

Effect of Weather Uncertainty on Model Predictive Control for Electrically Heated Canadian
Houses with PV–Battery Systems

Ahmed Khedr

A Thesis

In the Department of
Building, Civil, and Environmental Engineering

Presented in Partial Fulfillment of the Requirements

For the Degree of

Master of Applied Science in building engineering

at

Concordia University

Montréal, Québec, Canada

April 2026

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By: Ahmed Khedr

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Signed by the final examining committee:

Dr. Bruno Lee	Chair
Dr. Liangzhu (Leon) Wang	Examiner
Dr. Bruno Lee	Examiner
Dr. Andreas K Athienitis	Supervisor

Approved by _____

Dr. Chunjiang An, Graduate Program Director

04/28/2026

Date of Defense

Dr. Mourad Debbabi, Dean of Gina Cody School of Engineering and Computer Science

Abstract

Effect of Weather Uncertainty on Model Predictive Control for Electrically Heated Canadian Houses with PV–Battery Systems

Ahmed Khedr

Concordia University, 2026

In cold-climate regions, residential heating represents a major component of electricity demand, and electric heating systems combined with PV–battery integration offer significant potential for enhancing provision of load energy flexibility in response to grid needs; however, their operation remains strongly influenced by uncertainty in weather forecasts. This thesis develops and evaluates a predictive control framework to enhance heating energy flexibility in electrically heated residential buildings under weather forecast uncertainty. A control-oriented grey-box thermal model based on a three resistance–two capacitance (3R–2C) thermal network represents the thermal dynamics of a typical single-family house in Québec equipped with rooftop photovoltaic (PV) generation and battery storage. The model is integrated within a Model Predictive Control (MPC) framework that incorporates day-ahead weather prediction through scenario-based representations derived from historical variability. The controller coordinates heating operation, PV generation, battery charging and discharging, and grid electricity exchange under time-varying electricity pricing.

The proposed MPC demonstrates robust performance under weather uncertainty by maintaining total heating energy comparable to reactive control while keeping indoor air temperature within the predefined range close to the heating setpoint and reducing peak grid electricity import and operating costs. Under representative winter scenarios, peak-period grid electricity import and daily electricity operating costs are reduced by approximately 69–76% and 30–48%, respectively, compared with reactive control with PV–battery integration. Weather forecast uncertainty in one-day-ahead predictions leads to variations in peak grid import and daily electricity cost of approximately 46% and 38%, respectively. These findings demonstrate the potential of predictive control with PV–battery integration to enhance residential energy flexibility and reduce peak electricity demand in cold-climate regions while accounting for weather uncertainty.

Acknowledgements

I would like to express my sincere gratitude to my supervisor, Dr. Andreas K. Athienitis, for his continuous guidance, support, and encouragement throughout my graduate studies. I am deeply thankful and proud for the opportunity to conduct my research under his supervision and for the financial support that made this work possible. His knowledge, insight, and commitment to excellence have been invaluable throughout this journey.

I would also like to extend my sincere appreciation to Dr. Matin Abtahi, postdoctoral fellow, for his guidance, patience, and the time and effort he generously dedicated to supporting my research. His advice and technical insights greatly contributed to the development of my work.

My heartfelt thanks go to my wife, Kseniia Gavrilova, for her unconditional support, patience, and care during this demanding period. I am also deeply grateful to my beautiful daughter, Amalia Khedr, whose presence and joy gave me strength even during the most challenging moments.

I would like to acknowledge my colleagues and friends at the laboratory, including Dr. Navid Morovat, Postdoctoral Fellow and Dr. Jiwu Rao, Research Associate, for their support and for contributing to a stimulating and collaborative research environment. I am also thankful to all my lab mates with whom I had the pleasure of working and sharing this academic journey.

Finally, I would like to express my deepest gratitude to my parents, whose endless support, encouragement, and belief in me have always been the foundation of my achievements.

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Nomenclature

Symbol	Definition	Unit
T_k^{air}	indoor air temperature at time step k	°C
T_k^{env}	envelope temperature at time step k	°C
T_k^{out}	outdoor air temperature at time step k	°C
T_k^{sp}	temperature setpoint at time step k	°C
T_{min}, T_{max}	allowable indoor temperature bounds	°C
T_{Up}, T_{low}	upper and lower outdoor temperature scenarios	°C
C_{air}	thermal capacitance of indoor air	J/°C
C_{env}	thermal capacitance of building envelope	J/°C
U_1	heat transfer coefficient between indoor air and outdoor	W/°C
U_2	heat transfer coefficient between envelope and outdoor	W/°C
U_3	heat transfer coefficient between indoor air and envelope	W/°C
α	solar gain coefficient for indoor air node	m ²
β	solar gain coefficient for envelope node	m ²
P_k^{heat}	heating power at time step k	W
P_k^{batt}	battery power at time step k	W
P_k^{pv}	photovoltaic power generation at time step k (W)	W
P_k^{net}	net electrical power (W)	W
P_k^{imp}	grid import power (W)	W
SOC_k	battery state of charge	—
SOC_{min}, SOC_{max}	minimum and maximum battery state of charge	—
E_{batt}	battery energy capacity	kWh
η_{ch}, η_{dis}	battery charge and discharge efficiencies	-
GHI_k	global horizontal irradiance	W/m ²
GHI_k^{up}, GHI_k^{low}	upper and lower solar irradiance scenarios	W/m ²
CV_d	coefficient of variation at time step k	—
CV_{eff}	effective coefficient of variation	—
α_{type}	day-type scaling factor	—

Q_{sol}	solar heat gain	W
Δt	simulation time step	s
k	discrete time index	—
N	prediction horizon length	—
x_k	state vector	—
u_k	control vector	—
d_k	Inputs / disturbance vector	—
J	objective function	—
$J_{1,k}$	electricity import cost	—
$J_{2,k}$	net power regularization	—
$J_{3,k}$	temperature tracking	—
$J_{4,k}$	input smoothness	—
$J_{5,k}$	allowable temperature range	—
λ_k^{elc}	electricity pricing factor at time step k	—
ω_{trk}	temperature tracking penalty factor	—
ω_{range}	temperature range penalty factor	—
e_k	temperature tracking error	—
I_k	integral of temperature error	—

Abbreviations

Abbreviation	Definition
MPC	Model Predictive Control
PV	Photovoltaic
GHI	Global Horizontal Irradiance
SOC	State of Charge
RC	Resistance–Capacitance
DER	Distributed Energy Resources
PI	Proportional–Integral (controller)
HVAC	Heating, Ventilation, and Air Conditioning
RMSE	Root Mean Square Error
CVRMSE	Coefficient of Variation of RMSE

NMBE	Normalized Mean Bias Error
MBE	Mean Bias Error
COP	Coefficient of Performance
ASHRAE	American Society of Heating, Refrigerating and Air-Conditioning Engineers
IPMVP	International Performance Measurement and Verification Protocol
DR	Demand Response
BAU	Business As Usual

Chapter 1: Introduction

1.1 Background and Motivation

The electrification of space heating in residential buildings, especially single-family houses in cold-climate regions, has led to a significant increase in electricity consumption and peak electricity demand. In provinces such as Quebec, where electrically heated single-family houses represent a large share of the residential building stock, electric heating systems typically include electric resistance baseboard heaters, electric furnaces, and air-source heat pumps or hybrid combinations. In this context, approximately 70–75% of residential buildings rely on electric resistance heating systems (mainly baseboards), while heat pumps account for roughly 25–30% of installations (Hydro-Québec, 2022; Natural Resources Canada, 2022).

Winter peak demand periods place substantial stress on local power grids. These challenges are further exacerbated by extreme weather events, which increase heating loads, raise the likelihood of power outages, and threaten the reliability and resilience of the electricity system.

Residential space heating in Quebec relies heavily on electricity, making it a major contributor to both overall electricity consumption and winter peak demand. Electricity remains the dominant energy source for residential space heating in the province, supplying approximately 77% of total heating demand (Natural Resources Canada, 2022), reflecting the widespread adoption of electric heating systems as illustrated in Figure 1.1.

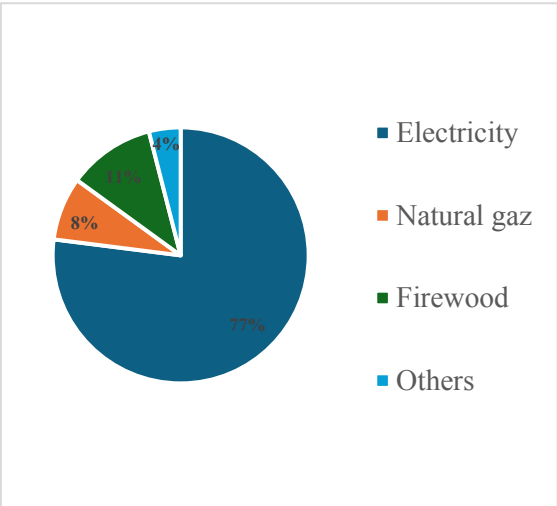


Figure 1.1: Residential Heating Energy by Source, 2022

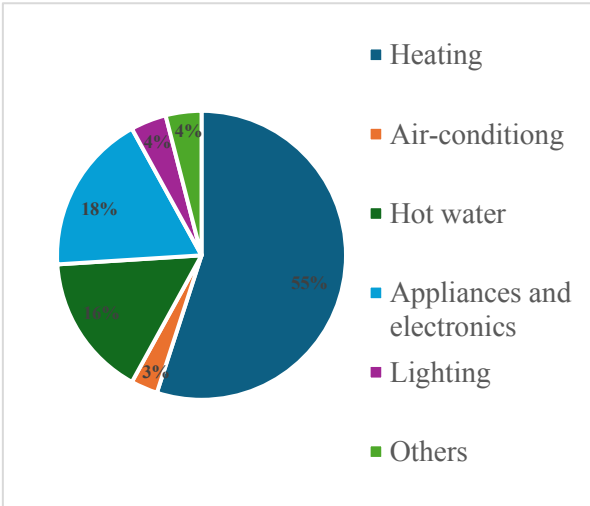


Figure 1.2: Breakdown of Average Electricity Use for a Detached House in Quebec, 2026

At the household scale, space heating is also the dominant driver of electricity consumption. Figure 1.2 shows that heating accounts for roughly 55% of total electricity use in a typical detached house, significantly exceeding the shares associated with hot water, appliances, and lighting (Hydro-Québec, 2026). Together, these observations explain the central role of residential heating in winter peak electricity demand and highlight the need for improved heating control strategies that can better manage demand and support energy flexibility.

Distributed energy resources (DERs), including rooftop photovoltaic (PV) systems and residential battery storage, have the potential to mitigate peak demand and reduce households' reliance on the grid. However, the effectiveness of these technologies is highly dependent on weather conditions. Uncertainty in outdoor temperature and solar irradiance directly affects both heating energy demand and on-site electricity generation, making it difficult to optimally coordinate heating systems, PV production, and battery operation using conventional control strategies.

Most existing residential heating control approaches rely on reactive or rule-based methods that do not explicitly anticipate future conditions. As a result, they are limited in their ability to reduce peak electricity demand, enhance energy flexibility, or strategically shift heating loads away from high demand periods. Furthermore, many studies neglect the impact of weather uncertainty, leading to control strategies that may perform well under ideal conditions but are less robust in real world operation.

Consequently, there is a clear need for an advanced control framework capable of managing residential heating systems equipped with PV and battery storage while accounting for weather uncertainty. Such a framework should reduce peak electricity demand to support grid stability while also lowering electricity costs for end users under local tariff structures such as Hydro-Québec's Rate Flex D. It also increases the utilization of on-site renewable energy, and decrease reliance on the grid. Addressing this gap is essential for improving residential energy flexibility and supporting the operational needs of power grids in cold-climate regions.

From a grid level perspective, the increasing electrification of residential space heating creates significant challenges for electricity grids, particularly during the winter peak season, where highly synchronized heating demand leads to pronounced load peaks, increased operational stress, and

heightened risks to grid reliability and resilience. Grid operators therefore seek strategies that can smooth electricity demand over the day, reduce electricity consumption during peak periods, and improve system robustness under extreme weather conditions.

On the other hand, from the household perspective, end users are concerned with the reliability and continuity of electricity supply, whether provided by the grid, on-site distributed energy resources, or a combination of both, especially during severe winter events, while also aiming to reduce electricity costs under time and demand-dependent pricing structures. In addition, households require stable indoor temperature regulation, where deviations from the heating setpoint are minimized despite external disturbances and uncertainty in operating conditions.

Together, these perspectives emphasize the need for advanced residential energy control frameworks that balance grid level reliability and peak demand reduction with household level operational performance under uncertain conditions.

1.2 Scope and Objectives

This thesis focuses on the development and evaluation of heating energy flexibility in electrically heated single-family residential buildings located in cold-climate regions. The study considers a typical detached house located in Shawinigan, Quebec equipped with rooftop photovoltaic (PV) generation and battery energy storage, representing an increasingly relevant configuration in residential distributed energy systems. Nevertheless, electric baseboard systems are increasingly being replaced or supplemented by heat pumps that are efficient down to -20°C . For example, systems such as the Mitsubishi Electric Zuba-Central heat pump can operate efficiently at temperatures as low as -30°C , with a coefficient of performance (COP) typically around 1.8–2.2 at -15°C under standard operating conditions.

The scope of the research includes the development of a simplified thermal model using a 3-resistance–2-capacitance (3R–2C) thermal network, capturing the dominant thermal dynamics of the building. This thermal model is integrated within a Model Predictive Control (MPC) framework to optimally coordinate space heating operation, battery charging and discharging, and grid electricity exchange.

For comparison in addition to the MPC approach, two reactive control strategies are implemented as reference cases:

- (a) A reactive control strategy for an electrically heated house without PV and battery systems.
- (b) A reactive control strategy for a house equipped with PV generation and battery storage.

These control strategies serve as baseline scenarios to assess the benefits of predictive control and distributed energy resources.

The effect of weather uncertainty is evaluated by comparing system performance across the range of upper, nominal, and lower weather scenarios for representative days, allowing the impact on key metrics such as peak grid import and electricity cost to be quantified.

A key focus of this work is the explicit consideration of weather uncertainty, particularly outdoor temperature and solar irradiance, and its impact on heating demand and PV electricity generation. Weather uncertainty is represented within the MPC framework using historical variability and scenario-based forecasts to enhance control robustness under uncertain conditions.

The analysis is conducted through simulation studies using representative residential and climatic data. System performance is evaluated by comparing the MPC strategy against the two reactive control cases in terms of heating energy consumption, peak electrical demand, solar self-consumption, and electricity cost. The scope of this thesis is limited to space heating and residential electrical energy management and does not include cooling systems, detailed occupant behavior modeling, or network-level grid constraints.

The main objective of this study is to develop and evaluate a robust Model Predictive Control (MPC) framework for residential heating systems that explicitly accounts for uncertainty in day-ahead weather forecasts. This objective is addressed through the following research goals:

- Develop a robust MPC strategy that maintains stable and effective performance under variations and uncertainty in predicted outdoor temperature and solar irradiance.
- Shift residential heating loads away from utility defined peak demand periods to reduce peak electricity demand.
- Minimize indoor temperature deviation from the heating setpoint during operation.

- Reduce total residential electricity costs through the optimal coordination of heating systems, PV generation, battery storage, and grid electricity exchange.
- Maintain overall heating energy consumption through predictive control strategies.

1.3 Thesis Organization and Outline

This thesis is organized into five main chapters, structured to progressively introduce the research problem, review relevant literature, describe the adopted methodology, present and discuss the results, and conclude with key findings and future research directions.

Chapter 1: Introduction. This chapter introduces the research background and motivation, highlighting the increasing electrification of residential space heating in cold climates and the associated challenges for energy systems. The research problem, objectives, and scope of the thesis are clearly defined, followed by an overview of the thesis structure.

Chapter 2: Literature Review. This chapter reviews existing literature on residential building thermal modeling and energy flexibility. It surveys modeling approaches used in residential buildings for energy analysis and control, and reviews studies on heating energy flexibility, demand response, and reactive and predictive control strategies, including Model Predictive Control (MPC). The chapter also examines the role of weather conditions and forecast uncertainty in residential energy modeling and control, as well as the contribution of photovoltaic (PV) systems and battery energy storage to residential energy flexibility.

Chapter 3: Methodology. This chapter presents the methodological framework used in this research. It begins with a description of the studied system and the data used, followed by the representation of weather uncertainty. The modeling of the residential building and associated energy systems is then introduced. The chapter subsequently describes the formulation and implementation of the Model Predictive Control (MPC) framework, including the coordination of heating control, PV–battery operation, and weather uncertainty. Finally, the baseline reactive control strategies used for comparison are presented.

Chapter 4: Results and Discussion. This chapter presents and analyzes the simulation results obtained from the implemented control strategies. The performance of the proposed MPC

approach is evaluated and compared against the reference reactive control in terms of heating energy consumption, peak load and cost reduction, and energy flexibility. In addition to the impact of PV-battery integration and weather uncertainty on system performance.

Chapter 5: Conclusions and Future Work. This chapter summarizes the main findings and contributions of the thesis. It discusses the implications of the results for residential heating control and energy flexibility in cold climate regions. Finally, limitations of the current study are identified, and recommendations for future research are proposed.

Chapter 2: Literature Review

In recent years, the role of residential buildings has expanded beyond passive energy consumption to include active participation in power system operation. Electrically heated buildings, particularly in cold-climate regions, offer significant potential to provide energy flexibility through the temporal shifting of heating demand and the coordinated use of on-site resources such as photovoltaic (PV) systems and battery storage. However, the ability of residential buildings to contribute effectively to grid objectives depends strongly on the adopted control strategies, the representation of building thermal dynamics, and the consideration of external drivers such as weather conditions and electricity pricing.

This chapter is structured to establish a coherent foundation for predictive control of residential energy systems under realistic operating conditions. The reviewed literature is organized to progressively connect building energy flexibility, energy modeling, control strategies, and external drivers influencing residential heating performance.

Section 2.1 introduces the concept of building energy flexibility and its relevance for electrically heated residential buildings in cold climates, highlighting the role of thermal inertia and load shifting in reducing grid dependency and operating costs.

Section 2.2 reviews building energy modeling approaches, with emphasis on control-oriented formulations that balance physical interpretability and computational efficiency. Section 2.3 then focuses on building thermal dynamics representation through thermal network models and calibration techniques that enable accurate and tractable predictive models for control applications.

Section 2.4 examines residential heating control strategies, progressing from reactive and heuristic rule-based approaches toward optimization-based Model Predictive Control (MPC). The distinction between simulation-based and applied MPC implementations is discussed in Section 2.5 to highlight the gap between theoretical performance and real-world feasibility.

Section 2.6 focuses on time-varying electricity pricing structures and their role in shaping residential energy management strategies. The reviewed literature highlights how dynamic tariffs

create economic incentives for load shifting and motivate the use of advanced control strategies such as MPC to reduce energy costs and peak electricity demand.

Finally, Section 2.7 addresses the impact of weather uncertainty on building energy performance and predictive control. This section emphasizes how inaccuracies in temperature and solar irradiance forecasts can degrade MPC performance and highlights the need for control frameworks that remain robust under uncertain operating conditions.

2.1 Building Energy Flexibility

Building energy flexibility has been increasingly framed as an operational capability that allows buildings to modify their energy use ahead of peak demand periods through anticipatory actions rather than purely reactive responses (Samson et al., 2024). In heating-dominated regions, the electrification of residential space heating amplifies the relevance of such flexibility, as electric heating loads constitute a major driver of winter peak electricity demand (Arteconi et al., 2013). In this context, energy flexibility enables electricity demand to be adjusted over time in response to external conditions such as weather variability, electricity prices, and grid constraints (International Energy Agency [IEA], 2021).

Within the framework of the IEA EBC Annex 67, an energy-flexible building is defined as one that can manage its energy demand and on-site generation in response to local climate conditions, occupant needs, and grid requirements without compromising indoor comfort or technical performance (Jensen et al., 2017). In residential buildings, the potential for energy flexibility is strongly influenced by the thermal behavior of the building envelope and interior elements, which can act as short-term thermal energy storage (Reynders, et al., 2014). By exploiting this inherent thermal inertia, heating demand can be shifted in time through controlled variations in indoor temperature setpoints while maintaining acceptable comfort levels (Le Dréau & Heiselberg, 2016). This approach has been shown to be particularly effective in cold climates, where pre-heating strategies can reduce peak electrical demand without causing immediate discomfort to occupants (Arteconi et al., 2013).

From a grid operation perspective, residential energy flexibility enables services such as peak load shifting and reserve provision through the temporary reduction or rescheduling of electricity consumption, thereby supporting grid stability during periods of high demand (Hydro-Québec, 2019). From a household perspective, energy flexibility allows occupants to actively manage their electricity use in response to time-varying prices and local generation availability, leading to reduced energy costs, increased photovoltaic self-consumption, and improved utilization of on-

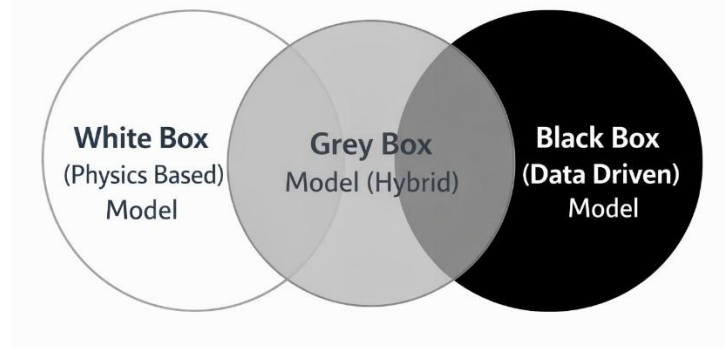


Figure 2.1: Building Energy Models' Classification

site energy resources. In addition, higher levels of energy flexibility can reduce household reliance on the electricity grid by enabling continued operation of critical loads during grid outages or supply interruptions, particularly in buildings equipped with photovoltaic and battery storage systems. Consequently, energy flexibility can be interpreted as being inversely related to grid dependency, since higher flexibility enables a greater share of household energy demand to be met through locally generated and stored electricity rather than instantaneous grid supply (International Energy Agency, 2021; Reynders et al., 2017). In residential buildings equipped with photovoltaic systems, flexibility performance is commonly assessed using self-generation and self-consumption indicators that quantify the extent to which locally generated renewable electricity is utilized within the dwelling (Wang, 2014; Salom et al., 2014).

2.2 Building Energy Modeling Approaches

Developing appropriate models and control frameworks is fundamental to research in building and energy engineering. Models serve as essential tools for understanding complex physical and operational behavior in buildings and enable engineers to analyze, predict, and optimize system performance under varying conditions (Date, 2015). In the residential sector, modeling and control

approaches are widely used to support design decision-making, improve system operation, reduce energy consumption and peak electricity demand, enhance the integration of distributed energy resources, and maintain acceptable levels of occupant thermal comfort (Valinejadshoubi et al., 2024).

Building thermal models are commonly classified into white-box (physics-based), black-box (data-driven), and grey-box (hybrid) approaches as shown in Figure 2.1. These paradigms differ in their reliance on physical knowledge versus measured data, their computational complexity, and their suitability for control-oriented applications.

2.2.1 White Box (Physics Based) Models

White-box models describe building thermal behavior using first principles formulations derived from conservation of energy and heat transfer theory. These models explicitly represent conduction through building envelopes, convective heat exchange between surfaces and indoor air, short-wave and long-wave radiation, as well as ventilation and infiltration processes. Early and foundational work by Athienitis and collaborators established the use of physically derived thermal networks and transient heat transfer formulations for modeling the dynamic thermal behavior of buildings, particularly in the context of passive solar and residential applications (Athienitis et al., 1985; Athienitis et al., 1990). These physics-based approaches have since been widely adopted and formalized in building performance simulation literature (Clarke, 2015; Hensen & Lamberts, 2011).

The thermal behavior of building components is governed by transient heat conduction, which is commonly described by diffusion-type partial differential equations. Accurate numerical solutions require detailed knowledge of building geometry, material properties, and boundary conditions, and rely on spatial and temporal discretization techniques such as finite difference or finite volume methods. Physically based discretized thermal network models derived from these governing equations were introduced early on by Athienitis and co-workers and later extended by other researchers for enhanced numerical efficiency and control-oriented formulations (Athienitis et al., 1985; Date et al., 2015; Wetter et al., 2019).

Physics-based models provide high physical transparency and form the backbone of whole-building simulation tools such as EnergyPlus and TRNSYS (Crawley et al., 2001). However, their high dimensionality and computational burden limit their applicability for real-time control and optimization, particularly in residential buildings where simplified and scalable representations are preferred. Consequently, white-box models are rarely used directly for advanced control applications, but instead serve as a theoretical foundation from which reduced-order and control-oriented models are derived (Athienitis & O'Brien, 2015; Killian & Kozek, 2016).

2.2.2 Black-Box (Data-Driven) Models

Black-box models rely exclusively on measured input–output data to describe building thermal behavior, without explicit representation of physical processes. Techniques such as linear regression, autoregressive models, artificial neural networks, and support vector machines have been widely applied for short-term prediction of indoor temperature and energy consumption (Mustafaraj et al., 2011; Jain et al., 2014).

Although black-box models can achieve high predictive accuracy under conditions similar to the training data, they lack physical interpretability and perform poorly when operating conditions change, such as during equipment upgrades or altered occupancy patterns (Foucquier et al., 2013). Furthermore, their limited ability to enforce physical constraints makes them less suitable for optimization-based control strategies such as MPC (Afram & Janabi-Sharifi, 2014). Consequently, black-box models are typically used as complementary tools rather than standalone models for predictive control.

2.2.3 Grey-Box (Hybrid) Models

Grey-box thermal models combine simplified physics-based representations with data-driven parameter identification, offering a balanced compromise between physical interpretability and computational efficiency. In contrast to purely white-box models, which rely exclusively on detailed physical descriptions, and black-box models, which are entirely data-driven, grey-box models retain a physically meaningful structure while allowing key parameters to be estimated from measured building data (Kristensen et al., 2004; Reynders et al., 2014).

In the context of building thermal modeling, grey-box models are typically formulated using low, medium, or higher order representations derived from fundamental heat transfer principles. These representations capture the dominant thermal dynamics of the building such as heat storage in the building mass and heat transfer between the indoor air and the envelope while neglecting higher frequency spatial details that have limited influence on overall thermal behavior at the building scale (Hazyuk et al., 2012). Unknown or uncertain parameters, including effective thermal resistances, capacitances, and aggregated heat gains, are then identified using measured operational data, allowing the model to reflect the actual behavior of the building under real operating conditions (Reynders et al., 2014; Sun et al., 2019).

A key advantage of grey-box models lies in their physical interpretability. Unlike black-box approaches, grey-box models preserve a direct connection between model parameters and physical building properties, enabling the enforcement of physical constraints and facilitating insight into system behavior. At the same time, the use of parameter identification allows these models to compensate for modeling simplifications, construction uncertainties, and unmodeled phenomena that are inevitable in real buildings (Kristensen et al., 2004; Fouquier et al., 2013).

Grey-box modeling has been shown to be particularly well suited for control-oriented applications, where repeated model evaluations and calibrations are required within an optimization framework. Model Predictive Control (MPC), in particular, places stringent requirements on model structure, computational efficiency, and numerical robustness. Grey-box models satisfy these requirements by providing sufficiently accurate predictions over the control horizon while maintaining low computational complexity, making real-time implementation feasible even in residential buildings (Hazyuk et al., 2012; Killian & Kozek, 2016; Drgoňa et al., 2020).

Candanedo and Athienitis (2011) employed a control-oriented grey-box model to enable predictive control of a radiant floor heating system in a solar house, achieving improved energy performance and load shifting capability. Subsequent studies extended this framework to more complex system configurations and operating conditions, highlighting the robustness of grey-box models when integrated within MPC schemes (Candanedo et al., 2015; Saberi Derakhtenjani & Athienitis, 2021).

More recent work has further reinforced the suitability of grey-box models for residential energy flexibility analysis. Vallianos and Athienitis (2022) demonstrated that cost-aware MPC based on grey-box thermal models can effectively reduce heating costs and peak electricity demand under dynamic pricing schemes. These studies illustrate that grey-box models provide a practical and scalable modeling framework that can be adapted to different residential buildings while maintaining sufficient accuracy for predictive control.

Another important characteristic of grey-box modeling is its compatibility with model order reduction. By selecting an appropriate level of abstraction, grey-box models can be tailored to the objectives of the study. For residential buildings operating under a common indoor temperature setpoint, low-order grey-box models are often sufficient to capture the dominant thermal behavior, enabling generalization across different dwellings and facilitating comparative studies of control strategies (Hazyuk et al., 2012; Reynders et al., 2017). This flexibility makes grey-box modeling particularly attractive for large-scale studies of residential energy flexibility and demand response.

2.3 Building Thermal Dynamics Representation

Building thermal network representations provide a structured and physically intuitive means of translating heat transfer processes in buildings into compact mathematical models suitable for analysis and control design. By representing heat transfer paths and thermal storage elements as interconnected nodes and links, such representations enable a clear mapping between building components, boundary conditions, and dynamic thermal behavior (Athienitis, 1985; Hensen & Lamberts, 2011). Thermal network representations have played a foundational role in the development of reduced-order building models, particularly in enabling control-oriented formulations that balance physical interpretability with computational efficiency (Athienitis & O'Brien, 2015; Oldewurtel et al., 2012).

2.3.1 Thermal Resistance–Capacitance (R-C) Modeling

Thermal resistance–capacitance (R–C) models are widely used for representing building thermal dynamics due to their simplicity, physical interpretability, and suitability for control-oriented applications. These models are based on the well-established analogy between heat transfer and electrical circuits, where thermal resistances represent heat transfer paths and thermal capacitances

represent energy storage in building components (Athienitis et al., 1985; Athienitis & O'Brien, 2015; Goyal & Barooah, 2012).

It is important to emphasize that R–C models are not intrinsically grey-box models, a distinction that is often overlooked in the literature. Rather, the classification of an R–C model as white-box, grey-box, or black-box depends on how the model parameters are derived, not on the R–C structure itself. When thermal resistances and capacitances are computed exclusively from known physical properties such as material thermal conductivity, density, specific heat capacity, layer thickness, and geometric configuration, the resulting R–C model constitutes a white-box, physics-based representation of the building thermal system. This physics-based formulation of R–C thermal networks has been employed extensively in early and contemporary work by Athienitis and collaborators for transient heat transfer analysis and building thermal simulation (Athienitis et al., 1985; Athienitis et al., 1990; Athienitis & O'Brien, 2015), as well as in classical building performance simulation literature (Clarke, 2015; Hensen & Lamberts, 2011).

When the same R–C structure is retained but selected parameters are identified or calibrated using measured operational data, the model becomes a grey-box representation, combining physical structure with data-driven parameter estimation (Kristensen et al., 2004; Reynders et al., 2014). In this sense, R–C modeling should be understood as a modeling framework rather than a modeling paradigm, capable of supporting white-box, grey-box, or black box, depending on the modeling objectives and data availability.

2.3.2 Model Order Selection and Scope of Application

The appropriate order of an R–C model strongly depends on the scope and objectives of the study. For buildings with multiple independently controlled thermal zones or studies focused on detailed spatial temperature distribution and occupant-related effects, higher-order R–C models are often recommended. Increasing the number of resistances and capacitances allows for a more detailed representation of envelope layers, inter-zonal heat transfer, and local thermal dynamics (Reynders et al., 2014; Zakula et al., 2014).

For example, Jennifer Date's work employed a high-order R–C thermal network derived from explicit finite-difference discretization of building envelope components. In her detailed modeling

framework, individual construction layers were discretized into multiple nodes, several R-C models were tested reaching a 25R–13C thermal network for a single building section as shown in Figure 2.2, enabling accurate representation of transient heat diffusion and nonlinear effects (Date et al., 2015; Date, Candanedo & Athienitis, 2021). While such high order models offer high fidelity, they are computationally intensive and less applicable on different buildings.

In contrast, when the objective is to study the aggregate thermal behavior of a residential house operating under a common indoor temperature setpoint, lower-order R–C models are generally more practical and widely recommended. These models capture the dominant thermal dynamics using a limited number of states, enabling easier calibration, improved numerical robustness, and generalization across different dwellings (Hazyuk et al., 2012; Killian & Kozek, 2016).

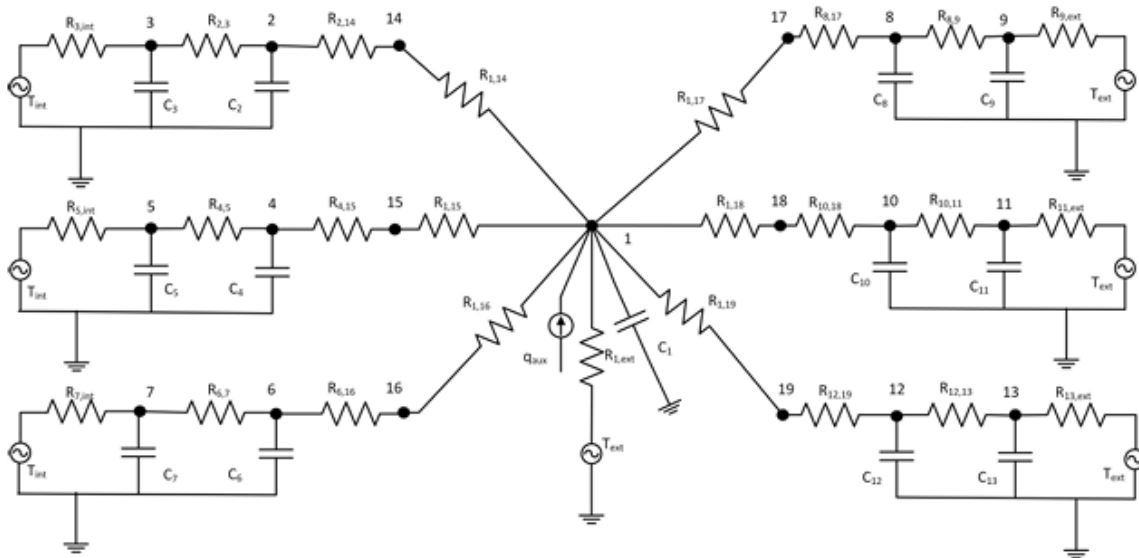


Figure 2.2: 25R – 13C Thermal Network (Date, 2015)

Abtahi (2021) adopted reduced-order 6R – 3C and 7R – 3C thermal network models as shown in Figure 2.3 to represent the thermal dynamics of a Canadian residential house for predictive load management and energy flexibility analysis. These models were utilized to provide an effective balance between accuracy and simplicity, allowing reliable prediction and real-time MPC implementation under dynamic electricity pricing (Abtahi, 2021). The progression from high-order R–C models to reduced order representations highlight the evolution of control-oriented building modeling toward scalable and generalizable solutions.

Accordingly, for residential buildings characterized by a single dominant heating zone and a common temperature setpoint, lower-order R–C models are widely recognized as the most suitable choice for MPC-based studies, particularly when the aim is to assess energy flexibility and demand-side management potential across a population of houses (Morovat et al., 2019; Vallianos & Athienitis, 2022).

2.3.3 Model Calibration and Parameter Identification

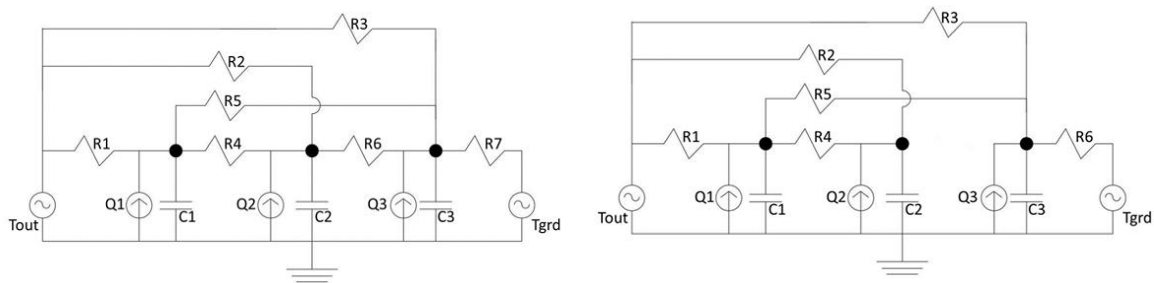


Figure 2.3: 7R – 3C and 6R – 3C Thermal Models (Abtahi, 2021)

Accurate calibration of R–C models is essential to ensure reliable predictive performance. Calibration involves estimating unknown parameters such as thermal resistances, capacitances, and effective heat gains using measured data. Common approaches include least-squares optimization, maximum likelihood estimation, and Bayesian inference (Kristensen et al., 2004; Reynders et al., 2014).

The accuracy of model predictions relative to measured data is commonly evaluated using statistical performance indices that quantify both the magnitude and the systematic nature of prediction errors. These indices provide objective criteria for assessing model accuracy and determining whether a calibrated model is suitable for subsequent analysis or control applications.

One of the most widely used indicators is the root mean squared error (RMSE), which measures the overall magnitude of the prediction error and reflects the dispersion of residuals between measured and predicted values:

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (M_i - P_i)^2} \quad (2.1)$$

where P_i and M_i denote the predicted and measured values, respectively, and N is the total number of observations.

To provide a normalized measure of model accuracy, the coefficient of variation of the RMSE (CVRMSE) is often employed. This dimensionless metric expresses the RMSE relative to the mean of the measured values:

$$\text{CVRMSE} = \frac{\text{RMSE}}{\mu} \times 100 \quad (2.2)$$

where μ represents the mean of the measured data.

Another commonly used indicator is the mean bias error (MBE), which captures the average tendency of the model to overpredict or underpredict measured values. A positive MBE indicates systematic underprediction, while a negative value reflects overprediction. For model calibration assessment, the normalized mean bias error (NMBE) is more commonly reported, as it expresses the bias relative to the mean of the measured values and provides a dimensionless metric consistent with ASHRAE Guideline 14. The NMBE is defined as:

$$\text{MBE} = \frac{1}{N} \sum_{i=1}^N (M_i - P_i) \quad (2.3)$$

$$\text{NMBE}(\%) = \frac{\sum_{i=1}^N (M_i - P_i)}{(N - 1)\bar{M}} \times 100 \quad (2.4)$$

Together, RMSE, CVRMSE, and NMBE provide complementary insight into both random and systematic model errors.

$$R^2 = 1 - \frac{\sum_{i=1}^N (M_i - P_i)^2}{\sum_{i=1}^N (M_i - \bar{M})^2} \quad (2.5)$$

The coefficient of determination (R^2) is also used to indicate how well the model follows the overall trend and variability of the measured data over time and is reported alongside the error-based metrics.

Guidelines for acceptable calibration accuracy have been established in the literature and by industry standards. These criteria are typically expressed using the coefficient of variation of the root mean square error (CVRMSE), which quantifies the normalized spread of prediction errors, and the normalized mean bias error (NMBE), which indicates systematic over- or under-prediction. ASHRAE Guideline 14 recommends that, when comparing whole-building energy consumption, a calibrated model should satisfy $\text{CVRMSE} \leq 15\%$ and $|\text{NMBE}| \leq 5\%$ on a monthly basis, or $\text{CVRMSE} \leq 30\%$ and $|\text{NMBE}| \leq 10\%$ on an hourly basis (Reddy, 2006). Similarly, the International Performance Measurement and Verification Protocol (IPMVP) suggests stricter thresholds of $\text{CVRMSE} \leq 5\%$ for monthly data and $\text{CVRMSE} \leq 20\%$ for hourly data when evaluating whole-building energy use (IPMVP Committee, 2002).

Several previous studies have demonstrated that proper calibration significantly improves MPC performance and robustness under real operating conditions (Candanedo et al., 2015; Vallianos & Athienitis, 2022). However, challenges such as parameter identifiability, measurement noise, and uncertainty in boundary conditions remain active research topics, particularly for residential applications with limited sensing infrastructure.

2.4 Building Energy Control Strategies

2.4.1 Reactive Control Strategy

Reactive control strategies are the most commonly implemented control approaches in residential heating systems. These strategies rely on feedback from measured variables, such as indoor air temperature, and adjust system operation in response to deviations from predefined setpoints.

Typical implementations include on–off thermostatic control and proportional–integral (PI) or proportional–integral–derivative (PID) controllers (Åström & Hägglund, 2006).

In reactive control, control actions are based exclusively on current or past system states, without explicit consideration of future disturbances such as weather variations or electricity price changes. As a result, reactive controllers are inherently unable to anticipate upcoming system conditions, which limits their effectiveness in environments characterized by time varying electricity tariffs and high levels of electrification (Oldewurtel et al., 2012).

Several studies have demonstrated that while reactive control strategies are robust and easy to implement, they often lead to suboptimal performance in terms of energy efficiency and peak demand reduction. In cold-climate residential buildings, reactive control tends to increase heating power demand during peak electricity periods, thereby exacerbating grid stress and increasing operating costs (Athienitis & O’Brien, 2015; Killian & Kozek, 2016). Consequently, reactive control strategies are typically used as baseline cases against which more advanced control approaches are evaluated.

Reactive heating control is approximated using proportional–integral (PI) control, which represents one of the most widely adopted feedback control strategies in residential building applications. In PI control, the control input is determined based on the instantaneous temperature error between the indoor air temperature and the desired setpoint, as well as the accumulated error over time. This formulation allows the controller to correct both short-term deviations and steady-state offsets, while maintaining a simple and robust control structure suitable for baseline performance evaluation.

The PI control law governing the heating input can be expressed as:

$$Q_{aux,n} = K_p e_n + K_i \sum_{i=1}^N e_i \Delta t \quad (2.6)$$

where $Q_{aux,n}$ is the auxiliary heating power at time step n , K_p is the proportional control gain, K_i is the integral control gain, e_n is the temperature tracking error at time step n , and Δt denotes the simulation time step.

The temperature error is defined as:

$$e_n = T_{sp,n} - T_{air,n} \quad (2.7)$$

where $T_{sp,n}$ is the indoor air temperature setpoint and $T_{air,n}$ is the measured indoor air temperature at time step n .

2.4.2 Heuristic (Near Optimal Rule-Based) Control Strategy

Heuristic control strategies extend reactive control by incorporating predefined operational rules and heuristic logic derived from expert knowledge or empirical observations. These approaches aim to approximate optimal system behavior without explicitly solving an optimization problem. The control rules are commonly implemented using conditional statements that adjust system operation based on variables such as time of day, outdoor temperature, or electricity price level (Afram & Sharifi, 2014).

In residential applications, rule-based control has been widely used to implement simple demand response strategies, such as pre-heating during off-peak periods or reducing heating power during predefined peak events. Compared to purely reactive control, rule-based approaches offer greater operational flexibility and can partially account for anticipated system conditions through scheduling (Maasoumy et al., 2014).

Despite these advantages, rule-based control strategies suffer from important limitations. The design of effective rules is highly system-specific and often requires extensive tuning, limiting their scalability and transferability across different buildings. Furthermore, rule-based controllers do not explicitly optimize system performance and may perform poorly under conditions that deviate from those assumed during rule development (Killian & Kozek, 2016). These limitations

motivate the adoption of optimization-based control approaches that can systematically account for system dynamics and future disturbances.

2.4.3 Model-Based Predictive Control (MPC) Strategy

Model-Based Predictive Control (MPC) represents a class of optimization-based control strategies in which control actions are obtained by solving a constrained optimization problem over a finite prediction horizon. At each control step, the optimization is repeated using updated measurements, allowing the controller to continuously adapt to changing operating conditions (Camacho & Bordons, 2013). Predictive control approaches can generally be classified into model-based and model-free formulations; in this study, the focus is placed on model-based MPC, where an explicit system model is used to predict future system behavior.

In building energy systems, the primary advantage of MPC lies not in basic feedback regulation which can be achieved using simpler control schemes, but in its ability to systematically incorporate system dynamics particularly anticipated thermal lag effect, operational constraints, and anticipated variations in the weather and occupancy effect within a unified optimization framework. This capability is particularly relevant for electrically heated residential buildings operating under time varying electricity prices and weather conditions, where control decisions must balance short-term performance with future system states (Oldewurtel et al., 2012).

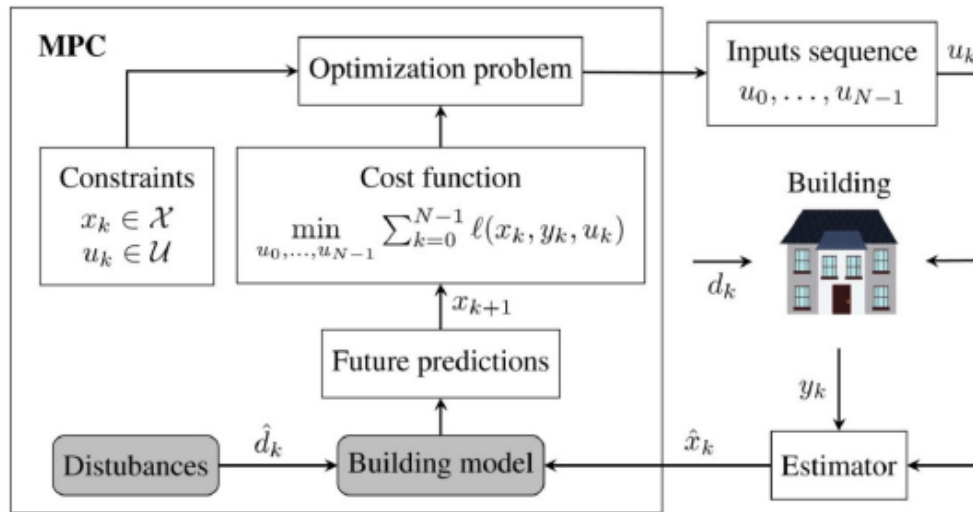


Figure 2.4: Schematic Representation of the Building Control using an MPC and State Estimator (Drgoňa et al., 2020).

Unlike reactive and rule-based control strategies, MPC determines control actions by minimizing a cost function subject to explicit constraints, rather than relying on predefined heuristics or schedules. This formulation enables transparent trade-offs between competing objectives such as energy cost reduction, peak power limitation, and operational feasibility, making MPC inherently more flexible and scalable under changing operating conditions (Abtahi, 2025; Killian & Kozek, 2016).

A critical requirement for successful MPC implementation in buildings is the availability of a predictive model that is both sufficiently accurate and computationally efficient. High-order physics-based models are generally impractical for real-time optimization, while purely data-driven models lack robustness and physical consistency. Consequently, low-order control-oriented thermal models, particularly resistance–capacitance (R–C) formulations, are widely adopted in building MPC applications due to their favorable trade-off between computational efficiency and physical interpretability (Hazyuk et al., 2012; Drgoňa et al., 2020).

Within this context, several applied and simulation-based studies have demonstrated the effectiveness of MPC when combined with carefully calibrated grey-box R–C models. Notably, Morovat developed and validated MPC frameworks for all-electric school buildings, showing that predictive control can significantly reduce energy consumption and peak electricity demand while remaining robust to model mismatch and operational constraints (Morovat et al., 2019; Morovat et al., 2022). These studies emphasized practical implementation aspects, including constraint handling, forecasting uncertainty, and scalability, which are critical for real-world deployment.

Similarly, as discussed earlier, Abtahi investigated MPC-based heating load management strategies for residential buildings using reduced-order R–C thermal models. His work demonstrated that low-order, control-oriented models can effectively support real-time MPC implementation while enabling meaningful energy flexibility under dynamic electricity pricing schemes (Abtahi, 2021; Abtahi et al., 2024). These results highlight that well-designed MPC formulations do not require overly complex models to achieve strong performance, particularly in residential buildings characterized by common temperature setpoints.

More broadly, an increasing number of experimental and applied MPC studies indicates that predictive control can consistently outperform conventional control strategies when the predictive model is properly calibrated and implementation constraints are explicitly addressed (Drgoňa et al., 2020; Vallianos & Athienitis, 2022). These findings reinforce the suitability of MPC as a core control strategy for residential energy flexibility studies.

2.5 Applied vs Simulation Based MPC

Model Predictive Control (MPC) for residential buildings has been widely investigated through simulation-based studies, which allow systematic evaluation of control formulations under controlled conditions and often assume perfect forecasts of weather, occupancy, and system disturbances (Oldewurtel et al., 2012; Reynders et al., 2017). While such studies are essential for controller development and benchmarking, they typically rely on idealized models and operating conditions, which can lead to optimistic estimates of achievable energy savings and peak demand reduction (Maasoumy et al., 2014).

Applied, or field-implemented, MPC addresses many of the limitations of simulation-based studies by evaluating predictive control strategies in real buildings under practical constraints, including forecast uncertainty, model mismatch, sensor noise, and integration with existing control infrastructure (Saber Derakhtenjani & Athienitis, 2021; Vallianos & Athienitis, 2022). As a result, applied MPC studies provide more realistic assessments of achievable performance and implementation feasibility.

A recent comprehensive review by Saloux et al. (2025) analyzed 91 field implementations of MPC in the built environment and found that only a small subset (10 studies) focused on residential buildings, highlighting the limited deployment of applied MPC in the residential sector compared to commercial and institutional buildings. The review further indicates that practical MPC applications primarily target thermal energy systems and thermal storage, with reported reductions in heating energy use, electricity cost, or peak demand typically ranging between 10% and 35%, depending on system configuration, tariff structure, and climatic conditions.

For cold-climate residential buildings, applied MPC has proven particularly effective when combined with control-oriented grey-box thermal models and heating systems that exploit building

thermal mass, such as radiant floors and envelope-based storage (Abtahi, 2021; Saberi Derakhtenjani & Athienitis, 2021). Compared to simulation-based investigations, field implementations provide critical insights into robustness, feasibility, and operational reliability that cannot be captured through simulation alone. Accordingly, simulation-based and applied MPC should be viewed as complementary approaches, where simulations support methodological development and applied studies validate real-world performance.

Table 2.1: Applied MPC studies in residential buildings reported in the literature.

Building type	Country / location	Reported outcomes	Reference
Residential dwelling	Denmark	9% energy cost reduction	(Pedersen et al., 2013)
Single-family house	Sweden	28% thermal energy savings	(Lindelöf et al., 2015)
Residential house	Canada	33% energy cost reduction	(Afram et al., 2017)
Residential house	Netherlands	5% energy cost reduction	(Finck et al., 2019)
Residential test house	Switzerland	25% thermal energy savings	(Bünning et al., 2020)
Residential house	United States	14% energy cost reduction	(Finck et al., 2020)
Residential house	United States	6% thermal energy savings	(Jain et al., 2020)
Residential test house	Switzerland	27% thermal energy savings	(Huber et al., 2021)
Single-family house	Denmark / Iceland	10% energy cost reduction	(Thorsteinsson et al., 2023)
Residential house	United States	24% energy cost reduction	(Wang et al., 2023)
Single-family	Canada (Québec)	30% reduction in daily heating energy cost	(Abtahi et al., 2024)

Based on the residential field implementations identified by Saloux et al. (2025), Table 2.1 summarizes the reported outcomes of applied MPC studies conducted on residential buildings.

2.6 Dynamic Electricity Market

Electricity pricing structures that vary with time and system conditions are increasingly adopted to encourage load shifting and reduce peak electricity demand in residential buildings (IEA EBC Annex 67, 2020). Time-varying tariffs introduce higher electricity prices during periods of elevated system stress and lower prices during off-peak periods, providing economic incentives for consumers to modify their consumption patterns. These pricing mechanisms are closely aligned with demand response concepts, in which electricity demand is adjusted in response to price or incentive signals to enhance grid reliability and reduce operational costs (Federal Energy Regulatory Commission [FERC], 2020).

MPC has been widely investigated as an effective control strategy for building operation under time varying electricity prices. By explicitly anticipating future system states, weather conditions, and pricing signals, MPC can shift heating demand away from high-cost periods while maintaining stable indoor temperature regulation. Previous studies have demonstrated that MPC can achieve significant reductions in both heating energy costs and peak power demand in residential buildings (Oldewurtel et al., 2012; Reynders et al., 2017; Abtahi, 2021). In the context of electrically heated buildings in cold climates, several studies have shown that incorporating electricity price signals into MPC formulations enhances energy flexibility and improves economic performance. Saberi Derakhtenjani and Athienitis (2021) demonstrated that price-responsive MPC can effectively reduce peak electricity demand, while Abtahi and Athienitis (2023) highlighted the robustness of MPC under uncertain weather and operating conditions (Abtahi et al., 2023). These findings indicate that advanced predictive control frameworks are well suited for operating residential heating systems under modern time-varying pricing schemes (Vallianos & Athienitis, 2022).

In this research, a time-varying electricity tariff is explicitly considered through the adoption of Hydro-Québec's Rate Flex D system in the province of Quebec, Canada, which is designed to encourage electricity consumption reduction during winter peak demand periods as shown in Figure 2.6.

During the winter season (December 1 to March 31), electricity is priced at a lower rate outside peak demand events (4.774 ¢/kWh for the first 40 kWh/day and 8.699 ¢/kWh thereafter), while a substantially higher price is applied during peak events (45.088 ¢/kWh). Peak demand events may occur between 6–10 a.m. and 4–8 p.m., with up to 30 events per winter season and a maximum total duration of 120 hours as shown in Figure 2.6.

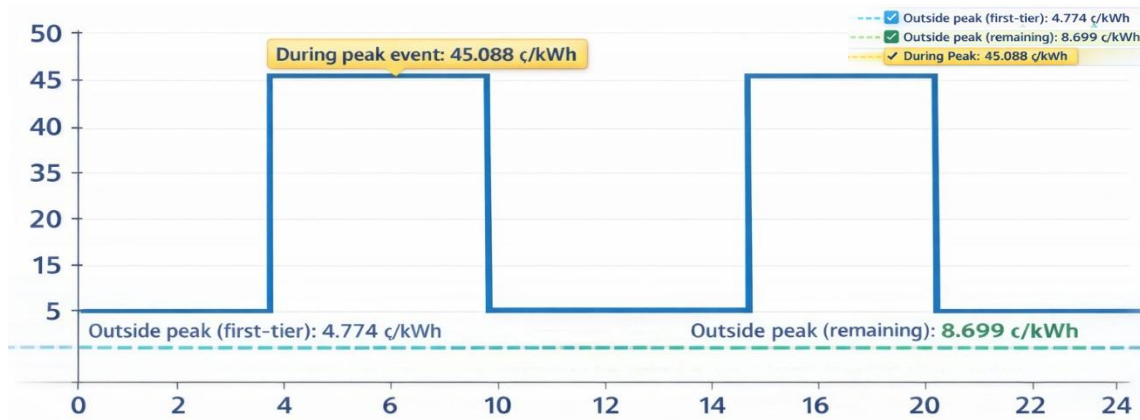


Figure 2.5: Hydro-Quebec Rate Flex D Energy Cost Structure

This pronounced price differential creates a strong economic incentive for advanced control strategies, such as MPC, to strategically shift heating operation and battery charging or discharging away from peak periods while maintaining indoor thermal comfort (Hydro-Québec, 2019).

In this context, Hydro-Québec’s Hilo program represents a practical implementation of demand response strategies, where households are incentivized to reduce or shift electricity consumption during peak periods through smart control of heating and other loads (Hilo, 2023). Such programs highlight the growing importance of intelligent energy management systems, aligning with the objectives of predictive control approaches investigated in this work.

Several electricity pricing structures are currently implemented worldwide to influence energy consumption patterns and facilitate peak load shifting, thereby reducing stress on electricity grids. Table 2.2 presents selected examples of such pricing schemes applied in Canadian provinces and some European countries.

Table 2.2: Selected Examples of Electricity Pricing Structure.

Region / Country	Utility / Market	Pricing Structure	Key Characteristics	Primary Objective
Inside Canada				
Quebec, Canada	Hydro-Québec ((Hydro-Québec, 2026)	Rate Flex D	Higher electricity prices during predefined winter peak hours and lower prices outside peak periods	Reduce winter peak demand and encourage load shifting
Ontario, Canada	Ontario Energy Board (Ontario Energy Board, 2023)	Time-of-Use (TOU) / Ultra-Low Overnight (ULO)	Multiple daily price periods, including very low overnight rates	Shift electricity use away from system peak hours
British Columbia, Canada	BC Hydro (BC Hydro, 2022)	Time-of-Day Pricing	Discounted overnight rates and surcharges during evening peak hours	Encourage off-peak electricity consumption
Nova Scotia, Canada	Nova Scotia Power (Nova Scotia Power, 2021)	Time-of-Day Rate	Differentiated peak and off-peak prices	Reduce peak demand and improve grid utilization
Internationally				
Norway / Nordics	Nord Pool (Nord Pool, 2023)	Dynamic Spot-Price Contracts	Hourly (or sub-hourly) electricity prices linked to wholesale market	Align demand with system conditions and renewable availability
France	EDF (EDF, 2022)	Tempo Tariff	Time-of-day pricing combined with limited high-price winter “event” days	Strong reduction of consumption during critical winter peaks
Germany / EU	European Commission (European Commission, 2019)	Dynamic Retail Tariffs	Electricity prices vary based on market conditions	Improve system flexibility and renewable integration

2.7 Impact of Weather Prediction Uncertainty on Predicted vs Actual Building Energy Performance

Weather conditions represent one of the dominant external inputs influencing building thermal behavior and energy consumption, particularly in heating dominated climates and electrically heated buildings. Outdoor air temperature directly affects transmission and ventilation heat losses, while solar irradiance governs passive solar gains and the availability of on-site renewable energy. In Model Predictive Control (MPC) frameworks, these variables are typically introduced as weather forecast over a finite prediction horizon. However, weather prediction models are

inherently uncertain due to imperfect initial conditions, model simplifications, and the stochastic nature of atmospheric processes, leading to inevitable discrepancies between forecasted and actual weather conditions, particularly on days with variable cloud cover (Henze & Schoenmann, 2003; Henze et al., 2004; Hou et al., 2022).

The existing literature has demonstrated that weather forecast uncertainty can significantly degrade building energy performance when predictive control strategies rely on inaccurate inputs. Oldewurtel et al. showed that deviations between predicted and actual outdoor temperature result in erroneous thermal load estimations, causing suboptimal heating schedules and reduced energy efficiency (Oldewurtel et al., 2012). Petersen and Bundgaard further reported that MPC performance is highly sensitive to weather forecast quality and that forecast errors can offset a considerable portion of the theoretical energy savings achievable through predictive control (Petersen & Bundgaard, 2012). These findings emphasize that assuming perfect weather forecasts is unrealistic and limits the applicability of MPC in real buildings.

The adverse effects of weather uncertainty have been quantified in terms of both energy consumption and operating cost. Lamoudi et al. observed that degradation in weather forecast quality could increase energy costs by up to 4%, even when advanced MPC strategies are employed (Lamoudi et al., 2012). Hedegaard et al. similarly found that forecast errors reduced heating cost savings by approximately 4% in low-energy buildings (Hedegaard et al., 2012). Hou et al. demonstrated that MPC systems without explicit mechanisms to handle forecast uncertainty may perform worse than conventional rule-based controllers, leading to increased indoor temperature violations and diminished cost savings (Hou et al., 2022). These studies collectively show that weather uncertainty can directly undermine both energy efficiency and thermal comfort.

Weather forecast uncertainty becomes even more critical when both outdoor temperature and solar irradiance are considered. Errors in solar radiation forecasts affect predicted solar gains and photovoltaic (PV) generation, which in turn influence heating demand, battery charging strategies, and grid interaction. Studies on PV-integrated building systems have shown that uncertainty in solar irradiance leads to inefficient utilization of renewable energy resources and reduced operational flexibility (Mathieu & Callaway, 2012; Bessa et al., 2012; Zhang et al., 2018). In

systems combining heating, PV, and battery storage, these effects are compounded, as inaccurate irradiance forecasts propagate through both thermal and electrical subsystems.

Several studies have explicitly examined the combined impact of temperature and solar irradiance uncertainty on MPC performance. Hu et al (2024) reported that conventional MPC experiences significant degradation in temperature tracking accuracy and energy efficiency under weather forecast errors, particularly in residential buildings equipped with PV-battery systems (Hu et al., 2024). Their results demonstrated that robust MPC formulations can improve temperature regulation accuracy by more than 15% and reduce seasonal energy consumption despite forecast inaccuracies. Similar conclusions have been reported in studies investigating uncertainty propagation in predictive building control, highlighting the need for controllers that remain effective under imperfect disturbance information (Chakraborty et al., 2016).

Several approaches have been proposed to address weather forecast uncertainty in MPC, including stochastic MPC, chance-constrained MPC, and min–max robust optimization. Chance-constrained MPC incorporates uncertainty by enforcing system constraints probabilistically, ensuring that comfort or operational limits are satisfied with a specified confidence level. While stochastic MPC can theoretically account for uncertainty, it often assumes Gaussian disturbances and linear system behavior, assumptions that are rarely valid for building thermal dynamics or weather processes (Oldewurtel et al., 2008; Zavala & Biegler, 2013). Min–max robust MPC formulations provide worst-case guarantees but typically require high computational effort and accurate uncertainty bounds, limiting their applicability in residential and real-time settings (Gao et al., 2019; Parisio et al., 2014).

Consequently, recent research increasingly emphasizes the need for MPC frameworks that are inherently robust to weather uncertainty rather than relying on highly accurate forecasts. Hou et al. demonstrated that incorporating simple error-handling mechanisms based on deviations between forecasted and actual weather conditions can recover most MPC benefits without increasing computational complexity (Hou et al., 2022). Hu et al. similarly showed that robust MPC architectures correcting forecast discrepancies significantly improve control reliability under uncertain temperature and solar irradiance conditions (Hu et al., 2024). These findings align with

broadier reviews of MPC for buildings, which identify robustness to uncertainty as a key requirement for practical deployment (Ma et al., 2012; Reynders et al., 2017; Drgoňa et al., 2020).

2.8 Summary of Research Gaps in MPC Applications for Houses

Previous studies have demonstrated the potential of MPC to improve energy efficiency and demand flexibility in residential buildings. However, important research gaps remain. Many existing MPC applications assume accurate weather forecasts, despite the significant influence of forecast uncertainty on heating demand and PV generation. Furthermore, limited studies have simultaneously considered weather uncertainty, dynamic electricity pricing, and coordinated PV–battery operation within a unified MPC framework for electrically heated houses in cold climates. Thus, this study develops a control-oriented MPC framework that explicitly incorporates weather forecast uncertainty while coordinating building thermal dynamics, PV generation, battery storage, and dynamic electricity tariffs.

In addition, most of the reviewed MPC applications have been implemented at the single-building level, while the extension of predictive control strategies to community scale or multi building energy systems remains relatively limited in the literature. Investigating coordinated control of multiple buildings represents an important direction for future research in enhancing system-level energy flexibility.

Chapter 3: Methodology

This study proposes a structured methodological framework to develop and evaluate a robust Model-based Predictive Control (MPC) strategy for enhancing heating energy flexibility in electrically heated residential buildings under weather uncertainty. The methodology integrates control-oriented thermal modeling, photovoltaic (PV)–battery coordination, dynamic electricity pricing, and explicit representation of forecast variability within a unified simulation framework.

Section 3.1 presents the case study building and the data acquisition process, including building characteristics, electrical heating configuration, PV–battery system specifications, and historical weather datasets used for model development and validation.

Section 3.2 describes the data preparation procedures and the representation of weather uncertainty. This includes processing of outdoor temperature and solar irradiance data, generation of scenario-based forecast inputs, and statistical characterization of variability to be incorporated within the predictive control framework.

Section 3.3 details the development of the control-oriented 3R–2C grey-box thermal network model. The model structure, state-space formulation, discretization approach, and parameter identification process are presented, ensuring a balance between physical interpretability and computational efficiency suitable for MPC implementation.

Section 3.4 introduces the applied building energy control strategies. The proposed MPC framework incorporating weather forecast variability is first presented, followed by the conventional reactive control approaches used for benchmarking. The control objectives include peak demand reduction, enhanced renewable self-consumption, electricity cost minimization under dynamic pricing, and minimization of setpoint temperature deviation.

Section 3.5 presents a sensitivity analysis of the MPC objective function weighting structure. By varying the temperature tracking weight under identical operating conditions, the analysis quantifies the trade-offs between setpoint temperature deviation, peak demand reduction, and overall energy flexibility.

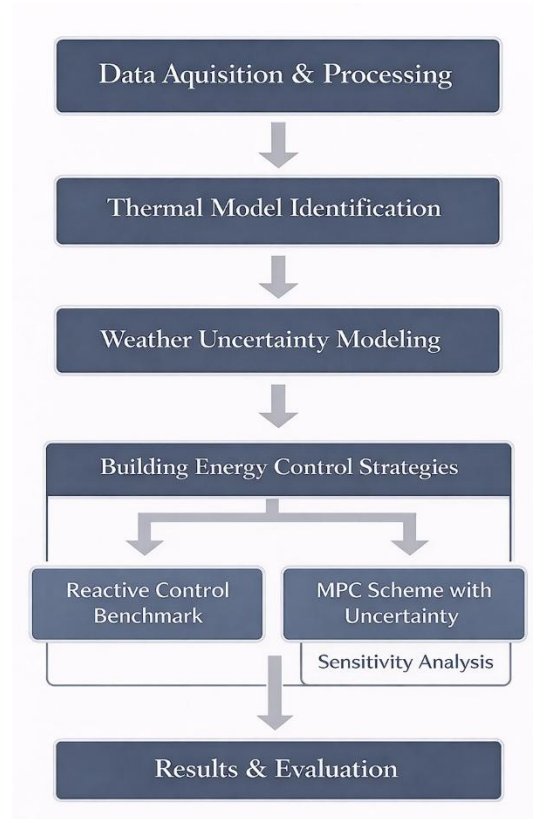


Figure 3.1: Overview of the Research Methodology and Control Strategy Evaluation Process

The overall methodological workflow of this study follows a structured sequence, illustrated in Figure 3.1. The process begins with data acquisition and preprocessing, including building operational data and historical weather records. This is followed by thermal model identification and calibration using a control-oriented 3R–2C grey-box framework. Subsequently, weather uncertainty is modeled through a scenario-based statistical representation of forecast variability. The calibrated model is then implemented under conventional reactive control for baseline benchmarking, while the uncertainty scenarios are incorporated within the proposed Model Predictive Control (MPC) framework. Finally, a sensitivity analysis of the MPC objective function weights is conducted to evaluate the trade-offs between setpoint tracking, peak demand reduction, and energy flexibility. This structured framework ensures methodological consistency from data-

driven model identification to predictive control evaluation under representative operating conditions.

3.1 Case Study and Data Acquisition

The Experimental House for Building Energetics (EHBE) is a two-storey, unoccupied residential research facility owned by Hydro-Québec and located in Shawinigan, Québec. The building operates exclusively on electricity and is designed to represent a typical Québec single-family dwelling constructed after 1980. It has a footprint of approximately 60 m². The ground floor consists of a living room, kitchen, dining area, and a guest bathroom, while the upper floor contains three bedrooms and a full bathroom. The house also includes an excavated basement and an



Figure 3.2: Hydro-Québec Experimental House, Shawinigan, Québec

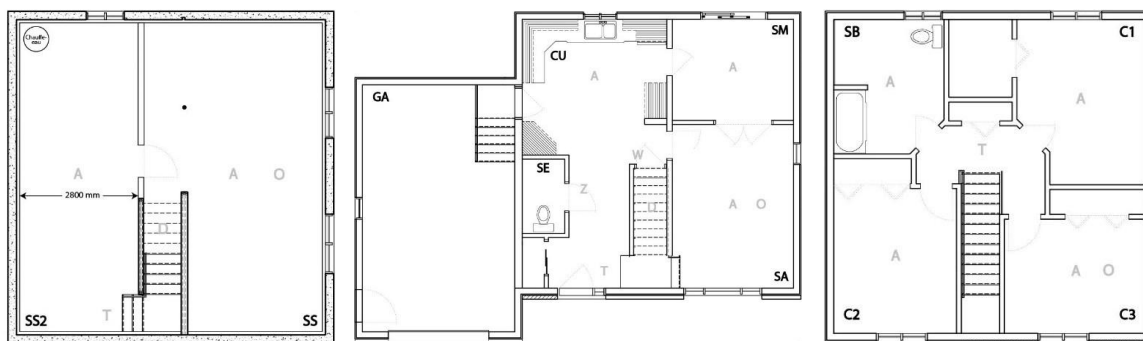


Figure 3.3: House Layout: Basement Floor (Left), Ground Floor and Garage (Center), and Second Floor (Right)

attached garage. Space heating is provided by electric baseboard heaters installed in each room and controlled through smart thermostats. In addition, the building is equipped with a 5-kW rooftop photovoltaic (PV) system and a 5 kW / 10 kWh lithium-ion battery, forming an integrated grid-connected residential energy system that supports load shifting, peak-period demand mitigation, and enhanced heating energy flexibility under dynamic electricity pricing schemes. A summary of the thermal zones, their corresponding volumes, and installed heating capacities is provided in table 3.1

Table 3.1: Location and Installed Capacity of each Electric Baseboard Heaters.

Abbreviation	Room	Location	Installed heating capacity (kW)
SS	Basement 1	Basement	2
SS2	Basement 2	Basement	2
CU	Kitchen	First Floor	1.5
SM	Dining Room	First Floor	1.25
SA	Living Room	First Floor	1.5
SE	Bathroom 1	First Floor	-
C1	Bedroom 1	Second Floor	1.25
C2	Bedroom 2	Second Floor	1.25
C3	Bedroom 3	Second Floor	1.25
SB	Bathroom 2	Second Floor	1
Total	-	-	13

The building is extensively instrumented with sensors installed indoors and in the surrounding soil. The monitoring system records 15-minute averaged measurements of zone-by-zone heating loads, plug loads, and lighting consumption, along with solar irradiance on both vertical and horizontal surfaces, outdoor air temperature, soil temperature, and zone air and surface temperatures.

The dataset used in this study covers the winter peak period identified by Hydro-Québec from December 1, 2023 to March 31, 2024, providing high-resolution measurements suitable for model development, parameter identification, and control performance evaluation.

3.2 Data Preparation and Weather Uncertainty Representation

Prior to model development and control implementation, the available datasets were subjected to a systematic preprocessing procedure. This included inspection of data consistency, identification and treatment of missing values, removal of anomalous measurements, and resampling to a unified 15-minute interval. These steps ensured reliable inputs for model calibration and predictive control development.

Two primary datasets were used in this study. The first dataset was obtained from Hydro-Québec and consists of detailed measurements collected from sensors installed within and around the experimental research house described in Section 3.1. The measurements include zone heating loads, plug and lighting loads, indoor air and surface temperatures, outdoor air temperature, solar irradiance, and soil temperature. Owing to its measured and high-resolution nature, this dataset was used for thermal model calibration and parameter identification.

The second dataset corresponds to a Typical Meteorological Year (TMY) dataset for Montreal. A TMY dataset represents a synthesized year of hourly weather data constructed from long-term historical records to statistically reflect typical climatic conditions of a given location. Unlike measured data from a specific year, TMY data provide representative weather conditions suitable for standardized control evaluation. In this work, the TMY dataset was used for MPC development and performance assessment to ensure that control strategies are evaluated under representative climatic conditions rather than a single observed winter season.

The TMY dataset includes outdoor air temperature T_{out} and global horizontal irradiance (GHI), which directly influence heating demand and photovoltaic generation. These variables form the basis for the structured representation of weather uncertainty within the MPC framework. In this study, historical weather data are used to quantify the typical variability of outdoor temperature and solar irradiance. These variability statistics are then applied to the nominal TMY profiles to generate a range of upper and lower forecast scenarios. Using different datasets for model calibration and for MPC implementation enhances the robustness of the framework, demonstrating that the proposed approach can operate reliably with datasets of different sources and temporal resolutions.

3.2.1 Temperature-Based Day Classification

Since temperature measurements are recorded at each time step, while classification is performed at the daily level, each day is first represented by aggregated daily statistics derived from the intra-day temperature profile. In this work, the daily mean temperature is computed and used as the primary feature for clustering.

$$\bar{T}_d = \left(\frac{1}{n_d}\right) \sum_{k=1}^{n_d} T_{d,k} \quad (3.1)$$

To represent variability in thermal demand, daily temperature profiles were classified using K-means clustering. Given a set of daily temperature feature vectors x_d , the clustering objective minimizes the within-cluster variance:

$$\min C \sum_{k=1}^K \sum_{\bar{T}_d \in C_k} \|\bar{T}_d - \mu_k\|^2 \quad (3.2)$$

where:

- C_k denotes cluster K ,
- μ_k is the centroid of cluster K ,
- $\bar{T}_d$ represents the daily mean outdoor temperature for day d .
- K is the number of clusters.

Days were grouped into representative categories such as moderate, cold, and very cold, enabling structured representation of temperature-driven heating variability.

3.2.2 Solar-Based Day Classification

Solar variability was characterized using the Hottel clear-sky model to estimate theoretical clear-sky irradiance GHI_{cs} . The clear-sky irradiance can be expressed as:

$$GHI_{cs} = I_0 \cdot \tau_b \cdot \cos(\theta_z) \quad (3.3)$$

where:

- I_0 is extraterrestrial solar radiation,
- τ_b is atmospheric transmittance,
- θ_z is the solar zenith angle.

A clearness index K_t was then defined as:

$$K_t = \frac{GHI}{GHI_{cs}} \quad (3.4)$$

This index quantifies the deviation of measured irradiance from clear-sky conditions. The clearness index is first computed at each time step. Since classification is performed at the daily level, the index is aggregated over all daylight time steps of each day. The daily mean clearness index is defined as:

$$\bar{K}_{t,d} = \left(\frac{1}{N_d} \right) \sum_{k=1}^{N_d} K_{t,k} \quad (3.5)$$

Based on threshold values of $\bar{K}_{t,d}$, days were classified into sunny, partly cloudy, and cloudy categories.

The combined temperature and solar classifications form the foundation for scenario-based weather representation, allowing explicit incorporation of forecast uncertainty within the predictive control framework.

3.2.3 Categorized Variability Representation

To represent weather forecast uncertainty in a structured and data-driven manner, a multiplicative uncertainty model is adopted for solar irradiance, while an additive bounded model is used for outdoor temperature. The approach is built in two layers. First, a baseline uncertainty level is computed at each time step using the historical variability of the corresponding month, which captures the typical dispersion of the signal at that time of year. Second, this baseline error is

adjusted using a day-type penalty factor derived from the categorized coefficient of variation (CV), so that partly cloudy days receive the largest uncertainty inflation, cloudy days receive a reduced inflation, and sunny days remain unpenalized.

Let y denote a generic weather variable ($y \in \{GHI, Tout\}$). For a month m and time step index k (15-min resolution), the baseline coefficient of variation is computed from the historical dataset as the ratio between the standard deviation and the mean:

$$CV_{m,k} = \frac{\sigma_{m,k}}{\mu_{m,k}} \quad (3.6)$$

This baseline CV captures seasonal variability but does not distinguish between different sky conditions. To distinguish between day types, categorized daily variability is computed. For each classified day d , the daily coefficient of variation of irradiance is computed

$$CV_d = \frac{\sigma_d}{\mu_d} \quad (3.7)$$

Then, the average variability within each sky-condition category is then evaluated as:

$$\overline{CV}_{type} = \left(\frac{1}{N_{type}} \right) \sum_{d \in type} CV_d \quad (3.8)$$

Based on these statistics, the day-type scaling parameters are defined relative to sunny-day variability:

$$\beta = \frac{\overline{CV}_{P.cloudy}}{\overline{CV}_{Sunny}}, \quad \gamma = \frac{\overline{CV}_{Cloudy}}{\overline{CV}_{Sunny}} \quad (3.9)$$

The day-type adjustment factor is defined as shown in equation (3.10)

$$\alpha_{type} \begin{cases} 1, & \text{sunny days} \\ \beta, & \text{partly cloudy days} \\ \gamma, & \text{Cloudy days} \end{cases} \quad (3.10)$$

This allows the uncertainty level to be adjusted, assigning higher variability to partly cloudy and cloudy days.

$$CV_{eff} = \alpha_{type}(d(t))CV_{m,k} \quad (3.11)$$

That allow getting the upper and lower scenarios for each time step through adding or subtracting the CV_{eff} term for each time step.

Weather uncertainty is ultimately represented through a scenario-based scaling approach in which the time-step coefficient of variation is adjusted using a category-dependent scaling factor α_{type} , then CV_{eff} is calculated as defined in Equation (3.11). For each 15-minute time step t . The upper and lower solar irradiance scenarios are then generated as in (3.12):

$$GHI_{up/low}(t) = GHI_{nom(t)} \cdot (1 \pm CV_{eff(t)}) \quad (3.12)$$

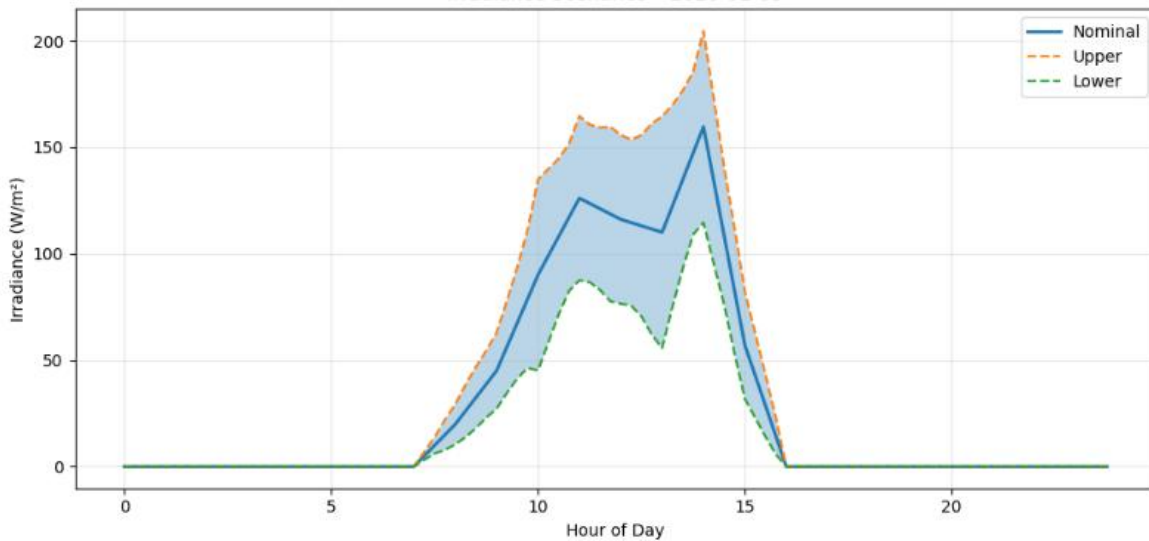


Figure 3.4: Representation of GHI Uncertainty for a Typical Cloudy Day

Figure 3.4 illustrates the representation of GHI uncertainty, where the blue curve denotes the nominal (forecasted) scenario, the orange curve represents the upper scenario, and the green curve corresponds to the lower scenario.

For outdoor temperature, the same uncertainty representation methodology is applied. However, to ensure physically realistic behavior, the additive deviation is bounded by a maximum limit of

± 3 °C. Forecast verification analyses (e.g., Pennsylvania State University, 2024) indicate that short-range temperature forecasts (1–3 days ahead) typically exhibit mean absolute errors within a few degrees Celsius. Since the present study considers one-day-ahead forecasts, which are generally more accurate than longer forecast horizons, the adopted ± 3 °C bound represents a conservative and physically realistic uncertainty range for outdoor temperature. This documented forecast accuracy supports the adoption of a bounded deviation in the order of a few degrees Celsius, making the ± 3 °C cap a realistic and physically consistent representation of temperature uncertainty for short-horizon predictive control applications. A minimum uncertainty of ± 1 °C is imposed to avoid collapse of proportional uncertainty near zero temperature and to maintain a realistic baseline level of variability. The resulting bounded uncertainty formulation is given in (3.13).

$$T_{up/low}(t) = T_{nom}(t) \pm \max(\min(CV_{eff}(t) \cdot |T_{nom}(t)|, 3^\circ C), 1^\circ C) \quad (3.13)$$

This formulation ensures bounded, regime-dependent uncertainty representation while preserving physical consistency for both solar irradiance and temperature inputs within the MPC framework.

3.3 Model Structure and Parameter Identification

3.3.1 Model Structure

To ensure a consistent and physically interpretable representation of the building thermal behavior across all investigated control strategies, a control-oriented grey-box resistance–capacitance (RC)

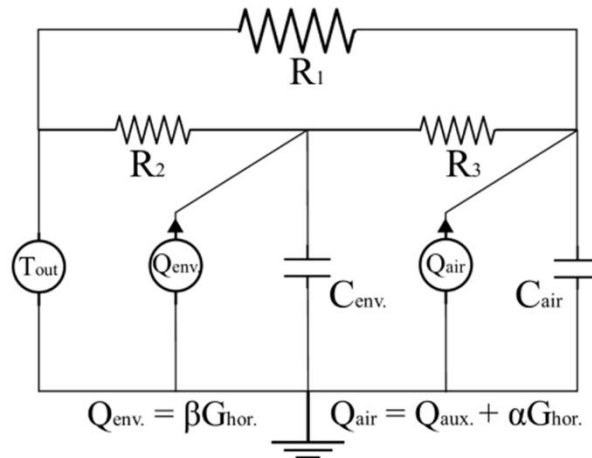


Figure 3.5: 3R-2C Thermal Model

model is adopted. The model follows a second-order 2C–3R structure, consisting of two thermal capacitances and three thermal resistances as shown in Figure 3.5. This configuration captures the dominant heat transfer mechanisms of the residential building while maintaining computational simplicity suitable for real-time control implementation.

The two capacitances represent the main thermal storage elements of the building:

- C_{air} : effective thermal capacitance associated with the indoor air node, representing the low thermal inertia components of the conditioned space, including the air mass and lightweight interior elements that respond rapidly to thermal disturbances.
- C_{env} : effective thermal capacitance associated with the building envelope node, representing the higher thermal inertia of structural and envelope components, including walls, floors, ceilings, and other massive elements that respond more slowly to thermal variations.

The three thermal resistances are represented in conductance form as:

- U_1 : conductance between indoor air and outdoor air
- U_2 : conductance between envelope and outdoor air
- U_3 : conductance between indoor air and envelope

The solar gains are represented by two lumped coefficients:

- α : effective solar gain area associated with the indoor air node
- β : effective solar gain area associated with the building envelope

These coefficients convert global horizontal irradiance (W/m^2) into equivalent thermal power (W) distributed between the indoor air and structural mass.

These parameters are treated as lumped (effective) parameters. They represent aggregated thermal storage and heat transfer effects rather than explicitly resolved physical elements of the building. This lumped representation enables a reduced-order description suitable for control-oriented modeling.

Since thermal conductance U is the inverse of resistance R , the model corresponds to a classical 2C–3R thermal network. The model is discretized using a constant sampling period Δt (900 s in this work) and forward Euler integration.

The 3R–2C grey-box thermal model is selected for this study as it provides a balanced trade-off between physical interpretability, predictive accuracy, and computational efficiency required for control-oriented applications. Unlike lower-order models (e.g., 1R–1C), it explicitly distinguishes between the fast dynamics of indoor air and the slower thermal response of the building envelope, enabling a more accurate representation of heat storage and release processes that are essential for preheating and load shifting. In contrast to higher-order models, the 3R–2C structure avoids over-parameterization, which can introduce identifiability challenges, increase calibration complexity, and reduce robustness when only limited or noisy data are available. This balance is particularly important for MPC implementation, where model simplicity directly affects real-time optimization performance. Therefore, the 3R–2C model is sufficiently detailed to capture the dominant thermal dynamics of residential buildings while ensuring reliable parameter estimation and practical applicability in real-world control systems.

Applying the energy balance to the indoor air node and expressing it in discrete time yields:

$$T_{k+1}^{air} = T_k^{air} + \frac{\Delta t}{C_{air}} \left(\alpha GHI_k + P_k^{heat} + U_1(T_k^{out} - T_k^{air}) + U_3(T_k^{env} - T_k^{air}) \right) \quad (3.14)$$

where:

- GHI_k [W/m²] is the global horizontal irradiance,
- α [m²] is the solar gain coefficient associated with indoor air,
- P_k^{heat} [W] is the electrical heating power input,
- T_k^{out} [°C] is the outdoor temperature,
- U_1 [W/°C] models heat exchange between indoor air and outdoors,
- U_3 [W/°C] models heat exchange between indoor air and envelope,
- C_{air} [MJ/°C] effective thermal capacitance associated with the indoor air node.

This equation represents the transient energy balance of the indoor air node, accounting for heating input, solar gains, and conductive heat exchange with both the outdoor environment and the building envelope. In a similar manner, applying the energy balance to the envelope node leads to the following discrete-time expression:

$$T_{k+1}^{env} = T_k^{env} + \frac{\Delta t}{C_{env}} \left(\beta GHI_k + U_2(T_k^{out} - T_k^{env}) + U_3(T_k^{air} - T_k^{env}) \right) \quad (3.15)$$

where:

- β [m²] represents the fraction of solar irradiance absorbed by the building structure,
- U_2 [W/°C] models heat exchange between envelope and outdoor air,
- C_{env} [MJ/°C] is the effective thermal capacitance of the building mass.

This second equation captures the slower thermal response of the building structure and enables representation of thermal inertia, which is essential for analyzing load shifting and energy flexibility.

3.3.2 Model Calibration and Parameter Identification

The calibration of the proposed 3R–2C grey-box thermal model was performed using measured indoor air temperature data over the selected calibration period.

The procedure aims to achieve two main objectives:

- (a) to assess the predictive accuracy of the model, and
- (b) to identify physically meaningful thermal parameters of the resistance–capacitance network.

The predictive performance of the calibrated model was assessed using three standard statistical indicators: RMSE, CVRMSE, and NMBE, which quantify overall prediction error and systematic bias between measured and simulated indoor air temperatures.

As summarized in Table 3.2, the model achieved training and testing RMSE values of 0.226 °C and 0.470 °C, respectively. The corresponding CVRMSE values were 1.13% and 2.5%, while the NMBE remained at –0.02% for both datasets, indicating negligible bias. All performance

indicators remain well within the acceptable limits recommended by ASHRAE Guideline 14 ($CVRMSE \leq 15\%$) and those discussed in Section 2.3.3. The moderate increase in testing error is expected and reflects satisfactory generalization without evidence of overfitting, confirming the robustness of the calibrated 3R–2C model for subsequent control and optimization analyses.

Table 3.2: Model Calibration and Parameter Identification Results

Model Calibration Evaluation				Parameter Identification		
Metric	Training	Testing	Unit	Parameter	Value	Unit
	Value	Value				
RMSE	0.23	0.47	(°C)	C_{air}	10.3	(MJ/K)
R2				C_{env}	49.39	(MJ/K)
CVRMSE	1.13	2.5	(%)	U_1	116.81	(W/K)
				U_2	42.06	(W/K)
NMBE	-0.02	-0.02	(%)	U_3	1106.26	(W/K)
				α	2.34	(m ²)
				β	5.38	(m ²)

In addition to performance evaluation, the calibration process provided estimates of the thermal parameters of the 3R–2C network. The parameter identification was conducted using a nonlinear least-squares optimization approach, where the objective function minimizes the sum of squared errors (SSE) between measured and simulated indoor temperature. The resulting calibrated parameters are also presented in Table 3.2.

The identified parameters exhibit physically consistent magnitudes. In particular, the envelope thermal capacitance C_{env} is substantially larger than the indoor air capacitance C_{air} , reflecting the dominant heat storage capacity of the building structure. The relatively large value of U_3 indicates the significant contribution of ventilation and direct heat transfer mechanisms in the overall heat balance.

3.4 Applied Building Energy Control Strategies

The calibrated 2C–3R grey-box thermal model described in Section 3.3 is employed consistently across all investigated control strategies. This ensures methodological consistency and enables a

transparent comparison between predictive and reactive approaches. The Model-Based Predictive Control (MPC) formulation constitutes the primary contribution of this work, while the reactive strategies serve as business-as-usual reference cases.

3.4.1 Model-Based Predictive Control

MPC is implemented to optimally coordinate heating power and battery operation over a 24-hour prediction horizon while explicitly accounting for forecasted weather inputs and time-varying electricity prices. At each 15-minute sampling instant, the optimization problem is solved over the full one-day horizon, and only the first control action is applied according to a receding-horizon strategy. (3.16)

$$x_k = \begin{bmatrix} T_k^{air} \\ T_k^{env} \\ SOC_k \end{bmatrix}, \quad u_k = \begin{bmatrix} P_k^{heat} \\ P_k^{batt} \end{bmatrix}, \quad d_k = \begin{bmatrix} T_k^{out} \\ GHI_k \end{bmatrix} \quad (3.16)$$

PV generation is computed from irradiance using (3.17) and treated as a known exogenous input over the prediction horizon.

$$P_k^{pv} = P_{rated} \frac{GHI_k}{GHI_{ref}} \quad (3.17)$$

where $P_{rated} = 5$ kW is the nominal PV capacity and $GHI_{ref} = 1000$ W/m² corresponds to Standard Test Conditions (STC). According to IEC 60904-3 (IEC, 2016), PV modules are rated at 1000 W/m² irradiance under reference conditions. This formulation ensures that the model reproduces the nominal 5 kW output at reference irradiance while maintaining dimensional consistency. The simplified linear representation is appropriate for the control-oriented scope of this study.

The battery energy storage system is modeled using an energy-based state-of-charge balance as shown in equation (3.18)

$$SOC_{k+1} = SOC_k + \frac{\Delta t}{E_{batt}} P_k^{batt} \quad (3.18)$$

where E_{batt} is the nominal battery energy capacity, defined as $E_{batt} = 10 \times 3.6 \times 10^6$ corresponding to a 10-kWh battery. This formulation allows the MPC to track energy accumulation in the battery and to schedule charging and discharging actions in response to PV availability and electricity prices. Positive P_k^{batt} denotes charging and negative values denote discharging. Additionally, battery efficiencies are neglected for simplicity.

Electrical interaction between heating, battery, PV and the grid is enforced through the power balance expressed in equation (3.19):

$$P_k^{net} = P_k^{heat} + P_k^{batt} - P_k^{pv} \quad (3.19)$$

$$P_k^{imp} = \max(P_k^{net}, 0) \quad (3.20)$$

Grid import is defined as the positive part of the net power exchange, such that electricity cost is incurred only when net power is imported from the grid.

The optimization is solved subject to actuator and storage constraints:

$$0 \leq P_k^{heat} \leq 15000 \quad (3.21)$$

$$-5000 \leq P_k^{batt} \leq 5000 \quad (3.22)$$

$$0.2 \leq SOC_k \leq 1 \quad (3.23)$$

The lower bound on the state of charge is introduced to avoid deep discharge of the battery, which can accelerate degradation and reduce battery lifetime. Maintaining a minimum SOC of approximately 20% is a common practice in lithium-ion battery operation to ensure long-term performance and reliability (Battery University, 2023).

The MPC optimization problem is initialized using measured and scheduled operating conditions at the beginning of each control horizon. Specifically, the indoor air temperature and the envelope

temperature are initialized at the baseline temperature setpoint as shown in equation (3.24), ensuring consistency with the scheduled setpoint conditions at the start of the optimization. The battery state of charge is initialized using the SOC value at the final time step of the previous day. A minimum SOC constraint of 20% is imposed to preserve battery capacity. These initial conditions are enforced as equality constraints within the optimization problem and guarantee a well-posed state trajectory over the prediction horizon.

$$T_0^{air} = T_0^{sp} \quad , \quad T_0^{env} = T_0^{sp} \quad , \quad SOC_0^d = SOC_{end}^{d-1} \quad (3.24)$$

To minimize deviations from the desired operating condition, a baseline indoor temperature setpoint T_k^{sp} is defined in equation (3.25). A symmetric allowable temperature band of ± 0.5 °C is imposed around this reference as

$$T_k^{min} = T_k^{sp} - 0.5 \quad , \quad T_k^{max} = T_k^{sp} + 0.5 \quad (3.25)$$

A time-varying electricity price signal is incorporated into the MPC formulation to represent off-peak and peak-event tariff periods. This price is denoted by λ_k^{elc} , where the superscript “elc” explicitly refers to electricity. During predefined peak events, electricity prices increase significantly, providing a strong incentive to reduce grid import. An additional peak indicator variable m_k is used to activate extra penalties during peak periods when required.

The MPC objective function minimizes a weighted sum of economic cost, setpoint deviation, grid interaction, and operational smoothness terms over the prediction horizon, as expressed in Equation (3.26).

$$\min_{x_k, u_k} J = \sum_{k=1}^{N-1} (J_{1,k} + J_{2,k} + J_{3,k} + J_{4,k} + J_{5,k}) \quad (3.26)$$

Where the objective components are:

$J_{1,k}$ represents electricity import cost (tariff-based)

$$J_{1,k} = \lambda_k^{elc} P_k^{imp} \quad (3.27)$$

$J_{2,k}$ represents net power regularization

$$J_{2,k} = |P_k^{net}| \quad (3.28)$$

$J_{3,k}$ represents setpoint tracking

$$J_{3,k} = \omega_{trk} (T_k^{air} - T_k^{sp})^2 \quad (3.29)$$

$J_{4,k}$ represents Input smoothness (heating and battery)

$$J_{4,k} = \|P_k^{heat} - P_{k-1}^{heat}\|_2 + \|P_k^{batt} - P_{k-1}^{batt}\|_2 \quad (3.30)$$

$J_{5,k}$ represents the allowable temperature range enforcement term

$$J_{5,k} = \omega_{range} [\max(T_k^{min} - T_k^{air}, 0) + \max(T_k^{air} - T_k^{max}, 0)] \quad (3.31)$$

The objective function balances economic and operational performance criteria. The electricity price term penalizes grid import according to the time-varying tariff structure. The net power term regularizes grid interaction to promote PV self-consumption and limit excessive exchange. The quadratic tracking term enforces temperature regulation around the prescribed setpoint, while the control variation term penalizes abrupt changes in heating and battery power to ensure smooth and physically realistic actuation.

Table 3.3: Active Weighting Coefficients of The MPC Objective Function and Their Implementation in the Optimization Model.

Symbol	Meaning	Code Variable	Value	Notes
λ_k^{elec}	Electricity price weight	price_flx[k]	1 (off-peak), 10 (peak)	Time-varying tariff signal
ω_{trk}	Temperature tracking weight	penalty_track[k]	40,000	Quadratic tracking penalty
ω_{range}	Temperature constraint violation penalty	penalty_range[k]	30,000	Linear violation penalty

Finally, a slack-variable penalty softly enforces setpoint bounds by assigning a cost to any temperature violation beyond the allowable range. Table 3.3 presents the active weighting coefficients included in the optimization model.

The MPC problem is formulated as a convex optimization problem, since it combines linear system dynamics with convex objective terms, including quadratic tracking and norm-based penalties, under linear constraints. Consequently, the resulting optimization problem is convex and admits a well-defined global optimal solution at each control step. The problem is implemented using the CVXPY optimization framework and solved using the ECOS solver at each control step. This MPC formulation constitutes the core methodological contribution of this thesis and forms the foundation for the performance evaluation presented in the subsequent chapters.

3.4.1.1 Sensitivity Analysis

A sensitivity analysis was conducted to examine the influence of the weighting structure in the MPC objective function and to characterize the interaction between its competing objectives. The controller simultaneously penalizes electricity import cost, temperature deviation from the setpoint, net power exchange, and control effort variation. The relative magnitude of these weights determines the balance between maintaining thermal setpoint conditions and enabling operational flexibility. The presented analysis in Figure 3.6 focuses on the tracking weight ω_{trk} indicates that an optimal range lies between 30,000 and 45,000. Within this range, the MPC achieves a balanced trade-off between maintaining indoor temperature close to the setpoint and preserving energy

flexibility. A value of 40,000 was selected as it provides near-optimal temperature tracking while avoiding excessive increases in peak-period grid import and total heating energy.

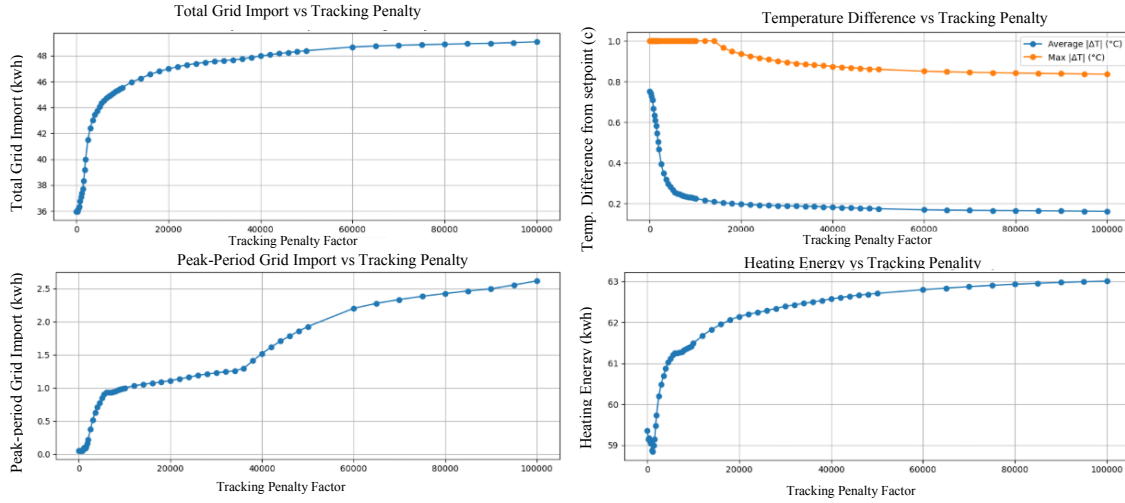


Figure 3.6: Sensitivity of Key Performance Indicators to Variations in the Tracking Weight (ω_{trk}).

The results demonstrate the intrinsic trade-offs within the multi-objective optimization framework. Increasing the tracking weight strengthens temperature regulation and reduces deviation from the setpoint. However, stricter tracking limits the controller’s ability to exploit building thermal inertia for load shifting, thereby increasing total grid import, peak-period import, and heating energy use. Conversely, lower tracking weights enhance operational flexibility and peak reduction capability while permitting controlled temperature deviations within the prescribed temperature bounds. This clearly illustrates the fundamental trade-off between adhering to the predefined setpoint temperature and energy flexibility within the proposed MPC framework.

Overall, the analysis quantitatively demonstrates how prioritizing temperature tracking influences energy consumption patterns and peak demand behavior, thereby enabling systematic selection of weighting coefficients according to desired performance priorities, such as setpoint preservation, peak demand mitigation, or overall energy reduction.

3.4.2 Reactive Control with PV and Battery

In the second scenario, indoor temperature is regulated using a classical Proportional–Integral (PI) controller without predictive optimization. The 2C–3R thermal model governs the building

dynamics, while PV generation and battery storage are integrated using the same relations introduced in the MPC formulation.

Equation (3.32) defines the temperature tracking error as:

$$e_k = T_k^{sp} - T_k^{air} \quad (3.32)$$

While equation (3.33) defines the integral term as:

$$I_k = I_{k-1} + e_k \Delta t \quad (3.33)$$

The heating command is presented in equation (3.34)

$$P_k^{heat} = K_p e_k + K_i I_k \quad (3.34)$$

The PI gains K_p and K_i were determined through simulation-based manual tuning to achieve stable closed-loop temperature regulation. The proportional gain was set to $K_p = 8000 \text{ W/}^\circ\text{C}$, corresponding to approximately half of the maximum heating capacity (15 kW), such that a 1°C temperature deviation results in a heating command of roughly 8 kW while avoiding actuator saturation. The integral gain was selected as $K_i = 0.1 \text{ W/}^\circ\text{C.s}$, based on simulation-based manual tuning to ensure stable closed-loop performance (Åström and Hägglund, 1995).

Battery dynamics follow the same SOC equation defined in the MPC formulation; however, battery dispatch is governed by a predefined rule-based strategy rather than optimal scheduling, where the battery charges during PV surplus and discharges during peak tariff periods.

3.4.3 Reactive Control Without PV and Battery

In the baseline scenario, heating is regulated using the same PI controller that was introduced in section (3.4.2) but without PV or battery integration, so grid import equals heating demand as shown in equation (3.35):

$$P_k^{grid} = P_k^{heat} \quad (3.35)$$

This purely reactive controller responds only to instantaneous temperature deviations and does not anticipate future forecast or price variations.

Finally, the proposed MPC framework and the business-as-usual reactive control strategies defined in this section provide the basis for the comparative performance evaluation presented in Chapter 4, where the impact of weather uncertainty and PV–battery coordination on heating energy flexibility is analyzed.

Chapter 4: Results and Discussion

The objective of this chapter is to evaluate the performance of the proposed Model Predictive Control (MPC) framework and compare it with conventional reactive control strategies under different weather conditions and simulation horizons. The evaluation focuses on the ability of the control strategies to manage heating demand, interact with PV–battery systems, and reduce grid electricity consumption during peak tariff periods. The chapter is organized as follows. Section 4.1 describes the weather representation used in the simulations, including the selected case-study days and the characterization of weather conditions used to evaluate the control strategies.

Section 4.2 presents the performance of the MPC strategy. This section first analyzes the one-day MPC operation for the selected case-study days, followed by a one-week simulation to evaluate the cumulative performance and operational behavior of the controller over a longer horizon.

Section 4.3 presents the results of the reactive control strategies, including both reactive control without PV–battery integration and reactive control with PV–battery integration. The section also includes a one-week evaluation to assess the performance of the reactive strategies over multiple consecutive days.

Finally, Section 4.4 provides a comprehensive comparison between MPC and reactive control strategies using the defined performance indicators. The comparison focuses on differences in heating energy use, grid import patterns, peak demand reduction, and electricity cost.

4.1 Weather Representation

To evaluate the performance of the proposed control strategies under different environmental conditions, three representative winter days were selected from the dataset. The selection aims to capture a diverse range of outdoor temperature and solar irradiance conditions that typically occur during the heating season in Montréal.

The selected days represent three distinct operating scenarios:

1. Moderate cold sunny day, characterized by relatively mild winter temperatures and significant solar availability.

2. Very cold cloudy day, representing extreme heating demand conditions with minimal solar gains.
3. Cold partly cloudy day, characterized by noticeable fluctuations in solar irradiance due to intermittent cloud cover.

These representative days allow the evaluation of the control strategies across different combinations of heating demand levels and solar availability, thereby reflecting the variability of real winter operating conditions.

The moderate cold sunny day provides an opportunity to assess the ability of the system to utilize available solar energy for battery charging and load shifting. In contrast, the very cold cloudy day represents a more conservative scenario in which heating demand is high while solar generation is minimal. Finally, the cold partly cloudy day introduces significant solar variability, allowing the evaluation of controller performance under fluctuating solar conditions. The selected representative days are summarized in Table 4.1.

Table 4.1:Categories of the Three Selected Representative Days

Date	Temperature Category	Solar Category	Peak Time
December 5	Moderate Cold	Sunny	Night
January 15	Very Cold	Cloudy	Night
March 22	Cold	Partly Cloudy	Morning

The selected days cover different periods of the winter season, including early winter (December), peak winter (January), and late winter (March).

In the Hydro-Québec Rate Flex D tariff structure, a given day contains only one peak period, which can occur either in the morning (06:00–10:00) or evening (16:00–20:00) depending on the system conditions defined by Hydro-Québec. In this study, December 5 and January 15 correspond to evening peak events, while March 22 was intentionally selected to represent a morning peak scenario, given that the morning time of March 22 was colder than its night in order to evaluate the controller performance under both peak configurations.

Figure 4.1 presents the outdoor air temperature and global horizontal irradiance (GHI) profiles for the three selected representative days. The shaded regions indicate the uncertainty range considered in the analysis, while the solid lines represent the nominal weather conditions used in the simulations. The very cold cloudy day (January 15) exhibits the lowest outdoor temperatures and minimal solar availability, representing a high heating demand scenario. In contrast, the moderate cold sunny day (December 5) shows higher solar irradiance levels and milder temperatures, allowing greater utilization of solar energy. The partly cloudy day (March 22) demonstrates noticeable fluctuations in solar irradiance due to intermittent cloud cover, providing a dynamic scenario for evaluating controller performance under variable solar conditions.

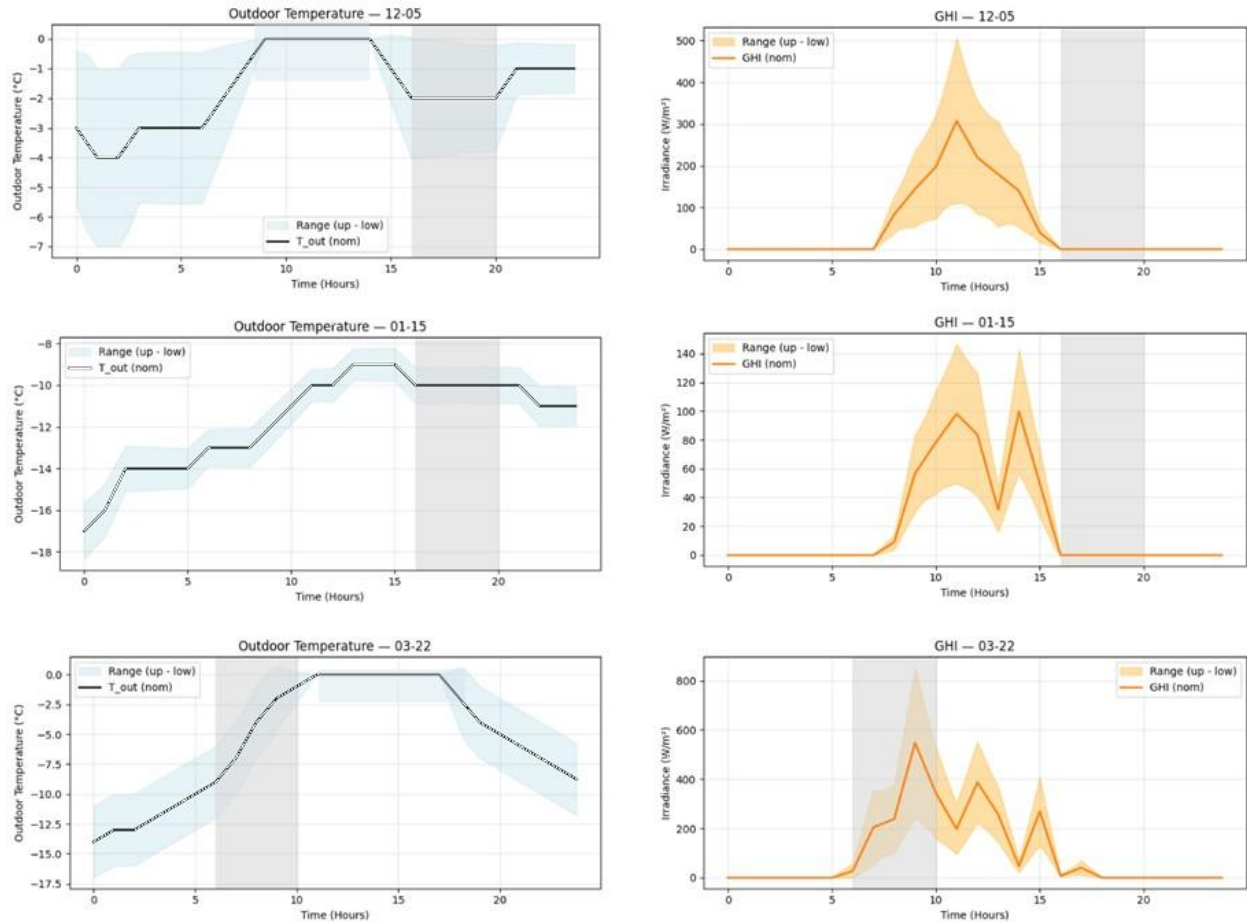


Figure 4.1: Outdoor Air Temperature and Global Horizontal Irradiance (GHI) Profiles for the Three Selected Days

In addition to the representative daily scenarios, the applicability of the proposed MPC framework was evaluated over a longer operational horizon. A one-week simulation was therefore performed

in order to assess the cumulative performance of the control strategies under continuously varying weather conditions.

The selected week spans from February 3 to February 9, including moderate cold, cold, and very cold temperature conditions together with cloudy, partly cloudy, and sunny solar conditions. This diversity allows the evaluation of the MPC framework under varying environmental conditions and enables the assessment of its stability and consistency over multiple consecutive days. The weather conditions of the selected week are summarized in Table 4.2.

Table 4.2: Days Categorization of the Selected Week

Date	Temperature	Solar Category
Feb 3	Moderate Cold	Cloudy
Feb 4	Moderate Cold	Partly Cloudy
Feb 5	Moderate Cold	Cloudy
Feb 6	Cold	Sunny
Feb 7	Cold	Sunny
Feb 8	Very Cold	Sunny
Feb 9	Very Cold	Sunny

The outdoor temperature and solar irradiance profiles for the selected week are illustrated in Figure 4.2. The figure shows how both temperature and solar availability vary significantly throughout the week, providing a realistic representation of winter weather dynamics in Montréal. The shaded regions represent the uncertainty ranges considered in the analysis, while the solid lines correspond to the nominal weather conditions used by the controller.

The simulations presented in the following sections apply both the proposed MPC strategy and the business-as-usual reactive control strategies to these representative weather conditions in order to analyze system behavior under different environmental scenarios. The resulting thermal dynamics, energy flows, and operational cost impacts are evaluated in detail in the subsequent sections.

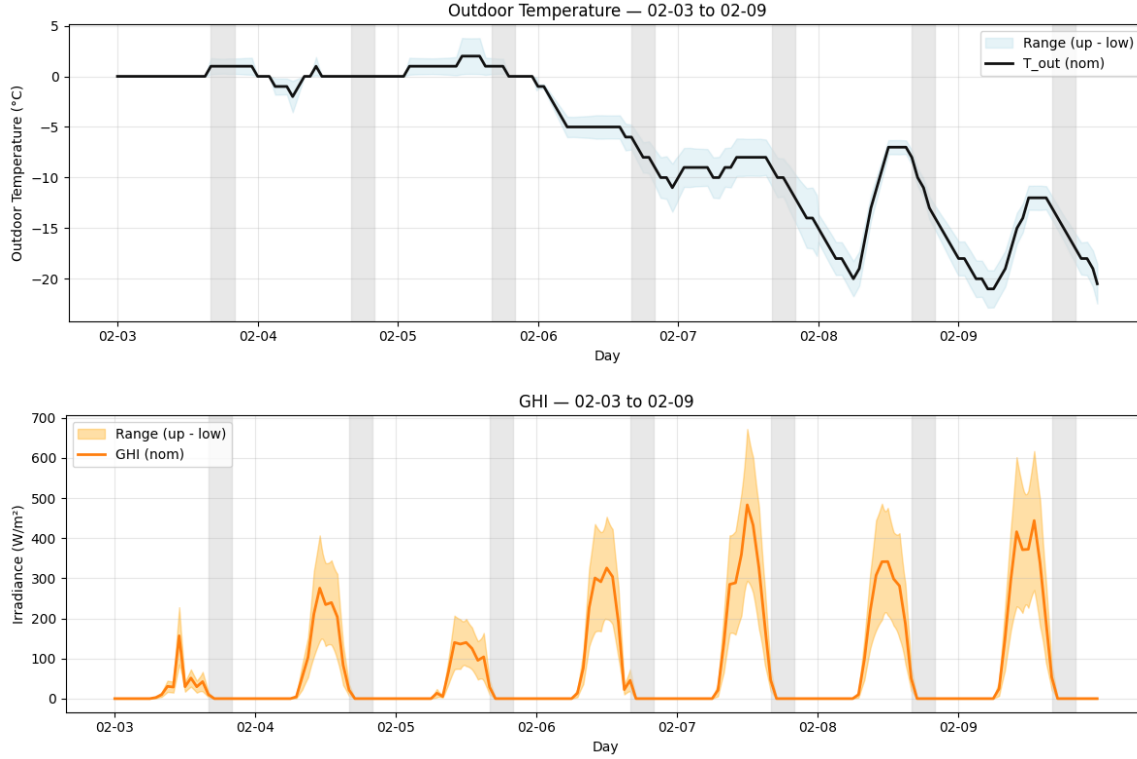


Figure 4.2: Outdoor Temperature and Solar Irradiance Profiles (03 – 09 Feb) Week

4.2 MPC Performance

4.2.1 Dec 05 (Night Peak)

The first representative scenario corresponds to December 05, which represents a moderate cold sunny winter day with a night peak event (16:00–20:00). This scenario allows the evaluation of the MPC controller performance under conditions where solar generation is available during the daytime, enabling potential battery charging and load shifting before the peak period.

The solar generation profile for the selected day is presented in Figure 4.3. The photovoltaic (PV) production begins in the morning and reaches its maximum value around midday, with nominal PV output approaching approximately 1.5 kW, while the uncertainty range extends up to approximately 2.5 kW due to variations in solar irradiance. As expected for winter conditions, PV production occurs only during daylight hours and decreases gradually toward the afternoon before reaching zero in the evening. The presence of this solar generation provides an opportunity for the controller to utilize locally generated electricity for both direct heating demand and battery charging.

The resulting heating power profile obtained from the MPC optimization is shown in Figure 4.4. Heating power varies dynamically throughout the day to maintain the indoor temperature within the predefined setpoint range while minimizing grid electricity consumption during the peak period. During the early morning hours, the heating demand increases briefly due to lower outdoor temperatures and thermal losses, leading to a short increase in heating power. After this period, heating power stabilizes at moderate levels during midday when solar gains and higher outdoor temperatures reduce heating demand. After this period, heating power stabilizes at moderate levels during midday when solar gains and higher outdoor temperatures reduce heating demand.

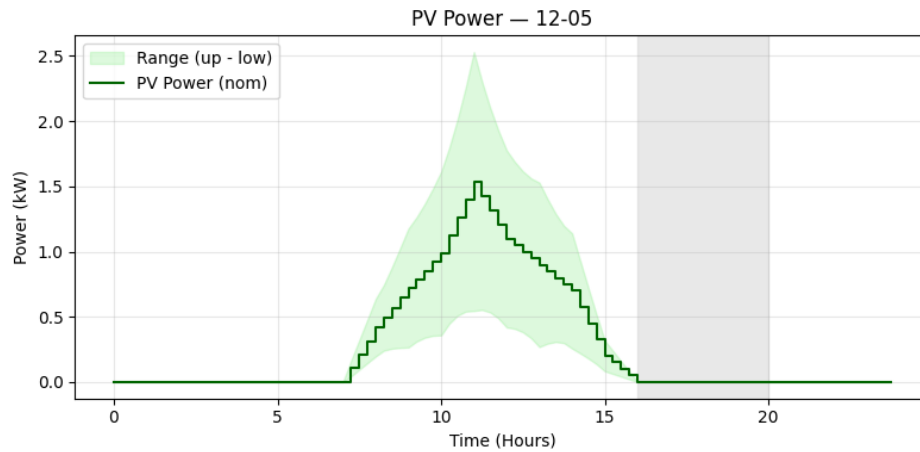


Figure 4.3: PV Power – 5 December

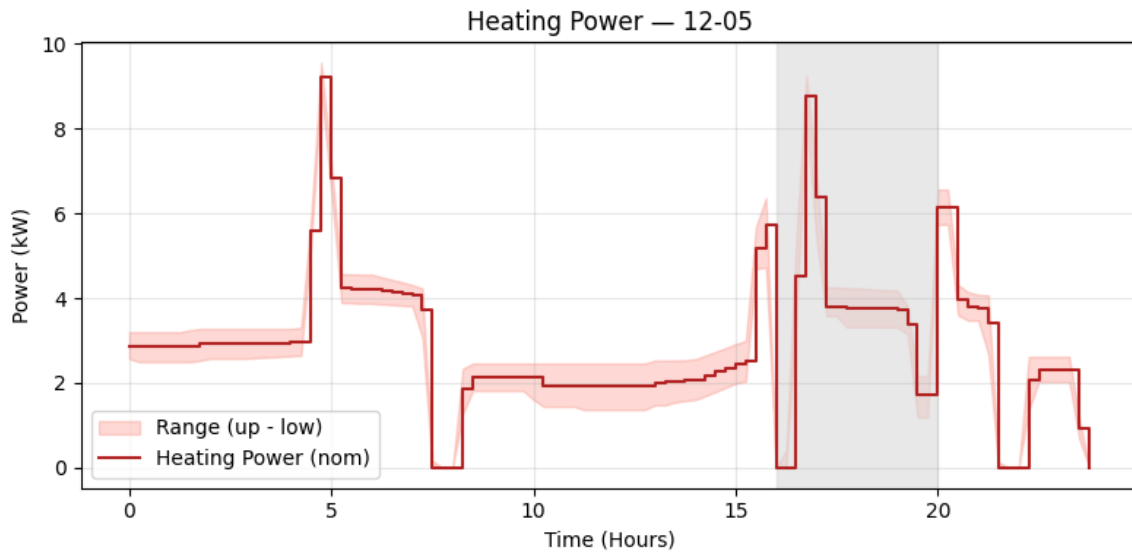


Figure 4.4: Heating Power – 5 December

A noticeable increase in heating power occurs shortly before the peak period, which indicates that the controller performs preheating of the building. This strategy increases the indoor thermal

energy stored within the building envelope and indoor air, allowing the heating system to reduce its operation during the peak tariff hours. During the 16:00–20:00 peak period, heating operation becomes more intermittent, reflecting the controller’s effort to limit electricity consumption while maintaining indoor temperature within the setpoint range.

The battery operation and the resulting state-of-charge evolution are illustrated in Figure 4.5. The battery gradually charges during the daytime when PV generation is available, reaching a maximum state of charge of approximately 100% shortly before the peak period. A minimum SOC limit of 20% is maintained to protect battery life and preserve an emergency energy buffer. This charging strategy ensures that sufficient stored energy is available to support the building loads during the peak event. During the peak hours, the battery discharges to supply part of the heating demand, thereby reducing the amount of electricity imported from the grid.

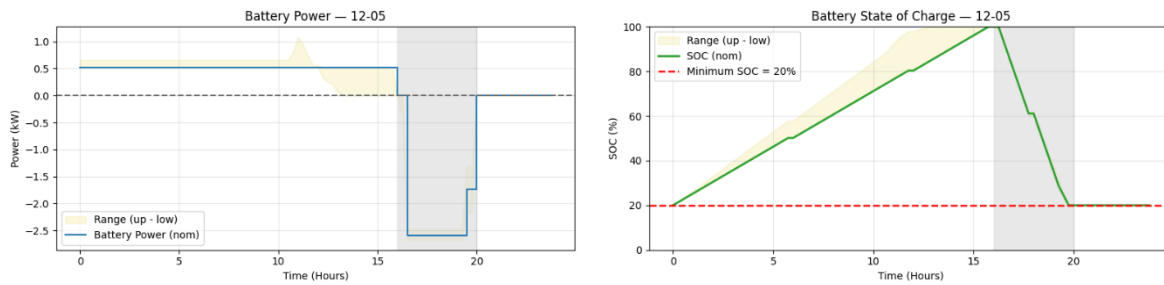


Figure 4.5: Battery Charge / Discharge and SOC – 5 December

The envelope temperature and the indoor air temperature evolution are shown in Figure 4.6. The controller maintains the indoor temperature within the defined setpoint bounds throughout the day, while slightly increasing the temperature before the peak period as part of the preheating strategy. This controlled increase in indoor temperature allows the building to utilize its thermal mass as a form of thermal energy storage, which helps reduce heating demand during the peak tariff window.

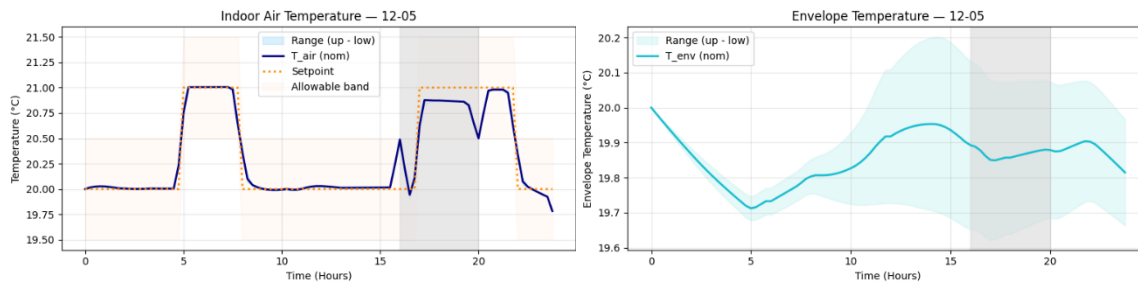


Figure 4.6: Indoor Air and Envelope Temperatures – 5 December

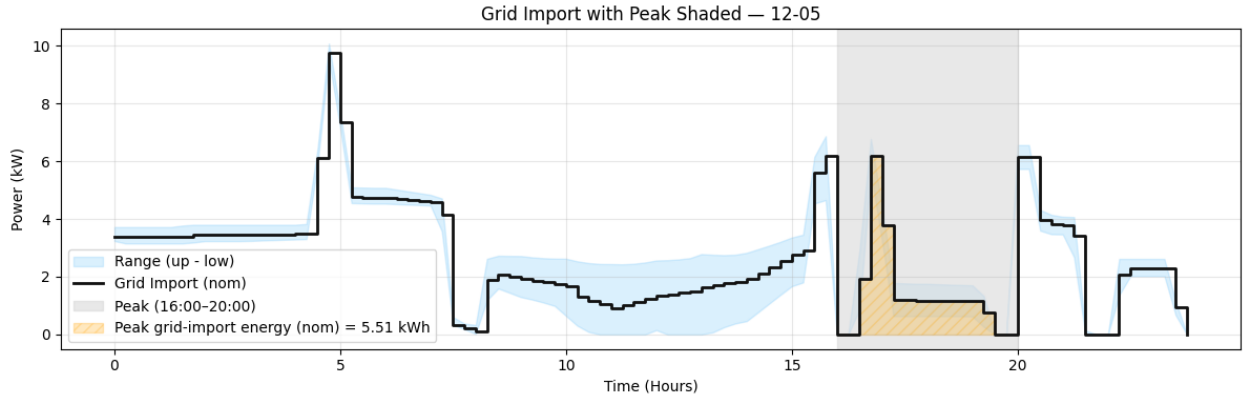


Figure 4.7: Peak Grid Import and Net Heating Power – 5 December

The resulting grid electricity import profile is presented in Figure 4.7. A clear reduction in grid import can be observed during the peak period due to the combined effect of building preheating and battery discharge. Across the considered scenarios, the total heating energy ranges from 60.18 to 78.68 kWh, while the total grid import ranges from 49.42 to 75.91 kWh. More importantly, the grid import during the peak period is reduced to 3.76–7.34 kWh, demonstrating the ability of the MPC controller to shift demand away from high price periods while maintaining satisfactory temperatures. This improvement can be clearly observed when comparing Figures 4.4 and 4.7.

The MPC setpoint temperature trajectory, shown in Figure 4.8, represents the temperature reference generated by the supervisory MPC controller and transmitted to the local heating regulators. Unlike the internal optimization variables used by the controller, this setpoint corresponds to the actual command applied to the heating system and therefore reflects the final control action implemented in practice. The setpoint is adjusted with a 0.5 °C resolution, which matches the typical control resolution of residential electric baseboard thermostats. As shown in the figure, the MPC slightly increases the setpoint temperature prior to the peak period in order to preheat the building and store thermal energy within the indoor air and building envelope. During the peak period, the setpoint is maintained close to the nominal reference temperature while allowing a small set-point deviation, which reduces heating demand and helps minimize electricity consumption during the peak tariff window. This behavior illustrates how the MPC translates the optimization results into practical temperature commands that enable demand shifting while limiting deviations from the desired temperature setpoint.

The numerical results obtained from the simulation further confirm the effectiveness of the MPC strategy for this scenario. The total daily grid import ranges between approximately 49.4 and 75.8 kWh, depending on the considered uncertainty scenario. However, the grid import during the peak

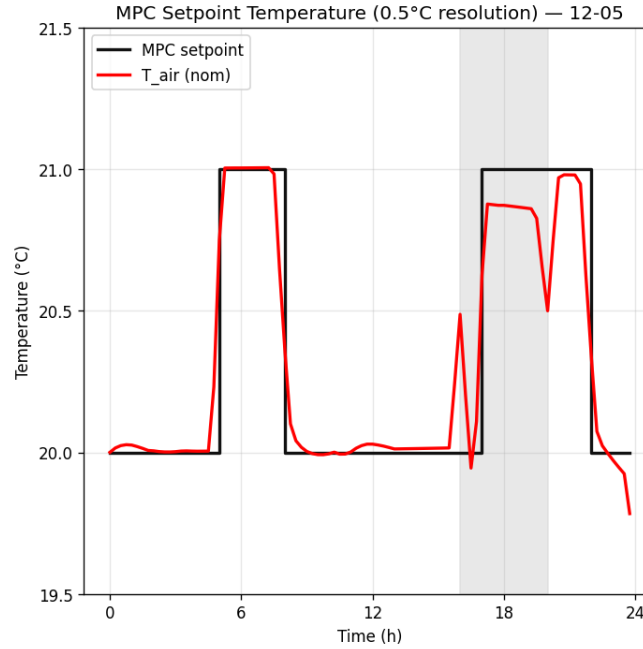


Figure 4.8: MPC Setpoint Temperature –5 December

period is significantly reduced, ranging only between 1.6 and 5.2 kWh. This represents a substantial reduction in peak electricity demand compared to typical uncontrolled heating operation.

The total heating energy required for the day varies between 60.2 and 78.7 kWh, reflecting the influence of the weather uncertainty scenarios on the heating demand. Meanwhile, PV generation ranges between 2.55 and 10.56 kWh, all of which is utilized locally within the system. As a result, the system achieves 100% PV self-consumption, meaning that all generated solar electricity is either used directly by the building or stored in the battery.

Overall, the results for the December 05 scenario demonstrate the ability of the MPC controller to effectively coordinate heating demand, battery operation, and PV generation in order to minimize grid electricity consumption during peak tariff periods while maintaining indoor temperature within the predefined setpoint range.

4.2.2 Jan 15 (Night Peak)

The second representative scenario corresponds to January 15, which represents a very cold cloudy winter day with a night peak event (16:00–20:00). This scenario is particularly important for evaluating the MPC controller under high heating demand conditions combined with limited solar availability, representing a more challenging operating case compared to the moderate cold sunny day discussed previously.

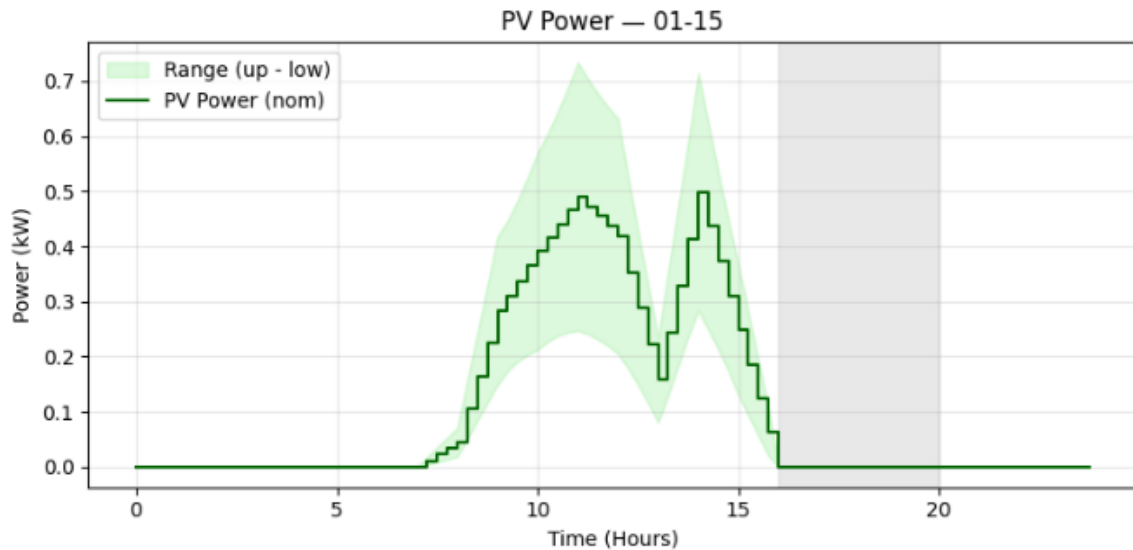


Figure 4.9: PV Power – 15 January

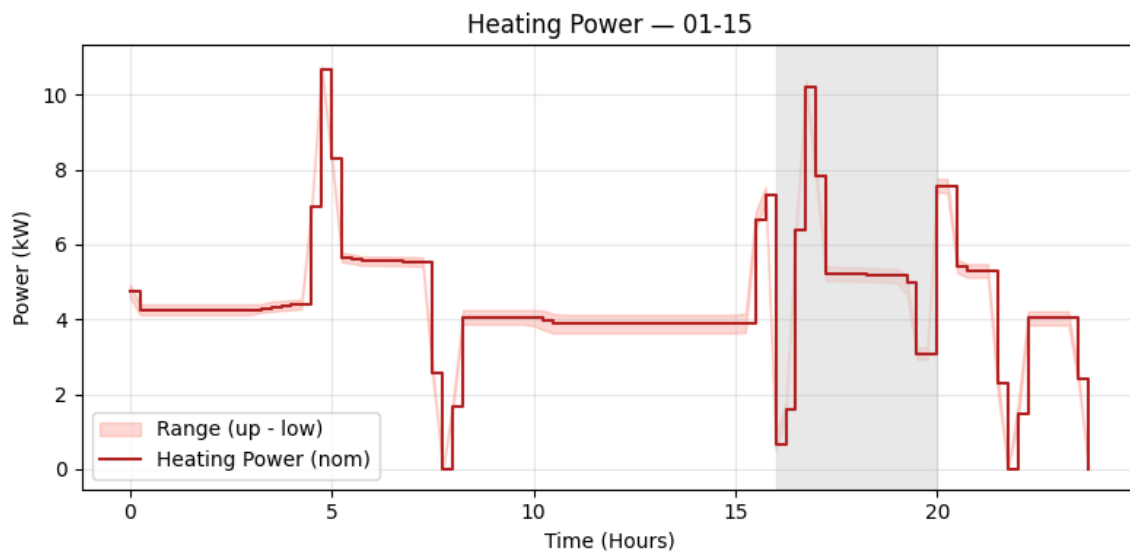


Figure 4.10: Heating Power – 15 January

The photovoltaic (PV) generation profile for this day is shown in Figure 4.9. Due to the cloudy sky conditions and the lower solar irradiance typically observed during very cold winter days, the PV output remains relatively limited throughout the day. The nominal PV power reaches values below approximately 0.6 kW, with a wider uncertainty band reflecting variations in solar irradiance. In comparison with the previous sunny-day scenario, the total available PV energy is significantly lower, ranging between approximately 1.35 and 3.72 kWh. Consequently, the contribution of PV generation to the building energy balance is reduced, and the system relies more heavily on grid electricity to meet the heating demand.

The resulting heating power profile obtained from the MPC optimization presented in Figure 4.10. Due to the lower outdoor temperatures on this day, the heating demand remains relatively high throughout the entire day. The heating power therefore operates at higher levels compared to the previous scenario, reflecting the increased thermal losses of the building envelope under very cold conditions. Despite this higher demand, the MPC controller still performs a preheating strategy prior to the peak period, increasing heating power shortly before the 16:00–20:00 peak window. This preheating allows the building to store thermal energy in the indoor air and building envelope, thereby reducing the heating requirement during the expensive peak tariff period.

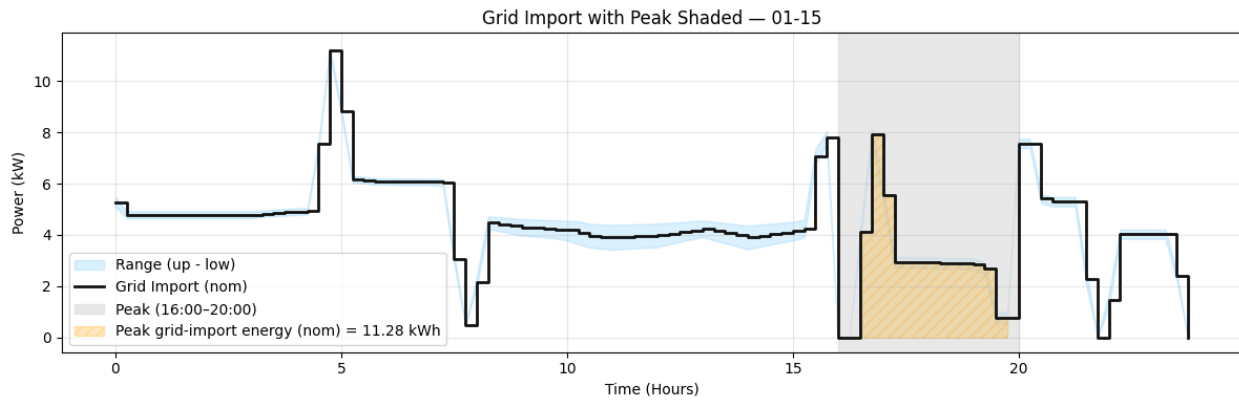


Figure 4.11: Peak Grid Import and Net Heating Power – 15 January

The resulting grid electricity import profile is illustrated in Figure 4.11. As expected for a very cold day with limited solar generation, the overall grid import is higher than in the previous scenario. The total daily grid import ranges between approximately 98.78 and 109.85 kWh, reflecting the increased heating energy demand under colder outdoor conditions, with total heating energy varying between 102.68 and 111.37 kWh across the considered scenarios. Nevertheless,

the MPC controller successfully reduces electricity consumption during the peak period through the combined effect of preheating and controlled heating reduction during the peak hours. The peak-period grid import remains limited to approximately 10.50–12.05 kWh, demonstrating the controller’s ability to shift a portion of the heating demand away from the peak tariff window even under severe weather conditions.

The MPC setpoint temperature trajectory, shown in Figure 4.12, represents the temperature reference generated by the supervisory MPC controller and transmitted to the local heating regulators. This setpoint corresponds to the actual control signal applied to the heating system, and is therefore directly implementable by the local thermostatic controllers. The setpoint is adjusted with a 0.5 °C resolution, consistent with the typical control resolution of residential electric baseboard thermostats. As observed in the figure, the MPC slightly increases the setpoint temperature prior to the peak period in order to perform building preheating and store thermal energy within the building mass. During the peak period, the setpoint is maintained close to the nominal reference temperature while allowing a limited set-point deviation, which helps reduce heating demand and consequently lowers electricity consumption during the peak tariff window.

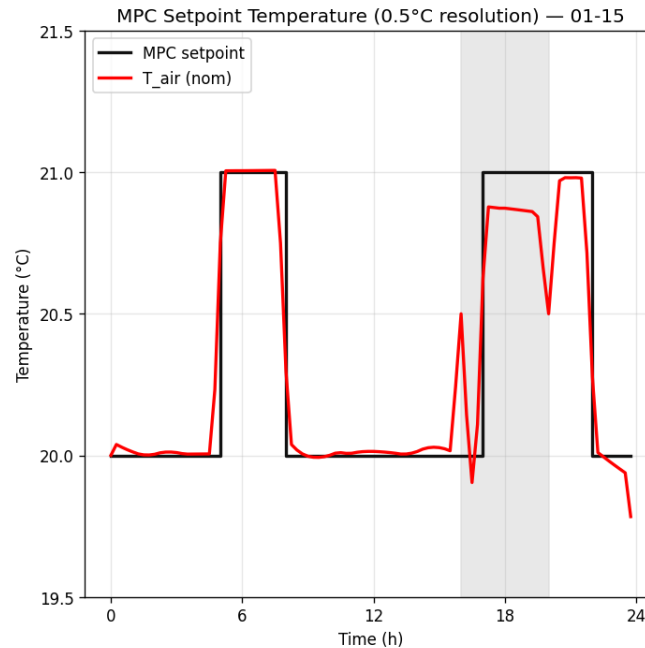


Figure 4.12: MPC Setpoint Temperature – 15 January

The numerical results further confirm the influence of the severe weather conditions on the system energy balance. The total heating energy required for the day ranges between approximately 102.7 and 111.4 kWh, which is significantly higher than the moderate cold scenario previously analyzed. Due to the limited solar availability, PV generation contributes only a small portion of the total energy supply. Nevertheless, all generated PV electricity is utilized locally within the system, resulting in 100% PV self-consumption. Overall, the results for the January 15 scenario demonstrate that the MPC controller remains effective in reducing peak-period electricity consumption and coordinating heating operation under extreme winter conditions, even when renewable energy availability is limited.

4.2.3 Mar 22 (Morning Peak)

The third representative scenario corresponds to March 22, which represents a cold partly cloudy winter day with a morning peak event (06:00–10:00). This scenario differs from the previous two cases because the peak electricity tariff occurs before the main solar generation period. As a result, the controller must rely primarily on thermal preheating of the building and battery operation during the early morning hours, while the available solar energy becomes beneficial later in the day.

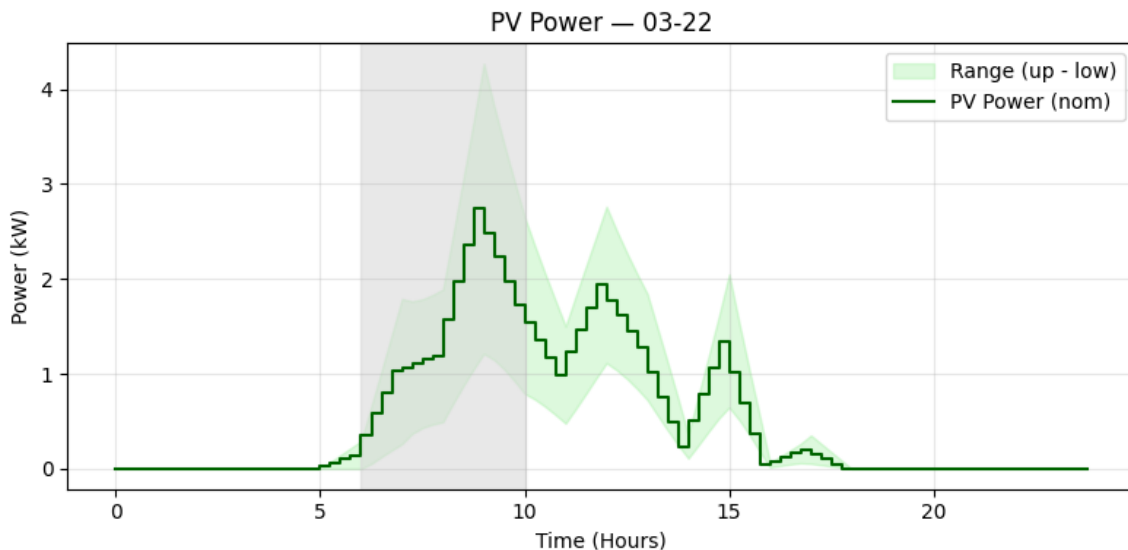


Figure 4.13: PV Power – 22 March

The photovoltaic (PV) generation profile for this day is illustrated in Figure 4.13. Due to the partly cloudy conditions, the PV production exhibits noticeable fluctuations throughout the daytime period. The nominal PV power reaches values close to 3 kW, with a wider uncertainty band reflecting the variability of solar irradiance caused by intermittent cloud cover. The total daily PV generation ranges between approximately 6.0 and 19.7 kWh, which is significantly higher than the cloudy January scenario and provides substantial renewable energy availability during the middle of the day. However, since the peak period occurs early in the morning, this solar energy cannot directly contribute to reducing grid consumption during the peak window.

The heating power profile obtained from the MPC optimization is shown in Figure 4.14. A noticeable increase in heating power occurs shortly before the morning peak period, indicating that the controller performs building preheating during the early hours of the day. This strategy increases the thermal energy stored within the indoor air and building envelope prior to the expensive tariff period. During the peak hours (06:00–10:00), the heating power decreases significantly as the controller temporarily reduces heating demand while relying on the previously stored thermal energy. After the peak period, the heating power gradually increases again to restore the building temperature and compensate for heat losses.

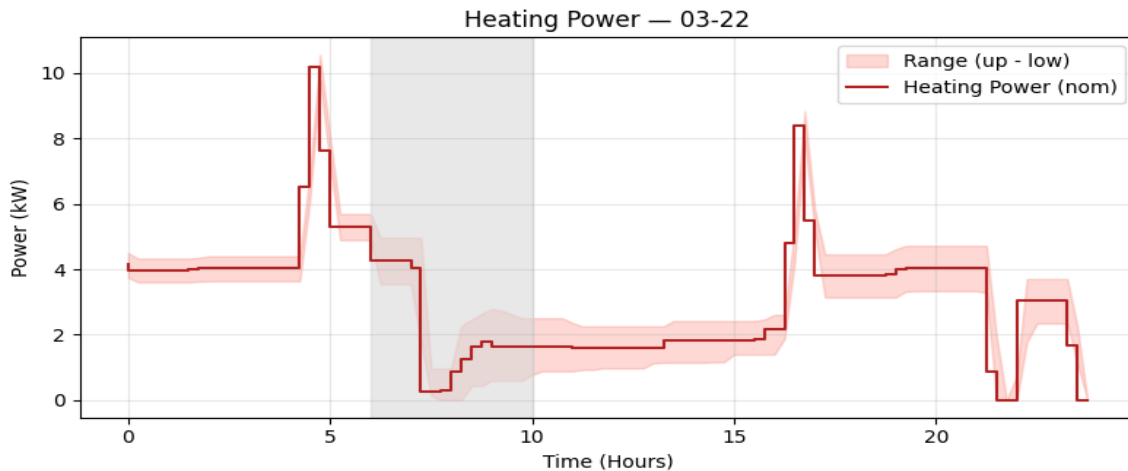


Figure 4.14: Heating Power – 22 March

The resulting grid electricity import profile is presented in Figure 4.15. The effectiveness of the MPC strategy is clearly visible in this figure. While grid import is relatively high during the early morning preheating phase, it drops substantially during the peak tariff window. The peak-period grid import remains extremely low, ranging between approximately 0.00 and 0.43 kWh, indicating

that the controller almost completely avoids electricity consumption during the peak hours. This demonstrates the capability of the MPC controller to shift heating demand away from the peak period even when solar generation is not available at that time.

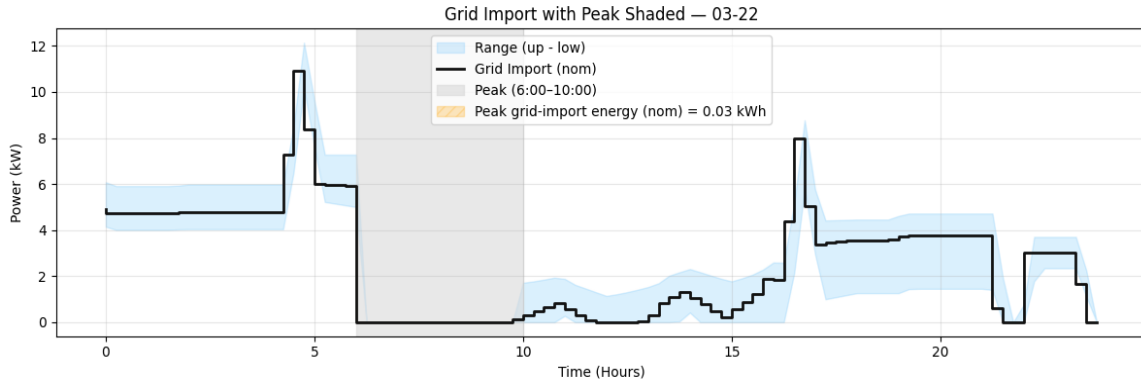


Figure 4.15: Peak Grid Import and Net Heating Power – 22 March

The MPC setpoint temperature trajectory, shown in Figure 4.16, represents the temperature reference generated by the supervisory MPC controller and transmitted to the local heating regulators. This setpoint corresponds to the actual control signal applied to the heating system, and therefore reflects the final implementable control action. The setpoint is adjusted with a 0.5 °C resolution, consistent with the typical control resolution of residential electric baseboard thermostats. As observed in the figure, the MPC slightly increases the setpoint temperature before

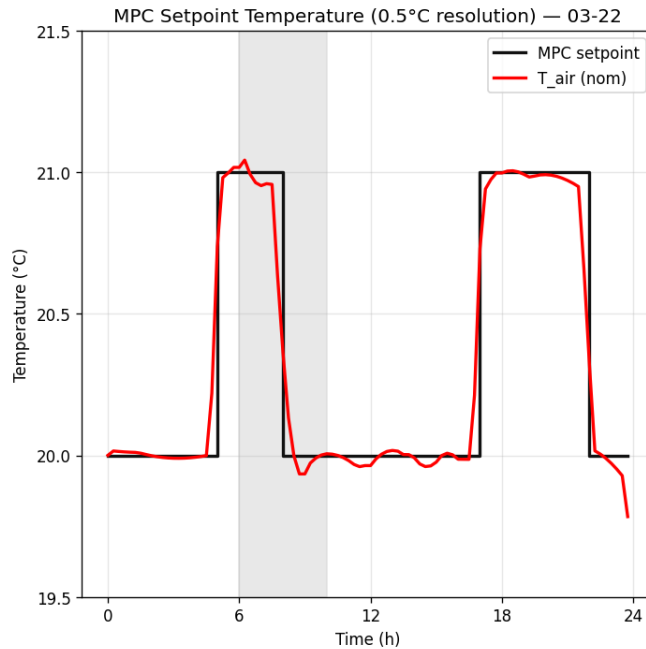


Figure 4.16: MPC Setpoint Temperature – 22 March

the peak period to initiate building preheating. During the peak hours, the setpoint is allowed to decrease slightly relative to the nominal reference temperature, introducing a controlled set-point deviation that reduces heating demand and helps minimize electricity consumption during the peak tariff window.

The numerical results further confirm the effectiveness of the MPC strategy under this morning peak scenario. The total daily grid import ranges between approximately 41.0 and 83.9 kWh, depending on the considered uncertainty scenario. However, the grid import during the peak period remains extremely low, demonstrating the controller's ability to nearly eliminate electricity consumption during the peak hours. The total heating energy required for the day varies between 60.3 and 89.2 kWh, reflecting the influence of weather uncertainty on heating demand. Due to the relatively high solar availability on this day, PV generation contributes significantly to the building energy balance. Nearly all generated PV electricity is utilized locally within the system, resulting in PV self-consumption levels between approximately 98.5% and 100%.

Overall, the results for the March 22 scenario demonstrate the flexibility of the MPC controller in adapting its control strategy to different peak tariff configurations. By performing early preheating and allowing controlled set-point deviations during the peak period, the controller effectively minimizes peak electricity consumption even when solar generation occurs after the peak window.

4.2.4 One-Week MPC Performance (Feb 03 – Feb 09)

To further evaluate the robustness and operational stability of the proposed MPC framework, the controller was applied over a continuous one-week simulation period from February 03 to February 09. Unlike the previously analyzed representative days, which illustrate specific weather scenarios, the weekly simulation allows the assessment of controller behavior under continuously varying environmental conditions. This extended horizon enables the evaluation of the MPC's ability to maintain consistent control performance while adapting to variations in outdoor temperature, solar availability, and heating demand over multiple consecutive days.

The photovoltaic (PV) generation profiles during the selected week are illustrated in Figure 4.17. As expected for winter conditions in Montréal, PV production occurs during daylight hours and

exhibits noticeable variability between days due to changing cloud conditions. The nominal PV generation peaks at values between approximately 1.0 and 3.0 kW, while the uncertainty bands capture the variability associated with forecast errors in solar irradiance. The total weekly PV production ranges between approximately 33.1 and 85.1 kWh, providing a non-negligible contribution to the building energy supply despite the relatively short winter daylight period.

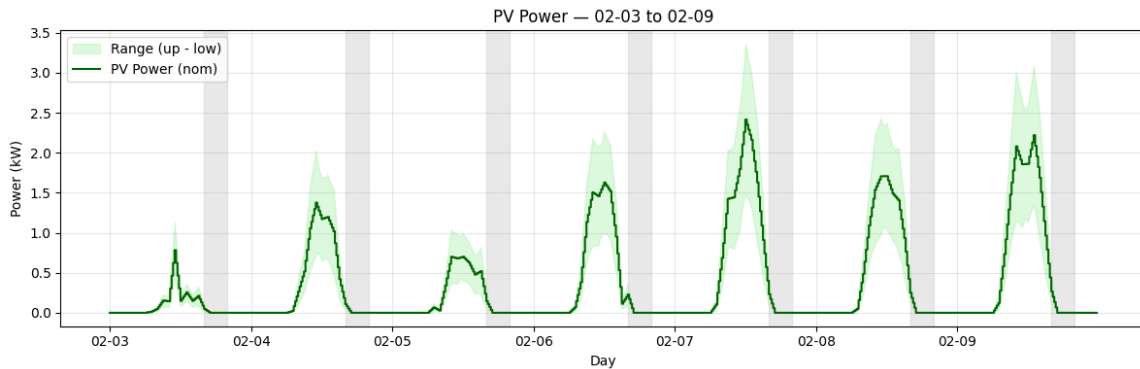


Figure 4.17: PV Power (03 – 09 Feb) Week

The resulting heating power profiles obtained from the MPC optimization are presented in Figure 4.18. The heating demand varies significantly throughout the week due to changes in outdoor temperature and solar gains. Higher heating power levels can be observed during colder days, while lower values occur when outdoor temperatures increase or when solar gains partially offset the building heat losses. Despite these variations, the controller consistently applies a preheating strategy prior to each peak tariff window, which is visible as temporary increases in heating power before the shaded peak periods. This behavior indicates that the MPC systematically stores thermal energy within the building mass in advance of the peak hours.

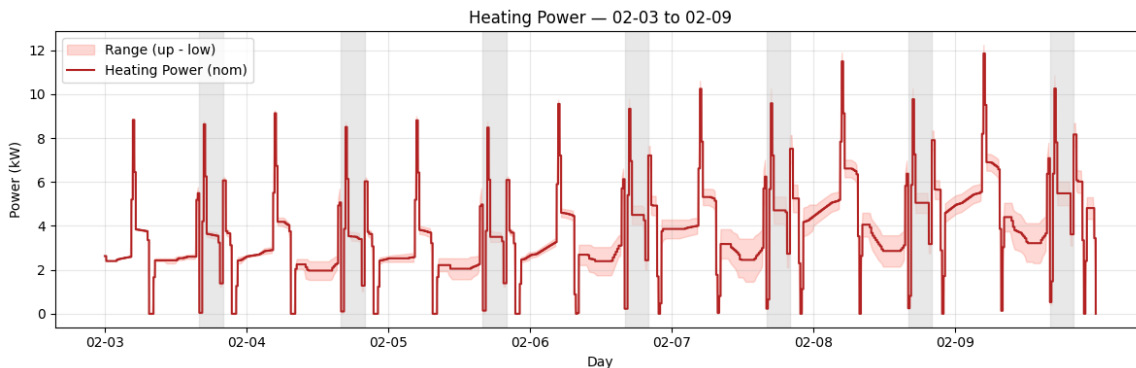


Figure 4.18: Heating Power (03 – 09 Feb) Week

The corresponding grid electricity import profile is shown in Figure 4.19. Similar to the individual daily scenarios, the MPC controller successfully reduces grid electricity consumption during the peak periods throughout the entire week. While grid import increases during the preheating phase

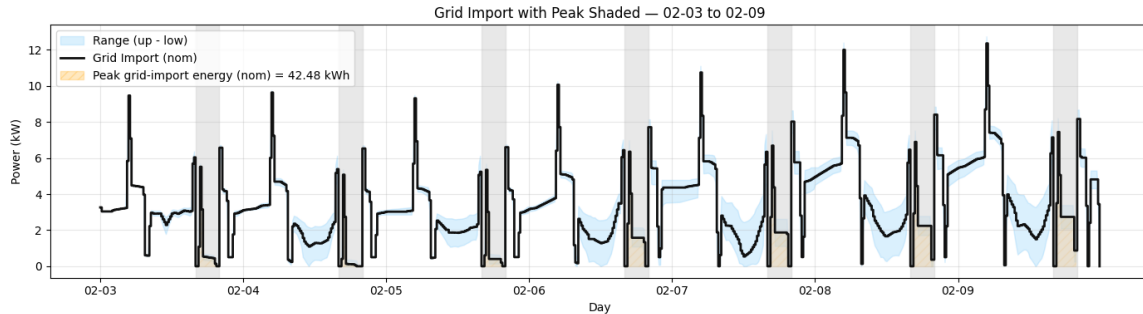


Figure 4.19: Peak Grid Import and Net Heating Power (03 – 09 Feb) Week

preceding each peak event, it decreases significantly during the peak hours as the controller temporarily reduces heating demand. Over the entire week, the total grid import ranges between approximately 460 and 639 kWh, depending on the considered uncertainty scenario. More importantly, the electricity consumed during peak periods remains relatively limited, ranging between approximately 31.8 and 53.5 kWh, demonstrating the effectiveness of the MPC strategy in shifting heating demand away from the peak tariff window across multiple consecutive days.

The MPC setpoint temperature trajectory, shown in Figure 4.20, represents the temperature reference generated by the supervisory MPC controller and transmitted to the local heating regulators. This trajectory corresponds to the actual control signal applied to the heating system, and is therefore directly implementable by the local thermostatic controllers. Throughout the week, the controller repeatedly increases the setpoint temperature before each peak period to initiate building preheating, followed by a slight reduction during the peak hours. This controlled modulation introduces a limited set-point deviation, which allows the controller to temporarily reduce heating demand while maintaining stable indoor temperature conditions.

The numerical results further confirm the stability of the MPC performance over the extended simulation horizon. The total heating energy required during the week ranges between approximately 545 and 672 kWh, corresponding to a variation of about 21%, reflecting the combined influence of weather variability and uncertainty scenarios. Despite the variability in heating demand, the controller maintains consistent operation and effectively coordinates heating

demand with available solar generation. Nearly all generated PV electricity is utilized locally within the system, resulting in PV self-consumption values close to 100%.

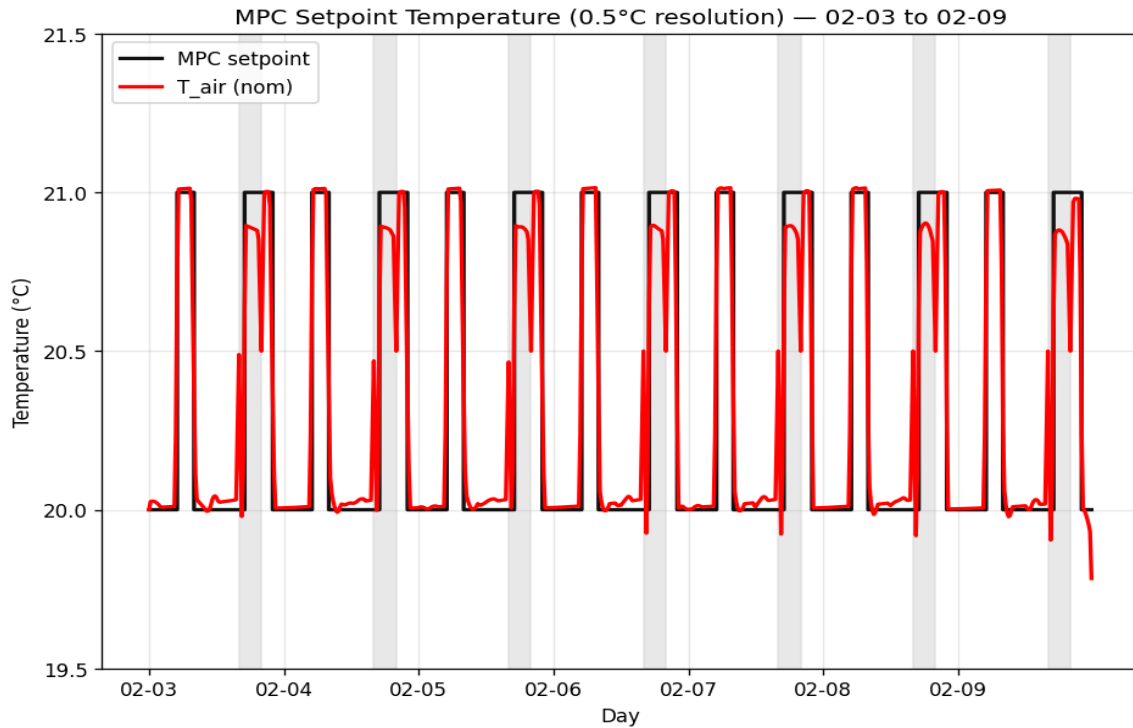


Figure 4.20: MPC Setpoint Temperature (03 – 09 Feb) Week

Overall, the weekly simulation results demonstrate that the proposed MPC framework maintains stable and consistent control performance over multiple consecutive days. By systematically applying preheating strategies and controlled set-point deviations during peak periods, the controller successfully reduces peak electricity consumption while adapting to changing weather conditions throughout the week.

4.3 Reactive Control Performance

4.3.1 Reactive Control without PV–Battery

To establish a baseline for system operation, the building was simulated under a reactive control strategy without PV–battery integration. In this configuration, the heating system operates based solely on the local temperature setpoint without predictive capabilities or energy storage. The

controller reacts directly to indoor temperature deviations from the setpoint, resulting in heating power adjustments only after temperature changes occur.

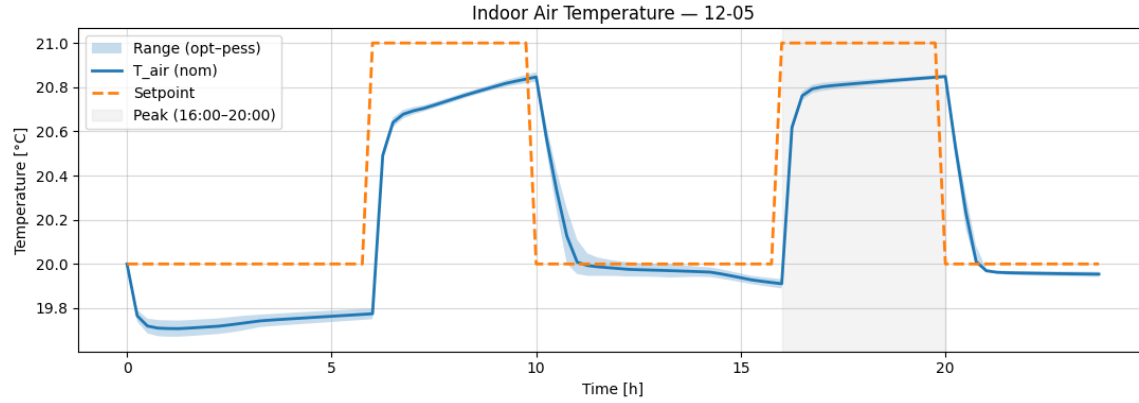


Figure 4.21: Indoor Setpoint and Air Temperatures – 5 December

The indoor air temperature and heating power profiles for December 05 are shown in Figure 4.21 and Figure 4.22. The temperature follows the prescribed setpoint schedule, with slight deviations due to the thermal dynamics of the building. When the setpoint increases, the heating system produces a short power spike as the indoor temperature rises toward the desired value. Once the temperature approaches the setpoint, the heating power decreases and stabilizes at a lower level required to compensate for the building heat losses.

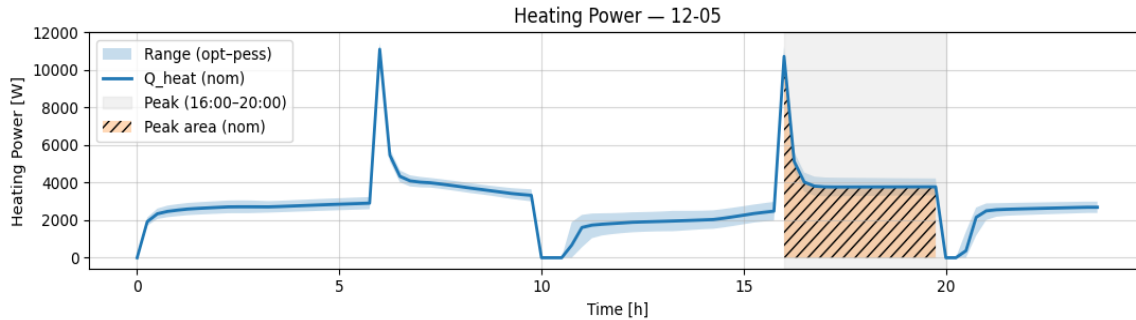


Figure 4.22: Heating Power & Peak Grid Import – 5 December

The corresponding heating power profile shows that significant heating demand occurs during the peak period (16:00–20:00). Because the reactive controller does not anticipate future peak events, heating operation continues normally during this period. As a result, a considerable portion of the daily heating energy is consumed during the peak window. For this scenario, the total daily heating energy ranges between 59.4 and 77.8 kWh, while the peak-period heating energy ranges between 15.3 and 19.2 kWh, corresponding to approximately 24.6–25.7% of the daily heating demand.

The heating power profiles for January 15 and March 22 are presented in Figure 4.23 and Figure 4.24, respectively. Similar reactive behavior can be observed in both cases. The heating system responds directly to temperature changes without any anticipation of the peak tariff periods.

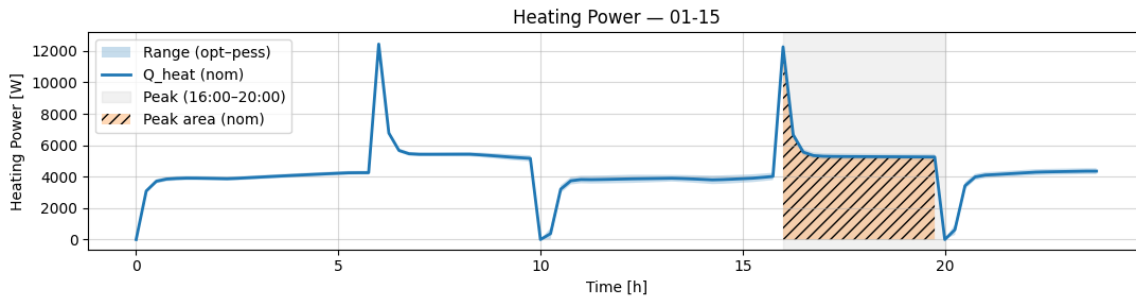


Figure 4.23: Heating Power & Peak Grid Import – 15 January

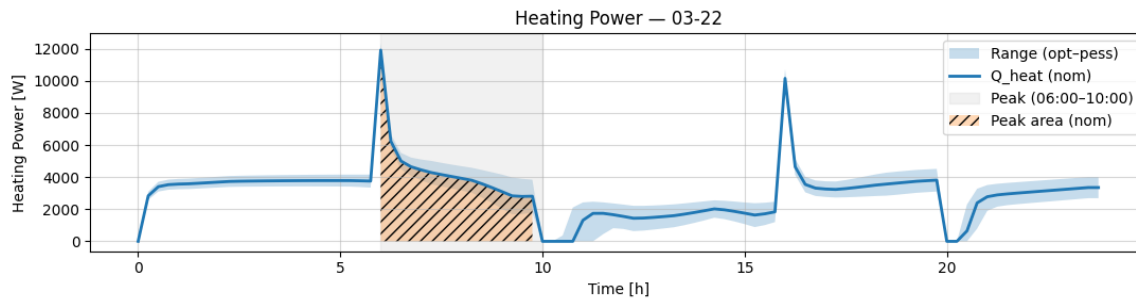


Figure 4.24: Heating Power & Peak Grid Import – 22 March

Consequently, heating power remains active during the peak windows. For the very cold day (January 15), the total heating energy increases to approximately 101.4–110.0 kWh, with 22.5–24.1 kWh consumed during the peak period, corresponding to roughly 21.9–22.1% of the total heating demand. In the morning peak scenario (March 22), the total heating demand ranges between 59.1 and 87.6 kWh, while 14.6–20.9 kWh is consumed during the peak window, representing approximately 23.8–24.7% of the daily heating energy.

Overall, the reactive control strategy maintains indoor temperature close to the scheduled setpoint but does not actively manage heating demand relative to the peak tariff periods. As a result, a substantial portion of the heating energy is consumed during the peak windows across all representative days.

4.3.2 Reactive Control with PV–Battery

The reactive control strategy was also evaluated with PV generation and battery storage integrated into the system. In this configuration, the heating system continues to operate purely based on the indoor temperature setpoint, while the PV–battery subsystem follows predefined rule-based charging and discharging rules. As a result, the indoor air temperature behavior remains identical to the reactive control case without PV–battery integration, and only the energy flows associated with PV generation and battery operation are discussed in this section.

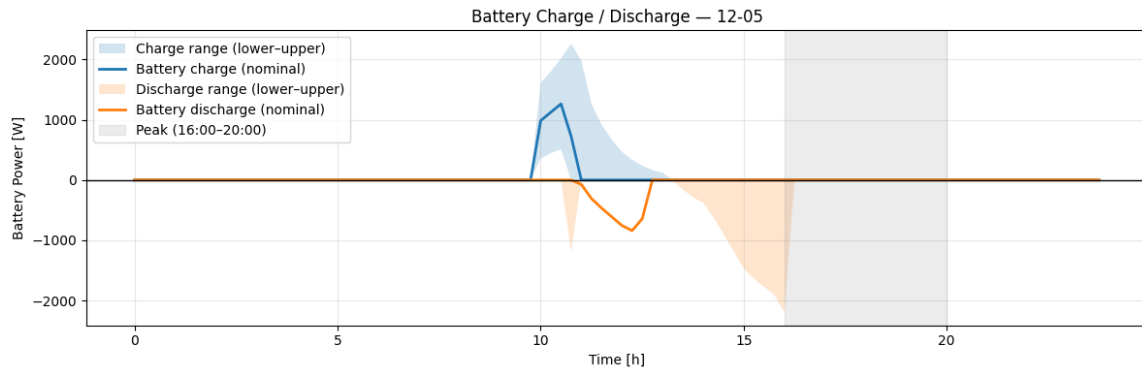


Figure 4.25: Battery Charging / Discharging – 5 December

For December 05, the heating and PV power profiles are presented in Figure 4.25. PV generation occurs during daytime hours and partially contributes to the building heating demand. Excess PV production is used to charge the battery as shown in Figure 4.26, which later discharges to support the load. However, since the heating control remains reactive, a significant portion of the heating demand still occurs during the evening peak period (16:00–20:00). The total heating energy ranges between 59.4–77.8 kWh, with 15.3–19.2 kWh consumed during the peak window. The battery

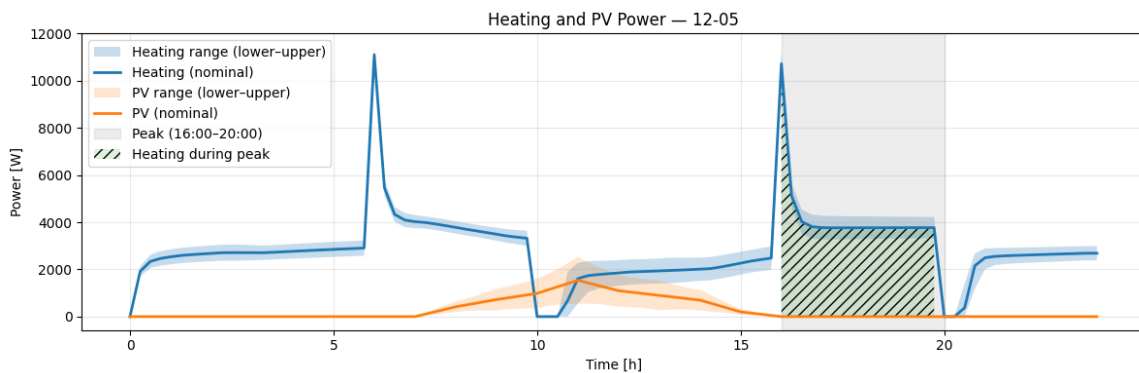


Figure 4.26: Heating Power, PPV, and Peak Grid Import – 5 December

activity remains moderate, with charging between 0.33–3.46 kWh and discharging between 0.30–3.12 kWh.

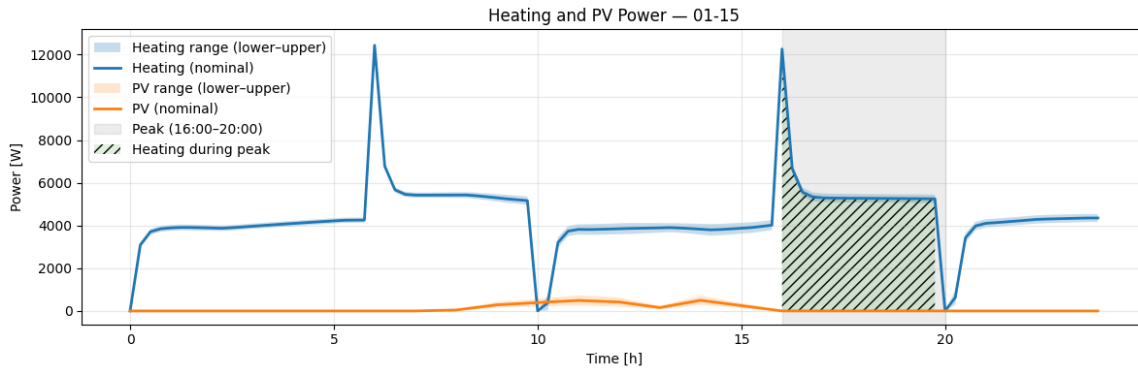


Figure 4.27: Heating Power, PPV, and Peak Grid Import – 15 January

For January 15, the heating and PV profiles are shown in Figure 4.27. Due to the very cold and cloudy conditions, PV production is limited (1.35–3.72 kWh), resulting in minimal battery operation. Consequently, most of the heating demand is supplied by the grid. The total heating energy increases to 101.4–110.0 kWh, with 22.5–24.1 kWh occurring during the peak period.

For March 22, the heating and PV power profiles are illustrated in Figure 4.28. In this case, the tariff peak occurs during the morning period (06:00–10:00), while PV generation mainly occurs later in the day. As a result, PV energy cannot directly assist during the peak hours. The battery behavior for this day is shown in Figure 4.29, where charging occurs primarily during midday PV production and discharging occurs afterward. The total heating demand ranges between 59.1–87.6

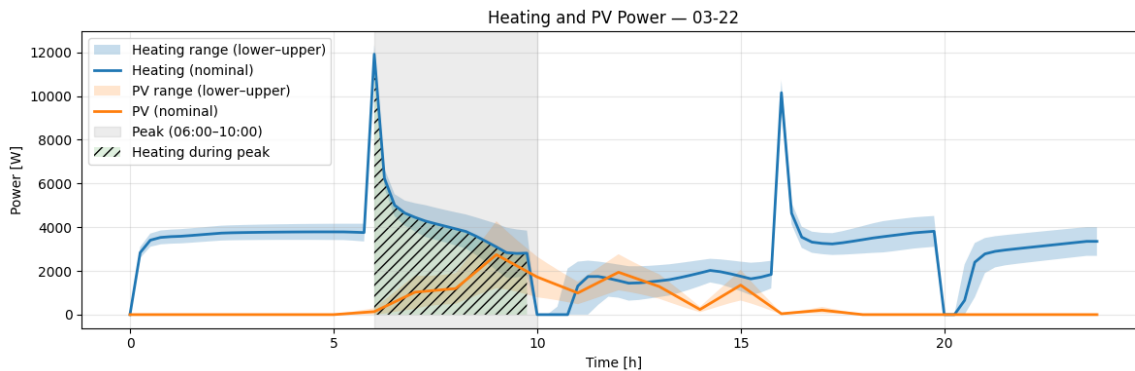


Figure 4.28: Heating Power, PPV, and Peak Grid Import – 22 March

kWh, with 14.6–20.9 kWh occurring during the peak window. PV generation varies between 6.02–19.70 kWh, leading to more noticeable battery activity compared to the other two scenarios.

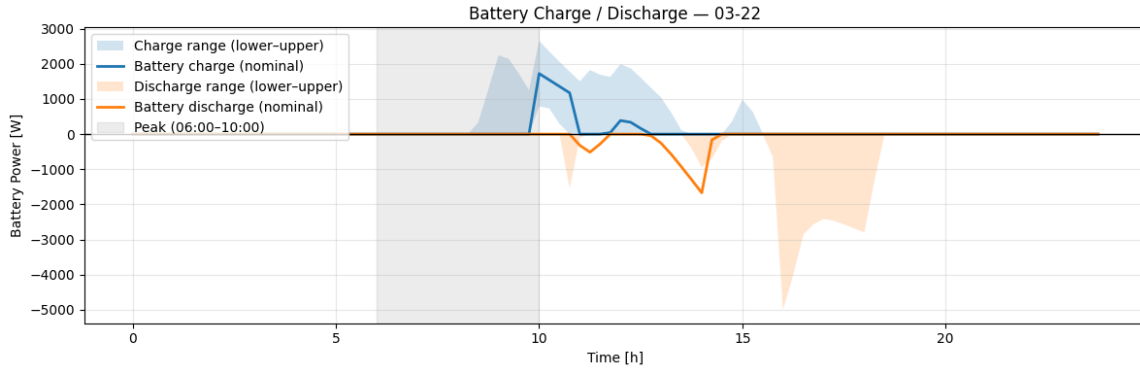


Figure 4.29: Battery Charge / Discharge - 22 March

Overall, the PV–battery system provides additional locally generated electricity depending on solar availability. However, because the heating operation remains purely reactive, a considerable portion of the heating demand continues to occur during the peak tariff periods.

4.3.3 One-Week Reactive Control Performance (Feb 03 – Feb 09)

The reactive control strategy was further evaluated over a one-week period (Feb 03–Feb 09) under continuously varying weather conditions. The heating power profile for the configuration without PV–battery is shown in Figure 4.30. Since the controller reacts only to indoor temperature deviations, heating demand frequently occurs during the tariff peak windows. Over the week, the total heating energy ranges between 545.9–672.5 kWh, with 123.4–146.2 kWh occurring during peak periods, corresponding to approximately 21.7–22.6% of the total heating demand.

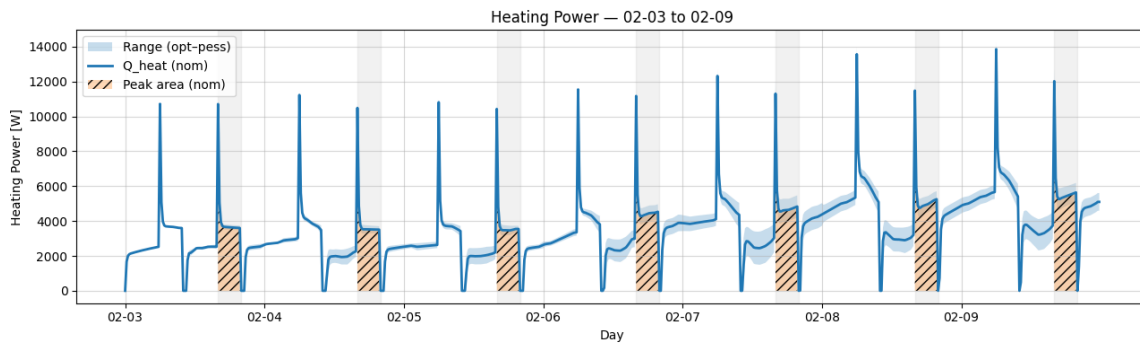


Figure 4.30: Heating Power & Peak Grid Import (03 – 09 Feb) Week

The heating and PV power profiles for the reactive control with PV–battery integration is presented in Figure 4.31. PV generation occurs during daytime hours and partially offsets the building load, producing 33.1–85.1 kWh over the week. The corresponding battery operation is illustrated in Figure 4.32, where the battery charges during PV availability and discharges later to support the load. Weekly battery charging ranges between 2.1–16.7 kWh, while discharging ranges between 1.9–15.1 kWh. Despite the presence of PV and storage, the heating control remains reactive, and a substantial portion of the demand still occurs during peak tariff periods.

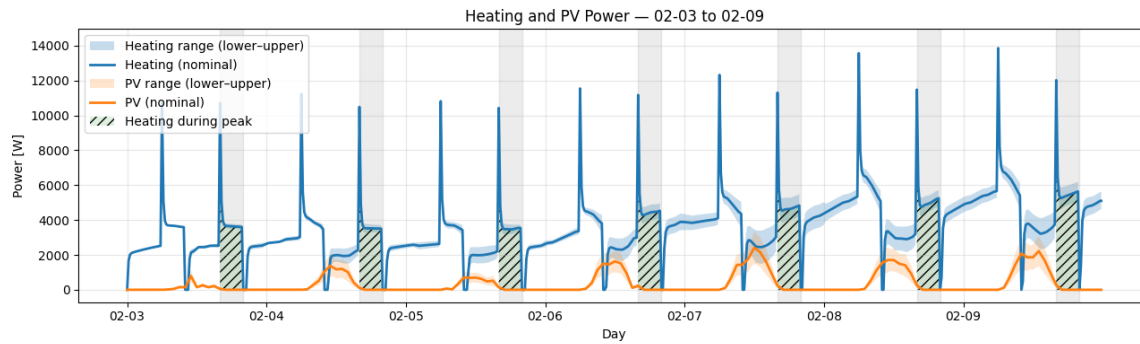


Figure 4.31: Heating Power & Peak Grid Import (03 – 09 Feb) Week

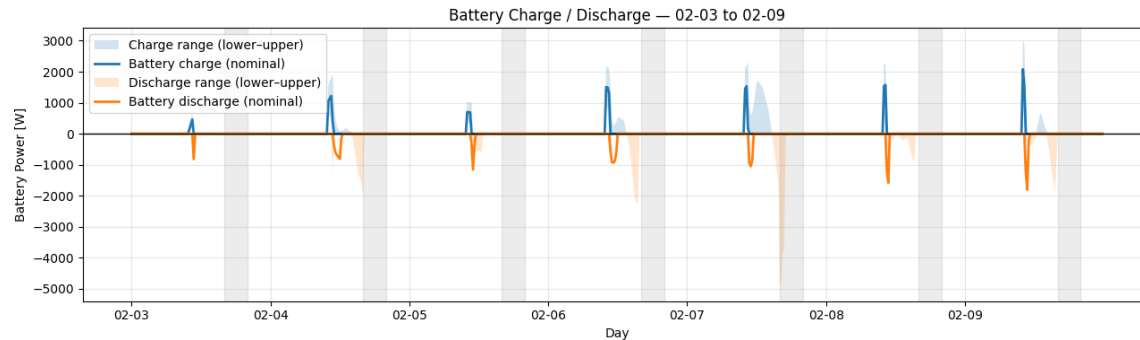


Figure 4.32: Battery Charge / Discharge (03 – 09 Feb) Week

4.4 Comparison of MPC and Reactive Control

Table 4.3 presents the simulation results for the selected daily case studies, comparing the performance of the model predictive control (MPC) strategy with the two reactive control strategies. The comparison includes total heating energy, grid electricity import, peak grid import, PV production, and daily electricity cost.

Table 4.3: Comparison of MPC and Reactive Control Simulation Results, One-Day Results

Day	Control Strategy	Total Heating Energy (kWh)	Grid Import (kWh)	Peak Grid Import (kWh)	PPV (kW)	Daily Energy Cost (\$)
Dec 5 (M. Cold – Sunny)	Reactive No PV & Batt	59 - 78	59 - 78	15 - 19	-	10 - 13
	Reactive PV & Batt	59 - 78	49 - 75	15 - 19	3 - 11	9 - 12
	MPC	60 - 79	49 - 76	4 - 7	3 - 11	5 - 8
Jan 15 (V. Cold - Cloudy)	Reactive No PV & Batt	101 - 110	101 - 110	23 - 24	-	16 - 17
	Reactive PV & Batt	101 - 110	98 - 109	23 - 24	1 - 4	16 - 17
	MPC	102 - 111	99 - 110	11 - 12	1 - 4	11 - 13
Mar 22 (Cold – P. Cloudy)	Reactive No PV & Batt	59 - 88	59 - 88	15 - 21	-	9 - 14
	Reactive PV & Batt	59 - 88	40 - 82	15 - 21	6 - 20	8 - 14
	MPC	60 - 89	41 - 84	0 - 0	6 - 20	2 - 7

The results indicate that total heating energy remains nearly identical for all control strategies. For example, on Dec 5 the total heating energy ranges between 59–78 kWh for the reactive strategies and 60–79 kWh for MPC. Similar behavior is observed for the other evaluated days. This confirms that MPC mainly shifts heating demand in time rather than reducing the total heating requirement of the building.

More significant differences are observed in peak grid import. On Dec 5, MPC reduces peak grid import from 15–19 kWh under reactive control to 2–5 kWh. On Jan 15 peak grid import decreases from 23–24 kWh to 8–10 kWh when MPC is applied. The most notable improvement occurs on Mar 22, where MPC completely eliminates grid import during the peak period. These results demonstrate the ability of MPC to shift heating demand away from peak tariff hours.

The reduction in peak grid import leads to corresponding reductions in electricity cost. On Dec 5, daily electricity cost decreases from 10–13 \$ under reactive control without PV–battery integration to 4–7 \$ with MPC. On Jan 15 the cost decreases from 16–17 \$ to 10–12 \$, while on Mar 22 it drops from 9–14 \$ to 2–7 \$.

Table 4.4: Comparison of MPC and Reactive Control Simulation Results, one-Week Results

	Control Strategy	Total Heating Energy (kWh)	Grid Import (kWh)	Peak Grid Import (kWh)	PPV (kW)	Weekly Energy Cost (\$)
Feb 3 – Feb 9	Reactive No PV & Batt	546 - 673	546 - 673	123 - 146	-	85 – 104
	Reactive PV & Batt	546 - 673	462 - 640	123 - 146	33 - 85	77 – 101
	MPC	546 - 672	460 - 639	32 - 54	33 - 85	44 – 67

Table 4.4 presents the results of the weekly evaluation. Similar to the daily results, total heating energy remains nearly identical for all strategies, ranging between 546–673 kWh. However, significant reductions are observed in peak grid import. Under reactive control, peak grid import ranges between 123–146 kWh, while MPC reduces it to 32–54 kWh. This reduction leads to a corresponding decrease in weekly electricity cost, from 85–104 \$ under reactive control without PV–battery integration to 44–67 \$ when MPC is applied.

Weather forecast uncertainty in one-day-ahead predictions introduces noticeable variability in system performance. Based on the reactive control case, this results in average variations of approximately 21% in peak grid import and 23% in daily electricity cost across the considered scenarios.

To better quantify the improvements achieved by MPC, Table 4.5 summarizes the percentage reductions in peak grid import and electricity cost relative to the reactive control strategies. The results show that MPC reduces peak grid import by 58–100% depending on the scenario.

Correspondingly, electricity cost reductions range between 29–78% compared to reactive control without PV–battery integration and 29–75% compared to reactive control with PV–battery integration. For the weekly evaluation, peak grid import reductions range between 56–78%, while weekly electricity cost reductions range between 13–58%.

Table 4.5: Comparison of MPC and Reactive Control Peak Grid Import and Cost Reduction Percentages.

Case	Comparison	Peak Grid Import Reduction (%)	Electricity Cost Reduction (%)
Dec 5 – Moderate Cold Sunny	MPC vs Reactive (No PV & Battery)	63 – 73	38 – 50
	MPC vs Reactive (PV & Battery)	63 – 73	33 – 44
Jan 15 – Very Cold Cloudy	MPC vs Reactive (No PV & Battery)	50 – 52	24 – 31
	MPC vs Reactive (PV & Battery)	50 – 52	24 – 31
Mar 22 – Cold Partly Cloudy	MPC vs Reactive (No PV & Battery)	100	50 – 78
	MPC vs Reactive (PV & Battery)	100	50 – 75
Feb 3–9 – Weekly Evaluation	MPC vs Reactive (No PV & Battery)	56 - 74	20 – 58
	MPC vs Reactive (PV & Battery)	63 - 78	13 – 43

4.5 Effect of Weather Uncertainty on MPC

Weather forecast uncertainty in one-day-ahead predictions introduces significant variability in the performance of the proposed MPC framework. Based on the analysis conducted over all winter days, the impact of uncertainty is quantified by comparing the upper, nominal, and lower scenarios. The results indicate that weather uncertainty leads to average variations of approximately 46% in peak-period grid import and 38% in daily electricity cost. These variations arise from deviations

in both outdoor temperature and solar irradiance, which directly influence heating demand and PV generation, and consequently affect the MPC scheduling decisions.

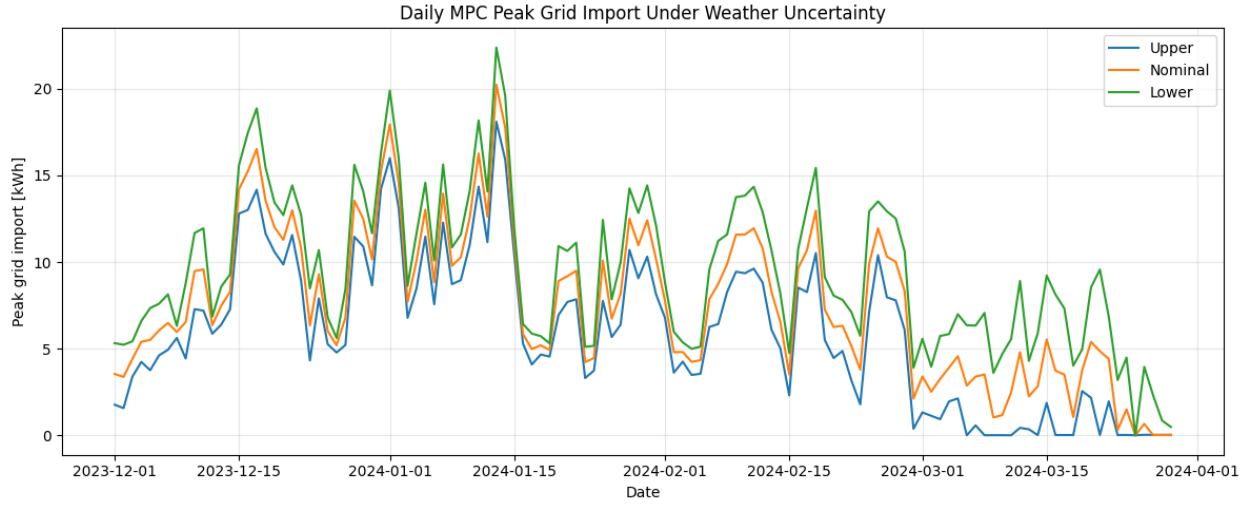


Figure 4.33: Effect of Weather Uncertainty on Daily MPC Peak Grid Import

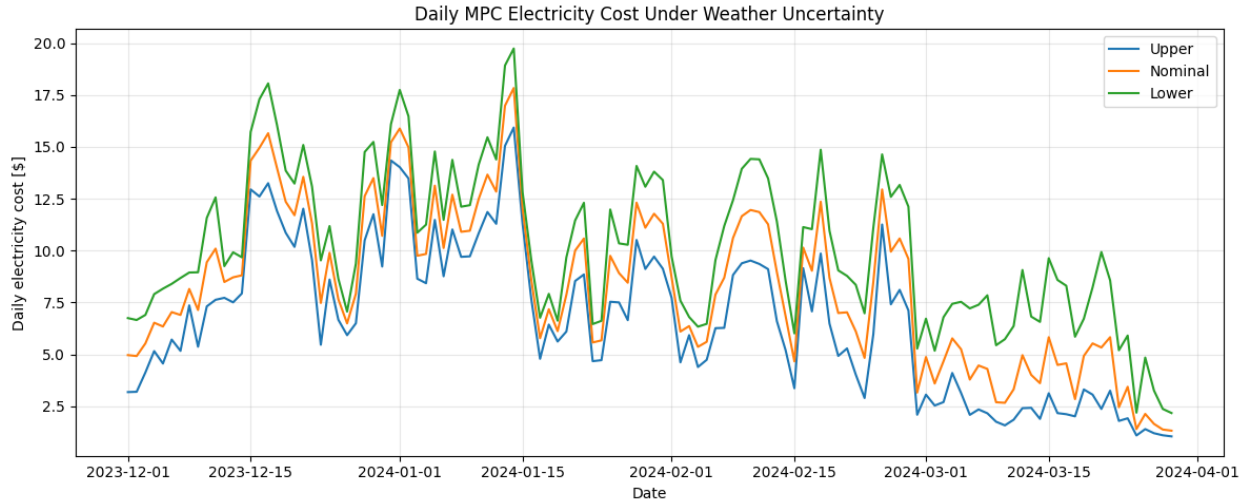


Figure 4.34: Effect of Weather Uncertainty on Daily MPC Electricity Cost

As illustrated in Figure 4.33 and Figure 4.34, the daily evolution of peak grid import and electricity cost exhibits consistent separation between the upper, nominal, and lower scenarios across the analyzed period. The upper scenario results in increased reliance on grid electricity during peak periods, while the lower scenario benefits from reduced heating demand and/or enhanced PV contribution. Despite these variations, the MPC controller maintains its ability to shift loads and mitigate peak demand, demonstrating robust performance under uncertain weather conditions.

These findings confirm that, although MPC effectively enhances energy flexibility, its performance remains sensitive to forecast accuracy, highlighting the importance of reliable short-term weather predictions in achieving optimal operation

In summary, the results demonstrate that the proposed MPC framework effectively coordinates heating demand, PV generation, and battery operation to reduce peak electricity consumption under varying weather conditions. While the total heating energy remains similar across the control strategies, the MPC successfully shifts energy consumption away from peak tariff periods, resulting in substantial reductions in peak grid import and electricity cost. Moreover, the results confirm that the MPC maintains stable performance under the considered weather uncertainty scenarios. By incorporating uncertainty into the control formulation, the controller adopts a more robust operational strategy that ensures reliable temperature regulation while maintaining effective peak demand reduction. The consistent performance observed across both daily and weekly simulations highlights the robustness and practical applicability of the proposed MPC framework for enhancing residential energy flexibility under uncertain weather conditions.

Chapter 5: Conclusions and Future Work

5.1 Summary of the Study

This thesis investigated the use of model predictive control (MPC) to enhance the operational flexibility of electrically heated residential buildings under uncertain weather conditions. A predictive control framework was developed for a detached house equipped with a photovoltaic (PV) system and battery storage. The building thermal dynamics were represented using a control-oriented grey-box model based on a 3R–2C resistance–capacitance structure. Weather uncertainty in outdoor temperature and solar irradiance was incorporated through scenario-based representations within the MPC formulation.

The proposed control strategy was evaluated under different winter operating conditions and compared with conventional reactive control approaches both with and without PV–battery integration. The analysis included both single-day and multi-day simulation periods in order to examine the controller behavior under varying weather conditions. In addition, sensitivity analyses were conducted to examine the influence of controller tuning parameters on system performance and operational flexibility.

5.2 Key Findings and Contributions

The results demonstrate that MPC can significantly improve the operational flexibility of electrically heated residential buildings compared with conventional reactive control strategies. By anticipating upcoming peak tariff periods, the MPC framework shifts a portion of the heating demand to off-peak hours through controlled preheating of the building thermal mass. This strategy reduces grid electricity consumption during critical peak periods while maintaining stable indoor temperature conditions. This work can be further extended to include other electric energy systems such as heat pumps and integrated electric vehicles, enabling a more comprehensive assessment of residential energy flexibility.

From the end-user perspective, this load shifting leads to lower electricity operating costs under dynamic pricing structures such as the Hydro-Québec Rate Flex D tariff. At the same time, reducing peak-period grid import improves building-level energy flexibility by decreasing reliance

on grid electricity during high demand periods. From a broader system perspective, mitigating residential peak loads, where space heating represents a dominant share of winter electricity demand, can support improved reliability and resilience of the electricity supply.

The integration of PV generation and battery storage further enhances system performance and energy flexibility when coordinated through the predictive controller. The MPC strategy schedules battery charging during periods of PV availability or low electricity cost and discharges stored energy during peak tariff periods to support heating demand. In contrast, the reactive control approach operates without predictive scheduling, charging the battery when excess PV generation is available and discharging it when heating demand occurs. As a result, it does not actively contribute to peak load shifting and is less effective in utilizing the available flexibility of the PV–battery system.

The analysis also shows that incorporating weather uncertainty within the predictive framework improves the robustness of control decisions. By considering conservative weather scenarios, the controller maintains reliable indoor temperature regulation while limiting peak-period grid import. Although this approach may occasionally result in slightly higher total heating energy due to precautionary preheating, it ensures stable system performance under varying meteorological conditions.

In addition, the study demonstrates that weather forecast uncertainty affects the performance of the MPC framework by introducing variability in both peak-period grid import and daily electricity cost. These variations arise from changes in heating demand and PV generation, which influence the controller’s scheduling decisions. Despite this, the MPC maintains robust operation by continuously adapting to updated forecasts while still achieving effective peak reduction and cost savings.

The main outcomes of this study can be summarized as follows:

- The proposed model predictive control (MPC) framework shifts a portion of heating demand from peak tariff periods to off-peak hours through anticipatory use of the building thermal mass, enabling effective temporal redistribution of heating loads.

- Peak-period grid electricity consumption is significantly reduced compared with conventional reactive control strategies, with peak grid import reductions ranging approximately from 67–75% for systems without PV–battery integration and 69–76% when PV–battery systems are included.
- This peak-demand reduction translates into lower electricity operating costs under the Hydro-Québec Rate Flex D tariff, resulting in daily electricity cost reductions ranging approximately from 33–54% without PV–battery systems and 30–48% with PV–battery integration.
- Coordinated operation of the PV–battery system within the MPC framework improves the utilization of locally generated solar energy and further supports peak demand reduction by optimally scheduling battery charging during low-cost or PV-generation periods and discharging during peak tariff periods.
- Explicit consideration of weather uncertainty enhances the robustness of control decisions and ensures stable indoor temperature regulation under varying meteorological conditions while maintaining reduced peak-period grid import.
- Controller tuning parameters influence the balance between temperature tracking and operational flexibility, highlighting the importance of appropriate weighting in predictive control design.
- The analysis quantifies the impact of one-day-ahead weather forecast uncertainty on system performance, resulting in average variations of approximately 46% in peak grid import and 38% in daily electricity cost, highlighting the importance of accounting for uncertainty in predictive control applications.

Overall, the thesis demonstrates that predictive control combined with PV–battery integration can effectively enhance the energy flexibility of residential heating systems in cold climates, reducing peak grid electricity dependence by approximately 67–76% and lowering electricity operating costs by approximately 30–54%, while maintaining stable indoor set point conditions.

5.3 Limitations of the Study

The analysis assumes that windows and external doors remain closed during the simulation period and does not explicitly consider occupant behavior. In practice, occupant interactions such as window opening, ventilation adjustments, occupancy schedules, and internal heat gains can influence indoor temperature conditions and heating demand. Since the study focused on a typical single-family detached house, these factors were neglected to simplify the analysis. However, in real applications, occupant presence and behavior may affect building energy use and heating system operation.

A simplified approach was adopted to estimate photovoltaic (PV) power generation. The PV output was calculated using a simplified relationship between the rated PV capacity and solar irradiance. While this method provides a reasonable approximation for system level analysis, it does not account for detailed PV performance factors such as temperature effects, inverter efficiency, or system losses.

Finally, PV power generation was estimated using global horizontal irradiance (GHI). In practice, PV panels are installed with a specific tilt angle and orientation, which means that the incident solar radiation on the panel surface may differ from the horizontal irradiance. This simplification was adopted to maintain a consistent and simplified comparison between the predictive and reactive control strategies.

5.4 Recommendations for Future Work

Future work could extend the present research by applying the proposed control framework to multiple houses or aggregated residential communities. Evaluating coordinated control of groups of buildings would allow assessment of the potential contribution of residential demand flexibility to distribution network peak reduction and grid stability.

In addition, future studies could incorporate occupant-related factors that were neglected in this work. Including occupancy schedules, internal heat gains, and window opening behavior would provide a more realistic representation of building operation and allow the evaluation of predictive control performance under varying occupant interactions.

Moreover, the integration of heat pump systems within the MPC optimization framework under weather forecast uncertainty represents a promising direction for future research, as it remains relatively underexplored in the existing literature, particularly for cold-climate residential buildings where both thermal dynamics and system performance are strongly affected by weather variability.

Finally, advanced predictive control formulations such as stochastic or adaptive MPC could be investigated to better account for uncertainty in weather forecasts and building operation. Moreover, extending the framework to year-round operation, including cooling demand in warmer seasons, would further broaden the applicability of predictive control strategies for residential buildings.

5.5 Research Outcomes

The research outcomes of this thesis have been disseminated through a peer-reviewed conference paper and a poster presentation, contributing to the field of building energy flexibility and predictive control under weather uncertainty.

Conference Paper:

Khedr, A., Abtahi, M., Athienitis, A. K., *Development of Heating Energy Flexibility in Canadian Houses with PV–Battery Systems Using Model Predictive Control under Weather Uncertainty*, accepted for presentation at the **eSim 2026 Conference**, Canada, 2026.

Poster Presentation:

Khedr, A., Abtahi, M., Athienitis, A. K., *Effect of Weather Forecast Uncertainty on Model Predictive Control for Electrically Heated Canadian Houses with PV–Battery Systems*, presented at the **Electrify Society Summit (Volt-Age)**, Canada, 2026.

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Appendix A: K-Means Clustering for Outside Temperature

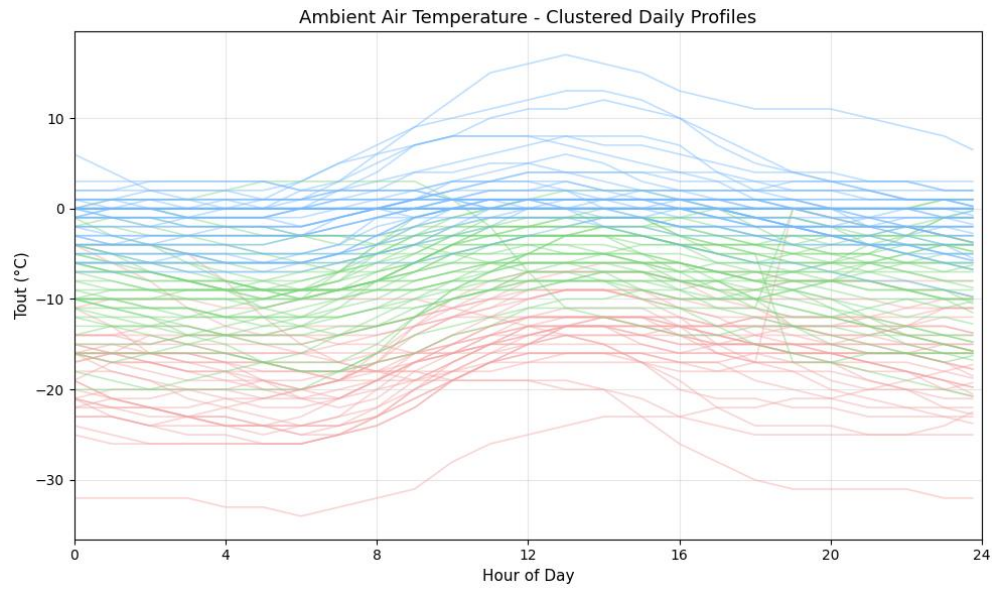


Figure A- 1: K-Means Clustering of The Outside Temperature

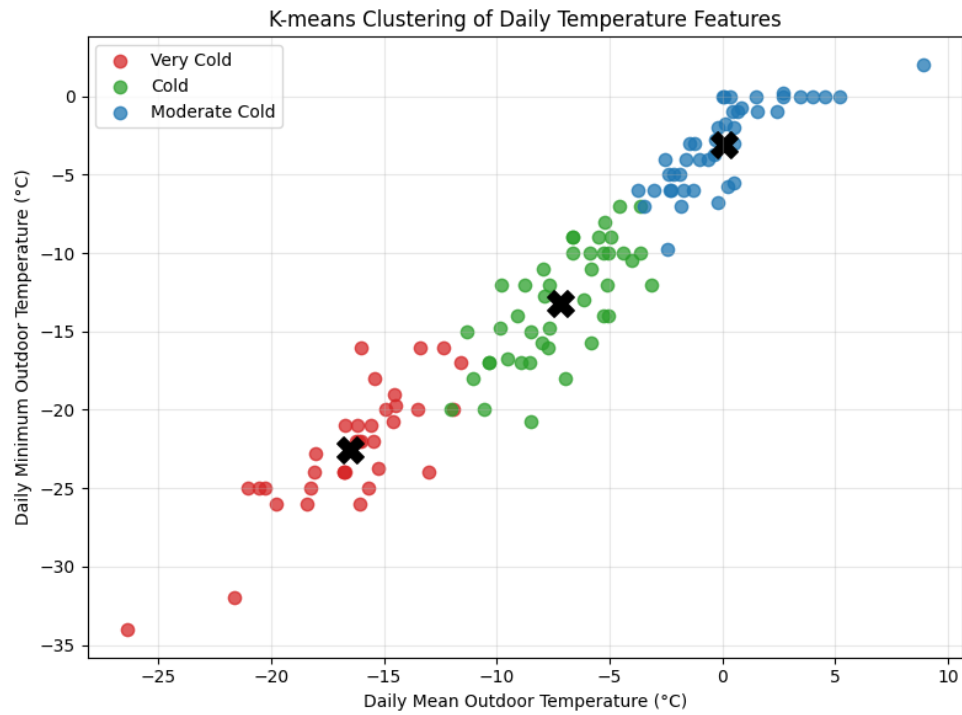


Figure A- 2: Scatter Representation K-Means Clustering of The Outside Temperature

Appendix B: Hottel Clear Sky Model for GHI based Day Classification

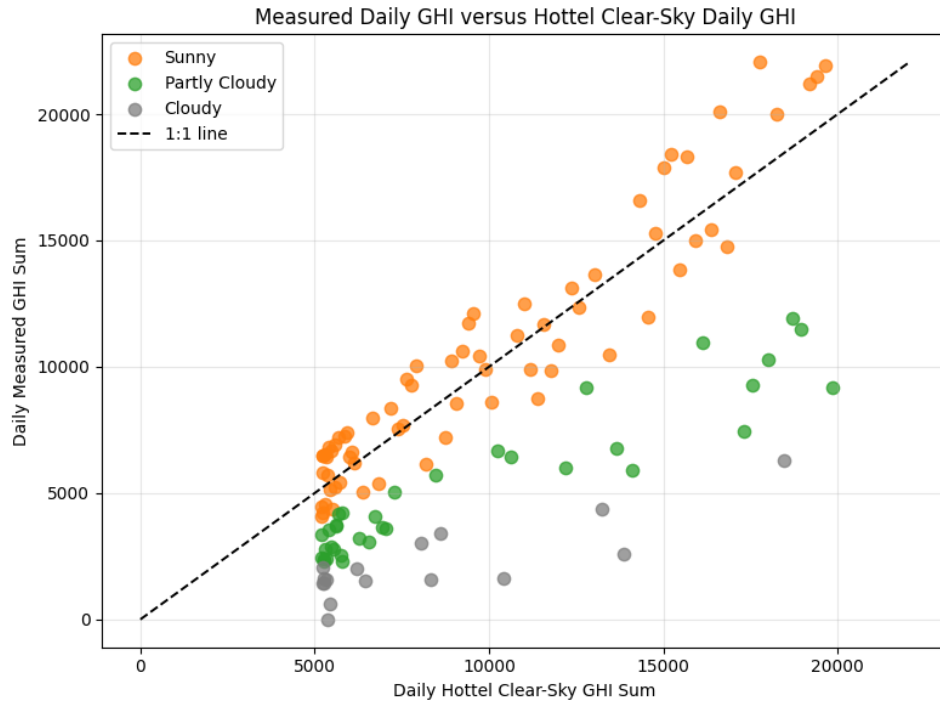


Figure B- 1: Measured Daily GHI Versus Hottel Clear Sky Daily GHI – Categorized

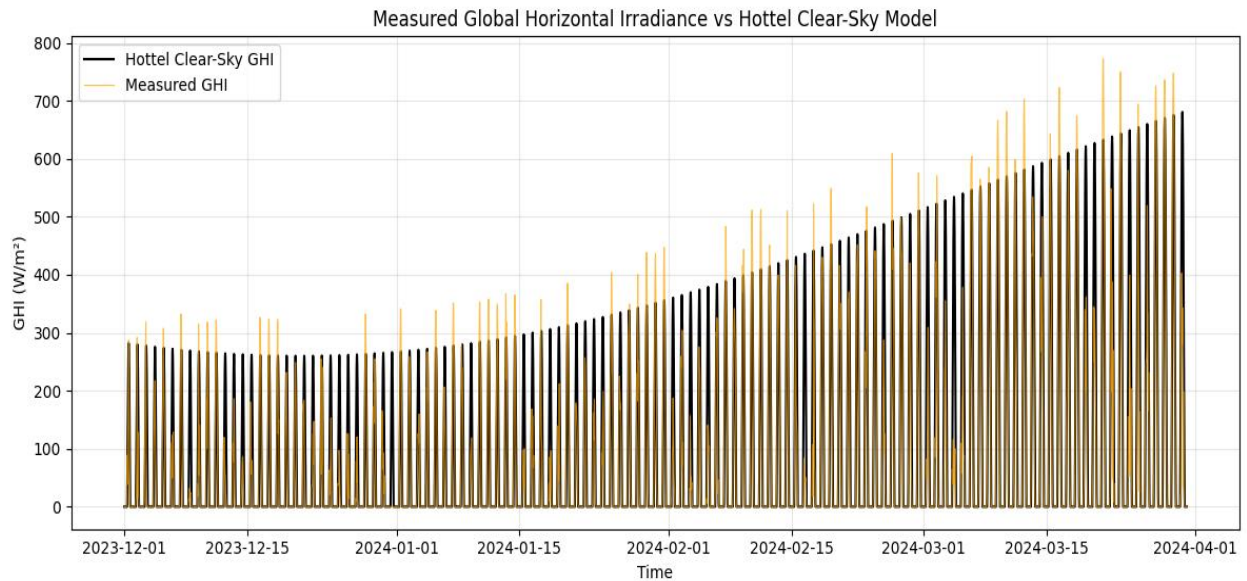


Figure B- 2: Measured Daily GHI Versus Hottel Clear Sky Daily GHI – Uncategorized

Appendix B: Hottel Clear Sky Model for GHI based Day Classification

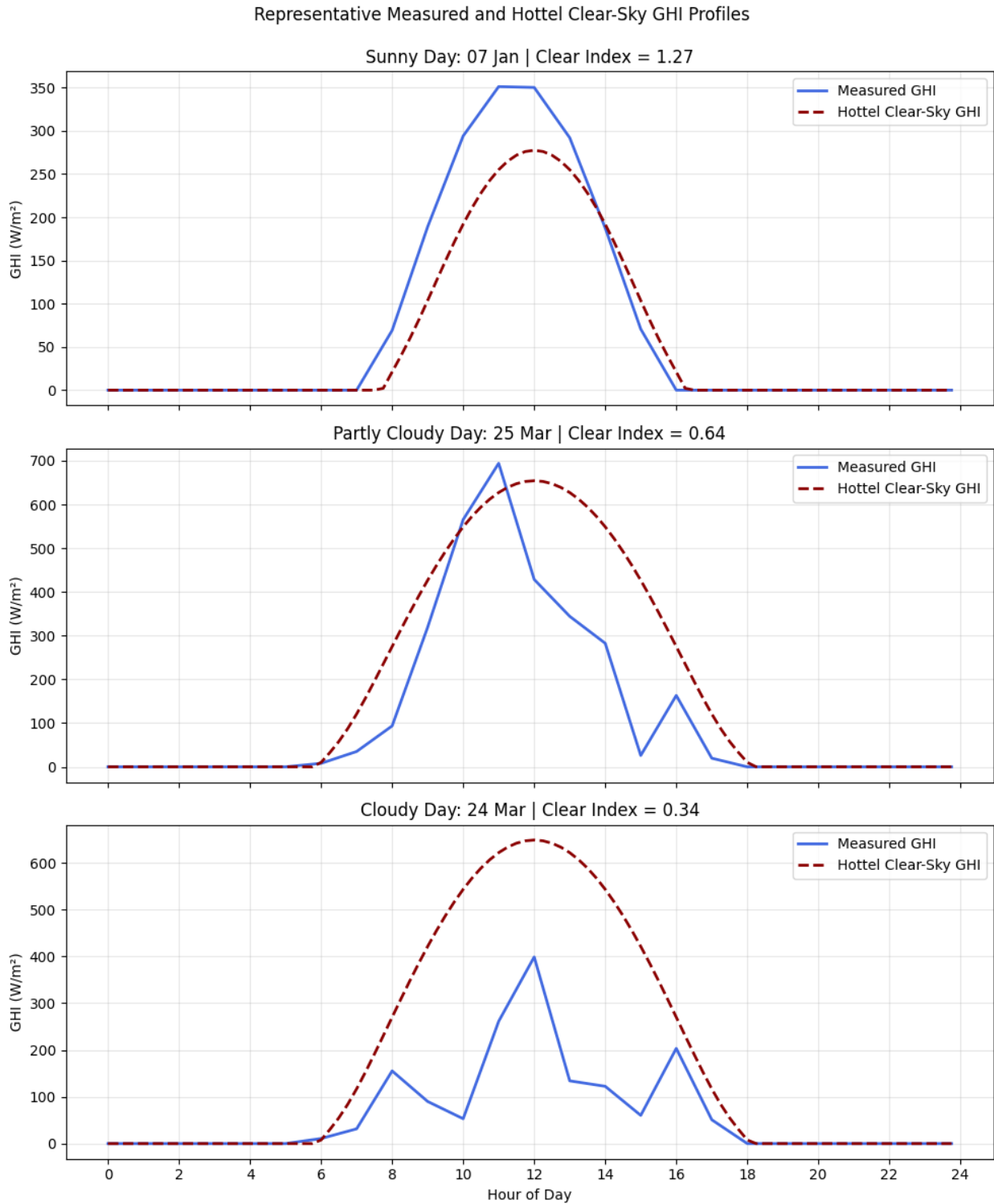


Figure B- 3: Representative Measured and Hottel Clear Sky GHI Profiles

Appendix C: Model Calibration (Training and Testing Results)

Training Results

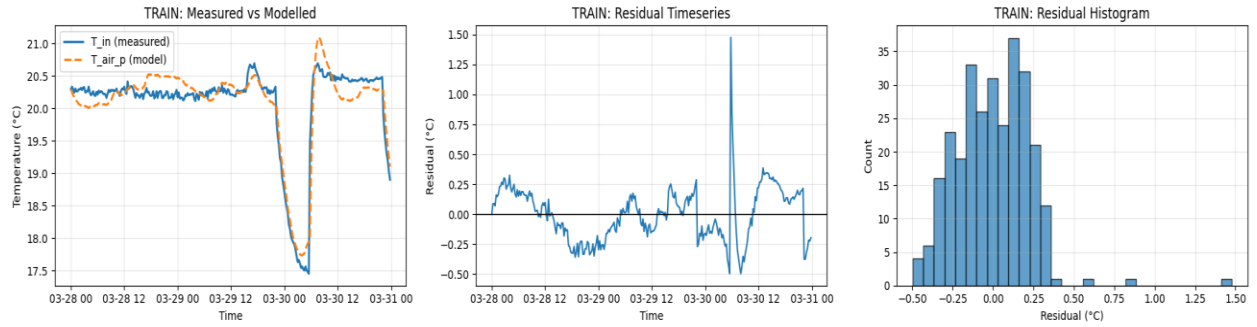


Figure C- 1: Model Calibration Training Period Results

Testing Results

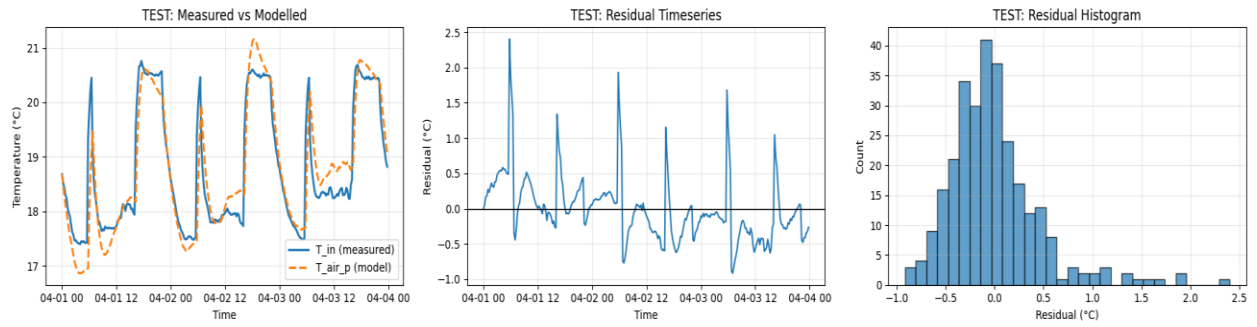


Figure C- 2: Model Calibration Testing Period Results

Appendix D: Model Calibration Code Sample

```
# Example excerpt of the grey-box thermal model calibration (2C–3R)

from lmfit import Parameters, Minimizer
import numpy as np

def simulate_2c3r(df, dt, p):

    GHI = df["GHI"]
    Tout = df["T_out"]
    Tin = df["T_in"]
    Q = df["Q_tot"]

    T_air = np.zeros(len(df))
    T_env = np.zeros(len(df))

    T_air[0] = Tin.iloc[0]
    T_env[0] = Tin.iloc[0]

    for k in range(len(df)-1):

        T_air[k+1] = T_air[k] + (dt/(p["Cair"]*1e6))*((
            p["α"]*GHI.iloc[k] + Q.iloc[k]
            + p["U1"]*(Tout.iloc[k] - T_air[k])
            + p["U3"]*(T_env[k] - T_air[k])
        ))

        T_env[k+1] = T_env[k] + (dt/(p["Cenv"]*1e6))*((
            p["β"]*GHI.iloc[k]
            + p["U2"]*(Tout.iloc[k] - T_env[k])
            + p["U3"]*(T_air[k] - T_env[k])
        ))

    return T_air

# Least-squares calibration
def objective(pars):
    p = pars.valuesdict()
    T_pred = simulate_2c3r(df_train, dt, p)
    return df_train["T_in"].values - T_pred

result = Minimizer(objective, params).minimize(method="least_squares")
```

Appendix E: MPC Code Sample

Example excerpt of the scenario-based MPC formulation

```
import cvxpy as cp
```

```
ph = 96  
dt = 900
```

```
x = cp.Variable((3, ph))    # [T_air, T_env, SOC]  
u = cp.Variable((2, ph))    # [P_heat, P_batt]  
d = cp.Variable((3, ph))    # [T_out, GHI, P_pv]
```

```
cost = 0  
constraints = []
```

```
for k in range(1, ph):
```

```
    constraints += [  
        x[0,k] == x[0,k-1] + (dt/(Cair*1e6))*(  
            alpha*d[1,k-1] + u[0,k-1]  
            + U1*(d[0,k-1] - x[0,k-1])  
            + U3*(x[1,k-1] - x[0,k-1])),  
  
        x[1,k] == x[1,k-1] + (dt/(Cenv*1e6))*(  
            beta*d[1,k-1]  
            + U2*(d[0,k-1] - x[1,k-1])  
            + U3*(x[0,k-1] - x[1,k-1])),  
  
        x[2,k] == x[2,k-1] + (dt*u[1,k])/(E_batt*3.6e6),  
        0 <= u[0,k], u[0,k] <= 15000,  
        -5000 <= u[1,k], u[1,k] <= 5000,  
        0 <= x[2,k], x[2,k] <= 1  
    ]
```

```
net = u[0,k] + u[1,k] - d[2,k]  
grid = cp.maximum(net, 0)
```

```
cost += price[k]*grid  
cost += w_track*cp.square(x[0,k] - T_set[k])  
cost += w_smooth*cp.square(u[0,k] - u[0,k-1])
```

```
problem = cp.Problem(cp.Minimize(cost), constraints)  
problem.solve(solver=cp.ECOS)
```

