REFORMULATED ASSET PRICING MODELS: THEORY AND TESTS

Zhongzhi Lawrence He

A Thesis

in

the John Molson School of Business

Presented in Partial Fulfillment of the requirements

for the degree of Doctor of Philosophy at

Concordia University

Montreal, Quebec, Canada

August 2002

CONCORDIA UNIVERSITY

0-612-73356-4

ABSTRACT

REFORMULATED ASSET PRICING MODELS: THEORY AND TESTS

Zhongzhi He, Ph.D.
Concordia University, 2002

The dissertation consists of three essays that address both the theoretical and empirical aspects of characteristics-based asset pricing models.

In the first essay, we reformulate a characteristics-based model to demonstrate why firm characteristics explain cross-sectional expected returns. The model is based on an economic setting where the fully-rational group of investors adopts contrarian strategies against the quasi-rational group of investors. The key result is a parsimonious cross-sectional equation that is not only specified by the risk-return relationship, but is also determined by both market-wide and firm-specific adjustments. We offer consistent explanations for the behaviors of growth and value stocks, and also for the prominent cross-sectional patterns such as book-to-market, earnings-to-price, and size effects.

In the second essay, we reformulate an asset pricing model where liquidity is an endogenous determinant of expected returns. The key result is that a firm's expected return can be explained by three components: an interest rate term that includes a market-average expected liquidity, a market risk term determined by a weighted average consumption beta, and a firm-specific term determined by a linear deviation of the firm's expected liquidity from that of the market portfolio. We test various empirical implications derived from the theory and find that the expected liquidity effect and the size effect are significant, but the risk-return relationship is flat in the Canadian market.

In the third essay, we propose a characteristics-based asset-pricing model from an *ex post* perspective. We examine the widely used empirical procedure that groups stocks into portfolios by sorting firm characteristics, showing that the exhibited systematic patterns may be largely due

to the way of forming portfolios. We design a new portfolio approach and perform robustness tests for the cross-sectional relationships between risk, liquidity, and returns using Canadian stock market data. We find a strong liquidity-return relationship and a significant risk-return relationship when conditioning on realized returns. Both the risk effect and the liquidity effect are highly robust across different portfolio formations.

ACKNOWLEDGEMENTS

This thesis is specially dedicated to my wife Nina, who is always in my accompany during the long journey of my Ph.D. study. No words are enough to express my gratitude to her. Without her continuous love and emotional support, I would not have finished my Ph.D. Special dedication also to my daughter Lucy, who has been with me since the thesis started. Her smile always let me forget all the distress during some difficult periods.

Wholehearted gratitude to my parents, whose hard work and belief in education have kindled my aspiration for science and knowledge, and motivated me to pursue advanced academic studies all the way to the Ph.D. degree.

I am very grateful to my thesis supervisor Dr. Lawrence Kryzanowski for his continuous encouragement and support to finish the arduous and long journey that is involved to complete a Ph.D. His supervisory style provided me the confidence to feel free to examine various topics and to accommodate them to my inquisitive needs. He wisely oriented and respected the changes to end this endeavor to my satisfaction.

I appreciate Dr. Dennis Kira and Dr. Simon Lalancette for their insightful comments that greatly improved the content of the thesis. Also thanks to my collogues Mohamed Ayadi and Skander Lazrak, who unselfishly share their ideas and provide positive feedback to the thesis.

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER 1

INTRODUCTION

Risk factor models, such as the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Black (1972), the Intertemporal Capital Asset Pricing Model (ICAPM) of Merton (1973), and the Arbitrage Pricing Theory (APT) of Ross (1976), form the foundation of asset pricing theories in financial research. These models are used to explain a number of systematic behaviors of cross-sectional portfolio returns. For example, the prominent Fama and French (1993) three-factor model finds that underlying distress factors successfully account for the cross-sectional variations exhibited by two prominent firm characteristics, namely, the size effect and the book-to-market effect.

However, recent empirical studies find compelling evidence that risk factor models do not explain fully cross-sectional expected returns. Lakonishok, Shleifer and Vishny (1994) examine the profitability of value strategies by using financial ratios such as earnings-to-price (E/P), cashflow-to-price (C/P), and book-to-market (B/M) as proxies for a firm's past and future performance. Their finding that high-ratio portfolios (value stocks) significantly outperform low-ratio stocks (glamour stocks) after controlling for risk does not support the risk-based explanation that value portfolios are fundamentally riskier. Instead, they offer an alternative behavioral explanation. When naïve investors overreact to firm fundamental information, more sophisticated investors earn higher returns by adopting contrarian strategies against the naïve strategies.

Daniel and Titman (1997) empirically investigate whether the cross-sectional patterns of firm characteristics such as size and book-to-market are consistent with the covariance structure of multi-factor models. They propose an ad hoc characteristics-based pricing model as an alternative hypothesis to the Fama-French three-factor model, and they form portfolios to distinguish between the two hypotheses. After controlling for firm characteristics, they show that the covariance model has little explanatory power. They conclude that cross-sectional expected returns appear to be mainly explained by firm characteristics that do not proxy for loadings on any risk factor. The authors suggest a few tentative explanations for their characteristics-based model. These include behavior-based, agency-based, and liquidity-based stories, and the prior that investors may consistently prefer certain characteristics to others. With regard to the liquidity-based explanations, Amihud and Mendelson (1987) and Brennan and Subrahmanyam (1996) provide evidence that asset returns include a significant premium for illiquidity after controlling for risk.

Based on theoretical arguments, MacKinlay (1995) points out that the *ex post* nature of empirical research makes it difficult to discriminate between multifactor models and characteristic models. He develops a theoretical framework which shows that CAPM deviations due to missing risk factors are very difficult to detect empirically, whereas deviations resulting from nonrisk-based sources such as characteristics are easily detectable.

Although supported by empirical evidence, characteristics-based models have not received formal treatment. No theoretical model has been developed, and little is known about the empirical implications of a characteristic model. This dissertation intends to fill the gap in the asset-pricing literature by focusing on the issues related to characteristics-based models. The theme of the dissertation is to theoretically propose and empirically examine characteristics-based models as an alternative to risk-based models to explain cross-sectional expected returns. The dissertation consists of three chapters that address both the theoretical and empirical aspects of characteristics-based models.

The second chapter, **"A Reformulated Asset Pricing Model Based on Contrarian Strategies"**, examines the characteristics-based model from a theoretical perspective. We propose a Reformulated Asset Pricing Model (RAPM) to demonstrate why firm characteristics explain expected returns. Our model is based on an economic setting where two groups of heterogeneous agents hold different beliefs about firm fundamental values which aggregate to the consumption growth in the economy. "Quasi's", the naïve group of investors, are subject to cognitive biases.

Thus, "quasi's" hold irrational expectations about consumption growth. "Rationals", the more sophisticated group, not only make unbiased expectations about consumption growth, they also possess perfect foresight of the cognitive biases inherent in "quasi's" psychology. "Rationals" attempt to make arbitrage profits by adopting contrarian strategies against "quasi's". We model the interactive behaviors between "rationals" and "quasi's" within the classic one-period consumption-investment framework. Our key result is a parsimonious cross-sectional equation that is composed of three terms: an interest term that captures the market-wide adjustment for "quasi's" misperception; a market risk term that specifies the traditional risk-return relationship; and a firm-specific term that is determined by the fundamental-to-price deviation between a given firm and the market portfolio. We offer consistent explanations for the behaviors of growth and value stocks. Essentially, if "quasi's" tend to overreact (underreact) to fundamental information, the contrarian strategies adopted by "rationals" make expected returns exhibit systematic patterns that are classified as value (growth) stocks. We also explain the prominent cross-sectional patterns such as book-to-market, earnings-to-price, and size effects. Both the book-to-market and earnings-to-price effects are considered the joint effects of "quasi's" overreaction behavior and positive cross-sectional correlation between fundamental variables (such as earnings and book values) and expected fundamental values. The size effect is shown to be induced by the joint effect of "quasi's" irrational behavior and misspecification of firm fundamental values. Our theoretical arguments are consistent with the characteristics-based explanations in the literature.

In the third chapter, **"A Reformulated Asset Pricing Model of Risk, Liquidity and Returns"**, we address both the theoretical and empirical aspects of a reformulated asset pricing model where liquidity is an endogenous determinant of expected returns. Our insight is to realize that trading costs should be cross-sectionally related to before-cost expected returns since after-cost returns should be equilibrated in informationally efficient markets. We solve a rational investor's portfolio problem within the classic one-period consumption-investment framework, and derive an equilibrium model that unifies risk, liquidity, and returns. The key result is a cross-sectional equation that specifies a firm's expected return as consisting of three components: an interest rate term where the market-average expected liquidity is an endogenous variable; a market risk term where the consumption beta is a weighted average of the fundamental beta and the liquidity beta; and a firm-specific term determined by the firm's liquidity deviation from that of the market portfolio. The key observation from the theoretical development is that the illiquidity premium has both a non-diversifiable risk effect and a firm-specific characteristic effect. We then discuss the empirical implications derived from the theoretical model and test the implications using the monthly Canadian market data sampling from March 1977 to December 1998. Our empirical results indicate that the fundamental beta plays a much more important role than the liquidity beta due to its predominant weight. Hence, the covariance liquidity (dynamic) is unlikely to explain before-cost returns within our monthly sample. We then test the liquidity-return relation and find that the expected liquidity (static) is significant in explaining before-cost cross-sectional returns. We attribute the different findings between our study and Chordia et al. (2000) to different data sampling frequencies. We also test the risk effect, and find it to be insignificant after controlling for the size effect. Our final test presents evidence that the expected liquidity effect and the size effect are both significant for explaining before-cost returns. We interpret the size effect as model misspecification of expected liquidity.

In the fourth chapter, **"An *Ex Post* Examination of a Characteristics-based Model in the Canadian Market"**, we propose a characteristics-based asset-pricing model from an *ex post* perspective and examine its cross-sectional relationships empirically. The *ex post* characteristic model is a cross-sectional relationship that combines the market risk premium and the characteristic deviation premium. The characteristic component is not related to any risk factor but is determined by the deviation between a given firm's characteristics and those of the market portfolio. We examine the widely used empirical procedure that groups stocks into portfolios by sorting firm characteristics, showing that the exhibited systematic patterns may be largely due to the way of forming portfolios. Based on the implications from the characteristic model, we design a new portfolio approach and perform robustness tests for the cross-sectional relationships between risk, liquidity, and returns using Canadian stock market data. Empirical evidence supports our argument that size premia change in a predictable way depending on how portfolios are formed. Our tests also find a significant risk-return relationship when conditioning on realized returns on the Canadian market portfolio, and a strong liquidity-return relationship. Both the risk effect and the liquidity effect are highly robust across different portfolio formations.

Finally, some concluding remarks and directions for future research are presented in the fifth chapter.

CHAPTER 2

A REFORMULATED ASSET PRICING MODEL BASED ON CONTRARIAN STRATEGIES

2.1 Introduction

Equilibrium asset pricing models, initially developed in the form of the CAPM (Sharpe 1964; Black 1972) and ICAPM (Merton, 1973), have a set of fully rational investors who trade off current consumption and investment to maximize their own utilities in competitive equilibrium. The economy is typically represented by a homogeneous agent who fully incorporates public information into asset prices. In equilibrium, the expectation of asset returns is attributed to a single (i.e., CAPM) or multiple (i.e., ICAPM) risk factors. Risk models are widely used to explain the cross section of expected returns. For example, in support of the multi-factor models, Fama-French (1992, 1993) find evidence that underlying distress factors successfully account for the cross-sectional patterns exhibited by two prominent firm characteristics, namely, the size effect and the book-to-market effect. However, recent empirical results, as exemplified below, present serious challenges to risk models and their explanations.

Lakonishok, Shleifer and Vishny (LSV, 1994) examine the profitability of value strategies by using financial ratios such as earnings-to-price (E/P), cashflow-to-price (C/P), and book-to-market (B/M) as proxies for a firm's past and future performance. The LSV finding that high-ratio portfolios (value stocks) significantly outperform low-ratio stocks (glamour stocks) after controlling for risk does not support the risk-based explanation that value portfolios are fundamentally riskier. Instead, LSV offer an alternative behavioral explanation. When naïve investors overreact to firm fundamental information, more sophisticated investors earn higher returns by adopting contrarian strategies against the naïve strategies.

Daniel and Titman (1997) empirically investigate whether the cross-sectional patterns of firm characteristics such as size and book-to-market are consistent with the covariance structure of multi-factor models. They first propose a characteristics-based pricing model as an alternative hypothesis to the Fama-French Three-Factor model, and then form portfolios to distinguish between the two hypotheses. After controlling for firm characteristics, they show that the covariance model has little explanatory power. They conclude that cross-sectional expected returns appear to be mainly explained by firm characteristics that do not proxy for loadings on any risk factor. The authors suggest a few tentative explanations for their characteristics-based model. These include behavior-based, agency-based, and liquidity-based stories, and the prior that investors may consistently prefer certain characteristics to others.

As noted by the proponents of each position,[1] empirically distinguishing between risk- and characteristics-based models is very difficult, if not impossible, due to the high correlation between factor loadings and characteristics. Furthermore, no attempts have been made to distinguish between them theoretically, and no theory has been developed to justify why firm characteristics should explain cross-sectional expected returns. Thus, while characteristics-based models are supported empirically as an alternative to risk-based models, a wide gap exists between empirical observations and theoretical justification. Although contrarian strategies are proposed by LSV (1994) as a plausible explanation for characteristics-based models, no rigorous formulation exists to demonstrate how contrarian behaviors lead to the systematic patterns as demonstrated by firm characteristics.

In this paper, we propose a Reformulated Asset Pricing Model (RAPM) with heterogeneous agents to demonstrate why cross-sectional returns should be explained by firm characteristics. The RAPM is a one-period equilibrium model that explicitly introduces two groups of heterogeneous investors who hold different beliefs about firm fundamental processes.

[1] We refer to the most recent "Covariance vs Characteristics" arguments between Davis, Fama and French (2000) and Daniel and Titman (1997).

The representative agents from the two groups are termed as "rationals" and "quasi's", respectively.[2] "Rationals" are fully rational investors in the information sense who accurately perceive the true process of fundamental values, hence fully adjust public information into firm prices. "Quasi's", in contrast, make biased expectations. However, "Quasi's" think that they behave fully rationally because they do not recognize their cognitive biases. Thus, "quasi's" form their portfolio strategies naively. "Rationals" are fully aware of these perception errors, and the rationals adopt contrarian strategies based on these perception errors. Given the above descriptions, we impose an aggregation condition that facilitates the separation of the firm characteristic effect from the market risk effect. Within the classic consumption-investment framework, we arrive at a parsimonious cross-sectional relationship that is jointly determined by both naïve and contrarian behaviors in market equilibrium. The main results of this paper are summarized next.

Firstly, the RAPM is an equilibrium model that explains cross-sectional expected returns given contrarian strategies followed by "rationals". The key result is a linear pricing relationship that is determined by both risk and adjustment terms. When the market weight or perception bias of "quasi's' is trivial, the RAPM reduces exactly to the CAPM, where expected return is determined only by the risk-return relationship. However, when "quasi's" play a significant role in price determination, a firm's expected return is subject to both market-wide and firm-specific adjustments due to contrarian strategies. The market-wide adjustment is endogenous in interest rates, and corrects for the misperceptions of "quasi's' about the expected fundamental growth of the overall market. The firm-specific adjustment term corrects for the misperceptions of "quasi's' about the difference in the expected fundamental-to-price ratios between the firm and the market portfolio. An intuitive cross-sectional implication of the firm-specific adjustment is that a firm's

[2] We adopt these terms from Thaler (1991).

expected return is not determined by the firm's own expected fundamental-to-price ratio, but depends on how much the ratio for the firm deviates from that of the market.

Secondly, we formally analyze the behaviors of growth stocks and value stocks. The firm is classified as a growth stock or a value stock by its expected fundamental-to-price ratios relative to that of the market portfolio. The movements of growth and value stocks are discussed under three behavioral scenarios followed by "quasi's"; namely, fully rational, overreaction, and underreaction. Under the last two scenarios, a firm's expected return is subject to both market-wide and firm-specific adjustments that are consistent with interest rate movements and cross-sectional observations.

Thirdly, the RAPM provides a theoretical framework within which prominent cross-sectional returns can be consistently explained by firm characteristics. We show that when the behaviors of "quasi's" are non-trivial, fundamental variables (such as book value and earnings) that are cross-sectionally correlated with firm fundamental values have predictive power in determining expected returns. The book-to-market and earning-to-price observations are the joint effect of fundamental specifications and the overreaction behaviors of "quasi's'. The size effect results from misspecification of expected fundamental values. We show that these firm characteristics do not proxy for loadings on risk factors that should conform to a quadratic covariance structure. Instead, the relationship between expected returns and the characteristic terms follow a linear structure. This result lends direct support to the characteristics-based model proposed by Daniel and Titman (1997).

Finally, we show that quasi-rational behaviors by some market participants imply that "rationals" can always apply contrarian strategies to make arbitrage profits. We discuss why it is hard to do this consistently by showing that the requirements are stringent. To identify arbitrage opportunities, an investor must know *ex ante* the "quasi's" proportional weight and sentiment parameter for any given period. No one is likely to possess such ability in practice.

The remainder of the paper is organized as follows. In the next section, we present the economic setting and discuss the individual and interactive behaviors of "rationals" and "quais's". In the third section, we derive the RAPM within the classic consumption-investment framework, and also discuss the cross-sectional implications of this model. The fourth section presents a formal analysis of the behaviors of growth and value stocks. In the fifth section, we explain the prominent cross-sectional anomalies associated with the book-to-market ratio, earnings-to-price ratio, and firm size. In the sixth and final section, our concluding comments include a discussion of the arbitrage implications of our model.

2.2 Heterogeneous Agents - The Behaviors of "Rationals" and "Quasi's"

The existing literature on rational models with heterogeneous agents focuses on different aspects of investor constraints or behaviors that ultimately determine aggregate consumption. Campbell and Mankiw (1989) place investment constraints on one group of agents who are prevented from trading in asset markets and simply consume their labor income each period. The other group of agents is unconstrained. The constrained group may play an important role in aggregate consumption, but its consumption does not determine asset equilibrium returns. It is also possible that investors exhibit heterogeneity in their labor income and hence in their consumption levels. Constantinides and Duffie (1996) present a model where individual consumption levels have a cross-sectional log-normal distribution. In such an economy, although individual consumption determines the expected returns for each investor, aggregate consumption is not a valid determinant of equilibrium returns. These above consumption models with heterogeneous investment constraints and income do not explain the cross section of expected returns.

Existing behavioral models feature heterogeneity from the perspective of cognitive biases inherent in investors' psychology. De Long, Shleifer, Summers and Waldmann (1990) set up an economy where a group of fully rational investors faces the risk of liquidating their portfolios within a limited time horizon and another group of "noise traders" who trade assets based on reasons other than fundamental information. Hong and Stein (1999) assume that there are two types of cognitively biased investors: "newswatchers" and "momentum investors". The interaction between these two groups produces systematic behaviors consistent with the observed patterns in stock returns. A critical limitation of existing behavioral models, as reviewed by Campbell (2000), is that they focus on one single asset, and thus do not consider general equilibrium. Specifically, if the behaviors of "noise traders" are firm-specific, then they are diversifiable and should not have any impact on expected returns in the aggregate. On the other hand, if cognitive biases are market-wide, then their impact on expected returns should be endogenously captured in interest rates that reflect market perceptions about aggregate consumption growth. However, none of the existing behavioral models accounts for the behaviors of "noise traders" at both the firm-specific and market-wide levels.

The model proposed in this section is a general equilibrium model with heterogeneous cognitive biases. Two groups of representative agents form heterogeneous expectations about firm fundamental values, which are equivalent to firm consumption values. We impose an aggregation condition such that the consumption expectations for the two heterogeneous groups are additive to the aggregate consumption that determines the market expected returns. For simplicity, but with no loss of generality, we consider a one-period economy consisting of time 0 (present) and time 1 (end of the period). The capital markets contain J risky assets ($j = 1, \ldots, J$), where each risky asset is produced by a different firm. Our model features two groups of heterogeneous investors. Their representative agents are referred to as "rationals" ($i = 1$) and "quasi's" ($i = 2$), respectively. The relative proportions of wealth are denoted by a market weight ϕ (i.e., $0 < \phi \leq 1$)[3] for "rationals" and $(1 - \phi)$ for "quasi's". At time 0, both "rationals" and

[3] We exclude the limiting case where $\phi = 0$ to ensure a finite solution.

"quasi's" incorporate the same set of public information about firm fundamental values to determine firm future prices. "Rationals" are fully rational investors who are themselves not subject to any cognitive biases, so they can accurately perceive fundamental processes. "Quasi's" try as hard as "rationals" to perceive fundamental values. However, since they are subject to cognitive biases, their perceived fundamental processes always contain systematic pricing errors. The interactions between "rationals" and "quasi's", or precisely, the contrarian strategies of "rationals' against "quasi's" formulate the expectations of firm returns in market equilibrium at the end of the period.[4]

Within this economic setting, we invoke a number of capital market and behavioral assumptions to develop our theory.

Market assumptions are:

(M.1) Markets are frictionless and there are no market imperfections such as taxes, restrictions on short selling, indivisibilities of assets, etc;

(M.2) Markets are always in competitive equilibrium; and

(M.3) Firm returns follow a joint normal distribution.

The above are standard market assumptions from the original CAPM. In addition, we introduce the following behavioral assumptions:

(B.1) Representative agents are price takers who assign strictly positive (negative) prices to more (less) values for every firm; and

(B.2) Representative agents are risk-averse investors who attempt to maximize their own expected utility of their end-of-period consumption; and

[4] In the real economy, "rationals" bear the name of "smart money" which represents big institutional investors, mutual fund and hedge fund managers whose profession requires them not only to evaluate firm fundamental values, but also to take advantage of the sentiment of small investors. "Quasi's" are typically noise traders who are more likely to be subject to cognitive biases and make biased expectations about consumption growth.

(B.3) "Rationals" are sophisticated investors who are fully aware of the cognitive biases of "quasi's". They attempt to implement contrarian strategies against the "quasi's" to earn arbitrage profits.

The price adjustment processes for the two groups at the end of the period are represented respectively as:

$$\tilde{P}_{1j} = \tilde{V}_j \quad (2.1)$$

$$\tilde{P}_{2j} - P_j = \lambda(\tilde{V}_j - P_j) \quad (2.2)^{5}$$

where

P_j is firm j's current price or market capitalization observed at the beginning of the period;

$\tilde{V}_j$ is firm j's true fundamental value process to be realized at the end of the period;

$\tilde{P}_{1j}, \tilde{P}_{2j}$ are firm j's end-of-period price processes determined by "rationals" (i = 1) and "quasi's" (i = 2), respectively; and

λ is the sentiment parameter that is inherently attached to the psychology of "quasi's".

Equation (2.1) formalizes the statement that "rationals" fully adjust fundamental information into firm prices so $\tilde{P}_{1j}$ is an unbiased process. It is a special case of equation (2.2) when λ = 1. Equation (2.2) is in the spirit of the price adjustment model of Amihud and Mendelson (1987, p.536), except that it has no additional error term to reflect asset demands due to market imperfections. Equation (2.2) explicitly introduces a sentiment parameter (λ) to reflect the "quasi's" information adjustment process. λ is inherently attached to the psychology of the

[5] We use upper case letter P_j to denote the firm's total price, or market capitalization, which equals its price per share times the number of shares outstanding. All the letters with (without) a tidal denotes random variables (constants).

"quasi's" who are subject to cognitive biases. Since "quasi's" do not fully incorporate information into prices, λ differs from 1 in equation (2.2), and $\tilde{P}_{2j}$ is a biased process.

We can decompose equation (2.2) into two terms:

$$\tilde{P}_{2j} = \tilde{V}_j + (\lambda - 1)(\tilde{V}_j - P_j) \tag{2.3}$$

The first term on the right hand side ($\tilde{V}_j$) is the true or unbiased fundamental process. The second term is the mispriced component that captures the divergence between a firm's fundamental value and its misadjusted price.

To investigate the properties of the sentiment parameter λ, we now show that λ is bounded from below and above, specifically, $-1 < \lambda - 1 < 1$.[6] The bounds can be directly inferred from assumption (B.1); namely, "quasi's" strictly prefer more value to less for every firm. If λ is out of this range, assumption (B.3) will not hold. To see this, observe that equation (2.3) can be written as:

$$E[\tilde{P}_j] = P_j + (E[\tilde{V}_j] - P_j) + (\lambda - 1)(E[\tilde{V}_j] - P_j) \tag{2.4}$$

If $\lambda \le 0$, "quasi's" are irrational with those firms whose fundamentals are expected to be greater than their current prices (i.e., $E[\tilde{V}_j] - P_j > 0$). In this case, "quasi's" assign a zero or negative price to a sure positive value. This results in $E[\tilde{P}_j] \le P_j$. On the other hand, if $\lambda \ge 2$, "quasi's" are irrational with those firms whose fundamentals are expected to be lower than their current prices (i.e., $E[\tilde{V}_j] - P_j < 0$). In this case, "quasi's" assign a zero or positive price to a sure negative value. This results in $E[\tilde{P}_j] \ge P_j$. When $\lambda \in (0, 2)$, the more-to-less preference order is always preserved for "quasi's".

[6] The upper and lower bounds are necessary for firm prices to have finite variance in an infinite horizon. See Amihud and Mendelson (1987).

As such, the individual behaviors of "quasi's" and "rationals" can be formalized as follows.

Proposition I:

"Rationals" are fully rational agents whose sentiment parameter satisfies $\{\lambda \mid \lambda = 1\}$; *"Quasi's" are quasi-rational agents whose sentiment parameter satisfies*

$\{\lambda \mid (\lambda - 1) \in (-1, 1) \text{ and } \lambda \neq 1\}$.

Proposition I implies an irrational behavior to be $\{\lambda \mid (\lambda - 1) \notin (-1, 1)\}$, so there is a clear boundary between "quasi's" behavior and a purely irrational one. Furthermore, for "quasi's", we follow the literature convention that a positive (negative) departure of λ from 1 is deemed over- (under-) reaction behavior because fundamental information is overly (partially) incorporated into the firm price processes. This leads to the following proposition.

Proposition II ("Quasi's Behavior):

Over-reaction Behavior: $\{\lambda \mid 0 < (\lambda - 1) < 1\}$

Under-reaction Behavior: $\{\lambda \mid -1 < (\lambda - 1) < 0\}$

Having described the individual behaviors for "rationals" and "quasi's", we now elaborate on their interactive behaviors based on assumption (B.3). Note that both "rationals" and "quasi's" need to assess firm fundamental values (through fundamental analysis) in order to make their asset allocation decisions. "Quasi's" are not aware of their cognitive biases but rather think that they behave with full rationality. Thus, their evaluation of fundamental values is identical to their misadjusted prices. On the other hand, "rationals" are sophisticated enough to recognize the other group's cognitive biases which they can take advantage of. Therefore, "rational" evaluation of fundamental values will not only include their unbiased adjustment of firm prices, but also incorporate the corrections for "quasi's" mispriced component.

2.3 Model Development

Each group of agents is subject to the following budget constraint:

$$W_i = C_i + \sum_j X_{ij} P_j \text{ , i = 1, 2} \tag{2.5}$$

where

i = 1 for "rationals" and i = 2 for "quasi's";

W_i is group i's initial wealth endowed at time 0;

C_i is group i's consumption at time 0;

X_{ij} is the group i's proportional wealth invested in firm j.

Since each group's portfolio decisions are based on their respective evaluations of fundamental values, which ultimately determine their respective expectations of future consumption, we formulate their portfolio choices respectively. Before proceeding, we express a firm's true consumption value ($\tilde{C}_j$), or equivalently, its true fundamental value process ($\tilde{V}_j$) as:

$$\tilde{C}_j = \tilde{V}_j = E[\tilde{V}_j] + \tilde{\varepsilon}_j \tag{2.6}$$

where $E[\tilde{V}_j]$ denotes firm j's expected fundamental value at the end of the period; and

$\tilde{\varepsilon}_j \sim N(0, \sigma_j^2)$ is a random shock to the fundamental process.

2.3.1 Portfolio Choice for "Quasi's"

"Quasi's" make naïve portfolio decisions. Since they do not recognize their cognitive biases and think they behave with full rationality, their assessment of the consumption value of firm j is equivalent to their misperceived prices as in equation (2.2), i.e.,

$$\tilde{C}_{2j} = \tilde{V}_{2j} = \tilde{P}_{2j} \tag{2.7}$$

where $\widetilde{C}_{2j}$ and $\widetilde{V}_{2j}$ denotes firm j's consumption value and fundamental value evaluated by "quasi's". In other words, the biased agent "quasi's" have quasi-rational expectations of consumption growth.

The expected value and variance of "quasi's" consumption are therefore:

$$E[\widetilde{C}_{2j}] = E[\widetilde{P}_{2j}] \tag{2.8}$$

$$Var[\widetilde{C}_{2j}] = \lambda^2 \sigma_j^2 \tag{2.9}$$

Then, the expected consumption value and variance for the portfolio of "quasi's" are:

$$E[\widetilde{C}_2] = \sum_j X_{2j} E[\widetilde{P}_{2j}] \tag{2.10}$$

$$Var[\widetilde{C}_2] = \sum_j X_{2j} \operatorname{cov}(\widetilde{P}_{2j}, \widetilde{P}_2) \tag{2.11}$$

where

$\widetilde{C}_2$ denotes the portfolio's consumption value evaluated by "quasi's"; and

$\widetilde{P}_2 \equiv \sum_j X_{2j} \widetilde{P}_{2j}$ denotes the portfolio's price process determined by "quasi's".

2.3.2 Portfolio Choice for "Rationals"

In contrast to "quasi's" naïve portfolio choice, "rationals" make their choices strategically. "Rationals" evaluation of future consumption is composed of two terms: the first is their unbiased perception of consumption , which equals the true fundamental value process as specified in equation (2.1); and the second reflects "rationals" contrarian strategy that make corrections for the mispriced component as captured by the second term on the right-hand-side of equation (2.3). In Appendix 2-A, we derive the portfolio choice for "rationals" as:

$$\widetilde{C}_{1j} = \widetilde{V}_{1j} = \widetilde{P}_{1j} - \frac{1-\phi}{\phi}(\lambda - 1)(\widetilde{V}_j - P_j) \tag{2.12}$$

where $\tilde{C}_{1j}$ denotes assessment of "rationals" of future consumption; and the two terms on the right-hand-side represent the true fundamental value and the contrarian strategy, respectively. From a consumption perspective, "rationals" not only perceive the true consumption value, but they also attempt to gain additional consumption that is misperceived by "quasi's". Note that the ability of "rationals" to apply the contrarian strategies depends on the values of the following two parameters:

1) The sentiment parameter $(\lambda-1)$; and
2) The relative proportional weight $(1-\phi)/\phi$.

The product term $\frac{1-\phi}{\phi}(\lambda-1)$ is a relative sentiment measure of "quasi's" since the "quasi's" sentiment parameter is factored by a relative proportional weight. For ease of subsequent illustrations, our model focuses on a subset of the ϕ and λ space such that the spread of the relative sentiment measure is less than unity, i.e.,

$$\left|\frac{1-\phi}{\phi}(\lambda-1)\right|<1 \qquad (2.13)^7$$

Condition (2.13) is not a very strong restriction. It simply requires that the economy not be too dominated by "quasi's". To explain, let us consider the following two situations that satisfy (2.13). First, when "rationals" have an equal or larger market weight ($\phi \geq 0.5$), (2.13) is automatically satisfied by Proposition I. Second, "rationals" may have a smaller weight ($\phi < 0.5$), but the "quasi's" sentiment parameter needs to be small enough for (2.13) to hold. Intuitively, condition (2.13) avoids the setting where the economy is predominately populated by "quasi's" whose λ deviates significantly from one. In the limit, the case $\phi \rightarrow 0$ must be excluded to ensure a finite variance. In addition, for the boundary condition of ϕ, the following relationship must hold:

[7] This condition is stronger than necessary, and is for ease of explanation only. We use the subset in the ϕ and λ space in order to present an intuitive scenario for the interactive behaviors between "rationals" and "quasi's". A rigorous discussion of the ϕ and λ relation is beyond the scope of this paper.

$\phi = 1 \Leftrightarrow \lambda = 1$, i.e., everyone is fully rational. (2.14)

For the rest of the paper, let us assume that both λ and ϕ are constant during the period and "rationals" know their values *ex ante* so that "rationals" can accurately apply contrarian strategies.

As such, the expected consumption value and variance of equation (2.12) becomes:

$$E[\tilde{C}_{1j}] = E[\tilde{P}_{1j}] - \frac{1-\phi}{\phi}(\lambda - 1)(E[\tilde{V}_j] - P_j) \tag{2.15}$$

$$Var[\tilde{C}_{1j}] = [1 - \frac{1-\phi}{\phi}(\lambda - 1)]^2 \sigma_j^2 \tag{2.16}$$

From (2.15) and (2.16), the expected consumption value and variance of the portfolio held by "rationals" are calculated as:

$$E[\tilde{C}_1] = \sum_j X_{1j} E[\tilde{P}_{1j}] - \frac{1-\phi}{\phi}(\lambda - 1)\sum_j X_{1j}(E[\tilde{V}_j] - P_j) \tag{2.17}$$

$$Var[\tilde{C}_1] = [1 - \frac{1-\phi}{\phi}(\lambda - 1)]^2 \sum_j X_{1j} \operatorname{cov}(\tilde{P}_{1j}, \tilde{P}_1) \tag{2.18}$$

where $\tilde{P}_1 \equiv \sum_j X_{1j} \tilde{P}_{1j}$ denotes the portfolio's price as perceived by "rationals".

2.3.3 The Consumption-Investment Problem for "Rationals" and "Quasi's"

Based on equations (2.5), (2.10), (2.11), (2.17) and (2.18), the portfolio choice problem can be solved within the classical consumption-investment framework. Both groups of agents attempt to maximize their own utility as parameterized by $(C_i, E[\tilde{C}_i], Var[\tilde{C}_i])$, i = 1, 2. Formally, each group of agents solves the following consumption-investment problem:

$$\underset{\{X_{ij}\}}{Max} \quad U_i(C_i, E[\tilde{C}_i], Var[\tilde{C}_i]), \qquad i = 1, 2 \qquad (2.19)^{8}$$

subject to

(2.5), (2.17) and (2.18) for "rationals" (i = 1); and

(2.5), (2.10) and (2.11) for "quasi's" (i = 2)

In Appendix 2-B, we show that the expected firm prices contributed by "rationals" (i = 1) and "quasi's" (i = 2) can be respectively expressed as:

$$E[\tilde{P}_{1j}] = \gamma_{01} P_j + \frac{2}{\tau_{11}}[1 - \frac{1-\phi}{\phi}(\lambda - 1)]^2 \operatorname{cov}(\tilde{P}_{1j}, \tilde{P}_1) + \frac{1-\phi}{\phi}(\lambda - 1)(E[\tilde{V}_j] - P_j) \qquad (2.20)$$

$$E[\tilde{P}_{2j}] = \gamma_{02} P_j + \frac{2}{\tau_{12}} \operatorname{cov}(\tilde{P}_{2j}, \tilde{P}_2) \qquad (2.21)$$

where

γ_{01} and γ_{02} measure the marginal rate of substitution between current consumption and expected portfolio value for "rationals" and "quasi's", respectively. Intuitively, γ_{01} and γ_{02} can be interpreted as the risk-free rate assigned by "rationals" and "quasi's".

τ_{11} and τ_{12} denote the risk tolerance measures for "rationals" and "quasi's", respectively, which capture the tradeoffs between expected value and variance. By the definition of τ_{11} and τ_{12} as the negative of the marginal rate of substitution between expected value and variance (as shown in Appendix 2-B), the higher the risk tolerance, the more risk the agent is willing to take.

2.3.4 The Aggregation Condition

Equations (2.20) and (2.21) represent a firm's expected price assigned by "rationals" and "quasi's", respectively. We now aggregate the two different expectations to obtain the market expectation of the firm's price. Aggregating across "rationals" (i = 1) and "quasi's" (i = 2) yields the market expectation as follows:

---

[8] We express the expected utility in terms of a certainty-equivalent measure.

$$E[\tilde{P}_j] = \gamma_{0m}P_j + \frac{2}{\tau_{11}}[1 - \frac{(1-\phi)(\lambda-1)}{\phi}]^2 \phi \operatorname{cov}(\tilde{P}_{1j}, \tilde{P}_1) + \frac{2}{\tau_{12}}(1-\phi)\operatorname{cov}(\tilde{P}_{2j}, \tilde{P}_2)$$
$$+ (1-\phi)(\lambda-1)(E[\tilde{V}_j] - P_j) \tag{2.22}$$

where

$E[\tilde{P}_j] \equiv \phi E[\tilde{P}_{1j}] + (1-\phi)E[\tilde{P}_{2j}]$ is the firm's expected price assigned by the market;

and

$\gamma_{0m} \equiv \phi\gamma_{01} + (1-\phi)\gamma_{02}$ is the risk-free rate assigned by the market.

Equation (2.22) reflects the market's assignment of a firm's expected price. In this equation, we refer to the two covariance terms as the individual risk effects, and the last linear term as the characteristic effect. Generally speaking, equation (2.22) is not a tractable expression since the covariance terms are not additive across the two individual groups. In other words, each group individually evaluates firm risks whose effects cannot be aggregated to reflect the market's evaluation of risk. Due to this non-additive nature of the individual risk terms, equation (2.22) keeps us from studying the asset-pricing roles attributed to firm characteristics alone. Fortunately, by imposing a specific covariance structure, the individual risks can be aggregated to the market risk term. This allows us to theoretically separate the firm characteristic effect from the market risk effect. The covariance structure is specified for the risk tolerance measures between "rationals" (τ_{11}) and "quasi's" (τ_{12}) as given by the following condition.

Aggregation Condition:

Individual firm risks, as evaluated separately by "rationals" and "quasi's", can be aggregated to the market risk if the risk tolerance measures between "rationals" (τ_{11}) and "quasi's" (τ_{12}) satisfy the following condition:

$$\frac{\tau_{11}}{\tau_{12}} = [1 - \frac{1-\phi}{\phi}(\lambda - 1)]^2 \tag{2.23}$$

The proof is presented in Appendix 2-C. Before proceeding, we illustrate the behavioral intuitions implied from this aggregation condition. As illustrated before, the term $\frac{1-\phi}{\phi}(\lambda - 1)$ is "quasi's" relative sentiment measure, and the "rationals'" sentiment measure is 1. Thus, the aggregation condition as in equation (2.23) measures the sentiment difference between "rationals" and "quasi's". This condition implies that "rationals" and "quasi's" have different attitudes towards risk. From condition (2.13), observe that when "quasi's" exhibit overreaction behavior, i.e., $0 < \lambda - 1 < 1$, "rationals" tend to be less risk tolerant, or equivalently, more risk averse than "quasi's". In this case, $\tau_{11} < \tau_{12}$. On the other hand, when "quasi's" exhibit underreaction behavior, i.e., $-1 < \lambda - 1 < 0$, "rationals" tend to be more risk tolerant, or equivalently, less risk averse than "quasi's". In this case, $\tau_{11} > \tau_{12}$. There is no risk attitude differential between the two groups (i.e., $\tau_{11} = \tau_{12}$) when "quasi's" are fully rational (i.e., $\lambda - 1 = 0$ or $\phi = 1$). To illustrate further, suppose that "quasi's" are overly optimistic about firm earnings and are therefore investing in highly risky portfolio holdings. "Rationals" will be less willing to choose highly risky portfolios. The converse also holds in an overly pessimistic situation where "rationals" will be more willing to invest in risky assets. The aggregate condition facilitates a closed-form solution that is demonstrated below.

Based on equations (2.22) and (2.23), and from Appendix 2-C, we obtain the following relationship in market equilibrium:

$$E[\tilde{P}_j] = \gamma_{0m} P_j + \frac{2}{\tau_{12}} \mathrm{cov}(\tilde{P}_j, \tilde{P}_m) + (\lambda - 1)(1-\phi)(E[\tilde{V}_j] - P_j), \; \mathrm{j} = 1,\ldots,\mathrm{J} \tag{2.24}$$

Equation (2.24) demonstrates that three components are incorporated in forming the expectation of a firm's end-of-period price. These three components are: the firm's current price (the first term), the firm's covariance with the price of the market portfolio (the second term), and an additional term that reflects the adjustment for "quasi's" misperception about the firm's fundamental value (the third term).

2.3.5 Expected Return for a Firm

By scaling a firm's expected price by its current market capitalization P_j, we derive the expression for expected return in closed form. Let $\tilde{R}_j$ denote the equilibrium return per dollar from investing in firm j. We apply the security valuation formula $E[\tilde{R}_j] = E[\tilde{P}_j]/P_j - 1$ to equation (2.24) and obtain:

$$E[\tilde{R}_j] = \gamma_{0m} + \frac{2P_m}{\tau_{12}}\mathrm{cov}(\tilde{R}_j, \tilde{R}_m) + (\lambda - 1)(1-\phi)(\frac{E[\tilde{V}_j]}{P_j} - 1) \qquad (2.25)$$

Since equation (2.25) holds for every firm j = 1,…,J, it must hold for the market portfolio as well.

$$E[\tilde{R}_m] = \gamma_{0m} + \frac{2P_m}{\tau_{12}}\mathrm{var}(\tilde{R}_m) + (\lambda - 1)(1-\phi)(\frac{E[\tilde{V}_m]}{P_m} - 1) \qquad (2.26)$$

Substituting the term $2P_m/\tau_{12}$ from equation (2.26) into equation (2.25), we obtain the following equation:

$$E[\tilde{R}_j] = \gamma_{0m} + \beta_{jm}\{E[\tilde{R}_m] - (\lambda - 1)(1-\phi)(\frac{E[\tilde{V}_m]}{P_m} - 1) - \gamma_{0m}\} + (\lambda - 1)(1-\phi)(\frac{E[\tilde{V}_j]}{P_j} - 1)$$
$$(2.27)$$

where $\beta_{jm} = \mathrm{cov}(\tilde{R}_j, \tilde{R}_m)/\mathrm{var}(\tilde{R}_m)$ denotes the market beta that gives the formal definition of a firm's risk.

Rearranging terms yields the key result of the RAPM:

$$E[\tilde{R}_j] = R_{0m} + \beta_{jm}(E[\tilde{R}_m] - R_{0m}) + (\lambda - 1)(1-\phi)(\frac{E[\tilde{V}_j]}{P_j} - \frac{E[\tilde{V}_m]}{P_m}) \qquad (2.28)$$

where

$$R_{0m} = \gamma_{0m} + (\lambda - 1)(1-\phi)(\frac{E[\tilde{V}_m]}{P_m} - 1) \qquad (2.29)$$

Theorem:

A firm's expected return is determined by the following three components:

(i) *An interest rate term that captures both the marginal rate of substitution between successive consumption and a market-wide adjustment for "quasi's" misperception about the overall market growth;*

(ii) *A market risk term determined by the covariance of the firm's return with the market portfolio and the reward for bearing each unit of market risk; and*

(iii) *A firm-specific return that adjusts for "quasi's" misperception about the deviation between the firm's and the market's expected fundamental-to-price ratios.*

2.3.6 Cross-Sectional Implications

The interest rate term R_{0m} is composed of two components. The first component maintains the original definition of the risk-free rate as the marginal rate of substitution between two successive consumptions. The second component, which adjusts for "quasi's" perception biases about the expected fundamental growth of the market, is another endogenous variable that determines the risk-free rate.

The second part of the theorem shows that the RAPM preserves the risk-return relationship as in the CAPM. In particular, a firm's market beta is exactly the same as that specified by the CAPM and is not affected by "quasi's" cognitive biases or by the contrarian strategies of "rationals". This result is due to the aggregation condition, which allows the covariance terms to be aggregated across the two groups of agents. In equilibrium, a firm's market risk reflects its covariance with only the market portfolio, and is thus independent of each individual's portfolio. Also, note that the market risk premium, as captured by $(E[\tilde{R}_m] - R_{0m})$, is affected by market-wide interest rate adjustments.

With regard to the third part of the theorem, equation (2.28) indicates that the firm-specific return is not determined cross-sectionally by the firm's own expected fundamental-to-price ratio, but instead by the deviation of its ratio from that of the market portfolio. This is analogous to the conventional understanding of risk, namely, the risk of an asset is not determined by its own variance, but is determined by its covariance with the market portfolio. Thus, the market portfolio plays a critical role as a benchmark for determining both the firm's market risk and the firm's specific return. On the other hand, while a firm's market risk is measured by its covariance structure (a quadratic term), its firm-specific return is measured by its expected fundamental-to-price deviation (a linear term). This implies that we still may observe cross-sectional variation in risk-adjusted expected returns that can be explained by firm characteristics which proxy for fundamental-to-price ratios. How firm characteristics are related to cross-sectional expected returns will be discussed later.

In addition, inspection of equations (2.28) and (2.29) reveals that a firm's expected return is subject to two levels of adjustments due to contrarian strategies. The first level of adjustment, which is one of the endogenous components of the interest rate, is market-wide and affects every individual asset in the market. The second level of adjustment, which measures the deviation between a firm's and the market's expected fundamental-to-price ratio, is firm specific. The larger the deviation, the greater is the firm-specific return. Notice that for those firms with the market-average fundamental-to-price ratio, i.e., $E[\tilde{V}_j]/P_j = E[\tilde{V}_m]/P_m \quad \forall j$, the deviation term is zero, and thus the non-existence of firm-specific expected returns. However, these firms are still affected by market-wide adjustments and their expected returns are determined by the CAPM-like equation. A critical implication is that, although firm-specific expected return is diversifiable by constructing a portfolio that possesses the market-average characteristics, such a portfolio is still subject to market-wide adjustment, for example, by central bank adjustments of interest rates.

2.4 The Behavior of Growth Stocks and Value Stocks

The performance of growth stocks and value stocks has received a great deal of attention in recent years. While general agreement exists that value stocks outperform growth stocks over a long-term investment horizon (e.g., 3 to 5 years), the explanation is still unclear. Fama and French (1995) argue that the superior performance of value stocks is due to their higher sensitivity to systematic risk factors. On the other hand, Lakonishok et al. (1994) find that the return discrepancy between value and growth stocks is too large to be explained by risk factors. They attribute the superior returns of value stocks to sophisticated investors who adopt contrarian strategies against naïve investors. While these two competing explanations are supported empirically, neither is supported by formal theoretical arguments.

In this section, we formalize a theoretical argument in support of the explanation of Lakonishok et al. within the RAPM framework. First of all, we need to formally distinguish between growth and value stocks by using a market benchmark. We adopt the classification scheme of growth stocks and value stocks used by Lakonishok et al. (1994).[9] According to their terminology, growth (value) stocks are those that have performed well (poorly) in the past and are expected to perform well (poorly) in the future. They use ratios such as earnings-to-price and book-to-market to proxy for past performance and expected future performance, where lower (higher) ratios imply higher (lower) past and future performance. While their classification is sufficient for empirical investigations, it is too vague for theoretical analysis because there is no clearly identified benchmark against which firms can be classified. Specifically, their classification scheme does not specify how low or high the ratios should be for a stock to qualify

[9] They use the term glamour stocks instead of growth stocks.

as a growth or as a value stock. In addition, their classification scheme does not specify what fundamentals these ratios proxy. The following classification scheme provides a formal treatment:

$$If\ \frac{E[\tilde{V}_j]}{P_j} = \frac{E[\tilde{V}_m]}{P_m},\ then\ the\ firm\ is\ a\ market\text{-}average\ stock\ (the\ benchmark); \tag{2.30.1}$$

$$If\ \frac{E[\tilde{V}_j]}{P_j} < \frac{E[\tilde{V}_m]}{P_m},\ then\ the\ firm\ is\ a\ growth\ stock; \qquad and \tag{2.30.2}$$

$$If\ \frac{E[\tilde{V}_j]}{P_j} > \frac{E[\tilde{V}_m]}{P_m},\ then\ the\ firm\ is\ a\ value\ stock \tag{2.30.3}$$

Having presented the classification for growth and value stocks, we investigate their behaviors in the following three behavioral scenarios.

2.4.1 Full Rationality ($\lambda - 1 = 0$ or $\phi = 1$)

When "quasi's" are fully rational, there should be no risk-adjusted abnormal returns. The RAPM equations (2.28) and (2.29) reduce exactly to the original CAPM, with R_{0m} interpreted as the risk-free rate or the return on a zero-beta portfolio. Thus, the risk-return relationship is a complete description of a firm's expected return. While growth stocks and value stocks still exist due to cross-sectional variation in expected fundamental-to-price ratios, there is no need to distinguish between growth investing and contrarian investing because their expected abnormal profits are zero. In addition, the risk-free rate is determined only by the marginal rate of substitution between successive consumptions, and is not subject to market-wide adjustments.

2.4.2 Overreaction Behavior ($0 < \lambda - 1 < 1$)

When "quasi's" tends to exhibit overreaction behavior, equation (2.28) predicts that growth (value) stocks are expected to earn negative (positive) risk-adjusted abnormal returns over the period. A positive (negative) risk-adjusted abnormal return can be interpreted as the positive (negative) compensation or reward for investing in value (growth) stocks. Empirical evidence presented by Lakonishok et al. (1994) suggests that the value strategy is profitable over a long investment horizon, e.g., 3 to 5 years.

Equation (2.29) suggests that the adjustment for overreaction behavior also affects the risk-free rate. First, observe that the risk-free rate is affected by overall market growth. If the market is expected to grow, i.e., $(\frac{E[\tilde{V}_m]}{P_m} - 1) > 0$, adjustment for overreaction behavior would induce an increase in the risk-free rate. On the other hand, the adjustment for overreaction to negative market growth, i.e., $(\frac{E[\tilde{V}_m]}{P_m} - 1) < 0$, would result in a decrease in the risk-free rate. This observation is consistent with the monetary policies of central banks that adjust interest rates accordingly to dampen over-heated economies and to stimulate over-cooled economies.

2.4.3 Underreaction Behavior ($-1 < \lambda - 1 < 0$)

Growth stocks and value stocks also behave differently when underreaction behavior is corrected. Specifically, growth (value) stocks are expected to earn positive (negative) abnormal returns. Here, a positive (negative) risk-adjusted abnormal return is interpreted as the positive (negative) compensation or reward for investing in growth (value) stocks. Thus, a momentum strategy that takes long and short positions in winners and losers, respectively, is profitable. Jegadeesh and Titman (1993) find that such a strategy is profitable over a short investment horizon, e.g., 3 to 12 months.

The impact of corrections for underreaction behavior on the risk-free rate can also be inferred from equation (2.29). The risk-free rate decreases in a market with growing fundamental values, and increases in a market with declining fundamental values.

2.5 Explanations of Characteristics-based Models

It is generally acknowledged that the CAPM fails to explain prominent cross-sectional regularities. Three firm characteristics (book-to-market (B/M) ratio, earnings-to-price (E/P) ratio, and market capitalization as a proxy for firm size) are found to exhibit systematic patterns. Specifically, empirical evidence suggests that E/P and B/M ratios are positively correlated with cross-sectional returns, and firm size has a significant negative correlation with firm returns.[10]

Pursuing a risk-based explanation, Fama and French (1992, 1993) view the B/M ratio and firm size as proxies for loadings on some unobserved distress factors. In their view, high B/M and small-sized portfolios are fundamentally riskier due to their higher covariance with the underlying factors. The Fama-French three-factor model is widely used to account for risks associated with these two characteristics. While empirically grounded, little theoretical grounding exists why the B/M ratio and firm size are risk related. Instead, Ferson, Sarkissian and Simin (1999) argue that these characteristic effects do not always imply a risk-based explanation. Using a parable, they show that characteristic-sorted portfolios may pick out factors uncorrelated to risk, and suggest that the Fama-French three-factor model shares this possibility. Moreover, MacKinlay (1995) argues that multifactor models may not explain CAPM deviations and calls for alternative explanations. He calculates the distribution of the *ex ante* Sharpe ratio implied from the three-factor portfolios, and concludes that its value is too high to be consistent with a risk-based explanation. On the empirical side, Ferson and Harvey (1999) reject a conditional version of the three-factor model by showing that the instrumental variables still have predictive power to explain abnormal returns.

The above studies suggest the need for an alternative explanation, which is provided within the RAPM framework below. In empirical research, it is common practice to use fundamental variables such as firm earnings, dividends, and book values to proxy for fundamental values. A formal representation of this is to express the RAPM as given by (2.28) and (2.29) in the *conditional* form as the following:

---

[10] Banz (1981) documents the size effect, and Basu (1983) documents the E/P effect. Fama and French (1992) find that the B/M ratio is the most powerful explanatory variable of cross-sectional returns.

$$E[\tilde{R}_j \mid F] = R_{0m} + \beta_{jm}(E[\tilde{R}_m \mid F] - R_{0m}) + (\frac{E[\tilde{V}_j \mid F]}{P_j} - \frac{E[\tilde{V}_m \mid F]}{P_m})(1-\phi)(\lambda - 1) \tag{2.31}$$

where

$$R_{0m} = \gamma_{m0} + (\frac{E[\tilde{V}_m \mid F]}{P_m} - 1)(1-\phi)(\lambda - 1) \tag{2.32}$$

$E[\tilde{V}_j \mid F]$ and $E[\tilde{R}_j \mid F]$ denote respectively the conditional expected fundamental value and conditional expected return given the *cross section* of observed fundamental variables F, a (J*K) vector where J is the total number of firms and K is the total number of fundamental variables.

Since a firm's fundamental process is unobservable, it is necessary to specify the conditional expected fundamental value in terms of the cross section of observed fundamental variables, such as:

$$E[\tilde{V}_j \mid F] = \alpha_0 + \sum_k f_{kj}\alpha_k \tag{2.33}$$

where

α_0 is the intercept;

f_{kj} is the kth observed fundamental variable of firm j; and

α_k is the coefficient on the kth fundamental variable.

Substitute equation (2.33) into (2.31) and denote $E[\tilde{R}_j^a \mid F]$ as the conditional risk-adjusted expected return, we obtain the following equation: [11]

[11] $\frac{f_{km}}{P_m}$ is the weighted-average of $\frac{f_{kj}}{P_j}$, where the weights are given by the proportion of wealth invested in firm j. Specifically, $\frac{f_{km}}{P_m} = \sum_j X_{jm} \frac{f_{kj}}{P_j}$.

$$E[\tilde{R}_j^a \mid F] = (\frac{1}{P_j} - \frac{1}{P_m})\gamma_0 + \sum_k (\frac{f_{kj}}{P_j} - \frac{f_{km}}{P_m})\gamma_k \tag{2.34}$$

where

$$E[\tilde{R}_j^a \mid F] \equiv E[\tilde{R}_j \mid F] - R_{0m} - \beta_{jm}(E[\tilde{R}_m \mid F] - R_{0m})$$

$$\gamma_0 \equiv \alpha_0(1-\phi)(\lambda-1),\ \text{k} = 0 \tag{2.35.1}$$

$$\gamma_k \equiv \alpha_k(1-\phi)(\lambda-1),\ \text{k} = 1,\ldots,\text{K} \tag{2.35.2}$$

Equations (2.34) and (2.35) are the empirical form of the RAPM. Thus, explaining the prominent characteristic effects becomes an identification exercise of those firm characteristics that are highly correlated with risk-adjusted expected returns, and thus have significant coefficients γ_k (k = 0,…,K). Furthermore, as indicated by equation (2.35), the characteristic coefficients γ_k (k = 0,…,K) are jointly determined by "quasi's" sentiment measure (as captured by $(1-\phi)(\lambda-1)$) and the cross-sectional relationship between fundamental variables and expected fundamental values (as captured by α_k, where k = 0,…,K).

Before proceeding, observe that if the behaviors of "quasi's" are trivial, which means that they are very close to being fully rational ($\lambda \rightarrow 1$) or their proportion of wealth is very small ($\phi \rightarrow 1$), then any *ex post* test of the RAPM should have insignificant intercept γ_0 and loading γ_k on any explanatory variables. Furthermore, firm characteristics should not have any systematic impact on risk-adjusted expected returns. However, numerous empirical studies have documented highly significant B/M, E/P and size effects, which are explained below.

2.5.1 The B/M and E/P Effects

Our explanation of the observed E/P and B/M patterns is based on the argument that firm earnings and book values are highly correlated with expected fundamental values.

For the B/M effect, observe that a firm's book value of equity can be viewed as a proxy for the firm's operating size, which positively determines the firm's cash flows which in turn determine its fundamental value. As such, one would expect firm book values to be *positively* correlated with their fundamental values cross-sectionally, i.e., the bigger a firm, the more cash flow is expected. Thus, the coefficient α_1 (k = 1) obtained from the specification based on book values will be highly significant and *positive*. Berk (1995) makes a similar argument in his explanation of the B/M effect. Suppose that book value is the only significant fundamental variable, (2.34) can be rewritten as:

$$E[\tilde{R}_j^a \mid F] = (\frac{B_j}{P_j} - \frac{B_m}{P_m})\gamma_2 \tag{2.36}$$

where B_j/P_j and B_m/P_m denote the B/M ratios for firm j and for the market portfolio, respectively.

To explain the E/P effect, it is a common practice in fundamental analysis to forecast firm fundamental values using earnings as one of the explanatory variables. A positive relationship between a firm's earnings (denoted by E_j) and its future fundamental value is normally expected, i.e., firms with strong current earnings are expected to have more fundamental value. As long as such a relationship holds cross-sectionally, the coefficient α_1 (k = 2) obtained from specification (2.34) based on firm earnings E_j will be significant and *positive*. Suppose that firm earnings are the only significant fundamental variable, we can rewrite (2.34) as:

$$E[\tilde{R}_j^a \mid F] = (\frac{e_j}{p_j} - \frac{e_m}{p_m})\gamma_1 \tag{2.37}$$

where the lower case letters e_j and p_j denote the firm's earnings per share and price per share, and e_j/p_j and e_m/p_m are the E/P ratios for firm j and for the market portfolio, respectively.

Based on equations (2.36) and (2.37), we now explain the observed B/M and E/P patterns, namely, firms with high ratios tend to have higher abnormal returns. Observe from equation (2.35.2) that the sign of γ_k is determined jointly by the specification coefficient α_k (k = 1, 2) and the "quasi's" sentiment measure $(1-\phi)(\lambda - 1)$. We have already discussed positive α_k for book values (k = 1) and earnings (k = 2), and we know that $1-\phi$ is strictly positive. Thus, given the restrictions on (λ -1) as demonstrated in Proposition I, one can verify that γ_k (k = 1, 2) is positive given "quasi's" overreaction behavior, negative given underreaction behavior, and zero given fully rational behavior. Then it follows from equations (2.36) and (2.37) that the observed E/P and B/M patterns are consistent only with "quasi's" overreaction behavior. More specifically, equations (2.36) and (2.37) show that under an overreaction market, firms with above- (below-) average E/P and B/M ratios earn positive (negative) risk-adjusted expected returns. We would observe the opposite pattern under market underreaction or no systematic pattern in a fully rational market. These observations are the results of contrarian strategies adopted by "rationals".

To summarize, our analysis demonstrates the essence of the B/M and E/P anomalies. Both of them are the joint effects of "quasi's" overreaction behavior and positive cross-sectional correlation between fundamental variables (such as earnings and book values) and expected fundamental values. Because the abnormal returns, as in equation (2.34), do not conform to a covariance structure, we therefore argue against the risk-based explanation that the B/M and E/P effects are due to underlying distressed factors. Instead, these effects conform to a linear structure that is consistent with a characteristics-based model.

2.5.2 The Firm Size Effect

Firm size plays a crucial and theoretical role in determining expected returns. An examination of the first term on the right-hand-side of equation (2.34) reveals that even if firm size does not explain fundamental values, it is *theoretically* related to cross-sectional expected returns. Therefore, unlike such fundamental variables as firm earnings and book value that are *empirically* specified to predict expected fundamentals, the size effect is a *theoretical* result. To elaborate further, we distinguish between operating characteristics and market characteristics. A firm's earnings and book value are the operating characteristics that result from its operating activities, whereas firm size as measured by its market value of equity is a market characteristic determined by the market perception of a firm's value.

To develop an explanation of the size effect, we assume that firm size P_j does not explain expected fundamental value $E[\tilde{V}_j]$. There is no reason to expect such a relationship since, unlike firm earnings and book value, the market value of equity is not an operating characteristic of the firm. This assumption is analogous to the one adopted by Berk (1995) that no presumed relation exists between firm size and risk. Inspection of the equation (2.35.1) reveals that the significance of the size effect is determined jointly by the intercept α_0 obtained from the specification equation (2.33) and the "quasi's" sentiment measure $(1-\phi)(\lambda -1)$. This implies that the firm size effect should not be observed under either one of the following situations: either the "quasi's" are fully rational (λ-1 = 0 or 1-ϕ = 0), or the expected fundamental value $E[\tilde{V}_j]$ is fully explained by the specified fundamental variables, and thus $\alpha_0 = 0$. In the second situation, the specified fundamental-to-price ratios fully capture the systematic variation in risk-adjusted expected returns, leaving the unexplained proportion as random noise. However, empirical studies identify the firm size effect as the most pronounced anomaly. Thus, it should be interpreted as the joint effect of quasi-rational behaviors and misspecification of fundamental value.

We now investigate the implications of the pattern that small firms earn higher abnormal returns than large firms. Observe from equation (2.33) that the intercept α_0 will be significantly *positive* if the specified fundamental variables such as earnings and book values only *partially* explain $E[\tilde{V}_j]$. This is very likely to be true since other unidentified factors may also determine fundamental values. Thus, by similar arguments, the observed size effect can only be consistent with "quasi's" overreaction behavior. To illustrate, if none of the significant fundamental variables are identified, then equation (2.34) reduces to:

$$E[\tilde{R}_j^a \mid F] = (\frac{1}{P_j} - \frac{1}{P_m})\gamma_0 \tag{2.38}$$

Given positive γ_0 under an overreaction behavior, we observe positive (negative) risk-adjusted expected returns for firms with market capitalizations below (above) the market average. We would observe the opposite pattern under market underreaction or no size anomaly in a fully rational market. Zarowin (1990) finds that the overreaction evidence appears to be another manifestation of the size effect. Our analysis argues that they are essentially the same phenomenon.

The above explanation of the firm size anomaly shares the same spirit as the Berk (1995) critique of the firm size effect. Our assumption that firm size does not explain $E[\tilde{V}_j]$ is analogous to his assumption. Both arguments demonstrate that the size effect is a theoretical result that is caused by model misspecification. Therefore, firm size plays a unique role in asset pricing as a detector of model misspecification. As Berk argues, firm size can be used as a natural yardstick by which all the asset-pricing models could potentially be measured. If a model claims to capture all of the variation in cross-sectional expected returns, it must leave firm size with no residual explanatory power. However, there is a fundamental difference between the argument of Berk and ours. Berk's argument is risk-based and he shows that firm size is correlated with any misidentified risk factors, and therefore is consistent with the risk-based explanations. In contrast, we argue that the firm size effect is not related to risk since it does not conform to a covariance structure. Instead, what is misspecified is the firms' expected fundamental value, so that firm size should be correlated with any misidentified fundamental variables. It follows that the fundamental reason for the observed firm size anomaly is "quasi's" overreaction behavior, which is adjusted by "rationals" contrarian strategies. Finally, we advocate the use of firm size as a detector of the joint effect of quasi-rational behaviors and model misspecification. This implies that the firm size pattern should not be observed if either effect is absent.

2.6 Conclusions and Implications

We have reformulated an equilibrium asset-pricing model that accounts for quasi-rational behaviors. The RAPM rests on an economic setting that describes the heterogeneous behaviors of "rationals" and "quasi's", is derived within the classical utility maximizing framework, and emits a cross-sectional relationship that is determined by both market risk and two levels of adjustments. Unlike traditional consumption models, such as those of Campbell and Mankiw (1989) and Constantinides and Duffie (1996), which focus on heterogeneous investment constraints and heterogeneous incomes, our model describes heterogeneous behaviors in terms of cognitive biases. Unlike existing behavior models that focus on one single asset, our model explains the cross section of expected returns in a general equilibrium setting.

We present a formal analysis of the classification and behaviors of growth and value stocks. The RAPM explains the three prominent cross-sectional anomalies (the B/M, E/P, and size effects), showing that they are the joint effects of quasi-rational behaviors and model misspecifications. Our explanations can be viewed as an alternative to the Fama and French (1996) multi-factor explanations, and as support to the Daniel and Titman (1997) characteristic explanations. In contrast to their contending arguments that are based on different empirical procedures, our explanations are grounded in equilibrium theory that describes investors' economic behaviors. Our theory illustrates why firm characteristics rather than additional risk factors are priced.

As a characteristics-based model, the RAPM generates rich implications that may take different views from the risk-based models. Perhaps the most disturbing implication is that "rationals" can always make arbitrage profits from their contrarian strategies against the "quasi's". How to justify this implication in the real world of investments is yet to be discussed. For this purpose, we intend to answer the following question within the RAPM framework: how to identify arbitrage opportunities *ex ante*; or why it is so difficult to do so?

Theoretically, the RAPM is a one-period static model that requires the group of "rationals" to accurately predict the proportion and sentiment parameters of the "quasi's". In other words, both ($1-\phi$) and ($\lambda-1$) must be known *ex ante* for any given period. "Rationals" can thus adopt contrarian strategies as given by equation (2.12) to make arbitrage profits, driving the expected returns to the equilibrium when the period ends. Such a group of agents must always exist, so that no permanent divergence exists between fundamental values and firm prices. However, it by no means implies that one can always belong to the "rational" group. To identify arbitrage opportunities, the RAPM in the form of equations (2.28) and (2.29) challenges any investor to meet the following criteria with increasing order of importance and difficulty:

(I) Identify firm *average* characteristics that proxy for fundamentals in order to distinguish between above-average and below-average firms;

(II) Estimate the proportional weight of "quasi's", i.e., the value of ($1-\phi$);

(III) Evaluate the sign and the value of "quasi's" sentiment parameter ($\lambda-1$); and

(IV) Determine the time frame where (I), (II) and (III) apply.

We elaborate on each criterion in turn while assuming the other criteria are satisfied.

For criterion (I), empirical evidence already documents that B/M, E/P, and so forth are significant fundamental variables. Even if none of the variables are identified, that explains the size effect according to our arguments, at least the firm market capitalization can be used as the characteristic for arbitrageurs. The market average characteristics can then be calculated as the cross-sectional mean of all the firms in the sample. Although this method is subject to the Roll's (77) critique that the market is unobservable, one can take long and short positions in extreme portfolios. In conclusion, even if criterion (I) may be subject to theoretical criticism, it may be feasible to implement empirically.

As to criterion (II), intuition tells us that ($1-\phi$) is not easy to estimate empirically. Let us look at its impact in practice. Inspection of equation (2.28) reveals that a misspecified $(1-\hat{\phi})$

will result in either over- (if $\hat{\phi} < \phi$) or under- (if $\hat{\phi} > \phi$) adjustment (with respect to exact adjustment) for "quasi's" rational behaviors, causing firm expected returns to deviate from their rational ones. However, since the sign of $(1-\phi)$ is strictly positive, the direction (i.e., long or short) of contrarian strategies is always correct, even though the investment amount (i.e., how much) may not be. In other words, the investor who misestimates $(1-\phi)$ will gain from the "quasi's" but lose to "rationals" in the arbitrage game.

For criterion (III), empirically evaluating the sign and value of $(\lambda-1)$ is equally difficult but it has more impact. First, misspecifing the amount of $(\lambda-1)$ shares the same effect as misspecifing $(1-\phi)$. Furthermore, since the sign of $(\lambda-1)$ can be either positive or negative, the sign of the abnormal return can in turn be either positive or negative. A contrarian strategy that misidentifies the sign of $(1-\lambda)$ will make completely opposite investments that result in negative risk-adjusted returns. For example, an investor who bets on and acts against overreaction behaviors when "quasi's" are actually underreacting is expected to become the biggest loser at the end of the period.

Finally, the investor must determine the investment holding period where (I), (II) and (III) apply. Behavioral models (e.g., Barberis, Shleifer and Vishny, 1998; and Daniel, Hirshleifer and Subramanyam, 1998) and empirical studies (e.g., on closed-end funds by Lee, Shleifer and Thaler, 1991) provide compelling evidence that investor sentiment $(\lambda-1)$ and proportion $(1-\phi)$ may change over time stochastically. As a static model that explains cross-sectional returns, the RAPM has little to say about the time-series patterns of "quasi's" behavior. However, our model should apply to any one period, however large or small, where both $(1-\phi)$ and $(\lambda-1)$ remains relatively stable. An investor can make arbitrage profits on a period-by-period basis using a process such as: For the first period, he applies a contrarian strategy based on his existing information on criterion (I) to (III). As soon as this period ends, he needs to apply a different contrarian strategy based on a new set of criterion (I) to (III), and so on. In other words, it requires that the investor consistently belong to the group of "rationals" over time. However, reality suggests that few people possess such ability. Though an investor may belong to the "rationals" group for any one period *by chance*, he is unlikely to belong to the group for all the periods due to the stochastic behaviors of (λ-1) and (1-ϕ). The public saying that "I know the market is overreacting but don't know when it's going to finish" is a common fallacy that fails to satisfy criterion (IV).

Finally, we emphasize that the RAPM offers just one of the possible explanations for the characteristics-based models. Other possibilities, such as the liquidity effect, agency problems, or the prior that investors may systematically prefer certain characteristics to other, still remain open for further investigation by us and other researchers.

CHAPTER 3

A REFORMULATED ASSET PRICING MODEL OF RISK, LIQUIDITY AND RETURNS

3.1 Introduction

As the most salient features of capital markets, the risk-return and liquidity-return models have been the central focus of many studies. Traditionally, the two relations are investigated in separate lines of research with different data sampling frequencies. While the risk-return models focus on the co-movement of a single asset or a portfolio of assets with the market portfolio using monthly data, most of the liquidity literature views transaction costs as an attribute of a representative homogenous asset based on high frequency transaction data. This is evident from the abundant market microstructure literature that measures the components of transaction costs that are attributed to information asymmetry and inventory costs (e.g., Glosten and Harris 1988; Stoll 1989; Hasbrouck 1991; George, Kaul and Nimalendran 1991).

There has been growing interest to examine the commonality of liquidity for a cross section of assets. Three empirical studies document the systematic component of liquidity based on different data sets and statistical techniques. Chordia, Roll and Subrahmanyam (2000) perform a "market model" time series regression, which is to regress the daily percentage changes in liquidity variables for an individual asset on the cross-sectional average of the same liquidity variables. Using a large transaction database consisting of more than one thousand NYSE stocks, they find a highly significant liquidity beta, which suggests a significant liquidity correlation among individual securities. Huberman and Halka (2001) obtain the daily quotes for 240 size-sorted NYSE stocks. They estimate time series models for size-sorted portfolios using bid-ask spreads and market depths as proxies for liquidity. They also document the existence of commonality in liquidity. The data sample of Hasbrouck and Seppi (2001) is limited to the intra-day trading quotes for the thirty Dow Jones stocks. They use principal components and canonical correlation analyses to identify the common factors in liquidity. In contrast to the findings of the previous two studies, they find that the common factors in liquidity are relatively small. All of these documented results are empirically based and exploratory in nature.

With regard to the liquidity-return relation, Amihud and Mendelson (1986) and Brennan and Subrahmanyam (1996) realize that transaction costs should be cross-sectionally related to before-cost or gross expected returns since the after-cost or net returns should be related only to risk in market equilibrium. Specifically, Amihud and Mendelson (1986) find evidence that asset returns include a significant premium for illiquidity. Brennan and Subrahmanyam (1996) also find a significant relation between monthly stock returns and illiquidity measures after controlling for risk. Both studies use prominent risk factor models to adjust for the risk-return effect. Amihud and Mendelson (1986) use the CAPM (Sharpe 1964, Black 1972) and Brennan and Subrahmanyam (1996) use the Fama-French three-factor model (Fama and French, 1993) to adjust for risk. In both studies, liquidity is treated as an exogenous variable to the risk-return models. The linkage between the risk-return and liquidity-return relations however is somewhat ad hoc.

Given the evidence that liquidity correlates across assets and the liquidity-return relation is significant, a fundamental issue arises as to whether the illiquidity premium is largely due to a non-diversifiable risk effect (stochastic or dynamic) or an expected characteristic effect (deterministic or static). This issue is highlighted by Chordia et al. (2000) for further research since both effects can explain their findings. This question gives rise to a theoretical issue, which is to identify the interrelationship between the covariance liquidity (risk) and the expected liquidity (characteristic), and a broader issue which is to develop the cross-sectional linkage between the risk-return relation and the liquidity-return relation. Without a cross-sectional theory, it is not clear how expected liquidity and covariance liquidity impact asset pricing, and how the liquidity-return relation interacts with the risk-return relation in a market-wide cross section.

In this paper, we reformulate an asset-pricing model that unifies risk, liquidity, and returns and present empirical evidence on the risk effect and liquidity effect. Our insight is to link a firm's before-cost fundamental process and the liquidation process with a firm's after-cost consumption process. We derive a rational investor's portfolio solution within the classic consumption-investment framework. The key result is a cross-sectional equation that specifies a firm's expected return as consisting of three components: an interest rate term where the market-average expected liquidity is an endogenous variable; a market risk term where the consumption beta is a weighted average of the fundamental beta and the liquidity beta; and a firm-specific term determined by the firm's liquidity deviation from that of the market portfolio. The key observation is that the illiquidity premium has both a non-diversifiable risk effect and a firm-specific characteristic effect.

We then discuss the empirical implications derived from our theoretical model and test the implications using the monthly Canadian market data sampling from March 1977 to December 1998. Our empirical results indicate that the fundamental beta plays a much more important role than the liquidity beta due to its predominant weight. Hence, the covariance liquidity (dynamic) is unlikely to explain before-cost returns within our monthly sample. We then test the liquidity-return relation and find that the expected liquidity (static) is significant in explaining before-cost cross-sectional returns. We attribute the different findings between our study and Chordia et al. (2000) to different data sampling frequencies. We also test the risk effect which is found to be insignificant after controlling for the size effect. Our final test presents evidence that the expected liquidity effect and the size effect are both significant for explaining before-cost returns. We interpret the size effect as model misspecification of expected liquidity.

The remainder of the paper is organized as follows. In the second section, we specify the fundamental, liquidity and consumption processes and derive the portfolio solution within the classic consumption-investment framework. The asset-pricing interpretations also are discussed in this section. In the third section, we present the key empirical implications derived from our theory. We describe the Canadian market data in the fourth section. In Section five, we discuss empirical procedures and present test results. We conclude the paper and offer further interpretations of the empirical results in the last section. The derivation of our theory is found in the Appendix 3-A.

3.2 Reformulation of the Consumption-Investment Problem for Risk, Liquidity and Returns

We develop a capital market equilibrium model within the classic consumption-investment framework. Consider a one-period economy where I risk-averse investors (i = 1,...,I) attempt to maximize their consumption utility derived from investing and consuming J risky assets (j = 1,...J).

Currently at time 0, investor i is endowed with an initial wealth W_i. He solves his consumption-investment problem by choosing a current consumption level denoted by C_i and then allocating his remaining wealth to J risky assets. His end-of-period wealth is a random variable determined solely by his time-0 investments. At time 1, he liquidates all the asset holdings in his portfolio for end-of-period consumption denoted by $\tilde{C}_i$. The investor's objective is to maximize his own utility that is ultimately derived from his consumption plan (C_i, $\tilde{C}_i$). Let X_{ij} be the proportion of his remaining wealth invested in the jth firm and V_j be the firm's current market value. Then the individual's consumption-investment problem can be generally formulated as:

$$\underset{\{C_i, X_{ij}\}}{Max} \quad U_i(C_i, \tilde{C}_i) \tag{3.1}$$

s.t.

$$W_i = C_i + \sum_j X_{ij} V_j \tag{3.2}$$

It is reasonable to assume that a firm's consumption process is determined by two independent processes: a before-cost or gross fundamental process that drives the firm's fundamental value before any transaction costs, and a liquidation process that bears transaction costs.

Suppose that a firm's before-cost fundamental process follows:

$$\tilde{V}_j = E[\tilde{V}_j] + \varepsilon_j \qquad \text{where } \varepsilon_j \sim N(0,\sigma_j^2), \qquad j = 1,\ldots,J \tag{3.3}$$

where

$E[\tilde{V}_j]$ denotes firm j's expected fundamental value at the end of the period; and

$\tilde{\varepsilon}_j \sim N(0,\sigma_j^2)$ is a random shock to the fundamental process with variance σ_j^2.

Our setting requires that a firm's consumption process is not only determined by its fundamental process, but is also affected by its liquidation process, which is uncorrelated with the firm's fundamental process.[12] Specifically, a firm's liquidation process is written as:

$$\tilde{L}_j = E[\tilde{L}_j] + \eta_j \qquad \text{where } \eta_j \sim N(0,\theta_j^2), \qquad j = 1,\ldots,J \tag{3.4}$$

where

$E[\tilde{L}_j]$ denotes firm j's expected transaction costs at the end of the period; and

$\eta_j \sim N(0,\theta_j^2)$ is the unexpected transaction costs with variance θ_j^2.

Thus, a firm's consumption process can be represented as the combination of the two processes:

$$\tilde{C}_j = \tilde{V}_j - \tilde{L}_j \tag{3.5}$$

[12] We emphasize that our choice of liquidity is for empirical purposes. In theory, any process could be chosen as long as it meets the following two conditions: 1) It is related to consumption; and 2) It is independent of the firm's fundamental process.

where $\text{cov}(\tilde{V}_j, \tilde{L}_j) = 0$ indicates independency between the fundamental process and the liquidation process. The minus sign suggests transaction costs due to liquidation have the effect of reducing the asset's consumption value.

Given the joint normality assumption of $\tilde{V}_j$ and $\tilde{L}_j$, investor i's end-of-period consumption is completely described by his portfolio's expected consumption value and consumption variance. Specifically,

$$E[\tilde{C}_i] = E[\tilde{V}_j] - E[\tilde{L}_j] = \sum_j X_{ij} E[\tilde{V}_j] - \sum_j X_{ij} E[\tilde{L}_j] \tag{3.6}$$

$$\begin{aligned} Var(\tilde{C}_i) &= \sigma_i^2 + \theta_i^2 = \sum_j \sum_{j'} X_{ij} X_{ij'} [\text{cov}(\tilde{V}_j, \tilde{V}_{j'}) + \text{cov}(\tilde{L}_j, \tilde{L}_i)] = \\ &\sum_j X_{ij} \text{cov}(\tilde{V}_j, \tilde{V}_i) + \sum_j X_{ij} \text{cov}(\tilde{L}_j, \tilde{L}_i) \end{aligned} \tag{3.7}$$

where $\tilde{V}_i = \sum_{j'} X_{ij'} \tilde{V}_{j'}$ and $\tilde{L}_i = \sum_{j'} X_{ij'} \tilde{L}_{j'}$ denote the random fundamental process and liquidation process of the investor's portfolio, respectively.

We then write the maximization problem in equation (3.1) as: [13]

$$\underset{\{C_i, X_{ij}\}}{Max} \quad U_i(C_i, E[\tilde{C}_i], \text{var}[\tilde{C}_i]) \tag{3.8}$$

subject to

Equations (3.2), (3.6) and (3.7).

The problem can be solved within the classic consumption-investment framework. In the Appendix 3-A, we derive the solution for the problem based on the seminal work of Fama (1976) as follows:

$$E[\tilde{R}_j] = R_{0m} + \beta_{jm}^C (E[\tilde{R}_m] - R_{0m}) + (\frac{E[\tilde{L}_j]}{V_j} - \frac{E[\tilde{L}_m]}{V_m}) \tag{3.9}$$

where

[13] This is the certainty-equivalent form of the expected utility objective function.

$R_{0m} = \gamma_{m0} + E[\frac{\tilde{L}_m}{V_m}]$ is interpreted as the interest rate term; (3.10)

$\beta^C_{jm} = \rho^V \beta^V_{jm} + \rho^L \beta^L_{jm}$ is interpreted as the consumption beta which is the weighted average of the fundamental beta and the liquidity beta; (3.11)

$\beta^V_{jm} = \frac{\mathrm{cov}[\tilde{R}_j, \tilde{R}_m]}{\mathrm{var}[\tilde{R}_m]}$ is defined as the before-cost fundamental beta; (3.12)

$\beta^L_{jm} = \frac{\mathrm{cov}[\frac{\tilde{L}_j}{V_j}, \frac{\tilde{L}_m}{V_m}]}{\mathrm{var}[\frac{\tilde{L}_m}{V_m}]}$ is defined as the liquidity beta; (3.13)

$\rho^V = \frac{\mathrm{var}[\tilde{R}_m]}{\mathrm{var}[\tilde{R}_m] + \mathrm{var}[\frac{\tilde{L}_m}{V_m}]}$ is the proportional weight of the fundamental beta; (3.14)

$\rho^L = \frac{\mathrm{var}[\frac{\tilde{L}_m}{V_m}]}{\mathrm{var}[\tilde{R}_m] + \mathrm{var}[\frac{\tilde{L}_m}{V_m}]}$ is the proportional weight of the liquidity beta; (3.15)

$\frac{E[\tilde{L}_j]}{V_j}$ is defined as the expected liquidity return of firm j; and

$\frac{E[\tilde{L}_m]}{V_m}$ is defined as the expected liquidity return of the market portfolio.

Given the above definitions, we hereby propose the following theorem to explain a firm's expected return.

Theorem:

A firm's expected return is composed of the following three components:

(i) *An interest rate term that captures both the marginal rate of substitution between successive consumptions and the expected liquidity of the market portfolio;*

(ii) *A market risk term determined by the weighted average consumption beta multiplied by one unit of risk premium, with the weights given by the proportional variance of the fundamental return and the proportional variance of the liquidity return; and*

(iii) *A firm-specific term that is captured by the deviation of a firm's expected liquidity from that of the market portfolio.*

R_{0m} is composed of two components. The first component maintains the original definition of the interest rate as the marginal rate of substitution between two successive consumptions. The second component, which captures the expectation of the market-average liquidity return, is another endogenous variable that determines the interest rate. Hence, this component has the interpretation of an interest rate premium that compensates for the adverse investment environment that reduces the overall consumption value due to illiquidity.

The second part of the theorem shows that the RAPM preserves the risk-return relationship as in the CAPM. The distinct feature of our theorem is that the beta is a weighted average of the fundamental beta and the liquidity beta (as defined in equations 3.12 and 3.13), with the weights given by the proportional variance of the fundamental return (as defined in equation 3.14) and the proportional variance of the liquidity return (as defined in equation 3.15) for the market portfolio. Whether the fundamental beta or liquidity beta has more weight in the determination of the consumption beta is an empirical issue to be examined in our empirical tests.

With regard to the third part of the theorem, equation (3.9) indicates that the firm-specific term is not determined cross-sectionally by the firm's own expected liquidity, but instead by the deviation from that of the market portfolio. A firm earns a higher (lower) illiquidity premium if it has a higher (lower) than average liquidity return. A firm with the market-average liquidity return is not required to earn any additional illiquidity premium. This feature is analogous to that of the liquidity beta of an asset, namely, the illiquidity risk of the asset is not determined by its own variance of liquidity return, but is determined by its covariance with that of the market portfolio. Thus, the market portfolio plays a critical role as a benchmark for determining the illiquidity premium in different ways. Specifically, a firm's sensitivity to illiquidity risk conforms to a covariance structure, whereas its liquidity deviation is determined by a linear technology. Hence, a firm's illiquidity premium may come from two sources: the covariance liquidity where illiquidity is an underlying risk factor and the expected liquidity where illiquidity is regarded as a firm characteristic. Whether the covariance or characteristic structure plays a more important role in determining the illiquidity premium is subject to empirical examination.

3.3 Empirical Implications

In this section, we discuss various empirical issues implied by equations (3.9) through (3.15).

3.3.1 Covariance Liquidity Versus Expected Liquidity

As mentioned earlier, the liquidity effect of an asset constitutes both a covariance effect determined by the asset's weighted liquidity beta and a characteristic effect determined by its linear deviation from the market portfolio. As long as appropriate liquidity proxies are identified (as are presented in the next paragraph), the liquidity beta can be estimated from time-series observations using the traditional procedure of estimating the fundamental beta. In addition, the significance of the covariance effect depends on the percentage of the market portfolio's liquidity variance in its consumption variance (i.e., consumption variance = fundamental variance + liquidity variance). The fundamental variance and liquidity variance of the market portfolio also can be estimated from time series data. Thus, examining the covariance liquidity effect renders two empirical implications. The first is a test of whether the weight of the liquidity beta is significant relative to that of the fundamental beta. The second is a test of whether the risk premium (i.e., $E[\tilde{R}_m]-R_{0m}$) is significant in a cross-sectional regression of asset returns $E[\tilde{R}_j]$ on consumption beta β^C_{jm}, j = 1,...,J.

We now discuss the empirical implications of the expected liquidity effect. Since a firm's liquidity process is unobservable, it is necessary to specify the firm's expected liquidity in terms of observed variables, such as:

$$E[\tilde{L}_j]=\alpha_0+\sum_k L_{kj}\alpha_k \tag{3.16}$$

where

α_0 is the intercept;

L_{kj} is the kth observed liquidity variable of firm j; and

α_k is the coefficient on the kth liquidity variable.

Substituting equation (3.16) into (3.9) yields the following equation: [14]

$$E[\tilde{R}_j]=R_{0m}+\beta^C_{jm}(E[\tilde{R}_m]-R_{0m})+\sum_k\left(\frac{L_{kj}}{V_j}-\frac{L_{km}}{V_m}\right)\alpha_k+\left(\frac{1}{V_j}-\frac{1}{V_m}\right)\alpha_0$$

(3.17)

where

$$R_{0m}=\gamma_{m0}+\sum_k\frac{L_{km}}{V_m}\alpha_k+\frac{\alpha_0}{V_m} \tag{3.18}$$

Empirical studies use various spread-related measures as the observable liquidity variables. For example, Amihud and Mendelson (1985) use the quoted bid-ask spread as a proxy for liquidity. Chordia, Roll and Subrahmanyam (2000) use five spread-related measures: the

---

[14] $\frac{L_{km}}{V_m}$ is the weighted-average of $\frac{L_{kj}}{V_j}$, where the weights are given by the proportion of wealth invested in firm j. Specifically, $\frac{L_{km}}{V_m}=\sum_j X_{jm}\frac{L_{kj}}{V_j}$.

quoted spread, the proportional quoted spread, depth, effective spread, and proportional effective spread as liquidity variables. In our empirical test, L_{kj} (where k = 1) in equation (3.16) is represented by the quoted bid-ask spread or the effective spread for firm j.[15] Thus, equations (3.17) and (3.18) can be simplified to:

$$E[\tilde{R}_j] = R_{0m} + \beta_{jm}(E[\tilde{R}_m] - R_{0m}) + (\frac{l_j}{p_j} - \frac{l_m}{p_m})\alpha_1 + (\frac{1}{V_j} - \frac{1}{V_m})\alpha_0 \tag{3.19}$$

where

$$R_{0m} = \gamma_{m0} + \frac{l_m}{p_m}\alpha_1 + \frac{\alpha_0}{V_m} \tag{3.20}$$

l_j denotes the quoted or effective spread per share, and p_j is the market price per share of firm j. Thus, l_j/p_j and l_m/p_m are interpreted as the proportional quoted spread (PQS) or proportional effective spread (PES) for firm j and for the market portfolio, respectively. Time series of l_j/p_j and l_m/p_m are observable.

Given equations (3.19) and (3.20), examining the expected liquidity effect is to test whether α_1 is statistically significant in a cross-sectional regression of asset returns on the proportional spread deviations from that of the market portfolio.[16] By construction, we expect α_1 to be positively related to expected return. If α_1 is significantly positive, we should observe both a market-wide and a firm-specific relationship. In an illiquid market, the market interest rate R_{0m} is always higher than in a perfect market where $l_j = 0$. The excess interest rate premium is required to compensate for the non-perfectly liquid investment environment, and it is a common

[15] L_j denotes the spread for firm j's total shares outstanding, i.e., $L_j = l_j * n_j$, where l_j is the spread per unit share and n_j is the total shares outstanding. Similarly, firm j's total market value is denoted by $V_j = p_j * n_j$, where p_j is the market price per share.

[16] We defer the discussion of the last terms of equations (19) and (20) to the next subsection. We assume $\alpha_0 = 0$ for the current discussion.

component to every firm in the market. The interest rate premium is positively related to the market-average proportional spread with coefficient $\alpha_1 > 0$.[17]

With respect to the cross-sectional relationship, the expected return of a given firm is compensated by its proportional spread deviation from that of the market average firm. If the risk level is fixed, then firms with higher (lower) than the market-average proportional spread, i.e., $l_j/p_j > l_m/V_m$ ($l_j/p_j < l_m/p_m$) require higher (lower) risk-adjusted returns. Firms with the market-average proportional spread, i.e., $l_j/p_j = l_m/p_m$, earn no risk-adjusted abnormal returns. Excess returns are considered illiquidity premium, and compensate investors who make investments in those firms with above-average trade costs. Similarly, investors expect a negative premium or a discount to invest in those firms with below-average trade costs.

3.3.2 A Critique of the Size Effect

Firm size plays a crucial and theoretical role in determining expected returns.[18] The above analysis of the cross-sectional relationships assumes that the observed liquidity variables (i.e., the quoted or effective spread) L_{kj} fully specify the expectations of liquidity so as to leave any unexplained component statistically insignificant. In other words, the intercept $\alpha_0 = 0$ in equation (3.16), and the last terms of equations (3.19) and (3.20) do not exist. This implies that the size effect should not be observed if the liquidity variables fully capture the cross-sectional variation of liquidity. However, if the liquidity variables misspecify the liquidity expectations in equation (3.16), then the intercept α_0 will be significant in the cross section. By examining the last terms of equations (3.19) and (3.20), we observe that the deviation of the reciprocal of a firm's market capitalization from that of the market portfolio explains cross-sectional expected returns. Thus, unlike such liquidity variables as quoted or effective spreads that are *empirically*

[17] This is consistent with the empirical findings by Black et al. (1972), Fama and MacBeth (1973), amongst others, that the estimated mean return on the zero-beta portfolio exceeds the risk-free rate of interest.

[18] The size effect was discovered by Banz (1981).

identified to explain expected liquidity, the size effect is a *theoretical* result that arises from model misspecification.

The significance of the size effect depends on the significance of the intercept α_0. Observe from equation (3.16) that the intercept α_0 will be significantly *positive* if the identified liquidity variables such as quoted or effective spreads only *partially* explain $E[\tilde{L}_j]$. This is very likely to be true since other unidentified factors may also determine cross-sectional liquidity. Given a fixed level of risk and illiquidity premium, inspection of equation (3.19) reveals that those firms with smaller (larger) than average market size, i.e., $1/V_j > 1/V_m$ ($1/V_j < 1/V_m$), earn positive (negative) expected returns. The excessive returns are interpreted as size premium for the size deviation between the firm and the market. The size premium accounts for the misspecification of cross-sectional expected liquidity.

The above explanation of the firm size anomaly is similar in spirit to the Berk (1995) critique of the firm size effect. Both arguments demonstrate that the size effect is a theoretical result that is caused by model misspecification. Therefore, firm size plays a unique role in asset pricing as a detector of model misspecification. As Berk argues, firm size can be used as a natural yardstick by which all the asset-pricing models potentially can be measured. If a model claims to capture all of the variation in cross-sectional expected returns, it must leave firm size with no residual explanatory power. However, there is a fundamental difference between the argument of Berk and ours. The Berk argument is risk-based as he argues that firm size is correlated with any misidentified risk factors, and therefore is consistent with the risk-based explanations. However, his argument does not explain why the size effect should be related to risk factors instead of the size characteristic itself. In contrast, we show theoretically that the firm size effect is not related to risk since it does not conform to a covariance structure. Our theory, as embodied in equations (3.19) and (3.20) shows that a firm's size is linearly related to that of the market portfolio. Because cross-sectional expected liquidity is misspecified, firm size should be correlated with any misidentified liquidity variables. The significance of the size effect is subject to the empirical tests performed below.

3.3.3 Testable Implications

To summarize, we test the four empirical implications associated with equations (3.19) and (3.20). First, we examine whether the fundamental beta β^V_{jm} or the liquidity beta β^L_{jm} is the dominant component of the consumption beta β^C_{jm} by examining the proportional variance of ρ^V relative to ρ^L. Second, we examine whether the consumption beta can explain expected returns. The null hypothesis is that the risk premium is zero in cross-sectional regressions. A rejection of the null supports the risk-return relationship. Third, we examine whether the expected liquidity effect is significant by testing the significance of α_k in a cross section, and discuss the relative significance between the characteristic effect and the covariance effect for liquidity. Fourth, we examine whether the size effect is significant by testing the significance of α_0, and whether the characteristic effect is still significant if jointly tested with the size effect.

3.4 Data Description

Our study examines monthly data on common stocks listed in the CFMRC database from March 1977 to December 1998, for a total of 262 time series observations. The monthly database records monthly prices, returns, betas, and shares outstanding for stocks listed on the Toronto Stock Exchange (TSE) since January 1950. The daily database contains the closing bid and closing ask quotes of listed stocks for each trading day starting from March 1, 1977. To convert the daily bid-ask quotes to monthly series, we take the averages of the daily closing bid and ask quotes over each month from March 1977 to December 1998. The monthly averaging is performed for each stock that passes through a screening process which controls for data error, adjusts for thin trading, and removes stale trades in the daily database. Specifically, the screening process filters out those firms with market values below CAD $2 million or firms with a market price per share less than CAD $2 or transactions with invalid bid and ask quotes. In addition, a stock must have at least three days of valid closing bid and ask quotes in any given month to be included in the sample. To extract information from the monthly database, firms with a share price below $2 and a market capitalization below $2 million are deleted. We then join the monthly average bid and ask quotes with the monthly returns, betas, and market values for each selected stock.[19] Proportional spreads are calculated for each stock for each month. By definition, the proportional quoted spread (PQS) is calculated as $(p_{jt}^{a} - p_{jt}^{b}) / p_{jt}^{m}$ where p_{jt}^{a} (p_{jt}^{b}) denotes the closing ask (bid) price, and $p_{jt}^{m} = (p_{jt}^{a} + p_{jt}^{b}) / 2$ is the mid point of the bid-ask spread of firm j at time t. The proportional effective spread (PES) is calculated as $2 \mid p_{jt}^{x} - p_{jt}^{m} \mid / p_{jt}^{x}$ where p_{jt}^{x} is the closing trade price of firm j for month t.

For the risk-free rate, we convert the 90-day Canadian Treasury bill series into a 30-day series for the same period. We use three proxies for the market portfolio. The first two proxies are the equally weighted market index (EWIR) and the value weighted market index (VWIR) reported in the CFMRC database. We create a new proxy for the market portfolio by taking the cross-sectional mean (CSMR) of the selected sample for each month t = 77:03 to 98:12.[20] This proxy is necessary to create the liquidity time series for the market portfolio because the CFMRC database does not have bid and ask spreads for the market indices. Time series statistics are reported in Table 3.1 for these three market portfolio proxies. We observe that the CSMR proxy has a time series mean close to that of the EWIR but has a lower variance than the EWIR. The lower variance is due to our screening process that removes some outliers from the sample data.

[Insert Table 3.1 about here]

[19] We make an inner join so as to make sure that a firm has valid data for return, beta, size, bid and ask for every entry.

[20] The empirical results reported in this paper are based on the cross-sectional mean (CRMR) proxy for the market portfolio. We also use the equally weighted and value weighted market indices as proxies. There is no material difference in the results between the three proxies.

3.5 Empirical Procedures and Results

3.5.1 Fundamental Beta Versus Liquidity Beta

We first test the relative significance between the fundamental beta and the liquidity beta by examining their proportional weights ρ^V and ρ^L given by equations (3.13) and (3.14). To calculate the variance, we need the time series of the fundamental returns and proportional spreads of the market portfolio. The time series of fundamental returns is proxied by the CSMR series. For the proportional quoted and effective spreads of the market portfolio, we use the cross-sectional mean of spreads for each month in our sample. Doing so, we create a time series of before-cost returns and proportional spreads from 77:03 to 98:12. The time-series mean, variance and proportional variance are reported in Table 3.2.

[Insert Table 3.2 about here]

Table 3.2 reports the percentage of fundamental variance (99.78%) and the percentage of liquidity variance (0.22%) in the total consumption variance. The results indicate that the before-cost return variance completely dominates the liquidity variance so that the consumption variance is almost entirely determined by the fundamental variance. This evidence makes us conclude that the fundamental beta β^V_{jm} is a predominant component of the consumption beta β^C_{jm} and that the impact of the liquidity beta β^L_{jm} is negligible. Thus, in the cross-sectional tests below, we estimate only the fundamental beta.

3.5.2 Cross-Sectional Test of the Size and Beta Effects

We adopt the Fama-MacBeth (1973) two-step portfolio approach. Fama and French (1992) find that the beta effect is highly correlated with the size effect. Thus, to control for the size effect, we use the Fama-French (1992) portfolio approach which first sorts stocks into 5 size portfolios each of which is then subdivided into 5 beta portfolios. The 5*5 size-beta portfolios are formed for each month t = 77:03 to 98:12. In the first step, we estimate the Dimson beta for each portfolio using time series regressions.[21] In the second step, we run cross-sectional tests by regressing the returns of the 25 portfolios on portfolio sizes and portfolio betas. The cross-sectional regressions are performed for each month from 77:03 to 98:12. The time series of size and beta coefficients are used to test the significance of the size and beta effects. Table 3.3 reports the test results.

[Insert Table 3.3 about here]

The results indicate that the size effect is significant at the 5% level (t-stat = 3.58), whereas the beta effect is insignificant (t = 1.58). This evidence is consistent with that of Elfakhani, Lockwood and Zaher (1998) who reach the same conclusion for their Canadian data sample (75:06 to 92:12) obtained from the same database. The evidence also concurs with the argument proposed by Fama and French (1992) that, after controlling for the size effect, the beta is flat in tests of the risk-return relationship. The positive sign of the size coefficient is consistent with empirical observations. Specifically, firms with below (above) average market capitalization earn positive (negative) size premia. Average-sized firms do not earn this characteristic premium.

3.5.3 Cross-Sectional Test of the Expected Liquidity Effect

Our first test results indicate that firm returns are unlikely to be explained by the co-movement in liquidity measures due to the extremely small liquidity variance of the market portfolio. The current test intends to examine whether the cross-sectional returns can be explained by the expected liquidity effect, which is determined by the linear deviation of a firm's liquidity return from that of the market portfolio. To this end, we construct 5*5 beta-liquidity portfolios. The first-pass portfolios are sorted on pre-ranking betas in order to test the risk effect and to control for it if it is significant. Then, each first-pass portfolio is sorted by PQS or PES into five

---

[21] To estimate the Dimson beta, we first perform the following OLS regression:

$$R_{i,t} = \alpha_i + \beta_{i,1} R_{m,t} + \beta_{i,2} R_{m,t-1} + \varepsilon_{i,t}$$

where $r_{i,t}$ is the return for firm i for month t, $R_{m,t}$ is the return on the market portfolio for month t, and $R_{m,t-1}$ is the return on the market portfolio for month t-1. The Dimson beta for firm i is then estimated as:

$$\hat{\beta}_i = \hat{\beta}_{i,1} + \hat{\beta}_{i,2}.$$

liquidity portfolios. To implement the Fama and MacBeth (1973) procedure, we first estimate the Dimson beta for each portfolio and calculate the cross-sectional mean as the PQS and PES for each portfolio. In the second step, portfolio returns are regressed on portfolio betas and portfolio PQS or PES. We run 262 (77:03 – 98:12) such cross-sectional regressions and obtain coefficients in a time series to test for significance.

Test results, which are reported in Table 3.4, strongly support the expected liquidity effect. This effect is highly significant at the 5% level for both the PQS and PES measures. However, the statistics on the beta effect again exhibit a flat risk-return relationship with insignificant t-statistics.

[Insert Table 3.4 about here]

3.5.4 Cross-Sectional Test of the Size and Liquidity Effects

Previous results suggest that the size effect and the liquidity effect are significant when tested individually with the beta effect. A natural question arises as to whether or not the two effects are correlated with each other. In other words, if the size effect is controlled for, can the expected liquidity effect still explain cross-sectional returns? The current test aims to answer this question using the Fama-MacBeth two-step procedure. To this end, five portfolios are sorted by firm size in the first pass, then each portfolio is divided into five liquidity portfolios based on the firms' PQS or PES measure.

The results, which are reported in Table 3.5, indicate that both the size and liquidity effects are highly significant at the 5% level. This means that, even after controlling for the size effect, the expected liquidity measures still explain cross-sectional before-cost returns.

[Insert Table 3.5 about here]

3.6 Conclusion and Interpretations

Our paper has both theoretical and empirical intents. The theory part of the paper reformulates an asset-pricing model that unifies risk, liquidity, and returns. Cross-sectional expected returns are explained by three components: an interest rate term that includes an endogenous market average expected liquidity; a market risk term determined by a weighted average consumption beta; and a firm-specific term determined by the expected liquidity deviation from that of the market portfolio. The most important observation of the model is that the illiquidity premium contains both a non-diversifiable covariance term and a firm-specific characteristic term. To determine the relative importance between the two terms, along with their other cross-sectional implications, we perform empirical tests using Canadian market data on a monthly basis.

Our empirical study confirms some existing findings observed from other empirical research, and generates new results that warrant further discussion. Firstly, our finding of a significant size effect and insignificant beta effect is consistent with the observed evidence in both the U.S. markets (Fama and French, 1992) and the Canadian markets (Elfakhani et al, 1998). Fama and French (1993) interpret the size effect as a systematic risk on some unobserved distress factors. However, other studies (e.g., Daniel and Titman, 1997) present alternative evidence that the size effect is not risk related but is due to the size characteristic itself. Our theory in the form of equations (3.19) and (3.20) lends support to the characteristic argument since a firm's size premium is determined by its linear deviation from an average-sized firm. The underlying reason for the observed size effect is due to the misspecification of the expected liquidity of firms. In fact, in our cross-sectional test of the beta and size effects, it is essentially to set the liquidity variables L_{kj} in equation (3.16) to be zero. This leaves the expectation of liquidity totally unspecified. The significance of the size effect implies that there exists a significant amount of before-cost return variation that is not explained.

Secondly, our theoretical model demonstrates that the impact of the liquidity beta is determined by its proportional weight relative to that of the fundamental beta. The empirical results show that the market portfolio exhibits very little liquidity variability over time compared to the before-cost return variance. Thus, the liquidity beta is unlikely to explain the cross-sectional variation in before-cost returns due to its negligible weight. Furthermore, our tests show that the expected liquidity (a firm's liquidity deviation from that of the market portfolio) plays a significant role in explaining before-cost returns. This finding provides the answer to the issue raised by Chordia, Roll and Subrahmanyam (2000). They call for further research to determine the relative importance between two channels of liquidity: a static channel influencing average trade costs (expected liquidity in our context) and a dynamic channel influencing risk (covariance liquidity in our context). We present evidence that the static channel is far more material than the dynamic channel in our monthly sample. In contrast, in the study of Chordia et al. (2000), liquidity measures such as the PQS and PES are found to exhibit substantial variability. We hypothesize that the different findings may be attributed to different data measurement frequencies between their study and ours. Chordia et al. (2000) use intra-day transaction data from the New York Stock Exchange (NYSE) for the year 1992. Since intra-day transaction data is highly noisy, they calculate *daily* liquidity measures by averaging valid *intra-day* trades for each stock. In contrast, our study calculates *monthly* liquidity measures by averaging valid *inter-day* bid and ask quotes for each stock. In addition, Chordia et al. (2000) create *daily* liquidity time series for the market portfolio by taking the cross-sectional mean, whereas we create the *monthly* liquidity time series for the market portfolio. Thus, it is reasonable to expect that their *daily* time series exhibits much more variability than our *monthly* time series. However, our theory implies that the significance of the liquidity beta is not only related to its own liquidity variance, but also depends on its *relative* weight with respect to the before-cost return variance. Therefore, in future research, it would be of interest to determine the *relative* importance between the liquidity variance and return variance using intra-day transaction data, as are used in Chordia et al. (2000). Thirdly, the results from our last cross-sectional test indicate that both the size effect and the liquidity effect are significant. This is essential to test model specification of the expected liquidity as in equation (3.16). As we argued previously, if the expectation of liquidity is fully and correctly specified by some observed liquidity variables, then firm size should not play a significant role in explaining cross-sectional before-cost returns. Therefore, the empirical evidence implies that the proportional quoted spread (PQS) and the proportional effective spread (PES), as liquidity measures, are insufficient (though significant) to capture the full cross-sectional variation in expected liquidity. Possible remedies are first is to identify other potential liquidity variables from the market microstructure literature. A second possible remedy is to identify an additional process that is related to the consumption process but is independent of a firm's fundamental process. The impact of tax on asset pricing, as examined by Brennan (1970), is a potential candidate that meets this requirement. We intend to address these research avenues in future research.

CHAPTER 4

AN *EX POST* EXAMINATION OF A CHARACTERISTICS-BASED MODEL IN THE CANADIAN MARKET

4.1 Introduction

It is generally acknowledged that the CAPM (Sharpe 1964, Black 1972) fails to explain cross-sectional expected returns. Deviations from the CAPM are typically explained by two competing categories of research: risk-based models and characteristics-based models. The risk-based category includes multifactor asset-pricing models that attribute the abnormal returns not explained by the CAPM to missing risk factors.[22] For example, the prominent Fama and French three-factor model (1993) claims to explain the abnormal returns by a size portfolio and a book-to-market portfolio that proxy for underlying risk factors. The nonrisk-based category includes characteristic models that explain abnormal returns by firm characteristics such as firm size, book-to-market ratios, and dividend yield. Daniel and Titman (1997) find empirical evidence that stock returns are mainly explained by firm characteristics rather than by risk factors. He and Kryzanowski (2002) develop *ex ante* theories to demonstrate that firm characteristics are related to quasi-rational behavior (2002a) and illiquidity (2002b). In empirical research, whether multifactors or characteristics should explain abnormal cross-sectional returns has triggered intense debate.[23] MacKinlay (1995) points out that the *ex post* nature of empirical research makes it difficult to discriminate between multifactor models and characteristic models. MacKinlay develops a theoretical framework which shows that CAPM deviations due to missing risk factors

[22] The theoretical foundation of multifactor models is built on the ICAPM (Merton 1973) and the APT (Ross 1976).

[23] We refer to the most recent "Covariance versus Characteristics" arguments between Davis, Fama and French (2000) and Daniel, Titman and Wei (2001).

will be very difficult to detect empirically, whereas deviations resulting from nonrisk-based sources such as characteristics are easily detectable.

In this paper, we do not intend to distinguish between the multifactor model and the characteristic model. Instead, we propose an alternative way of formulating an asset-pricing model from an *ex post* perspective and test its cross-sectional relations. The *ex post* formulated model is composed of a market risk component and a characteristic deviation component. We demonstrate how portfolio returns are consistent with the *ex post* characteristic model. In addition, we empirically examine the cross-sectional relations of the characteristic model by using an extended Fama and French (1992) portfolio approach. As implied from a characteristic model, if we change the way of forming portfolios, then portfolio returns are expected to exhibit systematic patterns in a predictable way. In particular, if we form portfolios not based on any priced characteristic, then these portfolios should not exhibit any characteristics-related returns. In other words, the systematic returns as exhibited by firm characteristics may not be related to any risk factors, but are merely the artifact of the empirical procedure for forming portfolios. Our arguments shed light on some recent studies that examine the asset-pricing implications of portfolio formation. Berk (2000) demonstrates that sorting data into groups and running cross-sectional tests within each group can lead to a false rejection of the asset-pricing model. Liang (2000) shows that the traditional portfolio formation procedure is biased against the CAPM when sorting on an *ex post* significant variable. He proposes a random sampling approach to form portfolios and to run cross-sectional tests. We support the random sampling process that essentially diversifies away measurement errors induced by firm characteristics.

We implement our empirical methodology using Canadian stock market data from March 1977 to December 1998. We group firms into 5*x*5 portfolios using a two-pass portfolio procedure. The first-pass procedure selects a given percentage of firms based on size and selects the remaining percentage of firms randomly. We form different panels of portfolios by changing the percentage of firms sorted on size in the first pass. This methodology is designed for two purposes. The first is to demonstrate the influence of portfolio formation on observed systematic returns. As the percentage chosen on size decreases from panel to panel, we expect a monotonically diminishing size effect in both values and significance. Secondly and more importantly, as portfolio composition is altered from panel to panel, our methodology provides a robustness test for the beta and liquidity effects in the Canadian markets. To this end, we form the second-pass portfolios by sorting firms into subportfolios based on pre-ranking betas and liquidity measures. Then in cross-sectional tests, we examine the risk-return and liquidity-return relationships under different first-pass portfolio formations. Due to the *ex post* nature of our model, we adopt the methodology of Pettengill, Sundaram and Mathur (1995) that accounts for the conditional relationship between beta and *realized* returns. The liquidity-return test is performed for two liquidity measures: Proportional Quoted Spread (PQS) and Proportional Effective Spread (PES). Our main finding is that the conditional beta-return and the liquidity-return relationships remain highly significant and robust across all panels of portfolios.

The remainder of this paper is organized as follows. The next section formulates the characteristic model from an *ex post* perspective. In the third section, we develop the argument that the size effect may be the artifact of the method chosen for portfolio formation. In the fourth section, we illustrate our new empirical methodology, describe the Canadian market data, and present the empirical results for the relations between risk, liquidity, and returns. In the fifth and final section, we complete the paper with concluding remarks.

4.2 An *Ex Post* Formulation of the Characteristic Model

We formulate the characteristic model in its *ex post* form. Consider an economy with N assets, each issued by a different firm. The return generating process for each asset i conforms to a market model:

$$r_{i,t} = E[r_{i,t}] + \beta_{i,t-1} f_t + \varepsilon_{i,t} \tag{4.1}$$

where

$r_{i,t}$ is the random return of asset i ($i = 1, \ldots, N$) at time t;

$E[r_{i,t}]$ is asset i's expected return in excess of the risk-free rate;

f_t is the random innovation of the market portfolio at time t;

$\beta_{i,t-1}$ is the loading of asset i's return on the market portfolio; and

$\varepsilon_{i,t}$ is the idiosyncratic return at time t and is specific to each asset i.

For expositional purposes, the expected excess return $E[r_{i,t}]$ and factor loading $\beta_{i,t-1}$ are assumed to be constant so that we can focus on a one-period economy setting, from time t-1 to time t. We adopt the standard assumptions for the market model:

$$E[f_t] = 0;\ E[f_t^2] = 1 \tag{4.2}$$

$$E[\varepsilon_{i,t}] = 0;\ E[\varepsilon_{i,t}^2] = \sigma_i^2;\ E[\varepsilon_{i,t}\varepsilon_{j,t}] = 0 \tag{4.3}$$

$$E[f_t\varepsilon_{i,t}] = 0 \tag{4.4}$$

By construction, equation (4.4) implies that the market portfolio realization f_t and idiosyncratic return $\varepsilon_{i,t}$ are uncorrelated. Equation (4.3) requires the *unconditional* idiosyncratic return to be a white noise.

In equilibrium, the original CAPM (Sharpe 1965) claims that asset i's expected excess return is determined by:

$$E[r_{i,t}] = \beta_{i,t-1}\lambda_1 \tag{4.5}$$

where

λ_1 is the risk premium for the market portfolio.

Numerous empirical studies indicate strong rejection of equation (4.5) in its *ex post* form. The evidence is usually classified into a number of "cross-sectional anomalies". These include the size effect (Banz 1981), price-to-earnings effect (Basu 1983), book-to-market effect (Fama and French 1992), and so forth. Firm characteristics such as firm size, price-to-earnings (P/E) and book-to-market (B/M) are found to explain a significant portion of idiosyncratic returns. He and

Kryzanowski (2002a, 2002b) develop *ex ante* theories in support of the characteristic model. Their theories explain why firm characteristics are related to expected returns *ex ante*. The focus of this paper is to propose an *ex post* formulation of the characteristic model and test some well-known empirical relationships in the Canadian markets.

To facilitate an *ex post* formulation, we express the idiosyncratic return $\varepsilon_{i,t}$ in its *conditional* form, conditioning on the realizations of observed firm characteristics. Specifically,

$$E[\varepsilon_{i,t} \mid I_{t-1}] = \alpha_0 + \theta_{i,t-1}\alpha_1 \qquad (4.6)^{24}$$

where

I_{t-1} is the information set observed at time t-1;

$\theta_{i,t-1}$ is the array (1*K) of firm characteristics realized at time t-1 for firm i;

K is the number of the characteristics being conditioned ($k = 1,\ldots,K$);

α_0 is the intercept of the cross-sectional specification; and

α_1 is the array (1*K) of characteristic coefficients (loadings).

Without loss of generality, we take $K = 1$ in the following analysis. Empirically, $\theta_{i,t-1}$ is the characteristic on which portfolios are formed. More concretely, consider $\theta_{i,t-1}$ as the logarithm of the inverse of market capitalization (as a proxy for firm size) for firm i at time t-1, i.e., $\theta_{i,t-1} \equiv \ln(1/MV_{i,t-1})$.[25] Size portfolios are formed by ranking firms based on the realizations of $\theta_{i,t-1}$.

Given the unconditional (4.3) and conditional (4.6) expectations, we now link the two forms for the idiosyncratic return $\varepsilon_{i,t}$. First we partition $\varepsilon_{i,t}$ into two components: the conditional term and an error term,

$$\varepsilon_{i,t} = E[\varepsilon_{i,t} \mid I_{t-1}] + \eta_{i,t} \qquad (4.7)$$

[24] The conditional model is a cross-sectional specification where cross-sectional information is conditioned. This differs from time-series conditioning that is commonly used in the literature.

[25] Firm size plays a theoretical role in asset pricing. As explained by He and Kryzanowski (2002b), the size effect is essentially induced by the misspecification of adverse characteristics such as illiquidity. So, the choice of firm size has a theoretical grounding regardless of empirical identification of firm characteristics. Therefore, we use firm size as the characteristic to develop our arguments.

where

$$E[\eta_{i,t} \mid I_{t-1}] = 0 \tag{4.8}$$

$\eta_{i,t}$ is the part of the idiosyncratic return with zero conditional expected return. It is associated with those characteristics that are not priced in the cross-section of asset returns.

We take the *unconditional* expectation of equation (4.7), and then use equation (4.6) to obtain:

$$E[\varepsilon_{i,t}] = \alpha_0 + \bar{\theta}_{t-1}\alpha_1 = 0 \tag{4.9}$$

where $\bar{\theta}_{t-1} \equiv E[\theta_{t-1}]$ is the cross-sectional mean for this priced characteristic.

To remove the intercept α_0 in equation (4.9), we subtract equation (4.9) from equation (4.6) and obtain:

$$E[\varepsilon_{i,t} \mid I_{t-1}] = (\theta_{i,t-1} - \bar{\theta}_{t-1})\alpha_1 \tag{4.10}$$

The term $(\theta_{i,t-1} - \bar{\theta}_{t-1})$ measures the characteristic deviation between firm i and the market. The coefficient α_1 can be interpreted as the unit characteristic premium.[26]

Substituting equations (4.7) and (4.10) into equation (4.1), we get:

$$r_{i,t} = E[r_{i,t}] + \beta_{i,t-1}f_t + (\theta_{i,t-1} - \bar{\theta}_{t-1})\alpha_1 + \eta_{i,t} \tag{4.11}$$

If *conditional* expectations are taken on both sides when conditioning on the information set I_{t-1}, then we can use equation (4.6) to express an asset's expected return in the *conditional* form:

$$E[r_{i,t} \mid I_{t-1}] = E[r_{it}] + (\theta_{i,t-1} - \bar{\theta}_{t-1})\alpha_1 = \beta_{i,t-1}\lambda_1 + (\theta_{i,t-1} - \bar{\theta}_{t-1})\alpha_1 \tag{4.12}$$

Equation (4.12) gives the result of an *ex post* formulated asset-pricing model. When conditioning on the observed information set I_{t-1}, a firm's excess return is determined by two

---

26 The characteristic premium α_1 has different economic meanings in different settings. He and Kryzanowski (2002a) show that α_1 captures the market overreaction sentiment given the existence of quasi-rational behaviors and contrarian strategies. In He and Kryzanowski (2002b), α_1 captures the market aversion to illiquidity. In both settings, α_1 can be interpreted as a unit characteristic premium which is strictly positive by construction.

components: the market risk component and the characteristic deviation component. The market risk component is the part of expected return determined by the covariance between a firm's return and that of the market portfolio. The characteristic deviation component is the part of expected return determined by the difference between a firm and the market for the priced characteristics. The unit characteristic premium α_1 is positive by construction.[27] Thus, a firm with higher than the market-average characteristics, i.e., $\theta_{i,t-1} > \overline{\theta}_{t-1}$, earns a positive characteristic premium. A firm with lower than the market-average characteristics, i.e., $\theta_{i,t-1} < \overline{\theta}_{t-1}$, earns a negative characteristic premium. A firm with the market-average characteristics, i.e., $\theta_{i,t-1} = \overline{\theta}_{t-1}$, does not earn any characteristic premium. An important implication is that a portfolio can earn a positive, negative, or zero characteristic premium depending on the way stocks are grouped into portfolios. We formalize this argument below by examining characteristic sorted portfolios.

4.3 An *Ex Post* Examination of Size-Sorted Portfolios

Suppose the economy has a total number of N assets. Denote x_i to be the weight of asset i in the constructed portfolio. Such a portfolio has a finite number of n assets. Without loss of generality, consider an equally-weighted portfolio indexed by p with n assets.[28] We shall obtain a return generating process for this portfolio as:

$$R_{p,t} = E[R_{p,t}] + B_{p,t-1} f_t + (\Theta_{p,t-1} - \overline{\Theta}_{t-1})\alpha_1 + \xi_{p,t} \tag{4.13}$$

where

$R_{p,t} = \sum_{i=1}^{n} x_i r_{i,t}$ is the portfolio random return;

[27] See note 5.

[28] Equally weighted portfolios are used for expositional purposes. Value-weighted portfolios can be used in the same way as long as asset weights are adjusted accordingly for the constructed portfolios and the market characteristics.

$B_{p,t-1} = \sum_{i=1}^{n} x_i \beta_{i,t-1}$ is the portfolio loading on the market portfolio;

$\Theta_{p,t-1} = \sum_{i=1}^{n} x_i \theta_{i,t-1}$ is the portfolio characteristic realized at time t-1;

$\overline{\Theta}_{t-1} = \sum_{i=1}^{n} x_i \theta_{i,t-1}$ is the value of the market characteristic realized at t-1;

$\xi_{p,t} = \sum_{i=1}^{n} x_i \eta_{i,t-1}$ is the part of the portfolio return not related to any characteristics.

Using equation (4.12), we derive the excess return for the constructed portfolio as:

$$E[R_{p,t} \mid I_{t-1}] = B_{p,t-1}\lambda_1 + (\Theta_{p,t-1} - \overline{\Theta}_{t-1})\alpha_1 \tag{4.14}$$

To be concrete, we consider size portfolios. Let $\Theta_{p,t-1} \equiv \ln(1/MV_{p,t-1})$ and $\overline{\Theta}_{t-1} \equiv \ln(1/MV_{m,t-1})$ where $MV_{p,t-1}$ and $MV_{m,t-1}$ denote the market value for portfolio p and for the market portfolio, respectively. Size portfolios are formed by first sorting $\theta_{i,t-1}$ over all the firms in the economy (i = 1,…,N) in a non-decreasing order. Cut-off values of $\theta_{i,t-1}$ are predetermined to make each size portfolio contain the same number of firms. Suppose we form a total number of P size portfolios, each containing n = N/P number of firms. Then the first portfolio (the largest size) will contain firms with size from $\theta_{1,t-1}$ to $\theta_{n,t-1}$, the second portfolio from $\theta_{n+1,t-1}$ to $\theta_{2n,t-1}$, etc., and the last portfolio (the smallest size) from $\theta_{(P-1)n+1,t-1}$ to $\theta_{N,t-1}$, where $\theta_{i,t-1}$ is ranked in a non-decreasing order for all the firms, i.e.,

$$\theta_{1,t-1} \le \theta_{2,t-1} \le \ldots \le \theta_{n,t-1} \le \ldots \le \theta_{2n,t-1} \le \ldots \le \theta_{N,t-1} \tag{4.15}$$

Condition (4.15) suggests that the following relation must be true:

$$\Theta_{1,t-1} \le \ldots \le \Theta_{p,t-1} \le \ldots \le \Theta_{P,t-1} \tag{4.16}$$

To examine the size effect, we use the result from equation (4.12), and write:

$$E[R_{1,t} \mid I_{t-1}] = B_{1,t-1}\lambda_1 + (\Theta_{1,t-1} - \overline{\Theta}_{t-1})\alpha_1$$

…

$$E[R_{p,t} \mid I_{t-1}] = B_{p,t-1}\lambda_1 + (\Theta_{p,t-1} - \overline{\Theta}_{t-1})\alpha_1 \tag{4.17}$$

...

$$E[R_{P,t} \mid I_{t-1}] = B_{P,t-1}\lambda_1 + (\Theta_{P,t-1} - \overline{\Theta}_{t-1})\alpha_1$$

where

$R_{p,t}$ is the random return for the pth portfolio; and

$B_{p,t-1}$ is the factor loading of the pth portfolio on market realization.

By construction of the size portfolios, α_1 is significantly positive *ex ante*.[29] Combining this fact with the condition (4.16), we see that in equation (4.17), the term $(\Theta_{p,t-1} - \overline{\Theta}_{t-1})\alpha_1$ exhibits a systematic pattern. Portfolios with size smaller than the market average earn a positive size premium. Portfolios with size larger than the market average earn a negative size premium. Portfolios with the market average size do not earn any size premium. However, the observed systematic patterns depend critically on the way portfolios are formed. In particular, if we modify the portfolio procedure that sorts on size, we expect the size effect to change in both values and significance. Based on this implication from the characteristics-based model, we design a new portfolio approach in the next section. Our methodology is also used as a robustness test for the risk, liquidity, and return relations in the Canadian stock markets.

4.4 An Empirical Examination of Risk, Liquidity and Returns

4.4.1 An Extended Portfolio Approach

To examine the risk, liquidity and return relationships, we design an extended two-pass portfolio approach which runs six panels of portfolio formation procedures. To describe the first-pass, in the first panel, our procedure is identical to that of Fama and French (1992), which sorts five portfolios based on firm size. From the second to the sixth panel of our first-pass procedure,

[29] See note 5.

we construct portfolios based on a decreasing percentage of stocks sorted on size. Specifically, in the second panel, each portfolio contains 80% of the stocks sorted on size, and 20% of the stocks selected from a random pool. In the third panel, each portfolio contains 60% of the stocks sorted on size, and 40% of the stocks from the random pool, and so on. In the sixth and final panel, none of the stocks in any of the 5 portfolios is selected on size, and each portfolio contains a fixed number of stocks that are randomly selected. This first-pass portfolio procedure is best illustrated through an example presented in the Appendix 4-A. We then perform the second-pass procedure as follows. Within each first-pass portfolio, we form five subportfolios based on pre-ranking betas as in Fama and French (1992) and liquidity measures to be discussed, thus generating 25 size-beta or size-liquidity portfolios. Finally, we run cross-sectional tests for the 25 portfolios within every panel and compare the values and significance of size, beta, and liquidity across all the six panels.

The design of the extended portfolio approach serves two purposes. Firstly, we intend to provide empirical evidence to support the theoretical argument that the well-documented size effect may not be related to any risk factors, but is merely the artifact of portfolio formation. Such evidence will be consistent with a characteristics-based model as developed in the second section. As the percentage of firms sorted on size successively is decreased by 20% from panel 1 to panel 6, we artificially reduce the size effect by diversifying away the size variations across the 25 portfolios. Thus, we expect a monotonically diminishing size effect in both its values and significance.

Secondly, our methodology provides a robustness test for the beta and liquidity effects in the Canadian markets for both theoretical and empirical reasons. Empirically, as portfolio compositions are changed from panel to panel, we strategically create multiple samples to test the significance of the beta and liquidity effects.[30] Furthermore, by altering portfolio compositions, we effectively control the correlation between size and beta/liquidity. In the first panel where all the first-pass portfolios are sorted on size, we intend to examine the beta/liquidity effects by controlling for the size effect. As the percentage of firms sorted on size is decreased, we gradually loosen the control for the size effect, then reexamine the beta/liquidity effects within each panel. Thus, the stability of the beta/liquidity effects *across* all the six panels provides the primary observation for a robustness test. In addition to the discussed empirical perspective, the fundamental and theoretical reason for our methodological design is the theoretical role that firm size plays in asset pricing. Berk (1995) and He and Kryzanowski (2002) argue that the size effect is induced by model misspecification. In essence, our strategy that reduces the size effect from panel 1 to panel 6 progressively diversifies away model misspecifications. In our specific context, it progressively diversifies away the measurement errors of beta and liquidity. In particular, panel 6 is essentially a random sampling approach for the test of the size/liquidity effects with measurement errors fully diversified away. Our argument shares the same spirit as that in Liang (2000) who proposes the random sampling approach for testing asset-pricing models.

4.4.2 Data Description

Our study examines monthly data on common stocks listed in the CFMRC database from March 1977 to December 1998, for a total of 262 time series observations. The monthly database records monthly prices, returns, betas, and shares outstanding for stocks listed on the Toronto Stock Exchange (TSE) since January 1950. The daily database contains the closing bid and closing ask quotes of listed stocks for each trading day starting from March 1, 1977. To convert the daily bid-ask quotes to monthly series, we take the averages of the daily closing bid and ask quotes over each month from March 1977 to December 1998. The monthly averaging is

[30] For the purpose of creating multiple samples, our first-pass procedure is similar to the bootstrapping methodology. The difference is that we do not form portfolios randomly, but instead follow a pre-determined strategy.

performed for each stock that passes through a screening process which controls for data error, adjusts for thin trading, and removes stale trades in the daily database. Specifically, the screening process filters out those firms with market values below CAD $2 million or firms with a market price per share less than CAD $2 or transactions with invalid bid and ask quotes. In addition, a stock must have at least three days of valid closing bid and ask quotes in any given month to be included in the sample. To extract information from the monthly database, firms with a share price below $2 and a market capitalization below $2 million are deleted. We then join the monthly average bid and ask quotes with the monthly returns, betas, and market values for each selected stock.[31]

For the risk-free rate, we convert the 90-day Canadian Treasury bill series into a 30-day series for the same period. We use three proxies for the market portfolio. The first two proxies are the equally weighted market index (EWIR) and the value weighted market index (VWIR) reported in the CFMRC database. We create a new proxy for the market portfolio by taking the cross-sectional mean (CSMR) of the selected sample for each month t = 77:03 to 98:12.[32] This proxy is necessary to create the liquidity time series for the market portfolio because the CFMRC database does not have bid and ask spreads for the market indices. Time series statistics are reported in Table 4.1 for these three market portfolio proxies. We observe that the CSMR proxy has a time series mean close to that of the EWIR but has a lower variance than the EWIR. The lower variance is due to our screening process that removes some outliers from the sample data.

[Insert Table 4.1 about here]

4.4.3 The Relation between Conditional Beta and Return

[31] We make an inner join so as to make sure that a firm has valid data for return, beta, size, bid and ask for every entry.

[32] The empirical results reported in this paper are based on the cross-sectional mean (CRMR) proxy for the market portfolio. We also use the equally weighted and value weighted market indices as proxies. There is no material difference in the results between the three proxies.

We first examine the relation between conditional betas and returns for the 25 cross-sectional portfolios. Empirically, we implement the methodology in the following way. At the end of each June, we calculate each firm's market value and group firms into quintile portfolios for the next 12 months. Each portfolio is composed of a certain percentage of stocks sorted on size and the remaining percentage selected randomly, depending on the percentage requirement for the given panel.[33] Then, within each portfolio, we classify firms into one of the five subportfolios based on pre-ranking betas. To estimate pre-ranking betas, a firm needs to have return data in at least 24 out of 60 months prior to the most recent June.[34] We estimate a firm's Dimson beta to adjust for bias due to non-synchronous trading.[35] Therefore, for any month t starting from March 1977 to December 1998, we construct 5*x*5 size-beta portfolios. For each portfolio p at month t, its return $R_{p,t}$ is the equally weighted returns of all the firms within the portfolio. The post-ranking beta $\hat{\beta}_p$ for each portfolio p is estimated from the time-series regression using 262 observations. $\hat{\beta}_p$ is assumed constant for the cross-sectional tests.

Cross-sectional tests are then performed for each month t. The purpose is to examine whether a systematic risk-return relationship exists for the Canadian stock markets. Due to the *ex post* nature of empirical tests, we adopt the methodology of Pettengill, Sundaram and Mathur (1995) that accounts for the conditional relationship between beta and *realized* returns.

---

[33] We implement two different methods to ensure randomness. In the first method, we pick the remaining percentage of stocks based on the alphabetical order of firm tickers – a characteristic that is not related with stock returns *ex ante*. In the second method, we generate a random seed that is randomly mapped to one firm in the population. Results from the two methods are almost identical, so we report the results generated from the second method.

[34] This means that the data used to estimate betas start as early as March 1972 for the beta portfolios formed on March 1977.

[35] To estimate the Dimson beta, we first perform the following OLS regression:

$$R_{i,t} = \alpha_i + \beta_{i,1} R_{m,t} + \beta_{i,2} R_{m,t-1} + \varepsilon_{i,t}$$

where $r_{i,t}$ is the return for firm i at month t, $R_{m,t}$ is the return on the market portfolio at month t, and $R_{m,t-1}$ is the return on the market portfolio at month t-1. The Dimson beta for firm i is then calculated as:

$$\hat{\beta}_i = \hat{\beta}_{i,1} + \hat{\beta}_{i,2}$$

Specifically, if the realized return on the market portfolio is above the risk-free rate, portfolio betas and returns should be positively related. If the realized return on the market portfolio is below the risk-free rate, portfolio betas and returns should be inversely related. This conditional relationship requires a cross-sectional test conditioned on the up and down of the realization of the market portfolio. This conditional test is consistent with the *ex post* nature of our proposed model. Thus, combining the tests for both the size effect and the conditional risk-return relationship, the empirical equation implied from equation (14) can be written as:

$$R_{p,t} = \gamma_{0,t} + \lambda_{1,t} * \delta * \hat{\beta}_p + \lambda_{2,t} * (1-\delta) * \hat{\beta}_p + \alpha_{1,t} * (\Theta_{p,t} - \Theta_{m,t}) + \varepsilon_{p,t}, \ \mathrm{p} = 1,\ldots,25 \qquad (4.18)$$

where

$R_{p,t}$ is the return on portfolio p in month t;

$\hat{\beta}_p$ is the post-ranking beta for portfolio p estimated from time-series regressions;

$\Theta_{p,t} \equiv \ln(1/MV_{p,t})$ is the natural logarithm of the inverse of the market value for portfolio p;

$\Theta_{m,t} \equiv \ln(1/MV_{m,t})$ is the natural logarithm of the inverse of the market value for the market index; and

δ is the dummy variable conditioning on up and down markets, where $\delta = 1$ if $(R_{m,t} - R_{f,t}) \geq 0$ and $\delta = 0$ if $(R_{m,t} - R_{f,t}) < 0$.

Finally, we average the estimated coefficients $\hat{\lambda}_{1,t}$, $\hat{\lambda}_{2,t}$ and $\hat{\alpha}_{1,t}$ over the monthly cross-sectional regressions and test for statistical significance. $\hat{\lambda}_{1,t}$, $\hat{\lambda}_{2,t}$ and $\hat{\alpha}_{1,t}$ are interpreted as the risk premia under up-market, risk premia under down-market, and size premia, respectively. To test significance, the t-statistics for the time-series averages of the lambdas and alphas are calculated.

Table 4.2 presents the test statistics for the cross-sectional regressions. As our portfolio composition changes gradually from a purely size-sorted portfolio in Panel 1 to a totally random portfolio in Panel 6, we observe monotonically decreasing size premia and statistical significance. The size premia decrease in magnitude from 0.204 in Panel 1 to virtually 0 in Panel 6. The t-statistics are highly significant at the 95% confidence level in Panel 1 (3.58), 2 (2.64), 3 (2.27), and become non-significant in Panels 4 (1.86), 5 (1.39) and 6 (-0.028). These test statistics confirm our theoretical argument that the observed size effect may be just an artifact of the portfolio formation procedure. Thus, a characteristics-based model explains the size effect in an alternative way that is not related to any risk factors.

[Insert Table 4.2 about here]

Examination of the beta premia across panels lends support to the conditional risk-return relationship. Beta plays an important role in asset pricing when conditioning on up and down markets. The market risk premia are all highly significant and vary little across panels of different portfolio constructions and compositions. The highly stable statistics across panels provide the compelling evidence that the conditional-beta and return relation as found in our tests is robust. As would seem plausible, risk premia and significance during down markets seem to be always higher than those during up markets.

A comparison between our results and those of Elfakhani et al (1998) is meaningful. Using the Canadian data from the same source from July 1975 to December 1992, Elfakhani et al (1998) implement the original Fama and French (1992) portfolio approach to examine the size effect in Canadian markets. They find a highly significant size effect and attribute the size effect to underlying risk factors. Using the extended portfolio approach, we instead demonstrate that the size effect can be reduced and even diversified away if we change the way of constructing portfolios. Hence, the risk-based explanation of the size effect observed in Canadian markets may be subject to a characteristic explanation. In addition, Elfakhani et al (1998) document no significant relationship between beta and return for the Canadian markets. Due to the *ex post* nature of empirical research, we instead perform a conditional test conditioning on realized market returns. We find a highly significant beta and return relationship in both up and down markets, and conclude that risk-return is an appropriate description of Canadian market movements.

4.4.4 The Relation between Liquidity and Return

The relations between liquidity and returns have been tested in many studies. Amihud and Mendelson (1986) find evidence that asset returns include a significant premium for illiquidity. Brennan and Subrahmanyam (1996) find a significant relation between monthly stock returns and illiquidity measures after controlling for risk. Both studies are conducted for the U.S. stock markets. He and Kryzanowski (2002b) examine the liquidity-return relation for the Canadian stock market and find a highly significant liquidity effect. The purpose of the current test is to examine the robustness of their results.

We first describe the liquidity measures used for empirical tests. Other studies use various spread-related measures as the observable liquidity variables. For example, Amihud and Mendelson (1985) use the quoted bid-ask spread as a proxy for liquidity. Chordia, Roll and Subrahmanyam (2000) use five spread-related measures: the quoted spread, the proportional quoted spread, depth, effective spread, and proportional effective spread as liquidity variables. In our tests, we use two proportional spreads that are calculated for each stock for each month. Specifically, the Proportional Quoted Spread (PQS) is calculated as $(p_{jt}^{a} - p_{jt}^{b}) / p_{jt}^{m}$ where p_{jt}^{a} (p_{jt}^{b}) denotes the closing ask (bid) price, and $p_{jt}^{m} = (p_{jt}^{a} + p_{jt}^{b}) / 2$ is the mid point of the bid-ask spread of firm j at time t. The Proportional Effective Spread (PES) is calculated as $2 \mid p_{jt}^{x} - p_{jt}^{m} \mid / p_{jt}^{x}$ where p_{jt}^{x} is the closing trade price of firm j for month t. We take the cross-sectional mean of PQS and PES as the liquidity measures for the market portfolio.

The model implementation is identical to that described in Section 4.4.2 for the first-pass procedure. In the second-pass, we sort the first-pass portfolios by liquidity measures PQS or PES within each panel of portfolios. Cross-sectional regressions are performed as below:

$$R_{p,t} = \gamma_{0,t} + \lambda_{1,t} * (L_{p,t} - L_{m,t}) + \alpha_{1,t} * (\Theta_{p,t} - \Theta_{m,t}) + \varepsilon_{p,t},\ p = 1,\ldots,25$$

where

$R_{p,t}$ is the return on portfolio p in month t;

$L_{p,t}$ is the liquidity measure PQS or PES for portfolio p;

$L_{m,t}$ is the liquidity measure PQS or PES for the market portfolio;

$\Theta_{p,t} \equiv \ln(1/MV_{p,t})$ is the natural logarithm of the inverted market value for portfolio p;

$\Theta_{m,t} \equiv \ln(1/MV_{m,t})$ is the natural logarithm of the inverted market value for the market index m.

We take the time-series averages to calculate the coefficients $\hat{\lambda}_{1,t}$ and $\hat{\alpha}_{1,t}$ over the observation period. $\hat{\lambda}_{1,t}$ and $\hat{\alpha}_{1,t}$ are interpreted as illiquidity premia and size premia, respectively. The mean values and their associated t-statistics for these estimated coefficients are reported in Table 4.3.

[Insert Table 4.3 about here]

Table 4.3 reports results for using PQS and PES as liquidity measures, respectively. As expected, size premia and significance – (in the second column (PQS) and the fourth column (PES)), diminish monotonically as portfolio construction shifts from a fully size-sorted to a randomly selected strategy. The fact that the size premium is significant only in the first panel indicates that the liquidity effect picks up much of the cross-sectional variations in not so strongly size-sorted portfolios. As we examine the illiquidity premia as reported in the first (PQS) and third (PES) columns, we find that all of them are highly significant at the 95% confidence level, and the values are relatively stable across all the six panels. As explained in Section 4.4.1, stability and significance of the illiquidity premia assure the robustness of the documented liquidity-return relations in the Canadian markets. [36]

4.4.5 The Relation between Conditional Beta, Liquidity and Return

In the final test of our empirical study, we perform a comprehensive analysis by regressing the returns of the 25 portfolios on conditional beta, liquidity, and firm size. The construction of the 25 portfolios follows the same strategy as in Section 4.4.3. Specifically, we run the following cross-sectional regression at each month t:

$$R_{p,t} = \gamma_{0,t} + \lambda_{1,t} * \delta * \hat{\beta}_p + \lambda_{2,t} * (1-\delta) * \hat{\beta}_p + \alpha_{1,t} * (L_{p,t} - L_{m,t}) + \theta_{1,t} * (\Theta_{p,t} - \Theta_{m,t}) + \varepsilon_{p,t}$$

$$p = 1, \ldots, 25$$

where

$R_{p,t}$ is the return on portfolio p in month t;

$\hat{\beta}_p$ is the post-ranking beta for portfolio p;

$L_{p,t}$ is the liquidity measure PQS or PES for portfolio p;

$L_{m,t}$ is the liquidity measure PQS or PES for the market portfolio;

$\Theta_{p,t} \equiv \ln(1/MV_{p,t})$ is the natural logarithm of the inverted market value for portfolio p;

$\Theta_{m,t} \equiv \ln(1/MV_{m,t})$ is the natural logarithm of the inverted market value for the market index m; and

δ is the dummy variable conditioned on up and down markets, where $\delta = 1$ if $(R_{m,t} - R_{f,t}) \geq 0$ and $\delta = 0$ if $(R_{m,t} - R_{f,t}) < 0$.

The cross-sectional estimated coefficients $\hat{\lambda}_{1,t}$, $\hat{\lambda}_{2,t}$, $\hat{\alpha}_{1,t}$ and $\hat{\theta}_{1,t}$ are time-series averages over the observation period. The mean values and their associated t-statistics for these estimated coefficients are reported in Table 4.4.

---

[36] We also test the conditional liquidity effect by conditioning on the returns of up and down markets and by conditioning on the variance of up and down markets in the same fashion as testing the conditional beta. Liquidity premia are mostly significant in both conditioning situations. However, the results do not exhibit a systematic pattern. Thus, we report unconditional liquidity effect in this section.

[Insert Table 4.4 about here]

Statistics are consistent with our previous individual findings. Size premium monotonically decreases from panel 1 to panel 6, but is significant only in the first panel where all the stocks are size sorted. Conditional beta premia are highly stable and significant in both up markets and down markets across all the panels. Liquidity as measured by PQS[37] is also a significant explanatory variable in explaining portfolio returns. The robustness of our test results is confirmed by the comprehensive analysis.

4.5 Conclusion

This paper has both theoretical and empirical intents. In the theoretical part, we formulate a characteristics-based asset-pricing model on an *ex post* basis. The *ex post* model is a cross-sectional relationship that combines the market risk premium and the characteristic deviation premium. The characteristic component is not linked to the variance-covariance structure but is determined cross-sectionally by how much a given firm's characteristics are different from those of the market portfolio. We examine the widely used empirical procedure that groups stocks into portfolios by sorting firm characteristics, showing that the exhibited systematic patterns may be largely due to the way portfolios are formed.

We must emphasize that our formulated model is *ex post* in nature. Our proposed empirical approach is conditional on *realized* firm characteristics and *realized* market returns. Thus, our results are subject to the well-known data snooping biases. However, such criticism equally plague other empirical studies due to the *ex post* nature of empirical research.[38] In addition, since our empirical procedure does not specify the alternative hypothesis to run against, we certainly do not rule out the risk-based explanations of any characteristic effects. Instead, our

[37] We also run the regression using PES as the liquidity measure and obtain the same findings.

[38] MacKinlay (1995) explains the *ex post* nature of empirical research using theoretical arguments.

purpose is to demonstrate that a nonrisk-based model can provide an alternative explanation of any characteristic effect.

Based on the implications of the *ex post* characteristic model, we design a new portfolio approach and perform robustness tests for the relationships between risk, liquidity, and returns in the Canadian stock markets. Empirical evidence supports our argument that size premia change in a predictable way depending on how portfolios are formed. Our results also find a significant risk-return relationship when conditioning on realized returns on the Canadian market portfolio, and a strong liquidity-return relationship. Both the risk effect and the liquidity effect are highly robust across different panels of portfolio formations.

CHAPTER 5

CONCLUSION

The central theme of this dissertation is to propose characteristics-based models as an alternative to the traditional risk-based models and to examine the empirical implications of characteristic models in Canadian stock markets. Although the theoretical development and empirical examinations are presented in three chapters, the contribution of this dissertation to the asset-pricing literature can be summarized from two perspectives.

First, as an alternative to risk-based models, the characteristic model is justified from both theoretical and empirical perspectives. As demonstrated in the second chapter, the interactive behavior between two groups of investors ("rationals" and "quasi's") is the underlying driving force that ultimately determines cross-sectional expected returns in market equilibrium. With the assumption of the aggregation condition, cross-sectional returns can be explained by parsimonious and intuitive components. We demonstrate that it is the firm characteristics, rather than the risk factors, that determine cross-sectional expected returns. In the third chapter, we set up an economy where the aggregate consumption is determined by two independent processes: the fundamental process and the liquidation process. We show that both the expected liquidity (related to firm characteristics) and unexpected liquidity (related to risk factors) impact on cross-sectional expected returns. The relative importance of the two effects is empirically tested. Using monthly Canadian market data, we present empirical evidence that the characteristic liquidity effect is far more significant than the liquidity effect related to risk. This finding lends direct support to characteristics-based models. In the fourth chapter, we propose a characteristic model from the *ex post* perspective, showing that the *ex post* model contains a linear characteristic term. The *ex post* model has important implications to the common practice of grouping stocks into portfolios by sorting on characteristics such as firm size. We demonstrate that the systematic patterns exhibited by firm characteristics such as the size effect may be largely due to the way portfolios are formed. The three chapters collectively provide comprehensive theoretical arguments and empirical evidence to support characteristics-based models as an alternative to risk-based models.

Secondly, the empirical part of this dissertation examines the cross-sectional relations between risk, size, liquidity and returns in the Canadian stock market. To our knowledge, this dissertation is among the first research to extensively test the liquidity-return relation in Canadian markets. In the third chapter, we perform some preliminary tests for the size, beta, liquidity and return relations. In the fourth chapter, we design a new portfolio approach to perform robustness tests for our preliminary findings. By forming the first-pass portfolios in such a way that the percentage of firms sorted on size systematically decreases, we create multiple testing samples where the correlation between the size effect and the liquidity or beta effect is effectively controlled. Some tentative conclusions can be reached from the robustness tests. The major finding is the relationship between expected returns and liquidity. In out tests, liquidity is measured by two spread-related proxies: the proportional quoted spread and the proportional effective spread. Liquidity is used as an explanatory variable of portfolio returns in conjunction with other variables such as size and/or beta. In brief, we find a highly significant liquidity effect in all the cross-sectional regressions. Furthermore, in our preliminary test, we find that the unconditional risk-return relationship is flat in the Canadian markets. However, in the *ex post* examinations, we test the risk-return relation conditioning on up and down markets and find that the conditional risk-return relation is very strong. Robustness tests indicate that our empirical findings are highly stable across all the different portfolio formations. The comprehensive evidence not only enriches the literature on the behavior of the Canadian stock market, but it also lends empirical support to characteristic models.

We must emphasize that characteristics-based models as proposed in the thesis just offer a preliminary alternative explanation of expected returns. We certainly do not rule out the mainstream risk-based theories. However, given the sheer absence of alternative theories in the existing asset-pricing literature, the proposed characteristic models can serve as a preliminary alternative to the risk-based models. For characteristic models to qualify as the formal alternative, we intend to conduct future research along both theoretical and empirical avenues. First of all, from the theory perspective, the models proposed in the dissertation are confined to a one-period economy. A significant theoretical contribution will be to extend the existing one-period characteristic models to multi-periods where investors face dynamic investment opportunities. Secondly, the characteristic models developed in the second and third chapters are based on two respective economic settings, namely, irrational behavior and liquidity based settings. There exists other explanations such as the agency-based setting where institutions and analysts exhibit systematic optimism for their own benefits, and the prior that investors may systematically prefer certain characteristics to others. A rigorous formulation of these alternative settings will enhance the explanatory power of characteristics-based models. From the empirical perspective, the empirical results presented in the thesis only provide some *alternative* evidence. These results do not have the power against risk-based models. In order to do so, it will be necessary to develop statistics that formally distinguish between the null (characteristic models) and the alternative (risk models). Secondly, we intend to investigate further the liquidity issues in the Canadian market using high frequency data. The behaviors of high frequency liquidity measures can be meaningfully compared against the monthly liquidity measures used in the thesis. We will pursue these issues in our future research.

BIBLIOGRAPHY

Amihud, Yakov, and Haim Mendelson, 1986, Asset pricing and the bid-ask spread, *Journal of Financial Economics* 17, 223-249.

Amihud, Yakov, and Haim Mendelson, 1987, Trading mechanisms and stock returns: An empirical investigation, *Journal of Finance* 42, 533-553.

Banz, Rolf W., 1981, The relationship between return and the market value of common stocks, *Journal of Financial and Quantitative Analysis* 14, 421-441.

Barberis, Nicholas, Andrei Shleifer, and Robert Vishny, 1998, A model of investor sentiment, *Journal of Financial Economics* 49, 307-343.

Basu, S., 1983, The relationship between earnings yield, market value, and return of NYSE common stocks, *Journal of Financial Economics* 12, 126-156.

Berk, Jonathan B., 1995, A critique of size-related anomalies, *Review of Financial Studies* 8, 75-286.

Berk, Jonathan B., 2000, Sorting out sorts, *Journal of Finance* 55, 407-427.

Black, Fischer, 1972, Capital market equilibrium with restricted borrowing, *Journal of Business* 45, 444-455.

Black, Fischer, Michael C. Jensen and Myron Scholes, 1972, The capital asset pricing model: Some empirical tests, in: Michael C. Jensen, ed., *Studies in Theory of Capital Markets*, Praeger, New York.

Brennan, J. Michael, 1970, Taxes, market valuation and corporate financial policy, *National Tax Journal*, December 1970, 417-427.

Brennan, J. Michael, and Avanidhar Subrahmanyam, 1996, Market microstructure and asset pricing: On the compensation for liquidity in stock returns, *Journal of Financial Economics* 41, 441-464.

Campbell, John Y., 2000, Asset Pricing at the Millennium, *Journal of Finance* 55, 1515-1567.

Campbell, John Y. and N. Gregory Mankiw, 1989, Consumption, income, and interest rates: Reinterpreting the time series evidence, in O.J. Blanchard and S. Fischer eds., *National Bureau of Economic Research Macroeconomics Annual* 4, 185-216.

Chordia, Tarun, Richard Roll, and Avanidhar Subrahmanyam, 2000, Commonality in Liquidity, *Journal of Financial Economics* 56, 3-28.

Constantinides, George and Darrell Duffie, 1996, Asset pricing with heterogeneous consumers, *Journal of Political Economy* 104, 219-240.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 1998, A theory of overconfidence, self-attribution, and security market under- and over-reactions, *Journal of Finance* 53, 1839-1886.

Daniel, Kent, and Sheridan Titman, 1997, Evidence on the characteristics of cross-sectional variation in stock returns, *Journal of Finance* 52, 1-33.

Daniel, Kent, Sheridan Titman, and K.C. John Wei, 2001, Explaining the cross-section of stock returns in Japan: Factors or characteristics?, *Journal of Finance* 55:2, 743-766.

Davis L., James, Eugene F. Fama, and Kenneth R. French, 2000, Characteristics, covariances, and average returns: 1929 to 1997, *Journal of Finance*, 55, 389-406.

De Long, J. Bradford, Andrei Shleifer, Lawrence Summers, and Michael Waldmann, 1990, Noise trader risk in financial markets, *Journal of Political Economy* 98, 703-738.

Elfakhani, Said, Larry Lockwood, and Tarek Zaher, 1998, Small firm and value effects in the Canadian stock market, *Journal of Financial Research* 21, 277-291.

Fama, Eugene F., 1976, Foundations of finance, Basic Books, Inc.

Fama, Eugene F., and James MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy* 71, 607-636.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427-465.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3-56.

Fama, Eugene F., and Kenneth R. French, 1995, Size and book-to-market factors in earnings and returns, *Journal of Finance* 50, 131-155.

Fama, Eugene F., and Kenneth R. French, 1996, Multifactor explanations of asset pricing anomalies, *Journal of Finance* 51, 55-84.

Ferson, Wayne E., and Campbell R. Harvey, 1999, Conditioning Variables and the cross-section of stock returns, *Journal of Finance* 54, 1325-1360.

Ferson, Wayne E., Sergei Sarkissian, and Timothy Simin, 1999, The alpha factor pricing model: A parable, *Journal of Financial Markets* 2, 49-68.

George, Thomas, Gautam Kaul, and M. Nimalendran, 1991, Estimating the components of the bid-ask spread: A new approach, *Review of Financial Studies* 4, 623-656.

Glosten, Lawrence and Lawrence Harris, 1988, Estimating the components of the bid-ask spread, *Journal of Financial Economics* 21, 123-142.

Hasbrouck, Joel, 1991, Measuring the information content of stock trades, *Journal of Finance* 46, 179-207.

Hasbrouck, Joel and Duane J. Seppi, 2001, Common factor in prices, order flows and liquidity, *Journal of Financial Economics* 59, 383-411.

He, Zhongzhi, and Lawrence Kryzanowski, 2002a, A reformulated asset pricing model based on contrarian strategies, Working Paper, Concordia University, revised.

He, Zhongzhi, and Lawrence Kryzanowski, 2002b, A reformulated asset pricing model of risk, liquidity and returns, Working Paper, Concordia University, revised.

Hong, Harrison and Jeremy C. Stein, 1999, A unified theory of underreaction, momentum trading, and overreaction in asset markets, *Journal of Finance* 54, 2143-2184.

Huberman, Gur and Dominika Halka, Summer 2001, Systematic Liquidity, *Journal of Financial Research*, 161-178.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65-91.

Lakonishok, Josef, Andrei Shleifer, and Robert W. Vishny, 1994, Contrarian investment, extrapolation and risk, *Journal of Finance* 49, 1541-1578.

Lee, Charles M., Andrei Shleifer, and Richard H. Thaler, 1991, Investor Sentiment, and the Closed-End Fund Puzzle, *Journal of Finance* 46, 75-109.

Liang, Bing, 2000, Portfolio formation, measurement errors, and beta shifts: A random sampling approach, *Journal of Financial Research* 23 (3), 261-284.

MacKinlay, A. Craig, 1995, Multifactor models do not explain deviations from the CAPM, *Journal of Financial Economics* 38, 3-28.

Merton, Robert C., 1973, An intertemporal asset pricing model, *Econometrica* 41, 867-887.

Pettengill, N. Glenn, Sridhar Sundaram, and Ike Mathur, 1995, The conditional relation between beta and returns, *Journal of Financial and Quantitative Analysis 30*, 101-116.

Roll, Richard, 1977, A critique on the asset pricing theory's tests, *Journal of Financial Economics* 4, 129-176.

Ross, Stephen A., 1976, The arbitrage theory of capital asset pricing, *Journal of Economic Theory* 13, 341-360.

Sharpe, William F., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425-442.

Stoll, Hans, 1989, Inferring the components of the bid and ask spread: Theory and empirical tests, *Journal of Finance* 44, 115-134.

Thaler, Robert H., 1991, *Quasi-Rational Economics*, Russell Sage Foundation, New York, N.Y.

Zarowin, Paul, 1990, Size, seasonality, and stock market overreaction, *Journal of Financial and Quantitative Analysis* 25, 113-125.

APPENDICES

Appendix 2-A

From the viewpoint of "rationals", a firm's fundamental value is jointly determined by "rationals" and "quasi's" in the following process:

$$\tilde{V}_j = \phi\tilde{V}_{1j} + (1-\phi)\tilde{V}_{2j} \tag{2-A1}$$

where

$\tilde{V}_j$ is the firm's true fundamental value as perceived by "rationals";

ϕ and $1-\phi$ are the proportional weight of wealth for "rationals" and "quasi's", respectively; and

$\tilde{V}_{1j}$ and $\tilde{V}_{2j}$ are the fundamental values evaluated by "rationals" and "quasi's", respectively.

Rearranging terms and substituting equations (2.1), (2.3) and (2.7), we obtain:

$$\begin{aligned} \phi\tilde{V}_{1j} &= \tilde{V}_j - (1-\phi)\tilde{V}_{2j} \\ &= \tilde{P}_{1j} - (1-\phi)\tilde{P}_{2j} \\ &= \tilde{P}_{1j} - (1-\phi)[\tilde{P}_{1j} + (\lambda-1)(\tilde{V}_j - P_j)] \\ &= \phi\tilde{P}_{1j} + (1-\phi)(\lambda-1)(\tilde{V}_j - P_j) \end{aligned} \tag{2-A2}$$

Dividing through by ϕ on both sides yields equation (2.12).

Appendix 2-B

To solve the consumption-investment problem in the form of equation (2.20) for "rationals", we first derive the first order condition as follows:

$$-\lambda_{01}P_j + \lambda_{11}\{E[\tilde{P}_{1j}] - \frac{1-\phi}{\phi}(\lambda-1)(E[\tilde{V}_j] - P_j)\} + 2\lambda_{21}[1 - \frac{1-\phi}{\phi}(\lambda-1)]^2 \operatorname{cov}(\tilde{P}_{1j}, \tilde{P}_1) = 0 \tag{2-B1}$$

where

$\lambda_{01} \equiv \dfrac{\partial U_1}{\partial C_1}$ is the marginal utility with respect to current consumption;

$\lambda_{11} \equiv \dfrac{\partial U_1}{\partial E[\tilde{C}_1]}$ is the marginal utility with respect to expected portfolio value;

$\lambda_{21} \equiv \dfrac{\partial U_1}{\partial Var[\tilde{C}_1]}$ is the marginal utility with respect to portfolio variance.

Equation (2-B1) can be rewritten as:

$$-\lambda_{01}P_j + \lambda_{11}E[\tilde{P}_{1j}] + 2\lambda_{21}[1 - \frac{1-\phi}{\phi}(\lambda-1)]^2 \operatorname{cov}(\tilde{P}_{1j}, \tilde{P}_1) - \frac{1-\phi}{\phi}\lambda_{11}(\lambda-1)(E[\tilde{V}_j] - P_j) = 0 \tag{2-B2}$$

By dividing each term of equation (2-B2) by λ_{11} and rearranging terms, we obtain the expression for firm j's price that is expected by "rationals":

$$E[\tilde{P}_{1j}] = \gamma_{01}P_j + \frac{2}{\tau_{11}}[1 - \frac{1-\phi}{\phi}(\lambda-1)]^2 \operatorname{cov}(\tilde{P}_{1j}, \tilde{P}_1) + \frac{1-\phi}{\phi}(\lambda-1)(E[\tilde{V}_j] - P_j)$$

where

$$\gamma_{01} \equiv \frac{\lambda_{01}}{\lambda_{11}} = \frac{\partial U_1 / \partial C_1}{\partial U_1 / \partial E[\tilde{C}_1]};$$

$$\tau_{11} \equiv -\frac{\lambda_{21}}{\lambda_{11}} = -\frac{\partial U_1 / \partial E[\tilde{C}_1]}{\partial U_1 / \partial Var[\tilde{C}_1]}$$

Following the same procedure, we can derive the expected price evaluated by "quasi's" as:

$$E[\tilde{P}_{2j}] = \gamma_{02} P_j + \frac{2}{\tau_{12}} \operatorname{cov}(\tilde{P}_{2j}, \tilde{P}_2)$$

where

$\gamma_{02} = \dfrac{\partial U_2 / \partial C_2}{\partial U_2 / \partial E[\tilde{C}_2]}$ is the marginal rate of substitution between consumption and the expected portfolio value for "rationals"; and

$\tau_{12} = -\dfrac{\partial U_2 / \partial E[\tilde{C}_2]}{\partial U_2 / \partial Var[\tilde{C}_2]}$ is the negative of the marginal rate of substitution between expected portfolio value and portfolio variance for "rationals".

Appendix 2-C

From equation (2.23), we write equation (2.22) more compactly as:

$$E[\tilde{P}_j] = \gamma_{0m} P_j + \frac{2}{\tau_{12}} \sum_i \phi_i \operatorname{cov}(\tilde{P}_{ij}, \tilde{P}_i) + (1-\phi)(\lambda - 1)(E[\tilde{V}_j] - P_j), \text{ i} = 1, 2 \qquad \text{(2-C1)}$$

where the proportional weight for "rationals" is $\phi_1 = \phi$ and for "quasi's" is $\phi_2 = 1-\phi$.

Aggregating across the two groups of agents (i = 1, 2), we obtain:

$$E[\tilde{P}_j] = \gamma_{0m} P_j + \frac{2}{\tau_{12}} \sum_i \sum_{i'} \phi_i \phi_{i'} \operatorname{cov}(\tilde{P}_{ij}, \tilde{P}_i) + (1-\phi)(\lambda - 1)(E[\tilde{V}_j] - P_j) \qquad \text{(2-C2)}$$

(2-C2) can be simplified to equation (2.24) to obtain:

$$E[\tilde{P}_j] = \gamma_{0m} P_j + \frac{2}{\tau_{12}} \operatorname{cov}(\tilde{P}_j, \tilde{P}_m) + (1-\phi)(\lambda - 1)(E[\tilde{V}_j] - P_j)$$

Appendix 3-A

To solve the consumption-investment problem, we form the Larangian equation from equation (3.8) as:

$$-\lambda_{i0}V_j + \lambda_{i1}(E[\tilde{V}_j] - E[\tilde{L}_j]) + 2\lambda_{i2}(\mathrm{cov}[\tilde{V}_j, \tilde{V}_i] + \mathrm{cov}[\tilde{L}_j, \tilde{L}_i]) = 0 \tag{3-A1}$$

where

$$\lambda_{i0} \equiv \frac{\partial U_i}{\partial C_i} \tag{3-A2}$$

$$\lambda_{i1} \equiv \frac{\partial U_i}{\partial E[\tilde{C}_i]} \tag{3-A3}$$

$$\lambda_{i2} \equiv \frac{\partial U_i}{\partial\, \mathrm{var}[\tilde{C}_i]} \tag{3-A4}$$

The investor's preferences are reflected in the multipliers λ_{ik} (k = 0, 1, 2), which, by definition, specify the marginal utility for the current consumption, expected consumption, and consumption variance. The sign of λ_{ik} (k = 0, 1, 2) can be pre-determined from the behavior assumptions that investors are rational and risk averse. Thus, $\lambda_{i0} > 0$, $\lambda_{i1} > 0$, $\lambda_{i2} < 0$.

Next, by dividing each term of (3-A1) by λ_{i1}, we define the individual's marginal rate of substitution between current consumption (k = 0), consumption variance (k = 2) and expected consumption (k = 1) as follows:

$$\tau_{i0} \equiv \frac{\lambda_{i0}}{\lambda_{i1}} > 0 \tag{3-A5}$$

$$\tau_{i2} \equiv -\frac{\lambda_{i1}}{\lambda_{i2}} > 0 \tag{3-A6}$$

τ_{i0} measures the individual's marginal rate of substitution between current consumption and future consumption, which is consistent with the definition of the interest rate evaluated by the individual. τ_{i2} measures the marginal rate of substitution between expected consumption and consumption variance. It means that the individual's willingness to take one more unit of variance must be rewarded by τ_{i2} more units of future consumption. Following the Sharpe (1991) notion, we say that τ_{i2} measures the consumption risk tolerance for the individual.

Given the above interpretations, equation (3-A1) becomes:

$$E[\tilde{V}_j] = \tau_{i0} V_j + E[\tilde{L}_j] + \frac{2}{\tau_{i2}}(\text{cov}[\tilde{V}_j, \tilde{V}_i] + \text{cov}[\tilde{L}_j, \tilde{L}_i]),\ j = 1,\dots,J \tag{3-A7}$$

To obtain an equilibrium equation, we require the market clearing condition that ensures that the demand for each asset is equal to its market supply, i.e., $\sum_i X_{ij} = 1$, j = 1,...,J. Aggregating equation (3-A7) across all the individuals (i = 1,...,I), we obtain the equilibrium equation for a representative agent as:

$$\sum_i X_{ij}\tau_{i2} E[\tilde{V}_j] = \sum_i X_{ij}\tau_{i0}\tau_{i2} V_j + \sum_i X_{ij}\tau_{i2} E[\tilde{L}_j] + 2\sum_i X_{ij}(\text{cov}[\tilde{V}_j, \tilde{V}_i] + \text{cov}[\tilde{L}_j, \tilde{L}_i]) \tag{3-A8}$$

which is simplified to:

$$E[\tilde{V}_j] = \gamma_{m0} V_j + E[\tilde{L}_j] + \frac{2}{\tau_{m2}}(\text{cov}[\tilde{V}_j, \tilde{V}_m] + \text{cov}[\tilde{L}_j, \tilde{L}_m]) \tag{3-A9}$$

where

$$\gamma_{m0} \equiv \sum_i X_{ij}\tau_{i0}\tau_{i2} / \tau_{m2} \tag{3-A10}$$

$$\tau_{m2} \equiv \sum_i X_{ij}\tau_{i2} \tag{3-A11}$$

$$\text{cov}[\tilde{V}_j, \tilde{V}_m] = \sum_i X_{ij}\,\text{cov}[\tilde{V}_j, \tilde{V}_m] \tag{3-A12}$$

$$\text{cov}[\tilde{L}_j, \tilde{L}_m] = \sum_i X_{ij}\,\text{cov}[\tilde{L}_j, \tilde{L}_m] \tag{3-A13}$$

τ_{m2} (k = 2) defines the representative agent's or the market risk tolerance, which is a weighted average of the τ_{i2} with the weights given by each individual's proportion of wealth invested in firm j. γ_{m0} is a weighted average of each individual's assessment of the interest rate with the weights given by each individual's consumption risk tolerance measure τ_{i2}. Hence, γ_{m0} reflects the equilibrium interest rate assigned by the market. $\text{cov}[\tilde{V}_j, \tilde{V}_m]$ denotes the covariance of firm j's fundamental value with that of the market portfolio. $\text{cov}[\tilde{L}_j, \tilde{L}_m]$ denotes the covariance of firm j's liquidity with that of the market portfolio.

Let $\tilde{R}_j$ denote the equilibrium return per dollar from investing in firm j. We apply the security valuation formula $E[\tilde{R}_j] = E[\tilde{V}_j]/V_j - 1$ to equation (3-A9) and obtain:

$$E[\tilde{R}_j] = \gamma_{m0} + E[\frac{\tilde{L}_j}{V_j}] + \frac{2V_m}{\tau_{m2}}(\text{cov}[\tilde{R}_j, \tilde{R}_m] + \text{cov}[\frac{\tilde{L}_j}{V_j}, \frac{\tilde{L}_m}{V_m}]) \tag{3-A14}$$

where

$\tilde{L}_j / V_j$ is interpreted as the liquidity return of firm j; and

$\tilde{L}_m / V_m$ is interpreted as the liquidity return of the market portfolio.

Aggregating across all the firms (j = 1,...,J), we obtain:

$$E[\tilde{R}_m] = \gamma_{m0} + E[\frac{\tilde{L}_m}{V_m}] + \frac{2V_m}{\tau_{m2}}(\text{var}[\tilde{R}_m] + \text{var}[\frac{\tilde{L}_m}{V_m}]) \tag{3-A15}$$

Substituting the term $2V_m/\tau_{m2}$ from equation (3-A15) into (3-A14), we get:

$$E[\tilde{R}_j] = \gamma_{m0} + E[\frac{\tilde{L}_j}{V_j}] + \frac{\text{cov}[\tilde{R}_j, \tilde{R}_m] + \text{cov}[\frac{\tilde{L}_j}{V_j}, \frac{\tilde{L}_m}{V_m}]}{\text{var}[\tilde{R}_m] + \text{var}[\frac{\tilde{L}_m}{V_m}]}(E[\tilde{R}_m] - \gamma_{m0} - E[\frac{\tilde{L}_m}{V_m}])$$

(3-A16)

Rearranging terms and making substitutions yields the following key results:

$$E[\tilde{R}_j] = R_{0m} + (\rho^V \beta^V_{jm} + \rho^L \beta^L_{jm})(E[\tilde{R}_m] - R_{0m}) + (\frac{E[\tilde{L}_j]}{V_j} - \frac{E[\tilde{L}_m]}{V_m}) \tag{3.9}$$

where

$$R_{0m} = \gamma_{m0} + E[\frac{\tilde{L}_m}{V_m}] \tag{3.10}$$

$$\beta_{jm}^{V} = \frac{\mathrm{cov}[\tilde{R}_j, \tilde{R}_m]}{\mathrm{var}[\tilde{R}_m]} \tag{3.12}$$

$$\beta_{jm}^{L} = \frac{\mathrm{cov}[\frac{\tilde{L}_j}{V_j}, \frac{\tilde{L}_m}{V_m}]}{\mathrm{var}[\frac{\tilde{L}_m}{V_m}]} \tag{3.13}$$

$$\rho^{V} = \frac{\mathrm{var}[\tilde{R}_m]}{\mathrm{var}[\tilde{R}_m] + \mathrm{var}[\frac{\tilde{L}_m}{V_m}]} \tag{3.14}$$

$$\rho^{L} = \frac{\mathrm{var}[\frac{\tilde{L}_m}{V_m}]}{\mathrm{var}[\tilde{R}_m] + \mathrm{var}[\frac{\tilde{L}_m}{V_m}]} \tag{3.15}$$

Appendix 4-A

An Example of the First-Pass Portfolio Procedure

Suppose that the total number of firms in the economy is 500. To form portfolios in the first pass, we first rank these firms based on $\theta_{i,t-1}$ in a non-decreasing order such that $\theta_{1,t-1} \geq \ldots \theta_{100,t-1} \geq \ldots \theta_{200,t-1} \geq \ldots \theta_{300,t-1} \geq \ldots \theta_{400,t-1} \geq \ldots \theta_{500,t-1}$. This means that the first firm in the list has the smallest size and the last firm has the largest size. We make 5 portfolios each of which contains exactly 100 firms. In Panel 1, portfolios are totally based on size. Thus, the first portfolio contains firms from $\theta_{1,t-1}$ to $\theta_{100,t-1}$, the second one from $\theta_{101,t-1}$ to $\theta_{200,t-1}$, and the last one from $\theta_{401,t-1}$ to $\theta_{500,t-1}$. In Panel 2, each portfolio is composed of 80% of firms sorted on size and 20% firms randomly selected. We implement the selection procedure as follows. We first construct a random selection pool that collects the bottom 20 firms from each of the 5 portfolios. This means that the random pool is composed of firms satisfying the set $\{\theta_{81,t-1}$ to $\theta_{100,t-1}\} \cup \{\theta_{181,t-1}$ to $\theta_{200,t-1}\} \cup \{\theta_{281,t-1}$ to $\theta_{300,t-1}\} \cup \{\theta_{381,t-1}$ to $\theta_{400,t-1}\} \cup \{\theta_{481,t-1}$ to $\theta_{500,t-1}\}$. The first portfolio is composed of 80 firms from $\theta_{1,t-1}$ to $\theta_{80,t-1}$, and the remaining 20 firms are randomly selected from the random pool. We delete the 20 already selected firms from the random pool. The second portfolio contains 80 firms from $\theta_{101,t-1}$ to $\theta_{180,t-1}$ and 20 firms randomly selected from the updated random pool. This selection procedure is continued until the fifth portfolio, which contains 80 firms from $\theta_{401,t-1}$ to $\theta_{480,t-1}$ and 20 firms from the random pool. We implement the first-pass procedure for Panel 3 (60%), Panel 4 (40%), Panel 5 (20%) and Panel 6 (0%).

Table 3.1 Time Series Statistics for the Market Portfolio Returns (77:03 – 98:12)

	Mean	Variance
EWIR	0.0172	0.0041
VWIR	0.0117	0.0024
CSMR	0.0183	0.0026

EWIR: Equal Weighted Index Return obtained from the CFMRC database;

VWIR: Value Weighted Index Return obtained from the CFMRC database;

CSMR: Cross-Sectional Mean Return created from the sample for each month;

Mean and variance are time series mean and variance from 77:03 to 98:12.

Table 3.2 Time Series Statistics for Liquidity Measures and Proportional Variance

	Mean	Variance (x100)	Proportional Variance
PQS	0.0186	0.000586	0.22%
PES	0.0047	0.000216	0.08%
CSMR	0.0183	0.260000	99.78% (99.92%)

PQS: Proportional Quoted Spread;

PES: Proportional Effective Spread;

CSMR: Cross-Sectional Mean Return created from the sample for each month;

Mean and variance are time series mean and variance from 77:03 to 98:12.

Proportional variance for PQS = var(PQS)/[var(PQS) + var(CSMR)];

Proportional variance for EQS = var(PQS)/[var(EQS) + var(CSMR)];

Proportional variance for CSMR = var(CSMR)/[var(PQS) + var(CSMR)], or
CSMR = var(CSMR)/[var(EQS) + var(CSMR)].

Table 3.3 Cross-Sectional Test of the Size and Beta Effects

	Constant	Size	Beta
Mean	0.012	0.002	0.006
Standard Deviation	0.035	0.008	0.061
t-Statistic	5.329*	3.581*	1.518

For each month t = 77:03 to 98:12, we run the following cross-sectional regression:

$$R_{p,t} = \gamma_{0,t} + \lambda_{1,t}\hat{\beta}_{pt} + \lambda_{2,t}(\Theta_{p,t} - \Theta_{m,t}) + \varepsilon_{p,t}, \qquad p = 1,\ldots,25$$

where

$R_{p,t}$ is the return on portfolio p for month t;

$\hat{\beta}_{pt}$ is the post-ranking beta for portfolio p for month t;

$\Theta_{p,t} \equiv \ln(1/MV_{p,t})$ is the natural logarithm of the inverted market value for portfolio p for month t;

$\Theta_{m,t} \equiv \ln(1/MV_{m,t})$ is the natural logarithm of the inverted market value for the market index m for month t.

The cross-sectionally estimated $\hat{\gamma}_{0,t}$, $\hat{\lambda}_{1,t}$ and $\hat{\lambda}_{2,t}$ form time-series of estimated coefficients over the observation period. The mean, standard deviation and the associated t-statistics for these estimated coefficients are reported in the table.

Table 3.4 Cross-Sectional Test of the Expected liquidity Effect

	Constant	Beta	Liquidity
Panel A: Coefficient statistics using the proportional quoted spread (PQS)			
Mean	0.014	0.004	0.214
Standard Deviation	0.033	0.062	1.145
t-Statistic	7.016*	1.011	3.014*
Panel B: Coefficient statistics using the proportional effective spread (PES)			
Mean	0.015	0.003	0.238
Standard Deviation	0.032	0.060	0.743
t-Statistic	7.531*	0.878	5.168*

For each month t = 77:03 to 98:12, we run the following cross-sectional regression:

$$R_{p,t} = \gamma_{0,t} + \lambda_{1,t}\hat{\beta}_{pt} + \lambda_{2,t}(S_{p,t} - S_{m,t}) + \varepsilon_{p,t}, \qquad p = 1,\ldots,25$$

where

$R_{p,t}$ is the return on portfolio p for month t;

$\hat{\beta}_{pt}$ is the post-ranking beta for portfolio p for month t;

$S_{p,t}$ is the proportional spread for portfolio p for month t;

$S_{m,t}$ is the proportional spread for the market index m.

The cross-sectionally estimated $\hat{\gamma}_{0,t}$, $\hat{\lambda}_{1,t}$ and $\hat{\lambda}_{2,t}$ form time-series of estimated coefficients over the observation period. The mean, standard deviation and the associated t-statistics for these estimated coefficients are reported in the table.

Table 3.5 Cross-Sectional Test of the Size and Liquidity Effects

	Constant	Size	Liquidity
Panel A: Coefficient statistics using the proportional quoted spread (PQS).			
Mean	0.015	0.002	0.556
Standard Deviation	0.048	0.008	1.318
t-Statistic	4.945*	4.207*	6.814*
Panel B: Coefficient statistics using the proportional effective spread (PES)			
Mean	0.016	0.001	0.297
Standard Deviation	0.049	0.009	1.493
t-Statistic	5.281*	2.657*	3.209*

At each month t = 77:03 to 98:12, we run the following cross-sectional regression:

$$R_{p,t} = \gamma_{0,t} + \lambda_{1,t}(\Theta_{p,t} - \Theta_{m,t}) + \lambda_{2,t}(S_{p,t} - S_{m,t}) + \varepsilon_{p,t}, \qquad p = 1,\ldots,25$$

where

$R_{p,t}$ is the return on portfolio p for month t;

$\Theta_{p,t} \equiv \ln(1/MV_{p,t})$ is the natural logarithm of the inverted market value for portfolio p for month t;

$\Theta_{m,t} \equiv \ln(1/MV_{m,t})$ is the natural logarithm of the inverted market value for the market index m for month t;

$S_{p,t}$ is the proportional spread for portfolio p for month t;

$S_{m,t}$ is the proportional spread for the market index m for month t.

The cross-sectionally estimated $\hat{\gamma}_{0,t}$, $\hat{\lambda}_{1,t}$ and $\hat{\lambda}_{2,t}$ form time-series of estimated coefficients over the observation period. The mean, standard deviation and the associated t-statistics for these estimated coefficients are reported in the table.

Table 4.1 Time Series Statistics for the Market Portfolio Returns (77:03 – 98:12)

	Mean	Variance
EWIR	0.0172	0.0041
VWIR	0.0117	0.0024
CSMR	0.0183	0.0026

EWIR: Equal Weighted Index Return obtained from the CFMRC database;

VWIR: Value Weighted Index Return obtained from the CFMRC database;

CSMR: Cross-Sectional Mean Return created from the sample for each month;

Mean and variance are time series mean and variance from 77:03 to 98:12.

Table 4.2 Results of the Cross-Sectional Regressions for the 5x5 Portfolio Returns on Conditional Betas and Size Deviations

Beta Premium in Up Markets (t value)	Beta Premium in Down Markets (t value)	Size (x100) (t value)
Panel 1. Percentage of firms sorted on size = 100%		
0.0344	-0.0683	0.204
(4.73*)	(-8.17*)	(3.58*)
Panel 2. Percentage of firms sorted on size = 80%		
0.0353	-0.0674	0.165
(4.98*)	(-8.23*)	(2.64*)
Panel 3. Percentage of firms sorted on size = 60%		
0.0317	-0.0696	0.114
(4.53*)	(-8.22*)	(2.27*)
Panel 4. Percentage of firms sorted on size = 40%		
0.0321	-0.0701	0.083
(4.57*)	(-8.38*)	(1.86)
Panel 5. Percentage of firms sorted on size = 20%		
0.0328	-0.0700	0.042
(4.64*)	(-8.02*)	(1.39)
Panel 6. Percentage of firms sorted on size = 0%		
0.0327	-0.0684	-0.004
(4.66*)	(-8.42*)	(-0.028)

Notes:

1. "*" indicates significance at the 5% level;

2. Each set of 25 (i.e., 5x5) portfolios are formed on size and betas as follows. The procedure in the first panel (percentage = 100%) follows the Fama-French (1992) approach where firms are first sorted and grouped on size, and then firms are ranked by pre-ranking betas and subdivided into beta groups within each size group. The procedure in the last panel (percentage = 0%) does not sort on size. Instead, all the stocks are randomly selected from the

whole population, and then firms are sub-allocated into beta groups within each randomly selected group. For the other Panels (2-5), the first-pass portfolio is a mixture of stocks sorted on size and stocks randomly selected. For example, each portfolio in the third Panel (percentage = 60%) is first chosen where 60% of the firms are sorted on size and 40% of the firms are selected randomly, and then these groups are sub-divided into beta subgroups as described before;

3. At each month t, we run the following cross-sectional regression:

$$R_{p,t} = \gamma_{0,t} + \lambda_{1,t} * \delta * \hat{\beta}_p + \lambda_{2,t} * (1-\delta) * \hat{\beta}_p + \alpha_{1,t} * (\Theta_{p,t} - \Theta_{m,t}) + \varepsilon_{p,t}, \; p = 1,\ldots,25$$

where

$R_{p,t}$ is the return on portfolio p in month t;

$\hat{\beta}_p$ is the post-ranking beta for portfolio p;

$\Theta_{p,t} \equiv \ln(1/MV_{p,t})$ is the natural logarithm of the inverted market value for portfolio p;

$\Theta_{m,t} \equiv \ln(1/MV_{m,t})$ is the natural logarithm of the inverted market value for the market index m; and

δ is the dummy variable conditioned on up and down markets, where $\delta = 1$ if $(R_{m,t} - R_{f,t}) \geq 0$ and $\delta = 0$ if $(R_{m,t} - R_{f,t}) < 0$.

The cross-sectionally estimated coefficients $\hat{\lambda}_{1,t}$, $\hat{\lambda}_{2,t}$ and $\hat{\alpha}_{1,t}$ are time-series averages over the observation period. The mean values and their associated t-statistics for these estimated coefficients are reported in each panel;

4. The number of months in up markets is 154 and in down markets is 108.

Table 4.3 Results of the Cross-Sectional Regressions for the 5x5 Portfolio Returns on Size and Liquidity

PQS (t value)	Size (x100) (t value)	PES (t value)	Size (x100) (t value)
	Panel 1. Percentage of firms sorted on size = 100%		
0.556 (6.81*)	0.219 (4.21*)	0.297 (3.21*)	0.139 (2.66*)
	Panel 2. Percentage of firms sorted on size = 80%		
0.314 (2.63*)	0.086 (1.30)	0.327 (3.39*)	0.074 (1.19)
	Panel 3. Percentage of firms sorted on size = 60%		
0.361 *(4.03*)*	0.081 *(1.21)*	0.262 *(2.55*)*	0.049 *(0.64)*
	Panel 4. Percentage of firms sorted on size = 40%		
0.447 *(4.93*)*	0.035 *(0.68)*	0.295 *(4.76*)*	0.038 *(0.40)*
	Panel 5. Percentage of firms sorted on size = 20%		
0.395 *(4.19*)*	0.033 *(0.51)*	0.332 *(5.44*)*	0.036 *(0.53)*
	Panel 6. Percentage of firms sorted on size = 0%		
0.318 (3.44*)	0.022 (0.40)	0.271 (3.96*)	-0.011 (-0.22)

Notes:

1. "*" indicates significance at the 5% level;

2. Each set of 25 (i.e., 5x5) portfolios are formed on size and liquidity measures as follows. The procedure in the first panel (percentage = 100%) follows the Fama-French (1992) approach where firms are first sorted and grouped on size, and then firms are ranked by PQS or PES and subdivided into liquidity groups within each size group. The procedure in the last panel (percentage = 0%) does not sort on size. Instead, all the stocks are randomly selected from the whole population, and then firms are sub-allocated into liquidity groups within each randomly selected group. For the other Panels (2-5), the

first-pass portfolio is a mixture of stocks sorted on size and stocks randomly selected. For example, each portfolio in the third Panel (percentage = 60%) is first chosen where 60% of the firms are sorted on size and 40% of the firms are selected randomly, and then these groups are sub-divided into liquidity subgroups as described before;

3. At each month t, we run the following cross-sectional regression:

$$R_{p,t} = \gamma_{0,t} + \lambda_{1,t} * (L_{p,t} - L_{m,t}) + \alpha_{1,t} * (\Theta_{p,t} - \Theta_{m,t}) + \varepsilon_{p,t}, \ p = 1,\dots,25$$

where

$R_{p,t}$ is the return on portfolio p in month t;

$L_{p,t}$ is the liquidity measure PQS or PES for portfolio p;

$L_{m,t}$ is the liquidity measure PQS or PES for the market portfolio;

$\Theta_{p,t} \equiv \ln(1/MV_{p,t})$ is the natural logarithm of the inverted market value for portfolio p;

$\Theta_{m,t} \equiv \ln(1/MV_{m,t})$ is the natural logarithm of the inverted market value for the market index m.

The cross-sectionally estimated coefficients $\hat{\lambda}_{1,t}$ and $\hat{\alpha}_{1,t}$ are time-series averages over the observation period. The mean values and their associated t-statistics for these estimated coefficients are reported in each panel;

4. Columns 1 and 2 report results when PQS is used as the liquidity measure $L_{p,t}$;
Columns 3 and 4 report results when PES is used as the liquidity measure $L_{p,t}$.

Table 4.4 Results of the Cross-Sectional Regressions for the 5x5 Portfolio Returns on Size, Conditional Beta and Liquidity

Beta Premium (Up) (t value)	Beta Premium (Down) (t value)	Size (x100) (t value)	PQS[a] (t value)
	Panel 1. Percentage of firms sorted on size = 100%		
0.022	-0.038	0.246	0.575
(4.70*)	(-9.20*)	(4.63*)	(7.11*)
	Panel 2. Percentage of firms sorted on size = 80%		
0.028	-0.038	0.077	0.306
(4.89*)	(-9.62*)	(1.53)	(3.02*)
	Panel 3. Percentage of firms sorted on size = 60%		
0.029	-0.035	0.057	0.350
(5.16)*	*(-7.63*)*	*(1.21)*	*(3.99*)*
	Panel 4. Percentage of firms sorted on size = 40%		
0.029	-0.034	0.059	0.344
(5.16)*	*(-8.28*)*	*(1.16)*	*(3.83*)*
	Panel 5. Percentage of firms sorted on size = 20%		
0.033	-0.036	0.044	0.320
(7.41)*	*(-7.98*)*	*(0.97)*	*(3.51*)*
	Panel 6. Percentage of firms sorted on size = 0%		
0.033	-0.035	0.040	0.260
(5.45*)	(-7.40*)	(0.62)	(2.99*)

Notes:

1. "*" indicates significance at the 5% level;

2. Reported results are for PQS. Results for PES are similar and not reported here;

3. Each set of 25 (i.e., 5x5) portfolios are formed on size and liquidity measures as follows. The procedure in the first panel (percentage = 100%) follows the Fama-French (1992) approach where firms are first sorted and grouped on size, and then firms are ranked by PQS and subdivided into liquidity groups within each size group. The procedure in the

last panel (percentage = 0%) does not sort on size. Instead, all the stocks are randomly selected from the whole population, and then firms are sub-allocated into liquidity groups within each randomly selected group. For the other Panels (2-5), the first-pass portfolio is a mixture of stocks sorted on size and stocks randomly selected. For example, each portfolio in the third Panel (percentage = 60%) is first chosen where 60% of the firms are sorted on size and 40% of the firms are selected randomly, and then these groups are sub-divided into liquidity subgroups as described before;

4. At each month t, we run the following cross-sectional regression:

$$R_{p,t} = \gamma_{0,t} + \lambda_{1,t} * \delta * \hat{\beta}_p + \lambda_{2,t} * (1-\delta) * \hat{\beta}_p + \alpha_{1,t} * (L_{p,t} - L_{m,t}) + \theta_{1,t} * (\Theta_{p,t} - \Theta_{m,t}) + \varepsilon_{p,t}$$

$$p = 1, \ldots, 25$$

where

$R_{p,t}$ is the return on portfolio p in month t;

$\hat{\beta}_p$ is the post-ranking beta for portfolio p;

$L_{p,t}$ is the liquidity measure PQS for portfolio p;

$L_{m,t}$ is the liquidity measure PQS for the market portfolio;

$\Theta_{p,t} \equiv \ln(1/MV_{p,t})$ is the natural logarithm of the inverted market value for portfolio p;

$\Theta_{m,t} \equiv \ln(1/MV_{m,t})$ is the natural logarithm of the inverted market value for the market index m; and

δ is the dummy variable conditioned on up and down markets, where $\delta = 1$ if $(R_{m,t} - R_{f,t}) \geq 0$ and $\delta = 0$ if $(R_{m,t} - R_{f,t}) < 0$.

The cross-sectionally estimated coefficients $\hat{\lambda}_{1,t}$, $\hat{\lambda}_{2,t}$, $\hat{\alpha}_{1,t}$ and $\hat{\theta}_{1,t}$ are time-series averages over the observation period. The mean values and their associated t-statistics for these estimated coefficients are reported in each panel.