Investigating the impact of thermal bridging calculation methods on building energy performance – a comparative study

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Abstract

Investigating the impact of thermal bridging calculation methods on building energy performance – a comparative study

Bahareh Jahangiri

Current energy building codes and standards commonly consider the impact of thermal bridging in assessing building envelope performance. This research examines four thermal bridging calculation methods and provides a comparative study of the challenges, limitations, and effectiveness of each method to evaluate heating and cooling energy demands.

Each calculation method is applied to 21 different residential buildings covering a variety of building archetypes in Montreal. The equivalent envelope thermal resistance values are reported, and the impact on annual heating and cooling demand is evaluated using building performance simulation.

The results show that the underestimation of annual heating demand could reach 37% when the impact of linear thermal bridges is ignored. The annual cooling demand is also shown to be overestimated by 14%. In addition, the variation in Window to Wall Ratio (WWR) and Vertical Surface Area per Floor Area Ratio (VFAR) are highly correlated to the heating and cooling energy demand deviation.

Keywords: Thermal Bridging Calculations; Energy Codes; Heating and Cooling Energy Demand; Building Energy Modeling; Energy Performance

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Contribution

This study highlights the importance of selecting the proper thermal bridging calculation method for policymakers and authors of building energy codes and standards. The results bring awareness to engineers and architects to consider building physical characteristics when applying different thermal bridging calculation methods as suggested by building energy codes.

Table of contents

List of Figures

List of Tables

Nomenclature

Symbols	Definition	Unit
A_{total}	The total opaque wall area	m^2
b	Width	m
d	Thickness	m
H_T	Transmission heat transfer coefficient	W/K
H_D	Direct transmission heat transfer coefficient	W/K
H_U	Transmission heat transfer coefficient through unconditioned space	W/K
L	Length	m
R	Thermal Resistance	m^2K/W
R_{se}	External surface resistance	m^2K/W
R_{si}	Internal surface resistance	m^2K/W
U	Thermal transmittance	$W/(m^2 \cdot K)$
θ	Celsius temperature	°C
λ	Design thermal conductivity	W/(m.K)
φ	Heat flow rate	W
ψ	Linear thermal transmittance	W/(m.K)
χ	Point thermal transmittance	W/K

Acronyms	**Definition**
ACEEE	American Council for an Energy-Efficient Economy
ASHRAE	American Society of Heating, Refrigeration and Air Conditioning Engineers
BETBG	Building Envelope Thermal Bridging Guide
DOE	U.S. Department of Energy
EPBD	Energy Performance of Buildings Directive
HP	Heat Pump
HVAC	Heating, Ventilation, and Air Conditioning
HWL	Hot Water Loop
MH	Morrison Hershfield
MNECB	Model National Energy Code for Buildings
MURBs	Multi-unit residential buildings
NCR	National Research Council of Canada
NECB	National Energy Code of Canada for Buildings
NBC	National Building Code
PkSZ	Package Single Zone
PkVarVol	Package Variable Volume
PTAC	Package Terminal Air Conditioning
VFAR	Vertical Surface Area per Floor Area Ratio
WWR	Window to Wall Ratio

1. Introduction

1.1 Background

The first building thermal insulation requirements date back to the 1950s in Europe (Smeds, 2004). However, most of the changes with an accelerated pace were put in place in the 1970s. By the 1980s the impact of these regulations on reducing energy consumption became evident. The first Energy Performance of Buildings Directive (EPBD) was published in 2002 and it was the first common legislative procedure that was adopted in Europe not only for energy saving but also for CO_2 emission reduction (Papadopoulos, 2016).

In Canada, the first model code was developed in 1997 by the National Research Council of Canada (NCR) named Model National Energy Code for Buildings (MNECB). In 2011 the last update came with the new name as National Energy Code of Canada for Buildings (NECB) that introduced the minimum energy efficiency level outline with 25% more savings compared to its predecessor (Government of Canada, 2019). The NECB elaborates on five building design key factors, the first one introduces the building envelope efficiency requirements by implementing the minimum envelope thermal resistance values for different climate zones and complimentary calculation methods (NRC, 2017). Up to now, building codes and standards are updating their requirements for building envelope efficiency especially in countries with cold climates and high Heating Degree Days. The most recent required values for envelope thermal transmittance of some cities with a comparatively high HDD are shown in table 1 (Boermans & Petersdorff, 2007).

Table 1: Maximum thermal transmittance values for cities in the cold climate

City	Country	Degree Days Heating/Cooling	U-Value requirements (W/m^2K)		
			Wall	Roof	Floor
Montreal	Canada	4470/143	0.25	0.18	0.18
Tallinn	Estonia	4760/14	0.25	0.16	0.25
Helsinki	Finland	4898/16	0.25	0.16	0.25
Riga	Latvia	4430/41	0.25	0.20	0.25
Vilnius	Lithuania	4339/50	0.20	0.16	0.25
Stockholm	Sweden	4210/43	0.18	0.13	0.15
Copenhagen	Denmark	3720/22	0.20	0.15	0.12

However, improvement of the thermal insulation levels of the building envelope to meet the building energy code requirements cannot ensure the envelope efficiency and might also increase the relative magnitude of heat flow through thermal bridges (Papadopoulos, 2016). For example, the linear thermal bridge can increase the heat transmittance through the envelope up to 30% and even higher (Theodosiou et al., 2015). In such a manner, incorporation of the impact of thermal bridges into the building codes are of great importance to achieve high accuracy in the estimation of heat flow through the building envelope (Theodosiou et al., 2021). Different methods have been implemented all around the world specially in Europe with a wide range of different assumptions, simplification, and details (Roels et al., 2011; ISO, 2017a; ISO, 2017b; ISO, 2017c; OIB, 2011; ÖNORM, 2019) and in some countries, several methods are accepted (Erhorn et al., 2010). Table 2 demonstrates different standards that are elaborating on thermal bridging calculation in some cities in cold climate. By comparing table 1 and 2, it can be deducted that even though there are similarities in maximum envelope thermal transmittance values in different countries in cold climate, their thermal bridges calculation methods are completely different (Eichhammer & Schlomann, 2005).

Table 2: Standards of thermal bridging calculation in different countries in the cold climate

Country	**Title**	**Building Types**	**Details**	**Flexibility**
Canada	Building Envelope Thermal Bridge Guide (MH, 2018a).	All types	> 1000	No
Estonia	EN ISO 14683: Thermal bridges in building construction – Linear thermal transmittance – simplified method and default values (ISO, 2017c).	Residential	76	No
Finland	National Building code of Finland Sections: C3, D3, and D5 (Finnish Government, 2017).	New Buildings	All	Yes
Lithuania	Building regulation STR 2.05.01: Thermal technique of the building envelope, Annex 7 (TAR, 2005).	All types	200	Yes
Sweden	EN ISO 10211: Thermal bridge in building calculation – Heat flows and surface temperature – Detailed calculation (ISO, 2017a).	All types	All	Yes
Denmark	Danish Standard 418. Calculation of heat loss from buildings (Dansk Standard, 2011).	All types	> 275	No

The variety among thermal bridging calculation methods terminates to inconsistency in equivalent thermal resistance values and consequently decreases the accuracy of building energy modeling by underestimating or overestimating the building peak loads and total energy demand.

In Canada, Individual provinces are responsible for setting their energy efficiency requirements based on model codes such as NECB, ASHRAE, and so on (Laustsen, 2008). Thus, thermal bridging calculation methods vary from one province to the other. Unfortunately, the number of comparative studies on these methods is limited

1.2 Problem Statement

Building energy codes and standards are considering adopting thermal bridging calculations in evaluating the building energy performance. The different calculation methods may offer a trade-off between accuracy and the effort required (Capozzoli et al., 2013). This is creating a challenge to select the most appropriate method considering the difference in climate, building type, common construction details, etc. At present, there is a lack of study to evaluate the impact of applying different thermal bridging calculation methods on different archetypes in different climates and to demonstrate the potential deviation in building energy performance as a result. This research conducts a comparative study to investigate the impact of the application of four main different methods on local buildings allows stakeholders to have an insight for further modifications and updates.

1.3 Research Objectives

The main objective of this research will be to investigate the impact of implementing different thermal bridging calculation methods on Multi-unit Residential Buildings (MURBs) energy performance in Montreal. Afterwards, conducting a comparative study to quantify the discrepancy among different calculation methods and identifying building physical characteristics where the discrepancies are more pronounced. At the end, assessing the deviation in annual building energy demand resulting in application of different thermal bridging calculation methods are of the other objectives.

1.4 Dissertation Outline

The rest of this study is as ordered as follows:

Chapter 1 provides a background about the importance of thermal bridge calculation methods and their application to building codes. This chapter also discusses the problems that need to be considered in this study and the objectives of this study.

Chapter 2 reviews the literature which outlines the impact of implementing the thermal bridge calculation methods by building codes and standards, differences among these methods with an emphasis on cold climate requirements. Lastly, the potential areas and scope of this research are presented.

Chapter 3 presents the proposed methodology for this research, identifies the design parameters and the prototype building models, and investigates the local construction detail designs.

Chapter 4 presents a comparative case study buildings for the application of different thermal bridge calculation methods on typical MURBs in Montreal including low-rise, mid-rise, and high-rises. The approach for including thermal bridge impact on energy modeling is also detailed here. Finally, the variables to be considered for each building type are presented.

Chapter 5 presents and discusses the results of the comparative case studies.

Chapter 6 provides a summary of this study with conclusions and future research opportunities.

2. Literature Review

2.1 Thermal Bridge Types

Thermal bridges are defined as areas with a higher thermal conductivity than their adjacent areas. Besides changes in heat flow rate, changes in inner surface temperature are the main consequences of thermal bridges. Thermal bridge occurs where:

- There is a temperature difference between inside and outside (ISO, 2017a).
- A material with higher thermal conductivity pass through the insulation layer, usually they are included in the overall clear field thermal resistance calculation.
- There is a punctual penetration of construction elements into the insulation layer (These point thermal bridges are ignored in many national energy performance calculation procedures).
- There is an air movement within the construction, between construction and inside, or between construction and outside but not all through inside to outside (Erhorn et al., 2010).
- Where the geometry of the building changes and the heat-absorbing surface is bigger than the heat-emitting surface, mostly at corners (Schöck, 2018).

The most typical thermal bridges are known as Linear Thermal Bridge and Point Thermal Bridge.

2.1.1 Linear Thermal Bridge

Linear thermal bridges can be defined as disturbances in building envelope along a linear length. The energy loss incurred by them is described as Linear Thermal Transmittance Coefficient (Ψ) in W/m.K (MH, 2018a). Common linear thermal bridges are located at:

- The junction between external surfaces (Corners of exterior walls, wall to roof, wall to floor).
- The junction of internal walls with external walls and roofs.

- The junction of intermediate floors and projection of slabs with external walls.
- The perimeter of openings (ISO, 2017c).

2.1.2 Point Thermal Bridges

Point thermal bridges are known as an individual punctual discontinuity in insulation layer that occurs by penetration of structural elements. Point Thermal Transmittance Coefficient (χ) in W/K is used to measure the energy loss through the point thermal bridges (MH, 2018a).

2.2 Influence of Thermal Bridging Calculation on Building Energy Performance

Many comparative studies have been done all around the world to investigate the impact of thermal bridge calculation on building energy performance. In this section the previous studies are presented in the following order:

First, primary comparative studies that are elaborating on including thermal bridge impact into building simulation are presented. In these studies, different simulation scenarios with and without thermal bridge impact are proposed (Pelss et al., 2010). For these studies, the method of thermal bridge calculation is not objective and that is why in some research they don't even mention the method. For example, in 2008, Theodosiou et al. (Theodosiou et al., 2008) made a research to investigate the impact of thermal bridges on building energy demand in Greece. They examined four different scenarios for wall composition once with including the thermal bridges and once without it. A typical three-story building has been chosen and as been discussed, intermediate floors had lower heating demand while the first floor by having more exposed area and less solar irradiation had the highest heating demand. For cooling, the last story presented the highest cooling demand by high solar irradiation received by flat roofs. The research showed an increase in annual heating demand of approximately 30% for high insulated buildings and around 16% for poorly insulated ones. Regarding the climate of central Europe, the impact of the thermal bridges on annual cooling demand was negligible.

After that, when the impact of the thermal bridge on building energy performance became evident, the limitation of analysis tools and simulation software for calculation and applying this impact became an interest of the researchers (Nagata, 2005). Most of the common energy simulation software cannot see the impact of thermal bridges where they happen and that's why most of the time the equivalent U-value in 1D is given to the whole wall instead of seeing the impact of each thermal bridge separately. Ge et al. (2015), conducted a comparative study between applying the equivalent U-value and the 3D dynamic simulation model. they record an 8% to 13% increase in annual heating load in a cold climate for 3D dynamic simulation compared to the equivalent U-value method. However, the 3D dynamic simulation method provides more accurate results (Quinten & Feldheim, 2016), because of time-consuming and complexity, the equivalent U-value method is more popular.

As the 3D dynamic simulation of thermal bridges cannot be done easily, many attempts and research are done to explore different calculation methods that can provide acceptable accuracy. By increasing the suggested methods for thermal bridge calculation in national and international codes and standards, comparing these methods and evaluating their impact on building energy performance became necessary. Bergero et al. (2018) studied more than forty types of thermal bridges. They compared the impact of implementing two different calculation methods. One, 2D numerical simulation by THERM which is based on detailed calculation method suggested by ISO 10211, and the other, the thermal bridge catalog based on Thermal Bridge National Abacus which has been modeled by EC700 software (Baba & Ge, 2016). The results showed about 8% more heat exchange through thermal bridges by using THERM software and up to 12% more energy demand per unit area. These comparative studies are usually done by modeling only one typical building which can help to exclude other variables and only focus on thermal bridge impacts (Theodosiou et al., 2021). However, the results cannot be a reference for other buildings with different physical characteristics and mechanical systems.

There are some other limitations regarding the thermal bridge calculation. For example. based on an overview of building code requirements for thermal bridges (Erhorn et al., 2010), it has been noticed that lots of codes are only applicable to new buildings and a few are concerning the existing buildings and renovation projects. Assessing retrofit solutions in thermal bridge reduction for existing buildings is of great importance which is a recent topic in this field (Aelenei et al., 2021; Bergero et al., 2017; Kotti, 2017). Moreover, all the above-mentioned research highlights the notable impact of thermal bridges, especially on annual heating demand. However, the impact should be investigated on a bigger scale such as carbon emissions, energy bills, construction cost increase, and payback periods than just the energy demand and this is how the real impact of thermal bridge reduction can be evaluated. In some cases, construction cost increase with a very long payback period might counterbalance the energy saving from thermal bridge reduction (Evola et al., 2011).

Another challenge in reviewing previous studies is the publication date. By increasing the concern about thermal bridge impact, many updates on building code requirements are released frequently and that is why for better understanding the previous works other than the methodology that has been used, the year of publication is important as well. As we see in table 6, not every standard has the same definition of thermal bridges. Moreover, the types of thermal bridges might change over time with each update. Thus, for publications, indicating every detail in the calculation process has high importance.

2.3 Review on Calculation Methods

Applying the impact of thermal bridges in building envelope thermal resistance calculation is part of building code requirements in most European countries however, in southern Europe it is only recommended, and simplified methods are accepted (Papadopoulos, 2016). In Canada, each province is responsible to set the regulations to calculate thermal bridges. British Columbia was the first province that established the regulations for thermal bridges. Ontario is using the same principle as BC and Quebec recently published a draft and suggested another method for calculating the impact of thermal bridges. As the thermal bridge calculation will be applied in Quebec in December 2021 and still few provinces don’t have any regulation concerning it, this section introduces main calculation methods and some of the national and international standards that are following these methods.

2.3.1 Default Value Method

This method has the advantage of using simplified calculations to estimate the direct heat transfer coefficient through the building envelope (H_D).

$$H_D = \Sigma_i A_i U_i + \Sigma_k l_k \Psi_k + \Sigma_j \chi_j \qquad (1)$$

Where:

A_i	Area of component i of the building envelope,	m^2
U_i	Thermal transmittance of component i of the building envelope,	$W/m^2.K$
l_k	The length of linear thermal bridge k,	m
Ψ_k	The linear thermal transmittance coefficient of linear thermal bridge k,	W/m.K
χ_j	The point thermal transmittance coefficient of point thermal bridge j,	W/K

The main difference of this method from the others is that instead of calculating the linear and point thermal transmittance coefficients (Ψ and χ), it uses the default values that are considering the worst-case scenarios hence, this method has the least accuracy among all others. However, by using default values of linear and point thermal transmittance where the details are not yet designed, but the dimensions and main geometry of the building are defined such as area of roofs, walls, and floors, a rough estimation of the contributions of thermal bridges to overall heat loss can be made easily. This method is not valid for the estimation of the surface temperature and condensation. ISO 14683 (ISO, 2017c) is the most popular international standard that suggests this method for heat transfer calculation through the building envelope. NECB 2015_amended by Quebec (*NRC, 2020)* also provides a similar method.

2.3.1.1. ISO 14683

ISO 14683 can increase the average U-value of the building from 8% to 30% depending on building geometries (Ge et al., 2013). This method ignores the impact of point thermal bridges and identifies the linear thermal bridges as:

- Junction between external components of the envelope (wall to roof, wall to floor, wall to wall or corners).
- Junction between internal partitions and external elements such as walls and roofs.
- Junction of intermediate floors and exterior walls.
- Where there is a column in exterior walls.
- The perimeter of windows and doors.

and for calculation of the length of linear thermal bridges consider three different scenarios:

- Internal dimensions (i): measured between the edge of the internal faces of each room (the thickness of interior partitions is not included).
- Overall internal dimensions (oi): measured between the edge of internal faces of the whole building (the thickness of interior partitions is included).

- External dimensions (e): measured between the edge of external faces of the building elements.

The default values for Ψ are given based on the location of the insulation layer in building envelope components. These values also are referring to the maximum effect of thermal bridging. Figure 1 presents the Ψ values for wall/projection junctions (Balconies):

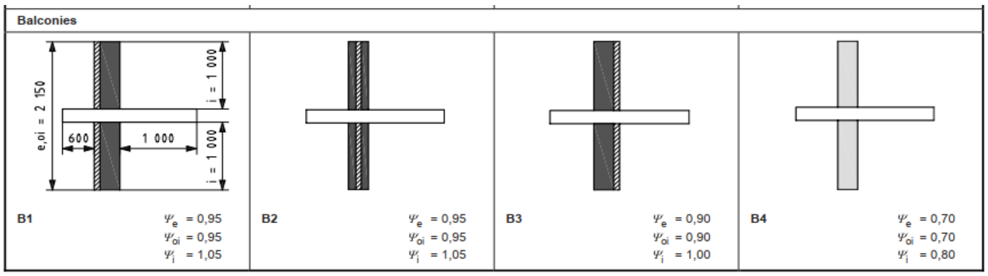


Figure 1: Default values of linear thermal bridge coefficient for balconies (ISO, 2017c)

2.3.1.2. NECB 2015_amended by Quebec

The method suggested by Quebec is even simpler than the method provided by ISO 14683. For example, to measure the length of the linear thermal bridge only the external dimensions are considered, the default values for junctions are not dependent on the location of the insulation layer, and only four main junctions of the building envelope are considered as thermal bridges:

- Junction between exterior wall and roof.
- Junction between exterior wall and intermediate floors.
- Junction between exterior wall and foundation.
- Junction between exterior wall and floor projection.

The equivalent thermal resistance of the above-ground opaque building elements is calculated using the equation below:

$$RSI_{Eq} = \frac{1}{\frac{\sum_{m=1}^{m}(\Psi_m . L_m) + \sum_{n=1}^{n}(\chi_n . N_n)}{A_i} + \frac{1}{RSI_{Ef}}} \tag{2}$$

Where:

RSI_{Eq}	Equivalent thermal resistance of opaque building component,	$(m^2.K)/W$
Ψ_m	The linear thermal transmittance coefficient of linear thermal bridge m,	W/m.K
L_m	The length of linear thermal bridge m,	m
M	Total number of junctions,	-
χ_n	The point thermal transmittance coefficient of point thermal bridge n,	W/K
N_n	Number of type n of point thermal bridges,	-
N	Total number of point penetration types,	-
A_i	Area of component i of the building envelope,	m^2
RSI_{Ef}	Effective thermal resistance of the opaque building component before applying the thermal bridge impact.	$(m^2.K)/W$

The value suggested for the point thermal bridge coefficient is 0.5 W/K and the values for linear thermal bridge coefficient is presented in table 3.

Table 3: Default linear thermal transmittance coefficients

Intersection	**Ψ, in W/(m.K)**
Wall / Roof	0.325
Wall / Intermediate Floor	0.300
Wall / Projection	0.500
Wall / Foundation	0.450

The effective thermal resistance of above-ground opaque parts should comply with the values in table 4 for each climate zone:

Table 4: Effective thermal resistance of above-ground opaque building assemblies

Above-Ground Opaque Building Assembly	Heating Degree-Days under 18°C of Building location in Celsius Degree-Days					
	Zone 4: < 3000	Zone 5: 3000 to 3999	Zone 6: 4000 to 4999	Zone 7A: 5000 to 5999	Zone 7B: 6000 to 6999	Zone 8: ≥ 7000
	Minimum Effective Thermal Resistance, RSI in $m^2.K/W$					
Walls	3.6	3.60	3.60	3.60	4.05	4.05
Roofs	5.46	5.46	5.46	5.46	6.17	6.17
Floors	5.46	5.46	5.46	5.46	6.17	6.17

2.3.2 Advanced Numerical Calculation

The advanced numerical calculation method is known as one of the most accurate ones that is valid for further calculation of condensation and surface temperatures. This method provides the most accurate value of Ψ when all construction details are known and there is precise information on junctions. ISO 10211 (ISO, 2017a) is the main international standard that provides detail about this method. 1D, 2D, and 3D heat transfer are taken into account to estimate total heat transfer through the building envelope. Figure 2 demonstrates this partitioning.

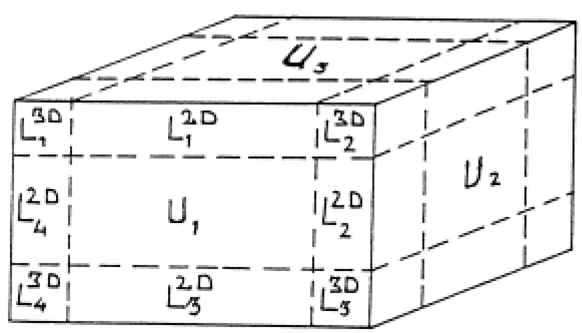


Figure 2: 1D, 2D, and 3D partitioning of the building envelope (ISO, 2017a)

Thermal coupling coefficient L_{ij} which is defined as heat flow per temperature difference of thermally connected environment of i and j is calculated using the equation below:

$$L_{i,j} = \sum_{n-1}^{N} L_{n(i,j)}^{3D} + \sum_{m-1}^{M} L_{m(i,j)}^{2D} . l_m + \sum_{k-1}^{K} U_{k(i,j)} . A_k \quad (3)$$

Where:

$L_{n(i,j)}^{3D}$	3D thermal coupling coefficient,	W/K
$L_{m(i,j)}^{2D}$	Linear (2D) thermal coupling coefficient,	W/m.K
l_m	The length of linear thermal bridge m,	m
$U_{k(i,j)}$	Thermal transmittance of the k opaque part of the envelope from 1D calculation	W/(m^2.K)
A_k	Area where the value U_k applies,	m^2
N	Total number of 3D intersections,	-
M	Total number of 2D intersections,	-
K	Total number of 1D assemblies	-

The software "THERM" developed at Lawrence Berkeley National Laboratory (LBNL, 2019) is using the same principles as ISO 10211 for its calculations. Given the complexity of this method, its application should be weighted against the building cooling load. For example, in the Mediterranean climate where cooling is dominant, the effort invested in the thermal bridging calculation should be in balance with the total energy savings (Capozzoli et al., 2013).

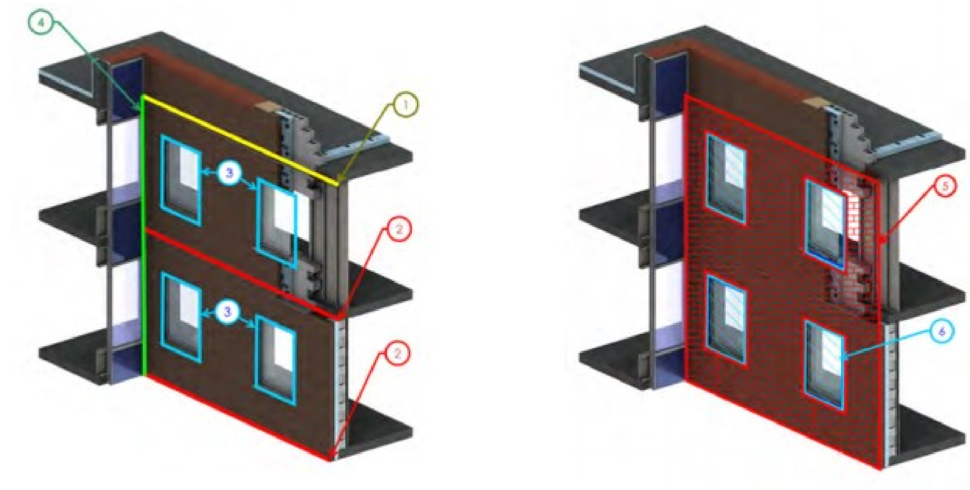
However this method is one of the most accurate ones among the others, it still has some limitations hence, some people developed more detailed calculation methods using nonlinear regression models (Larbi, 2005).

2.3.3 Catalog Method

The thermal bridge catalog provides a variety of construction details and their actual thermal transmittance values coming out of laboratory experimental analysis (Martin et al., 2012) or simulations. This method has a high accuracy only if the same construction detail has been used in the building otherwise the overall accuracy varies up to ±20%. On the other hand, this method is not flexible enough to be able to meet different requirements because it has been made for only specific materials and dimensions. Building Envelope Thermal Bridge Guide (BETBG) (MH, 2018a) is the most well-known thermal bridging catalog in North America that is based on 2D heat transfer simulation.

2.3.3.1. Building Envelope Thermal Bridging Guide (BETBG)

BETBG version 1.0 was published in October 2014 for the first time. The most recent update came out in December 2020. BETBG with more than hundreds of details and assemblies as the Canada-wide resource supports the industry acknowledging the impact of thermal bridging in building energy codes and in practice. This guide can be applied to all Canadian climates and includes details that are meeting the net-zero building requirements. The catalog provides a database including different construction details for wood, steel, and concrete structures, curtain walls, and glazing systems. The definition and calculation of the area and length of building elements are shown in figure 3.

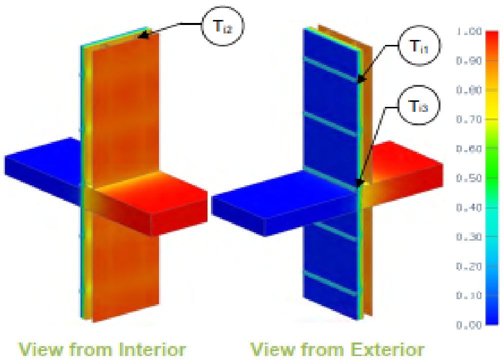


1. Parapet Length
2. Slab Lengths
3. Wall to Window Transition Lengths
4. Corner Length
5. Opaque Brick Wall Area
6. Glazing Area

Figure 3: Building envelope area and length takeoffs (MH, 2018a)

Detail 5.2.40

Exterior Insulated 3 5/8" x 1 5/8" Steel Stud (16" o.c.) Wall Assembly with Horizontal Z-girts (24" o.c.) Supporting Metal Cladding – Isokorb K65-V8 Thermally Broken Slab Projection with Insulated Curb



Thermal Performance Indicators

Assembly 1D (Nominal) R-Value	R_{1D}	R-3.2 (0.56 RSI) + exterior insulation
Transmittance / Resistance without Anomaly	U_o, R_o	"clear wall" U- and R-value, without slab
Transmittance / Resistance	U, R	U- and R-values for overall assembly
Surface Temperature Index[1]	T_i	0 = exterior temperature 1 = interior temperature
Linear Transmittance	ψ	Incremental increase in transmittance per linear length of slab

[1]Assumptions and limitations for surface temperatures identified in ASHRAE 1365-RP

Nominal (1D) vs. Assembly Performance Indicators

Exterior Insulation 1D R-Value (RSI)	R_{1D} ft²·hr·°F / Btu (m² K / W)	R_o ft²·hr·°F / Btu (m² K / W)	U_o Btu/ft² ·hr ·°F (W/m² K)	R ft²·hr·°F / Btu (m² K / W)	U Btu/ft² ·hr ·°F (W/m² K)	ψ Btu/ft hr °F (W/m K)
R-15 (2.64)	R-18.2 (3.20)	R-11.3 (1.99)	0.088 (0.50)	R-9.7 (1.71)	0.103 (0.58)	0.143 (0.248)

Temperature Indices

T_{i1}	0.75	Min T on sheathing away from slab, between studs at girts
T_{i2}	0.86	Max T on sheathing away from slab, between girts at studs
T_{i3}	0.78	Min T on slab exposed to interior air, at sheathing between studs

Figure 4: Thermal performance of wall/projection junction (MH, 2018a)

This method uses equation 2 for heat transfer calculation and the details have been modeled regarding the requirements of ISO 10211 by THERM software. Manufactures proprietary systems, ASHRAE 90.1 “Energy Standard for Building Except Low-Rise Residential” (ASHRAE, 2016), and ISO 14683 are the other information sources of this catalog. Figure 4. presents a detailed assessment of the thermal transmittance of each element of junction between an exterior wall and floor projection. This method by providing more than hundreds of details and taking advantage of the advanced numerical calculation method of ISO 10211 and 2D heat transfer modeling is considered the most accurate method and consequently the baseline in this study.

2.4 Summary

This chapter presented different calculation methods, due to this variety, knowing the advantages, disadvantages, and limitations of each method makes the direction of further studies clearer. Unfortunately, the number of studies for the cold climate is limited while this climate is more susceptible to thermal bridge impact not only on building energy performance but also on condensation and mold growth (Batty et al., 1984). Table 5 summarizes these methods and their specifications. It is worth mentioning that two different standards using the same calculation methods might have different definitions of thermal bridges and their types. Therefore, the equivalent thermal resistance values that have been calculated will be different. Table 6 shows the thermal bridge types of this study’s standards. Other than the above-mentioned methods, for some specific projects, full-scale experimental studies can be conducted to evaluate the actual thermal bridge behavior in the façade (Garay et al., 2014).

Table 5: Comparison between thermal bridging calculation methods

Method	Accuracy	Codes and Standards	Advantages	Disadvantages
Advanced Numerical Calculation / 2D Heat Transfer Modeling	±5%	ISO 10211 BETBG	More accurate Valid for surface temperature calculation Valid for condensation assessment Flexibility	Complexity Time-consuming Requires detailed information
Catalog	±20%	-	Accurate only if the same detail is used in building Quick No need to calculation	Not flexible Limited details only Need to be updated frequently
Default Values	0% - 50%	ISO 14683 NECB 2015-amended	Quick Simplified calculation Suitable for rough estimation at the early design stage	Does not consider all the thermal bridges Consider the worst-case scenario Does not consider efficient details
Manual Calculation	±20%	-	No need for any software or computer skills Efficient for simple details and 1D heat transfer calculation	Requires good knowledge of heat transfer calculations Time-consuming Only applicable for 1D and 2D analysis
Laboratory Assessment	<±5%	-	Accuracy Flexibility	Time-consuming Costly Requires advanced assessment tools

Table 6: Comparison of thermal bridge types in selected calculation methods

Thermal Bridge Types	No Linear Transmittance (NECB 2015)	Default Value Method (NECB 2015_amended)	Default Value Method (ISO 14683)	2D Heat Transfer Modeling (BETBG)
Structural elements (Studs, Anchors, etc.)	✓	✓	✓	✓
Wall / Roof	-	✓	✓	✓
Wall / Intermediate Floors	-	✓	✓	✓
Wall / Projection (Balcony)	-	✓	✓	✓
Wall / Foundation	-	✓	✓	✓
Opening Perimeter	-	-	✓	✓
Corners	-	-	-	✓
Internal Walls	-	-	✓	✓

Adding or ignoring each junction in thermal bridging calculations have a noticeable impact on final equivalent thermal resistance values. Some studies investigate the heat loss through a specific junction to determine its role in envelope efficiency. Lee S. et al. (2007), Coburn S. (2011), and Bienvenido-Huertas D. et al. (2019) evaluated the impact of corners, window related thermal bridges, and junction between pillar and wall respectively.

3. Methodology

Following the literature review and knowing the challenges and limitations of thermal bridging impact assessment, the design methodology is incorporated in a comparative case study of 21 MURBs in Montreal. All buildings are selected based on NECB 2015 requirements, including new residential buildings with more than three-floor stories, more than 600 m^2 area, enlargement projects, or major renovation. Thus, single-family detached houses are beyond the scope of this research.

Four different building energy codes and standards are employed for building envelope thermal resistance calculations. The first energy code only considers thermal bridges in the opaque clear field wall composition like studs, beams, anchors, and so on but ignores the linear and point thermal bridges concerning the geometry of the buildings (NECB 2015). In the second and third energy codes, the impact of linear and point thermal bridges are taken into account with the aid of the default value method (NECB 2015_amended and ISO 14683). However, both are representing the same method, their definitions and default values for each thermal bridge type are different. The last code takes all the details and possible thermal bridge types into consideration by providing a large number of 2D heat transfer modeling outputs as a catalog (BETBG). This method by providing more than hundreds of details and taking advantage of the advanced numerical calculation method of ISO 10211 and 2D heat transfer modeling is considered the baseline in this study.

3.1 Equivalent RSI Calculation

To estimate the equivalent RSI values of the envelope, it is necessary to see each individual thermal bridge detail. For all four scenarios, the clear field Effective RSI is calculated base on ISO 6946 (ISO, 2017d), this method considers the impact of structural elements where these elements with higher thermal conductivity pass through the insulation layer or interrupt its continuity. For example, columns, beams, studs, and façade fixtures are the most common thermal bridges in clear field effective RSI calculation. NECB 2015 provides the same methodology. However, this method cannot meet all the requirements for thermal bridging calculation as it only elaborates on thermal bridges in opaque clear field areas and does not consider all other thermal bridges that are located at the junctions and intersection of exterior surfaces. The impact of linear and point thermal bridges are added to the Effective RSI with the aid of equation (2). The linear (Ψ) and point (χ) thermal bridge coefficients are picked from the tables in energy codes for the default value method or are found in the catalog that represents the 2D heat transfer analysis results of the junction.

Table 7 demonstrates these differences for only one junction between the exterior wall and intermediate floor detail. The difference in equivalent RSI in a situation that only one junction has been taken into account is noticeable, this difference when all the thermal bridges are counted will be much higher than expected and will impact building energy performance markedly in a cold climate. Picking the right Ψ value for each detail can be challenging knowing that each method has limitations and does not cover all different construction details. For example, NECB 2015_amended only suggests one value for all junctions and does not see the impact of quality of each individual detail design, ISO 14683 and BETBG provide the Ψ value only for limited construction details that sometimes makes it hard to find the same detail. However, this challenge in BETBG is less significant as the number of details with different wall thermal resistance is provided which decreases the risk of discrepancy.

The type of junctions and their length is dependent on building geometry. In this study, the impact of building shape is taken into account by the Vertical Surface Area per Floor Area Ratio (VFAR) (MH, 2018b). VFAR is one of the important factors in the heating energy use of buildings especially when demand intensity is normalized per floor area. The heat loss per unit floor area is higher for buildings with complex and narrow shapes resulting in a higher VFAR. Single-family houses have a VFAR between 1.2 and 1.5, while this number for the high-rise MURBs is from 0.5 to 0.65. This value is calculated for the case study buildings and has the range of 0.29 to 0.84.

Table 7: An example of equivalent RSI calculation with different methods

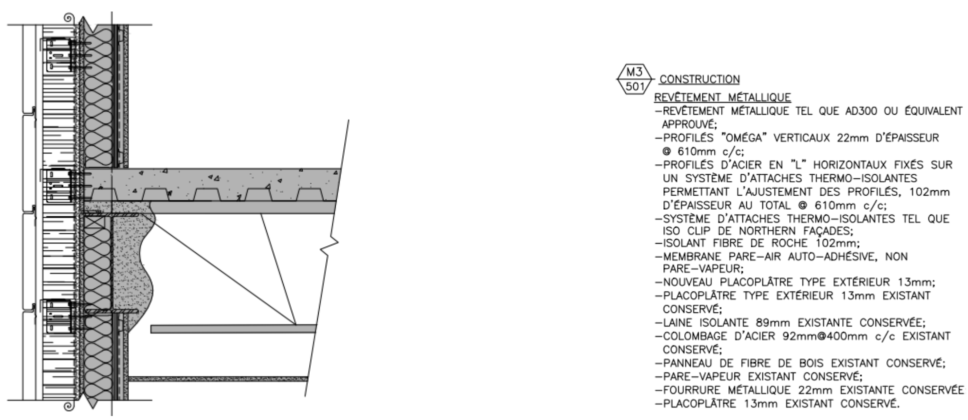
No Linear Transmittance (NECB 2015)

$RSI_{Ef} = 5.20\ m^2.K/W$

$\Psi = 0\ W/m.K$

$$RSI_{Eq} = \frac{1}{\frac{\sum_{m=1}^{m}(\Psi_m . L_m) + \sum_{n=1}^{n}(\chi_n . N_n)}{A_i} + \frac{1}{RSI_{Ef}}}$$

$RSI_{Eq} = 5.20\ m^2.K/W$



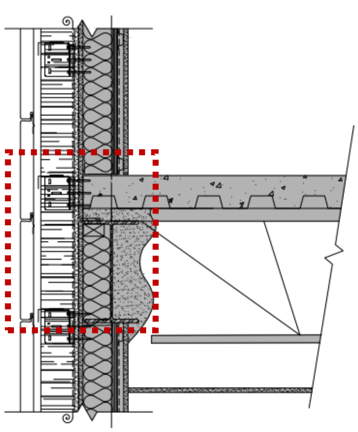
Default Value Method (NECB 2015_amended)

$RSI_{Ef} = 5.20\ m^2.K/W$

$\Psi_{wall/roof} = 0.300\ W/m.K$

$$RSI_{Eq} = \frac{1}{\frac{\sum_{m=1}^{m}(\Psi_m . L_m) + \sum_{n=1}^{n}(\chi_n . N_n)}{A_i} + \frac{1}{RSI_{Ef}}}$$

$RSI_{Eq} = 4.86\ m^2.K/W$



Intersection	Ψ, in W/(m.K)
Wall / Roof	0.325
Wall / Intermediate Floor	0.300
Wall / Projection	0.500
Wall / Foundation	0.450

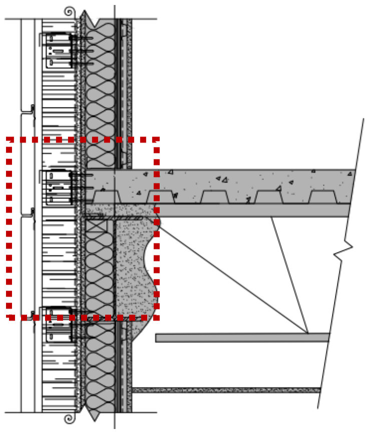
Default Value Method (ISO 14683)

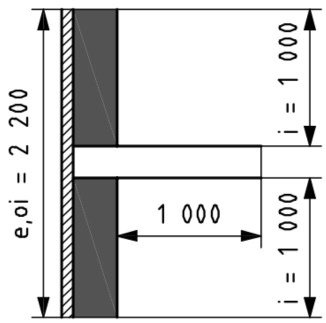
RSI $_{Ef}$ = 5.20 m^2.K/W

$\Psi_{wall/roof}$ = 0.00 W/m.K

$$RSI_{Eq} = \frac{1}{\frac{\sum_{m=1}^{m}(\Psi_m \cdot L_m) + \sum_{n=1}^{n}(\chi_n \cdot N_n)}{A_i} + \frac{1}{RSI_{Ef}}}$$

RSI$_{Eq}$ = 5.20 m^2.K/W





Ψ_e = 0,00

Ψ_{oi} = 0,00

Ψ_i = 0,10

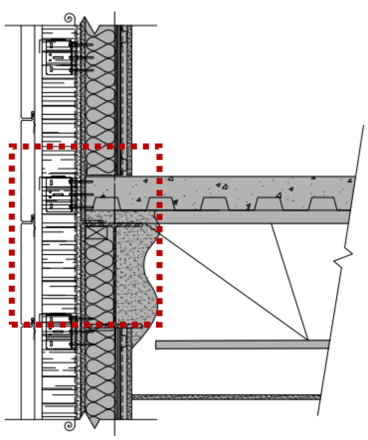
2D Heat Transfer Modeling (BETBG)

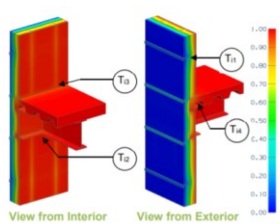
RSI $_{Ef}$ = 5.20 m^2.K/W

$\Psi_{wall/roof}$ = 0.162 W/m.K

$$RSI_{Eq} = \frac{1}{\frac{\sum_{m=1}^{m}(\Psi_m \cdot L_m) + \sum_{n=1}^{n}(\chi_n \cdot N_n)}{A_i} + \frac{1}{RSI_{Ef}}}$$

RSI$_{Eq}$ = 5.01 m^2.K/W





Exterior Insulation 1D R-Value (RSI)	R_{1D} ft^2·hr·°F / Btu (m^2 K / W)	ψ Btu/ft hr °F (W/m K)
R-0 (0)	R-14.2 (2.50)	0.487 (0.842)
R-5 (0.88)	R-19.2 (3.38)	0.177 (0.306)
R-10 (1.76)	R-24.2 (4.26)	0.121 (0.210)
R-15 (2.64)	R-29.2 (5.14)	0.093 (0.162)
R-20 (3.52)	R-34.2 (6.02)	0.079 (0.137)
R-25 (4.40)	R-39.2 (6.90)	0.067 (0.117)

4. Case Study

According to ASHRAE climate zone categories, Montreal is located in Zone 6 where heating degree day (HDD) is between 4000 to 4999. Among the few weather stations located in the region, Montreal International Airport (DOE, 2016) is used for this study as it is quite centrally located and yet not influenced by microclimate of the city center.

In Montreal, the electricity consumption of residential buildings accounts for more than 40% of the total consumption of the city. Figure 5 demonstrates the residential total electricity consumption in GWh from 2007 to 2011 (Ville de Montréal, 2017).

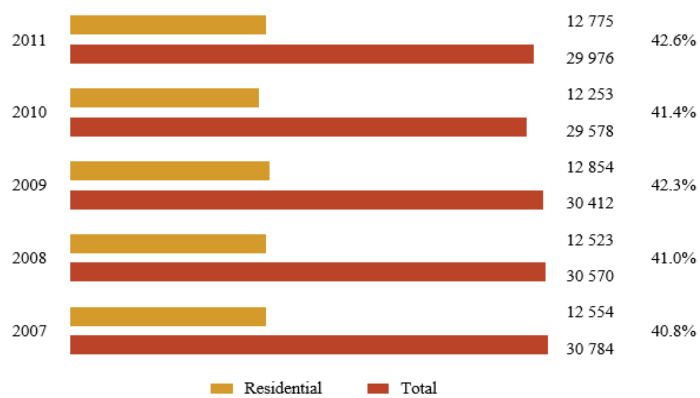


Figure 5: Electricity consumption of residential buildings in Montreal compared with total consumption in GWh

In residential buildings, there are so many factors that have an impact on electricity consumption. Regarding the climate zone of Montreal, many residential buildings do not have air conditioners, and as shown in the figure below for buildings with air conditioning only 3% of the electricity consumption of the building counts for cooling. The majority of consumption in residential buildings accounts for 45%-50% heating, 20% domestic hot water, and 20% electric appliances (Hydro Quebec, 2021).

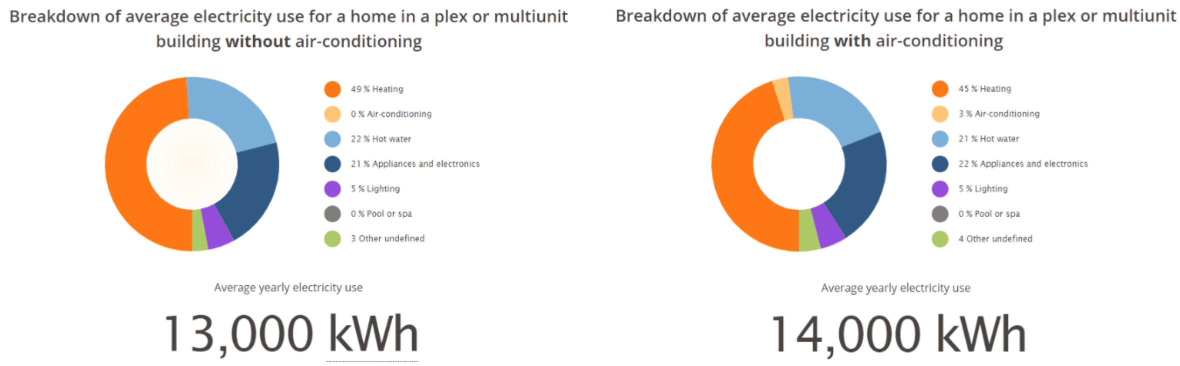


Figure 6: Electricity consumption breakdown of residential buildings (Hydro Quebec, 2021)

Moreover, the geometry of residential buildings and the fact that thermal bridges at balconies have a huge impact on heating peak load and demand (Kotti et al., 2017), are the other reasons for choosing residential buildings as a comparative case study. 21 MURBs in Montreal are chosen to represent common construction types with different areas, height, WWR, VFAR, mechanical system, and energy source. The energy modeling of the buildings is made and calibrated with eQuest (DOE-2, 2009) based on actual technical drawings and energy bills. These models are used to investigate the impact of different thermal bridging calculation methods on equivalent RSI values and consequently on annual energy demand.

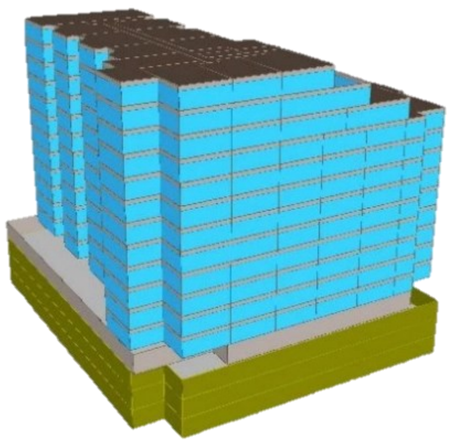
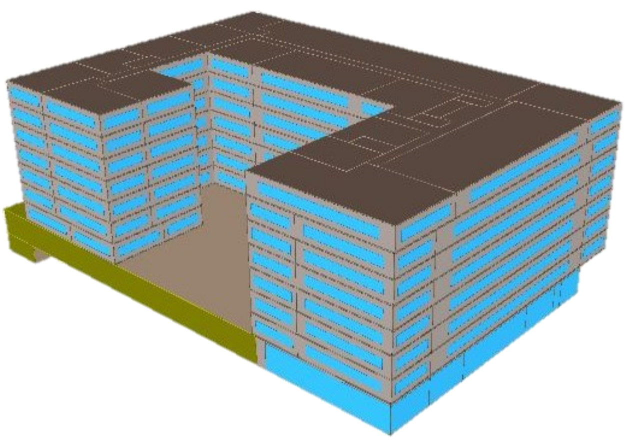
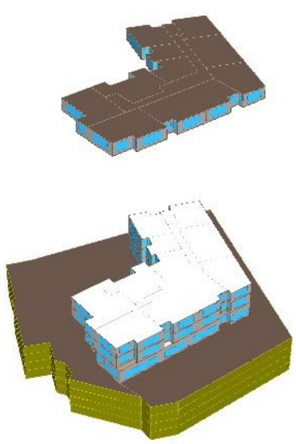
4.1 Whole Building Energy Analysis Using eQuest

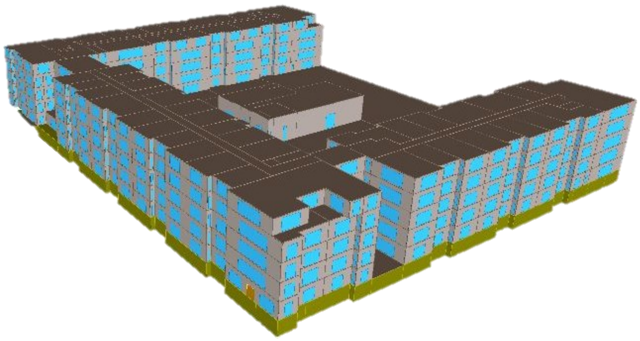
According to the last update of the NECB 2015_amended by Quebec, this code will only apply to MURBs which have more than three-floor stories or an area of 600 m^2 or more. This research respects these requirements and does not include the detached single-family houses. The comparative case study model includes 21 MURBs from two-floor story (with the area of more than 600 m^2) to 26 floor story, all located in Montreal regions, some are newly built, the others are part of enlargement projects, or they had a major renovation. Figure 7 represents Montreal MURBs where the NECB 2015_amended requirements are applicable.

Figure 7: Residential buildings with NECB 2015_amended requirement

Table 8 depicts the case study 3D energy models in the eQuest interface and their building characteristics. The buildings are chosen in a way to be a proper representative of MURBs in Montreal.

Table 8: Comparative case study building characteristics

3D Models	Building Characteristics.
	**Case Study 1:** New Residential Building Number of Floors: 13 WWR: 76% Area: 31380 m^2 VFAR: 29% Mechanical System: Package Terminal Air Conditioning + Baseboard Construction Type: Curtail wall system
	**Case Study 2:** New Residential Building Number of Floors: 8 WWR: 35% Area: 23436 m^2 VFAR: 29% Mechanical System: Package Terminal Air Conditioning + Baseboard Construction Type: Curtain wall system
	**Case Study 3:** New Residential Building Number of Floors: 21 WWR: 43% Area: 23533 m^2 VFAR: 30% Mechanical System: Fan coil Construction Type: Curtain wall system



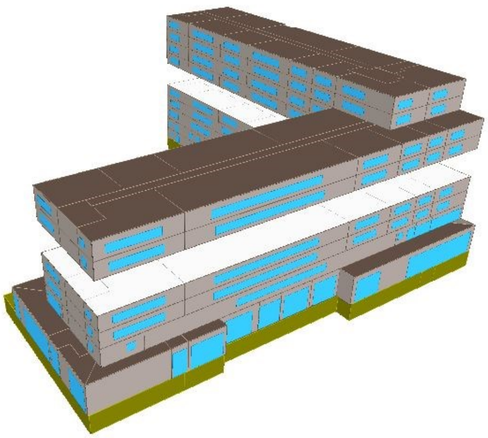
Case Study 4: New Residential Building

Number of Floors: 5 WWR: 28%

Area: 28545 m^2 VFAR: 33%

Mechanical System: Package Terminal Air Conditioning

Construction Type: Curtain wall system



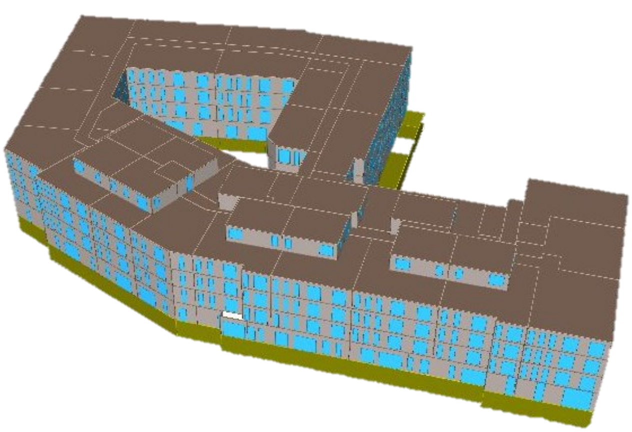
Case Study 5: New Residential Building

Number of Floors: 10 WWR: 33%

Area: 36402 m^2 VFAR: 33%

Mechanical System: Fan coil

Construction Type: Concrete



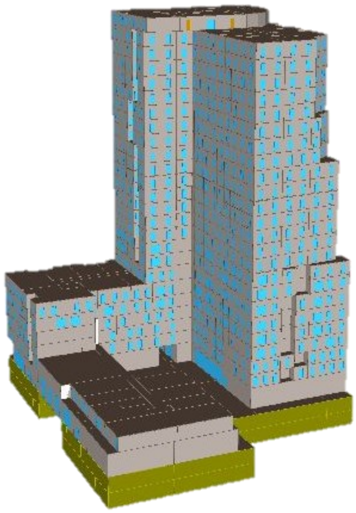
Case Study 6: New Residential Building

Number of Floors: 5 WWR: 32%

Area: 12766 m^2 VFAR: 36%

Mechanical System: Package Terminal Air Conditioning

Construction Type: Metal Frame



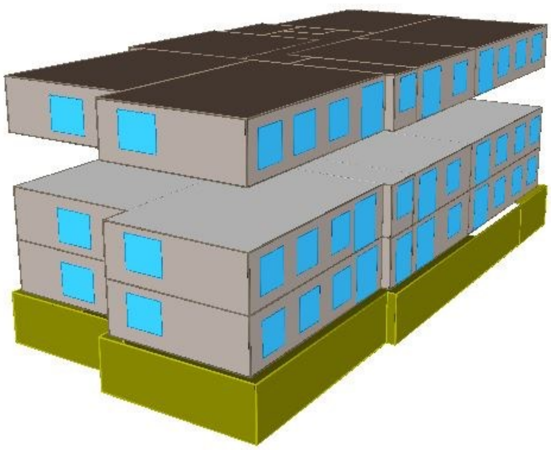
Case Study 7: New Residential Building

Number of Floors: 26 WWR: 23%

Area: 41807 m^2 VFAR: 37%

Mechanical System: Water Loop Heat Pumps

Construction Type: Curtain wall system



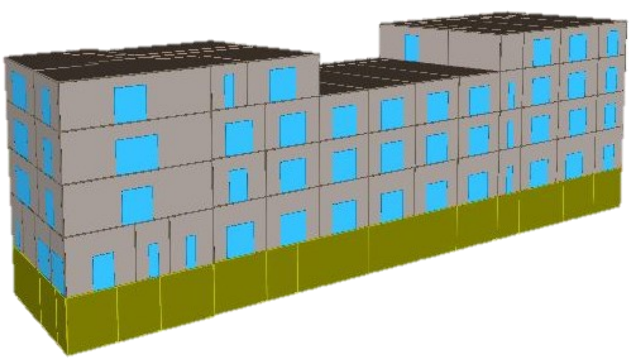
Case Study 8: New Apartment

Number of Floors: 4 WWR: 32%

Area: 3501 m^2 VFAR: 42%

Mechanical System: Package Single Zone + Baseboard

Construction Type: Metal Frame



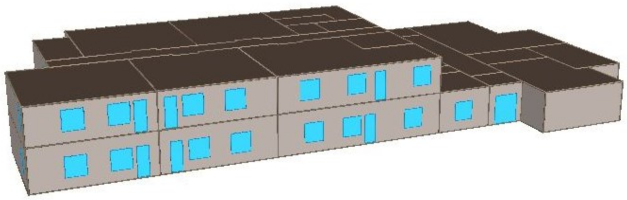
Case Study 9: New Residential Building

Number of Floors: 4 WWR: 28%

Area: 10004 m^2 VFAR: 45%

Mechanical System: Package Single Zone + Baseboard

Construction Type: Concrete



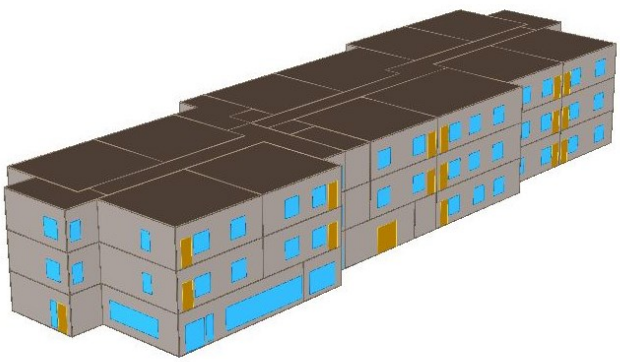
Case Study 10: Enlargement

Number of Floors: 2 WWR: 18%

Area: 1181 m^2 VFAR: 45%

Mechanical System: Electrical Baseboard

Construction Type: Wood



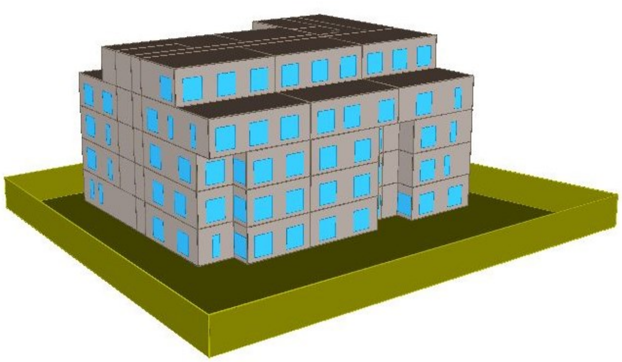
Case Study 11: New Residential Building

Number of Floors: 3 WWR: 15%

Area: 2968 m^2 VFAR: 50%

Mechanical System: Electrical Baseboard

Construction Type: Concrete



Case Study 12: New Residential Building

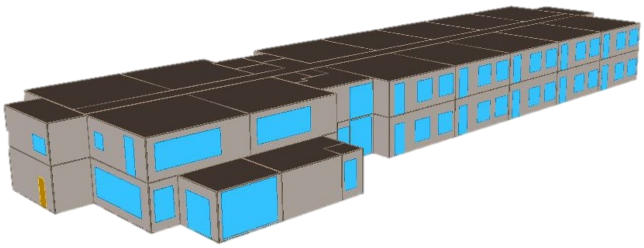
Number of Floors: 5 WWR: 20%

Area: 2243 m^2 VFAR: 51%

Mechanical System: Package Variable

Volume + Baseboard

Construction Type: Metal Frame



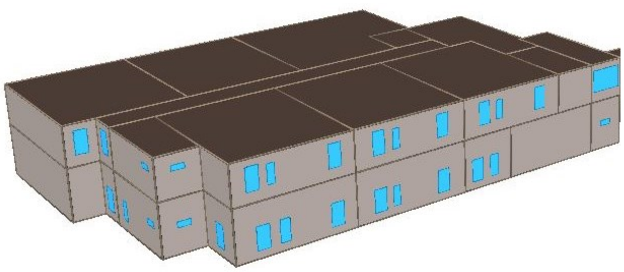
Case Study 13: Major Renovation

Number of Floors: 2 WWR: 13%

Area: 1320 m^2 VFAR: 55%

Mechanical System: Package Single Zone + Baseboard

Construction Type: Wood



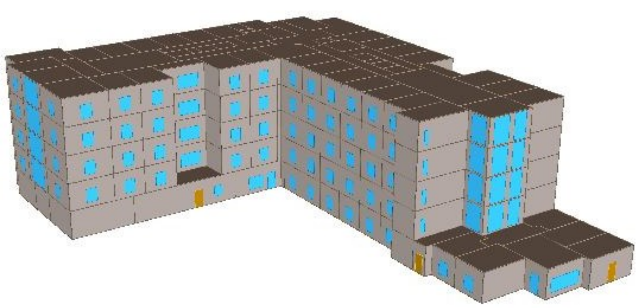
Case Study 14: Enlargement

Number of Floors: 2 WWR: 17%

Area: 7676 m^2 VFAR: 56%

Mechanical System: Package Variable Volume + Baseboard

Construction Type: Concrete



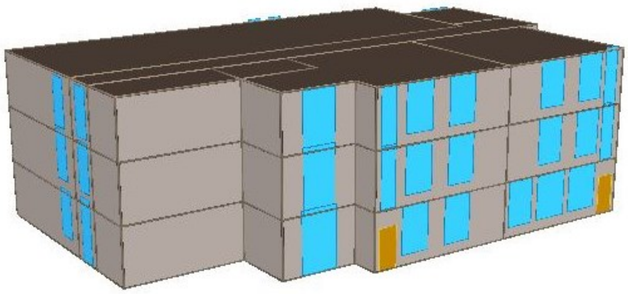
Case Study 15: New Residential Building

Number of Floors: 5 WWR: 25%

Area: 2967 m^2 VFAR: 56%

Mechanical System: Package Single Zone + Baseboard

Construction Type: Concrete



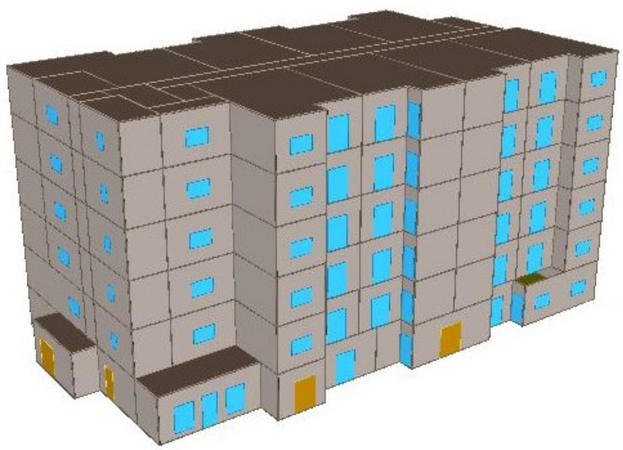
Case Study 16: Enlargement

Number of Floors: 3 WWR: 19%

Area: 1326 m^2 VFAR: 57%

Mechanical System: Package Single Zone + Baseboard

Construction Type: Metal Frame



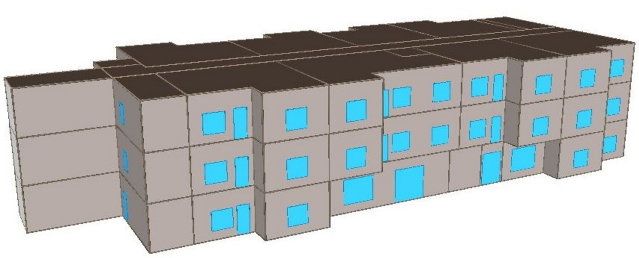
Case Study 17: New Residential Building

Number of Floors: 6 WWR: 14%

Area: 3769 m^2 VFAR: 60%

Mechanical System: : Package Variable Volume + Baseboard

Construction Type: Wood



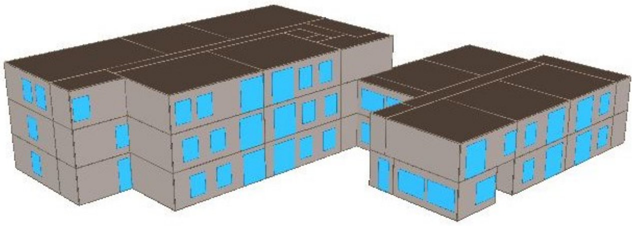
Case Study 18: Enlargement

Number of Floors: 3 WWR: 13%

Area: 680 m^2 VFAR: 62%

Mechanical System: Package Single Zone + Baseboard

Construction Type: Metal Frame



Case Study 19: Enlargement

Number of Floors: 3 WWR: 15%

Area: 2508 m^2 VFAR: 66%

Mechanical System: Package Variable Volume + Baseboard

Construction Type: Wood

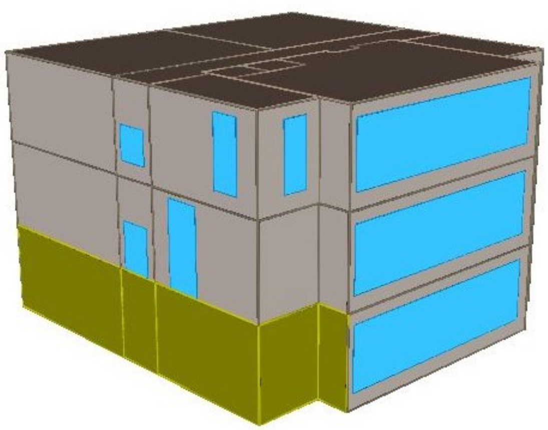
Case Study 20: New Residential Building

Number of Floors: 7 WWR: 17%

Area: 4763 m^2 VFAR: 70%

Mechanical System: Package Variable Volume + Baseboard

Construction Type: Concrete



Case Study 21: Major Renovation

Number of Floors: 3 WWR: 13%

Area: 2329 m^2 VFAR: 85%

Mechanical System: Package Single Zone + Baseboard

Construction Type: Wood

In terms of architectural point of view, the comparative case study buildings are chosen in a way to represent different building shapes. Figure 8 shows some of these buildings with different shapes other than rectangle and square. The impact of diversity in building shapes are seen by introducing the VFAR metric. Buildings with narrow and complex shapes have higher VFAR compared to buildings with simple shapes. As the VFAR is an influential factor in building heating energy use, buildings with higher VFAR tend to have a greater heat loss per unit floor area; thus, they require an improved envelope system to compensate for the impact of their complex shapes.

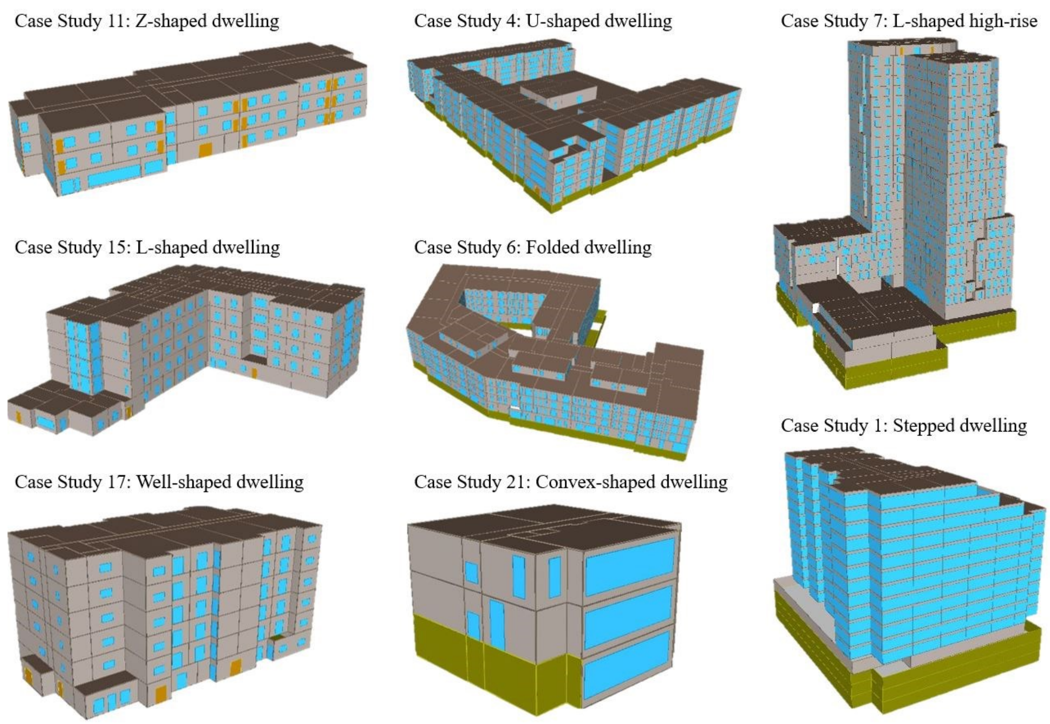
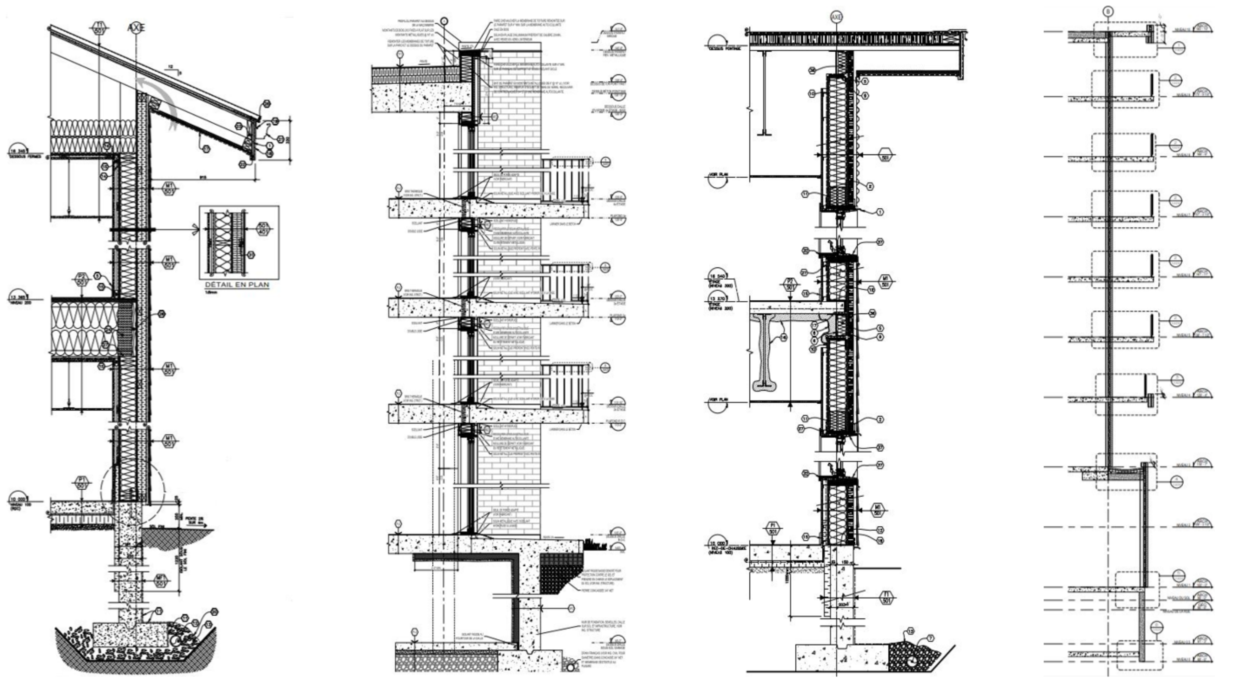


Figure 8: Different building shapes of the comparative case study buildings

On the other hand, in terms of construction types, the comparative case study also presents different common construction types in Montreal. Wooden construction, concrete, metal frame (LSF), and curtain wall system are the common construction types in low-rise, mid-rise, and high-rise MURBs, respectively. Figure 9 presents the wall section of different construction types of the case study buildings. More insightful examples for different construction types and wall section details of the comparative case study buildings are provided in table 12 of Appendix A.



Wooden Construction | Concrete Construction | Metal Frame Construction | Curtain Wall System

Figure 9: Different construction types of the comparative case study buildings

As curtain wall systems are commonly deployed for high-rise buildings, it can be observed that higher buildings are more likely to have higher WWR, with occasional exceptions. On the other hand, low-rise buildings usually do not have air conditioning and central mechanical systems. To evaluate the impact on annual energy demand, parametric simulation has been run for each building energy model with four different equivalent RSI values of the walls. The mechanical systems of buildings with air conditioner have been modeled according to the actual mechanical drawings. For the buildings without air conditioning, a direct expansion cooling coil has been added to the central ventilation system. By doing so, assessing the impact on applying different thermal bridging calculation methods on cooling peak load and annual cooling demand will be feasible. The heat recovery system is only modeled for buildings that meet the NECB 2015 requirements. Indoor air temperature of 22.2°C (72°F) is assigned for heating and 23.9°C (75°F) for cooling. The occupancy, lighting, and appliance loads are picked from table A-8.4.3.2.(2)-A & B, the operating schedule is based on schedule type G from table A-8.4.3.2.(1)-G for residential buildings, and roof and window thermal resistance are complying with the values suggested in tables 3.2.2.2. and 3.2.2.3., all provided by NECB 2015.

Although, eQuest is a user-friendly building energy analysis tool and provides accurate and high-quality results, it has some limitations (Rallapalli et al., 2010). The main limitation is that eQuest cannot model the heat loss through thermal bridges and the surface temperature around them. Length of the junctions have to be calculated manually in order to compute the equivalent RSI of the walls, which are input to eQuest to evaluate the impact of thermal bridges.

5. Results and Discussion

5.1 Scenario Generation

A comparative case study of 21 MURBs in Montreal is modeled separately in eQuest based on their actual technical drawings. The equivalent RSI for each building wall is calculated by the four above-mentioned methods. Table 9 demonstrates the equivalent RSI values for each building wall and the underestimation range that could happen by not considering the linear thermal bridges properly. The maximum underestimation occurs when only the thermal bridges in opaque clear field wall composition are calculated, and the linear and point thermal bridges are completely ignored. The most accurate method is the 2D heat transfer modeling that considers almost all linear thermal bridge types and provides a detailed analysis of the thermal behavior of junctions.

Table 9: Equivalent RSI calculation with four different methods

	Equivalent RSI-value (m^2.K/W)				
Case Studies	**No Linear Transmittance (NECB 2015)**	**Default Value Method (NECB 2015_amended)**	**Default Value Method (ISO 14683)**	**2D Heat Transfer Modeling (BETBG)**	**Under-estimation**
1	4.3	1.3	0.8	0.8	00% - 82%
2	2.8	1.5	1.1	0.9	19% - 70%
3	3.6	2.2	1.5	1.2	18% - 66%
4	3.7	2.2	1.6	1.3	17% - 65%
5	3.7	2.3	1.5	1.2	18% - 66%
6	4.3	2.6	1.7	1.4	17% - 67%
7	2.2	1.6	1.2	1.0	18% - 55%
8	5.3	2.4	1.5	1.4	09% - 74%
9	3.0	2.1	1.6	1.3	18% - 56%

10	5.5	2.5	1.6	1.5	06% - 73%
11	4.2	2.5	1.6	1.3	17% - 69%
12	4.9	1.7	0.9	0.8	09% - 83%
13	3.3	2.4	1.8	1.4	24% - 58%
14	3.7	2.6	1.9	1.4	25% - 61%
15	3.6	2.4	1.6	1.3	18% - 63%
16	3.5	2.2	1.6	1.2	19% - 65%
17	5.0	3.1	1.9	1.6	18% - 69%
18	3.7	2.4	1.7	1.4	19% - 63%
19	5.0	2.2	1.4	1.3	09% - 74%
20	6.3	3.2	2.0	1.8	12% - 72%
21	5.2	3.3	2.1	1.7	18% - 67%

The simulation model is run parametrically for each building with four different equivalent RSI values shown in table 9. Not applying linear thermal bridges on building thermal resistance calculation resulted in 55% to 83% underestimation in equivalent RSI values with respect to the baseline.

5.2 Simulation Results

The building energy models are run hourly for the whole year with the mechanical systems that work 24 hours a day. To be able to see the impact not only on heating but also on cooling demand, for buildings without cooling system, an air conditioner has been added to their mechanical system. The results for heating and cooling demand are picked from the report SS-D Building HVAC Load Summary of eQuest detailed simulation results. These values are divided by unit area to normalize the heating and cooling demand for different buildings with different size and area.

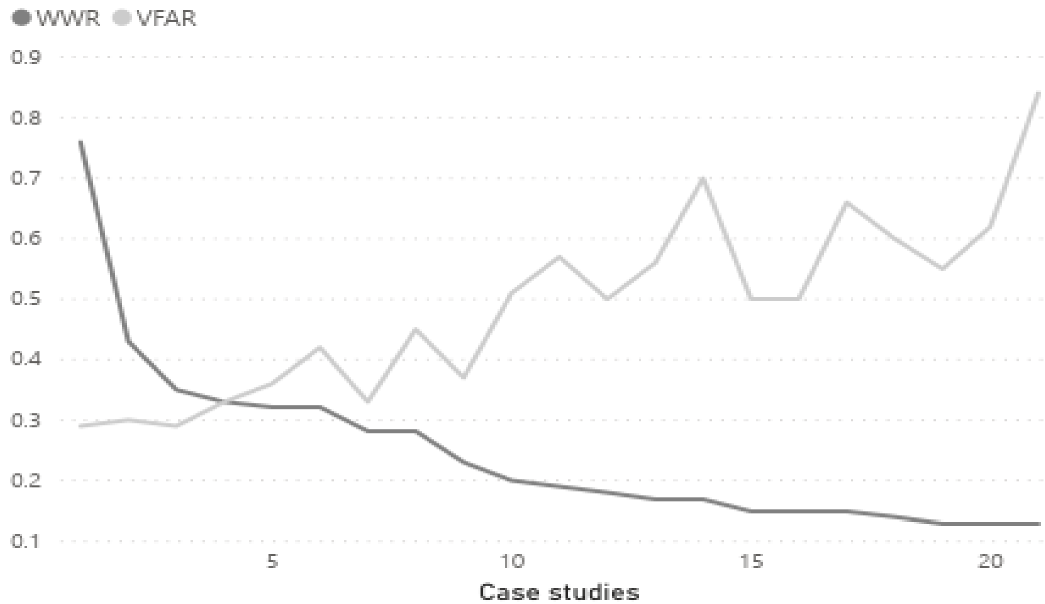


Figure 10: Negative correlation between WWR and VFAR

In this study Window to Wall Ratio (WWR) and Vertical Surface Area per Floor Area Ratio (VFAR) have been measured for all buildings. VFAR has been introduced as a metric that can replace the surface to volume ratio in building demand studies. This metric has a direct relationship with demand per floor area (MH, 2018b). As it is shown in figure 10, VFAR and WWR are, in general, inversely correlated. Both metrics are provided to observe the impact of thermal bridges to different types of buildings.

5.2.1 Annual Heating and Cooling Demand

After running the energy models parametrically, the annual heating and cooling demand per floor area are collected for all different scenarios and are shown in table 10. The comparative case study buildings cover a wide range of heating and cooling demands for buildings of different size, height, area, mechanical systems, and wall thermal resistance. Results show that by decreasing the equivalent wall RSI values, the annual heating demand increases and the annual cooling demand decreases. Thus, by not calculating the thermal bridge impacts properly, there will be a risk of underestimation of heating and overestimation of cooling demand compared to the baseline. BETBG method has been considered as the baseline for this study as it is shown in table 6 this method is the only one that considers all different junction types as thermal bridges compared to the other methods. Figure 11 depicts this deviation in annual heating and cooling demand for each building with different thermal bridging calculation methods compared to the baseline.

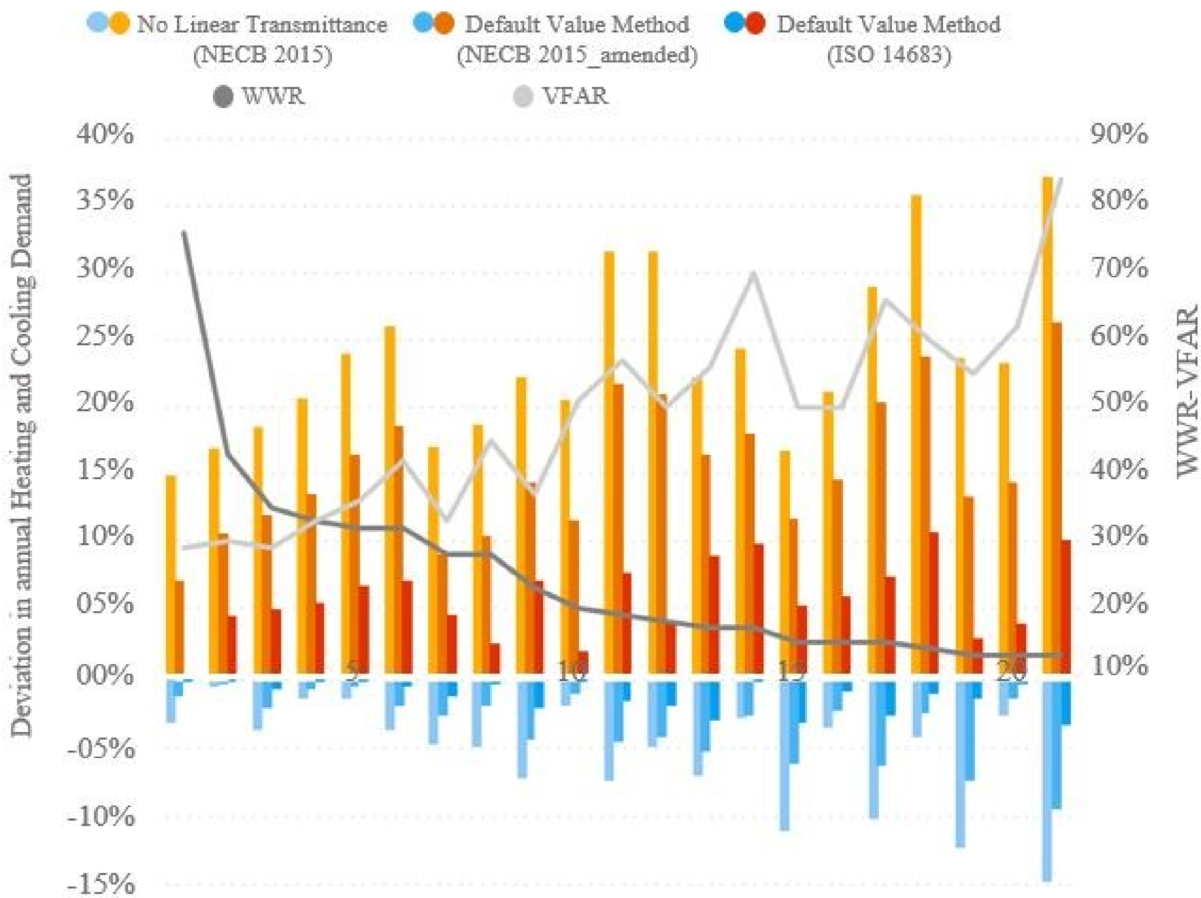


Figure 11: Deviation of heating and cooling demand under three calculation methods with respect to the baseline method

Table 10: The impact of different RSI on annual demand

Case studies	WWR	VFAR	Heating Demand (kWh/m^2)				Cooling Demand (kWh/m^2)			
			No Linear Transmittance (NECB 2015)	**Default Value Method (NECB**	**Default Value Method (ISO 14683)**	**2D Heat Transfer Modeling**	**No Linear Transmittance (NECB 2015)**	**Default Value Method (NECB 2015_amended)**	**Default Value Method (ISO 14683)**	**2D Heat Transfer Modeling**
1	76%	29%	61	66	71	71	40	39	39	39
2	43%	30%	73	79	84	88	39	39	39	39
3	35%	29%	52	56	60	63	19	19	19	19/
4	33%	33%	72	79	86	91	27	27	27	27
5	32%	36%	37	40	45	48	38	38	38	38
6	32%	42%	50	57	65	70	18	18	17	17
7	28%	33%	82	87	91	95	24	23	23	23
8	28%	45%	46	51	56	57	39	38	38	38
9	23%	37%	52	58	63	67	24	24	23	23
10	20%	51%	83	93	103	105	11	11	11	11
11	19%	57%	51	58	69	75	16	16	15	15
12	18%	45%	130	150	183	190	35	35	34	34
13	17%	56%	60	65	70	77	40	39	38	37
14	17%	70%	123	133	146	162	14	14	14	14
15	15%	50%	52	55	59	62	16	15	14	14
16	15%	50%	71	77	85	90	12	12	12	12
17	15%	66%	65	73	85	91	9	9	9	9
18	14%	60%	41	49	57	64	15	15	15	15
19	13%	55%	76	86	97	99	16	15	14	14
20	13%	62%	67	75	84	87	7	7	7	7
21	13%	84%	50	59	72	80	13	12	11	11

Moreover, results show that not only does the VFAR have a direct correlation with annual demand per floor area but also this metric has an impact on demand deviation. In another word, the higher the VFAR is, the impact of the envelope thermal resistance calculation method on annual heating and cooling demand estimation will be higher. Some methods consider perimeter of windows as thermal bridge while others do not. For those that do not consider the perimeter, energy demand deviation is inversely related to WWR. While, for those that consider the perimeter of windows, VFAR shows a better correlation with energy demand deviation. Regarding HDD and CDD of Montreal, the deviation will be much higher on annual heating demand than cooling demand. The maximum annual heating demand underestimation can reach up to 37% for the case that only opaque clear field thermal bridges are counted and all linear and point thermal bridges are ignored. Cooling demand reaches up to 14% of overestimation for the same case.

5.2.2 Heating and Cooling Peak Load

The results for heating and cooling peak load are picked from the report LS-C Building Peak Load Components of eQuest detailed simulation results. Building mechanical systems are designed and sized based on the peak loads. Thus, inaccuracy in peak load prediction can have a huge impact on mechanical system sizing, cost, energy consumption, and thermal comfort of occupants. This study shows a maximum deviation of 38% for heating and 20% for cooling compared to the baseline. Moreover, The average deviation based on different envelope thermal resistance is higher on peak load than demand. The average of underestimation when the linear thermal bridges are ignored is around 29% for heating peak load and 12% for cooling peak load, these numbers for heating and cooling demand are 24% and 5% respectively. As it is shown in figures 12 and 13, the general trend in heating and cooling peak load follows the trend in demand and increases with higher VFAR and lower WWR.

The actual values of heating and cooling peak load in W/m^2 for each scenario have been shown in table 13 in the Appendix B.

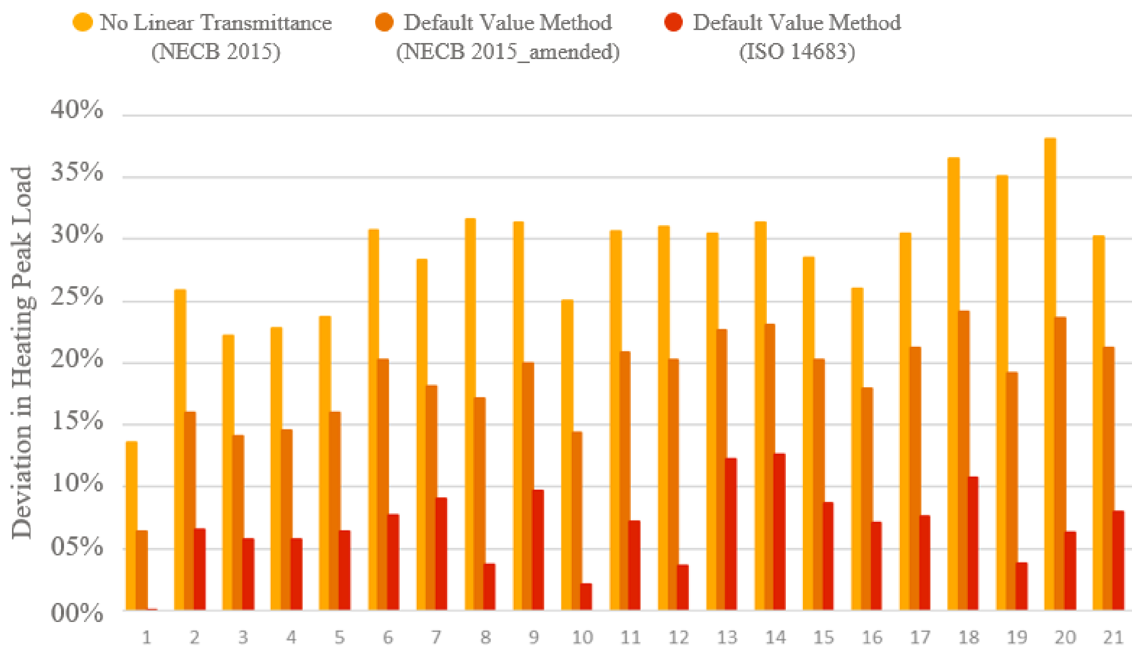


Figure 12: Deviation of heating peak load under three calculation methods with respect to the baseline method

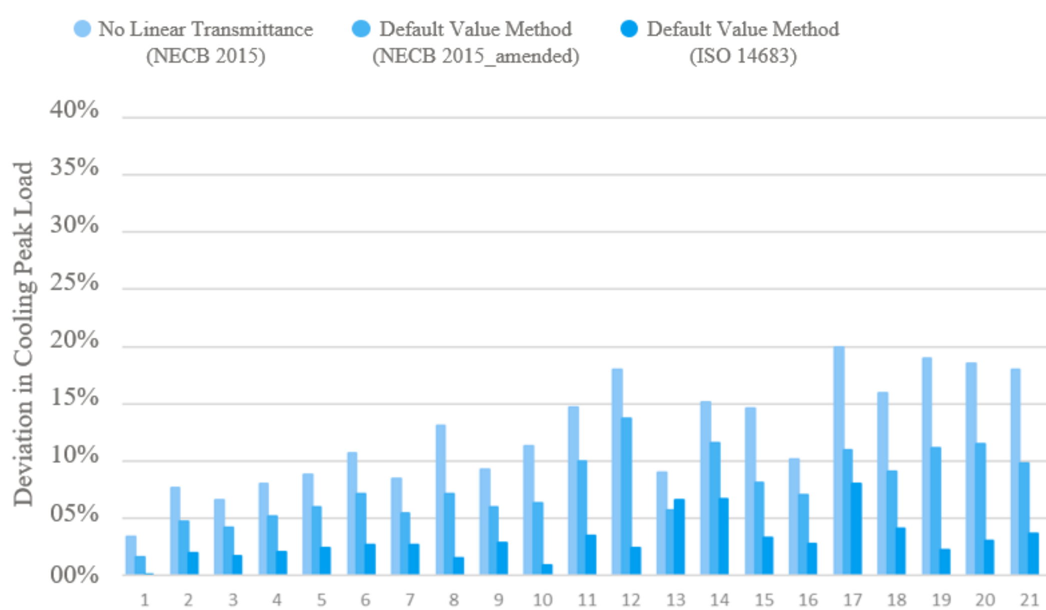


Figure 13: Deviation of cooling peak load under three calculation methods with respect to the baseline method

5.3 Summary

This research indicates that building physical characteristics play a critical role in the thermal bridging calculation method selection. As opposed to some research that shows a direct correlation between WWR and thermal bridging calculation method impact, this research adds VFAR as a more accurate metric especially when the calculation method does not apply the impact of the window perimeter junction and is not considering it as a thermal bridge. It is shown that the MURBs with higher VFAR and lower WWR are more impacted by thermal bridging calculation methods and show the highest deviation in annual demand. The construction type of this group of buildings is mainly wood and LSF, sometimes concrete and their wall assemblies include two layers of insulations which increase the envelope efficiency. As has been mentioned before, the impact of thermal bridging is more significant in well-insulated buildings. This research shows the deviation up to 37% of heating and 14% of cooling annual demand for this group of MURBs.

The MURBs that are less impacted by thermal bridging calculation methods have higher WWR and lower VFAR. These buildings have more floor stories, with concrete construction and curtain wall systems. The deviation in heating and cooling annual demand for this group of buildings is less than 15% and 10% respectively.

Because the buildings are located in a cold climate, the impact on annual heating demand is more noticeable than annual cooling demand. The impact on building peak load can have a deviation up to 38% for heating and 20% for cooling. This deviation will impact the HVAC sizing significantly. Thus, applying the thermal bridging impact by choosing a proper calculation method is of great importance to building energy performance and savings.

Table 11 summarise the metrics and characteristics of two main groups of buildings. The first group includes the buildings that are highly impacted by different thermal bridging calculation methods, and the second group presents the least impacted ones.

Table 11: MURB groups of the most and the least impacted by thermal bridging calculation methods

<table>
<tr><th colspan="2">Group 1: The most impacted MURBs</th></tr>
<tr><td></td><td>

- VFAR > 50%
- WWR < 20%
- Number of Floors < 5
- Construction type: Wood, LSF, Concrete
- Deviation in demand :
15% – 37% in Heating
10% – 15% in Cooling

</td></tr>
<tr><th colspan="2">Group 2: The least impacted MURBs</th></tr>
<tr><td></td><td>

- VFAR < 50%
- WWR > 20%
- Number of Floors > 5
- Construction type: Concrete, Curtain wall system
- Deviation in demand :
< 15% in Heating
< 10% in Cooling

</td></tr>
</table>

6. Conclusion

Different building energy codes and standards have different requirements in evaluating building energy performance. Some are most stringent in accounting for thermal bridging calculations, and some are relying on simplified methods and default values.

This study presented that a huge deviation can be expected in the annual heating and cooling demand of a residential building by applying different thermal bridging calculation methods. Within the comparative case study buildings presented, buildings with higher VFAR and lower WWR are more impacted by the thermal bridging calculation methods.

It was shown that the impact of adopting different methods can have a huge impact on building performance given the building massing and design characteristics. Out of 21 MURBs modeled in the research, the worst-case shown a deviation up to 37% and 14% in heating and cooling demand estimation respectively. The highest deviation occurs at the lowest WWR and at the highest VFAR values. Since some methods considers perimeter of windows as thermal bridge while others do not, for those that do not consider the perimeter, energy demand deviation is inversely related to WWR. While, for those that consider the perimeter of windows, it seems that VFAR shows a better correlation with energy demand deviation. To indicate the detailed correlation between building physical characteristics and thermal bridging impact a further systematic sensitivity analysis is required.

In addition to inaccuracy in the building annual demand prediction, adopting different thermal bridging calculation methods has an impact on building peak loads estimations. Inaccurate building peak load estimation will lead to oversizing or undersizing building mechanical systems and result in energy wastage or thermal discomfort of the occupants. However, quantifying the impact of deviation in peak load estimation on HVAC sizing is beyond the scope of this research.

Building energy codes are the main drivers for regulating and promoting energy-efficient building design. The adoption of suitable thermal bridging calculation methods in building energy codes are so important. This research work made the first step to demonstrate this importance.

6.1 Future Works

This study presented a significant deviation in building energy performance due to applying different thermal bridging calculation methods. This was the first step toward developing the regulations that include building physical characteristics in thermal bridging calculation requirements. However, providing the proper thermal bridging calculation method concerning building physical characteristics needs further investigation. For future work, conducting a sensitivity analysis to highlight all possible correlation between building physical characteristics and their susceptibility toward thermal bridging calculation methods is highly recommended.

Identifying the role of different thermal bridge types in building energy performance is another interesting topic to be investigated.

In addition, by comparing European codes and standards mentioned in table 2, with the commonly used codes in Canada (ASHRAE, 2016; NRC, 2017), some changes in Canadian regulations concerning thermal bridging impact are suggested and listed below:

- Realization of the building envelope detail design, especially where linear thermal bridges are located.
- Implementing quality assurance procedure in building envelope details at the construction phase.
- Providing design solutions to eliminate the impact of thermal bridges in retrofit and renovation projects.
- Introducing detailed energy-efficient envelope design for passive houses and high-performance buildings.
- Implementing the minimum requirements for building envelope inner surface temperature.

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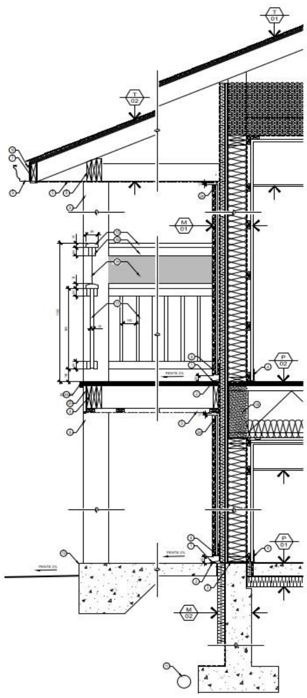
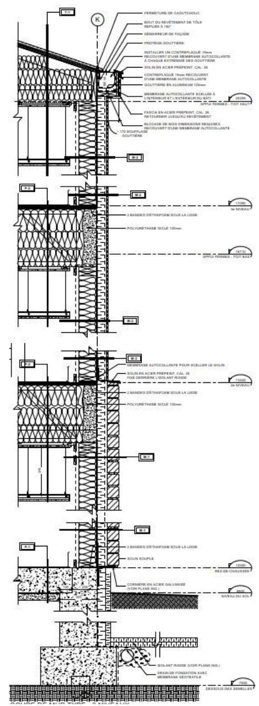
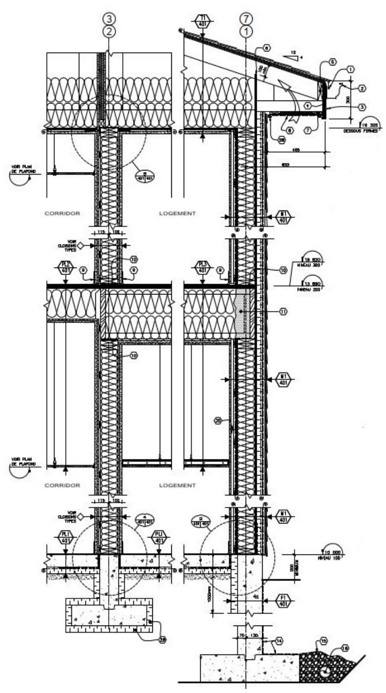
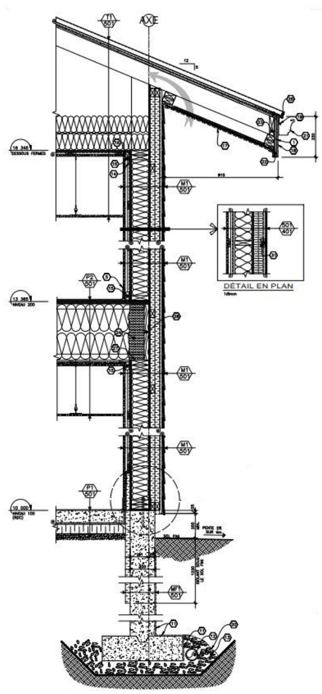
APPENDICES

Appendix A

The comparative case study buildings are presenting different common construction types in Montreal. The wooden constructions are used mostly for low-rise residential buildings. The wall assemblies of this group include two layers of insulations which increases the envelope efficiency and as it has been mentioned before, the impact of thermal bridging is more significant in well-insulated buildings.

The concrete constructions, LSFs, and curtain wall systems are mainly used for mid-rise and high-rise residential buildings. The wall section of these construction types is shown in table 12.

Table 12: Case study wall sections based on construction types

Wooden Construction	
Case Study 10	**Case Study 17**
	
Case Study 19	**Case Study 21**
	

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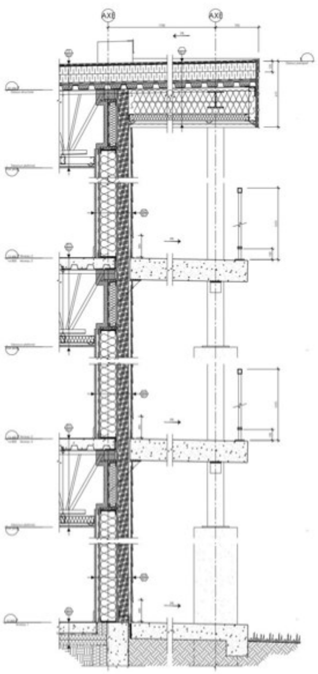
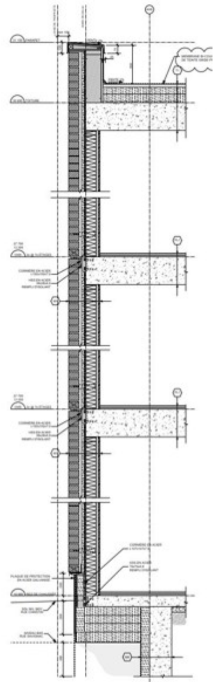
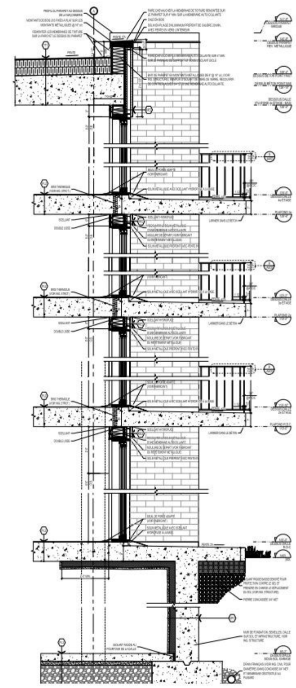
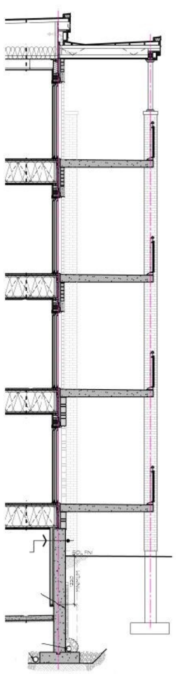
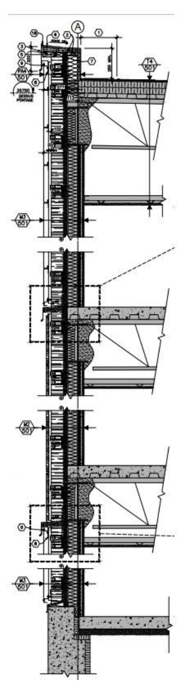
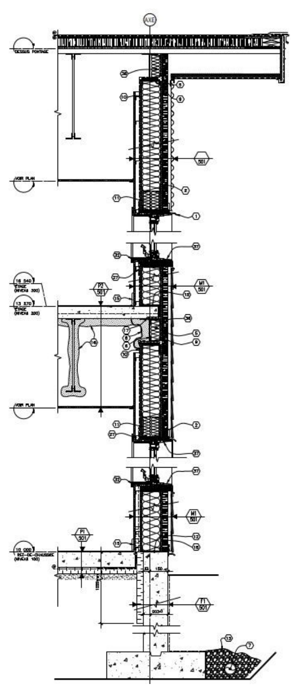
<tr><td></td><td></td><td></td></tr>
<tr><th colspan="3">Metal Frame Construction, LSF</th></tr>
<tr><th>Case Study 6</th><th>Case Study 16</th><th>Case Study 18</th></tr>
<tr><td></td><td></td><td></td></tr>
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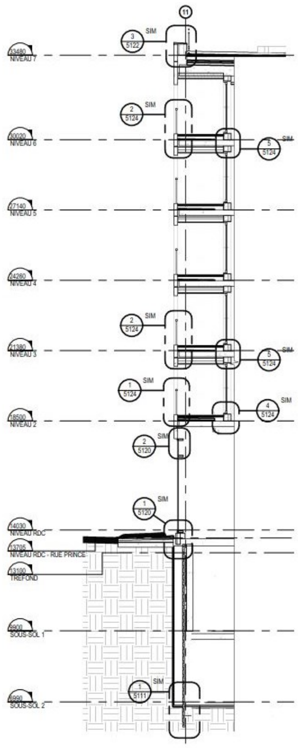
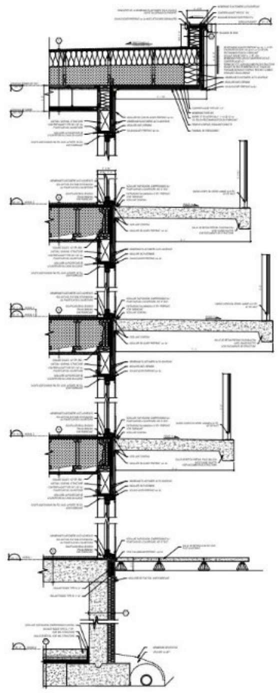
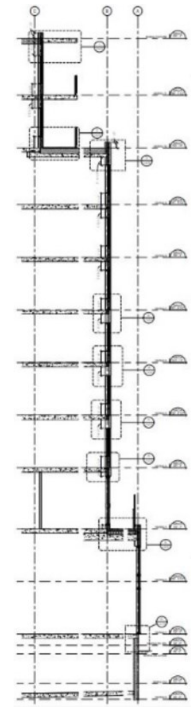
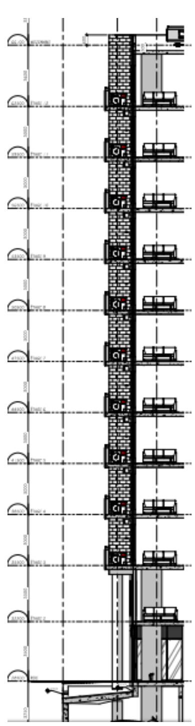
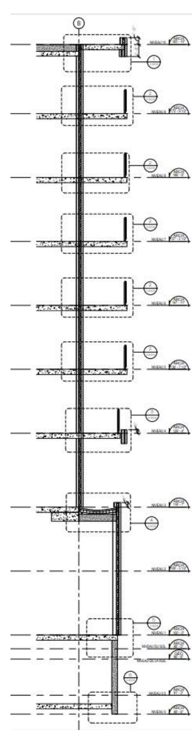
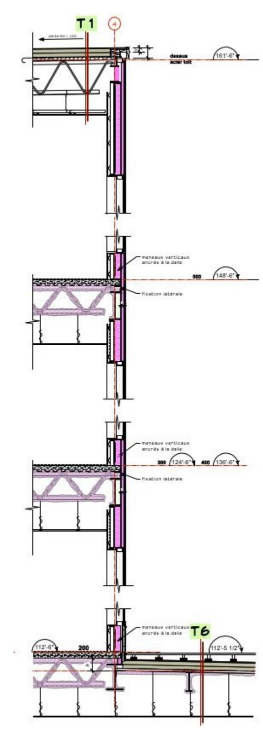
Curtain Wall System		
Case Study 1	**Case Study 2**	**Case Study 3**
		
Case Study 4	**Case Study 7**	**Case Study 15**
		

Appendix B

- Heating and cooling peak load values for the case study scenarios are shown in table below:

Table 13: Heating and cooling peak load (W/m^2)

Case studies	Heating Peak Load (W/m^2)				Cooling Peak Load (W/m^2)			
	No Linear Transmittance (NECB 2015)	Default Value Method (NECB 2015_amended)	Default Value Method (ISO 14683)	2D Heat Transfer Modeling (BETBG)	No Linear Transmittance (NECB 2015)	Default Value Method (NECB 2015_amended)	Default Value Method (ISO 14683)	2D Heat Transfer Modeling (BETBG)
1	27	29	31	31	42	43	43	43
2	20	23	26	28	37	38	39	40
3	18	20	22	23	23	24	25	25
4	21	23	25	27	27	28	29	30
5	22	24	27	28	25	25	26	27
6	20	23	26	28	29	30	31	32
7	15	17	19	21	23	24	25	26
8	14	17	19	20	19	20	21	22
9	13	16	18	20	22	23	23	24
10	25	29	33	33	28	30	31	32
11	22	25	29	32	26	27	29	30
12	26	32	38	40	22	25	28	29
13	18	20	22	25	20	21	21	22
14	31	35	40	45	29	30	32	34
15	21	24	27	30	29	31	33	34
16	22	25	28	30	28	29	31	31
17	23	27	31	34	25	26	27	33
18	16	19	23	26	22	23	24	25
19	21	26	31	32	22	24	26	27
20	15	18	23	24	18	20	21	22
21	27	31	36	39	29	30	32	33