The Effectiveness of Monetary Policy in a Regime-Switching Environment

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Abstract

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The role of money in guiding or predicting the business cycle is a recurring, if not a cyclical, theme in economics. Different econometric approaches, ranging from narrative to vector autoregressions, have been proposed to assess the validity of the money matters proposition in the economy. It is argued here that most empirical work on the effectiveness of monetary policy in explaining the business cycle is based on an incorrect premise that the business cycle can be adequately represented by a linear time-series model. This thesis demonstrates that the business cycle is inherently nonlinear and that this nonlinearity can be captured by regime-switching models and, in particular, by STAR models. STAR models applied to the logarithm of the U.S. real GNP have, however, not produced very satisfactory results. One reason advanced for this lack of success is that the growth rate of GNP does not possess enough variability. The position taken here is that the disappointing results generated by STAR models of real GNP are not due to insufficient variability but simply to the choice of the switching variable that defines the states of nature. My contention is that interest rate instruments and, more specifically, the federal funds rate describe the environment in which the economy functions. This thesis demonstrates that a STAR model of real GNP, with the regimes being determined by the quarterly difference in the funds rate, outperforms linear representations of real GNP by substantially reducing the standard error.

Furthermore, this thesis examines whether the growth rate of money explains the U.S. business cycle when the latter is modeled as a regime-switching process. It is shown that the growth rate of M1 and M2, which in the usual bivariate linear setting does not Granger-cause output for the 1960-1993 sample period, would '*cause*' output when the business cycle is modeled as a STAR process. This '*causality*' result holds for the difference stationary, trend stationary and Hodrick-Prescott representations of the business cycle. In particular, the growth rate of M1 has greater effects on the real economy in the low growth regime, while the alternate regime features money neutrality.

Finally, this thesis examines whether the Markov regime-switching model of Hamilton, based on an unobservable state of nature, outperforms STAR models based on an observable state of nature when applied to the growth rate of U.S. real GNP. It is shown that a STAR model of real GNP outperforms Hamilton's changing intercept model for the 1960-1993 sample period and compares favorably with further refinements of the Hamilton model.

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CHAPTER 1

Introduction

The role of money in guiding or predicting the business cycle is a recurring, if not a cyclical, theme in economics. Keynesians viewed the Great Depression as proof of the limitations and ineffectiveness of monetary policy in steering the economy out of a recession and promoted fiscal policy as the focal point of control for the economy at the expense of monetary policy. Moreover, the Keynesian revolution put aside preoccupations with the business cycle for the simpler problem of determining output, interest rates and prices at a point in time. The debate on the relevance of money in affecting the real economy was reanimated in part by Friedman and Schwartz (1963) *A Monetary History of the United States* and later by Lucas (1972,1973) which presented a model of misperception or monetary surprise. More recently, Real Business Cycle proponents have returned to models where money plays no role in explaining real variables (see, for example, Kydland and Prescott 1982; Long and Plosser 1983; King R.G. et al. 1988a, 1988b).

An important obstacle for proponents of the '*money matters*' view has been justifying empirically the importance of monetary aggregates in explaining the real economy. Different econometric approaches have been proposed to assess the validity of the '*money matters*' proposition in the real economy. Friedman's et al. (1963) and more recently Romer and Romer (1989) use a '*narrative*' approach which consists of a case by case study of each expansion and recession phase of the business cycle. Sims (1972) adopts a Granger-causality approach to test whether lagged values of money explain output in a system that includes lagged values of output. More recently, vector autoregressions have been proposed to ascertain the importance of money in the real economy (see, for example, (Sims 1980a, 1980b); Litterman and Weiss (1985); Stock and Watson (1989); Friedman and Kuttner (1993); Leeper et al. (1996)). These various empirical techniques have led to different conclusions on the relevance of money to the real economy, and consequently have not settled the *'money matters'* issue.

It is argued here that most empirical work on the effectiveness of monetary policy in explaining the business cycle is based on an incorrect premise that the business cycle can be adequately represented by a linear time-series model. While used extensively in economics, linear time-series models have been shown to be deficient in dealing with certain traits of some economic series. They are unable, for instance, to fully capture the high volatility of certain financial variables (Bollerslev et al. (1992) provides a review of ARCH models with regard to this issue) or the asymmetrical behavior of the business cycle (Hamilton (1989); Beaudry and Koop (1993); Potter (1995)).

This thesis will attempt to demonstrate that the business cycle is inherently nonlinear and that this nonlinearity can be captured by regime-switching models and, in particular, by the smooth transition autoregressive (STAR) model. These models stipulate that the process exhibits a different dynamic according to the prevailing state of nature or regime. STAR models applied to the logarithm of the U.S. real GNP have, however, not produced very satisfactory results. One reason advanced by Terasvirta and Anderson (1992) for this lack of success is that the growth rate of GNP does not possess enough variability. The position taken in this thesis is that the disappointing results generated by STAR models of GNP are not due to insufficient variability but simply to the choice of the switching variable that defines the states of nature.

My contention is that interest rate instruments and, more specifically, the quarterly difference in the federal funds rate determine or describe the environment in which the real economy functions. This thesis demonstrates that a STAR model of real GNP, with the regimes or states of nature being determined by the quarterly difference in the funds rate, outperforms linear representations of real GNP by substantially reducing the standard error. This result holds for the difference stationary, the trend stationary and the Hodrick-Prescott filtered series representation of the business cycle. It is further shown that a linear time-series model of the growth rate of real GNP would be unsuccessful in reproducing the sharp drops in economic growth associated with recessions whereas the fitted values generated by the STAR models would more closely resemble the actual data set for the 1960-93 sample period.

It has been argued by Friedman and Kuttner (1998) that every postwar U.S. recession can be associated with a tightening of monetary policy except for the 1990-91 recession; Friedman et al. point out that most of the monetary and financial indicators failed to anticipate this last recession. It is shown in this thesis that the transition function associated with the quarterly difference in the funds rate would have predicted the 1990-91 downturn in the U.S. economy.

Furthermore, it is shown that the growth rate of M1, which in the usual bivariate linear setting does not Granger-cause output (i.e., U.S. real GNP) for the 1960-1993 sample period, would '*cause*' output when the business cycle is modeled as a regime-switching process. This '*causality*' result holds for the difference stationary, trend stationary and Hodrick-Prescott filtered series representations of the business cycle. In particular, the growth rate of M1 has greater effects on the real economy in the low growth regime, while the alternate regime features money neutrality. This last result may provide an explanation for the apparent lack of correlation existing between the growth rate of M1 and the business cycle for the 1960-1993 sample period since the growth rate of money only affects the economy in the low growth regime.

Finally, the STAR model of real GNP, with the quarterly difference in the federal funds rate provoking the switches between regimes, performs as well as the model of Hamilton (1989), another regime-switching model based on an unobserved state of nature.

The following is a general outline of this thesis. Chapter 2 presents a selected overview of the interaction between monetary factors and the business cycle according to different schools of thought. The characterization of the business cycle by a representative series such as real GNP is considered, as is the ability of monetary aggregates and interest rate instruments to explain real GNP. Chapter 3 introduces the threshold and the Hamilton regime-switching models. Differences between these models are discussed, as are tests for their nonlinearity and methods for their estimation. The fourth chapter discusses selected regime-switching models for real GNP and the unemployment rate. Emphasis is also placed on the behavior of the quarterly difference in the funds rate in the context of a dual preoccupation of the Federal Reserve, that of output and inflation. Chapter 5 looks at Granger-causality of the growth rate of money (i.e., M1 and M2) with respect to real GNP when the business cycle is represented as a regime-switching process. The final chapter compares the STAR and the Hamilton models for the growth rate of real GNP.

CHAPTER 2

The Business Cycle and Monetary Policy

A number of stylized facts have been advanced regarding the interaction between the business cycle and certain economic series (Lucas (1977); Brunner and Meltzer (1993)). Of a particular interest to this thesis is a stylized fact regarding monetary aggregates, short-term interest rates and the real economy, namely that these variables are procyclical. The fact that monetary aggregates and short-term interest rates are procyclical does not imply a causality relation moving from these variables to the real economy; it is possible instead that money, interest rates and the economy are all driven by outside factors such as technological shocks or oil shocks. In addition, feedback from one variable to another could account for the observed co-movement between money and GNP.

One way to assess the importance of money on the economy is to construct an artificial economy and then test whether money plays any role in this system. A number of real business cycle (RBC) studies use a variant of this approach which excludes a priori money from the structural model and reproduces a number of stylized facts concerning the economy, thereby implying the irrelevance of money in explaining the economy.

An alternative and less ambitious approach is to associate with the business cycle a representative series such as GNP or the unemployment rate and then model this process. This is the approach taken in this thesis. This reduced form approach should, however, still be of interest to advocates of the structural approach since GNP or unemployment are often an integral part of most systems of equations.

This chapter presents a brief and selected review of the relation between the business cycle and monetary policy. More detailed surveys of this topic can be found in Blanchard (1990), Zarnowitz (1992a, 1992b) and Brunner and Meltzer (1993). At least two centuries of preoccupation with the role of money in the economy has fostered different schools of thought; from the Classical "*money is a veil*" theorists to the more recent Real Business Cycle proponents. The first part of this chapter deals with the representation of the business cycle in terms of real GNP, while the second part concerns the interaction between output and monetary policy, as defined in the narrow sense by monetary aggregates (i.e., M1 and M2), or in the larger sense by interest rate instruments.

2.1 The Business Cycle

The term business cycle could be misleading if one has in mind a process that moves in wave-like motion through time, since the length and severity of U.S. recessions and expansions have varied greatly throughout history (see Zarnowitz (1992a)). In the 1930's two pioneering works from Slutsky (1937) and Frisch (1933) were published which encompass the main principles of today's time-series approach to the modeling of economic series, and in particular the business cycle. Slutsky's contribution was to show that a moving average or impulse component could generate a cyclical process. Frisch attempted to reproduce the business cycle from a structural model using a propagation mechanism. Using a combination of differential and difference equations, Frisch showed that the solution would resemble cycles. Shocks or an impulse component can be added to the system to give it more life but the cycle itself emanates from the structural model. The multiplier accelerator model of Samuelson (1939) is a simpler model based on the same idea that the cycle originates from a propagation mechanism. Business cycles have consequently been explained or generated in terms of a propagation mechanism or an impulse component.

Real GNP will represent for the purpose of this thesis our main indicator of the business cycle. The representation of the business cycle in terms of real GNP has historically been expressed as the natural logarithm of real GNP minus a time trend, such as:

$$y_t^{TS} = \ln(GNP_t) - (\xi_0 + \xi_1 t) \qquad \textbf{(2.1)}$$

where $(\xi_0 + \xi_1 t)$ represents the trend component. The $\{ y_t^{TS} \}$ or simply $\{y_t\}$ series is then modeled as a linear time-series process[1] (i.e., trend stationary approach:TS). For instance, the series $\{y_t\}$ could be modeled as a AR(p) process, such as:

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \varepsilon_t$$

where y_t is defined by (2.1) and ε_t is a white noise process or the more general ARMA(p,q) model:

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \sum_{i=0}^{q} \xi_i \varepsilon_{t-i} \text{ where } \xi_0 = 1.$$

[1] The $\{y_t\}$ series follows a linear process if it has the representation $y_t = \sum_{i=-\infty}^{\infty} \psi_i \varepsilon_{t-i}$ for all t, where $\{\varepsilon_t\}$~$WN(0,\sigma^2)$ and $\sum_{i=-\infty}^{\infty} |\psi_i| < \infty$ (see Brockwell and Davis 1996).

The estimation of AR(p) and ARMA(p.q) processes, and other aspects of linear time-series models such as stationarity considerations, can be found in Box and Jenkins (1976) and Hamilton (1994).

The view of the business cycle as being represented by a series GNP minus its trend has been questioned throughout the 1980's, for example by Nelson and Plosser (1982) and Campbell and Mankiw (1987). An alternative approach to achieve stationarity is to take the first difference of the logarithm of real GNP which is approximately the growth rate of the series:

$$y_t^{DS} = \Delta \ln(GNP_t) \qquad \textbf{(2.2)}$$

and then to consider $\{ y_t^{DS} \}$ as the business cycle (i.e., difference stationary approach: DS). The different specification for the business cycle (i.e., TS or DS) will correspond to different properties of the cycle:

> ".... expansions are shorter and contractions longer in growth cycles than in business cycles. Growth cycles are more nearly symmetrical and less variable than business cycles with respect to both duration's and amplitudes of their phases. Growth cycle peaks tend to occur before the corresponding business cycle peaks, while the troughs of the matching growth cycles and business cycles tend to be roughly coincident." (Zarnowitz 1992b, p. 47).

This characterization by Zarnowitz of the two representations of the business cycle, namely the TS and DS versions, is for the most part still valid for the U.S. real GNP series for the sample period 1960-1993 (see chapter 4). Choosing TS over DS has repercussions in terms of forecasting errors and the persistence of an exogenous shock on the system. For instance, the effect of an innovation at time t on $\{y_t\}$ will eventually disappear if the series follows a TS process whereas it would remain in the system if the process is DS (Campbell and Perron (1991) present different issues in the unit roots debate).

Finally, empirical studies using the RBC framework often rely on the Hodrick and Prescott (HP) filter as a mean of decomposing the data, $\{y_t\}$, into a growth, $\{g_t\}$, and a cyclical, $\{y_t^{HP}\}$, component. The series $\{g_t\}$ is the solution to the following minimization problem:

$$\underset{\{g_t\}}{\mathrm{Min}} \left(\sum_{t=1}^{N} (y_t - g_t)^2 + \lambda \left[\sum_{t=1}^{N} ((g_{t+1} - g_t) - (g_t - g_{t-1})) \right]^2 \right) \quad \textbf{(2.3)}$$

One needs to assign a value to the parameter λ in order to solve this optimization problem; for example Kydland and Prescott (1982) set $\lambda = 1600$. The Hodrick-Prescott filtered series representation of the business cycle will therefore refer to the series $\{y_t^{HP}\}$ where $y_t^{HP} = y_t - g_t$.

2.2 Money and Output

The importance or irrelevance of money in explaining the real economy is often seen as a fundamental question in macroeconomics, yet the question remains largely unresolved. Representing thoroughly the different schools of thought, or even the nuances existing within each school is well beyond the scope of this thesis. The following presentation should therefore be seen only as an overview of the various viewpoints on the importance of money with respect to the real economy.

The classical view, which is often associated with Fisher, maintains that money does not affect real variables; the goods market determines relative prices while the money market determines the absolute price level (see Grandmont (1983)). The classical dichotomy between the real economy and the monetary sector was disputed by Patinkin on the grounds that it did not take into account the real balance effect (Niehans (1978); Gale (1982)). For Keynesians money can play a role in determining real output in the short-run due to the rigidities in nominal wages but in the long-run money is neutral. The business cycle or fluctuations in output levels are, however, mostly explained by an equilibrium schedule in the goods market or the IS-curve. By assuming essentially a horizontal LM-curve, fluctuation around the full employment level can be explained by the multiplier-accelerator model.

Friedman and Schwartz (1963), *A Monetary History of the United States*, reassess the role of money with respect to the Great Depression. The Keynesian position was that monetary policy had been ineffective in getting the economy out of the depression. The Friedman et al. interpretation of events suggests that the recession was prolonged precisely by monetary policy and that this policy was consequently a potent tool in managing the economy. According to Friedman et al. the Central bank's decision to increase the money supply affects the economy through its impact on relative prices; changes in the money supply brought about by open-market operations modifies individual and business portfolios. Friedman's contribution to the monetary debate is evidently much more sweeping than simply re-interpreting the events of the Great Depression. For instance, the natural rate hypothesis implies that in the long run the monetary authorities are not able to exploit any trade-off between inflation and unemployment on the grounds that economic agents '*cannot be fooled all of the time*' (Friedman (1991), p. 72). Friedman's advice to the monetary authorities is to target monetary aggregates directly instead of interest rates. This is in order to focus on one objective, price stability. According to Friedman, "*Experience and not theory has demonstrated that the first two strategies are not feasible, that monetary policy is not an effective instrument for achieving directly either full employment or economic growth*" (Friedman (1986), p. 13). In opposition to this view, this thesis demonstrates that interest rate instruments define the setting (high growth or low growth regime) in which the economy functions and in this sense monetary policy is a crucial instrument in achieving economic growth.

The emergence of Rational Expectations in the 1970's, and more specifically the Lucas (1972,1973) Island model, provided a new perspective on the role of money on the economy. Money can affect the real economy in the short-run because of incomplete information on the part of economic agents at the time of decision. Agents must decide if the observed change in the price of a commodity corresponds to a change in relative prices or is simply the consequence of overall inflation in the economy. An unanticipated change in the money supply can affect the real economy since a change in the absolute price is misperceived as a change in relative price. The debate no longer concerns whether M1, M2 or other monetary aggregates explain real GNP or industrial production, rather it examines whether *anticipated* money does. Barro (1977,1978), Barro and Rush (1980) found that only unanticipated money explained real GNP while Mishkin (1983) and Cecchetti (1986) showed that anticipated money affects the economy.

The real business cycle (RBC) approach also builds upon the concepts of general equilibrium and rational expectations, similar to the Lucas model, but where the latter emphasizes monetary factors as a possible source of cyclical fluctuations, RBC proponents stress technological shocks (see, for example, Kydland and Prescott (1982);

Long and Plosser (1983); King, R.G. et al. (1988a, 1988b)). The objective in most RBC models is to maximize the lifetime (infinite) expected utility function subject to certain constraints such as a production function that is prone to technological shocks. RBC models have generally been successful in generating co-movements between economic variables which resemble the actual data.

A number of econometric approaches based on single equations and systems of equations have been used in assessing the predictive power of monetary aggregates on the economy. One single equation approach has been to test if money explained output or the growth rate of output in a system that included lagged values of output (i.e., Granger-causality test). Sims (1972) found that money Granger-causes output (nominal GNP) whereas income does not Granger-cause money. An alternative approach to assessing the predictability of monetary aggregates on the economy is the introduction of VAR systems. VAR estimation has been extensively used in economics since Sims (1980a) seminal paper on the money-output relationship. It has taken various forms such as the Granger-causality test, impulse-response functions and variance decomposition. Sims (1980a, 1980b), using Vector Autoregression (VAR) systems applied to U.S. quarterly and monthly data for the period 1949-1975, showed that money no longer Granger-caused output once short-term interest rates were inserted into the system. There have subsequently been numerous and often contradictory studies on the role of money which have used the VAR approach. Christiano and Ljungqvist (1988) found causation from money to output in log levels. A further conclusion was that the apparent rejection of causality in the difference of the logs could be attributed to a lack of power in the causality test. Friedman and Kuttner (1992a, 1992b) focused on the predictive powers of different interest rates in explaining the business cycle and concluded that money did not cause output. The authors also stressed that even if money played a historical role in explaining output, this relation appears to have disappeared completely in the last decade. Stock and Watson (1989) modeled the growth rate of money with a time trend and found that money did Granger-cause output in the last three decades. They maintained that previous results using the money variable for the period before the 1960's may be prone to measurement errors. In addition to the usual issues raised regarding VAR systems, such as the importance of ordering the variables, the Choleski decomposition and the choice of horizon length, it should be pointed out that VAR estimation results are irrelevant if the business cycle follows a regime-switching process.

A final point regards a paper by Fernandez (1997) which is related to the topic in this thesis. Fernandez assumes that the business cycle can be modeled by a broken-detrended process; it is assumed that the generating process possesses a different trend component after a certain cutoff point. Using monthly industrial production, Fernandez shows that detrended M1 growth would Granger-cause output for the 1959-1994 sample period. This approach differs from the one used in this thesis in the sense that Fernandez still relies on a linear relation to analyze the interaction between money and output.

2.3 Interest Rate Instruments and Output

The lack of consensus on the effectiveness of monetary policy on the real economy may stem from an inappropriate representation of the monetary stance. McCallum (1983) argues that interest rate instruments might be more reflective of the monetary stance since the central bank could target the money stock through an interest rate instrument. It is also possible according to Goodfriend (1995) that the Federal Reserve directly targets interest rates rather than the money supply and hence movements in the amount of deposits are mostly brought about by changes on the demand side and not on reserve/deposit multiplier interaction.

The traditional monetary mechanism assumes that when the Central bank decides to tighten monetary policy, it sells government securities (open market operations). This results in a reduction by the Central Bank in the corresponding amount of reserves assigned to the commercial banks. One difficulty with linking monetary policy to interest rates is that the Fed controls directly only the discount rate which is the rate the Fed loans reserves to banks and other depository institutions. However, the Fed is such an important player that it can '*influence*' the level of other interest rates in the economy.

> "For example, if the public's demand for money and credit is substantial, due, perhaps, to strong growth in the general economy, a restrictive approach to the provision of reserves by the Fed tends to put immediate upward pressure on the Federal funds rate, which is the short-term interest charged for the use of reserves when they are sold (lent) and bought (borrowed) in the so-called Federal funds market. The rise in the Federal funds rate, in turn, causes other interest rates to rise, which acts to reduce both the supply and demand for money and credit and hence their growth." (Broaddus 1988, p. 26).

In the analysis presented here, the federal funds rate and the interest rate differential or the short-term spread will be used as the primary interest rate candidates for the prediction of the real business cycle. The short-term spread or the paper-bill spread as defined by Friedman et al. (1992a) and used throughout this thesis is the difference between the 6-month commercial paper rate and the 180-day Treasury Bill rate. Commercial paper is an unsecured promissory note with a short-term maturity issued by large firms and finance companies. It is consequently an alternative to bank loans for the finance of short-term projects, and has gained importance in the past two decades. While in theory commercial paper is an unsecured note, in practice few firms have defaulted. Most commercial paper is rated by the major rating companies such as Moody's and Standard & Poor's (Stigum 1990). The second component comprising the short-term spread is determined by a Federal Reserve auction. It is clear that a spread does exist between the two short-term interest rates. This observation can easily be accounted for by the fact that commercial paper is not as secure as a Treasury Bill. Moreover, gains from a Treasury Bill are not taxed and provide greater liquidity than commercial paper.

The second major interest rate instrument used throughout this thesis is the federal funds rate, which is essentially the rate at which banks loan each other funds in order to meet their reserve requirements. The funds rate has been, for the most part, since 1960's, a key instrument of monetary policy for the Federal Reserve (Cook and Hahn 1989). The one exception is for the 1979-1982 period where the Fed focused on a money supply target (see Stigum 1990).

There is little debate over the predictive power of the short-term spread and the federal funds rate systems (see, for example, Friedman and Kuttner (1992a, 1992b); Bernanke and Blinder (1992); Bernanke and Gertler (1995)). Rather the controversy concerns the reasons why these interest rate instruments perform so well in predicting the business cycle. The Friedman-Kuttner view is that movements in the short-term spread reflect, in part, the risk of default on the commercial paper. However, the default on commercial paper has historically been zero so this should not, according to Bernanke (1990), be a major factor in lenders' decisions. Bernanke's view is that the short-term spread and the funds rate both reflect the monetary stance. In order for monetary policy to affect the short-term spread, it is required that bank loans and commercial paper be imperfect substitutes, both for banks and firms, as asserted in Kashyap, A. K. et al. (1993). Friedman et al. (1992a) argue that if indeed the short-term spread reflects mostly monetary policy then the Fed's policy should also have repercussions on the Treasury Bill rate thus leaving the difference between the Treasury Bill and the commercial paper rate virtually unchanged.

The "credit channel" view provides, according to Bernanke, an alternative explanation of the predictability of the short-term spread on the economy. The traditional monetary mechanism is quite straightforward and operates in textbook fashion. A restrictive monetary policy leads to a reduction in bank deposits in order to satisfy the legal reserve requirement. This decrease in deposits implies a reduction in the money supply which in turn results in an increase in interest rates. This increase in interest rates will impact on the real economy. The credit channel view complements this traditional monetary mechanism channel supposedly reinforcing the effects of monetary policy on the economy. Bernanke and Blinder (1988, 1992) argue that '*lumping*' bank loans into a '*bonds*' market and then discarding this market via Walras' law negates the importance of bank loans for firms. The credit channel assumes that monetary policy will not only affect deposits, and hence the money supply, but also the supply of bank loans. The credit channel view assumes that some borrowers are dependent on these bank loans.

The credit channel is contested in several ways. Suppose the reserve level is reduced and therefore the volume of bank deposits. The supply of bank loans need not be affected as a result since banks can use certificates of deposits, which do not fall under the legal reserve requirement constraint, to offset the reduction in deposits (Romer et al. 1990). It is argued by Kayshap et al. (1993) that if the supply of bank loans is not affected by the tightening of monetary policy then the observed decline in loans must be the result of a shift in demand. One would therefore expect that the factors affecting the demand for bank loans would also influences the demand for commercial paper. What will be observed according to Kayshap et al. is a reduction in the volume of commercial paper issues, suggesting the existence of the credit channel. Since bank loans are just one possible source of finance for firms, the link made between bank loans and the real economy must be contrived, even if the supply of bank loans is affected by monetary policy.

The difficulty with the interpretation given in Bernanke et al. (1992) of the predictability of the short-term spread as a reflection of monetary policy, is that the federal funds rate, which is more closely associated with Federal Reserve monetary policy, does not outperform the short-term spread in single-equation estimation in terms of providing a better fit. In this thesis it is argued that the Bernanke-Blinder credit view is not undermined by the short-term spread in single-equation estimation, if we take into account the nonlinear aspects of the business cycle.

The previous arguments for and against the existence of a credit channel are based on the assumption that the legal reserve requirement lowered the level of deposits. It is more likely according to Goodfriend (1995) that the drop of deposits is the consequence of an increase in the interest rate, which by negatively affecting output eventually reduces the amount of deposits.

> "The Fed's reserve provision policy has usually sought to neutralize the effect of loan and deposit demand shocks on short-term interest rates by

adding or draining bank reserves in order to support the targeted Federal funds rate. The result is that it is nearer to the truth to say that deposits are demand determined from month to month at the level of short-term rates targeted by the Fed than to say that are directly controlled by way of reserve/deposit multiplier. Of course, from time to time the Fed deliberately changes its Federal funds rate target in the pursuit of macroeconomic stabilization policy. Even here, the Fed has been careful not to exert excessively disruptive liquidity disturbances. Whenever the Fed cuts non-borrowed reserves to support a higher Federal funds rate, it allows banks to continue to support the initial deposit stock by borrowing reserves at the discount window." (Goodfriend (1995), p. 199).

A final issue, pointed out by Bernanke et al. (1997), is that the 1973-75, 1980-82 and the 1990-91 recession were all preceded by oil price shocks. This is in contrast with most of the monetary indicators which failed to predict the 1990 recession. It is argued by Bernanke et al. that movement in the federal funds rate may be brought about by these oil price shocks: the Fed intervenes in the market in order to stem out expected inflationary pressures caused by the oil price shock. The oil shock incites the Fed to react to the expected inflationary situation and it is the subsequent tightening of monetary policy which induces the recession. The next chapters propose to re-analyze the interaction between monetary factors such as the growth rate of money and interest rate instruments and output in a regime-switching context instead of the customary linear time-series framework.

CHAPTER 3

Regime-Switching Models

A number of recent papers have concluded that the business cycle is asymmetrical (see, for example, Hamilton (1989); Beaudry and Koop (1993); Potter (1995)). Linear time-series models are not incompatible with asymmetry, with the exception that the asymmetry must be provided by the impulse or the error component which rules out the normality assumption. This thesis examines the threshold and the Hamilton regime-switching models which share the principle that the generating process of a series is function of some state of nature. Regime-switching models are consequently natural candidates for incorporating asymmetry since the process behaves differently according to the prevailing state of nature. For instance, the expansionary phase of the business cycle could be generated or characterized by a different process than the recessionary phase. The difficulty resides in defining the states of nature or conditions '*we are in the expansion phase of the business cycle*' and '*we are in the recession phase of the business cycle*'.

The first part of this chapter introduces regime-switching models. The second part concerns the estimation of smooth transition autoregressive (STAR) models. The third part deals with testing for nonlinearity. Part four discusses the Hamilton model.

3.1 Regime-Switching Models

The threshold, the STAR and the Hamilton models can all be written in the following form:

$$y_t = \sum_{i=1}^{r} x_t^T \beta_i F_i(S_t;\psi) + \varepsilon_t \quad \text{for } t = 1,\ldots,N \tag{3.1}$$

with $\sum_{i=1}^{r} F_i(S_t;\psi) = 1;\; F_i(S_t;\psi) \geq 0\ \forall i \text{ and } \forall t.$

β_i and ψ are respectively $(p\times 1)$ and $(k\times 1)$ vectors of parameters

r: the number of regimes

x_t: $(p\times 1)$ vector of 'observed' variables

S_t: $(q\times 1)$ vector of switching indicators

F_i: a function that indicates the state of nature or regime $(i=1,\ldots,r)$.

$\varepsilon_t \sim$ i.i.d. $(0,\sigma^2)$ noise component.

Given that all the above switching models can be written as (3.1), what will distinguish these models is essentially the function F_i. In the pure threshold model, we have a function F_i that takes on two discrete values $\{0,1\}$. The simplest case is to assume two regimes which enables us to express F_2 in terms of F_1 (i.e., $F_2 = 1-F_1$). We would then have the following model:

$$y_t = x_t^T\beta_2 + x_t^T(\beta_1-\beta_2)F_1(S_t;\psi) + \varepsilon_t \tag{3.2}$$

$$\text{with } F_1(S_t;\psi) = \begin{cases} 1 & \text{if } S_t \in A_1(\psi) \\ 0 & \text{otherwise} \end{cases}$$

where $A_1(\psi)$ is a set that defines regime 1; for example, $A_1(\psi)$ could be an interval with values of S_t inside this interval indicating we are in regime 1. Extensions to more than two regimes is also straightforward. Suppose, for instance, that there are three regimes and let:

$$F_1(S_t;\psi)=\begin{cases}1 & \text{if } S_t \in A_1(\psi)\\ 0 & \text{otherwise}\end{cases}$$

$$F_2(S_t;\psi)=\begin{cases}1 & \text{if } S_t \in A_2(\psi)\\ 0 & \text{otherwise}\end{cases}$$

and

$$F_3(S_t;\psi)=\begin{cases}1 & \text{if } S_t \in A_3(\psi)\\ 0 & \text{otherwise}\end{cases}$$

where A_1, A_2 and A_3 represent disjoint sets defining regimes 1 to 3 ; for example S_t is an observable variable and $A_1 = (-\infty, a1]$, $A_2 = (a1, a2]$ and $A_3 = (a3, \infty)$ where $a1 < a2 < a3$. Note that F_3 can be rewritten in terms of F_1 and F_2:

$$F_3(S_t;\psi) = 1 - F_1(S_t;\psi) - F_2(S_t;\psi)$$

We have in a more general setting:

$$F_i(S_t;\psi)=\begin{cases}1 & \text{if } S_t \in A_i(\psi)\\ 0 & \text{otherwise}\end{cases}$$

for $i = 1,\ldots,r$, where $A_i(\psi)$ are disjoint sets which define the r possible states of nature. We can obtain, conditional on the state of nature and other conditions, consistent estimates of β_i by ordinary least squares on each regime (Chan 1993). The challenge with threshold models is that it is not possible to know if the condition $S_t \in A_i(\psi)$ is the appropriate one. An extensive treatment of threshold models is found in Tong (1983, 1990).

An alternative approach to the previous threshold model is to use a continuous function $F_i : \Re \rightarrow [0,1]$ called the transition function. These types of models are called smooth transition autoregressive (STAR) models and have been mostly advocated by

Granger and Terasvirta (1993) and Terasvirta (1994,1995). In the threshold model, we have r possible regimes whereas STAR models have been up to now expressed solely in terms of two 'extreme' regimes (i.e., r = 2) with continuous fluctuations between these two extreme or limit regimes. The STAR model still allows observation of the state of nature while admitting an infinite number of intermediate regimes. Various functional forms for the transition function F have been proposed such as the logistic function:

$$F(S_t;\gamma,c) = \frac{1}{1+\exp(-\gamma(S_t - c))} \quad (\gamma > 0) \qquad \textbf{(3.3)}$$

and the exponential function:

$$F(S_t;\gamma,c) = 1-\exp(-\gamma(S_t - c)^2) \quad (\gamma > 0) \qquad \textbf{(3.4)}$$

The first functional form (3.3) defines a regime to exist when the switching variable is above a critical threshold value and another to exist when this variable is below the cutoff point. The second transition function (3.4) implies that a limiting regime is defined for values of the switching variable 'close' to the point c and the alternative regime when we move away from this point (either from above or below).

Finally, Hamilton (1989,1990) proposed a regime-switching model based on an unobservable state of nature S_t. The regime indicator F_i in equation 3.1 is still a function of S_t except that the variable S_t is no longer observable. Hamilton assumes that S_t follows a first-order Markov process which enables him to solve the maximization of the joint likelihood function of the economic series and the unobserved states of nature. Though the states are unobservable, we can nonetheless infer the probability of being in one state by looking at previous observations.

A simple example, given in relation to the business cycle, illustrates the difference between the regime-switching models used throughout the text. Suppose the observed value of last quarter's growth rate of real GNP indicated '*we are in a recession*'. For instance, if we observed a negative growth rate of real GNP in the previous quarter, a pure threshold model could stipulate that we have the regime '*recession*':

$$y_t = x_t^T\beta_2 + \varepsilon_t$$

A different linear time-series process would be used for the regime '*expansion*':

$$y_t = x_t^T\beta_1 + \varepsilon_t$$

A smooth transition autoregressive (STAR) model would also define the states of nature or regimes according to the value of the growth rate of real GNP at lag 1, but would allow the economy to fluctuate continuously between these two '*extreme*' regimes (i.e., $y_t = x_t^T\beta_1 + \varepsilon_t$ and $y_t = x_t^T\beta_2 + \varepsilon_t$); linear combinations of the two regimes are used with the weights being determined by a function of the growth rate of GNP_{t-1}. For instance, in the case of the logistic transition function (3.3), a high growth regime existed when we encountered a positive growth rate of real GNP in the previous quarter and a low growth regime existed otherwise.

Finally, the Hamilton model would also make use of a two regimes setup (i.e., $y_t = x_t^T\beta_1 + \varepsilon_t$ and $y_t = x_t^T\beta_2 + \varepsilon_t$) except that the state '*we are in a recession*' is no longer observable and the state '*recession*' must therefore be inferred by looking at previous observations of the growth rate of real GNP.

3.2 Estimation of STAR Models

A maximum likelihood approach can be used to estimate (3.1) since S_t is observable. The strategy is to concentrate out the likelihood function in order to reduce the optimization problem to a more tractable one. This will permit a grid-search approach to narrow the starting values of certain parameters.

Let:

$$z_t(\psi)^T = [\, x_t^T \times F_1(S_t;\psi) \ldots\; x_t^T \times F_r(S_t;\psi)\,]$$

Where: $z_t(\psi)^T$ is a $(1 \times rp)$ vector, x_t a $(p \times 1)$ vector of explanatory variables, r is the number of regimes and F_i is the transition function. We assume that the error term, ε_t, follows the same process regardless of the prevailing regime. We can express (3.1) in matrix form:

$$\underset{(N\times 1)}{y} = \underset{(N\times rp)(rp\times 1)}{Z(\psi)\beta} + \underset{(N\times 1)}{\varepsilon} \tag{3.5}$$

where $\beta = (\beta_1^T \ldots \beta_r^T)^T$ and ε is i.i.d. $N(0, \sigma^2 I)$.

The loglikelihood function can be written as:

$$\ell(\beta, \sigma^2, \psi) = \text{constant} - \frac{N}{2}\ln(\sigma^2) - \frac{(y - Z(\psi)\beta)^T (y - Z(\psi)\beta)}{2\sigma^2} \tag{3.6}$$

We can, in theory, find the values of β, ψ and σ that maximize the loglikelihood function (3.6). There are practical difficulties in the estimation procedure. The likelihood surface can be characterized by flat segments and numerous local maxima which make the starting values of the parameters critical. My approach is to abbreviate or simplify the optimization procedure by focusing on the ψ parameters since they seem to be the main culprits in the estimation difficulties. The optimization problem is simplified by concentrating out the β and σ^2 parameters. The first order conditions with respect to β and σ^2 give the familiar result:

$$\tilde{\beta}(\psi) = (Z(\psi)^T Z(\psi))^{-1} Z(\psi)^T y \qquad \text{(provided the inverse exists).} \tag{3.7}$$

and

$$\tilde{\sigma}^2(\psi) = \frac{(y - Z(\psi)\tilde{\beta}(\psi))^T (y - Z(\psi)\tilde{\beta}(\psi))}{N} \tag{3.8}$$

The optimization procedure consist in maximizing the concentrated loglikelihood function:

$$\ell^*_c = -\frac{N}{2}\ln(\tilde{\sigma}^2(\psi)) \tag{3.9}$$

with respect to ψ. We can therefore estimate β and σ^2 by inserting the solution of (3.9) (i.e., ψ_{MLE}) into (3.7) and (3.8). The concentrated likelihood approach permits us to substantially simplify our optimization problem by requiring that we limit our search via a hill-climbing method over the (ψ)-space. An additional advantage is that it reduces the parameter space sufficiently to permit a grid-search approach in the (ψ,σ^2)-space. The starting values of ψ may be crucial, in the hill-climbing method and in finding the global maximum given the often erratic shape of the likelihood surface associated with regime-switching models. We show in the appendix A that a STAR model could easily be incorporated into a VAR framework.

3.3 Testing for Nonlinearity in STAR Models

One reason for testing for nonlinearity is to avoid the estimation problems inherent to nonlinear models. A Lagrange multiplier (LM) approach would be preferred since this constrained version is simply a linear time-series model. The problem is that the LM approach is not directly applicable because some of the parameters are not identifiable under the null of a linear model. For example, suppose we assume that our series $\{y_t\}$ follows a logistic STAR model with two regimes:

$$y_t = x_t^T\beta_2 + x_t^T\phi F(S_t;\psi) + \varepsilon_t \tag{3.10}$$

with $\phi = \beta_1 - \beta_2$ and $\psi = \{\gamma, c\}$(see equation 3.2). Imposing the restriction $\phi = 0$ reduces our model to a linear time-series model but ψ is not identifiable under the null. Another way of achieving linearity is to set $\gamma = 0$ which implies that $F(S_t;\psi) = 0.5$ in the case of the logistic STAR model (3.3) and $F(S_t;\psi) = 0$ for the exponential STAR model (3.4). Unfortunately this leaves $\{\phi, c\}$ unidentifiable and consequently the inverse of the information matrix does not exist. Luukkonnen et al. (1988) derived a Lagrange multiplier test for STAR models based on the null H_0: $\gamma = 0$. The test requires that we approximate the transition function with a third order Taylor expansion evaluated at zero. An extensive discussion of the nonlinearity test is found in Granger and Terasvirta (1993).

3.4 The Hamilton Model

Hamilton (1989) proposed the following two regimes-switching model:

$$y_t = \alpha_0 + \alpha_1 S_t + z_t \tag{3.11}$$

where z_t follows an AR(p) process that includes the error term: i.e.,

$$z_t = \phi_1 z_{t-1} + \ldots + \phi_p z_{t-p} + \varepsilon_t$$

and

$$S_t = \begin{cases} 0 \text{ if regime } 0 \\ 1 \text{ otherwise} \end{cases} \quad \textbf{(3.12)}$$

represents the unobservable states of nature or regimes. This two regimes-switching model can be expressed in the form of (3.1) by assuming:

$$F(S_t;\psi) = S_t,\ \beta_1 = (\alpha_0 + \alpha_1, 1),\ \beta_2 = (\alpha_0, 1) \text{ and } x_t = (1, z_t - \varepsilon_t).$$

The interesting and innovative feature of the Hamilton approach is that the data generating mechanism is a function of the current but unobservable state of nature S_t. It is assume that S_t follows a first-order Markov process. We define the following transition probability matrix, $P = \begin{pmatrix} p_{00} & 1-p_{00} \\ 1-p_{11} & p_{11} \end{pmatrix}$ where p_{11} is the probability of being in regime 1 given we were already in this regime in the previous period (i.e., $P(S_t = 1|S_{t-1} = 1)$) and p_{00} is the probability of being in the other regime conditional on being in this same regime in the previous period (i.e., $P(S_t = 0|S_{t-1} = 0)$). In order to solve the Hamilton model, we need to rewrite (3.11) in terms of lagged values of y_t and lagged values of the states of nature S_t. In the initial form (3.11), we only require knowledge of S_t while for the estimation aspects of the model, we need to know $(S_t, \ldots, S_{t-p})$:

$$y_t = \alpha_0 + \alpha_1 S_t + \phi_1(y_{t-1} - \alpha_0 - \alpha_1 S_{t-1}) + \ldots + \phi_p(y_{t-p} - \alpha_0 - \alpha_1 S_{t-p}) + \varepsilon_t \quad \textbf{(3.13)}$$

Note that the Hamilton model is more general than simply allowing for different intercepts in both regimes. Extensions of the basic Hamilton model were elaborated in Lam (1990), Hansen (1992), Kim (1994) and Gordon (1997). A number of issues have been raised in regards to the Hamilton model and other regime-switching models.

Amongst these is testing for nonlinearity. The usual test statistics are not applicable because some parameters such as the transition probabilities and α_1 can not be identified under the null. Hansen (1992) derived a test statistic and concluded that he could not reject a linear specification, namely an AR(4) model, in favor of the Hamilton model. Goodwin (1993) estimated the Hamilton model for eight countries and concluded that he could reject the AR(4) model in favor of the regime-switching model. Aside from the linearity issue, the estimation of the Hamilton model is not an easy task since the likelihood surface usually contains numerous local maximums and singularities and, as pointed out by Hansen (1992), it is more than likely that the global maximum will be overlooked.

The switching is provoked in Hamilton's model via a variable, S_t, which is not directly observable. It is therefore different from the STAR models in the sense that it does not require that an observable variable such as GNP, the growth rate of money or some other financial variables to activate the passage from one regime to another. For example, the 'true' switching variable for the growth rate of GNP, if it exists, might never be chosen in the case of the STAR model. However, Hamilton's approach requires the first-order Markov structure assumption.

Hansen (1992) found evidence that a model in which all the coefficients are functions of the states of nature more aptly represents the growth rate of GNP:

$$y_t = \alpha_0 + \alpha_1 S_t + \phi_{1S_t} y_{t-1} + \ldots + \phi_{pS_t} y_{t-p} + \varepsilon_t \qquad \textbf{(3.14)}$$

where S_t is defined as before and

$$\phi_{iS_t} = \phi_{i0} + \phi_i S_t \quad \text{for } i=1,2,\ldots,p. \qquad \textbf{(3.15)}$$

The expression 'modified Hamilton model' will refer to a model in which all of the AR coefficients vary according to the states of nature. This is more in line with the STAR models where the AR coefficients are function of the prevailing regime. Let:

$$y_t = \alpha_0 + \alpha_1 S_t + z_t \qquad \textbf{(3.16)}$$

where

$$z_t = \phi_{1S_t} z_{t-1} + \phi_{2S_t} z_{t-2} + \ldots + \phi_{pS_t} z_{t-p} + \varepsilon_t \qquad \textbf{(3.17)}$$

and ϕ_{iS_t} is defined as in (3.15) and S_t is defined as in (3.12). We have:

$$y_t = \begin{cases} (\alpha_0 + \alpha_1) + (\phi_{10} + \phi_1) z_{t-1} + \ldots + (\phi_{p0} + \phi_p) z_{t-p} + \varepsilon_t & \text{in regime 1} \\ \alpha_0 + \phi_{10} z_{t-1} + \ldots + \phi_{p0} z_{t-p} + \varepsilon_t & \text{otherwise} \end{cases} \qquad \textbf{(3.18)}$$

This model differs from the Hansen approach in that it does not require that the series $\{y_t\}$ be a function of only its lagged values and the state of nature at time t. I estimate this model by using the same transformation as the one used in the changing intercept model, that is by rewriting everything in terms of lagged GNP and lagged states of nature:

$$y_t = \alpha_0 + \alpha_1 S_t + \phi_{1S_t}(y_{t-1} - \alpha_0 - \alpha_1 S_{t-1}) + \ldots + \phi_{pS_t}(y_{t-p} - \alpha_0 - \alpha_1 S_{t-p}) + \varepsilon_t \qquad \textbf{(3.19)}$$

The estimation aspects of Hamilton's changing intercept model can be found in Hamilton (1989) and will, therefore, be omitted from this thesis. The Hansen and the modified Hamilton model are simply extensions of Hamilton's original changing intercept model and can be estimated in a similar manner as Hamilton (1989). Chapter 6 will demonstrate that this modified Hamilton model outperforms the Hamilton changing intercept and the Hansen model.

CHAPTER 4

Regime-Switching Models of Real GNP

Regime-switching models have been, for the most part, successful in representing the U.S. real GNP, see for example Hamilton (1989) which uses a Markov switching model and Potter (1995) which features a pure threshold model. STAR models of U.S. GNP have not encountered much success since, according to Terasvirta and Anderson (1992), U.S. GNP lacks enough variability to be successfully captured by this type of regime-switching model. Consequently, Terasvirta et al. (1992) chose as a smooth transition autoregression application monthly industrial production, which they converted into quarterly data instead of the usual GNP series[2]. Here, quarterly GNP is chosen rather than industrial production because the latter might not appropriately reflect the premise for an asymmetrical business cycle: industrial production does not reflect all the sectors of the economy. The decision to use industrial production data for the quarterly measure of output instead of GNP may have been dictated by the fact that we would not reject linearity in favor of the STAR model when the switching variable is lagged GNP. This chapter will attempt to show that there is enough variability in quarterly real GNP to warrant the use of STAR models.

The first section of this chapter introduces the various representation of the business cycle used throughout this thesis. The second section examines the depiction of the business cycle as a linear time-series process. The third section presents the results of a nonlinearity test. The fourth section relates to the estimation of various STAR models of the business cycle. Section five considers the unemployment rate as an alternative measure of the business cycle. The final section looks at the behavior of the Fed with respect to inflation and output.

4.1 The Data

Real U.S. GNP features an annual growth rate of approximately 3% over the period 1960-1993, but many incongruities exist between decades. The 1960's with its growth rate of 3.7% and essentially one recession is drastically different from the tumultuous 1970's and part of the 1980's which had 'severe' recessions and a growth rate of 2.4%. Thereafter, there is a period similar to the 1960's with only one recession and a standard deviation of essentially the same order but with a lower annual growth rate of 2.7%.

As we saw in chapter 2, there is no agreement on the appropriate representation of the business cycle (i.e., difference stationary (DS), trend stationary (TS) or Hodrick-Prescott (HP)). The different specifications of the business cycle in terms of real GNP were derived in the following manner: The DS process was obtained by taking the natural logarithm of real GNP in 1982 dollars and then differencing the mentioned series (see figure 4.1). This new series is often interpreted as the growth rate of GNP since for small changes between periods the difference in logs is a good approximation of the growth rate. The TS formulation of the business cycle is simply the logarithm of real

[2] It should be pointed out that Terasvirta (1995) modeled the annual U.S. growth rate of GNP for the period 1889-1987 as an exponential STAR model with the growth rate of GNP at lag 2 switching the process.

GNP minus a linear time trend component (i.e., $\xi_0 + \xi_1 t$ where t = 1, 2, ... ,136). The estimated values of ξ_0 and ξ_1 by ordinary least squares were both significant and were equal to 7.4757 and 0.0071 respectively (see figure 4.2). The economy is expected to grow at an annual rate of 2.8% according to our simple representation of the linear trend component. The HP series was generated with λ = 1600 (see equation (2.3)).

The expansionary and recessionary phases of the business cycle, as determined by the National Bureau of Economic Research (NBER), are obtained by analyzing the movement of a number of economic indicators which lead, lag and coincide with this artifact defined as the business cycle.

> "Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; the sequence of changes is recurrent but not periodic; in duration business cycle vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own". (Burns and Mitchell 1946, p. 3).

One observes that the behavior of the DS, TS and HP series coincide reasonably well with the different recession and expansion phases of the business cycle as defined by the NBER. This obviously does not imply that the three series have the same empirical moments (see appendix B). We will use in this and the subsequent chapters the following notation: GNP (DS), GNP (TS) and GNP (HP) for the difference stationary, trend stationary and Hodrick-Prescott representation of the business cycle.

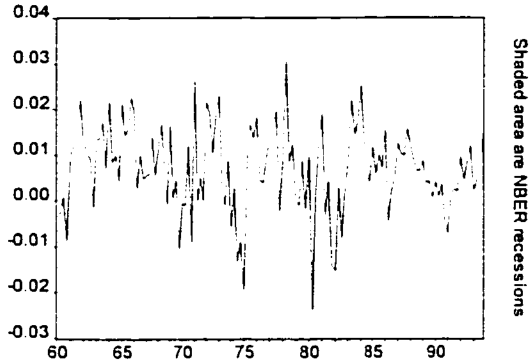

Figure 4.1 GNP (Difference Stationary)

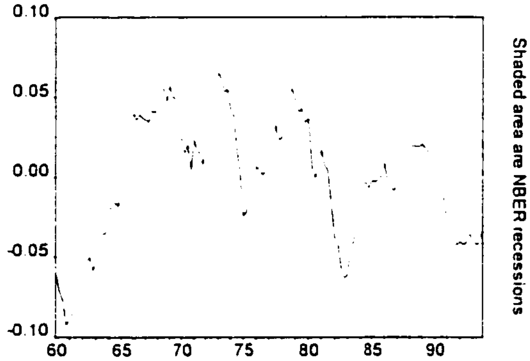

Figure 4.2 GNP (Trend Stationary)

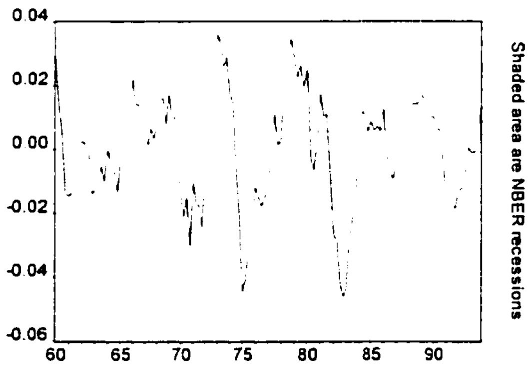

Figure 4.3 GNP (Hodrick-Prescott)

4.2 Linear Time-Series Model of Real GNP

This section presents a linear time-series model for real GNP which will serve as a benchmark to assess the performance of the smooth transition autoregressive (STAR) models and will represent the null in the Granger-Terasvirta nonlinearity test. There has been considerable debate over the last decade on the appropriate time-series representation of GNP. More precisely, the question is whether GNP is trend or difference stationary. An augmented Dickey-Fuller test indicates that we cannot reject difference stationarity at the 5% level[3]. It is shown, in this thesis, that the DS, TS and HP representations of the business cycle follow a nonlinear process. The acceptance or rejection of a unit root based on the augmented Dickey-Fuller test may be misleading given the nonlinear and possibly nonstationary behavior of the cyclical component. We will, therefore, not rule out a priori any of the three GNP representations of the business cycle (i.e., DS, TS and HP).

The estimation of the various linear representations of the business cycle was done with the Box-Jenkins procedure in RATS (see table 4.1). My approach adopts the autoregressive specification since this more readily permits a comparison with the STAR models. This approach is not in itself restrictive since a moving-average process could be converted to an autoregressive one provided the process is invertible.

[3] The Dickey-Fuller test consists in regressing $\ln(GNP_t)$ on a constant, trend, $\ln(GNP_{t-1})$, $\Delta\ln(GNP_{t-1})$ to $\Delta\ln(GNP_{t-5})$. H_0: $\rho=0$ versus H_1: $\rho<0$ (where ρ is the coefficient on $\ln(GNP_{t-1})$;
$P\left(\frac{\rho-1}{\sigma_\rho} \leq C_\alpha\right) = \alpha$ (critical value obtained from Hamilton (1994), table B6, case 4).

Table 4.1: Linear Time-Series Model of Real GNP: $y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \varepsilon_t$

Business cycle	$y_t = y_t^{DS}$ AR(2)	$y_t = y_t^{TS}$ AR(3)	$y_t = y_t^{HP}$ AR(3)
α_0 [4]	0.0075 (0.0014)	0.0050 (0.0118)	0.0002 (0.0032)
α_1	0.2302 (0.0812)	1.1676 (0.0853)	1.0302 (0.0851)
α_2	0.2018 (0.0858)	-0.0031 (0.1344)	0.0250 (0.0838)
α_3	---------	-0.2351 (0.0842)	-0.2450 (0.0838)
σ	0.0088	0.0085	0.0070
$\hat{\kappa}_3$ [a]	-0.1603 (0.4574)	0.0276 (0.8984)	-0.0504 (0.8151)
$\hat{\kappa}_4$	0.5880 (0.1728)	0.7304 (0.0904)	0.8034 (0.0625)
J-B test[b]	0.2966	0.2364	0.1717
ARCH test[c]	0.3718	0.6826	0.5661
Q(8)[d]	0.5046	0.4003	0.2208
Q(16)	0.6716	0.6319	0.1794
Q(24)	0.8353	0.8067	0.4100

a $\hat{\kappa}_3$ and $\hat{\kappa}_4$ are respectively the estimated skewness and excess kurtosis measure (figures in parentheses are the p-values).

b p-value of the Jarque-Bera (1987) statistic.

c p-value of the ARCH statistics.

d p-value of the Ljung-Box Q-statistics.

[4] The OLS estimates for the constant term, α_0, are respectively 0.0042, 0.0004, and 0.0001 for the DS, TS and HP representations of the business cycle. The large discrepancy between the Box-Jenkins and OLS estimated values of α_0 is due to the fact that the Box-Jenkins constant term represents the unconditionnal expected value of y_t.

The figures in parentheses below the estimated parameters in table 4.1 are the standard errors. The choice of an autoregressive (AR) model is based on weighting different indicators such as the partial and autocorrelation functions and the information criteria of Akaike and Schwarz. There are often numerous possibilities for modeling, and consequently one must choose the most concise model that maintains the main characteristics of the time-series. For the above reasons, I have chosen an AR(2) for the DS version of the business cycle and an AR(3) for both the TS and HP version.

The DS, TS and HP linear time-series specifications have all their roots outside the unit circle. The autocorrelation function and the portmanteau test on the residuals do not reveal any serious problems in the three specifications of the business cycle.

A number of financial series display isolated periods of volatility followed by more tranquil series behavior (see Bollerslev et al. (1992)). Autoregressive conditional heteroskedasticity (ARCH) models are able to produce series with periods of large volatility followed closely by periods of almost no volatility. For this reason, the three specifications of the business cycle are tested here for ARCH effects. The ARCH test regresses the estimated squared residuals, u_t^2, on a constant and p lagged values of u_t^2. The statistic $T \times R^2$ follows a chi-square distribution with p degrees of freedom. The test does not, for $p = 4$, suggest any ARCH effects in the three specifications of the business cycle. Finally, the skewness, excess kurtosis and the Jarque-Bera statistics do not lead to a rejection of the assumption of normality in all three cases.

It is therefore intriguing to note that none of the diagnostic tests used suggests that a linear specification for the three representations of the business cycle is inadequate. However, if one was to plot the fitted values obtained from the linear time-series specification of the business cycle with the actual data, one would observe that the linear time-series model is often unable to reproduce recessions. We will see in the next sections that we are able to reject linearity for a number of regime-switching indicators and that these switching models outperform the linear specification in terms of procuring a smaller standard error and in replicating the different phases of the business cycle.

4.3 Testing for Nonlinearity of GNP

A LM statistic proposed by Granger and Terasvirta (1993) is used to test for nonlinearity; the test statistic is presented in appendix C. The nonlinearity test was carried out for the following set of regime-switching indicators: GNP, growth rate of M1 and M2, short-term spread, quarterly difference in the federal funds rate, long-term spread[5] and the S&P stock price; the maximum number of lags used for the switching indicator is 8. The choice of the regime-switching indicators is based on the success they enjoyed in predicting the business cycle in the linear context; the graph of the different series in relation to the recession periods as defined by the NBER for the sample period 1960-1993 are presented in appendix D. Should linearity with the mentioned transition variable be rejected for specifications with more than one lag value, Granger and Terasvirta (1993) suggest that the lag with the lowest p-value be chosen. Table 4.2 therefore presents the lag value of the switching variable that gave the smallest p-value for each switching variable. For example, the first entry of column 2 states that we would not reject nonlinearity for GNP (DS) at the 1% significance level when the switching variable is the $\Delta \ln(GNP)$ at lag 5 .

The table of p-values indicates that nonlinearity is rejected for a large number of switching candidates if the suggested Granger et al. (1993) criteria of p-values of less than 0.01 is used. Only three variables, the quarterly difference in the funds rate, the short-term spread and the long-term spread, are significant at the 0.5% level for all three representation of the business cycle. Note that according to the Granger et al. criteria we would reject lagged GNP as a possible switching variable in the three cases.

Table 4.2 Nonlinearity Test: p-values of the LM- statistic for real GNP.

Business Cycle	**GNP (DS): AR(2)**	**GNP (TS): AR(3)**	**GNP (HP): AR(3)**
Switching Variables:			
GNP	Lag 5 **	Lag 7 *	NS
Funds rate (quarterly difference)	Lag 6 ***	Lag 6 ***	Lag 6 ***
Short-term spread	Lag 1 ***	Lag 2 ***	Lag 2 ***
Long-term spread	Lag 1 ***	Lag 1 ***	Lag 1 ***
S&P (growth rate)	Lag 2 ***	Lag 2 *	Lag 2 ***
M1 (growth rate)	Lag 6 *	Lag 8 *	NS
M2 (growth rate)	Lag 4 *	Lag 4 ***	NS

* SIGNIFICANT AT 5%
** SIGNIFICANT AT 1%
*** SIGNIFICANT AT 0.5%
NS NOT SIGNIFICANT (ABOVE 5%)

4.4 Smooth Transition Regression Models

This section sets out two objectives. The first objective is to present my estimates of several STAR models of real GNP with various switching variables as suggested by the nonlinearity test. The second is to present an analysis of the gains generated by adopting a regime-switching specification over a linear time-series one. The previous section showed that we would reject the linearity assumption for a large number of proposed switching variables if the 0.01 p-value criteria were used. This section looks in

[5] The long-term spread is defined as the difference between the 6-month and the 10 year + Treasury Bill.

turn at the different STAR models suggested by the previous test. Included are STAR models where financial variables only act to define particular regimes. In the next chapters this assumption will be relaxed allowing financial variables to enter the system as both a regressor and a switching catalyst or variable.

The STAR models estimated in this chapter correspond to the following specification:

$$y_t = \beta_2' x_t + (\beta_1 - \beta_2)' x_t F(S_t, \gamma, c) + \varepsilon_t$$

or simply

$$y_t = \alpha' x_t + \beta' x_t F(S_t; \gamma, c) + \varepsilon_t \quad \textbf{(4.1)}$$

with $\alpha = \beta_2$, $\beta = \beta_1 - \beta_2$ and where $x_t = (1, y_{t-1}, \ldots, y_{t-p})$, S_t is the transition variable and

$F(S_t; \gamma, c) = \dfrac{1}{1 + \exp(-\gamma(S_t - c))}$ is the logistic transition function.

We can interpret ($\alpha + \beta$) as the coefficients in regime one (i.e., $F \approx 1$) whereas α correspond to the coefficients in the alternative regime.

The criteria used to judge the validity or superiority of STAR models over the linear specifications will be whether the reduction in the standard deviation obtain from the nonlinear formulation is substantial and also if the transition function picks out the recessionary periods as defined by the NBER. Terasvirta and Anderson (1992) suggest that we should reject the STAR specification if the ratio of the residual variance, $\dfrac{\sigma^2_{STAR}}{\sigma^2_L}$, is greater than 0.9 or if the reduction in the standard deviation obtained by going from the linear time-series model to a STAR model is less than 10%; σ^2_L is the estimated residual variance for the linear model $y_t = \alpha' x_t + \varepsilon_t$ where y_t and x_t are defined as in (4.1) and

σ^2_{STAR} is the estimated residual variance obtain from the STAR model (4.1). We will used this rule of thumb throughout this section as a minimum requirement for the acceptance of a STAR model instead of the simpler linear specification.

Tests are also carried out on the residuals. The skewness and excess kurtosis measures as well as their significance levels, used throughout this thesis, are taken from Davidson and Mackinnon (1993, p. 569). The estimated autocorrelation functions of the residuals have been omitted from this thesis since for all the relevant STAR models, the values taken by the sample autocorrelation function were within the $\pm \frac{2}{\sqrt{N}}$ interval where N represents the number of observations. It is important to note that other criteria such as out of sample forecasting could have been used to assess the relative improvement of the STAR models over the linear ones.

The estimation of the STAR models is based on the concentrated likelihood function (see chapter 3). Having determined in the nonlinearity test the most likely lag value for the switching variable S_t, a grid-search technique is used in the $\{\gamma, c, \sigma\}$-space to orient the choice of starting values for γ and c for the maximization of the concentrated likelihood function. The optimization procedure used here follows the BFGS (Broyden, Fletcher, Goldfard, and Shanno) iterative hill-climbing algorithm. As a cautionary note, one must make use of the generalized inverse in the concentrated likelihood approach in order to ensure that the computer program does not crash due to rounding factors. STAR models of the business cycle which seems promising will be subsequently estimated by maximizing the likelihood function with respect to all of the parameters in equation (4.1) (i.e., α, β, γ and c). Note that the generalized inverse is not used in this approach. This optimization strategy enables us to get standard errors for the parameters of the STAR models[6].

I will make use of a 'switching region' which will indicate the values of S_t required, given the estimated γ and c parameters, to be in an 'intermediate or middle regime' as oppose to the limit or extreme regimes (i.e., $F \approx 0$ and $F \approx 1$). I define as an 'intermediate regime' the values of S_t for which the logistic transition function is between 0.1 and 0.9 (i.e., $F(S_t, \hat{\gamma}, \hat{c}) \in [0.1, 0.9]$).[7]

For example, in the following estimated STAR model of GNP(DS):

$$y_t = \underset{(0.0008)}{0.0061} + \underset{(0.0574)}{0.2156}\, y_{t-1} + \underset{(0.0563)}{0.0992}\, y_{t-2} + \left(-\underset{(0.0019)}{0.0126} - \underset{(0.1452)}{0.4350}\, y_{t-1} + \underset{(0.1621)}{0.5729}\, y_{t-2} \right) \times F(S_t, \gamma, c) + \hat{\varepsilon}_t$$

with

$S_t = funds_{t-6}$ (funds is the quart. diff. in the funds rate), $\hat{\gamma} = 216.78$, $\hat{c} = \underset{(0.0117)}{0.9772}$.

We would be in the intermediate regime when the quarterly difference in the funds rate is in the (0.9671,0.9873) range. This relatively 'small' interval for the quarterly difference in the funds rate implies that the STAR model is essentially a two-regime pure threshold model. In fact, most of the estimated STAR models in this chapter behave as a two-regime pure threshold model.

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[6] Terasvirta has remark on the difficulty of obtaining realistic standard errors for the γ parameter: "*a large number of observations in the neighborhood of c would be needed to estimate γ accurately*" (Terasvirta 1994, p. 213). For example, Terasvirta (1995) estimated logistic STAR model had a γ value of 145 with a standard deviation estimate of 7300. I have, therefore, decided to omit from this thesis the estimated standard errors for the γ parameter.

[7] I am looking for values of S for which $\varepsilon \le \frac{1}{1+e^{-\gamma(S-c)}} \le 1-\varepsilon$ or

$$c - \frac{1}{\gamma}\ln\left(\frac{1-\varepsilon}{\varepsilon}\right) \le S \le c + \frac{1}{\gamma}\ln\left(\frac{1-\varepsilon}{\varepsilon}\right) \text{ where } \varepsilon = 0.1.$$

4.4.1 Switching Variable is Lagged GNP

A natural candidate for the switching variable in threshold models has been the lagged series itself. Potter (1995) examined the asymmetrical behavior of the business cycle assuming a two-regime threshold model where the cutoff point delimiting each regime was a zero growth rate of GNP at lag 2. Potter, using a nonlinear impulse response function, shows that the business cycle is asymmetrical. The Terasvirta et al. nonlinear test would have rejected this STAR model in favor of a linear one at the 1% level for the DS, at the 5% level for the TS version and at the 12% significance level for the HP specification.

STAR model of GNP (DS): switching variable is the growth rate of GNP_{t-5}.

$$y_t = \underset{(0.0021)}{0.0131} - \underset{(0.1220)}{0.1336}\, y_{t-1} + \underset{(0.1400)}{0.1779}\, y_{t-2} + \left(-\underset{(0.0022)}{0.0102} + \underset{(0.1307)}{0.4416}\, y_{t-1} + \underset{(0.1467)}{0.0232}\, y_{t-2} \right) \times F(S_t, \gamma, c) + \hat{\varepsilon}_t$$

with $S_t = y_{t-5}^{DS}$, $\hat{\gamma} = 61504$, $\hat{c} = -\underset{(0.2784)}{0.0052}$, $\hat{\sigma} = 0.0084$, $\dfrac{\hat{\sigma}^2}{\sigma_L^2} = 0.9112$, $\hat{\kappa}_3 = -\underset{(0.2269)}{0.2606}$,

$\hat{\kappa}_4 = \underset{(0.1627)}{0.6021}$, J-B = 0.1819 and IR=(-0.00524,-0.00516).

σ_L^2 is the MLE estimate in the linear case, $\hat{\kappa}_3$ and $\hat{\kappa}_4$ are respectively the skewness and excess kurtosis measure (figures in parentheses below $\hat{\kappa}_3$ and $\hat{\kappa}_4$ are the p-values), J-B is the p-value of the Jarque-Bera statistic and IR is the intermediate regime. Figures in parentheses below the estimated parameters are the standard errors.

The values taken by the transition function for GNP (DS) when the switching variable is GNP_{t-5} do not clearly delineate the recession episodes as defined by the NBER (see figure 4.4). The TS version of the business cycle with GNP as the switching variable was rejected; we would require a deviation of ln(GNP) from trend to be greater than 0.0459 in order to switch to the alternate regime which corresponds to a 21% annual growth rate of GNP. In all three representations of the business cycle, we would reject the STAR model if the switching variable is lagged GNP.

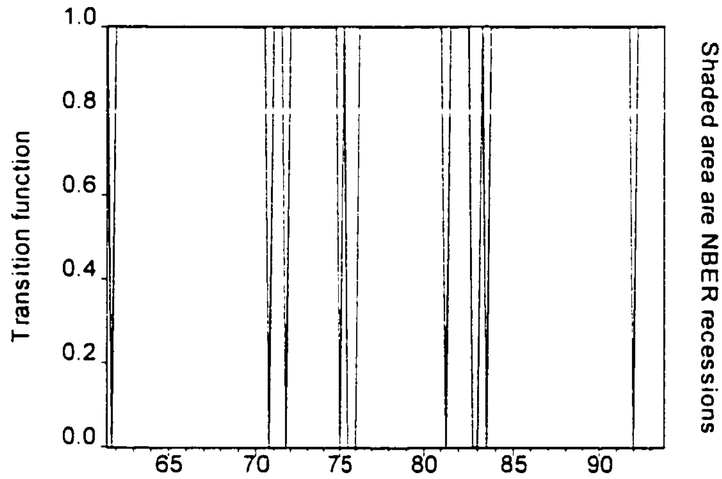
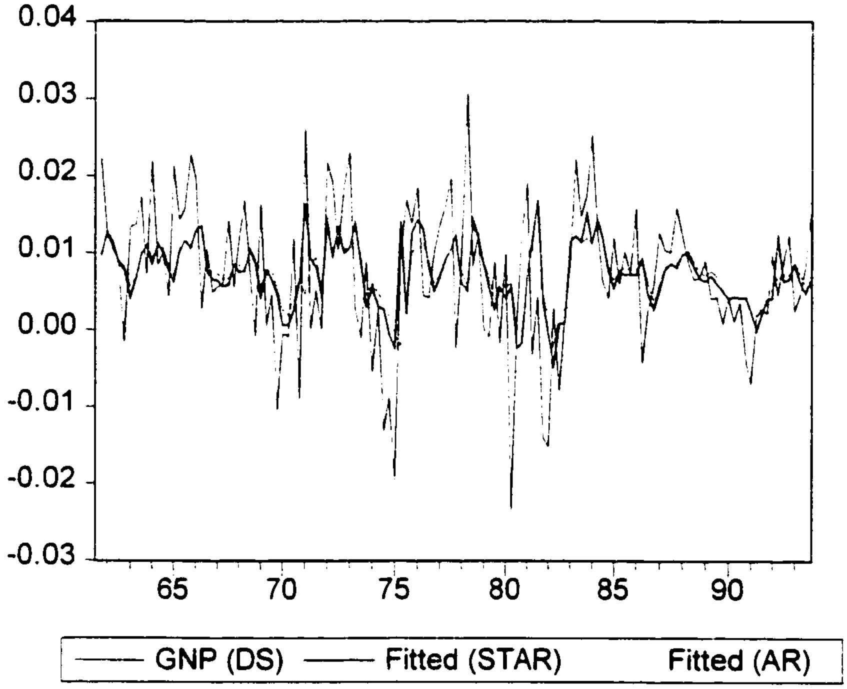


Figure 4.4 STAR model of GNP (DS)
(switching variable is lagged GNP)

The estimated STAR model of GNP (DS version), when the transition variable is GNP_{t-5}, would give an estimated standard deviation of 0.0085 compared to 0.0088 in the linear case. We would therefore reject the STAR model of GNP (DS) according to the rule of thumb of 0.9 suggested by Terasvirta et al (1992); the estimated STAR models obtained by maximizing the concentrated likelihood function have been relegated to appendix E. The variance ratio permits us to have a simple rule to accept or reject the STAR model in favor of the linear time-series model. An auxiliary and useful strategy is to plot the estimated STAR and linear models with the actual data set. Figure 4.4a compares the estimated or fitted values of the STAR and AR(2) model of GNP (DS) with the observed value of GNP for the 1960-1993 sample period. One observes that both the STAR and the linear model of GNP are frequently unable to capture or reflect the recessionary phases of the business cycle. The STAR and the linear model both behave in a similar manner which is reflected by the fact that the variance ratio is above the 0.9 threshold.

Figure 4.4a Actual and fitted model of GNP (DS) (switching variable is lagged GNP)[a]



a Fitted STAR: $\hat{y}_t = \hat{\alpha}^T x_t + \hat{\beta}^T x_t F(S_t; \hat{\gamma}, \hat{c})$

Fitted AR: $\hat{y}_t = \hat{\alpha}^T x_t$

Throughout this section two limiting regimes will be presented as if the STAR model was a simple two-regime pure threshold model. Due to the limited number of observations available for low growth regimes over the 1960-1993 sample period, one must be careful when attributing properties to these regimes. For instance, in the case of GNP (DS) with switching occurring when GNP_{t-5} crosses the -0.005 threshold value (annual negative growth rate of 2%) we would have a low growth regime corresponding to a 2.4% growth rate while the alternate regime would have a growth rate of 5.4% (see appendix F).

4.4.2 Switching Variables are the Growth Rate of M1 and M2.

One of the main topics in this thesis is the relevance of money in explaining part of the business cycle process. The growth rate of M1 and M2 does not suggest nonlinearities in the case of the HP series representation of the business cycle. The grid-search approach for the DS version of GNP revealed that for the growth rate of M1 the maximum of the likelihood function occurs for values of c that approach its observed minimum value. This implies that no switching between regimes actually takes place. We are then left with our original AR(2) model of GNP (DS). We would also reject, according to the ratio rule, a STAR model of GNP (DS) using the growth rate of M2. The no switching result for the growth rate of M1 in the TS version is the same as that of the DS case.

The only case where the STAR model of real GNP is not rejected is under a TS representation using the growth rate of M2 at lag 4 as the switching variable. However, only when the growth rate of M2 is above 11% (annual growth rate), would we be in the low growth regime, corresponding to a negative annual growth rate of 1.8% (deviation from trend). The alternate regime or high growth regime would lead to an annual growth rate of 47%. We would be "stuck" in this regime whenever the growth rate of money fell below 11%! If this model was truly reflective of the real economy, the Fed would clearly only choose monetary policies that achieved money growth of less than 11%. Inflation be damned if the real economy grows at a rate of 47%! Given these figures, this model must also be discarded as unrealistic. Note that the behavior of the transition function with the growth rate of M2 as the switching variable also does not reflect periods of recession as defined by the NBER (see figure 4.5). The value of the variance ratio suggests favoring the STAR model. Note that simply eyeballing the fitted values of the STAR and the linear models with the actual value of GNP (TS) would not lead to a clear choice since both models seem to reflect fairly well the observed data (see figure 4.5a). I will, for the most part, mainly rely on the variance ratio for the TS and HP specifications of the business cycle since it is often difficult to favor one model over another just by looking at the plot of the fitted values and the actual data.

STAR model of GNP (TS): switching variable is the growth rate of $M2_{t-4}$.

$$y_t = \underset{(0.0005)}{-0.0004} + \underset{(0.0551)}{1.1023}\,y_{t-1} + \underset{(0.0841)}{0.1054}\,y_{t-2} + \underset{(0.0539)}{-0.2924}\,y_{t-3} +$$

$$\left(\underset{(0.0020)}{0.0117} - \underset{(0.1376)}{0.0493}\,y_{t-1} - \underset{(0.1641)}{0.9223}\,y_{t-2} + \underset{(0.1382)}{0.9567}\,y_{t-3}\right) \times F(S_t, \gamma, c) + \hat{\varepsilon}_t$$

with $S_t = M2_{t-4}$, $\hat{\gamma} = 520327$, $\hat{c} = \underset{(00178)}{0.0270}$, $\hat{\sigma} = 0.0078$, $\dfrac{\hat{\sigma}^2}{\sigma_L^2} = 0.8421$, $\hat{\kappa}_3 = \underset{(0.5384)}{-0.1326}$,

$\hat{\kappa}_4 = \underset{(0.2651)}{0.4807}$, $J - B = 0.4448$ and IR=(0.02699,0.02700).

Based on these criticisms, the specifications based on the growth rate of money as the switching variable will be set aside in favor of a set of models based on interest rate instruments. These models outperform STAR models with the growth rate of money switching regimes, providing smaller standard deviations and closely following defined periods of recession via the transition function.

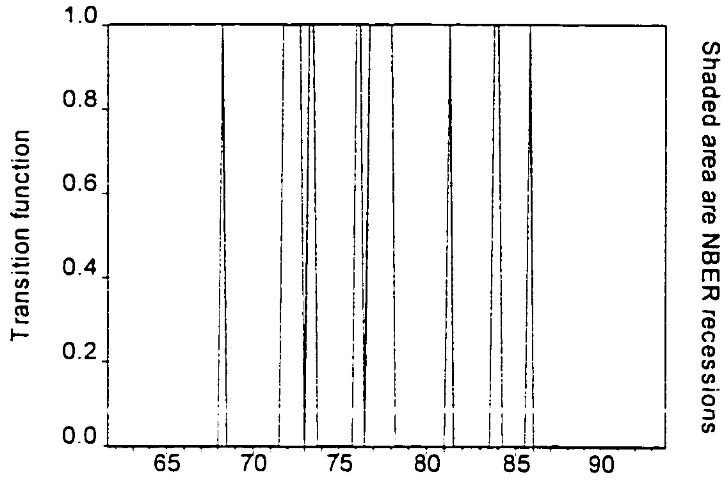
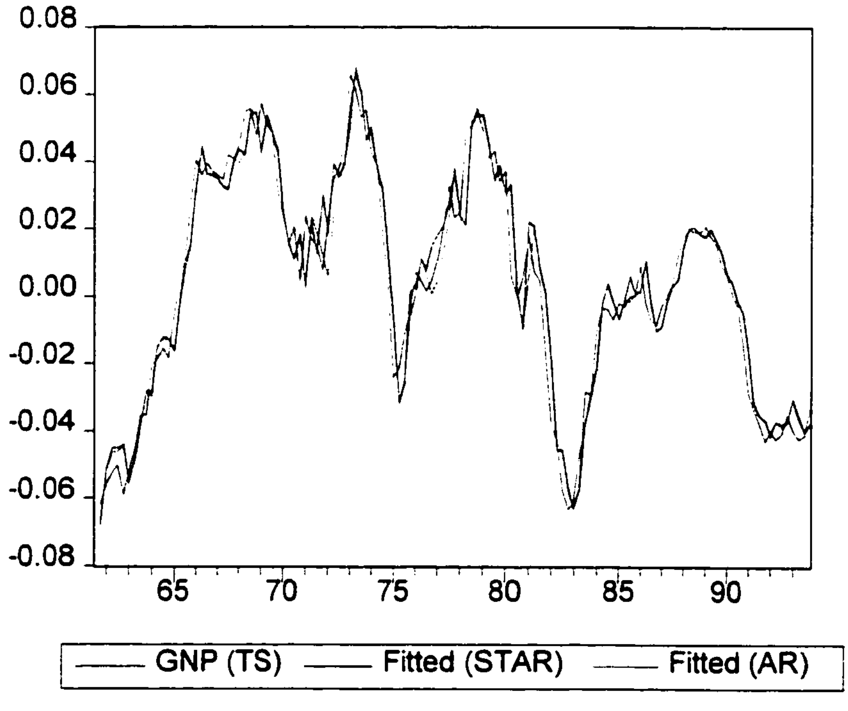


Figure 4.5 STAR model of GNP (TS)
(switching variable is the growth rate of M2)

Figure 4.5a Actual and fitted model of GNP (TS)
(switching variable is the growth rate of M2)



4.4.3 Switching Variable is the Short-Term Spread

Among the many interest rate instruments found in the literature, the short-term spread, the federal funds rate and the long-term spread have been highly successful in predicting the business cycle in a linear time-series framework. When one observes the relationship between a widening of the interest rate differential (i.e., the short-term spread) and the recessionary period the effect is quite striking. Whether we interpret movements in the short-term spread as a measure of default risk or a tightening of monetary policy, nearly all recessions are preceded by an increase in the short-term spread by more than 80 basis points. The notable exception is the 1990 recession where the short-term spread was decreasing prior to the economic downswing. This is in line with Friedman and Kuttners (1998) claim that all of the major leading business indicators including the short-term spread failed to predict the 1990 recession. We saw in the case of real GNP that the series could be characterized by three segments, each coinciding roughly with a decade, with the middle segment subject to much larger economic swings. Movements in the short-term spread did not emulate this same volatility in the real GNP series for part of the mid 70's and 80's; one notes that, apart the spike in 1973 and the subsequent negative values in the following quarters, each decade appears to be more or less the same (see Appendix D, figure 4).

The hill-climbing method combined with the grid-search approach pointed to a STAR model of GNP (DS) with $\gamma = 1562$ and c = 1.19 when the switching variable is the short-term spread lagged (t-1). A c of 1.19 (the cutoff point delimiting recessions) is essentially the mean of the short-term spread (1.10) during the recessions as defined by the NBER during the period 1960-1990 (see Friedman and Kuttner 1992a); The sample mean of the short-term spread for the entire period is 0.553 with a standard deviation of 0.443. The transition function coincides reasonably well with the expansion and contraction phase of the business cycle except for the 1990 recession (see figure 4.6). This observation was, according to Bernanke and Kuttner (1992), one justification for the use of the federal funds rate instead of the short-term spread since the latter failed to predict the last recession. We have, due to the large γ value, a system where values of the 6-month commercial paper rate above the Treasury Bill by more than 120 basis points would propel us into regime 1 (i.e., $F \approx 1$). The dynamics of the system are such that the high growth regime (i.e., $F \approx 0$) would be stationary only if we were to remain permanently stuck in this regime whereas the low growth is nonstationary. The high value of ϕ_2 in the low growth regime implies that two consecutive negative growth rates of the same magnitude would result in an important downturn in the economic activity of the following quarter. The process might still be stable, however, since shocks to the system may force us out of the low growth regime process. The fitted values of the STAR model of GNP (DS), with the short-term spread switching the process, more closely mimics the actual values of GNP associated with the 1970, 1975 and 1980 recessions (see figure 4.6a); we saw that the STAR model of GNP (DS) based on lagged values of GNP switching the process would not successfully mirror the actual data for the recessionary periods.

STAR model of GNP (DS): switching variable is the short-term spread$_{t-1}$

$$y_t = \underset{(0.0008)}{0.0051} + \underset{(0.0611)}{0.3002}\, y_{t-1} + \underset{(0.0568)}{0.1006}\, y_{t-2} + \left(-\underset{(0.0018)}{0.0097} - \underset{(0.1587)}{0.9419}\, y_{t-1} + \underset{(0.2345)}{0.9793}\, y_{t-2} \right) \times F(S_t, \gamma, c) + \hat{\varepsilon}_t$$

with $S_t = \text{spread}_{t-1}$ (spread is the short-term spread), $\hat{\gamma} = 1562.65$, $\hat{c} = \underset{(0.5723)}{1.1859}$,

$\hat{\sigma} = 0.0079$, $\frac{\hat{\sigma}^2}{\sigma_L^2} = 0.8059$, $\hat{\kappa}_3 = \underset{(0.8882)}{0.0303}$, $\hat{\kappa}_4 = \underset{(0.9906)}{0.0051}$, J-B = 0.9901 and

IR=(1.18449,1.187306).

A linear model which includes the short-term spread as a regressor performs equally well in terms of reducing the standard deviation, providing evidence to support the proposition that the information conveyed by the short-term spread is not dependant on the STAR formulation. In fact, a linear model with the short-term spread as a regressor leads to a virtually identical standard deviation value of 0.008: i.e.,

$$y_t = \underset{(0.0016)}{0.0113} + \underset{(0.0853)}{0.0475}\, y_{t-1} + \underset{(0.0787)}{0.1372}\, y_{t-2} - \underset{(0.0018)}{0.0095}\, S_t + \hat{\varepsilon}_t$$

with $\hat{\kappa}_3 = -\underset{(0.9785)}{0.0058}$, $\hat{\kappa}_4 = \underset{(0.2675)}{0.4836}$ and where y is GNP (DS) and $S_t = \text{spread}_{t-1}$ (spread is the short-term spread): figures in parentheses are standard errors. In this linear case, we would reject the null that the short-term spread lagged (t-1) plays no role in explaining GNP (DS). It will be shown that this result is in direct contrast with the quarterly difference in the funds rate and the long-term spread variables. It will not be possible to reject the analogous null:

$$H_0: \delta = 0 \text{ in the equation } y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \delta S_t + \varepsilon_t$$

where S_t is the financial variable in question (i.e., the funds rate and the long-term spread).

One justification for modeling the business cycle as a nonlinear model is the apparent asymmetry of the process. The skewness and excess kurtosis measures obtained from the estimated residuals are considerably reduced in the STAR model with the short-term spread as compared to the AR(2) model of GNP (DS); it is assumed that the error terms are drawn from the same distribution which is independent of the regimes. The TS version model with the short-term spread is accepted according to the ratio rule, but the transition function only depicts one brief stay in the low growth regime for the entire 1960-1993 period (see figure 4.7).

STAR model of GNP (TS): switching variable is the short-term spread$_{t-2}$.

$$y_t = \underset{(0.0005)}{0.0003} + \underset{(0.0685)}{1.2466}\, y_{t-1} - \underset{(0.1095)}{0.1282}\, y_{t-2} - \underset{(0.0652)}{0.1759}\, y_{t-3} +$$
$$\left(\underset{(0.0166)}{0.0388} - \underset{(0.2507)}{0.5012}\, y_{t-1} + \underset{(0.5472)}{1.6149}\, y_{t-2} - \underset{(0.7946)}{2.1518}\, y_{t-3}\right) \times F(S_t, \gamma, c) + \hat{\varepsilon}_t$$

with $S_t = \text{spread}_{t-2}, \hat{\gamma} = 15.60, \hat{c} = \underset{(0.0579)}{1.3650}, \hat{\sigma} = 0.0080, \dfrac{\hat{\sigma}^2}{\sigma_L^2} = 0.8858,$
$\hat{\kappa}_3 = -\underset{(0.4801)}{0.1521}, \hat{\kappa}_4 = \underset{(0.0186)}{1.0154}$, J-B = 0.0488 and IR=(1.22415,1.50585).

The transition function for the HP series, when the switching variable is the short-term spread, is essentially a carbon copy of the DS version (see figure 4.8). An identical result is obtained, whereby the short-term spread lagged (t-2) acting as a regressor in the linear specification provides the same standard deviation as a STAR model with the short-term spread lagged (t-2) playing the role of the switching indicator.

STAR model of GNP (HP): switching variable is the short-term spread$_{t-2}$.

$$y_t = \underset{(0.0005)}{0.0004} + \underset{(0.0684)}{1.1092}\, y_{t-1} - \underset{(0.1011)}{0.1598}\, y_{t-2} - \underset{(0.0645)}{0.1391}\, y_{t-3} +$$
$$\left(-\underset{(0.0027)}{0.0033} - \underset{(0.1572)}{0.4548}\, y_{t-1} + \underset{(0.2272)}{0.8582}\, y_{t-2} - \underset{(0.2294)}{0.7775}\, y_{t-3}\right) \times F(S_t, \gamma, c) + \hat{\varepsilon}_t$$

with $S_t = \text{spread}_{t-2}, \hat{\gamma} = 40.39, \hat{c} = \underset{(0.0623)}{1.2338}, \hat{\sigma} = 0.0075, \dfrac{\hat{\sigma}^2}{\sigma_L^2} = 0.9013, \hat{\kappa}_3 = -\underset{(0.6037)}{0.1119},$

$\hat{\kappa}_4 = \underset{(0.0451)}{0.8641}$, J-B = 0.1175 and IR=(1.17940,1.28820).

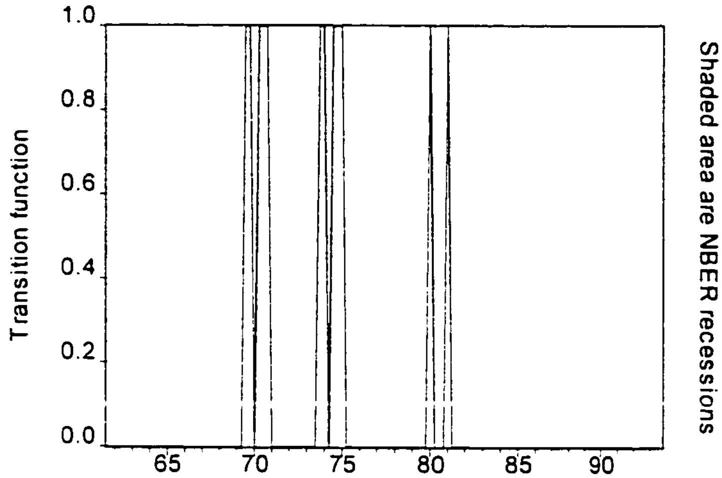
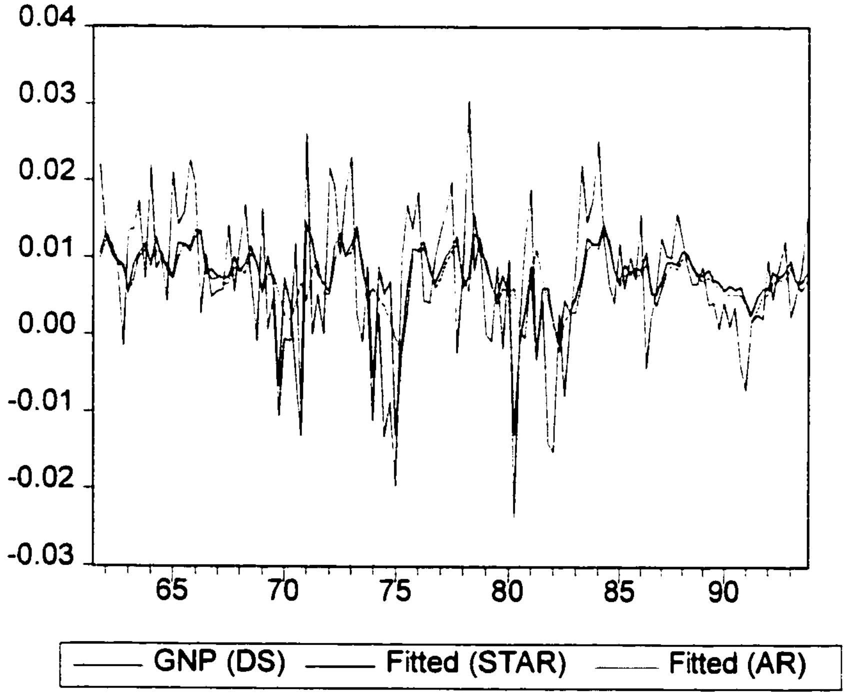


Figure 4.6 STAR model of GNP (DS)
(switching variable is the short-term spread)

Figure 4.6a Actual and fitted model of GNP (DS)
(switching variable is the short-term spread)



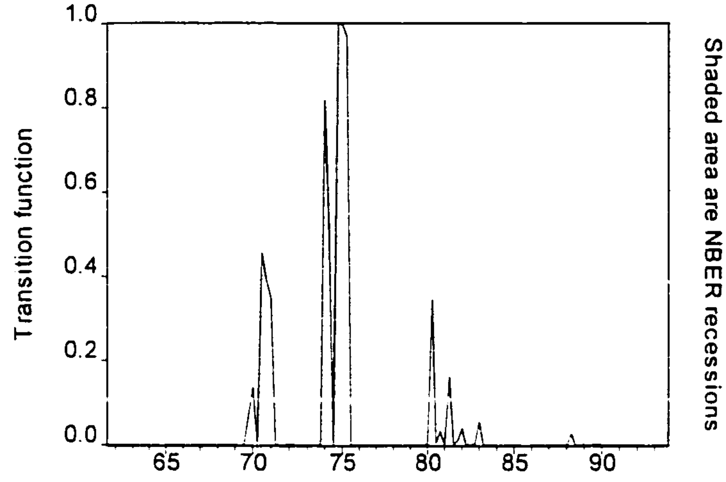
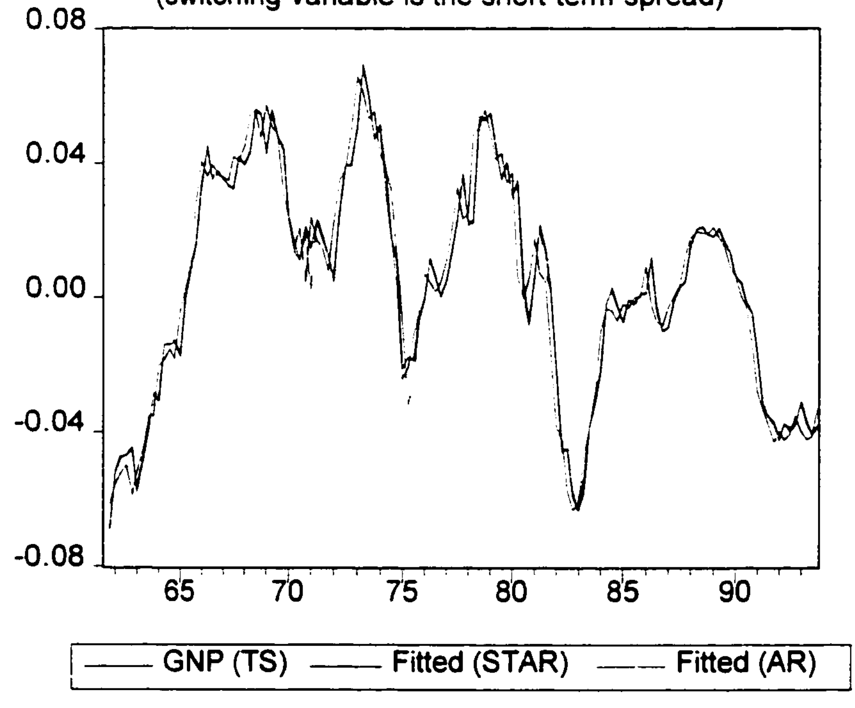


Figure 4.7 STAR model of GNP (TS)
(switching variable is the short-term spread)

Figure 4.7a Actual and fitted model of GNP (TS)
(switching variable is the short-term spread)



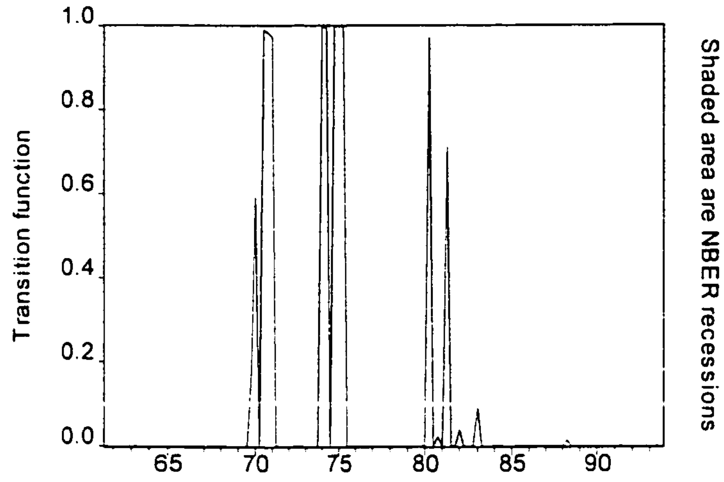
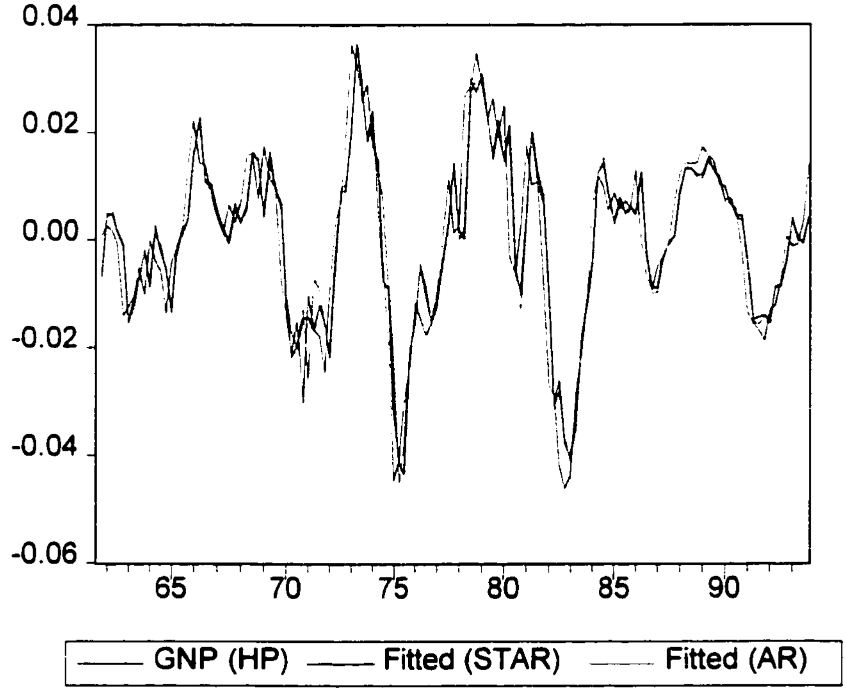


Figure 4.8 STAR model of GNP (HP)
(switching variable is the short-term spread)

Figure 4.8a Actual and fitted model of GNP (HP)
(switching variable is the short-term spread)



4.4.4 Switching Variable is the Federal Funds Rate

The relation between the quarterly difference in the federal funds rate and the economy is much more difficult to depict graphically than the previous short-term spread and business cycle relation. A visual inspection reveals that each start of a recession is not immediately preceded by an increase or a decrease in the funds rate (see Appendix D, figure 3). It will be argued below that the trigger for a recession, or more precisely a low growth regime, is the quarterly difference in the funds rate at a lag of a year and a half prior to the recession. It is interesting to note that, similar to the real GNP series, the federal funds rate also exhibited greater volatility over the middle period.

A STAR model with the quarterly difference in the funds rate as the switching variable gives essentially the same standard deviation as in the previous model with two lags of GNP (DS) and the short-term spread causing the switches between regimes; the standard deviation for the quarterly difference in the funds rate is 0.0080 compared to 0.0079 for the short-term spread. Still, there are important differences between these switching models even though they both rely on interest rate based instruments as the switching condition. The model is still essentially a two regimes pure threshold model with a switching cut off at a quarterly difference in the funds rate of 0.98 or 100 basis points; the sample mean of the quarterly difference in the funds rate is -0.007 with a standard deviation of 1.127.

STAR model of GNP (DS): switching variable is the quarterly difference in the funds rate$_{t-6}$.

$$y_t = \underset{(0.0008)}{0.0061} + \underset{(0.0574)}{0.2156}\,y_{t-1} + \underset{(0.0563)}{0.0992}\,y_{t-2} + \left(-\underset{(0.0019)}{0.0126} - \underset{(0.1452)}{0.4350}\,y_{t-1} + \underset{(0.1621)}{0.5729}\,y_{t-2}\right) \times F(S_t, \gamma, c) + \hat{\varepsilon}_t$$

with $S_t = \text{funds}_{t-6}$ (funds is the quarterly difference in the funds rate).

$$\hat{\gamma} = 216.78,\ \hat{c} = \underset{(0.0117)}{0.9772},\ \hat{\sigma} = 0.0080,\ \frac{\hat{\sigma}^2}{\sigma_L^2} = 0.8244,\ \hat{\kappa}_3 = \underset{(0.3451)}{0.2037},\ \hat{\kappa}_4 = \underset{(0.7536)}{0.1354},\ \text{J-B} =$$

0.6096 and IR=(0.96706,0.98734).

The second major difference is that the transition function for the funds rate picks up the 1990 recession for the DS representation of the business cycle whereas the transition function for the short-term spread depicted no movement towards a possible slowdown of the economy (see figure 4.9). It is possible, however, that the failure to reach the upper bound is explained by the fact the 1990 recession was notably less severe than previous downturns. The fitted values, implied by our estimated STAR model of GNP (DS) with the quarterly difference switching the process, replicates the results obtained with the short-term spread, namely of reproducing recessions (see figure 4.9a). The two limiting regimes associated with the quarterly difference in the funds rate would be stationary. We saw in the previous models that there was essentially no gain in terms of reducing the standard deviation from inserting the short-term spread as a regressor in the linear model or as the switching variable in the STAR framework. This is not the case with the quarterly difference in the funds rate where we would reject the regressor quarterly difference in the funds rate lagged (t-6) in the linear specification for the DS, TS and HP representation of the business cycle. The sample autocorrelation functions associated with the STAR specification for the three representations of the business cycle do not reveal any autocorrelation of the residuals. Furthermore, we would not reject the assumption of normality in all three cases.

Romer and Romer (1989) maintained that the Federal Reserve tried on several occasions to induce a recession in order to relieve the inflationary pressures on the economy; the dates are December 1968, April 1974, August 1978 and October 1979. It is therefore interesting to observe that our transition function for the DS series does pick out these policy changes and, except for one episode, it appears the Fed was successful according to our model in attaining their objectives. While April 1974 is not associated with a recession 6 quarters ahead, we nonetheless can observe a sharp drop in the growth rate of GNP. The TS and HP versions, with the quarterly difference in the funds rate as the switching indicator, give similar results in terms of the transition function and also in terms of having the smallest variance ratio amongst the different switching candidates. It is worth mentioning that the transition function does not depict the 1990 recession in both the TS and HP representation of the business cycle (see figure 4.10 and 4.11).

A final point regarding the difference between the STAR model based on the short-term spread and the quarterly difference in the funds rate is that we would not reject the assumption of normality at the 5% level when the switching variable is the quarterly difference in the funds rate, whereas we would reject normality for the TS and HP representation of the business cycle when the switching between regimes is provoked by the short-term spread.

STAR model of GNP (TS): switching variable is the quarterly difference in the funds rate$_{t-6}$.

$$y_t = \underset{(0.0005)}{0.0013} + \underset{(0.0574)}{1.1858}\, y_{t-1} - \underset{(0.0877)}{0.1012}\, y_{t-2} - \underset{(0.0558)}{0.1318}\, y_{t-3} +$$
$$\left(-\underset{(0.0026)}{0.0075} - \underset{(0.2256)}{0.3673}\, y_{t-1} + \underset{(0.4490)}{0.3282}\, y_{t-2} - \underset{(0.2456)}{0.4660}\, y_{t-3} \right) \times F(S_t, \gamma, c) + \hat{\varepsilon}_t$$

with $S_t = \text{funds}_{t-6}$, $\hat{\gamma} = 10.19$, $\hat{c} = \underset{(0.0789)}{1.3255}$, $\hat{\sigma} = 0.0075$, $\dfrac{\hat{\sigma}^2}{\sigma_L^2} = 0.7994$, $\hat{\kappa}_3 = \underset{(0.2488)}{0.2487}$, $\hat{\kappa}_4 = \underset{(0.1751)}{0.5849}$, J-B = 0.2051 and IR=(1.10987,1.54113).

STAR model of GNP (HP): switching variable is the quarterly difference in the funds rate$_{t-6}$.

$$y_t = \underset{(0.0005)}{0.0011} + \underset{(0.0545)}{1.0181}\, y_{t-1} - \underset{(0.0748)}{0.0552}\, y_{t-2} - \underset{(0.0537)}{0.1541}\, y_{t-3} +$$

$$\left(-\underset{(0.0017)}{0.0112} - \underset{(0.1188)}{0.3471}\, y_{t-1} + \underset{(0.1783)}{0.8249}\, y_{t-2} - \underset{(0.1264)}{0.6058}\, y_{t-3} \right) \times F(S_t, \gamma, c) + \hat{\varepsilon}_t$$

with $S_t = \text{funds}_{t-6}$, $\hat{\gamma} = 399.99$, $\hat{c} = \underset{(0.2784)}{1.0433}$, $\hat{\sigma} = 0.0070$, $\frac{\hat{\sigma}^2}{\sigma_L^2} = 0.7851$, $\hat{\kappa}_3 = \underset{(0.3100)}{0.2189}$,

$\hat{\kappa}_4 = \underset{(0.0754)}{0.7670}$, J-B = 0.1229 and IR=(1.03781,1.04789).

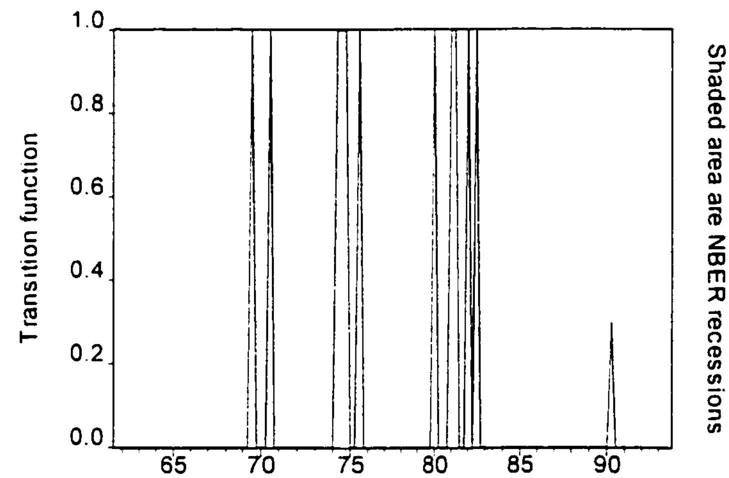
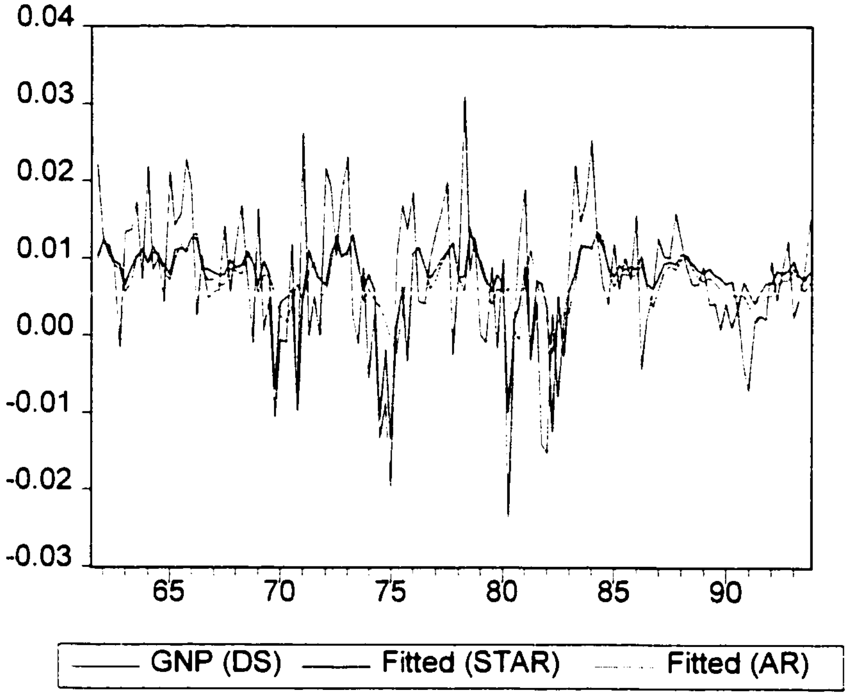


Figure 4.9 STAR model of GNP (DS)
(switching variable is the quart. diff. in the funds rate)

Figure 4.9a Actual and fitted model of GNP (DS)
(switching variable is the quart. diff. in the funds rate)



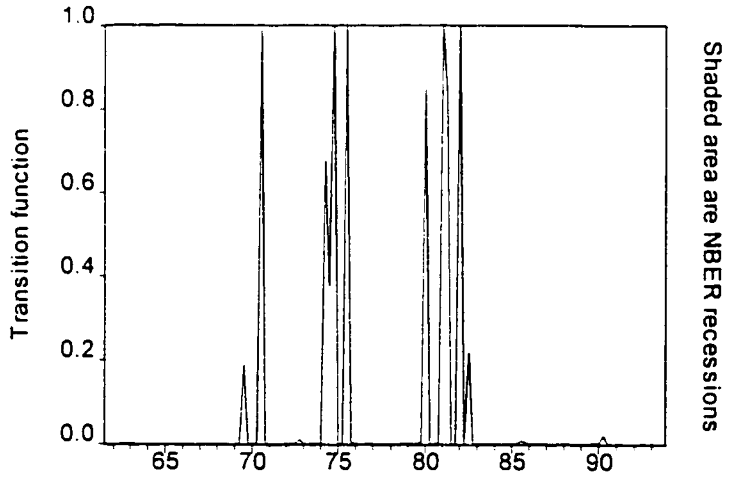
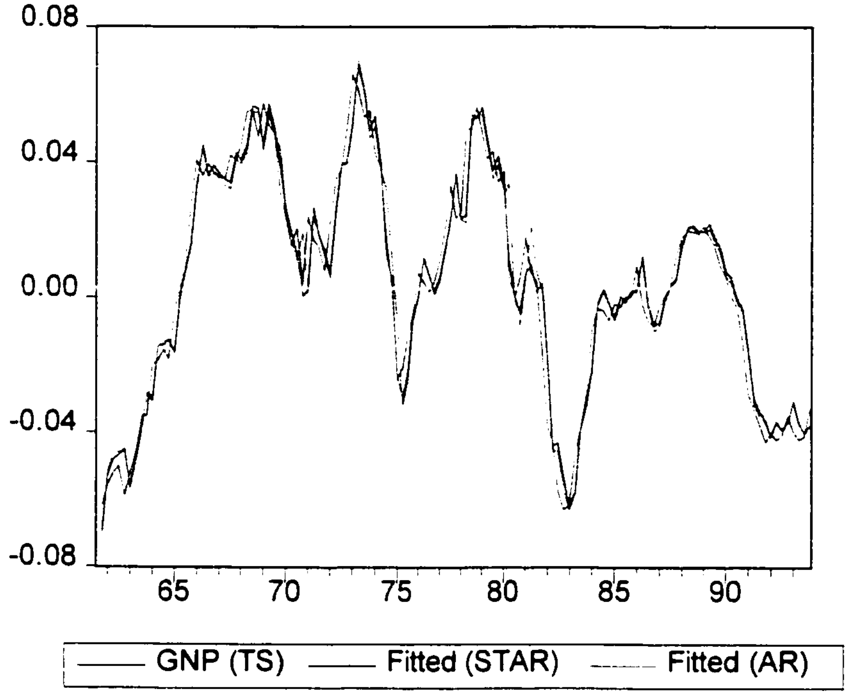


Figure 4.10 STAR model of GNP (TS)
(switching variable is the quart. diff. in the funds rate)

Figure 4.10a Actual and fitted model of GNP (TS)
(switching variable is the quart. diff. in the funds rate)



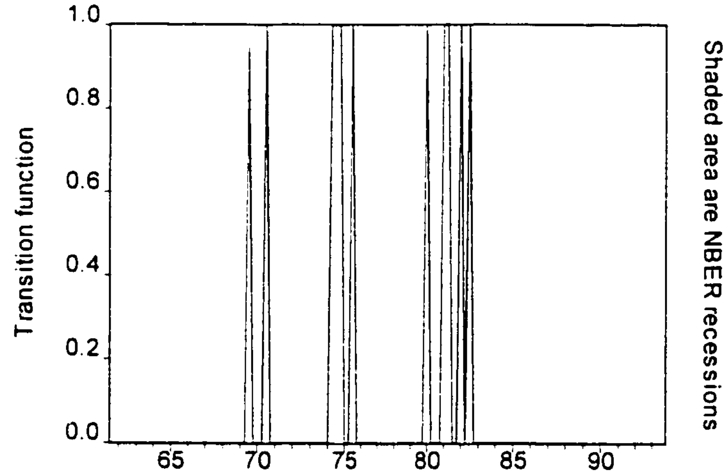
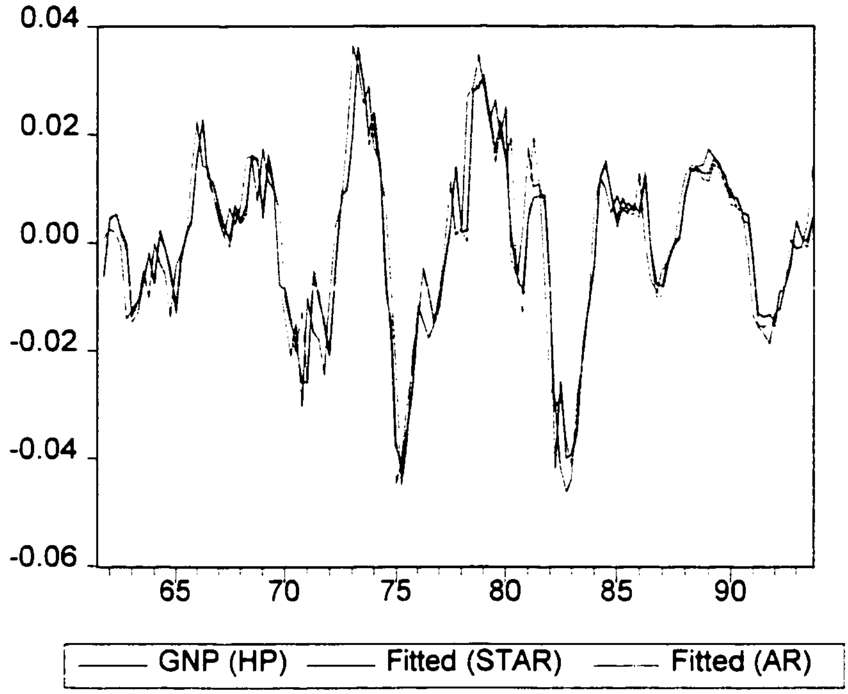


Figure 4.11 STAR model of GNP (HP)
(switching variable is the quart. diff. in the funds rate)

Figure 4.11a Actual and fitted model of GNP (HP)
(switching variable is the quart. diff. in the funds rate)



In attempting to explain the business cycle, I have relied on the predictive power of the quarterly difference in the nominal funds rate instead of the real interest rate. It could be the case that the quarterly difference in the 'nominal' funds rate is successful in predicting the business cycle because it essentially mimics the behavior of the quarterly difference in the real interest rate. To address this issue, my approach will be to use the difference in the ex-post real interest rate as the switching variable in the STAR model of real GNP (DS). A grid-search approach and subsequent estimation of a STAR model, with the 'observed' difference in the real interest rate lagged (t-6) as the switching variable, would be rejected according to the variance ratio rule. Furthermore, a STAR model with 4 lags of GNP (DS) and 4 lags of the ex-post real interest rate (in levels), with the ex-post real interest rate at lag 6 causing the switches between regimes, would also be rejected in favor of the simpler AR(4) model of GNP (DS) according to the likelihood ratio statistic. The STAR model based on the funds rate instead of the ex-post real interest rate will, therefore, be preferred, on the grounds that it leads to a smaller standard deviation and also to a behavior of the transition function which is more indicative of recessions.

4.4.5 Switching Variable is the Long-Term Spread

As for our last interest rate instrument, namely the long-term spread, a negative spread between a short-term interest rate, (the rate on a 6 month Treasury Bill) and a long-term yield (the rate on a Treasury Bill of 10 years or more) generally precedes a recession (see Appendix D, figure 5). But it is also the case that numerous episodes, most notably since the 1970's, can be found with negative values in the long-term spread of the same magnitude that did not lead to a recession.

The switching would be conducted, in the DS version, by the value of the first lag of the long-term spread. The transition function would barely suggest the 1990 recession (see figure 4.12). The fitted STAR model behaves in a similar manner as the other STAR models based on interest rates as the switching variable in terms of reflecting recessions except that the model would not performed as admirably for the 1988-1993 period. The estimated STAR model does not display the same abrupt movement from expansion to recession as in the two previous cases with the short-term spread and the quarterly difference in the funds rate. The high growth limiting regime would be stationary with an annual growth rate of 4%, while the other limiting regime would be nonstationary. Note that this last regime is never attained, in fact the economy does not even approach it, save perhaps for the last quarter of 1980. For the TS version of the business cycle, the first regime is again non stationary (see figure 4.13). The HP version did not lead to a satisfactory model.

STAR model of GNP (DS): switching variable is the long-term spread$_{t-1}$.

$$y_t = -\underset{(0.0093)}{0.0199} - \underset{(0.2202)}{0.2033}\,y_{t-1} + \underset{(0.4853)}{1.1743}\,y_{t-2} + \left(\underset{(0.0095)}{0.0279} + \underset{(0.2476)}{0.3393}\,y_{t-1} - \underset{(0.5063)}{1.1035}\,y_{t-2}\right) \times F(S_t, \gamma, c) + \hat{\varepsilon}_t$$

with S_t = lspread$_{t-1}$ (lspread is the long-term spread), $\hat{\gamma} = 2.1476$, $\hat{c} = -\underset{(0.3833)}{1.2974}$,

$\hat{\sigma} = 0.0076$, $\dfrac{\hat{\sigma}^2}{\sigma_L^2} = 0.7459$, $\hat{\kappa}_3 = \underset{(0.3440)}{0.2041}$, $\hat{\kappa}_4 = -\underset{(0.7156)}{0.1572}$, J-B = 0.5980 and IR=(-2.32051, -0.27429).

STAR model of GNP (TS): switching variable is the long-term spread$_{t-1}$.

$$y_t = -\underset{(0.0026)}{0.0113} + \underset{(0.0959)}{0.8969}\,y_{t-1} + \underset{(0.1683)}{0.6559}\,y_{t-2} - \underset{(0.1169)}{0.4229}\,y_{t-3} +$$
$$\left(\underset{(0.0026)}{0.0134} + \underset{(0.1197)}{0.2981}\,y_{t-1} - \underset{(0.1992)}{0.7817}\,y_{t-2} + \underset{(0.1328)}{0.3240}\,y_{t-3}\right) \times F(S_t, \gamma, c) + \hat{\varepsilon}_t$$

with S_t = lspread$_{t-1}$, $\hat{\gamma} = 300.50$, $\hat{c} = \underset{(0.9604)}{0.0655}$, $\hat{\sigma} = 0.0078$, $\dfrac{\hat{\sigma}^2}{\sigma_L^2} = 0.8421$, $\hat{\kappa}_3 = \underset{(0.1681)}{0.2972}$,

$\hat{\kappa}_4 = \underset{(0.8533)}{0.0797}$, J-B = 0.3802 and IR=(0.05818,0.07282).

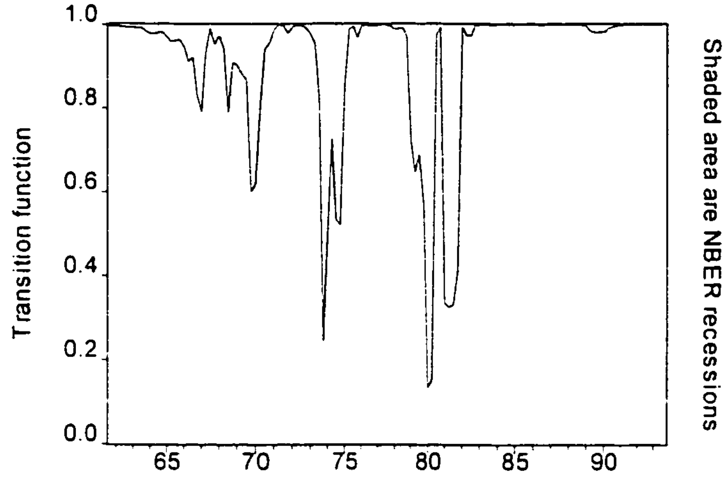
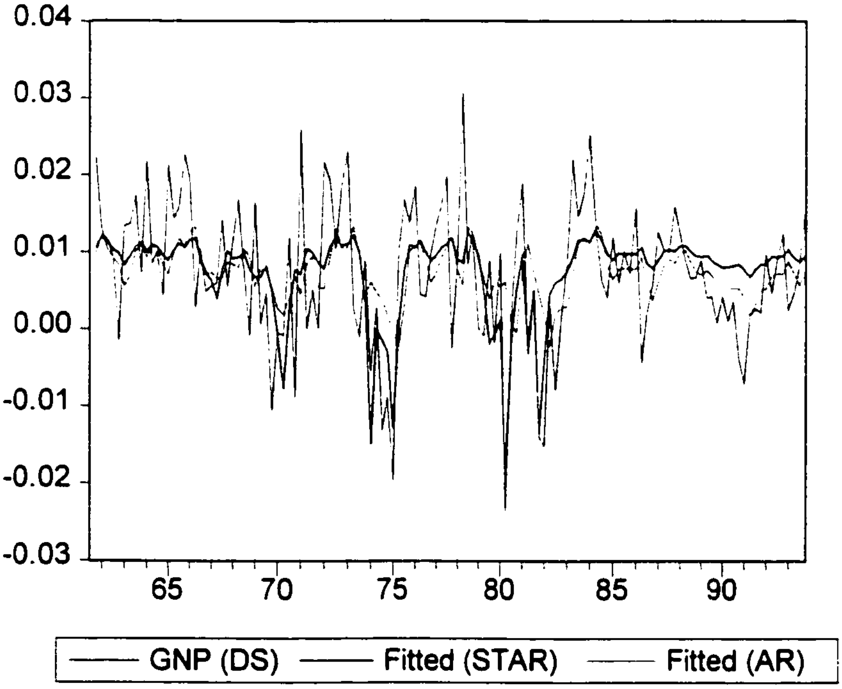


Figure 4.12 STAR model of GNP (DS)
(switching variable is the long-term spread)

Figure 4.12a Actual and fitted model of GNP (DS)
(switching variable is the long-term spread)



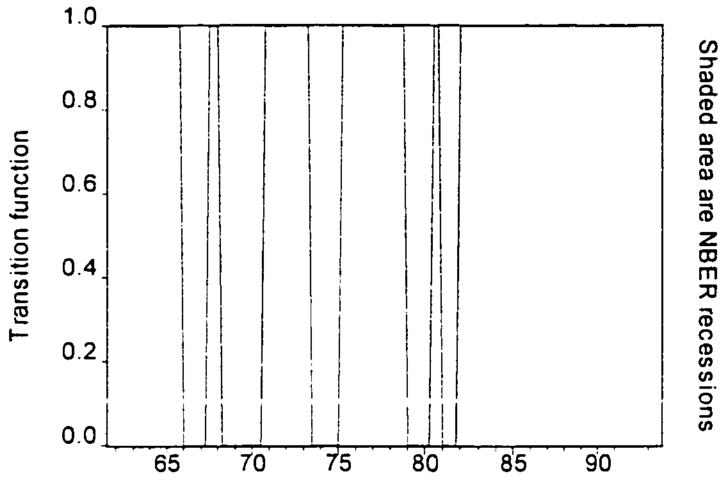
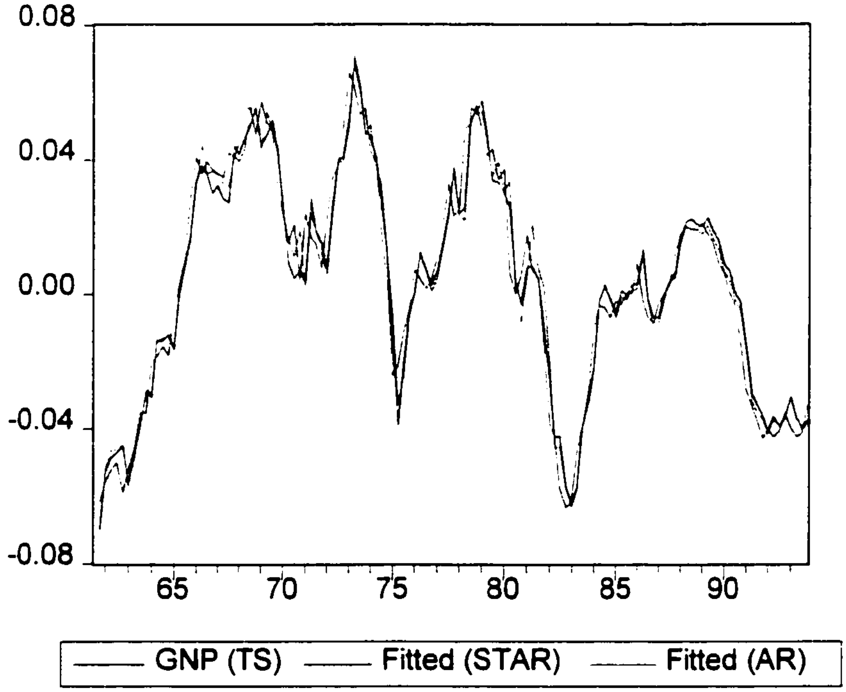


Figure 4.13 STAR model of GNP (TS)
(switching variable is the long-term spread)

Figure 4.13a Actual and fitted model of GNP (TS)
(switching variable is the long-term spread)



4.5 Unemployment

A number of switching indicators have been considered in the previous section for the modeling of the business cycle as a STAR process. It was shown that interest rate instruments and more precisely the quarterly difference in the funds rate outperformed other switching variables in terms of predicting the business cycle when the later is express in terms of real GNP. The representation of the business cycle in terms of GNP is obviously an oversimplification of the true business cycle as defined by the NBER. To expand the analysis the unemployment rate is also considered as a possible alternative measure of the status of the economy (see figure 4.14). I use an AR(1) representation for the quarterly difference in levels of the unemployment rate. This is not wholly satisfactory since we would reject, according to the Jarque-Bera test, the assumption of normality. Note that this non-normality results also holds for autoregressive processes up to order 6. The Terasvirta test indicated a nonlinear process for the quarterly difference in the unemployment rate when the switching variables are the quarterly difference in the funds rate, short-term spread and the inflation rate.

The previous results concerning the 'superiority' of the funds rate over the short-term spread are reinforced in the case of unemployment. The transition function for the short-term spread acting as the state of nature indicator only reveals one change of regime in 1975 for the entire sample period and leads to a standard deviation of 0.2326. A STAR model for unemployment with the quarterly difference in the funds rate at lag 6 as the switching variable reduces the standard deviation to 0.2200. Not surprisingly, the transition function (see figure 4.15) displays a similar pattern than in the GNP (DS) case since the critical c values are quite close. However, this STAR model, like the linear

AR(1) specification, would not be consistent with the assumption of normality. The fitted STAR model of unemployment would outperform the AR(1) specification in terms of more closely resembling the actual data (see figure 4.16a). For instance, the STAR model would almost perfectly replicate the 1975 sharp increase in the unemployment rate.[8]

STAR model of the quarterly difference in the unemployment rate: switching variable is the quarterly difference in the funds rate$_{t-6}$.

$$u_t = \underset{(0.0150)}{-0.0395} + \underset{(0.0424)}{0.4595}\, u_{t-1} + \left(\underset{(0.0423)}{0.1219} + \underset{(0.1114)}{1.0733}\, u_{t-1}\right) \times F(S_t, \gamma, c) + \hat{\varepsilon}_t$$

with S_t = funds$_{t-6}$ (funds is the quarterly difference in the funds rate),

$\hat{\gamma} = 61504$, $c = \underset{(0.2784)}{0.8132}$, $\hat{\sigma} = 0.2230$, $\frac{\hat{\sigma}^2}{\sigma_L^2} = 0.6756$, $\hat{\kappa}_3 = \underset{(0.3842)}{0.1876}$, $\hat{\kappa}_4 = \underset{(0.0000)}{1.9387}$, J-B = 0.0000 and IR=(0.81316,0.81323).

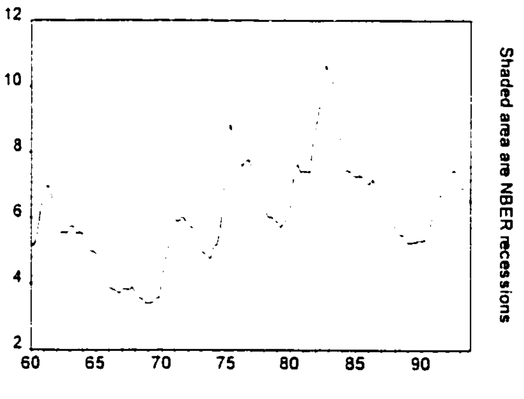


Figure 4.14 Unemployment rate

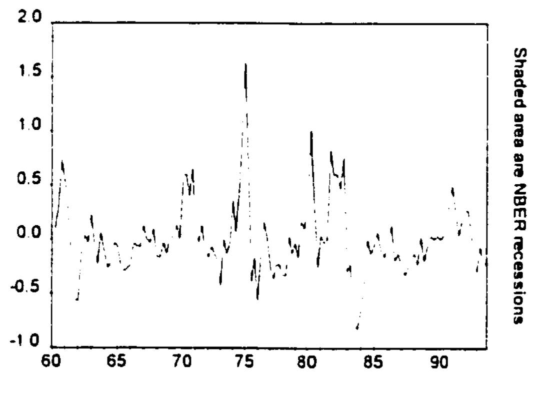


Figure 4.15 Quarterly difference in the unemployment rate

[8]Note that a STAR model of unemployment with the inflation rate provoking switches between regimes was estimated but the resulting model did not perform as well as the interest rate instruments in terms of reducing the standard deviation.

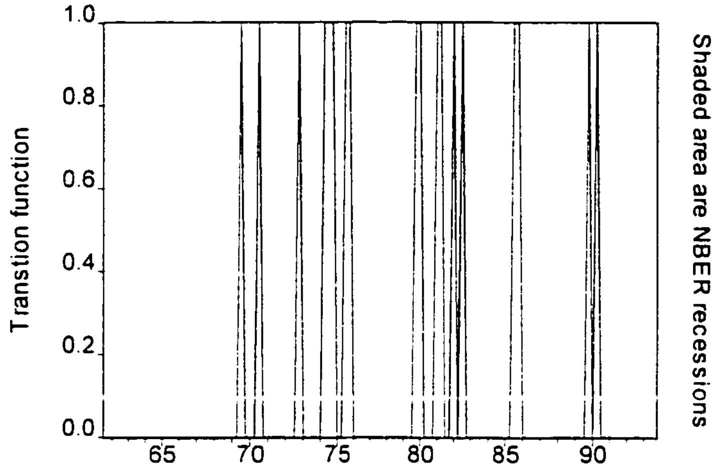
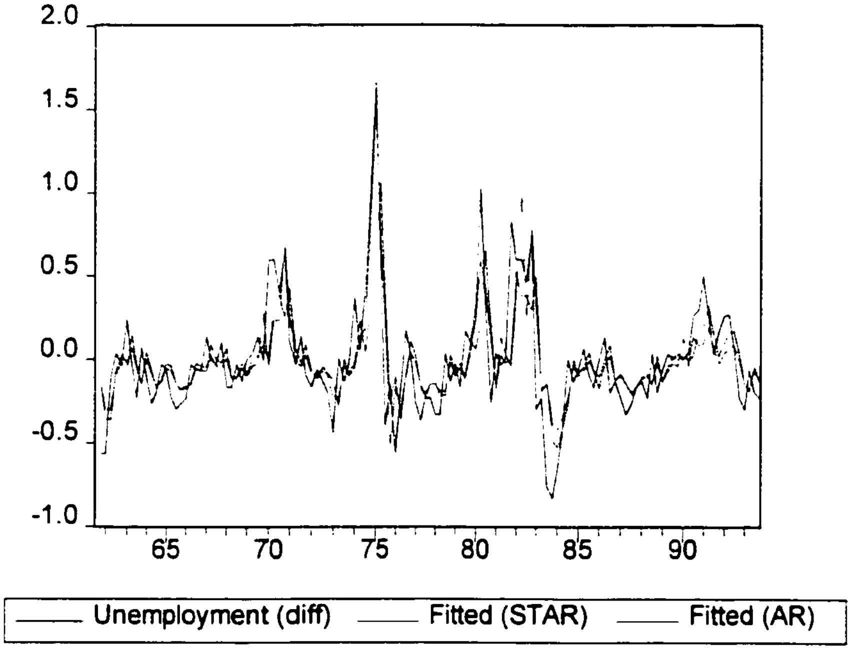


Figure 4.16 STAR model of the quart. diff. in the unemployment rate
(switching variable is the quart. diff. in the funds rate)

Figure 4.16a Actual and fitted model of unemployment
(switching variable is the quart. diff. in the funds rate)



4.6 Dual Objective for the Fed of Price Stability and Economic Growth

It has been shown that the business cycle or real GNP can be successfully portrayed as a smooth transition autoregression model with interest rate instruments initiating the switches from one regime to another. Of these STAR models, it was demonstrated that the quarterly difference in the funds rate gave the best fit. Changes in the quarterly difference in the funds rate of more than 100 basis points in the DS version with two lags of GNP implied a low growth regime one year and a half later. We have a similar result for the quarterly difference in the unemployment rate; changes in the funds rate of more than 90 basis points implied a regime where the unemployment rate would increase. One stated objective of the Federal Reserve is to 'encourage economic growth' but if this was its sole goal then following from this presentation of the STAR model the Fed could simply "fix" the quarterly difference in the funds rate below a critical c value to achieve this high growth objective (see Broaddus (1988) for a discussion on the objectives of the Federal Reserve). It seems plausible to assume that the Fed has enough influence over the funds rate to insure that the latter remains below the c value. The fact that this objective is not always achieved has two possible interpretations. Either 1^0 The Fed is unaware of this relation between the funds rate and real GNP or, more likely, 2^0 There are other objectives pursued by the Fed than simply aiming for a high growth rate of GNP or a low unemployment rate. It has been pointed out by Romer et al. (1990) and by others that the Fed has tried on several occasions to stem inflation by inducing recessions. This section is concerned with the interaction between the funds rate, the growth rate of GNP and the inflation rate as measured by the CPI index.

Our objective is to show that the Fed increases the funds rate in order to fight inflation. One cannot rule out, in the linear specification, that the quarterly difference in the funds rate explains part of the inflation process; the significance level is 0.0003 for the null: H_0: $\beta_i = 0$ for i = 1,....6 in the equation:

$$\pi_t = \alpha_0 + \sum_{i=1}^{6} \alpha_i \pi_{t-i} + \sum_{i=1}^{6} \beta_i \text{funds}_{t-i} + \varepsilon_t \qquad \textbf{(4.2)}$$

where π is the inflation rate (CPI index) and funds is the quarterly difference in the funds rate. Moreover, suppose we express inflation in terms of lagged values of the quarterly difference in the funds rate:

$$\pi_t = \xi_0 + \sum_{i=1}^{r} \xi_i \text{funds}_{t-i} + \varepsilon_t . \qquad \textbf{(4.3)}$$

Estimation of this equation with 12 lags of the quarterly difference in the funds rate indicates that all the coefficients are positive regardless of their significance level. This implies that $\frac{\partial \pi_t}{\partial \text{funds}_{t-i}} \geq 0$. In other words, increases in the quarterly difference in the funds rate lead to increases in inflation. VAR estimations based on systems that include interest rate and inflation often lead to similar conclusions; that is, increases in the interest rate leads to increases in inflation. For instance, Leeper et al. (1996), relying on VAR estimations and more precisely, the impulse response function, remarks that if one associates an increase in interest rates with a monetary contraction, then one is left with explaining a 'price puzzle'; that is, a monetary contraction that leads to an increase in the price level.

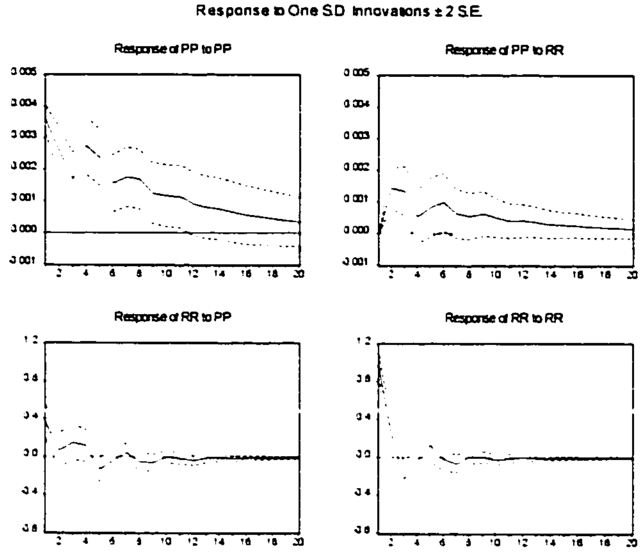


Figure 4.17 Impulse Response Functions (order of equations: pp, rr)
(pp: inflation rate (cpi), rr: quarterly difference in the funds rate)

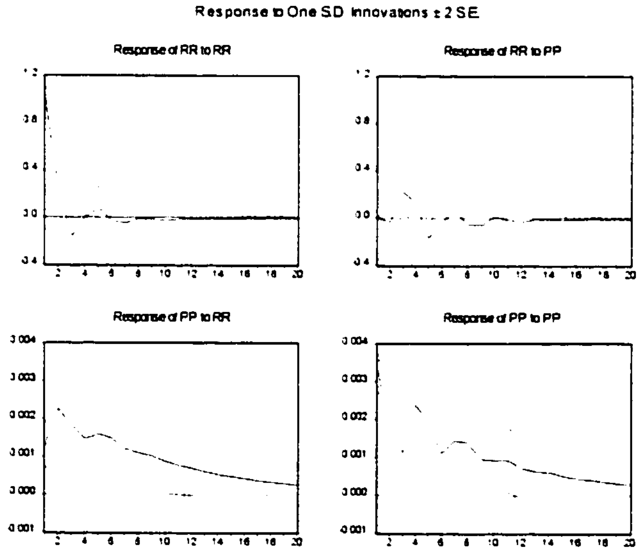


Figure 4.17a Impulse Response Functions (order of equations: rr,pp).
(pp: inflation rate (cpi), rr: quarterly difference in the funds rate)

This same price-interest rate pattern is also depicted with the impulse response function in our sample set for the 1960-1993 period[9] (see figure 4.17); a positive shock in the quarterly difference of the funds rate provokes an increase the inflation rate with the effect eventually dying out after one year. The order of equations for the impulse response function in figure 4.17 is inflation and the quarterly difference in the funds rate. The impulse response function is based on the Cholesky decomposition which implies that the order of the equations in the VAR system will have an impact on the behavior of the impulse response functions. For instance, if one uses the interest rate-inflation ordering then a positive shock in the quarterly difference in the funds rate would lead to an increase in the inflation rate except that the effect would last for 12 quarters as compared to 4 in the previous ordering (see figure 4.17a). It nonetheless remains the case that an increase in the quarterly difference in the funds rate would not lead to a decrease in the inflation and this regardless of the chosen ordering of the equations.

It could be that the Fed is unsuccessful in fighting inflation, but if this were the case then we would expect the Fed to eventually drop this price objective and concentrate entirely on the growth rate of the economy. One would then still be faced with explaining why the Fed allows the funds rate to increase by more than 100 basis points since this action would imply a low growth regime in 6 quarters.

One problem in explaining the movements in the quarterly difference in the funds rate in the context of a preoccupation of the Fed with the inflation rate and the growth rate of GNP is that it is more than likely that the Fed often shifts emphasis or objectives going from price stability to lowering the unemployment rate. It could be the case that

[9] The impulse response function is based on a VAR system that includes 4 lags of the quarterly difference in the funds rate and 4 lags of the inflation rate (CPI index).

the Fed is sucessful in fighting inflation if one analyses the relation between the real economy and the inflation rate; the Fed increases the quarterly difference in the funds rate above a certain critical c value which provokes a recession or a low growth regime and it is this recession environment that eventually causes a reduction in the price level. We would reject the null that GNP (DS) or the quarterly difference in the unemployment rate does not explain inflation in a system based on 4 lags of the regressors; the p-values for GNP and the unemployment rate were respectively 0.0328 and 0.0000. The impulse response functions associated to a VAR model with 4 lags of the inflation rate (CPI) and 4 lags of GNP (DS) indicates that a decrease in the growth rate of GNP (one standard deviation) would barely have an impact on the inflation rate, safe perhaps a small reduction in the inflation rate at lag 5 (see figure 4.18: Response of PP to YY). We have a similar result in the case of the unemployment rate (see figure 4.18: Response of PP to UU).

The period of high inflation and unemployment one observed in the 1970's suggested an end to a Phillips curve type relation or trade-off between inflation and unemployment. The impulse response function reinforces this belief that no trade-off exists between inflation and unemployment since an increase in the unemployment rate (one standard deviation) would not have a tremendous impact on the inflation rate. Though we observe the appropriate sign of an impact of a negative shock in the economy on the inflation rate, the overall effect on inflation resulting from a contraction in the economy appears quite limited. Given these results with GNP and unemployment, one is still left with the dilemma of explaining why the Fed induces recessions.

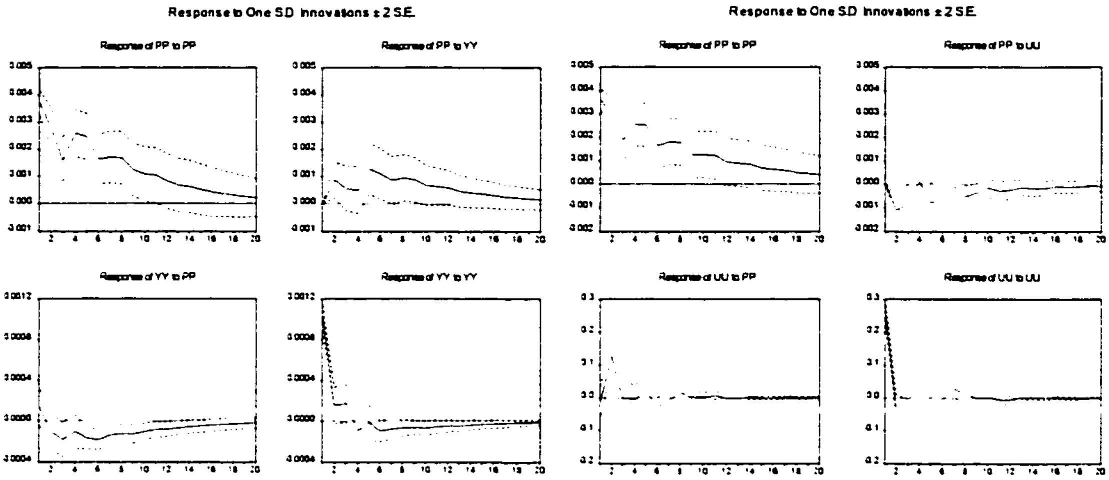


Figure 4.18 Impulse Response Functions(order of equations: pp.yy and pp.uu). (pp: inflation rate (CPI). yy: GNP (DS) uu: quarterly difference in the unemployment rate)

The last part of this section considers the impact of changes in the funds rate on inflation based on simulations of the estimated models. The objective is to measure the impact of an increase in the funds rate on the inflation rate by simulating the data implied by the STAR models of GNP or unemployment and then using these generated series to predict the inflation rate. For instance, suppose the Fed increases the quarterly difference of the funds rate by 100 basis point for a span of 10 quarters and then returns to a regime where the funds rate remains unchanged. Given this assumed behavior in the funds rate, it would then be possible to generate the growth rate of GNP or the quarterly difference in the unemployment rate series implied by the following STAR models estimated in the previous sections:

STAR model of GNP (DS): switching variable is the quarterly difference in the funds rate$_{t-6}$.

$$y_t = \underset{(0.0008)}{0.0061} + \underset{(0.0574)}{0.2156}\, y_{t-1} + \underset{(0.0563)}{0.0992}\, y_{t-2} + \left(-\underset{(0.0019)}{0.0126} - \underset{(0.1452)}{0.4350}\, y_{t-1} + \underset{(0.1621)}{0.5729}\, y_{t-2} \right) \times F(S_t, \gamma, c) + \hat{\varepsilon}_t$$

with y_t = GNP (DS), $S_t = \text{funds}_{t-6}$ (funds is the quarterly difference in the funds rate), $\hat{\gamma} = 216.78$, $\hat{c} = \underset{(0.0117)}{0.9772}$.

and

STAR model of the quarterly difference in the unemployment rate: switching variable is the quarterly difference in the funds rate$_{t-6}$.

$$u_t = -\underset{(0.0150)}{0.0395} + \underset{(0.0424)}{0.4595}\,u_{t-1} + \left(\underset{(0.0423)}{0.1219} + \underset{(0.1114)}{1.0733}\,u_{t-1}\right) \times F(S_t, \gamma, c) + \hat{\varepsilon}_t$$

with u_t = quarterly difference in the unemployment rate, $S_t = \text{funds}_{t-6}$ (funds is the quarterly difference in the funds rate), $\hat{\gamma} = 61504$, $c = \underset{(0.2784)}{0.8132}$.

It is assumed, in order to start the simulation, that the initial growth rate of GNP and the quarterly difference in the unemployment rate are zero. Note that no error term are added to this process. The objective is to show that a contractionary monetary policy, that is, an increase of the funds rate of more than 100 basis points brought about by the Fed will eventually lower the inflation rate. In order to show this result, one can first simulate the GNP (DS) or the unemployement series implied by the estimated STAR model and then construct the inflation series based on a linear time-series model of inflation with 4 lags of inflation and 4 lags of GNP (or unemployment), i.e.,

$$\pi_t = \alpha_0 + \sum_{i=1}^{4} \alpha_i \pi_{t-i} + \sum_{i=1}^{4} \beta_i w_{t-i} + \varepsilon_t \qquad \textbf{(4.4)}$$

where π is the inflation rate and w is GNP (DS) or the quarterly difference in the unemployment rate. I am assuming in this simulation exercise that inflation does not explain the GNP process or maybe more precisely that the information contained in lagged values of GNP is sufficient to predict the current value of GNP. It is assumed that the first 4 inflation rates are zero. The monetary contraction policy is initiated at period 20 and lasts for 10 quarters: i.e.,

$$S_t = \begin{cases} 100 \text{ for } t = 20, \ldots, 30 \\ 0 \quad \text{otherwise} \end{cases}$$

Note that in our STAR model, S_t is the quarterly difference in the funds rate at lag 6.

Figure 4.19 Relation between GNP (DS) and the inflation rate (cpi)

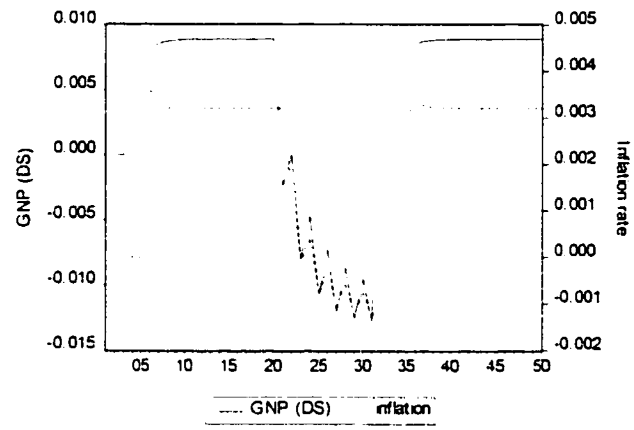
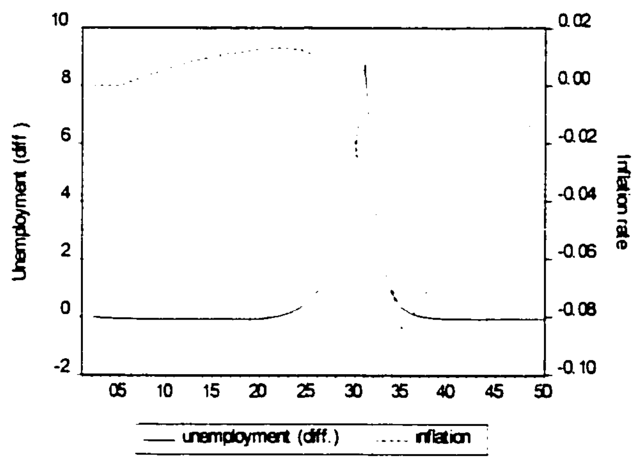


Figure 4.20 Relation between the quarterly difference in the unemployment rate and inflation (cpi).



We observe in both the GNP and unemployment cases that a contraction in monetary policy, as represented by an increase in the funds rate of 100 basis points, will result in a downturn in the economy (see figure 4.19 and 4.20). The impact of the monetary contraction is felt more quickly in the case of GNP (DS) since the downturn occurs almost immediately whereas unemployment would take approximately one years to increase by 1 point. However, the unemployment rate would increase by 15 points in the span of the 10 quarters of monetary contraction and would continue to increase for another 5 quarters. The growth of rate of GNP would, according to our simulation, return almost immediately to its growth rate level of approximately 3% after the 10 quarters contraction period.

The implication of this simulation is that inflation would decrease in both the GNP and the unemployment cases with the exception that the reduction would be relatively minor in the case of GNP whereas it would be quite substantial when one measures the economic downturn in terms of the unemployment rate; the inflation rate would go from a high of 0.01 to –0.085 in the case of unemployment rate. The inflation rate would attain, in the case of GNP, its pre-monetary contraction level almost immediately after the end of the monetary contraction. The Fed would therefore have no incentive, if the GNP-inflation relation is the relevant one, to induce a recession since it would only affect the price level temporarily. This is not the case when one considers the 'trade-off' between inflation and unemployment. We saw that the reduction in the price level would be much more severe in the case of unemployment. It is also the case that it would take a longer time to return to the inflationary levels existing prior to the 'induce' recessions. In fact, the price level would continue to drop for at least 20 quarters after the monetary contraction exercise.

A point can be made that the Fed reacted to reduce the "expected" inflation rate; the simulated inflation data shows that inflation is still increasing when the monetary contraction policy is initiated. The overall effect of the Fed's induce recession might consequently be much more important is one considers the period required to return to the expected inflation rate. Nonetheless, it remains that inflation would eventually return to its previous level which brings us to our original and main question " Why would the Fed induce a recession" since the monetary contraction is only a short-term remedy attain at the high cost of a recession.

It is possible that the apparent difficulty in finding a appropriate 'trade-off' between inflation and output is due to the fact that we are testing whether the quarterly difference in the funds rate explains part of the inflation process in a linear framework. Unfortunately, we might be facing the same situation as for the growth rate of GNP, namely that the process is nonlinear. The 1960's and the period following the mid-80's can be characterized as a low inflationary regime while the 1970's and part of the 1980's is viewed as a high inflation regime brought in part by the oil price shocks. If the inflation rate follows a regime-switching process then the VAR model, and more precisely the impulse response function associated with the VAR model, will be unable to reflect or capture this assumed characteristic in our inflation series. Note that the inflation rate has been successfully modeled as a Hamilton type regime-switching model by Garcia and Perron (1996) and Evans and Wachtel (1993).

A STAR model of inflation with the quarterly difference in the funds rate as the switching variable was estimated and subsequently rejected. I have, therefore, estimated the following regime-switching model based on an unobservable state of nature: i.e.,

$$\pi_t = \phi_{0S_t} + \phi_{1S_t}\pi_{t-1} + K + \phi_{4S_t}\pi_{t-4} + \delta_{1S_t}u_{t-1} + K + \delta_{4S_t}u_{t-4} + \varepsilon_t \qquad \textbf{(4.5)}$$

This is essentially the Hansen version of the Hamilton model that is presented in chapter 3. We would have a different Phillips curve type relation according to the unobservable state of nature S_t. The estimated Hansen and the linear model are presented in table 4.3. One would accept the Hansen model according to the rule of thumb proposed by Terasvirta since $\frac{\sigma^2_{Linear}}{\sigma^2_{Hansen}} = 0.5726$. The estimated Hansen model of inflation suggest that recessions as defined by the NBER are mostly associated with regime 1; this result is especially apparent when one considers the mid 1970 recession and the 1980-1982 recession (see figure 4.21).

The previous simulation exercise is carried out when the inflation process is governed by state 0 and state 1. The behavior of the inflation series in state 0 would be almost identical to the one found in the linear time-series model (see figures 4.20 and 4.22). This is not the case when the inflation process is in state 1. We would observed unrealistic and permanent drops in the inflation rate if we were to remain permanently in this regime (see figure 4.23).

Table 4.3 Hansen model of inflation

$$\pi_t = \phi_{0S_t} + \phi_{1S_t}\pi_{t-1} + K + \phi_{4S_t}\pi_{t-4} + \delta_{1S_t}u_{t-1} + K + \delta_{4S_t}u_{t-4} + \varepsilon_t$$

where π is the inflation rate, u is the quarterly difference in the unemployment rate, S_t is the unobservable state of nature, $\phi_{iS_t} = \phi_{i0} + \phi_i S_t$ for i=0,1,...,4 and $\delta_{iS_t} = \delta_{i0} + \delta_i S_t$ for i=1,...,4.

		Regime 0	**Regime 1**
	Linear model	**S(t) = 0**	**S(t) = 1**
ϕ_0	0.0011	0.0027	-0.0014
ϕ_1	0.8093	0.4532	1.2193
ϕ_2	0.0138	0.1242	-0.3554
ϕ_3	0.2884	0.2473	0.3986
ϕ_4	-0.2014	-0.1283	-0.1106
δ_1	-0.0043	-0.0041	-0.0051
δ_2	-0.0025	-0.0013	0.0007
δ_3	0.0021	0.0019	0.0019
δ_4	-0.0023	0.0004	-0.0082
σ_{Linear} **= 0.0037**			
σ_{Hansen} **= 0.0028**			
P_{00} **= 0.8300** [a]			
P_{11} **= 0.7757**			

a $P_{00} = P(S_t=0|S_{t-1}=0)$ and $P_{11} = P(S_t=1|S_{t-1}=1)$

Figure 4.21 Hansen model of inflation

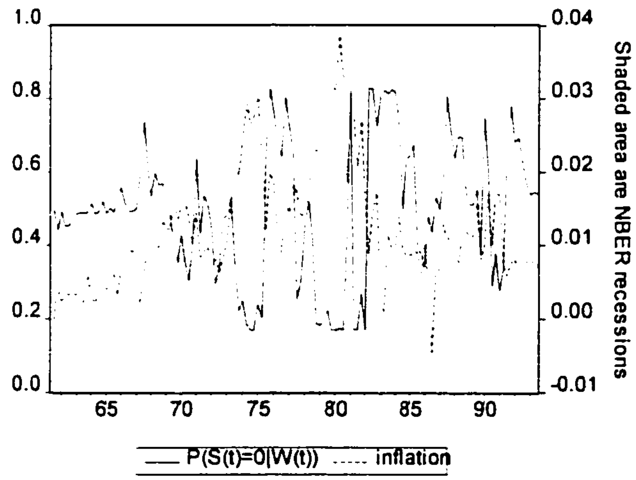


Figure 4.22 Relation between the quarterly difference in the unemployment rate and inflation when S(t)=0

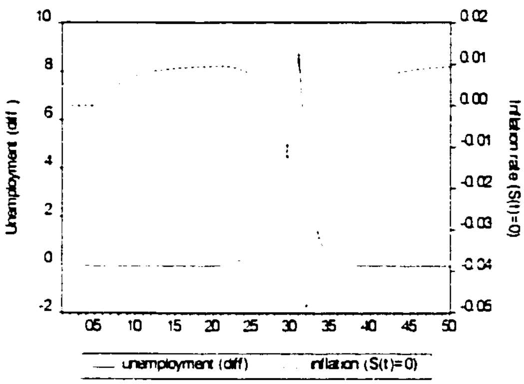


Figure 4.23 Relation between the quarterly difference in the unemployment rate and inflation (S(t)=1)

But this 'extreme' trade-off between inflation and the quarterly difference in the unemployment rate is under the assumption that we are "stuck" in regime 1. The estimated probability of being in regime 0 implied by the Hansen model suggests that we are often switching between the two regimes; the probability of being in regime 0 is frequently in the [0.4,0.6] range. The equilibrium probability of being in regime 0 is 0.56[10]. The expected duration of regime 0 given we are in regime 0 in the previous quarter is 5.9 quarters while the expected duration of regime 1 given we are already in regime 1 is 4.5 quarters. In one regime, we have inflation that returns almost immediately to its "pre-monetary contraction" period while the alternate regime is associated to permanent deflation.

Our final simulation exercise will therefore aim to show that a combination of these two regimes can "provoke" a permanent reduction in the inflation rate and thereby justify the Fed's behavior of inducing recessions. In order to proceed with this

[10] The equilibrium probability of being in regime 0 is given by $P(S_0=0) = (1-P_{11})/(2-P_{11}-P_{00})$ where $P_{00}=P(S_t=0|S_{t-1}=0)$ and $P_{11}=P(S_t=1|S_{t-1}=1)$.

simulation, I generate a vector, v, of uniform random variable and assign of value of 1 if $v_t > 0.5$ and zero otherwise, i.e.,

$$\theta(t) = \begin{cases} 0 \text{ if } v_t \leq 0.5 \\ 1 \text{ otherwise} \end{cases}.$$

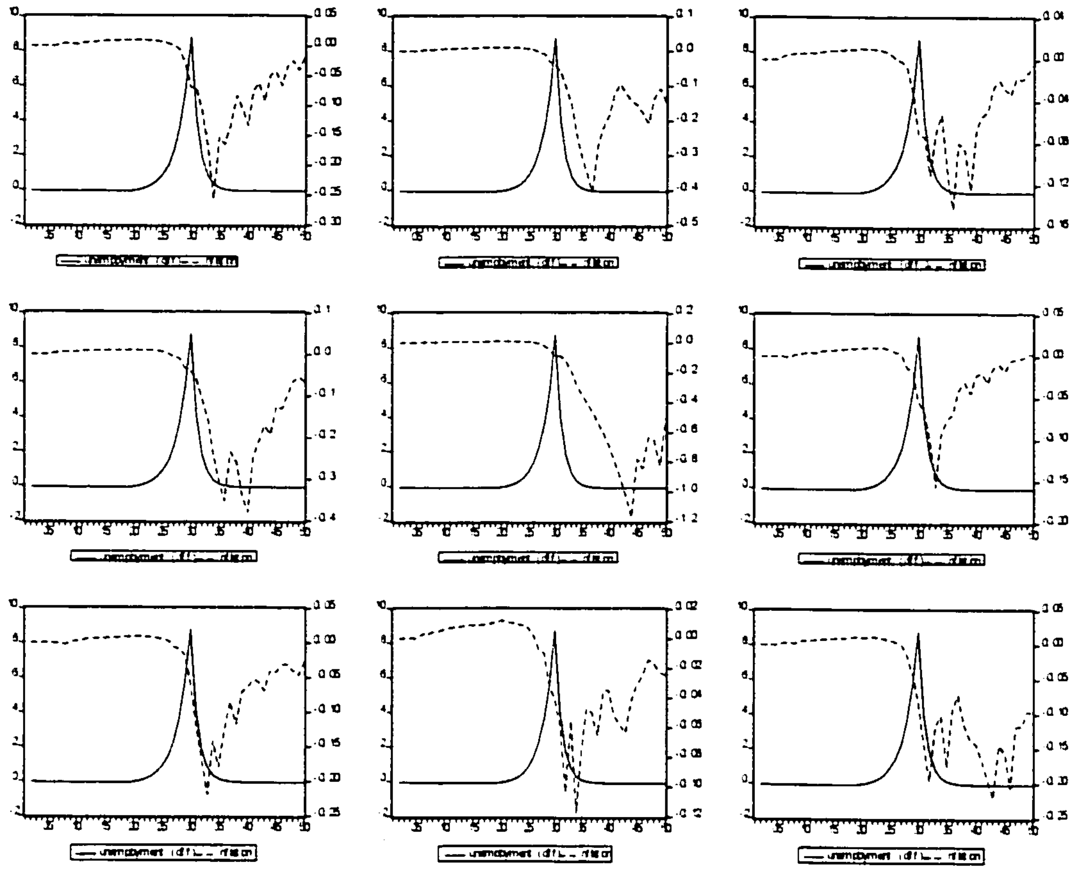
It is therefore possible to simulate the inflation series by using the same approach as before based on the following model of inflation:

$$\pi_t = \theta(t) \times \left(\phi_0^0 + \phi_1^0 \pi_{t-1} + K + \phi_4^0 \pi_{t-4} + \delta_1^0 u_{t-1} + K + \delta_4^0 u_{t-4}\right) + \\ (1 - \theta(t)) \times \left(\phi_0^1 + \phi_1^1 \pi_{t-1} + K + \phi_4^1 \pi_{t-4} + \delta_1^1 u_{t-1} + K + \delta_4^1 u_{t-4}\right)$$

I have included a number of simulated inflation series based on different realization of θ (see figure 4.24). In all cases, we observe that inflation decreased as one initiates the monetary contraction measured by the increase in the quarterly difference in the unemployment rate. But as before, inflation always returned to its previous levels, the mixture of the two regimes suggest that it is possible to attain a situation where inflation would remain at a lower level even after the monetary contraction is over.

The objective of these "simulated" exercise is to show that a contractionary monetary policy would bring about a reduction in the inflation rate and therefore explain why the Fed permits changes in the quarterly difference in the funds rate above a critical c value. Though both the GNP and unemployment series lead to the coveted conclusion, a Phillips curve type relation between the quarterly changes in the unemployment rate and the inflation rate makes a stronger case for the Fed of succesfully fighting inflation.

Figure 4.24 Relation between the quarterly difference in the unemployment rate and inflation Mixture of two regimes



The 'price puzzle' may therefore be solved if one assumes that an increase in the interest rate is not sufficient to be dubbed a monetary contraction. What is suggest by the STAR model of the business cycle, in terms of GNP (DS) or unemployment, is that an increase in the interest rate of a certain magnitude is required in order to propel the economy into a recession and eventually bring about a reduction in the inflation rate.

We have shown in this chapter that the business cycle is more aptly represented by a STAR model than a linear one and this result is irrespective of the particular representation of the business cycle (i.e., DS, TS or HP). Amongst the different STAR models of GNP considered in this thesis, the one based on the quarterly difference in the funds rate as the switching variable resulted in the best fit.

Assuming the Fed is aware of the interaction between the funds rate and the business cycle, as implied by the STAR model, one is then left with explaining why the Fed would permit changes in the funds rate above a certain critical value since this action would propel the economy into a low growth regime. A possible explanation is that the Fed is concerned not only with the growth rate of the economy but also with other factors such as the inflation rate. The difficulty one encounters with this interpretation is showing that increases in the funds rate will eventually lower the inflation rate since the impulse response function suggest the opposite effect. One possible strategy in resolving this 'price puzzle' is to focus on a 'trade-off' relationship between output and inflation instead of the relation between interest rates and prices. This strategy did not however lead to satisfactory results in a linear time-series framework. The simulated data from the STAR model of GNP (DS) and the unemployment rate suggest that a increase in the funds rate of more than 100 basis points would result in a contraction of the economy. Furthermore, that this contraction would bring about a reduction in the inflation rate. This last result is especially true when one "simulates" the relation between inflation and the quarterly difference in the unemployment rate. The impact of the induced recession on inflation would, however, only be a temporary solution if the relation existing between inflation and unemployment is linear since inflation eventually returns to its prior levels.

Stock and Watson remarked that a trade-off between inflation and unemployment still exists if one focuses on the cyclical components of these variables for the 1953-1996 sample period.

", while there is no stable relationship between the levels of inflation and unemployment, there is a clear and remarkably stable negative relation between the cyclical components of inflation and unemployment" (Stock and Watson 1998, p. 21).

The estimation of a regime-switching model of inflation based on an unobservable state of nature would further suggest that the "trade-off" mainly exist when we are in regime 1. The conclusion of a short-term remedy is therefore modified when one assumes a regime-switching model of inflation based on an unobservable state of nature.

CHAPTER 5

Does Money Matter?

One of the main issue in this thesis is whether the growth rate of money, or more broadly, whether monetary policy, plays any role in explaining the business cycle; it could be the case that interest rate instruments are more indicative of the monetary stance than the monetary aggregates M1 and M2. One of the stylized facts from Lucas (1977) is that monetary aggregates are procyclical with output. This in itself does not establish causality of money with respect to output or vice-versa. It does, however, suggest further investigation on the interaction between money and the real economy.

Milton Friedman (1968) claims that the growth rate of money affects the economy. This assertion seems to be corroborated graphically for M1 and M2 as a drop in the growth rate of money (see appendix D) precede each recession. Unfortunately for proponents of the 'Money Matters' proposition, there are other episodes which featured a declining growth rate of money of the same magnitude, but which did not lead to an immediate recession. There is also the possibility of feedback from output to money that must be considered.

A number of studies based on vector autoregression (VAR) systems concluded that money or monetary policy played no relevant role in explaining the economy. For example, Leeper et al. (1996) show that shifts in monetary policy do account, at best, for only a 'modest portion' of the variance in output or prices in the U.S. since the 1960's and, in some model specifications, for none of the variance. It was shown in the previous chapter that monetary policy expressed in terms of the funds rate would necessarily account for some of the variance in output since the quarterly difference in the funds rate describes the regime in which the economy functions. We also concluded in chapter 4 that the growth rate of money (i.e., M1 or M2) would be rejected as a switching indicator for the DS and HP version of the business cycle and would not lead to a satisfactory model in the TS case.

This chapters deals with testing if monetary policy as measured by the growth rate of M1 or M2 plays any role in explaining the business cycle when it enters as a regressor in a STAR formulation. A conclusion one often finds in recent "money" studies is that the growth rate of M1 plays no role in explaining the business cycle in the linear case. One has a sense that the empirical debate on the importance of money is mostly fought in terms of redefining the business cycle or adopting different monetary aggregates measures in order to achieve the desired results on the importance or irrelevance of money. A common thread in all of these approaches is to assume, once the output and money indicator are chosen, that the relation between the business cycle and money can be captured by a linear specification. We will show in this thesis that the growth rate of M1 would Granger-cause output in the DS, TS and HP case if the business cycle follows a STAR process. One therefore only needs to assume a nonlinear relation instead of constructing new output or money indicators to show the relevance of money on the real economy. Another implication of this last result is to negate or lessen the argument that the funds rate is mostly endogenous and therefore the fact that it defines the regimes in a STAR specification is not conclusive of the effectiveness of monetary policy. Further considerations, in this chapter, are whether the growth rate of money explains output mainly in one limit regime and is essentially neutral in the alternate regime. The results indicate that the growth rate of M1 is neutral in the high-growth regime.

The first section introduces the Granger-causality test. The second section test for money-output causality in the linear and nonlinear case. The last section looks at the money-output causality in the high and low growth regime.

5.1 Granger-Causality

Showing empirically the importance of money on the real economy has proven to be a formidable challenge. One encountered difficulty for proponents of the 'money matters' proposition resides in explaining the lack of correlation between the business cycle and the growth rate of M1 (see table 5.1). A positive correlation between the growth rate of M1 and GNP at k=0 (i.e., $corr(M1_t, GNP_{t+k})$) suggest that the series behave in a procyclical manner whereas a negative correlation would point to a countercyclical behavior (see Stock and Watson 1998). The growth rate of M1 and the DS representation of the business cycle would suggest that these series are procyclical for the 1960-1993 sample period. However, if one used the alternative TS and HP representation of the business cycle then one would conclude that the series are countercyclical. Maybe more importantly for the 'money matters' debate is that the correlation between the growth rate of M1 and GNP (i.e., DS, TS and HP) is not statistically different from zero for all k; the one exception is for k=-1 in the HP case which suggest that the growth rate of M1 tend to lag the business cycle.

Table 5.1 Cross Correlogram: Corr($M1_t$,GNP_{t+k})[1]

	K												
	-6	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
DS	-0.175	-0.104	-0.108	-0.129	-0.056	-0.101	0.155	0.151	0.149	-0.041	-0.026	-0.037	0.021
TS	-0.005	-0.020	-0.037	-0.057	-0.053	-0.071	-0.010	0.033	0.080	0.077	0.073	0.081	0.092
HP	-0.081	-0.107	-0.135	-0.180	-0.190	-0.215	-0.111	0.005	0.113	0.115	0.129	0.120	0.173

1 M1 is the growth rate of M1

The existence of correlation between the growth rate of M1 and the business cycle would not able us to conclude on the importance of money since it is possible to construct cases where correlation between two variables does not imply that one series causes another. Our task of showing the importance of money would seem rendered more difficult since we have to contend with the case of no correlation between the series. My results suggest that this no correlation result can be explained by the fact that the growth rate of money only explains or causes the business cycle in recessions.

Finding a leading indicator of the business cycle does not imply that this series explains or causes the business cycle. One definition of causality, which will be used throughout, arises in a test to determine if a variable X explains a variable Y when one takes into account the information contained in the lagged values of Y (see Sims 1972). The Granger-causality test in the linear representation consists in testing the hypotheses H_0: $\delta_i = 0$ for i = 1,...,p in the equation

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \sum_{i=1}^{p} \delta_i w_{t-i} + \varepsilon_t \quad \textbf{(5.1)}$$

The statistic $S = \dfrac{N \times (RSS_0 - RSS)}{RSS}$ follows asymptotically a $\chi^2(p)$. RSS_0 and RSS are respectively the restricted and unrestricted sum of squared residuals and N the number of observations (see Hamilton 1994).

A similar definition of causality is used in the case of a STAR model. The Granger-causality test in the STAR framework consists in testing

H_0: $\alpha_{2i} = \beta_{2i} = 0$ for i=1,2,...,p in the equation

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_{1i} y_{t-i} + \sum_{i=1}^{p} \alpha_{2i} w_{t-i} + \left(\beta_0 + \sum_{i=1}^{p} \beta_{1i} y_{t-i} + \sum_{i=1}^{p} \beta_{2i} w_{t-i} \right) F(S_t; \gamma, c) + \varepsilon_t \qquad \textbf{(5.2)}$$

The test statistic used is

$$LR_{stat} = N \times \ln\left(\frac{\sigma_R^2}{\sigma_U^2} \right) \qquad \textbf{(5.3)}$$

which follows a $\chi^2(2p)$; σ_R is the estimated standard deviation obtained from

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_{1i} y_{t-i} + \left(\beta_0 + \sum_{i=1}^{p} \beta_{1i} y_{t-i} \right) F(S_t; \gamma, c) + \varepsilon_t \qquad \textbf{(5.4)}$$

whereas σ_U is the estimated standard deviation obtained from (5.2).

5.2 Granger-Causality of Money-output in a linear and nonlinear framework

The single equation version of the Granger-causality test, for p = 4 in equation (5.1), reveals a number of similarities between the three GNP based representations of the business cycle and the growth rate of M1 and M2 (see Table 5.2). The growth rate of M1 would not Granger-cause output in a linear setting nor would the growth rate of M2 at the 5% level; the p-value for M2 in the HP case is in the order of 65%. These no causality results regarding the growth rate of money and the business cycle are not surprising given that we concluded there was no correlation between these variables.

Table 5.2 Granger-Causality of Money-Output (p-values).

The causality test for the linear and STAR specification is based on a system of 4 lags of GNP (i.e., DS, TS and HP) and 4 lags of the financial variables M1 and M2.

Business Cycle	DS	TS	HP
Linear Specification			
GNP and M1[1]	0.1755	0.1917	0.2076
GNP and M2	0.0917	0.0544	0.6479
STAR specification (switching variable is the funds rate (t-6))			
GNP and M1	0.0390	0.0024	0.0002
GNP and M2	0.0016	0.0012	0.0073

1 M1 and M2 are in terms of growth rates.

It has been pointed out by Stock and Watson (1989) and Friedman and Kuttner (1993) that causality results often depend on the sample period and also on the particular representation of the business cycle. For example, results may depend on whether we use DS or TS, and, in the case of TS, on the characterization of the trend component. Friedman et al. (1993), adopting the same methodology that Stock et al. (1989) used to find that money explained output, showed that extending the analysis of the Stock and

Watson paper by 5 years rendered money insignificant. Friedman et al. (1993) also showed that changing the interest rate instrument from a Treasury Bill to a commercial paper rate in the Stock and Watson paper changed the conclusion on the importance of money for the economy.

The approach taken is this thesis is that the growth rate of money may not Granger-cause output simply because one models incorrectly the business cycle as a linear process instead of a nonlinear one. This section therefore considers the role of money in explaining the business cycle under the assumption that a regime-switching model generates the business cycle. The first task is consequently to model the business cycle as a STAR process with lagged values of GNP and lagged values of the growth rate of money as the set of regressors and the quarterly difference in the funds rate at lag 6 as the switching variable (equation (5.2)). We can proceed with the estimation of this STAR model in the same manner as in chapter 4; first by maximizing the concentrated loglikelihood function with respect to γ and c and then maximizing the loglikelihood function with respect to the parameters $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma$ and c in equation (5.2).

The introduction of the growth rate of money as a regressor leads to a model that better represents the class of "true" STAR models by allowing for smooth transition between the two extreme or limit regimes. For the most part, the models in the previous chapter only mimicked two-regimes threshold models as revealed by the small interval defining the intermediate or middle regime; we are either in the low growth or high growth regime. Adding this element to the analysis eliminates the simple dichotomy in which values above a given c were associated with one of the limiting regimes. For example, in the case of a STAR model with 4 lags of GNP (DS) and 4 lags of the growth rate of M1, with the quarterly difference in the funds rate at lag 6 as the switching variable, the c value is 0.6745, but one requires a quarterly difference in the funds rate of 4 to be in regime 1 and -3 for the alternate regime (see appendix G for the estimated STAR models and appendix H for the transition functions).

This section looks at whether the growth rate of M1 or M2 explains part of the business cycle process when it enters solely as a regressor in the STAR formulation. The growth rate of M1 and M2 did not 'Granger-cause' output at the 5% significance level for the three specifications of the business cycle in the linear time-series case. This 'non-causality' result no longer holds when the business cycle is represented by a STAR model. Table 5.2 shows that the growth rate of M1 and M2 would 'Granger-cause' output in the STAR specification at the 5% level for the three representation of the business cycle.

The introduction of lagged values of the growth rate of M1 in the linear specification of GNP (DS) would not help predict, when one considers the fitted values, the various recessions occurring during our sample period (see figure 5.1). This is, however, not the case when one models GNP (DS) as a regime-switching process since most of the recessions would be replicated by this type of nonlinear model; the one exception is for the 1990 recession.

A STAR model with the quarterly difference in the funds rate acting as both regressor and switching variable is nonetheless preferred since it leads to a smaller standard deviation and almost the same transition function as in the previous case with lagged values of the growth rate of M1 acting as regressors[11]. Furthermore, with respect to discerning the 1990 recession via the transition function, the STAR model with 4 lags of GNP and 4 lags of the quarterly difference in the funds rate assigns, as compared to the STAR models in chapter 4, a greater weight of being in the low growth regime for the 1990 recession (see Appendix H). For example, the STAR model of GNP (DS) with the quarterly difference in the funds rate switching the process indicates F = 0.25 for the 1990 recession whereas we would have F = 0.75 in the case of the STAR model of GNP (DS) with 4 lags of both GNP and the quarterly difference in the funds rate. The fitted values derived from the STAR model for GNP (DS) would closely resemble the actual values for most of the recessions except for the last one. One would therefore conclude that monetary policy, whether defined in terms of growth rates of M1 and M2 or the funds rate, explains or causes the business cycle[12].

[11] The estimated standard deviation associated with this model is 0.0069 compared to 0.0078 for the linear specification: i.e., $y_t = \alpha^T x_t + \beta^T w_t + \varepsilon_t$ where $x_t = (1, y_{t-1}, \ldots, y_{t-4})$, $w_t = (funds_{t-1}, \ldots, funds_{t-4})$ and funds is the quarterly difference in the funds rate.

[12] Note that money causality tests were also carried out when the short-term and the long-term spread are the regime indicators instead of the quarterly difference in the funds rate. The assumption that money plays no role in the short-term case cannot be rejected at the 5% level whereas the opposite is true in the case of the long-term spread (see appendix I). A STAR model of GNP (DS) with the short-term spread as a regressor is rejected at the 5% when the switching is provoked by the short-term spread but not when lagged values of the growth rate of M1 acts as a regressor. The STAR model with the quarterly difference in the funds rate as the switching variable is preferred to the short-term spread and the long-term spread specification on the grounds that it provides a smaller standard deviation and a transition function which corresponds more closely to the defined NBER periods of recessions (see appendix J).

Figure 5.1 Actual and fitted model of GNP (DS) with M1 (switching variable is the quart. diff. in the funds rate)

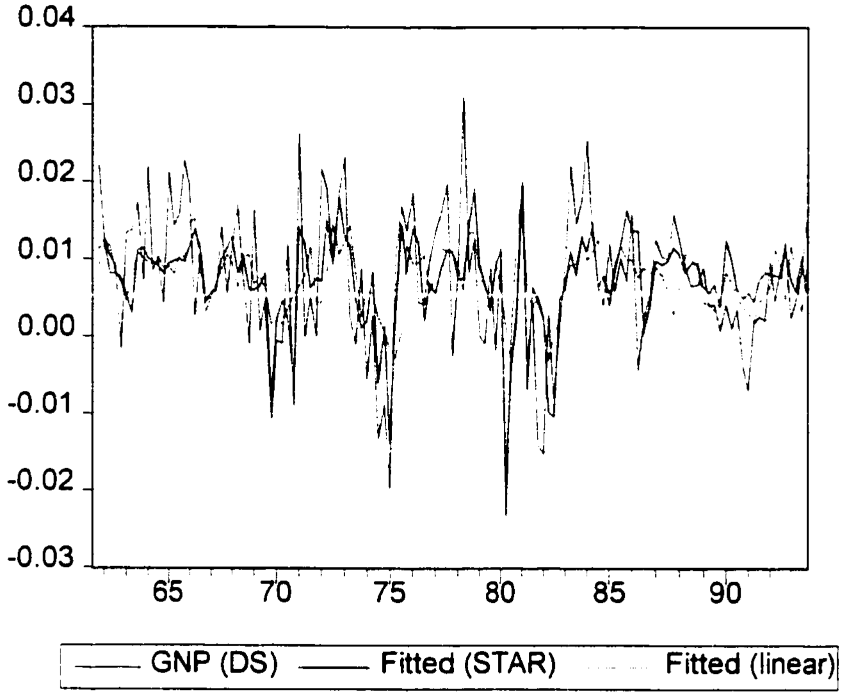


Figure 5.2 Actual and fitted model of GNP (TS) with M1 (switching variable is the quart. diff. in the funds rate)

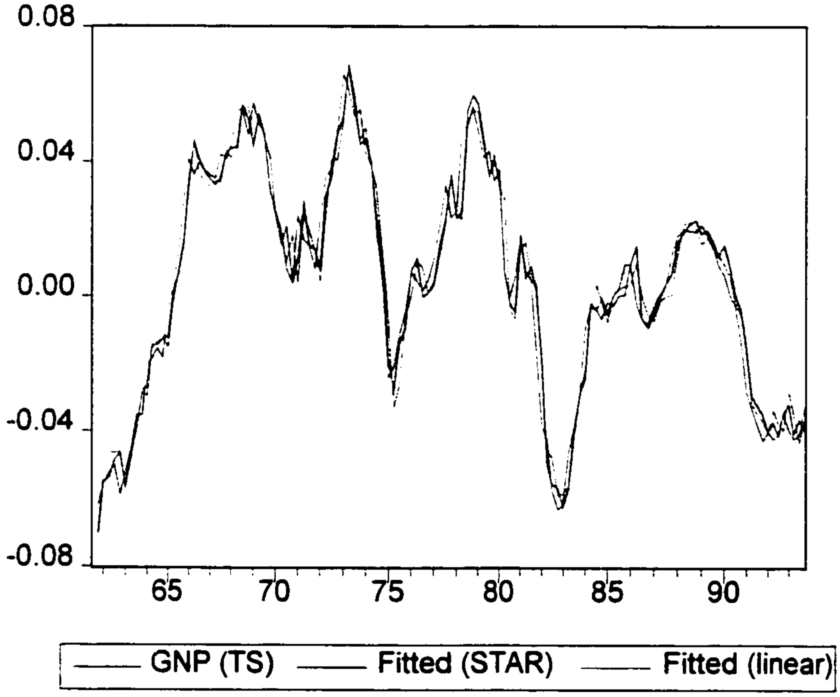


Figure 5.3 Actual and fitted model of GNP (HP) with M1
(switching variable is the quart. diff. in the funds rate)

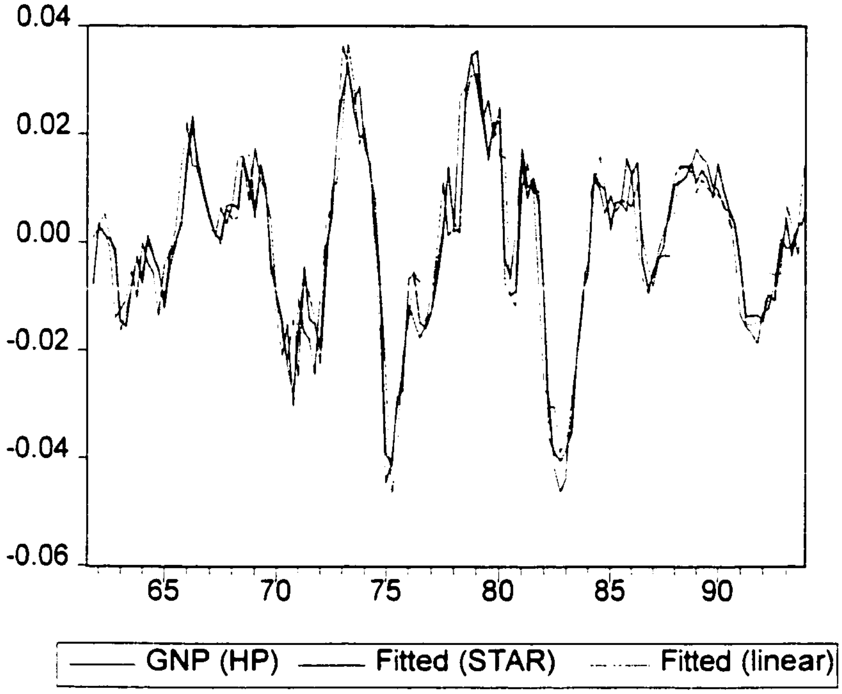


Figure 5.4 Actual and fitted model of GNP (DS) with the funds rate
(switching variable is the quart. diff. in the funds rate)

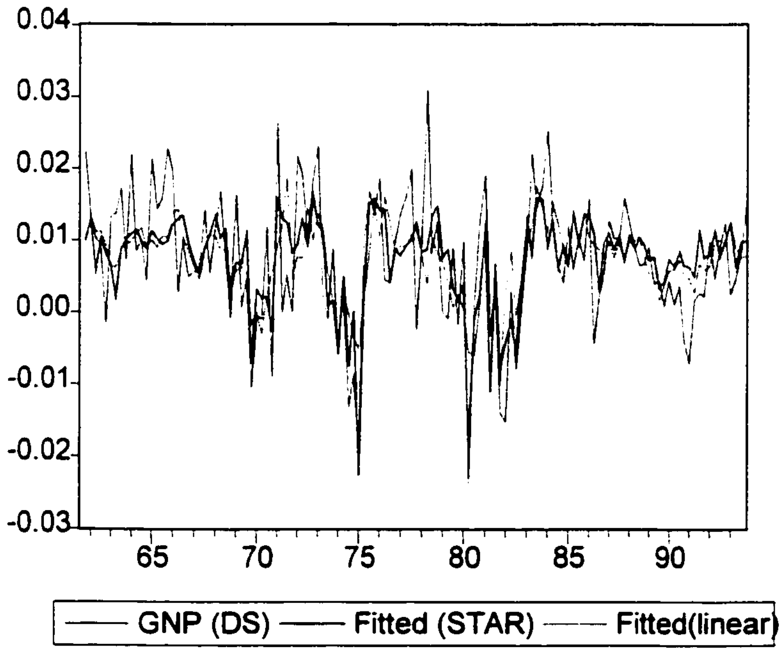
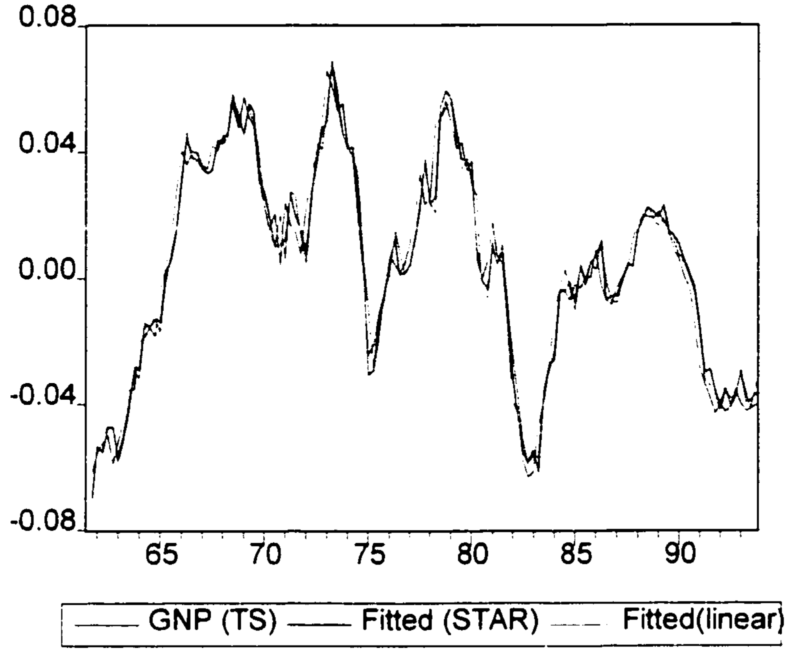
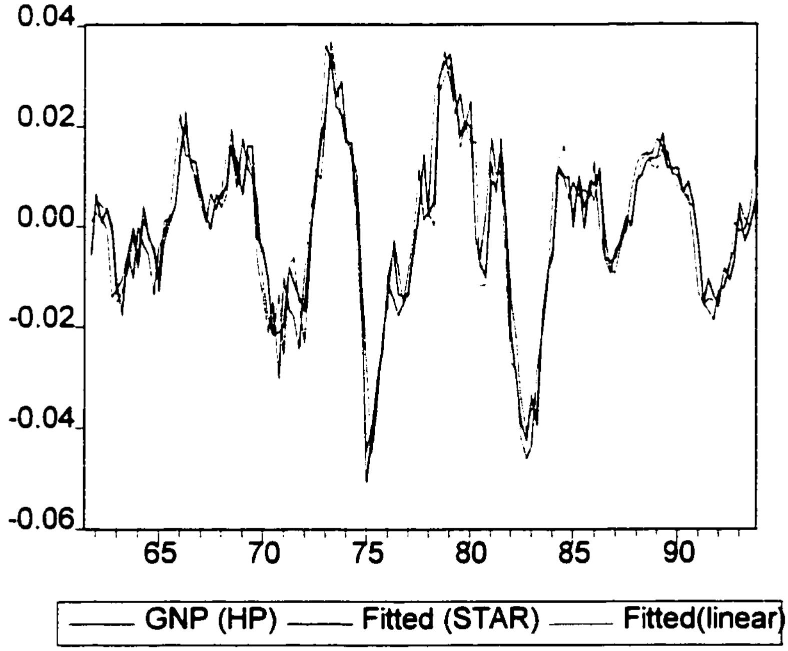


Figure 5.5 Actual and fitted model of GNP (TS) with the funds rate (switching variable is the quart. diff. in the funds rate)



igure 5.6 Actual and fitted model of GNP (HP) with the funds rate (switching variable is the quart. diff. in the funds rate)



5.3 The importance of Money in the expansionary and recessionary regimes.

This section looks at whether the growth rate of M1 explains the business cycle in one limit regime and is neutral in the alternate limit regime. Two approaches are considered to verify this assumption of money neutrality. The first approach treats the STAR model as a two-regime threshold model and test if the growth rate of M1 explains the business cycle in each regime. The second approach tests the hypotheses that lagged values of the growth rate of M1 only enter the STAR specification (equation (5.2)) as regressors that are multiplied by the transition function; if this is the case then the growth rate of M1 would be neutral when the transition function goes to zero.

Suppose the STAR models with lagged GNP and the growth rate of M1 as regressors were to behave as a threshold model with two regimes, displaying only well delineated periods in which the economy is in the high growth or low growth regime. It would then be possible, if the threshold condition was known, to test if the growth rate of money Granger-causes output in both, in only one or in none of the regimes. Here, the tests will proceed as if we have a two-regime threshold model even though the transition function shows that this is not always the case. The sample is divided according to whether the quarterly difference in the funds rate lagged (t-6) is above or below the estimated c value. For instance, in the DS case, there are 25 observations for which the quarterly difference in the funds rate is above 0.6745 (see table 5.3). One observes that the growth rate of M1 explains output at the 5% level for the DS and TS versions of the business cycle in one regime (i.e., $F \approx 0$) (above the critical c value) and does not Granger-cause output in the alternate regime. The significance level in the alternative regime is in the order of 0.8653 and 0.8825 for the DS and TS version respectively. The only exception is the HP representation of the business cycle in which the growth rate of M1 does not cause output in the alternate regime. This might be due to the fact that there are only 12 available observations in regime 1 and 9 coefficients to estimate. Strangely, we have a situation where the growth rate of M1 does not cause GNP (HP) in either regimes (if we treat the system as a two-regime threshold model) but does Granger-cause output in the STAR specification (5.1).

The alternate approach, in analyzing the importance of money in both limit regime, is to test H_0: $\alpha_{2i} = 0$ for i=1,2,...,p in equation (5.2). If the null is true then we have a STAR model where the role of money is subject to the value taken by the transition function.

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_{1i} y_{t-i} + \left(\beta_0 + \sum_{i=1}^{p} \beta_{1i} y_{t-i} + \sum_{i=1}^{p} \beta_{2i} ml_{t-i} \right) F(S_t;\gamma,c) + \varepsilon_t \qquad \textbf{(5.5)}$$

The growth rate of M1 would be neutral in the limit regime 0 (i.e., $F(S_t;\gamma,c) \to 0$) and would explain the business cycle in the alternate limit regime (i.e., $F(S_t;\gamma,c) \to 1$); regime 1 corresponds to the contractionary monetary case. A likelihood ratio statistic is used to test H_0: $\alpha_{2i} = 0$ for i=1,2,...,4 in equation (5.2) for p=4. We would, in the three representations of the business cycle, accept the null that the growth rate of M1 is neutral in the high-growth regime; the p-values for the DS, TS and HP representation of the business cycle are 0.6406, 0.6257 and 0.4507 respectively.

Table 5.3: The role of the growth rate of M1 in the two regimes.

Business Cycle	DS		TS		HP	
Regimes[a]	Regime 0	Regime 1	Regime 0	Regime 1	regime 0	Regime 1
α_0	-0.0040	0.0073	0.0017	0.0006	-0.0146	-0.0002
α_1	0.0801	0.1729	0.7682	1.1718	0.5169	0.9937
α_2	0.7094	0.1302	1.0334	-0.1397	1.4496	-0.0237
α_3	0.1501	-0.0519	-0.6990	-0.0854	-0.4546	-0.1295
α_4	0.0480	-0.1122	-0.2780	-0.0042	-0.7584	-0.0232
β_1	0.5046	0.0201	0.2395	0.0135	1.2382	0.0656
β_2	0.2126	-0.0130	0.3638	-0.0198	0.8734	0.0632
β_3	-0.4213	0.1070	-0.4902	0.1244	-0.5868	-0.0125
β_4	-0.0673	-0.1173	-0.0540	-0.0966	-0.3284	-0.0323
# of obs.	25	104	31	98	12	117
c	0.6745	0.6745	0.6105	0.6105	0.9708	0.9708
Signif. Level[b]	0.0517	0.8654	0.0034	0.8824	0.3430	0.7055
Average of GNP	0.0033	0.0083	0.0076	0.0032	-0.0150	0.0016

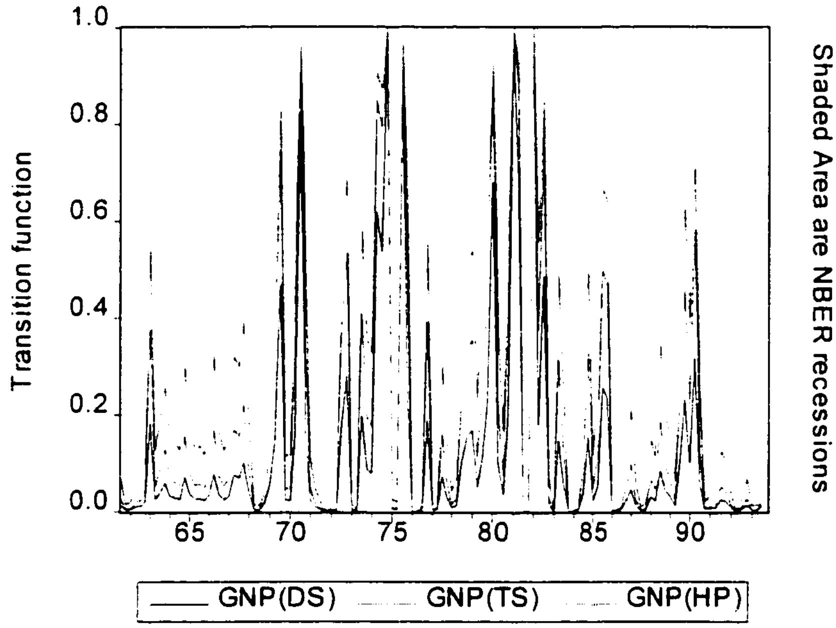
a The two regimes are modeled as $y_t = \alpha_0 + \sum_{i=1}^{4} \alpha_i y_{t-i} + \sum_{i=1}^{4} \beta_i ml_{t-i} + \varepsilon_t$ where y_t is GNP and $m1_t$ is the growth rate of M1. Regime 0 ($F \approx 0$) corresponds to the sample set for which $S_t < c$ (S_t is the quarterly difference in the funds rate at lag 6 and *c* is the maximum likelihood estimate from equation (5.1)).

b We are testing H_0: $\beta_i = 0$ for i=1,...,4 in the equation:

$$y_t = \alpha_0 + \sum_{i=1}^{4} \alpha_i y_{t-i} + \sum_{i=1}^{4} \beta_i ml_{t-i} + \varepsilon_t .$$

One notes that the behavior of the transition function associated with the three mentioned cases are all similar apart from the fact that the value taken by the transition function for the DS representation is always well below the TS and HP case (see figure 5.7). This, however, does not imply that the growth rate of M1 is less important in the DS case since the weights or the coefficients assign to the growth rate of M1 are often twice the ones found in the alternate representation of the business cycle (see the estimated coefficients β_{2i} in table 5.4).

Figure 5.7 STAR model of GNP with the growth rate of M1 (switching variable is the quart. diff. in the funds rate)



We have a situation where the growth rate of M1 is neutral when F tends to zero. Suppose we deem that $F \approx 0$ when $F=0.05$. This implies that the growth rate of M1 would be, in the DS case, neutral when the changes in the funds rate are less than 0.2403. If $F=0.05$ is a valid representation of money neutrality then one observes that money is neutral in 60% of our 1960-1993 sample period and is associated to a 3.4% annual growth rate of real GNP. Using the same definition of neutrality (i.e., $F=0.05$), we would require, for the TS and HP cases, drops in the funds rate in the order of 0.3371 and

0.0189 respectively. Changes in the funds rate of less than –0.3371 and –0.0189 appeared 30% and 44% of the time for the TS and HP cases.

Table 5.4 STAR models of GNP with GNP and the growth rate of M1 as regressors.

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_{1i} y_{t-i} + \left(\beta_0 + \sum_{i=1}^{p} \beta_{1i} y_{t-i} + \sum_{i=1}^{p} \beta_{2i} m1_{t-i} \right) F(S_t; \gamma, c) + \varepsilon_t$$

where y_t is GNP, $m1_t$ is the growth rate of M1 and S_t is the quarterly difference in the funds rate at lag 6.

Business Cycle	**DS**	**TS**	HP
α_0	0.0065	0.0007	0.0004
α_{11}	0.2493	1.2928	1.0624
α_{12}	0.0463	-0.4243	-0.2646
α_{13}	-0.0408	0.0234	-0.0250
α_{14}	-0.0482	0.0737	0.0274
β_0	-0.0271	0.0011	-0.0052
β_{11}	-1.2261	-0.8185	-0.8545
β_{12}	1.7596	2.0330	2.2567
β_{13}	0.7611	-0.8177	-0.6670
β_{14}	-0.3750	-0.5779	-0.8863
β_{21}	2.0668	0.4283	0.7049
β_{22}	0.9614	0.5249	0.8965
β_{23}	-1.0067	-0.6238	-0.7432
β_{24}	-0.6698	-0.2289	-0.3472
γ	2.9765	3.0158	3.3084
C	1.2295	0.6392	0.8710
σ	0.0074	0.0070	0.0063
S[1]	0.2403	-0.3371	-0.0189

1 Values of S_t for which F=0.05 (i.e., $S_t = c - \frac{1}{\gamma} \ln\left(\frac{1-\varepsilon}{\varepsilon}\right), \varepsilon = 0.05$).

Our two approaches would therefore lead to similar conclusion on the irrelevance of the growth rate of M1 in the expansionary regime. One cannot, however, make the same claim of neutrality in regime 0 for the growth rate of M2 and the quarterly difference in the funds rate; the p-values for the growth rate of M2 and the quarterly difference in the funds rate were all in the 0.02 range or below. This last result might simply reflect the fact that the M2 and the funds rate are more endogenous than M1.

Monetary policy is, therefore, ineffective in the high-growth regime only if one assumes that the growth rate of M1 is the relevant instrument for monetary policy. If this is the case then one is left with explaining why the growth of M1 does not play a role in the high growth regime. One possible explanation is that we essentially have, in the high growth regime, the 'classical' scenario of a vertical supply curve; the real wage adjusting automatically to equate the quantities of demand and supply in the labor market.

An important obstacle for proponents of the "money matters" proposition as been justifying the importance of money on empirical grounds. Numerous studies have concluded that money played no relevant role in explaining the economy or if it once did, its importance as greatly diminish when one adds the 1980's in the sample period. The position taken in this thesis is that the previous studies are based on examining the money-output relation in a linear framework, which is inconsequential, if the output series itself is intrinsically nonlinear. It is shown in this chapter that the growth rate of money does influence the business cycle when the later is modeled as a smooth transition autoregression process. Furthermore, this result is not dependent upon a particular business cycle representation (i.e., DS, TS or HP). We conclude that monetary policy, as measured by the growth rate of M1, is ineffective in the high growth regime.

CHAPTER 6

Is the Business Cycle Based on an Observable or Unobservable State of Nature?

It was shown in the previous chapters that the business cycle could be successfully modeled as a regime-switching process with an observable variable determining the regimes. This result differs with Hamilton's (1993, p. 234) claim that regime changes are mostly triggered by events that are unobservable and largely unrelated to past realizations of the series itself. In this chapter, I attempt to demonstrate that a STAR model of the growth rate of real GNP, using the quarterly difference in the federal funds rate as the observable switching variable, performs better than the changing intercept model of Hamilton (1989) and compares favorably with further refinements of the Hamilton approach.

The first part of this chapter estimates and compares the changing intercept model of Hamilton with a STAR formulation based on the funds rate as the switching variable. The second part looks at extensions of the Hamilton model, namely the Hansen and the modified Hamilton model and then compares these specifications with a STAR model.

6.1 Hamilton's changing intercept model.

Hamilton's 1989 estimated changing intercept model of the U.S. growth rate of GNP for the 1955-1984 period is based on a fourth order autoregressive process: i.e.,

$$y_t = \alpha_0 + \alpha_1 S_t + \phi_1(y_{t-1} - \alpha_0 - \alpha_1 S_{t-1}) + K + \phi_4(y_{t-4} - \alpha_0 - \alpha_1 S_{t-4}) + \varepsilon_t \qquad \textbf{(6.1)}$$

where

$$S_t = \begin{cases} 1 \text{ if regime } 1 \\ 0 \text{ otherwise} \end{cases}$$

is the unobservable state of nature, which follows a first order Markov process.

The objective of this section is to compare (6.1) with a similar specification based on a STAR model, namely

$$y_t = \alpha_0 + \sum_{i=1}^{4} \alpha_i y_{t-i} + \left(\beta_0 + \sum_{i=1}^{4} \beta_i y_{t-i} \right) \times F(S_t; \gamma, c) + \varepsilon_t \qquad \textbf{(6.2)}$$

where y_t is the growth rate of GNP, S_t is the quarterly difference in the funds rate at lag 6 and F is the logistic transition function (see equation (3.3)).

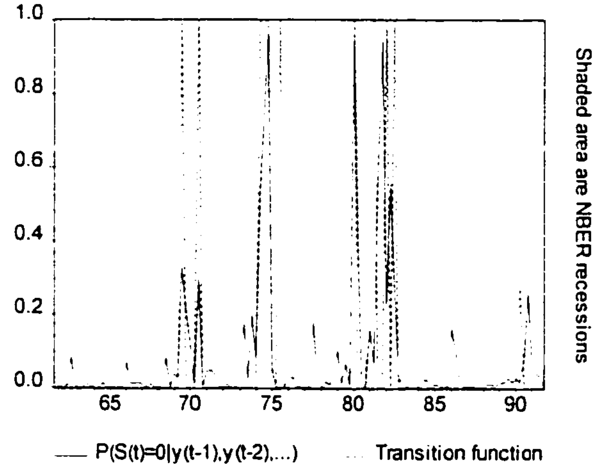
One of the criteria used to judge the superiority of a particular regime-switching model of the growth rate of real GNP over another will be whether the model leads to a smaller standard deviation. Another criteria looks at whether the probability of being in a recession in the Hamilton model and the transition function associated with the STAR model reflect the recessionary phase of the business cycle as defined by the NBER.

A STAR model of GNP (DS) was estimated for the 1955-1984 period with the quarterly difference in the funds rate at lag 6 acting as the switching variable. Given that the transition function of this STAR specification did not depict changes in regimes or recessions for the 1955-1960 period, it does not outperform the Hamilton model. This result is, however, in line with the view that the funds rate did not convey relevant information about the monetary stance prior to the 1960's.

> "The rate on Federal funds played only a limited role as an indicator of reserve availability during these years although it gained attention during the 1960s. The interbank market was not very broad as the 1960s began,

but activity was expanding. Until the mid-1960s, the Federal funds rate never traded above the discount rate." (Meulendyke 1989, p. 36)

Figure 6.1 Changing Intercept Model and the STAR Model
(growth rate of U.S. real GNP)



We will see that the superiority of the changing intercept model over the STAR specification no longer holds for the 1960-1993 period. The estimated changing intercept model of Hamilton for GNP (DS) for the 1960-1993 period does not perform as admirably as in the 1952-1984 sample. The changing intercept model depicts relatively well the 1973 and early 1980's recessions but only assesses a probability of 25% of being in a recession for 1969 and 1990 (i.e., $P(S_t = 1 | y_t, y_{t-1}, ...)$) (see figure 6.1).

Hamilton's Changing Intercept Model: 1960-1993.

$$y_t = \underset{(0.5600)}{-0.7899} + \underset{(0.5038)}{1.6878} S_t + \underset{(0.1095)}{0.1878} y_{t-1} + \underset{(0.1110)}{0.1914} y_{t-2} - \underset{(0.1026)}{0.0636} y_{t-3} + \underset{(0.1023)}{0.0519} y_{t-4} + \hat{\varepsilon}_t$$

$P(S_t = 1 | S_{t-1} = 1) = 0.964$, $P(S_t = 0 | S_{t-1} = 0) = 0.619$, $P(S_0 = 1) = 0.914$ and

$\hat{\sigma} = 0.7723$ (The figures in parentheses below the estimated parameters are the standard errors).

Let p_{11} be the probability of being in regime 1 given we were in regime 1 in the previous quarter (i.e., $P(S_t=1|S_{t-1}=1)$). Let p_{00} be the probability of being in the alternate regime given we were in this regime in the previous quarter (i.e., $P(S_t=0|S_{t-1}=0)$). For instance, the probability of being in a recession given that we were in a low growth regime in the previous quarter is, according to the Hamilton's changing intercept model, equal to 0.619. Conditional on being in a recession (or state 0), the expected duration of a recession is given by $1/(1-p_{00})$ (see Hamilton 1989). In the changing intercept case, the expected duration of a recession, conditional on already being in a recession, is 2.6 quarters; the average duration of a recession is 4.4 quarters for the 1960-1993 sample period.

It is possible to show that in a two-regime first-order Markov process that the equilibrium probability of being in regime 1 is equal to:

$$P(S_0 = 1) = \frac{1 - p_{00}}{2 - p_{11} - p_{00}} \qquad \textbf{(6.3)}$$

where $p_{00} = P(S_t = 0|S_{t-1} = 0)$ and $p_{11} = P(S_t = 1|S_{t-1} = 1)$.

A STAR model with 4 lags of GNP (DS) and a switching condition determined by the quarterly difference in the funds rates would outperform the Hamilton model in terms of indicating recessions via the transition function and would give essentially the same standard deviation (i.e., 0.7723 and 0.7789). Note that a linear AR(4) model would lead to a standard deviation of 0.8764; the GNP series was multiplied by 100 for computational purposes.

STAR model of GNP (DS): switching variable is the quarterly difference in the funds rate$_{t-6}$.

$$y_t = \underset{(0.0802)}{0.7078} + \underset{(0.0574)}{0.2242}\, y_{t-1} + \underset{(0.0577)}{0.1381}\, y_{t-2} - \underset{(0.0586)}{0.0382}\, y_{t-3} - \underset{(0.0585)}{0.1220}\, y_{t-4} +$$
$$\left(-\underset{(0.2100)}{1.4100} - \underset{(0.1521)}{0.5380}\, y_{t-1} + \underset{(0.1681)}{0.4731}\, y_{t-2} - \underset{(0.1476)}{0.3542}\, y_{t-3} - \underset{(0.1838)}{0.1141}\, y_{t-4}\right) \times F(\gamma, c) + \hat{\varepsilon}_t$$

with $\hat{\gamma} = 930.25, \hat{c} = \underset{(0.2784)}{1.1468}, \hat{\sigma} = 0.7789\ \hat{\kappa}_3 = \underset{(0.1835)}{0.2868}, \hat{\kappa}_4 = \underset{(0.5637)}{0.2491}$ and J-B = 0.3495.

(The figures in parentheses below the estimated parameters are the standard errors. $F(\gamma, c)$ is the logistic transition function. Figures below the skewness and excess kurtosis measure are the significance level. J-B is the p-value of the Jarque-Bera statistic).

A point of observation regarding the STAR and the Hamilton model is that the latter would imply, based on the annual growth rate, more severe recessions and higher growth rate in the expansion phase of the business cycle; the average of negative growth rates for the sample 1960-1993 period is –2.6% and 4.2% for the positive growth rate which is more in line with the values implied by the STAR model.

Table 6.1 Limiting Regimes

	Hamilton Model (eq. 6.1)		STAR Model (eq. 6.2)	
Regime	$S_t = 0$	$S_t = 1$	F = 0	F = 1
α_0	-0.7899	0.8979	0.7078	-0.7022
α_1	0.1878	0.1878	0.2242	-0.3138
α_2	0.1914	0.1914	0.1381	0.6112
α_3	-0.0636	-0.0636	-0.0382	-0.3924
α_4	0.0519	0.0519	-0.1220	-0.1334
Growth rate[1]	-5.0%	5.7%	3.5%	-2.3%

1 The annual growth rate measure is the unconditional mean $E[y_t]$

6.2 The Hansen and the Modified Hamilton model.

The Hansen model is an extension of the changing intercept model in which the coefficients associated to the lagged values of GNP (DS) also change according to the states of nature. This is expressed as:

$$y_t = \alpha_0 + \alpha_1 S_t + \phi_{1S_t} y_{t-1} + K + \phi_{4S_t} y_{t-4} + \varepsilon_t \qquad \textbf{(6.4)}$$

where S_t is defined as in the changing intercept model and $\phi_{iS_t} = \phi_{i0} + \phi_i S_t$ for i=1,...,4.

A grid-search approach over the (p_{00}, p_{11})-space is used to find starting values for the optimization problem. Hansen (1992) used U.S. real GNP quarterly data for the period 1952-1984 which led to the following estimates: $p_{00} = 0.388$, $p_{11} = 0.638$ and a standard deviation of 0.657. The estimated Hansen model for the sample period (1960-

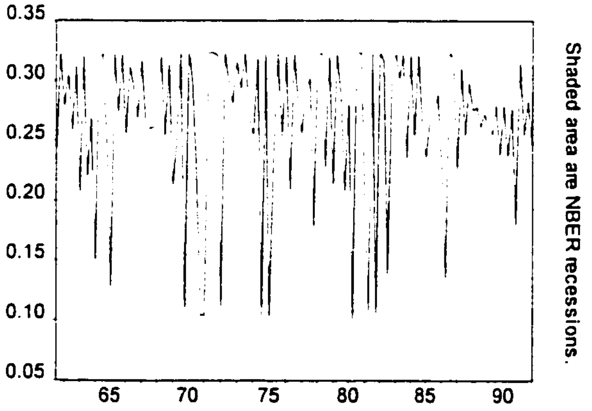
1993) gives essentially the same results as in Hansen (1992) in terms of p_{11} (0.676) and in terms of standard deviation (0.648), except that the probability of observing $S_t = 0$ given that $S_{t-1} = 0$ is now only 0.101. This implies that the expected duration of a recession, conditional on being in a recession, is only 1.11 quarters. Another obvious drawback is that the probability of being in a recession (i.e., $P(S_t=1|y_{t-1},...,y_{t-p})$) in the Hansen model is never above the 35% level for the entire 1960-1993 sample period (see figure 6.2).

Hansen's Model 1960-1993.

$$y_t = \underset{(0.1987)}{0.1680} - \underset{(0.1912)}{0.5751} S_t + (\underset{(0.2401)}{1.3112} + \underset{(0.2188)}{0.9478} S_t) y_{t-1} + (-\underset{(0.1692)}{0.5746} - \underset{(0.2522)}{1.3014} S_t) y_{t-2} +$$
$$(\underset{(0.1431)}{0.5080} + \underset{(0.2051)}{0.6182} S_t) y_{t-3} + (\underset{(0.2384)}{0.4803} - \underset{(0.1877)}{0.6632} S_t) y_{t-4} + \hat{\varepsilon}_t$$

$P(S_t = 1 | S_{t-1} = 1) = 0.676$, $P(S_t = 0 | S_{t-1} = 0) = 0.101$, $P(S_0 = 1) = 0.735$ and $\hat{\sigma} = 0.6481$.

Figure 6.2 Hansen's Model
(growth rate of U.S. real GNP)



The modified Hamilton model is simply the Hansen model with lagged values of GNP replaced by an unknown AR(p) process z_t that we can re-express in terms of lagged GNP and states of nature;

$$y_t = \alpha_0 + \alpha_1 S_t + \phi_{1S_t}(y_{t-1} - \alpha_0 - \alpha_1 S_{t-1}) + K + \phi_{4S_t}(y_{t-4} - \alpha_0 - \alpha_1 S_{t-4}) + \varepsilon_t \qquad \textbf{(6.5)}$$

The modified Hamilton model outperforms the previous two versions (the changing intercept and the Hansen model) in terms of procuring a smaller standard deviation and in terms of depicting recessions (see figure 6.3). The probability of being in the high growth regime conditional on being in a recession the previous quarter is 0.7 compared to 0.1 for the other regime.

Hamilton (1989) remarks on how well the probability function of being in a recession conditional on lagged GNP coincides with the periods of recession as defined by the NBER. One can make a similar claim in the case of the modified Hamilton model with the exception that we would assess a high probability of being in a recession for some events when the NBER deemed there was no recession. The modified Hamilton version would incorrectly predict four "recessions" if our criterion for deeming a low growth regime is a cut-off probability value greater than 75%. Only one of these four events (1986) is associated with a negative growth rate. The remaining three episodes, in 1963, 1964 and 1972, are not characterized by negative growth rates, but are nonetheless associated with sharp drops in the growth rate of GNP in the order of 50%. However, the expected duration of a recession associated with this modified Hamilton model would only be 1.13 quarters.

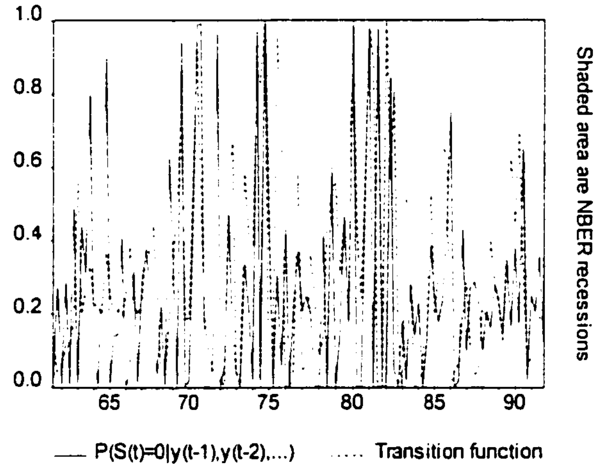
Modified Hamilton Model 1960-1993.

$$y_t = \underset{(0.2565)}{1.1516} - \underset{(0.1872)}{0.5397} S_t + (\underset{(0.1741)}{1.4888} + \underset{(0.2103)}{0.9006} S_t)(y_{t-1} - \underset{(0.2565)}{1.1516} - \underset{(0.1872)}{0.5397} S_t) +$$
$$(-\underset{(0.1872)}{0.4644} - \underset{(0.1874)}{1.4779} S_t)(y_{t-2} - \underset{(0.2565)}{1.1516} - \underset{(0.1872)}{0.5397} S_t) + (\underset{(0.1475)}{0.4780} + \underset{(0.2442)}{0.4846} S_t) \times$$
$$(y_{t-3} - \underset{(0.2565)}{1.1516} - \underset{(0.1872)}{0.5397} S_t) + (-\underset{(0.2343)}{0.2800} - \underset{(0.1842)}{0.6397} S_t)(y_{t-4} - \underset{(0.2565)}{1.1516} - \underset{(0.1872)}{0.5397} S_t) + \hat{\varepsilon}_t$$

$P(S_t = 1 | S_{t-1} = 1) = 0.681$, $P(S_t = 0 | S_{t-1} = 0) = 0.113$, $P(S_0 = 1) = 0.735$ and $\hat{\sigma} = 0.6361$.

We have in the case of the Hansen and modified Hamilton model an equilibrium probability of being in regime 1 of 0.733 and 0.735 respectively. These equilibrium probability values are quite close to the ones obtained in Hamilton (1989) for the sample period 1952-1984 (i.e., 0.720) whereas the changing intercept model for the period 1960-1993 implies an equilibrium probability of 0.914.

Figure 6.3 Modified Hamilton Model and the STAR Model
(growth rate of U.S. real GNP)



Suppose we define two 'limiting regimes' in the modified Hamilton version by the cases in which we remain permanently stuck in one of the states of nature (i.e., $S_t = S_{t-1} = ... = S_{t-p} = 0$ or $S_t = S_{t-1} = ... = S_{t-p} = 1$). The usefulness of such definitions is perhaps limited in that these regimes might never occur in practice. For instance, there would be 64 different unobservable states of nature in a system that includes two regimes and 4 lags in the autoregressive process z_t. The following two regimes represent the case where we are always in regime 1 or always in regime 0:

Regime 0 (i.e., $S_t = S_{t-1} = ... = S_{t-p} = 0$).

$$y_t = -0.256 - 1.489y_{t-1} - 0.464y_{t-2} + 0.478y_{t-3} - 0.280y_{t-4} + u_t$$

Regime 1 (i.e., $S_t = S_{t-1} = ... = S_{t-p} = 1$).

$$y_t = 0.311 + 2.390y_{t-1} - 1.942y_{t-2} + 0.963y_{t-3} - 0.919y_{t-4} + u_t$$

These two 'extreme' regime processes would be nonstationary.

Suppose instead of the Hamilton, we use a STAR model based on 4 lags of GNP and the funds rate with the funds rate at lag 6 switching the process; i.e.,

STAR model of the growth rate of real GNP with 4 lags of GNP and the funds rate; switching variable is the quarterly difference in the funds rate at lag 6.

$$y_t = \underset{(0.1693)}{0.7821} + \underset{(0.0976)}{0.2952}\, y_{t-1} - \underset{(0.1591)}{0.2923}\, y_{t-2} - \underset{(0.0925)}{0.0077}\, y_{t-3} + \underset{(0.1036)}{0.0674}\, y_{t-4} + \underset{(0.1036)}{0.2118}\, \text{funds}_t -$$
$$\underset{(0.0983)}{0.2954}\, \text{funds}_{t-2} + \underset{(0.0916)}{0.1499}\, \text{funds}_{t-3} - \underset{(0.0693)}{0.2574}\, \text{funds}_{t-4} + (-\underset{(0.2697)}{0.9967} - \underset{(0.2781)}{0.6111}\, y_{t-1} + \underset{(0.3899)}{1.6644}\, y_{t-2} +$$
$$\underset{(0.2358)}{0.1912}\, y_{t-3} + \underset{(0.2230)}{0.1667}\, y_{t-4} - \underset{(0.2011)}{0.3864}\, \text{funds}_{t-1} - \underset{(0.1315)}{0.1231}\, \text{funds}_{t-2} - \underset{(0.1731)}{0.4917}\, \text{funds}_{t-3} +$$
$$\underset{(0.1669)}{0.1780}\, \text{funds}_{t-4}) \times F(\gamma, c) + \hat{\varepsilon}_t$$

$\hat{\gamma} = 2.33$, $\hat{c} = 0.5973$, $\hat{\sigma} = 0.6921$, $\hat{\kappa}_3 = \underset{(0.0143)}{0.5279}$, $\hat{\kappa}_4 = \underset{(0.4554)}{0.3219}$ and J-B = 0.2908;

$F(\gamma, c)$ is the logistic transition function, y is the growth rate of GNP and funds is the quarterly difference in the funds rate. Figures below the skewness and excess kurtosis measure are the significance level. The figures in parentheses below the estimated parameters are the standard errors.

One observes that the standard deviation associated with this STAR model is essentially of the same magnitude with the one for the modified Hamilton model (0.69210 compared to 0.6361 for the modified Hamilton model). The modified Hamilton model is not particularly 'superior' to the STAR version based solely on the standard deviation and the transition function criteria; the behavior of the transition function is almost a carbon copy of the probability distribution derived from the modified Hamilton model. A case can be made that these two models behave almost identically. As a result it appears sensible to adopt the simpler STAR model based on an observable and somewhat "controllable" state of nature.

The objective of this chapter was to compare two types of regime-switching models of the growth rate of real GNP. We showed that the two regime-switching models gave essentially the same standard deviation and depicted recessions equally well. One type of regime-switching model is based on an unobservable state of nature and hence its main attraction but possibly also its main flaw. It might be useful to know that there is a probability of x% of being in a recession in the next quarter but maybe as important for policy makers would be how to avoid it. The fact that the state of nature, in the Hamilton model, is unobservable would render the policy makers task difficult since the generating mechanism governing the business cycle is based on the state of nature. In this sense, a STAR model based on the observable federal funds rate as the switching condition may provide a more interesting avenue for decision makers.

However, if the objective is mostly to forecast the business cycle then one would favor the Hamilton model since forecasting in the case of the Hamilton model is straightforward given the Markov assumption whereas one would need to model the funds rate in the case of the STAR model.

Finally, it is quite intriguing to observe that a STAR model that relies on the information of the Federal funds rate at lag 6 to provoke switches between regimes could compare favorably with the Hamilton model which uses a unobservable switching variable at time t!

Chapter 7

Conclusion

The debate on the relevance of money in the real economy is a longstanding one. A number of recent papers rely on VAR estimations to support claims about the importance or irrelevance of money or interest rate instruments to the real economy, for example Sims (1980a), Friedman and Kuttner (1993), Bernanke and Blinder (1992), Leeper et al. (1996). The appropriateness of VAR estimates rest upon the assumption that the system of equations is linear or at least that it can be reasonably approximated by a linear system.

This thesis has argued that the business cycle, whether defined in terms of DS, TS or HP, is intrinsically nonlinear and that STAR models can capture this nonlinearity. Of the various STAR models of the business cycle studied in this thesis, the one with the quarterly difference in the funds rate provides the strongest results. It was shown that a STAR model of GNP with the quarterly difference in the funds rate as the switching variable outperformed the linear time-series representation in terms of reducing the standard deviation; a similar result was also observed for the unemployment rate. This observation regarding the funds rate reinforces Bernanke and Blinder's (1992) claim that the short-term spread predicts the business cycle because it represents the monetary stance. The challenge with Bernanke et al. (1992) interpretation of the predictability of the short-term spread as a reflection of the monetary stance is explaining why the short-term spread outperforms the funds rate in terms of predicting the business cycle since the latter is more closely related to monetary policy. I have shown, in this thesis, that Bernanke and Blinder's interpretation is not undermined since the quarterly difference in the funds rate outperformed the short-term spread when the business cycle is modeled as a regime-switching process.

The structure of our STAR model of the business cycle implies an important role to the Federal Reserve since it could insure that we remain in the high growth regime by keeping the quarterly difference in the funds rate below a certain threshold. One is then left with explaining why the Fed would permit recessions. It has been stated by Romer et al. (1989) that the Fed induces recessions in order to fight inflation. The difficulty one encounters is showing empirically that increasing the interest rate will provoke a reduction in the inflation rate. Simulations of the STAR model suggest that a trade-off or Phillips curve exists between inflation and the unemployment rate thereby providing a plausible explanation for the behavior of the Fed; it is assume that the "Phillips curve" is generated by a Markov regime-switching process

Furthermore, it has been shown that the growth rate of money (M1 and M2) would Granger-cause output if the business cycle is modeled as a regime-switching model whereas the growth rate of money did not Granger-cause output in the single equation linear system. The growth rate of money would cause output mostly in the low growth regime and be neutral in the alternative regime. This last result provides an explanation for the apparent lack of correlation between the growth rate of M1 and the business cycle since M1 only explains the business cycle when we are in the low growth regime. One will nonetheless prefer a STAR of the business cycle based on the quarterly difference in the funds rate as both regressor and switching indicator since it provides a smaller standard deviation and the transition function better depicts the recessionary periods as defined by the NBER.

Lastly, based on smaller observed standard deviations and on more accurate depictions of recessions via the transition function, this thesis provided evidence that a STAR model based on an observable switching variable performs better than Hamilton's initial changing intercept model. Moreover, the STAR model compares favorably with further refinements of the Hamilton model.

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Appendix A (CHAPTER 3)

Regime-Switching VAR Model

We saw in previous chapters that VAR systems are often used to assess the importance of money or interest rates on the real economy. We wish to show in this appendix that STAR models could be easily incorporated into a VAR setting. Suppose we have a system of r-regime-switching equations instead of the 'single equation regime' system:

$$\begin{aligned} Y_t &= \sum_{i=1}^{r} \left(\Pi_{i1} Y_{t-1} + K + \Pi_{ip} Y_{t-p} \right) F_i(S_t; \psi) + \varepsilon_t \\ &= \sum_{i=1}^{r} \left(\Pi_{i1} X_{i,t-1}(\psi) + K + \Pi_{ip} X_{i,t-p}(\psi) \right) + \varepsilon_t \end{aligned} \tag{A.1}$$

where $X_{i,t-j}(\psi) = Y_{t-j} F_i(S_t; \psi)$ and Y_t is a vector of variables at time t and F is the transition function. We can write (A.1) in matrix form:

$$Y = X(\psi)\Pi + E. \tag{A.2}$$

We vectorize (A.2).

$$\begin{aligned} &\mathrm{Vec}(Y) = \mathrm{Vec}(X(\Psi)\Pi) + \mathrm{Vec}(E) \qquad \textbf{(A.3)} \\ &\Rightarrow \mathrm{Vec}(Y) = (I \otimes X(\Psi))\mathrm{Vec}(\Pi) + \mathrm{Vec}(E) \\ &\Rightarrow \tilde{y} = \tilde{X}(\Psi)\tilde{\pi} + \tilde{\varepsilon} \end{aligned}$$

where $\tilde{y}$, $\tilde{X}(\Psi)$, $\tilde{\pi}$ and $\tilde{\varepsilon}$ are respectively Vec(Y), $(I \otimes X(\psi))$, Vec(Π) and Vec(E).

We can, conditional on ψ, estimate Π and Ω by generalized least square and re-introduce $\Pi(\psi)$ and $\Omega(\psi)$ into the likelihood function. The generalized least square estimator of π and σ^2 conditional on ψ is

$$\tilde{\pi}(\Psi)_{GLS} = (\tilde{X}(\Psi)^T(\Omega \otimes I)^{-1}\tilde{X}(\Psi))^{-1}\tilde{X}(\Psi)^T(\Omega \otimes I)^{-1}\tilde{y}$$
$$= (I \otimes (X(\Psi)^T X(\Psi))^{-1} X(\Psi)^T)\tilde{y}$$

and **(A.4)**

$$\hat{\Omega}(\Psi) = \frac{\hat{E}(\tilde{\pi}(\Psi)_{GLS})^T \hat{E}(\tilde{\pi}(\Psi)_{GLS})}{N}$$

The optimization problem consist in finding the ψ that maximizes the concentrated likelihood function, i.e.,

$$\lambda_c(\Psi) = -\frac{N}{2}\ln|\Omega(\Psi)| \qquad \textbf{(A.5)}$$

Note that the number of parameters in this model would quickly 'explode' given the number of regimes and the order of the VAR system.

Appendix B (CHAPTER 4)

Summary Statistics 1960-1993

	Sample mean [a]	σ	$\hat{\kappa}_3$ [b]	$\hat{\kappa}_4$ [c]
GNP (DS)	0.0288	0.0094	-0.4248	0.69159
GNP (TS)	0.0000	0.0373	-0.4196	-0.5765
GNP (HP)	0.0000	0.01706	-0.3818	0.1278
M1 (growth rate)	0.0616	0.0103	0.5413	1.2344
M2 (growth rate)	0.0734	0.0086	0.3405	0.8261
Short-term spread	0.5529	0.4433	1.9954	6.9040
Funds rate (quarterly diff.)	-0.0069	1.1273	0.3748	7.3889
Long-term spread	0.9621	1.4075	0.0355	-0.4053

a The means are expressed in annual terms.

b $\hat{\kappa}_3$ is the skewness measure.

c $\hat{\kappa}_4$ is the excess kurtosis measure.

List of variables.

Real GNP(1982) SA (source: usaoecd.rat: usargnp)
M1 : money stock (monthly data converted to quarterly data) SA (source: usaoecd.rat: usam1s)
M2: money stock (monthly data converted to quarterly data) (source: citibase: FM2)
Federal Funds (effective) (% per annum) NSA (monthly data converted to quarterly data) (source: citibase FYFF; completed with Federal Reserve Bulletin)
Commercial paper, 6-month (% per annum) NSA (monthly data converted to quarterly data) (source: citibase FYCP)
U.S. Treasury Bills, second. market, 6-month (% per annum) NSA (monthly data converted to quarterly data) (source: citibase FYGM6)
U.S. Treasury Composite, 10 years + (long-term) (% per annum) NSA (monthly data converted to quarterly data) (source: citibase FYGL)
Standard and Poor index (source: citibase)
Unemployment rate: all workers 16 years and over (in %) SA (monthly data converted to quarterly data) (source: citibase LHUR)

Appendix C (CHAPTER 4)

Nonlinearity Test.

This section presents the LM statistic used in this thesis to test for nonlinearity. The nonlinearity test consists of the following:

Let $y_t = x_t^T \theta + u_t$ with $x_t = (1, x_t^*)$ and $x_t^* = (y_{t-1}, \dots, y_{t-p})$.

Case 1: If the transition variable S_t is a element of the set of regressors x_t^* then

1^0 Regress x_t on y_t to get the estimated residuals $\tilde{u}_t$

2^0 Estimate by ols:

$$\tilde{u}_t = x_t^T \xi_0 + x_t^{*T} \xi_1 S_t + x_t^{*T} \xi_2 (S_t)^2 + x_t^{*T} \xi_3 (S_t)^3 + \varepsilon_t$$

3^0 Construct the statistic

$$LM = (N) \times \frac{(SSR0 - SSR)}{SSR0}$$

where SSR0 is the residual sum of squares from the restricted model (step 1^0) and SSR is the residual sum of squares from step 2^0. The statistics LM follows a chi-square with 3p degrees of freedom where p is the number of lags in the autoregressive model (step 1^0).

Case 2: If the transition variable S_t does not belong to the set of regressors x_t^* then

1^0 Same step as case 1 ($\tilde{u}_t$)

2^0 Estimate by ols

$$\tilde{u}_t = x_t^T \xi_0 + x_t^T \xi_1 S_t + x_t^T \xi_2 (S_t)^2 + x_t^T \xi_3 (S_t)^3 + \varepsilon_t$$

3^0 Use the LM statistic defined in case 1 with the same degrees of freedom.

Appendix D (CHAPTER 4)

Regime-Switching Variables: Shaded area are NBER recessions.

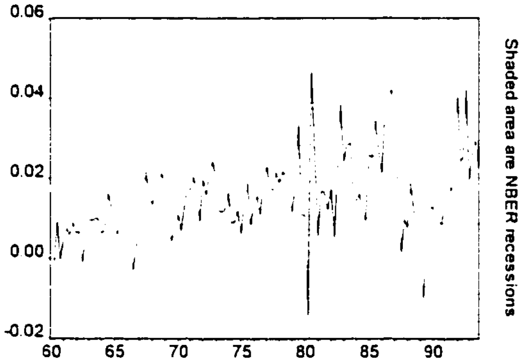

Figure 1 Growth rate of M1

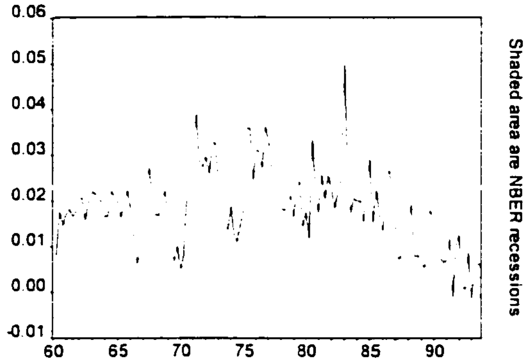

Figure 2 Growth rate of M2

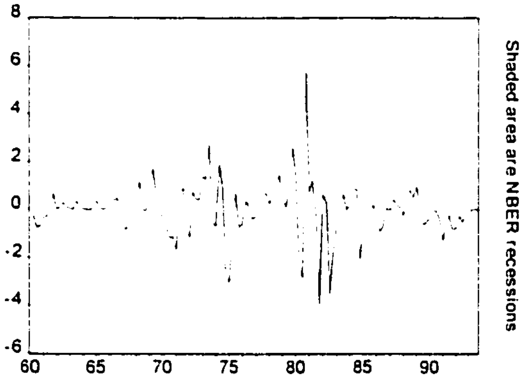

Figure 3 Quarterly difference in the federal funds rate

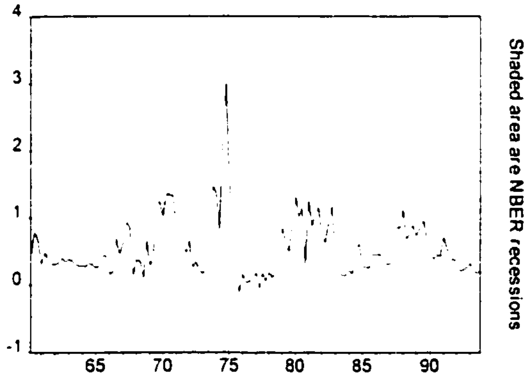

Figure 4 Short-term spread

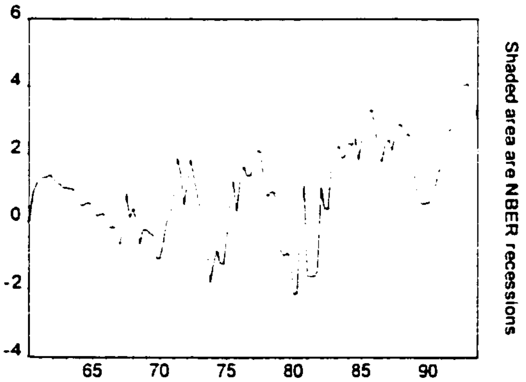

Figure 5 Long-term spread

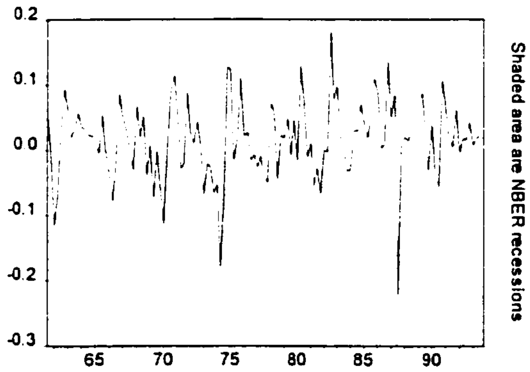

Figure 6 S&P stock index (quart. diff.)

Appendix E (CHAPTER 4)

STAR models of real GNP: Concentrated Likelihood Approach.

A: STAR MODELS of GNP (DS)

$$y_t = \alpha_0 + \sum_{i=1}^{2} \alpha_i y_{t-i} + (\beta_0 + \sum_{i=1}^{2} \beta_i y_{t-i}) F(S_t; \gamma, c) + \varepsilon_t$$

(the number of lags used in the STAR specification is based on the number of regressors in the linear counterpart; for example, GNP (DS) is modeled as a AR(2) process in the linear case).

Switching variable: S_t	Linear model AR(2)	GNP (t-5)	Short-term spread (t-1)	Funds (diff.) (t-6)	Long-term spread (t-1)	M2 (t-4)	S& P (t-2)
α_0	0.0042	0.0153	0.0051	0.0061	-0.0199	0.0068	-0.0030
α_1	0.2302	-0.2413	0.3002	0.2156	-0.2033	-0.1361	-0.7325
α_2	0.2018	0.1052	0.1006	0.0992	1.1742	0.0100	1.2027
β_0	--------	-0.0124	-0.0097	-0.0126	0.0279	-0.0043	0.0081
β_1	--------	0.5495	-0.9419	-0.4350	0.3393	0.4664	0.9833
β_2	--------	0.0964	0.9793	0.5730	-1.1034	0.2624	-1.0793
γ	--------	1550	1207	265	2.15	66739	356
C	--------	-0.0053	1.1867	0.9765	-1.2973	0.0144	-0.0667
Variance ratio	--------	0.9112	0.8059	0.8244	0.7459	0.9330	0.9112
Residuals							
σ	0.0088	0.0084	0.0079	0.0080	0.0076	0.0085	0.0084
$\hat{\kappa}_3$	-0.1911	-0.2756	0.0303	0.2037	0.2041	-0.1448	-0.2257
$\hat{\kappa}_4$	0.6036	0.6313	0.0051	0.1354	-0.1572	0.6264	1.0496
J-B	0.2996	0.1516	0.9901	0.6095	0.5983	0.2781	0.0299
ARCH test	0.3718	0.1263	0.4558	0.6228	0.7985	0.9489	0.5060

B: STAR MODELS of GNP (TS)

$$y_t = \alpha_0 + \sum_{i=1}^{3} \alpha_i y_{t-i} + (\beta_0 + \sum_{i=1}^{3} \beta_i y_{t-i}) F(S_t; \gamma, c) + \varepsilon_t$$

Switching variable: S_t	Linear model AR(3)	Short-term spread (t-2)	Funds (diff.) (t-6)	Long-term spread (t-1)	M2 (t-4)	S& P (t-2)
α_0	0.0004	0.0030	0.0013	-0.0113	-0.0004	-0.0180
α_1	1.1676	1.2466	1.1858	0.8969	1.1031	-2.7777
α_2	-0.0031	-0.1283	-0.1012	0.6559	0.1042	8.1858
α_3	-0.2351	-0.1759	-0.1318	-0.4230	-0.2921	-3.9022
β_0	--------	0.0388	-0.0075	0.0134	0.0117	0.0188
β_1	--------	-0.5012	-0.3673	0.2981	-0.0609	3.9534
β_2	--------	1.6148	0.3282	-0.7817	-0.9151	-8.2484
β_3	--------	-2.1516	-0.4660	0.3240	0.9657	3.7153
γ	--------	15	11	24	114060	59
C	--------	1.3650	1.3255	0.0804	0.0268	-0.1292
Variance ratio	--------	0.8858	0.7994	0.8421	0.8421	0.9081
Residuals						
σ	0.0085	0.0080	0.0076	0.0078	0.0078	0.0081
$\hat{\kappa}_3$	-0.0020	-0.1523	0.2487	0.2973	-0.1301	0.0347
$\hat{\kappa}_4$	0.7974	1.0154	0.5845	0.0800	0.4821	1.1701
J-B	0.2364	0.4915	0.2042	0.4103	0.4448	0.0251
ARCH test	0.6826	0.5018	0.5105	0.6873	0.1995	0.4389

C: STAR MODELS of GNP (HP).

$$y_t = \alpha_0 + \sum_{i=1}^{3} \alpha_i y_{t-i} + (\beta_0 + \sum_{i=1}^{3} \beta_i y_{t-i}) F(S_t; \gamma, c) + \varepsilon_t$$

Switching variable: S_t	Linear model AR(3)	Short-term spread (t-2)	Funds (diff.) (t-6)	Long-term spread (t-1)	S& P (t-2)
α_0	0.0001	0.0040	0.0012	-0.0922	-1.1492
α_1	1.0302	1.1092	1.0181	1.8109	73.035
α_2	0.0250	-0.1598	-0.0552	6.0024	89.291
α_3	-0.2450	-0.1391	-0.1541	-3.2921	-184.2
β_0	--------	-0.0033	-0.0113	0.0935	2.1827
β_1	--------	-0.4586	-0.3501	-0.8119	-136.8
β_2	--------	0.8583	0.8281	-6.0407	-169.3
β_3	--------	-0.7775	-0.6087	3.1572	349.3
γ	--------	40	589	2.79	0.0320
C	--------	1.2338	1.1884	-1.9739	-3.3365
Variance ratio	--------	0.9013	0.7851	0.7851	0.9255
Residuals					
σ	0.0079	0.0075	0.0070	0.0070	0.0076
$\hat{\kappa}_3$	-0.0493	-0.1119	0.2205	0.1902	0.3079
$\hat{\kappa}_4$	0.8661	0.8641	0.7687	0.7323	1.1120
J-B	0.1717	0.1173	0.1211	0.1604	0.0130
ARCH test	0.5661	0.2494	0.4119	0.9233	0.5079

Appendix F (CHAPTER 4)

Limiting Regimes for STAR models.

Let $y_t = x^T\beta_2 + x^T(\beta_1-\beta_2)F(S_t) + \varepsilon_t$.

We define Regime 0 when F=0: i.e.. $y_t = x_t^T\beta_2 + \varepsilon_t$ and

Regime 1 when F=1: i.e.. $y_t = x_t^T\beta_1 + \varepsilon_t$ where $x_t = (1, y_{t-1}, \ldots, y_{t-p})$

A: Limiting regimes for STAR models of GNP (DS):

S_t	GNP_{t-5}		Short-term spread$_{t-1}$		Funds$_{t-6}$ (diff.)		Long-term spread$_{t-1}$	
DS version	Regime 0	Regime 1	Regime 0	regime 1	Regime 0	Regime 1	Regime 0	regime 1
β_0	0.0153	0.0029	0.0051	-0.0046	0.0061	-0.0065	-0.0199	0.0080
β_1	-0.2413	0.3082	0.3002	-0.6420	0.2156	-0.2194	-0.2033	0.1330
β_2	0.1052	0.2016	0.1005	1.0790	0.0993	0.6722	1.1742	0.0708
β_3	--------	--------	--------	--------	-------	-------	--------	--------
station.	Yes	Yes	Yes	No	Yes	Yes	No	Yes
growth [a]	5.4%	2.4%	3.4%	---------	3.1%	-4.4%	--------	4.1%

a The annual growth rate indicator is the unconditional mean. E[y].

B: Limiting regimes for STAR models of GNP (TS):

S_t	$M2_{t-4}$		Short-term $spread_{t-2}$		$Funds_{t-6}$ (diff.)		Long-term $spread_{t-1}$	
TS version	Regime 0	Regime 1	Regime 0	regime 1	regime 0	Regime 1	Regime 0	regime 1
β_0	-0.0004	0.0113	0.0030	0.0418	0.0013	-0.0062	-0.0133	0.0021
β_1	1.1031	1.0422	1.2466	0.7454	1.1858	0.8185	0.8969	1.1950
β_2	0.1042	-0.8109	-0.1283	1.4865	-0.1012	0.2270	0.6559	-0.1258
β_3	-0.2921	0.6736	-0.1759	-0.6295	-0.1318	-0.5978	-0.4230	-0.0990
station.	Yes	Yes	Yes	No	Yes	Yes	No	Yes
growth	-1.8%	47%	6%	--------	11%	-4.5%	--------	11%

C: Limiting regimes for STAR models of GNP (HP):

S_t	Short-term $spread_{t-2}$		$Funds_{t-6}$ (diff.)		Long-term $spread_{t-1}$	
HP version	Regime 0	Regime 1	regime 0	Regime 1	regime 0	regime 1
β_0	0.0040	0.0007	0.0012	-0.0190	-0.0992	0.0013
β_1	1.1092	0.6506	1.0181	0.6680	1.8109	0.9990
β_2	-0.1598	0.6985	-0.0552	0.7729	6.0024	-0.0383
β_3	-0.1391	-0.9166	-0.1541	-0.7628	-3.2921	-0.1349
station.	Yes	Yes	Yes	Yes	No	yes
growth	8.4%	0%	2.5%	-12%	--------	3%

Appendix G (CHAPTER 5)

STAR model of GNP with 4 lags of GNP and 4 lags of the financial variable (i.e., the growth rate of M1, the growth rate of M2, and the quarterly difference in the funds rate : switching variable is the quarterly difference in the funds rate$_{t-6}$.

$$y_t = \alpha_0 + \sum_{i=1}^{4} \alpha_{1i} y_{t-i} + \sum_{i=1}^{4} \alpha_{2i} w_{t-i} + \left(\beta_0 + \sum_{i=1}^{4} \beta_{1i} y_{t-i} + \sum_{i=1}^{4} \beta_{2i} w_{t-i} \right) F(S_t; \gamma, c) + \varepsilon_t$$

where y_t: GNP (i.e., DS, TS and HP), w_t: the growth rate of M1 and of M2, and the funds rate, S_t: quarterly difference in the funds rate at lag 6.

	Growth rate of M1			Growth rate of M2			Quart. diff. in funds rate		
	DS	*TS*	*HP*	*DS*	*TS*	*HP*	*DS*	*TS*	*HP*
α0	0.0089	0.0005	-0.0008	0.0108	-0.0029	0.0039	0.0078	0.0012	0.0009
α11	0.3992	1.3276	1.1087	0.5027	1.5214	1.3231	0.2951	1.2199	0.9725
α12	-0.1895	-0.5299	-0.4189	-0.2410	-0.9376	-0.8647	-0.2923	-0.4825	-0.4172
α13	-0.1007	0.0706	0.0119	-0.2807	-0.0730	-0.0758	-0.0077	0.1820	0.0501
α14	-0.0613	0.1035	0.0833	-0.2421	0.4575	0.3635	0.0674	0.0437	0.0923
α21	-0.1751	-0.0801	-0.0919	0.0884	0.0069	-0.2095	0.0021	0.0029	0.0039
α22	-0.0501	-0.0915	-0.1015	-0.6756	-0.5937	-0.5549	-0.0030	-0.0025	-0.0009
α23	0.2237	0.2429	0.1822	0.2356	0.3246	0.2461	0.0015	0.0020	0.0031
α24	-0.0582	-0.0613	0.0687	0.3522	0.4704	0.3830	-0.0026	-0.0021	-0.0011
β0	-0.0150	0.0011	-0.0013	-0.0302	-0.0024	-0.0280	-0.0100	0.0043	0.0030
β11	-0.9564	-0.9001	-1.0771	-1.2005	-1.9474	-1.6339	-0.6111	-0.6476	-0.3651
β12	1.5061	2.2214	2.8874	1.2488	4.0725	3.9764	1.6644	2.5378	2.7374
β13	0.3565	-0.8941	-0.8372	0.6925	-0.3968	-0.4380	0.1912	-1.6086	-1.4451
β14	0.0447	-0.6078	-1.0546	0.6774	-1.8949	-1.5156	0.1667	-0.4137	-0.6201
β21	1.0997	0.5961	1.0753	0.6142	0.9056	1.5039	-0.0039	-0.0052	-0.0074
β22	0.4564	0.6533	1.0837	1.9660	2.5400	2.5329	-0.0012	-0.0011	-0.0041
β23	-0.8562	-0.9389	-1.1593	-0.4672	-1.3023	-1.0846	-0.0049	-0.0053	-0.0077
β24	-0.295	-0.198	-0.6324	-1.2195	-1.9887	-1.7079	0.0018	0.0046	0.0024
γ	2.0302	2.44	2.19	1.3307	1.0131	1.0256	2.3301	2.7439	3.3779
c	0.6745	0.6106	0.9706	0.6289	1.2450	1.3072	0.5973	0.7428	0.8559
σ	0.0073	0.0069	0.0062	0.0070	0.0068	0.0064	0.0069	0.0068	0.0062
$\hat{\kappa}_3$ [a]	0.1157 (0.5917)	0.2178 (0.3126)	0.2669 (0.2150)	0.0560 (0.7953)	0.1261 (0.5589)	0.2624 (0.2237)	0.3533 (0.1013)	0.5279 (0.0143)	0.3184 (0.1398)
$\hat{\kappa}_4$	0.2164 (0.6159)	0.8765 (0.0421)	1.3689 (0.0015)	0.5520 (0.2007)	0.9763 (0.0236)	1.0708 (0.0130)	0.0113 (0.9791)	0.3219 (0.4554)	0.2323 (0.5901)
J-B[b]	0.7636	0.0762	0.0030	0.4264	0.0650	0.0219	0.2612	0.0378	0.2908
IR[c]	-0.4078, 1.7568	-0.2899, 1.5111	-0.0327, 1.9739	-1.0223, 2.2801	-0.9238, 3.4138	-0.8352, 3.4496	-0.3457, 1.5403	-0.0580, 1.5436	0.2054, 1.5064

a Figures in parentheses for $\hat{\kappa}_3$ and $\hat{\kappa}_4$ are the p-values of the skewness and excess kurtosis statistics.

b p-value of the Jarque-Bera statistic.

c IR is the intermediate or middle regime.

Appendix H (CHAPTER 5)

Transition functions of the estimated STAR models

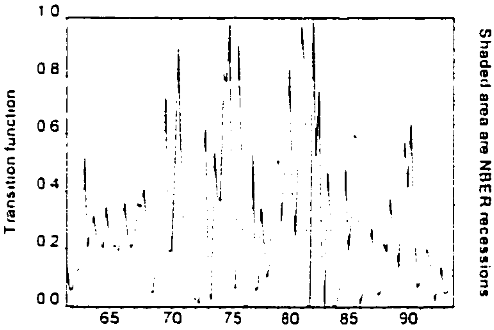

Figure 1 STAR model of GNP (DS) with 4 lags of GNP and M1
(switching vanable is the quart diff in the funds rate)

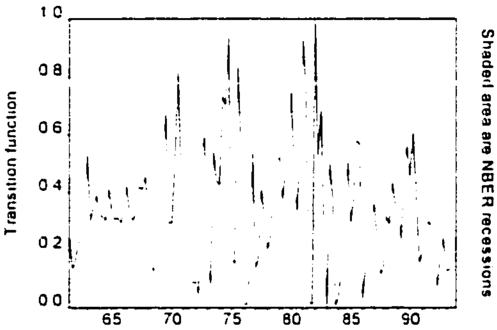

Figure 2 STAR model of GNP (DS) with 4 lags of GNP and M2
(switching vanable is the quart diff in the funds rate)

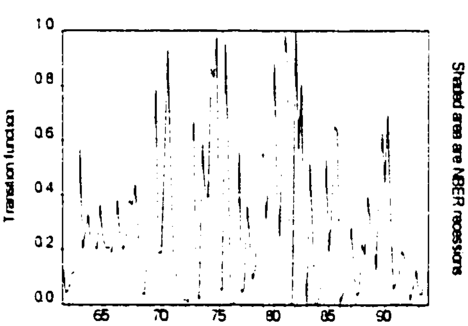

Figure 3 STAR model of GNP with 4 lags of GNP (DS) and the funds rate
(switching variable is the quart. diff in the funds rate)

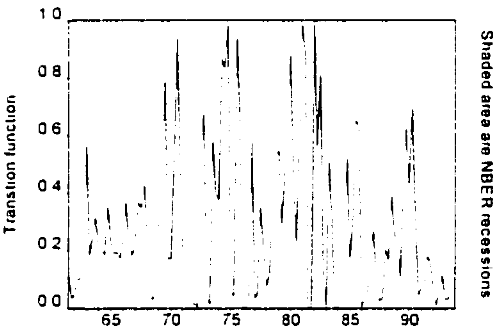

Figure 4 STAR model of GNP (TS) with 4 lags of GNP and M1
(switching vanable is the quart. diff in the funds rate)

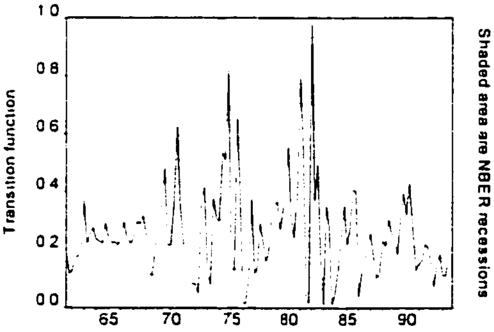

Figure 5 STAR model of GNP (TS) with 4 lags of GNP and M2
(switching vanable is the quart. diff in the funds rate)

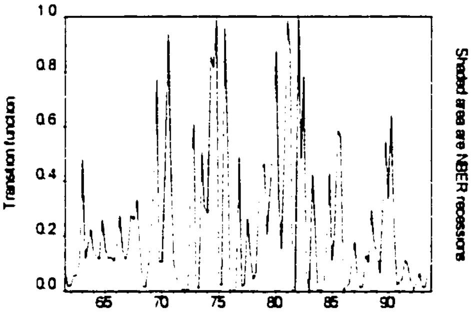

Figure 6 STAR model of GNP (TS) with 4 lags of GNP and the funds rate
(switching variable is the quart. diff. in the funds rate)

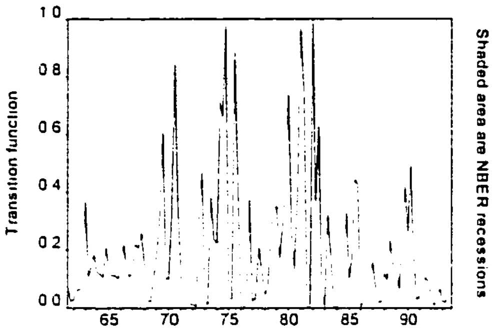

Figure 7 STAR model of GNP (HP) with 4 lags of GNP and M1
(switching variable is the quart. diff. in the funds rate)

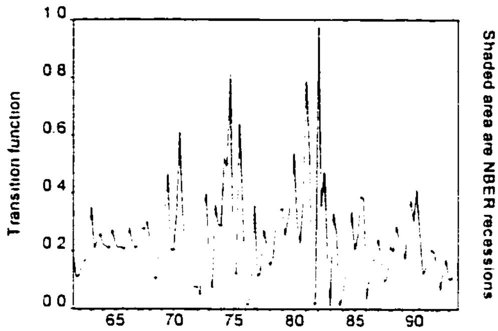

Figure 8 STAR model of GNP (HP) with 4 lags of GNP and M2
(switching variable is the quart. diff. in the funds rate)

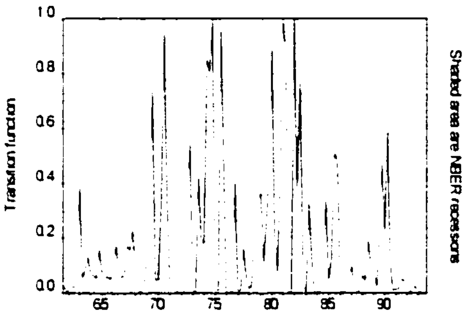

Figure 9 STAR model of GNP (HP) with 4 lags of GNP and the funds rate
(switching variable is the quart. diff. in the funds rate)

Appendix I (CHAPTER 5)

STAR model of GNP with the short-term and long-term spread as the switching variables.

$$y_t = \alpha_0 + \sum_{i=1}^{4}\alpha_{1i}y_{t-i} + \sum_{i=1}^{4}\alpha_{2i}w_{t-i} + \left(\beta_0 + \sum_{i=1}^{4}\beta_{1i}y_{t-i} + \sum_{i=1}^{4}\beta_{2i}w_{t-i}\right)F(S_t;\gamma,c) + \varepsilon_t$$

where y_t is GNP (DS) and w_t is the financial variable.

	(1)	(2)	(3)	(4)	(5)	(6)
α0	0.0055	0.0089	0.0049	-0.0207	0.0152	-0.0071
α11	0.3037	0.2036	0.2250	-0.1016	-0.4871	0.6887
α12	0.1183	0.0271	0.1428	1.2569	0.3717	1.3335
α13	-0.0224	-0.0280	-0.0296	-0.1549	-0.6935	-0.7306
α14	-0.0475	-0.0051	0.0123	0.1384	-0.4570	-0.2061
α21	-----	-0.0067	0.0694	-----	0.0049	-0.2326
α22	-----	-0.0021	0.0399	-----	0.0040	0.1719
α23	-----	-0.0002	0.0496	-----	0.0009	-0.6544
α24	-----	0.0019	-0.1150	-----	0.0110	0.3979
β0		-0.0332	-0.0361	0.0287	-0.0073	0.0131
β11	-0.0111	-1.8724	0.2216	0.2451	0.6593	-0.5730
β12	-0.5350	2.8108	1.7316	-1.1870	-0.2719	-1.2259
β13	0.9344	0.7199	-2.1934	0.1032	0.6601	0.7125
β14	-1.2896	5.5157	-1.0373	-0.0969	0.5434	0.2682
β21	-----	-0.1015	-1.7010	-----	-0.0043	0.3228
β22	-----	0.1130	2.9460	-----	-0.0058	-0.0886
β23	-----	-0.1335	-1.3529	-----	0.0013	0.6063
β24	-----	0.1799	2.9261	-----	-0.0126	-0.4690
γ	976	70	1526	2.14	6.9	3.69
c	1.1966	1.3000	1.1522	-1.3223	-0.2708	-1.2032
LR[a]	-----	0.0647	0.0333	-----	0.0861	0.6457
σ	0.0077	0.0072	0.0071	0.0076	0.0072	0.0074
σ_L[b]	0.0088	0.0081	0.0085	0.0088	0.0082	0.0085
$\hat{\kappa}_3$	0.0386 (0.8581)	0.0387 (0.8576)	0.1153 (0.5929)	0.2143 (0.3204)	0.1417 (0.5113)	0.2263 (0.2940)
$\hat{\kappa}_4$	0.4018 (0.3516)	0.0757 (0.8608)	0.3622 (0.4011)	-0.1134 (0.7925)	0.0383 (0.9292)	-0.0863 (0.8413)
J-B	0.6378	0.9690	0.6093	0.5897	0.8028	0.5652
Variance ratio	0.7656	0.7901	0.6977	0.7459	0.7710	0.7579

(1) STAR model with 4 lags of GNP: switching variable is the short-term spread$_{t-1}$.
(2) STAR model with 4 lags of GNP and the short-term spread: switching variable is the short-term spread$_{t-1}$
(3) STAR model with 4 lags of GNP and M1: switching variable is the short-term spread$_{t-1}$

(4) STAR model with 4 lags of GNP: switching variable is the long-term spread$_{t-1}$
(5) STAR model with 4 lags of GNP and the long-term spread: switching variable is the long-term spread$_{t-1}$
(6) STAR model with 4 lags of GNP and M1: switching variable is the long-term spread$_{t-1}$.

a) **LR represents the p-value (likelihood ratio statistic) for the null H_0: $\alpha_i = \beta_i = 0$ for i=5,...,8.**
b) **standard deviation from corresponding linear model (same set of regressors)**

Appendix J (CHAPTER 5)

STAR model of GNP (DS): Transition functions
Switching variables are the short-term and the long-term spread.

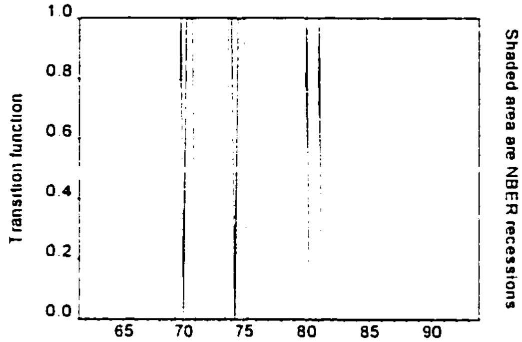

Figure 1 STAR model of GNP (DS)
(switching variable is the short-term spread)

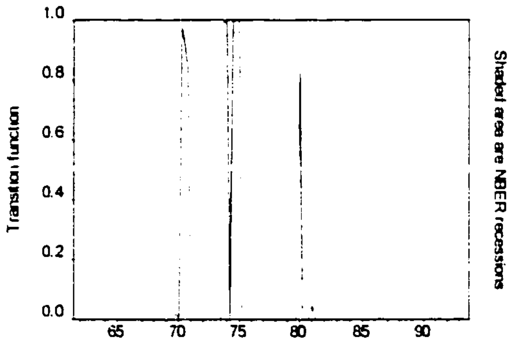

Figure 2 STAR model of GNP with 4 lags of GNP and the short-term spread
(switching variable is the short-term spread)

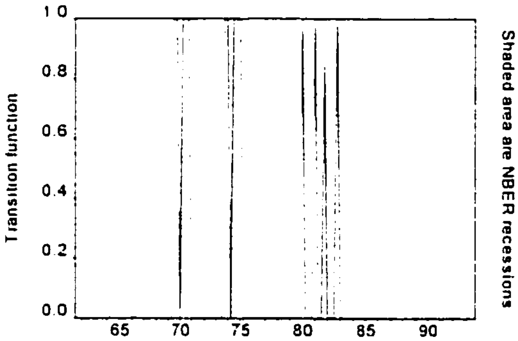

Figure 3 STAR model of GNP (DS) with 4 lags of GNP and M1
(switching variable is the short-term spread)

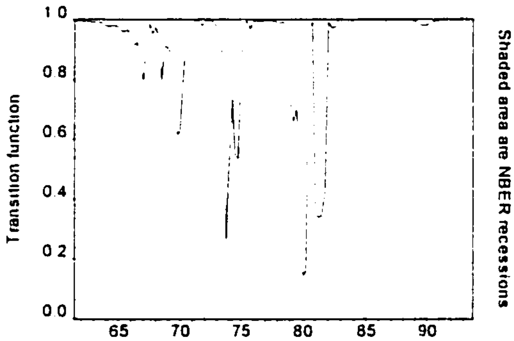

Figure 4 STAR model of GNP (DS) with 4 lags of GNP
(switching variable is the long-term spread)

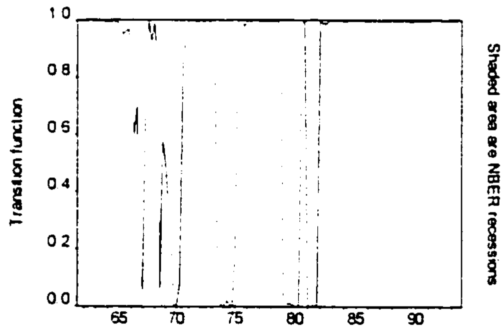

Figure 5 STAR model of GNP (DS) with 4 lags of GNP and the long-term spread
(switching variable is the long-term spread)

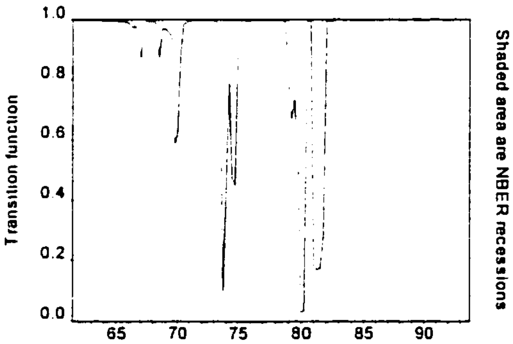

Figure 6 STAR model of GNP (DS) with 4 lags of GNP and M1
(switching variable is the long-term spread)