Quantitative Easing in an Open Economy Model with Financial Intermediaries

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Abstract

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This paper models quantitative easing in a small open economy with unrestricted bond markets in both countries to examine its impact on long-term interest rates and stimulating production growth. Bonds with short-term and long-term maturities, as well as domestic and foreign bonds, are modeled as imperfect substitutes reflecting investor preferences to hold a diversified portfolio. Following a quantitative easing shock that lowers the market supply of domestic long-term bonds, this bond segmentation causes long-term domestic bond yields to decrease, leading to higher domestic production. This model includes banks that receive deposits from households and invest them in domestic and foreign bonds. Households earn a rate of return that is a weighted average of the returns earned by banks. In contrast to models that include households holding bonds directly, models with banks allow traditional monetary policies of lowering short-term interest rates to continue to be impactful when short-term interest rates are zero. The results show that quantitative easing leads to lower long-term interest rates, increased domestic production, and higher inflation.

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1 Introduction

A relatively new and unconventional monetary stimulus tool used by many central banks over the past few decades, quantitative easing (QE) aims to boost economic growth during recessionary periods by lowering long-term interest rates through purchases of large quantities of domestic long-term bonds. This paper models the impact of a QE policy in a small open economy where each country issues bonds of two durations with unrestricted bond markets between both countries. This model in this paper is built upon the model in Kabaca (2016), which does not include financial intermediaries and where households hold bonds directly. The model in this paper is modified by adding banks that make investments in bonds from household deposits. Additionally, this model includes government efficiency costs, which represent political implications for the government when implementing quantitative easing, reflecting more representative policy setting.

An important component in models simulating quantitative easing is the modeling of bonds of different maturities and origins as imperfect substitutes. Large-scale asset purchases (LSAPs) of long-term bonds by the government reduce their market supply, funded through selling short-term government bonds, which increases their supply. The decrease in the long-term bond supply lowers their yields, causing investors to rebalance their bond portfolios by increasing the proportion of short-term bonds they hold relative to long-term bonds. However, preferences by investors to hold a certain proportion of both short-term and long-term bonds modeled through portfolio adjustment costs prevent complete bond portfolio rebalancing. As investors' bond portfolios increasingly deviate from their steady-state proportions, the higher the costs they incur. This causes investors to hold fewer short-term bonds and more long-term bonds than they otherwise would, lowering long-term yields and stimulating productivity. The scope of the QE policy follows the calibration in Kabaca (2016), who scales it relative to the size of the LSAP by the Federal Reserve in 2010.

In a closed economy, investors do not have the option to replace their decreased holdings of long-term bonds with those from a foreign country. QE policies are modeled in closed economies in works such as Chen, Cúrdia, and Ferrero (2012), Gertler and Karadi (2018), Karadi and Nakov (2021), and Carlstrom, Fuerst, and Paustian (2017). However, open economy models including open bond markets where investors in both countries have unrestricted access to foreign bonds, allow investors to purchase foreign long-term bonds following a domestic QE policy, potentially lowering the effectiveness of the policy. These portfolio adjustment costs prevent investors from fully rebalancing their long-term bond holdings with foreign bonds, leading them to demand more domestic long-term bonds. Chin, Filippeli, and Theodoridis (2015), Kolasa and Wesolowski (2020), and Bletzinger and Thadden (2021) are modeled in an open economy but with restrictions between domestic and foreign bond markets. Others such as Kabaca et al. (2023) and Alpanda and Kabaca (2015) include open bond markets with no holding strictions for both domestic and foreign. This paper's bond markets follow that in Kabaca (2016), where both countries have full access to each other's bonds of both maturity durations.

It is common for portfolio adjustment costs to be modeled as quadratic functions included in either the household or financial institutions' utility maximization problems. Bletzinger and Thadden (2021) and Kabaca (2016) contain portfolio adjustment cost terms for domestic short-term and long-term bonds as well as domestic and foreign long-term bonds. Chin, Filippeli, and Theodoridis (2015) include one portfolio adjustment cost term that consists of the steady-state ratio of domestic short-term bonds to total long-term bonds held. The portfolio adjustment cost terms in this paper are modeled in the same manner as in Kabaca (2016). Another method of including imperfect substitutability involves including elasticity of substitution terms in the household budget constraint for domestic and foreign bond holdings, as well as short-term and long-term bond holdings, such as in Kabaca et al. (2023) and Alpanda and Kabaca (2015). Additionally, some works including Chen, Cúrdia, and Ferrero (2012) and Kolasa and Wesolowski (2020) contain restricted and unrestricted agents that differ on which types of bonds they are able to trade.

The main difference between this paper and the model included in Kabaca (2016) is the introduction of banks, which accept deposits from households and in return hold domestic and foreign bonds of short and long durations. The modeling of banks follows the general setup used in Bletzinger and Thadden (2021). Bonds are commonly held in models that include QE policy shocks by either households directly or by financial intermediaries. When households hold bonds, there are two rates of return in the model, one for short-term bonds and another for long-term bonds. Conventional monetary policy generally leads to the lowering of short-term interest rates which becomes ineffective once they reach the zero-lower bound. When financial intermediaries or banks are included in the model, they accept deposits from households and then invest them in short- and long-term bonds. Households earn a return on their deposits equal to a weighted average of the returns banks earn on their investments. Thus, when short-term interest rates approach zero, the rate of return earned by households remains positive, and they continue to have an incentive to make deposits in banks. Households hold bonds directly in Chen, Cúrdia, and Ferrero (2012), Kolasa and Wesolowski (2020), Alpanda and Kabaca (2015), Kabaca et al. (2023), and Kabaca (2016). Other works that contain financial intermediaries include Chin, Filippeli, and Theodoridis (2015), Gertler and Karadi (2018), Karadi and Nakov (2021), Sims, Wu, and Zhang (2023), Carlstrom, Fuerst, and Paustian (2017), and Bletzinger and Thadden (2021).

Another difference between this model and that in Kabaca (2016) is the addition of an efficiency cost for the government when it implements a QE policy. It is included in the government budget constraint as a quadratic function following the setup in Kabaca et al. (2023) and Karadi and Nakov (2021). The efficiency cost can be considered a political repercussion or a barrier to implementing a QE policy.

The results in this paper show that a QE policy by one country leads to lower long-term interest rates and increased domestic production. Domestic banks increase their holdings of domestic short-term and foreign long-term bonds while reducing their holdings of domestic long-term bonds and foreign short-term bonds. The increase in domestic GDP results in inflationary pressure and an increase in the real exchange rate in the domestic country. The results are mostly consistent with Kabaca (2016), with similar results with the response in the market supply of bonds and the holdings of bonds by domestic agents. One difference is the domestic short-term bonds held by foreign agents. My paper shows a decrease while Kabaca (2016) has an increase. Responses in long-term interest rates, domestic production, inflation, and the exchange rate are similar between the two papers.

The paper is structured beginning with a description of the model. Then, the data used in the model is detailed. Next, the results of the QE policy including a comparison with my term paper results are discussed. Finally, I conclude in the last section.

2 Model

The first part of this section describes the household consumption model and how it contrasts with Kabaca (2016). Next, banks and their utility function are detailed, including the types of bonds that banks invest household deposits in and the role of portfolio adjustment costs. Then, the production model that includes final-goods and intermediate-goods producers is described, as well as how domestic and foreign New Keynesian Phillips Curves are derived. The next section covers monetary policy, including interest rates determined by a Taylor rule, and fiscal policy detailing the government budget constraint and bond supplies. Finally, the last section presents the market-clearing conditions for production and bond markets in both the domestic and foreign economies.

Variables that represent goods that are consumed or produced by the domestic country are indexed by h, while those for the foreign country are indexed by f. Variables that belong to the foreign country are indexed with a *, while those for the domestic country are without a *.

2.1 Household consumption model

Households are able to consume, work, and make deposits in banks. Instead of directly holding government bonds as in Kabaca (2016), banks make investments with the funds from household deposits. Households receive a return on these deposits that are a weighted average of the returns banks earn from their bond investments. Therefore, banks instead of households face portfolio adjustment costs when changing bond portfolio composition. The household consumption model includes two terms: one for consumption, c_t, and the other for labor, n_t, expressed as

$$E \sum_{t=0}^{\infty} \beta^t \epsilon_t^g \left[log[c_t - h_t] - \xi_n \frac{n_t^{1+\nu}}{1+\nu} \right] \tag{1}$$

where β serves as the time discount factor. The parameter ν represents the inverse of the Frisch elasticity of labor supply, which determines the sensitivity of labor supplied by households following changes in the wage level. Next, ξ_n denotes the weight assigned to labor in the model. Households receive disutility from working more hours, and this parameter determines the magnitude of the loss of utility for a particular quantity of labor supplied. External habit is given by h_t, where $h_t = \zeta c_{t-1}$, with ζ as a parameter. Its role is to represent the desire for households to consume at levels similar to the prior period. Larger fluctuations in consumption between periods result in greater disutility for households. Finally, ϵ_t^g is a preference shock that adheres to an autoregressive logarithmic process. This shock causes fluctuations in household preferences for desired consumption and labor supply levels.

Households face a budget constraint where their consumption and deposits in the current period are equal to the sum of their wages earned, returns received from bank deposits made in the prior period, producer and financial intermediary profits, and taxes paid to the government. It is written as

$$c_t + \frac{D_t}{P_t} \leq \frac{W_t}{P_t} n_t + \frac{R_{D,t-1} D_{t-1}}{P_t} + \frac{\Pi_{H,t}}{P_t} + \frac{\Pi_{R,t}}{P_t} + \frac{\Pi_{FI,t}}{P_t} - \frac{TAX_t}{P_t} \tag{2}$$

with P_t as the consumer price index and W_t representing the nominal wage level. Nominal wages are given by $W_t = w_t P_t$, where w_t is the real wage level. The real wage level follows Carlstrom, Fuerst, and Paustian (2017) for households that cannot set their wage level in a period, given by

$$w_t = \frac{\pi_{t-1}}{\pi_t} w_{t-1}. \tag{3}$$

Households can make deposits, D_t, in banks and earn a return on these deposits at the rate, $R_{D,t}$. The government collects lump-sum taxes, TAX_t, from households. Profits from domestic producers, $\Pi_{H,t}$, retailers, $\Pi_{R,t}$, and financial intermediaries, $\Pi_{FI,t}$ are distributed to households.

2.1.1 Household optimality conditions

The optimality conditions for households are found by taking the first-order conditions for each endogenous variable in the household consumption model subject to the budget constraint. The optimality condition for consumption is

$$c_t = \frac{\epsilon_t^g}{\lambda_t} + \zeta c_{t-1} \tag{4}$$

with λ_t denoting the Lagrange multiplier. Consumption in the current period is positively influenced by the external habit parameter and the preference shock. Similarly, this equation is updated for foreign consumption, c_t^*, as

$$c_t^* = \frac{\epsilon_t^g}{\lambda_t} + \zeta c_{t-1}^*. \tag{5}$$

Next, the optimality condition for labor is a function of real wages, given by

$$n_t = \left(\frac{\lambda_t W_t}{P_t \epsilon_t^g \xi_n}\right)^{1/\nu} \tag{6}$$

where the size of the reaction of the household labor supply response depends on the value of the Frisch elasticity of labor supply included in the exponent term.

The first-order condition with respect to deposits made by households is

$$\beta R_{D,t} E_t \left[\frac{\lambda_{t+1}}{P_{t+1}}\right] = \frac{\lambda_t}{P_t} \tag{7}$$

where the interest rate earned by households depends on the expected price level in the next period $t+1$, relative to that in the current period t. In addition, it depends on the marginal value of the Lagrange multiplier in the current period relative to its expected value in the next period.

2.2 Banks

This paper modifies the model in my term paper that replicated Kabaca (2016) by introducing banks, following the general model structuring for banks in Bletzinger and Thadden (2021). Banks accept deposits from households and invest them in four types of bonds: short-term and long-term, both domestic and foreign. They pay a return to households that is a weighted average of the returns they earn. Banks are perfectly competitive, and the only costs they incur are portfolio adjustment costs for when their portfolio holdings differ from their steady-state proportions, which restricts their response to quantitative easing. The banks aim to maximize the present value of their profits through the maximization problem

$$E\sum_{t=0}^{\infty}\left[\Lambda_{0,t}\Big(\frac{R_{t-1}B_{HS,t-1}}{P_t}+\frac{e_tR^*_{t-1}B_{FS,t-1}}{P_t}+\frac{(1+\kappa q_{L,t})B_{HL,t-1}}{P_t}+\frac{e_t(1+\kappa q^*_{L,t})B_{FL,t-1}}{P_t}-\frac{R_{D,t}D_t}{P_t}-\Theta_t\Big)\right] \tag{8}$$

where $B_{HS,t}$ are domestic short-term bonds, $B_{HL,t}$ are domestic long-term bonds, $B_{FS,t}$ are foreign short-term bonds, $B_{FL,t}$ are foreign long-term bonds, and e_t is the nominal exchange rate. The banks pay a return, $R_{D,t}$, on the deposits households make in the current period that is equal to the real returns earned on bank investment in government bonds in the previous period, less portfolio adjustment costs, Θ_t. The banks' maximization problem is discounted through the stochastic term $\Lambda_{t,t+1}$ following Gertler and Karadi (2018), and given by

$$\Lambda_{0,t}\equiv\beta^t\frac{U'(c_t)}{U'(c_0)} \tag{9}$$

where it includes the discount factor used in the household consumption model, β^t, and the marginal utility of consumption in period t, $U'(c_t)$, relative to the marginal utility in period 0, $U'(c_0)$. The discount factor $\Lambda_{0,t}$ ensures that banks discount earnings at the same rate as households.

The modeling of bond maturities and their returns follows the structure given in Chen, Cúrdia, and Ferrero (2012). Short-term bonds, both domestic and foreign, have a maturity of one period. Short-term domestic bonds earn return, R_t, while foreign short-term bonds earn a return, R^*_t. Both domestic and foreign long-term bonds are modeled as perpetuities that issue a coupon payment each period. This coupon payment decreases each period in perpetuity, where it is equal to κ^s in period $t+s+1$. Domestic long-term bonds are issued at a price $q_{L,t}$, while foreign long-term bonds are issued at a price $q^*_{L,t}$. The price of long-term bonds decreases each period in perpetuity by this coupon rate, following Woodford (2001). The price of a domestic long-term bond in period t can be represented by in a period subsequent to period t, denoted as period s, where $s>t$, by

$$q_{L,s}=\kappa^s q_{L,t} \tag{10}$$

and for foreign long-term bonds by

$$q^*_{L,s}=\kappa^s q^*_{L,t} \tag{11}$$

reflecting the decline in long-term bond prices over time. Given this structure, the prices of long-term bonds of different maturities can be expressed as functions of bond prices at time t. The return on domestic long-term bonds, $R_{L,t}$, is written as

$$R_{L,t} = \frac{1 + \kappa q_{L,t+1}}{q_{L,t}} \tag{12}$$

while the return on foreign long-term bonds, $R^*_{L,t}$, is similarly denoted by

$$R^*_{L,t} = \frac{1 + \kappa q^*_{L,t+1}}{q^*_{L,t}} \tag{13}$$

where the coupon payment received decreases each period by the coupon rate. This decreasing return reflects the decline in the value of long-term bonds across periods, resulting from their structure as a perpetuity with no maturity.

The deposits received from households are invested into government bonds and are given by

$$\frac{D_t}{P_t} = \frac{B_{HS,t}}{P_t} + \frac{e_t B_{FS,t}}{P_t}\epsilon_t^{cr} + \frac{q_{L,t} B_{HL,t}}{P_t}\epsilon_t^{dr} + \frac{e_t q^*_{L,t} B_{FL,t}}{P_t}\epsilon_t^{cr}\epsilon_t^{dr} \tag{14}$$

which includes four terms, one for each type of bond.

Banks pay transaction costs to financial intermediaries for the purchase of both long-term bonds and foreign bonds following their setup in Kabaca (2016) and Chen, Cúrdia, and Ferrero (2012). They represent the premiums paid for the additional risk of holding these types of securities. These costs are modeled exogenously that act to prevent potential arbitrage opportunities. The first type of transaction costs are on the purchase of long-term bonds, both domestic and foreign, which represent the premium investors demand for their longer maturity duration compared to short-term bonds. The longer-term durations increase the risk that interest rate fluctuations could occur during their term, altering their prices. Transaction costs are incurred on purchases of long-term bonds, given by ξ_t^{dr}, where $\epsilon_t^{dr} = 1 + \xi_t^{dr}$. The term ξ_t^{dr} follows a logarithmic autoregressive process and serves as changes in the premium demanded by investors for holding bonds with higher maturity durations. As long-term bonds that were issued in previous periods can be expressed in terms of long-term bonds at time t, taking transaction costs on long-term bond holdings at time t is equivalent to fees being applied to purchases of long-term bonds in period t. The second type of transaction costs are on purchases of foreign bonds, both short-term and long-term. These costs represent the premium investors require to hold foreign bonds due to risks not present when holding domestic bonds, including foreign exchange rate fluctuations. They are denoted by ξ_t^{cr}, where $\epsilon_t^{cr} = 1 + \xi_t^{cr}$. The term ξ_t^{cr} follows a logarithmic autoregressive process and serves as the premium domestic agents require for holding foreign bonds over domestic bonds.

The financial intermediaries that collect the fees are not modeled as profit-maximizing entities and distribute the fees to households to prevent the loss of resources from the economy. The profits for financial intermediaries follow Chen, Cúrdia, and Ferrero (2012), where they consist of the fees they collect given by

$$\Pi_{FI,t} = B_{HL,t}q_{L,t}\xi_t^{dr} + B_{FS,t}e_t\xi_t^{cr} + B_{FL,t}q_{L,t}^{*}e_t(\xi_t^{dr} + \xi_t^{cr}) \tag{15}$$

including fees on purchases of long-term bonds, both domestic and foreign, as well as foreign bonds, both short-term and long-term.

Banks incur portfolio adjustment costs, as modeled in Equation 16, when they modify their portfolio bond holdings in proportions that differ from their steady-state holdings. They have investment preferences to hold a diversified portfolio consisting of bonds of short- and long-term maturity durations as well as domestic and foreign issuances. Following Kabaca (2016), banks are incentivized to hold short-term bonds for their higher liquidity and lower interest rate risks, and desire to hold long-term bonds because of their higher returns. In addition, banks have higher accessibility to data for domestic bonds than their foreign counterparts making foreign bonds more risky. However, banks still have a preference to hold foreign bonds to maintain foreign currency liquidity for carrying out transactions in the foreign economy and to reduce risks from productivity shocks in the domestic economy.

A QE policy implemented by the government alters the composition of domestic bonds supplied in the market, leading banks to change the proportions of bonds they desire to hold. Following a QE policy, the market supply of domestic long-term bonds decreases, increasing their prices and lowering long-term domestic bond yields. Banks then desire to hold more short-term bonds relative to long-term bonds compared to before the QE policy. However, they are dissuaded from fully rebalancing their portfolios due to the adjustment costs they would incur. In addition, they attempt to rebalance their portfolios by buying an optimal quantity of bonds from the foreign country. However, due to these adjustment costs between domestic and foreign bonds, they are unable to completely rebalance their portfolios using foreign bonds. This restriction leads to an increase in the demand of domestic long-term bonds by domestic banks, reducing long-term domestic bond yields. The portfolio adjustment costs in this model are given by

$$\Theta_t = \frac{\xi_a}{2}\Big(\frac{1-\gamma_a}{\gamma_a}\frac{A_{S,t}}{A_{L,t}} - 1\Big)^2 + \frac{\xi_s}{2}\Big(\frac{1-\gamma_S}{\gamma_S}\frac{B_{HS,t}}{e_t B_{FS,t}} - 1\Big)^2 + \frac{\xi_L}{2}\Big(\frac{1-\gamma_L}{\gamma_L}\frac{q_{L,t}B_{HL,t}}{e_t q_{L,t}^{*} B_{FL,t}} - 1\Big)^2 \tag{16}$$

where

$$A_{S,t} = B_{HS,t} + e_t B_{FS,t} \tag{17}$$

and

$$A_{L,t} = q_{L,t}B_{HL,t} + e_t q_{L,t}^{*} B_{FL,t} \tag{18}$$

with $A_{S,t}$ representing a sub-portfolio consisting of domestic and foreign short-term bonds, while

$A_{L,t}$ denotes a sub-portfolio that contains domestic and foreign long-term bonds. Equation 16 includes three quadratic functions, one for each portfolio adjustment cost type. The first term on the right-hand side includes the adjustment costs between short-term and long-term bonds, both domestic and foreign. Here, γ_a is the proportion of short-term bonds held overall in the banks' portfolios in the steady state. Adjustment costs increase as banks' holdings of short-term bonds relative to long-term bonds differ from their desired amount, given by $\frac{1-\gamma_a}{\gamma_a}$. The parameter ξ_a represents the size of the portfolio adjustment costs associated with deviating between short-term and long-term bonds.

The next two terms contain portfolio adjustment costs between domestic and foreign bonds. The second term on the right-hand side in Equation 16 includes only domestic and foreign short-term bonds, while the third term includes domestic and foreign long-term bonds. The parameter γ_S represents the steady-state proportion of domestic short-term bonds relative to total short-term bonds held in banks' portfolios. Similarly, γ_L represents the proportion of domestic long-term bonds compared to the total long-term bonds held by banks. As these proportions differ from the ideal holdings given by the terms, $\frac{1-\gamma_S}{\gamma_S}$ and $\frac{1-\gamma_L}{\gamma_L}$, banks see an increase in the portfolio adjustment costs they face. The parameter ξ_S denotes the relative size of the portfolio adjustment cost between domestic and foreign short-term bonds, while ξ_L is the size of the costs between domestic and foreign long-term bonds.

2.2.1 Banks optimality conditions

The first-order conditions of the optimization problem for banks are taken with respect to both domestic and foreign short-term and long-term bonds. The optimality condition with respect to domestic short-term bonds, $B_{HS,t}$, is

$$\begin{aligned} E_t\left[\Lambda_{t,t+1}\frac{R_t}{P_{t+1}}\right] =& \xi_a\Lambda_{0,t}\left(\frac{(1-\gamma_a)A_{S,t}}{\gamma_a A_{L,t}}-1\right)\frac{1-\gamma_a}{\gamma_a A_{L,t}} \\ &+\xi_s\Lambda_{0,t}\left(\frac{(1-\gamma_s)B_{HS,t}}{\gamma_s e_t B_{FS,t}}-1\right)\frac{1-\gamma_s}{\gamma_s e_t B_{FS,t}}+\frac{\Lambda_{0,t}R_{D,t}}{P_t} \end{aligned} \tag{19}$$

with the real return on domestic short-term debt on the left-hand side. The first term on the right-hand side accounts for the adjustment cost from deviations of total short-term bonds relative to total long-term bonds from the steady-state allocation. The second term includes the adjustment costs for deviations of domestic short-term bonds over foreign short-term bonds. The third term is the real return on household deposits. It represents the cost that banks must pay to households for their investments.

Next, the first-order condition with respect to foreign short-term bonds, $B_{FS,t}$, is

$$
\begin{aligned}
E_t\left[\Lambda_{t,t+1}\frac{R_t^* e_{t+1}}{P_{t+1}}\right] =& \xi_a\Lambda_{0,t}\left(\frac{(1-\gamma_a)A_{S,t}}{\gamma_a A_{L,t}}-1\right)\frac{(1-\gamma_a)e_t}{\gamma_a A_{L,t}} \\
&-\xi_s\Lambda_{0,t}\left(\frac{(1-\gamma_s)B_{HS,t}}{\gamma_s e_t B_{FS,t}}-1\right)\left(\frac{(1-\gamma_s)B_{HS,t}}{\gamma_s e_t B_{FS,t}^2}\right)+\frac{\Lambda_{0,t}R_{D,t}e_t\epsilon_t^{cr}}{P_t}
\end{aligned} \tag{20}
$$

with the real return on foreign short-term debt on the left-hand side. The right-hand side includes terms for the adjustment costs for switching between short- and long-term bonds, adjustment costs between domestic and foreign short-term bonds, and the real household deposit interest rate.

The first-order condition with respect to domestic long-term bonds, $B_{HL,t}$, is given by

$$
\begin{aligned}
E_t\left[\Lambda_{t,t+1}\frac{1+\kappa q_{L,t+1}}{P_{t+1}}\right] =& \xi_a\Lambda_{0,t}\left(\frac{(1-\gamma_a)A_{S,t}}{\gamma_a A_{L,t}}-1\right)\cdot\frac{(1-\gamma_a)A_{S,t}q_{L,t}}{\gamma_a A_{L,t}^2} \\
&+\xi_L\Lambda_{0,t}\left(\frac{(1-\gamma_L)q_{L,t}B_{HL,t}}{\gamma_L e_t q_{L,t}^* B_{FL,t}}-1\right)\cdot\frac{(1-\gamma_L)q_{L,t}}{\gamma_L e_t q_{L,t}^* B_{FL,t}} \\
&+\frac{\Lambda_{0,t}R_{D,t}q_{L,t}\epsilon_t^{dr}}{P_t}
\end{aligned} \tag{21}
$$

with the expected real return on domestic long-term debt on the left-hand side. The right-hand side contains terms for the adjustment costs between short-term and long-term bonds, adjustment costs between domestic and foreign long-term bonds, and the real interest rate on household deposits.

Lastly, the first-order condition with respect to foreign long-term bonds, $B_{FL,t}$, is

$$
\begin{aligned}
E_t\left[\Lambda_{t,t+1}\frac{e_{t+1}\left(1+\kappa q_{L,t+1}^*\right)}{P_{t+1}}\right] =& \xi_a\Lambda_{0,t}\left(\frac{(1-\gamma_a)A_{S,t}}{\gamma_a A_{L,t}}-1\right)\frac{(1-\gamma_a)A_{S,t}e_t q_{L,t}^*}{\gamma_a A_{L,t}^2} \\
&-\xi_L\Lambda_{0,t}\left(\frac{(1-\gamma_L)q_{L,t}B_{HL,t}}{\gamma_L e_t q_{L,t}^* B_{FL,t}}-1\right)\frac{(1-\gamma_L)q_{L,t}B_{HL,t}}{\gamma_L e_t q_{L,t}^* B_{FL,t}^2} \\
&+\frac{\Lambda_{0,t}R_{D,t}e_t q_{L,t}^*\epsilon_t^{cr}\epsilon_t^{dr}}{P_t}
\end{aligned} \tag{22}
$$

with the expected return on foreign long-term debt on the left-hand side. Similar to Equation 21, the right-hand side includes terms for the adjustment costs between short-term and long-term bonds, adjustment costs between domestic and foreign long-term bonds, and the real return on household deposits.

2.3 Production model

The production model consists of domestic final-goods producers, domestic intermediate-good producers, and foreign intermediate-goods producers. Foreign-produced intermediate goods are imported into the domestic country by retailers. Domestic and foreign inflation follow a Calvo-style price stickiness model, where a portion of domestic and foreign intermediate-goods producers can optimally adjust prices each period. This results in reduced domestic and foreign price volatility. The production model used in this paper follows that of Kabaca (2016).

2.3.1 Final-goods production

Aggregate domestic production, Y_t, is the sum of production by domestic intermediate-goods producers, given by

$$Y_t = \left(\int_0^1 y_t(j)^{\frac{\epsilon_h - 1}{\epsilon_h}} \, dj \right)^{\frac{\epsilon_h}{\epsilon_h - 1}} \tag{23}$$

where $y_t(j)$ is the production by intermediate goods producer j, and ϵ_h is the elasticity of substitution between different types of domestic intermediate goods.

Final goods-producing firms are perfectly competitive and maximize profits through the maximization problem

$$\max P_t c_t - P_{h,t} c_{h,t} - P_{f,t} c^*_{f,t} \tag{24}$$

which is subject to the constraint

$$c_t = \left[\gamma_c^{\frac{1}{\lambda}} c_{h,t}^{\frac{\lambda-1}{\lambda}} + (1 - \gamma_c)^{\frac{1}{\lambda}} c^{*\frac{\lambda-1}{\lambda}}_{f,t} \right]^{\frac{\lambda}{\lambda-1}} \tag{25}$$

where $P_{h,t}$ is the price level of intermediate goods produced domestically, and $P_{f,t}$ is the price level of imported intermediate goods produced by the foreign country. The quantity of domestic intermediate goods produced and used in domestic final goods production is given by $c_{h,t}$, while the quantity of foreign intermediate goods produced and used in domestic final goods production is denoted by $c^*_{f,t}$. λ is a parameter representing the elasticity of substitution between domestically and foreign-produced intermediate goods. Its value influences the substitutability between the two types of goods, making them more substitutable when given a higher value. The parameter γ_c represents the home bias of intermediate goods, given by the ratio of domestically-produced intermediate goods relative to the total intermediate goods produced in both countries.

The demand function for domestic intermediate-goods producer j is a function of the price firm j sets relative to the aggregate price level, and aggregate domestic production. It is written as

$$y_s(j) = \left(\frac{P_{h,s}(j)}{P_{h,s}}\right)^{-\epsilon_h} Y_t \tag{26}$$

where $P_{h,s}(j)$ is the price set by firm j and $P_{h,s}$ is the aggregate price level on domestic intermediate goods. Production by firm j is expressed in future period s ($s > t$), being in terms of the domestic prices in period s and the aggregate domestic production in current period t. Aggregate domestic production, Y_t, is sold as domestically produced intermediate goods that are consumed domestically, $c_{h,t}$, and exported to the foreign country, $c^*_{h,t}$.

The maximization problem in Equation 24 is used to find the optimal quantity of domestic intermediate goods produced by

$$c_{h,t} = \left(\frac{P_{h,t}}{P_t}\right)^{-\lambda} \gamma_c c_t \tag{27}$$

and the optimal quantity of foreign intermediate goods produced as

$$c^*_{f,t} = \left(\frac{P_{f,t}}{P_t}\right)^{-\lambda} (1-\gamma_c) c_t. \tag{28}$$

The above equations are used in determining the price level in the domestic economy, P_t, given by

$$P_t = [\gamma_c P_{h,t}^{1-\lambda} + (1-\gamma_c) P^{*\,1-\lambda}_{f,t}]^{\frac{1}{1-\lambda}} \tag{29}$$

which is a weighted average of the price levels of intermediate goods produced domestically and in the foreign country. Similarly, the price level in the foreign country, P^*_t, is given by

$$P^*_t = [\gamma^*_c P^{*\,1-\lambda^*}_{h,t} + (1-\gamma^*_c) P_{f,t}^{1-\lambda^*}]^{\frac{1}{1-\lambda^*}} \tag{30}$$

where γ^*_c is the proportion of domestic intermediate goods used in the production of foreign final goods. The parameter λ^* is the elasticity of substitution between domestic and foreign-produced goods in the foreign economy. These parameters are assumed to have the same values as their domestic counterparts.

Log-linearization of the domestic price equation in Equation 29 gives the domestic inflation rate for final goods as

$$\hat{\pi}_t = \gamma_c \hat{\pi}_{h,t} + (1-\gamma_c) \hat{\pi}_{f,t} \tag{31}$$

where domestic inflation is determined by a weighted average of inflation on domestic and foreign intermediate goods. Log-linearized variables, represented by a ˆ, reflect the percentage difference between the variable and its value in the steady state. by Inflation in either country has a positive impact on final goods inflation in the domestic economy, with the impact by country determined by the parameter γ_c.

2.3.2 Domestic intermediate-goods production

The domestic country contains a continuum of intermediate-goods producers, indexed by j, that supply differentiated intermediate goods to final-goods producers. A portion of intermediate-goods producers, $1-\theta_h$, are able to optimally set prices in each period, while the remaining firms, θ_h set their prices based on an indexation of prices in previous periods.

Domestic intermediate-goods producer (j) earns profits by

$$\Pi_t(j) = P_{h,t}(j)y_t(j) - w_t n_t(j) \tag{32}$$

where firm j's profits in current period t, $\Pi_t(j)$, are equal to revenues collected by the firm less their expenses in that period. Revenues are composed of the price set by firm j for intermediate goods produced, $P_{h,t}(j)$, and the firm's production, $y_t(j)$. Expenses consist of the domestic wage level, w_t, and the labor employed by firm j, $n_t(j)$.

Domestic firms contain a production function of

$$y_t(j) = \epsilon_t^z n_t(j) \tag{33}$$

where the production level for firm j is equal to a productivity shock ϵ_t^z, which follows a logarithmic autoregressive process, and the labor utilized by firm j, given by $n_t(j)$.

In periods where domestic intermediate-goods producers are not able to optimally set prices, the price that firms set is determined by an indexation of prior-period prices. The demand function from Equation 26 can be written to incorporate this price indexation, giving

$$y_s(j) = \left(\frac{P_{h,t}^{new}(j)}{P_{h,s}}\left(\frac{P_{h,s-1}}{P_{h,t-1}}\right)^{\varphi_h}\right)^{-\epsilon_h}(c_{h,t} + c_{h,t}^*) \tag{34}$$

where the demand for firm $j's$ goods in future period s depends on the optimal price set in period t, the indexation of prices, and the demand for domestically-produced intermediate goods in current period t. The prices that firms optimally set in period t are set while considering the expected production in future period s resulting from this price and the indexation of prices for periods when they cannot set optimal prices. The impact of the prior-period inflation indexation on the production level is given through the power of the parameter φ_h. The aggregate demand in current period t is used by firms setting prices in period t to determine how this chosen price will impact its future production in future period s.

Domestic intermediate-goods producing firms have an objective function where they set optimal prices in period t, $P^{new}_{h,t}(j)$, given by

$$E_t\Sigma_{s=t}^{\infty}\theta_h^{s-t}\Lambda_{t,s}\Pi_s(j) \tag{35}$$

where θ_h^{s-t} is the probability that firms cannot set optimal prices between periods s and t. Π_s are profits earned by domestic intermediate-goods producers in period s, defined in Equation 36, and the $\Lambda_{t,s}$ is the discount factor, given in Equation 9.

The profits of firm j can be expressed as

$$\Pi_s(j) = y_s(j)(P_{h,s}(j) - P_{h,s}MC_s) \tag{36}$$

where they are equal to the revenue for firm j, consisting of the output by firm j multiplied by its price, less expenses incurred by firm j, given by its output, aggregate price index, and marginal costs, MC_s. Marginal costs are given by

$$MC_s = \frac{W_s}{P_{h,s}\epsilon_s^z} \tag{37}$$

where they consist of the domestic wage level expected in period s, expected domestic price in period s, $P_{h,s}$, and the productivity shock in period s, ϵ_s^z.

Equation 36 can be updated to include the optimal price setting and prior-period inflation indexing giving

$$\Pi_s(j) = y_s(j)(P^{new}_{h,t}(j)\Big(\frac{P_{h,s-1}}{P_{h,t-1}}\Big)^{\varphi_h} - P_{h,s}MC_s) \tag{38}$$

where firm $j's$ profits in future period s depend on the optimal price set by firms in period t, price indexation to prior-period inflation subject to φ_h, the price level in period s, and the marginal costs in period s.

The objective function in Equation 35 can be updated using the profit expression for firm j in Equation 38 to give the firm's maximization problem

$$E_t\Sigma_{s=t}^{\infty}\theta_h^{s-t}\Lambda_{t,s}y_s(j)\Big[P^{new}_{h,t}(j)\Big(\frac{P_{h,s-1}}{P_{h,t-1}}\Big)^{\varphi_h} - P_{h,s}MC_s\Big] \tag{39}$$

where θ_h^{s-t} is the probability that firms cannot set optimal prices between current period t and future period s. Firm j maximizes its profits by setting a optimal price in period t while considering the probability that it will not be able to adjust prices again between periods t and s, and discounting its future profits by the discount factor $\Lambda_{t,s}$.

The first-order condition of the firms' optimization problem in Equation 39 with respect to the optimal price set in period t, $P^{new}_{h,t}$, is

$$E_t \Sigma_{s=t}^{\infty} \theta_h^{s-t} \Lambda_{t,s} y_s(j) \left[P_{h,t}^{new}(j) \left(\frac{P_{h,s-1}}{P_{h,t-1}} \right)^{\varphi_h} - \frac{\theta_h}{\theta_h - 1} P_{h,s} MC_s \right] = 0 \tag{40}$$

where it determines the optimal price-setting level given the expected demand in the future period s, the probability it will not be able to adjust its price between period t and s, the expected domestic price level, $P_{h,s}$, and marginal costs, MC_s, in period s. The optimal price chosen by firms can be written as

$$P_{h,t}^{\text{new}}(j) = \frac{\theta_h}{\theta_h - 1} \frac{E_t \sum_{s=t}^{\infty} \theta_h^{s-t} \Lambda_{t,s} y_s(j) P_{h,s} MC_s}{E_t \sum_{s=t}^{\infty} \theta_h^{s-t} \Lambda_{t,s} y_s(j) \left(\frac{P_{h,s-1}}{P_{h,t-1}} \right)^{\varphi_h}} \tag{41}$$

where the numerator represents the discounted value of the firm's marginal costs while the denominator is the discounted value of the firm's output adjusted by the price indexation term. The price set by firm j is marked up by the ratio $\frac{\theta_h}{\theta_h - 1}$.

The domestic price level of intermediate goods is a weighted average of the optimal prices chosen by the portion of firms $1 - \theta_h$, and the portion, θ_h, that cannot optimally set prices. It is given by

$$P_{h,t} = \left[(1 - \theta_h) P_{h,t}^{new(1-\epsilon_h)} + \theta_h \left(P_{h,t-1} \left(\frac{P_{h,t-1}}{P_{h,t-2}} \right)^{\varphi_h} \right)^{1-\epsilon_h} \right]^{\frac{1}{1-\epsilon_h}} \tag{42}$$

where the firms that can optimally set prices set them at a price level, $P_{h,t}^{new}$, raised to the power of $1 - \epsilon_h$, As prices change, the substitutability of domestic goods varies based on ϵ_h, with this parameter positively influencing the substitutability among domestic goods when there is an increase in the domestic price level.

The first-order condition of the firms' maximization problem in Equation 40 can be combined with the equation for the domestic price level in Equation 42 and then log-linearized to give the inflation rate on intermediate domestically produced goods, $\hat{\pi}_{h,t}$, given by

$$\hat{\pi}_{h,t} = \frac{\varphi_h}{1 + \varphi_h \beta} \hat{\pi}_{h,t-1} + \frac{\beta}{1 + \varphi_h \beta} E_t \hat{\pi}_{h,t+1} + \frac{(1 - \theta_h)(1 - \beta \theta_h)}{\theta_h (1 + \varphi_h \beta)} (\widehat{mc}_t) \tag{43}$$

where the marginal costs are

$$\widehat{mc}_t = \nu \hat{Y}_t - (1 - \nu) \hat{\epsilon}_t^z + \frac{1}{1 - \zeta} (\hat{c}_t - \zeta \hat{c}_{t-1}) - \hat{p}_{h,t} \tag{44}$$

where the domestic inflation rate is the domestic price level for intermediate goods in the current period relative to the prior period given by $\pi_{h,t} = P_{h,t}/P_{h,t-1}$. The real domestic price level, $p_{h,t}$, is defined as the domestic price level for intermediate goods relative to the aggregate domestic price level given by $p_{h,t} = P_{h,t}/P_t$.

The inflation rate of domestically produced intermediate goods follows a New Keynesian Phillips Curve, including influences from prior-period inflation, expected inflation in future periods, and marginal costs incurred by firms. The first term on the right-hand side of Equation 43 gives the impact the prior-period inflation rate has on the current-period inflation rate. It depends on the parameter, φ_h, influencing the indexation of price-setting constraints for those firms that cannot optimally set prices. It is also impacted by the discount factor, β, which influences the degree of the impact of prior-period inflation rates. The second term represents the influence of inflation expectations in the next period. This term is influenced by the same parameters as the first term except that β also appears in the numerator. This implies that expectations of future inflation have a larger impact on current-period inflation levels than prior-period levels. The last term gives the weight of marginal costs firms incur on current-period inflation. Marginal costs have a positive relationship with inflation as higher input costs lead to higher prices for intermediate goods. Their impact on inflation is influenced by the proportion of firms unable to optimally set prices, θ_h, which affects the price-setting behavior of firms.

Marginal costs as shown in Equation 44, are impacted by domestic production, y_t, as higher production levels require the use of greater quantities of inputs, putting increased pressure on prices. The impact of domestic production in the first term is impacted by the value of the inverse of the Frisch elasticity, ν, as it influences the quantity of labor supplied by households to meet production needs. The second term includes the production shock, which impacts marginal costs by adjusting the production efficiency levels of firms. The third term represents the impact of the difference between current-period and prior-period consumption levels. Large discrepancies in consumption across periods can result in increased pressure on production, leading to higher marginal costs. This term is influenced by the external habit parameter, ζ, as it determines how smooth or rigid the response of consumption changes is on marginal costs. The final term, the real price of domestically-produced intermediate goods, $\hat{p}_{h,t}$, has a negative relationship with marginal costs because when prices increase, firms have a greater incentive to reduce costs.

The profits of domestic intermediate-goods producers are distributed to households following the structure in Chen, Cúrdia, and Ferrero (2012). They are given by

$$\Pi_{H,t} = (c_{h,t} + c^*_{h,t}) \left(P_{h,t} - \frac{W_t}{P_{h,t}\varepsilon^z_t} \right) \tag{45}$$

where the quantity of intermediate-goods produced domestically includes those consumed domestically, $c_{h,t}$, and exported to the foreign country, $c^*_{f,t}$. The quantity produced is multiplied by the difference between the domestic price level of intermediate goods, $P_{h,t}$, and the marginal costs for firms, as defined in Equation 37.

2.3.3 Imported intermediate-goods production

Domestic firms that import foreign intermediate goods, referred to as retailers, are modeled as a continuum where the goods they import are used in the production by final-goods producers.

Foreign intermediate-goods importers have profits given by

$$\Pi_t^*(j) = P_{f,t}(j)c_{f,t}^*(j) - w_t^* n_t^*(j) \tag{46}$$

where the profit for firm j, $\Pi_t^*(j)$, consists of its revenues less its expenses. Firm $j's$ revenues include the price it sets for foreign intermediate goods it imports, $P_{f,t}(j)$, and the goods it imports, $c_{f,t}^*(j)$. The firm's expenses are made up of the real foreign wage level, w_t^*, and the labor utilized by firm j, $n_t^*(j)$. Real foreign wages are given by

$$w_t^* = \frac{\pi_{t-1}^*}{\pi_t^*} w_{t-1}^* \tag{47}$$

where W_t^* are nominal foreign wages.

Retail firms have a production function of

$$c_{f,t}^*(j) = \epsilon_t^f n_t^*(j) \tag{48}$$

where firm j has a production level that consists of a productivity shock, ϵ_t^f, following a logarithmic autoregressive process, and the labor firm j uses.

Retail firms have a demand function of

$$c_{f,s}^*(j) = \Big(\frac{P_{f,t}^{new}(j)}{P_{f,s}}\Big(\frac{P_{f,s-1}}{P_{f,t-1}}\Big)^{\varphi_f}\Big)^{-\epsilon_f}(c_{f,t}^* + c_{f,t}) \tag{49}$$

where $c_{f,s}^*$ is the domestic demand for foreign intermediate goods expressed in future period s, where $s > t$. $P_{f,s}(j)$ is the price of foreign intermediate goods set by firm j, $P_{f,s}$ is the aggregate price level of foreign intermediate goods while aggregate foreign production is equal to the foreign-produced intermediate goods consumed in the foreign country, $c_{f,t}$, and exported to the domestic economy, $c_{f,t}^*$.

Profits for retail firms are given by

$$\Pi_s^*(j) = c_{f,s}^*(j)(P_{f,t}^{new}(j)\Big(\frac{P_{f,s-1}}{P_{f,t-1}}\Big)^{\varphi_f} - P_{f,s}MC_s^*) \tag{50}$$

where retail firms' profits reflect the demand for foreign intermediate goods in period s, the optimal price retail firms set in period t, the indexation of prior-period prices used to set prices for retail firms who cannot optimally set prices, and the marginal costs incurred by retail firms. Marginal costs incurred by retail firms are given by

$$MC_s^* = \frac{W_s^*}{P_{f,s}\epsilon_s^f} \tag{51}$$

where they include the nominal foreign wage level, W_s^*, and the productivity shock.

The optimization problem retail firms face is given by

$$E_t \Sigma_{s=t}^{\infty} \theta_f^{s-t} \Lambda_{t,s} c_{f,s}^*(j) \left[P_{f,t}^{new}(j) \left(\frac{P_{f,s-1}}{P_{f,t-1}} \right)^{\varphi_f} - P_{f,s}^* MC_s^* \right] \tag{52}$$

where θ_f^{s-t} is the probability that firms are unable to optimally set prices between period t and future period s. The discount factor is represented by $\Lambda_{t,s}$, given in Equation 9.

Taking the first-order condition of the optimization problem in Equation 52 with respect to $P_{f,t}^{new}(j)$ gives

$$E_t \Sigma_{s=t}^{\infty} \theta_f^{s-t} \Lambda_{t,s} c_{f,s}^*(j) \left[P_{f,t}^{new}(j) \left(\frac{P_{f,s-1}}{P_{f,t-1}} \right)^{\varphi_f} - \frac{\theta_f}{\theta_f - 1} P_{f,s} MC_s^* \right] = 0 \tag{53}$$

where retail firms choose the optimal price it sets in period t to maximize their profits while considering the demand for their goods, prices, and marginal costs.

The aggregate price level for foreign intermediate goods is given by

$$P_{f,t} = \left[(1-\theta_f) P_{f,t}^{new(1-\epsilon_f)} + \theta_f \left(P_{f,t-1} \left(\frac{P_{f,t-1}}{P_{f,t-2}} \right)^{\varphi_f} \right)^{1-\epsilon_f} \right]^{\frac{1}{1-\epsilon_f}} \tag{54}$$

with the portion of firms, $1-\theta_f$, setting optimal prices in period t, while the portion, θ_f, cannot optimally set prices. The elasticity of substitution, ϵ_f, represents the degree of substitutability between foreign goods at a given price level. The degree of resistance on foreign prices of intermediate goods is given by the parameter, φ_f.

The first-order condition in Equation 53 can be combined with the equation for the price of foreign-produced intermediate goods in Equation 54 and log-linearized to find the New Keynesian foreign inflation rate given by

$$\hat{\pi}_{f,t} = \frac{\varphi_f}{1+\varphi_f \beta} \hat{\pi}_{f,t-1} + \frac{\beta}{1+\varphi_f \beta} E_t \hat{\pi}_{f,t+1} + \frac{(1-\theta_f)(1-\beta\theta_f)}{\theta_f(1+\varphi_f \beta)} (\widehat{rer}_t - \hat{p}_{f,t}) + \hat{\epsilon}_t^f \tag{55}$$

where $\pi_{f,t} = P_{f,t}/P_{f,t-1}$ and the real foreign price level is $p_{f,t} = P_{f,t}/P_t$. The inflation rate of foreign-produced intermediate goods is structured similarly to that of domestic goods. The first term gives the influence of prior-period inflation on current-period inflation. The second term includes the impact of the expectation of foreign inflation in the next period for firms when setting prices. The third term gives the influence of the real exchange rate, which is defined as

$$rer_t = \frac{e_t P_t^*}{P_t} \tag{56}$$

as well as the price of foreign goods. Fluctuations in the real exchange rate affect how domestic prices impact foreign inflation. The final term, $\hat{\epsilon}_t^f$, is a shock stabilizing price fluctuations and follows a logarithmic autoregressive process.

Domestic retail firms who import foreign intermediate goods to be used in domestic final-goods production distribute their profits to domestic households, which are denoted by

$$\Pi_{R,t} = c_{f,t}^* \left(e_t P_{f,t} - e_t \frac{W_t^*}{P_{f,t} \varepsilon_t^z} \right) \tag{57}$$

where they are the sum of foreign-produced intermediate goods imported to the domestic country multiplied by the difference in the foreign price level, and the marginal costs for retail firms given in Equation 51.

2.4 Monetary and fiscal policy

Domestic and foreign short-term interest rates are set by a Taylor rule, where they are determined through two main elements, prior-period short-term interest rates and variations in variables representing other economic states. The domestic short-term interest rate is given by

$$R_t = R\left(\frac{R_{t-1}}{R}\right)^{\rho} \left(\left(\frac{\pi_t}{\pi}\right)^{r_\pi} \left(\frac{Y_t}{Y}\right)^{r_Y} \left(\frac{Y_t}{Y_{t-1}}\right)^{r_{\Delta Y}} \left(\frac{e_t}{e_{t-1}}\right)^{r_d} \right)^{1-\rho} \epsilon_t^r \tag{58}$$

where ρ is a smoothing parameter determining the impact of these two elements on the current-period interest rate. Variations between the prior-period interest rate, R_{t-1}, and the steady-state interest rate, R, are positively correlated with the current-period interest rate. In addition, greater differences between variables including current-period inflation, output, and the exchange rate with either the steady-state value or prior-period value increase short-term interest rates. These economic variables are each weighted by a smoothing parameter, with r_π giving the weight of current-period inflation, π_t, to steady-state inflation, π. r_Y gives the weight of current-period output, Y_t, to steady-state output, Y. $r_{\Delta Y}$ assigns the weight of current-period output, Y_t, to prior-period output, Y_{t-1}. r_d gives the weight of the current-period exchange rate, e_t, relative to the prior-period exchange rate e_{t-1}. A monetary policy shock, ϵ_t^r, is included and follows a logarithmic autoregressive process.

The short-term interest rate for foreign bonds is updated similarly by

$$R_t^* = R^*\left(\frac{R_{t-1}^*}{R^*}\right)^{\rho} \left(\left(\frac{\pi_t^*}{\pi^*}\right)^{r_\pi} \left(\frac{Y_t^*}{Y^*}\right)^{r_Y} \left(\frac{Y_t^*}{Y_{t-1}^*}\right)^{r_{\Delta Y}} \left(\frac{e_{t-1}}{e_t}\right)^{r_d} \right)^{1-\rho} \tag{59}$$

where R^* is the steady-state foreign short-term interest rate, π^* is the steady-state foreign inflation rate, π_t^* is the foreign inflation rate in the current period, Y^* is the foreign steady-state production level, and Y_t^* is foreign production in the current period. The parameter values weighting the economic variables were taken to be the same as in the domestic short-term interest rate equation in Equation 58.

The government issues short-term and long-term debt while also setting fiscal policy through taxation. The domestic government is given a budget constraint of

$$\frac{R_{t-1}}{\pi_t} b_{S,t-1} + \frac{R_{L,t-1}}{\pi_t} q_{L,t} b_{L,t-1} = \frac{TAX_t}{P_t} + b_{S,t} + q_{L,t} b_{L,t} - \Phi_t \tag{60}$$

where $b_{S,t}$ is real short-term domestic debt, given by $\frac{B_{S,t}}{P_t}$, and $b_{L,t}$ is real long-term domestic debt, given by $\frac{B_{L,t}}{P_t}$. The left-hand side consists of the cost of the real debt issued by the government in the previous period. The government pays a return R_t on short-term bonds it issues while paying a return $R_{L,t-1}$ on long-term bonds, given in Equation 12. The right-hand side includes the resource allocation in the current period, including lump-sum taxes collected from households, total debt issued, and less an efficiency cost, Φ_t. This efficiency cost is not included in Kabaca (2016) and follows a similar structure as in Kabaca et al. (2023). This cost represents certain institutional risks the government could face when enacting a QE policy. The efficiency cost is given by the quadratic function

$$\Phi_t = \frac{\tau_t}{2} \left(\frac{q_{L,t} b_{L,t}}{q_L b_L} \right)^2 \tag{61}$$

where τ_t is a parameter determining the size of the cost. The size of the efficiency cost is positively impacted by the current-period long-term bond price and quantity issued, relative to their steady-state values. This acts as a constraint on the government when enacting a QE policy, limiting its extent due to other economic and political forces.

The amount of debt accumulated by the domestic government is stabilized by the equation

$$\frac{TAX_t}{P_t} = \Xi Y \left(\frac{b_{S,t-1} + q_{L,t-1} b_{L,t-1}}{b_S + q_L b_L} \right)^{\tau_b} \epsilon_t^{\tau} \tag{62}$$

where it depends on the steady-state output, Y, and the change in government debt compared to its steady-state level. Higher amounts of debt in the previous period relative to steady-state debt balances require the government to increase the taxes it collects in the current period. This stabilizes the debt levels incurred by the government and prevents excessive debt issuance. The parameter, Ξ, scales the taxes collected relative to the production level in the domestic economy. The parameter, τ_b, represents the response in current-period tax levels set by the government following a change in government debt levels in the prior period. Finally, a shock in domestic government taxation relative to domestic long-term government debt, ϵ_t^τ, follows a logarithmic autoregressive process.

The proportion of domestic long-term and short-term government debt issued are set by

$$q_{L,t} b_{L,t} = \Gamma \epsilon_t^b b_{S,t} \tag{63}$$

where the value of long-term debt is equal to the quantity of short-term government debt issued, a parameter Γ representing the proportion of long-term government bonds compared to total government debt issued in the steady state, and a shock ϵ_t^b in the quantity of domestic long-term bonds relative to total domestic bonds following a logarithmic autoregressive process. This shock serves as the quantitative easing policy by the domestic government in the model.

2.5 Market clearing

This section includes the market clearing conditions for the production and bond markets for the domestic and foreign economies. In this model, aggregate bond and production markets clear, ensuring demand and supply are balanced.

2.5.1 Production goods

The aggregate production in the domestic economy follows as

$$Y_t = c_{h,t} + c_{h,t}^* \tag{64}$$

and is composed of goods produced in the domestic economy that are consumed by domestic consumers, $c_{h,t}$, and exported to and consumed in the foreign country, $c_{h,t}^*$. The exports of the domestically produced goods are expressed as

$$c_{h,t}^* = \left(\frac{P_{h,t}^*}{e_t P_t^*}\right)^{-\lambda^*} c_t^* \tag{65}$$

and depend on the ratio of the price of domestic intermediate goods exported to the foreign economy, $P_{h,t}^*$, relative to the price of final goods in the foreign economy, $e_t P_t^*$. The exponential term, λ^*, represents the degree of substitutability between goods in the foreign country. The foreign country demands more imports when the price of domestic intermediate goods is lower relative to foreign prices of final goods or when foreign consumption, c_t^*, is higher.

Similarly, aggregate production in the foreign economy follows

$$Y_t^* = c_{f,t} + c_{f,t}^* \tag{66}$$

where it is equal to imports to the domestic economy, $c_{f,t}^*$, and consumption of foreign-produced goods in the foreign economy, $c_{f,t}$. Imports to the domestic country are determined similarly to exports in Equation 65 and are given by

$$c^*_{f,t} = (\frac{e_t P^*_{f,t}}{P_t})^{-\lambda} c_t \tag{67}$$

where they are determined by a term including the ratio of prices of foreign intermediate goods used in production in the domestic economy relative to the price level of domestically produced final goods. Imports to the domestic country depend on the price paid for foreign intermediate goods and the domestic consumption level, c_t. The elasticity of substitution between goods in the domestic and foreign countries, denoted by λ and λ^*, is assumed to be equal in both countries.

2.5.2 Bond markets

Both the domestic and foreign governments issue short-term and long-term bonds. The short-term debt issued by the domestic government follows

$$B_{S,t} = B_{HS,t} + B^*_{HS,t} \tag{68}$$

where domestic short-term bonds are held by domestic banks, $B_{HS,t}$, and foreign banks, $B^*_{HS,t}$.

The demand for domestic short-term debt held by foreign banks is assumed to follow

$$\frac{B^*_{HS,t} P_t}{rer_t} = \left(\frac{R_t e_t}{R^*_t e_{t+1} \epsilon^{cr}_t} \right)^{-\eta^*_S} a^*_{S,t} \tag{69}$$

where it depends on the return of domestic to foreign short-term bonds as well as the real portfolio holdings of short-term debt by foreign banks, $a^*_{S,t}$. As this model represents a small open economy, the short-term bond portfolio of foreign agents is assumed to be equal to the real bonds issued by the foreign government, $b^*_{S,t}$. The ratio of the domestic to foreign returns of short-term bonds has an exponent of elasticity of substitution given by

$$\eta^*_S = \frac{(1-\zeta)c}{a} \frac{\xi_S}{\gamma_a \gamma_S (1-\gamma_S)} \tag{70}$$

where a is the domestic aggregate bond portfolio in the steady state and c is steady-state domestic consumption. Domestic bonds become more substitutable in foreign portfolios when portfolio adjustment costs between short-term domestic and foreign bonds decrease, aggregate domestic consumption decreases, the size of aggregate bond portfolios increases, and the proportion of short-term bonds in the steady state increases.

Similarly, long-term bonds issued by the domestic government, $B_{L,t}$, follow

$$B_{L,t} = B_{HL,t} + B^*_{HL,t} \tag{71}$$

and are equal to the long-term bonds it issues that are held by domestic banks, $B_{HL,t}$, and foreign banks, $B^*_{HL,t}$. The demand for domestic long-term bonds by foreign banks is assumed to have a demand function of

$$\frac{q_{L,t} B^*_{HL,t} P^*_t}{rer_t} = \left(\frac{R^e_{L,t} e_t}{R^{e*}_{L,t} e_{t+1} \epsilon^{cr}_t} \right)^{-\eta^*_L} a^*_{L,t} \tag{72}$$

which depends on the ratio of the return on domestic to foreign long-term bonds as well as the long-term bond portfolio held by foreign banks, $a^*_{L,t}$. Similar to the demand for domestic short-term bonds in the foreign country, the portfolio of long-term bonds held in the foreign country, $a^*_{L,t}$, can be replaced with the total real long-term bonds issued by the foreign country, $b^*_{L,t}$. The elasticity of substitution of domestic and foreign long-term bonds held by foreign agents follows a similar setup as Equation 70 given by

$$\eta^*_L = \frac{(1-\zeta)c}{a} \frac{\xi_L}{(1-\gamma_a)\gamma_L(1-\gamma_L)} \tag{73}$$

where the substitutability depends on steady-state domestic consumption, c, real steady-state aggregate portfolio holdings, a, portfolio adjustment costs between domestic and foreign long-term bonds, ξ_L, the proportion of domestic long-term bonds held among total long-term bonds, and the proportion of short-term bonds held relative to all bond maturities, γ_a.

The issuance of and demand for foreign short- and long-term bonds follow a similar setup as for the domestic bonds above. Foreign short-term bonds issued follow

$$B^*_{S,t} = B_{FS,t} + B^*_{FS,t} \tag{74}$$

which total the quantity of foreign short-term bonds issued that are held by domestic banks, $B^*_{FS,t}$, and by foreign banks, $B_{FS,t}$. The demand for foreign short-term bonds held domestically is assumed to follow

$$\frac{B^*_{FS,t} P^*}{rer_t} = \left(\frac{R^*_t e_t}{R_t e_{t+1} \epsilon^{cr}_t} \right)^{-\eta_S} a_{S,t} \tag{75}$$

where the elasticity of substitution for domestic and foreign short-term bonds for domestic banks is given by

$$\eta_S = \frac{(1-\zeta)c^*}{a} \frac{\xi_S}{\gamma_a \gamma_S (1-\gamma_S)} \tag{76}$$

which follows the elasticity of substitution for foreign agents given in Equation 70.

The quantity of foreign long-term bonds issued follows

$$B^*_{L,t} = B_{FL,t} + B^*_{FL,t} \tag{77}$$

where they are composed of foreign long-term bonds held by domestic banks, $B^*_{FL,t}$, and by foreign banks, $B_{FL,t}$. The demand for foreign long-term bonds held domestically is assumed to follow

$$\frac{q^*_{L,t} B^*_{FL,t} P^*_t}{rer_t} = \left(\frac{R^*_{L,t} e_t}{R_{L,t} e_{t+1} \epsilon^{cr}_t} \right)^{-\eta^*_L} a_{L,t} \tag{78}$$

with the elasticity of substitution for domestic and foreign long-term bonds for domestic banks given by

$$\eta_L = \frac{(1-\zeta)c^*}{a} \frac{\xi_L}{(1-\gamma_a)\gamma_L(1-\gamma_L)}. \tag{79}$$

which is consistent with the elasticity of substitution for foreign agents given in Equation 73.

3 Data

The parameter values in this model are taken from the posterior median parameter values used in Kabaca (2016). The small economy in Kabaca (2016) is modeled based on Canadian data and the large economy by data from the United States. The values for Canadian parameters are estimated using data from Statistics Canada while U.S. parameters are estimated using data from the National Income and Product Accounts and Flow of Funds Accounts. The timeframe of the data used spans from 1990 to 2015. Short-term bonds are classified as bonds with terms lasting up to three months, while all bonds with terms exceeding three months are classified as long-term bonds. Kabaca (2016) scales the quantitative easing shock by the domestic economy to the LSAP by the United States in 2010 when the Federal Reserve made purchases of \$600 billion in treasury securities. This resulted in the market supply of short-term bonds increasing by 27% relative to the supply of long-term bonds.

Table 1 in the appendix displays the parameter values utilized in this paper. The parameters calibrated by Kabaca (2016) include the discount factor, β, the portion of domestic bonds in the short-term bond portfolio, γ_S, the portion of domestic bonds in the long-term bond portfolio, γ_L, the portion of short-term bonds in the overall portfolio, γ_a, the decay rate of the coupon payments of long-term bonds, κ, and home bias of domestic goods, γ_c. The remaining parameters in Table 1 were estimated by Kabaca (2016). The process of estimating parameters begins by using prior values, which are values based on prior beliefs and without using observed data. Posterior values are then determined by updating these prior values by incorporating empirical data and applying Bayesian statistical methods. The median posterior values are the midpoint of parameter value distributions and are the values chosen for the estimated parameters in this paper.

The portfolio adjustment cost for switching between short-term and long-term bonds, ξ_a, is assigned a value of 0.3750. A higher adjustment cost parameter value increases the costs banks incur when modifying portfolios to hold short-term and long-term bonds in proportions different from the steady state. The adjustment cost for switching between domestic and foreign short-term bonds, ξ_S, has a value of 0.0042, while the adjustment cost for switching between domestic and foreign long-term bonds, ξ_L, is 1.3e-05. The adjustment costs between short-term and long-term bonds are substantially higher than the adjustment costs between domestic and foreign bonds. This implies that domestic and foreign bonds of the same maturity are significantly more substitutable than bonds of different maturity durations.

The composition of bond portfolios held by domestic banks consists mostly of domestically issued bonds. Domestic short-term bonds in domestically held portfolios, γ_S, have a value of 0.93, while domestic long-term bonds held in domestic portfolios, γ_L, have a value of 0.923. Meanwhile, short-term bonds make up slightly less than half of the total bonds held in banks' portfolios, γ_a, with a value of 0.42.

As described in the production section, only a portion of intermediate-goods producers can optimally set prices each period. The portion of domestic intermediate-goods producers that are unable to optimally set prices each period, θ_h, has a value of 0.5356, while for foreign intermediate-goods producers, θ_f, has a value of 0.3427. Therefore, a smaller proportion of domestic intermediate-goods producers can optimally set their prices in a given period compared to their foreign counterparts. The domestic price indexation parameter, φ_h, has a value of 0.2707 while the foreign price indexation parameter, φ_f, has a value of 0.3839. Domestic prices have a lower price indexation than foreign prices, meaning they are less tied to previous-period prices than foreign prices.

The smoothing parameters within the Taylor rule for the short-term interest rate determine the impact of changes in variables representing economic conditions. The parameters used in the Taylor rule in Equation 58 include the interest rate smoothing parameter, ρ, with a value of 0.7594, inflation, r_π, with a value 2.6169, output, r_Y given a 0.0713, output growth, $r_{\delta Y}$, with a value of 0.2533, the nominal exchange rate, r_d, with a value of 0.1108.

All the shocks in the model follow autoregressive logarithmic processes. The standard deviations and persistence values used for all shocks in the model are shown in Table 1.

4 Results

The government of the small economy implements a QE policy that changes the composition of the market supply of bonds and causes banks to rebalance their bond portfolios. The impact of this policy within an open bond market, where bonds are imperfect substitutes, is examined for the changes in bond holding proportions, interest rates, production, inflation, and other variables. The results of this model are discussed in the first part of this section followed by a comparison of the results from my term paper.

4.1 Model results

The QE policy shock is implemented by the domestic government through the purchase of domestic long-term bonds while issuing domestic short-term bonds. The intention of the government is to lower long-term interest rates and increase production. The size of the stimulus follows the calibration in Kabaca (2016), which is set to replicate the 2010 LSAP by the Federal Reserve.

Figure 1 shows the changes in domestic bond issuance and bond holdings by domestic banks. The QE policy by the government reduces the supply of domestic long-term bonds and increases the supply of domestic short-term bonds. The decrease in the supply of domestic long-term bonds raises their prices and lowers their yields, as shown in Figure 2. This decrease in long-term bond yields results in domestic banks demanding fewer domestic long-term bonds. Banks continue to desire to contain a portion of long-term bonds in their portfolios due to the term premium giving long-term bonds a higher yield than short-term bonds. Therefore, they replace some of the domestic long-term bonds they held previously with foreign long-term bonds. As domestic banks consider domestic bonds to be less risky than foreign bonds, the increase in the supply of domestic short-term bonds makes them more appealing to domestic banks. Domestic banks respond by holding more domestic short-term bonds and fewer foreign short-term bonds in their portfolios. These changes in domestic bond portfolios are consistent with the responses in Kabaca (2016).

The stimulating impacts of QE on productivity arise from modeling bonds as imperfect substitutes, with investor preferences reflected through portfolio adjustment costs. As the proportions of bonds held in bank portfolios differ from their steady-state bond holdings, banks incur higher adjustment costs. The presence of portfolio adjustment costs prevents banks from completely rebalancing their portfolios following a QE shock, leading to lower long-term yields and a boost in production. Portfolio adjustment costs are included between bonds of different maturities and country issuance.

The first type of portfolio adjustment costs included is between short-term and long-term bonds. Banks respond to the QE policy by desiring a higher proportion of short-term bonds but face resistance from portfolio adjustment costs for switching between short-term and long-term bonds. This causes banks to demand more long-term bonds than they otherwise would, lowering long-term interest rates and stimulating production. Harrison (2011) models QE in a closed economy containing short-term and long-term bonds that are imperfect substitutes. A scenario is included where there are no portfolio adjustment costs between short-term and long-term bonds, making them perfect substitutes. This results in complete rebalancing of agents' portfolios, leading to no changes in long-term interest rates and thus no resulting changes to GDP or inflation. In addition, Harrison (2011) includes a case where portfolio adjustment costs are increased. In this scenario, there is less rebalancing following the QE policy as deviating from steady-state bond holdings is more costly. The demand for domestic long-term bonds is stronger, lowering long-term yields further and leading to higher responses in GDP growth and inflation.

The second type of portfolio adjustment costs is between domestic and foreign bonds. These costs model domestic and foreign bonds as imperfect substitutes and make it more costly for banks to replace their holdings of domestic bonds with foreign bonds. Following a QE policy, lower yields on domestic long-term bonds cause domestic banks to demand more foreign long-term bonds. However, portfolio adjustment costs prevent complete rebalancing of domestic long-term bonds with foreign long-term bonds. This leads to higher domestic long-term bond prices and yields compared to an economy where domestic and foreign bonds are perfect substitutes. This is shown in Kabaca (2016) where a scenario is examined where no portfolio adjustment costs exist between domestic and foreign bonds. In this case, complete rebalancing occurs after the QE shock, with domestic agents optimally replacing their domestic long-term bonds with foreign ones. Thus, there is no change in long-term bond prices and interest rates, leaving GDP growth and inflation unchanged. In addition, another scenario is modeled that includes an increase to adjustment costs between domestic and foreign long-term bonds, resulting in less rebalancing and a larger response to long-term interest rates, GDP growth, and inflation.

Domestic short-term interest rates see a slight increase following the QE policy shock, as shown in Figure 2. Short-term interest rates experience a small decline shortly afterwards, once domestic inflation and GDP growth subside. Short-term interest rates are determined through a Taylor rule and thus only respond mildly from the change in bond supply from the QE shock. Many models studying QE include short-term interest rates that can vary freely and compare the results to when short-term rates are held at the zero-lower bound. This comparison is commonly done as it is a common practice for central banks to set short-term interest rates close to zero at the time of carrying out QE policies. Gertler and Karadi (2018) compare QE between scenarios where the government lets short-term interest rates fluctuate following a Taylor rule and when they are held near the zero-lower bound. When short-term interest rates are not restrained, they increase following the shock as agents require higher returns to hold the additional supply of short-term bonds. The increase in short-term interest rates significantly reduces the economic stimulus provided by QE, with the rise in GDP growth and inflation substantially reduced.

Foreign banks hold fewer domestic short-term bonds following the QE policy, as shown in Figure 2. The expectation is that foreign investors would move to purchase more domestic short-term bonds following their increased supply, which is the result in Kabaca (2016). Domestic banks in this model however increase their holdings of domestic short-term bonds more than their increase in supply resulting from the QE policy, as domestic long-term bonds become less appealing due to their decrease in yields.

The interest rate earned by households is a weighted average of the returns earned by domestic banks. The increase in short-term interest rates is smaller in magnitude than the decrease in the domestic long-term interest rates, causing the interest rate earned on deposits by domestic households to decrease following the shock. The decrease in the interest rate earned by households gives them a lower incentive to invest, resulting in a reduction in the deposits they make to banks, as seen in Figure 2. As a result, households allocate more of their resources to consumption following the shock instead of saving, as shown in Figure 3. The decrease in the household interest rate following the QE policy is consistent with Bletzinger and Thadden (2021).

The decrease in long-term interest rates stimulates domestic production, leading to an increase in domestic GDP, as shown in Figure 3. The rise in productivity creates increased pressure on domestic prices, increasing domestic inflation initially following the shock. As GDP approaches the steady state, the domestic economy experiences deflation until it reaches the steady state. The initial higher inflation causes the real exchange rate between the domestic and foreign countries to rise following the shock. As a result, domestic goods become cheaper for the foreign country, increasing exports to the foreign country. Furthermore, as domestic GDP increases, domestic households raise their consumption and import more goods from the foreign economy.

The modeling of a QE policy in an open economy is significant as responses in bond holdings following the shock differ due to the actions of the foreign bonds available and bond portfolios of foreign agents. Following a QE policy in a closed economy, agents respond by demanding fewer long-term bonds, resulting in lower yields. However, in an open economy, domestic agents have the option of replacing part of the domestic holdings of domestic long-term bonds with their foreign counterparts. Domestic agents thus demand fewer domestic long-term bonds, lowering their yields and the stimulus impact. Alpanda and Kabaca (2015) and Kabaca (2016) test the difference of a QE implemented in an open and closed economy and find significantly larger effects on long-term interest rate reductions and increases in GDP growth in a closed economy.

4.2 Bonds held by financial institutions vs. by households

This section includes a comparison between the results from my term paper that differs from the model in this paper by two ways. First, this paper includes banks that hold bonds that accept deposits from households, modeled based on the setup in Bletzinger and Thadden (2021), whereas in my term paper, households hold bonds directly without any financial intermediaries being present. This setup introduces an additional interest rate on the returns on deposits earned by households. Households in this model take into consideration only this interest rate when making optimization decisions instead of both short-term and long-term interest rates directly.

The second change in this model is the inclusion of an efficiency cost for the government when implementing a QE policy, following Kabaca et al. (2023). It reflects the political repercussions the government may incur when implementing a QE policy. It is modeled in Equation 61 that is included in the government's budget constraint that restricts the government's ability to carry out long-term bond purchases.

The appendix includes a graphical comparison of the results from both models. The results from the model including bonds held by households are amplified for easier comparison, as in most cases its results are significantly smaller.

In most cases, the reactions to QE were similar, but with larger results in the model including financial instituitions. The holdings of bonds by domestic agents in this model are included in the maximization problem for banks. The decision-making for banks is solely based on the return from their bond portfolios. This contrasts with the model where households hold bonds directly, with the household problem including these returns but also consumption and labor decisions. Banks thus respond more strongly to changes in bond yields compared to households that hold bonds directly, whose portfolio holdings are also impacted by consumption and labor decisions. This change leads to a higher responsiveness by domestic agents and their demand for domestic securities in a model including banks following the QE policy. This increased response led to larger impacts on the macroeconomic effects, including GDP growth, inflation, and the exchange rate.

Figure 4 compares the domestic bond issuance and domestic agents' bond holdings. Following the QE policy, domestic agents in both models act by lowering the quantity of domestic long-term bonds they hold while increasing the domestic short-term bonds in their portfolios. They replace a portion of their holdings of domestic long-term bonds with foreign long-term bonds, but are unable to completely rebalance their portfolios due to portfolio adjustment costs. In addition, the increase in supply of domestic short-term bonds increases their attractiveness to domestic agents, increasing their holdings following the shock.

The comparison of the changes in interest rates is shown in Figure 5. Both models show decreases in domestic long-term rates resulting from the decrease in supply of domestic long-term bonds. This increases production, resulting in GDP growth and inflation to occur, as shown in Figure 6. Short-term interest rates rise following the increase in supply of domestic short-term bonds, but decrease in later periods when economic growth and inflation fall.

5 Conclusion

Quantitative easing is implemented by monetary policymakers through large purchases of long-term bonds to lower long-term interest rates and stimulate domestic production during recessionary periods. The preferences of investors to hold a diversified portfolio of short-term and long-term bonds as well as domestic and foreign bonds are reflected in this model by portfolio adjustment costs. These costs discourage investors from fully rebalancing their portfolios from domestic long-term bonds to short-term bonds, leading to a stronger demand for long-term bonds and lowering their yields. Access to foreign bond markets reduces the effectiveness of the QE policy, as investors can replace their domestic long-term bond holdings with foreign equivalents. However, the inclusion of adjustment costs between domestic and foreign bonds prevents investors from fully replacing their domestic long-term bond holdings with foreign long-term bonds. The model in this paper takes place in a small open economy setting with open bond markets between both countries.

The impact of the QE policy rebalances the market supply of domestic bonds available to domestic and foreign banks. The decrease in the supply of domestic long-term bonds causes their price to increase and yields to decrease. Domestic banks respond by increasing their holdings of domestic short-term bonds while reducing the quantity of domestic long-term bonds they hold. In addition, they replace some of their holdings of domestic long-term bonds with foreign long-term bonds. The portfolio adjustment costs prevent banks from optimally rebalancing their bond portfolios, resulting in a larger demand for domestic long-term bonds. This increased demand further lowers domestic long-term interest rates, stimulating domestic production. The increase in domestic production results in higher domestic inflation and an appreciation of the domestic real exchange rate.

The model in this paper is based on the model in my term paper for International Finance with two main modifications. First, this paper introduces banks that accept household deposits and invest them in domestic and foreign bonds, while my term paper had households holding bonds directly. Second, it includes a government efficiency cost for when the government enacts a QE policy, reflecting the social costs it may incur from implementing QE. A potential extension would be to model QE in a scenario where the short-term interest rate is held constant at the zero lower bound, reflecting the typical behavior of short-term interest rates during periods when QE is implemented.

6 Use of Generative AI and AI-assisted tools

During the preparation of my thesis, I used ChatGPT for correcting grammatical errors and LaTeX troubleshooting. After using this tool, I reviewed and edited the content as needed and take full responsibility for the content of my thesis.

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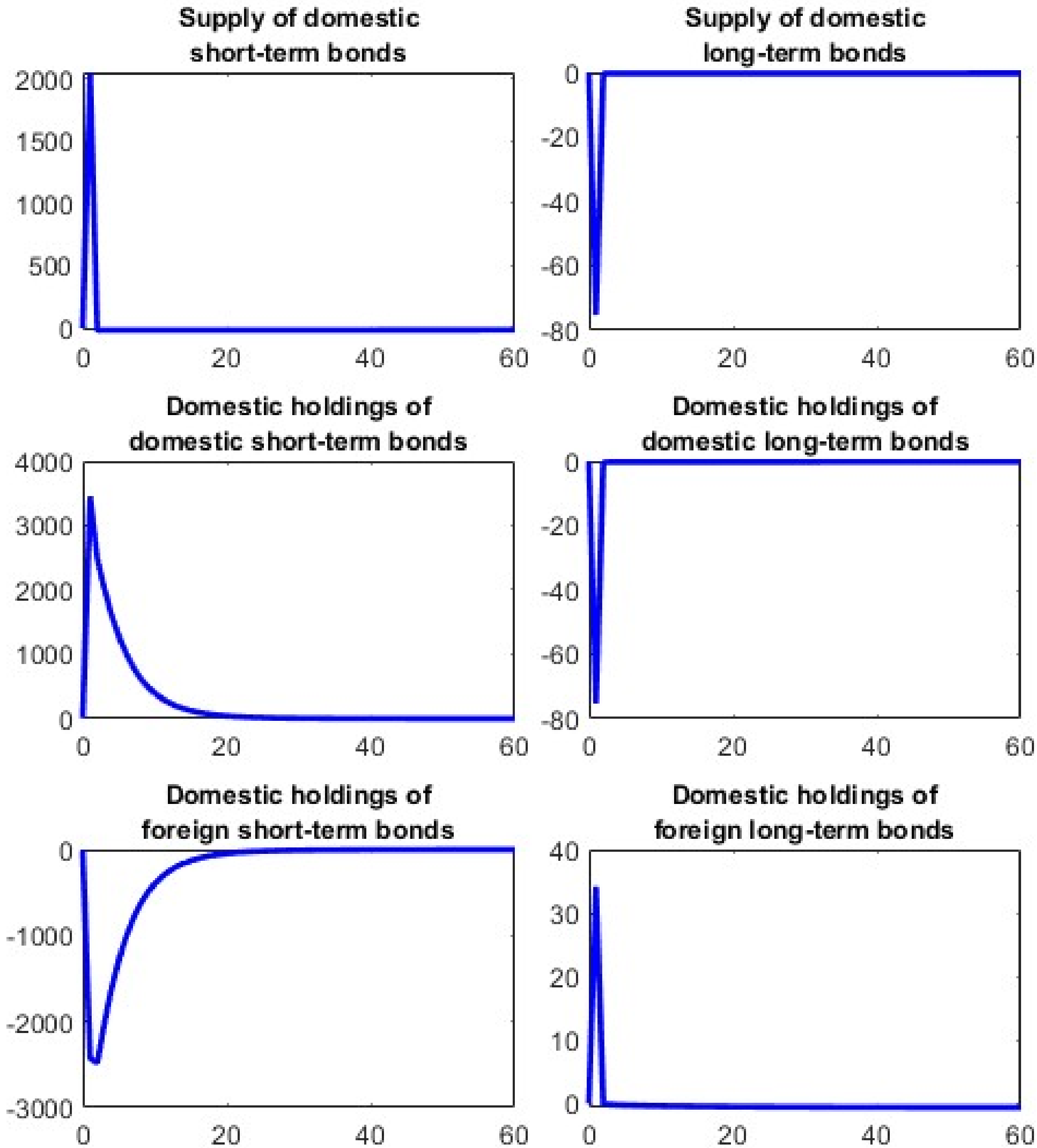
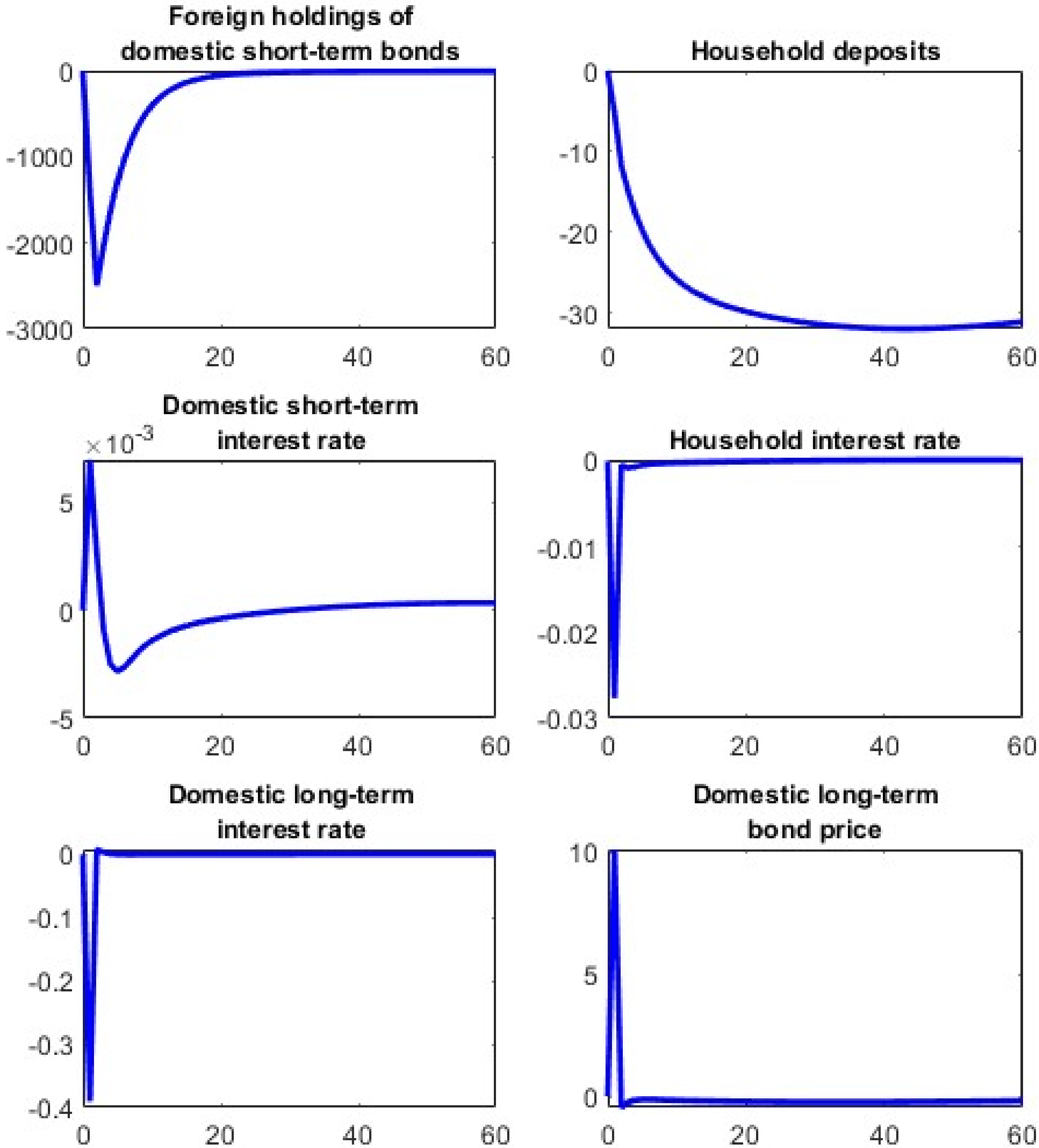
Appendix A: Figures and Tables

Figure 1: Changes in domestic bond market supply and bond holdings of domestic agents



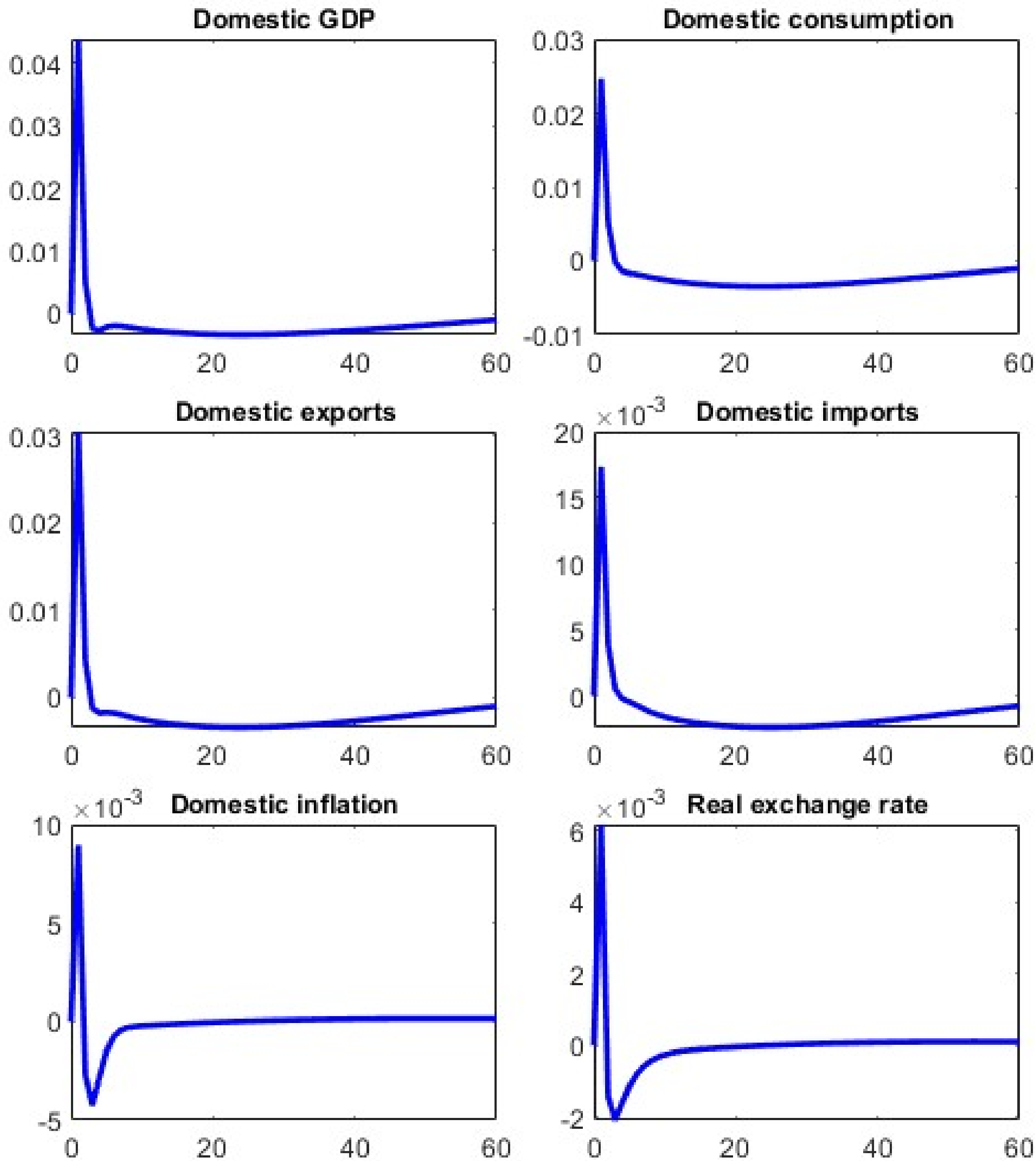
Note: The results are expressed in absolute changes from steady-state values. Domestic QE increases the domestic short-term bond supply and decreases the domestic long-term bond supply. Domestic agents hold more domestic short-term and foreign long-term bonds while holding fewer domestic long-term and foreign short-term bonds.

Figure 2: Foreign holdings of domestic bonds, deposits, interest rates, and bond prices



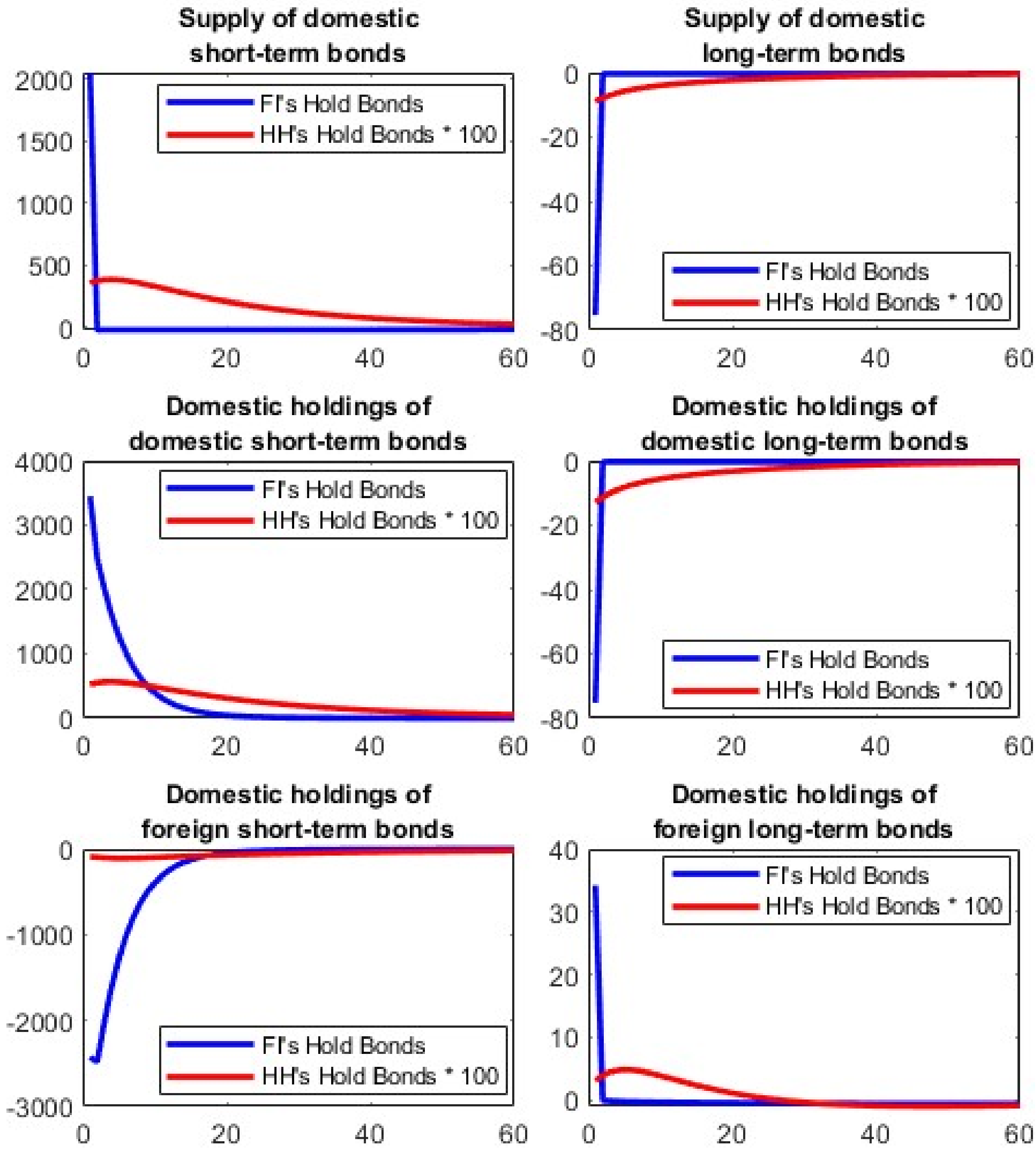
Note: The results are expressed in absolute changes from steady-state values. Households decrease their deposits following the QE policy. Domestic short-term interest rates slightly increase initially while domestic long-term interest rates decrease. The domestic household interest rate is a weighted average of the returns by banks, which decreases following the shock.

Figure 3: Changes in domestic GDP, consumption, trade, inflation, and exchange rate



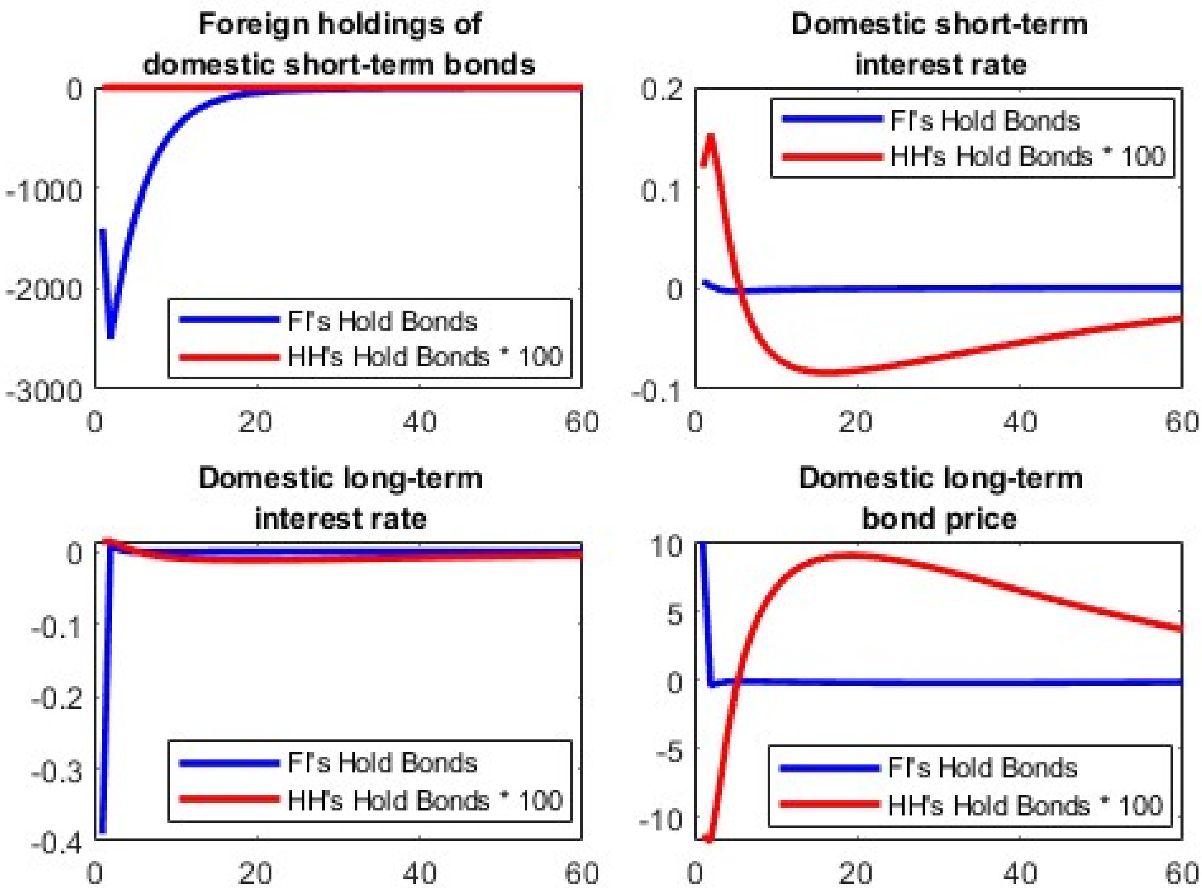
Note: The results are expressed in absolute changes from steady-state values. A domestic QE policy results in positive domestic production growth. This results in higher domestic consumption, imports, exports, inflation, and the real exchange rate.

Figure 4: Comparison of changes in market bond supply and domestic bank bond holdings



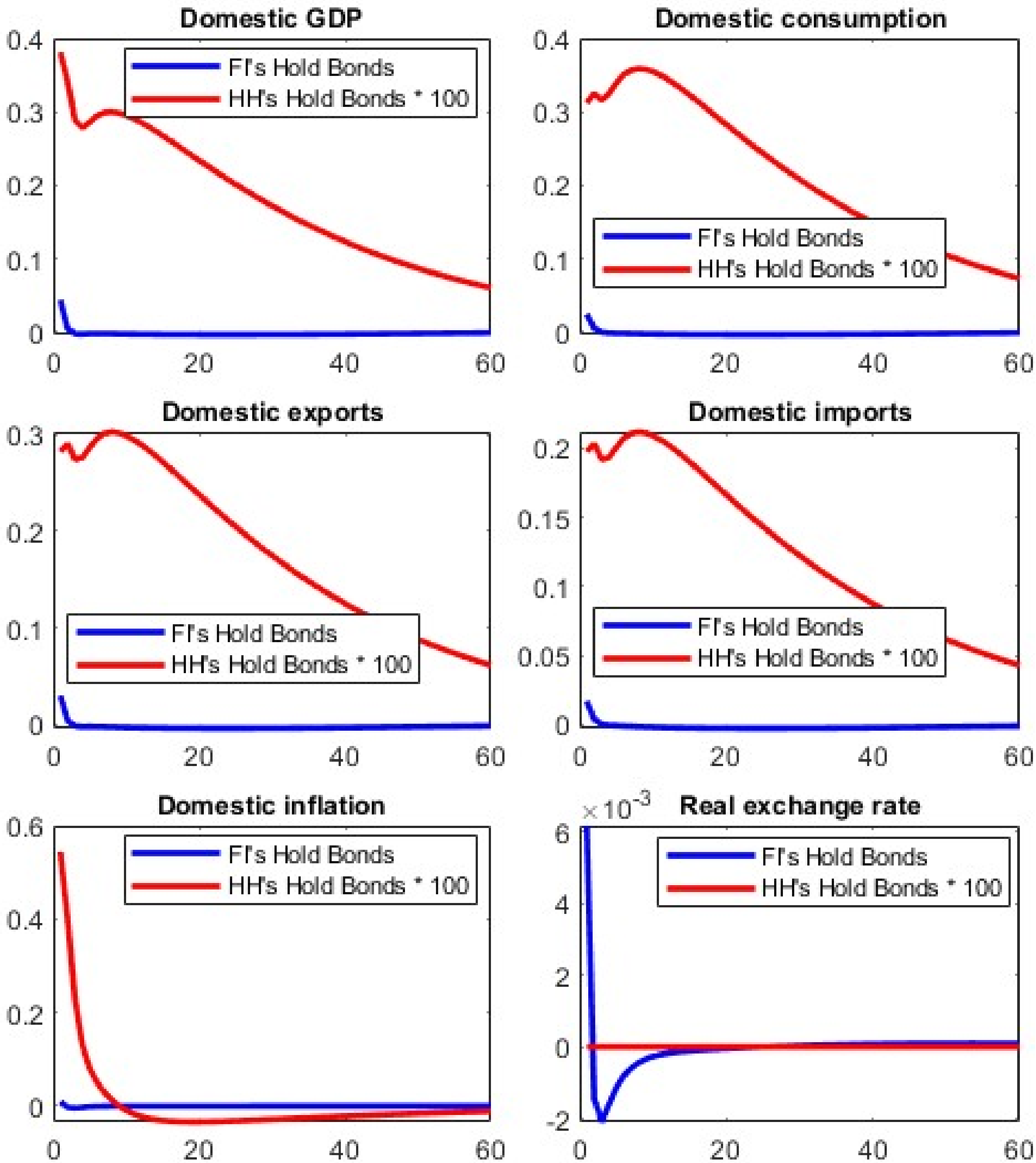
Note: The results are expressed in absolute changes from steady-state values. The response in the market supply of domestic short-term and long-term bonds as well as the domestic bond holdings are significantly larger in the model including financial intermediaries compared to the model where households hold bonds.

Figure 5: Comparison in interest rates and bond prices



Note: The results are expressed in absolute changes from steady-state values. The model including financial intermediaries has a large increase in the response of domestic short-term interest rates and a larger decrease in the domestic long-term interest rate.

Figure 6: Comparison in domestic GDP, consumption, trade, inflation, and exchange rate



Note: The results are expressed in absolute changes from steady-state values. The model including financial intermediaries experiences a significantly higher response in domestic production, consumption, exports, imports, inflation, and real exchange rate compared to model where households hold bonds.

Table 1: Model parameter values

Parameter Description	Parameter	Value
Discount factor	β	0.985
Home bonds in ST portfolio	γ_S	0.93
Home bonds in LT portfolio	γ_L	0.92
ST bonds in total portfolio	γ_a	0.42
Inverse elasticity between ST and LT bonds	ξ_a	0.3750
Inverse elasticity between domestic and foreign ST bonds	ξ_S	0.0042
Inverse elasticity between domestic and foreign LT bonds	ξ_L	1.3e-05
Decrease in coupon payment per period	κ	0.98
Home bias of domestic goods	γ_c	0.70
Inverse Frisch elasticity	ν	1.6908
External habit	ζ	0.3728
Proportion of domestic firms that optimally set prices	θ_h	0.5356
Proportion of foreign firms that optimally set prices	θ_f	0.3427
Domestic price indexation	φ_h	0.2707
Import price indexation	φ_f	0.3839
Domestic and foreign good elasticity of substitution	λ	0.5207
Interest rate smoothing	ρ	0.7594
Inflation smoothing	r_π	2.6169
Output smoothing	r_Y	0.0713
Output growth smoothing	$r_{\Delta Y}$	0.2533
Nominal exchange rate smoothing	r_d	0.1108
Elasticity of taxes to long-term debt	τ_b	1.5524
Productivity shock standard deviation	σ_z	0.9861
Preference shock standard deviation	σ_g	0.3221
Import price stabilization shock standard deviation	σ_f	3.0479
Country risk premium shock standard deviation	σ_{cr}	0.2178
Duration risk premium shock standard deviation	σ_{dr}	0.8112
LT bond supply shock standard deviation	σ_b	4.9175
Government tax to LTD shock standard deviation	σ_τ	0.3451
Monetary policy shock standard deviation	σ_r	0.2212
Productivity shock persistence	ρ_z	0.9512
Preference shock persistence	σ_g	0.7879
Import price stabilization shock persistence	σ_f	0.9712
Country risk premium shock persistence	σ_{cr}	0.7870
Duration risk premium shock persistence	σ_{dr}	0.8660
LT bond supply shock persistence	σ_b	0.9515
Government tax to LTD shock persistence	σ_τ	0.7568

Appendix B: Dynare code

```
close all

var c             % domestic consumption
    c_star        % foreign consumption
    D             % bank deposits
    R_D           % interest rate on bank deposits
    R             % domestic ST interest rate
    R_star        % foreign ST interest rate
    R_L           % domestic LT interest rate
    R_L_star      % foreign LT interest rate
    q_L           % domestic LT bond price
    q_L_star      % foreign LT bond price
    w             % wage rate
    w_star        % foreign wage rate
    w_real        % real domestic wage rate
    w_star_real   % real foreign wage rate
    Pi_H          % profits from domestic producers
    Pi_R          % profits from foreign retailers
    Pi_FI         % profits from financial intermediaries
    n             % labor supply
    eps_g         % preference shock
    eps_tau       % government tax shock
    eps_b         % QE shock
    eps_r         % monetary policy shock
    eps_cr        % transaction costs on purchases of foreign
       bonds
    eps_dr        % transaction costs on purchases of LT bonds
    P             % CPI price index
    P_star        % foreign CPI price index
    c_h           % domestic goods used in final-goods production
       for domestic consumption
```

```
c_f          % foreign goods used in final-goods production
    for domestic consumption
c_h_star     % foreign goods used in foreign final-goods
    production for consumption
c_f_star     % domestic goods used in foreign final-goods
    production for consumption
lambda       % Lagrange multiplier
b_HS         % real ST domestic debt held by domestic agents
b_HL         % real LT domestic debt held by domestic agents
b_FS         % real ST foreign debt held by domestic agents
b_FL         % real LT foreign debt held by domestic agents
B_HS         % nominal ST domestic debt held by domestic
    agents
B_HL         % nominal LT domestic debt held by domestic
    agents
B_FS         % nominal ST foreign debt held by domestic
    agents
B_FL         % nominal LT foreign debt held by domestic
    agents
b_S_star     % supply of foreign ST bonds
b_L_star     % supply of foreign LT bonds
A_S          % domestic held ST debt sub-portfolio
A_L          % domestic held LT debt sub-portfolio
a            % steady-state aggregate real porfolio holdlings
b_S          % domestic issued ST government debt
b_L          % domestic issued LT government debt
b_FS_star    % real foreign held domestic issued ST debt
b_FL_star    % real foreign held domestic issued LT debt
b_HS_star    % real domestic held foreign issued ST debt
b_HL_star    % real domestic held foreign issued LT debt
B_HS_star    % nominal domestic held domestic issued ST debt
B_HL_star    % nominal domestic held domestic issued LT debt
B_FS_star    % nominal foreign held domestic issued ST debt
B_FL_star    % nominal foreign held domestic issued LT debt
Lambda_dis   % bank stochastic discount factor
exch         % nominal exchange rate
```

```
    exch_r        % real exchange rate
    pi            % domestic inflation rate
    pi_hat        % log-linearized domestic inflation rate
    pi_star       % foreign inflation rate
    P_h           % nominal price of domestically produced
       intermediate goods
    p_h           % real price of domestically produced
       intermediate goods
    pi_h          % inflation rate of domestically produced
       intermediate goods
    P_f           % nominal price of imported intermediate goods
    p_f           % real price of imported intermediate goods
    pi_f          % inflation rate of imported intermediate goods
    pi_h_hat      % log-linearized inflation rate of domestically
       produced intermediate goods
    pi_f_hat      % log-linearized inflation rate of imported
       intermediate goods
    eps_z         % productivity shock
    eps_z_hat     % log-linearized productivity shock
    eps_f         % cost-push shock
    eps_f_hat     % log-linearized cost-push shock
    p_h_hat       % log-linearized real price of domestically
       produced intermediate goods
    p_f_hat       % log-linearized real price of imported
       intermediate goods
    c_hat         % log-linearized domestic consumption
    exch_r_hat    % log-linear real exchange rate
    TAX           % lump-sum domestic government taxes
    Gamma_eff     % Efficiency cost of central bank
    y             % domestic production
    y_hat         % log-linear domestic production
    y_star        % foreign production
    tc_cr         % credit premium risk transaction fees paid
    tc_dr;        % duration premium risk transaction fees paid

varexo e_g e_tau e_b e_r e_z e_f e_cr e_dr;
```

```
parameters
betta          % time discount parameter
v              % inverse of Frisch elasticity of labor supply
zeta           % external habit by households
kappa          % coupon payment decay rate
rho_g          % household shock persistence
rho_tau        % taxes to LTD shock persistence
rho_b          % LT bond supply shock persistence
rho_cr         % Country risk premium shock persistence
rho_dr         % Durantion risk premium shock persistence
rho_z          % Productivity shock presistence
rho_f          % Import price shock persistence
Gamma          % SS relative supply of long-term bonds
tau_b          % response of taxes to long-term government debt
rho            % Interest rate smoothing, Taylor rule
r_pi           % importance of domestic inflation, Taylor rule
r_y            % importance of output gap, Taylor rule
r_delta_y      % importance of output growth, Taylor rule
r_d            % importance of depreciation of currency, Taylor
   rule
gamma_a        % SS ratio of short-term bonds in overall portfolio
gamma_s        % SS ratio of home bonds in ST portfolio
gamma_L        % SS ratio of home bonds in LT portfolio
xi_a           % inverse elasticity ST-LT portfolios
xi_s           % inverse elasticity H-F ST bonds
xi_L           % inverse elasticity H-F LT bonds
lambda_star    % foreign elasticity of substitution across goods
   in the foreign economy
Xi             % Tax coefficient - determines SS ratio of tax to
   GDP
xi             % Labor coefficient - determines relative
   importance of labour in utility function
tau_eff        % resource cost of QE
gamma_c        % domestic share of domestic goods
gamma_c_star   % foreign share of domestic goods
```

```
lambda_P        % domestic elasticity of substitution between home
   and foreign goods
lambda_P_star % foreign elasticity of substitution between home
   and foreign goods
theta_h         % probability firms can will not be able to adjust
   its price
theta_f         % probability firms can will not be able to adjust
   its price
sigma_h         % degree of indexatioin to previous period's
   inflation rate for domestic good prices
sigma_f;        % degree of indexatioin to previous period's
   inflation rate for foreign good prices


betta           = 0.985;
v               = 1.6908;
zeta            = 0.3728;
xi              = 22.46;
Xi              = 0.17;
kappa           = 0.98;
rho_g           = 0.75;
rho             = 0.7594;
rho_b           = 0.9515;
tau_b           = 1.5524;
rho_tau         = 0.7568;
rho_cr          = 0.7870;
rho_dr          = 0.8660;
r_pi            = 2.6169;
r_y             = 0.0713;
r_delta_y       = 0.2533;
r_d             = 0.1108;
gamma_a         = 0.42;
gamma_s         = 0.93;
gamma_L         = 0.92;
xi_a            = 0.3750;
xi_s            = 0.0042;
```

```
xi_L            = 0.000013;
Gamma           = 0.5;
gamma_c         = 0.7;
gamma_c_star    = 0.7;
tau_eff         = 0.1;
lambda_P        = 0.5207;
lambda_P_star   = 0.5207;
theta_h         = 0.5356;
theta_f         = 0.3427;
sigma_h         = 0.2707;
sigma_f         = 0.3839;
rho_z           = 0.9512;
rho_f           = 0.9712;
lambda_star     = 0.5207;

model;
//Household consumption

//FOC wrt c (domestic consumption)
c = (eps_g / lambda) + zeta * c(-1);

//Foreign consumption
c_star = (eps_g / lambda) + zeta * c_star(-1);

//FOC wrt n (labor)
n = ((lambda * w)/(P * eps_g * xi))^(1/v);

//FOC wrt D (deposits)
R_D = ((lambda * P(+1)) / (P * lambda(+1) * betta));

//FOC wrt lambda (Lagrange multiplier)
c + (D/P) = ((w * n)/P) + ((R_D(-1) * D(-1)) / P) + (Pi_H / P) +
    (Pi_R / P) + (Pi_FI / P) - (TAX/P);

//Wages
w_real = (pi(-1) / pi) * w(-1);
```

```
w = w_real * P;

//Foreign wages
w_star_real = (pi_star(-1) / pi_star) * w_star(-1);

w_star = w_star_real * P_star;

//Bank deposits
D = B_HS + (exch * B_FS * eps_cr) + (B_HL * q_L * eps_dr) +
    exch * B_FL * q_L_star * eps_cr * eps_dr;

//Domestic producers profits
Pi_H = (c_h + c_f_star) * (P_h - (w / (P_h * eps_z)));

//Retailer profits
Pi_R = c_f * (exch * P_f - (exch * w_star / (P_f * eps_z)));

//Credit risk premium transaction fees
eps_cr = (1 + tc_cr);

//Duration risk premium transaction fees
eps_dr = (1 + tc_dr);

//Financial intermediary profits
Pi_FI = B_HL * tc_dr * q_L + B_FS * tc_cr * exch + B_FL * tc_dr *
    q_L_star * exch + B_FL * tc_cr * q_L_star * exch;

//Productivity shock
log(eps_g) = rho_g * log(eps_g(-1)) + e_g;

//Banks

//Stochastic discount factor
Lambda_dis = betta * (log(c(+1) - zeta * c) / (log(c - zeta * c
   (-1))));
```

```
//FOC wrt B_HS

R = ((((xi_a * ((((1 - gamma_a) * (B_HS + exch * B_FS)) / (
    gamma_a * (q_L * B_HL + exch * q_L_star * B_FL)))-1) *
            ((1 - gamma_a) / (gamma_a * (q_L * B_HL + exch *
                q_L_star * B_FL)))
            + xi_s * ((((1 - gamma_s) * B_HS) / (gamma_s * exch *
                B_FS))-1) *
            ((1 - gamma_s) / (gamma_s * exch * B_FS)) +
            R_D / P))) * Lambda_dis) *
            (P(+1) / Lambda_dis(+1));

//ROC wrt B_FS
R_star = ((((xi_a * (((1 - gamma_a) * (B_HS + exch * B_FS)) / (
    gamma_a * (q_L * B_HL + exch * q_L_star * B_FL)) - 1) *
            (((1 - gamma_a) * exch) / (gamma_a * (q_L * B_HL +
                exch *q_L_star * B_FL))) +
            xi_s * ((((1 - gamma_s) * B_HS) / (gamma_s * exch *
                B_FS)) - 1) *
            -(((1 - gamma_s) * B_HS * gamma_s * B_FS) / ((gamma_s
                * exch * B_FS) ^ 2)) +
            ((R_D * exch * eps_cr) / P)))) * Lambda_dis) *
            (P(+1) / (exch(+1) * Lambda_dis(+1)));

//FOC wrt B_HL
(((1 + kappa * q_L(+1)) * Lambda_dis(+1)) / P(+1)) = (xi_a *
    ((((1 - gamma_a) * (B_HS + exch * B_FS)) / (gamma_a * ((q_L *
    B_HL) + (exch * q_L_star * B_FL)))) - 1) *
            -(((1 - gamma_a) * (B_HS + exch * B_FS) * (gamma_a *
                q_L)) / ((gamma_a * (q_L * B_HL + exch * q_L_star *
                B_FL))^2)) +
            xi_L * ((((1 - gamma_L) * q_L * B_HL) / (gamma_L *
                exch * q_L_star * B_FL)) - 1) *
            (((1 - gamma_L) * q_L) / (gamma_L * exch * q_L_star *
                B_FL)) +
```

```
        ((R_D * q_L * eps_dr) / P)) * Lambda_dis;

//FOC wrt B_FL
((exch(+1) * Lambda_dis(+1)* (1 + kappa * q_L_star(+1))) / P(+1))
    = (xi_a * ((((1 - gamma_a) * (B_HS + exch * B_FS)) / (gamma_a
    * (q_L * B_HL + exch * q_L_star * B_FL))) - 1) *
        (((1 - gamma_a) * (B_HS + exch * B_FS) * (gamma_a *
          exch * q_L_star)) / ((gamma_a * (q_L * B_HL + exch *
           q_L_star * B_FL))^2)) -
        xi_L * ((((1 - gamma_L) * q_L * B_HL) / (gamma_L * exch
           * q_L_star * B_FL)) - 1) *
         ((1 - gamma_L) * (((q_L * B_HL) * (gamma_L * exch *
           q_L_star)) / ((gamma_L * exch * q_L_star * B_FL)^2)
           )) +
         ((R_D * exch * q_L_star * eps_cr * eps_dr) / P)) *
           Lambda_dis;

//Country risk premium shock
log(eps_cr(+1)) = rho_cr * log(eps_cr) + e_cr(+1);

//Duration risk premium shock
log(eps_dr) = rho_dr * log(eps_dr(-1)) + e_dr;

//Return on domestic LT bonds
R_L = (1 + kappa * q_L(+1)) / q_L;

//Return on foreign LT bonds
R_L_star = (1 + kappa * q_L_star(+1)) / q_L_star;

//Sub-portfolio containing domestic and foreign ST bonds
A_S = B_HS + exch * B_FS;

//Sub-portfolio containing domestic and foreign LT bonds
A_L = q_L * B_HL + exch * q_L_star * B_FL;

//Steady-state portfolio holdings
```

```
a = (A_S + A_L) / P;


//Exchange rate

//Nominal exchange rate
exch = pi_h / pi_f;

//Real exchange rate
exch_r = exch * P_star / P;


//Production

//Domestic consumer price index
P = (gamma_c * P_h^(1-lambda_P) + (1 - gamma_c) * P_f^(1-lambda_P
    ))^(lambda_P/(1- lambda_P));

//Foreign consumer price index
P_star = ((gamma_c_star * P_h^(1-lambda_P_star) + (1 -
    gamma_c_star) * P_f^(1-lambda_P_star))^(lambda_P_star/(1-
    lambda_P_star)));

//Domestic consumption of domestic produced goods
c_h = (P_h/P)^(-lambda_P) * gamma_c * c;

//Foreign consumption of foreign produced goods
c_f = (P_f/P)^(-lambda_P) * gamma_c_star * c;

//Domestic productivity shock
log(eps_z) = rho_z * log(eps_z(-1)) + e_z;

//Import price stabilization shock
log(eps_f) = rho_f * log(eps_f(-1)) + e_f;

//Log-linear domestic productivity shock
```

```
eps_z_hat = log(eps_z) - log(steady_state(eps_z));

//Log-linear import price stabilization shock
eps_f_hat = log(eps_f) - log(steady_state(eps_f));

//Log-linear inflation on domestically produced intermediate
   goods
pi_h_hat = (sigma_h * pi_h_hat(-1)) / (1 + sigma_h * betta) + (
   betta * pi_h_hat(+1)) / (1 + sigma_h * betta) +
           ((1 - theta_h)*(1 - betta * theta_h)) / (theta_h * (1
              + sigma_h * betta)) *
           (v * y_hat - (1 - v) * eps_z_hat + (1/(1-zeta))*(c_hat
              -zeta*c_hat(-1)) - p_h_hat);

//Log-linear inflation on domestically produced intermediate
   goods
pi_h_hat = log(pi_h) - log(steady_state(pi_h));

//Log-linear inflation on imported intermediate goods
pi_f_hat = (sigma_f * pi_f_hat(-1)) / (1 + sigma_f * betta) + (
   betta * pi_f_hat(+1)) / (1 + sigma_f * betta) +
           ((1 - theta_f)*(1 - betta * theta_f)) / (theta_f * (1
              + sigma_f * betta)) * (exch_r_hat - p_f_hat) +
              eps_f_hat;

//Log-linear inflation on imported intermediate goods
pi_f_hat = log(pi_f) - log(steady_state(pi_f));

//Log-linear real exchange rate
exch_r_hat = log(exch_r) - log(steady_state(exch_r));

//Log-linear domestic consumption
c_hat = log(c) - log(steady_state(c));

//Log-linear relative price of domestic goods
p_h_hat = log(p_h) - log(steady_state(p_h));
```

```
//Log-linear relative price of foreign goods
p_f_hat = log(p_f) - log(steady_state(p_f));

//Price of domestic goods
P_h = pi_h * P_h(-1);

//Price of foreign goods
P_f = pi_f * P_f(-1);

//Relative price of domestic goods
p_h = P_h / P;

//Relative price of foreign goods
p_f = P_f / P;

//Log-linear domestic inflation
pi_hat = gamma_c * pi_h_hat + (1 - gamma_c) * pi_f_hat;

//Log-linear domestic inflation
pi_hat = log(pi) - log(steady_state(pi));


//Interest rate

//Foreign inflation
pi_star = P_star / P_star(-1);

//Domestic short-term interest rate (Taylor rule)
R = steady_state(R) * (R(-1)/steady_state(R))^rho * ((pi/
   steady_state(pi))^r_pi * (y/steady_state(y))^r_y *
    (y/y(-1))^r_delta_y * (exch/exch(-1))^r_d)^(1 - rho) *eps_r;

//Foreign short-term interest rate (Taylor rule)
R_star = steady_state(R_star) * (R_star(-1)/steady_state(R_star))
   ^rho * ((pi_star/steady_state(pi_star))^r_pi *
```

```
        (y_star/steady_state(y_star))^r_y * (y_star/y_star(-1))^
          r_delta_y * ((1/exch)/(1/exch(-1)))^r_d)^(1 - rho);

//Monetary policy shock
log(eps_r) = rho * log(eps_r(-1)) + e_r;

//Fiscal policy

//Real domestic holdings of domestic ST bonds
b_HS = B_HS / P;

//Real domestic holdings of domestic LT bonds
b_HL = B_HL / P;

//Real domestic holdings of foreign ST bonds
b_FS = B_FS / P;

//Real domestic holdings of foreign LT bonds
b_FL = B_FL / P;

//Domestic government budget constraint
(R(-1) * b_S(-1)) / pi + (R_L * q_L * b_L(-1)) / pi = (TAX / P) +
    b_S + (q_L * b_L) - Gamma_eff;

//Efficiency cost of central bank
Gamma_eff = (tau_eff / 2) * (((q_L * b_L) / (steady_state(q_L) *
   steady_state(b_L)))^2);

//Domestic tax to LT government debt
TAX = P * ((Xi * steady_state(y)) * ((b_S(-1) + q_L(-1) * b_L(-1)
   )/ (steady_state(b_S) + steady_state(q_L) * steady_state(b_L))
   )^tau_b * eps_tau);

//LT domestic bond supply shock
log(eps_tau) = rho_tau * log(eps_tau) + e_tau;
```

```
//Monetary Policy

//LT domestic bond supply
b_L * q_L  = (Gamma * b_S * eps_b);

//LT domestic bond supply shock
log(eps_b) = rho_b * log(eps_b) - e_b;


//Market clearing

//real domestic ST bond supply
b_S = b_HS + b_FS_star;

//real domestic LT bond supply
b_L = b_HL + b_FL_star;

//real foreign ST bond supply
b_S_star = b_HS_star + b_FS;

//real foreign LT bond supply
b_L_star = b_HL_star + b_FL;

//Foreign demand for foreign ST bonds
b_HS_star = ((R_star / (R * (exch(+1)/exch) * eps_cr))^
            -(((1 - zeta) * steady_state(c_star) * xi_s)/(
              steady_state(a) * gamma_a * gamma_s * (1 - gamma_s
              )))*(A_S/P)) * exch_r;

//Foreign demand for foreign LT bonds
b_HL_star = ((R_L_star / (R_L * (exch(+1)/exch) * eps_cr))^
            -(((1 - zeta) * steady_state(c_star) * xi_L)/(
              steady_state(a) * (1 - gamma_a) * gamma_L * (1 -
              gamma_L)))*(A_L/P)) *
            (exch_r / q_L_star);
```

```
//Foreign demand for domestic ST bonds
b_FS_star = ((R / (R_star * (exch(+1)/exch) * eps_cr))^
            -(((1 - zeta) * steady_state(c) * xi_s)/(steady_state
               (a) * gamma_a * gamma_s * (1 - gamma_s)))*(
               b_S_star)) * exch_r;

//Foreign demand for domestic ST bonds
b_FL_star = ((R_L / (R_L_star * (exch(+1)/exch) * eps_cr))^
            -(((1 - zeta) * steady_state(c) * xi_L)/(steady_state
               (a) * (1 - gamma_a) * gamma_L * (1 - gamma_L)))*(
               b_L_star)) *
            (exch_r / q_L);

//Foreign holdings of foreign ST bonds
B_HS_star = b_HS_star * P;

//Foreign holdings of foreign LT bonds
B_HL_star = b_HL_star * P;

//Foreign holdings of domestic ST bonds
B_FS_star = b_FS_star * P;

//Foreign holdings of domestic LT bonds
B_FL_star = b_FL_star * P;

//Domestic production
y = c_h + c_f_star;

//Log-linear domestic production
y_hat = log(y) - log(steady_state(y));

//Foreign production
y_star = c_h_star + c_f;

//Domestic consumption of foreign produced goods
```

```
c_h_star = (P_h * exch / P)^(-lambda_star) * c;

//Foreign consumption of domeestic produced goods
c_f_star = (P_f / (exch * P_star))^(-lambda_star) * c_star;

end;

initval;
pi_hat = 0;
pi_h_hat = 0;
pi_f_hat = 0;
eps_z_hat = 0;
eps_f_hat = 0;
p_h_hat = 0;
p_f_hat = 0;
c_hat = 0;
exch_r_hat = 0;
y_hat = 0;
eps_g     = 1;
eps_tau   = 1;
eps_b     = 1;
eps_r     = 1;
eps_cr    = 1;
eps_dr    = 1;
eps_f     = 1;
eps_z     = 1;
pi_star   = 1;
pi_h      = 1;
pi_f      = 1;
P_h       = 1;
P_f       = 1;
q_L       = 27.391;
q_L_star  = 29.4565;
B_HS      = 15.320875;
B_HL      = 1;
B_FS      = 15;
```

```
B_FL       = 1;
lambda = 0.9;
w_real = 0.7996;
w      = 0.7996;
w_star_real = 0.80;
w_star      = 0.80;
R_D        = 1 / betta;
Gamma_eff = (tau_eff / 2);
tc_cr = eps_cr − 1;
tc_dr = eps_dr − 1;
c = eps_g / (lambda ∗ (1 − zeta));
c_star = eps_g / (lambda ∗ (1 − zeta));
Lambda_dis = betta ∗ (log(c − zeta ∗ c) / (log(c − zeta ∗ c)));
exch = pi_h / pi_f;
D = B_HS + (exch ∗ B_FS ∗ eps_cr) + (B_HL ∗ q_L ∗ eps_dr) + exch
   ∗ B_FL ∗ q_L_star ∗ eps_cr ∗ eps_dr;
P = (gamma_c ∗ P_h^(1−lambda_P) + (1 − gamma_c) ∗
     P_f^(1−lambda_P))^(lambda_P/(1− lambda_P));
P_star = ((gamma_c_star ∗ P_h^(1−lambda_P_star) +
         (1 − gamma_c_star) ∗ P_f^(1−lambda_P_star))^(
            lambda_P_star/(1− lambda_P_star)));
exch_r = exch ∗ P_star / P;
R_L = (1 + kappa ∗ q_L) / q_L;
R_L_star = (1 + kappa ∗ q_L_star) / q_L_star;
p_h = P_h / P;
p_f = P_f / P;
c_h = (P_h/P)^(−lambda_P) ∗ gamma_c ∗ c;
c_f = (P_f/P)^(−lambda_P) ∗ gamma_c_star ∗ c;
c_h_star = (P_h ∗ exch / P)^(−lambda_star) ∗ c_star;
c_f_star = (P_f / (exch ∗ P_star))^(−lambda_star) ∗ c_star;
Pi_H = (c_h + c_f_star) ∗ (P_h − (w / (P_h ∗ eps_z)));
Pi_R = c_f ∗ (exch ∗ P_f − (exch ∗ w_star / (P_f ∗ eps_z)));
Pi_FI = B_HL ∗ tc_dr ∗ q_L + B_FS ∗ tc_cr ∗ exch + B_FL ∗
        tc_dr ∗ q_L_star ∗ exch + B_FL ∗ tc_cr ∗ q_L_star ∗ exch;
y = c_h + c_f_star;
n = (((eps_g/(c−c∗zeta)) ∗ w)/(P ∗ eps_g ∗ xi))^(1/v);
```

```
b_HS = B_HS/P;
b_HL = B_HL/P;
b_FS = B_FS/P;
b_FL = B_FL/P;
A_S = B_HS + exch * B_FS;
A_L = q_L * B_HL + exch * q_L_star * B_FL;
a = (A_S + A_L) / P;
R = (((xi_a * ((((1 - gamma_a) * (B_HS + exch * B_FS)) / (gamma_a
    * (q_L * B_HL + exch * q_L_star * B_FL)))-1) *
            ((1 - gamma_a) / (gamma_a * (q_L * B_HL + exch *
               q_L_star * B_FL)))
            + xi_s * ((((1 - gamma_s) * B_HS) / (gamma_s * exch *
               B_FS))-1) *
            ((1 - gamma_s) / (gamma_s * exch * B_FS)) +
            R_D / P))) * P;
R_star = (((xi_a * (((1 - gamma_a) * (B_HS + exch * B_FS)) / (
   gamma_a * (q_L * B_HL + exch * q_L_star * B_FL)) - 1) *
            ((1 - gamma_a) / (gamma_a * (q_L * B_HL + exch *
               q_L_star * B_FL))) +
            xi_s * ((((1 - gamma_s) * B_HS) / (gamma_s * exch *
               B_FS)) - 1) *
            -(((1 - gamma_s) * B_HS * gamma_s * B_FS) / ((gamma_s
               * exch * B_FS) ^ 2)) +
            ((R_D * exch * eps_cr) / P)))) * P / exch;
b_HS_star = exch_r * ((R_star / (R * eps_cr))^
            -(((1 - zeta) * c * xi_s)/(a * gamma_a * gamma_s * (1
               - gamma_s)))*(A_S/P));
b_S_star = b_HS_star + b_FS;
b_HL_star = (exch_r/q_L_star)*(R_L_star / (R_L * eps_cr))^
            -(((1 - zeta) * c * xi_L)/(a * (1 - gamma_a) *
               gamma_L * (1 - gamma_L)))*(A_L/P);
b_L_star = b_HL_star + b_FL;
b_FS_star = exch_r * (R / (R_star * eps_cr))^
            -(((1 - zeta) * c * xi_s)/(a * gamma_a * gamma_s * (1
               - gamma_s)))*(b_S_star);
b_FL_star = (exch_r / q_L) * (R_L / (R_L_star * eps_cr))^
```

```
        -(((1 - zeta) * c * xi_L)/(a * (1 - gamma_a) *
          gamma_L * (1 - gamma_L)))*(b_L_star);
B_FS_star = b_FS_star * P;
B_FL_star = b_FL_star * P;
B_HS_star = b_HS_star * P;
B_HL_star = b_HL_star * P;
b_S = b_HS + b_FS_star;
b_L = b_HL + b_FL_star;
TAX = ((Xi * y) *
      ((b_S + q_L * b_L)/ (b_S + q_L * b_L))^tau_b * eps_tau) * P
        ;
y_star = c_h_star + c_f;
pi = ((R * b_S) + (R_L * q_L * b_L)) / ((TAX / P) + b_S + (q_L *
   b_L) - Gamma_eff);

end;

steady;
resid;

shocks;

//Preferences shock
var e_g;
stderr 0.3321;

//Tax to long-term debt shock
var e_tau;
stderr 0.3451;

//Monetary policy shock
var e_r;
stderr 0.2212;

//Productivity shock
var e_z;
```

```
stderr 0.9861;

//Cost-pull shock
var e_f;
stderr 0.9712;

//Shock to country risk premium
var e_cr;
stderr 0.2718;

//Shock to duration risk premium
var e_dr;
stderr 0.8112;

//Shock to domestic long-term bond supply
var e_b;
stderr 4.9175;

end;

model_diagnostics;
check;
stoch_simul(irf=60,order=1,periods=500,nograph) w w_star TAX
   y_star lambda R_D q_L_star R_star R_L_star B_HS B_HL B_FS B_FL
B_FS_star B_HL_star D B_HS_star B_FL_star b_S_star b_L_star b_S
   b_L R R_L q_L y P P_h P_f pi pi_star pi_h pi_f exch exch_r c_f
c_f_star c_h_star c c_star;


fig = figure;
set(fig, 'Units', 'centimeters', 'Position', [2, 2, 12.5, 14]);

tiledlayout(3,2, 'TileSpacing', 'compact', 'Padding', 'tight');

nexttile;
plot(0:options_.irf, [0 oo_.irfs.b_S_e_b], 'b', 'LineWidth', 2);
```

```
title({'Supply of domestic', 'short-term bonds'}, 'FontSize', 9,
   'FontWeight', 'bold');

nexttile;
plot(0:options_.irf, [0 oo_.irfs.b_L_e_b], 'b', 'LineWidth', 2);
title({'Supply of domestic', 'long-term bonds'}, 'FontSize', 9, '
   FontWeight', 'bold');

nexttile;
plot(0:options_.irf, [0 oo_.irfs.B_HS_e_b], 'b', 'LineWidth', 2);
title({'Domestic holdings of', 'domestic short-term bonds'}, '
   FontSize', 9, 'FontWeight', 'bold');

nexttile;
plot(0:options_.irf, [0 oo_.irfs.B_HL_e_b], 'b', 'LineWidth', 2);
title({'Domestic holdings of', 'domestic long-term bonds'}, '
   FontSize', 9, 'FontWeight', 'bold');

nexttile;
plot(0:options_.irf, [0 oo_.irfs.B_FS_e_b], 'b', 'LineWidth', 2);
title({'Domestic holdings of', 'foreign short-term bonds'}, '
   FontSize', 9, 'FontWeight', 'bold');

nexttile;
plot(0:options_.irf, [0 oo_.irfs.B_FL_e_b], 'b', 'LineWidth', 2);
title({'Domestic holdings of', 'foreign long-term bonds'}, '
   FontSize', 9, 'FontWeight', 'bold');

fig = figure;
set(fig, 'Units', 'centimeters', 'Position', [2, 2, 12.5, 14]);

tiledlayout(3,2, 'TileSpacing', 'compact', 'Padding', 'tight');

nexttile;
plot(0:options_.irf, [0 oo_.irfs.B_FS_star_e_b], 'b', 'LineWidth'
   , 2);
```

```
title({'Foreign holdings of', 'domestic short-term bonds'}, '
   FontSize', 9, 'FontWeight', 'bold');

nexttile;
plot(0:options_.irf, [0 oo_.irfs.D_e_b], 'b', 'LineWidth', 2);
title({'Household deposits'}, 'FontSize', 9, 'FontWeight', 'bold'
   );

nexttile;
plot(0:options_.irf, [0 oo_.irfs.R_e_b], 'b', 'LineWidth', 2);
title({'Domestic short-term', 'interest rate'}, 'FontSize', 9, '
   FontWeight', 'bold');

nexttile;
plot(0:options_.irf, [0 oo_.irfs.R_D_e_b], 'b', 'LineWidth', 2);
title({'Household interest rate'}, 'FontSize', 9, 'FontWeight', '
   bold');

nexttile;
plot(0:options_.irf, [0 oo_.irfs.R_L_e_b], 'b', 'LineWidth', 2);
title({'Domestic long-term', 'interest rate'}, 'FontSize', 9, '
   FontWeight', 'bold');

nexttile;
plot(0:options_.irf, [0 oo_.irfs.q_L_e_b], 'b', 'LineWidth', 2);
title({'Domestic long-term', 'bond price'}, 'FontSize', 9, '
   FontWeight', 'bold');

fig = figure;
set(fig, 'Units', 'centimeters', 'Position', [2, 2, 12.5, 14]);

tiledlayout(3,2, 'TileSpacing', 'compact', 'Padding', 'tight');

nexttile;
plot(0:options_.irf, [0 oo_.irfs.y_e_b], 'b', 'LineWidth', 2);
title('Domestic GDP', 'FontSize', 9, 'FontWeight', 'bold');
```

```
nexttile;
plot(0:options_.irf, [0 oo_.irfs.c_e_b], 'b', 'LineWidth', 2);
title('Domestic consumption', 'FontSize', 9, 'FontWeight', 'bold'
    );

nexttile;
plot(0:options_.irf, [0 oo_.irfs.c_f_star_e_b], 'b', 'LineWidth',
     2);
title('Domestic exports', 'FontSize', 9, 'FontWeight', 'bold');

nexttile;
plot(0:options_.irf, [0 oo_.irfs.c_f_e_b], 'b', 'LineWidth', 2);
title('Domestic imports', 'FontSize', 9, 'FontWeight', 'bold');

nexttile;
plot(0:options_.irf, [0 oo_.irfs.pi_e_b], 'b', 'LineWidth', 2);
title('Domestic inflation', 'FontSize', 9, 'FontWeight', 'bold');

nexttile;
plot(0:options_.irf, [0 oo_.irfs.exch_r_e_b], 'b', 'LineWidth',
    2);
title('Real exchange rate', 'FontSize', 9, 'FontWeight', 'bold');
```