MARKET THINNESS AND THE POTENTIAL BENEFITS OF DOMESTIC-ONLY PORTFOLIO DIVERSIFICATION IN CANADIAN EQUITY MARKETS

Ping Liu

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ABSTRACT

MARKET THINNESS AND THE POTENTIAL BENEFITS OF DOMESTIC-ONLY PORTFOLIO DIVERSIFICATION IN CANADIAN EQUITY MARKETS

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This study examines the benefits of diversification for stocks that are subject to varying degrees of market thinness for domestic-only investment in Canadian equities. The study also determines if the benefits of diversification are best examined using time-varying or static estimates of volatility.

This thesis has four major findings. First, about 30 securities are required, on average, to obtain most of the risk-reduction (variance-reduction) benefits from diversification for all-domestic diversification for Canadian equities. Second, the same level of diversification is achieved with less securities for the less stringent trading infrequency index. Third, both size and trading infrequency effects exist in the variances of the monthly returns and in the risk-adjusted performance ratios for the simulated portfolios. Fourth, the monthly total returns for the simulated portfolios do not exhibit heteroscedasticity based on the Portmanteau Q and Lagrange Multiplier tests for the time periods examined in this thesis. However, some evidence exists that thin trading induces some heteroscedasticity into the monthly returns of Canadian stocks.

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This thesis is dedicated to my parents, who have always encouraged me to pursue the highest education possible. I sincerely wish that they are proud of me.

Table of Contents

List of Tables

List of Figures

MARKET THINNESS AND THE POTENTIAL BENEFITS OF DOMESTIC-ONLY PORTFOLIO DIVERSIFICATION IN CANADIAN EQUITY MARKETS

1. INTRODUCTION

The number of securities required to obtain most of the benefits of diversification for domestic-only investment is a subject of ongoing interest to both academics and practitioners. Studies by Kryzanowski, Rahman and Sim (1985) and by Cleary and Copp (1999) finds that about 30 to 50 stocks are required to capture most of the benefits associated with domestic diversification in Canadian equity markets. Other studies, such as Fowler, Rorke and Jog (1979), suggest that trading infrequency is a characteristic of Canadian equity markets that may affect the number of securities required to obtain most of the benefits of domestic-only investment, and may affect whether or not the variances of individual security returns vary over time. However, studies that estimate time-varying variances for other markets (such as the U.S.) suggest that the variances of return are more likely to be time-varying for individual securities and for returns measured at high frequency, such as intraday or daily. Unfortunately, no study appears to assess the effect of market thinness on the potential benefits obtained from domestic-only diversification in equities.

Thus, the primary objective of this thesis is to examine the benefits of diversification for stocks that are subject to varying degrees of market thinness for domestic-only investment in Canadian equities. A secondary objective is to assess if the benefits of diversification are best examined using time-varying or static estimates of volatility. To this end, the monthly returns for all the stocks on the Toronto Stock Exchange (TSE) that are included on the 1998 version of the CFMRC database are examined over the ten-year period from January 1988 through December 1997. To test if diversification benefits are period dependent, the tests also are applied to two five-year sub-periods; namely, January 1987 through December 1992, and January 1993 through December 1997. Portfolios of size 1, 5, 10, 15, 20, 30 and 50 are examined in this thesis.

This thesis makes a number of important contributions to the literature. First, about 30 securities are required, on average, to obtain most of the risk-reduction (variance-reduction) benefits from diversification for all-domestic diversification for Canadian equities. The number of securities required to achieve a given level of diversification in this study is comparable to that obtained by Cleary and Copp (1999), and is higher than that required for the bigger U.S. market. The relatively greater number of securities required in Canadian markets compared to U.S. markets is expected due to the higher concentration of Canadian stocks within a few industries, and the smaller universe of investment possibilities in Canadian markets. Second, the same level of diversification is achieved with less securities when a less stringent trading infrequency index is used. Third, both size and trading infrequency effects exist in the variances of the monthly returns and in the risk-adjusted performance ratios for the simulated portfolios. Fourth, the monthly total returns for the simulated portfolios do not exhibit heteroscedasticity based on the Portmanteau Q and Lagrange Multiplier tests for the time periods examined in this thesis. However, thin trading does induce some heteroscedasticity into the monthly returns of Canadian stocks.

The remainder of the thesis is organized as follows. The literature is reviewed in the next section. The third section outlines the sample selection procedure used herein and describes the data. The fourth section presents the empirical procedure used in this thesis. The fifth section presents and analyses the empirical results on the benefits of portfolio diversification when thinness is held constant and portfolio size varies, and when portfolio size is held constant and thinness varies. The sixth section tests for possible heteroscedasticity in the monthly returns of the studied portfolios. The last section concludes the thesis by presenting the major findings, implications of the findings, and directions for future research.

2. REVIEW OF THE LITERATURE

In this section, three relevant strands of the literature are reviewed. The first strand deals with thin trading in Canadian markets. The second strand deals with the relationship between the benefits of portfolio diversification and the number of securities that investors hold in their portfolios. The third and final strand deals with whether or not Canadian stock returns exhibit heteroscedasticity. Each of these is now reviewed in turn.

2.1 Thin Trading on the Toronto Stock Exchange

Fowler et al. (1980) examine the frequency of trading at a monthly frequency for stocks on the TSE during the period 1970-1979. They examine three categories of trading frequencies; namely, "fat" securities that trade during the closing day of every month, "moderate" securities that trade at least once each month (but not necessarily on the last trading day); and "infrequent" securities that have one or more months without any trades. They find that the last category is by far the largest. The last category accounts for 42 to 59 percent of all the stocks, depending upon the type of securities considered.

Amihud and Mendelson (1986) find that the market-observed expected return is an increasing and concave function of the bid-ask spread.

2.2 Benefits of Diversification with Different-Size Portfolios

Only articles dealing with the diversification benefits of domestic-only equity investment in U.S. and Canadian markets are reviewed in this section of the thesis. Thus, the literature dealing with domestic-only equity investment in non-North American markets,[1] international diversification,[2] the use of derivatives for diversification,[3] the impact of diversification on trading rule profitability,[4] and the diversification benefits available across style categories[5] and in other asset classes, such as bonds,[6] real estate,[7]

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[1] This literature includes Allen and Sugianto (1994) for Australia, and Poon, Taylor and Ward (1992) for U.K.
[2] This literature includes Akdogan (1996), Athanasoulis (1996), Balkan and Erol (1995), Byers and Peel (1993), Cosset and Suret (1995), Denning and Chow (1992), De Santis and Gerard (1997), Errunza, Hogan and Hung (1999), Glassman and Riddick (1994), Jorion (1985), Olienyk (2000), Solnik (1974) and Stevenson (2000).
[3] This literature includes Jensen, Johnson and Mercer (2000).
[4] This literature includes Chelley-Steeley and Steeley (1997).
[5] To illustrate, Israelsen (1999) suggests that diversification across different market caps is required to create a diversified investment portfolio.
[6] This literature includes Riepe (2000), Hill and Schneeweis (1981), and McEnally and Boardman (1979).

currency[8] and mixed assets,[9] are not reviewed herein. The section begins with a review of the literature on the benefits of diversification in U.S. markets, and concludes with a review of the literature on the benefits of diversification in Canadian markets.

Evans and Archer (1968) regress portfolio standard deviation on the inverse of portfolio size. For 60 replications, they find that eight to ten securities are sufficient to form a well-diversified portfolio. Latane and Young (1969) verify the results of Evans and Archer. They examine the incremental reduction in standard deviation as the number of stocks in the portfolio increases. With 60 replications, they find that an eight-stock portfolio achieves 85 percent of the possible gains from diversification.

Fisher and Lorie (1970) examine the frequency distributions and dispersions of the wealth ratios of investments for different-sized portfolios consisting of stocks on the New York Stock Exchange for the period, 1926-1965. For portfolios formed with equal initial investments in each stock included in a portfolio, they identify the portfolio size that generates an 80 percent reduction in relative dispersion. They find that the opportunity to reduce dispersion by increasing the number of stocks in the portfolio is rapidly exhausted. Approximately 40%, 80% and 90% of the achievable reduction in diversifiable risk is obtained by holding two, eight, and 16 stocks, respectively.

Sharpe (1970) concludes that a portfolio containing fifteen or so securities may be considered well-diversified. Mokkelbost (1971) finds that a major portion of the achievable reduction in risk is accomplished when "relatively few" different securities are included in a portfolio. Wagner and Lau (1971) regress the coefficient of

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[7] This literature includes Cheng and Liang (2000), Stevenson (2000), Ori (1995), Lai, Wang, Chan and Lee (1992), and Kuhle (1987).
[8] This literature includes Ariff and Varghese (1990).
[9] This literature includes Giliberto (1999).

determination from a market model regression on portfolio size. They find that ten securities are needed to form a well-diversified portfolio.

Sharpe (1972) demonstrates that diversification reduces non-market risk. Generally, the likelihood that sufficient good fortune will balance off bad fortune increases as the number of securities in the portfolio increases.

Fielitz (1974) regresses the mean absolute deviation of portfolio returns on the inverse of portfolio size. He finds that eight securities are needed to form a well-diversified portfolio.

Klemkosky and Martin (1975) regress the market model residual variance on portfolio size. They find that eight to fourteen securities are needed to form a well-diversified portfolio. Fama (1976) identifies the portfolio size required to generate a 95 percent reduction in portfolio variance. He finds that twenty securities are needed to form such a well-diversified portfolio. Elton and Gruber (1977) conclude that a portfolio containing fifteen securities may be considered well-diversified based on an analytical solution of the relationship between risk and portfolio size.

Klein and Bawa (1977) consider the effect of limited information and estimation risk on optimal portfolio diversification. For situations of insufficient information and minimal prior information, they show that it is asymptotically optimal for an investor to limit diversification to a subset of the securities. Lloyd, Hand and Modani (1981) contend that there can be no presumption that equal weights are optimal, and the evidence on the relationship between portfolio size and effective diversification can not be conclusive unless optimal weights are used. They argue that modern portfolio theory shows that an equal weighting of securities is an inefficient method of forming portfolios.

Tole (1982) regresses the portfolio standard deviation on portfolio size, regresses market model coefficient of determination on portfolio size, and regresses market model residual variance on portfolio size. He found that 60 securities are needed to form a well-diversified portfolio.

Kryzanowski and Rahman (1985) examine the benefits of domestic-only diversification for U.S. equities when variance varies over time. They find that 81 and 90 percent of the benefits of diversification in terms of the mean and the variance of the intertemporal variance, respectively, are achieved, on average, with a portfolio consisting of five securities. They find that 95 and 98 percent of the benefits of diversification in terms of the mean and the variance of the intertemporal variance, respectively, are achieved, on average, with a portfolio consisting of fifteen securities.

Statman (1987) compares the return on portfolios of different sizes with the returns on a levered, diversified benchmark portfolio. He finds that a portfolio of randomly chosen stocks must include at least 30 and 40 stocks for a borrowing and a lending investor, respectively, to be considered well diversified.

Based on a survey of a number of U.S. investment textbooks and academic studies, Newbould and Poon (1993) conclude that the consensus view is that portfolios consisting of eight to 20 stocks are generally considered to be well diversified. Newbould and Poon (1993) argue that the standard recommendations to form a portfolio with between eight and 20 stocks are flawed, and that it may be desirable to have substantially more than 20 stocks in a portfolio to eliminate diversifiable risk.

Beck, Perfect and Peterson (1996) find that the testing methodology used affects the size of what is considered a well-diversified portfolio. They propose the use of two methodologies to determine the number of securities needed to obtain a well-diversified portfolio; namely, the calculation of the power curves of the statistical tests or the selection of a test statistic that is less sensitive to the number of replications (such as the modified Levene test). Based on the use of the first approach, approximately 48 securities are necessary to eliminate diversifiable risk. Based on the use of the second approach, 18 securities are needed to achieve a well-diversified portfolio.

For Canadian markets, Kryzanowski, Rahman and Sim (1985) examine the benefits from purely domestic (Canadian) and constrained foreign (U.S.) diversification in terms of the reduction of the mean and the variance of the time series of monthly portfolio return variances for various portfolio sizes. Unlike the case for the United States, they find that the benefits of diversification are not exhausted as quickly in Canadian equity markets. For purely domestic (Canadian) diversification, a portfolio of 30 securities is required to attain 95 and 98 percent of the benefits of diversification in terms of the mean and the variance, respectively, of the intertemporal return variance. Furthermore, it appears that a global portfolio, which is 50 percent invested in a portfolio containing 30 equally-weighted Canadian securities and 50 percent invested in a portfolio containing 15 equally-weighted U.S. securities, on average, attains 95 and 98 percent of the total benefits of global diversification in terms of the mean and the variance, respectively, of the intertemporal return variance.

More recently, Cleary and Copp (1999) use monthly arithmetic mean rates of return, and the monthly standard deviation of these returns to examine the benefits of diversification. They examine the 222 TSE-listed stocks that have complete total return information available over the period from January 1985 through December 1997. They also examine two equal sub-periods; the first from January 1985 through June 1991, and the second from July 1991 through December 1997. There are 236 stocks available for the first sub period, and 415 stocks available for the second period. Cleary and Copp find that 30 to 50 Canadian stocks are required to capture most of the benefits associated with diversification. However, substantial benefits occur by diversifying across as few as 10 stocks.

Canadian investment textbooks, such as Cleary and Jones (1999) and Bodie, Kane, Marcus, Perrakis and Ryan (1997), refer to the Statman (1987) study as a guide to determining what the benefits are from diversification. In contrast, Sharpe, Alexander, Bailey and Fowler (1997) suggest that 30 stocks is the 'magic' number for ensuring that a portfolio is likely to be well diversified without providing any references.

2.3 Heteroscedasticity on the Toronto Stock Exchange

Belkaoui (1977) examines biweekly price data for a sample of 45 randomly chosen TSE common stocks and the TSE industrial index as a market proxy. He reports evidence of heteroscedasticity in the market model residuals for 91 percent of the sampled firms based on the Spearman rank correlation coefficient, for 40 percent of the sampled firms using the Goldfeld and Quandt procedure, and for 62 percent of the sampled firms based on the Bartlett test. He concludes that heteroscedasticity is a serious problem in the market model for the majority of the Canadian stocks studied.

Dhingra (1978) tests for heteroscedasticity in the monthly closing returns for a sample of 251 TSE common stocks selected from the Financial Post Weekly Closing

Price tape. Four of the six tests for heteroscedasticity find that a large proportion of the securities exhibit heteroscedasticity at the 1% level.

Using the monthly closing prices, dividends and returns from the Laval file for the period, June 1965 to June 1976, Fowler, Rorke and Jog (1979) investigate the effects of trading frequency on the behavior of the residual variance of the market model for stocks traded on the TSE. They test their hypothesis of no heteroscedaticity using two different returns calculations (arithmetic and logarithmic), two different indices, and confidence levels of 1%, 5% and 10%. The reason for calculating returns in two different ways is to determine whether the use of logs improves the results as suggested by Fowler, Rorke and Riding (1979). They perform three tests of heteroscedasticity, namely, the Spearman rank correlation test, the modified Bartlett test, and the Goldfeld and Quandt test. Except for the Spearman rank order correlation test, all tests clearly indicate that the frequency of trading has an important effect on the homoscedasticity of the error term of the market model. However, their results show an inconsistent relationship between heteroscedasticity and thinness of trading. They find that the percentage of stocks exhibiting heteroscedasticity ranges from 30 to 93%. Heteroscedasticity in the TSE is clearly evident, but is not as serious for almost fat and fat securities. Fowler et al. note that the use of the logarithmic form of the market model reduces heteroscedasticity somewhat, and that the use of thin-trading adjustment procedures, such as those suggested by Scholes and Williams (1977) or Dimson (1979), may lead to improved estimates of the residuals with less evidence of heteroscedasticity. Fowler et al. caution that the detected phenomenon may not be true heteroscedasticity but simply an artifact caused by non-stationarity in the distribution of the residuals induced by thin trading.

3. SAMPLE SELECTION AND DESCRIPTION OF THE DATA

The initial sample consists of all the stocks traded on the Toronto Stock Exchange. Only stocks with prices greater than two dollars per share for the month prior to the period of study are retained in the sample. Thus, this screen is based on the per-share stock prices for December 1987 for the first five-year period and for the entire ten-year period, and for December 1992 for the second five-year period. The sample sizes are 785 stocks for the entire ten-year period and the first five-year sub-period, and 617 stocks for the second five-year sub-period.

Monthly and not daily returns are examined. The reason is that, since investors and portfolio managers are more likely to rebalance their portfolios monthly rather than daily, these market participants are more likely to be concerned with monthly portfolio volatility. The monthly returns are obtained from the Canadian Financial Markets Research Centre (CFMRC) database. Months for which returns are based on no trades (identified by a -9 in the database) or months where the stock is no longer included in the CFMRC are replaced by a zero return. The sensitivity of potential diversification benefits to this treatment of non-trading months is examined below.

4. EMPIRICAL PROCEDURE

4.1 Typical Measure of Diversification Benefits

The benefits of portfolio diversification are typically measured by the reduction in the standard deviation of returns for the portfolio compared to that for the market. This measure also is used herein. Specifically, diversification benefits are measured as the ratio of the average variance of all of the randomly-chosen portfolios of size n minus the average variance of all the randomly-chosen portfolios of size one, all divided by the average market variance minus the average variance of all the randomly-chosen portfolios of size one. A number of other measures of diversification benefits also are used and are described below in section 4.4.

4.2 Portfolio Formation Procedure

Since thin trading is allegedly a problem for the majority of Canadian securities (see the review in section 2.1 above), the portfolio diversification benefits obtainable by increasing the available investment opportunity set by including more infrequently traded securities is examined. To this end, a trading infrequency index is first calculated for each stock in the sample for each of the three holding periods studied herein. The index τ is obtained for each stock by dividing the number of trading months with no trades in that security by the number of trading months in that ten (five) year period, and then multiplying this decimal value by 100 to get a percentage value. Thus, an index value of 0% for a stock indicates that all of the months include at least one trade for the stock, and an index value of 100% indicates that none of the months include at least one trade for the stock.

Seven trading infrequency index cut-off values are used to simulate the impact of different levels of market thinness (trading infrequency) on the benefits of portfolio diversification. These cut-off infrequency index values used herein are 50%, 40%, 30%, 25%, 20%, 10%, and 5%.

The universe of stocks available for investment purposes for the seven values of the trading infrequency index for each of the three time periods are summarized in Table 1. As expected, more stringent trading frequency requirements (i.e., lower values of the trading infrequency index) lead to a substantial reduction in the universe of stocks available for investment purposes. For example, for the entire ten-year holding period, the universe of stocks ranges from 785 firms for a 100% trading infrequency index screen to 405 stocks for a 50% trading infrequency index screen to 183 stocks for a 5% trading infrequency index screen. To illustrate, the 50% and 5% values of the screen reduce the universe of securities available for investment by 48% and 77%, respectively.

For each combination of holding period, portfolio size n and thinness index τ, 500 simulated portfolios are formed and the results are averaged. For each portfolio of size n, where n = 1, 5, 10, 15, 20, 30 and 50, n stocks are randomly choose to enter an equally-weighted portfolio of size n. The implicit assumption is that each portfolio is not subsequently rebalanced over its planned holding period of five (or ten) years.

The specific procedure followed for a specific combination of portfolio holding period, trading infrequency index cut-off value τ, and portfolio size n consists of six steps. First, n securities without replacement are randomly drawn from the applicable universe of securities available for the chosen portfolio-holding period and trading infrequency index cut-off value τ. Second, an equally-weighted portfolio of the stocks chosen in step one is formed, and then is held until the end of the specific portfolio holding period being examined. Third, the monthly returns of the portfolio are calculated from the monthly returns of its constituent stocks for each of the months for the specific portfolio holding period being examined. Fourth, the variance of the monthly returns for the portfolio of stocks for the chosen size n and trading infrequency index τ are calculated. Fifth, the first four steps are repeated 500 times. And finally, the average variance and relative reduction of the variance are calculated for the 500 portfolios.

The above procedure is repeated for every combination of portfolio holding period (3 possibilities), trading infrequency index cut-off value τ (7 possibilities) and portfolio size n (7 possibilities).

4.3 The Benchmark for Assessing the Benefits of Diversification

The value-weighted TSE-Western index is used as the benchmark to assess the diversification benefits achieved by forming portfolios of different sizes from stock universes that exhibit different trade infrequency characteristics. This index is used because theory suggests that optimal diversification is achieved by holding the market portfolio of risky assets (i.e. all the assets according to their market weights) in an efficient market.

As a test of robustness, we also use the equally-weighted TSE-Western index as a benchmark in our simulations. Since an equally-weighted index places greater weight on smaller (and supposedly more risky) individual securities, we expect that the risk reduction for some of the larger portfolio sizes examined may exceed 100 percent when the equally-weighted index is used as the standard of comparison.

4.4 Other Measures of the Benefits of Diversification

While the relative benefits of diversification are generally measured as the relative reduction in risk (variance), some of the risk reduction can be at the expense of return if the systematic risks of the portfolios being compared are not held constant and/or markets are not truly efficient. The Sharpe (1966) measure examines the reward to total volatility trade-off. The Sharpe ratio is calculated by dividing average portfolio excess return over the sample period by the standard deviation of returns over that period. It is appropriate because it considers the total risk of the portfolios being examined.

5. THE EMPIRICAL RESULTS ON THE BENEFITS OF PORTFOLIO DIVERSIFICATION

5.1 Comparison of the Variance of the Portfolio Monthly Returns for Various Portfolios Sizes and Trading Infrequency Index Values

Standard deviations of the monthly returns are used instead of the variances since the standard deviation statistically behave better than do the variances.

5.1.1 Two-way ANOVA and One-way ANOVA

Risk reduction potentially depends upon two factors: portfolio size n and trading infrequency index τ. Since the standard deviations of the returns are potentially related to these two factors, the two-way ANOVA test is used first to examine if both factors significantly affect the standard deviation of return and if any interaction effect occurs from these two factors. The null hypothesises of two-way ANOVA test are that there is no size effect, there is no trading infrequency effect and there is no interaction effect. If the significant effect is found for one of the two factors, a one-way ANOVA can be used to examine the effect of one factor assuming another factor is held constant. The null hypothesises of one-way ANOVA test is that there is no size effect for comparison of across the different sizes, and there is no trading infrequency effect for comparison across the different trading infrequency effect. The significance level I used is 5%.

Analysis of variance models require that some assumptions be involved about the data; namely that the data are normally distributed and that the variances in groups are equal. Analysis of variance is quite robust and can withstand violations of normality, particularly if the sample size is large. If the cells have equal numbers of observations, as in this thesis, ANOVA is even more robust.

5.1.2 Two-way ANOVA of Standard Deviations of the Portfolio Monthly Returns

Before using the two-way ANOVA to examine if the portfolio size, the trading infrequency, and the interaction of size-trading infrequency affect the standard deviation of the monthly returns, the assumptions of the two-way ANOVA are first checked. The boxplots of the standard deviation of the monthly returns for all three periods, which are presented in Figure 1, allow for a visual examination of whether or not the data appear to be normally distributed and whether or not the sample variances are equal. The boxplots suggests that all of the groups of standard deviation of the monthly returns are fairly normally distributed. However, the boxplots shows that the variances of the standard deviations of the monthly returns tend to decrease as portfolio size increases. This violation of the equality of variances assumption may make the F statistic less reliable.

The two-way ANOVA results are summarized in Table 2. For the entire ten year period and the first five year period, portfolio size, trading infrequency and their interactive term significantly affect the standard deviations of monthly portfolio returns. The significance of the interactive term means that when the portfolios size changes, the trading frequency also changes. For the second five year period, there is both a size and trading infrequency effect, but no interaction effect of size and trading infrequency on the standard deviations of portfolio's monthly returns.

5.1.3 One-way ANOVA of Standard Deviations of the Portfolio Monthly Returns

The one-way ANOVA test is preceded by a visual inspection of the boxplots of the standard deviation of the portfolio's monthly returns that are presented in Figures 2 and 3. This is done to determine whether or not the data appear to be normally distributed and their variances are equal.

Based on a visual examination of the box plots for all three time periods, the variances of the data appear to be equal across the different trading infrequency for the same portfolio size. However, the variances of the data appear decrease as portfolio size increase for the same trading infrequency. This may affect the accuracy of the inferences drawn below of the size effect. The boxplots for the three periods also suggest that the data are distributed fairly symmetrically because the medians are basically located in the middle of the central box. So, generally, one can apply a one-way ANOVA to the data.

A one-way ANOVA is used first to test if the standard deviations of monthly returns are significantly different across portfolio sizes for the same trading infrequency index. Based on the results presented in Table 3, the standard deviations of the monthly returns across the seven portfolio sizes are significantly different for each trading infrequency index and time period. Thus, there is a size effect on the standard deviation of the monthly returns for all trading infrequencies and the time periods examined in this thesis.

The one-way ANOVA is then used to test if the standard deviations of monthly returns are significantly different across trading infrequency indexes for a fixed portfolio size. Based on the results presented in Table 4, generally, the standard deviations of the monthly returns are significantly different across the seven trading infrequency indexes for each portfolio size for all three periods expect the portfolio size of one in the second five year. Thus, there is a trading infrequency effect on the standard deviation of the monthly returns for all portfolio size and time periods examined in this thesis.

5.2 Relative Risk Reduction for Various Portfolio Sizes and/or Various Trading Infrequency Index Levels

In this thesis, securities that have at least one trade for at least 95 percent of the months (i.e., have a trading infrequency index of at least 5%) are deemed to be frequently traded or thick securities. Similarly, securities that have at least one trade for at least 50 percent of the months (i.e., have a trading infrequency index of 50%) are deemed infrequently traded or thin securities. The risk (variances) and relative risk reduction for the seven different portfolio sizes for the entire ten-year period and for each of two five-year sub-periods are reported in Table 5, respectively.

The relative reduction in the return variances of the portfolios are compared first across different portfolio sizes for a fixed infrequent trading index cut-off value, and then across the different infrequent trading index cut-off values for a fixed portfolio size for each of the three periods. For the entire ten-year period, the risk reductions are 78.1%, 88.3% and 92.8% for portfolios with sizes of 5, 10, and 20 securities, respectively, for a universe of securities based on our most stringent trading infrequency screen (i.e., the 5% cut-off value). The corresponding relative risk reductions are 77.7%, 89.3% and 94.7%

for portfolios of similar sized portfolios drawn from a universe of securities based on our least stringent trading infrequency screen (i.e., the 50% cut-off value). While the risk reduction benefits achievable at smaller portfolio sizes of five and ten stocks are similar across the universe of stocks screened by their trading infrequency indexes, the average risk reduction benefits achievable at larger portfolio sizes of 20 tend to increase with less stringent trading infrequency cut-off values. To illustrate, to achieve an average risk reduction benefit of about 95%, a portfolio size of 30 stocks is required for a universe of stocks obtained by applying a 5% trading infrequency cut-off value, and a portfolio size of 20 stocks is required for a universe of stocks obtained by applying a 50% trading infrequency index cut-off value.

For the first five-year period, the relative risk reductions are 73.6%, 85.4% and 90.8% for portfolios sizes of 5, 10, and 20, respectively, for a universe of securities based on our most stringent trading infrequency screen (i.e., the 5% cut-off value). The corresponding relative risk reductions are 78.8%, 87.1% and 93.5% for portfolio sizes of 5, 10 and 20 securities, respectively, for a universe of securities based on our least stringent trading infrequency screen (i.e., the 50% cut-off value). While the relative risk reduction benefits achievable at smaller portfolio sizes of five and ten stocks are similar across the universe of stocks screened by their trading infrequency indexes, the average relative risk reduction benefits achievable at the larger portfolio size of 20 tends to increase with a less stringent trading infrequency cut-off value. To illustrate, while a portfolio size of 50 stocks is required to achieve average relative risk reduction benefits of 93.9% by applying a 5% trading infrequency cut-off value, and a portfolio size of 20 stocks is required to achieve average risk reduction benefits of 93.5% by applying a 50% trading infrequency cut-off value.

For the second five-year period, the relative risk reductions are 81.7%, 91.2% and 94.4% for portfolios with sizes of 5, 10 and 15 securities, respectively, for a universe of securities based on our most stringent trading infrequency screen (i.e., the 5% cut-off value). The relative risk reductions are 84.4%, 93.9% and 97.6% for portfolios with sizes of 5, 10 and 15 securities, respectively, for a universe of securities based on our least stringent trading infrequency screen (i.e., the 50% cut-off value). While the relative risk reduction benefits achievable at smaller portfolio sizes of five and ten stocks are similar across the universe of stocks screened by their trading infrequency indexes, the average risk reduction benefits achievable at the larger portfolio size of 15 securities tends to increase with a less stringent trading infrequency cut-off value. To illustrate, portfolio sizes of 30 and 15 stocks achieve average risk reduction benefits of 97.7% and 97.6% for universes of stocks obtained by applying 5% and 50% trading infrequency cut-off values, respectively.

To summarize, less stocks are needed in a portfolio to obtain a fixed relative risk reduction for less stringent trading infrequency screens.

5.3 Comparison of the Return-to-Variability Ratios for Various Portfolio Sizes and Trading Infrequency Index Values

The trade-off between the reduction in risk and possible reduction in return from diversification is further examined in this section of the thesis by calculating and comparing the Sharpe ratios for various portfolio sizes and trading infrequency index cut-off values. The Sharpe ratio compares the average excess return on a portfolio (i.e., its average return minus the risk-free return) to the standard deviation of return for the portfolio. The Sharpe measure examines the reward to total volatility for a portfolio. More formally, the Sharpe measure is given by:

$$\text{Sharpe ratio} = (\overline{r_p} - \overline{r_f}) / \delta_p$$

Where $\overline{r_p}$ is the average portfolio return, $\overline{r_f}$ is the average risk-free rate, and δ_p is the standard deviation of portfolio return.

5.3.1 Two-way ANOVA and One-way ANOVA

Since the Sharpe ratios are potentially related to both portfolio size n and trading infrequency screen τ, a two-way ANOVA test is conducted first to determine if both factors, and their interactive term significantly affect the Sharpe ratios. The null hypothesises are there is no size effect, there is no trading infrequency effect and there is no interaction effect. If there is a significant effect are found of one of these two factors, then a one-way ANOVA test can be used to examine the one factor effect, while the other factor is held constant. The null hypothesises is there is no size effect for comparison of across the different sizes, and there is no trading infrequency effect for comparison across the different trading infrequency effect. The significance level I used is 5%.

ANOVA model require that the data are normally distributed and the variances in groups are equal.

5.3.2 Two-way ANOVA of the Sharpe Ratios of the Portfolio Monthly Returns

The assumptions of two-way ANOVA test are checked prior to conducting it to examine if there is a size effect, trading infrequency effect and interactive effect on the Sharpe ratios for the various portfolios for each of the three time periods.

Based on the boxplots, which are presented in Figure 4, all of the groups of Sharpe ratios appear to be normally distributed, but their variances appear to be unequal. The variances tend to decrease with an increase in portfolio size. As noted earlier, this violation of the equality of the variances assumption may make the F statistic less reliable.

The results, which are presented in Table 6, suggest that both a size and trading infrequency effect, but no interaction effect exist for the entire ten year period. For both of the five-year periods, all three effects are present. This means that for each of the sub-periods, the Sharpe ratios are affected not only by portfolio size and trading infrequency but also by the co-movement of size and trading infrequency.

5.3.3 One-way ANOVA of the Sharpe Ratios of the Portfolio Monthly Returns

The one-way ANOVA boxplots of the sharpe ratios for each of the three time periods are presented in Figures 5 and 6. A visual inspection of these boxplots suggests that all of the groups of Sharpe ratios groups are normally distributed. For testing the trading infrequency effect for each of the time periods, the variances of the Sharpe ratios are approximately equal. However, since the variances appear not to be equal for testing the size effect, the reliability of such tests is somewhat suspect.

The results from the use of a one-way ANOVA to test if the Sharpe ratios are significantly different across portfolio sizes for the same trading infrequency index cut-off value, and if they are significantly different across trading infrequency index cut-off values for the same portfolio size are presented in Table 7 and 8. Each of these findings is discussed in turn.

The comparisons of the Sharpe ratios across the 7 different portfolio sizes for each of the trading infrequency index cut-off values are designed to detect if any significant size effect exists in the Sharpe ratios. Based on the results presented in table 7, for the total period, a significant size effect is identified in the Sharpe ratios for only the 50% trading infrequency index cut-off value. A significant size effect is identified in the Sharpe ratios for the 5% and 25% trading infrequency index cut-off values for both sub-periods, and for the 50% trading infrequency index cut-off value for only the first sub-period. Thus, little consistent evidence exists for the existence of a size effect in the Sharpe ratios.

The comparisons of the Sharpe ratios across the various trading infrequency index cut-off values for each of the portfolio sizes are designed to detect if any significant trading infrequency effect exists in Sharpe ratios. Based on the results, which are presented in Table 8, trading infrequency has a significant effect on portfolio risk-adjusted performance. It has a significant effect for all seven portfolios sizes for the entire time period, for all but the two smallest portfolio sizes (i.e., sizes of one and five) for the first sub-period, and for all but the smallest portfolio size (i.e., size of one) for the second sub-period.

6. TESTS FOR HETEROSCEDASTICITY IN THE VARIANCES OF THE MONTHLY RETURNS OF THE STUDIED PORTFOLIOS

The literature surveyed in section 2.3 above suggests that Canadian stock market exhibit heteroscedasticity by using certain methodology. Whether or not this applies to the variances of total monthly return or periods being examined is unknown. Thus, in this section of the thesis, we examine the conditional variances of total monthly returns using a generalized autoregressive conditional heterocedasticity (GARCH) model developed by Bollerslev (1986). Before a GARCH (1,1) model is estimated, we conduct some initial tests on the data to determine if the monthly return series for the various simulated portfolios exhibit heteroscedasticity.

6.1 Examination for Data Stationarity

A sequence plot of each time series is first examined visually to assess if the monthly returns of each portfolio for each of the three time periods is stationary. Figure 7 contains the data series plots for the most stringent and least stringent trading infrequency screens for a portfolio size of one for the total time period. A visual examination of these plots suggests that the data series are stationary.

6.2 Tests For Heteroscedasticity in the Monthly Returns for the Simulated Portfolios

Before the GARCH models are estimated, the portmanteau (Q) and Lagrange multiplier (LM) tests are conducted to check if heteroscedasticity is present in the time-series of monthly returns for the simulated portfolios. The LM and Q statistics are computed from the OLS residuals assuming that the disturbances are white noise. The Q and LM statistics have an approximate $\chi^2_{(q)}$ distribution under the null hypothesis of homoscedasticity.

For nonlinear time-series models, the portmanteau test statistic, Q, uses the squared residuals to test the independence of the time series (McLeod and Li, 1983). The test statistic is given by:[10]

$$Q\ (q) = N(\ N + 2\ ) \sum_{i=1}^{q} \frac{r\left(i; \hat{\varepsilon}_t^2\right)}{(N - i)}$$

where

$$r\left(i; \hat{\varepsilon}_t^2\right) = \frac{\sum_{t=i+1}^{N} (\hat{\varepsilon}_t^2 - \hat{\sigma}^2)(\hat{\varepsilon}_{t-i}^2 - \hat{\sigma}^2)}{\sum_{t=1}^{N} (\hat{\varepsilon}_t^2 - \hat{\sigma}^2)}, \text{ and } \hat{\sigma}^2 = \frac{1}{N} \sum_{t=1}^{N} \hat{\varepsilon}_t^2$$

N is the number of observations, and q is the number of lags.

$\hat{\varepsilon}_t$ come form $y_t = u + \varepsilon_t$

The Q statistic is used to test if nonlinear (e.g. GARCH) effects are present in the residuals. Since the GARCH (p, q) process can be considered as being an ARMA [max(p, q), p] process, the Q statistic calculated from the squared residuals is used to identify the order of the GARCH process.

The Lagrange multiplier test for ARCH disturbances proposed by Engle (1982) is asymptotically equivalent to the test used by Breusch and Pagan (1979). The Lagrange multiplier test for the qth-order ARCH process is written as:[11]

[10] SAS /ETS User's Guide, version 6, second edition.

[11] SAS /ETS User's Guide, version 6, second edition.

$$\mathrm{LM}(q) = \frac{NW'Z(Z'Z)^{-1}Z'W}{W'W}$$

where $\mathrm{W} = \left(\frac{\hat{\varepsilon}_1^2}{\hat{\sigma}^2}, \ldots\ldots, \frac{\hat{\varepsilon}_N^2}{\hat{\sigma}^2} \right)'$, and

$$\mathrm{Z} = \begin{bmatrix} 1 & \hat{\varepsilon}_0^2 & \cdots & \hat{\varepsilon}_{-q+1}^2 \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \hat{\varepsilon}_{N-1}^2 & \cdots & \hat{\varepsilon}_{N-q}^2 \end{bmatrix}$$

The LM(q) test may have different finite sample properties depending on the pre-sample values used, although they are asymptotically equivalent regardless of the choice of the pre-sample values.

Based on the results presented in Table 9, the 90% cases do not reject the null hypothesis of homoscedasticity. Thus, we conclude that the monthly returns of the simulated portfolios do not exhibit heteroscedasticity. However, the monthly returns for the simulated portfolios based on the least stringent trading infrequency screen (i.e., the 50% cut-off value) exhibit more heteroscedasticity than those for the simulated portfolios formed using the most stringent trading infrequency screen (i.e., the 5% cut-off value). Thus, in general, these results provide little support for the use of a GARCH (1,1) to model the data.

However, the literature reviewed in section 2.3 find heteroscedasticity in the monthly returns of Canadian stocks, albeit using other statistical detection methods. Thus, in the interest of being prudent in drawing inferences, such a GARCH model is estimated. Its estimates are reported in section 6.3 of this thesis.

6.3 The Estimates of the GARCH Model for the Monthly Returns of the Simulated Portfolios

The conditional variances of the monthly returns of the simulated portfolios, $Y_{t,k}$, are obtained by estimating the following standard GARCH (p,q) model of Bollerslev (1986):[12]

$$y_{t,k} \mid \Psi_{t-1,k} \sim \mathrm{N}(0,\ h_{t,k})$$

where $\Psi_{t-1,k}$ denotes all the information available at time $t-1$ for portfolio k.

$$y_{t,k} = \mu_k + \varepsilon_{t,k}$$

$$\varepsilon_{t,k} = \sqrt{h_{t,k}}\, e_{t,k}$$

$$h_{t,k} = \alpha_{0,k} + \sum_{i=1}^{q} \alpha_{1,k} h_{t-i,k} e_{t-i,k}^{2} + \sum_{j=1}^{p} \alpha_{2,k} h_{t-j,k}$$

$e_{t,k} \sim$ N (0, 1), $\alpha_{0,k} > 0$, $\alpha_{1,k} \geq 0$, $\alpha_{2,k} \geq 0$, and $p = 1$, $q = 1$,

$h_{t,k}$ is the conditional variance.

$t = 1, \cdots\cdots, \mathrm{T}$. (T = 60 or 120).

$k = 1, \cdots\cdots, 500$. (the number of simulated portfolios).

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[12] SAS/ETS User's Guide, version 6, second edition.

The normality test of Bera and Jarque (1982) is used to check if the residuals are distributed normally.[13] Given skewness and kurtosis, Bera and Jarque (1982) calculate the following test statistic to test for normality:

$$T_N = \left[\frac{N}{6}b_1^2 + \frac{N}{24}(b_2 - 3)^2\right]$$

where

$$b_1 = \frac{\sqrt{N}\sum_{t=1}^{N}\hat{e}_t^3}{\left(\sum_{t=1}^{N}\hat{e}_t^2\right)^{\frac{3}{2}}}, \text{ and } b_2 = \frac{N\sum_{t=1}^{N}\hat{e}_t^4}{\left(\sum_{t=1}^{N}\hat{e}_t^2\right)^2},$$

where b_1 is skewness and b_2 is kurtosis. N is the number of observations.

The $\chi^2(2)$ distribution provides an approximation to the normality test T_N. When the GARCH model is estimated, the normality test is obtained using the standardized residuals $\hat{e}_t = \hat{\varepsilon}_t / \sqrt{h_t}$. The normality test is used to detect misspecification of the family of ARCH models.

The null hypothesis is that the error term of the GARCH model is normally distributed.

The procedure to obtain an estimate of the conditional variance, $h_{t,k}$, of the monthly returns for simulated portfolio k is now explained. Based on the assumption that the conditional variance of $Y_{t,k}$ is normally distributed, we have:

[13] SAS/ETS User's Guide, version 6, second edition.

$$h_{t,k} = \alpha_{0,k} + \sum_{i=1}^{q} \alpha_{1,k} h_{t-i,k} e_{t-i,k}^{2} + \sum_{j=1}^{p} \alpha_{2,k} h_{t-j,k} \qquad (1)$$

or:

$$h_{t,k} = \alpha_{0,k} + (\alpha_{1,k}\ e_{t-1,k}^{2} + \alpha_{2,k})\, h_{t-1,k} \qquad (2)$$

In the first step, the parameters, $\alpha_{0,k}$, $\alpha_{1,k}$, $\alpha_{2,k}$, in eq. (1) are estimated, and the 120 error terms ($e_{t,k}$) from the estimation are saved. In the second step, the estimates of $\alpha_{0,k}$, $\alpha_{1,k}$, $\alpha_{2,k}$ and the successive values of $h_{t-1,k}$ (i.e., the lagged values of the conditional variance of portfolio k) are used to calculate each of the 120 values of $h_{t,k}$ over the ten-year period.

Based on the estimates of the GARCH model (1,1), which are presented in Table 10, the monthly returns of all the simulated portfolios are not modeled well by the GARCH (1, 1). Therefore, there appears to be little support for examining the benefits of diversification using the conditional variances of the simulated portfolios.

The normality test results, which are presented in Table 11, suggest that the null hypothesis of normality is rejected for most cases. Thus, we can not apply the GARCH (1, 1) model, whose error term is normally distributed, to the portfolio monthly returns for the time periods examined herein.

7. MAJOR FINDINGS, IMPLICATIONS AND DIRECTIONS FOR FUTURE RESEARCH

7.1 Major Findings

This thesis has four major findings. The first major finding is that about 30 securities are required, on average, to obtain most of the risk-reduction (variance-reduction) benefits from diversification for all-domestic diversification for Canadian equities. The number of securities required to achieve a given level of diversification in this study is comparable to that obtained by Cleary and Copp (1999), and is higher than that obtained for the bigger U.S. market. The relatively greater number of securities required in Canadian markets compared to U.S. markets is intuitive because of the higher concentration of Canadian stocks within a few industries, and the smaller universe of investment possibilities in Canadian markets. The second major finding is that the same level of diversification is achieved with less securities as the universe of securities available for selection is extended to include stocks with infrequent trading. The third major finding is that both size and trading infrequency effects exist in the variances of the monthly returns and in the risk-adjusted performance ratios for the simulated portfolios. The fourth major finding is that the monthly total returns for the simulated portfolios did not exhibit heteroscedasticity based on the Portmanteau Q and Lagrange Multiplier tests for the time periods being examined in this thesis. It does find evidence that suggests that thin trading induces some heteroscedasticity into the returns of Canadian stocks. Whether or not this result is robust to alternative estimation method is left for future study.

7.2 Implications of the Research

The research presented in this thesis has a number of implications for investment professionals. The first implication is that Canadian portfolio managers need to consider both portfolio size and the trading infrequency of the universe of stocks from which they make selection decisions for portfolio formation purposes. The second implication is that, while expanding the universe of stocks considered for portfolio building purposes increases the diversification achievable for all portfolio sizes, it is likely to have an added cost of making the portfolio more illiquid. The third implication is that the impact of portfolio size on the benefits of portfolio diversification probably is best studied using net and not gross returns, at least for portfolios with shorter investment horizons.

7.3 Directions for Future Research

The results of this research may depend upon the time period and on the length of the time period examined herein. Thus, future research could use both a longer time period and different time periods to assess the robustness of the diversification and heteroscedasticity results reported herein. Such a study also could examine the role played by thin trading in whether or not Canadian stock returns exhibit heteroscedasticity. The results of this study also are likely to depend on the implicit investment horizon used herein. A study using other investment horizons would be of interest. Another avenue that requires further study is to assess the benefits of diversification using net and not gross returns. Such a study would incorporate the differences in liquidity or trade costs across securities when assessing the risk reduction achievable from portfolios of different sizes, and from portfolios of a fixed size that select from less thickly (more thinly) traded stocks. As part of this study, it would be interesting to examine the role of portfolio dollar value and investment horizon on potential portfolio benefits and performance.

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Table 1

Universe of Stocks Available for Investment Purposes for Various Trading Infrequency Screens

This table reports the number of stocks available for investment purposes for seven trading infrequency screens or cut-off values for three holding periods. The population of stocks available for investment purposes consists of all the stocks with per-share prices over $2 as reported on the CFMRC database for December 1987 for the ten-year holding period and the first five-year holding period, and for December 1992 for the second five-year holding period. The trading infrequency index is calculated as the number of months with no trades divided by the total number of months in the holding period times 100. Higher index values indicate less frequent trading.

	Trading Infrequency Index Cut-off Value (%) τ	Universe of Stocks Available for Investment
Ten Year (January 1988 – December 1997)	100%	785
	50%	405
	40%	369
	30%	322
	25%	296
	20%	269
	10%	210
	5%	183
First Five Years (January, 1988 - December 1992)	100%	785
	50%	459
	40%	403
	30%	371
	25%	345
	20%	318
	10%	264
	5%	229
Second Five Years (January, 1993 - December, 1997)	100%	617
	50%	428
	40%	386
	30%	356
	25%	340
	20%	323
	10%	277
	5%	245

Table 2

Two -Way ANOVA of Standard Deviations

This table reports if portfolio size, trading infrequency and their interaction term significantly affect the standards deviations of portfolio monthly returns.

Horizon / Source	Ten Years (Jan.1988 - Dec. 1997)		First Five Years (Jan.1988 - Dec. 1992)		Second Five Years (Jan.1993 - Dec. 1997)	
	F	Sig.	F	Sig.	F	Sig.
TRADING INFREQUENCY	6.190	.000	9.155	.000	20.414	.000
SIZE	2588.450	.000	2114.187	.000	4824.075	.000
TRADING INFREQUENCY * SIZE	2.812	.000	3.302	.000	.907	.628

Table 3

One - Way ANOVA of Standard Deviations (Size Effect)

One-way ANOVA is used first to test if the standard deviations of monthly returns are significantly different across portfolio sizes for the same trading infrequency index. The seven trading infrequency indexes range from 50% (i.e., 0.5) to 5% (i.e., 0.05).

	Ten Years (Jan.1988 – Dec. 1997)		First Five Years (Jan. 1988 – Dec.1992)		Second Five Years (Jan.1993 – Dec.1997)	
	F	Sig.	F	Sig.	F	Sig.
SD _ 0.5	465.547	.000	289.534	.000	696.429	.000
SD _ 0.4	356.048	.000	206.408	.000	693.854	.000
SD _ 0.3	397.160	.000	311.537	.000	719.225	.000
SD _ 0.25	280.673	.000	422.748	.000	803.374	.000
SD _ 0.2	384.051	.000	288.736	.000	708.181	.000
SD _ 0.1	352.584	.000	417.149	.000	610.316	.000
SD _ 0.05	422.981	.000	406.872	.000	628.294	.000

Table 4

One - Way ANOVA of Standard Deviations (Trading Infrequency Effect)

One-way ANOVA is then used to test if the standard deviations of monthly returns are significantly different across trading infrequency indexes for a fixed portfolios size. The seven portfolio sizes ranges from 1 to 50.

	Ten Years (Jan.1988 – Dec. 1998)		First Five Years (Jan. 1988 – Dec.1992)		Second Five Years (Jan.1993 – Dec.1997)	
	F	Sig.	F	Sig.	F	Sig.
SD _ 50	105.701	.000	4.264	.000	298.258	.000
S D _ 30	45.268	.000	4.848	.000	127.794	.000
SD _ 20	9.823	.000	7.460	.000	56.775	.000
SD _ 15	10.584	.000	5.772	.000	46.368	.000
SD _ 10	6.116	.000	5.506	.000	17.887	.000
SD _ 5	4.021	.001	2.787	.010	6.585	.000
SD _ 1	2.600	.016	4.130	.000	.435	.856

Table 5

Portfolio Relative Risk Reduction Summary

The relative benefits of diversification are measured in this table as the relative reduction in risk (variance). This is done first across different portfolio sizes for a fixed infrequency trading index cut-off value τ, and then across the different infrequency trading index cut-off values for a fixed portfolio size for each of the three time periods.

Ten Years (January 1988 – December 1997)

$\tau = 50\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0171	0.0000	0.0000	0.0351
5	0.0047	0.8997	0.7771	0.0475
10	0.0029	1.0334	0.8925	0.0633
15	0.0023	1.0747	0.9283	0.0665
20	0.0020	1.0963	0.9469	0.0642
30	0.0017	1.1219	0.9689	0.0789
50	0.0015	1.1364	0.9815	0.0766

$\tau = 40\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0196	0.0000	0.0000	0.0439
5	0.0045	0.9317	0.8221	0.0608
10	0.0031	1.0159	0.8964	0.0719
15	0.0023	1.0630	0.9380	0.0767
20	0.0021	1.0750	0.9485	0.0825
30	0.0018	1.0974	0.9684	0.0873
50	0.0015	1.1119	0.9812	0.0979

$\tau = 30\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0197	0.0000	0.0000	0.0461
5	0.0052	0.8889	0.7848	0.0585
10	0.0032	1.0122	0.8937	0.0711
15	0.0024	1.0584	0.9345	0.0801
20	0.0022	1.0702	0.9449	0.0838
30	0.0019	1.0922	0.9643	0.0869
50	0.0016	1.1047	0.9753	0.0926

$\tau = 25\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0194	0.0000	0.0000	0.0550
5	0.0049	0.9026	0.7953	0.0628
10	0.0032	1.0071	0.8874	0.0800
15	0.0024	1.0575	0.9318	0.0848
20	0.0022	1.0729	0.9454	0.0915
30	0.0020	1.0854	0.9564	0.0982
50	0.0017	1.1042	0.9729	0.1013

$\tau = 20\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0180	0.0000	0.0000	0.0481
5	0.0047	0.9053	0.7889	0.0674
10	0.0032	1.0121	0.8820	0.0771
15	0.0026	1.0482	0.9134	0.0828
20	0.0023	1.0730	0.9351	0.0890
30	0.0020	1.0914	0.9511	0.0899
50	0.0018	1.1085	0.9659	0.0992

$\tau = 10\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0159	0.0000	0.0000	0.0555
5	0.0043	0.9278	0.7909	0.0723
10	0.0031	1.0235	0.8725	0.0749
15	0.0025	1.0662	0.9089	0.0820
20	0.0023	1.0870	0.9266	0.0842
30	0.0020	1.1097	0.9459	0.0929
50	0.0018	1.1253	0.9593	0.0979

$\tau = 5\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0144	0.0000	0.0000	0.0617
5	0.0041	0.9330	0.7805	0.0681
10	0.0027	1.0561	0.8834	0.0830
15	0.0023	1.0942	0.9153	0.0895
20	0.0021	1.1101	0.9286	0.0884
30	0.0019	1.1345	0.9490	0.0939
50	0.0018	1.1448	0.9577	0.0988

First Five Years (January 1988 – December 1997)

$\tau = 50\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0209	0.0000	0.0000	-0.0624
5	0.0052	0.8400	0.7876	-0.1128
10	0.0035	0.9291	0.8712	-0.1402
15	0.0026	0.9796	0.9186	-0.1527
20	0.0023	0.9969	0.9348	-0.1623
30	0.0019	1.0161	0.9528	-0.1686
50	0.0016	1.0301	0.9659	-0.1797

$\tau = 40\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0249	0.0000	0.0000	-0.0572
5	0.0052	0.8688	0.8237	-0.1198
10	0.0034	0.9461	0.8969	-0.1340
15	0.0027	0.9790	0.9282	-0.1445
20	0.0023	0.9982	0.9464	-0.1459
30	0.0020	1.0074	0.9551	-0.1603
50	0.0017	1.0226	0.9695	-0.1671

$\tau = 30\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0178	0.0000	0.0000	-0.0506
5	0.0046	0.8495	0.7868	-0.1107
10	0.0031	0.9411	0.8716	-0.1313
15	0.0024	0.9864	0.9135	-0.1459
20	0.0022	1.0029	0.9288	-0.1575
30	0.0019	1.0206	0.9452	-0.1619
50	0.0017	1.0338	0.9574	-0.1658

$\tau = 25\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0170	0.0000	0.0000	-0.0632
5	0.0046	0.8360	0.7712	-0.1125
10	0.0029	0.9516	0.8778	-0.1428
15	0.0023	0.9918	0.9148	-0.1616
20	0.0021	1.0050	0.9270	-0.1673
30	0.0018	1.0307	0.9507	-0.1780
50	0.0016	1.0393	0.9586	-0.1856

$\tau = 20\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0208	0.0000	0.0000	-0.0500
5	0.0045	0.8763	0.8212	-0.1059
10	0.0030	0.9561	0.8961	-0.1364
15	0.0026	0.9816	0.9200	-0.1492
20	0.0022	0.9997	0.9369	-0.1577
30	0.0019	1.0148	0.9511	-0.1656
50	0.0017	1.0278	0.9633	-0.1752

$\tau = 10\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0145	0.0000	0.0000	-0.0624
5	0.0042	0.8366	0.7599	-0.1232
10	0.0030	0.9377	0.8517	-0.1522
15	0.0023	0.9910	0.9001	-0.1623
20	0.0021	1.0075	0.9150	-0.1692
30	0.0019	1.0269	0.9327	-0.1758
50	0.0017	1.0419	0.9462	-0.1861

$\tau = 5\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0130	0.0000	0.0000	-0.0458
5	0.0041	0.8203	0.7355	-0.1141
10	0.0027	0.9520	0.8535	-0.1300
15	0.0023	0.9901	0.8877	-0.1416
20	0.0021	1.0128	0.9081	-0.1535
30	0.0019	1.0306	0.9240	-0.1586
50	0.0017	1.0471	0.9388	-0.1674

Second Five Years (January 1993 – December 1997)

$\tau = 50\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0122	0.0000	0.0000	0.1209
5	0.0031	1.1716	0.8439	0.1884
10	0.0020	1.3036	0.9389	0.2370
15	0.0016	1.3554	0.9762	0.2519
20	0.0015	1.3779	0.9924	0.2737
30	0.0012	1.4048	1.0118	0.2881
50	0.0011	1.4261	1.0271	0.3123

$\tau = 40\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0134	0.0000	0.0000	0.1088
5	0.0033	1.1191	0.8376	0.1927
10	0.0021	1.2604	0.9433	0.2403
15	0.0017	1.3027	0.9750	0.2611
20	0.0015	1.3177	0.9863	0.2792
30	0.0013	1.3410	1.0037	0.2902
50	0.0012	1.3596	1.0176	0.3159

$\tau = 30\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0122	0.0000	0.0000	0.1244
5	0.0033	1.1358	0.8208	0.1870
10	0.0022	1.2808	0.9256	0.2368
15	0.0018	1.3338	0.9639	0.2516
20	0.0016	1.3546	0.9789	0.2670
30	0.0014	1.3784	0.9962	0.2855
50	0.0013	1.3983	1.0105	0.2991

$\tau = 25\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0120	0.0000	0.0000	0.1319
5	0.0032	1.1517	0.8245	0.2029
10	0.0023	1.2782	0.9151	0.2383
15	0.0018	1.3357	0.9563	0.2453
20	0.0016	1.3604	0.9740	0.2713
30	0.0015	1.3843	0.9911	0.2817
50	0.0013	1.4043	1.0054	0.3028

$\tau = 20\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0123	0.0000	0.0000	0.1347
5	0.0034	1.1219	0.8122	0.2009
10	0.0023	1.2634	0.9147	0.2358
15	0.0019	1.3191	0.9550	0.2649
20	0.0017	1.3344	0.9661	0.2701
30	0.0015	1.3637	0.9874	0.2903
50	0.0014	1.3833	1.0015	0.3008

$\tau = 10\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0127	0.0000	0.0000	0.1437
5	0.0036	1.0941	0.8035	0.2182
10	0.0023	1.2503	0.9182	0.2579
15	0.0020	1.2861	0.9444	0.2739
20	0.0018	1.3150	0.9656	0.2958
30	0.0016	1.3339	0.9795	0.3066
50	0.0014	1.3538	0.9941	0.3250

$\tau = 5\%$		Relative risk reduction attained in average variance		
Portfolio size	Average variance	Equally-weighted index	Value-weighted index	Sharpe ratio
1	0.0128	0.0000	0.0000	0.1303
5	0.0035	1.1086	0.8165	0.1965
10	0.0024	1.2380	0.9118	0.2400
15	0.0020	1.2818	0.9441	0.2516
20	0.0018	1.3069	0.9626	0.2705
30	0.0016	1.3267	0.9772	0.2828
50	0.0015	1.3450	0.9906	0.2928

Table 6

Two -Way ANOVA of Sharpe Ratios

This table reports if portfolio size, trading infrequency and their interaction term significantly affect the sharpe ratios of portfolio monthly returns.

	Ten Years (Jan.1988 – Dec.1992)		First Five Years (Jan.1988 – Dec.1992)		Second Five Years (Jan.1993 – Dec.1997)	
Source	F	Sig.	F	Sig.	F	Sig.
TRADING INFREQUENCY	46.751	.000	140.396	.000	167.249	.000
SIZE	219.057	.000	52.125	.000	454.429	.000
TRADING INFREQUENCY * SIZE	1.131	.271	23.514	.000	63.462	.000

Table 7

One - Way ANOVA of Sharpe Ratios (Size Effect)

One-way ANOVA is used first to test if the Sharpe ratios of monthly returns are significantly different across portfolio size for the same trading infrequency index. The seven trading infrequency indexes range form 50% (i.e., 0.5) to 5% (i.e., 0.05).

	Ten Years (Jan.1988 – Dec. 1997)		First Five Years (Jan. 1988 – Dec.1992)		Second Five Years (Jan.1993 – Dec.1997)	
	F	Sig.	F	Sig.	F	Sig.
SHARPE_0.5	2.404	.025	2.273	.034	.723	.631
SHARPE_0.4	.360	.905	.462	.836	2.025	.059
SHARPE_0.3	.156	.988	.606	.725	.565	.758
SHARPE0_.25	.953	.455	110.703	.000	22.565	.000
SHARPE_0.2	.837	.541	.218	.971	.792	.576
SHARPE_0.1	.395	.882	.341	.916	.303	.936
SHARPE_0.05	.580	.747	103.093	.000	14.711	.000

Table 8

One -Way ANOVA of Sharpe Ratios (Trading Infrequency Effect)

One-way ANOVA is then used to test if Sharpe ratios of monthly returns are significantly different across trading infrequency indexes for a fixed portfolio size. Portfolio sizes range from 1 to 50.

	Ten Years (Jan.1988 – Dec. 1997)		First Five Years (Jan. 1988 – Dec.1992)		Second Five Years (Jan.1993 – Dec.1997)	
	F	Sig.	F	Sig.	F	Sig.
SHARPE_50	31.331	.000	1056.353	.000	5231.319	.000
SHARPE_30	11.000	.000	352.419	.000	150.322	.000
SHARPE_20	16.175	.000	390.008	.000	101.805	.000
SHARPE_15	8.205	.000	277.529	.000	67.760	.000
SHARPE_10	5.175	.000	4.150	.000	6.552	.000
SHARPE_5	5.622	.000	1.131	.341	6.121	.000
SHARPE_1	4.062	.000	1.847	.086	1.918	.074

Table 9

Some Summary Results for Tests of Heteroscedasticity in Monthly Returns

Before estimating the GARCH model, the portmanteau (Q) and Lagrange multiplier (LM) tests are conducted to check if heteroscedasticity is present in the time-series of monthly returns for the simulated portfolios. The LM and Q statistics are computed from the OLS residuals assuming that the disturbances are white noise. The Q and LM statistics have an approximate $\chi^2_{(q)}$ distribution under the null hypothesis of homoscedasticity of returns. The returns for 16 simulated portfolios for various portfolio sizes for infrequency trading screen of 5% and 50% are presented herein.

Most stringent infrequency trading screen (5%)

HETEROSCEDASTICITY					
	lag	Q	Prob>Q	LM	Prob>LM
Portfolio 1	1	0.3224	0.5702	0.2685	0.6044
	2	0.8722	0.6466	0.8972	0.6385
	3	0.8778	0.8308	0.8972	0.8261
	4	2.3624	0.6694	2.5453	0.6366
	5	7.9989	0.1563	7.6979	0.1737
	6	8.0003	0.2381	7.8633	0.2483
	7	8.1492	0.3196	7.8639	0.3447
	8	8.2205	0.4122	7.8788	0.4454
	9	11.191	0.2628	9.7323	0.3726
	10	11.8441	0.2956	11.8687	0.2939
	11	11.8485	0.3751	11.887	0.3722
	12	13.0081	0.3685	13.0491	0.3655
Portfolio 2	1	1.2409	0.2653	1.1457	0.2844
	2	1.2419	0.5374	1.1539	0.5616
	3	1.2898	0.7316	1.1959	0.754
	4	1.3016	0.8611	1.2195	0.8749
	5	2.2299	0.8165	2.0596	0.8408
	6	2.2374	0.8966	2.0602	0.9141
	7	3.9159	0.7894	4.0908	0.7693
	8	3.995	0.8576	4.0931	0.8486
	9	4.1085	0.9041	4.2641	0.8932
	10	4.2676	0.9345	4.3997	0.9275
	11	5.1592	0.9232	5.3535	0.9128
	12	5.6419	0.933	5.6237	0.9339
Portfolio 3	1	0.9522	0.3292	0.9277	0.3355
	2	1.0246	0.5991	0.9664	0.6168
	3	1.1829	0.7571	1.2208	0.748
	4	3.4983	0.4781	3.3909	0.4947
	5	3.5196	0.6204	3.6081	0.6071
	6	4.9136	0.5549	4.7757	0.5729
	7	4.9465	0.6665	4.7852	0.6862
	8	5.0082	0.7567	5.1613	0.7402
	9	5.0593	0.8291	5.1652	0.8197
	10	8.2632	0.6031	8.3266	0.597
	11	9.2237	0.6013	9.8691	0.5422
	12	9.2501	0.6814	9.9542	0.62
Portfolio 4	1	0.068	0.7942	0.064	0.8003
	2	0.1347	0.9349	0.1302	0.937
	3	0.1573	0.9842	0.1501	0.9852
	4	0.1849	0.996	0.1735	0.9964
	5	0.2554	0.9984	0.2336	0.9987
	6	1.8989	0.9288	1.7652	0.94
	7	2.0207	0.9587	1.8389	0.9682
	8	2.0477	0.9795	1.8469	0.9854
	9	2.3359	0.985	2.0645	0.9904
	10	2.4672	0.9913	2.1546	0.995
	11	2.4826	0.996	2.1579	0.9979
	12	2.5024	0.9982	2.2547	0.9989

HETEROSCEDASTICITY					
	lag	Q	Prob>Q	LM	Prob>LM
Portfolio 5	1	0.1518	0.6968	0.1567	0.6923
	2	0.6396	0.7263	0.6974	0.7056
	3	1.7261	0.6311	1.9104	0.5912
	4	1.9684	0.7416	2.2693	0.6864
	5	2.5237	0.7729	2.4807	0.7794
	6	5.2172	0.5163	5.1508	0.5246
	7	10.2126	0.1768	10.0294	0.1869
	8	11.3283	0.1838	11.7008	0.1651
	9	11.363	0.2516	11.7173	0.2297
	10	13.0273	0.2222	13.4821	0.198
	11	13.2457	0.2776	13.4857	0.2628
	12	14.5122	0.2692	13.9833	0.3018
Portfolio 6	1	0.0228	0.8799	0.0235	0.8781
	2	0.2674	0.8749	0.2557	0.88
	3	0.3492	0.9505	0.32	0.9562
	4	0.3686	0.985	0.4333	0.9797
	5	0.3725	0.9961	0.4444	0.994
	6	1.7107	0.9443	7.0127	0.3197
	7	1.7623	0.9718	7.2786	0.4005
	8	1.8387	0.9856	7.3851	0.4957
	9	1.9694	0.9919	8.1115	0.523
	10	2.0066	0.9963	8.8086	0.5504
	11	2.9872	0.9909	14.4997	0.2066
	12	2.9874	0.9956	14.5598	0.2664
Portfolio 7	1	0.0326	0.8567	0.0343	0.8531
	2	0.119	0.9422	0.1147	0.9443
	3	0.2539	0.9685	0.2247	0.9735
	4	4.1431	0.387	4.0453	0.3999
	5	4.3834	0.4956	4.36	0.4988
	6	4.7426	0.5772	4.5698	0.6
	7	4.8935	0.673	4.8874	0.6737
	8	4.9373	0.7643	5.2548	0.73
	9	5.4083	0.7974	5.4391	0.7945
	10	5.4114	0.8621	5.4968	0.8556
	11	5.4345	0.9083	6.0702	0.8686
	12	5.4875	0.9397	6.0747	0.9123
Portfolio 8	1	0.0032	0.9548	0.003	0.9565
	2	5.6668	0.0588	5.5142	0.0635
	3	5.8262	0.1204	5.6485	0.13
	4	5.848	0.2108	5.7729	0.2168
	5	5.9029	0.3158	5.7777	0.3285
	6	6.0224	0.4207	5.8803	0.4367
	7	6.0323	0.536	5.9066	0.5507
	8	6.0355	0.6433	5.9164	0.6566
	9	6.2915	0.7104	6.099	0.73
	10	6.521	0.7698	6.3136	0.7883
	11	6.5603	0.8335	6.3137	0.8516
	12	6.7091	0.8762	6.3383	0.8981

HETEROSCEDASTICITY					
	lag	Q	Prob>Q	LM	Prob>LM
Portfolio 9	1	0.5319	0.4658	0.6991	0.4031
	2	0.7487	0.6877	1.1149	0.5727
	3	1.014	0.7979	1.1509	0.7648
	4	1.0266	0.9057	1.1575	0.885
	5	1.5143	0.9114	1.8511	0.8693
	6	1.532	0.9573	2.1274	0.9076
	7	2.0669	0.956	3.3744	0.8483
	8	2.0669	0.9789	3.5141	0.8981
	9	2.3647	0.9843	3.5416	0.9389
	10	2.5432	0.9902	3.5776	0.9644
	11	3.1088	0.9892	4.4774	0.9538
	12	5.4049	0.9431	8.2428	0.7659
Portfolio 10	1	0.1422	0.7061	0.1243	0.7244
	2	2.8355	0.2423	32.9197	0.0001
	3	2.8374	0.4174	32.9472	0.0001
	4	3.0185	0.5547	33.0513	0.0001
	5	3.0401	0.6938	33.662	0.0001
	6	3.1191	0.7938	34.9172	0.0001
	7	3.1252	0.8732	35.0083	0.0001
	8	3.2002	0.9212	35.0304	0.0001
	9	3.2412	0.954	35.0382	0.0001
	10	3.29	0.9738	35.0384	0.0001
	11	3.3399	0.9854	36.9383	0.0001
	12	3.4089	0.9919	37.7553	0.0002
Portfolio 11	1	13.9033	0.0002	14.496	0.0001
	2	13.9034	0.001	15.4431	0.0004
	3	14.3553	0.0025	15.691	0.0013
	4	14.3673	0.0062	15.8087	0.0033
	5	14.412	0.0132	15.9312	0.007
	6	14.4233	0.0252	16.0806	0.0133
	7	14.4443	0.0438	16.0838	0.0244
	8	14.6524	0.0663	16.4328	0.0366
	9	14.715	0.0991	16.5209	0.0568
	10	15.4779	0.1156	17.9295	0.0562
	11	15.659	0.1543	19.4921	0.0528
	12	17.0419	0.148	19.8387	0.0702
Portfolio 12	1	0.2714	0.6024	0.2908	0.5897
	2	0.2714	0.8731	0.2911	0.8645
	3	1.5552	0.6696	1.6012	0.6591
	4	1.5657	0.8149	1.6231	0.8046
	5	1.846	0.87	2.0375	0.8439
	6	2.1109	0.9092	2.2293	0.8974
	7	2.1466	0.9513	2.2309	0.946
	8	3.7411	0.8797	3.8458	0.8708
	9	20.4971	0.0151	19.2464	0.0232
	10	20.6428	0.0237	19.5397	0.0339
	11	22.125	0.0234	20.9912	0.0335
	12	27.3284	0.0069	23.8486	0.0213

HETEROSCEDASTICITY					
	lag	Q	Prob>Q	LM	Prob>LM
Portfolio 13	1	2.1431	0.1432	2.1883	0.1391
	2	4.3095	0.1159	3.6898	0.158
	3	11.8304	0.008	9.3572	0.0249
	4	12.5516	0.0137	9.3716	0.0525
	5	12.9829	0.0235	9.3781	0.0949
	6	13.31	0.0384	9.4002	0.1523
	7	19.0697	0.008	14.6297	0.0411
	8	19.42	0.0128	14.6357	0.0666
	9	19.7255	0.0197	15.5747	0.0763
	10	21.949	0.0154	16.0425	0.0984
	11	24.6702	0.0102	18.5311	0.07
	12	24.6909	0.0164	18.536	0.1004
Portfolio 14	1	3.4286	0.0641	2.9978	0.0834
	2	3.5266	0.1715	3.2249	0.1994
	3	3.6474	0.3021	3.7846	0.2857
	4	3.6518	0.4552	3.8598	0.4253
	5	3.6954	0.5941	3.8886	0.5656
	6	3.9097	0.6889	4.7398	0.5776
	7	3.9793	0.7822	4.9376	0.6676
	8	4.1599	0.8424	5.4471	0.7089
	9	4.2422	0.8948	5.7923	0.7605
	10	4.3317	0.9311	5.7926	0.8324
	11	4.428	0.9557	6.4848	0.8391
	12	5.4002	0.9433	8.0861	0.7784
Portfolio 15	1	4.4559	0.0348	4.4224	0.0355
	2	6.8412	0.0327	5.8007	0.055
	3	7.7508	0.0515	6.0518	0.1091
	4	9.0631	0.0595	6.6682	0.1545
	5	24.9499	0.0001	19.3469	0.0017
	6	37.5433	0.0001	24.8472	0.0004
	7	51.7974	0.0001	31.2922	0.0001
	8	52.7094	0.0001	31.314	0.0001
	9	53.7691	0.0001	31.3535	0.0003
	10	60.3379	0.0001	32.5615	0.0003
	11	66.9075	0.0001	32.8011	0.0006
	12	72.7891	0.0001	33.0226	0.001
Portfolio 16	1	0.1422	0.7061	0.1243	0.7244
	2	2.8355	0.2423	32.9197	0.0001
	3	2.8374	0.4174	32.9472	0.0001
	4	3.0185	0.5547	33.0513	0.0001
	5	3.0401	0.6938	33.662	0.0001
	6	3.1191	0.7938	34.9172	0.0001
	7	3.1252	0.8732	35.0083	0.0001
	8	3.2002	0.9212	35.0304	0.0001
	9	3.2412	0.954	35.0382	0.0001
	10	3.29	0.9738	35.0384	0.0001
	11	3.3399	0.9854	36.9383	0.0001
	12	3.4089	0.9919	37.7553	0.0002

Least stringent infrequency trading screen (50%)

HETEROSCEDASTICITY					
	lag	Q	Prob>Q	LM	Prob>LM
Portfolio 1	1	5.3274	0.021	4.9031	0.0268
	2	5.4281	0.0663	5.6078	0.0606
	3	6.4314	0.0924	6.1915	0.1027
	4	6.6187	0.1575	6.1953	0.185
	5	6.8327	0.2334	6.4308	0.2665
	6	9.2442	0.1603	9.914	0.1283
	7	9.6062	0.212	9.9455	0.1917
	8	9.9289	0.2701	10.389	0.2388
	9	11.1334	0.2667	10.7001	0.2968
	10	11.2112	0.3413	10.7497	0.3773
	11	14.2032	0.222	12.4328	0.332
	12	14.2036	0.2879	12.979	0.3706
Portfolio 2	1	3.0661	0.0799	3.0389	0.0813
	2	3.4618	0.1771	3.1623	0.2057
	3	4.4469	0.2171	3.8196	0.2816
	4	5.3783	0.2506	4.2532	0.3728
	5	5.4884	0.3592	4.254	0.5135
	6	5.5248	0.4785	4.2553	0.6422
	7	6.5225	0.4802	4.9787	0.6626
	8	6.525	0.5886	5.0498	0.7522
	9	6.5459	0.6843	5.0651	0.8286
	10	6.5538	0.7668	5.1077	0.8839
	11	6.5786	0.8321	5.1353	0.9244
	12	9.0751	0.6965	7.7633	0.8033
Portfolio 3	1	0.3644	0.5461	0.3233	0.5696
	2	0.4464	0.8	0.516	0.7726
	3	3.4322	0.3297	4.6826	0.1966
	4	3.8974	0.4201	5.7837	0.2159
	5	5.0763	0.4066	8.0816	0.1518
	6	5.1114	0.5296	8.2729	0.2188
	7	5.2077	0.6346	9.0758	0.2473
	8	5.9022	0.6582	9.3033	0.3174
	9	5.9178	0.7481	9.3039	0.4097
	10	6.1716	0.8006	9.8581	0.453
	11	6.2333	0.8574	9.8927	0.5401
	12	6.6017	0.8828	10.2075	0.5978
Portfolio 4	1	0.0032	0.9548	0.003	0.9565
	2	5.6668	0.0588	5.5142	0.0635
	3	5.8262	0.1204	5.6485	0.13
	4	5.848	0.2108	5.7729	0.2168
	5	5.9029	0.3158	5.7777	0.3285
	6	6.0224	0.4207	5.8803	0.4367
	7	6.0323	0.536	5.9066	0.5507
	8	6.0355	0.6433	5.9164	0.6566
	9	6.2915	0.7104	6.099	0.73
	10	6.521	0.7698	6.3136	0.7883
	11	6.5603	0.8335	6.3137	0.8516
	12	6.7091	0.8762	6.3383	0.8981

HETEROSCEDASTICITY					
	lag	Q	Prob>Q	LM	Prob>LM
Portfolio 5	1	0.2655	0.6063	0.2834	0.5945
	2	0.4092	0.815	0.4068	0.8159
	3	0.7763	0.8551	0.7413	0.8634
	4	1.014	0.9077	0.9875	0.9117
	5	2.6821	0.7489	2.8709	0.7199
	6	2.9039	0.8208	3.1586	0.7887
	7	3.6356	0.8207	3.5598	0.8288
	8	5.1687	0.7394	4.6106	0.7983
	9	5.674	0.772	5.1197	0.8237
	10	6.1627	0.8014	5.5039	0.8551
	11	6.1629	0.8623	5.5191	0.9035
	12	7.0183	0.8564	6.3994	0.8946
Portfolio 6	1	0.8192	0.3654	0.8566	0.3547
	2	1.0121	0.6029	0.9584	0.6193
	3	1.3119	0.7263	1.3894	0.708
	4	1.5179	0.8235	1.4043	0.8434
	5	1.5183	0.9109	1.4193	0.9222
	6	2.4139	0.878	2.8313	0.8297
	7	2.7689	0.9055	3.6509	0.819
	8	3.037	0.932	3.7221	0.8813
	9	3.9753	0.913	4.9461	0.839
	10	4.2003	0.9379	5.636	0.8449
	11	4.3169	0.9597	5.6902	0.8932
	12	5.3964	0.9434	6.1845	0.9065
Portfolio 7	1	0.4512	0.5018	0.4589	0.4982
	2	0.8781	0.6446	0.8909	0.6405
	3	8.5654	0.0357	8.8146	0.0319
	4	8.7485	0.0677	9.5915	0.0479
	5	9.5784	0.0881	9.7659	0.0821
	6	14.4943	0.0246	12.2843	0.0559
	7	23.3215	0.0015	21.0571	0.0037
	8	23.5217	0.0028	21.2025	0.0066
	9	23.7785	0.0047	21.2286	0.0117
	10	29.4706	0.001	22.6374	0.0122
	11	30.0883	0.0015	22.6393	0.0199
	12	30.353	0.0025	22.8505	0.029
Portfolio 8	1	0.0329	0.8562	0.0334	0.855
	2	0.0576	0.9716	0.0632	0.9689
	3	17.3201	0.0006	17.1283	0.0007
	4	17.5318	0.0015	17.4894	0.0016
	5	17.5356	0.0036	17.4961	0.0036
	6	21.2646	0.0016	17.8486	0.0066
	7	21.7544	0.0028	19.0707	0.008
	8	21.9092	0.0051	19.1895	0.0139
	9	23.337	0.0055	19.1991	0.0236
	10	23.7034	0.0084	20.9731	0.0213
	11	24.3866	0.0112	22.2109	0.0228
	12	24.5397	0.0172	22.2242	0.0351

HETEROSCEDASTICITY					
	lag	Q	Prob>Q	LM	Prob>LM
Portfolio 9	1	12.1854	0.0005	11.4185	0.0007
	2	13.2911	0.0013	11.4242	0.0033
	3	13.3037	0.004	11.7466	0.0083
	4	13.5974	0.0087	12.0724	0.0168
	5	15.4957	0.0084	13.4735	0.0193
	6	15.5755	0.0162	13.7847	0.0321
	7	16.0715	0.0245	14.277	0.0465
	8	16.1655	0.0401	14.277	0.0748
	9	16.2008	0.0628	14.2962	0.1122
	10	17.6728	0.0607	16.8293	0.0782
	11	17.7006	0.0888	17.0256	0.1071
	12	18.5098	0.1011	18.6756	0.0967
Portfolio 10	1	0.1559	0.693	0.1611	0.6881
	2	0.4443	0.8008	0.4317	0.8059
	3	0.4445	0.9309	0.4351	0.9329
	4	0.4448	0.9786	0.4351	0.9795
	5	0.8006	0.977	0.7177	0.982
	6	2.3375	0.8862	2.4095	0.8785
	7	2.4136	0.9335	2.4399	0.9316
	8	2.8493	0.9435	2.6622	0.9537
	9	12.06	0.2099	12.3033	0.1967
	10	12.9548	0.2262	12.666	0.243
	11	13.6611	0.2523	12.8595	0.3026
	12	14.0874	0.2952	13.0634	0.3644
Portfolio 11	1	0.3232	0.5697	0.3212	0.5709
	2	6.7247	0.0347	6.2643	0.0436
	3	7.0103	0.0716	6.3907	0.0941
	4	7.2901	0.1213	7.9732	0.0926
	5	7.3161	0.1982	8.0134	0.1555
	6	9.4958	0.1476	11.2651	0.0805
	7	9.5238	0.2172	11.2651	0.1275
	8	9.7532	0.2828	13.4568	0.0971
	9	9.7594	0.3703	13.4619	0.1428
	10	9.9624	0.4438	13.6044	0.1918
	11	10.5155	0.4847	14.5566	0.2037
	12	11.1019	0.5202	14.6265	0.2625
Portfolio 12	1	0.0551	0.8145	0.0538	0.8165
	2	3.3441	0.1879	3.2535	0.1966
	3	8.8076	0.032	8.3988	0.0384
	4	8.9725	0.0618	8.9014	0.0636
	5	17.4131	0.0038	13.5908	0.0184
	6	17.4211	0.0079	13.7472	0.0326
	7	18.0583	0.0117	15.4657	0.0305
	8	18.316	0.019	15.6353	0.0479
	9	18.3953	0.0309	15.7981	0.0712
	10	18.4291	0.0481	15.8021	0.1054
	11	18.665	0.0674	16.0269	0.1401
	12	18.6822	0.0965	16.5129	0.1689

HETEROSCEDASTICITY					
	lag	Q	Prob>Q	LM	Prob>LM
Portfolio 13	1	0.2017	0.6534	0.1438	0.7045
	2	0.8673	0.6481	0.7242	0.6962
	3	0.9146	0.8219	0.8006	0.8493
	4	0.9798	0.9128	0.8788	0.9276
	5	1.8608	0.8681	1.6597	0.8939
	6	1.9586	0.9235	1.7431	0.9417
	7	4.7576	0.6895	4.0882	0.7696
	8	4.8895	0.7693	4.0895	0.849
	9	5.9136	0.7485	4.3628	0.886
	10	8.452	0.5848	5.9128	0.8225
	11	9.8465	0.5442	9.2818	0.5959
	12	9.8468	0.6294	9.3348	0.6741
Portfolio 14	1	0.0005	0.9816	0	0.9947
	2	0.9409	0.6247	0.7985	0.6708
	3	2.0739	0.5572	1.7017	0.6366
	4	2.1104	0.7155	1.7561	0.7805
	5	2.3858	0.7936	2.0276	0.8453
	6	2.6306	0.8536	2.1675	0.9037
	7	2.6584	0.9147	2.3098	0.9407
	8	2.7333	0.95	2.3348	0.969
	9	2.7487	0.9734	2.3506	0.9846
	10	3.1652	0.9773	3.3634	0.9715
	11	3.3142	0.9859	3.7827	0.9758
	12	4.0293	0.9829	3.9432	0.9844
Portfolio 15	1	14.9024	0.0001	14.6094	0.0001
	2	26.9739	0.0001	19.6466	0.0001
	3	44.3884	0.0001	26.2184	0.0001
	4	47.9446	0.0001	26.5495	0.0001
	5	48.1782	0.0001	28.6231	0.0001
	6	49.1677	0.0001	28.6237	0.0001
	7	49.1803	0.0001	28.6348	0.0002
	8	49.3972	0.0001	29.4965	0.0003
	9	49.4757	0.0001	29.4979	0.0005
	10	49.4839	0.0001	29.6158	0.001
	11	49.4993	0.0001	29.6355	0.0018
	12	49.5227	0.0001	29.7021	0.0031
Portfolio 16	1	0.0038	0.9507	0.0032	0.9549
	2	0.1609	0.9227	0.1477	0.9288
	3	0.2987	0.9603	0.2747	0.9647
	4	0.3832	0.9838	0.3563	0.9859
	5	0.399	0.9954	0.3736	0.996
	6	1.4556	0.9624	1.3346	0.9697
	7	1.506	0.9821	1.378	0.9862
	8	1.5595	0.9917	1.3991	0.9943
	9	1.6819	0.9956	1.4704	0.9974
	10	1.7391	0.998	1.5011	0.9989
	11	1.8442	0.999	1.5826	0.9995
	12	1.8964	0.9995	1.6749	0.9998

Table 10

Some Summary Results for some representative GARCH model Estimation

The literature reviewed in section 2.3 finds heteroscedasticity in the monthly returns of Canadian stocks, albeit using other statistical detection methods. Thus, in the interest of being prudent in drawing inferences, GARCH model is estimated. This table presents some parameters of GARCH (1, 1) model (error term with normal distribution) of 16 randomly selected portfolios for a portfolio size 1 for trading infrequency screens of 5% and 50% for entire ten year period.

Most stringent infrequency screen (5%)

GARCH				
Portfolio	Variable	B Value	t Ratio	Approx Prob
Portfolio 1	α_0	0.007187	7.661	0.0001
	α_1	9.84E-23	0.000	1.0000
	α_2	0.000332	49.253	0.0001
Portfolio 2	α_0	0.004817	9.557	0.0001
	α_1	8.91E-23	0.000	1.0000
	α_2	0.000175	72.064	0.0001
Portfolio 3	α_0	0.000591	1.303	0.1927
	α_1	0.124119	1.346	0.1785
	α_2	0.724663	4.588	0.0001
Portfolio 4	α_0	0.037962	18.894	0.0001
	α_1	4.53E-23	0.000	1.0000
	α_2	0.003945	51.522	0.0001
Portfolio 5	α_0	0.004369	6.448	0.0001
	α_1	0.030376	0.207	0.8362
	α_2	-2.51E-23	0.000	1.0000
Portfolio 6	α_0	0.004227	6.222	0.0001
	α_1	0.703625	5.696	0.0001
	α_2	3.75E-23	0.000	1.0000
Portfolio 7	α_0	0.004727	5.506	0.0001
	α_1	0.553612	3.833	0.0001
	α_2	-7.19E-18	0.000	1.0000
Portfolio 8	α_0	0.004059	16.000	0.0001
	α_1	2.07E-24	0.000	1.0000
	α_2	0.000031951	31.033	0.0001

GARCH				
	Variable	B Value	t Ratio	Approx Prob
Portfolio 9	α_0	0.005836	8.963	0.0001
	α_1	3.83E-24	0.000	1.0000
	α_2	0.000195	51.433	0.0001
Portfolio 10	α_0	0.001995	0.570	0.5686
	α_1	0.343247	1.993	0.0463
	α_2	0.621056	1.969	0.049
Portfolio 11	α_0	0.008238	4.908	0.0001
	α_1	0.647735	3.357	0.0008
	α_2	-1.06E-18	0.000	1.0000
Portfolio 12	α_0	0.000179	0.508	0.6115
	α_1	0.081287	1.028	0.3038
	α_2	0.880291	6.937	0.0001
Portfolio 13	α_0	0.000716	0.739	0.4597
	α_1	0.101402	1.352	0.1765
	α_2	0.803821	4.502	0.0001
Portfolio 14	α_0	0.000481	1.745	0.0810
	α_1	0.346138	2.074	0.0381
	α_2	0.213834	0.700	0.4839
Portfolio 15	α_0	0.001362	2.849	0.0044
	α_1	0.303195	3.121	0.0018
	α_2	0.683491	10.324	0.0001
Portfolio 16	α_0	0.001995	0.57	0.5686
	α_1	0.343247	1.993	0.0463
	α_2	0.621056	1.969	0.049

Least stringent infrequency scmeen (50%)

GARCH				
Portfolio	Variable	B Value	t Ratio	Approx Prob
Portfolio 1	α_0	0.004259	1.605	0.1085
	α_1	0.108316	1.392	0.1639
	α_2	0.007143	0.013	0.9896
Portfolio 2	α_0	0.002455	2.245	0.0248
	α_1	0.441133	3.444	0.0006
	α_2	0.313136	1.507	0.1318
Portfolio 3	α_0	0.003271	0.681	0.4958
	α_1	0.136322	1.874	0.0609
	α_2	0.755584	3.515	0.0004
Portfolio 4	α_0	0.004059	16.000	0.0001
	α_1	2.07E-24	0.000	1.0000
	α_2	0.000031951	31.033	0.0001
Portfolio 5	α_0	0.01293	7.930	0.0001
	α_1	1.04E-22	0.000	1.0000
	α_2	0.000498	23.628	0.0001
Portfolio 6	α_0	0.025077	10.237	0.0001
	α_1	1.21E-23	0.000	1.0000
	α_2	0.000192	3.128	0.0018
Portfolio 7	α_0	0.023544	9.845	0.0001
	α_1	0.044278	0.848	0.3963
	α_2	5.54E-19	0.000	1.0000
Portfolio 8	α_0	0.000747	0.795	0.4267
	α_1	0.092798	1.338	0.1807
	α_2	0.756708	2.921	0.0035

GARCH				
	Variable	B Value	t Ratio	Approx Prob
Portfolio 9	α_0	0.01183	6.989	0.0001
	α_1	0.348519	1.945	0.0517
	α_2	-5.83E-19	0.000	1.0000
Portfolio 10	α_0	0.03351	12.103	0.0001
	α_1	0.031027	0.533	0.5937
	α_2	-3.96E-23	0.000	1.0000
Portfolio 11	α_0	0.010164	10.239	0.0001
	α_1	5.78E-23	0.000	1.0000
	α_2	0.000318	31.504	0.0001
Portfolio 12	α_0	1.05E-08	2.707	0.0068
	α_1	9.792411	5.935	0.0001
	α_2	0.235291	17.711	0.0001
Portfolio 13	α_0	0.003483	1.002	0.3162
	α_1	0.166872	1.329	0.1840
	α_2	0.295038	0.531	0.5954
Portfolio 14	α_0	0.005535	9.123	0.0001
	α_1	1.65E-23	0.000	1.0000
	α_2	-4.43E-23	0.000	1.0000
Portfolio 15	α_0	0.000258	3.029	0.0025
	α_1	0.51426	3.095	0.002
	α_2	0.603348	7.168	0.0001
Portfolio 16	α_0	0.004621	18.164	0.0001
	α_1	4.87E-23	0.000	1.0000
	α_2	0.000105	89.549	0.0001

Table 11

Some Summary Results for Tests of the Normality of the Residuals form a GARCH model

This table presents results from conducting the normality test of Bera and Jarque (1982) for checking if the residuals from the GARCH model, $e_t = \varepsilon / \sqrt{h_t}$, are normally distributed. Summary values for 16 randomly selected portfolios for a portfolio size of 1 for trading infrequency screens of 5% and 50% are presented herein.

Most Stringent Infrequency Trading Screen (5%)

Normality		
Portfolio	Normality	Prob>Chi-Sq
1	3.976	0.137
2	6.1198	0.0469
3	7.9143	0.0191
4	1669.987	0.0001
5	0.1043	0.9492
6	99.3866	0.0001
7	29.4276	0.0001
8	699.9737	0.0001
9	5.5872	0.0612
10	8.0181	0.0182
11	3.8824	0.1435
12	2.2489	0.3248
13	6.5491	0.0378
14	0.2414	0.8863
15	35.6391	0.0001
16	8.0181	0.0182

Least Stringent Infrequency Trading Screen (50%)

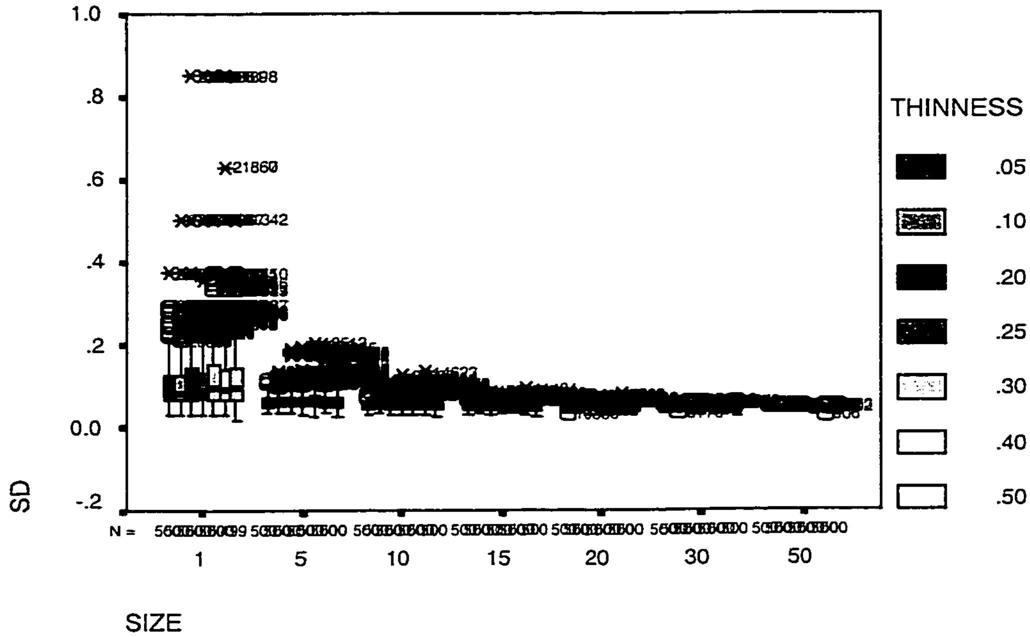
Normality		
Portfolio	Normality	Prob>Chi-Sq
1	3.8975	0.1424
2	4.5404	0.1033
3	3.5874	0.1663
4	699.9737	0.0001
5	15.3054	0.0005
6	14.1191	0.0009
7	152.8885	0.0001
8	49.8581	0.0001
9	1.8033	0.4059
10	258.4906	0.0001
11	11.2154	0.0037
12	2879.604	0.0001
13	2.979	0.2255
14	9.9986	0.0067
15	255.9221	0.0001
16	1171.3	0.0001

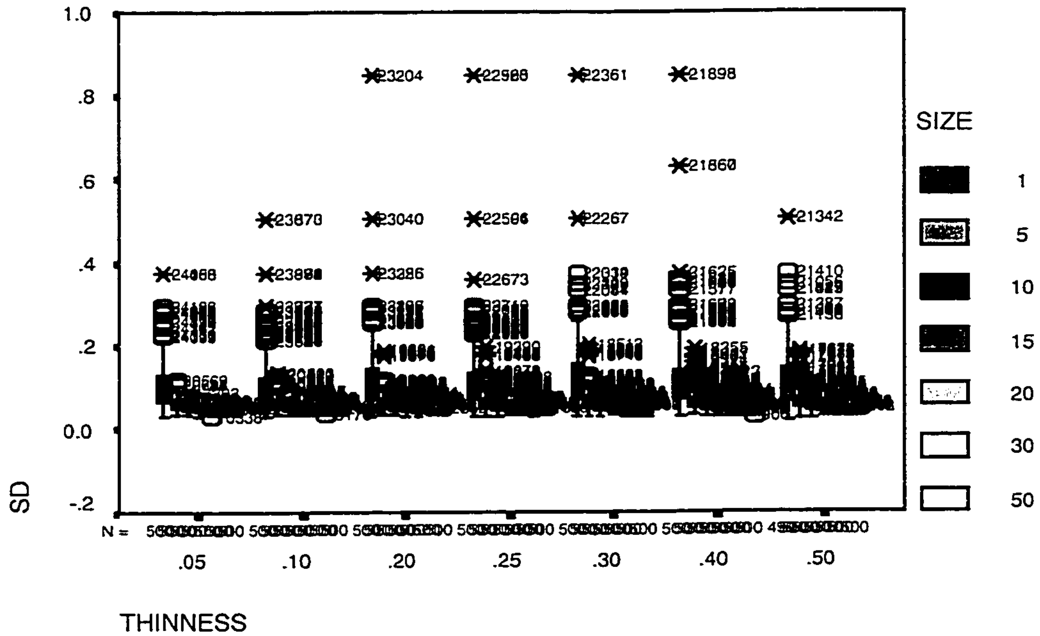
Figure 1

Two -Way ANOVA of Standard Deviations

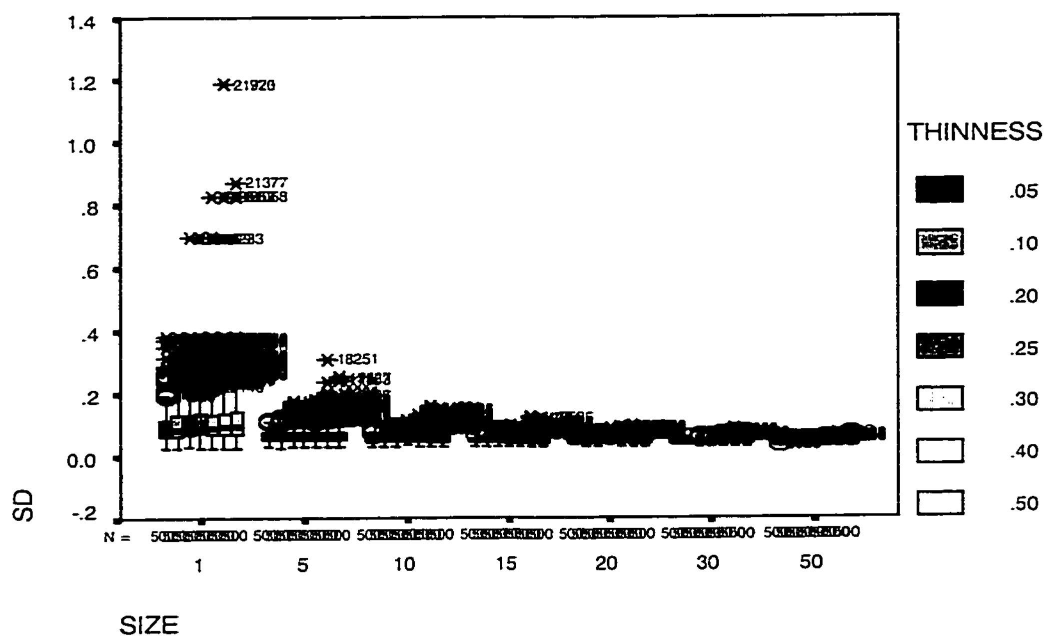
These Box Plots are used to examine whether or not the data fit the assumptions of the two-way ANOVA test. They examine whether or not the portfolio monthly returns are normally distributed and the sample variances are equal for various portfolio sizes and trading infrequency index cut-off values for each of the three time horizons.

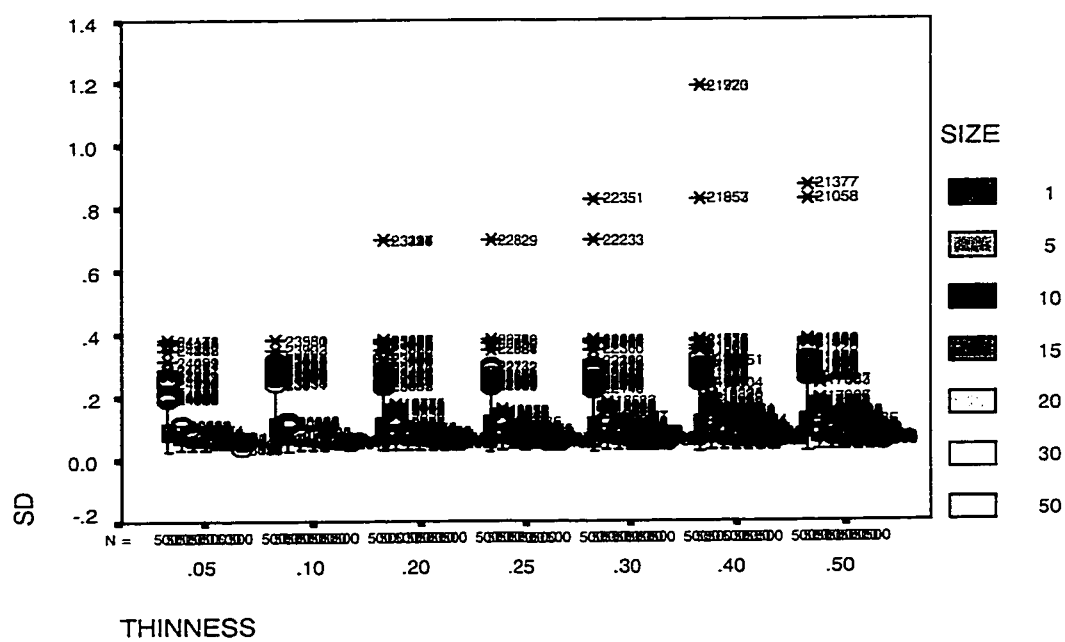
Ten Years (January 1988 – December 1997)



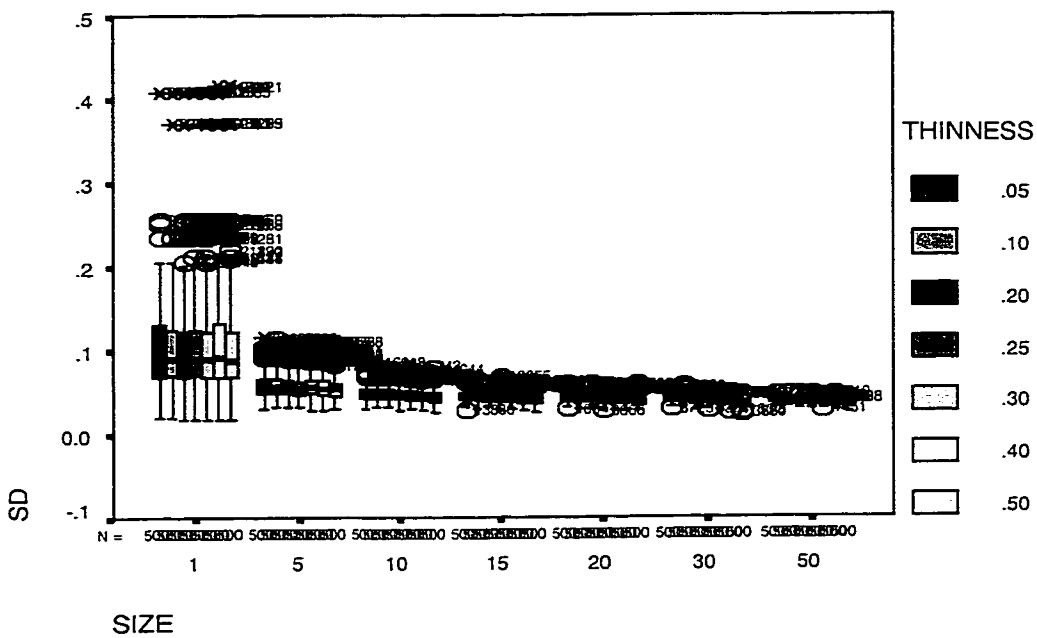


First Five Years (January 1988 – December 1992)





Second Five Years (January 1993 – December 1997)



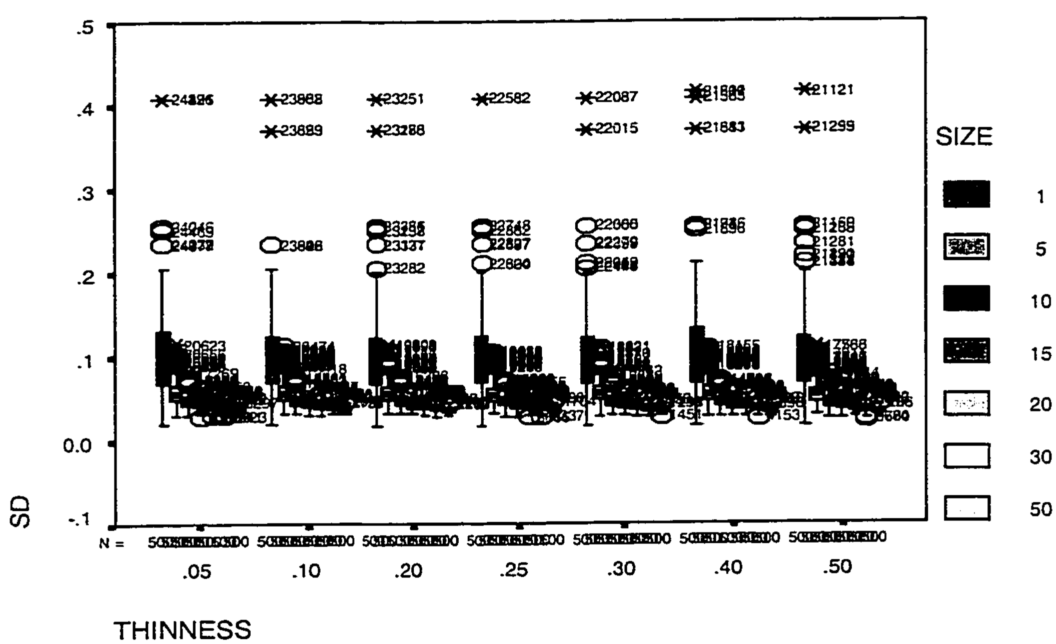
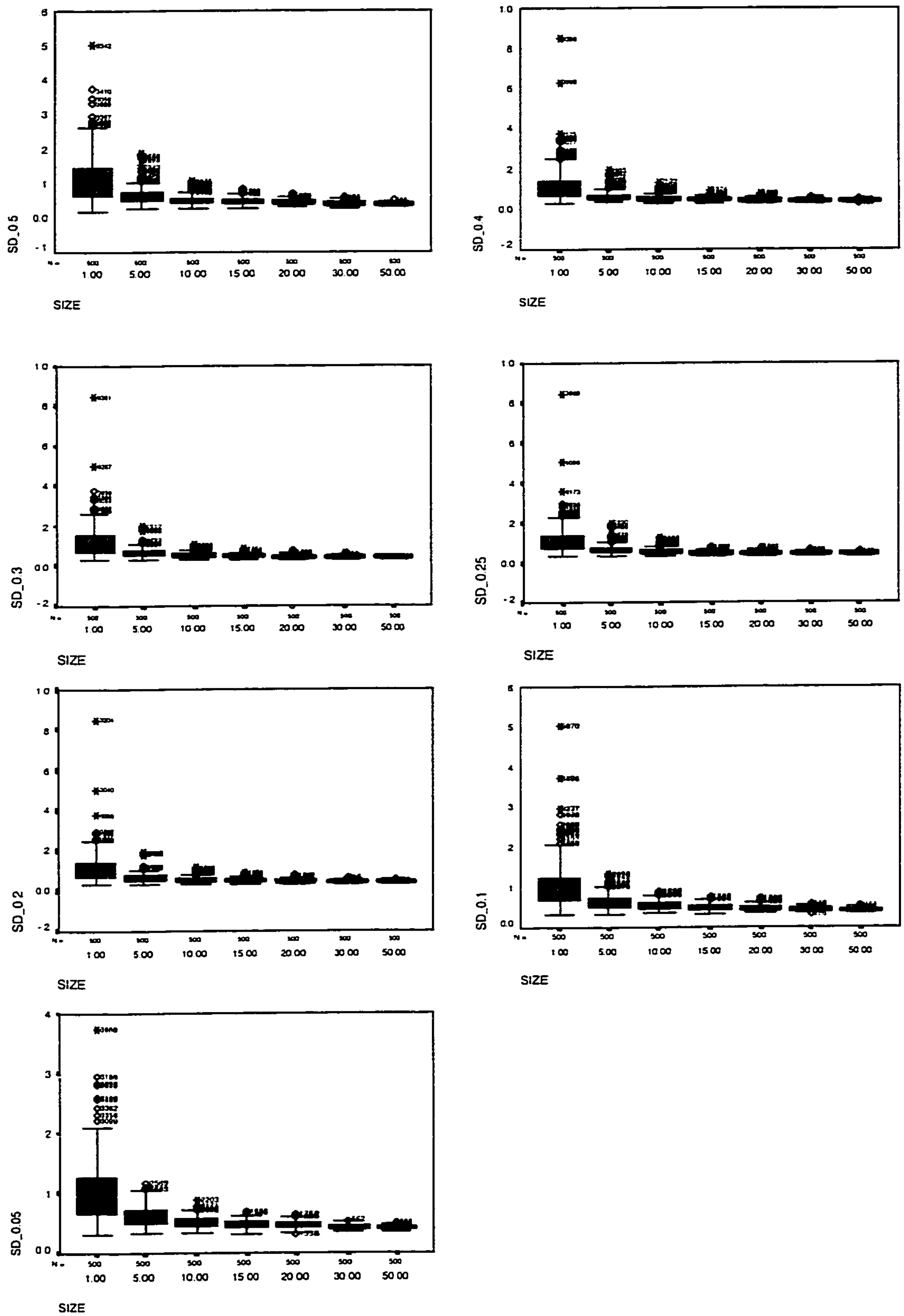


Figure 2

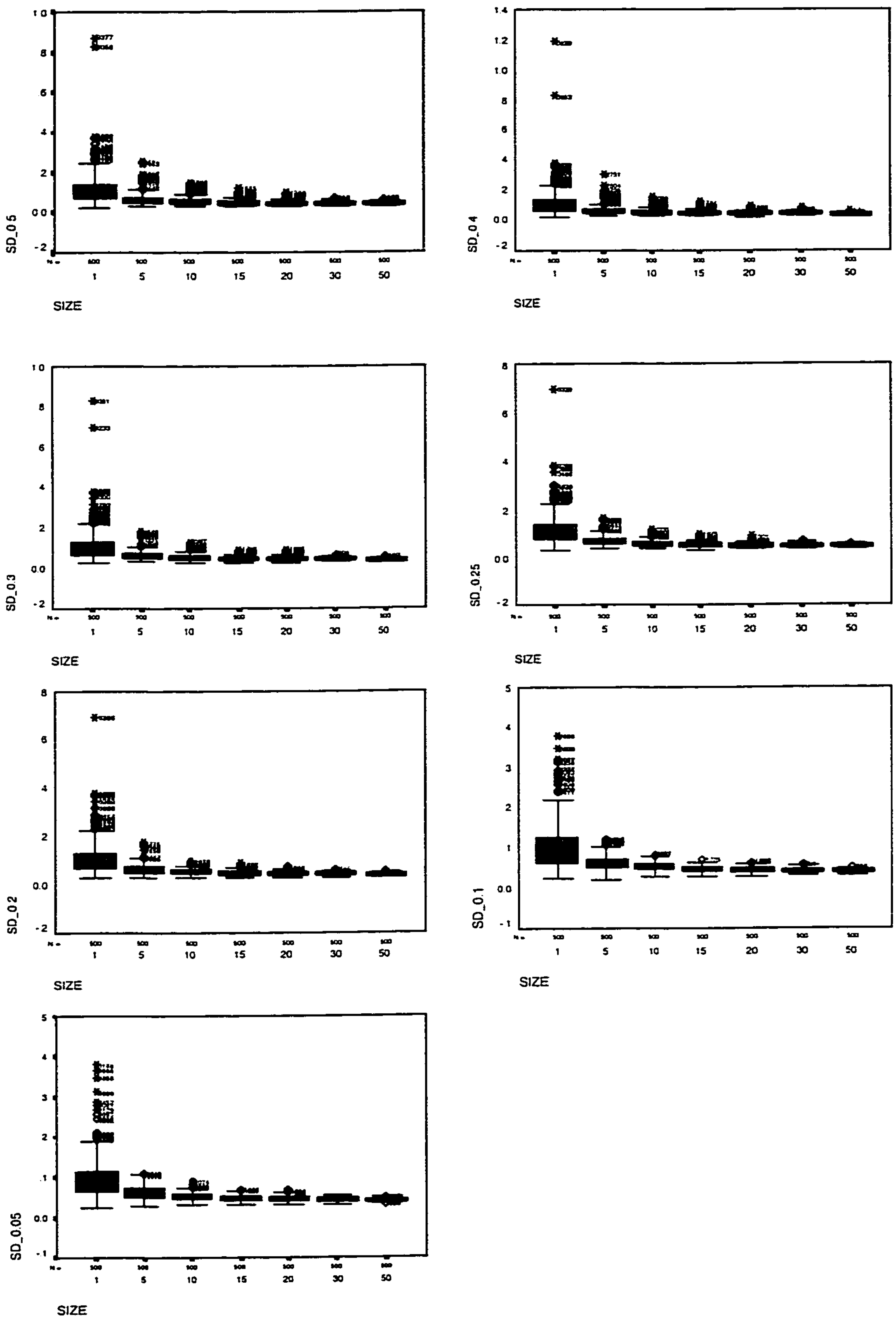
One - Way ANOVA of Standard Deviations (Size Effect)

These Box Plots are used to examine whether or not the data fit the assumptions of the one-way ANOVA test. They examine whether or not the portfolio monthly returns are normally distributed and the sample variances are equal across various portfolio sizes for the same trading infrequency index cut-off value for each of the three time horizons.

Ten Years (January 1988 – December 1997)



First Five Years (January 1988 – December 1992)



Second Five Years (January 1993- December 1997)

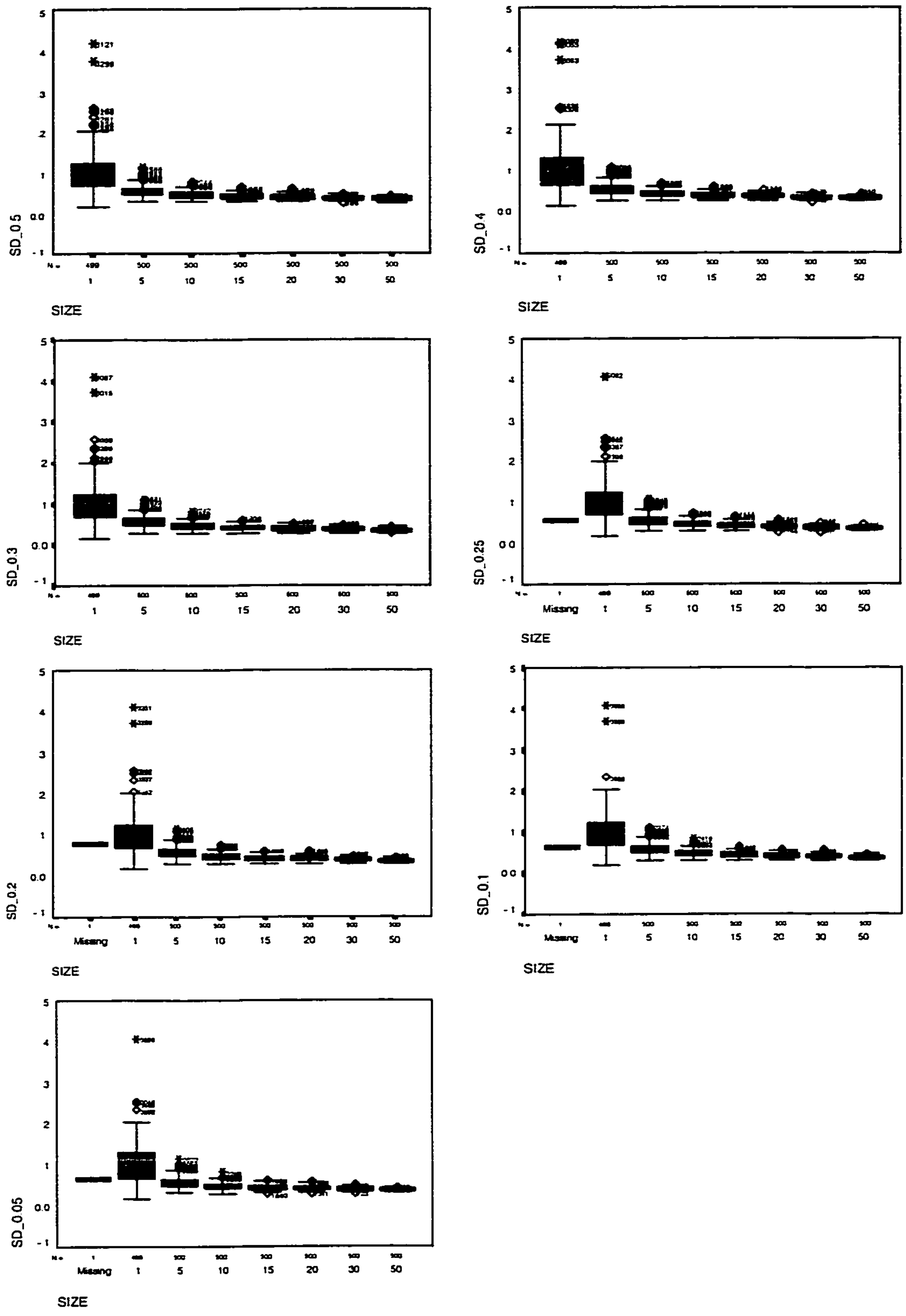
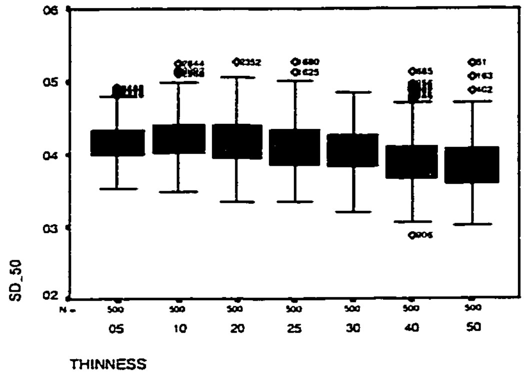


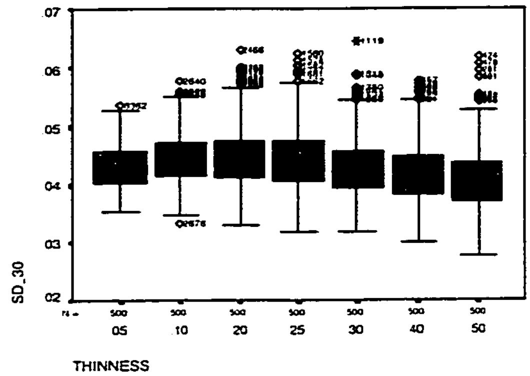
Figure 3

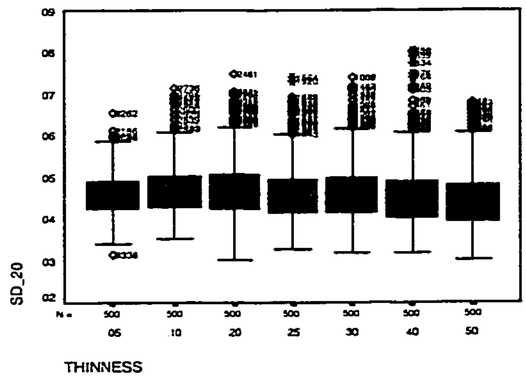
One -Way ANOVA of Standard Deviation (Trading Infrequency Effect)

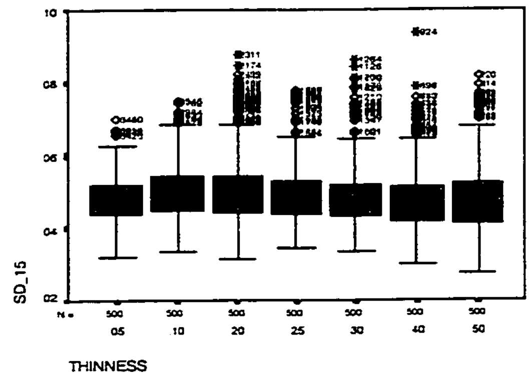
These Box Plots are used to examine whether or not the data fit the assumptions of one-way ANOVA test. They examine whether or not the portfolio monthly returns are normally distributed and the sample variances are equal across various trading infrequency index for the same portfolio size for each of the three time horizons.

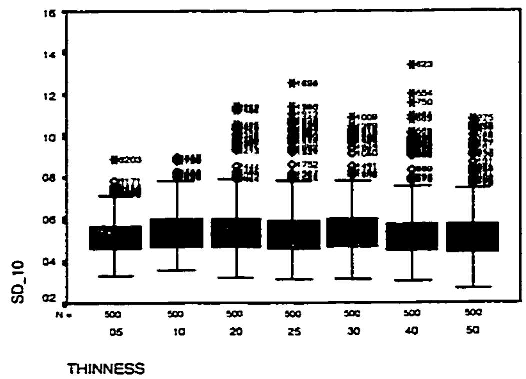
Ten Years (January 1988 – December 1997)

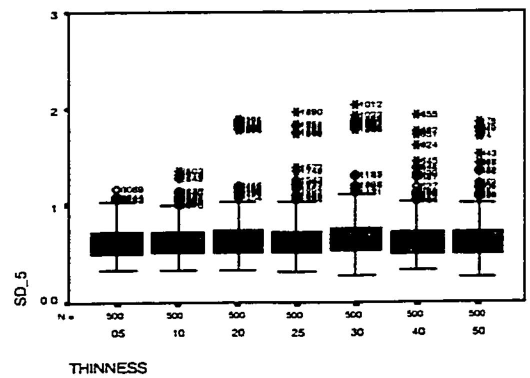


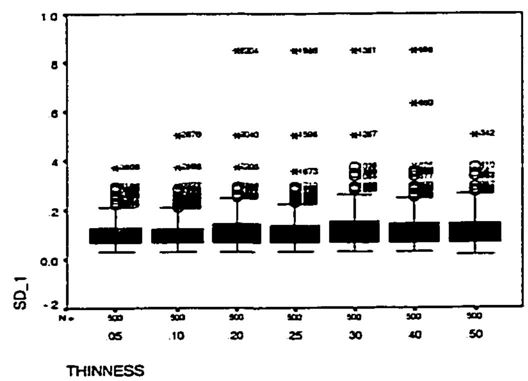




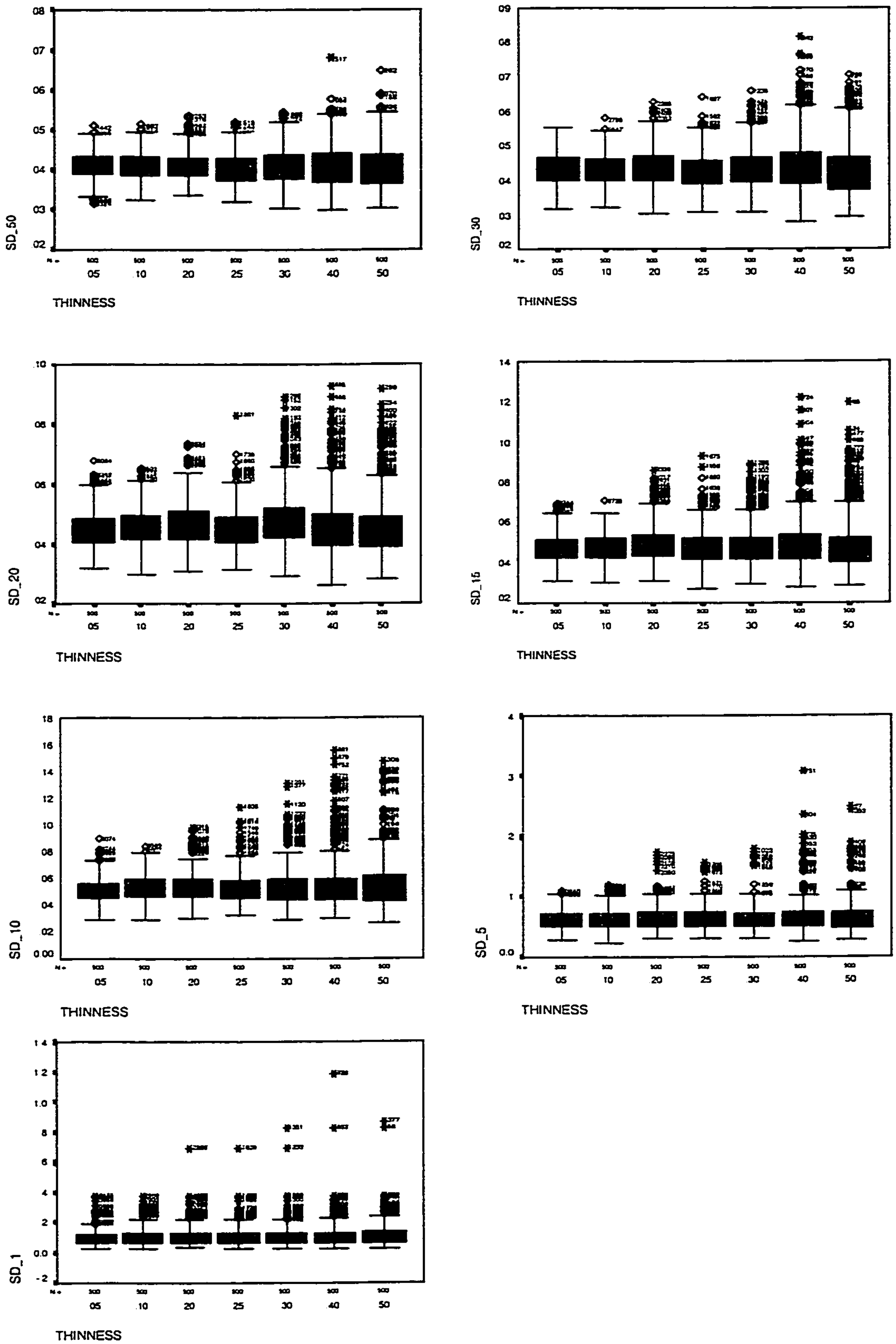




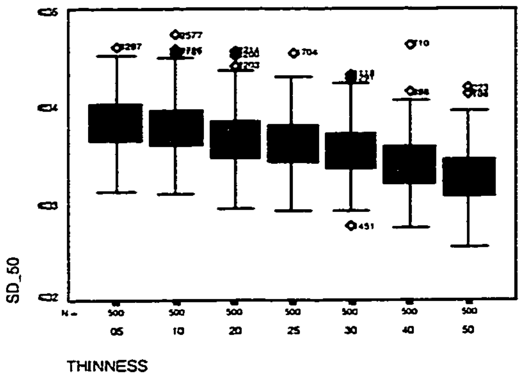


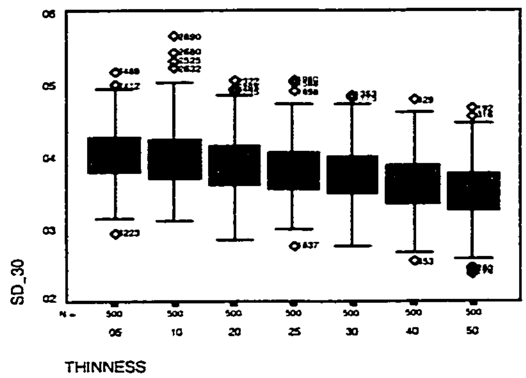


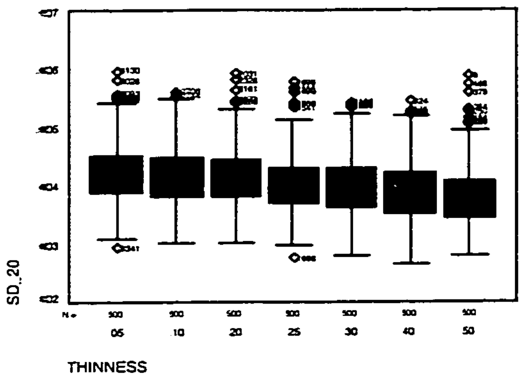
First Five Years (January 1988 – December 1992)

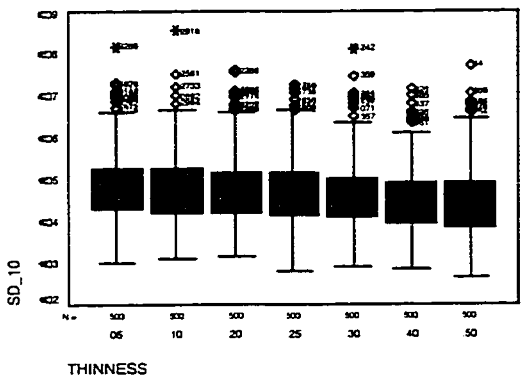


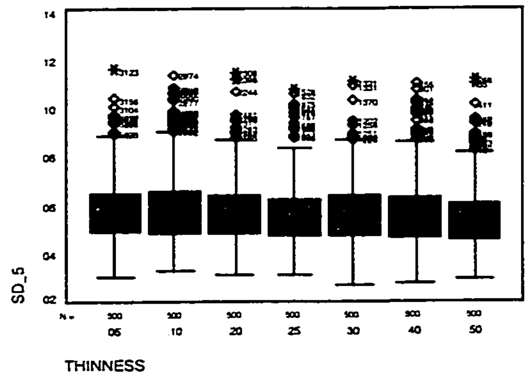
Second Five Years (January 1993 – December 1997)











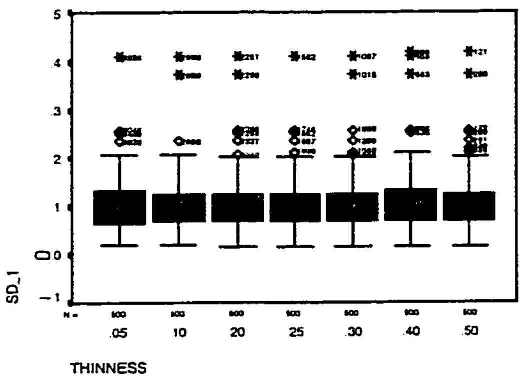
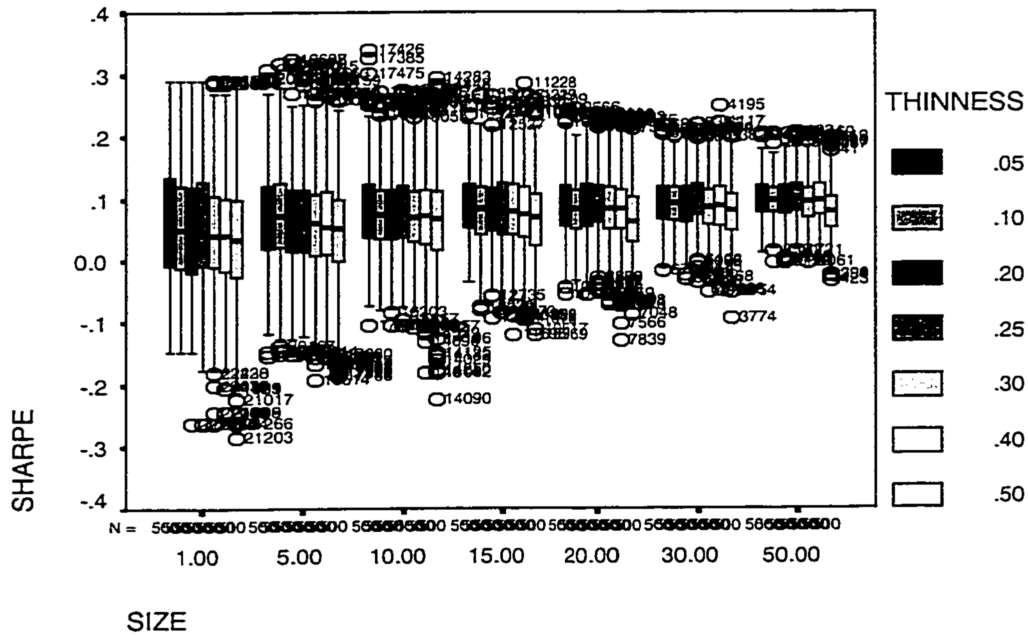


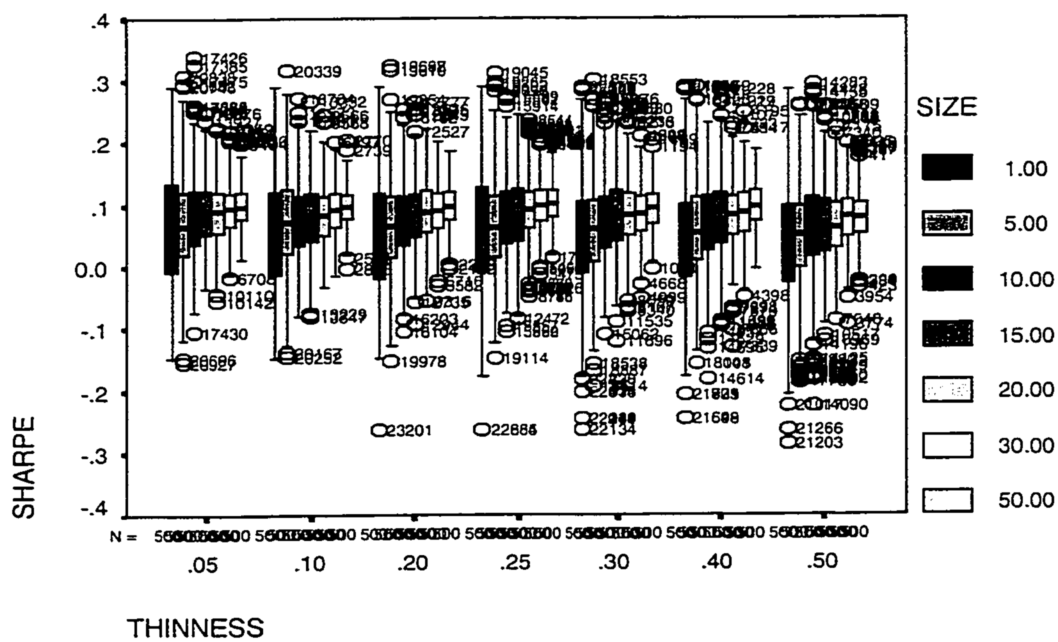
Figure 4

Two -Way ANOVA of Sharpe Ratios

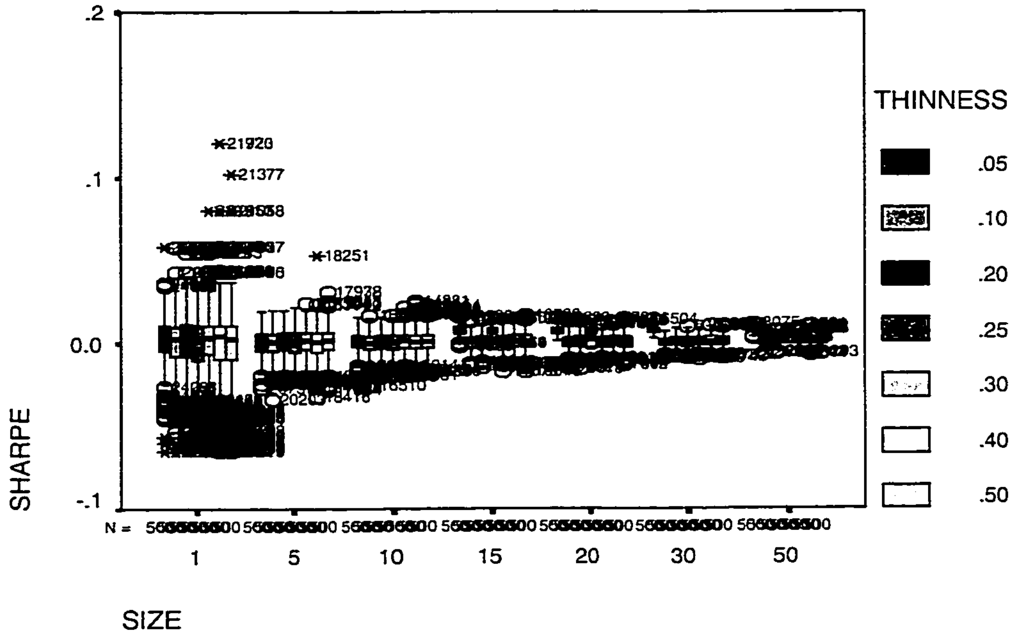
These Box Plots are used to examine whether or not the data are fit the assumptions of the two-way ANOVA test. They examine whether or not the portfolio monthly returns are normally distributed and the sample variances are equal for various portfolio sizes and the trading infrequency index cut-off values for each of the three time horizons.

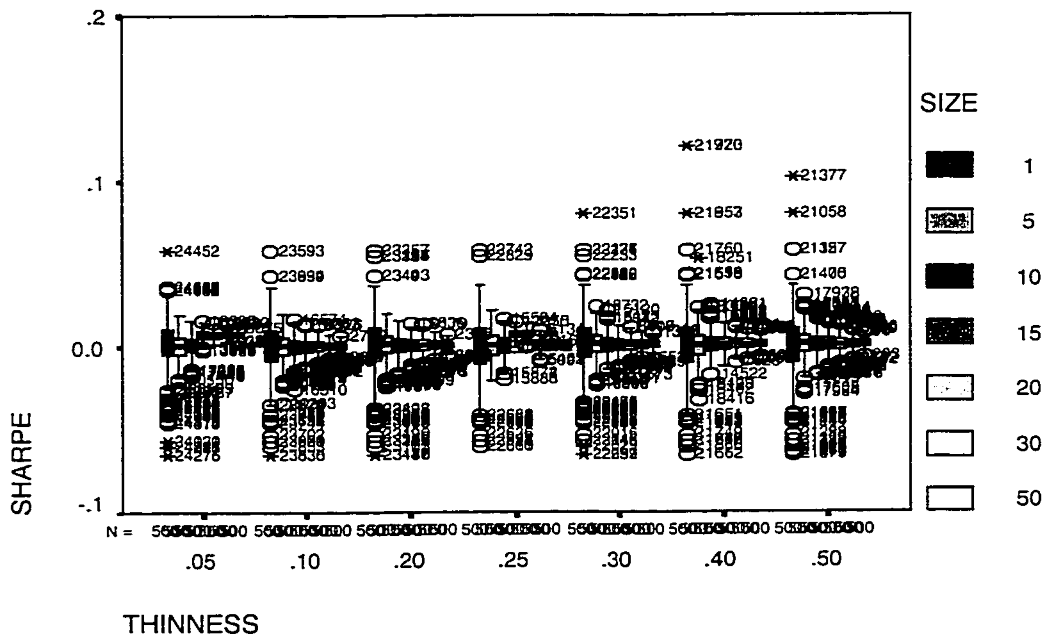
Ten Years (January 1988 – December 1997)



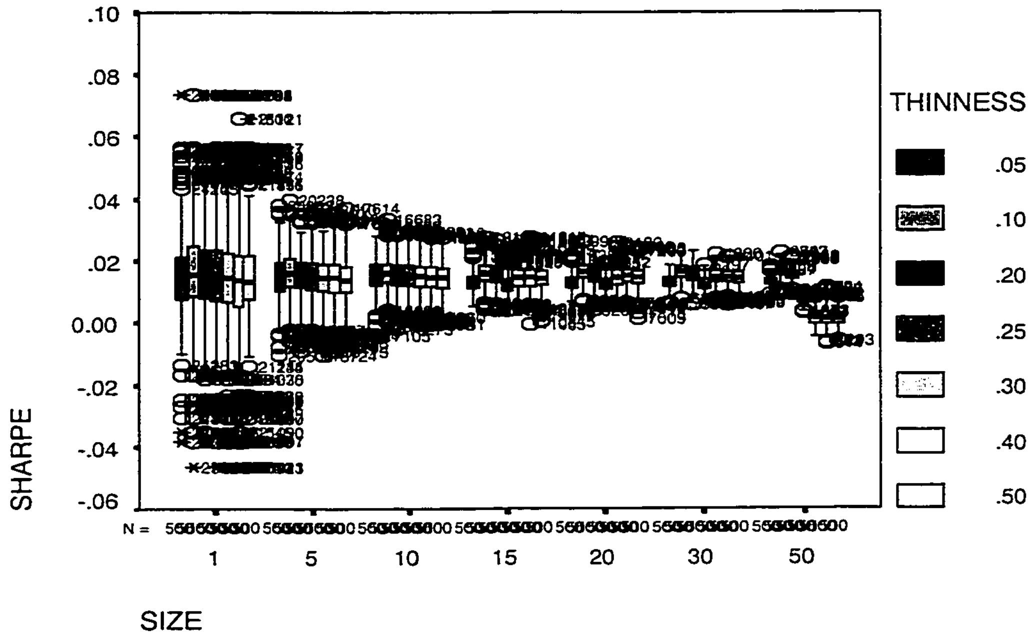


First Five Years (January 1988 – December 1992)





Second Five Years (January 1993 – December 1997)



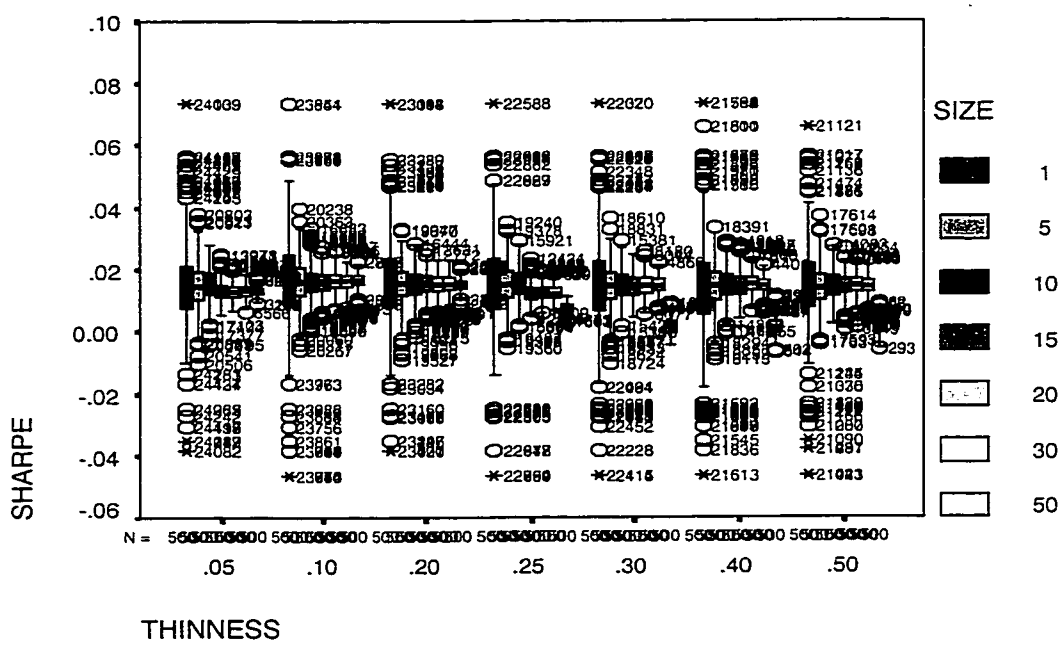
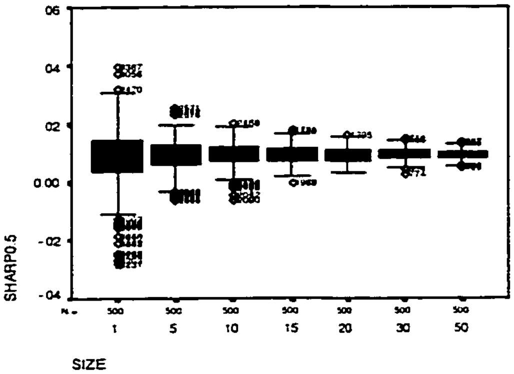


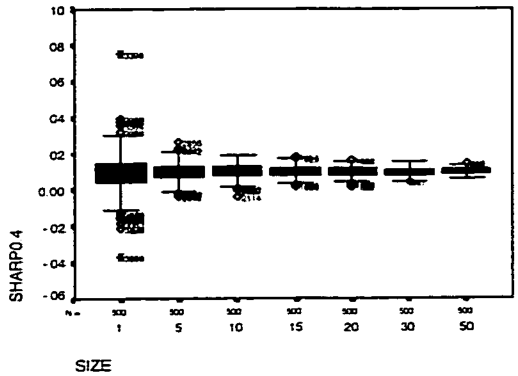
Figure 5

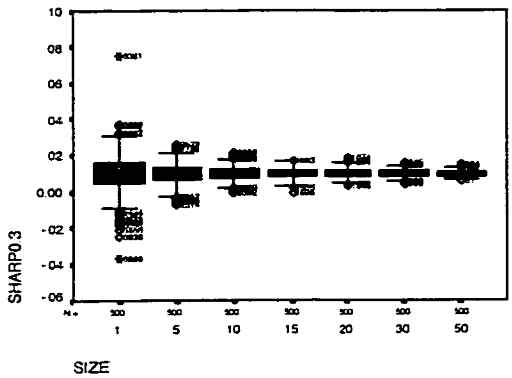
One - Way ANOVA of Sharpe Ratios (Size Effect)

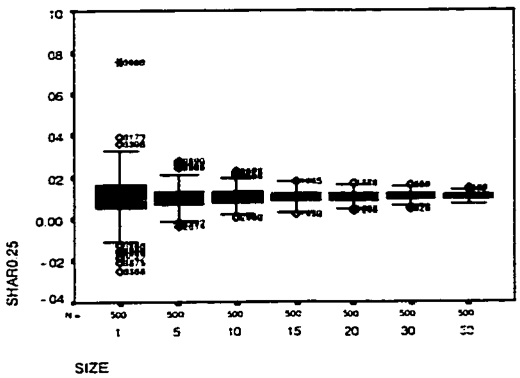
These Box Plots are used to examine whether or not the data are fit the assumptions of the one-way ANOVA test. They examine whether or not the portfolio monthly returns are normally distributed and the sample variances are equal across various portfolio sizes for the same trading infrequency index cut-off values for each of the three time horizons.

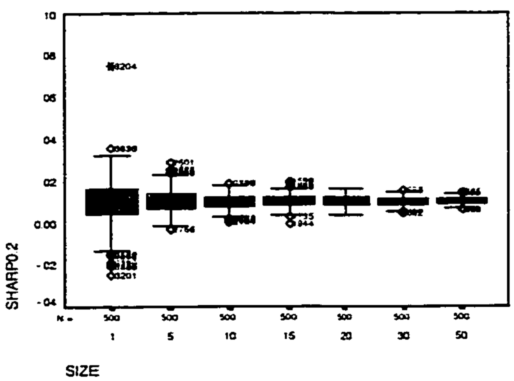
Ten Years (January 1988-December 1997)

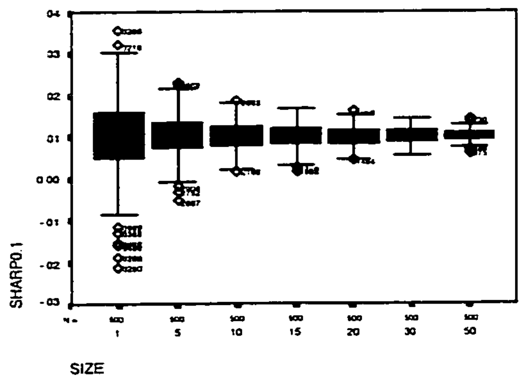


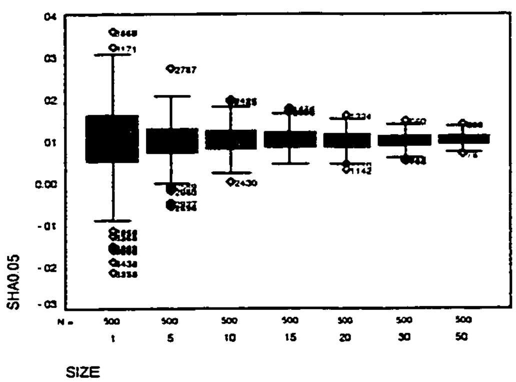




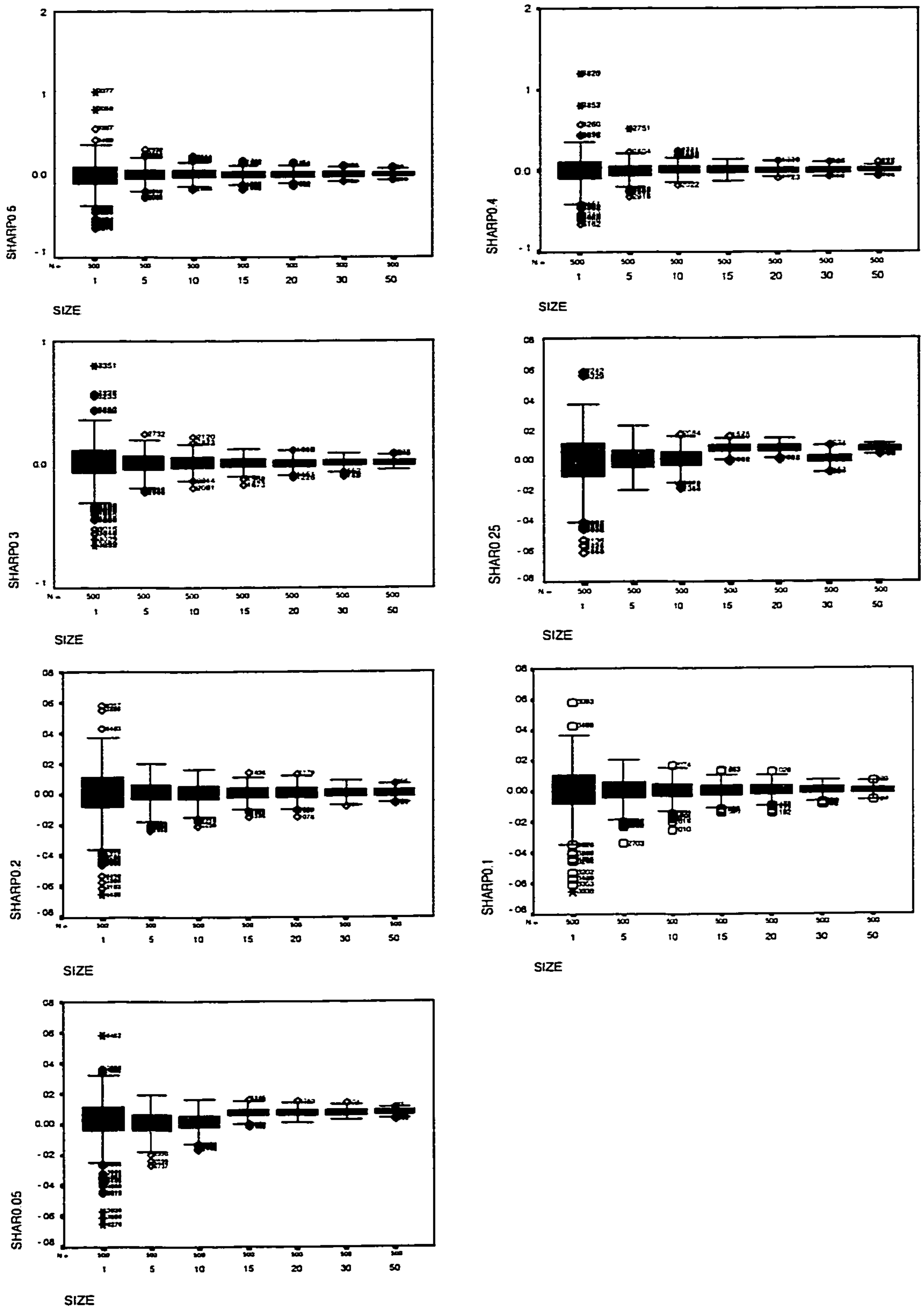








First Five Years (January 1988-December 1992)



Second Five Years (January 1993-December 1997)

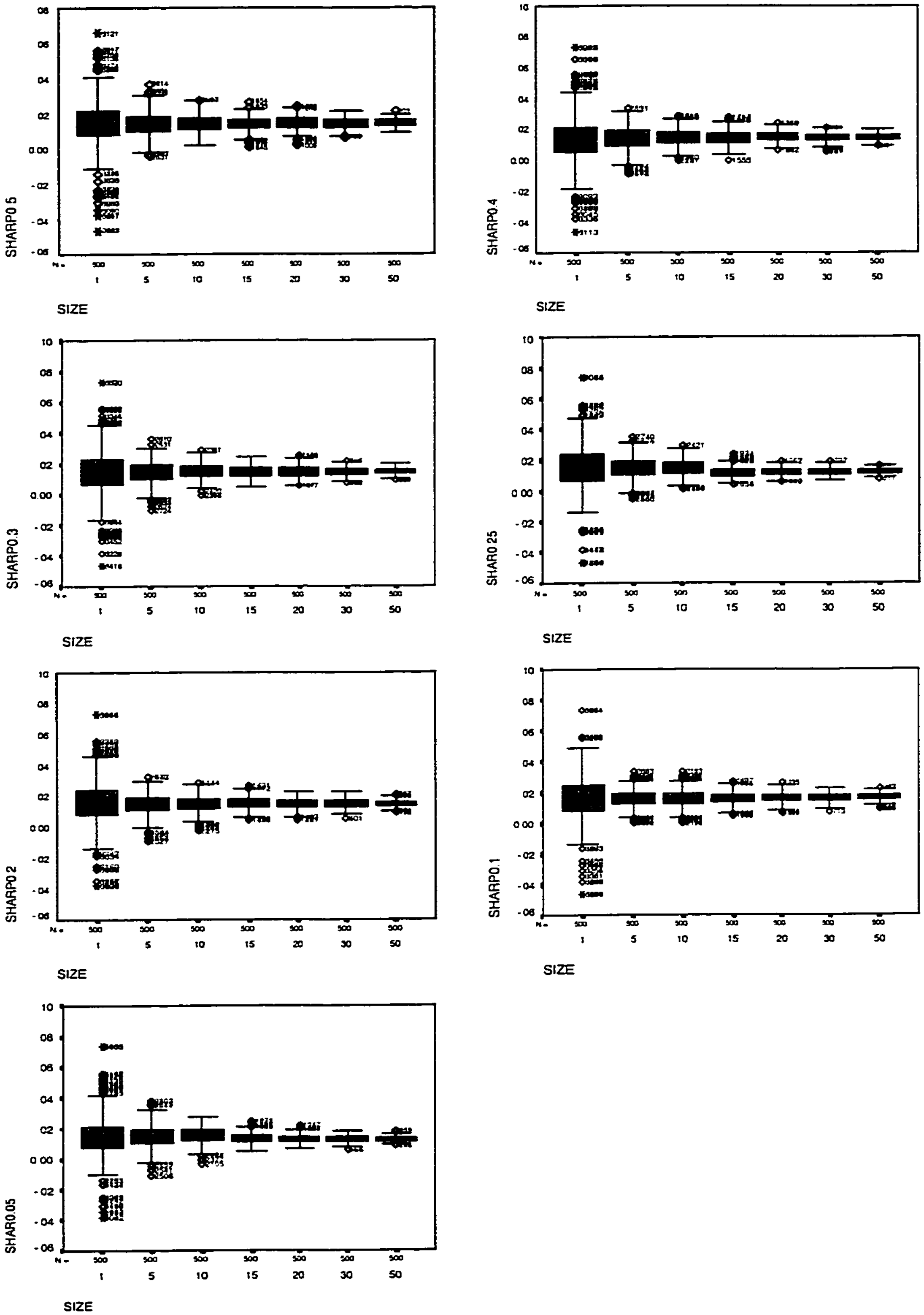
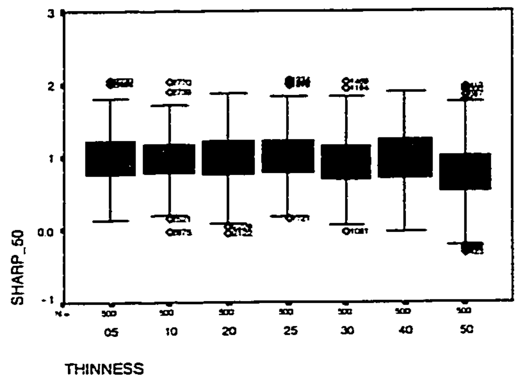


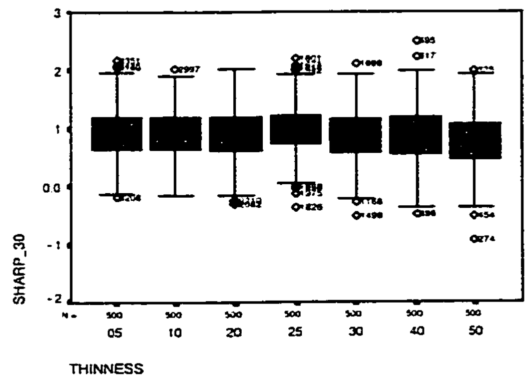
Figure 6

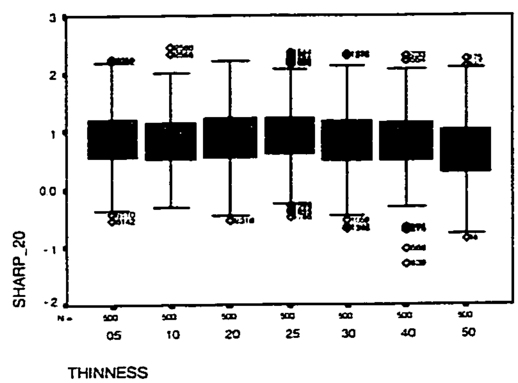
One - Way ANOVA of Sharpe Ratios (Trading Infrequency Effect)

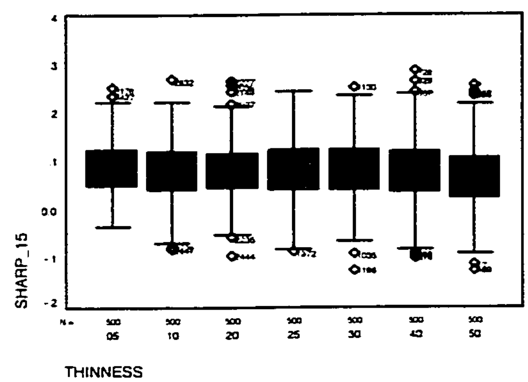
These Box Plots are used to examine whether or not the data are fit the assumptions of the one-way ANOVA test. They examine whether or not the portfolio monthly returns are normally distributed and the sample variances are equal across various trading infrequency index cut-off values for the same portfolio size for each of the three time horizons.

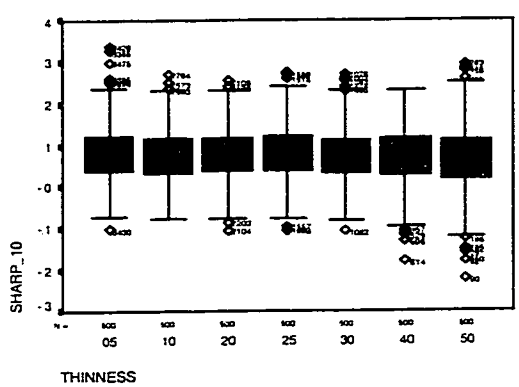
Ten Years (January 1988 – December 1997)

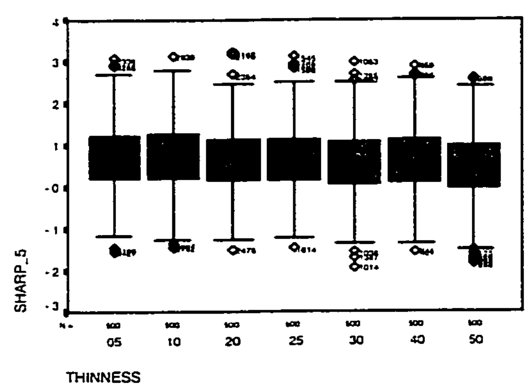


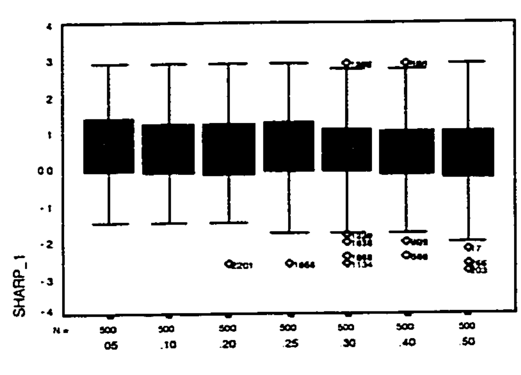




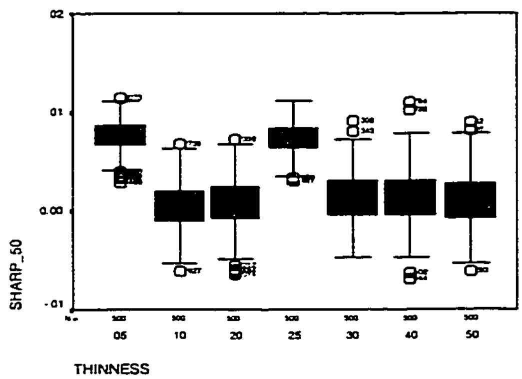


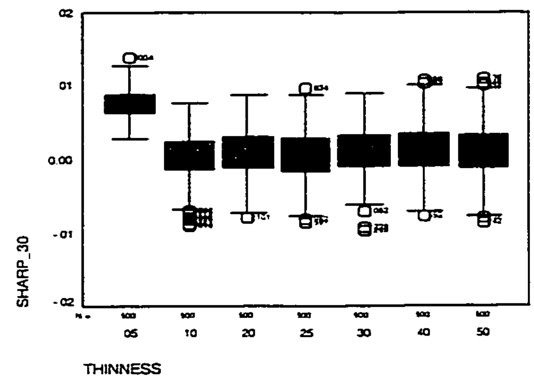


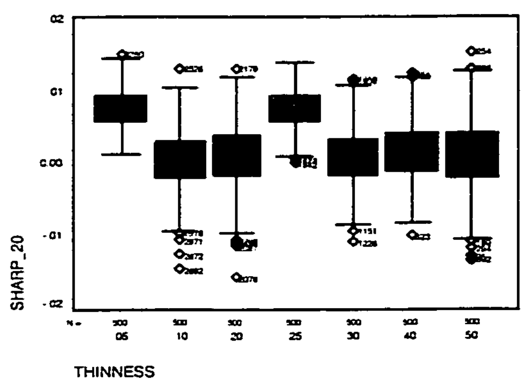


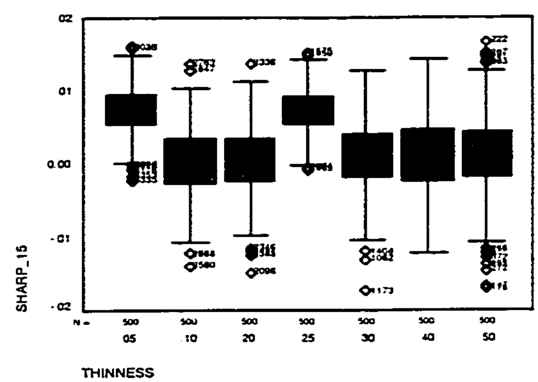


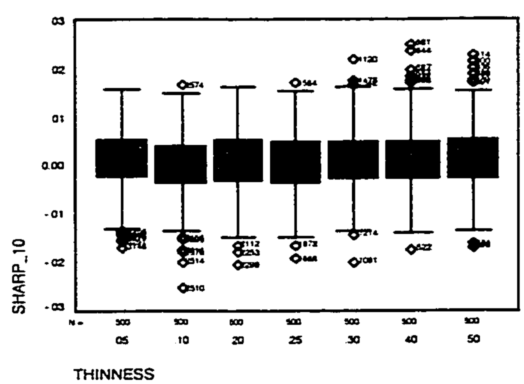
First Five Years (January 1988 – December 1992)

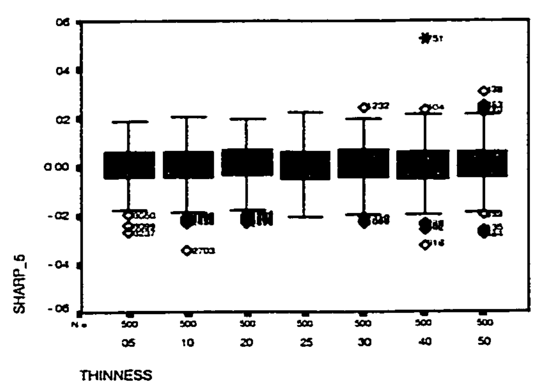


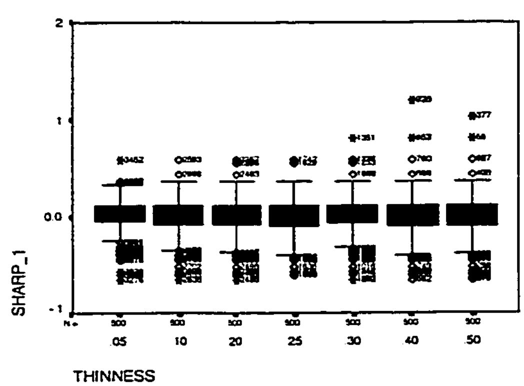












Second Five Years (January 1993 – December 1997)

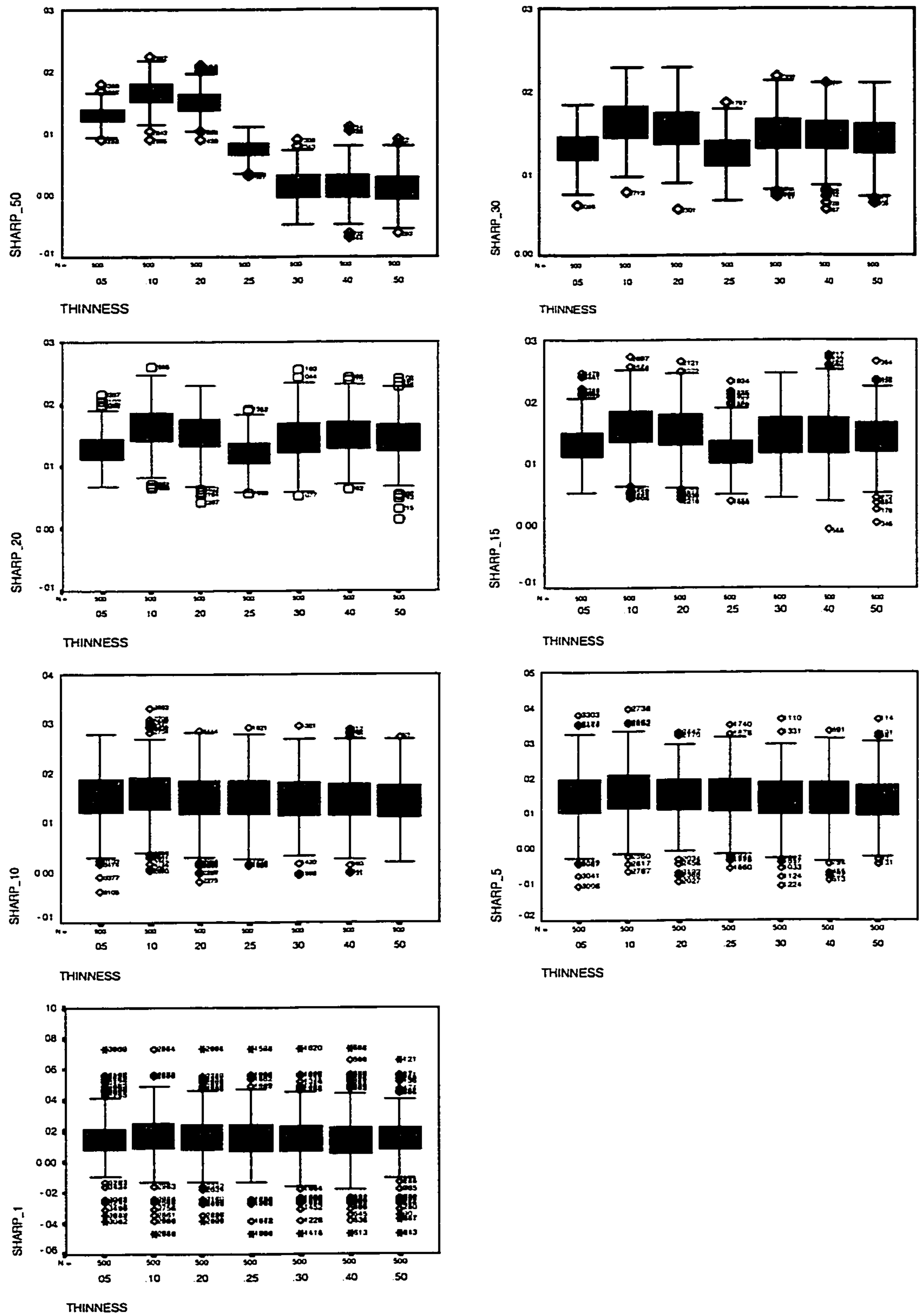
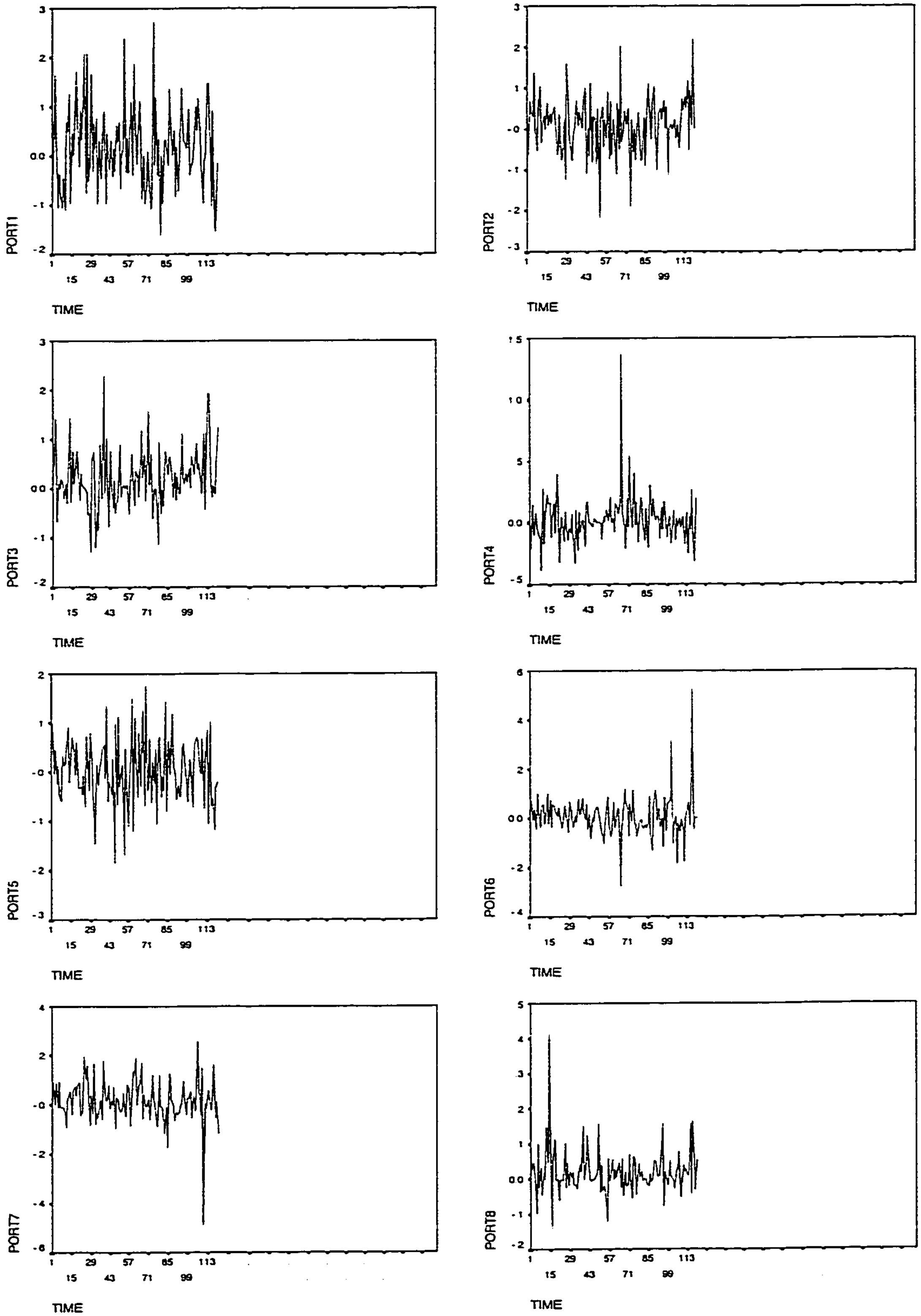


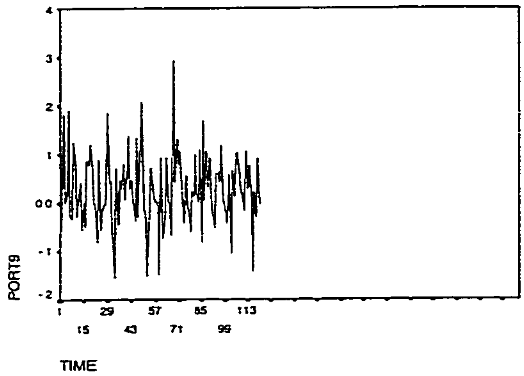
Figure 7

Sequence Plot to visually access stationary

A sequence plot of each time series is examined to assess if the monthly returns of each portfolio for each of three time periods are stationary. Figures of 16 randomly selected portfolios for a portfolio size of 1 for trading infrequency screens of 5% and 50% are presented herein.

Most Stringent Infrequency Trading Infrequency Index Cut off of 5%

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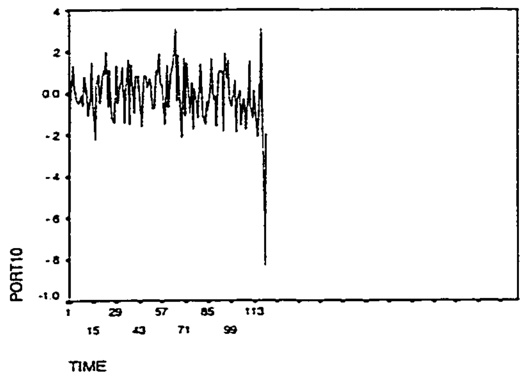
PORT10
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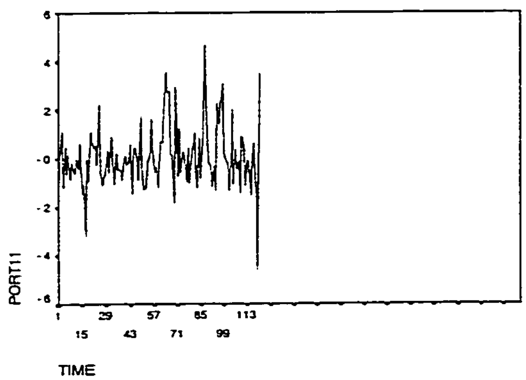
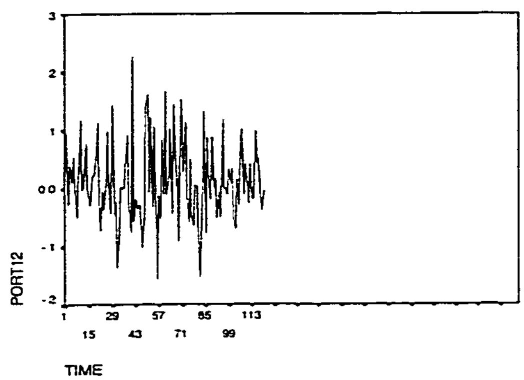
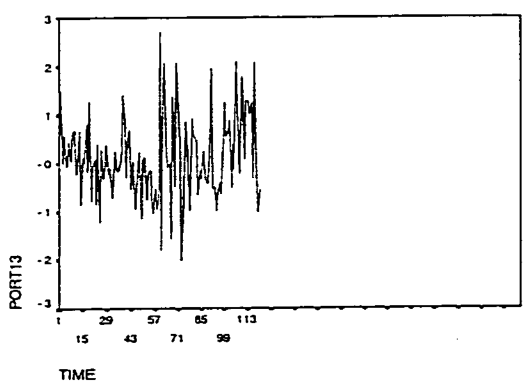

PORT11
TIME


PORT12
TIME


PORT13
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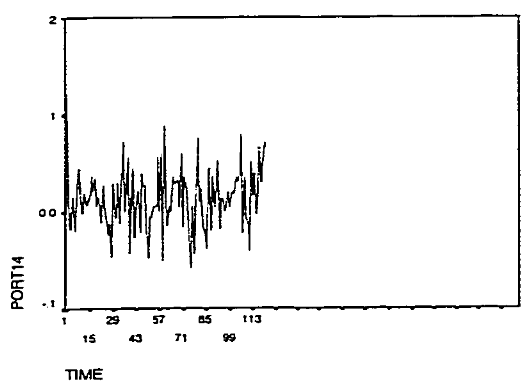
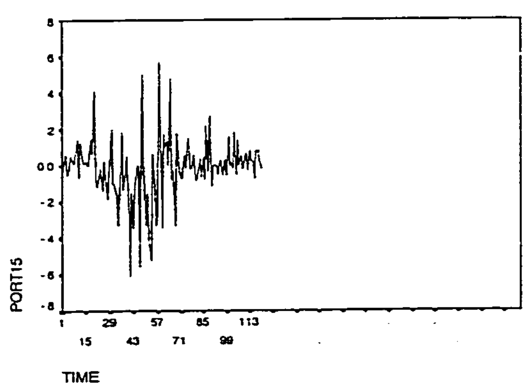
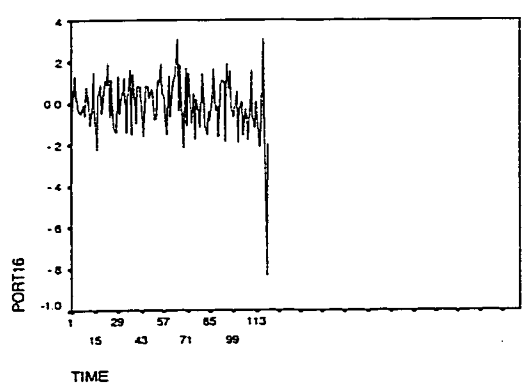
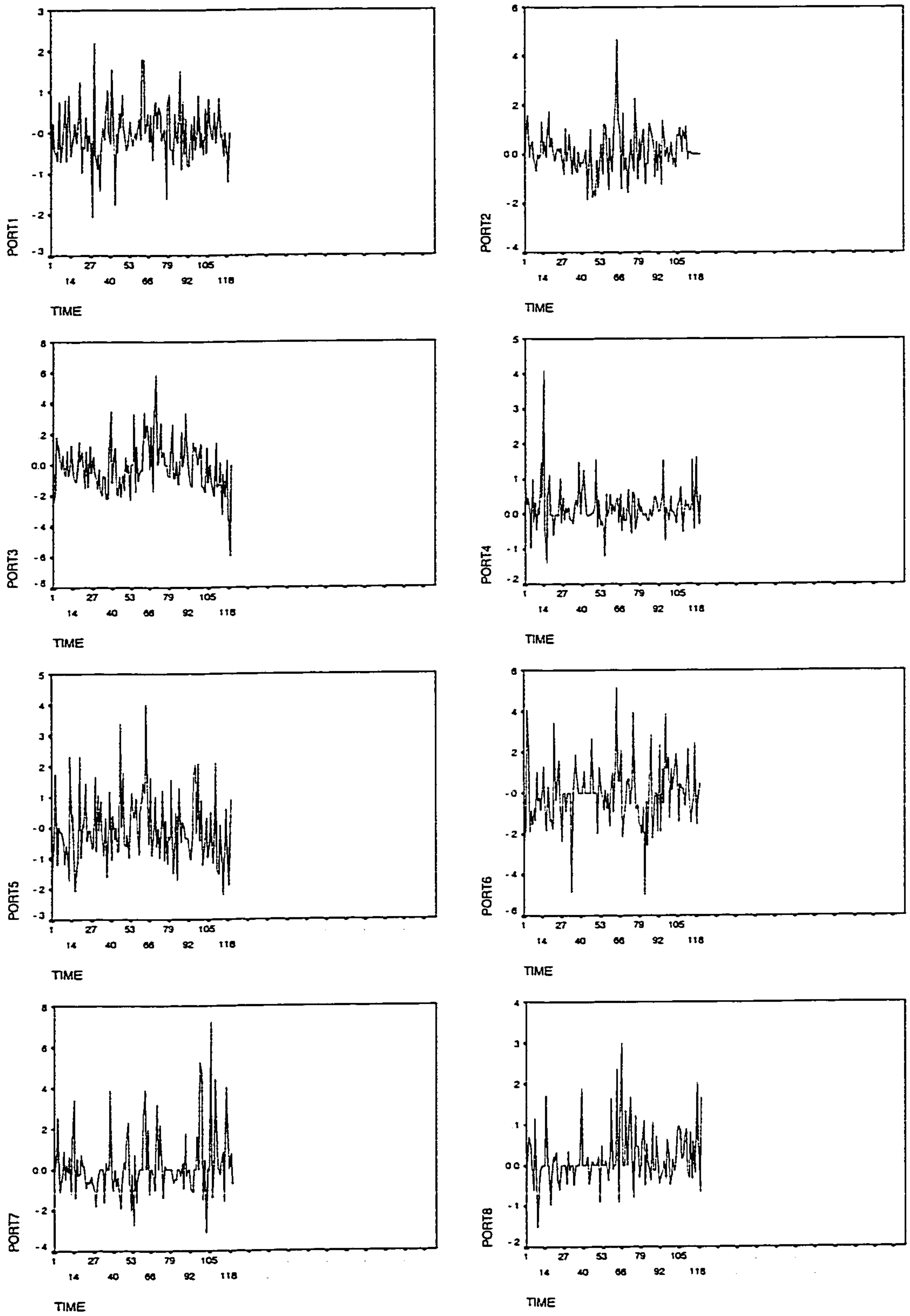
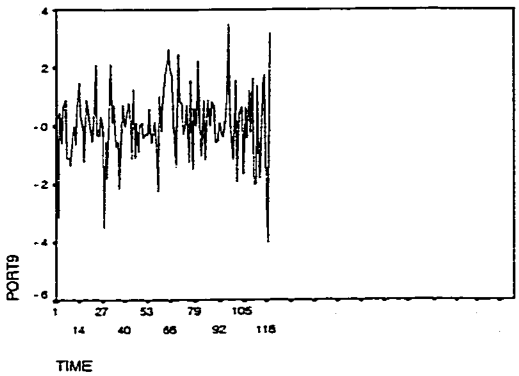

PORT14
TIME

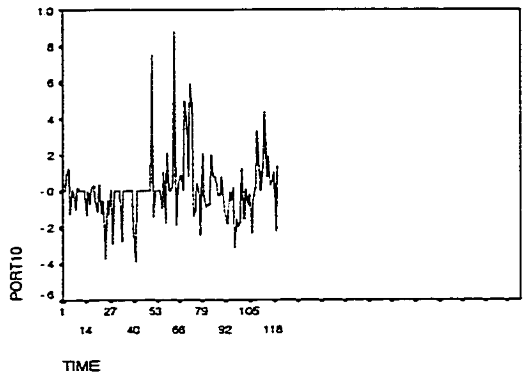

PORT15
TIME

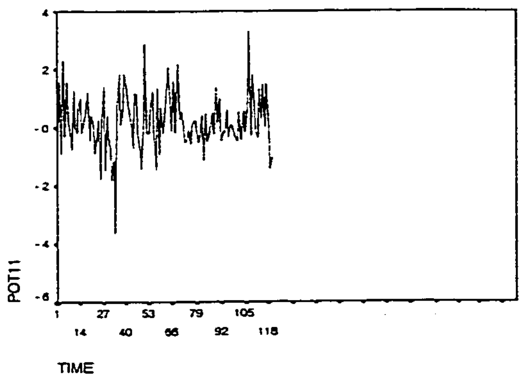

PORT16
TIME

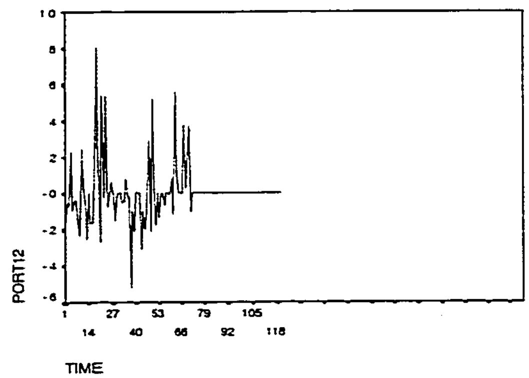
Least Stringent Trading Infrequency Index Cut-off of 50%




PORT9
4
2
-0
-2
-4
-6
1 27 53 79 105
14 40 66 92 118
TIME


PORT10
10
8
6
4
2
-0
-2
-4
-6
1 27 53 79 105
14 40 66 92 118
TIME


POT11
4
2
-0
-2
-4
-6
1 27 53 79 105
14 40 66 92 118
TIME


PORT12
10
8
6
4
2
-0
-2
-4
-6
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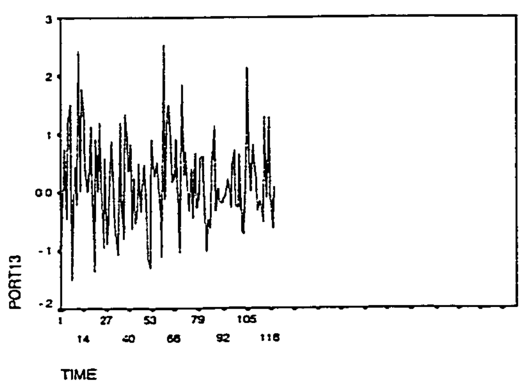
14 40 66 92 118
TIME


PORT13
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2
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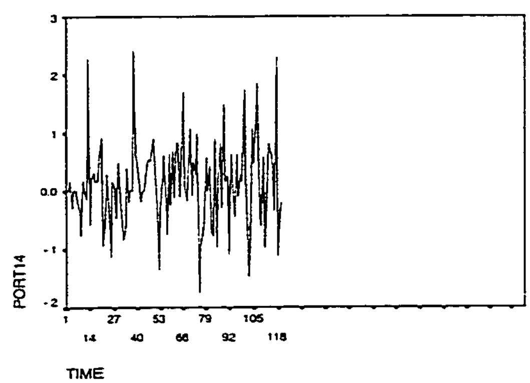
-1
-2
1 27 53 79 105
14 40 66 92 118
TIME


PORT14
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-1
-2
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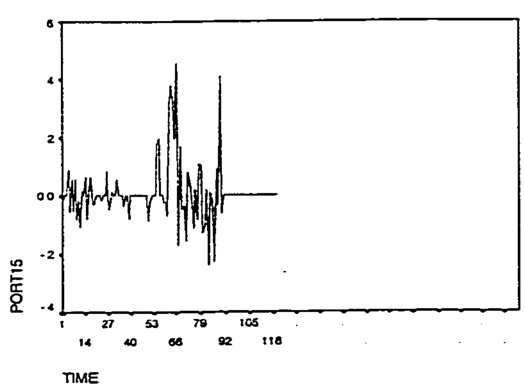
14 40 66 92 118
TIME


PORT15
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-4
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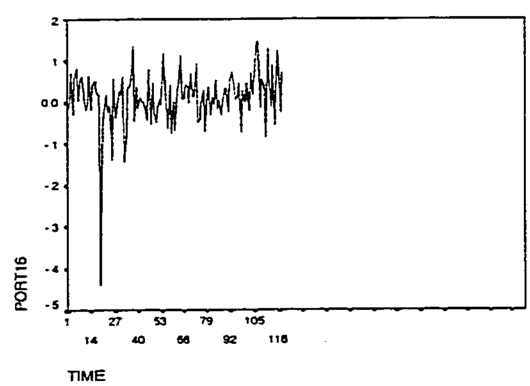
TIME


PORT16
2
1
0.0
-1
-2
-3
-4
-5
1 27 53 79 105
14 40 66 92 118
TIME