Information Revelation in Second Price Auctions

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Abstract

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In this paper, I model a second price auction with an informed and an uninformed bidder and find the optimal information revealing strategy followed by the informed bidder at equilibrium. The informed bidder knows the state of the world with certainty, but the uninformed bidder does not know it. The informed buyer can commit to a strategy to reveal information to the uninformed buyer. The uninformed buyer then updates his belief about the state of the world based on the information revealed. I characterize the information revealing strategy that maximizes the expected payoff of the informed bidder. The strategy depends upon the state of the world, valuation of the informed bidder, and the distribution of valuations of the uninformed bidder for the object in the auction. The social planner's optimal decision would be to reveal complete information to remove the inefficiencies in the auction due to information asymmetry.

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1 Introduction

Auctions are being widely used in today's world. Although the history of auctions goes back thousands of years, its usage has risen considerably with the exponential rise in the use of the internet during the last two decades. From extremely rare antique pieces to ubiquitous daily consumable goods on eBay, the popularity of auctions is increasing as a mechanism to buy or sell goods. Even the government uses the auction mechanism to sell mineral rights, telecommunication licenses, construction contracts, etc., the financial world uses it to sell the stocks of the company to the public, and many more.

One of the major characteristics of auctions is that it helps in finding the value of the object; it is used when the seller is unsure about the value of the object. Had the seller known the value of the object, he could have sold the object at that price but without surety in the valuation of the object, it cannot be priced at fair value. Also, the buyers might have different valuations for the object which is even greater than what the seller would have anticipated and auctions could become a way to find it. Auctions can also be used to gain insight into the distribution on how people value the object as shown in Jiang and Leyton-Brown (2005) that has a significant impact on informed decision making. The other important feature of auctions is the efficient allocation i.e. the object goes to the buyer who values the object the most. This explains why governments are increasingly using auctions in selling contracts.

Most commonly, auctions can be categorized on the basis of valuation and price. In terms of valuation, an auction can be private value or common value auction. In a private value auction, the bidders know their valuation for the object for sure but do not know the other bidders' valuations for the object. For example, the valuation of an antique piece is different for different bidders depending upon their personal preferences but the bidders know their personal value. In a common value auction, the valuation of the object is the same for all the bidders but they have different estimates. For example, in the bid of the oil reserve or mineral rights, the value of oil or minerals is the same for every bidder but they have different estimates of the value depending upon the level of information they possess.

An auction can be first price sealed bid auction or second price sealed bid auction with respect to the price the bidder pays conditional on winning. In the first price auction, the bidder with the highest bid wins the object and pays the amount of the bid. The bidder with the highest bid wins the object but pays the second highest bid in a second price sealed bid auction also known as Vickrey auction.

This paper focuses on an independent private value second price auction. One of the Nash equilibria (from several possible equilibria) of independent private value second price auction is that the bidders bid their own true valuation as it is a weakly dominant strategy for all the bidders. But would it be still an equilibrium if the level of information regarding the state of the object is different among the bidders? Information has a significant role in determining the strategy and bid of bidders in auction theory. Availability of new information changes the expected value of the object for bidders that ultimately changes the strategy and payoffs of the bidders leading to different auction mechanisms. A classic example would be the "Winner's Curse" in the case of common value auction from Capen, Clapp, and Campbell (1971). The winner of the auction finds that he overestimates the value of the object than all the other bidders and knows that he could have won the object for lesser than what he bids. If he gets such information before the bid, he surely could have lowered the bid and won the object with greater expected payoff but the information is known only after the winner is decided. This example shows how the information and more importantly the timing of the information has greater value in setting the strategy in auctions.

Going back to the question: is bidding true valuation still an equilibrium in second price sealed bid auction if there is some sort of information asymmetry among the bidders? In real life, two bidders might not have the exact same level of information regarding the object in auction. To elaborate the situation, let's take an example of an auction where a seller is selling a used vehicle through auctions. To simplify the case, let us consider the vehicle could be at high quality level (the owner used the vehicle with utmost care) or at low quality level (the owner used the vehicle roughly). Say two bidders are bidding for the vehicle; one is a car mechanic with years of experience whereas the other has just passed the driving license and would like to own a car. The level of information these two bidders would have regarding the quality of the car cannot be the same. Although both the bidders know their valuation to the vehicle provided that they are certain with the quality, only the mechanic is certain of the quality. If we assume the mechanic as an informed person while the other person as an uninformed person considering the level of information each possesses, it is a (weakly) dominant strategy to bid true valuation only for the informed bidder. The result does not hold for an uninformed bidder as he is not sure of the state of the world (high quality or low quality).

Huber (2020), in his working paper, models this game to find the Bayesian Nash Equilibrium (BNE) between two sets of bidders: one entrant in the market (uninformed bidder) and n incumbents in the market (informed bidders) provided that the informed bidders are committed to the truthful bidding. He claims that all the bids (except bidding true valuations in two states of the world) are strictly dominated by bidding the true valuations of the uninformed bidder. At equilibrium, the uninformed bidder would bid either his valuation at low state of the world or the valuation at the high state provided that all the incumbents bid truthfully and he concludes the paper by presenting a condition that if satisfied, the uninformed bidder bid the valuation at high state of world in equilibrium.

In this paper, I extend a simpler case of Huber (2020) model with one informed and one uninformed bidder and would like to answer the following research questions:

- Does the informed bidder have the incentive to reveal the extra information and gain from the information asymmetry? If yes, how can he reveal information to maximize his expected payoffs?
- What would be the social planner's decision if he had access to all the information of the game?

The uninformed bidder initially has a prior belief that the probability of the world being in a high quality state as p. To maximize the expected payoff, the informed bidder commits to an information revelation principle of revealing the true state of the world with probability q. The uninformed bidder updates his belief after the information revelation using Bayes' rule. Bayes' rule provides a mathematical formula to find the conditional probability i.e., to update the probability of occurrence taking the new information into account. In the paper, it is observed that the optimum information revelation of the informed bidder depends upon the prior belief of the uninformed bidder regarding the state of the world and the valuation of the informed bidder when the state of the world is of high quality. When the valuation of the informed bidder at a state of 'High Quality' is higher, this paper shows that the accuracy of the information revelation is lower at equilibrium than when the valuation of the informed bidder is lower. The accuracy of the information revelation is greater if the uninformed bidder's prior belief of the state of the world being at 'High Quality' is greater.

This paper is organized as follows: Section 2 provides a review of related literature, section 3 contains the model setup followed by the discussion of the solution in section 4. Finally, section 5 concludes the paper with concluding remarks.

2 Literature Review

The bidding strategy of the bidders in auctions mostly depends upon the type of auction and the information they possess. Krishna (2009) mentions that the behaviour of the bidders in equilibrium is complicated in first price auction than in second price auction. In second price auction with private valuation, it is a weakly dominant strategy to bid the true valuation of the bidders. Here, I focus on the second price auction and this result has an important role in this paper.

Information has great importance in auctions. Milgrom and Weber (1982) propose that the bidder can have positive profits when he has some private information in sealed bid auctions. The authors mention that the profit of the bidder is increased if he gathers extra information. More recent literature explores the cases where having more information is detrimental. Kim (2008) models a common value first price auction with two bidders where one bidder can learn the other bidder's signal. He illustrates that more information is not always beneficial. There is the possibility of an aggressive response from the opponent that ultimately leads to a decrease in the expected payoff of the informed bidder. The optimal information disclosure policy can be studied from the seller's point of view as well. The result varies with the assumptions and setup of the model. Ganuza and Penalva (2010) consider the incentives of the auctioneer to reveal private information and find that the auctioneer provides less than efficient level of information. The authors also propose that the more efficient allocation is obtained with more precise signal. Rayo and Segal (2010) analyze setup with a monopolist selling an object to risk-neutral buyers and mention that there is no additional information rent to the buyers if the signals are controlled by the seller. Bergemann et al. (2022) characterize the information disclosure policy that maximizes the seller's revenue. The authors conclude that the optimal information disclosure policy for the seller is to fully reveal the low values of bids whereas pool the high values where the size of the pool is dependent on the number of bidders.

Recent literature, starting from Kamenica and Gentzkow (2011), has explored optimal information disclosure strategy in a variety of sender-receiver settings. Kamenica and Gentzkow (2011) show how a sender can benefit from persuasion. Using persuasion, the sender can reveal the optimum signal regarding the information he has that could change the strategy of the receiver and increase the payoff of the sender. One of the important assumptions in the literature of Bayesian persuasion is the commitment to the information revealing strategy. The commitment assumption is what makes persuasion different from cheap talk. Although more recent literature such as Best and Quigley (2016) and Min (2021) relax the binding commitment in their model, I adopt the assumption that the sender can fully commit to an information revelation strategy.

Huber (2020) models an uninformed and several informed bidders in second price auction and mentions that his model is not covered in any existing literature: not in the main model or even as a special case. The setting of the model is unique and extends previous literature. This paper introduces the possibility by the informed bidder to reveal information to the uninformed bidder to Huber (2020)'s model. The result provides the optimal information revealing strategy for the informed bidder using Bayesian persuasion. In this paper, the informed bidder leverages his additional information to his benefit.

3 Model

3.1 Huber (2020)'s Model: Simplified two bidders case

Consider a second price sealed bid auction where two risk-neutral bidders (bidders) bid for an indivisible object. There are two possible states of the world of the object: the state of high quality denoted by H and the state of low quality L. One of the bidders knows the state of the world with certainty, say informed bidder denoted by $Bidder$ 1 whereas the other uninformed bidder denoted by $Bidder$ 2 has a prior belief of the probability of the object being in the state H. Let us denote such probability as $p \epsilon (0, 1)$ and assume that p is common a knowledge.

Let us denote the valuations by v_H and v_L at the high and low state of nature respectively. The valuations of both bidders are $i.i.d.$ drawn according to cumulative density functions (CDF) F_L, F_H and the density functions $f_L(v_L)$, $f_H(v_H)$ respectively. Also, let us assume that v_L has support $[\underline{v_L}, \overline{v_L}]$ and v_H has support $[\underline{v_H}, \overline{v_H}]$ with $\underline{v_H} \geq \overline{v_L}$ such that the valuation of the object conditional on high state of nature is always greater than or equal to the valuation of object conditional on the low state of nature. Bidders' valuations when the state of the world is at H or L are independent to each other. All the CDFs and density functions are assumed to be continuous on the supports of v_L and v_H.

The game can be summarized as:

- State of the world: H or L
- bidders: i, where $i = 1, 2$
- Strategy: b_i, where b_i is non-negative
- Payoffs: $u_i = v_i - b_i$ if $b_i > b_{-i}$, and 0 otherwise

Timeline of the game

1. Both bidders draw their valuations.
2. The informed bidder observes the quality of the object, H or L.
3. Both bidders privately submit a sealed bid to the auctioneer.
4. The bidder with the greater bid is announced as the winner and pays the opponent's bid.

First, we observe that bidding their true valuation is weakly dominant strategy for the informed bidder. We focus on equilibria where they use this strategy. Conditional on the informed bidder bidding his true valuation, for the uninformed bidder, Huber (2020) argues that every bidding strategy other than bidding his valuation when the quality is high or low (i.e., v_L^2, v_H^2) is dominated by these two bids.

Proposition 0: *The uninformed bidder bids v_H only if*

$$\frac{p}{1-p} \geq \frac{\int_{v_L^2}^{\overline{v_L^2}} f_L(v)(v - v_L^2)dv}{\int_{\underline{v_H}}^{v_H^2} f_H(v)(v_H^2 - v)dv}$$

is satisfied (Huber 2020).

Proof:

Given that *Bidder* 1 bids his true valuation, *Bidder* 2 wins the auction when *Bidder* 1's valuation is lower than v_L^2 or v_H^2 (depending upon the bid of *Bidder* 2 and has to pay Bidder 1's valuation for the object. Then, bidding v_L^2 gives *Bidder* 2 the expected payoff of

$$\begin{aligned} E[u_2(b_2 = v_L^2)] &= (1-p)P\{v_L^2 > v_L^1\}E[v_L^2 - v_L^1 | v_L^2 > v_L^1] \\ &= (1-p)F_L(v_L). \int_{\underline{v_L}}^{v_L^2} \frac{(v_L^2 - v)f_L(v)}{F_L(v_L)} dv \\ &= (1-p)\int_{\underline{v_L}}^{v_L^2} (v_L^2 - v)f_L(v)dv \end{aligned} \tag{1}$$

and bidding v_H^2, the expected payoff of *Bidder* 2 is

$$\begin{aligned} E[u_2(b_2 = v_H^2)] &= (1-p)E[v_L^2 - v_L^1] + p.P\{v_H^2 > v_H^1\}E[v_H^2 - v_H^1 | v_H^2 > v_H^1] \\ &= (1-p). \int_{\underline{v_L}}^{\overline{v_L}} f_L(v)(v_L^2 - v)dv + p. \int_{\underline{v_H}}^{v_H^2} f_H(v)(v_H^2 - v)dv \end{aligned} \tag{2}$$

Bidder 2 will bid v_H only if

$$\begin{gathered} E[u_2(b_2 = v_H^2)] \geq E[u_2(b_2 = v_L^2)] \\ p. \int_{\underline{v_H}}^{v_H^2} f_H(v)(v_H^2 - v)dv \geq (1-p)\int_{v_L^2}^{\overline{v_L^2}} f_L(v)(v - v_L^2)dv \\ G(v_H^2, v_L^2) \geq 0 \end{gathered} \tag{3}$$

Inequality 3 is satisfied and bid v_L otherwise.

3.2 Expansion of the model

Inequality (3) can be written as

$$\frac{p}{1-p} \geq \frac{\int_{v_L^2}^{\overline{v_L^2}} f_L(v)(v - v_L^2)dv}{\int_{\underline{v_H}}^{v_H^2} f_H(v)(v_H^2 - v)dv} \tag{4}$$

In the modified game, let us consider the case where the informed bidder has the option to commit to revealing the state of the world with certain accuracy. Let us denote the accuracy of the information by q_H and q_L when the state of the world is H and L respectively. Basically, this means that the informed bidder is committed to the information revelation principle of revealing the correct state of the world with probability q_H and q_L depending upon the true state of the world. The main idea is to explore if there are optimizing values of q_H and q_L to reveal the state of the world that maximizes the expected payoff of the informed bidder conditional on the uninformed bidder updating his belief of the state of the world using Bayes' rule.

In second price auction, the winner pays the second highest bid conditional on winning the auction. In our game with two bidders, the winner pays the bid of the other bidder. Thus, with the option of information revelation, the informed bidder is better off if he could reveal the information such that the uninformed bidder lowers his bid and bids v_L where he would have bid v_H without any information revelation. Also, in the game with base model, it is possible that the uninformed bidder wins the object when the state is L by bidding v_H that leads to the payoff of zero for the informed bidder. The informed bidder would be better off to minimize or remove such scenarios with optimized information revelation.

The summary of the game does not change but the timeline of the game changes. The game is still the same in a way that the informed bidder bids his truthful valuation and the uninformed bidder bids either v_L or v_H depending upon his belief of the state of the world. The winner of the auction still pays the bid of the opponent. The only change in the game is the step of information revelation by the informed bidder. After the bidders know the value of p which is common knowledge, the informed bidder reveals a signal $s\epsilon(h, l)$ representing the two states of the world: H and L. The informed bidder is committed to the information revelation strategy and the uninformed bidder then updates his prior belief regarding the state of the world and they both submit a sealed bid to the auctioneer. The change in timeline is discussed as follows.

Timeline of the modified game

1. Both bidders draw their valuations.
2. The informed bidder commits to an information strategy q_H and q_L conditional on the state of the world.
3. The informed bidder observes the state of the world.
4. The informed bidder reveals a signal $s\epsilon(h, l)$.
5. The uninformed bidder updates his posterior belief regarding the state of the world using Bayes' rule.
6. Both the bidders privately submit a sealed bid to the auctioneer.
7. The bidder with the greater bid is announced as the winner and pays the opponent's bid.

From Inequality 4, we see that the lower probability of H state leads the uninformed bidder to be more likely to bid v_L. The more likely the uninformed bidder bids v_L, the better off the informed bidder is because the informed bidder pays the bid of the uninformed bidder if he wins. So, the informed bidder's preferred default action is that the uninformed bidder bid v_L. Kamenica and Gentzkow (2011) state that the sender benefits from Bayesian persuasion when the receiver does not take the sender's preferred action by default. So, the informed bidder benefits from the persuasion when the state of the world is H. Thus, the optimized information revelation principle is to be committed to reveal l with probability one when the state of the world is L. Now, let us consider q as the accuracy of signal when the state of the world is H. The informed bidder's choice is to always reveal l when the state of the world is L and mix between l and h when the state of the world is H. Figure 1 shows the probability tree diagram.

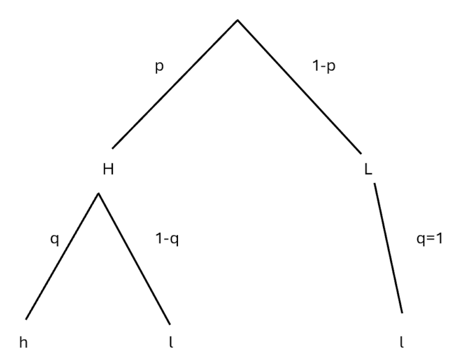

Figure 1: Updating belief using Bayes' rule

After the informed bidder releases the signal s, the uninformed bidder updates his belief of the state of the world being H using Bayes' rule. Let $P(H|h)$ denotes the posterior belief of the uninformed bidder about the state of the world being H given that signal s is high h and $P(H|l)$ denotes the posterior belief of being H given that signal is l.

Using Bayes' rule, we get,

$$P(H|h) = \frac{P(h|H).P(H)}{P(h)} = \frac{q.p}{p.q} = 1 \tag{5}$$

And

$$P(H|l) = \frac{P(l|H).P(H)}{P(l)} = \frac{(1-q).p}{p(1-q)+(1-p)} = \frac{p-pq}{1-pq} \tag{6}$$

Since the probability of the state of the world being H is 1 when the signal is h, the uninformed bidder will bid v_H^2 for sure in this case. On the other side, when the signal is l, the uninformed bidder bids v_H^2 only if the expected payoff of bidding v_H^2 is greater than that of bidding v_L^2.

Lemma 1: *In the extended model in the modified game, the uninformed bidder bids v_H only if*

$$\frac{p(1-q)}{1-p} \geq \frac{\int_{v_L^2}^{\overline{v_L^2}} f_L(v)(v - v_L^2)dv}{\int_{\underline{v_H}}^{v_H^2} f_H(v)(v_H^2 - v)dv}$$

is satisfied.

Proof:

Given that *Bidder* 1 bids his true valuation, *Bidder* 2 wins the auction when *Bidder* 1's valuation is lower than v_L^2 or v_H^2 (depending upon the bid of *Bidder* 2) and has to pay Bidder 1's valuation for the object. Then, if the signal is l, bidding v_L^2 provides *Bidder* 2 with an expected payoff of

$$\begin{aligned} E[u_2(b_2 = v_L^2)|l] &= (1 - P(H|l))P\{v_L^2 > v_L^1\}E[v_L^2 - v_L^1|v_L^2 > v_L^1] \\ &= (1 - P(H|l))F_L(v_L). \int_{\underline{v_L}}^{V_L^2} \frac{(v_L^2 - v)f_L(v)}{F_L(v_L)}dv \\ &= (1 - P(H|l) \int_{\underline{v_L}}^{V_L^2} (v_L^2 - v)f_L(v)dv \end{aligned} \tag{7}$$

and bidding v_H^2 provides *Bidder* 2 with an expected payoff of

$$\begin{aligned} E[u_2(b_2 = v_H^2)|l] &= (1 - P(H|l))E[v_L^2 - v_L^1] + P(H|l).P\{v_H^2 > v_H^1\}E[v_H^2 - v_H^1|v_H^2 > v_H^1] \\ &= (1 - P(H|l)).\int_{\underline{v_L}}^{\overline{v_L}} f_L(v)(v_L^2 - v)dv + P(H|l).\int_{\underline{v_H}}^{v_H^2} f_H(v)(v_H^2 - v)dv \end{aligned} \tag{8}$$

Provided that the signal is l, *Bidder* 2 will bid v_H^2 only if

$$\begin{gathered} E[u_2(b_2 = v_H^2)|l] \geq E[u_2(b_2 = v_L^2)|l] \\ P(H|l).\int_{\underline{v_H}}^{v_H^2} f_H(v)(v_H^2 - v)dv \geq (1 - P(H|l)\int_{v_L^2}^{\overline{v_L^2}} f_L(v)(v - v_L^2)dv \\ \frac{P(H|l)}{1 - P(H|l)} \geq \frac{\int_{v_L^2}^{\overline{v_L^2}} f_L(v)(v - v_L^2)dv}{\int_{\underline{v_H}}^{v_H^2} f_H(v)(v_H^2 - v)dv} \end{gathered} \tag{9}$$

where $P(H|l)$ is the posterior belief of the uninformed bidder regarding the state of the world being H when the signal s is l.

Since,

$$\begin{aligned} \frac{P(H|l)}{1 - P(H|l)} &= \frac{\frac{p-pq}{1-pq}}{1 - \frac{p-pq}{1-pq}} \\ &= \frac{p(1-q)}{1-p}, \end{aligned}$$

From Inequality 9, the condition for the uninformed bidder to bid v_H^2 at equilibrium provided that the signal is l is given by

$$\frac{p(1-q)}{1-p} \geq \frac{\int_{v_L^2}^{\overline{v_L^2}} f_L(v)(v - v_L^2)dv}{\int_{\underline{v_H}}^{v_H^2} f_H(v)(v_H^2 - v)dv} \tag{10}$$

Q.E.D.

For a state of the world $p \ \epsilon \ (0,1)$ with probability of revealing the state H with signal h as $q \ \epsilon \ (0,1)$, the uninformed bidder is indifferent between bidding v_L^2 and v_H^2 when the inequality sign in Inequality 10 is replaced by equal sign as shown in Equation 11.

$$\frac{p(1-q)}{1-p} = \frac{\int_{v_L^2}^{\overline{v_L^2}} f_L(v)(v - v_L^2)dv}{\int_{\underline{v_H}}^{v_H^2} f_H(v)(v_H^2 - v)dv} \tag{11}$$

3.3 Case of Uniform Distribution

Let us consider the case where the valuations $v_L \sim U(a,b)$ and $v_H \sim U(c,d)$, where $U(.,.)$ represents the uniform distribution at supports (a,b) and (c,d) respectively.

For the uniform distributions, Equation 10 and Equation 11 become (details shown in Appendix I),

$$\frac{p(1-q)}{1-p} \geq \frac{(d-c)(b-v_L^2)^2}{(b-a)(v_H^2-c)^2} \tag{12}$$

$$\frac{p(1-q)}{1-p} = \frac{(d-c)(b-v_L^2)^2}{(b-a)(v_H^2-c)^2} \tag{13}$$

The informed bidder's problem is to maximize his payoff for the case where the uninformed bidder mixes between bidding v_L^2 and v_H^2. The informed bidder bids v_L^1 only if the state of the world is L. In this case, the only way he can win the object and has to pay is when the uninformed bidder bids v_L^2 as well. So, there is no way he can maximize his payoff. The case where *Bidder* 1 is better off is by revealing signal s such that when he bids v_H^1 for the object, he maximizes his expected payoff given *Bidder* 2 can bid one of the two bids.

The payoff maximization for *Bidder* 1 can be seen in the following plot with the distribution of v_L and v_H along with the bid of *Bidder* 1's bid v_H^1.

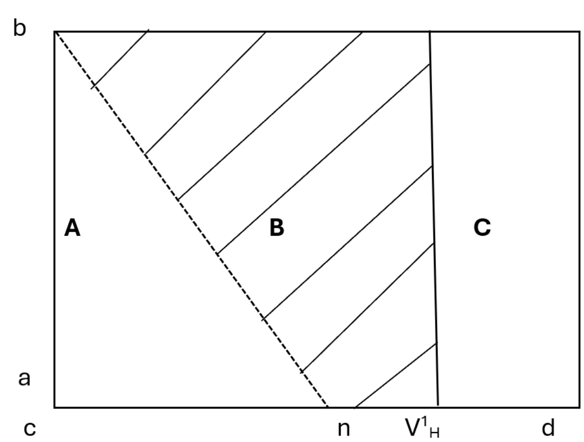

Figure 2: Distribution of v_L and v_H with indifference line for *Bidder* 2

The dotted curve $b - n$ is the indifference curve for *Bidder* 2 to bid v_L or v_H. *Bidder* 2 bids v_L^2 in area **A** whereas he bids v_H^2 in areas **B** and **C**. The payoff of the *Bidder* 1 conditional on *Bidder* 2 bidding v_L^2 is $(v_H^1 - v_L^2)$ and the payoff is $(v_H^1 - v_H^2)$ conditional on *Bidder* 2 bidding v_H^2 and $v_H^2 < v_H^1$. This means that, in the area **C**, the payoff for the informed bidder is zero as his bid is lower than the uninformed bidder.

Mathematically, the total expected payoff of the informed bidder conditional on the state of the world ϵ H is

$$EU_1 = \int_A (v_H^1 - v_L).dF_L(v_L)\, dF_H(v_H) + \int_B (v_H^1 - v_H).dF_L(v_L)\, dF_H(v_H) \tag{14}$$

If v_H be the X axis and v_L be the Y axis, then Equation 14 can be written with integral limits as

$$EU_1 = \frac{1}{(b-a)(d-c} \int_c^d \int_a^{v_L} (v_H^1 - v_L).dv_L\, dv_H + \frac{1}{(b-a)(d-c)} \int_a^{v_H^1} \int_{v_L}^b (v_H^1 - v_H).dv_L\, dv_H \tag{15}$$

From Equation 13, separating v_L, we can see that v_L is a function of q,

$$v_L = f(q) \tag{16}$$

Combining Equations 15 and 16, we get the expected utility of the informed bidder as

$$EU_1 = \frac{1}{(b-a)(d-c} \int_c^d \int_a^{f(q)} (v_H^1 - v_L).dv_L\, dv_H + \frac{1}{(b-a)(d-c)} \int_a^{v_H^1} \int_{f(q)}^b (v_H^1 - v_H).dv_L\, dv_H \tag{17}$$

and *Bidder* 1's problem is to find the optimal $q*$ that maximizes the EU_1 as

$$\begin{aligned} &\max_q EU_1 \\ &= \max_q \frac{1}{(b-a)(d-c} \int_c^d \int_a^{f(q)} (v_H^1 - v_L).dv_L\, dv_H + \frac{1}{(b-a)(d-c)} \int_a^{v_H^1} \int_{f(q)}^b (v_H^1 - v_H).dv_L\, dv_H \end{aligned} \tag{18}$$

subject to the indifference condition

$$\frac{p(1-q)}{1-p} = \frac{(d-c)(b-v_L^2)^2}{(b-a)(v_H^2-c)^2}$$

Taking FOC as $\frac{dEU_1}{dq} = 0$ and solving for q gives the optimum value of $q*$ which is adopted by $Bidder$ 1 to reveal the signal s at equilibrium.

4 Results and Discussion

Proposition 1:

If $v_L \sim U(0,1)$, $v_H \sim U(2,3)$, then the optimum information revelation strategy $q*$ for the informed bidder with his valuation of the object at high state of the world v_H is

$$q* = 1 - \frac{B^2}{4A^2}\frac{(1-p)}{p}$$

where,
$A = \frac{37}{6}$, $B = \left(\frac{5}{6} + \frac{v_H^1}{2} + (v_H^1)^2 - \frac{(v_H^1)^3}{6}\right)$, $C = v_H^1 - \frac{1}{2}$, and $k = \frac{p}{1-p}$.

Proof:

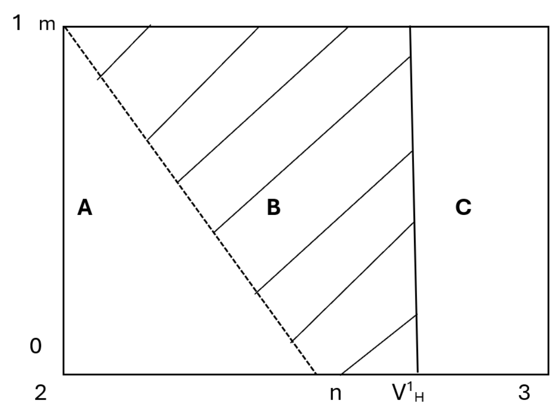

Figure 3: $v_L \sim U(0,1)$ and $v_H \sim U(2,3)$

The indifference curve (equation of curve $m - n$) for the case is

$$\frac{p(1-q)}{1-p} = \frac{1 - v_L^2)^2}{(v_H^2 - 2)^2}$$
$$or,\ (1 - v_L) = \sqrt{\frac{p(1-q)}{1-p}}(v_H - 2)$$

Assume $\sqrt{\frac{p(1-q)}{1-p}} = w$ then, $v_L = 1 \rightarrow v_H = 2$, and $v_L = 0 \rightarrow v_H = 2 + \frac{1}{w}$.

Now, the expected utility becomes (details in Appendix),

$$\begin{aligned} EU_1 &= \int_2^3 \int_0^{1-w(v_H-2)} (v_H^1 - v_L) dv_L dv_H + \int_2^{v_H^1} \int_{1-w(v_H-2)}^1 (v_H^1 - v_H) dv_L dv_H \\ &= \frac{37}{6} w^2 - \left(\frac{5}{6} + \frac{v_H^1}{2} + (v_H^1)^2 - \frac{(v_H^1)^3}{6} \right) w + (v_H^1 - \frac{1}{2}) \end{aligned} \tag{19}$$

Let Equation 19 be denoted as $EU_1 = A.w^2 - B.w + C = A.k.(1-q) - B.\sqrt{k}.\sqrt{1-q} + C$, where,
$A = \frac{37}{6}$,
$B = \left(\frac{5}{6} + \frac{v_H^1}{2} + (v_H^1)^2 - \frac{(v_H^1)^3}{6} \right)$,
$C = v_H^1 - \frac{1}{2}$, and
$k = \frac{p}{1-p}$

The FOC condition is $\frac{EU_1}{dq} = -A.k + \frac{B\sqrt{k}}{2\sqrt{1-q}} = 0$.
Solving for q, we get,

$$\begin{aligned} q* &= 1 - \frac{B^2}{4kA^2}, \\ i.e., q* &= 1 - \frac{B^2}{4A^2} \frac{(1-p)}{p} \end{aligned} \tag{20}$$

Q.E.D.

Special Case: When $p = \frac{1}{2}$,
If $v_H^1 = 2 \rightarrow q* = 0.86$, and
if $v_H^1 = 3 \rightarrow q* = 0.68$.

Similarly, the informed bidder can solve the maximization problem for any value of p and known distributions v_L and v_H to know the value of p that maximizes his expected utility.

Conjecture 1:

The optimal information revelation strategy of the social planner is to reveal the true state of the world to the uninformed bidder.

Let us consider a social planner who observes the state of the world of the object in the auction. The social planner's objective is to have an efficient allocation i.e., the object goes to the bidder with a greater valuation. He would like to remove the inefficiencies present in the auction due to differences in information between the bidders. Without proof, I would like to discuss what would be the social planner's optimal decision.

We can observe that whenever there is an asymmetry in information between the bidders, there is always a probability of inefficient distribution of the object when the object goes to the informed bidder although he has a lower valuation than the uninformed bidder. This happens when the state of the world is high but the uninformed bidder bids his lower valuation due to a lack of information (although his valuation is greater than the informed bidder in the high state of the world) and the auction is won by the informed bidder at the price of uninformed bidder's bid. The seller's revenue is also lower in such condition as the informed bidder would pay the valuation of the uninformed bidder at the low state of the world although the true state of the world is high. Thus, it seems clear that the social planner's optimal information revelation principle is to reveal the truth such that the game then follows the Vickrey auction and the dominating strategy for both the bidders is truthful bidding of their true valuation. This leads to the eradication of all the inefficiencies and the winner is the one with greater valuation of the object.

5 Conclusion

In a second price auction with asymmetric information: where one bidder has more information than the other bidder, the informed bidder can commit to an information revelation mechanism that maximizes his expected payoff. In our model with two states of the world, the informed bidder should reveal the state of the world with certainty if the state is low (that gives a higher payoff to the informed bidder) whereas he should mix between revealing the state of the world between high state and low state if the state is high (that gives a lower payoff to the informed bidder). The probability of mixing between the states depends upon the state of the world, the valuation of the informed bidder, and the distribution of valuations of the uninformed bidder for the object in the auction. If the belief of the uninformed bidder of the state of the world being high is greater, the accuracy of the signal sent by the informed bidder is also high at equilibrium. The accuracy of the signal is lower if the valuation of the object for the informed bidder is higher when the state of the world is high. From a social planner's view, the optimum decision is to reveal all the information regarding the state of the world.

As an extension to the paper, it would be interesting to evaluate the optimal information revelation strategy of the auctioneer as he knows the state of the world.

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A Derivation of Equations 11 and 12

The RHS is calculated as

$$
\begin{aligned}
\frac{\int_{v_L^2}^{\overline{v_L^2}} f_L(v)(v - v_L^2)dv}{\int_{\underline{v_H}}^{v_H^2} f_H(v)(v_H^2 - v)dv} &= \frac{\int_{v_L^2}^{b} \frac{1}{b-a}(v - v_L^2)dv}{\int_{c}^{v_H^2} \frac{1}{d-c}(v_H^2 - v)dv} \\
&= \frac{(d-c)\left[\frac{v^2}{2} - v.v_L^2\right]_{v_L^2}^{b}}{(b-a)\left[v.v_H^2 - \frac{v^2}{2}\right]_{c}^{v_H^2}} \\
&= \frac{(d-c)(\frac{b^2}{2} - b.v_L^2 - \frac{{v_L^2}^2}{2} + {v_L^2}^2}{(b-a)({v_H^2}^2 - \frac{{v_H^2}^2}{2} - c.v_H^2 + \frac{c^2}{2}} \\
&= \frac{(d-c)(b^2 - 2b.v_L^2 + {v_L^2}^2}{(b-a){v_H^2}^2 - 2c.v_H^2 + c^2} \\
&= \frac{(d-c)\,(b - v_L^2)^2}{(b-a)\,(v_H^2 - c)^2}
\end{aligned}
$$

B Derivation of Equation 19

Let us suppose $v_L = y$ and $v_H = x$.

$$
\begin{aligned}
EU_1 &= \int_2^3 \int_0^{1-w(v_H-2)} (v_H^1 - v_L) dv_L dv_H + \int_2^{v_H^1} \int_{1-w(v_H-2)}^1 (v_H^1 - v_H) dv_L dv_H \\
&= \int_2^3 \int_0^{1-w(x-2)} (v_H^1 - y) dy dx + \int_2^{v_H^1} \int_{1-w(x-2)}^1 (v_H^1 - y) dy dx \\
&= \int_2^3 \left[v_H^1 y - \frac{y^2}{2} \right]_0^{1+2w-wx} dx + \int_2^{v_H^1} \left[v_H^1 y - xy \right]_{1+2w-wx}^1 dx \\
&= \int_2^3 \left[v_H^1 + 2v_H^1 w - v_H^1 wx - \frac{1}{2}\left(1 + 4w + 4w^2 - 2wx - 4w^2 x + w^2 x^2\right) \right] dx + \\
&\int_2^{v_H^1} \left[v_H^1 - x - v_H^1 - 2v_H^1 + wv_H^1 x + x + 2wx - wx^2 \right] dx \\
&= \left[v_H^1 x + 2wv_H^1 x - v_H^1 w\frac{x^2}{2} - \frac{x}{2} - 2wx - 2w^2 x + w\frac{x^2}{2} + w^2 x^2 + w^2 \frac{x^3}{6} \right]_2^3 + \\
&\left[wx^2 - w\frac{x^3}{3} + w\frac{v_H^1}{2} - 2v_H^1 wx \right]_2^{v_H^1} \\
&= \left[\frac{w^2}{6} x^3 + \left(w^2 + \frac{w}{2} - v_H^1 \frac{w}{2} \right) x^2 + \left(v_H^1 + 2wv_H^1 - \frac{1}{2} - 2w - 2w^2 \right) x \right]_2^3 + \\
&\left[\frac{-w}{3} x^3 + \left(w + w\frac{v_H^1}{2} \right) x^2 - 2v_H^1 wx \right]_2^{v_H^1} \\
&= \frac{(27-8)}{6} w^2 + \frac{(9-4)}{2} (2w^2 + w - v_H^1 w) + (v_H^1 + 2wv_H^1 - 2w - 2w^2 - 1/2) - \\
&({v_H^1}^3 - 2^3)\frac{w}{3} + (2w + wv_H^1)\frac{{v_H^1}^2 - 4}{2} - 2v_H^1 w(v_H^1 - 2) \\
&= \frac{19}{6} w^2 + 5w^2 + \frac{5}{2} w - \frac{5}{2} v_H^1 w + v_H^1 + 2v_H^1 w - 2w - 2w^2 - \frac{1}{2} - \frac{{v_H^1}^3 - 2^3}{3} w + \\
&({v_H^1}^2 - 4)w + \frac{v_H^1}{2}({v_H^1}^2 - 4)w - 2{v_H^1}^2 w + 2v_H^1 w \\
&= \frac{37}{6} w^2 - \left[\frac{5}{6} + \frac{v_H^1}{2}(v_H^1)^2 - \frac{(v_H^1)^3}{6} \right] w + (v_H^1 - \frac{1}{2})
\end{aligned}
$$