The Effects of Using Motion Detector Technology to Develop Conceptual Understanding of Functions Through Dynamic Representation in Grade 6 Students

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ABSTRACT

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This thesis presents the effects of using technology to develop conceptual understanding of functions through graphical representations. It examines the effects of including dynamic representation in a conceptual approach to the teaching of functions.

The teaching experiment developed in this thesis has been implemented in one Grade 6 class (pupils 11 - 12 years of age). Participating students were observed during class discussions and their assignments were analysed over three sessions. Conclusions are drawn regarding the effects of using dynamic representation on the conceptual understanding of functions through graphical representations and suggested improvements to the introduction of the function concept in middle school instructional programs are given. The results of the research project showed that an appropriate use of motion detector technology did have a positive effect on the students' conceptual understanding of functional relationships between independent and dependent variables as well as the general mathematical notion of function.

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Introduction

Functions are used to model numerous scientific and social phenomena. Debates over such international issues as global warming, population control, disease prediction and control, radioactive waste, inflation rates and national debt often revolve around understanding the mathematical behaviour of functions. Their importance for an educated citizenry is indisputable. Furthermore, the study of functions offers a unique opportunity to explore the relationship between mathematics and nature and, in dóing so, can make mathematics both relevant and accessible – qualities currently demanded in most cries of curricular reform in mathematics reform.

The motion detector lesson presented in this thesis was conceived in response to many calls for proleptic research in developing understanding of the function concept in young children (Mevarech & Kramarsky, 1997; Harvey, Waits & Demana, 1995). It is based on the assumption that extra time taken in the early introduction of the function concept will greatly reduce the number of cognitive obstacles and misconceptions that our students presently encounter or create.

Presently, it is not until advanced Grade 10 mathematics that students in Quebec are taught functions. Only approximately 25% of the Grade 10 students are placed in this advanced stream, while 75% of the students are placed in the regular Grade 10 mathematics program. Therefore the majority of high school mathematics students in the regular stream will not receive any instruction on functions in Quebec throughout their high school education. It is the goal of this thesis to suggest that middle school instructional programs should include an emphasis on functions so that all students understand the mathematical behaviour of functional relationships.

In particular, I make the following three hypotheses based on the preliminary analyses presented in the theoretical framework of the research project (Chapter 1):

Hypothesis 1 Dynamic representation helps students to develop conceptual understanding of the relationship between distance and time in problems of motion.

Hypothesis 2 Dynamic representation facilitates the development of conceptual understanding of independent and dependent variables.

Hypothesis 3 The investigations of the relationship between distance and time in problems of motion help the students better understand other functional relationships between independent and dependent variables and the general mathematical notion of function in learning mathematics in higher grades.

The activities presented in this thesis were planned based on a social constructivist theory of learning. All activities were first accomplished individually by the students and later discussed in class. Therefore, class discussion formed an integral part of the teaching experiment; it is believed by many, including myself, to be the essence of teaching and learning mathematics (Lindquist & Elliott, 1996). It is important to realise that the environment within which the motion detector lesson was performed was conducive to the making of conjectures, to the posing of questions, and to a general feeling of security for the students.

The technology used in the described activity includes a graphing calculator and a sonic motion detector. Both of these tools are products of Texas Instruments Inc. However, other graphing calculators and sonic motion detectors may be used.

Both researchers and teachers agree that the most widely used representational systems for expressing functions are equations, tables, and graphs (NCTM, 1998). Modelling, interpreting, and the ability to move from one form of representation to another is no doubt essential in understanding the concept of function. However, students' understanding of relationships between variable quantities must be the first step in any didactic model for the teaching of functions. Sierpinska (1992) suggests that functions should first be seen as models of such relationships.

The motion detector lesson discussed in this study shows a fourth system for representing functions. The dynamic representation allows students to understand better the relationships between variable quantities. In fact, the student is physically in control of the situation and is receiving immediate feedback to his or her actions.

There exists disturbing new research that suggests students have difficulty with, and therefore avoid, graphic representations because they simply were never taught how to use them (Kaldrimidou & Ikonomou, 1998). The use of motion detector technology creates dynamic graphing situations which allow students to see relationships between dependent and independent variables as they occur. Such powerful dynamic teaching and learning tools also allow students to control these relationships physically, thus deepening their understanding of functions in the sense that the students become more conscious of what the variables in a relationship are, which variable controls which variable, and in what way.

This thesis is organised into four sections. First, the theoretical framework of the research project presents a detailed preliminary analysis for the teaching and learning of functions and variables. The second section outlines the methodology of the study which is that of teaching experiment. The description, realisation and analysis of the research project is undertaken in the third section. The final section of this thesis contains conclusions with regard to the effects of using dynamic representation on the conceptual understanding of function and suggests improvements to the introduction of the function concept. Many of these conclusions will be based on the information obtained from the global analyses of each session. However, conclusions will also be drawn implicitly from the findings of the preliminary analysis presented in the theoretical framework of the research project.

Chapter I:
The Theoretical Framework of the Research Project

Historically, the first graphical representations of relationships between two changing quantities represented change as a function of time. Evidence of this dates back to before 1361 when Oresme had the idea of drawing a graph (non-Cartesian) to describe the way quantities vary. It may be of interest to know that such a graphical representation of functions was known as the latitude of forms (Merzbach, 1991). It is also interesting to note that the first graphical representations expressed change as a function of time. In a study by Kieran (1993), it is suggested that using time as an independent variable has epistemological value in studying qualitative graphs to represent functions. Developing such a qualitative perspective of graphs is key in minimising the cognitive obstacles restricting students' conceptual understanding of functions. It is also essential to realise that understanding the concept of variable is fundamental to students' acquisition of the function concept. Early informal experiences with the concept of function can lead to a deeper understanding of functions throughout high school (NCTM, 1998). Modelling, interpreting, and the ability to move from one form of representation to another is essential in understanding the concept of function. These different modes of representation (equation, table, and graph) should be part of any research on the teaching and learning of functions.

The Concept of Function

The concept of function as a relationship between variables has proven to be more effective in the teaching of function than the modern set-theoretic definition of function found in many textbooks used by teachers (O'Callaghan, 1998; Vinner & Dreyfus, 1989). In fact, the Dirichlet-Bourbaki concept is referred to as this modern concept of function and is described as:

> a correspondence between two nonempty sets that assign to every element in the first set (the domain) exactly one element in the second set (the codomain).
>
> (Vinner & Dreyfus, 1989, p. 357)

This set-theoretic definition of function appears to be too abstract for students to understand. Historically, functions first appeared as models of relationships. As Sierpinska (1992) reasons, this is how they should first be introduced.

> If we assume that the meaning of a concept lies in the problems and questions that gave birth to it, and we wish that our students grasp the meaning of the notion of function, then this seems to be a quite reasonable claim to make (p. 32).

Consequently, for the purpose of this study, a function will be described as a relationship between two variables such that each value of the independent variable corresponds to exactly one value of the dependent variable.

Literature on the conceptions and misconceptions that students and teachers hold in relation to the notion of functions is not lacking (Kaldrimidou & Ilkonomou, 1998; Sierpinska, 1992; Dubinsky & Harel, 1992). Also, recent national and international tests suggest that students continue to have difficulties with the function concept (TIMMS & SAIP, 1997). Recent reforms in mathematics education (NCTM, 1998) call for a functional emphasis to be integrated throughout the school curriculum, beginning in the elementary grades. Unfortunately, the concept of function is still typically not introduced until the later years of high school. In the province of Quebec, students are first exposed to the concept of function in the advanced Grade 10 mathematics course (Curriculum, *Mathematics 436 Secondary School*, 1996). In Grade 7, the students acquire prerequisite skills to the study of algebra, such as working with patterns (Curriculum, *Mathematics 116 Secondary School*, 1993) and are informally introduced to variables as changing quantities. Grade 8 has as a general objective to help students develop the ability to use algebra to solve problems (Curriculum, *Mathematics 216 Secondary School*, 1996). Students learn to translate from one form of representation to another and to solve problems that can be translated as a first degree equation.

In Grade 9, the students are taught to use algebra to generalise situations. More specifically, the students learn:

> To determine the dependent variable and the independent variable in a given situation.
> To represent the rule that applies in a given situation, using a table of values.
> To represent a situation and its corresponding rule by means of a graph, given a table of values.
>
> (Curriculum, *Mathematics 314 Secondary School*, 1996, p. 17)

In addition, the students are also taught to solve problems related to situations in which a linear relationship exists between variables. Then, in advanced Grade 10 mathematics students in Quebec are finally and formally introduced to functions.

Now, Kaldrimidou & Ilkonomou (1998) explain that function can have many different conceptions depending on the context in which the concept is applied:

> For example, a function can be considered as a relation, as a transformation, as a mapping, or as an object. These conceptions are different from an epistemological point of view because the meaning of the notion is different in each case (p. 273).

Both the present *NCTM Standards* (1989) and the draft version of the *NCTM Standards 2000* document (1998) suggest that even primary school students should be able to identify and describe relationships between two quantities that vary together. For middle school mathematics, the *NCTM Standards 2000* draft suggests that mathematics instructional programs should pay particular attention to functions and models so that all students understand various functional relationships (p. 221).

The Concept of Variable

In light of this, it is important that sufficient time be allocated for developing understanding of the concept of variable prior to the implementation of the motion detector lesson. In order to minimise cognitive obstacles and misconceptions, it is essential that such a conception of variable emphasises relationships of dependence among variables and that variables are presented as quantities with changing values. It is this conception of variable that will allow students to model and to interpret situations of relationships.

Understanding the concept of variable is fundamental to students' acquisition of the function concept. The fact that many mathematics education researchers do not agree on a definition of variable leads to several interpretations of the concept of variable. As a result, curriculum designers responsible for the writing of the mathematics program in our province, state, or country are faced with different interpretations from which to choose. The concept of variable that these designers adopt can be responsible for the introduction of many didactic constraints on the teaching and learning of algebra. For example, only a very small percentage of these curricula, if any at all, is allocated to developing understanding of the concept of variable itself. As a result of this shortcoming, teachers themselves spend little, if any, time investigating the concept of variable with their students.

Presently, in the province of Quebec, students first encounter variables as unknowns (boxes or symbols) in elementary school. In Grade 7, students are exposed to letters and symbols as pattern generalisers and seem to learn about variable in a natural way as a relationship between rank and a specific attribute to a pattern. It is unfortunate, however, that the notion of variable is not addressed specifically at this point. In fact, the first place the word "variable" is officially used in our mathematics program is in Grade 8 when students learn to solve first degree equations, and the term "variable" here is only used as an unknown. It is not until Grade 9 that these students are introduced to the notions of dependent and independent variables (Curriculum, *Mathematics 314 Secondary School*, 1996).

Although our official program guides do not use the term "variable" until Grade 8, many of the textbook series presently being used in elementary schools often use the term variable when strictly referring to unknowns. For example, in *Challenging Mathematics 6 – Teaching and Activity Guide* (Mondia Éditeurs, 1996), variables are referred to as hidden numbers. Consequently, the students come to an understanding of variable as a hidden number or an unknown at an early age. By no means is the above situation unique to Quebec. Unfortunately, it is probably the reality in too many curricula in both Canada and the United States.

The research community (NCTM, 1998) seems to agree that the conceptual understanding of a variable is crucial in setting a firm foundation for other related algebraic concepts. However, very little, if any, time in such programs is devoted to developing the varying property of variables. This varying property of variables, which will be referred to as "variability" in this study, occurs in situations in which one variable takes on a variety of values resulting in corresponding values for a dependent variable. It appears that since understanding of the concept of variable is not an objective in itself, students are expected to acquire this concept while visiting other mathematical concepts such as patterns and sequences, first degree equations, and relations. Going from the concept of variable as a hidden number or unknown to the concept of variable that includes "variability" is no small step for students to make on their own. If the NCTM's *Curriculum Standards* (NCTM, 1989) are correct in stating that a major problem in students' efforts to understand and do algebra results from their narrow interpretation of variable, then it is important for us to study the epistemological and didactic aspects of the concept of variable. In the 1988 Yearbook, *The Ideas of Algebra, K-12* (NCTM, 1988), Usiskin explains that variables take on different concepts depending on the conceptions of algebra being studied. In

Usiskin's framework on conceptions of algebra, he explains the following uses of variable:

1. **Stand for quantities and have a feel of knowns.** For example, the length (L) and width (W) in A = LW, where, of course, A stands for area.

2. **As an unknown or constant.** For example, the variable x in the expression 2x = 10.

3. **As an argument or a parameter.** An argument can be defined as the independent variable of a function. For example, the sum of the measures of the interior angles of a polygon can be determined using the function $f(x) = 180(x - 2)$, where the variable x is the argument of the function. A parameter, on the other hand, is a constant that can have different values in an expression without changing the form of the expression or function. In each case, the graph of $y = mx$ is a straight line passing through the origin. The different slopes correspond to different values of the parameter, m (Fyfied & Blane, 1995).

4. **As an arbitrary object in a structure related by certain properties.** Such a conception of variable is normally reserved for college level students studying abstract algebra, who do not always go back to the level of referent when operating on variables. For example, students do not always think of numbers or ratios in triangles when asked to derive trigonometric identities.

Usiskin cautions that trying to fit everything into one single concept of variable distorts the purpose of algebra (Usiskin, 1988, p. 10). Perhaps we should be more concerned about the many misconceptions attributed to so many uses of variable. Quantities that have a feel of knowns, unknowns, constants, and arbitrary objects are all symbols that represent objects in one or more conceptions of algebra. Arguments and parameters differ from these symbols in that they contain "variability" and, unfortunately, it is this conception of variable that is often recognised or discovered to be the concept that blocks students' success in algebra. It should be of no surprise to anyone that students have great difficulty going from their concept of variable as an unknown (hidden number) to a new conception of variable that includes "variability". This is truly a difficult leap in their conceptual understanding of variable. It is difficult for students to break away from their conception that a variable is a letter standing for a particular number. Research suggests that the grouping of symbolic constants, parameters, unknowns, and unconstrained variables into one general conception of variable is responsible for many cognitive obstacles in children (Herscovics, 1989; Leitzel, 1989; Kieran & Wagner, 1989).

It is therefore essential that sufficient time in any mathematics program must be allocated for developing the understanding of the concept of variable. We should not use the term variable until we are in fact dealing with arguments or parameters that possess variability and that the introduction of the concept of variable should emphasise relationships of dependence. It is therefore equally important that we consider refraining from using the term "variable" when students investigate situations involving knowns, unknowns, or constants. Above all, we should not give students inaccurate or incomplete definitions of such important mathematical concepts. Let's consider allowing students to conceptualise the different uses of symbols as what they are: knowns, unknowns, and constants. Perhaps such an approach to the concept of variable can help students acquire the function concept and improve their success in algebra.

An example of problems associated with not doing this is presented in the *Mathematics Carrousel* series (Wilson & Lafleur, 1996-1997). *Mathematics Carrousel 3* is the only English mathematics textbook written in conjunction with the province of Quebec's mathematics curriculum. This textbook, which is founded in large part on the NCTM's *Curriculum Standards* (NCTM, 1989) is partly responsible for the improvement in the teaching and learning of mathematics in the province of Quebec. In fact, since the implementation of the new curriculum, and therefore these textbooks, the students of Quebec out-performed the majority of the students of other provinces on a recent national assessment (SAIP, 1997). However, much improvement is still needed in the teaching and learning of the concept of variable. If mathematics education researchers have such a muddled and confused understanding of the concept of variable, it should come to no surprise that this confusion trickles down to the authors of elementary and high school mathematics textbooks and to teachers themselves.

The authors of *Mathematics Carrousel 3*, a textbook used for teaching Grade 9 mathematics, chose to bypass the introduction of the concept of variable altogether. In fact, it is simply presented as knowledge that is to be easily assimilated by the student while visiting the concept of relation. The students are informed on the first page of the chapter on relations that the link between two numbers that vary is known as a **relation**. They are also told that "a relation is an association between two sets of data" (p. 110). The students are then given one activity to apply their interpretation of these rules before being given the following definitions.

> Quantities with changing values are called **variable quantities** or simply **variables**. If the value of a quantity does not change it is said to be **constant**.
>
> In most relations, one of the variables reacts to changes to the other. Such a variable is **dependent**, while the other is **independent** (p. 112).

Although their definition of variable as quantities with changing values is accurate, I object to the way that it is introduced. There are no investigations for students to develop an understanding of the concept of variable. They are simply expected to remember and understand that quantities with changing values are called "variables." For many of these students, this is an unnecessarily difficult leap in their conceptual understanding of variable because of a cognitive obstacle introduced by the same authors the previous year in Grade 8. In *Mathematics Carrousel 2* the author gives the students the following definition of variable: "A symbol used to replace a number is called a **variable**" (p. 170). It cannot be surprising, therefore, that it is difficult for students to break away from their initial conception that a variable is a letter standing for a particular number, when that is exactly what they are taught the previous year.

This cognitive obstacle caused by giving an incomplete or inaccurate definition of such an important mathematical concept is completely unnecessary. This cognitive obstacle is of didactic origin and is a result of inappropriate instruction. The definition of variable described by the author in *Mathematics Carrousel 2* is not even included in the definition of variable defined in *Mathematics Carrousel 3*.

As mentioned earlier, student understanding of the concept of variable is fundamental in their acquisition of the function concept. It is therefore essential that steps be taken to reduce the cognitive obstacles that impede children's learning.

There are two areas of students' conceptions associated with the present teaching of variable: one is related to the meaning of letters, the other to the idea of generalisation. These conceptions are described below, and examples of the consequences on the solving of some typical school problems are also provided.

I. The Meaning of Letters

Students' Conception 1: Treating letters as objects or labels.

Example: Contact lenses cost l dollars a pair and glasses cost g dollars a pair. If I buy 2 pairs of contact lenses and 2 pairs of glasses, what does $2l + 2g$ represent?

Solution Based on Conception: 2 pairs of contact lenses and 2 pairs of glasses

Correct Solution: $2l + 2g$ represents the total cost of purchasing 2 pairs of contact lenses and 2 pairs of glasses.

Example: If a represents the number of apples and each apple costs 25 cents, what does $25a$ represent?

Solution Based on Conception: 25 apples

Correct Solution: $25a$ represents the value of 25 apples.

Students' Conception 2: Thinking that letters always have one specific value.

Example: If $a + b = 14$ and a is an even number less than b, what can you say about a?

Solution Based on Conception: $a = 2$

Correct Solution: a may be equal to 2, 4, or 6 depending on the value of b.

Students' Conception 3: Believing that rules are used to determine which number a letter stands in connection to the alphabet.

Example: $A = 1$ because A is the first letter of the alphabet. D is equal to 7 because it is 2 more than B which is equal to 5.

II. Misconceptions about Generalisations

Students' Conception: Generalisations are incomplete answers.

Example: Perimeter, $P = 2l + 2w$, where l represents length and w represents width.

Solution Based on Conception: $2l + 2w$ can not be the final answer because there is still an addition to do.

Adapted from *Mathematics Programmes of Study, NCC Inset Resources*, National Curriculum Council for Great Britain (1992)

Having pointed out an example of serious weaknesses found in the introduction of variables, an alternative approach to the introduction of the concept of variable will be presented in this thesis.

Conceptual Understanding of Functions Through Graphical Representation

A didactic analysis regarding the learning and teaching of function puts forward the assumption that extra time taken in the early introduction of the function concept will greatly reduce the amount of cognitive obstacles and misconceptions associated with graphical representations that our students presently encounter. Examples of such misconceptions are the considering of a graph as a picture or a map and the conceiving of a graph as a construction composed of discrete points (Leinhardt et al., 1990). It is well-documented that the cognitive obstacle resulting from the first of these misconceptions is evident when a student interprets or constructs a literal picture of that situation (Mevarech, and Kramarsky, 1997). This phenomenon is often witnessed when asking middle school students to describe a situation that would be represented by a graph similar to the one in Fig. 1. Unfortunately, you can always count on at least one student describing the situation as one in which a person is moving up a hill until s/he reaches the top, and then walking on level ground for a while until s/he begins to descend the hill. It is the intention of the teaching experiment in this research project to minimise the frequency of this and similar responses to interpretations or constructions of graphs representing situations. The motion detector lesson in the present study also attempts to alleviate difficulties students have in identifying relationships between variables by having students realise that there is more to a graph than simply a few ordered pairs.

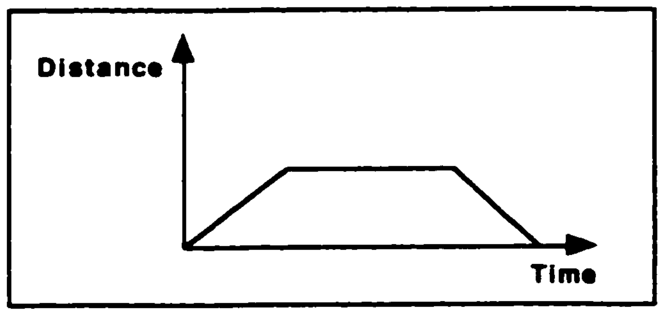

Fig. 1 Students are at times asked to describe situations which would be represented by such a graph.

All of the activities presented in this research project are planned with the social constructivist theory of learning in mind. Derived from constructivism, social constructivism stresses the collaboratory efforts of groups of learners as sources of learning (Boudourides, 1998). This means that students develop their understanding by comparing their perceptions with those of the teacher and those of other students. They can then confirm or adjust their understanding accordingly.

The notion of understanding is often overlooked in mathematics education or ignored. Researchers, philosophers, and psychologists are all quite concerned with what it means to understand. "Understanding" is now in fact an object of study in mathematics education. O'Callaghan (1998) describes a function model consisting of four component competencies: modelling, interpreting, translating, and reifying. The first component in understanding the function concept in this model is in fact *modelling* and can be described as the creation of a mathematical representation (an equation, a table, a graph) from a situation. The *interpretation* of functions in different modes is the second component. The ability to move from one mode of representation to another will be referred to as *translation*, the third component of understanding. The final component in understanding the function concept in this model is *reification*. Reification occurs when one is able to form a mental object from what was initially perceived as a process or procedure. On this conception of function, Sfard (1991) states that:

> ... the ability of seeing a function or a number both as a process and as an object is indispensable for deep understanding of mathematics, whatever the definition of "understanding" is (p. 5).

The motion detector lesson presented in this thesis will attempt to help students acquire the competencies associated with the first two components of the model in understanding the function concept as well as overcoming difficulties or obstacles associated with the concept.

Recent studies report that students have a poor conceptual understanding of functions (Martinez-Cruz, 1998). Conceptual understanding is the acquiring of knowledge that is rich in relationships and equated with connected networks (Hiebert, J. & Carpenter, T.P., 1992). Such reports show that a major cognitive obstacle in student understanding of functions is, in fact, the inability to conceive relationships between variables. New research by Kaldrimidou & Ilkonomou (1998) confirms an earlier claim made by Vinner & Dreyfus (1989) that students have difficulties with, and consequently avoid, graphic representation because they have, in fact, not learned how to use them.

Even in students who were first exposed to functions using graphs, there exists evidence to show that there may still be serious didactic obstacles (Martinez-Cruz, 1998). One of which is caused by the ever so popular vertical line test. Many students end up simply memorising this test to determine whether or not certain graphs represent functions. In applying this rule, they get confused with a horizontal line test. These students know that a line must not intersect a graph at more than one place for that graph to be considered a function. However, they are uncertain if it is a vertical line or a horizontal line or both. Martinez-Cruz (1998) suggests that this confusion with horizontal and vertical lines creates an obstacle in learning the vertical line test. This should not be surprising. Procedural knowledge is often easier to attain but it is often the easiest to forget as well.

Also, as stated in the draft version of the NCTM *Standards 2000* (1998):

> More than any other single factor, teachers influence what mathematics students learn and how well they learn it (p. 30).

The Impact of the Teacher's View

Research indicates that teachers and prospective teachers do not separate their ideas about a particular concept from notions about how to teach that particular concept (Lloyd & Wilson, 1998). It is therefore implied that teacher's ideas about the function concept have a strong impact on their teaching methods. The didactic situations designed by teachers are influenced by their knowledge of the subject matter (Even, 1993). The role of the teacher must not be overlooked when studying the function concept as it plays a significant factor in both the didactic and cognitive analyses regarding the teaching and learning of functions. There are preservice teachers and practising teachers who believe that a mathematics teacher must teach specific rules and procedures in a very structured way, explaining exactly which procedures students are expected to use (Wilson, 1994). Justification given for doing this is to avoid confusing the students.

In research done by Lloyd and Wilson (1998), it was discovered that a teacher's formal definition of a function concept does not always correspond to that teacher's image of the function concept. In fact, in their study, there was a teacher who felt that his formal definition of the function concept (i.e. the Dirichlet-Bourbaki definition, see above, p. 6) was the most appropriate. It was interesting, however, that this teacher continued to rely on his understanding of his informal definition of a function as a relationship and on the practicality of graphical representations when solving problems. This finding suggests the claim by Vinner and Dreyfus (1989), that a person can hold a concept image for a function that does not directly correspond to a formal mathematical definition.

Another significant factor in this present analysis is the fact that teachers all too often present students with easy procedures that overemphasise procedural knowledge, neglecting conceptual knowledge (Even, 1993). Procedures such as the vertical line test are given to students as a rule to follow without any understanding of the rule. Such actions can be responsible for some of the obstacles already described.

Chapter II:

The Methodology of the Research Project

The research methodology that I plan to be guided by in my study is that of teaching experiment. It is one of the few research techniques used to examine teaching from a constructivist perspective (Schatz, Koehler & Grouws, 1992). The teaching experiment is a qualitative research methodology that has as a goal to "catch processes in their development and to determine how interaction can optimally influence these processes" (Kantowski, 1978). According to Kieran (1985) a teaching experiment necessarily involves three steps: the establishment of hypotheses concerning the learning process from preliminary work, the construction of a teaching sequence based on these hypotheses, and the execution of the teaching sequence along with the analysis of its effects. I have chosen the teaching experiment as my research methodology over comparative experiments for very specific reasons.

The objective of comparative experiments is to determine if desired outcomes are caused by specific actions. These comparisons are usually made between two groups, an experimental group treated by a set of actions and a control group receiving no such treatment (Romberg, 1992). Although subjects are often randomly assigned to each group in an attempt to control for internal invalidity, such experiments use both external instruments such as questionnaires, interviews, and tests, and external validation statistically comparing both experimental and control groups. Also, external validation can too often unintentionally be designed to show desired results in such comparative experiments. For example, O'Callaghan (1998) used specific criteria for forming his experimental and control groups. The researcher focused on 3 classes of 40 students. The experimental group was comprised of volunteering students, taught by the researcher. The other two classes, forming the control group (one taught by the researcher and the other by another instructor), were given no such choice. I object to such a research method because often students who volunteer for special projects are those with open minds. Often they are keener and positive students. Therefore, I question the results that state the experimental group ended up enjoying mathematics more that the control group. I feel that the way the students were placed had a lot to do with the results. Most of the results of this experiment were based on the data collected from two sets of interviews with 6 students from these classes. Once again, the students volunteered for these interviews. Out of all the students who volunteered to be part of the experimental group, 6 of them volunteered to be part of the interviews: the keenest of the keen.

It is my opinion that there are too many variables that could lead to invalid results using comparative experiments as displayed in O'Callaghan's (1998) criteria for forming experimental and control groups. It is for reasons such as these that I have chosen to use the teaching experiment as research methodology for this thesis.

The validation of the hypotheses concerned in this research is essentially internal in the sense that it is based on the confrontation of the *a priori* and *a posteriori* analyses, rather than external, based on the statistical comparison of the achievements of experimental and control groups.

Chapter III: Description, Realisation and Analysis of the Research Project

The teaching experiment developed in this thesis has been implemented in one Grade 6 class (pupils 11 - 12 years of age). Participating students were observed during class discussions and their assignments over three sessions. There were 30 students in the Grade 6 class.[1] Also, each session was audio-taped to ensure that no part of any class discussion was missed.

The first session consisted of two activities. Each of these activities, including discussion, took 25 minutes to complete. There was a 15-minute break between activities. A motion detector lesson was undertaken in the second session on the next day and lasted 45 minutes. The two activities of the last session were carried out on the third day and lasted 15 and 25 minutes respectively. A TI-73 graphing calculator overhead set-up and CBR™ motion detector (Appendix CBR), along with a link cable connecting these two tools were used throughout the motion detector lesson. Also, the blackboard and regular chalk was used in class discussions when referring to different aspects of the graphs.

Recall that the following hypotheses were made in the introduction:

[1] One of the students was an integrated student coded as severely learning disabled and functioning at a Grade 4 level in mathematics using the *Mini-Battery of Achievement* (Woodcock, McGraw and Werder, 1994).

Hypothesis 1 Dynamic representation helps students to develop conceptual understanding of the relationship between distance and time in problems of motion.

Hypothesis 2 Dynamic representation facilitates the development of conceptual understanding of independent and dependent variables.

Hypothesis 3 The investigations of the relationship between distance and time in problems of motion help the students better understand other functional relationships between independent and dependent variables and the general mathematical notion of function in learning mathematics in higher grades.

The following are the didactic choices that are pertinent and related to these hypotheses:

(1) Functions as relations (rather than as a mapping, translation, or set).

(2) Introduction of function as relation between distance and time (rather than between two other variables).

(3) The use of motion detector technology in the introduction of the function concept.

Justification for the first two of these choices is largely epistemological. It is stated in the theoretical framework of the research project that the first graphical representations of relationships between two variables represented change as a function of time. Also, as stated in Chapter 1, it was found that research suggesting using time as an independent variable has epistemological value in studying qualitative graphs to represent functions and that developing such qualitative perspectives of graphs is crucial in reducing the cognitive obstacles restricting students' conceptual understanding of functions. The choice to use motion detector technology was made because these devices create dynamic graphing situations which allow students to see and to control physically relationships between variables as they occur.

Overview of the Activities

Below are the specific aims of each activity.

Session 1

Activity 1: Investigating Changing Quantities

In this activity, the students will be able to:

- develop an understanding of the variable as a quantity with varying values;
- develop an understanding of the relationships of dependence and independence that exist between two given variables; and
- express the relationship between variables in a given situation in their own words.

Activity 2: Investigating Variables

In this activity, the students will be able to:

- understand relationships of dependence between two given variables; and
- identify dependent and independent variables in the context of specific situations.

Session 2

Motion Detector Lesson: Are You Up to the Match?

In this motion detector lesson, the students will be able to:

- develop an understanding of the relationship that exists between the x- and y-axes;
- understand the concept of position;
- understand how a change in one quantity affects another quantity;
- develop an understanding of the concept of variable and a notion of dependence between two variables;
- develop an understanding of the notion of function as a relationship; and
- interpret a graphical representation of motion data.

Session 3

Activity 3: Describing Motion

In this activity, the students will be able to:

- interpret a graphical representation of motion data involving a change in speed.

Activity 4: Interpreting Qualitative Graphs

In this activity, the students will be able to:

- interpret qualitative graphs and determine the relationship that exists between the water level and time in each graph; and
- match a functional relationship with its corresponding graph.

Session 1

Description and Analysis of Activity 1:

Investigating Changing Quantities — Appendix S1

In each of the 5 situations presented in this activity (Appendix S1), the result of one quantity depends on changes made to another quantity. The students are given the following example at the beginning of the activity:

Example:
- *number of hours spent babysitting*
- *amount of money earned*

Solution:

The amount of money earned depends on the number of hours spent babysitting.

The students are asked to write a sentence for each of the 5 situations indicating how one quantity is dependent on another quantity. The activity is done individually and later discussed as a class. Also, following this activity, the researcher summarises the discussion and teaches the students about dependent and independent variables. At the end of the activity, the worksheets are collected and further analysed.

Recall that the aim of this activity is for the students to be able to:

- develop an understanding of the variable as a quantity with varying values;

- develop an understanding of the relationships of dependence and independence that exist between two given variables; and
- express the relationship between variables in a given situation in their own words.

The situations that constitute this activity have common mathematical analysis, expected correct behaviours and anticipated difficulties. As a result, I will present an *a priori* analysis for the activity in general.

A good understanding of dependence in general is required for students to write a sentence for each situation indicating how one quantity is dependent on another quantity. The expected correct behaviours would therefore be for student to write such sentences. The anticipated difficulties associated with these situations would be the inability to find a dependent variable, the inability to see the relationship of dependence, a different interpretation of the situation and seeing a different relationship of dependence.

The first and fifth situations presented in this activity are more difficult than the others simply because there are two possible interpretations of the situations. This was done in order to emphasise the importance of interpretation of problem situations and to give students opportunities to interpret situations in more than one way.

For example, the students are given the following two quantities in the first situation.

- number of people watching a movie
- number of available seats at the cinema

The following behaviours would be considered correct:

- The number of people watching a movie depends on the number of available seats at the cinema.
- The number of available seats at the cinema depends on the number of people already watching a movie.

Both interpretations of the situation would be accepted by the teacher and following the activity, discussed with the class.

Activity 1: Investigating Changing Quantities **Appendix S1**

Local Analysis

Problem 1 • **number of people watching a movie**

• **number of available seats at the cinema**

Of the 30 students, 14 chose the *number of people watching a movie* as the dependent quantity and 14 students chose the *number of available seats at the cinema* as the dependent quantity. The remaining 2 students simply transcribed the question instead of answering it.

Upon reading this question, many of the students claimed that either sentence should be acceptable. It was explained by the students that, very often, the number of available seats depends on the number of people already watching a movie. As part of the expected behaviours, it is a relationship of dependence. It is also a relationship that many students have been part of when hoping to get tickets for a show that is about to sell out.

Problem 2 • **wear on bicycle tires**

• **distance travelled**

Of the 30 students, 22 displayed the expected correct behaviour by choosing *wear on bicycle tires* as the dependent quantity and 6 students chose *distance travelled* as the dependent quantity. 2 of the students displaying the expected correct behaviour asked for the meaning of the word wear and once again, the same 2 students from question 1 simply transcribed the question instead of answering it.

Upon reading this question, many of the students claimed that either sentence is correct. Many of the students who wrote down the expected behaviour explained that the distance one could travel could depend on the wear already on bicycle tires. This can also be seen as a relationship of dependence and is therefore accepted.

Problem 3 • **amount of fallen snow**
• **number of hours spent shovelling**

Of the 30 students, 22 displayed the expected correct behaviour by choosing the *number of hours spent shovelling* as the dependent quantity and 7 students chose *amount of fallen snow* as the dependent quantity.

Two of these 7 students interpreted the question differently. They wrote that the amount of fallen snow remaining depends on how many hours were spent shovelling. These statements do show a relationship of dependence. However, the remaining five solutions were rejected. One student did not answer this question.

Problem 4 • **steepness of a hill**
• **force required to push a barrel up the hill**

Of the 30 students, 24 displayed the expected correct behaviour by choosing the *force required to push a barrel up the hill* as the dependent quantity and 5 students chose the *steepness of the hill* as the dependent quantity. One student did not answer this question.

Problem 5
- **number of hours lights are on in a year**
- **number of light bulbs used**

Of the 30 students, 25 displayed the expected correct behaviour by choosing the *number of light bulbs used* as the dependent quantity and 5 students chose the *number of hours lights are on in a year* as the dependent quantity.

Although none of these 5 students volunteered any explanation for their answer, it is possible that some interpreted the situation as follows: The number of hours the light bulbs may be on depends of the number of light bulbs available for the year.

Description and Analysis of Activity 2: Investigating Variables

Appendix S2

In this activity, the student is asked to analyse the 5 different situations in Activity 1 and to identify the dependent variable by placing a check mark in the appropriate area. The students are given the following example at the beginning of the activity:

Example: ___ *number of hours spent babysitting*

_√_ *amount of money earned*

The activity is first done individually and later discussed as a class. Following the activity, the worksheets are collected for further analysis.

It was expected that, in this activity, the students would start developing an understanding of relationships of dependence between two given variables. They would also be able to identify dependent and independent variables in the context of specific situations.

The situations that make up this activity also have common mathematical analysis, expected correct behaviours and anticipated difficulties. As a result, I will present a general *a priori* analysis for the entire activity.

In order to identify the dependent variable in each situation, the students must understand the concept of variable as quantities with changing values. Also, a good understanding of dependence in general is required in order to choose the dependent and independent variable. The expected correct behaviour for each situation is for the students to place a check mark next to the dependent variable. The anticipated difficulties associated with this activity are the same as those anticipated for the first activity. Namely, the inability to determine a dependent variable, the inability to see the relationship of dependence, a different interpretation of the situation and seeing a different relationship of dependence.

In Activity 1, variables were only referred to as changing quantities. Some teaching occurred prior to this activity. The researcher gave the class the definition of a variable as a quantity with changing values. The researcher was then able to have students explain the difference between independent and dependent variables. Therefore the students are now familiar with the term variable.[2]

[2] It should be noted that the integrated student coded as seriously learning disabled was not successful in answering any of the questions of Activity 1.

Activity 2: Investigating Variables **Appendix S2**

Local Analysis

Problem 1 ____ **number of people watching a movie**

____ **number of available seats at the cinema**

Of the 30 students; 12 chose the *number of people watching a movie* as the dependent variable and 8 students chose the *number of available seats at the cinema* as the dependent variable. As described in activity 1, the students explained that the number of available seats often depends on the number of people already watching a movie. Therefore, both of the choices are expected correct behaviours.

Problem 2 ____ **wear on bicycle tires**

____ **distance travelled**

Of the 30 students; 25 displayed the expected correct behaviour by choosing *wear on bicycle tires* as the dependent variable. 5 students chose the *distance travelled* as the dependent variable. Based on the discussion that took place during activity 1, it is possible that some chose *distance travelled* because they interpreted the situation as follows: The distance one could travel depends on the wear already on the bicycle tires. Such an interpretation can also be seen as a relationship of dependence and is therefore accepted.

Problem 3 ____ **amount of fallen snow**

____ **number of hours spent shovelling**

All of the students displayed the expected correct behaviour in this situation by choosing the *number of hours spent shovelling* as the dependent variable.

Problem 4 ____ **steepness of a hill**

____ **force required to push a barrel up the hill**

Of the 30 students; 25 displayed the expected correct behaviour by choosing the *force required to push a barrel up the hill* as the dependent variable. 5 of the students chose the *steepness of a hill* as the dependent variable. Some of these 5 students may still not understand relationships of dependence. They may still confuse the meaning of a dependent variable. Once again, the researcher discussed relationships of dependence with the class. In particular, he discussed dependent and independent variables.

Problem 5 ____ **number of hours lights are on in a year**

____ **number of light bulbs used**

All of the students displayed the expected correct behaviour in this situation by choosing the *number of light bulbs used* as the dependent variable.

Global Analysis of Session 1

Following the analyses of the two activities of Session 1, it is clear that the students have developed an understanding of variables as quantities with varying values. The results obtained from the worksheets as well as through class discussion show that the students were able to understand relationships of dependence and independence that exist between two variables. Also, the students were able to identify dependent and independent variables in specific situations as well as express the relationships between variables in their own words. The integrated student has yet to show any real understanding of relationships of dependence between two variables.

Session 2

Description and Analysis of Motion Detector Lesson:

Are You Up to the Match? Appendix S3

In this motion detector lesson, the students are asked to answer a series of 14 questions as well as make demonstrations and/or observations concerning movement in front of a motion detector as a function of time. Sufficient time was given for students to answer each question. Also, class discussicns regarding the relationship between distance and time took place following the generation of a graph resulting from a student volunteer's movement in front of a motion detector. At the end of the lesson, the worksheets were collected and further analysed.

Recall that the aim of this motion detector lesson is for the students to be able to:

- develop an understanding of the relationship that exists between the x- and y-axes;
- understand the concept of position;
- understand how a change in one quantity affects another quantity;
- develop an understanding of the concept of variable and a notion of dependence between two variables;
- develop a notion of function as a relationship; and
- interpret a graphical representation of motion data.

The questions that constitute this motion detector lesson do not have common mathematical analysis, expected correct behaviours, or anticipated difficulties. As a result, I will give a detailed *a priori* analysis for each question of the motion detector lesson.

Motion Detector Lesson: Are You Up to the Match? **Appendix S3**

Problem 1 **In the viewscreen window provided below, reproduce the graph that is on the overhead screen.**

The students are asked to reproduce a graph (Fig.2) that is projected onto an overhead screen in the viewscreen window provided to them in question 1 (Appendix S3). In order to answer the question correctly, the students need to know the important components of a graph (i.e. labels, identification of units, and graduations on the axes).

The expected correct behaviours would be for students to display a graph similar to the one in Fig. 2 in the space provided to them on their answer sheets.

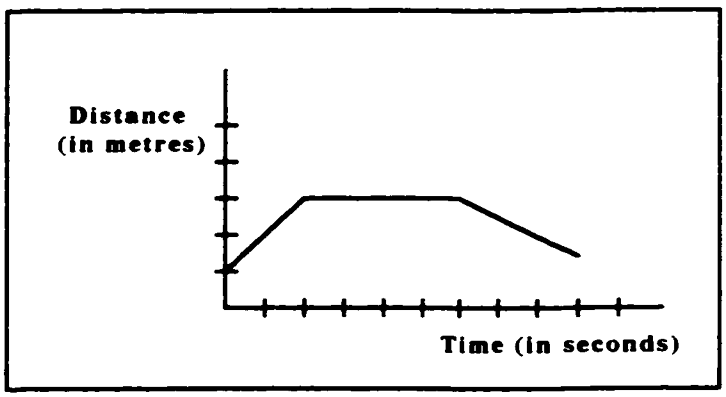

Fig. 2

The anticipated difficulties associated with this question would be for the students to omit labelling the axes; to not place importance on the units on both axes; to neglect to include graduation on either axis; and to ignore the values on the x- and y-axes when sketching their graphs. These difficulties are anticipated because the only previous exposure that these students had with graphs this year concerned the stock exchange in the Challenging Mathematics programme (*Challenging Mathematics 6*, Mondia Éditeurs, 1995). In fact, the first graph presented in their textbook has neither axes labelled nor any graduation along the axes (Fig. 3).

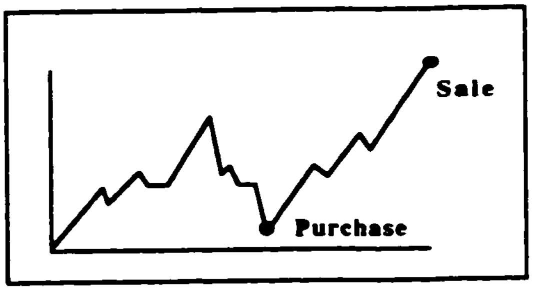

Fig. 3

Local Analysis

Of the 30 students, only 10 students drew a good sketch of the graph along with all the important features and information. Of the remaining 20 students, many were missing important features such as the labels on the axes. 16 students omitted to show graduations along the x-axis, while only 10 students omitted showing graduations along the y-axis.

It is interesting to note that there were more students (38%) omitting to show graduations along the x-axis than on the y-axis. Perhaps the greater importance allocated to the y-axis is a result of their previous exercises with graphs. On page 56 of their textbook, the students are told that investors buy shares when the price is low and sell when the price is high and are given the graph in Fig. 3. It is perhaps this focus on vertical movement as opposed to horizontal movement that leads to a greater number of students omitting to show graduation along the x-axis than on the y-axis.

Problem 2 Explain what you think is happening in the above graph.

This question is also referring to the graph displayed in Fig. 2 (question 1). The students are asked to interpret the graph by writing a brief description in a space provided to them (Appendix S3). The students must understand the concept of position and the relationship that exists between the x- and y-axes in order to answer this question correctly. They must also understand how movement in front of the motion detector affects the graph.

Some students may explain that, at first, the distance away from the motion detector increases. After about 3 seconds, there is no change in distance for about 3 1/2 seconds, at which time the distance away from the motion detector decreases. This is the expected correct behaviour.

Many wrong answers were expected here. Many of the students will not yet understand the relationship between the distance away from the motion detector and the time elapsed. Students may not understand the relationship that exists between the x- and y-axes. It is anticipated that some students may disregard the units on both axes and resort back to situations that are familiar to them (i.e. the stock market as described in *a priori* analysis for question 1). Therefore it should not be surprising to find that some students resort back to situations involving stocks when interpreting graphs and that many students view graphs as literal pictures.

Local Analysis

Of the 30 students, only 6 students displayed the expected correct behaviour by describing the graph as a relationship between distance and time. 3 other students explained that the graph was representing movement in general. 12 students described the graph as representing the value of stocks in a stock exchange (literal picture) and 9 students described the graph as a literal picture of other situations. For example: walking up a hill then walking on level ground before coming down the hill.

These results were expected. However, it is interesting to note that no students mentioned the terms "distance" and "time" in their explanations. It is clear from these results that the majority of the students do not understand the relationship between the x- and y-axes.

The results show that 60% of the students described the graph as a literal picture of a situation, completely ignoring the units on both axes, as was anticipated.

Following a brief class discussion on several responses to their question, a volunteer student was asked to come to the front of the class and try reproducing the graph while walking in front of the CBR™ (Appendix T3). The students were given very little information as to how the CBR™ works. They were told that a motion detector works by sending out an ultrasound pulse and then measuring how long it takes for the pulse to return after bouncing off an object. The volunteer student (and the rest of the class) can view the graph representing his or her motion projected on a screen as he or she is moving in front of the CBR™. At first some students tried to walk in many directions in front of the CBR™. However, it was expected that after a few attempts the class would realise that they must walk in-line with the CBR™.

Problem 3 Were you or your classmates able to reproduce the graph while moving in front of the CBR™? If not, describe the way you would move in front of the CBR™ in order to produce the appropriate graph.

The students must understand the concept of position and the relationship that exists between the x- and y-axes in order to answer this question correctly. They must also understand how movement in front of the motion detector affects the graph.

Most likely, the answer will be "no" for this question. Students may give the following responses:

- "I would start farther from / closer to the CBR™."
- "I would have started off walking away from / towards the CBR™."

Students will also suggest that they would go faster or slower on different parts of the graph. They should now be aware that time in seconds is represented by the x-axis and that distance away from the motion detector is represented by the y-axis. Students who previously ignored the units, resorting back to situations that were familiar to them (eg. stocks) will now be able to understand the context of the situation.

It is anticipated that some students will still not yet understand the relationship that exists between the two axes. Therefore, these students will not be able to describe how one would have to walk in front of the motion detector to reproduce the graph. Another anticipated difficulty associated with this question would be considering the graph as a picture out of the range of the motion detector. This would indicate a lack of understanding of how the motion detector works as well as a cognitive obstacle created by seeing the graph as a literal picture.

Note: Following a class discussion on several responses to this question, the students were asked to think about where one should be placed in front of the motion detector at the beginning of the exercise and at the end of the exercise. Another volunteer student was then asked to try to reproduce the same graph and the students were once again asked to think about their responses to question 3. (Appendix T3)

Students can be helped to overcome the difficulty created by seeing the graph as a literal picture by analysing their own involvement in front of the motion detector.

Local Analysis

2 students were able to describe the movement as a function of time, eg. "distance away from the CBR™ increases. After about 3 seconds there is no change in the distance for about 3 1/2 seconds at which time the distance away from the motion detector decreases." In spite of the awkwardness in their expression, it is clear that they are beginning to understand.

20 other students were able to describe the above situation without being as specific with respect to time. Of the remaining 8 students, 5 students incorporated the notions of distance and time in their solutions. However, they did so incorrectly. For example, their starting and ending positions were wrong. 2 of the 8 students were only able to identify where they would begin and one of the 8 students indicated a wrong starting position.

It is interesting to note that 20 students were now able to describe, although in a very general way, the situation represented by the graph. An example of such a situation is as follows: "I would start close to the CBR™, move away, stop for a while and then come back towards the CBR™."

Problem 4 Sketch the graph that is on the overhead screen in the viewscreen window below.

It is expected that more of the students will understand the context of the situation modelled by the graph and consequenty accord more importance to the units on the axes. More of the sketches will include the units.

The anticipated difficulties associated with this question would be for the students to omit labelling the axes; to not place importance on the units on both axes; and to neglect including graduation on either axis when sketching their graphs.

The students were asked to reproduce a graph (Fig. 4) that is projected onto an overhead screen in the viewscreen provided to them in question 4 (Appendix S3). In order to answer the question correctly the students needed to know the important components of a graph.

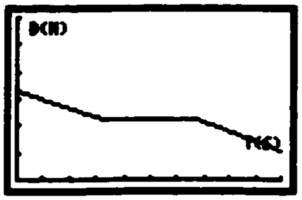

Fig. 4

The expected correct behaviour would be for students to display a graph similar to the one in Fig. 5 in the space provided to them on their answer sheets.

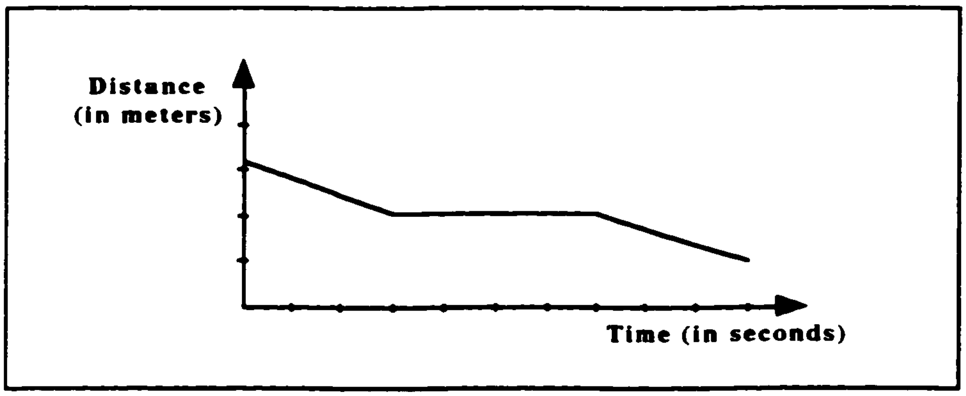

Fig. 5

Local Analysis

Of the 30 students, 15 drew a good sketch of the graph along with all of the important features and information. Of the remaining 15 students, 9 students omitted to show divisions along both axes. 4 additional students omitted to show divisions along the x-axis only.

More students sketched a graph containing all the important information. There is still a greater emphasis on the y-axis. Perhaps this is a result of their previous exposure to graphs dealing with the stock exchange. This is still evident in the fact that a greater number of students omit to show divisions along the x-axis than along the y-axis.

Problem 5 **How would you have to move in front of the CBR™ in order to reproduce this graph? Think about how far you should be away from the CBR™ just before starting.**

The students must understand the concept of position and the relationship that exists between the x- and y-axes in order to answer this question correctly. They must also understand how movement in front of the motion detector affects the graph.

Students should now begin to understand the relationship that exists between the distance away from the motion detector and the time elapsed. Students should also know where to stand at the beginning of the exercise and where to be standing at the end. Other students will focus more on the relationship between distance and time.

It is anticipated that some students will still not yet understand the relationship that exists between the two axes. Some students may not have placed importance on a starting and finishing point. Therefore they would begin their graphs at any location along the y-axis.

Local Analysis

Of the 30 students, 2 were able to describe the movement as a function of time, eg. "I would start 3 metres away from the CBR™ and start walking towards it at a constant rate for about (a given number) seconds. I would then stop and remain still for (a given number) seconds and then start walking towards the CBR™ again at a slightly faster rate." Both of these students mentioned speed in their answers. For example, they explained that they would begin walking slowly towards the CBR™. The remaining 28 students were able to describe the above situation without being as specific with respect to time. 2 of these students also mentioned speed in their answers.

All of these results were anticipated. In fact, the entire class was now able to describe, in a general way, a situation involving a relationship between distance and time represented by a graph. There were no students who described the graph as a literal picture of a situation. It is interesting to note that the student who is supposedly functioning at a Grade 4 level is also beginning to show an understanding of the relationship between distance and time. This became evident in the student's oral response to question 5. The student was able to describe the movement in front of the CBR™ as a function of time (in a general way).

Following a discussion of the class's explanations for question 5, a volunteer student is asked to try to reproduce the graph while walking in front of the motion detector (Appendix T3). With the exception of the first section, the graph was reproduced well.

Problem 6 Was it a perfect match? If not, describe how you would improve each segment of the graph.

This question refers to the graph presented in the previous question (Fig. 3) and the graph generated by a volunteer student attempting to match it.

The students must understand the concept of position and the relationship that exists between the x- and y-axes in order to answer this question correctly. They must also understand how movement in front of the motion detector affects the graph.

The match will probably be much closer now. However, it will not be perfect. It is expected that students will make suggestions similar to those in question 3. They may also suggest other ways to help them better represent the graphs, such as using a measuring tape / meter stick and masking tape to mark off specific distances away from the motion detector. This will help them know where to change directions and/or speed.

Some students may not understand completely the relationship between the axes. Therefore, they may be confused as to whether they should be changing direction or speed. In a class discussion prior to the beginning of the match, several students suggested using metre sticks to find the actual starting point.

Local Analysis

Of the 30 students, 28 now incorporated speed in their solution. They explained that they would walk slowly in the first section and faster in the last section. 2 students began to describe the movement as a function of time, eg. "There was no movement for (a given number) seconds." The majority of the students focused on the first section of the graph because it was the least well represented by the movement of the volunteer student.

Note: Following a discussion of the students' explanations for question 6, another volunteer was asked to try to reproduce the same graph while walking in front of the motion detector. The students were once again asked to reflect on question 6 during a class discussion (Appendix T3). The graph was well reproduced by the volunteer student and the class was able to explain how the match could have been even better (eg. "He should have started a 1/2 metre closer.").

Problem 7 Make up a story to describe an event that may be represented by the following graph.

The students are asked to describe an event that may be represented by the graph displayed in Fig. 6.

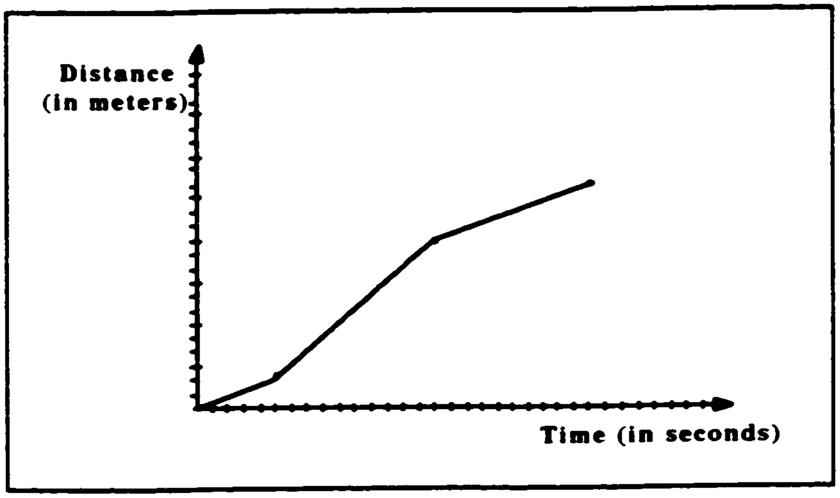

Fig. 6

A good understanding of the following is required to answer these questions:

- the relationship that exists between the x-axis (time in seconds) and the y-axis (distance away from the motion detector in metres)
- the concept of variable and a notion of dependence between two variables
- interpretation of graphical representation of motion data

Many possible stories or events should be expected. For example, Sue was slowly walking towards the bus. She started to run as she noticed the bus was leaving. After running for ten seconds, Sue returned to walking, realising that she now had to walk to school.

It is anticipated that there may be some students who do not see the connection between this question and the previous exercises in front of the motion detector. Other students may once again consider the graph as a literal picture as a result of their prior experience with graphs in their textbook as explained in the anticipated difficulties of question 1. It may therefore be necessary to encourage these students to think about this situation in terms of distance away from the motion detector.

Local Analysis

Of the 30 students, 17 made up stories displaying a good understanding of the relationship between distance and time represented on this graph, 5 students considered the graph as a literal picture representing their story, and 8 students completely disregarded the units and resorted back to describing situations involving stocks in a stock exchange.

Of the 28 students who were able to describe situations represented by graphs involving movement in front of a motion detector, 17 students (61%) were able to transfer that knowledge to this question. The fact that this question was a little different from the previous ones caused some (5) students to resort back to considering a graph as a literal picture. Other students (8) who were uncertain as to how to answer this question resorted back to situations that they were once comfortable with involving the stock market.

Problem 8 **Describe how you would walk in front of the CBR™ if you were to try producing a graph resembling a mountain peak. Think about this carefully. Your teacher might actually ask you to try it.**

The students must understand the relationship that exists between the distance away from the motion detector and the time elapsed in order to answer this question. They must also be able to represent mathematical relationships using graphs.

Expected correct behaviours would be for students to explain that they would start close to the motion detector and walk quickly away from the motion detector for a few meters and then immediately return to where they started.

The anticipated difficulties associated with this questions would be students starting at the wrong place (far away from the motion detector).

Local Analysis

Of the 30 students, 28 described the initial position, the final position and the movement correctly. 2 students incorrectly described the initial position. These 2 students described movement that would produce a graph resembling a valley. Perhaps they were not able to visualise what should be happening.

Note: Following a discussion on question 8, a volunteer student was asked to reproduce a graph resembling a mountain peak while walking in front of the motion detector (Appendix T3). The student was successful.

Problem 9 Describe how you would walk in front of the CBR™ if you were to try reproducing a graph resembling a valley.

The students must understand the relationship that exists between the distance away from the motion detector and the time elapsed in order to answer this question. The students must also be able to represent mathematical relationships using graphs.

The expected correct behaviours would be for the students to explain that they would start away from the motion detector, walk quickly towards the motion detector for a few meters and then immediately return to where they started.

The anticipated difficulties associated with this question would be students starting at the wrong place (very close to the motion detector).

Local Analysis

All 30 students described the initial and final positions correctly and 28 of these students described the movement correctly.

It should be noted that the integrated student appeared to be getting excited by the fact that he was beginning to understand. This was apparent by his eagerness to answer questions as well as by the many new questions he posed (eg. "How far do you have to walk away before the graph goes off the screen?").

Note: Following a discussion on question 9, a volunteer student was asked to reproduce a graph resembling a valley while walking in front of the motion detector (Appendix T3). Once again the student was successful.

Problem 10 **Think about how you would walk in front of the CBR™ if you were to produce a graph resembling the first letter of your name. Is it possible? If so, describe how you would move in front of the CBR™ and draw a sketch of the graph you would expect to see. If not, explain why it is not possible.**

The students must understand the relationship that exists between the distance away from the motion detector and the time elapsed in order to answer this question. They must also be able to represent mathematical relationships using graphs. The students must also have an understanding of the concept of variable and a notion of dependence between two variables. In order to answer this question correctly, the students will have developed an understanding of the concept of function as a relationship between distance and time.

Students who understand the relationship between the distance away from the motion detector and time elapsed will successfully answer this question.

An anticipated difficulty associated with this question is that motivated students may want to try at all costs. Also, some students may not completely understand the relationship between the distance away from the motion detector and the time elapsed.

Local Analysis

Of the 30 students:

- 7 had first letters that could be reproduced by motion in front of a motion detector (W, M, N, V).
- 23 had first letters that could not be reproduced by motion in front of a motion detector.
- 6 of the 7 students with a first letter being reproducible described the movement correctly.
- 1 of the 7 students with a first letter being reproducible described the movement incorrectly.
- 4 of the 23 students with first letters that could not be reproduced attempted to describe movement.
- 11 of the 23 students with first letters that could not be reproduced correctly explained why it was not possible (eg. time cannot go backwards, or you can't be in two places at once.)

- 4 of the 23 students with first letters that could not be reproduced correctly did not answer this question.
- 4 of the 23 students whose first names began with a J explained how they would reproduce the symbol J. They altered the symbol J to , which could be reproduced by motion in front of a motion detector.

Note: Before any discussion of the students' responses, a volunteer student whose first initial can be produced as a relationship between distance and time (eg. Nancy, Mike or Victoria) was asked to reproduce a graph resembling the first letter of his or her name. Another such volunteer was asked to reproduce the first letter of his or her name. Afterwards, a volunteer student whose first initial cannot be produced as a functional relationship between distance and time was asked to use the motion detector to produce a graph resembling the first letter of his or her first name (Appendix T3). The volunteer student attempted to reproduce the letter **P**. By altering the slant of the letter, the student was able to reproduce the first section. However, the student soon realized that he was unable to complete the letter no matter how he moved. Another student in the class explained that time could not go backwards.

Problem 11 Describe the graph that you would expect to see if you were to stand in front of the CBR™ without moving.

Understanding of relationships that exist between distance from the motion detector and time elapsed is required to answer this question. The realisation of time as the independent variable is also crucial here.

It is expected that students would say one or all of the following:

- "You would see a horizontal line."
- "The CBR™ would record no change in the distance away from the CBR™ for every second elapsed."
- "For each second elapsed, I would be at exactly the same place."

As anticipated difficulties, some students may explain that there would be a blank graph or just one point. This would show a lack of understanding of time as an independent variable.

Local Analysis

Of the 30 students, 26 answered the question correctly, 2 students wrote that there would only be one point on the graph and 2 students wrote that the graph would appear empty. 87% of the students have come to the realisation that time is an independent variable in this case. They also have an understanding of how no movement in front of a motion detector would be represented graphically.

Problem 12 This problem is composed of 5 parts (a, b, c, d and e). All five parts of this question refer to the graph displayed in Fig. 7.

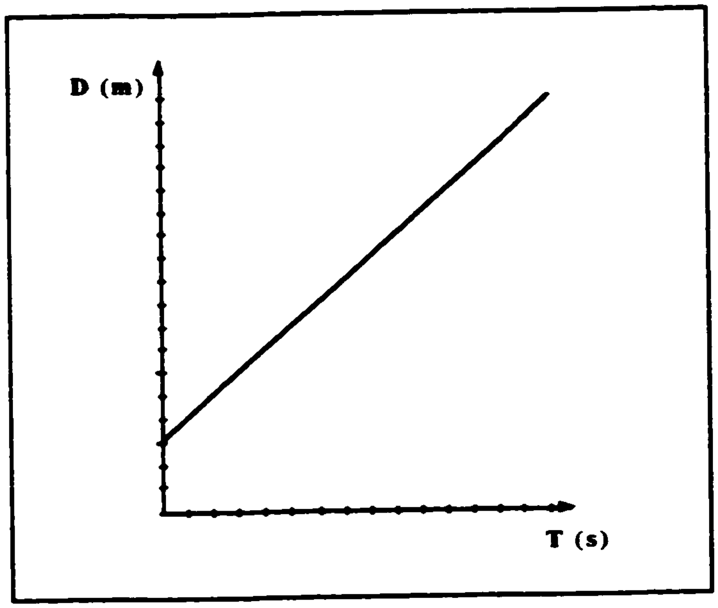

Fig. 7

An understanding of relationships that exist between distance from the motion detector and the time elapsed is required to answer all parts of this question. The realisation of time as the independent variable is also essential here. Finally, in order to answer all parts of this question correctly, interpretation of graphical representation of motion data is required.

Problem 12 (a) Describe how you would walk in front of the CBR™ in order to reproduce the above graph.

It is expected that students will explain that they would start 3 metres in front of the motion detector and walk away at a steady rate. It is also expected that most students will realise that the speed is constant here.

It is anticipated that some students will not pay attention to the fact that the starting point is 3 metres away from the motion detector.

Local Analysis

Of the 30 students, 10 displayed the expected behaviour, which was to explain that they would start 3 metres in front of the motion detector and walk away at a steady rate. 17 students explained that the graph represented the motion of someone walking at a steady rate away from the motion detector without mentioning the starting point. Only 1 student described the motion as someone walking towards the motion detector. Almost all students are showing an understanding of the relationships that exist between the motion detector and the time elapsed.

Problem 12 (b) What do you notice about the distance away from the CBR™ as time elapses?

The expected correct behaviour is that the students will notice that the distance away from the motion detector increases as the time elapses.

It is expected that some students will not understand this question because they will feel it is too vague.

Local Analysis

Of the 30 students, 27 were able to answer this question correctly. 3 students did not answer the question. Perhaps they felt the question was too vague.

Problem 12 (c) What is the maximum distance from the CBR™ after 15 seconds?

Approximately 18 metres would be considered as an acceptable solution. However, it is anticipated that some students may have difficulty interpreting such specific information from a graph because it is their first experience with such a detailed question.

Local Analysis

Of the 30 students, 22 had solutions between 17 and 19 metres. 5 students had solutions between 16 and 20 metres. The same 3 students who did not answer part (b) did not answer the question.

Problem 12 (d) **How far did the person get after 5 seconds? 10 seconds?**

An acceptable solution would be approximately 8 metres and 13 metres respectively.

Some students may still have difficulty interpreting such specific information from a graph.

Local Analysis

Of the 30 students, 10 had solutions between 7 and 9 metres for the question at 5 seconds and between 12 and 14 metres at 10 seconds. 7 students had solutions between 6 and 10 metres for the question at 5 seconds and between 11 and 15 metres at 10 seconds. 4 students had solutions between 6 and 10 metres for the question at 5 seconds and between 11 and 20 metres at 10 seconds. 9 students did not answer this question. As anticipated, some students may still have difficulty interpreting such specific information from a graph.

Problem 12 (e) **If the CBR™ had continued collecting information for another 15 seconds, how far do you think the person would have gone? Explain your answer.**

The acceptable solution would be approximately 33 metres. (It is expected that the students will have different responses.)

An anticipated difficulty is that some students may forget about the context of this situation and ignore the fact that the starting point is 3 metres away from the motion detector and give the following explanation: "If after 15 seconds, I am 18 meters away from the CBR™, then after 15 more seconds I would be 18 more meters away from the CBR™ (18 + 18 = 36)." Therefore it is expected that some students may have difficulties making these kinds of predictions. The actual answer of 33 metres is not important here. The goal is to have the students begin to realise that the distance away from the motion detector is dependent on the time elapsed. (Appendix 3T)

Local Analysis

Of the 30 students, no student had solutions between 32 - 34 metres. Of the 15 students who explained their answers, 4 obtained their solution by doubling the value they put down for 12 (c). This was anticipated in the a priori analysis.

A greater number of students had difficulties with question 12 than any other question. This is evident by the fact that so many parts went unanswered. Parts (d) and (e) received no explanations from 9 and 15 students respectively.

Perhaps the graph itself was not clear enough (eg. more spacing between units). Also, question 12 (e) may have been premature for this lesson. The students have not yet been asked to make these kinds of predictions in their mathematics education.

Problem 13

This problem is composed of 4 parts (a, b, c and d). All 4 parts of this question refer to the graph displayed in Fig. 8.

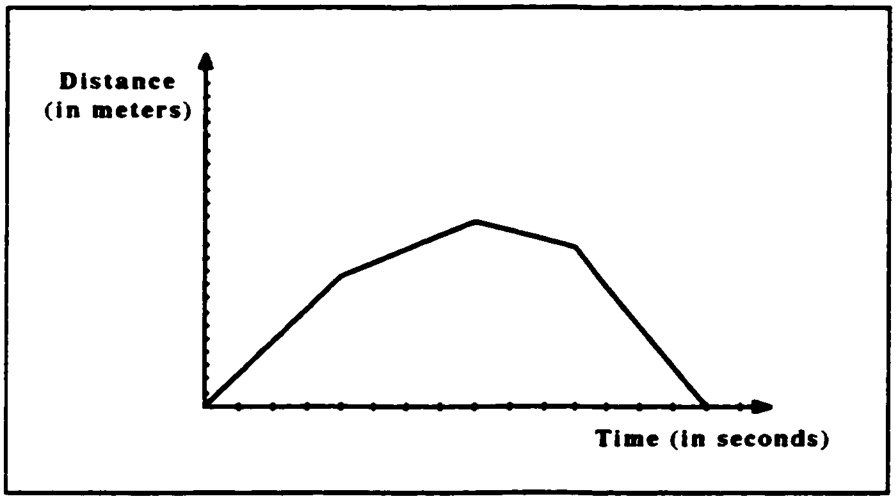

Fig. 8

An understanding of relationships that exist between distance from the motion detector and the time elapsed is required to answer the 4 parts of this question. The realisation of time as the independent variable is also essential and the ability to interpret graphical representation of motion data is necessary to answer this question.

Problem 13 (a) How is the distance away from the CBR™ changing as time elapses?

An example of an expected correct behaviour is the following: "At the beginning, the distance away from the CBR™ increases at a fast rate. After 4 seconds, the distance away from the CBR™ continues to increase but at a slower rate until 8 seconds have elapsed. After 8 seconds, the distance away from the CBR™ decreases at a slow rate until 11 seconds have elapsed, where the distance away from the CBR™ decreases at a quicker rate." It is also expected that some students will give less accurate explanations.

It is anticipated that some students may choose to ignore the different rates of change and instead only focus on movement towards and away from the motion detector.

Local Analysis

Of the 30 students, only 2 were able to describe the motion represented by the graph correctly and were specific with regards to time. 25 students were able to describe correctly the motion represented by the graph, but omitted to mention exact times. 3 students left it blank.

Problem 13 (b) How is the time changing as the distance away from the CBR™ changes?

It is expected that the students will explain that the time is not affected by the changes in the distance away from the motion detector. However, it is also expected that some students will not understand this question because they will feel it is too vague.

Local Analysis

Of the 30 students, 14 displayed the expected behaviour, which was to notice that time is not affected by the changes in the distance away from the motion detector. 11 students explained that they did not understand the question and 5 students left the question blank.

The results of this question were anticipated. However, after a class discussion, it became clear that the students understood the independent role of time. Some even suggested that they did not find it was an appropriate question because it was "obvious." Too obvious to be an answer.

Problem 13 (c) **Is it possible to be at a specific distance away from the CBR™ at more than one time over an interval of time? Explain your answer.**

An example of expected correct behaviour is the following: "Yes, you could be 2 meters away from the CBR™ as you begin walking away for a certain time and then you could return to 2 meters away from the CBR™. You could therefore be at a specific distance away from the CBR™ at more than one time."

The anticipated difficulties associated with this question would be that some students may feel it is too complex. Further assistance may be required by the researcher, such as suggesting the students focus on individual sections of the graph at first. This may reduce the complexity of the question. It should be anticipated that some students may not be familiar with the term "interval." If this is the case, the researcher would explain what is meant by "interval of time" (eg. a specific period of time).

Further explanation of this question may be required. It may be useful to refer back to the original graph in #13 a). It may also be helpful to make a graph similar in shape to the one in #13 a) while walking in front of the motion detector.

Local Analysis

Of the 30 students, 12 answered this question correctly. An example of a correct response is: "Yes, you could be 2 meters away from the CBR™ as you begin walking away for a certain time and then you could return to 2 meters away from the CBR™. You could therefore be at a specific distance away from the CBR™ at more than one time." 5 students answered this question incorrectly and many students asked for clarification of the term "interval."

The researcher encouraged the students to refer back to 13 a) in considering this question to help them realise that it is possible to be at a specific distance away from the motion detector at more than one time over an interval of time.

The fact that so many students (13) left this question blank implied that they were not sure of what the question was asking or that they did not know the answer. A demonstration in front of the motion detector resulted in the clarification of the question for many students. Some students explained that they could be at the same distance away from the motion detector many times over an interval of time.

Problem 13 (d) **Is it possible to be more than one distance away from the CBR™ at a specific time? Explain your answer.**

An example of an expected correct behaviour is the following: "No, one cannot be in two places at once. If I am 4 meters away from the CBR™, I cannot also, at the same time, be 6 meters away from the CBR™."

The anticipated difficulties for part (d) are the same as those outlined in part (c).

Local Analysis

Of the 30 students, 23 answered this question correctly. 2 students answered this question incorrectly and 5 students left the question blank.

During the class discussion, one of the students who answered this question correctly suggested that perhaps the question could have been rephrased as follows: "Is it possible to be at two different places at the same exact time?" This simplification of the question clarified it for those students who answered this question incorrectly or left the question blank.

Problem 14

This problem is composed of 3 parts (a, b and c). The 3 parts of this question refer to graphs generated by motion detectors in general.

In order to answer these questions intelligently the students must understand the concept of variable as a quantity with changing values. Also a good understanding of dependence is required in order to identify the dependent and independent variable.

Problem 14 (a) **Identify the two variables that are present in graphs produced by the CBR™.**

The expected correct answer is "distance and time."

It should be anticipated that some students may have forgotten what variables are.

Note: The researcher may choose to remind students of activities in Session 1, where one of the objectives was for the students to develop an understanding of the variable as a quantity with varying values.

Local Analysis

Of the 30 students, 21 were able to answer the question correctly. The remaining 9 students were able to identify correctly only one of the variables (distance). Among the 9 students, 5 included another factor affecting the graph produced by movement in front of a motion detector (speed).

There was a class discussion following this question to ensure what was meant by variable. This was done because it was discovered that some students considered speed as one of the variables of this situation, which is not the independent or dependent variable.

Problem 14 (b) Describe the relationship that exists between these two variables.

The expected correct behaviour is for the students to explain that the distance away from the CBR™ depends on how much time has gone by.

It should also be anticipated that some students may have forgotten what is meant by relationships of dependence. Once again, the researcher may choose to remind students of activities in Session 1. Another objective of the activities in Session 1 was for the students to develop an understanding of the relationships of dependence and independence that exist between two given variables.

Local Analysis

Of the 30 students, 28 displayed the expected behaviour by explaining that the distance away from the motion detector depends on how much time has gone by, and 2 students left the question blank. These 2 students forgot what was meant by relationship.

Problem 14 (c) Identify the dependent and independent variables in graphs produced by the CBR™.

The expected correct behaviours are for the students to identify the dependent variable as the distance away from the CBR™ and the independent variable as time.

Some students may confuse dependent with independent variables. However, very few difficulties are anticipated here. Following this motion detector lesson and the other two previous activities, it is expected that the students will have developed an understanding of the concept of variable and a notion of dependence between two variables.

Local Analysis

Of the 30 students, 28 displayed the expected behaviour. 2 students identified the wrong variables as dependent and independent.

Note: The integrated student answered correctly each part of question 14 and is therefore showing signs of understanding the relationship of dependence that exists between distance away from the motion detector and time elapsed.

Global Analysis of Session 2

The two hypotheses (H1 and H2) were made prior to the implementation of the motion detector lesson.

Hypothesis 1 Dynamic representation helps students to develop conceptual understanding of the relationship between distance and time in problems of motion.

Hypothesis 2 Dynamic representation facilitates the development of conceptual understanding of independent and dependent variables.

Based on the analyses of each question and the results, both of these hypotheses were shown to be true. The aim of this motion detector lesson described in the introduction of Session 2 (p. 43) had to be met in order for these hypotheses to be true. The motion detector lesson was successful in developing the ability to interpret a graphical representation of motion data in Grade 6 students.

In the first question (question 2) that asked them to describe motion represented by a specific graph, only 20% of the students were able to give an appropriate description. A few students then had the opportunity to walk in front of the motion detector while simultaneously viewing the results of their actions graphically. At the same time, the other students in the class had the opportunity to observe these students and simultaneously view the results of their actions. As a result, the second time the students were asked to describe motion represented by a specific graph (question 5), 93% of the students were able to give an appropriate description included as an expected behaviour.

Following the observations made by the students after one dynamic representation of motion data (question 3), one-third indicated where they would begin in relation to the motion detector when interpreting a graph. By question 5, after only one other dynamic representation of motion data, the majority (93%) of the students included a starting position in their interpretation of the graph. These students showed an understanding of the significance of position.

The fact that all of the students were able to describe, in a general way, a situation involving a relationship between distance and time represented by a graph (question 5), indicated that students were successful in developing an understanding of the relationship that exists between the x- and y-axes. It is also very interesting to note that no students described the graph as a literal picture of a situation. The cognitive obstacles created by this misconception are reduced very quickly after dynamic representation of motion data. The immediate feedback provided by the motion detector to the students is very helpful in understanding the relationship between distance and time.

It is clear from the analyses that much of the motion detector lesson was responsible for helping the student develop an understanding of the notion of function as a relationship. However, the results obtained from question 10 demonstrated this the best.

In question 10, most (86%) of the students who had letters that could be reproduced by motion in front of a motion detector described the movement as a relationship between distance and time correctly. Two-thirds of the students were able to explain why it was not possible to produce a graph resembling the first letter of their names or gave a good explanation of how to produce a graph resembling a modified version of the first letter of their name. The most common points made by these students in their explanations were that one cannot go back in time and that it is impossible to be in two places at once. Four good problem-solvers with first names beginning with the letter J were able to modify their letter so that it could be produced by movement in front of the motion detector (). These results show that the students have acquired an understanding of function as a relationship of dependence between distance and time. Many of the students now understand that time is the independent variable. Such a conceptual understanding acquired in realising why or why not such a graph could be produced is more important than the memorisation of any vertical-line test that is so easily forgotten (Martinez-Cruz, 1998). A greater number of students were able to understand these relationships by the end of the motion detector lesson. It should also be noted that the integrated student coded as functioning at Grade 4 level also displayed a good understanding of the functional relationship between distance and time by answering many of the questions presented in the motion detector lesson correctly.

The questions in the motion detector lesson as well as the activities in Session 1 were successful in helping the students understand how a change in one quantity affects another quantity and helping them develop an understanding of the concept of variable and a notion of dependence. The results obtained in question 14 of this motion detector lesson revealed that almost all of the students (93%) had a good understanding of the concept of variable and a notion of dependence between two variables.

Session 3

Recall that, in Session 2, the graphical representation of motion data was predominantly linear in nature. The majority of the activities presented in Session 2 required the students to walk at constant speeds over specific intervals of time. In the first activity of Session 3 (Activity 3) the students will have to interpret graphical representations of motion data involving a change in speed.

Description and Analysis of Activity 3:

Describing Motion Appendix S4

In this activity, consisting of three parts, (a), (b) and (c), the students were asked to describe two graphs as relationships between distance and time. The activity was done individually and later discussed in class. At the end of the activity, the worksheets were collected and further analysed.

Recall that the aim of this activity is for the students to be able to interpret a graphical representation of motion data involving a change in speed.

A good understanding of the following is required to answer these questions:

- the relationship that exists between the x-axis (time in seconds) and the y-axis (distance away from the motion detector in metres)
- the concept of variable and a notion of dependence between two variables
- interpretation of graphical representation of motion data

How would you move in front of the motion detector in order to reproduce the following graphs?

(a) The students are asked to discuss how they should move in front of a motion detector in order to reproduce the graph in Fig. 9.

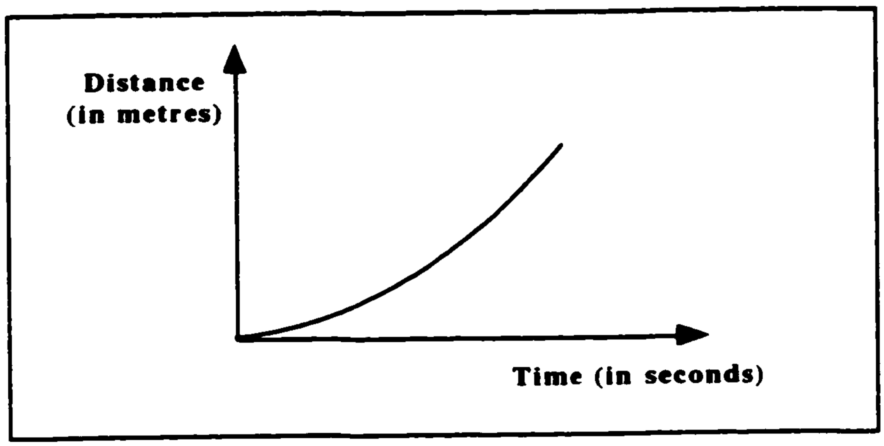

Fig. 9

The expected correct behaviour is for the students to explain that, at first, they are moving slowly away from the motion detector and then they begin to walk away faster and faster.

The anticipated difficulties associated with this question would be that the students may confuse the change in speeds. For example, they may suggest to start quickly and then slow down. Others may neglect to mention any change in speed.

Local Analysis

Of the 30 students, 17 displayed the expected behaviour by explaining that they would start by moving slowly away from the motion detector and then begin to walk away faster and faster. Of the remaining 13 students, 5 students explained that they would first walk away quickly from the motion detector before slowing down, and 8 students described the movement but did not mention any change in speed.

This question was difficult because until this point, all of the graphs considered in Sessions 1 and 2 were linear in nature. In fact, this was anticipated in the *a priori* analysis.

(b) The students are asked to discuss how they should move in front of a motion detector in order to reproduce the graph in Fig. 10.

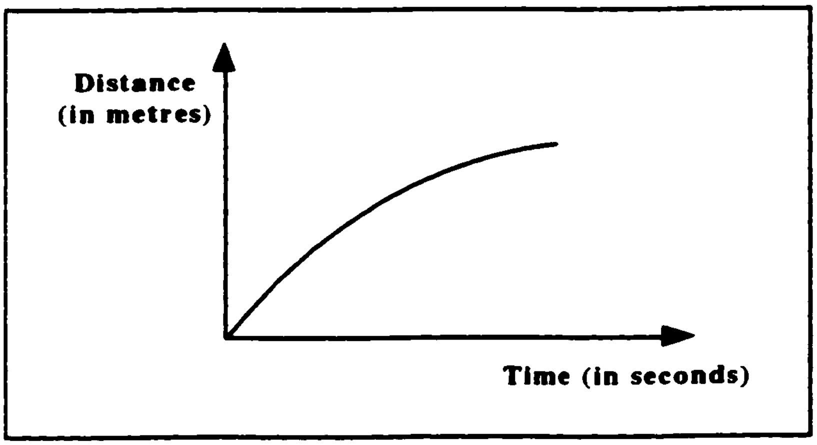

Fig. 10

The expected correct behaviour is for the students to explain that, at first, the distance away from the motion detector increases very rapidly, and then the distance away from the motion detector begins to slow down.

The anticipated difficulties associated with this question would be that the students may once again confuse the change in speeds. Also, students may have difficulty in describing this graph because the majority of the graphs looked at during Session 1 and 2 were linear in nature and, again, this one is a curve.

Local Analysis

Of the 30 students, 28 answered the question correctly. Only 2 students did not describe any decrease in speed. This explanation was limited to "moved away."

Following these questions, students reproduced both graphs while moving in front of the motion detector. The importance of changing speed became important to everyone.

Description and Analysis of Activity 4:

Interpreting Qualitative Graphs Appendix S5

Recall that hypothesis 3 stated that the investigations of the relationship between distance and time in problems of motion help the students better understand other functional relationships between independent and dependent variables and the general mathematical notion of function in learning mathematics in higher grades. Activity 4 aims to test this hypothesis in relation to depth of water as a function of time.

In this activity, the students are asked to select graphs that best represent specific situations. All of the situations involve depth of water as a function of time. This activity is also done individually and later discussed in class. As with all other activities, the worksheets were collected and further analysed.

The aim of this activity is for the students to be able to interpret qualitative graphs and determine the relationship that exists between the water level and the time in each graph, and to match a functional relationship with its corresponding graph.

A good understanding of the relationship that exists between the x-axis and y-axis and the notion of dependence is required to answer the questions in this activity. Interpretation of graphs of functional relationships is also required.

Problem 1 **A young child's swimming pool has a cylindrical shape. A hose is left running into the pool at a constant rate. Which of the three graphs below best shows how the depth of the water in the wading pool changes with time? Draw a circle around the correct graph.**

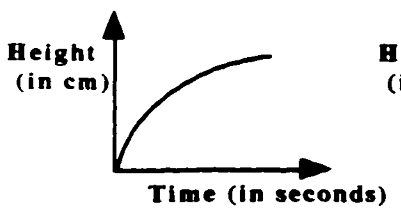

Fig. 11

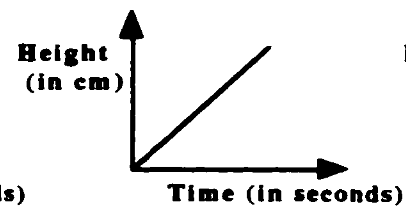

Fig. 12

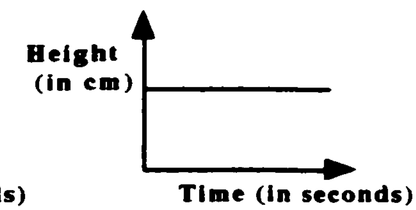

Fig. 13

The students were asked to identify which of three graphs, Fig. 11, Fig. 12, or Fig. 13 best shows how the depth of the water in a cylindrical shaped pool changes over time when a hose is left running in the pool at a constant rate. The students are instructed to draw a circle around the correct graph on their worksheets (Appendix S5).

It is expected that students will choose the middle graph (Fig. 12).

It is anticipated that students may have difficulty with the term "constant rate" because it may not be familiar. The researcher would then have to explain its meaning. Some students may have difficulty visualising what is happening.

It should also be anticipated that some students may have difficulty transferring their understanding of functional relationships between distance away from a motion detector and time elapsed to this new relationship. It is very important to discuss with the students this new functional relationship. The researcher may choose to ask the students to describe the relationships represented in each graph in the context of the situation.

Local Analysis

Of the 30 students, 23 selected the correct graph (Fig. 12). 4 students selected the first graph. These students chose this graph because they felt it most resembled a cylindrical shape, resorting back to viewing a graph as a literal picture. 3 students selected the third graph. These students chose this graph because they felt it represented water coming out at the same rate. They did not understand that the graphs represented relationships between the height (in cm) of the water level and the time elapsed (in seconds). They did not yet make the transfer from the previous activities.

Problem 2 **A water goblet has a shape like the one shown. It is placed under a tap with water running at a constant rate.**

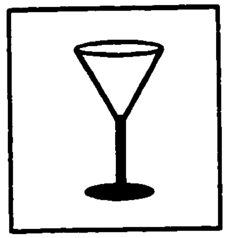

Fig. 14

(a) Which of the three graphs best shows how the height of the water in the goblet changes with time? Draw a circle around the correct graph.

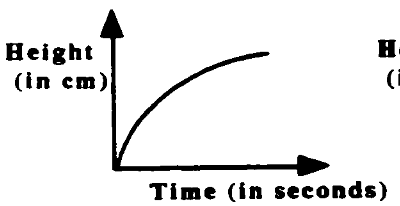

Fig. 15

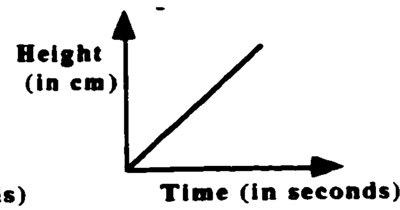

Fig. 16

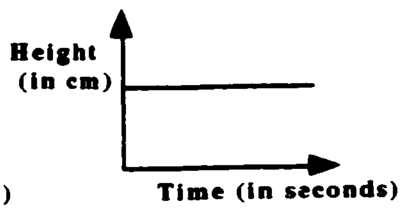

Fig. 17

The students were asked to identify which of the following three graphs, Fig. 15, Fig. 16, or Fig. 17 best shows how the height of the water in a goblet (Fig. 14) changes with time, if it is placed under a tap with water running at a constant rate. The students are once again instructed to draw a circle around the correct graphs on their worksheets.

It is expected that the students will choose the first graph.

It is anticipated that some students may resort back to viewing the graph as a literal picture of a situation. For example, some students may choose the second graph as an answer because it best models one of the sides of the glass. Other students may have difficulty transferring their understanding of functional relationships between distance away from a motion detector and time elapsed in the previous activities to this new relationship.

Local Analysis

Of the 30 students, 16 selected the correct graph (Fig. 15). 13 students selected the second graph and 1 student selected the third graph. Several students explained that they chose the second graph because they felt it resembled the side of a water goblet the best. These students resorted back to viewing a graph as a literal picture.

Problem 3 **Using a hose with a constant flow, you fill the containers below with water. As the time (in seconds) goes by, examine the height of the water in the containers.**

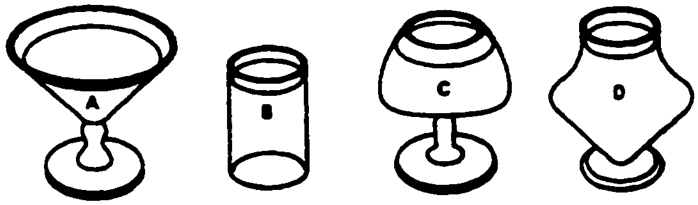

Fig. 18 *Fig. 19* *Fig. 20* *Fig. 21*

Which graph corresponds to each container?

G1

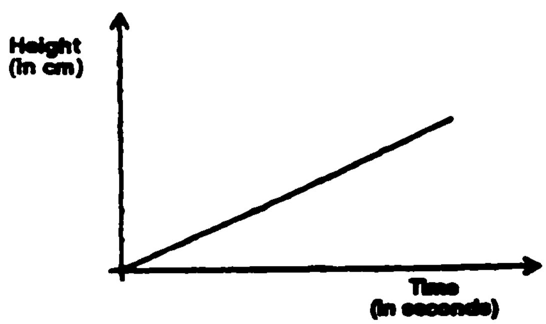

Fig. 22

G2

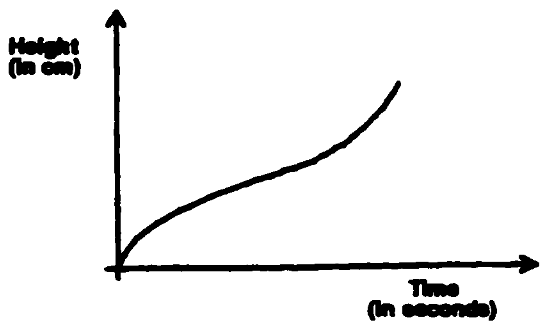

Fig. 23

G3

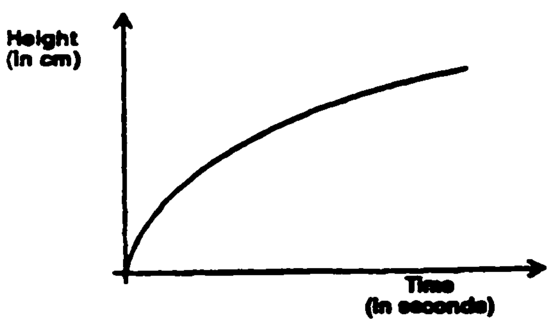

Fig. 24

G4

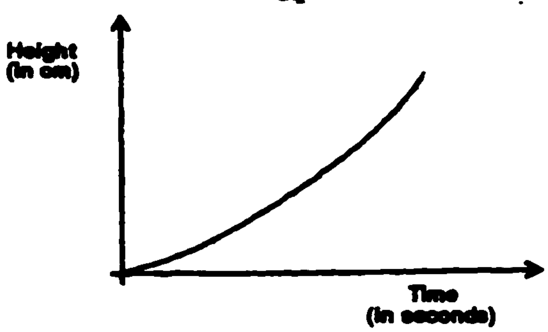

Fig. 25

The students were asked to match a series of containers (Fig. 18, Fig. 19, Fig. 20, and Fig. 21) with the appropriate graphs (Fig. 22, Fig. 23, Fig. 24, and Fig. 25) on their worksheets (Appendix S5).

It is expected that the students will match the following containers with their graphs:

Containers	**Graphs**
A	G_3
B	G_1
C	G_4
D	G_2

The anticipated difficulties associated with this question would be that some students may still have difficulty visualising what is happening. It is also anticipated that some students may match containers with graphs based on the shape of each. For example, container D has the most curves therefore it should be represented by graph G_2.

Local Analysis

Of the 30 students:

- 16 matched container A with its appropriate graph G_3.
- 18 matched container B with its appropriate graph G_1.
- 8 matched container C with its appropriate graph G_4.
- 23 matched container D with its appropriate graph G_2.
- 7 matched container A with graph G_4 and container C with graph G_3.

Many of the difficulties that the students had with this question were revealed in the class discussion. Some of the students who matched container D with graph G_2 did so based on change. Container D and G_2 both show the greatest amount of change. Also, it was discovered that students matched containers A and C with graphs G_4 and G_3 respectively, because they best represented literal pictures of the sides of the containers. It's amazing that they are still doing this! During the class discussion the researcher asked every student to imagine that a hose with a constant flow was placed in each container. They were then asked to imagine the speed at which the water level would rise. The researcher then asked the students to apply their imagination in observing the effects of filling each container. The results of the discussion showed that a greater number of students were now able to match containers with their appropriate graphs. In other words, the researcher guided the students step by step in order to achieve a visualisation of the event.

Global Analysis of Session 3

The results obtained in Activity 3 (Describing Motion) showed that the students were able to interpret a graphical representation of motion data involving a change in speed. Therefore, Activity 3 was successful in meeting its aim.

Comparing the *a priori* analysis of the questions in Activity 4 and the results obtained, the aim of the activity does not appear to have been met very successfully. It was not until the class discussion which ensued that it became clear that the students did, in fact, meet the aim of the activity. Although the written results obtained from the students showed some common misconceptions, such as the matching of a container with a graph that physically represented one of the sides of the container, the class discussion is what showed the third hypothesis (H3)[3] to be true for over 80% of the students. The teacher asked the students to imagine each container being filled with water and to describe what they were visualising.

[3] H3 The investigations of the relationship between distance and time in problems of motion help the students better understand other functional relationships between independent and dependent variables and the general mathematical notion of function in learning mathematics in higher grades.

Chapter IV: Conclusions and Recommendations

The goal of this study was to explore the effects of using motion detector technology to develop conceptual understanding of graphical representations of function in middle school students. The global analysis revealed that dynamic representation is effective in allowing Grade 6 students to develop conceptual understanding of the relationship between distance and time in problems of motion (H1). It was also shown that students were able to develop a good understanding of independent and dependent variables (H2). The class discussion in Activity 4 showed the third hypothesis to be true. The investigations of the relationship between distance and time in problems of motion help the students better understand other functional relationships between independent and dependent variables, water level and time in this case, and the general mathematical notion of function in learning mathematics in higher grades. However, many of the written responses from the students brought forward the known and persistent misconception, that is the matching of a container with a graph that best physically represented one of its sides. A recommendation for future research would be to use a physical model to allow the students to conceptualise the relationship between the water level and the time elapsed.

Subsequent to the motion detector lesson, the students quickly realised that time was the independent variable and learned how to control the dependent variable (distance) by his or her movement (direction and speed). Using a motion detector allows students to reflect on how they are moving as they are moving. This reflection is significant in achieving understanding of functions through dynamic representation. The investigations in the motion detector lesson also encouraged the students to visualise relationships, further deepening their understanding of function. The misconception associated with considering a graph as a literal picture of a situation was corrected effectively. The students were able to represent given situations graphically as well as describe situations by interpreting graphs. For example, it is clear that the reflections, conjectures, and experimentations resulting from trying to determine whether or not it is possible to produce a graph resembling the letter **B** was a key element in the students' conceptual understanding of function. The most common response from the middle school students in this study as a reason for not being able to complete this task was the realisation that time (the independent variable) cannot go backwards. These students will have much less difficulty to identify and to interpret qualitative graphs of functions than those students who rely on familiar procedures such as the vertical-line test. This procedural knowledge is often easier to attain, but it is often the easiest to forget as well and it does not promote understanding of the concept of function.

The global analysis does suggest that extra time taken in the early introduction of the function concept using dynamic representation through the use of motion detector technology is effective in reducing the number of cognitive obstacles and misconceptions that students presently encounter. The results suggest that middle school instructional programs should include attention to functions so that all students can understand the behaviour of functional relationships. Such a recommendation would have a great impact on the mathematics curriculum in Quebec, where the majority of students do not receive any instruction on the concept of function throughout high school. The results of this study have shown that an introduction to this concept need not be reserved for advanced Grade 10 and 11 mathematics. The use of motion detector technology creates dynamic graphing situations which allow students to see relationships between dependent and independent variables as they occur. Such powerful dynamic teaching and learning tools also allow students to control these relationships physically, thus deepening their understanding of functions.

Researchers (NCTM, 1998) agree that a conceptual understanding of variable is fundamental to students' success in algebra. Such understanding is equally important in developing understanding of the function concept. In order to avoid the great difficulty that students have in going from a concept of variable as an unknown to a new conception of variable as a quantity with changing values, it is critical that we do not give students inaccurate or incomplete definitions of this concept. Doing so unnecessarily creates cognitive obstacles, hindering a conceptual understanding of functions for many students. It is equally important for curriculum designers to be conscious of the didactic constraints imposed by their present curricula. The concept of variable is quite complex (NCTM, 1998). Consequently, it is important that sufficient time be allotted to developing the understanding of the concept of variable itself in future curricula. Assuming that students will come to an understanding of this concept while visiting other concepts, as is presently the case, is no small assumption. Such a method of instruction is certainly not pedagogically sound nor conducive to the learning of mathematics. If, in fact, a conceptual understanding of variable is critical in setting a firm foundation for other related algebraic concepts such as that of function, then it is imperative for future research to focus on the epistemological and didactic aspects of the concept of variable as well.

An early introduction to the concept of function through dynamic representation will allow students to observe concrete relationships of dependence in everyday life through mathematical eyes and perhaps begin to develop their understanding of the concept earlier. We often hear educators express that it is very difficult to motivate children and that it is therefore laborious to teach for conceptual understanding. Encouraging these educators to incorporate the use of motion detector technology in conjunction with an appropriate lesson when introducing the concept of function will motivate children to learn. Dynamic representations of real-world data makes mathematics relevant, accessible, and exciting for many students – attributes we strive to promote in middle school mathematics.

The investigations presented in the motion detector lesson were structured in such a way as to help children experience the joy of discovering special relationships of dependence in a truly dynamic environment. It was shown that students (even with severe learning difficulties) can develop a conceptual understanding of functions through dynamic representation. The sense of achievement derived from these activities was shown to be very motivating for students and may even provide an incentive to continue learning mathematics. These activities were shown to make students feel good and excited about understanding mathematics. This was evident in their willingness to answer questions, to volunteer to move in front of the motion detector, and in the many exploratory type questions they had for the teacher.

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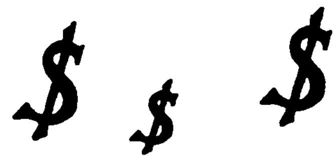
Appendix S1

Investigating Changing Quantities

In each of the following situations, the result of one quantity depends on changes to another quantity.

Write a sentence for each of these situations indicating how one quantity is dependent on another quantity.

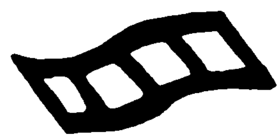
Example:
- number of hours spent babysitting
- amount of money earned



Solution:

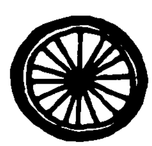
The amount of money earned depends on the number of hours spent babysitting.

1.
- number of people watching a movie
- number of available seats at the cinema



Solution:

2.
- wear on bicycle tires
- distance travelled



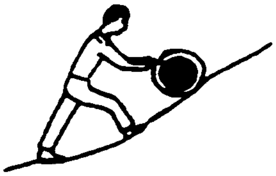
Solution:

Appendix S1 (continued)

3.
- amount of fallen snow
- number of hours spent shovelling

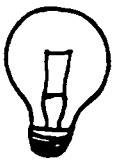
Solution:

4.
- steepness of a hill
- force required to push a barrel up the hill



Solution:

5.
- number of hours lights are on in a year
- number of light bulbs used



Solution:

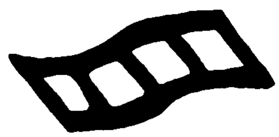
Appendix S2

Investigating Variables

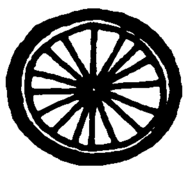
In each of the following situations, identify the dependent variable by placing a check mark in the appropriate area.

Example: ____ number of hours spent babysitting
__√__ amount of money earned

1. ____ number of people watching a movie
____ number of available seats at the cinema

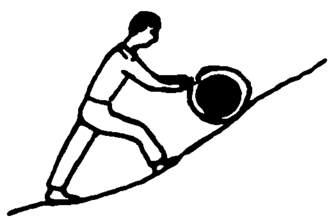


2. ____ wear on bicycle tires
____ distance travelled

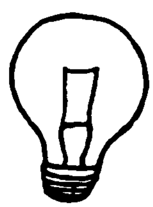


3. ____ amount of fallen snow
____ number of hours spent shovelling

4. ____ steepness of a hill
____ force required to push a barrel up the hill



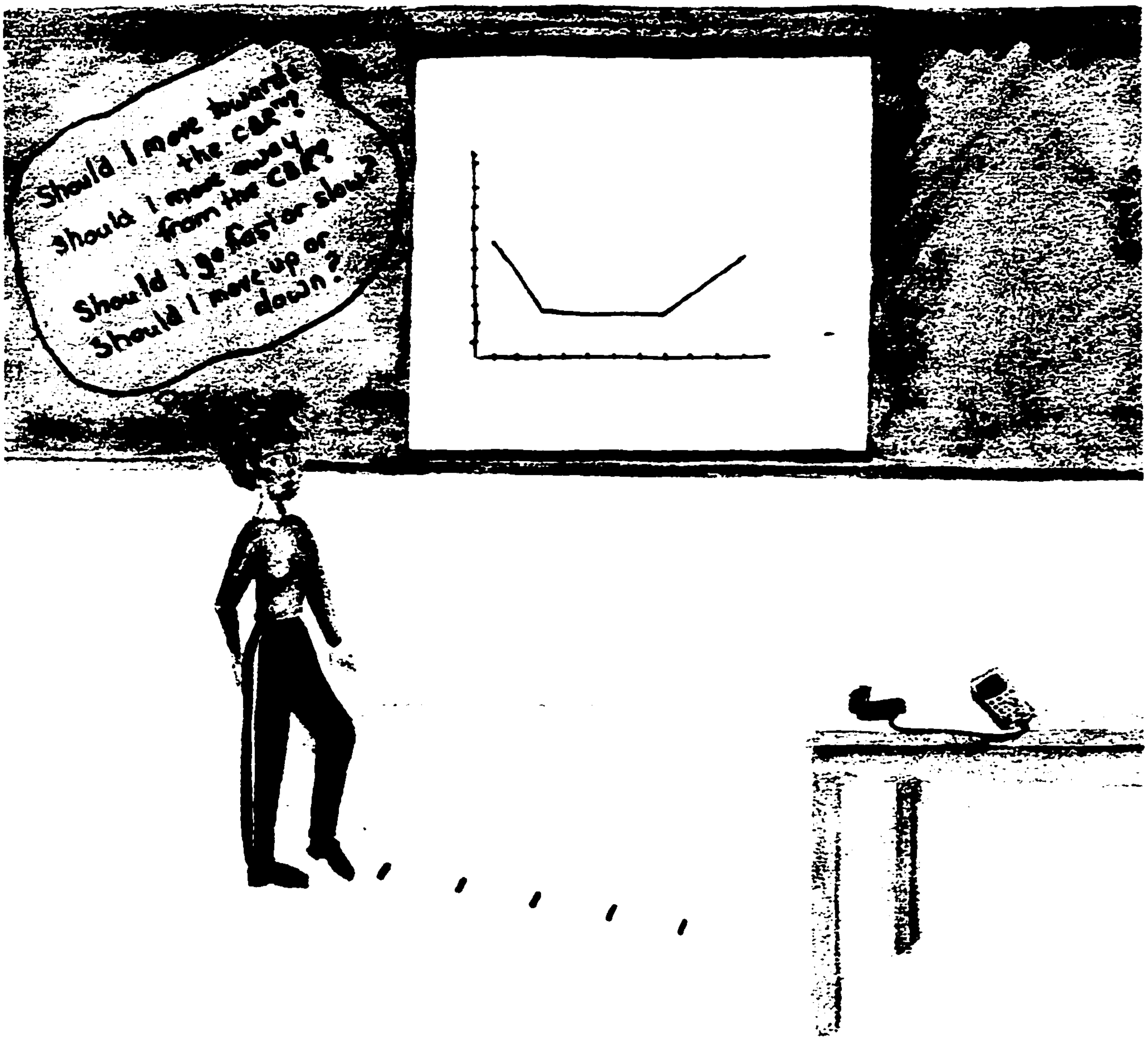
5. ____ number of hours lights are on in a year
____ number of light bulbs used



Appendix S3

Are You Up to the Match?

You are about to experience the Calculator Based Ranger (CBR™). It's a lot of fun! Try to figure out how it works.



Appendix S3 (continued)

1. In the viewscreen window provided below, reproduce the graph that is on the overhead screen.

2. Explain what you think is happening in the above graph.

3. Were you or your classmates able to reproduce the graph while moving in front of the CBR™?

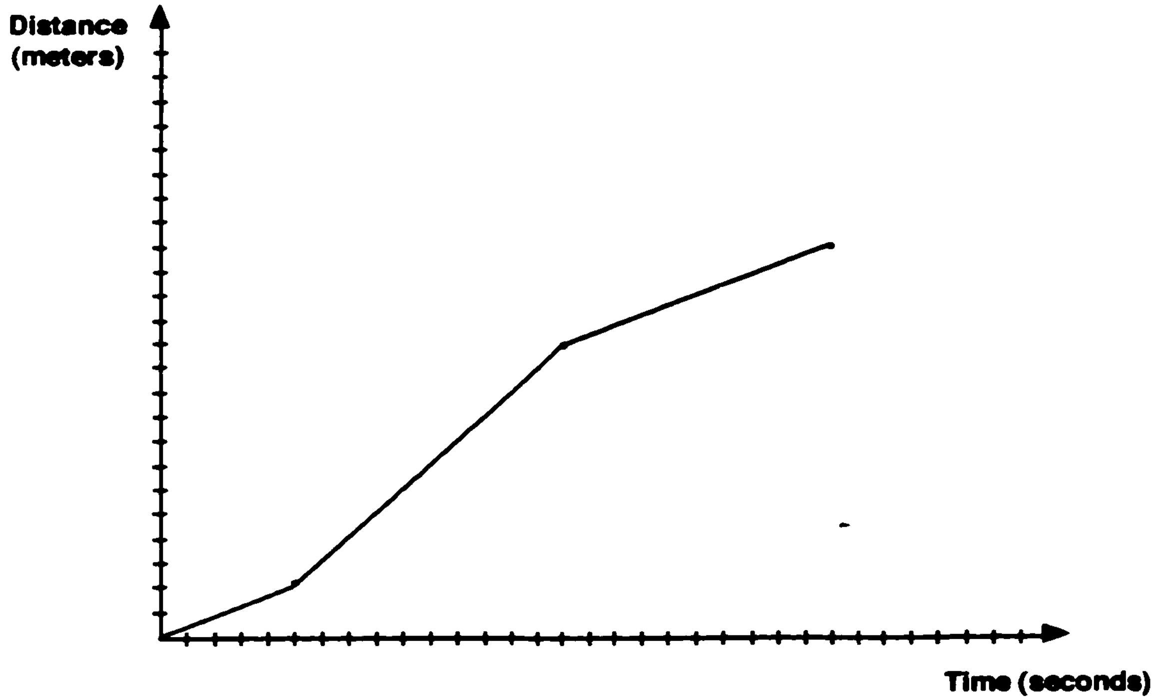
Appendix S3 (continued)

4. Sketch the graph that is on the overhead screen in the viewscreen window below.

5. How would you have to move in front of the CBR™ in order to reproduce this graph? Think about how far you should be away from the CBR™ just before starting.

6. Was it a perfect match? If not, describe how you would improve each segment of the graph.

7. Make up a story to describe an event that may be represented by the following graph.



8. Describe how you would walk in front of the CBR™ if you were to try producing a graph resembling a mountain peak. Think about this carefully. Your teacher might actually ask you to try it.

Appendix S3 (continued)

9. Describe how you would walk in front of the CBR™ if you were to try reproducing a graph resembling a valley.

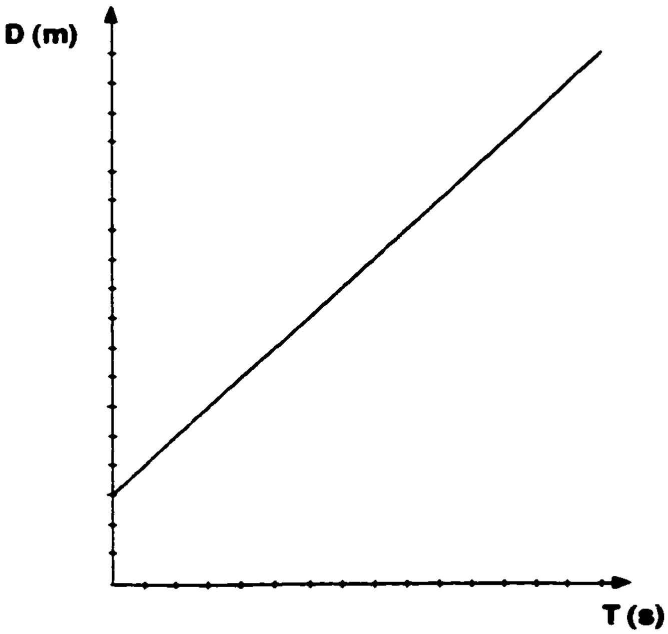
10. Think about how you would walk in front of the CBR™ if you were to produce a graph resembling the first letter of your name. Is it possible? If so, describe how you would move in front of the CBR™ and draw a sketch of the graph you would expect to see. If not, explain why it is not possible.

Explanation

Sketch

11. Describe the graph that you would expect to see if you were to stand in front of the CBR™ without moving.

12.



(a) Describe how you would walk in front of the CBR™ in order to reproduce the above graph.

Appendix S3 (continued)

(b) What do you notice about the distance away from the CBR™ as time elapses?

(c) What is the maximum distance from the CBR™ after 15 seconds?

(d) How far did the person get after 5 seconds?

10 seconds?

(e) If the CBR™ had continued collecting information for another 15 seconds, how far do you think the person would have gone? Explain your answer.

13.

(a) How is the distance away from the CBR™ changing as time elapses?

(b) How is the time changing as the distance away from the CBR™ changes?

(c) Is it possible to be at a specific distance away from the CBR™ at more than one time over an interval of time? Explain your answer.

(d) Is it possible to be more than one distance away from the CBR™ at a specific time? Explain your answer.

14. (a) Identify the two variables that are present in graphs produced by the CBR™.

(b) Describe the relationship that exists between these two variables,

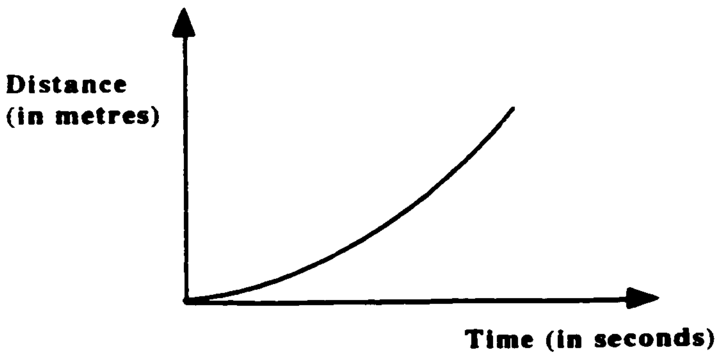
(c) Identify the dependent and independent variables in graphs produced by the CBR™.

Appendix S4

Describing Motion

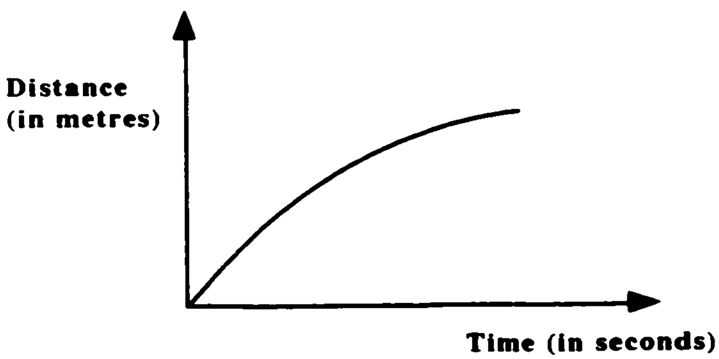
How would you move in front of the motion detector in order to reproduce the following graphs?

a)



Explanation:

b)

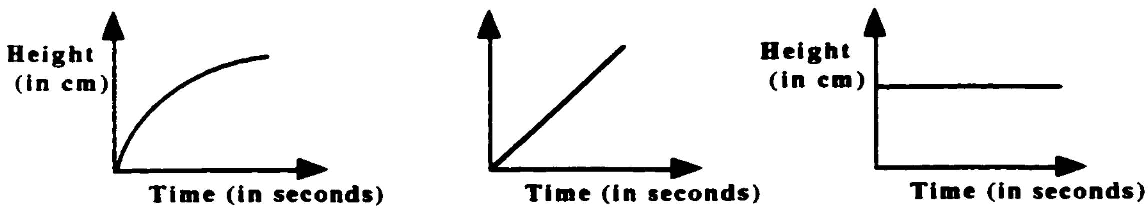


Explanation:

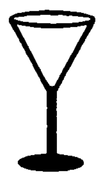
Appendix S5

Interpreting Qualitative Graphs

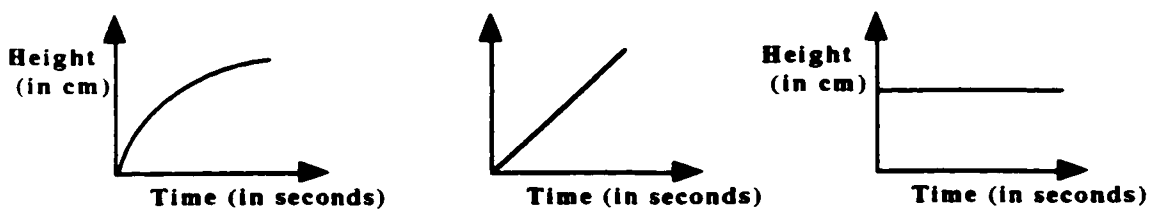
1. A young child's swimming pool has a cylindrical shape. A hose is left running into the pool at a constant rate. Which of the three graphs below best shows how the depth of the water in the wading pool changes with time? Draw a circle around the correct graph.



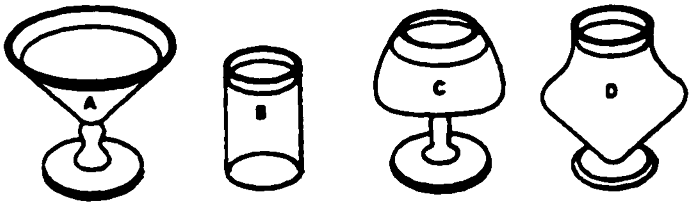
2. A water goblet has a shape like the one shown.
It is placed under a tap with water running at a constant rate.



(a) Which of the three graphs best shows how the height of the water in the goblet changes with time? Draw a circle around the correct graph.

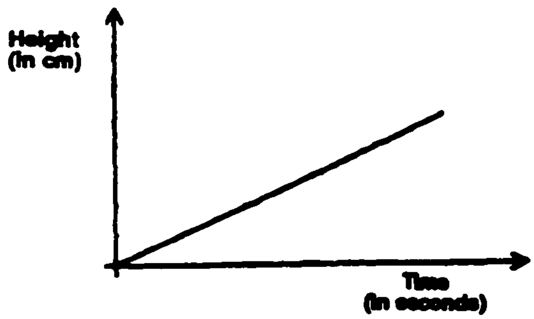


3. Using a hose with a constant flow, you fill the containers below with water. As the time (in seconds) goes by, examine the height of the water in the containers.

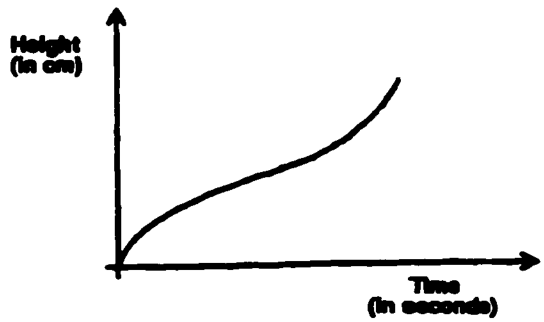


Which graph corresponds to each container?

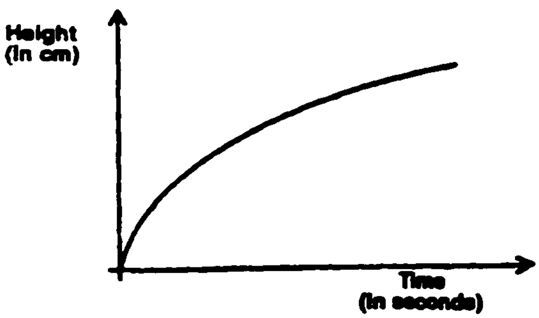
G_1



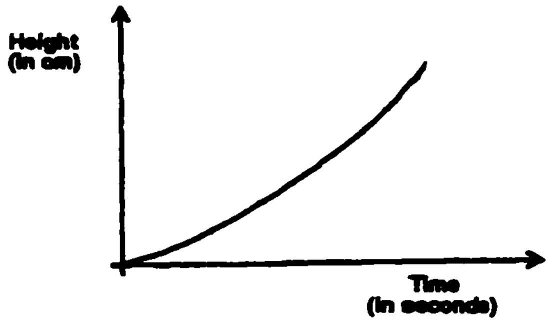
G_2



G_3



G_4



Appendix T1

Investigating Changing Quantities

In each of the following situations, the result of one quantity depends on changes to another quantity.

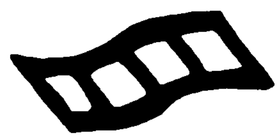
Write a sentence for each of these situations indicating how one quantity is dependent on another quantity.

Example:
- number of hours spent babysitting
- amount of money earned

Solution:

The amount of money earned depends on the number of hours spent babysitting.

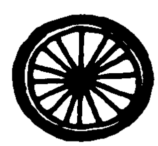
1.
- number of people watching a movie
- number of available seats at the cinema



Solution:

The number of people watching a movie depends on the number of available seats at the cinema.

2.
- wear on bicycle tires
- distance travelled



Solution:

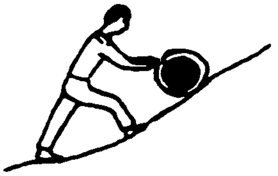
The amount of wear on bicycle tires depends on the distance travelled.

Appendix T1 (continued)

3. • **amount of fallen snow**
   • **number of hours spent shovelling**

Solution:

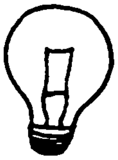
The number of hours spent shovelling depends on the amount of fallen snow.



4. • **steepness of a hill**
   • **force required to push a barrel up the hill**

Solution:

The force required to push a barrel up the hill depends on the steepness of a hill.



5. • **number of hours lights are on in a year**
   • **number of light bulbs used**

Solution:

The number of light bulbs used depends on the number of hours lights are on in a year.

Appendix T2

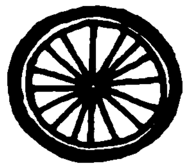
Investigating Variables

In each of the following situations, identify the dependent variable by placing a check mark in the appropriate area.

Example:
____ number of hours spent babysitting
_√_ amount of money earned

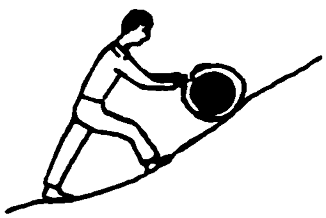
1. _√_ number of people watching a movie
____ number of available seats at the cinema

2. _√_ wear on bicycle tires
____ distance travelled

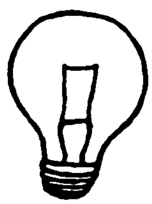


3. ____ amount of fallen snow
_√_ number of hours spent shovelling

4. ____ steepness of a hill
_√_ force required to push a barrel up the hill



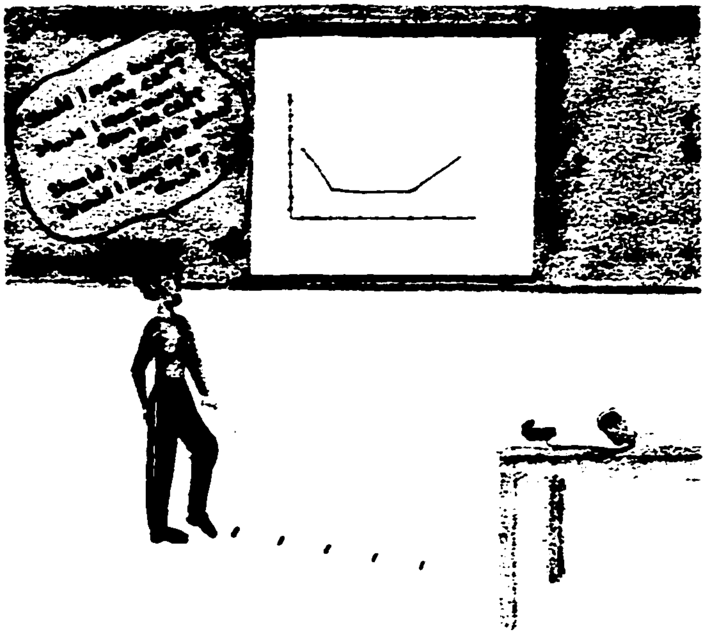
5. ____ number of hours lights are on in a year
_√_ number of light bulbs used



Appendix T3

Are You Up to the Match?

You are about to experience the Calculator Based Ranger (CBR™). It's a lot of fun! Try to figure out how it works.



Requirements

- TI-73 overhead set-up
- Calculator Based Ranger (CBR™)
- link cable (to connect CBR™ to TI-73)
- measuring tape / meter stick
- masking tape / marking tape

Objectives of Activity

In this activity, the students will be able to:

- develop an understanding of the relationship that exists between the x and y axes;
- understand the concept of position;
- understand how a change in one quantity affects another quantity;
- develop an understanding of the concept of variable and a notion of dependence between two variables;
- develop a notion of function; and
- represent and describe mathematical relationships using graphs.

Remember: The CBR™ works best when objects are between 1/2 meter and 6 meters away from it.

1. In the viewscreen window provided below, reproduce the graph that is on the overhead screen.

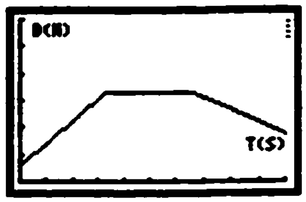
Run the CBR™ application **DIST MATCH** .

- ***Connect the CBR™ to your TI-73 via the link cable.***
- ***Press*** APPS 2 ENTER 3 ENTER 3 1 1

  and follow instructions on your screen.

Have the students reproduce the graph that is now on the overhead screen in the viewscreen window provided to them in question 1.

Example of a graph that may be displayed:



2. Explain what you think is happening in the above graph.

Answers will vary here due to the different graphs that may be generated by this application. Expect many wrong answers here. That's okay. Many of your students may not yet understand the relationship that exists between the x and y axes.

Note: Without judging the students' responses for question 2, ask for a volunteer to come to the front of the class and try reproducing the graph while walking in front of the CBR™.

Press ENTER

3. Were you or your classmates able to reproduce the graph while moving in front of the CBR™? If not, describe the way <u>you</u> would move in front of the CBR™ in order to produce the appropriate graph.

Most likely, the answer will be " no " here. Hopefully, you will get some of the following responses from your students: • I would start further from / closer to the CBR™. • I would have started off walking away from / towards the CBR™. Students will also suggest that they would go faster or slower on different parts of the graph.

Appendix T3 (continued)

The students should now be aware that* TIME IN SECONDS *is represented by the x axis and that* DISTANCE <u>AWAY</u> FROM THE CBR™ *is represented by the y axis. Have the students think about where one should be placed in front of the CBR™ at the beginning of the exercise and at the end of the exercise.

Now, ask for another volunteer to try reproducing the same graph. Afterwards, have another class discussion on question 3.

Press ENTER (1) ***(pause)*** ENTER

4. Sketch the graph that is on the overhead screen in the viewscreen window below.

Press ENTER (2) ***and follow instructions on your screen.***

Have the students reproduce the graph that is now on the overhead screen in the viewscreen window provided to them in question 4.

5. How would you have to move in front of the CBR™ in order to reproduce this graph? Think about how far you should be away from the CBR™ just before starting.

Answers will vary here due to the different graphs that may be generated by this application. Students should now begin to understand the relationship that exists between the distance away from the CBR™ and the time elapsed.

Note: Following a discussion of the students' explanations for question 5, ask for a volunteer to come to the front of the class and try reproducing the graph while walking in front of the CBR™.

Press ENTER

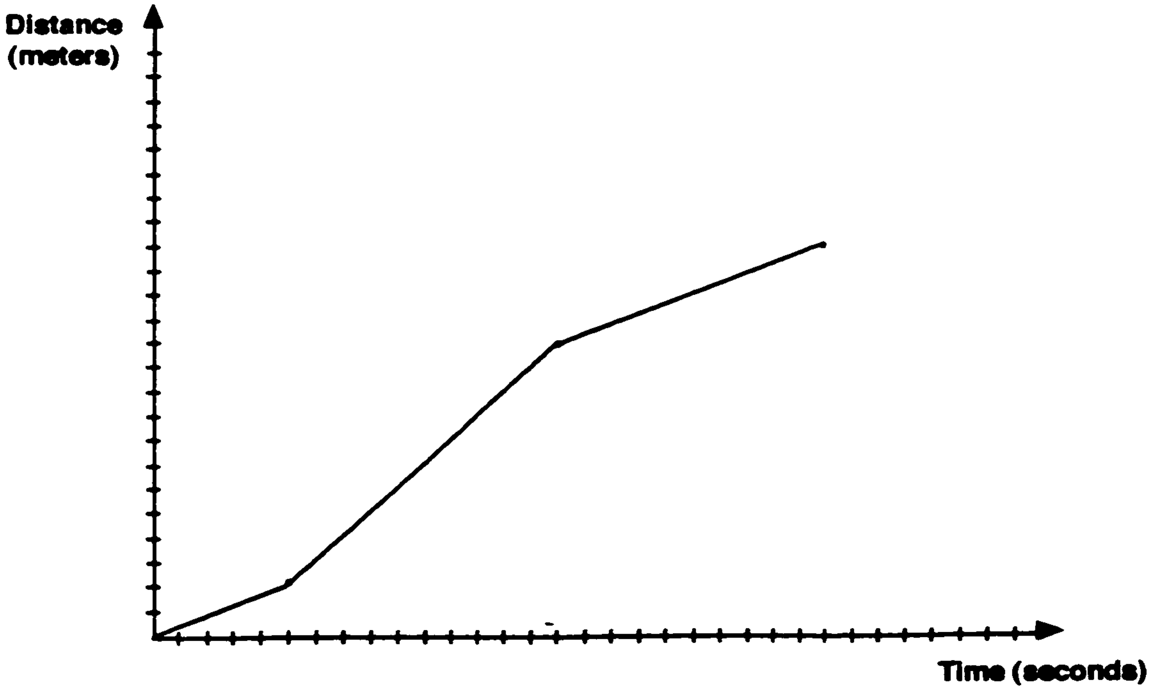
6. Was it a perfect match? If not, describe how you would improve each segment of the graph.

The match will probably be much closer now. However, it will not be perfect. Students will most likely make suggestions similar to those in question 3. They may also suggest other ways to help them better represent the graphs, such as using a measuring tape / meter stick and masking tape to mark off specific distances away from the CBR™. This will help them know where to change directions and/or speed.

Note: Now, ask for another volunteer to try reproducing the same graph. Afterwards, have another class discussion on question 6.

Press ENTER (1) ***(pause)*** ENTER

7. Make up a story to describe an event that may be represented by the following graph.



Many possible stories or events. Example: Sue was slowly walking towards the bus. She started to run as she noticed the bus was leaving. After running for ten seconds (need not be this specific), Sue returned to walking, realizing that she now had to walk to school.

8. Describe how you would walk in front of the CBR™ if you were to try producing a graph resembling a mountain peak. Think about this carefully. Your teacher might actually ask you to try it.

It would be important for students to note that they would start close to the CBR™. The students would then explain that they would walk quickly away from the CBR™ for a few meters and then immediately returned to where they started.

Note: Have a student actually try to make a graph resembling a mountain peak using the CBR™.

Press ENTER 4 1 ▲ ENTER *and follow the instructions on your screen.*

9. Describe how you would walk in front of the CBR™ if you were to try reproducing a graph resembling a valley.

> It would be important once again for students to note where they should begin (away from the CBR™). The students would then explain that they walked quickly towards the CBR™ for a few meters and then immediately returned to where they started.

Note: Have a student actually try to make a graph resembling a valley using the CBR™.

Press ENTER 4 1 ▲ ENTER ***and follow the instructions on your screen.***

10. Think about how you would walk in front of the CBR™ if you were to produce a graph resembling the first letter of your name. Is it possible? If so, describe how you would move in front of the CBR™ and draw a sketch of the graph you would expect to see. If not, explain why it is not possible.

> Insist that the students write down their explanations as to whether or not it is possible. Also, have them sketch the graph if it is possible to do so in the space provided for them.

Note: Before any discussion ask Nancy, Mike, or Victoria (or any other student whose first initial can be reproduced as a relationship between distance and time) to go to the front of the class and use the CBR™ to produce a graph resembling the first letter of his or her first name.

Press ENTER 4 1 ▲ ENTER ***and follow the instructions on your screen.***

Have another student whose first initial can be reproduced go to the front of the class and use the CBR™ to produce the graph.

Press ENTER 4 1 ▲ ENTER ***and follow the instructions on your screen.***

Now, ask Pat, Brenda or Ryan (or any other student whose first initial cannot be reproduced as a relationship between distance and time) to go to the front of the class and use the CBR™ to produce a graph resembling the first letter of his or her first name.

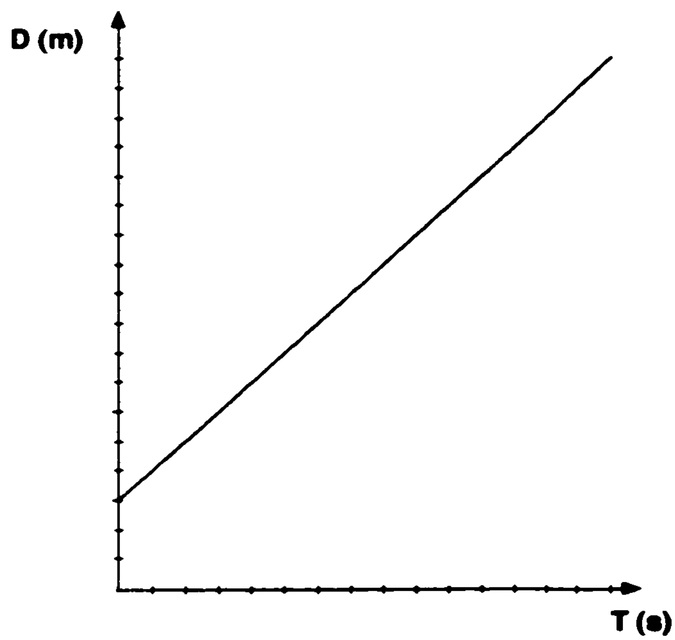
Try holding off discussions until someone has tried the exercise, therefore, ask someone who thinks that they can do it.

> Why is it not possible? Why can't you make the graph of a P or B representing a relationship between distance and time? Most common answers: You can't make time go backwards. You can't be in two places at once.

11. Describe the graph that you would expect to see if you were to stand in front of the CBR™ without moving.

Possible answers: You would see a horizontal line. The CBR™ would record no change in the distance away from the CBR™ for every second elapsed. For each second elapsed, I would be at exactly the same place.

12.



I would start 3 meters in front of the CBR™ and walk away at a steady rate.

Appendix T3 (continued)

(b) What do you notice about the distance away from the CBR™ as time elapses?

The distance away from the CBR™ increases as the time elapses.

(c) What is the maximum distance from the CBR™ after 15 seconds? $\boxed{18\text{ m}}$

(d) How far did the person get after 5 seconds? $\boxed{8\text{ m}}$

10 seconds? $\boxed{13\text{ m}}$

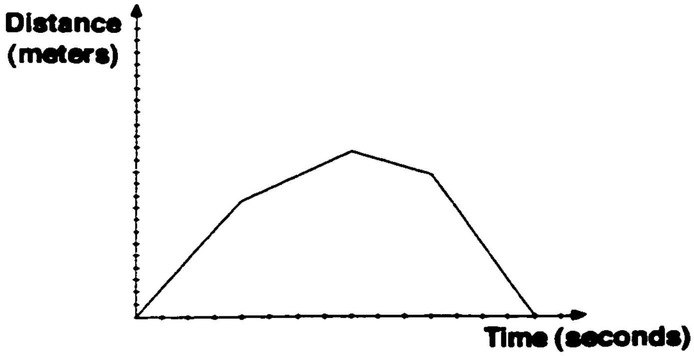
(e) If the CBR™ had continued collecting information for another 15 seconds, how far do you think the person would have gone? Explain your answer.

Example of student response: 36 meters Expect different responses if you are giving this problem to younger students. Explanation: If after 15 seconds, I was 18 meters away from the CBR™, then after 15 more seconds I would be 18 more meters away from the CBR™. 18 + 18 = 36

***Note:* The actual answer of 33 meters is not important here. The goal is to have the students begin to realize that the distance away from the CBR™ is dependent on the time elapsed.**

Appendix T3 (continued)

13.



(a) How is the distance away from the CBR™ changing as time elapses?

Possible Answer: At the beginning, the distance away from the CBR™ increases at a fast rate. After 4 seconds, the distance away from the CBR™ continues to increase but at a slower rate until 8 seconds have elapsed. After 8 seconds, the distance away from the CBR™ decreases at a slow rate until 11 seconds have elapsed, where the distance away from the CBR™ decreases at a quicker rate.

(b) How is the time changing as the distance away from the CBR™ changes?

Possible Answer: The time is not affected by the changes in the distance away from the CBR™.

(c) Is it possible to be at a specific distance away from the CBR™ at more than one time over an interval of time? Explain your answer.

Possible Answer: Yes, you could be 2 meters away from the CBR™ as you begin walking away for a certain time and then you could return to 2 meters away from the CBR™. You could therefore be at a specific distance away from the CBR™ at more than one time.

(d) Is it possible to be more than one distance away from the CBR™ at a specific time? Explain your answer.

Possible Answer: No, one cannot be in two places at once. If I am 4 meters away from the CBR™, I cannot also, at the same time, be 6 meters away from the CBR™.

14. (a) Identify the two variables that are present in graphs produced by the CBR™.

distance and time

(b) Describe the relationship that exists between these two variables,

The distance away from the CBR™ depends on how much time has gone by.

(c) Identify the dependent and independent variables in graphs produced by the CBR™.

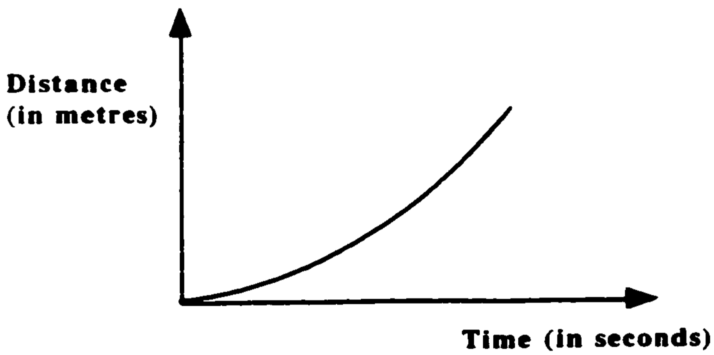
dependent variable: distance away from the CBR™
independent variable: time

Appendix T4

Describing Motion

How would you move in front of the motion detector in order to reproduce the following graphs?

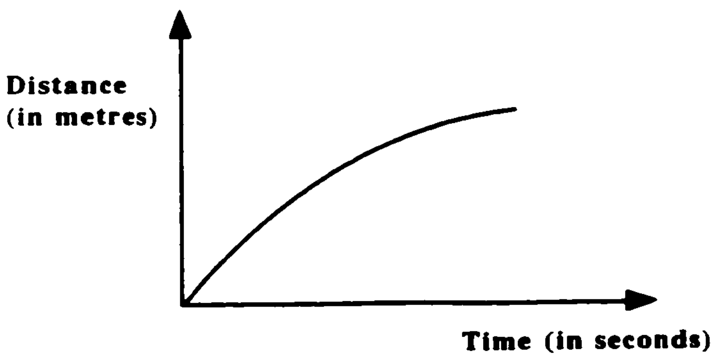
a)



Explanation:

The students will explain that, at first, they are moving slowly away from the motion detector and then they begin to walk away faster and faster.

b)



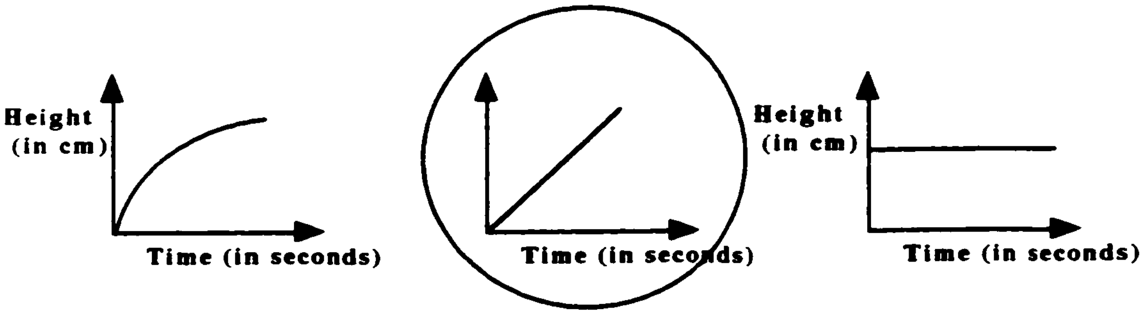
Explanation:

The students will explain that at first the distance away from the motion detector increases very rapidly and then the distance away form the motion detector begins to slow down.

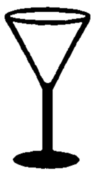
Appendix T5

Interpreting Qualitative Graphs

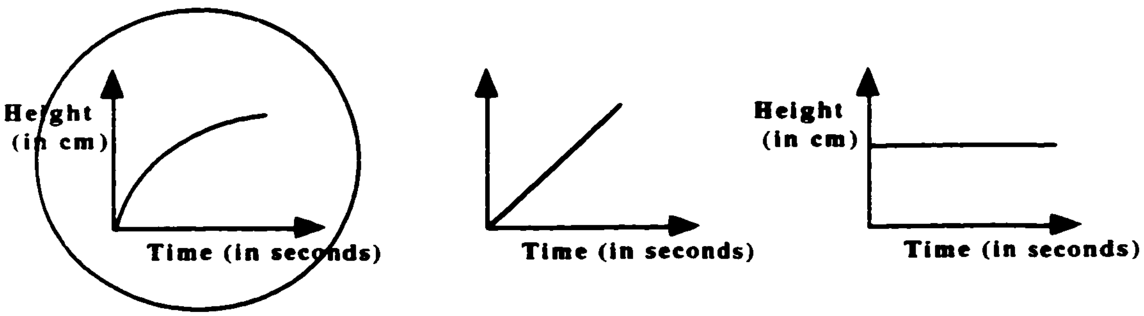
1. A young child's swimming pool has a cylindrical shape. A hose is left running into the pool at a constant rate. Which of the three graphs below best shows how the depth of the water in the wading pool changes with time? Draw a circle around the correct graph.



2. A water goblet has a shape like the one shown.
It is placed under a tap with water running at a constant rate.

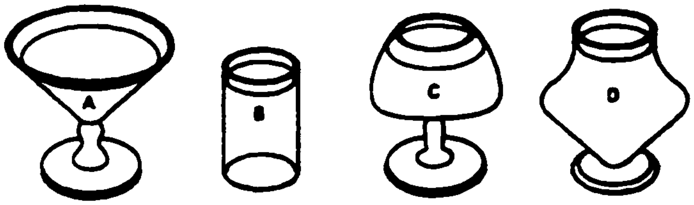


(a) Which of the three graphs best shows how the height of the water in the goblet changes with time? Draw a circle around the correct graph.

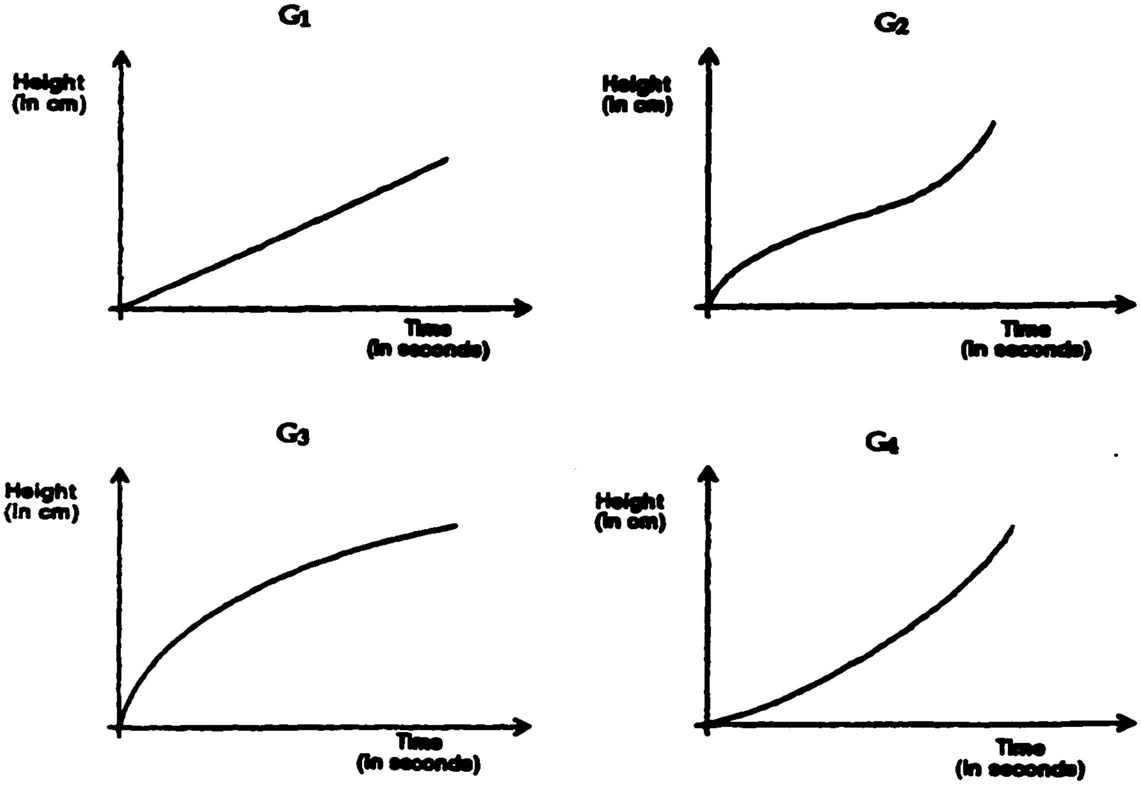


Appendix T5 (continued)

3. Using a hose with a constant flow, you fill the containers below with water. As the time (in seconds) goes by, examine the height of the water in the containers.

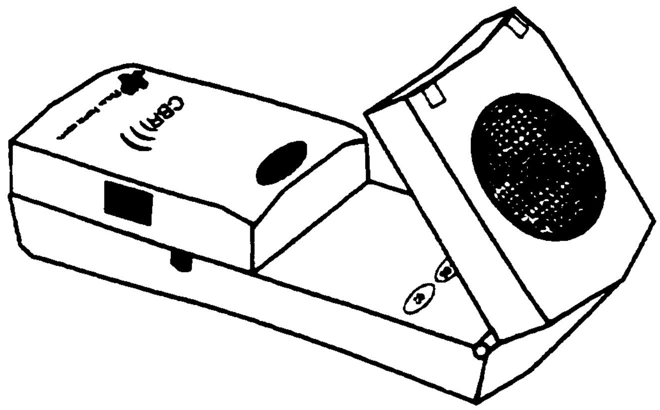


Which graph corresponds to each container?



A - G_3 B - G_1 C - G_4 D - G_2

Appendix CBR



Calculator-Based Ranger™ (CBR™)

The CBR™ is a sonic motion detector which, when used with a graphing calculator, allows students to collect, view and analyse motion data without tedious measurements and manual plotting. Also, this motion detector can be used in such a way as to allow students to see and even physically control relationships between variables as they occur,