

Electric Bus Operation Planning Optimization and Charging Station Location Planning

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Abstract

Electric Bus Operation Planning Optimization and Charging Station Location Planning

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Greenhouse gas-emitting energy sources are a major driver of global warming. As cities pursue sustainability, electrifying public transit has become a key strategy to reduce urban air pollution. Electric buses (EBs) are increasingly adopted as energy-efficient, zero-emission alternatives to diesel fleets. However, EBs have limited range, long charging times, and depend on charger locations, so traditional transit planning models do not apply directly. New optimization models and solution methods are needed to operate EBs efficiently while maintaining a high level of service.

In this thesis, we address the emerging optimization challenges of integrating EBs into urban transit systems. First, we conduct a comprehensive literature review in this field to critically assess and classify the existing works and to identify potential areas for future research. The goal is to develop more realistic and applicable models and solution approaches for EB transit systems.

Second, we develop a mixed-integer linear programming (MILP) model that jointly optimizes bus timetabling and scheduling to minimize total operational costs while accounting for passenger waiting time and seat availability. Using a normalized weighted-sum approach to balance cost and service quality, we determine optimal headways and demonstrate, in a Montreal university shuttle case study, that the model significantly reduces waiting times and improves seat availability compared to the current schedule.

We also address the role of charging infrastructure in EB operations by proposing an integer linear programming model that jointly optimizes bus scheduling and fast-charging station locations, minimizing scheduling, travel, energy, and charger installation costs. Because of the problem's complexity, we design and test several branch-and-price algorithms with different branching strategies to find the most efficient approach. The model is applied to a real Montreal case study, and a

sensitivity analysis is conducted to identify the most cost-effective EB type for the transit authority.

The case studies and computational experiments show that the proposed models and solution methods support a smooth transition to a fully electric public transit system. By offering transit agencies practical and effective models, this research advances sustainable urban mobility and contributes to global efforts to address climate change.

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To the ones I cherish most

With endless gratitude and love

Contribution of Authors

This dissertation is presented under the "manuscript-based format". It contains three articles and one book chapter where two of articles and the book chapter have been published and one is submitted. The first article, "Electric bus scheduling and timetabling, fast charging infrastructure planning, and their impact on the grid: A review", is published in *Energies* in 2022. The book chapter, "Electrification of Buses", as a Word Scientific Encyclopedia of Next-Generation Cities Institute (NGCI), Volume 3, is published in World Scientific Publishers. The second manuscript, "Enhancing Electric Shuttle Bus Efficiency: A Case Study on Timetabling and Scheduling Optimization", is published in *Energies* in 2024. The third paper, "Joint optimization of electric bus scheduling and fast charging infrastructure location planning", is submitted to *Transportation Research Part C: Emerging Technologies* in 2025.

All the three papers were co-authored with Professor Ursula Eicker as the supervisor. The second and third papers and the book chapter involved Professor Frédéric Quesnel, who provided research guidelines, contributed to the development of the methodology, and reviewed the papers and chapter prior to submission. Additionally, Professor Chunyan Lai contributed to editing the final version of the first paper. The author of this thesis served as the principal researcher, taking responsibility for tasks such as developing initial ideas, formulating the research, designing algorithms, coding solution methods, analyzing computational results, and writing the first drafts.

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List of Abbreviations

Abbreviation	Definition
EB	Electric Bus
HB	Hybrid Bus
DB	Diesel Bus
EU	European Union
B&P	Branch-and-Price
MILP	Mixed-Integer Linear Programming
ILP	Integer Linear Programming
PBM	Path-Based Model
VS	Vehicle Scheduling
TOU	Time-of-Use
TT	Timetabling
V2G	Vehicle to Grid
PHEV	Plug-in Hybrid Electric Vehicles
FCILP	Fast Charging Infrastructure Location Planning
BPSO	Binary Particle Swarm Optimization
EHDG	Enhanced Heuristic Descent Gradient
B2G	Bus to Grid
PHA	Progressive Hedging Algorithm
GA	Genetic Algorithm
DF	Deficit Function
ACO	Ant Colony Optimization
NSGA-II	Non-dominated Sorting Genetic Algorithm II
AGA	Adaptive Genetic Algorithm
UBPM	Uncertain Bi-Level Programming Model
DOD	Depth of Discharge
VSP-TW	Vehicle Scheduling Problem with Time Windows
SA	Simulated Annealing
LNS	Large Neighborhood Search
ITTVS	Integrated Timetabling and Vehicle Scheduling
TCO	Total Cost of Ownership
SOC	State of Charge
FCLP-EBS	Fast-Charging Location Planning and Electric Bus Scheduling
BITF	Bound and Inter-Task Fixing
MCF	Multiple Column Fixing
CGI	Column Generation Iterations
SPS	Shortest Paths Solved
CG	Columns Generated
GTFS	General Transit Feed Specification
EBSP	Electric Bus Scheduling Problem

Chapter 1

Introduction

1.1 Motivation and Background

Energy sources that release greenhouse gases are a major contributor to global warming. Therefore, replacing such sources of energy with clean and renewable ones has become vital in recent decades. Electrifying vehicles is a solution to air pollution and the energy dilemma. The world is seeing a significant rise in the use of electric vehicles and electric buses due to increasing concerns about the environment, air quality, and energy consumption. In recent years, electric buses have become more popular in cities worldwide. According to recent research by Bloomberg New Energy Finance, the number of electric buses in service is expected to triple, growing from 386,000 in 2017 to 1.2 million by 2025 ([An, 2020](#)). As of 2024, approximately 780,000 electric buses are already in operation globally. Figure 1.1 depicts the year-over-year increase in the European Union (EU) battery-electric bus fleet from 2021 to 2022 (*European Alternative Fuels Observatory, 2022*). The European electric bus market is projected to increase by 18.6% between 2022 and 2027, indicating a shift towards electric buses from conventional buses.

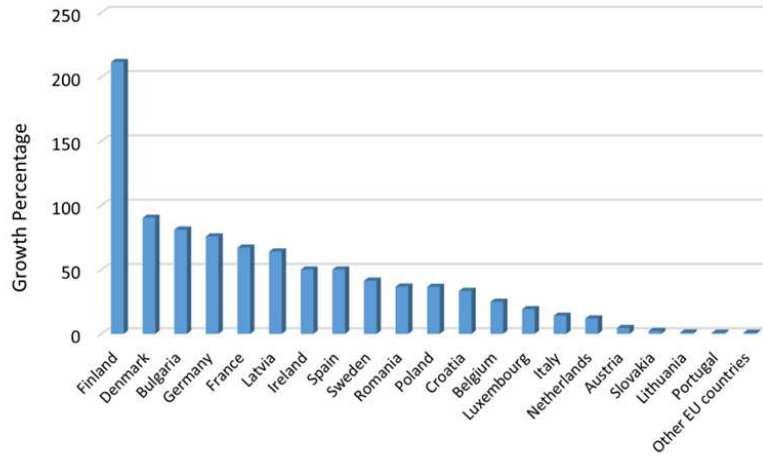


Figure 1.1: Growth percentage of battery electric buses in EU

One of the most important factors affecting the effectiveness of the bus transportation system is operational processes (L. Kang & Meng, 2017). Optimizing bus operation planning is essential to reduce operational costs and decrease passengers waiting time. Bus scheduling is one of the most vital processes in bus operations and has a considerable impact on operational costs. Bus/vehicle scheduling (VS) is the process of assigning buses to the trips of a given timetable and aiming to use the minimum number of buses while minimizing operating costs. Electric buses have battery limitations and their scheduling and charging processes differ from those of conventional buses (Yao, Liu, Lu, & Yang, 2020). The operating range of EBs is less than conventional buses and their charging time is longer than refueling diesel buses. Thus, a reasonable charging strategy is necessary to improve bus scheduling (Bie, Hao, & Guo, 2021).

The limited traveling range of EBs has prompted new research into the literature; the problem of locating charging stations for electric buses. This problem tries to determine the ideal places for fast-charging infrastructures to be installed on the bus transportation network, as well as the optimal number of such stations. To solve the long charging time and restricted driving range issue of electric buses, public transportation agencies adopted fast-charging technology with high-voltage electricity to recharge EBs in several minutes. Using such technology makes EBs scheduling more complex due to the need to account for frequent charging events. These events must be considered in both the bus scheduling and charging schedules, which depend on charger availability and time-of-use

electricity pricing. (Y. He, Liu, & Song, 2020). However, bus transit systems that use fast-charging technologies are gaining popularity. Also, charging stations at bus terminals are less costly than depot charging and better suited to bus electrification throughout the life cycle (Lajunen, 2018a).

Transitioning from traditional buses to electric buses requires further research on public transit operation planning steps. These steps are network route design, timetable development, vehicle scheduling, and crew scheduling. First, the network route design outlines the bus routes and their bus stops based on the regions' geographical layout and travel demand. Finding the best spots to locate charging stations or continuous charging infrastructures is another aspect of the network design step for electric buses. Next, the timetable development sets bus departure frequencies in order to cover the travel demand of passengers. When using fast chargers as the chosen charging technology, their short charging duration needs to be factored into the timetable. This means that the charging time for electric buses with pantograph chargers should be included as a new element in the timetable development. The timetable should be adjusted to account for the charging time, ensuring the correct departure and arrival times for the electric buses. In the vehicle scheduling step, buses will be assigned to bus routes based on the timetables, aiming to reduce operational costs. At this stage, another important problem arises: the charging scheduling of electric buses. This problem involves determining the location, timing, and amount of charging needed for the buses, with the goal of minimizing the total recharging costs. Bus scheduling and charging scheduling should be integrated to achieve a more cost-effective and efficient vehicle scheduling process. Finally, the last step determines the schedule for bus drivers (Häll, Ceder, Ekström, & Quttineh, 2019).

1.2 Problem Statement

By rapidly replacing conventional buses with electric ones, new challenges to the operation planning of EBs emerge. Due to the operational differences between conventional buses and electric buses, such as the limited travel range and long charging times of electric buses, bus transit operations need to be customized to enhance the level of service for the bus users while reducing operational costs. Thus, the network design of the bus transit system, bus timetabling, and bus scheduling necessitate rigorous reassessment. The adjustments required for the operational planning

steps after introducing EBs depends on the specifications of the adopted EBs and different charging technologies. Existing mathematical optimization models for conventional buses are not directly applicable to EBs due to their different operational constraints. Also, different charging technologies introduce new challenges to the operational planning of electric buses that current models cannot address. Fast charging technology, which is the focus of this research, has a significant impact on the charging station location planning, electric bus timetabling, and bus scheduling. According to the literature, there is limited research on integrating charging location planning with electric bus scheduling, as well as on combining bus timetabling with bus scheduling. Thus, new optimization models need to be developed to ensure that EBs operate efficiently while still providing a high level of service to public bus riders.

1.3 Research Questions

There are three research questions that we are trying to answer during this thesis. These three questions are as follows:

- (1) How does the operation planning of electric buses differ from conventional buses, and what are the key optimization challenges?
- (2) How can bus timetabling be adjusted to account for charging events by integrating timetabling and scheduling optimization?
- (3) How to integrate electric bus scheduling and fast charging station location planning problems efficiently?

1.4 Objectives and Contributions

This thesis tackles the optimization problems in operational planning arising in bus transit systems when transitioning to EBs. It identifies research gaps and explore potential directions for future studies, helping to develop more realistic and practical models and solution approaches for fully electric bus transit systems. Also, we propose mathematical optimization models to find the

optimum or near optimum operation plan for EBs, considering their charging schedules, limited travel range, and high purchase costs. To efficiently solve this problem, we adopt a state-of-the-art solution approach that reduces execution time. Furthermore, we conduct a sensitivity analysis and explore various scenarios to assess the model's robustness for improved decision-making.

The objectives that we aim to achieve are listed below:

- (1) Identifying the research gaps and exploring future directions by evaluating and classifying existing studies on EB operation planning to develop more realistic and practical models to operate electric buses more efficiently.
- (2) Integrating EB scheduling and timetabling problems for a shuttle bus service to balance passenger comfort and cost-effectiveness.
- (3) Proposing an integrated approach that combines EB scheduling with fast-charging infrastructure location planning. This approach incorporates a multi-depot setup and a partial charging policy, supported by a mathematical optimization model. The model efficiently manages EB charging schedules while considering the capacity constraints of fast-charging stations, ensuring seamless and cost-effective operations.
- (4) Developing a branch-and-price (B&P) algorithm to solve the EB scheduling and charging station location planning problem while considering multiple depots, partial charging, interlining, and time of use electricity pricing.
- (5) Applying the model on a real case study of Montreal and performing sensitivity analysis on different EB types based on original equipment manufacturers' real data.

Most of the studies have been conducted only on one step of the bus transit system operational and strategical scheme, i.e., scheduling and charging station planning. The main goal of this research is to integrate the steps involved in planning public electric bus transit operations. By doing so, we aim to improve the overall efficiency and performance of operating electric buses and fleet management. In this thesis, we seek to minimize the total operational costs of EBs and the number of required vehicles to cover all the given trips. Also, we seek to find the best locations and the

optimum number of fast chargers to be placed on the bus network to meet the charging demand for electric buses with minimum cost.

A list of contributions of this thesis is as follows:

- Conduct a comprehensive literature review on the operational planning of EBs to critically assess and classify existing studies, identify research gaps, and outline potential directions for future work. The aim is to support the development of more realistic and applicable models and solution approaches for fully electric bus transit systems.
- Introduce a new approach that integrates timetabling and bus scheduling to both enhance service levels and minimize operational costs. Service quality is improved by reducing student waiting times and ensuring seat availability for better in-vehicle comfort, while scheduling focuses on lowering total operational costs, including travel, electricity consumption, and electric bus usage.
- Complete integration of fast-charging infrastructure location planning problem and bus scheduling to come up with a better solution set compared to the sequential integration approach. We extend the previous electric bus scheduling problem presented by [J.-Q. Li \(2014\)](#) by taking into account various factors like the deadhead costs, usage cost of electric buses, establishment cost of charging stations, and the cost of waiting time of EBs.
- Developing an exact solution approach based on a B&P method to solve the proposed NP-hard problem to achieve optimum or near optimum solution even for large-scale problems, considering the charging scheduling of EBs based on the bus schedule, charging station capacity, time-of-use (TOU) electricity pricing, multi-depot setup and partial charging policy.

1.5 Thesis Organization

This thesis comprises a chapter of introduction, three peer-reviewed journal papers, and a chapter of analysis of different scenarios with different parameters and integration of the research with the Tools4Cities platform. Chapter 1 describe the motivation and necessity of focusing on the operational planning of electric buses. This chapter also arises research questions, objectives,

and main contributions of this thesis. Chapter 2, which includes both the first paper and the book chapter, provides a critical review and classifies the works done in electric bus operation planning problems and the potential future studies that could have been done. This chapter aims to highlight the research gaps and potential directions for future studies in this domain to develop more realistic and applicable models and solution approaches for fully electric bus transit systems. Chapter 3 proposes a new approach to integrate timetabling and bus scheduling to enhance the level of service and minimize operational costs, using a case study of a university shuttle bus service in Montreal, Canada. The proposed approach ensures a smooth transition to a fully electric transit system for shuttle bus services. Chapter 4 presents a path-based optimization model, formulated as an Integer Linear Program, for the joint optimization of EB scheduling and fast-charging infrastructure location planning. Four heuristic branch-and-price algorithms are presented to solve the problem and determine the optimal bus schedules, charging station locations, and EB charging timetables, all while minimizing costs and achieving reasonable computation times in extensive experiments. The proposed model is also applied to a real-world case study in Montreal to demonstrate its effectiveness and practical applicability. Finally, Chapter 5 presents the conclusions of this dissertation and highlights potential directions for future research.

Chapter 2

Electric Bus Scheduling, Timetabling, Fast Charging Infrastructure Planning, and their Impact on the Grid: A Review¹; Electrification of Buses²

2.1 Overview

Public transportation is essential for making urban areas greener and more sustainable. Having a completely electric and efficient public transportation network would reduce greenhouse gas emissions within cities. Transit agencies are increasingly embracing electric buses (EB) as an energy-efficient and emission-free alternative to conventional bus fleets. They are rapidly replacing conventional buses with electric ones. As a result, emerging challenges of electrifying public transportation bus networks in cities should be addressed. Introducing electric buses to the bus transit system would affect the public transit operation planning steps. The steps are network design, timetabling, bus scheduling, and crew scheduling. Regarding the functional and operational differences between

¹Alamatsaz, K., Hussain, S., Lai, C., & Eicker, U. (2022). Electric bus scheduling and timetabling, fast charging infrastructure planning, and their impact on the grid: A review. *Energies*, 15(21), 7919.

²Alamatsaz, K. & Quesnel, F. (2024). Electrification of Buses, “A Word Scientific Encyclopedia of Next-Generation Cities Institute (NGCI), Volume 3”, for World Scientific Publishers.

conventional buses and electric buses, such stages should be changed and optimized to enhance the level of service for the users while reducing operating costs for service providers. Many mathematical optimization models have been developed for conventional buses. However, such models would not fit the electric buses due to EBs' limited traveling range and long charging time. Therefore, new mathematical models should be developed to consider the unique features of electric buses. Thus, a comprehensive literature review has been done in this field to critically review and classify the works done in this scope and the potential future studies that could have been done. The considered papers cover the integration of timetabling and vehicle scheduling, recharging scheduling planning, fast charging infrastructure location planning. The main goal of this research is to highlight the research gaps and potential directions for future studies in this domain to develop more realistic and applicable models and solution approaches for fully electric bus transit systems.

2.2 Introduction

Greenhouse gas emitting energy sources are responsible for global warming. Thus, replacing such energy sources with clean and renewable sources of energy has become crucial during the past few decades. According to Figure 2.1, transportation is one of the significant emission sectors, contributing 22% of the total CO₂ emissions. Road transport accounts for three-quarters of transport emissions and 15% of total CO₂ emissions (Nejat et al., 2015). That is why it has become an attractive area for potential emission reduction (Miles & Potter, 2014). Electrifying transportation is a solution to address both urban air pollution and the energy crisis. We are witnessing a rapid increase in electric vehicles and electric buses because of the increasing concerns about air quality, greenhouse gas emissions, and energy demand. In the UK alone, buses produce approximately 4.3 million tons of emissions, underscoring the urgent need for a transition to cleaner energy sources. The Economic Consequences of Outdoor Air Pollution reported by OECD ("The Economic Consequences of Outdoor Air Pollution", n.d.) highlights the severe impact of air pollution, which caused nearly three million early deaths in 2010, with projections indicating that annual deaths could rise to between 6 and 9 million by 2060. This alarming data underscores the critical need for a shift from pollutant energy sources to clean and renewable energy options. Research findings from Lie,

Synnevåg, Lamb, and Lien (2021) suggest that adopting biofuel and electric buses could reduce the carbon footprint of bus fleet by 37%, while a full shift to electrification could cut it by up to 52%. In response to these challenges, China has made a significant move by transitioning its bus fleets to fully electric vehicles. By 2019, China's public transport had integrated around 400,000 electric buses, making up 99% of the global EB fleet, demonstrating a proactive approach to reducing emissions and combating air pollution (Margolis, 2019).

The electrification of buses could significantly reduce environmental concerns, decrease the exploitation of natural resources, and provide better fuel economy and greater energy efficiency (Deng, Liu, Chen, & Liang, 2019). According to Miles and Potter (2014), electrifying public buses will improve living conditions in metropolitans. Other advantages of using electric buses are their low noise level and regenerative braking system for recovering energy (J.-Q. Li, 2016). On the other hand, EBs' operational range is shorter than diesel buses, and their recharging process for depot charging is considerably more time-consuming than conventional ones. Furthermore, the cost of electric buses and their batteries, charging infrastructures and their establishment cost are significantly higher than conventional buses.

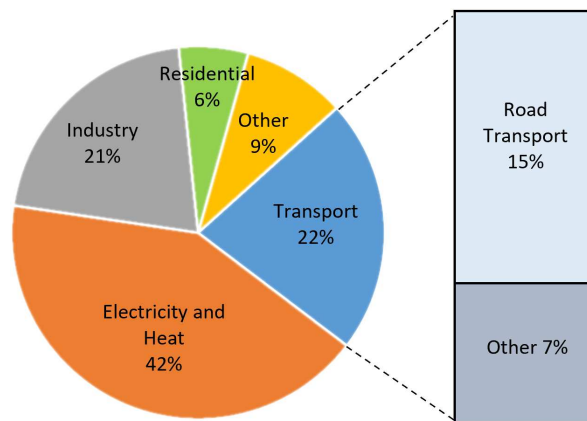


Figure 2.1: Total CO₂ emission of different sectors and the portion of road transport Nejat et al. (2015).

Cities are struggling to improve their public transport systems' efficiency, especially bus transit systems. Operational processes are one of the most critical aspects of the bus transit system's performance (J. Kang et al., 2017). Bus timetabling and scheduling are among the most vital processes in

bus operations. Bus timetabling (TT) aims to determine the departure and arrival times for all trips and routes in the network. It seeks to maximize passengers' satisfaction through minimizing the waiting time, transferring time, and increasing seat availability (Y. Zhang, Hu, Meng, & Ralescu, 2020). The process of assigning vehicles to the trips of a specified timetable is known as vehicle scheduling (VS). It aims to use the minimum number of vehicles while minimizing operational costs. Bus scheduling has a notable impact on operational costs and passenger travel times. With the increase of electric buses, a new set of scheduling and timetabling problems of public electric bus transit systems have emerged. In such new problems, electric buses' limited driving range and long-lasting recharging should be considered. For example, charging during off-peak hours could reduce both the fleet size and impact on the grid. Thus, timetabling and bus scheduling should be coordinated and changed regarding the new constraints of electric buses to satisfy both bus operators' and public users' interests. Furthermore, to improve bus schedules, a reasonable charging strategy is required (Bie et al., 2021).

The limited traveling range of EBs has prompted new research into the literature; the problem of locating charging stations for electric buses. This problem aims to find the best locations for fast charging infrastructures to establish on the bus transit network while determining the optimum number of such stations. Public transportation agencies introduced fast charging technology with high voltage power to recharge e-buses in several minutes to address long charging times and limited driving range issues. On the other hand, fast-charging station location planning make battery-electric bus scheduling more complex (Y. He et al., 2020). However, bus transit systems that use fast charging technologies are gaining popularity. This approach needs an extensive infrastructure to install charging stations along bus routes. Moreover, compared to depot charging, charging stations at bus terminals are less expensive and more suited to bus electrification throughout the life cycle (Lajunen, 2018b).

Most studies deal with public transit operation planning steps sequentially. This means that the output of an operational planning step would be the input of the subsequent step. The drawback of this approach is the inefficient public transit operation compared to the complete integrated approach. The complete integration approach investigates the problem as a whole integrated problem that simultaneously considers each step of public transit planning. For instance, slight changes in

the timetable of buses could result in a better vehicle schedule, and determining the location of fast charging infrastructures based on the bus schedules could reduce the operational costs of vehicle scheduling in a few years.

2.2.1 The Surveying Method

We wrote this review paper based on a methodological framework by choosing several keywords to look for papers that fit the scope of this article. The selected keywords were *electric bus scheduling*, *electric bus timetabling*, *fast charging location*, and *charging schedule fast charging infrastructure*, each searched individually. Also, two other keywords, *Vehicle to Grid (V2G)* and *impact on the grid*, were searched along with the mentioned keywords. After the articles' titles and abstracts were initially reviewed, the relevant papers were thoroughly examined, their contents were analyzed in detail, and each study's methodology was explained. Google scholar was the scientific database used to track the articles that fall within the purview of this study. Table 2.1 illustrates the comparison of this survey with other review papers in this scope.

This literature review has been structured as follows: Different types of electric buses and their charging technologies will be discussed in section 2.3, the theoretical background and related works of fast charging infrastructure location planning, scheduling, and timetabling of electric buses are reviewed in section 2.4. Section 2.5 describes the challenges and limitations of electric buses planning. Finally, section 2.6 addresses the future research, potential research directions, and gaps in earlier studies.

Table 2.1: Comparing different review papers with the current study

Review Paper	Comparing Charging Technologies	VS VS	TT- VS	Charging Infrastructure Location	Charging Scheduling	Impact on the Grid	Remark
J.-Q. Li (2016)	✓					✓	Analyzing the vehicle cost, energy cost, and emissions of buses powered by different sources.
Mahmoud et al. (2016)	✓						Reviewing environmental, economic, and energy efficiency of electric buses.
DaSilva and Mohamed (2020)		✓					Categorizing planning, case studies, and simulation of electric buses.
Deng et al. (2019)	✓				✓	✓	Reviewing power management, travel range limitation, energy storage system sizing.
Perumal et al. (2021)		✓		✓	✓		An overview of strategic, tactical and operational problems.
Manzolli et al. (2022b)	✓						Future research on EBs will be strategies for energy and fleet management and sustainability.
Rong et al. (2021)		✓			✓		Integration of EB charging scheduling and vehicle scheduling for improving economic attractiveness.
This work	✓	✓	✓	✓	✓	✓	Public transit operation planning integration and impact of charging stations on the grid.

2.3 Different Types of EBs and Charging Technologies

Electric buses are mainly divided into three groups: hybrid electric, fuel cell electric, and fully electric, as shown in Figure 2.2. Hybrid electric buses are further categorized as parallel, series, and series-parallel. In a parallel design, both the combustion engine and electric motors could propel the bus, unlike the series type in which only the electric motor is used for propulsion. The combustion engine supplies the energy of the electric motor. The combination of these two types is known as series-parallel, which benefits from the advantages of both types (Mahmoud et al., 2016). Hybrid electric buses could travel longer compared to fully electric ones, and they have very little emission. The issues related to this kind of bus are managing the sources of their energy and optimizing the size of engines and batteries. Fully electric buses only rely on electric power stored in their batteries to operate. They are not dependent on oil, so they have no emission, and the travel range of such

EBs is dependent on the capacity of the batteries. On the other hand, the high price batteries and long charging time along with sparse charging stations are the main issues of this kind of buses.

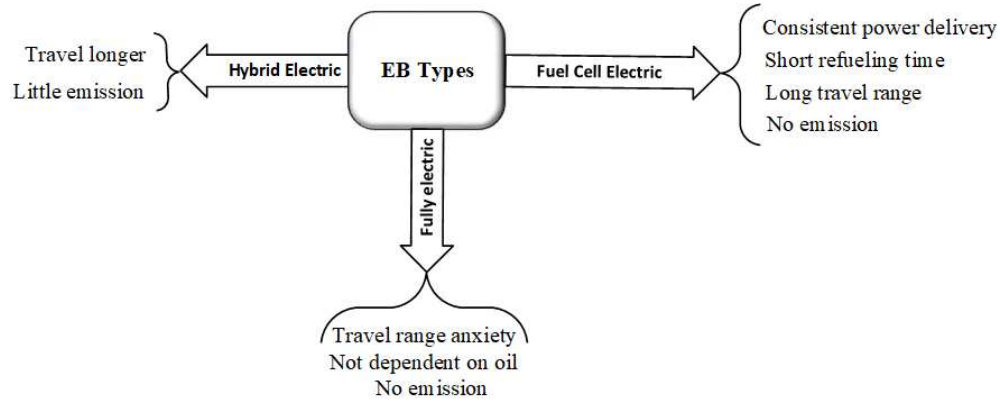


Figure 2.2: Different types of EBs

Table 2.2 outlines the environmental, economic, and energy-related distinctions between EBs, hybrid buses and diesel buses (DB). It provides a comparative overview of different types of buses, outlining their differences in purchase and operational costs, energy efficiency, CO₂ emissions, operational range, and noise levels. It serves as a guide for transportation authorities to assess the economic and environmental impacts of integrating these different types of buses into their fleets. Table 2.2 is designed to assist decision-makers in considering the short-term and long-term benefits and potential trade-offs associated with each bus type, facilitating informed choices in pursuing sustainable and cost-effective public bus transportation solutions.

Table 2.2: Functional variations between diesel, hybrid and EBs (Hallmark & Sperry, 2012; Mahmoud et al., 2016; Quarles et al., 2020; Ranganathan, 2012)

Bus type	Purchase cost (€)	Operation cost vs DB	Energy efficiency	CO ₂ (gr/km)	Range (km)	Noise
Electric	550,000	80% less than DB	6.76 MJ/km	0	72-150	Less
Hybrid	400,000	39% less than DB	10.81 MJ/km	417	175-350	More than EB
Diesel	250,000	-	20.66 MJ/km	822	400-1100	Most

Table 2.3 presents a comparison of several renowned EB models from various manufacturers. It outlines three key parameters of EBs: energy consumption, battery capacity, and maximum travel range. It is worth mentioning that the ranges given for BYD and Proterra account for the variations

across their respective models for these parameters. Table 2.3 highlights the range of specifications offered by the EB market, designed to meet a variety of transit requirements and efficiency standards.

Table 2.3: Comparative features of various well-known electric bus models (Aamodt et al., 2021; An, 2020; Bi et al., 2017, 2015; Fang et al., 2017; Mahmoud et al., 2016; Manzolli et al., 2022a; Shin et al., 2013)

Original Equipment Manufacturer	Energy Consumption (kWh/km)	Battery Capacity (kWh)	Travel Range (km)	Charging Technology
BYD	0.64-1.75	128-591	200-322	Vary
Proterra	1.09-1.17	440-660	404-563	Vary
New Flyer	1.09	454	418	Depot charging & battery swapping
Designline	1.31	261.8	200	Vary except fast charging
Chongqing Hengtong	1.55	77.6	50	Fast charging & wireless charging
Irizar	1.50	376	250	Depot charging & battery swapping
EBusco	1.04	311	300	Depot charging & battery swapping

Different charging strategies for electric buses are fast/quick charging, depot/overnight charging, battery swapping, and continuous charging. Quick or fast charging requires a large amount of voltage to recharge buses in a short time (a few minutes). Depot or overnight charging refers to the charging poles which use less voltage to recharge buses but in a longer time. In battery swapping, electric buses' batteries will be replaced with new charged ones. Finally, continuous charging includes wireless charging and overhead lines. With this type, buses are charged all the time during their trips. Table 2.4 summarizes the information about different charging technologies and bus features for each charging type (For more detailed information, readers are referred to Aamodt et al. (2021)).

Table 2.4: Recommended features of buses and charging stations as a function of charging technologies (Aamodt et al., 2021; Ahmad et al., 2020; Alwesabi et al., 2021; An, 2020; Bi et al., 2017; Fang et al., 2017; J.-Q. Li, 2016; Manzolli et al., 2022b; Mathieu, 2018)

		Depot Charging	Fast Charging	Wireless Charging	Battery Swapping
Bus	Type	Hybrid/fully electric	Fully electric	Fully electric	Hybrid/fully electric
	Battery capacity (kWh)	>200	40-120	18.1, 123-201	324, 320-590
	Range (km)	175-350, 322	40-110, 32-48	unlimited theoretically	72-150
	Weight	More	Less	Less	Depending on the battery size
	Cost (bus + battery) (\$1000)	Hybrid: 650, 624, BEB: 806, 729	806	591	>650
Charger	Power (kW)	40-50, 65-150, 30-50	Up to 600, 200-500 350-450 350-600, 100-600	60, typically 100	9, 60
	Time	4-6 hours typically, 2-4 hours	5-10 minutes	3-4 hours, no time for en-route charging	15 min
	Transformer	No need to upgrade	Need to upgrade	-	-
	Grid stability	High	Low	-	High
	Location	Bus depot	Certain bus stops	Certain bus stops/bus depot	Specific spots

Depot charging:

It has high efficiency with multiple levels. This charging technique could help reduce the grid power loss by using the vehicle to grid (V2G) configuration. This technology allows EBs to send stored energy from their batteries back to the grid, helping to meet energy demand, particularly during peak times, and supporting grid stability. Compared to fast charging technology, this charging technology does not require upgrading in its lifetime. Depot chargers use lower power and often take place overnight, making them less demanding on the grid and infrastructure. In contrast, fast chargers demand significantly higher power levels compared to depot charging. This increased power requirement necessitates the installation of more advanced charging infrastructure, including higher-capacity transformers and more robust power distribution systems. Also, fast charging can place more strain on the local grid, requiring grid upgrades to handle the higher loads. Depot chargers provide higher grid stability and higher profit for bus depot operators. On the other hand, they have a complex infrastructure and are highly dependent on the electricity grid restrictions. The weight and cost of buses that use this charging technique are higher in comparison to other charging

technologies. The recharging procedure is time consuming, as it takes 4 to 6 hours to completely charge a battery. Moreover, if bus depot operators use a vehicle-to-grid configuration, the lifetime of batteries will decrease.

Fast charging:

This charging technique is the most efficient way to recharge electric buses in terms of time. Their charging time is somewhere between 5 to 10 minutes. As the capacity of electric buses which use this charging technology is lower than buses that use depot or battery swapping technology, their weight and cost are much lower. However, it will provide a limited travel range. Additionally, fast charging can strain the local grid, requiring upgrades to handle the higher power demands, which can lead to reduced grid stability. For more information and explanations readers are referred to [Yoldaş, Önen, Muyeen, Vasilakos, and Alan \(2017\)](#) and [Habib et al. \(2018\)](#). Fast chargers provide several key advantages for electric buses. They greatly reduce charging time, allowing buses to quickly recharge during short stops. This makes it possible to operate on longer routes compared to buses that rely solely on overnight depot charging. Because the charging process takes very little time, service schedules face minimal disruption. Moreover, buses using fast chargers can be equipped with smaller batteries, reducing both vehicle weight and overall costs while still meeting operational needs.

Pantograph charging and plug-in DC charging are the two main types of fast charging. Fast chargers are typically more costly than depot charging. To establish charging stations along the routes, agencies might need to rent or purchase land and right of way. Transit agencies may face issues such as deliberate vandalism, damage from road accidents, complaints from nearby residents who dislike having chargers close to their homes, and technical limitations, since chargers shut down in temperatures below -20°F. Also, it may be more difficult for agencies to repair or maintain fast chargers when these or other issues arise, since maintenance specialists often need to travel long distances to reach them. Additionally, if one fast charger is not working, it might occasionally affect the dependability of the transit service when relying only on fast charging infrastructure ([Lepre, Burget, & McKenzie, 2022](#)). The cost of pantograph charging stations for battery electric buses is much higher than stationary overnight depot charging. However, the battery cost for buses that use overnight charging is higher than fast-charging electric buses. Plug-in DC fast chargers are less

costly to install than pantograph systems, but they are typically located at depots and require manual connection for each charging session, unlike the automated pantograph chargers.

Battery swapping:

Quick battery replacement, benefit from V2G technology, and extended battery life due to slow charging are the main advantages of battery swapping technology. Nevertheless, the high initial cost and large land requirements for setting up such charging stations are major drawbacks of this technology. It requires a huge investment to rent a large area to store the batteries, large number of expensive batteries and equipment.

Wireless charging:

The recharging procedure is safe, convenient, and there is no need to stop for recharging. Furthermore, there is no need for a standard connector compared to conductive charging. On the other hand, it does not provide strong power and it has a low range of power transmission. Moreover, it requires a huge amount of investment for en-route charging infrastructure on the roads.

The advantages and disadvantages of each charging technology are as follows (see Table [2.5](#)):

Table 2.5: Advantages and disadvantages of different charging technologies

Charging Technology	Advantages	Disadvantages
Depot charging (Tan, Ramachandaramurthy, & Yong, 2016), (Yong, Ramachandaramurthy, Tan, & Mithulananthan, 2015)	Multiple charging level Providing V2G configuration High efficiency Less grid loss No need to lease property around service area The upfront capital cost is often cheaper	Batteries' lifetime will decrease due to V2G operation Long recharging time Increases deadhead trips to/from depot Larger battery packs, more weight and cost Route restrictions due to EBs' range limits
Fast charging (Yoldaş et al., 2017), (Habib et al., 2018)	Less recharging duration Covers longer routes than depot charging Little time loss for recharging Requires smaller batteries	Voltage instability High infrastructure cost Difficult to place chargers in crowded areas
Wireless charging (Tran, Choi, et al., 2017), (D. Patil, McDonough, Miller, Fahimi, & Balsara, 2017)	Safe and convenient recharging No need for sockets or connectors Can recharge while moving	High infrastructure cost Low power transmission range Weak power transfer
Battery swapping (T. Zhang, Chen, Yu, Zhu, & Shi, 2018), (Sun, Tan, & Tsang, 2017), (Martínez-Lao, Montoya, Montoya, & Manzano-Agugliaro, 2017)	Quick battery replacement Prevents battery degradation from slow charging Supports V2G for demand balancing	Require high budgets for the facility Requires a large budget for battery purchases Requires significant space for battery swapping equipment

2.4 Theoretical Background and Related Works

In this section, first, a brief background regarding different types of charging station location planning approaches will be described. Then, a detailed review of literature on the fast charging infrastructure location and recharging planning will be mentioned. In the next part, a concise overview of bus scheduling and timetabling will be presented, followed by a comprehensive investigation of studies focusing on the integration of bus scheduling and timetabling.

2.4.1 Charging Station Location Planning

There are several main approaches for locating charging infrastructures for electric vehicles as follows:

Node-based approach:

This approach would satisfy the charging demand by finding the best locations/nodes for chargers among the potential set of nodes. P-median and p-center are among the most common models to deal with node-based approaches. The p-median model minimizes the average distance between the charging demand nodes and the closest charging infrastructures (Jung, Chow, Jayakrishnan, & Park, 2014). Charging demand nodes are defined as the centroids of predetermined zones, each representing a different level of charging demand. In comparison, p-center aims to minimize the maximum distance between demand nodes and charging infrastructure location (Eisel, Schmidt, & Kolbe, 2014). Baouche, Billot, Trigui, and El Faouzi (2014) optimized the charging station locations concerning the p-dispersion constraint. The p-dispersion constraint selects p charging stations from a set of candidate sites while keeping them as far apart as possible. In this model, no two stations can be located closer than a predefined minimum distance. Other classic facility location approaches are maximal covering and set covering. The former seeks to cover the most charging demand of EVs (Asamer, Reinthaler, Ruthmair, Straub, & Puchinger, 2016), and the latter tries to minimize the total installed charging infrastructures while satisfying all the charging demands (S. Y. He, Kuo, & Wu, 2016).

Flow-based approach:

The flow-based approach is more suitable than the node-based approach for vehicles since the cars move along the roads instead of remaining idle at specific nodes. Kuby and Lim (2005) developed a flow refueling location model based on the flow-capturing approach to find the best locations for refueling stations. Maximizing the total volume of flow was the objective of this study.

Path-based approach:

Unlike the past two approaches, which focused on flows or nodes, the path-based focuses on vehicles paths. Path refueling location model and sub-path location model are the two branches of this approach. The objective is to find the best places for the charging infrastructures on a path to ensure that vehicles can complete their trips on that path (Mak, Rong, & Shen, 2013; H. Zhang, Moura, Hu, Qi, & Song, 2017).

Equilibrium-based approach:

This approach analyzes the impact of charging stations' locations on the travel behavior of

electric vehicle drivers (Z. Chen, He, & Yin, 2016; F. He, Yin, & Zhou, 2015). An equilibrium modeling framework was presented by F. He, Wu, Yin, and Guan (2013) to maximize the social benefit regarding the plug-in hybrid electric vehicles (PHEV) recharging routing. The objective of the study was to find the best places for charging stations for PHEVs. They considered the availability of charging stations, route choice of electric vehicles, and electricity price to formulate a mathematical allocation model.

According to Lin, Zhang, Shen, Ye, and Miao (2019), the path-based approach is the most common approach for solving charging station location problems for electric buses. Such problems are classified into two categories: charging lanes and charging stops. The former has attracted significant attention recently and is the most advanced charging technology. In Ko and Jang (2013), wireless charging location planning for lanes and optimizing battery capacity is studied and particle swarm optimization were used to find the best location and optimum battery capacity. Z. Liu and Song (2017) studied the same problem while considering the uncertain nature of electric buses' energy consumption. They dealt with the uncertainty of buses' energy consumption and travel time by adopting a robust optimization approach followed by the affinely adjustable robust counterpart (AARC). In a more recent study, Helber, Broihan, Jang, Hecker, and Feuerle (2018) investigated the wireless charging location planning for airport's electric shuttle buses. For the latter, electric buses are supposed to be charged only on several specific spots on the road network. Kunith, Mendelevitch, and Goehlich (2017) presented a capacitated set covering problem to solve a joint fast-charging infrastructure location and battery capacity problem. The authors developed a mixed-integer linear programming (MILP) formulation to find the optimum number of chargers and their optimal locations. For inductive and conductive charging infrastructure planning, Xylia, Leduc, Patrizio, Kraxner, and Silveira (2017) studied a case study in an urban context of Stockholm. The objective was to minimize the total energy consumption and operational costs of the system.

In the overnight or depot charging strategy, the location of depots is a critical factor. Charging impacts on the grid (Mohamed, Farag, El-Taweel, & Ferguson, 2017), establishing costs of charging stations, and operational costs such as deadhead trips would be affected by charging station locations (Y. Liu, Feng, Ding, Hua, & Ruan, 2020). Another important issue addressed by Uslu and Kaya (2021) and Rogge, Van der Hurk, Larsen, and Sauer (2018) is the trade-off between the accessibility

of overnight charging stations and the total establishment cost of such stations. Moreover, the possibility of upgrading existing depots with charging infrastructure should be considered (Lin et al., 2019). Also, the location where the continuous charging infrastructure is located plus the electrified distance of the road affect the network planning of bus lines, too. There is no paper considering the effects of wireless charging on the network route design of bus lines. By introducing fast chargers for electric buses, a group of new location planning problems has emerged in the scope of the network design problem. Fast charging infrastructures are new inputs for the network route design, and the best location for bus stops and such infrastructures should be considered simultaneously (W. Zhang, Zhao, & Song, 2021).

2.4.1.1 Fast Charging Infrastructure Location Planning (FCILP)

Electric buses have not yet gained widespread adoption compared to other electric vehicles due to their high ownership and operating costs, long charging times, and uneven spatial distribution of charging infrastructure. Furthermore, dynamic environmental factors like unanticipated traffic congestion, varying travel demand, and even different weather conditions can considerably impact electric buses' charging efficiency (G. Wang et al., 2020). Another critical factor is the lack of adequate charging infrastructure for large-scale EB fleets. As a result, studying the location planning of charging infrastructure for electric buses is essential. Pantograph chargers can decrease the number of deadhead trips to and from the depots, decrease the battery capacity, and minimize energy usage compared to standard overnight/depot charging. Although fast charging technologies can shorten the time it takes for buses to charge, they will significantly increase operational costs. As a result, the cost of installing and location of fast-charging stations should be factored into the planning process.

Kunith, Goehlich, and Mendelevitch (2014) performed one of the first studies of fast charging infrastructure planning in 2014. They aimed to find the optimum number of fast chargers and their best location on the network, considering the operational constraints. The distribution and bus operating networks have been considered for this problem in (X. Wu et al., 2021). The model's goals were to reduce the total installation costs of fast chargers, their operating and maintenance

expenses, travel cost to the chargers, and the cost of their power loss. To find the number of required fast charging infrastructure, the Affinity Propagation method was used and the best locations of chargers and their optimum capacities were obtained by the Binary Particle Swarm Optimization (BPSO). Another optimization method to deal with the fast charging infrastructure location planning is Enhanced Heuristic Descent Gradient (EHDG). [Othman, Gabbar, Pino, and Repetto \(2020\)](#) used this algorithm to find the optimum locations for placing fast chargers. [Csonka \(2021\)](#) solved the problem for both static and dynamic charging to find the best locations for charging infrastructures. Static charging and dynamic charging refer to conductive charging stations and overhead wire charging lanes, respectively. The amount of charging in each charging event was a decision variable in this study.

Bus systems that use fast charging technologies are gaining popularity. This trend necessitates extensive studies on optimizing the location planning of establishing charging station infrastructures along bus routes. [Kunith et al. \(2017\)](#) proposed a mixed-integer linear programming model to simultaneously optimize electric buses' fast-charging infrastructure planning and battery capacity for each line. Through a set covering problem, the best locations and the optimum number of chargers were determined. [Y. He, Song, and Liu \(2019\)](#) addressed the same problem by considering installing an energy storage system to store the energy in off-peak hours and supply it to fast charging infrastructures in on-peak hours. A reduction of 9.2% in the total system costs was the result of this study compared to [Kunith et al. \(2017\)](#) work. [Olmos, López, Gaztañaga, and Herrera \(2019\)](#) investigated the problem of locating opportunity charging infrastructure for hybrid and fully electric buses to find the best location for such facilities. The authors also aimed to determine the best power rate of opportunity charging infrastructures and the size of energy storage systems.

Battery aging and partial charging for the problem of charging station location have been considered in [\(Rohrbeck, Berthold, & Hettich, 2018\)](#). The objective was to reduce total costs, which include the price of establishing charging stations and the costs of purchasing the vehicles that will be used. [Z. Liu, Song, and He \(2018\)](#) dealt with the problem while considering uncertain energy consumption for battery-electric buses. They developed a mixed-integer linear programming model based on a robust optimization approach to find the minimum total implementation cost. The combination of charging station location planning and the power grid is studied in the paper of [Lin et al.](#)

(2019) in 2019. Other extensions of this problem are determining the best capacity for e-buses (Kunith et al., 2017), designing transit route networks (W. Zhang et al., 2021), and charging schedule for each fast-charging infrastructure (Y. He et al., 2020).

In a recent study by Hu, Du, Liu, and Perez (2022), the combination of opportunity charging location and charging scheduling problem of EBs is studied. They considered time-of-use electricity pricing and added the waiting time of passengers due to the charging process during the trips as a penalty cost. They aimed to minimize the cost of purchasing both opportunity chargers and electric buses' batteries, reduce the total charging costs and minimize the passengers' extra waiting time. To address the uncertainties related to trip time and passenger travel demand, a robust optimization technique was suggested.

The combination of depot charging planning and electric bus scheduling (EBS) has been considered in the paper of Olsen and Kliewer (2022). The objective was to minimize the total cost of installing depot chargers, vehicle costs, and operating costs. A metaheuristic solution approach based on variable neighborhood search (VNS) was developed to solve this problem. They showed that simultaneously optimizing these two problems would yield better results than sequential planning. The multi-depot and multi-vehicle type version of this problem for refueling charging stations was studied by L. Li, Lo, and Xiao (2019) in 2019. They aimed to minimize the number of required buses and refueling stations, maintenance costs, energy consumption, and external emission costs. They developed an integer linear program to solve the small-scale problems and proposed a time-space bus flow network to deal with large-scale problems. Alwesabi et al. (2021) presented a mixed-integer linear programming formulation to simultaneously find the optimum fleet size, battery capacity, and dynamic wireless charging locations. The combination of electric bus scheduling and fast charging infrastructure location planning has not been widely studied in the literature. The closest research to this topic is the work of Stumpe, Rößler, Schryen, and Kliewer (2021). The authors studied the simultaneous optimization of electric bus scheduling and opportunity charging location planning. They proposed a new mixed-integer linear formulation and solved it using VNS.

Some public transit operators cooperate with companies that provide operational and strategic planning services. M. Li, Tang, Lin, and He (2022) addressed the fast charging infrastructure location planning in Chengdu by developing an MILP model to maximize the net present value of

investment for the service provider and minimize the present value of cost, including deadheading cost and charging services costs. The authors addressed the location problem in a multistage scenario due to the gradual transport electrification. [Tzamakos, Iliopoulou, and Kepaptsoglou \(2022\)](#) studied the fast wireless charging infrastructure location planning, considering the impact of delays caused by buses queuing up to charge at charger locations. They developed a MILP model to minimize the total the expenses of building wireless charging. The details of each study is presented in Table 2.6.

Since the electric power demand of fast charging infrastructures is high and charging during peak hours would put much pressure on the grid, charging scheduling of EBs using such charging technology is challenging. Without paying attention to charging scheduling, the energy cost of this charging type would heighten and the economic viability of switching to electric bus transit systems would lessen. [Y. He et al. \(2020\)](#) addressed this problem by proposing a network modeling framework. They minimized the total charging costs, including energy and electricity demand charges. (For more information, see ([Y. He et al., 2020](#))) Another critical issue of implementing fast charging infrastructures for electric buses is the congestion of EBs at the charging stations due to the lack of available stations. [Abdelwahed, van den Berg, Brandt, Collins, and Ketter \(2020\)](#) addressed this issue by presenting two mixed-integer linear programming models. They also considered the impact of the recharging schedule on the grid by considering off-peak and on-peak charging periods. One way to deal with the congestion of electric buses at fast charger stations is bus holding strategy. Bus holding is used when a bus arrives early or too close to the one ahead; in such cases, the bus is held at a stop for a short time before continuing its trip. [Gkiotsalitis \(2020\)](#) determined the best departure times of buses based on bus holding strategy while considering the scheduled charging times. Minimizing the charging time of EBs during the service time is studied by [R. Patil, Rahegaonkar, Patange, and Nalavade \(2022\)](#).

[Y. Wang, Huang, Xu, and Barclay \(2017\)](#) proposed an optimization model to find the best recharging schedule for electric buses while considering planning decisions such as finding the best number of chargers, their locations, and station capacity. The combination of optimal charging scheduling of electric buses and drivers' mealtime time window has been discussed in ([Y. Jiang & He, 2022](#)). Battery degradation and bus-to-grid (B2G) technology was studied by [Manzolini et al.](#)

(2022a) for battery electric bus charging optimization. The objective was to minimize the charging cost of a real case study in Portugal. They provided a mixed-integer linear programming model for this problem and solved it with IBM ILOG CPLEX. There are other approaches such as Lagrangian relaxation for depot charging (Huang, Wang, Jia, Liu, & Wang, 2022), Progressive Hedging Algorithm (PHA) for both fast charging and battery swapping (Huang & Wang, 2022), and Genetic Algorithm (GA) (Hu, Du, & Perez, 2021) that could be adopted to deal with charging scheduling of electric buses.

2.4.2 Bus Timetabling

From the passengers' perspective, the definition of a good bus timetable may vary. One may recognize a good timetable with its regularity, while one prefers a timetable with the exact headway or frequency that s/he wants, or a good quality timetable may be represented by the minimum difference between the actual and desired frequency. Thus, determining a timetable that satisfies everyone would be challenging due to several objectives that should be satisfied. Minimizing the waiting time of passengers and maximizing the synchronization of bus line departures are the main objectives of timetabling. Improving comfort and the reliability of the service are the other objectives of the bus timetabling problem.

Researchers addressed the timetabling problem by taking various angles; Multi-objective models, which simultaneously reduce the passengers' waiting time and increase bus utilization (A. A. Ceder, Hassold, & Dano, 2013; Teng, Chen, & Fan, 2020; S. Zhang, Ceder, & Cao, 2020), maximizing bus frequencies to satisfy the passengers' travel demand (Shang, Huang, & Wu, 2019), and balancing average even passenger loads on buses to reduce overcrowding (A. Ceder & Philibert, 2014). According to Häll et al. (2019), depot charging does not affect electric bus timetables. Thus, the timetabling approaches and their methods for conventional buses could be implemented on EBs without any modifications. However, fast-charging affects the development of bus timetabling. The time of charging, which is about 5 to 10 minutes, should be considered for generating the timetable. Although fast charging technology affects the timetabling problem (the recharging time should be added as an input to the problem), there is no study in the literature modeling the problem of electric bus timetabling specifically. Note that overnight and continuous wireless charging have no impact

Table 2.6: Comparison of different studies on EBs' charging infrastructure location planning

Paper	Objectives and Decision Variables	Charging type	Model	Algorithm	Case study	Remarks
Kunith et al. (2014)	Optimum number of fast charging stations; min construction cost	Fast charging ¹	MILP	standard solver	-	Considering battery charging behavior and operational constraints
Kunith et al. (2017)	Best locations and the optimum number of chargers and battery capacity	Fast charging ¹	MILP	standard solver	Berlin, Germany	Capacitated set covering problem
Rogge et al. (2018)	Min the total cost of ownership	Depot charging	MILP	Grouping genetic algorithm	Germany & Denmark	Min vehicle investment, charger investment, operational costs, and energy expenses
Rohrbeck et al. (2018)	Min total costs: the price of establishing charging stations and purchasing cost of vehicles	Opportunity charging	MILP	standard solver	Mannheim, Germany	Considering battery aging, traffic congestion, and partial charging
Z. Liu et al. (2018)	Min the cost of installing fast-chargers and batteries	Fast charging ²	MILP	AARC	Utah, United States	Considering uncertain energy consumption for battery-EB
Y. He et al. (2019)	Minimizing the total cost of installing fast chargers, ESS, and EB batteries	Fast charging ²	MILP	standard solver	Utah, United States	Showing how the ESSs may save system costs by lowering demand charges
Olmos et al. (2019)	Best location of chargers, power rate of charging infrastructures, and size of ESS	Opportunity charging	-	Iterative sequence	Donostia, Spain	Minimizing the total cost of ownership
Lin et al. (2019)	Min the total operating, establishing, and grid power loss costs	Fast charging ¹	MISOCP	Spatial-temporal approach	Shenzhen, China	Multistage planning model
T. Liu and Ceder (2020)	Min the required number of EBs and fast charging infrastructure	Fast charging ³	DF and IP	Adjusted max-flow	Singapore	lexicographic method-based two-stage construction-and-optimization solution was adopted to solve the bi-objective problem
Othman et al. (2020)	Min the operational costs and energy consumption	Fast charging ⁴	EHDG	Voronoi diagram	Toronto, Canada	The proposed EHDG algorithm is based on genetic algorithm and gradient descent technique
Y. Liu et al. (2020)	Min operating costs: all backup buses, drivers, maintenance, energy consumption cost, and construction costs of charging depots	Depot charging	MILP	Artificial fish swarm algorithm	China	Optimizing the layout of bus routes, the service frequency, and the location of charging depots
W. Zhang et al. (2021)	Total costs of user and operator unsatisfied demand, passengers' travel time, and operator cost	Fast charging ³	MINLP	& Modified genetic algorithm	Swiss	Bi-level programming framework
Uslu and Kaya (2021)	Min the total cost; optimal locations and capacities of EB charging stations	Depot charging	MILP	standard solver	Turkey	Driving range has the highest effect for selecting locations and capacities of charging stations at minimum cost
X. Wu et al. (2021)	Min maintenance, fast charging station construction, travel to charging stations costs and power loss of fast chargers	Fast charging ³	BPSO	Mathematical program	Yangjiang, China	The bus terminuses clustered by the Affinity Propagation approach in order to estimate the approximate number of charging stations
Olsen and Kliewer (2022)	Min the total cost of installing depot chargers, vehicle, and operating costs	Opportunity charging	VNS	-	-	They proved that complete integration of BEB scheduling and charging station location planning is better than sequential planning
Alwesabi et al. (2021)	Min the battery cost, system inverters, and total cable cost	Wireless charging	MILP	standard solver	Binghamton University	Finding the optimum fleet size, battery capacity, and dynamic wireless charging locations, simultaneously
Stumpe, Rößler, Schryen, and Kliewer (2021)	Min the number of required vehicles, personnel and energy consumption costs	Opportunity charging	VNS	MILP	-	They performed sensitivity analysis for different input parameter uncertainties
Tzamakos et al. (2022)	Min the number of wireless chargers	Wireless charging	MILP	standard solver	-	M/M/1 queueing model was used for bus recharging queuing
M. Li et al. (2022)	Min the deadhead trips and charging services	Fast charging ³	MILP	standard solver	Chengdu, China	The fast charging infrastructure location planning was investigated under the BOT model
Hu et al. (2022)	Min the total cost of system; buying new chargers, EBs' batteries, charging cost and passengers' extra waiting time	Opportunity charging	MILP	standard solver	Sydney, Australia	Robust optimization technique was used to deal with the uncertain passengers' travel demand and trip time
Y. Wang, Liao, and Lu (2022)	Min the fleet size, BEBs; batteries, and installing pan-topograph chargers costs	Opportunity charging	MILP	standard solver	Oslo, Norway	Time-dependent dwelling time, ridership, and travel time of BEBs were considered

¹ In depot. ² At bus stop. ³ In terminal. ⁴ In specific station.

on generating timetables. According to [Häll et al. \(2019\)](#), the methods to deal with the timetabling problem for conventional buses could also be implemented on EBs.

[A. Ceder, Golany, and Tal \(2001\)](#) presented a mixed-integer model to create timetables with the objective of synchronizing bus arrivals at transfer stops. Since the suggested mixed-integer programming model belongs to NP-hard set of problems, a heuristic approach was presented to solve the large-scale problems. [Zhigang, Jinsheng, Haixing, and Wei \(2007\)](#) solved this problem using the Nesting Tabu Search algorithm. [Ibarra-Rojas and Rios-Solis \(2012\)](#) solved this problem while avoiding bus bunching of bus lines. Minimizing the passengers' waiting time has been addressed in [Saharidis, Dimitropoulos, and Skordilis \(2014\)](#) through optimizing the synchronization of bus arrivals at the connection nodes in the network. The authors solved the problem by formulating a MILP model. [Parbo, Nielsen, and Prato \(2014\)](#) investigated the maximal synchronization problem to minimize the passengers' waiting time by focusing on the social benefits of the created timetable.

[A. A. Ceder et al. \(2013\)](#) developed a multi-objective model to reduce the expected waiting time of randomly arriving passengers while increasing bus utilization from the operator's perspective. [S. Zhang et al. \(2020\)](#) in 2020 proposed a decomposition heuristic algorithm to solve the multiple vehicle-types scheduling problem and introduced a bi-objective optimization model for a feeder bus line to reduce operating costs and passenger waiting times with consideration of three different types of buses.

[Shang et al. \(2019\)](#) proposed a timetabling problem to determine the best bus frequency and headway. The timetabling is optimized by striking a balance between customer satisfaction and bus transit performance, taking the load factor into account. [A. Ceder and Philibert \(2014\)](#) developed an approach for creating even-load transit timetables to achieve a uniform maximum load on vehicles and seamless transfers. Because of the low load discrepancy, such even-load timetables would improve vehicle utilization and minimize empty seat minutes. However, such timetables would prolong the waiting period for passengers at bus stops. [Gkiotsalitis and Alesiani \(2019\)](#) presented a robust timetable using a bus movement mathematical model which incorporates travel times and passenger demand uncertainty to reduce the potential loss in worst-case scenarios by minimizing the deviation between actual departure time of buses and the desired ones.

2.4.3 Bus Scheduling

The process of assigning buses to the trips of a specified timetable is known as vehicle scheduling (VS). The set of trips from the timetable is fed into vehicle scheduling problem, aiming to cover them as efficiently as possible while meeting all operational constraints, typically by reducing the required bus numbers and deadhead trips. Both exact and heuristic/metaheuristic solution approaches are adopted to solve this problem (Kliewer, Mellouli, & Suhl, 2006; Wen, Linde, Ropke, Mirchandani, & Larsen, 2016). Vehicle scheduling with one depot and multi depots are the two main categories of bus scheduling. The single-depot vehicle scheduling problem can be solved in polynomial time (Bertossi, Carraraesi, & Gallo, 1987), while multi-depot vehicle scheduling belongs to the class of NP-hard problems (Carpaneto, Dell'Amico, Fischetti, & Toth, 1989a). The time-space network is another framework for modeling vehicle scheduling problems. In this framework, nodes represent the departures and arrivals of buses at a certain time and location and arcs represent the travel between nodes by a vehicle (Bertossi et al., 1987).

Another approach to deal with bus scheduling problems is the Deficit Function (DF) modeling. Deficit Function modeling is a technique used in bus scheduling to find the minimum number of buses needed to cover the given trips in a timetable. It works by tracking, at each departure and arrival from a terminal, the difference between the buses leaving and those returning. This creates a curve that shows how the deficit changes over time. The Deficit Function's visual aspect is its key benefit. When the curve rises, it means more buses are on the road than have returned, signaling a greater fleet requirement. When it drops, buses are coming back. The highest point on the curve indicates the maximum deficit during the day, which directly corresponds to the minimum number of required buses. Gertsbach and Gurevich (1977), A. Ceder and Stern (1981), and A. Ceder (2016) used this method to find the optimum required number of vehicles and assign them to the predetermined timetable based on DF theory. The applications of this theory to network design and crew scheduling of public transit operation planning were investigated by A. Ceder (2016). Readers are directed to T. Liu and Ceder (2017a) for a thorough explanation of the fundamental theory and significant advancements in DF modelling and applications.

Driving range limitation and long time recharging process for electric buses are the two factors

which alter the conventional models and solution approaches of vehicle scheduling problems. EBs have range limitations and should be recharged at bus depots, terminals, or bus stops, depending on the charging technology, a few times a day. This would necessitate studying electric bus scheduling from the economic and operational point of view. [J.-Q. Li \(2014\)](#) showed the changes to the bus scheduling process after introducing electric buses.

2.4.3.1 EB Scheduling with Heuristic/Metaheuristic Solution Approaches

[H. Wang and Shen \(2007\)](#) paper is one of the first studies in electric bus scheduling with a limited travel range and minimum recharging time. The objectives were to minimize the number of buses and decrease the total time of deadhead trips. The authors solved the problem by Ant Colony Optimization (ACO) algorithm. [Chao and Xiaohong \(2013\)](#) addressed this problem in a single depot VSP form. They considered a battery swapping strategy for charging electric buses. Minimizing the cost of ownership of buses and their extra batteries and minimizing the charging costs were the two objectives of this study. The solution approach was based on Non-dominated Sorting Genetic Algorithm II (NSGA-II) to present optimal Pareto fronts for a case study in China. [Paul and Yamada \(2014\)](#) addressed the charging and operation scheduling of electric buses and the operating of conventional buses for four bus lines. The paper aims to maximize the EBs' total travel distance and decrease the amount of CO₂ emissions and fuel cost. The authors solved the proposed problem by a k-greedy algorithm and validated it through a simulation process using real data from a case study in Japan. [Sung, Chu, Chang, Yeh, and Chou \(2022\)](#) seek to minimize the cost of charging stations, batteries, buses, and electricity consumption using a simulation model and a heuristic algorithm was developed to solve the problem. This study's main outputs are buses and charging stations' optimum number and type. Other metaheuristic solution methods and their combinations are used to determine the optimum departure time of buses. [Ke, Fang, and Lai \(2022\)](#) combined GA, PSO, and SA to minimize the electricity costs and total emission of greenhouse gases. Joint optimization of EB scheduling and crew scheduling for a heterogeneous bus fleet (traditional and electric buses) was studied in [J. Wang, Wang, Chang, and Song \(2022\)](#). A bi-level programming model was provided to minimize the economic and operation cost of CO₂ in the upper level and minimize the drivers' wage while increasing the bus-driver specificity (the same driver operates the

same bus) in the lower level. To solve this problem, a PSO algorithm was adopted based on an epsilon constraint method.

Adaptive Genetic Algorithm (AGA) is another approach to solve vehicle scheduling problem. [X. Li et al. \(2020\)](#) used this method to tackle the integration of electric bus scheduling and stationary charger deployment for a real case study in Anting Town, Shanghai. They considered partial charging and time-varying electricity price and tried to minimize the total construction and maintenance cost of electric bus scheduling and charging infrastructure. Uncertainty in the number of arriving passengers, their waiting time and energy consumption of electric buses were considered in [Guo, Xue, and Guan \(2022\)](#) to find the optimum departure intervals of EBs. The authors developed an uncertain bi-level programming model (UBPM), which the upper level seeks to minimize the passenger's travel costs and the lower level tries to minimize the energy consumption of electric buses. GA was adopted to solve a real case study in Nanchang, China. Also, reducing power consumption and in-service costs were included in the objective function. A multiple-vehicle-type version of electric bus scheduling was studied by [Yao et al. \(2020\)](#) in 2020. They aimed to reduce the required investment for buying electric buses and charging infrastructures while minimizing the operational costs of deadhead trips. Since the problem was an NP-hard one, a heuristic approach was adopted to solve it in a reasonable time. A bi-objective integer programming approach was developed by [T. Liu and Ceder \(2020\)](#) to minimize both the total number of EBs and required fast charging infrastructures in a multiple bus line transit system with a partial charging policy. To solve this problem, the authors suggested two methods: lexicographic and modified max-flow approach. They implemented their model in a real case study in Singapore. Multi-depot electric bus scheduling with partial charging policy for a heterogeneous fleet was investigated by [A. Zhang et al. \(2022\)](#), in 2022. The objective function was to minimize the purchase cost of EBs and operational costs. They proposed an MILP and solved it using an adaptive large neighborhood search algorithm. The collaborative optimization of electric bus scheduling and charging scheduling is studied in [G.-J. Zhou, Xie, Zhao, and Lu \(2020\)](#). This study provided a multi-objective bi-level programming model to minimize carbon emissions and operating costs, including deadhead trip costs, passenger trip costs, and power consumption costs. Iterative neighborhood search and a greedy dynamic search strategy were adopted to solve the electric bus scheduling and charging scheduling, respectively.

2.4.3.2 EB Scheduling with Exact Solution Approaches

[Alwesabi, Wang, Avalos, and Liu \(2020\)](#) proposed a mixed-integer model for the electric bus scheduling problem and solved it using the Gurobi solver. Their objective was to minimize total costs by determining the optimal fleet size while accounting for charging station availability and battery capacity constraints. [J.-Q. Li \(2014\)](#) developed an MILP model scheduling of electric buses which use fast chargers or battery swapping technology and solved it using column generation algorithm. The author also examined this problem with the limited travel range for buses with different sources of energy. The author believes that the proposed model could be used for both charging types since the battery swapping time is approximately equal to the fast-charging time. [Rinaldi, Parisi, Laskaris, D'Ariano, and Viti \(2018\)](#) studied the scheduling problem of electric buses with service factor and charging factor constraints to determine the sequence of electric and hybrid buses departing from a multi-line bus terminal. They presented a mixed integer linear program to minimize the total operation costs. In 2020, [Tang, Lin, and He \(2019\)](#) applied a branch-and-price algorithm to minimize the operating costs of the electric bus scheduling problem. They addressed the problem by presenting static and dynamic scheduling models. The static model uses a buffer-distance strategy to mitigate the effects of trip time uncertainty, while the dynamic model leverages real-time traffic updates to periodically reschedule the electric bus fleet throughout the day.

According to [A. Ceder \(2016\)](#), multi-depot vehicle scheduling problems are split into two parts: determining the trip chains and assigning buses to each trip. The introduction of EBs has more impact on the former since the trip chains should be modified regarding the limited travel range of EBs. [M. Jiang and Zhang \(2022\)](#) developed an MILP and solved this problem using a branch-and-price algorithm. They considered this problem with a partial charging policy and vehicle-depot constraint (the starting point and end point of EBs trips should be at the same depot). [Gkiotsalitis, Iliopoulou, and Kepaptsoglou \(2022\)](#) considered time windows for the starting and ending times of trips in the multi-depot electric bus scheduling problem. They aimed to minimize buses' costs and waiting times by proposing a mixed-integer nonlinear program. They linearized the model and solved it using a branch-and-cut algorithm based on valid inequalities. Multi electric vehicle-type scheduling problem with limited battery capacity was discussed by [Reuer, Kliewer, and Wolbeck](#)

(2015). They extend a classical time-space network based optimization approach and proposed six decomposition strategies to solve the problem. Although the authors claim to have adopted an exact solution approach, the decomposition only guarantees the feasibility of the schedules, not global optimality. [W. Wu, Lin, Liu, and Jin \(2022\)](#) formulated the problem as an MILP on a time-expanded network and solved it with a branch-and-price algorithm. Their model accounts for time-of-use pricing and grid peak load, aiming to minimize both peak charging demand and total operating costs.

[van Kooten Niekerk, Van den Akker, and Hoogeveen \(2017\)](#) adopted mixed integer linear programming to solve the battery electric bus scheduling problem and demonstrated that their model could be solved efficiently for small and medium-scale instances. For large-scale problems, they developed two additional techniques based on a column generation approach to obtain near-optimal solutions. The study considered two variants of the problem: the first made simplifying assumptions, such as linear battery chargeability, ignoring time-of-use electricity prices, and neglecting depth-of-discharge effects; the second was more realistic, allowing different charging processes, incorporating actual electricity prices, and accounting for battery depreciation costs. [Y. Wang et al. \(2017\)](#) proposed a MILP model to determine the optimal recharging strategy for addressing the limited travel range of electric buses. They solved the model using the CPLEX solver and applied it through a real-world case study in California. Similarly, [Y. Wang et al. \(2022\)](#) developed an MILP model to jointly optimize pantograph charger location planning, electric bus scheduling, and battery capacity. The model aimed to minimize the fleet's total annual cost.

2.4.4 Integration of Bus Scheduling and Timetabling

This section of the literature discusses the integration of timetabling and vehicle scheduling problems. [Häll et al. \(2019\)](#) examined how timetabling and vehicle scheduling are affected after the arrival of EBs. They also optimized different charging strategies, including continuous, overnight, and fast charging. The authors investigated the effects of introducing EBs on public transportation's operational planning (transit network, timetabling, and scheduling). Integration of electric bus timetabling and vehicle scheduling is required to meet passenger demand, enhance social benefits, and reduce operator costs [Teng et al. \(2020\)](#). Hence, finding the optimal timetable and vehicle

schedule for electric buses is a key factor in reaching a more sustainable transit system. Most previous studies considered bus scheduling and bus timetabling individually and separately. [A. Ceder \(2001\)](#) and [Chakroborty, Deb, and Sharma \(2001\)](#) were the first researchers who studied the integration of timetabling and vehicle scheduling. Ceder et al. addressed an approach for combining timetables and vehicle scheduling from both the customer and the operator's perspective to reduce the fleet size. Their approach was based on a four-step sequential method with a feedback loop. Chakroborty et al. addressed the problem without the interlining option and minimized the fleet size while reducing passengers' transfer and waiting time, based on a genetic algorithm approach. Most of the studies in the literature mainly discuss the minimization of passengers' waiting and travel time for timetabling and minimization of bus operator costs (purchase costs of new buses, deadhead trip costs, scheduling, and recharging costs).

The combination of bus timetabling and scheduling is a bi-objective problem. On the one hand, it should optimize the passengers' satisfaction by minimizing waiting time, travel time, seat availability, and so on; on the other hand, it should consider operational costs from the bus companies' point of view. These two objectives are in conflict, and to solve and handle such issues, several strategies could be adopted: Shifting, Weighting, Pareto front, Bi-level programming, and Reordering.

In the shifting strategy, vehicle scheduling is solved by making small adjustments to the timetable that shift arrival and departure times within allowable time windows to improve compatibility with vehicle scheduling. These minor shifts help reduce scheduling costs, enhance vehicle circulation, and decrease the number of buses required. [Kliwer et al. \(2006\)](#) were the first scholars who introduced shifting strategy for vehicle scheduling problem, which is called vehicle scheduling problem with time windows (VSP-TW). They used a time-space network to find out the feasible combinations of trips and solved the problem using heuristic and column generation methods. [Fleurent, Lessard, and Séguin \(2004\)](#) and [van den HEUVEL, van den AKKER, and Van Kooten \(2008\)](#) extended the model presented by Kliwer et al. [Kliwer et al. \(2006\)](#) and proposed a hierarchical approach to solve the vehicle scheduling problem and used mathematical programming models to find the best number of vehicles and optimize the type of vehicles based on a simulated annealing

(SA) solution approach. A combination of shifting and weighting strategies based on a metaheuristic approach was developed by [Fonseca, van der Hurk, Roberti, and Larsen \(2018a\)](#). They aimed to minimize the total operational costs while minimizing the passengers' transfer times.

The weighting strategy is one of the most straightforward strategies to deal with the bi-objective nature of the problem, but the issue is determining the weights' best values to describe the preference of passengers and public transportation service providers. [Petersen, Larsen, Madsen, Petersen, and Ropke \(2013\)](#) introduced a new problem to simultaneously address passengers waiting time and total resource costs for an integrated TT and VS problem. They solved the model based on the large neighborhood search (LNS) metaheuristic solution approach and used the weighted sum method to deal with the bi-objective nature of the problem. [Carosi, Frangioni, Galli, Girardi, and Vallese \(2019\)](#) proposed a multicommodity flow style mixed-integer linear programming model and used the weighting approach to balance service provider costs and customer satisfaction optimally. In another paper, [Guihaire and Hao \(2010\)](#) suggested an iterated local search method to solve the integrated timetabling and vehicle scheduling (ITTVS) problem. The objectives of the problem were minimizing the operational costs of vehicles and maximizing the service quality. The latter objective was measured by the evenness of headways, and the former was calculated with respect to the length of the deadhead trips and fleet size. [Schmid and Ehmke \(2015\)](#) addressed the ITTVS problem with a degree of flexibility to change the timetable and balanced departure times and solved it based on the Large Neighborhood Search (LNS) approach. The two objectives were improving the quality of timetable and reducing the operating costs which were optimized using weighted sum approach.

Another strategy is using the Pareto front. This strategy aims to find the Pareto optimal front, which any other solutions would not dominate. [Ibarra-Rojas, Giesen, and Rios-Solis \(2014\)](#) solved timetabling and vehicle scheduling individually by introducing two integer linear programming formulations. Then, they combined the two problems into a bi-objective integrated problem. The method used to deal with such a bi-objective model was based on the epsilon constraint method. The authors showed that their proposed model and solution approach can solve the problem of up to 50 bus lines. A combination of timetabling and VS was also suggested by [Weiszer, Fedorko, and Čujan \(2010\)](#) in 2010. The suggested model included two objective functions: minimizing

passengers' waiting time at each bus stop and minimizing the required number of buses to cover all the trips mentioned in the timetable. Finally, the authors solved the problem by proposing a NSGA-II optimization technique. [T. Liu and Ceder \(2016\)](#) studied the impact of schedule deviations on public transport users' routing choices. They developed a collaborative approach for timetabling, vehicle scheduling, and demand assignment simultaneously. For the electric buses, [Teng et al. \(2020\)](#) addressed the single-line bus timetabling and vehicle scheduling problem by proposing a multi-objective particle swarm optimization (PSO) algorithm. This paper aims to minimize the number of electric buses and simultaneously their charging cost.

Bi-level programming solves the problem in two different stages: leader and follower. Leaders optimize their objective regardless of followers' objective; then, followers solve the problem and optimize their objective function based on the output of the first stage optimization of leader. [T. Liu and Ceder \(2017b\)](#) proposed a bi-level integer programming model that considers the interests of public transport users and service providers to optimize public transport timetable integrated with vehicle scheduling. A novel deficit function (DF)-based sequential search method is proposed to solve the problem and obtain Pareto fronts. [T. Liu, Ceder, and Chowdhury \(2017\)](#) formulated an integer programming model to address the integrated bus timetabling and vehicle scheduling. Their research aims to minimize the number of vehicles required for the total trips and maximize the number of vehicles that arrived simultaneously at the transfer nodes. This bi-objective model was solved based on a two-stage deficit function approach to generate Pareto optimal fronts. It is worth mentioning that the last two mentioned papers showed how changes in bus timetable and even vehicle scheduling could affect passengers' trip mode choice. [Z.-g. Liu and Shen \(2007\)](#) presented a bi-level programming formulation to solve bus timetabling and vehicle scheduling. At the first level, the number of required vehicles to assign to each trip is minimized. Then, at the second level, passengers' transfer time in connection stops is minimized concerning the solution from the first level.

The last strategy is reordering, which means considering public transportation planning as one integrated problem. [Michaelis and Schöbel \(2009\)](#) considered four stages of the public transportation planning system as one integrated problem. The four stages were: network design (line planning), timetabling, vehicle scheduling, and crew scheduling. The authors proposed new order for

the public transportation system planning. First, the vehicle routes are designed; in the second stage, the designed routes split into bus lines, and at the final stage, a periodic timetable would be assigned for each line. [Pätzold, Schiewe, Schiewe, and Schöbel \(2017\)](#) proposed three ways to include VS in different planning stages of public transportation systems, such as timetabling and line planning.

Table 2.7 represents an overview of the objective functions of various studies on bus scheduling and timetabling. This table shows the potential research area for future studies regarding simultaneous optimization of bus scheduling and timetabling objectives. Also, we can conclude that there is a significant lack of research on integrating electric bus scheduling and charging TOU and combined optimization of EBS and timetabling. Table 2.8 represents the method and algorithms adopted to solve electric bus scheduling and timetabling problems. The table shows that the number of studies using exact solution methods is remarkably less than papers using heuristic or metaheuristic solution approaches. Although EBS with multi depot is an NP-hard problem, there is a research gap in investigation reformulating such problems and solving them by the exact methods.

Table 2.7: Different studies related to the objectives of bus scheduling and timetabling

Paper	EB	VS	TT	Objective					
				Purchase Cost/No. Vehicles	Deadhead Trips	Fuel/Electricity Cost	TOU	Waiting Time	Transferring Time
Ibarra-Rojas et al. (2014)		✓	✓	✓	✓			✓	✓
Schmid and Ehmke (2015)		✓	✓		✓			✓	
Wen et al. (2016)	✓	✓		✓	✓				
Fonseca, van der Hurk, Roberti, and Larsen (2018b)		✓	✓		✓				✓
Cao and Ceder (2019)		✓	✓	✓		✓		✓	✓
L. Li et al. (2019)	✓	✓		✓		✓			
S. Zhang et al. (2020)	✓	✓	✓	✓				✓	
Yao et al. (2020)	✓	✓		✓	✓	✓			
Teng et al. (2020)	✓	✓	✓	✓		✓	✓	✓	
G.-J. Zhou et al. (2020)	✓	✓			✓	✓			
T. Liu and Ceder (2020)	✓	✓		✓					
X. Li et al. (2020)	✓	✓		✓		✓	✓		

Table 2.8: The characteristics of electric bus scheduling, timetabling, and solution approaches

Paper	Single/Multi Depot	Single/Multi Line	Vehicle Type	Model	Method/Algorithm	Case Study
Ibarra-Rojas et al. (2014)	Single	Multi	Homogeneous	MILP	ϵ -constraint	Monterrey, Mexico
Schmid and Ehmke (2015)	Single	Multi	Homogeneous	MIP	LNS	Göttingen, Germany
Wen et al. (2016)	Multi	Multi	Homogeneous	MIP	Adaptive LNS	-
Fonseca et al. (2018b)	Multi	Multi	Homogeneous	MILP	Matheuristic	Copenhagen, Denmark
Cao and Ceder (2019)	Single	Single	Homogeneous	MILP	GA	Auckland, New Zealand
X. Li et al. (2020)	Multi	Multi	Heterogeneous	ILP	Heuristic approach	Hong Kong
L. Li et al. (2019)	Multi	Multi	Heterogeneous	ILP	Time-space-energy network and time-space	Hong Kong
S. Zhang et al. (2020)	Single	Single	Heterogeneous	MIP	GA	Beijing, China
Yao et al. (2020)	Multi	Multi	Heterogeneous	MILP	GA	Beijing, China
Teng et al. (2020)	Single	Single	Homogeneous	MIP	Multiobjective PSO	Shanghai, China
G.-J. Zhou et al. (2020)	Single	Multi	Heterogeneous	Bi-level programming	Iterative neighborhood search	Beijing, China
T. Liu and Ceder (2020)	Multi	Multi	Homogeneous	DF and IP	Adjusted max-flow	Singapore
X. Li et al. (2020)	Single	Multi	Homogeneous	Nonconvex mathematical model	AGA	Anting Town, Shanghai
Bie et al. (2021)	Single	Single	Homogeneous	ILP	Branch-and-price	-
Sung et al. (2022)	Multi	Multi	Heterogeneous	Simulation	Heuristic	Kaohsiung, Taiwan
Gkiotsalitis et al. (2022)	Multi	Multi	Homogeneous	MILP	Branch-and-cut	-
Y. Wang et al. (2022)	Single	Multi	Homogeneous	MILP	MILP	Oslo, Norway
Y. Jiang and He (2022)	Multi	Multi	Homogeneous	MILP	Branch-and-price	-
A. Zhang et al. (2022)	Multi	Multi	Heterogeneous	MILP	ALNS	Nanjing, China
Guo et al. (2022)	Single	Single	Homogeneous	UBPM	GA	Nanchang, China
W. Wu et al. (2022)	Multi	Multi	Homogeneous	MILP	Branch-and-price	Guangzhou, China

2.5 Challenges and Limitations

Implementing a fully electric bus transit system faces several challenges. The first challenge is the energy power supply issue of the high energy demand of electric buses, especially in big cities.

Not all cities benefit from renewable energy sources for generating electricity; thus, satisfying the energy demand of a fully electric bus transportation system will be a big challenge for the energy side. Another challenge is the high purchasing price of fully electric buses compared to conventional or hybrid ones. Convincing cities' authorities, municipalities and governments to switch to such a high-cost transit system is complicated and that is why many cities have not decided to use electric buses. Last but not least is the range limitation of EBs and researchers are trying to solve this issue by proposing different charging technologies or producing buses with longer travel range. However, such solutions are costly for transit agencies. So, still, there is a vast research area to address this problem more sustainably.

Another group of challenges is operational planning challenges. This group relates to the recharging duration, battery degradation, and low efficiency of pantograph chargers. The refueling process of conventional buses does not take long and can be done every hour a day. On the other side, the electric bus recharging process is a long-lasting task that should be done during off-peak hours to balance the load on the distribution grid. EB batteries will degrade slowly over time, depending on the frequency and type of charging used. Electric bus batteries are considered to have reached their end of life at 80% capacity or when they lose 5% of their charge per hour without use. Generally, EB batteries are warrantied for 8-12 years. This lifetime is much less than traditional buses. Cold temperatures will affect battery charge primarily due to the use of heating, which greatly impacts the amount of charge used, reducing range by up to 41%. Therefore charge needs to be maintained above 20% to reduce the risk of stalling in the winter.

Switching from a conventional bus system to a wholly electric one for cities lacking required infrastructures is very complicated. On the other hand, integrating bus depot operators and energy sectors may be feasible for only small towns trying to improve or design a new transit system while considering the impact of charger loads on the grid. That is because managing and optimizing the combination of transit authorities and energy manager agencies for big cities is complex. Moreover, the impact of installing pantograph chargers on the neighborhood is a challenge for transit authorities and urban planners. Safety is another challenging issue of using high voltage chargers for EBs. Although there is no report regarding the injuries or death related to charging infrastructures low safety measures, such infrastructures should be examined and checked regularly to reduce the risk

as much as possible.

The electricity generation finite capacity is an important factor preventing wide distribution of electric buses in cities around the world. The power generators might not be able to meet the demand of a fully electric transit system and transforming from oil-base system to electricity-base system would take time. This is the reason that most cities switch from conventional buses to electric one, gradually. In many cases, transit authorities have decided to use hybrid buses as a first step and then gradually replace them with electric ones.

The accelerated deployment of EBs will place a heavy load on the grid, affecting the operation of utilities and power systems. Public transit operators typically lack the infrastructure required to address such a problem. Extending the distribution system's capacity is possible, although doing so would be expensive and take a long time to complete. Therefore, creating innovative methods to reduce severe grid stress by fleet charging becomes a crucial problem ([Dietmannsberger, Schumann, Meyer, & Schulz, 2017](#); [Mohamed et al., 2017](#); [Z. Wu, Guo, Polak, & Strbac, 2019](#)). Smart coordinated charging techniques should be regarded as one of the most important issues to be handled for this goal. In addition, public transit authorities must consider how the grid and bus systems interact. The best strategies for dealing with hierarchical decisions and decisions containing various and competing factors to optimize are bi-level and multi-objective optimization. Another attractive study area is the subject of Bus to Grid (B2G) interactions. For example, public transport authorities can sell energy back to the grid by taking advantage of intra-day changes in the price of electricity.

2.6 Future Direction

We divided the literature gaps in electric bus operation planning into three categories: electric bus scheduling and timetabling, fast charging infrastructure location planning, and impacts on the grid.

Although electric bus scheduling, timetabling, and charging station location problem have been the subject of various studies, more work is still needed to bridge the gap between theory and practice. If we explore the literature, we can find that only a few works have been done on the influence of electric bus scheduling on the location of fast charging infrastructure and vice versa. A

few papers currently deal with the location problem of fast charging infrastructures. According to these papers, the charging station location problem is treated as an individual optimization problem.

To the best of our knowledge, there are very few studies on rescheduling and planning robustness for scheduling electric buses in the literature. Thus, future research is expected to focus on the design of recovery techniques that facilitate the use of electric buses. The integration of public transit operation planning steps for electric buses has not been investigated comprehensively and deeply in the literature. Although considering joint optimization of planning problems would increase the computational complexity, it may improve the efficiency and level of service of EBs transit systems. Therefore, such integration could be an interesting area of research direction in the future.

Charging scheduling of electric buses should be studied for heterogeneous bus types with mixed-type charging technology such as depot charging and fast charging. Also, to dynamically modify the charging schedule, monitoring the real-time information of EBs such as state of charge (SoC) batteries, traffic conditions, passenger travel demand during the day, and buses' earliness and tardiness should be considered in future works (Hu et al., 2022). Moreover, the stochastic behavior of electric bus operations has not been well examined up to this point. As a result, using stochastic or robust optimization models seems to be a promising area for further research (Y. He et al., 2019).

Another gap that should be filled is the impact of charging station location and electric bus scheduling on the grid. If we want to categorize such impacts, it will result in negative and positive impacts. Negative impacts refer to the charging load of electric buses, especially during peak hours. Reducing the grid's stability, generating harmonics, and reducing the transformer service life are several negative impacts on the grid. Such issues should be addressed in future studies, and the effects of the EBs charging load should be taken into account for each step of public transportation planning. The positive side refers to using V2G or, in this context B2G configuration. This technology will help the grid to be more stable and supply energy through ancillary services.

In most studies, the charging locations of electric buses are assumed to be at depots or terminals (Rogge et al., 2018). By eliminating this assumption, the location problem for charging stations could be studied, too. With the rapid increase in fully electric buses, research on the integration of bus scheduling and fast-charging infrastructure location problem would be necessary. Integration of crew scheduling with electric bus scheduling and timetabling has significant research value that

should be considered in future studies.

In a real-world setting, unpredictable circumstances, including road, weather conditions, and driving habits, may impact how much energy BEBs use ([Y. Chen, Zhang, & Sun, 2021](#)). Also, estimating EBs' energy consumption involves an unanticipated inaccuracy due to the uncertainty around passenger volume. These would affect the scheduling process of EBs and their recharging procedure. Thus, the uncertain energy consumption of EBs and more accurate energy consumption models should be taken into account to improve the bus scheduling and their charging scheduling. Adding more charging strategies and partial charging choices might further boost the BEB system's operating effectiveness. As a result, future studies could investigate the hybrid strategy that combines various charging techniques and further look into the possibility of partial charging at the central terminal in mixed-type technologies of chargers.

It is important to explore how charger location planning affects the expansion or modification of network design. In other words, the impacts of fast charging infrastructure location should also be taken into account from the bus network design point of view. Furthermore, operational features such as traffic conditions, energy consumption, charging schedules, and time-of-use pricing strategy could be considered in future research ([M. Li et al., 2022](#)).

All in all, Figure 2.3 represents different solution approaches to deal with electric bus scheduling, timetabling, charging scheduling, fast charging infrastructure location planning, and their impacts on the grid. The red dots indicate that the integration of these research areas has not been studied yet. Note that several other possibilities for integrating these problems are not shown in this figure and have not been investigated. The main purpose of this figure is to show the adopted methods for solving integration of such problems with each other. As it is represented in the figure, a few papers discussed such optimization problems with exact solution approaches. So, focusing on developing exact methods to solve these problems would be another research direction for future studies. Another point of this figure is to show the possible future direction for research in this scope. For example, electric bus scheduling, FCILP, and impacts on the grid have not been investigated yet. Thus, this could be an interesting area of research in the future.

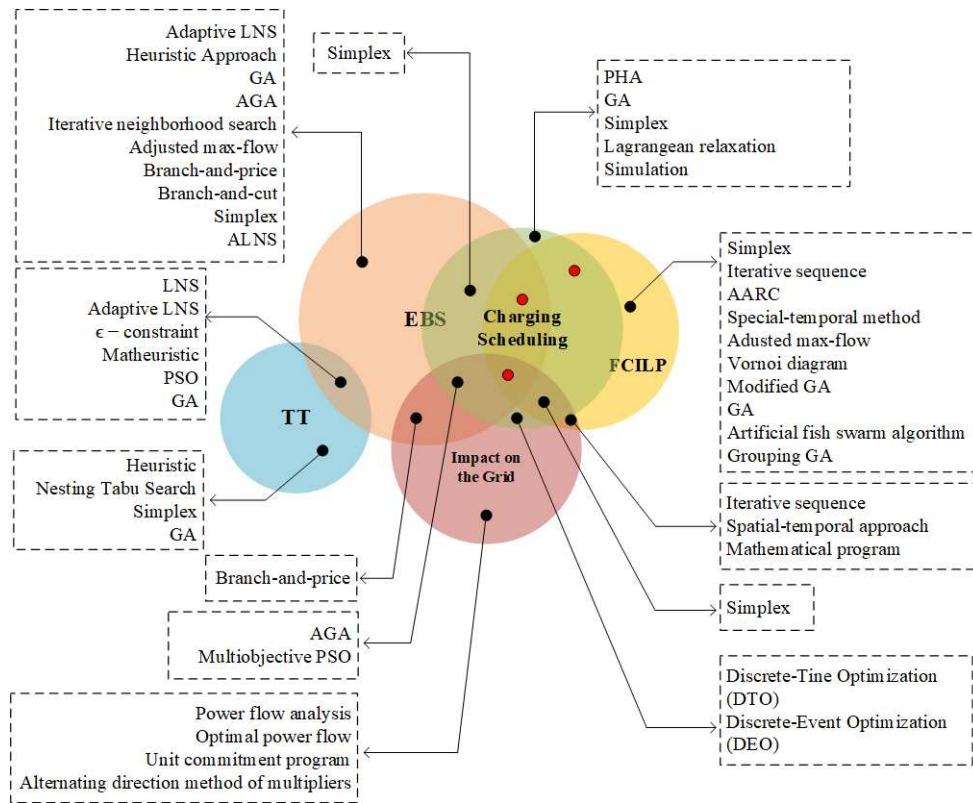


Figure 2.3: Integration of EB optimization problems and possibilities of future studies.

Chapter 3

Enhancing Electric Shuttle Bus Efficiency: A Case Study on Timetabling and Scheduling Optimization¹

3.1 Overview

As transit authorities increasingly adopt electric buses (EBs) to mitigate air quality concerns and greenhouse gas emissions, new challenges arise in bus scheduling and timetabling. Unlike traditional buses, EBs face operational obstacles due to their shorter range and extended charging times. Existing mathematical optimization models for operation planning of traditional buses must be revised to address these unique characteristics of EBs. This study introduces a new approach to integrate timetabling and bus scheduling to enhance the level of service and minimize operational costs, using a case study of a University shuttle bus service in Montreal, Canada. The level of service will be enhanced by reducing students waiting time and improving their in-vehicle comfort through seat availability. The scheduling aspect seeks to reduce the total operational costs, which include travel, electricity consumption, and usage costs of EBs. The proposed algorithm calculates the waiting time and seat availability for different headway values and addresses the scheduling problem

¹Alamatsaz, K., Quesnel, F., & Eicker, U. (2024). Enhancing Electric Shuttle Bus Efficiency: A Case Study on Timetabling and Scheduling Optimization. *Energies*, 17(13), 3149.

using a mixed-integer linear programming (MILP) model with an arc-based approach, solved using the Cplex Optimization Studio software version 12.8. A normalized weighted sum technique is then applied to select the optimal headway, balancing waiting time, seat availability, and operational costs. The effectiveness of our approach was tested through a case study of Concordia University's shuttle bus service. Comparative analysis of the current and proposed schedules shows that our approach significantly improves service quality by decreasing waiting times and increasing seat availability while optimizing cost-effectiveness compared to the existing timetable of the Concordia shuttle bus. The proposed approach ensures a smooth transition to a fully electric transit system for shuttle bus services.

3.2 Introduction

Sources of energy that release greenhouse gases are major contributors to global warming and atmospheric pollution. Shifting to electric transit systems is crucial for developing greener and sustainable urban environments. In the United Kingdom, buses are responsible for emitting roughly 4.3 million tons of CO₂, emphasizing the critical necessity for shifting towards more sustainable energy sources. Electric buses (EBs) emerge as a viable strategy to reduce urban transport's carbon footprint, which is responsible for 15% of worldwide CO₂ emissions ([Nejat et al., 2015](#)). In light of this, China has taken a substantial step by converting its bus fleets to electric vehicles. As of 2019, approximately 400,000 electric buses were operating in China's public transport system, constituting 99% of the worldwide total of EBs ([Margolis, 2019](#)). Also, numerous cities have embarked on ambitious plans to electrify their public bus fleets ([Pelletier, Jabali, Mendoza, & Laporte, 2019](#)). Similarly, academic institutions, including those in the United States, Japan, Turkey, and Singapore, are adopting EBs for campus shuttle operations ([Saner et al., 2022](#)). Research conducted by [Lie et al. \(2021\)](#) indicates that the adoption of biofuels and electric buses could diminish the carbon footprint of bus fleets by 37%, with a complete transition to electric buses potentially reducing emissions by as much as 52%.

The shift toward electric transportation systems, particularly electric buses, involves significant challenges, primarily the high initial costs. This includes the cost of the buses, their batteries, and the essential charging infrastructure, which is about 63% higher than conventional diesel buses (Quarles et al., 2020). However, electric buses offer lower operational and maintenance expenses, around CAD 700,000 annually compared to CAD 1.1 million for diesel buses, potentially resulting in savings over their lifetime (Johnson, Nobler, Eudy, & Jeffers, 2020). Despite the advantages of lower fuel and maintenance costs, electric buses face challenges such as limited range, long charging times, and performance declines in cold climates, which significantly reduce their operational range. These issues complicate their integration into existing transportation systems.

Public transit agencies are actively working to enhance the efficiency of their transportation systems, focusing particularly on bus operations. The effectiveness of these systems largely relies on how they are managed, particularly in aspects such as bus timetabling and scheduling. Bus timetabling (TT) is the process of setting departure and arrival times for all bus trips to improve service quality (Y. Zhang et al., 2020). The goal is to minimize waiting and transfer times for passengers, ensure more available seats, and coordinate bus arrivals at major transfer hubs and intersections. On the other hand, The vehicle scheduling problem (VSP), or bus scheduling problem (BSP) in this context, involves assigning buses to trips based on a predetermined timetable. The aim is to reduce the number of buses needed and minimize operational costs. These costs include travel expenses, electricity and refuelling costs, expenses related to bus waiting times, and maintenance costs (Bie et al., 2021).

Since switching from hybrid or conventional buses to fully electric buses affects the operational and economical aspects of the bus service, a reconsideration of the current timetable and schedule of EBs is required. This study focuses on improving the timetable and schedule of EBs by developing a new integrated approach specifically designed for the operational needs of electric buses, with objectives to increase the level of service and minimize EB operational costs. By finding the best headways, the intervals between consecutive bus departures from the same stop, for buses and optimum schedules of fully electric shuttle bus services, we seek to facilitate a smoother transition to a fully electric transit system and improve operational efficiency and reduce students' waiting time. Additionally, strategic choices regarding the best models of EBs underscore the importance

of this research in reducing total operational costs and improving the service level.

The remainder of this paper is structured as follows. The theoretical background and related works of electric bus timetabling, scheduling, their integration, and challenges of electrifying the bus transit systems are reviewed in Section 3.3. Section 3.4 describes a detailed description of the integration of electric bus scheduling and timetabling for shuttle bus services. Also, it outlines the proposed solution approach, the mathematical optimization model, and the algorithm to solve the problem. Section 3.5 provides a detailed explanation of the Concordia University shuttle bus service as a case study and reports the computational results and sensitivity analysis. Finally, conclusions and potential research direction are drawn in Section 3.6.

3.3 Literature Review

This literature review examines significant contributions that address the complexities of creating effective timetables and schedules, emphasizing the necessity of integrating these elements to better support electric bus operations.

3.3.1 Bus Timetabling

Bus timetabling involves several key decisions to improve transit services. One is the departure times of buses, which involves setting the time buses leave terminals and stops, including the first and last departures for each route. Frequency setting is another important aspect, where intervals of buses are determined based on the time of day, like peak or off-peak hours, to match passenger demand and prevent overcrowding or underutilization. Headway management then focuses on managing the time gaps between consecutive buses on a route to minimize passenger waiting times (A. Ceder, 2016).

Studies have examined the TT phase from various angles, focusing on multi-objective models designed to minimize waiting times, enhance bus utilization (S. Zhang et al., 2020), and adjust bus frequencies to align with passenger demand (Shang et al., 2019). Enhancing the synchronization of bus arrivals at transfer points and bus intersections enables EBs to utilize idle time for recharging, necessitating the incorporation of charging times into the timetabling process. This charging time

takes about 5 to 15 min. Even though fast-charging technology influences timetable adjustments, research on timetabling for EBs remains limited (Häll et al., 2019).

A. A. Ceder et al. (2013) formulated a multi-objective model aimed at minimizing the expected wait times for passengers who arrive randomly while also enhancing bus utilization from the operators' perspective. S. Zhang et al. (2020) introduced a decomposition heuristic algorithm to tackle a bi-objective optimization model for feeder bus lines that aimed to minimize passenger waiting time costs while considering the total operational costs and budget constraints for bus operators. Shang et al. (2019) suggested a timetabling strategy to maximize bus frequency and optimize headway values by focusing on customer satisfaction. This approach seeks to find a balance between satisfying bus users' demands through minimal waiting time and seat availability and achieving efficient bus transit operations by focusing on increasing income with more passengers. However, their strategy did not address the scheduling of buses based on these optimized headways, an issue our research intends to tackle. A. Ceder and Philibert (2014) developed a methodology for creating transit timetables that aim for an even distribution of load across vehicles to facilitate smooth transfers and achieve a high vehicle load without discrepancies, thereby enhancing vehicle use and reducing the time vehicles spend with empty seats. However, this approach could lead to longer wait times for passengers at stops that are not predetermined. Gkiotsalitis and Alesiani (2019) introduced a robust timetable through a mathematical model of bus movements, aiming to minimize the impact of worst-case scenarios by reducing the discrepancies between actual and planned bus departure times.

3.3.2 EB Scheduling

The EBSP can encompass various goals, including minimizing the number of EBs needed, reducing the total acquisition cost of EBs, reducing deadhead trips (empty trips to relocalize a bus), and lowering travel cost, electricity, and maintenance costs associated with EBs (M. Jiang & Zhang, 2022). Key decisions in EBSP include assigning buses to specific trips, charging scheduling of EBs, and determining the required number of EBs. The charging technology and the EB specifications will significantly impact these decisions.

There are different types of EBSP, primarily influenced by the fleet's characteristics and the structure of the bus networks. These problems can be specific to operations involving a single bus

line (Teng et al., 2020) or multiple lines (W. Wu et al., 2022), and similarly, they may concern single or multiple bus depots (Guo, Xue, & Guan, 2023; A. Zhang et al., 2022). Another vital consideration is the type of vehicles in the fleet. For a more comprehensive understanding of electric bus scheduling and its variations, readers are directed to the detailed reviews provided in the recent articles by Perumal, Lusby, and Larsen (2022) and Alamatsaz, Hussain, Lai, and Eicker (2022).

3.3.3 Integrating TT and Scheduling

A. Ceder (2001) and Chakroborty et al. (2001) were pioneers in examining the combination of bus timetabling and bus scheduling. A. Ceder (2001) proposed a method to merge timetables and bus schedules, looking at both bus users' satisfaction and operational efficiency to decrease the number of buses needed. Their strategy utilized a four-step sequential process with a loop for feedback. On the other hand, Chakroborty et al. (2001) were the first to incorporate the decision to determine the optimal fleet size into the timetable development problem. The objectives were minimizing the fleet size and reducing passengers' waiting time simultaneously. They also demonstrated their approach using a test instance involving three bus lines over a total scheduling period of four hours. The majority of research in this field primarily focuses on minimizing passenger waiting and travel times for timetabling and reducing the costs faced by bus operators, including the expenses associated with acquiring new buses and the costs of deadhead trips.

Balancing the different objectives of TT and BSP requires the use of various approaches, including adjusting timing (shifting), prioritizing certain goals over others (weighting), utilizing Pareto optimization to find the best compromise (Pareto front), applying a hierarchical decision-making process (bi-level programming), and rearranging processes or priorities (reordering) (Carosi et al., 2019).

In the *shifting* method, BSP is addressed with minor adjustments to the timetable, such as slight shifts in arrival and departure times, to lower the operational costs associated with scheduling. This method optimizes a selected objective function while ensuring that a second objective does not fall below or exceed a certain threshold (Kliewer et al., 2006). Kliewer et al. (2006) were the first to introduce this method for the vehicle scheduling problem. A. A. Ceder (2011) also used the shifting technique to enhance the service level by increasing passenger comfort and minimizing wait times.

To address the scheduling problem, the authors utilized the deficit function. Their objective was to reduce the number of vehicles required to cover the trips planned during the timetabling phase. However, Ceder's research is specifically focused on conventional buses and is not applicable to EBs.

Another approach to address this type of problem is the *weighting* method. [Guihaire and Hao \(2010\)](#) presented an iterative local search algorithm with the goal of optimizing service quality and decreasing bus operating expenses. Service quality was evaluated based on the evenness of headways, and operational costs were assessed by considering the lengths of deadhead trips and fleet size. Another application of the weighting technique for this problem is detailed in ([Petersen et al., 2013](#)), which focuses on balancing passenger waiting times with the overall cost of resources. The problem was also addressed by [Schmid and Ehmke \(2015\)](#), who took a flexible approach to schedule modifications and balanced departure times. Their objectives included minimizing operational expenses and optimizing the timetable's quality by minimizing deviations from desired headways. [Carosi et al. \(2019\)](#) developed a MILP model to optimally balance the costs for service providers and passengers' waiting times for conventional buses. However, this model does not take into account the seat availability metric. The key challenge of studies using the weighting method is determining the weight values that best represent the desires of both transit authorities and bus users.

Another method for addressing such integrated problems is the *Pareto front* technique. This approach aims to identify the optimal Pareto front, a set of solutions where no other solutions can outperform any in the optimal set without compromising another criterion. Essentially, it enables decision makers to visualize trade-offs and make choices that best balance competing objectives, ensuring that every solution on the Pareto front represents an optimal balance of these objectives. [Weiszer et al. \(2010\)](#) introduced a model focused on two goals, which were minimizing passenger wait times at each bus stop and reducing the number of buses required to cover all scheduled trips. In order to address TT and BSP, [Ibarra-Rojas et al. \(2014\)](#) employed two mathematical formulations, yielding optimal headways and schedules. After that, they combined the two problems into a single bi-objective integrated problem. [Teng et al. \(2020\)](#) developed a multi-objective particle swarm optimization (PSO) approach to address single-line bus timetabling and the bus scheduling problem

for EBs. The authors focused on reducing both the total number of electric buses needed and the costs associated with charging them.

The bi-level programming technique is another approach that tackles problems on two levels: leader and follower. In this method, leaders optimize their objectives independently of the followers. After that, followers address the problem by optimizing their own objectives, utilizing the outcomes from the leader's initial optimization stage ([Lachhwani & Dwivedi, 2018](#)). [T. Liu and Ceder \(2017b\)](#) developed a bi-level integer programming model to determine the departure time of buses aimed at minimizing passenger waiting times and ensuring seat availability, while also reducing the number of buses needed. However, unlike our research, their study did not consider EBs, deadhead trips, and the fuel costs of buses. [Z.-g. Liu and Shen \(2007\)](#) applied this method to address bus TT and BSP. At the first level, they focused on minimizing the number of buses required for each trip. Subsequently, at the second level, they aimed to reduce passengers' transfer times at bus intersections and transfer hubs, taking into account the solutions derived from the first level.

The reordering method is the last approach discussed, and it considers public bus transit planning as a unified, integrated problem ([T. Liu & Ceder, 2017b](#)). This method is named "reordering" because it rearranges the traditional sequence of bus operational planning steps. [Xu, Yu, and Long \(2023\)](#) developed a framework based on a time-space network to combine electric bus TT and EBSP. This framework takes into account various factors such as minimum and maximum headway times, deadhead trips, and the battery capacities of the vehicles. The authors aimed to maximize operational profit by increasing the number of service trips and reducing operational costs. This reduction in costs is achieved by minimizing the number of EBs used and the travel time of deadhead trips. [Quttineh, Häll, Ekström, and Ceder \(2023\)](#) introduced a new mathematical model that combines the TT and BSP. However, they did not consider any passenger's point of view, including factors like waiting time, transfer time, and seat availability, in developing the timetable.

The current research focuses on integrating the timetabling and scheduling of electric shuttle buses, aiming to address multiple objectives. We aim to reduce the number of EBs required for each shift with varying travel times, decrease deadhead trips and electricity costs, and simultaneously lower student waiting times while improving seat availability. This study seeks to find a balance

between minimizing operational costs for Concordia University Facility Management and enhancing service quality for shuttle bus users. Unlike many existing studies, we employ and solve an MILP model to identify the most cost-effective schedule. To our knowledge, this dual focus on cost efficiency and service quality in the context of electric buses has not been previously explored.

3.3.4 Shuttle Bus Operations Planning

Shuttle bus service operation planning and public bus transportation planning serve different purposes and address the needs of distinct user groups, resulting in different operational characteristics. Shuttle buses are often designed for specific tasks, such as transporting employees from transit hubs to workplaces, moving passengers between airport terminals, or moving students between campuses. These services are flexible, typically operate on a smaller scale with limited routes, and adjust routes and schedules based on specific needs or events. They are usually privately funded or managed by specific institutions, focusing on efficiency and convenience for targeted users.

Shuttle bus services are typically used for short-distance commuting at university campuses (Hulagu, Atasayar, & Celikoglu, 2019), tourist sites (Hulagu & Celikoglu, 2020), and airports (Wei, Yang, & Liu, 2022). A simulation-based optimization modelling approach was developed by Z. Liu et al. (2023) to help airport shuttle operators efficiently deploy EBs. The authors used an event-driven simulation model to optimize battery capacity, charging power, and the number of chargers to minimize the capital cost and emissions. Similarly, academic institutions across various countries, including the United States, Canada, Japan, Turkey, and Singapore, are increasingly adopting electric buses for their campus shuttle services. This shift underscores a broader commitment to sustainability and modernization within university transportation systems. By integrating electric buses, these institutions are not only improving the efficiency of their transport services but also playing a significant role in reducing carbon emissions on their campuses. Thus, it is necessary to study the operational planning of shuttle bus services that are transitioning to fully electric transit systems.

To ensure cost-effective and practical operations of campus shuttle bus services, a comprehensive framework was developed by Saner et al. (2022), with the objective of distributing trips among EBs optimally to balance daily energy usage. This framework also tackles the charging scheduling

problem to minimize electricity costs, taking into account both demand and time-of-use rates, significantly reducing charging costs and battery degradation. The authors applied their model on the shuttle bus service of the National University of Singapore. Building on these efficiencies, another integrated optimization method was devised to improve bus timetable and BSP based on real-time passenger demand (Cao & Ceder, 2019). Further enriching the operational planning of campus transportation systems, a multi-objective integer programming model was created by (Hulagu & Çelikoglu, 2019) to incorporate power usage and variable electricity rates within university zones. The objectives were minimizing operational, recharging, and total charging station installation costs and determining the most economical routes for buses to adequately meet the transport demands of the entire campus without exceeding the EB's range before requiring a recharge. Finally, the implementation of autonomous vehicles for shuttle bus services has been studied by Iclodean, Cordos, and Varga (2020). The authors investigated the social aspects of implementing autonomous vehicles and the legal framework of autonomous driving for shuttle services. Also, the comparison between EBs and modular autonomous vehicles to reduce the energy consumption per passenger has been studied in (Oargă, Prunean, Varga, Moldovanu, & Micu, 2024).

3.3.5 Challenges of Bus Electrification: Vehicle Scheduling and Timetable Adjustments

Electric buses represent a major step forward in sustainable urban transportation. However, their adoption and integration into existing operations pose unique challenges that are not faced with traditional diesel buses. These challenges stem primarily from the distinct characteristics of electric vehicles, such as limited battery capacity, longer charging times, and fewer charging stations, which differ significantly from conventional fuel vehicles (Alamatsaz et al., 2022). Consequently, transitioning from diesel to electric buses impacts both operational and economic aspects, requiring a comprehensive reevaluation of timetables and schedules for electric buses (Häll et al., 2019).

EBs are limited by the distance they can travel on a single charge, known as their range. This range is particularly limited under specific operational conditions, including passenger loads, driving profiles, the number of stops, climate conditions, road gradients, and the use of air conditioning

or heating systems. For more detailed information on how these factors affect the energy consumption and travel range of EBs, readers are directed to ([Al-Ogaili et al., 2021](#); [Papa, Santo Zarnik, & Vukašinić, 2022](#); [Suh, Lee, Kim, Oh, & Won, 2015](#); [B. Zhou et al., 2016](#)). The successful deployment of EBs is also heavily dependent on the availability of adequate charging infrastructure. Essential to this infrastructure are sufficient charging stations and their strategic locations. Also, the type of charging plays a critical role in the operational efficiency of EBs ([Alamatsaz et al., 2022](#)).

One of the main challenges with adopting EBs is the high initial investment required. The costs of purchasing EBs and setting up the necessary charging infrastructure are significantly higher than those for traditional diesel buses ([Aamodt et al., 2021](#)). While the operating costs of EBs are lower than conventional buses, the initial investment represents a substantial barrier for many transit agencies considering the switch to EBs. Also, it is crucial to plan for the recycling and procurement of batteries and charging infrastructure, which tend to have shorter lifecycles, to ensure a cost-effective and efficient bus transit system. In response to these challenges, [Gao et al. \(2017\)](#) developed a tool that optimizes the battery sizing and configuration of EBs, aiming to enhance their service reliability. Another challenge lies in the need to standardize EBs and the charging technologies to promote wider adoption and compatibility ([Pereirinha et al., 2018](#)). Moreover, electric buses require specialized maintenance practices that differ from those needed for diesel buses. Consequently, transit agencies must invest in training their technical staff to handle electric-specific issues and perform routine maintenance, which is essential for ensuring the long-term operational stability and efficiency of the fleet.

Pantograph chargers provide opportunity charging along bus routes, reducing the need for large batteries and consequently lowering energy consumption. This technology seamlessly integrates into existing bus networks by installing fast chargers at specific stops. This arrangement maintains continuous bus service, reduces maintenance costs, enhances bus availability, and allows for around-the-clock operations ([Al-Saadi et al., 2022](#)). This charging technology is highly flexible and can charge electric buses during the day at rates from 150 kW to 600 kW. Importantly, pantographs are designed to be compatible with buses from various manufacturers. They adhere to international standards and have undergone extensive interoperability testing to ensure this compatibility. Although pantograph systems are more costly upfront, they have the potential to lower the total cost

of ownership (TCO) and operational costs for EBs over time (Y. Zhou, Wang, Wang, Yu, & Tang, 2024).

The introduction of electric buses introduces specific challenges for BSP and TT, mainly because of their limited range and lengthy charging times (Quttineh et al., 2023). EBs must make trips to charging stations as part of their daily routines to ensure they can operate throughout the day. By strategically scheduling charging during off-peak hours and aligning with time-of-use (TOU) electricity rates, it is possible to reduce charging costs through dynamic electricity pricing and lessen the load on the electrical grid (L. Zhang et al., 2024). Timetables may require modifications to allow longer stops at charging stations, especially if on-route pantograph charging has been implemented. Consequently, adjustments in timetabling and scheduling are necessary to cater to the specific requirements of EBs, ensuring they serve the objectives of both the service providers and the needs of the passengers. Such adjustments could lead to more waiting time for passengers, impacting the overall appeal of EBs. Incorporating charging times into bus timetables without substantially extending service hours requires careful planning and often encounters challenges from peak traffic periods and high passenger demand (Alamatsaz et al., 2022).

Numerous mathematical optimization models have been formulated for the BSP and TT of traditional buses. Nonetheless, these models fall short when applied to EBs. Therefore, there is a need to create new mathematical models that account for the distinctive characteristics of EBs. This requirement highlights the importance of further research into the various phases of bus operation planning, especially TT and BSP. As indicated in Table 3.1 and according to Häll et al. (2019), fast-charging, wireless charging, and battery swapping are the charging strategies that influence both TT and electric BSP (EBSP). That is why studying the integration of TT and EBSP for bus transit systems that use such charging technologies is necessary. Depot charging only influences the BSP phase and continuous charging has no significant impact on TT and BSP.

Table 3.1: Indicating the effects of different charging technologies on BSP and TT

	Depot Charging	Fast- Charging	Wireless Charging	Continuous Charging	Battery Swap- ping
TT	-	✓	✓	-	✓
BSP	✓	✓	✓	-	✓

In the timetabling step, the duration of charging for electric buses, which depends on the charging technology used, plays a critical role. To improve the alignment of bus arrival times at intersections, EBs may use the downtime during service connections to recharge. This requires explicitly integrating charging durations into the timetabling process. Fast-charging technologies significantly influence the way timetables are designed. For instance, opportunity charging lasts approximately 30 seconds and can occur at designated stops without passengers alighting. On the other hand, pantograph charging, conducted when buses are not carrying passengers, demands strategic location planning to prevent interference with other traffic flows. Given that buses often have extended wait times at transfer points compared to regular stops, leveraging these intervals for charging is a feasible strategy. Thus, timetables should detail not just the bus schedules but also the timing and locations for charging, including identifying optimal locations for charging infrastructure. If the charging process is brief, less than 15 min, it directly impacts the timetable configuration (Häll et al., 2019). Conversely, longer charging times, ranging from 2 to 6 hours, as seen with depot charging, do not affect the timetable. Similarly, during the BSP phase, the specifications of the bus fleet, including capacity, range, and energy consumption, are vital for developing an optimal schedule, underscoring the importance of these considerations in planning an efficient bus operation schedule.

Typically, vehicle timetabling and scheduling for public transit systems are handled as two separate processes, where the timetable is created first and then used to inform the scheduling. A major limitation of this sequential approach is that it often overlooks the interactions between the timetable and the schedule. This oversight can lead to suboptimal outcomes. For instance, a well-constructed timetable might require using many buses, which increases operational costs. Conversely, a well-designed schedule might compromise the timetable's effectiveness by restricting vehicle availability. Thus, the solution is a complete integration of timetabling and bus scheduling. The complete integration approach treats the problem as a fully integrated problem, simultaneously addressing both TT and BSP of public transit planning. For example, minor adjustments to the bus timetable could lead to a more efficient vehicle schedule. This integration is more essential for EBs, where the traditional sequential approach, which is designing the timetable first and then scheduling, can significantly increase costs due to the EBs' limited range and longer charging times. For instance, an overly strict timetable may make the optimal schedule infeasible for EBs due to their charging

requirements. Therefore, a fully integrated approach to timetabling and scheduling is necessary for transit systems using electric buses.

3.4 Methodology

In this section, we explain the problem description and the methodology used to solve the problem. We propose an algorithm to determine the optimal headways for different passenger volumes and find the most efficient schedule for the EBs. The proposed methodology aims to reduce passengers' waiting times and increase their comfort by ensuring seat availability, and optimizing the schedule of shuttle buses to minimize the total operational costs. First, we will discuss timetable development. Then, we will present a mathematical optimization model for the shuttle bus scheduling problem. Finally, we will describe the proposed workflow.

3.4.1 Problem Description

This study aims to improve the timetable and schedule of fully electric shuttle bus services at Concordia University that use pantograph chargers to charge EBs. By optimizing bus headways, we aim to minimize waiting times and ensure seat availability for student comfort. Simultaneously, we seek to optimize the EBs' schedule to lower operational costs while ensuring all scheduled trips are covered. This includes minimizing the number of required EBs, deadhead trips (empty trips to relocalize buses), and electricity costs for charging EBs.

The problem is centered around a real-world case study of Concordia University in Montreal, Canada, focusing on its shuttle bus service. The service includes two bus stops, $S = \{S_1, S_2\}$, which serve as the starting and ending points of the shuttle route. Additionally, there is a depot, denoted as O , from which the electric buses begin their daily operations. The EBs are short-range electric buses that must be charged frequently during their operation with pantograph chargers located at the Loyola campus, represented by F . Since charging with pantograph chargers takes 5 to 10 min to fully charge an EB and they could only charge one EB at a time, we should also consider the charging scheduling and its impact on the shuttle bus frequency settings and scheduling.

The operation time of the shuttle bus service is denoted by T and it is divided into J intervals or time spans. The total number of daily trips made by the shuttle bus is represented by I , and the number of trips within each specific time span is denoted by I_j , where $j \in J$. This number depends on the headway value h . For instance, if $h = 20$ min and the total time $|T_j| = 100$ min, then there will be five trips for time span $j \in J$. Headway between shuttle buses varies between a minimum of h_{min} and a maximum of h_{max} , and it is assumed to be constant throughout a time span. The travelling distance and travelling time between the bus stops are known during the planning and denoted by d_{ij} and τ_{ij} , where $i, j \in S$, respectively. Similarly, the travel distance and travel time from the depot to the bus stops and vice versa are fixed and given. The ridership datasets of the shuttle bus are available, and we assume the number of passengers will not be affected by the headway of shuttle buses.

3.4.2 Waiting Time and Passenger Comfort

Designing an effective and practical timetable is complex due to balancing multiple objectives. According to [Shang et al. \(2019\)](#), the primary goals of frequency setting and TT are minimizing passenger waiting times and in-vehicle seat availability. The interaction between bus headways, passenger waiting time, and seat availability is essential in optimizing public transport systems. Shorter headways typically reduce waiting times and improve seat availability, leading to higher passenger satisfaction but potentially increasing operational costs. On the other hand, longer headways can result in longer waiting times and crowded buses, decreasing seat availability. Effective transit management requires carefully balancing these factors by adjusting headways based on passenger demand to enhance both service quality and operational efficiency.

According to [McLeod \(2007\)](#) research, the average waiting time of bus users during a specific time span $j \in J$, denoted by AWT_j , is calculated as follows:

$$AWT_j = \frac{1}{2} \left(\frac{\sigma_j^2 + \mu_j^2}{\mu_j} \right) \quad (1)$$

where μ_j and σ_j^2 are the mean and variance of the bus headways for time span $j \in J$, respectively.

The average passenger waiting time consists of two components. The first component, $\frac{1}{2}\mu_j$, is influenced by the bus service frequency. The second component, $\frac{1}{2}\left(\frac{\sigma_j^2}{\mu_j}\right)$, is impacted by the variability in headways. Typically, the component associated with bus service irregularity holds greater significance in performance measurement since the other component remains constant for a specified bus frequency. That is why we focus on the average interval, or headway, between shuttle buses. In this study, the headway value is consistent across all time spans, resulting in a variance of zero. Therefore, given a headway value h , the average waiting time is given by

$$AWT_{jh} = \frac{h}{2} \quad (2)$$

The pattern of passengers' arrival is assumed to follow a Poisson distribution based on the ridership dataset from the existing timetable of the shuttle bus service. The Poisson distribution serves as a robust statistical model for analyzing passenger arrivals at bus stops, given its ability to accurately represent events that occur independently and maintain a consistent average rate. This model assumes that each passenger's arrival is an independent event unaffected by the arrivals of others, which aligns well with the random and uncoordinated nature of passengers arriving at bus stops. Characterized by a constant mean rate of passengers arriving, denoted by λ , the Poisson distribution captures the average rate of passenger arrivals per unit of time. The value of λ is determined using the available ridership data. The total waiting time of passengers for trip $i \in I_j$ where $j \in J$ at a bus stop is determined by both the departure time of trip i at time span j and the number of passengers waiting at a bus stop, denoted as ρ_{ij} . The total waiting time for trip $i \in I_j$ is expressed through the equation:

$$w_{ij} = \rho_{ij} \times AWT_j \quad (3)$$

The total waiting time for time span $j \in J$, denoted as γ_j , is calculated as follows:

$$\gamma_j = \sum_{i \in I_j} w_{ij} \quad (4)$$

Passenger comfort is hard to quantify, and according to [Shang et al. \(2019\)](#), in-vehicle bus comfort through seat availability is one of the most important attributes of passenger comfort. To

assess passenger satisfaction, we start by calculating the load factor, denoted as L . Load factor is equal to l_{ij}/ϕ , which l_{ij} is the number of passengers riding on trip $i \in I_j$, where $j \in J$ and ϕ is the number of seats in the shuttle bus. Let Q be the bus capacity and $L_{max} = Q/\phi$ be the maximum load factor. If $L \leq 1$, all passengers have seats; otherwise, if $L > 1$, ϕ passengers have seats while $(L - 1)\phi$ passengers experience crowding and must stand. Based on the load factor values, we calculate the passengers' comfort through seat availability by quantifying this criterion. As a result, passengers' comfort, η_{ij} , for trip $i \in I_j$ can be quantified and calculated by

$$\eta_{ij} = \begin{cases} 1 & l_{ij} \leq \phi \\ \frac{L_{max}\phi - l_{ij}}{L_{max}\phi - \phi} & \phi < l_{ij} \end{cases} \quad (5)$$

and the total comfort of passengers for time span $j \in J$ is as follows:

$$\delta_j = \sum_{i \in I_j} \eta_{ij} \quad (6)$$

3.4.3 Shuttle Bus Scheduling

This section explains the electric shuttle bus scheduling problem in more detail. We present an arc-based mathematical formulation to model the scheduling of electric shuttle buses with the objective of minimizing the total operational costs of EBs. The operational costs include the usage cost of each EB, or, in this case, the annual leasing fees for each bus that Concordia University must pay for each shuttle bus shift, the travelling costs of EBs, the deadhead trips, and the cost of charging EBs with the pantograph chargers. We assume buses can be leased on a per-shift basis, so the number of leased buses can vary between time spans.

In this paper, the term *trip* is used to describe each scheduled travel within the shuttle bus service timetable. Assume the shuttle bus operates daily starting at 07:20 a.m., running at 50-min intervals until noon. This yields a timetable with six trips for the morning time span, specifically at 07:20 a.m., 08:10 a.m., 09:00 a.m., 09:50 a.m., 10:40 a.m., and 11:30 a.m. To address the shuttle bus scheduling problem, we consider a set of trips, represented as I , which are defined by their start times s_i and end times e_i . This set of trips is derived from the timetabling process outlined in

section 3.4.4. Each electric shuttle bus is allocated to only one trip at any given time. Also, an electric bus can begin a new trip if it completes its previous one with sufficient time and energy remaining to start the next trip.

Deadhead trips, which involve EBs travelling empty in order to relocate for subsequent trips and charging, is an essential component of the operational logistics of EB scheduling. These trips typically occur between the depot and the starting points of the bus line or between the charging station and the starting points of the bus line. Let d_{ij} represent the distance of a deadhead trip from i to j , where i and j can be a depot, a charging station, or the start/end location of a trip. The trip distance $i \in I$, is denoted by $\tilde{d}_i$ and Δ_i represents the duration of trip $i \in I$. The maximum distance an EB can travel on a fully charged battery is indicated by D and the annual leasing or usage cost of each EB is represented by C^v . The cost per unit distance travelled by an electric shuttle bus is denoted by c^d . Lastly, the cost of using a fast-charging station at time t is given by C^t .

Electric shuttle buses begin their routes fully charged from a depot. As long as they have enough energy to return to the same depot or a pantograph charger, they may run the whole schedule. At the pantograph charging station, only one EB can be charged at a time, and a fixed charging period, denoted by U , is assumed for each charging event. We conceptualize the charging station's capacity using time-expanded station nodes. This involves dividing the possible times for charging EBs into distinct, one-minute intervals. The set of time periods, denoted by P , is defined as $\{1, 2, \dots, |P|\}$, representing the planning horizon for charging EBs at the charging station. Furthermore, P' represents the sequence of time periods $\{1, 2, \dots, |P| - U\}$, ranging from the first period to the last, adjusted by subtracting the fixed charging duration of electric buses.

Assuming a fixed charging duration for EBs and charging them to full capacity simplifies their operational planning. Fully charging the batteries, rather than maintaining them at mid-level charges, helps preserve their health and longevity. Battery management systems are designed to optimize full charging cycles, and avoiding partial charges can extend battery life. Additionally, fully charging the batteries results in predictable energy usage patterns, which are essential for effective energy management and planning. This allows the university to more accurately predict electricity costs and manage demand on the electrical grid.

Our arc-based formulation is based on the following network $G = (N, A)$ where N includes

the daily trips of the shuttle bus, the depot, O , and the time-expanded nodes of the pantograph charger, and A represents the feasible connecting arcs. The arcs in A are categorized into five types: *depot-to-trip* arcs from O to $i \in I$; *trip-to-trip* arcs from $i \in I$ to $j \in I$ if $s_i + \Delta_i + \tau_{ij} \leq e_j$ (i.e., if the corresponding connection is feasible); *trip-to-charger* arcs from $i \in I$ to the time-expanded node $t \in P$ at the pantograph charger located at the Loyola campus, represented by F if $s_i + \Delta_i + \tau_{it} \leq |P| - U$; *charger-to-trip* arcs from $t \in P$ to $j \in I$ if $t + \tau_{tj} \leq s_j$; and *trip-to-depot* arcs from $j \in I$ to O . Additionally, A' represents all potential arcs between trips and the charging station. The cost of an arc (i, j) is defined as follows: depot-to-trip arcs have cost $c_{ij} = c_d d_{ij} + C_v$; trip-to-charger arcs have cost $c_{it} = c_d d_{it} + C_t$ where $t \in P$; and trip-to-trip arcs have cost $c_{ij} = c_d d_{ij}$. As an example, an arc-based network for three time spans is illustrated in Figure 3.1. The proposed mathematical model optimizes one time span individually, using the list of trips derived from the headway value. In this model, we focus only on feasible trips between nodes, removing any infeasible connections between trips, the charging station, and the depot. This is carried out by considering the travel times between nodes and the duration of the trips.

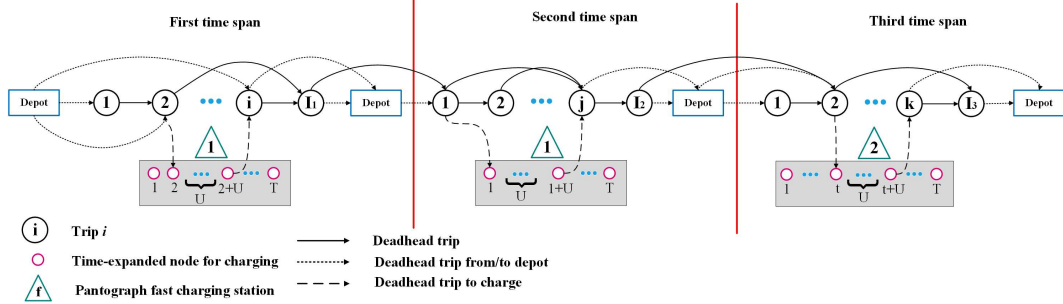


Figure 3.1: Arc-based network for three time spans.

We now introduce a mathematical optimization model for scheduling electric shuttle buses. The following defines the model's decision variables: Let x_{ij} be a binary decision variable, where $x_{ij} = 1$ indicates that a shuttle bus takes arc (i, j) , and $x_{ij} = 0$ otherwise. Additionally, let r_i be a continuous decision variable, indicating the cumulative distance travelled from the depot to the endpoint of trip $i \in I$ since the last charging event. Let v_{ij} be a variable equal to $x_{ij}r_i$. The purpose of v_{ij} is to make the model linear.

The bus scheduling problem is modelled as follows:

$$Z = \min \left(\sum_{(i,j) \in A} c_{ij} x_{ij} \right) \quad (7)$$

s.t.

$$\sum_{(i,j) \in A} x_{ij} = 1 \quad \forall j \in I \quad (8)$$

$$\sum_{(i,j) \in A} x_{ij} - \sum_{(j,i) \in A} x_{ji} = 0 \quad \forall i \in N \quad (9)$$

$$r_j = \sum_{(i,j) \in A} (v_{ij} + (\tilde{d}_j + d_{ij}) x_{ij}) \quad \forall i \in S \quad (10)$$

$$r_t \leq (1 - x_{it})D \quad \forall (i, t) \in A' \quad (11)$$

$$r_j \leq D \quad \forall j \in S \cup P \quad (12)$$

$$v_{ij} \leq D x_{ij} \quad \forall (i, j) \in A \quad (13)$$

$$v_{ij} \leq r_i \quad \forall i \in I \quad (14)$$

$$v_{ij} \geq r_i - (1 - x_{ij})D \quad \forall (i, j) \in A \quad (15)$$

$$\sum_{(i,t \in P) \in A'} x_{it} + \sum_{(i,t \in P) \in A'} \sum_{s=1}^U x_{i,t+s} \leq 1 \quad \forall t \in P' \quad (16)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in A \quad (17)$$

$$r_j \geq 0 \quad \forall j \in I \cup P \quad (18)$$

$$v_{ij} \geq 0 \quad (i, j) \in A \quad (19)$$

The objective function (7) calculates the total operational costs of shuttle bus scheduling. The costs include the usage cost of each electric bus, the charging costs, and the travel costs between trips and the charging station. Constraints (8) and (9) ensure trip coverage and flow conservation, respectively. Constraint (10) measures the accumulative distance travelled since the most recent

charging event. Constraint (11) specifies that the total distance travelled by an electric bus is reset to zero whenever it recharges at a station. Constraint (12) requires that the total distance travelled by each electric bus must not surpass its maximum range. Constraints (13) ensure that the value of v_{ij} does not exceed the maximum travel range of EBs whenever the corresponding decision variable $x_{ij} = 1$. If $x_{ij} = 0$ then v_{ij} is forced to zero. Similarly, constraints (14) guarantee that the auxiliary variables cannot exceed the total travelled distance of each EB. Constraints (15) indicate the exact value of v_{ij} . If $x_{ij} = 1$, then v_{ij} is at least r_i . If $x_{ij} = 0$, then v_{ij} should be equal to zero. Finally, constraint (16) requires adherence to the station capacity constraint. Constraints (17) to (19) determine the domain of variables.

3.4.4 Solution Approach

To determine the optimal timetable and headway for the shuttle bus service, we developed and implemented an algorithm using a time-span-based decomposition. This algorithm assesses different headway values to develop an effective timetable that minimizes waiting times and improves passenger comfort. Simultaneously, it optimizes the shuttle bus schedule to reduce operational costs. The inputs for this algorithm include the shuttle bus service data, EBs' specifications, designated time spans, student ridership data, and the weighted importance of each optimization objective. The optimization process begins with the initial time span $j = 1$. For this time span, we start with the smallest possible headway, h_{min} , and calculate both the average waiting time of passengers and in-vehicle bus comfort for passengers, based on Equations (4) and (6), respectively, represented in Section 3.4.2. Based on the selected headway, we then determine the list of departure times for that time span, which is then incorporated into the mathematical optimization model presented in Section 3.4.3 to generate a bus schedule. The proposed algorithm calculates the costs associated with each headway, considering the weights assigned to the waiting time, ω_γ , comfort ω_δ , and cost ω_Z . We collect the outcomes as the normalized weighted sum of the objectives, represented by the

variable ψ_j for each time span $j \in J$. The formula used to calculate ψ_j is as follows:

$$\begin{aligned} \psi_{jh} = & \left(\frac{(\gamma_{jh} - \min_h(\gamma_{jh}))}{\max_h(\gamma_{jh}) - \min_h(\gamma_{jh})} \right) \omega_\gamma - \left(\frac{(\delta_{jh} - \min_h(\delta_{jh}))}{\max_h(\delta_{jh}) - \min_h(\delta_{jh})} \right) \omega_\delta \\ & + \left(\frac{(Z_{jh} - \min_h(Z_{jh}))}{\max_h(Z_{jh}) - \min_h(Z_{jh})} \right) \omega_Z \end{aligned} \quad (20)$$

where γ_{jh} , δ_{jh} , and Z_{jh} are, respectively, the waiting time, seat availability metric, and total operational cost for time span j under headway h , and $\psi_h = \sum_{j=1}^J \psi_{jh}$ is the total normalized weighted sum of the objectives across all the time spans.

This process is then repeated for each possible value of h , ranging from $h_{\min} = 15$ minutes to $h_{\max} = 40$ minute with $\Delta h = 1$. The optimal headway is the value that minimizes ψ_{jh} . We denote this optimal value as $h_j^* = \arg \min_h(\psi_{jh})$ and the corresponding minimized value as $\psi_j^* = \min_h(\psi_{jh})$. Once h_j^* is determined, the corresponding schedule is fixed and the algorithm then optimizes time span $j + 1$. Note that the bus scheduling optimization model takes into account the bus state (location, remaining charge, etc.) from the previous window; thus, the overall schedule remains consistent. Once the last time span is optimized, a full bus schedule is obtained by merging each time span's optimal schedule. Finally, the total weighted objective value is given by $\psi = \sum_{j=1}^J \psi_j^*$. The detailed steps of our method are outlined in the pseudo-code in Algorithm 1 and Figure 3.2.

Algorithm 1 Pseudo-Code of Integrating TT with EBSPhr/>

```
1: function TT-EBSPhr/>2:   Input:hr/>3:     Ridership datasethr/>4:     Shuttle bus network datahr/>5:     Electric bus specificationshr/>6:     Other required parameters and thresholdshr/>7:   Output:hr/>8:     Best balance between bus timetabling and schedulinghr/>9:    $j \leftarrow 1$ hr/>10:   $h \leftarrow h_{min}$ hr/>11:  while  $j \leq |J|$  dohr/>12:    while  $h \leq h_{max}$  dohr/>13:      Obtain the number of passengers for  $h$  based on Poisson distribution and the rider-  
ship datasethr/>14:      Calculate total average waiting time for  $h$ ,  $\gamma_{jh}$ hr/>15:      Obtain passenger comfort for  $h$ ,  $\delta_{jh}$ hr/>16:      Collect the list of trips for each headwayhr/>17:      Optimize the scheduling of shuttle buses for  $h$ hr/>18:      Collect  $Z_{jh}$ hr/>19:      Compute  $\psi_{jh}$ hr/>20:       $h \leftarrow h + 1$ hr/>21:    end whilehr/>22:    Determine  $h_j^*$  and  $\psi_j^*$ hr/>23:    Update the SOC of electric buses for the next time slothr/>24:    Update the starting and ending points of the first and last tripshr/>25:     $j \leftarrow j + 1$ hr/>26:  end whilehr/>27:  return  $\{h_j^*, \forall j \in J\}, \psi$ , and optimal schedulehr/>28: end function
```

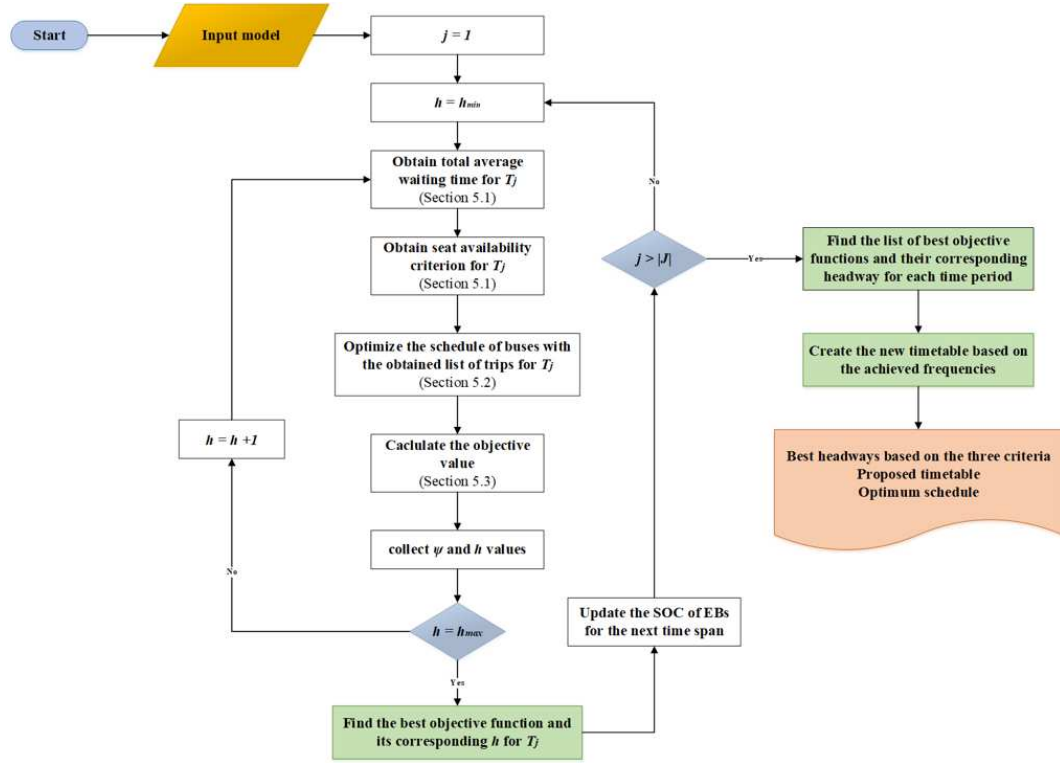


Figure 3.2: Flowchart of integrating TT with EBSP

The reason to decompose the problem is mainly to find a coherent way to calculate headways. We start by optimizing the first time span, after which we update the necessary inputs for the subsequent time span and proceed to solve the scheduling problem for the next iteration. Key parameters that require updating include the state of charge of the electric buses, which our model represents as the total travel distance of each bus. Updating these parameters is essential for the optimization model to resume effectively from where the last time span concluded, ensuring continuity and efficiency in our approach.

3.5 Case Study and Results

3.5.1 Case Study

Concordia University, located in Montreal, Canada, is home to two campuses: Sir George William (SGW) and Loyola. SGW Campus houses 40 buildings and Loyola Campus comprises

27 buildings. During the 2022–2023 academic year, the university hosted 35,404 undergraduates and 10,084 graduates, totalling 45,488 students (*Concordia in numbers*, 2023). Many students rely on public transportation, the university’s shuttle bus, or personal vehicles for their daily commute. The shuttle bus service, free for Concordia’s students and staff, facilitates travel between the SGW and Loyola campuses from Monday to Friday according to a set schedule. The route of the shuttle buses, the real-time location of buses, and trip details provided by Concordia University’s Facility Management are illustrated in Figure 3.3.

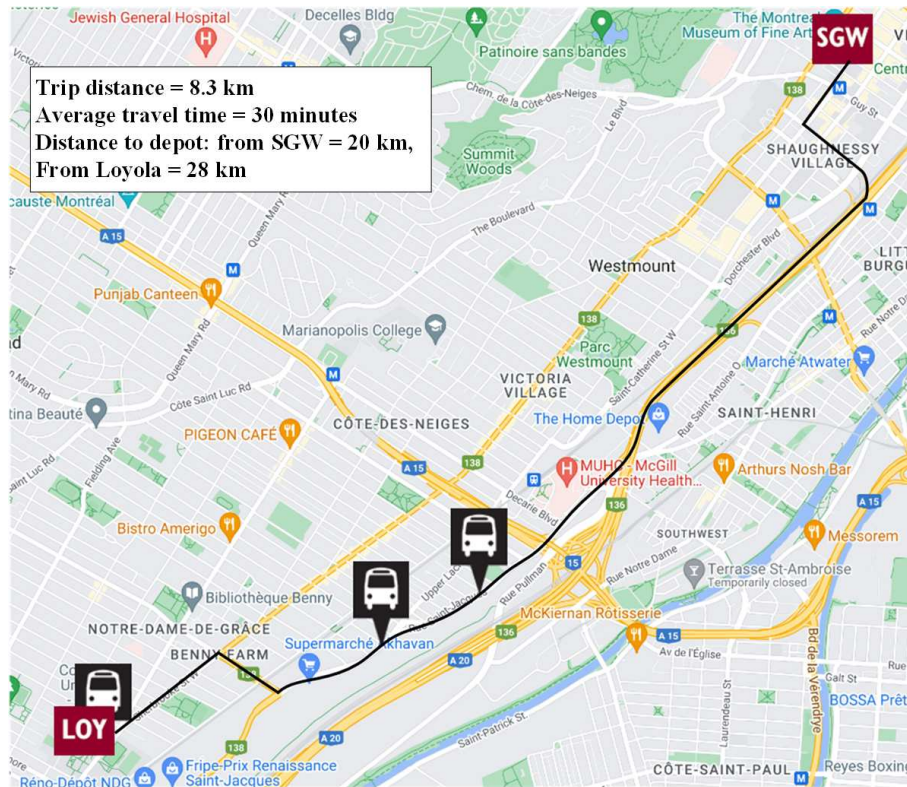


Figure 3.3: Concordia shuttle bus route and the real-time location of buses.

Concordia University is promoting sustainable transportation through its Climate Action Plan, which targets reducing the university’s carbon footprint and achieving climate neutrality within 20 years. This includes a commitment to fully electrify its campus transportation system by 2040, a move expected to cut 25% of its total emissions from 2014–2015 levels. The focus is on converting the shuttle bus fleets to electric, significantly lowering CO₂ and greenhouse gas emissions. This study specifically addresses the Concordia University shuttle bus system, covering the operational

and economic aspects of the service between SGW and Loyola campuses. It presents a strategic plan to transition to electric buses and optimize their schedule for efficiency and student convenience. The goal is to enhance service timetables, reduce student wait times, increase comfort, and minimize operational costs.

Facility Management at Concordia University is focused on developing operational strategies to ensure a smooth and efficient transition to electric buses for their shuttle service. This research aims to provide a method to establish the most effective timetables and schedules for the shuttle, determine the best type of EBs, identify the optimal number of EBs to lease, and outline the best headways to seamlessly integrate electric buses into the existing service.

In an effort to evaluate the effectiveness and satisfaction levels associated with the Concordia shuttle bus service, a student-run social campaign conducted a detailed survey, gathering insights into student satisfaction, seat availability, and waiting times (*Improve Concordia University Shuttle Bus*, 2024). The survey data underscore the urgent need for operational improvements in the Concordia shuttle bus service. Addressing the issues of seat availability and wait times could greatly enhance user satisfaction. The findings reveal that only a small portion of students expressed full satisfaction with the shuttle service, suggesting considerable potential for improvement. According to Figure 3.4, a majority reported dissatisfaction and a notable fraction remained indifferent.

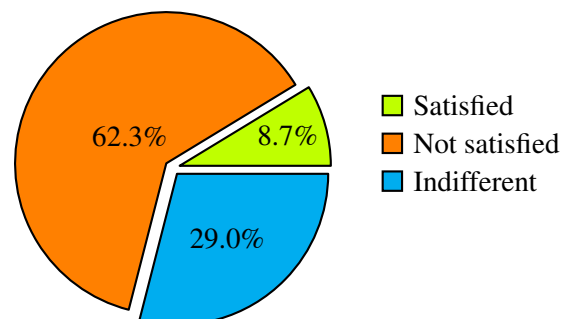


Figure 3.4: Students' satisfaction with Concordia Shuttle bus service.

As shown in Figure 3.5, concerning seat availability, a substantial number of respondents indicated that they never find seats, highlighting a major issue with capacity that likely affects overall satisfaction and comfort. The distribution of responses presented in Figure 3.6, regarding waiting times shows that while most students wait for a moderate duration, a considerable percentage

experience longer waits, and only a few enjoy minimal waiting times.

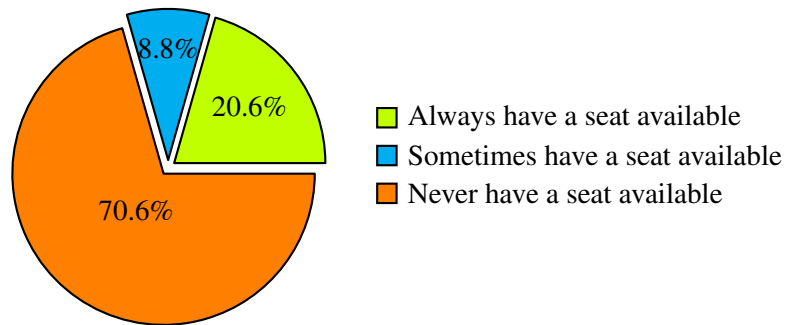


Figure 3.5: Students' response regarding seat availability.

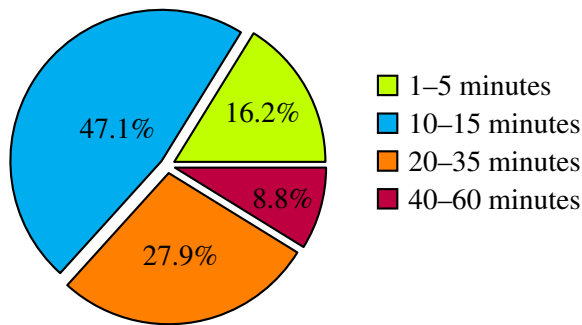


Figure 3.6: Students' waiting time for the shuttle bus.

Concordia University has provided access to its 2022 shuttle bus ridership data. This dataset records the number of students who used the shuttle service throughout the year, detailing usage at different times of the day according to the current timetable. Given that the class schedule remains relatively consistent from year to year, the average ridership can be effectively used for planning in subsequent years. After receiving these data, we implemented our proposed solution approach to determine the optimal headways, aiming to reduce waiting times and enhance passenger comfort. Simultaneously, we optimized the schedule for electric buses to minimize total operational costs. This methodology enabled us to create and implement a more efficient and cost-effective timetable and schedule for the shuttle bus service.

3.5.2 Model Results Analysis

In this section, we detail the outcomes of optimizing the Concordia shuttle bus service and compare these results with the existing timetable. Additionally, we perform a sensitivity analysis to determine which type of EBs, with various characteristics, would be most suitable for the Concordia shuttle bus service.

The starting and ending points of the trips are the two main campuses of Concordia which are SGW and Loyola campus. Both the conventional shuttle buses and the new electric buses have the same seating capacity, which is 40 seats. The depot's coordinates are provided, and the pantograph charging station is located at the Loyola campus near the bus stop. The annual usage cost for an EB is CAD 53,266 ([Bloomberg New Energy Finance, 2018a](#)), which translates to approximately CAD 17,775 per shift. Based on ([Bloomberg New Energy Finance, 2018a](#)), the capital cost of pantograph chargers is CAD 230,000. We assumed that the estimated lifespan of the buses is 15 years. Additionally, the annual usage cost of conventional shuttle buses, specifically the Nova Bus model currently used for the Concordia shuttle bus, is CAD 23,333, or approximately CAD 7,777 per shift. According to [J.-Q. Li \(2014\)](#), the operational cost per kilometer is CAD 0.662 for an EB and CAD 0.67 for conventional buses. Additionally, the maximum travel range for EBs is assumed to be 78 km. Note that a minimum layover must be scheduled at the end of each trip to allow drivers time to rest and use the washroom. In this case study, this layover time could be added to the trip times. It is crucial to understand that the considered operational costs are based on the number of working days in a year, which are then annualized to facilitate direct comparison with the yearly costs associated with using new EBs. The weights assigned to waiting time, seat availability, and costs are 0.25, 0.25, and 0.5, respectively. Concordia University's Facility Management has the flexibility to adjust these weights based on their preferences and can rerun the model as needed.

According to data provided by Concordia University's Facility Management, the shuttle bus service annually consumes approximately 90,026.21 litres of fuel. The greenhouse gas emissions from this volume of fuel, estimated using emission factors of 2.681 kg/L for CO₂, 5.1×10^{-5} kg/L for CH₄, and 2.2×10^{-4} kg/L for N₂O, result in annual emissions of 241.36 tonnes of CO₂, 0.0046 tonnes of CH₄, and 0.0198 tonnes of N₂O. To account for their higher global warming potentials,

methane and nitrous oxide emissions are converted to CO₂ equivalent emissions using conversion factors of 86 kg CO₂ for methane and 298 kg CO₂ for nitrous oxide, resulting in additional emissions of 0.40 tonnes CO₂ from CH₄ and 5.90 tonnes CO₂ from N₂O ([Environment & Canada, 2020](#)). The total emissions are then summed to yield 247.66 tonnes CO₂ per year from the combustion of diesel fuel for the shuttle bus service. Also, the unit cost of diesel in Montreal is assumed to be 1.865 CAD/litre.

The comparison between the current Concordia shuttle bus service timetable and the proposed timetable from our study is detailed in Table 3.2. Note that the reported results are on an annual basis. The results of the current timetable are based on the assumption that the existing timetable and schedule of buses are being performed by current diesel buses. According to this table, our analysis shows that the proposed timetable offers improvements in both waiting times and seat availability across the given time spans when compared to the existing schedule. The computational time for the proposed algorithm across the three intervals totals 4690 s, which is approximately 78 min.

Table 3.2: Comparative analysis of current TT using conventional buses vs. proposed TT

	Current Timetable			Proposed Timetable		
	Morning	Afternoon	Evening	Morning	Afternoon	Evening
Number of buses	4	4	4	4	4	4
Bus usage cost (CAD)	31,108	31,108	31,108	71,100	71,100	71,100
Electricity/fuel cost (CAD)	64,120	77,861	41,220	4409	10,471	4409
Travel cost (CAD)	38,516	46,770	24,760	18,396	38,106	15,768
Total cost (CAD)	133,744	155,739	97,088	93,905	119,677	91,277
Waiting time (min)	6292	9255	810	7125	5292	361
Seat availability	10.9	14.3	8.0	6.3	29.0	12.0

According to Table 3.2, both timetables maintain the same number of buses throughout the day. While the bus usage cost shows a substantial increase under the proposed timetable due to higher bus leasing costs, the fuel cost for current diesel buses is significantly higher than the electricity cost for EBs. It is important to note that the fuel costs for diesel buses include Quebec's carbon tax, which is CAD 0.17 per litre of diesel ([Fuel Charge Rates - Canada.ca, n.d.](#)). Thus, the total operational costs for the proposed timetable using EBs are lower at different times of the day compared to the existing timetable with diesel buses. In terms of service quality, as shown in Figure 3.7, the proposed timetable slightly increases the waiting time in the morning but substantially reduces it in

the afternoon and slightly in the evening, improving overall passenger satisfaction. Seat availability is significantly enhanced in the afternoon, suggesting better accommodation for peak-time ridership. Notably, the proposed timetable achieves zero carbon dioxide emissions, marking a significant step towards environmental sustainability. As a result, the proposed timetable and schedule not only aligns with environmental goals but also attempts to balance cost with improved level of service.

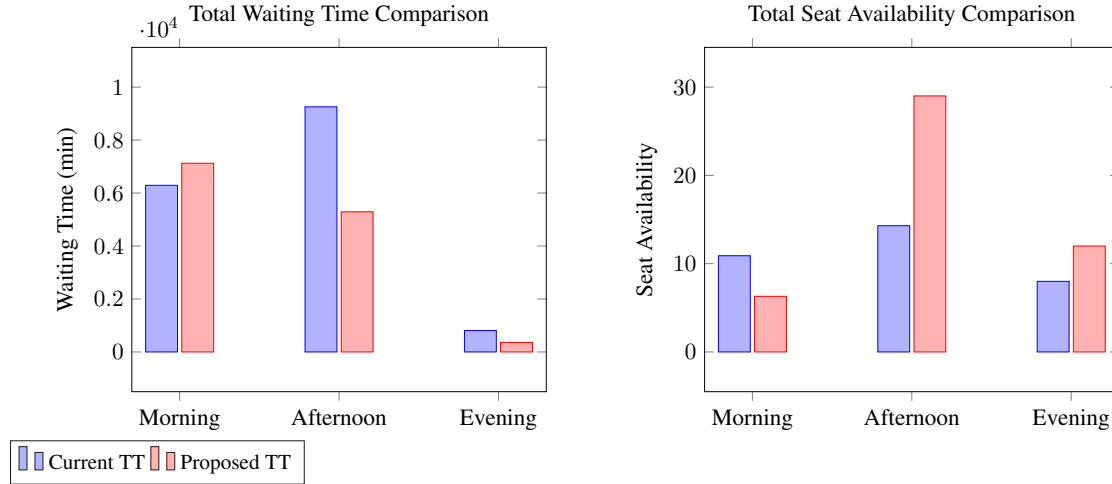


Figure 3.7: Comparing the current and proposed TT on waiting time and seat availability metrics

We can now compare the modified version of the current timetable to the proposed timetable. In this modified schedule, we still follow the current timetable but have optimized the shuttle bus scheduling to accommodate electric buses. The findings from this comparison are outlined in Table 3.3. This analysis demonstrates the enhancements offered by the proposed timetable, highlighting its effectiveness even when transitioning to EBs while maintaining the existing timetable.

Table 3.3: Comparative analysis of current TT using EB vs. proposed TT.

	Current TT with Modified Schedule			Proposed Timetable		
	Morning	Afternoon	Evening	Morning	Afternoon	Evening
Number of buses	5	3	3	4	4	4
Bus usage cost (CAD)	88,875	53,325	53,325	71,100	71,100	71,100
Electricity cost (CAD)	2204	7716	6062	4409	10,471	4409
Travel cost (CAD)	10,512	23,652	22,338	18,396	38,106	15,768
Total cost (CAD)	101,591	84,693	81,725	93,905	119,677	91,277
Waiting time (min)	6292	9255	810	7125	5292	361
Seat availability	10.9	14.3	8.0	6.3	29.0	12.0

The comparative analysis between the current and proposed timetables reveals several important differences. As shown in Table 3.3, the proposed timetable consistently utilizes four EBs across all three time spans, morning, afternoon, and evening, unlike the current timetable, which varies the number of EBs. This consistency in the number of buses simplifies fleet management and scheduling of drivers. While the total costs associated with the existing timetable are lower, the proposed timetable offers significant improvements in the total annual waiting time for students and seat availability. Notably, the waiting time for the morning shift under the existing modified schedule is the only period that shows better performance compared to the proposed timetable. However, for the afternoon and evening shifts, the proposed timetable achieves a substantial reduction in passenger waiting times, which outweighs the longer wait times experienced in the morning. This pattern of improvement is mirrored in the seat availability metric, where the proposed TT enhances the overall passenger experience significantly, suggesting better resource allocation during peak hours. These differences highlight the need to account for the EBs' characteristics when generating the timetable. Indeed, using a standard timetabling approach, which does not account for the need for buses to recharge, can lead to suboptimal schedules, both in terms of costs and passenger satisfaction.

The headways from the current timetable of Concordia University's shuttle bus service, alongside the headways derived from our proposed solution, are shown in Table 3.4. To make the timetable more practical, we have also included suggested headways, as three different scenarios rounded to the nearest multiple of 5. While rounding the headways from our optimization algorithm might slightly reduce efficiency and affect results, it simplifies the university's scheduling and timetabling process. Furthermore, rounded headways make it easier for students to plan their trips with precise, consistent departure times.

Table 3.4: Comparing headways values for the current, optimum and suggested scenarios

Time Span	Headway				
	Current	Optimum	Scenario 1	Scenario 2	Scenario 3
Morning	15 min	19 min	20 min	20 min	15 min
Afternoon	30 min	18 min	20 min	15 min	15 min
Evening	30 min	19 min	20 min	20 min	15 min

The comparison of the considered objectives for the suggested and current headways, shown in

Table 3.4, is detailed in Table 3.5.

Table 3.5: Comparing the objective functions of the current, optimum and suggested headways.

Timetable	Cost (CAD) (Z)	Waiting Time (γ)	Seat Availability (δ)	ψ
Current	386,571	16,357 min	33.24	0.68
Optimum	304,859	12,778 min	47.33	0.39
Scenario 1	297,188	13,340 min	40.96	0.50
Scenario 2	332,300	12,007 min	48.96	0.53
Scenario 3	381,669	10,027 min	60.83	0.46

The comparative analysis between the current and optimum timetables reveals significant improvements in key operational metrics. According to Table 3.5, the optimum timetable achieves a cost reduction of about 21% compared to the current setup. More notably, it reduces the waiting time by approximately 22%, significantly enhancing passenger convenience. Additionally, seat availability sees a substantial increase of approximately 42%, reflecting an improvement in service quality and capacity utilization. These enhancements contribute to a lower ψ value of 0.39 for the optimum timetable, compared to 0.68 for the current one, indicating a more efficient and balanced operation that better meets the objectives of cost efficiency, reduced waiting times, and improved passenger comfort. These percentages underscore the effectiveness of the optimum timetable in optimizing performance across multiple dimensions.

The comparison of the optimum and suggested timetables highlights the various trade-offs involved in balancing cost, waiting time, and passenger comfort. The goal is to find the headway scenario closest to the optimum timetable, which has the lowest ψ value and offers the best balance among these factors. Scenario 1 results in higher costs and longer waiting times, yielding a moderate ψ value. Scenario 2 improves seat availability and reduces waiting time but at a higher cost, leading to the highest ψ value of 0.53. In contrast, Scenario 3, despite having the highest total cost, provides the lowest waiting time and best seat availability, achieving a ψ value of 0.46, making it the closest to the optimum. This analysis shows that while the optimum timetable is the most efficient, Scenario 3 offers a practical alternative that comes closest to optimizing performance without significant compromises.

According to Table 3.5, the third scenario shows significant improvements in waiting time and seat availability compared to the current timetable. Although the transition to EBs involves high

initial costs, the proposed timetable offers the most cost-efficient solution for Concordia University's shuttle bus service after switching to EBs.

We now provide a Pareto front to illustrate the trade-off between operational costs and passenger satisfaction. The passenger satisfaction metric represents a normalized weighted sum of students' waiting times and seat availability as $\left(\frac{(\gamma_{jh} - \min_h(\gamma_{jh}))}{\max_h(\gamma_{jh}) - \min_h(\gamma_{jh})} \right) \omega_\gamma$ and $\left(\frac{(\delta_{jh} - \min_h(\delta_{jh}))}{\max_h(\delta_{jh}) - \min_h(\delta_{jh})} \right) \omega_\delta$, respectively. This metric is crucial for evaluating the quality of service provided to passengers, with higher values indicating greater satisfaction. The Pareto front of the considered EB is depicted in Figure 3.8. On the x-axis, the cost represents the total expenses associated with operating EBs across optimum headways determined through our optimization algorithm. The Pareto front displays results from nine distinct combinations of weights, ranging from $\omega_\gamma + \omega_\delta = 0.9$ and $\omega_Z = 0.1$ to $\omega_\gamma + \omega_\delta = 0.1$ and $\omega_Z = 0.9$, increasing by increments of 0.1. In all cases, ω_γ and ω_δ are equal.

Figure 3.8 effectively demonstrates how different scheduling strategies impact both cost and passenger satisfaction. Points closer to the bottom left of the graph indicate lower costs and lower passenger satisfaction, while points towards the top right suggest higher costs but improved passenger satisfaction. The point depicted in red is the solution that we have compared with the existing timetable and schedule of the shuttle bus. This visualization aids in identifying potential optimal points where the balance between minimizing operational costs and maximizing passenger satisfaction can be achieved, providing valuable insights for the Facility Management of Concordia University for its shuttle bus service.

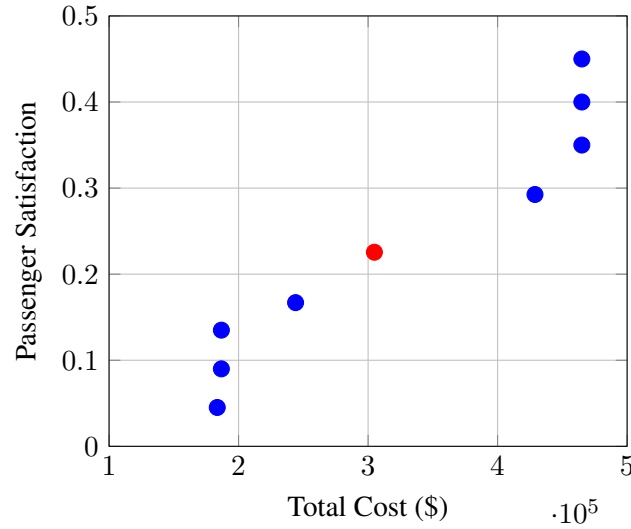


Figure 3.8: Pareto front illustrating the trade-off between scheduling cost and passenger satisfaction.

In the Pareto front graph, a cluster of points near the bottom left corner represents scenarios where operational costs are minimized but at the expense of relatively low passenger satisfaction. However, a notable feature of this segment of the graph is that a slight increase in cost can lead to a significant improvement in passenger satisfaction. This suggests that small adjustments in budget allocation towards operating costs can yield substantial benefits in terms of service quality. Additionally, as we move away from these closely clustered points and ascend the curve, the data indicates that achieving even higher levels of passenger satisfaction requires progressively larger increases in operational costs. This portion of the graph illustrates the diminishing returns of investing in passenger satisfaction; while improvements can certainly be achieved, they come at an increasingly higher cost. This insight is crucial for Concordia University shuttle bus planners when considering optimizations that balance cost against service quality outcomes.

3.5.3 Sensitivity Analysis

3.5.3.1 EB Types Analysis

In this section, we evaluate the model's performance and accuracy by examining the results for different types of electric buses. The variations among EB types stems from differences in their usage costs, number of seats, maximum travel ranges, and charging times. It is assumed that all

EBs have a lifespan of 15 years. Table 3.6 shows the features of four real-world EB kinds. We run the model for each type of EB and report the findings. This analysis helps the Facility Management of Concordia University select the most suitable EB type for their shuttle bus service, taking into account factors such as cost-effectiveness and student satisfaction.

Table 3.6: Characteristics of four well-known electric bus types (Gallo et al., 2014; Nova Bus LFSe, n.d.; Proterra Electric Vehicle Technology Manufacturer, 2016b).

Brand	Model	Annual Usage Cost (CAD)	Maximum Travel Range (km)	Charging Time (min)	Number of Seats
Proterra	40 ft. Catalyst	53,266	78	10	40
Proterra	40 ft. Catalyst+	66,666	100	13	40
Nova Bus	40 ft	80,000	37	5	37
New Flyer	40 ft	86,666	72	6	40

The results of comparing different EB models for the Concordia University shuttle bus service, considering total cost, waiting time, seat availability, and optimum headways, are presented in Table 3.7.

Table 3.7: Comparison of results for various types of electric buses used in shuttle bus services.

Brand	Total Cost (CAD)	Total Waiting Time	Seat Availability	ψ	Optimum Headways
Proterra 40 ft	371,556	12,778 min	47.33	0.39	19, 18, 19
Proterra 40 ft Catalyst+	378,352	12,833 min	47.00	0.39	19, 19, 16
Nova Bus	406,592	12,465 min	42.78	0.42	18, 18, 18
New Flyer	430,832	12,465 min	46.93	0.43	18, 18, 18

According to Table 3.7, Proterra buses exhibit superior overall performance relative to Nova Bus and New Flyer EB types based on their ψ values, where lower ψ values indicate better level of service and efficiency. The Proterra 40 ft and 40 ft Catalyst+ stand out with the lowest ψ value of 0.39, indicating top performance. While the Proterra 40 ft slightly outperforms the Catalyst+ in the three evaluated objectives, the differences are not significant enough to affect the ψ value, resulting in the same overall performance rating for both models. According to Figure 3.9, the Proterra 40 ft has the lowest total costs compared to other EB types, but it offers a relatively higher total waiting time compared to Nova Bus and New Flyer. In contrast, although Nova Bus and New Flyer models

provide satisfactory waiting times and suggest stable service throughout the day, their significantly higher total costs and relatively lower seat availability make them less ideal for shuttle services. This analysis provides Concordia University with a valuable tool for determining the best EB type for their shuttle bus service. By applying the proposed model and conducting comprehensive evaluations, they can gain crucial insights to support informed decision-making when purchasing or expanding their bus fleet. This method also allows other institutions and organizations that use shuttle buses to choose an EB type that best meets their specific needs, leading to a more efficient, effective, and cost-efficient transportation system.

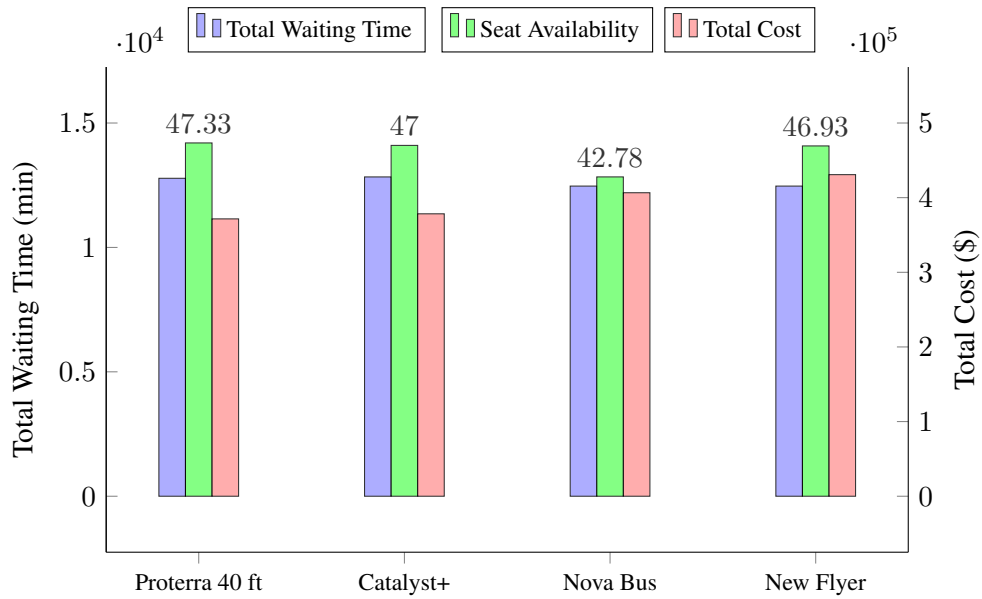


Figure 3.9: Comparison of the considered objectives for different EB types.

3.5.3.2 Pareto Fronts for EB Types

In this section, we assess the performance of various electric bus types detailed in Table 3.6, considering different weights for the three objectives. We will do this by adjusting the weights of the three main objectives. It is important to note that the results presented in Section 3.5.3.1 were based on specific weights for the objectives. Changing these weights could significantly affect the outcomes and influence which EB model is preferred. The Pareto fronts of the considered EB types are demonstrated in Figure 3.10. The Pareto fronts illustrate the results from nine different combinations of weights, varying from $\omega_\gamma + \omega_\delta = 0.9$ and $\omega_Z = 0.1$ to $\omega_\gamma + \omega_\delta = 0.1$ and

$\omega_Z = 0.9$, in increments of 0.1. In all scenarios, ω_γ and ω_δ are kept equal.

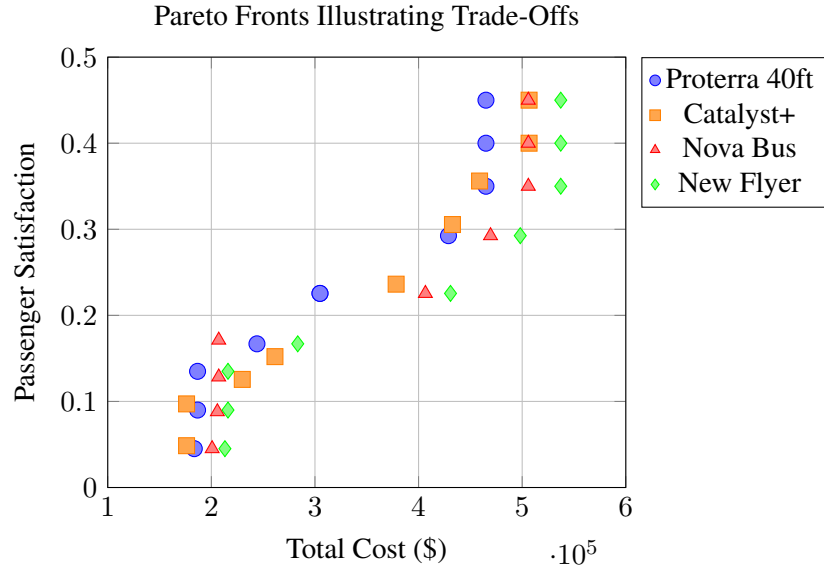


Figure 3.10: Pareto fronts showing scheduling cost and passenger satisfaction trade-offs for EB types.

The Pareto front shown in Figure 3.10 illustrates the balance between scheduling costs and passenger satisfaction across different types of EBs, each marked with unique identifiers for easy comparison. Notably, the Catalyst+ and Proterra 40ft models generally provide higher passenger satisfaction at lower costs in most scenarios, whereas the Nova Bus and New Flyer models are less cost-effective. A critical observation is made at the fourth and fifth data points for each EB type, which represent significant trade-offs. These points demonstrate that increasing passenger satisfaction significantly raises costs. At the fourth data point, the Nova Bus stands out by offering higher passenger satisfaction at lower costs than other EB types. On the other hand, at the fifth data point, where costs increase substantially, the Proterra model tends to perform better than others.

This analysis is critical for the Facility Management of Concordia University as it assists in evaluating which bus models provide the best balance between cost efficiency and passenger satisfaction. The visual representation helps the university quickly assess which EB types meet specific operational goals or budget constraints, informing more strategic decision making in fleet selection and management. The analysis also opens the door to considering other electric vehicle options, such as trolley buses and trams. However, while these alternatives might have lower initial purchase

costs compared to battery electric buses, they necessitate substantial infrastructure investment for overhead lines and tracks. Furthermore, given that Montreal lacks existing infrastructure for trolley or tram buses, and considering the regulatory and permitting challenges in the downtown area, these options have been excluded from the potential choices for upgrading the university's shuttle bus service.

3.6 Conclusions and Future Studies

In conclusion, this study introduced a new algorithm that integrates timetabling and scheduling for electric shuttle buses using pantograph chargers to minimize operational costs and improve service levels. The research focuses on a real-world case study of a University shuttle bus service in Montreal, Canada, which aims to switch to a fully electric transit system. Our approach seeks to reduce students' waiting times and enhance in-vehicle comfort by increasing seat availability, while offering a cost-efficient solution for the university's shuttle bus service after switching to electric buses. We developed an integrated algorithm that calculates waiting times and seat availability across different headway values and uses a MILP model, solved with the Cplex solver, to optimize the schedule of EBs to minimize total costs. These costs include travel expenses, electricity consumption, and usage costs associated with electric buses. A normalized weighted sum technique was applied to address the multi-objective problem. The effectiveness of our methodology was demonstrated through significant improvements, including a 22% reduction in student waiting times, a 42% improvement in the seat availability criterion, and a 21% improvement in the total cost of scheduling the shuttle buses compared to the existing timetable. Additionally, our approach included the strategic selection of optimal electric bus models and was capable of accommodating varying bus travel times during peak and off-peak hours, further enhancing operational efficiency. By adjusting the timetable for fully electric buses, this research promotes a more sustainable approach to university transportation systems, setting a benchmark for similar initiatives at other institutions and contributing to the broader adoption of EBs in public transit networks. This initiative represents a significant advancement in adapting transportation infrastructure to support environmental sustainability and operational excellence.

The proposed study offers the potential to be improved in several ways. One critical enhancement is adapting the model to accommodate different charging technologies, which vary in establishment costs, charging power, and charging times. Further improvements could focus on integrating network design as a decision variable. This would involve optimizing shuttle bus routing and including it in the model. Additionally, future studies could integrate decisions regarding charging technology and the strategic placement of charging stations. Given that pantograph chargers can significantly impact electricity loads, it is crucial to assess the effects of these charging technologies on neighborhood energy supplies and the power grid during peak hours. Investigating power loss associated with integrating such infrastructure and its impact on energy sources could provide valuable insights for more sustainable electric bus operations. Introducing uncertainty into the model regarding the travel time and energy consumption of EBs is another promising area for improvement. Various factors can influence EB travel time, including passenger volume, driving behavior, traffic congestion, road conditions, and weather. By incorporating stochastic travel times, we can make the model more robust and reflective of real-world scenarios. Similarly, the energy consumption of EBs can vary based on factors like passenger load, battery life, and road conditions. Future research could focus on including these variables to enhance the model's accuracy.

Chapter 4

Joint Optimization of Electric Bus Scheduling and Fast Charging Infrastructure Location Planning¹

4.1 Overview

Transit authorities are transitioning from conventional buses to electric buses (EBs) due to growing concerns about air quality and greenhouse gas emissions. Many mathematical optimization models have been developed for scheduling conventional buses. However, such models would not fit EBs due to their limited travelling range and long charging time. Such operational differences have prompted new research into the literature on the charging station location problem. This study combines EB scheduling with fast-charging infrastructure location planning to minimize total scheduling and charger installation costs. We propose an Integer Linear Programming (ILP) formulation for a path-based model, solved using four developed branch-and-price algorithms, and test their performance across various instances. Our computational experiments show which branching strategy is computationally efficient in terms of execution time and optimality gap. Finally, we conduct a real case study and perform a sensitivity analysis to identify the most cost-effective type of

¹Alamatsaz, K., Quesnel, F., & Eicker, U. (2024). Joint optimization of electric bus scheduling and fast charging infrastructure location planning. Les Cahiers du GERAD ISSN, 711, 2440.

electric bus, considering the specific characteristics of different EB types.

4.2 Introduction

Greenhouse gas-emitting energy sources are responsible for global warming. Thus, replacing such energy sources with clean and renewable sources has become crucial during the past few decades. Aligned with this, transit agencies are progressively transitioning towards electrifying their transit systems. They are increasingly embracing electric buses (EB) as an energy-efficient and emission-free alternative to conventional bus fleets. Thus, there has been a significant increase in the utilization of electric vehicles and electric buses. According to the recent research by [Bloomberg New Energy Finance \(2018b\)](#), EBs are projected to replace more than 47% of the world's city bus fleet by 2025. The European electric bus market is projected to increase by 18.6% between 2022 and 2027, indicating a shift towards electric buses from conventional buses.

Transit authorities are increasingly focused on enhancing the efficiency of their public bus transit systems and one of the most important factors affecting that is operational processes ([L. Kang & Meng, 2017](#)). Therefore, optimizing bus operation planning has become essential to reduce operational costs and decrease passengers waiting time. Bus scheduling is one of the most vital processes in bus operations and has a considerable impact on operational costs. Bus/vehicle scheduling (VS) is the process of assigning buses to the trips of a given timetable and aiming to use the minimum number of buses while minimizing operating costs. EBs have battery capacity limitations and their scheduling and charging processes differ from those of conventional buses ([Yao et al., 2020](#)). The operating range of EBs is less than conventional buses and their charging time is longer than refueling diesel buses. These differences necessitate the development and adoption of new plans and strategies for scheduling EBs effectively. [J.-Q. Li \(2014\)](#) demonstrates how the introduction of electric buses affects the bus scheduling procedure.

To overcome the challenges of limited driving range and extended charging times, public transportation agencies are increasingly adopting fast-charging (FC) technology, which utilizes high-voltage electricity to recharge electric buses within minutes. This innovative solution has made fast-charging systems an increasingly popular choice for bus transit networks. Also, charging stations at

bus terminals are less costly than depot charging and better suited to bus electrification throughout the life cycle ([Lajunen, 2018a](#)). The limited travelling range of EBs has prompted new research into the literature, which is the problem of locating charging stations for electric buses. This problem identifies the ideal places for installing fast-charging infrastructures on the bus transportation network, as well as the optimal number of such stations to minimize the total establishment costs. However, using such technology makes EBs scheduling more complex ([Y. He et al., 2020](#)). For example, the location of fast chargers affects the travelling time of charging trips of electric buses; hence, EB scheduling should be customized to consider this charging trip time. Also, the time of charging due to the limited availability and capacity of charging stations should be coordinated. Thus, as highlighted by [Bie et al. \(2021\)](#), a reasonable charging strategy is necessary to improve bus scheduling, and it is crucial to study the fast-charging infrastructure location planning problem for electric buses.

Transit authorities opting for fast-charging technologies employ pantograph chargers capable of delivering up to 350 kW of power. Although initially more costly, pantograph systems could ultimately decrease the total cost of ownership (TCO) and operational expenses for EBs as they allow continuous opportunity charging along routes ([Alamatsaz et al., 2022](#)). This reduces the need for large battery capacities and energy consumption. Moreover, pantograph charging enables an automatic connection from the bus roof, which can be integrated into existing bus networks by installing fast chargers at particular stops. This integration supports continuous bus operation with lower maintenance costs and higher availability, allowing buses to run around the clock ([Al-Saadi et al., 2022](#)). This mechanism is a well-established method for linking electric buses with their power sources. A special inverted pantograph automatically lowers when a bus arrives at the charging station. After completing the safety checks, the system provides the bus with a swift and powerful recharge.

Transitioning from traditional buses to electric buses requires further research on public transit operation planning steps. These steps are network route design, timetable development, vehicle scheduling, and crew scheduling. First, the network route design outlines the bus routes and their bus stops based on the regions' geographical layout and travel demand. Next, the timetable development sets bus departure frequencies in order to cover the travel demand of passengers. In the vehicle

scheduling step, buses will be efficiently assigned to bus routes based on the timetables. Finally, the last step determines the schedule for bus drivers. The limited range and longer charging time of EBs necessitate a reassessment of these steps. The majority of studies discuss public transportation operation planning in a sequential manner, where the output of one planning phase is used as input for the next stage. This approach is less effective compared to a fully integrated strategy. The fully integrated method analyzes the problem as a whole, considering all phases of public transportation planning simultaneously. For instance, making minor adjustments to the bus timetable might lead to better vehicle scheduling, and locating fast-charging infrastructure based on bus schedules could lower the operational costs of bus scheduling. Thus, in this research, we aim to integrate charging station location planning and bus scheduling steps of electric bus transit operation planning to improve the overall efficiency of the transit system (Olsen & Kliewer, 2022).

In the network design step, the travel demand of each region is analyzed to design the bus routes and their bus stop locations based on the geographical features and travel demand of that region. It also includes determining the length of bus routes ensuring the routes are of a length that EBs can complete their cycles without battery depletion. Finding the best spots to locate charging stations or continuous charging infrastructures is another aspect of the network design step for EBs. Also, this step includes choosing the most suitable charging technology based on the EBs' specifications and bus network features. The vehicle or bus scheduling step involves assigning the buses to each route based on the vehicle's range, charging requirements, given timetable, and route's length. The aim is to cover all the given trips in timetables with minimum operational costs. In this step, planners seek to minimize the total number of required EBs, total deadhead trips, and charging costs while covering all the given trips. Also, the charging scheduling of EBs will be analyzed in this step.

In this research, we tackle a problem that integrates elements of the network design step through fast-charger location planning with the electric bus scheduling step. This problem is called fast-charging location planning and electric bus scheduling (FCLP-EBS). We seek to minimize the total operational costs, including deadhead trips, electricity and ownership costs of EBs while covering all the given trips. Also, we find the best locations and the optimum number of fast chargers to be placed on the bus network to meet the charging demand for EBs with minimum cost. By doing so, we improve the overall efficiency and performance of EBs' operation and fleet management.

We introduce a path-based model (PBM) for the FCLP-EBS problem and use a branch-and-price algorithm with different branching strategies to solve the problem.

The contributions of this study are as follows:

- Proposing a novel integrated model for the FCLP-EBS problem that includes a multi-depot setup, interlining of bus trips, and partial charging. The model also accounts for the impact of time-of-use (TOU) electricity pricing on charging decisions, encouraging off-peak charging to reduce electricity costs and improve overall scheduling efficiency.
- Crucially, our model considers the charging schedule of buses and the capacity of charging stations. This is in contrast to recent studies (e.g. [Y. Wang et al. \(2022\)](#) and [Davatgari, Verbas, Cokyasar, and Mohammadian \(2024\)](#)), which suppose fast chargers are always available. Considering charger availability and energy demand is important to properly determine the number and locations of fast chargers. However, this added complexity makes the problem significantly more challenging to solve.
- Developing a branch-and-price (B&P) algorithm to solve the FCLP-EBS problem. The pricing subproblems are constrained shortest path problems. We propose an innovative network structure to reduce the number of network arcs and speed up the solution process.
- Comparing the integrated approach with the sequential approach (fast charger location, followed by bus scheduling) and showing the benefits of the former.
- Applying the model on a real case study of Montreal and performing sensitivity analyses on different EB types based on original equipment manufacturers' real data.

We adopt an exact solution approach for small and medium-sized problems to achieve the global optimum. For large-scale problems, we employ heuristic branch-and-price methods to obtain near-optimal solutions with an average optimality gap of less than 1%.

The remainder of this article is structured as follows. A literature review on charging station location planning for EBs, electric bus scheduling problems, and the combination of them are presented in Section 4.3. In Section 4.4, we provide a detailed description of the fast-charging location planning and electric bus scheduling problem. Section 4.5 describes the proposed path-based model

model. This section also provides an overview of the branch-and-price algorithm used to solve the model and the branching strategies. Then, Section 4.6 presents the computational results and includes a real-life case study to demonstrate the effectiveness and efficiency of the proposed solution approach. Finally, conclusions are drawn in Section 4.7.

4.3 Literature review

The following sections will provide a brief overview of the related works and theoretical background of charging station location planning and EB scheduling. Section 4.3.1 describes the different charging station location models found in the literature. In Section 4.3.2, we review the literature on EB scheduling problems, focusing on exact and heuristic methods used to address these problems. Finally, the integration of charging station location planning and EB scheduling problems are briefly discussed in Section 4.3.3.

4.3.1 Charging Station Location

Adopting fast chargers for electric buses to recharge them in a short time is becoming more common. Due to this growth, extensive research is required to determine the best locations for installing charging station infrastructure along bus routes. Kunitz et al. (2014) conducted one of the initial studies on the planning of fast-charging stations. The study aimed to identify the optimal locations for fast-chargers throughout the bus network to meet the daily energy demand of EBs. Csonka (2021) addressed the problem while considering partial charging, i.e. how much to charge EBs during each charging event. In another study, Olmos et al. (2019) looked into the problem of locating opportunity charging infrastructure for hybrid and fully electric buses to cover the required charging activities that fulfill the energy demand of EBs.

Alwesabi et al. (2021) provided a mixed-integer linear programming (MILP) formulation for determining the battery capacity and dynamic wireless charging locations. Tzamakos, Iliopoulou, and Kepaptsoglou (2023) investigated fast-wireless-charging infrastructure location planning, taking into account the charging scheduling. Considering the optimal capacity for EBs (Kunitz et al., 2017), the optimal number and types of EB chargers (Y. Zhou, Ong, Meng, & Cui, 2023), battery

aging (Berthold, Förster, & Rohrbeck, 2017), the power distribution network to reduce the power loss costs (X. Wu et al., 2021), and added energy storage system (ESS) to store the energy during off-peak hours and sending it to fast-charging infrastructure during peak hours (Y. He et al., 2019) are other extensions of this problem. Hu et al. (2022) addressed the opportunity charging location problem while taking into account the charging scheduling of EBs. The authors aimed to minimize the cost of procuring opportunity chargers and EB batteries, as well as the total cost of charging and the extra passengers' waiting time by proposing a robust optimization model. The main distinction between their study and ours is that they neglect to consider the EB scheduling problem and the number of buses required to cover all trips. In addition, their model focuses on a single depot and does not incorporate TOU electricity pricing and interlining of buses.

In earlier literature, most studies treat the energy demand for EBs as a constant. Furthermore, these studies assume that the bus schedule is predetermined. Consequently, the primary focus was identifying the best locations for charging stations and their optimum number, ensuring EBs can complete their trips based on fixed schedules. However, our research introduces flexibility by treating the bus schedule as a variable that could be changed according to the charging station locations and the number of such facilities. Thus, this research seeks to find the optimum solution for the fast charging station location planning while optimizing the scheduling of electric buses. This integrated approach promises more effective solutions than the traditional sequential methods where the electric bus schedules are set in advance (See Section 4.6.2.2 for more details).

4.3.2 Electric Bus Scheduling

Vehicle scheduling (VS) is the process of assigning buses to the routes of a given timetable. Bus scheduling aims to cover the given timetable efficiently while satisfying all operational constraints by finding the necessary number of buses and reducing the deadhead trip costs. The number of possible solutions for EB scheduling is especially large when the EBs are based at multiple depots or terminals. The problem of electric bus scheduling across multi-depots falls within the category of NP-hard problems (Carpaneto, Dell'Amico, Fischetti, & Toth, 1989b). As a result, much of the research in this area focuses on computational challenges. Perumal et al. (2022) and Y. Zhou et al. (2024) provided detailed literature reviews of the constraints associated with the electric bus

scheduling problems and the methodologies proposed to address them. A well-known approach to model vehicle scheduling problems was introduced by [Bertossi et al. \(1987\)](#) and is known as the time-space network framework. In this framework, nodes represent the trips and their starting and ending times, while arcs represent the routes taken by vehicles to get from one node to another. We use this framework to model the integration of EB scheduling and fast charging location planning problem.

The literature on electric bus scheduling problems offers a wide variety of models and methods. [Yao et al. \(2020\)](#) solved the EB scheduling problem using a Genetic Algorithm (GA) for scheduling heterogeneous types of electric buses. [Teng et al. \(2020\)](#) integrated bus timetabling with EB scheduling and aimed to optimize the overall efficiency of the bus line, reducing operational costs while ensuring timely service. [G.-J. Zhou et al. \(2020\)](#) integrated the EB scheduling problem with the charging management of electric buses by proposing a bi-level programming approach. The model simultaneously addresses vehicle scheduling and charging management, optimizing the overall operational efficiency of the fleet. Similarly, [G.-J. Zhou, Zhao, Zhu, and Xie \(2025\)](#) considered the impact of EB charging schedules on the power grid within the context of bus scheduling. They employed the NSGA-II algorithm, a multi-objective evolutionary approach, to solve the problem. [Sung et al. \(2022\)](#) proposed a heuristic solution approach to tackle the EB scheduling problem by considering a mix of heterogeneous buses and chargers for multi-depots and multi-bus line networks. [Y. Wang, Chen, Tang, and Liu \(2024\)](#) addressed the same problem while considering restricted charging facilities. The authors proposed an adaptive large neighborhood search (ALNS) algorithm that incorporates innovative acceleration techniques to handle large-scale problems efficiently. Their proposed method was validated through a case study in Lianyungang, China. As these studies rely on heuristic or metaheuristic methods to solve the electric bus scheduling problem, they do not guarantee optimal solutions.

On the other hand, there are a few studies focusing on exact solution approaches to deal with EB scheduling problems. Notably, [L. Li et al. \(2019\)](#) developed a scheduling model for mixed bus fleets, addressing the challenges of electric buses' range limitations and refueling needs. They have adopted a time-space-energy network approach and developed an integer linear programming model to solve the problem. [Gkiotsalitis, Iliopoulou, and Kepaptsoglou \(2023\)](#) presented an exact

approach based on a branch-and-cut (B&C) algorithm for the multi-depot electric bus scheduling problem, to address operational constraints like time windows and charging requirements. [Tang et al. \(2019\)](#) addressed the single-depot problem by robust scheduling strategies for electric buses, focusing on the uncertainties introduced by stochastic traffic conditions to minimize total operating costs. The authors used a branch-and-price method to tackle the problem. [Bie et al. \(2021\)](#) proposed an optimal scheduling strategy for electric buses by combining all-stop and short-turning methods. Their model aims to balance the operational efficiency of electric buses with energy consumption constraints, enhancing overall system performance for a single bus line. [M. Jiang and Zhang \(2022\)](#) developed an advanced branch-and-price algorithm to solve the large-scale multi-depot electric bus scheduling problem, which is more complicated compared to the single-depot problem. [Lu, Xing, and Li \(2025\)](#) proposed a mixed-integer programming (MIP) model that integrates both vehicle assignment and charging decisions while considering the constraint of TOU pricing that penalize peak-hour charging. The authors solve the problem by using a branch-and-price algorithm and benchmark against heuristic methods.

A nonlinear optimization model is proposed in the study of [M. Zhang, Yang, Li, and Huang \(2024\)](#) to address the electric bus scheduling problem while accounting for constraints like limited charging stations and power availability. The approach combines bus scheduling with the optimization of charging times and power usage to minimize total system costs, including vehicle operation and charging expenses. A genetic algorithm was used to solve the model. Similarly, [van Kooten Niekerk et al. \(2017\)](#) presented two methods for scheduling EBs with limited travel ranges. The first method uses a simplified model assuming linear charging regardless of time-of-use, while the second adjusts this to enhance realism. Both models were solved for small to medium-sized problems using integer linear programming. For larger-scale problems, the authors proposed two near-optimal solutions using the column generation method. In another study by [A. Zhang et al. \(2022\)](#), the authors seek to optimize the use of EBs by allowing flexible routing and partial recharging strategies, which helps to overcome limitations in battery range and charging infrastructure. Further advancing the field, [W. Wu et al. \(2022\)](#) addressed the multi-depot electric vehicle scheduling problem, incorporating power grid characteristics into the scheduling process. Additionally, electric bus scheduling under energy consumption and travel time uncertainty have been studied by

[Avishan, Yanikoğlu, and Alwesabi \(2023\)](#).

4.3.3 Integrated Fast-charging Station Location and Bus Scheduling

Recently, there has been a growing trend in integrating charging station location planning with electric bus scheduling. [Olsen and Kliewer \(2022\)](#) studied the combination of depot charging location planning with EB scheduling for a single-depot setup. The goal was to reduce the total cost, which included operational costs, installation costs of chargers in the depot, and vehicle costs. To address this problem, a metaheuristic solution method based on Variable Neighborhood Search was proposed. They proved that optimizing these two problems simultaneously outperforms the sequential planning approach. However, their study did not develop an exact solution method to tackle the problem and did not account for time-of-use electricity pricing. [L. Li et al. \(2019\)](#) explored the multidepot and mixed fleet version of this problem. For small-scale problems, they proposed an arc-based model and formulated it as an integer linear program, while for large-scale problems, they introduced a time-space bus flow network to find approximate solutions. They did not account for deadhead trip costs, partial charging, and time-of-use electricity pricing. Additionally, the maximum number of trips they solved was limited to 288, which is significantly lower than in our study.

[X. Li et al. \(2020\)](#) presented a joint optimization model for EBs, addressing both the scheduling of EBs and the deployment of chargers. The study incorporates a partial charging policy and time-of-use electricity prices to balance operational efficiency with energy costs. By optimizing the timing of bus charging and strategically placing chargers using an adaptive genetic algorithm (AGA), the model aims to minimize total operational and electricity expenses. While the model demonstrates the capability to handle up to 1,260 trips, the authors do not report key performance metrics such as the optimality gap or solution runtime, leaving the quality of the solutions unclear. In contrast, our research proposes an exact solution approach that guarantees optimal solutions. Additionally, the study focuses on depot-based charger deployment, whereas we address the more operationally challenging case of fast chargers. Fast chargers significantly reduce charging time but require more frequent detours to charging stations due to the limited travel range of EBs. This added complexity increases the difficulty of managing charging schedules and routing, which is a

key focus of our work.

[Stumpe, Rößler, et al. \(2021\)](#) investigated the simultaneous optimization of opportunity charging location planning and electric bus scheduling. They suggested an innovative mixed-integer linear formulation and used Variable Neighborhood Search metaheuristic to solve it. However, they considered single-depot problem and neglected to consider multiple depots for scheduling EBs. Also, opportunity charging is different from fast charging pantograph chargers. The pantograph chargers are usually located at the beginning or end of bus lines and their total number is typically less than opportunity chargers that are located at specific bus stops of bus lines. The charging duration of pantograph chargers, 5 to 15 minutes, is longer than that of opportunity chargers, which is only a couple of minutes. Also, the charging with opportunity chargers can occur when passengers are on the board, but for pantograph chargers, the EBs should be emptied. The opportunity chargers of each bus line are typically for the buses running that bus line. However, pantograph chargers are shared with the EBs running different bus lines. That is why the charging scheduling of pantograph chargers is more challenging and requires a different formulation and methodology to deal with. In this study, one aspect is to address the charging scheduling of EBs with pantograph chargers to smooth the charging events for EBs with the minimum waiting time of EBs at the charging stations. An additional advantage of our research is that we use an exact solution approach to achieve the optimal solution, whereas [Stumpe, Rößler, et al. \(2021\)](#) employed a metaheuristic approach, which does not guarantee to find the global optimum.

One of the closest works to our research is the work of [T. Liu and Ceder \(2020\)](#). They proposed a bi-objective integer programming method for the problem with the objective of reducing the number of EBs and fast-charging infrastructures. The authors proposed two approaches to address this problem: a lexicographic approach and a modified max-flow technique. However, their study does not consider time-of-use electricity pricing, which is critical for realistic and cost-effective EB scheduling. Furthermore, the scale of their model, in terms of the number of trips and potential charger locations, is significantly smaller than that addressed in our research.

[Y. Wang et al. \(2022\)](#) optimized the deployment of pantograph chargers for electric buses while also optimizing the scheduling of buses. They developed a mixed-integer linear programming model to minimize overall fleet costs. However, their work did not address multi-depot operations, partial

charging, travel costs, charging costs, and the scheduling of EB charging. Additionally, their results, obtained using a commercial solver, report an optimality gap of 4% for a case study with only 100 trips. In contrast, our study adopts an exact solution method for small to medium-sized problems to ensure global optimal solutions. For larger instances, we incorporate heuristic branching strategies, enabling us to handle up to 1,000 trips while maintaining an optimality gap of less than 1%, demonstrating both scalability and solution quality.

[Davatgari et al. \(2024\)](#) addressed a similar problem by developing a mixed-integer programming model that determines the optimal assignment of EBs to bus lines while accounting for battery constraints and identifying the best locations for charging station. Their approach significantly enhances operational efficiency and charger utilization by reducing the total number of chargers needed and minimizing vehicle idle time. However, the scalability of their model is limited as it can only handle instances with up to 60 trips, making it unsuitable for larger real-world transit networks. Additionally, their model does not account for several key operational features, such as multi-depot setups, TOU electricity pricing, charging scheduling, and interlining, all of which are addressed in the current research. [L. Zhang, Guo, Wang, Hu, and Han \(2025\)](#) introduced a mixed-integer nonlinear programming model that addresses the same problem, focusing on key operational challenges such as battery capacity degradation and scheduling uncertainties. Their goal was to minimize total operational costs, and they demonstrated the economic and operational benefits of en-route charging through a real-world case study. However, they did not consider partial charging for EBs. The primary distinction between their research and ours lies in scope. While their experiments were limited to 84 trips, a single depot, and a single bus line, our model extends to multiple depots and multiple bus lines, offering a more realistic and comprehensive solution for large-scale transit systems. This broader scope improves the applicability of the model, but also introduces significantly greater computational complexity, which our solution methodology is designed to handle effectively.

Another relevant study is the work by [Najafi et al. \(2025\)](#), which tackles a similar problem while incorporating renewable energy resources into the model. Their objectives include minimizing the total cost of purchasing and maintaining EBs, chargers, and renewable energy systems, as well as reducing time-of-use electricity costs. However, several practical considerations are missing

from their model. Notably, they neglect to address the multi-depot setting, which is essential for capturing the operational complexity of real-world transit systems. Their model is also limited to only three bus routes, whereas depot operators typically manage a much larger network of bus lines. In addition, travel costs and waiting time costs are not included, which are key components in accurately assessing fleet performance. Most importantly, the model does not allow for partial charging of EBs, a realistic and often necessary strategy in high-demand operations. In contrast, our research addresses all these critical factors, providing a more comprehensive and practical solution for electric bus scheduling and infrastructure planning.

Although electric bus scheduling, charging scheduling, and charging station location problems have been the subject of various studies, there is still more work to be done to bridge the gap between theory and practice. A review of the literature reveals that few studies have explored the influence of electric bus scheduling and charging scheduling on the location of fast-charging infrastructure and vice versa. However, these studies did not consider the aggregation of practical factors such as multi-depots, time-of-use (TOU) electricity pricing, interlining, and partial charging of electric buses with pantograph chargers, factors that we address in this study. Also, a review of the literature reveals that most existing studies overlook the use of exact solution methods and struggle to address large-scale problems effectively. The absence of exact solution methods in the literature limits the ability to benchmark heuristic approaches effectively and restricts insights into the true potential of optimal scheduling and infrastructure design. Additionally, the inability of the current methods to scale effectively to real-world scenarios highlights the urgent need for innovative strategies that can offer a balance between computational efficiency and solution quality. To address these shortcomings, we propose the use of a branch-and-price algorithm to solve the FCLP-EBS problem. This exact solution approach not only guarantees optimality for small and medium-sized instances but also provides a robust framework to address the complexity of real-world transit systems.

4.4 Problem Description

This paper addresses the joint optimization of fast charging station location planning and electric bus scheduling. The aim is to schedule a homogeneous fleet of EBs that start their trip from

multiple depots and return to the same depots after completing their daily trips with minimum total operational costs and simultaneously find the best locations for fast-chargers to be placed on predetermined spots on the bus network to satisfy the energy demand of EBs. Since EBs have a limited travel range, they need to be charged at charging stations frequently throughout the day to complete their daily trips. The objective is to minimize the required number of EBs needed to fulfill the given trips and their total operational costs by reducing their deadhead trips and electricity costs while minimizing the total installing cost of fast chargers. Figure 4.1 illustrates a schematic sample example of the problem. The diagram displays a network of bus routes, depicted in different colors, representing various bus lines that EBs follow. The buses start and end their trips at multiple depots based on the given timetables. Also, the network includes a variety of predetermined potential spots for charging stations, illustrated as circles with charger icons. Two types of arcs represent the deadhead trips between depots, bus lines, and charging stations.

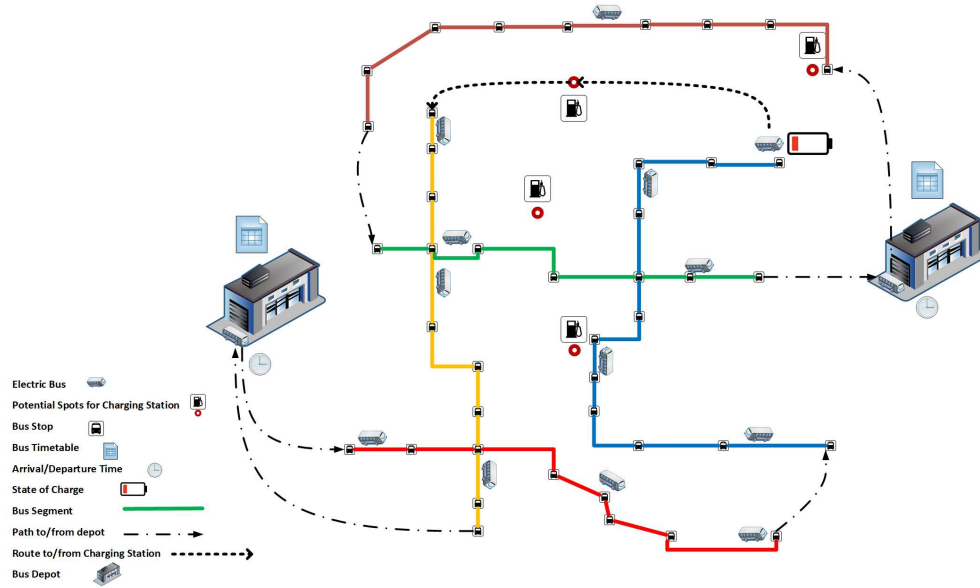


Figure 4.1: A schematic view of the FCLP-EBS problem with two depots, five bus lines, and five candidate locations for installing charging stations

Henceforth, the term *trip* refers to each journey with a predetermined departure and ending time in the given timetable. Consider a predetermined timetable listing the trips associated with each route. For instance, a bus route that begins its first journey at 06:00 AM with a frequency of 30 minutes until 08:00 AM, with each trip lasting 45 minutes. For this bus route, there would be a

series of six trips, starting at 06:00, 06:30, 07:00, 07:30, and 08:00, and correspondingly ending at 06:45, 07:15, 07:45, 08:15, and 08:45. For the FCLP-EBS problem, we are given a set of depots as O , set of trips denoted as S and a set of potential locations for locating fast-charging stations as B . Trip $i \in S$ has a starting time a_i and an ending time e_i . Each bus can only be assigned to one trip at a time. An EB can start a new trip if it finishes its previous one with enough time to travel to the starting point of the next trip as a deadhead trip and still has enough energy. Following a trip, an EB can also proceed to a charging station for recharging. Deadhead trips refer to the transit between trips, in which buses travel without any passengers. They occur mainly to relocate buses for their next trip to travel to and from charging stations. Deadhead trips can also be from the depots to bus routes and vice versa, or between charging stations and bus routes and vice versa. The depots, charging station locations, and bus trip start and end points are collectively referred to as *locations*.

Let d_{ji} denote the deadhead trip distance from location j to location i . Let $\tilde{d}_i$ be the distance of trip $i \in S$. Let $\hat{D}$ be the set of all deadheads. Similarly, let τ_{ji} denote the travel time from location j to location i . Let Δ_i denote the travel time of trip $i \in S$. Let w_{ji} denote the waiting time between location j and i . Let d_{max} indicate the maximum distance that a fully charged battery can travel. A bus route incurs a cost of C_d per unit of distance travelled, and C_w per unit of time spent waiting. Meanwhile, the cost of charging per minute at a fast-charging station at time t is indicated by C_t , and C_v denotes the fixed annual cost of each EB. By strategically scheduling charging during off-peak hours and taking advantage of time-of-use electricity rates, charging costs can be minimized while also reducing the strain on the electrical grid. We account for two distinct electricity rates, one for peak hours and another for off-peak hours, and aim to optimize the charging schedule of EBs to lower overall charging costs. The details of TOU calculation is presented in Appendix A, Section A.1.

In the FCLP-EBS problem, we incorporate interlining for electric buses. Interlining allows a single bus to operate on multiple routes or trips sequentially, without returning to the depot between trips. Instead, the EB transitions directly from one route to another, optimizing resource utilization and reducing costs like deadhead trips. Electric buses, starting fully charged from a depot, can operate along a series of feasible connecting trips as long as they have sufficient energy to return to a charging station. At each charging station, a maximum of W buses can be charged simultaneously;

in this study, the capacity of each charger is $W = 1$. Additionally, the maximum charging time for any charging event is limited to U . Let V_b denote the establishment cost of installing a fast charger at location $b \in B$. To model the charging station capacity, we discretize the planning horizon into a set of time periods T . Each bus is assumed to be charged for one or more consecutive full time periods.

We assume that electric buses charge linearly using pantograph chargers. While this assumption may not fully reflect reality, it is suitable for the purposes of this study. The primary reason for this simplification is that incorporating a nonlinear charging process based on the State of Charge (SoC) significantly slows down the proposed solution method. Additionally, the main objective is to determine the optimal locations for fast-charging stations and a nonlinear charging assumption is unlikely to affect this outcome. If necessary, a traditional bus scheduling problem with nonlinear charging could be run afterward to fine-tune the bus schedules.

4.5 Methodology

This section provides a detailed explanation of the fast-charging location planning and electric bus scheduling (FCLP-EBS) problem, along with the methodology used to address it. In Section 4.5.1, we introduce a path-based model for the FCLP-EBS problem. Due to the high number of variables in the path-based formulation, standard commercial solvers are not suitable for solving it. To address this challenge, we propose a branch-and-price algorithm, which is described in detail in Section 4.5.2.

4.5.1 Path-Based Model (PBM)

We introduce a path-based model that features a large number of variables but a reasonable number of constraints. This structure allows the optimization model to effectively utilize the branch-and-price algorithm. We denote P as the set of all possible paths starting from the depots and returning to the same depots. Such paths or schedules should satisfy the maximum distance constraint prior to battery renewal. Let $\varpi_{ji}^p = 1$ if deadhead (j, i) is included in path p . Similarly, let $\delta_i^p = 1$ if trip i is covered by path p . Let $\sigma_{tb}^p = 1$ if path p includes charging during time period $t \in T$ at

charging station b . Let w_p represents the total waiting time in path p , which includes the waiting time between trips, trips to/from chargers, and waiting time at the charging stations. The cost of path p , denoted as c_p , is then given by: $c_p = C_v + \sum_{(j,i) \in \hat{D}} \varpi_{ji}^p (C_d d_{ji}) + \sum_{i \in S} \delta_i^p (C_d \tilde{d}_i) + C_w w_p + \sum_{b \in B} \sum_{t \in T} \sigma_{tb}^p C_t$.

Let binary variable $y_p = 1$, if path p is selected in the solution, otherwise $y_p = 0$. Let z_b be a binary decision variable with $z_b = 1$ if a fast-charger is located at location $b \in B$, and $z_b = 0$ otherwise. The path-based formulation of FCLP-EBS problem is as follows:

$$\min Z = \left(\sum_{p \in P} c_p y_p + \sum_{b \in B} V_b z_b \right) \quad (21)$$

s.t.

$$\sum_{p \in P} \delta_i^p y_p = 1 \quad \forall i \in S \quad (22)$$

$$\sum_{p \in P} \sigma_{tb}^p y_p \leq W z_b \quad \forall b \in B, \forall t \in T' \quad (23)$$

$$y_p \in \{0, 1\} \quad \forall p \in P \quad (24)$$

$$z_b \in \{0, 1\} \quad \forall b \in B \quad (25)$$

The objective function (21) aims to minimize the total annual operational costs of EBs and the total annual establishment costs of fast-chargers. The first term is the total annual purchase and operational costs of EBs. The second term is the annual cost of establishing fast chargers along the network. Constraint (22) ensures that each trip is served once. Constraint (23) states that to charge an EB at a charging station, a charger should have been installed at that point and ensures that the charging station capacity limitation must always be met. Constraints (24) and (25) determine the domain of variables.

4.5.2 Branch-and-Price

Branch-and-price is a powerful optimization method used to solve combinatorial optimization problems, which involve a large number of variables and constraints. This method combines two well-known techniques: branch-and-bound and column generation.

Column generation is an optimization method used to solve large-scale linear programming problems. This technique involves iteratively generating columns, in this context paths, by solving the *Pricing Problem* and adding them to the *Restricted Master Problem* (RMP) until an optimal solution is reached. The RMP consists of the linear relaxation of the model (1)-(5), with $\bar{P} \subseteq P$, representing a subset of all feasible schedules. At each iteration, $\bar{P}$ is expanded by adding new paths with negative reduced costs, identified by solving the subproblems.

To generate these new columns, a pricing problem is solved to find negative reduced cost variables (also called columns) to add to the master problem, which is then solved again. The process of generating new EB schedules and adding them to the problem formulation continues until there are no other schedules that can be added to master problem and improve the solution. In this case, the optimum solution is reached. It is not guaranteed that the optimal solution obtained is integral even if the linear relaxation of the path-based model is solved to optimality via column generation. Thus, the column generation algorithm is embedded into a branch-and-bound method, forming a branch-and-price algorithm.

The branch-and-price algorithm generates columns dynamically at each node of the branch-and-bound tree. At each node, the linear relaxation of the problem is solved using column generation to obtain a lower bound on objective function (1). Nodes are pruned if the restricted master problem is infeasible, produces an integer solution, or has a bound greater than the best-known integer solution. The overall procedure of the proposed solution approach is summarized in Algorithm 2. While exact branching schemes can theoretically provide optimal results, they often lead to an overwhelming number of nodes. To address this, heuristic branching techniques can be employed as a practical alternative.

Algorithm 2 Branch-and-Price Algorithm for FCLP-EBS

```
1: Input: Set of trips  $S$ , candidate stations  $B$ , time horizon  $T$ 
2: Parameters: Distances  $d_{ij}$ , travel times  $\tau_{ij}$ , costs  $C_d, C_w, C_t, C_v$ 
3: Initialize restricted master problem (RMP) with feasible subset  $\bar{P} \subseteq P$ 
4: Set incumbent best solution to  $\infty$ 
5: Initialize branch-and-bound tree with root node containing RMP
6: while branch-and-bound tree is not empty do
7:   Select node  $n$  from tree
8:   repeat
9:     Solve linear relaxation of RMP at node  $n$ 
10:    Extract dual variables
11:    for all depots  $o \in O$  do
12:      Solve Pricing Subproblem( $o$ )
13:      if a path with reduced cost  $\bar{c}_p < 0$  is found then
14:        Add path to RMP
15:      end if
16:    end for
17:  until no negative reduced cost path is found
18:  if RMP is infeasible then
19:    Prune node  $n$ 
20:  else if RMP gives integer solution then
21:    if Objective  $<$  incumbent best then
22:      Update incumbent best
23:    end if
24:    Prune node  $n$ 
25:  else if lower bound  $\geq$  incumbent best then
26:    Prune node  $n$ 
27:  else
28:    Branch on fractional variable (heuristic or exact rule)
29:    Add child nodes to the tree
30:  end if
31: end while
32: return Best integer solution
```

To apply the branch-and-price technique to the FCLP-EBS problem, we model the pricing subproblems as a constrained shortest-path problem on an acyclic network. There is one subproblem for each depot and we explain the network for a single-depot $o \in O$. This subproblem network is shown in Figure 4.2 and is described as follows. Consider network $G = (N, A)$, where N stands for the node sets and A for the arc sets. The set of nodes contains four different nodes: origin and destination nodes, trip nodes, time-expanded nodes, and dummy trip nodes. There is one *origin* node for depot $o \in O$ and one *destination* node for the same depot. There is one *trip* node for each trip

in S . For each fast-charging location, we create two sets of node: there is one *time-expanded* node per fast charger and per time period, signifying available charging intervals at five-minute samples, and one *dummy* trip node per fast charger and per trip in S , which is a replica of the actual trips. The set of all dummy nodes is denoted by S' and the set of dummy nodes associated with each fast charger $b \in B$ is denoted by S'_b . The set of arcs contains five different sets: beginning and ending arcs, deadhead arcs, charging arcs, and waiting time arcs for fast chargers. The *beginning* trip arcs connect the depot to the trip nodes, S , to start a path for an EB and the *ending* trip arcs connect each trip node to the destination depot to finish the path for that EB. Deadhead arcs are divided into three groups: First, the arcs linking *trip nodes to each other*. A deadhead arc exists between the trip node of trip $i \in S$ and trip $j \in S$ if $e_j + \tau_{ji} \leq a_i$. Second, the arcs linking *trip nodes to the time-expanded nodes* of fast chargers. Let max_T denote the maximum or the latest available time for charging EBs. Let t_{ib}^e be the earliest time an EB can be at charging station $b \in B$ after completing trip $i \in S$. A deadhead arc links trip node i to the time node of station b that corresponds to t_{ib}^e if $e_i + \tau_{ib} \leq max_T$. If t_{ib}^e falls between two time intervals, the EB is required to select the arc leading to the first time interval after t_{ib}^e , with the arc's consumption accounting for the waiting time. Third, the arcs linking each dummy trip node to its corresponding trip node, enabling the transition to the next scheduled trip. When a bus arrives at a charging station, it can wait before charging, charge for a specific duration, and potentially wait again before starting its next trip. To reflect these possibilities, time-expanded nodes are arranged chronologically, with *waiting arcs* connecting them to represent waiting before charging. Then, for each time-expanded node, there are several *charging arcs* connecting it to dummy trip nodes. Each charging arc represents a specific charging duration and ends at the first dummy trip node that aligns with a feasible trip after charging. Dummy trip nodes are also connected by waiting arcs to account for additional waiting time after charging.

In the network illustrated in Figure 4.2, the charging stations are depicted by a block containing a group of two sets of node types: time-expanded nodes and dummy trips. This is created to model the charging scheduling of EBs at these stations based on the capacity of the charging stations. The time-expanded nodes indicate the arrival time of EBs at each charging station. Also, the waiting time of EBs at charging stations could be calculated using time-expanded nodes in the given network structure. To demonstrate the functionality of the developed network, Figure 4.2 also shows a

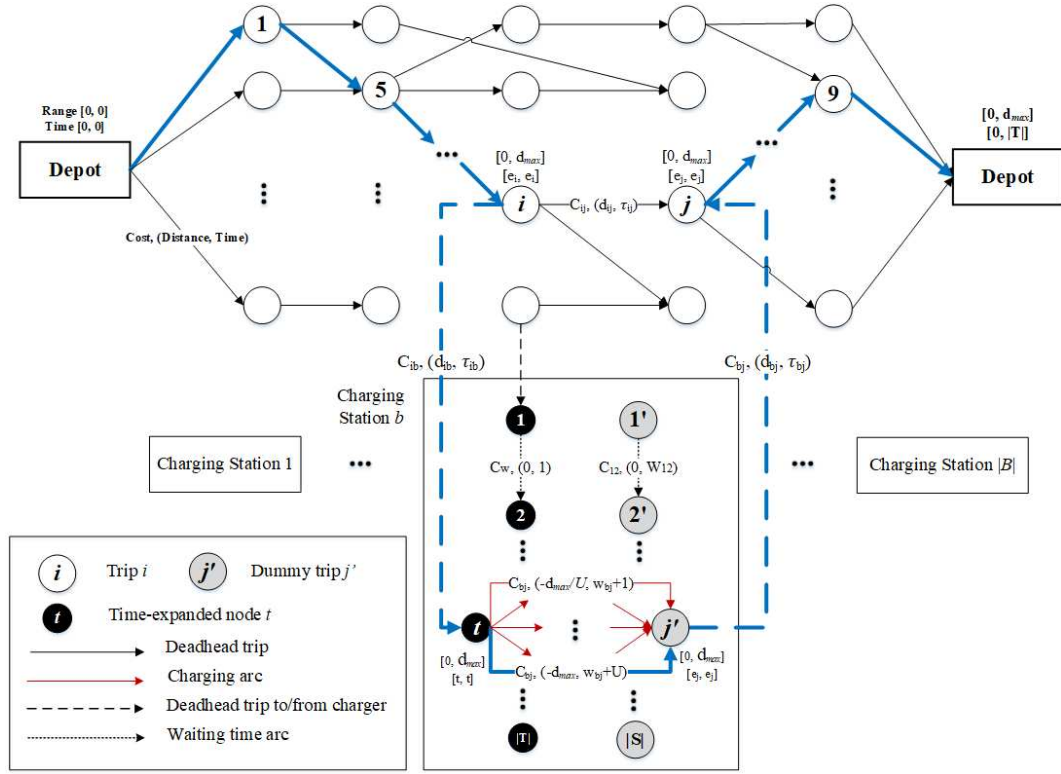


Figure 4.2: Acyclic network of the problem to deploy the branch-and-price method

sample path highlighted by bold blue arrows. In this example, an electric bus starts its route from the depot, first serving Trip 1, then moving on to Trip 5. After completing several other trips, it reaches Trip i . At this point, the bus no longer has enough energy to proceed to the next scheduled trip. Therefore, it heads to the charging station b , arriving at its time-expanded node at t . The bus begins charging for U minutes. Once fully recharged, it continues its route, completing Trips j and 9, before returning to the same depot.

The pricing subproblem, in our case, is the problem of finding a feasible route with the minimum reduced cost from the origin depot $o \in O$ to the destination depot $d \in D$. The pricing subproblem can be classified as a type of constrained shortest-path problem, and this particular problem has been shown to be NP-hard by [Garey Michael and Johnson David \(1979\)](#). Thus, it can be concluded that the suggested pricing subproblem is also NP-hard. Note that the subproblem network is acyclic. We employ a label correcting algorithm to address the shortest path problem. This algorithm is especially effective for acyclic network problems because it can topologically order nodes. This

ordering allows each node to be processed sequentially without needing to revisit any nodes. This linear processing eliminates the need for backtracking, as there are no cycles to navigate or potential revisits to optimize further.

The dual variables associated with the restricted master problem are outlined as follows. λ^1 is the vector of dual variables associated with constraints (22), and λ^2 is the vector of dual variables associated with constraints (23). Thus, the reduced cost of path p is as follows:

$$\bar{c}_p \equiv c_p - \sum_{i \in S} \delta_i^p \lambda_i^1 - \sum_b \sum_t \sigma_{tb}^p \lambda_{bt}^2 \quad (26)$$

The reduced cost of path p is decomposed to the reduced cost of the arcs composing that path. Each arc (i, j) has a reduced cost, denoted $\bar{c}_{ji}$, that depends on the origin and destination nodes, includes the travelling, waiting time, and electricity costs of EBs and dual variables. For a beginning arc connecting the depot to trip node $i \in S$, the reduced cost is $\bar{c}_{oi} = C_d d_{oi} + C_d \hat{d}_i + C_v - \lambda_i^1$. Similarly, for the ending arcs connecting trip node i to the depot, the reduced cost is $\bar{c}_{jo} = C_d d_{jo}$. For trip-to-trip arc (i, j) , the reduced cost is $\bar{c}_{ji} = C_d d_{ji} + C_d \hat{d}_i + C_w w_{ji} - \lambda_i^1$. For a deadhead arc connecting trip node i to the time-expanded node t^b (of charging station $b \in B$), the reduced cost is $\bar{c}_{it^b} = C_d d_{ib} + C_w w_{it^b}$. The reduced cost of charging arc (t, i) (corresponding to charging u' minutes at station b , starting at time period t) is $\bar{c}_{tb_i} = u' C_t + C_w w_{tb_i} - \sum_{t'=t}^{t+u'} \lambda_{bt'}^2$, which consists of the waiting time of an EB before the charging event and the charging cost of the EB. Deadhead arcs connecting dummy trips $j \in S'$ at charging station b , denoted as j^b , to other trips have a reduced cost of $\bar{c}_{j^b i} = C_d d_{j^b i} + C_d d_i - \lambda_i^1$. Finally, waiting time arcs incur a reduced cost equal to the total waiting time of an EB multiplied by the unit cost of waiting.

Path restrictions are enforced in the network via resources. Our problem has two limited resources, which are *time* and *range*. Traversing arcs in the network consumes resources. Every constrained resource at each node must lie within a specified range, known as the resource window. Let R represent the set of resources. For each node $i \in N$ and each resource $r \in R$, the resource window is defined by $[l_i^r, u_i^r]$, where l_i^r and u_i^r represent the lower and upper bounds, respectively, of resource r for node i .

The time resource keeps track of the time corresponding to a given schedule. Each arc consumes

time according to the travelling or waiting time it represents. The time resource windows for the origin and destination node are respectively $[0, 0]$ and $[0, |T|]$. The time resource window at a trip node $i \in S$ and all dummy trip nodes associated with i is set to $[e_i, e_i]$. Finally, the time resource window for time-expanded node corresponding to time t is $[t, t]$. Note that the lower bound of each resource window is soft, meaning a node can be visited even when the arrival time falls below the lower bound. In that case, the value of the resource gets updated to the lower bound.

The range resource keeps track of the range remaining for the EB. Because of the soft lower bound, the value of this resource is defined as $d_{max} - \xi$, where ξ is the actual range remaining. The range resource is initialized at 0, increases when the EB travels, and decreases when the EB recharges. The range resource window is $[0, 0]$ for the origin and $[0, d_{max}]$ for all other nodes. On each arc (i, j) where the bus travels (beginning, ending, and deadhead arcs), the resource consumption is d_{ij} . For a charging arc corresponding to charging u' minutes, the resource consumption is set as $-d_{max}u'/U$ (the range gained by the recharging activity). No range resource is consumed on waiting arcs.

The pricing subproblem is formulated as a constrained shortest-path (CSP) problem over a time-expanded, acyclic network that incorporates operational and energy constraints. The algorithm employs a label-setting method with topological sorting to efficiently evaluate paths from the origin to the destination depot. It maintains labels for cost, time, and remaining range at each node and ensures that resource windows for both time and range are satisfied.

Note that the proposed network structure is not trivial. In particular, the way charging is modelled is designed to minimize the number of arcs in the network. A more trivial implementation would instead include charging decisions on trip-to-trip arcs. Several arcs would connect trip node i to trip node j , each corresponding to a different charging time and duration. However, such a network structure would yield $\mathcal{O}(|S|^2|T||B|)$ arcs, whereas the proposed structure has $\mathcal{O}(|N||B| + |T||B|)$ arcs (Each trip has one arc going to and coming from each charging station, and each charging station has $\mathcal{O}(|T|)$ arcs connecting the time-expanded nodes and dummy nodes among themselves). Since the time required to solve the constrained shortest-path problem grows roughly quadratically with the number of arcs, this network reduction represents significant time savings.

4.5.3 Branching Heuristics

Although the branch-and-price method presented above is optimal, its runtime can be very high for large instances. In those cases, heuristic strategies can be used to speed up the solution process while sacrificing little in terms of solution quality. As such, we use three branching strategies, such as bound and inter-task fixing (BITF), multiple column fixing (MCF), and the combination of MCF and BITF. The first one is an exact method that branches on static variables and inter-tasks. The second strategy is a column fixing heuristic, which sets one or more path variables to 1 at each node in a diving fashion. The third strategy mixes the exact method with column fixing. Also, we implement an acceleration strategy based on the solution improvement during CG process and its number of iterations threshold. The aim is to find a strategy that offers a good compromise between objective value and CPU time.

4.5.3.1 Exact Branching (BITF)

Our exact branching method works by branching on static variables (i.e., the z variables determining charging station locations) and inter-tasks (network arcs that link two tasks). Consider the linear relaxation of Equations (1) to (5) at a given node. The BITF method prioritizes branching on static variables with fractional values, z , if there are any. The score for a static variable is calculated based on its fractional value at the time of evaluation. Specifically, let $\hat{z} = z - \lfloor z \rfloor$, and the score is given by: $2|\hat{z} - 0.5|$. The variable with the highest score is then selected for branching.

If all static variables are integer, the algorithm instead branches on an inter-task arc in the subproblem network. The arc with cumulative flow closest to 0.5 is selected. Suppose arc (i, j) is the one selected. Two child nodes are then created, in which this inter-task arc is either forbidden or imposed. The former decision is enforced by removing arc (i, j) from all networks, while the latter is enforced by removing all arcs leaving i and all arcs entering j , except arc (i, j) . In both child nodes, all columns that contradict the decision are also removed (Desaulniers et al., 1997).

4.5.3.2 Multiple Column Fixing (MCF)

The column fixing algorithm fixes one of several path variables at each node without backtracking. To enforce path p on one branch, it is necessary to eliminate all paths that include the trips covered by p from the RMP, and to remove all arcs or nodes that represent those trips from the subproblem networks. The method selects which variables to fix based on their value in the previous node's fractional solution. It fixes first the variables with the highest fractional value. This selection method is used because columns with larger fractional values tend to have a lesser impact on the objective function when fixed at 1, compared to those with smaller values (Saddoune, Desaulniers, & Soumis, 2013). This technique, known as the diving heuristic, follows a depth-first approach, continuing down a single branch of the tree until it finds a solution where all variables are integers. The primary benefit of this branching strategy is its ability to find feasible solutions quickly. However, the quality of these solutions may decrease, particularly in smaller instances, where fixing a variable can significantly affect the lower bound.

At each node, columns to be fixed are selected in decreasing order of their values, subject to the following rules. If no columns have a fractional value greater than or equal to a predefined threshold $\eta \in]0, 1[$, only the column with the largest value is selected. Otherwise, only columns with fractional values greater than or equal to η are considered for fixing. The choice of η plays a crucial role in the solution process: a large value of η significantly restricts the number of variables fixed at once, which can slow down the process, whereas a small value allows more variables, even those with relatively small fractional values, to be fixed simultaneously, which may sometimes lead to poor-quality solutions. In this research, if a column has a fractional value greater than or equal to 0.5, column fixing branching is applied. Otherwise, a combination of strong branching and column fixing is used (Quesnel, Desaulniers, & Soumis, 2017).

To further control this process, the complementary value of a path variable y_p is defined as $1 - y_p$. The sum of the complementary values of the fixed columns must not exceed a maximum value $\psi \geq (1 - \eta)$, ensuring that the number of variables fixed near η remains manageable. Additionally, the total number of columns that can be fixed at a time is restricted to a maximum of μ .

4.5.3.3 Column Fixing and Exact Branching Combination

The third branching strategy, known as MCF-BITF, combines the MCF and BITF branching methods. In this heuristic approach, both strategies are applied simultaneously, and the one with the higher score is selected for branching. This process continues until all nodes are explored. The score for MCF is determined by the highest fractional value obtained. The score obtained from exact branching (BITF) is normalized and mapped within the range of $[b_l, b_u]$. Finally, the branching method with the highest score is selected.

4.5.3.4 Acceleration Strategy

Column generation typically converges slowly as it approaches the optimal value, a phenomenon known as the tailing effect. At this stage, adding paths with negative reduced costs to the model often has little effect on the value of the RMP. To counter this issue, we choose to stop column generation prematurely if there is less than κ percent improvement in the RMP's optimal value over the last β iterations. The downside of this acceleration strategy (AC) is the loss of the optimality guarantee normally offered by CG. However, in our case, this has a negligible effect on the optimality gap. In our test, we consider using this acceleration strategy in combination with MCF-BITF branching strategy.

4.6 Results and Analysis

We provide details on the experimental protocols used for our computational results in Section 4.6.1 and present comparative results of the proposed branching methods and acceleration strategy for different sizes of the problem and a comparison of sequential and integrated approaches for solving the problem, in Section 4.6.2. Then, in Section 4.6.3, the Montreal case study will be discussed to further assess the effectiveness of the proposed model. A brief sensitivity analysis with different types of EBs will be presented to observe the impacts of changing such parameters' values on the number of required EBs, charging stations and total costs. In addition, the impact of timestep resolution on the objective value and the runtime will be discussed. Finally, Section 4.6.4 presents a comparison between the developed branch-and-price algorithm and the arc-based model.

4.6.1 Experimental Protocol

For our computational experiments, most of the parameters used to generate instances of varying sizes are drawn from (J.-Q. Li, 2014) and (Gkiotsalitis et al., 2023). The starting and ending points of the trips, the coordinates of the depots, and the coordinates of potential spots for locating fast-chargers are located in a 10 km to 10 km square on the Euclidean plane, with an even distribution. The travel time between any two points is proportional to the Euclidean distance between them. Based on Bloomberg New Energy Finance (2018b), the capital cost of pantograph chargers and electric buses are \$230,000 and \$530,000, respectively. The lifetime of both EBs and pantograph chargers is 15 years. Thus, the annualized purchase cost of an EB and the annualized installation cost of fast chargers will be around \$35,333 and \$15,333, respectively. The actual cost may vary depending on the location where the charger is installed, and it will be multiplied by a factor within the range of [1, 1.25]. Also, the maximum travel range of EBs is assumed to be 120 km. This information and other required parameters are given in Table 4.1. It is important to note that the operational costs shown in Table 4.1 will be multiplied by the number of service days in a year. This will convert them into annual costs, allowing for a direct comparison with the yearly expenses of acquiring a new bus and installing a fast-charging station.

Table 4.1: Parameters settings for generating random instances

C_v	Annual cost of using a new bus	\$ 35,333	Bloomberg New Energy Finance (2018b)
V_b	Annual cost of establishing fast-chargers	\$ 15,333	Bloomberg New Energy Finance (2018b)
c_{ji}	Unit travel cost	0.06 \$/km	J.-Q. Li (2014)
U	Charging time	10 min	J.-Q. Li (2014)
C_w	Cost of waiting time	0.01 \$/min	
d_{max}	Maximum travel range of EBs	120 km	
T	Number of time periods	$\max_{i \in S} \{e_i\}$	

To solve the proposed model, we utilized the specialized Gencol library version 4.5, which is specifically designed for the branch-and-price algorithm on a Linux computer with an Intel i7-8700 CPU clocked at 3.20 GHz without parallel computing. All RMPs are solved using the primal simplex algorithm of CPLEX 22.1.1. The time limit for solving large-scale instances is 48 hours.

The parameter settings for the branching heuristics are as follows: $\eta = 0.5$, $\psi = 0.7$, $\mu = 5$, $b_l = 0.8$, and $b_u = 1$. For the acceleration strategy, the parameters are $\kappa = 0.2\%$ and $\beta = 15$.

4.6.2 Results on Synthetic Instances

Each instance is encoded according to its number of trips, S , potential spots to locate fast-chargers, B , and the number of time periods, T . For example, S30-B5-T80 indicates that there are thirty trips, five potential places to install fast-chargers, and 80 time periods for each charging station. The timestep for time-expanded nodes is set to 5 minutes, meaning that the start charging time must be in multiples of this interval. We solve instances of different sizes for two depots. Instances are categorized based on the number of trips: those with fewer than 100 trips are labelled as small, instances with more than 100 trips and less than 300 trips are categorized as medium-scale, and, finally, instances that involve more than 300 trips are classified as large-scale problems. They are important to this study because smaller instances generally result in larger integrality gaps.

4.6.2.1 Comparison of Branching Strategies

In this section, we analyze and compare the performance of the four proposed methods for addressing the FCLP-EBS problem. To evaluate the methods, several problem instances of varying scales were generated and solved using each approach. The results of these experiments are summarized in Table 4.2 and Table 4.3, which provide a detailed comparison of the effectiveness of the branching and acceleration strategies. The comparison focuses on two key metrics: CPU time and optimality gap. In Table 4.2, we compare the solutions obtained from each branching and acceleration strategy for the generated instances. The best relaxation cost is represented by Z_{relax} , while the best feasible solution is denoted by Z_{best} . The integrality gap is calculated as $|Z_{\text{best}} - Z_{\text{relax}}| / Z_{\text{relax}}$. The gap reported in Table 4.3 reflects the range of the optimality gap based on the calculated integrality gap. Since the optimality gap is smaller than or equal to the integrality gap, we can conclude that the obtained solutions have an optimality gap at most equal to the integrality gap. Table 4.3 highlights the strengths and weaknesses of each method, offering insights into their computational efficiency and solution quality. By comparing these methods, we aim to identify the most practical and efficient approach for solving the FCLP-EBS problem across a range of scenarios.

Table 4.2: Solutions comparison of different branching and acceleration strategies

Instance	BITF		MCF		MCF-BITF		MCF-BITF-AC	
	Z_{relax}	Z_{best}	Z_{relax}	Z_{best}	Z_{relax}	Z_{best}	Z_{relax}	Z_{best}
S30-B5-T80	740441.3	750976.1	740441.3	750976.1	740441.3	750976.1	740687.0	751007.8
S30-B10-T80	672122.4	683552.6	672122.4	684436.1	672122.4	683552.6	672465.9	683604.2
S50-B5-T88	1010939.3	1018008.5	1010939.3	1018028.5	1010939.3	1018008.5	1011328.7	1018008.5
S50-B10-T88	945149.5	967333.9	945149.5	967838.8	945149.5	967375.3	945659.0	967375.3
S100-B5-T88	1759183.6	1824293.4	1759183.6	1774536.9	1759183.6	1773434.1	1759977.0	1773421.9
S100-B10-T88	1754937.6	1757930.7	1754937.6	1755036.1	1754937.6	1755065.0	1754998.0	1754998.0
S100-B20-T88	1756736.9	1772896.7	1756736.9	1775952.2	1756736.9	1772892.4	1757795.5	1772937.1
S200-B10-T108	-	-	2703269.6	2752619.4	2703269.6	2718164.6	2705545.1	2716704.8
S200-B20-T108	-	-	2637489.7	2707974.8	2637489.7	2654578.1	2638686.8	2655389.4
S300-B10-T161	-	-	3131098.5	3206195.3	3131098.5	3153739.5	3134787.2	3115536.9
S300-B20-T161	-	-	3194333.3	3221348.1	-	-	3197752.1	3231564.4
S400-B10-T161	-	-	3817862.9	3890008.3	3817862.9	3837648.6	3822527.1	3840047.7
S500-B10-T161	-	-	4383302.7	4414700.0	4383302.7	4396899.5	4385375.6	4400656.4
S500-B20-T161	-	-	4314582.9	4416383.4	-	-	-	-
S1000-B10-T229	-	-	6161936.3	6175474.7	-	-	-	-

Table 4.3: Comparison of different branching and acceleration strategies in terms of CPU time and optimality gap

Instance	BITF		MCF		MCF-BITF		MCF-BITF-AC	
	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)
S30-B5-T80	0.3	0.000	0.3	< 1.423	0.3	< 1.423	0.2	< 1.427
S30-B10-T80	2.1	0.000	2.3	< 1.832	1.2	< 1.701	0.5	< 1.708
S50-B5-T88	0.9	0.000	0.8	< 0.701	1.8	< 0.699	0.8	< 0.699
S50-B10-T88	36.1	0.000	10.3	< 2.401	7.3	< 2.296	5.9	< 2.351
S100-B5-T88	124.6	0.000	6.0	< 0.873	3.4	< 0.810	2.4	< 0.809
S100-B10-T88	77.8	0.000	16.4	< 0.007	16.2	< 0.007	5.3	< 0.003
S100-B20-T88	1864.0	0.000	26.2	< 1.094	943.2	< 0.920	188.3	< 0.922
S200-B10-T108	-	-	52.6	< 1.826	828.2	< 0.551	312.5	< 0.496
S200-B20-T108	-	-	166.6	< 2.672	8931.0	< 0.648	2296.6	< 0.678
S300-B10-T161	-	-	401.8	< 2.398	5706.0	< 0.723	2858.0	< 0.497
S300-B20-T161	-	-	1547.9	< 0.846	-	-	74146.5	< 0.581
S400-B10-T161	-	-	690.0	< 1.890	34171.0	< 0.518	6560.0	< 0.395
S500-B10-T161	-	-	1451.0	< 0.716	59604.0	< 0.310	17083.0	< 0.395
S500-B20-T161	-	-	4220.0	< 2.359	-	-	-	-
S1000-B10-T229	-	-	36102.0	< 0.220	-	-	-	-
Average		0.000		1.417		0.883		0.843

Table 4.3 shows the computational results used to compare the performance of the four methods. For each instance, we report the computational time and the optimality gap range, which is calculated through the integrality gap between the best feasible solution and the relaxed solution. Since all methods produce identical linear relaxation solutions, these gaps allow us to compare the quality of solutions obtained by each heuristic. However, as noted in Table 4.2, the best relaxed

solutions for MCF-BITF-AC differ from those of the other methods. To ensure consistency, the integrality gap is calculated using the relaxed solution from the other methods. In Table 4.3, the branching strategy with the lowest gap in each row is highlighted in bold. Additionally, the last row of the table shows the average optimality gap for each branching strategy.

The BITF strategy consistently achieves a zero optimality gap in small-sized instances, demonstrating its effectiveness in finding optimal solutions for smaller problem sizes. However, its runtime becomes excessive as the problem size increases, since BITF only branches on charging station variables and inter-task network arcs, rather than directly on columns or paths. In contrast, MCF has the lowest runtime but also the highest average gap (1.417%), making it less suitable for small and medium instances where solution precision is important. This is because MCF quickly finds feasible solutions, but fixing variables too early can significantly affect the lower bound, especially in smaller instances. MCF-BITF offers a balanced trade-off, with improved accuracy (average gap of 0.883%) and moderate runtime. As a hybrid of exact branching and MCF, it is more computationally intensive than MCF alone, especially in medium and large instances. The MCF-BITF-AC strategy applies an acceleration technique to MCF-BITF, achieving a similar level of accuracy (average gap 0.843%) while reducing runtime, making it the most effective choice for large-scale problems. These results suggest that BITF is ideal for small instances, MCF-BITF-AC is preferable for large-scale problems where runtime is a concern, and MCF is suitable for very large problems where fast, approximate solutions are acceptable.

The summary of the solving process of the proposed branch-and-price algorithm for different instances is shown in Table 4.4. It highlights the efficiency and computational performance of four strategies, detailing key metrics such as the number of Solved Linear Relaxations (SLR), Column Generation Iterations (CGI), and Columns Generated (CG).

The number of solved linear relaxations reflects the depth of the search process under each branching strategy. As shown in Table 4.3, the BITF strategy solves only small instances efficiently and it becomes infeasible as the problem size increases. In contrast, the MCF strategy solves more linear relaxation problems, suggesting broader but less focused search behavior. The MCF-BITF strategy significantly increases the number of solved relaxations, indicating deeper exploration that improves solution quality but leads to longer runtimes. Notably, MCF-BITF-AC records the highest

Table 4.4: Solving process results. CGI: Column Generation Iterations, SPS: Shortest Paths Solved, CG: Columns Generated

Instance	BITF			MCF			MCF-BITF			MCF-BITF-AC		
	SLR	CGI	CG	SLR	CGI	CG	SLR	CGI	CG	SLR	CGI	CG
S30-B5-T80	7	184	586	7	184	586	7	184	586	15	110	492
S30-B10-T80	15	1386	2349	7	344	915	42	737	1489	42	386	1018
S50-B5-T88	9	369	1603	29	469	1272	9	369	1603	18	311	1421
S50-B10-T88	49	1958	4570	22	717	2075	83	2101	4518	65	1047	3658
S100-B5-T88	5	235	1959	90	896	2709	73	798	2902	66	539	2334
S100-B10-T88	23	1391	8418	66	1250	4948	85	2290	8791	78	834	4692
S100-B20-T88	185	30878	191296	30	2135	10494	316	30878	191296	181	2287	24404
S200-B10-T108	-	-	-	31	1830	14150	207	3999	41390	152	1876	19453
S200-B20-T108	-	-	-	32	2879	23395	350	77286	839240	734	11835	161163
S300-B10-T161	-	-	-	41	2692	22062	114	20960	227037	748	33388	196083
S300-B20-T161	-	-	-	61	12888	78035	-	-	-	2527	506937	1309127
S400-B10-T161	-	-	-	50	3823	36720	183	47249	757314	356	7998	137977
S500-B10-T161	-	-	-	43	5918	51572	404	49268	772068	802	15486	325566
S500-B20-T161	-	-	-	59	8689	87047	-	-	-	-	-	-
S1000-B10-T229	-	-	-	67	24700	253413	-	-	-	-	-	-

SLR in large instances, demonstrating its ability to thoroughly explore complex solution spaces while keeping runtime manageable, thanks to the adopted acceleration mechanism. Importantly, the number of SLR rises sharply with an increase in potential charger locations, indicating that the branching strategies are more sensitive to the number of potential spots for locating chargers than to the total number of trips.

The number of columns generated is an indicator of the method’s approach to refining the solution. MCF-BITF-AC and MCF-BITF tend to produce more columns compared to the MCF strategy as the instance size increases, suggesting that these strategies use iterative refinement intensively to converge on the optimal solution. The MCF-BITF-AC strategy generally performs well even with lower CGI and CG values on larger instances compared to the BITF and MCF-BITF methods, indicating its efficiency in handling large-scale problems. BITF tends to have higher values in small instances, especially in CG. The reason the number of CGI and CG in all branching strategies except MCF-BITF-AC are the same for the first instance, S30-B5-T80, is that the root node is already integer. Another point is that the effect of increasing the potential spots for locating chargers is much more than increasing the number of trips in terms of CG.

4.6.2.2 Sequential Vs. Integrated Approach

We evaluate the benefits and potential improvements of the integrated approach compared to the sequential approach. The sequential approach solves two separate problems: first, identifying charging station locations, and second, addressing the standard EB scheduling problem. In practice, this initial step is often performed manually by an experienced planner, relying on intuition rather than optimization. To simulate the decisions of a skilled but imperfect planner, we first solve the integrated model to determine the optimal charging station locations. Next, we intentionally modify these locations in one of three ways: first, adding a charging station, second, removing a charging station, and finally, relocating a charging station to a new selected location with the minimum installation cost. For each altered configuration, we resolve the EB scheduling problem using the proposed FCLP-EBS model, keeping the charging station decisions fixed. For all modification types, we test all possibilities, but only present the results for the one which has the least impact on the overall solution cost. This simulates the decision process of a nearly-perfect planner. This process allows us to assess the robustness and efficiency of the integrated model compared to sequential decision-making. This experiment was conducted on four instances of varying scales, and the resulting percentage cost deviations for each instance and scenario are summarized in Table 4.5.

Table 4.5: Comparing the integrated with the sequential approach for EB scheduling and charging location planning

Instance		Scheduling Cost	No. EB	Total Cost	Cost Deviation (%)
S50-B5-T88	Integrated	1,001,709	15	1,018,008	-
	Add	1,001,672	15	1,033,726	1.54
	Remove	1,725,778	26	1,757,832	72.67
	Change	1,200,288	18	1,216,043	19.45
S100-B5-T88	Integrated	1,769,232	26	1,803,274	-
	Add	1,766,721	26	1,817,062	0.76
	Remove	1,900,889	28	1,916,644	6.28
	Change	1,900,073	28	1,932,127	7.14
S70-B15-T88	Integrated	1,337,548	20	1,371,100	-
	Add	1,337,039	20	1,386,005	1.08
	Remove	1,469,999	21	1,486,574	8.42
	Change	1,403,437	21	1,435,426	4.96
S100-B20-T88	Integrated	1,742,675	26	1,775,744	-
	Add	1,337,039	20	1,386,005	1.08
	Remove	1,809,305	27	1,824,910	2.76
	Change	1,807,751	27	1,838,770	3.54

The results of Table 4.5 illustrate the substantial impact of charging station location planning on operational costs across various instances. Removing charging stations often leads to notable cost increases; for example, the first instance experienced a nearly 73% cost rise, highlighting the critical role of infrastructure in maintaining operational efficiency. In contrast, planning too many charging stations has a limited impact. This is expected since, in the worst case, the charging station is never utilized and the only added cost of the solution is the establishing cost, V_b . Modifying charging station locations resulted in mixed outcomes, with some instances showing moderate cost increases. This variability emphasizes the complexity of optimal placement, indicating that even minor adjustments can have significant financial consequences. Careful planning and analysis are therefore essential to optimize electric bus operations effectively.

4.6.3 Case Study

This section evaluates the performance of the proposed model and algorithm through a real-life case study in Montreal, Canada. We focus on two major bus depots within the city, Garage Stinson and Garage Legendre, and twelve bus lines are assigned to these depots for potential electrification.

Using data from the General Transit Feed Specification (GTFS) of Montreal, each bus line's operational characteristics, such as departure frequency, route length, and travel time, are presented in Table 4.6. Figure 4.3 illustrates the layout of the transit network under study, which features the bus routes and eight potential locations for charging stations. Electric buses on these routes are initially stationed at the two depots, marked by black icons on the map. EBs can serve trips on different routes if they remain cost-efficient.

The total number of trips considered across all bus lines is 507. Electric buses are allowed to recharge at any point during the day, from the beginning to the end of their service hours. The cost of purchasing and installing a charging station is estimated at 500,000 CAD, although this cost may vary depending on the station's location. The assumed lifespan of each charger is 12 years. In terms of operational expenses, buses incur a cost of 0.17 CAD per kilometre travelled. The charging cost with fast chargers is set at 15 CAD per hour during peak hours and 30 CAD per hour during off-peak hours. Peak hours are defined as 7:00 AM to 11:00 AM and 5:00 PM to 7:00 PM.

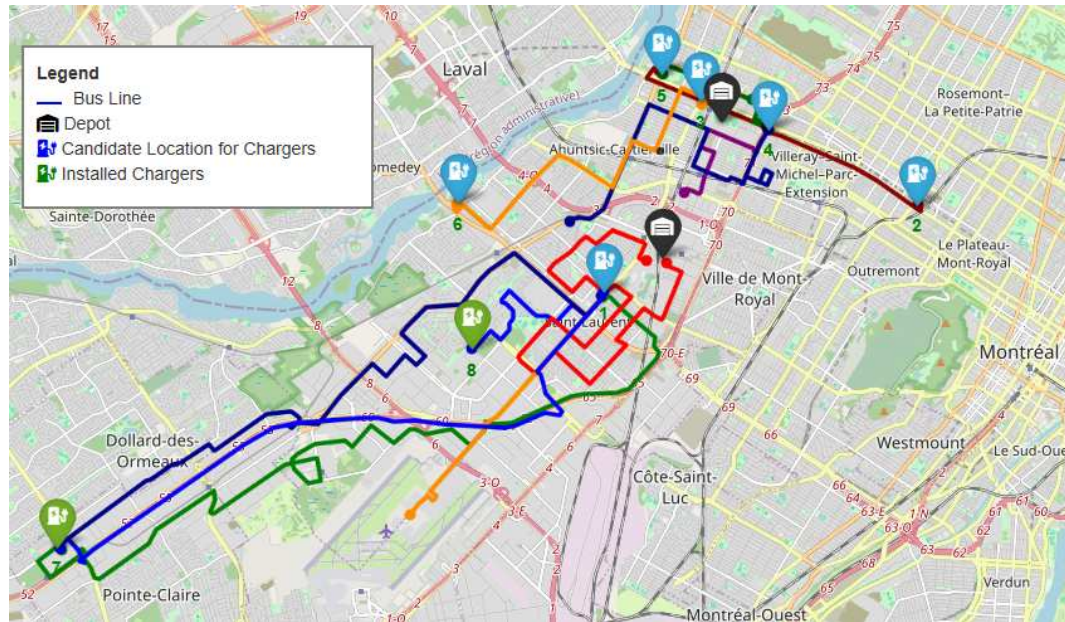


Figure 4.3: Geographical distribution of the selected bus lines departing from two depots and candidate spots for installing charging stations

Table 4.6: Information about transit routes and trips

Bus Line	Travel Time (min)	Route Length (km)	Departure Frequency (min)
70	28	9.4	20
72	51	17.0	15
19	16	5.4	20
128	63	21.0	30
174	21	7.0	25
31	26	8.7	30
135	33	11.0	30
180	31	10.4	20
56	8	2.7	15
146	26	8.7	30
215	49	16.4	25
216	31	10.4	27

Table 4.7 illustrates the characteristics of four real-world electric bus types according to (Proterra Electric Vehicle Technology Manufacturer, 2016a) and (Gallo et al., 2014). The differences in electric bus types are based on their purchasing cost, maximum travel range, and charging time. Note that the lifetime of all EBs is assumed to be 12 years. We conduct individual model runs for each electric bus type for the Montreal case study and present the corresponding results. This analysis can assist transit authorities of Montreal in determining the most suitable electric bus type for their fleet, in terms of cost-effectiveness.

Table 4.7: Characteristics of four well-known electric bus types Gallo et al. (2014); Proterra Electric Vehicle Technology Manufacturer (2016b).

Electric Bus	Price (CAD)	Annual EB Usage Cost (CAD)	Maximum Travel Range (km)	Charging Time (min)
New Flyer	852,500	71,041	90	6
Proterra Catalyst	958,500	79,875	116	10
Proterra Catalyst+	1,035,000	86,250	140	13
Proterra Catalyst XR	1,110,000	92,500	263	13

The results of the case study obtained using the MCF-BITF-AC method for different types of electric buses are presented in Table 4.8. It highlights the optimum number of EBs required to fulfill the given trips and the index of the selected locations for installing chargers. The table also reports the total capital expenditures (CAPEX) for purchasing EBs and chargers, as well as the

annualized cost of these investments. The operational expenditures (OPEX) column indicates the total travelling cost, waiting time cost, and cost of charging. The last column presents the objective value. The objective value includes the total operational costs, electricity costs, and the annual cost of purchasing EBs and installing fast-chargers. The aim is to find the EB type with the lowest total costs among the given electric bus types. By conducting these comparisons, we can identify the most cost-effective electric bus type for the case study. The results of this analysis are represented in Table 4.8.

Table 4.8: Comparison of results for different EB types for the case study

Electric Bus	Charger Location(s)	No. EB	CAPEX (CAD)	Annual Bus and Charger Cost (CAD)	OPEX (CAD)	Obj (CAD)
New Flyer	7,8	29	25,722,500	2,143,542	1,743,032	3,886,574
Proterra Catalyst	6,8	28	27,838,000	2,319,833	1,697,705	4,017,538
Proterra Catalyst+	7	27	28,445,000	2,370,417	1,637,034	4,007,451
Proterra Catalyst XR	-	27	29,970,000	2,497,500	1,613,328	4,110,828

According to Table 4.8, the New Flyer bus type emerges as the most cost-efficient option, achieving the lowest objective cost among all alternatives. Although its shorter travel range increases the need for more frequent recharging, which requires a larger fleet and additional chargers to maintain service levels, its lower procurement cost significantly reduces the overall capital expenditure. Additionally, the New Flyer benefits from a shorter charging time, allowing EBs to recharge more quickly and return to service sooner. This added flexibility improves scheduling efficiency and increases the number of trips that each bus can cover, further strengthening its overall cost-effectiveness. In contrast, the Proterra Catalyst XR, despite offering the longest travel range, results in the highest objective cost. This is primarily driven by its substantially higher purchase price, which increases both CAPEX and the annualized bus and charger costs. While the extended range reduces the operational costs and the number of recharging trips and potentially the need for charging infrastructure, the cost savings do not offset the high investment required. The Proterra Catalyst and Proterra Catalyst+ fall between these two extremes. They offer a balance between up-front investment and operational efficiency, yielding moderate objective values. Both require fewer chargers than the New Flyer, but still carry higher CAPEX due to their bus prices.

Another noteworthy observation is the variation in charger locations across EB types. For instance, the optimal charger placement for New Flyer is at spots 7 and 8, while Proterra Catalyst

places them at 6 and 8, and Catalyst+ requires only a single location at location 7. In contrast, when deploying Proterra Catalyst XR buses, no fast chargers are needed on the network, as these buses have sufficient range to be fully charged overnight at the depots, eliminating the need for mid-day charging infrastructure. This variation underscores the importance of considering bus characteristics, such as range and charging duration, when planning charging infrastructure. Charger location planning must be tightly integrated with fleet selection to ensure efficient operation across the entire network. Overall, these findings highlight the trade-offs between initial investment, operational flexibility, and infrastructure requirements. Longer-range buses reduce recharging frequency and infrastructure needs but come at a higher upfront cost. Shorter-range buses may require more logistical coordination but offer lower total costs when supported by efficient charging and scheduling strategies. This analysis emphasizes the critical role of travel range and charging efficiency in optimizing the deployment of electric buses in public transportation networks.

Figure 4.4 shows the breakdown of deadhead time, waiting time, and charging time for each type of electric bus. The figure highlights how operational time varies across different EB types. It is important to note that the total in-service time for all buses is 32,550 minutes.

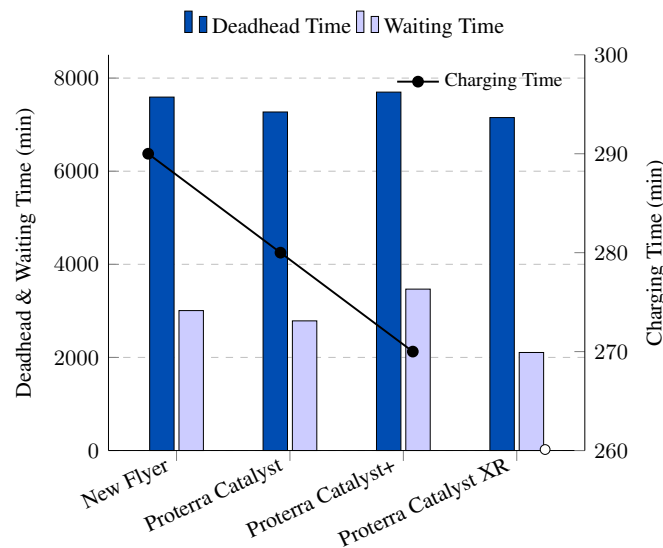


Figure 4.4: Comparison of deadhead time, waiting time, and charging time for different EB types

The analysis of deadhead, waiting, and charging times reveals important differences among the electric bus types. The New Flyer has the highest charging time and a relatively high deadhead

time, likely due to its shorter range requiring more frequent trips to charging stations. However, its lower procurement cost and faster individual charging sessions can improve scheduling flexibility. In contrast, the Proterra Catalyst XR shows the lowest deadhead and waiting times, indicating higher operational efficiency due to its longer travel range and reduced need for frequent charging. With no charging time at all, it is particularly well-suited for minimizing downtime. Proterra Catalyst+ experiences the highest waiting time, suggesting less efficient vehicle utilization. Overall, the results highlight the trade-offs between travel range, charging needs, and total operational time, which should be carefully balanced when selecting an EB type.

This experiment was conducted in a specific region of Montreal, considering a fixed number of bus lines and candidate locations for installing chargers. It is important to highlight that the results may differ if the scale of the problem, such as the number of bus lines or candidate locations, changes. Therefore, it is essential to first determine the scale of the bus network, consider the number of given trips, and identify potential locations for fast chargers. Once these factors are established, transit authorities can then assess which electric bus type best suits their fleet. By utilizing the proposed model and conducting a thorough analysis, transit authorities gain valuable insights to make well-informed decisions before purchasing or expanding their electric bus fleet. This approach empowers them to select the most suitable electric bus type that aligns with their specific requirements, resulting in a more efficient and cost-effective public transportation system.

4.6.3.1 Effect of Timestep Resolution on Total Cost and CPU Time

The time-expanded network in our model uses a default timestep of 5 minutes. To evaluate how changing this assumption affects both the solution quality and computational performance, we conduct an experiment using the New Flyer electric bus, which was identified in Table 4.8 as the most cost-effective option. We solve the problem using the MCF-BITF-AC branching strategy for two different timestep values: 5 minutes and 2 minutes. The goal is to understand the impact of a smaller timestep on the objective value and CPU time, and to determine whether the added computational effort is justified. Table 4.9 reports the total costs and runtime for both timestep settings. By comparing these results, we can assess whether the marginal gains in solution accuracy are worth the additional computation time, providing valuable insight into the trade-off between

model precision and efficiency.

Table 4.9: Comparison of solution quality and runtime for timesteps 2 and 5

EB Type	Objective Value (CAD), Timestep = 2	Time (s), Timestep = 2	Objective Value (CAD), Timestep = 5	Time (s), Timestep = 5	Change in Total Costs (%)	Change in Runtime (%)
New Flyer	3,847,443	10,948	3,886,574	2,321	1.00	-371.96

The results of Table 4.9 indicate that reducing the timestep from 5 minutes to 2 minutes leads to a slight improvement in solution quality, shown by a 1.00% decrease in the total costs. This gain reflects the finer modelling precision achieved with shorter timesteps, which allows more accurate scheduling of EBs and charging times. However, this modest improvement comes at a significant computational cost. The runtime increases sharply, from 2,321 seconds at the 5-minute timestep to 10,948 seconds at the 2-minute timestep, representing a 372% increase in CPU time. This substantial rise in CPU time illustrates the higher complexity and size of the time-expanded network when a finer resolution is used. Overall, these findings illustrate a clear trade-off between accuracy and efficiency. Although the 2-minute timestep produces a slightly better solution, the significant rise in computational time may not be justified, especially in large-scale or time-sensitive scenarios. In practical terms, the default 5-minute timestep strikes a better balance, offering near-optimal results while keeping computational effort at a manageable level.

4.6.4 Results Comparison: PBM vs. ABM

We propose a mixed-integer linear programming (MILP) formulation based on an arc-based model as another method to tackle the FCLP-EBS problem. The details of the arc-based model are provided in Appendix A, Section A.2. We solve the arc-based model with the CPLEX solver and compare its results in terms of runtime with the results of the developed path-based model solved using the B&P method. We solve instances of varying sizes for two depots. For each instance, we provide information on the number of trips, potential spots for fast-charger installations, and the number of time-expanded nodes, representing the available time for charging using pantograph chargers. A total of 21 instances are generated, and their results are presented in Table 4.10. The second and third columns show the total number of installed chargers and the total number of required EBs, respectively. The next two columns present the objective value and execution time of

CPLEX and the B&P method, respectively.

Table 4.10: Comparison of the results of CPLEX and branch-and-price method

Instance	No. Chargers	No. EB	Objective Value	Time (s) CPLEX	Time (s) B&P
S10-B3-T250	1	4	173,445.1	1.5	0.1
S10-B5-T250	1	4	172,568.3	2.3	0.2
S10-B8-T250	1	4	172,699.7	4.3	0.7
S20-B3-T300	1	7	297,467.8	84.8	1.7
S20-B5-T300	1	6	263,147.5	32.5	1.2
S20-B8-T300	1	6	263,059.9	65.4	4.1
S30-B3-T404	2	8	368,904.5	117.5	2.8
S30-B5-T404	2	8	366,700.4	205.4	16.4
S30-B8-T404	1	8	353,743.2	255.9	76.5
S40-B3-T404	3	10	472,012.9	91.8	9.3
S40-B5-T404	3	10	468,976.1	193.7	31.7
S40-B8-T404	3	10	468,740.8	393.9	152.8
S50-B3-T444	1	14	596,099.5	604.3	53.3
S50-B5-T444	1	14	592,956.9	615.2	124.2
S50-B8-T444	2	12	535,677.5	1,256.8	553.0
S60-B3-T444	2	15	659,801.9	###	394.9
S60-B5-T444	2	15	656,897.0	###	781.8
S60-B8-T444	2	15	654,719.7	###	5,043.5
S70-B3-T444	3	16	719,046.7	###	564.2
S70-B5-T444	4	16	737,987.1	###	12,194.4
S70-B8-T444	3	16	721,388.7	###	37,105.6

A comparison of the results reveals that the branch-and-price algorithm outperforms the arc-based model solved with CPLEX in terms of computational efficiency. In every case, the computational time for the branch-and-price method is shorter than the execution time for the CPLEX solver. The difficulty of solving the instances is greatly influenced by changes in their parameters. For instance, when three additional locations for fast chargers were added to the S40-B5-T404 case, resulting in the S40-B8-T404 instance, the CPU processing time doubled, indicating an increase in complexity. In a more extreme case, adding ten additional trips to the S50-B3-T444 instance made it so challenging that the CPLEX solver could not find the optimal solution within a day. It is important to note that the optimality gap was set to zero for all instances. Also, according to [J.-Q. Li \(2014\)](#), the cost of waiting time is 0.1\$/min, and the unit cost of EBs travelled distance is 6.6 cent/km. Also, the maximum travel range of EBs is assumed to be 70 km. Other parameters are

detailed in Table 4.1.

4.7 Conclusion

By rapidly replacing conventional buses with electric ones, new challenges to the operation planning of EBs will emerge. Given the limited travel range of EBs and their long charging durations, public transit operation planning necessitates rigorous reassessment. Electric bus scheduling will be challenging due to their different characteristics. Additionally, location planning of charging stations and scheduling the charging events further complicates decision-making. To address these complexities, this study introduced a path-based model through an integer-linear programming formulation and solved it with a branch-and-price algorithm. We evaluated the performance of the proposed branch-and-price algorithm through four branching strategies using instances randomly generated in a practical setting. Extensive computational studies were conducted and showed the most effective branching techniques in terms of run time and optimality gap. Additionally, a real case study in Montreal, Canada has been conducted to demonstrate the real-world applicability of the model. We solved the case study problem with real existing types of electric buses to help the transit authority of Montreal select the most suitable EB type with specific parameters for fleet electrification. This analysis highlights the model's potential to support transit agencies in identifying the EB type that best fits their transit system, ultimately reducing operational costs over the long term.

The proposed study has the potential for several extensions. One key area for improvement is to incorporate uncertainty into the model for EBs travel time and energy consumption. Travel time for EBs is influenced by various factors such as passenger volume, driving behavior, traffic congestion, road conditions, weather, and more. By considering stochastic travel time, the model can become more robust and better suited to real-world situations. Similarly, energy consumption for electric buses also varies depending on factors like passenger load, battery life, and road conditions. Future research could consider these factors to improve the accuracy of the model. Another area of interest to explore is the inclusion of different vehicle types in the proposed model. Currently, the model assumes that all the EBs in the fleet are homogeneous, while many cities have public bus

transportation systems with heterogeneous fleets. Additionally, the model can be adapted to account for different charging technologies with varying establishment costs, charging power, and charging times.

To enhance the public transportation operation planning for electric buses, several additional elements could be added to the proposed model in future studies. For example, the second stage of public transportation planning, which is timetable development could be included as a decision variable, like the first and third stages, which are network design and bus scheduling, respectively. This means that departure times and trip frequencies could be optimized rather than being fixed parameters. Furthermore, since pantograph chargers have high electricity loads, it is important to consider the impacts of locating such facilities in neighborhoods. Future works could investigate the power loss incurred in connecting such infrastructure to the grid and its effects on energy sources.

Chapter 5

Conclusion and Future Direction

5.1 Integration with Tools4Cities

In this section, we aim to show how the developed models could be integrated into the platform of Tools4Cities. Tools4Cities, created by Concordia University's Next-Generation Cities Institute (NGCI), is a free collection of digital tools designed for practical urban challenges designed to support the creation of more sustainable, resilient, and livable urban environments. The platform is built on a shared city-wide data model, ensuring coherence and integration across applications while serving the distinct needs of various stakeholders. For policy makers and planners, CITYlayers provides a browser-based interface to visualize urban data and run one-click simulations of energy use and emissions at the city scale. Together, these applications embody the institute's vision of putting the power of science in the hands of decision-makers and citizens alike, bridging technical research with practical urban transformation.

In this platform, our developed work under the name of "Electric Bus Planning" is integrated as a service that allows users to explore the operation planning of electric buses alongside charging station location optimization. As illustrated in Figure 5.1, users can access this service by selecting it from the menu and then choosing a specific region from the predetermined areas to investigate the planning process. Each area or use-case includes one or more bus depots, existing bus lines network, and several potential sites for installing fast-charging stations. For example, Figure 5.2 shows region B, where four alternative scenarios are available. In this case, Scenario 2 has been

selected, which includes five bus routes, three depots, and eleven candidate locations for chargers. Once the user clicks on “Calculate Optimal Plan,” the platform zooms into the selected region and opens a decision window that enables the user to choose the type of electric bus to operate within that network. Each bus type is characterized by specific features, including purchase cost, operational cost, maximum driving range, and charging duration. The results of this optimization are displayed in Figure 5.3, where the platform provides an optimal deployment plan for New Flyer electric buses and identifies the most suitable charging station locations. In this example, four fast-charging stations have been selected, 39 buses are required to operate the transit system at minimum cost, and the tool calculates the total annual expenditure, which includes bus acquisition, charging station installation, scheduling, and operating costs. Additionally, the platform quantifies the expected CO₂ emission reductions associated with the plan.

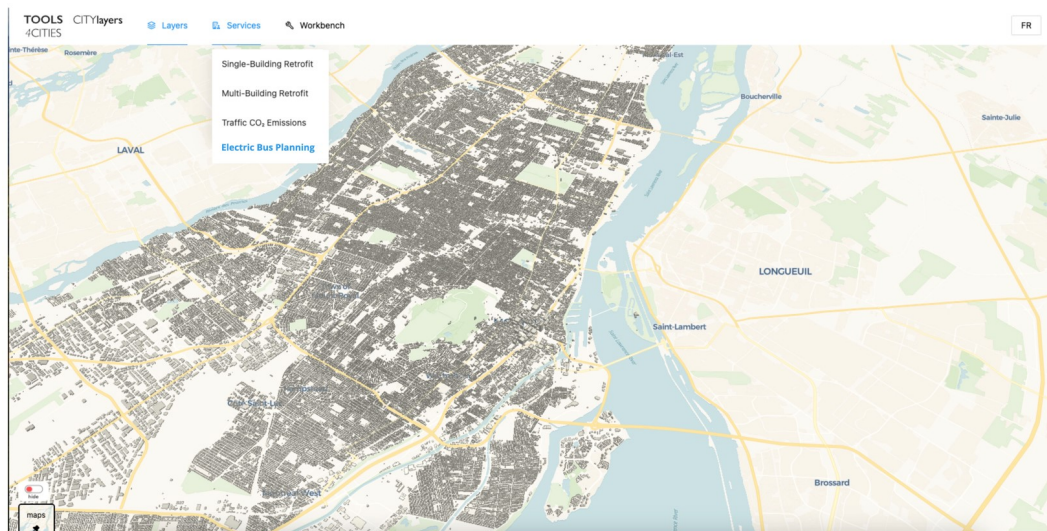


Figure 5.1: Tools4Cities interface with Electric Bus Planning as a service

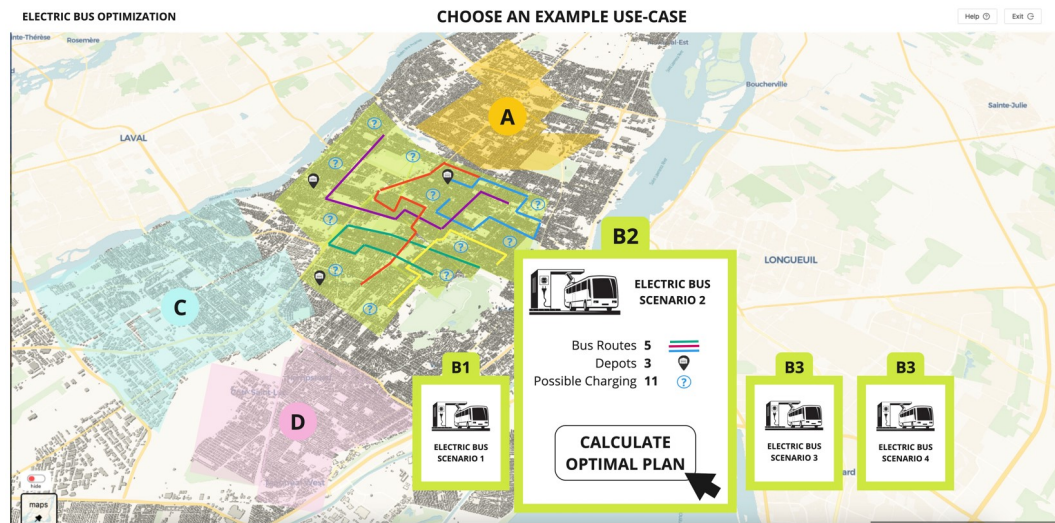


Figure 5.2: Region B, Scenario 2: five routes, three depots, eleven spots for lovsing chargers

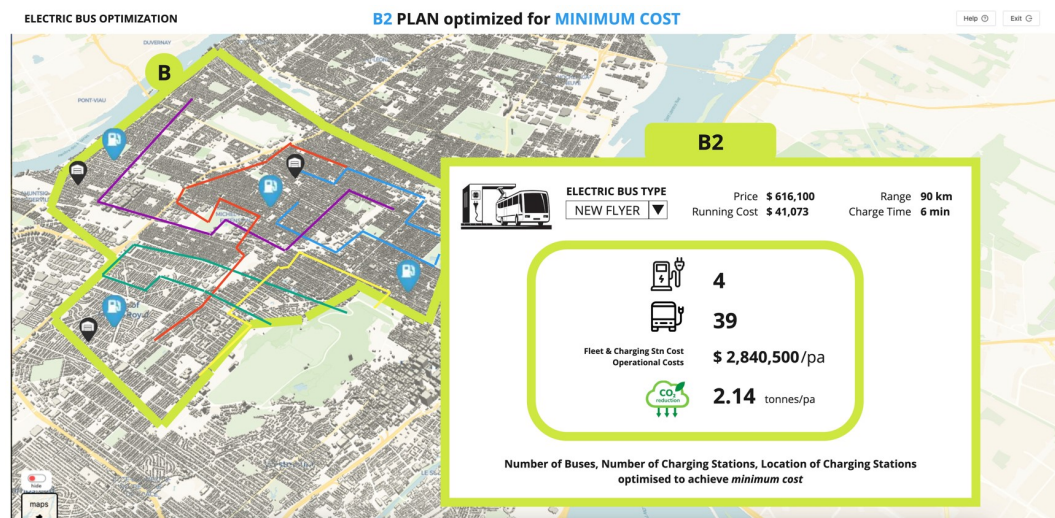


Figure 5.3: Optimized plan for New Flyer bus types

The Electric Bus Planning service in the Tools4Cities platform directly supports decision makers by integrating operational planning and infrastructure investment into a single interactive tool. Instead of relying on separate analyses for bus scheduling, fleet sizing, and charging infrastructure, the platform combines these dimensions into one optimization framework. Decision makers can test multiple scenarios for a specific region, each with different depots, bus routes, and potential charging locations, and then calculate the optimal plan. This ensures that the number of buses purchased, their operating schedules, and the location of fast chargers are all jointly optimized to

achieve minimum overall cost. By providing detailed outputs such as fleet size, charger placement, purchase costs, annual operational costs, and associated CO₂ emissions, the platform enables users to compare scenarios and make informed trade-offs between cost, service quality, and environmental impact. For instance, the tool can show how selecting a certain type of electric bus can lower lifetime operating costs despite a higher initial purchase price. This integrated, data-driven decision support system helps city planners and transit authorities minimize total costs while designing more efficient and sustainable electric bus operations.

5.2 Conclusion

This thesis has addressed the complex optimization challenges arising in operational planning for bus transit systems transitioning to electric buses (EBs). Given the key differences between conventional buses and electric buses, including shorter travel range, longer charging times, and higher purchase costs, we designed advanced optimization models and solution approaches that address the specific requirements of electrified transit fleets. This thesis also aimed to support the transition to electric public transportation by developing cost-optimized planning frameworks for electric bus fleets, integrating fleet scheduling, infrastructure planning, and operational cost management to enhance the feasibility of large-scale electrification. The research aimed not only to bridge the gap between existing theoretical models and real-world operational needs but also to provide actionable insights for transit agencies navigating the shift toward sustainable public transportation.

The review paper presented in Chapter 2 confirmed the need for new mathematical models that accurately reflect the unique operational characteristics of EBs, while maintaining service quality for passengers and cost efficiency for operators. The analysis of existing studies revealed clear research gaps and promising directions, particularly in the integration of timetabling, vehicle scheduling, recharging schedules, and fast-charging infrastructure location planning. Although these elements have been studied individually, their interdependencies remain largely unexplored. Notably, the two-way relationship between EB scheduling and charging infrastructure placement has been

largely overlooked, as most existing studies treat charging station location as a separate optimization problem. The findings highlight that addressing this gap requires integrated models and solution approaches capable of jointly optimizing scheduling and infrastructure planning. Such approaches would enable transit agencies to make better-informed decisions, improve operational performance, and facilitate a smoother transition to fully electric transit systems, ultimately contributing to both environmental sustainability and urban mobility efficiency.

Chapter 3 explored the operational challenges of EB scheduling and timetabling, driven by their shorter driving ranges and longer charging times compared to conventional buses. To address these issues, an integrated approach was developed that combines timetabling and scheduling for electric shuttle buses equipped with pantograph chargers. The model, formulated as a mixed-integer linear program (MILP) and solved using CPLEX, optimizes headways to balance passenger waiting time, seat availability, and total operational costs, which include travel, electricity, and EB usage expenses. A normalized weighted sum technique was applied to evaluate trade-offs and select the optimal schedule.

The methodology was validated through a case study of Concordia University's shuttle bus service, currently transitioning to a fully electric fleet. Results show significant improvements: a 22% reduction in passenger waiting times, a 42% increase in seat availability, and a 21% decrease in total costs compared to the existing schedule. The model also enables strategic EB type selection and accommodates peak and off-peak travel time variations, further enhancing operational efficiency. These findings demonstrate that integrating timetabling and scheduling for EBs can substantially improve service quality and cost-effectiveness. The approach provides a practical framework that supports a smooth and sustainable transition to fully electric shuttle operations and can be adapted for broader use in institutional and municipal transit systems.

Chapter 4 examined the operational challenges of EB scheduling and the placement of fast-charging infrastructure. An integrated path-based Integer Linear Programming (ILP) model was developed and solved using a branch-and-price algorithm. Four branching strategies were tested through computational experiments, identifying the most efficient approaches in terms of runtime and optimality gap. The model was validated through a real case study of Montreal's bus network, using operational data and specifications from existing EB types. Results showed its ability to guide

transit agencies in selecting the most cost-effective EB models and optimizing charging infrastructure placement. Sensitivity analysis highlighted how variations in EB characteristics impact total costs and infrastructure needs, offering practical insights for long-term fleet electrification planning. Overall, the chapter's findings demonstrate that integrating scheduling with charging station location planning can significantly improve operational efficiency and reduce costs, supporting a smoother and more cost-effective transition to fully electric public transit systems.

This thesis contributes to the fight against climate change and promotes sustainability by facilitating the transition to energy-efficient EBs in Montreal, thereby reducing greenhouse gas emissions and improving air quality. Transit agencies will experience enhanced efficiency and service through an optimized operational model. Additionally, local communities will benefit from cleaner air and a move towards sustainable urban living. In summary, this research not only supports the provincial green initiatives but also paves the way for future sustainable endeavors, ensuring a cleaner, greener, and more eco-friendly Montreal for generations to come.

5.3 Future Direction

Significant gaps remain in EB scheduling research, particularly in rescheduling strategies and the integration of transit operation planning steps. Future work should explore joint optimization of network design, timetabling, vehicle scheduling, and charging scheduling. While computationally complex, such integration could improve service quality and operational efficiency. Charging scheduling for heterogeneous fleets with mixed charging technologies should be studied, incorporating real-time data such as battery state of charge, traffic conditions, and passenger demand. Stochastic and robust optimization approaches are promising for addressing operational uncertainty in passenger load, energy use, and travel times. Travel time can be influenced by passenger volume, driving behavior, traffic congestion, road conditions, weather, and other operational factors. Similarly, energy consumption varies with passenger load, battery life, and road conditions. Incorporating stochastic elements for these variables would produce a more robust and realistic model that better reflects real-world operating environments. The interaction between EB operations and the power grid also requires attention, addressing both negative impacts, such as peak-load stress

and reduced transformer lifespan, and positive opportunities like vehicle-to-grid (V2G) services. Charging station location models should move beyond depot or terminal assumptions, integrating location planning with scheduling, infrastructure design, and network expansion. Additional research should improve energy consumption models, investigate nonlinear charging behaviors, and assess their operational drawbacks and benefits. Finally, few studies have tackled these integration challenges using exact optimization methods, highlighting an opportunity to develop scalable, exact approaches that jointly optimize EB scheduling, charging infrastructure placement, and grid impact.

Another important improvement is adapting the model to accommodate different charging technologies, which differ in establishment costs, charging power, and charging times. Future research could also integrate network design as a decision variable, optimizing shuttle bus routing alongside scheduling, and combining these with decisions on charging technology and strategic station placement. Given the potential impact of pantograph chargers on electricity loads, further investigation should assess their effects on neighborhood energy supplies and the power grid during peak hours, including potential power losses and impacts on energy sources. Another promising direction is incorporating uncertainty in EB travel time and energy consumption.

The model could also be expanded to accommodate heterogeneous fleets, as many transit systems operate with multiple vehicle types rather than a homogeneous EB fleet. Furthermore, integrating different charging technologies each with unique establishment costs, charging power, and charging times would improve the model's applicability to diverse operational contexts. Another important extension would be the inclusion of timetable development as a decision variable, alongside network design and bus scheduling. Optimizing departure times and trip frequencies rather than treating them as fixed parameters could significantly enhance service quality and operational efficiency. Additionally, future research could integrate charging technology selection with the strategic placement of charging stations, while assessing the impact of high-load infrastructure such as pantograph chargers on local energy supplies and the power grid during peak hours. Investigating potential power losses from connecting such infrastructure to the grid, as well as their effects on energy sources, could support more sustainable deployment strategies. Finally, the prediction of EB discharge rates using data-driven models represents another valuable research direction. Such models could improve understanding of maximum travel ranges under various conditions, including

passenger load, weather, battery age, charging technology, driving patterns, and traffic conditions, thereby enhancing both scheduling and charging strategies for electric transit systems.

Appendix A

A.1 TOU pricing

Let L be the set of time intervals during which the electricity price varies. The start and end times of each interval, where the electricity cost changes, are denoted by S_l and E_l , respectively. The electricity cost rate for time interval l is represented by π_l . The calculation of charging cost based on the time of charging, t , and TOU pricing is described below:

$$C_t = \begin{cases} \pi_l \times u & \text{if } S_l \leq t + u \leq E_l - u, \quad \forall l \in L \\ \pi_l \times (E_l - t) + \pi_{l+1} \times (u - (E_l - t)) & \text{o.w., } \quad \forall l \in L \end{cases}$$

A.2 Arc-Based Model (ABM)

This section explains an arc-based mathematical formulation for the fast-charging location planning and electric bus scheduling (FCLP-EBS) problem. We model the charging station capacity, which is denoted by W as time-expanded station nodes, by discretizing the range of service start times into a set of discretized nodes. The time period set T consists of $\{1, 2, \dots, |T|\}$, for the planning horizon for charging electric buses at charging stations and T' be the set of time periods, $\{1, 2, \dots, |T| - U'\}$, from the first period to the maximum time period reduced by the number of periods of a charging event, U' , of electric buses. The set of time-expanded nodes of a potential spot $b \in B$ for locating a charging station is denoted by T_b .

The set of arcs A is divided into five categories. There are depot-to-trip arcs between O and

$j \in S$. There are trip-to-trip arcs between $j \in S$ and $i \in S$ if $e_j + \tau_{ji} \leq a_i$. There are trip-to-charger arcs from $j \in S$ to time-expanded node t for potential spot $b \in B$ if $e_j + \tau_{jb} \leq \max T - U$, and charger-to-trip arcs from time-expanded node t for potential spot $b \in B$ to $j \in S$ if $t + \tau_{bj} \leq a_j$. Finally, there are trip-to-depot arcs between $j \in S$ and D . A' is the set of all possible arcs between trips and charging stations and vice versa. The cost of arc ji is defined as: (1) $c_{ji} = C_d d_{ji} + C_v$ if $j = O$; (2) $c_{ji} = C_d d_{ji} + C_w w_{ji} + C_t$ if $i \in B$; and (3) $c_{ji} = C_d d_{ji} + C_w w_{ji}$ for other arcs.

The decision variables for the problem are as follows. Let z_b be a binary decision variable taking value 1 if a fast-charger is located at location $b \in B$, and 0 otherwise. Let x_{ji}^o be a binary decision variable, taking value 1 if an electric bus is assigned to node i immediately after trip j starting at depot $o \in O$, and 0 otherwise. Similarly, x_{jt}^o is a binary decision variable taking value 1 if a vehicle starting at depot $o \in O$ assigned to charger b at time expanded node $t \in T_b$ directly after node j , and 0 otherwise. Let g_i^o be a continuous decision variable, representing the accumulated distance travelled from depot $o \in O$ to the ending point of trip $i \in S$ since the latest battery renewal. Lastly, g_t^o is a continuous decision variable, representing the accumulated distance travelled from depot $o \in O$ or the latest charging event to the time-expanded node $t \in T_b$.

The arc-based model can be formulated as a mixed-integer programming model as follows:

$$\min \left(\sum_{o \in O} \sum_{(j,i) \in A \setminus A'} C_{ji}^o x_{ji}^o + \sum_{o \in O} \sum_{(j,t) \in A'} C_{jt}^o x_{jt}^o + \sum_{b \in B} V_b z_b \right) \quad (27)$$

s.t.

$$\sum_{o \in O} \sum_{\substack{j \\ (j,i) \in A}} x_{ji}^o = 1 \quad \forall i \in S \quad (28)$$

$$\sum_{\substack{i \\ (j,i) \in A}} x_{ji}^o - \sum_{\substack{j \\ (i,j) \in A}} x_{ij}^o = 0 \quad \forall j \in S, \forall o \in O \quad (29)$$

$$g_i^o = \sum_{\substack{j \\ (j,i) \in A \setminus A'}} (g_j^o + d_{ji}) x_{ji}^o \quad \forall i \in S, \forall o \in O \quad (30)$$

$$g_t^o \leq (1 - x_{jt}^o) d_{max} \quad \forall (j, t) \in A', \forall t \in T_b, \forall b \in B, \forall o \in O \quad (31)$$

$$g_i^o \leq d_{max} \quad \forall i \in S \cup T, \forall o \in O \quad (32)$$

$$\sum_{o \in O} \sum_{(i, t \in T_b) \in A'} x_{it}^o + \sum_{o \in O} \sum_{(i, t \in T_b) \in A'} \sum_{s=1}^U x_{i, t+s}^o \leq W \quad \forall b \in B, \forall t \in T' \quad (33)$$

$$\sum_{(i, t \in T_b) \in A'} x_{it}^o \leq z_b \quad \forall b \in B, \forall o \in O \quad (34)$$

$$x_{ji}^o \in \{0, 1\} \quad \forall (j, i) \in A, \forall o \in O \quad (35)$$

$$z_b \in \{0, 1\} \quad \forall b \in B \quad (36)$$

$$g_j^o \geq 0 \quad \forall j \in S, \forall o \in O \quad (37)$$

The objective function (27) includes three terms. The first term calculates the total annual travelling and waiting costs between trips plus the annual purchase cost of each electric bus, the second term determines the annual charging cost and travelling cost between trips and charging stations, and the last term indicates the total annual establishment cost of fast-chargers. Constraints (28) and (29) represent the covering and flow conservation of trips. Constraint (30) determines the accumulative distance travelled since the latest battery renewal. This particular constraint introduces nonlinearity to the mathematical model. A linear version of our model can be found in Section [A.3](#). Constraint (31) ensures that when an EB recharges at a station, its total travelled distance resets to zero. Constraint (32) forces that the total travelled distance of each EB cannot exceed its maximum travel range. Constraint (33) ensures that the station capacity constraint must be satisfied. Constraints (34) ensure that EBs can be charged only at the locations where a charger has been

installed. Constraints (35) to (37) determine the domain of variables.

A.3 Linearization

The arc-based formulation in Section A.2 is nonlinear due to the multiplication of the binary variable, x_{ji}^o to the continuous variable g_j^o . We convert such formulations to linear ones by introducing a new variable denoted by h_{ji}^o , which is equal to $x_{ji}^o g_j^o$. Thus, the linear formulation of the first model will be as follows.

$$\min \left(\sum_{o \in O} \sum_{(j,i) \in A \setminus A'} C_{ji}^o x_{ji}^o + \sum_{o \in O} \sum_{(j,t) \in A'} C_{jt}^o x_{jt}^o + \sum_{b \in B} V_b z_b \right) \quad (38)$$

s.t.

$$\sum_{o \in O} \sum_{\substack{j \\ (j,i) \in A}} x_{ji}^o = 1 \quad \forall i \in S \quad (39)$$

$$\sum_{\substack{i \\ (j,i) \in A}} x_{ji}^o - \sum_{\substack{i \\ (i,j) \in A}} x_{ij}^o = 0 \quad \forall j \in S, \forall o \in O \quad (40)$$

$$\sum_{\substack{j \\ (j,t \in T_b) \in A'}} x_{jt}^o = \sum_{\substack{j \\ (j,t \in T_b) \in A'}} x_{tj}^o \quad \forall b \in B, \forall o \in O \quad (41)$$

$$g_i^o = \sum_{j: (j,i) \in A} (h_{ji}^o + d_{ji} x_{ji}^o) \quad \forall i \in S, \forall o \in O \quad (42)$$

$$h_{ji}^o \leq d_{max} x_{ji}^o \quad (j,i) \in A, \forall o \in O \quad (43)$$

$$h_{ji}^o \leq g_j^o \quad \forall j \in S, \forall o \in O \quad (44)$$

$$h_{ji}^o \geq g_j^o - (1 - x_{ji}^o) d_{max} \quad (j,i) \in A, \forall o \in O \quad (45)$$

$$g_t^o \leq (1 - x_{jt}^o) d_{max} \quad (j, t) \in A', \forall o \in O \quad (46)$$

$$g_i^o \leq d_{max} \quad \forall i \in S, \forall o \in O \quad (47)$$

$$\sum_{o \in O} \sum_{(i, t \in \overset{i}{T_b}) \in A'} x_{it}^o + \sum_{o \in O} \sum_{(i, t \in \overset{i}{T_b}) \in A'} \sum_{s=1}^U x_{i, t+s}^o \leq W \quad \forall b \in B, \forall t \in T', \forall o \in O \quad (48)$$

$$\sum_{(i, t \in \overset{i}{T_b}) \in A'} x_{it}^o \leq z_b \quad \forall b \in B, \forall o \in O \quad (49)$$

$$x_{ji}^o \in \{0, 1\} \quad \forall (j, i) \in A, \forall o \in O \quad (50)$$

$$z_b \in \{0, 1\} \quad \forall b \in B \quad (51)$$

$$g_j^o \geq 0 \quad \forall j \in S, \forall o \in O \quad (52)$$

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