GAME-THEORETICAL ANALYSIS OF NETWORK FORMATION AND DRUG PRICING

Noémie Cabau

A thesis
in
The Department
of
Economics

Presented in Partial Fulfillment of the Requirements
For the Degree of Doctor of Philosophy
Concordia University
Montréal, Québec, Canada

September 2020

Concordia University

School of Graduate Studies

This is to certify that the thesis prepared

By: **Ms. Noémie Cabau**

Entitled: **Game-theoretical analysis of network formation and drug pricing**

and submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy (Economics)

complies with the regulations of this University and meets the accepted standards with respect to originality and quality.

Signed by the final examining commitee:

Dr. Xiao Huang ______________________ Chair

Dr. Francis Bloch ______________________ External Examiner

Dr. Nicolas Klein ______________________ External Examiner

Dr. Françoise Forges ______________________ Examiner

Dr. Szilvia Pápai ______________________ Examiner

Dr. Sidartha Gordon ______________________ Co-supervisor

Dr. Ming Li ______________________ Co-supervisor

Approved by Dr. Christian Sigouin ______________________

Chair of Department or Graduate Program Director

September 2, 2020

Dr. PASCALE SICOTTE

Dean of Faculty of Art & Science

Abstract

Game-theoretical analysis of network formation and drug pricing

Noémie Cabau, Ph.D.

Co-tutelle between Concordia University & Université Paris Dauphine PSL

2020

This thesis consists of three chapters. Chapter 1 offers a model of endogenous network formation where agents form connections strategically to maximize information diffusion in their group. The model builds on the idea that groups of individuals who are better informed take better collective decisions. The equilibrium and efficient networks have simple architectures: e.g., loops and flowers that may or may not include all agents. If frictions happen during information transmission, optimal architectures centralize the connections around a single agent. Chapters 2 and 3 aim at rationalizing the use of secret rebates during the negotiations on drug prices. Secret rebates enable to hide the price a country pays for a pharmaceutical product from other countries. Two models of the interactions between public payers (countries) and a monopolist pharmaceutical firm are presented to rationalize the use of secret rebates. We reach the following conclusions. Manufacturers benefit from secret rebates because they avoid price interdependencies across markets; and reference countries gain from hiding the details of their deals when other countries would otherwise

base their offers to the supplier on the prices they observe other countries pay. However, the use of secret rebates has mitigated social effects. In particular, they benefit the countries which negotiate first and hurt those which negotiate last.

Acknowledgments

I wish to thank the Fonds de Recherche du Québec en Société et Culture, Concordia University and Paris Dauphine for their financial support, without which I could not have pursued this Ph.D. degree.

My deepest gratitude goes to Dr. Sidartha Gordon and Dr. Ming Li, who guided and encouraged me through my doctoral thesis and during the Job Market. To Sidartha Gordon, thank you for all you taught me. You allowed me to better my research beyond what I could have expected, and I feel very grateful for this opportunity to co-write with you. To Ming Li, thank you for your enthusiasm regarding my research, your guidance and support throughout these years. I remember the time I was taking my Microeconomics course with you, and how I knew right away that I wanted to work under your supervision.

My appreciation extends to my thesis committee for spending their precious time on reviewing this work, and for their support during the Job Market: Dr. Francis Bloch, Dr. Françoise Forges, Dr. Nicolas Klein and Dr. Szilvia Pápai.

I wish to thank Dr. Faye Diamantoudi, Dr. Rohan Dutta and Dr. Licun Xue for their outstanding courses in game theory.

Many thanks to the administrative personnel: Élise Mélançon, Émilie Martel, Kelly Walsh and Irene Grigoropoulos at Concordia; Valérie Fleurette, Nathalie Ferragu, Chantal Charlier and Karène Alarich-Val at Dauphine. I wish to thank Dr. Gabrielle Fack, Dr. Frédéric Loss and Dr. Philippe De Vreyer at Dauphine for their guidance and goodwill during the Job Market, as well as Dr. Ève Caroli, LEDa and DSR for financial support.

I have been incredibly lucky to have as PhD colleagues Dilek, Fadi, Olya, Ribal and Roozbeh at Concordia; and Aimée, Alexis, Charlotte, Dio, Eugénie, Leslie, Maroua and Morgan at Dauphine. I am very grateful to Fabien and Mia, for making Montreal a place I could call home.

Finally, I would like to thank my family, John-Luigi and my friend Julie for their encouragements. My partner, for his wisdom and support. Also, thanks to those I forgot to mention (I shall hasten to add them to this page if they point it out to me).

Contribution of Authors

Chapter 2 is solo-authored.

Chapters 3 is a joint work with my co-supervisor, Dr. Sidartha Gordon.

Chapters 4 is a joint work with my co-supervisor, Dr. Sidartha Gordon.

Contents

List of Figures

Chapter 1

Introduction

1.1 Background common to all chapters

Coordination for achieving efficient outcomes is the unifying theme of this research. The coordination of economic activities can be seen as a harmonious arrangement of various individual plans to avoid inefficient interference, which would result in Pareto dominated outcomes. Coordination is a source of mutual benefits when there exists a common ground between the agents. All economic problems approached in this thesis are instances where agents have *mixed interests*: typically, these instances allow for several Pareto optimal outcomes, and agents disagree on their preferred one.

A coordination failure occurs when agents do not regret their individual choice (the action profile is an equilibrium), however they regret the collective outcome produced by these choices (they miscoordinate on an equilibrium that is Pareto dominated by another equilibrium). Potential sources of coordination failures are the impossibility to communicate prior to taking a decision, or the existence of different ways of achieving the common interest. In *static non-cooperative games*, coordination failures may arise in a Nash equilibrium because of the strategic uncertainty caused by the simultaneity in decisions. Incorporating dynamics in the players' decision process (such games are called *dynamic games*) may change the incentive structure. This can be done by making the same set of players play a game repeatedly (*repeated games*), or by making different players move at different times (*sequential games*). As decisions are made over time, players may get the incentive to coordinate on a cooperative strategy profile that produces a Pareto optimal outcome. Assuming that they are farsighted and have perfect recall of all previous moves, cooperation can be sustained in an equilibrium only if the players can credibly commit to punish those who adopt a non-cooperative behavior, and reward those who behave cooperatively. Indeed, credibility is required in an equilibrium, as a player is deterred by a threat of sanction only if he believes that it is in the others' best interest to punish him, should he not follow a cooperative strategy. (The same goes with rewards.) A player moving at some time t then takes into account the reactions of those who will move later in time to his own action. The former reasons backwards, considering all scenarios for the rest of the game starting from the decision node he is currently at (this reasoning is called *backward induction*). When agents have perfect recall, a strategy profile forms an equilibrium if it is a Nash equilibrium of the game that satisfies the property of *subgame perfectness*: in every subgame, players' decisions must be part of a Nash equilibrium of the subgame.[1]

[1]A *perfect equilibrium* is defined by Selten (1975, [64]) as a combination of mixed strategies, and each strategy is affected a strictly positive probability, such that if i's

In some finite dynamic games, players are not expected to take a stand for the collective interest. For the players who move last, defecting does not have any consequence, as there is no subsequent subgame in which a punishment can be carried on. The players moving in the second to last subgame anticipate this, and may be reluctant to adopt a cooperative behavior knowing that the favor will not be reciprocated in the next subgame. This reasoning can be stretched up to the very first subgame, and coordination on a cooperative strategy profile may not be achieved in a subgame perfect Nash equilibrium.[2] A remedy is then to make the chain of decisions infinite: as there is no last subgame where defection can go unpunished, players may sustain cooperation in a subgame perfect Nash equilibrium.

In a game, coordination failures can be avoided if players have access to a commitment device. In *cooperative games*, agreements on a cooperative strategy profile can be made fully binding (through the making of a contract, for example). The focus is on the stability of these coalitions of players who cooperate in achieving a certain outcome, and not on the details of the procedure via which the outcome is achieved (as it is the case for non-cooperative games).

---

strategy s_i' is not a best-response to s_{-i}, then the weight on s_i' in i's mixed strategy is infinitesimally small. (Players' strategies are affected a non-infinitesimaly probability weight only if they are best-responses.)

[2]In the centipede game for example, the unique subgame perfect Nash equilibrium is such that the first player to move keeps all the money for himself. The outcome of the centipede game is Pareto dominated by that which would be produced if players were (cooperating on) waiting until the end of the game. A similar problem arises in finitely repeated prisoner's dilemma game.

1.2 Specific background and Overview on Chapter 2

Chapter 2 studies a static non-cooperative game of network formation. I offer an overview on game-theoretic models related to networks. I then introduce Bala and Goyal's (2000, [6]) seminal work on the topic, which is of particular importance for understanding my objectives as well as my contribution to the field.

1.2.1 State of the art on strategic games in network economics

Networks have interested economists because of their impact on socio-economic outcomes. Many economic activities are influenced by social ties: the crafting and passing of legislations (Canen, Jackson and Trebbi, 2019 [16]), persisting "silver-spoon" effect across generations (Joshi, Mahmud and Sarangi, 2020 [51]), the structure of criminal organizations (Ballester, Calvó-Armengol and Zenou, 2006 [7]), job search (Lalanne and Seabright, 2016 [54]), research output (Ductor, Fafchamps, Goyal and van der Leij, 2014 [26]), and R&D collaborations (Goyal and Moraga, 2001 [39]) among others. Game-theoretic models about networks can be divided into two groups: *games of network formation*, to which Chapter 2 belongs, and *games played on networks.*

Games of network formation study the specific instances where the nodes in a network (the players) form connections strategically. Examples of such instances can be alumni and professional networks, labor markets, political networks, etcetera. The objectives of these models are usually twofold: to characterize the equilibrium networks that stem from the nodes' decisions in link formation, and to compare them with the efficient networks of the game.[3]

Depending on the environment studied, a link between two agents can be *two-sided*, i.e. the link is formed only if the two agents consent (as it is the case with friendship and professional collaborations), or *one-sided* if consent is not required (a researcher can cite a peer without his consent).

One of the first formalizations of a strategic network formation game is due to Jackson and Wolinsky (1996, [48]).[4] Their connections model is based on the idea that the personal connections of an individual are a source of benefits, however forming and maintaining a relationship is costly. Their model is a static game where players can form two-sided links: a link is paid by the two players who agreed on forming it, and allows access to any player who is reachable from that link. An indirect connection to another agent, through a sequence of links, is supposed to be a source of benefits that deplete with the distance (a friend of friend may be less helpful than a direct friend.) Jackson and Wolinsky define an equilibrium network as one which is robust to one-link deviations: no two players must want to form a link nor a player must want to sever one (severance can be unilateral). This solution concept is called *pairwise stability*. Bala and Goyal (2000, [6]) take on the model of Jackson and Wolinsky, and study it in a context where

[3]For the sake of clarity, I review the most influential **static** models of network formation. Jackson (2005, [47]) has a survey of strategic games of network formation that encompasses dynamic games.

[4]In the economic literature, Aumann and Myerson (1988, [5]) were the very first to model network formation as a game. Theirs is an extensive form game, where the network serves as a communication device for forming coalitions in a cooperative game.

links are one-sided: a player can form a link with another agent without his consent and pays for all links he initiates. This change allows to formulate the network formation game as purely non-cooperative. As Bala and Goyal consider, like Jackson and Wolinsky, that the players' decisions in links are simultaneous, an equilibrium network of their model is a structure in which the players' decisions in links form a Nash equilibrium.

When games are played on a fixed network, the focus shifts on the relation between the players' best responses and the network structure. Two instances are worth distinguishing: pure complementary in the players' actions (which presupposes the existence of peer-effects: investment games, technology adoption), and pure substitutability in the players' actions (like private contributions to a public good).

In static games played on networks, two features of the network are of particular interest: the eigenvalues of its adjacency matrix, and the players' respective Bonanich centralities.[5][6] Bonacich's (1987, [12]) approach to centrality is related to the influence of a node (i.e. player) in the network. It is measured as a function of the number of paths that pass through a node, and the relative contribution of each path depends on its length (longer paths accounting for less). The strength of externalities in equilibrium actions and payoffs between two players depends on their relative location in the network: direct neighbors having the largest influence; and as themselves are impacted by the decisions of their own direct neighbors, a player's equilibrium action depends as well on that of its second degree neighbors, and so on. In other words, an exogenous perturbation on

[5]See Bloch, Jackson and Tebaldi (2019, [11]) for a detailled review of centrality measures in networks.

[6]The adjacency matrix of a network is a (0,1)-squared matrix with zeros as diagonal elements, and whose (i, j)th entry is equal to one if there exists a link between i and j.

a player's equilibrium action propagates along the paths in the network that starts at this player's position, and impacts potentially all players' choices of action. The absolute values of the largest and lowest eigenvalues of an adjacency matrix capture the strength of complementarity and substitutability in the network, respectively. Existence and uniqueness of a stable Nash equilibrium is guaranteed only for low magnitudes of these eigenvalues, thus for relative small network effects.[7] When this is satisfied, a player's Nash equilibrium action depends on his Bonacich centrality (Ballester, Calvó-Armengol and Zenou 2006, [7], Bramoullé, Kranton and D'amours 2014, [14] and Allouch 2015, [1]).

1.2.2 Aims and objectives

The aim of Chapter 2 is to apply network formation à la Bala and Goyal to an environment where the network is a source of common benefits. In some practical contexts, the value of informational exchanges is gauged by the collective outcomes they produce. Examples could be a group of researchers co-writing an article, a committee of experts working on a report, or even team workers collaborating on a joint project. In these environments, what is a source of benefits to each individual is the return from their joint endeavour. My modeling approach builds on the idea that individuals with a good ability to diffuse information in their community can achieve better collective outcomes.

[7] In the context of games played on networks, the criterion of stability refines the set of Nash equilibria and select those that are robust to small changes in agents' actions. In games with continuous actions, a Nash equilibrium a is stable if, starting from a and changing the players' actions by a little bit, the best responses lead back to the original vector a.

My model is a static non-cooperative game of network formation. The set of players is thought of as a group of individuals who are involved in a collective action. I assume that the success of their joint endeavour depends on the group's ability to share information. Prior to taking their collective action, the agents get the possibility to form one-way directed links. As in Bala and Goyal, every player pays for the links he initiates, and the players choose their links simultaneously. Once formed, the network serves as communication platform: a player can talk to any other player he can reach in the network, i.e. there exists a *path* (a sequence of links) that connects the former to the latter. Once communication is over, the agents take their collective action. Its return is the same for all, and it depends positively on the number of interactions that each of them had during the communication stage. The distinctive aspect of my model is that the network is a public good: every agent receives the same return from the collective action, regardless of his private contribution in links to building the network.

In this game, both coordination and cooperation problems may arise in an equilibrium. Because links are a source of positive externalities (one's own links may help connect an agent to another one, like j's link to k allows i to access k in the network on the left of Figure 1), the problem that the players who invest in links face is to arrange them in a way that maximizes the common benefit from the network. This is a pure coordination problem. Figure 1 provides an example.

$$i \longrightarrow j \longrightarrow k \qquad\qquad i \longrightarrow k \longleftarrow j$$

Figure 1: Two consecutive links versus two links pointing towards the same agent

For simplicity, assume that the return from the collective action is given by the number of ordered pairs of players such that the first player in the pair can reach (the latter has a path to) the second one. The network on the left hand side allows i to reach j and k, and j to reach k; hence, its return equals 3. The network on the right hand side allows i and j to reach k, and its return equals 2. In the network on the right, agent i can increase the collective return by one unit if he redirects his link towards j (which would give the network on the left side). Because the network produces non-excludable benefits, agents may have the incentive to free ride on the others' contributions in links. In Figure 1, player k enjoys a return of 3 and 2 in the networks on the left and right, respectively, without having contributed to building the network.

The objectives of this first chapter are threefold. First, I seek to characterize the network architectures that are supported in a strict Nash equilibrium, and to identify the differences between the equilibrium networks in my game and Bala and Goyal's. Throughout, I refer to a strict Nash network as a network that is shaped by a strategy profile in link formation which is a strict Nash equilibrium. In my model, each player trades off the social benefits of his links and the costs he incurs for forming them: taking the network on the left side of Figure 1 as an example, j maintains his link towards k if the worth from him reaching k, and i reaching k, makes up for the cost of the link and if there is no better link to form.[8] Second, I investigate if coordination problems and free riding arise in a strict Nash equilibrium. The problem of free riding is particular to my game, as

[8]In Bala and Goyal's set-up, benefits from links are private: the network benefit to player i is the number of players he can reach. Therefore, if we go back to the example provided in Figure 1, j would disregard the positive externality his link has on agent i in the computation of his benefit from maintaining the link towards k.

I suppose that the network produces non-excludable benefits. Coordination failures happen in a strict Nash network if, given the cost of forming links, the players could have formed another strict Nash network that gives each of them a larger payoff. In my game, coordination failures in equilibrium may be caused by the players' incentive to free ride. Third, I seek to characterize the network architectures that are efficient.

1.2.3 Contribution

Chapter 2 is a direct contribution to the literature on endogenous network formation. A body of works studies specifically the provision of public goods on an endogenously formed network. This literature was initiated by Galeotti and Goyal (2010, [32]), who combine a local public good game played on a network built from the players' strategic decisions in link formation. Galeotti and Goyal's paper extends the literature on local public good games played on fixed network structures (Bramoullé and Kranton 2007, [13] and Bramoullé, Kranton and D'Amours 2014, [14]) by endogenizing the network on which the game is played. In Galeotti and Goyal's set-up, homogeneous agents choose to acquire costly information and to form links with others to access the information they acquire. In equilibrium networks, the authors find what they call the *law of the few*: a small set of agents, the influencers, acquire information for the entire network, and the rest free-ride on their efforts in information acquisition (they simply connect to one of these well-informed influencers). This result seems robust even when players are heterogeneous in terms of their efficiency in producing the public good or their valuation of the public good (Kinateder and Merlino 2017, [52]). In my model, the network can be understood as a public good which is produced from every agent's private contribution in links. In the models in line with Galeotti and Goyal's, an agent's contribution to the public good generates positive externalities on his direct neighbors, exclusively. In my set-up, an agent's contribution in links generates positive payoff externalities on all players in the network, not just on his direct neighbors. For example, the benefits of a publication in a prestigious journal are common to all authors, while maybe just some have actively contributed to writing the article; on platforms like Discord, the knowledge created by discussions between the members of a group is publicly accessible to all of them, etc.

My model is the closest to Bala and Goyal's. Both of our games of network formation are static and non-cooperative. A distinctive feature of my game compared to theirs is that the network generates non-excludable benefits. This causes one major difference in the architectures of our equilibrium networks: in my model, some strict Nash networks are disconnected, with some players who free ride by not forming any links at all. Such a configuration never happens in a Nash equilibrium of Bala and Goyal's game. Apart from this difference, the patterns of links in our equilibrium networks are the same. When the network benefit does not depend on the distance between the agents, the links in an equilibrium network form wheels; and when it does, equilibrium networks have the architecture of flowers.

From a technical point of view, Chapter 2 is related to the literature on network formation and potential games (Tardos and Wexler 2007, [66]). In my game, a maximum of the potential has the property of being a Nash equilibrium with an efficient architecture. When the network benefit does not depend on the distance between the agents, I find that the set of maxima of the potential coincides with the set of strict Nash equilibria in which no player free rides.

1.3 Specific background and Overview on Chapters 3 and 4

This section aims at introducing the theoretical analyses of Chapters 3 and 4. Both chapters are co-written with my co-supervisor, Sidartha Gordon.

1.3.1 State of the art

Chapter 3 and 4 study two theoretical models on the negotiations of drug prices. The focus of this research is on drugs listed for reimbursement. The aim of these chapters is to understand the outcomes of negotiations. Despite their lack of transparency, the three well-known facts listed below provide some information on the negotiation procedure and the particularities of drug pricing.

> *Secret rebates*
>
> Drug manufacturers and payers, whether they are private insurers like in the US or public health officials, negotiate two prices. There is first the official price at which the drug is listed for reimbursement, called the *list price*. This price is made public as soon as the drug is ready for launch on the domestic market, and what patients pay

out of their pocket (in case the drug is not fully reimbursed by their insurance plan) is a percentage of the list price. The second price negotiated is a *secret rebate:* money that the manufacturer gives back to the payer, and whose amount is known to no one but the payer and the manufacturer. Rebates vary in terms of schemes and magnitudes. Rebate schemes seem to be mostly volume-based, and the rebate may be applied on a bundle of products (Valverde and Pisani 2016, [68]). About the magnitude of rebates, Morgan et al. (2017, [60]) estimate that they range from 40% to 70% for specialty pharmaceuticals, and from 10% to 50% for primary care drugs across North America, Europe, and Australasia. These figures suggest that a list price may be completely uninformative about the real price a country pays for a given drug.

Price interdependencies

There are price interdependencies between certain countries. These are driven by two phenomena. The first one is the use by some institutional payers of international price referencing in their negotiation with a drug manufacturer. *International price referencing* (IPR) is a price cap for a medicine that is calculated as a function of the list prices of similar treatments in other countries. According to Vogler, Paris and Panteli (2018, [69]), the majority of European countries, China, Japan, Canada and Brazil, among others, apply IPR for some drugs. The second phenomenon is the possibility of *parallel imports* between countries in a same economic community. In the EU, the parallel import of a medicine involves importing the product into one member state from another, and distributing it outside the distribution network set up by the manufacturer. The rationale behind parallel imports is to arbitrage away international price discrimination on medicines. Danzon (1997, [20] and 2018, [21]) notes that for the EU members, contracts with differential list prices and no rebate were replaced by confidential contracts including a rebate off a common list price because of parallel imports.

Strategic timing of negotiations

In the EU, drug manufacturers need first to obtain a market authorisation from the European Medicines Agency. Once granted, each country negotiates a price with the manufacturer. These negotiations between countries and manufacturers are sequential. The empirical studies by Danzon et al. (2005, [23]) and Kyle (2007, [53]) suggest that, among European countries that use IPR, manufacturers launch their products first in higher-priced markets.

These three facts have been subject to different research questions. A first body of research studies the implications of price interdependencies between markets. Jelovac and Bordoy (2005, [50]) look specifically at the effect of parallel imports on countries' welfare, and find that the effect is positive only if the trading partners have needs for different types of drugs. Marinoso, Jelovac and Olivella (2011, [33]) study instead the relation between IPR and list prices in reference countries (i.e. these countries that come first in the order of the negotiations). The authors find that a country benefits from using IPR when the co-payment it offers is relatively larger than that of a reference country, and that the benefit is lower the larger the country's market size.[9] Houy and Jelovac (2015, [46]) study the effect of IPR on the launch dates across countries. The authors find that the firm chooses a timing of launches that follows the ordering of the countries' willingness to pay and market sizes, where countries which represent higher sources of revenues get access to the medicine earlier.

Another part of the literature studies the relation between list prices and social insurance policies. Jelovac (2002, [49]) focuses on the effect of co-payment levels on list prices. The author predicts that list prices are negatively correlated with the degree of coverage, and highlights a series of implications that rationalizes this result: first, lower co-payments imply a relatively more inelastic demand for the drug, thereby increasing the firm's opportunity cost of failing its negotiation; this reinforces (endogenously) the country's bargaining power to influence the outcome of the negotiation towards its preferred one; namely, an agreement on a low list price.

1.3.2 Aims and objectives

The joint aim of Chapters 3 and 4 is to provide rational justifications for the use of secret rebates and IPR, and to highlight possible functions of secret rebates. From a supplier's perspective, the benefits of secret rebates are straightforward: given that countries negotiate in turn and use IPR, secret rebates allow to isolate each negotiation from the others. For countries, the impact of secret rebates on welfare is not clear at all. Our approach consists in proposing hypotheses that rationalize an offer of secret rebate

[9] A co-payment is a fixed amount that a healthcare beneficiary pays for medical expenditures covered by his or her health insurance plan. The remaining balance is paid by the insurance company.

by a country.

In Chapter 3, we take as given that countries which do not come first in the order of the negotiations will use IPR. For simplicity, we suppose that these countries never agree on paying a price larger than the list price they observe in other countries. We seek to understand how this affects the Pareto optimal outcomes of the negotiation, in terms of list price and rebate, between a reference country (i.e. one that negotiates first) and the manufacturer. In particular, we want to identify necessary and sufficient conditions for which a Pareto optimal and individually rational (PO-IR) contract has a strictly positive rebate, and a reference country prefers a PO-IR contract with a large rebate.

To this end, we propose a model with two countries and a monopolist pharmaceutical firm. The first of our assumption is that the firm accepts to include a rebate in a deal only for concealing the real price a country is paying to countries that negotiate later. Second, we assume that the country which negotiates first sets up a social insurance plan for reimbursing the drug. Specifically, the country chooses the level of social contributions it will levy on its population for funding the expenditures tied to the reimbursement scheme, given the deal with the manufacturer. We take the reimbursement scheme as a fixed parameter of the model, and it consists of the percentage of the list price that is covered by the social security.

We refer to a contract between the reference country and the firm as a list price - rebate pair, where the list price is the component of the contract that the second country can observe, while the net (rebated) price (i.e., the price paid by the country to the firm) is unknown to the second country. As mentioned earlier on, we assume that the second country to negotiate applies IPR: it never agrees to pay a price larger than the list price it observes in the first country.

Our objective is to characterize the set of PO-IR contracts between the firm and the first country. From the analysis of this set, three results stand out. First, the relative profitability of the two markets has an incidence on the occurrence of secret rebates in the PO-IR contracts, as well as on the possibility of mutually advantageous trades between the reference country and the firm. In particular, we find that strictly positive rebates are part of PO-IR contracts where the net (rebated) price is relatively low. As the firm would never agree on leaving such a low net price propagate in the second negotiation, the net price that the first country pays is hidden by a secret rebate off the list price. Second, in the set of PO-IR contracts with rebates, larger list prices are associated with larger rebates and lower rebated prices. Meaning, the first country and the firm trade large secret rebates against high list prices. Third, the country's payoff is the largest for the contract that has the largest rebate, the largest list price and the lowest rebated price. Therefore, large rebates should be expected when the negotiating power of the reference country is important.

In Chapter 4, we propose to rationalize the use of IPR and of secret rebates by an asymmetry of information between the countries and the manufacturer. The hypothesis we put forward is that a monopolist pharmaceutical firm may have knowledge about the time-lapse before the market release of a superior substitute: meaning, the latter has some private information that affects its willingness to accept low offers from the countries against earlier deals. To the end of rationalizing the use of secret rebates, we compare the optimal list prices in two regimes: an opaque regime, where the transaction price negotiated by each country with the firm is confidential; and a transparent regime, where countries can observe the prices that others pay. By comparing the equilibrium transaction prices in both regimes, we can assess if a reference country benefits from negotiating a secret rebate.

The model takes the form of a sequential game with asymmetric information between two countries and a monopolist firm. The private information held by the firm, which is the launch date of a superior substitute, is interpreted as the firm's *type*. We assume that the two countries have the same willingness to pay for the drug, and share a prior belief about the firm's type. The two countries negotiate in turn; and within a same negotiation, there are two rounds of offers: a country is the first to make an offer, that the firm either accepts or rejects; and in case of rejection, the firm makes a counter-offer to the country, that the latter either accepts or rejects. A deal is sealed as soon as one party accepts the offer of the other. In this game, the firm's decision to accept or reject an offer releases information about its type. Low types, which believe in the imminent entry of a substitute, have the incentive to seal a deal as soon as possible; while high types, confident in the duration of their monopoly, prefer to wait until the second round of the negotiation to extract a greater surplus from a country.

When negotiations are transparent, the second country to negotiate observes the price paid by the first country. This price is informative about the firm's willingness to accept a low price against an earlier deal. Altough partial, this information provides some indication about the firm's type. Because of this, the subgame that starts at the first decision node of the firm, where the latter decides whether to accept or reject the first country's offer, is a signaling game between the firm and the second country. The firm sends a signal about its type to the second country, through its decision regarding the first country's offer, and the second country formulates an offer that is optimal given the signal. When negotiations are opaque, the price paid by each country to the manufacturer is kept secret; therefore, the second country in the order of the negotiations does not learn any additional information about the firm's type than the information its prior belief provides it already.

We characterize the set of weak perfect Bayesian equilibria (PBE), for both opaque and transparent regimes. When negotiations are transparent, we find that the second country uses a form of IPR in equilibrium: it formulates an offer that depends positively on the transaction price in the first country. If it is farsighted, the firm is therefore more inclined to reject a same offer made by the first country than in an opaque regime, where the second country would not observe the price the first country pays. In our model, the combination of the firm's farsightedness and the use of IPR by the second country penalizes the country which negotiates first. Confidentiality about the transaction price through secret rebates are a means for the first country to cancel out this penalty, and to get the same payoff as in the opaque regime.

1.3.3 Contribution

In the theoretical literature on drug pricing, no model has yet proposed a rationalization of the use of secret rebates and of IPR. Importantly, we propose hypotheses under which a country initiates an offer of secret rebate. The empirical literature suggests that the appeal of secret rebates to countries is mostly budgetary, in the sense that rebates help contain public expenditures on pharmaceuticals (Vogler et al. (2012, [72])). Regarding this point, we find in Chapter 3 that a country prefers contracts with large rebates, because they are associated with low net prices (i.e. the net price is the price the country pays the manufacturer) and a large quantity traded. Still in this chapter, we conclude that the firm uses a secret rebate to hide the price a country pays from those which negotiate later. When the market in the country which negotiates last is sufficiently profitable, we further find that the variation between the list prices that are PO-IR for the firm and the reference country is low, however the variation between the net prices is high. This suggests that the negotiation between a reference country and a pharmaceutical firm is mostly about the secret rebate.

Chapter 4 contributes to the ongoing discussion on price transparency on pharmaceuticals and its potential effect on countries' welfare. Scholars seem to be divided on the matter. Transparency entails the use of differential pricing, which is opposite to the current system based on a uniform pricing (comparable list prices) and price discrimination among countries through confidential rebates. Danzon and Towse (2003, [22]) argue that differential pricing is unsustainable, and may be detrimental to poor countries for the following reason. As countries use IPR and might engage in parallel trade, manufacturers may have the incentive to charge a single price between the rebated prices that the countries would have gotten, had markets been separated and secret rebates allowed.[10] This would ultimately benefit high-income countries and hurt low-income ones. Based on the results in Chapter 4, where we assume that countries are identical in terms of market size and willingness to pay, we find that transparency has mitigated social effects: it is detrimental to the countries which negotiate first and beneficial to the countries which negotiate last. Vogler and Paterson (2017, [70]) disagree with the conclusion of Danzon and Towse, and believe that transparency can enhance both the accessibility and the affordability of medicines. They argue that the abolition of secret rebates in favor of differential pricing might lead countries to collaborate in leading joint negotiations with pharmaceutical companies.[11]

Chapters 3 and 4 contribute to the theoretical literature on quantity discount pricing and rebates. Rebate schemes are used in other industries than the pharmaceutical one, notably in the automobile, wholesale and electronics industries. For these industries, the literature identifies coupons and rebates as ways through which the manufacturer can price discriminate among end consumers and control retailers' incentive to hoard inventories. See Lee and Rosenbalt (1986, [55]), Gerstner and Hess (1991 [35], 1995 [36]), Ault et al. (2000, [2]) and Chiu, Cho and Tang (2011, [18]) among others. For the case of pharmaceuticals, the function of secret rebates that is mostly put forward by the literature is their preventing price interdependencies across countries, and therefore benefit manufacturers. We are able

[10] In the EU, the parallel import of a medicine involves importing the product into one member state from another, and distributing it outside the distribution network set up by the manufacturer.

[11] Examples of such collaboration is BeneluxA (Belgium, the Netherlands, Luxembourg and Austria and formed in 2015), as well as the Nordic Pharmaceuticals Forum (Sweden, Norway Iceland and Denmark). For more details, see the policy brief by Espin et al. (2016, [30])

to rationalize this function of secret rebates in Chapter 3, as we find that the firm agrees on making the first country pay a low price only if it is concealed from the second country by a large secret rebate off a high list price.

Chapter 2

Collective action on an endogenous network

2.1 Introduction

In collective actions, coordination is key for achieving better social outcomes. The ability of a group of activists to speak with one voice is crucial for their credibility and political influence. A team working on a joint project is more efficient if tasks are clearly allocated, and the workers are given feedback and updates on a regular basis. The types of collective actions that will interest us in this chapter are those that are communicative in nature, in the sense that their outcomes are shaped by the interpersonal interactions of the group members.

Collaborative networks that promote information sharing have aspects of a public good. Forming links, through which information is transmitted to others, costs the individual agent time, resources and efforts, while their benefits (the outcome from the collective knowledge generated by interactions) are publicly accessible. As with any public good, the ability of people to free ride on the efforts of others poses a threat to its provision. Theoretical development is needed to understand (i) Which network architectures stem from non-cooperative decisions in link formation? (ii) What network features make groups better or worse at generating common knowledge?

I propose a non-cooperative model of network formation where the network allows a group of agents to exchange information prior to taking a collective action. It is assumed that the success of the collective action depends positively on how well informed the group is. To this end, the players get the possibility to form a communication network.[1] The network formation stage is a one-shot game: every player decides for himself the links he wants to establish in the network without observing the others' decisions, and pays for all links he initiates. The network that is built from these decisions then serves as communication platform. An agent can talk to all those he can reach in the network; specifically, agent i can talk to agent j only if there exists a path (a sequence of links) from the former to the latter. The return from the collective action is the same for all agents (i.e., the network produces both non-rival and non-excludable benefits), and it is a function of some network statistics.

I consider two assumptions regarding the relation between the network and the collective return it produces. Under the benchmark assumption, the collective benefits from communication depend solely on the number

[1] In this paper, communication is not strategic. Papers that study strategic information sharing and persuasion in networks are those by Hagenbach and Koessler (2010, [42]), Bloch, Demange and Kranton (2018, [9]), Egorov and Sonin (2019, [28]) among others.

of interactions that each agent has in the network. The alternative assumption is that the collective return depends positively on the number of interactions, and direct interactions generate higher benefits than indirect (i.e. distant) ones. This second assumption accounts for possible informational distortions during retelling, transmission delays, etc. For this reason, the benchmark assumption is synonymous with *frictionless communication*, while the alternative assumption presupposes the existence of frictions during information transmission.

When communication is frictionless, the network structures that are supported in a strict Nash equilibrium are all wheels, that may or may not include all agents (See Figure 2). The interactions in any of these structures are exclusively reciprocal: agent i talks to agent j if and only if j talks to i. Some agents free ride on the efforts in link formation of some others when the wheel does not encompass all agents, and the occurrence of free riding in a strict Nash equilibrium depends positively on the group size. I show that these network structures where free riding happens, namely all wheels that do not include the entire group of agents, are sub-efficient from a utilitarian perspective. Yet, these network structures that allow certain players to free ride are not necessarily Pareto dominated by other strict Nash networks.

Figure 2: A wheel on all agents and a non-exhaustive wheel

The existence of frictions during communication gives the players an additional incentive to form links. Links are established primarily for connecting individuals to others, but also for cutting distances that would otherwise be too long. When the number of agents is relatively small, I find that some equilibrium candidates have a flower structure, that may or may not encompass all agents. (See Figure 3.) A flower is characterized by a cyclic pattern of links: links are organized in wheels, and the wheels communicate among each other through the central agent in the flower. This result suggests that optimal communication structures are highly centralized, which seems particularly efficient for shortening distances. Another result that stems from this analysis is that an efficient network architecture balances the benefits from shortening distances against the benefits from including more agents, for a fixed total number of links. In Figure 3, the network on the left allows every agent to interact with the rest of the group; in the network on the right, fewer agents can communicate, however the benefit from an interaction is higher (the distances between the agents who can reach each other are shorter).

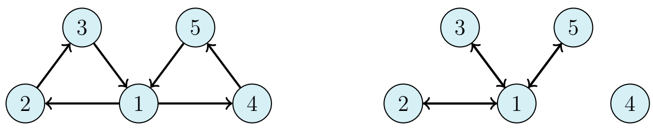


Figure 3: For 5 agents and 6 links, two possible efficient architectures

The paper contributes to the literature on network formation games. In the earliest models on the topic, it is assumed that the sole concern of the players is the number of agents they can reach via their own connections (Jackson and Wolinsky 1996, [48] and Bala and Goyal 2000, [6]). Participation in link formation has also been studied in a continuous model, where agents choose how they allocate fixed resources on several links (Bloch and

Dutta 2009, [10]) instead of choosing a discrete set of links. My set-up is close to Bala and Goyal's because we model network formation as a static non-cooperative game. In their model, as in mine, the Nash equilibrium is too permissive an equilibrium criterion, and motivates a stronger equilibrium concept: the strict Nash equilibrium. This stronger concept is very effective for narrowing down the set of network structures that arise in an equilibrium. A marked difference between our two models is the way in which the agents value their links. In Bala and Goyal, a player trades off the private benefits of his links (the number of agents a player can reach from his links) with the costs he incurs for forming them. In my set-up, a player trades off instead the social benefits of his links (the number of agents one's own links help connect across the network) and their costs. This difference affects our results about the connectedness of equilibrium networks: in Bala and Goyal, a strict Nash network that is not connected (i.e. it is connected if a player can reach any other player in the network) is necessarily empty, which is not the case for my game. As I consider that the network produces non-excludable benefits, disconnected networks happen in equilibrium because of the players' incentive to free ride. Apart from this difference, we find similar patterns of links in equilibrium networks: wheels when the network benefits do not depend on the distance between the agents, and flowers when the network benefit depends negatively on the distance.

A body of works studies network formation embedded within a coordination game (Goyal and Vega-Redondo 2005, [40], Herman 2014, [43]), an anti-coordination game (Bramoullé et al. 2004, [15]), a local public good game (Galeotti and Goyal 2010, [32], Kinateder and Merlino 2017, [52]), or a cooperative game (Dutta et al. 1998, [27], Slikker and van den Nouweland 2000, [65]). A player's links in the network influences his behavior and may determine his partners in the game subsequent to the network formation, as well as potential externalities in actions and payoffs. When the game that follows network formation is cooperative, the players' strategic decisions in link formation determine which coalitions they can or cannot join; and when it is non-cooperative, the network structure determines the scale of interdependencies between the players' equilibrium actions.

Related to collective action and networks, Chwe (2000, [19]) studies a model where agents' decisions to participate or not in a collective action depend on what their direct neighbors in a fixed network choose to do. My approach is markedly different from his because I consider that the network structure is endogenous. However, Chwe offers a richer analysis of the collective action problem. In my model, the outcome of the collective action is directly derived from the communication network the agents form.

This paper also contributes to the literature on potential games in the context of network formation (Tardos and Wexler 2007, [66]). Usually, when a game is a finite potential game, the strategy profiles that are potential maximizers can be used as an equilibrium refinement (Monderer and Shapley 1996, [58], Sandholm 2010, [62]). In my game, the argmax set of the potential function consists of networks that have a socially efficient architecture: they optimize the social cost of building the network, by maximizing the network benefit (in the context I study, the network benefit is the return from the collective action) per link. When communication is frictionless, these networks are the wheels that encompass all agents; if instead the benefits from communication decline with the distance, these networks are flowers (this is proved only for a small number of agents).

The rest of the paper is organized as follows. Section 2.2 features the benchmark model where communication is frictionless. Section 2.3 characterizes the set of strict Nash equilibria of the game. Section 2.4 characterizes the set of efficient networks and contains a discussion on equilibrium refinements. Section 2.5 features an alternate version of the model where the network benefit depends on the distances in the network. Section 2.6 concludes.

2.2 The model under frictionless communication

I first introduce the model, then I review some key properties of the network formation game.

2.2.1 Set-up

There is a group of individuals, $N = \{1, \ldots, n\}$ with $n \geq 3$. At some point in the game, the n individuals will take a collective action. Its return is determined by the communication network the agents form first. A communication network is a strategy profile $g = (g_1, \ldots, g_n)$, where g_i denotes player i's strategy in link formation. A strategy g_i for player i gives the set of agents towards whom i forms links, and it is a subset of $N \setminus \{i\}$. In this model, a link is *directed*, i.e. it is an ordered pair ij of two players, represented as $i \to j$, and it allows the first player in the pair, i, to talk to the second one, j. A link has a fixed cost $c \geq 0$, and a player pays for all links he initiates.[2] I restrict attention to pure strategies. The set of all pure strategies for agent i is denoted by G_i, and the space of pure strategy profiles is G.

The n players choose their strategies simultaneously. Once the network formation stage completed, agents communicate. In a network g, an agent talks to all those he can reach in the network. Meaning, i talks to j if and only if there exists a path from i to j in the network g. A *path* from agent i to agent j is a sequence of links $i \to i_1 \ldots \to i_k \to j$ along which all agents are distinct. A path is interpreted as a communication channel. An interaction of i with j can be direct, if there exists a link from i to j, or indirect, if i has a path to j that includes at least two links. I assume that communication is frictionless: the worth of i's communications depends solely on their number, $\kappa_i(g)$, which is referred to as i's *reach*, and it is an integer between 0 and $n-1$. Note that $\kappa_i(g) = 0$ if and only if $g_i = \varnothing$, and $\kappa_i(g) = n-1$ if i can reach all other players. In Section 2.5, I consider an alternative version of this model where the benefits from communication decline with the length of a path.

Once the network formed and communication is over, the players take their collective action. The network influences their collective decision, and groups that communicate well achieve better social outcomes.[3] The return from the collective action, $v(g)$ in some network g, is a reduced form of a

[2] In my model, a link is *one-way*, i.e. i does not need j's consent for establishing the connection $i \to j$.

[3] One could think that the agents who can communicate elaborate a social norm, and those who did not participate in its elaboration are unaware of it, and thus they may cause miscoordination in the collective action. Or more simply, agents use the communication network for exchanging social-relevant information that affects positively the outcome of their collective action.

possibly complicated decision process. The return is written alternatively as a function of the vector $\kappa(g) = (\kappa_1(g), \ldots, \kappa_n(g))$ of reaches in the network g:

$$v(g) = \Phi(\kappa(g))$$

where Φ is symmetric and strictly increasing in all players' reaches:

$$\Phi(\kappa(g)) > \Phi(\kappa(g'))$$

if $\kappa_i(g) \geq \kappa_i(g')$ for all $i \in N$, and there is at least one player $j \in N$ for which the inequality is strict, for any $j \in N$. The payoff of player i in some communication network $g \in G$ is the return from the collective action minus i's expenditure in links:

$$u_i(g_i, g_{-i}) = v(g) - c|g_i| \tag{1}$$

where $|g_i|$ is the cardinality of g_i, and it corresponds to the number of links that i forms in the network g. Note that the network is a public good: all agents get the same collective return from the network.

I shed light on two properties of v, which are direct implications of the assumptions on Φ.

Remark 2.2.1. *The function v is increasing in strategies:*

$$g_i \subseteq g_i' \quad \Rightarrow \quad v(g_i, g_{-i}) \leq v(g_i', g_{-i}),$$

$\forall g_i, g_i' \in G_i, g_{-i} \in G_{-i}, \ \ \forall i \in N$, *and it is anonymous (it is invariant under* *permutations of the players' labels):*

$$v(g) = v(g^{\pi})$$

$\forall g \in G$ and for any permutation π of N, where g and g^{π} are two isomorphic networks, and are said to have the same architecture.[4]

Anonymity implies that the collective return is determined by the patterns of links and not by the labels of the players who are in given positions in the network.

2.2.2 Equilibrium and efficiency concepts

In the remainder, I solve for the strategy profiles g that are strict Nash equilibria of the network formation game. I refer to such networks as strict Nash networks. In Section 2.3.3, I justify my choice of equilibrium concept.

Definition 2.2.1 A strategy profile $(g_1^*, \ldots, g_n^*)$ is a *strict Nash equilibrium* if, for every player $i \in N$:

$$u_i(g_i^*, g_{-i}^*) > u_i(g_i', g_{-i}^*)$$

$\forall g_i' \in G_i \setminus \{g_i^*\}$ with $g_{-i}^* \in G_{-i}$. The network g^* is *strict Nash* if $(g_1^*, \ldots, g_n^*)$ is a strict Nash equilibrium.

[4] Two networks g and g^{π} are isomorphic if there is a permutation π of the set of players N such that any link $i \to j$ exists in g if and only if the link $\pi(i) \to \pi(j)$ exists in g^{π}.

The players' non-cooperative decisions in links can be studied in an exact potential game. A game is an exact potential game if there exists a function $P : G \to \mathbb{R}$ such that:

$$P(g_i', g_{-i}) - P(g) = u_i(g_i', g_{-i}) - u_i(g_i, g_{-i})$$

$\forall i \in N, \forall g_i, g_i' \in G_i$. The game presented in Section 2.2.1 is an exact potential game, whose potential function is:

$$P(g) = v(g) - c \sum_{i \in N} |g_i| \tag{2}$$

and it can be re-written as:

$$P(g_i, g_{-i}) = u_i(g_i, g_{-i}) - c \sum_{j \neq i} |g_j|$$

As the potential function is only different from a player's payoff by a constant, its argmax set refines the set of Nash equilibria of the network formation game. In the potential game, all players optimize the potential function in (2). A game that is an exact potential game admits the existence of a Nash equilibrium in pure strategies, as the potential maximizer is a Nash equilibrium (Monderer and Shapley, (1996, [58])). Furthermore, a potential maximizer has the property of being the Nash equilibrium that is the most robust to possible perturbations of the game. In this game, we shall see (in Section 2.4) that the argmax set of the potential refines the set of strict Nash equilibria, and selects the architectures in which no player free rides on the others' contributions in links.

Let an efficient network be a network that maximizes a utilitarian social welfare function of the agents' payoffs. The welfare function is given by:

$$W(g) = nv(g) - c \sum_{i \in N} |g_i| \tag{3}$$

which can be re-arranged as:

$$W(g) = n \left(v(g) - \frac{c}{n} \sum_{i \in N} |g_i| \right)$$

Therefore, g is a maximum of the potential for some value c of the cost of a link if and only if g is efficient for a value of the cost that is n times larger than c.

2.3 Strict Nash networks

In this section, I characterize the set of strict Nash networks, and I justify my choice of equilibrium concept.

2.3.1 Strict Nash candidates

Consider the strategy g_i of any player $i \in N$ in some network g. I distinguish between two kinds of deviations:

> *Type I deviations:* all strategies $g_i' \in G_i$ for player i such that $g_i' \neq g_i$ and $|g_i'| = |g_i|$. If i deviates to such strategies, then his expenses in links remain constant, however the collective return may change.
>
> *Type II deviations:* all strategies $g_i' \in G_i$ for player i such that $g_i' \neq g_i$

and $|g'_i| \neq |g_i|$.

In this section I propose a test on the strategy profiles in G. The test consists of checking, for a given strategy profile g, if there is a player who could deviate by forming fewer links, and the deviation weakly improves the reach of every agent in N. If the test is positive, then the strategy profile is never strict Nash; if the test is negative, then the strategy profile may be a strict Nash equilibrium. This test allows me to restrict attention to four possible architectures for a strict Nash network. Lemma 2.3.1 opens the ways, with a result about the properties of the paths in a strict Nash network. First, I need to introduce a definition.

Definition 2.3.1 A path $\rho_{i_0 \to i_h} : i_0 \to \ldots \to i_h$ from i_0 to i_h is *included* in the path $\rho_{j_0 \to j_m} : j_0 \to \ldots \to j_m$ if there are integers k such that $0 \leq k \leq m-(h+1)$, and for all q with $0 \leq q \leq h$, $i_q = j_{k+1+q}$. Meaning,

$$j_0 \to \ldots j_k \to i_0 \to \ldots \to i_h \to j_{k+h+2} \to \ldots \to j_m$$

The relation between the two paths is denoted as $\rho_{i_0 \to i_h} \subseteq \rho_{j_0 \to j_m}$.

Lemma 2.3.1. *If $g \in G$ is a strict Nash network, then any two paths $\rho_{i \to k}$ and $\rho_{j \to k}$ directed towards agent k satisfy either $\rho_{i \to k} \subseteq \rho_{j \to k}$ or vice-versa, with $i, j \neq k$.*

Proof. The proof is by contradiction. Suppose that the strategy profile $g = (g_1, \ldots, g_n)$ is a strict Nash equilibrium, and there are two paths directed towards some player k that are not included in one another. Let me set $\rho_{i \to k} : i_0 \to \ldots i_l \to k$, where $i = i_0$ and $k = i_{l+1}$. And $\rho_{j \to k} : j_0 \to \ldots j_m \to$

k, where $j = j_0$ and $k = j_{m+1}$. If the conclusion of the lemma is false, then there exist two players, i_h along $\rho_{i \to k}$ and j_f along $\rho_{j \to k}$, such that: $i_h \neq j_f$ yet $i_{h+1} = j_{f+1} \in g_{i_h}, g_{j_f}$. Take any of these two; I will procede through the proof by considering i_h. (1) If i_h does not have a link towards j_0, then consider the deviation $g'_{i_h} = g_{i_h} \setminus \{i_{h+1}\} \cup \{j_0\}$. (2) Otherwise, consider the deviation $g'_{i_h} = g_{i_h} \setminus \{i_{h+1}\}$. In case (1) g'_{i_h} costs the same as g_{i_h} while in case (2) g'_{i_h} is strictly cheaper than g_{i_h}. Let me set $g' = (g'_{i_h}, g_{-i_h})$. It is immediate that any player a who has a path to any player b in g still has a path to b in g'. (If the path was including the link $i_h \to i_{h+1}$ in g, then the path from a to b in g' now inlcudes the sequence $i_h \to j_0 \ldots \to k$.) If i_h does not have a path to j_0 in g, then there are ordered pairs ab of players such that a can reach b in the network g', and a cannot reach b in the network g. Therefore the deviation is at least weakly profitable. A contradiction to the fact that g is strict Nash. □

Lemma 2.3.1 has strong implications on the architectures of the strict Nash candidates. These implications are stated in the three corollaries below.

Corollary 2.3.1. *If $g \in G$ is a strict Nash network, then there is at most one path from i to j in g, for any $i, j \in N$.*

Proof. Recall that all players along a path are distinct. The result is immediate by setting $i = j$ in Lemma 2.3.1. □

The next corollary characterizes the architecture of a component of a strict Nash network. A set $C \subseteq N$ is called a *component* if for every ordered pair ij of agents in C, i has a path to j and there is no strict superset C' of C for which this is true. A *singleton* is a component that has one element. A network that has one component is said to be *connected*; and a network that has strictly more than one component is referred to as *disconnected*. Consider a component C and denote the agents in C as $\{j_0, ..., j_m\}$, where $n \geq m > 1$. A wheel component has an architecture that is defined by the sequence of links $j_0 \to j_1 \ldots j_m \to j_0$, i.e. $C \cap g_{j_i} = j_{i+1}$ for all $i \in \{0, \ldots, m\}$ and $m + 1 = 0$. See Figure 4 below for a wheel component on three players.

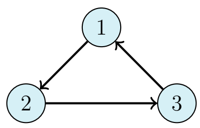


Figure 4: A wheel on three agents

Corollary 2.3.2. *A component of a strict Nash network is either a singleton or a wheel.*

Proof. To avoid trivialities, consider a component C formed by at least 3 distinct agents, i, j and k. By the definition of a component and Corollary 2.3.1, any of the three agents has one path to any of the two others. Thence, either (a) the path from i to j passes through k or (b) the path from i to k passes through j. If (a) is true, then by Lemma 2.3.1 the path from j to k passes through i. If (b) is true, then by Lemma 2.3.1 the path from k to j passes through i. Iterating the process for all agents in C, the result follows. $\square$

An immediate implication of Corollary 2.3.2 is that all connected strict Nash networks are wheels that encompass the whole set of players. The next corollary gives a property on a relation between the components in a disconnected strict Nash network. I use the relation $\mathcal{R}$ for comparing the components of a network, where $C \mathcal{R} D$ is read as: "any player who belongs to the component C has a path to any player in the component D". In short, C has access to D. The next corollary provides a key property of the partial order on the components in a strict Nash network.

Corollary 2.3.3. *If $g \in G$ is a strict Nash network and (1) g has two components C and D such that $C \mathcal{R} D$, then D is a singleton; (2) g has three components C, D and E such that $C \mathcal{R} E$ and $D \mathcal{R} E$, then either $C \mathcal{R} D$ or $D \mathcal{R} C$.*

Proof. For statement 1, assume by contradiction that D is not a singleton. Hence D is a wheel by Corollary 2.3.2. Take any $i \in C$, $k \in D$; and let $j \in D$ such that $j \in g_k$. By Lemma 2.3.1, there is one path from i to k in g; and one path from j to k in g. But note that neither $\rho_{j \to k} \not\subseteq \rho_{i \to k}$ nor $\rho_{i \to k} \not\subseteq \rho_{j \to k}$. Therefore g is not strict Nash by Lemma 2.3.1. For statement 2, set $i \in C$, $j \in D$ and $k \in E$. Again, if the conclusion is false, then neither $\rho_{j \to k} \not\subseteq \rho_{i \to k}$ nor $\rho_{i \to k} \not\subseteq \rho_{j \to k}$. Which contradicts that g is strict Nash by Lemma 2.3.1. □

In the next lemma, I further narrow down the set of disconnected architectures that qualify for strict Nash. The result complements the statement in Corollary 2.3.2.

Lemma 2.3.2. *If $g \in G$ is a strict Nash network, then there is at most one wheel component in g.*

For further details, see Appendix A.1. From Lemmas 2.3.1 and 2.3.2, we can identify another class of strict Nash candidates, that I will refer to as *disconnected flat architectures*. Networks that belong to this class have several components, none of them are comparable via $\mathcal{R}$, and they have at most one wheel component and the rest are isolated singletons. A component $C = i$ is an *isolated singleton* if i has no link adjacent to him, i.e. $g_i = \varnothing$ and $i \notin g_j$ for all $j \in N$. (See agent 4 in Figure 5.) Note that the wheel component in a disconnected flat architecture must count at least three agents if it is strict Nash.[5] The empty network of no links is a special case of these architectures. The rest of the networks in this class are nonempty, and I call them *non-exhaustive wheels*. Any non-exhaustive wheel has one wheel component on n_w agents, with $3 \leq n_w < n$, and $n - n_w$ isolated singletons. See Figure 5 below.

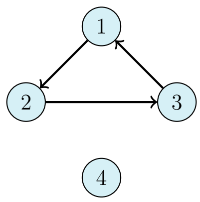


Figure 5: A non-exhaustive wheel, with $n_w = 3$ and one isolated singleton

The last lemma of this section complements Corollary 2.3.3, and highlights a key property of the partial order on the components in a strict Nash network. In a network g, the greatest element of the partial order is a component C such that $C \mathcal{R} D$ for all components D in g.

[5] If g has 1 wheel component on 2 players i and j, and the rest of the components are isolated singletons, then i (or j) can profitably deviate by severing his link to j (i) and adding a link towards any $k \neq j$ (or any $k \neq i$). i's expenditure stay constant, however the latter improve his reach by one, and the other players' reach remain unchanged. A contradiction that g is strict Nash.

Lemma 2.3.3. *If $g \in G$ is a strict Nash network and g has two components C and D such that $C \mathcal{R} D$, then the partial order on the components of g has a greatest element. Furthermore, the greatest element is a wheel if g has strictly less than n components.*

Proof. The second statement follows directly from Corollary 2.3.3 statement 1 and Corollary 2.3.2. The proof is by contradiction. Suppose g is strict Nash. Let C and D be maximal and minimal elements of the partial order on the components of g, respectively, and $C \mathcal{R} D$. Assume further that the partial order does not have a greatest element. Then there exists a component E such that C and E are not comparable via $\mathcal{R}$. By Corollary 2.3.3 statement 2, E and D are not comparable via $\mathcal{R}$ either. Let i_C be any agent in C, and let i_E be any agent in E. As C is a maximal element, no agent who has a path to i_C in g has a path to i_E, and the converse is true. Assume that $\kappa_{i_C}(g) \geq \kappa_{i_E}(g)$. If further $\kappa_{i_E}(g) > 0$, then i_E has a profitable deviation such that i_E severs any one of his links and adds simultaneously a link to i_C. A similar argument holds for the case where $\kappa_{i_C}(g) < \kappa_{i_E}(g)$ (as $C \mathcal{R} D$, note that $\kappa_{i_C}(g) > 0$ always holds). If $\kappa_{i_E}(g) = 0$, consider any player j who has a link towards D. Note that as $C \mathcal{R} D$, D is a singleton by Corollary 2.3.3 statement 1. Thus player j has a payoff equivalent strategy such that j severs his link to D and adds simultaneously a link towards i_E. A contradiction that g is strict Nash. $\square$

Lemma 2.3.3 reveals another class of disconnected strict Nash candidates: the out-trees of components, or *out-tree networks* for short. An out-tree network is a directed rooted-tree whose vertices are the components of the network and whose edges, which each connects one component to another one, are oriented away from the root. The root of an out-tree network is the greatest element of the partial order on the components. The root of a strict Nash network is a singleton if the network has only singleton components, or else it is a wheel. The rest of the vertices of the out-tree are all singletons. Note that an out-tree network implies a hierarchical ranking of the components. At the top of the hierarchy are the players in the root, who can reach everybody in the network; belong to the second layer of the out-tree the players who can reach anyone but those located in the root, and this reasoning can be stretched down to the bottom layer of the out-tree, which consists of players who cannot reach anyone. See Figure 6. Below are gathered the results of the section.

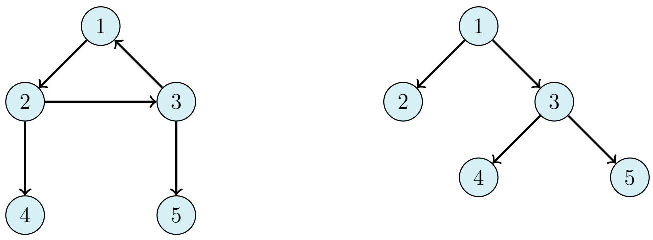


Figure 6: Two out-tree networks

Proposition 2.3.1. *A network $g \in G$ is strict Nash only if it has the architecture of one of the below:*

1. *a* ***wheel network*** *g^w (connected)*

2. *the* ***empty network*** *g^e (disconnected, flat)*

3. *a* ***non-exhaustive wheel*** *$g^{n.e.w}$ (disconnected, flat)*

   There is one wheel component on n_w agents, with $3 \leq n_w < n$; the rest of the components are all isolated singletons.

4. *an **out-tree network** (disconnected, hierarchical)*

   The root component is a wheel or a singleton, and the rest of the components are all singletons. Each agent has a link adjacent to him, i.e. either $g_i \neq \varnothing$ or $g_i = \varnothing$ and there exists $j \in N$ such that $i \in g_j$.

In an out-tree network, any of the root members can reach anyone in the network. Starting from the root and progressing down the tree, the larger an agent's reach, the fewer agents can reach him. Interactions between the components in an out-tree network are exclusively unilateral: if agent i in some component C can reach agent j in some other component D, then j cannot reach i. While in all other architectures, interactions are exclusively reciprocal: agent i can reach agent j if and only if j can reach i.

2.3.2 Existence of strict Nash networks

This section is devoted to proving the existence of strict Nash networks. Let me consider the following assumption on the collective return.

Assumption A: Φ *is additive-separable:*

$$v(g) = \sum_{i \in N} \phi(\kappa_i(g)) \tag{4}$$

with ϕ strictly increasing concave in all agents' reaches.

Note that the function v associated with any function Φ that satisfies Assumption A is both anonymous and increasing in the players' strategies. In what follows, I show that Assumption A rules out the out-tree networks from the set of strict Nash candidates.

Lemma 2.3.4. *(1) If Φ is additive separable, then an out-tree of singletons (i.e. the root is a singleton) is never strict Nash. (2) If further ϕ is concave, then no out-tree network is strict Nash.*

Proof. The proof is by contradiction. For statement 1: Assume g is an out-tree network, g is strict Nash, and the root component is a singleton, i. Thus i is the greatest element of the partial order on the components of g. If g is strict Nash, then any of i's links is worth maintaining: $c < v(g)-v(g_i', g_{-i}) \leq \phi(n-1)-\phi(0)$, for $g_i' = g_i \backslash \{j\}$ for any $j \in g_i$, and the last inequality holds only if $g_i' = \varnothing$. Let k be any minimal element of the partial order on the components of g; hence $g_k = \varnothing$. Consider $g_k' = i$ the deviation of player k such that k forms a single link towards the root, i. Note that: $u_k(g_k', g_{-k}) - u_k(g) = v(g_k', g_{-k}) - v(g) - c \geq \phi(n-1) - \phi(0) - c > 0$. Hence k has a strictly profitable deviation, a contradiction that g is strict Nash. For statement 2: assume that the root of g is a wheel on n_w agents. Let me first define the height of component C has the length of the longest path from C to a leaf, where a leaf is a minimal element of the partial order on the components (i.e. any agent who does not have any link). Consider any leaf k; and let j a player who has a link towards k. The link from j to k allows anyone along the path from the most distant player in the wheel to k to reach the latter. If g is strict Nash, the link is worth maintaining: for $g_j' = g_j \setminus \{k\}$,

$$c < v(g) - v(g_j', g_{-j}) \leq n_w[\phi(n-1) - \phi(n-2)] + \sum_{h=0}^{n-n_w-2} (\phi(h+1) - \phi(h))$$

The last inequality holds for the following reason. Because ϕ is concave, the largest variation $\phi(\kappa_l) - \phi(\kappa_l - 1)$ obtains for the lowest value of κ_l which is h_l, the height of agent l in the out-tree. The maximum number of agents who access k in g is $n-1$. (This is equivalent to assuming that all singletons are along the path from the root to k.) The above inequality can be re-written in a more compact way as:

$$c < n_w[\phi(n-1) - \phi(n-2)] + \phi(n - n_w - 1) - \phi(0) \quad (*)$$

Consider leaf k; by the definition of a leaf, $g_k = \varnothing$. Let k deviate to $g'_k = i$, where i is any agent in the root. Then $u_k(g'_k, g_{-k}) - u_k(g) \geq \phi(n-1) - \phi(0) - c$. Note that:

$$\begin{aligned} u_k(g'_k, g_{-k}) - u_k(g) \geq &[\phi(n-1) - \phi(n - n_w - 1)] + [\phi(n - n_w - 1) - \phi(0)] - c \\ &> [\phi(n-1) - \phi(n - n_w - 1)] - n_w[\phi(n-1) - \phi(n-2)] \\ &\geq 0 \end{aligned}$$

where the first strict inequality holds by $(*)$, and the last weak inequality because ϕ is concave. Therefore g'_k is strictly profitable; a contradiction that g is strict Nash. $\square$

Next, I show that in the rest of the networks featured in Proposition 2.3.1, there are deviations for the players that could never be best-responses to the strategies of the others. The next lemma yields information about the properties of these inferior deviations.

Lemma 2.3.5. *Let g be a network whose architecture is that of a non-exhaustive wheel. Under Assumption A, if $g'_i \in G_i$ is a deviation for player* *i such that:*

$$|g'_i| \geq |g_i| \quad \textit{and} \quad g'_i \not\subseteq g_i$$

then g'_i is never a best-response, for any $i \in N$.

See Appendix A.2 for the proof. Note that Lemma 2.3.5 is trivially satisfied in the empty network and the wheel network.[6] There are two

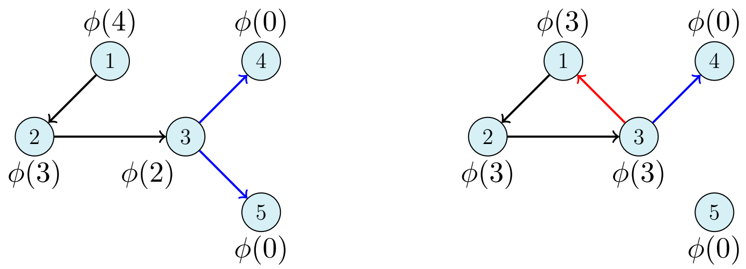


Figure 7: Type II deviations in a non-exhaustive wheel

important results in this lemma. The first one is that no deviation of type I is weakly profitable for any player in a wheel or a non-exhaustive wheel. If a player in a wheel component deviates to a strategy of type I such that the latter redirects his link towards another agent in the wheel, then the reach of only one agent stays constant, and all others' strictly decreases. As for the rest of the deviations of types I and II in the non-exhaustive wheels, the intuition goes as follows. Suppose that any player in the wheel component of a non-exhaustive wheel considers switching strategy to one that has weakly more links (say x links, with $1 \leq x \leq n-1$). A best-response can only be one of the following: to direct all x links towards isolated singletons, like in the network on the left of Figure 7; or to direct

[6]Note that no player in a wheel network gains from adding links, as the reach of the players cannot be further increased (the reach of any player is $n-1$ in any connected architecture); and no deviation of type I can be profitable, as such deviations disrupt the components (hence the collective return from the network strictly declines).

only $x-1$ of them towards singletons, and maintain the link in the wheel component, like in the network on the right side. The proof in Appendix A.2 shows that, when ϕ is concave, the second deviation always yields a larger payoff than the first one. The next lemma completes the analysis of type II deviations.

Lemma 2.3.6. *Let g be either the empty network or a non-exhaustive wheel, and let g_i be the strategy of any player i in g. Consider the set of deviations for player i:*

$$G_i^+ = \{g'_i \in G_i \ : \ g_i \subset g'_i\}$$

for which i adds links to his set of links in g. Under Assumption A:

$$\hat{g}_i \in \arg max_{g'_i \in G_i^+} \frac{v(g'_i, g_{-i}) - v(g)}{|g'_i| - |g_i|} \quad \Rightarrow \quad |\hat{g}_i| = |g_i| + 1$$

See Appendix A.3 for the proof. Gathering the results in Lemmas 2.3.5 and 2.3.6, it follows that the wheel, the non-exhaustive wheels and the empty network are strict Nash if (i) the link that, if added in the network, maximizes the increase in the collective return is not worth forming; and (ii) the link that, if removed from the network, minimizes the decrease in the collective return is worth maintaining. This allows me to find a parameter range for c such that for any cost within this range, no agent can profitably deviate towards a strategy of type II. These bounds are presented in the next proposition.

Proposition 2.3.2. *Let the payoffs be given by expression* (1)*, with v given by expression* (4)*. A strict Nash network is either a wheel network, the* *empty network or a non-exhaustive wheel. In particular,*

1. *the* ***wheel network*** g^w *is strict Nash if and only if:*

$$c < n\phi(n-1) - \sum_{h=0}^{n-1} \phi(h) = \overline{c}^w$$

2. *the* ***empty network*** g^e *is strict Nash if and only if:*

$$c > \phi(1) - \phi(0) = \underline{c}^e$$

3. *a* ***non-exhaustive wheel network*** $g^{n.e.w}$ *on* n_w *agents is strict Nash if and only if:*

$$c \in \left(\phi(n_w) - \phi(0)\ ,\ n_w\phi(n_w - 1) - \sum_{h=0}^{n_w-1} \phi(h) \right) = (\underline{c}^{n_w}\ ,\ \overline{c}^{n_w})$$

and this interval is never empty, for any $3 \leq n_w < n$.

The bounds can be directly verified. The parameter range for c in statement 3 is non-empty when ϕ is concave. All intervals of values in statement 3 are strictly included in the interval of values for which the wheel and the empty network are both strict Nash. Meaning, if any non-exhaustive wheel is strict Nash for some value c of the cost, then so are the wheel network and the empty network. Note that each of the bounds in statement 3 is an increasing function of n_w, and that the length of the interval is increasing in n_w when ϕ is concave. Finally, the intervals of values for which the non-exhaustive wheels on n_w and $n_w + 1$ agents are strict Nash overlap if $n_w > 3$. The results in Proposition 2.3.2 can be summarized as below. (The superscripts and subscripts of the bounds $\underline{c}_{n_w}$ and $\overline{c}^{n_w}$ give the number of agents in the wheel component of a non-exhaustive wheels: e.g., $\underline{c}_3$ is the lower bound on the range of costs for which the non-exhaustive wheel on $n_w = 3$ agents is strict Nash.)

Corollary 2.3.4. *Consider the parameter ranges of the cost c of a link in Proposition 2.3.2. Assume for simplicity that $\overline{c}^3 \geq \underline{c}^4$, i.e. $\phi(2) - \phi(1) \geq \phi(4) - \phi(2)$. (1) If $c \leq \underline{c}^e$, then the wheel network is the unique strict Nash equilibrium. (2) If $c \in (\underline{c}^e, \underline{c}^3] \cup [\overline{c}^{n-1}, \overline{c}^w)$, then only the wheel network and the empty network are strict Nash equilibria. (3) If $c \in (\underline{c}^3, \overline{c}^{n-1})$, then the wheel network, any non-exhaustive and the empty network are strict Nash equilibria. (4) If $c \geq \overline{c}^w$, then the empty network is the unique strict Nash.*

2.3.3 Discussion on the equilibrium concept

If I am less restrictive and use the concept of the Nash equilibrium, the test that I should have run instead is the following: given a strategy profile $(g_1, \ldots, g_n)$, is there a player i who could play an alternate strategy that costs the same as g_i and that weakly improves the reach of each agent, and improves strictly the reach of at least one them? Or, is there a player who could switch to a strategy that entails strictly fewer links and that weakly improves the reach of each agent? If the test is positive, then it is immediate that the strategy profile is never a Nash equilibrium; and if the test is negative, the strategy profile may be a Nash equilibrium.

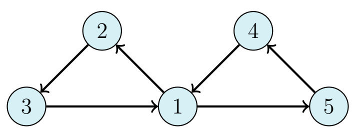


Figure 8: A Nash equilibrium candidate that is not strict Nash

An example of a network that may be Nash but that is never strict Nash is provided in Figure 8. Note that this network satisfies Corollary 2.3.1. In particular, no player can deviate by forming strictly fewer links without disrupting the component (which would therefore decrease the collective return from the network). However, this architecture violates Corollary 2.3.2. Corollary 2.3.2 restricts the architecture of a component in a strict Nash network to that of a wheel. In the network in Figure 8, player 3 is indifferent between maintaining his link with agent 1 and replacing it by a link directed towards either 4 or 5. These strategies are payoff-equivalent for agent 3: they imply the same costs of link formation, as well as the same collective return (regardless of whether 3 has a link towards 1, 4 or 5, the resulting network is connected i.e. all agents have a reach equal to 4).

The refinement of strictness is very effective in my setting, for two reasons. First, almost all network architectures are eliminated by considering just a few deviations. The strict Nash concept also eases the highlight on the out-trees and the non-exhaustive wheels, which raise richer questions regarding the relation between the components in an equilibrium network. Second, the strict Nash concept eliminates the architectures in which players have multiple best-responses to the others'. Such architectures are less stable, as some agents may be tempted to switch to a payoff-equivalent strategy.

Generally, any Nash equilibrium that forms a connected network exists for some positive values of the cost of a link, i.e. $c \geq 0$. Indeed, adding links in any connected architecture is worthless, as the players' reach cannot be further increased. Consider the network in Figure 8 and a wheel network on 5 agents. Suppose that the cost of a link is almost null, yet strictly positive. Note that if there exists a function v for which both architectures can be supported in a Nash equilibrium for this cost, then the wheel Pareto-dominates the architecture in Figure 8: players 2, 3, 4 and 5 get the same payoff in both networks, however agent 1's payoff is strictly larger in a wheel network.

2.4 Equilibrium selection and efficient networks

Recall that this game is an exact potential game, and that a network that is a maximum of the potential for some value c of the cost is efficient for a cost n times larger than c. Below, I characterize the argmax set the potential function, for all values of c.

Proposition 2.4.1. *Consider the expression of the potential function in* (2)*, with v given by expression* (4)*. A maximum of the potential function is either a wheel network or the empty network. (1) If $c < \phi(n-1) - \phi(0)$, then the maximum of the potential is achieved in a wheel network. (2) If $c \geq \phi(n-1) - \phi(0)$, then the maximum of the potential is achieved in the empty network.*

See Appendix A.4 for the proof. An important result in Proposition 2.4.1 is that the non-exhaustive wheels never maximize the potential function. Note that if the cost of link formation is below the threshold in Proposition 2.4.1 (and this threshold is lower than the upper bound $\bar{c}^w$ on the range of costs for which a wheel network is strict Nash), then the wheel network is both strict Nash and maximum of the potential. Hence, any architecture that can be supported in a strict Nash equilibrium for values of c less than the threshold in Proposition 2.4.1 and that is not a wheel network is sub-optimal, in the sense that it is under-connected (the wheel network offers a higher collective return per link than any other nonempty strict Nash network). On the contrary, if the cost of link formation is strictly above the threshold in Proposition 2.4.1, then the empty network is strict Nash and it is the unique maximum of the potential function. Any nonempty strict Nash network that exists for such values of the cost is sub-optimal, in the sense that it is over-connected.

Corollary 2.4.1. *Let the welfare function be given by expression* (3)*, with* v *given by expression* (4)*. An efficient network is either a wheel network or the empty network. (1) If* $c < n\,(\phi(n-1) - \phi(0))$*, then the wheel network is the unique efficient network. (2) If* $c > n\,(\phi(n-1) - \phi(0))$*, then the empty network is the unique efficient network.*

Proof. Recall that the argmax set of $W(g)$ for $c' = nc$ is the same as the argmax set of $P(g)$ for c. □

The argmax set of the potential function refines the set of strict Nash equilibria, and it sets aside all strict Nash networks in which some players free ride (these players are the isolated singletons in the non-exhaustive wheels). Yet, these architectures may be Pareto optima of the network formation game. A non-exhaustive wheel never Pareto dominates a wheel network, because any agent in a wheel component is always strictly better-off in a wheel network than in a non-exhaustive wheel regardless of the value of the cost. However, a wheel network may Pareto dominate a non-exhaustive wheel, if an isolated singleton would rather pay for a link in a wheel network. Below, I propose to refine the set of strict Nash equilibria to those that are Pareto optimal. Abusing language, I refer to a *payoff dominant* network as a network that is associated with a strict Nash equilibrium that Pareto dominates all other strict Nash equilibria.

Proposition 2.4.2. *Consider the parameter ranges of the cost c of a link in Proposition 2.3.2. (1) For any value of the cost c such that the wheel network and a non-exhaustive wheel on n_w agents, with $n_w \leq \lfloor \frac{n}{2} \rfloor$, are both strict Nash, the wheel network Pareto dominates the non-exhaustive wheel. (2) For any $c \in (\underline{c}^e, \overline{c}^w)$, the empty network is strict Nash and it is Pareto dominated. (3) There exists a parameter range for c and values of n_w between 3 and $n-1$ such that a non-exhaustive wheel is both strict Nash and Pareto optimal only if:*

$$\sum_{k=1}^{n-3} \left(\phi(n-2) - \phi(k)\right) \geq n \left(\phi(n-1) - \phi(n-2)\right)$$

Meaning, the largest payoff an isolated singleton can earn in a non-exhaustive wheel is larger than the lowest payoff any agent can earn in a wheel network, provided that both networks are strict Nash.

(4) If the above condition does not hold, then the wheel network is payoff dominant whenever it is strict Nash, i.e. for any $c < \overline{c}^w$.

The proof is provided in Appendix A.5. The players' incentive to free ride on the others' efforts in link formation may lead to coordination failures in equilibrium: the wheel network is Pareto superior to the empty network and to any non-exhaustive wheel that fails to encompass at least half of the agents, provided that these networks are strict Nash. Also, note that the maximum of the potential and the criterion of Pareto dominance refine the set of strict Nash networks differently: for costs in the range $[\phi(n-1)-\phi(0), \bar{c}^w)$, the empty network is a strict Nash equilibrium that is both a potential maximizer and Pareto dominated.

2.5 When the distance matters

In many practical contexts, the distance between agents in a network plays a role. Communicating with another person through many intermediaries may cause delay or informational distortions. In this type of context, it seems natural that a direct link from an agent to another one yields more benefits than a lengthy path. Then how would the predictions change if the benefits from communication decline with the distance? The remainder of the paper offers a partial answer to this question.

2.5.1 Payoffs

In this section, I assume that there are frictions occurring during the communication stage. Let me interpret a path as a communication channel through which the first agent along the path talks to the last agent along the path. Suppose that each time an agent relays a message, its informational content decays. I use the geodesic distance from an agent to another one for measuring the worth of their interaction. The *geodesic distance* from i to j in a network g is the number of links along the shortest path from the former to the latter, and I denote this distance as $d(i, j; g)$. If i has a path to j in some network g and $j \neq i$, then $1 \leq d(i, j; g) \leq n-1$. If

i does not have a path to j, then I use the notation $d(i,j;g) = \infty$; and I set $d(i,i;g) = 0$. In this version of the game, I suppose that the collective return associated with a network is a function of its distance matrix. The *distance matrix* is a $n \times n$ matrix, whose (i,j)th entry gives $d(i,j;g)$, and it is denoted as $D(g)$ for some network g. Note that the reach of agent i is given by the number of non-infinite entries on the ith row of D. The collective return associated with some network $g \in G$ is:

$$v(g) = \Phi(D(g)),$$

and Φ is decreasing in each entry of the distance matrix. For some given strategy profile $g \in G$, the payoff of any player i is written as:

$$u_i(g_i, g_{-i}) = \Phi(D(g)) - c|g_i|$$

The expression of the potential function is now:

$$P(g) = \Phi(D(g)) - c\sum_{i \in N} |g_i| \tag{5}$$

In this version of the model, the closeness of the agents determines the collective return.

2.5.2 Assumption

The aim of this section is to provide a criterion that allows to compare the collective return in different networks. I first introduce a definition.

Definition 2.5.1 The *cumulative distance distribution* $\Gamma(g)$ of a network g is a $(1 \times (n+1))$ vector, whose jth entry γ_j gives the number of distances less than or equal to $j-1$ in g divided by n^2, for any $0 \leq j \leq n$. The last entry γ_{n+1} is always equal to 1. The nth entry γ_n provides information on the number of finite distances in g; and $\gamma_{n+1} - \gamma_n$ provides information on the number of infinite distances in g.

Definition 2.5.2 For any two networks $g, g' \in G$, g' *dominates* g if the cumulative distribution $\Gamma(g)$ of distances in g first order stochastically dominates the cumulative distribution $\Gamma(g')$ of distances in g'.

Note that if some network g' dominates some other network g then, on average, an individual is closer to the rest of the group in g' than in g. The next assumption is based on Definition 2.5.2, and it offers a criterion for comparing the collective return in several networks.

Assumption B: *If g' dominates g, then $v(g') \geq v(g)$.*
Network architectures that bring the agents closer to each other yield larger collective benefits. Assumption B implies that v is anonymous.

Example. Consider the two networks below.

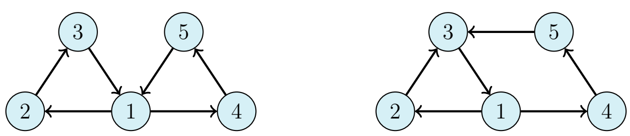


Figure 9: The network g on the left dominates the network g' on the right

	0	1	2	3	4	∞
$\Gamma(\mathrm{g})$	$\frac{5}{25}$	$\frac{11}{25}$	$\frac{19}{25}$	$\frac{23}{25}$	1	1
$\Gamma(\mathrm{g}')$	$\frac{5}{25}$	$\frac{11}{25}$	$\frac{18}{25}$	$\frac{23}{25}$	1	1

According to Assumption B, we have that $v(g) \geq v(g')$.

2.5.3 Analytical results

In this version of the game where frictions occur in the communication stage, characterizing the strict Nash equilibria of the game is more difficult. The objective of the rest of the chapter is to give insight on how the equilibrium predictions change when distances matter.

To this end, I change the equilibrium concept to the maximum of the potential, for the expression of the potential function in (5). I refer to an *equilibrium network* as a network that maximizes the potential function, given a value c of the cost of a link. Recall that this game admits the existence of a Nash equilibrium in pure strategies (because this game is a potential game), and that a strategy profile that maximizes the potential function is a Nash equilibrium. The analytical analysis of the argmax set of the potential function is limited. I am only able to characterize a relation between the components in a disconnected equilibrium network. This makes the object of the next proposition.

Proposition 2.5.1. *Let the potential function be given by expression* (5)*, and assume that v satisfies Assumption B. If $g \in G$ is an equilibrium network, then there is at most one component C in g such that $|C| > 1$*

(i.e., C is not a singleton).

A formal proof can be found in Appendix A.6; the intuition goes as follows. Assume that a network g has two components that are not singletons.

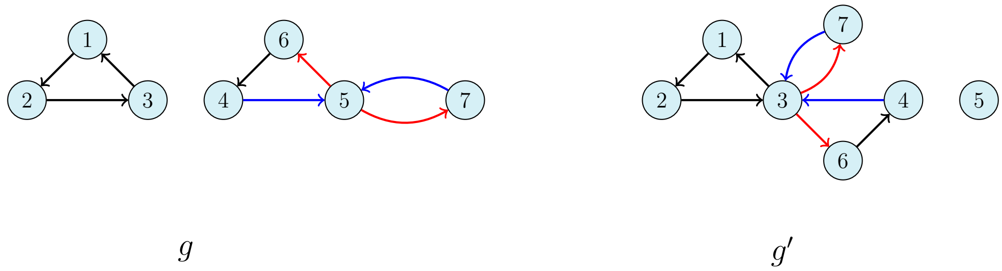


Figure 10: Inefficient architecture with several components

Let the components be merged as follows: some player i in one of the two components takes on the links of any player j in the other component, and maintains those he already has in g. Player j severs all of his links. Next, all players who have links towards j in g redirect them towards i. The rest of the links in g stay intact. The operation has merged the two components, except for j who is left with no links. Note that the resulting network has weakly fewer links than g.[7] See Figure 10: agent 3 takes on the links of agent 5 in addition to the ones he has in g, and the agents who have links towards 5 switch by connecting to 3 instead. Take players 3 and 5: aggregating the distances from these two agents, we obtain the same thing in both networks.[8] The distances between agents 4, 6 and 7 do not change; and the distances from them to 5 in g are equal to the distances from them to 3 in g'. In g', our three agents have access to agents 1 and 2. And agents

[7]The total number of links stays constant only if $|g_i|+|g_j| = |g_i \cup g_j|$ i.e. $g_i \cap g_j = \varnothing$, and strictly decreases otherwise.

[8]Between the two of them, and in both networks g and g' there are: 2 distances equal to 0, 3 distances equal to 1, 2 distances equal 2; no distance equal to $3, 4, 5$ or 6; and 7 distances equal to ∞.

1 and 2 gain access to agents 4, 6 and 7. Aggregating the distances in both networks, the network g' dominates the network g. Therefore, g is never a maximum of the potential. In this example, note that the two components of g are not comparable via $\mathcal{R}$. A similar argument holds when they are.

The second analytical result is related to the diameter of the equilibrium candidates that have many links. The *diameter* of a network g is the longest geodesic distance in g.

Proposition 2.5.2. *Let the potential function be given by expression* (5), *and suppose that* v *satisfies Assumption B. If a strategy profile* $g = (g_1, \ldots, g_n)$ *maximizes the potential function, and if further* $\sum_{i \in N} |g_i| \geq 2(n-1)$, *then the diameter of the network* g *is at most equal to* 2.

Proof. The proof is by contradiction. Assume g maximizes the potential function, g has a number of links x such that $2(n-1) \leq x \leq n(n-1)$ however the diameter of g is strictly larger than 2. Consider some network g' that has the same number x of links, and g' has a subgraph that is a star on all agents. See Figure 11: in a star, there is one central agent who forms links towards all other agents; and any other agent than the center has one link towards the center. Since such subgraph necessitates $2(n-1)$ links and $x \geq 2(n-1)$, g' exists. In g', there are n distances equal to 0, x distances equal to 1 and $n(n-1) - x$ distances equal to 2. But then g' dominates g. As g and g' have the same number of links, it follows that g is not a maximum of the potential. A contradiction. $\square$

When the cost c of a connection is fairly low, an equilibrium network may have many links. If the number of links is larger than $2(n-1)$, then it is always possible to form an architecture whose diameter is less than 2. For example, any network that has a subgraph which is a star on all n agents satisfies this restriction on the diameter. In a star, there is one central agent who forms links towards all others, and any agent who is not the center has a link towards the central agent. A star network is represented in Figure 11 below.

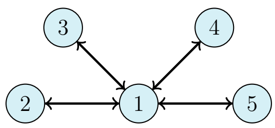


Figure 11: A star network

For low values of the cost, the set of equilibrium networks is restricted to the architectures that have a certain diameter (by Proposition 2.5.2). Consider relatively larger values of the cost c. Which architecture should we expect a component of an equilibrium network to have? First, the architecture of a wheel network may no longer be optimal for some values of the cost, and adding links may be worthwhile for the sake of cutting distances. Let me consider the group of $n = 6$ agents in Figure 12. Assume that we are in an environment where communication is frictionless, and that our agents have formed a wheel network. Suddenly, the benefits from interactions slightly decay with the distance, and agent 1 forms an additional link towards agent 4. This gives the network g_1. The architecture of g_1 does not enable the group to make the most out of the extra link from agent 1 to agent 4. To see why, let agent 3 redirect his link towards agent 1; this gives the network g_2. Agents 1, 4, 5 and 6 are not affected by the change operated by agent 3, however the latter is now strictly closer to his peers. Although the distances from agent 2 change, the average distance from this agent stays constant. Aggregating the distances, g_2 dominates g_1. The same argument holds for any number of links that is added to a wheel component.

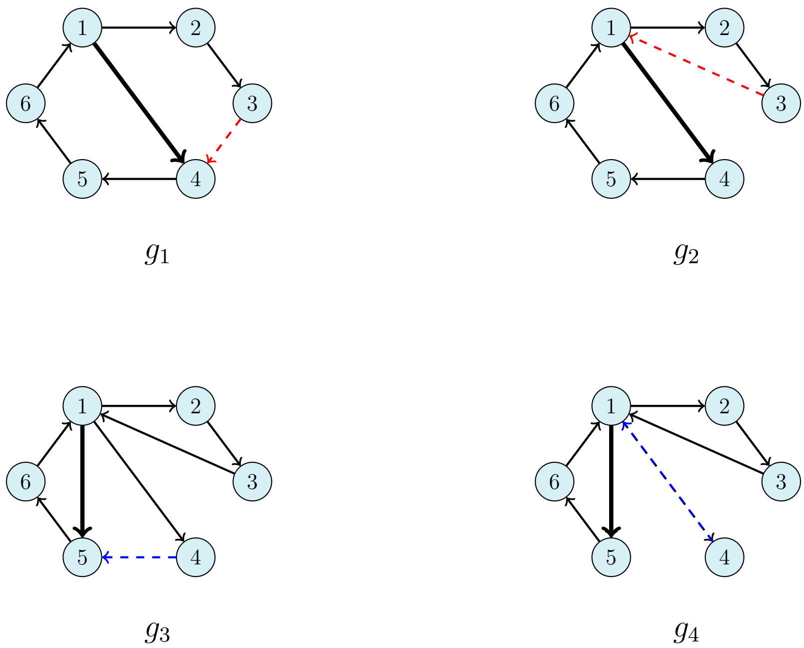


Figure 12: Starting from a wheel network, and adding links

By naively adding links into a wheel component, I just showed that the resulting architectures are not stable because some players, like agent 3 in the network g_1 in Figure 12, have a strictly profitable deviation. However, this is not possible in an equilibrium network, as a maximum of the potential is a Nash equilibrium of the game. The deviation that I just highlighted leads to centralize communication around one central agent, like agent 1 in the networks in Figure 12.

In the remainder, I characterize the architectures that belong to the argmax set of the potential function, for fixed values of the cost c of a link and for a small number n of agents. The results seem to validate the intuition just presented: for a fixed value of the cost c, equilibrium networks all have a central agent, and they exhibit the same patterns of links as in the networks g_2 and g_4 in Figure 12.

2.5.4 Results on the architectures of the equilibrium candidates

By Assumption B, a network g^* that has x links in total is an equilibrium candidate only if no other network with the same number of links produces a larger collective return. My objective is to provide a characterization of the architectures of equilibrium candidates for this version of the game.

First, let me consider the following set:

$$G_{n,x} = \left\{ (g_1, \ldots, g_n) = g \in G \ : \ \sum_{i \in N} |g_i| = x \right\}$$

Given a number n of agents, this is the set of all networks in G that have x links, with $0 \leq x \leq n(n-1)$. Next, let me define the following subset of $G_{n,x}$:

$$G^*_{n,x} = \{(g_1, \ldots, g_n) = g \in G_{n,x} \ : \ \nexists g' \in G_{n,x} \ s.t. \ g' \ dominates \ g\}$$

Given a number n of agents, I refer to $G^*_{n,x}$ as the set of equilibrium candidates with x links. Note that:

$$(g^*_1, \ldots, g^*_n) = g^* \in \text{argmax}_{g \in G} \ P(g) \text{ and } \sum_{i \in N} |g^*_i| = x \quad \Rightarrow g^* \in G^*_{n,x}$$

Therefore, the set:

$$G^* = \cup_{x=0}^{n(n-1)} G^*_{n,x}$$

is a superset that contains all of the equilibrium networks of the game.

This section is devoted to characterizing the architectures of the equilibrium candidates, and to identifying their key properties. Equilibrium networks that have many links (i.e. more than $2(n-1)$, so that there are enough links to form a star on all agents) do not have any architectural particularities except for their diameter. For this reason, I do not seek to characterize them. Instead, I focus on the equilibrium networks that have weakly less than $2(n-1)$ links. In particular, I make explicit the elements of the set G^* for small numbers n of agents.

The results featured below are obtained through a computer assisted proof. For $n = 5, 6$, I solve for the architectures in the sets $G^*_{n,x}$, and I vary the number x of links from 0 to $2(n-1)$.[9] Before introducing my results, let me first give an informal definition of a flower network. (For a formal definition, see Appendix A.7.) A *flower network* has one central agent, and the rest forms wheels (that I will call petals) of roughly the same size around the central agent. To be precise, either all petals have the same number of agents or else the maximum difference is one. In Figure 12, the networks g_2 and g_4 are flowers with 2 and 3 petals, respectively. The wheel network and the star network are special cases of flower architectures (with one petal and $n-1$ petals, respectively).

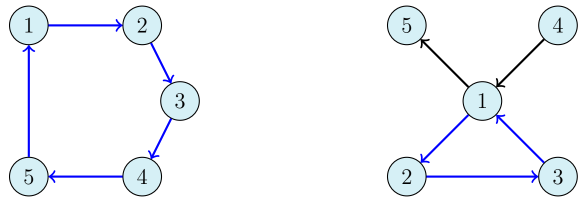
Proposition 2.5.3. *(1) If the network $g \in G$ is an equilibrium network, then there are at least* 3 *links in g. (2) For $n \in \{5, 6\}$, if the network $g \in G$ is an equilibrium network and g has x links in total, with $3 \leq x \leq 2(n-1)$, then g has one component that has the architecture of a flower, and the rest of the components (if any) are all singletons.*

[9] For $n = 3, 4$, all equilibrium candidates with x links, for $n \leq x \leq 2(n-1)$ are connected flowers. The analysis for $n = 5, 6$ players is richer in terms of results.

For statement 1, the proof is the same as that given in Appendix A.4 (see Claim 1). The proof of statement 2 is computer assisted (the software I use is R), and the code is provided in Appendix A.8. In words, my approach is first to fix a pair (n, x) for the number of agents and links in a network. For this pair, I generate the set $G_{n,x}$ of all networks on n agents with x links. Then, I look for the networks in $G_{n,x}$ that are not dominated by any other network in $G_{n,x}$. These networks constitute the elements of the set $G^*_{n,x}$ of equilibrium candidates that have x links. I repeat the same procedure for all pairs (n, x) that I consider. See the figures below for the architectures in the set G^*, for $n \in \{5, 6\}$ and a number of links larger than n. In blue, the flower component of each architecture.

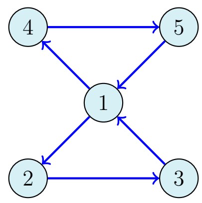
The results suggest the following. First, that the optimal architecture for a component is that of a flower. Flowers trade-off the higher costs of more links (as compared to the wheel network) against the benefits of shorter distances. Second, an equilibrium network may be disconnected, although it has more than n links. Fixing the total number of links in a network, there seems to be a trade-off between maximizing the agents' reach and minimizing distances: in a connected flower, the reach of the agents is maximized; in a disconnected flower, the agents who can reach each other are relatively closer than in a connected flower that has the same number of links. This shows at the level of the tails of the distance distributions of the equilibrium candidates that have the same number of links. Typically, in any set $G^*_{n,x}$, disconnected equilibrium candidates have distance distributions that put more weight on short distances and have shorter tails than the distance distribution of the connected equilibrium candidate. Third, the occurrence of disconnected architectures in a set $G^*_{n,x}$

increases as n gets larger. When the number of agents rises, it becomes less likely that incorporating all the agents in one component is efficient. In some disconnected equilibrium candidates, the singletons are linked to the flower component via the central agent. A link to (or from) the central agent gives short access to the rest of the network.



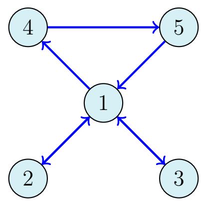
$\Gamma = \left\{\frac{1}{5}, \frac{2}{5}, \frac{15}{25}, \frac{20}{25}, 1, 1\right\}$ $\qquad$ $\Gamma = \left\{\frac{1}{5}, \frac{2}{5}, \frac{16}{25}, \frac{18}{25}, \frac{18}{25}, 1\right\}$

Figure 13: Architectures in $G^*_{5,5}$



$$\Gamma = \left\{\frac{1}{5}, \frac{11}{25}, \frac{19}{25}, \frac{23}{25}, 1, 1\right\}$$

Figure 14: Architecture in $G^*_{5,6}$

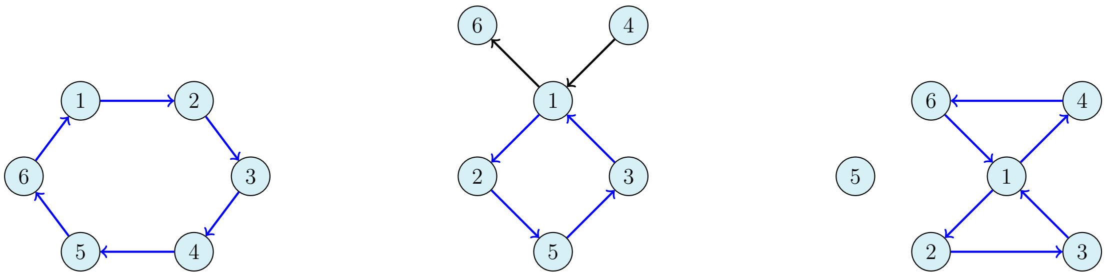


$$\Gamma = \left\{\frac{1}{5}, \frac{12}{15}, \frac{21}{25}, 1, 1, 1\right\}$$

Figure 15: Architecture in $G^*_{5,7}$

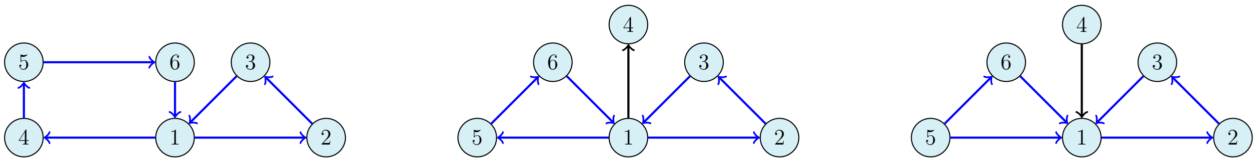
2.6 Conclusion

I presented a static non-cooperative game of network formation where the network is assumed to generate non-excludable benefits. The network helps connect a set of agents who use the network as a communication platform prior to taking a collective action. The outcome of the collective action is assumed to depend positively on the number of communications allowed by



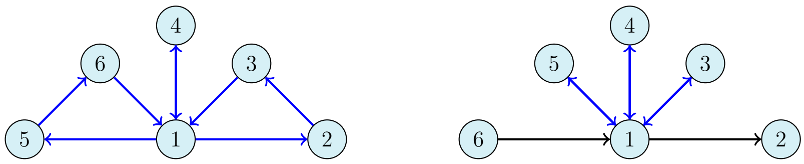
$\Gamma = \left\{\frac{1}{6}, \frac{12}{36}, \frac{18}{36}, \frac{24}{36}, \frac{30}{36}, 1, 1\right\}$ $\Gamma = \left\{\frac{1}{6}, \frac{12}{36}, \frac{19}{36}, \frac{25}{36}, \frac{27}{36}, \frac{27}{36}, 1\right\}$ $\Gamma = \left\{\frac{1}{6}, \frac{12}{36}, \frac{20}{36}, \frac{24}{36}, \frac{26}{36}, \frac{26}{36}, 1\right\}$

Figure 16: Architectures in $G^*_{6,6}$



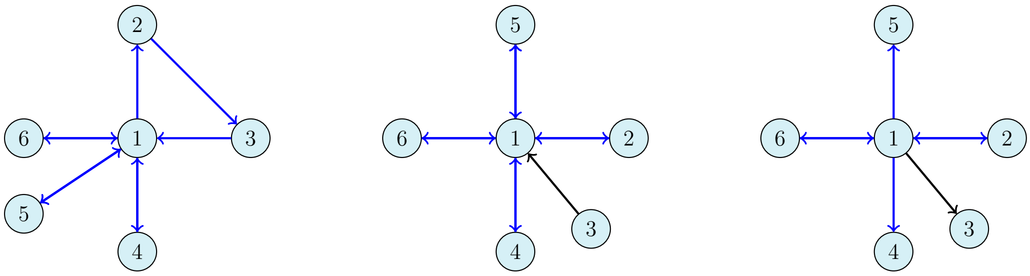
$\Gamma = \left\{\frac{1}{6}, \frac{13}{36}, \frac{22}{36}, \frac{30}{36}, \frac{34}{36}, 1, 1\right\}$ $\Gamma = \left\{\frac{1}{6}, \frac{13}{36}, \frac{23}{36}, \frac{28}{36}, \frac{30}{36}, \frac{30}{36}, 1\right\}$

Figure 17: Architectures in $G^*_{6,7}$



$\Gamma(\mathrm{g}) = \left\{\frac{1}{6}, \frac{14}{36}, \frac{26}{36}, \frac{34}{36}, 1, 1, 1\right\}$ $\Gamma = \left\{\frac{1}{6}, \frac{14}{36}, \frac{27}{36}, \frac{27}{36}, \frac{27}{36}, \frac{27}{36}, 1\right\}$

Figure 18: Architectures in $G^*_{6,8}$



$\Gamma = \left\{\frac{1}{6}, \frac{15}{36}, \frac{30}{36}, 1, 1, 1, 1\right\}$ $\Gamma = \left\{\frac{1}{6}, \frac{15}{36}, \frac{31}{36}, \frac{31}{36}, \frac{31}{36}, \frac{31}{36}, 1\right\}$

Figure 19: Architectures in $G^*_{6,9}$

the network structure. A network structure results from each individual's contribution in link formation, and an agent's decision is shaped by the trade-off between the social benefits of his links and the private cost he incurs for forming them. This network formation game has the property of being a potential game.

I studied two versions of the model. First, I assume that the return from the collective action is increasing in all players' reach in the network; meaning, on the total number of agents with whom each player talks. This implies that the benefit from an interaction of an agent with another one does not depend on the properties of the path they use for communicating, i.e. communication is *frictionless*. When communication is frictionless, the social benefit from a link is measured by the number of agents across the network that the link helps connect. In an alternative version, I assume that the shortest distance, i.e. the length of the communication channel an agent uses to talk to another one, affects the worth of an interaction. In particular, indirect (distant) connections are supposed to contribute less to the success of the collective action than direct ones. The social benefit of a link in this case is measured by the number of shortest paths that pass by the link, and longer paths account for less.

The objectives were to characterize the set of equilibrium networks and to compare them with the efficient architectures of the game. For the first version of the model, where communication is frictionless, I characterize the set of strict Nash equilibria of the game. The corresponding equilibrium networks have simple architectures: they are wheels that may or may not include all agents. Among these equilibrium networks, only the wheel on all agents and the empty network have an efficient architecture. In these structures, no player free rides on the others' contributions in links: either all contribute or none. For the second version of the game, where I assume that frictions occur along a path, I study the maximum of the potential. A maximum of the potential has the property of being a Nash equilibrium of the network formation game, and I show that the potential function is always maximized in a network that has an efficient architecture. I find that the networks that have a flower subgraph all qualify as equilibrium candidates (this is proved for a limited number of players). In a flower, the higher costs of link formation are balanced against the benefits of shorter distances, which is made possible by centralizing connections around one single agent who mediates most of the communications in the network.

In this game where the network is a public good, I assumed that all agents are homogeneous with respect to their valuation of the public good and their private costs for forming links. A possible extension of this game could be to introduce heterogeneity between the agents, along the lines just mentioned.

Chapter 3

Secret Rebates and List Prices in Negotiations between Countries and Pharmaceutical Firms

3.1 Introduction

In the negotiations on drug prices between a pharmaceutical firm and institutional payers (countries), secret rebates conceal the price each country effectively pays the manufacturer. For the case of a medicine listed for reimbursement in a country, the price the country pays corresponds to the list price of the drug (i.e. the official price at which it is listed for reimbursement, and this price is publicly observable), minus the secret rebate, which is money that the manufacturer gives back to the payer, and whose amount is kept confidential. During a negotiation are therefore determined two prices, the list price and a secret rebate. The use of a secret rebate makes the price each country pays confidential, in the sense that it is known to no one but the parties who negotiate it. The list price of a drug seems to be a poor indicator of the real price paid: estimations of the values of rebates range from 40% to 70% for specialty pharmaceuticals, and from 10% to 50% for primary care drugs across North America, Europe, and Australasia according to the figures of Morgan et al. (2017, [60]).

From the supplier's perspective, the use of secret rebates can be easily rationalized. Given that countries negotiate deals in turn, and that many of them apply *international price referencing* (IPR) (i.e. payers base their price offers to the manufacturer on the list prices of equivalent medicines in other countries), secret rebates makes it possible to isolate each negotiation from the others, and to avoid a low rebated price to propagate in subsequent negotiations.[1] Another source of price interdependencies between countries which also rationalizes the benefits of secret rebates to suppliers is the parallel import of medicines between countries in a same economic community, like in the EU. This involves importing a product into one member state from another, transacted at the list price in the exporting country.[2] In the related empirical literature, Danzon (1997, [20]) finds evidence of a strategic use of secret rebates, with confidential contracts including a rebate off a common list price replacing contracts with differential list prices and no rebate in most European countries.

From a country's perspective, the benefits of secret rebates seem mitigated. The main appeals are budgetary: rebates help in achieving financial

[1]See Vogler et al. (2018, [69]) and Towse et al. (2015, [67]).

[2]The possibility of parallel imports benefits countries, because it allows to arbitrage away differences in list prices.

goals and managing health budgets (see Scherer 1997, [63], Morgan et al. 2017, [60]). According to a recent work of Espin et al. (2018, [31]), rebates accorded to the countries in the EU5 have enabled to cap the growth of pharmaceutical expenditures to 2% over the past years. Yet, rebates are generally resource intensive to implement, especially for complicated rebate contracts; and they cause inefficiencies in terms of accessibility, as what individuals pay out of their pocket is a function of the list (unrebated) price (see Morgan et al. 2013, [59]).

This chapter focuses primarily on rationalizing the use of secret rebates from a country's perspective, and on characterizing conditions under which a country benefits from negotiating a secret rebate. To this end, we seek to understand the relation between the value of a list price and that of its associated rebate; and how these levels affect the demand and the health insurance policies in a country. As far as we know, no theoretical model has yet studied these relations. We offer a model where a monopolist firm has the opportunity to sell its drug to two countries. A key feature of our model is that we assume that a country and the firm agree on a contract that is both Pareto optimal and individually rational for them. A trade is settled through a contract that features two prices. The first one is the *list price*, which is public information; the second price is a *secret rebate*, known only to the parties who negotiate it. The countries are assumed to be welfare maximizers, and they balance the benefits from the availability of the drug on their respective market and the costs tied to acquiring it. Once the first negotiation is over, the list price in the first country is made public, and the second country starts its negotiation with the firm. The only assumption we make about this second country is that it applies a simple form of IPR: it never accepts to pay more than the list price it observes in the first country. As we consider that the firm is farsighted, the latter anticipates that the list price in the first country will affect the transaction price with the second one.

To understand how the outcome of price negotiations affects social insurance policies, we suppose that the first country to negotiate sets up a reimbursement scheme for the drug. The social insurance is funded by contributions levied on all citizens, and it reimburses a fixed share of the list price to the patients who can afford the drug. The reimbursement rate is fixed, however the social security charges are determined endogenously based on the deal with the manufacturer. This choice of ours is inspired by the French procedure through which the social coverage of medicines is determined. In France, the reimbursement rate is chosen by the Union des Caisses de l'Assurance Maladie (UCAM), which is composed of health professionals. Their choice is based on the therapeutic value added of the drug compared to already existing treatments within the same therapeutic class, the severity of the concerned disease and the degree of undesirable effects. The price of the product however does not appear to be a key determinant of the reimbursement rate. Economic and financial considerations (like the price, the finance scheme) are taken care of by another institution, in France called the Comité Économique des Produits de Santé (CEPS), after that the therapeutic benefit of the drug has been assessed by the UCAM.

We seek to determine the optimal level of social contributions put in place by the first country, and to characterize the set of Pareto optimal and individually rational (PO-IR) contracts between this country and the firm. We find that the first country always redistributes the money of the rebate towards lowering social charges, as this allows a maximum of patients to get access to the drug. Also, when social contributions are set to their optimal level, the first country has preference towards low list prices and low rebates for a fixed rebated price. These preferences are rationalized by the deadweight loss associated with a contract that has a secret rebate: the social security fund returns the money of the rebate to all citizens, even to those who do not need the drug, which is less efficient than an analogous decrease in the list price, which targets specifically the sick population. We identify the relative market sizes in the two countries as a key determinant of the outcome of the negotiation between the firm and the first country. If the market size in the second country is sufficiently small, the firm and the first country may find common ground on not using a secret rebate. In this instance, the firm prefers to sell more in the first country, even if this entails a low list price to propagate in the second negotiation. If, on the other hand, the second country represents a large and profitable market, then no deal is ever worth sealing with the first country from the firm's perspective. We conclude that the use of rebates in the first negotiation is conditional on the two countries being more or less balanced in terms of market sizes. If this condition is satisfied, the PO-IR contracts between the firm and the first country all include a strictly positive rebate; specifically, larger rebates are associated with larger list (unrebated) prices and lower rebated prices. This result suggests that rebates are used as currency between the first country and the firm. We reach the conclusion that the deal that maximizes the country's payoff is that with the largest rebate, because this deal is associated with the largest quantity traded and the lowest rebated price.

The work that is the most related to ours is that by Jelovac (2002, [49]). Jelovac studies the relation between the patients' co-payment for buying drugs and the list price of a patented pharmaceutical.[3] The author finds that countries which offer good social coverage have more bargaining power in their negotiation, and are able to obtain a lower list price from the manufacturer. Our paper contributes to this literature by giving a global insight onto the dynamics between health budget related decisions and the levels of both list prices and secret rebates: higher list prices and rebates are associated with lower social contributions. This relation is rationalized by the fact that contracts that feature higher list prices and rebates also correspond to lower rebated prices; hence, the social security needs levy less social charges. An interesting result is that overall, contracts with larger list prices are associated with a greater quantity demanded (i.e., the mass of individuals who can afford the treatment increases), which suggests that the country succeeds in turning off the negative effect on the demand from a larger list price with a lowering of social contributions.

The rest of the paper is organized as follows. Section 3.2 presents the model. Sections 3.3 and 3.4 feature our results about the PO-IR contracts between the first country and the firm. Section 3.5 concludes.

[3] A co-payment is a fixed amount that a healthcare beneficiary pays for medical expenditures covered by his or her health insurance plan. The remaining balance is paid by the insurance company.

3.2 Model

In this section, we present the model as well as the payoff functions of the countries and the firm. We set clear the assumptions we use, and we describe the social security system in the first country.

3.2.1 Set-up

Two countries, indexed by $i \in \{1, 2\}$, seek to purchase a drug sold by a monopolist pharmaceutical firm, F. The two countries negotiate in turn. We shall draw the attention of the reader on the fact that we do not assume anything about how the countries negotiate with the firm, except that they will choose a contract that is Pareto optimal and individually rational for both of them, i.e. for country i and the firm. A successful negotiation between country $i \in \{1, 2\}$ and the firm results in a contract (p_i, r_i). The contract specifies the *list price* p_i of the drug in country i, which is publicly observable. The contract may also include a unitary *rebate* r_i, which is money the firm pays back to the country on each unit purchased. The amount of the rebate is only known to the parties which negotiate it, country i and F. We call the *net price* the rebated price paid by country i to the manufacturer. It is equal to $p_i - r_i$, and we denote it by y_i. We further assume that the firm is farsighted: it anticipates that the outcome of the first negotiation impacts the price at which it can sell its product in the second country. We suppose that the firm initiates an offer a secret rebate only for avoiding that a low net price propagates across markets. Thence,

$r_2 = 0$. From now on, we denote by r the secret rebate that country 1 obtains.

Given that the two countries negotiate sequentially with the firm, country 2, prior to sit at the negotiation table, knows the list price p_1 of the drug in country 1. However, the former does not know the secret rebate the latter got. In the following assumption, we restrict country 2's behavior during its negotiation with the firm. We do not seek to rationalize this behavior.

Assumption A

While negotiating with the firm, country 2 uses international price referencing (IPR): it never accepts to pay a price greater than the list price in country 1, i.e. $p_2 \leq p_1$.

Assumption A can be understood as country 2's participation constraint in its negotiation with the firm.

3.2.2 Payoffs

Country 1

Country 1 has a population size normalized to 1, and a proportion α of agents who would benefit from getting access to the drug. Every sick agent is associated with a marginal disutility of one. This unit can be recovered if the individual gets treated. We assume that individual wealth ω is distributed according to some cumulative distribution F on the support $[0, 1]$. Furthermore, we suppose that the country sets up a social insurance plan for reimbursing the drug. The social security levies a contribution $\tau(p_1, r)$ on all citizens. Its amount is chosen endogenously by the country after that it knows its contract with the firm. The social insurance reimburses some fixed share $1 - \gamma \in (0, 1]$ of the list price to a sick individual who can afford the drug. We assume that all citizens can afford the contribution τ, however not all can afford the payment of γp_1 after reimbursement.[4] If the country has a deal with rebate, the total rebate (the unitary rebate times the quantity traded) goes directly into the funds of the social security. The demand function of the country is:

$$q_1(p_1, r) = \alpha \left[1 - F\left(\tau(p_1, r) + \gamma p_1\right)\right],$$

which is the share of the sick population who can afford the treatment. The quantity demanded is decreasing in the share of the list price the patient must pay, and it is increasing in the size of the sick population. Holding the private contribution constant, the demand is decreasing in the list price ($\frac{dq_1}{dp_1} \leq 0$); and for a fixed list price, the quantity demanded is decreasing in the private contribution. Aggregating the utilities of all citizens, the welfare in country 1 for a given contract (p_1, r) is:

$$W(p_1, r) = q_1(p_1, r)\left(1 - \gamma p_1\right) - \tau(p_1, r) - \alpha.$$

[4]Meaning, the country rises τ of social contributions in total. All agents whose wealth is greater than τ can afford the private contribution, which amounts to $\tau(1-\tau)$. As for the remaining τ^2 that cannot be levied on those whose wealth is less than τ, we suppose that the country levies it on healthy individuals who do not need to purchase the drug. In other words, we assume the existence of a redistributive tax scheme that we do not model here. Note that this redistributive scheme does not have any incidence on the quantity demanded.

This is the sum of all agents' utilities: every sick and treated individual has a utility of $-\gamma p_1 - \tau(p_1, r)$; that of a sick and untreated individual is $-1 - \tau(p_1, r)$; and the rest has utility $-\tau(p_1, r)$. Note that the quantity demanded is strictly positive if and only if $p_1 < \frac{1}{\gamma}$. At last, we assume the following.

Assumption B

The social security fund cannot run a deficit:

$$B(p_1, r) = \tau(p_1, r) - q_1(p_1, r)\left[(1-\gamma)p_1 - r\right] \geq 0 \tag{6}$$

for any contract (p_1, r)*.*

The first term in the expression of the budget constraint gives the total revenue from social contributions, and the last term is the net reimbursement cost on the social security. Country 1's payoff is denoted by v_1, and equals the sum of the welfare and the balance of the social security fund:

$$v_1(p_1, r) = W(p_1, r) + B(p_1, r)$$

If the negotiation with the firm is successful, the payoff corresponds to the following expression:

$$v_1(p_1, r) = q_1(p_1, r)[1 - (p_1 - r)] - \alpha \tag{7}$$

The product of the two first terms is the net social gain from the treated agents, and the last term is the social disutility from the sick individuals. If the negotiation fails, the payoff of the country is simply:

$$v_1 = -\alpha$$

No sick agent gets the treatment, and no insurance mechanism is set up. A first observation is that the country never accepts a contract that has a net price greater than the social marginal gain of treating an individual, i.e. $y_1 \leq 1$ must hold if the contract is individually rational for the country.

Country 2

We do not specify anything particular for this country. Recall that country 2 never obtains a rebate, and that it uses IPR. We do not model the negotiation between country 2 and the firm. We simplify the outcome of this negotiation along the lines described in the assumption stated below.

Assumption C

The outcome of the second negotiation is a list price p_2 such that:

$$p_2 \leq \min \left\{ p_1, p_2^{\mathcal{M}} \right\}$$

where $p_2^{\mathcal{M}}$ denotes the monopoly price in country 2.

The firm

We assume that the cost of producing the drug is null and that $R\&D$ costs are sunk. The firm can earn a profit on each market. For simplicity, let us write the firm's payoff on the second market as:

$$\theta \pi_2(\bar{p}_2)$$

where $\bar{p}_2 = \min\{p_1, p_2^{\mathcal{M}}\}$ is the maximum price the firm can charge country 2, and $\theta \in [0, 1]$ is a parameter that gauges the firm's negotiating power in its second negotiation.[5] Furthermore, we assume that the profit function π_2 is concave in the price. If trade happens with the first country, then the firm's total profit is written as:

$$\pi(p_1, r, p_2) = q_1(p_1, r)(p_1 - r) + \theta\pi_2(\bar{p}_2)$$

Note that the last term is the firm's profit on the second market, and it is always weakly less than the monopoly profit in country 2, that we denote by $\pi_2^{\mathcal{M}}$ throughout. If no trade happens with the first country, the firm's total profit is simply:

$$\pi = \theta\pi_2^{\mathcal{M}}$$

The firm makes no profit in country 1, and it earns a share of its monopoly profit in the second country.

3.2.3 Timing

We present the timing. First, country 1 and the firm negotiate their contract. If the negotiation succeeds, country 1 determines the level of the private contribution $\tau(p_1, r)$. If the negotiation fails, country 1 does not get the drug and does not set up an insurance plan. At last, country 2 negotiates with the firm. If the first negotiation with country 1 succeeded, then country 2 only accepts list prices lower than p_1. If country 1 and the

[5] The parameter θ can be understood as a measure of the market size in country 2.

firm could not reach an agreement, the list price in country 2 is less than or equal to the monopoly price, $p_2^{\mathcal{M}}$.

3.3 Pareto optimal and Individually rational contracts

We solve using backward induction. We first present country 1's optimal choice of private contribution for a given contract. Then, we characterize the contracts that are both Pareto-optimal and individually rational for the first country and the firm. Abusing language, we refer to these contracts as PO-IR.

3.3.1 The optimal contribution

Consider some contract (p_1, r) country 1 and the firm agreed on. Notice that maximizing the country's payoff in (7) is equivalent to minimizing the private contribution τ. Yet, the social security fund is budget-constrained. Therefore, the optimal contribution saturates the budget constraint in (6). Its expression is:

$$\tau^*(p_1, r) = \alpha\left[(1-\gamma)p_1 - r\right]\left[1 - F\left(\tau^*(p_1, r) + \gamma p_1\right)\right]$$

Note that the optimal contribution is negative (i.e., the country subsidizes the drug) when the marginal reimbursement cost on the social security, $(1-\gamma)p_1$, is less than the rebate, r. For a fixed list price, an increase in the rebate always leads to a decrease in the private contribution. This further causes an increase in the quantity demanded. For a fixed rebate, an increase in the list price has an ambiguous effect on the private contribution. Two forces are at play. First, the net cost of treating a sick person on the social security increases. Second, the rise in the list price impacts negatively the quantity demanded by the sick population. Therefore, the marginal reimbursement cost rises, however fewer individuals need a reimbursement. Depending on which effect dominates the other, the optimal contribution may either increase or decrease.

What is certain is that the welfare of the country is larger with social security than without.[6] For a given list price, more agents get treated with than without social security.

Remark 3.3.1. *When private contributions are set at their optimal level, country 1 is better-off with than without social security, for any net price less than* 1.

Proof. Recall that the country never accepts a contract that has a net price greater than 1, irrespective of whether the country offers social insurance or not. If the country does not offer social insurance, its payoff is given by:

$$v_1(y) = \alpha[1 - F(y)](1 - y) - \alpha.$$

[6] In our model, as it is the case for countries like France, subscription to social insurance is compulsory. We find that if the individuals were given the choice to enroll or not, they would all choose to subscribe to the social insurance scheme, i.e. pay τ and receive an expected reimbursement of $\alpha\gamma p_1$.

With social security, the country's payoff is:

$$v_1(p_1, r) = \alpha \left[1 - F\left(\tau^*(p_1, r) + \gamma p_1\right)\right](1 - y) - \alpha$$

Therefore the statement in the remark is true only if $\tau^*(p_1, r) \leq y - \gamma p_1$ i.e., more sick individuals can afford the treatment with than without social security. Let us set $y = p_1 - r$. The last inequality is re-arranged as $\tau^*(p_1, r) \leq (1 - \gamma)p_1 - r$. This always holds given that the quantity demanded is less than 1 in the expression of the optimal contribution. $\square$

In the remainder, we set F to be the uniform distribution. The quantity demanded is:

$$q_1(\tau^*(p_1, r), p_1, r) = \begin{cases} \dfrac{\alpha(1 - \gamma p_1)}{1 + \alpha(p_1 - r) - \alpha\gamma p_1} & \text{if } p_1 \leq \frac{1}{\gamma} \\ 0 & \text{if } p_1 > \frac{1}{\gamma} \end{cases} \tag{8}$$

which is always less than α. The numerator and the denominator of this expression are positive. The demand is increasing in α and decreasing in the share γ of the list price patients pay. Holding the list price constant, the quantity demanded is increasing in the rebate and decreasing in the net price; holding the rebate constant, the quantity is decreasing in the list price. The optimal contribution can be written as:

$$\tau^*(p_1, r) = \frac{\alpha\left[(1 - \gamma)p_1 - r\right](1 - \gamma p_1)}{1 + \alpha(p_1 - r) - \alpha\gamma p_1}, \tag{9}$$

When social contributions are at their optimal level, the country's payoff is given by the following expression:

$$v^*(p_1, r) = \frac{\alpha(1-\gamma p_1)}{1+\alpha(p_1-r)-\alpha\gamma p_1}[1-(p_1-r)]-\alpha. \tag{10}$$

Holding the rebate constant, the payoff is decreasing in the list price; and holding the list price constant, the payoff is increasing in the rebate, hence decreasing in the net price.

For a fixed net price, the country has preference towards a low list price and a low rebate. In fact, there exists a deadweight loss associated with a contract that has a strictly positive rebate: the country returns the money of the rebate to all individuals (even to those who do not need the drug), which is less effective than an equivalent reduction in the list price. In other words, for any two contracts that feature a same net price, the quantity traded is always larger for the contract that has the lowest list price, thus the smallest rebate.

We now study the firm's profit. If the first negotiation is successful, the firm's profit is given by:

$$\pi^*(p_1, r) = \begin{cases} \dfrac{\alpha(1-\gamma p_1)}{1+\alpha(p_1-r)-\alpha\gamma p_1}(p_1-r)+\theta\pi_2(p_1) & \text{if } p_1 \leq \min\left\{\frac{1}{\gamma}, p_2^{\mathcal{M}}\right\} \\ \dfrac{\alpha(1-\gamma p_1)}{1+\alpha(p_1-r)-\alpha\gamma p_1}(p_1-r)+\theta\pi_2^{\mathcal{M}} & \text{if } p_1 \in \left[p_2^{\mathcal{M}}, \frac{1}{\gamma}\right] \end{cases} \tag{11}$$

For a fixed rebate, the firm's profit is concave in the list price; and for a fixed list price, the firm's profit is increasing in the net price and decreasing in the rebate.[7] Also, the profit is increasing in α and decreasing in γ.

[7] To recover the result about the concavity of π^* in p_1, note that the sign of the second derivative of the profit with respect to p_1 is that of $-\gamma\left(1+\alpha(1-\gamma)p_1-\alpha r\right)(1-\alpha\gamma r)-2\alpha(1-\gamma)(1-\gamma p_1)(1-\alpha\gamma r)$, which is negative whenever $p_1 \leq \frac{1}{\gamma}$.

3.3.2 List price and rebate

The firm and country 1 must both gain from trading. In order to gauge these gains, we shall first clearly describe the parties' disagreement payoffs. For the country, failure to reach an agreement implies that the drug is not accessible. The country's payoff is then $-\alpha$. If the firm does not trade with the first country, it does not make any profit on the first market and earns some share of its monopoly profit in the second one. The disagreement payoffs are then $(-\alpha, \theta\pi_2^{\mathcal{M}})$. Below, we provide a formal definition of a PO-IR contract for both country 1 and the firm.

Definition 3.3.1 A contract between the firm and country 1 is *PO-IR* if each party earns at least its disagreement payoff (individual rationality), and there is no other contract that gives both parties a higher payoff (Pareto optimality).

A party agrees on a contract only if it satisfies its participation constraint. We make them explicit below.

Participation constraints

1. *Country 1 has two participation constraints:*

$$\gamma p_1 \leq 1 \text{ and } y_1 \leq 1 \tag{12}$$

i.e., the marginal cost on an individual, γp_1, and the maginal cost on society, which is the net price y_1, must be both lower than the *marginal benefit from treating an individual,* 1.

2. *The firm's participation constraint is:*

$$r \leq \min\left\{p_1\ ,\ p_1 - \frac{\theta[\pi_2^{\mathcal{M}} - \pi_2(p_1)](1-\alpha\gamma p_1)}{\alpha[1-\gamma p_1 - \theta(\pi_2^{\mathcal{M}} - \pi_2(p_1))]}\right\} \tag{13}$$

i.e., the profit on the first market is positive, and this profit compensates the loss of profit on the second market when $p_1 < p_2^{\mathcal{M}}$.

Proof. For the country: the quantity purchased must be positive, which is ensured by $p_1 \leq \frac{1}{\gamma}$; and given this, v_1^* is larger than $-\alpha$ if and only if $y_1 \leq 1$. For the firm, we show that the net price must be positive. By contradiction, suppose that country 1 and the firm signed a contract with $y_1 < 0$. But then the firm's profit is always less than its disagreement payoff. The second expression is obtained by rearranging $\pi^*(p_1, r) \geq \theta\pi_2^{\mathcal{M}}$ for the case where $p_1 < p_2^{\mathcal{M}}$. □

The indifference curve of country 1, expressed as the rebate in function of the list price, is of equation:

$$r = \frac{(\bar{v}+\alpha)[1+\alpha(1-\gamma)p_1] - \alpha(1-\gamma p_1)(1-p_1)}{\alpha[1-\gamma p_1 + (\bar{v}+\alpha)]}, \tag{14}$$

where $\bar{v} \in [-\alpha, 0]$ is a level of payoff for country 1. Note that when the country's participation constraint on the list price is satisfied, the denominator is positive. In the best scenario from the country's perspective, and given the firm's participation constraint, all patients get access to the drug and the net price is null. The above expression for the indifference curve is increasing convex in the list price p_1. See Appendix B.1 for the proof. Note that the country's payoff increases for north-west shifts of its indifference curve.[8] The marginal rate of substitution between the rebate and the list price is always negative: the country trades larger list prices against larger rebates. Since the indifference curve is convex in the list price, the increase in the rebate must be larger than the increase in the list price in order to maintain the country's payoff.

For any list price in country 1 that is less than the monopoly price in country 2, the equation of the firm's isoprofit curve is:

$$r = p_1 - \frac{(\bar{\pi} - \theta\pi_2(p_1))(1 - \alpha\gamma p_1)}{\alpha[1 - \gamma p_1 - (\bar{\pi} - \theta\pi_2(p_1))]} \tag{15}$$

where $\bar{\pi}$ is some profit level in $[\theta\pi_2^{\mathcal{M}}, \alpha + \theta\pi_2^{\mathcal{M}}]$. The lower bound guarantees that the firm does not loose from trading with country 1. To obtain the upper bound, note that the maximum quantity traded is α and that the largest net price the country may agree on is equal to 1. Also, note that both the numerator and the denominator of the ratio are positive when both parties' participation constraints are satisfied.[9] The firm's profit increases when its isoprofit curve shifts downwards. If instead the price in country 1 is above the monopoly price in country 2, the equation of the firm's isoprofit curve is:

$$r = p_1 - \frac{\left(\bar{\pi} - \theta\pi_2^{\mathcal{M}}\right)(1 - \alpha\gamma p_1)}{\alpha\left[1 - \gamma p_1 - (\bar{\pi} - \theta\pi_2^{\mathcal{M}})\right]} \tag{16}$$

with $\bar{\pi} \in [\theta\pi_2^{\mathcal{M}}, \alpha + \theta\pi_2^{\mathcal{M}}]$ for the same reason as aforementioned. The numerator and the denominator of the ratio are both positive.[10]

[8] For a same value of the rebate, an indifference curve that passes by a lower list price corresponds to a higher payoff. And for a same list price, an indifference curve that passes by a larger rebate corresponds to a higher payoff.

[9] If all participation constraints are satisfied, then we have $p_1 \leq \frac{1}{\gamma}$, $r < p_1$, and $\bar{\pi} \geq \theta\pi_2(p_1)$ for any $\bar{\pi} \in [\pi_2^{\mathcal{M}}, \alpha + \pi_2^{\mathcal{M}}]$, and $p_1 \in [0, p_2^{\mathcal{M}}]$.

[10] If all participation constraints are satisfied, then $p_1 \geq r$, $p_1 \leq \frac{1}{\gamma}$ and $\bar{\pi} \geq \theta\pi_2^{\mathcal{M}}$.

The profit that the firm makes on the first market is larger, given a fixed net price, for lower list prices and rebates. However, a low list price propagates on the second market and reduces the firm's profit there. When its monopoly profit in country 2 is sufficiently large, the firm may gain from trading exclusively with this country. In the next proposition, we give a sufficient condition on the demands of the two countries that precludes trade between country 1 and the firm.

Proposition 3.3.1. *Country 1 and the firm never trade if:*

$$\left(1 - \left|\varepsilon_2\left(p_2\right)\right|\right) q_2\left(p_2\right)\Big|_{p_2=\frac{1}{\gamma}} > \frac{\alpha\gamma}{\theta}$$

where q_2 is the demand in country 2 *and ε_2 its price elasticity.*

See Appendix B.2 for the proof. If the condition in the above proposition holds, then the firm's participation constraint is violated for all list price-rebate pairs that satisfy country 1's participation constraints. In other words, no trade is worthwhile sealing with country 1 if the monopoly price in country 2 exceeds the maximal list price country 1 may agree on.

The possibility of mutually advantageous trades depends negatively on how profitable the second market is. In what follows, we focus on the parameters of our model such that the firm serves both markets. Before providing a characterization of the PO-IR contracts, we first study the relation between a list price, its associated rebate and net price, and the quantity traded. For this, we provide a series of lemmas that study each of these relations separately.

Lemma 3.3.1. *If p_1^* is a PO-IR list price, then $p_1^* \leq p_2^{\mathcal{M}}$ and the following* *condition holds:*

$$\theta\frac{\partial\pi_2(p_1^*)}{\partial p_1}(1-\gamma p_1^*)(1-\alpha\gamma p_1^*) \geq \gamma\left(\bar{\pi}-\theta\pi_2(p_1^*)\right)\left[1-\alpha+\alpha\left(\bar{\pi}-\theta\pi_2(p_1^*)\right)\right]$$

i.e., the firm's isoprofit curve in expression (15) *is increasing at* p_1^*.

Proof. If p_1 such that $p_1 > p_2^{\mathcal{M}}$ is PO-IR, then country 1's indifference curve in expression (14) is tangent to the firm's isoprofit curve. Let $\frac{dr_C}{dp_1}$ be the country's marginal rate of substitution between list price and rebate. This is:

$$\frac{dr_C}{dp_1} = 1 + \frac{\gamma\left(\bar{v}+\alpha\right)\left[1-\alpha\left(\bar{v}+\alpha\right)\right]}{\alpha\left[1-\gamma p_1+\left(\bar{v}+\alpha\right)\right]^2}$$

with $\bar{v} \in [-\alpha, 0]$ in a PO-IR contract. It follows that the numerator of the ratio is positive. Therefore $\frac{dr_C}{dp_1} > 1$. Let $\frac{dr_F}{dp_1}$ be the firm's marginal rate of substitution between list price and rebate. Since $p_1 > p_2^{\mathcal{M}}$, the equation of the isoprofit corresponds to (16), which gives:

$$\frac{dr_F}{dp_1} = 1 - \frac{\left(\bar{\pi}-\theta\pi_2^{\mathcal{M}}\right)\left[\gamma(1-\alpha)+\alpha\gamma\left(\bar{\pi}-\theta\pi_2^{\mathcal{M}}\right)\right]}{\alpha\left[1-\gamma p_1-\left(\bar{\pi}-\theta\pi_2^{\mathcal{M}}\right)\right]^2}$$

with $\bar{\pi} \in \left[\theta\pi_2^{\mathcal{M}}, \alpha+\theta\pi_2^{\mathcal{M}}\right]$ in a PO-IR contract. Thus the numerator of the ratio is positive. But then $\frac{dr_F}{dp_1} \leq 1$, and it is never equal to $\frac{dr_C}{dp_1}$. Therefore, a contract (p_1, r) is PO-IR only if $p_1 \leq p_2^{\mathcal{M}}$.

If (p_1, r) with $p_1 \leq p_2^{\mathcal{M}}$ is PO-IR, then country 1's indifference curve in expression (14) is tangent to the firm's isoprofit curve:

$$\frac{\gamma(\bar{v}+\alpha)\left[1-\alpha(\bar{v}+\alpha)\right]}{\left[1-\gamma p_1+(\bar{v}+\alpha)\right]^2} = \frac{\theta\frac{\partial\pi_2(p_1)}{\partial p_1}(1-\gamma p_1)(1-\alpha\gamma p_1)-\gamma\left(\bar{\pi}-\theta\pi_2(p_1)\right)\left[1-\alpha+\alpha\left(\bar{\pi}-\theta\pi_2(p_1)\right)\right]}{\left[1-\gamma p_1-\left(\bar{\pi}-\theta\pi_2(p_1)\right)\right]^2}$$

The term on the left side of the equality sign is strictly positive, as $\bar{v} \in [-\alpha, 0]$ if p_1 is individually rational. Therefore, the right side of the equality sign must be positive. As the denominator is obviously positive, the numerator must also be positive valued. $\square$

Note that a PO-IR contract between the firm and the first country enables the second one to pay less than the monopoly price on its market, thanks to IPR. Next, we highlight the relation between the list price and the net price in a PO-IR contract.

Lemma 3.3.2. *Consider some contract (p_1^*, r^*), and let $\bar{\pi}$ the firm's profit for this contract. Let $\hat{p}_1(\bar{\pi})$ be the list price that minimizes the net price along the firm's isoprofit curve $\bar{\pi}$. If (p_1^*, r^*) is PO-IR, then:*

$$p_1^* \leq \hat{p}_1(\bar{\pi})$$

Furthermore, $\hat{p}_1(\bar{\pi})$ is decreasing in $\bar{\pi}$.

Proof. Consider a PO-IR contract (p_1^*, r^*) and let $\pi^*(p_1^*, r^*) = \bar{\pi}$. The net prices that give the firm the same level $\bar{\pi}$ of profit are along the curve of equation:

$$y(p_1, \bar{\pi}) = \frac{(\bar{\pi} - \theta\pi_2(p_1))\,(1 - \alpha\gamma p_1)}{\alpha\,[1 - \gamma p_1 - (\bar{\pi} - \theta\pi_2(p_1))]}$$

and (p_1^*, r^*) belongs to this curve. The first derivative of $y(p_1, \bar{\pi})$ wrt p_1 is:

$$\frac{\partial y(p_1, \bar{\pi})}{\partial p_1} = \frac{\left(-\theta\dfrac{\partial \pi_2(p_1)}{\partial p_1}(1 - \gamma p_1)(1 - \alpha\gamma p_1) + \gamma(\bar{\pi} - \theta\pi_2(p_1))[1 - \alpha + \alpha(\bar{\pi} - \theta\pi_2(p_1))] \right)}{\alpha^2[1 - \gamma p_1 - (\bar{\pi} - \theta\pi_2(p_1))]^2}$$

The sign is that of the numerator. The numerator is an increasing function of p_1, since its first derivative with respect to p_1 can be expressed as:

$$-\theta\frac{\partial^2\pi_2(p_1)}{\partial p_1^2}(1-\gamma p_1)(1-\alpha\gamma p_1)+2\alpha\gamma\theta\frac{\partial\pi_2(p_1)}{\partial p_1}[1-\gamma p_1-(\bar{\pi}-\theta\pi_2(p_1))\geq 0$$

The inequality holds because $p_1 \leq p_2^{\mathcal{M}}$ by Lemma 3.3.1, $r \geq 0$ in expression (15) if and only if $1-\gamma p_1-(\bar{\pi}-\theta\pi_2(p_1)) \geq 0$, and we assumed that π_2 is concave in the price. Let us go back to the numerator of $\frac{dy}{dp_1}$. We just established that it is increasing in p_1, for all $p_1 \in [0, p_2^{\mathcal{M}}]$. Note that $\frac{dy}{dp_1} \geq 0$ at $p_1 = p_2^{\mathcal{M}}$. Also, $\frac{dy}{dp_1} \leq 0$ at $p_1 = p_1^*$, by Lemma 3.3.1. Therefore, a global minimum of $y(p_1, \bar{\pi})$ exists. Let $\hat{p}_1(\bar{\pi}) = \text{argmin } y(p_1, \hat{\pi})$. This price solves:

$$-\theta\frac{\partial\pi_2(p_1)}{\partial p_1}(1-\gamma p_1)(1-\alpha\gamma p_1)+\gamma(\bar{\pi}-\theta\pi_2(p_1))[1-\alpha+\alpha(\bar{\pi}-\theta\pi_2(p_1))]\Big|_{p_1=\hat{p_1}(\bar{\pi})}=0$$

The function on the left side of the equality sign is increasing in $\bar{\pi}$; and recall that it is increasing in p_1. It follows that $\hat{p}_1$ is decreasing in $\bar{\pi}$.

The rest of the proof is by contradiction. Assume that $p_1^* > \hat{p}_1(\bar{\pi})$. Consider $\hat{r}(\bar{\pi})$ such that: $\pi(\hat{p}_1(\bar{\pi}), \hat{r}(\bar{\pi})) = \bar{\pi}$. The firm is indifferent between (p_1^*, r^*) and $(\hat{p}_1(\bar{\pi}), \hat{r}(\bar{\pi}))$. By the definition of $\hat{p}_1(\bar{\pi})$, $\hat{p}_1(\bar{\pi}) - \hat{r}(\bar{\pi}) < p_1^* - r^*$. Note that the country's payoff can be written as:

$$v^*(p_1, y) = \frac{\alpha(1-\gamma p_1)}{1+\alpha y-\alpha\gamma p_1}(1-y)-\alpha$$

and this expression is such that $\frac{dv^*}{dp_1} < 0$ and $\frac{dv^*}{dy} < 0$. But then country 1 strictly prefers $(\hat{p}_1(\bar{\pi}), \hat{r}(\bar{\pi}))$ over (p_1^*, r^*). A contradiction that (p_1^*, r^*) is Pareto optimal. $\square$

The lemma shows that, for a given level of profit for the firm, the contract that maximizes the country's payoff is not necessarily the one with the lowest net price. Such a contract involves a list price high enough for that the country prefers instead a contract with a larger net price and a lower list price. Next, we study the relation between the list price and the quantity traded.

Lemma 3.3.3. *Consider a PO-IR contract (p_1^*, r^*), and let $\bar{\pi}$ be the firm's profit for this contract. Let $\tilde{p}_1(\bar{\pi})$ be the list price that maximizes the quantity traded along the firm's isoprofit curve $\bar{\pi}$. If (p_1^*, r^*) is PO-IR, then:*

$$p_1^* \geq \tilde{p}_1(\bar{\pi})$$

Furthermore, $\tilde{p}_1(\bar{\pi})$ is decreasing in $\bar{\pi}$, and $\hat{p}_1(\bar{\pi}) \geq \tilde{p}_1(\bar{\pi})$.

Proof. Consider a PO-IR contract (p_1^*, r^*), and let $\pi^*(p_1^*, r^*) = \bar{\pi}$. The quantities q_1 demanded by country 1 that give the firm the same level of profit $\bar{\pi}$ are along the curve of equation:

$$q_1(p_1, \bar{\pi}) = \frac{\alpha[1 - \gamma p_1 - (\bar{\pi} - \theta\pi_2(p_1))]}{1 - \alpha\gamma p_1}$$

and (p_1^*, r^*) belongs to this curve. Note that:

$$\frac{\partial q_1(p_1, \bar{\pi})}{\partial p_1} = \frac{\alpha\left(-\gamma(1-\alpha) - \alpha\gamma(\bar{\pi} - \theta\pi_2^{\mathcal{M}}) + \left[\theta\frac{\partial \pi_2(p_1)}{\partial p_1}(1 - \alpha\gamma p_1) - \alpha\gamma\theta(\pi_2^{\mathcal{M}} - \pi_2(p_1))\right]\right)}{[1 - \alpha\gamma p_1]^2}$$

The sign of $\frac{dq_1}{dp_1}$ is that of its numerator. The function between squared brackets is decreasing in p_1, and it equals zero at $p_1 = p_2^{\mathcal{M}}$. Therefore, either $q_1(p_1, \bar{\pi})$ is single-peaked in p_1, or it is strictly decreasing in p_1. In any case, a global maximum of $q_1(p_1, \bar{\pi})$ exists on $[0, p_2^{\mathcal{M}}]$. We set $\tilde{p}_1(\bar{\pi}) =$ argmax $q_1(p_1, \bar{\pi})$. If $\tilde{p}_1(\bar{\pi}) > 0$, then it is given by:

$$\theta \frac{\partial \pi_2(p_1)}{\partial p_1}(1 - \alpha\gamma p_1) + \alpha\gamma\theta\pi_2(p_1)\Big|_{\tilde{p}_1(\bar{\pi})} = \alpha\gamma\bar{\pi} + \gamma(1-\alpha)$$

The left side of the equality is a decreasing function of p_1.[11] It follows that $\tilde{p}_1$ is decreasing in $\bar{\pi}$. We now show that $\hat{p}_1(\bar{\pi}) \geq \tilde{p}_1(\bar{\pi})$. Note that:

$$\theta \frac{\partial \pi_2(p_1)}{\partial p_1}(1 - \alpha\gamma p_1) - \gamma[1 - \alpha + \alpha(\bar{\pi} - \theta\pi_2(p_1))]\Big|_{\tilde{p}_1(\bar{\pi})} = 0$$

where the function on the left of the equality sign is decreasing in p_1 (see the proof of Lemma 3.3.2), and:

$$\theta \frac{\partial \pi_2(p_1)}{\partial p_1}(1-\gamma p_1)(1-\alpha\gamma p_1) - \gamma(\bar{\pi} - \theta\pi_2(p_1))[1-\alpha+\alpha(\bar{\pi}-\theta\pi_2(p_1))]\Big|_{\hat{p}_1(\bar{\pi})} = 0$$

where the function on the left of the equality sign is decreasing in p_1. Recall that $1 - \gamma \geq \bar{\pi} - \theta\pi_2(p_1)$, as the denominator of expression (15) is positive. And $\bar{\pi} \geq \theta\pi_2^{\mathcal{M}} \geq \theta\pi_2(p_1)$, where the first inequality holds as the firm's participation is satisfied if the contract is PO-IR; and $p_1 \leq p_2^{\mathcal{M}}$ by Lemma 3.3.1. It follows that $\hat{p}_1(\bar{\pi}) \geq \tilde{p}_1(\bar{\pi})$.

The rest of the proof is by contradiction. Assume that $p_1^* < \tilde{p}_1(\bar{\pi})$. Consider the contract $(\tilde{p}_1(\bar{\pi}), \tilde{r}(\bar{\pi}))$ with $\tilde{r}(\bar{\pi})$ such that $\pi(\tilde{p}_1(\bar{\pi}), \tilde{r}(\bar{\pi})) = \bar{\pi}$. The firm is indifferent between (p_1^*, r^*) and $(\tilde{p}_1(\bar{\pi}), \tilde{r}(\bar{\pi}))$. If p_1^* is part of a PO-IR contract, then $p_1^* \leq \hat{p}_1(\bar{\pi})$ by Lemma 3.3.2. Thus, we have

[11] The derivative of this function wrt p_1 is $\theta \frac{\partial^2 \pi_2(p_1)}{\partial p_1^2} \leq 0$, since we assume $\pi_2(.)$ is concave in the list price and $p_1 \leq p_2^{\mathcal{M}}$.

$p_1^* < \tilde{p}_1(\bar{\pi}) \leq \hat{p}_1(\bar{\pi})$; hence, we can infer that $\tilde{p}_1(\bar{\pi}) - \tilde{r}(\bar{\pi}) < p_1^* - r^*$. Therefore, the net price is lower and the quantity traded is larger with contract $(\tilde{p}_1(\bar{\pi}), \tilde{r}(\bar{\pi}))$ than with (p_1^*, r^*). The country's payoff can be written as:

$$v^*(y, q_1) = (1-y)q_1 - \alpha$$

and note that $\frac{dv^*}{dy} < 0$ and $\frac{dv^*}{dq_1} > 0$. But then country 1 strictly prefers $(\tilde{p}_1(\bar{\pi}), \tilde{r}(\bar{\pi}))$ over (p_1^*, r^*). Which contradicts that (p_1^*, r^*) is Pareto optimal.

□

For a fixed level of profit for the firm, the country's payoff is not necessarily the largest for the contract that maximizes the quantity traded. The net price associated with such a contract is large enough for that the country prefers a contract with a lower net price, a larger rebate and a larger list price. The two lemmas together imply the following relation between the rebate and the list price in a PO-IR contract.

Proposition 3.3.2. *Consider two contracts (p_1, r) and (p_1', r'). If $p_1 > p_1'$ and both contracts are PO-IR, then it must hold that $r \geq r'$.*

The proof can be found in Appendix B.3. In the set of PO-IR contracts, larger list prices are associated with larger rebates. If two contracts fail to satisfy this relation, then the one that has the largest list price (hence the lowest rebate) is Pareto dominated. A Pareto improvement can be achieved with a contract that has a larger net price, a lower list price and a lower rebate, and that involves a greater quantity traded. Below, we characterize the set of PO-IR contracts between the first country and the firm.

Proposition 3.3.3. *A contract (p_1, r) between country 1 and the firm is PO-IR if it is on the Pareto frontier of equation:*

$$r^*(p_1) = (1-\gamma)p_1 + \frac{1}{\alpha} - \frac{\alpha\gamma(1-\gamma p_1)}{\alpha\gamma - \theta\frac{\partial\pi_2(p_1)}{\partial p_1}(1+\alpha-\alpha\gamma p_1)} \tag{17}$$

and if it further satisfies:

$$p_1 \leq \min\left\{p_2^{\mathcal{M}}\ ,\ \frac{1}{\gamma}\right\}$$

and:

$$p_1 - \frac{\theta[\pi_2^{\mathcal{M}} - \pi_2(p_1)](1-\alpha\gamma p_1)}{\alpha[1-\gamma p_1 - \theta(\pi_2^{\mathcal{M}} - \pi_2(p_1))]} \geq r \geq p_1 - 1$$

The quantity traded for such a contract is given by the expression:

$$q_1^*(p_1) = 1 - \theta\frac{\partial\pi_2(p_1)}{\partial p_1}\left[\frac{1}{\alpha\gamma} + \left(\frac{1}{\gamma} - p_1\right)\right] \tag{18}$$

and it is strictly increasing in p_1. The optimal contribution for some PO-IR contract can be expressed as:

$$\tau^*(p_1, r) = \frac{\theta\frac{\partial\pi_2(p_1)}{\partial p_1}(1+\alpha-\alpha\gamma p_1) - \alpha\gamma(1-\alpha+\alpha\gamma p_1)}{\alpha^2\gamma}$$

and it is strictly decreasing in p_1.

Proof. Expression (17) gives the equation of the Pareto frontier. It is obtained by solving:

$$\frac{\partial v_1^*(p_1, r)/\partial p_1}{\partial v_1^*(p_1, r)/\partial r} = \frac{\partial\pi^*(p_1, r)/\partial p_1}{\partial\pi^*(p_1, r)/\partial r}$$

Note that the firm's participation constraint is the firm's isoprofit $\bar{\pi} = \theta\pi_2^{\mathcal{M}}$. This isoprofit curve intersects the Pareto frontier for some value of p_1 less than $p_2^{\mathcal{M}}$. Also, note that as p_1 approaches $p_2^{\mathcal{M}}$, the rebate in expression (17) approaches $p_1 + \frac{1-\alpha}{\alpha}$. Given that $p_1 - r \geq 0$ by the firm's participation constraint, it follows that $r = p_1$ if and only if $p_1 = p_2^{\mathcal{M}}$ and $\alpha = 1$. □

Further details are provided in Appendix B.4. The area of mutually advantageous trades, which is delimited by the parties' participation constraints, shrinks as the firm's disagreement payoff increases. Also, a larger disagreement payoff for the firm shifts the Pareto frontier to the right: for a fixed secret rebate, its associated PO-IR list price is higher.

Corollary 3.3.1. *There exists a PO-IR contract* (p_1^*, r^*) *such that* $r^* = p_1^*$ *if and only if* $p_1^* = p_2^{\mathcal{M}}$, $p_2^{\mathcal{M}} \leq \frac{1}{\gamma}$ *and* $\alpha = 1$.

The result follows directly from Proposition 3.3.3. Unless the whole population in country 1 is sick, the country pays the firm a strictly positive price. The next theorem clarifies the relation between the list price, the rebate and the net price in the set of PO-IR contracts.

Theorem 3.3.2. *Let* $\mathcal{X}_1$ *be the set of all PO-IR contracts. In this set, larger list prices are associated with larger rebates and lower net prices.*

The theorem above completes the analysis initiated in Proposition 3.3.2, and makes explicit the relation between secret rebate, list price and net price in the set of PO-IR contracts. More details are provided in Appendix B.5. In what follows, we give precision on each party's preferred PO-IR contract.

Corollary 3.3.3. *In the set $\mathcal{X}_1$ of PO-IR contracts, country 1's payoff is maximized for the contract that has the largest list price, the largest rebate and the lowest net price; and the firm's profit is maximized for the contract that has the lowest list price, the lowest rebate and the highest net price.*

Proof. In the set of PO-IR contracts, the contract that gives the firm the lowest profit, $\theta\pi_2^{\mathcal{M}}$, can be found at the intersection of the Pareto frontier in (17) and the firm's participation constraint in (13). Let us call this contract (p_1', r'). Take any other PO-IR contract (p_1, r) such that $p_1 < p_1'$. By Theorem 3.3.2, we have $r < r'$ and $y' < y$. As (p_1, r) satisfies the firm's participation constraint, and as the Pareto frontier is strictly increasing in p_1, it follows that $\pi(p_1, r) > \pi_2^{\mathcal{M}}$. Since (p_1, r) is Pareto optimal, it must be that the country prefers (p_1', r') over (p_1', r'). The result follows. $\square$

Note that some PO-IR contracts may include a negative rebate when the country's lowest indifferent curve, $\bar{v} = -\alpha$ (and whose equation is $r = p_1 - 1$), intersects the Pareto frontier at a list price less than 1. A negative secret rebate means that the country subsidizes a low list price. In the proposition below, we identify a necessary and sufficient condition for the existence of PO-IR contracts with negative rebates.

Proposition 3.3.4. *There exist PO-IR contracts (p_1, r) with $r \leq 0$ if and only if:*

$$\left.(1 - |\varepsilon_2(p_2)|)\, q_2(p_2)\right|_{p_2=1} \leq \frac{\alpha\gamma}{\theta\,(1 + \alpha(1-\gamma))^2},$$

where q_2 is country 2's demand and ε_2 its price elasticity.[12]

[12]If this condition holds, then the monopoly price in the second country is less than 1.

3.4 Results

We gather our results in the next propositions. First, let us define $p^{\#}$ as the list price along the Pareto frontier such that $r(p^{\#}) = 0$. We define $\underline{p}$ as the lowest PO-IR list price.[13] As the Pareto frontier is strictly increasing in p_1, note that $\underline{p} < p^{\#}$ if and only if $p^{\#} < 1$. Note that $p^{\#} \leq 1$ is equivalent to the condition in Proposition 3.3.4, and that this condition always holds when the monopoly price in country 2 is less than 1.[14] Also, $\underline{p} > \frac{1}{\gamma}$ is equivalent to the no-trade condition in Proposition 3.3.1, and it implies that the monopoly price in country 2 is larger than $\frac{1}{\gamma}$, the list price for which country 1's demand is null.

Proposition 3.4.1. *(1) If $p^{\#} \leq 1$, then there exist PO-IR contracts that have a non-positive rebate. (2) If $p^{\#} > 1$ and $\underline{p} \leq \frac{1}{\gamma}$, then all PO-IR contracts have a strictly positive rebate.*

When the monopoly price in country 2 is sufficiently low (see statement 1), PO-IR contracts that have the largest net prices include negative rebates. We conclude that the firm uses a positive rebate to hide the net price paid by country 1 from country 2, and the country uses a negative rebate as a subsidy in exchange for a low list price. Note that country 1 pays more for the drug than country 2 whenever the list price in the first country is less than $p^{\#}$ (i.e., when the rebate is negative); and the converse holds when the list price in country 1 is larger than $p^{\#}$. Therefore, the

[13]This is the list price at the intersection between the iso-net price curve of equation $r = p_1 - 1$ and the curve whose equation is given by expression (17).

[14]To see why: if $p_2^{\mathcal{M}} \leq 1$, then condition in Proposition 3.3.4 is always satisfied, as $\frac{\partial \pi_2(p_1)}{\partial p_1}$ (which is equivalent to expression on the left side of the quality sign) is negative.

rebate enables the firm to price discriminate the two countries. When both positive and negative rebates are possible, the first market is sufficiently profitable relative to the second one, and the firm prefers to charge country 1 a higher price than country 2 (as the firm's preferred contract has the lowest rebate, which is negative in this case).

For larger values of the monopoly price in country 2 (see statement 2), country 1 never agrees on a negative rebate, as this would violate its participation constraint ($y_1 \geq 1$). Rebates are then used exclusively for hiding the price paid by country 1 from country 2, and the first country always pays less for the drug than the second country. In Figure 20, we provide a graphical representation of this case. The purple point represents the contract that the country prefers (that with the largest rebate, the largest list price and the lowest net price), and the blue point corresponds to the contract the firm prefers (that with the lowest rebate, the lowest list price and the largest net price).

Proposition 3.4.2. *If $\underline{p} > \frac{1}{\gamma}$, then country 1 and the firm do not trade.*

For even larger values of the monopoly price in country 2, the firm never gains from trading with the first country.

We see two necessary conditions for that the negotiation leads to an agreement on a large rebate. The first condition is that the market in country 2 is sufficiently profitable, which precludes trades with country 1 that involve relatively low list prices (thus low rebates by Proposition 3.3.2). The second necessary condition is that country 1 has an important negotiating power, that allows it to influence the outcome of the negotiation towards the one it prefers: namely, a contract with a large rebate, a high list price and a low net price.

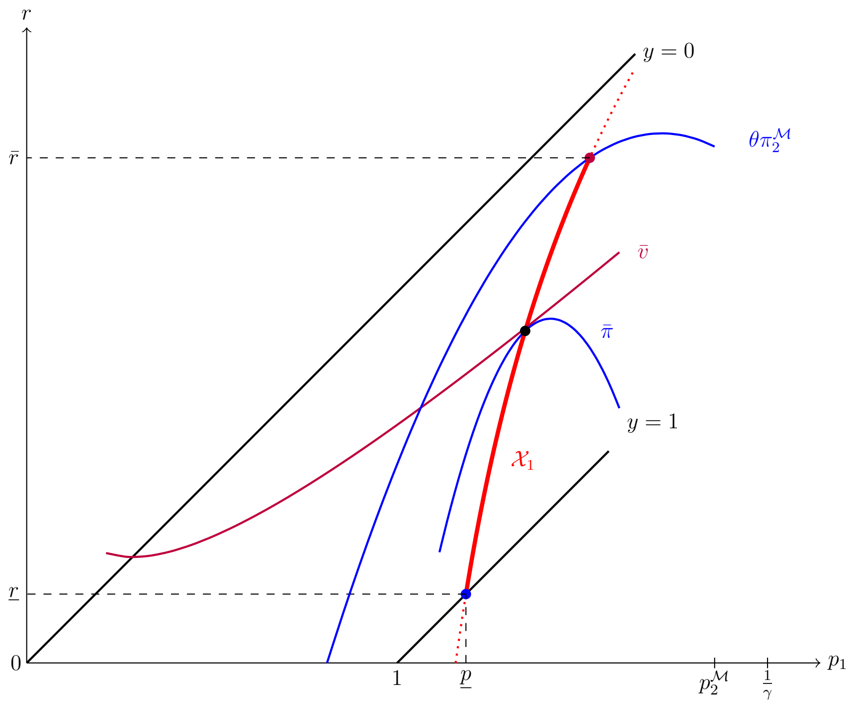


Figure 20: The set of PO-IR contracts is $\mathcal{X}_1$, in red on the graph

3.5 Conclusion

We presented a model where two countries negotiate in turn with a monopolist pharmaceutical firm, and we suppose that a country and the firm settle on a contract that is both Pareto optimal and individually rational for them. The second country to negotiate is assumed to apply a simple form of international price referencing, i.e. the latter never accepts to pay more than the list price it observes in the other country. The interdependence between the prices paid by the two countries can be turned off if the first country and the firm negotiate a secret rebate, which enables to hide the real price the latter pays for the drug.

We solve for the contracts that are both Pareto optimal and individually rational for the firm and the first country. We find that larger list prices are associated with larger rebates, lower rebated prices, lower social charges and greater quantities traded. Also, the relative profitability of each market taken in isolation influences the possibility of mutually advantageous trades between the first country and the firm, as well as the amount of secret rebate they can negotiate. In particular, we show that large rebates are to be expected when the second market is relatively profitable and when the first country has an important negotiating power. The firm benefits from using a secret rebate because it prevents low rebated prices to propagate in the second negotiation; as for the first country, the benefits of larger list prices and rebates are tied to their association with greater quantities traded and lower rebated prices.

Chapter 4

Asymmetric Information in Markets for Pharmaceuticals : International Price Referencing and Secret Rebates

4.1 Introduction

In this chapter, we rationalize the use of international price referencing and secret rebates in the negotiations on drug prices between public payers (countries) and a monopolist pharmaceutical firm. We put forward the hypothesis that the firm's propensity to accept offers from countries depends on its private information about the launch date of a superior substitute. To understand better why countries may gain from negotiating a secret rebate, we analyze a transparency regime where the details of the deals are made public. We show that countries base their offers on the transaction prices they observe in other countries. We find that the countries which negotiate first are always strictly better-off when they can avoid this information leakage. Finally, we argue that a natural arrangement between the firm and these countries is to use a secret rebate that conceals the real price they pay.

The pharmaceutical industry distinguishes itself from other industries on many grounds. When a medicine is listed for reimbursement, health authorities negotiate a price with the manufacturer, and the price at which the drug is listed for reimbursement, called the *list price*, is made public at the time of the market release. An institutional payer uses *international price referencing* (IPR) when it bases its price offer to the firm on the list price of similar medicines in other countries. According to Vogler, Paris and Panteli (2018, [69]), the vast majority of European countries uses IPR for at least some drugs, as well as China, Japan, Canada and Brazil among others. IPR creates price interdependencies across countries, which makes the order of launches a strategic variable for the manufacturer (Vogler and al. 2019, [71], Grepperud and Pedersen 2020, [41], Marinoso and Olivella 2005, [34]).

A particularity of the pricing of medicines is its lack of transparency. The (official) list price at which a drug is available in a country may give little to no information about the real price paid to the manufacturer, because of the use of secret rebates. Generally, a *rebate* is a subsidy offered to the buyer, which can be seen as money that the manufacturer pays back to the retailer under certain conditions like sales volume. A rebate is *secret* when it is only known to the parties involved in its negotiation. Rebates are used in other industries than the pharmaceutical one, notably in the automobile and electronics industries. Yet, rebates on pharmaceuticals and rebates on other types of goods do not seem to fill the same functions. In other industries, manufacturers offer rebates to retailers primarily for avoiding inventory hoardings.[1] In the pharmaceutical industry, it is argued that rebates are offered by manufacturers in reaction to IPR, as a way to conceal the real price a country pays to those which will negotiate later on.[2]

Secret rebates create informational asymmetries between payers about the price each of them pays for a same product. Informational asymmetries between a drug manufacturer and a payer seem to exist as well. In the theoretical literature, Marinoso and Olivella (2005, [34]) investigate the effect of an asymmetry of information about the production cost of a drug on official prices and the timing of launches in different countries. We take on a different approach, for the reason that it is now common knowledge that the production cost of most drugs is close to null (see Hill, Barber and Gotham 2018, [44]). A potential source of informational asymmetry, that we think relevant for the case of innovative drugs, involves the duration of the manufacturer's monopoly. A manufacturer may know whether its product can be easily improved upon, thus how long it may take before the launching of a superior therapeutic substitute (often referred to as *me-too* or *follow-on* drugs).[3] In many cases, me-too drugs are the result of parallel

[1] See Goodman and Moody (1970, [38]), Chevalier and Curhan (1976, [17]), Blattberg and Levin (1987, [8]), Gerstner and Hess (1991 [35], 1991b [37], 1995 [36]) and Ault et al. (2000, [2]) for more details about the reasons for and use of rebates in other industries.

[2] This argument can be found notably in Vogler, Paris and Panteli (2018, [69]).

[3] What has been studied is the effect of me-too drugs on price competition. See DiMasi and Paquette (2004, [25]), Lu and Comanor (1998, [57]), Ekelund and Persson (2003, [29]), Lichtenberg and Philipson (2002, [56]), DiMasi, Hansen, and Grabowski, (2003, [24]) and Régnier (2013, [61]) among others.

development, and have more or less identical clinical outcomes to pioneer drugs.[4] Despite their lack of therapeutic added value, me-too drugs are granted market authorization for the sake of broadening the range of alternatives to patients and stimulating price competition. Lichtenberg and Philipson (2002, [56]) find that competition by me-too products decrease incumbents' profits as much as competition from generics after patent expiry; however, the authors attribute the loss in innovators' profits to a reduction in their market share, and not to price competition. Lu and Commanor (1998, [57]) corroborate this finding, and show for the US that the average effect of adding an extra competitor is a price reduction on the order of 2%.[5]

We believe that the knowledge of the manufacturer of a pioneer drug about the duration of its monopoly influences its willingness to accept lower offers against earlier deals during the negotiations with countries. Assuming that our premise is true, we have three research questions. First, we want to understand how transparency on the prices that countries pay for a same product would affect the outcomes of the negotiations. Second, we intend to rationalize the use of IPR by countries. Third, we wish to rationalize the use of secret rebates, particularly from a country's perspective.

We present a theoretical model where two countries negotiate sequentially with a monopolist pharmaceutical firm. A negotiation unfolds with first the country making an offer, then the firm. A deal is sealed as soon

[4]Me-too drugs do not infringe on the patent of pioneer drugs. According to Régnier (2013, [61]), me-too drugs need only be marginally differentiated from a pioneer drug to be granted a market authorization.

[5]The average number of substitutes in their data is around 3 to 4. This suggests that the transition from monopoly to four to five firms with products that are not too differentiated triggers a price reduction of the order of 6%.

as one of the parties accepts the offer from the other one. We suppose that the firm holds private information about the launch date of a superior substitute, which affects its willingness to accept a lower price in exchange for an earlier deal. We first examine the case of price transparency, i.e. the country which negotiates last knows, before starting its own negotiation with the firm, the transaction price in the other country. In this case, we find that IPR is produced endogenously in an equilibrium of our model; in particular, we show that the optimal offer made by the last country depends positively on the price paid by the first one. As a consequence, if it is farsighted, the firm gets an additional incentive to reject the offer made by the country which negotiates first. Next, we investigate the outcome of the negotiations when the first country can offer a secret rebate. We find that the latter is strictly better-off in this instance: the first country to negotiate uses a secret rebate that prevents any informational spillover about the firm's type, which cancels the firm's incentive to reject on the ground that the second country offers a higher offer the larger the transaction price with the first country. The second country is strictly worse-off, as the price in the first country is uninformative about the firm's type. We succeed in highlighting a clear series of implications that rationalizes both the use of IPR and secret rebates: informative prices lead the last country to base its offer to the firm on the price it observes in the reference country. By anticipation, the firm is tougher in its first negotiation. For the first country that negotiates, secret prices are a means to cancel out the negative effect caused by the combination of IPR and the firm's farsightedness, and is therefore strictly better-off if it can use a secret rebate.

Our result about the mitigated social effects of price transparency contributes to the related literature. Danzon and Towse (2003, [22]) argue that transparency and differential pricing cannot be compatible, and may be detrimental to poor countries. The authors argue that, as countries use IPR and may engage in parallel trade, manufacturers would have the incentive to set a single price between the rebated prices that the countries would have gotten, had secret rebates been allowed. This would ultimately benefit high-income countries and hurt low-income ones. We find that transparency is detrimental to the countries which negotiate first and beneficial to those which negotiate last.

The work presented in this chapter complements that in Chapter 3. There, we take as given that the country which negotiates in second position uses IPR. Here, we offer to rationalize this behavior. In the set-up we propose, the firm holds information that is relevant for a country to know when formulating its offer (which is the firm's type). As this information is not directly observable, the second country uses the transaction price in the first country as an indicator of some partial information about the firm's type. We show that the optimal offer of the second country depends positively on the price that the first country pays: IPR is produced endogenously in an equilibrium of our model.

The rest of the paper is organized as follows. Section 4.2 features the model for just one country and a monopolist pharmaceutical firm. Section 4.3 extends the model to the case of two countries and a myopic firm. In Section 4.4, we solve the game for two countries and a farsighted firm. Section 4.5 studies a version of the game where secret rebates are allowed. Section 4.6 concludes.

4.2 Negotiation between a country and a firm under asymmetric information

In this section, we model the negotiation between just one country and an incumbent pharmaceutical firm. We consider for now that the negotiation is only about the list price, i.e. there is no rebate on the table, as there is just one country therefore nobody to hide the details of the deal from.

The negotiation between the firm and the country lasts for up to two periods, that we shall denote by τ^1 and τ^2. The market for the drug sold by the firm ends at some date t posterior to the end of the negotiation, and we define t as the launch date of a superior therapeutic substitute, also called me-too or follow-on drug. This date arrives before the expiry date L of the incumbent's patent. Generally, a me-too drug may not infringe on the incumbent's patent, and might be granted a market authorization before the patent on the pioneer medicine expires, which is what we assume here.[6] We suppose that t is privately known by the incumbent. Throughout, we refer to t as the firm's type, and t is an element of the set $\mathcal{T} = [0, 1]$ of all possible types. The country has a prior belief that the firm's type follows the cumulative distribution G on the support $\mathcal{T}$. We restrict attention to cumulative distributions that are concave.

Assumption 1

The country's prior belief about the distribution of the firm's type, $G(t)$, is

[6] The therapeutic substitute we are referring to cannot be a generic, as it is common knowledge that generics enter once the incumbent's patent on the pioneer drug expires.

continuous, twice-differentiable and $G'' < 0$ *everywhere on* $[0, 1]$.

The timeline is:

$$\tau^1 < \tau^2 < 0 \leq t \leq 1 < L.$$

We suppose that the country's willingness to pay for a fixed exogenous quantity of the drug for one period is commonly known to equal 1. The quantity demanded is exogenous and set to 1. For the sake of simplicity, we assume that neither the firm nor the country discount time.

The negotiation between the incumbent and the country occurs as follows. The country moves first by offering a price p_C in period τ^1. Then, the firm accepts or rejects this offer. If accepted, the deal is sealed and the negotiation ends. If the firm rejects, the negotiation continues in period τ^2. This time, the firm makes a counter-offer p_F to the country. The country either accepts or rejects. If the firm and the country found common ground on a price $p \in \{p_C, p_F\}$ at date $\tau \in \{\tau^1, \tau^2\}$, then the former receives p and the latter receives $1 - p$ at each period between τ and t. Otherwise, both parties get a zero payoff until t.[7]

At date t, a competitor is ready to launch a me-too drug. We do not model explicitly the negotiation process through which prices are determined past t. Instead, we make the following simplifying assumptions about the continuation payoffs of the incumbent and the country.

[7] In reality, when negotiations fail, the drug may be sold in the country however it is not listed for reimbursement. In most European markets, failure to list a drug for reimbursement dampens the companies' profits (see Marinoso et al. (2011, [33]): "*being excluded from the public funding may be almost as bad as not being authorized to sell the drug at all.*"). For a country, failure to list a drug for reimbursement limits drastically its accessibility.

First, we assume that the competitor overtakes the incumbent. One could think that the me-too drug offers superior therapeutic benefits, thereby making the pioneer drug completely obsolete.[8] The firm's payoff is given by the net present value of its market profits. If the negotiation was successful, the firm receives p at each period between the date at which the agreement was reached, τ, up until t, which gives:

$$\pi_F = p(t - \tau) \tag{19}$$

If no deal was sealed with the country, then the firm's payoff is null.

Second, we suppose that the pricing of the me-too drug is linked to that of the pioneer drug. This assumption of ours is motivated by the empirical findings of Lu and Comanor (1998, [57]), diMasi and Paquette (2004, [25]), Lichtenberg and Philipson (2002, [56]) and Régnier (2013, [61]), who show that the price paid to the manufacturer of a pioneer drug determines the long-run trajectory of prices of comparable medicines. In particular, Lu and Comanor (1998, [57]) find that the market release of a me-too drug, over the period where the pioneer drug is still patented, leads to a price reduction on the order of 2%.[9] For simplicity, we consider that the country buys the me-too drug at the same price as the pioneer drug. If the country did not trade with the incumbent firm prior to t, then we suppose that the competitor makes a take-it-or-leave-it offer which results in the price of 1.

[8] In reality, the profit made by the incumbent after the launch of a me-too drug by a competitor is not necessarily null, however it decreases significantly. According to Lichtenberg and Philipson (2002, [56]) competition between therapeutic substitutes reduces the incumbent's profit as least as much as competition from generics. Also, additional costs must be spent in marketing and promotion, which further reduces the incumbent's profit after the launch of a therapeutic substitute (see Hollis (2004, [45])).

[9] Similar results are found by diMasi and Paquette (2004, [25]), Lichtenberg and Philipson (2002, [56]) among others. Régnier (2013, [61]) shows that me-too drugs may limit the penetration of generics after the pioneer drug is no longer patent protected.

After date L, the country can buy the generic version of the pioneer drug for free. The country's objective is then to maximize its surplus over the period that starts from the date at which an agreement is reached with the incumbent, τ, until the date at which its generic version is accessible, L. The country's payoff is given by the following expression:

$$V_C = (L - \tau)(1 - p), \tag{20}$$

for some agreement on a price $p \in \{p_C, p_F\}$ at date $\tau \in \{\tau^1, \tau^2\}$. If the country does not trade with the incumbent, then its payoff is null.

4.2.1 Strategies and beliefs

This version of the model features a sequential Bayesian game with continuous type t for the firm and both continuous and discrete sets of actions for the two parties. We first make explicit the strategies of the firm and the country, as well as the beliefs of the latter about the former's type. Then, we define what a Perfect Bayesian equilibrium is in our game.

Let $P = [0, 1]$ be the set of all possible values for the price p. A price is an element of P that implies a partition of the country's per period willingness to pay, where p is the firm's per period profit and $1-p$ is the country's per period surplus. Let S_C be the set of all strategies for the country. Formally, S_C is the set of all pairs of functions:

$$s_C = \left(s_C^{\tau^1}, s_C^{\tau^2}\right)_{\forall t \in \mathcal{T}}$$

when for $\tau = \tau^1$, $s_C^\tau : P \to P$ and for $\tau = \tau^2$, s_C^τ is the response of the country to the firm's counter-offer. Without loss of generality, we assume throughout that the country always accepts if it is indifferent between accepting and rejecting. Similarly, $S_F(t)$ is the set of all pairs of functions:

$$s_F(t) = \left(s_F^{\tau^1}(t), s_F^{\tau_2}(t)\right)$$

for the firm of type t, when for $\tau = \tau^1$, $s_F^\tau(t) : P \to \{A, R\}$ is a response to the offer made by the country; and for $\tau = \tau^2$, $s_C^\tau(t)$ is the firm's counter-offer $p_F(t)$. A strategy profile s determines expected payoffs for each player, where the expectation is taken over the set of all possible types for the firm with respect to the country's beliefs.

There are two information sets encountered each time the country moves. At $\tau = \tau^1$, the country's belief that the firm's type is less than or equal to t is its prior belief G. If the country reaches its second information set, the country updates its belief by Bayes' rule.

Definition 4.2.1 In a *weak Perfect Bayesian Equilibrium* (PBE) $s^* = (s_C^*, s_F^*(t))$ of this game, strategies and beliefs must satisfy the two following conditions:

1. *sequential rationality*: s^* specifies the best-response at every information set, and it is optimal in expectation given the country's beliefs;

2. *consistency*: if the second information set is reached, i.e. the firm plays R at $\tau = \tau^1$, the country's belief at $\tau = \tau^2$ about the firm's type is updated using Bayes' rule.

4.2.2 Perfect Bayesian equilibria for the one country case

Consider the second information set where the country chooses to accept or reject the firm's counter-offer. It is immediate that the country always plays A. Given this, the firm's optimal strategy at τ^2 is to counter-offer the price of 1, regardless of its type. Next, we show that the optimal offer of the country at τ^1 is a price p in the interval:

$$P^* = \left(\frac{\tau^2}{\tau^1}, \frac{1-\tau^2}{1-\tau^1}\right]$$

Any price offer from the country that is outside the interval is strictly dominated. Any price offer that is strictly less than the lower bound is always rejected thus it gives the country a null payoff. Such a price offer is strictly dominated by any offer larger than the upper bound of P^* and strictly less than 1: the firm accepts it with probability one and the country's per period surplus is strictly positive. Any price strictly above the upper bound is accepted for sure; therefore such a price is strictly dominated by any price offer which is strictly lesser, yet greater than the upper bound of the interval.

A price offer p in the relevant interval is accepted by the firm of type t at τ^1 if and only if:

$$t \leq \frac{\tau^2 - \tau^1 p}{1-p} = T(p), \tag{21}$$

where $T(p)$ is monotone increasing in p, and it is increasing in the time-lapse between the two rounds of the negotiation, $\tau^2 - \tau^1$. We call $T(p)$ the *threshold type* below which the firm accepts p and above which it rejects it.

An offer p in the appropriate interval is accepted with probability $G(T(p))$, which is increasing in the value of the offer. The country's problem is:

$$\max_{p \in P^*} \mathbb{E}(V_C) = G\left(\frac{\tau^2 - \tau^1 p}{1-p}\right)(L - \tau^1)(1-p) \tag{22}$$

where $\mathbb{E}(V_C)$ denotes the country's expected payoff. The expected payoff is concave in the country's price offer, p, by Assumption 1.[10] Our next assumption guarantees the existence of an interior solution to the country's problem.

Assumption 2

The prior belief G and the date τ^1 satisfy:

$$1 > G'(1)(1 - \tau^1)$$

The country's problem is similar to that of a monopsonist seeking to maximize a payoff of the form $(1-p)S(p)$, facing a supply whose equation is:

$$S(p) = G\left(\frac{\tau^2 - \tau^1 p}{1-p}\right)(L - \tau^1)$$

with elasticity:

$$e_S(p) = \frac{S'(p)}{S(p)}(1-p)$$

($S'(.)$ is the first derivative of the supply function S with respect to p.) The

[10]The country's expected payoff expressed as a function of the type t, i.e. $G(t)\left(\frac{\tau^2\tau^1}{t-\tau^1}\right)$ is single-peaked. Concavity in t would require a stronger assumption than that in Assumption 1: $G(t)G''(t)(t-\tau^1) + G'(t)G(t) - (G'(t))^2(t-\tau^1) < 0$. We do not find necessary to impose this more stringent assumption.

formula of the elasticity can be expressed as a function of the firm's type:

$$e_S(t) = \frac{G'(t)}{G(t)}(t - \tau^1)$$

where the price is parameterized by the firm's type via the relation in (21). The elasticity is decreasing in τ^1, and it is monotone decreasing in t if and only if:

$$G''(t)G(t)[t - \tau^1] - (G'(t))^2[t - \tau^1] + G(t)G'(t) \leq 0 \quad \forall t \in [0, 1]$$

The condition above is more stringent than Assumption 1. Assumption 1 guarantees that the elasticity is decreasing in t for values of t that are not too large. [11] Note that Assumption 2 is equivalent to assuming that $e_S(t = 1) < 1$. We present our results in the proposition below.

Proposition 4.2.1. *The country's problem in* (22) *has a unique interior solution, p^*, which solves:*

$$e_S(p^*) = 1$$

Let t^* be the type of firm which is indifferent between accepting and rejecting p^*. The firm accepts p^* if its type belongs to $[0, t^*]$, and it rejects p^* if its type belongs to $(t^*, 1]$.

See Appendix C.1 for the proof. For this version of our game, there exists a unique PBE. We start with the best-responses at each round of

[11] The elasticity $e_S(p(t))$ is decreasing in t if and only if $G''(t)G(t)[t-\tau^1]-(G'(t))^2[t-\tau^1]+G(t)G'(t) \leq 0$, i.e. the country's payoff as a function of the firm's type t is concave in t.

the negotiation. At τ^1, the country's optimal offer is p^*; the firm's best-response is to accept any offer $p_C \geq \frac{t-\tau^2}{t-\tau^1}$ and to reject any offer $p_C < \frac{t-\tau^2}{t-\tau^1}$. At τ^2, the firm's optimal counter-offer is 1, for all possible histories. The country accepts any counter-offer, for all possible histories.

The belief system is the following. At the first information set, the country's belief is its prior G. If the second information set is reached, the country updates its belief via Bayes' rule, which gives:

$$G_R(t) = \frac{G(t) - G(t^*)}{1 - G(t^*)}$$

If the second information set is not reached, then any belief is consistent.

4.3 Negotiations between two countries and a myopic firm

Assume there are now two countries, C and D. Let us denote by ε the relative market size of country D. In this section, we make the simplifying assumption that the firm is *myopic* in that it does not take into account the future profits made in country D when carrying out the negotiation with country C. In the next section, we relax this assumption. We still suppose that only list prices are negotiated, and not secret rebates. For country D, this is without loss of generality: for the last country negotiating with the firm, secret rebates have no use, as there is nobody to hide the details of the deal from. For country C, it is also without loss of generality, to the extent that the latter achieves the same outcome with or without rebate since the firm is myopic.

Country C first negotiates with the firm, then country D. Each has potentially two periods of negotiation with the firm. Country $i \in \{C, D\}$ negotiates with the firm at dates τ_i^1 and τ_i^2. Both countries have the common prior belief that the firm's type t is distributed according to cdf G on the support $\mathcal{T}$. The time line is now:

$$\tau_C^1 < \tau_C^2 < \tau_D^1 < \tau_D^2 < 0 \leq t \leq 1 < L.$$

The market for the drug sold by the incumbent firm ends at some date t posterior to the end of both negotiations. Each of the two negotiations unfolds in the same way as the negotiation between C and F in the previous section, and payoffs are defined in the analogous way.

There are two information sets where information is useful: I_1 is the first of them, at which country C at date τ_C^1 makes an offer to the firm; and I_2 is analogous to I_1 but for country D. The countries always accept the firm's counter-offer if their own offer has been rejected. Note that at the information set I_1, C's belief is its prior G; while at I_2, D's belief is obtained by updating its prior using the information released by the outcome of the first negotiation. Due to the sequential rationality criterion, in equilibrium country D at date τ_D^1 either observes the price p^* or the price of 1 in country C (see Proposition 4.2.1).

As there are now two countries, we relabel the threshold type function in (21) as $T_C(p)$, whose equation is:

$$T_C(p) = \frac{\tau_C^2 - p\tau_C^1}{1 - p}$$

We start with the case where the firm accepted country C's offer, p^*. Country D updates its belief about the distribution of the firm's type by Bayes' rule, which gives:

$$G_A(t) = \frac{G(t)}{G(t^*)} \tag{23}$$

on the support $[0, t^*]$ for the firm's type t, where t^* is the type of firm indifferent between accepting and rejecting country C's offer p^*. If instead the firm rejected country C's price offer p^*, country D's updated belief is given by:

$$G_R(t) = \frac{G(t) - G(t^*)}{1 - G(t^*)} \tag{24}$$

on the support $(t^*, 1]$.

Consider some offer p extended by country D. This offer is accepted by the firm of type t if and only:

$$t \leq \frac{\tau_D^2 - p\tau_D^1}{1-p} = T_D(p) \tag{25}$$

The function $T_D(p)$ is interpreted in the same way as in the previous section: for a given price offer p, it gives the type which is indifferent between accepting and rejecting p. For the same reason as in the previous section, an optimal offer for country D must lie in the interval:

$$P_A = \left(\frac{\tau_D^2}{\tau_D^1}, \frac{t^* - \tau_D^2}{t^* - \tau_D^1} \right] \quad \text{or} \quad P_R = \left(\frac{t^* - \tau_D^2}{t^* - \tau_D^1}, \frac{1 - \tau_D^2}{1 - \tau_D^1} \right]$$

depending on whether the country believes that the firm's type is above or below t^*. The supply faced by country D is:

$$S_j(p) = G_j(T_D(p))\varepsilon(L - \tau_D^1)$$

with $j \in \{A, R\}$. The elasticity of these supplies are:

$$e_{S_A}(t) = \frac{G'(t)}{G(t)}[t - \tau_D^1] \quad \text{and} \quad e_{S_R}(t) = \frac{G'(t)}{G(t) - G(t^*)}[t - \tau_D^1],$$

where the price p is parameterized by t via the relation in (25). Note that the supply faced by country D following a rejection of C's offer, S_R, is relatively more elastic than S_A, the supply country D faces when the firm accepted country C's offer. Note as well that the supply country C faces is more elastic than S_A, as $-\tau_C^1 > -\tau_D^1$.

If the price of the drug in country C is p^*, then country D's problem is:

$$\max_{p \in P_A} \mathbb{E}(V_D) = \left(\frac{G\left(\frac{\tau_D^2 - p\tau_D^1}{1-p}\right)}{G(t^*)} \right) \varepsilon(L - \tau_D^1)(1-p)$$

If instead the price of the drug in country C is 1, country D's problem is written as:

$$\max_{p \in P_R} \mathbb{E}(V_D) = \left(\frac{G\left(\frac{\tau_D^2 - p\tau_D^1}{1-p}\right) - G(t^*)}{1 - G(t^*)} \right) \varepsilon(L - \tau_D^1)(1-p)$$

In both cases, D's expected payoff is concave in its offer, due to Assumption 1. The results about D's optimal offer in each case are presented in the proposition below.

Proposition 4.3.1. *Following an acceptance of country C's offer, country D's problem has an interior solution, p_a, that solves:*

$$e_{S_A}(p_a) = 1$$

The type of firm which is indifferent between accepting and rejecting p_a is

t_a. *Any type* $t \in [0, t_a]$ *accepts* p_a*, and any type* $t \in (t_a, t^*]$ *rejects it. Following a rejection of country C's offer, country D's optimal offer* p_r *is interior to* P_R *if and only if:*

$$e_{S_R}(p_r) = 1 \quad \textit{and} \quad p_r < \frac{1 - \tau_D^2}{1 - \tau_D^1}$$

Otherwise, $p_r = \frac{1-\tau_D^2}{1-\tau_D^1}$. *The type of firm which is indifferent between accepting and rejecting* p_r *is* t_r. *Both* p_r *and* t_r *are increasing in the value of* t^*. *Any type* $t \in [t^*, t_r]$ *accepts* p_r*, and any type* $t \in (t_r, 1]$ *rejects it.*

See Appendix C.2 for the proof. When the firm accepts country C's offer, the second country does not base its offer on the information released by the first negotiation. Only when country C got its offer rejected does country D take the information about the firm's type into account. Should country D not observe the outcome of the first negotiation, the offer it would have formulated (which is equal to p_a) would have been rejected with probability one. The outcome of the first negotiation signals country D that it must adjust its offer upwards if it wants to avoid rejection. Country D is always strictly better-off when it can observe the outcome of the first negotiation, and country C is neither positively nor negatively impacted by the second negotiation.

This version of our model where the firm is myopic can be thought of as a game with two pharmaceutical companies whose types are perfectly correlated, and one operates in country C and the other in D. Such a game has a unique PBE, that we characterize below. We begin with each party's best-response at every date. At τ_C^1, country C offers p^* (Proposition 4.2.1). The firm accepts any price $p_C \geq \frac{t-\tau_C^2}{t-\tau_C^1}$, and rejects any price $p_C < \frac{t-\tau_C^2}{t-\tau_C^1}$.

At τ_C^2, the firm counter-offers $p_F = 1$, for all possible histories. Country C always accepts $p_F = 1$, for all possible histories. At τ_D^1, country D offers p_a if the history of the game is $h_1 = (p^*, A, 1, A)$, and p_r if the history of the game is $h_2 = (p^*, R, 1, A)$. And for any history different from h_1 and h_2, country D offers $p_D = \frac{\tau_D^2}{\tau_D^1}$. The firm accepts any price $p_D \geq \frac{t-\tau_D^2}{t-\tau_D^1}$, and rejects any price $p_D < \frac{t-\tau_C^2}{t-\tau_C^1}$. At τ_D^2, the firm counter-offers $p_F = 1$, for all possible histories. Country D accepts $p_F = 1$, for all possible histories.

The belief system is the following. At I_1, country C's belief is its prior belief G. If the second information set of country C is reached, i.e. the history of the game at τ_C^2 is (p^*, R), the country's belief is G_R in (24). If the second information set is not reached, then any belief of the country is consistent. At I_2, country D's belief is G_A in (23) if the history of the game at τ_D^1 is $h_1 = (p^*, A, 1, A)$; G_R in (24) if the history of the game at τ_D^1 is $h_2 = (p^*, R, 1, A)$; and the country believes that the firm is of type 0 with probability one for any other history. If the second information set of country D is reached, i.e. the firm has rejected both countries' offers, its belief is:

$$G_{R,R}(t) = \frac{G(t) - G(t_r)}{1 - G(t_r)},$$

If the second information set of country D is not reached, then any belief is consistent.

In conclusion, country D applies a form of IPR in equilibrium. The latter bases its offer on the transaction price it observes in country C when C's offer was rejected. Therefore, country D uses IPR for increasing the probability of acceptance of its offer to the firm, which enables it to pay a lower price on expectation.

4.4 Two countries and a farsighted firm

Let us now assume that the firm is farsighted: it takes into account that the outcome of its negotiation with the first country influences the offer made by the second one. While negotiating with country C, the firm now seeks to maximize its total payoff:

$$\Pi_F = p(C)[t - \tau(C)] + \varepsilon p(D)[t - \tau(D)],$$

where $p(C)$ is the price paid by the first country and $\tau(C)$ the date at which they reach an agreement if they indeed do; $p(D)$ and $\tau(D)$ are similarly defined for the second country.

The subgame that folllows country C's offer is a signaling game played by the firm and country D. Through its decision to accept or reject the first country 's offer, the firm releases partial information about its type, and the second country's offer must be optimal given this signal in an equilibrium. We refer to the subgames that start at the firm's first decision node as "subgames p". In the next section, we provide sufficient conditions for the existence of equilibria in these subgames.

4.4.1 Sufficient conditions for existence

Throughout, we shall refer to p as country C's offer, and to t_f as the type of firm which is indifferent between accepting and rejecting p. For the case where $t_f = 0$, we call $\underline{p}_D$ country D's optimal offer given the signal $t_f = 0$, and we denote by $\underline{t}_D$ the threshold type associated with this offer. Abusing language, we may refer to $\underline{t}_D$ as D's best response to $t_f = 0$. Let $\bar{t}_f$ the lowest value of t_f to which country D's best response is $t_D = 1$.[12] Then country D's best response, in terms of threshold type, maps $t_f \in \left[0, \bar{t}_f\right]$ to $[\underline{t}_D, 1]$. Recall that D's best response following a rejection of p is strictly larger than t_f, and that for $t_f = 1$, we have $t_D = 1$.

In what follows, we look for conditions on the parameters of our model such that for all offers p from country C, there exists $t_D \in [0, 1]$ and a strategy for the firm in the first negotiation such that the firm and country D best respond to each other.

Sufficient conditions for pooling equilibria in subgames

In a subgame that follows a price offer p from country C, there are two possible pooling equilibria: all types accept or all types reject. We provide sufficient conditions for the existence of each of these two types of pooling equilibria in subgames.

First, note that country D's belief in a PBE equals its prior belief G in the unique subgame p (either accept or reject the offer). In a pooling equilibrium, the outcome of the first negotiation is uninformative to country D; hence, its best response, in terms of threshold type, is $\underline{t}_D$. In a PBE, country D's belief is not restricted in an off the equilibrium path subgame. From the firm's perspective, the worst possible case is that country D believes with probability 1 its type is 0, and thus offers the price:

$$p_D = \frac{\tau_D^2}{\tau_D^1}$$

[12] The type $\underline{t}_D$ solves: $G'(\underline{t}_D)(\underline{t}_D - \tau_D^1) - G(\underline{t}_D) = 0$, and $\underline{t}_D$ is equal to t_a in Proposition 4.3.1. Recall that $t_a < 1$. And $\bar{t}_f$ solves: $G'(1)(1 - \tau_D^1) - 1 = G(\bar{t}_f)$.

which the firm is (at least weakly) better-off rejecting, regardless of its type.

Pooling equilibrium where all types reject C's offer

Following the rejection of some price $p < 1$, country D's best response is $\underline{t}_D$. We now elucidate the conditions under which the firm is always better-off rejecting country C's offer, regardless of its type.

Let $\Delta(t)$ be the payoff gain from rejecting country C's offer p; this is the firm's payoff when it rejects p minus its payoff when it accepts p. By our assumption on D's belief off the equilibrium path, the latter extends the offer $\frac{\tau_D^2}{\tau_D^1}$ if it observes that the firm accepted p, and extends the offer $\underline{p}_D$ otherwise. We get:

$$\begin{aligned}\Delta(t) &= t - \tau_C^2 + \varepsilon(t - \tau_D^1)\underline{p}_D - p(t - \tau_C^1) - \varepsilon(t - \tau_D^2) \\ &= t\left(1 - p - \varepsilon(1 - \underline{p}_D)\right) - \left(\tau_C^2 - p\tau_C^1\right) + \varepsilon\left(\tau_D^2 - \underline{p}_D\tau_D^1\right)\end{aligned}$$

for any type $t \leq \underline{t}_D$, and:

$$\begin{aligned}\Delta(t) &= (t - \tau_C^2) - p(t - \tau_C^1) \\ &= (1 - p)(t - T_C(p))\end{aligned}$$

for any $t > \underline{t}_D$. The payoff gain from rejecting, Δ, is unambiguously increasing everywhere on $[\underline{t}_D, 1]$. The function Δ is monotonic in t on $[0, \underline{t}_D]$, and whether it is increasing or decreasing depends on the sign of:

$$1 - p - \varepsilon(1 - \underline{p}_D)$$

We need make sure that no type prefers to accept p. If the payoff gain function from rejecting, Δ, is monotone increasing in t, then the condition is the most stringent on the lowest type, $t = 0$. This type prefers to reject C's offer if and only if:

$$p \leq \frac{\tau_C^2}{\tau_C^1} + \varepsilon \frac{\underline{t}_D(\tau_D^2 - \tau_D^1)}{-\tau_C^1(\underline{t}_D - \tau_D^1)}$$

Suppose now that the payoff gain function from rejecting, Δ, is decreasing in the firm's type on the interval $[0, \underline{t}_D]$. Then the type which gains the most from accepting C's offer is type $\underline{t}_D$, and it prefers to reject if and only if:

$$p \leq \frac{\underline{t}_D - \tau_C^2}{\underline{t}_D - \tau_C^1}$$

We gather our results in the proposition below.

Proposition 4.4.1. *A pooling equilibrium in the subgame following offer p by country C where the firm always rejects regardless of its type exists if and only if either:*

$$p \leq \min \left\{ \frac{\tau_C^2}{\tau_C^1} + \varepsilon \frac{\underline{t}_D(\tau_D^2 - \tau_D^1)}{-\tau_C^1(\underline{t}_D - \tau_D^1)} \ , \ 1 - \varepsilon \frac{(\tau_D^2 - \tau_D^1)}{\underline{t}_D - \tau_D^1} \right\}$$

i.e., the payoff gain from rejecting, Δ, is monotone increasing in the firm's type, and all types prefer to reject p; or:

$$p \in \left[1 - \varepsilon \frac{(\tau_D^2 - \tau_D^1)}{\underline{t}_D - \tau_D^1} \ , \ \frac{\underline{t}_D - \tau_C^2}{\underline{t}_D - \tau_C^1} \right]$$

i.e., the payoff gain from rejecting, Δ, is single-peaked in the firm's type and achieves a minimum at $t = \underline{t}_D$, and all types prefer to reject p.

Note that the larger country D's offer $\underline{p}_D$, the larger the range of prices

p that allow the existence of a pooling equilibrium where all types reject.

Pooling equilibrium where all types accept C's offer

Following the acceptance of some price p, country D' best-response, in terms of threshold type, is $\underline{t}_D$. We investigate the conditions under which always accepting p is optimal for the firm, regardless of its type.

Consider the firm's payoff gain from rejecting C's offer p. Recall that country D's offer off the equilibrium path is $\frac{\tau_D^2}{\tau_D^1}$. The payoff gain from rejecting p must be negative, for all types. For any $t \leq \underline{t}_D$, the payoff gain from rejecting is:

$$\begin{aligned}
\Delta(t) &= (t-\tau_C^2)+\varepsilon(t-\tau_D^2)-p(t-\tau_C^1)-\varepsilon\underline{p}_D(t-\tau_D^1) \\
&= t\Big(1-p+\varepsilon(1-\underline{p}_D)\Big)-(\tau_C^2-p\tau_C^1)-\varepsilon(\tau_D^2-\underline{p}_D\tau_D^1)
\end{aligned}$$

For any $t > \underline{t}_D$, this is:

$$\begin{aligned}
\Delta(t) &= (t-\tau_C^2)-p(t-\tau_C^1) \\
&= (1-p)\left(t-T_C(p)\right)
\end{aligned}$$

Note that the function Δ is monotone and strictly increasing in the firm's type. Therefore, the type that has the greatest incentive to reject C's offer is type 1. And:

$$\Delta(1) \leq 0 \quad \Leftrightarrow \quad p \geq \frac{1-\tau_C^2}{1-\tau_C^1}.$$

We gather the results in the proposition below.

Proposition 4.4.2. *A pooling equilibrium where all types accept p exists* *if:*

$$p \geq \frac{1-\tau_C^2}{1-\tau_C^1}.$$

Our choice of beliefs for country D off the equilibrium path may be disputable in the context of an all-accept pooling equilibrium. We try to remedy to this by considering the following beliefs: following a rejection of p, country D believes that the firm is of type 1. This off the equilibrium path belief leads to the smallest set of prices p for which an all-accept pooling equilibrium exists. In such a profile, the net gain from rejecting p equals, for any $t \leq \underline{t}_D$:

$$\begin{aligned}\Delta(t) &= (t-\tau_C^2) + \varepsilon(t-\tau_D^1)\left(\frac{1-\tau_D^2}{1-\tau_D^1}\right) - p(t-\tau_C^1) - \varepsilon \underline{p}_D(t-\tau_D^1) \\ &= t\left(1-p+\varepsilon\left(\frac{1-\tau_D^2}{1-\tau_D^1}-\underline{p}_D\right)\right) - (\tau_C^2 - p\tau_C^1) - \varepsilon\tau_D^1\left(\frac{1-\tau_D^2}{1-\tau_D^1}-\underline{p}_D\right)\end{aligned}$$

which is increasing in t. For types larger than $\underline{t}_D$, the payoff gain function from rejecting is:

$$\begin{aligned}\Delta(t) &= (t-\tau_C^2) + \varepsilon(t-\tau_D^1)\left(\frac{1-\tau_D^2}{1-\tau_D^1}\right) - p(t-\tau_C^1) - \varepsilon(t-\tau_D^2) \\ &= t\left(1-p-\varepsilon\frac{(\tau_D^2-\tau_D^1)}{1-\tau_D^1}\right) - (\tau_C^2 - p\tau_C^1) + \varepsilon\frac{(\tau_D^2-\tau_D^1)}{1-\tau_D^1}\end{aligned}$$

If the above function is increasing in t, then the type which has the greatest incentive to reject C's offer is type 1. If instead the function is decreasing in t, then type $\underline{t}_D$ is the type that has the greatest incentive to reject; and this type prefers to accept if and only if:

$$p \geq \frac{\underline{t}_D - \tau_C^2}{\underline{t}_D - \tau_C^1} + \varepsilon\frac{(\tau_D^2-\tau_D^1)(1-\underline{t}_D)}{(1-\tau_D^1)(\underline{t}_D-\tau_C^1)}$$

We state our result below.

Proposition 4.4.3. *A pooling equilibrium where all types accept p, and country D believes that $t = 1$ with probability 1 following a rejection of p, exists if and only if either:*

$$p \in \left[\frac{1-\tau_C^2}{1-\tau_C^1} \ , \ 1 - \varepsilon \frac{(\tau_D^2 - \tau_D^1)}{1-\tau_D^1} \right]$$

i.e., the payoff gain from rejecting, Δ, is increasing monotone in the firm's type, and all types prefer to accept p; or:

$$p \geq \max \left\{ 1 - \varepsilon \frac{(\tau_D^2 - \tau_D^1)}{1-\tau_D^1} \ , \ \frac{\underline{t}_D - \tau_C^2}{\underline{t}_D - \tau_C^1} + \varepsilon \frac{(\tau_D^2 - \tau_D^1)(1-\underline{t}_D)}{(1-\tau_D^1)(\underline{t}_D - \tau_C^1)} \right\}$$

i.e., the payoff gain from rejecting, Δ, is single-peaked in the firm's type and achieves a maximum at $\underline{t}_D$, and all types prefer to accept p.

In Appendix C.3, we show that there exists an all-accept pooling equilibrium that satisfies the intuitive criterion.

A characterization of interior equilibria in subgames p

We first clarify our definition of an interior equilibrium.

Definition 4.4.1 An equilibrium in subgame p is *interior* if the set of types that accept p and the set of types that reject p are both non-empty.[13]

Let p_A and p_R be the prices offered by country D, upon observing that offer p has been respectively accepted or rejected. Let t_A and t_R be the

[13]This definition carries a slight abuse of terminology, since while non-empty, either set may be of measure zero.

threshold types that are indifferent between accepting p_A and p_R, respectively. In any equilibrium of a subgame p, we may, without loss of generality, restrict attention to prices p_A and p_R in the interval:

$$\left[\frac{\tau_D^2}{\tau_D^1}, \frac{1-\tau_D^2}{1-\tau_D^1}\right]$$

Recall that any offer from D that is strictly less than the lower bound is strictly dominated by any offer larger than the upper bound, yet strictly less than one; and any offer strictly larger than the upper bound is dominated by any lesser offer that is yet greater than the upper bound.

Below, we provide a series of lemma that enables us to assess sufficient conditions for the existence of interior equilibria in sugbgames.

Lemma 4.4.1. *In any interior equilibrium of a subgame* p*, it must be that:*

$$\Delta(t_A) \leq 0 \leq \Delta(t_R)$$

i.e, type t_A *accepts* p *and type* t_R *rejects* p*.*

Proof. The proof is by contradiction. If $\Delta(t_A) > 0$, then a firm of type t_A rejects C's offer p. By the continuity of Δ, so did the types in a neighohorhood of t_A. Therefore, one of the two following deviations is profitable to country D. If p_A is accepted with some positive probability, then a slighlty lower offer p'_A is accepted with the exact same probability as p_A. Thence D's payoff is strictly larger, which makes p'_A a strictly profitable deviation. If p_A is accepted with probability zero, then any price offer:

$$p'_A \in \left(\frac{\tau_D^2}{\tau_D^1}, 1\right)$$

is accepted with some strictly positive probability, and therefore constitutes a strictly profitable deviation. Thus, it must be that $\Delta(t_A) \leq 0$. A similar argument holds for proving that $\Delta(t_R) \geq 0$. $\square$

Lemma 4.4.2. *In any interior equilibrium of a subgame p, the inequality $t_A < t_R$ holds.*

Proof. We first prove that $t_A \leq t_R$. Suppose by contradiction that $t_R < t_A$. The equation of the payoff gain from rejecting, Δ, for types in the interval $[t_R, t_A]$ is then:

$$\Delta(t) = (1 - p + \varepsilon(1 - p_A))\, t - (\tau_C^2 - p\tau_C^1) - \varepsilon(\tau_D^2 - p_A\tau_D^1)$$

Since $p_A \leq \frac{1-\tau_D^2}{1-\tau_D^1} < 1$, Δ is strictly increasing in t on $[t_R, t_A]$. In an interior equilibrium, $\Delta(t_R) \geq 0$ and $\Delta(t_A) \leq 0$ must hold by our previous lemma. Yet $t_R < t_A$; a contradiction. Next, suppose by contradiction that $t_A = t_R$, so that $p_A = p_R$. Since $\Delta(t_A) \leq 0$ and $\Delta(t_R) \geq 0$ must hold in an equilibrium, it must be that $t_A = t_R = T_C(p)$. Therefore country C's offer is the same as when the firm is myopic. In particular, $p < 1$, so that:

$$\Delta(t) = (1 - p)\, t - \left(\tau_C^2 - p\tau_C^1\right)$$

is strictly increasing in t, for any $t \in [0, 1]$; and it is negative for any $t < t_A$ and strictly positive for any $t > t_A$. Hence the set of types that accept includes $[0, t_A)$ and is included in $[0, t_A]$; also, the set of types that reject includes $(t_R, 1]$ and is included in $[t_R, 1]$. This in turn implies that $p_A < p_R$, a contradiction. $\square$

The next result follows immediately from Lemmas 4.4.1 and 4.4.2.

Lemma 4.4.3. *In any interior equilibrium of a subgame p, the inequality $t_R \geq T_C(p)$ holds.*

Proof. Note that $\max\{t_A, t_R\} = t_R$, and $\Delta(t_R) \geq 0$ if and only if $t_R \geq T_C(p)$. $\square$

We can establish the following result.

Lemma 4.4.4. *If condition A:*

$$\varepsilon \leq \frac{\tau_C^2 - \tau_C^1}{\tau_D^2 - \tau_D^1}$$

and condition B:

$$\underline{t}_D > \frac{\tau_D^1(\tau_C^2 - \tau_C^1) - \varepsilon\tau_C^1(\tau_D^2 - \tau_D^1)}{\tau_C^2 - \tau_C^1 - \varepsilon(\tau_D^2 - \tau_D^1)},$$

hold, then the payoff gain from rejecting, Δ, is monotone and strictly increasing in the firm's type, i.e. the following relation holds:

$$t_R > t_D^{\circ}(p) = \tau_D^1 + \varepsilon\frac{(\tau_D^2 - \tau_D^1)}{1-p}$$

Meaning, no subgame p has an interior equilibrium where the firm plays a mixed strategy at its first decision node, where it decides whether to accept or reject p.

Proof. In an interior equilibrium, we know from Lemma 4.4.2 that the payoff gain from rejecting, Δ, is of equation:

$$\Delta(t) = \begin{cases} t\,(1-p+\varepsilon(p_R-p_A)) - (\tau_C^2 - p\tau_C^1) - \varepsilon\tau_D^1(p_R-p_A) & \text{if } t \leq t_A \\ t\,(1-p-\varepsilon(1-p_R)) - (\tau_C^2 - p\tau_C^1) + \varepsilon(\tau_D^2 - p_R\tau_D^1) & \text{if } t \in [t_A, t_R] \\ t(1-p) - (\tau_C^2 - p\tau_C^1) & \text{if } t \geq t_R \end{cases}$$

Still by Lemma 4.4.2, Δ is strictly increasing in t for any $t \leq t_A$, and it is unambiguously strictly increasing in t for any $t \geq t_R$. Hence, Δ is strictly monotone only if Δ is strictly increasing in t over the interval $[t_A, t_R]$. Note that by Lemmas 4.4.1 and 4.4.2, Δ cannot be strictly decreasing in t over the aforementioned interval. Hence it must be that $1-p-\varepsilon(1-p_R) \geq 0$. We now show that when the two conditions stated in the lemma hold, then Δ cannot be a constant of t, for any $t \in [t_A, t_R]$. Suppose by contradiction that it is. Then:

$$\begin{aligned} \Delta(t) &= -(\tau_C^2 - p\tau_C^1) + \varepsilon(\tau_D^2 - p_R\tau_D^1) \\ &= -(\tau_C^2 - p\tau_C^1) + \tau_D^1(1-p) + \varepsilon(\tau_D^2 - \tau_D^1) - \tau_D^1\,(1-p-\varepsilon(1-p_R)) \\ &= -(\tau_C^2 - p\tau_C^1) + \tau_D^1(1-p) + \varepsilon(\tau_D^2 - \tau_D^1) \\ &= (1-p)\left(-T_C(p) + \tau_D^1 + \varepsilon\frac{(\tau_D^2 - \tau_D^1)}{1-p}\right) \end{aligned}$$

for any $t \in [t_A, t_R]$. Furthermore, it must be that $\Delta(t) = 0$ for all $t \in [t_A, t_R]$ according to Lemma 4.4.1, which is equivalent to:

$$p = \frac{\tau_D^1 - \tau_C^2 + \varepsilon(\tau_D^2 - \tau_D^1)}{\tau_D^1 - \tau_C^1}$$

and $p \in [0,1]$ by our first condition on the parameter value of ε. By Lemma

4.4.3, it must be that $t_R = T_C(p)$. By transitivity, it follows that:

$$t_R = \tau_D^1 + \varepsilon \frac{(\tau_D^2 - \tau_D^1)}{1-p}$$

Since we are in an interior equilibrium, note that $t_R \geq \underline{t}_D$ must hold. Otherwise, country D would not play a best-response following a rejection of p. Replacing p by its value when $1 - p - \varepsilon(1 - p_R) = 0$ in the expression of t_R above, we get the following alternate expression of t_R:

$$t_R = \frac{\tau_D^1(\tau_C^2 - \tau_C^1) - \varepsilon\tau_C^1(\tau_D^2 - \tau_D^1)}{\tau_C^2 - \tau_C^1 - \varepsilon(\tau_D^2 - \tau_D^1)} < \underline{t}_D$$

where the inequality holds by condition B. A contradiction to the fact that $t_R \geq \underline{t}_D$. □

In Appendix C.4, we show that condition B is necessary for the existence of PBE in pure strategies in some subgames p. So far, our results imply that when conditions A and B in Lemma 4.4.4 hold, the payoff gain from rejecting is increasing in the firm's type: meaning, an offer from country C is accepted by low types and rejected by higher types. In particular, in an interior equilibrium of a subgame p, it must be that the firm of type 1 prefers strictly to reject C's offer (otherwise, all types would prefer to accept p). We obtain the following result.

Lemma 4.4.5. *If*

$$p > \frac{1 - \tau_C^2}{1 - \tau_C^1} \equiv \bar{p} \tag{26}$$

then no interior equilibrium exists in subgame p.

In any interior equilibrium of subgame p, there exists a type $t_f \in [0,1]$ such that all types in $[0, t_f)$ accept and all types in $(t_f, 1]$ reject. Since:

$$\Delta(0) = -\left(\tau_C^2 - p\tau_C^1\right) - \varepsilon\tau_D^1 \left(p_R - p_A\right),$$

a necessary condition for which some types accept is that $\Delta(0) \leq 0$, i.e.

$$p \geq \frac{\tau_C^2}{\tau_C^1} + \varepsilon\frac{\tau_D^1}{\tau_C^1}\left(p_R - p_A\right)$$

When $p \leq \bar{p}$, $\Delta(0) \leq 0$ and $t_R \geq \max\left\{t_D^\circ(p), T_C(p)\right\}$ hold, we define a type t_f as follows:

$$t_f = BR_F(p_A, p_R) \quad \Leftrightarrow \quad \Delta(t_f) = 0$$

and it is such that the firm's best reponse is to accept p if $t \leq t_f$, and to reject p if instead $t \geq t_f$. From our previous analysis, there exists a unique type t in $[0, t_R]$ such that $t = t_f$. For all p such that $\Delta(t_f = 0) < 0$, we extend BR_F to $BR_F(p_A, p_R) = 0$. Overall, BR_F is well-defined, as long as $p \leq \overline{p}$ for all $(p_A, p_R) \in [0,1]^2$, with:

$$p_R \geq \max\left\{p_A \ , \ \frac{\max\left\{T_C(p) \ , \ t_D^\circ(p)\right\} - \tau_D^2}{\max\left\{T_C(p) \ , \ t_D^\circ(p)\right\} - \tau_D^1}\right\}$$

This condition on the price p_R result from the conditions in Lemmas 4.4.3 and 4.4.4. Let $P_A(t_f)$ and $P_R(t_f)$ be the best response functions of country D, when it believes that the firm's type is distributed according to cdf:

$$G_A(t) = \frac{G(t)}{G(t_f)} \text{ on } [0, t_f] \tag{27}$$

and

$$G_R(t) = \frac{G(t) - G(t_f)}{1 - G(t_f)} \text{ on } [t_f, 1] \tag{28}$$

respectively. Consider the composite best response function:

$$\Phi(t_f) \equiv BR_F(P_A(t_f), P_R(t_f))$$

from $[0,1]$ to $[0,1]$. For any $p \in [0,1]$, and given our result in Lemma 4.4.3, the function Φ is well defined on the interval $[\underline{t}_f, 1]$, where $\underline{t}_f$ is given by:

$$\underline{t}_f = 0 \text{ if } \frac{\tau_D^2 - P_R(0)\tau_D^1}{1 - P_R(0)} \geq T_C(p)$$

or

$$\underline{t}_f = P_R^{-1}\left(\frac{T_C(p) - \tau_D^2}{T_C(p) - \tau_D^1}\right) \text{ otherwise.}$$

We can now assess the existence of interior equilibria in subgames given some conditions on the values of C's offer, p.

Proposition 4.4.4. *The threshold type t_f is part of an interior equilibrium of subgame p if and only if it is a fixed point of the mapping Φ and:*

$$p \geq \underline{p} \equiv \frac{\tau_C^2}{\tau_C^1} + \varepsilon \frac{\tau_D^1}{\tau_C^1}\left(P_R(0) - \frac{\tau_D^2}{\tau_D^1}\right) \tag{29}$$

Moreover, it must be that $t_f \geq \underline{t}_f$. Under conditions A and B in Lemma 4.4.4, for all $p \leq \overline{p}$, the mapping Φ is well defined on $\left[\underline{t}_f, 1\right]$ and admits a unique fixed-point in this interval.

Proof. If p_R and p_A are best responses to t_f and t_f is a best response to (p_A, p_R), then t_f must be a fixed-point of Φ. By Lemma 4.4.3, $t_R \geq T_C(p)$

must hold in an interior equilibrium; hence, $t_f \geq \underline{t}_f$ must be satisfied in an interior equilibrium. Last, if the fixed-point represents an interior equilibrium, type zero must accept p:

$$p \geq \frac{\tau_C^2}{\tau_C^1} + \varepsilon \frac{\tau_D^1}{\tau_C^1}[P_R(0) - P_A(0)] = \underline{p},$$

where $P_A(0) = \frac{\tau_D^2}{\tau_D^1}$ is the offer made by D when it observes that $\underline{p}$ was accepted (in such a case, D infers that the firm's type is 0 with probability 1). If $p < \underline{p}$, then $t_f = 0$ is a fixed point, however it is not part of an interior equilibrium because for such prices, even type 0 prefers to reject p. Under conditions A and B and $p \leq \bar{p}$, the inequality $t_f \geq \underline{t}_f$ guarantees that:

$$t_R \geq \max\{t_D^\circ(p) \ , \ T_C(p)\}$$

so that Φ is well defined on $[\underline{t}_f, 1]$. Since:

$$\Phi(\underline{t}_f) \geq \underline{t}_f \quad \text{and} \quad \Phi(1) \leq 1,$$

then by the intermediate values theorem, Φ has at least one fixed-point in this interval. Since P_A and P_R are weakly increasing in t_f, and BR_F is weakly decreasing in p_R, it follows that Φ is weakly non-increasing on its domain. Thus the fixed-point is unique. □

4.4.2 Country D's optimal offers

Let us first assume that the firm accepted country C's offer p, i.e. country D observes the price of p in country C. Then country D believes that the firm's type is distributed according to cdf G_A in expression (27), on the support $[0, t_f]$. Recall the alternative interpretation of our model. Country D is a monopsonist facing the supply curve of equation:

$$S_A(p) = \frac{G(T_D(p))}{G(t_f)}(L - \tau_D^1),$$

for T_D the threshold function in (25), and is trying to maximize a payoff of the form $(1-p)S_A(p)$ which is defined on the interval of prices:

$$\left(\frac{\tau_D^2}{\tau_D^1} , \frac{t_f - \tau_D^2}{t_f - \tau_D^1} \right]$$

The elasticity of S_A is:

$$e_{S_A}(t) = \frac{G'(t)}{G(t)}[t - \tau_D^1]$$

where $t \in [0, t_f]$. Let us now consider the case where the firm rejects country C's offer. Country D believes that the firm's type is distributed according to cdf G_R in expression (28) on the support $(t_f, 1]$. The supply that country D faces is:

$$S_R(p) = \frac{G(T_D(p)) - G(t_f)}{1 - G(t_f)}(L - \tau_D^1),$$

defined on the interval of prices:

$$\left(\frac{t_f - \tau_D^2}{t_f - \tau_D^1} , \frac{1 - \tau_D^2}{1 - \tau_D^1} \right]$$

and the elasticity is written as:

$$e_{S_R}(t) = \frac{G'(t)}{G(t) - G(t_f)}[t - \tau_D^1]$$

The results about the optimal offers of country D are gathered below.

Proposition 4.4.5. *Following an acceptance of country C's offer, country D's problem has an interior solution if and only if $t_f > \underline{t}_D$, with t_f the type of firm which is indifferent between accepting and rejecting country C's offer. Country D's optimal offer p_A then solves:*

$$e_{S_A}(p_A) = 1$$

Otherwise, country D's optimal offer is $p_A = \frac{t_f - \tau_D^2}{t_f - \tau_D^1}$. Let t_A be the threshold type associated with country D's offer p_A. The firm of type t accepts p_A if $t \leq t_A$, and rejects p_A otherwise.
Following a rejection of country C's offer, country D's problem has an interior solution if and only if $t_f < \bar{t}_f$. In this case, country D's offer p_R solves:

$$e_{S_R}(p_R) = 1$$

Otherwise, country D's optimal offer is $p_R = \frac{1 - \tau_D^2}{1 - \tau_D^1}$. Let t_R be the threshold type associated with country D's offer p_R. The firm of type t accepts p_R if $t \leq t_R$, and rejects p_R otherwise.

The proof follows the same lines as that of Proposition 4.3.1, therefore it is omitted. Next, we clarify the effect of t_f on country D's expected payoff.

Remark 4.4.1. *(1) If $t_f \leq \underline{t}_D$, then country D's expected equilibrium* *payoff is strictly decreasing in t_f if and only if:*

$$G'(t_f)(p_R - p_A) < G(t_f)\frac{(1 - p_A)}{(t_f - \tau_D^1)}$$

i.e., the marginal cost on country D from decreasing the number of types which accept C's offer is less than the marginal benefit to country D of paying a lower price in the case where C's offer is accepted.

(2) If $t_f > \underline{t}_D$, then country D's expected equilibrium payoff is strictly decreasing in t_f.

The proof is provided in Appendix C.5. In this version of the game where the firm is farsighted, country D uses a form of IPR in equilibrium: it bases its offer to the firm on the price it observes country C pays. Country D may use IPR for different ends, depending on whether the firm rejects or accepts C's offer. In the first case, IPR allows country D to increase the probability of acceptance of its offer to the firm. Should the price paid by country C be confidential, country D would have offered p_a (see Proposition 4.3.1), and the firm would have rejected it for sure. The second case happens in equilibrium only if the threshold type in country C, t_f, is sufficiently low (lower than $\underline{t}_D$). By observing the price paid by country C, country D can revise its offer downwards without decreasing the probability of acceptance.

4.4.3 Country C's optimal offer

In this section, we determine country C's optimal offer. From our previous analysis, we know that when conditions A and B in Lemma 4.4.4 hold, an equilibrium in a subgame p is monotone: the firm's payoff gain from rejecting, Δ, is strictly increasing in t for all $t \in [0,1]$).

Country C's optimal offer lies in the interval:

$$P_f^* = \left(\frac{\tau_C^2}{\tau_C^1} + \varepsilon \frac{\tau_D^1}{\tau_C^1} \left(P_R(0) - \frac{\tau_D^2}{\tau_D^1} \right) \ , \ \frac{1-\tau_C^2}{1-\tau_C^1} \right]$$

Any offer p strictly larger than the upper bound is dominated by an offer strictly lower than p, yet greater than the upper bound. And any offer strictly less than the lower bound is strictly dominated by an offer larger than the upper bound, yet strictly lower than 1. The type of firm which is indifferent between accepting and rejecting $p \in P_f^*$ is:

$$T_f(p) = \frac{(\tau_C^2 - p\tau_C^1) - \varepsilon\,(\tau_D^2 - P_R(t_f)\tau_D^1)}{1 - p - \varepsilon\,(1 - P_R(t_f))} \tag{30}$$

with $P_R(t_f)$ country D's best response following a rejection of p. Recall that for some fixed offer p and associated threshold type t_f, D's best-response is an increasing function of C's offer p, as:

$$\frac{dp_R}{dp} = \frac{(t_f - \tau_C^1)(1-p_R)}{(t_R - \tau_D^1)(1-p-\varepsilon(1-p_R))} \left(\frac{dt_R}{dt_f} \right) > 0$$

The denominator in expression (30) is strictly positive by Lemma 4.4.4. The numerator is positive for any price offer in P_f^*. An alternate expression of the threshold type t_f associated with some offer p is:

$$t_f = T_C(p) - \varepsilon \frac{(t_f - \tau_D^2) - P_R(t_f)(t_f - \tau_D^1)}{1-p}$$

and note that $t_f \leq T_C(p)$ for any p in the relevant interval.[14] Country C's problem is to choose a price offer that maximizes its payoff:

$$(1-p)S_f(p)$$

where $S_f(p)$ is the supply faced by country C, and whose equation is:

$$S_f(p) = G\left(T_f(p)\right)\left(L - \tau_C^1\right)$$

for $T_f(p)$ the threshold type function in (30). Note that country C faces a shorter supply when the firm is farsighted than when it is myopic. In the farsighted case, the elasticity of the supply faced by C is:

$$e_{S_f}(t) = e_S(t)\left(1 + \frac{\varepsilon(t-\tau_C^1)(\tau_D^2-\tau_D^1)}{Z(t_R)}\right)\left(1 - \frac{\varepsilon(t-\tau_C^1)(\tau_D^2-\tau_D^1)}{Z(t_R)}\left[\frac{dt_R}{dt}\frac{(t-\tau_D^1)}{(t_R-\tau_D^1)}\right]\right)$$

where $e_S(t)$ is the elasticity of the supply S that C faces in the myopic case, and:

$$Z(t_R) = (\tau_C^2 - \tau_C^1)(t_R - \tau_D^1) - \varepsilon(\tau_D^2 - \tau_D^1)(t_R - \tau_C^1)$$

The value of the elasticity is positive, for any type $t \in [0,1]$, and any $t_R \in (t,1]$. Z is always positive in an equilibrium, and it is strictly increasing in t (by condition A in Lemma 4.4.4 and the fact that t_R is an increasing function of t.) More details are provided in Appendix C.6. The relative elasticity of the supply S_f compared to S, the supply faced by C in the myopic case, depends on the relation between t_R and t_f. Note that for all

[14]We always have $P_R(t_f) > \frac{t_f-\tau_D^2}{t_f-\tau_D^1}$ by country D's best response. Therefore, $T_f(p) \leq T_C(p)$ for any price offer p.

values of $t \in [\bar{t}_f, 1]$, country D's optimal offer p_R is constant at:

$$\frac{1-\tau_D^2}{1-\tau_D^1}$$

For such values of t, the supply S_f is relatively more elastic than S. The next remark presents some comparative statics on the threshold type t_f.

Remark 4.4.2. *The threshold function in expression* (30) *is increasing in p and decreasing in ε. Moreover, the supply function $S_f(p)$ converges uniformly to the supply function $S(p)$ in the myopic case. Also, its first derivative with respect to p, $S_f'(p)$, converges uniformly to $S'(p)$, the derivative of the myopic supply function.*

Proof. Given that $\frac{\partial \Delta}{\partial p} < 0$ and $\frac{\partial \Delta}{\partial \varepsilon} > 0$ and $\frac{\partial \Delta}{\partial t_f} > 0$, it follows that $\frac{\partial \Phi}{\partial p} > 0$ and $\frac{\partial \Phi}{\partial \varepsilon} < 0$. Since $\frac{\partial \Phi}{\partial t_f} < 0$, it follows that t_f^* is increasing in p and decreasing in ε. □

Our results about country C's optimal offer are gathered in the proposition below.

Proposition 4.4.6. *Under conditions A and B, country C's problem has a unique solution $p_f^* \in P_f^*$, and it is interior if and only if:*

$$e_{S_f}(p_f^*) = 1 \quad \textit{and} \quad p_f^* < \frac{1-\tau_C^2}{1-\tau_C^1}$$

Otherwise, $p_f^* = \frac{1-\tau_C^2}{1-\tau_C^1}$. Let t_f^* be the type of firm which is indifferent between accepting and rejecting country C's equilibrium offer p_f^*. For t^* the analogous threshold type for the myopic case, and t_r the threshold type associated *with country D's best response $P_R(t^*)$:*

$$t_f^* \geq t^* \quad \Leftrightarrow \quad \left.\frac{dt_R}{dt}\right|_{t=t^*} \leq \frac{t_r - \tau_D^1}{t^* - \tau_D^1}$$

For p^* country C's optimal offer in the myopic case:

$$p_f^* \geq p^* \quad \Leftrightarrow \quad \left.\frac{dt_R}{dt}\right|_{t=t^*} \leq \frac{t_r - \tau_D^1}{t^* - \tau_D^1}\left(1 + \frac{\varepsilon(t^* - \tau_C^1)(\tau_D^2 - \tau_D^1)}{(\tau_C^2 - \tau_C^1)(t_r - \tau_D^1) - \varepsilon(\tau_D^2 - \tau_D^1)(t_r - \tau_C^1)}\right)$$

and $(\tau_C^2 - \tau_C^1)(t_r - \tau_D^1) - \varepsilon(\tau_D^2 - \tau_D^1)(t_r - \tau_C^1) > 0.$

See Appendix C.7 for the proof. We compare country C's payoff between the farsighted case and the myopic case.

Remark 4.4.3. *Country C's expected payoff is lower in the farsighted case than in the myopic case.*

Proof. Consider some fixed price offer p. At the beginning of this section, we established that: $T_f(p) \leq T_C(p)$ for any p in the relevant interval. Meaning, a same price offer is always more likely to be accepted in the myopic case than in the farsighted one. For some given offer p, country C's expected payoff in the myopic case is: $G(T_C(p))(1-p)$, and in the farsighted case this is: $G(T_f(p))(1-p)$. By our later argument, we have $G(T_C(p))(1-p) \geq G(T_f(p))(1-p)$ for all p in the relevant interval. Then $G(T_C(p^*))(1-p^*) \geq G(T_C(p_f^*))(1-p_f^*) \geq G(T_f(p_f^*))(1-p_f^*)$, where p^* is country C's optimal offer in the myopic case. $\square$

4.4.4 A characterization of the weak Perfect Bayesian Equilibria

We summarize our results from the two previous sections below.

Proposition 4.4.7. *Under conditions A and B, for every $p \in [0,1]$, an essentially unique equilibrium exists in subgame p. Consider $\underline{p}$ and $\overline{p}$ given by expressions (29) and (26), respectively.*

1. *If $p < \underline{p}$, it is a pooling equilibrium where all types reject p. If the firm rejects p, country D believes that t is distributed according to its prior belief G and offers $\underline{p}_D = P_R(0)$. If the firm accepts p, country D may believe the firm's type is 0 with probability one, and offers $P_A(0) = \frac{\tau_D^2}{\tau_D^1}$.*

2. *If $p \in \left[\underline{p}, \overline{p}\right]$, it is an interior equilibrium, characterized by a threshold type $t_f^* \in \left[\underline{t}_f, 1\right]$, with:*

$$\underline{t}_f = 0 \quad \textit{if} \quad \frac{\tau_D^2 - P_R(0)\tau_D^1}{1 - P_R(0)} \geq T_C(p)$$

   and

$$\underline{t}_f = P_R^{-1}\left(\frac{T_C(p) - \tau_D^2}{T_C(p) - \tau_D^1}\right) \quad \textit{otherwise,}$$

   such that $\Phi\left(t_f^*\right) = t_f^*$. In an interior equilibrium, all types in $\left[0, t_f^*\right)$ accept p and all types in $\left(t_f^*, 1\right]$ reject p. Country D offers $p_A = P_A\left(t_f^*\right)$ and holds belief G_A in (27) when p is accepted, and offers $p_R = P_R\left(t_f^*\right)$ and holds belief G_R in (28) when p is rejected.

3. *If $p > \overline{p}$, it is a pooling equilibrium where all types accept p. Upon*

observing that p is accepted, country D's belief is its prior belief G and offers $\underline{p}_D = P_A(1)$. If the firm rejects p, country D may believe that the firm's type is 0 with probability one, and offers $P_R(0) = \frac{\tau_D^2}{\tau_D^1}$. Other beliefs are also possible, and it is possible to support this equilibrium with beliefs that satisfy the intuitive criterion.

4.5 Two countries when the firm is farsighted, with a secret rebate

In the previous sections, we showed that the price in country C releases information about the firm's type, and influences country D's offer. In particular, country D's offer depends positively on the price that country C pays. If the firm is farsighted and the details of the deal with country C cannot be hidden from D, the firm gets an additional incentive to reject the first country's offer. Indeed, we showed that any given price offer that country C may extend is more likely to be rejected when the firm is farsighted, which leads to a reduction in C's payoff.

In order to avoid this penalty in the presence of a farsighted firm, the first country could make the price that country D observes completely uninformative about the firm's type. To this end, country C may offer a pair of prices (p, r) such that: p is the component of the contract that is observable to country D if the firm accepts the proposal, and we refer to p as the list price; and r is the secret rebate that D never knows. An offer from C that consists of a list price equal to 1 and a secret rebate permits to isolate the two negotiations completely: whether the firm accepts or rejects this contract, country D always observes the price of 1 in the first country, therefore the latter does not learn anything about the firm's type.

Under the restriction that the list price must equal 1, country C's optimal offer is $r = 1 - p^*$, where p^* is the optimal offer in the myopic case (characterized in Proposition 4.2.1). If the firm rejects, then it counter-offers a list price of 1 without a rebate. Note that the firm, even if it is farsighted, accepts this offer with rebate under the same condition as when it is myopic (Section 4.2). Indeed, country D's offer is the same regardless of the firm's decision to accept or reject the contract $(p = 1, r^*)$ that country C proposes. It follows that allowing the use of secret rebates makes country C as well-off as when the firm is myopic and prices are transparent (Section 4.3), and strictly better-off than in the case where the firm is farsighted and prices are transparent (Section 4.4). Overall, what penalizes country C is the combination of the firm's farsightedness and country D using a form of IPR in its negotiation.

Irrespective of whether the deal with rebate is accepted or rejected by the firm, country D learns nothing about the firm's type, and is therefore worse-off compared to the two previous sections (myopic or farsighted firm with price transparency). This finding suggests that price transparency is beneficial to country D. Since there is no informational spillover when the first country conceals its true offer with a secret rebate, the second country's belief in the subgame that follows country C's proposal is its prior belief. Note that D's optimal offer in this case is equal to the price p_a in Proposition 4.3.1, and the associated threshold type is equal to t_a.

It is worthwhile noting that the firm is weakly worse-off than in Section

4.3 (myopic case).[15] First, the firm accepts country C's offer with rebate if and only if its type is less than t^* in Section 4.3. Next, all firms whose type belongs either to $[0, t^*]$ or $[t_r, 1]$ are indifferent between an offer made of a list price of 1 and a rebate equal to $1 - p^*$, and a direct offer p^* without rebate (Section 4.3). (Recall that t_r is the threshold type defined in Proposition 4.3.1 in the myopic case). The types less than t_a accept the two countries' offers, and those between t_a and t^* accept C's and reject D's. The types in the interval $[t_r, 1]$reject both countries' offers, thus they get the same profit as in Section 4.3 as well. However, the types in the interval $(t^*, t_r]$ are strictly worse-off than in Section 4.3. These types strictly prefer to accept the offer p_r that D would have made, had rebates not been allowed, than to wait until period τ_D^2 and earn the price of 1.

In conclusion, there exists a PBE of our game with rebates that precludes any informational spillover on the second country. The best-responses of each party in this PBE are the following. At τ_C^1, country C offers a pair of two prices, $(p = 1, r = 1 - p^*)$, where the rebated price p^* corresponds to the country's optimal offer in Section 4.2. The firm of type t accepts any offer such that:

$$p_C - r \geq \frac{t - \tau_C^2}{t - \tau_C^1}$$

and rejects any offer such that:

$$p_C - r < \frac{t - \tau_C^2}{t - \tau_C^1}$$

[15] Almost nothing can be said about the difference in the firm's expected payoff in the farsighted case and the secret case. Note that for very small ε, our result that the firm is better-off with price transparency than under secrecy holds.

At τ_C^2, the firm counter-offers the list price of 1 and no rebate, for all possible histories. Country C accepts the counter-offer, for all possible histories. At τ_D^1, country D offers p_D, which solves:

$$G'(t_D)(t_D - \tau_D^1) = G(t_D)$$

and this price is equal to p_a, if the history of the game is either $h_1 = ((p = 1, r = 1 - p^*), A, 1, A)$ or $h_2 = ((p = 1, r = 1 - p^*), R, 1, A)$. For any other history, country D offers $p_D = \frac{\tau_D^2}{\tau_D^1}$. The firm of type t accepts any price offer $p_D \geq \frac{t-\tau_D^2}{t-\tau_D^1}$, and rejects any price offer $p_D < \frac{t-\tau_D^2}{t-\tau_D^1}$. At τ_D^2, the firm counter-offers the price of 1, for all possible histories, and country D accepts the counter-offer, for all possible histories.

The belief system is as follows. At I_1, country C's belief is its prior belief G. If the second information set of country C is reached, its belief is $G_R(t)$ in (24). If the second information set is not reached, then any belief of the country is consistent. At I_2, country D's belief is its prior belief G if the history of the game is either $h_1 = ((p = 1, r = 1 - p^*), A, 1, A)$ or $h_2 = ((p = 1, r = 1 - p^*), R, 1, A)$. Country D believes that the firm is of type 0 with probability one for any other history. If the second information set of country D is reached, its belief is:

$$\frac{G(t) - G(t_a)}{1 - G(t_a)}$$

on the support $(t_a, 1]$, where t_a is the threshold type associated with country D's optimal offer. If the second information set of country D is not reached, then any belief is consistent.

4.6 Conclusion

We studied the outcomes of the sequential negotiations of two countries with a monopolist pharmaceutical firm. We assumed that the firm holds private information about the duration of its monopoly, which affects its willingness to accept lower offers against earlier deals. The model highlights four main ideas.

First, that a form of international price referencing (IPR) emerges naturally when prices agreed with countries that reached earlier agreements carry information that is valuable to subsequent countries. In particular, we find that a country which can observe the transaction price in another country formulates an offer that depends positively on this transaction price.

Second, that IPR is beneficial to the countries which negotiate last and is detrimental to those which negotiate first. In an equilibrium of our model, the countries that come last in the order of the negotiations benefit from IPR, and apply it to increase the probability of acceptance of their own offers. As the corresponding optimal offers depend positively on the prices obtained by those which negotiated first, the firm has an additional incentive to reject these countries' offers.

Third, that in order to avoid this penalty, secret rebates emerge as a natural arrangement between the countries that negotiate first and the firm. Fourth, that when the firm is farsighted, secret rebates benefit the countries which negotiate first and are detrimental to those which negotiate last.

Bibliography

[1] Nizar Allouch. On the private provision of public goods on networks. *Journal of Economic Theory*, 157:527–552, 2015.

[2] Richard W Ault, T Randolph Beard, David N Laband, and Richard P Saba. Rebates, inventories, and intertemporal price discrimination. *Economic Inquiry*, 38(4):570–578, 2000.

[3] Robert J Aumann. Subjectivity and correlation in randomized strategies. *Journal of mathematical Economics*, 1(1):67–96, 1974.

[4] Robert J Aumann. Correlated equilibrium as an expression of bayesian rationality. *Econometrica: Journal of the Econometric Society*, pages 1–18, 1987.

[5] Robert J Aumann and Roger B Myerson. Endogenous formation of links between players and of coalitions: An application of the Shapley value. In *Networks and Groups*, pages 207–220. Springer, 2003.

[6] Venkatesh Bala and Sanjeev Goyal. A noncooperative model of network formation. *Econometrica*, 68(5):1181–1229, 2000.

[7] Coralio Ballester, Antoni Calvó-Armengol, and Yves Zenou. Who's who in networks. wanted: The key player. *Econometrica*, 74(5):1403–1417, 2006.

[8] Robert C Blattberg and Alan Levin. Modelling the effectiveness and profitability of trade promotions. *Marketing Science*, 6(2):124–146, 1987.

[9] Francis Bloch, Gabrielle Demange, and Rachel Kranton. Rumors and social networks. *International Economic Review*, 59(2):421–448, 2018.

[10] Francis Bloch, Bhaskar Dutta, et al. Communication networks with endogenous link strength. *Games and Economic Behavior*, 66(1):39–56, 2009.

[11] Francis Bloch, Matthew O Jackson, and Pietro Tebaldi. Centrality measures in networks. *Available at SSRN 2749124*, 2019.

[12] Phillip Bonacich. Power and centrality: A family of measures. *American journal of sociology*, 92(5):1170–1182, 1987.

[13] Yann Bramoullé and Rachel Kranton. Public goods in networks. *Journal of Economic Theory*, 135(1):478–494, 2007.

[14] Yann Bramoullé, Rachel Kranton, and Martin D'amours. Strategic interaction and networks. *American Economic Review*, 104(3):898–930, 2014.

[15] Yann Bramoullé, Dunia López-Pintado, Sanjeev Goyal, and Fernando Vega-Redondo. Network formation and anti-coordination games. *International Journal of game Theory*, 33(1):1–19, 2004.

[16] Nathan Canen, Matthew O Jackson, and Francesco Trebbi. Endogenous networks and legislative activity. *Available at SSRN 2823338*, 2019.

[17] Michel Chevalier and Ronald C Curhan. Retail promotions as a function of trade promotions: A descriptive analysis. *Sloan Management Review (pre-1986)*, 18(1):19, 1976.

[18] Chun-Hung Chiu, Tsan-Ming Choi, and Christopher S Tang. Price, rebate, and returns supply contracts for coordinating supply chains with price-dependent demands. *Production and Operations Management*, 20(1):81–91, 2011.

[19] Michael Suk-Young Chwe. Communication and coordination in social networks. *The Review of Economic Studies*, 67(1):1–16, 2000.

[20] Patricia M Danzon. Price discrimination for pharmaceuticals: Welfare effects in the US and the EU. *International Journal of the Economics of Business*, 4(3):301–322, 1997.

[21] Patricia M Danzon. Differential pricing of pharmaceuticals: theory, evidence and emerging issues. *PharmacoEconomics*, 36(12):1395–1405, 2018.

[22] Patricia M Danzon and Adrian Towse. Differential pricing for pharmaceuticals: reconciling access, R&D and patents. *International journal of health care finance and economics*, 3(3):183–205, 2003.

[23] Patricia M Danzon, Y Richard Wang, and Liang Wang. The impact of price regulation on the launch delay of new drugs evidence from

twenty-five major markets in the 1990s. *Health economics*, 14(3):269–292, 2005.

[24] Joseph A DiMasi, Ronald W Hansen, and Henry G Grabowski. The price of innovation: new estimates of drug development costs. *Journal of health economics*, 22(2):151–185, 2003.

[25] Joseph A DiMasi and Cherie Paquette. The economics of follow-on drug research and development. *Pharmacoeconomics*, 22(2):1–14, 2004.

[26] Lorenzo Ductor, Marcel Fafchamps, Sanjeev Goyal, and Marco J van der Leij. Social networks and research output. *Review of Economics and Statistics*, 96(5):936–948, 2014.

[27] Bhaskar Dutta, Anne Van den Nouweland, and Stef Tijs. Link formation in cooperative situations. *International Journal of Game Theory*, 27(2):245–256, 1998.

[28] Georgy Egorov and Konstantin Sonin. Persuasion on networks. *CEPR Discussion Paper No. DP13723*, 2019.

[29] Mats Ekelund and Björn Persson. Pharmaceutical pricing in a regulated market. *Review of Economics and Statistics*, 85(2):298–306, 2003.

[30] Jaime Espín, Joan Rovira, Antoinette Calleja, Natasha Azzopardi-Muscat, Erica Richardson, Willy Palm, and Dimitra Panteli. How can voluntary cross-border collaboration in public procurement improve access to health technologies in Europe. *Policy brief*, 21, 2016.

[31] Jaime Espín, Michael Schlander, Brian Godman, Pippa Anderson, Jorge Mestre-Ferrandiz, Isabelle Borget, Adam Hutchings, Steven Flostrand, Adam Parnaby, and Claudio Jommi. Projecting pharmaceutical expenditure in EU5 to 2021: adjusting for the impact of discounts and rebates. *Applied health economics and health policy*, 16(6):803–817, 2018.

[32] Andrea Galeotti and Sanjeev Goyal. The law of the few. *American Economic Review*, 100(4):1468–92, 2010.

[33] Begoña Garcia Mariñoso, Izabela Jelovac, and Pau Olivella. External referencing and pharmaceutical price negotiation. *Health Economics*, 20(6):737–756, 2011.

[34] Begoña Garcia-Mariñoso and Pau Olivella. Informational spillovers and the sequential launching of pharmaceutical drugs. Technical report, Mimeo City University, London, 2005.

[35] Eitan Gerstner and James D Hess. A theory of channel price promotions. *The American Economic Review*, pages 872–886, 1991.

[36] Eitan Gerstner and James D Hess. Pull promotions and channel coordination. *Marketing Science*, 14(1):43–60, 1995.

[37] Eitan Gerstner, James D Hess, et al. Who benefits from large rebates: Manufacturer, retailer or consumer. *Economics Letters*, 36(1):5–8, 1991.

[38] David A Goodman and Kavin W Moody. Determining optimum price promotion quantities. *Journal of Marketing*, 34(4):31–39, 1970.

[39] Sanjeev Goyal and Jose Luis Moraga-Gonzalez. R&D networks. *Rand Journal of Economics*, pages 686–707, 2001.

[40] Sanjeev Goyal and Fernando Vega-Redondo. Network formation and social coordination. *Games and Economic Behavior*, 50(2):178–207, 2005.

[41] Sverre Grepperud and Pedersen Pål Andreas. Positioning and negotiations: The case of pharmaceutical pricing. *European Journal of Political Economy*, page 101853, 2020.

[42] Jeanne Hagenbach and Frédéric Koessler. Strategic communication networks. *The Review of Economic Studies*, 77(3):1072–1099, 2010.

[43] Peter Herman. Endogenous network formation and the provision of a public good. *Available at SSRN 2690322*, 2014.

[44] Andrew M Hill, Melissa J Barber, and Dzintars Gotham. Estimated costs of production and potential prices for the WHO essential medicines list. *BMJ global health*, 3(1):e000571, 2018.

[45] Aidan Hollis. Me-too drugs: is there a problem. *WHO report*, 2004.

[46] Nicolas Houy and Izabela Jelovac. Drug launch timing and international reference pricing. *Health economics*, 24(8):978–989, 2015.

[47] Matthew O Jackson. A survey of network formation models: stability and efficiency. *Group formation in economics: Networks, clubs, and coalitions*, 664:11–49, 2005.

[48] Matthew O Jackson and Asher Wolinsky. A strategic model of social and economic networks. *Journal of economic theory*, 71(1):44–74, 1996.

[49] Izabela Jelovac. On the relationship between the negotiated prices of pharmaceuticals and the patients co-payment. In *VI EHEW meeting, Marseille*, volume 8, 2002.

[50] Izabela Jelovac and Catalina Bordoy. Pricing and welfare implications of parallel imports in the pharmaceutical industry. *International journal of health care finance and economics*, 5(1):5–21, 2005.

[51] Sumit Joshi, Ahmed Saber Mahmud, and Sudipta Sarangi. Network formation with multigraphs and strategic complementarities. *Journal of Economic Theory*, page 105033, 2020.

[52] Markus Kinateder and Luca Paolo Merlino. Public goods in endogenous networks. *American Economic Journal: Microeconomics*, 9(3):187–212, 2017.

[53] Margaret K Kyle. Pharmaceutical price controls and entry strategies. *The Review of Economics and Statistics*, 89(1):88–99, 2007.

[54] Marie Lalanne and Paul Seabright. The old boy network: The impact of professional networks on remuneration in top executive jobs. 2016.

[55] Hau L Lee and Meir J Rosenblatt. A generalized quantity discount pricing model to increase supplier's profits. *Management science*, 32(9):1177–1185, 1986.

[56] Frank R Lichtenberg and Tomas J Philipson. The dual effects of intellectual property regulations: within-and between-patent competition

in the US pharmaceuticals industry. *The Journal of Law and Economics*, 45(S2):643–672, 2002.

[57] Z John Lu and William S Comanor. Strategic pricing of new pharmaceuticals. *Review of economics and statistics*, 80(1):108–118, 1998.

[58] Dov Monderer and Lloyd S Shapley. Potential games. *Games and economic behavior*, 14(1):124–143, 1996.

[59] Steven Morgan, Jamie Daw, and Paige Thomson. International best practices for negotiating reimbursement contracts with price rebates from pharmaceutical companies. *Health Affairs*, 32(4):771–777, 2013.

[60] Steven G Morgan, Sabine Vogler, and Anita K Wagner. Payers experiences with confidential pharmaceutical price discounts: a survey of public and statutory health systems in North America, Europe, and Australasia. *Health Policy*, 121(4):354–362, 2017.

[61] Stephane Régnier. What is the value of me-too drugs? *Health care management science*, 16(4):300–313, 2013.

[62] William H Sandholm. *Population games and evolutionary dynamics*. MIT press, 2010.

[63] F Michael Scherer. How US antitrust can go astray: the brand name prescription drug litigation. *International Journal of the Economics of Business*, 4(3):239–256, 1997.

[64] R SELTEN. Reexamination of the perfectness concept for equilibrium points in extensive games. *International Journal of Game Theory*, 4:25–55, 1975.

[65] Marco Slikker and Anne Van Den Nouweland. Network formation models with costs for establishing links. *Review of Economic Design*, 5(3):333–362, 2000.

[66] Eva Tardos and Tom Wexler. Network formation games and the potential function method. *Algorithmic Game Theory*, pages 487–516, 2007.

[67] Adrian Towse, Michele Pistollato, Jorge Mestre-Ferrandiz, Zeba Khan, Satyin Kaura, and Louis Garrison. European Union pharmaceutical markets: a case for differential pricing? *International Journal of the Economics of Business*, 22(2):263–275, 2015.

[68] José Luis Valverde and Eduardo Pisani. *Globalisation of the Pharmaceutical Industry*. IOS Press, 2016.

[69] Sabine Vogler, Valérie Paris, and Dimitra Panteli. *Ensuring access to medicines: How to redesign pricing, reimbursement and procurement?* World Health Organization, Regional Office for Europe, 2018.

[70] Sabine Vogler and Kenneth R Paterson. Can price transparency contribute to more affordable patient access to medicines? *PharmacoEconomics Open*, 1(1):145–147, 2017.

[71] Sabine Vogler, Peter Schneider, and Nina Zimmermann. Evolution of average European medicine prices: implications for the methodology of external price referencing. *PharmacoEconomics Open*, 3(3):303–309, 2019.

[72] Sabine Vogler, Nina Zimmermann, Claudia Habl, Jutta Piessnegger, and Anna Bucsics. Discounts and rebates granted to public payers for medicines in European countries. *Southern med review*, 5(1):38, 2012.

Appendices

Appendix A

Appendix Chapter 2

A.1 Proof of Lemma 2.3.2

The proof is by contradiction. The network g is strict Nash, and there are two components C and D in g that are wheels. By Corollary 2.3.3 statement 1, C and D are not comparable via $\mathcal{R}$; and a player j has a path to anyone in X if and only if $j \in X$, for any component $X \in \{C, D\}$. Note that all players in X have the same reach, for any component $X \in \{C, D\}$. Let i be any agent in C; and let j be any agent in D. If $\kappa_i(g) \geq \kappa_j(g)$, then it follows immediately that $g'_j = i$ is a strictly profitable deviation, as: $\kappa_l(g'_j, g_{-j}) = \kappa_i(g) + 1 > \kappa_j(g)$, and $|g'_j| \leq |g_j|$. Following a similar argument for the case where $\kappa_i(g) < \kappa_j(g)$, the deviation $g'_i = j$ for player i is strictly profitable. A contradiction that g is strict Nash.

A.2 Proof of Lemma 2.3.5

This is a direct proof. I denote the agents in the wheel component as $N_w = \{i_1, \ldots, i_{n_w}\}$ where $3 \leq n_w \leq n$, and the wheel is characterized as $i_1 \to i_2 \ldots \to i_{n_w} \to i_1$. In a non-exhaustive wheel, the statement holds trivially for any singleton. Because players in a wheel component are symmetric, let me consider agent i_1. Assume that i_1 has a deviation g'_{i_1} such that $|g'_{i_1}| \geq |g_{i_1}|$ and $g'_{i_1} \not\subseteq g_{i_1}$. Let $g' = (g'_{i_1}, g_{-i_1})$ be the network that results from i_1's deviation to g'_{i_1}. It follows from the definition of g'_{i_1} that $i_2 \notin g'_{i_1}$. (Recall that $g_{i_1} = i_2$ in g.) Note that the players whose reach is affected by i_1's deviation all belong to N_w. I distinguish between two cases:

1. $g'_{i_1} \subset N_w$. It follows immediately that g_{i_1} yields a larger payoff than g'_{i_1}, as $\kappa_j(g') \leq \kappa_j(g)$ for all $j \in N$, and the inequality holds strictly for all agents in $N_w \setminus \{i_2\}$.

2. $g'_{i_1} \not\subset N_w$, i.e. $\exists k \in g'_{i_1}$ such that $g_k = \varnothing$. Let k any such element of g'_{i_1}, and consider $\tilde{g}_{i_1} = (i_1 \cup g'_{i_1}) \setminus \{k\}$ an alternate deviation for player i_1. Let me denote $(\tilde{g}_{i_1}, g_{-i_1})$ as $\tilde{g}$. Note first that $|\tilde{g}_{i_1}| = |g'_{i_1}|$. Second, note that: $\kappa_{i_2}(\tilde{g}) = \kappa_{i_2}(g') - 1$, $\kappa_{i_3}(\tilde{g}) = \kappa_{i_3}(g')$, and $\kappa_{i_m}(\tilde{g}) > \kappa_{i_m}(g')$ for all $i_m \in N_w \setminus \{i_2, i_3\}$. The variation in the collective return is:

$$\begin{aligned} v(\tilde{g}) - v(g') = &- \left[\phi\left(n_w + |g'_{i_1}|\right) - \phi\left(n_w - 1 + |\tilde{g}_{i_1}|\right)\right] \\ &+ \sum_{h=2}^{n_w+1} \left(\phi\left(n_w - 1 + |\tilde{g}_{i_1}|\right) - \phi\left(n_w - (h-1) + |g'_{i_1}|\right)\right) \end{aligned}$$

and this is always positive when ϕ is concave and $n_w \geq 3$. These two conditions are met by Proposition 2.3.1 and if Assumption A holds.

Therefore g'_{i_1} is never a best-response of i_1.

A.3 Proof of Lemma 2.3.6

The proof is by contradiction. Let me denote as G_i^{+1} the set of deviations for player i such that i adds one link to the set of links i has in g:

$$G_i^{+1}(g) = \{g'_i \in G_i \ : \ |g'_i| = |g_i| + 1\}$$

Assume that the conclusion of Lemma 2.3.6 is false. Therefore, there exists $\hat{g}_i \in G_i^{+} \backslash G_i^{+1}$ that maximizes the ratio in Lemma 2.3.6; and let me re-write $\hat{g}_i$ has $g_i \cup L$, with $|L| = l > 1$. Let me first analyze the case where g is a non-exhaustive wheel. I call N_w the set of agents who belong to the wheel component of g. Given the architecture of g and if $i \in N_w$, the value of the ratio is at most:

$$A = \frac{n_w}{l}[\phi(n_w + l - 1) - \phi(n_w - 1)]$$

(The value of the ratio is strictly less than A if $\exists\ k \in L$ such that $k \in N_w$.) Next, consider any deviation $\tilde{g}_i \in G_i^{+1}$ of the form $\tilde{g}_i = g_i \cup \{j\}$ with $j \neq N_w$. The value of the ratio in Lemma 2.3.6 is:

$$B = n_w[\phi(n_w) - \phi(n_w - 1)]$$

As ϕ is concave, A is always less than B, and the result follows.

Assume instead that $i \notin N_w$. For any strategy $\hat{g}_i = g_i \cup L$ with $|L| = l >$

1, the ratio in Lemma 2.3.6 is at most equal to:

$$C = \frac{\phi(n_w - 1 + l) - \phi(0)}{l}$$

and it is equal to C if $\exists k \in L$ such that $k \in N_w$. Take any deviation $\tilde{g}_i = j \in N_w$. The ratio is equal to:

$$D = \phi(n_w) - \phi(0) > C$$

by the concavity of ϕ. The result follows. The proof for the empty network follows the same line, and is therefore omitted.

A.4 Proof of Proposition 2.4.1

Claim 1. *Consider a fixed value c of the cost of a link. A maximum of the potential function for c either corresponds to a wheel network, a non-exhaustive wheel or the empty network.*

Proof. First, consider all strategy profiles $g = (g_1, \ldots, g_n)$ such that $\sum_{i \in N} g_i \geq n$. In any such network g, $v(g) = n\phi(n-1)$. The architecture that minimizes the total cost of network formation is a wheel network.

Next, consider all strategy profiles g such that $\sum_{i \in N} g_i = m$, where m is an integer such that $n > m \geq 3$. I segregate the set of all networks g that have the aforementioned property into two sets: the set G_1, that contains all networks with strictly less than n components; and the complement of this set is G_2, and it contains all nonempty networks with n components. In any network in the set G_1, the maximum reach of an agent is $m-1$. An architecture that gives this maximal reach to a maximum number of players (which is constrained by the number of links m) is the non-exhaustive wheel with $n_w = m$, and note that for this network, $v(g^{n.e.w}) = m\phi(m-1) + (n-m)\phi(0)$. Note that this architecture is that which minimizes the cost of network formation. Hence if a maximum of the potential belongs to G_1, then it is a non-exhaustive wheel. Among all networks in the set G_2, it is immediate that the architecture that maximizes v is a chain network on $m+1$ agents: $i_1 \rightarrow i_1 \ldots \rightarrow i_{m+1}$. In this network, note that $v(g) = (n-m)\phi(0) + \sum_{h=1}^{m} \phi(h)$. Fixing m, it is immediate that the non-exhaustive wheel on m agents yields a larger collective return than the chain on $m+1$ agents when ϕ is concave and $m \geq 3$.

Now, consider all networks that have either 1 or 2 links in total. For 2 links, it is immediate that the architecture that maximizes the collective return is a chain on 3 agents. I show that no network that has either 1 or 2 links can maximize the potential function. For this, recall first that a maximum of the potential is a Nash equilibrium of the game. Take any chain network on 2 or 3 agents. If it is Nash, then $c \leq \phi(h) - \phi(0)$ with $h \in \{1, 2\}$, for h the reach of the root component. Let me first consider the case of the chain on 3 agents. Take any player i who has no link in this network. If i forms a link to the root of the chain, then the collective return increases at least by $\phi(h+1) - \phi(0)$, and the deviation is strictly profitable. If the chain is just on 2 agents, there is just one link in the network, that I refer to as $i \rightarrow j$. If this is a Nash equilibrium then $c \leq \phi(1) - \phi(0)$. Recall that $n \geq 3$. Consider the strategy $g_j = k$, for any $k \neq i$, for agent j. The collective return increases by $\phi(2) - \phi(0)$. Hence the deviation is strictly profitable given the value of c, and the network is not Nash - hence it is not a maximum of the potential for the value c of the cost of a link. $\square$

Claim 2. *For any value of the cost c, a maximum of the potential is either a wheel network or the empty network.*

Proof. I take on the notations introduced in Proposition 2.3.1 for referring to the wheel network (g^w), non-exhaustive wheels ($g^{n.e.w}$) and the empty network (g^e). First, $P(g^w) = n\phi(n-1) - nc$, $P(g^{n.e.w}) = n_w\phi(n_w - 1) + (n - n_w)\phi(0) - n_w c$ with $3 \leq n_w < n$ and $P(g^e) = n\phi(0)$. Note that:

$$P(g^w) \leq P(g^e) \quad \Leftrightarrow \quad c \geq \phi(n-1) - \phi(0)$$

Assume that $c \geq \phi(n-1) - \phi(0)$. Note that:

$$P(g^e) - P(g^{n.e.w}) = n_w\left(c - [\phi(n_w - 1) - \phi(0)]\right) \geq n_w\left(\phi(n-1) - \phi(n_w - 1)\right) > 0$$

Hence the maximum of the potential is achieved in the empty network when $c \geq \phi(n-1) - \phi(0)$.

Next, consider the values of c such that $c < \phi(n-1) - \phi(0)$. Note that:

$$P(g^w) - P(g^{n.e.w}) = n\left(\phi(n-1) - \phi(0)\right) - n_w\left(\phi(n_w - 1) - \phi(0)\right) - (n - n_w)c$$

and

$$c < \phi(n-1)-\phi(0) \quad \Rightarrow \quad P(g^{w})-P(g^{n.e.w}) > n_w\left(\phi(n-1)-\phi(n_w-1)\right) > 0$$

Hence the maximum of the potential is achieved in a wheel network when $c < \phi(n-1)-\phi(0)$. □

A.5 Proof of Proposition 2.4.2

Statement 1. Consider any non-exhaustive wheel on n_w agents. Take the payoff of any isolated singleton i in this network; this is:

$$u_i(g^{n.e.w}) = n_w\phi(n_w-1)+(n-n_w)\phi(0)$$

Recall that if this non-exhaustive is strict Nash, then the wheel network is strict Nash as well. If i had been part of a wheel network instead, then his payoff would be:

$$u_i(g^{w}) = n\phi(n-1)-c$$

If $g^{n.e.w}$ is strict Nash, then:

$$c < n_w\phi(n_w-1)-\sum_{h=0}^{n_w-1}\phi(h)$$

by Proposition 2.3.2. For such values of the cost:

$$u_i(g^{w})-u_i(g^{n.e.w}) > n_w[\phi(n-1)-\phi(n_w-1)]+(n-n_w)[\phi(n-1)-\phi(0)]-\sum_{h=0}^{n_w-1}[\phi(n_w-1)-\phi(h)]$$

If $n_w \leq \lfloor \frac{n}{2} \rfloor$, then:

$$u_i(g^w) - u_i(g^{n.e.w}) > n_w[\phi(n-1) - \phi(n_w-1)] + n_w[\phi(n-1) - \phi(0)] - \sum_{h=0}^{n_w-1}[\phi(n_w-1) - \phi(h)]$$

and the right side of the inequality is strictly positive, as $\phi(n-1) - \phi(0) > \phi(n_w - 1) - \phi(h)$, for any h such that $0 \leq h \leq n_w - 1$ and any n_w such that $3 \leq n_w \leq n - 1$.

Statements 2 and 4. Consider any non-exhaustive wheel on n_w agents. If it is strict Nash, then $3 \leq n_w \leq n - 1$. In a non-exhaustive wheel, those in the wheel component get a strictly lower payoff than any of the isolated singletons. The lowest payoff of an agent in the network equals:

$$n_w\phi(n_w - 1) + (n - n_w)\phi(0) - \overline{c}^{n_w} = (n - n_w)\phi(0) + \sum_{k=0}^{n_w-1} \phi(k)$$

For the value of $\overline{c}^{n_w}$, see Proposition 2.3.2. The expression above is increasing in n_w. Hence, the lowest payoff an agent can earn in a non-exhaustive wheel is when $n_w = 3$ and $c = \overline{c}^3$, which is:

$$(n - 2)\phi(0) + \phi(1) + \phi(2)$$

Recall that if a non-exhaustive wheel is strict Nash, so is the empty network. ($c^e < \underline{c}^3$, and recall that $\underline{c}^{n_w}$ is increasing in n_w.) Any agent in the empty network earns the payoff of $n\phi(0)$. The result follows. A similar argument holds for proving that the wheel network Pareto dominates the empty network for any $c \in (\underline{c}^e, \overline{c}^w)$.

Statement 3. Consider any strict Nash network that has the architecture of a non-exhaustive wheel on n_w agents. Consider any isolated singleton

i in this network. His current payoff is when $n_w = n - 1$, and the payoff differential in this case is:

$$n_w \phi(n_w - 1) + (n - n_w)\phi(0)$$

Recall that if any non-exhaustive wheel is strict Nash, then the wheel network is also strict Nash. The minimum payoff that i could get in a wheel network, given the values of the cost for which the non-exhaustive wheel is strict Nash, is:

$$n\phi(n-1) - \bar{c}^{n_w} = n\phi(n-1) - n_w\phi(n_w - 1) + \sum_{h=0}^{n_w-1} \phi(h)$$

The payoff differential for agent i is equal to:

$$\begin{aligned} & n_w\phi(n_w - 1) + (n - n_w)\phi(0) - (n\phi(n-1) - \bar{c}^{n_w}) \\ = {} & n_w[\phi(n_w - 1) - \phi(0)] + \sum_{h=0}^{n_w-1} [\phi(n_w - 1) - \phi(h)] - n[\phi(n-1) - \phi(0)] \end{aligned}$$

Note that this is an increasing function of n_w. Hence, the largest payoff gain that i can have in being in a non-exhaustive wheel instead of being in a wheel network is:

$$-n[\phi(n-1) - \phi(n-2)] + \sum_{h=1}^{n-2}[\phi(n-2) - \phi(h)]$$

For some non-exhaustive wheels not to be Pareto dominated by the wheel network, the above expression needs to be positive.

A.6 Proof of Proposition 2.5.1

The proof is by contradiction. The strategy profile $g = (g_1, \ldots, g_n)$ is a maximum of the potential, however there exist two components in g, C and D, such that $|C|$ and $|D|$ are both strictly larger than 1. Note that either $C \mathcal{R} D$ or C and D are not comparable. In any case, there is at least one of these two components, that I designated as being D, that does not have access to the other: $d(i, j; g) = \infty$ for any $i \in D$ and $j \in C$. Consider any player $i \in D$ and any player $j \in C$. Let the network g' be given by the following strategy profile:

1. $g'_i = \varnothing$: in g', agent i has no link at all,
2. $g'_j = g_i \cup g_j$: in g', agent j forms the same links as in g, and forms the links that i has in g,
3. for all k such that $i \in g_k$, $g'_k = g_k \setminus \{i\} \cup \{j\}$: all of the agents who form a link towards i in g redirect their link towards j in g',
4. $g'_k = g_k$ for the rest of the agents.

In g', there are strictly less infinite distances than in g. Let me just focus on the distances between the agents in C and D. In g, if C and D are not comparable via $\mathcal{R}$, there are at least $2|C| \times |D|$ infinite distances, as none of the two components can access the other. In g', there are $2[|C| + |D| - 1]$ infinite distances: no agent in C have access to i, i does not have access to anyone in $C \cup D \setminus \{i\}$, and none of the agents in $D \setminus \{i\}$ have acess to i. Note that $2|C| \times |D| - 2[|C| + |D| - 1]$ is always positive when $|C|, |D| > 1$.

The rest of the distances between the agents in C and D are weakly shorter in g'. Hence $v(g') \geq v(g)$. If $C \mathcal{R} D$ in g, then there are $|D| \times |C|$ infinite distances, due to the agents in D who cannot access those in C. In g', there are $|C| + |D| - 1$ infinite distances, as i is isolated from the agents in C and D, Therefore the number of infinite distances is lower in g' than in g; and the rest of the distances are weakly shorter in g' than in g. Again, we have $v(g') \geq v(g)$. Note that the network g' are weakly less links than the network g, as $g'_j \leq g_j$ (the inequality holds strictly if $g_j \cap g_i \neq \varnothing$). Thus the potential is strictly higher in g' than in g. A contradiction.

A.7 Definition of flower networks

My definition of a flower network is inspired from that of Bala and Goyal (2000, [6]). The main difference with theirs is about the size of the petals.

Definition A.7.1 A *flower network* $g^f(n, x)$ on n agents and x links partitions the set N into a central individual, say agent n, and a collection $\mathcal{P} = \{\mathcal{P}_1, \ldots, \mathcal{P}_q\}$ where $\mathcal{P}_k \in \mathcal{P}$ is nonempty. A set $\mathcal{P}_k \in \mathcal{P}$ of agents is referred to as a petal. Let m be the number of petals, $l = |\mathcal{P}_k|$ the cardinality of petal $\mathcal{P}_k$ and denote the agents in $\mathcal{P}_k$ as $\{k_1, \ldots, k_l\}$. A flower network is then defined by setting $n \to k_1 \to k_2 \ldots \to k_l \to n$ for each petal $\mathcal{P}_k \in \mathcal{P}$, and no other agent than k_j has a link towards k_{j+1}, where (k_j, k_{j+1}) belong to the same petal $\mathcal{P}_k$, for any $0 \leq j \leq l$ given that $k_0 = n = k_{l+1}$. The number of petals is $m = x - (n-1)$ and the maximum difference in the petals cardinalities is 1. The cardinality of the smallest petal is $s = \lfloor \frac{n}{m} \rfloor$. There are $m(s+1) - x$ petals whose cardinality is $l = s$ and the rest of the

$x - ms$ petals have cardinality $l = s + 1$.

A.8 Proof of Proposition 2.5.3 (Code)

The software is R. The list of packages to download is:

```
library(combinat)
library(compiler)
library(igraph)
library(abind)
library(iterpc)
library(Matrix)
library(graph)
library(PairViz)
library(adagio)
library(iterpc)
library(data.table)
library(lattice)
library(generalCorr)
library(mltools)
library(dplyr)
```

It suffices to copy paste this code and to make it run on R.

```
# 1. n denotes the number of agents, and K the number of links
n = 5 #6
K = 5 #from 3 to 2(n-1)
# Other variables:
```

```
m=n-1
L= 0.5*n*m
H= L+1
P=n*m

# 2. Get all adjacency matrices that have K entries equal to 1.
func <- function(n, m) t(combn(P, K, function(a){z=integer(n);z[a]=1;z}))
Z <- func(P,K)
get_A <- function(n,i){ M <- matrix(NA, n,n)
diag(M) <- 0
M[lower.tri(M, diag=F) & is.na(M)] <- as.vector(Z[i,1:L])
M[upper.tri(M, diag=F) & is.na(M)] <- as.vector(Z[i, H:P])
return(M)}

# Compile.
g <- cmpfun(get_A)
g
A <- list()
for(i in 1:nrow(Z)){
        A[[i]] <- graph.adjacency(g(n,i))}

#4. For each adjacency matrix, get the distance matrix.
# Then, get the number of distances having length L,
# for L between 0 and n-1. Add an entry for infinite distances.
dist <- function(n,i) {G <- shortest.paths(A[[i]], mode="out")
        freq <- as.vector(as.data.table(table(G)))
        return(freq)}
# Compile:
```

```
dt <- cmpfun(dist)
dt
M <- list()
R <- list()
for(i in 1:nrow(Z)){ R[[i]]<- t(dt(n,i))
if(R[[i]][1,length(R[[i]][1,])]=="Inf")
  { M[[i]] = matrix(data= c(R[[i]][2, 1:length(R[[i]][2,])-1 ] ,
  rep(0,n+1-length(R[[i]][2,]) ),R[[i]][2,length(R[[i]][2,])] ),
  nrow=1, ncol=n+1)}
  else
{ M[[i]] = matrix(data=c(R[[i]][2,], rep(0, n+1-length(R[[i]][2,]))),
  nrow=1 , ncol=n+1)}
  colnames(M[[i]]) <- c(1:n-1, "Inf")
  rownames(M[[i]]) <- c("Number of distances of length y")
 M[[i]] <- mapply(M[[i]][1,], FUN=as.numeric) }

#6. In order to cut the computing time, the remaining code is run
# on the networks that have different distance distribution.
# Get the distance distributions.
Diff_dt <- unique(M)
CDF <- list()
Cumulative_Distrib <- function (i){ C =Diff_dt[[i]][1]
     for(k in 1:n+1) C[k] = sum(C[k-1]+Diff_dt[[i]][k])
     return(C)}
# Compile
 CD <- cmpfun(Cumulative_Distrib)
 CD
for(i in 1:length(Diff_dt)){CDF[[i]] <- CD (i) }
```

```
Matrix_CDF <- t(matrix(data=unlist(CDF), nrow=n+1, ncol=length(CDF)))
colnames(Matrix_CDF) <- c(1:n-1, "Inf")
rownames(Matrix_CDF) <-  c(1:length(CDF))
MCDF <- Matrix_CDF

#7. I first compute the distance distribution of the flower.
#As n=K=5, this is (5,10,15,20,25,25).
#Then I get the difference between each row of MCDF,
#and the distance distribution of the flower.
#All results are gathered in the matrix "Compare".

X=rep(c(5, 10 , 15 , 20 , 25,25), nrow(MCDF))
Benchmark <-  matrix(data=c(X), byrow=T, nrow=nrow(Matrix_CDF))
colnames(Benchmark) <- c(1:n-1, "Inf")
rownames(Benchmark) <-  c(1:length(CDF))
Compare <- MCDF - Benchmark

#8. Now we can see which networks are candidates for equilibrium.
# Keep the networks that are not dominated by the wheel.
select <- Compare[!apply(Compare, 1, function(x) all(0 >= x) &
sum(x) <0 ),]
select
#For n=K=5, "select" has length 1.

# 9. Find back the adjacency matrices of the equilibrium candidates.
r1 <- which(apply(Compare, 1, function(x) all.equal(x, select)) == "TRUE")
r2 <- which(sapply(M,
  function(x)identical(x,Diff_distrib[[return1[[1]]]])))
```

I plot the networks that are not isomorphic to each others.

Appendix B

Appendix Chapter 3

B.1 Indifference curve of country 1

<u>*Claim:*</u> *The indifference curve of country 1 in expression* (14) *is increasing convex in* p_1.

Proof. Let us take the first derivative of country 1's indifference curve wrt p_1. This is:

$$\frac{dr}{dp_1} = \frac{\alpha[1-\gamma p_1 + (\bar{v}+\alpha)][(\bar{v}+\alpha)(1-\gamma)+(1-\gamma p_1)] + \gamma(\bar{v}+\alpha)[1+\alpha(1-\gamma p_1)]}{\alpha[1-\gamma p_1 + (\bar{v}+\alpha)]^2}$$

The sign of the derivative is given by the sign of the numerator. As long as $p_1 \leq \frac{1}{\gamma}$, i.e. the quantity demanded is strictly positive, all terms in the expression above are positive. We get the second derivative wrt p_1. We find:

$$\frac{d^2r}{dp_1^2} = \frac{2\gamma^2(\bar{v}+\alpha)[1-\alpha(\bar{v}+\alpha)]}{\alpha[1-\gamma p_1 + (\bar{v}+\alpha)]^3}$$

The sign is that of the numerator. It is positive if country 1's payoff $\bar{v}$ does not exceed $\frac{1}{\alpha} - \alpha$. We show that this is always true for any contract that satisfies $p_1 - r \geq 0$. The expression of country 1's payoff is:

$$v^*(p_1, r) = \frac{\alpha(1 - \gamma p_1)}{1 + \alpha(p_1 - r) - \alpha\gamma p_1}[1 - (p_1 - r)] - \alpha.$$

The contract where $p_1 = r = 0$ maximizes country 1's welfare: $v^*(0, 0) = 0$. For any payoff level $\bar{v}$ that country 1 can achieve by contracting with the firm, we have that: $\bar{v} \leq v^*(0, 0) \leq \frac{1}{\alpha} - \alpha$. The result follows. $\square$

B.2 Proof of Proposition 3.3.1

First, we specify the largest profit the firm could earn by trading with country 1. By Lemma 3.3.1, $p_1 \leq p_2^{\mathcal{M}}$ in any PO-IR contract. Given this condition on the list price, the firm's profit is given by expression (11). Note that $\frac{d\pi^*}{dy} \geq 0$ for all values of $y \in [0, 1]$. Therefore, the highest profit the firm could get is when $y = 1$; hence $p_1 \geq 1$ as we do not allow for negative rebates. Consider the firm's total profit if the latter trades with both countries, fixing $p_1 - r = 1$. The expression of the firm's profit for $y = 1$ is:

$$\pi^*(p_1; y = 1) = \frac{\alpha(1 - \gamma p_1)}{1 + \alpha - \alpha\gamma p_1} + \theta\pi_2(p_1).$$

The first derivative wrt p_1 is:

$$\frac{\partial \pi^*(p_1)}{\partial p_1} = -\frac{\alpha\gamma}{(1 + \alpha - \alpha\gamma p_1)^2} + \theta\frac{\partial \pi_2(p_1)}{\partial p_1}.$$

The first term is negative, while the right term is weakly positive because $p_1 \leq p_2^{\mathcal{M}}$. Re-arranging this right term by setting $\pi_2(p_1) = p_1 q_2(p_1)$ for $q_2(p_1)$ the demand function on the second market, one gets:

$$\frac{\partial \pi^*(p_1; y=1)}{\partial p_1} = -\frac{\alpha\gamma}{(1+\alpha-\alpha\gamma p_1)^2} + \theta \left[(1 - |\epsilon_2(p_1)|) q_2(p_1) \right].$$

Let us call $\bar{p}$ the price that equalizes the first derivative to zero. If the sign of the above derivative is strictly positive for $p_1 = \frac{1}{\gamma}$ (thus $\bar{p} > \frac{1}{\gamma}$), then it means that the firm would do actually better by not selling anything on the first market.

B.3 Proof of Proposition 3.3.2

Claim: If $A = (p_1^A, r^A)$ and $B = (p_1^B, r^B)$ where $p_1^A < p_1^B \leq p_2^{\mathcal{M}}$ are two PO-IR contracts, then it must be that $r^A \leq r^B$.

Proof. The proof is by contradiction. Assume that A and B are two Pareto optimal contracts. WLOG, we set $p_1^B > p_1^A$. By contradiction, we consider that $r^A > r^B$. First, note that $y^A = p_1^A - r^A$, the net price associated with contract A, is strictly lower than $y^B = p_1^B - r^B$, the net price associated with contract B. It is immediate that country 1 prefers A over B. Hence it must be that the firm's profit is strictly larger under the terms of contract B, otherwise A would Pareto dominate B. Other things being equal, the firm's profit in expression (11) is increasing in the list price and decreasing in the rebate. Therefore, $\bar{\pi}_B = \pi(p_1^B, r^B) > \bar{\pi}^A = \pi(p_1^A, r^A)$. Also, the quantity traded is larger under contract A than under contract B (i.e. $q_1(p_1^A, r^A) > q_1(p_1^B, r^B)$), as:

$$q_1(p_1, r) = \frac{\alpha(1 - \gamma p_1)}{1 + \alpha y - \alpha\gamma p_1}$$

is both decreasing in the list price and the net price, holding the rest of the variables constant. Let $C = (p_1^B, r^C)$ be the contract such that the list price is the same as in contract B, and the rebate r^C is such that the firm is indifferent between contracts C and A (i.e. $\pi(p_1^B, r^C) = \bar{\pi}^A$). In other words,

$$r^C = p_1^B - \frac{(\bar{\pi}^A - \theta\pi_2(p_1^B))(1 - \alpha\gamma p_1^B)}{\alpha[1 - \gamma p_1^B - (\bar{\pi}^A - \theta\pi_2(p_1^B))]}$$

where the function corresponds to the expression of the firm's isoprofit in (15) for the level of profit $\bar{\pi}^A$. Let y^C be the net price associated with contract C, i.e. $y^C = p_1^B - r^C$. If contract A is Pareto optimal, then it must be that the country strictly prefers A over C, as the firm is indifferent between both. This is equivalent to saying that:

$$(1 - y^A)q_1(p_1^A, r^A) > (1 - y^C)q_1(p_1^B, r^C) \tag{31}$$

By Lemma 3.3.3, A is Pareto optimal if $p_1^A \geq \tilde{p}(\bar{\pi}^A)$. Since $p_1^B > p_1^A$, this means that the quantity traded is lower with contract C than with contract A (i.e. $q_1(p_1^B, r^C) \leq q_1(p_1^A, r^A)$). Also, as $\bar{\pi}^A < \bar{\pi}^B$, we have by Lemma 3.3.2 that $\hat{p}_1(\bar{\pi}^A) \geq \hat{p}_1(\bar{\pi}^B)$. Now if B is Pareto optimal, then it must be that $p_1^B \leq \hat{p}_1(\bar{\pi}^B)$ by Lemma 3.3.2. Hence we get that: $p_1^A < p_1^B \leq \hat{p}_1(\bar{\pi}^B) \leq \hat{p}_1(\bar{\pi}^A)$. Thus that $y^C < y^A$.

Next, consider contract $D = (p_1^A, r^D)$ such that the list price is the same as in contract A, and the firm is indifferent between contracts B and D. ($\pi(p_1^A, r^D) = \pi(p_1^B, r^B) = \bar{\pi}^B$.) It follows that r^D is given by:

$$r^D = p_1^A - \frac{(\bar{\pi}^B - \theta\pi_2(p_1^A))(1 - \alpha\gamma p_1^A)}{\alpha[1 - \gamma p_1^A - (\bar{\pi}^B - \theta\pi_2(p_1^A))]}$$

If B is Pareto optimal, then the country must strictly prefer B over D as the firm is indifferent between both:

$$(1 - y^B)q_1(p_1^B, r^B) > (1 - y^D)q_1(p_1^A, r^D) \tag{32}$$

where $y^B = p_1^B - r^B$ and $y^D = p_1^A - r^D$. Note that the quantity traded is greater under the terms of contract D than under those of contract B (i.e. $q_1(p_1^A, r^D) \geq q_1(p_1^B, r^B)$). To see why, recall from Lemma 3.3.3 that if B is Pareto optimal, then $p_1^B \geq \tilde{p}_1(\bar{\pi}^B)$; and that if A is also Pareto optimal, then $p_1^A \geq \tilde{p}_1(\bar{\pi}^A)$. Also by Lemma 3.3.3, we know that $\tilde{p}_1(\bar{\pi}^A) \geq \tilde{p}_1(\bar{\pi}^B)$, as $\tilde{p}_1(.)$ is a decreasing function of the firm's profit level and $\bar{\pi}^A < \bar{\pi}^B$. By transitivity, we have: $\tilde{p}_1(\bar{\pi}^B) \leq \tilde{p}_1(\bar{\pi}^A) \leq p_1^A < p_1^B$. Note that the net price associated with contract D is greater than that associated with contract B. Since $p_1^A < p_1^B$ and $p_1^B \leq \hat{p}_1(\bar{\pi}^B)$ if B is PO-IR, it follows that $y^D > y^B$.

We go back to the inequalities in (31) and (32). We use the expression of the firm's isoprofit curve in (15) to express all net prices y^A, y^B, y^C and y^D, as well as all quantities $q_1(p_1^A, r^A), q_1(p_1^B, r^B), q_1(p_1^B, r^C)$ and $q_1(p_1^A, r^D)$. First,

$$y(p_1, \bar{\pi}) = \frac{(\bar{\pi} - \theta\pi_2(p_1))(1 - \alpha\gamma p_1)}{\alpha[1 - \gamma p_1 - (\bar{\pi} - \theta\pi_2(p_1))]}.$$

Hence:

$$
\begin{aligned}
y^A &= \frac{(\bar{\pi}^A - \theta\pi_2(p_1^A))(1-\alpha\gamma p_1^A)}{\alpha[1-\gamma p_1^A - (\bar{\pi}^A - \theta\pi_2(p_1^A))]} \\
y^B &= \frac{(\bar{\pi}^B - \theta\pi_2(p_1^B))(1-\alpha\gamma p_1^B)}{\alpha[1-\gamma p_1^B - (\bar{\pi}^B - \theta\pi_2(p_1^B))]} \\
y^C &= \frac{(\bar{\pi}^A - \theta\pi_2(p_1^B))(1-\alpha\gamma p_1^B)}{\alpha[1-\gamma p_1^B - (\bar{\pi}^A - \theta\pi_2(p_1^B))]} \\
y^D &= \frac{(\bar{\pi}^B - \theta\pi_2(p_1^A))(1-\alpha\gamma p_1^A)}{\alpha[1-\gamma p_1^A - (\bar{\pi}^B - \theta\pi_2(p_1^A))]}
\end{aligned}
$$

Now, in the expression of the traded quantity in (8), we replace the rebate r by the expression in (15). For some level of profit $\bar{\pi}$, this gives:

$$
q_1(p_1, \bar{\pi}) = \frac{\alpha[1-\gamma p_1 - (\bar{\pi} - \theta\pi_2(p_1))]}{1-\alpha\gamma p_1} \tag{33}
$$

Compare this expression with that of the net price above. Note that:

$$
q_1(p_1, \bar{\pi}) = \frac{1}{y(p_1, \bar{\pi})}[\bar{\pi} - \theta\pi_2(p_1)].
$$

Henceforth:

$$
\begin{aligned}
q_1(p_1^A, r^A) &= \frac{1}{y^A}[\bar{\pi}^A - \theta\pi_2(p_1^A)] \\
q_1(p_1^B, r^B) &= \frac{1}{y^B}[\bar{\pi}^B - \theta\pi_2(p_1^B)] \\
q_1(p_1^B, r^C) &= \frac{1}{y^C}[\bar{\pi}^A - \theta\pi_2(p_1^B)] \\
q_1(p_1^A, r^D) &= \frac{1}{y^D}[\bar{\pi}^B - \theta\pi_2(p_1^A)]
\end{aligned}
$$

If country 1 strictly prefers A over C, then the inequality in (31) can be re-written as follows:

$$q_1(p_1^A, r^A) - q_1(p_1^B, r^C) > \theta[\pi_2(p_1^B) - \pi_2(p_1^A)] \geq 0 \tag{34}$$

where the last inequality holds as the profit on the second market is strictly increasing in the list price in country 1, this because both p_1^A and p_1^B are less than $p_2^{\mathcal{M}}$ if A and B are PO-IR (by Lemma 3.3.1). We re-write the expression in (32) along the same lines: if country 1 strictly prefers B over D, then:

$$0 \leq q_1(p_1^A, r^D) - q_1(p_1^B, r^B) < \theta[\pi_2(p_1^B) - \pi_2(p_1^A)] \tag{35}$$

where the first inequality is here to remind the reader that the quantity that would be traded under the terms of contract D is larger than the quantity that is traded with contract B. Gathering the inequalities in (34) and (35), we get that if country 1 prefers A over C and prefers B over D then:

$$q_1(p_1^A, r^A) - q_1(p_1^A, r^D) > q_1(p_1^B, r^C) - q_1(p_1^B, r^B) \geq 0 \tag{36}$$

(The last inequality to zero is always verified as for the same list price p_1^B, the rebate is larger in contract C than in contract B, and we know that holding all other variables constant, the quantity traded as expressed in (8) is increasing in the value of the rebate.)

Now, let us re-formulate each side of the inequality in (36), by expressing the quantity traded as in (33). For the expression on the left side of the inequality in (36), this is:

$$q_1(p_1^A, r^A) - q_1(p_1^A, r^D) = \alpha \frac{(\bar{\pi}^B - \bar{\pi}^A)}{1 - \alpha\gamma p_1^A}.$$

and the denominator is strictly positive, as $\bar{\pi}^B > \bar{\pi}^A$ and $q_1(p_1^A, r^A) > q_1(p_1^A, r^D)$ by (35). As for the right side, this is:

$$q_1(p_1^B, r^C) - q_1(p_1^B, r^B) = \alpha \frac{(\bar{\pi}^B - \bar{\pi}^A)}{1 - \alpha\gamma p_1^B}.$$

and the denominator is strictly positive, as $\bar{\pi}^B > \bar{\pi}^A$ and $q_1(p_1^B, r^C) > q_1(p_1^B, r^B)$ by (35). Since we set that $p_1^B > p_1^A$ and that $1 - \alpha\gamma p_1^A > 1 - \alpha\gamma p_1^B > 0$, we get: $\frac{1}{1-\alpha\gamma p_1^B} > \frac{1}{1-\alpha\gamma p_1^A}$, thus $q_1(p_1^B, r^C) - q_1(p_1^B, r^B) > q_1(p_1^A, r^A) - q_1(p_1^A, r^D)$. Which contradicts the (36). Therefore, if country 1 prefers A over C, then it must prefer D over B. Note that contract D, in comparison with contract B, has a larger net price however the quantity traded is greater. □

B.4 Proof of Proposition 3.3.3

For country 1, we find:

$$\frac{\partial v^*/p_1}{\partial v^*/r} = -1 - \frac{\gamma(1 - p_1 + r)[1 - \alpha + \alpha(p_1 - r)]}{(1 - \gamma p_1)[1 + \alpha(1 - \gamma p_1)]}$$

The ratio is negative valued: the country trades larger list prices against higher rebates. For the firm, we obtain:

$$\frac{\partial\pi^*/p_1}{\partial\pi^*/r} = -1 - \frac{\dfrac{\theta}{\alpha}\dfrac{\partial\pi_2(p_1)}{\partial p_1}[1+\alpha(p_1-r)-\alpha\gamma p_1]^2 - \gamma(p_1-r)[1-\alpha+\alpha(p_1-r)]}{(1-\gamma p_1)(1-\alpha\gamma p_1)}$$

A contract (p_1, r) is PO-IR iff:

$$\frac{\partial v^*/p_1}{\partial v^*/r} = \frac{\partial\pi^*/p_1}{\partial\pi^*/r}$$

equivalent to:

$$\frac{\gamma(1-p_1+r)[1-\alpha+\alpha(p_1-r)]}{(1+\alpha-\alpha\gamma p_1)}$$
$$= \frac{\dfrac{\theta}{\alpha}\dfrac{\partial\pi_2(p_1)}{\partial p_1}[1+\alpha(p_1-r)-\alpha\gamma p_1]^2 - \gamma(p_1-r)[1-\alpha+\alpha(p_1-r)]}{(1-\alpha\gamma p_1)}$$

The first ratio is positive valued, as $p_1 \leq \frac{1}{\gamma}$ and $1 \geq p_1 - r \geq 0$. The denominator of the second ratio is positive. It follows that its numerator must be positive:

$$\theta\frac{\partial\pi_2(p_1)}{\partial p_1} \geq \frac{\alpha\gamma[1-\alpha+\alpha(p_1-r)]}{[1+\alpha(p_1-r)-\alpha\gamma p_1]^2}$$

Expressing r as a function of p_1 and the rest of the variables:

$$r = \frac{\alpha\gamma[1-\alpha+\alpha p_1] - \theta\dfrac{\partial\pi_2(p_1)}{\partial p_1}(1+\alpha-\alpha\gamma p_1)[1+\alpha(1-\gamma)p_1]}{\alpha\Big(\alpha\gamma - \theta\dfrac{\partial\pi_2(p_1)}{\partial p_1}(1+\alpha-\alpha\gamma p_1)\Big)}$$

Next, we express the quantity traded as a function of the rebate given in expression (17). First, we re-arrange the expression of the quantity in (8) as:

$$q_1(p_1, r) = \frac{1 - \gamma p_1}{\frac{1}{\alpha} - r + (1-\gamma)p_1}$$

Then, in the above expression, we replace r by (17). This gives :

$$q_1(p_1) = 1 - \theta \frac{\partial \pi_2(p_1)}{\partial p_1} \left[\frac{1}{\alpha\gamma} \left(\frac{1}{\gamma} - p_1 \right) \right]$$

Given that $p_1 \leq \min\{\frac{1}{\gamma}, p_2^{\mathcal{M}}\}$ in a PO-IR contract, it follows that $q_1(p_1) \leq 1$ if p_1 is PO-IR. Last, we show that $q_1(p_1)$ is increasing in p_1. For this, let us take the first derivative of $q_1(p_1)$ wrt p_1. This is:

$$\frac{\partial q_1(p_1)}{\partial p_1} = -\theta \frac{\partial^2 \pi_2(p_1)}{\partial p_1^2} \left[\frac{1}{\alpha\gamma} + \left(\frac{1}{\gamma} - p_1 \right) \right] + \theta \frac{\partial \pi_2(p_1)}{\partial p_1} \geq 0$$

The first term is positive by our assumption that π_2 is concave in the price and $p_1 \leq \frac{1}{\gamma}$ by country 1's participation constraint. The second term is also positive since $p_1 \leq p_2^{\mathcal{M}}$ if p_1 is PO-IR, by Lemma 3.3.1. To sum up, $q_1(p_1)$ is increasing in p_1, and it is positive for any p_1 that is PO-IR. We now make explicit the equation of the optimal private contribution for a PO-IR contract. In (9), we replace the rebate by its expression in (17). We obtain:

$$\tau^*(p_1) = \frac{\theta \frac{\partial \pi_2(p_1)}{\partial p_1} (1 + \alpha - \alpha\gamma p_1) - \alpha\gamma (1 - \alpha + \alpha\gamma p_1)}{\alpha^2 \gamma}$$

To prove our claim that $\tau^*(p_1)$ is decreasing in p_1, we take its first derivative wrt p_1. This is:

$$\frac{\partial \tau^*(p_1)}{\partial p_1} = \frac{1}{\alpha^2\gamma}\left(\theta \frac{\partial^2 \pi_2(p_1)}{\partial p_1^2}(1+\alpha-\alpha\gamma p_1) - \alpha\gamma\theta\frac{\partial \pi_2(p_1)}{\partial p_1} - \alpha^2\gamma^2\right) < 0$$

with all three terms within brackets that are negative (recall the firm's profit on the second market is assumed to be concave in the price, and that $p_1 \leq \frac{1}{\gamma}$ by country 1's participation constraint on the list price).

B.5 Proof of Theorem 3.3.2

The Pareto frontier is of equation:

$$r = p_1(1-\gamma) + \frac{1}{\alpha} - \frac{\alpha\gamma(1-\gamma p_1)}{\alpha\gamma - \theta\frac{\partial \pi_2(p_1)}{\partial p_1}(1+\alpha-\alpha\gamma p_1)}$$

From the expression of country 1's demand in (8):

$$(1-\gamma)p_1 - r + \frac{1}{\alpha} = \frac{1-\gamma p_1}{q_1} \geq 0$$

since $p_1 \leq \frac{1}{\gamma}$ and since the quantity traded is positive. It follows that:

$$-\frac{\alpha\gamma(1-\gamma p_1)}{\alpha\gamma - \theta\frac{\partial \pi_2(p_1)}{\partial p_1}(1+\alpha-\alpha\gamma p_1)} \geq 0$$

Since $p_1 \leq p_2^{\mathcal{M}}$ if p_1 is PO-IR, and since $1+\alpha-\alpha\gamma p_1 \geq 0$, it follows that the denominator is increasing in p_1 and positive. The numerator is decreasing in p_1 and positive. Thus, the ratio is decreasing in p_1 and positive. As a consequence, a PO-IR rebate is increasing in p_1.

We still need to prove that the net price is a decreasing function of the list price p_1. If a contract (p_1, r) is PO-IR, then the net price is expressed as:

$$p_1 - r = \gamma p_1 - \frac{1}{\alpha} + \frac{\alpha\gamma(1-\gamma p_1)}{\alpha\gamma - \theta\frac{\partial \pi_2(p_1)}{\partial p_1}(1+\alpha-\alpha\gamma p_1)}$$

The remainder shows that the derivative of

$$\frac{\alpha(1-\gamma p_1)}{\alpha\gamma - \theta\frac{\partial \pi_2(p_1)}{\partial p_1}(1+\alpha-\alpha\gamma p_1)}$$

is smaller than -1. The following statements are equivalent:

$$\frac{\alpha}{\left[\alpha\gamma - \theta\frac{\partial \pi_2(p_1)}{\partial p_1}(1+\alpha-\alpha\gamma p_1)\right]^2}\Bigg(-\gamma\left[\alpha\gamma - \theta\frac{\partial \pi_2(p_1)}{\partial p_1}(1+\alpha-\alpha\gamma p_1)\right]$$
$$-\left[-\theta\frac{\partial^2 \pi_2(p_1)}{\partial p_1^2}(1+\alpha-\alpha\gamma p_1) + \alpha\gamma\theta\frac{\partial \pi_2(p_1)}{\partial p_1}\right](1-\gamma p_1)\Bigg) < -1$$

Assuming that the relation is true, one can re-arrange it as:

$$\alpha\gamma\left[\alpha\gamma - \theta\frac{\partial \pi_2(p_1)}{\partial p_1}(1+\alpha-\alpha\gamma p_1)\right] - \left[\alpha\gamma - \theta\frac{\partial \pi_2(p_1)}{\partial p_1}(1+\alpha-\alpha\gamma p_1)\right]^2$$
$$> -\alpha(1-\gamma p_1)\left[-\theta\frac{\partial^2 \pi_2(p_1)}{\partial p_1^2}(1+\alpha-\alpha\gamma p_1) + \alpha\gamma\theta\frac{\partial \pi_2(p_1)}{\partial p_1}\right]$$

or equivalently as:

$$\theta \frac{\partial \pi_2(p_1)}{\partial p_1} \left[\alpha\gamma - \theta \frac{\partial \pi_2(p_1)}{\partial p_1}(1+\alpha-\alpha\gamma p_1) \right]$$
$$> -\alpha(1-\gamma p_1) \left[-\theta \frac{\partial^2 \pi_2(p_1)}{\partial p_1^2}(1+\alpha-\alpha\gamma p_1) + \alpha\gamma\theta \frac{\partial \pi_2(p_1)}{\partial p_1} \right]$$

On the right side of the inequality, the term within squared brackets is strictly positive, by our assumption that the firm's profit on the second market is concave in the price (and this price is p_1 as $p_1 \leq p_2^{\mathcal{M}}$ in any PO-IR trade between country 1 and the firm, by Lemma 3.3.1). Therefore the right side is negative. The left side is positive since the term between squared brackets is positive. Thus the inequality is always verified.

Appendix C

Appendix Chapter 4

C.1 Proof of Proposition 4.2.1

The country's problem is:

$$\max_{p \in P^*} \mathbb{E}(V_C) = G\left(\frac{\tau^2 - p\tau^1}{1-p}\right)(1-p)(L-\tau^1)$$

The FOC is given by:

$$\frac{\partial \mathbb{E}(V_C)}{\partial p} = (L-\tau^1)\left\{\left(\frac{\tau^2-\tau^1}{1-p}\right)G'\left(\frac{\tau^2-p\tau^1}{1-p}\right) - G\left(\frac{\tau^2-p\tau^1}{1-p}\right)\right\}$$

The SOC is:

$$\frac{\partial^2 \mathbb{E}(V_C)}{\partial p^2} = (L-\tau^1)\frac{(\tau^2-\tau^1)^2}{(1-p)^3}G''\left(\frac{\tau^2-p\tau^1}{1-p}\right) < 0$$

by Assumption 1 and $p \in P^*$. Thus the country's expected payoff is concave in p, and has a unique global maximum on P^*. The global maximum is interior iff:

$$G'(1)(1-\tau^1)-1<0$$

which is guaranteed by Assumption 2. Let us call p^* the solution of the country's problem; p^* solves:

$$[T(p^*)-\tau^1]G'(T(p^*))=G(T(p^*)) \quad \Leftrightarrow \quad e_S(p^*)=1$$

where $T(p^*)=t^*$. Note that $e_S(p)$ crosses 1 only once at p^*. Also, notice that at $t=t^*$, the price-elasticity expressed as a function of t is decreasing:

$$G''(t)G(t)[t-\tau^1]-(G(t))^2[t-\tau^1]+G(t)G'(t)\Big|_{t=t^*}=G''(t^*)G(t^*)[t^*-\tau^1]<0 \quad \forall\tau^1$$

by Assumption 1.

C.2 Proof of Proposition 4.3.1

We start with the case where country C pays p^* for the drug. Country D's problem is written as:

$$\max_{p\in P_A} \mathbb{E}(V_D)=\frac{G\left(\frac{\tau_D^2-p\tau_D^1}{1-p}\right)}{G(t^*)}(1-p)\varepsilon(L-\tau_D^1)$$

The FOC is:

$$\frac{\partial\mathbb{E}(V_D)}{\partial p}=\frac{\varepsilon(L-\tau_D^1)}{G(t^*)}\left[\left(\frac{\tau_D^2-\tau_D^1}{1-p}\right)G'\left(\frac{\tau_D^2-p\tau_D^1}{1-p}\right)-G\left(\frac{\tau_D^2-p\tau_D^1}{1-p}\right)\right]$$

and the SOC is:

$$\frac{\partial^2 \mathbb{E}(V_D)}{\partial p^2} = \frac{\varepsilon(L-\tau_D^1)}{G(t^*)}\left[\frac{(\tau_D^2-\tau_D^1)^2}{(1-p)^3}G''\left(\frac{\tau_D^2-p\tau_D^1}{1-p}\right)\right] < 0$$

by Assumption 1 and $p \in P_A$. Thus the country's expected payoff is concave in p, and has a unique global maximum on P_A. The global maximum is interior iff:

$$G'(1)(1-\tau_D^1) - 1 < G'(1)(1-\tau_C^1) - 1 < 0$$

which is always true by the fact that $\tau_D^1 > \tau_C^1$ and Assumption 2. Let us call p_a the solution of the country's problem; p_a solves:

$$[T_D(p_a) - \tau_D^1]G'(T_D(p_a)) = G(T_D(p_a)) \quad \Leftrightarrow \quad e_{S_a}(p_a) = 1$$

where $T_D(p_a) = t_a$. Note that $e_{S_a}(p)$ crosses 1 only once at p_a, and that $t_a < t^*$.

Assume instead that country C pays the price of 1 for the drug. Country D's problem is written as:

$$\max_{p \in P_R} \mathbb{E}(V_D) = \left(\frac{G\left(\frac{\tau_D^2-p\tau_D^1}{1-p}\right) - G(t^*)}{1-G(t^*)}\right)(1-p)\varepsilon(L-\tau_D^1)$$

The FOC is given by the expression:

$$\frac{\partial \mathbb{E}(V_D)}{\partial p} = \frac{\varepsilon(L-\tau_D^1)}{1-G(t^*)}\left[G'(T_D(p))(T_D(p)-\tau_D^1) - [G(T_D(p)) - G(t^*)]\right]$$

The SOC is:

$$\frac{\partial^2 \mathbb{E}(V_D)}{\partial p^2} = \frac{\varepsilon(L-\tau_D^1)}{1-G(t^*)}\left[\frac{(\tau_D^2-\tau_D^1)^2}{(1-p)^3}G''\left(\frac{\tau_D^2-p\tau_D^1}{1-p}\right)\right] < 0$$

for the same reasons as those evoked previously. Country D's optimal offer is interior to P_R iff:

$$G'(1)(1-\tau_D^1) < 1 - G(t^*)$$

By Assumption 1, a sufficient condition for which country D's optimal offer is interior is:

$$G'(1)(1-\tau_D^1) - [1 - G(t^*)] < G'(1)(1-\tau_C^1)$$

In this case, note that both p_r and $t_r = T_D(p_r)$ are increasing function of t^*:

$$\frac{dt_r}{dt^*} = \frac{G'(t^*)}{-G''(t_r)(t_r - \tau_D^1)} > 0$$

by Assumption 1. Note also that $p_a < p_r$. To see why:

$$\left.\frac{\partial \mathbb{E}(V_D)}{\partial p}\right|_{p=p_a} = \frac{G(t^*)}{1 - G(t^*)} > 0$$

and $\frac{\partial \mathbb{E}(V_D)}{\partial p}$ is a decreasing function of p everywhere on P_R. This result is intuitive: the offer p_a is accepted by types strictly lower than t^*. If the firm rejected p^*, then its type is strictly larger than t^*. In this instance, the offer p_a is strictly dominated by any offer $p \in \left[\frac{1-\tau_D^2}{1-\tau_D^1}, 1\right)$. The offer p_a is accepted with probability zero, which gives a null payoff to country D. The offer p is accepted with probability one, and the country's payoff is strictly larger than zero.

C.3 Beliefs that satisfy the intuitive criterion

One may wonder the conditions under which an all-accept pooling equilibrium that satisfies the intuitive criterion exists. We propose to tackle this question here. The set of *reasonable types* which would gain from rejecting p all verify that:

$$\Delta(t) = \left[(1-p) + \varepsilon\left(\frac{1-\tau_D^2}{1-\tau_D^1} - \frac{\underline{t}_D - \tau_D^2}{\underline{t}_D - \tau_D^1}\right)\right] t - (\tau_C^2 - p\tau_C^1) - \varepsilon\tau_D^1\left(\frac{1-\tau_D^2}{1-\tau_D^1} - \frac{\underline{t}_D - \tau_D^2}{\underline{t}_D - \tau_D^1}\right) \geq 0$$

Since the expression is increasing in t, the set of reasonable types is of the form $[\hat{t}, 1]$, with $\hat{t} \in [0, 1]$. We distinguish between two cases, depending on whether type $\underline{t}_D$ is in the set of reasonable types or not.

If $\underline{t}_D$ is reasonable, then an all-accept pooling equilibrium can simply be sustained by the belief that the firm's type is $\underline{t}_D$ with probability 1. In this case, country D offers the same price regardless whether p was accepted or rejected. And accepting p is indeed optimal for all types.

For the case where $\underline{t}_D$ is unreasonable, we show that the belief for which D assigns probability 1 on the firm's type being 1 following a rejection of p sustains an all-accept pooling equilibrium. (Note that type 1 is always reasonable). Consider such a profile. To show that no type gains from deviating, it suffices to prove that Δ is non-positive at $t = \underline{t}_D$ and $t = 1$. This is immediately verified for $t = \underline{t}_D$, as we consider here that type $\underline{t}_D$ is unreasonable. For the firm of type 1, note that:

$$\Delta(1) = (1-p) - (\tau_C^2 - p\tau_C^1) \leq 0$$

for all $p \geq \frac{1-\tau_C^2}{1-\tau_C^1}$. As a result, no type has any incentive to deviate. Hence we can claim the following:

Proposition C.3.1. *For all*

$$p \geq \frac{1-\tau_C^2}{1-\tau_C^1},$$

there exists an all-accept pooling equilibrium that satisfies the intuitive criterion.

C.4 Non existence of PBE in pure strategies when condition B does not hold

Proposition C.4.1. *Suppose that condition B in Lemma 4.4.4 does not hold. Then, there are subgames p such that $\underline{t}_D \leq T_C(p) \leq t_D^{\circ}(p)$ and $p < \frac{1-\tau_C^2}{1-\tau_C^1}$. Such subgames do not admit an equilibrium in pure strategies; therefore, the game as a whole does not admit any PBE in pure strategies.*

Proof. Let p_R and p_A be the (deterministic) prices offered by D following rejection of p or acceptance of p, respectively. Let t_R and t_A be the types indifferent between accepting or rejecting p_R and p_A, respectively. First, note that a pooling profile where all types reject p is not an equilibrium in this case. This profile would be an equilibrium only if $t_R = \underline{t}_D$. However the payoff gain from rejecting in this instance is:

$$\Delta(t) = (1-p)\,t - \left[\tau_C^2 - p\tau_C^1\right],$$

and it is negative for all $t < T_C(p)$, as $p < \frac{1-\tau_C^2}{1-\tau_C^1}$. Similarly, there is no pooling equilibrium where all types accept, because the payoff gain from rejecting is strictly positive at $t = 1$.

We now look for interior equilibria, in pure strategies. Suppose first that $p_R = p_A$. Then the continuation value of accepting or rejecting p to the firm is the same. Thus the myopic behavior regarding acceptance or rejection of price p is optimal. Therefore the firm accepts p if $t \leq T_C(p)$ and rejects otherwise. But this induces optimal prices p_A and p_R such that $p_A < p_R$, contradicting our initial assumption. Therefore, this case is never produced in an interior equilibrium. Next, suppose that $p_A < p_R$. For all $t \in [t_A, t_R]$, the payoff gain from rejecting is given by:

$$\begin{aligned}\Delta(t) &= (t-\tau_C^2) + \varepsilon p_R(t-\tau_D^1) - p(t-\tau_C^1) - \varepsilon(t-\tau_D^2) \\ &= \frac{1-p}{t_R-\tau_D^1}\left([t_R - t_D^\circ(p)]t - T_C(p)(t_R-\tau_D^1) + t_R[t_D^\circ(p)-\tau_D^1]\right)\end{aligned}$$

Suppose first that $t_R > t_D^\circ(p)$. For any $t > t_R$, we have:

$$\Delta(t) = (1-p)\,t - \left[\tau_C^2 - p\tau_C^1\right].$$

But then all types $t \geq t_A$ prefer to reject p, as:

$$[t_R - t_D^\circ(p)]t > 0 \geq t_R[T_C(p) - t_D^\circ(p)] + \tau_D^1[t_R - T_C(p)]$$

where the last inequality holds as $T_C(p) < t_D^\circ(p)$ and $t_R > T_C(p)$ under our current assumption. Yet, type t_A must have accepted p, otherwise it would be best for D to offer a strictly lower price. A contradiction follows; hence $p_A < p_R$ and $t_R > t_D^\circ(p)$ is never part of an interior equilibrium.

We continue to assume that $p_A < p_R$, however suppose now that $t_R \in [T_C(p), t_D^{\circ}(p)]$. Note that:

$$\Delta(t) < 0 \quad \Leftrightarrow \quad [t_R - t_D^{\circ}(p)]t - t_R[T_C(p) - t_D^{\circ}(p)] + \tau_D^1[T_C(p) - t_R] < 0$$

thus $\Delta(t) < 0$, as each of three terms are negative. Still for the case where $p_A < p_R$, suppose last that $t_R < T_C(p)$. Then:

$$\Delta(t_R) = (1-p)\,t_R - \left[\tau_C^2 - p\tau_C^1\right] = (1-p)[t_R - T_C(p)] < 0$$

which is a contradiction. Next, suppose that $p_A > p_R$. Then all types $t > t_A$ reject both p_A and p_R. The payoff gain from rejecting p to any firm of type $t > t_A$ is:

$$\Delta(t) = (1-p)\,[t - T_C(p)],$$

which is strictly positive for $t > T_C(p)$. Since it must be that $\Delta(t_A) \leq 0$, therefore $t_A \leq T_C(p)$ must hold. For all $t \in [t_A, T_C(p))$, we still have:

$$\Delta(t) = (1-p)\,[t - T_C(p)],$$

which is strictly increasing in t. And for all $t \in [t_R, t_A]$, the payoff gain from rejecting is written as:

$$\begin{aligned}\Delta(t) &= (t-\tau_C^2) + \varepsilon(t-\tau_D^2) - p(t-\tau_C^1) - \varepsilon p_A(t-\tau_D^1) \\ &= \frac{[((1-p)(t_A-\tau_D^1) + \varepsilon(\tau_D^2-\tau_D^1))\,t - (\tau_C^2 - p\tau_C^1)(t_A-\tau_D^1) - \varepsilon t_A(\tau_D^2-\tau_D^1)]}{t_R - \tau_D^1}\end{aligned}$$

which is also strictly increasing in t. Note that:

$$\Delta(t_A) = \frac{(1-p)(t_A - \tau_D^1)}{(t_R - \tau_D^1)}[t_A - T_C(p)] \leq 0$$

if the condition $t_A \leq T_C(p)$ for which all types larger than t_A prefer to accept p holds. But then $\Delta(t) < 0$, which is impossible. So there cannot be such an equilibrium. In conclusion, there is no equilibrium in pure strategies in such subgames. This also implies that if condition B does not hold, the game as a whole does not admit any PBE in pure strategies. □

C.5 Proof of Remark 4.4.1

If $t_f > \underline{t}_D$, then $p_A < \frac{t_f - \tau_D^2}{t_f - \tau_D^1}$, and D's offer does not depend on t_f. Also, $t_A < t_f$. The expected equilibrium payoff of country D is:

$$\mathbb{E}(V_D(t_f)) = \varepsilon(L - \tau_D^1)\left\{G(t_f)\left[\frac{G(t_A)}{G(t_f)}(1-p_A)\right] + f(t_f)\right\},$$

where:

$$f(t_f) = [G(t_R(t_f)) - G(t_f)](1 - P_R(t_f))$$

with $p_R = P_R(t_f)$ D's best response. Taking the first derivative of $\mathbb{E}(V_D)$ wrt t_f:

$$\frac{\partial \mathbb{E}(V_D)}{\partial t_f} = \varepsilon(L - \tau_D^1)\left\{\frac{\partial f}{\partial t_f} + \frac{\partial f}{\partial p_R}\frac{\partial P_R(t_f)}{\partial t_f}\right\}$$

If $p_R < \frac{1-\tau_C^2}{1-\tau_C^1}$, then by the Envelop theorem $\frac{\partial f}{\partial p_R} = 0$. If $p_R = \frac{1-\tau_C^2}{1-\tau_C^1}$, then p_R is a corner solution of D's problem hence $\frac{\partial P_R(t_f)}{\partial t_f} = 0$. Therefore:

$$\frac{\partial \mathbb{E}(V_D)}{\partial t_f} = -\varepsilon(L - \tau_D^1)G'(t_f)(1 - p_R) < 0$$

We conclude that D's expected equilibrium payoff is decreasing in t_f for all $t_f \in [\underline{t}_D, 1]$. Let us consider now $t_f \in [0, \underline{t}_D]$. Country D offers $p_A = \frac{t_f - \tau_D^2}{t_f - \tau_D^1}$ (this is a corner solution) following the acceptance of C's offer, hence it always holds that:

$$G'(t_f)(t_f - \tau_D^1) - G(t_f) > 0 \quad \forall t_f \in [0, \underline{t}_D]$$

For these values of t_f, country D's expected equilibrium payoff is written as:

$$\begin{aligned}
\mathbb{E}(V_D(t_f)) &= \varepsilon(L - \tau_D^1)\left\{G(t_f)\left(1 - \frac{t_f - \tau_D^2}{t_f - \tau_D^1}\right) + f(t_f)\right\} \\
&= \varepsilon(L - \tau_D^1)\left\{G(t_f)\left(\frac{\tau_D^2 - \tau_D^1}{t_f - \tau_D^1}\right) + f(t_f)\right\}
\end{aligned}$$

Taking the first derivative wrt t_f, we get:

$$\begin{aligned}
\frac{\partial \mathbb{E}(V_D)}{\partial t_f} &= \varepsilon(L - \tau_D^1)\left\{G'(t_f)(1 - p_A) - G(t_f)\frac{(\tau_D^2 - \tau_D^1)}{(t_f - \tau_D^1)^2} - G'(t_f)(1 - p_R)\right\} \\
&= \varepsilon(L - \tau_D^1)\left\{G'(t_f)(1 - p_A) - G(t_f)\left(\frac{1 - p_A}{t_f - \tau_D^1}\right) - G'(t_f)(1 - p_R)\right\}
\end{aligned}$$

On the second line, $G'(t_f)(1 - p_A) - G(t_f)\frac{1-p_A}{t_f - \tau_D^1} > 0$ for any t_f in the appropriate interval.

C.6 Elasticity of the farsighted supply

The expression of the price-elasticity is:

$$e_{S_f} = \frac{S_f(p)'}{S_f(p)}(1-p)$$

with $S_f(p) = G(T_f(p))$ where $T^f(p)$ corresponds to the expression in (30). Therefore:

$$e_{S_f} = \frac{G'(T_f(p))}{G(T_f(p))}(1-p)\frac{\partial T_f(p)}{\partial p}$$

Recall that $T_f(p)$ is increasing in p. Note that:

$$\frac{\partial T_f(p)}{\partial p} = \frac{T_f(p) - \varepsilon\frac{dp_R}{dp}(T_f(p) - \tau_D^1)}{1 - p - \varepsilon(1-p_R)}$$

with $p_R = P_R(t_f)$, and both the numerator and denominator are positive valued by Lemma 4.4.4. The expression of $\frac{dp_R}{dp}$ is:

$$\frac{dp_R}{dp} = \frac{(t-\tau_C^1)(1-p_R)}{(t_R - \tau_D^1)(1-p-\varepsilon(1-p_R))}\frac{dt_R}{dt}$$

We want to express the elasticity as a function of the firm's type; for this, we parametrize the prices p, p_R by types, using the relation in (30) for expressing p as a function of t, and the relation in (25) for expressing p_R as a function of t_R. We get the following:

$$1 - p = \frac{\tau_C^2 - \tau_C^1}{t - \tau_C^1} - \varepsilon\frac{(t_R - t)(\tau_D^2 - \tau_D^1)}{(t_R - \tau_D^1)(t - \tau_C^1)}$$

$$1 - p_R = \frac{\tau_D^2 - \tau_D^1}{t_R - \tau_D^1}$$

and

$$1-p-\varepsilon(1-p_R)=\frac{\tau_C^2-\tau_C^1}{t-\tau_C^1}-\varepsilon\frac{(\tau_D^2-\tau_D^1)(t_R-\tau_C^1)}{(t-\tau_C^1)(t_R-\tau_D^1)}$$

Replacing, we find that $\frac{\partial T^f(p)}{\partial p}(1-p)$ can be written as a function of t and t_R:

$$\begin{aligned}&(t-\tau_C^1)\left(\frac{(\tau_C^2-\tau_C^1)(t_R-\tau_D^1)-\varepsilon(t_R-t)(\tau_D^2-\tau_D^1)}{(\tau_C^2-\tau_C^1)(t_R-\tau_D^1)-\varepsilon(t_R-\tau_C^1)(\tau_D^2-\tau_D^1)}\right)\\ &\times\left[1-\frac{\varepsilon(t-\tau_D^1)(t-\tau_C^1)(\tau_D^2-\tau_D^1)G'(t)}{-G''(t_R)(t_R-\tau_D^1)^2\left[(\tau_C^2-\tau_C^1)(t_R-\tau_D^1)-\varepsilon(t_R-\tau_C^1)(\tau_D^2-\tau_D^1)\right]}\right]\end{aligned}$$

This expression is positive for any value of t and t_R, as $\frac{\partial T^f(p)}{\partial p}(1-p)$ is positive for any value of p. Recall that we set:

$$Z(t_R)=(\tau_C^2-\tau_C^1)(t_R-\tau_D^1)-\varepsilon(t_R-\tau_C^1)(\tau_D^2-\tau_D^1)$$

and $Z(t_R)$ is the denominator of each term in the large parenthesis. Also, recall that:

$$\frac{dt_R}{dt}=\frac{G'(t)}{-G''(t_R)(t_R-\tau_D^1)}>0$$

by Assumption 1. We now study $Z(t_R)$ in more details. Note that $\frac{Z(t_R)}{(t-\tau_C^1)(t_R-\tau_D^1)}$ is an alternative expression of the denominator of $\frac{\partial T^f(p)}{\partial p}$, where p parametrized by t via the relation in (30), and p_R via that in (25). Therefore, $Z(t_R)$ is strictly positive, for any value of t_R. Note that:

$$\frac{dZ(t_R)}{dp}=\left[(\tau_C^2-\tau_C^1)-\varepsilon(\tau_D^2-\tau_D^1)\right]\frac{dt_R}{dt}$$

the sign is this expression is that of the term between squared brackets, and it is positive when condition A holds.

C.7 Proof of Proposition 4.4.6

We express country C's problem as a function of the type t, by relating a price to its corresponding threshold type via the expression in (30). This gives:

$$
\begin{aligned}
\max_{t\in[t_A,t_R]} \mathbb{E}(V_C) &= G(t)\left(1-\left(\frac{t-\tau_C^2}{t-\tau_C^1}\right)-\varepsilon\frac{(t_R-t)(\tau_D^2-\tau_D^1)}{(t_R-\tau_D^1)(t-\tau_C^1)}\right)(L-\tau_C^1)\\
&= G(t)\left(\frac{\tau_C^2-\tau_C^1}{t-\tau_C^1}-\varepsilon\frac{(t_R-t)(\tau_D^2-\tau_D^1)}{(t_R-\tau_D^1)(t-\tau_C^1)}\right)(L-\tau_C^1)\\
&= (L-\tau_C^1)\left\{G(t)\left(\frac{\tau_C^2-\tau_C^1}{t-\tau_C^1}\right)-\varepsilon G(t)\frac{(t_R-t)(\tau_D^2-\tau_D^1)}{(t_R-\tau_D^1)(t-\tau_C^1)}\right\}
\end{aligned}
$$

The first derivative of the country's expected payoff wrt t is:

$$
\begin{aligned}
&\frac{\partial\mathbb{E}(V_C)}{\partial t} = \frac{(L-\tau_C^1)(\tau_C^2-\tau_C^1)}{(t-\tau_C^1)^2}\left[G'(t)(t-\tau_C^1)-G(t)\right]\\
&-\varepsilon\frac{(L-\tau_C^1)(\tau_D^2-\tau_D^1)}{(t-\tau_C^1)^2}\left\{G'(t)(t-\tau_C^1)\frac{(t_R-t)}{(t_D-\tau_D^1)}\right.\\
&\left.-G(t)\left[\frac{(t_R-\tau_D^1)(t_R-\tau_C^1)-\frac{dt_R}{dt}(t-\tau_C^1)(t-\tau_D^1)}{(t_R-\tau_D^1)^2}\right]\right\}
\end{aligned}
$$

We express the FOC as:

$$
\begin{aligned}
&\left[G'(t)(t-\tau_C^1)-G(t)\right]-\varepsilon\left(\frac{\tau_D^2-\tau_D^1}{\tau_C^2-\tau_C^1}\right)\left\{G'(t)(t-\tau_C^1)\frac{(t_R-t)}{(t_R-\tau_D^1)}\right.\\
&\left.-G(t)\left[\frac{(t_R-\tau_D^1)(t_R-\tau_C^1)-\frac{dt_R}{dt}(t-\tau_C^1)(t-\tau_D^1)}{(t_R-\tau_D^1)^2}\right]\right\}=0
\end{aligned}
$$

The SOC condition is:

$$
\begin{aligned}
&G''(t)(t-\tau_C^1)\left[1-\varepsilon\left(\frac{\tau_D^2-\tau_D^1}{\tau_C^2-\tau_C^1}\right)\left(\frac{t_R-t}{t_R-\tau_D^1}\right)\right] \\
&-\varepsilon\left(\frac{\tau_D^2-\tau_D^1}{(\tau_C^2-\tau_C^1)(t_R-\tau_D^1)^2}\right)\left\{2G'(t)(t-\tau_C^1)\left[\frac{dt_R}{dt}(t-\tau_D^1)-(t_R-\tau_D^1)\right]\right. \\
&\left.+G(t)\left[\frac{d^2t_R}{dt^2}(t-\tau_C^1)(t-\tau_D^1)-2\frac{dt_R}{dt}(t_R-t)+2\left(\frac{dt_R}{dt}\right)^2\frac{(t-\tau_C^1)(t-\tau_D^1)}{(t_R-\tau_D^1)}\right]\right\}
\end{aligned}
$$

The function on the first line is negative valued for all $t \in [0,1]$, as $G''(.) < 0$ by Assumption 1 and the term between squared brackets is positive (see Appendix C.6: $Z(t_R)$ is positive in equilibrium). We conclude that for a sufficiently small ε, the FOC is a decreasing function of t, i.e. the country's expected payoff is single-peaked in t. Let us re-arrange the FOC as:

$$
\left[G'(t)(t-\tau_C^1)-G(t)\right]=\frac{\varepsilon(\tau_D^2-\tau_D^1)(t-\tau_C^1)G(t)\left[\frac{dt_R}{dt}\left(\frac{t-\tau_D^1}{t_R-\tau_D^1}\right)-1\right]}{(\tau_C^2-\tau_C^1)(t_R-\tau_D^1)-\varepsilon(\tau_D^2-\tau_D^1)(t_R-t)}
$$

The denominator of the ratio is positive (see Appendix C.6 $Z(t_R)$ is positive). Thus the sign of the ratio is that of the expression between the squared brackets on the numerator. Note that for any $t \geq \bar{t}^f$, we have $t_R(t) = 1$ thus $\frac{dt_R}{dt} = 0$. Recall that t^* is the type of firm which is indifferent between accepting and rejecting country C's optimal offer p^* in the myopic case. t^* solves:

$$
G'(t^*)(t^*-\tau_C^1)-G(t^*)=0
$$

Recall that $G'(t)(t-\tau_C^1)-G(t)$ is a decreasing function of t. Let us refer to t_f^* as the threshold type that maximizes C's payoff in the farsighted case.

Note that:

$$t_f^* > t^* \text{ iff } \frac{dt_R}{dt_f}|_{t=t^*} < \left(\frac{t_r - \tau_D^1}{t^* - \tau_D^1}\right)$$

where t_r is the threshold type associated with D's best-response $P_R(t^*)$. We compare p^* and p_f^*, C's optimal offers in the myopic and farsighted cases, respectively. Recall that for small parameter values of ε, the elasticity $e_{S_f}(t)$ equals one for one value of t, let us call it t_1, and that $e_{S_f}(t)$ is decreasing in t for all $t \leq t_1$. Note that $p_f^* \geq p^*$ iff:

$$e_{S_f}(t)\Big|_{t=t^*} \geq 1$$

equivalent to:

$$\left(1 + \frac{\varepsilon(t^* - \tau_C^1)(\tau_D^2 - \tau_D^1)}{Z(t_r)}\right)\left(1 - \frac{\varepsilon(t^* - \tau_C^1)(\tau_D^2 - \tau_D^1)}{Z(t_r)}\left[\frac{dt_R}{dt}\Big|_{t=t^*}\frac{t^* - \tau_D^1}{t_r - \tau_D^1}\right]\right) \geq 1$$