Forecasting Volatility In Canadian Markets

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ABSTRACT

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This study applies the methodology of Guan and Ederington (1998) to Canadian data. Historical volatility is estimated then forecast to examine the comparative performance of various time series models in forecasting volatility. Statistically significant regressions were provided that showed that forecasts of volatility produced low forecast errors, comparable to those found in Guan and Ederington. These errors, reported as the Root Mean Squared Forecast Error (RMSFE), were calculated using in-sample and out-of-sample data. Both static and dynamic forecasts were used. Static forecasts of volatility consistently produced lower forecast errors than dynamic forecasts.

ACKKNOWLEDGMENTS AND DEDICATIONS

I wish to dedicate this Master's Thesis to my family, who has shown me a great level of support throughout all my research. I wish to thank my friends for their understanding and patience. I wish to thank the members of my thesis committee, as well as all my former professors at Concordia University for having taught me so well. Lastly, I wish to thank all my former classmates for their help and generosity.

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Introduction

Volatility, with respect to financial products, can be thought of as the degree by which a financial security's price fluctuates around its average value. Volatility with respect to financial equities/indexes, is the degree by which a stock price or index value fluctuates about its average price or value (over a certain period of time). One measure of volatility is the standard deviation of returns. Others have defined volatility differently: "…not as a parameter but as a process which evolves over time in random but predictable ways (Engle and Mezrich, 1995)."

Significance of Measuring Volatility

Forecasting volatility is important, since it affects many areas of finance. Areas affected are at the macro-economic level down to the level of individual securities. Randolph (1991) states that there are several uses for short-term forecasts of volatility. They can be of use to option traders in discriminating between cheap and expensive options. Volatility is also an input into stock and commodity technical trading decision models. Finally, market timers can use the negative correlation between volatility and prices as a decision input. Other examples of situations which are affected by the accuracy of measurement of volatility are: 1) trading which is based on comparison of a fair value of the option to its market value; 2) reporting of fair values of an over-the-counter option position; it is possible to inflate the fair value of OTC option positions by increasing the volatility parameter used to calculate the value of the option. Traders can artificially increase their bonuses and create the impression that their losing option positions are actually winning option positions; 3) option pricing models can be used to measure the value of insurance provided by bodies like the Federal Deposit Insurance Corporation to estimate the exposure of the FDIC and the premium that should be paid by different banks. The volatility estimate affects this value of insurance premium; 4) others attribute the importance of volatility on a macroeconomic level. They explain why policymakers should care about asset price volatility, how asset price volatility affects the economy, and how monetary policy should respond to changes in asset prices. As an example, if on the macro level, volatility is consistently trending upwards, the economy's exposure to credit risk may be affected (e.g. government loans). A monetary policy review is required in order to account for such factors.

There are many other reasons why volatility plays such an important role on a global economic level. The following paragraphs focus on how volatility affects the economy, how fundamental asset-pricing models are affected, and how the hedging area is affected.

Asset Pricing Models

All asset pricing models are based on the assumption of risk aversion. Consequently, a risk parameter will be present in all models. For instance, the Beta in the Capital Asset Pricing Model (Sharpe, 1964) is a ratio comparing the standard deviation of the asset to the variance of the asset's underlying market. Both the standard deviation and variance components can be thought of as volatility components. Even risk-neutral or arbitrage-based models require the identification of perfect risk substitutes in order for the parity relationships to be identified. In the case of risk-neutral valuation (Hull, 2000a), all individuals are indifferent to risk. They require no compensation for risk, and the expected return on all securities is the risk-free interest rate. Using a binomial tree approach to price securities based on risk-neutral valuation, a probability component is introduced. This probability of an upward stock movement (or downward stock movement) is used as a risk measure. A third example is the arbitrage-based European Put-Call Parity (Hull, 2000b). It shows that the value of a European call with a certain exercise price and exercise date can be deduced from the value of a European put with the same exercise price and date (and vice versa). In the European Put-Call Parity formula, the option prices are calculated using a volatility component.

Economic Significance

Schwert (1989) relates volatility to the economy. Although many economic series were more volatile during the 1929-1939 Great Depression, stock volatility increased by a factor of two or three (compared with the usual level of the series). It was also shown that many aggregate economic series were more volatile during recessions. This was true for financial asset returns and for measures of real economic activity. One possible explanation is that "operating leverage" increases during recessions. The example given shows that if firms have large fixed costs, net profits will fall faster than revenues if demand falls. Along similar lines, since common stocks reflect claims on future profits of corporations, it is plausible that the volatility of real economic activity is a major determinant of stock return volatility. Therefore, the volatility of future expected cash flows, as well as discount rates, changes if the volatility of real economic activity changes. Schwert states that there is weak evidence that macroeconomic volatility can help predict stock and bond return volatility. The evidence is stronger that financial asset volatility helps to predict future macroeconomic volatility. This seems natural since the prices of speculative assets should react quickly to new information about economic events. Fama and French (1988b) have shown that variables such as a corporation's dividend yield or earnings yield predict stock returns as far as five years in the future. Keim and Stambaugh (1986) and Fama and French (1989) show that spreads between the yields on low versus high-grade long-term corporate debt also predict stock returns. However, Schwert (1989) reports that the relationship between stock volatility with either dividend yield or earnings yields is sometimes positive and sometimes negative. It is therefore difficult to find a stable relation. Lastly, Schwert shows that financial leverage affects volatility. When stock prices fall relative to bond prices, or when firms issue new debt securities in larger proportion to new equity than their prior capital structure, stock volatility increases. This effect explains only a small proportion of the changes in stock volatility over time.

Hedging

The ability to forecast volatility affects hedging strategies. Consider a portfolio of derivatives such as a portfolio of European call options. Calls are valued using a volatility component, amongst other factors[1]. If volatility is forecasted to change significantly, then hedgers (given their hedging objectives) can take an appropriate position to protect their portfolio. One method is to hedge based on the different dimensions of risk existing within an option position. These risk dimensions are referred to as the "Greek Letters" (see Hull, 2000d). One particular risk dimension is called the vega. The vega of a portfolio of derivatives measures the rate of change of the value of the portfolio with respect to the volatility of the underlying asset. If the vega is high, the portfolio is very sensitive to small changes in volatility. Based on set objectives, a hedger will therefore forecast volatility and then adjust the vega portfolio to compensate. There are other risk dimensions that are used by hedgers. For a complete discussion, refer to Hull (2000d). Another reason why hedging requires a good estimate of volatility has to do with a trader's hedge ratio. The hedge ratio depends on the estimated correlation between the spot and futures markets, and their

[1] Factors such as time to expiration, current stock price, strike price, risk-free rate and dividends.

volatilities (Ederington, 1979). Previous empirical research assumed that these volatilities and correlations remained constant over time. However, Grammatikos and Saunders (1983) showed that the hedge ratio does not remain constant over time. This was shown by examining the foreign currency futures market and the hedging of foreign exchange risk. Kroner and Sultan (1993) also point out that the existing literature assumes that the risk in spot and futures markets is constant over time. This implies that the minimum risk hedge ratio will be the same regardless of when the hedging is undertaken. In reality, as new information is received by the market, the riskiness of each affected asset changes. This implies that the risk-minimizing hedge ratio is time varying therefore conventional models cannot produce risk-minimizing hedge ratios. This raises important concerns regarding the risk reduction properties of conventional models. Kroner and Sultan demonstrate a method of calculating the risk-minimizing futures hedge that addresses this issue, and apply the method to several different currencies.

Literature Review

The previous section introduced volatility, and discussed its significance in the finance field. The next logical step is to comprehensively review the existing literature and summarize key findings on volatility.

Properties of Volatility

Mean Reversion

Volatility has certain properties. Hull (2000) states that volatility is mean-reverting. One proxy for volatility is the Chicago Board Option Exchange's VIX Volatility Index. This Index captures the implied volatilities of the S&P 100 Index across time[2]. Specifically, the VIX Index, based on the implied volatilities of eight different S&P 100 options series, represents a market consensus forecast of stock market volatility over the next thirty calendar days. A plot of the CBOE VIX Index over time would quickly show the periods of high volatility, as well as a long run mean reversion trend. Fleming, Ostdiek and Whaley (1995) also study the VIX Index and report that, for weekly changes, significant mean reversions in the index are detected. Further support for the mean-reversion property of volatility is shown in Bollerslev & Mikkelsen (1999). Their findings suggest that the long-run dependence in US stock market volatility is described by a slowly mean-reverting fractionally integrated process. Their study finds that the degree of mean reversion in the volatility process (implicit in the prices of the securities they studied) is best described by a fractionally integrated EGARCH model. Bookstaber & Pomerantz (1989) develop a model based on the relationship between volatility and information flows. This model leads to the specification of a stochastic process for volatility. A key characteristic of emerging from the model is volatility's mean reversion. The authors also characterize this mean-reversion as the tendency for high volatility to decline and for low volatility to increase. Mean-reversion is also apparent in Randolph (1991), who uses the Mean Reversion Model to forecast stock market volatility. The Mean Reversion Model is the same model considered by Cox, Ingersoll, and Ross (1985) for the short term interest rate. Randolph also

[2] The actual VIX Index is constructed using the implied volatilities of eight different S&P 100 option series. More detailed information can be obtained by visiting the CBOE Website at http://www.cboe.com

mentions that the Mean Reversion Model is also used by Hull and White (1988) in simulating results for the stochastic volatility option pricing model.

Heteroskedasticity

Some researchers report that stock returns exhibit clear signs of heteroskedasticity or "non-constant variance". French, Schwert, and Stambaugh (1987) use daily returns of the Standard & Poor's Composite portfolio to estimate monthly volatility from 1928 to 1984. They report that the standard deviation of aggregate monthly returns was four times larger in the 1929-1933 period than in the 1953-1970 period. Schwert and Seguin (1990) predict aggregate stock return variances from daily data to estimate time-varying monthly variances for size-ranked portfolios. They conclude that the failure to account for predictable heteroskedasticity can lead to the misleading conclusion that the conditional distribution of security returns is much more fat-tailed than a normal distribution. Hull (2000) and Luenberger (1998) provide evidence that the stock return distributions have "fatter tails" than the normal distribution.

Asymmetry

Other researchers introduce new properties of volatility. Conrad and Gultekin (1991) show a distinct asymmetry in the predictability of the volatilities of large versus small firms. Shocks to larger firms are important to the future dynamics of their own returns as well as the returns of smaller firms. The opposite however does not hold true – shocks to smaller firms have no impact on the conditional mean and variance of the returns of larger firms. French and Roll (1986) report that asset returns display a "puzzling difference" in volatility between exchange trading hours and non-trading hours. Three possible explanations are given. First, the arrival of public information may be more frequent during the business day. Second, private information may be much more likely to affect prices when the New York exchanges are opened. Third, the process of trading may induce volatility. They also find that between 4% and 12% of the daily variance is caused by security mispricing.

Volatility Clustering

According to Engle (1993), volatility clustering is one of the oldest noted characteristics of financial data. It describes the predictability of volatility. If large changes in financial markets tend to be followed by more large changes, in either direction, then volatility must be predictably high after large changes.

Implied Volatility

Due to the importance and complexity of implied volatility, the following discussion will focus solely on relating implied volatility to the context of this thesis. Implied volatility is the volatility implied by an option price observed in the market. Implied volatilities can be used to monitor the market's opinion about the volatility of a particular stock. Analysts often calculate implied volatilities from actively traded options on a certain stock and use them to calculate the price of a less actively traded option on the same stock. There is mixed evidence indicating whether or not implied volatility is a better proxy for true volatility, compared to historical volatility. Vasilellis and Meade (1996) examine forecasts drawn from both historical returns and the option market, compare them and explore the advantages of combining these forecasts. For an investment horizon of three months, it was shown that an option based implied volatility is the best individual forecasting model of actual volatility. For volatility forecasts based on historical time series of returns, the GARCH model performed better than the more simple unweighted and exponentially weighted models, although not significantly so. The authors also report that their most interesting finding was that the combination of time series forecast with the implied volatility forecast was shown to be significantly better than either of its components (for a three month horizon). Further studies on combining forecasts have been summarized in Clemen (1989).

Other Properties

Stoll and Whaley (1990) distinguish between the open-to-open and close-to-close stock market return volatility. The ratio of variance of open to open stock returns to the variance of close to close returns is shown to be consistently greater than 1 for NYSE common stocks (over the period of 1982 – 1986). A possible explanation appears to be attributable to private information revealed in trading and to temporary price deviations induced by specialists and other traders.

How to Measure Volatility

The return on a stock at time "t" R_t is defined as:

$$R_t = \ln\left(\frac{p_t + d_t}{p_{t-1}}\right)$$

Equation 1 - Stock return equation

Where "p_t" and "d_t" are defined as the price and dividend of the stock at time "t" respectively. Hull (2000) and Luenberger (1998) find stock returns to be lognormally distributed. Others, such as Bookstaber and McDonald (1987) present a return-measurement model (dubbed the "GB2") that has some attractive properties, making it useful in empirical estimation of security returns and in some specialized theoretical work in which the specification of the distribution is of vital importance. Furthermore, for the sample of stocks chosen in their study, the GB2 model provided a significantly better fit for returns. In short, the GB2 model provides a generalized distribution for describing security returns. The distribution has the feature of being flexible, and it includes a large number of well-known distributions, such as the lognormal, log-t, and log-Cauchy distributions, as special or limiting cases. This flexibility allows a direct representation of different degrees of fat tails in the distribution. This is one of the main differences between the standard lognormal and GB2 distributions. This thesis will consistently use the lognormal return distribution (instead of more complicated models such as the 'GB2' model) as the method of measuring stock returns.

According to Hull[3], the standard approach to estimating volatility is by assuming the stock price follows a generalized wiener process, calculating the stock return as shown in equation 1 above and applying the regular standard deviation formula to the returns. This is a relatively simple method. If the objective is to monitor the current level of volatility, a more sophisticated technique adds a weighting term which will place more emphasis on recent data. Extending this idea of adding weights to observations brings forth the ARCH, GARCH and EWMA (exponentially weighted moving average) models. Choosing between various models depends on the researcher's objectives. A GARCH(1,1) model incorporates mean-

[3] Hull, John C., 2000a, *Options, Futures & Other Derivatives – Fourth Edition*, Prentice Hall, USA, pp. 241-244, pp. 368-381

reversion whereas an EWMA model does not. There are other methods used to model volatility but the goal of this section was to discuss the basics. The next paragraph extends the discussion on the methodology used in estimating volatility.

Discussion on Volatility-Measurement Methodologies

The method by which volatility is calculated determines how effective the volatility component will be in financial models. An example was given in the introduction section that mentioned traders can artificially inflate the value of options by increasing the value of the volatility parameter used in their calculations. Before discussing this matter further, it is important to outline what causes volatility.

Some analysts have claimed that the volatility of a stock price is caused solely by the random arrival of new information about the future returns from the stock. Others have claimed that volatility is caused largely by trading. It would be interesting to determine if the volatility of an exchange-traded asset is the same when the exchange is open or closed. Both Fama (1965) and French (1980) have tested this empirically. They calculated the variance of stock price returns between the close of trading on one day and the close of trading on the next day when there were no intervening nontrading days. They also calculated the variance of the stock price returns between the close of trading on Fridays and the close of trading on Mondays. Their results suggested that volatility is larger when the exchange was open than when it was closed. This is further evidenced in French and Roll (1986).

The implications of all of this for the measurement of volatility are summarized by Hull (2000e). If daily data are used to measure volatility, then the literature suggests that days when the exchange is closed should be ignored. Therefore, the use of 252 trading days per year should be used in volatility calculations instead of 365 days per year. The use of 252 days is generally used by practitioners and has also been used in the current study. In short, there is evidence that says that the way volatility is calculated makes a difference.

Forecasting Volatility

There are two types of forecasts evaluated in this thesis – dynamic and static forecasts. Dynamic forecasting, also called multi-step forecasting, utilizes the previously forecasted value in the current forecast. Static forecasting uses the actual past value in the current forecast. The actual forecasting equations are presented in the research methodology section. Dominguez and Novales (1999, 2000) comment on the use of both static and dynamic forecasting. Their 1999 study evaluates the extent to which the explanatory power detected in the term structure in different markets and countries can actually be used to produce sensible forecasts of future short-term interest rates. Using monthly data on interest rates, they compare forecasts obtained from forward rates to those obtained from univariate autoregressions. By themselves, forward rates produce better static and dynamic forecasts of 1-month interest rates over a full year horizon, compared to those obtained from the actual past interest rates. Interestingly, the gain in static forecasting disappears for longer maturities, although forward rates still produce better once-and-for all predictions of 3 and 6 month interest rates (rather an univariate autoregressions) for a number of currencies. In other words, forward rates anticipate possible changes better than the actual past interest rate values.

Detailed objectives

This study is based on the working paper by Guan and Ederington (1998). Guan and Ederington forecast future volatility (over varying time lengths) based on actual historical volatility (based on a varying time period). In particular, attempts are made to focus primarily on measuring and estimating equity and index volatility. The objectives of this thesis is to apply the methodology of Guan and Ederington (1998) to Canadian data. Historical volatility is estimated then forecast to examine the comparative performance of various time series models in forecasting volatility. The volatilities of the following financial securities will be examined:

Table 1 - List of Securities Examined

Country	Stock Market	Security	Dividend Adjusted	Description
Canada	Toronto Stock Exchange	Nortel	Y	High-Tech Telecom Stock
Canada	Toronto Stock Exchange	TSE 300 Stock Index	N	Broad-based Index
USA	New York Stock Exchange	Dow Jones Industrials Average	N	Broad-based Index
Canada	Toronto Stock Exchange	Air Canada	N	"Old Economy" Stock

Nortel Networks

Nortel Networks is of particular interest to examine for the following reasons:

1. Nortel currently represents a relatively large weight in the TSE300 Stock Index.
2. Nortel stock is currently traded on both Canadian and American stock exchanges.
3. Nortel was recently part of a corporate spin-off by Bell Canada Enterprises.
4. Nortel's return volatility has increased in the past few years.

These characteristics make this particular stock a good candidate for further research and will contribute positively to the study.

Air Canada

Air Canada is classified as an "old economy" stock. Although various definitions of "old economy" stocks exist, one general definition is: An equity whose underlying process is governed by traditional theories. The opposite of "old economy" stocks are "new economy" stocks such as the "Dot Com" companies. Air Canada is a traditional company, whose stock price is valued by traditional methods such as the P/E Ratio, Book Value, etc. The company does not pay dividends and has been in existence for a relatively long time period (compared to "new economy" stocks). The stock is also not as volatile as some non-traditional stocks. These characteristics clearly distinguish this stock from others (such as Nortel). The Air Canada stock is included to determine whether volatility may be estimated more accurately for an "old economy" or a "high technology" stock.

Dow Jones Industrials Average Index & TSE 300 Index

Both indexes are generally accepted as being representative of the U.S. and Canadian economies and serve as good securities to study from an index point of view.

Data Description

Stock split-adjusted, dividend-adjusted returns are provided by Commodity Systems Inc. Each time series is described below.

Table 2 - Time Series Descriptions

Security	Data Date Range	Dividend-Adjusted	Number of Daily Returns
Air Canada (TSE)	12/Jan/1995 – 29/Dec/2000	N	1502
Nortel Networks (TSE)	12/Jan/1995 – 29/Dec/2000	Y	1503
TSE 300 Index	27/Dec/1995 – 29/Dec/2000	N	1265
Dow Jones Industrials Average	12/Jan/1995 – 29/Dec/2000	N	1506

Research Methodology

In this thesis, historical volatility will be used to forecast future volatility. Specifically, defining t_0 as today's date, the historical volatility from t_m to t_{-1} will be utilized to forecast the volatility from t_1 to t_n. The period t_m to t_{-1} is called the estimation period. The length of the estimation period as well as the length of the forecast period will be modified in order to determine which length of periods will produce better results. Note that a restriction was placed on the length of the estimation period – it had to be equal or larger than the length of the forecast period. The primary reason for this is as follows. There is a certain amount of information embedded in the estimation period. The greater the amount of historical information available, the better the forecast will be. In mathematical terms, the forecast error is sensitive to the size of the sample used in the estimation process. Ceteris paribus, as the sample size increases, the forecast error decreases. As an example, the simple linear regression forecast error is presented in the equation below:

$$\sigma_f^2 = \sigma^2\left[1 + \frac{1}{T} + \frac{\left(X_{T+1} - \overline{X}\right)^2}{\left(\sum\left(X_T - \overline{X}\right)^2\right)}\right]$$

Equation 2 - Forecast Error

Where "T" is the sample size. Clearly, as T increases, the forecast error decreases. Another reason for restricting the time series' lengths had to do with white noise. 5 days represents a trading week. Weekly data is less noisy than higher frequency data. This restriction seems irrelevant, given the efficient market hypothesis. However, the purpose of this study is not to predict a stock price outright. This study tries to forecast a stock price's volatility within a range. A second motivation governing this restriction is that more recent data over a shorter time period is deemed as being a better predictor of volatility over a short time period whose length is comparable to the length of the period of data in the estimation period. The following table summarizes the various lengths of estimation periods that will be used to forecast volatility in the future:

Table 3 - Historical & Forecast Periods

Length of Estimation Period (days)	Length of Forecast Period (days)
5	5
10	5
20	5
30	5
10	10
20	10
30	10
20	20
30	20

There are certain reasons for choosing these periods of time. 5 days represents one trading week. 20 days represents approximately one trading month. The 30 day estimation period was included to determine if extending the estimation period would improve results.

Serial Correlation

In time series analysis, it is observed that the regression error terms are correlated with their own lagged values. This serial correlation violates the basic assumption of regression theory that the error terms are independently and normally distributed. The main concerns of the problems associated with serial correlation are:

1. Ordinary Least Squares is no longer efficient among linear estimators. If there are lagged dependent variables on the right-hand side, OLS estimates are biased and inconsistent.
2. Standard errors of the regression coefficients are not correct, and are generally understated.

Since prior residuals help to predict current residuals, better prediction techniques can be used to take advantage of this information.

The First-Order Autoregressive Model

The First-Order Autoregressive Model or AR(1), is the simplest of the AR. The AR(1) model incorporates the residual from the past observation into the regression model for the current observation. There exists higher order AR models, but they generally did not improve the regression results of the current study and therefore will not be discussed. The AR(1) model is specified by the following equation:

$$y_t = x_t^{'}\beta + u_t$$
$$u_t = \rho u_{t-1} + \varepsilon_t$$

Equation 3 - First-Order Autoregressive Model

Relating this equation to the current study, y represents the volatility over the forecast period, x' represents the volatility over the estimation period, β is the coefficient of x', u is the disturbance term, ρ is the first-order serial correlation coefficient, and ε is the innovation of the disturbance.

The Durbin-Watson Statistic

The Durbin-Watson Statistic (or DW) is a test for first-order serial correlation. If no serial correlation is present, the DW value will be approximately equal to 2. If DW is less than 2, positive serial correlation exists. Conversely, negative serial correlation occurs when DW is between 2 and 4. With 50 or more observations and only a few independent variables, a DW statistic below about 1.5 is a strong indication of positive first order serial correlation. Though commonly used, there are limits to what the DW test can do. First, if the DW statistic falls outside a certain "data-dependent" boundary, the results are inconclusive. Second, if there are lagged dependent variables on the right hand side of the regression equation, the test is no longer valid. Third, one can only test the null hypothesis of no serial correlation against the alternative hypothesis of first-order serial correlation.

ARIMA

ARIMA is the acronym for AutoregRessive Integrated Moving Average. ARIMA models are generalizations of the AR model previously discussed. They are composed of three parts: 1. The autoregressive term, 2. The Integration Order term, and 3. The Moving Average term. Refer to Equation 4 below. Relating this to the current study, y represents the volatility over the forecast period, x' represents the volatility over the estimation period, β is the coefficient of x' , u is the disturbance term, ρ is the serial correlation coefficient, the ε's are the values of the forecast error, and the θ are the coefficients of the ε's. The autoregressive term is the same term as previously described in the AR model – the past period residual is incorporated into the current observation. Each integration order term corresponds to the number of times a series is differenced. A first-order integrated component means that the forecasting model is designed for the first difference of the original series. The Moving Average component uses lagged values of the forecast error to improve the current forecast. The theory behind Autoregressive Moving Average (ARMA) models is that the underlying process governing the times series is stationary. The generalized ARMA (p,q) model is represented by the following equation:

$$
\begin{aligned}
&y_t = x'_t\,\beta + u_t \\
&u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \ldots + \rho_p u_{t-p} \\
&+ \varepsilon_t + \vartheta_1 \varepsilon_{t-1} + \vartheta_2 \varepsilon_{t-2} + \ldots + \vartheta_q \varepsilon_{t-q}
\end{aligned}
$$

Equation 4 - ARMA(p,q) Model

Similar to the First-Order Autoregressive model described earlier on in the chapter, this equation is a generalization of the AR(1) model and was not used in the current study.

Stationarity of Time Series

Generally speaking, a series is said to be stationary if its mean and autocovariances do not depend on time. If this is not the case, then the series is automatically classified as nonstationary. Stationarity is a matter of concern because stationarity of regressors is assumed in the derivation of standard inference procedures for regression models. Nonstationary regressors invalidate many standard results and require special treatment.

The formal statistical test for stationarity is the Unit Root Test. A unit root is used to determine if a particular series is stationary or nonstationary.

Consider the following AR(1) process:

$$\Delta y_t = \mu + \rho y_{t-1} + \varepsilon_t$$

Equation 5 - AR(1) Process to Describe a Unit Root

Both μ and ρ are parameters and the errors ε_t 's are assumed to be independent and identically distributed with zero mean and equal variance. The AR(1) process is stationary if $-1<\rho<1$. If $\rho=1$ then the equation above defines a random walk (with drift) and y is then nonstationary. When $\rho=1$, it is said that a unit root exists.

The Unit-Root Test

The Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) statistics are used to test for the presence of a unit root (and consequently, the stationarity of the time series). Suppose Y_t can be described by the following equation:

$$Y_t = \alpha + \beta t + \rho Y_{t-1} + \varepsilon_t$$

Equation 6 - Unit Root Test Regression Equation

In order to perform the DF Test, one first runs the following unrestricted regression using OLS:

$$Y_t - Y_{t-1} = \alpha + \beta t + (\rho-1)Y_{t-1} + \varepsilon_t$$

Equation 7 - Unit Root Test Unrestricted Regression Equation

and then the following restricted regression (also using OLS):

$$Y_t - Y_{t-1} = \alpha + \varepsilon_t$$

Equation 8 - Unit Root Test Restricted Regression Equation

Next, the standard F-Test is used to test whether the restrictions $\beta = 0$ and $\rho = 1$ hold. The F-Statistic is calculated from the following equation:

$$F = (N-k)(ESS_R - ESS_{UR})/q(ESS_{UR})$$

Equation 9 - Unit Root Test F-Statistic Calculation

where ESS_R and ESS_{UR} is the error sum of squares from the restricted (R) and unrestricted (UR) regressions, N is the number of observations, k is the number of estimated parameters in the unrestricted regression, and q is the number of parameter restrictions. Note that the DF F-Ratio is not based on the standard F distribution under the null hypothesis. One must instead use the DF calculated distribution to test for the null hypothesis of a random walk[4].

One problem with the DF Test is that it assumes no serial correlation in the error term. A modified version called the *Augmented* Dickey-Fuller Test (ADF) takes serial correlation of the residuals into account. The test is carried out by including lagged changes in Y_t in both the unrestricted and restricted regression equations above. The number of lags is usually specified by intuition and trial and error.[5] Equation 6 above is expanded to:

$$Y_t = \alpha + \beta t + \rho Y_{t-1} + \sum_{j=1}^{p} \lambda_j \Delta y_{t-j} + \varepsilon_t$$

Equation 10 - ADF Test Equation

Where $\Delta Y_t = Y_t - Y_{t-1}$ and P represents the number of lags to include. The Unit Root Test is then run in a similar manner as before using OLS on an unrestricted regression:

$$Y_t - Y_{t-1} = \alpha + \beta t + (\rho - 1) Y_{t-1} + \sum_{j=1}^{p} \lambda_j \Delta Y_{t-j}$$

Equation 11 - ADF Unrestricted Regression Equation

and then the restricted regression:

$$Y_t - Y_{t-1} = \alpha + \sum_{j=1}^{p} \lambda_j \Delta Y_{t-j}$$

Equation 12 - ADF Restricted Regression Equation

Afterwards, a F ratio is calculated to test whether the restrictions ($\beta = 0$, $\rho = 1$) hold. The Dickey-Fuller distributions[6] must be used instead of the standard F-Distribution.

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[4] Refer to Dickey and Fuller, op. cit., Table IV, p. 1063, 1981.
[5] For details on the DF and ADF tests, refer to Pindyck, Robert S. and Rubinfeld, Daniel L. Econometric Models and Economic Forecasts, Fourth Edition; Irwin McGraw-Hill, 1998, pp. 507-510.
[6] Refer to Dickey and Fuller, op. cit., Table IV, p. 1063, 1981.

ARCH and GARCH Models

While studying stock market returns, evidence shows that the variance of the error term is not a function of an independent variable but instead varies over time in a way that depends on how large the errors were in the past. There is a particular kind of heteroscedasticity (non-constant variance) present in which the variance of the regression error term depends on the volatility of the errors in the recent past. Three models in particular will be discussed in this paper, namely ARCH, GARCH and EGARCH.

ARCH was developed by Robert Engle (1982). If one assumes the following regression equation:

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + \varepsilon_t$$

Equation 13 - ARCH Regression Equation

then a second equation is used to model the variance of the error term:

$$\sigma^2_t = \alpha_0 + \alpha_1 \varepsilon^2_{t-1}$$

Equation 14 - ARCH Variance Equation

The variance equation has 2 terms: a constant and "last period's news about volatility", which is modeled as last period's squared residual (the ARCH term). In this model, the ARCH term is heteroscedastic and conditional on ε_{t-1}. This leads to more efficient estimates of the original regression parameters β_1 , β_2 and β_3. When the variance equation is specified as such, the ARCH model is referred to as the ARCH(1) model, since only 1 past period's worth of information is taken into account. A more general ARCH(p) model exists in which more historical lags are entered as variables in the variance equation.

GARCH is a generalized version of ARCH and was first modeled by Tim Bollerslev (1986). Although higher order GARCH(p,q) models exist, the simplest is the GARCH(1,1) model whose variance is represented by:

$$\sigma^2_t = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \lambda_1 \sigma^2_{t-1}$$

Equation 15 - GARCH Variance Equation

where the first two terms are the same as in ARCH and the last term is last period's variance (the GARCH term). The last term introduces a new notion – the variance today depends on all past volatilities, but with geometrically declining weights.

EGARCH was proposed by Daniel Nelson (1991). Engle and Ng (1993) describe a phenomenon in which downward movements in the market are followed by higher volatilities than upward movements of the same magnitude. This "asymmetry" can be tested for using asymmetric models. One of these models is Nelson's EGARCH(1,1) model. When evidence of asymmetry is produced, it is said that there is a "leverage effect" in the conditional variance equation in EGARCH. Nelson's specification for the conditional variance is:

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \alpha\left(\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| - \sqrt{\frac{2}{\pi}}\right) + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$$

Equation 16 - EGARCH Conditional Variance Equation (Nelson Specification)

This differs slightly from the conditional variance specification defined in the software used in this study:

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \alpha\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$$

Equation 17 - EGARCH Conditional Variance Equation

Nelson assumes that the errors follow a generalized distribution while the software assumes normally distributed errors. Estimating under the assumption of normal errors will yield identical estimates except for the software's intercept term ω will which differ by:

$$\alpha\sqrt{\frac{2}{\pi}}$$

The following three models will be used as the models of choice in the current study:

1. ARCH (1)
2. GARCH (1,1)
3. EGARCH (1,1)

These three models were discussed in Guan and Ederington.

Forecasting with Autoregressive Terms

The first-order autoregressive model was described in the preceding paragraphs (refer to the discussion of Serial Correlation above) and was given as:

$$y_t = x_t'\beta + u_t$$
$$u_t = \rho u_{t-1} + \varepsilon_t$$

Where the fitted residuals are given as: $\varepsilon_t = y_t - x_t'b$.

Forecasts with an autoregressive term include a forecast of the residuals from the equation to the forecast of the structural model that is based on the right-hand side variables. Dynamic forecasting uses the lagged *forecasted* residuals. Static forecasting uses the *actual* lagged residuals. The only exception lies in the very first forecast – the results will be identical. Suppose this model was estimated with data up to t = S-1. Then, provided that the x_t values are available, the static forecasts for t=S, S+1, and S+2 are given by:

$$\hat{y}_S = x_S'b + \hat{\rho}_1\varepsilon_{S-1}$$
$$\hat{y}_{S+1} = x_{S+1}'b + \hat{\rho}_1\varepsilon_S$$
$$\hat{y}_{S+2} = x_{S+2}'b + \hat{\rho}_1\varepsilon_{S+1}$$

Equation 18 - Static Forecasting

In terms of dynamic forecasts, a similar set of equations is presented:

$$\hat{y}_S = x_S'b + \hat{\rho}_1\varepsilon_{S-1}$$
$$\hat{y}_{S+1} = x_{S+1}'b + \hat{\rho}_1\hat{\upsilon}_S$$
$$\hat{y}_{S+2} = x_{S+2}'b + \hat{\rho}_1\hat{\upsilon}_{S+1}$$

Equation 19 - Dynamic Forecasting

Where $\hat{\upsilon}_t = \hat{y}_t - x_t'b$. For subsequent observations, the dynamic forecast will always use the residuals based upon their previous forecast, while the static forecasts will use the actual values.

Calculations

The following calculations are defined:

1. Daily Log Returns:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

Equation 20 - Daily Log Returns

This is slightly modified to include the effects of dividends:

$$R_t = \ln\left(\frac{P_t + d_t}{P_{t-1}}\right)$$

Equation 21 - Dividend-Adjusted Daily Log Returns

2. Daily Return Deviations:

$$r_t = R_t - \mu$$

Equation 22 - Daily Return Deviations

where μ is measured as the mean of R_t over the entire sample period.

3. Actual Annualized Standard Deviation of Returns:

$$AS(s)_t = \sqrt{252\sum_{j=1}^{s}\frac{r_{t+j}^2}{s}}$$

Equation 23 - Actual Annualized Standard Deviation of Returns

where "s" is the horizon length.

4. Root Mean Squared Forecast Error (RMSFE):

$$RMSFE = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left(AS(s)_m - FSTD(s)_m\right)^2}$$

Equation 24 - Root Mean Squared Forecast Error

where FSTD is the annualized forecast standard deviation for an "s" day horizon beginning on day "m" using one of the various forecasting methods discussed above. "M" represents the number of forecast periods defined in Table 1 above.

Note that detailed expansions of the above equations are available in the Appendix. These equations are the same as in Guan and Ederington.

Analysis

In the previous section, detailed time-series analysis of the stock return volatilities were made. Refer to the table below for the description of the terminology used throughout the study.

Table 4 - Terminology Description

Term	Description
AS(x)	<u>Actual</u> standard deviation (starting at time t+1 measured over a period of "x"). For example, AS(5) describes the series as being "the actual standard deviation measured over 5 days".
STD(x)	<u>Historical</u> standard deviation (starting at time t measured over a period of "x"). For example, STD(10) describes the series as being "the historical standard deviation measured over the past 10 days".
ARCH1AR1	Standard ARCH(1) model with term correcting for 1st order autocorrelation
GARCH11AR1	Standard GARCH(1,1) model with term correcting for 1st order autocorrelation
EGARCH11AR1	Standard EGARCH(1,1) model with term correcting for 1st order autocorrelation

There were three main steps that were followed in the analysis: 1. Investigation of the time series properties of each independent variable, 2. Fitting the appropriate regression model, and 3. Measurement of the in-sample forecasting ability of each regression model chosen in 2.

<u>Step 1 – Investigation of the Time Series Properties of Each Dependent Variable</u>

Example: Dow Jones Industrials Index

Stock Exchange: NYSE
Frequency: daily data
Dividends: no
Time period: 28-Dec-1981 to 27-Dec-2000
Series : STD(5)

The STD(5) independent variable is chosen in this example. This variable measures the historical standard deviation of the Dow Jones Industrials Average Index over a period of 5 days. The first test performed was the Unit Root Test, in order to test for stationarity. Refer to the table below for the output of the Unit Root Test.

Table 5 - Unit Root Test Output

ADF Test Statistic	-16.20251	1% Critical Value*	-3.4349
		5% Critical Value	-2.8627
		10% Critical Value	-2.5674

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(STD5)
Method: Least Squares
Date: 02/01/01 Time: 15:47
Sample(adjusted): 2 4794
Included observations: 4793 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
STD5(-1)	-0.103998	0.006419	-16.20251	0.0000
C	0.000905	6.80E-05	13.32086	0.0000

R-squared	0.051948	Mean dependent var	9.89E-07
Adjusted R-squared	0.051750	S.D. dependent var	0.002757
S.E. of regression	0.002685	Akaike info criterion	-9.001986
Sum squared resid	0.034535	Schwarz criterion	-8.999284
Log likelihood	21575.26	F-statistic	262.5213
Durbin-Watson stat	1.842481	Prob(F-statistic)	0.000000

To perform a Unit Root Test, the Dickey-Fuller and Augmented Dickey-Fuller tests are used on the independent variable (i.e. historical standard deviation). Recall that H_o: $\rho=1$ (unit root present therefore series is nonstationary), H_a: not H_0. It is simpler to test for a unit root by respecifying the null hypothesis. For example, consider a simple AR(1) equation:

$$Y_t = \alpha + \rho Y_{t-1} + \varepsilon_t$$

Equation 25 - DF and ADF AR(1) Equation

This equation is modified to:

$$\Delta Y_t = \alpha + \gamma Y_{t-1} + \varepsilon_t$$

Equation 26 - Respecified DF and ADF AR(1) Equation

Where $\gamma = \rho - 1$. The null hypothesis now becomes H_0: $\gamma = 0$. The results are reported in the table above. The ADF Test Statistic (-16.20) is less than the critical values even at the 1% level of significance (-3.43). Ho is rejected at the 1% level. This signifies that the series does not have a unit root problem, the series is stationary, and the series does not have to undergo any additional adjustments. All other coefficients are statistically significant, and the DF and ADF regressions can be relied upon because the Durbin-Watson value of 1.84 is reasonably close to the value of 2.0 (refer to the previous chapter's discussion of the DW statistic for more information). The results of the analysis of each independent variable were used in the next step: fitting a proper regression model.

Step 2 – Fit Proper Regression Model

As an example, volatility will be forecasted over a 5-day period using the actual 5-day historical average. The independent variable will be called STD(5) (historical volatility) and the dependent variable will be called AS(5) (actual future volatility). First, simple OLS was used:

$$AS(5) = c + \beta\ STD(5)$$

Equation 27 - Simple OLS Equation

In all cases, OLS produced evidence of serial correlation (using the DW statistic), and evidence of non-constant variance. The OLS regression model was then modified to include a new independent variable, a 1st order autoregressive term:

$$AS(5) = c + \beta\ STD(5) + AR(1)$$

Equation 28 - Simple OLS Equation With AR(1) Term

Immediately, the statistics improved (especially the DW statistic). The table below outlines the typical DW statistic behaviors before and after adding the AR(1) term:

Table 6 - OLS DW Statistic Behavior

Security	Series	Original OLS Equation DW Value	OLS Equation With AR(1) Term DW Value
Dow Jones Industrials Average Index (1989-2000)	A5S5	0.428	1.906
Air Canada	A5S5	0.383	1.848
Nortel Networks	A5S5	0.456	1.871
TSE 300 Index	A5S5	0.395	1.812

Note: additional results are available upon request.

However, based on White's Heteroscedasticity Test[7], evidence of non-constant variance in the residuals was produced. This led to the abandonment of the OLS model altogether.

Next, the effectiveness of the autoregressive models mentioned in Guan and Ederington was examined. This study focused on 3 in particular: ARCH (1), GARCH (1,1), and EGARCH (1,1). Based on various statistical tests, the validity of each model was determined. To summarize, the following were used to determine the appropriateness of the model in question:

1. Significant P-Values of each term
2. R^2 value
3. DW Statistic
4. Normality of the residuals & Jarque-Bera Test Statistic

Regarding point 4 above, the notion of residual normality is important. Residual normality is used to test how well one's model "fits" the data. One statistical test is the Jarque-Bera Test (JB) that measures the difference of the skewness and kurtosis of the series with those from the normal distribution. It is calculated as follows:

$$JB = \frac{N-k}{6}\left(S^2 + \frac{1}{4}\left((K-3)^2\right)\right)$$

Equation 29 - Jarque-Bera Statistic

Where S is the skewness, K is the kurtosis, and k represents the number of estimated coefficients used to create the series. The null hypothesis is H_0: series is normally distributed, H_a: not H_0. The JB statistic is distributed as a χ^2 distribution with 2 degrees of freedom. If the JB P-Value < α, reject H_0.

Given the model constraints, the factors listed above and through trial and error experimentation, the models having comparatively the best results among all attempted trials were noted. Generally speaking, these results had the lowest JB statistic. The forecasting ability of these choice models was then measured.

[7] White, Halbert (1980) "A Heteroskedasticity-Consistent Covariance Matrix and a Direct Test for Heteroskedasticity," Econometrica, 48, 817–838.

Step 3 – Calculate the Forecasting Ability of the Regression Model

The forecasting abilities of the regressions were measured using the Root Mean Squared Forecast Error (RMSFE). This measurement was the primary forecast measurement in Guan and Ederington. The RMSFE's scale depends on the scale of the dependent variable. It can be used as a relative measure to compare forecasts for the same series across different models - the smaller the error, the better the forecasting ability of that model according to that criterion.

In-Sample vs. Out-Of-Sample

The effectiveness of the regression models based on out-of-sample data was also tested. Steps 1-3 listed above were performed twice -- once for the in-sample data and once for the out-of-sample data. Out-of-sample forecasts were consistently made on 10% of each series' data. In other words, 90% of the observations were used to fit the regression models and the rest were used to forecast[8]. The actual percentage used for the out-of-sample forecasts (i.e. the 10%) has some relevance. All the data series consisted of 5 years of daily data. This translates to approximately 1260 daily data points (252 trading days X 5 years). 10% of 1260 is 126 days, which represents approximately 6 months of data (126 / 20 trading days per month). 6 months was deemed a reasonable starting point for out-of-sample forecasts, because it represents two business quarters.

[8] For more information on forecasting, refer to Pindyck, Robert S., and Rubinfeld, Daniel L., 1998, Econometric Models and Economic Forecasts, Fourth Edition, Chapter 13, pp. 379-412

Results

This section reports the key findings from the analysis. Three sections are provided which correspond to the three steps listed in the analysis section: 1. Observations on the historical time series, 2. Observations during the regressions, 3. Observations on forecasting.

Observations on the Historical Time Series

- The longer the historical volatility period, the greater the chance the series was non-stationary (and consequently, had a unit root problem).
- Most historical volatility series failed to produce evidence of a unit root problem, even at the 1% level of significance.

Observations During the Regressions

- Inclusion of an AR(1) term consistently improved the regression model
- Exclusion of an intercept term consistently deteriorated the model
- Reducing the estimation period led to better regression results (only tested on TSE and DJ Indexes)
- The sign of the independent variable in <u>all</u> regressions was negative
- Keeping the forecast period constant, lengthening the estimation period deteriorated the model
- Analyzing OLS residuals produced evidence of heteroscedasticity
- In all cases, the residuals failed the normality test
- It was more difficult producing statistically significant regression results fitting models on the individual stocks, than fitting models on the indices

Forecast Observations – In-Sample

- Generally speaking, forecasting volatility using a short estimation time period yielded a lower dynamic RMSFE. This was not the case for static RMSFEs – there was no apparent trend.
- Generally speaking, forecasting volatility over a short time period in the future yielded a lower dynamic RMSFE. This was not the case for static RMSFEs – there was no apparent trend.
- Overall, considering only Arch(1), Garch(1,1) and Egarch(1,1) and based on lowest dynamic RMSFE:
    - Egarch(1,1) Model produced the lowest RMSFE 11/18 times - 61%
    - Arch(1) Model produced the lowest RMSFE 3/18 times - 17%
    - Garch(1,1) Model produced the lowest RMSFE 4/18 times - 22%
- Overall, considering only Arch(1), Garch(1,1) and Egarch(1,1) and based on lowest static RMSFE:
    - Egarch(1,1) Model produced the lowest RMSFE 7/18 times - 39%
    - Arch(1) Model produced the lowest RMSFE 6/18 times - 33%
    - Garch(1,1) Model produced the lowest RMSFE 5/18 times - 28%
- Lowest dynamic RMSFE by security type (i.e. Stock or Index):
    - Index (Dow Jones and TSE 300)
        - Egarch(1,1) produced the lowest RMSFE 8/12 times – 67%
        - Arch(1) produced the lowest RMSFE 2/12 times – 17%
        - Garch(1,1) produced the lowest RMSFE 2/12 times – 17%
    - Stock (Nortel and Air Canada)
        - Egarch(1,1) produced the lowest RMSFE 3/6 times – 50%
        - Garch(1,1) produced the lowest RMSFE 2/6 times –33%
        - Arch(1) produced the lowest RMSFE 1/6 times – 17%
- Lowest static RMSFE by security type (i.e. Stock or Index):
    - Index (Dow Jones and TSE 300)
        - Egarch(1,1) produced the lowest RMSFE 6/12 times – 50%

        - Arch(1) produced the lowest RMSFE 4/12 times –33%
        - Garch(1,1) produced the lowest RMSFE 2/12 times – 17%
    - Stock (Nortel and Air Canada)
        - Garch(1,1) produced the lowest RMSFE 3/6 times – 50%
        - Arch(1) produced the lowest RMSFE 2/6 times – 33%
        - Egarch(1,1) produced the lowest RMSFE 1/6 times – 17%
- Lowest dynamic and static RMSFE was obtained while analyzing the Dow Jones Industrials Average Index

Forecast Observations – Out-Of-Sample

- The only security who passed the cutoff criteria was the Dow Jones Industrials Average Index
- Overall, considering only Arch(1), Garch(1,1) and Egarch(1,1) and based on lowest <u>dynamic</u> RMSFE:
    - Egarch(1,1) Model produced the lowest RMSFE 6/7 times - 86%
    - Arch(1) Model produced the lowest RMSFE 1/7 times - 14%
    - Garch(1,1) Model produced the lowest RMSFE 0/7 times - 0%
- Overall, considering only Arch(1), Garch(1,1) and Egarch(1,1) and based on lowest <u>static</u> RMSFE:
    - Egarch(1,1) Model produced the lowest RMSFE 2/7 times - 29%
    - Arch(1) Model produced the lowest RMSFE 1/7 times - 14%
    - Garch(1,1) Model produced the lowest RMSFE 5/7 times - 71%

Discussion of Results

In order for a particular regression to be included in the results section, two conditions had to have been met. First, statistically significant (at the 0.10 level) regression coefficients had to exist for all ARCH1AR1, GARCH11AR1, and EGARCH11AR1 models for each series. This allowed for direct comparison from different angles. For instance, grouping by security type (stock & index) and then by regression model. Second, the Durbin-Watson Statistic had to be greater than 1.5. Strong serial correlation is present if the DW statistic falls below 1.5. Ensuring the DW statistic was above 1.5 in all cases provided more reliable regressions. Given four securities, nine time series for each security and three statistical models, the total possible number of regression results is 4 x 9 x 3 = 108.

Explanation of the Results Tables

Table 11, Table 12, Table 13, and Table 14 (available in the appendix) summarize the results of this study. Table 11 and Table 12 refer to the in-sample results, while Table 13 and Table 14 refer to the out-of-sample results. For example, Table 11 has 6 columns. The first column is labeled "Security" and simply lists the security. The second column "Series" outlines which series' results passed the cutoff criteria (statistically significant regression coefficients for all autoregressive models and a DW statistic above 1.5 – refer to previous paragraph). For example, only Air Canada's A10S10 and A10S20 series[9] met the cutoff criteria. Out of the 108 possible series, only 18 met the 2 conditions presented above. The third, forth and fifth columns report the sign of the independent variable in the ARCH1AR1, GARCH11AR1, and EGARCH11AR1 regression equations. Finally, the sixth column reports the sign of the EGARCH11AR1's "leverage effect" term (refer to Equation 16 and Equation 17 in the Research Methodology Section). A statistically significant negative sign indicates the presence of "leverage effects". This table description also holds for Table 13. As for Table 12 and Table 14, column 1 lists the security. Column 2 list the series

[9] Recall that A10S10 refers to: actual volatility over a period of 10 days (the A10 part) and a historical volatility period of 10 days (the S10 part). Refer to the Analysis Section for clarification.

that passed the cutoff criteria. Columns 3-5 list the dynamic RMSFEs, and columns 6-8 list the static RMSFEs.

Observations on the Historical Time Series

One observation was that the chance of a unit root (i.e. series was nonstationary) problem increased as the length of the historical time period increased. As the time period increased, the more likely the series followed a random walk. It is more likely that as the time period increases, the greater the chance that an unforeseen market shock will occur. Unforeseen shocks are random occurrences that could model volatility as a random walk. The chance of an unforeseen shock occurring over a period of 5-10 trading days is rarer than a shock occurring over a period of 30 trading days.

Observations During the Regressions

Correction for 1st Order Autocorrelation

Early on in the analysis, evidence of serial correlation was observed. Most Durbin-Watson statistics fell below 1.0. This led to the inclusion of an autoregressive AR(1) term – "lag" term correcting for 1st order serial correlation. This improved all regression models. The R^2, Durbin-Watson Statistic and P-values also improved. Adding an AR(2) term, deteriorated the results and, in many instances, led to insignificant regression coefficients. The fact that the AR(1) term consistently improved the model in all cases only adds to the theory that volatility is past-dependent on itself.

The Regression Intercept Term

In general, removing the intercept term from the regression equation deteriorated the model. In some instances, it even led to the underlying regression process being nonstationary. In numerous forecasts, it was not uncommon for the RMSFEs to jump 100% or more if the intercept term was excluded from the regression model. In the context of this study, the intercept term can be interpreted in one of 2 ways. Assume a simple regression equation:

$$AS(x) = c + \beta\ STD(x)$$

If the historical volatility was equal to 0, the forecast of the future volatility would be always equal to the value of the intercept term. Can volatility truly be equal to zero? If so, then an extraordinary event in the stock market would have to have occurred. One case would be a security resuming trading after a halt. In the case of an index, this would be a lot more serious. So serious in fact that perhaps the models presented here might not produce meaningful results. Therefore, the intercept term was dropped. The regression equation was reduced to:

$$AS(x) = \beta\ STD(x)$$

This implied that if the historical volatility was zero, then the future volatility was zero. This assumption was not unreasonable. If anything, it would have not allowed itself to be used in such extenuating circumstances. As was shown in the results, the presence of the intercept term only added positively to the model (perhaps for the reason given above).

Choice in Regression Models – OLS vs. ARCH vs. GARCH vs. EGARCH

Regressions early in the analysis were performed using OLS. OLS allows for the quick understanding of the characteristics of the data. OLS quickly produced evidence of serial correlation and heteroscedasticity. Results from OLS were, consistent regardless of the security chosen or time period. The basic regression equation was then modified to:

$$AS(x) = c + \beta\ STD(x) + AR(1)$$

ARCH(1), GARCH(1,1) and EGARCH(1,1) methodologies where then applied. For the first groups of series, regression without an intercept term was conducted, leading to the following equation:

$$AS(x) = \beta\ STD(x) + AR(1)$$

This was a trial and error analysis that eventually would signal the importance of the intercept term.

Regressions were conducted using ARCH(1) because the conditional variance of ARCH includes 2 terms: the mean and news about volatility from the previous period. It seemed reasonable to assume that the "previous period's news" component of ARCH would suffice in providing efficient regression estimates. It has already been demonstrated how the inclusion of the AR(1) term improves the model so, naturally, this "news" component should contribute positively.

Investigation using GARCH(1,1) followed. GARCH model is a generalized version of ARCH, in which a new term is introduced into the regression: last period's forecast variance (the GARCH term). The GARCH term consists of a lagged forecast variance present in the conditional variance equation. In the original Guan and Ederington study, the authors mention that GARCH(1,1) was widely used in forecasting. Perhaps one reason is that in GARCH, the observation weights decline exponentially. Recent observations contain more information about likely future volatility (compared to older observations) - GARCH's exponentially declining weighting scheme seems appropriate.

Using EGARCH, the presence of leverage effects can be tested by investigating the sign of the leverage term in the conditional variance equation. If the term is negative, then leverage effects are present. Using EGARCH, it would be possible to determine if the asymmetry phenomenon carried over to the securities used throughout this study.

Regression Observations - ARCH vs. GARCH vs. EGARCH

Residual Normality

Based on the Jarque-Bera statistic (Equation 29), none of the error terms passed the normality test. Consequently, a summary table was not provided but is available upon request. In terms of residual normality, no one model produced perfectly normally distributed residuals.

Leverage Effects

The concept of "leverage effects" was discussed earlier in the paper (see Research Methodology section). Equation 17 models EGARCH's conditional variance. The last term in Equation 17 is used to test for the presence of leverage effects:

$$\gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$$

Equation 30 - EGARCH(1,1) Leverage Component

If $\gamma < 0$, then there is a "leverage effect" present.

A summary of the in-sample leverage effects measured by the EGARCH model follows:

Table 7 - Presence of In-Sample EGARCH Leverage Effects

Series	$\gamma<0$ (i.e. leverage effects present)	Security Type
TSE 300	¼	Index
Dow Jones Industrial Average	2/8	Index
Nortel	¼	New Economy Stock
Air Canada	2/2	Old Economy Stock

Table 11 in the appendix shows the details (see column named "EGARCH Leverage Term Sign"). In both cases, the old economy stock Air Canada seems to have leverage effects present. Table 13 summarizes the leverage effects for the out-of-sample data.

Modification of the Estimation Period

The Guan and Ederington study used a long estimation period consisting of a maximum of 8,523 daily data points. This study began analyzing the Dow Jones Industrials Average Index from December 28, 1981 until December 27, 2000. Wanting to determine if the length of sample period impacted this study, it was then shortened to 10 years' worth (from 27-Dec-89 until 31-Dec-00). The statistical results greatly improved, especially the distribution of the error terms. A typical result was that the skewness and kurtosis of the residuals were cut by one half (ex. from Skewness=3.3, Kurtosis=40 to Skewness=1.8, Kurtosis=18). This signified that the model produced more efficient estimates. All time series analyzed and reported in this study were based on a 5-year period. The in-sample calculations used all data in the entire 5-year period. The out-of-sample calculations reserved 10% of the data for the forecasts.

Sign of Coefficient of Independent Variable

In all cases, the sign of the independent variable in the regression equation was statistically significant and negative. This is indicative of mean-reversion - an inverse relationship between the volatilities of subsequent time periods. For example, if the volatility was positive during one period, then it was negative the next. Since the original data was based on squared deviations (i.e. no negative values were present), this finding is particularly interesting. Once again, Table 11 and Table 13 in the appendix provide summaries of the details (refer to the columns listed under "Independent Variable Sign").

Fitting Regression Models Based on Security Type

Regarding the in-sample investigation, fitting regression models for the stocks produced more cases of statistically insignificant coefficients compared to fitting regression models for the indexes. This occurred more often as the historical sample period increased. One possible explanation lies in the composition of the index. For the most part, indexes are comprised of multiple stocks. Therefore, the behavior of all the other stocks in the index would buffer a shock affecting one particular stock.

In the case of regressions with out-of-sample data, it was difficult to fit statistically significant regression models for all securities except the Dow Jones Industrials Average Index. In fact, none of the Canadian securities passed the cutoff criteria. Further investigation is required to see if this observation carries over to other securities.

Forecasting Observations

The analysis performed in this section focused on obtaining the lowest forecast error using the RMSFE. Forecasting was based on both in-sample and out-of-sample data. The out-of-sample forecasts utilized 10% of the observations. In addition to in-sample and out-of-sample forecasting distinction, RMSFEs were recorded based on static and dynamic forecasts.

Modification of the Estimation Period

Originally, a large time period was chosen for the series. In the case of the Dow Jones Industrials Average, the original sample period was from Dec-1981 to Dec-2000. It was then reduced by an arbitrary amount (to 10 years). The result was a lowering of the dynamic RMSFE in all series (the procedure was not performed for static forecasts). In the regression observations, it was reported that shortening the sample period produced better-distributed residuals. Perhaps, there is a relation between normality and forecasting. Maybe more recent data is better at forecasting. Another possibility is that the first sample period included the market crash of 1987. This event could have seriously impacted certain time series models. For instance, in GARCH, the variance today depends on all past volatilities, but with geometrically declining weights. Therefore, the effect of the 1987 crash would not have been forgotten by the model. On the other hand, Guan and Ederington use over 40 years worth of historical data. Perhaps since they have an additional 20 years of previous return data, the effect of the 1987 crash was buffeted.

Modification of the Forecast Period (In-Sample Data)

In general, shortening the forecast period yields lower dynamic forecast errors. These observations are summarized in Table 12 (see Appendix). As the time period increases, the greater the chance that an unforeseen market shock will occur. The chance of an unforeseen shock occurring over a period of 5-10 trading days is rarer than a shock occurring over a period of 20 trading days. On the other side, using data that has too high a frequency (e.g. intra-day data, or even the average over 1 or 2 days) can present other problems. A balance must exist somewhere in between. Static forecasting produces lower RMSFEs (see Table 12). Contrary to dynamic forecasting, shortening the forecast period yields higher RMSFEs. This is apparent in the Dow Jones and TSE Indexes. This observation is also consistent in the ARCH, GARCH and EGARCH models. This may imply that static forecasting produces lower errors than dynamic forecasting. It seems that actual historical volatility (rather than forecasts of past volatility) is a more meaningful input for forecasting.

Modification of the Estimation Period (In-Sample Data)

Keeping the forecast period constant, lengthening the estimation period deteriorates the model's dynamic forecasting ability. The models forecast better if they forecast volatility over 5 days using 5 days of historical data, compared to forecasting volatility over 5 days using 30 days of historical data. This observation is not true for static forecasts. Modifying the estimation period seems to have no effect on the RMSFEs. Refer to Table 12 for the data.

Modification of the Forecast Period (Out-Of-Sample Data)

Comparing the RMSFEs of the in-sample and out-of-sample forecasts is straightforward except that this can only be accomplished for the Dow Jones Industrials Average Index, since this is the only security that passed the cutoff criteria. These results are available in Table 14. Based on dynamic forecasts, there does not seem to be a pattern. Static forecasts produce RMSFEs that are lower. Another observation is that the RMSFEs of the series containing a forecast period of 10 days are lower than the forecasts of the 5-day series.

Modification of the Estimation Period (Out-Of-Sample Data)

Lengthening the estimation period produces higher dynamic RMSFEs. These results are available in Table 14. Static forecasts' RMSFEs increase as the estimation period is lengthened, except if the series contains an estimation period of 20 days (refer to the A5S20 and A10S20 series in Table 14).

Forecasting Observations - ARCH vs. GARCH vs. EGARCH (In-Sample)

Overall and based on dynamic RMSFE, the EGARCH11AR1 model produced lower errors than both the ARCH1AR1 and GARCH11AR1 models. The table below summarizes these findings (refer to Table 12 in the appendix for the details).

Table 8 – Overall RMSFE Summary (In-Sample)

Lowest RMSFE Rank	ARCH1AR1	GARCH11AR1	EGARCH11AR1
Dynamic	3/18	4/18	11/18
Static	6/18	5/18	7/18

The EGARCH11AR1 model had the lowest dynamic RMSFE in 11 of 18 forecasts. This large discrepancy can be used as a starting point for further analysis. This raises additional questions. It was previously reported that one of the main reasons to use the EGARCH model was to test for asymmetry. Since few asymmetric or leverage effects were present (refer to Table 11 in the Appendix), why would EGARCH yield lower RMSFEs than GARCH or ARCH? One property of the EGARCH model is that it tends to place more weight on the most extreme observations than GARCH does. In terms of stock markets, periods of extreme volatility should be treated differently. For example, during periods of high volatility, volatility can spill over from one time period into the next. Interpreting Table 8 based on static forecasts, there seems to be a relatively even distribution across the three models (although EGARCH11AR1 still produced the lowest RMSFEs in the majority of cases).

The table below summarizes the RMSFEs by security type. In the case of Indexes, EGARCH11AR1 produced the lowest RMSFE 8 out of 12 cases. In the case of the stocks, EGARCH11AR1 produced the lowest RMSFE in 3 out of 6 cases.

Table 9 – RMSFEs Summary by Security Type (In-Sample)

Indexes	Arch1 AR1	Garch11 AR1	Egarch11 AR1		Stocks	Arch1 AR1	Garch11 AR1	Egarch11 AR1
Dynamic	2/12	2/12	8/12		Dynamic	1/6	2/6	3/6
Static	4/12	2/12	6/12		Static	2/6	3/6	1/6

When individual stocks are compared to indexes, the shock that one stock has on an index is buffered by all the other stocks in the index. This isn't the case when analyzing an individual stock. Perhaps this buffering effect explains why EGARCH yields lower errors in the case of the 2 stocks presented in this study. Perhaps in the case of individual stocks, placing more weight on extreme observations does not help forecasts. One observation is that EGARCH11AR1 produced the lowest static RMSFE in only 1 out of 6 cases for stocks.

Forecasting Observations - ARCH vs. GARCH vs. EGARCH (Out-Of-Sample)

Overall and based on dynamic RMSFE, the EGARCH11AR1 model produced lower errors in 6 out of 7 cases. The table below summarizes these findings (refer to Table 14 in the appendix for the details). In terms of static forecasting, the GARCH11AR1 model produced the lowest RMSFEs in 5 of 7 cases.

Table 10-Overall RMSFE Summary (Out-Of-Sample)

Lowest RMSFE Rank	ARCH1AR1	GARCH11AR1	EGARCH11AR1
Dynamic	1/7	0/7	6/7
Static	1/7	5/7	2/7

Note: results do not sum to unity due to ties in ranks

RMSFE Discussion Dynamic vs. Static

Throughout this study, static RMSFEs were lower than dynamic RMSFEs. This was consistent in the in-sample and out-of-sample forecasts. This may signify that estimation of volatility using actual past data is more relevant than using past forecasts of data (in the current forecast).

Why Forecasting Volatility May or May Not Be Practical

Given the RMSFEs observed in this thesis, can volatility be forecasted to the point where consistent profits can be made? If one believes in the efficient market hypothesis, then the answer may be no. After all, if volatility can be forecasted, arbitragers would be able to earn consistent riskless profits by trading securities that are highly sensitive to volatility, such as at-the-money call options. However, if one wanted to systematically control their portfolio risk exposure, then attempting to forecast volatility could be a useful tool. The smallest in-sample RMSFE was produced by estimating the Dow Jones Industrials Average Index using an estimation period of 30 days and a forecast period of 20 days (refer to Table 12 in the appendix). The lowest out-of-sample RMSFE was also obtained while analyzing the Dow Jones Industrials Average Index except that it occurred on a different time series (see Table 14 in the appendix). Recall that the scale of the RMSFE depends on the scale of the dependent variable. It should be used as relative measure to compare forecasts for the same series across different models; the smaller the error, the better the forecasting ability of that model according to that criterion. Since the dependent variable is a measure of volatility, a RMSFE of 0.0094 (in-sample, static RMSFE of Dow Jones Index – see Table 12) is a relatively good (small) error. There is no definite answer to the above question. However, this research opens the door for further analysis.

Suggestions for Future Research and Conclusion

Though only four securities were examined in the current study, certain notable highlights were discovered which could be used as a starting point for future research. Some suggestions include testing the use of EGARCH on stocks vs. indexes, testing for volatility asymmetry effects using autoregressive models such as EGARCH, continuing the methodology on other Canadian securities, testing for arbitrage opportunities using volatility forecasts, and forecasting in-sample and out-of-sample using varying sample periods. As previously observed, in the case of regressions with out-of-sample data, it was difficult to fit statistically significant regression models for all securities except the Dow Jones Industrials Average Index. None of the Canadian securities passed the cutoff criteria. Further investigation is required to see if this observation carries over to other securities. One final suggestion would be to build upon the existing forecasting techniques which combine historical and implied volatilities.

This study is based on the working paper by Guan and Ederington (1998). In particular, attempts are made to focus on measuring and estimating equity and index volatility. The objective of this thesis is to apply the methodology of Guan and Ederington to Canadian data. Historical volatility is estimated then forecast to examine the comparative performance of various time series models in forecasting volatility. Statistically significant evidence was provided that showed that forecasts of volatility produced low RMSFEs. These errors were calculated using in-sample and out-of-sample data.

REFERENCES

Bollserslev, Tim, 1986, Generalized Autoregressive Conditional Heteroscedasticity, *Journal of Econometrics*, Vol. 31, pp. 307-327

Bollerslev, Tim and Mikkelsen, Hans O., Long-Term Equity Anticipation Securities and Stock Market Volatility Dynamics, *Journal of Econometrics*, Vol. 92, No. 1, pp. 75-99

Bookstaber, Richard M. and Pomerantz, Steven, 1989, An Information-Based Model of Market Volatility, *Financial Analyst's Journal*, Vol 45, No. 6, pp. 37-47

Bookstaber, Richard M., and McDonald, James B., 1987, A General Distribution for Describing Security Price Returns, *Journal of Business*, Vol. 60, No. 3, pp. 401-424

Clemen, Robert T., 1989, Combined Forecasts: A Review and Annotated Bibliography, *International Journal of Forecasting*, Vol. 5, No. 4, pp. 559-583

Conrad, Jennifer, and Gultekin, Mustafa N., 1991, Asymmetric Predictability of Conditional Variances, *The Review of Financial Studies*, Vol. 4, No. 4

Cox, J. C., Ingersoll, J. E., and Ross, S. A., 1985, A Theory of the Term Structure of Interest Rates, *Econometrica*

Dominguez, Emilio, and Novales, Alfonso, 1999, Can Forward Rates Be Used To Improve Interest Rate Forecasts?, *Working Paper*-accepted for publication in the Journal of Applied Financial Economics

Dominguez, Emilio, and Novales, Alfonso, 2000, Dynamic Correlations and Forecasting of Term Structure Slopes in Eurocurrency Markets, *International Journal of Finance*, Vol. 12, No. 3, pp. 1807-1822

Ederington, Louis H., 1979, The Hedging Performance of the New Futures Market, *Journal of Finance*, 34, pp. 157-170

Ederington, Louis H. and Guan, Wei, 1998, Forecasting Volatility, *Working Paper*, http://papers.ssrn.com/paper.taf?ABSTRACT_ID=165528

Engle, Robert, 1982, Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation, *Econometrica*, Vol. 50, pp. 987-1008

Engle, Robert, 1993, Statistical Models for Financial Volatility, *Financial Analysts Journal*, January-February, Technical Note

Engle, Robert and Mezrich, Joseph, 1995, Grappling With GARCH, *Risk*, Vol. 8 #9

Engle, Robert F. and Victor K. Ng, 1993, "Measuring and Testing the Impact of News on Volatility," *Journal of Finance*, Vol. 48, pp. 1022–1082

Fama, Eugene, 1965, The Behavior of Stock Market Prices, *Journal of Business*, 38, pp. 34-105

Fama, Eugene F., French, Kenneth R., 1988b, Dividend Yields and Expected Stock Returns, *Journal of Financial Economics*, 22, pp. 3-25

Fama, Eugene F., French, Kenneth R., 1989, Business Conditions and Expected Returns on Stocks and Bonds, *Journal of Financial Economics*, Forthcoming

Fleming, Jeff, Ostdiek, Barbara, and Whaley, Robert E., 1995, Predicting Stock Market Volatility: A New Measure, *The Journal of Futures Markets*, Vol. 15, No. 3, pp. 265-303

French, Kenneth R., Stock Returns and the Weekend Effect, *Journal of Financial Economics*, 8, pp. 55-69

French, Kenneth R., and Roll, Richard, 1986, Stock Return Variances – The Arrival of Information and the Reaction of Traders, *Journal of Financial Economics*, 17, pp. 5-26

French, Kenneth R., Schwert, William G. , Stambaugh, Robert F., 1987, Expected Stock Returns and Volatility, *Journal of Financial Economics*, Vol. 19, Iss. 1

Grammatikos, Theoharry, and Saunders, Anthony, 1983, Stability and the Hedging Performance of Foreign Currency Futures, *Journal of Futures Markets*, Vol. 3, No. 3, pp. 295-306

Keim, Donald B. and Stambaugh, Robert F., 1986, Predicting Returns in the Stock and Bond Markets, *Journal of Financial Economics*, 17, pp. 357-390

Kroner, Kenneth F., and Sultan, Jahangir, 1993, Time-Varying Distributions and Dynamic Hedging With Foreign Currency Futures, *Journal of Financial and Quantitative Analysis*, Vol. 28, No. 4, pp. 535-551

Hull, John C., 2000, *Options, Futures & Other Derivatives – Fourth Edition*, Prentice Hall, USA

Hull, John C., 2000a, *Options, Futures & Other Derivatives – Fourth Edition*, Prentice Hall, USA, pp. 205-206

Hull, John C., 2000a, *Options, Futures & Other Derivatives – Fourth Edition*, Prentice Hall, USA, pp. 174-175

Hull, John C., 2000d, *Options, Futures & Other Derivatives – Fourth Edition*, Prentice Hall, USA, Ch. 13

Hull, John C., 2000e, *Options, Futures & Other Derivatives – Fourth Edition*, Prentice Hall, USA, Ch. 11, pp. 255-257

Hull, J., and White, A., 1988, An Analysis of the Bias in Option Pricing Caused by Stochastic Volatility, *Advances in Futures and Options Research*

Luenberger, David G., 1998, *Investment Science*, Oxford, USA

Nelson, Daniel B., 1991, "Conditional Heteroskedasticity in Asset Returns: A New Approach," *Econometrica*, Vol. 59, pp. 347–370

Randolph, William L., 1991, Use of the Mean Reversion Model in Predicting Stock Market Volatility, *Journal of Portfolio Management*, Vol. 17, No. 3, pp. 22-26

Schwert, William G., 1989, Why Does Stock Market Volatility Change Over Time, *The Journal of Finance*, Vol. XLIV, No. 5, pp. 1115-1153

Schwert, William G., and Seguin, Paul J., 1990, Heteroskedasticity in Stock Returns, *The Journal of Finance*, Vol. XLV, No. 4

Sharpe, W. F., 1964, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance*, Vol 19, pp. 425-442

Stoll, Haans R., and Whaley, Robert E., 1990, Stock Market Structure and Volatility, *The Review of Financial Studies*, Vol. 3, No. 1, pp. 37-71

Vasilellis, George A., and Meade, Nigel, 1996, Forecasting Volatility for Portfolio Selection, *Journal of Business Finance and Accounting*, Vol. 23, No. 1, pp. 125-143

APPENDIX

Equation Expansions

Actual Annualized Standard Deviation of Returns

Assume that s = 10, t = 0

$$AS(s)_t = AS(10)_0 = \left[252\sum_{j=1}^{s}\frac{r_{t+j}^2}{s}\right]^{\frac{1}{2}} = \left[\frac{252}{s}\sum_{j=1}^{s}r_{t+j}^2\right]^{\frac{1}{2}} = \left[\frac{252}{10}\sum_{j=1}^{10}r_{t+j}^2\right]^{\frac{1}{2}} =$$

$$\left[\frac{252}{10}\left(r_{0+1}^2 + r_{0+2}^2 + r_{0+3}^2 + \cdots + r_{0+10}^2\right)\right]^{\frac{1}{2}} = \left[\frac{252}{10}\left(r_1^2 + r_2^2 + r_3^2 + \ldots + r_{10}^2\right)\right]^{\frac{1}{2}}$$

Historical Standard Deviation

Assume that n = 10, t = 0

$$STD(n)_t = STD(10)_0 = \left[\sum_{j=0}^{n-1}\frac{r_{t-j}^2}{n}\right]^{\frac{1}{2}} = \left[\sum_{j=0}^{10-1}\frac{r_{t-j}^2}{10}\right]^{\frac{1}{2}} = \left[\frac{1}{10}\sum_{j=0}^{9}r_{t-j}^2\right]^{\frac{1}{2}} =$$

$$\left[\frac{1}{10}\left(r_{0-0}^2 + r_{0-1}^2 + r_{0-2}^2 + \ldots + r_{0-9}^2\right)\right]^{\frac{1}{2}} = \left[\frac{1}{10}\left(r_0^2 + r_{-1}^2 + r_{-2}^2 + \ldots + r_{-9}^2\right)\right]^{\frac{1}{2}}$$

Table 11 – In-Sample Regression Leverage Term & Sign Results Summary

		Independent Variable Sign			
Security	Series	ARCH1AR1	GARCH11AR1	EGARCH11AR1	EGARCH Leverage Term Sign
Air Canada	A10S10	-	-	-	-
	A10S20	-	-	-	-
Nortel	A5S10	-	-	-	-
	A5S30	-	-	-	+
	A10S10	-	-	-	+
	A10S30	-	-	-	+
DJIA	A5S5	-	-	-	+
	A5S10	-	-	-	+
	A5S20	-	-	-	+
	A5S30	-	-	-	+
	A10S10	-	-	-	-
	A10S20	-	-	-	+
	A10S30	-	-	-	+
	A20S30	-	-	-	-
TSE300	A5S10	-	-	-	+
	A5S20	-	-	-	+
	A5S30	-	-	-	+
	A10S20	-	-	-	-

Legend:

Column 1 (Security) lists the security. Column 2 (Series) lists the time series. For example, A10S20 indicates that this time series uses volatility over the last 20 days to forecast over a period of 10 days in the future. Columns 3-5 report the sign of the independent variable used in the regressions. Column 3 reports the sign of the independent variable using the ARCH1AR1 regression model (i.e. the Arch (1) model with a 1st order autocorrelation term). Column 4 reports the sign of the independent variable using the GARCH11AR1 regression model (i.e. the Garch (1,1) model with a 1st order autocorrelation term). Column 5 reports the sign of the independent variable using the EGARCH11AR1 regression model (i.e. the Egarch (1,1) model with a 1st order autocorrelation term). The last column (EGARCH Leverage Term Sign) reports on the sign of the leverage term present in the EGARCH11AR1 model. If the sign is negative, leverage effects are present.

All results are statistically significant at the 10% level.

Note: Series listed in table above were chosen after passing the cutoff criteria. Refer to "Discussion of Results" Section for the details.

Table 12-In-Sample Dynamic & Static RMSFE

		In-Sample Dynamic RMSFE			In-Sample Static RMSFE		
Security	Series	ARCH1AR1	GARCH11AR1	EGARCH11AR1	ARCH1AR1	GARCH11AR1	EGARCH11AR1
Air Canada	A10S10	0.2740	0.2706	0.2794	0.0619	0.0615	0.0611
	A10S20	0.3208	0.3103	0.3286	0.0616	0.0614	0.0614
Nortel	A5S10	0.3988	0.4432	0.3905	0.0997	0.0999	0.0997
	A5S30	0.5860	0.6832	0.6563	0.1012	0.1013	0.1020
	A10S10	0.3235	0.3272	0.3050	0.0554	0.0542	0.0549
	A10S30	0.4290	0.4723	0.4022	0.0555	0.0547	0.0556
DJIA	A5S5	0.1059	0.1047	0.0959	0.0330	0.0327	0.0326
	A5S10	0.1172	0.1253	0.1162	0.0328	0.0328	0.0328
	A5S20	0.1718	0.1629	0.1479	0.0327	0.0326	0.0326
	A5S30	0.1766	0.1814	0.1679	0.0331	0.0331	0.0331
	A10S10	0.0950	0.0924	0.0995	0.0174	0.0174	0.0174
	A10S20	0.1137	0.1051	0.1064	0.0178	0.0176	0.0177
	A10S30	0.1330	0.1221	0.1165	0.0178	0.0176	0.0176
	A20S30	0.1134	0.0907	0.0836	0.0095	0.0095	0.0094
TSE300	A5S10	0.1454	0.1570	0.1455	0.0352	0.0352	0.0354
	A5S20	0.1935	0.2047	0.1554	0.0350	0.0350	0.0355
	A5S30	0.2259	0.2326	0.2150	0.0350	0.0351	0.0356
	A10S20	0.1318	0.1388	0.2373	0.0195	0.0195	0.0196

Legend:

Column 1 (Security) lists the security. Column 2 (Series) lists the time series. For example, A10S20 indicates that this time series uses volatility over the last 20 days to forecast over a period of 10 days in the future. Columns 3-5 list the In-Sample Dynamic Root Mean Squared Forecast Errors. Column 3 reports the RMSFE of the ARCH1AR1 regression model (i.e. the Arch (1) model with a 1st order autocorrelation term). Column 4 reports the RMSFE of the GARCH11AR1 regression model (i.e. the Garch (1,1) model with a 1st order autocorrelation term). Column 5 reports the RMSFE of the EGARCH11AR1 regression model (i.e. the Egarch (1,1) model with a 1st order autocorrelation term). Similarly, columns 6-8 report the In-Sample Static Root Mean Squared Forecast Errors.

All results are statistically significant at the 10% level.

Note: Series listed in table above were chosen after passing the cutoff criteria. Refer to "Discussion of Results" Section for the details.

Table 13-Out-of-Sample Regression Leverage Term & Sign Results Summary

		Independent Variable Sign			
Security	Series	ARCH1AR1	GARCH11AR1	EGARCH11AR1	EGARCH Leverage Term Sign
DJIA	A5S5	-	-	-	+
	A5S10	-	-	-	+
	A5S20	-	-	-	+
	A5S30	-	-	-	+
	A10S10	-	-	-	+
	A10S20	-	-	-	+
	A10S30	-	-	-	-

Legend:

Column 1 (Security) lists the security. Column 2 (Series) lists the time series. For example, A10S20 indicates that this time series uses volatility over the last 20 days to forecast over a period of 10 days in the future. Columns 3-5 report the sign of the independent variable used in the regressions. Column 3 reports the sign of the independent variable using the ARCH1AR1 regression model (i.e. the Arch (1) model with a 1st order autocorrelation term). Column 4 reports the sign of the independent variable using the GARCH11AR1 regression model (i.e. the Garch (1,1) model with a 1st order autocorrelation term). Column 5 reports the sign of the independent variable using the EGARCH11AR1 regression model (i.e. the Egarch (1,1) model with a 1st order autocorrelation term). The last column (EGARCH Leverage Term Sign) reports on the sign of the leverage term present in the EGARCH11AR1 model. If the sign is negative, leverage effects are present.

All results are statistically significant at the 10% level.

Note: Series listed in table above were chosen after passing the cutoff criteria. Refer to "Discussion of Results" Section for the details.

Table 14-Out-Of-Sample Dynamic & Static RMSFE

		Out-Of-Sample Dynamic RMSFE			Out-Of-Sample Static RMSFE		
Security	Series	ARCH1AR1	GARCH11AR1	EGARCH11AR1	ARCH1AR1	GARCH11AR1	EGARCH11AR1
DJIA	A5S5	0.0860	0.0848	0.0800	0.0298	0.0296	0.0296
	A5S10	0.0948	0.1054	0.0932	0.0313	0.0312	0.0313
	A5S20	0.1287	0.1293	0.1150	0.0304	0.0304	0.0307
	A5S30	0.1847	0.2030	0.1807	0.0345	0.0346	0.0343
	A10S10	0.0782	0.0791	0.0752	0.0154	0.0154	0.0153
	A10S20	0.0866	0.0864	0.0823	0.0152	0.0150	0.0150
	A10S30	0.1019	0.1157	0.1746	0.0170	0.0169	0.0174

Legend:

Column 1 (Security) lists the security. Column 2 (Series) lists the time series. For example, A10S20 indicates that this time series uses volatility over the last 20 days to forecast over a period of 10 days in the future. Columns 3-5 list the In-Sample Dynamic Root Mean Squared Forecast Errors. Column 3 reports the RMSFE of the ARCH1AR1 regression model (i.e. the Arch (1) model with a 1st order autocorrelation term). Column 4 reports the RMSFE of the GARCH11AR1 regression model (i.e. the Garch (1,1) model with a 1st order autocorrelation term). Column 5 reports the RMSFE of the EGARCH11AR1 regression model (i.e. the Egarch (1,1) model with a 1st order autocorrelation term). Similarly, columns 6-8 report the In-Sample Static Root Mean Squared Forecast Errors.

All results are statistically significant at the 10% level.

Note: Series listed in table above were chosen after passing the cutoff criteria. Refer to "Discussion of Results" Section for the details.