

## Information and Abbreviations of DLSM Algorithms

	Group	Abbreviation	Algorithm name	Authors
1	RPCA	RPCA	Robust Principal Component Analysis	De la Torre and Black [1]
2	RPCA	PCP	Principal Component Pursuit	Candes et al. [2]
3	RPCA	FPCP	Fast PCP	Rodriguez and Wohlberg [3]
4	RPCA	R2PCP	Riemannian Robust Principal Component Pursuit	Hintermüller and Wu [4]
5	RPCA	AS-RPCA	Active Subspace: Towards Scalable Low-Rank Learning	Liu and Yan [5]
6	RPCA	ALM	Augmented Lagrange Multiplier	Tang and Nehorai [6]
7	RPCA	EALM	Exact ALM	Lin et al. [7]
8	RPCA	IALM	Inexact ALM	Lin et al. [7]
9	RPCA	IALM-LMSVDS	IALM with LMSVDS	Liu et al. [8]
10	RPCA	IALM-BLWS	IALM with BLWS	Lin and Wei [9]
11	RPCA	APG-PARTIAL	Partial Accelerated Proximal Gradient	Lin et al. [7]
12	RPCA	APG	Accelerated Proximal Gradient	Lin et al. [7]
13	RPCA	DUAL	Dual RPCA	Lin et al. [7]
14	RPCA	SVT	Singular Value Thresholding	Cai et al. [10]
15	RPCA	ADM	Alternating Direction Method	Yuan and Yang [11]
16	RPCA	LSADM	LSADM	Goldfarb et al. [12]
17	RPCA	L1F	L1 Filtering	Liu et al. [13]
18	RPCA	DECOLOR	Contiguous Outliers in the Low-Rank Representation	Zhou et al. [14]
19	RPCA	RegL1-ALM	Low-Rank Matrix Approximation under Robust L1-Norm	Zheng et al. [15]
20	RPCA	GA	Grassmann Average	Hauberg et al. [16]
21	RPCA	GM	Grassmann Median	Hauberg et al. [16]
22	RPCA	TGA	Trimmed Grassmann Average	Hauberg et al. [16]
23	RPCA	STOC-RPCA	Online Robust PCA via Stochastic Optimization	Feng et al. [17]
24	RPCA	MoG-RPCA	Mixture of Gaussians RPCA	Zhao et al. [18]
25	RPCA	noncvxRPCA	Robust PCA via Nonconvex Rank Approximation	Kang et al. [19]
26	RPCA	NSA1	Non-Smooth Augmented Lagrangian v1	Aybat et al. [20]
27	RPCA	NSA2	Non-Smooth Augmented Lagrangian v2	Aybat et al. [20]
28	RPCA	PSPG	Partially Smooth Proximal Gradient	Aybat et al. [21]
29	RPCA	flip-SPCP-sum-SPG	Flip-Flop version of Stable PCP-sum solved by Spectral Projected Gradient	Aravkin et al. [22]
30	RPCA	flip-SPCP-max-QN	Flip-Flop version of Stable PCP-max solved by Quasi-Newton	Aravkin et al. [22]
31	RPCA	Lag-SPCP-SPG	Lagrangian SPCP solved by Spectral Projected Gradient	Aravkin et al. [22]
32	RPCA	Lag-SPCP-QN	Lagrangian SPCP solved by Quasi-Newton	Aravkin et al. [22]
33	RPCA	FW-T	SPCP solved by Frank-Wolfe method	Mu et al. [23]
34	RPCA	BRPCA-MD	Bayesian Robust PCA with Markov Dependency	Ding et al. [24]
35	RPCA	BRPCA-MD-NSS	BRPCA-MD with Non-Stationary Noise	Ding et al. [24]
36	RPCA	VBRPCA	Variational Bayesian RPCA	Babacan et al. [25]
37	RPCA	PRMF	Probabilistic Robust Matrix Factorization	Wang et al. [26]
38	RPCA	OPRMF	Online PRMF	Wang et al. [26]
39	RPCA	MBRMF	Markov BRMF	Wang and Yeung [27]
40	RPCA	TFOCS-EC	TFOCS with equality constraints	Becker et al. [28]
41	RPCA	TFOCS-IC	TFOCS with inequality constraints	Becker et al. [28]
42	RPCA	GoDec	Go Decomposition	Zhou and Tao [29]
43	RPCA	SSGoDec	Semi-Soft GoDec	Zhou and Tao [29]
44	RPCA	GreGoDec	Greedy Semi-Soft GoDec Algorithm	Zhou and Tao [30]
45	ST	GRASTA	Grassmannian Robust Adaptive Subspace Tracking Algorithm	He et al. [31]
46	ST	GOSUS	Grassmannian Online Subspace Updates with Structured-sparsity	Xu et al. [32]
47	ST	pROST	Robust PCA and subspace tracking from incomplete observations using L0-surrogates	Hage and Kleinsteuber [33]
48	ST	ReProCS	Provable Dynamic Robust PCA or Robust Subspace Tracking	Narayanamurthy and Vaswani [34]
49	ST	MEDRoP	Memory Efficient Dynamic Robust PCA	Narayanamurthy and Vaswani [35]
50	MC	PG-RMC	Nearly Optimal Robust matrix Completion	Cherapanamjeri et al. [36]
51	MC	FPC	Fixed point and Bregman iterative methods for matrix rank minimization	Ma et al. [37]
52	MC	GROUSE	Grassmannian Rank-One Update Subspace Estimation	Balzano et al. [38]
53	MC	IALM-MC	Inexact ALM for Matrix Completion	Lin et al. [7]
54	MC	LMaFit	Low-Rank Matrix Fitting	Wen et al. [39]
55	MC	LRGeomCG	Low-rank matrix completion by Riemannian optimization	Bart Vandereycken, 2013 [40]
56	MC	MC-logdet	Top-N Recommender System via Matrix Completion	Kang et al. [41]
57	MC	MC-NMF	Nonnegative Matrix Completion	Xu et al. [42]
58	MC	OP-RPCA	Robust PCA via Outlier Pursuit	Xu et al. [43]
59	MC	OptSpace	Matrix Completion from Noisy Entries	Keshavan et al. [44]
60	MC	ORIMP	Orthogonal rank-one matrix pursuit for low rank matrix completion	Wang et al. [45]
61	MC	RPCA-GD	Robust PCA via Gradient Descent	Yi et al. [46]
62	MC	ScGrassMC	Scaled Gradients on Grassmann Manifolds for Matrix Completion	Ngo and Saad [47]
63	MC	SVP	Guaranteed Rank Minimization via Singular Value Projection	Meka et al. [48]
64	MC	SVT	A singular value thresholding algorithm for matrix completion	Cai et al. [49]
65	LRR	EALM	Exact ALM	Lin et al. [7]
66	LRR	IALM	Inexact ALM	Lin et al. [7]
67	LRR	ADM	Alternating Direction Method	Lin et al. [50]
68	LRR	LADMAP	Linearized ADM with Adaptive Penalty	Lin et al. [50]
69	LRR	FastLADMAP	Fast LADMAP	Lin et al. [50]
70	LRR	ROSL	Robust Orthonormal Subspace Learning	Shu et al. [51]
71	TTD	3WD	3-Way-Decomposition	Oreifej et al. [52]
72	TTD	MAMR	Motion-Assisted Matrix Restoration	Ye et al. [53]
73	TTD	RMAMR	Robust Motion-Assisted Matrix Restoration	Ye et al. [53]
74	TTD	ADMM	Alternating Direction Method of Multipliers	Parikh and Boyd [54]
75	NMF	NMF-MU	NMF solved by Multiplicative Updates	unknown
76	NMF	NMF-PG	NMF solved by Projected Gradient	unknown

77	NMF	NMF-ALS	NMF solved by Alternating Least Squares	unknown
78	NMF	NMF-ALS-OBS	NMF solved by Alternating Least Squares with Optimal Brain Surgeon	unknown
79	NMF	PNMF	Probabilistic Non-negative Matrix Factorization	unknown
80	NMF	ManhNMF	Manhattan NMF	Guan et al. [55]
81	NMF	NeNMF	NMF via Nesterov's Optimal Gradient Method	Guan et al. [55]
82	NMF	LNMF	Spatially Localized NMF	Li et al. [56]
83	NMF	ENMF	Exact NMF	Gillis and Glineur [57]
84	NMF	nmfLS2	Non-negative Matrix Factorization with sparse matrix	Ji and Eisenstein [58]
85	NMF	Semi-NMF	Semi Non-negative Matrix Factorization	unknown
86	NMF	Deep-Semi-NMF	Deep Semi Non-negative Matrix Factorization	Trigeorgis et al. [59]
87	NMF	iNMF	Incremental Subspace Learning via NMF	Bucak and Günsel [60]
88	NMF	DRMF	Direct Robust Matrix Factorization	Xiong et al. [61]
89	NTF	betaNTF	Simple beta-NTF implementation	Antoine Liutkus [62]
90	NTF	bcuNTD	Non-negative Tucker Decomposition by block-coordinate update	Xu and Yin [63]
91	NTF	bcuNCP	Non-negative CP Decomposition by block-coordinate update	Xu and Yin [63]
92	NTF	NTD-MU	Non-negative Tucker Decomposition solved by Multiplicative Updates	Zhou et al. [64]
93	NTF	NTD-APG	Non-negative Tucker Decomposition solved by Accelerated Proximal Gradient	Zhou et al. [64]
94	NTF	NTD-HALS	Non-negative Tucker Decomposition solved by Hierarchical ALS	Zhou et al. [64]
95	TD	HoSVD	Higher-order Singular Value Decomposition (Tucker Decomposition)	unknown
96	TD	HoRPCA-IALM	HoRPCA solved by IALM	Goldfarb and Qin [65]
97	TD	HoRPCA-S	HoRPCA with Singleton model solved by ADAL	Goldfarb and Qin [65]
98	TD	HoRPCA-S-NCX	HoRPCA with Singleton model solved by ADAL (non-convex)	Goldfarb and Qin [65]
99	TD	Tucker-ADAL	Tucker Decomposition solved by ADAL	Goldfarb and Qin [65]
100	TD	Tucker-ALS	Tucker Decomposition solved by ALS	unknown
101	TD	CP-ALS	PARAFAC/CP decomposition solved by ALS	unknown
102	TD	CP-APR	PARAFAC/CP decomposition solved by Alternating Poisson Regression	Chi et al. [66]
103	TD	CP2	PARAFAC2 decomposition solved by ALS	Bro et al. [67]
104	TD	RSTD	Rank Sparsity Tensor Decomposition	Yin Li [68]
105	TD	t-SVD	Tensor SVD in Fourier Domain	Zhang et al. 2013 [69]
106	TD	OSTD	Online Stochastic Tensor Decomposition	Sobral et al. [70]

#### Author details

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