Optimization Model for Production-Distribution Planning in the Cosmetic Industry: The Case of Cosmetics Company Canada

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A Thesis

in the Department of

Supply Chain Management

Presented in Partial Fulfillment of the

Requirements for the Degree of

Master of Supply Chain Management

at Concordia University

Montreal, Quebec, Canada

July 2024

CONCORDIA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

This is to certify that the thesis prepared

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Entitled: Optimization Model for Production-Distribution Planning in the Cosmetic Industry: The Case of Cosmetics Company Canada

and submitted in partial fulfillment of the requirements for the degree of

Masters of Science Supply Chain Management at John Molson School of Business

complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

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ABSTRACT

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Jonathan Weisbecker

This study conducts an in-depth analysis of the short-term variable transportation and warehousing costs at the Horace Plant (HP). The primary goal is to develop cost-saving strategies that enhance operational efficiency while reducing overall costs. The analysis focuses on three main cost components: trucking costs, pallet movement costs, and warehousing costs, which are incurred during shipments between suppliers, warehouses, and production facilities, as well as the movement and storage of raw materials and components in a short horizon. The study employs Linear Programming (LP) techniques, specifically a Multistage Multi-echelon Multiproduct Mixed Integer Linear Programming (MILP) model, to capture the complexity of Cosmetics Company's supply chain network. The model, including multiple products, suppliers, warehouses, production warehouses, and periods, offers a robust framework for optimization, instilling confidence and reassurance about its effectiveness in supply chain management. Results from the model reveal cost-saving opportunities and operational improvements. Sensitivity analysis provides insights into key cost drivers and potential areas for cost reduction. The practical application of this study lies in its ability to offer real-time, actionable insights for daily supply chain operations, which is crucial for handling demand fluctuations and ensuring cost efficiency in the beauty industry. The study enhances visibility into goods flow and potential short-term shortages by providing deeper managerial insights into the optimal routing and storage of pallets. This supports strategic and tactical planning, driving continuous improvement in supply chain performance and instilling a sense of optimism about the future of supply chain management. Ultimately, the study demonstrates the practical benefits of advanced optimization models in complex, dynamic environments, contributing valuable insights to the field of supply chain management.

Keywords: *Production Distribution Planning, Inventory Routing Problem, Multistage Mixed Integer Programming, Multiproduct Mixed Integer Linear Programming, Multi-Echelon Mixed Integer Linear Programming, Inventory Management*

ACKNOWLEDGEMENTS

I would like to express my great appreciation to my supervisor, Dr. Navneet Vidyarthi, for his guidance and navigation during my thesis process. The past year has been full of ups and downs, yet here we are at the tail end of the journey. Cheers to that.

Next, I would like to thank my classmates and support system for acting as additional eyes and cheerleaders throughout the thesis process. Being surrounded by so many intelligent, motivated peers within the industry is a special gift. Another special thanks to professors Alexandra M. Newman and Ed Klotz of the University of Colorado School of Mines for giving direct and prudent technical support at pivotal moments of the thesis process.

Finally, I would be remiss not to thank my family, particularly my mother, who has been so excited to see me succeed in this chapter of life and has never wavered in her support. I love you, Mom.

Table of Contents

List of Figures

List of Tables

Chapter 1: Introduction

Within the contemporary global manufacturing landscape, optimizing supply chain logistics is a necessity for sustaining competitive performance. This study focuses on an in-depth analysis of the short-term variable transportation and warehousing costs associated with the materials handled at Cosmetics Company Canada’s Horace Manufacturing Plant (HP). The objective is to devise cost-saving strategies and recommendations, enhancing operational efficiency and reducing overall costs. The analysis encompasses three primary cost components: trucking costs, pallet movement costs, and warehousing costs. These costs are incurred for trucking shipments between suppliers, warehouses, and production warehouses, the movement of pallets into and out of internal and external warehouses, and the storage of pallets across various locations. The costs exhibit variable components and notably vary among the external warehouses. All cost considerations are expressed on a per-day basis, aligning with the objective of developing a practical tool for daily use. The study employs linear programming (LP) techniques to explore the problem and formulates it as a multistage multi-echelon multiproduct mixed integer linear programming (MILP) model. The model captures the complexity of the supply chain network, including multiple products, suppliers, warehouses, production warehouses, and periods, thus providing a robust framework for optimization.

In their seminal work, Chopra and Meindl (2016) highlight the importance of strategic supply chain management in achieving cost efficiency and customer satisfaction. They argue that balancing cost and service levels is crucial to a company’s success in the marketplace. This study aims to apply these principles to the context of Cosmetics Company Canada’s Horace Plant, providing a detailed analysis of transportation and warehousing costs to identify cost-saving opportunities.

The decision variables, parameters, and constraints are defined, and the objective function is formulated. The model is then solved to obtain the optimal solution, which minimizes the total cost of supply chain operations. The model results are analyzed to identify cost-saving opportunities and operational improvements. Sensitivity analysis is conducted to understand the impact of various parameters on the model outcomes, providing valuable insights into key cost drivers and potential areas for cost reduction.

This study's practical application lies in its ability to provide real-time, actionable insights for daily supply chain operations. The model's design allows for flexibility in warehousing, both internal and external, which is crucial for handling demand fluctuations and ensuring cost efficiency. This approach is particularly relevant in the cosmetics industry, where demand volatility and product variety necessitate agile and cost-effective supply chain solutions.

Beyond providing a discrete solution to inventory routing problems, this study offers deeper managerial insights into the optimal routing and storage of pallets to minimize costs. It also highlights the importance of visibility into the flow of goods and potential short-term shortages, enabling supply chain managers to make informed decisions that enhance overall efficiency. The insights from this study are intended to support the organization's strategic and tactical planning processes, driving continuous improvement in supply chain performance.

The remainder of this chapter goes on to introduce Cosmetics Company and the context of the paper. The remainder of the paper is organized as follows. Chapter 2 provides literature on inventory routing and production distribution problems, comparing these to general optimization models. Chapter 3 establishes the definition of the problem, including a problem definition and model formulation. Chapter 4 describes a review of the results and analysis of our experiments. Finally, Chapter 5, conclusions covers Managerial Insights, limitations and future research.

1.1. Project Objective:

The primary objective of this study is to conduct an in-depth analysis of the transportation and warehousing costs associated with the materials handled at Cosmetics Company Canada’s Horace Plant (HP). The analysis encompasses three major costs: trucking, pallet movement, and warehousing. Costs are incurred for trucking shipments conducted between suppliers, warehouses, and production warehouses, the movement of pallets into and out of internal and external warehouses, and the storage of pallets across various locations. These costs exhibit a mix of fixed and variable components. Notably, they vary among the external warehouses, though fixed warehousing costs are not factored in due to the nature of the model. All cost considerations are expressed per day, aligning with the objective of developing a practical tool for daily use.

The study utilizes linear programming (LP) techniques to explore the problem, and it can be considered a multistage multi-echelon multiproduct mixed integer linear programming model.

Whereas many past studies go on to develop theoretical frameworks that are impractical for company use, this case study focuses on short-term production planning demand within a closed supply chain network to best utilize flexible warehousing, internal and external, similar to that of Moradi and Sangari (2021). Yet, to my knowledge, there have been no studies within the beauty industry field that look at this type of optimization model

The utility of heuristics is explored in depth to determine potential strategies for developing cost-saving methods. Furthermore, because of its utility in a real-world situation, the research contains limitations that help prioritize solving within a specific timeframe based on the company's request.

More than providing a discrete solution to inventory routing problems, the study provides deeper managerial insights into where pallets should be routed and stored to minimize costs. It also indicates further visibility into flows of goods and potential short-term shortages within the team.

1.2. Cosmetics Company

Cosmetics Company, a globally recognized multinational corporation specializing in the cosmetics and beauty sector, has emerged as a prominent figure in the cosmetics industry. Founded by Eugène Schueller in France in 1909, Cosmetics Company expanded its reach internationally, establishing itself as the world's largest cosmetics conglomerate. Renowned for its extensive portfolio of products, encompassing skincare, haircare, makeup, and fragrances under various well-established brands, the organization has continuously featured in the Global 500 from 1996 until 2024. As of 2024, the group has a worldwide presence in 150 countries, and its activity is divided into six zones: North America, Latin America, Western Europe, Eastern Europe, North Asia, South Asia/Pacific/Middle East/North Africa, and Sub-Saharan Africa. Cosmetics Company's commitment to scientific research and development is an integral component of its operations, with a primary focus on advancing the beauty and well-being of individuals on a global scale.

Since 2018, Cosmetics Company has maintained a market share of between 8.5% and 9.2%, while industry revenue grew by 1%. (IBIS, 2024b). Cosmetics Company's recent strategic endeavors have been characterized by an aggressive acquisition approach to fortify its market position and diversify its product portfolio. Notably, the USD 2.53 billion acquisition of Aesop, an Australian powerhouse, in 2023 is a pivotal development in the company's expansion strategy.

(IBIS, 2024b). The acquisition of Aesop exemplifies this strategy, as it facilitated the company's entry into the Australian market and contributed significantly to its global footprint.

Table 1 Cosmetics Company Industry Revenue, Market Share, and Profit Margin Over Time

Source: (IBIS, 2024b)

Year	**Industry Revenue ($ million)**	**Market Share (%)**	**Profit Margin (%)**
2018	31782	8.5	18.3
2019	33457	8.8	18.6
2020	31924	8.7	18.6
2021	38285	9.2	19.1
2022	40227	9.0	19.5
2023	42736	9.2	19.5
2024	42736	8.9	19.5

Amidst the dynamic business landscape, Cosmetics Company has embarked on a comprehensive data transformation initiative, moving towards a cloud-based SAP storage system. The endeavor aims to streamline and unify disparate SAP systems across its global operations. By embracing a unified data infrastructure, Cosmetics Company seeks to enhance operational efficiency, optimize resource allocation, and foster greater agility in responding to market dynamics. The initiative underscores Cosmetics Company's commitment to adaptation to drive innovation and maintain its competitive edge in the cosmetics industry.

1.3. Cosmetics Company Supply Chain

The organization contains 35 brands, categorized within one of four divisions (Cosmetics Company Dermatological Beauty, Consumer Products Division, LUXE Division, and Professional Products Division. Each separate country is known as an affiliate; each division is run as a separate business model within an affiliate. Affiliates are tasked with internally transporting finished goods and all other business aspects, such as marketing and sales. The Global Sales & Operations Team partners with teams at the affiliate level to ensure that each country receives the requisite amount of finished goods needed, manages potential global shortages, and provides visibility on Cosmetics Company’s manufacturing plants.

Cosmetics Company produces finished goods across 40 manufacturing facilities worldwide, most of which are in Europe. Cosmetics Company transports goods from the manufacturing facilities to an International Distribution Center (IDC), tasked with group-level forecasting, allocating, and distributing products across affiliates. Finished goods are transported from manufacturing facilities to IDCs to affiliate warehousing for distribution; there are instances in which goods are shipped directly to affiliates. Affiliates are tasked with forecasting within a particular market and developing distribution strategies alongside significant clients. While Cosmetics Company brands like Keihl's have brick-and-mortar stores, the company generally relies on other major retailers (Amazon et al., etc.) for consumer sales.

1.4. Cosmetics Company Canada:

Cosmetics Company maintains a substantial presence in Canada, contributing significantly to the country's cosmetics and beauty sector. Cosmetics Company Canada was first based in Hamilton, Ontario, in 1958 under the name Cosmair before transferring to Montreal, Quebec. It was not until 2000 that the company officially rebranded Cosmair to become Cosmetics Company Canada.

Cosmetics Company Canada, headquartered in Montreal, is a critical nexus for the company's activities in North America. Comparatively, to its global footprint, Cosmetics Company Canada has an outsized market share within the country, hovering around 44% of the Canadian market share between 2018 and 2024 (IBIS, 2024a).

Table 2 Cosmetic & Beauty Manufacturing in Canada (IBIS, 2024a)

Company	Market Share (%) 2024	Revenue ($m) 2024	Profit ($m) 2024	Profit Margin (%) 2024
Cosmetics Company Canada Inc.	44.1	1,835.8	364.5	19.9
The Estee Lauder Companies Inc.	14.6	609.4	68.3	11.2
Groupe Marcelle Inc.	2.6	107.1	N/A	N/A

Montreal is a manufacturing and distribution hub, utilizing the Saint Lawrence River to access raw materials and components from North America and Europe. In Canada, Cosmetics Company's manufacturing plant, the Horace Plant in Montreal, specializes in producing haircare and hair coloration products distributed globally. Additionally, the flagship distribution center of the Canadian affiliate, Bois-De-Laisse (BDL), houses the entirety of Cosmetics Company Canada's catalog across all four divisions before being redistributed to clients. While a small contingent of sales representatives and client-facing supply chain team members are based in Toronto and Vancouver, the remainder of Cosmetics Company Canada employees are based in the head office in downtown Montreal.

1.4.1. Montreal Manufacturing Facility and Warehousing Infrastructure:

The Horace Plant (HP) in Montreal, QC, operates as a multifaceted manufacturing facility, boasting 27 distinct production lines. In 2023, the Horace Factory produced over 174 million units of various hair care formulas in 185 formats. This includes 1457 product formulas for 14 distinct brands under the Cosmetics Company umbrella.

Figure 1 Horace Production Warehouse map

These production lines are linked to a warehousing infrastructure that can effectively store up to 3000 components and raw material pallets. However, the operational challenge arises from the sheer magnitude of materials encompassing over 5500 distinct SKUs required for the intricate final product assembly. Cosmetics Company has strategically leveraged the services of four external warehouses to mitigate these complexities, each specializing in accommodating specific material types. The pivotal caveat lies in the disparate capacity and categorization constraints exhibited by each of these external warehouses. Because of the nature of our optimization model, we chose to allocate 1500 of the 3000 pallet spaces to RM and the other 50% to AC. It should also be noted that while there are spaces for 3000 pallets, which we use in the baseline of our model, the warehouse director prefers to retain a saturation level of 85% or less to account for unexpected deliveries or pivots in the production schedule. Our analysis looks at the potential benefits of increasing warehousing space at the production facility and how limiting the Horace Plant's internal capacity affects the costs outlined in the model.

1.4.2. Material Categorization:

The expansive array of 5,500 SKUs is categorized into three distinct classes, with SKUs allocated into specific classes. These classes encompass Raw Materials (RM), Components (AC), and Hazardous Materials (HM). Material use, storage limitations, and complexity primarily drive SKU categorization into these three distinct classes.

It became necessary to curtail the model's comprehensive scope to achieve a more streamlined and manageable analytical approach. Because of the complexity of HM materials, those SKUs have been eliminated from the SKU count. If a SKU was not currently available within the warehousing system or did not have an upcoming demand within the recognized scope, it was also removed from the dataset. Finally, components stored using unorthodox methods, such as labels in the production warehouse's specialty locations, were also removed from the model. This reduction in scope involved delineating a defined time interval, which, in turn, acted as a determinative factor for the inclusion of individual SKUs within the model. To elaborate, if a SKU exhibited a quantifiable level of demand during the specified period, it was subsequently integrated into the model as an 'Individual Flow SKU,' thereby indexing it as a unique product. SKUs that were not needed for granularity to develop a working tool were grouped as a single product, though the product type for each was still considered. After the initial data cleaning, we reduced the number of individual products from 5,500 to 1990, with one additional product index being utilized as the combined products within the index.

1.4.3 External Warehouses:

The quartet of external warehouses, BDL, XTL, La Chine, and VSL, serves distinct functions in the material storage hierarchy. BDL and XTL can accommodate RMs and ACs. La Chine, the principal component warehouse, exclusively stores ACs. VSL emerges as a unique warehousing asset, capable of housing materials from any category, with the salient distinction of being the sole external warehouse suitably equipped to host Hazardous Materials (HMs). Below are the storage capacities of the four warehouses based on the material type:

Table 3 External Warehouse Pallet Capacities

Warehouse	RM	AC	HM
BDL	1281	1281	0
XTL	1000	1000	0
La Chine	2500	0	0
VSL	500	500	500

In addition to the varying storage capacity of the different warehouses, each HP and warehouse has contractual agreements that dictate costs based on pallet movement and monthly storage costs. Because our analysis tool is meant to be updated daily, we converted variable warehouse storage costs to daily units. The four warehouses are so close that moving from a specific warehouse to HP is insignificant because the cost does not vary in a particular way. Our costs were determined by the logistics team, which would estimate the loading and unloading costs of trucks as a whole and the movement of individual pallets, which can be seen in specific warehousing contracts.

Each XTL, La Chine, and VSL all develop annual contracts in partnership with the production warehouse, where the pallet movement cost is determined. BDL acts independently of these annual contracts due to its utility as the HQ Distribution center in Canada and therefore relies on handshake deals and flexible conversations regarding storage capacity; at the moment of the experiment, the BDL transportation team limited the production facility to 2562 pallets for a single day, though it is likely that the number will decrease over time.

1.4.3. Interactions of Horace Teams

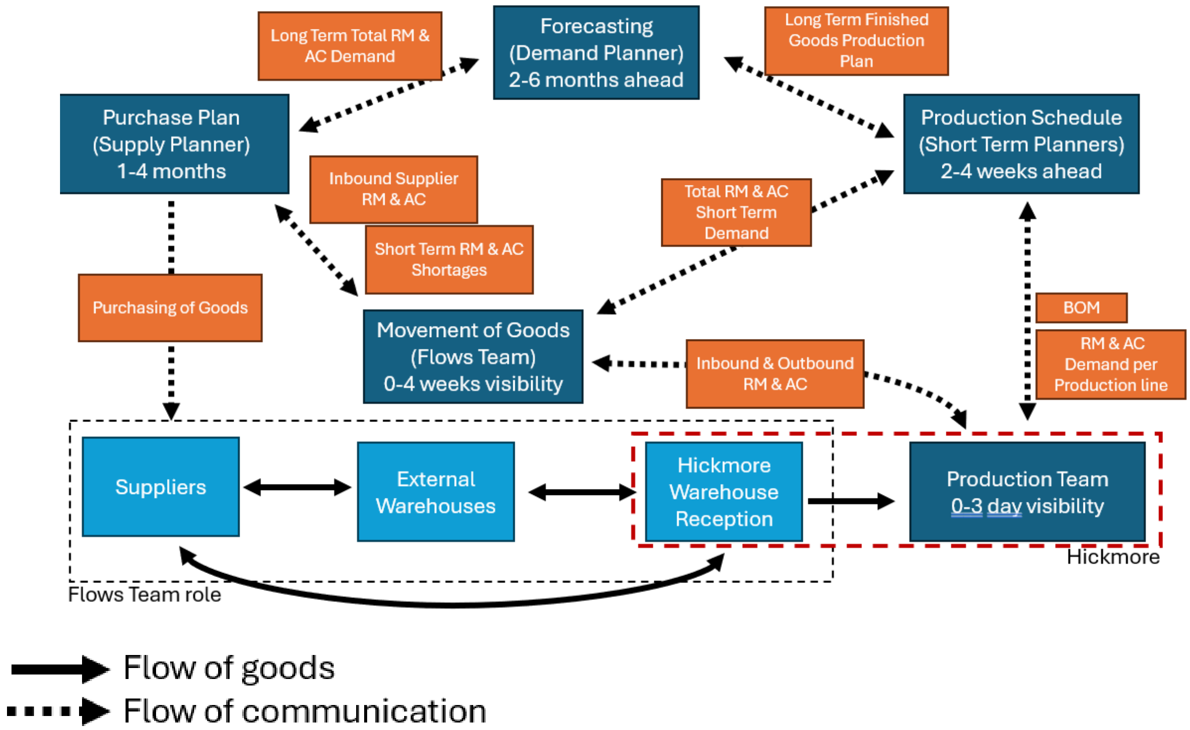
The movement of goods from forecasting to production is essential for meeting customer demand and maintaining competitiveness in today's global market. In this context, interdisciplinary collaboration among various teams within the supply chain is imperative. This collaborative process includes five significant teams at Cosmetics Company Canada – Demand Planners, Production Planners, Production Team, Supply Planners, and Flows Team – in facilitating the seamless flow of goods through the supply chain.

Demand Planners develop the long-term production planning schedule in collaboration with the Cosmetics Company S&OP team. Their forecasts provide visibility into long term demand. This long-term forecast guides the production planning process, ensuring all country affiliates receive the required finished goods throughout the year. The Demand Planning team disseminates the long-term production plan to Production Planners at Cosmetics Company Horace, who then develop the short-term production plan. The short-term production plan is subject to frequent changes due to last-minute disruptions such as missing materials or machine breakdowns. Production Planners collaborate closely with the Production Team to provide a breakdown of the Bill of Materials for each production line within the manufacturing facility. This collaborative effort ensures that production schedules align with inventory availability and capacity. The Production Team works with the Flows Team to manage the movement of raw materials and components within the plant, ensuring seamless production operations.

The Flows Team manages the movement of raw materials and components from suppliers to all storage locations and vice versa. They collaborate with the Production Planning team to determine short-term supply and demand of raw materials, which informs short-term supply planning. Working directly with suppliers and external warehouses, the Flows Team ensures timely delivery of raw materials and manages stock across storage locations. Moreover, they collaborate with the Supply Planning team to gain visibility on inbound materials and storage availability. The Supply Planning Team is responsible for purchasing raw materials from suppliers based on the long-term forecast provided by Demand Planners. Using the finished product forecast and Bill of Materials, they break down long-term demand for raw goods.

This collaborative effort ensures that the procurement of raw materials aligns with production requirements and demand forecasts, thus optimizing inventory levels and minimizing supply chain disruptions. Interdisciplinary collaboration among Demand Planners, Production Planners, Production Team, Supply Planners, and Flows Team is essential for ensuring the swift and consistent production of finished goods at Cosmetics Company Canada. By leveraging their expertise and collaborating effectively, these teams synchronize production schedules, optimize inventory management, and mitigate supply chain risks.

Figure 2Interactions of Teams



1.4.4. Daily Production Planning, Bulk Production and Bill of Materials (BOM):

Daily, the short-term production planners choreograph a meticulously devised production schedule for HP's manufacturing lines. This schedule generates a comprehensive Bill of Materials (BOM). This document details the requisite quantities of Raw Materials (RMs), Auxiliary Components (ACs), and Hazardous Materials (HMs) needed for finished good production that day. The BOM is paramount for final product manufacturing, providing an 'exploded view' of materials required for production. The BOM is linked to the inventory status of materials at HP and the specific demands of the short-term production horizon, which generally spans two to four. The utility of the BOM in developing demand is highlighted by Quetschlich, Moetz, and Otto (2020), who propose an integrated mathematical optimization model for multi-item, multi-echelon supply chains with nested multi-level products; their model emphasizes the importance of detailed planning and real-time data integration for efficient supply chain management, as demonstrated in the automotive industry using realistic industry data. This approach aligns with HP's use of the BOM to ensure precise material requirements and seamless production schedules (Quetschlich et al., 2020). Bulk Goods manufacturing, where raw materials are pre-mixed, and the resultant amalgamation is earmarked for deployment in subsequent recipes in the foreseeable future, acts as another demand. Bulk Goods manufacturing and final goods manufacturing signifies the total daily demand, thereby becoming the locus of our attention. The production lines responsible for creating Bulk Goods and final goods are deemed as 'customers' in the ensuing analytical framework, which provides the demand for our analysis.

1.4.5. Collaborative Forecasting and Material Acquisition:

Weekly to monthly collaboration to anticipate HP's material requirements based on production forecasts unfolds between the logistics and flows team. Beyond material procurement and transportation orchestration, they encompass the allocation of inbound materials from diverse Suppliers. Because of the role of the Generic SKUs in our model, it would be prudent for the logistics team and Flows team to collaborate to determine which Generic SKUs should be moved based on a more extended forecast. An additional critical facet of HP's material procurement entails the role of Suppliers. Suppliers are accorded the prerogative to dispatch materials directly to HP or route materials to the designated external warehouses. The Flows Team retains visibility of inbound supplier goods and can direct the flows of goods accordingly. Because of cross docking at a single central supplier depot, Cosmetics Company can quickly determine which pallets should be shipped to which warehouse, assuming visibility of over ten days or more.

All materials are amenable to simultaneous transportation within a single truck. However, this transport mode imposes variable limitations contingent upon the material category, with a capacity ceiling of 52 pallets for ACs and 26 for HC due to stacking infeasibility for the latter categories. There is no hard limitation on how many trucks can be received at the production warehouse nor how many trucks can be utilized for short-term planning trips from Horace to an external warehouse; the number of trucks received per day is generally no more than 12. The number of trucks from the external warehouse to the production warehouse and vice versa rarely exceeds two per warehouse each period.

As noted, forecasting and material acquisition are collaborative endeavors. While forecasting and truck reception have no hard limitations, we will consider soft limitations in our models, most notably the truck reception limitations, which will be noted later in the modeling sections of the paper.

Chapter 2: Literature Review

This literature review (LR) focuses on inventory routing problems (IRP) and Production Distribution Problems (PDP), which are a part of supply chain optimization with an emphasis, however, on Mixed Integer Linear Programming (MILP) models. The LR covers the importance of these tools for Cosmetics Company Canada's logistical problems, which are mainly associated with cost reduction and process flow enhancement. This chapter scopes existing studies and highlights the limitations this project aims to address. This leads to an in-depth discourse on enhancing Cosmetics Company Canada's Supply Chain Logistics.

2.1. Inventory Routing Problems (IRPs)

Most models' complexity, computational intensity, and specific assumptions limit their scalability and adaptability to actual supply chains. To bridge this gap, it is imperative to design flexible, scalable, and evidence-based intelligent models that address current supply chains' dynamic and multidimensional nature. This study focuses on developing a practical and robust optimization model tailored to Cosmetics Company Canada's supply chain network. By understanding and incorporating the principles of Inventory Routing Problems (IRPs), our model aims to synchronize procurement, inventory, and distribution processes to achieve the lowest cost in Cosmetics Company's raw material supply network. Drawing on methodologies highlighted by Moradi and Sangari (2021) and Geevers et al. (2023), who demonstrated practical applications of multi-echelon approaches, our multi-period, multi-echelon Mixed Integer Linear Programming (MILP) model integrates various aspects of supply chain management, including initial inventory levels, storage capacities, transportation costs, and truck availability. Unlike many theoretical frameworks, our model emphasizes real-world applicability by addressing short-term production planning within a closed supply chain network that leverages internal and external flexible warehousing, similar to the approaches discussed by Bell et al. (1983) and Daskin (1985). By minimizing total costs, which include transportation, storage, and truck usage expenses, our model aims to enhance economic efficiency while adhering to constraints that ensure balanced flows of goods, adherence to supply limits, efficient use of storage space, and realistic truck availability and utilization. Incorporating elements from seminal works on IRPs by Cannas et al. (2024), our model utilizes mixed-integer programming to manage multiple products and constraints related to vehicle capacity and customer service levels, further reinforced by meta-heuristic techniques as demonstrated by Ahsen & Moshref-Javadi (2024). This comprehensive approach addresses the complexities and limitations inherent in real-world operations. It significantly contributes to the academic literature on supply chain management and optimization, ultimately providing actionable insights for daily operations at Cosmetics Company Canada's Horace Plant. To our knowledge, while multi-product approaches have been viewed before, the granularity of our work, which looks at individual SKUs for Inventory Routing, has yet to be explored, which has practical, real-world implications, including short-term routing of products from suppliers to external warehouses to production facilities, the indication of shortages of goods within the production plan, and utilization and saturation of internal and external storage facilities.

Later, methodological approaches were used to address the inventory-routing problem in a single-period formulation rather than long-term iterations, thereby enabling the application of standard routing algorithms to IRPs (Dror & Ball, 1987). This paper ensures that we consider the problem discreetly and look at various subproblems that could be used to determine the solution to a problem. (Bertazzi & Speranza, 2012) Whereas a Vehicle Routing Problem (VRP) focuses on a discreet number of vehicles needed to visit routes within a given period, the IRP fixates on the inventory demanded and delivered by specific customers. In our scenario, we can consider the production warehouse as the customer-driving demand. "Inventory routing problems: An introduction" gives a baseline formulation of the IRP approach and cites Bell (1983) as a historical benchmark of this area of study (Archetti, Bertazzi 2012). This paper ensures that we consider the problem discreetly and look at various subproblems that could be used to determine the solution to a problem. An exact IRP approach was first introduced in 2007 by Archetti, Bertazzi Laport, and Speranz (2007). Archetti surveyed metaheuristics in 2014, identifying relaxation approaches as a primary way to discover feasible solutions (Archetti et al., 2014). This model was later expanded by comparing aggregated and disaggregated variables, examining routes and delivered quantities of each vehicle (Archetti & Ljubic, 2021).

The research delves into the Inventory Routing Problem (IRP) and its fusion with transportation and inventory management. The research findings of Sainathuni et al. (2014) suggest a nonlinear integer programming model to solve the Warehouse-Inventory-Transportation Problem (WITP), considering total distribution costs. According to Mostafa and Eltawil (2015), the integrated approach involves production, inventory, distribution, and vehicle routing planning. They highlighted the limitations of focusing on isolated function optimization without discussing pragmatic, scalable solutions for ample and dynamic supply chains. The framework lacks empirical validation, thus represents a gap between conceptual frameworks and practical actions. Prakash and Mukherjee (2023) designed a multi-period inbound inventory routing model, considering supply failure risks and demand uncertainty. Their mixed integer linear programming model determines the optimal sourcing strategies and delivery routes, indicating that supply chain risks and demand variation significantly affect cost performance. Similar to this thesis, the study mainly pertains to inbound logistics processes, as our model derives demand from the production plan and does not investigate the distribution or production success of finished goods. Zhang et al. (2017) put forth the Multiscale Production Routing Problem (MPRP), where production, inventory, distribution, and routing integration occur at multicommodity supply chains. Their MILP model and iterative heuristic provide high-quality solutions but are too costly in computational power. Models may be complex and require detailed operational constraints, making large-scale implementation involving product ranges and fluctuating demand patterns complex, particularly in supply chains. Conversely, Arab et al. (2020) introduced a multi-period, multi-product IRP with a two-objective model to minimize costs and transportation risks.

The literature reviewed in this study highlights the evolution and application of Inventory Routing Problems (IRPs) and Mixed Integer Linear Programming (MILP) in supply chain optimization, focusing on practical, real-world applications within Cosmetics Company Canada's supply chain. This study builds upon the foundational work of Bell (1983) and Daskin (1985) by integrating multi-echelon, multi-period MILP models to manage transportation, warehousing, and inventory costs, reflecting the methodologies and empirical approaches of Moradi and Sangari (2021) and Geevers et al. (2023). While incorporating advances from Cannas (2024) and Ahsen & Moshref-Javadi (2024) in the use of metaheuristics and modern technological integrations, our model maintains practicality by addressing the computational challenges noted by Zhang et al. (2017) and Arab et al. (2020). Archetti et al.'s (2007, 2012, 2014, 2021) work provides the theoretical underpinning for our model's relaxation techniques and multi-product considerations. Overall, this research bridges the gap between theoretical frameworks and practical implementation, offering a robust optimization model that enhances operational efficiency within Cosmetics Company's production logistics, as emphasized by the empirical validations of Sainathuni et al. (2014), Mostafa & Eltawil (2015), and Prakash & Mukherjee (2023).

Table 4 Inventory Routing Problems (IRPs) related Papers

Citation	Aim	Period	Type of Formulation	Echelon	Method Used	Findings
Sainathuni et al. (2014)	Minimize total distribution cost in WITP	Multi-period	Heuristic Iterative Local Search	Single-echelon	Nonlinear integer programming model and heuristic iterative local search	Reduced workload variance at warehouses and total distribution cost; sensitive to warehouse tech and productivity.
Mostafa & Eltawil (2015)	Integrate production, inventory, distribution, and routing	Multi-period	Mixed-integer programming	Multi-echelon	Literature review and mathematical model	Existing models are too simple; propose a more integrated model for realistic supply chain management.
Bertazzi et al. (2015)	Minimize total expected cost in the supply chain with stochastic demand	Multi-period	Mixed-integer linear programming	Single-echelon	Stochastic dynamic programming and metaheuristic approach	Policy based on average demand is less effective; optimal policies are provided for small instances.
Prakash & Mukherjee (2023)	Develop an inbound inventory routing model with supply risks and demand uncertainty.	Multi-period	Mixed-integer linear programming	Single-echelon	Mixed-integer linear programming and simulation	Lower demand variation and higher supply capacity improve cost performance; the model includes supply risks and demand uncertainties.
Malladi & Sowlati (2018).	Incorporate sustainability in IRPs	Multi-period	Single and multiple-objective models	Multi-echelon	Content analysis of 40 journal articles	Identifies gaps in sustainability research; calls for more multi-objective models, including environmental and social aspects.
Minsi et al. (2020).	Integrate strategic, tactical, and operational planning in LIRP	Multi-period	Mixed-integer programming	Multi-echelon	Hybrid Harmony Search-Simulated Annealing algorithm	The hybrid algorithm outperforms standard approaches and focuses on reverse logistics costs.
Nambirajan et al. (2020)	Optimize VMI systems with inventory routing	Multi-period	Mixed-integer linear programming	Multi-echelon	Mixed-integer linear program and three-phase heuristic (CAR)	Significant improvements in solution quality and computational efficiency; challenges in practical implementation due to collaboration needs.
Onggo et al. (2019).	Optimize perishable inventory routing with stochastic demand	Multi-period	Mixed-integer programming	Single-echelon	Mixed-integer program and simheuristic algorithm	Effective cost minimization for perishables; limited generalizability due to focus on single supplier and warehouse.
Zhang et al. (2017).	Coordinate production, inventory, distribution, and routing in MPRP	Multi-period	Mixed-integer linear programming	Multi-echelon	MILP model with iterative heuristic	High-quality solutions but with high computational cost; challenges in large-scale implementation.
Arab et al. (2020).	Minimize costs and transportation risks in multi-product IRP	Multi-period	Multi-objective optimization	Multi-echelon	ε-constraint method, NSGA-II, and MOICA algorithms	Effective bi-objective optimization, computational intensity, and assumptions limit practical application.

2.2. Production Distribution Problems

The combination of production and distribution has become a significant topic of supply chain management in current supply chain literature. The main goal of production and distribution integration is to improve the productivity and performance of supply chains in order to save costs and improve services. The research study undertaken by Goodarzian et al. (2021) proposed a new multi-objective model for production distribution in multi-echelon supply chains. The results validate the model's practical applications and contribute to improving supply chain network design under uncertainty. Manupati et al. (2020) developed the application of blockchain technology in the context of supply chain performance measurement and management. Under a carbon taxation policy in a multi-echelon setting, their use of distributed ledgers reduced operational costs and carbon emissions and had implications for policymakers and supply chain managers.

Govindan et al. (2016) proposed a multi-objective mixed-integer model for reverse supply chains. Their model improved the flow of products, parts, and materials and demonstrated how sustainable manufacturing could be made. Numerical examples provided and sensitivity analysis of the study proposed some recommendations to the decision-makers. Taxakis and Papadopoulos (2016) indicated two models for designing and managing a supply chain network. Their mixed integer linear and non-linear programming models illustrate the tangible impact of strategy and operations in the supply chain. Sakalli (2017) also researched a production distribution problem under stochastic and fuzzy environments. They also suggested a new way of finding the solution to possibility and chance-constrained programming, which can deal with several uncertainties and be applied at the tactical level. On the other hand, the study by Bank et al. (2020) was conducted in a two-stage food supply chain system for fresh food, which developed methodologies to improve distribution performance and lower operation costs while managing infeasibility. Nourifar et al. (2018) presented multi-period decentralized supply chain models under uncertainty. The authors also suggested a bi-level mixed integer linear programming model with stochastic parameters and presented heuristic strategies for managing decentralized supply chains practically. Amirtaheri et al. (2017) also examined an integrated multi-echelon supply chain under price and advertising-dependent demand. The development of Stackelberg's game theory and bi-level programming models indicated the use of genetic algorithms and particle swarm optimization to develop better and more efficient solutions that focus on strategy and tactics. Camacho-Vallejo et al. (2015) developed a bi-level model for production distribution planning and used the scatter search method. The literature review revealed that supply chain management requires the consideration of both production and distribution decisions. Future research should also be focused on robust and scalable solutions as supply chains continue to be characterized by growing complexity and dynamism.

Integrating production and distribution in supply chain management is critical for enhancing efficiency and reducing costs. The literature reviewed provides substantial contributions to this field, directly informing the development of my thesis on optimizing Cosmetics Company Canada's supply chain network. Goodarzian et al. (2021) demonstrate the efficacy of multi-objective MILP models in achieving high-quality solutions under real-world constraints, an approach mirrored in my work. Manupati et al. (2020) illustrate the benefits of integrating technology in improving supply chain transparency and reducing operational costs, reinforcing the importance of innovative technologies in my model. Diabat et al. (2019) and Nourifar et al. (2018) emphasize the need for robust and decentralized supply chain management, integrating stochastic and fuzzy parameters to handle uncertainties—a crucial aspect of my thesis. Govindan et al. (2016) and Taxakis and Papadopoulos (2016) provide foundational models for closed-loop supply chains and mixed-integer programming, respectively, aligning with my goal of cost minimization and strategic planning. Sakalli (2017) and Bank et al. (2020) offer insights into handling feasibility issues and shortages, ensuring my model's robustness and reliability. Finally, Amirtaheri et al. (2017) and Camacho-Vallejo et al. (2015) contribute by highlighting bi-level programming approaches and cost-effective transportation strategies, which are integral to my comprehensive optimization framework. Collectively, these studies underpin the methodological rigor and practical relevance of my research, advancing the field of supply chain optimization.

Table 5 Production Distribution Problems related Papers

Citations	Aim	Method Used	Period	Type of Formulation	Echelon	Findings
Goodarzian et al. (2021)	Develop a multi-objective model for production distribution in an uncertain supply chain network.	MILP model, GFLP, NSGA-II, Fast PGA	Multi	Mixed Integer Linear Programming	Multi-echelon	The novel formulation and algorithms effectively address the uncertain nature of costs, demands, and capacities.
Manupati et al. (2020)	Develop a blockchain-based approach for supply chain performance, optimizing emissions and costs.	Distributed ledger, MINLP, NSGA-II	Multi	Mixed Integer Nonlinear Programming	Multi-echelon	Blockchain minimizes costs and emissions, showing feasibility and supporting policy and managerial decisions.
Diabat et al. (2019)	Design a resilient supply chain network for humanitarian aid during disasters.	Bi-objective robust optimization, Lagrangian relaxation, ε-constraint	Single	Bi-objective optimization	Multi-echelon	A robust model minimizes delivery time and cost, handling multiple disruptions effectively.
Govinda et al. (2016)	Improve manufacturing sustainability in a closed-loop supply chain.	Multi-objective mixed integer mathematical problem	Single	Mixed Integer Programming	Closed-loop	The model maximizes profit, minimizes costs, and meets environmental regulations efficiently.
Taxakis & Papadopoulos (2016)	Optimize supply chain network design and operational planning.	MILP, MINLP, steady-state genetic algorithms	Single	Mixed Integer Linear/Nonlinear Programming	Multi-echelon	Models improve supply chain design and operational planning and are validated with MATLAB and GAMS comparisons.
Sakalli (2017)	Handle production distribution with stochastic and fuzzy uncertainties.	Possibilistic programming, chance-constrained programming	Multi	Deterministic, fuzzy, stochastic modeling	Multi-echelon	Proposed approaches successfully handle uncertainties and produce robust solutions.
Bank et al. (2020)	Integrate production and distribution in a seasonal, perishable goods supply chain.	MIP, Hybrid Simulated Annealing (HSA), Genetic Algorithm (GA)	Multi	Mixed Integer Programming	Two-stage	HSA and GA effectively solve the integrated problem, improving delivery before the specified
Nourifar et al. (2018)	Propose a decentralized supply chain model with uncertainty.	Bi-level MILP, heuristic algorithm, fuzzy and chance constraint approach	Multi	Bi-level Mixed Integer Linear Programming	Multi-echelon	The model handles uncertainty, demonstrating applicability through a numerical example and sensitivity analysis.
Camacho-Vallejo et al. (2015)	Plan production and distribution in a supply chain to minimize costs.	Bilevel mathematical problem, heuristic algorithm, Scatter Search	Single	Bilevel optimization	Multi-echelon	The algorithm improves existing solutions, effectively balancing transportation and operational costs.

Chapter 3: Problem Definition and Model Formulation

3.1. Problem Statement:

The core issue revolves around optimizing material flow within the complex supply chain, underpinned by evaluating variable warehousing and transportation costs. The nature of this challenge is compounded by the array of Stock Keeping Units (SKUs), rendering a product-level analysis difficult. As discussed earlier, SKUs are classified into two predefined categories: Raw Materials (RMs) and Auxiliary Components (ACs), each with unique characteristics.

The inflow of materials into the system emanates from suppliers, who undertake daily deliveries by logistical teams' forecasts and requisitions. Warehouses and the Horace Production Warehouse maintain daily starting inventories in conjunction with storage capacities that dictate the maximum volume of a specific material type, measured in pallets, that can be accommodated at a given facility. Notably, Horace serves as a distinctive entity, the sole location that can furnish materials for production lines. These production lines utilize Raw Materials (RMs) and Auxiliary Components (ACs) to manufacture finished goods and Bulk Goods. Suppliers can distribute materials to any warehouse or HP based on capacity constraints. The material interchange between warehouses and from HP to warehouses occurs freely in both directions. The demand for the model is determined by the Bill of Materials (BOM) for the finished and Bulk Goods, making these entities the focal point of our analysis and categorizing them as our 'customers' within the framework of our investigation.

The project seeks to explore optimization material flow techniques within the Horace Plant's internal supply chain. It will consider the nuances of transportation and warehousing costs and adhere to a structured classification of SKUs, facilitating practical and efficient decision-making processes. The ultimate objective is to provide actionable recommendations that enhance cost-effectiveness and operational efficiency and flag potential risks.

3.2. Data Sources:

The project leverages access to essential data, including the daily starting inventory and inbound orders from suppliers. These data sources collectively represent the total volume of incoming goods. Materials departing the warehouses for transfer to other warehouses, including HP and goods dispatched to third-party subcontractors, are regarded as outbound materials. HP assumes a distinctive position, as it functions as a warehouse and a production facility. In this context, the project considers pallets directed towards production, guided by Bill of Materials

(BOMs), as part of the outbound materials under scrutiny. Notably, the analysis does not encompass the calculation of finished goods, and the capacity computations for the HP warehouse are limited to the designated space allocated.

The Apriso Internal and External Warehouse documents provide a snapshot of the visibility of the Cosmetics Company warehousing ecosystem. Hence, we can utilize these as the starting inventories of our materials in various locations. Similarly, a delivery schedule, pulled outside of the SAP ecosystem, gives us upcoming deliveries of goods into the Cosmetics Company warehousing ecosystem. These are combined with the pallet conversion document developed for this project, noting the maximum UPC on a singular pallet, which is then used to standardize all UPCs to pallets. In this way, we speak about all UPCs in the same language, which would be impossible if we were to maintain the baseline units of UPCs. The Flows teams provided Warehouse Contracts. They are not available for public viewing and were used with me to calculate the shipping and storage costs. Additionally, the contracts indicate the number of pallets that can be stored at each warehousing location. The Warehouse contracts are renegotiated annually.

Table 6 Data Sources

Document	Bill of Material	Apriso Internal Warehouse	Apriso External Warehouse	Deliver Schedule	Pallet Conversion	Warehouse Contracts	Master Data
Source	SAP	SAP	SAP	Expo	SAP	Flows Team	SAP
Use	Demand	Starting Inventories	Starting Inventories	Incoming Goods	Unit Standardization	Shipping costs, storage costs, capacity	UPC Indexing

3.3. Data Organization

Converting organizational data into a format suitable for optimization software, specifically IBM ILOG CPLEX involves a series of methodical steps to program the mathematical modeling and subsequent user-friendly interpretation of results for end users. Traditional organizational data structures lack the simplicity and indices to integrate with optimization software. Therefore, data preprocessing is required before programming the model and presenting the outcomes in a comprehensible industry-specific format.

To begin, we amalgamate information from eight disparate sources. The incoming delivery data delineates suppliers' daily inbound goods by SKU. The requirements dataset offers insights into the daily demand. The Bill of Materials (BOM) provides a unit breakdown of UPCs to meet the daily demand for each finished product. This breakdown aids in retroactively determining the requisite quantity of units needed to fulfill demand. The BOM for bulk planning and finished goods are summarized together to give a period for each UPC rather than giving demand for each production line within the Production Warehouse.

External and internal warehouse data from Apriso furnish details on the inventory levels of individual products at external and production warehouses. This is the starting amount for period 0 for warehouses and production warehouses.

It is essential to convert the entire dataset from units to pallets, as pallets are the standard unit for the model. To do this, we need to refer to a supplementary datasheet that provides the number of units on a pallet for each specific product in both production and external warehouses.

Each SKU is indexed from 1 to n for enhanced interpretability by CPLEX. Analogous indexing is essential for warehouses and dates. The model's dynamism, particularly regarding date-related parameters, necessitates configuring the dataset to update based on varying dates dynamically. Flexibility to alter the calculation period, such as transitioning from daily to weekly calculations, is contingent upon proficient coding and dataset conversion. We have calculated the current model's time or period index for daily production. Returning to the SKU section of our paper, we have reduced the over 5500 SKUs in the dataset to 1990 with an additional index of 1, indicating the combined products that do not have demand during the period.

Dedicated sheets have been created to systematically index dates, products, and warehouses. When only one supply and production warehouse is considered, a uniform index assumption of one is applied. A scalable approach for multiple suppliers or production warehouses would replicate this indexing convention. As mentioned earlier, distinct Excel sheets based on the indices are created to allow data integration into CPLEX.

3.4. Costs

Three types of costs are considered, though only three are within the model. The three costs are pallet shipping, storage, and trucking. Pallet movement costs are a singular cost of receiving or shipping a pallet from one location to another. Storage costs signify the cost of storing a single pallet at a location in each period. Trucking cost represents the cost of using a truck going from one location to another. Regardless of the number of pallets in the truck, this cost occurs. Discreet trucking contracts are unavailable for Cosmetics Company and its partners, as they are variable daily. With the expertise of the Flows Team, we landed on a fixed figure to hire a truck for a single trip from one storage location to another.

Table 7 Trucking Costs

Going from X to Y	The cost of a single Truck
Supplier to External Warehouse	$150
Supplier to Production Warehouse	$100
External Warehouse to Production Warehouse	$250
Production Warehouse to External Warehouse	$250

Because of the financial structure of Cosmetics Company Canada and its partnership with Cosmetics Company's S&OP team, the cost of suppliers delivering to an external warehouse or Horace is considered as a warehouse reception price rather than a truck price, which makes it consistent across warehouses, though different between supplier to the production warehouse. It is only when goods are moved between external warehouses or Horace, and vice versa, that the flows team hires a short-term local truck team, which is calculated as an average given by the flow teams, as it does fluctuate in the real world. Pallet movement costs are built into the contracts of some suppliers, and a small labor cost is considered for any single pallet entering or exiting the production warehouse. These costs occur as a general deterrent of unnecessary movements by external partners and are used by the production warehouse to track employee labor use. These costs are symmetric; therefore, we have given the costs from the perspective of the Supplier and Production Warehouse. It should be noted that goods going from the external warehouse to the production warehouse are at the same price as the reversed.

Table 8 Supplier Pallet Movement Cost

Shipping from Supplier	Cost
XTL	0
BDL	$11.50
VSL	$5.87
La Chine	0
Horace Production Warehouse	$2

Table 9 Production Warehouse Pallet Movement Cost

Receiving Cost at Horace:	**Cost**
XTL	$2
BDL	$13.50
VSL	$7.87
La Chine	$2
Supplier	$2

Variable warehouse storage costs are based on the breakdown of contracts between the Cosmetics Company Horace plant and its external warehouse counterparts. This is the daily storage cost of a pallet, as these contracts are more prone to short-term storage at VSL and La Chine. In contrast, La Chine and BDL are unique because XTL is intended to host pallets for extended periods, and BDL does not require a contract as it is part of the organization.

Table 10 Variable Pallet Storage Costs

Warehouse	**Daily Pallet Storage Cost**
VSL	$2
XTL	$1.52

3.5. Assumptions

The Pallet Conversion assumption (1) indicates that we identified the number of units per pallet for all SKUs and converted each SKU appropriately. This standardization across supply, storage, and demand allows for the more straightforward calculation of per-pallet storage costs, shipping costs, and trucking limits. By using pallets as the unit of measure throughout the model, we ensure uniformity and comparability of data. Supplier Storage assumption (2) asserts that all incoming goods from suppliers must be transferred to either a warehouse or a production warehouse by the end of each day, resulting in zero-ending inventory for suppliers. This ensures that suppliers do not retain any goods and eliminates the possibility of goods being returned.

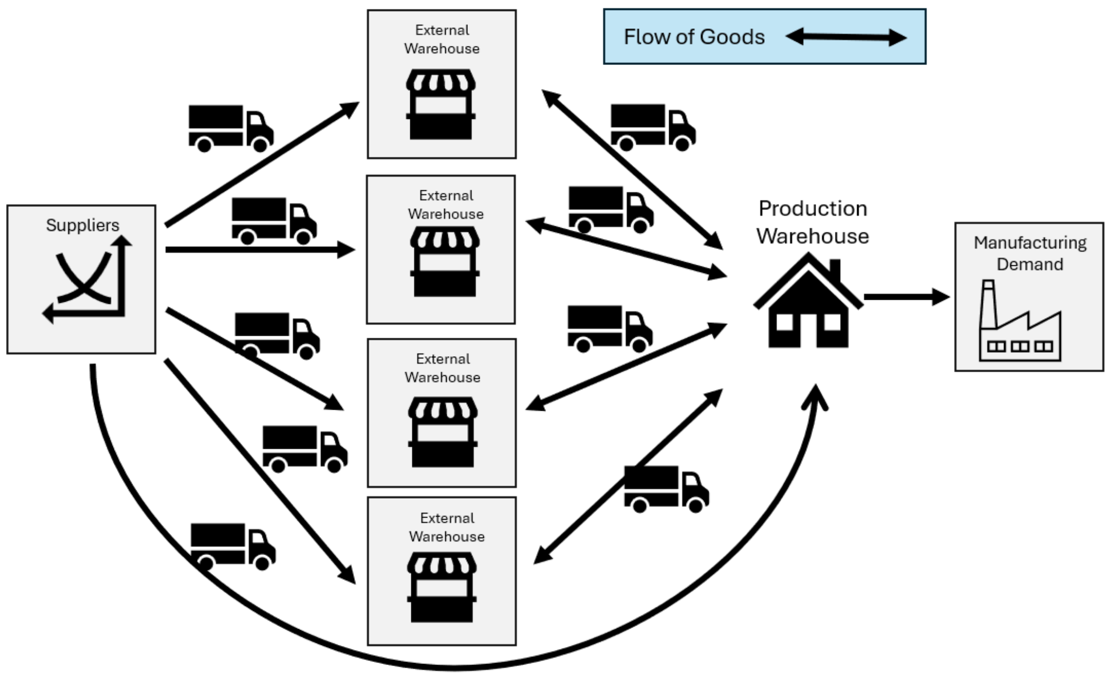
With the Starting Inventories assumption (3), we can specify that the initial inventories of each SKU at each location are known. After the initial period (T=0), starting inventories are determined based on flow constraints governing the movement of goods. The assumption of demand for SKUs (4) relies on the upcoming production schedule and the volume of each SKU needed to meet demand, with demand quantified in pallets. This is derived using the Bill of Materials (BOM) for finished and bulk production schedules. The Truck Reception assumption (5) posits that there is no restriction on the volume of goods moving in and out of an external warehouse, provided that capacity and supply constraints are met. In the case of the production warehouse, it is assumed that a limited number of trucks can be received in a single period, though the number of pallets is not indicated. While this model assumes an unlimited flow of trucks to external warehouses, certain facilities may encounter flow limitations. However, no external warehouse reception capacities have been recorded in this study. The Trucking Capacity assumption (6) states that all trucks can carry up to 52 AC pallets or 26 RM pallets, both being transportable on the same truck. A constraint is included to ensure that the requisite number of trucks is available to support the flow of goods. The Storage Capacity assumption (7) outlines that each warehouse and production warehouse has a defined maximum pallet storage capacity, which dictates the maximum ending inventory for each location and is split by product type for our purposes. This ensures that storage constraints are respected in the inventory management process.

3.6. Model Formulation

The production warehouse is involved with the transportation logistics from the incoming raw materials and components to the distribution of finished goods to wholesale customers, which includes a significant range of hair coloration and other beauty products. For the sake of our problem, we limit the scope of the incoming raw materials and components from outside suppliers until they reach the production lines. The demand developed by the BOM from the production planners drives pallets of goods from suppliers and external warehouses to the production warehouse. Products are broken down on the SKU level based on the production schedule developed by short-term planners, and the raw materials and components differentiate the family of products, which have different parameters associated with them, most notably pallet storage and truck pallet capacity.

The production warehouse acts as a staging center to meet demand, and four external warehouses can hold additional pallets of products at varying capacities. Each storage location has a pallet capacity based on product category and a cost associated with storing individual pallets at the end of a single period. For the sake of our problem, we consider our period t as a single day, and our t ranges from 1 to 10. Units of goods for supply, demand, storage, capacity, and shortage are expressed as pallets. Trucks can move from suppliers to external warehouses or directly to production warehouses. Similarly, trucks can move from external warehouses to production warehouses, and the production warehouse can send pallets back to warehouses to create space for products in the production demand. Trucks moving across arcs can contain a maximum number of pallets, though the maximum number depends on the material type. Pallets of differing product types can travel on the same truck, given that the sum of product types going from one location to another divided by the capacity of that product type does not exceed the number of trucks along that arc. Trucks moving along a given arc have a price associated with a single trip, plus there is a separate pallet movement cost, which is charged when a pallet is moved from one location to another, according to contracts negotiated with external warehouses.

Figure 3 Structure of Production Distribution Network



There are three types of flow balance constraints in the equation. The first ensures that all starting inventory from suppliers on any given day is shipped to one of the five storage locations. The second ensures that the starting inventory at a warehouse for any single product in any given period minus the outflow of the product plus the inflow from suppliers is equal to the starting inventory of the next day. The same can be said for production warehouses, though demand is considered an outflow, and if there is a shortage, that is considered an incoming flow. Shortages are considered in the model to ensure that we can meet the demand for any product in any production warehouse during any given period. Our shortages are associated with a high cost value in order to ensure that existing stock is first used within the model, prior to filling demand with a shortage value. In a more practical sense, a shortage indicates a short-term risk to the flows team that needs to be addressed based on the parameters set. Supply constraints indicate that the number of pallets of a particular product leaving a storage location and going to all other locations must be less than or equal to the starting inventory of that product during that time. The capacity constraints ensure that the total ending inventory on any given day based on a product type and storage location does not exceed the storage capacity of a specific storage location and production type. Regarding trucking constraints, we have truck reception constraints, which limit the number of trucks a single facility can receive on any given day. This applies to only our production warehouse in this model, though the concept could be added to the external warehouses using similar logic. Finally, we include truck requirement constraints that ensure the number of pallets from each product category will have enough trucks to ship from one starting location to another.

The model ends at the production line, though the demand is indicated through the BOM finished material requirements. The objective function considers four significant costs which it works to minimize. First, it takes on trucking costs from one location to another, which are indicated by Y variables (number of trucks) and TC variables (trucking costs). Similarly, palette flow is indicated by variables starting with X as the decision variable. The cost of a single pallet moving along the indicated is noted by variables that begin with C. Warehouse costs are indicated with the notation WC, which is multiplied by the sum of ending inventories of each storage location from all periods starting from 1..n in the model, and is indicated with variables beginning with S. Finally, the cost of shortages is added to the objective function, with the cost of a single missing pallet being notated by G while the missing pallet decision variable is given as L. The full notation is described, and the model is presented in the following subsections.

3.6.1. Notation Table

Table 11 Index

i	Index for suppliers, $i = 1, 2, 3, \dots, m$
j	Index for warehouses, $j = 1, 2, 3, \dots, n$
k	Index for production warehouse $k = 1..,a$
p	Index for product number $p = 1,2,3\ldots,q$
t	index for time period $t = 1,2,3\ldots,r$
u	Index for product category $u = 1,2\ldots v$

Table 12 Parameters

S_{puit}	The number of pallets of product *p* in category *u* in stock at supplier *i* at the end of period *t*
S'_{puj0}	The number of pallets of product *p* in category *u* stock at warehouse *j* at end of period *t=0*
S''_{puk0}	The number of pallets of product *p* in category *u* stock production warehouse *k* at end of period *t=0*
H_{ui}	The maximum number of pallets of category *u* stored at supplier *i*
H'_{uj}	The maximum number of pallets of category *u* stored at warehouse *j*
H''_{uk}	The maximum number of pallets of category *u* stored at production warehouse *k*
D_{pukt}	The number of pallets of product *p* in category *u* demanded at production warehouse k on day *t*
Cap_u	The number of pallets of category *u* a single truck can carry
N_k	Number of trucks able to be received by production warehouse *k*

Table 13 Costs

WC_{uj}	Cost of storing one pallet at warehouse *j*
WC'_{uk}	Cost of storing one pallet at production warehouse *k*
C_{uij}	Cost of shipping one pallet in category *u* from supplier *I* to warehouse *j*
C'_{uik}	Cost of shipping one pallet in category *u* from supplier *I* to production warehouse k
C''_{ujk}	Cost of shipping one pallet in category *u* from warehouse *j* to production warehouse *k*
C'''_{ukj}	Cost of shipping one pallet in category *u* from production warehouse *k* to warehouse *j*
TC_{ij}	Cost of a single truck going from supplier I to warehouse j
TC'_{ik}	Cost of a single truck going from supplier I to production warehouse k
TC''_{jk}	Cost of a single truck going from warehouse j to production warehouse k
TC'''_{kj}	Cost of a single truck going from production warehouse k to warehouse j
G	Cost of a pallet shortage

Table 14 Decision Variables

X_{puijt}	The amount of product *p* in category *u* that flows from supplier *i* to warehouse *j* on day *t*
X'_{puikt}	The amount of product *p* in category *u* that flows from supplier *i* to production warehouse *k* on day *t*
X''_{pujkt}	The amount of product *p* in category *u* that flows from supplier *j* to customer *k* on day *t*
X'''_{pukjt}	The amount of product *p* in category *u* that flows from production warehouse *k* to customer *j* on day *t*
Y_{ijt}	Number of trucks going from supplier i to warehouse j in period t
Y'_{ikt}	Number of Trucks going from supplier *i* to production warehouse k in period t
Y''_{jkt}	Number of Trucks going from warehouse j to production warehouse k in period t
Y'''_{kjt}	Number of Trucks going from production warehouse k to warehouse j in period t
S'_{pujt}	The number of pallets of product *p* in category *u* starting from warehouse *j* on day *t*
S''_{pukt}	The number of pallets of product *p* in category *u* starting from production warehouse *k* on day *t*
L_{pukt}	The number of pallets of product *p* in category *u* at production warehouse *k* in period *t* that is not available to meet demand.

3.6.2. Objective Function

The mixed integer programming model's objective function (1) seeks to minimize the total cost associated with transporting pallets among suppliers, warehouses, and production facilities. This cost encompasses the expenses related to truck utilization for transportation between suppliers, warehouses, and production facilities, as well as the overnight storage costs for each pallet held at the warehouses, as mentioned earlier, and production facilities.

$$Min\ Cost = \sum_{p=1}^{q}\sum_{u=1}^{v}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{r} C_{uij}X_{puijt} + \sum_{p=1}^{q}\sum_{u=1}^{v}\sum_{i=1}^{m}\sum_{k=1}^{a}\sum_{t=1}^{r} C'_{uik}X'_{puikt}$$
$$+ \sum_{p=1}^{q}\sum_{u=1}^{v}\sum_{j=1}^{n}\sum_{k=1}^{a}\sum_{t=1}^{r} C''_{ujk}X''_{pujkt} + \sum_{p=1}^{q}\sum_{u=1}^{v}\sum_{j=1}^{n}\sum_{k=1}^{a}\sum_{t=1}^{r} C'''_{kj}X'''_{pukjt}$$
$$+ \sum_{p=1}^{q}\sum_{u=1}^{v}\sum_{j=1}^{n}\sum_{t=1}^{r} WC_{uj}S'_{pujt} + \sum_{p=1}^{q}\sum_{u=1}^{v}\sum_{k=1}^{a}\sum_{t=1}^{r} WC'_{uk}S''_{pujkt} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{r} TC_{ij}Y_{ijt}$$
$$\sum_{i=1}^{m}\sum_{k=1}^{a}\sum_{t=1}^{r} TC'_{ik}Y'_{ikt} + \sum_{j=1}^{n}\sum_{k=1}^{a}\sum_{t=1}^{r} TC''_{jk}Y''_{jkt} + \sum_{k=1}^{a}\sum_{j=1}^{n}\sum_{t=1}^{r} TC'''_{kj}Y'''_{kjt} + G\sum_{p=1}^{q}\sum_{u=1}^{v}\sum_{k=1}^{a}\sum_{t=1}^{r} L_{pukt}$$

(1)

Transportation costs arise from moving pallets of products between suppliers, warehouses, and production warehouses. The total transportation cost comprises the expenses of shipping products across different routes. Specifically, the model accounts for the cost of shipping pallets from suppliers to warehouses, suppliers to production warehouses, warehouses to production warehouses, and from production warehouses back to warehouses. These transportation costs are represented in the model by the variables *C* and *X,* where *C* values represent the per-pallet shipping costs for various routes, and *X* values indicate the number of pallets shipped along those routes over a period of time.

Storage costs are incurred for holding inventory at both warehouses and production warehouses. The total storage cost is calculated based on the cost of storing pallets at these locations over time. The storage cost components are represented by *WC* and *S* values, where *WC* values denote the per-pallet storage costs, and *S* values represent the inventory levels of products at warehouses and production warehouses.

Truck usage costs encompass the fixed expenses associated with operating trucks to transport pallets between different locations. The total truck usage cost includes the costs of using trucks on various routes and during each period. The model captures these costs by *TC* values, which indicate the fixed costs of operating a truck for specific routes, and *Y* values, which denote the number of trucks used for transporting products.

In summary, the objective function comprises transportation, storage, and truck usage costs to provide a holistic view of the total cost to be minimized. This detailed cost breakdown facilitates an understanding of the economic impact of each decision within the supply chain network, enabling informed and strategic decision-making. By optimizing these cost components, the model aims to enhance the overall efficiency and effectiveness of supply chain operations, ensuring that resources are utilized most cost-effectively.

3.6.3. Constraints

The supply chain optimization model incorporates several key constraints to ensure the feasibility and efficiency of operations. These constraints address the flow of goods, supply limitations, storage capacity, and truck availability, each contributing to the overall balance and functionality of the system.

Firstly, the flow balance constraints are crucial for maintaining equilibrium within the supply chain. For suppliers, constraint (2) ensures that the volume of product p in category u starting from supplier i minus the volume delivered to warehouses j and production warehouses k equals zero for each period t. This guarantees that suppliers do not retain any excess product at the end of each day. For warehouses, constraint (3) balances the starting inventory, inbound shipments from suppliers, and inbound returns from production warehouses against the outbound shipments to production warehouses and the ending inventory. This ensures that all inbound and outbound flows are accounted for, maintaining a balanced warehouse inventory. Similarly, for production warehouses, constraint (4) balances the starting inventory and inbound shipments from suppliers and warehouses against the outbound shipments to warehouses, the demand for each product, and the ending inventory. This ensures that production warehouses can meet demand while maintaining a balanced flow of goods.

Supply constraints are essential for defining the initial inventory levels and the maximum outflow from each location. These constraints ensure that suppliers, warehouses, and production warehouses do not ship more than their available inventory. For instance, the warehouse supply constraint (5) dictates that the outbound volume of pallets must not exceed the starting inventory, ensuring that warehouses do not deplete their stock below sustainable levels. The production warehouse supply constraint (6) limits the outbound volume to prevent over-shipping from production warehouses.

Storage capacity constraints ensure that warehouses and production warehouses do not exceed their storage limits. Each location has a maximum storage capacity, and the constraints ensure that the sum of the starting inventory, inbound shipments, and outbound shipments does not surpass this capacity. This prevents overstocking and ensures efficient utilization of storage space. The warehouse capacity constraint (7) ensures that the total inventory held at each warehouse remains within its storage limits. In contrast, the production warehouse capacity constraint (8) similarly restricts the inventory at production warehouses.

Trucking constraints address the availability and utilization of trucks for transporting goods. Truck Reception constraints (9) ensure that the number of trucks the production warehouse receives in a single period does not exceed the reception team's limitation. This constraint is critical for maintaining realistic and feasible transportation operations. The truck required constraints for suppliers (10), warehouses (11), and production warehouses (12) ensure that the total number of trucks deployed does not exceed the available fleet, preventing overuse of transportation resources.

In summary, the constraints within the supply chain optimization model collectively ensure the balanced flow of goods, adherence to supply limits, efficient use of storage space, and realistic truck availability and utilization. These constraints are fundamental for maintaining the operational integrity of the supply chain, facilitating effective decision-making, and optimizing overall performance.

3.6.3.1. Flows Balance Constraints

Suppliers:

$$S_{pui(t-1)} - \sum_{j=1}^{n} X_{puijt} - \sum_{k=1}^{a} X'_{puikt} = 0 \qquad \forall\, p, u, i, t$$

(2)

Constraint set (2) is the Suppliers Flow Constraint, which ensures that the total volume of product p in category u that starts with a supplier i in a particular period t minus the total volume of product p in category u delivered to warehouses j and minus the amount of product delivered to production warehouses k is equal to zero. This ensures that no product is left with the suppliers at the end of the day.

Warehouses:

$$S''_{puj(t-1)} + \sum_{i=1}^{m} X_{puijt} + \sum_{k=1}^{a} X'''_{pukjt} - \sum_{k=1}^{a} X''_{pujkt} - S'_{pujt} = 0 \qquad \forall\, p, u, j, t$$

(3)

Constraint set (3) is the Warehouses Flow Constraint, which ensures that the starting amount of each product p in category, u at each warehouse j during each period t plus the inbound volume from all suppliers I minus the outflow volume to all production warehouses k minus the ending inventory is equal to 0. This ensures that each warehouse's starting amount and inbound and outbound volumes are accounted for and balanced.

Production Warehouses

$$S''_{puk(t-1)} + \sum_{i=1}^{m} X_{puikt} + \sum_{j=1}^{n} X''_{pujkt} - \sum_{j=1}^{n} X'''_{pukjt} - D_{pukt} - S''_{pukt} = 0 \qquad \forall\, p, u, k, t$$

(4)

Constraint set (4) is the Production Warehouses Flow Constraint, which ensures that the number of pallets of product p in category u at each production warehouse k during each period t plus the inbound number of pallets of each product p in category u from all suppliers i plus the inbound number of pallets each product p in category u from all warehouses j minus the number of pallets of product p in category u needed to meet demand on a given day minus the ending inventory of number of pallets of each product p in category u at each production warehouse k during period t is equal to 0. This ensures that the starting amount, inbound volume, outbound volume, and demand for the production warehouses are accounted for and balanced.

3.6.3.2. Storage Capacity Constraints

Starting Inventory constraints are given for the initial inventory levels of each product in each location during a given period. for t=0. All S_{pit}, S'_{pjt}, and S''_{pkt},. S_{pit} is given via the delivery schedule, whereas S'_{pjt} & $S''_{pkt}$are given by the Apriso Warehouse and Apriso Production Warehouse, respectively. The starting inventory of any product in any location cannot exceed the amount of product outflowing from that location in any period. Regarding outflow from our suppliers i, there is a set amount X_{pijt} & X'_{pikt}That will be delivered into the warehousing system, regardless of where the pallets will be stored.

Warehouses:

$$\sum_{k=1}^{a} X''_{pujkt} \leq S'_{puj(t-1)} \quad \forall\, p, u, j, t$$

(5)

Constraint set (5) is the Warehouse Supply Constraint, which states that the outbound number of pallets of each product p in category u going from warehouse j to production warehouse k in period t must be less than or equal to the starting number of pallets of product p in category u at supplier i in period t. This ensures that a warehouse does not ship out more pallets of a product than it has in the inventory.

Production Warehouses

$$\sum_{j=1}^{m} X'''_{pukjt} \leq S''_{puk(t-1)} \quad \forall\, p, u, k, t$$

(6)

Constraint set (6) is the Production Warehouse Supply Constraint, which states that the outbound number of pallets of each product p in category u going from production warehouse k to warehouse j in period t must be less than or equal to the starting number of pallets of product p in category u at supplier i in period t. This ensures that a production warehouse does not ship out more pallets of a product than it has in the inventory.

3.6.3.3. Storage Capacity Constraints

Storage Capacity Constraints ensure that a maximum quantity for p products can be held. Both the Warehouses and Production Warehouses have individual parameters that define H_j& H_k. The Storage Capacity Constraints consider the ending inventory of storage locations does not exceed the defined capacity limitations.

The Flows Balance Constraints dictate that there is no ending inventory for suppliers. Therefore, there is no need to include additional constraints for the storage capacity.

Warehouses

$$\sum_{p=1}^{q} S'_{pujt} \leq H_{uj} \, for \, \forall \, u, j, t$$

(7)

Constraint set (7) is the Warehouse Capacity Constraint, which indicates that sum the starting inventory of all pallets of products p in category u at each warehouse j during each period t plus the inbound number of pallets of all products p from production supplier i to warehouse j in each period t minus the sum of outbound pallets of all products going from warehouse j to production warehouse k must be less than or equal to the capacity of each warehouse. This ensures that the warehouse does not exceed palate capacity.

Production Warehouses

$$\sum_{p=1}^{q} S''_{pukt} \leq H_{uk} \qquad \forall \, u, k, t$$

(8)

Constraint set (8) is the Production Warehouse Capacity Constraint, which indicates that sum the starting inventory of all pallets of products p in category u at each warehouse j during each period t plus the inbound number of pallets of all products p from production supplier i to warehouse j in each period t minus the sum of outbound pallets of all products going from warehouse j to production warehouse k must be less than or equal to the capacity of each warehouse. This ensures that the warehouse does not store more pallets than it is able to store.

3.6.3.4. Trucking Constraints

The Truck Requirement Constraints ensure efficient allocation of trucking resources across the supply chain. For suppliers, the constraints ensure that the total amount of products transported from a supplier to a destination within a given period does not exceed the available truck capacity, thus determining the number of trucks needed. For warehouses, the constraints ensure that the total products shipped from a warehouse to a destination in a specific period fit within the truck capacity, ensuring the correct number of trucks are used. Similarly, for production warehouses, the constraints ensure that the products moving from a production warehouse to a destination in a given period do not exceed the truck capacity, calculating the necessary trucks for each route. These constraints collectively optimize the use of trucks across suppliers, warehouses, and production facilities.

Production Warehouse Truck Reception Constraints

$$\sum_{k=1}^{u} Y''_{jkt} + \sum_{k=1}^{u} Y'_{ikt} \leq N_{kt} \qquad \in \quad \forall\, j, t$$

(9)

In formulating a transportation model for product shipment, we account for the constraints introduced by the use of trucks. Each truck is assumed to possess a uniform and consistent pallet capacity, taking into account the product category, which represents a constraint type within the model. Temporally, the model operates across discrete periods denoted by *t*, where the truck reception at the production warehouse across arcs is consistently available along all *t*. Notably, the model does not utilize a truck-specific index, reflecting a deliberate abstraction that prioritizes focusing on broader truck fleet characteristics over individual vehicle details. In other words, all trucks are assumed to have the same capacity. Capacity is a given parameter with an index of u, where it is assumed that each vehicle can carry the same amount of pallets of a category type across any given arc.

Furthermore, the model is not concerned with truck loading or packing mechanisms, opting for an aggregate representation of capacity and availability attributes. These choices ensure that the model remains within the scope of the project. While there is no discrete limitation at the production warehouse, there is a general limitation on how many trucks can be received by said production warehouse in this problem based on the Number of Trucks available for reception across the sum of said arcs over a given period t. This constraint acknowledges that the number of trucks required is invariably an integer for practical considerations. A breach of this constraint, wherein the number of trucks received at a particular k in a single day surpasses the product the number of trucks allowed by reception, would signify a lack of reception capacity in that period. This constraint forms the foundational basis for more specific constraints associated with the various arcs of the transport model.

Truck Requirement Constraints

Suppliers:

$$\sum_{u=1}^{v} \frac{\sum_{p=1}^{q} X_{puijt}}{Cap_u} \leq Y_{ijt} \in \ \forall\, i, j, t$$

(10a)

$$\sum_{u=1}^{v} \frac{\sum_{p=1}^{q} X'_{puikt}}{Cap_u} \leq Y'_{ikt} \qquad \in \ \forall\, i, k, t$$

(10b)

The Supplier Trucks Needed Constraints (10) asserts that the sum of a product p in category u going from supplier i across a specific arc (j or k) in a period t is less than or equal to the number of trucks needed for a specific arc starting from supplier i going to a destination (j or k) in period *t*.

Warehouses:

$$\sum_{u=1}^{v} \frac{\sum_{p=1}^{q} X''_{pujkt}}{Cap_u} \leq Y''_{jkt} \qquad \in \ \forall\, j, k, t$$

(11)

The Warehouse Trucks Needed Constraints (11) asserts that the sum of a products p in category u going from warehouse j across a specific arc (k) in a period t is less than or equal to the number of trucks needed for a specific arc starting from warehouse j going to a destination (k) in period t.

Production Warehouses:

$$\sum_{u=1}^{v} \frac{\sum_{p=1}^{q} X'''_{pukjt}}{Cap_u} \leq Y'''_{kjt} \qquad \in \ \forall\, k, j, t$$

(12)

The Production Trucks Needed Constraints (12) assert that the sum of products p in category u going from production warehouser k across a specific arc (j) in a period t is less than or equal to the number of trucks needed for a specific arc starting from production warehouse k and going to a destination (j) in period t.

Chapter 4: Results & Analysis

In our results and analysis chapter we first introduce our baseline model. From there we presented three sets of experiments based on parameters to understand how changes within the model would affect total transportation costs. First, we tested the production warehouse capacity, looking at how increasing or decreasing potential reception would impact saturation, transport, and storage costs. We then looked at how fluctuations in supply and demand may affect the model's outcome. Finally, we set limitations on truck reception at the production warehouse to see how changing reception capacity may have an impact on the flow of goods and costs.

4.1. Baseline

We ran our baseline model with the parameters presented in our model formulation. Looking at our summary of the baseline model, we see that our cost breakdown, not include shortages; we see that trucking accounts for 31% of the total cost, pallet movement costs represent 27% of the total, and storage costs represent 39% of the total costs. While the shortage costs do represent a potential cost, we chose to exclude it from our baseline figure, as it acts as a visibility factor to indicate what should be changed within the production schedule rather than the actual costs within the model.

Table 15 Summary of Baseline Model

Trucking Costs	Pallet Movement Costs	Storage Costs	Cost without Shortage
$20,050	$17,919	$25,259	$65,279

In looking at saturation levels across storage locations, we see that the model initially pushes goods to the Horace plant, before prioritizing XTL, La Chine, VSL, and final BDL. We will discuss this more thoroughly during our capacity tests. Out of the total 132 trucks that are used in the baseline model, the truck movement from supplier directly to Horace represented 28% of the truck movements, the most of any route, followed by supplier to BDL and Horace to XTL , 18.2% and 12.9% respectively.

Figure 4 Average % Saturation of Baseline Results

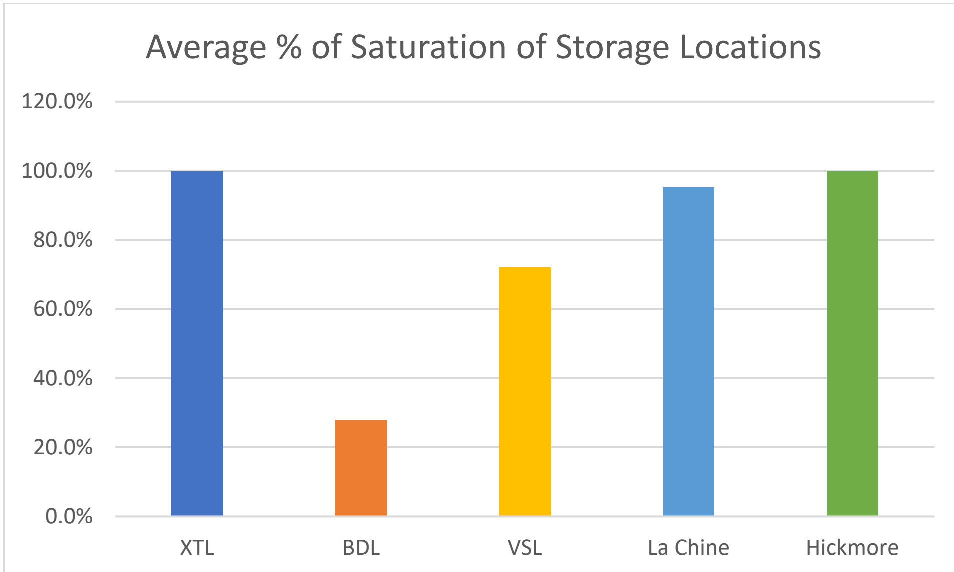


Table 16 Baseline Summary of Truck & Pallet Movement

From	To	Total Truck movement	Total Pallet movement	% of Truck Movement	% of Pallet Movement
Supplier	**Horace**	37	897	28.0%	21.7%
Supplier	**XTL**	8	136	6.1%	3.3%
Supplier	**BDL**	24	563	18.2%	13.6%
Supplier	**VSL**	10	482	7.6%	11.6%
Supplier	**La Chine**	11	440	8.3%	10.6%
XTL	**Horace**	3	19	2.3%	0.5%
BDL	**Horace**	2	14	1.5%	0.3%
VSL	**Horace**	6	151	4.5%	3.6%
La Chine	**Horace**	2	12	1.5%	0.3%
Horace	**XTL**	12	577	9.1%	13.9%
Horace	**BDL**	0	0	0.0%	0.0%
Horace	**VSL**	0	0	0.0%	0.0%
Horace	**La Chine**	17	851	12.9%	20.5%

4.2. Impact of Changing Production Warehouse Capacity:

In our first experiment, we investigated the impact of varying saturation levels of production warehouse capacity on the overall costs of the supply chain model. The baseline model assumes full utilization of the production warehouse capacity, while our analysis considered a preferred saturation level of 85%, as recommended by the flows team. To explore the potential costs associated with underutilizing production warehouse storage and the effects of expanding production warehouse capacities, we adjusted our H_{uk} value using a range of alpha values (0.8, 0.85, 0.9, 0.95, 1.05, 1.1, 1.15, and 1.2). This allowed us to compare the cost and saturation levels of external warehousing relative to the original production warehouse capacity. Our findings indicate that reducing the production warehouse capacity increases overall model costs, primarily due to the backflow of trucks to external warehouses. Despite the changes in capacity, the number of pallet shortages remained constant across all experiments. However, we observed a notable increase in ending inventories at the production warehouse and a corresponding decrease in saturation levels at external warehouses as production warehouse capacity increased. Table: Saturation of Warehouse with Change shows that 72% of the external warehouse capacity is utilized at the baseline, whereas when an additional 20% of production warehouse capacity is added, that number is reduced to 65% saturation of external warehouse space. This suggests that for short-term production planning, it is advantageous for suppliers to deliver raw materials or components directly to the production warehouse rather than external warehouses.

Table 17 Production Warehouse Saturation and Capacity

Capacity of Production WH	Storage Location Saturation			
	Total Ending Warehouse Inventory	**Total Ending Inventory**	**Average Saturation of Warehouses**	**Average Saturation of Production Warehouse**
80%	64179	23990	80%	80%
85%	62679	25500	78%	85%
90%	61179	26990	76%	90%
95%	59679	28500	74%	95%
Baseline	58179	30000	**72%**	**100%**
105%	56684	31496	70%	105%
110%	55199	33000	68%	110%
115%	53702	34495	67%	115%
120%	52209	35995	65%	120%

The analysis highlights the importance of optimizing flow management in the production warehouse to minimize last-minute truck movements between production warehouses and external warehouses. The current practice of moving 2-5 trucks last minute indicates inefficiencies and a potential misalignment in flow scheduling. Significant cost savings can be achieved by ensuring that products are shipped directly to the production warehouse, especially for known products in the production schedule. While the flows team prefers to maintain production warehouse saturation at around 85%, our results demonstrate a financial incentive to keep saturation levels at least 90% (Table Cost Comparison Overview). There is a substantial cost increase, approximately 15%, when moving from 90% capacity to 85% capacity compared to the baseline model. This underscores the importance of maintaining higher saturation levels to optimize costs and improve overall supply chain efficiency.

Table 18 Cost Comparison Overview for Production Warehouse Capacity

Capacity of Production WH	Cost without Shortage	Cost Savings	Cost Compared to Baseline Model
80%	$79,707	$(14,427.28)	122%
85%	$ 81,151	$(15,871.28)	124%
90%	$72,107	$(6,827.28)	110%
95%	$68,497	$(3,217.44)	105%
Baseline	$65,279	-	100%
105%	$63,638	$1,641.48	97%
110%	$60,764	$4,515.18	93%
115%	$57,074	$8,205.64	87%
120%	$53,825	$11,454.32	82%

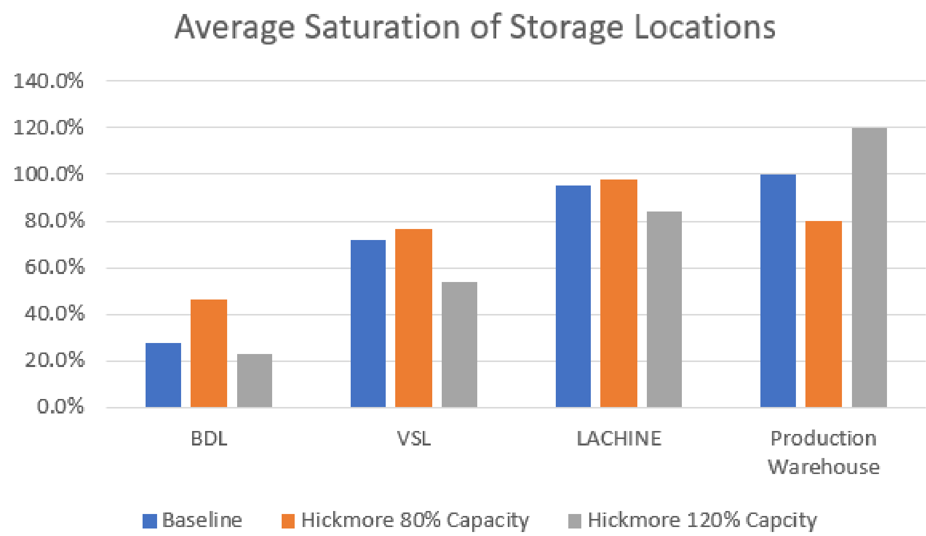
When production warehouse capacity is modified, the fluctuation in cost can be accounted for via the trucks returning from the production warehouse to the external warehouse, the flow of goods, and the ending inventory at warehouse costs. When warehouse capacity drops to 80%, we see a significant increase in trucks and pallets moving from production warehouses to warehouses as the model makes additional space for the requisite pallets to meet demand (Table Warehouse Capacity Comparison).

Table 19 Warehouse Capacity Comparison of Truck & Pallet Movement

Capacity of Production WH	# of Truck along Routes				# of Pallets along Routes			
	Supplier to Warehouse	Supplier to Production Warehouse	Warehouse to Production Warehouse	Production Warehouse to Warehouses	Supplier to Warehouse	Supplier to Production Warehouse	Warehouse to Production Warehouse	Production Warehouse to Warehouses
80%	56	38	15	39	1628	890	196	2021
85%	57	38	15	38	1611	907	218	1910
90%	56	38	15	34	1628	890	196	1721
95%	56	38	15	31	1628	890	196	1571
Baseline	**53**	**37**	**13**	**29**	**1621**	**897**	**196**	**1428**
105%	55	39	15	28	1627	891	202	1277
110%	52	43	15	27	1525	993	202	1230
115%	52	43	14	24	1512	1006	202	1093
120%	51	42	15	22	1512	1006	202	943

Looking further into how costs fluctuate, we can start with storage at external warehouses. Below, we see the saturation levels of the various warehouses over time when the production warehouse capacity is modified. XTL had no significant change, as it was filled at 100% capacity throughout these tests, so it is not included in the graphics. This signifies that XTL is a preferred partner, followed by La Chine, which reaches capacity the second most quickly. In contrast, VSL reaches 100% capacity on day 8 of our baseline model but does not fill above 60% if our production warehouse capacity is 120%. BDL is used the least, a testament to high variable storage costs.

Figure 5 Average Saturation of Storage Locations



When we look at the change of costs, whereas the increase of production capacity to 120% leads to a savings of $11,454 and a production warehouse capacity of 80% leads to an additional $14,427 in costs, the change of cost is primarily from trucks and pallets flowing from the production warehouse to external warehouses, and the warehouse storage costs at the external warehouses (Table 20: Change of Costs).

Table 20 Change of Costs Comparing Horace Saturation Level

	Truck Costs				**Pallet Movement Costs**				**Storage Cost**	
Model	**Supplier to Warehouses**	**Supplier to Production Warehouse**	**Warehouses to Production Warehouse**	**Production Warehouse to Warehouses**	**Supplier to Warehouses**	**Supplier to Production Warehouse**	**Warehouses to Production Warehouse**	**Production Warehouse to Warehouses**	**Ending Inventory at Warehouses**	**Total Cost**
Baseline	$7,950	$3,700	$2,600	**$5,800**	**$8,126**	$1,794	$2,199	$7,851	**$25,259**	$65,279
80%	$450	$100	$400	**$2,000**	**$(1,109)**	$(14)	$0	$2,295	**$10,305**	$14,427
120%	$(300)	$500	$400	**$(1,400)**	**$(1,811)**	$218	$12	$(3,805)	**$(5,268)**	$(11,454)

Regarding production warehouse reception, we tested to see if a limited number of trucks would change daily warehousing and trucking costs compared to accounting for an unlimited number of truck reception. While not a hard limit, current production warehouse practices suggest the number of trucks allowed to be received in a single period to 12. Though we had hypothesized that limiting the number of trucks received in a period would adversely affect meeting demand, we found no significant changes in shortages or significant cost savings by increasing or decreasing the truck reception limit. However, we found a change in when the production warehouse received pallets; as the number of trucks allowable to be received in each period decreased, outbound trucks also decreased during the initial period while increasing later. This signifies that while goods received at the production warehouse were utilized to meet demand early on, pallets outbound from the production warehouse were mainly utilized to clear space for pallets later in the production period.

4.3. Impact of Changing Demand

We then covered sensitivity analysis, running the baseline model without constraints on reception while changing demand. This was conducted with dynamic demand levels at 85%, 90%, 95%, 103%, 110%, and 120% original demand levels. We highlighted the difference at 85% and 103% due to their variances from the objective function. Based on the hour limit of our model, we saw an increase in cost, even at the inflection point after a 103% increase in demand. The cost is mainly derived from the expected increase in shortages in the model. Comparing the 90% demand and other alphas less than 100%, we see a dramatic decrease in shortage and an overall decrease in cost associated with the reduction of shortages. This tells us that an increase in demand in the production plan has a significant cost associated with our model, mainly because of the cost of shortages involved in the increase. In contrast, decreased demand, based on the parameters of the model, does induce increased cost savings, as additional supply can house prior to its need for demand. This indicates that the tight just in time function of the production schedule and the consistent reshuffling of production priorities may lead to increased costs, particularly as the large majority of shortages could be prevented by changing the production schedule.

Table 21 Demand Comparison of Shortages

Demand Alpha	Shortage	Obj Function	Cost without Shortage	Difference due to Shortage	Objective Function Compared to baseline
90%	434	$283,118	$65,279	$(44,500)	83%
Baseline	523	$326,779	$66,118	N/A	100.0%
103%	617	$373,425	$65,120	$47,000.00	118%

4.4. Impact of Changing Supply

Looking at Table 22 for Supply Tests, we again compare total costs, along with how costs are distributed across the model. When Supply increases, we see a significant increase in the number of trucks and pallets that move from Supplier to Warehouse. In contrast, our other trucks along various arcs remain relatively static. Surprisingly, the decrease of incoming pallets to 80% of the original supply does not cause a significant fluctuation in overall costs compared to the baseline model, with a 1.2% cost savings. Similarly, while there is an increase in costs when incoming pallets increase, the cost is not proportional to the increase of pallets, whereas even at a 120% increase in pallets, we only see a 13% increase in overall costs, the majority of which stems from increased truck costs from suppliers to warehouses which accounts for 31% of the cost increase (Table 22 Change of Costs).

Table 22 Change of Costs with Varying Supply Compared to Baseline

Supply Alpha	Trucks from Supplier to Warehouses	Trucks from Supplier to Production Warehouse	Trucks from Warehouses to Production Warehouse	Trucks from Production Warehouse to Warehouses	Ending Inventory at Warehouses	Cost Without Shortage
80%	98%	103%	85%	97%	99%	99%
90%	102%	100%	115%	103%	100%	101%
105%	111%	97%	131%	97%	107%	105%
120%	128%	95%	131%	97%	120%	112%

Trucks moving from Supplier to Warehouse signify the significant fluctuation when incoming pallets are increased, as additional pallets are stored at external warehouses. We see that while trucks going from supplier to production warehouses and from warehouses to production warehouses increase slightly, there is a more significant volume of trucks, which increases by 15 trucks over the period. Although the warehouse-to-production warehouses increase by 131% compared to the original cost and volume, this only signifies a 4-truck increase along said route (Table 23: Truck Usage with Incoming).

Table 23 Truck Usage with Incoming Pallets Change

Alpha	# of Truck along Routes			
	Supplier to Warehouse	Supplier to Production Warehouse	Warehouse to Production Warehouse	Production Warehouse to Warehouses
80%	**52**	38	11	28
90%	**54**	37	15	30
Baseline	**53**	37	13	29
105%	**59**	36	17	28
120%	**68**	35	17	28

The fluctuations of cost when accounting for change of supply occur within two significant costs: trucking costs, precisely the number of trucks from suppliers to warehouses, and the storage costs of variable storage costs for ending inventory at warehouses (Table 24: Cost with Fluctuation). We can deduce from this that with an increase in supply, additional trucks are routed directly to warehouses. In contrast, only the demand initially needed from suppliers over a shorter time is shipped directly to the production warehouse. Conversely, when the supply fluctuates, we also see a large percentage of changes in trucks from warehouses to production warehouses, mostly from BDL to the production warehouse. This illustrates the positive nature of flexible warehousing, a short-term holding place for pallets in demand before moving to the production warehouse.

Table 24 Costs with Fluctuation of Incoming Supply

	Truck Costs				Pallet Movement Costs				Storage Cost	
Model	Supplier to Warehouses	Supplier to Production arehouse	Warehouses to Production Warehouse	Production Warehouse to Warehouses	Supplier to Warehouses	Supplier to Production Warehouse	Warehouses to Production Warehouse	Production Warehouse to Warehouses	Ending Inventory at Warehouses	Total Cost
Baseline	**$7,950**	$3,700	$2,600	$5,800	$8,126	$1,794	$2,199	$7,851	**$25,259**	$65,279
80%	**$150**	$(100)	$400	$200	$194	$(52)	$0	$(246)	**$309**	$855
90%	**$(150)**	$0	$(400)	$(200)	$0	$0	$0	$0	**$113**	$(637)
105%	**$(900)**	$100	$(800)	$200	$65	$16	$(4)	$(53)	**$(1,649)**	$(3,025)
120%	**$(2,250)**	$200	$(800)	$200	$52	$58	$(40)	$30	**$(5,091)**	$(7,641)

4.5. Results Summary

The study analyzes the effects of varying production warehouse capacities, demand fluctuations, and supply levels on the total variable transportation costs within Horace's production distribution framework. Our findings reveal that maintaining a production warehouse saturation of at least 90% is crucial for cost optimization, as reducing capacity below this threshold significantly increases overall costs due to the increased reliance on external warehousing and the resulting backflow of trucks. The data indicates that reducing production warehouse capacity leads to a 15% rise in costs from 90% to 85% capacity, primarily due to additional storage and transport expenditures. Specifically, maintaining a production warehouse capacity at 85% increases costs by approximately $15,871 compared to the baseline. The inefficiencies observed in current practices, such as the need for last-minute truck movements between production and external warehouses, underscore the necessity for improved flow management. Direct deliveries to production warehouses, particularly for known products in the production schedule, could yield substantial cost savings and enhance operational efficiency by minimizing unnecessary truck movements and storage costs.

The sensitivity analysis on demand variations highlights the critical cost implications of just-in-time inventory practices and dynamic production scheduling. Increased demand beyond 103% notably raises costs due to the surge in shortages, resulting in a cost increase of approximately $47,000 and conversely, reducing demand levels below the baseline results in significant cost savings, primarily through the reduction of shortages and better utilization of available supply, with a decrease of 90% in demand, reducing costs by $44,500. The analysis of supply variations further emphasizes that increasing supply levels predominantly affects trucking and storage costs, with a marked increase in trucks and pallets moving from suppliers to warehouses. For instance, increasing supply to 120% of the baseline level raises overall costs by 13%, primarily due to the 31% increase in truck costs from suppliers to warehouses. These findings underscore the importance of efficient supply routing and strategic inventory management to mitigate additional costs. Implementing these strategies can improve supply chain efficiency, significantly reduce costs, and optimize production logistics operations for Cosmetics Company.

Chapter 5: Conclusions

5.1. Managerial Insights

The findings from this study highlight significant cost savings in variable transportation and warehousing costs at the production warehouse level. This optimization tool enables Cosmetics Company to pinpoint short-term cost savings and make rapid decisions regarding optimized pallet movements and the required number of trucks down to the UPC level. Unlike most optimization models that emphasize high-level cost savings and optimization, this model demonstrates the practical use of granular results, potentially reducing external warehousing costs while maintaining production levels across all lines.

Our analysis tested the impact of varying the number of trucks received per day on cost savings. It revealed that while the scheduling of pallets, particularly outbound from the production warehouse, affects cost savings and the storage of pallets at external warehousing partners, the optimization model effectively limited incoming trucks without showing significant cost savings. This finding underscores the importance of pallet scheduling in optimizing overall costs.

Considering fluctuations in supply and demand, the study indicates that during increased demand, Cosmetics Company Horace should consider enhancing the saturation level above its preferred 85% capacity of the production warehouse to maximize cost savings. Furthermore, the flow of pallets into the warehousing system favored storage at specific locations (XTL, La Chine, VSL, and BDL), suggesting the potential realignment of contracts for short-term warehouse storage savings.

This study maintained static variable warehousing and trucking costs over time. Future research should investigate fluctuating variable costs to develop a more dynamic model, offering more profound insights into prioritizing warehousing on a short-term basis. Moreover, exploring meta-heuristics and genetic algorithms, which have proven effective in Inventory Routing Problems (IRP) and Pickup and Delivery Problems (PDP), could enhance this model. Incorporating additional periods would provide greater visibility and help further reduce variable transportation and warehousing costs.

5.2. Limitations & Future Research

Regarding limitations, while our model does increase visibility and provides insights on the prioritization of warehousing locations based on short term variable logistics costs, the study does not take into account fixed costs in external warehousing contracts, which could provide more insights into inflections points of fixed warehousing costs. Additionally, while there is potential to expand the Horace warehousing facilities deeper MILP cost analysis could allow us to more deeply understand what the cost benefits would be of warehouse expansion over longer horizons.

The model does not use advanced technologies, such as API or blockchain to track specific pallets, and is limited by discrete supply and demand values, which does not take into account fluctuations of risk within the system. Whereas our tool is intended to be utilized using discrete parameters, the lack of risk analysis in the model limits our thesis to these parameters. While we do have more visibility of incoming trucks over the 10 day period, we do have forecasting, expected delivery dates over extended periods, and production forecasting. We purposefully chose more predicable periods for our model. Adding risk components and uncertainty to our model could provide more cost savings benefits.

Future research should also consider including additional variable types not used in this study due to their unique transportation and storage requirements, especially concerning specialized trucks. This could provide a more comprehensive understanding of the logistics involved. Additionally, many papers we looked at provided bi-objective MILP models that looked at sustainability efforts within IRP and PDP models. If Cosmetics Company were to want to look more into sustainability within the model framework, creating a multi-objective optimization problem would be relevant to this study.

The study confirms the viability of using optimization for daily decision-making. A granular model using MILP techniques can provide significant insights if paired with a robust decision-making dashboard. The following steps in integrating this model into daily or weekly operations involve converting the model into a Python-based application and developing a Power Query to aggregate various datasets. Post-model execution, the results should be displayed in a data visualization dashboard to facilitate informed decision-making. These dashboard features could include remaining spaces within trucks, upcoming shortages, flows from storage locations, and cost savings. Though the tool gives an overview of how costs are reduced, it is greater power is to give a specific understanding of how to make specific detailed decisions, which the suggested additions to the dashboard would assist in organizing the data in a more instantaneously helpful way.

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Appendix:

CPLEX .dat Code

```
// Define the parameters
int P = ...; // Number of products
int U = ...; // Product Category
int I = ...; // Number of suppliers
int J = ...; // Number of warehouses
int K = ...; // Number of production warehouses
int T = ...; // Number of time periods



range Products = 1..P;
range Category = 1..U;
range Suppliers = 1..I;
range Warehouses = 1..J;
range ProductionWarehouses = 1..K;
range Time = 1..T;

//int C_uij[Suppliers][Warehouses] = ...; // This will hold cost values directly

// Set parameter values

//Starting Inventories
tuple s_puit
{
  int p;
  int u;
  int i;
  int t;
  int S_puit;
}
{s_puit} S_puit =...;

tuple sa_pujt
{
  int p;
  int u;
  int j;
  int t;
  int SA_pujt;
}
{sa_pujt} SA_pujt =...;
```

```
tuple sb_pukt
{
 int p;
 int u;
 int k;
 int t;
 int SB_pukt;
}
{sb_pukt} SB_pukt =...;

// Storage CAP_uacity

tuple h_ui
{
 int u;
 int i;
 int H_ui;
}
{h_ui} H_ui =...; // The maximum number of pallets stored at supplier i

tuple ha_uj
{
 int u;
 int j;
 int HA_uj;
}
{ha_uj} HA_uj=...; // The maximum number of pallets stored at warehouse j

tuple hb_uk
{
 int u;
 int k;
 int HB_uk;
}
{hb_uk} HB_uk=...; // The maximum number of pallets stored at production warehouse k

// Demand Constraint

tuple d_pukt
{
 int p;
 int u;
 int k;
 int t;
 float D_pukt;
```

```
}
{d_pukt} D_pukt=...;

// Storage Cost

tuple w_uj
{
 int u;
 int j;
 float W_uj;
}
{w_uj} W_uj=...; // Cost of storing one pallet of product p at warehouse j

tuple wa_uk
{
 int u;
 int k;
 float WA_uk;
}
{wa_uk} WA_uk=...; // Cost of storing one pallet of product p at warehouse j


// Transport Cost

tuple c_uij
{
 int u;
 int i;
 int j;
 float C_uij;
}
{c_uij} C_uij=...; // Cost of shipping one pallet from supplier i to warehouse j


tuple ca_uik
{
 int u;
 int i;
 int k;
 float CA_uik;
}
{ca_uik} CA_uik=...; // Cost of shipping one pallet from supplier i to production warehouse k

tuple cb_ujk
{
 int u;
```

```
  int j;
  int k;
  float CB_ujk;
}
{cb_ujk} CB_ujk=...; // Cost of shipping one pallet from supplier j to customer k

tuple cc_ukj
{
  int u;
  int k;
  int j;
  float CC_ukj;
}
{cc_ukj} CC_ukj=...; // Cost of shipping one pallet from production warehouse k to warehouse j


// Trucking Cost

tuple tc_ij
{
  int i;
  int j;
  float TC_ij;
}
{tc_ij} TC_ij=...; // Cost of a single truck going from supplier i to warehouse j

tuple tca_ik
{
  int i;
  int k;
  float TCA_ik;
}
{tca_ik} TCA_ik=...; // Cost of a single truck going from supplier i to warehouse k

tuple tcb_jk
{
  int j;
  int k;
  float TCB_jk;
}
{tcb_jk} TCB_jk=...; // Cost of shipping one pallet from supplier j to customer k

tuple tcc_kj
{
  int k;
  int j;
```

```
  float TCC_kj;
}
{tcc_kj} TCC_kj=...; // Cost of shipping one pallet from production warehouse k to warehouse j


// Truck Availability
tuple n_i
{
  int i;
  int N_i;
}
{n_i} N_i=...; // Number of Trucks available to go from supplier i

tuple na_j
{
  int j;
  int NA_j;
}
{na_j} NA_j=...; // Number of Trucks available to go from supplier j

tuple nb_k
{
  int k;
  int NB_k;
}
{nb_k} NB_k=...; // Number of Trucks available to go from production warehouse k

// Truck Capacity
tuple cap_u
{
  int u;
  int CAP_u;
}
{cap_u} CAP_u=...;// Number of Pallets each truck can carry

// Define the maximum bounds for decision variables
int MaxTrucks = 20; // Maximum number of trucks
int MaxInventory = 3000; // Maximum inventory capacity
int MaxPallets = 1120; // Maximum number of pallets that can be moved
int MaxShortage = 12; // Maximum allowable shortage

// Define decision variables with bounds
dvar int+ Y_ijt[Suppliers][Warehouses][Time] in 0..MaxTrucks;
dvar int+ YA_ikt[Suppliers][ProductionWarehouses][Time] in 0..MaxTrucks;
dvar int+ YB_jkt[Warehouses][ProductionWarehouses][Time] in 0..MaxTrucks;
dvar int+ YC_kjt[ProductionWarehouses][Warehouses][Time] in 0..MaxTrucks;
```

```
// Ending Inventories with bounds
dvar int+ E_puit[Products][Category][Suppliers][Time] in 0..MaxInventory;
dvar int+ EA_pujt[Products][Category][Warehouses][Time] in 0..MaxInventory;
dvar int+ EB_pukt[Products][Category][ProductionWarehouses][Time] in 0..MaxInventory;

// Pallet Flows with bounds
dvar int+ X_puijt[Products][Category][Suppliers][Warehouses][Time] in 0..MaxPallets;
dvar int+ XA_puikt[Products][Category][Suppliers][ProductionWarehouses][Time] in
0..MaxPallets;
dvar int+ XB_pujkt[Products][Category][Warehouses][ProductionWarehouses][Time] in
0..MaxPallets;
dvar int+ XC_pukjt[Products][Category][ProductionWarehouses][Warehouses][Time] in
0..MaxPallets;

// Shortage with bounds
dvar int+ L_pukt[Products][Category][ProductionWarehouses][Time] in 0..MaxShortage;

//Objective function

minimize
  // Trucking costs between suppliers and warehouses
  sum(i in Suppliers, j in Warehouses, t in Time)
    (sum(tc_ij in TC_ij: tc_ij.i == i && tc_ij.j == j) tc_ij.TC_ij) * Y_ijt[i][j][t]

  // Trucking costs between suppliers and production warehouses
+ sum(i in Suppliers, k in ProductionWarehouses, t in Time)
    (sum(tca_ik in TCA_ik: tca_ik.i == i && tca_ik.k == k) tca_ik.TCA_ik) * YA_ikt[i][k][t]

  // Trucking costs between warehouses and production warehouses
+ sum(j in Warehouses, k in ProductionWarehouses, t in Time)
    (sum(tcb_jk in TCB_jk: tcb_jk.j == j && tcb_jk.k == k) tcb_jk.TCB_jk) * YB_jkt[j][k][t]

  // Trucking costs between production warehouses and warehouses
+ sum(k in ProductionWarehouses, j in Warehouses, t in Time)
    (sum(tcc_kj in TCC_kj: tcc_kj.k == k && tcc_kj.j == j) tcc_kj.TCC_kj) * YC_kjt[k][j][t]

  // Storage costs at warehouses
+ sum(p in Products, u in Category, j in Warehouses, t in Time)
    (sum(wu_j in W_uj: wu_j.u == u && wu_j.j == j) wu_j.W_uj) * EA_pujt[p][u][j][t]

  // Storage costs at production warehouses
+ sum(p in Products, u in Category, k in ProductionWarehouses, t in Time)
    (sum(wa_uk in WA_uk: wa_uk.u == u && wa_uk.k == k) wa_uk.WA_uk) *
EB_pukt[p][u][k][t]
```

```
  // Transport costs between suppliers and production warehouses
+ sum(p in Products, u in Category, i in Suppliers, k in ProductionWarehouses, t in Time)
    (sum(ca_uik in CA_uik: ca_uik.u == u && ca_uik.i == i && ca_uik.k == k) ca_uik.CA_uik) *
XA_puikt[p][u][i][k][t]

  // Transport costs between warehouses and production warehouses
+ sum(p in Products, u in Category, j in Warehouses, k in ProductionWarehouses, t in Time)
    (sum(cb_ujk in CB_ujk: cb_ujk.u == u && cb_ujk.j == j && cb_ujk.k == k) cb_ujk.CB_ujk) *
XB_pujkt[p][u][j][k][t]

  // Transport costs between production warehouses and warehouses
+ sum(p in Products, u in Category, k in ProductionWarehouses, j in Warehouses, t in Time)
    (sum(cc_ukj in CC_ukj: cc_ukj.u == u && cc_ukj.k == k && cc_ukj.j == j) cc_ukj.CC_ukj) *
XC_pukjt[p][u][k][j][t]

  // Penalty for shortages at production warehouses
+ 500 * sum(p in Products, u in Category, k in ProductionWarehouses, t in Time)
L_pukt[p][u][k][t];



// Define the constraints
subject to

{
//FLOW CONSTRAINTS

// Suppliers Flow Constraint From ChatGPT
forall (p in Products, u in Category, i in Suppliers, t in Time) {
    sum(s_puit in S_puit: s_puit.p == p && s_puit.u == u && s_puit.i == i && s_puit.t == t)
    s_puit.S_puit
    - sum(j in Warehouses) X_puijt[p, u, i, j, t]
    - sum(k in ProductionWarehouses) XA_puikt[p, u, i, k, t]
    == 0;
}

// Warehouses Flow Constraint
WarehousesFlowConstraint:
forall (p in Products, u in Category, j in Warehouses, t in 1..1){
      (sum(sa_pujt in SA_pujt: sa_pujt.p == p && sa_pujt.u == u && sa_pujt.j == j && sa_pujt.t
== 1) sa_pujt.SA_pujt)
       + (sum(i in Suppliers) X_puijt[p, u, i, j, t])
       - (sum(k in ProductionWarehouses) XB_pujkt[p, u, j, k, t])
       + (sum(k in ProductionWarehouses) XC_pukjt[p, u, k, j, t])
       - EA_pujt[p, u, j, t] == 0;
}
```

```
// Warehouses Flow Constraint for t = 2..T
WarehousesFlowConstraint2:
forall (p in Products, u in Category, j in Warehouses, t in 2..T){
    EA_pujt[p, u, j, (t-1)]
    +(sum(i in Suppliers) X_puijt[p, u, i, j, t])
    - sum(k in ProductionWarehouses) XB_pujkt[p, u, j, k, t]
    + sum(k in ProductionWarehouses) XC_pukjt[p, u, k, j, t]
            - EA_pujt[p, u, j, t] == 0;
}

// Production Warehouses Flow Constraint
ProductionWarehousesFlowConstraint:
forall (p in Products, u in Category, k in ProductionWarehouses,t in 1..1){
    (sum(sb_pukt in SB_pukt: sb_pukt.p==p && sb_pukt.u == u && sb_pukt.k == k && sb_pukt.t
== 1) sb_pukt.SB_pukt)
   +  sum (i in Suppliers) XA_puikt[p, u, i, k, t]
   +  sum (j in Warehouses) XB_pujkt[p, u, j, k, t]
   -  sum (j in Warehouses) XC_pukjt[p, u, k, j, t]
   -  sum(d_pukt in D_pukt: d_pukt.p==p && d_pukt.u == u && d_pukt.k==k && d_pukt.t == t)
d_pukt.D_pukt
   +  L_pukt [p,u,k,t]
   - EB_pukt[p, u, k, t]  == 0;
}

// Production Warehouses Flow Constraint for t = 2..T
ProductionWarehousesFlowConstraint2:
forall (p in Products, u in Category, k in ProductionWarehouses, t in 2..T) {
    EB_pukt[p, u, k, (t-1)]
   + (sum(i in Suppliers) XA_puikt[p, u, i, k, t])
   + (sum(j in Warehouses) XB_pujkt[p, u, j, k, t])
   - (sum(j in Warehouses) XC_pukjt[p, u, k, j, t])
   - (sum(d_pukt in D_pukt:d_pukt.p==p && d_pukt.u == u && d_pukt.k == k && d_pukt.t == t)
d_pukt.D_pukt)
   +  L_pukt [p,u,k,t]
   - EB_pukt[p, u, k, t] == 0;
 }

// SUPPLY CONSTRAINTS

// Supplier Constraint
//SupplierSupplyConstraint:
//forall (p in Products, u in Category, i in Suppliers, t in Time)
    //sum (j in Warehouses) X_puijt[p, u, i, j, t] + sum (k in ProductionWarehouses) XA_puikt[p,
u, i, k, t] == sum(s_puit in S_puit: s_puit.p==p && s_puit.i == i && s_puit.t == t) s_puit.S_puit;

// Warehouse Supply Constraint for t = 1
```

```
WarehouseSupplyConstraint1:
forall (p in Products, u in Category, j in Warehouses, t in 1..1)
   sum (k in ProductionWarehouses) XB_pujkt[p, u, j, k, 1] -
   sum(sa_pujt in SA_pujt: sa_pujt.p==p && sa_pujt.u==u && sa_pujt.j==j && sa_pujt.t == 1)
sa_pujt.SA_pujt <= 0;

// Warehouse Supply Constraint for t = 2..T
WarehouseSupplyConstraint2:
forall (p in Products, u in Category, j in Warehouses, t in 2..T)
   sum (k in ProductionWarehouses) XB_pujkt[p, u, j, k, t] - EA_pujt[p,u,j,(t-1)] <= 0;

// Production Warehouse Supply Constraint for t = 1
ProductionWarehouseSupplyConstraint1:
forall (p in Products, u in Category, k in ProductionWarehouses, t in 1..1)
   sum (j in Warehouses) XC_pukjt[p, u, k, j, 1] -
   sum(sb_pukt in SB_pukt:sb_pukt.p==p && sb_pukt.k == k && sb_pukt.t == 1)
sb_pukt.SB_pukt <= 0;

// Production Warehouse Supply Constraint for t = 2..T
ProductionWarehouseSupplyConstraint2:
forall (p in Products, u in Category, k in ProductionWarehouses, t in 2..T)
   sum (j in Warehouses) XC_pukjt[p, u, k, j, t] -
   EB_pukt[p,u,k,(t-1)] <= 0;

// STORAGE CAPACITY CONSTRAINTS

 //Warehouse Storage Capacity Constraint for t = 1
WarehouseStorageCapacityConstraint1:
forall (j in Warehouses, u in Category, t in Time)
sum (p in Products) EA_pujt [p,u,j,t] <= sum(ha_uj in HA_uj:ha_uj.u==u && ha_uj.j==j)
ha_uj.HA_uj;

// Production Warehouse Storage Capacity Constraint
ProductionWarehouseStorageCAP_uacityConstraint1:
forall (k in ProductionWarehouses, u in Category, t in Time)
sum (p in Products, u in Category )  EB_pukt [p,u,k,t] <= sum(hb_uk in HB_uk:hb_uk.u==u &&
hb_uk.k==k) hb_uk.HB_uk;


//Trucking Usage Constraint

SuppliertoWarehouseTruckUsageConstraint:
forall (i in Suppliers, j in Warehouses, t in Time)
   Y_ijt[i, j, t] -
   (sum(p in Products) X_puijt[p, 1, i, j, t] / sum(cap_u in CAP_u: cap_u.u == 1) cap_u.CAP_u) -
```

```
   (sum(p in Products) X_puijt[p, 2, i, j, t] / sum(cap_u in CAP_u: cap_u.u == 2) cap_u.CAP_u)
>= 0;

SuppliertoProductionWarehouseTruckUsageConstraint:
forall (i in Suppliers, k in ProductionWarehouses, t in Time)
   YA_ikt[i, k, t] -
   (sum(p in Products) XA_puikt[p, 1, i, k, t] / sum(cap_u in CAP_u: cap_u.u == 1) cap_u.CAP_u)
-
   (sum(p in Products) XA_puikt[p, 2, i, k, t] / sum(cap_u in CAP_u: cap_u.u == 2) cap_u.CAP_u)
>= 0;

WarehousetoProductionWarehouseTruckUsageConstraint:
forall (j in Warehouses, k in ProductionWarehouses, t in Time)
   YB_jkt[j, k, t] -
   (sum(p in Products) XB_pujkt[p, 1, j, k, t] / sum(cap_u in CAP_u: cap_u.u == 1) cap_u.CAP_u)
-
   (sum(p in Products) XB_pujkt[p, 2, j, k, t] / sum(cap_u in CAP_u: cap_u.u == 2) cap_u.CAP_u)
>= 0;

ProductionWarehousetoWarehouseTruckUsageConstraint:
forall (k in ProductionWarehouses, j in Warehouses, t in Time)
   YC_kjt[k, j, t] -
   (sum(p in Products) XC_pukjt[p, 1, k, j, t] / sum(cap_u in CAP_u: cap_u.u == 1) cap_u.CAP_u)
-
   (sum(p in Products) XC_pukjt[p, 2, k, j, t] / sum(cap_u in CAP_u: cap_u.u == 2) cap_u.CAP_u)
>= 0;


// TRUCKs RECEPTION CONSTRAINTS

// Truck Availability for Suppliers

//SupplierTruckAvailabilityConstraint:
//forall (i in Suppliers, t in Time)
  //sum(j in Warehouses) Y_ijt[i,j,t]  <= sum (ni in N_i: ni.i==i) ni.N_i;


// Truck Reception for Production Warehouses
//forall (k in ProductionWarehouses, t in Time)
  //sum(j in Warehouses) YB_jkt[j,k,t]+ sum(i in Suppliers) YA_ikt[i,k,t] <= sum (nbk in NB_k:
nbk.k==k) nbk.NB_k;

// Truck Availability for Production Warehouses
//ProductionWarehouseTruckAvailabilityConstraint:
//forall (k in ProductionWarehouses, t in Time)
  //sum(j in Warehouses) YC_kjt[k,j,t] <= sum (nbk in NB_k: nbk.k==k) nbk.NB_k;
```

```
}

execute {
   var f = new
IloOplOutputFile("C:\\Users\\j_weisbe\\Downloads\\SupplierFlowConstraintDebug.csv");

   // Write header
   f.writeln("Product,Category,Supplier,Time,S_puit,X_puijt,XA_puikt,ConstraintSatisfied");

   for (var p in Products)
      for (var u in Category)
         for (var i in Suppliers)
            for (var t in Time) {
               var supply = 0;
               for (var s in S_puit)
                  if (s.p == p && s.u == u && s.i == i && s.t == t)
                     supply += s.S_puit;

               var flowToWarehouses = 0;
               for (var j in Warehouses)
                  flowToWarehouses += X_puijt[p][u][i][j][t];

               var flowToProductionWarehouses = 0;
               for (var k in ProductionWarehouses)
                  flowToProductionWarehouses += XA_puikt[p][u][i][k][t];

               var constraintSatisfied = (supply == flowToWarehouses +
flowToProductionWarehouses);

               f.writeln(p + "," + u + "," + i + "," + t + "," + supply + "," + flowToWarehouses + ","
+ flowToProductionWarehouses + "," + constraintSatisfied);
            }

   f.close();
}
```