The Impact of Different Types of Hundreds Charts on Place Value and Number Sense in Kindergarten and First-Grade

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Abstract

The Impact of Different Types of Hundreds Charts on Place Value and Number Sense in Kindergarten and First-Grade

Vera Wagner

The present study examined the spatial affordances of hundreds charts in the context of place value knowledge, number sense, and use of the chart for addition and subtraction. Kindergarten and first-grade students (*N* = 47) participated in online interviews with a researcher. Participants were randomly assigned to one of three conditions in which they received a lesson on how to use the hundreds chart to solve addition word problems. Each condition used a different chart: (a) top-down, where the numbers begin in the top row and increase with downward movement; (b) bottom-up, where the numbers begin in the bottom row and increase with upward movement, (c) explicit analogy, where a bottom-up chart was accompanied by visual cues to highlight the structure of the bottom-up chart. Children were assessed on their place and number sense before and after the lesson. After the lesson, learning and transfer tasks were administered to assess place value and the use of the chart. Results showed that the top-down and explicit analogy conditions were better able to move through the chart in the appropriate vertical direction to solve subtraction problems than the bottom-up condition. The top-down chart was more effective for place value knowledge than the other two conditions. No effects were found for number sense. The present study contributes to the literature on the affordances of hundreds charts and has implications for teachers on the use of the chart in mathematics instruction.

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Table of Contents

List of Figures

List of Tables

Chapter 1: Statement of the Problem

Mathematics is a topic with many abstract concepts and all children study this sometimes-challenging subject in school. To make it more accessible, teachers often use external knowledge representations (e.g., pictures, graphs, counting chips) to concretize abstract ideas. A "hundreds chart" is one such representation. Hundreds charts are tools that are commonly recommended in early numeracy and elementary mathematics classrooms to illustrate and support concepts such as the number sequence, arithmetic, and place value (Conklin & Sheffield, 2012; Ginsburg & Ertle, 2008; Reys et al., 2010).

Hundreds charts are 10 x 10 grids, with one number per cell, and the numbers in these charts typically range from 1 to 100. In the traditional version of the hundreds chart (see Figure 1, Panel a; Vacc, 1995), the "top-down" chart, the number 1 is in the top left corner and the numbers increase by one from left to right in each row and increase by ten descending each row. The greatest number, 100, is in the bottom right corner. Other formats include the range of numbers from 0 to 99 or from 1 to 120 or more (Brandenburg, 2018; Russo, 2019). Some hundreds charts rotate the structure 90 degrees such that numbers in the rows become numbers in the columns (Vacc, 1995). Finally, another variation, the "bottom-up" chart, places the smallest numbers at the bottom of the chart (see Figure 1, Panel b; Randolph & Jeffers, 1974). When Bay-Williams and Fletcher (2017) asked teachers in the United States why they used the traditional version of the chart (i.e., the top-down version), they said because it matches the direction used to read written text, the calendar presents numbers in similar configurations, and because existing resources support this version of the hundreds chart.

Figure 1

Different Types of Hundreds Charts

(a)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

(b)

91	91	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

Note. (a) Top-down hundreds chart, (b) Bottom-up hundreds chart.

Teachers are commonly encouraged to use hundreds charts because of the perception that they can support place value understanding (Reys et al., 2010). There is speculation that the configuration of number in the chart, whether it starts with 0 or 1, can help draw children's attention to place value and embedded patterns. For example, the spatial configuration of the numbers facilitates counting up from a given number. Starting from 24, for example, a student can add 30 by moving up or down in the rows, depending on the chart, three times: 34, 44, 54. Subtraction by groups of 10 is likewise possible by moving in the opposite direction through the rows, each of which represents a group of 10. Students may also count by up or down by ones using horizontal movement in the chart. It is thus possible to use the hundreds chart as a computational tool and as a step to mental computation. Furthermore, the spatial arrangement of the numbers in the hundreds chart also facilitates operations on numbers close to ten, like adding 56 and 19. In this example, 19 is 1 less than 20. Students can count by 10 twice from 56 (i.e., 66,

76) by moving through the rows two times and subtract 1 with a leftward movement to arrive at the correct answer, 75. This could promote flexible thinking about numbers (Thornton, 1995). Similarly, the hundreds chart encourages seeking patterns in number. Teachers may ask students to identify missing values based on the numbers in the surrounding cells.

From a teacher's perspective, hundreds charts seem like the perfect tool to communicate mathematics to children (Conklin & Sheffield, 2012). The sequence of the numbers and the place value patterns in the rows and columns may seem logical to someone with good number sense. Despite the strong encouragement for teachers to use hundreds charts in mathematics instruction (e.g., Conklin & Sheffield, 2012; Reys et al., 2010) and the existing varieties of hundreds charts (Brandenburg, 2018; Randolph & Jeffers, 1974; Vacc, 1995), there is no empirical evidence to suggest that one configuration of number in the chart works better than another for specific aspects of mathematics learning, or even that specific versions promote learning in intended ways. Moreover, anecdotal evidence suggests that teachers use a variety hundreds charts with different configurations of numbers (Bay-Williams & Fletcher, 2017), such as the variations described above, but little is known about how the physical structure of hundreds charts affects student learning. One way of presenting numbers may be more conducive to learning one concept, but less helpful for another. In other words, different configurations of the hundreds chart may afford different types of mathematics knowledge in children. For this reason, research on the configuration of hundreds charts is helpful in revealing ways that teachers can use these tools appropriately in instruction.

Chapter 2: Literature Review

Mathematical Representations and the Role of Affordances

Teachers often use external knowledge representations to make abstract mathematical ideas, such as place value and number sense, more accessible to their students (Belenky & Schalk, 2014). Mathematical representations can take various forms and are present at all levels of education. In elementary school, some examples include pictures, graphs, and concrete objects, such as counters. Another commonly recommended representation for instruction is the hundreds chart (Conklin & Sheffield, 2012). Hundreds charts are popular in elementary school classrooms (Bay-Williams & Fletcher, 2017), particularly in the early elementary grades. Teachers use hundreds charts to introduce several foundational mathematical concepts, among them number names and counting, operations like addition and subtraction, and place value (Vacc, 1995).

Affordances permit certain functions of an object (Gibson, 1979). For example, the specific round shape of car tires allows vehicles to move forward and backward, but not side to side. This idea of affordance applies to mathematical representations as well (Osana et al., 2018; Lafay et al., in press), and presumably to hundreds charts specifically. Certain features of the hundreds chart may facilitate better place value understanding, but offer weaker support for other mathematical concepts (Siegler & Ramani, 2009).

Affordances enable certain skills or strategies to blossom, but they can also hinder learning (Gibson, 1979). For instance, the spatial properties of a collection of physical objects can allow for immediate recognition (or "subitizing," Sarama & Clements, 2009) of the represented quantity (Clements, 1999). Furthermore, different arrangements can make subitizing easier or more difficult: Linear presentations are easier to recognize than circular ones, for instance, and identifying the quantity of a scattered display of a group of items can be more effortful than when the same group is in a rectangular formation (Clements, 1999). Similarly, one arrangement of the numbers in a hundreds chart may afford better learning than another. The configuration of numbers in the chart may allow learners to reason differently about numeration and place value.

The affordances of representations can facilitate better ways of reasoning about mathematical ideas in domains other than cardinality. For example, the way fractions are represented (e.g., ½) tends to be better for thinking about discrete or discretized quantities, while decimals' representations (e.g., 0.5) tend to be better for thinking about continuous quantities. DeWolf et al. (2015) ran a series of experiments looking at the applications of fractions and decimals to ratio. In one of the authors' experiments, ratios were represented pictorially as continuous, discretized, or discrete area models. Continuous models were represented by an area separated into two colors. Discretized models were the same as the continuous models, but the colored parts were further divided into several sections, demarcated with black lines, thus rendering the sections countable. The discrete models were represented by a series of distinct items, like individual squares or stars. Given one of these three types of models, the adult participants determined whether it best matched a fraction (e.g., 2/5 or 2/3) or a decimal (e.g., 0.4 or 0.6) notation. The results indicated that the participants showed a preference for fractions when the model was countable (i.e., discretized and discrete quantities) and for decimals when the model was continuous. The authors suggested that, relative to decimals, the relation between a fraction's numerator and denominator lends itself better to thinking about ratios. Decimals' one-dimensional nature seems to hinder thinking about ratio as a relationship that juggles two quantities. In brief, symbols, and other representations, afford more or less ease in thinking about the quantities they represent.

Analogical Reasoning in Mathematics

Overview of Analogical Reasoning

A context, idea, or representation can act as an analog to another when there is an underlying structural similarity between the two (Gentner & Colhoun, 2010). On the surface, however, two situations or concepts may appear very different. For instance, leaves and solar panels have different physical properties (e.g., material make-up, context in which they are found), but they have a similar function: turning light into energy. This structural similarity is what makes these two contexts analogous.

Humans' ability to abstract the structural similarities between ideas is called analogical reasoning (Gentner & Colhoun, 2010). In school, a lesson may indeed use solar panels to explain photosynthesis in leaves. The source analog is the concept or representation that is used to explain the novel idea, which is the target analog. When children are familiar with the source analog (in the present example, the solar panels), they are better able to reason about the target analog (i.e., the leaves). Novice learners, however, are prone to think about how the analogs' superficial features relate (Gentner & Colhoun, 2010; Vendetti et al., 2015), such as how both items are flat. Teachers must draw the students' attention to the structural similarities between solar panels and leaves (i.e., how both use light to produce energy). Effective teaching uses one (i.e., the source) to teach the other (i.e., the target) and makes clear how they are related (Vendetti et al., 2015).

This abstraction of a common schema from the process of structural mapping between the source analog and target analog is at the heart of analogical reasoning (Gick & Holyoak, 1983). It is also a vital skill in learning mathematics (English, 1998, 2004). Under certain conditions, the use of analogy in instruction magnifies student learning on two accounts (Richland & Simms, 2015): It promotes deep understanding of the instructional concept and it develops students' ability to use abstraction as a tool for more sophisticated thinking. Experts in various disciplines, including mathematics, connect ideas on a structural level (Richland et al., 2012); as such, introducing this kind of reasoning in mathematics classrooms should, theoretically, also foster higher-order thinking.

The ability to connect concepts in terms of their structure enables learning that cuts below the surface (English, 1998; Richland & Simms, 2015; Richland et al., 2012). For example, abstracting the structure of a word problem can assist students in solving unfamiliar word problems with the same structure, which constitutes a more effective and meaningful solution than solving the problems procedurally and examining keywords in the text (e.g., "altogether" means add; English, 1998; Richland et al., 2012). Teaching mathematics solely as procedures to memorize hinders problem solving and may prevent students' success (Osana & Pitsolantis, 2013; Richland et al., 2012; Rittle-Johnson & Schneider, 2015).

Structure-mapping theory suggests that analogical mapping creates a structural alignment between abstract relations (Gentner et al., 2001; Gentner & Colhoun, 2010). The theory forwarded by Gentner and her colleagues proposes that the represented objects and the relations between them comprise a structural alignment. For instance, by juxtaposing a golden retriever and a terrier, a viewer may abstract the structure mapping of "dog." Elements in the source and target relations have one-to-one correspondence and are parallel to one another. This means that each element of the source analog uniquely maps to another element in the target analog, like how golden retrievers and terriers both have four legs. Structurally comparable elements merge until an overall mapping is finally established. Gentner and colleagues also put forward that structure-mapping theory assumes that further inferences and higher-order relations (e.g., causality) can be made based on the mapping.

Analogical Reasoning in Mathematics Instruction

Manipulatives are another type of mathematical representation used in classrooms to concretize abstract concepts (Osana et al., 2018). These hands-on tools can be used to convey a variety of concepts and may look very different. Teachers may use counters, for instance, as representations to explain place value ideas; geometric nets may be employed to show students how solids are constructed; and colored, interlocking cubes may represent parts of a fraction. Children may practice regrouping 10 plastic counters into 1 group of ten. These manipulatives allow learners to experience how single units can be regrouped and tens can be decomposed. Students' physical interactions with the counters map to the place value idea that 10 units can be regrouped into 1 group of ten (Osana & Pitsolantis, 2019). The same creation of a mapping between representations and concepts applies to geometric nets and solids. The nets subsume the edges, faces, and vertices that map to solids. Colored cubes that fit together map to fractional pieces. Ultimately, these pedagogical tools structurally map to the concepts they represent.

Richland et al. (2012) posited that analogical reasoning can improve students' perception of mathematics and how they think about the relations it contains. The authors point to a lack or misuse of comparisons in mathematics instruction in the American education system as a fault. Teachers must make comparisons between analogs explicit for learners to be able to abstract conceptual relations. A seminal study by Gick and Holyoak (1983) provided evidence to this effect. The authors examined how readily high school students could abstract and apply a schema of a problem solution in a story to an analogous situation. Students were assigned to one of three conditions: analog recall, analog summary, and control summary. In the analog conditions, students read a text describing a logical dilemma about floating party balloons whose ribbons must be tied together. Holding the ribbons in both hands at the same time was not possible. The story included some irrelevant details and a "pendulum" solution whereby a weight could be tied to one ribbon and swung toward the other ribbon. Students in the control summarize group were given a different story that contained no such solution. After the participants read their respective texts, they were asked, depending on condition, to either recall details or summarize important points in the stories. Then, the students read another text about a cord problem where two ropes suspended from the ceiling must be attached together. An analogous solution to the party balloon story could be applied. The study's final task was for participants to produce a solution to the cord conundrum and to indicate how relevant and familiar the new story was. A hint was provided, suggesting to the students that they could use the previous story to solve the problem in the new text.

Gick and Holyoak (1983) found that the students in the analog recall and analog summarize conditions were equally as successful at finding the pendulum solution to the cord problem. They both performed better than the control group. In other words, students who read the text containing an analogous solution to the novel problem outperformed the control group whose text contained no analogy to the novel problem. An implicit analogy was helpful. However, after the researchers hinted to the students that they could use one story to solve the problem in the other, the success rate of finding the pendulum solution—one that used the analogous situation—increased. Consequently, the analogy was most useful for the participants when the relation between the analogs was made explicit.

Analogies that connect abstract mathematical concepts with tangible mathematical representations have been called "pedagogical analogies" (English, 2004). In a study by Blondin et al. (2017), six second-grade teachers introduced their students to a novel type of manipulative called Digi-Blocks. Digi-Blocks are plastic blocks, ten of which fit into a bigger case that closes and looks identical to the smaller blocks, but ten times larger. The teachers used the Digi-Blocks to explain place value concepts in their classrooms. One of the six teachers in Blondin et al.'s study created a structural mapping between the properties of the old and new representations by physically aligning the new materials (i.e., the Digi-Blocks) to materials that the students had previously used (i.e., Base ten blocks). In doing so, she provided an explicit cognitive support for comparisons between analogous mathematical representations.

The presence of analogous materials in instruction is not enough on its own, however, because even when a mapping between a source analog and target analog is possible, students do not always make it themselves (Gick & Holyoak, 1983; Richland et al., 2007). Having students come to their own conclusions about how manipulatives map to the intended instructional concept (e.g., how base-ten blocks and double-digit numerals relate) may lead students to make faulty connections or ones too weak to be useful for further learning (Mix et al., 2019).

To make the analogy effective for learning, students must tune in to the relevant features of each analog and notice how the corresponding relevant features map onto one another (Vendetti et al., 2015). A teacher using the solar system as a source analog to teach students about the atom, for example, can map the sun to the atom's nucleus. Teachers could also relate the planets to the electrons that rotate about the sun and nucleus, respectively (Vendetti et al., 2015). On the other hand, protons and neutrons comprise the nucleus, a feature unexplained by the source analog. For the purposes of one lesson, this detail may not be important, but teachers must be sure to highlight the features relevant to the instructional goal (e.g., smaller object rotating about a larger body) for the students to abstract their common structure.

Richland et al. (2007) found that American teachers indeed use analogies in the classroom.

They found, however, that the absence of instructional supports that mapped the source analog to the target analog diluted their value. Comparatively, teachers in Hong Kong and Japan who made the mapping explicit promoted their students' success in mathematics. A variety of pedagogical supports can be useful in highlighting the relations needed for analogical reasoning. Some such supports include employing language that focuses on spatial relations (Loewenstein & Gentner, 2005); using gesture to highlight structural similarities in the analogs (Alibali et al., 2013; Richland & McDonough, 2010; Valenzeno et al., 2003); reducing irrelevant details such that connections between the analogs are more obvious (Kaminski et al., 2013; Gerjets et al., 2008); and using analogs that are known to the learners (Goswami & Pauen, 2005; Vendetti et al., 2015).

The spatial configuration of instructional materials can also affect the quality of the mapping to the concepts they are meant to teach and consequently, their effectiveness in the classroom. Indeed, spatial configuration is one way to support analogical reasoning in mathematics (Richland et al., 2007). Aligning source and target analogs in space in Richland and McDonough's (2010) study allowed participants to apply their learning to novel mathematics problems and improved their ability to ignore irrelevant problem features. These results continued to hold one week later. Naverrete et al. (2018) also found that numbers' spatial arrangement on board games benefitted children's knowledge of numeration.

Supporting Place Value and Number Sense with Instructional Representations

Place Value and Number Sense

In elementary school, place value and number sense are at the crux of mathematics learning. One way to assess place value knowledge is to determine whether a student can correctly indicate the value of each digit in a numeral. For example, grasping the idea that the 5 in 56 represents 5 groups of ten, or 50, and that the 6 represents 6 single units would qualify as having understood a central idea in place value. Place-value knowledge is an important component of base-ten numeration (Ross, 1989) that the value of the digits increases by a power of ten from right to left in a number.

Children's place value knowledge can also be assessed by examining their invented strategies for adding and subtracting multidigit numbers. In particular, children who understand that ten ones can become one group of ten, thereby shifting its place in the number (Wearne & Hiebert, 1994), can use this knowledge to invent conceptually-grounded intuitive strategies. In fact, the understanding and ability to manipulate groups of ten as if they were ones in the context of addition and subtraction demonstrates a key idea in place value (Carpenter et al., 2014; Carpenter et al., 1997).

Empirical evidence has shown that place value knowledge and understanding the structure of base-ten numeration go hand in hand. For instance, Ho and Cheng (1997) worked with 69 first-grade students for one hour per week over five weeks. With the goal of improving the participants' place value skills, they practiced oral counting and enumeration, and regrouping bundles of 10 and 100 straws. As the participants manipulated these bundles, they were asked to select cards showing different values of tens and units using numerals to represent their quantity of straws. Ho and Cheng (1997) found that strengthening the participants' place value comprehension improved their ability to calculate sums and differences on paper using numerals with and without regrouping. Meanwhile, learning these algorithms without place value understanding reduces their meaningfulness (Wearne & Hiebert, 1994) and taxes students' working memory (Chan et al., 2014).

Place value also enables the learner to match the written and spoken symbols used to represent quantities (Mix et al., 2019). To make sense of the symbols' meaning, children must align them with their referents. The number symbols themselves are arbitrary – there does not appear to be anything intrinsically obvious about how the symbols in the number 16 (i.e., the "1" and the "6"), for example, map to 16 discrete objects or that 16 can be conceptualized as one group of ten and six ones. This lack of an evident connection may pose a challenge for students trying to relate these representations. To exacerbate the challenge in learning place value concepts, numbers are used in many contexts and their roles may differ from one to the other. The order of digits in a telephone number does not change their value; one telephone number is not worth more than another based on the sequence of their digits. In contrast, the digits' order in the number indicating the price of a house is non-arbitrary. The value of a house's price depends on the number of digits and their sequence. Furthermore, the rules of representing a number symbolically may test students' working memory and understanding of the base-ten system. For example, students must understand that denominations "collapse" so that the number, "five hundred twenty-three" uses three places (i.e., 523), not six, as 500203 would. Mix et al. (2019) advocated for the proper use of instructional material and teacher guidance to navigate these potential obstacles in students' learning.

Number sense is a broad category of mathematics learning with several distinct definitions in the literature (Jordan et al., 2006). Subitizing, knowledge of number patterns, number magnitude, estimation, counting, and number transformations (e.g., nonverbal calculations) are some of the most agreed-upon elements of number sense (Jordan et al., 2006). This study will focus on one specific aspect of number sense described in the literature: number magnitude. In the present study, I define number magnitude as the ability to compare quantities through the correct interpretation of their symbolic forms. Each distinct number represents a different magnitude, and magnitudes can be arranged in increasing or decreasing order.

Number sense is important not only for children's success with more advanced mathematics in school, but also for understanding numbers in society. It would be difficult to understand the gravity of large numbers in the news, budgets, or populations, for example, without proper number sense (Ronau, 1988). Being able to estimate the number of people in two respective lines in a store and judge which is longer, for instance, requires a sense of numerical understanding (Siegler, 2016).

Despite their importance in children's mathematics understanding, place value and number sense themselves can be elusive to novice learners. The former requires, in part, that children relate their understanding of written and spoken number symbols (Mix et al., 2019). Children must also grapple with how a digit's position in a numeral changes its value when the digit falls in a different denomination (e.g., ones place, tens place; Wearne & Hiebert, 1994). Parts of the latter can be challenging even for adults: In fact, the difficulty some adults have with large numbers springs from a lack of understanding of the decimal system (Siegler, 2016). While newborn babies appear to show some capacity to discriminate between differing quantities, much of children's number sense seems to necessitate experience over time for its development (Siegler, 2016). Siegler (2016) underscored the importance of number magnitude knowledge, stating that it "… correlates with, predicts, and is causally related to arithmetic and overall mathematics achievement" (p. 354). Thus, finding ways to foster children's number sense and place value knowledge is significant to the development of their numeration understanding.

Spatial Supports for Place Value and Number Sense

Place value and number sense indeed pose challenges, but targeted instruction can effectively support children's learning. Teachers' use of space in instructional material and explicit mappings to mathematics concepts promotes proficiency of place value and number sense in students. Mathematics is a domain where learners must form connections, or relations, between symbols, quantities, and referents (Richland et al., 2012), and instruction that supports or fosters such relations is likely to be particularly effective (Gick & Holyoak, 1983; Vendetti et al., 2015). Mix et al. (2019) looked at three distinct elementary mathematics curricula to see how they might support relational learning. They examined how different materials and accompanying instructional guidelines focused on relational thinking in the context of place value. Mix et al. (2019) highlighted four components of instruction that can support relational learning: Co-occurrence is the idea that associations form between concepts or materials that frequently occur together; alignable elements, which dictates that the more similar two elements are, the easier the comparisons between them; multiple examples support the need for several different representations of the same concept; and, scaffolding alignment, where pointed instruction (e.g., gesture, spatial arrangement) accompanies students' comparison of elements.

The first curriculum reviewed by Mix et al. (2019), *Developmentally Appropriate Mathematics* (Van de Walle et al., 2010), fostered comparison of representations. For instance, some activities aimed for students to compare a double-digit number by representing it in alternate ways using base-ten blocks or ten-frames. This supported the development of place value by showing various "faces" of the same number and having students draw associations between them. The authors highlighted the importance of scaffolding in these activities to ensure that students notice these similarities and make appropriate observations.

In the *Number Talks* curriculum (Parrish, 2014), teachers use problems to stimulate discussion among the students. Children discuss how they arrive at solutions to mentally add double-digit numbers. Mix et al. (2019) observed a class that went over different versions of specific solutions, calling on decomposition of the numbers and examining the tens' and ones' values individually. The spatial arrangement of the equations as the class reviewed the solutions was an important structural aspect of the curriculum. The expanded forms of the equations were presented either below or beside others for the same problem, thus facilitating comparison between the numbers. Color or connecting lines that mapped between the different representations would further augment the chances that children pick up on the similarities.

Finally, the *Montessori Method* (Montessori, 1934) uses specific concrete materials that are intended to convey instructional concepts to students as they manipulate them according to specific guidelines. One exercise reviewed by Mix et al. (2019) incorporated "golden beads" to represent different denominations: ones, tens, hundreds, and thousands. Students are meant to match the beads with layered cards that show the expanded form of the represented number. Moreover, there is a predetermined sequence for the introduction of the activities, designed to show students different representations of the same concept. The sequence also focuses on how the numbers form and decompose into different denominations. Mix et al. (2019) argued that the *Montessori Method* (Montessori, 1934) curriculum best adhered to the relational learning theory the authors put forward.

Spatial similarities between visual representations in instruction and mathematical concepts can also support number sense. In one study, Booth and Siegler (2008) found that first-grade students' practice with estimating a number's location on a number line from 0 to 100 helped them develop a sense of approximate sums. This held even for novel, double-digit addition problems. The authors attributed the children's gains in arithmetic to working with a visual representation that accurately depicts number magnitude. Understanding number magnitude in terms of the number line allows children to narrow the sums' possible positions on the number line relative to the magnitudes of the addends. The authors also credit the mental number line as a structural organizer for numerical concepts.

The spatial configuration of linear board games is especially conducive to learning about number magnitude. Siegler and Ramani (2009) found that playing a linear number board game for one hour, spread over four sessions, improved preschool children's performance on number magnitude tasks (i.e., number magnitude comparison and number line estimation; see also Ramani & Siegler, 2008). Siegler and Ramani's (2009) study had three conditions: linear board game; circular board game; and numerical control, in which the participants in engaged in numeracy activities, such as counting and numeral identification. The participants in the linear board game not only showed better aptitude for number magnitude, but they were more successful at solving arithmetic problems than the other two groups. Siegler and Ramani (2009) attributed the number magnitude understanding that comes from playing with the linear board game to the idea that the numbers' spatial arrangement maps to the mental number line.

Siegler and Booth (2004), who also found that linear board games help support children's number sense development, suggested the participants' improvement in number magnitude relates to the practice that comes from moving the board game tokens away from the game's starting position. The kinesthetic movement synchronizes with the sequence of the number names the player may utter aloud, the length of time taken to move the token, and the distance traveled from the start. Siegler and Booth (2004) speculated that the players' experience moving their tokens along the board contributes to their development of number magnitude understanding.

Navarrete et al. (2018) tested how the spatial properties of instructional materials influence number magnitude understanding. During an intervention, preschool children observed a race between two toy animals along a linear board game. The board game mats, meant to act as an analog to the mental number line, were partitioned with the numbers 0 to 10 in increasing order. A control group used board mats with non-numerical images (e.g., tree, boat) in the partitioned spaces instead of numbers. The participants in the experimental conditions observed the board games from either the left side or the right side. Importantly, their position relative to the mat changed their perspective of the orders of the numbers: The group of participants on the right saw the numbers increasing from left to right, as is typical of positive integers on a number line, but the group on the left saw the numbers' absolute values increase from right to left, similar to the order of negative integers on a number line.

The results revealed that, of the three groups, the participants on the right who saw the numbers as presented on the number line that increases from left to right performed best on a number line estimation task. The fact that the best performance came from conditions where the spatial arrangement of the numbers on the mat matched the way that numbers are arranged on a number line suggests that the spatial configuration of instructional materials matters. The distance travelled by the toy animals in the race mapped to number magnitude because of the spatial alignment with the mental number line. Having to mentally reverse the sequence of numbers seemed to have complicated the task for the participants who saw the numbers increase from right to left as it was incongruent with the mental number line's spatial configuration. The authors concluded that space is an important component in the instruction of number as it seems to influence children's understanding of how numbers are spatially represented. In arranging instructional materials such that they match the mathematical concept's arrangement in space, students will more easily relate the representation and the target concept, thus promoting their understanding of the mathematical ideas using analogical reasoning.

Navarrete et al. (2018) also found that the numbers' configuration in space had different affordances for learning. The numbers' spatial arrangement was less predictive of the number counting task (i.e., reciting the number sequence to 10), the number identification task (i.e. recognizing number symbols), and number magnitude comparison task (i.e., comparing two side-by-side numbers) than performance on the number line estimation task.

With respect to hundreds charts specifically, Vacc (1995) argued that the spatial configuration of the numbers can align nicely with the instruction of the concepts it supports. She contended that the language for arithmetic (e.g., "more") and the language for navigating the space in the hundreds chart (e.g., "moving up") is important for students' ability to use the tool to make sense of numbers, especially for learners with disabilities. To better align the language for arithmetic and for describing movement in the chart, she suggested a new configuration of number in the chart such that the rows correspond with those in the standard algorithms for addition and subtraction. The spatial arrangement of number in her recommended hundreds chart fits well with the concepts for which teachers use it, such as place value and arithmetic. Alluding to research on how inconsistencies in instructional materials and language can create obstacles in children's learning, Vacc (1995) claimed that her version of the chart would better serve its instructional purpose.

In one of the few studies documenting children's mathematical activity with hundreds charts, Bay-Williams and Fletcher (2017) reported anecdotal evidence that the spatial configuration of number in the charts plays an important role in children's numeration. They described children's confusion when the numbers increased with downward movement in the chart. These thoughts echoed Randolph and Jeffers (1974), who also argued that the numbers' placement in space, specifically that the numbers get larger as they go down in the chart, can create conflict for children. Bay-Williams and Fletcher’s (2017) proposed solution was to use a hundreds chart that places the smallest numbers in the bottom row such that the numbers increase with upward movement. They suggested that this bottom-up configuration is more conducive to learning place value, addition, and subtraction. Bay-Williams and Fletcher (2017) speculated that the match between the language students and teachers used to discuss hundreds charts in class and the direction of the numbers in the chart was responsible for their learning. They also suggested that a bottom-up configuration is more consistent with other learning tools (e.g., number lines, Cartesian plane) and contexts experienced outside the classroom (e.g., stacking blocks) than the traditional top-down chart.

The hundreds chart grounds the structure of number in space. Arithmetic operations using the chart map to movement within it. In other words, the spatial structure of the chart enables a mapping between increases or decreases by 10 and groupings of 10. As such, when students abstract this relation, they will be better able to recognize groupings of 10 in place value and the base-10 numeration system. As Bay-Williams and Fletcher’s (2017) anecdotal evidence suggests, the children who use the bottom-up hundreds chart, where the structure of number mirrors their everyday experiences of upward growth (e.g., increasing the quantity of milk in a glass moves the fill line upward, people grow taller with time; Bay-Williams & Fletcher, 2017), will benefit more from the tool than the children who use the traditional hundreds chart with the smallest numbers in the top row. The implicit “up is more” relation in the bottom-up hundreds chart may thus promote superior place value and number sense understanding than a chart with a top-down structure. Making this relation explicit by making direct comparisons to upward growth during instruction will improve students’ chances of abstracting this schema.

The Present Study

The present study aimed to determine how the spatial configuration of the numbers in the hundreds chart influences kindergarten and first-grade students' number sense and place value understanding. The hundreds chart organizes 100 numbers in 10 rows and 10 columns, each of them containing one number per cell in increasing order. Increases in number in the chart occur with movement to the right and vertically, either increasing by ten in the upward or downward direction depending on the chart type. Participants used the chart to distinguish between tens and ones as they moved right for increases by units and up or down, depending on condition, for increases by tens. As such, I anticipated that they would come to perceive the value of the tens' digit in a number as distinct from the value of the units' digit. While horizontal and vertical movements in the chart by themselves do not have any inherent quantitative value, when numbers are placed in the chart, vertical movement has a different quantitative meaning than horizontal movement (i.e., increases and decreases by ten versus increases and decreases by one). I predicted that the participants who use the bottom-up hundreds chart would experience more success than those who use the top-down chart because the upward movement for increases by ten maps to children's experience where "more" indicates growth in the upward direction, frequently observed in the real world.

I hypothesized that one configuration of the hundreds chart is particularly amenable to the processes involved in analogical reasoning. When the smallest numbers start at the bottom of the chart, they relate to real world ways of thinking about increase. For instance, like plants, humans, and buildings that "grow" upward, the numbers in the chart get larger with each row moving toward the top of the chart. In other words, there is the abstract relation of up is more inherent in the chart. I hypothesized that this relation could help children use the hundreds chart as a tool more effectively for addition and subtraction. When using the bottom-up chart for addition, for example, the sum being larger than the starting amount is mapped onto the physical upward movement in the chart, reflecting the up is more relation. In other words, the spatial configuration of the hundreds chart facilitates a mapping between the upward movement in the chart and increases in number magnitude. A chart that embeds the up is more relation better matches real-world experiences, and as such, may be more conducive to place value understanding and number magnitude knowledge.

Kindergarten and first-grade students were randomly assigned to one of three conditions: top-down, bottom-up, or explicit analogy. The top-down condition used the top-down hundreds chart that begins with 1 in the top left corner and 100 in the bottom right corner. In other words, the numbers increase by ones from left to right and by tens moving down the rows. The bottom-up and explicit analogy conditions used a bottom-up hundreds chart where the number 1 is in the bottom left corner and 100 is in the top right corner. Unlike the top-down hundreds chart, the bottom-up chart increases by ten with upward movement by row. The explicit analogy condition highlighted the up is more abstract relation present in the chart during the instructional intervention with a visual illustration of water rising in a container.

The study began with a pretest composed of a number sense task and a place value task. In the number sense task, participants selected the larger of two double-digit numbers, some of which were presented horizontally and some of which were presented vertically. Following the pretest, the researcher delivered an instructional intervention where she formally introduced the hundreds chart to the participants and demonstrated how to use it to solve addition word problems. The participants were then given three similar word problems for practice with corrective feedback. After the intervention, a learning task, which contained problems isomorphic to the practice problems given during the intervention, was administered. The learning task assessed whether the participants learned how to use the hundred charts to solve addition problems with double-digit numbers. I also evaluated place value knowledge by documenting the participants' strategies on the task. Participants completed a transfer task after the learning task, which required them to use the hundreds chart to solve subtraction word problems. If participants could use the hundreds chart in a new way (i.e., to solve subtraction problems), it would indicate that they had internalized the structure of number in the chart. Finally, the participants completed a posttest composed of isomorphic versions of the number sense and place value tasks administered at pretest.

The present study had three research questions:

1. Does the structure of the hundreds chart have an effect on participants' (a) movement in the chart, as assessed by the direction in which they moved, (b) understanding of place value, and (c) number sense?
2. Does an explicit visual analogy of the up is more relation provide additional benefits in terms of participants' (a) movement in the chart, (b) understanding of place value, and (c) number sense?
3. Does the placement of the larger number in the vertical items of the number sense task moderate the effects of condition on participants' performance?

To answer questions 1a) and 2a), I performed a chi-square test to check for condition differences in the direction in which the participants moved in the chart to solve the problems on the learning and transfer tasks. The predicted outcome for direction was that the explicit analogy condition would have the strongest results, and the bottom-up condition would perform better than the top-down condition. Drawing from structure-mapping theory (Gentner et al., 2001;

Gentner & Colhoun, 2010), the explicit analogy condition's explicit pedagogical support (i.e., the container) and verbal hint to focus on this support would likely help these participants to best abstract the relation present in the bottom-up chart (Vendetti et al., 2015). The cue was expected to propel the explicit analogy condition to do better than the bottom-up condition where participants must pick up on the relation on their own (Blondin et al., 2017; Gick & Holyoak, 1983). I expected that the bottom-up condition would perform better than the top-down condition because the implicit up is more relation in the bottom-up hundreds chart matches upwards growth in children's everyday experiences (Bay-Williams & Fletcher, 2017). Moreover, unlike in the top-down chart where numbers get larger with downward movement, the upward movement in the bottom-up chart is consistent with the language describing number increases (e.g., greater, bigger; Bay-Williams & Fletcher, 2017). For the same reasons, I predicted the transfer task would replicate the outcomes of the learning task. Specifically, the downward movement in the bottom-up chart matches the language used for quantities getting smaller, as with subtraction on the transfer task.

Questions 1b) and 2b) concerned place value and were addressed in two ways. First, I conducted a one-way analysis of covariance (ANCOVA) using posttest scores on a place value measure as the dependent variable, condition as the between-groups factor, and pretest scores as the covariate. I predicted condition differences since, like the direction variable, I expected the bottom-up chart to be more helpful, given its inherent up is more relation, than the top-down chart. The participants in the bottom-up and explicit analogy conditions, who worked with the bottom-up chart, would perform better than the top-down condition on the place value measure. I also predicted that the explicit analogy condition would perform better than the bottom-up condition since the up is more relation is explicitly highlighted for the former group.

I evaluated place value knowledge in a second way, namely by examining the participants' strategies for solving the word problems on the learning and transfer tasks. Using item as the unit of analysis, I conducted a chi-square test to look at strategy as a function of condition. Similarly, I predicted that the bottom-up and explicit analogy conditions would better support the use of more sophisticated strategies than its top-down counterpart. I also predicted that since the up is more relation is made explicit for the explicit analogy condition, this condition's performance would exceed that of the bottom-up condition on strategy use.

Question 1c) and 2c) examined number sense, which was addressed using two separate ANCOVAs for horizontal and vertical items, respectively. I assessed horizontal items using a 2 (placement: larger on left, larger on right) x 3 (condition: top-down, bottom-up, explicit analogy) mixed-design ANCOVA using posttest accuracy scores and response times, respectively, with pretest results as covariate. The horizontal items were included as a test of discriminant validity, so I predicted no effects. Regarding the vertical items on the number sense task, I addressed questions 1c), 2c), and 3 using a 2 (placement: larger on top, larger on bottom) x 3 (condition: top-down, bottom-up, explicit analogy) mixed-design ANCOVA using posttest accuracy scores and response times, respectively, with pretest results as covariate. I predicted an interaction between condition and the larger number's position placement (larger on top or on the bottom). On the items where the larger number is on the bottom, the top-down condition was expected perform the best relative to the explicit analogy and bottom-up conditions because the top-down hundreds chart positions larger numbers below smaller numbers. I predicted that the bottom-up condition would also perform better than the explicit analogy condition because the up is more relation is only implicit in the structure of the chart. The reverse pattern was expected for the items in which the larger number is on the top.

Chapter 3: Method

Participants

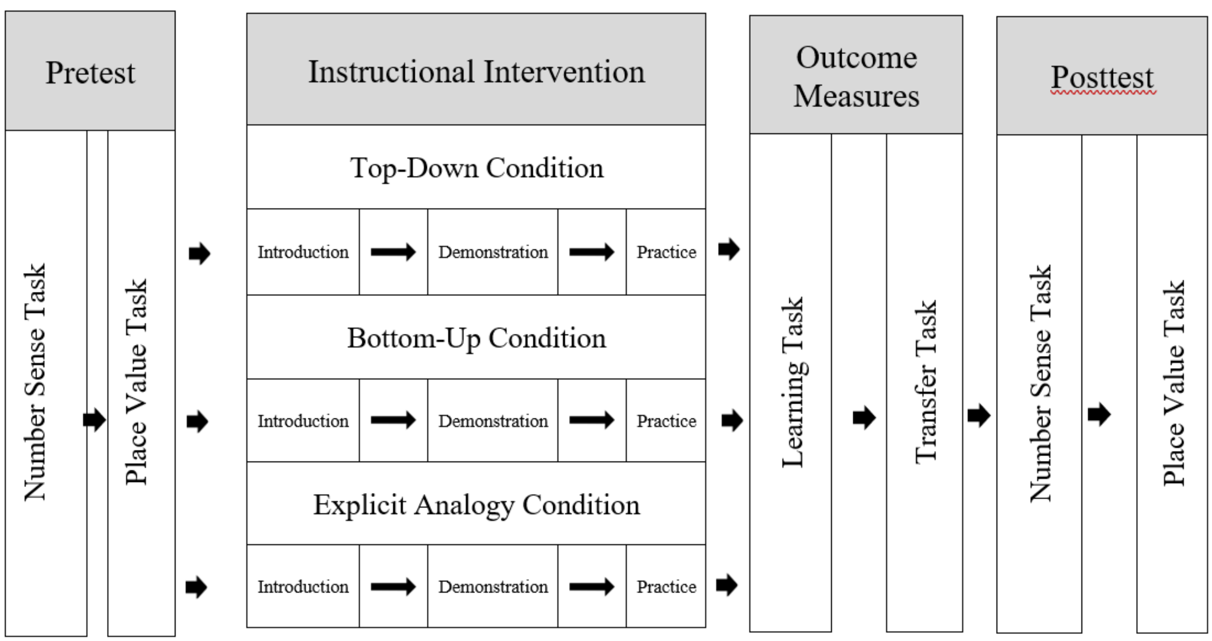
Fifty-one (N = 51) children in kindergarten and first-grade participated in the study. Eligible participants included students in kindergarten (n = 9) at the end of the school year (4 in May, 3 in June, 2 in July) and first-grade (n = 38) at the start (2 in September, 2 in October, 6 in November) and end (14 in April, 6 in May, 6 in June, 1 in July, 1 in August) of the school year. The mean age in years was 6.3 (SD = .59). Four participants were excluded from the final sample: Two children exhibited difficulty identifying numbers and number names, one child was stressed and stopped the interview early, and one child's performance was influenced by a parent. The participants were English-speaking children recruited from Canada and the United States. Participants were recruited through flyers advertised on social media, by contacting schools, and through word of mouth. Interested parents communicated with me using the email address provided on the flyer. Participants had to have access to an electronic device with a webcam and stable internet connection, and those who did not understand and speak English were excluded from the sample.

Design

This study employed a 2 (time: pretest, posttest) x 3 (condition: chart type) experimental design (see Figure 2). Pretest measures examined participants' number sense and place value understanding. During the instructional intervention, participants completed an addition activity with the researcher using the hundreds chart corresponding to condition. Immediately after the instructional intervention, participants completed a learning task and a transfer task. The learning task assessed whether the participants learned how to use the hundreds chart assigned to them in their condition to add double digit numbers. The transfer task assessed the degree to which the participants encoded the inverse abstract relation corresponding to up is more, namely, "down is less," to compute subtraction problems. The strategies participants used to solve the arithmetic problems on the learning and transfer tasks also shed light onto their understanding of place value. The posttest measures were isomorphic versions of those administered at pretest. I tested for condition differences in posttest scores on place value understanding and number magnitude accuracy and response time, respectively, using pretest scores as a covariate. I also tested for condition differences on the direction in which the participants moved in the chart and the types of strategies used on the learning and transfer tasks.

Figure 2

Study Design



Participants were randomly assigned to one of three conditions: (a) top-down, (b) bottom-up, or (c) explicit analogy. Although participants in all conditions completed the same pretest, posttest, learning, and transfer tasks, they used different hundreds charts during the instructional intervention and while completing the learning and transfer measures; the direction of the numbers in the hundreds chart used in the top-down condition differed from that used in the bottom-up and explicit analogy conditions. A bottom-up chart was used in the explicit analogy condition and participants were given a second, explicit visual analogy during instruction, which presented an image of a two-dimensional cylindrical container holding liquid. The rise of liquid in the container served to highlight the upward increase of the numbers in the hundreds chart. Otherwise, the instruction was identical across conditions. Participants met with the researcher individually in one online session during which all the data were collected.

Instructional Intervention

The purpose of the instructional intervention was to teach double digit addition using a hundreds chart. The intervention consisted of three phases: introduction, demonstration, and practice. The three phases of the instructional intervention took approximately ten minutes to complete. All the problems in each phase are presented in Table 1.

Table 1

Items for the Phases of the Instructional Intervention

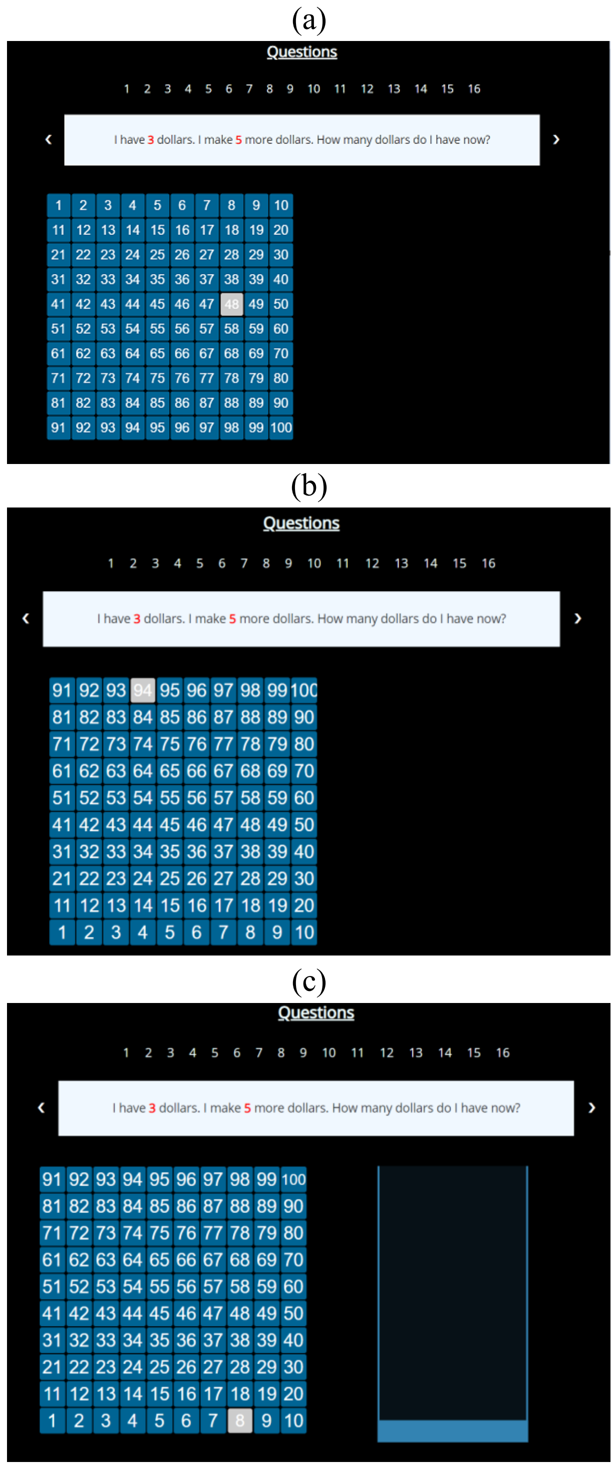
Phases	Addends	Items
Introduction	Not applicable	Explanation of the hundreds chart, no items
Demonstration	3 + 5	I have 3 dollars. I make 5 more dollars. How many dollars do I have now?
	24 + 10	There were 24 cows in a field. The farmer let 10 more cows into the field. How many cows then in the field?
	51 + 38	My friend has 51 stickers. I gave him more 38 stickers. How many stickers does my friend have now?
Practice	57 + 40	There were 57 carrots in a vegetable garden. The next week, 40 more carrots grew in the garden. How many carrots were there at the end of the second week?
	63 + 14	A bakery had 63 sweets on display. Then the baker came out and put 14 more sweets on display. How many sweets were on display?
	31 + 27	There were 31 parents early to watch a school concert. 27 more parents came in after. How many parents were watching the school play?

Materials

The researcher and participants used the Virtual Hundreds Chart (VHC; see Figure 3; Osana et al., 2021a, 2021b, 2021c), an electronic hundreds chart designed specifically for the present study, in all three conditions. The VHC is an online hundreds chart accessible to anyone with the URL. The participants and the researcher used a link to access it. The cells of the hundreds charts are blue with white numbers. The screen background is black. The word problems are presented above the chart in black text in a white rectangular box. The quantities in the word problems are presented in red. The VHC is navigable using the computer mouse or arrow keys on the keyboard. The cells light up as the user moves around the chart. There are two ways for users to select a number in the chart to state their answer to a problem: double clicking on the cell with the mouse or using the arrow keys on the keyboard and pressing the “Enter” key.

Figure 3

Virtual Hundreds Chart (VHC) Display



Note. (a) VHC Display for top-down condition, (b) VHC Display for bottom-up condition, (c) VHC Display for explicit analogy condition.

The numbers in the hundreds charts in all conditions increased by one from left to right in each row. The charts differed by condition in terms of the vertical direction in which the decades increase and decrease. In the top-down condition, participants used a top-down hundreds chart (see Figure 3, Panel a). In this case, the first row of cells at the top of the chart contained the numbers from one to ten, and 100 is in the bottom right corner; moving down each row indicates an increase by one decade. In the chart used in both the bottom-up and explicit analogy conditions, the numbers one through ten are in the lowest row and 100 is in the upper right corner (see Figure 3, Panels b and c). Climbing up the bottom-up hundreds charts by row reflects increases by ten. Both conditions involve an analogy where up is more. The analogy is implicit in the chart, and the container in the explicit analogy condition (Figure 3, Panel c) acted as an explicit visual emphasis on the up is more analogy that is inherent in the structure of the chart.

Introduction Phase

The introduction phase began by showing the participants their assigned hundreds chart via the researcher's shared screen on an online meeting platform (the hundreds charts in Figure 1 were used in the first part of the introduction phase). The researcher formally introduced the condition-specific Virtual Hundreds Chart to the participants (see Figure 3). With sweeping gestures of the cursor along the edges of the hundreds chart, the researcher explained that there are 10 squares across and 10 squares down in the chart. The cursor's movement over the cells lights them up, calling attention to those cells. The cursor's motion across the rows and columns matched the direction of the increase in numbers (i.e., moving from left to right along the top row in all conditions), moving from top to bottom in the top-down condition and from bottom to top in the bottom-up and explicit analogy conditions.

Next, the researcher hovered over the 1 in the top or bottom left corner, depending on condition, showing where the numbers start. With a sweeping motion of the cursor, highlighting the cells, she explained that the numbers get larger when moving to the right. In the top-down condition, she made a left-to-right motion over the starting row, then a downward movement along the rightmost column in the chart to show that the numbers increase in this pattern. She explained, “The numbers get bigger by one as we move to the right, and they go up by ten as we move down with every row.” She repeated the same gesture and explanation for the bottom-up and explicit analogy conditions, moving left to right and from bottom to top in each row. In all conditions, the researcher ended by highlighting 100 and explaining that it is the largest number in the chart. Then, she asked the participants to use the chart to count from 1 to 15. The purpose of having them count was to help the participants familiarize themselves with the direction in which the numbers increase within rows.

Demonstration Phase

With the exception of the specific hundreds chart used, the demonstration phase was identical across all conditions. It began with an explanation of an addition activity based on a modified version of the game, “Catch Me If You Can” (Bay-Williams & Fletcher, 2017) in which participants move across the cells of the hundreds chart. The objective of the activity is for participants to learn how to use the hundreds chart to add double-digit numbers. Addition problems were couched in narratives that reflect Join Result Unknown (JRU) problem structures (Carpenter et al., 2014; see Table 1). JRU problems are those that describe a joining action with the result as the unknown (e.g., I have 3 dollars. I make 5 more dollars. How many dollars do I have now?). The starting amount and the change are given in the problem.

The researcher modeled the solution strategy with three different problems, listed in Table 1. The solution to the first demonstration problem, 3 + 5, modeled the lateral movement on the chart, and was contained within the top or bottom row, depending on the condition's hundreds chart. The second example, 24 + 10, demonstrated the up or down direction of numbers in the chart, depending on the condition, by adding only decades to the first addend. By excluding the addition of units in the second addend, participants only moved in one direction, either up or down. This emphasized the idea that numbers increase in the up or down direction, depending on the hundreds chart assigned to condition. The third problem, 51 + 38, involved units and decades in the second addend, therefore combining movement in both vertical and horizontal directions. The researcher demonstrated the solution by adding the tens before the units. Like the first problem, the second and third demonstration problems were contextualized in JRU narratives (see Table 1 for the contexts for each problem).

The researcher started by modeling the solution to the first demonstration problem on the hundreds chart. First, she read the entire problem to the child, and then again, sentence-by-sentence, as she worked through the solution. After she reread the first sentence, she double clicked on 3, the starting amount. She clarified that she was beginning on this number because she is starting with three dollars. Upon rereading the second sentence out loud, she explained that she needed to move five spaces in the direction in which the numbers increase. The researcher used the arrow key to highlight the numbers as she counted five cells out loud, "1 unit, 2 units, 3, units, 4 units, 5 units." In all conditions, adding units corresponded to movements to the right. Finally, she explained that the last highlighted number is the number of dollars she had now, the solution to the problem (in this case, "The answer is 8. I have eight dollars now.").

For the second demonstration problem (i.e., 24 + 10), she again explained that she starts with 24 and double clicked that cell. Then, she explained that in 10, there is one group of ten, pointing to the 1 in the 10 in the problem with her cursor. She explained that since each row is a group of ten, she moves up one row (or down one, depending on condition). The last problem (i.e., 51 + 38) in the demonstration required adding decades and ones. She again explained that because she starts with 51, she double clicked that cell. Next, she pointed out that since they are adding 38, she needs to add three groups of ten. She moved up (or down) three rows. Lastly, she explained that they have to add 8 since there are eight units in 38. She moved eight squares to the right. Each time she finished a demonstration problem, she repeated that the number in the cell on which she ends is her solution to the problem.

Each demonstration problem was displayed in sentences above the hundreds chart on the computer screen (see Figure 3). The quantities in the problem text were written in a contrasting color (i.e., red). The different color highlighted the numbers within the sentences to draw attention to the quantities involved. The researcher read the problem out loud as many times as the participants wanted to hear it.

In the explicit analogy condition, the demonstration and practice phases of the instructional intervention included an explicit visual analogy of a container of liquid that highlighted the up is more relation inherent in the bottom-up hundreds chart. There was an image of a container on the right side of the hundreds chart (see Figure 3, Panel c), represented by a portrait-oriented rectangle with no top. For each problem, the container appeared to hold a liquid filled to match the starting amount in the problem, and the liquid rose when the second addend was added and the sum was selected on the chart. The height of the container reflected the range of numbers in the chart (i.e., 1 – 100).

Consider the problem 3 + 5. The researcher drew the participants' attention to the fill level by saying, "I will start on 3 because I start with 3 dollars. Look at what happened to the water in the container." She double clicked on first addend in the VHC which filled the container to this starting amount. When the researcher finished adding 5 (i.e., the amount of the second addend), she drew the participants' attention to the container again, "Did you see what happened to the water in the container?" Then the researcher stated the final answer, "My answer is 8. I have 8 dollars now."

Practice Phase

In the practice phase, the participants completed three items on their own (see Table 1). The researcher asked them to use the assigned chart in the same way she had for the three previous demonstration problems. Similar to the demonstration, the participants practiced one example where the second addend involves decades only. The second and third practice problems required two movements: vertical movement for the decades and horizontal movement for the units. In the explicit analogy condition, like in the demonstration, the researcher drew students' attention to the container beside the hundreds chart as the participants solved the practice problems. All sums were less than 100, and none of the problems involved regrouping. The researcher offered corrective feedback if participants made errors. The feedback focused on the procedures, without direct instruction about number structure or place value. To prompt the participants, the researcher directed their attention to the digits individually in the problem and reminded them of the direction in which to move on the hundreds chart.

For the practice problems, participants used the VHC that matched their condition and shared their screen with the researcher. For each practice problem, the researcher read the entire word problem, then reread the first sentence. Then, she asked the participants to double click the number to demonstrate their starting position. She stated, "Which number do we start on? Find that number and click on it twice." The researcher read the next sentence of the problem and asked the participants to use the arrow keys on their keyboards to move through the chart. This showed where they were moving in the chart and how they counted to arrive at their solution. The researcher also asked that they show their final answer by double clicking: “Please double click on your answer, which will be the number you land on when you’re done.”

Measures

Demographic Information and Exposure to Hundreds Charts in School

Participants’ parents completed a short demographic questionnaire prior to the interview for statistical purposes. The participant’s age, gender, language spoken at home, home province or state were among some of the questions on the questionnaire (see Appendix).

During communication with schools for participant recruitment, kindergarten and first-grade teachers were invited to complete a brief questionnaire on their use of hundreds charts in their classroom. The questionnaire asked if the teacher used a hundreds chart for teaching and for what learning objectives. Teachers were also asked to fill in the first and last 10 numbers in a 10 by 10 grid to illustrate what type of hundreds chart they used.

To gauge participants’ familiarity with the hundreds charts and whether they had prior experience with it, they saw another version of the hundreds chart on the screen following the pretest and before the researcher’s formal introduction of the condition-specific hundreds chart. This novel configuration turned the top-down hundreds chart by 90 degrees clockwise (see Figure 4). The researcher asked the participants if they had “seen things that looked like this in their class” before, and what they did with them.

Figure 4

Hundreds Chart Reference for Assessing Participant Exposure

91	81	71	61	51	41	31	21	11	1
92	82	72	62	52	42	32	22	12	2
93	83	73	63	53	43	33	23	13	3
94	84	74	64	54	44	34	24	14	4
95	85	75	65	55	45	35	25	15	5
96	86	76	66	56	46	36	26	16	6
97	87	77	67	57	47	37	27	17	7
98	88	78	68	58	48	38	28	18	8
99	89	79	69	59	49	39	29	19	9
100	90	80	70	60	50	40	30	20	10

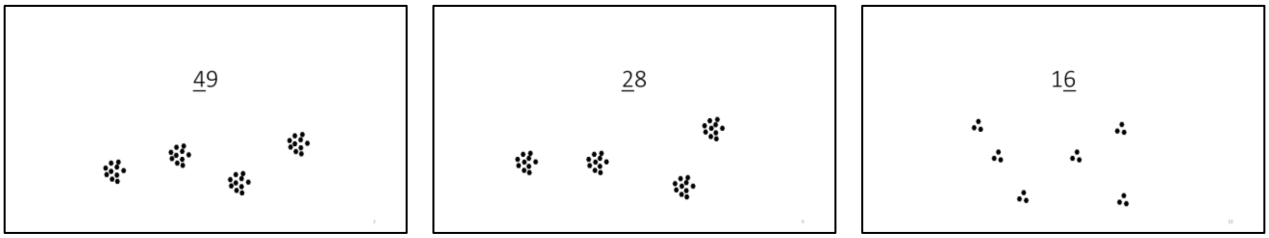
Place Value Task

Participants worked on a modified version of the Picture Place Value Task (PicPVT; Osana & Lafay, 2019). They identified whether the underlined digit in a two-digit number matched the pictorial representation depicted below it (see sample item in Figure 5). When the test trials began, each item was prefaced by the researcher asking the question, "Does the picture match the underlined part of the number?" The child answered "yes" or "no." The test took about five minutes to complete.

Figure 5

Sample PicPVT Items

(a) (b) (c)



Note. (a) Sample “yes” item, (b) Sample “no” item with a groups error, (c) Sample “no” item with a denominations error.

The pictorial representation on each item consisted of sets of hexagons. One hexagon was meant to represent one unit; the clusters represented groups of units. To accurately assess the representation, participants must understand what each place in a two-digit number means: A five in the tens’ place does not mean five units or five clusters of anything other than groups of ten units. Similarly, a five in the ones’ place could only be correctly represented with five single hexagons.

There were 18 items on the PicPVT. Eight of the items had a correct representation for the underlined digit (i.e., “yes” items). The remaining 10 items had incorrect representations (i.e., “no” items). There were two types of errors on the “no” items: A denomination error (5 items), where the number of groups was correct, but the group size was incorrect (e.g., clusters of 6 instead of 10 hexagons), and a groups error (5 items), where the number of groups was incorrect, but the group size was correct. For example, representing the 9 in the number 69 by 9 groups of 2 qualifies as a size error. Four groups of 10 hexagons to represent the 2 in the number 28 is a groups error.

To introduce the task, the researcher showed the child the picture used for the units and the picture used for the tens and explained that this is one unit and one group of 10 units, respectively. The next slide showed two groups of three hexagons. The researcher asked the participant how many groups are on the screen and how many hexagons are in each group. Irrespective of the participant's responses, the researcher gave the correct response and counted the number of groups and hexagons in each group, respectively. This process was repeated for four groups of two. Then, the participants completed one practice trial for an item with a correct representation (i.e., a "yes" item: the numeral <u>5</u>4, represented by 5 groups of 10 hexagons), followed by one practice trial with an incorrect representation (i.e., a "no" item with a groups error: the numeral 3<u>8</u>, represented by 5 groups of 1 hexagon). The researcher provided corrective feedback on both items. On the posttest, the researcher repeated a shorter version of this introduction. It was similar to the pretest, except there was only one item for the introduction (i.e., two groups of three) and one practice trial item (i.e., a "no" item with a groups error: the numeral <u>3</u>8, represented by 2 groups of 10 hexagons).

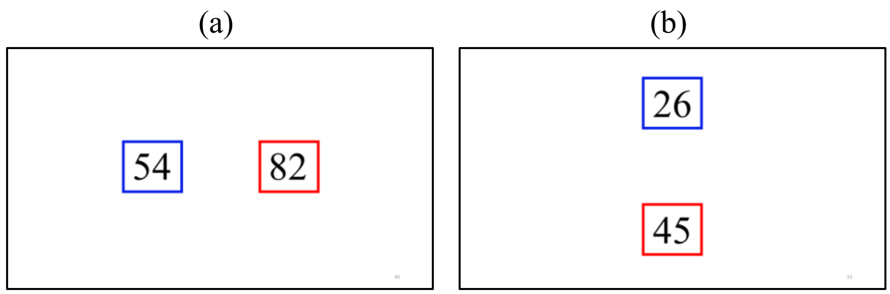
The corresponding items on the pretest and the posttest were isomorphic. On the isomorphic items, the underlined digit in the numeral increased or decreased by 1 or 2 and the other digit in the numeral stayed the same. Each correct answer was assigned 1 point and each incorrect answer, 0 points. The PV denomination score was computed by taking mean number of correct responses on all 8 items that were correctly represented by the hexagons and all 5 items where the size of the group was incorrect; the PV group score was computed by taking the mean number of correct responses on all 8 correct items and all 5 items where the number of groups was incorrect.

Number Sense Task

The Number Sense task assessed participants' number comparison through the identification of the larger of a pair of two-digit numbers presented symbolically on a white background (see Figure 6 for sample items). The task assessed whether participants were able to identify the larger of two numbers, requiring knowledge of decades and units (i.e., place value).

Figure 6

Sample Items for Number Sense Task



Note. (a) Horizontal number pair, (b) Vertical number pair

There were 24 items on the number sense task. The decomposed model (Pletzer et al., 2016) for processing multi-digit numbers posits that people process units and tens separately. Pletzer et al. (2016) had participants compare the magnitude of compatible and incompatible pairs of numbers. When judging the larger of the two numbers, the pair is compatible when the correct judgment is made regardless of whether the tens or the ones are compared. For example, in the compatible pair 56 and 23, participants can determine the larger quantity by comparing the ones (i.e., 6 vs. 3) or the tens (i.e., 5 vs. 2). In an incompatible pair, the larger number has fewer ones than the number of ones in the smaller number (e.g., 53 vs. 26). If participants judged magnitude based on the ones in this case, they would make the wrong magnitude judgment. Pletzer et al. (2016) found that numbers are processed more quickly and accurately when they are compatible than incompatible (called the unit-decade compatibility effect, Pletzer et al., 2016). On the number sense task, all items contained incompatible numbers because responses to these items are more indicative of whether the participants understand how place value determines number magnitude.

In number magnitude comparison tasks, numbers are often placed side by side (i.e., horizontally, see Figure 5, Panel a; Brankaer et al., 2017). On the number sense task used in this study, one third of the items (i.e., 8 items) presented the numbers horizontally, and two-thirds of the number pairs (i.e., 16 items) were vertically aligned. The vertical positioning reflected the structure of the decades in the hundreds charts. Although the placement of the items (i.e., horizontal or vertical) has no impact on the unit-decade compatibility effect (Pletzer et al., 2016), participants' experience with either the top-down or bottom-up hundreds chart in the intervention may have influenced their selection of the larger number when they were placed vertically. The horizontal positioning acted as a measure of discriminant validity because the structure of the hundreds chart should not impact their judgements on the horizontal items.

Among the items configured horizontally (8 items), half had the larger number on the right side in the pair, and half had the larger number on the left side in the pair. Among the items configured vertically (16 items), half had the larger number above the other number in the pair; half had the larger number below the other number in the pair. The difference between the number pairs was between 13 and 39 across all the items. As shown in Table 2, in the administration of the test at both pretest and posttest, there were 12 items with a difference between 13 and 19, 24 items with a difference between 24 and 29, and 12 items with a difference between 34 and 39. The number pairs that have differences in these ranges are considered in the small range and I predicted no effects based on the size of these differences (Pixner et al., 2009). The pretest and posttest had isomorphic items keeping the difference and the decades of the numbers in the pairs consistent.

Table 2

Number Sense Items

	Pretest (24 items)		Posttest (24 items)	
	Larger on Left	Larger on Right	Larger on Left	Larger on Right
Horizontal	Item A	Item A	Item A	Item A
	Item B	Item B	Item B	Item B
	Item C	Item C	Item C	Item C
	Item D	Item D	Item D	Item D
	Larger on Top	Larger on Bottom	Larger on Top	Larger on Bottom
Vertical	Item A	Item A	Item A	Item A
	Item B	Item B	Item B	Item B
	Item C	Item C	Item C	Item C
	Item D	Item D	Item D	Item D
	Item A′	Item A′	Item A′	Item A′
	Item B′	Item B′	Item B′	Item B′
	Item C′	Item C′	Item C′	Item C′
	Item D′	Item D′	Item D′	Item D′

Note. The differences between the numbers in each item are 24-27 for A and A′ items, 13-19 for B and B′ items, 24-29 for C and C′ items, and 34-39 for D and D′ items.

The researcher presented the items using a PowerPoint presentation by sharing her computer screen. The numbers on the left of the horizontal pairs and the top numbers in the vertical pairs were in blue boxes, and the numbers on the right and those on the bottom were in red boxes. Participants answered by stating the color of the box that contained their response. It took roughly 10 minutes to complete the task. Participants earned 1 point for correctly identifying the larger number. Incorrect answers were assigned a score of 0. Unanswered items also scored 0 points. I calculated an accuracy score for the horizontal and vertical items, respectively, based on the mean number of correct answers. Participants' response times were also collected for analysis. Response times counted the seconds from the time the item appeared on screen to the participants' verbalization of their final answers. I used a cellphone stopwatch to record the response times to the hundredth of a second.

Learning Task*: *Addition

The learning task included six JRU problems that required participants to add two numbers together with sums less than 100. Table 3 presents these items in the order in which they appeared on the task and the narrative contexts in which they were situated. On three items, the second addend was a number that involves tens, but no units. As such, participants only needed move vertically in the chart to solve them. The other three items required participants to add decades and units, necessitating both vertical and horizontal movement in the chart. No corrective feedback was provided.

Table 3

Items on the Learning Task

Addends	Items
47 + 20	My friend had 47 marbles. She found 20 more marbles under her bed. How many marbles does my friend have now?
60 + 35	There were 60 flowers outside my neighbour's house. Over the summer, 35 more flowers grew. How many flowers were outside my neighbour's house then?
36 + 20	I had 36 comic books. My brother gave me 20 comic books. How many comic books do I have now?
51 + 24	There were 51 ants on the sidewalk. 24 more ants came over. How many ants were on the sidewalk then?
42 + 10	In the morning, I found 42 shells at the beach. In the afternoon, I found 10 more shells. How many shells did I find at the beach?
73 + 15	There were 73 jelly beans in a jar. My grandma added 15 jelly beans to the jar. How many jelly beans were in the jar?

The task was administered using the VHC that matched the participants' conditions and on which the participants could move around using arrow keys on their keyboards or their mouse. As in the instructional intervention, the problem text was displayed above the chart, with the addends in a different color from the text.

The procedure for reading and solving the problems was similar to the practice phase of the intervention. The researcher read the entire problem out loud, then reread the first sentence.

The researcher said, “What number do we start on? Where is that number in the chart?” The researcher then said, “Now we’re ready to solve the problem,” which was followed by the researcher reading the remaining problem text out loud. The researcher also reminded the participants to double click on their final answers. The participants share their screen during the learning task, and their movements in the hundreds chart to solve the problems were recorded using the Zoom recording feature. The task took about 10 minutes to complete.

Coding and scoring. Participants’ performance on the learning task was scored in two ways, which yielded two dependent variables: (a) direction, which was a measure of the participants’ movements in the chart, and (b) strategy use, which was used as a second assessment of place value understanding.

The direction variable assessed whether participants moved in the correct vertical direction, according to their conditions’ assigned hundreds chart. Direction was coded as correct or incorrect vertical direction. On the learning task (i.e., addition problems), participants in the top-down condition who moved downward from their starting point in the chart were coded as travelling in the correct direction. In contrast, participants in the bottom-up and explicit analogy conditions were coded for correct direction when they moved upwards from their starting position.

The second variable derived from the learning task was strategy use. This variable examined the type of strategy participants used to arrive at their solution. They were coded as mathematically appropriate and mathematically inappropriate strategies. Mathematically appropriate strategies must have used the correct direction, according to the problem (learning, transfer) and the hundreds chart configuration used during the intervention (i.e., top-down, bottom-up). Two types of mathematically appropriate strategies were documented: (a) moving by tens, and (b) moving by ones. Moving by tens encompassed strategies that correctly used vertical movement for the tens and horizontal movement for the ones. Moving by ones strategies counted forward by ones horizontally from the starting point to the solution.

It was possible for students' strategies to be coded as mathematically appropriate if they committed one or more of the following errors. There was no penalty for participants who landed on the correct answer but said the wrong number name. Starting on the wrong addend, answers off by one (one unit or one ten or both), moving in increments, and skipping a row were forgiven. To distinguish answers off by one from all other types of errors, the difference between the tens digit and the ones digit in the addends was greater than one. The number 21, for instance, was not in the list of addends given that the small difference between the tens and units (i.e., 1) would make it difficult to identify whether a participant erroneously miscounted or confused the tens and units. In contrast, one of the addends in the practice phase of the intervention is 47, where the difference between 4, the number of decades, and 7, the number of units, is 3, which greater than one. If participants mistakenly counted 3 tens, it would be clear that they had miscounted because the number of units in the numeral (i.e., 7) was far enough away from the tens digit (i.e., 4) on the number line that swapping the units and digits would not seem like a plausible explanation for the error. Selecting digits in the numbers such that they have a difference greater than one helped to disentangle counting errors from mathematically inappropriate strategies.

Mathematically inappropriate strategies included one or more of the following errors: moving in the wrong direction for tens and/or ones based on problem type and hundreds chart configuration, confusing the tens' and ones' digits, moving too many or too few spaces (more than one), only moving the value of one digit, or using a serpentine action to navigate the hundreds chart.

Transfer Task: Subtraction

The transfer task consisted of four items that required participants to subtract quantities rather than add them, necessitating inverse directions through the chart. The items are displayed in Table 4 in the order in which they appeared on the task.

Table 4

Items on the Transfer Task

Number sentence	Items
72 – 10	There were 72 birds on a telephone wire. 10 birds flew away. How many birds were left on the telephone wire?
35 – 20	There were 35 beads on a table. I used 20 beads for a necklace. How many beads were left on the table?
53 – 40	My classroom had 53 new markers in a bin. After some time, 40 markers dried up. How many good markers were left in the bin?
48 – 30	There were 48 cars in a parking lot. 30 cars drove away. How many cars were left in the parking lot?

The items only required subtracting decades. They were contextualized in Separate Result Unknown (SRU) problem narratives (Carpenter et al., 2014). SRU problems involve a separating action where the start and change amounts are known, and the unknown quantity is the result (e.g., My friend has 48 cookies. He gives 30 cookies away. How many cookies does my friend have now*?*). Like the instructional intervention and the learning task, the word problems were presented on the screen above the chart, with the numbers in the problem shown in red. Given the seamless transition in interface from the learning task to the transfer task, the researcher advised participants, “These next problems are a little different, but they’re still using the hundreds chart.” The researcher read the problem text out loud in the same way as on the learning task. The task took roughly 10 minutes to complete.

Like the learning task, the transfer task yielded two dependent variables. First, the direction variable was coded in the same way as for the learning task, examining vertical direction from the starting point, according to the type of hundreds chart. However, since the transfer task involved subtraction problems, the correct direction was the inverse of that travelled on the learning task. Participants needed to move upwards in the chart from their starting point in the top-down condition and downwards in the bottom-up and explicit analogy conditions. Participants’ strategies were coded as correct or incorrect direction. The second dependent variable using the transfer data was the strategy use variable, like that of the learning task. It also involved coding participants’ strategies for solving the subtraction word problems using the same coding rubric as for the learning task.

Procedure

The entire interview took place on Zoom, an online meeting platform. Participants and the researcher shared their screens with each other. Participating parents contacted me using the email I provided on the recruitment flyer. I sent them an email with a schedule indicating available times to meet with their child virtually and access to a consent form and the demographics questionnaire. Parents were given the option of completing their responses on the digital document or printing and scanning their responses to send back via email.

Once a meeting time was established, I sent the parent a Zoom link. Participants accessed the virtual space using this link. I also included the URL link to the web-based VHC that the child manipulated during the intervention and the learning and transfer tasks. I resent the links in a reminder email the day before the meeting. Each meeting included the researcher, the child, and the child's parent. Parents were asked to remain in the room during the interview to assist with any technical difficulties. They were also asked not to interfere with responses as I was interested in what the children had to say.

During the Zoom meetings with the participants, I shared my screen and read the assent form out loud, displayed on a PowerPoint slide, to the child. I explained, in child-friendly language, what the study entailed and what would be asked of them. Before continuing, the children agreed orally that they had understood and wanted to participate in the study. The children's verbal assent was recorded using the Zoom recording feature.

Data collection was conducted entirely on Zoom with either the researcher or the child sharing her screen with their cameras and microphones on. The number sense task and the PicPVT were presented first. Both tasks were administered using PowerPoint slides, shared by the researcher. The VHC was used during the instructional intervention. The researcher shared her screen during the demonstration phase, and the participants shared their screens for the practice phase and for the learning and transfer tasks. The parent was asked to help with the sharing feature in Zoom and with accessing the VHC, if necessary. The researcher recorded the participants' answers on a printed scoresheet for the pretest, posttest, and learning and transfer tasks. Between tasks, the interviewer verified with the participants whether they needed to take a break.

Twelve short physical activities, called "brain breaks," took place intermittently throughout the Zoom meeting (i.e., about every 10 items). These moments allowed the participants to take a break from thinking mathematically. Moreover, it helped create a more relaxed atmosphere and forge a participant-researcher connection over a digital platform.

At the end of the session, I sent an email thanking the parent and the child for their time, effort, and contribution to the project. In addition, a participation diploma for the child was attached to the email. Starting in August, I offered small incentives to the participants because responses for participation had stopped. As such, the final 14 participants were offered digital gift cards as a token of thanks.

Chapter 4: Results

The first objective of the present study was to determine whether the configuration of number in the hundreds chart influence children's use of the chart for addition and subtraction. Learning of place value, and number sense. Given the up is more analogy present in the bottom-up hundreds chart, it was hypothesized that participants who worked with this chart would outperform participants who worked with the top-down chart.

The second objective was to determine whether the explicit analogy condition indeed had an advantage in chart use, place value, and number sense. The container at the side of the hundreds chart with which they worked filled as participants selected numbers higher in the chart. Since this feature was meant to help participants detect the pattern in the numbers, this condition was expected to do better than the top-down and bottom-up conditions. Thus, I anticipated that the participants in the explicit analogy condition would perform best relative to the bottom-up condition given the supplemental visual cues highlighting the up is more analogy.

The study's final objective was to determine whether there was a condition effect based on the larger number's position in the vertical items of the number magnitude comparison task. In this task, the vertical items included two double-digit numbers placed below and above one another. This configuration aligns with the hundreds chart where double-digit numbers in different decades fall in different rows. I hypothesized that the top-down condition would perform better than the other two conditions when the larger number was on the bottom as this configuration mimics the structure of the top-down hundreds chart. The bottom-up condition was expected to do better than the explicit analogy condition because participants who worked with the bottom-up chart might be less swayed by the up is more analogy. Similarly, the bottom-up and explicit analogy conditions were expected to outperform the top-down condition on items where the larger number was on top because this reflects the structure of the bottom-up hundreds chart. The explicit analogy condition was expected to do better than the bottom-up condition on these items because of the visual cue highlighting the up is more analogy. No condition differences were expected on the horizontal items.

This section will showcase my results and whether the configuration of number in the hundreds charts influences participants' directional movement in the chart, place value understanding, and number sense (i.e., research questions 1 and 2). I will begin by looking at participants' use of the hundreds chart by analyzing direction as dependent variable. Next, I will examine the participants' place value knowledge, first using the results from the PicPVT, then by considering a strategy use variable on the learning and transfer tasks. Finally, I will analyze number sense using performance accuracy and response times on the number magnitude comparison task, looking at horizontal, then vertical items separately. Here, I will consider whether the larger number's position in vertical items impacted participants' accuracy and response times on this number sense task (i.e., research question 3).

Demographic Information

Data were collected from 47 participants (49% male) from three Canadian provinces and four American states. Seventy-eight percent of the sample reported having seen a hundreds chart in their classroom before. Demographic characteristics of the sample are presented in Table 5.

Table 5

Characteristics of the Sample (N = 47)

Characteristics	Participants	
	n	%
Home Province or State		
Quebec, CAN	33	70.21
Ontario, CAN	2	4.26
Alberta, CAN	2	4.26
California, USA	6	12.77
Colorado, USA	2	4.26
Nevada, USA	1	2.13
Ohio, USA	1	2.13
Family Income ($)		
20 000 – 40 000	2	4.26
40 000 – 60 000	2	4.26
60 000 – 80 000	0	0
80 000 – 100 000	4	8.51
100 000 +	39	82.98
Language Spoken at Home		
English	31	65.96
English & French	6	12.77
English, French, & Another	5	10.64
English & Another (Not French)	4	8.51
Other (Not English nor French)	1	2.13
Language of Math Instruction		
English	32	68.08
French	11	23.40
English & French	3	6.38
Cree	1	2.13

The participants were randomly assigned to one of three conditions (top-down, $n = 15$; bottom-up, $n = 16$; explicit analogy, $n = 16$). A one-way ANOVA revealed that there was no difference in age (in months) between the three conditions, $F(2, 44) = .68$, $p = .51$. Tests of chi-square showed no significant difference in condition by gender, $\chi^2(2, N = 47) = 1.08$, $p = .58$, nor in condition by grade, $\chi^2(2, N = 47) = .68$, $p = .71$, nor in condition by language of math instruction, when comparing English and a collapsed category of French & English and French, $\chi^2(2, N = 46) = .17$, $p = .92$. Cree was excluded from this chi-square test since only one participant listed it as language of math instruction. Other demographic characteristics contained too few people in the possible categories for any statistical analysis.

When the participants were recruited through school boards, I asked if any kindergarten or first-grade teachers would complete a questionnaire on their use of hundreds charts in their classroom. Twelve teachers from eight Canadian schools in two provinces (i.e., Quebec, $n = 6$; Ontario, $n = 2$) and two American schools in two States (i.e., West Virginia, $n = 1$; Washington, $n = 3$) returned the questionnaire. A majority of teachers (83.3%) reported using the top-down hundreds chart in their classroom (see Table 6). Although only a small number of teachers provided these data, and the sample was not random, more teachers used the top-down hundreds chart in their classroom than other chart types.

Table 6

Self-Reported Data on Teachers' Type of Hundreds Chart Used in Their Classrooms

Hundreds Charts	First-grade (n = 11)		Kindergarten (n = 1)		Total (n = 12)	
	n	%	n	%	n	%
Top-down	9	81.8	1	100.0	10	83.3
Bottom-up	1	9.1	0	0	1	8.3
Does not use a hundreds chart	1	9.1	0	0	1	8.3

Direction

Data from the learning and transfer tasks were used to generate a dependent variable that assesses the ways in which participants used the hundreds chart they were provided. The direction variable examined whether participants moved in correct vertical direction in the hundreds chart on the learning and transfer tasks, i.e., on addition and subtraction word problems. A downward movement counted as a correct vertical direction for the top-down condition for addition problems on the learning task and an upward movement was correct for subtraction problems on the transfer task; an upwards vertical direction on addition problems was correct for the bottom-up and explicit analogy conditions, and a downward movement was correct for these conditions for subtraction problems. Participants who solved the word problems mentally were not included in analysis.

For direction, there was no variability on the learning task. That is to say, 100% of participants went in the correct vertical direction for their respective condition. As such, data from the learning task were not analyzed for this dependent variable. On the transfer task (see Table 7), a condition by correct vertical direction (yes, no) chi-square test using item as unit of analysis revealed differences, χ^2 (2, N = 151) = 8.62, p = .01. In particular, the proportion of items on the transfer task where the participant went in the incorrect direction in the top-down condition (i.e., 10 %) was significantly lower than in the bottom-up condition (30%; z = 2.58, p < .05). There were no significant differences between the top-down and explicit analogy conditions (z = .28, p > .05), nor between the bottom-up and explicit analogy conditions (z = -2.31, p > .05).

Table 7

Direction Frequencies on Transfer Task by Condition (N = 188)

	Top-down		Bottom-up		Explicit Analogy	
	n	%	*n*	%	*n*	%
Correct Direction	45	90.0	35	70.0	45	88.2
Incorrect Direction	5	10.0	15	30.0	6	11.8
Total	50	100.0	50	100.0	51	100.0

Note. Data were missing on 12 items on the transfer task (n = 151).

Effects of Chart Structure on Place Value Knowledge

PicPVT

The PicPVT was used to assess place value knowledge on the pretest and posttest. On the PicPVT, participants were asked to judge whether a display of small hexagons accurately represents an underlined digit in a double-digit number. Two scores were computed to assess participants' place value knowledge, and each one was sensitive to a different aspect of place value: the size of the denomination (PV-denomination score) and the number of groups in the position of the digit (PV-group score).

No statistical analyses were conducted on the group score given that they were at ceiling (see Table 8). A one-way analysis of covariance (ANCOVA) was conducted on the posttest denominations scores as the dependent variable, with condition as the between-groups factor and pretest scores as the covariate. A main effect of condition was found on the denominations score, $F(2, 41) = 3.68$, $p = .03$. Unadjusted means are presented in Table 8. Least Significant Difference post hoc analyses indicated that the top-down condition ($M_{adj} = .83$; SE = .03) outperformed both the bottom-up ($M_{adj} = .74$; SE = .03), $t(41) = 2.22$, $p = .03$, and explicit analogy ($M_{adj} = .73$; SE = .03), $t(41) = 2.47$, $p = .02$ conditions. There was no significant difference between the bottom-up and explicit analogy, $t(41) = .35$, $p = .73$ conditions.

Table 8

PicPVT Mean (Standard Deviation) for Accuracy Scores on the Denomination and Group Subscales at Pretest and Posttest

Score	Top-down (n = 15)		Bottom-up (n = 15)		Explicit Analogy (n = 15)	
	Pretest	Posttest	Pretest	Posttest	Pretest	Posttest
Denomination	.82(.17)	.87(.18)	.75(.18)	.74(.19)	.71(.13)	.69(.13)
Group	.96(.07)	.98(.04)	.95(.12)	.95(.10)	.98(.05)	.96(.06)

Note. One participant in each of the bottom-up and explicit analogy conditions did not complete the PicPVT at posttest, and were thus excluded from the analysis.

Strategy Use

Participants' place value knowledge was also assessed using the strategy use measure, which examined the type of strategy participants used on the learning and transfer tasks. Specifically, a chi-square test using item as unit of analysis was conducted to examine the relation between strategy (moving by tens, moving by ones, mathematically inappropriate) as a function of condition (top-down, bottom-up, explicit analogy; see Table 9). Strategies that used mental computation to solve the problems were excluded from the analyses. The results revealed a significant association between strategy type and condition on the learning task, $\chi^2(4) = 21.1$, $p < .001$, and on the transfer task, $\chi^2(4) = 18.5$, $p < .001$. On the learning task, the top-down condition presented a greater proportion (85.6%; $z = 4.34$, $p < .05$) of items on which the moving by tens strategy was used than the bottom-up condition. There were no significant differences between the proportion of items in the explicit analogy condition who used this strategy and the bottom-up ($z = -1.68$, $p > .05$) nor top-down ($z = 2.60$, $p > .05$) conditions. On the transfer task, 42% of the items in the bottom-up condition involved the moving by tens strategy, which was a significantly lower proportion ($z = 5.69$, $p < .05$) than the top-down condition. The bottom-up condition also had a significant lower proportion ($z = -3.48$, $p < .05$) than the explicit analogy condition. There was no significant difference in the proportions in the explicit analogy and top-down conditions ($z = 2.00$, $p > .05$).

Table 9

Strategy Use Frequencies for the Learning and Transfer Tasks within Condition

Strategy	Top down		Bottom up		Explicit analogy	
	n	%	n	%	n	%
	Learning (n = 240)					
Moving by tens	71	85.6	43	55.8	55	68.8
Moving by ones	7	8.4	10	13.0	12	15.0
Mathematically inappropriate	5	6.0	24	31.2	13	16.2
	Transfer (n = 151)					
Moving by tens	41	82.0	21	42.0	35	68.6
Moving by ones	2	4.0	9	18.0	6	11.8
Mathematically inappropriate	7	14.0	20	40.0	10	19.6

Note. Data were missing on 20 items on the learning task and 12 items on the transfer task.

Effects of Chart Structure on Number Magnitude

To assess number sense, participants completed a number magnitude comparison task at pretest and posttest, where they selected the larger of two double-digit numbers.

Accuracy

Scores for accuracy on the number sense task were not analyzed statistically because there were ceiling effects at pretest and posttest on both horizontal and vertical items (see Table 10).

Table 10

Number Magnitude Comparison Task Mean (Standard Deviation) for Accuracy Scores

Score	Top-down (n = 15)		Bottom-up (n = 15)		Explicit analogy (n = 16)	
	Pretest	Posttest	Pretest	Posttest	Pretest	Posttest
			Horizontal			
Larger on Left	.95(.19)	.93(.26)	.97(.09)	1.00(.00)	.94(.11)	1.00(.00)
Larger on Right	.95(.19)	.93(.26)	1.00(.00)	.98(.06)	1.00(.00)	1.00(.00)
			Vertical			
Larger on Top	.92(.23)	.94(.22)	.98(.04)	.99(.04)	.95(.10)	.99(.04)
Larger on Bottom	.96(.16)	.93(.23)	.98(.05)	.98(.10)	.98(.04)	.99(.03)

Note. One participant in the bottom-up condition did not complete Number Magnitude task at posttest.

Response Times

Participants' response times were collected in seconds from when the item appeared on the screen to the moment when the participants verbalized their final answer. To analyze the results of this task, a two-way ANCOVA was performed using posttest response time as the dependent measure, with placement of the larger number (left, right for horizontal items; top, bottom for vertical items) as the within-group measure, condition (top-down, bottom-up, explicit analogy) as the between-group measure, and pretest response time as the covariate. Horizontal and vertical items are analyzed separately.

On horizontal items, where the two numerals were presented side-by-side, there were no condition differences (see Tables 11 and 12). There was no main effect of placement (right, left), $F(1, 39) = 1.96$, $p = .17$, no main effect of condition, $F(2, 39) = .59$, $p = .56$, nor a placement by condition interaction, $F(2, 39) = .31$, $p = .73$. Similar results were found on the vertical items, where the numerals were presented above and below one another (see Tables 11 and 12). The ANCOVA test revealed no main effect of condition, $F(2, 39) = .43$, $p = .66$), or placement, $F(1, 39) = .45$, $p = .51$, nor a placement by condition interaction, $F(2, 39) = 1.08$, $p = .35$.

Table 11

Number Magnitude Comparison Task Mean (Standard Deviation) of Response Times, in Seconds

Response Times	Top Down (n = 15)		Bottom Up (n = 14)		Explicit Analogy (n = 15)	
	Pretest	Posttest	Pretest	Posttest	Pretest	Posttest
			Horizontal			
Larger on Left	2.84(.78)	3.42(1.40)	3.75(1.13)	4.76(2.51)	3.68(1.70)	4.58(2.53)
Larger on Right	2.72(.67)	3.40(1.45)	3.63(1.22)	5.00(4.12)	3.83(1.78)	4.30(2.53)
			Vertical			
Larger on Top	2.80(.60)	3.38(.95)	3.75(1.09)	4.10(2.49)	3.64(1.29)	4.63(3.40)
Larger on Bottom	2.83(.76)	3.06(.98)	3.55(1.35)	4.15(1.92)	3.82(1.67)	4.26(2.19)

Note. One participant in the bottom-up condition and one participant in the explicit analogy did not complete the task at posttest. Response time for another participant in the bottom-up condition was not recorded due to a technology malfunction.

Table 12

Number Magnitude Comparison Task Adjusted Mean (Standard Error) of Response Times, in Seconds

Response Times	Top Down (n = 15)	Bottom Up (n = 14)	Explicit Analogy (n = 15)
		Horizontal	
Larger on Left	3.42(1.40)	4.76(2.51)	4.58(2.53)
Larger on Right	3.40(1.45)	5.00(4.12)	4.30(2.53)
		Vertical	
Larger on Top	3.38(.95)	4.10(2.49)	4.63(3.40)
Larger on Bottom	3.06(.98)	4.15(1.92)	4.26(2.19)

Note. One participant in the bottom-up condition and one participant in the explicit analogy did not complete the task at posttest. Response time for another participant in the bottom-up condition was not recorded due to a technology malfunction.

Chapter 5: Discussion

The present study aimed to explore the affordances of different types of hundreds charts, namely the top-down and the bottom-up charts, on kindergarten and first-grade students' use of the chart to solve problems and the development of their place value and number sense knowledge. In particular, the first objective was to determine whether the structure of the hundreds chart impacts students' use of the chart for addition and subtraction and the development of their place value and number sense. The second objective sought to determine if an explicit visual analogy of the up is more relation in the bottom-up chart provided an advantage in participants' performance. Thirdly, the present study aimed to determine whether the larger number's position in vertical items on the number magnitude comparison task influenced participants' number sense performance by condition.

Kindergarten and first-grade students first completed a pretest, composed of two tasks: a number magnitude comparison task as a measure of number sense and the PicPVT, a test of place value knowledge. An instructional intervention ensued whereby the researcher showed participants how to use a hundreds chart to solve addition problems. The configuration of the hundreds chart differed by condition: the top-down condition worked with the top-down hundreds chart; the bottom-up and explicit analogy conditions worked with the bottom-up hundreds chart. The explicit analogy condition featured an additional visual cue on the screen during the instructional intervention, which was designed to draw participants' attention to the up is more relation in the bottom-up chart. Next, participants solved six addition problems on the learning task without corrective feedback. On the transfer task, all participants were asked to use their respective hundreds charts to solve subtraction problems. The posttest followed, with isomorphic versions of the PicPVT and number sense tasks to the pretest.

I looked for condition differences on the PicPVT and the number sense measure at posttest using pretest performance as a covariate. Learning and transfer tasks were used to further assess chart use, specifically the (a) direction in which they moved, and (b) the application of place-value concepts in the strategies the students used to solve arithmetic problems with the chart. These results addressed the present study's first objective. The second objective was to test for condition differences on all variables between the explicit analogy condition and the bottom-up condition. This was explored simultaneously with the first objective: Any condition differences were followed up with pairwise comparisons to contrast the explicit analogy and bottom-up conditions. An examination of how each condition fared on the number sense measure responded to the third objective. I considered accuracy scores and response times at posttest using pretest scores as a covariate.

I predicted that the participants who worked with the bottom-up chart, namely the bottom-up and explicit analogy conditions, would perform better on the direction variable on the learning and transfer tasks than the top-down condition. Support for this prediction comes from the analogical reasoning literature where the presence of the up is more relation in the bottom-up hundreds chart, even without a visual cue, was expected to help participants match upward increases in everyday life with upward number increases in the chart (Gentner & Colhoun, 2010). Further, the explicit analogy condition was expected to perform better than the bottom-up condition since the up is more analogy was made explicit for these participants (Gick & Holyoak, 1983). Predictions for place value and number sense were the same as for the direction variable, with similar justifications.

Summary of Findings

The results revealed that, as expected, the configuration of the hundreds chart impacted the students' performance, but the hypotheses about the direction of the effects were not supported. With respect to the direction variable, participants reached ceiling on the direction variable on the learning task: All the participants moved in the correct direction on this task. It is possible that the participants learned in which direction to move on the learning task only because of the explicit instruction they received in each condition (i.e., up in the bottom-up groups and down in the top-down group). These data suggest that, at the very least, they learned the procedure demonstrated in the instruction, regardless of any possible structural shortcomings of the hundreds charts used in any condition.

Results from the transfer task presented more variability in the vertical direction they used to move through the chart. The overall direction performance was relatively high in all three conditions: Between 70 and 90 percent of the responses on the transfer task were in the correct direction. This finding suggests that many participants knew to move in the opposite direction for subtraction problems. The participants' movements may have been based on direction only, however, with little attention paid to the numbers in the charts. If the participants recognized that they had learned to use the chart for addition and that the transfer task problems were asking them to perform the inverse operation, they may have understood that they needed to go in the opposite direction from what they had been doing to that point. In other words, examining the numbers in the cells may have been secondary, or perhaps entirely unrelated, to understanding that a switch in direction was required given the switch in arithmetic operation. Anecdotal evidence from the participants' utterances showed that some indeed recognized that the transfer task problems called for subtraction. This could have occurred without thinking about place value or other characteristics of the numbers involved.

Despite the overall strong performance on direction, the bottom-up condition performed relatively worse than the top-down condition. That is, there were more items in the bottom-up condition that contained movement in the wrong vertical direction than in the top-down condition. In contrast to my expectation, the bottom-up configuration was not as helpful as the top-down configuration at either promoting a downward movement for subtraction or prompting a change in direction necessitated by the change in operation. Finally, also contrary to predictions, comparisons with the explicit analogy condition and the other conditions on this variable were not significant. Perhaps with more statistical power from a larger sample, a greater number of significant differences would have emerged between conditions.

Place value knowledge was assessed using the PicPVT and strategy use data. On the PicPVT items that tested students' knowledge of the number of groups in each place, the results were at ceiling. The second subset of the items on the PicPVT provided a more valid test of place value knowledge by asking the participants to determine whether the size of the denominations was correctly represented: Analyses of these denominations items on the PicPVT revealed that the top-down condition outperformed both the bottom-up and explicit analogy conditions, and with no significant difference between the latter groups. Together, the discrepant results on the groups and denominations items types may have occurred because understanding the number of groups represented by a digit demands less advanced place value knowledge than understanding the size of the denominations. Participants needed only to count the number of groups once they attended to the face value of the underlined digit, while the groups' value (i.e., related to the size of the denomination) could be overlooked. Further, the finding counters the predicted outcome for this measure, where the top-down condition was expected to do worse than the bottom-up and explicit analogy conditions on the PicPVT.

The strategy use variable on the learning and transfer task documented the strategies that used movement by tens and ones and other strategies (i.e., counting by ones, mathematically inappropriate). Of the mathematically appropriate strategies, those that used movement by tens and ones were considered more sophisticated than only counting by ones because the former encompasses use of place value concepts, namely that ten ones are the same as one bundle of ten (Carpenter et al., 2014). The moving by tens strategy demonstrates a more developed place value understanding than the counting by ones strategy as it sums the tens and ones separately and more concisely than counting on by ones from the starting addend. Counting groups of ten as if they were ones is a hallmark of place value understanding and therefore, a more sophisticated strategy than moving by ones (Carpenter et al., 1997; Fuson, 1990). Carpenter et al. (1997) found that children who used invented strategies that incorporated this understanding of tens and ones were better able to transfer their numeracy understanding to novel contexts than the children who used the standard algorithm as taught to them in school.

Results showed that the structure of the chart caused different patterns of strategy use by condition on the learning and transfer tasks. Specifically, the top-down condition demonstrated the moving by tens strategy more on the learning and transfer tasks than the bottom-up condition, a finding that diverges from expectations. Previous exposure to the hundreds chart could help explain why the top-down condition performed best overall on using the chart to solve problems and in demonstrating place value understanding: Data from the teachers who completed the questionnaire on their use of hundreds charts suggest that the top-down configuration is more popular in classrooms than bottom-up and no chart at all. Because only a small number responded to the questionnaire, the results are likely not representative of the population, but they may nevertheless suggest that the participants may have been exposed more to top-down charts in their school experiences. It is possible that previous exposure to the top-down hundreds chart countered any negative effects that this configuration of number may have presented.

Furthermore, the increase in number in the top-down hundreds chart matches the direction for reading. While row increases run from left to right in both types of hundreds chart in the present study, only the top-down chart aligned with moving down a paragraph of text while reading. The familiarity with the direction in reading may have been a stronger relation than up is more for these young participants; in the early years, children are exposed to writing and reading text in this sequence in space (e.g., Ministère de l'Éducation et de l'Enseignement supérieur, 2009). This alignment between the structure of number in the top-down chart and the direction for reading may have primed the participants for better comprehension of the concepts demonstrated in the instruction. Specifically, it can be difficult for children, especially young children, to pay attention to structural similarities, even when the relation is explicitly highlighted, without the working memory resources to do so (Richland et al., 2006). By reducing the cognitive load with the use of a familiar hundreds chart configuration, the top-down participants may have used more of their resources to attend to the instruction. A misalignment with the reading direction may have set both bottom-up conditions for greater attention to be paid to an unfamiliar route in which to travel, and thus drawing attention away from the structure of number as highlighted in the chart relative to the top-down condition.

The analyses revealed mixed results from the explicit analogy condition. Specifically, participants in the explicit analogy condition used equally sophisticated strategies on the learning and transfer tasks to those in the top-down condition, but their performance was significantly lower on the PicPVT relative to the top-down condition. These findings do not provide support for any of the predictions related to the explicit analogy condition. As such, they prohibit definitive conclusions about the explicit analogy condition, and future research is needed to assess the conditions under which explicit cues to the spatial structure of mathematics representations may or may not be beneficial.

Predictions on the participants' number sense performance by condition were dependent on the type of item used on the number sense task. On horizontal items, I predicted no main effect of time nor condition given that the structure of the charts does not impact the position of the larger number on the task. These predictions were the same for number sense accuracy scores and response times. On vertical items where the larger number was on top of the smaller number, I predicted that the bottom-up and explicit analogy conditions would perform better than the top-down condition. By the same token, when the larger number was below the smaller number on the vertical items, the top-down condition was expected to perform better than the bottom-up and explicit analogy conditions. These predictions were borne of the fact that the positions of the larger and smaller numbers in the number pairs align with the structure of the hundreds charts used in each condition.

The results indicated that, as predicted, there were no condition differences on the horizontal items on accuracy nor response times. Looking at accuracy on the vertical items, where I expected condition differences, there was a ceiling effect across all conditions. It is possible that the size of numbers included were too easy for participants of this age group. Examining response times for this task revealed more variation among the data, but there were no condition differences regardless of the placement of the larger number. One possibility is that there may not have been enough variability in the response times to discriminate between weak and strong performers on this measure. Another possibility is that the number magnitude may not have been affected by the spatial configuration of number using this tool. Recall the study by Navarrete et al. (2018), where preschool students viewed a race between animals on a large-scale number line. The participants' perspective from either the left or right of the number line changed their view of the direction of increase of the numbers. While the spatial configuration of number on the number line was predictive of the participants' success on a number line estimation task, the authors found that it was less predictive of outcomes on other number sense tasks, namely number counting, number identification, and number magnitude comparison. As such, an explanation for the absence of condition effects on response times for the number magnitude task in the present study is that number magnitude may not be a number sense concept that was influenced by movements in the hundreds chart, regardless of its configuration.

Moreover, the absence of condition differences as a function of the larger number's position could be explained if the participants were simply as comfortable selecting the larger number regardless of where it was on the screen. The ceiling effects on accuracy for this measure attest to the level of proficiency with the skill of finding the larger of two double-digit numbers. The placement of the number seems not to have posed an additional challenge.

Contributions to Literature

Previous literature on hundreds charts predominantly guides teachers on ways to use it in the classroom (e.g., Quane, 2021; Zambo, 2008). In terms of research, the focus is on curricula or interventions that involve, but do not directly examine the effects of hundreds charts (e.g., Chard et al., 2008; Jordan & Dyson, 2016). Other investigations have focused on other types of mathematical structures (e.g., recognizing patterns in number or space), but not hundreds charts specifically (e.g., Gronow et al., 2020; Mulligan et al., 2011), and their impacts on children's thinking and learning. The primary contribution of the present study to the research literature is that the results allow for causal conclusions to be drawn about the spatial configuration of hundreds charts and children's numeration.

What little previous research exists on children's learning with hundreds charts is largely descriptive, with little to no systematic evidence to attest to the direct effects of the tool on specific mathematical concepts. Bay-Williams and Fletcher (2017) advocated for the bottom-up hundreds chart following the second author's classroom observations where he noted that first-grade students appeared to have an easier time working with a bottom-up hundreds chart than the top-down chart. These observations could be an example of the authors' reliance on availability heuristics, whereby humans tend to overestimate how often an event occurs generally when they see it happen frequently themselves (Tversky & Kahneman, 1973). As such, people are wont to overestimate the probability of an event happening that may not indeed be as likely as it looks. Bay-Williams and Fletcher's (2017) suspicion that the bottom-up hundreds chart is better for children's learning may not have been a systematic pattern. On his blog, Fletcher also describes his visits to elementary schools, where he noticed hundreds charts hanging on many classroom walls (Fletcher, 2014). He argues that the bottom-up hundreds chart seemed more intuitive than the top-down chart given that the language associated with the arithmetic matches the direction in the chart (e.g., increasing upwards) when adding. Fletcher also challenged why the direction of reading should necessarily transfer to mathematics. Importantly, he noted that his observations are "unscientific" in terms of research (Fletcher, 2014), and that more systematic research is needed. Similarly, Randolph and Jeffers (1974) provide a descriptive account that the top-down hundreds chart poses a "directional conflict" (p. 203), but offered no empirical evidence to support this argument.

Finally, Vacc (1995) alleged that a disjoint between language and mathematical representations challenges students with disabilities. Since language and mathematics are closely related (e.g., LeFevre et al., 2010; Xu et al., 2021), Vacc is likely correct in her position. She recommended a new configuration of hundreds chart where the numbers range from 0 to 99 and its structure rotates the top-down chart 90 degrees clockwise, such that the ones' place in the standard addition and subtraction algorithm lines up with the row of single-digit numbers in the chart. Her recommendations lack support from any systematic studies, however, nor does she present any evidence herself.

The spatial arrangement of the numbers in the hundreds chart is unique because it structures 100 numbers in increasing or decreasing order and highlights groups of 10. Unlike a number line, its grid configuration spatially organizes numbers by sets of ten. Grouping sets of ten together is a keystone to the number system that children must grasp for numeracy development (Fuson, 1990). Furthermore, the superimposed groups of ten invite the observation that numbers in different decades can align vertically when the ones' denomination matches. An unexpected finding of the present study is that the top-down hundreds chart was better for place value and direction for vertical movement than the bottom-up chart. This is an important contribution to the literature on place value, spatial configuration, and to mathematics teaching practices; physical affordances of mathematical representations matter for children's thinking and learning in mathematics and teachers must be attuned to the ways students intuitively move these tools' space for different learning objectives.

In contrast to the hundreds chart's configuration, traveling along a linear tool like the number line draws attention to the distance from its starting point (Siegler & Booth, 2004). The length of time it takes to reach a number and the physical linear distance from 0 to the number differentiate one magnitude from another (Siegler & Booth, 2004). Meanwhile, the stacked nature of the hundreds chart permits shortcuts between numbers that may be far from one another on a number line; it collapses the distance between groups of tens, arguably hiding the difference in two numbers' magnitudes. In other words, the chart's representation of number magnitude is not perfectly analogous to all number properties because it shortens the distance between groups of ten: Moving one cell vertically has a different numerical value than moving one cell horizontally. The number, five, for example, is physically close to 15 in the hundreds chart, but it is the same physical distance from 6 as it is from 15. This is perhaps where the number line may be more beneficial for number magnitude.

Although there were no condition differences in number magnitude, the spatial configuration of the hundreds chart, namely its physical chunking of groups of ten, may explain the participants' performance on the place value measures. The hundreds chart's configuration imposes on students to understand that the same movement has different values, depending on direction. Perhaps the hundreds chart constrains learning in such a way as to reinforce numeration concepts (Carbonneau & Marley, 2015; McNeil et al., 2009). Developing an understanding that the movements in the chart correspond to different numerical values may facilitate or reinforce children's understanding of place value and other numeration concepts, such as the size of the denominations. Children must come to see value of vertical movements in the chart as greater (or "farther") than horizontal movements. Future research should directly compare the hundreds chart with the number line on different measures.

The concept of linearity could be important for numeracy: When preschool children worked with different configuration of a board game, Siegler and Ramani's (2009) found that those who worked with the linear board game performed better on number magnitude tasks and arithmetic problems than those who worked with a circular board game or control. It would be interesting to compare to what extent the number line and hundreds chart support children's place value knowledge. The authors posit that an alignment with the mental number line could be important in explaining their results. While the left to right order of the mental number line (Navarrete et al., 2018) is present in the hundreds charts' rows, the way the charts segment groups of ten deviates from the mental representation of the number line. Perhaps this divergence from the mental number line is a critical point in how helpful hundreds charts are in children's understanding of number magnitude and arithmetic. Interestingly, this account places more importance on the grid configuration of hundreds charts than the upwards or downwards direction of number increase in them. Thus, the spatial arrangement of the groups of ten, which differs considerably from the mental number line, may have more weight developing numeracy skills than the movement in the chart. Use of the chart guides thinking about groups of ten and ones, which would foster underlying concepts of place value. Nevertheless, the present study revealed condition differences, meaning that direction within the hundreds chart matters. Future research is needed to examine the linearity of the tool (i.e., linear, grid) as compared to the movement required when using it.

Strengths and Limitations

The present study is the first to systematically study the potential effects of hundreds charts, a tool that is prevalent in instruction content for teachers (Reys et al., 2010), on place value, counting, and arithmetic. These findings consequently assume an important place in the teacher training literature. Although Naverrete et al.'s (2018) study examined the effects of the spatial configuration of the number line on preschool children's number sense, the present study is the first to explore how two-dimensional space impacts numeracy development. Grounding the present study in the theory of analogical reasoning strengthens it by using existing literature to inform the study design and to make predictions. Finally, the experimental design allowed for causal conclusions.

The top-down hundreds chart's prevalence in schools is a confound in the present study since participants may have been previously exposed to this version of the chart. Future research should consider examining the hundreds chart with younger children, perhaps at the beginning of kindergarten or slightly younger, before they have been exposed to hundreds charts in their classrooms. Alternatively, using a novel and unfamiliar configuration of the hundreds chart in all conditions could eliminate previous exposure as a confound. This would also reduce any effect the direction of reading may have on performance.

Furthermore, the sample at hand was not representative of the general population's income distribution. The small sample size was also a limitation. A worldwide pandemic prevented further recruitment within a reasonable timeframe. A larger sample, however, could have yielded greater power for other differences to emerge. For the direction variable and place value measures, the pattern in the data were in the opposite direction to what was predicted – that is, the top-down condition performed best. Perhaps with a larger sample, more pronounced differences may emerge. The present sample delivered ceiling effects on the number sense measure, meaning a more sensitive measure is required for participants of this age group.

The PicPVT contained correct and incorrect representations. The graphics could have been misrepresented in one of two ways. The first type of error, the groups error, represented the wrong number of groups, but the size of the groups was correct. This was arguably a weaker test of place value as participants needed to match the number of groups with the digits, without considering the whether the group size was appropriately represented. As a result, the data on this measure reached ceiling, and was thus unusable in the context of the present study. This limitation in assessing place value on the PicPVT was compensated by the inclusion of the denominations error where the correct number of groups of the wrong size was represented. This type of error yielded more variability in the data as it proved to be more sensitive to the participants' place value understanding than the groups items.

The feature in the explicit analogy condition that distinguishes it from the bottom-up condition appears to be a possible confound. The visual cue in the explicit analogy condition may have captured participants' attention more than the other conditions. The container with the moving water has a certain "cool" factor to it. To illustrate, some participants wondered aloud if a problem would sum to 100 and they could see the water fill to the top; one participant asked if there would be any fish in the water. Future replications of the project should control the level of engagement to make the conditions more equal in this respect.

The results of the explicit analogy condition appeared peripheral to the central trend that the top-down condition was better than the bottom-up condition. Future research should consider a 2(presence of visual cue: with, without) by 2(condition: top-down, bottom-up) study design to disentangle whether the configuration of the chart or the presence of a visual cue is more helpful for children's learning with the hundreds chart. This design would also help expose interactive effects between chart configuration and visual cues. It would be interesting to assess how the top-down configuration would improve or not with a visual emphasis on its direction, and how might this differ from a bottom-up condition with and without a cue.

The online method of delivery had both strengths and limitations. Participants could come from anywhere in the world. The digital format of the present study allowed it to proceed in the time of a worldwide pandemic. Participants may have felt more comfortable in their own homes with a parent nearby than at school or in a university lab. On the other hand, this setting also gave parents the opportunity to intervene if their child appeared to be struggling to answering questions. Although many of the participants seemed very comfortable maneuvering the digital platform and VHC, it arguably required more fine motor skills than a physical version of the same task.

Educational Implications

The present study's findings showcased the use of hundreds charts for number sense, place value, and use of the chart for addition and subtraction. The top-down configuration appeared more helpful for direction and place value than the bottom-up configuration. It is important to note, however, that the results of the present study are preliminary and further research is needed. As such, definitive recommendations for practice are premature.

One conclusion that can be drawn from the present study is that configuration matters and that teachers must be aware of the spatial configuration of the instructional materials they use. More specifically, teachers should pay attention to what students are saying while using the chart to solve problems, how they are moving through the space, and the strategies they are using. For example, counting by ones in the hundreds chart indicates a more limited place value understanding than counting by tens. Using a hundreds chart in a direction that clashes with the direction of number also indicates a weakness in children's grasp of numeration. The present study also suggests that instructional materials afford better learning for certain learning objectives, and teachers should select them accordingly. The hundreds chart appears to have benefits for place value understanding and numeration knowledge, but future research is needed to determine whether the top-down configuration is indeed most helpful for place value and to uncover what other mathematical concepts the hundreds chart best supports.

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Appendix

Parent Demographic Questionnaire

Date of completion : ___ / ___ / 2021

Questionnaire

1. What is your relationship to the child? (Check the appropriate box)
   ☐ Mother ☐ Father ☐ Other. Specify : ________________

2. Code given by the research assistant by email: ________________

3. Date of birth of the child: ________________

4. Age of the child: ________________

5. School: __________________________

6. Class: ________________

7. Gender of the child (check the appropriate box): ☐ Boy ☐ Girl

8. Country / Province or State: ________________

9. Phone number: ________________

10. E-mail: ________________

11. What is your family income? (Check the appropriate box)
    ☐ < 20 000 ☐ 20 000 – 40 000 ☐ 40 000 – 60 000
    ☐ 60 000 – 80 000 ☐ 80 000 – 100 000 ☐ > 100 000

12. Does your child show: (Check the correct box/boxes)
    - ☐ No difficulties in mathematics
    - ☐ Some difficulties in mathematics
    - ☐ A diagnosis of Mathematics Learning Disability (dyscalculia)

13. Does your child show: (Check the correct box/boxes)
    - ☐ A sensory impairment
    - ☐ A visual impairment
    - ☐ An intellectual disability
    - ☐ A motor impairment
    - ☐ A neurological disorder
    - ☐ A psychosocial disorder
    - ☐ An attention deficit disorder: ☐ Medicated ☐ Non-Medicated
    - ☐ A disgnosed developmental language disorder
    - ☐ A learning disorder in reading/spelling (dyslexia)
    - ☐ Difficulty identifying colours

14. Which language(s) is/are spoken at home?
    ☐ French ☐ English ☐ Other. Specify : ________________

15. Comments :

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This is the end of the questionnaire. Thank you for your time and your participation!