Fairness in Dynamic Networks

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Abstract

Fairness in Dynamic Networks

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The main focus of this research is directed towards fairness in a dynamic network. Two specific applications are mathematically formulated: heating, ventilation, and air conditioning (HVAC), and code division multiple access (CDMA). In the first problem, namely, fair power allocation for temperature regulation of a multi-unit building, the temperature of each unit is described by a discrete-time dynamic equation. The formulation considers the effect of outside temperature and heat transfer between the adjacent rooms. Temperature regulation is then described as a constrained optimization problem, where the objective is to maintain the temperature of each unit within a prescribed thermal comfort zone with a limited amount of power. An optimal control strategy is presented to minimize the maximum mutual temperature difference between different units (long-term fairness) while maintaining the temperature of each unit in the comfort zone or close to it at all times, as much as possible (short-term fairness). Simulations demonstrate the effectiveness of the proposed control strategy in regulating the temperature of every unit in a building. Regarding the second application, an optimization-based fair reverse-link rate assignment strategy is proposed for fair resource allocation in a CDMA network. The network is modeled in a star topology, where the nodes represent either the base station (BS) or access terminals (ATs). The BS at every instant computes the fair rate for each AT by minimizing the maximum disparity in users' rates. Then, the BS sends a single bit to all ATs at every instant. It is shown that if each AT could compute a specific variable, called the coordinating variable, it can find its fair rate, which means the decision-making strategy is distributed. The proposed method is computationally efficient, and simulations confirm its efficacy in different scenarios.

To my mother, father, and Amir.

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Chapter 1

Introduction

1.1 Preliminaries

Fairness is an old concept, but its root (”fair”) was formally described for the first time in 11th century in the meaning of ”pleasing to the eye or mind especially because of fresh, charming, or flawless quality.” In the current era, ”fair” means ”just”, ”equitable”, ”impartial”, ”unbiased”, and ”objective”. Each of the previously-mentioned definitions is described below [1]:

- ”fair” means a proper balance of conflicting interests; for example, a fair decision.
- ”just” means an exact following of a standard of what is right and proper; for example, a just settlement of territorial claims.
- ”equitable” is a less rigorous notion than ”just”, implying equal treatment of all concerned; for example, the equitable distribution of a property.
- ”impartial” means no favor or prejudice; for example, an impartial third party.
- ”unbiased”, implies in a stronger way than ”impartial” an absence of all prejudice; for example, your unbiased opinion.

- ”objective” means a tendency to view events or persons as apart from oneself and one’s own interest or feelings; for example, not being objective about one’s own child.

Fairness has been studied in several problems in science and engineering such as:

- Algorithmic fairness in computer science [2, 3].
- Machine learning, where sources of unfairness and methods to mitigate them are studied[4, 5, 6, 7].
- The concepts of ”fair value” and ”fair price” in economics, which makes financial statements easier to compare and balance sheets more reflective of real values [8].
- Network engineering, where the efficacy of the resource allocation methodologies is assessed using fairness measures [9, 10, 11].
- Game theory problems, where players are not self-interested [12, 13, 14, 15].

Fairness has appeared in many researchers. Nevertheless, the essence of the current study is fairness in resource allocation. Although all people can comprehend fairness and distinguish between unfair and fair acts (or resource allocation in the current context), there is a tremendous variation in defining and quantifying fairness. In other words, conceptually, fairness is trivial but mathematically is challenging to define. Therefore, different indices are used for quantifying fairness. Fairness is closely related to resource allocation, which is the distribution of limited resources to multiple users. Therefore, fairness and resource allocation are described mathematically, and the relationship between the two is specified. Once fairness is achieved, increasing resources corrupts it. In this context, agreement on a definition of fairness is essential. Although all people perceive fairness, this comprehension is affected by personal preferences and other factors. Thus, even in the case of two people, they might have a skirmish, but both might believe they are acting fair. These complexities do not exist in our problems, as personal preferences are excluded in our formulation.

Now, we turn to depict the mentioned concepts and their relationships by the following examples:

Example 1. *Let four apples be given to two friends. They can divide four apples among themselves in many ways. For example, one of them might favor the other one over oneself. In this case, friend 1 might get one apple and friend 2 three apples. However, if they are asked to divide the apples fairly, the ground truth for fairness is equal distribution. Therefore, one common ground for fairness is equality. However, in the following example, we will show that this is not always the case. In this example, we only had one constraint: the number of apples was limited, which is a coupled constraint because it constrains all users.*

Example 2. *Let an apple pie be given to two people. One of them is more hungry than the other. Moreover, they are told to distribute the pie fairly, and if anything remains from it after distributing, it would be a waste. In this example, we face two constraints: one individual constraint and one coupled constraint. The individual constraint is that one of the individuals is less hungry than the other. The coupled constraint is the amount of apple pie, which is limited. In this example, consider that person 1 cannot eat more than one slice, the other can eat any amount, and no waste is allowed. Here, fairness means that the lower capacity individual should get as much pie as they want, and the other would get all remaining slices.*

The above examples present some aspects of fairness in the current study with both coupled and individual constraints. The users cannot get more than their capacity and cannot collectively exceed the amount specified in the coupled constraint. Note that fairness is meaningful only when resources are limited (coupled constraint), and restrictions on individual capacities only make the problem more complex. If we only had coupled constraints and no individual constraints, then fairness would mean equality. Individual constraints transform the fairness problem into a distribution problem that is as equal as possible.

The distribution of resources to users can either follow particular dynamics or be performed at once. These correspond to two types of fairness problems, dynamic and static. An example of dynamic fairness is the fair distribution of heat to every apartment in a building. On the other hand, the distribution of apples in the previous examples is a static fairness problem.

Dynamic fairness is a more challenging problem because it involves multiple interacting users with possible connections among their dynamics. Thus, the problem is to distribute resources fairly in a dynamic network, and the notions of short- and long-term fairness in such settings are beneficial. Note that, as mentioned earlier, quantifying fairness is not straightforward. Furthermore, defining short- and long-term fairness is even more challenging. These notions will be further discussed in the sequel.

As mentioned earlier, static fairness corresponds to a one-time allocation of resources, and dynamic fairness is a continuous allocation of resources over a specific or infinite time horizon. Also, equality is the ideal fair distribution, but in the presence of constraints, we might not reach equality, and we may need to distribute the resources as equally as possible. Mathematically, we desire to minimize $|x_i - x_j|$ or $(x_i - x_j)^2$, where $i,\ j \in N_n := \{1, \ldots, n\}$, and n is the number of users, and x_i and x_j are allocated resources to user i and j, respectively. In this function, there are a combination of n users taken two at a time, i.e. $C(n, 2)$, which can be a large number. Therefore, it is desired to minimize the maximum of $C(n, 2)$ values with any prescribed constraints. To describe this mathematically, one can write:

$$
\begin{aligned}
\underset{x \in R^n}{\text{minimize}} \ \max_{i,j \in N_n} \quad & (x_i - x_j)^2, \\
\text{subject to} \quad & p(x) < 0 \\
& q(x) = 0
\end{aligned}
\tag{1.1}
$$

in which x_i and x_j are resources allocated to user i and j, respectively, x is the vector of all users' resources, $p(x)$ is the inequality constraint, and $q(x)$ is the equality constraint. In particular, when there is only a single coupled constraint and upper bound for each individual, we have:

$$
\begin{aligned}
&\underset{x \in R^n}{\text{minimize}} \max_{i,j \in N_n} && (x_i - x_j)^2, \\
&\text{subject to} && \sum_i x_i \leq \bar{x}, \\
& && 0 \leq x_i \leq \bar{x}_i,
\end{aligned} \tag{1.2}
$$

for which x_i, x_j, and x have the same meaning as (1.1). The constant $\bar{x}$ denotes the upper bound of the total resources, and $\bar{x}_i$ represents the upper bound for individual resources of each user.

For fairness in a dynamic network, resource allocation needs to be considered in a prespecified time horizon, with the network dynamics. The users in the network can be interacting or non-interacting. For simplicity, we consider linear coupled constraints along with upper- and lower-bound restrictions. Interaction among users can be represented by a matrix similar to the adjacency matrix in graphs. We will investigate this problem in our research.

In controlling a group of competing agents with limited resources, fairness is a central problem. Some real-world examples of this type of problem include heating, ventilation, and air conditioning (HVAC) systems, code-division multiple access (CDMA) systems, and sensor networks, to name only a few. For instance, in HVAC systems, the objective is to perform a fair distribution of the available energy among all the units in a building to heat them. On the other hand, in CDMA systems, it is aimed to allocate rates to all subscribed users in a fair manner in the presence of some constraints. To incorporate a fair strategy in these systems, it is essential to mathematically describe the governing network dynamics, which is unique in each application. In this study, we provide a formulation for fairness that can be applied to systems with different dynamics.

1.2 Applications of Fairness

In the proceeding sections, two applications of fairness are introduced, and a brief description for each of them is provided.

1.2.1 Heating, Ventilation, and Air Conditioning Systems

Consider a building with several units, and, for simplicity, assume it has only one floor. There would be dynamic coupling between units through the common walls. There would also be dynamic interaction with the outside environment. We aim to heat every unit to the desired temperature for its occupant, subject to the limited energy resources. Note that energy limitation is a realistic condition concerning the existing energy restrictions. Based on the previous discussions, we deal with the resource allocation problem subject to some constraints. One can define various optimization indices and allocate the resources accordingly. For this study, we consider fairness as the performance index. It might not be possible to achieve equal temperatures for all units because some might have lower power for the heating appliances. However, we allocate resources as equally as possible. We use a control strategy to reach the optimization objective. The heat-transfer model between the units is based on well-known thermodynamics laws; therefore, we can use model-based control methods such as model predictive control (MPC). The objective is then to design an MPC rule to achieve both short- and long-term fairness.

1.2.2 Code-Division Multiple Access Systems

The reverse link rate in a code-division multiple access (CDMA) network can be selected arbitrarily considering three constraints:

- Rise over thermal (RoT) constraint
- Upper-bound limit for rates
- Single bit feedback signal from the base station (BS) to access terminals (ATs)

The RoT constraint acts as a coupled constraint because it is related to the rates of all ATs. If this constraint is satisfied, the signals sent from ATs to the BS can be decoded successfully. The upper-bound limits are, in fact, bounds on rates. In other words, the rates of the ATs' communication links can not exceed a predefined limit. Moreover, all ATs will receive single-bit feedback (0 or 1) from the BS. Therefore, this can be formulated as a constrained optimization problem, where the resource allocation is to be optimized in terms of fairness, with limited resources (collective and individual). Therefore, we need to use binary feedback to generate an optimal control signal in the context of the bang-bang strategy.

1.2.3 Other Potential Applications

Some of the emerging areas where fairness in resource allocation can be crucial are listed below:

- Cable-driven parallel robots (CDPRs) in tele-operation applications
- Smart grids
- Social networks (SN)

Note that although the mathematical foundation of all applications is similar, the corresponding optimization problem and its solution are different. We will discuss this issue for the two first applications in detail in the next chapters.

1.3 Thesis Contributions

The novelties of the proposed approach can be summarized as follows:

- Using the min-max formulation for fairness in dynamic systems
- Providing an extendable formulation for fairness which can be used for a variety of systems
- Incorporating the dynamic coupling between the subsystems for a more realistic modeling in the HVAC problem
- Using the proposed fairness formulation in the rate allocation problem in wireless networks

1.4 Thesis Layout

This thesis is prepared in manuscript-based style according to the Student's Guide to Thesis Preparation, Examination Procedures and Regulations of the School of Graduate Studies, Concordia University. It includes four chapters as described below.

In Chapter 2, an algorithm is developed for achieving fairness in a dynamic network, with a special focus on the HVAC system. First, the temperature of different rooms in a multi-unit building is formulated as a dynamic model, using the heat-transfer equations between the adjacent rooms as well as that between exterior rooms and outside. Then, a multi-objective optimization problem is defined, whose solution results in a long-term equitable strategy for a fair power allocation for the users solving an optimal resource allocation. A control strategy is proposed by using model predictive control (MPC) and solving an optimal resource allocation problem along with the constraints that exists in a heating (or cooling) system. Two algorithms are developed to solve the problem. Solution to this problem provides both short- and long-term fairness, as well as user comfort in all building units. The theoretical findings are verified by simulations for realistic building models with different parameters.

Chapter 3 investigates fairness in a CDMA system. This chapter starts by providing a mathematical background for a general CDMA system. Then, a multi-objective optimization problem is defined, whose solution results in a fair strategy for reverse-link rate allocation from the base station (BS) side. Several lemmas and theorems are presented to solve this optimization problem. The describing function approach is used for stability analysis and characterizing the resultant limit cycles. Finally, the efficacy of the proposed method is evaluated by simulation.

Chapter 4 concludes the thesis and provides ideas for future research directions.

Chapter 2

Heating, Ventilation, and Air Conditioning Systems

2.1 Summary

Fair power allocation for temperature regulation in the heating, ventilation, and air conditioning (HVAC) system of a multi-unit building is investigated in this section. The temperature of each unit is described by a discrete-time dynamic equation, taking into account the effect of outside temperature as well as heat transfer between the adjacent rooms. Temperature regulation is then formulated as a constrained optimization problem, where the objective is to maintain the temperature of each unit within a prescribed thermal comfort zone with a limited amount of power. An optimal control strategy is presented to minimize the maximum mutual temperature difference between different units (long-term fairness) while maintaining the temperature of each unit in the comfort zone or close to it at all times, as much as possible (short-term fairness). Simulations demonstrate the effectiveness of the proposed control strategy in regulating the temperature of every unit in a building.

2.2 Introduction

Commercial and residential buildings have considerable proportion of the global energy consumption [16]. One of the primary energy uses in buildings is heating, ventilation, and air conditioning (HVAC) systems. HVAC systems are widely used for thermal comfort in modern buildings and have a 50% share of total energy usage in buildings [17]. According to [18], approximately 75% of heating and cooling is still generated from fossil fuels. Such fossil fuel consumption accounts for 10% of greenhouse gas emissions and 25% of the total energy usage in the United States [19]. Thus, there has been a growing interest in recent years in efficient power allocation for temperature control of both residential and commercial buildings [20, 21, 22]. Large HVAC facilities require constant supervision of technicians to adjust settings for optimal performance of the overall system [23, 24]. Efficient power allocation in HVAC systems can be considered from a different perspective, such as energy consumption efficiency and occupant thermal comfort. One important challenge in this type of system is to allocate the power to different units or rooms in the building as fairly as possible, given that allocating more heating power for one unit can come at the cost of possible discomfort to people in another unit. Therefore, it is desired to keep the temperature of different units close to each other and within the thermal comfort zone at all times as much as possible. On the other hand, the power allocated to each unit and also the total available power at any point in time are limited, adding to the complexity of the problem.

Different control strategies are proposed in the literature for thermal regulation in buildings. These control methods include classical control, hard control, soft control, hybrid control, and other control techniques, as demonstrated in Fig. 2.1 [25]. One of the well-established control methods for HVAC systems is model predictive control (MPC) [26, 27, 28] because it can incorporate weather forecast and occupancy profiles in real-time for decision-making [29]. In HVAC system, the goal of MPC is to fined a control actions that minimize a cost function or multiple cost functions, while ensuring the comfort of the occupants and respecting the constraint of the problem [30]. In HVAC systems, MPC methods offers more efficient control, higher energy savings and better indoor environment, compared to classical control methods [31]. Optimization-based methods are employed in [32, 33] to achieve the above objective using the model predictive control (MPC) framework. In [34, 35], robust MPC techniques are used for temperature regulation to account for uncertainty in the model of the building. In [36], authors proposed a mixed-integer linear programming-based MPC strategy to improve the operation of the HVAC systems in buildings. A multi-objective genetic algorithm is presented in [37] to optimize a number of set-points, including unit air temperatures taking into account energy consumption and thermal comfort. The authors in [38] introduce an HVAC control strategy whose objective is to achieve a human-sensation-based indoor thermal comfort as opposed to maintaining a constant indoor air temperature. The interested reader is referred to [25] for a more detailed survey on control techniques for the HVAC systems, especially MPC-based methods. Lexicographic fairness for bandwidth allocation and data collection is discussed in detail in [39, 40]. In [41], a computationally efficient lexicographic minimax algorithm is developed to achieve fairness by properly allocating resources. However, the method cannot be effectively used for thermal regulation using an HVAC system as the resource allocation scheme in [41] is fixed and not suitable for a dynamic system. On the other hand, there are some recent advances in fair resource allocation in buildings. Fair heat distribution is of particular interest in HVAC systems due to its impact on energy efficiency and user comfort [42]. Fair heat distribution in a system consisting of several heating sources is addressed in [43]. The authors in [44] propose an optimal heat allocation scheme by considering the heat flow through walls, investigating both deterministic and statistical approaches. Moreover, in [45], fair cost distribution for a smart building is studied, and a lexicographic minimax approach is provided for solving the problem.

While existing HVAC control techniques, including the ones cited in the previous paragraph, are useful in regulating the temperature in some buildings, they are not as effective for buildings with several units as they often ignore the heat transfer between adjacent units. Furthermore, most of the existing methods fail to consider both short-term and long-term fairness in power allocation for thermal regulation. In the present work, the temperatures of different units in the building are modeled by a dynamic equation, where the temperature of each unit is affected by not only the power allocated to that unit but also by the heat transfer to/from its adjacent units. The problem is then formulated as a constrained optimization, where fairness in power allocation is represented by an appropriate cost function, and the maximum available power for each unit as well as the total available power is described by inequality constraints. One of the key features of the proposed method is that it uses the prescribed thermal comfort zone as a design specification in the optimization process. The efficacy of the proposed optimal power allocation method is verified by simulations.

2.3 Problem Formulation

Consider a building with $n \in N$ units (rooms). The temperature of each unit is a function of the amount of heating provided to that unit by the HVAC system as well as heat transfer between that unit and its neighboring units. Thus, the temperature of different units is modeled as a dynamic, interconnected network, where the state of each node at any instant is the temperature of that node. Each link, on the other hand, represents the effect of heat transfer between two neighboring units at any time. A unit whose neighbors are all inside the building is called an *interior unit*. In contrast, a unit that is adjacent to at least one outside wall or window is referred to as an *exterior unit*.

Let the temperatures of different units evolve according to the following dynamic equation:

$$t(k+1) = (I - L)t(k) + B_1 t^{out}(k) + B_2 q(k), \tag{2.1}$$

where $t(k) \in R^n$ is a column vector containing the temperatures of all units at time k, with its i-th element $t_i(k)$ denoting the temperature of unit $i \in N_n := \{1, \ldots, n\}$ at time $k \in N$. Also, the matrix $L \in R^{n \times n}$ is used to describe the adjacency of each unit to its neighboring units inside

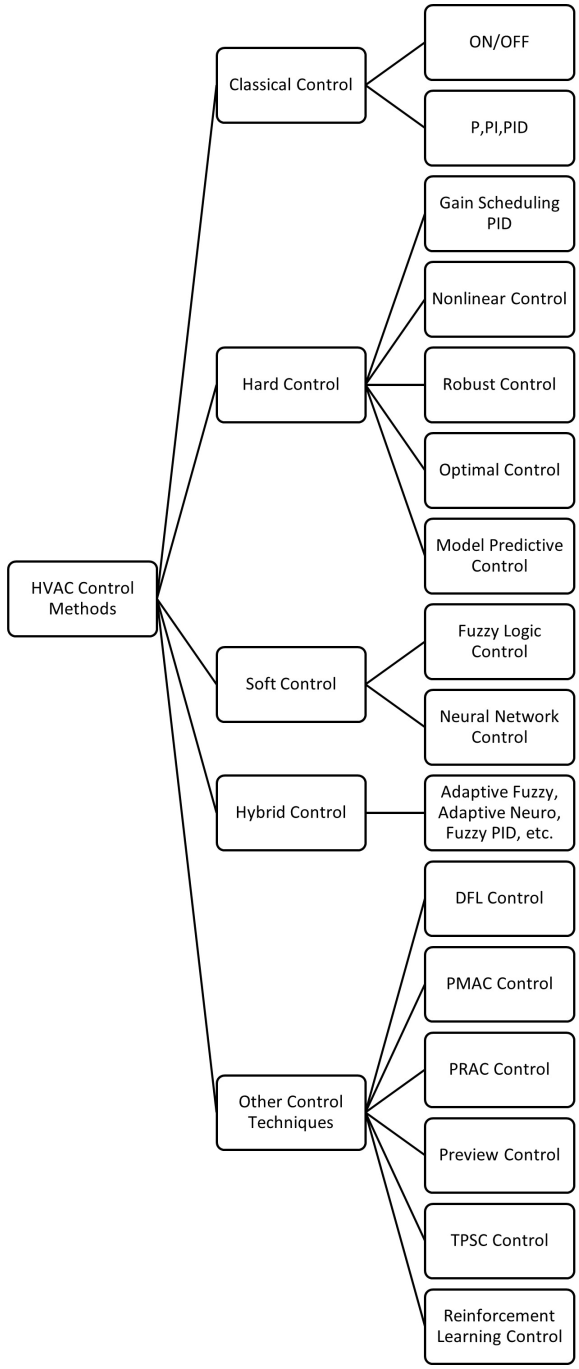


Figure 2.1: HVAC control methods

the building, and $t^{out} \in R$ is the outside temperature at time k (which is assumed to be the same everywhere around the building). Moreover, B_1 is a column vector representing energy transfer between the exterior units and outside. Note that for any interior unit, the corresponding term in vector B_1 is, by definition, set to zero. Furthermore, $q(k) \in R^m$, $m \leq n$, is a vector of the allocated heating source powers at time k, and matrix $B_2 \in R^{n \times m}$ describes the propagation of heat from the source to units, affecting their temperatures.

Remark 1. *The temperature dynamics in equation (2.1) is a linear approximation of the nonlinear heat transfer dynamics.*

Remark 2. *The coefficient of matrix L corresponding to a pair of adjacent rooms is dependent on the material within the wall separating those rooms as well as the area of the wall.*

Remark 3. *It is important to note that the notion of adjacency matrix in the context of heat transfer in a multi-unit building is different from that used in graph theory.*

Assumption 1. *For simplicity and without loss of generality, it is assumed in this research that every unit has one heating source, i.e., $m = n$. This means that the i-th element of vector $q(k)$, denoted by $q_i(k)$, is the power allocated to unit i at time k, and B_2 is diagonal.*

For the case when the outside temperature has been constant and the heating power for every unit has been fixed for a sufficiently long time, one can derive the steady-state temperature for every unit of the building through the following equation:

$$t_{ss} = B_1' t^{out} + B_2' q_{ss}, \tag{2.2}$$

where $t_{ss} \in R^n$ is the vector of steady-state temperatures of all units, and t^{out} and q_{ss} are fixed quantities as noted above. It is straightforward to verify that $B_1' = L^{-1}B_1$ and $B_2' = L^{-1}B_2$ in the above equation, for the case when L is non-singular (in practice L is always nonsingular because it is the sum of a positive-definite matrix and a positive semi-definite matrix).

Remark 4. *In the special case, if the adjacency of different units is neglected, matrix L in equation (2.1) becomes a diagonal matrix, leading to the following simplified dynamic equation for the unit temperatures:*

$$t(k+1) = At(k) + B_1 t^{out}(k) + B_2 q(k), \tag{2.3}$$

where $A = I - L$. Since A and B_2 are diagonal matrices, one can write the equation for the temperature of each unit separately as:

$$t_i(k+1) = \alpha_i t_i(k) + \beta_{1i} t^{out}(k) + \beta_{2i} q_i(k), \tag{2.4}$$

where α_i is the (i,i)-th element of matrix A, and β_{1_i} and β_{2_i} are, respectively, the i-th element of vector B_1 and the (i,i)-th element of matrix B_2, for any $i \in N_n$. It is implied from the above equation that if no power is allocated to an interior unit (i.e. $q_i = 0$ for some $i \in N_n$), the temperature of that unit will not change over time, and the outside temperature will only affect the exterior units. In practice, however, it is known that due to the adjacency of the units and the resulting heat transfer between them, the outside temperature will eventually affect every interior unit as well.

Assumption 2. *The power allocated to unit i at any time instant is upper-bounded by a pre-specified value $\bar{q}_i$ for any $i \in N_n$. Moreover, the total power provided to all units in the building cannot exceed a prescribed value denoted by $\bar{q}$, i.e., $\Sigma_{i=1}^{n} q_i \leq \bar{q}$.*

Remark 5. *From the above assumption, it can be easily verified that the maximum achievable steady-state temperatures vector for different units (when the outside temperature has been constant for a long time) is upper-bounded by $L^{-1}B_1 t^{out} + L^{-1}B_2\bar{q}_{ss}$, where $\bar{q}_{ss} = [\bar{q}_1, \ldots, \bar{q}_n]^T$.*

Remark 6. *While the main focus of this study is on the heating mode of the HVAC systems, the results can be easily extended to the cooling mode as well. The only difference in the formulation of* *the cooling mode is that matrix B_2 is multiplied by -1. If a unit does not have any power source, the corresponding coefficient in B_2 will be zero.*

Definition 1. *A comfort zone is defined for each unit, which is characterized by an upper temperature t_{i_u} and a lower temperature t_{i_l} for the i-th unit. For simplicity and with no loss of generality, it is assumed that all units have the same comfort zone with the upper and lower temperatures denoted, respectively, by t_u and t_l.*

Definition 2. *In controlling the temperature of a multi-unit building, it is typically desired to minimize the maximum mutual difference between the unit temperatures in steady-state and also minimize the maximum instantaneous temperature in transient. The former objective will hereafter be referred to as* long-term fairness*, and the latter will be called* short-term fairness*.*

To proceed with the controller design, it is required to determine a long-term fair temperature for every unit. Thus, a two-stage control algorithm is sought, where the first stage aims at regulating the temperature while achieving short-term fairness, and the second stage is concerned with guaranteed long-term fairness. Note that in the first stage, the desired temperature of the units is neglected, and only the constraint on the total available power is taken into account (because the desired temperature region is translated into the required power for achieving them). Algorithm 1 addresses the objectives of both stages, as discussed later.

To formulate fairness for the underlying interconnected network as an optimization problem, the maximum mutual difference between unit temperatures is to be minimized (long-term fairness). For a building with n units, this is mathematically described as:

$$
\begin{aligned}
&\underset{q_{ss}\in R^n}{\text{minimize}}\ \max_{i,j\in N_n} && (t_{i,ss}-t_{j,ss})^2, \\
&\text{subject to} && \sum_i q_{i,ss} \le \bar{q}, \\
& && q_{i,ss} \le \bar{q}_i, \\
& && t_{ss} = B_1' t^{out} + B_2' q_{ss},
\end{aligned}
\tag{2.5}
$$

where $t_{i,ss}$ denotes the temperature of unit i in steady state, and q_{ss} is the vector of steady-state allocated powers.

Remark 7. *The optimization problem above aims to minimize the difference between each room's temperature and that room's desired temperature. The constraints in this optimization problem reflect the limited heating resources.*

The cost function described in (2.5) takes into account the mutual temperature differences between all units. It is to be noted that one can use the absolute value of temperature differences in the objective function in (2.5), and squared values are used instead to simplify mathematical proofs. Note that because of the coupled inequality condition, the constrained optimization problem (2.5) may have multiple solutions. Therefore, without loss of generality, the coupled inequality is considered as equality for future results.

It is desired now to find a control law to achieve fair steady-state temperatures from any initial conditions. To this end, a model predictive control (MPC) approach is to be used to tackle the following optimization problem:

$$
\begin{aligned}
&\underset{q_{ss}\in R^n}{\text{minimize}} \max_{i,j\in N_n} && \sum_{k=1}^{P}(t_i(n+k)-t_j(n+k))^2, \\
&\text{subject to,} && min(\bar{q}_l,\bar{q}) \leq \sum_i q_i(k) \leq min(\bar{q}_u,\bar{q}), \\
& && q_i \leq \bar{q}_i, \\
& && t(k+1) = At(k) + B_1 t^{out}(k) + B_2 q(k),
\end{aligned}
\tag{2.6}
$$

where $\bar{q}_l$ and $\bar{q}_u$ are, respectively heating powers which yield temperatures t_l and t_u (the thermal comfort zone), obtained from Algorithm 2, and P is the prediction horizon.

Algorithm 1 Fair power allocation strategy for thermal comfort.

```
update t^out
initialize t_l, t_u
q_l ← LONGTERM_ALLOC(t_l, t^out)
q_u ← LONGTERM_ALLOC(t_u, t^out)
while true do
    if |Δt^out| > t^out_threshold then
        update t^out
        q̄_l ← LONGTERM_ALLOC(t_l, t^out)
        q̄_u ← LONGTERM_ALLOC(t_u, t^out)
    end if
    q ← solve optimization (2.6)
end while
```

2.4 Main Results

A procedure is developed in Algorithm 1 for allocating power to different units such that short- and long-term fairness is achieved. In this algorithm, at the beginning of each iteration, if the change in outside temperature is greater than a prescribed threshold, new upper and lower bounds for the total power should be derived according to Algorithm 2, because when outside temperature changes, $\bar{q}_l$ and $\bar{q}_u$ are subject to change as well. After this step, power allocation for the subsequent interval

Algorithm 2 The steady-state fair power allocation for thermal comfort

function LONGTERM_ALLOC(t_{sp}, t^{out})
  $t_{b,new} \leftarrow [0\ 0\ \dots\ 0]^T$
  $A_{eq} = \mathbf{1}$
  $b_{eq} = \bar{q}$
  while $(!(sum(t_{b,new}) = sum(t_b)$ and $counter! = 0))$ **do**
    $t_b \leftarrow t_{b,new}$
    $q \leftarrow$ long-term fair allocation, considering $A_{eq}q = b_{eq}$.
    $t \leftarrow$ new steady-state temperature according to q
    $t_{b,new} \leftarrow ((t - t_{sp}) \geq 0) || t_{b,new}$
    if all units have reached set-point **then**
      $t_b \leftarrow t_{b,new}$
      $flag \leftarrow true$
      break
    end if
    if q_i's are greater than $\bar{q}_i$ for some i **then**
      $flag \leftarrow true$
      break
    end if
    reinitialize A_{eq} and b_{eq}
    for i = all elements in $t_{b,new}$ **do**
      $A_{eq} \leftarrow [A'_{eq}\ A(i,:)']'$
      $b_{eq} \leftarrow [b_{eq}\ t_{sp}]'$
    end for
    counter++
  end while
  $A_{eq}, b_{eq} \leftarrow []$
  if flag **then**
    for i = 1:n **do**
      if $t_b(i)$ or $(!t_b(i)$ and $t_{max}(i) > t_{sp})$ **then**
        $A_{eq} \leftarrow [A'_{eq}\ A(i,:)']'$
        $b_{eq} \leftarrow [b_{eq}\ t_{sp}]'$
      else
        $I^i := zeros(1, n)$
        $I^i(i) \leftarrow 1$
        $A_{eq} \leftarrow [A_{eq} I^{i'}]$
        $b_{eq} \leftarrow [b_{eq}\ \bar{q}(i)]$
      end if
    end for
  end if
  $q \leftarrow$ optimization (2.5) with $A_{eq}q = b_{eq}$.
end function

is obtained in line 11.

In Algorithm 2, given the upper and lower bounds of the thermal comfort zone t_l and t_u, the set-point temperature t_{sp} is set to t_l and t_u in lines 8 and 9 of Algorithm 1, respectively. More precisely, in line 8 of Algorithm 1, t_l is passed to Algorithm 2 to compute $\bar{q}_l$, the minimum required power for reaching temperature t_l for all units. Then in line 9 of Algorithm 1, the same procedure is repeated, this time considering t_u instead of t_l, resulting in the minimum required power $\bar{q}_u$. The two values obtained as the minimum required power are subsequently used in line 11 of Algorithm 1 to achieve the desired thermal comfort zone. Note that Algorithm 2 uses the optimal strategy obtained by solving (2.5) and applies it to the system. If the unit's steady-state temperature is greater than the set-point, then the temperature of that unit is regarded as t_{sp} in the next iteration.

The algorithm stops when all units reach the set-point temperature (Line 10) or when there is no improvement in fair allocation of power (Line 5). By computing the total power at the end of the algorithm, the minimum required power for reaching either t_l or t_u can be obtained, which is subsequently used in Algorithm 1. Note that since $\bar{q}_l$ is the minimum required power for all units to reach t_l, if the total available power is chosen higher than this value, the temperatures of units will be higher than t_l. In other words, the minimum required power will be used first to raise the temperatures of all units to t_l, and then the excess power will be fairly distributed among all units to further increase their temperature within the thermal comfort zone. Similarly, if the total available power is less than $\bar{q}_u$, then the temperatures of all units will be less than t_u, and hence all unit temperatures are maintained within the desired range.

Assumption 3. *It is assumed that if unit $i \in N_n$ has a heater, the effect of that heater on that unit's temperature will be greater than that of any other unit. In other words, $\partial t_i / \partial q_i > \partial t_j / \partial q_i$, , where $j \in N_n$, $j \neq i$. A similar assumption holds for the case of cooling sources.*

Definition 3. *Given a column or row vector, the superscript m^- is used hereafter to generate another column or row vector whose elements are the first $m-1$ elements of the original vector.*

The superscript m^+, on the other hand, is used to generate another column or row vector consisting of elements $m, \ldots, n$ of the original vector. For example, $q_{ss}^{m^-}$ is a vector of the steady-state values of the allocated power of the heating system for units 1 to $m-1$, and $q_{ss}^{m^+}$ is a vector of the steady-state power of the heating system for units m to n.

Definition 4. *For any matrix $B \in R^{n\times n}$, B^{m^+} is a matrix consisting of rows m to n of B. Also, the short-hand notations of $B^{m^+_-}$ and $B^{m^+_+}$ denote matrices consisting of the first $m-1$ columns and columns m to n of B^{m^+}, respectively.*

Definition 5. *P is a $(n-m)\times(n-m+1)$ matrix with diagonal elements of all 1, and the elements to the immediate right-hand side of the diagonal elements are all -1.*

The minimax problem in (2.5) is equivalent to the following minimization problem:

$$\begin{aligned} \underset{q_{ss}\in R^n}{\text{minimize}} \quad & \gamma \\ \text{subject to} \quad & (t_{i,ss}-t_{j,ss})^2 \leq \gamma,\ i,j \in N_n \\ & \sum_i q_{i,ss} = \bar{q}, \\ & q_{i,ss} \leq \bar{q}_i, \\ & t_{ss} = B_1' t^{out} + B_2' q_{ss}. \end{aligned} \tag{2.7}$$

Theorem 1. *Given a multi-unit building, let the units be numbered based on their steady-state temperatures such that $t_{1,ss} < ... < t_{r,ss} = \ldots = t_{n,ss}$. Let m be the largest integer between r and n for which $q_{ss}^{m^-} = \bar{q}^{m^-}$. Then, the solution for (2.5) can be obtained as:*

$$q_{ss}^{m^+} = \begin{pmatrix} PB_2'^{m^+_+} \\ \mathbf{1}^T \end{pmatrix}^{-1} \begin{pmatrix} -PB_1'^{m^+} t^{out} - PB_2'^{m^+_-}\bar{q}^{m^-} \\ \bar{q} - \sum_{i=1}^{m-1} \bar{q}_i \end{pmatrix}, \tag{2.8}$$

where $\mathbf{1}^T$ is a row vector of all ones with appropriate dimension.

Proof. It is assumed that units with lower long-term optimal temperatures (which are units $1, \dots, r-1$) do not have the same exact steady-state temperature, which is a realistic assumption in practice. Using Lagrange multipliers for the constrained optimization problem described by (2.7) yields:

$$\begin{aligned} L =& \gamma + \sum_{\forall i,j} \lambda_{i,j}((t_{i,ss} - t_{j,ss})^2 - \gamma) \\ &+ \zeta(\sum_{i=1}^{n} q_{i,ss} - \bar{q}) + \sum_{i=1}^{n} \sigma_i(q_{i,ss} - \bar{q}_i). \end{aligned}$$

Since the desired minimum is a regular point, the first Karush-Kuhn-Tucker (KKT) condition is given by:

$$\frac{\partial L}{\partial \gamma} = 0,$$

which results in $\sum_{\forall i,j} \lambda_{i,j} = 1$; therefore, one can simply eliminate γ in the Lagrangian formulation. Removing γ and writing KKT condition for the new problem yields:

$$\begin{aligned} &\frac{\partial L}{\partial \zeta} = 0,\ \frac{\partial L}{\partial q_{i,ss}} = 0, \\ &\sigma_i(q_{i,ss} - \bar{q}_i) = 0,\ i \in N_n \\ &\lambda_{i,j}((t_{i,ss} - t_{j,ss})^2 - \gamma) = 0,\ \forall i,j \in N_n \\ &\lambda_{i,j} \geq 0,\ \sigma_i \geq 0,\ \forall i,j \in N_n. \end{aligned}$$

According to *Assumption 3* it can be proved that $\zeta \neq 0$. Units are renumbered using the scheme described earlier, $t_{i,ss} \neq t_{j,ss}$, and hence

$$\begin{cases} \lambda_{i,j} = 0,\ i \in N_{r-1},\ j \in N_n \\ \frac{\partial L}{\partial q_{i,ss}} = 0, \end{cases}$$

yields $q_{i,ss} = \bar{q}_i$. If there exists any other unit for which $q_i = \bar{q}_i,\ i \in \{r, \dots, n\}$ it is assigned a number between r and $m-1$, i.e. $q_{ss}^{m^-} = \bar{q}_{ss}^{m^-}$. Thus, for $i \geq m$, the corresponding σ_i is zero. For units $m,\ldots,n$ since $t_m = \ldots = t_n$ hence $Pt^{m^+} = 0$. Thus,

$$
\begin{aligned}
&P(B_1^{\prime m^+} t^{out} + B_2^{\prime m^+} q_{ss}) = 0 \\
\Rightarrow\ &PB_2^{\prime m^+} q_{ss} = -PB_1^{\prime m^+} t^{out}, \\
\Rightarrow\ &\begin{pmatrix} PB_2^{\prime m^+_+} \\ \mathbf{1}^T \end{pmatrix} q_{ss}^{m^+} = \begin{pmatrix} -PB_1^{\prime m^+} t^{out} - PB_2^{\prime m^+_-} \bar{q}^{m^-} \\ \bar{q} - \sum_{i=1}^{m-1} \bar{q}_i \end{pmatrix}.
\end{aligned}
$$

From the above result and noting that that for units $1,\ldots,m-1$ we have $q_i = \bar{q}_i$, it results that:

$$
q_{ss}^{m^+} = \begin{pmatrix} PB_2^{\prime m^+_+} \\ \mathbf{1}^T \end{pmatrix}^{-1} \begin{pmatrix} -PB_1^{\prime m^+} t^{out} - PB_2^{\prime m^+_-} \bar{q}^{m^-} \\ \bar{q} - \sum_{i=1}^{m-1} \bar{q}_i \end{pmatrix} \tag{2.9}
$$

□

Using Theorem 1, it is straightforward to obtain the steady-state temperature of every unit in the special case when there is no heat transfer between them. This special case is addressed in the next proposition.

Proposition 1. *Neglecting the effect of heat transfer between the adjacent units and reordering units as proposed in Theorem 1, the following solution is obtained for the optimization problem described by (2.5):*

$$
\begin{aligned}
t_{i,ss} &= \frac{\bar{q} + \sum_{k=m}^{n} \frac{\beta_{1k}}{\beta_{2k}} t^{out} - \sum_{k=1}^{m-1} \bar{q}_k}{\sum_{k=m}^{n} \frac{\beta_{1k}}{\beta_{2k}}},\ i = m,\ldots,n \\
t_i &= t_i^{out} + \frac{\beta_{2i}}{\beta_{1i}} \bar{q}_i,\ i = 1,\ldots,m-1.
\end{aligned} \tag{2.10}
$$

Proof. By neglecting the heat transfer between adjacent units (i.e., using equation (2.4)), it is straightforward to show that $B_1' = I$ and $B_2' = diag(\frac{\beta_{21}}{\beta_{11}},\ldots,\frac{\beta_{2n}}{\beta_{1n}})$ in equation (2.2). Therefore, one can decompose the following matrix into the product of a lower triangular matrix and upper triangular matrix, as follows:

$$\begin{pmatrix} PB_2'^{m_+^+} \\ \mathbf{1}^T \end{pmatrix} = \left(\begin{array}{cccc|c} I & & & & \mathbf{0} \\ \hline \frac{\beta_{1m}}{\beta_{2m}} & \frac{\beta_{1m}}{\beta_{2m}} + \frac{\beta_{1(m+1)}}{\beta_{2(m+1)}} & & \dots & \sum_{i=m}^{n} \frac{\beta_{1i}}{\beta_{2i}} \end{array}\right) \left(\begin{array}{c} PB_2'^{m_+^+} \\ \hline 0 \dots 0 \ \frac{\beta_{2n}}{\beta_{1n}} \end{array}\right)$$

The inverse of the above matrix can be obtained as:

$$\begin{pmatrix} PB_2'^{m_+^+} \\ \mathbf{1}^T \end{pmatrix}^{-1} = \begin{pmatrix} \frac{\beta_{1m}}{\beta_{2m}} & \frac{\beta_{1m}}{\beta_{2m}} & \dots & \frac{\beta_{1m}}{\beta_{2m}} \\ 0 & \frac{\beta_{1(m+1)}}{\beta_{2(m+1)}} & \dots & \frac{\beta_{1(m+1)}}{\beta_{2(m+1)}} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{\beta_{1n}}{\beta_{2n}} \end{pmatrix} \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\frac{\beta_{1m}}{\beta_{2m}}}{\sum_{i=m}^{n} \beta_{1i}/\beta_{2i}} & -\frac{\frac{\beta_{1m}}{\beta_{2m}} + \frac{\beta_{1(m+1)}}{\beta_{2(m+1)}}}{\sum_{i=m}^{n} \beta_{1i}/\beta_{2i}} & \dots & \frac{1}{\sum_{i=m}^{n} \beta_{1i}/\beta_{2i}} \end{pmatrix}. \tag{2.11}$$

Considering equation (2.3) in steady state and substituting in it the steady-state allocated power obtained from (2.8), one arrives at:

$$t_{ss}^{m^+} = \mathbf{1}t^{out} + B_2'^{m_+^+} \begin{pmatrix} PB_2'^{m_+^+} \\ \mathbf{1}^T \end{pmatrix}^{-1} \begin{pmatrix} -Pt^{out} \\ \bar{q} - \sum_{i=1}^{m-1} \bar{q}_i \end{pmatrix}. \tag{2.12}$$

The proof is completed by replacing the inverse matrix in the above equation from (2.11). □

Remark 8. *Proposition* 1 *implies that the optimal strategy obtained by solving (2.5) allocates power to units with the lowest maximum achievable temperature first, then the ones with the second-lowest*

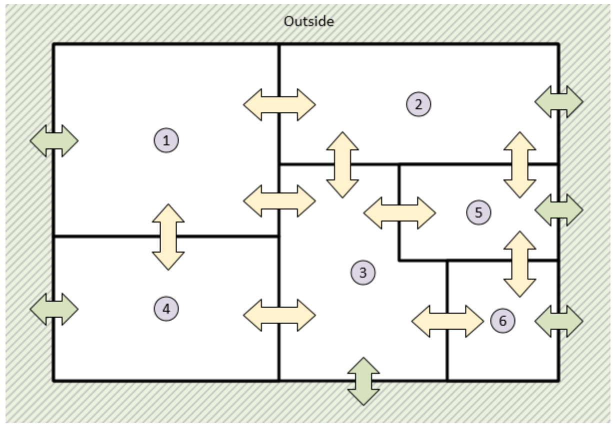


Figure 2.2: The floor map of the building in Example 3. The double-sided arrows represent heat transfer between two neighboring units (yellow arrows) or between a unit and outside (green arrows).

maximum achievable temperature, and so on. All other units with higher achievable temperatures reach consensus in their temperature.

2.5 Simulation Results

Three examples are presented in this section to illustrate the theoretical findings.

Example 3. *Consider a multi-unit building with the floor map depicted in Fig. 2.2, and let the parameters related to the building and environment be* $n = 6$, $\beta_1 = 0.1$, $B_1 = [\beta_1, ..., \beta_1]^T$, $B_2 = diag([0.1\ 0.18\ 0.09\ 0.11\ 0.12\ 0.095])$, $\bar{q}_1 = 20$, $\bar{q}_2 = 10$, $\bar{q}_3 = 25$, $\bar{q}_4 = 25$, $\bar{q}_5 = 20$, $\bar{q}_6 = 20$, $\bar{q} = 110$, $t(0) = [5\ \ 2\ \ 7\ \ 1\ \ 0\ \ 0]^T$*, and* $t_{out} = 0$*. Assume there is no heat transfer between different units. The results of the optimal strategies obtained by solving the optimization problem (2.6) are demonstrated in Fig. 2.3. Note that it is assumed in this example that there is no predefined thermal comfort zone for the units. In other words, lines 8 and 9 of Algorithm 1 are replaced by* $\bar{q}_l \leftarrow \bar{q}$ *and* $\bar{q}_u \leftarrow \bar{q}$*, respectively (this means that Algorithm 2 is not executed in this case). The simulations confirm that this approach exhibits short-term fairness.*

Example 4. *To verify short-term fairness property of the proposed method, consider a building with* *two units, and let the building parameters be* $\beta_1 = 0.1$, $B_1 = [\beta_1, \beta_1]^T$, $B_2 = diag([0.20, 0.20])$, $\bar{q}_1 = \bar{q}_2 = 14$, $\bar{q} = 25$, $t(0) = [2 \;\; 28]^T$, *and* $t^{out} = 0$. *The results are presented in Fig. 2.4, which show that both short-term and long-term fairness are achieved using this approach.*

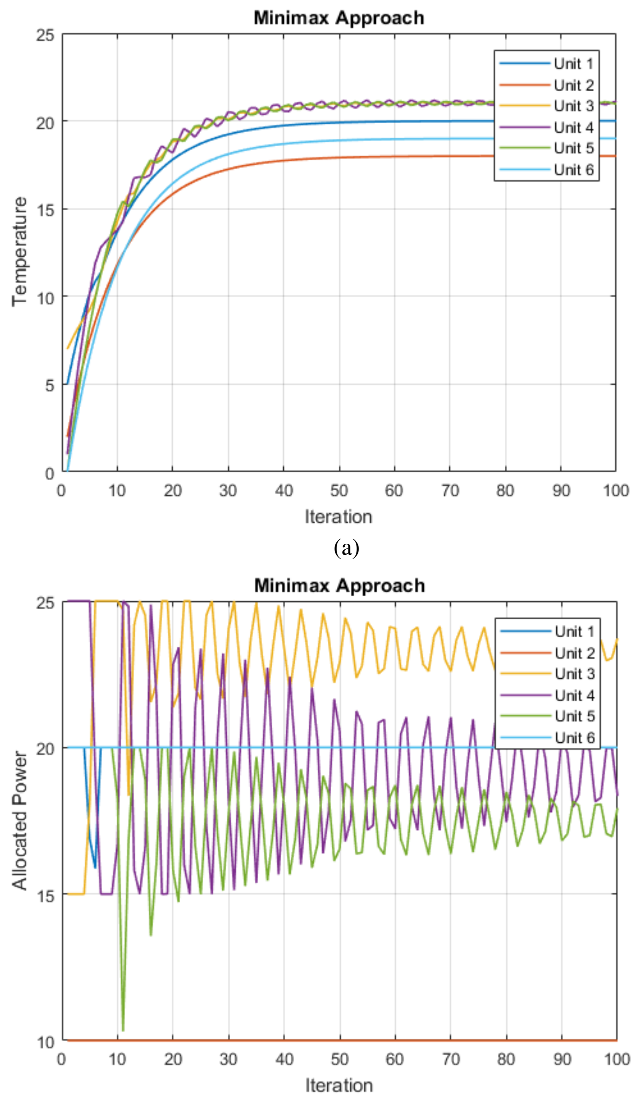
Example 5. *Consider a multi-unit building with the same floor map as in Example 3 (i.e., the one given in Fig. 2.2). Assume that the heat transfer between two adjacent units is not negligible, and let the adjacency matrix be given by:*

$$L = \begin{bmatrix} 7\beta_1 & -2\beta_1 & -\beta_1 & -3\beta_1 & 0 & 0 \\ -2\beta_1 & 8\beta_1 & -2\beta_1 & 0 & -3\beta_1 & 0 \\ -\beta_1 & -2\beta_1 & 9\beta_1 & -2\beta_1 & -2\beta_1 & -2\beta_1 \\ -3\beta_1 & 0 & -2\beta_1 & 6\beta_1 & 0 & 0 \\ 0 & -3\beta_1 & -2\beta_1 & 0 & 8\beta_1 & -2\beta_1 \\ 0 & 0 & -2\beta_1 & 0 & -2\beta_1 & 5\beta_1 \end{bmatrix}.$$

The simulation results, in this case, are provided in Fig. 2.5, which demonstrates the effectiveness of the proposed technique in achieving long-term fairness. This figure shows that under the optimal strategy developed in this work, the temperatures of all units settle within the thermal comfort zone while preserving short- and long-term fairness.

2.6 Conclusions

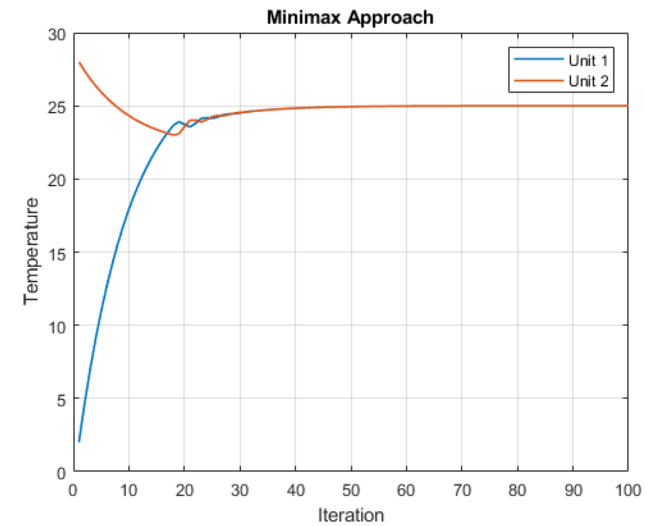
In this chapter, fair allocation of energy resources in a multi-unit building is investigated where it is desired to reach a thermal comfort zone for every unit with limited energy resources. Unlike existing strategies that often ignore the heat transfer between adjacent units, the proposed method uses an adjacency matrix to model the heat exchange dynamics. The Problem is then formulated as a constrained optimization where it is desired to achieve short-term and long-term fairness. Analytical solutions are provided for the optimization problem with and without heat transfer dynamics, and



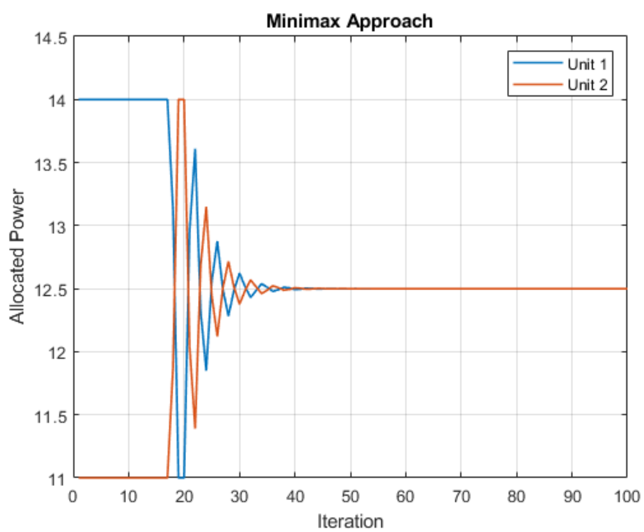
(a)

(b)

Figure 2.3: Temperatures and power allocation for the units in the building of Example 3 without taking the heat transfer between the units into consideration. (a) Room temperatures obtained by using the minimax approach, and (b) power allocation for the units obtained by using the minimax approach.



(a)



(b)

Figure 2.4: Temperatures and power allocation for the units in the building of Example 4 without taking the heat transfer between the units into consideration. (a) Room temperatures obtained by using the minimax approach, and (b) power allocation for the units obtained by using the minimax approach.

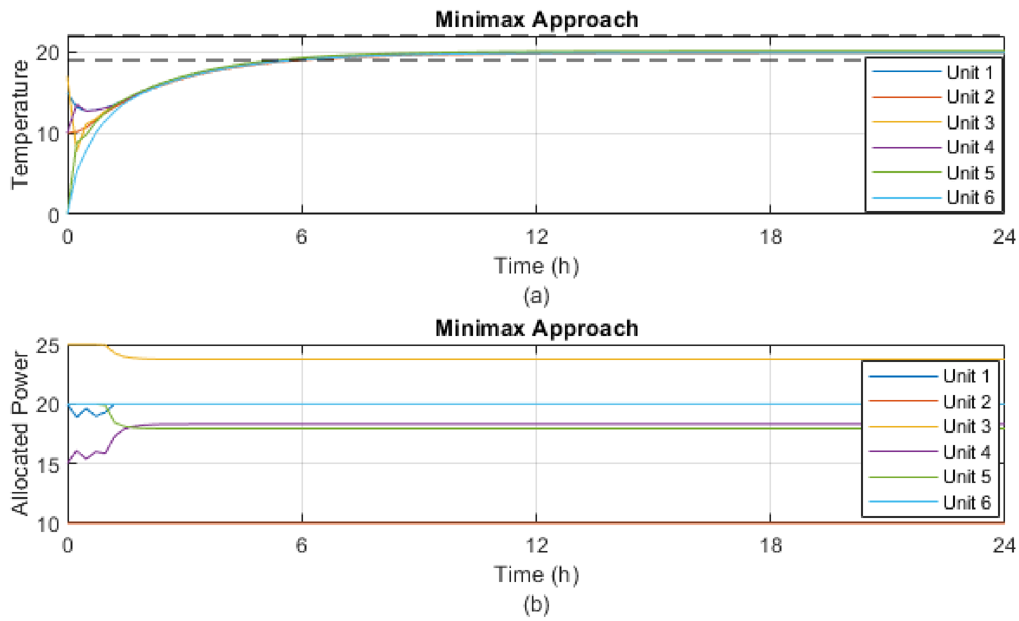


Figure 2.5: Allocated power and resulting temperature over 24 hours in the system of Example 5. (a) Temperatures of different units obtained by using the proposed method, and (b) power allocated to each unit using the proposed method.

fair steady-state temperatures of all units are derived in long term. Simulations for three different building models demonstrate the efficacy of the proposed optimal strategy in achieving short- and long-term fairness with limited resources.

Chapter 3

Fair Rate Assignment in Reverse-Link CDMA

3.1 Summary

This chapter studies fair dynamic rate allocation in a CDMA network. An optimization-based fair reverse-link rate assignment strategy is proposed. The network is modeled in a star topology, where the nodes represent either the base station (BS) or access terminals (ATs). The BS at every time instant computes the fair rate for each AT by minimizing the maximum disparity in user rates. Then, the BS sends a single bit to all ATs at every time instant. It is shown that if each AT could compute a specific variable, called the coordinating variable, it can find its fair rate, which means the decision-making strategy is distributed. The proposed method is computationally efficient, and simulations confirm its efficacy in different scenarios.

3.2 Introduction

Code-division multiple access (CDMA) 2000 standard was developed for high-speed wireless communication networks. One of the primary advantages of this technology is that it does not limit the user's frequency spectrum, hence optimizing the bandwidth utilization. In the past two decades, there have been several improvements in this type of network, and the technology continues to be dominant in many mobile telephone standards.

Fairness is one of the critical problems in wireless communication networks. The authors in [46] presents two categories of fairness measures in rate assignment: quantitative and qualitative. Two quantitative fairness measures are introduced in [47] and [48], respectively, based on Jain's index [49] and entropy [50]. The two main qualitative fairness measures, on the other hand, are max-min fairness [51, 52, 53, 54, 55] (which is often defined based on the notion of bottlenecks [56]), and proportional fairness (which is applied to the rate per unit charge). Lexicographic fairness for bandwidth allocation and data collection is discussed in detail in [39], [40]. The authors in [41] develop a computationally efficient lexicographic minimax algorithm to allocate resources appropriately. Two types of fairness, namely dynamic and static, are studied in [39], [40], [41]. The concept of fairness has also been applied to other applications, such as HVAC systems [57].

Authors in [58] provide two reverse-link algorithms in CDMA are proposed. They formulated the problem as a utility maximization problem. It is shown that both MAC algorithms solve this utility maximization problem and therefore show fair behavior. Koksal et al. [59] provide two metrics for measuring fairness for MAC protocols. The focus of their research was more on short-term fairness rather than considering fairness in an extended period, long-term fairness. In [60], a distributed rate control algorithm is proposed, which controls reverse-link rate for all ATs using a pricing mechanism to guarantee short- and long-term fairness. The authors in [61] provide two algorithms for optimal reverse link rate assignment using only a single-bit feedback signal. The rate assignment problem is formulated in the context of resource allocation in [62], where the stability criteria are developed in the presence of time delay. The authors in [63] develop models by analysis and simulation to evaluate network fairness. Four bandwidth allocation methods are developed in [64], and their fairness properties are analyzed for variable bit rate traffic. Moreover, there have been some valuable advancements in fairness in wireless sensor networks recently. For example, the authors in [65] investigate spectrum competition among users in a wireless network. This becomes a more prevalent problem when dealing with an exponentially increasing quantity of intelligent terminals. The work [66] proposes a fair and energy-efficient resource allocation in a specific wireless network setting. Furthermore, there is often a conflict between two objectives: energy efficiency and spectral efficiency. The above article transforms this conflict into a solvable minimization problem that results in a fairness-aware spectral and energy-efficient output. Furthermore, as a result of slower flow convergence during the loss recovery phase and flat-rate reduction during congestion control, the multi-hop wireless network shows poor throughput stability and flow fairness performance.. In [67], the authors propose feedback-assisted recovery to address this problem. In [68], the user cooperation method in wireless networks is investigated. The proposed method can mitigate inherent user unfairness issues in wireless network settings, resulting in more satisfaction for all users. The authors in [69] study resource allocation under a fairness-energy-throughput. This will lead to an increase in spectral efficiency and higher quality of service.

While the network performance in this type of network can be formulated as a classical control problem, most of the methods described in the previous paragraphs do not tackle the problem from a control-theoretic perspective. Such a viewpoint could help improve the network performance using powerful closed-loop control techniques. A constrained optimization strategy is developed in this work to enhance fairness in a wireless communication network. The analysis and design methods are based on a classical closed-loop control system, where the binary feedback signal from the BS to each user or AT is modeled as a nonlinear two-stage controller. The solution to the problem is obtained using the Karush-Kuhn-Tucker (KKT) conditions. It is also shown how the describing function method can be used to identify permanent oscillations in the assigned rates.

The remainder of this chapter is planned as follows. In Section III, the problem statement along with some useful background information is presented. Then, in Section IV, the main results are given. Next, simulations are provided in Section V for different scenarios, which support the efficacy of the theoretical results. Finally, the concluding remarks are presented in Section VI.

3.3 Problem Formulation

Now, we get back to a CDMA network. Consider a CDMA network with n ATs and one BS, as shown in Fig. 3.1. ATs cannot communicate with each other, and only receive one single bit from the BS. This is, in fact, the feedback signal and is the same for all ATs [60].

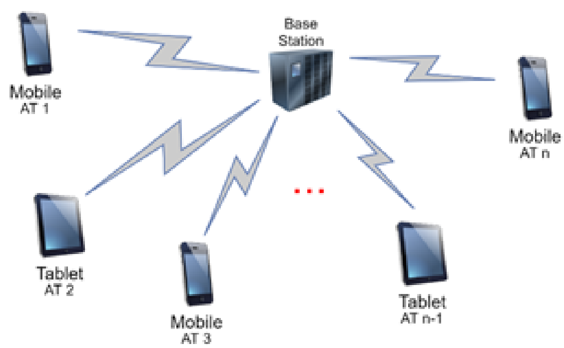


Figure 3.1: An example of a wireless communication network with one BS and n ATs (users)

Each AT can transmit at rates belonging to a prespecified finite set Γ. Let the selected rate for the i-th AT at time instant t be $R_i(t)$. Denote the ratio of transmitted power to the pilot power by $T_i(t)$. The relation between $R_i(t)$ and $T_i(t)$ is described by a function as follows:

$$T_i(t) = F(R_i(t)) \tag{3.1}$$

The function $F(.)$ can be derived from the IS-856 standard, depicted in Fig. 3.2 [60]. According to this figure, $F(.)$ could be approximated with a first-order or third-order fit. For simplicity, we consider a first-order approximation.

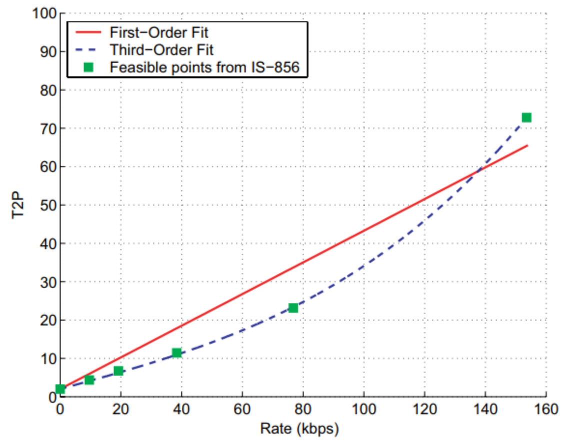


Figure 3.2: Graphical representation of function $F(.)$

For the BS to properly decode the received signal from the ATs, the rise-over-thermal (RoT) criteria given below needs to be satisfied [60]:

$$Z(t) = 10\log_{10}(1 + \sum_{i=1}^{n} \frac{T_i(t)P_{pilot}}{N_0 W}) \tag{3.2}$$

$$Z(t) \leq Z_{th} \tag{3.3}$$

where $Z(.)$ is the RoT at the BS, P_{pilot} is the received pilot power from the ATs, N_0W is the power of noise and interference, and Z_{th} is the RoT threshold to be met for the BS to decode the signals. Using (3.2), (3.3) and substituting T_i's from (3.1) yields:

$$\sum_{i=1}^{n} F(R_i(t)) \leq N_0 W(10^{Z_{th}/10} - 1)/P_{pilot} \tag{3.4}$$

It is straightforward to show that by using the first-order approximation for $F(.)$ one obtains:

$$\sum_{i=1}^{n} R_i(t) \leq R_t \tag{3.5}$$

where the parameter R_t is a threshold that can be easily obtained from (3.4). Let g_i be the long-term average rate guarantee for AT i. Let also $\rho_i(k)$ be an exponentially weighted average rate with the time constant $m \geq 2$, satisfying the following relation:

$$\rho_i(k) = \frac{1}{m}((m-1)\rho_i(k-1) + R_i(k)) \tag{3.6}$$

Definition 6. *The long-term fairness means a scalar c exists such that*

$$\lim_{k \to \infty} \rho_i(k) = cg_i \tag{3.7}$$

except when cg_i exceeds the maximum feasible transmission rate [60].

In the next section, it will be shown that the notion of fairness in a CDMA setting is equivalent to that of max-min fairness.

3.4 Main Results

Consider the following optimization problem:

$$\begin{aligned} &\text{minimize} \max_{i,j \in N_n} && (\frac{R_i}{g_i} - \frac{R_j}{g_j})^2 \\ &\text{subject to:} && R_i \leq \bar{R} \\ & && \sum_{i=1}^{n} R_i(t) \leq R_t \end{aligned} \tag{3.8}$$

in which R_i is the steady-state transmission rate of the i-th AT, g_i is its guaranteed rate, $\bar{R}$ is the maximum rate for the users, and R_t is the threshold defined based on the coupled inequality reflecting the maximum allowable interference.

Remark 9. *It is to be noted that the strict inequality optimization (3.8) has a trivial solution, which* *is zero for all variables; that is why in the optimization problem, the coupled constraint should be considered as equality.*

Remark 10. *In the performance index given in (3.8), a quadratic term is used instead of the absolute value because for the analytical solution this function needs to be differentiable.*

Now, reformulate the problem defined by (3.8) as follows:

$$\begin{aligned}
&\underset{\gamma, R_i, R_j}{\text{minimize}} \quad \gamma \\
&\text{subject to:} \quad (\frac{R_i}{g_i} - \frac{R_j}{g_j})^2 \leq \gamma \\
&\qquad R_i \leq \bar{R} \\
&\qquad \sum_{i=1}^{n} R_i(t) \leq R_t
\end{aligned} \tag{3.9}$$

To solve the above problem, one can write the Karush–Kuhn–Tucker (KKT) conditions. To this end, define the Lagrangian as:

$$\begin{aligned}
L(\gamma, R) = \gamma + \sum_{i,j} \xi_{ij}((\frac{R_i}{g_i} - \frac{R_j}{g_j})^2 - \gamma) \\
+ \sum_{i} \sigma_i(R_i - \bar{R}) + \zeta(\sum_{i=1}^{n} R_i(t) - R_t)
\end{aligned} \tag{3.10}$$

The KKT conditions are then given by:

$$\begin{aligned}
&\xi_{ij}((\frac{R_i}{g_i} - \frac{R_j}{g_j})^2 - \gamma) = 0 \\
&\sigma_i(R_i - \bar{R}) = 0 \\
&\xi_{ij} \geq 0, \sigma_i \geq 0 \\
&\frac{\partial L}{\partial \gamma} = 0, \frac{\partial L}{\partial R_i} = 0, \frac{\partial L}{\partial \zeta} = 0
\end{aligned} \tag{3.11}$$

The results derived in the sequel are obtained based on the above formulation.

Lemma 1. *There exists at least one ordered pair* (i_0, j_0) *for which the coefficient* $\xi_{i_0 j_0}$ *in (3.11) is strictly positive.*

Proof. It is known that the derivative of the Lagrangian in (3.10) with respect to γ should be zero. In other words:

$$\frac{\partial L}{\partial \gamma} = 0 \Rightarrow \sum_{ij} \xi_{ij} = 1, \xi_{ij} \geq 0 \tag{3.12}$$

From the above equation, it can be concluded that at least one of the ξ_{ij}'s is strictly positive. □

Let R_i/g_i's be ordered in descending sequence, i.e., $R_1/g_1 \geq \ldots \geq R_m/g_m > \ldots > R_{m'}/g_{m'} \geq \ldots \geq R_n/g_n$.

Lemma 2. *There exist* m *and* m' *such that for all* $i \neq 1, \ldots, m$ *or* $j \neq m', \ldots, n$ *, the coefficient* ξ_{ij} *is absolutely zero.*

Proof. The proof follows immediately by using the above ordered sequence in the KKT conditions (3.11). □

Lemma 3. *The value of* ζ *in (3.10) is strictly negative.*

Proof. We know that $\partial L / \partial R_t = 0$, for $t = 1, \ldots, N$. From Lemma 2 one can write:

$$2\frac{\frac{R_1}{g_1} - \frac{R_N}{g_N}}{g_t} \sum_i \xi_{i,t} + \sigma_t + \zeta = 0 \Rightarrow \zeta = -2\frac{\frac{R_1}{g_1} - \frac{R_N}{g_N}}{g_t} \sum_i \xi_{i,t} - \sigma_t \tag{3.13}$$

According to Lemma 1, there exists at least one pair (i_0, j_0) for which $\xi_{i_0,j_0} > 0$ and for all other pairs $\xi_{i,j} \geq 0$. Therefore, by selecting $t = i_0$ or $i = j_0$ in the above equation, it is guaranteed that $\zeta < 0$. □

Lemma 4. *For* $i = m+1, \ldots, N$*, we have* $R_i = \bar{R}$*.*

Proof. For all $i = m+1, \ldots, m'-1$, it follows from Lemma 2 that $\xi_{i,j} = 0$ for any $j = 1, \ldots, N$. . Thus, it results from (3.13) that $\sigma_i = -\zeta > 0$. From the KKT conditions in (3.11) we know that

$\sigma_i(R_i - \bar{R}) = 0$, and that $\sigma_i > 0$. This results in $R_i = \bar{R}$ for $i = m+1, \ldots, m' - 1$.

We now prove the same result for R_i with $i = m', \ldots, N$. For this, we know that based on (3.9), the function γ is minimized. Assume $R_i < \bar{R}$; then, we can increase R_i by a sufficiently small value δR and decrease this amount from R_j for $j = 1, \ldots, m' - 1$. Now, all constraints are satisfied but we have a lower value for γ which contradicts with the assumption γ was minimized. This completes the proof. □

Theorem 2. *For the set of equations and inequalities in (3.11), we have:*

$$\begin{aligned} &R_i = \bar{R}, \;\; m+1 \leq i < N \\ &R_i = \frac{g_i}{\sum_{i=1}^{m} g_i}(R_t - (N-m)\bar{R}), \;\; 1 \leq i \leq m \end{aligned} \tag{3.14}$$

where m is the largest value for which:

$$R_m \leq \bar{R} \tag{3.15}$$

Proof. From Lemma 4, we know that $R_i = \bar{R}$ for all $i = m+1, \ldots, N$, proving the first part of the theorem. As a result:

$$\sum_{i=1}^{m} R_i = R_t - (N-m)\bar{R} \tag{3.16}$$

From the sorting order discussed before, we have:

$$\frac{R_i}{g_i} = \frac{R_j}{g_j}, \;\; i = 1, \ldots, m, \; j = 1, \ldots, m \tag{3.17}$$

Equations (3.16) and (3.17), yields:

$$R_i = \frac{g_i}{\sum_{i=1}^{m} g_i}(R_t - (N-m)\bar{R}), \; 1 \leq i \leq m \tag{3.18}$$

Again, from the special sorting order, we know that m is the largest value for which $R_i = R_j$ for all $i, j = 1, \ldots, m$. This completes the proof. $\square$

So far, the fairness and maximum allowable interference constraints are satisfied. It is desired now to use a binary signal transmitted from the BS to each user to generate a bang-bang control law for every AT. To this end, equation (3.14) can be rewritten as:

$$R_i = \min\{\bar{R}, \frac{g_i}{\sum_{i=m} n g_i}(R_t - (N-m)\bar{R})\} \tag{3.19}$$

Let $C = \frac{1}{\sum_{i=m} n g_i}(R_t - (N-m)\bar{R})$, which can be computed at the BS. Thus:

$$R_i = \min\{\bar{R}, C g_i\} \tag{3.20}$$

According to the above formula, if the value of C was available to an AT, it could calculate the steady-state fairness by itself. Consider the network depicted in Fig. 3.1. Initially, all ATs consider the parameter C to be equal to zero. A bang-bang control strategy is then proposed as follows. At each time instant, the BS obtains ideal value of C as $C_{ideal} = \frac{1}{\sum_{i=m} n g_i}(R_t - (N-m)\bar{R})$. If the ATs' estimated value of C is greater than the value for C_{ideal}, the BS sends $S = 1$ to all ATs; otherwise, it sends $S = -1$ to them, and the ATs update the value of C using the following formula:

$$\begin{aligned} &\text{If } S = 1 \qquad \text{then } C_i(t) = C_i(t-1) + \Delta C, \;\; 1 \leq i \leq n \\ &\text{If } S = -1 \quad \text{then } C_i(t) = C_i(t-1) - \Delta C, \;\; 1 \leq i \leq n \end{aligned} \tag{3.21}$$

Once the value of C is updated for all ATs, they can calculate their long-term fair rate.

At the next step, we aim at using the describing function analysis to the above-mentioned bang-bang controller. To this end, assume that the sinusoidal input $A(t) = A\sin(t)$ is applied as an input to the ideal switch. Therefore, the output would be:

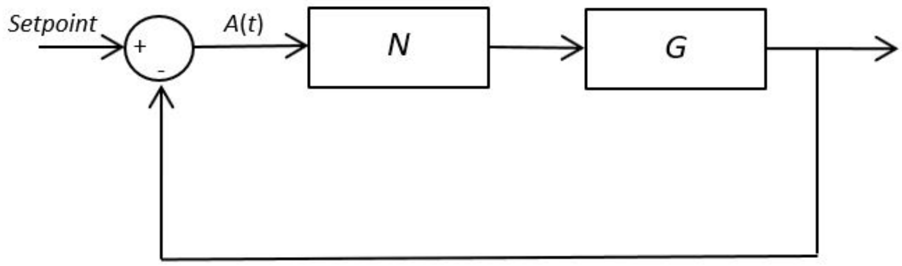


Figure 3.3: The nonlinear closed-loop system with on-off control N and linear system G, for describing function analysis.

$$B(t) = \Delta C, \;\; 0 \leq \omega t \leq \pi$$
$$B(t) = \Delta C, \;\; \pi \leq \omega t \leq 2\pi \tag{3.22}$$

As the output is an odd function, the cosine coefficients of its Fourier transform are zero, and in particular, $a_1 = 0$. As for the sine coefficients, we have:

$$b_1 = \frac{4\Delta C}{\pi} \tag{3.23}$$

Hence, the phase angle of the describing function is:

$$\arctan(a_1/b_1) = 0 \tag{3.24}$$

Consequently, the describing function can be written as:

$$N(A, \omega) = \frac{b_1}{A} 0^{\circ} = \frac{4\Delta C}{\pi A} \tag{3.25}$$

The feedback law described in equation (3.21) can be decomposed into an ideal switch (on-off controller) plus a discrete-time integrator. Note that in a limit cycle, the following relation holds:

$$G(i\omega) = -\frac{1}{N(A, \omega)} \tag{3.26}$$

where $G(i\omega)$,is the Fourier transform of a discrete integrator defined in (3.21). This means that:

$$\frac{1}{1-e^{i\omega}} = -\frac{\pi A}{4\Delta C} \Rightarrow A = -\frac{4\Delta C}{\pi}\frac{1}{1-e^{i\omega}} \tag{3.27}$$

In order for the above equation to have a real solution for A, we should have $\omega = (2k+1)\pi$ for any integer k, which yields $A = \frac{2\Delta C}{\pi}$. This means that the amplitude of the limit cycle is proportional to the value of the parameter ΔC, and hence, one can attenuate the oscillations arbitrarily by a proper choice of increments in (3.21). It is to be noted that by decreasing ΔC convergence time increases accordingly.

3.5 Simulation Results

Example 6. *Consider a wireless network with six users and the following parameters:* $\bar{R} = 10$*,* $R_t = 50$*,* $g = [2, 4, 5, 7, 9, 11]$*, and* $\Delta C = 0.05$*. The results of fair long-term rate assignment using a distributed scheme are depicted in Fig. 3.4. The results show that the long-term fairness is achieved by using the proposed binary data as feedback signal from the BS to ATs. The figure also confirms the existence of limit cycles. We have observed that the amplitude of these oscillations depend on* ΔC*, as expected, and can be adjusted accordingly.*

Example 7. *Consider a wireless network with two users, and let* $\bar{R} = 10$*,* $R_t = 18$*,* $g = [10, 9]$*, and* $\Delta C = 0.05$*. The results in this case, analogously to the previous example, are depicted in Fig. 3.5. Similar to the previous example, the results show that the fair rate assignment objective is achieved in this case too, with permanent oscillations with relatively small amplitude in the steady-state. If the same guaranteed rate is considered for both users, both rates will converge to the same value.*

The proposed algorithm proves effective for the case of a varying number of users too. For example, consider the case where at time $t = 25$, the number of users in Example 6 changes from 6 to 8. Suppose that: $\bar{R} = 10$, $R_t = 60$, $g = [2, 4, 5, 7, 9, 11, 12, 13]$, and $\Delta C = 0.05$. The results are

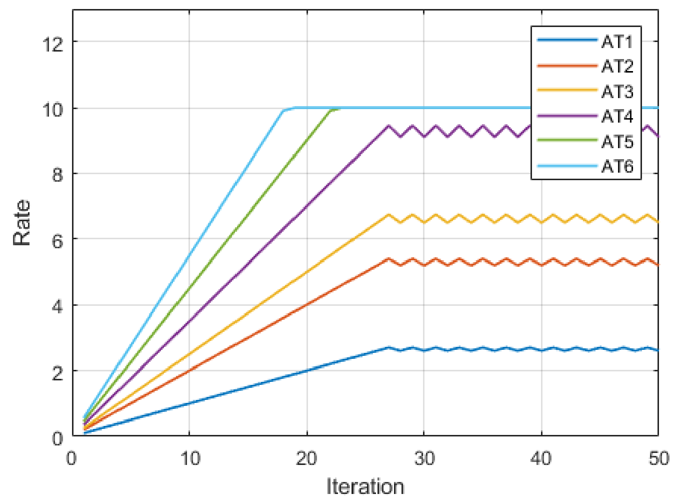


Figure 3.4: The results obtained by using the proposed algorithm with six users in Example 6

given in Fig 3.6. The results show that the fairness objective is achieved in this case as well.

3.6 Conclusions

In this chapter a novel approach to achieve fairness in a CDMA network reverse-link rate assignment was studied. The solution is formulated as a first-order update rule. The approach is computationally simple, with a closed-form solution. The method can also be extended to the case where the network variables are coupled. The users experience fairness when the maximum mutual differences of their rates are minimized. It is shown that, as expected intuitively, the optimal solution corresponds to the case where the users with lower rates are considered critical. Simulations confirm the effectiveness of the method.

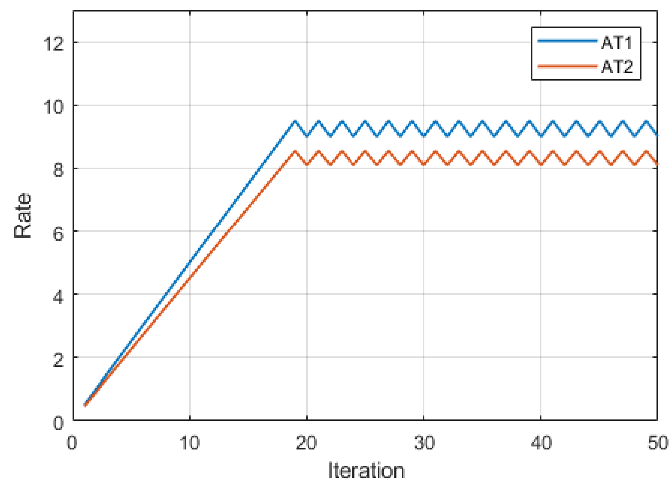


Figure 3.5: The results obtained by using the proposed algorithm with users in Example 7

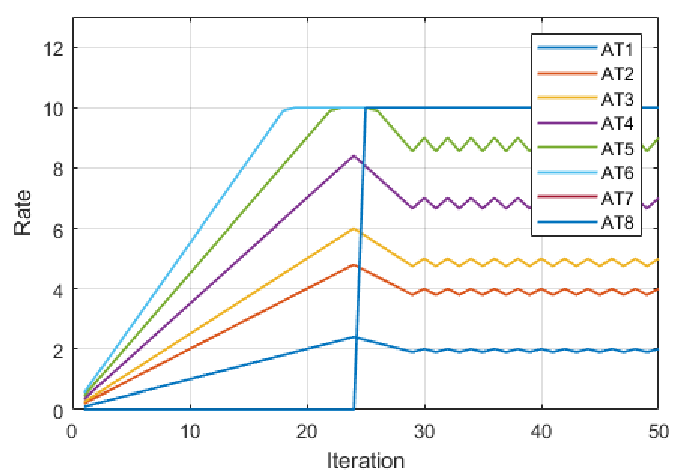


Figure 3.6: The results obtained by using the proposed algorithm in a network with a varying number of users.

Chapter 4

Conclusions and Future Work

In this work, we investigated fairness in a dynamic network with multiple users and limited resources. The problem was formulated in a constrained optimization framework, and effective methods were proposed to solve it. In particular, we studied two applications: heating, ventilation, and air conditioning (HVAC) and code-division multiple access (CDMA). In the first application, it is desired to reach a fair steady-state temperature in different units of a building. There is a thermodynamic coupling between each pair of adjacent units due to the heat transfer. An analytical solution for the optimization problem was provided, verified by simulations. As for the second application, it is aimed to assign data rates to different users connected to a base station in a wireless communication network. Practical limitations on the individual and collective rates in the network were considered, and fairness was subsequently formulated as a constrained optimization problem. An analytical solution was presented for this problem as well, and simulation results were provided to confirm the findings. In future work, one can investigate the application of balanced resource allocation in parallel cable robots to increase the system's lifetime.

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