The Book-to-Market Ratio and Schwert-Seguin Type Tests of Volatility

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Abstract

This thesis integrates 2 areas of financial research; research on the book-to-market (BM) anomaly and research on time-varying capital asset pricing models (CAPM). Fama and French (1992) introduced the BM anomaly to the academic literature and suggested that it might be driven by changes in economic variables missed by the static CAPM. Using the methodology developed in Schwert and Seguin (1990) this thesis directly tests the possibility that the BM is driven by changes in equity market volatility. This thesis does not find evidence to support the hypothesis that the BM effect is driven by changes in volatility.

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1. Introduction

In the past decade, the book-to-market ratio (BM) anomaly has received a great deal of attention from academics. This anomaly states that firms with a low BM ratio (value firms) earn larger returns than predicted by the capital asset pricing model (CAPM). Despite this anomaly's persistent significance in many different data samples, there is not an accepted theoretical reason for its existence. In their well known article on the BM anomaly, Fama and French (1992) suggest that "Examining the relationship between the returns on these portfolios and economic variables that measure variation in business conditions might help expose the nature of the economic risks captured by size and book-to-market..." (pg. 450). Daniel and Titman (1997) suggest that a possible reason for the existence of the BM anomaly is that it "proxies for distress and that distressed firms may be more sensitive to certain business cycle factors" (pg. 2). This thesis directly tests the relationship between BM-sorted portfolios and one of the business cycle variables, aggregate stock market volatility, that could potentially explain some of the economic risks captured by the BM variable.

This thesis tests the relationship between the BM-sorted portfolios and aggregate market volatility using the methodology developed in Schwert and Seguin (1990) and further modified in Koutmous, Lee and Theodossiou (1994) and Episcopos (1996). Schwert and Seguin (1990) studies, 1) the relationship between aggregate volatility and the variances/covariances of various size-sorted stock portfolios and 2) the effect of changing market volatility on test of the CAPM, specifically whether non-proportional heteroskedasticity implies that beta, the CAPM's measure of risk, varies over time. They study both questions for the sample period 1927-1986.

There are a number of contributions that this thesis makes to the literature. First, this thesis extends the evidence on both questions for size-sorted NYSE portfolios into the period 1987-1997. This allows us to compare and contrast the earlier results with those for the 1990s. This is especially important given the unusual and strong bull market during the 1990s.

Second, the relationship between aggregate market volatility and the variances/ covariances is tested for BM-sorted portfolios of all CRSP stocks for the period 1990-1997. Size-sorted portfolios are also studied from the same stocks for the purpose of comparison. This is the first volatility study on BM-sorted portfolios and represents an original contribution to the literature.

Third, one possible explanation for the BM anomaly is that it represents time-varying risk missed by the static CAPM. This test shows whether there is any relationship between time-varying risk and the BM anomaly. This has not been previously tested in the literature.

Fourth, to the author's knowledge, there are no previous studies of volatility and the variances/covariances of BM-sorted portfolios or of conditional capital asset pricing models, such as the one developed in Schwert and Seguin (1990), and BM-sorted portfolios. Consequently, this study will be the first to examine how the BM ratio is related to equity volatility as well as extending previous work on size. Previous work on the anomaly such as Chan, Hamoa and Lakonishok (1991), Fama and French (1992) and Kothari, Shanken and Sloan (1995) focus primarily on the expected risk adjusted returns instead of the effect of time-varying aggregate market volatility on the volatility patterns and pricing as we do. Thus, this is the first study to consider the second moment of returns, rather than the first moment, in the BM anomaly literature.

A number of articles have shown the importance of taking time-varying volatility and covariances into consideration when testing the CAPM for a variety of different types of portfolios. Empirical estimates of pricing relationships are found to improve for many types of portfolio sorts such as asset class sorted portfolios (Bollerslev, Engle and Woolridge (1988)), size-sorted portfolios (Schwert and Seguin (1990)), industry-sorted portfolios (Episcopos (1996)) and country-sorted portfolios (Koutmous, Lee and Theodossiou (1995) and Ferson and Harvey (1991)). Based on the pricing improvements found in these articles, there is reason to believe that using a conditional CAPM will result in better asset pricing results.

Finally, this paper studies the sensitivity of the empirical results to the choice of the estimation methodology. This thesis uses a variety of methods to control for statistical problems found in the data such as heteroskedasticity and autocorrelation. Specifically, the thesis estimates all models for the different data samples, using 1) Glesjer (1969) regressions for both weighted least squares (WLS) and Hansen's (1982) t-statistic correction, 2) generalized autoregressive conditional heteroskedasticity (GARCH) and 3) exponential generalized autoregressive conditional heteroskedasticity (E-GARCH). Econometrically, this thesis adds to the literature as it is the first opportunity for direct comparison between these different conditional volatility methods. Schwert and Seguin (1990) use the Glesjer regressions, but not the GARCH and E-GARCH methods. Koutmous, Lee and Theodossious (1994) use the GARCH method and Episopos (1996) uses the E-GARCH method. To the author's knowledge, there has not been any comparison between these methods and their effectiveness for studies of asset pricing models while holding the variation in the data and data sorting criterion constant.

Section 2 examines the related literature for this project. Section 3 introduces the data sets used and Section 4 examines the methodology. Section 5 contains the results of tests and Section 6 concludes.

2. Literature Review

2.1 Size and Book-to-Market (BM) Anomalies

Size is one of the best-known and oldest anomalies in the finance literature. Banz (1981) is the first article to examine the size effect. This article finds that small stocks earn significant abnormal returns. The article also shows that it is possible to earn abnormal arbitrage returns by selling short large or medium capitalization stocks and investing the proceeds in a portfolio of small cap stocks. Using leverage, the betas of the short portfolio and the portfolio of small cap stocks are made to be identical. Even with identical betas, the portfolio of small cap stocks provides significantly higher returns. While this study presented the size effect to the academic community, it did not try to explain possible economic reasons for its existence.

Rosenberg, Reid and Lanstein (1985) find that there are significant abnormal returns to a strategy of buying value stocks (high BM ratio) and selling growth (low BM ratio). They examine a zero investment portfolio that is funded through short selling growth stocks and investing the proceeds in value stocks. They control for betas, momentum, size, volume, earnings per share, price earnings ratio, leverage, volatility of earnings and several other factors. This paper, published in a practitioner journal, was largely ignored until substantively the same findings were published in an academic journal by Fama and French (1992).

Chan, Hamao and Lakonishok (1991) examine BM, size and other factors in the Japanese equity markets. They find that the BM ratio is a significant predictor of stock returns while controlling for market effects. Their model of returns is estimated using seemingly unrelated regressions as well as the Fama and Macbeth (1973) methodology. Interestingly, they find weak evidence of a reverse size effect, but any significance of this effect disappears when the BM ratio is introduced into the model.

Serious academic attention to the BM factor began with Fama and French (1992). This paper examines the predictive power of beta, the BM ratio, earnings per share (EPS) and leverage. They find that their empirically estimated beta has almost no predictive power for stock returns in contrast to the predictions of the CAPM. Fama and French (1992) conclude by recommending that a 2-factor model of stock returns, based on size and the BM ratio, be used instead of the CAPM. They suggest several possible explanations for their findings. One of which is that BM proxies for business cycle specific risk. This thesis explicitly tests one version of this hypothesis.

In a continuation of their earlier article, Fama and French (1993) show that there are common factors in the return on stocks and bonds. Differences in returns for size and the BM-sorted portfolios have explanatory power not only for stock returns, but also for bond returns. From this, they postulate that there are common risk factors in the returns on stocks and bonds. In this article, they suggest the use of a 3-factor model based on size, BM and an overall market factor.

Fama and French (1995) show that size and BM ratio are important predictors of firm earnings. They find that small firms and low BM firms have higher earnings growth than large firms and high BM firms. This suggests a rational basis for the predictive power of size and BM in stock returns. Because these factors are capable of predicting real economic factors pertinent to the firm, they should be able to predict the stock price, which should be a reflection of these real economic factors.

Fama and French (1996) examine a large number of asset pricing anomalies. They test for asset pricing anomalies while controlling for the Fama-French 3-factor model. They find that, after controlling for these 3-factors only the short-term momentum strategy of Jegadeesh and Titman (1993) remains an anomaly. Many of the other anomalies disappear as they are based on variables that are highly correlated with either size or BM.

In their articles on the BM, effect Fama and French (1992, 1993) state that BM must proxy for risk factors not measured by the CAPM. Daniel and Titman (1997) directly test this hypothesis. The authors develop and test 3 models. First is a factor model that extracts mathematical factors from the stock returns and also has a BM variable included. This tests if the BM factor proxies for risk factors missed by covariance-based measures of systematic risk. Second is a model identical to their first model except without the inclusion of the BM ratio. Finally the authors create a "characteristics based" model; this model prices a stock's visible characteristics, such as accounting ratios, and ignores its covariances with other stocks. The authors find that after controlling for characteristics, loadings on risk based factors do not contribute significant information to the pricing or equities. However, they find that after controlling for risk based factors characteristics still provide significant information for pricing. On the basis of this the authors conclude that prices are driven primarily by characteristics, not systematic risk.

Elfkhani, Lockwood and Zaher (1998) examine the size and BM effect in the Canadian stock market. They find that there is not a significant relationship between returns and beta during the period 1975-1992 or in any of the sub periods that they test. However, they find a strong size effect, both in January and the rest of the year. They also find that there is a significant relationship between the BM ratio and returns. However, the results are not significant outside of January in all sub periods.

Other international evidence on the BM effect is presented in Fama and French (1998). In this article, the authors show that there is a BM premium in the equity returns of 13 developed countries. They also show that there is value premium in the returns of stocks in 16 developing countries. The authors argue that this supports the argument that the BM anomaly is not sample specific and is not the consequence of data mining.

Kothari, Shanken and Sloan (1995) is one of the first serious critiques of Fama and French (1992). In this article, the authors argue that the results of Fama and French are inflated due to a survivorship bias and the sample used. Fama and French conduct their tests on data extracted from the COMPUSTAT database. However, COMPUSTAT is known to have a high survival bias in its earliest years, as these were backfilled using data only on firms that survived. Another survival issue is that because COMPUSTAT is a commercial database, it does not include information on firms that it considers to be too small to be worth following. Using industry group data from Standard and Poors, the authors find that BM ratio is only weakly correlated to returns. Unlike Fama and French (1992), the authors find that beta has explanatory power, but they cannot reject that there are other factors such as size and BM ratio that help to explain the cross section of returns.

Loughran (1997) questions the practical importance of the BM ratio for finance practitioners. He examines the predictive power of the BM ratio after controlling for firm size, exchange and the January effect. He finds that the BM ratio has no predictive power for the largest size quintile that represents the vast majority of stock capitalization. He also finds that the BM ratio only has significant predictive power in the month of January.

2.2 Volatility and Asset Pricing

Shiller (1981) is the article that begins the volatility literature for equity prices. He argues that stock market volatility is too high to represent rational behavior. Assuming a constant discount rate, the expectations of future dividends would have to vary dramatically over short periods of time and often be highly inaccurate, to justify the volatility of equity prices. Based on this analysis, it is suggested that the high volatility of equity prices is evidence against market efficiency.

French, Schwert and Stambough (1987) empirically examine the relationship between volatility and stock prices. By regressing the market risk premium on expected market volatility, calculated using an autoregressive integrated moving average (ARIMA) process, and on the unexpected component of market volatility (the error term from their ARIMA estimates), they find that there is a strong negative relationship between unexpected volatility and returns. They do not find a statistically significant relationship between expected volatility and returns. To further validate their findings, the authors also estimate the risk premiums and volatility using a generalized autoregressive model in mean (GARCH-M). The results support their findings with the ARIMA model.

Schwert (1989) examines the reasons why equity volatility changes over time. He tests the relationship between volatility and such variables as inflation, various monetary variables, industrial production, recessions (measured with a dummy variable), leverage, lagged values of volatility and trading volume. The results show that all of the above variables affect stock volatility, but the relationship with industrial production is weak. Overall the most important factor is lagged equity volatility.

Haugen, Talmor and Torous (1991) use a model that identifies specific days on which volatility shifts. By measuring the returns before and after the shifts, they show that when market volatility increases, prices go down and when volatility decreases, prices go up. Their work supports the basic intuition behind much of the volatility literature. If stock prices are equal to the present values of future dividends and if an increase in volatility is equivalent to an increase in risk and discount rates, then prices must fall so that the shares provide the higher returns demanded by investors.

2.3 Time-Varying Asset Pricing

In the late 1980s and early 1990s, a number of authors began to consider various models that would, ideally, allow for improved forecasts of asset prices by allowing for time-varying risk premiums or covariances. Bollerslev, Engle and Woolridge (1988) is one of the first articles to model changes in beta. The authors derive a multivariate GARCH-CAPM, in which both the error terms and the beta are constrained to be autoregressive conditional. Testing this model on 3 asset classes, bonds, stocks and bills, they find that it is an improvement in terms of mean variance efficiency over a constant beta CAPM.

Harvey (1989) develops an asset-pricing model with time-varying covariances. His model shows that covariances are time-varying and predicTable. This model holds the expected return on market constant and allows the beta terms to vary. He finds evidence in favor of his model, but he does not directly test its performance against the static CAPM.

Harvey (1991) refines the model developed in Harvey (1989) and uses it to measure time-varying betas in financial markets throughout the world. He finds that a single source of risk, the non-diversifiable exposure to a worldwide equity index, adequately accounts for the cross sectional variations in returns. However, size and BM ratio are not included in the tests. He finds that there is a consistent, but time-varying, price for risk throughout the world with the exception of Japan. Japan appears to have had a much higher price of risk throughout the period.

Ferson and Harvey (1991) examine changes in the market risk premium throughout time. What differentiates this article from much of the work in this area is that the authors allow for both time-varying covariances, as in the two articles by Harvey discussed above, but they also allow the market risk premium to vary over time. They find that allowing for time-varying risk premiums is more important in explaining asset price performances than time-varying betas.

2.4 Volatility and Time-Varying Asset Pricing

Schwert and Seguin (1990) examine the effect of heteroskedasticity in equity markets. They begin by testing if the covariances between stocks are related to aggregate market volatility. Using NYSE size-sorted portfolios, they find that the covariances between the returns of portfolios of size-sorted stocks are positively related to aggregate market volatility. As the covariances between asset returns are a major input into determining prices through the CAPM, the authors develop a model that takes into account changes in aggregate market volatility. This article effectively ties together the literature on volatility in asset pricing and the literature on time-varying risk premiums through the development of a model that explicitly prices changes in aggregate market volatility.

Koutmous, Lee and Theodossiou (19994) further refine the heteroskedastic CAPM, developed in Schwert and Seguin (1990). Instead of using weighted least squares (WLS), generalized method of moments (GMM) and autoregressive estimates of market volatility to control for heteroskedasticity and autocorrelation, this paper uses a generalized autoregressive conditional heteroskedasticity (GARCH) model. The main advantage of using the GARCH methodology is that it is not necessary to use generated regressors in the estimation of the model. Thus, there is less measurement error. The authors use this methodology to test the reactions of various international stock markets to changes in the volatility of an international stock index. They find that the relative price of risk increases for Australia, Germany and Switzerland and decreases for the United States and Japan during periods of high World volatility.

Episcopos (1996) uses the model developed by Schwert and Seguin (1990), but implements the exponential GARCH (E-GARCH) methodology to control for heteroskedasticity and autocorrelation. In addition to not requiring the use of generated regressors, the E-GARCH methodology allows for asymmetrical volatility.[1] This model is used to test the reactions of industry based sub indices of the Toronto Stock Exchange 300 (TSE 300) index. He finds differences between the reactions of different industry groups to changes in aggregate market volatility, indicating that differences in the price of risk between industry groups increase during periods of high market volatility. However, there is no clear relationship between the industry groups, whose relative price of risk increases and those whose price of risk decreases.

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[1] In a study of different models of stock volatility, Pagan and Schwert (1990) find that the E-GARCH model approximates market volatility better than any other model in out of sample tests.

2.5 Relationship Between the Literature and This Thesis

This thesis connects the literature on the BM anomaly with the literature on volatility dependent conditional CAPMs. From the work on time-varying betas and covariances and the literature on volatility, Schwert and Seguin (1990) developed a CAPM that takes into account the changes in pricing arising from changes in volatility. Later articles further refine and improve this model.

We use this model to test the suggestion of Fama and French (1992), who state that the BM ratio may proxy for business cycle related variables. Although there are numerous business cycle related variables, this thesis concentrates on one of them, aggregate market volatility, and tests to see how the pricing of BM-sorted portfolios is changed when time-varying equity volatility is explicitly accounted for.

3. Data

This thesis uses three data samples. The first sample presents portfolios[2] taken from the intersection of the COMPUSTAT and Center for Research in Securities Prices (CRSP) databases. COMPUSTAT is used to obtain market value and BM ratio information about the firms. CRSP is used to obtain the returns of these firms. This sample is created to test the importance of using a conditional CAPM for the pricing of BM-sorted portfolios. Size-sorted portfolios are also created using the same stocks for the purposes of comparison. Second and third samples of New York Stock Exchange (NYSE) size-sorted portfolios are also formed during the periods 1987-1997 and 1980-1986. These samples are used to extend the work of Schwert and Seguin (1990) into the 1990s and ensure that our results are comparable with their results.

3.1 Sample 1: BM-Sorted Portfolios and Size-Sorted Portfolios from the COMPUSTAT Database for the Period 1990-1997

Our sample includes all stocks traded on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and National Association of Securities Dealers Automated Quotation System (NASDAQ) that have data available on both COMPUSTAT and CRSP except for financial firms (all firms with SIC codes in the

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[2] Lo and MacKinlay (1990) show that sorting portfolios into groups on the basis of a variable that is known to be correlated with returns introduces a bias in favor of rejecting the asset pricing model being tested. The bias increases with the number of portfolios created. This point should be remembered when viewing our results.

600s). The financial firms are removed because they have unusually high leverage and are consequently not directly comparable with non-financial firms. This will help keep results comparable with Fama and French (1992) and Loughran (1997). This sample is taken from the same data as Fama and French (1992), but begins immediately after the sample used in their article ends.

BM ratio is taken from the COMPUSTAT database and is calculated, as in Loughran (1997), as fiscal year end book value of equity (COMPUSTAT item # 60) plus fiscal year end book value of deferred taxes (item # 74). BM ratio is calculated only annually while returns are calculated monthly. This is because accounting data, such as book value, are not available on a monthly basis.

On the basis of their BM ratio, firms are placed into one of five different portfolios. Each portfolio contains the same number of firms. Thus, the first portfolio contains the 20% of firms with the lowest BM ratio, the second portfolio contains the 20% of firms with the next lowest BM ratios, and so on. This procedure is accomplished using the "Categorize" function in SPSS.

Market value is taken as the firm's market value at fiscal year end. The use of fiscal year end data is consistent with Fama and French (1992) and further with Kothari, Shanken and Sloan (1995). Other articles typically use December 31 market capitalization. However, both Kothari, Shanken and Sloan (1995) and Fama and French (1992) find that the use of fiscal year end versus calendar year end is irrelevant as both measures lead to essentially identical results. Firms that are missing either market value or book value of equity in the COMPUSTAT database are excluded from the study. Firms that do not have a value for book value of deferred taxes, but have both other COMPUSTAT variables are included in the study because deferred taxes are usually very small in comparison to the other variables. The sorting of these portfolios is accomplished in the same way as for the BM-sorted portfolios, but with market capitalization used in place of the BM ratio.

After sorting into portfolios, the issue CUSIP numbers, taken from COMPUSTAT, were entered into CRSP's msxport and dsxport functions, where equally weighted portfolio returns, including dividends, were calculated on a daily and monthly basis. There are a total of 91 monthly observations in this sample.

3.2 Sample 2: NYSE Size-Sorted Portfolios for the Period 1987-1997

This sample is composed of NYSE stocks divided into five equal size portfolios on the basis of their year-end market value. The market value figure is taken from the CRSP database along with their returns. The December 31 year-end market value figures are used as the basis for forming portfolios for which returns are calculated in the following year. Equally weighted returns, including dividends, for these portfolios are calculated through the msxport program of CRSP. Excess returns are calculated by subtracting the 30-day T-bill rate from the CRSP output. T-bill rates are obtained from the Ibbotson (1998) yearbook.

This sample is calculated in the same way as the Schwert and Seguin (1990) sample and begins in 1987 immediately following the end of their sample in 1986 for a total of 120 observations. This sample will be the basis for extending their work.

3.3 Sample 3: NYSE Size-Sorted Portfolios for the Period 1980-1986

This sample is created in the same way as Sample 2, but covers the period from 1980-1986 for a total of 60 observations. The primary purpose of this sample is to be used as a basis for direct comparison with the Schwert and Seguin (1990) results to ensure that all models have been programmed correctly. As this sample is primarily a check sample and the results for it are covered in Schwert and Seguin (1990), there will be little discussion of it in the results section.

3.4 Summary Statistics For the Samples

Summary statistics of the three samples are given below.

<u>Sample 1 –NYSE, AMEX and NASDAQ 1990-1997</u>

Panel A: Size-sorted Data

	Small	2	3	4	Large
Mean Return	0.0324	0.01569	0.0129	0.0105	0.0116
Standard Deviation	0.0589	0.0532	0.0512	0.0467	0.0522

Panel B: BM-Sorted Portfolios

	Growth	2	3	4	Value
Mean Return	0.0120	0.0118	0.0142	0.0160	0.0212
Standard Deviation	0.0550	0.0469	0.0416	0.0411	0.0427

As can be seen above returns decrease with size and increase with BM ratio. Volatility decreases with size except for the portfolio of largest stocks, which breaks this pattern by having quite a high standard deviation. The standard deviations of the BM-sorted portfolios are decreasing as the BM ratio increases except for the value portfolio, which shows a slight increase. The standard deviations are smaller for the BM-sorted portfolios than the size-sorted portfolios, indicating that there is more non-systematic risk diversified away within the BM-sorted portfolios than within size-sorted portfolios.

Sample 2 –NYSE 1987-1997

	Small	2	3	4	Large
Mean Return	0.01261	0.0069	0.0070	0.0077	0.0077
Standard Deviation	0.0616	0.0452	0.0453	0.0444	0.0415

Sample 3 – NYSE 1980-1986

	Small	2	3	4	Large
Mean Return	0.019	0.0119	0.010	0.010	0.009
Standard Deviation	0.054	0.048	0.047	0.046	0.045

Both Samples 2 and 3 show the same pattern of higher returns and higher volatility for smaller stocks. They differ from the size-sorted portfolios of Sample 1 in that the portfolio of largest stocks does not have higher volatility and returns than the next largest portfolio.

Except for the smallest size portfolios, Sample 3 has higher standard deviations and higher mean returns than Sample 2. Thus, the conclusions of the portfolio theory hold for portfolios 2 through Large in comparing both samples. The small stock portfolio, on the other hand, presents inconsistent results with the predictions of the portfolio theory. This inconsistency, however, appears to be consistent with the well-documented size effect. Overall, these tables provide casual evidence in favor of time-varying returns and volatility.

3.5 Portfolio Sort Information

As can be seen from the table below, value stocks are also likely to be smaller stocks. This indicates that much of the size effect and BM anomaly may be driven by the same firms. These findings are consistent with Loughran (1997), who finds that many small firms are also value firms. A year by year breakdown of the portfolios' sorts is presented in Appendix 1.

Average Number of Firms in Groups for Sample 1 for Period 1990-1997

	Value	1	2	3	Growth
Small	258	125	96	90	160
2	204	132	121	122	158
3	166	157	143	131	139
4	109	162	161	166	135
Large	75	158	179	180	127

4. Methodology

4.1 The Models

This thesis estimates two models, introduced in Schwert and Seguin (1990). The first model is concerned with estimating the relationship between conditional portfolio return covariances and the market's variance, σ^2_{mt}. After introducing this model, we develop the second model, which is a version of the CAPM that takes the relationship between the variance of the market portfolio returns and asset return covariances into account, and then discuss the implications of this model for asset pricing.

4.1.1 Time-Varying Covariances

The first model is given as:

$$Cov_{t-1}(R_{it},R_{jt}) = \xi + \psi\, \sigma^2_{mt-1} \qquad (1)$$

Equation (1) states that the conditional covariance between asset i and j, calculated as in section 4.2, depends on a constant ξ and a slope coefficient, ψ, that relates to the market variance, σ^2_{mt}. Economically, this states that covariances between asset returns vary with the aggregate market variance. This relationship has been hypothesized by a number of authors, including Black (1976) and Christie (1982). They suggest that assets with exposure to the market experience greater changes in covariances than other firms, although they do not directly test this intuition. Schwert and Seguin (1990) test this intuition for size-sorted stocks, but there is not any research on covariance changes between BM-sorted portfolios.

In the CAPM, the beta term represents the amount of non-diversifiable risk for an asset. It allows an analyst to find the additional return that an investor can expect to receive for assuming an additional unit of risk. Beta is given by the covariance of an asset's returns with the returns on the market portfolio divided by the variance of the returns on the market portfolio. Thus, the second model is a version of the CAPM that takes into account the covariance relationship shown in equation (1). If both covariances and the market variance vary through time, but in a proportional manner, i.e. in equation (1), ξ is insignificant and ψ is significant, then beta is not time-varying. If both ξ and ψ are significant, then the relationship is not proportional and the traditional CAPM can be improved upon by taking the relationship in equation (1) into account. The underlying intuition here is that although both the variance and the covariance are changing through time, if they move through time proportionally, they will maintain a proportional relationship. Beta, the relationship between the covariance between asset returns and market returns and market variance, is constant if this relationship is proportional. The model is developed following the method shown in Schwert and Seguin (1990 pg.1139-1140).

4.1.2 Time-Varying Capital Asset Pricing Model

In the traditional CAPM:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it} \qquad (2)$$

The beta for asset i is given as:

$$\beta_i = \frac{\sigma_{im}}{\sigma^2_{mt}} \tag{3}$$

where σ_{im} is the covariance between the return on asset i and the return on the market portfolio.

As shown in Luenberger (1998, pg. 150), equation (3) can be written as:

$$\beta_i = \frac{\sum_{j=1}^{N} w_j \sigma_{ij}}{\sigma^2_{mt}} \tag{4}$$

where there are j other assets in the market portfolio, and w_j represents the market weight of asset j.

If the covariance is as given in equation (1) then equation (4) can be rewritten as:

$$\beta_{it} = \frac{\sum_{i=1}^{N} w_i (\xi + \psi \sigma^2_{mt})}{\sigma^2_{mt}} \tag{5}$$

A key point to notice here is that the beta in equation (5) is now time-varying and is different in each period t.

If the sum of the weights is constrained to equal one and the average weighted beta is also constrained to equal one, then as shown in Appendix 3, equation (5) reduces to:

$$\beta_{it} = \frac{\xi}{\sigma^2_{mt}} + \psi \tag{6}$$

Thus, if equation (1) is assumed to hold, the beta is composed of two portions; a constant part, similar to the traditional beta, and a second part that varies with the level of volatility. If, however, the constant term in equation (1) is equal to 0, then the first term of equation (6) disappears and the traditional CAPM holds. The conditional CAPM, given by this model, as shown in Appendix 3, is:

$$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt} / \sigma^2_{mt}) + \varepsilon_{it} \tag{7}$$

As shown in Appendix 3, the β_2 coefficient should, if the model holds perfectly, be identical to the ξ in the first model. If ξ is significant, it indicates that covariances and aggregate market variance do not change through time proportionally. Thus, the β_2 term adjusts the traditional CAPM to take into account the changes to the measure of risk that occurs through time due to changes in variances and covariances.

4.2 Standard Deviation and Covariance Calculations

For all data sets, the daily returns of stocks in all portfolios were calculated. They are used as input for calculating estimates of monthly standard deviations of the overall market. These estimates of monthly deviations were calculated using the method of French, Schwert and Stambough (1987):

$$s_{mt}^2 = \sum_{i=1}^{Nt} r_i^2 + 2 \sum_{i=1}^{Nt-1} r_i \, r_{i+1} \tag{8}$$

where r_i is the return on the CRSP market portfolio on day i, N_t is the number of trading days in the month and s_{mt}^2 is the estimated monthly variance of returns on the CRSP market portfolio.

Schwert and Seguin (1990) use daily returns, where they are available, arguing that more frequent sampling results in more accurate estimates of volatility. Where daily returns are not available, they estimate aggregate monthly standard deviation.

$$s_{mt} = \sqrt{\pi / 2}\, \{ |r_{it} - r_{ft}| - \mu_i \} \tag{9}$$

where μ_i is the mean of the series $|r_{it} - r_{ft}|$, where r_{it} is the return on portfolio i in month t, and r_{ft} is the risk free rate in month t. Tests, not reported in this thesis show that equations (8) and (9) give substantially the same results. The risk free rates used in this calculation are taken from Ibbotson Associates (1998).

The conditional covariances between the returns on the size-sorted and BM-sorted portfolios are a necessary input in this thesis. These conditional covariances are estimated following the method in Schwert and Seguin (1990). It is given as:

$$Cov_{t-1}(R_{it}, R_{jt}) = [(r_{it} - r_{ft}) - \mu_{Ri}] * [(r_{jt} - r_{ft}) - \mu_{Rj}] \tag{10}$$

where R_{it} represents the excess return to portfolio i in month t and μ_{Ri} represents the mean of value of R_i for all times t.

4.3 Controls for Heteroskedasticity and Autocorrelation

This thesis uses four estimation procedures to control for the problems of autocorrelation and heteroskedasticity in different ways. Heteroskedasticity may be a problem in studies of this type. Although it results in unbiased parameter estimates, the variance estimates may be biased upwards, which results in unreliable t-statistics. That is, the t-statistics are biased downwards. Autocorrelation may also be a problem While it does not bias parameter estimates, it can cause the estimated variance to be substantially smaller than the true variance. This inflates the F-statistics and t-statistics.

Multiple methodologies are used to control for these statistical problems for a number of reasons. First, using a number of methodologies allows for comparisons between methods, which, in turn, ensures that relationships identified exist and are not methodologically dependent. Second, related studies use a variety of methodologies; using all of the methods employed in related studies, while holding the variation in the data constant, enhances comparability between studies. Finally, and most importantly, as Davidian and Carroll (1987) argue, the assumed variance function contains important information in addition to being a means of controlling problems of heteroskedasticity.

4.3.1 GARCH and E-GARCH Estimations

The first two estimation procedures belong to the autoregressive conditional heteroskedasticity (ARCH) family introduced by Engle (1982). ARCH models simultaneously control for heteroskedasticity and autocorrelation. Both the generalized ARCH model (GARCH) and the exponential GARCH (E-GARCH) model are used.

The GARCH model was introduced in Bollerslev (1986). GARCH models assume that the heteroskedasticity is dependent on lagged values of the squared error term and lagged values of the estimated variance. This thesis uses a GARCH (1,1) model, which means that one lag of the squared error term and one lag of the estimated variance are included in the estimation procedure. The two models to be estimated using the GARCH (1,1) estimation method are:

$$Cov(R_{it}, R_{jt}) = \xi_{ij} + \psi_{ij} s^2_{mt} + \varepsilon_{ijt} \quad (11a)$$

$$h_{ijt} = \varpi_{ij1} + \phi_{ij1} \varepsilon^2_{ijt-1} + \gamma_{ij1} h_{ijt-1} + \eta_{ijt} \quad (11b)$$

$$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt}/s^2_{mt}) + e_{it} \quad (12a)$$

$$s_{it} = \varpi_{i2} + \phi_{i2} e^2_{it-1} + \gamma_{i2} s_{it-1} + v_{it} \quad (12b)$$

where h_t and s_t are the estimated variances at time t of equations (11a) and (12a). The data for market variance, $\hat{\sigma}^2_{mt}$, are obtained by squaring the results of equation (8). In the

GARCH equations (11b) and (12b), the first parameter represents the constant portion of volatility. The second parameter shows the amount of h_t or s_t that is determined by the previous time period's squared error term and the final parameter shows the amount of the present time period's estimated variance that can be predicted from the previous time period's estimate of the variance. The final terms, η_t and v_t, represent the error in estimation of the variances of the two models. The terms in equations (11a) and (12a) are as defined in discussing equations (1) and (7) in Section 4.1.

The model iterates between the two equations, minimizing the joint variance of part a and b of the equations, until further iterations become unnecessary. For the remainder of the paper, a reference to the GARCH or E-GARCH model indicates a (1,1) model.

The E-GARCH methodology, developed by Nelson (1991), is also used to estimate the two models. Similar to the GARCH method, the E-GARCH model also assumes that the variance of the error term at time t is dependent on lagged values of the squared error term and lagged estimates of the variance. However, the E-GARCH assumes that the variance of the function is an asymmetric function of lagged squared error terms and variances. This is consistent Christie (1982) and Black (1976), who argue that stocks prices react more strongly to volatility increases than volatility decreases. The two models, estimated using the E-GARCH procedure, are as follows:

$$\mathrm{Cov}(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\, s^2_{mt} + \varepsilon_{ijt} \quad (13a)$$

$$\ln(h_{ijt}) = \varpi_{ij1} + \phi_{ij1}\, g(z_{ijt-1}) + \gamma_{ij1} \ln(h_{ijt-1}) + \eta_{ijt} \quad (13b)$$

$$g(z_{ijt}) = \theta_{ijt} + [|z_{ijt}| - E|z_{ijt}|] \quad (13c)$$

$$z_{ijt} = \varepsilon_{ijt} / \sqrt{h_{ijt}} \quad (13d)$$

$$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt}/s^2_{mt}) + e_{it} \quad (14a)$$

$$\ln(s_{it}) = \varpi_{i2} + \phi_{i2}\, g(z_{it-1}) + \gamma_{i2} \ln(s_{it-1}) + v_{it} \quad (14b)$$

$$g(z_{it}) = \theta_{it} + 1 * [|z_{it}| - E|z_{it}|] \quad (14c)$$

$$z_{it} = \varepsilon_{it} / \sqrt{h_{it}} \quad (14d)$$

where h_t and s_t are the variances of the models at time t. ϖ is the constant portion of volatility. ϕ is the portion of variance that is dependent on lagged values of error terms, but through a different process than with the GARCH model. γ represents the part of variance at time t that is dependent on one lag of the estimate of the previous periods variance. The expected value of z_t, $|z_t|$, is equal to $(2/\pi)^{1/2}$ if z_t follows a standardized normal distribution. When z_t is positive the function $g(z_t)$ is equivalent to (θ-1); when it is negative, $g(z_t)$ equals ($\theta + 1$). Because the function gives different values for positive and negative surprises in volatility, the model allows for asymmetrical responses to these two types of volatility changes.

Pagan and Schwert (1990) find that the E-GARCH model is the best parametric model for modeling stock prices. While there are non-parametric models that outperform the E-GARCH model within sample, the E-GARCH model outperforms all of these models out-of-sample. It should be noted that Pagan and Schwert (1990) do not examine the weighted least square (WLS) method and OLS with Hansen's (1982) t-statistic correction that are also used in this thesis.

Similarly, Koutmous et. al. (1994) use GARCH estimation, but do not consider the WLS and Hansen corrected estimation. Episcopos (1996) uses E-GARCH estimation, but does not compare his results with other methodologies.

4.3.2 Methods Using Generated Regressors

In the remaining two methods, WLS and Hansen's (1982) correction, generated regressors are used to control for autocorrelation. WLS and Hansens's (1982) t-statistic correction are used to control for heteroskedasticity. Following Schwert and Seguin (1990), we use a 12th order WLS autoregressive model, and obtain estimates of the aggregate market standard deviation:

$$\hat{\sigma}_{mt} = \alpha_0 + \sum_{i=1}^{12} \beta_i s_{t-i} + \varepsilon_t \quad (15a)$$

$$|\varepsilon_t| = \gamma_0 + \gamma_1 \hat{\sigma}_{mt} + u_t \quad (15b)$$

where if γ_1 is significant, it indicates that heteroscedasticity exists and is dependent on the level of aggregate market standard deviations. If γ_0 is also significant, it indicates that the heteroscedasticity is non-proportional.

The standard deviation at time t is estimated using ordinary least squares (OLS). The absolute values of the error terms from this regression are estimated in the Glesjer (1969) regression, where they are assumed to be dependent on the aggregate level of market standard deviation. The inverse of the predicted values from this regression at time t are multiplied against all terms on the right side of the equation at time t to weight these terms. This process is repeated for a total of three iterations.

The disadvantage of using generated regressors is that it introduces estimation error from the first stage into the second stage, that is, in estimating either equation (1) or equation (7). This may lead to the variance of the second stage being underestimated. This biases the t-statistics upwards. To eliminate this problem, all t-statistics for the WLS estimates and Hansen's (1982) method are adjusted using the methods of Murphy and Topel (1985).

4.3.3 WLS and OLS with Hansen's Correction Estimations

WLS estimation begins with estimating the models using ordinary least squares. Then, the square of the error term is estimated using Glesjer (1969) regressions. In the Glesjer regressions, the error term is dependent on the aggregate market variance. The predicted error term is used as a weight for all terms in the original model, which is then re-estimated. This process is repeated for a total of three iterations. The WLS equations for the two models are as follows:

$$\mathrm{Cov}(R_{it}, R_{jt}) = \xi_{ij} + \psi_{ij} s^2_{mt} + \varepsilon_{ijt} \tag{16a}$$

$$\varepsilon^2_{ijt} = \varpi_{i1} + \delta_{ij1} s^2_{mt} + \eta_{ijt} \tag{16b}$$

$$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt}/s^2_{mt}) + e_{it} \tag{17a}$$

$$e^2_{it} = \varpi_{i2} + \delta_{i2} s^2_{mt} + v_{it} \tag{17b}$$

Equations (16a) and (17a) are as given in equations (1) and (7). Part b of the equations are similar to the correction discussed for equation (15b), where heteroskedasticity is assumed to be time-varying and dependent on the aggregate market variance.

Neter, Kutner, Nachsteim and Wasserman (1996) show that heteroskedasticity can be dependent on any of the underlying variables. Tests, not reported in this thesis, using the covariances and portfolio returns (the dependent variables) instead of aggregate market variance (the independent variable) in the Glesjer regressions showed that there is no relationship between heteroskedasticity and the level of covariance.

GMM estimation, developed by Hansen (1982), is a non-linear estimation method that gives consistent parameter estimates, but does not require that the variables be normally distributed. The version of this method used in this thesis and in Schwert and Seguin

(1990) gives the same parameter estimates as OLS estimation, but it adjusts the t-statistics. Heteroskedasticity results in estimates of the conditional variance that are larger than the true variance. This results in the t-statistics being biased downwards. By taking into account this heteroskedasticity, Hansen's (1982) correction results in reliable t-statistics.

The tables reporting the WLS and OLS results with the Hansen correction, the corrected t-statistics are presented in parenthesis beneath the parameter estimates. Beneath these t-statistics, are new t-statistics calculated using the adjustment of Murphy and Topel (1985) are reported. The Murphy and Topel (1985) adjustment follows immediately in the next section.

4.4 The Murphy Topel Adjustment

Murphy and Topel (1985) present a method of adjusting two stage econometric models, such as the WLS and GMM estimations in this thesis. In our model, the first stage is the 12^{th} order autoregressive model of monthly standard deviations in equation (15). The second step is the estimation of the portfolio covariances or the conditional CAPM. Because the first stage estimates contain sampling error, the second stage estimated standard errors are biased downwards. This results in inflated t-statistics that must be adjusted downwards to be properly interpreted.

Their article presents two methods of adjustments. The first method assumes that the first stage and second stage error terms are uncorrelated. However, as Murphy and Topel (1985) point out, when the two stages are estimated from the same or contemporaneous data, this assumption may not be correct. Because of this possibility, they derive a generalized method that allows for correlations between each stage's error terms. If there is no correlation between error terms, the second model gives results that are identical to the first model. Since the market index is derived from data that includes the individual portfolio data used for returns and covariance estimates, we use the second model. Estimates using the first model, not reported here, are essentially the same.

Their adjustment, as shown in Murphy and Topel (1985, pg. 376), is as follows:

$$\Sigma = \sigma^2 Q_0^{-1} + Q_0^{-1}[Q_1 R^{-1}(\theta) Q_1^{'} - Q_1 R^{-1}(\theta) Q_2^{'} - Q_2 R^{-1}(\theta) Q_1^{'}] Q_0^{-1} \qquad (18)$$

where;

Σ = the adjusted variance/covariance matrix

$Q_0 = n^{-1} Z^{`}Z$

$Q_1 = n^{-1} ZF^{*}$

$Q_2 = n^{-1} \sum_{i=1}^{n} Z_i^{'} \hat{u}_i l^{'}(\hat{\theta}; x_{1i})$

X = the data used in the first step estimation

Z = predictions from the first stage equation used in the second step

F = the predictions from the second stage model

F*=the first derivative of the F matrix

n = the number of observations

$\hat{u}$ =estimated residual from the second step

R= the information matrix

$\hat{\sigma}^2$ = 2nd stage variance before adjustment

The first term on the right is the unadjusted variance/covariance matrix and the remainder is the adjustment.

4.5 Information Content of the Assumed Variance Function

There are three assumed variance functions estimated using three different methods, the GARCH, E-GARCH and Glesjer regressions. Davidian and Carroll (1987) argue that the variance function is itself of interest, not only for its ability to control heteroskedasticity. In the GARCH regressions, the error term is assumed to be dependent on its own squared lagged values and lagged estimates of the variance of the error terms. The E-GARCH model is similar, but assumes that increases in volatility are of more significance than decreases in volatility. The Glesjer regression used here, assumes that the error term is dependent on the level of market variance in the present period.

The Glesjer method is rarely used in studies of financial markets. Its statistical performance and properties relative to the ARCH family estimation procedures have not been examined. While Schwert and Seguin (1990) benefit extensively from the Glesjer regressions, they do not contrast its performance and properties, while holding the variation in data constant, against the ARCH family estimations. While this thesis does not study the statistical performance property issues in detail, it gives a first glimpse of the differences in the estimation results between different procedures, while holding the variation in the data constant. This is another contribution of the thesis to the literature.

5. Results and Interpretation

5.1 Estimates of Market Standard Deviations

The results of the estimates of market standard deviations, estimated using WLS as in Schwert and Seguin (1990, pg. 1133), and subsequently used as generated regressors in the OLS and WLS estimates are shown in Table 1. For all samples the number of observations is considerably lower than that in the Schwert and Seguin (1990) sample. They have over 600 observations, while the samples used in this thesis have between 60 and 120 observations. Panels A and B of Table 1 show that there is far more autocorrelation for the NYSE sample during the period 1987-1997 than for the NYSE, AMEX and NASDAQ sample covering the period 1990-1997. This difference is likely driven either because the years 1987-1990 have higher autocorrelation or else because the NYSE has higher autocorrelation than the other exchanges. There is higher serial correlation for the period 1980-1986 than for the 1987-1997 period for NYSE stocks. This is consistent with the findings of Schwert and Seguin (1990), who note that autocorrelation is declining over time.

5.2 Variance Dependent Covariances

These models test the relationship between the conditional covariances of portfolios and the aggregate market variance. If the ψ term is significant, it indicates that the covariance at time t is dependent on the aggregate market variance. If the ξ is also significant, it indicates that while the covariance is dependent on the market variance, it is not a completely proportional relationship and that the market variance alone does not completely explain the covariance.

5.2.1 Conditional Covariances of Size-Sorted NYSE Portfolios for the Period 1987-1997

There is a significant positive relationship between market variance and covariances, shown in Tables 2-5. The Glesjer regression results, shown in Tables 4 and 5, have insignificant constants, and significant or marginally significant ψ after the Murphy-Topel adjustment. This indicates that the covariances and variances of the portfolios are fully dependent on the aggregate market variance. The GARCH and E-GARCH results, shown in Tables 2 and 3, show highly significant constants, ξ, and highly significant market variance coefficients, ψ. The R^2 are much higher than for the Glesjer regressions.

Although the estimation methods give different significances, the general pattern is similar. The coefficient estimated for ψ for small stock portfolios is larger than for other portfolios whose coefficients decrease as size increases. This is also consistent with the predictions of Black (1976). The covariances of smaller stocks also increase by a greater amount than for larger stocks. The major implication of this is that small stocks are more sensitive to changes in market variance.

As discussed in the methodology section, if the intercept term in equation (l) is significant, then there is evidence in favor of using the conditional CAPM. Based on these results, it is expected that the conditional version of the CAPM will improve estimates of portfolio returns for the GARCH and E-GARCH methods, but not the Glesjer methods.

Schwert and Seguin (1990, pg. 1134) show that a significant relationship between portfolio returns and market variance could cause the regressions in this section to be misspecified. To ensure that this is not a problem, regressions of portfolio returns on market variance were run using OLS. They show that there is not a significant relationship between portfolio returns and market variance.

The GARCH and E-GARCH results, shown in Tables 2 and 3, show that there is strong autocorrelation during this period. The squared, lagged error term coefficient, ϕ, is significant for all portfolios. The lagged estimated variance coefficient, γ, is significant in most, but not all, of the regressions. For the Hansen-corrected and WLS estimates the constant portion of volatility, ϖ, is insignificant in all cases. The market variance coefficient, δ, is significant in all cases. This shows that much of the heteroskedasticity in the portfolio return covariances is driven by changes in the market variance.

Tables 14-17 show results for size-sorted NYSE stock portfolios, covering the period 1980-1986. The GARCH and E-GARCH results show a significant relationship between the portfolios' return covariances however the coefficients are smaller than for the Schwert and Seguin (1990) sample and the 1987-1997 sample. The WLS and Hansen estimates of ψ, the relationship between covariances and market variance, are all insignificant. A possible explanation for these findings is the relatively small sample size.

5.2.2. Conditional Covariances for BM-Sorted Portfolios for the Period 1990-1997

All estimation procedures, shown in Tables 6-9, suggest that there is a significant, positive relationship between portfolio covariances and market volatility. The E-GARCH methodology, shown in Table 7, did not work well with numerous regressions, failing to converge even when the maximum number of iterations was increased to 150. Thus, the results will not be discussed here.

The Glesjer results, in Tables 8 and 9, show that some ξ are significant until the Murphy-Topel adjustment. After the adjustment the significance disappears in all cases. The GARCH results, shown in Table 6, show that the constant terms are not significantly different from 0, indicating that covariances vary through time proportionally to market volatility. The fact that the constants are not significant and that the covariances are proportional to market volatility suggests that the time-varying coefficient in the CAPM model will not be significant.

The results in Tables 6-9 support the intuition of Black (1976) who stated that when volatility changes, all stocks move in the same direction and that the higher beta stocks, such as value stocks, move by a greater amount. The ψ coefficients are generally larger for value stocks' covariances and decline for the growth stock covariances such as covariances (4,5) and (3,5). The variance of value stocks increases by a much greater amount than for other portfolios which generally decline as the BM ratio decreases.

As shown in Table 1, Panels A and B, there is less autocorrelation for this sample than for the sample in the previous section. As a result, the GARCH terms, ϕ and γ, which depend on squared lagged error terms and lagged estimates of the error term's variance respectively, are less significant than in the previous sample. The Glesjer regressions show, that, after the Murphy-Topel adjustment, the current aggregate market variance mainly determines the current period error term, as the constant term is generally insignificant.

5.2.3 Volatility Dependent Covariances Size-Sorted Portfolios for the Period 1990-1997

The volatility dependent covariances for this sample, shown in Tables 10-13, are similar to the BM results. For all estimation methods, the covariances are time-varying and are positively related to market variance. Consistent with the arguments of Black (1976) and the empirical findings of Schwert and Seguin (1990), the smaller stock portfolios' covariances generally increase by a larger amount when market variance increases.

For this sample, the WLS estimates, shown in Table 13, have intercepts that are significant and negative. None of the other estimation methods produce significant intercepts. Because of this, as shown in the methodology section, the time-varying term in the WLS estimates of the conditional CAPM should be significant. The fact that only one estimation method results in a significant intercept coefficient indicates that this finding may be methodologically dependent.

5.2.4 Comparisons Between Size-Sorted and BM-Sorted Portfolios for the Period 1990-1997

The general pattern of results for size-sorted and BM-sorted portfolios is quite similar. The portfolios that report anomalously high returns, the value stocks and small caps, have greater sensitivity to changes in market variance. However, there is a greater spread in the size of the ψ coefficients, the measure of sensitivity to changes in market variance, for value versus growth stocks than for small caps versus large caps.

5.3 Conditional CAPM

In the conditional version of the CAPM, shown in equation (7), the α term should be not significantly different from 0 if the CAPM adequately explains returns. If the CAPM does not adequately explain returns, there will be either abnormal gains to that portfolio, represented by a positive α, or abnormal losses, represented by a negative alpha. The β_1 term is the traditional CAPM beta that represents the amount of risk, assuming constant covariances and constant market variance. The β_2 term represents the part of the asset's return that is time-varying and dependent on the market's variance.

5.3.1. Conditional CAPM for Size-sorted NYSE Stocks for the period 1987-1997

This sample is a direct extension of Schwert and Seguin (1990), which ends in 1986 while this sample begins in 1987. All estimation methods give quite similar results. The conditional CAPM results, in Tables 18-21, show that only the portfolio of largest firms and the next to smallest have a significant β_2 coefficient. If the model developed in the methodology held perfectly, we would expect to see significant coefficients for all portfolios estimated using GARCH and E-GARCH methods. The largest stocks have a positive coefficient, while the next to largest have a negative coefficient, as shown in Tables 18 and 19. This provides some evidence that, consistent with Schwert and Seguin (1990), the beta spread for size-sorted portfolios increases during periods of high market volatility with the smaller stocks becoming relatively more risky. However, because only two of the five portfolios have significant time-varying coefficients, this evidence must be regarded as weak.

This sample provides weak evidence of a reverse size effect as small stocks consistently have negative alphas, while the large stock portfolio consistently has positive alphas. However, there is no significance to these results. As with the earlier size-sorted sample, β_1 successfully explains the vast majority of returns providing, evidence in support of the CAPM.

The Hansen-corrected betas for the 1987-1997 period are less significant and smaller than those in the Schwert-Seguin study. The WLS estimates are also smaller and less significant in our study, but not by a great margin. In general, the results suggest that covariances and asset prices are less dependent on the aggregate market variance than they were during previous decades.

5.3.2 Conditional CAPM for BM-Sorted Portfolios for the Period 1990-1997

This model directly tests one version of the Fama and French (1992) suggestion that BM proxies for some type of business cycle risk, specifically it tests for the relationship between BM-sorted portfolios and market variance. The results for the conditional CAPM, shown in Tables 22-25, does not work well for BM-sorted portfolios as can be seen from the β_2 column in these tables. For all estimation methods, the time-varying term has no consistent significance through the various estimation methods. There is also no discernable pattern of negative or positive signs associated with this term. This is to be expected, given the insignificant alphas in the tests of the covariance model. Although both covariances and variance are time-varying, their movements through time are proportional to one another. Therefore, although they both change, their relationship remains constant through time and the static CAPM captures virtually all of the pricing information.

The Glesjer estimation methods do not show a value premium, but the GARCH and E-GARCH methods do. For all methods, β_1 has a large amount of explanatory power in contrast to the findings of Fama and French (1992). Although the value premium, shown in the significant alphas, is evidence against the CAPM holding perfectly, the large R^2

and highly significant β_1 term contradict the findings of Fama and French (1992) who claim that beta has little explanatory power. In general, our results are more consistent with Fama and French (1993) that states that the traditional CAPM beta provides useful information, but can be supplemented with the additional factors of size and BM.

We do not find evidence that suggests the BM effect is driven by time-varying volatility. However, market volatility is only one of many business cycle variables; so, these findings cannot be considered evidence against the suggestion of Fama and French (1992). They merely fail to provide evidence for one specific version of this hypothesis.

5.3.3 Conditional CAPM for Size-Sorted Portfolios for the Period 1990-1997

Interestingly, only the GARCH method, shown in Table 26, shows a significant size premium. Other methodologies show no evidence of this size premium. A possible explanation for this is the findings of Shumway and Warther (1999), who report that, after controlling for the survival bias in the CRSP tapes, there is no evidence of a size premium at any time in history. Their article notes that the survival bias in CRSP data has been reduced since 1987. Consequently, our sample should not have a significant survival bias.

With the GARCH and E-GARCH methods, shown in Tables 26 and 27, respectively, the small stock portfolio time-varying coefficient is negative. The larger stock coefficients are positive; this is consistent with the findings of Schwert and Seguin (1990). However, the WLS estimates in Table 29 have the opposite pattern; large stocks' β_2 coefficient is negative, while the small stock portfolio coefficient estimate is positive. Overall, the results suggest that there is not a reliable β_2 coefficient for any of the portfolios. Any significance that arises seems to be mostly a product of methodology.

The WLS estimates of this model, shown in Table 29, show some significance in the time-varying risk premium coefficient, however other estimation methods, shown in Tables 26-28, are generally not significant. This is consistent with or earlier findings that show only WLS estimates of the covariance relationship find a significant intercept term.

Consistent with the BM-sorted portfolios, we find that β_1 has a great deal of explanatory power. The lowest R^2 is over 0.70 and the highest is over 0.97. This strongly contradicts the findings of Fama and French (1992), who claim that beta has no explanatory power. Overall, the findings of the size-sorted portfolios and BM-sorted portfolios suggest that the traditional CAPM generally works during this sample period.

6. Conclusion

This thesis has examined the relationship between aggregate market variance and the covariances of BM and size-sorted portfolios. It has also examined the pricing of these portfolios when this relationship between portfolio covariances and the market variance is explicitly modeled. We find that for BM-sorted portfolios and size-sorted portfolios during the 1990s, formed from the intersection of the COMPUSTAT and CRSP databases, there is evidence of time-varying covariances in response to changes in aggregate market variance. However, for these portfolios, the changes to covariances and volatility are proportional through time and, consequently, the static CAPM performs well.

Fama and French (1992) suggest that the BM anomaly could represent a relationship between time-varying economic risk and equity returns that is missed by the static CAPM. We test one version of this possibility by estimating returns to BM-sorted portfolios taking into account the time-varying nature of volatility. The results do not support the possibility that changes in volatility drive the BM effect.

This thesis also extends the work of Schwert and Seguin (1990) into the 1990s. We find that market volatility continues to drive covariance changes. However, we do not find evidence of a size effect.

This thesis has also compared a variety of methods for controlling common statistical problems that arise in financial studies. While GARCH and E-GARCH models are the most common methods for controlling autocorrelation and heteroskedasticity simultaneously, our results indicate that autocorrelation is a less of a problem in the past decade than it has been previously. The aggregate level of market volatility drove much of the heteroskedasticity in the 1990-1997 sample. In this situation, WLS appears to be a more effective method of controlling heteroskedasticity.

This thesis suggests a number of interesting ideas for future research. First, as noted in the thesis, market variance is only one business cycle related variable that could drive the BM premium. There are numerous other variables such as interest rates, GDP or monetary conditions that could be driving the BM effect. Secondly, the different estimation procedures used in this thesis gave differing results for various tests. A study of different estimation methods and their effects on tests of asset pricing models would be of interest for empirical financial studies.

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Appendix 1: Size and Book-to-Market Sorts by Year

1990	Value	1	2	3	Growth
Small	97	101	122	153	129
2	178	135	114	89	87
3	167	145	128	97	55
4	123	146	125	117	66
Large	38	76	114	147	92

1991	Value	1	2	3	Growth
Small	222	94	65	66	120
2	149	86	83	97	152
3	113	145	102	97	109
4	74	126	142	117	110
Large	65	119	145	158	80

1992	Value	1	2	3	Growth
Small	245	87	80	61	124
2	173	92	79	102	151
3	133	116	124	102	121
4	63	145	122	143	123
Large	54	119	152	166	105

1993	Value	1	2	3	Growth
Small	270	104	78	59	142
2	166	103	112	118	155
3	136	139	137	100	142
4	87	153	130	164	120
Large	64	131	175	165	118

1994	Value	1	2	3	Growth
Small	283	127	91	77	91
2	187	133	125	115	168
3	137	151	149	138	154
4	104	150	155	171	147
Large	73	158	189	172	134

1995	Value	1	2	3	Growth
Small	306	147	107	94	167
2	191	154	138	149	189
3	167	163	159	147	184
4	126	168	181	194	151
Large	93	188	190	204	143

1996	Value	1	2	3	Growth
Small	344	181	112	93	236
2	268	173	162	170	191
3	217	194	173	186	196
4	131	193	222	219	200
Large	83	224	240	234	182

1997	Value	1	2	3	Growth
Small	296	158	109	120	272
2	317	182	158	138	171
3	256	205	169	184	152
4	163	218	214	205	160
Large	128	246	230	195	162

Appendix 2

Table 1 Estimates of Market Standard Deviations Based on 12 Lags

$$\hat{\sigma}_{mt} = \alpha_0 + \sum_{i=1}^{12} \beta_i s_{t-i} + \varepsilon_t$$

$$|\varepsilon_t| = \gamma_0 + \gamma_1 \hat{\sigma}_{mt} + u_t$$

This model shows that $\hat{\sigma}_{mt}$, the autocorrelation free estimated market standard deviation at time t, is a function of 12 lags of the market deviation as measured by equation (8). The error term is assumed to be a function of a constant, γ_0, and the estimated market standard deviation at time t. The autocorrelation free estimated market standard deviation is estimated using WLS where the inverse of the $|\varepsilon_t|$ are used as weights for three iterations between the equations.

Panel A: Estimates of the market standard deviation based on 12 lags used for BM and size-sorted portfolios for the period 1990-1997.

	Beta	t-statistic	R^2	Overall Model Significance
Intercept	0.026487	2.893	0.1970	0.1105
Lag1	0.230188	1.092		
Lag2	0.095147	0.872		
Lag3	0.079449	0.722		
Lag4	-0.133077	-1.968		
Lag5	-0.023110	-0.253		
Lag6	0.069623	0.672		
Lag7	-0.143289	-1.569		
Lag8	-0.093418	-1.134		
Lag9	-0.066758	-0.782		
Lag10	0.205856	1.881		
Lag11	-0.115841	-1.311		
Lag12	0.106585	0.967		

Panel B: Estimates of the market standard deviation based on 12 lags used for NYSE size-sorted portfolios for the period 1987-1997.

	Beta	t-statistic	R^2	Overall Model Significance
Intercept	0.010788	1.468	0.4893	0.0001
Lag1	0.231062	1.936		
Lag2	-0.145176	-1.603		
Lag3	0.289052	2.305		
Lag4	0.010545	0.105		
Lag5	0.047690	0.511		
Lag6	-0.112425	-3.781		
Lag7	0.011600	0.152		
Lag8	0.027707	0.330		
Lag9	0.246363	2.032		
Lag10	0.015698	0.172		
Lag11	0.113445	0.965		
Lag12	0.015522	0.162		

Panel C: Estimates of the market standard deviation based on 12 lags used for the NYSE size-sorted portfolios during the period 1980-1986.

	Beta	t-statistic	R^2	Overall Model Significance
Intercept	0.013723	1.538	0.6898	0.0001
Lag1	-0.080655	-0.640		
Lag2	0.311868	2.083		
Lag3	0.266214	2.105		
Lag4	0.101914	0.766		
Lag5	0.002865	0.0.026		
Lag6	-0.027636	-0.238		
Lag7	0.246584	1.749		
Lag8	-0.066217	-0.570		
Lag9	-0.040282	-0.400		
Lag10	-0.175244	-2.528		
Lag11	0.110627	0.845		
Lag12	-0.007287	-0.064		

Table 2

Time-varying covariances for NYSE stock size-sorted portfolios during the period 1987-1997, using GARCH estimation.

$$Cov(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\hat{\sigma}^2_{mt} + \varepsilon_{ijt}$$

$$h_t = \varpi_{ij} + \phi_{ij}\,\varepsilon^2_{t-1} + \gamma_{ij}\,h_{t-1} + \eta_{ijt}$$

$Cov(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term. The 1 indicates the smallest stock portfolio and the 5 indicates the largest stock portfolio. Thus, Cov14 would be the covariance between the small stock portfolio and the next to largest portfolio.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	ϕ_{ij}	γ_{ij}	R^2
Cov11	-0.005246 (-18.714)	0.278743 (31.071)	0.000001 (1.879)	2.630098 (6.180)	0.145069 (6.299)	0.5261
Cov12	-0.003925 (-17.527)	0.205026 (26.157)	0.000001 (3.422)	3.818365 (6.168)	0.002024 (0.150)	0.7092
Cov13	-0.005516 (-28.233)	0.265731 (52.047)	0.000002 (2.468)	2.538332 (5.757)	0.001966 (0.207)	0.8034
Cov14	-0.004799 (-20.595)	0.242695 (37.739)	0.000002 (4.390)	1.970734 (4.971)	0.010760 (0.404)	0.7944
Cov15	-0.003713 (-9.870)	0.193655 (23.799)	0.000001 (3.475)	0.820510 (3.639)	0.353369 (4.280)	0.7878
Cov22	-0.005027 (-32.223)	0.237918 (56.878)	0.000001 (3.341)	2.946785 (6.794)	0.000000 (0.000)	0.8055
Cov23	-0.004916 (-18.878)	0.238964 (40.402)	0.000002 (4.450)	2.063586 (5.362)	0.017002 (0.373)	0.8106
Cov24	-0.004384 (-22.833)	0.217671 (41.213)	0.000001 (3.351)	2.533654 (5.879)	0.025727 (0.718)	0.7991
Cov25	-0.003930 (-16.752)	0.199696 (34.840)	0.000001 (2.244)	1.382857 (3.936)	0.234490 (2.916)	0.8060
Cov33	-0.004882 (-13.256)	0.248309 (30.356)	0.000001 (3.243)	1.086559 (3.087)	0.299588 (3.154)	0.8110
Cov34	-0.004258 (-16.450)	0.222014 (34.109)	0.000001 (1.737)	1.541579 (4.160)	0.282926 (4.243)	0.8016
Cov35	-0.004054 (-16.342)	0.206512 (36.857)	0.000000 (1.697)	0.952500 (2.905)	0.378930 (4.388)	0.8074
Cov44	-0.003766 (-13.931)	0.200630 (28.091)	0.000000 (1.299)	1.674367 (4.180)	0.293374 (4.430)	0.7901
Cov45	-0.004129 (-13.726)	0.198636 (37.320)	0.000000 (0.704)	0.221212 (2.544)	0.795803 (12.318)	0.8115
Cov55	-0.003388 (-12.968)	0.169018 (41.305)	0.000000 (1.134)	0.148839 (2.751)	0.847986 (21.928)	0.7924

Table 3

Time-varying Covariances for size-sorted NYSE stocks for the period 1987-1997, using E-GARCH estimation.

$$Cov(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\,\hat{\sigma}^2_{mt} + e_{ijt}$$

$$\ln(h_{ijt}) = \varpi_{ij} + \phi_{ij}\, g_{ij}(z_{t-1}) + \gamma_{ij}\ln(h_{ijt-1}) + \eta_{ijt}$$

$$g_{ij}(z_{ijt}) = \theta_{ijt} + [\,|z_{ijt}| - E|z_{ijt}|\,]$$

$$z_{ijt} = e_{ijt} / \sqrt{h_{ijt}}$$

$Cov(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term. However, the variance of the error term is assumed to respond asymmetrically to changes in volatility with past increases in the functions variance causing a greater reaction than decreases in the variance function. The 1 indicates the smallest stock portfolio and the 5 indicates the largest stock portfolio. Thus, Cov14 would be the covariance between the small stock portfolio and the next to largest portfolio.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	ϕ_{ij}	γ_{ij}	θ_{ij}	R^2
Cov11	-0.004118 (-0.025)	0.208811 (1.25)	-3.955061 (-23.676)	2.276709 (13.629)	0.617438 (3.696)	-0.426474 (-2.553)	0.4445
Cov12	-0.005300 (-0.053)	0.251649 (2.516)	-5.648018 (-56.480)	1.547464 (15.475)	0.485099 (4.851)	-0.126983 (0.2041)	0.8085
Cov13	-0.005762 (-16.414)	0.262856 (28.901)	-5.486022 (-3.543)	0.773617 (4.429)	0.533011 (4.042)	-0.935461 (-4.773)	0.8048
Cov14	-0.005009 (-16.291)	0.237676 (26.339)	-4.543096 (-3.995)	0.853575 (5.041)	0.619256 (6.440)	-0.904345 (-4.864)	0.8224
Cov15	-0.004129 (-11.419)	0.190660 (16.120)	-5.916508 (-4.082)	0.529037 (3.676)	0.507467 (4.255)	-1.450121 (-3.814)	0.7924
Cov22	-0.005302 (-17.193)	0.237657 (32.360)	-7.958385 (-4.468)	1.018393 (5.032)	0.323535 (2.135)	-0.761222 (-6.531)	0.8079
Cov23	-0.00522 (112.613)	0.240050 (129.090)	-6.368413 (-3.625)	0.889026 (4.364)	0.462998 (3.166)	-0.763263 (-5.080)	0.8159
Cov24	-0.004626 (-15.371)	0.218679 (26.115)	-5.074874 (-3.789)	0.968126 (4.489)	0.577545 (5.143)	-0.766442 (-4.285)	0.8039
Cov25	-0.004158 (-12.664)	0.198926 (20.481)	-3.866558 (-2.629)	0.904566 (3.159)	0.683419 (5.698)	-0.527138 (-2.150)	0.8276
Cov33	-0.004143 (-11.842)	0.191286 (18.163)	-9.987838 (-9.298)	1.902118 (6.683)	0.138018 (1.610)	-0.763348 (-14.617)	0.7589
Cov34	-0.004514 (-15.729)	0.220190 (27.080)	-3.78458 (-3.594)	0.958117 (4.504)	0.670682 (7.297)	-0.727822 (-4.346)	0.8074
Cov35	-0.004196 (-13.038)	0.203256 (21.061)	-3.358180 (-2.381)	0.828566 (2.870)	0.726333 (6.276)	-0.561457 (-2.432)	0.8137
Cov44	-0.004130 (-24.984)	0.20388 (30.291)	-3.314539 (-3.030)	1.001029 (4.387)	0.723246 (7.883)	-0.717787 (-5.468)	0.7979
Cov45	-0.003870 (-11.649)	0.192116 (18.298)	-0.875817 (-0.834)	0.485707 (2.357)	0.927666 (10.907)	-0.323863 (-1.039)	0.8072
Cov55	-0.003702 (-11.815)	0.175280 (19.879)	-18.70206 (-12.809)	0.376977 (3.176)	-0.52430 (-4.476)	-0.954692 (-2.596)	0.7952

Table 4

Time-varying covariances of size-sorted NYSE portfolios during the period 1987-1997, using Hansen's (1982) t-statistic correction.

$$\mathrm{Cov}(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\hat{\sigma}^2_{mt} + \varepsilon_{ijt}$$

$$\varepsilon^2_{ijt} = \varpi_{ij} + \delta_{ij}\hat{\sigma}^2_{mt} + e_{ijt}$$

$\mathrm{Cov}(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. The first t-statistics, directly below the parameter estimates, are estimated using Hansen's (1982) t-statistic correction while the t-statistics below these are Hansen's (1982) t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem. The 1 indicates the smallest stock portfolio and the 5 indicates the largest stock portfolio. Thus, Cov14 would be the covariance between the small stock portfolio and the next to largest portfolio.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	δ_{ij}	R^2
Cov11	0.0018488 (2.24) (0.85)	0.981846 (1.93) (1.89)	0.0000493 (0.90) (0.45)	0.053385 (1.35) (1.34)	0.0132
Cov12	0.0009758 (2.42) (0.60)	0.75926 (1.75) (1.72)	$-3.553E^{-7}$ (-0.02) (-0.00)	0.040631 (1.20) (1.19)	0.0178
Cov13	0.000998 (2.56) (0.69)	0.668198 (1.59) (1.57)	$-4.568E^{-7}$ (-0.02) (-0.01)	0.037401 (1.16) (1.15)	0.0134
Cov14	0.000873 (2.42) (0.66)	0.625607 (1.57) (1.56)	$-1.216E^{-6}$ (-0.07) (-0.02)	0.033631 (1.15) (1.14)	0.0138
Cov15	0.000861 (2.92) (0.82)	0.488400 (1.59) (1.57)	$1.353E^{-6}$ (0.13) (0.03)	0.019815 (1.12) (1.11)	0.0177
Cov22	0.000827 (2.39) (0.63)	0.617854 (1.57) (1.55)	$-2.191E^{-6}$ (-0.13) (-0.03)	0.033129 (1.14) (1.13)	0.145
Cov23	0.000869 (2.44) (0.72)	0.570108 (1.47) (1.45)	$-1.849E^{-6}$ (-0.12) (-0.03)	0.031321 (1.13) (1.12)	0.0119
Cov24	0.0008546 (2.40) (0.75)	0.532080 (1.44) (1.42)	$-1.742E^{-6}$ (-0.08) (-0.03)	0.028114 (1.12) (1.11)	0.0114
Cov25	0.000718 (2.39) (0.71)	0.471432 (1.48) (1.46)	$-9.112E^{-8}$ (-0.01) (-0.00)	0.01968 (1.11) (1.10)	0.0149
Cov33	0.0009949 (2.67) (0.87)	0.536712 (1.40) (1.39)	$-1.340E^{-6}$ (-0.09) (-0.02)	0.029805 (1.12) (1.11)	0.0099

Cov34	0.000984 (2.65) (0.92)	0.500169 (1.37) (1.36)	$-5.194E^{-7}$ (-0.04) (-0.01)	0.026697 (1.11) (1.10)	0.0096
Cov35	0.000841 (2.60) (0.86)	0.451482 (1.43) (1.41)	$4.553E^{-7}$ (0.05) (0.01)	0.018798 (1.11) (1.10)	0.0128
Cov44	0.00105107 (2.77) (1.05)	0.463697 (1.33) (1.32)	$5.739E^{-7}$ (0.05) (0.00)	0.23831 (1.10) (1.09)	0.0090
Cov45	0.000936 (2.80) (1.02)	0.419439 (1.39) (1.38)	$1.407E^{-6}$ (0.16) (0.04)	0.016753 (1.10) (1.09)	0.0118
Cov55	0.00095198 (3.09) (1.11)	0.388435 (1.44) (1.42)	$2.041E^{-6}$ (0.33) (0.08)	0.011814 (1.12) (1.11)	0.0161

Table 5

Time-varying covariances of NYSE size-sorted portfolios data for the period 1987-1997, using WLS estimators.

$$Cov(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\hat{\sigma}^2_{mt} + \varepsilon_{ijt}$$

$$\varepsilon^2_{ijt} = \varpi_{ij} + \delta_{ij}\hat{\sigma}^2_{mt} + e_{ijt}$$

$Cov(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. WLS estimation involves using the predicted values of the second equation as weights for the terms in the first equation and then re-estimating the first equation. We iterate between the equations 3 times. The first t-statistics, directly below the parameter estimates, are estimated using WLS while the t-statistics below these are WLS t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem. The 1 indicates the smallest stock portfolio and the 5 indicates the largest stock portfolio. Thus, Cov14 would be the covariance between the small stock portfolio and the next to largest portfolio.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	δ_{ij}	R^2
Cov11	-0.00019 (-0.229) (-0.039)	2.317089 (2.814) (2.704)	0.001296 (1.060) (0.562)	0.954429 (5.527) (4.810)	0.0629
Cov12	-0.000074 (-0.191) (-0.026)	1.398764 (3.363) (3.180)	0.000599 (0.661) (0.265)	1.01642 (4.825) (4.326)	0.0875
Cov13	-0.000050 (-0.144) (-0.018)	1.316975 (3.342) (3.162)	0.000493 (0.564) (0.265)	1.040476 (4.829) (4.329)	0.0865
Cov14	-0.00000875 (-0.025) (-0.004)	1.160313 (3.064) (2.923)	0.000429 (0.512) (0.186)	1.050405 (4.508) (4.093)	0.0737
Cov15	0.000342 (0.830) (0.200)	0.804581 (2.249) (2.192)	0.000347 (0.474) (0.162)	0.986980 (3.678) (3.442)	0.0411
Cov22	0.000103 (0.268) (0.048)	1.036664 (2.747) (2.644)	0.000271 (0.317) (0.116)	1.068764 (4.100) (3.721)	0.0601
Cov23	0.000132 (0.335) (0.063)	1.002975 (2.649) (2.557)	0.000229 (0.272) (0.097)	1.077136 (4.097) (3.778)	0.0561
Cov24	0.000174 (0.437) (0.089)	0.925036 (2.463) (2.695)	0.000231 (0.285) (0.098)	1.075286 (3.951) (3.662)	0.0489
Cov25	0.000178 (0.505) (0.108)	0.792081 (2.401) (2.014)	0.000295 (0.426) (0.136)	1.005908 (3.736) (3.489)	0.0466

Cov33	0.000251 (0.610) (0.123)	0.976421 (2.537) (2.455)	0.000144 (0.170) (0.061)	1.075189 (4.063) (3.751)	0.0517
Cov34	0.000311 (0.736) (0.165)	0.896690 (2.328) (2.265)	0.000170 (0.207) (0.072)	1.075830 (3.917) (3.634)	0.0439
Cov35	0.000265 (0.718) (0.159)	0.797092 (2.351) (2.286)	0.000277 (0.399) (0.118)	1.005227 (3.843) (3.397)	0.0447
Cov44	0.000438 (0.978) (0.251)	0.824993 (2.115) (2.067)	0.000183 (0.225) (0.078)	1.069782 (3.758) (3.772)	0.0365
Cov45	0.000392 (0.974) (0.246)	0.748524 (2.162) (2.111)	0.000289 (0.421) (0.134)	0.992680 (3.736) (3.489)	0.0381
Cov55	0.000471 (1.197) (0.319)	0.687192 (2.207) (2.153)	0.000357 (0.591) (0.184)	0.896276 (3.643) (3.370)	0.396

Table 6

Time-varying covariances for BM-sorted portfolios during the period 1990-1997, using GARCH estimation.

$$Cov(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\hat{\sigma}^2_{mt} + \varepsilon_{ijt}$$

$$h_t = \varpi_{ij} + \phi_{ij}\,\varepsilon^2_{t-1} + \gamma_{ij}\,h_{t-1} + \eta_{ijt}$$

$Cov(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term. The 1 indicates value and the 5 indicates growth. Thus, Cov14 would be the covariance between the value portfolio and portfolio 4.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	ϕ_{ij}	γ_{ij}	R^2
Cov11	0.000774 (0.537)	4.212632 (4.156)	0.000028 (11.657)	$3.67E^{-23}$ (0.000)	$2.378E^{-24}$ (0.000)	0.2587
Cov12	0.000177 (0.421)	3.058695 (9.145)	0.000002 (2.118)	1.314981 (5.432)	0.227726 (4.152)	0.3311
Cov13	0.000425 (0.464)	3.151135 (4.425)	0.000012 (9.967)	$-1.540E^{-23}$ (-0.000)	$-3.898E^{-23}$ (-0.000)	0.3102
Cov14	0.000280 (0.474)	2.357571 (3.527)	0.000007 (10.611)	0.386621 (2.546)	$-4.726E^{-19}$ (-0.000)	0.2688
Cov15	0.00066 (0.419)	2.665738 (2.613)	0.000025 (13.330)	$3.722E^{-23}$ (0.000)	0.0000787 (0.000)	0.1344
Cov22	0.000546 (0.962)	2.743067 (8.363)	0.000006 (5.839)	0.424491 (3.185)	$7.2874E^{-13}$ (0.170)	0.3817
Cov23	0.000397 (0.809)	2.460031 (9.935)	0.000005 (6.217)	0.307200 (3.042)	$5.2085E^{-13}$ (0.001)	0.3674
Cov24	0.000394 (0.935)	2.142629 (8.246)	0.000004 (6.750)	0.26759 (2.784)	$3.188E^{-12}$ (0.012)	0.3335
Cov25	0.000402 (0.600)	1.858604 (3.511)	0.000008 (12.914)	0.329929 (2.538)	$6.435E^{-22}$ (0.000)	0.1990
Cov33	0.00045 (0.905)	2.222848 (9.644)	0.000005 (6.680)	0.252551 (2.839)	$-5.484E^{-22}$ (-0.000)	0.3364
Cov34	0.000449 (1.018)	1.934772 (8.322)	0.000004 (7.522)	0.196269 (2.637)	$2.039E^{-12}$ (0.023)	0.3096
Cov35	0.000617 (0.742)	1.973426 (4.019)	0.000010 (12.900)	$1.644E^{-23}$ (0.000)	$-5.098E^{-23}$ (-0.000)	0.1716
Cov44	0.00055 (1.309)	1.654221 (6.818)	0.000003 (7.396)	0.15241 (1.898)	$2.750E^{-12}$ (0.150)	0.2650
Cov45	0.000680 (0.918)	1.522766 (3.107)	0.000007 (11.410)	0.144975 (1.754)	$2.0192E^{-12}$ (0.265)	0.1495
Cov55	0.001030 (0.660)	1.77212 (1.949)	0.000025 (1.949)	$-6.752E^{-23}$ (-0.000)	$3.453E^{-23}$ (0.000)	0.0655

Table 7

Time-varying Covariances for BM-sorted portfolios during the period 1990-1997, using E-GARCH estimation.

$$Cov(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\,\hat{\sigma}^2_{mt} + e_{ijt}$$

$$\ln(h_{ijt}) = \varpi_{ij} + \phi_{ij}\, g_{ij}(z_{t-1}) + \gamma_{ij}\ln(h_{ijt-1}) + \eta_{ijt}$$

$$g_{ij}(z_{ijt}) = \theta_{ijt} + [\,|z_{ijt}| - E|z_{ijt}|\,]$$

$$z_{ijt} = e_{ijt} / \sqrt{h_{ijt}}$$

$Cov(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term. However, the variance of the error term is assumed to respond asymmetrically to changes in volatility with past increases in the functions variance causing a greater reaction than decreases in the variance function. The 1 indicates value and the 5 indicates growth. Thus, Cov14 would be the covariance between the value portfolio and portfolio 4.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	ϕ_{ij}	γ_{ij}	θ	R^2
Cov11	0.000582 (1.665)	3.501688 (10.660)	-13.71116 (-8.884)	-0.44898 (-1.375)	-0.200614 (-1.454)	-3.239723 (-1.358)	0.2392
Cov12	Did not converge						
Cov13	0.000462 (1.476)	2.901327 (11.291)	-17.31158 (-13.258)	-0.82828 (-3.199)	-0.466323 (-4.216)	-.336197 (-3.695)	0.3072
Cov14	Did not converge						
Cov15	0.000082 (-0.001)	2.117487 (21.175)	-19.22280 (-192.228)	-1.48344 (-14.834)	-0.584805 (-5.848)	-1.319718 (-13.197)	0.0863
Cov22	0.000495 (1.311)	2.891196 (7.923)	-11.64014 (-3.815)	0.291482 (0.880)	0.005859 (0.023)	0.944087 (0.651)	0.3864
Cov23	0.000386 (1.176)	2.601092 (8.178)	-15.12801 (-7.673)	0.002814 (0.190)	-0.264354 (-1.595)	176.47266 (0.200)	0.3727
Cov24	0.000446 (1.560)	2.209701 (7.488)	-14.40853 (-6.443)	0.196994 (1.841)	-0.183332 (-1.010)	1.695118 (1.888)	0.3372
Cov25	Did not converge						
Cov33	0.000387 (1.334)	2.359286 (7.586)	-17.11313 (-8.381)	0.093224 (0.301)	-0.423711 (-2.542)	4.262233 (0.276)	0.3398
Cov34	Did not converge						
Cov35	Did not converge						
Cov44	Did not converge						
Cov45	Did not converge						
Cov55	0.000300 (1.855)	1.055184 (19.223)	-21.72495 (-194.162)	-3.29682 (-32.739)	-0.734172 (-14.437)	-1.006535 (-11.763)	- 0.0011

Table 8

Time-varying covariances of BM-sorted portfolios for the period 1990-1997, using Hansen's (1982) t-statistic correction.

$$\mathrm{Cov}(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\hat{\sigma}^2_{mt} + \varepsilon_{ijt}$$
$$\varepsilon^2_{ijt} = \varpi_{ij} + \delta_{ij}\hat{\sigma}^2_{mt} + e_{ijt}$$

$\mathrm{Cov}(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. The first t-statistics, directly below the parameter estimates, are estimated using Hansen's (1982) t-statistic correction while the t-statistics below these are Hansen's (1982) t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem. The 1 indicates value and the 5 indicates growth. Thus, Cov14 would be the covariance between the value portfolio and portfolio 4.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	δ_{ij}	R^2
Cov11	-0.001706 (-0.91) (-0.07)	4.663705 (2.27) (2.12)	-0.0000876 (-1.75) (-0.16)	0.106417 (1.89) (1.80)	0.0812
Cov12	-0.00121599 (-0.92) (-0.06)	3.592723 (2.52) (2.32)	-0.000035 (-2.31) (-0.15)	0.045669 (2.70) (2.46)	0.0935
Cov13	-0.00161548 (-1.31) (-0.08)	3.649244 (2.68) (2.44)	-0.000037 (-2.16) (-0.15)	0.045682 (2.39) (2.22)	0.1066
Cov14	-0.0012969 (-1.25) (-0.08)	3.133038 (2.72) (2.66)	-0.000025 (-2.00) (-0.14)	0.032902 (2.30) (2.14)	0.0998
Cov15	-0.00229 (-1.39) (-0.10)	4.14658 (2.24) (2.83)	-0.0000769 (-1.45) (-0.16)	0.090088 (1.51) (1.46)	0.0840
Cov22	-0.00085668 (-0.81) (-0.05)	3.008087 (2.64) (2.62)	-0.0000186 (-2.18) (-0.13)	0.027110 (2.74) (2.49)	0.0922
Cov23	-0.00114484 (-1.23) (-0.07)	2.953941 (2.89) (2.58)	-0.0000173 (-2.39) (-0.14)	0.023833 (2.82) (2.54)	0.1094
Cov24	-0.000917 (-1.20) (-0.07)	2.552783 (3.02) (2.38)	-0.00001 (-2.09) (-0.12)	0.016051 (2.75) (2.49)	0.1043
Cov25	-0.00147 (-1.46) (-0.09)	3.043274 (2.70) (2.70)	-0.00002456 (-1.96) (-0.15)	0.030792 (2.18) (2.04)	0.1085
Cov33	-0.001311 (-1.53) (-0.08)	2.951288 (3.10) (2.59)	-0.00001595 (-2.42) (-0.14)	0.021406 (2.79) (2.52)	0.1259

Cov34	-0.0011295 (-1.59) (-0.08)	2.622935 (3.33) (2.46)	$-9.907E^{-6}$ (-2.14) (-0.13)	0.014414 (2.69) (2.45)	0.1285
Cov35	-0.001734 (-1.65) (-0.10)	3.214922 (2.73) (2.85)	-0.0000275 (-1.68) (-0.16)	0.033531 (1.81) (1.73)	0.1191
Cov44	-0.0009597 (-1.60) (-0.08)	2.388204 (3.63) (2.25)	$-5.605E^{-6}$ (-1.70) (-0.11)	0.00967 (2.58) (2.37)	0.1279
Cov45	-0.0015057 (-1.65) (-0.10)	2.915515 (2.86) (2.71)	-0.000019 (-1.61) (-0.15)	0.024592 (1.80) (1.72)	0.1220
Cov55	-0.00247273 (-1.51) (-0.11)	4.091624 (2.23) (2.05)	-0.00007 (-1.29) (-0.16)	0.081916 (1.33) (1.30)	0.0924

Table 9

Time-varying covariances of BM-sorted portfolios during the period 1990-1997, using WLS estimation.

$$Cov(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\hat{\sigma}^2_{mt} + \varepsilon_{ijt}$$

$$\varepsilon^2_{ijt} = \varpi_{ij} + \delta_{ij}\hat{\sigma}^2_{mt} + e_{ijt}$$

$Cov(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. WLS estimation involves using the predicted values of the second equation as weights for the terms in the first equation and then re-estimating the first equation. We iterate between the equations 3 times. The first t-statistics, directly below the parameter estimates, are estimated using WLS while the t-statistics below these are WLS t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem. The 1 indicates value and the 5 indicates growth. Thus, Cov14 would be the covariance between the value portfolio and portfolio 4.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	δ_{ij}	R^2
Cov11	-0.002053 (-2.699) (-0.075)	5.186712 (4.701) (3.678)	-0.000899 (-0.935) (-0.147)	1.145078 (5.268) (3.930)	0.1989
Cov12	-0.002037 (-3.903) (-0.085)	4.535145 (5.568) (4.050)	-0.000277 (-0.504) (-0.052)	1.005984 (6.687) (4.425)	0.2584
Cov13	-0.001438 (-2.571) (-0.078)	3.512612 (4.447) (3.552)	-0.000552 (-0.891) (-0.090)	1.160825 (5.523) (4.033)	0.1818
Cov14	-0.001711 (-5.238) (-0.091)	3.573222 (6.684) (4.424)	-0.000173 (-0.348) (-0.032)	1.016474 (5.690) (4.097)	0.3342
Cov15	-0.002178 (-4.079) (-0.010)	4.094881 (4.712) (3.683)	-0.000467 (-0.575) (-0.073)	1.212979 (4.481) (3.569)	0.1997
Cov22	-0.000895 (-1.320) (-0.056)	3.035281 (3.758) (3.170)	-0.000207 (-0.348) (-0.039)	1.013189 (4.870) (3.756)	0.1369
Cov23	-0.000711 (-1.555) (-0.054)	2.506255 (4.169) (3.405)	-0.000351 (-0.620) (-0.065)	1.115158 (4.705) (4.135)	0.1634
Cov24	-0.000828 (-1.567) (-0.064)	2.447893 (3.923) (3.267)	-0.0000367 (-0.078) (-0.007)	0.963293 (4.552) (3.605)	0.1474
Cov25	-0.000434 (-1.646) (-0.044)	1.890234 (4.135) (3.387)	-0.000434 (-1.646) (-0.044)	1.890234 (4.135) (3.387)	0.1612
Cov33	0.000285 (0.933) (0.042)	1.297909 (2.723) (2.473)	-0.001044 (-1.146) (-0.125)	1.582120 (3.219) (2.826)	0.0769

Cov34	-0.000471 (-1.065) (-0.046)	1.952137 (3.615) (3.083)	-0.000142 (-0.278) (-0.027)	0.999333 (3.795) (3.192)	0.1280
Cov35	-0.001344 (-3.535) (-0.095)	2.685607 (4.839) (3.742)	-0.000191 (-0.356) (-0.032)	1.127495 (4.289) (3.470)	0.2083
Cov44	-0.000863 (-1.682) (-0.072)	2.278128 (3.973) (3.296)	0.000163 (0.419) (0.039)	0.780695 (3.957) (3.287)	0.1506
Cov45	0.0000569 (0.193) (0.008)	1.289263 (2.846) (2.564)	-0.001107 (-1.326) (-0.118)	1.777803 (3.457) (2.983)	0.0834
Cov55	-0.001556 (-3.994) (-0.098)	3.001458 (4.660) (3.658)	-0.000306 (-0.351) (-0.047)	1.233796 (3.166) (2.790)	0.1961

Table 10

Time-varying covariances for size-sorted portfolios during the period 1990-1997, using GARCH estimation.

$$Cov(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\hat{\sigma}^2_{mt} + \varepsilon_{ijt}$$
$$h_t = \varpi_{ij} + \phi_{ij}\,\varepsilon^2_{t-1} + \gamma_{ij}\,h_{t-1} + \eta_{ijt}$$

$Cov(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term. The 1 indicates the smallest stock portfolio and the 5 indicates the largest stock portfolio. Thus, Cov14 would be the covariance between the small stock portfolio and the next to largest portfolio.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	ϕ_{ij}	γ_{ij}	R^2
Cov11	0.002584 (0.510)	3.430492 (1.210)	0.000184 (13.108)	$-7.858E^{-23}$ (-0.000)	$1.778E^{-11}$ (5188.78)	0.0345
Cov12	0.001486 (0.543)	3.092958 (1.706)	0.000073 (13.670)	$2.12E^{-23}$ (0.000)	$-6.9276E^{-23}$ (-0.000)	0.0680
Cov13	0.001139 (0.543)	2.96188 (2.048)	0.000047 (13.908)	$6.39E^{-23}$ (0.000)	$-2.595E^{-23}$ (-0.000)	0.0934
Cov14	0.00068 (0.526)	2.608022 (2.481)	0.000021 (12.962)	$4.4151E^{-23}$ (0.000)	$1.39E^{-13}$ (3.635)	0.1482
Cov15	0.000556 (1.128)	1.932527 (3.620)	0.000002 (2.552)	1.022730 (2.358)	0.197405 (1.029)	0.2058
Cov22	0.00097 (0.616)	3.103713 (2.555)	0.000032 (12.089)	$4.3634E^{-23}$ (0.000)	$1.6038E^{-13}$ (10.838)	0.1438
Cov23	0.000727 (0.598)	3.11588 (3.558)	0.000021 (12.396)	$-7.6514E^{-24}$ (-0.000)	$2.2756E^{-13}$ (15.304)	0.2023
Cov24	0.000477 (0.529)	2.59943 (3.287)	0.000009 (7.267)	0.308734 (2.098)	$-1.091E^{-18}$ (-0.000)	0.2869
Cov25	0.000207 (0.540)	2.05666 (6.330)	0.000004 (8.363)	0.438705 (3.238)	$-2.35E^{-18}$ (-0.000)	0.3433
Cov33	0.000701 (0.757)	3.225565 (6.284)	0.000015 (12.180)	$1.5096E^{-23}$ (0.000)	$7.775E^{-23}$ (0.000)	0.2705
Cov34	0.000515 (0.687)	2.872728 (7.200)	0.000008 (7.692)	0.13635 (2.125)	$1.274E^{-12}$ (0.004)	0.3408
Cov35	0.000445 (0.843)	2.299194 (7.854)	0.000005 (6.241)	0.220682 (2.238)	$-1.218E^{-21}$ (-0.000)	0.3679
Cov44	0.000534 (0.700)	2.954613 (6.225)	0.000009 (9.804)	$-7.031E^{-23}$ (-0.000)	$2.433E^{-15}$ (1.135)	0.3522
Cov45	0.000481 (0.761)	2.452646 (5.282)	0.000006 (12.136)	$1.851E^{-23}$ (0.000)	$1.050E^{-15}$ (0.334)	0.3368
Cov55	0.000629 (1.086)	2.015875 (4.304)	0.000005 (12.526)	$-3.133E^{-23}$ (-0.000)	0.0000041 (0.001	0.2790

Table 11

Time-varying covariances for size-sorted portfolios during the period 1990-1997, using E-GARCH estimation.

$$Cov(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\hat{\sigma}^2_{mt} + e_{ijt}$$

$$\ln(h_{ijt}) = \varpi_{ij} + \phi_{ij}\, g_{ij}(z_{t-1}) + \gamma_{ij}\ln(h_{ijt-1}) + \eta_{ijt}$$

$$g_{ij}(z_{ijt}) = \theta_{ijt} + [\,|z_{ijt}| - E|z_{ijt}|\,]$$

$$z_{ijt} = e_{ijt} / \sqrt{h_{ijt}}$$

$Cov(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term. However, the variance of the error term is assumed to respond asymmetrically to changes in volatility with past increases in the functions variance causing a greater reaction than decreases in the variance function. The 1 indicates the smallest stock portfolio and the 5 indicates the largest stock portfolio. Thus, Cov14 would be the covariance between the small stock portfolio and the next to largest portfolio.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	ϕ_{ij}	γ_{ij}	θ_{ij}	R^2
Cov11	0.000342 (0.003)	2.25377 (22.524)	-1.407022 (-14.070)	4.943747 (49.437)	0.69118 (6.912)	-0.525855 (-5.259)	0.0185
Cov12	-0.000208 (-0.928)	2.693452 (12.095)	0.519043 (1.165)	0.519043 (1.165)	0.952739 (21.385)	-1.014834 (-11.288)	0.0176
Cov13	0.000666 (4.471)	2.527296 (12.264)	-3.025375 (-3.828)	2.293873 (8.918)	0.704435 (10.226)	-0.134612 (-1.546)	0.0800
Cov14	0.000355 (2.021)	2.123109 (62.747)	-2.718293 (-2.861)	1.603610 (7.998)	0.731500 (8.409)	-0.373683 (-3.794)	0.1261
Cov15	0.000179 (8.762)	1.899310 (1674.046)	-2.998942 (-2.252)	1.087724 (3.852)	0.727492 (6.409)	-0.424624 (-3.606)	0.1959
Cov22	0.000710 (1.010)	2.633192 (3.891)	1.597414 (4.182)	0.144467 (8.086)	1.148478 (30.035)	-4.356073 (-10.475)	0.1312
Cov23	0.000509 (0.005)	2.234422 (22.344)	-0.435422 (-4.354)	1.827567 (18.276)	0.906285 (9.063)	-0.672933 (-6.729)	0.1607
Cov24	Did not converge.						
Cov25	Did not converge.						
Cov33	Did not converge.						
Cov34	Did not converge.						
Cov35	0.000381 (1.277)	2.413644 (7.596)	-13.677536 (-3.795)	0.006053 (0.081)	-0.132315 (-0.443)	57.21335 (0.088)	0.3701
Cov44	-0.000259 (-0.003)	3.144026 (31.440)	-7.019190 (-70.192)	-1.900759 (-19.008)	0.463285 (4.633)	-0.592316 (-5.923)	0.3191
Cov45	0.000468 (1.345)	2.489664 (7.504)	-19.689638 (-7.875)	-0.000058 (-0.142)	-0.646546 (-3.106)	3833.2456 (0.164)	0.3367
Cov55	Did not converge.						

Table 12

Time-varying covariances for size-sorted portfolios during the period 1990-1997 using Hansen's (1982) t-statistic correction.

$$Cov(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\hat{\sigma}^2_{mt} + \varepsilon_{ijt}$$

$$\varepsilon^2_{ijt} = \varpi_{ij} + \delta_{ij}\hat{\sigma}^2_{mt} + e_{ijt}$$

$Cov(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. The first t-statistics, directly below the parameter estimates, are estimated using Hansen's (1982) t-statistic correction while the t-statistics below these are Hansen's (1982) t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem. The 1 indicates the smallest stock portfolio and the 5 indicates the largest stock portfolio. Thus, Cov14 would be the covariance between the small stock portfolio and the next to largest portfolio.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	δ_{ij}	R^2
Cov11	-0.004297 (-0.91) (-0.89)	8.008736 (1.53) (1.45)	-0.000532 (-1.13) (-1.12)	0.617956 (1.16) (1.22)	0.01362
Cov12	-0.0028519 (-0.98) (-0.95)	5.603728 (1.74) (1.62)	-0.000204 (-1.20) (-1.18)	0.240961 (1.25) (1.20)	0.0577
Cov13	-0.002423 (-1.06) (-1.03)	4.854167 (1.92) (1.76)	-0.0001286 (-1.22) (-1.21)	0.153901 (1.29) (1.24)	0.0645
Cov14	-0.001809 (-1.20) (-1.14)	3.713342 (2.24) (2.00)	-0.000054 (-1.37) (-1.19)	0.067147 (1.51) (1.43)	0.0770
Cov15	-0.0011443 (-1.23) (-1.14)	2.567592 (2.50) (1.43)	-0.000019 (-1.76) (-1.75)	0.024855 (2.09) (1.89)	0.0867
Cov22	-0.001850 (-0.97) (-0.94)	4.29994 (2.06) (1.87)	-0.000085 (-1.41) (-1.38)	0.103747 (1.53) (1.45	0.0696
Cov23	-0.001676 (-1.09) (-1.03)	3.942405 (2.34) (1.87)	-0.000056 (-1.50) (-1.46)	0.069491 (1.67) (1.56)	0.0815
Cov24	-0.001413 (-1.26) (-1.16)	3.335623 (2.72) (2.32)	-0.000028 (-1.91) (-1.88)	0.037055 (2.23) (1.99)	0.0973
Cov25	-0.001064 (-1.34) (-1.27)	2.551569 (2.91) (2.43)	-0.000013 (-2.17) (-1.26)	0.018311 (2.64) (2.27)	0.1062
Cov33	-0.001405 (-1.07) (-1.01)	3.750872 (2.63) (2.26)	-0.000038 (-1.72) (-1.64)	0.049782 (2.00) (1.82)	0.0920

Cov34	-0.0013701 (-1.24) (-1.16)	3.45126 (2.91) (2.43)	-0.0000238 (-2.51) (-2.20)	0.032326 (3.02) (2.49)	0.1115
Cov35	-0.0010959 (-1.22) (-1.14)	2.771651 (2.87) (2.41)	-0.0000142 (-2.43) (-1.38)	0.019564 (3.00) (2.48)	0.1171
Cov44	-0.001475 (-1.27) (-1.17)	3.50403 (2.87) (2.41)	-0.0000227 (-2.25) (-2.12)	0.030119 (2.81) (2.37)	0.1287
Cov45	-0.00126475 (-1.20) (-1.14)	2.976417 (2.69) (2.30)	-0.0000177 (-1.85) (-1.70)	0.022797 (2.27) (2.02)	0.1313
Cov55	-0.00092 (-0.92) (-0.88)	2.545885 (2.46) (2.15)	-0.000014 (-1.58) (-1.36)	0.018424 (1.98) (1.81)	0.1200

Table 13

Time-varying covariances of size-sorted portfolios during the period 1990-1997, using WLS estimation.

$$Cov(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\hat{\sigma}^2_{mt} + \varepsilon_{ijt}$$

$$\varepsilon^2_{ijt} = \varpi_{ij} + \delta_{ij}\hat{\sigma}^2_{mt} + e_{ijt}$$

$Cov(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. WLS estimation involves using the predicted values of the second equation as weights for the terms in the first equation and then re-estimating the first equation. We iterate between the equations 3 times. The first t-statistics, directly below the parameter estimates, are estimated using WLS while the t-statistics below these are WLS t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem. The 1 indicates the smallest stock portfolio and the 5 indicates the largest stock portfolio. Thus, Cov14 would be the covariance between the small stock portfolio and the next to largest portfolio.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	δ_{ij}	R^2
Cov11	0.000789 (0.399) (0.398)	1.478634 (0.476) (0.473)	-0.007806 (-1.282) (-1.275)	4.924749 (2.071) (1.875)	0.0025
Cov12	-0.001747 (-1.929) (-1.661)	4.446581 (3.435) (2.712)	-0.000849 (-0.479) (-0.479)	1.395789 (3.107) (2.542)	0.1171
Cov13	-0.002455 (-3.239) (-2.647)	4.821425 (4.504) (3.155)	-0.000194 (-0.165) (-0.163)	1.159520 (3.576) (2.780)	0.1856
Cov14	-0.001755 (-3.995) (-2.767)	3.612531 (5.202) (3.369)	0.000136 (0.174) (0.017)	1.088478 (3.889) (2.920)	0.2331
Cov15	-0.001107 (-2.369) (-2.026)	2.521371 (4.225) (3.054)	0.0000976 (0.196) (0.196)	1.055233 (4.358) (3.103)	0.1670
Cov22	-0.002143 (-3.255) (-2.526)	4.542441 (4.840) (3.264)	-0.000334 (-0.349) (-0.348)	1.163698 (4.333) (3.094)	0.2084
Cov23	-0.001957 (-6.362) (-3.217)	4.174312 (6.677) (3.686)	-0.000089 (-0.113) (-0.113)	1.046811 (4.382) (3.11)	0.3337
Cov24	-0.001489 (-4.523) (-2.834)	3.370515 (5.984) (3.556)	-0.000065 (-0.113) (-0.112)	1.044950 (4.970) (3.303)	0.2869
Cov25	-0.000845 (-2.297) (-1.974)	2.316452 (4.592) (3.185)	-0.000151 (-0.334) (-0.323)	1.082764 (4.878) (3.276)	0.1916

Cov33	-0.002070 (-6.121) (-3.265)	4.414091 (7.025) (3.741)	-0.000123 (-0.194) (-0.191)	0.977629 (5.331) (3.403)	0.3567
Cov34	-0.001683 (-5.764) (-2.973)	3.771865 (6.881) (3.719)	-0.000194 (-0.386) (-0.345)	1.005318 (6.098) (3.579)	0.3473
Cov35	-0.001032 (-3.926) (-1.823)	2.706740 (5.922) (2.669)	-0.000224 (-0.510) (-0.482)	1.029982 (5.492) (3.444)	0.2827
Cov44	-0.001264 (-4.786) (-2.440)	3.296046 (6.491) (3.654)	-0.000296 (-0.556) (-0.053)	1.021314 (5.467) (3.437)	0.3213
Cov45	-0.000537 (-1.262) (-1.270)	2.256960 (4.057) (2.989)	-0.000436 (-0.785) (-0.771)	1.128876 (4.566) (3.176)	0.3213
Cov55	-0.00003211 (-0.061) (-0.019)	1.687457 (2.792) (2.367)	-0.000606 (-0.955) (-0.932)	1.206778 (3.848) (2.902)	0.0805

Table 14

Time-varying covariances for size-sorted portfolios of NYSE stocks during the period 1980-1986, using GARCH estimation.

$$Cov(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\hat{\sigma}^2_{mt} + \varepsilon_{ijt}$$

$$h_t = \varpi_{ij} + \phi_{ij}\,\varepsilon^2_{t-1} + \gamma_{ij}\,h_{t-1} + \eta_{ijt}$$

$Cov(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term. The 1 indicates the smallest stock portfolio and the 5 indicates the largest stock portfolio. Thus, Cov14 would be the covariance between the small stock portfolio and the next to largest portfolio.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	ϕ_{ij}	γ_{ij}	R^2
Cov11	-0.003604 (-1.994)	0.177641 (5.894)	0.0000129 (8.770)	$-1.768E^{-23}$ (-0.000)	$-3.753E^{-23}$ (-0.000)	0.3198
Cov12	-0.003112 (-2.419)	0.151420 (6.637)	0.000008 (8.756)	$-3.877E^{-23}$ (-0.000)	$-1.776E^{-22}$ (-0.000)	0.3571
Cov13	-0.003203 (-2.948)	0.151108 (7.601)	0.0000063 (7.761)	$-6.493E^{-23}$ (-0.000)	$-1.819E^{-22}$ (-0.000)	0.4139
Cov14	-0.003098 (-3.084)	0.142819 (7.597)	0.0000056 (7.592)	$3.464E^{-23}$ (0.000)	$-1.330E^{-22}$ (-0.000)	0.4141
Cov15	-0.002766 (-3.140)	0.126118 (7.013)	0.0000042 (5.839)	$1.716E^{-23}$ (-0.000)	$-2.506E^{-22}$ (-0.000)	0.4257
Cov22	-0.002958 (-2.755)	0.142626 (6.679)	0.0000061 (6.754)	$4.632E^{-23}$ (0.000)	0.0000028 (0.001)	0.3935
Cov23	-0.003213 (-3.199)	0.147835 (7.138)	0.0000049 (5.792)	$-2.564E^{-23}$ (-0.000)	0.000075 (0.079)	0.4628
Cov24	-0.003211 (-2.941)	0.145116 (6.429)	0.0000046 (0.058)	0.008303 (0.051)	0.001071 (0.000)	0.4647
Cov25	-0.003387 (-4.120)	0.153816 (8.173)	0.0000020 (2.990)	0.519801 (1.640)	$4.493E^{-21}$ (0.000)	0.4597
Cov33	-0.003454 (-3.619)	0.155511 (7.968)	0.0000043 (5.268)	$1.716E^{-23}$ (0.000)	0.0000648 (0.082)	0.5188
Cov34	-0.003642 (-3.951)	0.164837 (8.314)	0.0000032 (2.915)	0.25366 (0.927)	$-1.321E^{-18}$ (-0.000)	0.5137
Cov35	-0.003690 (-5.591)	0.166361 (11.064)	0.0000012 (1.151)	0.410918 (1.628)	0.291195 (1.116)	0.4899
Cov44	-0.003643 (-4.659)	0.168864 (9.500)	0.0000023 (2.947)	0.593602 (1.777)	$1.401E^{-17}$ (0.000)	0.4874
Cov45	-0.003448 (-5.641)	0.163034 (10.977)	0.0000006 (1.078)	0.489367 (2.030)	0.404825 (2.196)	0.4671
Cov55	-0.003340 (-6.374)	0.157423 (11.560)	0.0000003 (0.597)	0.292639 (1.279)	0.648517 (2.549)	0.4505

Table 15

Time-varying Covariances for size-sorted portfolios of NYSE stocks for the period 1980-1986, using E-GARCH estimation.

$$Cov(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\,\hat{\sigma}^2_{mt} + e_{ijt}$$

$$\ln(h_{ijt}) = \varpi_{ij} + \phi_{ij}\,g_{ij}(z_{t-1}) + \gamma_{ij}\ln(h_{ijt-1}) + \eta_{ijt}$$

$$g_{ij}(z_{ijt}) = \theta_{ijt} + [\,|z_{ijt}| - E|z_{ijt}|\,]$$

$$z_{ijt} = e_{ijt} / \sqrt{h_{ijt}}$$

$Cov(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term. However, the variance of the error term is assumed to respond asymmetrically to changes in volatility with past increases in the functions variance causing a greater reaction than decreases in the variance function. The 1 indicates the smallest stock portfolio and the 5 indicates the largest stock portfolio. Thus, Cov14 would be the covariance between the small stock portfolio and the next to largest portfolio.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	ϕ_{ij}	γ_{ij}	θ_{ij}	R^2
Cov11	Did not converge.						
Cov12	0.005190 (-339.570)	0.189609 (38.588)	-18.515341 (-6.550)	-1.118721 (-2.488)	-0.547330 (-2.368)	0.083869 (0.433)	0.2962
Cov13	0.005231 (-10.268)	0.190148 (14.752)	-17.907598 (-7.721)	-0.998351 (-3.333)	-0.473192 (-2.504)	0.219418 (0.949)	0.3516
Cov14	0.004240 (-30.699)	0.161973 (38.174)	-8.834083 (-3.935)	-1.211993 (-3.249)	0.28323 (1.581)	0.467340 (2.255)	0.3858
Cov15	Did not converge.						
Cov22	Did not converge.						
Cov23	0.001618 (-0.016)	0.098027 (0.980)	-6.386317 (-63.863)	-1.589353 (-15.894)	0.498928 (4.989)	-0.012640 (-0.126)	0.4051
Cov24	0.001832 (-6.039)	0.107120 (29.412)	-17.25045 (-6.528)	1.291948 (2.733)	-0.382928 (-1.861)	-0.266424 (-1.972)	0.4339
Cov25	0.003223 (-0.032)	0.142586 (1.426)	-18.935587 (-189.356)	0.779491 (7.795)	-0.503407 (-5.034)	0.14316 (0.143)	0.4833
Cov33	Did not converge.						
Cov34	0.002328 (-0.015)	0.121650 (0.795)	-20.650672 (-134.962)	0.815284 (5.328)	-0.647565 (-4.232)	-0.172791 (-1.129)	0.5024
Cov35	0.003423 (-6.377)	0.157725 (10.475)	-4.848172 (-1.526)	0.656628 (2.345)	0.616533 (2.448)	-0.145937 (-0.465)	0.5010
Cov44	0.003395 (-1.096)	0.151137 (2.677)	-23.119459 (-231.195)	0.86448 (8.684)	-0.851378 (-8.554)	0.161527 (1.583)	0.5118
Cov45	Did not converge.						
Cov55	Did not converge.						

Table 16

Time-varying covariances for size-sorted portfolios of NYSE stocks during the period 1980-1986, using Hansen's (1982) t-statistic correction.

$$Cov(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\hat{\sigma}^2_{mt} + \varepsilon_{ijt}$$

$$\varepsilon^2_{ijt} = \varpi_{ij} + \delta_{ij}\hat{\sigma}^2_{mt} + e_{ijt}$$

$Cov(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. The first t-statistics, directly below the parameter estimates, are estimated using Hansen's (1982) t-statistic correction while the t-statistics below these are Hansen's (1982) t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem. The 1 indicates the smallest stock portfolio and the 5 indicates the largest stock portfolio. Thus, Cov14 would be the covariance between the small stock portfolio and the next to largest portfolio.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	δ_{ij}	R^2
Cov11	0.001735 (1.85) (1.58)	0.599099 (1.10) (0.82)	0.000013 (1.38) (0.41)	0.000549 (0.22) (0.22)	0.0036
Cov12	0.001259 (1.55) (1.30)	0.629061 (1.24) (0.95)	$8.122E^{-6}$ (1.32) (0.26)	0.00103 (0.66) (0.66)	0.0084
Cov13	0.0013986 (1.90) (1.76)	0.446992 (1.01) (0.82)	$7.581E^{-6}$ (1.66) (0.24)	0.000444 (0.34) (0.34)	0.0037
Cov14	0.001179 (1.69) (1.50)	0.449586 (1.05) (0.86)	$5.752E^{-6}$ (1.58) (0.18)	0.000707 (0.65) (0.64)	0.0058
Cov15	0.001234 (1.92) (1.79)	0.219885 (0.58) (0.49)	$5.211E^{-6}$ (2.20) (0.16)	-0.000108 (-0.11) (-0.10)	0.0008
Cov22	0.000953 (1.26) (1.06)	0.716252 (1.44) (1.11)	$5.210E^{-6}$ (1.30) (0.16)	0.001453 (1.29) (1.27)	0.0185
Cov23	0.00113465 (1.67) (1.46)	0.526213 (1.27) (1.05)	$5.975E^{-6}$ (1.79) (0.19)	0.000512 (0.49) (0.49)	0.0094
Cov24	0.000937 (1.44) (1.34)	0.570493 (1.43) (1.20)	$4.789E^{-6}$ (1.77) (0.15)	0.00059 (0.66) (0.66)	0.0155
Cov25	0.00109 (1.80) (1.63)	0.292262 (0.85) (0.74)	$4.860E^{-6}$ (2.30) (0.15)	-0.000245 (-0.29) (-0.29)	0.0035
Cov33	0.001414 (2.12)	0.338026 (0.90)	$6.893E^{-6}$ (2.09)	-0.000075 (-0.07)	0.0024

	(2.06)	(0.77)	(0.22)	(-0.07)	
Cov34	0.001232 (1.91) (0.18)	0.384905 (1.05) (0.90)	$5.728E^{-6}$ (2.10) (0.18)	0.000055 (0.06) (0.06)	0.0055
Cov35	0.001434 (2.18) (2.12)	0.121591 (0.34) (0.29)	$6.096E^{-6}$ (2.48) (0.19)	-0.000717 (-0.75) (-0.75)	0.0003
Cov44	0.001128 (1.71) (1.64)	0.43612 (1.15) (0.98)	$5.200E^{-6}$ (1.99) (0.16)	0.00005 (0.05) (0.05)	0.0097
Cov45	0.001433 (2.07) (1.90)	0.133839 (0.36) (0.31)	$6.106E^{-6}$ (2.21) (0.19)	-0.00090 (-0.77) (-0.76)	0.0001
Cov55	0.001854 (2.36) (1.23)	-0.315186 (-0.38) (-0.22)	$7.786E^{-6}$ (2.24) (0.25)	-0.002003 (-1.26) (-1.26)	0.0031

Table 17

Time-varying covariances for size-sorted NYSE portfolios during the period 1980-1986, using WLS estimation.

$$Cov(R_{it},R_{jt}) = \xi_{ij} + \psi_{ij}\hat{\sigma}^2_{mt} + \varepsilon_{ijt}$$

$$\varepsilon^2_{ijt} = \varpi_{ij} + \delta_{ij}\hat{\sigma}^2_{mt} + e_{ijt}$$

$Cov(R_{it},R_{jt})$ is the covariance, as calculated in equation (10), of the excess returns to portfolios i and j at time t. The model shows that this covariance is dependent on the aggregate market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. WLS estimation involves using the predicted values of the second equation as weights for the terms in the first equation and then re-estimating the first equation. We iterate between the equations 3 times. The first t-statistics, directly below the parameter estimates, are estimated using WLS while the t-statistics below these are WLS t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem. The 1 indicates the smallest stock portfolio and the 5 indicates the largest stock portfolio. Thus, Cov14 would be the covariance between the small stock portfolio and the next to largest portfolio.

	ξ_{ij}	ψ_{ij}	ϖ_{ij}	δ_{ij}	R^2
Cov11	0.001665 (1.228) (0.821)	0.651027 (0.723) (0.404)	0.001502 (0.614) (0.575)	0.416742 (0.451) (0.418)	0.0112
Cov12	0.000839 (0.747) (0.224)	0.93222 (1.168) (0.707)	0.000924 (0.756) (0.496)	0.554414 (1.035) (0.761)	0.0288
Cov13	0.001296 (1.245) (0.380)	0.521132 (0.745) (0.502)	0.000913 (0.568) (0.449)	0.486227 (0.560) (0.501)	0.0119
Cov14	0.000943 (0.996) (0.303)	0.619980 (0.941) (0.638)	0.000749 (0.590) (0.396)	0.575465 (0.857) (0.803)	0.0189
Cov15	0.001368 (1.708) (0.531)	0.12878 (0.256) (0.197)	0.001750 (0.433) (0.437)	-0.042592 (-0.016) (-0.016)	0.0014
Cov22	0.000368 (0.383) (0.116)	1.143803 (1.592) (1.028)	0.000668 (0.793) (0.229)	0.566615 (1.485) (0.895)	0.0522
Cov23	0.000943 (1.001) (0.306)	0.663819 (1.022) (0.698)	0.000866 (0.696) (0.494)	0.472217 (0.742) (0.619)	0.0222
Cov24	0.00074 (0.857) (0.262)	0.711931 (1.186) (0.843)	0.000853 (0.865) (0.612)	0.443378 (0.825) (0.792)	0.0297
Cov25	0.001245 (1.654) (0.514)	0.188166 (0.404) (0.321)	0.001773 (0.677) (0.665)	-0.147595 (-0.086) (-0.086)	0.0035
Cov33	0.001534 (1.645) (0.508)	0.255170 (0.431) (0.309)	0.001503 (0.475) (0.473)	0.150672 (0.091) (0.091)	0.0040

Cov34	0.001367 (1.590) (0.493)	0.291913 (0.541) (0.404)	0.001671 (0.711) (0.682)	0.01681 (0.013) (0.013)	0.0063
Cov35	0.001908 (2.583) (0.807)	-0.19499 (-0.496) (-0.416)	-0.001123 (-0.397) (-0.240)	1.707912 (0.983) (0.739)	0.0053
Cov44	0.001254 (1.534) (0.476)	0.349543 (0.684) (0.523)	0.001655 (0.899) (0.955)	-0.016339 (-0.016) (-0.016)	0.0101
Cov45	0.001908 (2.674) (0.839)	-0.182546 (-0.488) (-0.415)	-0.001420 (-0.474) (-0.277)	1.839358 (1.011) (0.751)	0.0052
Cov55	0.002774 (4.934) (1.550)	-0.71298 (-4.087) (-3.928)	0.000018 (0.023) (0.006)	0.952297 (2.147) (0.994)	0.2664

Table 18

Heteroskedastic CAPM for NYSE size-sorted portfolios during the period 1987-1997, using GARCH estimation.

$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt} / \hat{\sigma}^2_{mt}) + e_{it}$

$s_{it} = \varpi_i + \phi_i e^2 + \gamma_i s_{t-1} + v_{it}$

This model shows the return on a portfolio, i, at time t is a function of the return on the market and the return on the market divided by the estimated market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term.

	α_i	β_{i1}	β_{i2}	ϖ_i	ϕ_i	γ_i	R^2
Small	-0.001428 (-1.392)	1.111597 (21.700)	0.000011 (0.299)	0.000150 (3.607)	1.098462 (4.908)	0.011517 (0.188)	0.7865
Size 2	-0.000407 (-0.475)	1.018999 (68.102)	-0.000044 (-2.374)	0.000050 (5.604)	0.066188 (0.972)	$2.794E^{-12}$ (0.000)	0.9732
Size 3	-0.001624 (-2.131)	0.997895 (70.325)	0.000012 (0.716)	0.000032 (4.761)	0.483319 (2.075)	$-8.985E^{-18}$ (-0.000)	0.9703
Size 4	-0.000081 (-0.123)	0.997937 (59.318)	0.000008 (0.407)	0.000042 (4.020)	0.751436 (3.650)	$5.712E^{-18}$ (0.000)	0.9392
Large	0.000884 (0.639)	0.863062 (28.968)	0.000068 (2.434)	0.000141 (5.207)	0.467188 (2.965)	$-2.602E^{-18}$ (-0.000)	0.8452

Table 19

Heteroskedastic CAPM for size-sorted NYSE stocks during the period 1987-1997, using E-GARCH estimation.

$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt}/\sigma^2_{mt}) + e_{it}$

$\ln(h_{it}) = \varpi_i + \phi_i g_i(z_{t-1}) + \gamma_i \ln(h_{it-1}) + \eta_{it}$

$g_i(z_{it}) = \theta_{it} + [|z_{it}| - E|z_{it}|]$

$z_{it} = e_{it} / \sqrt{h_{it}}$

This model shows the return on a portfolio, i, at time t is a function of the return on the market and the return on the market divided by the estimated market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term. However, the variance of the error term is assumed to respond asymmetrically to changes in volatility with past increases in the functions variance causing a greater reaction than decreases in the variance function.

	α_i	β_{i1}	β_{i2}	ϖ_i	ϕ_i	γ_i	θ_i	R^2
Small	-0.002014 (-1.567)	1.06163 (28.240)	0.000013 (0.382)	-3.304310 (-2.807)	0.965194 (5.667)	0.570479 (3.818)	-0.358928 (-3.185)	0.7770
Size 2	-0.000349 (-0.463)	1.00444 (44.989)	-0.00003 (-2.383)	-17.4315 (-10.772)	0.23712 (1.204)	0.769739 (-4.694)	-0.706014 (-0.892)	0.9732
Size 3	-0.001203 (-0.012)	0.99055 (9.906)	0.000014 (0.000)	-4.099177 (-40.992)	0.763557 (7.636)	0.584544 (5.845)	0.257103 (2.571)	0.9703
Size 4	0.000200 (0.267)	0.99180 (40.190)	0.000021 (1.076)	-5.900650 (-2.126)	0.899318 (3.284)	0.371042 (1.262)	0.299167 (1.663)	0.9387
Large	0.000417 (0.332)	0.85354 (27.241)	0.000079 (3.011)	-8.196912 (-4.118)	0.670693 (3.574)	0.027352 (0.116)	0.147072 (0.804)	0.8464

Table 20

Heteroskedastic CAPM estimates for NYSE size-sorted portfolios during the period 1987-1997, using Hansen's (1982) t-statistic correction.

$$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt} / \hat{\sigma}^2_{mt}) + e_{it}$$
$$\varepsilon^2_{it} = \varpi_i + \delta_i \hat{\sigma}^2_{mt} + e_{it}$$

This model shows the return on a portfolio, i, at time t is a function of the return on the market and the return on the market divided by the estimated market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. The first t-statistics, directly below the parameter estimates, are estimated using Hansen's (1982) t-statistic correction while the t-statistics below these are Hansen's (1982) t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem.

	α_i	β_{i1}	β_{i2}	ϖ_i	δ_i	R^2
Small	-0.00037 (-0.17) (-0.12)	1.22210 (12.75) (12.19)	0.903490 (0.94) (0.94)	0.00073 (3.57) (3.46)	0.011255 (0.27) (0.27)	0.7980
Size 2	-0.00141632 (-1.60) (-1.58)	0.988368 (39.45) (39.42)	0.031951 (0.11) (0.11)	0.0000629 (6.03) (5.95)	-0.000687 (-0.34) (-0.34)	0.9720
Size 3	-0.00116045 (-1.29) (-1.29)	0.990683 (37.56) (37.51)	-0.043455 (-0.11) (-0.11)	0.0000631 (4.07) (3.16)	0.0045683 (1.22) (1.20)	0.9705
Size4	0.0000279 (0.03) (0.03)	0.955264 (30.27) (30.15)	-0.086572 (-0.19) (-0.19)	0.000126 (3.16) (2.81)	0.000525 (0.07) (0.07)	0.9416
Large	.00292002 (2.13) (1.17)	0.843585 (19.09) (16.41)	-0.805415 (-1.39) (-1.39)	0.000276 (3.97) (3.81)	0.003516 (0.26) (0.26)	0.8450

Table 21

Heteroskedastic CAPM for NYSE size-sorted portfolios during the period 1987-1997, using WLS estimation.

$$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt}/\hat{\sigma}^2_{mt}) + e_{it}$$

$$\varepsilon^2_{it} = \varpi_i + \delta_i \hat{\sigma}^2_{mt} + e_{it}$$

The model shows that this covariance is dependent on the aggregate market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. WLS estimation involves using the predicted values of the second equation as weights for the terms in the first equation and then re-estimating the first equation. We iterate between the equations 3 times. The first t-statistics, directly below the parameter estimates, are estimated using WLS while the t-statistics below these are WLS t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem.

	α_i	β_{i1}	β_{i1}	R^2
Small	0.002211 (0.886) (0.886)	1.10938 (33.029) (33.014)	0.030404 (0.050) (0.050)	0.9090
Size 2	-0.002166 (-2.387) (-2.096)	1.033392 (114.155) (92.673)	0.199725 (0.679) (0.679)	0.9916
Size 3	-0.002459 (-3.107) (-1.799)	1.048097 (154.044) (66.608)	0.434952 (1.765) (1.765)	0.9951
Size 4	-0.001580 (-1.550) (-0.853)	0.953580 (68.258) (39.552)	0.602366 (2.057) (2.057)	0.9780
Large	0.003097 (1.964) (0.869)	0.863793 (41.515) (19.075)	-1.233037 (-2.792) (-2.792)	0.9385

Table 22

Heteroskedastic CAPM for BM-sorted portfolios in during the period 1990-1997, using GARCH estimates.

$$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt}/\hat{\sigma}^2_{mt}) + e_{it}$$

$$s_{it} = \varpi_i + \phi_i e^2 + \gamma_i s_{t-1} + v_{it}$$

This model shows the return on a portfolio, i, at time t is a function of the return on the market and the return on the market divided by the estimated market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term.

	α_i	β_{i1}	β_{i2}	ϖ_i	ϕ_i	γ_i	R^2
Growth	-0.007299 (-4.332)	1.271473 (41.328)	$5.048E^{-10}$ (0.108)	0.000157 (4.798)	0.199396 (1.117)	$-1.217E^{-24}$ (-0.000)	0.9439
2	-0.004425 (-3.828)	1.086335 (56.881)	$2.244E^{-9}$ (0.525)	0.000051 (2.094)	0.589462 (2.762)	0.116147 (0.598)	0.9458
3	0.000482 (0.604)	0.933086 (63.643)	$2.724E^{-9}$ (2.185)	0.000027 (4.584)	0.487888 (9.231)	$2.878E^{-18}$ (0.000)	0.9760
4	0.003567 (3.038)	0.848546 (45.909)	$-1.496E^{-9}$ (-0.696)	0.000074 (5.793)	$9.099E^{-24}$ (0.000)	$-1.417E^{-23}$ (0.000)	0.9521
Value	0.011128 (7.399)	0.849356 (23.306)	$-5.504E^{-9}$ (-1.291)	0.00009 (3.070)	0.853486 (2.038)	$-1.015E^{-24}$ (-0.000)	0.8766

Table 23

Heteroskedastic CAPM of BM-sorted portfolios for the period 1990-1997, using E-GARCH estimation.

$$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt}/\sigma^2_{mt}) + e_{it}$$
$$\ln(h_{it}) = \varpi_i + \phi_i g_i(z_{t-1}) + \gamma_i \ln(h_{it-1}) + \eta_{it}$$
$$g_i(z_{it}) = \theta_{it} + [|z_{it}| - E|z_{it}|]$$
$$z_{it} = e_{it} / \sqrt{h_{it}}$$

This model shows the return on a portfolio, i, at time t is a function of the return on the market and the return on the market divided by the estimated market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term. However, the variance of the error term is assumed to respond asymmetrically to changes in volatility with past increases in the functions variance causing a greater reaction than decreases in the variance function.

	α_i	β_{i1}	β_{i2}	ϖ_i	ϕ_i	γ_i	θ_i	R^2
Growth	0.00729 (-4.089)	1.237332 (10.509)	5.436E^{-10} (0.064)	8.492083 (-85.352)	0.001609 (0.015)	0.002407 (0.039)	0.000414 (0.035)	0.9439
2	0.00453 (-0.045)	1.058379 (10.584)	1.853E^{-9} (0.000)	8.898199 (-88.982)	0.000001 (0.000)	0.000001 (0.000)	0.000001 (0.000)	0.9467
3	0.00057 (-0.006)	0.931798 (9.318)	3.573E^{-9} (0.000)	-9.98675 (-99.867)	0.000004 (0.000)	0.000186 (0.002)	-0.000180 (-0.002)	0.9765
4	0.00355 (0.036)	0.850788 (8.508)	-1.751E^{-9} (-0.000)	9.479362 (-94.794)	0.000024 (-0.000)	0.002628 (0.026)	0.000466 (0.005)	0.9521
Value	0.01242 (0.124)	0.890391 (8.904)	-9.028E^{-9} (-0.000)	8.450076 (-84.501)	0.780757 (7.808)	0.004270 (0.043)	-0.403615 (-4.036)	0.8745

Table 24

Heteroskedastic CAPM for BM-sorted portfolios during the period 1990-1997, using Hansen's (1982) t-statistic correction.

$$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt} / \hat{\sigma}^2_{mt}) + e_{it}$$

$$\varepsilon^2_{it} = \varpi_i + \delta_i \hat{\sigma}^2_{mt} + e_{it}$$

This model shows the return on a portfolio, i, at time t is a function of the return on the market and the return on the market divided by the estimated market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. The first t-statistics, directly below the parameter estimates, are estimated using Hansen's (1982) t-statistic correction while the t-statistics below these are Hansen's (1982) t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem.

	α_i	β_{i1}	β_{i2}	ϖ_i	δ_i	R^2
Growth	-0.003532 (-0.64) (-0.54)	1.187011 (32.91) (19.84)	-3.655186 (-0.74) (-0.74)	-0.000080 (-0.74) (-0.06)	0.252147 (2.43) (2.25)	0.9385
2	0.0000734 (0.01) (0.01)	0.980077 (16.63) (12.49)	-3.971943 (-0.73) (-0.72)	-0.000294 (-1.25) (-0.11)	0.490729 (1.85) (1.77)	0.9117
3	0.0038565 (1.13) (0.79)	0.873783 (22.86) (14.41)	-3.607587 (-1.21) (-1.20)	-0.000064 (-0.87) (-0.08)	0.146828 (1.81) (1.73)	0.9477
4	0.00389057 (1.29) (1.26)	0.779203 (21.10) (20.45)	-0.719193 (-0.27) (-0.27)	-0.000106 (-1.23) (-0.10)	0.199524 (2.11) (1.99)	0.9293
Value	0.0018054 (0.31) (0.24)	0.827204 (17.64) (10.41)	4.914907 (0.93) (0.93)	-0.000259 (-1.37) (-0.10)	0.387736 (1.99) (1.99)	0.9056

Table 25

Heteroskedastic CAPM for BM-sorted portfolios during the period 1990-1997, using WLS estimation.

$$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt} / \hat{\sigma}^2_{mt}) + e_{it}$$
$$\varepsilon^2_{it} = \varpi_i + \delta_i \hat{\sigma}^2_{mt} + e_{it}$$

The model shows that this covariance is dependent on the aggregate market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. WLS estimation involves using the predicted values of the second equation as weights for the terms in the first equation and then re-estimating the first equation. We iterate between the equations 3 times. The first t-statistics, directly below the parameter estimates, are estimated using WLS while the t-statistics below these are WLS t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem.

	α_i	β_{i1}	β_{i2}	R^2
Growth	-0.003248 (-0.658) (-0.524)	1.18731 (36.592) (19.644)	-3.905807 (-0.961) (-0.960)	0.9385
2	-0.005403 (-1.158) (-1.157)	1.097598 (49.017) (48.824)	0.152720 (0.040) (0.040)	0.9693
3	0.003816 (1.141) (0.757)	0.905226 (42.925) (16.321)	-3.929096 (-1.441) (-1.440)	0.9549
4	0.004199 (1.242) (1.167)	0.805846 (37.830) (26.649)	-1.292874 (-0.468) (-0.460)	0.9425
Value	0.010190 (2.505) (2.501)	0.776024 (36.685) (23.669)	-1.918677 (-0.582) (-0.581)	0.9443

Table 26

Heteroskedastic CAPM for size-sorted portfolios during the period 1990-1997, using GARCH estimation.

$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt} / \hat{\sigma}^2_{mt}) + e_{it}$

$s_{it} = \varpi_i + \phi_i e^2 + \gamma_i s_{t-1} + v_{it}$

This model shows the return on a portfolio, i, at time t is a function of the return on the market and the return on the market divided by the estimated market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term.

	α_i	β_{i1}	β_{i2}	ϖ_i	ϕ_i	γ_i	R^2
Small	0.018651 (6.018)	1.024532 (21.660)	$-1.177E^{-8}$ (-1.710)	0.000439 (3.879)	0.634994 (3.095)	$5.573E^{-19}$ (0.000)	0.7826
Size 2	-0.001301 (-0.771)	1.086859 (43.900)	$-4.678E^{-9}$ (-0.979)	0.000165 (6.752)	$4.136E^{-24}$ (0.000)	0.00006 (0.000)	0.9426
Size 3	-0.005238 (-5.748)	1.062123 (73.835)	$1.992E^{-9}$ (0.974)	0.000056 (5.961)	$-6.659E^{-23}$ (-0.000)	$-2.329E^{-21}$ (-0.000)	0.9796
Size 4	-0.006333 (-2.774)	0.949130 (23.376)	$5.097E^{-9}$ (0.633)	0.000147 (0.202)	0.037867 (0.242)	0.383166 (0.131)	0.8967
Large	-0.003315 (-1.322)	0.763328 (18.170)	$1.040E^{-8}$ (1.540)	0.000363 (5.330)	0.240211 (1.803)	$-9.694E^{-27}$ (-0.000)	0.7421

Table 27

Heteroskedastic CAPM for size-sorted portfolios during the period 1990-1997, using E-GARCH estimation.

$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt}/\sigma^2_{mt}) + e_{it}$

$\ln(h_{it}) = \varpi_i + \phi_i g_i(z_{t-1}) + \gamma_i \ln(h_{it-1}) + \eta_{it}$

$g_i(z_{it}) = \theta_{it} + [|z_{it}| - E|z_{it}|]$

$z_{it} = e_{it} / \sqrt{h_{it}}$

This model shows the return on a portfolio, i, at time t is a function of the return on the market and the return on the market divided by the estimated market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term. However, the variance of the error term is assumed to respond asymmetrically to changes in volatility with past increases in the functions variance causing a greater reaction than decreases in the variance function.

	α_i	β_{i1}	β_{i2}	ϖ_i	ϕ_i	γ_i	θ_i	R^2
Small	0.01701 (0.170)	0.994140 (9.941)	-1.52297 (-0.00)	7.079535 (-70.795)	0.839552 (8.396)	0.014202 (0.142)	-0.486678 (-4.867)	0.7579
Size 2	0.00130 (-0.013)	1.086859 (10.869)	$-4.678E^{-9}$ (-0.000)	8.678738 (-86.787)	0.00001 (0.000)	0.00001 (0.000)	0.00001 (0.000)	0.9426
Size 3	0.00523 (-0.052)	1.062108 (10.621)	$1.937E^{-9}$ (0.000)	9.749335 (-97.493)	0.000245 (0.002)	0.000133 (0.001)	-0.000242 (-0.002)	0.9796
Size 4	0.00637 (-0.064)	0.945497 (9.455)	$5.210E^{-9}$ (0.000)	8.243756 (-82.438)	0.000001 (0.000)	0.000001 (0.000)	0.000001 (0.000)	0.8967
Large	0.00277 (-0.028)	0.732534 (7.253)	$1.035E^{-8}$ (0.000)	7.602994 (-76.030)	0.000001 (0.000)	0.000001 (0.000)	0.000001 (0.000)	0.7439

Table 28

Heteroskedastic CAPM for size-sorted portfolios during the period 1990-1997, using Hansen's (1982) t-statistic correction.

$$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt}/\hat{\sigma}^2_{mt}) + e_{it}$$

$$\varepsilon^2_{ijt} = \varpi_{ij} + \delta_{ij}\hat{\sigma}^2_{mt} + e_{ijt}$$

This model shows the return on a portfolio, i, at time t is a function of the return on the market and the return on the market divided by the estimated market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. The first t-statistics, directly below the parameter estimates, are estimated using Hansen's (1982) t-statistic correction while the t-statistics below these are Hansen's (1982) t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem.

	α_i	β_{i1}	β_{i2}	ϖ_i	δ_i	R^2
Small	0.002159 (0.18) (0.14)	1.125047 (9.88) (6.15)	9.971379 (0.91) (0.91)	-0.001288 (-1.39) (-0.20)	1.190579 (1.84) (1.76)	0.7775
Size 2	-0.004462 (-1.02) (-0.78)	1.067766 (29.84) (17.45)	2.090386 (0.55) (0.55)	0.0000599 (1.17) (0.14)	0.079275 (1.67) (1.61)	0.9413
Size 3	-0.000928 (-0.35) (-0.28)	1.071846 (52.27) (32.17)	-3.480462 (-1.73) (-1.73)	0.0000929 (2.75) (0.56)	-0.030959 (-1.18) (1.16)	0.9799
Size 4	-0.00184057 (-0.29) (-0.22)	0.967305 (17.08) (10.31)	-3.21049 (-0.56) (-0.55)	-0.000277 (-1.26) (-0.12)	0.454357 (1.92) (1.83)	0.8950
Large	0.005071 (0.60) (0.46)	0.768036 (10.48) (6.30)	-5.370812 (-0.72) (-0.72)	-0.000361 (-1.03) (-0.09)	0.750863 (1.93) (1.83)	0.7346

Table 29

Heteroskedastic CAPM for size-sorted portfolios during the period 1990-1997, using WLS estimation.

$$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt}/\hat{\sigma}^2_{mt}) + e_{it}$$

$$\varepsilon^2_{ijt} = \varpi_{ij} + \delta_{ij}\hat{\sigma}^2_{mt} + e_{ijt}$$

The model shows that this covariance is dependent on the aggregate market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. WLS estimation involves using the predicted values of the second equation as weights for the terms in the first equation and then re-estimating the first equation. We iterate between the equations 3 times. The first t-statistics, directly below the parameter estimates, are estimated using WLS while the t-statistics below these are WLS t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem.

	α_i	β_{i1}	β_{i2}	R^2
Small	0.008223 (0.900) (0.781)	0.961036 (23.638) (11.745)	5.438959 (0.726) (0.726)	0.8853
Size 2	-0.006118 (-1.468) (-1.091)	1.025907 (43.182) (18.233)	3.90666 (1.145) (1.144)	0.9566
Size 3	-0.000613 (-0.247) (-0.141)	1.069923 (65.939) (20.882)	-3.722458 (-1.817) (-1.815)	0.9804
Size 4	0.004518 (1.056) (0.480)	0.979126 (34.642) (8.332)	-8.736970 (-2.427) (-2.427)	0.9418
Large	0.005273 (0.776) (0.578)	0.931225 (31.265) (10.581)	-6.342947 (-1.170) (-1.169)	0.9321

Table 30

Heteroskedastic CAPM of NYSE size-sorted portfolios for the period 1980-1986, using GARCH estimates.

$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt}/\hat{\sigma}^2_{mt}) + e_{it}$

$s_{it} = \varpi_i + \phi_i e^2 + \gamma_i s_{t-1} + v_{it}$

This model shows the return on a portfolio, i, at time t is a function of the return on the market and the return on the market divided by the estimated market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term.

	α_i	β_{i1}	β_{i2}	ϖ_i	ϕ_i	γ_i	R^2
Small	0.005602 (1.522)	1.140759 (16.535)	-0.000094 (-0.738)	0.000359 (0.000)	$-2.734E^{-23}$ (-0.000)	0.017704 (0.000)	0.8720
Size 2	-0.000283 (-0.235)	1.020110 (23.313)	-0.000015 (-0.263)	0.000069 (4.201)	0.012839 (0.067)	$-2.531E^{-21}$ (-0.000)	0.9685
Size 3	-0.002012 (-3.074)	1.022381 (56.553)	-0.000012 (-0.400)	0.000024 (5.170)	$-2.275E^{-23}$ (-0.000)	$5.635E^{-23}$ (0.000)	0.9890
Size 4	-0.001905 (-1.338)	0.964660 (30.463)	0.000022 (0.394)	0.000068 (6.288)	$7.569E^{-23}$ (0.000)	$9.306E^{-24}$ (0.000)	0.9674
Large	-0.001382 (-0.541)	0.853138 (15.393)	0.000099 (1.089)	0.000268 (5.468)	$-5.615E^{-23}$ (-0.000)	$-9.513E^{-24}$ (-0.000)	0.8676

Table 31

Heteroskedastic CAPM on Size-sorted NYSE stocks during the period 1980-1986, using E-GARCH estimation procedure.

$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt}/\sigma^2_{mt}) + e_{it}$

$\ln(h_{it}) = \varpi_i + \phi_i g_i(z_{t-1}) + \gamma_i \ln(h_{it-1}) + \eta_{it}$

$g_i(z_{it}) = \theta_{it} + [|z_{it}| - E|z_{it}|]$

$z_{it} = e_{it} / \sqrt{h_{it}}$

This model shows the return on a portfolio, i, at time t is a function of the return on the market and the return on the market divided by the estimated market variance at time t. The variance of the error term at time t, h_t, is assumed to be dependent on the lagged squared error terms and lagged variances of the error term. However, the variance of the error term is assumed to respond asymmetrically to changes in volatility with past increases in the functions variance causing a greater reaction than decreases in the variance function.

	α_i	β_{i1}	β_{i2}	ϖ_i	ϕ_i	γ_i	θ_i	R^2
Small	0.00694 (0.069)	1.10799 (11.080)	0.0000834 (0.001)	7.298387 (-72.984)	1.526406 (-15.264)	0.100661 (1.007)	0.2513 (2.513)	0.8613
Size 2	0.00077 (-0.792)	1.01778 (39.868)	-0.0000065 (-0.153)	7.914634 (-4.377)	0.927172 (-1.929)	0.176703 (0.938)	0.6820 (1.953)	0.9684
Size 3	0.00200 (-3.653)	1.02141 (47.438)	-0.0000285 (-1.100)	-6.15570 (-2.473)	0.564662 (-1.910)	0.426630 (1.833)	0.6621 (1.843)	0.9888
Size 4	0.00236 (-2.149)	0.96357 (20.057)	-0.0000214 (-0.394)	9.855831 (-1.320)	0.764006 (-2.019)	0.019980 (-0.026)	0.1276 (-0.327)	0.9660
Large	0.00188 (-0.019)	0.90488 (9.049)	0.0000665 (0.001)	9.593754 (-95.938)	0.431582 (-4.316)	0.162984 (-1.630)	0.03096 (-0.310)	0.8663

Table 32

Heteroskedastic CAPM on NYSE size-sorted stocks for the period 1980-1986, using Hansen's (1982) t-statistic correction.

$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt}/\hat{\sigma}^2_{mt}) + e_{it}$

$\varepsilon^2_{ijt} = \varpi_{ij} + \delta_{ij}\hat{\sigma}^2_{mt} + e_{ijt}$

This model shows the return on a portfolio, i, at time t is a function of the return on the market and the return on the market divided by the estimated market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. The first t-statistics, directly below the parameter estimates, are estimated using Hansen's (1982) t-statistic correction while the t-statistics below these are Hansen's (1982) t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem.

	α_i	β_{i1}	β_{i2}	ϖ_i	δ_i	R^2
Small	0.004636 (1.36) (1.32)	1.086024 (15.36) (14.33)	0.689496 (0.37) (0.37)	0.0003396 (2.37) (2.31)	0.019431 (0.30) (0.26)	0.8704
Size 2	-0.000568 (-0.37) (-0.36)	1.011105 (40.09) (38.60)	0.178777 (0.20) (0.20)	0.0000427 (2.09) (1.18)	0.017703 (1.35) (0.51)	0.9684
Size3	-0.001749 (-1.77) (-1.71)	1.014844 (60.26) (56.58)	-0.15616 (-0.44) (-0.44)	0.000028 (4.71) (3.89)	-0.00249 (-1.33) (-0.51)	0.9890
Size 4	-0.00028 (-0.19) (-0.14)	0.975889 (37.86) (20.06)	-1.043720 (-1.17) (-1.17)	0.000047 (2.45) (1.79)	0.012815 (0.94) (0.48)	0.9677
Large	0.0020367 (-0.63) (-0.62)	0.912138 (15.40) (15.04)	0.331608 (0.20) (0.20)	0.0000255 (3.61) (1.14)	0.01093 (0.34) (0.29)	0.8656

Table 33

Heteroskedastic CAPM for NYSE size-sorted portfolios for the period 1980-1986, using WLS estimation.

$$R_{it} = \alpha_i + \beta_{i1} R_{mt} + \beta_{i2} (R_{mt} / \hat{\sigma}^2_{mt}) + e_{it}$$

$$\varepsilon^2_{ijt} = \varpi_{ij} + \delta_{ij} \hat{\sigma}^2_{mt} + e_{ijt}$$

The model shows that this covariance is dependent on the aggregate market variance at time t. The squared error term at time t is assumed to be dependent on the aggregate market variance at time t. WLS estimation involves using the predicted values of the second equation as weights for the terms in the first equation and then re-estimating the first equation. We iterate between the equations 3 times. The first t-statistics, directly below the parameter estimates, are estimated using WLS while the t-statistics below these are WLS t-statistics adjusted using the method of Murphy and Topel (1985) to control for the generated regressor problem.

	α_i	β_{i1}	β_{i2}	R^2
Small	0.005620 (1.917) (1.910)	1.097690 (27.783) (26.340)	0.182914 (0.153) (0.153)	0.9350
Size 2	0.000058 (0.035) (0.035)	1.00890 (40.988) (38.915)	-0.205888 (-0.264) (-0.264)	0.9673
Size 3	-0.001649 (-1.668) (-1.582)	1.015905 (69.833) (59.401)	-0.227508 (-0.491) (-0.491)	0.9885
Size 4	-0.000636 (-0.439) (-0.358)	0.966980 (41.118) (24.543)	-0.799586 (-1.205) (-1.205)	0.9689
Large	-0.003805 (-1.299) (-1.102)	0.920901 (20.055) (12.738)	1.412484 (1.016) (1.016)	0.8761

Appendix 3

From the Methodology section equation 7 is:

$$\beta_{it} = \frac{\sum_{j=1}^{N} w_j(\xi + \psi\sigma_{mt}^2)}{\sigma_{mt}^2}$$

This can be rewritten as:

$$\beta_{it} = \frac{\sum_{j=1}^{N} (w_j\xi + w_j\psi\sigma_{mt}^2)}{\sigma_{mt}^2}$$

Since the sum of the weights must equal one, this reduces to:

$$\beta_{it} = \frac{\xi + \psi\sigma_{mt}^2}{\sigma_{mt}^2}$$

$$\beta_{it} = \frac{\xi}{\sigma_{mt}^2} + \frac{\psi\sigma_{mt}^2}{\sigma_{mt}^2}$$

$$\beta_{it} = \frac{\xi}{\sigma_{mt}^2} + \psi$$

The risk premium given by equation 5 is;

$$R_{it} = (R_{mt})(\frac{\xi}{\sigma_{mt}^2} + \psi)$$

From equation 6 it is easy to see that the traditional CAPM, by substituting β_1 for ψ and β_2 for ξ, can be rewritten as:

$$R_{it} = \alpha + \beta_{i1} R_{mt} + \beta_{i2} \frac{R_{mt}}{\hat{\sigma}_{mt}^2} + \varepsilon$$