman_viktor/.
[55] See description of the program at http://www.concordia.ca/academics/graduate/individualized-program-phd.html

The MTM program through the eyes of some faculty members:

Fred Szabo, Bill Byers, Joel Hillel and Anna Sierpinska

In 2018-20, Anna Sierpinska interviewed Fred Szabo, Bill Byers and Joel Hillel on their experience with the MTM program and wrote her own story. This section reports on these interviews and contains Anna's story.

Interview with Fred Szabo

Figure 3. Fred Szabo in 2012.

The photo of Fred Szabo (Figure 3) comes from an article about him published, in November 2012, on Concordia's website[56], the Concordia University News page.

At the time of writing this part of the document (September 2019), Fred Szabo is still an active member of the department, teaching mathematics, supervising graduate students, and publishing. He supervised the project of the last MTM student still in the program after admissions had been suspended in January 2016, Mandana Mir, who graduated in May 2019.

[56] http://www.concordia.ca/cunews/main/stories/2012/11/27/fred-szabos-classroom-of-the-future.html

Having joined the department in 1964, Fred Szabo would have been an active member for 55 years. He was not involved in writing the proposal for the creation of the MTM program, but he was asked to design and teach the first Analysis course for the MTM students. Here is how he remembers his experience in an interview with Anna Sierpinska, December 12th, 2018:

> *F.Sz.: It was in many ways, the most enjoyable and exciting things I've ever done. Because the students were mature, they were interested in ideas, not techniques of calculating – they haven't done this in a long time. They were more interested in history and the conceptual side. So, we explored things and they were writing projects, etc. I was a young PhD student at McGill at that time and I was taking all these graduate courses myself, and I thought I knew what we should be teaching. Calculus and Analysis is the mathematics of motion, there is some physics involved, geometry involved. It was a wonderful mix. And so, I got myself a whole stack of books on topology, on advanced calculus – that's what we called it at that time – and ordinary calculus and I picked three books that we found exciting. The book on topology was really fun because we took some concepts like convergence, and we asked questions like what could go wrong if we redefined convergence that way or that way. We did a comparative study and people found that really interesting. Instead of taking a definition and a few examples and doing exercises, like; "Does this converge to that?"; we asked, "Why not?" And, there was always the human side: Who thought of this? What did they want to achieve by taking that definition?*
>
> *A.S.: So, comparing the topological definition in terms of neighborhoods with the epsilon-delta definition?*
>
> *F.Sz.: Yes, absolutely. My background is in logic. I studied logic at Oxford, I studied logic at McGill, and so I was interested in that whole idea of infinitesimals, of nonstandard analysis, even at that time, and we took it all the way from Leibniz and Newton up to Robinson and it was a sort of vindication of Leibniz that you could do this with logic and this was set theory, which was very difficult of course, but we did that. So, as a result of that work, it became a really interesting course.*
>
> *A.S.: What was the name of that book on Topology you mentioned?*
>
> *F.Sz.: Wolfgang J. Thron's "Topological structures" (1966). But I didn't use a single book for the course as a textbook. I had my own ideas. I used some books as an inspiration. For example, there was this book by Purcell[57], a beautiful book. But instead of doing regular exercises, we moved on to projects. What I did most successfully was to get people involved in projects.*

Fred Szabo directed a good number of MTM projects. He could not recall all of them, but he wrote down a list of 15 projects that he supervised. Here it is:

L. Beaulieu, (graduated in the fall of 1978), "A guide to connectedness"

[57] See the Contents of "Calculus", 9th edition, by Dale Varberg, Edwin Purcell and Steve Rigdon at https://www.scribd.com/document/357686103/Calculus-9rd-Edition-Dale-Varberg-Edwin-Purcell-and-Steve-Rigdon-pdf

F. Lovasco, (Spring 1978), “Geometry in Euclidean space”

J. Leung, (date of graduation unknown), “Minimization of convex functions”

J. Gaspar, (Fall 1977), “Logic, and its relationship to function sets, and arithmetic”

M. S. Whight, (Fall 1977), “The trisection problem”

E. Chee, (Spring 1975), “The trigonometric functions”

L. Lambert, (Spring 1975), “Programmation linéaire”

F. Gennaoui, N. Mardirossian, and N. Rayes, (Spring 1975), “L’enseignement de la parabole”

J. Nelson, (Fall 1975), « L’enseignement des treillis au niveau collégial »

J. Block, (Spring 1974), ‘The binomial theorem”

W. Colpitts, (Spring 1974), “Computational complexity”

R. Purchio, (Spring 1974), “Bilinear transformations”

B. M. Rhodes, (Fall 1974), “Compactness”[58]

Y. Younglai, (Fall 1974), “The Fundamental Theorem of Algebra”

Fred Szabo remembers particularly well one MTM student, Jill Britton. This is what he said about her in the interview:

> *F.Sz.: She may have changed her name after she got married. Her maiden name was Guadagni[59]. Jill had influence on me and on the department in some ways. We started working and we became interested in tessellations. We had a lot of fun with space filling shapes and patterns and mathematics as a science of patterns and so on. She became a bit of a world expert on tessellations and she wrote a book[60] on it and she moved out west and she created an educational fund at Camosun College in Victoria, BC. Unfortunately, she died (on 29 February 2016)[61]. She had tremendous influence on teaching tessellations at the college level. I consider the relationship and friendship that we developed and the enthusiasm was one of the high points of my work with the MTM program. It was fantastic.*

The story Fred Szabo tells in the interview of how he got hired at the SGW University and what was the atmosphere in the department back then shows how the culture of this university was different from the present one at Concordia:

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[58] In 2019, Barbara Rhodes was still a part-time teacher at Concordia, see https://www.concordia.ca/artsci/math-stats/faculty.html?fpid=barbara-rhodes

[59] She is not on the list of MTM graduates under either name.

[60] Dale Seymour and Jill Britton, 1986, “Introduction to tessellations”, Dale Seymour Publications

[61] See her obituary at http://www.legacy.com/obituaries/montrealgazette/obituary.aspx?n=jill-britton&pid=177916230

> *F.Sz.: When I came to Montreal, I got a job in a high school and I taught math and Latin. A colleague there told me "You really should be teaching at Sir George". At that time Norm Smith was chairman of the math department[62], so I came to the math department at Sir George, and I asked Norm Smith if he could give me some teaching and he said "Yes, why not" and he hired me, a young guy. I didn't even have a PhD yet, I got one in 1970 or 1971. It was really that simple at that time. He put me up in some office in the Norris building that I had to share with some poet who smoked pot in his office. That's how it started in 1964. The real mathematicians were elsewhere. Jean Turgeon was there and there was an Englishman who taught differential equations, rather from an engineering point of view. He was always wearing a lab coat and was quite funny.*
>
> *At that time, we talked about teaching. How do you teach quadratic functions, how do we teach trigonometry, should we go with triangles, with circles…. Norm Smith who was the chairman at the beginning was an amazing man. He was laid back, was a musician, played the piano, we sang a lot, and we drank a lot, but at the same time he allowed us to explore and I could always go to Norm's office and say, "Norm, I have this sequential convergence, what does it mean and how would you approach that?" He loved matrices and he did a second PhD at McGill with Lambek. And he would say, "I don't know but I will look it up and we can talk about it later." Norm and I talked a lot about how to teach certain things. Now it's all compartmentalized, every course has its course outline and there is no room for discussion and exploration.*
>
> *At this time, SGW was basically a teaching university. We had very few graduate programs. The CEGEPs were being established. And Sir George Williams University had this tradition since WWII or maybe from before of taking people and giving them a second chance at life. It was either the soldiers who came back from the war and wanted to get a degree or it was high school teachers who wanted to become college teachers.*

At the beginning, the MTM curriculum focused mostly on mathematical content; the goal was to brush up the teachers' knowledge of more advanced mathematics and engage them in mathematical explorations. This changed in the 1980s when the department hired David Wheeler and Nick Herscovics, who were interested in research into the psychological aspects of learning and teaching mathematics. Fred Szabo deplores this turn in the direction that the program took, because as he states:

> *Now we are at the point where the M.T.M, although it has made tremendous contributions, is no longer good enough [for students in the program] to become college professors. At the beginning these people were trained mathematically, not pedagogically. Then, the pedagogical side became maybe the raison d'être for the MTM. Maybe that was appreciated at some point but now we are at a point where we were before [focusing on the mathematical content by merging the MTM within the*

[62] See tribute to Norman Smith on Concordia's webpage at http://www.concordia.ca/cunews/main/stories/2014/09/10/a-pioneering-academicandgiftedalgebrateacherleavesaninspiringleg.html

> *MSc in mathematics]. I think there is room for both and need for both [mathematics and pedagogy], but I can see this shift. I don't see how the MTM program could continue.*

In the years 1986-92, Fred Szabo acted as the Dean of Graduate Studies. He talked about it in the interview, and also at a Long Term and Retiree Employees luncheon at Concordia in December 2015. As a CU News published at that time reports:

> *Szabo (…) supervised the establishment of many new graduate degrees, including the PhD in Biology, PhD in Mathematics, MSc in Administration and MSc in Aerospace Engineering.*
>
> *"I fought tooth and nail for Concordia to create graduate programs at a time when this was really a big deal," says Szabo. "These things are now taken for granted. In the late '80s they required finesse, tact and inspiring collaboration."*[63]

In the interview, he added:

> *Had I not been involved with the MTM as a young PhD student and then as a professor, I probably wouldn't have made the contributions that I made to the graduate studies at the university because it really got me into this.*

Fred Szabo is now known as an expert on the use of technology in mathematics education. He taught the course on this topic in the MTM program many times. But his interest in this area started when he came back to the department from his deanship. This is the story he told in the interview:

> *When I took over grad studies there was basically no technology. There was one IBM computer, where they entered data for student admissions and another one from Texas Instruments for producing the graduate calendar. That was it. And in the basement they had a large room full of student files, nobody knew what was what because they haven't been catalogued. I said, "If we want to have a graduate school we need to have an infrastructure." We had a budget, but we could be a bit liberal with it, so we bought computers. (…)*
>
> *When I came back from my deanship, Joel Hillel asked me if we could have Maple*[64]*, so we started to do some experiments and workshops but nobody came. So, I said, having Maple as an add-on will never work. If students don't have to do it, they won't. And I am going to teach with technology, that's how I started getting interested. So, I learned about Maple, and about APL*[65]*. That's when Hal [Harald Proppe] and others started the Actuarial program, that's another idea, but there was nobody to teach the*

[63] See http://www.concordia.ca/cunews/main/stories/2015/12/10/alfie-plenzich-fred-szabo-reflect-on-their-years-at-concordia-long-service.html
[64] A computer algebra system, see https://www.maplesoft.com/
[65] https://en.wikipedia.org/wiki/APL_(programming_language)

programming, so for several years I taught this course MATH 232 which I think we still have[66]. I taught Maple to math majors and APL to Actuarial students.

Later, Fred Szabo developed the first computer lab version of the undergraduate Linear Algebra I and II courses[67], and an online version of the college level Vectors and Matrices course.

Interview with William P. Byers

William P. (Bill) Byers joined Concordia's mathematics department in 1972 (when Concordia was still the SGW University) and remained its member until his retirement in 2008.

Figure 4. William P. Byers sometime in 2007-2008.

The photo in Figure 4 shows Bill Byers in his office at the university, in the Library Building, at 1455 De Maisonneuve blvd. West, in Montreal. On his desk, there is his first book, "How mathematicians think: Using ambiguity, contradiction and paradox to create mathematics", published in 2007, by Princeton University Press.

William P. Byers was one of those "real mathematicians" that Fred Szabo mentions in his description of the mathematics department in the first years of his work there. At the time Bill Byers joined the department, there was a process of transformation of the department from what was an essentially an undergraduate teaching institution into a research and teaching mathematics department. The university was hiring mathematicians with doctoral degrees,

[66] It is now called MAST 232 Mathematics with Computer Algebra. It used to be based on Maple, but now it is done in the Matematica environment. See the Winter 2019 course outline at https://www.concordia.ca/content/dam/artsci/math-stats/docs/Outlines%202018-2019/MAST232_4_18.pdf

[67] F Szabo wrote a textbook for this course, titled "Linear Algebra with Maple, Lab Manual: An Introduction Using Maple".

many of them from McGill University but also from universities in the U.S.A. In the interview, Bill says:

> *Who do I remember? Joel [Hillel] came before me, and there was Hal [Harald Proppe}, and Maurice Cohen[68]… These were the times of the process that I called later the "professionalization" of Canadian universities, especially mathematics. SGWU was at the time a "farm" for McGill University. That is to say, it was a place for McGill professors, like Lambek[69], to put his PhD students – McGill graduates got jobs at Sir George. So many people at Sir George had PhDs from McGill. Elie Cohen (who joined the department in 1964) was one of them. Lambek was a serious researcher and he was a good ring theorist. Abraham Boyarsky also had a PhD from McGill; he came in 1971. They also started to hire people who had degrees from the better American schools, people like Joel [Hillel], and myself and Maurice [Cohen], and Hershy Kisilevsky later on. They started to bring in people who had something I call mathematical culture because a few years before, we had people who essentially had at most master's degrees in mathematics and were just teaching it. So, it was an essentially an undergraduate department that was weak but which in a relatively short period of time became a more or less serious mathematics department.*

When asked how he remembers the period of transition from SGW to Concordia (1974-~1979, or mid-70s), W.P. Byers said:

> *I was chair, Maurice was Dean of Science. We had 20 odd faculty members and Loyola College had 7 faculty members in their department of mathematics, and we aspired to build a mathematics department, what we considered a mathematics department and they had a very primitive department, that is to say no one knew very much mathematics. So, we basically took over the Loyola math department. So, things didn't change much for us. We were the same department with adding a few people. One was Ron Smith who was interested in mathematics education and he basically left us to go to the education department and became actually a reasonable educator. Another one was Srivastava, who had a PhD but there was no one at Loyola who contributed to the MTM. Many of the people who were there left, some were fired.*

Bill Byers was not involved in the creation of the MTM program but he taught mathematics courses for MTM students – he even taught one such course after he retired, the Topology course –, he (co)supervised a few MTM theses – in particular the thesis of Elaine M. Landry, presently (2019) a professor in the University of California at Davis[70] – and was a reviewer for several MTM theses. About the MTM courses, Bill Byers says in the interview:

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[68] The only trace of Maurice Cohen I could find on the internet is his brief review of a 1972 book titled "Lectures on Algebraic Topology", by A. Dold, in the Canadian Mathematical Bulletin dated 1975 and signed Maurice Cohen, Concordia University. The Author of the review paid attention to the didactical organization of the book and not only to its mathematical organization and content. M. Cohen concludes his review by saying that it is a good reference book for some parts of Algebraic Topology but a poor textbook for a course on the topic.

[69] https://en.wikipedia.org/wiki/Joachim_Lambek

[70] See Elaine M. Landry's autobiographical note at https://philosophy.ucdavis.edu/people/emlandry

WPB: I made up a course that I taught a number of times in qualitative theory of differential equations; something that was not taught in the undergraduate curriculum but it was something that I picked up at Berkeley and that was a new way of looking at differential equations, and it was accessible to people who had some Calculus and Linear Algebra. So, that was one of my contributions to the MTM program. Or, I would teach Advanced Calculus but from a point of view which was more modern. We looked at the derivative as a linear approximation rather than… etc. So, there was something a little bit different about… I also remember teaching Analysis to them. Analysis was a stretch for many of the MTM students. Their qualifications were somewhere between first year university and second year university. They hadn't necessarily mastered what you'd call rigorous mathematics. You couldn't really teach an epsilon-delta course without being very careful. Joel [Hillel] taught linear algebra, also some math courses that he had invented for the MTM program.

AS: Coding theory. So, these were special courses for MTM students when there were enough MTM students to fill a class. More recently if an MTM student wanted to take Analysis they had to register in a regular undergraduate Analysis course and do some extra work for it to count as an equivalent to an MTM [graduate] course. There were not enough MTM students to offer an Analysis course just for them. So, back then you had many students in the MTM program? Like 30 students?

WPB: Yeah, sometimes. Like these differential equations course, I have this memory of this course, which I taught in the evening, 6 to 8, the usual slot, and the students, who were all teachers, were all smart. I remember people like Barbara [Maude Jean] Rhodes, who became a part-time teacher later on. I remember some very intelligent people but they weren't… they had very little mathematics education… They had no conceptual background. They didn't really know what a derivative was, they didn't understand continuity, limits, all the foundational concepts of mathematics. They could calculate some limits but they've never been faced with the challenge of understanding mathematics. By the way, it was not just the MTM program. All of Sir George was at that level. It took many, many years before the curriculum became more modern, where you asked students to master a more theoretical way of looking at mathematics.

W. P. Byers was the chair of the department when I (Anna Sierpinska) was hired in 1990. He mentions this as he reminisces about Nick Herscovics:

At the beginning, Nick Herscovics only had a master's degree in mathematics. Later on he went on and did his PhD in math education. He is connected to you. I was chair, he came to me and he said: "We have to hire Anna Sierpinska". He thought you were wonderful. He said: "Wow, we have a chance to hire Anna Sierpinska!" We had that position that was open, and here you are.

Bill recounts his view of the history of the MTM program:

Let me tell you a little bit about the history of the MTM. MTM essentially was a "perfectionnement" degree, it wasn't an initial degree. It wasn't a degree that could get you into teaching if you had not already been accepted. But there was a time when the province moved to classify all teachers on a 20-point scale and it depended on the number of years you had done graduate work. I didn't really understand all that but I

> *remember that a PhD made you a 20-point teacher. But, there were people already in the system who ran the risk of being declassified and there was a way to raise your classification by going back to school. So, at one point many, many of the CEGEP teachers, on the anglophone side, went through the MTM program, as a kind of professional development program, and essentially, this was a way of raising their 'scolarity' so that their salaries went up. And then, we probably ran out [of candidates to the program] because at some point everyone [either already had the MTM degree or didn't need it].*

In the interview, Bill was asked if he remembered any themes that were hotly debated at the departmental council meetings and if yes, if there were issues of teaching and learning mathematics among those hotly debated themes. His response was:

> *I don't remember hotly debated issues of teaching mathematics. My memory of the department, and this includes Concordia, was that there never was an intense engagement with curriculum development.*
>
> *Even with my close friend Hershy Kisielevsky, we never had a discussion about education. As the department became more and more research oriented there was a positive antagonism towards mathematics education. I remember when I was at Berkeley and there was one well-known algebraic topologist named Emery Thomas [who] gave me some advice as I was graduating: 'You spend as little time as you can teaching, so you can devote yourself totally to your research if you want to succeed in this profession'. That was the standard advice that was given and it was such a powerful force that…. Just to take a specific example, let me…. I remember spending hundreds of hours talking to Joel about the teaching of linear algebra, how badly it was taught and how it should be changed. But the two of us, in spite of being influential, never succeeded in forcing the change how linear algebra was taught. Some people continue to teach linear algebra the way they always taught it, that is, axiomatically, in a sort of rote kind of way. So, we failed even in specific areas, in bringing about an approach to the teaching of mathematics which is closer to what I believe in.*
>
> *[At the beginning] what happened was that you had a bunch of professors and those professors did their own thing. [On the wave of the New Math movement] we taught some courses in a very different way. I remember scrapping final exams in one course and just giving them multiple kinds of midterms that they could retake. We did the usual in the sixties kinds of experiments with education and no one in the department objected. In a sense we were in a privileged position…. The professor controlled his classroom. There was no overarching [policy] …. Remember, the people who were in charge of the department like my father [Victor Byers] knew much less mathematics than the younger people who were assistant professors. And, they knew they knew much less mathematics. My father, I used to tell him that regularly. And he knew so he would ask me. So there was a whole generation, Norm Smith, Martin Harrow, who was involved in the MTM, maybe had even been its first director and Nick [Herscovics] too, at the beginning, but he professionalized himself. He went out and got a PhD because there was a time when, and I think it was due to a large extent to Victor Byers, that people were fired from the department for not being qualified. And many people went to get PhDs.*

Nick Herscovics was at the root of the process of professionalization of the MTM program, in the sense of it becoming a graduate program, specializing in research in mathematics education. Mary Brian especially, was active in creating and running a diploma in mathematics education.

> *She [Mary Brian] ran the diploma in secondary math education and Alberta Boswell ran a certificate, or something like that. Those diplomas and certificates were also "perfectionnement" programs for teachers, not for CEGEP teachers but for high school teachers who wanted to upgrade their salaries. So, depending on their level, we would teach them first year mathematics and then second year mathematics and so some of them were then able to teach CEGEP mathematics.*
>
> *Joel will tell you how much in terms of mathematics education went on in the MTM. The interesting question is, how did mathematics education [research] enter into the MTM? Who brought it in? Was it Nick, was it David Wheeler? But it was clear that when David Wheeler arrived, things changed. David was a serious math educator and so it is my memory that Joel got into math education because of David. He made it a kind of accessible discipline for other people and he had connections nationally and internationally. [He started the journal "For the learning of mathematics".] And he was an editor and a brilliant editor. David was not only inspirational to the local community but to the national Canadian mathematical community; the crucial thing was the Canadian Mathematics Education Study Group. David had different relationships with different people. When David taught a course, it was a math education course, and Joel, too but I don't remember the courses that they taught. I just remember going to lectures with David and of course he had a lot of influence on me which took many, many years to mature. He had this notion of mathematizing, and the idea that you could talk about mathematics as process and not structure stayed with me. So, I think that David had a good way of thinking not only about mathematics education but about mathematics in general. He was a bit of a philosopher. I think that part of the department changed substantially when David came.*

Asked if he remembers anything about Stanley Erlwanger, Bill Byers said:

> *Oh, of course I remember him very well from the years at Loyola [campus][71]! He was a fascinating man. He came from Zimbabwe. He was the first African American to get a PhD in math education. He was a highly intelligent man who wrote quite a notable PhD thesis. [But he did not like the process a faculty member had to go through if they wanted to get a promotion.] He said, 'I am not going to start writing nice things about myself and go up for promotion'. He said – there was something very "English gentleman" about him – he said, 'I don't like to write nice things about myself and solicit opinions, etc.'. Actually, I think he had a very powerful case but he did not want to go through that ordeal and maybe he was correct. At that stage, I felt that he started to withdraw from the department. There may have been other reasons…*

[71] Until 2005 the department had offices both at the Loyola and the SGW campuses.

In spite of what Bill said that he did not manage to have any influence on teaching mathematics in the department, he and Joel Hillel developed the course MAST 217 Introduction to Mathematical Thinking – the course continues to be taught to this day (Sierpinska & Gontijo-Lauar, 2019) – which embodied some of the ideas about mathematics teaching that he believed in. But, in the interview, he said that he does not consider this contribution a success:

> *I was responsible for this, but I was wrong. I made many mistakes. It's clear… I had this notion that we should be introducing epsilon-delta definition of limit, that is to say a rigorous Analysis course earlier on for the better students. And I remember talking to a guy who graduated from Berkeley who was a professor at Cornell and he was telling me that he didn't do it until the junior year and I was convinced that we should do it for even younger students. I was wrong and the proof of that was, I remember teaching this Analysis course to our students in the Actuarial program and I remember I taught a section, Kisilevsky taught a section and John Harnad taught a section, and we did very badly. The students were hostile, they didn't understand what they were doing and why they were doing it. We had a post mortem after that and we concluded that (a) the students were very bright, that they did understand what we were doing and (b) that this kind of students were never going to appreciate this kind of approach; it was just totally wrong for them. We identified rigorous mathematics with intelligence. It didn't occur to us that you could be intelligent and have sort of a really difficult time. It's not that these students didn't try to learn; it's just that there was something fundamentally alien about this way of thinking that they were not ready for… It was very abstract and quite meaningless to these students and we forced them to prove silly theorems using epsilon-deltas. It was meaningless so why were we doing this?*
>
> *I didn't know that before but if we started developing this Introduction to Mathematical Thinking [it was to prepare these students to appreciate proving things rigorously]. But this was also a failure because it was taught to the students in the Math Major program instead of being taught to students who were in specialization and honors programs. It was the right course for the wrong group of students. What was the point? Students in the Major weren't going to do rigorous mathematics. We did manage to teach these students the elements of proofs and they were fine with it…. It's very accessible to them, but this was the one and only time that we had a course in mathematics which was focused on methodology as opposed to content.*

Another contribution of Bill Byers to mathematics education in the department was an undergraduate course on great mathematical ideas (MATH 215 Great Ideas in Mathematics[72]), which, unfortunately, was not taught after his retirement:

> *Hershy Kisilevsky and I also invented a course, Mathematical Ideas, which was a kind of introduction to mathematics through geometry, and number. Because there was a*

[72] The calendar description of the course says: "Mathematics is used to unravel the secrets of nature. This course introduces students to the world of mathematical ideas and mathematical thinking. Without being overly technical, that is, without requiring any formal background from the student other than high school mathematics, the course delves into some of the great ideas of mathematics. The topics discussed range from the geometric results of the Ancient Greeks to the notion of infinity to more modern developments."

> *book at that time, Great Ideas in Mathematics (Singh, 1959), a sort of historical account. I found also that there was a great hunger, on the part of mathematics students to know something about the culture and the history of their subject and that was not to be found in this formalist approach to teaching mathematics. I used to teach mathematics like this. I used to teach differential equations from a book by Simmons[73] which was based on the historical development that discussed the kind of problems that led to these differential equations, situate them in a context, etc. But for the most part, this formalist philosophy, which people don't even realize they have because they don't understand they have a philosophy of mathematics, is so powerful that it was very hard to resist. If you fought it, it was as though you were defined as an inferior mathematician; because if you were a real mathematician you would be concerned about proving theorems. So, it was a kind of vicious circle into which you would get into with this kind of debate. It was hard to find a niche… I don't think, by the way, that the colleagues, such as Peter Taylor, had such an easier time with it when they eventually got into math education themselves. It's hard, it's just intrinsically difficult in those math departments to have a reform….*

Bill acknowledged, however, that various mathematics departments did engage in interesting experiments with their curricula,…

> *for example, Harvard had a course where they would pick a theme, let's say quotients, and they would do it right across mathematics. So that's interesting. When I was at Berkeley… we would start with a problem like the problem of heat conduction and developed all the mathematics from whatever field that contributed to it. In other words, mathematics can be organized in many, many, ways. It doesn't have to be organized axiomatically, and [there is] the idea of having interdisciplinarity within mathematics courses where you teach, for example, advanced calculus and linear algebra simultaneously. They fit very well together. Instead of 3 credits for this, 3 credits for that and then…*

But…,

> *Anyhow, we never succeeded in bringing a broader philosophy to the mathematics curriculum as a whole…. It was actually a personnel problem because Concordia gave people credit for teaching so many credits and for publishing so many papers. It didn't matter how good the papers were or how good the teaching was. As long as you did those things, you didn't have to worry about your career. So, there were people who taught the easiest way possible, that it to say, with the least effort and the least effort means axiomatically. So, it would have taken a real revolution… I maybe didn't do enough but certainly…. I considered a great success to have involved Hershy Kisilevsky in teaching the kind of course that we taught together. But the only way he would do it was if I was in the classroom with him all the time.*

W.P. Byers firmly believes in the value of the knowledge of history of mathematics in the upbringing of a well-rounded research mathematician and academic teacher:

[73] Simmons, George F., "Differential equations with applications and historical notes" (Simmons, 1972)

The MSc program should have a mandatory history course. Every undergraduate who gets a degree, especially a specialization degree, should have a course in history of mathematics. The kids that we exposed to history, they loved it! Students crave having a context. It makes set theory so much more interesting to them if they have heard stories about Cantor and others. But now they are not even exposed to the fact that there are mathematical controversies! That people take their subject seriously and that there are no controversies. But there were controversies. When you learn about non-Euclidean geometries you should also learn that their discovery had incredible social implications! Educated people were affected by the question [about] whether there was one geometry or many geometries, and that bothered lots of people. To teach differential equations without talking about the physical problems that led to them is… is… So what I am saying is that there was a very impoverished intellectual climate around and that affected our capability to do educational work, because education seems trivial to such people. It merely consists in writing axioms on the board and producing theorems from these axioms. That's all that there is to it. Not that they don't have powerful intuitions about the problems they work on but just like in Victorian English there was no talking about sex, in this culture there is no talking about how you thought about something. So, when you teach metric topology you usually think about two-dimensional examples, that is to say, you check your theorems against this two-dimensional model. If you don't mention this geometric model, if the student can't draw a picture, then there is no place for the student to get their intuition. How do they guess if the statement is true or not? They have no ability to do that because they have no source of examples. And when it comes to defining a real number that's such a complex object and it's "just a point on the line"… So that whole issue of where that intuition comes from which these people undoubtedly have it's never discussed, it's never put on the table and discussed. Of course there are exceptional people. Think about Paul Halmos's autobiography. Spanier, he was a professor at Berkeley. He gave a course, which was very good, but he wrote a book… He had his notes and he abstracted them and abstracted and 10 years later he published it. It was the most abstract book ever written. He had the most general and abstract version of each theorem and it was written in such a language that you couldn't figure out what was going on. It was a way of hiding the thinking person behind the mathematics. It's some sort of a combination of Platonism with formalism. Whereas I and other mathematicians writing about mathematics, particularly Reuben Hersh, fight for some kind of humanistic mathematics.

Interview with Joel Hillel

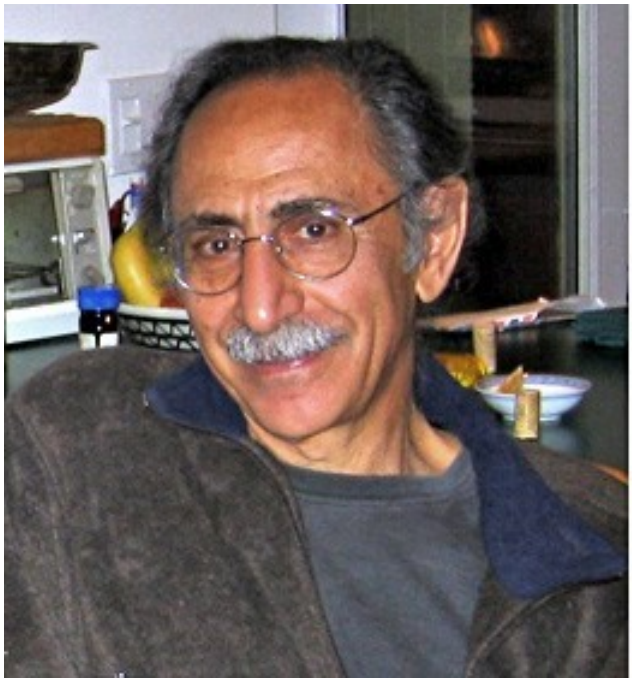

Figure 5. Joel Hillel.

The source of Joel Hillel's photo in Figure 5 is the departmental web page[74].

Joel Hillel joined the Department of Mathematics of Sir George Williams University in 1970, and stayed on after it became the Department of Mathematics and Statistics at Concordia University, until his retirement in 2009. He served as the director of the MTM program several times in the years 1973 - 1992, and was its director from 2002 to 2009. Here is how he remembers his beginnings in an interview with Anna Sierpinska, October 17, 2019:

> *When I came to the department, in 1970, the MTM was already an on-going program. It's been there for several years. I was one of a group of young PhDs at the department and I thought about math, research and changing the department, but in fact the only graduate program, at the time, was this master's program in teaching mathematics and even though it talked about teaching mathematics, it was really just math; math courses for very good mathematically-competent teachers who were all going into the CÉGEP system. They had good training in mathematics, but it was all the classical math, it wasn't the 'new math'[75] program. Early on, I had contact with those teachers, teaching courses such as Abstract Algebra or Topology. They were an interesting group*

[74] https://www.concordia.ca/artsci/math-stats/profiles/joel-hillel.html

[75] "New Math" refers here to what may be called a structural perspective on mathematical content, in terms of algebraic structures such as groups, fields, modules, vector spaces, and in the language of set theory; for example, seeing linear algebra not as a theory of solving linear equations but as a theory of vector spaces over arbitrary fields; or, seeing geometries as theories of invariants of groups of transformations. In the years 1960-70, this perspective was quite widespread in curricular reforms from elementary school to university (Moon, 1986).

> *because they liked it that they were part of a math department and not of the education department. They weren't interested in having any education or psychology courses. They really were mathematicians, they wanted to act as math professors, and that's what the program was back then. I remember them being quite a big group of students – around 120, actually. A vast majority of them were in the French or English CÉGEP system. Many of the students worked part-time and therefore had time to complete the program. They were interested and really liked math. I remember one time I was teaching the students about Category Theory and about a dozen of them came to ask me for lectures to learn more on the subject. This was that caliber of students.*
>
> *The only non-mathematical content course given in the MTM program that I remember was History of Mathematical Ideas, taught by Paul Rosenbloom from the Teacher's College of Columbia University. He was originally a mathematician from Minnesota. He got involved in mathematics education in the United States and had contacts in Montreal, which gave him the chance to come and offer this course during the summer, shortly after the program started. (Joel Hillel, interview, October 17, 2019)*

By 1973, the CEGEP system in Montreal was saturated with teachers of mathematics and the supply of candidates interested in taking courses in abstract algebra or topology within the MTM program dried out. Still, the professional development system in Quebec at that time encouraged elementary and secondary school teachers to enter certificate, diploma or master's programs and rewarded them by higher classification and better salaries. The department was asked to design programs for elementary and junior high-school teachers. Mary Brian and Alberta Boswell got involved with this task and a Diploma in the Teaching of Mathematics, a 30-credit program, was born. Classes and workshops in this program were to be held not at the university but in schools, which required collaboration and coordination with schools and schoolboards. Graduation records show that a total of 13 students obtained the diploma, 5 in spring 1976, 3 in fall 1977, 4 in fall 1979 and 1 in spring 1984.

"As for the MTM program", Joel Hillel said,

> *… we were getting high school math teachers that weren't as sophisticated in mathematics nor as interested in taking just mathematics courses as the students we had at the beginning of the program. It was to the point where they would ask us why they needed to take these courses in the first place and how this helps them handle situations with their students. We noticed the number of people in the program was dropping and we started thinking about what skills we could teach for the high school teachers to feel better equipped in terms of teaching mathematics correctly. We needed to re-evaluate which math courses would be best to teach them and what tools to give them. (Joel Hillel, ibid.)*

This situation created a need for more knowledge in mathematics education among the faculty. Nick Herscovics, who was already working in some capacity in the department, had a master's degree in mathematics, but did not have a PhD, decided to do his PhD in mathematics education, and went to do it with Jacques C. Bergeron at the University of Montreal. He obtained it in 1979 and specialized in research in mathematics education. In the meanwhile, to

save the program, Joel Hillel had the idea to attract teachers from other provinces by offering a 6-week summer program:

> *When I had just started my MTM program director career in 1974 and the number of students was decreasing, we suddenly thought: 'Why don't we get some teachers here from out-of-province by doing MTM as a summer program for 3 summers where you do about 6 weeks,' and we did that for about 3 years. We actually received many people from the Maritimes and from Ontario, even our own students sometimes preferred doing their courses in the summer too. (Joel Hillel, ibid.)*

But there remained the more fundamental problem of making MTM a program better suited for high school teachers, with which Joel Hillel was tasked as the director of the program.

> *It happened, in the winter of 1973 or 74, that Paul Rosenbloom was in Montréal and I asked him what the minimal requirements for a high school math teacher would be in terms of math, and Paul said that they should really have Calculus, and Analysis, and Topology and Linear Algebra and he ended up listing all the MTM courses again. He didn't cut one course from our MTM courses, maybe Logic. So that was not very helpful. At the same time, he said he was sorry, but he could no longer come to Montréal to give the History of Mathematical Ideas course. (Joel Hillel, ibid.)*

The department needed an instructor to teach the History of Mathematical Ideas course and this is how David Wheeler came into the picture:

> *I then spoke to Claude Gaulin from the University of Laval in Québec who was always around and helping us with the program, about getting someone to the MTM program to teach the course. He told me, 'There is a David Wheeler, who is in New York right now, working with Caleb Gategno, why don't you write to him and ask if he would come.' I didn't know David Wheeler at that time, but I wrote to him, asking if he would be interested in coming to Concordia in the summer to teach this course. And he said yes, and came to teach the course in the summer of 1974. I wasn't there at the time, so I never met him, but apparently, he left enough of an impression to be invited to come back in the fall. At this time, there was already talk in the department of hiring somebody in math education, so he came in the fall, and taught a class called Heuristics and Problem Solving. He was hired in the winter of 1975 as a full-time professor and his presence changed the MTM completely. Very early in the game, he taught a course on psychology in math education. In fact I am not sure if it was a course or a seminar for the faculty, because a lot of us came and he introduced us to (George) Pólya, (Jean) Piaget, (Karl) Popper, (Imre) Lakatos, and all these people about whom, except perhaps for Pólya, we didn't know much. (Joel Hillel, ibid.)*

The presence of David Wheeler changed the character of the MTM program completely. It was no longer just "mathematics for teachers":

> *So that was the big change in terms of mathematics education. We started to introduce more courses. Nick Herscovics started giving a course on Psychology in Mathematics Education. To me, that's really the beginning of how the MTM program became a master's in the teaching of mathematics. (Joel Hillel, ibid.)*

David Wheeler also opened up the program to discussions of trends and issues moving the international mathematics education community, by bringing guests from abroad to speak to the students.

> *In the past, we also brought in guests, like Jean Dieudonné, who argued that you don't need geometry in linear algebra and Peter Hilton, who showed that you can teach category theory in high school, and this kind of nonsense but David (Wheeler) immediately invited Alan Bell[76]. He basically invited people from the real math education community…. I remember the visit of Eleanor Duckworth, who was Piaget's student, as a speaker to talk about Piaget's work and she had asked some people that were attending to bring children and made everyone see that even children of math professors couldn't conserve number. This turned out to be a very interesting lesson. Very effective. (Joel Hillel, ibid.)*

In search of inspiration for incorporating pedagogy in the MTM courses, Joel Hillel spent his 1976-77 sabbatical year in the Center for Learning and Teaching of Mathematics at the University of Georgia, Athens. This was a hotbed of new approaches to research in mathematics education at the time, and Claude Gaulin strongly encouraged him to go there. Several people at the Centre worked on research in mathematical problem solving[77], and this area interested him particularly. Here is how he remembers his experience.

> *There was Jeremy Kilpatrick, Larry L. Hatfield, indeed a whole group interested in problem-solving. They spent a lot of time worrying about methodology and how to do a clinical interview, and is it legitimate, and that kind of stuff. I also attended Jeremy's course on research in math education that ultimately taught students what bad research was. He would pick articles from the NCTM publications that had wrong statistics or badly worded hypothesis or conclusions. Unfortunately, I didn't find the course to be very interesting except at the end of the course he brought in the Benny studies of Stanley Erlwanger to raise the question, Is it research?, because it was all through interviews and clinical interviews, and, for me, this was a big discovery. I read that study and I really loved it. I really loved reading about what one kid was thinking when doing math. It was quite ironic that 3 years later, Stanley Earlwanger was also hired in a math education position in the math department and gave courses in the MTM program. May I add, Tom Kieren was there, too, on sabbatical, at least in the first I didn't know him, I didn't know anything about him. He was very busy at the beginning because he was working on a research project with Les Steffe, but towards the end of his stay I got to see him more, talk to him more and started to appreciate some of his work and thinking. And, when he was leaving, he left me with his monograph on rational number. This work really struck me; not that it contained anything new, but it had reminded me of just how much is involved in rational numbers, how many things you have to put together, and how complex this notion can be. So,*

[76] Alan Bell is known for his method of teaching and doing research called "diagnostic teaching". See a story of Alan Bell's life and contributions to mathematics education written by Hugh Burkhart, his collaborator at the Shell Centre, at https://www.nottingham.ac.uk/education/documents/news-events/alan-bell.pdf

[77] There exists a document, dated 1978, containing papers from a workshop on research on mathematical problem solving, organized by the Georgia Center.

> *he influenced me too, and when I came back to the department, I could contribute to the education aspect of the MTM program, by teaching different courses, a lot on problem solving…. (Joel Hillel, ibid.)*

After his stay in the Georgia Center, Joel Hillel got involved in research in mathematics education and started to obtain funding for it. His first project was on problem solving; it was a qualitative research consisting of an in-depth analysis of task-based interviews with students. He conducted this research together with David Wheeler, for whom it was also his first time doing this kind of empirical research in mathematics education. The study was funded by FCAR[78] and lasted 3 years, 1978-81.

> *I wrote the application with David [Wheeler]. {Dieter] Lunkenbein who was on the committee said it was a proposal they haven't seen in years, because David wrote so well. We had it translated [into French] by Liliane [Beaulieu[79]]. At the time, they were very impressed. (Joel Hillel, ibid.)*

The problem with this type of research is that it is difficult to squeeze its results into a 20-page article; its value is in the detailed analyses of individual students' thinking. It is amazing, and indeed quite eye-opening, to see how other people, not mathematicians but students, understand a problem and how many different ways they can think about it. If we synthesize the results into correct/incorrect solutions, or even a few categories of solutions, all this richness is gone. Joel Hillel and David Wheeler wrote a 250-page report from their research (Hillel & Wheeler, 1982). Earlier however, David Wheeler wanted to present partial results at a conference:

> *After 2 years, David said there was a math education conference in Grenoble [the PME 1981] and suggested we present a bit of our findings. I thought we shouldn't because it was 3-year project and it wasn't finished, but David presented our findings on behalf of both of us anyway, although it wasn't at that meeting. Once our project was over, I had told David we couldn't write a paper, there was a lot of material to cover. We ended up writing a very elaborate research report. We took our data and reanalyzed it with many math educators who were around, like Alan Bell and David Tall, and others. We even included their analyses in the report, as well as our protocol and had it transcribed. We sent it to the community so they could have their own interpretation and of course we sent it to all the people that we knew. It was never really published. (Joel Hillel, ibid.)*

Joel Hillel talked about this study at the PME 1983 in Israel in a presentation of a 6-page "research report" (Hillel, 1983), focusing on the methodology but a full journal article never came out of this research.

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[78] Québec provincial funding agency : « Formation de Chercheurs et l'Aide à la Recherche ». This grant now bears the name of FRQNT, « Fonds de recherche du Québec – Nature et technologies ».
[79] Liliane Beaulieu was an MTM graduate, fall 1978.

An interesting by-product of Joel Hillel's sabbatical year at the University of Georgia was the development of a course on Coding Theory that he taught many times in the MTM program as one of the MATH 601 Topics in Mathematics courses. Here is the story of how it happened:

> *When I was in the University of Georgia, sometimes I'd be a bit lost in the math education department; they taught a lot of things about teacher training, pre-service training, which oddly wasn't very relevant for me, but I had to keep myself busy. I was reading an article in the MAA Monthly, it was about coding theory and I stopped to read it, only to find out it spoke of something I talked about in my doctoral thesis. It was a completely different topic, but with the same structures and I thought: 'That's a good course to teach high school teachers, because you can do it simply, it's relevant, it will answer the students' questions to know 'what this is for''. I had actually taught Coding Theory at many different levels to many different groups, undergraduate math students, MSc students. For MTM students, I actually had a different version of Coding Theory that I thought was kind of a nice addition to the program. (Joel Hillel, ibid.)*

Starting in the 1980s, Joel Hillel became interested in technology in mathematics education; he did research in this area, published many papers on the subject and directed several MTM theses or projects about it. (Some of his published papers are co-authored with Carolyn Kieran, an MTM graduate (Fall 1979) who then went on to do a PhD in Psychology at McGill and became a professor at UQAM). His interest was triggered by the popularity, among mathematics educators, of the computer program "LOGO", based on Turtle geometry (Abelson & diSessa, 1981), and promoted by Seymour Papert in his book "Mindstorms". There was also another factor, namely the rapid development, in the 1980s, of personal computers, and especially of the Macintosh computer, with its user-friendly interface, based on icons to which one could point with a mouse, instead of writing commands in code. And, last but not least, mathematics education research community was bursting with energy. International conferences abounded; there was even a special conference on LOGO. The PME[80] annual conferences were gaining recognition as high standards scientific meetings; Nick Herscovics actually contributed to the PME's coming into being (Bergeron, 1994). New journals were started, open to new trends and approaches. One of these new journals – *For the Learning of Mathematics* – was born at Concordia; it was started, in 1980, by David Wheeler. Doing research in mathematics education was exciting.

David Wheeler also inspired the creation of the Canadian Mathematics Education Study Group (CMESG/GCEDM) which held annual conferences at different Canadian universities, bringing together mathematicians, mathematics teachers of all levels and mathematics educators, mostly from Canada but also some from the US, UK, and other countries.

It is at one of the CMESG conferences by the end of the 1980s that Joel Hillel heard about MAPLE, the computer algebra system, developed by Canadian mathematicians from the University of Waterloo. He remembers saying then, 'This is going to change undergraduate

[80] Psychology of Mathematics Education.

teaching', and, to some extent, it did. It triggered a reform of the undergraduate program in mathematics at Concordia – Joel Hillel talked about these changes in an interview for the Concordia's Thursday Report (Ramsey, 1997). In the revised program for math majors, Linear Algebra was taught in a computer lab equipped with Maple, and a new special course on using computer algebra systems for mathematical computations was added. Throughout the 1990s, experimenting with the new technology raised many questions which, for Joel Hillel and Anna Sierpinska, who joined the department in 1990, triggered several research projects, SSHRC and FCAR funded. The projects involved the participation of a few MTM students who then wrote their theses on the learning of linear algebra: Luis Saldanha (graduated in Fall 1995), Michael Haddad (Fall 1999), and Richard Masters (Fall 2000). The thesis of Andreea Pruncut (Spring 2008) was also on learning linear algebra in a Maple environment but it was not related to a funded project; Andreea used the data obtained from a Maple-assisted lab-based Linear Algebra course taught by Anna Sierpinska.

In 2017, at the 41st meeting of the CMESG group, held in Montreal, at the McGill University, Joel Hillel was asked to deliver the "Elder talk" (Hillel, 2018). He told the audience how he got into mathematics education. Here is how he remembered it in the interview:

> *Two years ago [in 2017], I had to give an Elder talk to the Canadian Math Study Group when they met here [in Montreal]. The only people I recognized among those who came to my talk [were] David Reid[81] and Peter Taylor. All others were new to me. I was just trying to tell them how I got into math education. I started to talk about the department and the MTM program. I talked about the contribution of David Wheeler, about students with whom I did research, and I talked about the conferences I went to.*
>
> *I started to go to a few MAA meetings thinking they knew a little bit more about math education. Then I went to an NCTM meeting and I was totally lost. However, the first time I went to a real math education conference, it was one of the annual CIEAEM[82] meetings. The conference was mostly in French. It was just a show, from my perspective; somebody would talk and then the French contenders would orate, it was like a game, it was like a stage. I wasn't used to this, because I'd often go to math conferences and it wasn't like this at all. One talk that captured my attention was given by Georges Glaeser[83]. It was about problem-solving and he spoke about how the solvers speak and it may sound like nonsense, but somehow they manage to establish*

[81] MTM graduate, fall 1992. David Reid did his PhD in mathematics education under the supervision of Tom Kieran. He then worked as faculty member at the Memorial University of Newfoundland, and at Acadia University in Nova Scotia (see his page at http://www.acadiau.ca/~dreid/). At the time of writing this document, he lives in Germany and works at the University of Bremen.

[82] Commission Internationale pour l'Etude et l'Amélioration de l'Enseignement des Mathématiques (http://www.cieaem.org/)

[83] Georges Glaeser was a mathematician who turned to mathematics education later in his life and contributed to a collection of interesting and challenging mathematical problems for high school students, to research on problem solving, to research on epistemological obstacles and to theory of mathematics education (see (Pluvinage, 2002); also https://en.wikipedia.org/wiki/Georges_Glaeser).

> *communication with each other. He showed examples of some very good problems that we used in our research with David Wheeler later on.*
>
> *Then I went to the big ICME congress in Karlsruhe, with David Wheeler. It was my first international congress of mathematics education. First of all, many of the main speakers were mathematicians too, like Michael Atiyah[84], and somebody from England who was just trying to sell his book about applied mathematics. It took me a while to really get into the culture. Even if it was a congress on math education, there was a lot of hard mathematics, and talk about courses, undergraduate courses. So, it was an interesting beginning for me. It took me a while to really understand what was going on. (Joel Hillel, ibid.)*

Joel Hillel's path to mathematics education led through undergraduate and graduate mathematical studies. In 1960-1980, it was the usual path for mathematics educators, especially in Europe. One studied mathematics, did some research, got a teaching job, became intrigued by students' learning problems with mathematics, sometimes to the point of wanting to research these problems more deeply and ended up becoming a mathematics educator. The passage from mathematical culture to mathematics education culture is always a bit difficult, as Joel describes it.

There are other paths to mathematics education, but anybody hired as a professor to support the MTM program at Concordia had to have at least a master's degree in mathematics. A professor in the department has to teach at least 4 courses per year, and they could not all be pedagogical courses in the MTM program. At most 2 such courses were offered in a year. (Even in the pedagogical courses the mathematics discussed had to contain some college level mathematical content because most MTM students aspired to become college teachers.) Any faculty member must, therefore, teach some university mathematics courses, and to teach these at least a master's in mathematics is necessary. There are not very many people with this type of background in mathematics who have done and published research in mathematics education, which made hiring for MTM difficult.

[84] One of the top mathematicians of the 20th century (see https://www.theguardian.com/science/2019/jan/15/sir-michael-atiyah-obituary; https://en.wikipedia.org/wiki/Michael_Atiyah)

MTM through Anna Sierpinska's eyes

This section is written in the 1st person singular, by Anna Sierpinska.

How it began

I joined the department in December 1990, to support the MTM program, after David Wheeler retired. I didn't know David as a colleague but I met him in 1988 as a member of the scientific program committee for the International Congress of Mathematical Education which was to take place in 1992 in Quebec City; David Wheeler was the chairman of this committee.

It was Nick Herscovics who talked me into applying for the position at Concordia when it was opened in 1990. We had met at the 11th International Conference of the Psychology in Mathematics Education group which was taking place on the premises of the Université de Montréal during the hot and humid summer of 1987, the year remembered in the history of the city as the "year of the Montreal flood".[85] On July 14, in just about 2 hours, between noon and 2:30 pm, torrential rains poured over 100 mm of water onto the city, flooding basements, highways and disrupting the electrical power system. I landed in Montreal – the Mirabel airport – about one hour after the storms, flying in from Europe. I was staying at a friend's house; her basement was flooded, and she had no power. Until the night fell, we were carrying out buckets of water from the basement, trying to save whatever we could from the collection of books that she kept there. Such was my first encounter with Montreal: Dealing with a natural disaster the way Montrealers do, "toughing it out". By July 19, electricity in the Université de Montréal buildings was restored and the PME conference was held without major disruptions.

Nicolas Herscovics was among the organizers of this conference, and, together with Carolyn Kieran and Jacques C. Bergeron, an editor of its proceedings. Carolyn Kieran was affiliated with the Université du Québec à Montréal, Jacques Bergeron – with Université de Montréal, and Nick Herscovics – with Concordia University. The kind of research presented in the PME conferences at that time had representatives from three out of four universities in Montréal, all except for McGill University. I had an opportunity to meet a number of them at the PME in Montreal, in particular, Nadine Bednarz, Claude Janvier, Gisèle Lemoyne, etc. The next year, 1988, Nadine and Claude were organizing an international conference[86] on the subject of

[85] https://en.wikipedia.org/wiki/Montreal_flood_of_1987

[86] The 1988 conference on obstacles and conflicts was organized with the institutional and financial support of C.I.R.A.D.E. (Centre Interdisciplinaire de Recherche sur l'Apprentissage et le Développement en Education). The centre is described this way in « L'UQAM branché » of October 25, 1999 (Vol. 26, no. 4, at http://www.journal.uqam.ca/99-2000/JOURNAL/4/4E.htm): « Créé en 1980, le CIRADE est un centre de recherche en éducation dans lequel les chercheurs s'intéressent à des problématiques concernant l'appropriation des savoirs dans des contextes différents (milieux de l'enseignement, de la santé, du travail, etc.). Cette problématique renouvelée et élargie de l'éducation est abordée dans une perspective résolument interdisciplinaire. Les chercheurs du CIRADE sont formés dans diverses disciplines et proviennent de différentes institutions (UQAM, UQAC, UQAH, Université Laval, Université de Montréal). » At the time of the conference, and for several years after I came to Concordia, Nadine Bednarz was the director of this centre. During her directorship, C.I.R.A.D.E.

epistemological obstacles and cognitive conflicts, a hot topic at that time. They invited me to participate in the conference which I eagerly accepted since my research at this time was totally devoted to epistemological obstacles to mathematical concepts (such as infinity, limits, functions) (Sierpinska, 1988a; 1988b; 1989). It is at that conference that I came to know better the members of the Montreal mathematics education community and appreciate their understanding of mathematics and of the problems of its teaching and learning and the originality of their research approaches. These people were familiar with the approaches developed in France under the influence of Guy Brousseau's Theory of Didactic Situations, as well as with approaches of various German, British and American groups of researchers, but did not fall into any kind of rigid orthodoxy by following one or the other of them. Rather, they used them as a source of inspiration to develop their own ways of doing research, letting the problems of practice of teaching mathematics guide their methodologies and theoretical frameworks. I would call their approach to research in mathematics education "pragmatic" and it appealed to me very much. So, when Nick Herscovics wrote to me in April 1990 urging me to apply for the position at Concordia, I did so. I had a chance to be part of that vibrant community of open-minded and original researchers.

My first semester at Concordia, winter 1991, with visitors from abroad, Colette Laborde, Liora Linchevski, Shlomo Vinner, and colleagues active in mathematics education research, Joel Hillel and Nick Herscovics, made me feel as if I was at an international conference. It was quite exciting. Teaching was also exciting. I taught an MTM course on the philosophy of mathematics and an undergraduate course in Linear Algebra. I was surprised both by the freedom I was given in choosing the content and approach in the graduate course, and by the quasi complete lack of it in teaching the undergraduate one. The content of the Linear Algebra course was prescribed in a detailed manner, with sections of the textbook to "cover" and exercises to do in class and to assign for homework each week specified by a professor in the role of the "course examiner". I was free to write my own class test. Here, I learned an important element of the North American undergraduate mathematics culture the hard way. In that test, I failed to ask the students explicitly to "justify your response", but expected them to do so (because a response without a justification did not count for much in the mathematical culture of the country I came from, Poland). So, I took away marks for lack of justification. After receiving their grades, students complained that this was unfair. I remember one Chinese student who patiently explained to me that if I want a justification I should explicitly say so in the formulation of the problem. This and other experiences made me realize that, in North America, "mathematics" at the university level can refer to knowledge of some problem types and techniques of their solution without awareness of the reasons of the validity of these techniques or the ability to formulate them in a more general terms. In brief, this is the idea

hosted several seminars where the keynote speakers were prominent researchers in mathematics education or scholars often quoted by mathematics educators. This international presence and interdisciplinary breadth of discussions was very stimulating intellectually.

that in learning mathematics, one can do without theoretical thinking. I spent the rest of my working life at Concordia trying to understand the phenomenon, identify its consequences and to convince students of the importance of theoretical thinking for learning and doing mathematics.

The next year, 1991/92, was a bit sad. Nick Herscovics was diagnosed with cancer. Stanley Erlwanger left Concordia and academia altogether and went back to his native Zimbabwe. By 1994, we were left just the two of us, Joel Hillel and me, to support the MTM program, teaching courses and alternating as directors of the program.

The MTM students

Time and again, we had to justify the program's very existence to other faculty and the university's higher administration[87]. For me, the raison d'être of this program was always the students it attracted, some highly original personalities, with all sorts of life experiences, all curious and passionate about mathematics and its teaching. Some were already teachers. These came to advance their careers (e.g., in teaching at higher level, or in research, planning to do a PhD in mathematics education after MTM), and/or in search of inspirational ideas for teaching. Some worked in other professions but were not happy with them. There was an engineer who found his work routine and boring. He believed that teaching mathematics, although constrained by curricula and syllabuses, still leaves a lot of room for the teacher's creativity, and, creativity, he was bursting with it. There was an actuary (with a BSc in mathematics from Concordia) who could not stand working another day in an insurance company, alone in a cubicle, feeding numbers into a computer program that she could not change or even question. This was a very well-paid job yet she quit it. Somehow, she started helping children with math, and discovered that she had a passion for teaching. She decided to learn some more about it and chose to apply to the MTM program. In this program, she could focus on the teaching and learning of mathematics (rather than study theories of teaching in general and of learning in general, which she feared she would have to do in a master's program in education), and she could take more advanced courses in mathematics to complete the program which she was interested in and which would help her to obtain a position of a mathematics teacher in a CEGEP.

Every MTM student was special in their own, interesting way. Their life experiences enriched classroom discussions in the MTM courses and could be used in the choice and conduct of their research projects. For example, in his thesis, the above-mentioned engineer attempted to characterize the mathematics taught to engineering students, comparing it to mathematics taught to mathematics students on the one hand, and, on the other, to the mathematics

[87] There is more on this topic in the section, "The struggle to keep MTM alive".

actually used in engineering practice (Pearce, 2015) but which might be ignored in the traditional mathematics content courses.

I enjoyed accompanying the MTM students in their research and guiding them in writing their theses (MATH 654 Thesis) and projects (MATH 603 Extended Project). Altogether, in the MTM program, I supervised 28 theses and 23 projects.

Although I was pleased when some of them went on to do doctoral degrees, I was very happy when, after graduation, students wrote to me that they got a job (I could stop worrying about their financial situation), and most happy when this was a mathematics teaching job in a CEGEP, for this was a job I knew they wanted.

Some MTM students came to visit me at Concordia on my retirement day (June 1, 2018), or sent a note. There were flowers and a cake and we took a picture (Figure 6).

Figure 6. Anna Sierpinska's "retirement party", June 1, 2018. From left to right: Analía Bergé, Anna Sierpinska, Richard Masters, Sabrina Giovanniello, Sandi Mak, Sabrina De Cicco, Carol Beddard, Krista Day, Dalia Challita, Adrian Bradley, Judy Thykootathil and Geraldine Ford.

On people in the picture taken at my retirement party (June 1, 2018) and some other MTM students

The event was organized by Sandi Mak, who graduated from MTM in spring 2006, and was, at the time of the event, coordinator of the mathematics department at Vanier College. She was helped, on the side of the Concordia University's department of mathematics and statistics, by Judy Thykootathill (the front desk administrative assistant) and Geraldine Ford (the administrative assistant to the Chair). I must mention that these two ladies, who appear in picture on the far right, have also been of great help to me in fulfilling my teaching and administrative duties at Concordia. In the picture, besides Sandi Mak, the MTM graduates are: Richard Masters (fall 2000), Sabrina Giovanniello (fall 2017), Sabrina De Cicco (spring 2016), Carol Beddard (spring 2012), Krista Day (spring 2018), Dalia Challita (fall 2013), and Adrian Bradley (spring 2014).

Richard Masters works at John Abbott College, along with two other MTM graduates: Kevin Flood (spring 2009), Caroline Lefèbvre (spring 2010)[88]. Dalia Challita (fall 2013), who is on the photo, also worked at John Abbott for a couple of years before moving to Italy.

Richard Masters graduated with a thesis (Masters, 2000). The thesis was about students' understanding of linear algebra. Along with several other MTM students, he had been a research assistant in one of our projects, with Joel Hillel, related to linear algebra teaching and learning. The thesis was, however, based on empirical data grounded in Richard's own teaching experience.

Let me at least name here the MTM students who helped us in our research with Joel Hillel on linear algebra, since we owe them a lot: Astrid Defence (graduated fall 1994), Tsolaire Khatcherian (withdrew from the program in 1997 and moved to Toronto), Luis Saldanha (fall 1995), and Michael Haddad (fall 1999). Astrid, Tsolaire and Luis are my co-authors in a chapter about this research in a book edited by Jean-Luc Dorier (Sierpinska, Defence, Khatcherian, & Saldanha, 1997). Michael Haddad wrote a thesis on students' difficulties in the learning of linear algebra, but, like Richard, he used his own empirical data collected when he was teaching the MATH 204 Vectors and Matrices course (Haddad, 1999).

Sandi Mak, Carol Beddard and Adrian Bradley were teaching at the Vanier College at the time of the picture being taken, and still do when I am writing these words (May, 2020). Looking at the list of mathematics teachers employed at Vanier I recognized the names of seven more MTM graduates.

I worked quite closely with Carol Beddard. She was one of those unorthodox individualities that MTM seemed to attract. She had a BSc in chemistry but for various reasons had not worked in the domain. In 2008, when her children were in high schools or colleges, she decided to get a

[88] See members of the John Abbott College mathematics department (in 2020) at https://johnabbott-messagerie-luka.omnivox.ca/mgvc/Annuaire/Annuaire.ovx?Ref=001703625845&C=JAC&L=ANG

degree in mathematics and applied to study mathematics at Concordia. She was admitted to the Mathematics Major program. After a year of courses in this program, she applied to and was accepted into the Pure and Applied Mathematics which is a specialization program. In 2010, when she had completed the required 21 credits in undergraduate mathematics, she applied to the MTM program and was accepted on the basis of her BSc in chemistry and these 21 credits in mathematics with very good grades. She graduated from the MTM two years later, with a thesis (Beddard, 2012), which I had the pleasure to supervise. The thesis was based on her experience of teaching a college level algebra course at Concordia, to a class of mostly "mature entry"[89] students, where she tried different ways of motivating them to study more systematically. She had an impressively mature scientific approach to her data and research in general, which made me see her more as a colleague than a student. She also worked as a "marker" for one of the linear algebra courses I taught at the undergraduate level, but she did more than just grade students' homework. I had the idea that grading students' solutions manually is only worthwhile when the problem requires a justification or a proof. Grading such problems takes a lot of time so a weekly homework assignment cannot have more than 1 or 2 such problems. Still, homework should touch upon all the important content taught in class. So, questions about this content must be included but if justifications are not asked for, these questions can be programmed to be graded electronically. This programming became Carol's task. She programmed the questions in Moodle, learning how to do so and teaching me what she learned while she was doing it. I used this skill in all my subsequent courses[90]. With all that, she did not abandon her undergraduate program in mathematics. She completed it, "with distinction", in 2014.

Sabrina Giovanniello graduated in the fall of 2017, with a thesis (Giovanniello, 2017) whose title was a question: "What algebra [college level] Calculus students need to know?" She built a critical answer to that question by an extremely thorough analysis of (a) typical final examination problems in one such course offered at Concordia and (b) approximately 60 students' solutions to these problems. Based on this analysis, she created an inventory of students' difficulties, misconceptions and false rules in algebra and showed how they can be used in the construction of multiple-choice items on a "placement test" for the course. This was a palpable, practical result of a research that started off as highly theoretical, and I was quite proud of Sabrina for having had the stamina and patience to achieve it. She worked diligently on her thesis despite physical pain in her arms after a car accident she was in in December 2016. Then, in May 2017, I broke my hip, had a surgery, then a month of rehabilitation, so we couldn't see each other in person. We communicated by email, phone and Skype, and she could defend her thesis in September 2017 as planned. Oh, and I forgot to mention that all this

[89] See definition of "mature entry" at http://www.concordia.ca/academics/undergraduate/calendar/current/sec14/14.html#b14.1

[90] Had I been still teaching at Concordia at the time of the covid-19 pandemic, when all courses s tarted to be taught online, this skill and my bank of programmed problems in Moodle would have come in handy.

occurred while she was working as a research assistant to Professor Jennifer McGrath in the Department of Psychology. She had a BSc degree in Psychology, and the professor tried to talk her into doing a PhD in this domain. She might yet do it, who knows! For now, she is listed as part-time instructor at Concordia[91], teaching mathematics online at eConcordia. She teaches also at LaSalle College in Montreal.

Sabrina De Cicco did an MTM project with Nadia Hardy as her supervisor, on online mathematics courses and the development of virtual tutorials for a basic college algebra course.[92] She was a student in at least one of my MTM courses and I remember her strong and clear voice; she would not have to use a microphone to be heard even in the last rows of a hall the size of an opera house!

Krista Day chose the course option in the MTM program. She was admitted on the basis of a BA Major in mathematics and statistics from Concordia obtained in 2013. She started the program in the winter of 2014 as a part-time student – she worked a full 9 am – to – 5 pm job at a company – and finished it in 2014. In the last year of her studies, Fred Szabo took her under his wings and offered her a reading course in the summer of 2017 on online learning strategies. (By the way, how timely this topic sounds in 2020, when schools closed and universities went online because of the covid-19 pandemic!) He had discovered Krista's technological flair in the MTM course he was teaching in the winter of 2017, MATH 639 Topics in Technology in Mathematics Education: Visualization as an Instructional Tool.

Adrian Bradley entered the program in the winter of 2012 and finished it May 2014, a record time for the Course Option which he chose. He was in two MTM courses that I gave in those years. He had a deep understanding of the mathematics he knew and a good understanding of other matters, pedagogical and philosophical as well.

The lady that stands on my right in the picture is Analía Bergé, the visiting professor from Université du Québec à Rimouski, mathematician and mathematics educator, whose cheerful presence and meaningful conversation brightened my last semester at Concordia before retirement. I write about her some more in the section "Visiting professors and postdoctoral fellows associated with MTM".

From MTM to PhD in mathematics education

And now a few words about those of my students who, after MTM, went on to a PhD program in mathematics education.

The first was David A. Reid. He shared my interest in understanding "epistemological difficulties" in learning mathematics, that is, difficulties that are specific to the *meaning* of

[91] See https://www.concordia.ca/artsci/math-stats/faculty.html?fpid=sabrina-giovanniello

[92] For a long time already, there has been much research and development going on at Concordia in relation to online courses. No wonder Concordia was able to go online already one week after Prime Minister Legault announced the closure of all schools and universities (March 13, 2020).

mathematical concepts, theories and methods, rather than difficulties stemming from, e.g., students' cognitive limitations or teachers' ways of teaching mathematics. He was particularly interested in students' understanding and doing mathematical proofs, and he carried this interest through to his doctoral studies (with Tom Kieren[93] in Alberta) and further research as a university professor (Reid & Knipping, 2010). For his master's thesis, he chose to study students' difficulties with proofs by mathematical induction. His thesis' title was, **"**Mathematical induction: an epistemological study with consequences for teaching" (Reid, 1992).

Five more of my MTM thesis students went on to do their PhD in mathematics education:

- Luis A. Saldanha, graduated from MTM in 1995, with a thesis titled, "The notions of linear independence/ dependence: a conceptual analysis and students' difficulties" and he did his PhD with Pat Thompson[94], in the U.S.A. Presently (May 2020), he is a professor in the department of mathematics at Université du Québec à Montréal[95].

- Georgeana Bobos-Kristof, graduated in 2006, with a thesis titled, "Effect of weekly quizzes on the development of students' theoretical thinking". After a master degree in mathematics under the supervision of Galia Dafni[96], she did her PhD with me (Anna Sierpinska) at Concordia University, in the INDI[97] program. She teaches college level mathematics at Dawson College in Montreal and as a part-time faculty member at Concordia. At Concordia, she also taught a graduate course in mathematics education, the MTM course MATH 624 Topics in Mathematics Education: Teaching Calculus in College, cross-listed with the MSc course MAST 652 Topics in Research in Mathematics Education Research, in 2016. We did some research together[98].

- Shiva Gol Tabaghi, graduated in 2007, with a thesis titled, "APOS analysis of students' understanding of logarithms"; went to Simon Fraser University and did her PhD with Nathalie Sinclair[99]. Presently (May 2020) she works at Capilano University[100].

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[93] See a video of a talk by Tom Kieren at https://www.youtube.com/watch?v=dsy4W197aAQ in 2018.
[94] See information about Pat Thompson at https://math.asu.edu/people/patrick-thompson or at http://pat-thompson.net/
[95] See Luis Saldanha's page at http://profmath.uqam.ca/~saldanha_l/Welcome.html .
[96] See Galia Dafni's page at https://www.concordia.ca/artsci/math-stats/faculty.html?fpid=galia-dafni
[97] See http://www.concordia.ca/academics/graduate/individualized-program-phd.html .
[98] See Georgeana Bobos page at https://www.researchgate.net/scientific-contributions/79081592_Georgeana_Bobos.
[99] See Nathalie Sinclair's page at https://www.sfu.ca/education/faculty-profiles/nsinclair.html .
[100] See Shiva Gol Tabaghi's page at https://www.capilanou.ca/programs--courses/search--select/explore-our-areas-of-study/faculty-profiles/shiva-gol-tabaghi/ .

- Nicolas Boileau, graduated in 2012, with a thesis titled "Two secondary school mathematics teachers' use of technology through the lens of instrumentation theory". He went on to do his PhD in mathematics education[101] with Patricio Herbst[102] at the University of Michigan.

- Maria-Josée Bran-Lopez, graduated in 2015, with a thesis titled, "Sources of Mature Students' Difficulties in Solving Different Types of Word Problems in Mathematics". She went on to do her PhD in mathematics education at the McGill University, under the supervision of Limin Jao[103]. More precisely, the name of the program she is in is Doctor of Philosophy (Ph.D.) Educational Studies: Mathematics and Science Education.

I have not listed here Tyler Marghetis[104], who graduated in the spring of 2009, with a thesis written under my supervision, titled, "When a proof is a satisfactory explanation for the reader?" Although he also went on to do a PhD, his doctorate was not in mathematics education. He did his PhD in cognitive science with Rafael Núñez at the University of California at San Diego, after obtaining a master's degree in this domain.

I also did not include in the above list Hong Yue who, after graduating from the MTM program in the fall of 2003, with a project titled, "Cultural background and mathematics achievement", went on to do a PhD in mathematics, in the area of Analysis, also at Concordia, under the supervision of Galia Dafni. She already had one PhD, in [mathematical] physics, from China, but she believed that a PhD in mathematics from a North-American university would give her a better chance of obtaining a university position in Canada or the USA. It eventually did, and she now holds an assistant professor's position in the Department of Mathematics of the university, Georgia College, U.S.A.

The MTM students whom I supervised

I remember fondly all my students; each project and thesis was an adventure on its own. Each would have an interesting story to tell but we must keep this document down to a reasonable size. So, I will honor the students I supervised by just mentioning their name and the year of graduation, in chronological order. The titles of their theses or projects can be found in the section which lists all MTM graduates.

[101] See Nicolas Boileau's pages at http://legacy.soe.umich.edu/people/profile/boileau_nicolas/; https://www.researchgate.net/profile/Nicolas_Boileau, and https://www.linkedin.com/in/nicolas-boileau-7a604656.
[102] See Patricio Herbst's page at https://soe.umich.edu/directory/faculty-staff/patricio-g-herbst
[103] See Limin Jao's page at https://www.mcgill.ca/dise/Jao.
[104] See Tyler Marghetis' bio at https://www.tylermarghetis.com/about.

David Alexander Reid, 1992; Michael Richard Woodall, 1994; Astrid Defence, 1994; Enikö Eva Kiefer, 1995; Luis A. Saldanha, 1995; Elazar Meroz, 1997[105]; Stéphanie Ouellette, 1998; Michael Haddad, 1999; Sonia Manago, 1999; Richard A. Masters, 2000; Steve Mazerolle, 2000; Edouard Saidah, 2000; Nathalie Prévost, 2001 ; Krishpa Kotecha, 2002; Valérie Lebel, 2002; Varda Levy, 2002; Viktor Freiman, 2003; Hong Yue, 2003; Nadia Kamar, 2004; Estelle Fainsilber; 2004; Georgeana Bobos-Kristof, 2006; Shiva Gol Tabaghi, 2007; Laura Anne Gauthier, 2007; Michael Besner, 2007; Linda Agoune, 2007; Angela Michelle Smart, 2008; Andreea Pruncut, 2008; Jitesh Patel, 2008; Gul Yayli, 2008; Pak Kei Wong, 2008; Hasmig Keropian, 2008; Edit Baloghne Farago, 2008; Tyler Marghetis, 2009; Aline Djerrahian, 2009; Alnusayan Iman, 2009; Mahmoud Saleha, 2010; Ivats Boyanov Dakov, 2011; Nicolas Boileau, 2012; Carol Beddard, 2012; Zoltan Lazar, 2012; Maria Tutino, 2013; Julie Shapin Thésée, 2014; Maria-Josée Bran-Lopez, 2015; David Pearce, 2015; Jean-Marc Miszaniec, 2016; Rima Taki, 2017; Rawan Bashmail, 2017; Sabrina Giovanniello, 2017; Romel Ekladios, 2018; Natalia Vasilyeva, 2018; Iulian Frasinescu, 2018.

The MTM courses I developed and taught

I also enjoyed planning and teaching the MTM courses. The first MTM course I taught was on philosophy of mathematics. I taught it in the lounge of the department, on the Loyola campus, on the first floor of Hingston Building, sitting around in comfortable armchairs and couches and discussing philosophical questions in an informal manner. Nick Herscovics taught his MTM classes in this lounge, and I thought that I should do this, too. Later on, I taught in more regular classrooms.

I don't remember all the courses I taught, but I kept materials from some of them, and even shared a part of those materials online, so I can mention these courses here.

In the fall of 1999, I taught MATH 645 Topics in Mathematics Education Research on the topic of the Theory on Didactical Situations in Mathematics. This theory, developed in France by Guy

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[105] I cannot resist saying a few words about my collaboration with Elie Meroz. My early research in mathematics education was on epistemological obstacles related to limits. Some mathematicians claim that the mathematical notion of limit solves the paradoxes of infinity such as Dichotomy or Achilles and the Tortoise. Does the concept of limit, however, *explain* the paradoxes? This can be regarded as an empirical question or a philosophical one. In Poland, I directed a master's thesis which explored this issue by a group of technically oriented students:

Cała, B. (1989): Spontaniczne wyjasnienia paradoksów Achilles i Dychotomia przez uczniów oraz akceptacja przez nich wyjasnien opartych na sumowaniu szeregów. (Spontaneous explanations of the paradoxes of Achilles and Dichotomy by students and their acceptance of explanations by summing of series) - in Polish.

Here, in Montreal, I suggested exploring this question to an MTM student who seemed to be philosophically inclined, Eli Meroz. He was familiar with the Jewish religion and also happened to tutor in mathematics some students who attended a Jewish religious school. This was a golden opportunity. I asked him if he could present the paradox of Achilles and the Tortoise to the students and ask for their spontaneous explanations. Assisting Eli in his research and his writing of the thesis was a fascinating journey. (Meroz, 1997).

Brousseau and his collaborators, was becoming better known in the English speaking community of mathematics educators, thanks to the publication of a translation into English of Brousseau's papers presenting the fundamental tenets and concepts of the theory (Brousseau, 1997). Since my entry into research in mathematics education took place through reading Brousseau's papers, and, more generally, through the "French didactique"[106], I thought I understood the theory well enough to be able to teach it. I wrote lectures[107] for the course, where I explained the concepts of the theory by examples and applications and sometimes by trying to reveal their sources in the intellectual climate of the times when they were invented. I devoted a whole course to this theory only once in my life. Later, the theory became part of the course, MATH 624 Topics in Mathematics Education; Topic: *Instructional design in mathematics education*[108].

The last time I taught this course, in the fall of 2014, the first assignment I gave to the students, was to choose a misconception in mathematics and write a Socratic dialogue modelled on Plato's "Meno" where the teacher first elicits the misconception in a student and then helps them to overcome it. Students wrote beautiful and witty dialogues and we had a lot of fun sharing them in class.

I knew that the assignment would be a success; I had already previously used the same assignment in a course on the history of mathematics education that I taught in the fall of 2012: MATH 624L Topics in Mathematics Education; Topic: *History of Mathematics Education*. I received some very interesting examples of students' creative writing from this course. (I have posted the readings and oral and written assignments from the course on Academia.edu[109].)

To write such a dialogue, one has to be aware of some of the more common students' misconceptions, such as the one addressed in "Meno": that to double the area of a square, it is enough to double its side. More generally, to design a lesson on a mathematical idea, one has to be aware of the difficulties that students may have in grasping and learning it. It therefore

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[106] Thanks are due largely, to Nicolas Balacheff, whom I met in 1980, in Poland, when he was visiting Madame Zofia Krygowska, a leading figure on the Polish mathematics education scene in those times. It was Nicolas who introduced me to empirical research in mathematics education, based on observation of small groups of students solving problems. He sent me Brousseau's papers and invited me to visit the institute IMAG in Grenoble in 1981. I also went to two "Ecoles d'été de didactique des mathématiques" where Brousseau's theory was discussed and further developed.

[107] My lectures for the course can be viewed and downloaded from my personal webpage at: https://annasierpinska.rowebca.name/index.php?page=courses (9 lectures in separate files) or from Academia.edu at https://www.academia.edu/27542930/Lecture_notes_on_the_Theory_of_Didactic_Situations_in_mathematics (all lectures in one single pdf file)

[108] The course outline of the fall 2014 version of the course is available from the department's webpage at https://www.concordia.ca/content/dam/artsci/math-stats/docs/Outlines%202014-2015/MATH624M_2_14.pdf (or, at least, WAS available on May 31, 2020).

[109] Materials for teaching a course on the history of mathematics education can be found at https://concordia.academia.edu/AnnaSierpinska/Course-Notes

made sense to develop a course on students' difficulties in mathematics, which, ideally, an MTM student would take before the course on instructional design. This became the course, MATH 630 Topics in the Psychology of Mathematics Education; Topic: *Research into students' difficulties in mathematics*. My MTM courses always addressed the problem of students' difficulties, but this course was explicitly focused on them. In my last decade of working at Concordia, I taught it three times, in 2011, 2013 and 2017[110]. After I retired, I compiled my materials for the course and posted them on Academia.edu[111]; I thought they might be useful for other teachers.

The last course I want to talk about here is the course on popularization of mathematics that I developed and taught (with great joy!) twice, in the fall terms of 2010 and 2015, as the course MATH 624J Topics in Mathematics Education; Topic: *Popularization of Mathematics.* In 2015, the course outline described the purpose of the course as follows:

> *Graduates of preservice teacher education programs are expected to become competent and professional teachers. An MTM graduate should have the ambition to become a primus inter pares, a leading figure inspiring his or her colleagues to go beyond the mere call of duty in exercising their profession. For example, it is well known that students' attitudes towards mathematics and their learning of it can be strongly influenced by the poor public image and poor understanding of mathematics in the society at large. No subject is as often hated or feared as mathematics. Parents, whether knowingly or not, transfer their attitudes – positive or negative – towards mathematics to their children, and this has an impact on their learning. It makes sense to believe, therefore, that, by reaching out to students' parents, and, more generally, to the communities in which the students live, engaging them in activities aimed at improving their appreciation and understanding of mathematics, may have a beneficial effect on students' learning, attention and behavior in the mathematics class.*
>
> *Reaching out with mathematical activities to the public beyond the school walls is part of what has been called 'popularization of mathematics'. To design and conduct an engaging and useful popularization activity is not an easy task. There are many examples of popularization, but there seem to be no techniques, no methods or guidelines on how to do it. There has been little reflection, so far, on what distinguishes popularization of mathematics from other mathematical activities, and what makes a popularization activity "good" and how can we distinguish between a well-conceived and poorly conceived popularization activity. One of the few steps in this direction has been made by one of my doctoral students, Klara Kelecsenyi. Her dissertation – available from the Spectrum Research Repository in the library – will be one of the main*

[110] On June 1, 2020, the course outline for the 2017 version of MATH 630 Topics in the Psychology of Mathematics Education; Topic: Research on students' difficulties in mathematics was still on the department's website at https://www.concordia.ca/content/dam/artsci/math-stats/docs/Outlines%202017-2018/MAST652AA_MATH630AA_2_17.pdf

[111] Materials for the course on students' difficulties posted on Academia.edu at https://concordia.academia.edu/AnnaSierpinska/Course-Notes

> *resources for us in this course. Like her, we will, in this course, study popular works in mathematics, devise criteria of analyzing and assessing them, design popular activities and seek to evaluate their effects on our audiences.*

There were three assignments in the course.[112]

In the first assignment, students were asked to "Choose ten popularization works of different categories (books about mathematics, recreational mathematics, websites, public lectures, movies, works of art such as paintings or sculptures, etc.) and construct hypothetical responses of their authors to a survey question such as, 'Complete the sentence, Math is…'. They were also asked to compare these authors' responses.

Assignment 2 was given as follows: "Describe your criteria of 'good popularization'. Take two examples of popularization, belonging to the same category (e.g. both from the category of recreational mathematics, or both from the category of popular writing, or two web sites, or two works of art, etc.), one that you would qualify as 'good popularization' and the other - as 'poor popularization' and justify your assessment."

The third assignment asked students to conduct a popularization activity and to evaluate its effect on the audience. In 2010, each student conducted an individual activity, independently from others. In 2015, the whole class worked as a team to organize a "Math trail" activity in Montreal, in a walk from Dawson College to the Hall Building at the Concordia University campus downtown. This was a much more complex affair, and I think we all learned a lot from it, experiencing the joys of creation of mathematical activities linked to remarkable places in that walk, and the frustration of the realization of the trail when only a few Dawson students showed up to take the walk.

[112] A detailed description of the 2010 version of the course is available on my personal webpage in the "rowebca" domain (https://annasierpinska.rowebca.name/index.php?page=courses); it contains the formulation of the three assignments in the course and responses to them of those students who gave me permission to publish their work online.

More information about D. A. Wheeler, N. Herscovics, S. H. Erlwanger and N. Hardy

David H. Wheeler

David Wheeler was a professor at Concordia from 1976 until his retirement in 1990. He is credited for changing the profile of the MTM from a program aiming only at upgrading or brushing up teachers' mathematical knowledge to one also preparing interested and capable students to engage in empirical and theoretical research in mathematical education, mainly of the kind reported in annual conferences of the International Group for the Psychology of Mathematics Education (PME). This research seeks to understand, represent and influence "how students learn mathematics, how teachers teach mathematics, and how mathematicians, teachers and students do mathematics"[113]. He did not adhere to any particular school of thought, ideology or theory in mathematics education. He promoted reflection and critical thinking. His goals for MTM students were probably the same as for the journal he started, *For the Learning of Mathematics* in whose description he wrote:

> *The journal aims to stimulate reflection on and study of the practices and theories of mathematics education at all levels; to generate productive discussion; to encourage enquiry and research; to promote criticism and evaluation of ideas and procedures current in the field.*[114]

The photo of David Wheeler in Figure 7 is taken from the archives of Simon Fraser University[115].

[113] The quote is taken from the webpage of the PME organization at http://www.igpme.org/
[114] Under "Journal info" on the page of the journal at https://www.jstor.org/journal/forlearningmath.
[115] At http://www.sfu.ca/sfublogs-archive/research/davidwheeler/91_about-david-wheeler.html

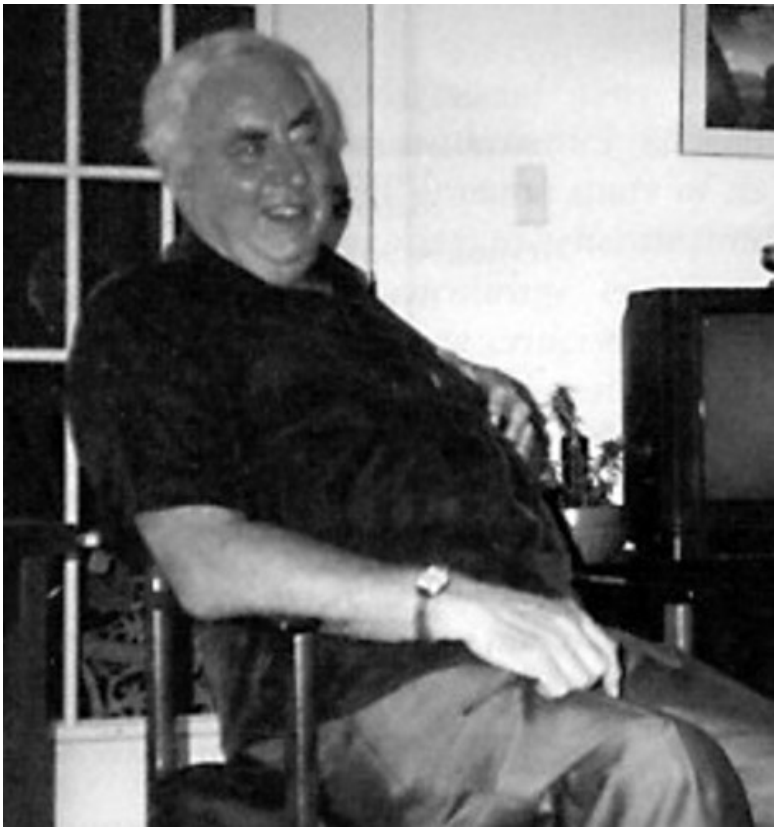

Figure 7. David H. Wheeler

David Wheeler played an important role in the development of the mathematics education research community in Canada. He inspired the creation of the Canadian Mathematics Education Study group[116]. There is a story of how he did that in the piece titled, ”Some aspects of David Wheeler’s career in Canada” that William Higginson and Joel Hillel wrote in 1997, for a tribute to David Wheeler that was published in the bulletin[117] of the International Commission on Mathematical Instruction[118].

He was an international figure in the area of mathematics education. He chaired the international program committee for the ICME congress in Quebec City in 1992, and was a member of such committees for two other ICME’s. He knew many people in the international mathematics education community and was able to bring some of them to visit Concordia. As Joel Hillel and William Higginson wrote after David Wheeler’s death in 2000 in a CMESG newsletter,

> *When David Wheeler came to Concordia, the Mathematics Department's main commitment to education was through the Master's in the Teaching of Mathematics*

[116] http://www.cmesg.org
[117] https://www.mathunion.org/fileadmin/IMU/Organization/ICMI/bulletin/42/icmi.wheeler.html
[118] https://www.mathunion.org/icmi

> *programme (MTM). At the time, the MTM consisted essentially of content courses in mathematics and did not provide a broader based vision of mathematics education. Wheeler brought a wider perspective to the programme by weaving in the pedagogical, psychological, historical and philosophical connection to mathematics education. He introduced faculty and students alike to Piaget's work in developmental psychology, to Pólya's classical writing on heuristics and problem solving, to Lakatos' perceptive insights of the process of mathematization and proof. He brought the international mathematics education community to Concordia by attracting visiting scholars and lecturers. By co-directing the first FCAR (Fonds pour la formation de chercheurs et l'aide a la recherche du Quebec) three year research project on problem-solving, he helped launch the research aspect of the mathematics education group. Within five very short years, the group has achieved an international reputation, with a very high research profile and an active role in many national and international organizations.*[119]

The CMESG Newsletter of December 2000 contains also memories of other people about David Wheeler. Further memories can be found in *For the Learning of Mathematics* 21(20), 2001.

The Brenda Carter and David Wheeler Memorial Award[120] at Concordia bears his name; Brenda was David's wife and a mathematics teacher. The award is open to all Concordia students, graduate and undergraduate. It is enough to satisfy one of the following criteria: 1. Having a proven interest in the teaching of mathematics; 2. Having returned to academic study after some interruption; 3. Having overcome some particular handicap or obstacle; 4. Having been active as a volunteer in some social, political or professional organization.

Nicolas Herscovics

Nicolas Herscovics became a member of the department in 1965, before MTM was established (1967) and before SGWU became Concordia University (1974) and the department of mathematics was renamed "department of mathematics and statistics". He had a BSc in Physics and a Master's degree in Mathematics, and, in SGWU, he was promoted to the rank of Associate Professor in 1973 (Concordia's Thursday Report of January 18, 1990[121]). After SGWU became Concordia University, he went to the Université de Montréal and, in 1979 he obtained a PhD in mathematics education. His supervisor was Jacques C. Bergeron and the title of his thesis was "La compréhension de la droite au niveau secondaire".

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[119] http://www.sfu.ca/sfublogs-archive/research/davidwheeler/uploads/2010/09/David_Wheeler_CMESG_Dec2000.pdf

[120] http://www.concordia.ca/students/financial-support/scholarships-awards/internal/call-for-applications--the-brenda-carter-and-david-wheeler-memor.html

[121] https://www.concordia.ca/content/dam/concordia/offices/archives/docs/concordia-thursday-report/1989-90/Thursday-Report-1990-January-18.pdf

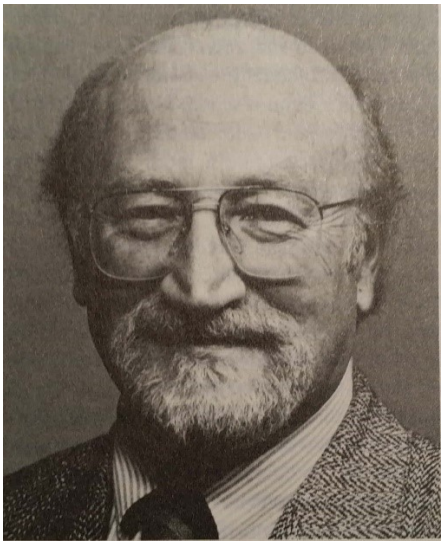

Figure 8. Nicolas Herscovics[122]

He devoted his professional life entirely to this area, working at it with passion and unparalleled energy, until his untimely death in 1994. He was always bursting with ideas, and working on papers, brief research reports for conference proceedings, journal articles, and longer review papers synthesizing large areas of research findings (such as the now classic paper on cognitive obstacles related to learning algebra at the secondary school level (Herscovics, 1989)). He participated in creating organizations that supported the dissemination of research in mathematics education internationally (PME and PME-NA), and in the organization of their conferences. In particular, he was a member of the organizing committee of the PME conference in 1987 in Montreal.

He supervised several students (in MTM and in other universities) not abandoning them even when he was already very ill with cancer. Among the MTM students who wrote a thesis under his supervision, were, Carolyn Kieran, Louise Chalouh, and Pat Lytle:

Carolyn Kieran, Fall 1979, Thesis: "Constructing meaning for the concept of equation"

Louise Chalouh, Spring 1986, Thesis: "The construction of meaning for algebraic expressions"

Patricia Lytle, Fall 1992, Thesis: "Use of a neutralization model to develop understanding of integers and of the operations of integer addition and subtraction"

He supervised MTM projects as well, for example, the project of Dona Marceline Rita Gajasinghe (Fall 1993), on the topic of teaching fractions in the context of mixing solutions.

He also supervised or co-supervised doctoral theses of students from other universities, among them, those of Nicole Nantais and Bernard Héraud:

[122] https://www.concordia.ca/content/dam/concordia/offices/archives/docs/concordia-thursday-report/1989-90/Thursday-Report-1990-January-18.pdf

Nicole Nantais, 1989, PhD at Université de Montréal, co-supervision with J.C. Bergeron, Dissertation : « La mini-entrevue, un nouvel outil d'évaluation de la compréhension mathématique au primaire »

Bernard Héraud, 1992, PhD at Université de Montréal, co-supervision with J.C. Bergeron, Dissertation : « Genèse de la notion de mesures spatiales: Construction de la mesure bilineaire »

Nicole Nantais' doctoral thesis was awarded the Dieter Lunkenbein prize for the best doctoral thesis in Quebec in 1989[123].

More information about Nicolas Herscovics can be found in the *Obituary* written by his colleagues in the department for Concordia's Thursday Report January 13, 1994[124], and in a moving *In Memoriam* written by his mentor, doctoral dissertation supervisor and colleague Jacques Bergeron (Bergeron, 1994), and published in *Educational Studies in Mathematics*.

Nicolas Herscovics helped students financially with his grant money when alive, and he continues to do so after his death with a scholarship established in his name, The Nick Herscovics Memorial Scholarship. This scholarship was dedicated to MTM students until 2019. After the suspension of the program, the award was extended to all graduate students pursuing full-time studies in the area of mathematics education in the Department of Mathematics and Statistics at Concordia University[125].

Stanley Erlwanger

Stanley Erlwanger was a professor at Concordia from 1980 until his leave in 1991.

Figure 9. Stanley H. Erlwanger[126]

[123] See interview with N. Nantais published in 1991 in Bulletin de l'AMQ at https://www.amq.math.ca/ancien/archives/1991/2/1991-2-part10.pdf

[124] https://www.concordia.ca/content/dam/concordia/offices/archives/docs/concordia-thursday-report/1993-94/Thursday-Report-1994-January-13.pdf

[125] https://www.concordia.ca/artsci/math-stats/programs/awards-scholarships.html

[126] This photo of Stanley H. Erlwanger is reproduced from his portrait in the History of ICMI online document at https://www.icmihistory.unito.it/portrait/erlwanger.php

He died in 2003, in Zimbabwe. His obituary in Concordia's Thursday Report[127]reads as follows:

> *Professor Emeritus Stanley H. Erlwanger, who taught mathematics at Concordia from 1980 to 1991, died of cancer on June 18 in Harare, Zimbabwe.*
>
> *He was known internationally for his insightful work on children's conceptions and misconceptions of mathematics. Before joining the Department of Mathematics and Statistics in 1980, he had a rich career as a high school math teacher and Senior Inspector of Schools (Zimbabwe), Head of the Department of Science Education and Dean of the Faculty of Education at the University of Botswana and Swaziland, and a director of major projects on the mathematical development of children at Florida State University and at the Institute for Advanced Study at Princeton.*
>
> *While at Concordia he was also actively involved with mathematics teachers at all levels and in initiating curricular changes….* (Concordia's Thursday Report of September 11, 2003 (Vol. 28, no. 1)[128]

His contributions to mathematics education are discussed in a paper by fellow researchers Bob Speiser and Chuck Walter in (Speiser & Walter, 2004)**.** There is also an entry about him on the History of ICMI webpage[129]. Note that on 12 Erlwanger's publications listed on this page, six are co-authored with members of the Mathematics and Statistics at Concordia University. S. H. Erlwanger supervised at least two MTM theses: those of Elaine Rhona Wisenthal (graduated in Spring 1985), titled, "High school students' understanding of percent", and of David Malcolm Grant Salusbury (Spring 1990), titled, "Structural spaced review: A case study 1988/89".

Nadia Hardy

Nadia Hardy joined the Department of Mathematics & Statistics of Concordia University in 2009. Her research area at the doctoral level was Mathematics Education; earlier that year, she defended her doctoral thesis, titled "Students' models of the knowledge to be learned about limits in college level Calculus courses. The influence of routine tasks and the role played by institutional norms"[130] in the department. She did her master's degree at McGill University, in mathematics, in the area of graph theory. Thus, being trained both as a mathematician and a mathematics educator, she was a perfect fit for a position supporting research in mathematics education in the Department: She could teach both strictly mathematics courses and mathematics education courses and supervise graduate students in mathematics education.

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[127] http://ctr.concordia.ca/2003-04/sept_11/13-in-memoriam/index.shtml
[128] https://www.concordia.ca/content/dam/concordia/offices/archives/docs/concordia-thursday-report/2003-2004/Thursday-Report-2003-09-11.pdf
[129] http://www.icmihistory.unito.it/portrait/erlwanger.php
[130] Nadia Hardy's PhD thesis is available from the Concordia University research repository Spectrum at https://spectrum.library.concordia.ca/976385/.

Figure 10. Nadia Hardy in 2009[131].

In the MTM program, Nadia Hardy developed and taught several mathematics education courses:

MATH 641 Survey of Research in Mathematics Education – Winter 2011; Winter 2014

MATH 652 Seminar in Mathematics Education; Topic: Theoretical frameworks in mathematics education – Winter 2012; Winter 2015

MATH 618J Topics in the Applications of Mathematics; Topic: Graph theory and its applications – Fall 2011

MATH 640A Topics in Logic; Topic: Introduction to mathematical logic from Euclid to Gödel – Fall 2012; Winter 2014; Winter 2016

Nadia Hardy's research was based in the Anthropological Theory of the Didactic (Chevallard, 1992; 1999) which looks at mathematics education as an "organization" within an "institution", in a technical sense of these terms. Around 2015, she became interested in experiencing the institutional aspects of university education as a practitioner, not only as a theoretician and researcher, and not only as a subject of the institution who has to follow its rules and regulations but as a member who can actively participate in the making of these rules. So, in 2014, she ran for the position of the chair of the department. She was successful, and occupied this position from July 2014 until July 2016, when she was offered the position of Vice-Provost, Faculty Development and Inclusion. She accepted and has held this role since August 2016. Since July 1, 2019, she also serves as Interim Deputy Provost[132]. (This is being written in March 2020).

Nadia Hardy supervised nine MTM students, three theses and six projects, listed below:

[131] This photo of Nadia Hardy is taken from the departmental website at https://www.concordia.ca/artsci/math-stats/faculty.html?fpid=nadia-hardy/

[132] See https://www.concordia.ca/provost/about/areas/faculty-development.html

1. Spring 2010 Caroline Lefèbvre, Project: “Probability vs typicality: Making sense of misconceptions”
2. Spring 2011 Catherine Poisson, Project: “Mathematical and didactical organization of Calculus textbooks”
3. Fall 2012 Christy Lyons, Thesis: “The role of blogs in secondary mathematics education”
4. Spring 2013 Megan Danielle Tremblay, Project: “Mathematics education and vocational training programs in Quebec”
5. Fall 2013 Dalia Challita, Thesis: “Providing college level calculus students with opportunities to engage in theoretical thinking”
6. Spring 2014 Natalie Donfrancesco, Project: “Mathematical and didactic inconsistencies in typical college algebra textbooks”
7. Spring 2015 Steven Pileggi, , Thesis: “Breaking customs in an Algebra classroom for mature students and providing them with opportunities to engage in theoretical thinking”
8. Spring 2016 Sabrina De Cicco, Project: “Online mathematics courses and the development of virtual tutorials for MATH 200”
9. Spring 2018 Kyle Duelund, Project: “Do you control your emotions or do they control you?” (co-supervision with A. Sierpinska)

She has also supervised several MSc/MA and PhD students’ theses in the area of mathematics education. Two of these MSc students started their graduate studies at Concordia as MTM students: Erin Murray and Hadas Brandes. Erin entered MTM in 2013 and withdrew from it in December 2015, with all her coursework completed, and only the thesis left to be done. She entered the MSc in mathematics program in January 2016, and with 12 credits worth of pure mathematics courses transferred from MTM to MSc, she took two extra pure mathematics courses, completed her thesis and graduated in the spring of 2017. The thesis was titled, “Exploring Narrative Inquiry in an Introduction to Mathematical Thinking Course”. Hadas’ story was analogous: she started the MTM in 2014, then switched to the MSc in mathematics program in January 2017, with 12 credits, took two pure mathematics courses and graduated in the same year with a thesis, titled “The bare necessities for doing undergraduate Multivariable Calculus”. She then went on to do her PhD in mathematics in the department, also with Nadia Hardy as her supervisor. Nadia Hardy also supervised the doctoral work of Sarah Mathieu-Soucy and Laura Broley. Sarah was enrolled in the INDI program; she was funded by SSHRC with a Joseph-Armand Bombardier scholarship; her thesis was titled “Becoming a Teacher: An Inquiry into the Experiences of Novice Teachers of Mathematics in CEGEP” and she successfully defended it in 2019. Laura Broley is currently (August 2020) enrolled in the doctoral program in

Mathematics & Statistics. Her studies were funded by a SSHRC Vanier scholarship. She defended her thesis on 27 August 2020, successfully and with brio[133].

So even now, as vice-provost, Nadia Hardy continues to supervise several masters and doctoral students in mathematics education, helping to keep alive the flame of research in this area in the department.

In the next, and final, section our intention was to list all students who completed the MTM degree.

[133] Achieving this kind of vivacity of performance was amazing given that the defense took place in a Zoom meeting.

MTM graduates

Our intention here is to list all MTM graduates, from the beginning of the program's existence till the suspension of admissions in 2016 and the graduation of the last remaining MTM student in 2019. It is based on the archived convocation booklets from the years 1969-2004, graduated student files kept in the department of mathematics and statistics for the years 2005-2018, and the student information system (SIS) of the university. Some information about the graduates is missing, for example, the name of the supervisor, the title of a project…. We just hope we haven't omitted any names.[134]

Fall 1969

1. Denise Théorice Bouchard
2. Carole Margaret Channer
3. Melinda Wild El-Guindy
4. Enid Helen Lofthouse
5. Gabriel Macoosh
6. Robert Austin Cook Mearns
7. David Morris
8. Abraham Schwartz
9. Bernard Shoub
10. Donald Anthony Clive Tavares
11. Edward Zegray

Spring 1970

12. Reynold Lee Stanley Clarke
13. Gilles C. Des Ruisseaux
14. Lucy A. Mackinnon
15. Raymond Paquin
16. Oksana Shulakewych
17. Stephen Walter Whitty

Fall 1970

18. Arthur Anshel Abramovitch
19. Richard Allaire
20. George Bashconji
21. André Brodeur

[134] If you think we have omitted a name of an MTM graduate and know this name, please write to Anna Sierpinska at anna.sierpinska@concordia.ca or to the chair of the Department of Mathematics and Statistics and somebody will verify the information and, if true, the name will appear in the next edition of this document.

22. Nicolino Carmosino
23. Edward Entus
24. George Daniel Leighton
25. Raymond Monette
26. Pritam Singh Phull
27. Frank Ricci
28. Souad Sewiha Tadros
29. Giuseppe Vannelli
30. Wilbert Wesley Ward
31. Guy Zanaitis

Fall 1971

32. Sidney Brown
33. Anita Cadoch
34. Thérèse Proulx Doherty
35. Roger Doran
36. Patrick Dunn
37. Kevin James Gallacher
38. Jean Denis Groleau
39. Doreen Holmes
40. Shiraz Hosein
41. Nourhan Ilter
42. Robert Kurys
43. Smittar Phull
44. Rhona Pinsky
45. Robert Taylor

Spring 1972

46. Robert Austin Adams
47. Eva Fortunée Alalouf
48. Albert Joseph-Théodore Barrette
49. Andrée Marie Monique Brunette
50. Arthur Patrick Burke
51. Roderick Paterson Hart
52. Gerald Hecht
53. Michael Joseph McNally
54. Vincenzo Nichilo
55. John Walter Pichovich
56. John Lorne Walsh

Fall 1972

57. Sigitas Vincas Barsauskas
58. Harold Clamen
59. Clément Dassa
60. Monaldo Carmelino d'Orazio
61. Gaetano Garofalo
62. Kurt Vincent Keipert
63. Ignatius Simeon Kemdirim
64. Jean Kessous
65. Yi-Yuin Lin
66. Horace Carlton Mahabir
67. Hugh Victor Mahabir
68. Robert Francis Shaughnessy
69. Gerald Anthony Smith
70. Claude Teasdale
71. Yvon Vautour

Spring 1973

72. Edith Annette Clark
73. John Robert Dick
74. Lloyd Matthew Elder
75. Rita Gagnon
76. Norman Gingras
77. Yvon Hadd
78. Marcel St. Jean
79. Ian Randall Smith

Fall 1973

80. Paul John d'Antoni
81. René Pierre Bureau
82. Robert George Butler
83. Joseph Gilles Chabot
84. Alma Noreen Dobson
85. Jayakar Sugandharaju James
86. Laurence Claire Juneau-Ouellet
87. Louis Mariano
88. Robert Michel McKenzie
89. Ian William Rennie
90. Giovanni Rossi
91. Helmy Tewfik Said

92. Bruno Santaguida
93. Cornelius Stehouwer
94. Peter Charles William Woodwark
95. Miroslaw Julien Zdanowicz

Spring 1974

96. John Leonard Block, Project: “The binomial theorem” (Supervisor F. Szabo)
97. Wendy Colpitts, Project: “Computational complexity” (Supervisor F. Szabo)
98. George Alexander Tony Kirkwood
99. Janet Elisabeth Klein
100. Roland Vincenzo Purchio, Project: “Bilinear transformations” (Supervisor F. Szabo)

Fall 1974

101. Joseph Paul Marino
102. Barbara Maude Jean Rhodes, Project: “Compactness” (Supervisor F. Szabo)
103. Philip Younglai, Project title : “The Fundamental Theorem of Algebra” (Supervisor F. Szabo)

Spring 1975

104. Céline Archambault
105. Eugene Frederick Chee, Project: “The trigonometric functions” (Supervisor F. Szabo)
106. Jean Claude Côté
107. Louis André Del Castilho
108. Farag Gennaoui, Project: « L’enseignement de la parabole » (Supervisor F. Szabo)
109. Lise Lambert, Project: « Programmation linéaire » (Supervisor F. Szabo)
110. Nigoghos Mardirossian, Project: « Programmation linéaire » (Supervisor F. Szabo)
111. Nabil Elias Rayes, Project: « Programmation linéaire » (Supervisor F. Szabo)
112. Sidney David Shtern

Fall 1975

113. Lomer Rolland Barriault
114. Angelo Salvatore Clemente
115. Arthur Jacob Hammerschmidt
116. Penelope Sue Harfield
117. Michael di Maio
118. Jean-Claude Morin

119. Jean-Pierre Nelson, Project: « L'enseignement des treillis au niveau collégial » (Supervisor F. Szabo)
120. Kevin Francis O'Connor
121. Ira Roth
122. Danielle Scheier
123. Carlo Sollazzo
124. Robert Philip Spencer
125. Armand John Vergil

Spring 1976

126. Monique Jeanne Baril
127. Albert Salvatore Benghiat
128. Leonard Jacob Cohen
129. Marguerite Andre De Serres
130. Cornelia Lucie Martha Droge
131. Bruno Marie Louis Garin
132. Michael William Hennessy
133. Tulsi D. Majumdar
134. Joseph Rabinovich
135. Richard Taylor

Spring 1977

136. Marcel Elbaz
137. Gabriel Giangi
138. Peter Kerklaan
139. André Laperrière
140. William John Cameron McDougall
141. Eileen Mary-Alice McGurk
142. Saverio Mirarchi
143. Mordechai Michael Perl
144. Richard Michael Wyjad

Fall 1977

145. Tammy Bailis
146. Anne Couto
147. John Di Domenico
148. Kevin Edmund Gearey
149. Rolf Willi Richter
150. Roger Warmoes
151. Margaret Susan Whight, Project: "The trisection problem" (Supervisor F. Szabo)

Spring 1978

152. John Gaspar, Project: “Logic, and its relationship to function sets, and arithmetic” (Supervisor F. Szabo)
153. Edelmera Novlet Harrison
154. Frank Lovasco, Project: “An introduction to vectors” (supervisor F. Szabo)
155. Domenico Elio Manzo
156. Rocco Antonio Polifroni, Project: “Part I - Study of integral equations of the form [x**n+x**n=z**n]. Part II - Geometric constructions with straightedge and compass” (Supervisor: F. Szabo)

Fall 1978

157. Liliane Beaulieu, Project: “A guide to connectedness” (Supervisor: F. Szabo)
158. Desmond Peter Conrad, Project: “MATH 552 Vector geometry: A set of transparencies”
159. John Joseph Andrew Greene, Project: “Informal geometry and mensuration: A mathematics program for grade seven”
160. Gary George Millward, Project: “MacDonald manufacturing company: A simulation for general mathematics classes”
161. Rampersad Rameshwar
162. Wallace George Richardson, Project: “Probability and Pi”
163. Robert Harold Schuster

Fall 1979

164. Harry Stephens Fischer
165. Norman Leo Hannan, Project: “Old MacDonald builds a fence: a tape-slide presentation on area and perimeter”
166. Patricia Claire Hayes
167. Carolyn Kieran, Thesis: “Constructing meaning for the concept of equation” (Supervisor: N. Herscovics)
168. Brian Patrick Kirlin, Thesis: “Set theory in high school: an error in timing?” (Supervisor: Victor Byers)
169. Louise Maurice, Project: “MATH 110 Expérience au Collégial”
170. Ellen Rhoda Rabin, Project, title unknown

Spring 1980

171. Claude Avakian, Project: “Probability and Pi (videotape)”
172. Victor Douglas Cunningham
173. Allan A. Harvey, Project: “Integer test”
174. Albertha A. Henry, Project: “’Why Johnny can’t add?’ The failure of the New Math”

175. Robert Kin-Wah Lam, Project: “The Pythagorean”
176. Deborah Ship Litvack, Project: “The teaching of the quadratic function to enriched students”
177. Paul William Rhodes, Project: “The going up, the going down and the lying over theorems”
178. Peter Gavin Williams

Fall 1980

179. Joanne M. Campbell, Project: “MATHEMATICS 522/532 – A course, an exam”
180. Shuk Han Lai, Project: “Math avoidance”
181. Gershon Mandel
182. Suzanne McCoy, Project: “Orthogonal transformations in Euclidean space : A vector approach”
183. Jean-Marc Tourigny

Spring 1981

184. Luc Amyotte
185. Arthur Jr. Philip Galardo
186. Ramchand Janack, Project: “Mathematical refreshments: Polygonal numbers”

Spring 1982

187. Carol Mary Côté
188. Ivan Gombos
189. Francesco Mario Grosso
190. Sayed Hassan Navab Kashani
191. Serge Monosiet
192. Marcel Roy
193. Allan Arthur Seddon
194. Shan-How Tsao
195. Wai Har Wong

Spring 1983

196. Catherine Pik-Wah Fai
197. Pierre Goyette
198. Lesley Olive Lee, Thesis: “A study of the problem solving behavior of twenty four students” (Supervisor: D. H. Wheeler)
199. John William Samuel McClintock
200. Richard Marian Orzechowski
201. Robert Roy
202. Anne Helen Elisabeth Self
203. Regina Solomon

Spring 1984

204. Robert Benjamin
205. Celine Jeannine Boislard
206. William Joseph Bromley
207. Leonard Allan Savage
208. Norman Saxe
209. Mary Angela Wells

Spring 1985

210. Marlene Bissessar
211. Louise Chalouh, Thesis: “The construction of meaning for algebraic expressions” (Supervisor: N. Herscovics)
212. William Errol Brent Corrigan, Thesis: “Report of LOGO project: third grade students Allancroft school, 1983- 84” (Supervisor: J. Hillel)
213. Nuncio Falco
214. Bruce William Glasspoole, Thesis: “Relations between mathematical pedagogy and the history of mathematics”[135]
215. Alexander Thomas Kerr, Thesis: “An alternative textbook for the slow-stream mathematics curriculum”[136]
216. Maria Cécile Lavoie, Thesis: “An exploratory study in LOGO: Grade four students” (Supervisor: J. Hillel)
217. Anne Winnifred Luckow, Thesis: “Mathematics and programming concepts used by fifth grade students in a LOGO turtle graphics environment” (Supervisor: J. Hillel)
218. Palakuzhivil Varughese Oommen
219. Tina Antonina Ottoni
220. Helton Pachake
221. Daniel Alexander Sipos
222. Shanti Sarup Soni
223. Edith-Margit Weberbauer
224. Ellen J. Wernecke
225. Elaine Rhona Wisenthal, Thesis: "High school students’ understanding of per-cent” (Supervisor: S. Erlwanger)

Spring 1986

226. Morris Barcan
227. Maria Judite Pereira Gonçalves
228. Olasseril Joseph Joseph

[135] Thesis not available in Spectrum. Supervisor unknown.
[136] Thesis not available in Spectrum. Supervisor unknown.

229. Peter James Leesinsky, Thesis: "Problem solving with the calculator"[137]
230. James Mc Diarmid
231. Angela Maria Monteferrante-Orsi
232. Louise Robert, Thesis: "L'ordinateur dans nos classes: un exemple en Calcul Integral"[138]
233. Diane Elisabeth Roszkowski

Spring 1987

234. Dunbar Philip Ferdinand
235. Krystyna S. Kurcin
236. Mohammad Wasi, Thesis: "Historical development of non-Euclidean geometries"[139]

Fall 1987

237. Marion Barfurth, Thesis: "Learning math through LOGO: a procedure- based approach" (Supervisor: J. Hillel)
238. Farida Ismail

Fall 1988

239. Debbie Ann Cheifetz
240. Deanna Mendelson, Thesis: "Using LOGO as a tool to develop the concept of angle in secondary I" (Supervisor: J. Hillel)

Spring 1989

241. Ellen May Cooper
242. Robert Anthony Henderson
243. Stephen Matthew Lau

Fall 1989

244. Man Ching Vong

Spring 1990

245. Marie-France Lavallée
246. David Malcolm Grant Salusbury, Thesis: "Structural spaced review: A case study 1988/89" (Supervisor: S. Erlwanger)
247. B. James Sutcliffe

[137] Thesis available in Spectrum. Supervisor not acknowledged.
[138] Thesis not available in Spectrum.
[139] Thesis not available in Spectrum.

Spring 1991

248. Donald Castle

Fall 1991

249. Craig Sterrett Bowers, Thesis: **"**On teaching and learning the concept of fractal" (Supervisor: W.P. Byers)

Spring 1992

250. Elaine Marie Landry, Thesis: "An inquiry into students' obstacles (cognitive, epistemological, and ontological) affecting the understanding of mathematical infinity." (Supervisor: W.P. Byers)
251. David Alexander Reid, Thesis**: "**Mathematical induction: an epistemological study with consequences for teaching" (Supervisor: A. Sierpinska)

Fall 1992

252. Marino Discepola
253. Patricia Ann Lytle, Thesis: "Use of a neutralization model to develop understanding of integers and of the operations of integer addition and subtraction" (Supervisor: N. Herscovics)
254. Lorna Marjorie Townson

Spring 1993

255. Joanne Avraam
256. Theresa Mae Galarneau-Bagshaw
257. Japheth Gumbo Ododa Origa, Thesis title: "Learning difficulties encountered in the teaching and learning of linear programming" (Supervisor: J. Hillel)

Fall 1993

258. Dona Marceline Rita Gajasinghe, Extended project on teaching fractions in the context of mixing solution (exact title unknown; Supervisor: N. Herscovics)
259. Jean Pichette, Extended project, title unknown

Spring 1994

260. Frances Ann Lortie
261. Michael Richard Woodall, Thesis: "The design, implementation, and evaluation of a self-paced calculus course" (Supervisor: A. Sierpinska)

Fall 1994

262. Astrid Defence, Project: "The readability of the mathematics textbook: with special reference to the mature student" (Supervisor: A. Sierpinska)

Spring 1995

263. Enikö Eva Kiefer, Project: Designing a test for identifying mathematically gifted students (exact title not known; supervisor: A. Sierpinska

Fall 1995

264. Luis A. Saldanha, Thesis title: “The notions of linear independence/dependence: a conceptual analysis and students' difficulties” (Supervisor: A. Sierpinska)

Spring 1996

265. Nancy Braganza
266. George Lerakis, Project
267. Marina Padamadan, Project

Fall 1996

268. Alemu Bekele, Project
269. Susan Elizabeth Broden, Project
270. Sadik Kadhim

Spring 1997

271. Wesley Mark Sanders

Fall 1997

272. Marc J. Corbeil, Project
273. Elazar Meroz, Thesis title: “Mathematical, philosophical, religious and spontaneous students' explanations of the paradox of Achilles and the tortoise” (Supervisor: A. Sierpinska)

Spring 1998

274. Abbas Abdulridha, Project: “Extraction of roots and binomial expansion – the work of Al-Kashi” (Supervisor: M. Malik)
275. Anne Marie Hegney, Thesis: “Teaching transformational geometry using Cabri geometry" (Supervisor: J. Hillel)
276. Hana Joudi, Project: “On geometrical constructions & the work of abu'l wafa" (Supervisor: M. Malik)
277. Pankaj Kumar Kamthan, Thesis title: “Dynamical systems education on the www” (Supervisor: E.J. Doedel)
278. Stéphanie Ouellette, Project: “Teaching elementary probability: how to overcome common misconceptions” (Supervisor: A. Sierpinska)

Fall 1998

279. Rakesh Rajan Singh Bisen, Thesis: “Matrix inequalities and their applications to statistics” (Supervisor: T.N. Srivastava)

Spring 1999

280. William Nevin
281. Alhan Zahrai

Fall 1999

282. Michael Haddad, Thesis: “Difficulties in the learning and teaching of linear algebra - a personal experience” (Supervisor: A. Sierpinska)
283. Sonia Manago, Thesis: “Students’ difficulties in the transition from arithmetic to algebra” (Supervisor: A. Sierpinska)

Spring 2000

284. Peter Balyta, Thesis: “The effects of using motion detector technology to develop conceptual understanding of functions through dynamic representation in grade 6 students” (Supervisor: G. Boulet)
285. Zareh Iskenderian

Fall 2000

286. Richard A. Masters, Thesis: “The effect of students' physics background on their understanding of linear algebra” (Supervisor: A. Sierpinska)
287. Steve Mazerolle, Thesis: « Les traces de la psychologie de l'apprentissage dans l'évolution des curriculums et des manuels scolaires, au Québec, de 1982 à 1997. Recherche historique » (Supervisor: A. Sierpinska)
288. Edouard Saidah, Thesis: “The role of the graphing calculator in teaching and learning of the concept of function: observations in a mathematics classroom” (Supervisor: A. Sierpinska)

Spring 2001

289. Sarkis Guleserian
290. Pasquale Pietrantonio

Fall 2001

291. Alfred Anaeto Nnadozie, Thesis: “Application of statistics to the study of cognitive processes in mathematics education - a case study of theoretical thinking as a factor of students' success in linear algebra” (Supervisor: T.N. Srivastava)
292. Nathalie Prévost, Thesis: « Utilisation de la théorie des processus de la communication d’Alex Mucchielli dans l'étude des enjeux occasionnés par l'introduction

de la réforme (actuelle au Québec) dans nos écoles primaires » (Supervisor: A.Sierpinska)

Fall 2002

293. Anthony Garone, Project: “Mathematical modeling: an observation” (Supervisor: J. Hillel)
294. Atef Guirguis, Project: “Students' understanding of geometric transformations” (Supervisor: J. Hillel)
295. Krishpa Kotecha, Thesis: “A pilot study on the phenomenography of problem solving” (Supervisor: A. Sierpinska)
296. Valérie Lebel, Project: “Utilisation de Cabri-geomètre pour enseigner les homothéties“ (Supervisor: A. Sierpinska)
297. Varda Levy, Project: “Understanding symmetry” (Supervisor: A. Sierpinska)

Spring 2003

298. Viktor Freiman, Thesis: “Identification and fostering of mathematically gifted children at the elementary school” (Supervisor: A. Sierpinska)
299. Foroozan Rokni, Project: “Adult understanding of fractions to decimal conversion and conversion facilitation via manipulative aids” (Supervisor: J. Hillel)

Fall 2003

300. Hong Yue, Project: “Cultural background and mathematics achievement” (Supervisor: A. Sierpinska)

Spring 2004

301. N. Kamar, Project: “History of algebra & algebra in modern schools: cognitive obstacles” (Supervisor: A. Sierpinska)
302. Estelle Fainsilber, Project: “Gender and achievement in mathematics” (Supervisor: A. Sierpinska)

Fall 2005

303. Zuneira Gopaul
304. Chantal Desrosiers, Thesis: “The connection between student understanding of composite functions and their subsequent understanding of the chain rule" (Supervisor: J. Hillel)

Spring 2006

305. Leslie Bennett
306. Ingrid Kupka
307. Sandi Mak
308. Qing Wen Zhang

Fall 2006

309. Xinzhou Ning
310. Georgeana Bobos-Kristof, Thesis: “Effect of weekly quizzes on the development of students' theoretical thinking” (Supervisor: A. Sierpinska)

Spring 2007

311. June Saunders
312. Melina Vigorito
313. Stephane Beauregard
314. Karen Paige Husbands

Fall 2007

315. Kenneth J.A. Macdougall, Thesis: “The residential school experience: residual effects upon first nations students in their understanding and mastery of tasks within the mathematics curriculum (Supervisor: J. Hillel)
316. Shiva Gol Tabaghi, Thesis: “APOS analysis of students' understanding of logarithms” (Supervisor: A. Sierpinska)
317. Laura Anne Gauthier, Thesis: “Relations between confidence and achievement in adult education students” (Supervisor: A. Sierpinska)
318. Michel Besner, Project: “Can an Algebra Pretest be an accurate predictor of success in a social science Calculus class?” (Supervisor: A. Sierpinska)
319. Linda Agoune, Project: “A study of web-based mathematics courses” (Supervisor: A. Sierpinska)

Spring 2008

320. Katya Blagoeva Terziyska Nikolova
321. Angela Michelle Smart, Thesis: “Introducing angles in grade four: a realistic approach based on the van Hiele model (Supervisor: A. Sierpinska)
322. Andreea Pruncut, Thesis: “A study of students' theoretical thinking in a technology-assisted environment” (Supervisor: A. Sierpinska)
323. Nadine Anne Pomilio
324. Jitesh Patel, Project: “Teaching logarithms in an adult education centre and in high school” (Supervisor: A. Sierpinska)
325. Bahaa Bu Dargham, Project: “Integrating technology to pre-university math classes” (Supervisor: J. Hillel)
326. Giulia Grace Chiappetta

Fall 2008

327. Gul Yayli, Project: Teaching and learning limits (Supervisor: A. Sierpinska)

328. Pak Kei Wong, Project: “Sexuality profiling and queer students' experiences in mathematics classrooms” (Supervisor: A. Sierpinska)
329. Hasmig Keropian, Project: “Design of an effective pre-calculus assessment test” (Supervisors: A. Sierpinska and Harald Proppe)
330. Edit Baloghne Farago, Project: “The design and the analysis of tasks for assessment tests” (Supervisor: A. Sierpinska and Harald Proppe)

Spring 2009

331. Kelly Giesen
332. Maximilien Ohayon
333. Tyler Marghetis, Thesis: “When a proof is a satisfactory explanation for the reader?” (Supervisor: A. Sierpinska)
334. Saeed Ghafouri, Project: “History and current explorations in Operations Research” (Supervisor: Sun Wei)
335. Emmanuel Georgantakis
336. Kevin Flood
337. Aline Djerrahian, Project: “Online Calculus course with special reference to limits and continuity” (Supervisor: A. Sierpinska)

Fall 2009

338. Alnusayan Iman, Project: “A study of two Linear Algebra textbooks from the point of view of LACSG Principles: Concreteness, Necessity and Generalizability” (Supervisor: A. Sierpinska)
339. Hawarneh Shahnaz

Spring 2010

340. Michèle Ansha Lalanne
341. Caroline Lefèbvre, Project: “Probability vs typicality: Making sense of misconceptions” (Supervisor: N. Hardy)
342. Mahmoud Saleha, Project : ”Mathematics in the workplace: The case of calculating the future value of an investment” (Supervisor: A. Sierpinska)

Spring 2011

343. Ivats Boyanov Dakov, Project: “An evaluative study of technology in mathematics education from 1960 to 2010” (Supervisor: A. Sierpinska)
344. Calvin Grant
345. Catherine Poisson, Project: “Mathematical and didactical organization of Calculus textbooks” (Supervisor: N. Hardy)
346. Sujata Saha
347. Lee Zentner

Spring 2012

348. Carol Beddard, Thesis: “Motivating adult students taking a basic algebra course in a university setting” (Supervisor: A. Sierpinska)
349. Nicolas Boileau, Thesis: “Two secondary school mathematics teachers' use of technology through the lens of instrumentation theory” (Supervisor: A. Sierpinska)

Fall 2012

350. Zoltan Lazar, Thesis: “Teaching the singular value decomposition of matrices: a computational approach” (Supervisor: A. Sierpinska)
351. Christy Lyons, Thesis: “The role of blogs in secondary mathematics education” (Supervisor: N. Hardy)

Spring 2013

352. Nelly Khachatryan
353. Megan Danielle Tremblay, Project: “Mathematics education and vocational training programs in Quebec” (Supervisor: N. Hardy)
354. Maria Tutino, Thesis: “Experimenting with discursive and non-discursive styles of teaching absolute value inequalities to mature students” (Supervisor: A. Sierpinska)

Fall 2013

355. Dalia Challita, Thesis: “Providing college level calculus students with opportunities to engage in theoretical thinking” (Supervisor: N. Hardy)

Spring 2014

356. Adrian Bradley
357. Natalie Donfrancesco, Project: “Mathematical and didactic inconsistencies in typical college algebra textbooks” (Supervisor: N. Hardy)
358. Julie Shapin Thésée, Project: “Does solving a situational problem involving inequalities engage students in theoretical thinking?” (Supervisor: A. Sierpinska)

Spring 2015

359. Steven Pileggi, Thesis: “Breaking customs in an Algebra classroom for mature students and providing them with opportunities to engage in theoretical thinking” (Supervisor: N. Hardy)

Fall 2015

360. Maria-Josée Bran-Lopez, Thesis: “Sources of mature students’ difficulties in solving different types of word problems in mathematics” (Supervisor: A. Sierpinska)
361. David Pearce, Thesis: “Mathematics for engineers and engineers’ mathematics” (Supervisor: A. Sierpinska)

Spring 2016

362. Sabrina De Cicco, Project: "Online mathematics courses and the development of virtual tutorials for MATH 200" (Supervisor: N. Hardy)
363. Daniel Gernazian
364. Jean-Marc Miszaniec, Thesis: "Designing effective lessons in probability: A pilot study focused on the illusion of linearity" (Supervisor: A. Sierpinska)
365. Kristina Perella

Spring 2017

366. Rima Taki, Project: "Activities to promote behaviors characteristic of mathematically gifted children in all children" (Supervisor: A. Sierpinska)
367. Rawan Bashmail, Project: "Students' difficulties with algebraic notation" (Supervisor: A. Sierpinska)

Fall 2017

368. Sabrina Giovanniello, Thesis: "What algebra do Calculus students need to know?" (Supervisor: A. Sierpinska)

Spring 2018

369. Matthew Chan
370. Krista Day
371. Kyle Duelund, Project: "Do you control your emotions or do they control you?" (Supervisors: N. Hardy and A. Sierpinska)
372. Romel Ekladios, Project: "Identifying mathematically gifted students" (Supervisor: A. Sierpinska)
373. Natalia Vasilyeva, Thesis: "How do students know they are right and how does one research it?" (Supervisor: A. Sierpinska)
374. Mahmoud Yassin, Project: "On the Riemann Zeta function and the consequences of its truth" (Supervisor: Hershy Kisilevsky)

Fall 2018

375. Iulian Frasinescu, Thesis: "Understanding inquiry, an inquiry into understanding: A conception of inquiry-based learning in mathematics" (Supervisors: N. Hardy, A. Sierpinska)

Spring 2019

376. Mandana Mir, Project: "It's about time" (Supervisor: Fred Szabo)

We also list here the graduates of the **Diploma in the Teaching of Mathematics**:

Spring 1976

Victor Douglas Cunnigham

Alexandra Gamulka

James McDiarmid

Cathy J. O'Brien

Lily Petros

Fall 1977

Gianfranco Buffa

Patrick Joseph Kelly

Tawfik Zaki Tadros

Fall 1979

Gwenyth Mary Doucet

Huguète Picard-LeBlanc

Judith Rosenswig

George Michael Steszyn

Fall 1980

Chaim Adler

Spring 1984

Myrna Blanche Brien

References

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Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education , 59*(5), 389-407.

Beddard, C. (2012). *Motivating adult students taking a basic algebra course in a university setting.* Montreal: Concordia University. Retrieved from https://spectrum.library.concordia.ca/973724/

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Bobos, G., & Sierpinska, A. (2017). Measurement approach to teaching fractions: A design experiment in a pre-service course for elementary teachers. *International Journal for Mathematics Teaching and Learning, 18*(2), 203-239. Retrieved July 3, 2017, from http://www.cimt.org.uk/ijmtl/index.php/IJMTL/article/view/65/34

Bobos-Kristof, G. (2007). *The development of the modern integration theory from Cauchy to Lebesgue : a historical and epistemological study with didactical implications. Masters thesis.* Montreal: Concordia University. Retrieved from https://spectrum.library.concordia.ca/975476/

Bobos-Kristof, G. (2015). *Teaching fractions through a Measurement Approach to prospective elementary teachers: A design experiment in a Math Methods course. A PhD Thesis.* Montreal : Concordia University.

Brousseau, G. (1997). *Theory of didactical situations in mathematics.* Dortrecht: Kluwer.

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Chevallard, Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en Didactique des Mathématiques 19(2)*, 221-266.

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Freiman, V. (2003). *Identification and fostering of mathematically gifted children at the elementary school. Master's thesis.* Montreal: Concordia University. Retrieved from https://spectrum.library.concordia.ca/2099/

Giovanniello, S. (2017). *What algebra do Calculus students need to know?* Montreal: Concordia University. Retrieved from https://spectrum.library.concordia.ca/982929/

Haddad, M. (1999). *Students' difficulties in the learning of linear algebra: A personal experience.* Montreal: Concordia University. Retrieved from https://spectrum.library.concordia.ca/view/creators/Haddad=3AMichael=3A=3A.html

Hardgrove, C. (1963). CUPM Report on the training of teachers of elementary school mathematics. *American Mathematical Monthly, 70*(8), 870-877.

Herscovics, N. (1989). Cognitive obstacles encountered in the learning of algebra. In S. Wagner, & C. Kieran, *Research issues in the learning and teaching of algebra* (pp. 60-86). Reston, Virginia: Erlbaum.

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Appendix – Support letters for the MTM program

Support letter from Nicolas Balacheff

Dr. Gail Valaskakis
Dean, Faculty of Art and Science
Concordia University
L - AD 324

Fax : 514 848 2877

le 17 février 1997

Monsieur le Recteur,

C'est avec étonnement et regret que j'apprends que l'existence du Master in the Teaching of Mathematics de l'Université de Concordia est remise en question.

Votre Université a acquis au cours des vingt dernières années une réputation internationale dans le domaine de la recherche en Education Mathématique, certains membres de votre département de mathématiques sont ou ont été des pionniers remarqués et respectés du développement de ce domaine.

L'éducation est une préoccupation des nations et une priorité de la communauté internationale affichée par ses diverses instances. Cependant il reste tentant de réaliser des économies au détriment de ce domaine plus vulnérable que d'autre et dont le retour d'investissement paraît moins spéctaculaire. Pourtant les progrès de l'enseignement des mathématiques auquel a largement participé le département de mathématiques de votre Université est une des conditions du développement scientifique et économique. Une formation avancée sur l'enseignement des mathématiques est un moyen de diffusion des connaissances issues de la recherche, c'est aussi une source de forces humaines pour avancer les travaux.

La suppression du Master in the Teaching of Mathematics de l'Université de Concordia serait un coup fatal porté à une équipe excellente dont je peux témoigner ici du rayonnement international et de la qualité des collaborations.

Je formule le voeux, Monsieur le Recteur, que vous puissiez revenir sur cette décision et contribuer ainsi à préserver un capital que vous avez contribué à constituer et dont j'ai toutes les raisons de penser que les fruits futurs bénéficieront à l'image de l'Université de Concordia.

N. Balacheff
Directeur de recherche CNRS
Ancien Président du groupe International
"Psychology of Mathematics Education"

Support letter from Jean Dionne, co-signed by Lucie DeBlois

FACULTÉ DES SCIENCES DE L'ÉDUCATION
Département de Didactique,
Psychopédagogie et Technologie éducative

Cité universitaire
Québec, Canada G1K 7P4

Le 19 février 1997

Dr Gail Valaskakis
Dean of the Faculty of Arts and Science
Concordia University
7141 Sherbrooke street west
Montréal, Québec
H4B 1R6

Madame Valaskakis,

Nous venons d'apprendre avec désolation la menace de fermeture qui pèse sur votre programme de maîtrise en didactique des mathématiques (Master Teaching of Mathematics). Cette désolation s'explique par la conscience nette qu'une telle fermeture constituerait un appauvrissement pour la communauté québécoise en général et plus particulièrement pour la communauté des gens préoccupés par les questions et problèmes touchant l'enseignement et l'apprentissage des mathématiques.

Cet appauvrissement serait d'autant plus évident que l'on connaît la qualité des personnes qui oeuvrent dans ce programme, plusieurs d'entre elles étant reconnues au plan international comme chercheurs et professeurs. Cette qualité du programme est aussi très justement soulignée par celle des étudiants qui en sont issus. Elle a été récemment mise en évidence par l'attribution du prix Dieter-Lunkenbein qui a couronné le mémoire de maîtrise de l'un d'entre eux, Monsieur Luis Saldanha. Il a travaillé sous la direction de Mme Sierpinska. Son étude portait sur les notions de dépendance et indépendance linéaires. Enfin, cette qualité du programme est aussi largement révélée par celle des cours qui sont offerts. La cosignataire de cette lettre, au moment où elle complétait sa scolarité de doctorat à l'Université Laval, a même choisi de voyager de Québec à Montréal chaque semaine pendant toute une session de manière à pouvoir suivre un de ces cours alors donné par le regretté professeur Nicolas Herscovics.

Ce ne sont là que quelques arguments qui militent en faveur du maintien du programme. Nous espérons que les administrateurs de l'Université sauront faire preuve d'imagination pour trouver ailleurs les ressources financières qui permettront d'assurer le maintien d'un programme qui apparaît comme un service au Québec.

Veuillez agréer, madame, l'expression de nos sentiments les meilleurs.

Lucie DeBlois, Ph.D. Jean J. Dionne, Ph.D.

Support letter from Jean-Luc Dorier

Laboratoire LEIBNIZ
Dr. Jean-Luc DORIER (Maître de Conférences).
UMR 5522 CNRS-UJF-INPG
Institut IMAG
46, ave Félix Viallet
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Grenoble 20/2/97

To Dr; Gail Valaskakis,
Dean, Faculty of Arts and Science,
Concordia University.

Dear Colleague,

I have been informed by Pr. Hillel of some worrying events that occured recently in your university: massive cutbacks have put some projects in danger.

I had the privilege of being invited by Pr. Hillel in your university in March 1993, I had then appreciated this staying which offered me the opportunity to have very constructive exchanges with researchers in Concordia. I had especially appreciated the work made in the Master in the Teaching of Mathematics, offering a unique place for in-service trainning at a research level for teachers.

Since then, my collaboration with Pr. Hillel and Dr. Sierpinska is very active, I am part of a project entitled " Meaning and communication in linear algebra: the case of the teaching and learning of linear operator and its eigenvalues with the use of the computer algebra system Maple", supported by the Social sciences and Humanities research Coucil (File n° 410-96-0741).

I hope that the financial situation will not put in danger the activities of my colleagues, especially with regard to the MTM.

I would like to express my support to my colleagues, and hope that you will take it into account in your future decisions.

With best reagrds, sincerely yours,

Dr. Dorier.

Support letter from Tommy Dreyfus

Dear Dr. Lightstone

You may remember that during 1996/97, I am spending my sabbatical year as a visiting professor in Concordia's Department of Mathematics and Statistics where I am associated with the Master's in Teaching Mathematics (MTM) program. It is not without reason, that I have chosen Concordia, after having been in contact with several other places including the University of California at Berkeley, the University of Massachusetts in Dartmouth, and the University of Sidney. Due to its MTM program Concordia has attracted faculty members and developed a research program in advanced mathematical thinking which is outstanding. The opportunity to collaborate with some of the world's top researchers in the field, within the MTM framework was the main reason for me to come to Concordia. And as you know, I am not alone. Just this month, Dr. Laurinda Brown from Bristol University in England, Dr. Gerald Goldin from Rutgers University in New Jersey, USA, and Dr. David Reid, from MNU in St. John, NFLD are visiting here as well.

This is not the place to detail the academic achievements of the MTM faculty; suffice it to point to the great honor bestowed upon Dr. Anna Sierpinska by the International Congress for Mathematics Education by inviting her to give a plenary lecture in front of their 1996 Congress in Seville, Spain which was attended by over 4000 participants. (Every four years, no more than four mathematics educators are being so honored.)

Not only the faculty of the MTM program contribute to Concordia's name but also its graduates and students who teach and do research in universities, colleges and schools throughout Quebec (and beyond). In fact, several of them teach at Concordia's mathematics department itself. This is largely due to the uniqueness of the program: Its association with mathematics department led to a strong emphasis on college and undergraduate mathematics education; this stress feeds back into an improvement of the teaching of mathematics at Concordia itself (in addition to the CEGEPs and schools. The MTM program thus makes an invaluable contribution to the province's education system in general, and to Concordia in particular.

Considering the remarkable academic level of the MTM program and of the associated faculty members, I was extremely surprised to read that you have proposed halting admissions to the program and thus effectively closing it. I

understand that the prime motive for your proposal is not academic but financial. But why should financial reasons lead to the closure of one of Concordia's best programs, and to the risk of losing two internationally renowned faculty members? This question obtains added urgency since it is quite obvious that the MTM program does not cost the university very much, if anything at all. I am not well versed in the financial details but the fact speak for themselves: This year, four courses are being taught in the framework of the program, of which one is accessible (for credit) also to students from other graduate mathematics programs. Even taking into account some cost for administration etc., this program can hardly cost more than one position, and this for almost 30 enrolled students: An excellent ratio for any graduate program! Now consider on top of this the research grant money brought in by the faculty members associated with the program for research which is directly related to the program - and even a good part of the cost of that single position should be covered.

For the reasons set out above, I am urging you to retract your proposal to halt admissions to the MTM program on March 1, 1997, and thus to preserve one of the factors which contribute to the renown of Concordia University throughout Canada and the world.

Sincerely,

Dr. Tommy Dreyfus, Visiting Professor
Professor at the Center for Technological Education Holon
Tel Aviv University, Israel

Support letter from Colette Laborde

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Dr Colette Laborde

à Dr. Gail Valaskakis
Dean, Faculty of Arts and Science
Concordia University
L - AD 324
Fax 00 1 514 848 2877

Dear Colleague,

Je me permets de vous écrire en français que je manie mieux que l'anglais. Je peux ainsi m'exprimer avec plus de nuances. L'objet de cette lettre est la surprise et la consternation qui m'ont saisie, lorsque j'ai appris les menaces de fermeture qui planent sur le Master in the Teaching of Mathematics du département de Mathématiques et Statistiques de votre université.

Ayant eu le privilège d'être professeur invité à ce même département du 1er Janvier 1991 au 30 Juin 1991, et d'avoir fait deux cours à ce master, je voudrais vous expliquer les raisons de ma consternation. En France, nous défendons depuis plus de 20 ans, l'importance pour les recherches sur l'enseignement des mathématiques d'être conduites au sein des départements de mathématiques. Les carrières des universitaires engagés dans ce type de recherche sont gérées par les commissions relevant des mathématiques. Notre expérience nous renforce dans notre point de vue. Il est très important pour la qualité des recherches dans ce domaine de se faire en lien avec la discipline mère, qui offre une garantie épistémologique. J'avais été très intéressée par la place du MTM à l'université Concordia qui, contrairement à la plupart des universités nord-américaines, se situe dans le département de mathématiques. Il offre ainsi aux étudiants une formation mathématique solide alliée à une formation approfondie en didactique des mathématiques. Les deux composantes de la formation en bénéficient: ouverture et prise de distance par rapport aux mathématiques qu'offre une formation à la recherche sur l'enseignement des mathématiques, connaissances solides en mathématiques nécessaires pour étudier les problèmes de leur apprentissage.

Lors de mon séjour à l'université Concordia et de mon enseignement au MTM, j'avais pu apprécier le fonctionnement de cet enseignement: importance de la population étudiante, assiduité, sérieux et investissement de ces étudiants, lien avec la recherche de par l'assistance au séminaire et la préparation de thèse. J'avais pu constater que le MTM attire des étudiants d'horizons divers, enseignants en exercice, étudiants étrangers et étudiants venant d'autres universités de la ville de Montréal. Le programme proposé m'avait semblé particulièrement attractif de par l'équilibre qu'il offre entre une formation aux mathématiques, à l'épistémologie, et à la recherche en didactique des mathématiques.

J'avais connu le MTM bien avant ma venue, en particulier grâce à Joël Hillel et David Wheeeler, chercheurs de renommée internationale.1991 a aussi été l'année où Anna Sierpinska a été nommée professeur au département de mathématiques de l'université Concordia. Son renom international n'est plus à vanter, elle est vice-présidente du comité exécutif de l'International Commission on Mathematical Instruction, commission de l'Union Mathématique Internationale, cette même commission qui décerne les médailles Fields en mathématiques. J'ai toujours pensé que le département de mathématiques et de stastistiques de l'Université Concordia avait une grande chance de posséder en son sein de tels membres et c'est une des raisons qui m'avait poussée à venir y effectuer un séjour. Ce n'est pas un hasard si des personnalités de l'enseignement des mathématiques connues internationalement y ont aussi effectué des séjours.

Une activité de recherche sans possibilité de formation est stérile, elle est sujette à se fermer, et à se scléroser. Ne craignez vous pas que les chercheurs de renom en didactique des mathématiques de ce département ne le quittent pour chercher un environnement plus dynamique à la hauteur de leur travail ? La fermeture du MTM porterait certainement atteinte au rayonnement du département de mathématiques et de statistiques de l'Université Concordia et par là à l'université même. Ce serait un suicide et je suis sûre que la réflexion et l'examen attentif des données du problème vous amèneront à la conclusion de l'intérêt pour l'université Concordia de maintenir le MTM.

Avec l'expression de mes meilleurs sentiments.

C. Laborde

Support letter from Richard Noss

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Dr Gail Valaskakis
Dean, Faculty of Arts and Science
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Fax: 514 848 2877

Dear Dr Valaskakis

I have just received word that your mathematics education programme, MTM, has been proposed as a candidate for closure. I am writing to express my surprise, and to ask you to reconsider the matter.

Despite the tragic death of Nick Herscovics, the Concordia mathematics education group has continued to maintain the highest profile within the international research community. I will not single out individuals, but it is no exaggeration to claim that recent congresses and conferences I have attended would have had a very different character without one or more representatives from your institution. Concordia's work in the field has a well-deserved reputation for intellectual integrity and innovative theoretical and empirical investigation.

One particularly important feature of Concordia's contribution, is the strong links that have always existed between mathematics and mathematics education. This may not be unique in the field, but it is rare; and where such a relationship does exist, it seldom does so as productively as at Concordia. I do not want to overstate my case: but if the department were unable to attract research grants, or to continue to contribute to the international mathematics education research, it would be nothing short of disastrous for our community.

As an senior academic at a British research university, I assure you that I am familiar with the rigours of financial constraints, and I sympathise with the difficulties you and your colleagues must be facing. I urge you, however, to consider carefully the long-term damage that might be done by closing the MTM programme: these are not always easy to quantify as they involve international prestige, the status of your institution and other measures which take time to emerge. They will emerge in time. When they do, I think you will find that rebuilding what is lost can be considerably more difficult and expensive than any saving in the short term.

My request that you reconsider is, in essence, a selfish one: if Concordia's role in my field were to be diminished, the field would be the poorer. As academics we have a vested interest in nurturing scholarship of the highest quality wherever we find it: I urge you to recognise the quality of the mathematics education group in Concordia, and to search for constructive ways to resolve matters which will not involve irreparable damage to such a productive and scholarly group.

Yours sincerely,

Richard Noss, PhD
Professor of Mathematics Education

Support letter from Renan Samurçay

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Cognition en situation de travail et de formation
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Renan SAMURÇAY
Chargée de recherche au CNRS

Saint-Denis, 18-Oct-19

Dr. Gail VALASKAKIS
Dean, Faculty of arts and Science
Concordia University
L-AD 324
Montreal PQ
CANADA

Monsieur Le Doyen,

J'ai appris par mes collègues du Département de Mathématique de l'Université de Concordia que le programme de Didactique des Mathématiques allait être supprimé à cause de restriction budgétaire. Dans cette lettre que je vous adresse je voudrais d'une part, argumenter les raisons pour lesquelles je trouve déplorable une telle décision et d'autre part vous inviter en tant que scientifique et non seulement en tant qu'administrateur à faire votre possible pour le maintien de ce programme dont dépend la survie de l'équipe de recherche en didactique de mathématiques de l'Université de Concordia qui bénéficie d'une grande reconnaissance scientifique internationale par la communauté.

De Août au Décembre 1984, j'ai eu la chance d'être invitée comme chercheur associé pour réaliser un programme de recherche avec le Prof. Joël Hillel sur "La genèse d'apprentissage de la notion de variable informatique par les élèves de 8 à 12 ans". A cette occasion j'ai bénéficié d'un environnement scientifique exceptionnel avec l'équipe de chercheurs en didactique de mathématique, notamment avec Nicolas Herscovics, David Wheeler, Joël Hillel et tant d'autres... Cette période a été pour moi très productive, à la fois en termes de production scientifique (des rapports de recherche, des communications écrites aux congrès et des articles ont été produits) et en termes de construction commune de concepts et méthodes grâce au foisonnement des outils de pensée venant des cultures scientifiques différentes.

Je voudrais ci-dessous souligner quelques arguments importants en faveur du maintien du programme de didactique des mathématiques dans le département des mathématiques de votre université :

1. La didactique en tant que science de l'action a besoin d'une part d'être encrée dans la discipline dont elle étudie et analyse la transmission et d'autre part, de s'appuyer sur des recherches de terrains qui nécessite la formation des acteurs. Donc c'est en formant les enseignants de mathématique à la recherche par la recherche que la discipline pourrait produire des résultats scientifiques crédibles.

2. La didactique, comme d'autres disciplines scientifiques, a besoin d'avoir des lieux de production de connaissances. Elle ne peut pas être réduite seulement à la pratique qui n'aura pas les moyens de porter des regards critiques sur elle-même.

3. Le développement et la qualité de production d'une discipline dépendent de l'existence et de la pérennité des équipes de recherche, des jeunes en formation et des moyens qui assurent la continuité des programmes engagés. La mise en avant des critères économiques seules ne peut pas être acceptée par les scientifiques comme un argument valable, ni par la société d'ailleurs.

4. Enfin, les disciplines scientifiques comme les mathématiques, la physique, etc...ne peuvent pas démissionner de s'interroger sur les problèmes de transmission des savoirs qu'elles construisent. C'est aussi un devoir de citoyen pour les scientifiques de ces disciplines d'oeuvrer pour l'accès à l'éducation scientifique de bonne qualité des générations futures non seulement dans leur pays mais également dans des pays en voie de développement. La formation des futurs formateurs et enseignants de ces pays fait partie des missions des programmes de didactique des mathématiques.

Le programme de didactique de mathématique de l'Université de Concordia doit continuer à exister pour produire comme avant à la fois des connaissances scientifiques de qualité (la revue Learning of Mathematics joue de ce point de vue un rôle important dans la diffusion) et des idées nouvelles qui ne peuvent provenir d'une augmentation de potentiel humain qualifié et moyens de production de la recherche.

Convaincue de la juste lutte que vous allez mener pour défendre le maintien et le développement du programme de didactique des mathématiques, je vous prie, Monsieur Le Doyen, d'agréer mes sentiments respectueux.

R.Samurçay

Support letter from Heinz Steinbring

UNIVERSITÄT DORTMUND
Fachbereich Mathematik
Institut für
Didaktik der Mathematik
Prof. Dr. Heinz Steinbring

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Zu erreichen mit der
S-Bahn-Linie S1
Haltestelle "Dortmund-Universität"
H-Bahn im Universitätsbereich

Dr. Gail Valaskakis
Dean, Faculty of Arts and Science
Concordia University,
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Kanada

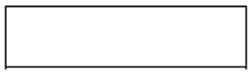
Ihre Nachricht vom Mein Zeichen Dortmund, den
27. 2. 97

Betr.: Intention to eliminate MTM

Dear Dr Valaskakis,

I have heard about the intentions to eliminate the mathematics education program "Mater in the Teaching of Mathematics (MTM)" at the mathematics departement of Concordia University. I am very upset about these plans. Since some years I am now collaborating very closely with mathematics education researchers at Concordia University; these cooperations have been very stimulating and effective. As far as I know, there is been quite a number of other cooperative work between other internationally renowned researchers in mathematics education and researchers from Concordia. The Research projects and the education programs in mathematics conducted at Concordia University have a very strong international reputation. I think, that with the intention to close the MTM program, the Concordia University really risks a very serious loss of international reputation, which up to now has been connected with mathematics education at Concordia.

Dear Dr Valaskakis, I hope that the responsible institutions and that you could reconsider the intention of closing the MTM program and that you could base your definite decision about MTM really on a serious base of evaluating the scientific quality and the international reputation of the mathematics education research at Concordia University. I strongly hope that you could really come to a better decision about the future of the MTM program.
Yours sincerely



(Prof. Dr. Heinz Steinbring)

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[i] The recommendations of the MAA whose influence can be detected in the proposed MTM curriculum seem to be the following:
For high school teachers (called "Level III teachers"):
"[T]he minimum requirements for high school teachers should consist of the following:
A. Three courses in Analysis...
B. Two courses in abstract algebra…
C. Two courses in geometry beyond analtic geometry…
D. Two courses in probability and statistics…
E. Two upper class elective courses, e.g., introduction to real variables, number theory, topology, history of mathematics, or numerical analysis (including use of high-speed computing machines).
One of these courses should contain an introduction to the language of logic and sets, which can be used in a variety of courses. (…)
[For college level teachers:] For teachers of the elements of calculus, linear algebra, proabability, etc., we recommend a Master's deggree, with at least two thirds of the courses being in mathematics, and for which an undergrauate program at least as strong as Level III training is a prererquisite."
(The Mathematical Association of America, 1960, p. 635)