

Finite Element Simulation and Reliability Assessment of Mode-I and Mode-II Delamination Testing of Composite Laminates

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Abstract

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Mohammadreza Rabiei

During recent decades, the use of composite laminates has been increasing due to their desirable characteristics such as high stiffness, high strength, low density, corrosion resistance, and fatigue life. Delamination, as the most common failure in composite laminates, occurs due to the degradation of stiffness and interlaminar strength of composite materials. In the present study, numerical simulation and analysis of the Double Cantilever Beam (DCB) and the End Notched Flexure (ENF) tests are carried out using the Cohesive Zone Modeling (CZM) approach and the Finite Element Modelling and Analysis using the software ANSYS®. The results obtained are verified using relevant works in the literature. There are certain sources of uncertainty in the ply material properties, manufacturing, property measurement, and testing of composite laminates, which should be considered in the delamination testing and the design process. To this end, first, a parametric study is carried out to determine the influences of material and geometric properties together with the stacking sequence of the composite laminate specimens for the DCB and ENF tests and their variabilities on the mode-I and mode-II fracture toughnesses and their variabilities. Second, a probabilistic study is conducted to determine the non-deterministic characteristics such as the second-moment information of the mode-I and mode-II energy release rates and the probability of delamination growth in mode-I and mode-II in the DCB and ENF tests, respectively. Next, the sensitivity analysis is conducted to specify a ranking of important random variables for both the DCB and ENF tests. The reliability assessment is conducted using the First-Order Second-Moment (FOSM) method, histogram sampling, First-Order Reliability Method (FORM), Second-Order Reliability Method (SORM), Monte Carlo Sampling (MCS), and importance sampling using associated software Rtx®. Based on the parametric analyses, it is established that the variabilities of the thickness, length, and longitudinal elastic modulus of the DCB and ENF test specimens have the most significant effects on the variabilities of the mode-I and mode-II fracture toughnesses. Based on the reliability analysis it is shown that the probabilities of mode-I delamination growth in the DCB test and mode-II delamination growth in the ENF test, are 50.27% and 49.51%, respectively.

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Chapter 1

Introduction

1-1 Background and motivation

In recent years, the application of composite materials has been increasing in the engineering industries. Composite materials consist of two or more materials and can be categorized into different types, including particle-reinforced, fiber-reinforced, and structural composites. The usage of composite materials has been growing due to their desirable characteristics such as high stiffness, strength, low density, corrosion resistance, and fatigue life [1]. As the most common type of composites, laminated composites have been used in industries due to their high strength and lightweight. However, there are some limitations in composite structures which should be recognized and considered in engineering design. For instance, in composite laminate structures, the most typical drawback is that the stiffness and strength through their thickness are low, particularly at the interfaces of composite layers, which can result in interlaminar fracture or delamination. Delamination growth ensues failure in the composite laminate because of the reduced stiffness. Delamination is the most common catastrophic failure in composite laminates. Three common delamination failure modes, including mode-I, mode-II and mixed-mode I/II, have been studied through numerical and experimental approaches in the literature.

In recent decades, different composite laminate specimens with various material properties and in different dimensions subjected to mode-I, mode-II, and mixed-mode I/II delamination tests have been studied by several researchers and scientists, as discussed later in the literature survey. However, potential uncertainties in the delamination property tests of composite materials have been rarely investigated. There are usually some sources of uncertainties in manufacturing, curing, and property measurement and testing of composite materials. These uncertainties probably cause variations in the mechanical and geometric properties of composite specimens and also in their fiber orientation angles. Since in standard tests for obtaining fracture toughness, such as delamination testing of composites, specific values for geometric parameters of the test specimens have not been provided, one of the most dominant motivations of the present thesis is to investigate the influence of the values of geometric parameters of test specimens and their variability on the fracture toughness and its variability. In addition, uncertainty in the applied load or displacement in the delamination property tests is essential to be considered. This uncertainty stems from the standard testing machines used for mode-I and mode-II failure tests, which have an allowable error range, leading to variations in the applied load or displacement. All the variations mentioned above may affect the fracture toughness of composite specimens subjected to mode-I and mode-II or mixed-mode failure tests. Thus, it is vital to consider these variations in investigating the fracture behavior of composite laminates. In the present numerical study, the bi-linear Cohesive Zone Model (CZM), discussed in chapter 2, has been used based on FEA using ANSYS®. This approach allows predicting the delamination initiation and propagation. The main motivation of the present study is performing a numerical parametric investigation to find out the individual effects of every dependent and independent engineering constants and their variabilities, in addition to geometric parameters of a composite specimen and their variabilities, as well as the lay-up sequence of the composite laminate specimen, on the fracture toughnesses of the composite laminate specimens and their variabilities. In this way, uncertainties that cause variations in the variables of fracture mechanics investigation have been taken into account in the numerical analysis. The parametric analyses of the present thesis as virtual experiments study the influences of the aforementioned variations rather than conducting many actual experiments. The variations in material properties due to the in-situ variability of composite materials, the variations in geometric parameters because of the different proposed geometries in the standard tests, and the variations in the lay-up sequence

of composite laminates due to the manufacturing and curing processes have been investigated in the present study.

Furthermore, a probabilistic study is required to consider the mentioned uncertainties and variations in terms of non-deterministic points of view. A reliability study can determine the probability of delamination growth in mode-I and mode-II failure tests, as well as to determine the most sensitive variables in these tests. Moreover, the probabilistic study provides the second-moment information of the fracture toughnesses in mode-I and mode-II delamination tests. The non-deterministic approach would make it possible for future works to design composite structures according to their reliability level in mode-I and mode-II delamination.

1-2 Mode-I and mode-II delamination property tests of composites

Delamination failure modes are attributed to out-of-plane failure, which occurs in macro-scale between plies. In contrast to in-plane cracks, delamination is associated with the interlaminar stresses. Thus, in order to identify the responses of composite materials to delamination failure modes, the classical laminated theory is not adequate. Scientists have proposed different experimental property testing of composites to obtain fracture behavior regarding mode-I and mode-II delamination of composite structures.

Delamination due to normal stress, in opening or peel mode, has been studied through the Double Cantilever Beam (DCB) test first proposed by Kanninen [2]. It is the most common test used to determine mode-I fracture toughness. DCB test is designed to characterize fracture toughness of composites in tension mode, and ASTM D5528 is the standard used for this test which explains how to obtain mode-I fracture toughness by conducting DCB test [3]. In this test, upper and lower beams are loaded by edge force until the delamination propagates, and mode-I failure occurs. The DCB specimen was modified in the work of Lee et al. [4] width tapered DCB (WTDBC) test. Using this test specimen allows one to compute the mode-I energy release rate independent of the crack length since the compliance is changing with the crack length. Also, another specimen that has been used to characterize mode-I fracture toughness is the thickness tapered DCB (TDCB) test proposed by Qiao et al. [5]. TDCB specimen has customized-stiffness property and allows for a

stiffer specimen that is still light in weight. These specimens make it possible to compute the mode-I fracture toughness independent of the crack length.

Delamination due to shear stress, in sliding and shear modes, has been studied through different experiment configurations. End Notched Flexure (ENF) test is the most common one first proposed by Russel et al. [6]. This test is standardized in ASTM D7905 [7]. It is the standard test for mode-II interlaminar fracture toughness of unidirectional composites. The ENF test has a simple specimen geometry and its closed-form solution to compute the mode-II fracture toughness is straightforward. Wang and Williams, and Wang et al. [8], [9] studied the developed End-Loaded Split (ELS) specimen configuration to test mode-II delamination. The biggest drawback of the ELS test is that large displacements happen during the test. Afterward, four-point bend end-notched flexure (4ENF) test was proposed by Schuecker and Davidson [10]. Pure mode-II bending was ensured by this configuration, and large displacements were eliminated, but the fixture of the test is complicated.

In the present study, DCB and ENF tests, as well-known configurations for mode-I and mode-II delamination, respectively, have been considered. All the numerical and reliability analyses are performed regarding these test specimens.

1-3 Finite element method

Analytical solutions are complex and experimental studies may be expensive and time-consuming; the Finite Element Method (FEM) is a desirable approach to analyzing and solving complex designs and structures. FEM makes solving a problem containing a complex or huge structure or system easier by considering simple and small parts called finite elements as representatives of the structure or system. Mesh construction leads to several finite elements that make up the structure. Mesh sizing is a determinant factor in this approach since it defines the element size and number of elements that can affect the solution time, accuracy, and even convergence.

Finite Element Analysis (FEA) is a practical form of FEM used in engineering analysis. FEA simulations help design complicated engineering structures or engineering structures that require to be repeatedly examined empirically. As an illustration, in the present study, parametric analyses

need to be conducted several times to achieve reliable and safe results. To this end, FEA allows one to obtain accurate structure response without consuming a lot of time or conducting costly experiments.

In the present study, commercial Finite Element Analysis (FEA) software ANSYS® is used to simulate the Double Cantilever Beam (DCB) and End Notched Flexure (ENF) tests of unidirectional composites with delamination.

1-4 Uncertainty and reliability studies of composite structures

There are a lot of sources of uncertainty and variability in composite materials and structures. Various variabilities are inherent in materials, manufacturing, curing, and molding processes. Variability because of thermal distortion is another source of uncertainty stemming from the mismatch of coefficients of thermal expansion between tool and composite laminate surfaces. When variability becomes uncontrollable and causes degraded structural performance, it can be named as a defect. There are various types of defects in composites based on their manufacturing method. These defects may stem from materials or design decisions, manufacturing, and curing processes. Some commonplace defects in composites include delamination, void, resin rich zones, and in-plane fiber misalignment [11]. For instance, delamination can originate from variability in design decisions, such as delamination property testing, in materials, in processing, or in manufacturing errors that may cause total failure. In addition, fiber misalignment may be inherent in the manufacturing and curing processes. Variability due to thermal distortion, or variability in materials, or manufacturing errors may also cause fiber misalignment in composites. Understanding the sources of variability and defects in composite materials makes it possible to reduce the risk of composite material failure. Considering all possible variabilities and defects, from design decisions to post-processing of composite materials, results in high quality and less expensive composite products.

As mentioned, there is a wide range of variability and uncertainty in composite structures. When it comes to delamination property tests of composites, designers and engineers should consider these uncertainties due to variability in manufacturing, processing, and materials. For this purpose, the reliability and probabilistic studies can be used. These non-deterministic approaches allow one to consider uncertainty and variability in the design of engineering products. Different reliability

analysis methods can reveal the statistical information about a critical function essential for a structure. These can show how the scattered data of material properties, geometric parameters, and stacking sequence of composite specimen are distributed considering the uncertainties of the structure. In addition, the probability of failure for a physical phenomenon can be obtained through a reliability study. In the present study, the reliability of Graphite-epoxy composite laminates in the presence of delamination under the DCB and ENF tests is investigated. The probability of delamination growth in mode-I and mode-II is determined and discussed.

1-5 Literature survey

This section provides important contributions available in the literature in three categories. First, worthwhile works in the delamination growth field are presented. Next, few studies about composite material defects are provided. Finally, essential studies on the reliability of composite structures are reviewed.

1-5-1 Delamination growth studies

The increasingly widespread use of composites in industries has motivated researchers to study different aspects in this field of study. Delamination is one of the most probable failure modes in composites that has attracted a lot of attention. Double Cantilever Beam (DCB) test as the most common mode-I delamination test has been studied experimentally and numerically by many researchers. It is first developed by Kanninen [2]. He studied unstable crack propagation and arrest by proposing an improved analytical model of DCB specimen. Few primary computational results were provided for unstable crack propagation. Whitney et al. [12] examined four different composite materials subjected to mode-I delamination testing. They conducted the experiments on specimens of different initial crack sizes and used different data reduction approaches to calculate the critical energy release rate. Influence of the adhesive thickness on the mode-I fracture toughness of carbon fiber reinforced plastic were investigated by Sela et al. [13]. They conducted mode-I and mode-II delamination tests by using two types of adhesive. The changes in the interlaminar fracture toughness of composite specimen caused by changing the adhesive thickness were compared. Caprino [14] studied three different measurement methods of mode-I interlaminar fracture toughness of composite laminates using DCB test specimens. He indicated that the data

reduction method based on linear elasticity brings about more accurate results in an experimental study due to its measurable variables in the DCB test. He also conducted a DCB test for thin samples of graphite-epoxy composite to confirm theoretical predictions. Derivation of a closed-form solution for compliance and mode-I energy release rate for isotropic and orthotropic materials with an adhesive layer was proposed by Penado [15]. He used MSC NASTRAN finite element analysis software to verify the closed-form results. FEA results for different material properties and crack lengths illustrated a good agreement. Ducept et al. [16] conducted an experimental study of the influences of material properties, specimen thickness, and initial crack size in the DCB, ENF, and Mixed-Mode Bending (MMB) tests. Delamination resistance of a hand laid-up unidirectional glass/epoxy was determined in mode-I, mode-II and mixed-mode delamination.

Prediction of delamination growth has been made through numerical studies by using finite element modeling. To this end, different techniques have been introduced, namely the Virtual Crack Closure Technique (VCCT) [17-19] and the J-integral method [20]. In recent years, the Cohesive Zone Model (CZM) has been widely used for finite element modeling of crack propagation and delamination in composites. First it was developed by Barenblatt et al. [21], [22] for studying fracture mechanics of brittle materials. Dugdale [23] extended the same idea for plastic materials by considering the crack tip plastic zone. He conducted experiments on two types of specimens consisting of an edge slit and internal slit subjected to tensile loading and measured and compared the lengths of the plastic zone under different loadings. Williams and Hadavinia [24] studied analytical solutions for five different forms of cohesive zone traction laws based on the elastic foundation model for cantilever beam specimens. They discussed the conditions of using these solutions to analyze delamination propagation and peeling. Alfano and Crisfield [25] used interface elements and interface damage law, initially proposed by Crisfield et al. [26-28], to simulate mode-I DCB test and for the mixed-mode delamination analyses of the laminated composite specimens. They compared their numerical results of DCB analysis and mixed-mode multiple delamination analysis with experimental results. Possible issues in numerically solving the debonding problem by bi-linear cohesive zone law were argued, and the effects of some important parameters in finite element analysis of delamination such as interfacial strength of the interface on convergence of the solution and final numerical outcomes were investigated. Alfano [29] studied the numerical response of four models under pure mode-I and mode-II tests using four different shapes of the interface law, including bilinear, linear-parabolic, exponential and

trapezoidal laws. He indicated that the shape of the interface law has a significant effect on the numerical analysis results. Moura et al. [30] experimentally studied interlaminar and intralaminar mode-I fracture toughnesses of laminated composite specimen and validated the results with a numerical analysis. They used data reduction based on beam theory to evaluate the fracture toughness after fiber bridging phenomenon to observe the effect of bridging in the measured fracture toughness. According to the results of their study, the bridging phenomenon was more significant on intralaminar tests. Marzi et al. [31] investigated the effect of ply thickness on the fracture properties of one batch of epoxy adhesive. They used DCB and two versions of TDCB specimens to evaluate mode-I fracture toughness. In addition, two versions of the ENF test and end-loaded shear joint (ELSJ) test specimens were used to obtain the mode-II fracture toughness. For thin layer specimens in mode-I and mode-II failure tests, all test results were in good agreement; however, thick layers result in larger variations in the values of mode-I and mode-II fracture toughness.

Russel et al. [6] first proposed the end-notched flexure (ENF) test to perform the mode-II fracture toughness. Three-point bending load was applied to a double cantilever beam specimen to obtain the interlaminar fracture toughness in mode-II delamination. Compston et al. [32] presented an experimental study of the effects of matrix toughness, fiber volume fraction, and loading rate on mode-II fracture toughness of unidirectional glass-fiber composites. A more detailed experimental investigation about the influence of varying crosshead displacement rate in the ENF test on the mode-II fracture properties of a range of composite materials was conducted by Cantwell [33]. Results illustrated that the increase of crosshead displacement rate leads to the increase of mode-II fracture toughness in most of the composite materials.

De Moura and De Morais [34] used three-dimensional finite element models based on interface finite elements to analyze ENF and End Loaded Split (ELS) tests. In the study, two equivalent crack methods, including Corrected Beam Theory with Effective Crack Length (CBTW) and Compliance-Based Beam Method (CBBM), were evaluated and compared to the classical beam theory. De Morais [35] proposed a novel cohesive beam model for the mode-II interlaminar fracture of the ENF test specimen. He compared the finite element analyses with the cohesive beam model with a bilinear cohesive law to indicate a good agreement between these analyses.

Blackman et al. [36] conducted an experimental study about mode-II fracture toughness of various carbon-fiber-reinforced-polymer (CFRP) and two types of epoxy-adhesives commonly used in automotive structures. They used different loading modes, including mode-I, mode-II, and mixed-mode I/II, and a wide range of loading rates. Results illustrated that the highest mode-II energy release rate was in HTS-SIA joints. It was observed that the mode-I fracture toughness is more sensitive to the loading rate than mode-II fracture toughness. Krueger [37] indicated the application of benchmark examples in the DBC, ENF, Single-Leg Bending (SLB), and MMB tests. He discussed the benefits of benchmarking process to tackle FEM issues like selecting proper input parameters for the VCCT implementations in ABAQUS/Standard®. The mode-II fatigue delamination behavior of carbon-epoxy laminates by using ENF tests was studied by Sousa et al. [38]. A new approach was used to obtain the crack propagation rates considering different stress ratios. Woven fiber composite and laminate with offset center plies indicated more sensitivity to the loading rate compared with the unidirectional composites.

Recently, some studies in the fracture mechanics field have been performed to take manufacturing and curing induced defects into account. Carraro et al. [39] investigated the influence of the presence of void content in carbon-epoxy laminates due to the manufacturing through liquid infusion molding on fracture properties. Incremental static and DCB tests were carried out to define the effect of voids on damage onset and propagation. Taleb et al. [40] studied the effect of transverse interfacial defects in the DCB test specimen on the fracture properties. Tests were conducted on samples consisting of homogenous adhesion with either a strong or weak interface. Artificial adhesion defects or weaken of zone were created by spraying Poly Tetra Fluor Ethylene (PTFE) particles onto the surface. They also performed numerical modeling via 2D finite element simulations to investigate the dynamic transition between crack onset and crack arrest using ABAQUS/CAE®. The research was conducted on the effects of post-curing parameters on the mode-II fracture toughness of structural adhesive using the ENF test by Han et al. [41]. A combined experimental-numerical approach was used to obtain the optimal set of adhesive parameters in a specimen that underwent different curing processes. Kumar [42] recently studied the effect of void defects at the ply interfaces on mode-I interlaminar fracture toughness of Polymer Matrix Composites (PMCs) and Ceramic Matrix Composites (CMCs). Two DCB and Interlaminar Tension (ILT) tests were simulated using cohesive zone finite element models in which randomly distributed defects were modeled on a critical fracture plane. Numerical results indicated that both

mode-I fracture toughness and ILT strength are sensitive to defects. It was shown that when the defect size is larger than the fracture process-zone of the delamination crack, it affects the mode-I fracture propagation.

1-5-2 Composite material defects

Variability and defects in the composite materials due to different uncertainty sources such as manufacturing, curing process, and materials have been studied in the literature. Various sources of uncertainty and types of defect were explained and classified by Potter [11]. He provided a taxonomy of possible defects in composites manufactured by autoclave molding or Resin Transfer Molding (RTM). Distortion of fibers as a manufacturing defect of composite which is caused by variability in thermal distortion may appear in the forms of in-plane fiber waviness, out-of-plane fiber waviness, and wrinkles, is reviewed in some studies. For instance, Khan et al. [43] investigated the mechanical behavior of woven carbon fiber laminates containing defects such as in-plane and out-of-plane fiber waviness, resin-rich layers on the surface, and folding of all fibers. Some compression and tension tests were carried out to observe the effects of the defects mentioned above on the compressive strength and ultimate tensile strength of laminates. It was concluded that 30° in-plane waviness of one-half of laminate plies leads to a compressive strength reduction of 35%. Also, compressive strength declined by 71.5% when a 29° out-of-plane fiber waviness happened in all the plies of the laminate. Experimental outcomes of the study showed that the ultimate tensile strength of the laminate fell by 32%.

Moreover, Mukhopadhyay et al. [44] investigated carbon fiber epoxy laminate containing wrinkles under tension-tension fatigue loading. FEM modeling was carried out in the study to observe the onset of fatigue delamination growth. Also, their experimental works verified the 3D finite element modelling of fatigue delamination initiation and growth in the presence of out-of-plane fiber waviness. It was shown that the embedded wrinkles have adverse effects on the high-cycle fatigue life of composites. An investigation of IM7/8552 graphite-epoxy composites with different stacking sequences due to the in-plane fiber misalignment caused by using a U-shape aluminum tool was carried out by Lightfoot et al. [45]. They offered a new wrinkle formation mechanism in the graphite-epoxy composite manufactured by the hand lay-up process. Placing 0-degree plies in the tool brings about prepreg bridging in the round edges of the U-shape tool called instability

sites. Applying the cure cycle at a specified temperature and pressure increases the shear stress between the tool and the composite. Besides, mismatches in the coefficients of thermal expansion (CTE) of the composite and tool result in interfacial shear stress. It has been shown that using a release film between the first ply and the tool surface may result in wrinkle formation in that ply. Decreasing the slippage between the composite and the tool leads to a growth in the shear forces. Based on the results of the study of Lightfoot et al. [45], it has been observed that the graphite-epoxy composite manufactured in the U-shape tool exhibits no wrinkle in the plies but a considerable in-plane misalignment of the fibers, which arises from the formation of buckled and bridged plies during consolidation in the tool. The in-plane waviness is recorded by examination of high-resolution micrographs. Using the Yurgartis method [46], in-plane misalignment of fibers was color-coded and quantified by a color recognition algorithm in MATLAB® [47].

1-5-3 Reliability studies of composite structures

Non-deterministic approaches have been used in many engineering fields such as aerospace engineering, civil engineering, and Micro-Electro-Mechanical Systems (MEMS) engineering. Reliability assessment methods are used to quantify and model the uncertainties. When it comes to different composite engineering structures, these methods are commonly used in fracture mechanics. This section briefly presents some statistical, probabilistic, and reliability studies that have been made in some fields of composite structures.

In the early stage of reliability studies of composite structures, some studies were conducted about fatigue crack propagation. Lin and Yang [48] proposed a new statistical theory for the probabilistic analysis of fatigue crack propagation. They investigated the randomness of time during the crack propagation and observed the crack size growth. In 1991, Besterfield et al. [49] offered an approach made up of a combination of the probabilistic finite element method (PFEM) and reliability analysis to study the probabilistic fatigue crack growth. They also proposed an approach to determine the probability of fatigue failure for mixed-mode cyclic loading. In the same field of study, the general fatigue crack growth reliability problem was studied by Liu et al. [50]. They examined the failure probability by using FORM and MCS reliability methods.

In 1995, Rahman [51] developed a probabilistic model for nonlinear fracture mechanics analysis of through-wall-cracked (TWC) pipes subjected to bending loads. He evaluated the failure

probability of the cracked pipe based on different failure criteria by using reliability methods. The probabilistic characteristics of failure load computed using the FORM, SORM, and MCS methods were accurate. Chen et al. [52] presented a new continuum-based method for analyzing a crack sensitivity to a mixed-mode loading condition in a homogenous, isotropic, and linear-elastic body. By solving two numerical examples, they indicated that the proposed method provides sensitivity analysis results that were close to that of the analytical method. The same authors [53] also presented similar study in which a new method was proposed to perform reliability and sensitivity analyses of the same crack body as that in their previous study subjected to mode-I loading. They carried out a probabilistic fracture mechanics analysis by using FORM. Their proposed method was accurate compared with the numerical results, and at worst, there were less than three percent differences. Nobile and Gentilini [54] presented a probabilistic analysis of the response of cracked structures containing uncertainty in crack depth and location. They investigated the effect of crack depth and location on damaged structures by considering them as random variables in the probabilistic analysis. Their study indicated that crack trajectory, location, and depth affect the linear response of the damaged structure.

The effect of correlation between changes in the random variables on the reliability was studied by Shaw et al. [55]. The transformation of micro-scale uncertainty to macro-scale uncertainty was quantified, and a multi-scale analysis was carried out. They used the Tsai-Hill failure criterion to formulate limit state equations containing two sets of random variables corresponding to micro-scale and macro-scale analyses. The results of the study illustrated a good agreement between the failure probabilities obtained from micro and macro-scale analyses. In 2010, António [56] and Hoffbauer presented an approach based on optimal inverse design to conduct reliability analysis of composite laminated structures. In this study, through an artificial neural network-based MCS approach, the ply angle of laminates was used as a parameter to model the structural reliability index. It was also used to simulate the sensitivity of the critical Tsai number and the structural reliability index. Weleman and Dehmous [57] presented a coupled approach consisting of a reliability method and micromechanics to estimate the failure of composite material. By linking micro and macro-scales, they offered a comprehensive analysis of the effect of various uncertainties. They studied the failure of unidirectional carbon-fiber-reinforced composites using the simulation, and the results were compared with some experimental data.

More studies concerning the failure probability of structures in the presence of crack, considering random loading, have been conducted. Kunaporn et al. [58] investigated the crack growth in a thin-walled beam subjected to stochastic loading. Three different pre-crack locations on the beam were assumed to evaluate the effect of the initial crack location on the obtained probability of failure. They concluded that the crack propagation is most significant when the pre-crack is located close to the fixed end of the beam. The random location and size of a longitudinal crack in the thin-walled beam subjected to random loading were considered by Kunaporn et al. [59]. They used FORM analysis to compute the reliability index and failure probability of the structure.

There have been reliability studies of composite structures based on different failure criteria in the most recent studies. Zhang et al. [60], using the Tsai-Hill criterion, conducted a reliability assessment of fiber-reinforced plastic composite structures. They surveyed the effect of statistical correlation between ply mechanical properties on the reliability evaluation. The effect of ply level thickness uncertainty on the reliability of laminated composite was investigated in another work by Zhang et al. [61]. They applied the Tsai-Hill criterion, and the MCS and Monte Carlo Markov-Chain approaches were used to evaluate the probability of failure of laminate panels subjected to uniaxial and multi-axial loads. A wind turbine blade subjected to static bending loads was studied by Kimiaefar et al. [62]. In their study, reliability assessments of failure in composite layers and failure in adhesive layers were conducted using Tsai-Wu and maximum principal stress failure criteria, respectively. Two limit states based on the first buckling mode and Tsai-Wu first ply failure criteria were used to evaluate the reliability of composite panels by Díaz et al. [63]. They quantified the uncertainties of the composite panels using stochastic expansion and limit state approximation methods which were validated using MCS method. According to the work of Zhou et al. [64], the multi-scale analysis can be used in the reliability analysis of composite structures based on FORM and MCS considering different failure criteria such as Tsai-Wu, Tsai-Hill and maximum stress criteria. In another study, the same authors [65] presented a new reliability analysis framework consisting of a novel stochastic multi-scale finite element method combined with FORM and SORM methods. The new approach allowed one to determine the stochastic structural response with regard to random variables encompassing microscopic and macroscopic uncertainties.

1-6 Objectives and scope of the thesis

As mentioned earlier, uncertainties in composite manufacturing, curing, loading, and material property measurement experiments cause variations in the fiber orientation angles, engineering properties, and geometric parameters of the composite laminate specimen. In order to consider these uncertainties in the mode-I and mode-II failure tests, the present study focuses on the effects of essential variables, including material and geometric properties and their variabilities, as well as fiber orientation angles of composite laminates, on the results of the DCB and ENF tests. In the present thesis, parametric numerical study of virtual DCB and ENF tests has been conducted several times to obtain the sensitivities of the values of mode-I and mode-II fracture toughness to each critical variable. Moreover, in-plane fiber misalignment is considered as a probable manufacturing and curing induced defect in the ENF test specimen, then the obtained mode-II energy release rate and its variability are studied. Finally, a probabilistic study has been conducted to consider the uncertainties through a non-deterministic approach. The reliability analyses of both DCB and ENF tests have been carried out to determine the probability of delamination growth under mode-I and mode-II loadings. Sensitivity analysis has indicated a ranking of important variables in each delamination property testing.

The major objectives of the present study are given in the following:

1. To investigate the effects of material properties and geometric parameters of the DCB and ENF test specimens and their random variabilities on the mode-I and mode-II fracture toughness parameters based on FEA using ANSYS®.
2. To investigate the effects of applied displacement and its variability in the DCB and ENF tests on the mode-I and mode-II fracture toughnesses and their variabilities, respectively, based on FEA using ANSYS®.
3. To investigate the effects of random fiber misalignments of unidirectional laminated composite on the mode-II fracture toughness in the ENF test based on FEA using ANSYS®. Developed models have been considered to determine the influences of the severity of the in-plane fiber misalignments, and the location of the misaligned ply group on the mode-II fracture toughness.

4. To determine the probabilities of mode-I and mode-II delamination growth in the DCB and ENF tests, respectively, and to obtain the sensitivities of the reliability index and the probability of the delamination growth in the DCB and ENF tests to the material properties and geometric parameters of the test specimens through reliability analyses.

1-7 Layout of the thesis

This dissertation contains five chapters. The present chapter provides a brief introduction and literature survey on delamination property testing of composite materials, possible defects in composite structures, and reliability studies of composite materials. The objectives and the scope of the thesis are also presented.

Chapter 2 is divided into two parts. The first part presents finite element modeling of composite laminates with delamination under mode-I and mode-II failure tests. The double cantilever beam (DCB) and end notched flexure (ENF) tests simulated using FEA tool ANSYS® are discussed. The modeling specifications are thoroughly explained, and load versus displacement curves obtained from numerical analysis are used to calculate the energy release rate of the composite materials. The chapter reviews data reduction approaches to get the interlaminar fracture toughnesses in mode-I and mode-II for the DCB and ENF test, respectively. Finally, the load-displacement curves, critical load and displacement, and the energy release rates are determined for both delamination tests, and the results have been compared with the available results in the literature. The second part of chapter 2 provides all theoretical formulations of the reliability study and different approaches of reliability analysis used in chapter 4 to predict the probability of delamination growth in both mode-I and mode-II. Different reliability methods including first-order second-moment (FOSM), first/second-order reliability methods (FORM/SORM), and samplings including histogram sampling, Monte Carlo Sampling (MCS), and importance sampling are reviewed, and the concept of each method has been discussed.

Chapter 3 firstly provides a parametric study of the effects of the engineering properties and geometric parameters of composite specimen and their variabilities on the mode-I energy release rate and its variability in the DCB test based on finite element modeling using ANSYS®. Afterward, a parametric study about the influence of engineering properties and geometric parameters of composite specimen and their variabilities on the mode-II interlaminar fracture

toughness and its variability is investigated using ANSYS®. Then, composite laminate models with in-plane fiber misalignment defects are described and simulated under the ENF mode-II delamination property test. Twenty composite laminate samples with different in-plane fiber misalignment severities and different layup sequences subjected to the ENF test are simulated using ANSYS®. Performed numerical analyses illustrate the influence of the in-plane fiber misalignment of the ENF test specimen on the mode-II fracture toughness. The last section of this chapter is a numerical analysis of the effect of the location of the misaligned ply group on the mode-II energy release rate developed in ANSYS® using the FEA method.

In chapter 4, reliability analyses of the mode-I and mode-II failure tests are carried out using Rtx®, a software with reliability/risk/resilience analysis tools. Methodology of reliability analysis is introduced and discussed through sections 4-1 to 4-3. Histogram sampling, first-order second-moment (FOSM), first/second-order reliability methods (FORM/SORM), Monte Carlo Sampling (MCS), and importance sampling are used to compute the probability of delamination growth. Reliability results present the distribution of the energy release rate and the second-moment information. The probabilities of failure determined using different reliability methods are presented and compared. In addition, the influences of the variations in the material properties and geometric parameters of the composite specimen and the applied displacement, as random variables, on the likelihood of failure of laminate models under the DCB and ENF tests are investigated. The sensitivities of the random variables consisting engineering properties and geometric parameters of the composite specimen are also studied and compared in this chapter.

Chapter 5 presents the overall conclusions of the present study considering all numerical and reliability analyses. Few recommendations are also provided for future studies.

Chapter 2

Finite element simulation of mode-I and mode-II delamination tests, formulations and methods of reliability assessment

2-1 Introduction

The widespread use of composite materials in several engineering structures has brought about several studies about the behavior of composite materials. Nowadays, composites have been widely used in industries to fabricate structures, including aircraft, ships, vehicle body, and wind turbines. Finding out the material behavior of the most commonly used composites in industries subjected to different loading modes is essential. Considering mechanical failure in composite material, degradation of stiffness and strength usually causes crack formation between adjacent plies in composites known as delamination. Delamination is one of the most probable failure modes in laminated composites stemming from interlayer stresses. Fracture mechanics is a way to take imperfection and defects such as cracks into account for designing structures to predict the response of structure to loading encountered in real-life applications. The weakness at layer interface may lead to the separation of layers that can occur under normal or shear stresses. There are three different independent modes of delamination, that are, mode-I, mode-II and mode-III. Also, combinations of these three modes may occur, a possible one is mixed-mode I/II. Few experimental mechanical testing procedures have been designed to study the failure modes. For characterizing the mode-I delamination, the Double Cantilever Beam (DCB) test, for mode-II delamination, the End Notched Flexure (ENF) test, and for mixed-mode I/II delamination, the Mixed-Mode Bending (MMB) test are performed. Several investigations have been conducted during recent decades to study the behavior of composites undergoing loading with existing delamination.

In this chapter, primary models of numerical analysis for simulating both DCB and ENF tests are presented and verified using available studies in the literature. The FEA aspects of these models such as meshing, element type, number of elements, element geometry, and material properties of

elements are discussed. Then, the numerical analysis results are provided, discussed, and compared with the references. The numerical analysis provided has been proven as a trustable and accurate analysis. It has been used frequently to investigate the effects of different parameters on mode-I and mode-II delamination later in chapter 3.

Nowadays, scientists strongly believe that non-deterministic approaches such as probabilistic approach need to be applied widely in various types of engineering applications. These methods have already attracted significant attention in engineering fields such as aerospace, automotive, civil, industrial, and ocean engineering industries. Commonly, in real-life engineering problems, there are numerous sources of uncertainty in the manufacturing and design of products. An engineering application may consist of several sources of uncertainty that have their own individual effects and compound effects of uncertainties. A comprehensive framework seems to be essential so as to consider all the possible risks to a system. Non-deterministic methods define reliability as a safety measure of a structure or application instead of the safety factor in deterministic methods. With regard to the uncertainty sources, probabilities, and consequences, system reliability can be investigated through computing failure probability. Unified theories of uncertainty and mathematical tools for modeling different types of uncertainty pave the way to compute the failure probability in a system and to realize to what extent the system is reliable in a specific application. How a modelled uncertainty in the input variables may lead to the uncertainty of the response can also be observed. A tool for propagating uncertainty through a component or a system is needed for this purpose. This chapter provides the concepts and governing theoretical equations of reliability analyses. In chapter 4, the reliability analysis is performed using the Rtx[®] software in order to evaluate the probability of delamination growth in composite materials subjected to mode-I and mode-II delamination tests.

2-2 Finite Element Analysis (FEA) in ANSYS[®]

The finite element method is a numerical approach used at various stages of design of structures. So as to perceive the failure mechanism of composite structures, simulation of delamination testing is required. Different modeling approaches such as Virtual Crack Closure Technique (VCCT), Continuum Damage Models (CDM), Linear Elastic Fracture Mechanism (LEFM), Extended Finite

Element Method (XFEM), and Cohesive Zone Model (CZM) have been used in various studies to study the delamination behavior of different materials particularly composite laminates. In the present study, the CZM approach in conjunction with the FEA tool ANSYS® is used to model failure testing of composite laminates. In the following, the geometries and characteristics of different element types used in 2D and 3D simulations of DCB and ENF tests are briefly provided.

Figure 2-1 shows the geometry of the PLANE183 element. This finite element is defined by eight nodes as indicated in the figure or by six nodes. This type of element can be used in irregular meshes since it has quadratic displacement behavior [66]. The element can have plasticity, hyper-elasticity, creep, stress stiffening, large deflection, and large strain [66]. It is used in the plane-stress 2D modeling of the present work.

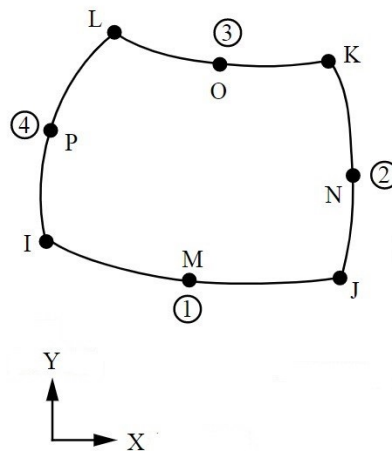


Figure 2-1 Geometry of finite element - PLANE183 [66]

Figure 2-2 illustrates the CONTAC172 finite element geometry. This element is used to assign contact and sliding between 2D target surfaces (TARGE169), as shown in figure 2-3, and a deformable surface. It can be used for both pair-based contact and general contact [67]. In general contact, CONTAC172 elements can be used for the deformable target surface [67].

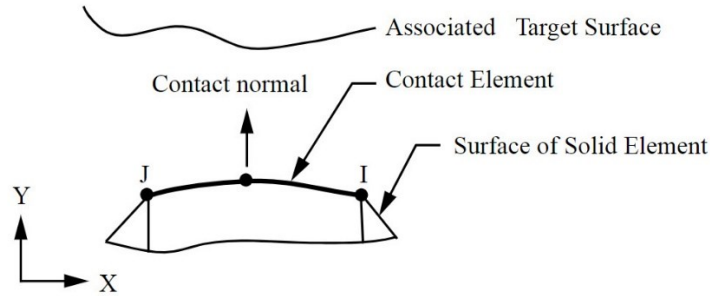


Figure 2-2 Geometry of finite element - CONTAC172 [67]

TARGE169 is an element type used to present 2D target surfaces only for rigid bodies for the associated contact element such as CONTAC172 described in the previous paragraph [68]. Figure 2-3 illustrates the geometry of this finite element. The target segment element can be subjected to rotational displacement, temperature, voltage, pore pressure [68]. This element, as well as CONTAC172, are used to describe the contact between the beams in the DCB and ENF test specimens that are simulated in the present study.

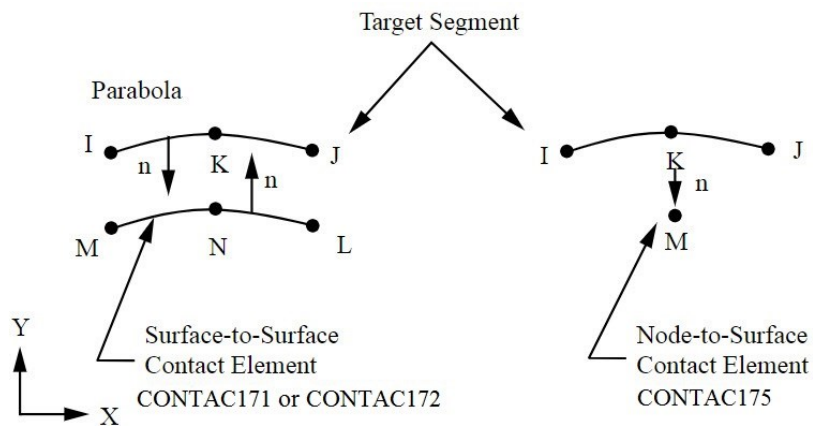


Figure 2-3 Geometry of finite element - TARGE169 [68]

SOLID185 is a 3D finite element used for solid structure modeling shown in figure 2-4. It has eight nodes with three degrees of freedom at each node [69]. The element is capable of having

plasticity, hyper-elasticity, stress stiffening, creep, large deflection, and large strain [69]. The capability to have large deflection makes this finite element suitable to be used in the delamination property tests. It is used as the dominant element type in the 3D FEA of the present work.

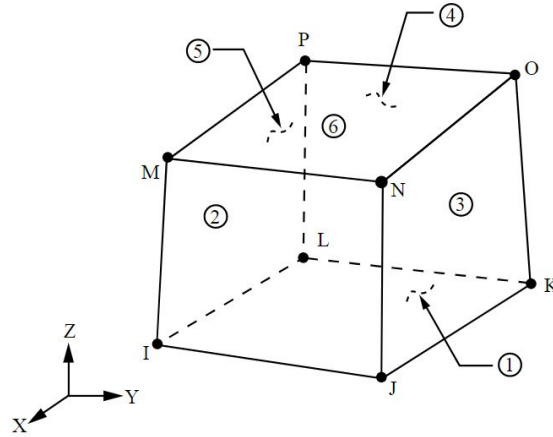


Figure 2-4 Geometry of finite element - SOLID185 [69]

Figure 2-5 illustrates the geometry of the CONTAC174 element. This element is used for defining contact or sliding between two surfaces of 3D solid parts. It is applicable for both pair-based contact and general contact [70]. In pair-based contact, TARGE170 elements, introduced later on, determine the target surface. Nonetheless, CONTAC174 elements can define the deformable target surface in general contact, and TARGE170 assigns the target surface of only rigid bodies [70].

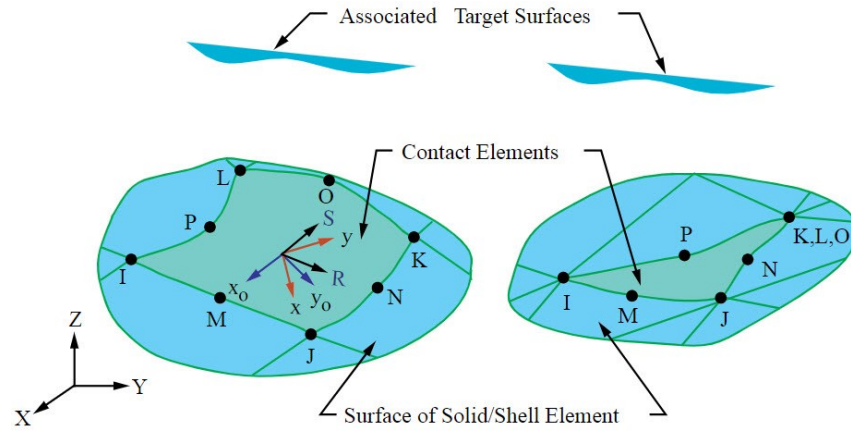


Figure 2-5 Geometry of finite element - CONTAC174 [70]

In addition, CONTAC175 element is the other type that can be used to represent contact and sliding between two surfaces in 2D or 3D modeling [71]. As illustrated in figure 2-6, it also can define contact between a node and a surface or between a line and a surface [71]. When there is a pair-based contact, and the CONTAC175 element defines contact surface, the target surface can be defined by TARGE169 and TARGE170 in 2D and 3D modeling, respectively.

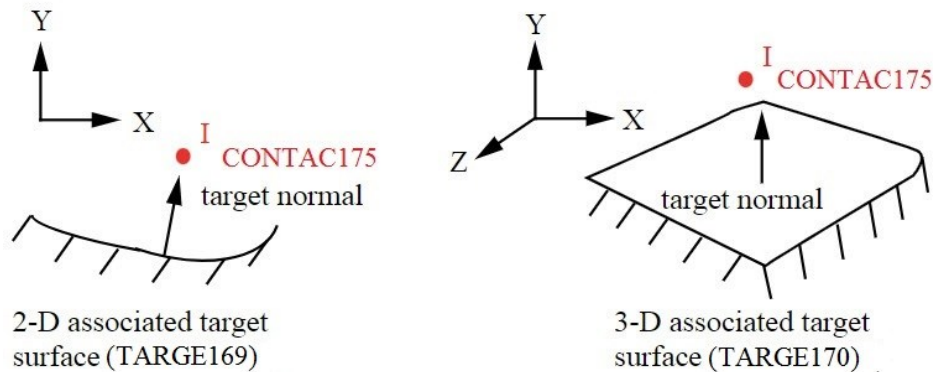


Figure 2-6 Geometry of finite element - CONTAC175 [71]

TARGE170 is a finite element used to determine the target surface in contact with the deformable solid, shell, or line, defined by contact elements such as CONTAC174 and CONTAC175 described above [72]. Figure 2-7 illustrates the geometry of TARGE170 in contact with different geometries and contact elements. As the element type of TARGE169 in 2D structures, TARGE170 on the target surface in 3D models can be subjected to rotational displacement, temperature, voltage, pore pressure, concentration, forces, and moments [72].

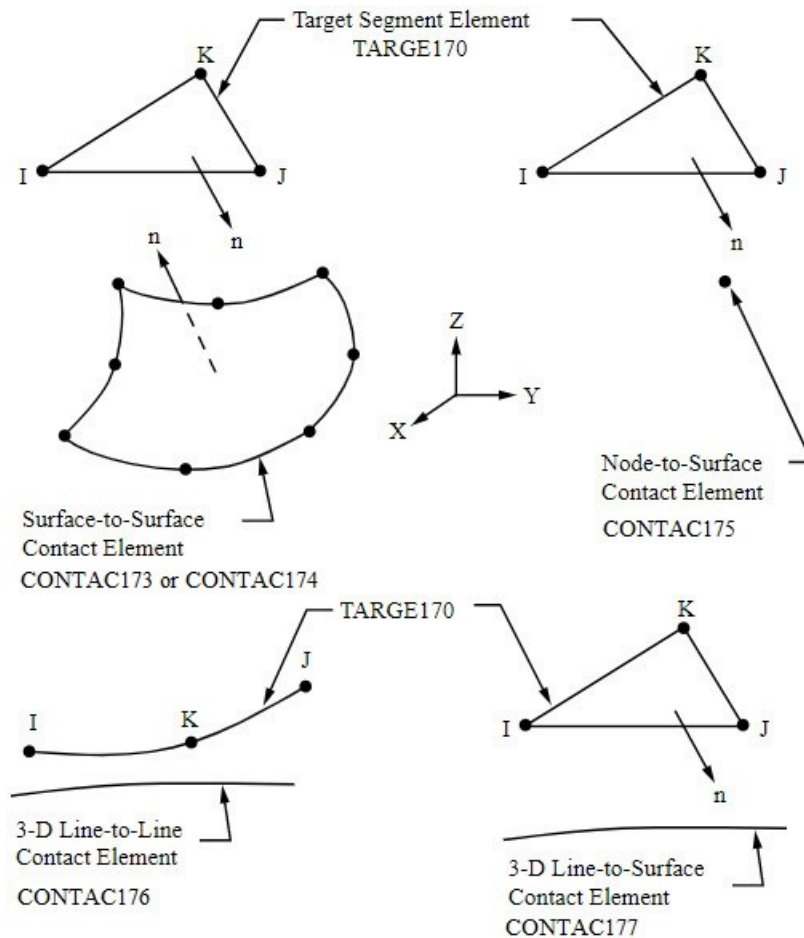


Figure 2-7 Geometry of finite element - TARGE170 [72]

In the present study, 2D modeling and 3D modeling are frequently used to meet the required outcomes for a parametric study. In 2D simulations, PLANE183, CONTAC172, and TARGE169 are used to describe the composite and cohesive elements. Besides, in 3D modeling, using

SOLID185, CONTAC174, CONTAC175, and TARGE170 makes it possible to simulate the models accurately.

Besides, other element types can be used to simulate delamination testing. In some available studies in the literature, ABAQUS/Standard® software has been used to simulate the delamination test specimens. For instance, in the study of Soroush et al. [100], in 2D modeling, CPE4R, which is the four nodes continuum element, has been used to describe the composite laminate. Also, COH2D4, which is the four nodes element, has been used to define the cohesive interface.

2-3 Cohesive Zone Model

The CZM approach is used to predict damage initiation and propagation. The cohesive zone model assumes that when damage initiates, the stress transfer capacity between the two separating faces of delamination is not lost suddenly. Still, it degrades gradually until the total fracture happens. Plastic deformations are transferred to a surface ahead of the crack tip. In this way, the damage propagation ahead of the crack tip can be identified. As mentioned in the literature survey in chapter 1, there are different forms of interface law. Bi-linear constitutive law is considered to predict the delamination onset and propagation in this study. This law consists of three parts, including linear-elastic stage, loss of stiffness, and total failure. CZM is a straightforward method that can simulate quasi-brittle fracture and make a connection between traction and separation of the crack tip. Since cohesive elements are of zero thickness, the engineering stress-strain relationships are replaced with a bilinear traction-separation law, as shown in figure 2-8. The mode-I and mode-II damage propagations can be investigated using bilinear constitutive law in the same way. The deformation of cohesive material in laminated composites is formulated similarly for both damage modes. In the figure below, the vertical axis shows the cohesive interface traction, and the horizontal axis represents the displacement between two separating faces at the damage interface.

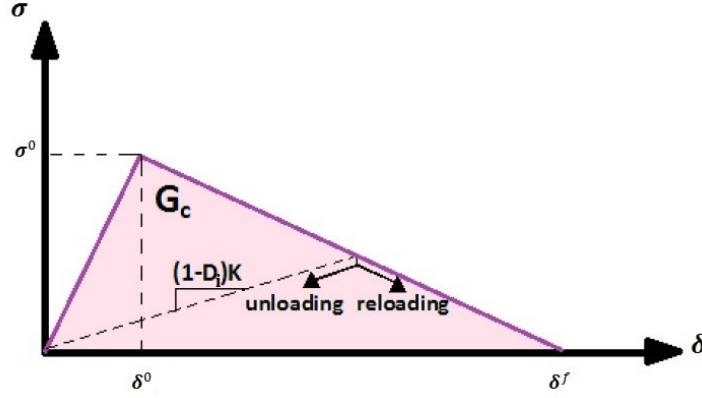


Figure 2-8 Bilinear constitutive law of cohesive element [73]

In figure 2-8, σ^0 is the maximum interfacial strength of cohesive material under nominal and shear stress for mode-I and mode-II loadings, respectively. Besides, δ^0 denotes the corresponding separation between the crack faces at the damage initiation, and δ^f denotes the displacement of the separating faces of the crack at failure. Damage initiation and mode-I and mode-II energy release rates are attributed to the interfacial strength of the material and delamination crack propagation, respectively. The area under the triangular curve is the energy release rate. The bilinear constitutive equation of the cohesive finite element is expressed as follows [25], [74]:

$$\sigma_i = (1 - D_i)K\delta_i \quad (2-1)$$

where σ_i denotes the element traction, D_i is damage variable, K is penalty stiffness, and δ_i denotes the separation of two faces of the delamination. Penalty stiffness indicates the stiffness of the interface materials, which is specific for each loading mode. It is worth noting that this parameter is different from elastic or shear modulus because the bilinear constitutive law is based on zero-thickness cohesive elements. Different assumptions for the interface stiffness proposed by some authors were provided in the study of Turon [74]. Equation (2-1) can be rewritten according to the three different stages of the bilinear law. Linear-elastic, interface stiffness degradation, and total failure stages make up the bilinear constitutive law and the corresponding traction-separation equations are illustrated based on the three different parts in the equation below [74]:

$$\sigma_i = \begin{cases} K\delta_i & 0 \leq \delta_i \leq \delta^0 \\ \left(\frac{\delta^f - \delta_i}{\delta^f - \delta^0}\right)\sigma^0 & \delta^0 < \delta_i < \delta^f \\ 0 & \delta_i \geq \delta^f \end{cases} \quad (2-2)$$

It is evident that from the beginning of opening or sliding loading to the delamination initiation in mode-I or mode-II, where the traction in the interface, σ^0 , is maximum, the behavior of material is linear-elastic. Based on the equation above, at the damage initiation, maximum interfacial strength of the cohesive i.e., maximum traction without failure, is obtained by $\sigma^0 = K\delta^0$.

The damage variable, D_i , measures the loss of cohesive stress transfer, that is, the interface stiffness degradation. It is clear that the damage variable is zero before the damage initiation because there is not any stiffness degradation in that stage. However, the following condition is true during the loss of cohesive stiffness stage: $0 < D_i < 1$. By increasing the separation of two beams at the crack tip, the damage variable value approaches unity. At the failure point, it is equal to unity since the cohesive stiffness degrades completely. With regard to the equations (2-1) and (2-2), the damage variable (D_i) is obtained for the three stages of the damage law as in the equation below [74].

$$D_i = \begin{cases} 0 & 0 \leq \delta_i \leq \delta^0 \\ \left(\frac{\delta_i - \delta^0}{\delta_i}\right)\left(\frac{\delta^f}{\delta^f - \delta^0}\right) & \delta^0 < \delta_i < \delta^f \\ 1 & \delta_i \geq \delta^f \end{cases} \quad (2-3)$$

As it can be seen from figure 2-8, at the end of the linear-elastic stage, the cohesive material reaches its maximum traction without failure, followed by a moderate loss of its stiffness. The traction of the damaged interface decreases gradually to zero where the failure happens.

2-4 Characterization of delamination

Fracture of composite laminates may occur due to several reasons. Intralaminar problems such as fiber fracture and matrix cracking may lead to the failure of composite materials. Delamination or interlaminar cracking is the separation of layers. It is known as the most common interlaminar damage mechanism that may cause the fracture of composite materials. The basic modes of delamination failure basic modes are: opening mode, mode-I, sliding and shearing mode, mode-II, and tearing mode, mode-III, as shown in figure 2-9. A combination of different modes, particularly mixed-mode I/II, may happen under certain loading and boundary conditions.

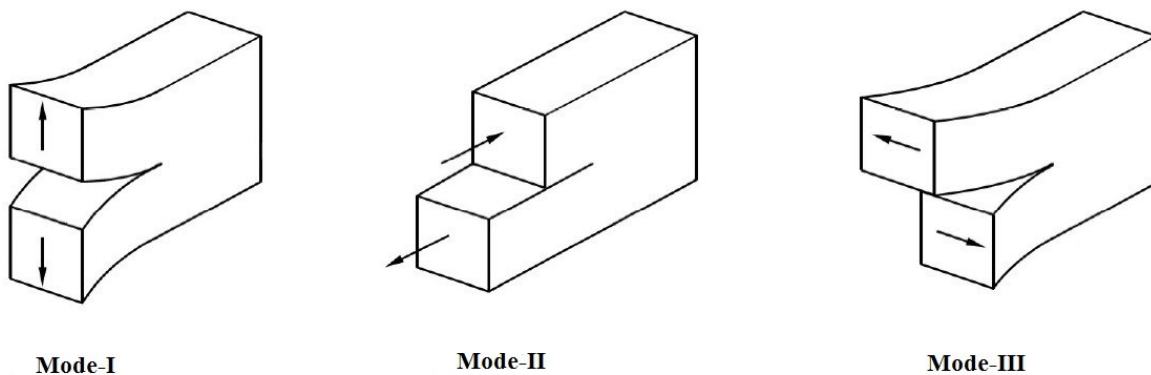


Figure 2-9 Delamination modes in composite materials

Delamination in any mode may happen under different conditions because the interface of adjacent layers is the weakest zone in a composite laminate. In the case of each delamination, the composite material experiences a significant reduction in material stiffness and load-carrying capability. In the present study, mode-I and mode-II delamination failures in composite materials have been investigated using DCB and ENF tests, respectively.

In order to figure out the delamination property of composites, some important tests that are in compliance with worldwide standards, such as ASTM, are conducted. Based on the literature survey in chapter 1, the most updated standards for the DCB test, mode-I delamination test, and the ENF test, mode-II delamination test, are ASTM D5528 [3] and ASTM D7905 [7], respectively.

These standards propose test configurations and data reduction methods to obtain the fracture toughness of the unidirectional fiber-reinforced polymer matrix composites subjected to mode-I and mode-II delamination tests. In an investigation of delamination in composite materials, delamination initiation and propagation are essential concepts. In the present study, the CZM approach in conjunction with FEA tool ANSYS® is used to simulate delamination onset with regard to the interlaminar stresses and delamination propagation considering bilinear constitutive law and loss of stiffness. The DCB and ENF tests, used frequently in the next chapter, are described and verified with the available results in the literature in the following sections.

2-5 Finite element modeling and simulation of DCB test and verification

The CZM approach has been used in conjunction with finite element analysis to model and simulate the DCB test in commercial finite element analysis tool ANSYS®. The DCB test specimen consists of two identical beams of similar geometry that are attached by using a cohesive material. A pre-crack is considered at the left side of the specimen between the upper and lower beams. The DCB test configuration and the specimen geometry are illustrated in figure 2-10. A pure mode-I delamination test reveals the behavior of the composite material under opening or peel delamination mode.

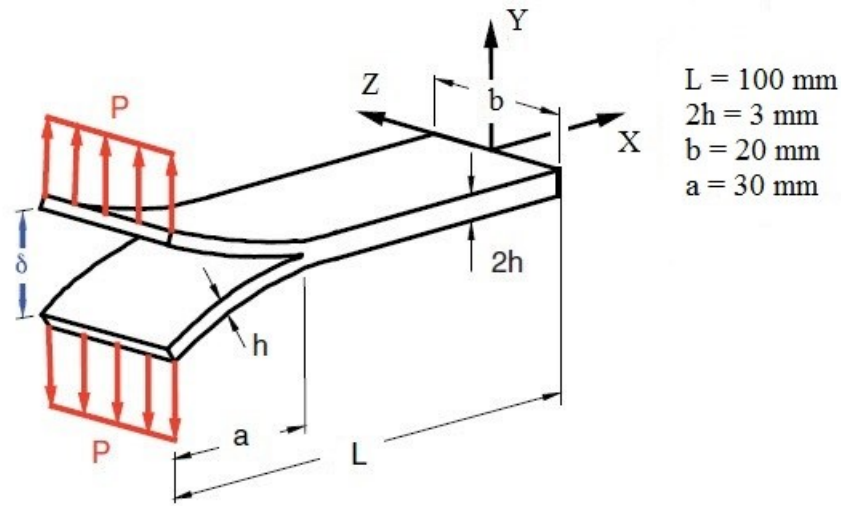


Figure 2-10 Configuration and geometry of the DCB test specimen [25]

The length of specimen is 100 mm, and its whole thickness is 3 mm with a width of 20 mm. The delamination length considered at the free end of the beam is 30 mm. The DCB test specimen used in this study is a prepreg composite laminate with a stacking sequence of $[0]_{20}$. Engineering properties of the composite ply, as well as cohesive properties of the interface, are provided in tables 2-1 and 2-2, respectively.

Table 2-1 Engineering properties of the composite material ply in the DCB test specimen [25]

E_{11} (MPa)	E_{22} (MPa)	E_{33} (MPa)	ν_{12}	ν_{13}	ν_{23}	G_{12} (MPa)	G_{13} (MPa)	G_{23} (MPa)
135300	9000	9000	0.24	0.24	0.46	5200	5200	5200

Table 2-2 Cohesive interface properties of the DCB test specimen [25]

σ_n^0	G_{Ic}	η
1.7 MPa	0.28 N/mm	$10 \times 10^{-7} s$

In table 2-2, σ_n^0 denotes the interfacial strength under nominal stress or maximum traction without failure, G_{Ic} is mode-I fracture toughness, and η represents artificial damping coefficient.

Modeling the DCB test is conducted by imposing opening displacement to both free ends of the specimen. An upward and a downward displacement of 10 mm are applied to the edges of the upper and lower beams, respectively. The opening displacement makes the delamination propagate until the fracture of the composite laminate. During the simulation, the load value of the free end of the upper beam is recorded and plotted versus the corresponding displacement. Figure 2-10 shows the load-displacement curve of the free end of the DCB test specimen.

The DCB test is simulated in 2D and 3D modeling. In 2D modeling, edge sizing and face meshing methods are used to mesh the DCB test specimen [100]. The element size of the longitudinal edges of the model is 0.5 mm. Also, for edge meshing of the four edges of the specimen, which are through its thickness, the element size is defined as 0.75 mm. Afterward, 4381 nodes and 1985 elements are created by face meshing. A total of 1985 elements, including 1191 PLANE183 elements, 397 CONTAC172 elements, and 397 TARGE169 elements, are used. As described earlier, CONTAC172 and TARGE169 elements are the contact elements used to define contact and target surfaces between two identical beams in the modeling of DCB test specimen. The edge sizing approach is also used for meshing 3D models. Element size is defined as 1.125 mm in all edges through the thickness of the beams. For the eight longitudinal edges of the specimen, the element size is assigned 1 mm, and it is defined as 2 mm for edges through the width of model. In 3D modeling, a total of 23062 nodes and 3880 elements are created as a result of the meshing, which a pretty fine is meshing that is trustworthy for precise 3D modeling. A total of 2000 SOLID185, 940 CONTAC174, and 940 TARGE170 elements make up the DCB test specimen.

The force-displacement curve of the free end of the DCB test specimen is the main outcome of this simulation shown in figure 2-11. It can be seen that the critical load is 50.56 N and its

corresponding critical displacement is 1.48 mm. These values can be used to evaluate the mode-I critical energy release rate.

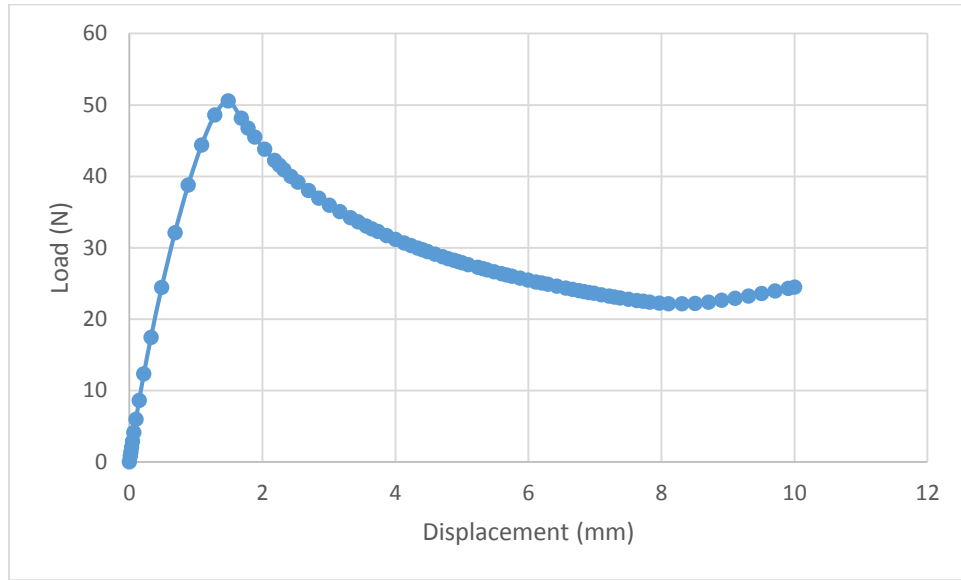


Figure 2-11 Load-displacement curve of the free end of the DCB test specimen

Since the geometry, boundary conditions, and material properties of the specimen, as well as the test configuration, are selected according to the reference numerical study [25], the load-displacement results are verified with the outcomes of the reference study as illustrated in figure 2-12. Comparing figures 2-11 and 2-12 indicates that there is a good agreement between the results of the present study and the study of Alfano et al. [25].

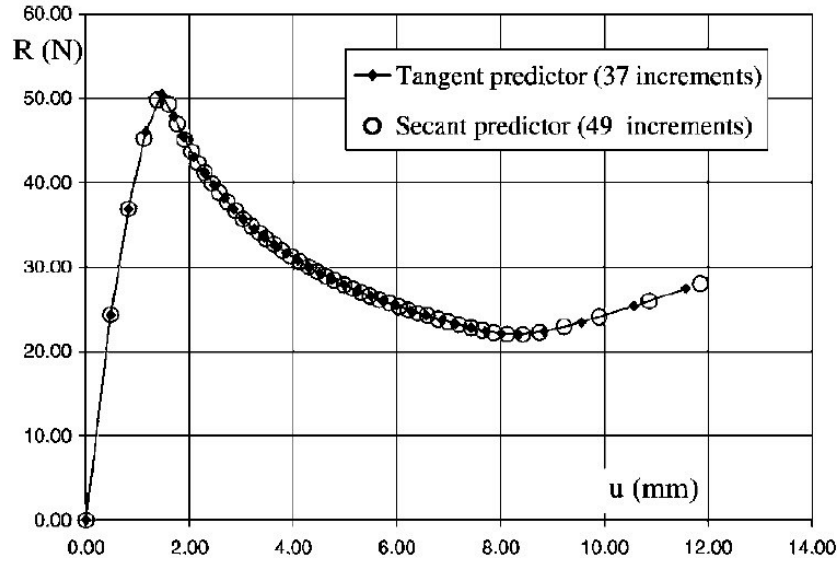


Figure 2-12 Load-displacement curve of the free end of the DCB test specimen in reference study [25]

Figures 2-13 and 2-14 show the DCB test specimen after undergoing the opening displacement, leading to mode-I delamination. In the parametric study of this test concerning the mechanical properties and geometric parameters of the test specimen, provided in the next chapter, since several models are simulated in most of the cases, 2D modeling is used. In chapter 3, DCB test models are primarily simulated in 2D because the plane-stress assumption shows a good agreement and helps save time and cost of simulations.

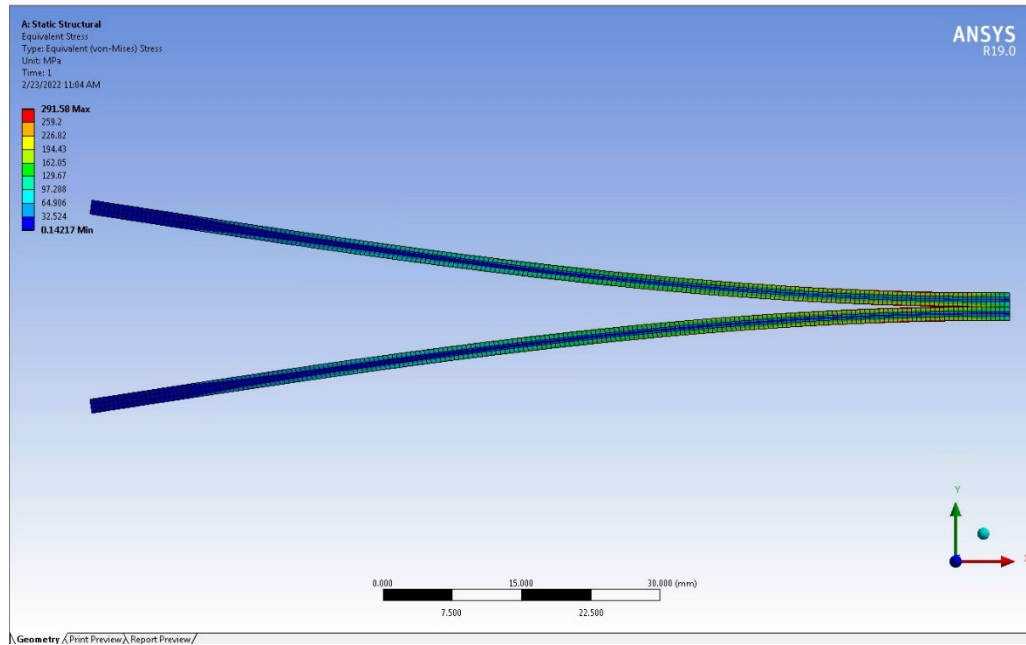


Figure 2-13 Contour of deformed DCB test specimen in 2D modeling

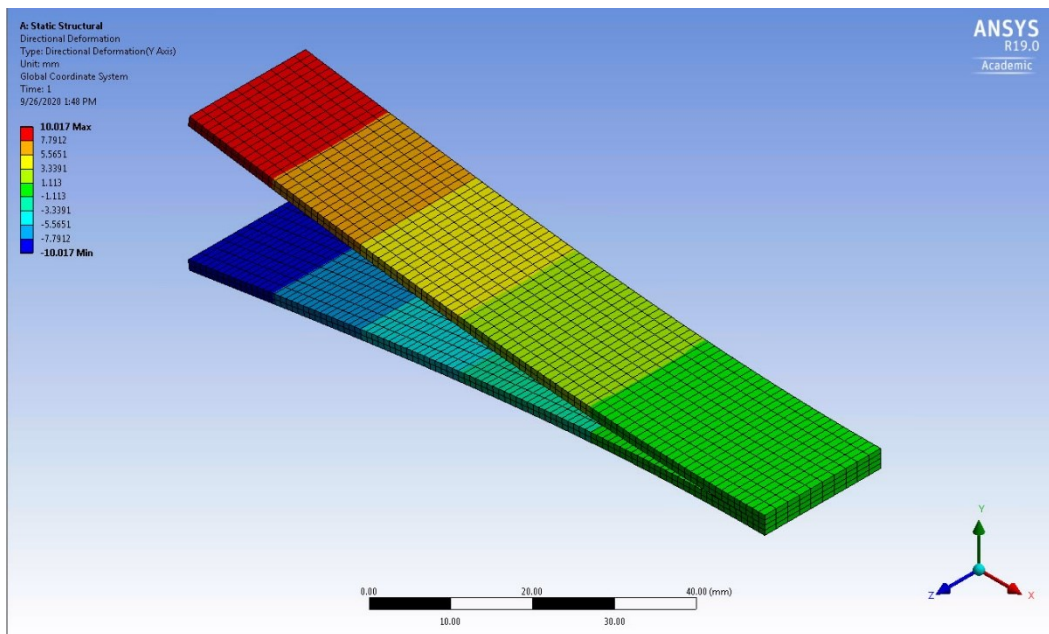


Figure 2-14 Contour of deformed DCB test specimen in 3D modeling

2-6 Finite element modeling and simulation of ENF test and verification

The End Notched Flexure (ENF) test is the most common mode-II delamination test widely used to evaluate the behavior of the composite material under pure mode-II loading. Modeling of the ENF test is performed using the FEA tool ANSYS® in conjunction with the CZM approach for simulating crack onset and propagation. The test specimen consists of two identical beams of 101.6 mm length, 25.4 mm width, and 2.25 mm thickness. The test specimen thickness is 4.5 mm, as illustrated in figure 2-15. The pre-crack length is 25.4 mm at the left side of the test specimen. The test configuration and geometric parameters of the test specimen are selected according to the study of Krueger [37].

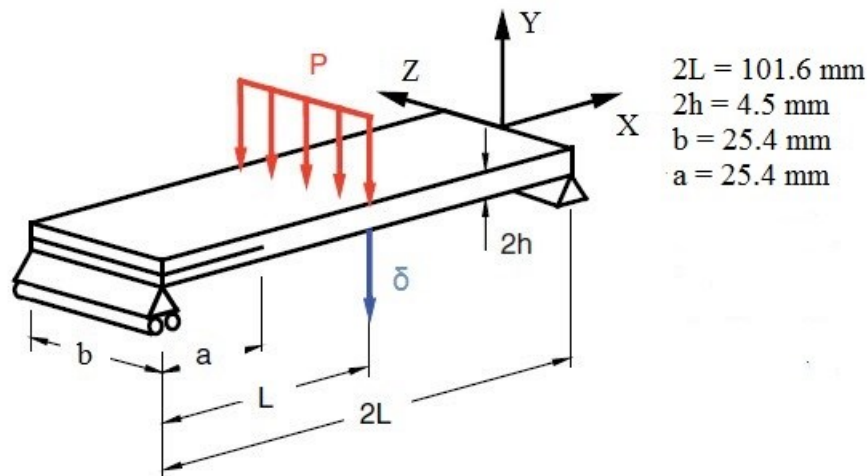


Figure 2-15 Configuration and geometry of the ENF test specimen [37]

The test specimen is made of IM7/8852 unidirectional graphite-epoxy composite laminate with $[0]_{24}$ stacking sequence. Table 2-3 provides the engineering properties of the composite laminate undergoing the ENF test modeling. The properties of the cohesive interface of the two beams are provided in table 2-4. These properties are imported to the engineering data option in the static structural analysis system in ANSYS®.

Table 2-3 Engineering properties of the composite material ply in the ENF test specimen [37]

E_{11} (MPa)	E_{22} (MPa)	E_{33} (MPa)	ν_{12}	ν_{13}	ν_{23}	G_{12} (MPa)	G_{13} (MPa)	G_{23} (MPa)
161000	11380	11380	0.32	0.32	0.48	5200	5200	3900

Table 2-4 Cohesive interface properties of the ENF test specimen [37]

σ_s^0	G_{IIc}	η
26 MPa	0.78 N/mm	0.0007 s

As for the cohesive interface properties, σ_s^0 denotes the interfacial strength or maximum traction without failure under shear stress, G_{IIc} is mode-II fracture toughness, and η is the artificial damping coefficient.

Specimen boundary conditions are according to the work used for validation and they are shown in figure 2-15. The ENF test specimen is subjected to a displacement of 4.5 mm at the mid-point of the upper beam. In the 2D modeling, the specimen is meshed by edge sizing and face meshing methods [100]. The element size for all the longitudinal edges is defined as 1 mm, and for the through-thickness edges, the element size is determined at 1.125 mm. Different element types were used to model the specimen, including 406 PLANE183 elements, 203 CONTAC172, and 203 TARGE169 elements. Two later element types define contact between the two beams based on the CZM approach and cohesive interface material properties. In 3D modeling of the ENF test specimen, a total of 26911 elements, including 22848 SOLID185, 1904 CONTAC174, 1905 TARGE170, and 45 CONTAC175 elements are used.

The load and the displacement of the mid-point are recorded during the modeling until the central displacement approaches 4.5 mm. According to the recorded data, the load-displacement curve of the mid-point of the ENF test specimen is obtained after the simulation, as illustrated in figure 2-16. It can be seen that the value of the critical load is 1528.54 N and the corresponding

displacement is 1.35 mm. Comparing the result with that of the validation work proves that the present numerical study offers trustable results. As shown in figure 2-17, the critical values of load and displacement are 1526 N and 1.42 mm, respectively, which are in good agreement with the outcome of the present simulation work.

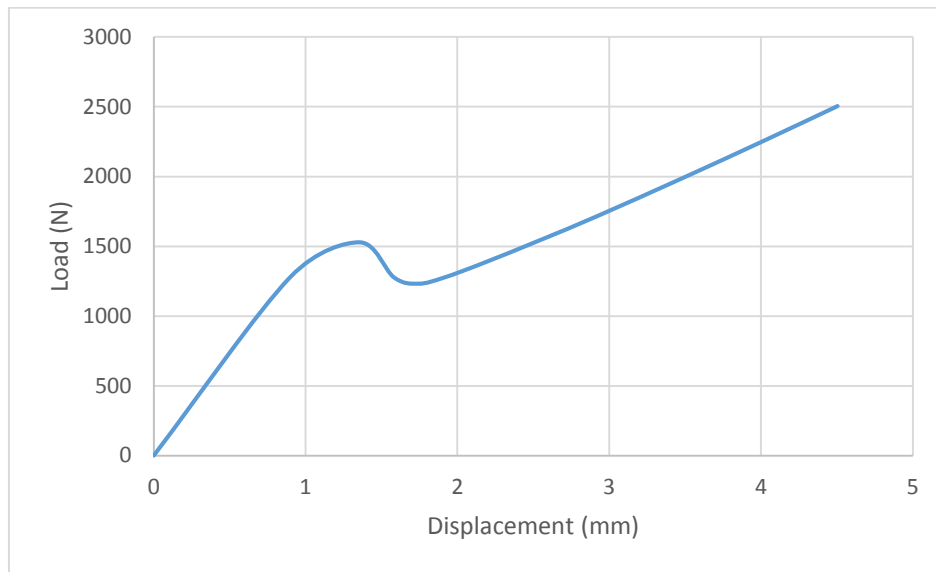


Figure 2-16 Load-displacement curve of the mid-point of the ENF test specimen

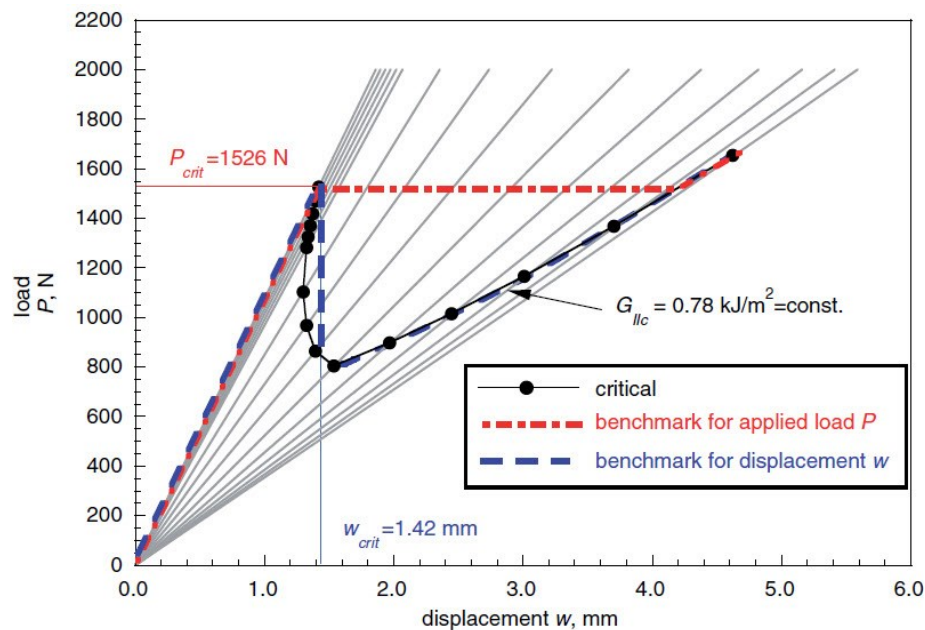


Figure 2-17 Load-displacement curve of the mid-point of the ENF test specimen in the study of Krueger [37]

After subjecting to the mode-II delamination test loading, in 2D and 3D modeling, the deformed specimen is as illustrated in figures 2-18 and 2-19, respectively. It can be seen how different parts of the test specimen undergo deflections due to the central displacement. In the parametric study of this test concerning the mechanical properties and geometric parameters of the test specimen, provided in the next chapter, since several models are simulated in most of the cases, 2D modeling is used. In the investigation of the effects of fiber misalignment and its location in the laminate in the ENF test, presented in chapter 3, the 3D modeling was conducted using ACP (Pre) module in ANSYS® in addition to the static structural analysis system.

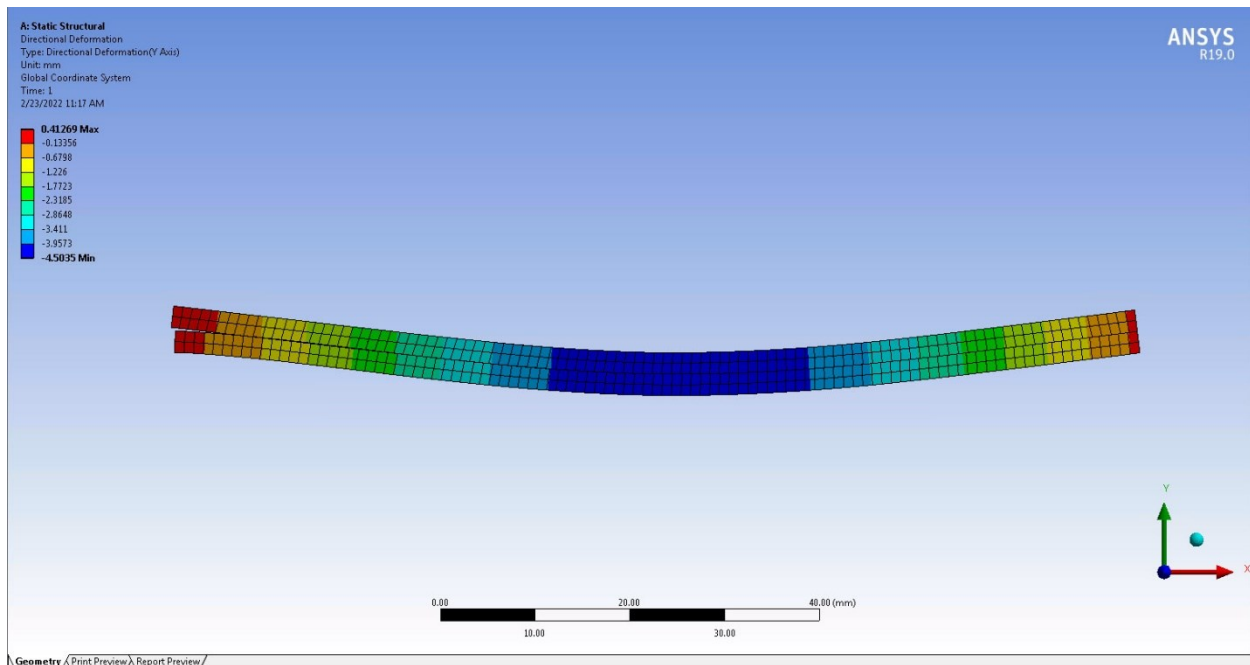


Figure 2-18 Contour of deformed ENF test specimen in 2D modeling

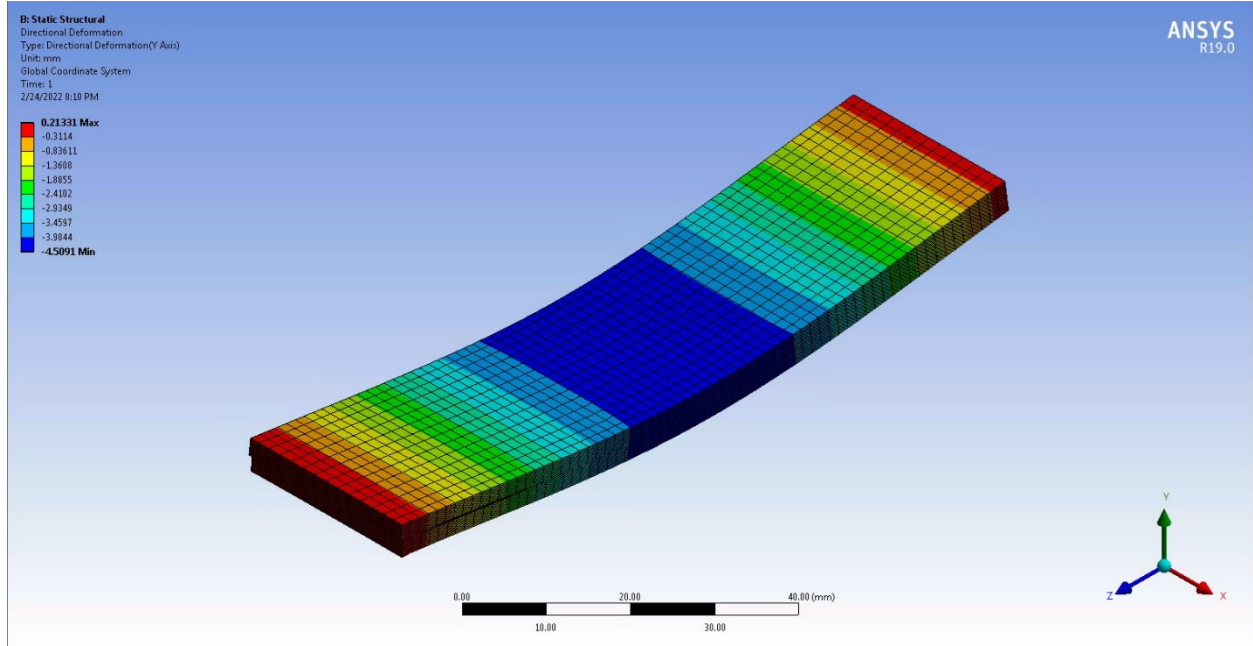


Figure 2-19 Contour of deformed ENF test specimen in 3D modeling

2-7 Data reduction method for calculating fracture toughness

There are different methods of data reduction to obtain the interlaminar fracture toughness with the existence of initial delamination. The most important ones can be named as Direct Beam Theory (DBT), Modified Beam Theory (MBT), Compliance Calibration (CC), Modified Compliance Calibration (MCC), and area method. In ASTM D5528, MBT, CC, and MCC are provided and discussed [3], and the area method is presented and discussed by few authors [75], [76]. In the present study, DBT is used to determine the compliance, and then the energy release rate is obtained for each loading mode. Compliance is the ratio of the displacement at the load point over the applied load ($C = \frac{\delta}{P}$).

As for the DCB test, using DBT results in the following equation for the test compliance [77]:

$$C = \frac{8a^3}{bh^3E_{11}} \quad (2-4)$$

where C is the DCB test compliance, E_{11} is the longitudinal elastic modulus, and a , b and h denote the crack length, width, and half-thickness of the DCB test specimen, respectively (figure 2-10).

The energy release rate can be derived as a function of the geometric parameters of test specimen, crack length, and loading, according to the Irwin-Kies expression as follows [78]. This equation can be used for different loading modes.

$$G_I = \frac{P^2}{2b} \frac{\partial C}{\partial a} \quad (2-5)$$

Based on the derivation of the compliance, and the expression to compute the compliance provided in equation (2-4), the interlaminar energy release rate in the DCB test can be calculated by the equation provided below.

$$G_I = \frac{12P^2 a^2}{b^2 h^3 E_{11}} \quad (2-6)$$

Using direct beam theory for the ENF test leads to an expression to calculate the compliance as follows [77], [79]:

$$C = \frac{3a^3 + 2L^3}{8bh^3 E_{11}} \quad (2-7)$$

where C is the ENF test compliance, E_{11} is longitudinal elastic modulus. Also, a , L , b and h denote the crack length, length, width, and half-thickness of the test specimen, respectively (figure 2-15).

Applying the Irwin-Kies expression for the pure mode-II delamination test, the ENF test, and substituting equation (2-7) in this expression yields the equation to calculate mode-II energy release rate as follows:

$$G_{II} = \frac{9P^2a^2}{16b^2h^3E_{11}} \quad (2-8)$$

2-8 Theoretical concepts and formulations of reliability study

In the following sub-sections, the key aspects of reliability analysis are elaborated. There are few known approaches to calculate failure probability. On one hand, few methods are founded on first-order approximation, and others deal with second-order approximation which are provided in the upcoming sub-sections. On the other hand, few sampling methods are used to predict the probability of failure which are described in individual sub-sections in the present chapter. Essential contents of reliability study, mathematical formulations, theories, and the description of the methodology used in the present study are provided.

2-8-1 Basic reliability problem

In a basic structural reliability problem, there is a capacity or resistance, R , of the component that resists the effect of load, S . R and S can be described by random variables with two distinct marginal probability density functions f_R and f_S , respectively [80]. Limit state function, G , is defined according to a comparison between load and capacity that is formulated as in the following:

$$G = R - S \quad (2-9)$$

If the load exceeds the resistance, the structural element will fail. Therefore, the probability of failure is as follows:

$$P_f = P(R < S) = P(G \leq 0) \quad (2-10)$$

Considering the limit state function with a joint probability density function that is named as f_{RS} , the probability of failure can also be stated as [81]:

$$P_f = \iint_{R < S} f_{RS}(r, s) dr ds \quad (2-11)$$

Assuming that R and S are independent, the above equation can be written as [81]:

$$P_f = \int_{-\infty}^{\infty} \int_{-\infty}^{s \geq r} f_R(r) f_S(s) dr ds \quad (2-12)$$

The following integral indicates the cumulative distribution function definition for any random variable, X:

$$F_X(x) = P(X \leq x) = \int_{-\infty}^x f_X(y) dy \quad (2-13)$$

By substituting equation (2-13) in equation (2-12), the probability of failure can be written in a single integral form as:

$$P_f = P(G \leq 0) = \int_{-\infty}^{\infty} F_R(x) f_S(x) dx \quad (2-14)$$

By assuming that R and S are independent random variables which are normally distributed with corresponding mean values of μ_R and μ_S , and standard deviations of σ_R and σ_S , respectively, then the mean and variance of the limit state function, G, can be calculated as the following [81]. The limit state function divides the failure zone and safe zone.

$$\mu_G = \mu_R - \mu_S \quad (2-15)$$

$$\sigma_G^2 = \sigma_R^2 + \sigma_S^2 \quad (2-16)$$

Considering the joint cumulative distribution function, as well as the mean and standard deviation of the limit state function, the failure probability equation can be written as following [81]:

$$P_f = P(G \leq 0) = F_{RS}(0) = \Phi\left(\frac{0 - \mu_G}{\sigma_G}\right) = \Phi\left(-\frac{\mu_G}{\sigma_G}\right) = \Phi(-\beta) \quad (2-17)$$

where Φ is the cumulative standard normal distribution of the limit state function, and $\beta = \mu_G/\sigma_G$ is named as reliability index or safety index. The safety index indicates how much the distance is between the mean value of the function and the boundary of the failure zone or origin ($G = 0$), figure 2-20. The greyish region in this figure shows the failure zone. It can also be declared that the safety index expresses how many times of standard deviation is the mean value of the function far away from the failure domain. A high-reliability index means that the component is safer and failure probability is low.

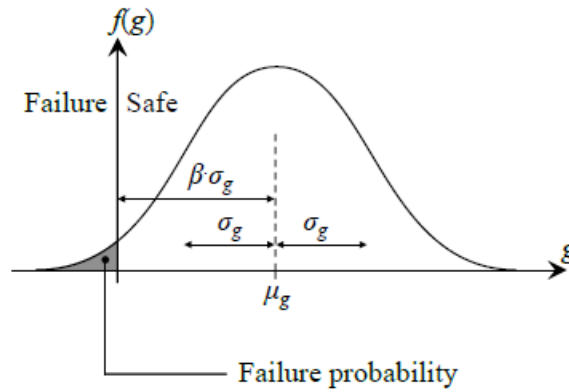


Figure 2-20 Normal probability distribution for the basic limit-state function [83]

Reliability index and failure probability are roughly inversely proportional to each other. According to the study of Der Kiureghian [82], for the typical ranges of β and P_f values, the exact relationship between the reliability index and failure probability, $P_f = \Phi(-\beta)$ as well as the approximation relationship between these two parameters, $P_f = 10^{-\beta}$ can be plotted [83]. The approximation approach is not practical since it can only predict the failure probability for the values of the reliability index around $\beta \sim 3$. Figure 2-21 illustrates the difference between the exact relationship between the reliability index and failure probability and the approximation of their relationship in the case that the value of reliability index is 3, $\beta = 3$. However, in both approaches, a rise of the reliability index brings about diminishing failure probability.

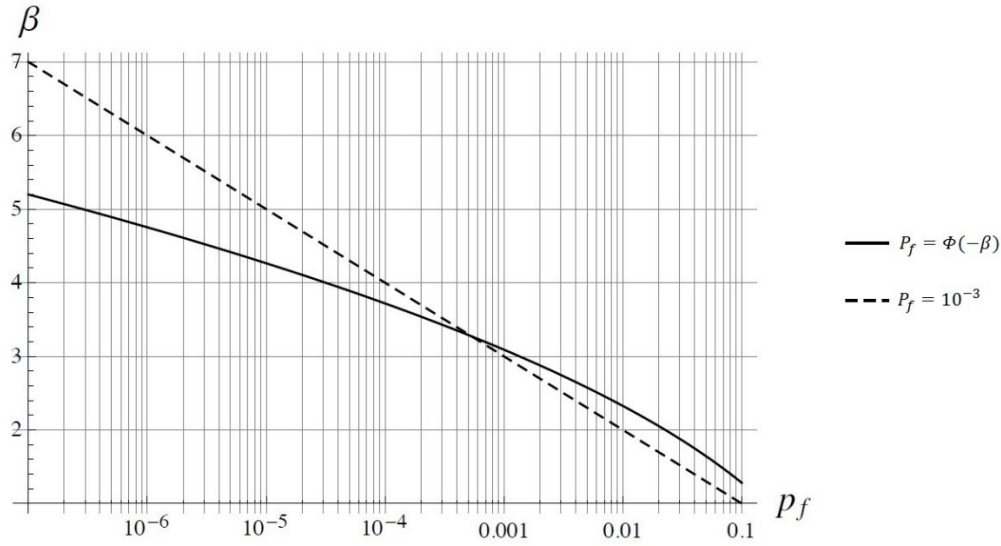


Figure 2-21 Relationship between β and P_f [83]

Reliability can be interpreted as the probability of safety of a structure over a period of time. In other words, reliability shows the possibility under which a component or a system will perform appropriately without failure. Structural reliability is commonly determined as the complement of the probability of failure of the structure:

$$Reliability = 1 - P_f \quad (2-18)$$

In fact, the probability of failure illustrates the chance of the occurrence of a predefined event. In the case that there are trials and observations of the event, the probability of the occurrence of the event can be obtained. Consequently, the reliability would be defined for the structure. In designing engineering applications, the structural reliability can help designers to make precise decisions and design more reliable products.

2-8-2 Second-moment concepts

When there is a function comprising two or more random variables, the mean and variance of the function can be derived using expectation and variance operators. These operators work differently according to the function type. For instance, mean value, μ_Y , and variance, σ_Y^2 , of Y which is a linear function including n random variables as shown in equation (2-19), are calculated using equations (2-20) and (2-21), respectively [80].

$$Y = h(\underline{X}) = a + \underline{b}^T \underline{X} = a + b_1X_1 + b_2X_2 + \cdots + b_nX_n \quad (2-19)$$

$$\mu_Y = a + \underline{b}^T \underline{M}_X \quad (2-20)$$

$$\sigma_Y^2 = \underline{b}^T \underline{\Sigma}_{XX} \underline{b} \quad (2-21)$$

where $\underline{X}$ is a random variables vector, $\underline{b}$ is a factors vector, $\underline{M}_X$ is the means of random variables vector, and $\underline{\Sigma}_{XX}$ is the covariance matrix of random variables.

On the other hand, generally limit state functions such as the limit state function of delamination growth problem in the present study, discussed in chapter 4, as Y indicated in equation (2-22) are non-linear functions in which random variables have non-linear correlations. In this case, a linear approximation of the Taylor series about the mean point is needed to linearize the function. Afterward, mean μ_Y and variance σ_Y^2 of the function are obtained and expressed as follows [80]:

$$Y = h(\underline{X}) \approx h(\underline{M}_X) + \nabla h^T(\underline{M}_X)(\underline{X} - \underline{M}_X) \quad (2-22)$$

$$\mu_Y = h(\underline{M}_X) \quad (2-23)$$

$$Var [h(\underline{X})] = \sigma_Y^2 = \nabla h^T(\underline{M}_X) \underline{\Sigma}_{XX} \nabla h(\underline{M}_X) \quad (2-24)$$

where ∇ is a gradient operator. These mean and variance are not precise because these are obtained based on linear approximation. In fact, the linearized function is replaced with the non-linear function. The second-moment information of the linearized function may not be calculated precisely, since the first-order approximation is not always accurate for non-linear functions.

2-8-3 Probability transformation

Generally, reliability analyses are carried out in standard normal space due to its specific characteristics. The most remarkable is radial and rotational symmetry in this dimensionless space. Probability density function exponentially falls with distancing from the origin of standard normal space. Due to these characteristics, reliability analysis should be carried out in this space [82]. Likewise, in the present reliability study, all random variables need to be transformed to standard normal variables. The standard normal variables are all normally distributed with zero mean and unit standard deviation in standard normal space. For the purpose of standardizing random variables, standardization of the second-moment parameters is required. Equation (2-25) is used to transform the original variable to a standard variable [81]:

$$Y_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \quad (2-25)$$

It is evident that the mean and standard deviation of each standard normal variable are: $\mu_{Y_i} = 0$ and $\sigma_{Y_i} = 1$, respectively. Random variables x with known second-moment information, correlation, and distribution are transformed to independent standard normal variables with zero mean and unit standard deviation. In the present study, for different types of reliability methods used in chapter 4, Nataf transformation is used regularly.

2-8-4 FOSM approach and importance vector

In the first-order second-moment (FOSM) method, linearizing a function is carried out by a first-order approximation of the function about the mean of the random variables. Hence, it is also known as mean value first-order second-moment method, MVFOSM. This method is usually used when only the second-moment information about random variables, including means, standard deviations, and correlation coefficients, is provided. Due to the lack of information about the distribution type of random variables, this solution is only able to compute the mean and standard deviation of the function. When the function is a linear function of random variables, the exact values of mean and standard deviation as well as an accurate reliability index can be computed. The following equation shows the relationship between the reliability index, which is analogous

to the reciprocal of the coefficient of variation, the mean value and the standard deviation value of the linearized limit state function.

$$\beta = \frac{\mu_g}{\sigma_g} \quad (2-26)$$

Nonetheless, when the limit state function is nonlinear, according to the Taylor's first-order approximation of limit state function about mean values, the mean and standard deviation of the function would be computed based on the second-moment concept as given below [80]:

$$\mu_g \approx g(\underline{M_X}) \quad (2-27)$$

$$\sigma_g \approx \sqrt{\nabla g^T(\underline{M_X}) \underline{\Sigma_{xx}} \nabla g(\underline{M_X})} \quad (2-28)$$

In this reliability method, since the available data is all about second-moment information of random variables, only the outputs about the second-moment of the limit state function can be derived. In other words, due to the lack of information about probability density, computing failure probability is not the case in this method. Moreover, in the present study, reliability analysis for the mode-I delamination test comprises a nonlinear limit state function. Therefore, FOSM is only able to determine the second-moment information but the accuracy of this approximation of the limit state function is questionable. In this respect, more trustable results can be obtained by using other algorithms that are introduced in the upcoming sub-sections.

After doing reliability analyses, important facts about the influence of different random variables on the failure probability are still uncovered. In this respect, random variables can be ranked in the order of importance in reliability problems. The linearized limit state function at the mean point is taken into account as follows [83]:

$$g(\underline{x}) \approx g(\underline{M_X}) + \nabla g^T(\underline{M_X})(\underline{x} - \underline{M_X}) \quad (2-29)$$

According to the analysis of functions and second-moment concept, the expression below can be used to calculate the variance of the linearized limit state function [83].

$$\begin{aligned}
 Var[g(x)] &= \sigma_g^2 = \nabla g^T(\underline{M}_X) \underline{\Sigma}_{XX} \nabla g(\underline{M}_X) \\
 &= (\nabla g_1 \sigma_1)^2 + (\nabla g_2 \sigma_2)^2 + \dots + (\nabla g_n \sigma_n)^2 + \sum_{i=1}^n \sum_{\substack{j=1 \\ j \neq i}}^n \nabla g_i \nabla g_j \sigma_i \sigma_j \rho_{ij}
 \end{aligned} \tag{2-30}$$

where n is the number of random variables and ρ_{ij} is the correlation coefficient between two random variables. Regardless of the last term of the equation above, which shows the incorporated contribution of each pair of random variables, every other term individually indicates the direct contribution of the corresponding random variable to the total variance of the linearized limit state function. Therefore, every term in the equation above except the last term can be interpreted as the importance factor as, $\nabla g^T \underline{D}_X$. It is common to use a normed vector with a negative sign as the FOSM importance vector as provided below [83]:

$$\omega = - \frac{\nabla g^T \underline{D}_X}{\|\nabla g^T \underline{D}_X\|} \tag{2-31}$$

The larger the absolute value of ω_i , the more important the random variable, x_i is. In this way, random variables can be ranked after performing the FOSM method. The sign of the FOSM importance components matters. If it is negative it implies that the corresponding random variable acts as a capacity variable. Whereas, a positive sign points that the corresponding random variable acts as a load variable. After performing the FOSM, the value of ω_i for every random variable, x_i can be observed individually. Later in the results and discussion section of chapter 4, FOSM importance outcomes are reviewed.

2-8-5 FORM, sensitivity parameters and importance vectors

This method is based on making a linear approximation of the limit state function about a design point. This method assumes that the limit state function has to be continuous and differentiable in

the region close to the design point. This method resolves the invariance problem in the FOSM approach by changing the approximation point to a point on the limit state function. That is, through using this algorithm, a linear approximation is made at the design point rather than at the mean point. It is discussed in detail in the following. Considering a component reliability problem with a probability of density function $f(x)$ and failure zone of $g(x) \leq 0$. Transformation of original random variables into standard normal variables according to what is already explained above yields the following equation:

$$P_f = \int_{g(x) \leq 0} f(x) dx = \int_{G(y) \leq 0} \varphi(y) dy \quad (2-32)$$

where φ is the probability density function and $G(y)$ is the limit state function in standard normal space.

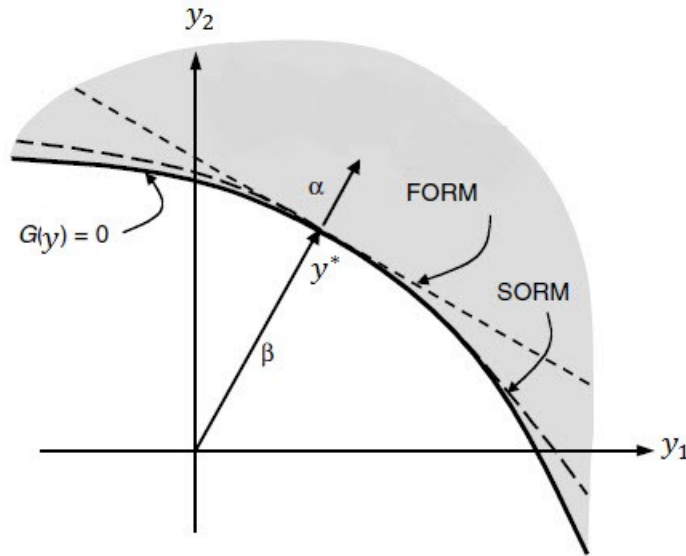


Figure 2-22 FORM and SORM approximations [82]

Since linearizing about mean has its shortcomings such as invariance problem, this method offers a different approach to avoid this kind of problem. As shown in figure 2-22, linearizing is done on the surface of the limit state function where $G(y) = 0$. The checkpoint has to be located on $G(y) = 0$ and at the closest point to the origin. A hyperplane is applied to fit the limit state surface at the design point. This is indicated as a dashed line in the 2D coordinate system of two standard normal variables in figure 2-22. Due to the normal space characteristics, it is the most probable point. By

moving away from this point, probability density drops exponentially. Therefore, approximation about the Most Probable Point (MPP) leads to the highest level of accuracy. In the FORM algorithm, the reliability problem is simplified to finding the MPP or design point, shown as y^* in figure 2-22. This point is determined by the constrained optimization problem as follows [82]:

$$y^* = \operatorname{argmin} \{ \|y\| \mid G(y) = 0 \} \quad (2-33)$$

The equation (2-33) is an optimization for y^* by minimizing the value of the norm of vector, $\|y\|$, under the condition that $G(y) = 0$. It yields the design point through which the probability of failure can be calculated. Firstly, it is required to linearize the limit state function. Through the use of the Taylor series, linear approximation about the design point results in the below equation [82]:

$$G(y) \cong G(y^*) + \nabla G^T(y^*)(y - y^*) \quad (2-34)$$

By knowing $G(y^*) = 0$ and the definition of normalized negative gradient vector at the design point provided in equation (2-35), the linearized limit state function can be simplified [82].

$$\alpha = - \frac{\nabla G(y^*)}{\|\nabla G(y^*)\|} \quad (2-35)$$

Here, α is a unit vector normal to the limit state surface at the design point with the opposite direction of the gradient, i.e., toward the failure zone. Because the gradient vector is normal to the limit state surface as well as the position vector of the design point because it has the minimum distance from the origin, α and y^* are parallel vectors, that is, the magnitude of the vector y^* is equal to reliability index, β : $\beta = \alpha^T y^*$.

Now linearized limit state function can be rewritten as the following equation:

$$G(y) \cong \|\nabla G(y^*)\| (\beta - \alpha^T y) \quad (2-36)$$

It may be seen from figure 2-20 that the failure domain is where $G(y) \leq 0$. It will be interpreted when the inequality $\beta - \alpha y \leq 0$ is true, a component or a system would fail thanks to a low-reliability index. Therefore, these two safe and failure domains are divided by linearized limit state function determined by the distance β from the origin of the standard normal space. Generally, in reliability problems, limit state function in standard normal space is not available by problem information. Thus, it is necessary to obtain equivalent random variables and limit state function in a standard normal state at an early stage. Afterward, it makes the way to find the design point and solve the problem. The first-order approximation indicates that the probability of failure is determined by the reliability index, β , which is the distance of the design point on the limit state surface from the origin of the standard normal space. The equation below illustrates the failure probability where P_{f1} denotes the failure probability obtained using first-order approximation:

$$P_f \cong P_{f1} = \Phi(-\beta) \quad (2-37)$$

In this reliability algorithm, a convergence problem may occur due to the optimization process for finding the design point introduced earlier. Moreover, this method may fail to converge when the limit state function is strongly nonlinear or nonflat. SORM, described in the next section, can solve this problem using the second-order approximation. Searching for the design point is carried out using a defined algorithm. It is a step-by-step procedure that commences with an initial point, for instance, mean, and checks whether the current point is the MPP or not. The sequence of points is computed by the recursive equation which is provided below [83]:

$$y_{m+1} = y_m + s_m d_m \quad (2-38)$$

where y_{m+1} is the coordination of secondary point, y_m is the coordination of primary point, s_m is a step size, and d_m is a search direction vector. Concerning the earlier mentioned optimization equation (2-33) as well as the definition of the unit vector normal, α in equation (2-35), step direction can be computed as it is expressed in the equation below [83].

$$d_m = \left[\frac{G(y_m)}{\|\nabla G(y_m)\|} + \alpha y_m \right] \alpha^T - y_m \quad (2-39)$$

It is worthwhile to mention that by choosing a unit step size, this algorithm does not work if

$1 \leq |\beta\kappa_m|$, where κ_m is a principal curvature of the limit state surface at the design point, step size should be appropriately selected to avoid convergence problems. In this respect, a merit function can be used. Zhang and Der Kiureghian have suggested a merit function which is provided as [84].

$$m(y) = \frac{1}{2} \|\underline{y}\|^2 + c|G(y)| \quad (2-40)$$

where c is a penalty parameter that is selected individually at every step to meet the below-mentioned condition:

$$c \geq \frac{\|\underline{y}\|}{\|\nabla G(y)\|} \quad (2-41)$$

In using the merit function, the penalty parameter has a crucial role. It can be determined as [84].

$$c = \gamma \frac{\|\underline{y}_m\|}{\|\nabla G(y_m)\|} + \eta \quad (2-42)$$

where η , adder, is a constant and γ is a coefficient which is known as the multiplier. These are user-defined determinant variables in solving the problem. Assigning different numbers to these two parameters has an effect on the convergence probability or the pace of convergence of solving the reliability problem utilizing the FORM algorithm in Rtx[®] software, which is discussed more in chapter 4. As for choosing the step size, it is suggested to select as “b” a number between zero and one by choosing arbitrary numbers for “k” in the following:

$$s_m = b^k \quad (2-43)$$

where $0 < b < 1$ and $k = 0,1,2,3, \dots$. Step size needs to be varied to make sure that the design point is not being passed and ensure that it is being approached through the searching process. Based on the Armijo rule [84], for checking the appropriateness of the step size, it is recommended to compare merit function for the primary and secondary points according to the following equation:

$$m(y_{m+1}) - m(y_m) \leq as_m(\nabla m^T(y_m)d_m) \quad (2-44)$$

where “a” is known as the factor and, according to literature recommendation, usually $a = 0.5$ to have a proper criterion according to which the above-mentioned inequality is satisfied [84]. Factor number also can affect the possibility of convergence in finding MPP. This parameter number, together with adder and multiplier, are user-defined parameters.

According to the FORM algorithm, when the limit state surface is not a plane, and it may be a parabolic surface, the second point is computed as the following [82]:

$$y_{m+1} = \left[\frac{G(y_m)}{\|\nabla G(y_m)\|} + \alpha^T y_m \right] \alpha \quad (2-45)$$

Generally, in addition to the conditions mentioned above, there are two essential convergence criteria for FORM problems. Firstly, through searching points, the selected point must be located on the limit state function, $G(y) = 0$, or very close to it. This condition can be interpreted as the following inequality:

$$\left| \frac{G(y_m)}{G_0} \right| \leq \varepsilon_1 \sim 0.001 \quad (2-46)$$

where G_0 denotes the value of limit state function at the initial point of searching, which can be selected as the mean value. By setting $\varepsilon_1 \sim 0.001$, the requirement for being on limit state function is met since a smaller distance than ε_1 is negligible and can be considered zero.

Secondly, at each step, it is vital to determine whether the current point is the closest to the origin or not through the inequity (2-47) [83].

$$\left\| \frac{y_m}{\|y_m\|} - (\alpha_m \frac{y_m}{\|y_m\|}) \alpha_m \right\| \leq \varepsilon_2 \sim 0.001 \quad (2-47)$$

These two fundamental conditions in the FORM algorithm need to be checked at every step.

It is evident that to find the design point in the aforementioned optimization formula, it is necessary to take some steps and investigate whether the point is being obtained or not through some computation. For this aim, a good pattern is required to keep in the accurate direction throughout step searching. The iHLRF algorithm introduced by Zhang and Der Kiureghian in 1994 [84] is an improved method of HLRF algorithm developed by Hasofer and Lind in 1974 [85], and Rackwitz and Fiessler in 1978 [86]. This algorithm directs the optimization problem solver to find the most probable point to take steps properly in the standard normal space. Improved HLRF is an algorithm through which merit function and step direction checker are determined to solve the optimization problem appropriately. Once the convergence criteria are fulfilled, the last step of point searching offers the design point in standard normal space, $\underline{y}^*$. The magnitude of vector $\underline{y}^*$ yields the reliability index, β .

Finding the design point as the main objective of the FORM solution needs to search for the different points at each step and calculate the limit state function and its gradient in standard normal space at that point. Nataf transformation is required in the whole process to transform the current point in each step to the corresponding original point. Considering $y = h(x)$, using the following equation assists to transform points in standard normal space:

$$X_i = h^{-1}(Y_i) \quad (2-48)$$

Afterward, by means of the Jacobian inverse matrix, $J_{\underline{y},\underline{x}}^{-1}$, the gradient of the limit state function is convertible at each step from standard normal space to original one or vice versa to evaluate the function and its gradient at every point as expressly provided in the equation below [82].

$$\nabla G(Y_i) = \nabla g(X_i) J_{\underline{y},\underline{x}}^{-1}(X_i) \quad (2-49)$$

where $J_{\underline{y},\underline{x}}$ is the Jacobian matrix or derivative transformation computed with regard to $f_i(x_i)$, $\varphi(y_i)$ that denote PDF of $\underline{x}$ and $\underline{y}$, respectively. The following equation indicates how the Jacobian matrix is computed [82]:

$$\underline{J}_{\underline{y}, \underline{x}} = \underline{L}^{-1} \begin{bmatrix} \frac{f_1(x_1)}{\varphi(y_1)} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \frac{f_n(x_n)}{\varphi(y_n)} \end{bmatrix} \quad (2-50)$$

where $\underline{L}^{-1}$ is the inverse matrix of correlation obtained from cholesky decomposition, a lower-triangular matrix.

It has been highly worthwhile to distinguish the most important random variable in a reliability study. Ranking random variables can describe the effect of each variable on failure probability. In this way, the most important random variable or variables come more into attention in reliability analysis. Regarding FORM analysis, a set of sensitivity measures can be obtained that indicates the sensitivity of failure probability or reliability index to probability distribution or limit state function parameters. Regarding linearized limit state function about design point as following [82]:

$$G(\underline{y}) = \|\nabla G(y^*)\|(\beta - \underline{\alpha}^T \underline{y}) \quad (2-51)$$

Having in mind that $\underline{y}$ is a vector of standard normal variables with zero mean and unit standard deviation, that is $\underline{\Sigma}_{yy} = I$, then mean and variance of linearized $G(\underline{y})$ can be computed as follows:

$$\mu_G = \|\nabla G\|\beta \quad (2-52)$$

$$\sigma_G^2 = \|\nabla G\|^2 \|\alpha\|^2 = \|\nabla G\|^2 (\alpha_1^2 + \alpha_2^2 + \dots + \alpha_n^2) \quad (2-53)$$

Since α is a unit vector, the last equation can be simplified to $\sigma_G^2 = \|\nabla G\|^2$. However, the equation above illustrates that the contribution of the standard normal random variable y_i to the variance of the linearized limit state function is proportional to α_i^2 . In other words, when the original random variables are independent, the magnitude of α_i^2 shows the importance of the corresponding variable. If α_i is greater than zero, y_i would be a load type of variable. Whereas the negative proves that y_i is a capacity type of variable. Alpha importance vector indicates which input variable in standard space has the most significant effect on failure probability while original random

variables are considered statistically independent. Therefore, when original random variables are statistically dependent, the Alpha vector may not be that accurate because the primary assumption becomes invalid.

In the case that there is a statistical correlation among random variables, by defining a new vector, $\hat{\underline{x}}$ as equivalent normal of $\underline{x}$, which is a linear function of $\underline{y}$, and have a normal joint distribution, new importance vector Gamma represents the importance of random variables. It is computed through the Alpha vector, Jacobian matrix of design point in both the normal and original spaces, $\underline{J}_{y^*,x^*}$ as well as standard deviation matrix of equivalent normal of original random variables, $\hat{\underline{D}}$ as follows [82]:

$$\gamma = \frac{\underline{\alpha} \underline{J}_{y^*,x^*} \hat{\underline{D}}}{\|\underline{\alpha} \underline{J}_{y^*,x^*} \hat{\underline{D}}\|} \quad (2-54)$$

The covariance matrix of equivalent random variables in original space generally depends on the design point and is computed as follows [82]:

$$\underline{\Sigma}_{\hat{\underline{x}}\hat{\underline{x}}} = \underline{J}_{y^*,x^*}^{-1} \underline{J}_{y^*,x^*}^{-T} \quad (2-55)$$

Gamma importance vector can rank dependent random variable due to taking variables correlation into account. Thus, once a reliability problem contains statistically dependent variables, this measure assists in finding out the importance of every random variable.

With assuming $\underline{\theta}$ as a vector of distribution parameters of the joint PDF of $\underline{x}$, each θ_i could be any parameters including statistical moments of the marginal distribution of the corresponding X_i , while $\underline{x}$ includes dependent random variables. As a result of FORM analysis, some important by-products can be obtained as Alpha and Gamma importance vectors which have already been discussed. In addition, some sensitivity measures can be computed, among which two of them are brought up here. The gradient of failure probability and reliability index computed in the FORM with respect to the distribution parameters of random variables as following expression proposed by Hohenbichler and Rackwitz [87]:

$$\nabla_{\theta}\beta = J_{y^*\theta} \underline{\alpha} \quad (2-56)$$

Delta sensitivity vector has been defined to observe how the mean values of random variables affect the failure probability. For this aim, the gradient of reliability index with respect to mean among distribution parameters vector is multiplied by random variables standard deviation matrix. The result of this operation would be a delta sensitivity vector, as shown in equation (2-57). Each element of the delta sensitivity vector can be expressed as equation (2-58) [82], [87].

$$\underline{\delta} = \nabla_{\mu}\beta \underline{D}_x \quad (2-57)$$

$$\delta_i = \frac{\partial \beta}{\partial \mu_i} \sigma_i \quad (2-58)$$

where $\underline{D}_x$ is the standard deviation matrix of random variables and σ_i is the standard deviation of the corresponding random variable X_i .

Eta sensitivity vector is another important sensitivity vector that indicates how the variations of every random variable influence the failure probability. This time, a computed gradient of reliability index with respect to standard deviation multiplied by random variables standard deviation matrix would result in eta sensitivity vector (equation 2-59). Accordingly, equation (2-60) indicates how each vector element can be derived.

$$\underline{\eta} = \nabla_{\sigma}\beta \underline{D}_x \quad (2-59)$$

$$\eta_i = \frac{\partial \beta}{\partial \sigma_i} \sigma_i \quad (2-60)$$

The larger the magnitude of eta sensitivity array, the more important the random variable is in terms of standard deviation variation. Varying the standard deviation of the random variable, reliability index and failure probability experience more changes. In Rtx[®] software, by-products

of FORM analysis, which shows the sensitivity vector, can be observed after the analysis, which is discussed later in chapter 4.

2-8-6 SORM

This method has similar procedures as the FORM method but with a different approach. Two distinct limit state functions are considered, including a nonlinear limit state function, as shown in figure 2-22, and a linear limit state function. The first-order approximation for both limit states will be the same linearized dashed line in figure 2-22 by applying FORM. Therefore, the calculated failure probability for both limit states would be identical because the estimated failure domain for both of them is alike based on FORM linearization. However, it is undoubted that the failure domain of the nonlinear limit state function should be larger than that of a linear one. This means that these two limit states should have different failure probabilities and accordingly distinguish reliability. Therefore, as mentioned earlier, it can be concluded that first-order approximation is not as accurate as SORM, particularly for nonlinear limit states with a high degree of nonlinearity. The second-order reliability method, SORM, is able to tackle this matter by fitting a hyper paraboloid to the limit state surface at the design point. It brings in a second-order approximation of the Taylor series of the limit state function. As shown in figure 2-22, the second-order linearized function is closer to the primary nonlinear limit state function. In the early stage of SORM, the design point has to be found by taking similar steps as FORM.

2-8-6-1 Curvature fitting SORM

In the following, the curvature fitting SORM approach in which calculating the second derivative matrix is required is presented. According to the Taylor series, second-order approximation of the limit state function expression would be as follows [83]:

$$\begin{aligned}
G(\underline{y}) &\cong G(\underline{y}^*) + \nabla G^T(\underline{y}^*) (\underline{y} - \underline{y}^*) + \frac{1}{2} (\underline{y} - \underline{y}^*)^T \underline{H} (\underline{y} - \underline{y}^*) \\
&= \|\nabla G(\underline{y}^*)\| (\beta - \underline{\alpha}^T \underline{y}) + \frac{1}{2} (\underline{y} - \underline{y}^*)^T \underline{H} (\underline{y} - \underline{y}^*)
\end{aligned} \tag{2-61}$$

where obviously because the design point is on the limit state surface, $G(\underline{y}^*)=0$, and

$$\underline{H} = \begin{bmatrix} \frac{\partial^2 G(\underline{y}^*)}{\partial y_1^2} & \dots & \frac{\partial^2 G(\underline{y}^*)}{\partial y_1 \partial y_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 G(\underline{y}^*)}{\partial y_n \partial y_1} & \dots & \frac{\partial^2 G(\underline{y}^*)}{\partial y_n^2} \end{bmatrix}$$

is named as the Hessian matrix, which is the second-derivative matrix

at the design point [83]. The first term of the equation above is derived from the first-order derivative, and the second term is concluded from a second-order partial derivative of the limit state function. Now, it is required to rotate the coordinate system so that one of the random variables is aligned with the Alpha vector in standard normal space to have the hyper paraboloid at the design point. Figure 4-4 illustrates this axis rotation for an example consisting of two random variables.

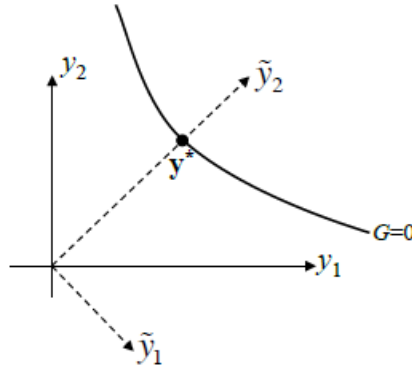


Figure 2-23 Rotation of the standard normal space [83]

The rotation is done by means of a matrix, $\underline{P}$, which is an n-dimensional square matrix with $\underline{\alpha}$ as its last row, that n is the number of random variables. Due to the characteristics of $\underline{P}$ ($\underline{P}^T \underline{P} = \underline{I}$), the following expressions indicate how the axis rotation is carried out [83]:

$$\underline{\tilde{y}} = \underline{P} \underline{y} \tag{2-62}$$

$$\underline{y} = \underline{P}^T \underline{\tilde{y}} \quad (2-63)$$

where $\underline{\tilde{y}}$ denotes the rotated coordinate system. $\underline{P}$ can be constructed by using algorithms such as Gram-Schmidt. By substituting axis rotation formula in the last obtained equation of linearized limit state, the second-order Taylor's approximation about the design point in the rotated coordinate system would be as follows [83]:

$$\begin{aligned} G(\underline{\tilde{y}}) &= \|\nabla G(\underline{y}^*)\| (\beta - \underline{\alpha}^T (\underline{P}^T \underline{\tilde{y}})) + \frac{1}{2} (\underline{P}^T \underline{\tilde{y}} - \underline{P}^T \underline{\tilde{y}}^*)^T \underline{H} (\underline{P}^T \underline{\tilde{y}} - \underline{P}^T \underline{\tilde{y}}^*) \\ &= \|\nabla G(\underline{y}^*)\| (\beta - \tilde{y}_n) + \frac{1}{2} (\underline{\tilde{y}} - \underline{\tilde{y}}^*)^T \underline{P} \underline{H} \underline{P}^T (\underline{\tilde{y}} - \underline{\tilde{y}}^*) \end{aligned} \quad (2-64)$$

where $\tilde{y}_n = \underline{\alpha}^T (\underline{P}^T \underline{\tilde{y}})$. The limit state function equation is simplified by dividing two sides of the equation by the norm of the gradient at the design point [83].

$$\tilde{G}(\underline{\tilde{y}}) \approx \beta - \tilde{y}_n + \frac{1}{2} (\underline{\tilde{y}} - \underline{\tilde{y}}^*)^T \underline{A} (\underline{\tilde{y}} - \underline{\tilde{y}}^*) \quad (2-65)$$

where $\tilde{G}(\underline{\tilde{y}}) = \frac{G(\underline{\tilde{y}})}{\|\nabla G(\underline{y}^*)\|}$, $\underline{A} = \frac{\underline{P} \underline{H} \underline{P}^T}{\|\nabla G(\underline{y}^*)\|}$. Knowing that all elements of the last row and column of $\underline{A}$ are zero because of rotation assists in simplifying the equation above. Also, it is assumed that the design point is on the last random variable axis in the rotating system and its distance from the origin point is reliability index; so, it is a vector matrix with n rows and one column, and its coordination is as below:

$$\underline{\tilde{y}}^* = \begin{Bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \beta \end{Bmatrix} \quad (2-66)$$

If it is considered that subscript “cut” for a vector illustrates that the last vector element is eliminated and likewise for a matrix tells that the last row and column are eliminated, the second-order approximation considering matrix multiplication can be written as follows [82]:

$$\tilde{G}(\tilde{\underline{y}}) \approx \beta - \tilde{y}_n + \frac{1}{2} \left[\tilde{\underline{y}}_{cut}^T \underline{A}_{cut} \tilde{\underline{y}}_{cut} + 2(\tilde{y}_n - \beta) \underline{A}_{cut,n}^T \tilde{\underline{y}}_{cut} + A_{nn}(\tilde{y}_n - \beta)^2 \right] \quad (2-67)$$

where $\underline{A}_{cut,n}$ is a column vector matrix with (n-1) rows and one column, $\underline{A}_{cut}$ is an (n-1)×(n-1) square matrix, and A_{nn} is a number. In this equation, it is worth noting that when $\tilde{\underline{y}}$ approaches $\tilde{\underline{y}}^*$, all terms of $\tilde{G}(\tilde{\underline{y}})$ approach zero which certifies that the design point in the rotated coordinate system is on the limit state surface. Two terms in the equation approach zero by order of two; consequently, those can be neglected and removed from the equation. Then the more simplified equation is provided below:

$$\tilde{G}(\tilde{\underline{y}}) \approx \beta - \tilde{y}_n + \frac{1}{2} \cdot \tilde{\underline{y}}_{cut}^T \underline{A}_{cut} \tilde{\underline{y}}_{cut} \quad (2-68)$$

The equation above indicates a second-order approximated limit state function rotated in the coordinate system. It can be rewritten based on eigenvalues of $\underline{A}_{cut}$ as follows [83]:

$$\tilde{G}(\tilde{\underline{y}}) \approx \beta - \tilde{y}_n + \frac{1}{2} \sum_{i=1}^{n-1} \kappa_i y_i^2 \quad (2-69)$$

where κ_i is the eigenvalue of $\underline{A}_{cut}$, which denotes the principal curvatures of the hyper paraboloid. Now bringing up the concept of using SORM, the region outside the hyper paraboloid shows failure domain. Therefore, putting $\tilde{G}(\tilde{\underline{y}}) = 0$ yields the limit state function, which is the boundary between the safe and failure region. It can be interpreted that the last standard normal variable in the rotated coordinate is a function of other variables considering a coefficient of their principal curvature of the hyper paraboloid, and the relationship between this random variable in the rotated system and other variables can be expressed as below:

$$\tilde{y}_n = \beta + \frac{1}{2} \sum_{i=1}^{n-1} \kappa_i y_i^2 \quad (2-70)$$

This paraboloid is the limit state surface on which the design point is located and is at a distance β from the origin. It is noteworthy that if $\kappa_i > 0$, the limit state curves away from the origin when the reliability index is positive. In contrast, $\kappa_i < 0$ indicates that the limit state surface curves towards the origin. Every principal curvature has to have a minimum value in order to have the same and accurate design point on the surface. So, the inequality of $\kappa_i > -\frac{1}{\beta}$ has to be checked for every principal curvature. Under the above inequality condition, Tvedt [88] has offered an exact expression for failure probability. However, concerning the obtained hyper paraboloid formula and computed failure probability in FORM, the failure probability in SORM, P_{f2} based on asymptotic approximations derived by Breitung [89], is expressed as follows:

$$P_{f2} \approx \Phi(-\beta) \prod_{i=1}^{n-1} \frac{1}{\sqrt{1 + \psi(\beta)\kappa_i}} \quad (2-71)$$

where $\psi(\beta) = \frac{\varphi(\beta)}{\Phi(-\beta)}$ and it can be considered equal to β . In fact, in this formula, the FORM approximation is corrected by being multiplied by an operator containing correction factors that represent the principal curvatures of the limit state surface at the design point, κ_i .

As mentioned earlier, this approach of SORM is called curvature fitting SORM since the approximated hyper paraboloid determines the limit state function by fitting a hyper paraboloid to the principal curvatures at the design point. In this approach, evaluating the Hessian matrix is the most challenging problem. It is not the case when the limit state function is expressed as a function of standard normal random variables. Nonetheless, in real-life problems, mostly limit state function is expressed according to experimental data or finite element analysis, which is a set of numbers and numerical responses obtained from physical equations in original space. Thus, for solving this kind of reliability problem with SORM, calculation of Hessian matrix requires chain rule of differentiation, more details of which are available in the literature [82].

Since transforming the second-derivative matrix from original space to standard normal space is too complicated and time-consuming, in practice, curvature fitting SORM is limited to explicit limit state functions.

2-8-6-2 Point fitting SORM

Addressing difficulties of computing transformation of Hessian matrix or generally high-order derivatives as well as limit state function noise while computing due to truncation errors, point fitting SORM was developed by Der Kiureghian et al. [90]. In this method, two points roughly far away from the design point at either side of it have been chosen on the limit state surface. Now two semi parabolas are fitted to the limit state surface at these two points, as indicated in figure 2-24. In the following, $\tilde{y}_i$, $i = 1, 2 \dots n-1$, is an axis in the rotated coordinate system. Searching for the points is carried out through a dashed line as shown in figure 2-24. The path comprising $\tilde{y}_i = \pm b$ and the semicircle of radius b that is centered at the design point [82]. In this method, “ b ” is chosen with regard to the value of β to ensure that the fitting points have a proper distance from the design point.

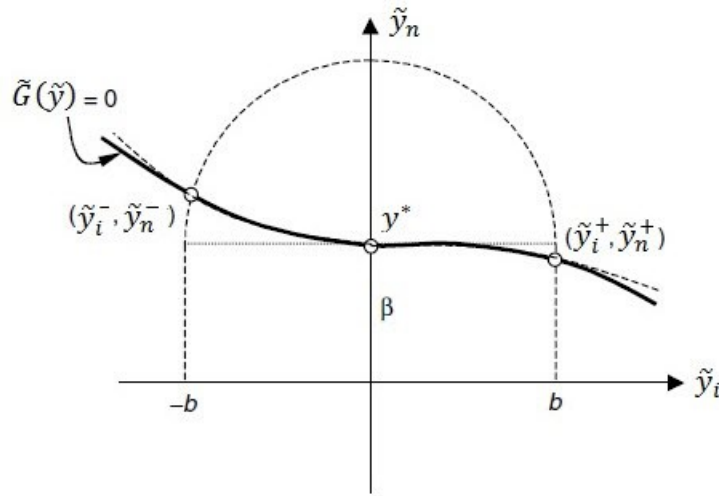


Figure 2-24 Definition of fitting points in point fitting SORM method [82]

The curvature of each semi parabola can be expressed by the following equation [82]:

$$a_i^{sgn(\tilde{y}_i)} = \frac{2(\tilde{y}_n^{sgn(\tilde{y}_i)} - \beta)}{(\tilde{y}_i^{sgn(\tilde{y}_i)})^2} \quad (2-72)$$

where $sgn(\tilde{y}_i)$ means the sign of the coordinate on the $\tilde{y}_i$ axis. By using this method, the limit state function is approximated as follows [82]:

$$\tilde{G}(\tilde{\mathbf{y}}) \approx \beta - \tilde{y}_n + \frac{1}{2} \sum_{i=1}^{n-1} a_i^{sgn(\tilde{y}_i)} (\tilde{y}_i)^2 \quad (2-73)$$

In this expression, approximated limit state function in the rotated coordinate system is determined by the reliability index and the curvature coefficients. The derivation of the piecewise paraboloid is zero at the design point; the limit state surface and the paraboloid are tangent to each other at this point. This approach can be used for all component reliability problems; however, there are limitations to using this method for system reliability. It can facilitate the second-order reliability solution and prevent calculation difficulties of the chain rule of differentiation. Point fitting SORM is available in Rtx[®] and is used in the present study; the results are discussed in chapter 4.

In summary, the SORM solution comprises certain steps briefly described in this paragraph. First, the defined solver analyzes the limit state function and transforms random variables with regard to their correlation. Then, it initiates to search for the design point by taking proper steps according to Armijo and iHLRF algorithms [84]. After the design point is obtained, the point fitting solver commences to find two points on the limit state surface at each side of the design point. These two points are on an axis that is orthogonal to the axis of the first principal curvature. Additionally, the plus and minus curvature vectors are computed based on equation (2-73). Finally, by calculating the limit state function in the rotated coordinate system and reliability index, the probability of failure in SORM can be obtained according to Breitung's formula shown in equation (2-71).

2-8-7 Monte Carlo sampling

There are sampling methods in addition to first-order and second-order approaches to solve component or system reliability problems. These simple simulation techniques can be used for both implicit and explicit functions. In the previously described methods for solving reliability problems, all random variables are transformed to standard normal space. First-order or second-order approximation of Taylor simplifies the probability integration by approximation of limit state

function. In sampling methods, numerical approximations are used to solve reliability problems. One of the most common interpretations of probability is that as the number of samples increases, sample averages tend to stabilize. So, by having a sufficiently large number of samples, the failure probability integral, given below in equation (2-74), can be interpreted as the mean value of all simulation samples. The most commonly used sampling method is Monte Carlo Sampling (MCS). Various available software such as Rtx[®], used in the present study, make it easy to program this algorithm. This method is used to evaluate the accuracy of the FORM and SORM methods in computing the failure probability.

Consider a random variable vector, $\underline{x}$, which is used to model uncertainties. Generating random numbers for these variables based on their joint probability density function, $f(\underline{x})$, results in a sample. Many samples need to be generated using random numbers. In every sample, by substituting variables in the limit state function, $g(\underline{x})$, it can be evaluated if the failure happens in the sample, $g(\underline{x}) \leq 0$, or the sample remains safe. The duplication of the aforementioned steps provides a significant number of samples as well as information about their failure or being safe. The failure region, Ω_f , is determined in a random variable coordinate system where the limit state function is smaller than zero. It can be expressed as the following expression: $\Omega_f = \{g(\underline{x}) \leq 0\}$. Main failure probability is defined as the integral expressed in the equation below.

$$P_f = P(g \leq 0) = P(\Omega_f) = \int \dots \int_{\Omega_f} f(\underline{x}) d\underline{x} \quad (2-74)$$

In the MCS method, this equation can be expressed by introducing a new function known as the indicator function, which is expressed in equation (2-75). The function allows to distinguish failed and safe samples and eliminate the probability density of the safe samples from the above integral.

$$I(\underline{x}) = \begin{cases} 1 & \text{if: } g(\underline{x}) \leq 0 \\ 0 & \text{if: } g(\underline{x}) > 0 \end{cases} \quad (2-75)$$

With regard to the indicator function illustrated in equation (2-75), equation (2-74) can be rewritten as follows [93].

$$P_f = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} I(\underline{x}) f(\underline{x}) d\underline{x} \quad (2-76)$$

This equation also holds for standard normal variables. As a matter of fact, in the standard normal space, this integral with respect to standard normal variables, joint standard normal PDF $\varphi(\underline{y})$, and the identical indicator factor expresses failure probability as shown in the equation below.

$$P_f = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} I(\underline{y}) \varphi(\underline{y}) d\underline{y} \quad (2-77)$$

where the indicator function itself is a discrete random variable that can have two values of unity and zero with the probability of P_f and $1 - P_f$, respectively. As mentioned earlier, the failure probability integral can be interpreted as the mean value of the stochastic experiment in the MCS simulation technique. Thus, if sufficient number of samples are generated, an accurate estimation of the reliability problem is expressed as below [81].

$$P_f = \frac{1}{N} \sum_{i=1}^N I(x_i) = \frac{1}{N} \sum_{i=1}^N q_i(x_i) \quad (2-78)$$

where $q_i(x_i) = I(x_i) \frac{\varphi(x_i)}{h(x_i)}$ that is equal to $I(x_i)$ in the MCS sampling. Here $\varphi(x_i)$ and $h(x_i)$ denote the standard normal density function and density function, respectively, and N is enough number of samples, which is quantified by the coefficient of variation of outcomes of samples, and the indicator function is typically shown as $q_i = I(x_i)$, as a literature notation. By using MCS for a reliability problem, failure probability is expressed by a proportion of the number of samples in the failure region, the number of failed samples, to the total number of samples.

An accurate estimation of the failure probability through the MCS technique is achieved if there are sufficient samples. It can be explained by the coefficient of variation of failure probability. In order to calculate that, the mean value and variance of the failure probability are required, which are expressed as follows based on equation (2-78).

$$\mu_{P_f} = \mu_q \quad (2-79)$$

$$\sigma_{P_f}^2 = \frac{1}{N} \sigma_q^2 \quad (2-80)$$

Which regard to expectation and variance operators and the fact that notation “q” can have two values of one and zero with the probability of P_f and $1 - P_f$, respectively, the above two equations can be rewritten as given below.

$$\mu_{P_f} = \mu_q = E[q] = \sum_{i=1}^2 q_i P(q_i) = P_f \quad (2-81)$$

$$\sigma_{P_f}^2 = \frac{1}{N} \sigma_q^2 = \frac{1}{N} Var[q] = \frac{1}{N} \sum_{i=1}^2 (q_i - \mu_q)^2 P(q_i) = \frac{P_f}{N} (1 - P_f) \quad (2-82)$$

Now, the coefficient of probability variation can be computed based on enough samples and the failure probability. Ayyub and Hadler [91] suggested a formula expressing this relationship provided in the following equation.

$$\delta_{P_f} = \sqrt{\frac{1 - P_f}{NP_f}} \quad (2-83)$$

According to the literature review, the acceptable range of the coefficient of variation for the probability in order to have fair estimation of the probability of failure is $\delta_{P_f} < 5\%$. It is discussed that $\delta_{P_f} < 2\%$ provides the accurate failure probability [82], [92].

The desired aim is to calculate N, enough number of samples, so as to reach the intended coefficient of variation. In this way, the failure probability estimation through the sampling is acceptable and accurate. Conducting a simple calculation, equation (2-83) can be written as follows.

$$N = \frac{1}{\delta_{P_f}^2} \left(\frac{1 - P_f}{P_f} \right) \quad (2-84)$$

According to the equation above, the smaller the failure probability is, the more samples are required to meet the proper range of probability coefficient of variation.

2-8-8 Histogram sampling

In this section, another sampling technique is introduced. In this method, sampling is conducted as a function of original random variables instead of a limit state function. Many samples are created based on different values generated for random variables comprising the function. This technique is attributed to the analysis of functions that yields the probability density function and relative frequency, together with cumulative distribution function or cumulative frequency [81]. This sampling also needs a random number generator to generate standard normal variables as MCS. Moreover, the Nataf transformer must convert the variables to the original space to compute the function for every sample. The outcomes of using this sampling method are narrated in chapter 4.

2-8-9 Importance sampling

So far, two different techniques of sampling have been discussed. In this section, another sampling method is introduced, which is a variance reduction technique. In order to have more efficient sampling, especially in solving reliability problems in engineering application, it is essential to develop sampling methods that need fewer simulations. As for Monte Carlo Sampling (MCS), most samples are placed around the mean since the highest probability density is in the neighborhood of mean. In other words, by increasing the distance from the mean point, the occurrence probability of samples decreases. Therefore, a number of samples are required to estimate failure probability appropriately, so this method may be time-consuming. Importance sampling is one of the variance reduction methods in which sampling initiates from the design point rather than the origin point in standard normal space. The design point obtained by the FORM technique is the most probable point on the limit state surface. In this way, more samples occur in the failure region. The distribution function of this sampling is different from that of MCS. Since the start point in importance sampling is the design point, if $h(\underline{y})$ denotes the distribution function, it can be expressed as:

$$h(\underline{y}) = \frac{1}{(2\pi)^{\frac{n}{2}}} e^{-\frac{1}{2}(\underline{y}-\underline{y}^*)^T(\underline{y}-\underline{y}^*)} \quad (2-85)$$

where n is the number of samples. Failure probability is computed according to equation (2-77). Indicator function in importance sampling is different from “ q ” function due to the different distribution function given by equation (2-85). Multiplying and dividing the expression inside the probability integral indicated in equation (2-77) by the distribution function yield integral below [81].

$$P_f = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} I(\underline{y}) \frac{\varphi(\underline{y})}{h(\underline{y})} h(\underline{y}) d\underline{y} \quad (2-86)$$

where $\varphi(\underline{y})$ is the standard normal distribution function, as mentioned earlier. Considering that

$q(\underline{y}) = I(\underline{y}) \frac{\varphi(\underline{y})}{h(\underline{y})}$, the equation above can be rewritten as follows.

$$P_f = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} q(\underline{y}) h(\underline{y}) d\underline{y} \quad (2-87)$$

This probability integral can be estimated by the importance sampling. As in MCS, the proportion of the number of samples that occur in the failure region to the total number of samples expresses the failure probability [80], [81].

$$P_f = \frac{1}{N} \sum_{i=1}^N q_i = \frac{1}{N} \sum_{i=1}^N I(\underline{y}) \frac{\varphi(\underline{y})}{h(\underline{y})} \quad (2-88)$$

Importance sampling is a robust simulation with the advantages of both FORM and sampling analysis. This method has a high degree of accuracy since FORM importance measures are attainable and the design point is used. Additionally, sampling commencement from the design point reveals the most probable state, and the failure probability is obtained fast. Chapter 4 presents the use and outcome of this method in solving for the delamination growth probability.

2-9 Summary

This chapter contains two main parts. The first part of the present chapter provides simulation of the mode-I and mode-II delamination of composite laminate specimens subjected to the DCB and ENF tests, respectively. First, the numerical analysis has been carried out using the FEA tool ANSYS® to simulate mode-I and mode-II delamination in the DCB and ENF tests. Each modeling setting, element type, and meshing have been discussed, and the same studies in the literature have been used for verification. Using DBT as a data reduction method allows computing the energy release rate both in mode-I and mode-II based on the recorded displacements and loads during the test simulation. This data reduction approach is how mode-I and mode-II interlaminar fracture toughnesses have been evaluated in the numerical analysis provided in chapter 3. According to the validated numerical analyses provided in this chapter, the next chapter presents further numerical analyses of the effect of mechanical properties and geometric parameters of the test specimen and fiber misalignment on the interlaminar fracture toughness.

In the second part of this chapter, the fundamentals of the reliability analysis have been discussed. FOSM and FORM have been presented as two important first-order methods, and expressions for obtaining sensitivity parameters and importance vectors based on these approaches have been provided. Then, two main approaches of the second-order reliability method have been presented. Point fitting SORM in the present study is used to compute the reliability of the composite material subjected to the DCB and ENF tests. Three different sampling methods, including MCS, importance sampling, and histogram sampling, have been also discussed. The two former methods are used to determine the probability of delamination growth in the composite material. The latter method is used to define the mean values, standard deviations, and the coefficients of variation of mode-I and mode-II energy release rates, in chapter 4. All the reliability methods that are used in chapter 4 are based on a limit state function. For the DCB and ENF tests, the limit state function has been defined based on resistance to delamination propagation, which is embedded in the definition of interlaminar fracture toughness in mode I and mode II, respectively. As mentioned in this chapter, the energy release rate in each delamination property test is defined using the DBT method. All the reliability analyses that are carried out, provided in chapter 4, are according to the principles, formulations and methods that were presented in this chapter.

Chapter 3

Parametric analysis of mode-I and mode-II fracture toughnesses in the DCB and ENF tests

3-1 Introduction and problem statement

Sources of uncertainties are inherent in ply properties, manufacturing processes such as hand lay-up, Resin Transfer Molding (RTM), and curing process such as autoclave curing process [11]. The graphite-epoxy composites considered in the present numerical study are transversely-isotropic. Their independent engineering constants can be measured by techniques such as tensile tests with a gauge or digital image correlation and ultrasonic techniques. For instance, for measuring out-of-plane Poisson's ratio of the composite materials, the ultrasonic characterization is used. The resultant data of all the experimental measurement methods have some deviations [94]. The variation in the engineering properties has to be taken into consideration in the delamination property testing of composites. The following sections of this chapter are on the parametric studies which illustrate the influence of the variations in engineering constants on the energy release rate. Although there are few standardized mode-I and mode-II failure tests for composite specimens, researchers use different test specimens to experimentally and numerically study delamination in composite materials. In the present study, the DCB specimen used in the work of Alfano and Crisfield [25] is considered so as to figure out and characterize the importance of mechanical properties and geometric parameters of the DCB test specimen when obtaining the mode-I interlaminar fracture toughness. Besides, the influence of engineering properties and geometric parameters of ENF test specimen on the mode-II energy release rate is investigated through numerical analysis.

Moreover, there have been significant studies about flaws and defects in composite materials due to the manufacturing and curing processes. According to previous works, as reviewed in chapter 1, common manufacturing defects of composite materials can be named delamination, void, pore, resin rich zones, wrinkles, and in-plane misalignment of fibers [11]. These defects can be identified by imaging and non-destructive identification techniques such as X-ray and ultrasonic inspections.

In this chapter, defective composite specimens that have ply orientation misalignment cured using the U-shape tool subjected to the ENF test are modeled. The in-plane waviness is defined as observed in the work of Lightfoot et al. [45]. The last section of the present chapter investigates the location of the ply misalignment in the stacking sequence of composite laminate. Numerical analysis of different specimens brings out important conclusions in this regard.

3-2 Influence of the material properties of DCB test specimen on mode-I failure

Studying the effects of mechanical properties of composite laminate on the DCB test results through numerical analysis is the approach of the present study in this section. Several experiments must be designed to track the influence of every mechanical property of the composite specimen. Each time, different samples with distinct material properties have to be prepared to undergo the DCB test. It is probably very time-consuming and expensive due to the labor, experiment, and materials costs. The virtual analysis or computer experiment presented in this section is conducted to avoid carrying out plenty of actual physical trials. More than 1700 models are designed in ANSYS®, and each one has a specific set of material properties. As mentioned earlier, the deviations in composite material properties is inevitable, meaning that there is some variations in the stiffness properties of a composite material. Therefore, in this parametric study, every engineering property of the composite laminate is examined individually to record its own influence on the outcomes of mode-I failure test.

In the following, first, the variation in the longitudinal Young's modulus is considered. Five modeling groups represent 268 models with different DCB test specimens. In all samples, mechanical properties are kept as constant and deterministic except E_{11} . More than fifty models are created for every modeling group for which the longitudinal elastic modulus of the composite material is determined according to the normal distribution with a coefficient of variation between 4.47% (for the models of the first modeling group) and 9.96% (for the models of the fifth modeling group) (Table 3-1). This variation in longitudinal elastic modulus is considered based on an experimental study on the elastic properties of carbon fiber composite material available in literature [94]. According to the mentioned study, the variation in the longitudinal elastic modulus

can occur with a COV of about 4% to 7.5% with regard to three different measurement tests. In the present study, a wider range is considered with respect to the experimental results obtained from the previously-mentioned work [94].

Table 3-1 Modeling groups corresponding to the longitudinal elastic modulus of the DCB test specimen

Modeling group	Number of test models, n	COV of E_{11}
1	51	0.044725
2	55	0.045095
3	55	0.068635
4	53	0.083897
5	54	0.09962

A total of 268 DCB test models with different E_{11} have been used to determine how the mode-I fracture toughness varies by changing the longitudinal Young's modulus of the composite laminate specimen as illustrated in figure 3-1. The best fit curve showing the correlation between the coefficient of variation of E_{11} and that of G_{Ic} is a polynomial equation of order three which is represented in the curves in figure 3-1. The legend of the figure contains a line with a solid point and a dotted line that represent the correlation between the COV of E_{11} and that of G_{Ic} in the modeling groups, and the best fit curve of this correlation, respectively. It is worth mentioning that the coefficient of variation of the mode-I fracture toughness witnesses a rise from 2.95% to 6.27% over the whole range of coefficient of variation of the longitudinal elastic modulus.

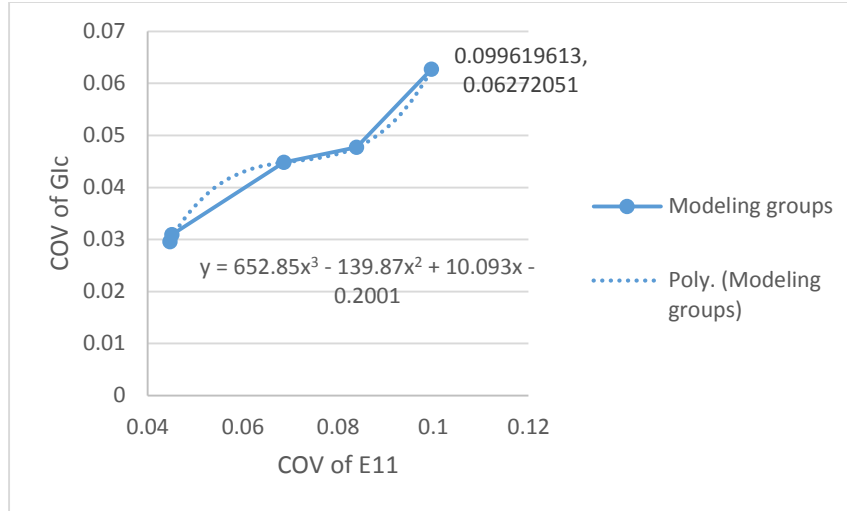


Figure 3-1 Effect of the variability in longitudinal Young's modulus on G_{Ic}

Tensile test of composite materials in a direction transverse to the fiber direction results in transverse Young's modulus of the composite. The test outcomes usually have a standard deviation [94] which is considered in the mode-I failure simulation as described in the following. Modeling groups 6 through 10 are designed to observe the influence of the variation in E_{22} on the mode-I failure test. As given in table 3-2, five different normal distributions with distinct coefficients of variation and a constant mean value of 9000 MPa are considered to generate five sets of numbers representing the magnitude of the transverse elastic modulus. In this way, every specimen possesses a specific value of E_{22} and then undergoes the simulated DCB test. The variations in the transverse elastic modulus are considered based on the work of Munoz et al. [94].

Table 3-2 Modeling groups corresponding to the transverse elastic modulus of the DCB test specimen

Modeling group	Number of test models, n	COV of E_{22}
6	51	0.044725
7	53	0.05291
8	53	0.07005
9	53	0.08318
10	56	0.1059

A new command as a subroutine is written in ANSYS® to record the outputs of all models making up a modeling group. It is done in the mechanical static structural module in ANSYS®. Inserting a specific command in APDL tool of ANSYS® which is written by the author of this thesis allows one to record all the outputs of models in every step after the parametric simulation. According to this written command, the critical load corresponding to the initiation of delamination and the corresponding displacement corresponding to the mode-I failure based on bilinear traction separation law are marked as parameters to be shown in the parameter set tool. Additionally, the transverse Young's modulus is selected, and the generated numbers are assigned to it in the parameter set. The energy release rate in mode-I is computed for all 266 models, and the coefficient of variation of the results is calculated for every modeling group individually.

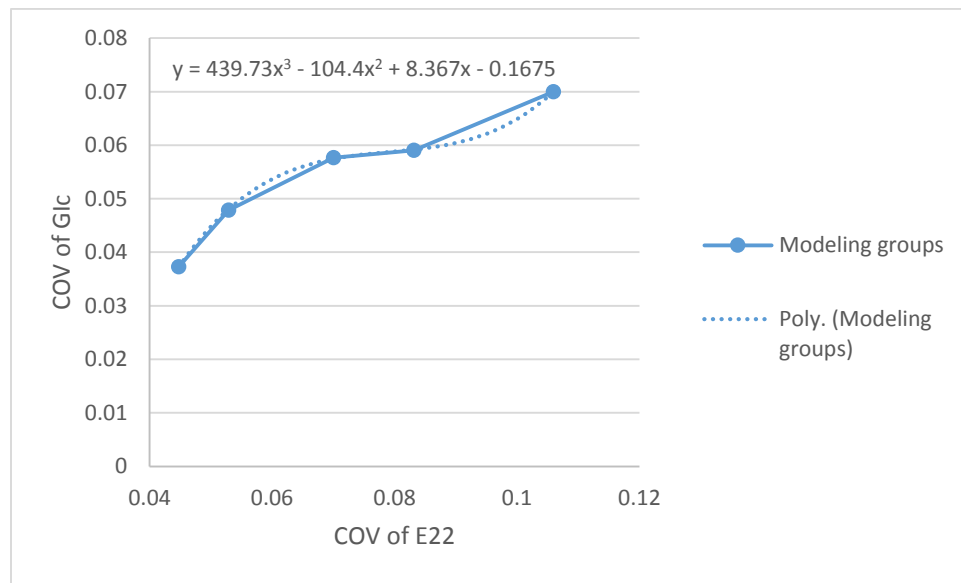


Figure 3-2 Effect of the variability in transverse Young's modulus on G_{Ic}

Figure 3-2 demonstrates the relationship between the coefficient of variation of transverse elastic modulus and that of the mode-I energy release rate. The legend of the figure contains a line with a solid point and a dotted line that represent the correlation between the COV of E_{22} and that of G_{Ic} in the modeling groups, and the best fit curve of this correlation, respectively. It is observed that by increasing the COV of E_{22} , the COV of G_{Ic} experiences a corresponding non-linear increase. There is a significant increase in the coefficient of variation of mode-I fracture toughness with an increase in the COV of E_{22} .

Modeling groups 11 through 15 have been used to investigate the influence of out-of-plane modulus, E_{33} , on the DCB test results. Modeling specimens are only different in one dependent engineering constant, the out-of-plane modulus. Since the composite material used in this parametric study is transversely-isotropic, the variation in E_{33} is determined in the same range that is as mentioned for E_{22} based on the work of Munoz et al. [94]. For every group, fifty-eight values are generated based on normal distributions with COVs from approximately 5% to 10.5% to be given as the COV of E_{33} of the composite laminate specimen (Table 3-3). Every virtual test result is collected after each model is subjected to the mode-I opening displacement at either end of the beam.

Table 3-3 Modeling groups corresponding to the out-of-plane elastic modulus of the DCB test specimen

Modeling group	Number of test models, n	COV of E_{33}
11	58	0.044913
12	58	0.05209
13	58	0.065037
14	58	0.09027
15	58	0.10553

The simulation outcomes are used to compute mode-I fracture toughness. According to the results, it is concluded that the variation in the out-of-plane Young's modulus does not affect considerably the mode-I energy release rate. Therefore, changing the COV of E_{33} has no considerable influence on the COV of the mode-I energy release rate obtained from the DCB test. Since in the present parametric study several models have been simulated, the 2D FEA models are used. The elements used in the numerical analyses are plane-stress elements. The 2D modeling-based observation indicates no significant influence of out-of-plane elastic modulus variability on the DCB test results because in the plane-stress analysis only in-plane stresses are considered.

The effect of G_{12} as an independent engineering constant on the mode-I failure test is assessed through numerical analysis. The simulation includes 220 models of different G_{12} with the same

other engineering properties and specimen geometric properties. The fluctuations in input data corresponding to the in-plane shear modulus are defined according to the test results of the work of Munoz et al. [94]. Considering a range for that experimental deviation, table 3-4 is provided. This table illustrates the four different modeling groups designed to determine different variations of the in-plane shear modulus.

Table 3-4 Modeling groups corresponding to the shear modulus of the DCB test specimen

Modeling group	Number of test models, n	COV of G_{12}
16	53	0.04949
17	55	0.07301
18	55	0.09095
19	57	0.108

After simulating all the models, the mode-I fracture toughness of the specimen is calculated 220 times. The coefficient of variation of the mode-I fracture toughness for each modeling group is computed and provided versus the coefficient of variation of the shear modulus as shown in figure 3-3. The legend of the figure contains a line with a solid point and a dotted line that represent the correlation between the COV of G_{12} and that of G_{Ic} in the modeling groups, and the best fit curve of this correlation, respectively.

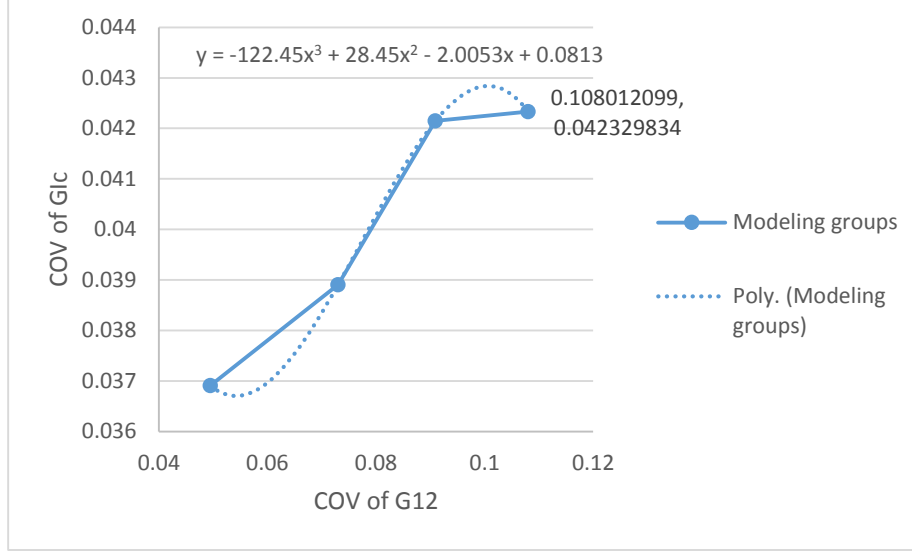


Figure 3-3 Effect of the variability in shear modulus on G_{Ic}

It may be seen in figure 3-3 that as the coefficient of variation of G_{12} increases, so does the COV of the computed mode-I energy release rate. Nonetheless, the rise is not substantial. i.e., the coefficient of variation of G_{Ic} increases by about 0.55% when there is nearly 5.85% of increase in COV of shear modulus.

On the other hand, G_{23} is a dependent engineering property that in transversely-isotropic materials is computed according to the equation below:

$$G_{23} = \frac{E_{22}}{2(1 + \nu_{23})} \quad (3-1)$$

Based on this equation, changing the values of transverse elastic modulus and out-of-plane Poisson's ratio affects the magnitude of G_{23} . Thus, the changes in E_{22} and ν_{23} cause variation in the out-of-plane shear modulus. As mentioned before, the fluctuations in the transverse elastic modulus and out-of-plane Poisson's ratio of the composite material are determined based on the experimental study of Munoz et al. [94]. Considering equation (3-1) and the coefficient of variation of E_{22} and ν_{23} , four COVs are considered for out-of-plane shear modulus. Table 3-5 shows a classification of four modeling groups, each containing fifty-five models with different out-of-plane shear moduli but constant in all other engineering properties. Four normal distributions with different COVs are taken into consideration when generating the 220 values for the shear modulus.

Table 3-5 Modeling groups corresponding to the out-of-plane shear modulus of the DCB test specimen

Modeling group	Number of test models, n	COV of G_{23}
20	55	0.052653
21	55	0.081586
22	55	0.090493
23	55	0.105341

All the DCB test models are designed in ANSYS® for numerical analysis. Since all the simulation settings, such as applied mesh size, meshing techniques, boundary conditions, and specimen geometry are similar among models, selecting the shear modulus YZ, the value of G_{23} is changed by importing the generated numbers in the parameter set. After calculating the energy release rate in mode-I for all models, it is observed that fluctuation of G_{23} values or changing its COV has no significant influence on the mode-I energy release rate or its COV in this statistical modeling.

Based on the tensile test carried out in the study of Munoz et al. [94], there is about 0.03 standard deviation in measured in-plane Poisson's ratio. In order to consider a wider range of data variation compared with the results of the above-mentioned experimental study, two hundred twenty models are considered in four groups of different coefficients of variation of in-plane Poisson's ratio to see the influence of this variation on the mode-I fracture toughness, as shown in table 3-6.

Table 3-6 Modeling groups corresponding to the major Poisson's ratio of the DCB test specimen

Modeling group	Number of test models, n	COV of ν_{12}
24	53	0.05197
25	55	0.07356
26	55	0.09025
27	57	0.1055

According to the above table, 220 specimens with only differences in major Poisson's ratio, otherwise with the same other engineering constants and geometric parameters, are designed so that distinctive values of the mode-I energy release rate would be calculated after simulating the DCB test for every specimen. The coefficient of variation of the mode-I fracture toughness is computed for all modeling groups. It is presented versus the coefficient of variation of the major Poisson's ratio in figure 3-4. Overall, the mode-I fracture toughness variation is not as notable as that of ν_{12} . As an illustration, in the 27th modeling group, the computed coefficient of variation of G_{Ic} is 0.0024% when there is a COV of 10.55% for the Poisson's ratio. However, it is worthwhile to mention the linear relationship between the COV of G_{Ic} and that of ν_{12} .

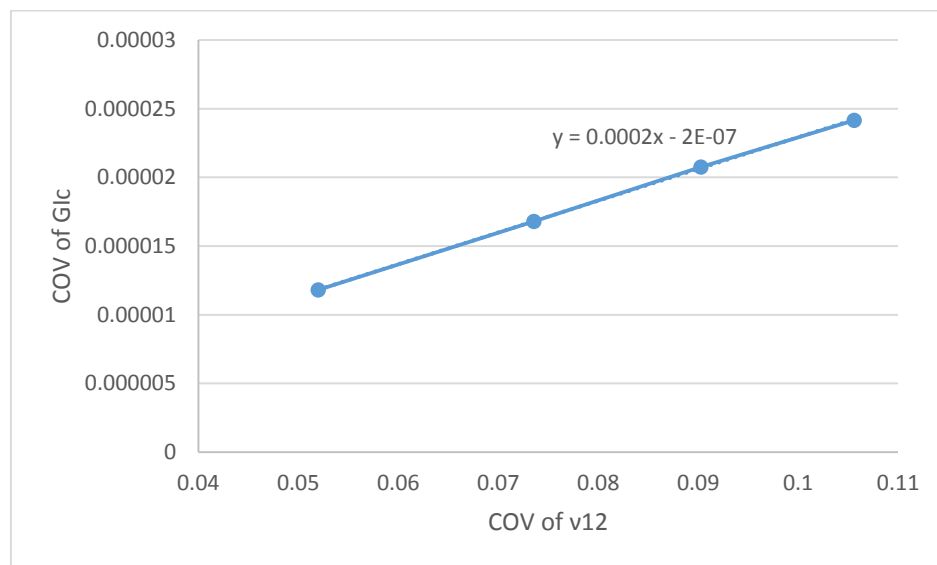


Figure 3-4 Effect of the variability in major Poisson's ratio on G_{Ic}

Furthermore, out-of-plane Poisson's ratio as one of the independent engineering properties of the composite material is essential to be considered. Since it is determined by testing, there may be some variation in the value of out-of-plane Poisson's ratio of a composite material [94]. Table 3-7 lists four different modeling groups; each one contains a distinct normal distribution with a different coefficient of variation with regard to the experimental results of the mentioned study [94] considered so that a set of numbers is generated for determining the out-of-plane Poisson's ratio of different values.

Table 3-7 Modeling groups corresponding to the out-of-plane Poisson's ratio of the DCB test specimen

Modeling group	Number of test models, n	COV of ν_{23}
28	53	0.0555
29	55	0.0652
30	55	0.0877
31	57	0.1136

All the models are simulated in ANSYS® by using the parameter set tool. It is shown that in the same analysis setting of the DCB test, a distinctive value of mode-I fracture toughness is computed for every model. Figure 3-5 represents how the coefficient of variation of G_{Ic} varies with the change in the COV of the out-of-plane Poisson's ratio of the composite material. It is evident from figure 3-5 that by increasing the COV of ν_{23} from 5.55% to 6.52%, the COV of the mode-I fracture toughness increases dramatically; between modeling groups 2 and 4, however, the rise in COV of G_{Ic} is more moderate, from 1.11% to just over 2%.

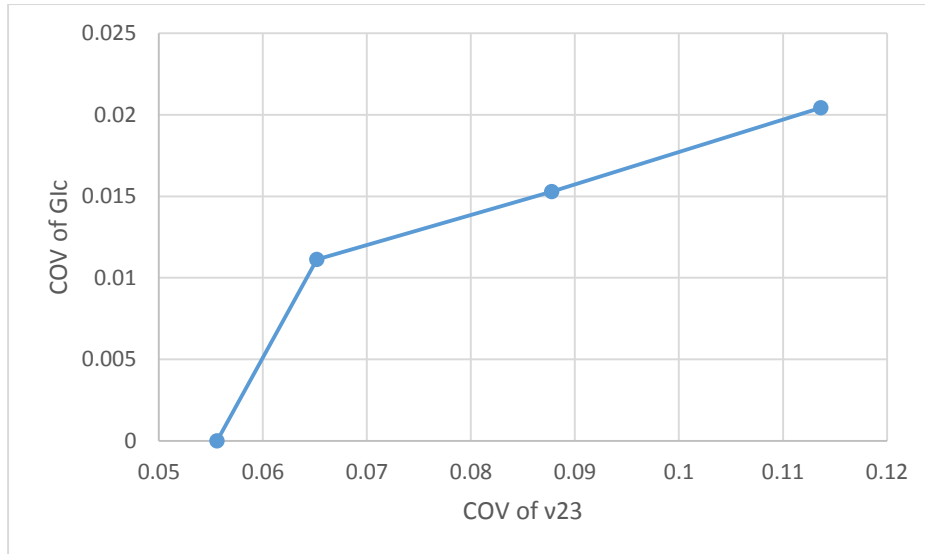


Figure 3-5 Effect of the variability in out-of-plane Poisson's ratio on G_{Ic}

3-3 Influence of the geometric parameters of DCB test specimen on mode-I failure

This section investigates the influence of the geometric properties of the DCB test specimen on the mode-I energy release rate. Several models are created concerning every geometric property, including the length, width, thickness, and pre-crack length of the composite laminate specimen individually. The parametric study presented in this section observes the variation of the energy release rate in the mode-I failure test with variations in the geometric parameters of the specimens. The numerical model of the delaminated beam is developed in ANSYS® using the FEA method.

The applied numerical analysis examines the influence of the length of the mode-I failure test specimen on mode-I fracture toughness. In this regard, four groups of random numbers, each containing fifty normally distributed numbers, were generated. The COVs of the length of specimen are determined based on the work of Ducept et al. [16]. In the first model group, the coefficient of variation of the specimen length is 0.64%. Different laminate lengths correspond to fifty models subjected to the mode-I failure test. The four model groups with varying coefficients of variation of the length of specimen are given in table 3-8.

Table 3-8 Modeling groups corresponding to the length of the DCB test specimen

Model group	Number of test models, n	COV of the length of specimen
1	50	0.0064
2	50	0.0089
3	50	0.0166
4	50	0.0233

Two hundred models are designed to be solved using FEA in ANSYS®. In order to prevent modifying specimen length in the model in every model, the parameters module of the static structural environment is used. The energy release rate in mode-I is calculated fifty times for each model group. Subsequently, the coefficient of variation of the energy release rate in mode-I is computed for every model group. Figure 3-6 illustrates the correlation between the COVs of the mode-I fracture toughness and the length of specimen.

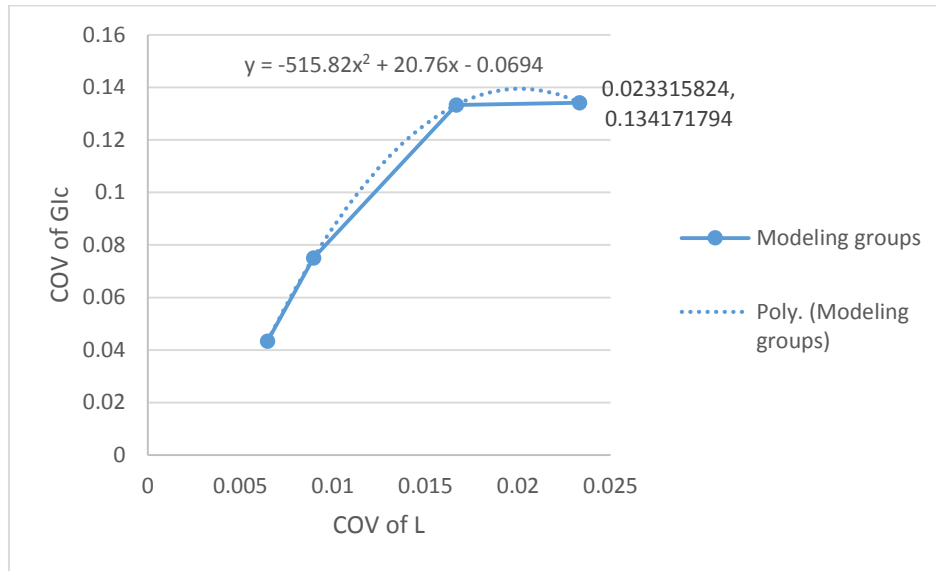


Figure 3-6 Effect of the variability in the length of specimen on G_{Ic}

A polynomial curve is obtained as the best fit curve to the plot, shown in the dotted line in the legend of the figure, describing the relationship between the coefficient of variation of the length of specimen as input and the coefficient of variation of the mode-I energy release rate as the output of the modeling.

Numerical analysis of the variability in the thickness of DCB specimen is carried out using the parameters module in ANSYS®. While keeping other geometric properties constant, parameterizing the thickness of the beam allows for an individual investigation of the effect of the thickness of composite laminate. It lets one to consider the variation of the thickness of specimen because of the variations in the thicknesses of composite plies in the mode-I failure test, regardless of the number of plies present. Table 3-9 represents four modeling groups, each containing fifty models. The COVs of the thickness of specimen are defined according to the study of Ducept et al. [16]. A wider range of deviation is considered to see the correlation between the COV of the thickness of specimen and the COV of the mode-I energy release rate. The values of thickness of beam among models are selected by random number generation of the normal distribution with a constant mean value of 4.5 mm and different coefficients of variation for the modeling groups seen in table 3-9.

Table 3-9 Modeling groups corresponding to the thickness of the DCB test specimen

Modeling group	Number of test models, n	COV of the thickness of specimen
5	50	0.0089
6	50	0.0175
7	50	0.0245
8	50	0.032

The thickness of specimen in every model is imported in the table of design points in the parameters set in ANSYS®. The outcomes for all models are computed, and the coefficient of variation of the mode-I fracture toughness through fifty models of every modeling group is obtained. Four coefficients of variation of the thickness of DCB test specimen and their corresponding coefficients of variation of mode-I energy release rate are plotted in figure 3-7. The legend of the figure contains a line with a solid point and a dotted line that represent the correlation between the COV of the thickness of specimen and that of G_{Ic} in the modeling groups, and the best fit curve of this correlation, respectively. The figure below shows how changing the coefficient of variation of the thickness of specimen from 0.89% to 3.2% increases by 1.15% the coefficient of variation of the energy release rate in mode-I.

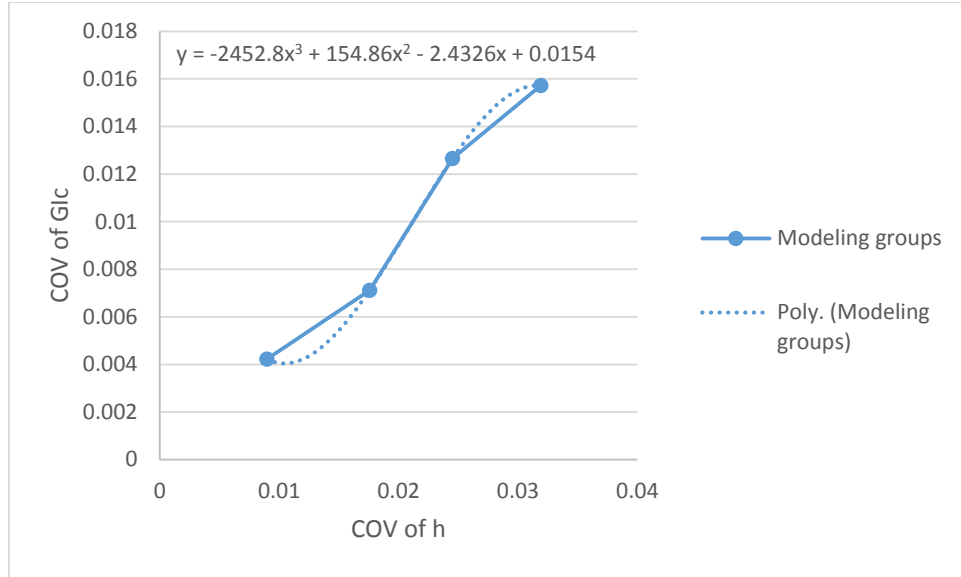


Figure 3-7 Effect of the variability in the thickness of specimen on G_{Ic}

Modeling groups 9 to 12 aim to detect the influence of the variation of the width of specimen on the mode-I failure test results. Four different coefficients of variation values are given to the normal distribution of the width of the specimen with a constant mean value of 20 mm. The coefficient of variation of the width of specimen is determined based on the study of Ducept et al. [16]. As shown in table 3-10, two hundred numbers are generated according to which two hundred models with different widths of specimen are created undergoing the DCB test.

Table 3-10 Modeling groups corresponding to the width of the DCB test specimen

Modeling group	Number of test models, n	COV of the width of specimen
9	50	0.0063
10	50	0.0089
11	50	0.018
12	50	0.0263

The mode-I energy release rate is calculated for every model, and the coefficient of the variation of each modeling group containing fifty models is computed. Figure 3-8 illustrates the relationship

between the COV of the width of specimen and the COV of the G_{Ic} . The legend of the figure contains a line with a solid point and a dotted line that represent the correlation between the COV of the width of specimen and that of G_{Ic} in the modeling groups, and the best fit curve of this correlation, respectively. As can be seen, the correlation is almost linear for the first three modeling groups; however, there is a dramatic increase in the COV of the mode-I fracture toughness, after the coefficient of variation of the width of specimen exceeds 1.8%.

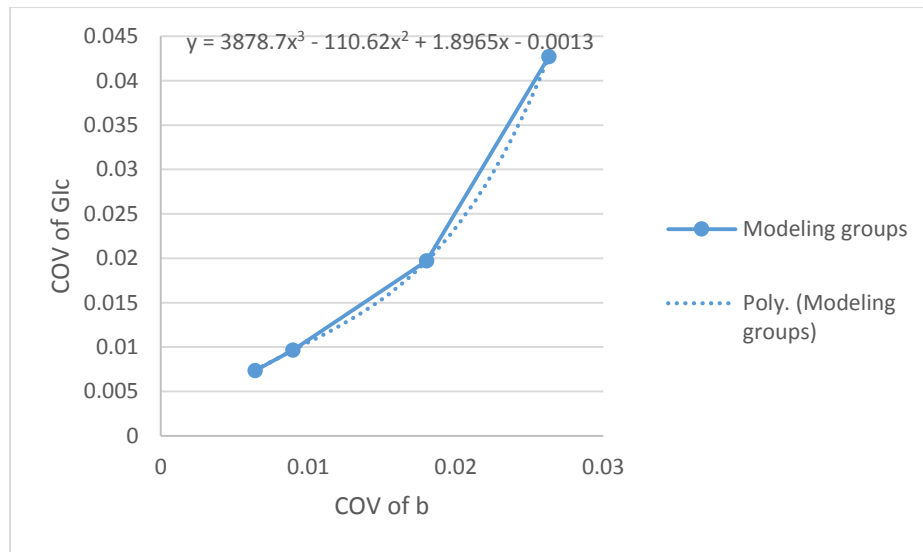


Figure 3-8 Effect of the variability in the width of specimen on G_{Ic}

Four modeling groups of fifty models are considered to see the effect of the pre-crack length of the DCB test specimen on the mode-I energy release rate as given in table 3-11. All the specimen dimension parameters remain the same in every model, and only the magnitude of the pre-crack length in the delaminated specimen is varied based on the work of Ducept et al. [16] to record the change in the DCB test outcomes.

Table 3-11 Modeling groups corresponding to the crack length of the DCB test specimen

Modeling group	Number of test models, n	COV of the crack length of specimen
13	50	0.0108
14	50	0.0171
15	50	0.0268
16	50	0.0293

Allocating fifty values to the pre-crack length of specimen is done in the table of design points in the parameters module in ANSYS®. A simulation of fifty models is carried out four times with the intention to calculate the coefficient of variation of the mode-I fracture toughness of modeling groups from 13 to 16. The results are summarized in figure 3-9 as a bi-linear trend and its best fit curve.

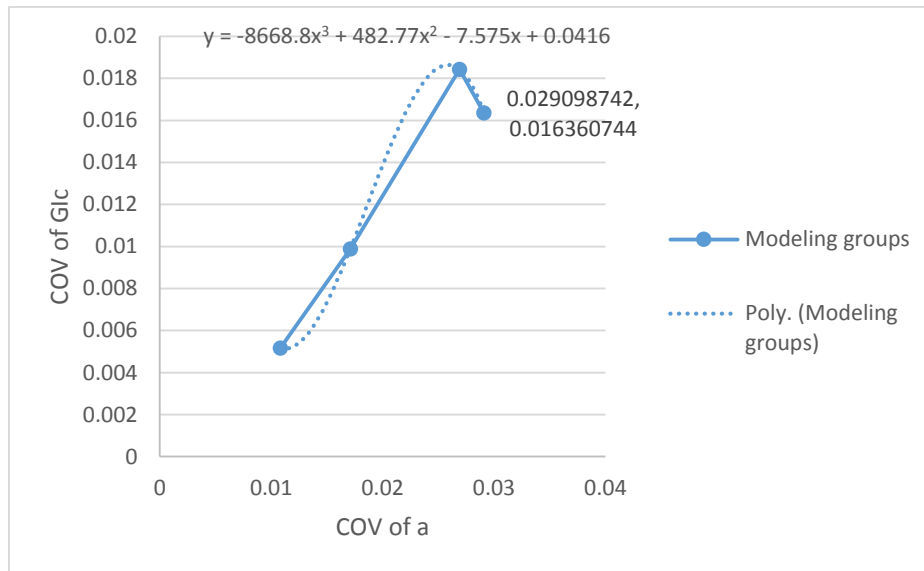


Figure 3-9 Effect of the variability in the crack length of specimen on G_{Ic}

As can be seen from figure 3-9, by increasing the COV of the crack length, there is an upward trend in COV of the mode-I fracture toughness from 0.5% to nearly 1.85%. Whereas the COV of G_{Ic} experiences a decline after the COV of the pre-crack length exceeds 2.68%.

Last but not least, the effect of simultaneous variation of all geometric properties of a delaminated DCB test specimen is investigated. A total of ten sets of different specimen dimensions and distinguished pre-crack lengths are considered. The geometric properties of all sets are given in table 3-12. The mean values of the length, thickness, width, and pre-crack length of the composite laminate are varied together by keeping the individual coefficient of variation constant at about 3%.

Table 3-12 Geometric parameters of ten DCB test specimens

Set numbers	L (mm)	h (mm)	b (mm)	a (mm)
Set 1	96	1.44	19.2	28.8
Set 2	97	1.455	19.4	29.1
Set 3	98	1.47	19.6	29.4
Set 4	99	1.485	19.8	29.7
Set 5	100	1.5	20	30
Set 6	101	1.515	20.2	30.3
Set 7	102	1.53	20.4	30.6
Set 8	103	1.545	20.6	30.9
Set 9	104	1.56	20.8	31.2
Set 10	105	1.575	21	31.5

In figure 3-10, the computed energy release rates in mode-I for each set are compared. The dimension of specimen in every set together with the crack length is illustrated in clustered columns, while the line with markers indicates the energy release rate in mode-I. In this figure, the x-axis represents the number of sets, and the left side y-axis shows the magnitude of the geometric parameters all in mm.

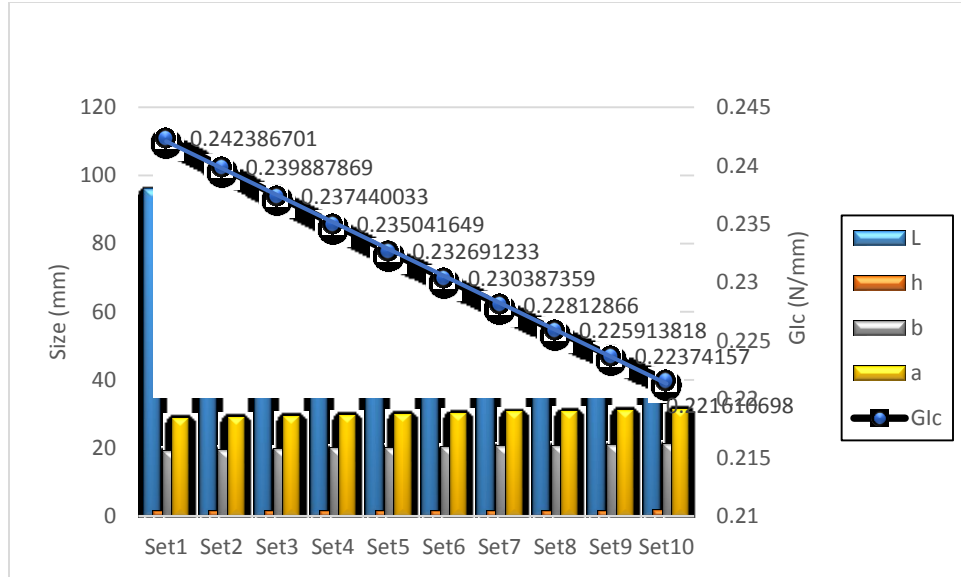


Figure 3-10 Effect of the variabilities in the geometric properties of specimen on G_{Ic}

As shown in figure 3-10, the rising mean value of geometric properties results in a moderate fall in the mode-I energy release rate. The first set having the smallest specimen dimension and pre-crack length indicates the largest mode-I fracture toughness after the DCB test. The relationship between the concurrent changes in geometric properties and the magnitude of the energy release rate is linear.

3-4 Influence of applied displacement on the DCB test result

This section assesses the influence of the variation of applied displacement on either side of the beams in the DCB test on mode-I energy release rate. According to the ASTM standard test method for mode-I interlaminar fracture toughness [3], a testing machine in displacement control mode is designed according to standard practices of testing machines [95]. This standard indicates that the testing machine error or deviation from the correct value may be within the range of $\pm 1\%$. In the present work, the variation in applied displacement is considered in a wider range. In order to see the variation of G_{Ic} of the mode-I failure test caused by change in the applied displacement, the same deviations as that of the geometric parameters, which are based on the study of Ducept et al. [16], are considered. A normal distribution of the magnitude of the applied displacement with the mean value of 10 mm based on the work of Alfano and Crisfield [25], and COV of 0.89% is

created, according to which 50 random numbers are generated. In the same way, a further three modeling groups of fifty models are considered with different coefficients of variation of applied displacement, as given in table 3-13.

Table 3-13 Modeling groups corresponding to the applied displacement of the DCB test

Modeling group	Number of test models, n	COV of applied displacement
1	50	0.0089
2	50	0.0169
3	50	0.0231
4	50	0.0268

Four simulations are conducted in ANSYS® using the parameters module to import different values of applied displacement for each model. Modeling outcomes result in fifty distinct values of mode-I fracture toughness for every modeling group. Then, the coefficient of variation of mode-I fracture toughness is computed for all modeling groups, and the results are presented in figure 3-11. The legend of the figure contains a line with a solid point and a dotted line that represent the correlation between the COV of applied displacement and that of G_{Ic} in the modeling groups, and the best fit curve of this correlation, respectively. As a result, the increase of the coefficient of variation of the applied displacement results in a higher coefficient of variation of the mode-I energy release rate. The increase is more significant when the COV of the displacement increases from 1.69% to 2.31%.

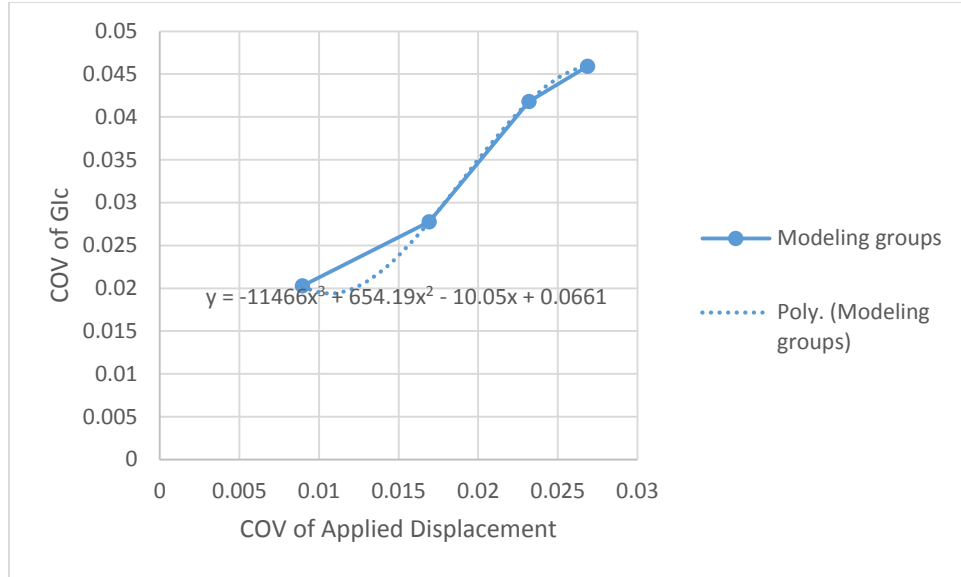


Figure 3-11 Effect of the variability in applied displacement on G_{Ic}

3-5 Influence of the material properties of ENF test specimen on mode-II failure

As mentioned earlier, composite property testing and measurement can cause changes in the engineering constants of the composite material. This section is a parametric study of the individual influence of each composite material property and its variability on the mode-II energy release rate and its variability as a result of the ENF test. The numerical analysis provided below assesses the influence of the variation of E_{11} on the outcomes of the ENF test. Table 3-14 lists five modeling groups of fifty models categorized based on the normal distributions of different coefficients of variation determined based on the experimental study of Munoz et al. [94]. All the generated numbers are assigned to the axial elastic modulus of 250 models.

Table 3-14 Modeling groups corresponding to the longitudinal elastic modulus of the ENF test specimen

Modeling group	Number of test models, n	COV of E_{11}
1	50	0.06055
2	50	0.06981
3	50	0.08135
4	50	0.09468
5	50	0.1057

According to this table, sets of fifty models are simulated together by using the parameter set tool in ANSYS®. Five normal distributions of the values of mode-II fracture toughness make up the results of the simulations. By calculating the coefficient of variation of every group, the influence of the variability in axial elastic modulus of ENF test specimen on the mode-II fracture toughness variability is concluded. Figure 3-12 indicates how the COV of G_{IIc} varies by the change in that of E_{11} . As can be seen, the correlation between the COV of G_{IIc} and that of E_{11} is linear, and there is an increase of 3.66% in the COV of the mode-II energy release rate while the COV of the E_{11} rises by 4.52%.

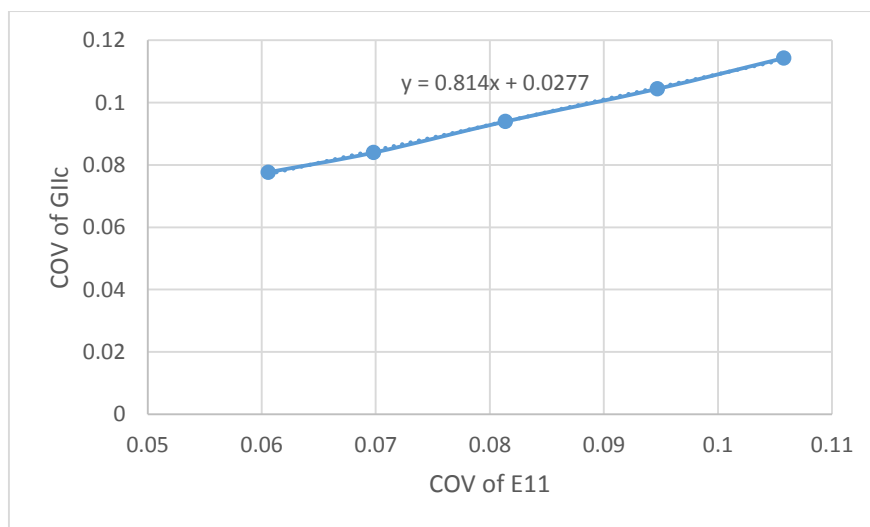


Figure 3-12 Effect of the variability in longitudinal Young's modulus on G_{IIc}

Elastic modulus in the fiber matrix plane and transverse to the direction of the fibers is one of the essential engineering properties. As mentioned earlier, since composite properties testing is a source of uncertainty in composite materials, there would be variations in the values of E_{22} . The COVs of the transverse elastic modulus are defined based on the experimental study of Munoz et al. [94]. In order to simulate the effect of the variation of transverse elastic modulus on the results of ENF test, four groups of models are considered as provided in table 3-15. Changing the coefficient of variation of E_{22} from 5.27% to 8.91% allows the different models to have different transverse elastic moduli.

Table 3-15 Modeling groups corresponding to the transverse elastic modulus of the ENF test specimen

Modeling group	Number of test models, n	COV of E_{22}
5	50	0.05273
6	50	0.06872
7	50	0.07599
8	50	0.08916

All the specimens are simulated as subjected to the mode-II failure displacement, and mode-II fracture toughness is computed based on the simulation outcomes for every model. The outcomes of all models of every group have the normal distribution with different coefficients of variation. The trend of varying the coefficient of variation of G_{IIc} with changing the COV of the transverse elastic modulus is indicated in figure 3-13. The legend of the figure contains a line with a solid point and a dotted line that represent the correlation between the COV of E_{22} and that of G_{IIc} in the modeling groups, and the best fit curve of this correlation, respectively. The coefficient of variation of the mode-II fracture toughness increases moderately among modeling groups due to an increase in the COV of E_{22} .

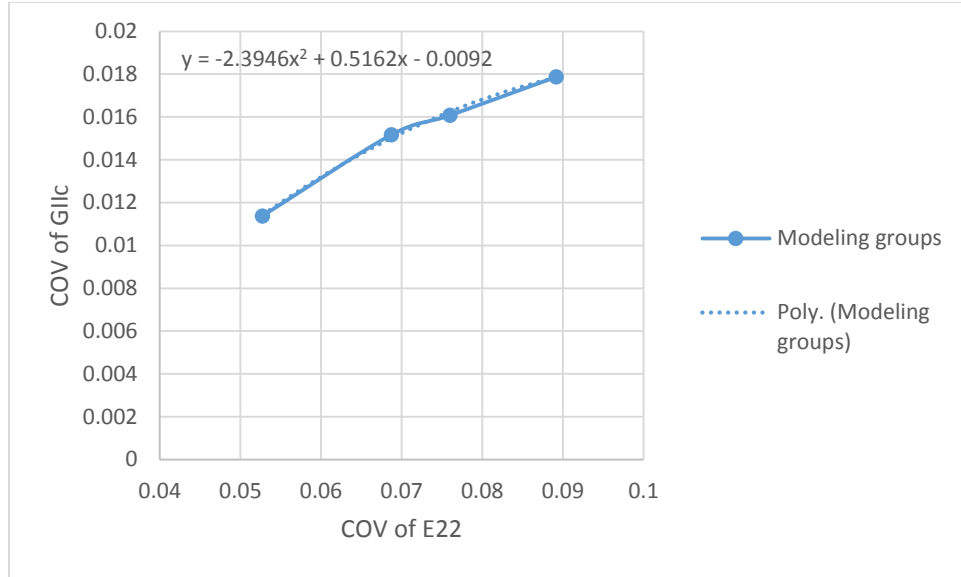


Figure 3-13 Effect of the variability in transverse Young's modulus on G_{IIc}

Moreover, the variation in out-of-plane elastic modulus is also taken into account. Although in transversely-isotropic material, values of the transverse and out-of-plane elastic moduli are equal, the influences of their inconsistency in material behavior would be different. The four normal distributions data almost similar to what has been considered for transverse elastic modulus have been given to E_{33} of 200 specimens with different out-of-plane elastic moduli to determine how they would respond subjected to the ENF test. (Table 3-16).

Table 3-16 Modeling groups corresponding to the out-of-plane elastic modulus of the ENF test specimen

Modeling group	Number of test models, n	COV of E_{33}
9	50	0.05365
10	50	0.07148
11	50	0.0893
12	50	0.0953

Performing a total of 200 ENF models in ANSYS® leads to two hundred different values of mode-II energy release rate. These values are normally distributed, and the modeling outcomes are computed in four groups. Assessing the results reveals that by changing the COV of E_{33} , the coefficient of variation of mode-II energy release rate remains constant among all models with the mean value of about 0.79 N/mm. The 2D modeling-based observation indicates that the variation of out-of-plane elastic modulus has no significant influence on the ENF test results because of the plane-stress elements used in this 2D analysis.

IM7/8552 graphite-epoxy composite is a transversely-isotropic material such that the shear moduli of the fiber-matrix plane have the same value ($G_{12} = G_{13}$). Based on the determination of elastic properties in the study of Munoz et al. [94], measured axial shear modulus of the composite material may have a standard deviation of 300 to 800 MPa. Regarding the mean value of 5200 MPa, the coefficient of variation of shear modulus is assigned between about 6% and 11%. The introduced modeling groups to simulate the influence of the variation of axial shear modulus on the ENF test outcomes are presented in table 3-17. Defining normal distributions with different coefficients of variation allows one to give various meaningful values to the G_{12} of the specimen.

Table 3-17 Modeling groups corresponding to the shear modulus of the ENF test specimen

Modeling group	Number of test models, n	COV of G_{12}
13	50	0.0616
14	50	0.0754
15	50	0.0904
16	50	0.1112

A total of 200 models simulated and subjected to the ENF test. The coefficients of variation of the mode-II fracture toughness for every modeling group are computed. Figure 3-14 shows the energy release rate in mode-II versus the coefficient of variation of G_{12} . The COV of G_{IIc} increases steadily with increasing the COV of the axial shear modulus. It is evident that in-plane shear modulus of composite material affects the mode-II fracture toughness; the influence, however, is not dramatic.

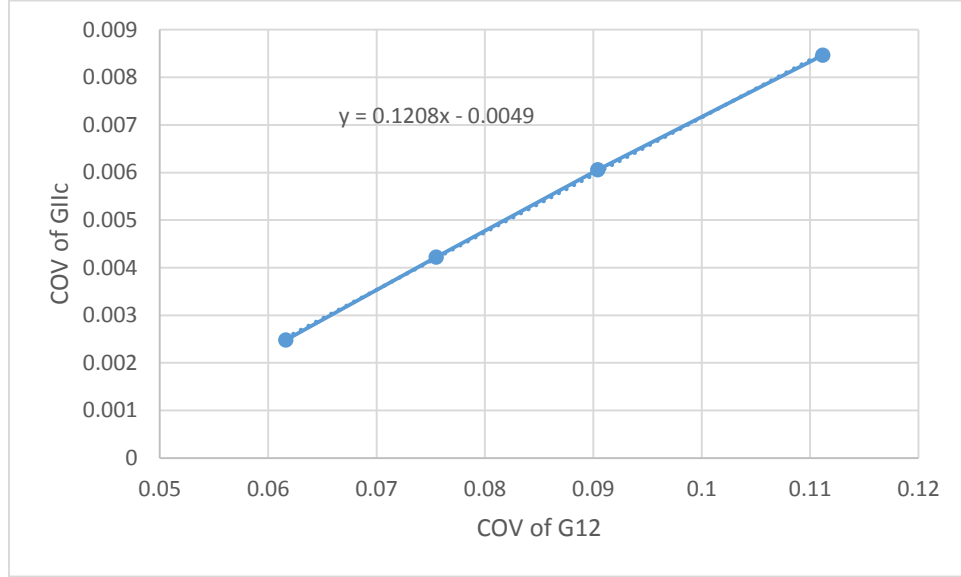


Figure 3-14 Effect of the variability in in-plane shear modulus on G_{IIc}

On the other hand, a total of 220 models are considered, as shown in table 3-18, to simulate how the out-of-plane shear modulus may affect the results of the ENF test. As mentioned earlier, G_{23} for transversely-isotropic materials is obtained using equation (3-1). This dependent engineering property varies with E_{22} and ν_{23} . With regard to the variations in transverse elastic modulus and out-of-plane Poisson's ratio of the composite material, four sets of numbers are generated by using four distinct normal distributions of different coefficients of variation and a constant mean value of 3900 MPa. Every generated number stands for the out-of-plane shear modulus of the ENF test specimen for each model. After completing all simulations, the mode-II fracture toughness is computed for all models individually. Results are too close to each other and show a negligible standard deviation. Therefore, the variation of the G_{23} of the specimen has no considerable influence on the magnitude of the mode-II energy release rate in the ENF test.

Table 3-18 Modeling groups corresponding to the out-of-plane shear modulus of the ENF test specimen

Modeling group	Number of test models, n	COV of G_{23}
17	55	0.0538
18	55	0.0686
19	55	0.0891
20	55	0.10534

In-plane Poisson's ratio is one of the engineering properties that has indicated variations in the different property measurement methods [94]. Since in the present study, all possible variations in the engineering properties of composite materials are taken into account, in the following, the effects of in-plane and out-of-plane Poisson's ratio of specimen subjected to the ENF test are studied. First, four groups of models, including fifty values generated from four normal distributions of a constant mean value of 0.32 and different coefficients of variation from 5.29% to 10.5%, are created (Table 3-19). All numbers are given as the values of ν_{12} for different models to analyze the influence of the major Poisson's ratio of the composite material.

Table 3-19 Modeling groups corresponding to the major Poisson's ratio of the ENF test specimen

Modeling group	Number of test models, n	COV of ν_{12}
21	50	0.0529
22	50	0.0670
23	50	0.0856
24	50	0.105

The ENF test simulation of 200 specimens with different major Poisson's ratio of the specimen is carried out, and the outcomes are compared one to another. After that, the mode-II energy release rate is computed for every model. Results show that the obtained values of the mode-II energy release rate are statistically distributed normally with different coefficients of variation among all

modeling groups. Figure 3-15 plots the coefficient of variation of mode-II fracture toughness versus the coefficient of variation of inputs, the major Poisson's ratio of the composite specimen. The legend of the figure contains a line with a solid point and a dotted line that represent the correlation between the COV of ν_{12} and that of G_{IIc} in the modeling groups, and the best fit curve of this correlation, respectively. Although the fluctuation of the COV of G_{IIc} is not significant in whole, the variation of the ν_{12} influences the coefficient of variation of the mode-II energy release rate.

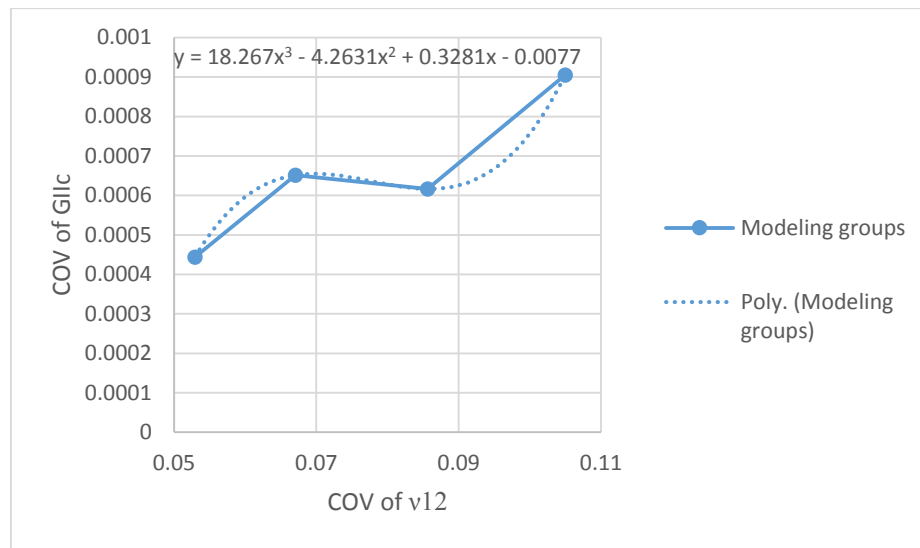


Figure 3-15 Effect of the variability in major Poisson's ratio on G_{IIc}

Second, Out-of-plane Poisson's ratio of the composite material is discussed. Table 3-20 presents four modeling groups of fifty models created to determine different values of ν_{23} of the composite laminate specimen. The deviations are determined according to the study of Munoz et al. [94]. Since the only non-deterministic value among the material properties of these models is the out-of-plane Poisson's ratio, this parametric analysis allows one to track the effect of ν_{23} on the energy release rate in mode-II as the principal outcome of the ENF test.

Table 3-20 Modeling groups corresponding to the out-of-plane Poisson's ratio of the ENF test specimen

Modeling group	Number of test models, n	COV of ν_{23}
25	50	0.0539
26	51	0.0703
27	50	0.0866
28	51	0.0985

All simulation results in 202 different values of the mode-II energy release, which are normally distributed with varying coefficients of variation among four modeling groups. As represented in figure 3-16, there is a linear relationship between the coefficient of variation of G_{IIc} and the COV of out-of-plane Poisson's ratio. An increase of about 4.5% in the COV of the out-of-plane Poisson's ratio doubles the COV of the mode-II fracture toughness. This change is more substantial compared with the variation caused by the in-plane Poisson's ratio.

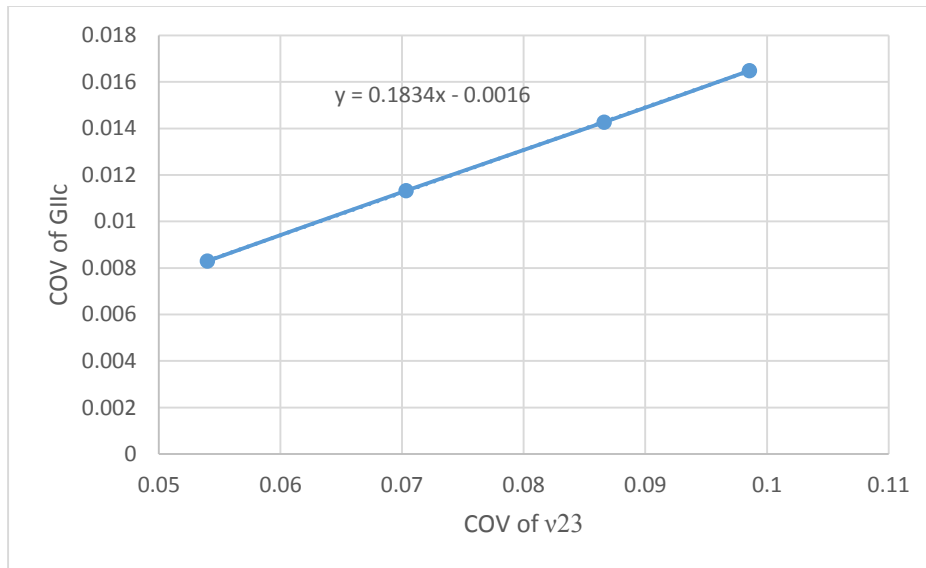


Figure 3-16 Effect of the variability in out-of-plane Poisson's ratio on G_{IIc}

3-6 Influence of the geometric parameters of the ENF test specimen on mode-II failure

This section presents a parametric study of the geometric parameters of the ENF test specimen. To this end, a total of sixteen modeling groups of fifty models are considered. The first four modeling groups intend to assess the influence of the length of specimen and its variability on the mode-II fracture toughness and its variability. Four deviations are considered for the length of specimen based on the study of Ducept et al. [16]. As a result, four normal distributions with a constant mean value of 101.6 mm and different COVs are considered to generate fifty distinct values of the length of specimen for each modeling group. Table 3-21 provides the modeling group and the coefficient of variation of the length of specimen for each group.

Table 3-21 Modeling groups corresponding to the length of the ENF test specimen

Modeling group	Number of test models, n	COV of the length of specimen
1	50	0.00835
2	50	0.01332
3	50	0.01843
4	50	0.02329

The length of specimen differs for all two hundred models. The parameter set tool is used in ANSYS® to prevent the repetition of designing and performing many simulations and prepare the ground for entering different length values of the specimen and extracting the outputs efficiently. In order to determine the coefficient of variation of the mode-II energy release rate, it is calculated individually for every modeling group. Figure 3-17 compares the COV of the mode-II energy release rate in all modeling groups with different COVs of the length of specimen. There is an ascending linear relation between the coefficient of variation of the length of specimen and that of G_{IIc} . The COV of the mode-II fracture toughness rises increasingly by increasing the coefficient of variation of the length of specimen.

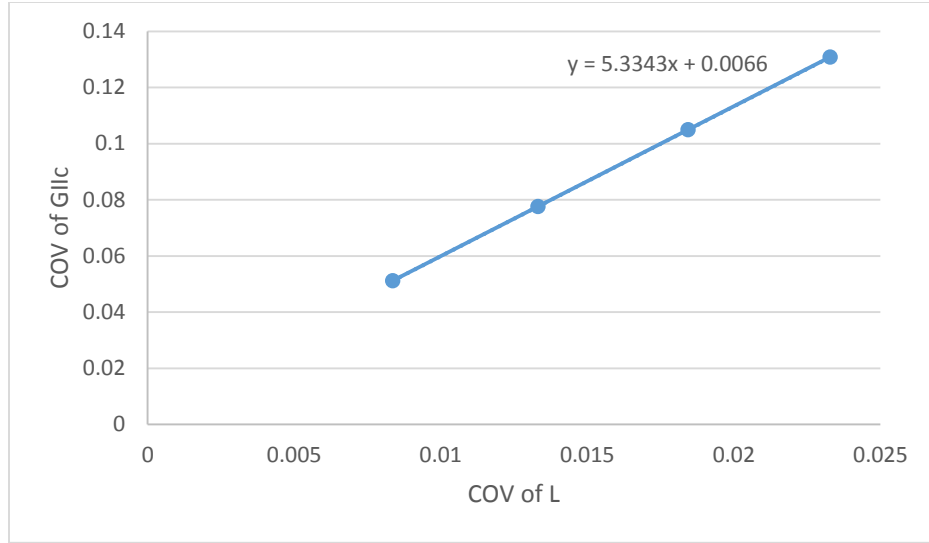


Figure 3-17 Effect of the variability in the length of the specimen on G_{IIc}

Four different modeling groups are designed to analyze the effects of the thickness of composite laminate specimen on the mode-II failure test. Deviations in the values of the thickness of specimen is considered based on the study of Ducept et al. [16]. Four normal distributions with different COVs result in four different modeling groups with fifty models, as shown in table 3-22.

Table 3-22 Modeling groups corresponding to the thickness of the ENF test specimen

Modeling group	Number of test models, n	COV of the thickness of specimen
5	50	0.00907
6	50	0.01796
7	50	0.02327
8	50	0.0279

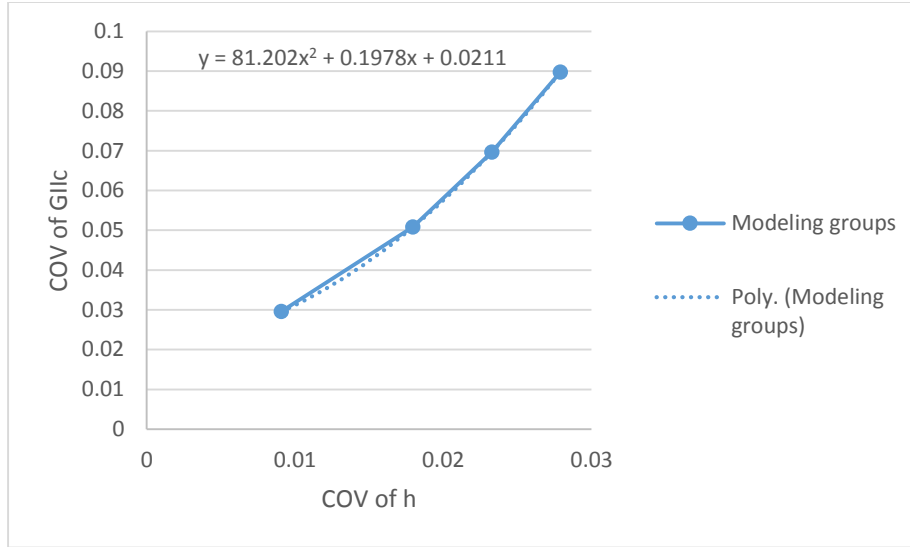


Figure 3-18 Effect of the variability in the thickness of specimen on G_{IIc}

Figure 3-18 shows that the relationship between the coefficient of variation of the thickness of specimen and the COV of the mode-II fracture toughness among these two hundred models best fits to a polynomial equation of order two, as shown in the dotted line in the legend of the figure. It is evident from the figure above that the coefficient of variation of mode-II energy release rate witnesses a substantial growth with an increase of the COV of the thickness of specimen.

As for assessing the influence of the width of specimen on the outcomes of the ENF test, modeling groups of different models considering deviations in the width of specimen are developed. The COVs of the width of specimen are considered based on the study of Ducept et al. [16]. A total of two hundred models are created that the values of the width of specimen are assigned based on four normal distributions with a constant mean value of 25.4 mm. Table 3-23 provides the number of the ENF test models, as well as the COV of modeling groups 9 through 12.

Table 3-23 Modeling groups corresponding to the width of the ENF test specimen

Modeling group	Number of test models, n	COV of the width of specimen
9	50	0.00895
10	50	0.0175
11	50	0.02108
12	50	0.0249

The energy release rate in mode-II is calculated for all 200 models individually. The coefficient of variation of the results is computed for every group and is given in figure 3-19. It is evident from the figure below that the COV of the mode-II fracture toughness has a linear relationship with that of the width of specimen. The coefficient of variation of mode-II fracture toughness increases moderately with the increase of the COV of the width of specimen. The increase is less significant compared with the rise in the COV of G_{IIc} caused by varying the coefficients of variation of the length and thickness of specimen.

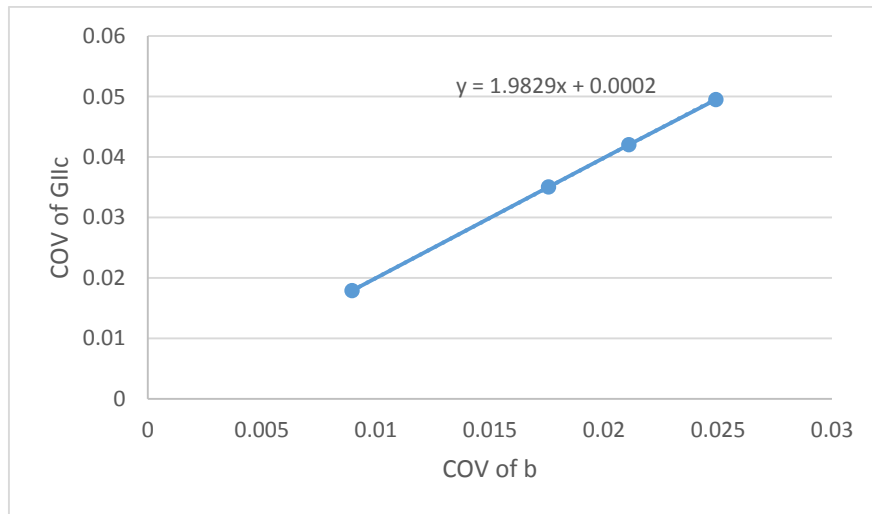


Figure 3-19 Effect of the variability in the width of specimen on G_{IIc}

In the present investigation, the last geometric property studied is the delamination or pre-crack length of the ENF test specimen. One of the most critical inputs of the numerical analysis of the ENF test is the pre-crack length of specimen. It affects the results of the simulation of mode-II failure tests. Table 3-24 lists four groups of fifty models that each one has a different coefficient of variation of the pre-crack length considered based on the study of Ducept et al. [16]. This categorization allows one to observe how varying the coefficient of variation of the pre-crack length of specimen affects the COV of the mode-II fracture toughness.

Table 3-24 Modeling groups corresponding to the crack length of the ENF test specimen

Modeling group	Number of test models, n	COV of crack length of specimen
13	50	0.00895
14	50	0.01668
15	50	0.0265
16	50	0.0365

The mode-II energy release rate is calculated based on the modeling results obtained for each model. Subsequently, the coefficient of variation of mode-II fracture toughness is calculated for all modeling groups individually. It is, therefore, possible to track whether changing the COV of the pre-crack length affects the coefficient of variation of G_{IIc} (Figure 3-20). The legend of figure 3-20 contains a line with a solid point and a dotted line that represent the correlation between the COV of the crack length of specimen and that of G_{IIc} in the modeling groups, and the best fit curve of this correlation, respectively. It can be cited that increasing the COV of the crack length results in a gradual increase in the coefficient of variation of mode-II fracture toughness. An upward trend of the coefficient of variation of the crack length from 0.89% to 3.65% brings about an increase in the COV of mode-II energy release rate from 3.3% to 4.69%.

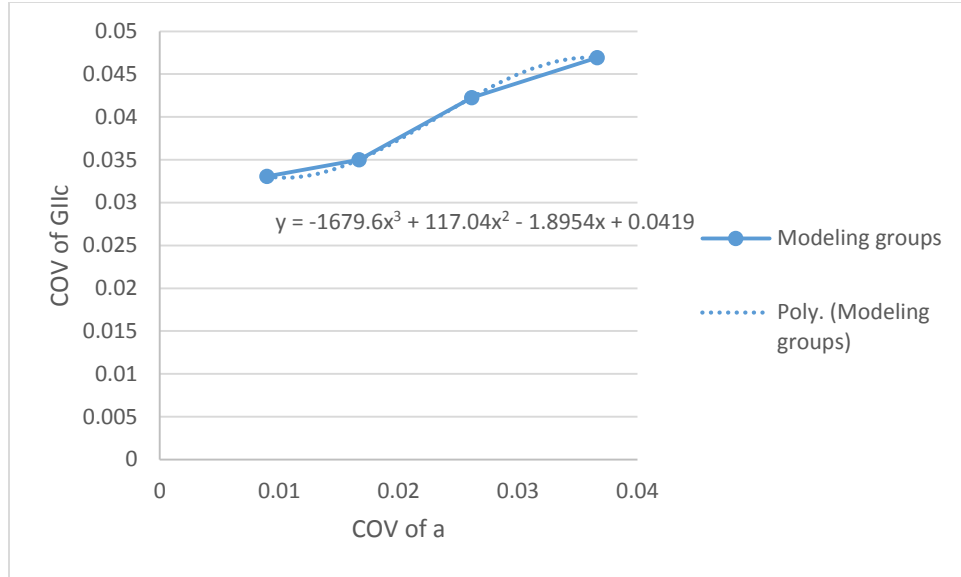


Figure 3-20 Effect of the variability in the crack length of specimen on G_{IIc}

Aiming at tracking the influences of the dimension characteristics and pre-crack length of the ENF test specimen on the mode-II fracture toughness, simultaneous variation of these parameters are considered by modeling described in the following. Set 6 is the primary specimen of graphite-epoxy with the same geometric properties as the principal specimen considered in chapter 2. All the mechanical and cohesive properties of the specimen are kept as constant. In contrast, the main values of the geometric parameters of specimen vary between -5% and 5% of the primary specimen, from set 1 to set 11. Every parameter has a normal distribution and a constant coefficient of variation of about 3%. Table 3-25 shows the geometric parameters of eleven sets in the ENF test.

Table 3-25 Geometric parameters of eleven ENF test specimens

Set numbers	L (mm)	h (mm)	b (mm)	a (mm)
Set 1	48.26	2.1375	24.13	24.13
Set 2	48.768	2.16	24.384	24.384
Set 3	49.276	2.1825	24.638	24.638
Set 4	49.784	2.205	24.892	24.892
Set 5	50.292	2.2275	25.146	25.146
Set 6	50.8	2.25	25.4	25.4
Set 7	51.308	2.2725	25.654	25.654
Set 8	51.816	2.295	25.908	25.908
Set 9	52.324	2.3175	26.162	26.162
Set 10	52.832	2.34	26.416	26.416
Set 11	53.34	2.3625	26.67	26.67

The results of the mode-II failure test simulation are plotted versus different sets in figure 3-21. In this figure, x-axis presents different sets, and y-axis shows the values of different geometric parameters all in mm. There is a linear relationship between mode-II fracture toughness and geometric variant models, which may be seen in the figure. The mode-II energy release rate increases when the geometric properties of the ENF test specimen increase. Set 11 with the largest geometric parameters values indicates the lowest mode-II fracture toughness after the ENF test modeling. On the contrary, set 1, in which the mean values of the length, thickness, and width and the pre-crack length of the test specimen are 5% less than the primary model, indicates the highest value of mode-II fracture toughness at nearly 0.85 N/mm.

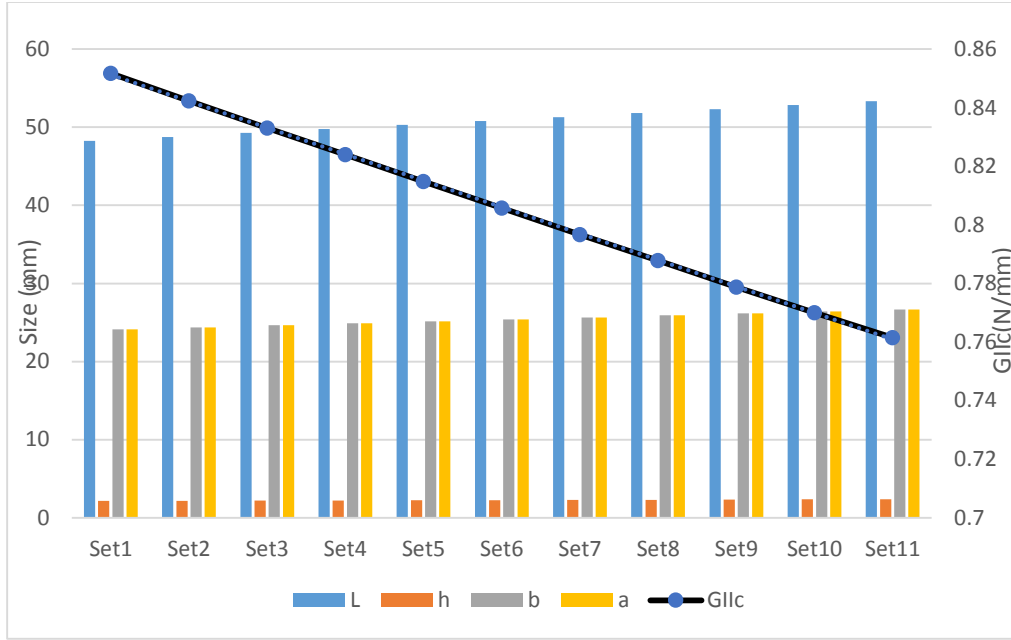


Figure 3-21 Effect of the variabilities in the geometric properties of specimen on G_{IIc}

3-7 Influence of applied displacement on the ENF test result

The influence of varying applied displacement on the middle of the test specimen on mode-II fracture toughness is investigated. Varying applied displacement in the ENF test changes the mode-II energy release rate. Considering a testing machine is designed according to standard practices of testing machines [95], there may be $\pm 1\%$ error in the test machine. In the present work, the variation in applied displacement is considered in a bit wider range. In order to assess this effect, normal distributions of applied displacement with the constant mean value of 4.5 mm and different COVs of 0.95%, 1.7%, 2.1%, and 2.55% are considered that would result in four modeling groups, each containing 50 models with different applied displacements. These deviations are selected similar to the deviations in the geometric parameters of the specimen which were considered based on the study of Ducept et al. [16]. Table 3-26 lists different modeling groups among which the COV of applied displacement is distinctive.

Table 3-26 Modeling groups corresponding to the applied displacement of the ENF test

Modeling group	Number of test models, n	COV of applied displacement
1	50	0.0095
2	50	0.017
3	50	0.0211
4	50	0.0255

ANSYS® is used to run four simulations of the modeling groups in the parameters module. Varying displacement values for each model is considered while all geometric parameters and mechanical properties of the specimens are kept as constant. Fifty distinct values of mode-II fracture toughness for each group of models result from the modeling specimens. The coefficient of variation of the mode-II energy release rate is calculated for each modeling group, and the results are represented in figure 3-22.

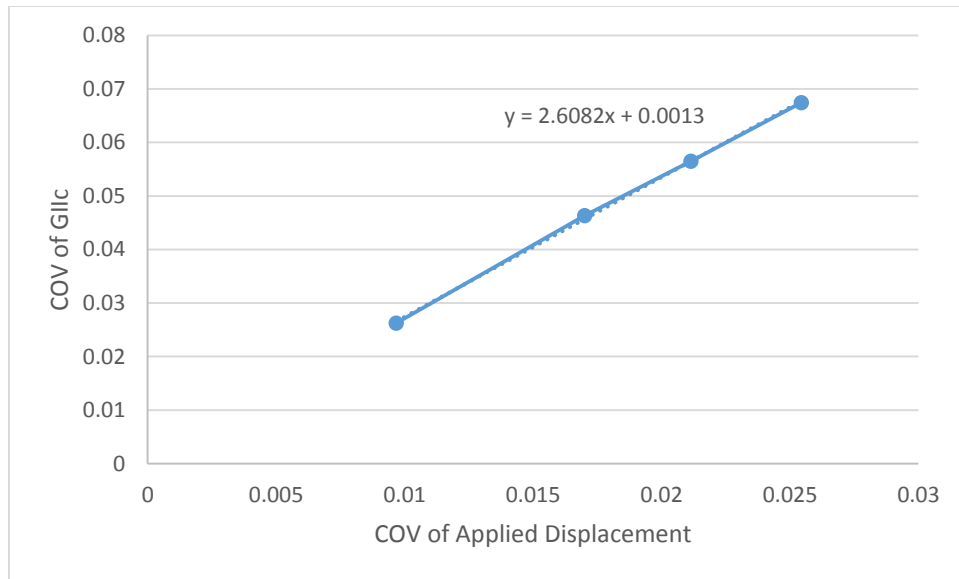


Figure 3-22 Effect of the variability in applied displacement on G_{IIc}

The figure above illustrates almost a linear relationship between the coefficient of variation of the applied displacement and the COV of mode-II fracture toughness in the ENF test. The slope of this linear equation is approximately 2.6, which means that by rising the COV of the applied displacement, the COV of mode-II fracture toughness increases more considerably.

3-8 Influence of in-plane fiber misalignment on mode-II failure

As mentioned earlier, the existence of defects and flaws in composite structures is inevitable. Defects such as the void, pore, resin rich zones, wrinkles, and in-plane fiber misalignment or waviness are formed due to manufacturing and curing processes. This section investigates the effect of in-plane fiber misalignment on the mode-II fracture toughness in the ENF test. To this end, a numerical analysis of twenty samples consisting of in-plane fiber misalignment subjected to the ENF test is carried out in ANSYS®. Based on the study of Lightfoot et al. [45], using a U-shaped aluminum tool for manufacturing the graphite-epoxy laminate during hand layup is a source of uncertainty, which results in wrinkles and in-plane fiber misalignment. They used an imaging technique and an image processing algorithm that was already written in MATLAB® and used by Yurgartis [46] to quantify the in-plane misalignment of the 0° plies. According to the results of their experimental study, the in-plane misalignment was observed particularly in 0° plies within the first four plies up to about 40°. The proportions of the in-plane misalignment of the 0° plies are indicated in six intervals, as shown in table 3-27.

Table 3-27 Proportions of in-plane fiber misalignment [45]

Interval	1	2	3	4	5	6
Fiber misalignment θ	$0^\circ \leq \theta < 3^\circ$	$3^\circ \leq \theta < 10^\circ$	$10^\circ \leq \theta < 19^\circ$	$19^\circ \leq \theta < 29^\circ$	$29^\circ \leq \theta < 40^\circ$	$\theta \geq 40^\circ$
Proportions (%)	55%	27%	7%	4%	5%	2%

In order to see the effect of in-plane fiber misalignment, twenty samples with different lay-up sequences are considered. The lay-up sequence of these samples after the curing process is considered according to the proportions of in-plane fiber misalignment observed in the experimental study of Lightfoot et al. listed in table 3-27 [45]. The first four plies on the tool side in the U-shape tool used for the curing process experienced more significant waviness due to the effect of the in-plane compression [45]. According to the six intervals of in-plane fiber misalignment and their proportions, uniform distribution is used to define fiber angles of different plies in every $[0]_{12}$ graphite-epoxy composite laminate sample after the curing process. Twelve fiber angles are arranged from the largest to the smallest to assign to the plies of sample from the first ply to the last ply to obtain the lay-up sequences of all the samples. The above approach is used twenty times to create twenty samples with different lay-up sequences as illustrated in table 3-28. All these samples are simulated and subjected to the ENF test using 3D modeling-based in ANSYS®.

Table 3-28 Lay-up sequences of the ENF test samples

Sample number	Lay-up sequence after the curing process
1	[25/14/9/8/2/1/1/0/0/0/0/0]
2	[31/11/8/7/3/2/1/1/0/0/0/0]
3	[18/18/9/7/4/2/2/1/0/0/0/0]
4	[8/1/1/1/1/1/1/1/0/0/0/0]
5	[32/9/4/3/3/2/2/2/1/1/0/0]
6	[29/11/11/5/3/2/2/1/1/1/0/0]
7	[12/9/8/4/3/3/3/2/2/2/1/0]
8	[12/8/7/7/6/5/4/4/4/1/1/1]
9	[6/5/4/3/2/2/1/1/0/0/0/0]
10	[33/7/6/2/1/1/1/1/1/0/0/0]
11	[24/8/6/4/4/3/3/2/1/1/1/0]
12	[34/14/8/6/3/2/2/2/1/0/0/0]
13	[39/8/5/4/4/3/2/1/1/0/0/0]

14	[30/9/7/7/4/4/1/1/1/0/0/0]
15	[16/11/5/4/3/2/2/2/2/1/0]
16	[23/8/6/6/5/3/2/1/1/0/0/0]
17	[9/7/6/6/4/3/2/2/1/1/1/1]
18	[15/7/7/5/5/3/2/2/1/1/1/0]
19	[8/7/4/3/3/2/2/1/1/0/0/0]
20	[3/3/2/0/0/0/0/0/0/0/0/0]

Numerical analysis of the effect of the in-plane fiber misalignment on the mode-II energy release rate is explained briefly in the following. In the ANSYS® toolbox, ACP (Pre) is selected after expanding the component systems tab. This module is for designing and composite laminate modeling. Building-up of the shell composite is performed in the geometry module, and the solid composite data are determined in the setup tool of ACP (Pre) by defining the composite laminate plies thickness and lay-up sequence. The model module of ACP (Pre), compatible with the static structural analysis system, allows one to define mesh to the part. In order to have a refined mesh, two meshing methods are applied. First, all the model edges are assigned to edge size meshing. The longitudinal edges of the model are selected as edge meshed with an element size of 1.5 mm. Besides, 2 mm element size is used to mesh the width edges of the model. Then, face meshing has been used to mesh the whole part. The quadrilaterals method is used to mesh the shell parts. In addition, in this module, faces and edges are added to the named selections so as to be readily referred to in other analysis steps. The composite material and cohesive material properties are determined in the engineering data module. Table 3-29 illustrates the resin epoxy property set chosen from the ANSYS® library. It has isotropic properties with two independent engineering properties.

Table 3-29 Resin epoxy properties

E ₁	E ₂	E ₃	ν_{12}	ν_{13}	ν_{23}	G ₁₂	G ₁₃	G ₂₃
3780 MPa	3780 MPa	3780 MPa	0.35	0.35	0.35	1400 MPa	1400 MPa	1400 MPa

The ply thickness of 0.1875 mm is defined through the fabric tool in the ACP (Pre) module. The direction of fibers within the plies is determined in the same direction of the length of the model through the rosette tool. Six plies have been created for the upper half of the composite laminate, and six more plies for the lower half of the laminate. On one hand, an upward vector is defined as the orientation for the top composite and crack areas, and the previously described rosette is also selected. On the other hand, a downward vector is specified as the orientation for the bottom composite and crack areas with the same rosette. Subsequently, two main modeling groups are created for defining the composite plies. The bottom ply group contains the first to sixth plies, whereas the top ply group contains the six last plies.

Every ply is created individually by determining the proper pre-defined orientation selection sets, ply material, and the fiber angle according to table 3-28 for each sample. Finally, the solid model tool in this module makes it possible to create a solid composite laminate part with all the characteristics that are already defined. At this point, ACP (Pre) modeling is completed, and the static structural analysis system is added to the project schematic. Dragging the setup option of the ACP (Pre) window to the model option of the static structural and selecting transfer solid composite data accomplishes the workflow. All the imported plies with their characteristics can be seen in the outline window of the model. Every ply containing 952 elements is noteworthy since all ply elements have the same thickness, length, and width regardless of the fiber angle. The contact interaction between the top and bottom beams is defined through two distinguished contact methods containing bonded contact and frictionless contact. The former is applied between the top composite interface of the bottom beam and the bottom composite interface of the top beam. The latter is used at the cracked side of the top and bottom beams where the initial delamination is assigned. The fracture tool in this module environment simulates the debonding occurrence by selecting the CZM method. Also, the pre-defined bonded contact between composite interfaces of the beams needs to be chosen as the contact region. Some necessary changes in auto time stepping in the analysis setting tool are made to prevent convergence issues in the problem solution. A total of fifteen thousand two hundred seventy-eight elements are used in this modeling, where solid components account for 11424 elements, and contact elements make up 3854 elements, respectively. The solid elements name is SOLID185, and the contact elements types are

CONTAC174 and CONTAC175. Additionally, the element shape of solid components is HEX8, and that of contact elements is QUAD8.

Each simulation took almost fourteen hours in a Corei7 with 16 GB RAM pc system, depending on the lay-up sequence of the sample. One of the deformed specimens as a result of subjecting to the ENF test is indicated in figure 3-23. Equivalent Von-Mises stress distribution through the model is specified by color-coding appearing at the left of the figure. As predicted, the highest stress happens at the middle of the top beam, where the downward displacement of 4.5 mm is applied. Slippage of the beams at the cracked side is significant. It is probably due to the fiber misalignment that the sliding is more considerable.

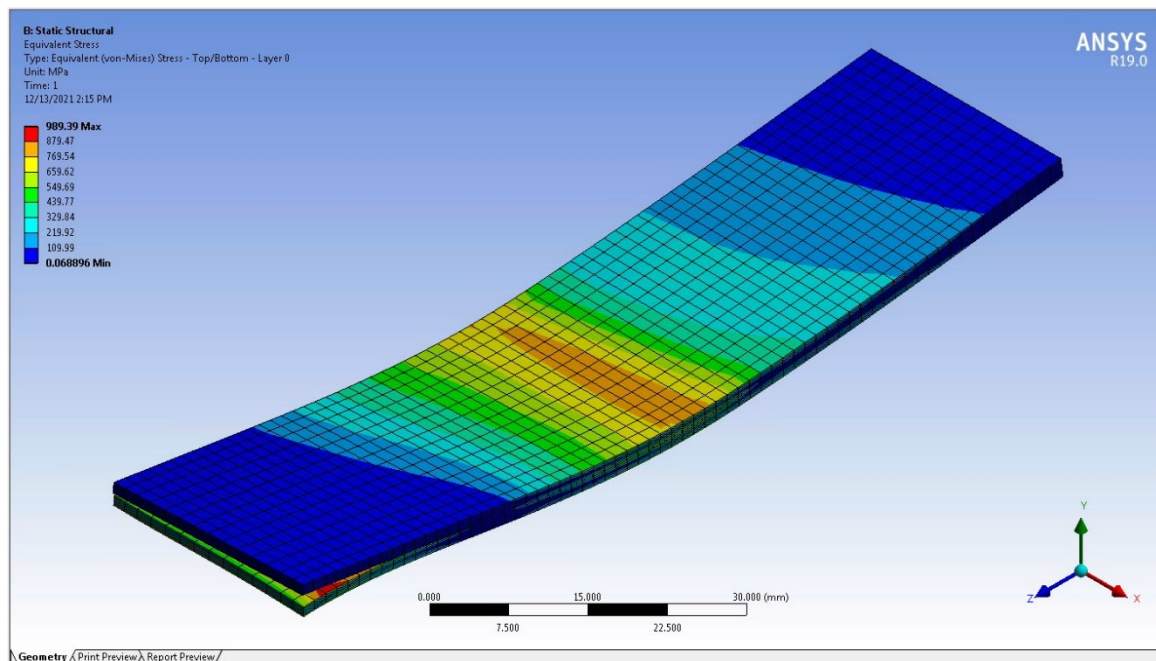


Figure 3-23 Deformed specimen under mode-II delamination test in ANSYS®

The simulation is carried out 20 times for all the 20 different samples. Each time modeling the ply groups needs to be varied. All fiber angles of the composite laminate are defined for every sample according to table 3-28. After performing simulation, every sample modeling results in specific outcomes. For instance, figure 3-24 demonstrates the results for model 20 as a load-displacement plot.

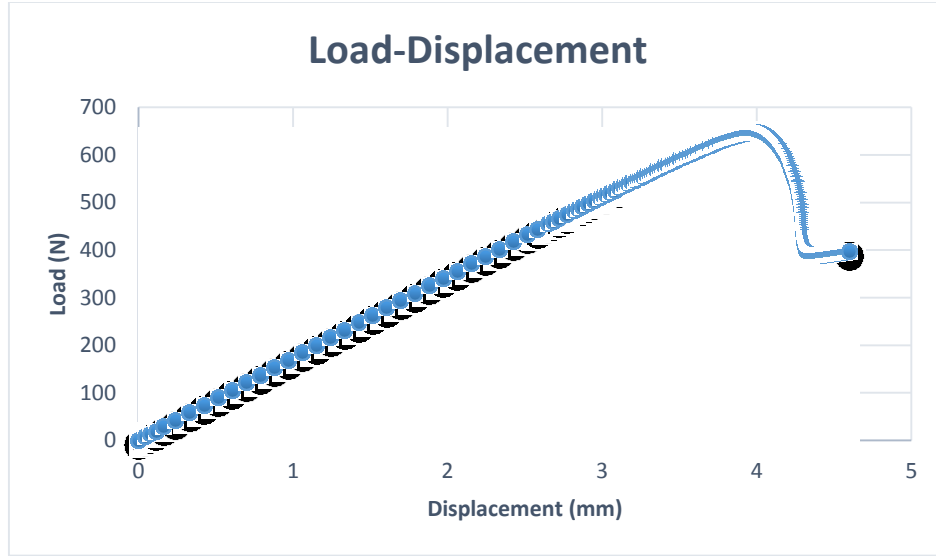


Figure 3-24 Curve of load-displacement for ENF test of specimen 20

It can be seen that the delamination occurs when the displacement approaches about 4 mm. At this point, the critical load is almost 647 N. According to the Direct Beam Theory (DBT) for the ENF test discussed in chapter 2, the energy release rate is calculated for every sample. Figures 3-25 compares the mode-II energy release rate for all the models. Due to mode-II delamination in sample 12, the highest energy release rate at just over 1 N/mm is observed. Sample 6 mode-II energy release rate is also the second-highest at approximately 1 N/mm. On the contrary, samples 20 and 4 demonstrate the lowest mode-II fracture toughness at 0.64 and 0.66 N/mm, respectively.

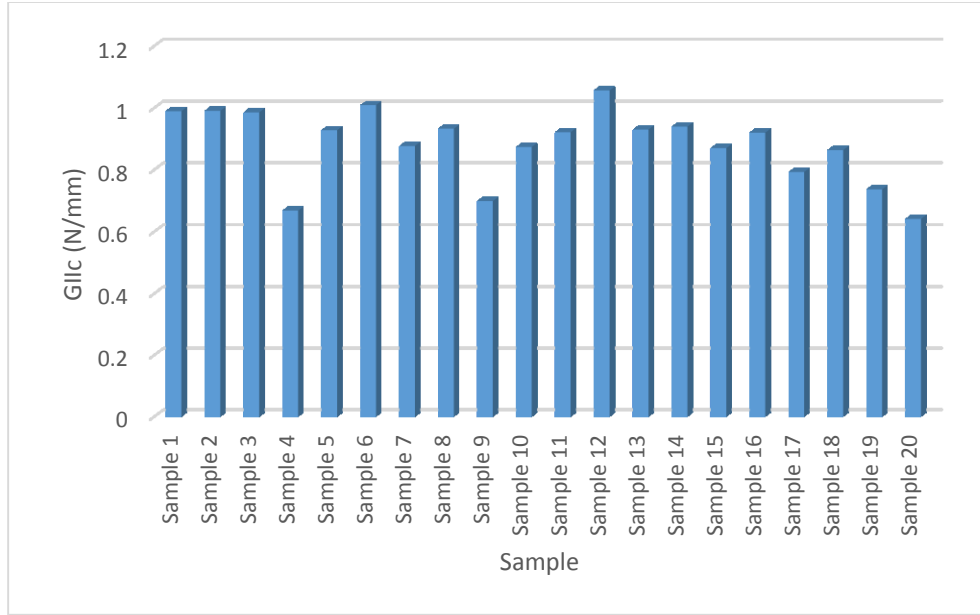


Figure 3-25 Mode-II energy release rates of 20 samples consisting of in-plane fiber misalignment

Comparing these samples indicates that the lowest mode-II energy release rate belongs to sample 20 with the slightest in-plane fiber misalignment. This sample has in-plane misalignment only in the first, second, and third plies, with the largest waviness located in the first ply, whose fiber angle is 3° . Nevertheless, sample 12 has the most substantial waviness in its plies. The first ply orientation angle is 34° , and the orientation angles of the second and third plies are 14° and 8° , respectively. Although the largest fiber angle is attributed to the first ply of sample 13 at 39° , the mode-II energy release rate in this sample was 0.93 N/mm , which is lower than that of samples 12 and 6. Comparing the second and third ply orientation angles of sample 13 with those of samples 12 and 6 demonstrates that in addition to the orientation angle of the first ply, the orientation angles of the second and third plies play an important role in the mode-II fracture toughness. By reviewing the values of the mode-II fracture toughness, with regard to the lay-up sequences of the samples, it can be noted that the higher misalignment severity of one-ply in a composite laminate does not necessarily lead to a higher mode-II energy release rate. However, composite laminate samples with higher in-plane fiber misalignment of the first three or four plies demonstrate higher mode-II energy release rate values. Therefore, the location of the misaligned plies is also important in addition to the misalignment severity, and this is discussed further in the next section. Furthermore, it can be expressed that the in-plane fiber misalignment higher than 5° is likely to affect the energy release rate of the composite material.

3-9 Influence of in-plane fiber misalignment location on mode II failure

This section concerns with the misalignment location in the stacking sequence and the specific effect that it may have on the mode-II interlaminar fracture toughness. A ply group including three misaligned plies is assumed in every sample concerning the in-plane fiber misalignment location in the composite laminate. According to the study of Lightfoot et al. [45], average misalignments for the plies of the graphite-epoxy composite laminate are considered. The plies in the ply group are assumed to be together with ply orientation angles of 3°, 5°, and 7° from top to bottom within the ply group. The composite laminate only has a defective part where the ply group is located; i.e., in all samples, other 0° plies are flawless. Since in the study of Lightfoot et al. [45], the in-plane misalignment defect under 8° was observed for all plies, here the defective ply group location is considered to vary by moving from the first plies at the bottom of the composite laminate to the last plies at the top. The central ply of the ply group, the fiber angle of which is 5°, is chosen to be moved from the second ply to the eleventh ply among different models. The central ply of the misalignment ply group may be located between the 2nd and 11th plies. Fifty samples of 12-ply graphite-epoxy composite laminate, [0]₁₂, are created to be examined and simulated subjected to the ENF test. Since the 12-ply laminate specimens are considered, the location of central ply of the defective ply group can move between the second and eleventh plies. The location of the central ply of the defective ply group is determined according to a uniform distribution between two and eleven and rounding the generated numbers. Table 3-30 shows all the created samples with different positions for the central ply of the ply group, which indicates the location of the ply group. Observing from the table below, after rounding the uniformly distributed values which represent the location of the central ply of the ply group in the stacking sequence of laminate specimens, there is not any model with a misaligned central ply situated on the 11th ply.

Table 3-30 Fifty ENF test samples with different locations of the misaligned ply group

	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7	Set 8	Set 9	Set 10
Number of samples	2	4	6	6	8	4	6	7	7	0
Ply number of central ply	2	3	4	5	6	7	8	9	10	11

The Finite Element modeling is conducted to complete the modeling of ply group location in ANSYS®. ACP (Pre) is used to design the modeling and the relevant steps have been described in section 3-8. Nevertheless, the orientation angles of plies must be defined in the modeling group tool. As an illustration, two models of set 1 have the same ply group arrangement, according to which the first, second and third ply orientation angles are 7°, 5° and 3°, respectively. In the same way, all models are created based on their ply group locations. Meshes with edge sizes of 1.5 mm and 2 mm are used throughout the length and width of the specimen. Every ply of the model contains 952 SOLID185 elements, each of which has a thickness of 0.1875 mm. A total of 11424 SOLID185, 3808 CONTAC174 and 46 CONTAC175 elements make up the entire specimen model with a total of 15278 elements.

Based on the simulation results, the mode-II fracture toughness is calculated. By numerical analysis, the defect-free composite laminate has a computed mode-II energy release rate of 0.6345 N/mm after undergoing the ENF test. Due to the similar misaligned position of ply groups across all models of the same set, it is evident that they all produce the same results. Figure 3-26 shows the figures for all fifty models of sets one to ten. The mode-II energy release rate is influenced by the location of the defective ply group, as described in this section. As compared to the pristine model, the calculated mode-II energy release rate increases in the models of all sets except sets 6, 7, and 8.

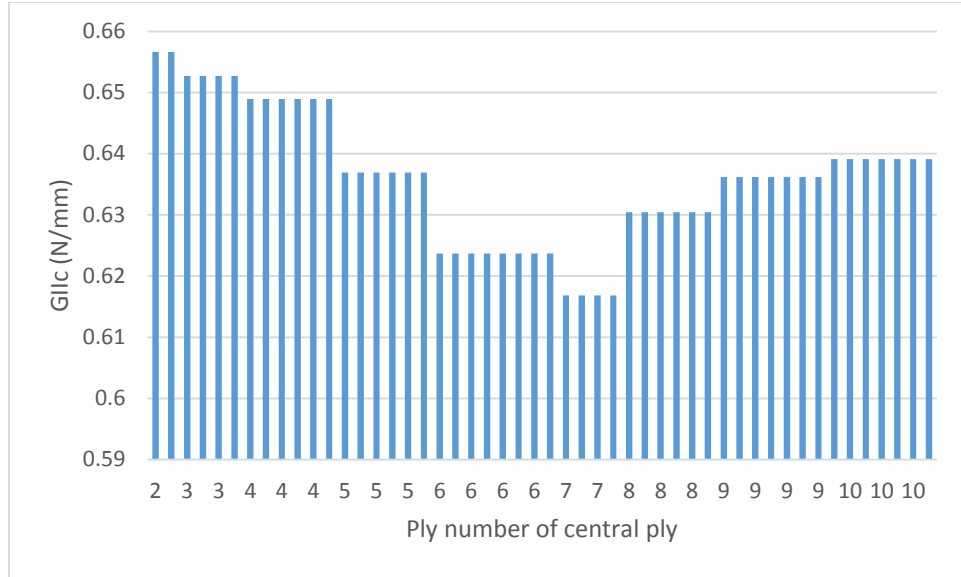


Figure 3-26 Mode-II energy release rates of 50 specimens

According to the figure above, four models of set 6 with the misaligned central ply located as 7th ply reveal the lowest mode-II fracture toughness at nearly 0.61 N/mm; whereas the mode-II energy release rate in two models of set 1 in which the misaligned ply group is at the very bottom of the composite laminate is the highest at well over 0.65 N/mm. A wide u-form shaped pattern in figure 3-26 indicates that the existence of a misaligned ply group at the bottom and top of the composite laminate brings about higher mode-II fracture toughness while undergoing the ENF test. Besides, by moving the ply group to the central plies of the composite laminate, the energy release rate in mode-II experiences a decline.

3-10 Conclusion

In this chapter, the effects of engineering properties of composite laminate and their variability on the mode-I energy release rate and its variability have been investigated using the FEA tool ANSYS®. Deviations in engineering constants have been considered based on an existing experimental study. Results indicate that the longitudinal elastic modulus has the most profound influence on the obtained mode-I energy release rate in the DCB test among all engineering properties. The coefficient of variation of the mode-I fracture toughness increases considerably with the fluctuation of E_{11} . However, numerical analysis reveals that the out-of-plane elastic modulus, E_{33} , and out-of-plane shear modulus, G_{23} , have no significant influence on the mode-I energy release rate in DCB testing. Further, the numerical and probabilistic analyses of the effects of the geometric parameters of the DCB test specimen have been carried out. Results show that the deviation of data in all parameters causes some variation in the mode-I energy release rate results. It can be concluded that the mode-I energy release rate decreases by increasing the values of the geometric parameters of the DCB test specimen. The largest variability of mode-I fracture toughness has been seen where there is a deviation in the length of the DCB specimen. The modeling group, in which the COV of the length of specimen is 2.33%, has resulted in the COV of mode-I fracture toughness of 13.4%. Taking the effects of the variabilites in all the inputs into account, the variation of geometric parameters of the DCB test specimen causes more variability in the mode-I fracture toughness compared with the variation of the material properties of specimen. For instance, the variability in the length of DCB test specimen with the COV of 2.33% leads to the COV of 13.41% in the mode-I fracture toughness values; while , the variability in the longitudinal elastic modulus of the DCB test specimen with the COV of 9.96% results in the COV of 6.27% in the mode-I fracture toughness values. It has been seen that the effect of the variability in the length of the specimen on the variability in the mode-I fracture toughness is 2 times more considerable than the variability in the E_{11} of the composite material specimen. Tables 3-31 and 3-32 provide the ranking of the material properties and geometric parameters of the DCB test specimen, respectively, based on their influence on the variability in the mode-I fracture toughness.

Table 3-31 Ranking of the influential material properties of the DCB test specimen

Rank	Material properties
1	E_{11}
2	E_{22}
3	G_{12}
4	ν_{23}
5	ν_{12}
6	E_{33}
7	G_{23}

Table 3-32 Ranking of the influential geometric parameters of the DCB test specimen

Rank	Geometric parameters
1	L
2	b
3	h
4	a

Moreover, numerical analysis of the ENF test has been conducted repeatedly to observe the effects of the engineering properties and the geometric parameters of test specimen and their variabilities on the mode-II energy release rate and its variability using the FEA tool ANSYS®. The numerical analysis illustrates that there is a linear correlation between the COV of G_{IIc} and that of E_{11} . The COV of the mode-II energy release rate increases moderately to just over 11.4%, with the increase of the COV of E_{11} to 10.57%. Longitudinal Young's modulus has the most significant effect on G_{IIc} among all material properties. Nonetheless, variations in E_{33} and G_{23} have not significantly affected the mode-II energy release rate. As for the geometric parameters of the ENF test specimen, numerical analysis results indicate that the thickness and length of specimen significantly influence the mode-II energy release rate. The COV of the mode-II fracture toughness increases to approximately 9% with the increase of COV of the thickness of specimen. Besides, the increase of the COV of the length of specimen to 2.32% results in an increase of about 13% in the COV of

the mode-II energy release rate. Altogether, it has been observed that the effect of the variations of the geometric parameters of the ENF test specimen on the variability of the mode-II fracture toughness is more significant than the variations of the engineering properties of the specimen. Tables 3-33 and 3-34 provide the ranking of the material properties and geometric parameters of the ENF test specimen, respectively, based on their influence on the variability in the mode-II fracture toughness.

Table 3-33 Ranking of the influential material properties of the ENF test specimen

Rank	Material properties
1	E_{11}
2	E_{22}
3	ν_{23}
4	G_{12}
5	ν_{12}
6	E_{33}
7	G_{23}

Table 3-34 Ranking of the influential geometric parameters of the ENF test specimen

Rank	Geometric parameters
1	L
2	h
3	b
4	a

It can be seen that the rankings of the influential material properties of test specimens for both the DCB and ENF tests are alike except the out-of-plane Poisson's ratio which is placed in the third

place among material properties of the ENF test specimen, whereas for the DCB test it is placed in the fourth place of ranking. That is, the out-of-plane Poisson's ratio has more significant influence in the variability in mode-II fracture toughness than mode-I fracture toughness. On the other hand, the influence of the in-plane shear modulus on the mode-I fracture toughness is more significant than its influence on the mode-II fracture toughness.

The transverse elastic modulus, E_{22} , ranked as the second most influential parameter among all material properties in the DCB and ENF tests. It can be interpreted that the material properties of resin used in composite laminates play an essential role in the mode-I and mode-II fracture toughnesses. If the resin of the composite laminates used in the DCB and ENF test specimens varies, the mode-I and mode-II fracture toughnesses of the composite laminates change accordingly. It has been proven that in addition to the material properties of the selected fiber, the material properties of the chosen resin significantly influences the fracture toughness of composite laminates.

As for the geometric parameters, in both the DCB and ENF tests, the length of the specimens has the most significant influence on the mode-I and mode-II fracture toughnesses and their variabilities. However, the influence of the thickness of specimen is more considerable on the mode-II fracture toughness and its variability compared with the mode-I fracture toughness and its variability. Thus the thickness of the specimen is ranked in the second place in the ranking of influential geometric parameters for the ENF test, however, it is ranked in the third place for the DCB test.

Furthermore, the influence of in-plane misalignment as a curing defect in ENF test specimen on the mode-II interlaminar fracture toughness has been investigated through numerical analysis using ANSYS®. Twenty specimens with different in-plane misalignment severities have been simulated under the mode-II failure test. FEA outcomes indicate that the specimen with the largest misalignments in all the first four plies results in the largest mode-II energy release rate. Besides, a ply group consisting of three misaligned plies is considered to study the effect of the location of misalignment on the mode-II energy release rate in the ENF test. Numerical analysis on a laminate specimen with 12 plies illustrates that the largest mode-II energy release rate is obtained when the center of the misalignment ply group is located in the 2nd ply. In contrast, the lowest mode-II energy release rate corresponds to the specimen where the center of the misalignment ply group is

located in the 7th ply. Overall, when moving the misaligned ply group of test specimen models from the middle plies toward the top and bottom plies, the specimen experiences a higher mode-II energy release rate. Also, the largest value of mode-II energy release rate has been obtained to be well over 0.65 N/mm in the models in which the central ply of the misaligned ply group is located in the second ply. On the contrary, the models in which the misaligned ply group is located in the 7th ply has indicated the lowest mode-II energy release rate value of 0.61 N/mm.

Chapter 4

Reliability assessment and sensitivity analysis of the composite laminates in the DCB and ENF tests

4-1 Introduction

In chapter 2, the reliability concept and its necessity in engineering industry were emphasized. Also, a few common approaches of reliability analysis were briefly explained. This section describes the reliability analysis for the delamination growth in the DCB and ENF tests. Six reliability approaches are used through using the software Rtx[®], and the outcomes are discussed later in this chapter. In order to embark on a reliability analysis of delamination property testing, one must determine the random variables and the limit state function of the delamination growth problem. As discussed in chapter 3, the deviations in the engineering properties and geometric parameters of composite laminate specimen profoundly affects delamination property of composite materials. Uncertainties in the engineering properties and geometric parameters of composite specimens may be caused by manufacturing, curing, or practical issues described in chapters 2 and 3. In this chapter, carried out are the static reliability analyses of mode-I and mode-II delamination tests. All random variables have been considered to change in defined ranges. It is worthwhile to mention that the random variables such as that of the material properties of the composite laminates may vary in time, particularly when specimens are subjected to fatigue loading. However, in the present probabilistic approach, since the static reliability assessment has been considered, the variabilities of the random variables with respect to time are neglected.

All these sources of uncertainties need to be considered in the reliability analysis of composite testing. Given the two necessary delamination property tests, DCB and ENF tests, characterization of the above effects based on these two delamination tests on the reliability of the delamination growth in the composite materials is one of the principal goals of the study presented in this chapter. Besides, based on the sensitivity analysis results, a ranking of random variables in order of their importance in defining the probability of delamination growth in both the mentioned tests is determined and provided.

The software Rtx[®] [96-98] is used to assess the probability of delamination growth in the DCB and ENF tests. This computer software used for reliability and risk analysis is a multi-model analysis software developed with C++ using Qt's cross-platform development tool [96-98]. It is available on Windows, Mac OS X, and Linux. It has a library of probabilistic models. The software can conduct reliability and risk analysis [96-98]. Rtx[®] consists of two main modules: Models and Methods. The former lets users access different modeling types such as random variables, decision variables, time and location variables, and another user-defined modeling. Every tool of the software can share data among themselves and save or process. In the Methods module, the analyzer option helps select the analysis approach, and subsequently, necessary tools for every analysis can be selected in the Objects pane. All objects in conjunction with each other operate to complete the analysis, which is the reliability of delamination growth in the present study. The following sections narrate the utilization of the software in the reliability analysis of the composite materials under two main modes of delamination.

4-2 Reliability analysis of mode-I delamination

Considering the specimen with an initial crack and pre-set dimensions, as illustrated in figure 4-1, for the DCB test, the engineering constants and geometric parameters of the test specimen are random variables of the problem. First, a limit state function comprising random variables has to be defined. According to the basic reliability problem, discussed in chapter 2, every limit state function has capacity and load components. Mode-I delamination occurs when the mode-I energy release rate exceeds the mode-I critical energy release rate. At this point, delamination grows, and failure occurs. Thus, the limit state function for mode-I can be formulated as given below.

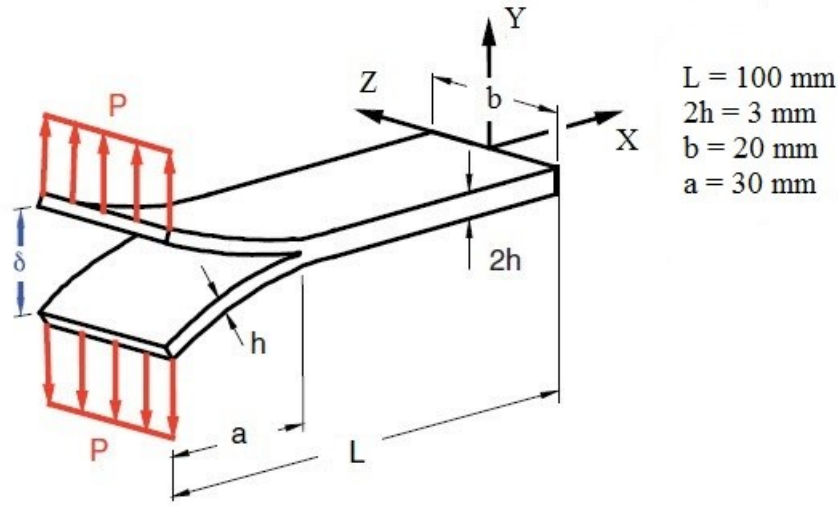


Figure 4-1 Configuration and geometry of the DCB test specimen [25]

$$g_I(x) = G_{Ic} - G_I \quad (4-1)$$

G_{Ic} is the critical energy release rate in mode-I, also known as mode-I interlaminar fracture toughness, and G_I is the mode-I energy release rate as a result of the DCB test. The former is the capacity component of the reliability problem. Based on the compliance definition in the Direct Beam Theory (DBT) previously discussed in chapter 2, the energy release rate in mode-I can be computed as follows.

$$G_I = \frac{12P^2a^2}{b^2h^3E_{11}} \quad (4-2)$$

where P is the applied load at the end of the beam, E_{11} is the longitudinal Young's modulus, and a , b , and h denote the crack length, width, and half of the thickness of the test specimen, respectively, as seen in figure 4-1. Regarding the compliance definition for the DCB test, which has been described in chapter 2, equation (4-2) can also be expressed as follows in terms of the displacement at the end of the specimen.

$$G_I = \frac{3\delta^2h^3E_{11}}{16a^4} \quad (4-3)$$

where δ denotes the displacement of the beam as shown in figure 4-1. Since the specimen undergoes an applied displacement in this work for numerical analysis of mode-I delamination, the last equation is used to calculate the energy release rate. By substituting the equation above in equation (4-1), the limit state function for the DCB test or mode-I delamination test can be rewritten as a function of the opening displacement at the free ends of the beam specimen as given below.

$$g_I(x) = G_{Ic} - \frac{3\delta^2 h^3 E_{11}}{16a^4} \quad (4-4)$$

As explained earlier, the limit state function contains five variables which represent different sources of uncertainties of the DCB test arising from manufacturing, measurement, and defects. For instance, the value of critical energy release rate in mode-I obtained from the DCB test have variation [16]. Numerical simulations based on FEA has been conducted to investigate how variations in the material properties and geometric parameters of composite laminates may affect the DCB and ENF test outcomes have been discussed in chapter 3. Here, a non-deterministic approach is adopted by evaluating the limit state function to observe the possibility of the delamination growth. First of all, five random variables are determined with their second-moment and distribution information to assess the limit state function. All random variables are assumed to be distributed normally in this study. Table 4-1 shows the variables, including engineering properties of a prepreg composite material used in the work of Alfano and Crisfield [25] and geometric parameters of the test specimen variables. The statistical information such as the coefficients of variation of variables are selected based on related previous studies [16]. In chapter 3, the variation in the mechanical properties such as E_{11} due to the deviation of experimental measurements was taken into account. The variables had been considered to vary with the coefficients of variation from nearly 0.04 to about 0.11 according to the extent of standard deviation seen in experimental studies [16]. Moreover, the coefficients of variation of geometric parameters of the specimen, such as laminate thickness, length, and crack size, changed between approximately 0.006 and 0.03. Here, these variations are chosen constant based on the study of Ducept et al. [16] experimental study. The intervals of the coefficients of variation of the variables are considered extensive enough to record how altering the COV values of inputs affects the COV values of outputs.

Table 4-1 Statistical information of random variables of DCB specimen

Random Variable	Mean Value	Unit	COV
Longitudinal Young's modulus, E_{11}	135300	MPa	0.1
Transverse Young's modulus, E_{22}	9000	MPa	0.1
In-plane shear modulus, G_{12}	5200	MPa	0.1
Out-of-plane shear modulus, G_{23}	3900	MPa	0.1
In-plane Poisson's ratio	0.24	-	0.1
Out-of-plane Poisson's ratio	0.46	-	0.1
Laminate length, L	100	mm	0.02
Laminate thickness, $2h$	3	mm	0.02
Laminate crack length, a	30	mm	0.02

It is assumed that the coefficients of variation of all engineering properties of the composite specimen are equal to 0.1 while the coefficients of variation of all of the geometric parameters of the composite specimen are equal to 0.02 [16]. The critical energy release rate is available according to an experimental study comparing the static and dynamic fracture properties of a range of composite materials [99]. Moreover, the coefficient of variation of the interlaminar fracture toughness in mode-I is determined given the crack length and based on previous experimental work on different composites [16]. Briefly noting, the coefficient of variation of the mode-I fracture toughness can be selected between 3% and 7% with regard to the type of the composite laminate. In the present study, G_{Ic} for the prepreg composite material used in the work of Alfano

and Crisfield [25] is assigned a mean value of 0.28 N/mm and a coefficient of variation of about 5% based on the experimental results of the study of Ducept et al. [16]. The collection of random variables expressing uncertainties describes the limit state function. Since the material properties and geometric parameters are the determinant variables in defining the energy release rate, considering random variables in which the uncertainties are embedded allows the non-deterministic approach to be used when determining the mode-I energy release rate in the DCB test. Once the limit state function is achieved, earlier mentioned reliability analysis methods, including FOSM, FORM, SORM, MCS, histogram, and importance sampling methods, are used to predict the probability of delamination growth under double cantilever beam test, as well as the second-moment information of the mode-I energy release rate. The outcomes of these analyses are discussed later on.

4-3 Reliability analysis of mode-II delamination

Mode-II delamination in the ENF test happens when the critical energy release rate in mode-II is surpassed by mode-II energy release rate. It corresponds to the initiation of mode-II delamination growth in the laminate. From a reliability point of view, the exceedance can be interpreted as a failure in the component. Therefore, the limit state function is formulated as given below.

$$g_{II}(x) = G_{IIc} - G_{II} \quad (4-5)$$

The ENF test specimen is shown in figure 4-2. As mentioned in chapter 2, a closed-form expression for mode-II energy release rate is available [77-79] based on the direct beam theory. Based on this theory, the energy release rate is expressed as a function of load, axial elastic modulus, crack length, and beam width, and beam thickness, as presented below.

$$G_{II} = \frac{9P^2a^2}{16b^2h^3E_{11}} \quad (4-6)$$

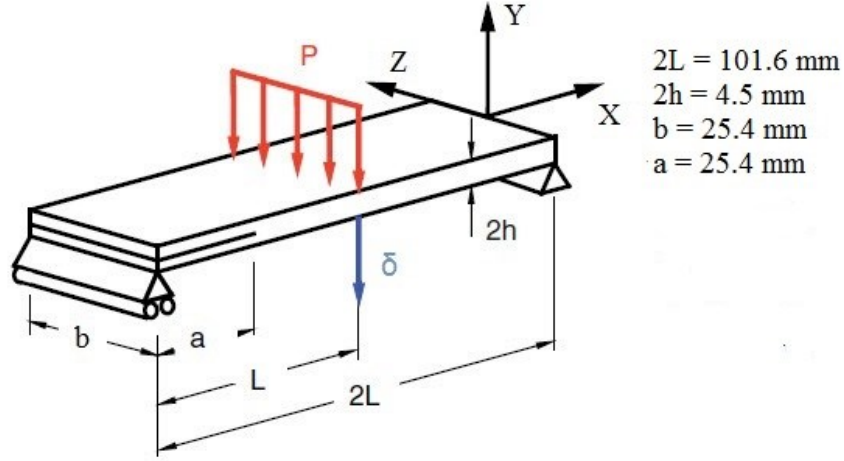


Figure 4-2 Configuration and geometry of the ENF test specimen [37]

where as shown in figure 4-2, P denotes the applied load at the middle of the beam, and a , b , and h denote the crack length, width and half of the thickness of the specimen. E_{11} is the longitudinal Young's modulus of the composite material. End notched flexure test is designed to apply displacement at the center of the beam. So it is desirable to formulate the limit state function with respect to the beam displacement rather than load. According to the compliance definition and based on the correlation between the load and displacement in the ENF test [77-79], reviewed earlier in chapter 2, the energy release rate in mode-II can also be written as a function of the central displacement of the beam as follows.

$$G_{II} = \frac{36\delta^2 h^3 a^2 E_{11}}{(3a^3 + 2L^3)^2} \quad (4-7)$$

where, δ denotes the displacement at the center of specimen and L denotes half of the length of the specimen. By substituting the equation above into the equation (4-5), the limit state function for the ENF test is expressed as follows.

$$g_{II}(x) = G_{IIc} - \frac{36\delta^2 h^3 a^2 E_{11}}{(3a^3 + 2L^3)^2} \quad (4-8)$$

This equation considers six variables that represent uncertainties arising from manufacturing, measurement, and defects. Six random variables of Gaussian distribution should be considered to represent the uncertainties. The interlaminar fracture toughness in mode-II is assigned a value of

0.78 N/mm for IM7/8852 unidirectional graphite-epoxy composite laminate according to the study of Krueger [37], with the coefficient of variation of 3% which is considered based on an experimental study [99]. As an initial step in evaluating the limit state function, data sets of inputs are generated as what has been done in the analysis of characteristics of different material properties and geometric parameters of the composite laminate specimen on mode-II delamination in ENF tests in chapter 3. Table 4-2 illustrates the random variables with specific probabilistic characteristics for IM7/8852 unidirectional graphite-epoxy composite laminate. In the generation of random numbers, all random variables have a normal distribution. ENF test specimen is the same as the one given in chapter 2, with the same mechanical and geometric properties. As can be seen, the mechanical properties of the composite laminate are permitted to vary with the coefficient of variation of 0.1, while geometric properties change with the coefficient of variation of 0.02. This COVs are selected based on the study of Ducept et al. [16] with a small modification to see the effects of the variabilities in the material properties and geometric parameters on the probability of the delamination growth more clearly.

Table 4-2 Statistical information of random variables of ENF specimen

Random Variable	Mean Value	Unit	COV
longitudinal Young's modulus, E_{11}	161000	MPa	0.1
Transverse Young's modulus, E_{22}	11380	MPa	0.1
In-plane shear modulus, G_{12}	5200	MPa	0.1
Out-of-plane shear modulus, G_{23}	3900	MPa	
In-plane Poisson's ratio	0.32	-	0.1
Out-of-plane Poisson's ratio	0.45	-	0.1
Laminate length, $2L$	101.6	mm	0.02

Laminate thickness, 2h	4.5	mm	0.02
Laminate crack length, a	25.4	mm	0.02

The limit state function and random variables are the main factors of the reliability analysis. Therefore, first-order and second-order reliability methods, and sampling reliability approaches are the applicable methods to estimate the probability of the delamination growth under mode-II loading. In the sub-sections of section 4-5, all the results of the reliability analysis are investigated and compared.

4-4 Reliability analysis on Rtx[®]

The reliability analysis approaches introduced in chapter 2 are used, and critical outcomes are discussed in the following sections. FOSM, FORM, and SORM methods together with Monte Carlo Sampling (MCS), histogram sampling, and importance sampling are used in the present study. These methods are implemented in Rtx[®] software developed with C++ by Mahsuli et al. [96-98] in three studies. Here, the implementation procedure was followed to define the random variables and limit state functions for both DCB and ENF tests and computing the probability of the delamination growth is followed. Rtx[®] as the multi-model analysis software makes it possible for users to import all sources of uncertainty consisting of the different random variables and a given limit state function that accounts for random variables and constants. The correlation between each pair of variables has to be defined so that the software can calculate the covariance matrix in the computation process. One of the defined algorithms in the objects pane in Rtx[®] for performing reliability analysis is FOSM. By selecting the energy release rate as a limit state function, in the present work, this algorithm can readily be used to perform the reliability analysis and the function mean and standard deviation can be obtained.

Moreover, FORM can be selected under the analyzer tab in Rtx[®] to start FORM analysis. A nonlinear single constraint solver in FORM option tool in Rtx[®] ensures that the searched point is

on the limit state function surface. Nataf transformer is required to transform the variables from original space to standard normal space for calculating the reliability index in the standard normal space. A convergence checker is created to check whether the FORM optimization expression (equation 2-46 in chapter 2) converges. This option determines two essential conditions of FORM. Equations (2-59) and (2-60) fulfill these conditions, as described in chapter 2. Referring to what has been discussed in chapter 2 about finding the design point, the step size and direction have to be defined in Rtx[®]. To this end, a merit function provided in equation (2-53) is applied. Multiplier, adder, and factor, introduced in equations (2-55), (2-57), are three user-defined constants in merit function that are assigned based on the convergence of FORM through trial and error in Rtx[®]. Besides, the convergence checker ensures that the final point obtained after searching for the design point is on the limit state surface and is the closest point on the surface to the origin of the standard normal space; i.e., the design point is achieved. At the end of the FORM analysis, the design point about which the first-order approximation is made in FORM analysis is achieved in the original and standard normal space. Sensitivity analysis, the principles of which were discussed in chapter 2, is also done after FORM analysis. All the sensitivity indices of variables are computed according to the obtained reliability index and failure probability. All the mentioned outcomes are reviewed in the following sections. The iHLRF algorithm [84], through which merit function and step direction checker are determined to solve the optimization problem of FORM analysis, discussed earlier in chapter 2, is used in the FORM approach. The algorithm may reach convergence in a few steps; however, in some cases, it probably does not converge as is probable in other optimization algorithms. Changing the solver algorithm factors, including the user-defined factors introduced earlier, can solve the convergence issue.

The SORM approach as in the case of FORM, requires the limit state function, nonlinear solver, and Nataf transformer to perform the approach. The analysis method is point-fitting which is previously introduced in chapter 2. First, a first-order process utilizes the nonlinear solver to obtain the design point. Second, two points on either side of the design point are found on one axis. Subsequently, the corrected failure probability is obtained by computing curvature vectors at the two points and using the Breitung's formula, equation (2-71).

The concepts of sampling methods have been discussed in chapter 2. MCS works according to random number generation by using the limit state function and Nataf transformation. After

completing the solution, it can be tracked how the probability of failure varies by using several samples. The coefficient of variation of the computed delamination growth probability indicates the accuracy of the sampling. Higher accuracy is achieved by increasing the simulation cycling number which results in a low COV of the delamination growth probability. At the end of MCS, the accuracy is assessed by computing COV of the obtained probability by using equation (4-9). As shown in figure 4-8 later, the trend of the changes in the COV of the probability of the delamination growth versus sampling cycle numbers is obtained. Like other sampling approaches, histogram sampling deals with the primary function of the limit state function, which is the energy release rate in this investigation. A total of 100000 samples through random number generation are created in histogram sampling, and the function is evaluated for every model. At the end of the solution, the mean and standard deviation of the function are obtained. In addition, the probability density function, cumulative distribution function, and coefficient of variation of the function are plotted, which are provided in the next section. Finally, importance sampling as a practical approach is done using the design point achieved from FORM as a starting point for random number generation. It also needs the limit state function and the Nataf transformer is to be used. The method gives an improved percentage for the probability of failure with a different number of samples from the MCS. Figure 4-3 shows the results of five separate analysis approaches for computing reliability and their corresponding output, details, and plots. The object pane and property window in models and methods modules account for all required options and tools to determine the analysis inputs.

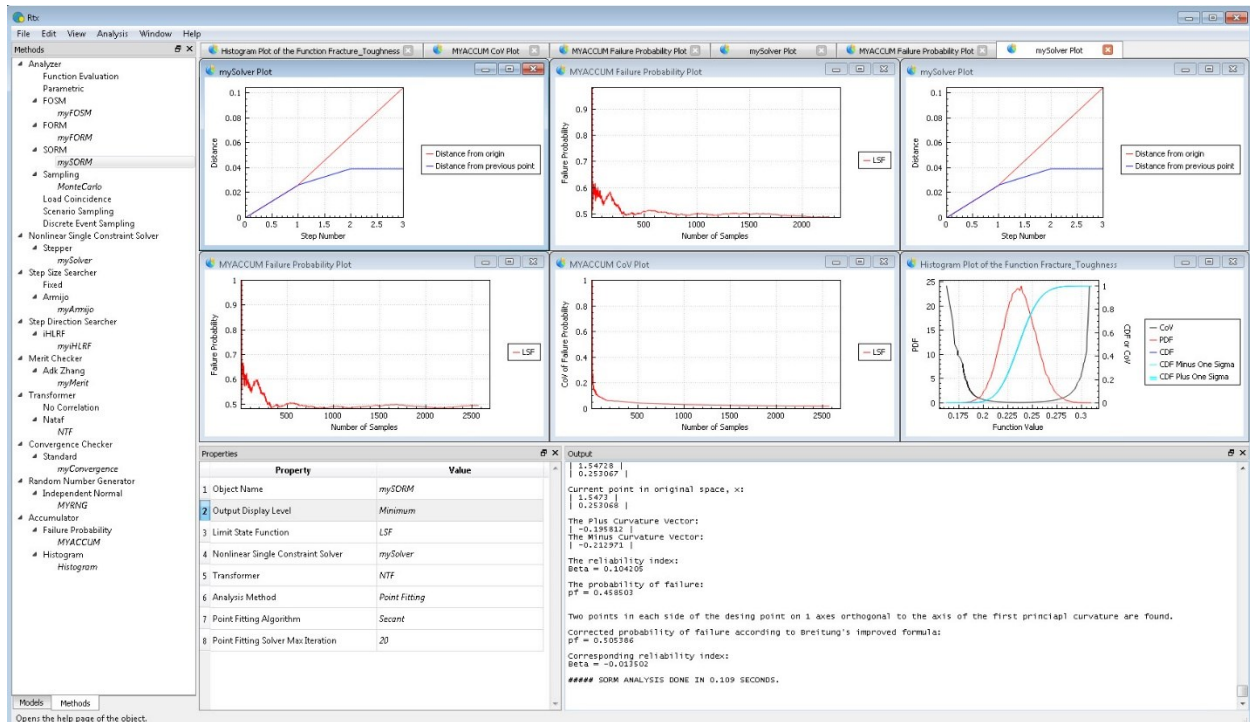


Figure 4-3 Different analyzers in Rtx®

In the following sections, outcomes of the different reliability analysis methods are represented and compared for two significant problems of the present study.

4-5 Results and discussion

All the reliability analysis methods are applied for both DCB and ENF tests using the Rtx® software. Results obtained using each method are presented and compared in the next two sub-sections. In addition, the effects of all crucial variables in each test on the probability of the delamination growth in the corresponding mode are indicated individually. Furthermore, the sensitivity of the probability of the delamination growth to all variables in both DCB and ENF tests are compared and illustrated in the following sub-sections.

4-5-1 Probabilistic analysis of mode-I delamination

First, the utilization of histogram sampling for the primary function of the DCB test, which is mode-I energy release rate, is presented. According to the data in table 4-1, the primary function is formulated. Through a random number generator and a transformer, a total of 100000 samples

are generated to evaluate the probabilistic characteristics of the primary function. Figures 4-4 to 4-6 illustrate the probability density, cumulative distribution, and coefficient of variation of the mode-I energy release rate obtained from the sampling.

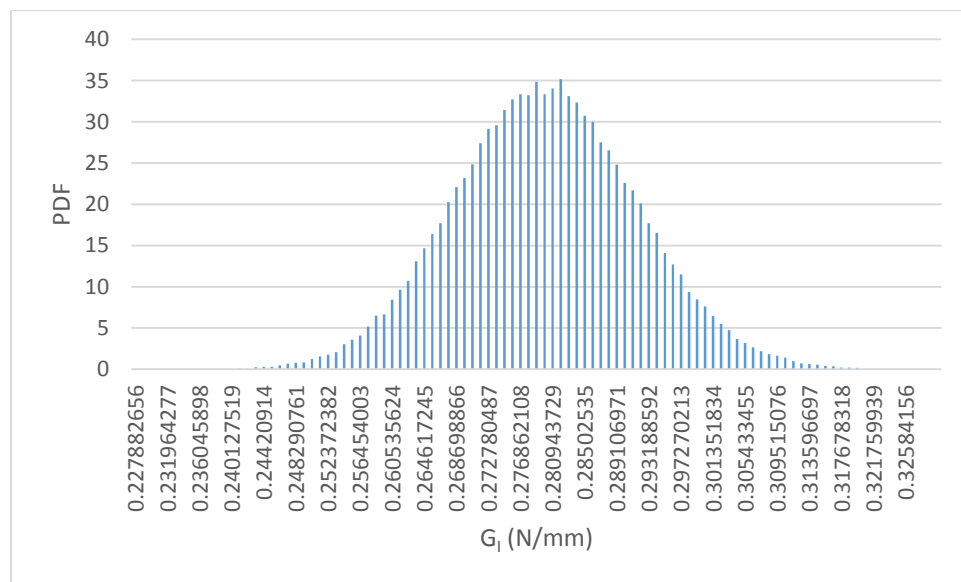


Figure 4-4 Probability density function of mode-I energy release rate

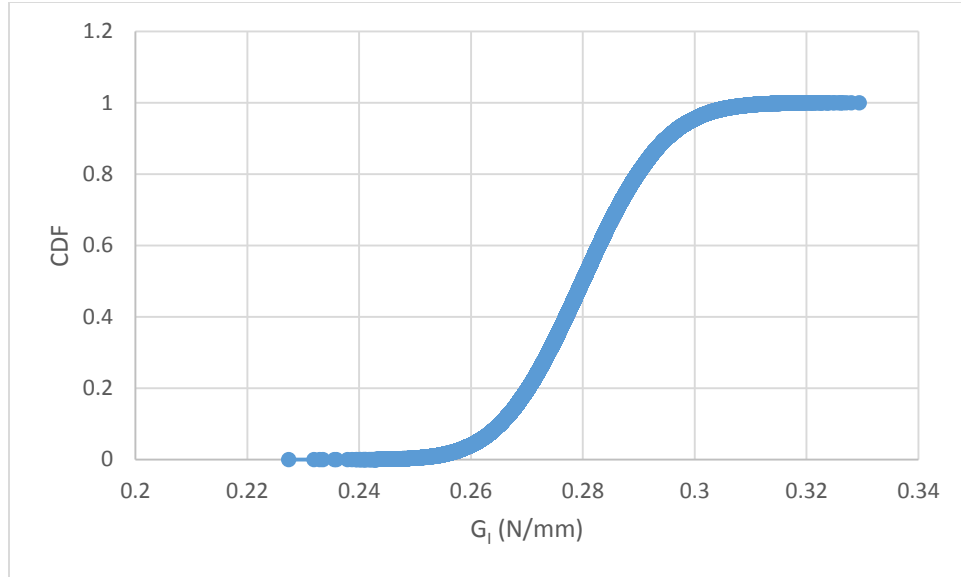


Figure 4-5 Cumulative distribution function of mode-I energy release rate

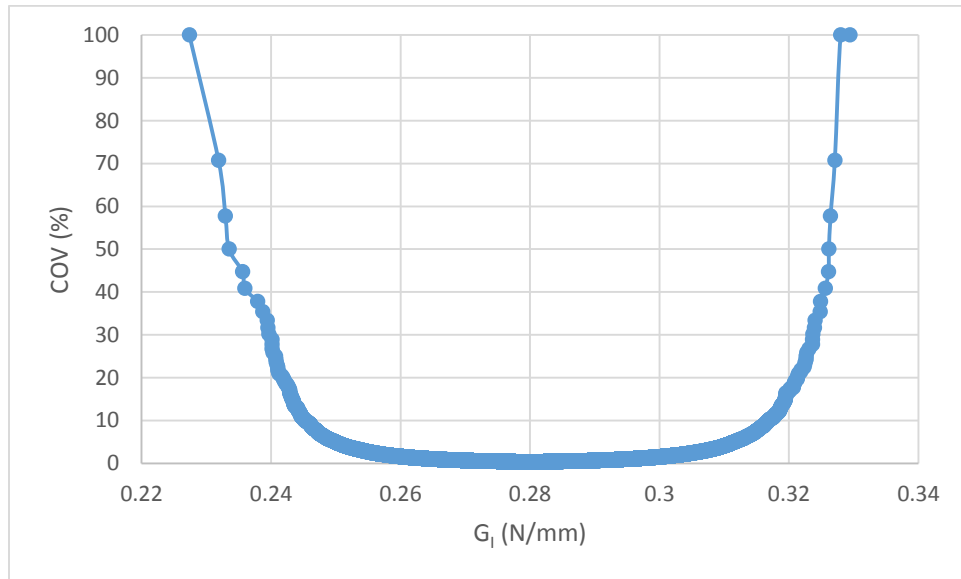


Figure 4-6 Coefficient of variation of mode-I energy release rate

Based on histogram sampling, the mean value of the mode-I energy release rate in the DCB test mean value is 0.2825 N/mm, while its standard deviation is 0.056 N/mm.

FOSM analysis is also used to evaluate the mean value and standard deviation of the mode-I energy release rate. Regarding equation (4-3) and random variables that the energy release rate equation

comprises, the FOSM method is used. Outcomes of the method indicate that the mean value and standard deviation of the mode-I energy release rate are 0.2799 and 0.0559 N/mm, respectively. Besides, the FOSM importance vector for this solution is obtained based on equation (2-44). Initial crack length and composite laminate thickness are the most effective random variables with 0.73 and -0.54 indices, respectively. It confirms that initial crack length acts as load in the analysis, whereas thickness is the capacity of the energy release rate function as the most important one among all variables. Comparing the computed second-moment values of the energy release rate in FOSM and in histogram sampling results proves that the results obtained using these two methods are in good agreement and are much closer. Although the first-order approximation about mean is not trustable particularly for nonlinear functions such as the mode-I energy release rate, histogram sampling results show that FOSM approach is also applicable for determining the second-moments for the mode-I energy release rate.

FOSM may introduce an invariance problem, which means that different but equivalent functions with identical surfaces $g = 0$ in random variables space will result in different probabilities of failure results. It is because of nonlinear limit state functions with respect to variables. Similar limit state functions may result in different reliability indexes due to the imprecise first-order approximation about the mean value in nonlinear functions. Thus, this method may lead to inaccuracies, particularly for nonlinear functions. The FORM algorithm addresses this issue by approximating the limit state function about a point on the surface of $g = 0$ such as a design point. The principles of this method were already discussed in chapter 2. The FORM algorithm is used to determine the probability of delamination growth in mode-I under the DCB test for the prepreg composite laminate used in the work of Alfano and Crisfield [25]. The design point is achieved, and accordingly, the probability of failure is computed at 49.98%. Several reliability analyses of FORM and SORM are conducted through the present study. Every trial includes random variables as input with different COVs and mean values. Based on all possible random variables of the problem, the SORM algorithm calculates the delamination growth probability to be 49.98%. A perfect match between the two approaches is observed. Comparing these two methods together with MCS is presented below.

Furthermore, Monte Carlo Sampling (MCS), is performed to evaluate the delamination growth probability. Computing delamination growth probability using 2474 samples shows that for the

composite laminate specimen with defined mechanical properties and geometric parameters used in the DCB test, the likelihood of delamination growth is 50.28%. Comparing this percentage to those calculated using FORM and SORM algorithms verifies the results. Failure probability stabilizes at a certain level by increasing the number of samples, as shown in figure 4-7.

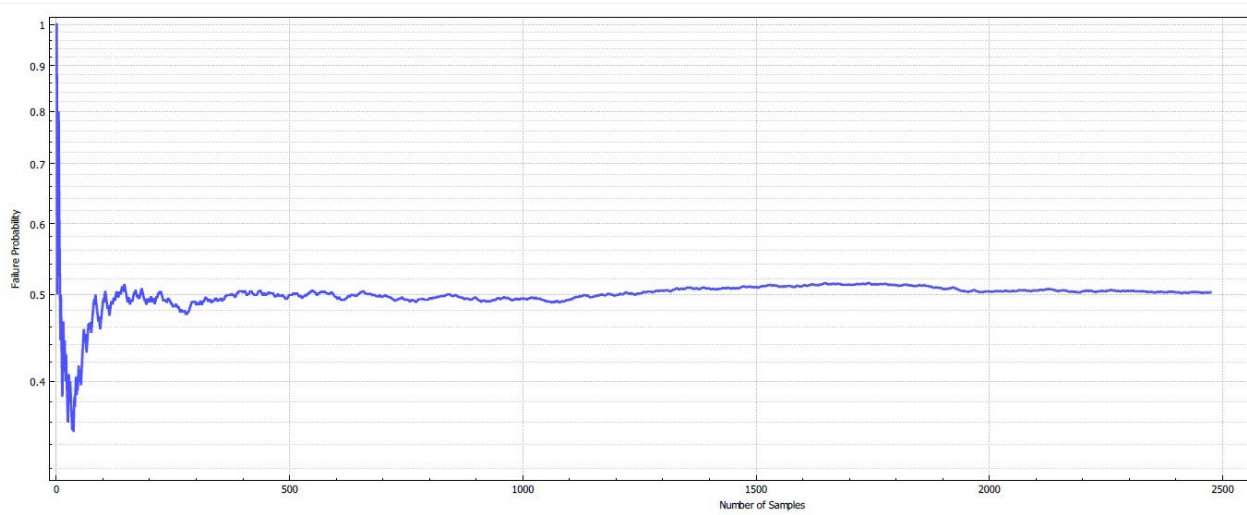


Figure 4-7 Monte Carlo Sampling (MCS) for DCB test

The coefficient of variation of the probability of delamination growth is an important factor in MCS. In order to make sure that the sampling estimation is accurate, the COV of the computed probability has to be minimized. The number of simulation cycling and the computed likelihood are essential for the coefficient of variation of the calculated probability. Ayyub and Haldar [91] recommended a way to evaluate the variance of the calculated probability that is provided below:

$$COV_{(P_f)} \approx \sqrt{\frac{1 - P_f}{N * P_f}} \quad (4-9)$$

where P_f denotes the probability of failure, which is the probability of delamination growth here, and N is the number of simulation cycling. At the end of sampling, the coefficient of variation of delamination growth computed using MCS reaches just below 2%, which is an acceptable rate (figure 4-8).

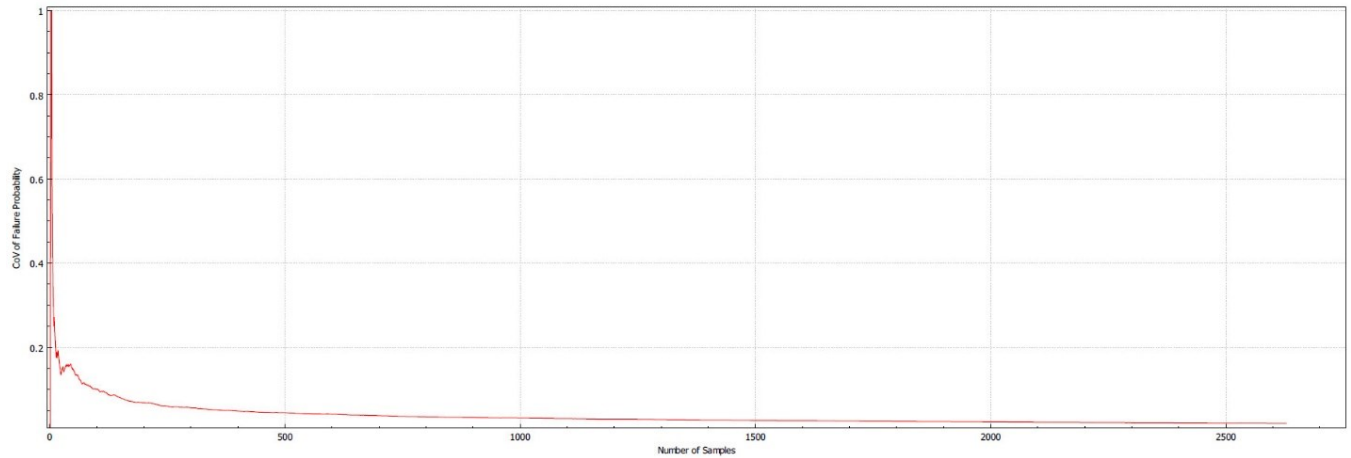


Figure 4-8 COV of the probability of the delamination growth in MCS

In addition, importance sampling that is considered as the most effective approach is conducted. The design point calculated in FORM as the initial point of random number generation makes this method different from MCS. The importance sampling analysis results in the probability of delamination growth of 50.27% in mode-I. It is very close to the percentage of the likelihood of delamination growth obtained in MCS and verifies that both sampling methods are accurate and trustable for the present study.

Several reliability analyses are carried out to evaluate the sensitivity of the delamination growth probability to the coefficient of variation of random variables. All the failure probability outcomes are obtained through importance sampling. Figure 4-9 indicates how the dispersion of every random variable influences the delamination growth probability. Increasing COV of the initial crack length to a value more than 0.7 leads to a significant decline in the probability of delamination growth. However, as the COV of the applied displacement (δ) increases beyond 0.7, the possibility of delamination rises to 52.55%. It can be seen that the dispersion of mode-I fracture toughness, G_{Ic} , has not had that significant effect on delamination probability. At most, it may cause a 0.25% variation in the probability of delamination growth.

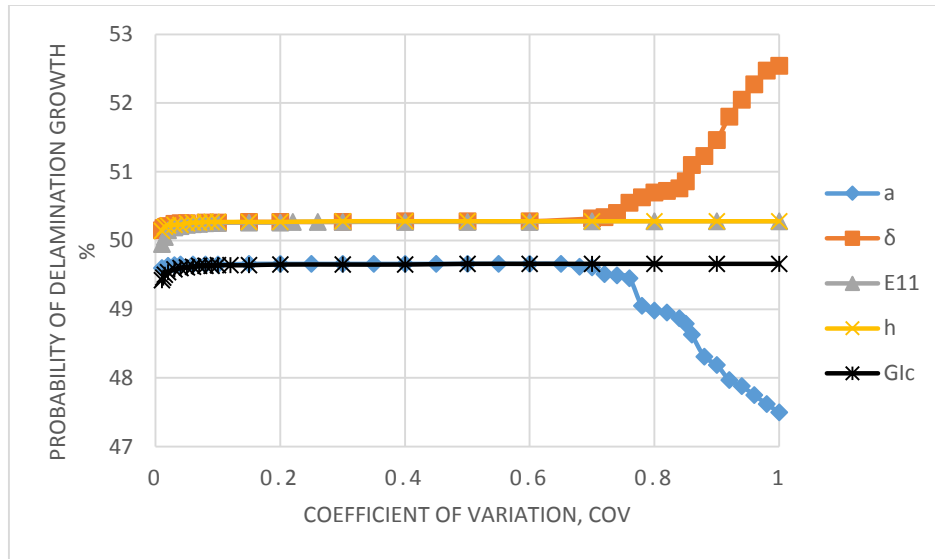


Figure 4-9 Sensitivity of probability of delamination growth to COV of different variables

Figure 4-10 illustrates the effect of applied displacement on mode-I delamination growth probability evaluated by using FORM, SORM, and MCS. According to the equation (4-4), delamination in the composite laminate occurs once the applied displacement exceeds the critical displacement. The value of critical displacement is obtained to be 1.6275 mm, through using equation (4-4) at the boundary of the safe zone and failure zone.

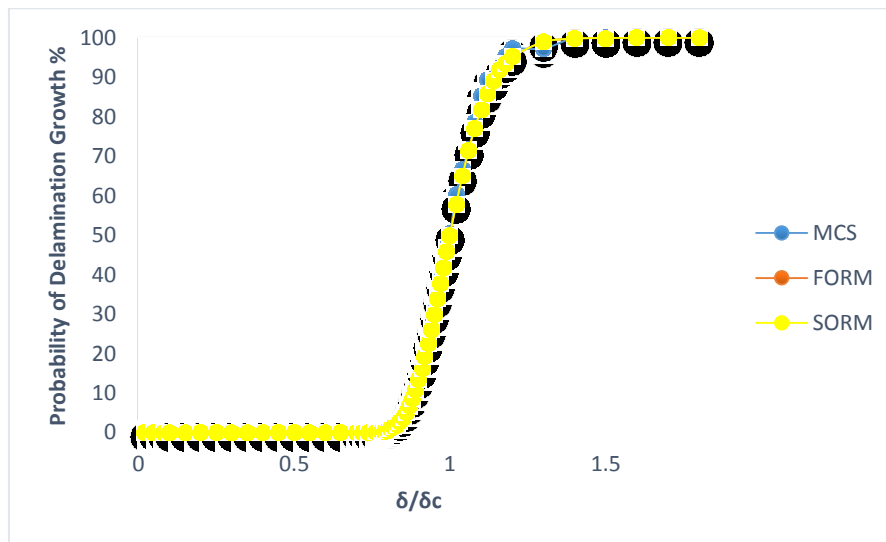


Figure 4-10 Effect of applied displacement in the DCB test on the probability of mode-I delamination growth

The influence of the displacement normalized with respect to the critical displacement on the probability of delamination growth is investigated in figure 4-10. An increase of the probability from zero to unity by increasing the normalized displacement can be observed. Results of the three reliability analyses are very relative, with a slight discrepancy at some points on the curves. It is worth noting that at the unit amount of normalized displacement, the corresponding probability of delamination growth is about 50% in all the three approaches and is equal to the likelihood of delamination growth mentioned earlier in this section. Nevertheless, in the deterministic study, as discussed in chapter 3, delamination does occur at the critical amount of displacement. Whereas, reviewing outcomes reveals that in the probabilistic study, once the applied displacement approaches 1.6 times of critical displacement, mode-I delamination growth definitely happens.

Based on the definition of sensitivity and importance vectors discussed in chapter 2, sensitivity analysis is carried out for the reliability analysis of the DCB test. Given the random variables involved in the DCB testing, the derivative of reliability index, β , with respect to mean values and standard deviations of variables reveals which random variables are more effective in the probability of delamination growth. Figure 4-11 represents the outcomes of sensitivity analysis for all random variables of mode-I delamination test. It is noteworthy that the initial crack length and laminate thickness are the most sensitive random variables. A negative sensitivity index for the initial crack length and fracture toughness in mode-I indicates that these two variables act as capacity variables. Nevertheless, load variables with a positive sensitivity index are laminate thickness, axial elastic modulus, and displacement.

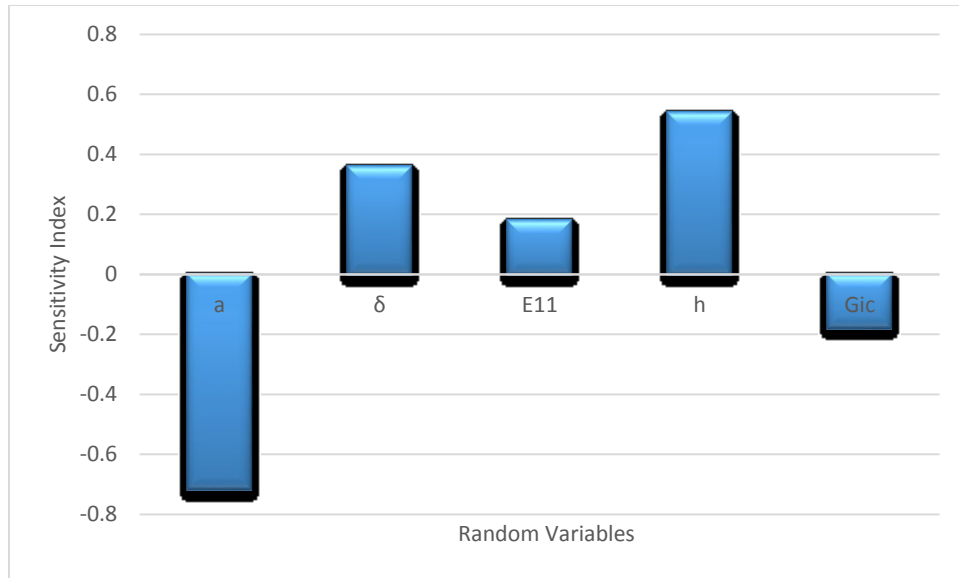


Figure 4-11 Sensitivity measure of DCB test

The information is beneficial in the probabilistic design of the mode-I delamination test. Ranking random variables makes it possible to make better decisions about composite manufacturing and curing processes, as well as improving the property testing and measurements accuracy, to lower the probability of delamination growth. The most sensitive variables have to be taken into account more precisely. In contrast, in some cases wherein the sensitivity index of a variable is far less than others, it can be eliminated from the probabilistic design and replaced with a deterministic value. In this analysis, all the random variables directly affect the system reliability to some extent, so it is crucial to consider all of them in studying the problem.

4-5-2 Probabilistic analysis of mode-II delamination

According to the data provided in table 4-2, the limit state function of the ENF test can be defined, and the reliability analyses can be carried out.

Histogram sampling shows how the mode-II energy release rate, defined in equation (4-7), is distributed. It determines the second-moment information together with the probability distribution of the primary function. By evaluating a total of 100000 different samples, the probability density function, cumulative distribution function, and coefficient of variation of the mode-II energy release rate in the ENF test are computed, which are represented in figures 4-12 to

4-14, respectively. Histogram sampling outcomes show that the mean value and standard deviation of mode-II energy release rate are 0.7945 and 0.0224 N/mm, respectively.

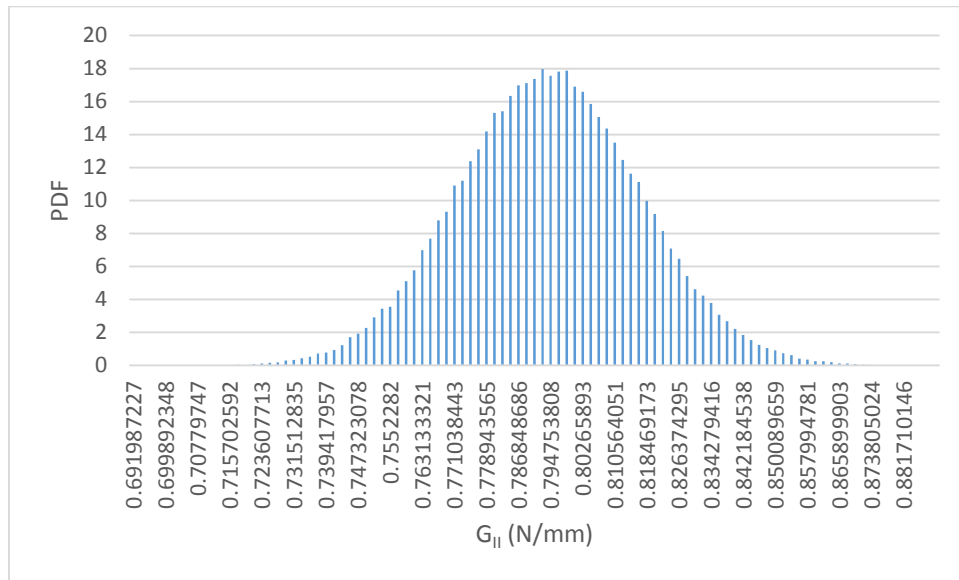


Figure 4-12 Probability density function of mode-II energy release rate

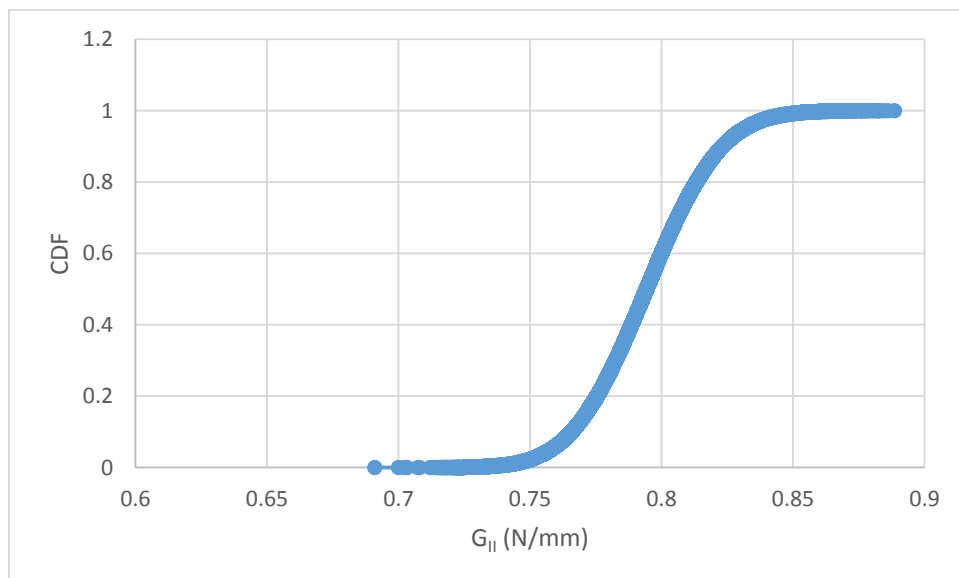


Figure 4-13 Cumulative distribution function of mode-II energy release rate

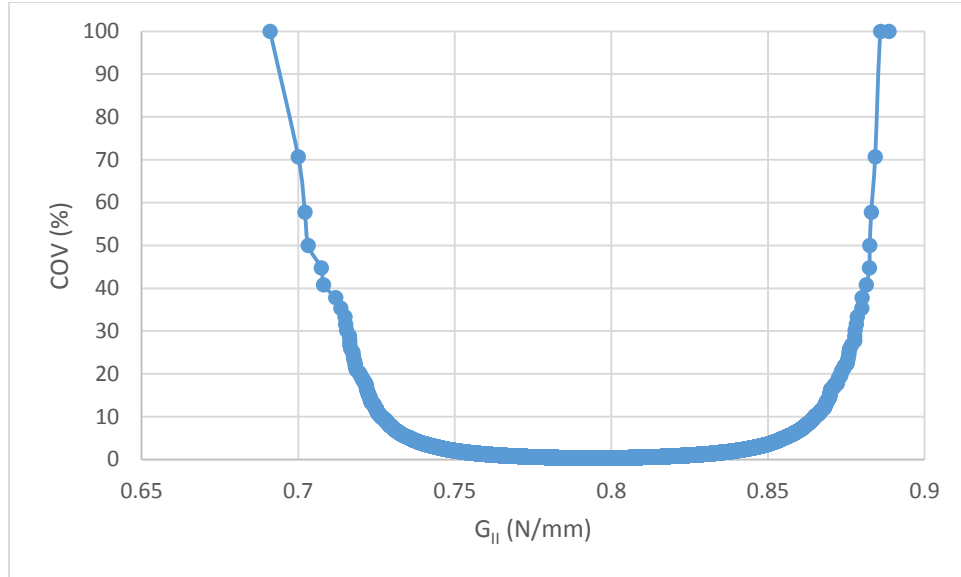


Figure 4-14 Coefficient of variation of mode-II energy release rate

Second-moment parameters of the mode-II energy release rate are also evaluated by the FOSM method. Given the table 4-2 data of random variables, the results of the FOSM approach illustrate that the mean value and standard deviation of the energy release rate in mode-II are 0.7943 and 0.0224 N/mm, respectively. Comparing this outcome to that of histogram sampling confirms that both approaches estimate the function probabilistic characteristics similarly. Histogram sampling results are very accurate since these are obtained by plenty of samples. However, FOSM analysis allows determining the FOSM importance vector of all variables. The essential random variables are length and thickness of the composite laminate regarding FOSM importance vector calculation. FOSM importance index of the length of composite laminate variable is 0.79, the greatest index among all variables. The index of the thickness of laminate is -0.47, the second important variable in this analysis. The positive and negative signs of these indices note that the length and thickness of laminate act as load and capacity variables of the FOSM analysis, respectively.

The reliability of the ENF test is also investigated through FORM and SORM methods. That is the probability of mode-II delamination growth for the graphite-epoxy composite laminate under the ENF test. After obtaining the design point used in both algorithms, the FORM reliability index is computed. Accordingly, the failure probability is achieved, which is the probability of the delamination growth in mode-II. Among all FORM attempts, in the most general case, the computed probability of the delamination growth is 48.35%. Afterward, applying the Breitung

equation (equation 2-71) in the SORM point fitting approach results in the probability of the delamination growth of 48.65%. The results of the two reliability methods are highly matched in their ability to predict the probability of delamination growth in mode-II.

In order to evaluate first and second-order reliability analyses, two sampling approaches are used. MCS can compute the probability of mode-II delamination growth accurately by performing a total of 2518 samples. Given the random variables determined in table 4-2 and the random number generator, the mode-II delamination growth probability is obtained at 49.84%. The coefficient of variation of the sampling is computed using equation (4-9). It is less than 2% which is accepted as an appropriate rate. Figure 4-15 shows that after significant oscillation over the first 500 samples, the failure probability smoothly hovers around 50%. In the end, it remains steady at 49.84%, which endorses that the FORM and SORM estimations are powerfully accurate in assessing the probability of mode-II delamination growth.

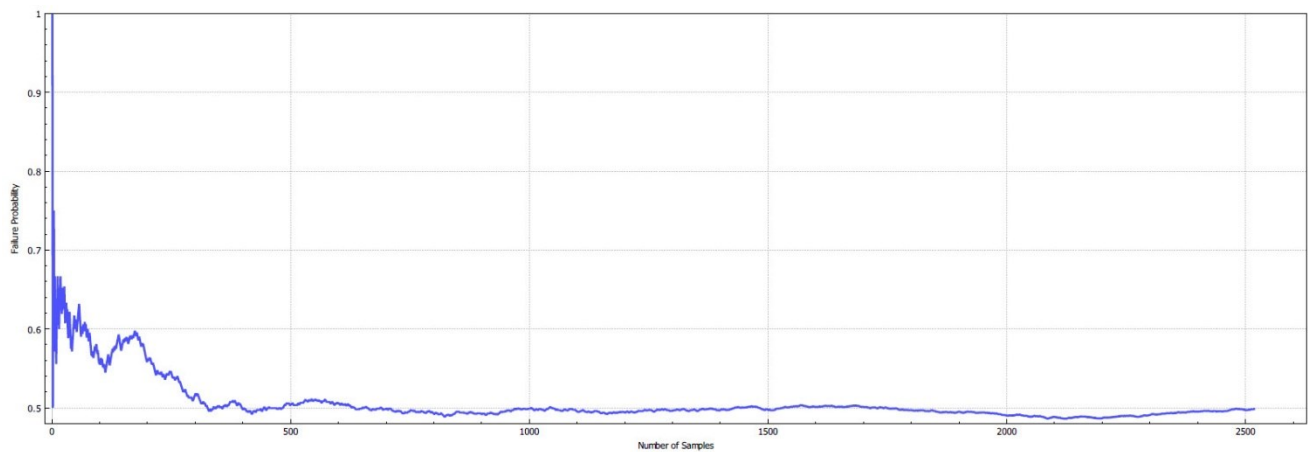


Figure 4-15 Monte Carlo Sampling (MCS) for ENF test

Moreover, varying the initial point of the random number generator leads to different sampling. Importance sampling is conducted by defining the design point as the start point of random number generation. According to this sampling solution, there is a 49.51% probability that the mode-II delamination grows under the ENF test. As this approach combines sampling and FORM advantages, further attempts are made based on the importance sampling to analyze the reliability and the sensitivity.

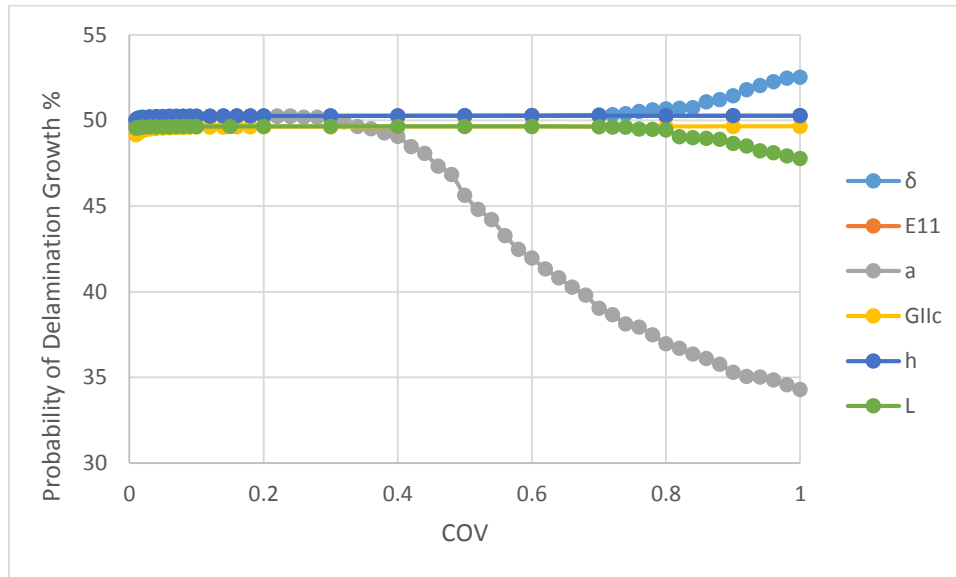


Figure 4-16 Sensitivity of probability of delamination growth to COV of different variables

Figure 4-16 compares how the dispersion of different random variables in the ENF test influences the mode-II delamination growth probability. Worthy of mentioning that the change in the coefficient of variation of the crack length of composite laminate has the most substantial effect on the failure probability. By increasing the COV of crack length above 0.4, the likelihood of delamination diminishes significantly. At the endpoint of the trend, where the COV of the crack length is unity, the probability of the delamination growth approaches 34.3%. In addition, an increase of the COV of the length of composite laminate leads to a decrease in the probability. The change in COV of the length of the composite laminate specimen from zero to unity results in an almost 2% decline in the mode-II delamination growth probability. However, by increasing the COV of applied displacement to the unity, the probability of mode-II delamination growth increases by about 2.5%. It is noteworthy that varying the COV of longitudinal elastic modulus and mode-II fracture toughness does not considerably affect the probability of failure.

Applied displacement at the middle of the beam is essential in defining the reliability of the ENF test. Figure 4-17 represents the effect of the normalized displacement with respect to the critical displacement on the probability of mode-II delamination growth. The value of critical displacement is obtained to be 1.3323 mm, through using equation (4-8) at the boundary of the

safe zone and failure zone. The probability of the delamination growth in mode-II is evaluated in the FORM, SORM, and MCS approaches. Three smooth trend lines are almost matched, whereas a few data points of MCS results slightly differ from that of FORM and SORM approaches on the curves. It can be seen that once the critical displacement, 1.3323 mm, is applied, the computed probability of the delamination growth in all the three reliability methods is about 50%. It was also discussed a bit earlier at the beginning of this sub-section.

Nevertheless, it is seen in chapter 3 that from a deterministic viewpoint, the critical displacement results in the composite mode-II delamination. Figure 4-17 indicates that the delamination growth probability is escalated dramatically by increasing the displacement to a higher value than critical displacement. It reaches about a hundred percent once the displacement rises to 1.6 times of critical amount. At this point, mode-II delamination occurs with 100% probability, and afterward, the reliability approaches zero. Comparing these results to those of the DCB test indicates that the influences of the applied displacement in both the DCB and ENF tests on the probability of the delamination growth in mode-I and mode-II are very similar. The only small difference that can be mentioned is how the probability of delamination growth increases after exceeding the critical displacement according to FORM method. Based on the FORM approach, the pace of increasing the probability of the delamination growth in mode-I is larger than in mode-II. For instance, where the normalized displacement is 1.2 in the ENF test, there is a probability of 93.56% that the mode-II delamination occurs. However, once the normalized displacement is 1.2 in the DCB test, the probability of the mode-I delamination is computed to be 97.26%.

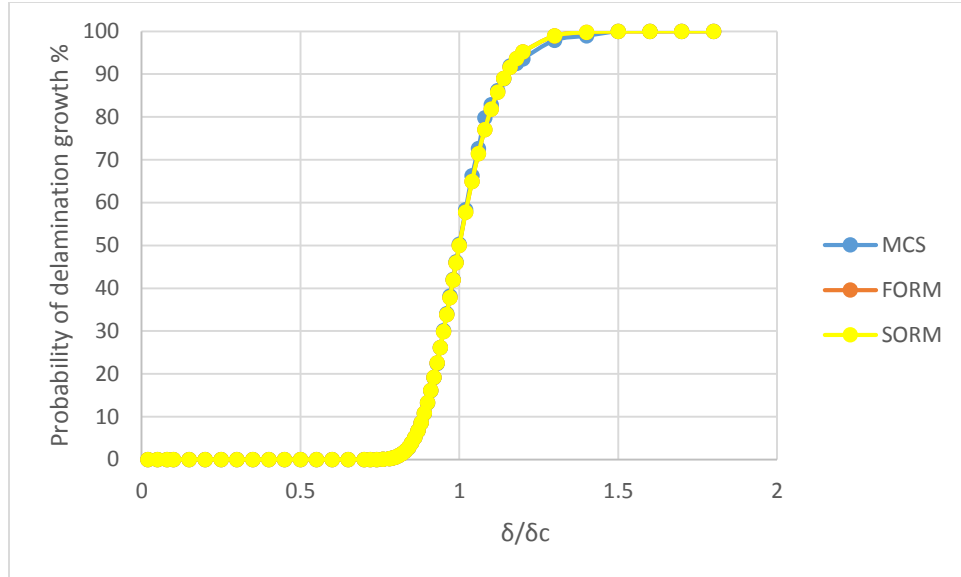


Figure 4-17 Effect of applied displacement in the ENF test on the probability of mode-II delamination growth

By-products of FORM analysis are the parameters through which the sensitivity of the random variables is defined. It shows to what extent the reliability of the system depends on each random variable. Figure 4-18 illustrates the outcomes of sensitivity analysis for mode-II delamination test. It can be argued that the laminate length and laminate thickness are the most important random variables. Nonetheless, the sensitivity of composite reliability is less attributed to the mode-II fracture toughness and longitudinal elastic modulus of the composite laminate.

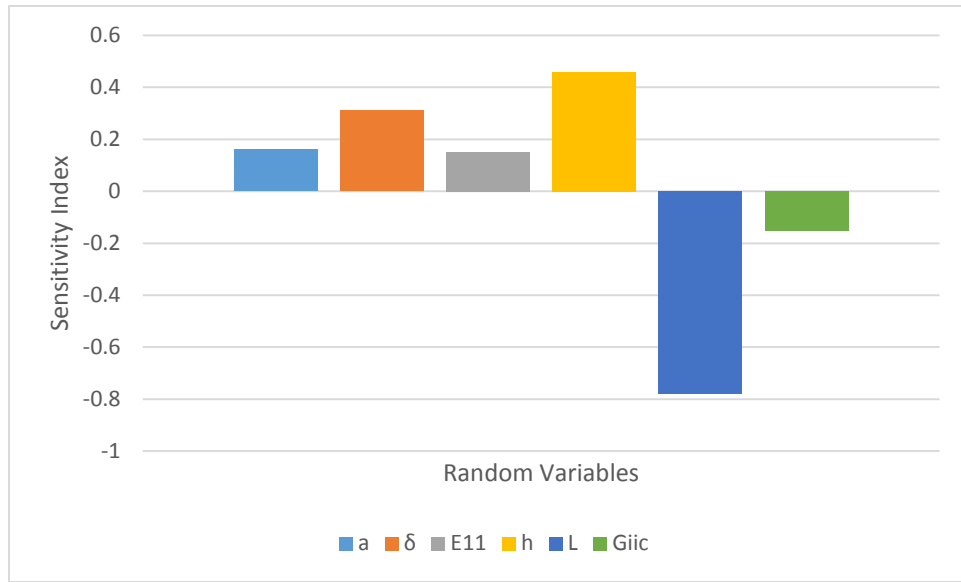


Figure 4-18 Sensitivity measure of ENF test

It can be observed that the sensitivity indices of mode-II fracture toughness and laminate length are negative. Interpreting these results verifies that these random variables act as capacity variables of the limit state function, as can be seen in equation (4-8). In contrast, positive sensitivity indices of laminate thickness, axial elastic modulus, initial crack length, and displacement at the middle of the beam indicate that these random variables act as load variables in mode-II delamination reliability assessment. This ranking of random variables is worthwhile in the probabilistic study of the ENF test. It can offer ideas to designers to be aware of more sensitive variables. In this way, the more sensitive variables need to be definitely taken into account in designing the manufacturing and curing processes to develop more competitive and reliable products.

4-6 Conclusion

Non-deterministic methods of investigation of the DCB and ENF composite property tests were presented in this chapter. The reliability analysis of the delamination growth in the composite laminate with a pre-crack in mode-I and mode-II under the DCB and ENF tests have been conducted. Few random variables have been considered for the reliability analyses of the tests. Although the transverse elastic modulus, E_{22} , which represents the resin property in the composite materials, affects the fracture toughness, this parameter is not involved in the reliability assessment of the present study. It is due to the fact that the Direct Beam Theory (DBT) that provides the closed-form solutions to calculate the mode-I and mode-II fracture toughnesses from the DCB and ENF tests is used to formulate the limit state functions. Such closed-form solutions are not functions of the transverse elastic modulus, as can be observed from equations 4-4 and 4-8. Six distinct approaches, including FOSM, FORM, SORM, MCS, importance sampling, and histogram sampling have been used. It has been shown that the probability of the delamination growth in mode-I delamination testing obtained using the above-mentioned reliability methods is higher than the likelihood of the delamination growth in mode-II delamination testing. That is, the reliability of the DCB test is lower than the reliability of the ENF test. According to the sensitivity analysis, the laminate thickness and pre-crack length are the most influential variables on the reliability of both tests. In the ENF test, the laminate length is also an important variable. Therefore, the fluctuations of these variables have to be considered carefully in designing the DCB and ENF tests. An increase of the applied displacement has an identical effect on the delamination growth in mode-I and mode-II. The reliability of the DCB and ENF tests decreased significantly by increasing the applied displacement in each test. It has been indicated that once the applied displacement becomes equal to the critical displacement, the reliability of both the DCB and ENF tests is about 50% which means that the probability of the delamination growth is also about 50%.

Chapter 5

Conclusion

5-1 Contributions

5-1-1 Contributions on parametric study

In the present thesis, a parametric study of the delamination in composite laminates by using Finite Element Modeling and Analysis has been conducted. The Double Cantilever Beam (DCB) and End Notched Flexure (ENF) tests have been modeled using FEA tool ANSYS® based on Cohesive Zone Modeling (CZM) to obtain the energy release rates in mode-I and mode-II. The energy release rate has been computed using the Direct Beam Theory (DBT) approach and the Irwin-Kies expression. The effects of the engineering properties and geometric parameters of the test specimens and their variability on the energy release rate and its variability have been investigated. Fluctuations of different material properties and geometric parameters have been considered to observe their influences on the fracture toughness and its variability.

Moreover, a numerical study has been conducted to investigate the influence of in-plane fiber misalignment and its location in the laminate on the mode-II fracture toughness in the ENF test. The results of the FEA modeling in ANSYS® reveal how different severities of fiber misalignment in the composite laminate may affect the outcomes of the mode-II delamination test. Also, other FEA models have been used to survey the influences of the location of the misaligned ply group on the mode-II energy release rate.

All the parametric analyses of the present study have been presenting virtual experiments. In order to prevent conducting a number of actual tests on different specimens in terms of the mechanical properties, laminates size, and stacking sequence of composite materials, the virtual experiments consider the in-situ variability of composite materials as well as the variation in the geometric parameters of the test specimen. Results of the present parametric study can be extended for the actual delamination testing on larger scales. The variabilities in the mode-I and mode-II fracture toughnesses due to the variations in material properties, geometric parameters, and lay-up

sequence of composite laminates, which have been obtained in the present thesis, have to be taken into account to design a practical delamination testing.

5-1-2 Contributions on reliability assessment

Furthermore, a reliability analysis of delamination growth in the composite laminate subjected to the DCB and ENF tests has been carried out. According to the DBT and the Irwin-Kies expression, the limit state functions have been formulated for each test as a function of the engineering properties and geometric parameters of the test specimen. Two first-order reliability approaches, including First-Order Second-Moment (FOSM) method, and First-Order Reliability Method (FORM) have been used to evaluate the values of the second-moment parameters of the energy release rate, and the probability of delamination growth, respectively. The Second-Order Reliability Method (SORM) has been used to estimate the probability of delamination growth in mode-I and mode-II in the DCB and ENF tests, respectively. Three sampling approaches have been used to verify the results of FOSM, FORM, and SORM approximation. Histogram sampling has been applied to compute and plot the Probability Density Function (PDF), the Cumulative Distribution Function (CDF), and the Coefficient of Variation (COV) of the energy release rates in mode-I and mode-II in the DCB and ENF tests. Also, Monte Carlo Sampling (MCS) and importance sampling have been conducted to compute the probabilities of delamination growth in mode-I and mode-II in the DCB and ENF tests. All the reliability analyses have been conducted using the Rtx[®] software, and the results of different analyses have been compared.

5-2 Conclusions

The present thesis contains three major studies named numerical modeling and analysis, parametric study, and reliability analysis. Since the numerical analyses have been conducted repeatedly to fulfill the requirements of parametric study, many results have been obtained. Also, in reliability analyses some important outcomes have been gained for both the DCB and ENF tests. Major conclusions of the parametric study and reliability assessment are presented in the two following sub-sections.

5-2-1 Conclusions of parametric study

- 1- The longitudinal elastic modulus of the composite material has the most profound influence on the obtained mode-I energy release rate in the DCB test among all material properties.
- 2- The coefficient of variation of the mode-I fracture toughness increases by more than 3.3% with the increase in the COV of E_{11} from 4.47% to about 10%.
- 3- The transverse elastic modulus, E_{22} , is the second most influential variable on mode-I fracture toughness of the DCB test among all material properties showing the importance of resin properties in defining mode-I fracture toughness of composite materials.
- 4- The out-of-plane elastic modulus, E_{33} , and out-of-plane shear modulus, G_{23} , have no significant influence on the mode-I fracture toughness in the DCB test.
- 5- The mode-I critical energy release rate is more sensitive to the fluctuation of the out-of-plane Poisson's ratio than the fluctuation of the in-plane Poisson's ratio.
- 6- Mode-I fracture toughness decreases by increasing the values of geometric parameters of the DCB specimen. Based on the numerical analysis, the smallest test specimen results in the mode-I fracture toughness of 0.242 N/mm for the prepreg composite material used for the DCB validation; however, the mode-I fracture toughness has been computed to be 0.221 N/mm for the largest test specimen of the same composite material.
- 7- The length of the DCB specimen causes the most variability in the mode-I critical energy release rate. By fluctuation the value of the length of specimen by COV of 2.33%, the COV of mode-I fracture toughness varies up to 13.4%.
- 8- In the ENF test, mode-II fracture toughness is significantly sensitive to the fluctuation of the longitudinal elastic modulus. The COV of the mode-II energy release rate increases moderately to just over 11.4%, with the increase of 4.5% in the COV of E_{11} .
- 9- The influence of the variation in transverse elastic modulus, E_{22} , on mode-II fracture toughness of the ENF test has been significant, which indicates the importance of resin properties in the variation of mode-II fracture toughness of composite materials.

10- Fluctuations of E_{33} and G_{23} have no significant influence on the mode-II fracture toughness.

11- The effect of the variations in the geometric parameters of the ENF test specimen on the variability of the mode-II fracture toughness is more significant than the fluctuations of the engineering properties of the test specimen.

12- Among the geometric parameters of the ENF test specimen, the thickness of specimen has the most remarkable effect on the mode-II fracture toughness variability. The COV of the mode-II fracture toughness increases to approximately 9% with the increase of the COV of the thickness of test specimen to 2.8%.

13- There is a linear correlation between the COV of the length of ENF test specimen and the COV of the mode-II fracture toughness. Increasing the COV of the length of test specimen by about 1.5%, from 0.8% to 2.32%, results in an increase of about 7.9% in the COV of the mode-II fracture toughness.

14- Mode-II fracture toughness decreases by increasing the values of the geometric parameters of the ENF test specimen. The mode-II fracture toughness for the largest and the smallest specimens subjected to the ENF test have been obtained to be 0.76 N/mm and 0.85 N/mm for IM7/8552 graphite-epoxy composite $[0]_{24}$, respectively.

15- FEA modeling of the ENF test considering in-plane fiber misalignment in the composite laminate has indicated that the specimen with the most severe in-plane fiber misalignment in its first four plies leads to the highest mode-II energy release rate at 1.05 N/mm for graphite-epoxy composite $[0]_{12}$.

16- The lowest mode-II fracture toughness at 0.64 N/mm has been observed in the specimen with in-plane misalignment up to 3° in its first three plies.

17- Investigation of the misalignment location considering a ply group of three misaligned plies has shown that the largest mode-II fracture toughness at 0.65 N/mm is obtained when the center of the misaligned ply group is located in the 2nd ply; nonetheless, the sample in which the central ply of the ply group is located in the 7th ply has illustrated the lowest mode-II fracture toughness at 0.61 N/mm.

5-2-2 Conclusions of reliability assessment

1- Histogram sampling of a total of hundred thousand DCB test samples has been conducted and the results illustrated that the mean value of mode-I energy release rate is 0.2825 N/mm, while its standard deviation is 0.056 N/mm for the prepreg composite material used in the work of Alfano and Crisfield [25].

2- The probability of the mode-I delamination growth in the DCB test has been computed using FORM, SORM, MCS, and importance sampling. The results of all analyses were in very good agreement, and according to the importance sampling, the probability of mode-I delamination growth in the DCB test has been obtained to be 50.27%.

3- The sensitivity analysis has revealed that the crack length and thickness of the DCB test specimen are the most sensitive and essential variables in the probability of the mode-I delamination growth.

4- Second-moment parameters of the ENF test have been determined using FOSM and histogram sampling. Results of a total of hundred thousand samples have shown that the mean value and standard deviation of the mode-II energy release rate are 0.7945 and 0.0224 N/mm for IM7/8552 graphite-epoxy composite [0]₂₄, respectively.

5- The reliability analyses, including FORM, SORM, MCS, and importance sampling have been conducted to obtain the probability of the mode-II delamination in the ENF test. All the outcomes of these methods were in good agreement, and based on the importance sampling, there is 49.51% probability for the mode-II delamination growth in the ENF test specimen to occur.

6- According to the sensitivity analysis, the most sensitive variables in the ENF test aside from the applied displacement, are the length and thickness of the test specimen.

7- It can be concluded that the probability of the mode-I delamination growth in the DCB test is higher than the likelihood of the mode-II delamination growth in the ENF test.

5-3 Recommendations for future studies

According to the content of the present study, there can be some further numerical studies about the delamination onset and propagation in composite laminates. The future numerical analysis or parametric study can be carried out based on the proposed suggestions and recommendations as follows:

- 1- Numerical analyses used in this study to investigate the effects of the engineering properties and geometric parameters of the test specimens can be extended for thickness-tapered and width-tapered composite laminates.
- 2- Numerical analyses of mixed-mode I/II delamination tests such as SLB and MMB tests can be conducted to investigate the influences of the engineering properties and geometric parameters of the test specimens on fracture toughness.
- 3- Numerical analyses provided in this thesis considering the in-plane fiber misalignment in composite laminates subjected to the ENF test can be conducted considering models that have other manufacturing defects such as a gap and resin-rich areas.
- 4- Numerical analysis of the DCB and ENF tests considering out-of-plane waviness in the plies can be conducted.

Besides, based on the observations of the present thesis, more probabilistic studies can be conducted based on the following recommendations:

- 1- Reliability analysis of delamination growth in the laminate models with two or more delaminations can be conducted.
- 2- Dynamic reliability study of the delamination growth considering the influence of fatigue loading.
- 3- Probabilistic analysis of the influence of the crack location in the DCB and ENF test specimens on the probability of delamination growth.

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