Teaching Inequalities to Young Children Using Visual Representations: Perceptual Richness and Concreteness

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Abstract

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Perceptual Richness and Concreteness

Arielle Orsini

Pictures are often used in the mathematics classroom to depict quantities in problems. Their physical properties can differ by perceptual richness, the degree to which they are visually stimulating, and by concreteness, the degree to which they evoke prior knowledge of real-world contexts. In the present study, I examined the effects of perceptual richness and concreteness of visual representations on children's ability to learn about inequalities online. Twenty-eight five- to six-year-olds were randomly assigned to one of three visual representation conditions for all presented sets: bland abstract, rich concrete, and rich abstract. Participants number knowledge was tested at the start of the study. They then participated in an interactive lesson about inequalities and engaged in five tasks assessing different conceptual components of inequalities: three learning tasks and two transfer tasks. There was no effect of perceptual richness or concreteness on children's performance on any of the tasks. Children with greater number knowledge received higher scores on the transfer tasks. Qualitative analysis of participants reading of inequalities revealed three types of readers: Direction readers were able to correctly read the inequalities based on the direction of the inequality symbol. Magnitude readers ignored the direction of the inequality symbol but were able to correctly read the magnitude relationship of the inequality. Incorrect readers failed to learn how to read inequalities. The study's small sample did not afford the opportunity to investigate whether reader type differed by condition. The study's limitations and directions for future research are discussed.

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Chapter 1: Statement of the Problem

A common mathematics teaching method involves the use of visual representations where mathematical concepts are highlighted through graphics, diagrams, illustrations, and other images (Arcavi, 2003). For example, young children may practice magnitude comparison between sets by counting the quantity of apples to oranges (6:4) on a given worksheet whereby the apples and oranges are visually represented with drawings. The set of fruits are thus visual representations used to help students compare quantities; it is easier to see the difference between six apples and four oranges with pictures than with numerical symbols when children have not yet acquired knowledge of symbolic notation, such as numerals. Visual representations can thus be useful learning tools to promote students' mathematical comprehension (Múñez et al., 2013; Urban et al., 2017; Yung & Paas, 2015). Furthermore, visual representations come in such a wide variety that there are many ways researchers can classify and study them: They can be categorized them by their function (Elia & Philippou, 2004), their degree of seductive details (Harp & Mayer, 1997; Rey, 2012), and their use, such as the effects of multiple representations (Ainsworth et al., 2002) and dynamic representations (e.g., Ainsworth & VanLabeke, 2004), all of which can impact student learning.

One recent factor of interest is concreteness, a spectrum on which visual representations may systematically differ in their appearance. At one end of this spectrum exists concrete visual representations, sometimes called grounded representations, which are defined as visually stimulating images that aim to represent a particular real-world context (Belenky & Schalk, 2014; Kaminski et al., 2013). At the other end of the spectrum exists abstract visual representations, sometimes referred to as idealized or generic representations, which are simple images that are more conceptually abstract in nature and do not evoke an individual's existing knowledge of any one specific context (Belenky & Schalk, 2014; Kaminski et al., 2013). For example, a monochromatic picture of circle is a more abstract visual representation compared to an image or drawing of a pizza.

Multiple researchers have studied the effects of concreteness on children's abilities to learn and transfer their knowledge to novel mathematical situations and problems, but the results from one study to the next are inconsistent (see De Bock et al., 2011; compared to Siler & Willows, 2014). These mixed results may be in part due to an incomplete perspective on concreteness that confounds potential independent variables. Petersen and McNeil (2013) indeed found an interaction between perceptual richness (i.e., the degree of visual stimulation) and established knowledge (i.e., the degree to which a representation evokes an individual's prior knowledge about the phenomenon, object, or animal it aims to represent). This provides empirical evidence that perceptual richness and concreteness are two different variables and should be operationalized as such.

Most definitions of concreteness in the literature, however, do not distinguish between perceptual richness and the prior knowledge evoked by concreteness. Abstract visual representations tend be both perceptual bland in appearance and evoke little prior knowledge. Meanwhile, concrete visual representations tend be perceptually rich in appearance because they are trying to evoke the established knowledge an individual has of a particular real-world context. That said, abstract representations can indeed be perceptually rich and concrete representations can be perceptually bland. Thus, the current conception of the concreteness spectrum regarding visual representations confounds perceptual richness with prior knowledge.

The current study aims to address this issue by individually assessing the effects of perceptual richness and concreteness (i.e., the degree of prior knowledge that is evoked by visual representations) on children's ability to learn about inequalities and transfer their understanding to related mathematical problems. Mathematical inequalities present an unequal comparison of magnitudes where one quantity is greater than or less than a second quantity. Inequalities can be represented symbolically with numerals. When teaching inequalities, teachers may use manipulatives so that students can physically interact with the quantities, or they can use visual representations where images are used to represent the quantities on a chalkboard or digitally. This study will use visual representations to represent the sets of quantities, whereby the visual representations will vary in both perceptual richness and concreteness. Doing so will not only expand on the current literature, adding more nuance to definitions of concreteness, but may also provide practical advice for designing visual representations, be it for research or educational purposes. Specifically, this research might help provide recommendations for teachers to choose the most appropriate kinds of visual representations based on their learning goals.

Chapter 2: Literature Review

Pictures, illustrations, graphs, and diagrams are common visual representations used in mathematics education (Arcavi, 2003). Their function is to help elucidate the underlying abstract mathematical concepts and relationships for learners to increase their comprehension and problem-solving skills (Arcavi, 2003; Urban et al., 2017; Yung & Paas, 2015). In their didactic exploration of visual representations in early mathematics education, Urban and colleagues (2017) stated that visual representations are especially critical for promoting understanding of novel or unfamiliar concepts. Visual representations may also help reduce children's cognitive load compared to mathematics problems without external representations, thus making problems easier for them to solve (Yung & Paas, 2015). They can also help children solve problems more quickly and reduce errors (Múñez et al., 2013). Because of their common use and potential benefits to mathematics learning, it is critical to investigate the affordances visual representations may have.

Visual Representations: Pictures or No Pictures?

To date, most of the literature on visual representations has focused on the impact of illustrations on students' mathematical problem-solving (Clinton & Walkington, 2019; Cooper et al., 2018; Dewolf et al., 2014; Kulm et al., 1974; Yung & Paas, 2015). Illustrations are images that accompany word problems that could be solved without the pictures; thus, illustrations act as add-ons to the mathematics problems. Additionally, illustrations are often meant to be realistic and convey real-world phenomenon or objects, such as the distance between a helicopter and the Statue of Liberty or a cartoon dog chewing on a length of rope (see Clinton & Walkington, 2019; Cooper et al., 2018).

Illustrations are often added to word problems to either help students solve the problems or increase student interest in the problem, but studies show mixed results regarding their effectiveness. Compared to word problems without illustrations, studies found that illustrations helped students' problems solving (Yung & Paas, 2015), had no effect on problem-solving (Berends & van Lieshout, 2009; Clinton & Walkington, 2019; Dewolf et al., 2014; Klum et al., 1974), or even hindered problem-solving for students with lower cognitive skill (Klum et al., 1974). The moderating effect of students' mathematics skills on the impact of illustrations was also demonstrated in a trigonometry study, where illustrations helped students with higher mathematics abilities but hindered those with lower abilities (Cooper et al., 2018). Therefore, illustrations can differentially impact students' learning based on their level of prior knowledge.

Visual representations in mathematics are not always presented as illustrations, as is the case with picture problems. Unlike standard word problems, picture problems are arithmetic problems created with images; the pictures contain all (or most) of the mathematical information necessary for problem-solving, such as representing the relevant quantities with pictures (van Lieshout & Xenidou-Dervou, 2018). When comparing students' performance on word problems with no visual representations to picture problems, Hoogland et al. (2018) found that students scored better on picture problems. A study by van Lieshout and Xenidou-Dervou (2018) showed similar results when comparing picture problems to word problems delivered auditorially and combined picture-auditory problems; students' problem-solving was better with pictures than on standard word problems. In general, picture problems were more helpful than standard word problems, while word problems with or without illustrations differentially impacted students based on their prior mathematics knowledge.

A critical difference between illustrations added to word problems compared to picture problems are that picture problems deliver all the mathematical information through the same medium, namely pictures, while illustrations on word problems create mixed mediums: information is partially symbolic with written language and numerals, and partially illustrative, with images. In 2020, Elia assessed how mixing pictures and symbols in picture problems impacted students' performance. In Elia's research, there were three conditions: (a) picture-picture problems, where the quantities in the problem were represented by pictures only (e.g., six monkeys and two new monkeys at the zoo), (b) picture-symbol problems, where one quantity was represented with pictures and the other was represented with a numeral (e.g., 2 new lollipops being added to a box, and inside the box is a lollipop marked by the number 6 to indicate 6 lollipops in the box), and (c) symbol-symbol problems, where both quantitates were represented with numerals (e.g., a toy car marked with the number eight). The researcher found a main effect of picture type, where children had higher accuracy scores on picture-picture problems than on picture-symbol and symbol-symbol problems, thus further demonstrating that pictures can help students' performance on arithmetic problems.

Additionally, Elia (2020) found that students in the mixed picture-symbol condition performed worse compared to the other conditions depending on the type of arithmetic problem. These results demonstrate that mixed media for delivering information can negatively impact students' performance and may partially explain why illustrations added to word problems produce mixed results whereas pictures in pictorial problems are consistently more beneficial. In short, the effectiveness of visual representations on children's mathematics learning may differ based on whether word problems are presented as picture-picture problems, picture-symbol problems, or with added illustrations.

Affordances of Representations on Learning and Transfer

While learning has been defined as internalizing a schema in one context and being to recall and apply it under similar conditions, transfer is the ability to recall and apply the learned concept to novel situations (Salomon & Perkins, 1989). Transfer can further be subdivided into near-transfer and far-transfer, which refers to the distance from which the task is removed from the original learning conditions; near-transfer being closer and far-transfer being farther.

In representation research, transfer is typically assessed by modifying the representations used in the learning context. In near-transfer, the task would typically use a different representation than that used at learning to assess whether individuals have abstracted their understanding to other representations. For example, in one condition, Kaminski and colleagues (2013) used black shapes during the learning task, but used clouds that changed in size and color during the transfer task. In far-transfer, the task would typically eliminate the use of concrete or pictorial representations to assess whether individuals have abstracted the underlying concept to more generalized representations. In their study, for example, Siler and Willows (2014) included the same representations as Kaminski and colleagues during their learning conditions, but at transfer they eliminated the visual representations and used numbers instead.

The properties of visual representations can additionally carry different affordances for both learning and the transfer of conceptual understanding to novel problems (Kaminski & Sloutsky, 2013; Menendez et al., 2020). Perceptual richness refers to how visually interesting a representation is and exists on a theoretical continuum from bland (i.e., boring and unalluring) to rich (i.e., attractive and stimulating; Petersen & McNeil, 2013). Some studies have assessed the effects of visual representations' perceptual richness—typically framed as the presence or absence of extraneous information—on participants' learning and transfer (Kaminski & Sloutsky, 2013; Menendez et al., 2020). In these studies, there was typically no difference in participants' ability to learn based on whether they were shown perceptually rich or bland visual representations. However, on transfer tasks, participants shown bland visual representations outperformed those shown perceptually rich visual representations.

Other studies have assessed the effects of visual representations' the degree of concreteness on learning and transfer (Belenky & Schalk, 2014; De Bock et al., 2011; Kaminski et al., 2008, 2013; Siler & Willows, 2014). According to Kaminski et al. (2013), concreteness can refer to either perceptual concreteness or conceptual concreteness. Perceptual concreteness is defined in much the same way as perceptual richness (i.e., the visual details or lack thereof), but with the added notion that perceptual concreteness triggers a viewer's prior knowledge about the context to which the representation refers. Conceptual concreteness is defined as the degree to which a concept is grounded in reality and more easily perceived through the senses. Kaminski et al. gave the example of cats versus the concept of infinity, whereby cats are more easily perceived and thus concrete, and infinity is more abstract. Kaminski and colleagues' research has predominately focused on perceptual concreteness. Regardless of the type of concreteness being assessed, it exists on a spectrum from concrete to abstract or generic, whereby concrete representations provide information that elicits an individual's knowledge of the context and abstract representations do not activate such information.

Other authors have developed similar understandings of visual representations. Belenky and Schalk (2014) refer to this spectrum as grounded versus idealized. The authors defined grounded representations as concrete representations that trigger a person's knowledge of specific instantiations of real-world objects, and idealized representations as abstract representations that activate little to no prior knowledge. Urban and colleagues (2017) similarly distinguished between object models and schematic models, whereby objects models are representations of real-world objects in mathematics problems and schematic models replace real-world objects with abstract shapes.

Under the concreteness spectrum, research has shown that there was either no difference in learning when using abstract or concrete visual representations (Kaminski et al., 2008, 2013) or that concrete representations were better than abstract ones for learning (Goldstone & Son, 2005). On transfer tasks, participants performed better when they had learned with abstract representations compared to concrete representations (Goldstone & Son, 2005; Kaminski et al., 2008, 2013). Belenky and Schalk (2014) insisted that concrete representations were more effective for learning tasks because the concrete features could help students activate pertinent internal representations, so long as the representations contained few irrelevant details (i.e., low perceptual richness). They also argued that abstract representations were more effective for transfer because their lack of concrete features made it easier for students to abstract the underlying mathematical concepts in the problem, which are more easily transferred to novel contexts. Similarly, Urban and colleagues (2017) stated that object models (i.e., concrete representations) were suitable for learning and that schematic models (i.e., abstract representations) promoted the abstraction of mathematical concepts, as is the goal of transfer.

In an extension of Kaminski et al.'s (2008) study, De Bock and colleagues (2011) supported Belenky and Schalk's (2014) and Urban and colleagues' (2017) arguments in their research. They also argued that the Kaminski and colleagues' transfer task more closely matched their abstract representation condition than their concrete representation condition, which may be why they found an advantage of abstract over concrete. Thus, De Bock and colleagues (2011) varied whether participants were shown abstract or concrete visual representations during the learning and transfer tasks, creating four conditions: (a) AA, abstract representations at learning and transfer, (b) AC, abstract representations at learning and concrete at transfer, (c) CA, concrete during learning and abstract at transfer, and (d) CC, concrete during learning and transfer. They found a partial advantage of concrete representations on the learning task, where participants in the concrete learning groups outperformed those in the AC group, but not the AA group. Regarding transfer, they found that CC participants scored higher than the AC participants, and the AA participants scored higher than the CA participants. Thus, the researchers concluded that participants performed better on transfer tasks when their visual representations matched at learning and transfer.

Siler and Willows (2014) extended Kaminski et al. (2008), De Bock et al. (2011), and others' research by assessing the effects of concreteness when the transfer task did not use visual representations. They changed the transfer task to be number-based rather than picture-based to better assess whether students abstracted the arithmetic concepts from the lesson. They argued that the use of picture-based representations at transfer in prior studies failed to appropriately capture whether students were truly understanding the underlying mathematics concepts, which transfer tasks are meant to assess.

Siler and Willows created three conditions: (a) abstract condition, where the visual representations were basic black shapes, (b) abstract-relevant condition, where the representations were rectangles with shaded parts in thirds, and (c) concrete-relevant condition, where the representations were measuring cups with shaded parts in thirds. In the relevant conditions, the shaded proportions were associated with the underlying fraction arithmetic concept to be learned. They found that there was no effect of visual representation on participants' learning, similar to Kaminski and colleagues (2008, 2013). Regarding the near-transfer task, however, they reported that participants in the concrete-relevant condition outperformed those in the abstract-relevant condition, who outperformed those in the abstract condition. On the far-transfer task, the concrete-relevant condition was better than both abstract conditions. Thus, Siler and Willows' study (2014) indicated that concrete representations might be better for transfer than generic ones. They posited that these results occurred because the transfer task from prior studies, such as that used in Kaminski et al. (2008), made the features of the concrete representations irrelevant, whereas the design of Siler and Willows' (2014) study made the same features relevant.

Similar findings were demonstrated in research involving concrete objects as mathematical representations (also known as manipulatives). Carbonneau and Marley (2015) for example, assessed learning by asking preschoolers to compare quantities using manipulatives, whereby the sets in the problem were either represented with green chips (bland manipulatives that were more abstract) or frog counters (rich manipulatives that were more concrete). Concerning the learning task, the students were more accurate when shown bland manipulatives compared to rich manipulatives. On the transfer task, the opposite effect was found: Students who used rich manipulatives performed better than those who used bland manipulatives. The meta-analysis by Carbonneau and colleagues (2013) came to a similar conclusion: Perceptually bland (and therefore abstract) manipulatives were better for learning and perceptually rich (and therefore concrete) manipulatives were better for transfer.

Interestingly, although the manipulatives research supports Siler and Willows (2014) in terms of the effects of perceptual richness on children's transfer, manipulatives research reports different effects on learning. Visual representation research either indicated no effect (Kaminski & Sloutsky, 2013; Kaminski et al., 2008, 2013; Menendez et al., 2020; Siler & Willows, 2014), or that concrete representations were superior to abstract representations for learning (Belenky & Schalk, 2014; De Bock et al., 2011; Goldstone & Son, 2005). Some manipulatives research, on the other hand, reported an effect of abstract (i.e., bland manipulatives) over concrete (i.e., rich) manipulatives on learning (Carbonneau & Marley, 2015; Carbonneau et al., 2013).

Perceptual Richness vs. Prior Knowledge of Representations

A potential reason for the aforementioned mixed results may be because definitions of concreteness, and the way it is typically operationalized, confound perceptual richness and prior knowledge. Representations that tend to be more concrete also tend to incorporate more extraneous information—that is, they tend to be more perceptually rich. Representations that are more abstract tend to be more perceptually bland, displaying little to no extraneous information. This is not always necessarily the case, however. Concrete representations, those that evoke children's prior knowledge of specific contexts and real-world entities, can be more or less perceptually rich. Take as an example the bug life-cycle pictures Menendez et al. (2020) used to teach students about the biological concept of metamorphosis. The pictures of bugs were either black-and-white outlines of real-life bug species or colored drawings of real-life bug species. Both were drawings of the same bug life cycle; as such, both were concrete because students typically have prior knowledge of bugs. The black-and-white representations, however, were perceptually bland, whereas the colored representations were perceptually rich. Thus, abstract visual representations can similarly vary in their degree of perceptual richness.

Kaminski and Sloutsky's study (2013) presented a perfect example of confounding perceptual richness and prior knowledge. In their study, participants in one condition were presented graphs composed of monochromatic bars—that is, the bars were perpetually bland and abstract (i.e., unlikely to trigger participants' prior knowledge of any specific contexts). Participants in the other condition were shown bars containing pictures of shoes or flowers, and as such, the bars were perceptually rich and concrete (i.e., likely to trigger participants' prior knowledge of shoes and flowers). While the authors found that the monochromatic bars were more effective than the bars with pictures, it is impossible to decipher whether students were hindered by the degree of extraneous information in the representations (i.e., the degree of perceptual richness), or by the potential activation of students' prior knowledge about certain contexts, or both.

Petersen and McNeil (2013) attempted to delineate the potential confound between perceptual richness and prior knowledge with concrete representations. In their first study, the researchers selected eight different objects from a teaching supply store: Two were classified as having rich physical features and high prior knowledge (e.g., colorful, plastic animal toys), two were classified as having rich features but low prior knowledge (e.g., colorful, sparkly pom-poms), another two were classified as having bland features and high prior knowledge (e.g., uniformly-colored wooden pencils), and the last two were classified as having bland features and low prior knowledge (e.g., uniformly-colored wooden pegs). The participants were then given the manipulatives to perform counting tasks and a knowledge by perceptual richness interaction was found. With low prior knowledge objects, children performed better with rich manipulatives than bland ones. With high prior knowledge objects, however, there was no difference in children's performance between using rich or bland objects. As such, rich manipulatives were tentatively better than bland manipulatives, but only when children had low prior knowledge of the objects.

The authors performed a second study, creating their own manipulatives to have greater control over prior knowledge and physical richness. The rich manipulatives were shiny red and green shapes, and the bland manipulatives were dull shades of grey. Children in the high prior knowledge condition were introduced to the manipulative a week before the counting tasks, whereas the children in the low prior knowledge condition never saw them before the counting tasks. With greater control, the researchers found that there was no difference in children's performance with bland or rich manipulatives when they had low prior knowledge of the objects. When the children had high prior knowledge, the bland manipulatives were better than rich manipulatives. Thus, bland manipulatives may have been better than rich manipulatives only when children had high prior knowledge of the objects. Petersen and McNeil (2013) thus provided evidence that prior knowledge and perceptual richness are two separate variables.

In sum, the literature on the affordances of representations indicates different effects between learning and transfer. For learning, manipulatives demonstrated an effect of bland over rich (Carbonneau & Marley, 2015; Carbonneau et al., 2013; McNeil et al., 2009), and this is perhaps solely for high prior knowledge manipulatives, according to Petersen and McNeil's (2013) more controlled second study. Meanwhile, visual representations research either shows no effect on learning (Kaminski & Sloutsky, 2013; Kaminski et al., 2008, 2013; Menendez et al., 2020; Siler & Willows, 2014), or that concrete representations are better for learning than abstract representations (Belenky & Schalk, 2014; De Bock et al., 2011; Goldstone & Son, 2005). Interestingly, Petersen and McNeil's first study similarly found that rich manipulatives may be more beneficial than bland manipulatives, but only when children had low prior knowledge of the objects.

For transfer, some studies suggested that abstract or bland visual representations were better than concrete or rich visual representations (Belenky & Schalk, 2014; Goldstone & Son, 2005; Kaminski & Sloutsky, 2013; Kaminski et al., 2008, 2013; Menendez et al., 2020), while others reported that concrete was better than abstract, or rich was better than bland (Carbonneau & Marley, 2015; Carbonneau et al., 2013; Siler & Willows, 2014). However, aside from Petersen and McNeil (2013), these studies either confounded or did not account for the differences between perceptual richness and prior knowledge, a gap in the literature that will be addressed in the present study.

Present Study

Theoretically, visual representations can vary in their degree of perceptual richness and the degree to which they activate children's prior knowledge. In the case of visual representations, perceptual richness more narrowly refers to the visual properties of the image, including the colors, visual textures (e.g., shading), dimensions (3D vs. 2D), and degree of detail they contain. Prior knowledge of visual representations is then defined as the knowledge a representation triggers in the individual about real-world phenomena, concepts, and personal experiences. The literature on visual representations views concreteness as a function of established knowledge, where concrete (or grounded) representations activate rich networks of prior knowledge, while abstract (idealized or generic) representations activate little to no prior knowledge.

As previously mentioned, research on representations (whether they were manipulatives or visual representations) demonstrated effects on learning and transfer based on their degree of concreteness. Several conceptualizations of concreteness in previous research, however, confounded perceptual richness and prior knowledge; concrete representations were typically also perceptually rich and abstract representations were usually bland. Thus, the question arises whether perceptual richness and prior knowledge evoked by the concreteness of visual representations differentially impact children's mathematics comprehension and performance.

This study extricates perceptual richness from its operationalization of concreteness. As such, concreteness is defined solely by the degree to which the representations are likely to evoke children's prior knowledge. Perceptual richness is defined as the visual properties of an image, including color and other details. The present study aims to investigate the impact of perceptual richness and concreteness associated with visual representations on children's learning and understanding of inequalities. I have adapted and expanded Carbonneau and Marley's (2015) study, which demonstrated the effects of perceptual richness on learning and transfer. I modified their study to assess the affordances of visual representations rather than manipulatives, and expanded it to assess the effects of concreteness and perceptual richness as two separate factors. Carbonneau and Marley (2015) also assessed the impact of guidance level and found that children performed better with high guidance compared to low guidance across both learning and transfer tasks. Additionally, there was no interaction between guidance and perceptual richness, except on the transfer task: Participants with high guidance performed equally well regardless of manipulative type, but with low guidance there was an advantage for rich manipulatives over bland ones. The high guidance condition better mimics authentic classroom teaching and was thus used in the present study for the lesson on inequalities.

Participants were randomly assigned to one of three visual representations conditions: (a) bland abstract representations, (b) rich abstract representations, and (c) rich concrete representations. The perceptually bland representations were uniform in color while the perceptually rich representations were multi-colored. The abstract representations were shapes because, according to Kaminski and colleagues (2013), shapes would not evoke much specific knowledge of any one physical, real-world context. The concrete representations were designed to activate children's prior knowledge of a specific, real-world animal (i.e., frogs). I also avoided mixing representation types between learning and transfer by ensuring that participants assigned to a condition were shown the same representations at learning and at transfer when applicable

(De Bock et al., 2011).

The children's number knowledge was assessed at the start of the study to obtain a baseline. They then participated in an interactive, online lesson on mathematical inequalities with a researcher, where pictures were used to represent the quantities in the inequalities (i.e., picture problems; van Lieshout & Xenidou-Dervou, 2018). After the lesson, the children engaged in five tasks (also designed as picture problems), each assessing different conceptual components of mathematical inequalities. The first three tasks were learning tasks, all picture-picture problems, and the last two were transfer tasks, a near-transfer, picture-symbol problem task and a far-transfer, symbol-only problem task.

Two specific research questions guided this study: (a) How does the perceptual richness of visual representations impact participants' performance on learning and transfer tasks about inequalities? (b) How does the degree of concreteness (i.e., the extent to which the representation activates prior knowledge) impact children's performance on learning and transfer tasks about inequalities?

With regards to perceptual richness, I predict that children shown bland representations will outperform those shown rich representations on the learning task. The prediction is based primarily on Petersen and McNeil (2013)—the only study I know of to have disentangled perceptual richness from prior knowledge. I also predict that children shown rich representations will outperform those shown bland representations on the transfer tasks. While Petersen and McNeil did not assess transfer, studies that similarly found an effect of bland over rich on learning demonstrated an effect of rich over bland on transfer (Carbonneau & Marley, 2015; Carbonneau et al., 2013).

With regards to concreteness, I predict no performance difference between children shown abstract representations compared to concrete representations on the learning tasks because the majority of studies that explicitly assessed the effects of concreteness found no effect on learning (Kaminski et al., 2008, 2013; Siler & Willows, 2014). For the near-transfer task, where pictures are still included, I predict that participants shown abstract representations will outperform those shown concrete representations. Studies whose transfer tasks included visual representations reported an advantage of abstract over concrete (Goldstone & Son, 2005; Kaminski et al., 2008, 2013). For the far-transfer task, where there are no pictures, I predict that participants shown concrete representations will outperform those shown abstract representations. This is because studies with symbol-only transfer problems reported an advantage of concrete over abstract (Carbonneau & Marley, 2015; Siler & Willows, 2014).

Answering this study's questions is particularly critical because although the use of pictures is promoted in mathematics instruction, not all visual representations are created equal. Some are much more colorful and detailed, while others are more simplistic and bland. Some are more reminiscent or real-world objects and are thus more likely to evoke children's prior knowledge, while some are more abstract or novel to children. The literature on the affordances of pictures have yet to differentiate between perceptual richness and prior knowledge, which may differentially impact students' learning and their ability to transfer their understanding to novel, but conceptually similar, mathematics problems. As has been shown with manipulatives, perceptual richness and prior knowledge—qualities associated with the objects themselves—influence children's problem-solving (Petersen & McNeil, 2013). It is possible that these same features of static images may also impact children's mathematics performance. Therefore, when teachers create pictorial representations, it is critical to determine which kinds of images are best suited for improving children's mathematics comprehension.

Chapter 3: Method

Participants

The participants were recruited from Canada, the United Kingdom, or unspecified. A recruitment flyer was distributed across various social media platforms and contacts, including school principals and school boards. Parental consent for children's participation was obtained prior to their one-on-one interviews, and only children who provided assent before their interview took part in the study. Fifty parents reached out about the study, but only 29 consented (one with twins) for a retention rate of 58%. Of the 30 participants, one was excluded for having failed the screening measure and one was excluded for being the only four-year-old in the sample; all others were in kindergarten. This resulted in a final sample of 28 participants, where 61% were recruited from Quebec school boards (n = 17), 3% from Facebook (n = 1), and 11% from the Children Helping Science recruitment platform (n = 3). It is unknown from where the remaining 25% were recruited (n = 7). Students had a mean age of 70.25 months (SD = 5.49), and 14 were female (50%). Parents also reported their children's ethnicity on a demographic survey whereby 21 were Caucasian (75%), 3 were Asian (11%), and 4 were multi-ethnic (14%).

Design

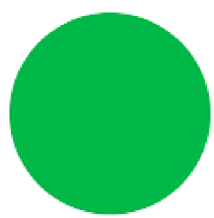
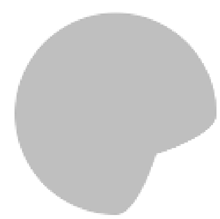
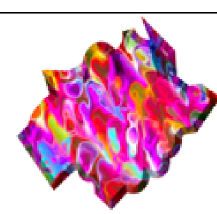
The study used a 3(condition: bland abstract, rich abstract, rich concrete) between-groups design. Participants were randomly assigned one of three visual representation conditions: (a) perceptually bland and abstract visual representations (n = 12), (b) perceptually rich and abstract visual representations (n = 8), or (c) perceptually rich and concrete visual representations (n = 8). The bland abstract condition could be compared to the rich abstract condition to test for effects of perceptual richness while controlling for concreteness. The rich abstract could be compared to the rich concrete condition to test for effects of concreteness while controlling for richness.

Experimental Manipulation

All visual representations were used to represent sets of quantities during the inequalities lesson and applicable measures. Participants in the bland abstract condition saw visual representations that were either complete green circles or grey circles with a divot, and were 3.35 cm by 3.35 cm on the PowerPoint slide. Participants in the rich abstract condition were shown rainbow-colored, unusual shapes (all identical), which were 3.56 cm by 3.56 cm on the PowerPoint slide. Participants in the rich concrete condition saw green cartoon frogs that were 3.63 cm by 3.35 cm. Each visual representation was shaped to appear as the same size as the others, and all quantities were presented in identical configurations on the slides.

Figure 1

The Visual Representation Conditions

Bland Abstract	 
Rich Concrete	
Rich Abstract	

Note. Participants in the Bland Abstract condition were only shown one of the two.

The Inequalities Lesson

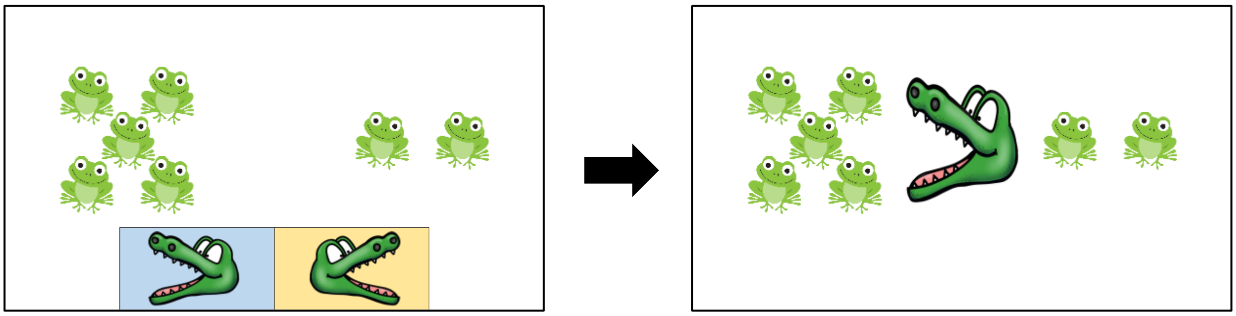
The interactive inequalities lesson taught children about the concept of inequalities (see protocol in Appendix A). This lesson was modified for online delivery from Carbonneau and Marley's (2015) high-guidance instruction condition. The purpose was to determine how the different visual representations used in the lesson impacted students' performance on subsequent learning and transfer task that assessed different conceptual components of inequalities.

Participants were first introduced to the concept of inequalities through a crocodile analogy in which participants were shown the picture of a crocodile head with its mouth open and were told that crocodiles want to eat the larger quantities. All quantities were represented using the pictures assigned to the participants' condition. Students were then shown two examples of the crocodile with its open mouth facing the larger of two quantities. In the first example the crocodile faced the larger quantity and in the second example participants were asked to select one of two crocodiles (left facing and right facing) that would eat the larger quantity. The researcher corrected children if they selected the wrong facing crocodile. For each example, the researcher read the inequality to the participants (e.g., "four is more than two," "three is less than five").

The students were then guided through three practice inequalities with the crocodile. For each practice item, they first selected the crocodile that would eat the bigger pile and then read the inequality. Corrective feedback was provided when necessary. Next, participants were introduced to the inequality symbol and the crocodile as an analogy—both representations open towards the larger of the two presented quantities, as if eating the larger quantity. Participants were shown one example of an inequality with the inequality symbol and how to read it before being presented with three practice inequalities using the inequality symbol. As with the crocodile practice inequalities, participants selected the proper inequality symbol and were asked to read the inequality. Corrective feedback was again provided when necessary.

Figure 2

Example of a Practice Item Using the Crocodile During the Inequalities Lesson



Note. The example uses the visual representations from the Rich Concrete condition. Aside from the crocodiles, the visual representations differ by condition. Participants were shown the first slide and were asked: "Which crocodile eats the bigger pile? Blue or Yellow?". After corrective feedback, they were shown the second slide and were asked: "Can you read this inequality?"

Measures

Screening Measure

Most of the tasks in the subsequent measures were presented in a multiple-choice format. The possible answers were embedded in colored boxes and participants had to name the color that held their answer. Thus, a screening measure was used to assess whether participants were capable of adequately responding to this online testing format considering their young age (see Appendix B).

The screening measure was comprised of two components. The first part was a 10-item training session to familiarize the participants with the names of the five different possible color options for the multiple-choice questions. Participants were asked to name the color and the researcher corrected them when necessary. They were presented each of the five colors twice to ensure that they used the proper color name. The second component consisted of the actual screening test where participants were asked to name the color of the box with the rabbit inside. There were three items for a total of three points. Participants were deemed capable of responding to the online format only if they scored all three items correctly.

Number Knowledge Measures

The number knowledge measures were inspired by Carbonneau and Marley (2015), who assessed ordinality, cardinality, arithmetic, and number recognition. Ordinality, cardinality, and arithmetic were assessed in this study using the Preschool Early Numeracy Scales (PENS; Purpura & Lonigan, 2015). Specifically, ordinality was assessed using the Set Comparison subscale, cardinality was assessed using the Set-to-Numerals subscale, and arithmetic was assessed using items 55, 56, 58, and 59 of the Story Problems subscale to best match Carbonneau and Marley’s (2015) method. The number recognition measure was based on Carbonneau and Marely’s (2015) number recognition test because the PENS did not have such a measure. All tasks were modified for online data collection. See Appendix C for sample items of each number knowledge measure.

Ordinality. The ordinality measure had six items assessing children’s ability to identify most and least quantities, three items for each. The ordinality measure had acceptable internal consistency (α = .76).

Cardinality. The cardinality measure had five items assessing children’s ability to match a set of dots to a numeral and numerals to a set of dots: three set-to-numeral and two numeral-to-set. The cardinality measure had good internal consistency (α = .80).

Arithmetic. The arithmetic measure had four items assessing children’s ability perform basic mental arithmetic presented in a word problem. The internal consistency of the arithmetic measure was low (α = .25).

Number Recognition. The number recognition measure had four items assessing children's ability to identify numerals. The first two items asked participants to identify a numeral among three other non-numeral symbols, and the last two items asked participants to identify the non-numeral symbol amongst three other numerals.

Coding. For each number knowledge measure, participants received 1 point for correct responses and 0 points incorrect responses. A mean number knowledge composite score was then created by summing the total points across all number knowledge measures and dividing by the total number of pretest items attempted.

Learning and Transfer Tasks

There was a total of five tasks, (see Appendix D for the protocol and sample items on each task). Four tasks—judging inequalities, completing inequalities using symbols, completing inequalities using numerals, and number line—were inspired by Carbonneau and Marley's (2015) measures. All tasks were modified from the original for online data collection and to account for differences between the use of manipulatives and visual representations. The fifth task—reading inequalities—was an additional task I created for this study to capture a more complete picture of children's conceptual understanding of inequalities.

All tasks with inequalities showed quantities, not numerals, and the quantities were represented using the visual representations of the participants' assigned condition. During the administration of each task, the researcher wrote down the participants' responses on a scoring sheet along with their score on each item.

Judging Inequalities. The judge task was adapted from Carbonneau and Marley's (2015) first conceptual task, which was a learning task that assessed participants' ability to recognize whether an inequality is correct or incorrect. Correct inequalities were those where the open face of the inequality symbol was directed towards the larger of the two quantities (e.g., "4 > 1"). Incorrect inequalities were those where the open face of the inequality symbol was towards the smaller of the two quantities (e.g., "5 > 9").

The judge task had six items: three items were correct and three items were incorrect. Additionally, the first three items presented greater than inequalities (i.e., >) and the last three items presented less than inequalities (i.e., <). For each item, participants were asked to show a "thumbs up" to the camera if they judged the inequality to be correct, and a "thumbs down" to the camera if they judged the inequality to be incorrect. Participants scored 1 point for each correctly-judged item and 0 points for each incorrectly-judged item. A mean judge score was calculated by summing the total points and dividing by the total number of items completed.

Completing Inequalities using Symbols. The symbol task was adapted from Carbonneau and Marley's (2015) second conceptual task that assessed participants' ability to choose the correct inequality symbol (greater than or less than) to produce a correct inequality. On this task, two different quantities were presented on the slide with the appropriate visual representation with a vacant space outlined in the center for the symbol (e.g., 8 __ 6). The greater than symbol was presented in a blue box and the less than symbol was presented in a yellow box below the missing symbol sentence. The children were asked to select the symbol that would make a good inequality by naming the color of the box.

The symbol task had six items. Half of the items had larger numbers on the left-hand side of the screen, such that a good inequality would require selecting the greater than symbol (>). The other half of the items had larger numbers on the right-hand side of the slide, such that a good inequality required selecting the less than symbol (<). Participants scored 1 point for each correct response and 0 points for each incorrect response. A mean symbol score was calculated by summing the points and dividing by the total number of items completed.

Reading Inequalities. The read task was created because of the emphasis in the Inequalities Lesson on children's ability to read inequalities. This task assessed participants' ability to (a) correctly read greater than and less than inequalities (i.e., the ability to read the direction of the inequality symbol), and (b) correctly read the magnitude relationship between the quantities -- that is, reading which quantity is either larger or smaller in relation to the other, regardless of the direction of the inequality symbol. Participants were shown a slide with an inequality and asked to read the inequality out loud. There were six items in the read task, half of the items were greater than inequalities and the other half were less than inequalities.

Participants received two scores for the read task. The first score is the direction read score, which assessed whether participants correctly or incorrectly read the inequality based on the direction of the inequality symbol. Participants received 1 point for correctly-read inequalities and 0 points for incorrectly-read inequalities.

The second score is the magnitude read score, which assessed whether participants could correctly read the magnitude of the inequality, regardless of the direction of the inequality symbol. For example, on item 1 (i.e., $4 > 3$), participants may have read it as "three is less than four," which preserves the correct magnitude of the relationship, or "four is more than three." The magnitude was counted as incorrect if they read "four is less than three" or "three is more than four." Participants received 1 point for correctly reading the magnitude, and they received 0 points for incorrectly reading the magnitude.

For both direction and magnitude reading, items were still counted as correct if participants incorrectly identified a quantity in the problem, as long as they were not off by more than one. For example, if a participant said, “five is more than three” instead of “four is more than three” for 4 > 3, they still received a full point. Items were also considered correct if they used correct variations of “more than,” including “bigger” or “greater than,” and “less than” or “smaller.” The mean direction read score and the mean magnitude read score were each calculated by summing the total points and dividing by the total number of items completed.

Completing Inequalities using Numerals. The numeral task was adapted from Carbonneau and Marley’s (2015) procedural task, a near-transfer task that assessed participants’ ability to create correct inequalities using numerals. The task assessed near-transfer because participants had only been presented inequalities with visual representations of quantities (i.e., sets of dots) during the lesson and prior measures. The numeral task, however, created a numeral-to-set comparison (i.e., a picture-symbol problem instead of a picture-picture problem).

Participants were presented with an inequality missing a number on the left-hand side (e.g., ____ > 7; the right-hand side presented a set of seven visual representations). They were asked to select a numeral from three possible choices to create a correct inequality, where only one response out of three would create a correct inequality and the two other possibilities would create incorrect inequalities. Each choice was presented in a blue, yellow, or pink box at the bottom of the slide, and the participant named the color of the box that contained the correct answer.

The numeral task had six items in total. Participants received 1 point for each correctly-answered item and 0 points for each incorrect answer. A mean numeral score was calculated by summing the total points and dividing by the total number of items completed.

Number Line. The number line task was the last task and was adapted from Carbonneau and Marley's (2015) transfer task. The task was a far-transfer task that assessed participants' abilities to complete magnitude comparisons on a number line. It was considered a far-transfer task because it presented magnitude comparisons on a number line rather than in a number sentence. Participants were first given a brief explanation of the number line, after which they were presented with a number line from one to ten, with a target number bolded and in red.

There were six items on the number line task. Three items asked participants to select two numbers on the number line that were more than the target number and the remaining three items asked them to select two numbers that were less than the target number. Correct answers were any selected number that correctly fit the direction of the magnitude comparison in relation to the target number. For each item, participants received 2 points if both numbers named were correct, 1 point if only one of the two numbers was correct, and 0 points if both numbers named were incorrect. The number line score was the mean number of points on each item, then divided by two so that the mean score would be comparable to the mean scores of the other measures.

Missing data. For all scored measures, the participants' answers were marked as missing if the participants did not want to answer the items, but they received 0 points if they said they did not know the answer. In the cases where a parent or guardian attempted to assist the child, participants were scored based on the answer they gave prior to any assistance and were given a missing score if a parent or guardian assisted prior to letting the child answer on their own. Lastly, when there was poor internet connection and responses were inaudible, participants received a missing score.

Procedure

Children were interviewed one-on-one with a researcher online via Zoom. Video recordings of each interview were obtained, and each interview lasted between 20 to 40 minutes.

One participant completed the session on two consecutive days, while the rest did the study in one sitting. The interview tasks were administered by screen sharing PowerPoint slides controlled by the researcher.

After the participants' assent was obtained at the start of the interview, they underwent the two-minute screening measure. Participants who passed the screening then moved on to the number knowledge measures, which lasted about four to five minutes in total, at which point they completed the rest of the study. Children who failed the screening were only administered the first two number knowledge measures (ordinality and cardinality) before the interview was terminated.

After the number knowledge measures, students were given an interactive lesson on inequalities that took approximately seven minutes. From there, the children completed the first three learning tasks and the two transfer tasks (one near- and one far-transfer). The tasks were completed in the following order: judging inequalities, completing inequalities using symbols, reading inequalities, completing inequalities using numerals, and the number line task. Once the tasks were completed, the researcher thanked the child for their participation and ended the interview.

Chapter 4: Results

Descriptive Statistics

The means and standard deviations for each number knowledge measure are reported in Table 1. Notably, there was next to no variance in the mean number recognition because all participants except one received a perfect score. As such, it was excluded from the mean number knowledge score. The total number knowledge score was thus a composite of the ordinality, cardinality, and arithmetic scores.

Table 1

Means and Standard Deviations of the Number Knowledge Measures

Mean Score	*M*	*SD*
Ordinality	.95	.13
Cardinality	.87	.25
Arithmetic	.79	.23
Number Recognition	.99	.05
Total Number Knowledge	.87	.14

Note. $N = 28$.

Participants' learning and transfer tasks scores by condition are presented in Table 2. Overall, children in the rich abstract condition had higher mean scores on the judge task and the numeral task than those in the bland abstract condition, who in turn had higher mean scores than participants in the rich concrete condition. Participants in the bland abstract condition had higher mean scores on symbol task, direction reading, and number line task than those in the rich abstract condition, who in turn had higher mean scores than those in the rich concrete condition. Additionally, participants in the bland abstract condition had higher mean scores on magnitude reading than those in the rich concrete condition, who in turn had higher mean scores than those in the rich abstract condition. Notably, across all three conditions, participants had higher magnitude reading scores than direction reading scores.

Table 2

Descriptives of Age, Total Number Knowledge, Learning, and Transfer Task Scores by Condition

Variable	Bland Abstract [a]		Rich Concrete [b]		Rich Abstract [c]	
	M	*SD*	*M*	*SD*	*M*	*SD*
Age (mo)	70.08	5.50	68.25	6.74	72.50	3.59
Number Knowledge	.88	.11	.85	.20	.88	.13
Judge	.79	.23	.75	.24	.96	.12
Symbol	.90	.29	.71	.32	.81	.35
Direction	.57	.34	.52	.37	.53	.36
Magnitude	.82	.39	.74	.39	.71	.45
Numeral	.47	.23	.35	.30	.48	.24
Number Line	.91	.10	.79	.29	.90	.18

[a] $n = 12$. [b] $n = 8$. [c] $n = 8$.

Correlations between year of testing, age, total number knowledge score, and mean learning and transfer task scores across the entire sample were assessed to check for possible covariates (see Table 3).

There was a large time gap between testing certain participants, which may have impacted performance on the learning and transfer tasks because of the different amounts of formal education they had received. Nine participants were tested at the beginning of the school year in 2020 and 19 participants were tested at the end of the school year in 2021. Year of testing was significantly and positively correlated with participants' age in months, mean total number knowledge score, mean judge score, and mean number line score. In contrast, age in months was not significantly correlated with any of the variables. Total number knowledge was significantly and positively correlated with numeral score and number line score.

Regarding the correlations between the learning and transfer scores, participants' direction reading score was significantly and positively correlated with their magnitude reading score and their number line score. None of the other learning and transfer task scores were correlated.

Table 3

Correlation Matrix Between Year of Testing, Age, Total Number Knowledge Score, and Learning and Transfer Task Scores

Variable	*M*	*SD*	1	2	3	4	5	6	7	8	9
1. Year	–	–	–								
2. Age (mo)	70.25	5.49	.51**	–							
3. Number Knowledge	.87	.14	.66**	.32	–						
4. Judge	.83	.22	.46*	.27	.37	–					
5. Symbol	.82	.31	.02	.24	.25	.27	–				
6. Direction	.55	.34	.24	.10	.30	.07	.05	–			
7. Magnitude	.76	.39	.26	.02	.18	.01	-.09	.82**	–		
8. Numeral	.44	.25	.30	.33	.44*	.18	.17	.13	.09	–	
9. Number Line	.87	.19	.38*	.26	.78**	.33	.15	.40*	.24	.24	–

Note. Year of testing is a point-biserial correlation; two groups, 2020 and 2021. $N = 28$. *$p < .05$. **$p < .01$.

Condition Effects

To test for potential effects of perceptual richness and concreteness on the learning tasks and transfer tasks, several analyses of variance were performed. Year of testing was correlated with participants mean judge score, but the point-biserial correlations were not close to equal within the three conditions (bland abstract: $r = .36$, $p = .25$; rich concrete: $r = .68$, $p = .06$; rich abstract: $r = -.14$, $p = .74$). Thus, year of testing was not used as a covariate for the analysis of variance by condition on the judge learning task. Similarly, the point-biserial correlations between year of testing and the number line task were not close to equal within the three conditions (bland abstract: $r = .09$, $p = .79$; rich concrete: $r = .44$, $p = .28$; rich abstract: $r = .91$, $p = .002$). Thus, year of testing was not used as a covariate for the analysis of variance by condition on the number line transfer task.

The total number knowledge score was correlated to both the numeral and number line scores (see Table 3). Further correlations by condition revealed that total number knowledge was approximately equally correlated to numeral score in all three conditions (bland abstract: $r = .58$, $p = .047$; rich concrete: $r = .31$, $p = .45$; rich abstract: $r = .46$, $p = .25$), and was thus used as a covariate. Total number knowledge was also about equally correlated with the number line score in all three conditions (bland abstract: $r = .73$, $p = .01$; rich concrete: $r = .83$, $p = .01$; rich abstract $r = .76$, $p = .03$), and was thus used as a covariate.

Learning Tasks

A one-way ANOVA between the three conditions was performed with the mean judge score as the dependent variable. There was no statistically significant difference between conditions on their mean judge score, $F(2,25) = 2.39$, $p = .11$. Across all three conditions, participants appeared to perform equally well on judging whether inequalities were correct or incorrect, regardless of condition. This indicates that there was no effect of perceptual richness nor concreteness on the judge task.

A second one-way ANOVA between the three conditions was performed with the mean symbol score as the dependent variable. There was no statistically significant difference between conditions on their mean symbol score, $F(2,25) = 0.92$, $p = .41$. Participants seemed to perform equally well on selecting the appropriate inequality symbol to make a correct inequality, regardless of condition. Thus, there appears to have been no effect of perceptual richness or concreteness on the symbol task.

A third 2 (reading: direction, magnitude) x 3 (condition: bland abstract, rich concrete, rich abstract) mixed ANOVA was performed to assess whether difference in reading scores interacted with condition. A main effect of reading was observed, $F(1,25) = 22.65$, $p < .001$, where participants had higher mean scores on magnitude reading ($M = .76$, $SD = .39$) than on direction reading ($M = .55$, $SD = .34$). Regardless of condition, participants were better at reading the magnitude relationship of inequalities than the direction of inequalities. That said, there was no main effect of condition, $F(2,25) = 0.12$, $p = .88$. Across all three conditions, participants performed equally well on the reading measures. Additionally, there was no interaction effect between reading and condition, $F(2,23) = 0.25$, $p = .78$. This means that perceptual richness and concreteness did not impact the differences between magnitude reading and direction reading; participants in all conditions had higher magnitude reading scores than direction reading scores.

Transfer Tasks

A one-way ANCOVA between the three conditions was performed with the mean numeral score as the dependent variable and the mean total number knowledge score as the covariate. There was a main effect of total number knowledge, $F(1,24) = 5.40$, $p = .03$, whereby the greater the participants' total number knowledge, the better they performed on the numeral task. When controlling for total number knowledge, there was no statically significant difference between conditions on their mean numeral score, $F(2,24) = 0.45$, $p = .65$. Regardless of condition, participants appeared to perform equally well when asked to choose a numeral to correctly complete the inequalities. This indicates that there was no effect of perceptual richness or concreteness on the near-transfer task.

A second one-way ANCOVA between conditions was performed with the mean number line score as the dependent variable and the mean total number knowledge score as the covariate. Again, there was a main effect of total number knowledge, $F(1,24) = 36.67$, $p < .001$, whereby the greater the participants' total number knowledge, the better they performed on the number line task. When controlling for participants' total number knowledge, there was no significant difference between conditions on their mean number line score, $F(2,24) = 1.10$, $p = .35$. Participants in each condition seemed to perform equally on magnitude comparisons using the number line. As such, there was no effect of perceptual richness or concreteness on the participants' performance on the far-transfer task.

Reading Profile Analysis

The prior analysis that assessed the difference between participants' magnitude reading and direction reading across the three conditions used means scores that combined participants' correct responses with their incorrect responses. To better understand how participants were correctly reading the inequalities between the conditions, I looked at participants who correctly read direction and correctly read magnitude, and those who made errors when reading the inequalities.

I classified participants into a reading profile based on their performance on the read task. Participants were assigned to one of three reading profiles: (a) direction readers were those whose total score on direction reading was greater than or equal to their total score on magnitude reading; (b) magnitude readers were those whose total magnitude score was greater than their total direction score; and (c) incorrect readers were participants whose number of incorrect and missing scores on the magnitude items was greater than their total direction score and their total magnitude score, separately.

Table 4 illustrates the number of participants in each reading profile by condition. Of the 28 participants, half were magnitude readers, but there were more direction readers than incorrect readers. Thus, almost 80% of the participants in the sample were able to read inequalities in some capacity after the lesson. Notably, there was a higher proportion (almost 60%) of magnitude readers (than other types of readers) in the bland abstract condition than in the other two conditions. Additionally, there was a slightly higher proportion of direction readers (than other types of readers) in the rich concrete condition than in the other conditions. Lastly, there was a slightly higher proportion of incorrect readers (than other types) in both rich conditions compared to the bland abstract condition.

Table 4

Frequencies and Proportions of Reading Profiles by Condition and Total

Reader	Condition						Full Sample [d]	
	Bland Abstract [a]		Rich Concrete [b]		Rich Abstract [c]			
	n	%	*n*	%	*n*	%	*n*	%
Direction	3	25.0	3	37.5	2	25.0	8	28.6
Magnitude	7	58.3	3	37.5	4	50.0	14	50.0
Incorrect	2	16.7	2	25.0	2	25.0	6	21.4

[a] $n = 12$. [b] $n = 8$. [c] $n = 8$. [d] $N = 28$

Magnitude readers likely interpreted the task as a magnitude comparison task, ignoring the inequality symbol, but correctly identifying the magnitude relationship in the inequality. However, it is unclear whether direction readers merely learned the procedure for reading inequalities or if they, too, internalized the magnitude relationship in inequalities. I thus compared direction readers to magnitude readers on their number line task performance, which assessed participants' ability to correctly compare magnitudes. If magnitude readers have greater mean number line scores than direction readers, this may indicate that direction readers learned the procedure for reading inequalities without learning to interpret the magnitude relationship in the inequalities.

The independent-samples t-test demonstrated no significant difference between direction readers and magnitude readers on their mean number line scores, $t(20) = 0.93$, $p = .36$. This lack of difference suggests that direction readers were equally capable of interpreting magnitude relationships between two numbers in an inequality ($M = .94$, $SD = .18$) as magnitude readers ($M = .87$, $SD = .16$). Thus, direction readers likely had a superior understanding of how to read inequalities: Not only could they interpret the magnitude relationship, but they could also read the direction of the inequality symbol.

Chapter 5: Discussion

The goal of this study was to investigate the influence of visual representations' perceptual richness and concreteness on children's ability to understand and solve inequalities. Children were randomly assigned to one of three visual representation conditions: bland abstract, rich abstract, and rich concrete. A researcher gave them a short interactive lesson about inequalities before administering five tasks assessing different conceptual components of inequalities. The first three tasks were learning tasks, the fourth was a near-transfer task, and the fifth was a far-transfer task. The visual representation assigned to the conditions was shown throughout the inequalities lesson and all applicable tasks. For the most part, I did not find what I had predicted because of the study's small sample, which resulted in an insufficient amount of power to detect differences (see my discussion of the study's limitations below). As such, the pattern of results summarized below must be interpreted with caution.

Regarding the effects of perceptual richness, I had predicted that participants shown bland visual representations would outperform those shown rich representations on the learning tasks and that those show rich representations would outperform those in the bland condition on the transfer tasks. These predictions were not supported. I found no difference between participants in the bland abstract condition and the rich abstract condition (or the rich concrete condition) on any of the learning or transfer tasks. As such, the data show no effect of perceptual richness on children's learning and transfer of knowledge regarding inequalities.

A handful of studies have similarly failed to find an effect of perceptual richness on learning (Kaminski & Sloutsky, 2013; Menendez et al., 2020; Osana et al., 2018). In some of these studies, however, the authors' operationalization of perceptual richness was questionable. They compared bland abstract representations to rich concrete representations, thus confounding perceptual richness and concreteness. In one notable exception, Menendez et al. (2020) compared bland concrete to rich concrete visual representations in a study on biology learning, thus controlling for concreteness, and found no difference in students' learning. Taken together with the present study's failure to detect differences on the learning tasks between participants exposed to bland abstract and those exposed to rich abstract visual representations, there may be tentative evidence that perceptual richness does not impact students' learning when concreteness is taken into account. However, both Kaminski and Sloutsky (2013) and Menendez and colleagues (2020) reported that bland representations were better than rich ones for transfer, whereas my study found no such difference.

Regarding the effects of concreteness, I had predicted that there would be no difference between the performance of participants who were shown concrete or abstract visual representations on the learning tasks. I had also hypothesized that participants in the abstract condition would outperform those in the concrete condition on the near-transfer task, and the reverse on the far-transfer task. While no differences were found on the learning tasks, as predicted, neither were differences found between conditions on either of the transfer tasks. Together, the observed results suggest that there is no effect of concreteness on learning or transfer.

The fact that participants performed equally well at transfer regardless of visual representation type may be due to the lesson having a high level of guidance. Carbonneau and Marley (2015) also found no differences between conditions on transfer among children who received high-guidance instruction. A difference between bland and rich manipulatives only emerged for participants in the low guidance condition. As such, the impact of the properties of visual representations might be off-set by the explicit guidance provided during instruction.

There are several possible explanations for the sum of the results. For one, some prior research suggests that the effects of rich/realistic details (i.e., perceptually rich and concrete) on transfer depends on students' prior knowledge of the subject at hand. Magner and colleagues (2014) found that seductive details only impacted students with very low or very high prior knowledge on a near-transfer measure, but that when participants had near average levels of prior knowledge, there was no difference between the groups. On far-transfer, prior knowledge was the only effect the Magner et al. found, with no effect of seductive illustrations at all. They concluded that seductive details might only negatively impact students with low prior knowledge, but not those with high prior knowledge. These findings may suggest that prior knowledge moderates the effects of perceptual richness and concreteness on transfer. My study supports this conclusion because children's total number knowledge was positively correlated with both the near-transfer and far-transfer tasks, such that the greater their number knowledge, the better they performed on both transfer measures.

On the methodological side, participants in the bland abstract condition were shown one of two different types of bland abstract visual representations, whereas participants in the rich concrete and rich abstract conditions were each shown only one type of visual representation. One of the bland abstract visual representations was green circles and the other was grey circles with a divot. According to existing operationalizations in the literature (Kaminski et al., 2013), both visual representations can be considered perceptually bland and abstract, but perceptual richness and concreteness exist on a spectrum. Thus, one of the representations used in my study may have been slightly more perceptually bland or abstract, or both, than the other, enough to potentially obscure the effects of the blandness and abstractness within the condition itself. Anecdotally, one participant called the grey divot-circles a pizza, mentally concretizing the visual representation in their mind. It is unknown what the other participants thought of their visual representations and whether their internal perceptions of the visual representations impacted how concrete the representations actually were or the children's mathematics performance, a consideration left for future research. Some preliminary evidence exists to show that the nature of children's mental representations does indeed impact their mathematical problem-solving (Osana, Adrien, et al., 2021), much remains unanswered with respect to the impact of the perceptual features of visual representations on young children's numeracy development.

Another methodological explanation for the observed results is that visual representations were also used in the number knowledge measures to display sets, and these visual representations were different from those used in each of the conditions. De Bock and colleagues (2011) demonstrated that students performed better on the learning and transfer tasks when there was a match between visual representations at learning and transfer, and performed worse when there was a mismatch. In my study, the visual representations in the number knowledge measures were black dots, thus bland and abstract. This means that participants in the bland abstract condition had visual representations during the lesson and tasks that matched the number knowledge measures, but this was not the case for participants in either of the rich conditions. This may have produced an unfair advantage for participants in the bland abstract condition or hindered participants in the rich conditions, or both, potentially obscuring differences on task performance between conditions. In short, it is possible that the type of visual representations used in the number knowledge measures acted as a confound.

There are also several differences between my study and others that assessed the perceptual affordances of visual representations that may account for my results. Critically, my study was designed to disentangle the effects of concreteness from perceptual richness. Petersen and McNeil (2013) created the only other study I know of to have attempted the same. Their study used manipulatives, not visual representations, and they found interaction effects. I attempted to create a similar study using visual representations to assess whether the same effects hold, but I found no effect of either perceptual richness or concreteness.

There are two possible explanations for the difference between my study and that of Petersen and McNeil. First, Petersen and McNeil only found interaction effects, but no main effects; my study was not designed to test for possible interaction effects (only main effects while controlling for the other factor). Thus, perceptual richness and concreteness might impact visual representations, but only under certain conditions, and future research designed to test for interactions with larger samples is needed. Second, perceptual richness and concreteness might only impact physical representations and not visual ones, a finding supported by the embodied cognition literature. Embodied cognition posits that children learn best through action and perception, suggesting that manipulating concrete objects leads to better understanding (Glenberg et al., 2007; Martin & Schwartz, 2005; Pouw et al., 2014) than if no physical manipulation occurs. By the same token, properties of physical representations may be more likely to impact performance than properties of visual representations because it is not possible to interact with static visual representations in the same way as with manipulatives (Osana, Lafay, et al., 2021).

Beyond perceptual richness and concreteness, interesting patterns emerged in how participants learned to read inequalities. The data appear to suggest that children were more likely to adopt a “magnitude reading” of inequalities, ignoring the direction on the inequality symbol, than a “direction reading,” based on the direction of the symbol. Additionally, the proportion of direction readers, magnitude readers, and incorrect readers, was not the same in each condition. The majority of participants in both abstract conditions were magnitude readers, whereas in the concrete condition, the proportions of all three types of readers were approximately equal. While visual representations' perceptual richness and concreteness might not influence children's ability to solve inequalities, this might not be the case when it comes to learning how to read inequalities. Unfortunately, the sample sizes were too small to assess whether reading type differed by condition.

Limitations

The results of this study must be interpreted with caution because of several limitations. Most egregious is the small sample size ($N = 28$), which severally diminished the power to detect statistically significant effects. Thus, using these data, it is difficult to draw a conclusion about the effects of perceptual richness and concreteness of visual representations on learning and transfer. Not only did the small number of participants in each condition reduce power, but it also increased the risk of non-normally distributed data. Additionally, the sample sizes varied between conditions (Bland Abstract $n = 12$; Rich Concrete $n = 8$; Rich Abstract $n = 8$), thereby potentially violating the assumption of homogeneity of variance. While the analysis of variance is robust against both violations, the F-test still has its limitations (Rheinheimer & Penfield, 2001; Schmider et al., 2010). Thus, there was a large risk that the cell sizes were not big enough to trust the results of an analysis of variance.

Another factor that hindered the overall strength of the study was the large time gap between participant testing. All the participants were in the same grade, but some were tested at the beginning of the school year in 2020 and others were tested at the end of the school year in 2021. The possible effect of the time gap is evident, in part, from the relation between time of testing and performance on the number knowledge measure: Participants tested in 2021 had greater total number knowledge than those tested in 2020. It is possible that no effects of perceptual richness or concreteness were found because by the end of the school year children had enough number knowledge that these features of visual representations no longer hindered their performance. As previously stated, prior knowledge can impact the effects of visual representations (Magner et al., 2014).

In a similar vein, the tasks may have been too easy for my participants, as indicated by their high total number knowledge scores—especially those tested in 2021. The study was adapted from Carbonneau and Marley's (2015) manipulatives research, who tested 3- to 4-year-old participants. In the present study, most participants were tested at the end of the school year, meaning they received almost a whole year's worth of formal education. Thus, the tasks may have no longer been developmentally appropriate, resulting in ceiling effects that obscured potentially real impacts of the representations' perceptual qualities on their learning and transfer.

Another limitation was that the total number knowledge measure was a composite score of three subtasks: ordinality, cardinality, and arithmetic. The number recognition subscale was not included because there was almost no variance. Performance on the ordinality subscale was at or near ceiling and the arithmetic subscale did not have good internal consistency, resulting in a composite score that was possibly not sensitive enough to detect true differences in participants' number knowledge. It is possible the arithmetic subscale did not have good internal consistency because I only used four out of the seven items in the PENS Story Problem subscale. Additionally, it may be that children who participated in 2020 drastically underperformed compared to those who participated in 2021 due to their age difference.

Lastly, the COVID-19 pandemic negatively affected participant recruitment. The pandemic made it difficult to recruit participants from Quebec school boards that were busy adapting to the pandemic, and it slowed online recruitment through the Children Helping Science research recruitment website, which became backlogged from the increased movement towards online research. Additionally, the study had to be put on hold for several months because of recurring waves of infection, which partly contributed the large time gap between participant testing.

More critically, it is unknown which of the participants, and how many, were familiar with online learning. Participants were recruited from several locations, each of which may have delivered online schooling at different times or not all. Greater familiarity with online education may have made the study's online format easier for those participants and harder for others. Additionally, not all students received the same mathematics curriculum since they were sampled from various locations, which could have also impacted their performance. The random assignment to conditions served to mitigate this limitation, however.

Future Directions

The results from this study were inconclusive, and as such, should be replicated with modifications to reduce the study's current weaknesses. Participants should be preschoolers and kindergarteners, tested solely at the beginning of the school year to ensure that the tasks are developmentally appropriate and to eliminate gaps in testing. The number knowledge measure should use a more advanced number recognition measure, such as the one in the PENS perhaps, which has been validated with Kindergarteners and preschoolers (Purpura & Lonigan, 2015). Additionally, the number knowledge measure should only use numerals and not sets to avoid the potential confound of exposure to visual representations prior to the inequalities lesson.

Changes should also be made to the conditions. First, the visual representation design for the bland abstract condition should be uniform. Future replications should either use green circles or the grey circle with a divot, or a new representation entirely. If the green circle is used, I would perhaps choose a duller shade of green to make the representation more perceptually bland in color, thus increasing the difference between the bland representations and the rich representations. If the grey divot-circle is used, I would choose a darker shade of grey to increase the contrast between the grey circle and the white background of the screen. That said, perhaps the best visually bland and abstract representation would be black dots, but a consideration for future research would be to find a representation that reduces as much as possible the participants' visualization of real-world items, such as pizzas.

Second, a fourth bland concrete condition should be added in order to test for potential interaction effects between perceptual richness and concreteness. The bland concrete representations should be a black-and-white version of the frog used in the rich concrete condition, similar to Menendez and colleagues' (2020) approach. Participants should also be asked how they perceive the representations to ensure that their internal construction of the representation matches the condition's intent.

Most importantly, the study should be replicated with a much larger sample, with at least 15 participants per condition, to have enough power to detect differences between conditions. All these changes are necessary to more adequately determine if a visual representation's perceptual richness and concreteness impacts students' ability to understand and solve inequalities. Until research can confidently state how perceptual richness and concreteness impact both learning and transfer in mathematics education, future research on the affordances of visual representations must continue to account for both variables.

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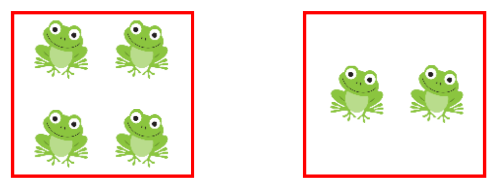
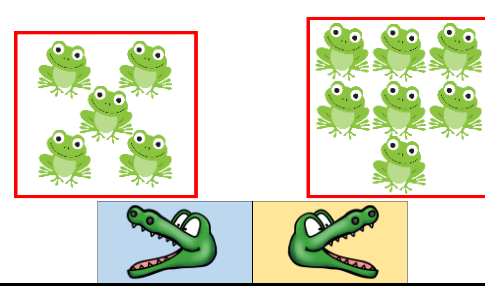
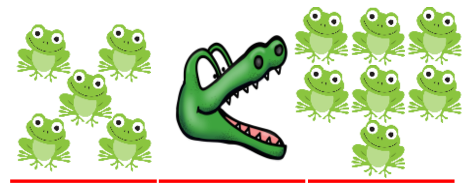
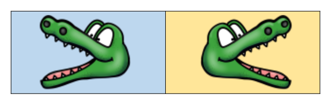
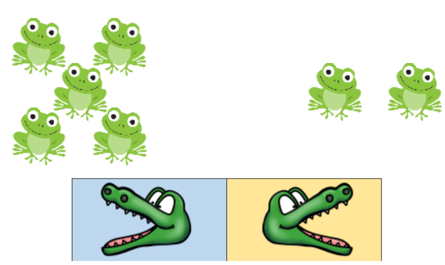
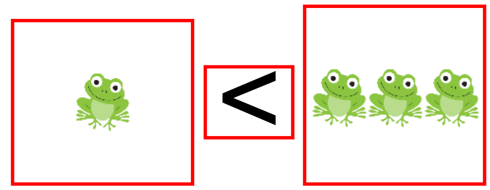
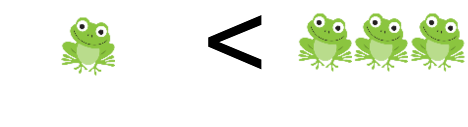
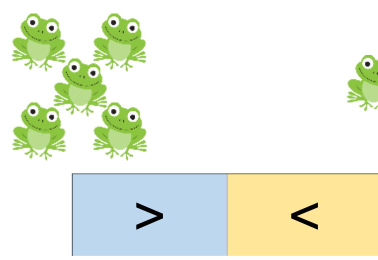
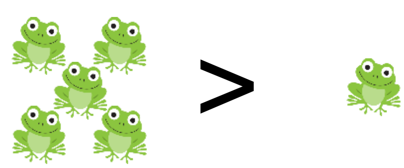
Appendix A

Inequalities Lesson Protocol

Crocodile Game Introduction:	
Say, **We are going to play a game with this crocodile. Crocodiles are very hungry and want to eat big piles.**	
For example, (click) **if I have 4 over here** (click) **and 2 over here,**	
I want my crocodile to eat the pile of four because it is bigger. **This makes an inequality, can you say inequality?** If Incorrect: repeat the word slowly and ask again. Say with participant if needed. **Great, then I can read:** (click) **4** (click) **is more than** (click) **2.** (click) **When the crocodile is facing this way, we say "more than". This inequality lets us know which quantity is bigger. Let's try a different one.**	
What if (click) **I have 5 on this side** (click) **and 7 on this side? Which color has the crocodile that would eat the bigger pile, Blue or Yellow?**	

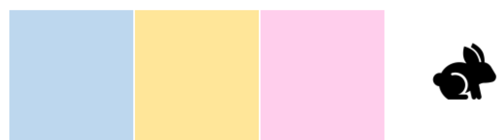
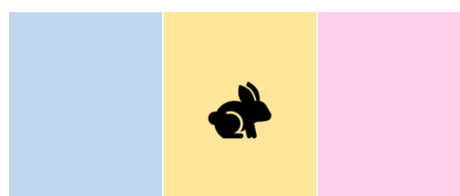
After participant responds: If Correct: **Good, I would want my crocodile to eat the pile of 7 because 7 is bigger.** If Incorrect: **I want my crocodile to eat the pile of 7 because 7 is bigger.** **I can read my inequality:** (click) **5** (click) **is less than** (click) **7.** (click) **When the crocodile is facing this way, we say "less than". This inequality lets us know which quantity is smaller.** (click) **Can you read this inequality?** If Correct: **Good.** If Incorrect: **This inequality reads: 5 is less than 7.**	
Say, **I am going to start the inequality by showing two piles and you are going to be in charge of the crocodile by picking the crocodile that eats the bigger pile. Ready?** After participant responds, **Great let's try a few inequalities.**	
<u>Inequalities with Crocodile</u>:	
*Repeat for each practice inequality presented (5 > 2 ; 4 < 6 ; 3 > 1): Say, **Which crocodile eats the bigger pile? Blue or Yellow?** If Correct: **Good.** If Incorrect: **The [*color*] crocodile eats the bigger pile. See-**	
Ask, **Can you read this inequality?** If Correct: **Good, this inequality reads [*answer*].** If Incorrect: **This inequality reads [*answer*].** *Answer format = "# is more than #" or "# is less than #"*	

Inequalities with Symbol:	
Say, **Great work! Let's try some more inequalities. For example,** (click) **I can have 1 on this side and** (click) **3 on this side** (click) **and use this symbol to make my inequality. This is called the inequality symbol and it works just like our crocodile. Do you remember what the job of our crocodile was?** After participant responds, **Good, the crocodile and the inequality symbol always eat the bigger number.**	
So this inequality reads: (click) **1** (click) **is less than** (click) **3.**	
*Repeat for each practice inequality presented (5 > 1 ; 3 < 6 ; 4 > 2): Say, **Which way should the inequality symbol go? Blue or Yellow?** If Correct: **Good.** If Incorrect: **The [color] symbol eats the bigger pile. See-**	
Ask, **Can you read this inequality?** If Correct: **Good, this inequality reads [*answer*].** If Incorrect: **This inequality reads [*answer*].** *Answer format = "# is more than #" or "# is less than #"*	

Note. Used rich concrete condition in above protocol for the example slides. "Click" (in red) corresponds to activating an animation within the slide.

Appendix B

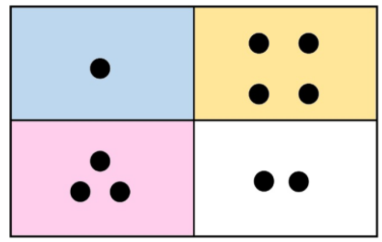
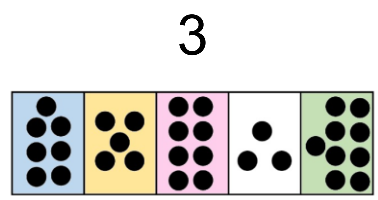
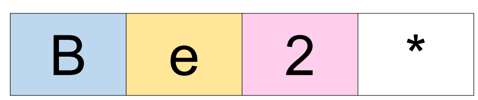
Screening Measure

Training Session:	
Repeat twice for each color (blue, yellow, pink, white, green): Say, **What color do you see?** *Feedback:* *Correct*: Say, **Good. This is [color].** *Incorrect*: Say, **We're going to call this [color]. What color is this?** *Correct*: Say, **Good.** *Incorrect*: Say, **This color is [color].**	
Screening:	
Say, **Now we are going to play a game. You're going to see 3 colored boxes: Blue, Yellow, and Pink. A bunny is going to hop inside 1 of the boxes and I want you to tell me the color of the box that has the bunny.**	
Say, **Which color has the bunny?** *Record response*	
Say, **Which color has the bunny?** *Record response*	

Say, **Which color has the bunny?** *Record response*	

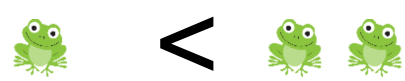
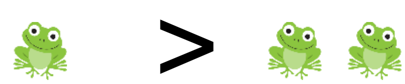
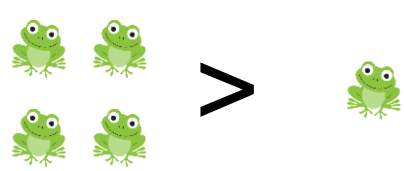
Appendix C

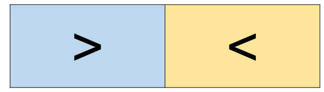
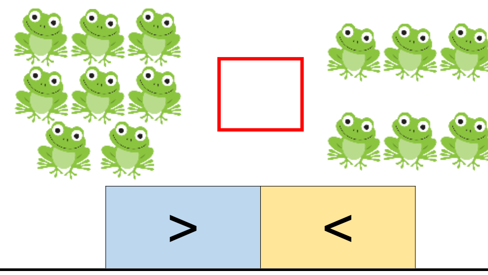
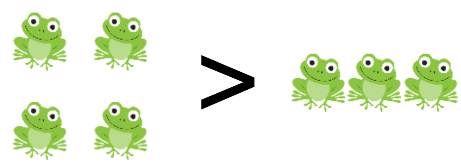
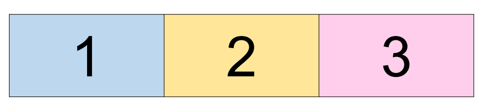
Number Knowledge Measure: Example Items

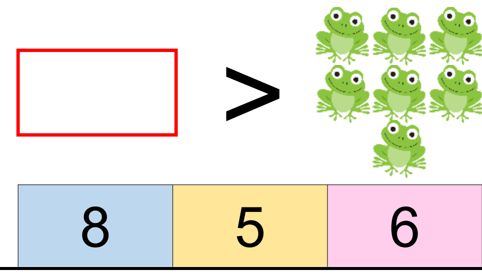
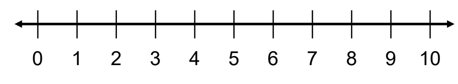
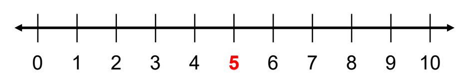
Ordinality measure: "Say, **Which box has the MOST dots?**" *Record response*	
Cardinality measure: "Say, **Which of these groups of dots means the same thing as the number on top?**" *Record response*	
Arithmetic measure: "Say, **Which of these groups of dots means the same thing as the number on top?**" *Record response*	*Blank slide
Number recognition measure: "Say, **Which one is a number?**" *Record response*	

Appendix D

Learning and Transfer Tasks: Protocol and Example Items

Judging Inequalities:	
Say, **Now we are going to play a game using our thumbs. I want you to tell me if the inequality makes sense or not.** (click) **Give a thumbs up to the camera when you see an inequality where the inequality symbol is eating the bigger number.** (click) **And give a thumbs down to the camera when you see an inequality symbol eating the smaller number. Let's try one before we begin.**	
2 is bigger and the inequality symbol is eating 2. This makes sense so we give a thumbs up. Can you show me a thumbs up? After participant responds, **Great.**	Example 
Here, 2 is still bigger but the inequality symbol is eating 1. This does not make sense so we give a thumbs down. Can you show me a thumbs down? After participant responds, **Great. Let's start.**	Example 
Example Item: Say, **Thumbs up or thumbs down?**	
Completing Inequalities using Symbols:	

Say, **Now we'll play another game. Just like we did earlier, I want you to pick the inequality symbol that makes a good inequality. A good inequality is one where the inequality symbol is eating the bigger number. Again, name the colour that has your answer. Let's start.**	
Example Item: Say, **Pick the inequality symbol that goes in the red box?**	
Reading Inequalities:	
Say, **Now I am going to show you some more inequalities and I'd like you to read them, just like we did before. Let's start.**	*Blank slide
Example Item: Say, **Read the inequality.**	
Completing Inequality using Numbers:	
Say, **For this next math game, I am going to show you some inequalities that are missing numbers. I want you to pick the number that makes a good inequality. Just like before, name the colour that has your answer. Let's start.**	

Example Item: Say, **Which one makes a good inequality?**	
<u>Number Line:</u>	
Say, **You're doing great work. We are going to use this number line to play one last math game. This is a number line and it shows all of the numbers from 0 to 10.** (click) **On this side we start with small numbers like zero, one, two,** (click) **and as we move up our number line we get bigger and bigger numbers** (click). **Use the number line to help you answer my questions.**	
Example Item: Say, **Name two numbers on this number line that are more than 5?**	

Note. Used rich concrete condition in above protocol for the example slides. All example items are the first items of each measure. "Click" (in red) corresponds to activating an animation within the slide.