Mixed Integer Programming Models for Supply Chain Integrated Planning

Devender Mohan Gupta

A Thesis

in

The Department

of

Mechanical and Industrial Engineering

Presented in Partial Fulfillment of the Requirements

for the Degree of Master of Applied Science (Mechanical Engineering) at

Concordia University

Montreal, Quebec, Canada

April 2003

Your file Votre référence
ISBN: 0-612-83882-X
Our file Notre référence
ISBN: 0-612-83882-X

Abstract

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This thesis is intended to highlight the methodology of integrated planning for coordinating the supply chain in order to improve a system's overall performance. For that, a typical system is considered in which several products must be produced and distributed over several time periods. This work is done on a particular system that includes several suppliers, raw material processing plants, finishing plants and customers. Two main models have been developed based on different approaches, and they have been implemented in two different contexts. Various processes such as raw material procurement, capacity utilization, inventory, and distribution are integrated and optimized. More specifically, the following stages are considered: supply of raw material; production, at raw material processing plants, of product families; production, at finishing plants, of finished goods; and distribution of finished goods to customers. The finished goods are grouped into product families; setups are incurred at the raw material processing plants for product families, and at the finishing plants for individual products. Mixed Integer Program (MIP) formulations are utilized for optimizing the system. Test cases include both small sized and large sized problems. Analysis is done to gain insights into the workings of the models and systems like these in general. In the end, we have successfully demonstrated the utility of managing the supply chain effectively by integrating various processes along with the power and utility of MIP for representing such systems.

Acknowledgements

The author is greatly indebted to Dr. Jean-Marie Bourjolly and Dr. Mingyuan Chen for their invaluable guidance, support and encouragement through all the phases of graduate study and thesis preparation. Financial assistance received from CAE, through its award, Excellence in Engineering, and from the Department of Mechanical and Industrial Engineering, Concordia University is highly appreciated.

Finally, the author wishes to thank his family and friends without whose support and encouragement this thesis work would not have been possible.

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Chapter One

Introduction to Supply Chain Management

In recent times, the area of Supply Chain Management (SCM) has become a hot topic for researchers in Logistics, Production/Operations Management and Management Science all around the world and the terms Supply Chain Management and Value Chain Management (Oliver and Webber, 1982; Porter, 1985) have gained significant popularity. So much so that SCM and VCM are now considered as prime factors for success in business management. And due to its ever-increasing popularity researchers/practitioners have defined SCM according to their convenience. Before defining SCM, one should briefly take a look into a basic manufacturing system. Such a system consists of a network of suppliers, factories, distribution centers and customers. Through this network raw material is passed from suppliers to the factories, where it is converted to finished goods that are then sent to the distribution centers where they are packaged and sent to customers according to various specifications. The conversion of raw material can either be done through actual processing or by assembling it with other materials, or a combination of both. If one takes a closer look at such a system, one can identify various processes it comprises. Some of them can be listed as follows:

- Raw Material Procurement
- Material Processing
- Distribution and Transportation
- Capacity Allocation
- Demand Management
- Scheduling
- Quality and Maintenance
- Product Planning and Inventory Management, etc

It can be concluded that in a manufacturing system, there are a host of issues that have to be taken care of in order to have a complete control of the system and, more importantly, derive profit. Most of these processes have an intricate connection to each other, and planning for one cannot be done exclusively.

Supply Chain is a series of links and shared processes that exist between the suppliers, manufacturers and customers. These links and processes involve all the activities from acquisition of raw material to the delivery of finished goods to the end customer. Raw materials enter into a manufacturing organization via a supply system and are transformed into finished goods. The finished goods are then supplied to consumers through a distribution system. Generally, several companies are linked together in this process, each adding value to the product as it moves through the supply chain (Clarkston Group, Internet Working Paper).

One prominent researcher in Supply Chain Management (see J. Shapiro, 2001) defines supply chain as "consisting of geographically dispersed facilities where raw materials, intermediate products, or finished products are acquired, transformed, stored, or sold, and transportation links connecting the facilities along which products flow. There is a distinction between plants, which are manufacturing facilities where physical product transformations take place and distribution centers, which are facilities where products are received, sorted, put away in inventory, picked from inventory, and dispatched, but not physically transformed. The company may operate these facilities, or vendors, customers, third-party providers or other firms with which the company has business arrangements may operate them. The company's goal is to add value to its products as they pass through its supply chain and transport them to geographically dispersed markets in the correct quantities, with the correct specifications, at the correct time, and at a competitive cost."

Shapiro further defines SCM as the crystallization of these concepts of integrated business planning that have been espoused in the past few years by logistics experts, strategists, and operations research practitioners. A correlation between Supply Chain Management and Integrated Planning is put in place in the following manner, by defining SCM as consisting of

a) Functional integration of purchasing, manufacturing, transportation and warehousing activities
b) Spatial integration of these activities across geographically dispersed vendors, facilities and markets
c) Inter-temporal integration of these activities over strategic, tactical and operational planning horizon.

Simchi-Levi (2003) does not distinguish between SCM and Logistics Management and he defines SCM as "a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses and stores so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize the system wide costs while satisfying the service level requirements".

As businesses become larger and more complex, it becomes increasingly difficult to control costs and meet ever-higher expectations for customer service. Cost reduction and customer service improvements are shown as the primary drivers of Supply Chain Management (Table 1.1). Studies show that these are complementary areas, and improvements in one of them are often accompanied by improvements in the other (Supply Chain Consultants, 2001).

Table 1.1: Supply Chain Planning Delivers Significant Benefits (SC consultant, 2001)

10%	Reduction in total supply chain cost
15%	Improvement in on-time delivery performance
15%-20%	Improvement in asset utilization
20%-30%	Reduction in inventory
25%-35%	Reduction in order fulfillment lead times
40%-65%	Advantage in cash-to-cash cycle time over average companies

Summing up, the key factors in recent times can be concluded as below

- Realization by both researchers and practitioners of the enormous potential benefits arising out of identifying the numerous procedures/processes taking place in a business (useful for analyzing them so that they can be managed)
- Set of approaches that can be utilized to improve/optimize them (to achieve customer service level and to reduce cost)
- Massive leaps in computer/technology advancement (to manage and analyze the huge amounts of information effectively)

The major benefits to effective supply chain management can be summarized as follows:

1. Improved customer service: having the right products, available for delivery when requested, at a good price.
2. Reduction of costs across the supply chain and more efficient management of the working capital
3. More efficient management of raw materials, work-in-process, and finished goods inventory
4. Increased efficiency in the transactions between supply chain partners
5. Better manufacturing resource management
6. Optimized manufacturing schedules
7. Optimal distribution of existing inventory across the supply chain
8. Enhanced customer value, often in the form of lower prices (Clarkston Group, Internet Working Paper)

In the end the following statement can sum up the relevance of managing one's supply chain effectively:

"To stay alive and competitive, organizations will increasingly face the need to integrate their supply chains. In the past, companies that wanted to differentiate themselves in the marketplace and achieve operational excellence undertook supply chain management projects. Today, however, these projects are becoming a necessity for many of the big [and] middle-market...organizations due to their growing need for collaboration and integration."(Kavanagh, 2001)

1.1 Research Background

This thesis generalizes and extends earlier results obtained by Chen and Wang (1997). The model in that case was a Linear Program (LP). Here, due to the presence of setups, one has to work with both continuous and 0-1 variables. Thus, the models become Mixed Integer Programs (MIPs). Further, the planning horizon increases to incorporate multiple time periods. Also the number of facilities in terms of raw material processing plants and is being increased. The models also formulate inventories both at raw material and at finished goods processing centers. Therefore a substantial extension is being made to the original formulation. All the models have been formulated and coded with the commercial software LINGO™ on a Pentium-4 machine. A variety of problem instances have been tested with the models. Some of them (e.g. 3 time periods, 2 raw material processing plants, 3 finishing plants, 3 customers, 3 product families, 6 products) have been solved to optimality. Larger ones have been solved to 1% optimality. Problem difficulty has been analyzed with respect to the size and tightness of the constraints and data.

1.2 Thesis Topic

The topic is the integrated planning of production and distribution in a manufacturing system. The system includes several suppliers, raw material processing plants (RMPs), finishing plants (FPs) and customers. Two main MIPs have been developed in the study. They represent two distinct methodologies:

1) No provision is made for tracking back the products through the chain.
2) Product identity is maintained throughout the chain (This is useful, e.g., in the pharmaceutical industry, where it must be possible to trace back all the constituents of a given medicine, enabling easier tracking of defects and avoiding unnecessary shutdowns).

These methodologies have been studied in two different contexts:

A. Demand is pooled for each finishing plant; this particular case is relevant to a franchising environment in which demand from individual outlets is either aggregated and received at a distribution center from a dedicated plant or at least globally managed by a central authority.
B. Demand from each customer is to be met individually.

In all, four models have been formulated and developed (Two different situations for each methodology). Besides corresponding to different real-life situations, Models 1 and

2 serve as a verification tool for each other since they must provide the same global optimal value for the objective function, and their solutions, though different, must agree.

1.3 Organization of the Thesis

Following the introductory Chapter 1, in Chapter 2 we will review the literature for the recent and earlier work done in this area,

Chapter 3 presents the overall system under study,

Chapter 4 presents the models developed for the system,

Chapter 5 contains the problem-instances used for testing the models. All four models are analyzed, and results are presented,

Chapter 6 presents scenario-analyses for the models, to test the models' robustness in differing situations,

In Chapter 7 we conclude and we define future research work that can be done in this area.

Chapter Two

Literature Review

Over the last few decades, significant progress has been made in the area of Integrated Planning and more recently newer technologies have helped the researchers to look into the fertile area of Supply Chain Integrated Planning. This chapter reviews some of the past and present papers written in this area, specifically with respect to the modeling aspect of the supply chain. The papers have been broadly categorized by the number of echelons and locations examined. Table 2.1 presents the categorization. Echelons refer to the levels of the supply chain that are being considered for integrated planning (each level would indicate a specific portion of the supply chain such as supplier level, manufacturer level, etc) while locations refer to the number of individual entities present in each of those levels (such as the number of suppliers, etc).

Table 2.1: Categorization of Literature

Index	Echelons	Authors (Year)
1	Bi-echelon and multiple locations	Anily and Federgruen (1990)*, Bell *et al.* (1983)*, Chandra and Fisher (1994)*, Chien *et al.* (1989)*, Federgruen and Zipkin (1984)*, Özdamar and Yazgac (1999), Vishwanathan and Mathur (1997)*, Zuo *et al.* (1991)
2	Tri-echelon and single location	Cohen and Pyke (1993), Cohen and Pyke (1994)
3	Tri-echelon and multiple locations	Blumenfeld *et al.* (1987), Chen *et al.* (1994), Dhaenens-Flipo and Finke (2001), Folie and Tiffin (1976), Geoffrion and Graves (1974), Glover *et al.* (1979),
4	Four or more echelons and multiple locations	Chen and Wang (1997), Cohen and Lee (1988), Klingman *et al.* (1988)
5	Other interesting papers	Chen *et al.* (2001), Graves (1980), Geoffrion (1976), Sox and Muckstadt (1996), Thomas and Griffin (1996)
6	Classical references	Dantzig (1967), Ford and Fulkerson (1958), Graves and Schwarz (1977), Shannon and Buckles (1980)

*: Vehicle routing decisions also incorporated in the modeling

2.1 Bi-echelon and Multiple Locations

2.1.1. Anily and Federgruen (1990): In this paper the authors considered distribution systems, with a depot and many geographically dispersed retailers, with external demands occurring at constant, deterministic, but retailer specific rates. They determined the feasible replenishment strategies (i.e. inventory rules and routing patterns) for minimizing long-term average transportation and inventory costs. The discussion was restricted to a class of strategies in which a collection of regions was specified to cover all outlets. The discussion also included a class of low complexity heuristics and the authors demonstrated that under mild probabilistic assumptions the generated solutions were asymptotically optimal. The authors also found lower and upper bounds on the system wide costs. They presented moderately-sized problem instances; solved them using the heuristics to verify their performance, and calculated the bounds for those.

2.1.2. Bell *et al.* (1983): In this paper the authors presented a methodology and its application to the distribution of industrial gases. The methodology integrated the inventory management decisions with vehicle scheduling and dispatching decisions. They designed an advanced decision support system for modeling the system. This decision support system includes on-line data entry functions, customer usage forecasting, a time/distance network with a shortest path algorithm to compute the inter-customer travel times and distances, a mathematical optimization module to produce daily delivery schedule and an interactive schedule change interface. The mathematical optimization routine was an MIP that was solved suboptimally using Lagrangian relaxation. The authors adopted a bottom-up approach in which the bottom level modules fed their optimal solutions to the overall MIP. They reported huge savings of 10-15 % in the annual distribution costs, and predicted further savings in capital expenditure, and increase in productivity. The MIP formulation contained two types of 0-1 variables. A set of 0-1 variables to select the routes and a separate 0–1 variable for each vehicle and its possible starting hour for dispatching. The other variables were continuous. By using Lagrangian relaxation, the authors were able to decompose the problem into a set of knapsack problems, one for each vehicle. Due to the interactive nature of the system, it was able to respond in real-time to changes in the scenarios. The authors reported that the system was implemented in eight out of eleven possible depots that were being considered in the integrated system.

2.1.3. Chandra and Fisher (1994): In this paper the authors discussed the integration of production scheduling and vehicle routing decisions for a single plant, producing several products over several time periods. The plant maintained an inventory of finished goods. These products were distributed through a fleet of trucks to the retailers, with deterministic demand. The authors presented two methodologies for managing such an operation: one in which the decisions regarding production scheduling and vehicle routing were taken independently and another one in which the decisions were coordinated within a single model. They modeled the operations of the plant as a capacitated lot size problem. The fleet operation was modeled as a standard multi-period local delivery routing problem. They tested their methodologies on 132 problem instances and reported a reduction of 3-20%

in the total operating cost from the integration of the above-mentioned decisions. The authors utilized improvement heuristics from the literature to solve the problems separately, while they developed a heuristic to coordinate the decisions of production scheduling and vehicle routing. They found the value of coordinating production and distribution to be high when used in the right conditions. To illustrate, three sets of data were presented signifying the change in the value of the coordination with varying parameters in the problem instances.

2.1.4. Chien *et al.* (1989): In this paper the authors discussed the problem of distributing a limited amount of inventory among customers, using a fleet of vehicles, so as to maximize profit. The problem essentially was to determine how to allocate the available inventory at the depot/warehouse to different customers/retailers, subject to the demand constraint and how to route each vehicle. The system understudy had a single warehouse and multiple retailers. The problem was formulated as an integrated problem and modeled as an MIP. Lagrangian relaxation was used to develop heuristics that generated both good bounds and solutions. The problem was studied for a single period but multi-period problems could be formulated by slightly modifying the algorithm so as to decompose it into several single-period problems, and linking them appropriately. The Lagrangian procedure decomposed the NP-hard problem into one inventory allocation sub-problem and one customer assignment/vehicle utilization sub-problem. The latter was further decomposed into a by-node and a by-vehicle problem. Subgradient search method was used to improve the bounds. Finally the authors suggested extending the research to multiple depots and heterogeneous vehicles.

2.1.5. Federgruen and Zipkin (1984): In this paper the authors incorporated the inventory costs into a single depot vehicle routing problem. They considered stochastic demand and nonlinear inventory costs and suggested a nonlinear integer programming formulation for the combined problem. Their approach decomposed the main problem into a nonlinear inventory allocation subproblem and a number of traveling salespersons (TSP) problems. Starting with an initial inventory allocation and corresponding vehicle routes, they iteratively applied heuristics for constructing a better set of TSP tours and improving the inventory allocation. Their algorithm terminated when no further improvement in the total inventory and routing costs was possible. They also compared the solutions for the combined inventory/routing problems and pure vehicle routing problems. The results showed that about 6-7% savings in operating costs could be achieved by using the combined approach.

2.1.6. Vishwanathan and Mathur (1997): In this paper, which is a generalization and extension of Anily and Federgruen (1990), the authors considered the same problem of integrating routing and inventory decisions in a single depot and multiple retailers/warehouses distribution systems, as was presented in the literature. However, the problem now had multiple products to be distributed to the geographically dispersed retailers. The authors formulated this problem and developed a new heuristic based on stationary nested joint replenishment policy, i.e., optimal replenishment quantities for the products at the retailers, as well as the vehicle routes to deliver these quantities so as to minimize the average inventory and transportation cost over an infinite horizon. The new heuristic was tested and compared with the one developed by Anily and Federgruen (1990). The authors found that their heuristics performed better on a few instances of single product problems. However, it was not universally guaranteed.

2.1.7. Zuo *et al.* (1991): This paper addressed an actual production planning problem for a large seed-production company. The problem involved both allocation of products to available production plants and transportation of the products to where they are needed by the customers. LPs were developed in the paper to represent and optimize the system. Those models were coded in a computer program that combined both commercially available optimizers and a specifically designed heuristic to solve the problem. The solutions obtained and an accompanying sensitivity analyses were reported to provide the management with an insight into the system's operation and the potential cost savings.

2.2 Tri-echelon and Single Location

2.2.1. Cohen and Pyke (1993): The authors discussed performance characteristics for a three-echelon network, consisting of a single station model of factory, a finished goods stockpile (inventory), and a retailer, to expose the impact of integrating the decisions corresponding to each echelon. The system produced a single product with stochastic demand. The authors developed a sequential approach for obtaining steady state distributions for key random variables in this Markov chain model. The performance of this system is evaluated focusing on the finished goods replenishment cycle. A heuristic that provided near optimal solutions and its test results on a variety of problem instances illustrated the insights one might get from this research, namely the tradeoffs associated with the various processes in an integrated supply chain. A multiple product simulation model was used to test the accuracy of the single product approximation. The authors found that the latter works well when the utilization is not very high, and the number of periods containing the expedite-order is relatively small.

2.2.2. Cohen and Pyke (1994): An extension of an earlier paper by the same authors, this paper analyzed the management of material flow in an integrated supply chain. The system modeled is the same, with the exception of modeling multiple products each having independent stochastic demand, thereby generating significant savings by minimizing the overall production and distribution costs. The authors also modeled the possibility of expediting an order (this was also modeled in the earlier paper), on account of a stock decreasing below a certain level (expedite reorder point). They approximated the distribution of key random variables to compute costs and service levels for all products across the supply chain. The stochastic demand at the retailers defined the demand process at the inventory level, and that further generated the order process for the plant. While the queue process at the factory determined the lead-time for the product in the factory (as seen by the inventory) that in turn defined the lead-time for the retailer. This lead-time affected the level of customer service. A three-way tradeoff was shown to exist between replenishment batch sizes, inventory at the finished goods level and inventory at the retailer, through the test scenarios solved by the authors. The algorithm developed was shown to converge. They also noted that a small replenishment batch size effectively reduced the production capacity (due to more setups), thus increasing the production lead times, thereby increasing the downstream inventories.

2.3 Tri-echelon and Multiple Locations

2.3.1. Blumenfeld *et al.* (1987): In this paper the authors showcased the utility of management science models to present the integrated logistics model developed to solve the planning problem faced by GM during the 1980's decade. The authors developed a decision support system (DSS), TRANSPART, to demonstrate a 26% logistics cost savings opportunity. The system incurred inventory costs, at the warehouses and at plants, and inbound and outbound transportation costs. The objective was to minimize the overall cost of inventory and transportation costs. Because of the presence of a large number of plants and warehouses, the system on the whole was a complex network. The authors decomposed it into various sub-networks involving a single plant. Each sub-network was then solved by enumerating the possible route options (as they were less in number) and using the EOQ model to calculate the optimal shipment sizes. For the network in general, it allowed an optimal shipping strategy (shipment sizes and routes) to be determined quickly and easily for routing options involving a combination of direct and warehouse shipping. While developing the new heuristic, the authors also utilized the notion of composite product that was simply a proportionate mixture of the various products that could be shipped together on a link, introduced by Graves (1980). By the time the authors published the paper the DSS was being utilized in 40 of the GM plants.

2.3.2. Chen *et al.* (1994): In this paper two non-linear models were developed for solving a production planning problem of minimizing total cost composed of transportation costs, processing costs and inventory costs. The system understudy consisted of a central factory having multiple satellite factories. The satellite factories were each responsible for producing parts/ subassemblies of the product, the final assembly for which was taking place at the central factory. The satellite factories also passed the product to other satellite factories, depending on the exploded BOM and the material flow network. Based on different methodologies, two models were formulated and solved. The first model aggregated the raw material ordering policy based on the type of products that were requested by the factories, while the second model ordered raw materials according to various factory demands. However both models aggregated the rest of the planning processes, such as order size determination, material-handling functions etc., to define a cost effective production plan. The models were optimized using heuristics that decomposed the nonlinear model into two sub-models. While the first sub-model was linear, the formulation for the second was nonlinear. Based on the test results the authors concluded that the performance of model II was better than model I.

2.3.3. Dhaenens-Flipo and Finke (2001): In this paper the authors presented a multi-facility, multi-product, and multi-period industrial problem. The objective was the integration of production and distribution decisions in a model to be optimized simultaneously as the decisions were interrelated. The system was formulated as a network flow problem with the MIP code written in commercial optimizer CPLEX. The authors decomposed the network into three linked networks, the PL network (production-line network; allocated products to production line in a plant), the LS network (Line-stock network; allocated the products for inventory), and the SC network (Stock-customer network; allocated the products to the customers/warehouses). Each sub-network provided a set of constraints and a term in the objective function of the model formulation. Thus, all the sub-networks were solved simultaneously. The authors first developed a single period model that was extended to a multi-period with appropriate links between the periods. The authors in the end proposed to develop heuristics to solve larger problem instances.

2.3.4. Folie and Tiffin (1976): A multi-product production and distribution was formulated and implemented in the paper. The logistics problem was to allocate production of commodities in a factory and then to distribute those to the customers with intermediate halts at the warehouses for stock-keeping. The new heuristic based on an arc-chain formulation (Ford, 1958) with improvements from incorporating the generalized upper bound algorithm (Dantzig, 1967) was compared with the traditional Node-arc formulation code and Arc-chain formulation. The resulting algorithm was, as indicated by the tests, significantly more efficient than the basic Ford and Fulkerson method with regard to the storage requirements and computational time.

2.3.5. Geoffrion and Graves (1974): In the paper the authors presented a typical problem that occurs in the distribution systems design, i.e. the problem of optimal location of intermediate distribution facilities between the plants and customers. This was formulated as a multi-commodity capacitated single period problem to be modeled as an MIP. Heuristics based on Benders decomposition technique were developed to solve the problem. The decisions to be achieved using this formulation were to determine the optimal locations of the DCs; their capacity and outgoing links (which customers should be served by which DC). Supply constraints were formulated for each plant-product combination, effectively fixing the production mix at each plant. This decomposition separated the problem, at each iteration, into several easily solvable LPs (one for each commodity). The Bender's decomposition method was shown to be very effective for solving problems such as these, although no clear reason as to why they are so effective was given.

2.3.6. Glover *et al.* (1979): In this paper the authors presented a successful implementation of an MIP formulation at Agrico firm. The system, called PDI (Production Distribution and Inventory), integrated the decisions regarding supply (consisting of production, purchases), storage and customer distribution (involving sizing and locating bulk distribution centers), and demand (involving customer demand and locations where the product must be supplied to). The authors designed and implemented the PDI system using network formulation techniques. Apart from the extensive optimization, the system reportedly gave the capability to provide planners with an insight into the system-wide ramifications of their decisions. The authors also stated that its integrated framework allowed planners to make long-range decisions. In terms of long range planning, the system was reportedly being used primarily for sizing and configuration of the distribution system, i.e., it was indicating where the DCs should be located and how much long term investment should be made. The PDI system, according to the authors, made the most substantial impact on short-range planning and operational decisions. In terms of short range planning the system dealt with questions of allocating a product to a plant and delivering it through specific distribution channel. In essence, it was used to decide what, where, and how much should be produced as well as when, where and how much should be transported. The development of the system also led to improved coordination and information flow between key departments. The system was essentially developed as a network model and was implemented using MIP formulation. Typical inputs were production rates, capacity limitations, transportation costs, fixed and variable costs, demand mode of shipment etc. Due to the large size of the problem, a new LP solution, PNET/LP, was developed. In the end the authors reported substantial savings in distribution and energy costs, and in production and inventory costs, due to the implementation of the system.

2.4 Multi-echelons (4) and Multiple Locations

2.4.1. Chen and Wang (1997): The paper, whose results are being extended in the present research, illustrated an integrated production-distribution planning problem for a system having a single factory, multiple finishing factories and multiple customers. The material flowed from the supplier; to the central factory; to the finishing factories; and finally to the customer. A single period problem was formulated keeping in mind the production and distribution planning decisions to be taken simultaneously. An LP model solved the problem using a commercial optimizer. Computational results with the planning problem revealed that high benefits could be realized by integrating the above-mentioned functions. The model was applied in a steel manufacturing industry.

2.4.2. Cohen and Lee (1988): A comprehensive model framework for linking decisions and performance in a typical supply chain was discussed in this paper. The supply chain network was divided into 4 sub-networks, namely, material control (supply), production, finished goods stockpile (inventory), and distribution network. All the sub-models were linked appropriately, so that an integrated methodology was followed and the tradeoffs between the various decisions were accounted in the optimal solution. The material control sub-model formulated the randomness of both the demand process and the re-supply times from the vendors. So it considered the setup and inventory costs in determining the ordering policies. The production sub-model, considering the tradeoff between the cost of WIP in the production process and the queuing relationship, formulated the production lot sizes for each production line. The inventory sub-model formulated the link between the production sub-model and the distribution sub-model. The distribution network took care of the demand for the products. The lead-time distribution, on account of the differing transportation times, illustrated an effect on the distribution system stocking policy. Instead of optimizing the entire network, the authors developed a decomposition approach, so that each sub-model could be optimized, subject to some service target defined for it. These service targets essentially worked as links between the sub-models. So, the optimal solution of one sub-model drove the other one, and so forth, in the end giving a "good" solution that may or may not be globally optimal. The importance of the paper lay in the integration of a broad range of processes for a supply chain, including performance measures. Since the authors were considering service level requirements as well as cost, they proposed its utilization as an important strategic analysis tool.

2.4.3. Klingman *et al.* (1988): The authors described an optimization-based logistics planning system developed at a chemical manufacturing firm. The mathematical model underlying the system included production and distribution of multiple commodities over several time periods. The monolithic problem was first decomposed into a generalized network component and small non-network component. The network component was then transformed into a pure network component and the non-network component was incorporated in the network as bounds for the network. Using Lagrangian procedures, the rest of the model was incorporated in the objective function of the model. The resulting model was solved using efficient pure network solution techniques to obtain an advanced starting solution for a basis-partitioning algorithm developed in the literature (Glover and Klingman, 1981). The users were reported using insights from the solutions to make strategic decisions involving millions of dollars.

2.5 Miscellaneous Papers

2.5.1. Chen *et al.* (2001): In this paper, the authors studied a fundamental distribution channel, where a supplier supplied a single product to the retailer, who in turn sold the product to the customers. The demand was based on some known demand function. The authors argued that an optimal strategy (maximizing total system wide profit) for a centralized system was achieved by a decentralized system, provided that coordination was reached through periodically charged fixed fees and a nontraditional discount pricing scheme given to the retailer. They further illustrated that traditional discount schemes based on quantity discounts do not suffice to optimize the system when there are non-identical retailers.

2.5.2. Geoffrion (1976): In this paper the author argued the use of an auxiliary model (simpler closed form scaled down models of the full version) to offer insights into the system's workings, for strategic applications, to guide the development of effective plans and decisions. The author further stated the importance of the "whys" behind the solutions rather than the "whats". The approach was illustrated with a facility location problem.

2.5.3. Graves (1980): In this paper the author developed a heuristic for a multi-product production cycling problem (MPCP). The problem was to determine the production and inventory policies for a family of products, each of which required processing on a single capacitated plant and had a stochastic demand. Because of the inherent computational difficulty in solving a multi-product model as a Markov chain problem, unlike the single product problem that was formulated over a two dimensional state space consisting of the product's inventory level and the machine's status, a heuristic was developed to find "good" solutions. The heuristic was tested against four other heuristics, based on traditional inventory theory, and it performed significantly better than the others.

2.5.4. Sox and Muckstadt (1996): In this paper the authors presented a finite horizon capacitated production planning problem. The problem was formulated excluding the setup costs and times for the products, but including the backorder costs. The authors utilized a Subgradient optimization algorithm from the literature to optimize the model. The solutions obtained were 1% off the lower bound obtained from the Lagrangian dual. The main advantage for the approach, according to the authors, was that realistic problem instances were solved quickly, and "good" solutions were obtained in a reasonable amount of time although the optimal solutions to these problems were difficult to obtain computationally.

2.5.5. Thomas and Griffin (1996): In this paper the authors provided an extensive review on coordinated supply chain management strategies related to production and distribution systems, as well as mathematical models developed in literature. They broadly categorized their survey into papers dealing with operational planning and papers dealing with strategic issues. The former was further categorized into buyer-vendor coordination, production-distribution coordination and inventory-distribution coordination, while the latter was divided into papers that discussed the methodologies, papers that discussed the case studies for the successful implementation of the SCM solutions, and papers that dealt with issues that are imperative in a coordinated supply chain management function.

Most of the above mentioned literature has common points with the problem studied in this thesis. On the other hand, the existing research in this area does not bring together all the characteristics of multiple time periods, multiple echelons, multiple products (with significant setups and product aggregation), and multiple locations. The mathematical models developed in this thesis research address these issues.

Chapter Three

Problem Introduction and Definition

This chapter will describe, in detail, the manufacturing and distribution network considered. This includes an explanation on the various processes and their relationship to each other. We also explain the entities/facilities considered in the network. An MIP model framework will be presented.

3.1 Investigation of the Manufacturing System

The considered system can be thought of as a manufacturing firm producing a variety of products to satisfy the demand of customers over several time periods. These products are produced from a number of product families, which themselves are produced from the raw material. The raw material is bought from several suppliers. The vast array of products, coupled with a number of manufacturing facilities and the presence of multiple suppliers and customers make the system complex. This complexity makes the system ideal for studying how integrated planning can be utilized to coordinate the supply chain in the system. As discussed in the previous chapters, two main models based on two distinct approaches are considered that are modeled in two different settings. The following section highlights the main models and settings under study.

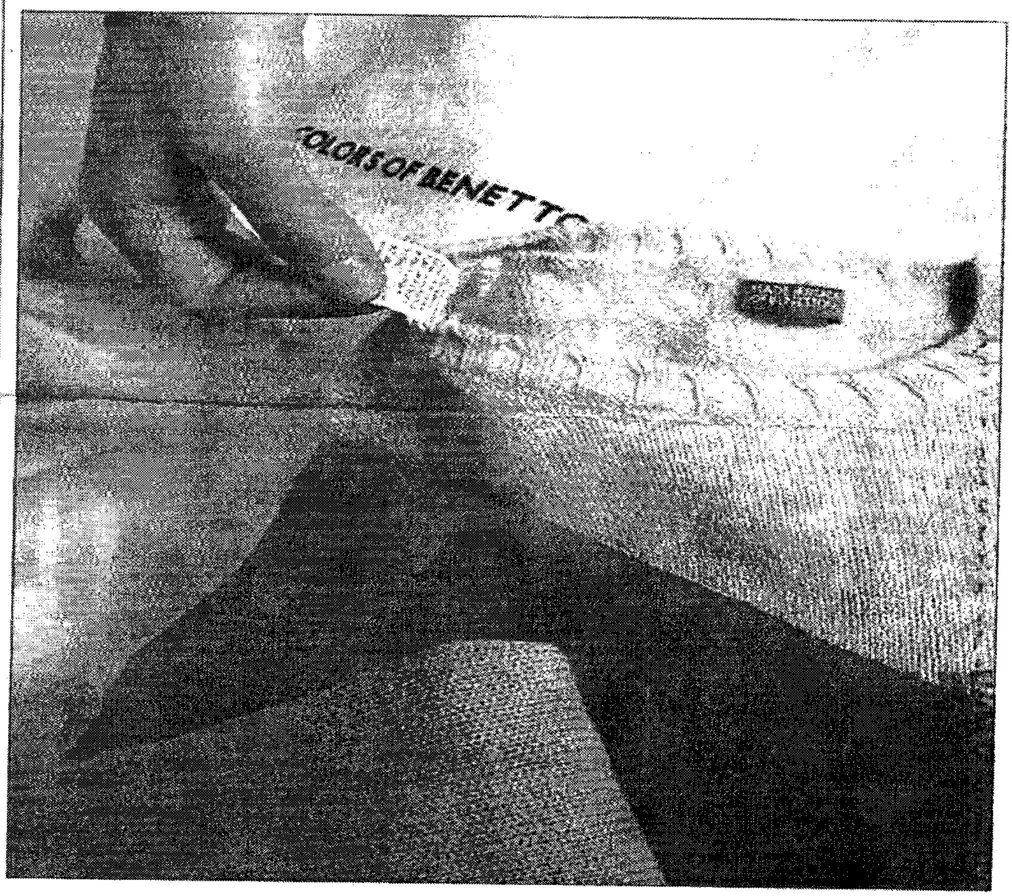
3.1.1 Model Types

In the first type of formulations (See Chapter 1, Section 1.2), there is no provision for backtracking the products. As will be demonstrated later, this can be achieved by appropriately defining the model variables to be optimized. This formulation applies to the general case where being able to trace back the inputs to a product is not a requirement. This means that given a finished product at a customer, there is no provision of determining from which RMP, it was processed or which supplier supplied the resources for its manufacturing. Otherwise stated, once a product reaches a destination, it "forgets" its history and is mixed with other products from different plants. The main advantage of having such a formulation/system is the significant reduction in the model size and thereby computational time for optimization.

In the second type of formulation, by appropriately defining the model variables, one is able to incorporate the provision of tracking the products. Formulations such as these can help a firm pinpoint all the supplies and the subassemblies involved in the defective product. For example, that will enable a pharmaceutical firm to identify all the stocks involved in the manufacturing of the defective medicine so as to rapidly take corrective measures while avoiding unnecessary shutdowns. Nowadays, the World Health Organization (WHO) recommends putting in place such a tracking system in the food industry. This comes at a price of increased complexity and higher computational time. Obviously, there are two distinct aspects for effective planning. One is information gathering while the other is the actual modeling. Figure 3.1 shows the incorporation of tracking the products by placing a chip on the shoulder of the clothes currently being implemented by Benetton Inc. to keep track of its sales thereby introducing the information gathering system in place (The Gazette, 2003).

THE GAZETTE, MONTREAL, **WEDNESDAY,** MARCH 12, 2003

Chip on shoulder



ASSOCIATED PRESS

Displayed on a Benetton top is a smart tag. Clothes sold at Benetton stores will soon contain microchip transmitters that allow the Italian retailer to track its garments from their point of manufacture to the moment they're sold in any of its 5,000 shops.

Figure 3.1: Benetton Tracking its Products

3.1.2 Formulation Settings/Contexts

In the formulations for setting A (see Chapter 1, Section 1.2), the demand is pooled for each finishing plant (FP). This particular case is relevant to a franchising environment, where the demand from individual outlets is either aggregated at the headquarters and received at a distribution center (DC) from a dedicated plant or at least globally managed by a central authority. The main advantage for systems, such as these, is that they allow risk-pooling to take place. As a result, better quality of service can be achieved with lower inventory (Simchi Levi, 2003).

In the latter context (B), the demand for each customer has to be met individually. Modeling for this type of scenario was found to be slightly more complex than the former due to the presence of extra variables to be taken into account (See Chapter 4). In this case, the FPs are free to produce for any customer (most generic case).

The following sections highlight the individual processes in the system including

- Raw Material Sources and Procurement
- Production and Product Structure
- Facility Capacity Utilization
- Inventory
- Distribution
- Setups
- Business Demand

3.1.3 Raw Material Sources and Procurement

There are various suppliers that can provide the raw material to the RMPs for the production. Emphasis will be on purchasing the raw material from the closest one (for the purpose of transportation costs incurred). However due to the presence of capacity for each supplier and differing price of the raw material in the time periods, optimizing the model should provide the best estimate on the following relevant decisions:

- The suppliers to purchase from.
- The amount to purchase.
- The time to purchase.

3.1.4 Production and Product Structure

Two types of production facilities are considered in the system:

1) Raw material processing facilities (RMP) that convert the raw material to product families, and
2) Finishing plants (FP) that take those product families to convert them to individual finished products.

As will be seen subsequently (Figure 3.3) the number of facilities, for both raw material processing and finished products processing, in the system is not restricted. With regards to the product structure, the system produces various types of finished products, each having a certain deterministic demand. The finished products are aggregated into various product families. An example from steel manufacturing can be illustrated here for further clarification. The raw material in the system could be scrap steel that can be sent to the

RMPs for producing differently sized ingots (product families). Thee ingots can be sent to the FPs for further processing into either pipes or utensils (individual products). Summing it up, FPs transforms the product families into individual products. Product families are manufactured at RMPs. RMPs process these product families by the transformation of raw material, bought from the suppliers. Figure 3.2 illustrates the logic of material flow as explained.

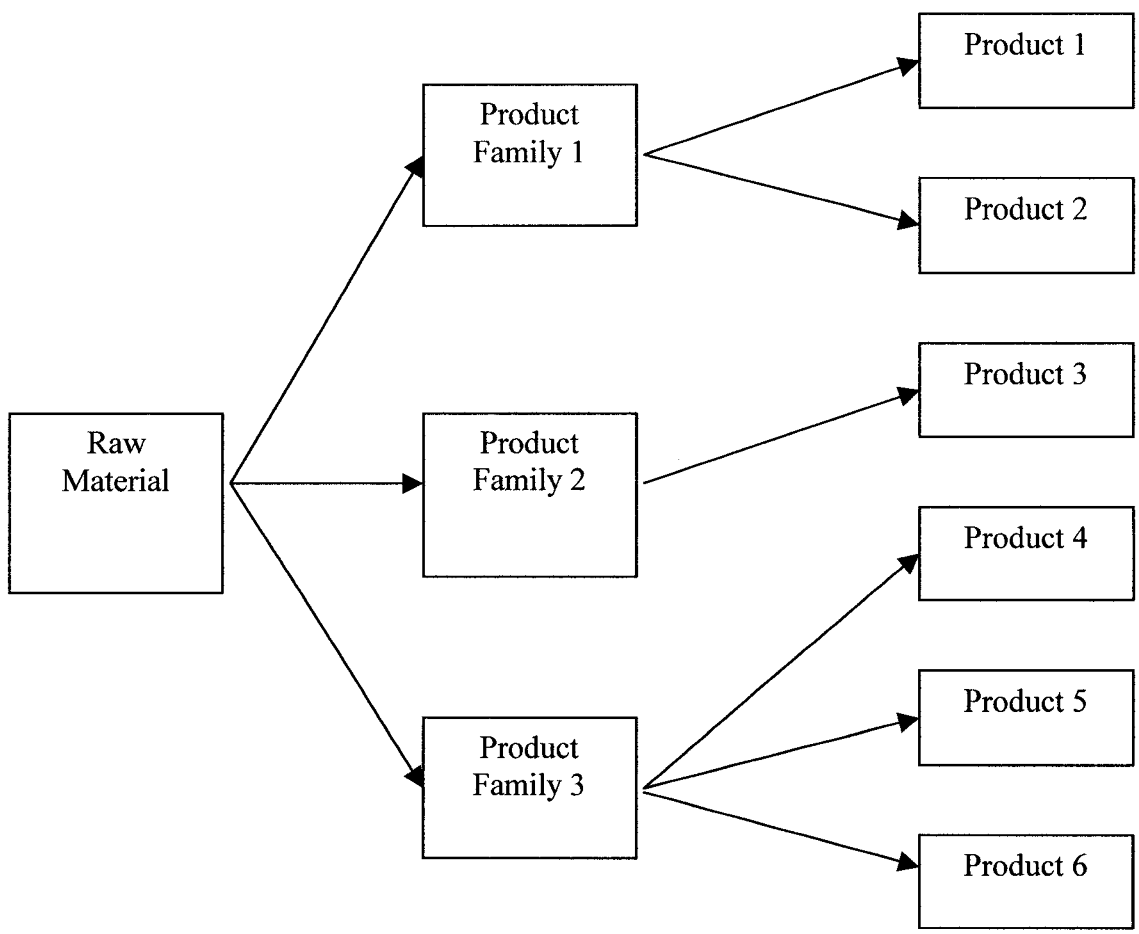


Figure 3.2 Material Flow Logic

3.1.5 Facility Capacity Utilization

In each time period of a planning horizon, the available production time in the production facilities (both at RMPs and at FPs) is limited. Therefore this time has to be judiciously distributed among the product setup and actual production times. In addition, due to the different production rates for the finished products and their corresponding product families, the economical value of each scheduled period of time is different among different products. This can be illustrated by the hypothetical data shown in Table 3.1

Table 3.1: An Example for Product Comparison

	Production Rate (tons/hour)	**Unit Profit ($/ton)**	**Profit per Hour ($/Hour)**
Product A	125	53	6,625
Product B	94	87	8,178

This example illustrates that apart from taking into account the production rate, both the cost and profit information are necessary for adequate planning. Care also has to be taken with regards to the inventory cost of the product. The product whose inventory cost is less can be produced and stored in larger batches compared to the product that has a higher inventory cost. The demand has to be met in all time periods. However, there is variability in the capacities at the plants and the demands of the customers. In light of this, inventory cost plays an important role in the planning. In the actual production environment the number of facilities and finished product (or product families) is quite large. Computerized planning and models have to be used to help the manager to make adequate decisions. There is no limit in the model formulations on the number of facilities/entities in the system. However, since these problems are known to be NP-hard, the computational time is the limiting factor on the size of the instances. Instances larger than those tested in this thesis exist in the world. Those problems can also be tested on our generic model formulations to assess their robustness and efficiency.

3.1.6 Inventory

The purpose of inventory in the system is to compensate for the limited capacity in the facilities and for the suppliers. There are two types of inventories being considered in modeling the manufacturing system.

1) Raw material inventory
2) Finished products inventory

3.1.6.1 Raw Material Inventory

This inventory would be kept at each RMP. Each plant has its own inventory rather than a global inventory. It is assumed that no inventory crossover is allowed, that is inventory at one plant cannot be sent to another plant. This inventory is a result of predetermined fixed capacities of both raw material suppliers and RMPs. This fixed capacity necessitates the inventory in periods of high demand. The inventory will be modeled appropriately to indicate the different approach and contexts.

3.1.6.2 Finished Goods Inventory

This inventory would be kept at each FP. Again each plant will have its own inventory rather than a global inventory. Again, no inventory crossover is allowed.

These two types of inventories should serve the system well. The RMPs send all the product families to the FPs for further processing. The FPs will process all the received product families to finished products. Each inventory type has an associated cost to represent its carrying and handling cost. They may be different for different product families and finished products. No inventory for product families is considered.

3.1.7 Distribution

Figure 3.3 demonstrates the layout of the supply chain understudy and also indicates the material flow from the supplier to the customer. Each link shown the figure is associated with a cost. This cost is assumed to be linear with respect to the quantity of material being sent.

3.1.8 Customer Demand

There are various customers present in the system. Each customer has a deterministic demand over the time horizon. These customers do not necessarily represent individuals but can be firms themselves. Recall that for situation A, the demand is being pooled. In real life this is done for risk pooling (to deal with uncertainty). Here we are considering pooling of demand although the data is deterministic.

3.2 Problem Identification

It is often seen that in a complex manufacturing environment, each department does its own planning. This leads to the creation of walls between various departments thus creating islands of information (Shapiro, 2001). These walls between the departments create misinformation, high variability and multiplicity in the company data (Bull-Whip

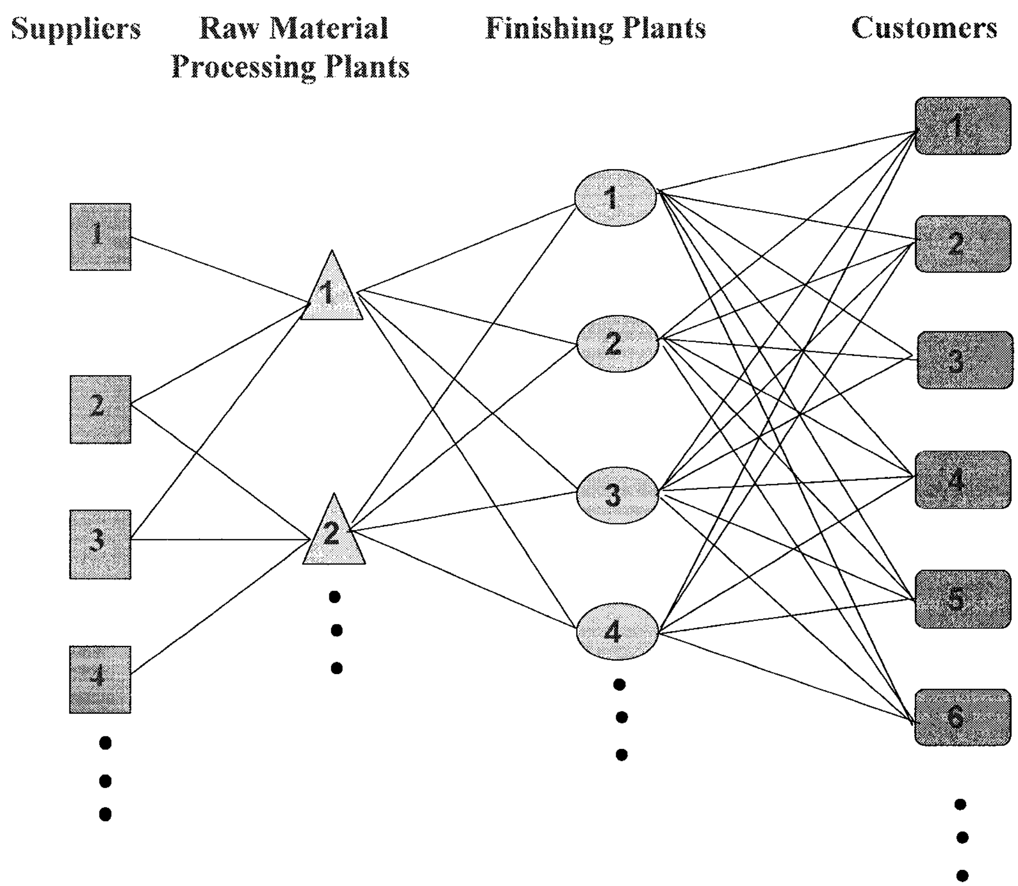


Figure 3.3: Structure of the Supply Chain

effect, Simchi-Levi, 2003). Therefore it becomes imperative to integrate the planning function not only to optimize the various processes but also to have a control on those functions by a full knowledge of the vital information on those processes. This will in the end lead to an improvement in the efficiency of the production.

Chapter Four

Integrated Models for Coordinating the Supply Chain

This chapter presents the two types of MIP models, representing two different methodologies 1 and 2, for situations A and B. Several critical issues involved in the models and problem-solving strategy are discussed. Assumptions made in formulating the models are also presented and discussed.

4.1 Models Development for Situation A

The problem of coordinating the supply chain in the manufacturing environment has been formulated as MIP models. These models are intended to integrate the planning procedure regarding raw material supply, production planning (at both RMPs and FPs) and distribution to several customers across a time horizon (divided into several individual time periods). The raw material is transported to the RMPs. This material is then transformed to the product families according to the requirements of the finished products, as the demand is pulled through the chain, at the RMPs. These product families are then transported to the FPs, are transformed to appropriate finished products, and are finally transported to the right customers in the right quantity in the right time period.

For representing this system mathematically, certain assumptions had to be made. The following section is devoted to the presentation of the general assumptions made for the models. Any changes for a specific model formulation will be explicitly mentioned.

4.1.1 Assumptions

The assumptions can be divided into four categories.

4.1.1.1 Material Supply and Transportation

a) Raw material from each supplier is limited in a time period. The associated purchasing cost may be different for each supplier. Transportation cost is assumed to be linear.

b) Any supplier can supply its raw material to any RMP.

c) Any RMP can supply its end product (product families) to any FP with associated transportation costs. Similarly any FP can supply the finished products to any customer with associated freight costs. This assumption is at work, particularly for situation B models.

4.1.1.2 Production

d) The production process considered can consist of multiple setups for a product, both at RMP, for product family, and at FP, for an individual product. However a product is considered processed only if it passes through all the production setups and thus the production process is represented as a single binary variable signifying all the setups at a facility for a product. In other words, the setup times are aggregated for the production process for a product family processed at an RMP and for a particular product processed at an FP.

e) A setup is incurred whenever production moves from one family/product to another family/product. Setup carryover is not considered.

f) Each product family has a unique production rate, transformation rate, setup time and setup cost. This transformation rate or yield can be aggregated in the similar fashion as setup times.

g) Similarly each finished product has its own yield, production time, setup time and setup cost.

h) Each plant has a production capacity expressed as total available production time.

4.1.1.3 Inventory

i) An RMP only keeps inventory of the raw material and therefore ships all the product families to the FPs.

j) An FP only keeps inventory of the finished products, i.e. it transforms all it receives from the RMPs.

k) No backlog is allowed.

l) In the type 1 models representing methodology 1, inventory at the RMPs is defined with respect to plant c and time period t, while for models representing methodology 2, inventory at the RMPs is defined with respect to plant c, at time period t, of the raw material received from supplier j.

m) Inventory at the end of time horizon is equal to the inventory at the beginning of the time horizon and is equal to a specific number – a real life situation that can be determined by a company's policy.

4.1.1.4 Demand

n) Demand has to be met in each time period. If it is not, then the computational results will indicate infeasibility.

o) Demand is deterministic.

p) The selling price for each product differs from one customer to another.

q) For the situation A models, demand for product n, by customer l, is to be satisfied by plant f, at time t. For the situation B models, demand for product n, by customer l must be met at time t by any of the FPs either individually or combined.

4.1.2 Cost Factors

The costs at different production stages in the modeling include:

1. Raw Material Purchasing Cost.
2. In-Plant Production Costs:
    a. Fixed costs incurred for production at RMP and at FP.
    b. Variable costs incurred for production at RMP and at FP.
    c. Setup costs incurred at RMPs and at FPs.
    d. Inventory costs incurred at RMPs and at FPs.
3. Freight Costs:

a. Raw material transportation (from the supplier to the RMPs) cost.

b. Product family transportation (from the RMPs to the FPs) cost.

c. Finished products transportation (from the FPs to the customers) cost.

4. Revenue from selling the products to the customers.

4.2 MIP Model Formulations

The following subsection will present the models developed in the study.

4.2.1 Notation and Variable Definitions

In presenting the models the following notation and variables are defined.

4.2.1.1 Indices

j = Index of the raw material suppliers, $j \in \{1 \ldots J\}$.

c = Index of the raw material processing plants, $c \in \{1 \ldots C\}$.

f = Index of the finishing plants, $f \in \{1 \ldots F\}$.

l = Index of the customers, $l \in \{1 \ldots L\}$.

m = Index of the product families, $m \in \{1 \ldots M\}$.

$n(m)$ = Index of the product items in a product family, $n(m) \in \{1 \ldots N(M)\}$.

t = Index of the time periods in a planning horizon, $t \in \{1 \ldots T\}$.

4.2.1.2 Parameters

1. Material Supply and Transportation

   a. Costs:

RC_{jct} = Unit purchasing cost of raw material from supplier j at time period t for RMP c.

TR_{jct} = Unit transportation cost of raw material from supplier j to the RMP c, at time period t.

TS_{cfmt} = Unit transportation cost of product m from RMP c to FP f at time period t. Type 1 models use the above representation for the transportation costs.

TS_{jcfmt} = Unit combined cost of transportation of raw material from the supplier to the RMP and the transportation of product family from the RMP to the FP. Models of type 2 utilize this representation.

$TF_{fln(m)t}$ = Unit transportation cost of product n of family m from FP f to customer l at time period t.

b. Capacities:

L_{jt} = Supply capacity for supplier j at time period t.

2. Production

a. Costs:

$FC_{cn(m)}$ = Fixed costs for the manufacturing of product n at RMP c.

$VC_{cn(m)}$ = Variable costs for the manufacturing at RMP c for product n.

$FF_{fn(m)}$ = Fixed cost for the manufacturing of product n at FP f.

$VF_{fn(m)}$ = Variable cost for the manufacturing of product n at FP f.

PS_m = Production rate of product family m in terms of raw material units/time units.

$PF_{n(m)}$ = Production rate of finished product *n* in terms of product family units/time units.

YM_m = Transformation rate for product family *m* in terms of product family units/raw material units.

$YF_{n(m)}$ = Transformation rate for finished product *n* in terms of finished product units/product family units.

CST_{cm} = Unit cost associated with the setup at RMP *c* for product family *m*.

$CST_{fn(m)}$ = Unit cost associated with the setup at FP *f* for product *n* of family *m*.

ST_{cm} = Setup time at RMP *c* for a product family *m*.

$ST_{fn(m)}$ = Setup time at FP *f* for a product *n* of family *m*.

b. Capacities:

ΓC_{ct} = Available production capacity (time) at RMP *c* at time period *t*.

ΓF_{ft} = Available production capacity (time) at FP *f* at time period *t*.

3. Inventory

IC_{tc} = Inventory cost at RMP *c* at time period *t*.

$IF_{fln(m)t}$ = Inventory cost at FP *f* at time period *t* for customer *l* of product *n*. This type of representation is for models implementing situation A.

4. Demand

$DC_{fln(m)t}$ = Demand for product n by customer l at time t to be satisfied by FP f. This type of representation is utilized by model for situation A.

$PR_{fln(m)t}$ = Unit selling price of product n for customer l in time period t produced by FP f. Models for situation A utilize this kind of representation.

4.2.2 Decision Variables

Table 4.1 presents a comparison between the decision variables for Models 1 and 2. These variables are only for the situation A. As will be seen later, situation B has slightly different variables.

Table 4.1: Decision Variables for Situation A

Model 1		Model 2	
Decision Variable	**Description**	**Decision Variable**	**Description**
X_{cfmt}	Amount of product family *m*, produced by RMP *c*, for FP *f*, at time *t*.	X_{jcfmt}	Amount of product family *m*, produced by RMP *c*, from material by supplier *j*, for FP *f*, at time *t*.
$X_{fln(m)t}$	Amount of product *n*, of product family *m*, produced by FP *f*, for customer *l*, at time *t*.	$X_{cfln(m)t}$	Amount produced by FP *f*, of a product *n*, of product family *m*, for customer *l*, from the material sent to *f*, by RMP *c* (As is evident, there is an additional index in the variable).
$I_{fln(m)t}$	Inventory at FP *f* for customer *l* of product family *m*, for product *n* at time *t*.	$I_{fln(m)t}$	Inventory at FP *f* for customer *l* of product family *m*, for product *n* at time *t*.
I_{ct}	Inventory of raw material at RMP *c* at time period *t*.	I_{jct}	Inventory of raw material at RMP *c* at time period *t* sent by supplier *j*.
W_{jct}	Amount of raw material purchased from supplier *j* at time *t* for RMP *c*.	W_{jct}	Amount of raw material purchased from supplier *j* at time *t* for RMP *c*.
S_{cmt}	Binary variable signifying setup for the product family *m* at RMP *c* at time *t*.	S_{cmt}	Binary variable signifying setup for the product family *m* at RMP *c* at time *t*.
$S_{fn(m)t}$	Binary variable signifying setup for the product *n* family *m* at FP *f* at time *t*.	$S_{fn(m)t}$	Binary variable signifying setup for the product *n* family *m* at FP *f* at time *t*.

4.2.3 Constraints

One can follow the same procedure in illustrating the general constraints for both types of models representing situation A. Any modifications with respect to a methodology will be explicitly mentioned.

Constraint on Raw Material Supply

The planned raw material purchasing quantity should not exceed the capacity of supplier j in time period t in the time horizon. This can be represented by

$$\sum_{c=1}^{C} W_{jct} \leq L_{jt} \qquad \forall j,\ \forall t$$

With this quantity of received raw material, an RMP transforms it to the various product families. Therefore, an RMP cannot transform more than what it has received. This quantity includes any inventory it has from the previous time periods. This constraint can be represented by,

For Model 1,

$$\sum_{j=1}^{J} W_{jct} + I_{ct} = \sum_{f=1}^{F} \sum_{m=1}^{M} (X_{cfmt}/YM_m) + I_{c(t+1)} \qquad \forall c,\ \forall t$$

For Model 2,

$$\sum_{j=1}^{J} W_{jct} + I_{jct} = \sum_{j=1}^{J} \sum_{f=1}^{F} \sum_{m=1}^{M} (X_{jcfmt}/YM_m) + I_{jc(t+1)} \qquad \forall c, \forall t$$

Constraint for the Production Capacity in an RMP

The production in RMP c is also constrained by the available capacity (time) in time period t. Therefore the total production time should not exceed the total available time for an RMP. This production time also includes the setup time. It can be formulated as,

For Model 1,

$$\sum_{f=1}^{F} \sum_{m=1}^{M} (X_{cfmt}/ YM_m) / PS_m + \sum_{m=1}^{M} S_{cmt} \times ST_{cm} \leq \Gamma C_{ct} \qquad \forall c, \forall t$$

For Model 2,

$$\sum_{j=1}^{J} \sum_{f=1}^{F} \sum_{m=1}^{M} (X_{jcfmt}/ YM_m) / PS_m + \sum_{m=1}^{M} S_{cmt} \times ST_{cm} \leq \Gamma C_{ct} \qquad \forall c, \forall t$$

Constraint for the Production Capacity in an FP

The production at an FP is constrained by two factors; available production time, and available product family quantity. The constraints for those can be formulated in such a manner,

For Model 1,

$$\sum_{l=1}^{L} \sum_{n=1}^{N} (X_{fln(m)t} / YF_{n(m)}) / PF_{n(m)} + \sum_{n=1}^{N} S_{fn(m)t} \times ST_{fn(m)} \leq \Gamma F_{ft} \qquad \forall f, \forall t$$

The total finished product quantities cannot exceed the quantity of the corresponding product family available to an FP during a time period. So,

$$\sum_{l=1}^{L} \sum_{n=1}^{N} (X_{fln(m)t} / YF_{n(m)}) = \sum_{c=1}^{C} X_{cfmt} \qquad \forall f, \forall m, \forall t$$

For Model 2,

$$\sum_{c=1}^{C} \sum_{l=1}^{L} \sum_{n=1}^{N} (X_{cfln(m)t} / YF_{n(m)}) / PF_{n(m)} + \sum_{n=1}^{N} S_{fn(m)t} \times ST_{fn(m)} \leq \Gamma F_{ft} \qquad \forall f, \forall t$$

Note the extra index c used in representing the finished product (because of the need to track the product). Also,

$$\sum_{l=1}^{L} \sum_{n=1}^{N} (X_{cfln(m)t} / YF_{n(m)}) = \sum_{j=1}^{J} X_{jcfmt} \qquad \forall c, \forall f, \forall m, \forall t$$

Constraints for the Customer Demand

The quantity produced by FP f at time period t along with the inventory from the earlier time periods should be equal or exceed demand of customer l for that time period. It can be represented as,

For Model 1,

$$X_{fln(m)t} + I_{fln(m)t} = DC_{fln(m)t} + I_{fln(m)(t+1)} \qquad \forall f, \forall l, \forall m \ \forall n, \forall t$$

For Model 2,

$$\sum_{c=1}^{C} (X_{cfln(m)t}) + I_{fln(m)t} = DC_{fln(m)t} + I_{fln(m)(t+1)} \qquad \forall f, \forall l, \ \forall m, \forall n, \ \forall t$$

Constraint for Production Setup

Production for product family (m) or product (n) entails the presence of setup for that family or product at an RMP (c) or FP (f) in time period t. Introducing a variable big M in the following formulation to ensure the above relation.

For Model 1 at the RMP,

$$\sum_{f=1}^{F} X_{cfmt} \leq M \times S_{cmt} \qquad \forall c, \ \forall m, \forall t$$

And at the FP,

$$\sum_{l=1}^{L} X_{fln(m)t} \leq M \times S_{fn(m)t} \qquad \forall f, \forall m, \ \forall n, \ \forall t$$

For Model 2 at the RMP,

$$\sum_{j=1}^{J} \sum_{f=1}^{F} X_{jcfmt} \leq M \times S_{cmt} \qquad \forall c,\ \forall m,\ \forall t$$

And at the FP,

$$\sum_{c=1}^{C} \sum_{l=1}^{L} X_{cfln(m)t} \leq M \times S_{fn(m)t} \qquad \forall f,\ \forall n,\ \forall t$$

The "big *M* formulation" ensures that only when the setup variable assumes the value 1 can there be any value/quantity associated with the production variables at the RMPs and at the FPs.

End Inventory Constraint

It is assumed that the end inventory is 0 so that no extra production takes place for the time horizon and inventory costs are kept to minimum. This can be expressed as,

For Model 1 at the FP,

$I_{fln(m)(T+1)} = 0$, where T is the ending time period in the horizon $\qquad \forall f,\ \forall l,\ \forall m,\ \forall n$

Or it can be assumed to be equal to the initial inventory (inventory at the beginning of the time horizon). It can then be represented as,

$I_{fln(m)(T+1)} = I_{fln(m)t}$ where T = ending time horizon, t = 1 (beginning of the time horizon)

Similarly for the RMP,

$I_{c(T+1)} = 0$, where T is the ending time horizon $\forall c,\ \forall t$

Or $I_{c(T+1)} = I_{ct}$ $\forall c,\ \forall t$

For Model 2 at the RMP,

$I_{jc(T+1)} = 0$, where T is the ending time horizon $\forall j, \forall c,\ \forall t$

And at the FP,

$I_{fln(m)(T+1)} = 0$, where T is the ending time horizon $\forall f, \forall m,\ \forall n$

4.2.4 The Objective Function

The objective function for these models is to maximize the total profit that can be expressed as:

MAXIMIZE TOTAL PROFIT = REVENUE – TOTAL COST

For this study, the revenue is simply the total selling income (Chen and Wang, 1997). Thus, For Model 1,

$$REVENUE = \sum_{f=1}^{F} \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{t=1}^{T} PR_{fln(m)t} \times X_{fln(m)t}$$

For Model 2,

$$REVENUE = \sum_{c=1}^{C} \sum_{f=1}^{F} \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{t=1}^{T} PR_{fln(m)t} \times X_{cfln(m)t}$$

Various costs involved can be calculated from the factors as discussed below.

- **Total Raw Material Purchasing Cost**

  For Model 1, it can be calculated by multiplying the quantity of raw material ordered with the price paid for that region/supplier

$$RMCOST = \sum_{j=1}^{J} \sum_{c=1}^{C} \sum_{t=1}^{T} (RC_{jct} \times W_{jct})$$

  This cost would be the same for Model 2 as well.

- **Total Fixed Cost**

  The fixed cost is assumed to be dependent on the total quantity of material produced in a particular time period (Chen and Wang, 1997). Therefore it can be represented as,

  For Model 1,

$$FIXCOST = \sum_{f=1}^{F} \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{t=1}^{T} (FC_{cm} + FF_{n(m)}) \times X_{fl(m)t}$$

  For Model 2,

$$FIXCOST = \sum_{c=1}^{C} \sum_{f=1}^{F} \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{t=1}^{T} (FC_{cm} + FF_{n(m)}) \times X_{cfln(m)t}$$

Note the extra index for RMP *c*

- **Total Variable Cost**

The variable cost is assumed to be dependent on the total quantity of material produced in a particular time period (Chen and Wang, 1997). Therefore it can be represented as,

For Model 1,

$$VARCOST = \sum_{f=1}^{F} \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{t=1}^{T} (VC_{cm} + VF_{n(m)}) \times X_{fln(m)t}$$

For Model 2,

$$VARCOST = \sum_{c=1}^{C} \sum_{f=1}^{F} \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{t=1}^{T} (FC_{cm} + FF_{n(m)}) \times X_{cfln(m)t}$$

- **Total Raw Material Freight Cost**

It is the total cost incurred in the time horizon for transporting the raw material from the suppliers to the RMPs. It can be expressed as,

For Model 1,

$$RMTRPCST = \sum_{j=1}^{J} \sum_{c=1}^{C} \sum_{t=1}^{T} TR_{jct} \times W_{jct}$$

For Model 2 this cost is not calculated explicitly. Instead, the combined cost of transporting the raw material from supplier j to RMP c and transporting the product family m from the RMP to FP f is calculated. It can be done in this manner,

$$RMTRPCST = \sum_{j=1}^{J} \sum_{c=1}^{C} \sum_{f=1}^{F} \sum_{m=1}^{M} \sum_{t=1}^{T} TS_{jcfmt} \times X_{jcfmt}$$

- **Total Product Family Transportation Cost from the RMPs to the FPs**

It is the total cost incurred for transporting the product family m from RMP c to FP f in the time horizon. This cost is relevant for only the models representing methodology 1. It can be expressed as,

$$SFCPTRCT = \sum_{c=1}^{C} \sum_{f=1}^{F} \sum_{m=1}^{M} \sum_{t=1}^{T} TS_{cfmt} \times X_{cfmt}$$

- **Total Finished Products Transportation Cost**

These transportation costs are incurred in the time horizon for transporting the finished products, from the FPs, to the customers. It can be expressed as,

For Model 1,

$$FNTRPCST = \sum_{f=1}^{F} \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{t=1}^{T} TF_{fln(m)t} \times X_{fln(m)t}$$

For Model 2,

$$FNTRPCST = \sum_{f=1}^{F} \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{t=1}^{T} TF_{fln(m)t} \times \sum_{c=1}^{C} X_{cfln(m)t}$$

- **Setup Costs at RMPs**

These setup costs are incurred in the time horizon by having setups for product families at the RMPs. It can be expressed as,

For both Models 1 and 2,

$$SETCPCST = \sum_{c=1}^{C} \sum_{m=1}^{M} \sum_{t=1}^{T} S_{cmt} \times CST_{cmt}$$

- **Setup Costs at FPs**

These setup costs are incurred in the time horizon by having setups for final products at the FPs. It can be expressed as,

For both Models 1 and 2,

$$SETFFCST = \sum_{f=1}^{F} \sum_{n=1}^{N} \sum_{t=1}^{T} S_{fn(m)t} \times CST_{fn(m)t}$$

- **Inventory Cost at RMP**

It is the total cost in the time horizon incurred for keeping raw material inventory at RMPs. It can be expressed as,

For Model 1,

$$INVCPCST = \sum_{c=1}^{C} \sum_{t=1}^{T} I_{ct} \times IC_{ct}$$

And for Model 2,

$$INVCPCST = \sum_{j=1}^{J} \sum_{c=1}^{C} \sum_{t=1}^{T} I_{jct} \times IC_{ct}$$

- **Inventory Cost at FP**

  It is the total cost in the time horizon incurred for keeping finished product inventories at the FPs. It can be expressed as,

  For Model 1 and 2,

$$INVFFCST = \sum_{f=1}^{F} \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{t=1}^{T} I_{fln(m)t} \times CI_{fln(m)t}$$

Thus from the above equations

Profit (Z) = *REVENUE* – (*RMCOST* + *FIXCOST* + *VARCOST* + *RMTRPCST* + *SFTRPCST* + *FNTRPCST* + *SETCPCST* + *SETFFCST* + *INVCPCST* + *INVFFCST*)

And the objective function is,

MAXIMISE *(Z = Profit)*

4.3 Models Development for Situation B

The important difference associated with situation B is the representation of demand. As was expressed in the previous chapters, for situation A, the demand is pooled at the FP and received by the customer/distribution center from a dedicated plant. This situation is representative of the franchising environment. Situation B, however, represents an environment where the demand is taken directly from the customers. As will be seen subsequently, this type of formulation is slightly more complex than the former one, and has a larger number of variables for a similar problem. In order to simplify the presentation only the most relevant assumptions different from the ones presented for Situation A will be presented. However, for the sake of exhaustive analysis, all the cost formulations and the constraints will be presented.

4.3.1 Assumptions:

4.3.1.1 Material Supply and Transportation:

a) Any RMP can supply its end product to any FP with associated transportation costs. Similarly any FP can supply the finished products to any customer with associated freight costs.

4.3.1.2 Inventory

b) Inventory at the FPs is defined with respect to a particular plant, a particular product and the time period. This is different from the earlier models where the inventory is also indexed by the customer for which it is kept. This assumption will come in light for Model 1.

4.3.1.3 Demand

c) For the situation B models, demand for product *n*, by customer *l* must be met at time *t* by any of the FPs. The difference from the former assumption is the absence of the plant index *f*.

4.4 MIP Model Formulations

The following subsection will present the model developed in the study.

4.4.1 Notation and Variable Definition

In presenting the models the following notation and variables are defined.

4.4.1.1 Indices

They are same as presented previously for Situation A.

4.4.1.2 Parameters

1. Material Supply and Transportation

   a. Costs:

      RC_{jct} = Raw material cost from supplier *j* at time *t* for RMP *c*.

      TR_{jct} = Transportation cost of raw material from supplier *j* to the RMP *c*, at time *t*.

      TS_{cfmt} = Transportation cost of product *m* from RMP *c* to FP *f* at time *t*. The models of type 1 use the above representation for the transportation costs.

TS_{jcfmt} = Unit combined cost of transportation of raw material from the supplier to the RMP and the transportation of product family from the RMP to the FP. Models of type 2 utilize this representation.

$TF_{fln(m)t}$ = Unit transportation cost of product *n* from FP *f* to customer *l* at time period *t*.

a. Capacities:

L_{jt} = Supply capacity for supplier *j* at time period *t*.

2. Production

b. Costs:

$FC_{cn(m)}$ = Fixed costs for manufacturing at RMP *c*.

$VC_{cn(m)}$ = Variable costs for manufacturing at RMP *c*.

$FF_{fn(m)}$ = Fixed cost for the manufacturing of product *n* at FP *f*.

$VF_{fn(m)}$ = Variable cost for the manufacturing of product *n* at FP *f*.

PS_m = Production rate of product family *m* raw material unit/time unit.

$PF_{n(m)}$ = Production rate of finished product *n*. product family unit/time unit.

YM_m = Transformation rate for product family *m* product family unit/raw material unit.

$YF_{n(m)}$ = Transformation rate for finished product *n* finished product unit/ product family unit .

CST_{cmt} = Cost associated with the setup at RMP *c* for product family *m*.

$CST_{fn(m)t}$ = Cost associated with the setup at FP *f* for product *n* of family *m*.

ST_{cm} = Setup time at RMP *c* for a product family *m*.

$ST_{fn(m)}$ = Setup time at FP f for a product n of family m.

c. Capacities:

ΓC_{ct} = Available production capacity (time) at RMP c at time t.

ΓF_{ft} = Available production capacity (time) at FP f at time t.

5. Inventory

IC_{tc} = Inventory cost at RMP c at time t.

$IF_{fn(m)t}$ = Inventory cost at FP f at time t of product n.

6. Demand

$DC_{ln(m)t}$ = Demand for product n by customer l at time period t.

$PR_{fln(m)t}$ = Unit selling price of product n for customer l in time t produced by FP f.

4.4.2 Decision Variables

Table 4.2 presents a comparison between the decision variables for Models 1 and 2. These variables are for the situation B.

4.4.3 Constraints

The same procedure will be followed, of presenting the general constraints for both types of models for situation B. Any modifications will be explicitly mentioned.

Constraint on Raw Material Supply

The planned raw material purchasing quantity should not exceed the suppliers' capacity in the time horizon. This can be represented by

$$\sum_{c=1}^{C} W_{jct} \leq L_{jt} \qquad \forall j,\ \forall t$$

With this quantity of raw material received an RMP transforms it to product families. Therefore, an RMP cannot transform more than what it received. This constraint can be represented by,

For Model 1,

$$\sum_{j=1}^{J} W_{jct} + I_{ct} = \sum_{f=1}^{F} \sum_{m=1}^{M} (X_{cfmt}/YM_m) + I_{c(t+1)} \qquad \forall c,\ \forall t$$

For Model 2,

$$\sum_{j=1}^{J} (W_{jct} + I_{jct}) = \sum_{j=1}^{J} \sum_{f=1}^{F} \sum_{m=1}^{M} (X_{jcfmt}/YM_m) + I_{jc(t+1)} \qquad \forall c, \forall t$$

Table 4.2: Decision Variables for Situation B

Model 1		Model 2	
Decision Variable	**Description**	**Decision Variable**	**Description**
$X_{fn(m)t}$	Amount of product *n*, of product family *m*, produced by FP *f*, at time *t*.	$X_{cfln(m)t}$	Amount produced by FP *f*, of a product *n*, of product family *m*, for customer *l*, from the material sent to *f*, by RMP *c*
X_{cfmt}	Amount of product family *m*, produced by RMP *c*, for FP *f*, at time *t*.	X_{jcfmt}	Amount of product family *m*, produced by RMP *c*, from material by supplier *j*, for FP *f*, at time *t*.
$Y_{fln(m)t}$	Amount of product *n*, of product family *m*, produced by FP *f*, **sent** to customer *l*, at time *t*.	$Y_{fln(m)t}$	Amount of product *n*, of product family *m*, produced by FP *f*, **sent** to customer *l*, at time *t*.
$I_{fn(m)t}$	Inventory at FP of product family *m*, for product *n* at time *t*.	$I_{fn(m)t}$	Inventory at finishing factory *f* of product family *m*, for product *n* at time *t*.
I_{ct}	Inventory of raw material at RMP *c* at time period *t*.	I_{jct}	Inventory of raw material at RMP *c* at time period *t* sent by supplier *j*.
W_{jct}	Amount of raw material purchased from supplier *j* at time *t* for RMP *c*.	W_{jct}	Amount of raw material purchased from supplier *j* at time *t* for RMP *c*.
S_{cmt}	Binary variable signifying setup for the product family *m* is there at RMP *c* at time *t*.	S_{cmt}	Binary variable signifying setup for the product family *m* is there at RMP *c* at time *t*.
$S_{fn(m)t}$	Binary variable signifying setup for the product *n* family *m* is there at FP *f* at time *t*.	$S_{fn(m)t}$	Binary variable signifying setup for the product *n* family *m* is there at FP *f* at time *t*.

Constraint for the Production Capacity in an RMP

The total production time should not exceed the total available time for an RMP. It can be represented as,

For Model 1,

$$\sum_{f=1}^{F} \sum_{m=1}^{M} (X_{cfmt} / YM_m) / PS_m) + \sum_{m=1}^{M} S_{cmt} \times ST_{cm} \leq TC_{ct} \qquad \forall c,\ \forall t$$

For Model 2,

$$\sum_{j=1}^{J} \sum_{f=1}^{F} \sum_{m=1}^{M} (X_{jcfmt} / YM_m) / PS_m + \sum_{m=1}^{M} S_{cmt} \times ST_{cm} \leq TC_{ct} \qquad \forall c,\ \forall t$$

Constraint for the Production Capacity in an FP

The production at an FP is constrained by two factors; available production time and available product family quantity. The formulation for those can be represented by,

For Model 1,

$$\sum_{n=1}^{N} (X_{fn(m)t} / YF_{n(m)}) / PF_{n(m)} + \sum_{n=1}^{N} S_{fn(m)t} \times ST_{fn(m)} \leq TF_{ft} \qquad \forall f,\ \forall t$$

The total finished product quantities cannot exceed the quantity of the corresponding product family available to an FP during a time period. Thus,

$$\sum_{n=1}^{N} (X_{fn(m)t})/YF_{n(m)} = \sum_{c=1}^{C} X_{cfmt} \qquad \forall f,\ \forall m,\ \forall t$$

For Model 2,

$$\sum_{c=1}^{C} \sum_{l=1}^{L} \sum_{n=1}^{N} (X_{cfln(m)t} / YF_{n(m)}) / PF_{n(m)} + \sum_{n=1}^{N} S_{fn(m)t} \times ST_{fn(m)} \leq \Gamma F_{ft} \qquad \forall f,\ \forall t$$

Also,

$$\sum_{l=1}^{L} \sum_{n=1}^{N} (X_{cfln(m)t})/YF_{n(m)} = \sum_{j=1}^{J} X_{jcfmt} \qquad \forall c,\ \forall f,\ \forall m,\ \forall t$$

Constraints for the Customer Demand

The quantity produced by the FPs at any given time period along with inventory from earlier time periods should meet or exceed the customers' demand for that time period. Introducing a variable $Y_{fln(m)t}$ representing the quantity that is transported to the customers from the FPs. The constraint can be represented by,

For Model 1,

$$(X_{fn(m)t} + I_{fn(m)t}) = Y_{fln(m)t} + I_{fn(m)(t+1)} \qquad \forall f,\ \forall l,\ \forall m,\ \forall t$$

Therefore in order to meet the demand we would have,

$$\sum_{f=1}^{F} Y_{fln(m)t} = DC_{ln(m)t} \qquad \forall l,\ \forall n,\ \forall t$$

For Model 2,

$$\sum_{c=1}^{C} (X_{cfln(m)t}) + I_{fln(m)t} = Y_{fln(m)t} + I_{fn(m)(t+1)} \qquad \forall t,\ \forall f,\ \forall l,\ \forall m$$

Where again, $Y_{fln(m)t}$ represents the quantity that is transported to the customers from the FPs (Refer Table 4.2). In order to satisfy the demand of the customers the total quantity transported from all the FPs combined to it should be equal to the demand. Or,

$$\sum_{f=1}^{F} Y_{fln(m)t} = DC_{ln(m)t} \qquad \forall l,\ \forall n,\ \forall t$$

Constraint for Production Setup

Production for a particular product family/product requires presence of a setup for that family/product at RMPs/FPs. Therefore utilizing the "big *M* formulation", as introduced in the previous section, can ensure the above case,

For Model 1 at the RMP,

$$\sum_{f=1}^{F} X_{cfmt} \leq M \times S_{cmt} \qquad \forall m,\ \forall c,\ \forall t$$

And at FP,

$$X_{fn(m)t} \leq M \times S_{fn(m)t} \qquad \forall f, \forall m, \forall n, \forall t$$

And for Model 2 at the RMP,

$$\sum_{j=1}^{J} \sum_{f=1}^{F} X_{jcfmt} \leq M \times S_{tmc} \qquad \forall c, \forall m, \forall t$$

And at the FP,

$$\sum_{c=1}^{C} \sum_{l=1}^{L} X_{cfln(m)t} \leq M \times S_{fn(m)t} \qquad \forall f, \forall m, \forall n, \forall t$$

End Inventory Constraint

It is assumed that the end inventory is equal to 0 so that no extra production takes place. This can be expressed as,

For Model 1 at the FP,

$I_{fn(m)(T+1)} = 0$, where T is the end time period in the time horizon $\qquad \forall f, \forall l, \forall m, \forall n, \forall t$

Or it can be assumed to be equal to the initial inventory (inventory at the beginning of the time horizon). It can then be represented as,

$I_{fln(m)(T+1)} = I_{fln(m)t}$ where T = ending time horizon, $t = 1$ (beginning of the time horizon)

Similarly for the RMP,

$I_{c(T+1)} = 0$, where T is the ending time horizon $\forall c,\ \forall t$

For Model 2 at the RMP,

$I_{jc(T+1)} = 0$, where T is the ending time horizon $\forall j, \forall c,\ \forall t$

And at the FP,

$I_{fln(m)(T+1)} = 0$, where T is the ending time horizon $\forall f,\ \forall l, \forall m,\ \forall n,\ \forall t$

4.4.4 The Objective Function

The objective function for these models is to maximize the total profit. It is the same as illustrated for Situation A. Thus,

For Model 1 and 2,

$$REVENUE = \sum_{f=1}^{F} \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{t=1}^{T} PR_{fln(m)t} \times Y_{fln(m)t}$$

The various costs can be calculated from the factors as discussed below.

- **Total Raw Material Purchasing Cost**

  For Model 1, it can be calculated by multiplying the quantity of raw material ordered with the price paid for that region/supplier

$$RMCOST = \sum_{j=1}^{J} \sum_{c=1}^{C} \sum_{t=1}^{T} (RC_{jct} \times W_{jct})$$

For Model 2, this cost will be the same.

- **Total Fixed Cost**

This cost is assumed to be dependent on the total quantity of material produced in a particular time period. Therefore it can be represented as,

For Model 1,

$$FIXCOST = \sum_{f=1}^{F} \sum_{n=1}^{N} \sum_{t=1}^{T} (FC_{cm} + FF_{n(m)}) \times X_{fn(m)t}$$

For Model 2,

$$FIXCOST = \sum_{c=1}^{C} \sum_{f=1}^{F} \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{t=1}^{T} (FC_{cm} + FF_{n(m)}) \times X_{cfln(m)t}$$

- **Total Variable Cost**

This cost is assumed to be dependent on the total quantity of material produced in a particular time period. Therefore it can be represented as,

For Model 1,

$$VARCOST = \sum_{f=1}^{F} \sum_{n=1}^{N} \sum_{t=1}^{T} (VC_{cm} + VF_{n(m)}) \times X_{fn(m)t}$$

For Model 2,

$$VARCOST = \sum_{c=1}^{C} \sum_{f=1}^{F} \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{t=1}^{T} (FC_{cm} + FF_{n(m)}) \times X_{cfln(m)t}$$

- **Total Raw Material Freight Cost**

  This is the total cost incurred in the time horizon for transporting the raw material from the suppliers to the RMPs. It can be expressed as,

  For Model 1,

$$RMTRPCST = \sum_{c=1}^{C} \sum_{j=1}^{J} \sum_{t=1}^{T} TR_{jct} \times W_{jct}$$

For Model 2 the combined cost of transporting the raw material from the supplier to the RMP and transporting the product family from RMP to the FP is calculated in the following manner,

$$RMTRPCST = \sum_{c=1}^{C} \sum_{j=1}^{J} \sum_{f=1}^{F} \sum_{m=1}^{M} \sum_{t=1}^{T} TS_{jcfmt} \times X_{jcfmt}$$

- **Total Product Family Transportation Cost from the RMPs to the FPs**

  This is the total cost incurred for transporting the said material in the time horizon. It can be expressed as,

  For Model 1,

$$SFCPTRCT = \sum_{c=1}^{C} \sum_{f=1}^{F} \sum_{m=1}^{M} \sum_{t=1}^{T} TS_{cfmt} \times X_{cfmt}$$

- **Total Finished Products Transportation Cost**

  This is the total cost incurred in the time horizon for transporting the finished products, from the FPs, to the customers. It can be expressed as,

  For both Models 1 and 2,

$$FNTRPCST = \sum_{f=1}^{F} \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{t=1}^{T} TF_{fln(m)t} \times Y_{fln(m)t}$$

- **Setup Costs at RMPs**

  This is the total cost incurred in the time horizon by having a setup for product families at the RMPs. It can be expressed as,

  For both Models 1 and 2,

$$SETCPCST = \sum_{c=1}^{C} \sum_{m=1}^{M} \sum_{t=1}^{T} S_{cmt} \times CST_{cmt}$$

- **Setup Costs at FPs**

  This is the total cost incurred in the time horizon by having a setup for the final products at the FPs. It can be expressed as,

  For both Models 1 and 2,

$$SETFFCST = \sum_{f=1}^{F} \sum_{n=1}^{N} \sum_{t=1}^{T} S_{fn(m)t} \times CST_{fn(m)}$$

- **Inventory Cost at RMP**

  This is the total cost in the time horizon incurred for keeping inventory of raw material at the RMPs. It can be expressed as,

  For Model 1,

$$INVCPCST = \sum_{c=1}^{C} \sum_{t=1}^{T} I_{ct} \times IC_{ct}$$

  And for Model 2,

$$INVCPCST = \sum_{j=1}^{J} \sum_{c=1}^{C} \sum_{t=1}^{T} I_{jct} \times IC_{ct}$$

- **Inventory Cost at FP**

  It is the total cost in the time horizon incurred for keeping inventory of products at the FPs. It can be expressed as,

  For Model 1,

$$INVFFCST = \sum_{f=1}^{F} \sum_{n=1}^{N} \sum_{t=1}^{T} I_{fn(m)t} \times CI_{fn(m)t}$$

And for Model 2,

$$INVFFCST = \sum_{f=1}^{F} \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{t=1}^{T} I_{fln(m)t} \times CI_{fln(m)t}$$

Thus from the above equations

Profit (Z) = *REVENUE* – (*RMCOST* + *FIXCOST* + *VARCOST* + *RMTRPCST* + *SFTRPCST* + *FNTRPCST* + *SETCPCST* + *SETFFCST* + *INVCPCST* + *INVFFCST*)

And the objective function for the models would be,

MAXIMISE *(Z = profit)*

4.5 Optimizing the Models

The models have been coded in LINGO™ resident on a Pentium-4 machine under Windows-XP operating system. The test cases/problem instances studied, for the above-explained models, are presented in Chapter 5.

Chapter Five

Numerical Example and Analysis

This chapter presents problem instances to illustrate the integrated planning methodologies introduced and developed in the previous chapters. The models developed on those methodologies have been tested over a variety of problem instances. The results from some of them will be presented in this chapter. Some of the problem instances (e.g. three time periods, two raw material processing plants, three finishing plants, three customers, two product families, six products) have been solved to optimality. Larger ones have been solved to 1% optimality.

Problem instances from Situation A will be first discussed. Situation A is the case when the demand is pooled at the FPs and received by the customers from a dedicated plant.

5.1 Example Problem

To demonstrate the application of the MIP models presented in the previous chapters, consider a planning problem with three raw material suppliers (J = 3). The system has two RMPs (C = 2) and three FPs (F = 3). The number of product-families are two (M = 2), and each has three finished products (N_1 = 3, N_2 = 3). There are three customers in the system (L = 3), whose demand has to be satisfied in the time horizon divided into three time periods (T = 3). The number of variables and constraints can estimate the size of the problem. In this case the total number of variables will be, for the first model, 406, while the number of constraints, for the first model, will be 289. The problem size, for the first model, is 289 X 406. Similarly for the same situation the size of Model 2 is 480 X 809. The details for the problems are summarized in Table 5.1. As is observed from the table, two problem instances have been presented and optimized. Example 1 is a relatively small problem and will be presented here for the sake of illustration. The second one is larger. It is important to mention that even the smaller sized problem is actually realistic. Few other details of the problem are summarized in Tables A1 to A4 in the Appendix.

Table 5.1: Report on two Instances

	Index	Model 1A	Model 2A
Example 1	*j*	3	3
	c	2	2
	f	3	3
	l	3	3
	m	2	2
	n	6	6
	t	3	3
	Size (constraints X variables)	289 X 406	480 X 809
	Binary variables	66	66
	Time	5 sec	9 sec
Example 2	*j*	3	3
	c	2	2
	f	3	3
	l	3	3
	m	4	4
	n	8	8
	t	5	5
	Size	643 X 979	1108 X 1997
	Binary variables	160	160
	Time	260 sec	45 min

5.2 Determining the Optimal Production and Distribution Plan

The information presented in the tables in the Appendix was the input to the MIP model and the optimal plan was found from the model solution. It is discussed below.

5.2.1 Optimal Raw Material Purchasing Plan

Table 5.6 in the Appendix, provides the raw material purchasing plan obtained from the MIP programming model. The total purchasing cost is $110980 in the entire time horizon for both models. The total cost of transporting the raw material from the suppliers to the RMPs is $193580 for Model 1. Figure 5.1 shows the relationship between the suppliers' territory and the optimal plan in each territory for Model 1 in time period 2. Similarly Figure 5.2 shows the relationship for Model 2 in the same time period. As can be seen in Tables A5 and A6 in the Appendix, the number of units in time period $t = 2$ and $t = 3$ are different for the two models; however, the total quantities are the same; for Model 1 it is: 4510 + 5000+ 4110 + 490 + 10000 + 1850+ 5000 + 300 = 31260; for Model 2 it is 3900 + 1100 + 5000+ 5000 + 10000 + 1260 + 5000 = 31260.

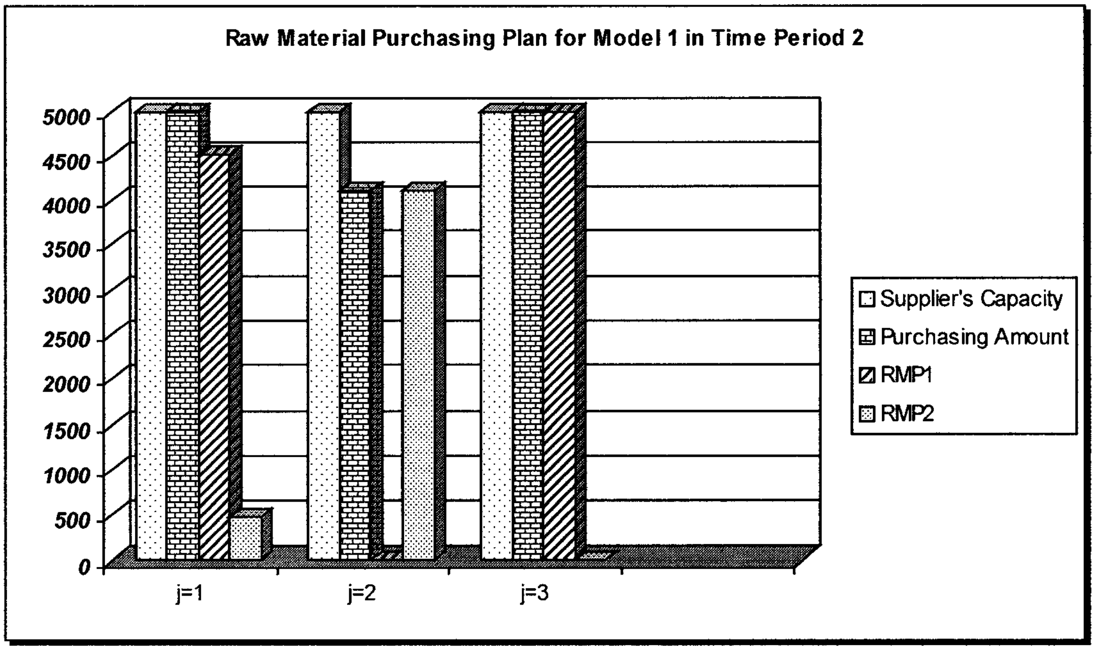


Figure 5.1 Raw Material Purchasing Plan

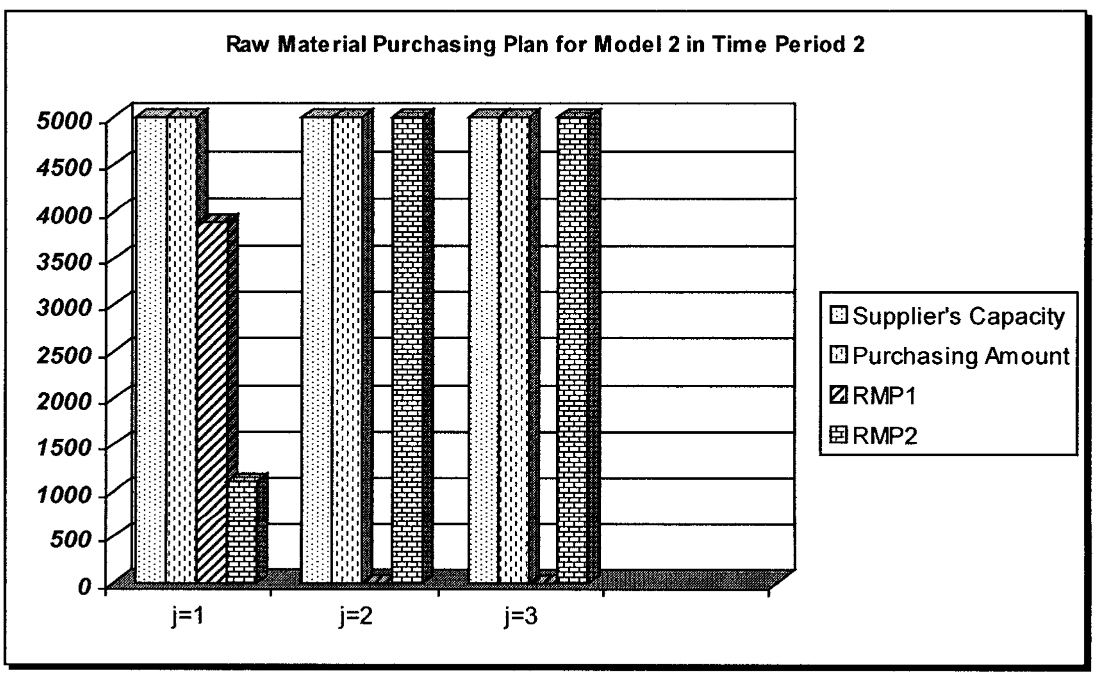


Figure 5.2 Raw Material Purchasing Plan

5.2.2 Optimal Product Families Production Plan

Tables A7 to A9, in the Appendix, show the quantities of product families produced by the RMPs to be distributed to the FPs. Although the quantities processed by the FPs in each individual time period are different for the two models, the aggregate quantity is the same. The same was observed earlier for the quantities sent to the RMPs by the suppliers. This is because both Models 1 and 2 are formulating the same system albeit differently. Therefore the global value of the objective function should be same although the optimal solution *per se* might be different. As can be seen from the tables, the individual quantities processed by the FPs in time periods $t=2$ and $t=3$ as provided by the optimal solution for both models are different, but the aggregate quantity is the same; for Model 1 it is 9510 + 4600 + 11850 + 5300 = 31260; for Model 2 it is 3900 + 11100 + 11260 +5000 = 31260.

5.2.3 Optimal Finished Products Distribution Plan

The values for the quantities produced for the customers from the different FPs for Model 1 and 2 are given in Tables A10 to A18 in the Appendix. These quantities can be different from the actual demand. But that difference would only be on the positive side. In other words the difference will lead to inventory. As mentioned in the assumptions, any solution in which demand is not met in any of the time periods will be infeasible. Tables A19 to A21 in the Appendix show demand versus production for the products in the time horizon for Model 1.

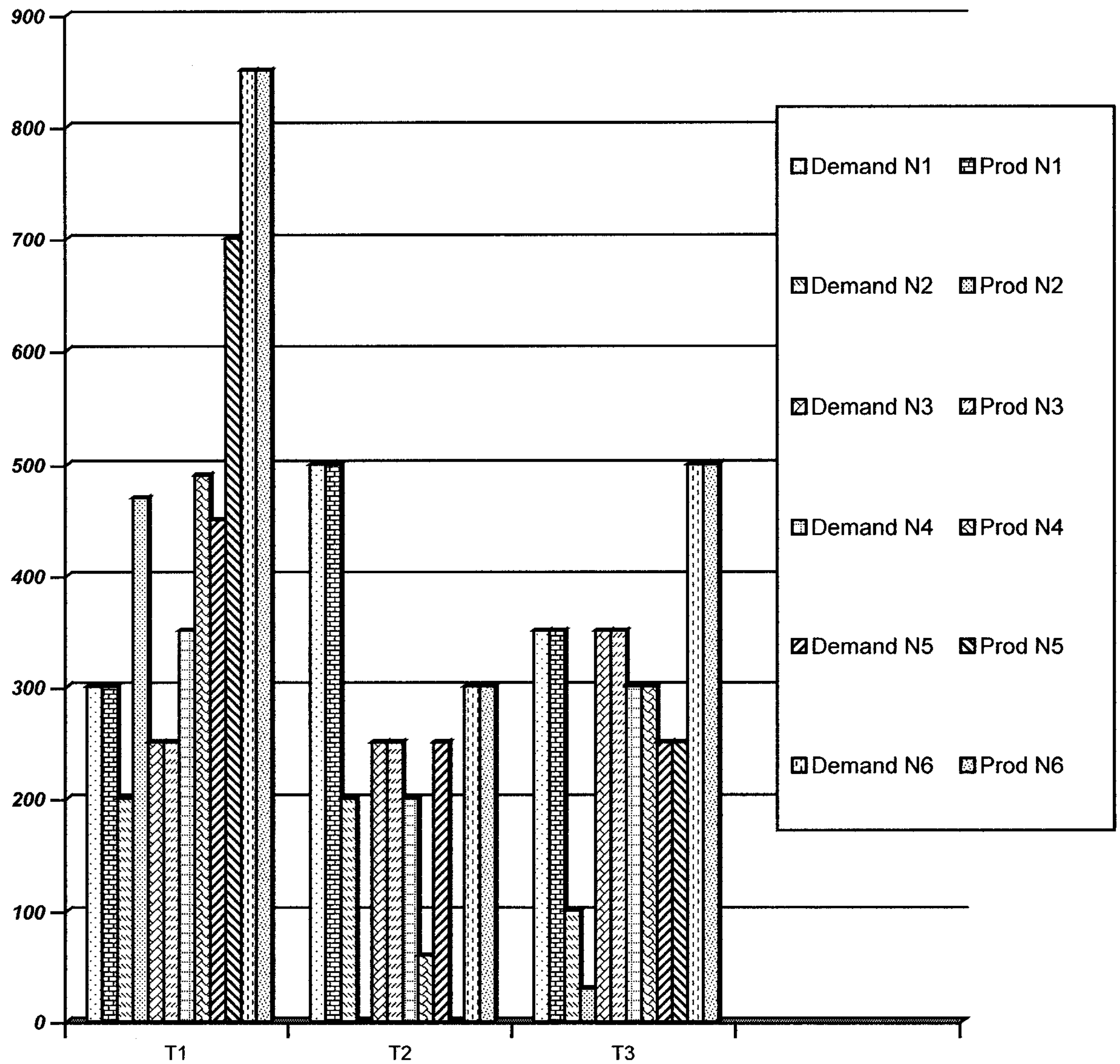


Figure 5.3: Relationship Between the Business Demands for $f = 1$ and $l = 1$

As can be seen from Tables A19 to A21, and A22 to A24 in the Appendix, for a particular customer, the demand is being met in both Models 1 and 2 for all time periods. Figure 5.3 shows the relationship between the production plan and the business demand as planned for customer $l = 1$ for Model 1. Figure 5.4 shows the relationship between the demand of customer $l = 2$ and the production as obtained by optimizing Model 2.

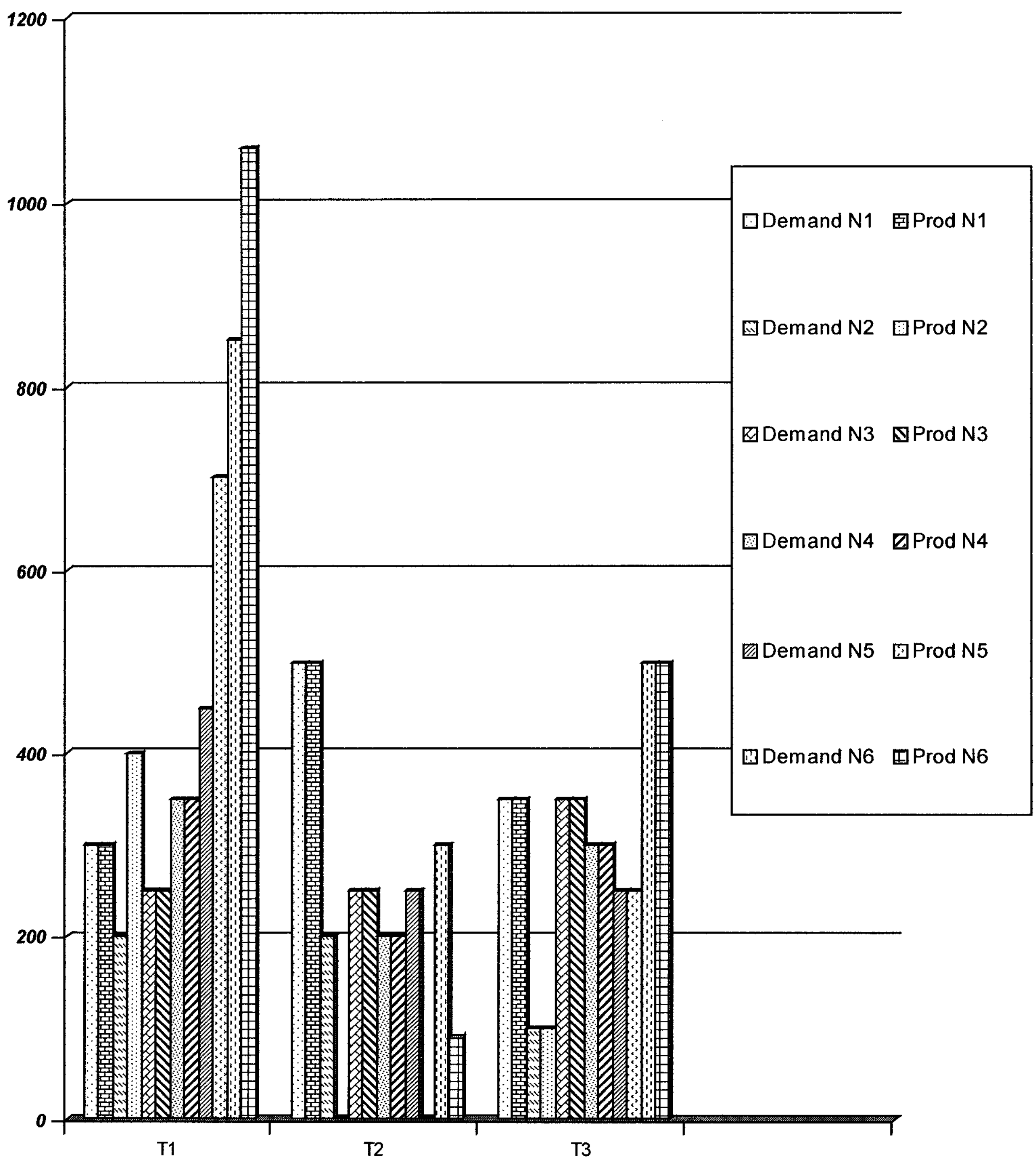


Figure 5.4: Relationship Between Demand and Production for $f = 1$ and $l = 1$

5.3 Overview of Revenue, Costs and Profit

According to the optimal solution of Model 1, the revenue in this example is \$ 4,100,800, while the profit is \$ 3,516,030. The total cost is \$ 584,770. Detailed cost factors are presented in Table 5.2. A comparison between the costs obtained by Model 1 and Model 2 is also shown in the table. As is evident, the objective function *Profit* to be maximized for Models 1 and 2 is the same, providing the evidence that both models are conceptually correct. However the exploded costs tell a different story. The important figures to notice are the raw material purchase costs and the inventory costs at RMP. There is a significant difference between Models 1 and 2 showing that the models took different paths to arrive at the same solution. Figures 5.5 and 5.6 provide breakdown of costs.

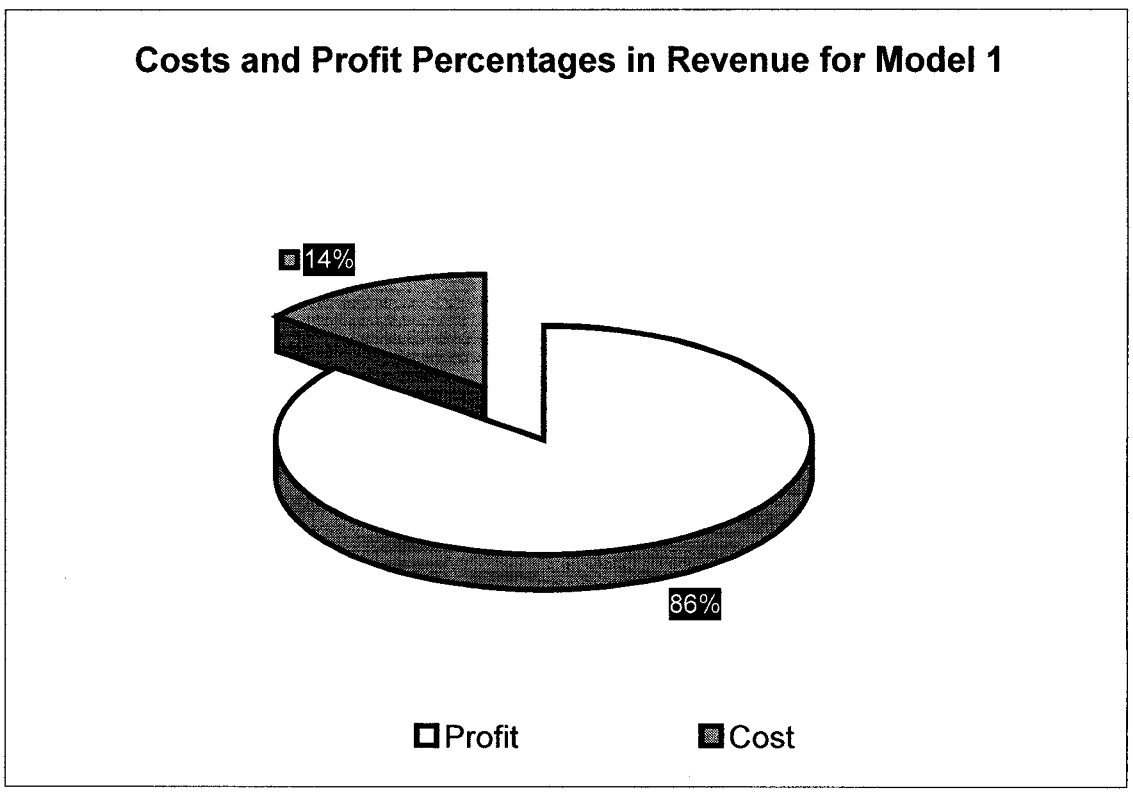


Figure 5.5: A Pie Breakdown of the Revenue

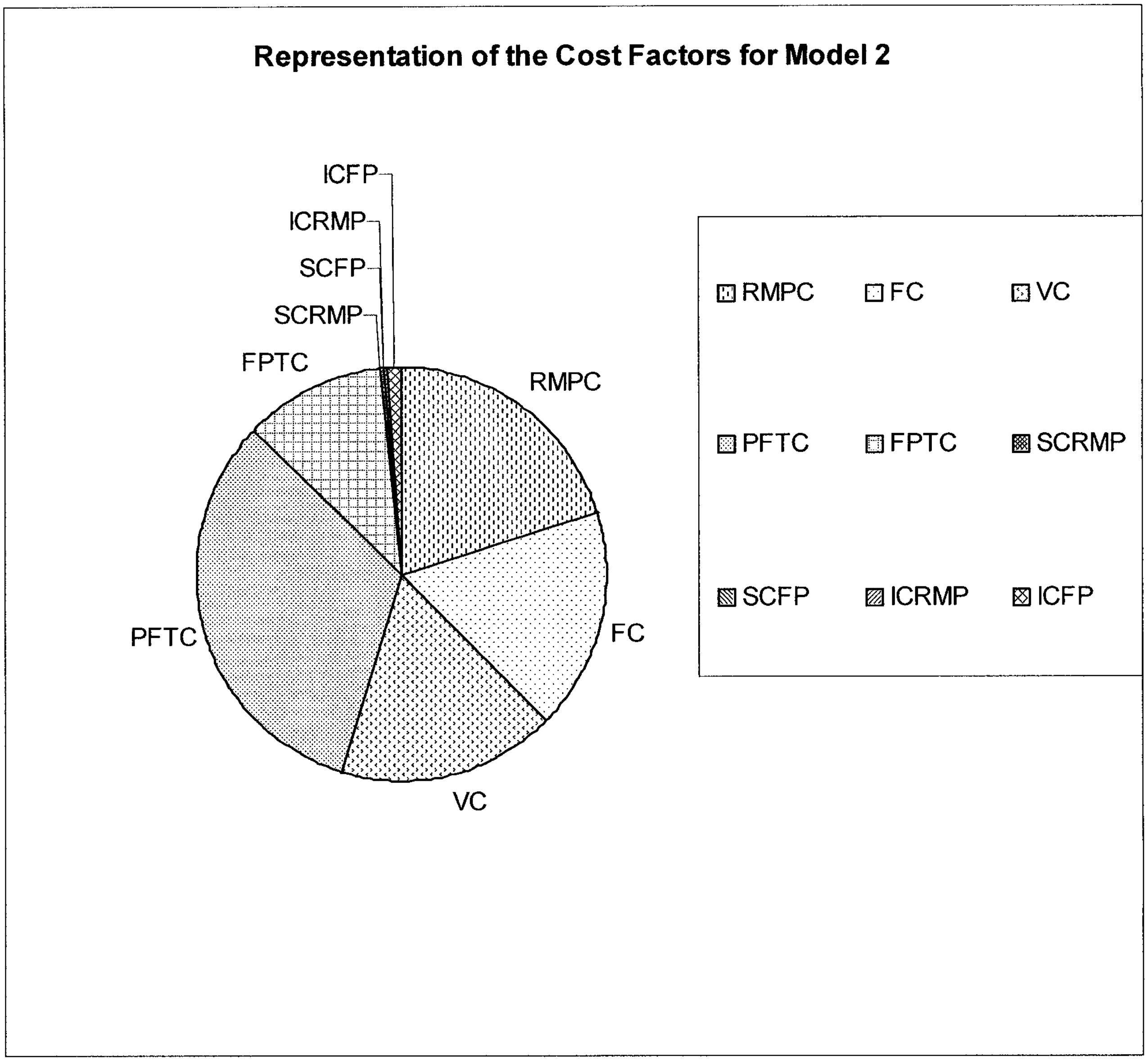


Figure 5.6: Breakdown of the Costs Incurred for Model 2

RMPC: Raw Material Purchase Costs

FC: Fixed Costs

VC: Variable Costs

PFTC: Product Family Transportation Costs

FPTC: Finished Product Transportation

SCRMP: Setup Cost at RMPs

SCFP: Setup Cost at FPs

ICRMP: Inventory Cost at RMPs

ICFP: Inventory Cost at FPs

Table 5.2: Breakdown of Total Profit (Small Example)

Model 1		Model 2	
Type of Cost	**Description**	**Type of Cost**	**Description**
Profit	Total profit from the products = 3,516,030.	*Profit*	Total profit from the products = 3,516,030.
Raw material purchase cost	Total cost of raw material purchased in the time horizon = 119,760.	*Raw material purchase cost*	Total cost of raw material purchased in the time horizon = 126,180.
Raw material transportation cost	Cost of transporting the raw material to the RMPs in the time horizon = 117,800.	*Raw material and product family transportation cost*	Cost of transporting the product families and the raw material that is used to manufacture them, in the time horizon = 178,380.
Product family transportation cost	Cost of transporting product family *m*, from the RMPs to the FPs in the time horizon = 68,780.	*(See above)*	This cost is not calculated explicitly and is taken into account in the combined formulation for raw material and product family transportation cost
Finished product transportation cost	= 65,840.	*Finished product transportation cost*	= 65,840.
Setup cost at RMPs	Setup costs incurred at the RMPs in the time horizon = 300.	*Setup cost at RMPs*	Setup costs incurred at the RMPs in the time horizon = 300.
Setup cost at FPs	Setup costs incurred at the FPs in the time horizon = 730.	*Setup cost at FPs*	Setup costs incurred at the FPs in the time horizon = 730.
Inventory cost at RMPs	Inventory costs incurred at the RMPs in the time horizon = 0.	*Inventory cost at RMPs*	Inventory costs incurred at the RMPs in the time horizon = 1780.
Inventory cost at FPs	Inventory costs incurred at the FPs in the time horizon = 6,520.	*Inventory cost at FPs*	Inventory costs incurred at the FPs in the time horizon = 6,520.

The next section will deal with the problem instance for situation B

5.4 Problem Instance for Situation B

Now an instance corresponding to situation B will be presented. Recall that formulation B refers to the situation where the demand from each customer has to be met individually. Thus instead of a pooled demand, we simply have a demand corresponding to each customer. The input parameters have been provided in Tables A25 to A27 in the Appendix.

5.5 Determining the Optimal Production and Distribution Plan

The information presented in Table A25 to Table A27 was input for the MIP model and the optimal plan was found from the model solution. It is discussed below. Table 5.3 gives the summary of the problems presented.

5.5.1 Optimal Raw Material Purchasing Plan

This plan will be illustrated using tables as was done for situation A. As evident from Table A28 in the Appendix, both models arrive at the same exact solution for raw material supply. Taking time period $t = 2$, one notices that RMP 1 purchases 5000 units from supplier 1 and 3000 units from supplier 2. The same is observed for Model 2.

Table 5.3: Report on two Instances for Situation B

	Index	Model 1B	Model 2B
Example 1	j	3	3
	c	2	2
	f	3	3
	l	3	3
	m	2	2
	n	2	2
	t	3	3
	Size	181 X 286	324 X 627
	Binary variables	48	48
	Time	120 sec	60 sec
Example 2	j	3	3
	c	2	2
	f	3	3
	l	3	3
	m	4	4
	n	8	8
	t	5	5
	Size	383 X 652	708 X 1477
	Binary variables	110	110
	Time (1% optimal)	40 sec	50 sec

5.5.2 Optimal Production of Product Families Plan

As will be subsequently seen, the exploded solution numbers might be different for a same problem instance, however the aggregated values are the same. Refer Tables A29 and A30 in the Appendix.

5.5.3 Optimal Finished Products Distribution Plan

As was seen earlier, for situation B both models have supplied the RMPs with the same quantity of raw material from the suppliers in a time period. Furthermore those RMPs have produced and distributed the same quantity of product families for the FPs. Now one can analyze the production and distribution plan for the finished products. For the situation B, a variable will be introduced representing the quantity of a particular product transported to the customers in each time period. Because of the assumption (demand has to be met in each time period), it is obvious that the value of shipments from FPs should be such that they would, individually or collectively, meet the demand of a product for a customer in a time period. The excess production from the shipment would be inventory. By comparing the values for the two models, it was observed that the solutions obtained by the models are the same. Almost all of the values in the exploded solution are the same. Note that the end solution in terms of optimum value of the objective function is again the same for both models. Table A31 in the Appendix provides the figures for the finished products production from the product families. As is observed, because of not keeping any inventory for the product families, an FP converts all of it into individual products. An interesting facet to this inventory management that might be explored in future research is the push-pull methodology incorporation. The preliminary thinking behind this will be explained briefly now. Given a long-term forecasted demand, the product families can be pushed in the chain and be kept in the inventory. However, only on having an actual demand would one commence the production of the individual products, thereby pulling the product from that stage onwards. This methodology might be explored in the future to understand more on the push-pull system. Table A32 and A33 in the Appendix shows the values of the shipment and the amount produced. The underlined figures represent the presence of inventory.

5.6 Overview of Revenue, Costs and Profits

According to the optimal solution for Model 1, the revenue in this example is $ 4,520,000, while the profit is $ 3,867,600. The total cost is $652,400. Detailed cost factors are presented in Table 5.4. A comparison between the costs obtained by Model 1 and Model 2 is also shown in the table.

Larger instances were also tested and the parameters for which are shown in the form of Example 2 in Tables 5.1 and 5.3. With the integrated planning methodology, various analyses can be taken based on MIP models. This next chapter will highlight several critical issues using scenario analysis.

Table 5.4: Comparison of Costs for Model 1 and Model 2

Model 1		Model 2	
Type of Cost	**Description**	**Type of Cost**	**Description**
Profit	Total profit from the products = 3,867,600.	*Profit*	Total profit from the products = 3,867,600.
Raw material purchase cost	Total cost of raw material purchased in the time horizon = 132,500.	*Raw material purchase cost*	Total cost of raw material purchased in the time horizon = 132,500.
Raw material transportation Cost	Cost of transporting the raw material to the RMPs in the time horizon = 132,500.	*Raw material and product family transportation cost*	Cost of transporting the product families and the raw material that is used to manufacture them, in the time horizon = 202,500.
Product family transportation cost	Cost of transporting product family *m*, from the RMPs to the FPs in the time horizon = 77,700.	*(See above)*	
Finished product transportation cost	= 62,000.	*Finished product transportation cost*	= 61,000.
Setup cost at RMPs	Setup costs incurred at the RMPs in the time horizon = 1500.	*Setup cost at RMPs*	Setup costs incurred at the RMPs in the time horizon = 1500.
Setup cost at FPs	Setup costs incurred at the FPs in the time horizon = 1500.	*Setup cost at FPs*	Setup costs incurred at the FPs in the time horizon = 1500.
Inventory cost at RMPs	Inventory costs incurred at the RMPs in the time horizon = 0.	*Inventory cost at RMPs*	Inventory costs incurred at the RMPs in the time horizon = 0.
Inventory cost at FPs	Inventory costs incurred at the FPs in the time horizon = 19,000.	*Inventory cost at FPs*	Inventory costs incurred at the FPs in the time horizon = 27,000.

Chapter Six

Integrated Planning Analysis

With the integrated planning methodology, various analyses can be taken based on MIP models. This section will highlight several critical issues on using scenario analysis. This analysis can provide information in the changing scenario. This is particularly useful in a dynamic production and marketing environment.

6.1 Scenario Analysis

Because the optimal solution from the models is based on the input parameters, the variation in the data may change the optimal solution. Scenario analysis can identify the significance of impact of parameters on the objective function. For example, how changes in factors such as material cost, transportation cost, etc. will affect the total profit.

6.1.1 Observation of Experiments

We carry out the following experiments for the scenario analysis on:

1. Inventory costs

   a. Raw material.
   b. Finished products.
2. Setup Costs
   a. At an RMP *c*.
   b. At an FP *f*.
3. Raw material purchase price
4. Transportation costs
   a. For raw material transportation.
   b. For finished products transportation.

Results presented below are for Model type 1 formulated for situation A. Other factors can also be analyzed for sensitivity.

6.1.1.1 Inventory Costs

Since the model formulates two types of inventory in the system, raw material and finished products, one can analyze the impact of changing costs on the total profit for both of them. Inventory cost is a significant factor in the experiments. Inventory is present in the system formulations because of

1. Production Capacity at the RMPs and FPs.
2. Capacitated Supply.

The model tries to first meet the demand of the customer with the inventory at hand before moving to making a setup for the product. Since inventory comes with its cost, the model always makes a balance between the inventory costs and setup costs. Therefore any change in the inventory cost can have a significant impact on the overall profit. While conducting experiments it was found that by changing the inventory costs by 5-10%, the production and distribution plan changed in order to reflect a new optimal solution. For both raw material inventory and finished products inventory it was observed that for some instances a 5% change would cause a large impact on the total profit, while for others a change of 10% would cause a significant impact in the optimal solution. See Tables A34 and A35 in the Appendix.

6.1.1.2 Setup Costs

As expressed earlier, the models in all the time periods of the horizon have to strike a fine balance between the setup costs, for making a product, and the inventory costs, for keeping that product produced in the previous time periods. Needless to say, any change in setup costs can have a significant impact on the optimal solution. Two possible situations can arise:

1. Setup cost is so high compared to inventory cost that one is enticed to produce, as mush as possible each time a setup is perfumed.
2. Setup costs is so low compared with inventory cost that one is enticed to perform setups repeatedly just to meet the demand.

Apart from this since the system has setups at both RMPs and FPs, any changes in the setup costs have impact on the inventory at the plants. See Tables A36 and A37, in the Appendix.

6.1.1.3 Raw Material Purchase Price

The raw material price too plays an important role in the determination of the optimal solution. The simple reason for that is if the supplier is forecasted to supply the raw material at a much higher price in the coming time periods, then it might be a better option to stock up, provided the raw material inventory costs are not that significant and the supplier can provide the required quantity. The experiments again revealed a range of 5-10% change in the raw material price could lead the model to a new optimal solution. Refer Table A38, in the Appendix.

6.1.1.4 Transportation Costs

The transportation costs are also an important factor in determining the optimal solution. And if significant changes are made to it, which can happen on account of introducing a new mode of transportation, there is a possibility of a different optimal solution. Using this a manager can thus foresee any changes that may arise in the integrated plan on account of introducing a potential new mode of transportation in the future. There are three types of transportation costs being used in models. One reflects the material transportation between the supplier and RMPs, one between the RMPs and FPs and the last between the FPs and the customers. Refer Tables A39 and A40, in the Appendix.

Figure 6.1 illustrates the range of changes for these factors for the optimal solution i.e. the integrated plan proposed, to remain the same although the corresponding objective function value might change. Any larger change would take the model to a new optimal solution and thus the integrated plan would change.

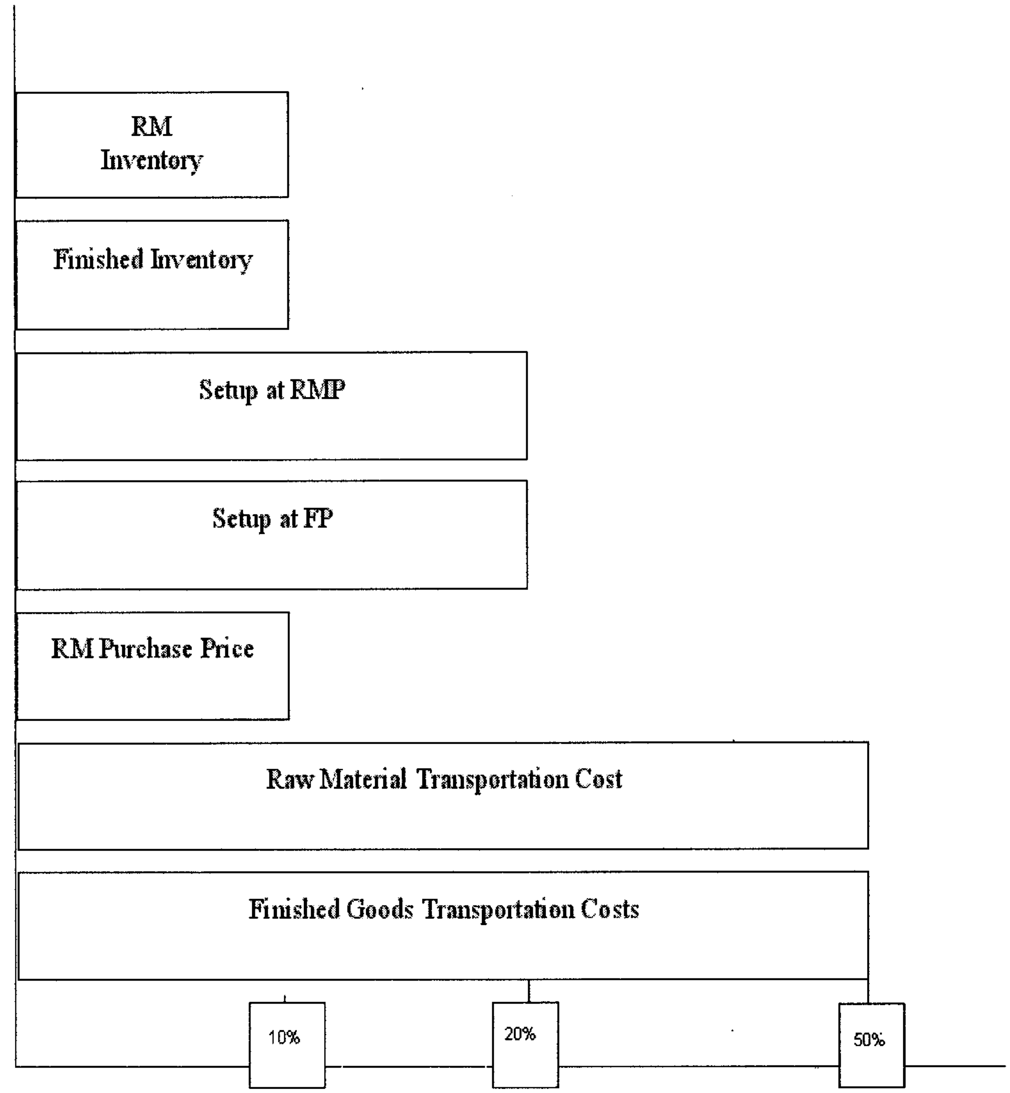


Figure 6.1: Scenario Analysis Summary

Chapter Seven

Summary and Conclusions

This chapter provides concluding remarks about the integrated planning methodology. Future research is also discussed.

7.1 Concluding Remarks

In addition to proposing an integrated methodology to a production-distribution system, the research also formulates two basic types of mathematical models to represent the system. The models were implemented in two different contexts prevalent in industry. These models were studied using both relatively small-size and large-size problems. It is important to point out that even the small instance is quite realistic. Those instances were solved to optimality. For larger instances, 1% optimal solutions were readily obtained. These 1% solutions refer to the closeness of the objective function to the upper bound for the formulation as calculated by LINGO and presented by the LINGO's interactive dialogue box. Critical analysis was conducted. In view of inherent inaccuracies involved in data collection, solutions that are optimal up to 1% may in many cases be quite acceptable. With quickly obtainable sub-optimal (1%) solutions, an analyst can simulate several scenarios before making a final decision.

With integrated method for planning, many benefits can be realized in a large scale manufacturing system (Wang, 1995). In order to implement the integrated supply chain planning methodology, apart from modeling the system, it is imperative to structure the information necessary for the modeling, although it is not necessary to integrate the information to coordinate the planning between the various departments of the organization (Shapiro, 2001). However the task of planning can be accomplished much more easily in the presence of streamlined data. Therefore, many authors have also bifurcated the utilization of information technology in supply chain planning into Transactional IT (that takes care of the information handling) and Analytical IT (that takes care of the actual modeling and optimization).

7.2 Contribution to the Research

This research was aimed at extending the work of Chen and Wang (1997); in the horizontal direction, by including multiple time periods; in the vertical direction, by including multiple locations. The contribution, along with the proposition of the methodologies, also includes the development of those methodologies into four distinct models. The conditions, i.e. multiple products, multiple locations, multiple types of locations, multiple setups, multiple time periods, modeled by the formulations were not seen in the literature that has been surveyed. Also the approach of verification, by formulating two models (Models 1 and 2) that give the same global solution, seems to be unique in the literature surveyed. This type of verification not only makes sure that the models correctly formulate the conditions in the system, but also makes sure that the results obtained by the models are the results obtained in real life, subject to certain tolerances.

7.3 Future Research

In this research a holistic view of the manufacturing system was undertaken to realize the models. However there are plenty of intra-plant processes, such as scheduling, that need to be carefully analyzed to fully integrate the system. The present modeling does not consider those. However the results that are obtained in optimization can be the input for algorithms, such as scheduling algorithms that can then take over, to schedule the required production in the plants effectively; and distribution algorithms, to adequately deliver the products to the customers. Much research is being done, however much still needs to be done in order to completely understand the intricacies involved in such systems. The author would consider the following aspects for the future research of this study:

- The mixed integer programming formulations can be strengthened by the addition of valid inequalities thereby reducing the computational time.
- Meta-heuristics can be designed for providing "good" solutions to large instances of these problems.
- Integrated planning can be further generalized so as to include scheduling and vehicle routing.
- Stochastic demand rather than deterministic demand can be incorporated to enhance the models applicability to the real life scenario.

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Appendix

Tables Utilized in Presenting the Discussion

Table A1: A Few Input Data Parameters

Product Family	1			2		
Production Rates at RMPs	50			50		
Setup Time at RMPs	250			250		
Setup Costs at RMPs	30			20		
Product Item	1	2	3	1	2	3
Production Rates at FPs	30	30	30	30	30	30
Setup Times at FPs	150	150	150	150	150	150
Setup Costs at FPs	10	20	15	15	20	10

Table A2: Input Parameters for RMPs

RMPs	1			2		
Time Periods	t_1	t_2	t_3	t_1	t_2	t_3
Available Production Time	1000	1000	1000	1000	1000	1000

Table A3: Input Parameters for FPs

FPs	$f=1$			$f=2$			$f=3$		
Time Periods	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3
Available Production Time	1200	1000	1100	1200	1000	1100	1200	1000	1100

Table A4: Input Parameters for Suppliers

Suppliers	Periods	Supplier Capacity (units)	RMPs	Purchase Price ($/unit)
j_1	t_1	10000	c_1	2
			c_2	4
	t_2	5000	c_1	2
			c_2	4
	t_3	10000	c_1	2
			c_2	4
j_2	t_1	10000	c_1	3
			c_2	3
	t_2	5000	c_1	3
			c_2	3
	t_3	10000	c_1	3
			c_2	3
j_3	t_1	10000	c_1	4
			c_2	2
	t_2	5000	c_1	4
			c_2	2
	t_3	5000	c_1	4
			c_2	2

Table A5: Model 1: Raw Material Purchase Plan

RMPs	Time Periods	Suppliers	Quantity Supplied
$c=1$	$t=1$	j_1	10000
		j_2	0
		j_3	0
	$t=2$	j_1	4510
		j_2	0
		j_3	5000
	$t=3$	j_1	10000
		j_2	1850
		j_3	0
$c=2$	$t=1$	j_1	0
		j_2	0
		j_3	10000
	$t=2$	j_1	490
		j_2	4110
		j_3	0
	$t=3$	j_1	0
		j_2	300
		j_3	5000

Table A6: Model 2: Raw Material Purchase Plan

RMPs	Time Periods	Suppliers	Quantity Supplied
$c=1$	$t=1$	j_1	10000
		j_2	0
		j_3	0
	$t=2$	j_1	3900
		j_2	0
		j_3	0
	$t=3$	j_1	10000
		j_2	1260
		j_3	0
$c=2$	$t=1$	j_1	0
		j_2	0
		j_3	10000
	$t=2$	j_1	1100
		j_2	5000
		j_3	5000
	$t=3$	j_1	0
		j_2	0
		j_3	5000

Table A7: Model 1: Production of Product Families

$c_1t_1 = 10000$	$f=1$	$m_1 = 3340$	$c_2t_1 = 10000$	$f=1$	$m_1 = 0$
		$m_2 = 4890$			$m_2 = 0$
	$f=2$	$m_1 = 0$		$f=2$	$m_1 = 3020$
		$m_2 = 1770$			$m_2 = 930$
	$f=3$	$m_1 = 0$		$f=3$	$m_1 = 3750$
		$m_2 = 0$			$m_2 = 2300$
$c_1t_2 = 9510$	$f=1$	$m_1 = 2050$	$c_2t_2 = 4600$	$f=1$	$m_1 = 0$
		$m_2 = 1510$			$m_2 = 0$
	$f=2$	$m_1 = 1850$		$f=2$	$m_1 = 0$
		$m_2 = 4100$			$m_2 = 0$
	$f=3$	$m_1 = 0$		$f=3$	$m_1 = 1900$
		$m_2 = 0$			$m_2 = 2700$
$c_1t_3 = 11850$	$f=1$	$m_1 = 2850$	$c_2t_3 = 5300$	$f=1$	$m_1 = 0$
		$m_2 = 3150$			$m_2 = 0$
	$f=2$	$m_1 = 2150$		$f=2$	$m_1 = 0$
		$m_2 = 3700$			$m_2 = 0$
	$f=3$	$m_1 = 0$		$f=3$	$m_1 = 2600$
		$m_2 = 0$			$m_2 = 2700$

Table A8: Model 2: Production of Product Families

c_1t_1 (10000 units)	j_1	m_1	f_1 =3270	f_2 = 0	f_3 = 0
		m_2	f_1 = 4960	f_2 = 1770	f_3 = 0
	j_2	m_1	f_1 = 0	f_2 = 0	f_3 = 0
		m_2	f_1 = 0	f_2 = 0	f_3 = 0
	j_3	m_1	f_1 = 0	f_2 = 0	f_3 = 0
		m_2	f_1 = 0	f_2 = 0	f_3 = 0
c_1t_2 (3900 units)	j_1	m_1	f_1 = 2120	f_2 = 0	f_3 = 0
		m_2	f_1 = 1440	f_2 = 0	f_3 = 0
	j_2	m_1	f_1 =0	f_2 = 0	f_3 = 0
		m_2	f_1 = 0	f_2 = 0	f_3 = 0
	j_3	m_1	f_1 =0	f_2 = 0	f_3 = 0
		m_2	f_1 = 0	f_2 = 0	f_3 = 0
c_1t_3 (11260 units) + 340 (inventory)	j_1	m_1	f_1 = 1590	f_2 = 1900	f_3 = 0
		m_2	f_1 = 3150	f_2 = 3700	f_3 = 0
	j_2	m_1	f_1 = 1260	f_2 = 0	f_3 = 0
		m_2	f_1 = 0	f_2 = 0	f_3 = 0
	j_3	m_1	f_1 = 0	f_2 = 0	f_3 = 0
		m_2	f_1 = 0	f_2 = 0	f_3 = 0

Table A9: Model 2: Production of Product Families

c_2t_1 (10000 units)	j_1	m_1	$f_1=0$	$f_2=0$	$f_3=0$
		m_2	$f_1=0$	$f_2=0$	$f_3=0$
	j_2	m_1	$f_1=0$	$f_2=10$	$f_3=0$
		m_2	$f_1=0$	$f_2=0$	$f_3=0$
	j_3	m_1	$f_1=0$	$f_2=3020$	$f_3=3750$
		m_2	$f_1=0$	$f_2=930$	$f_3=2300$
c_2t_2 (11100 units)	j_1	m_1	$f_1=0$	$f_2=1100$	$f_3=0$
		m_2	$f_1=0$	$f_2=0$	$f_3=0$
	j_2	m_1	$f_1=0$	$f_2=400$	$f_3=1900$
		m_2	$f_1=0$	$f_2=0$	$f_3=2700$
	j_3	m_1	$f_1=0$	$f_2=350$	$f_3=0$
		m_2	$f_1=0$	$f_2=4100$	$f_3=0$
c_2t_3 (5000 units) + 550 (inventory)	j_1	m_1	$f_1=0$	$f_2=0$	$f_3=0$
		m_2	$f_1=0$	$f_2=0$	$f_3=0$
	j_2	m_1	$f_1=0$	$f_2=0$	$f_3=0$
		m_2	$f_1=0$	$f_2=0$	$f_3=0$
	j_3	m_1	$f_1=0$	$f_2=250$	$f_3=2600$
		m_2	$f_1=0$	$f_2=0$	$f_3=2700$

Table A10: Model 1: Production by FP $f = 1$ in the Time Horizon

$f_1m_1t_1$ (3340 units), $f_1m_2t_1$ (4890)	l_1	m_1	n_1 =300	n_2 = 470	n_3 = 250
		m_2	n_4 = 490	n_5 = 700	n_6 = 850
	l_2	m_1	n_1 = 500	n_2 = 620	n_3 = 100
		m_2	n_4 = 500	n_5 = 400	n_6 = 350
	l_3	m_1	n_1 = 500	n_2 = 400	n_3 = 200
		m_2	n_4 = 300	n_5 = 700	n_6 = 600
$f_1m_1t_2$ (2050 units), $f_1m_2t_2$ (1510)	l_1	m_1	n_1 = 500	n_2 = 0	n_3 = 250
		m_2	n_4 = 60	n_5 = 0	n_6 = 300
	l_2	m_1	n_1 =250	n_2 = 0	n_3 = 350
		m_2	n_4 = 250	n_5 = 0	n_6 = 250
	l_3	m_1	n_1 =250	n_2 = 0	n_3 = 450
		m_2	n_4 = 250	n_5 = 0	n_6 = 400
$f_1m_1t_3$ (2850 units), $f_1m_1t_3$(3150)	l_1	m_1	n_1 = 350	n_2 = 30	n_3 = 350
		m_2	n_4 = 300	n_5 = 250	n_6 = 500
	l_2	m_1	n_1 = 200	n_2 = 720	n_3 = 200
		m_2	n_4 = 800	n_5 = 100	n_6 = 250
	l_3	m_1	n_1 = 500	n_2 = 200	n_3 = 300
		m_2	n_4 = 100	n_5 = 200	n_6 = 650

Table A11: Model 1: Production by FP $f = 2$ in the Time Horizon

$f_2m_1t_1$ (3020 units), $f_2m_2t_1$ (2700)	l_1	m_1	$n_1 = 300$	$n_2 = 450$	$n_3 = 350$
		m_2	$n_4 = 300$	$n_5 = 500$	$n_6 = 300$
	l_2	m_1	$n_1 = 300$	$n_2 = 370$	$n_3 = 400$
		m_2	$n_4 = 150$	$n_5 = 350$	$n_6 = 300$
	l_3	m_1	$n_1 = 300$	$n_2 = 300$	$n_3 = 250$
		m_2	$n_4 = 300$	$n_5 = 350$	$n_6 = 150$
$f_2m_1t_2$ (1850 units), $f_2m_2t_2$ (4100)	l_1	m_1	$n_1 = 300$	$n_2 = 0$	$n_3 = 250$
		m_2	$n_4 = 350$	$n_5 = 450$	$n_6 = 850$
	l_2	m_1	$n_1 = 500$	$n_2 = 0$	$n_3 = 100$
		m_2	$n_4 = 500$	$n_5 = 200$	$n_6 = 350$
	l_3	m_1	$n_1 = 500$	$n_2 = 0$	$n_3 = 200$
		m_2	$n_4 = 300$	$n_5 = 500$	$n_6 = 600$
$f_2m_1t_3$ (2150 units), $f_2m_1t_3$(3700)	l_1	m_1	$n_1 = 100$	$n_2 = 250$	$n_3 = 150$
		m_2	$n_4 = 300$	$n_5 = 400$	$n_6 = 550$
	l_2	m_1	$n_1 = 300$	$n_2 = 100$	$n_3 = 300$
		m_2	$n_4 = 550$	$n_5 = 300$	$n_6 = 250$
	l_3	m_1	$n_1 = 550$	$n_2 = 150$	$n_3 = 250$
		m_2	$n_4 = 500$	$n_5 = 550$	$n_6 = 300$

Table A12: Model 1: Production by FP $f = 3$ in the Time Horizon

***$f_3m_1t_1$* (3750 units), *$f_3m_2t_1$* (2300)**	l_1	m_1	$n_1 = 500$	$n_2 = 450$	$n_3 = 250$
		m_2	$n_4 = 200$	$n_5 = 250$	$n_6 = 300$
	l_2	m_1	$n_1 = 250$	$n_2 = 750$	$n_3 = 350$
		m_2	$n_4 = 250$	$n_5 = 200$	$n_6 = 250$
	l_3	m_1	$n_1 = 250$	$n_2 = 500$	$n_3 = 450$
		m_2	$n_4 = 250$	$n_5 = 200$	$n_6 = 400$
***$f_3m_1t_2$* (1900 units), *$f_3m_2t_2$* (2700)**	l_1	m_1	$n_1 = 300$	$n_2 = 0$	$n_3 = 350$
		m_2	$n_4 = 300$	$n_5 = 500$	$n_6 = 300$
	l_2	m_1	$n_1 = 300$	$n_2 = 0$	$n_3 = 400$
		m_2	$n_4 = 150$	$n_5 = 350$	$n_6 = 300$
	l_3	m_1	$n_1 = 300$	$n_2 = 0$	$n_3 = 250$
		m_2	$n_4 = 300$	$n_5 = 350$	$n_6 = 150$
***$f_3m_1t_3$* (2600 units), *$f_3m_1t_3$*(2700)**	l_1	m_1	$n_1 = 300$	$n_2 = 250$	$n_3 = 350$
		m_2	$n_4 = 300$	$n_5 = 500$	$n_6 = 300$
	l_2	m_1	$n_1 = 300$	$n_2 = 250$	$n_3 = 400$
		m_2	$n_4 = 150$	$n_5 = 350$	$n_6 = 300$
	l_3	m_1	$n_1 = 300$	$n_2 = 200$	$n_3 = 250$
		m_2	$n_4 = 300$	$n_5 = 350$	$n_6 = 150$

Table A13: Model 2: Production by $f = 1$ from Material by c_1 in the Time Horizon

$c_1f_1t_1$ (8230 units), m_1 = 3270, m_2 = 4960	l_1	m_1	n_1 =300	n_2 = 400	n_3 = 250
		m_2	n_4 = 350	n_5 = 700	n_6 = 1060
	l_2	m_1	n_1 = 500	n_2 = 620	n_3 = 100
		m_2	n_4 = 500	n_5 = 400	n_6 = 350
	l_3	m_1	n_1 = 500	n_2 = 400	n_3 = 200
		m_2	n_4 = 300	n_5 = 700	n_6 = 600
$c_1f_1t_2$ (3560 units), m_1 = 2120, m_2 = 1440	l_1	m_1	n_1 = 500	n_2 = 0	n_3 = 250
		m_2	n_4 = 200	n_5 = 0	n_6 = 90
	l_2	m_1	n_1 =250	n_2 = 0	n_3 = 350
		m_2	n_4 = 250	n_5 = 0	n_6 = 250
	l_3	m_1	n_1 =250	n_2 = 0	n_3 = 520
		m_2	n_4 = 250	n_5 = 0	n_6 = 400
$c_1f_1t_3$ (6000 units), m_1 = 2850, m_2 = 3150	l_1	m_1	n_1 = 350	n_2 = 100	n_3 = 350
		m_2	n_4 = 300	n_5 = 250	n_6 = 500
	l_2	m_1	n_1 = 200	n_2 = 720	n_3 = 200
		m_2	n_4 = 800	n_5 = 100	n_6 = 250
	l_3	m_1	n_1 = 500	n_2 = 200	n_3 = 230
		m_2	n_4 = 100	n_5 = 200	n_6 = 650

Table A14: Model 2: Production by $f = 1$ from Material by c_2 in the Time Horizon

$c_2f_1t_1$ (0 units), $m_1 = 0$, $m_2 = 0$	l_1	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_2	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_3	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
$c_2f_1t_2$ (0 units), $m_1 = 0$, $m_2 = 0$	l_1	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_2	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_3	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
$c_2f_1t_3$ (0 units), $m_1 = 0$, $m_2 = 0$	l_1	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_2	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_3	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$

Table A15: Model 2: Production by $f = 2$ from Material by c_1 in the Time Horizon

$c_1f_2t_1$ (1770 units), $m_1 = 0$, $m_2 = 1770$	l_1	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 170$	$n_5 = 0$	$n_6 = 0$
	l_2	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 150$	$n_5 = 350$	$n_6 = 300$
	l_3	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 300$	$n_5 = 350$	$n_6 = 150$
$c_1f_2t_2$ (0 units), $m_1 = 0$, $m_2 = 0$	l_1	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_2	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_3	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
$c_1f_2t_3$ (5600 units), $m_1 = 1900$, $m_2 = 3700$	l_1	m_1	$n_1 = 100$	$n_2 = 0$	$n_3 = 150$
		m_2	$n_4 = 300$	$n_5 = 400$	$n_6 = 550$
	l_2	m_1	$n_1 = 300$	$n_2 = 100$	$n_3 = 300$
		m_2	$n_4 = 550$	$n_5 = 300$	$n_6 = 250$
	l_3	m_1	$n_1 = 550$	$n_2 = 150$	$n_3 = 250$
		m_2	$n_4 = 500$	$n_5 = 550$	$n_6 = 300$

Table A16: Model 2: Production by $f = 2$ from Material by c_2 in the Time Horizon

$c_2f_2t_1$ (3950 units), $m_1 = 3030$, $m_2 = 930$	l_1	m_1	$n_1 = 300$	$n_2 = 450$	$n_3 = 350$
		m_2	$n_4 = 130$	$n_5 = 500$	$n_6 = 300$
	l_2	m_1	$n_1 = 300$	$n_2 = 370$	$n_3 = 400$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_3	m_1	$n_1 = 300$	$n_2 = 300$	$n_3 = 250$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
$c_2f_2t_2$ (5950 units), $m_1 = 1850$, $m_2 = 4100$	l_1	m_1	$n_1 = 300$	$n_2 = 0$	$n_3 = 250$
		m_2	$n_4 = 350$	$n_5 = 450$	$n_6 = 850$
	l_2	m_1	$n_1 = 500$	$n_2 = 0$	$n_3 = 100$
		m_2	$n_4 = 500$	$n_5 = 200$	$n_6 = 350$
	l_3	m_1	$n_1 = 500$	$n_2 = 0$	$n_3 = 200$
		m_2	$n_4 = 300$	$n_5 = 500$	$n_6 = 600$
$c_2f_2t_3$ (250 units), $m_1 = 250$, $m_2 = 0$	l_1	m_1	$n_1 = 0$	$n_2 = 250$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_2	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_3	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$

Table A17: Model 2: Production by $f = 3$ from Material by c_1 in the Time Horizon

$c_1f_3t_1$ (0 units), $m_1 = 0$, $m_2 = 0$	l_1	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_2	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_3	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
$c_1f_3t_2$ (0 units), $m_1 = 0$, $m_2 = 0$	l_1	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_2	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_3	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
$c_1f_3t_3$ (0 units), $m_1 = 0$, $m_2 = 0$	l_1	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_2	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$
	l_3	m_1	$n_1 = 0$	$n_2 = 0$	$n_3 = 0$
		m_2	$n_4 = 0$	$n_5 = 0$	$n_6 = 0$

Table A18: Model 2: Production by $f = 3$ from Material by c_2 in the Time Horizon

$c_2f_3t_1$ (6050 units), $m_1 = 3750$, $m_2 = 2300$	l_1	m_1	$n_1 = 500$	$n_2 = 450$	$n_3 = 250$
		m_2	$n_4 = 200$	$n_5 = 250$	$n_6 = 300$
	l_2	m_1	$n_1 = 250$	$n_2 = 750$	$n_3 = 350$
		m_2	$n_4 = 250$	$n_5 = 200$	$n_6 = 250$
	l_3	m_1	$n_1 = 250$	$n_2 = 500$	$n_3 = 450$
		m_2	$n_4 = 250$	$n_5 = 200$	$n_6 = 400$
$c_2f_3t_2$ (4600 units), $m_1 = 1900$, $m_2 = 2700$	l_1	m_1	$n_1 = 300$	$n_2 = 0$	$n_3 = 350$
		m_2	$n_4 = 300$	$n_5 = 500$	$n_6 = 300$
	l_2	m_1	$n_1 = 300$	$n_2 = 0$	$n_3 = 400$
		m_2	$n_4 = 150$	$n_5 = 350$	$n_6 = 300$
	l_3	m_1	$n_1 = 300$	$n_2 = 0$	$n_3 = 250$
		m_2	$n_4 = 300$	$n_5 = 350$	$n_6 = 150$
$c_2f_3t_3$ (5300 units), $m_1 = 2600$, $m_2 = 2700$	l_1	m_1	$n_1 = 300$	$n_2 = 250$	$n_3 = 350$
		m_2	$n_4 = 300$	$n_5 = 500$	$n_6 = 300$
	l_2	m_1	$n_1 = 300$	$n_2 = 250$	$n_3 = 400$
		m_2	$n_4 = 150$	$n_5 = 350$	$n_6 = 300$
	l_3	m_1	$n_1 = 300$	$n_2 = 200$	$n_3 = 250$
		m_2	$n_4 = 300$	$n_5 = 350$	$n_6 = 150$

Table A19: Demand vs. Model 1: Production by FP $f = 1$ in the Time Horizon

f_1t_1	l_1	*Dem*	$n_1 = 300$	$n_2 = 200$	$n_3 = 250$	$n_4 = 350$	$n_5 = 450$	$n_6 = 850$
		Prod	$n_1 = 300$	$n_2 = 470$*	$n_3 = 250$	$n_4 = 490$*	$n_5 = 700$*	$n_6 = 850$
	l_2	*Dem*	$n_1 = 500$	$n_2 = 120$	$n_3 = 100$	$n_4 = 500$	$n_5 = 200$	$n_6 = 350$
		Prod	$n_1 = 500$	$n_2 = 620$*	$n_3 = 100$	$n_4 = 500$	$n_5 = 400$*	$n_6 = 350$
	l_3	*Dem*	$n_1 = 500$	$n_2 = 100$	$n_3 = 200$	$n_4 = 300$	$n_5 = 500$	$n_6 = 600$
		Prod	$n_1 = 500$	$n_2 = 400$*	$n_3 = 200$	$n_4 = 300$	$n_5 = 700$*	$n_6 = 600$
f_1t_2	l_1	*Dem*	$n_1 = 500$	$n_2 = 200$	$n_3 = 250$	$n_4 = 200$	$n_5 = 250$	$n_6 = 300$
		Prod	$n_1 = 500$	$n_2 = 0$	$n_3 = 250$	$n_4 = 60$	$n_5 = 0$	$n_6 = 300$
	l_2	*Dem*	$n_1 = 250$	$n_2 = 500$	$n_3 = 350$	$n_4 = 250$	$n_5 = 200$	$n_6 = 250$
		Prod	$n_1 = 250$	$n_2 = 0$	$n_3 = 350$	$n_4 = 250$	$n_5 = 0$	$n_6 = 250$
	l_3	*Dem*	$n_1 = 250$	$n_2 = 300$	$n_3 = 450$	$n_4 = 250$	$n_5 = 200$	$n_6 = 400$
		Prod	$n_1 = 250$	$n_2 = 0$	$n_3 = 450$	$n_4 = 250$	$n_5 = 0$	$n_6 = 400$
f_1t_3	l_1	*Dem*	$n_1 = 350$	$n_2 = 100$	$n_3 = 350$	$n_4 = 300$	$n_5 = 250$	$n_6 = 500$
		Prod	$n_1 = 350$	$n_2 = 30$	$n_3 = 350$	$n_4 = 300$	$n_5 = 250$	$n_6 = 500$
	l_2	*Dem*	$n_1 = 200$	$n_2 = 720$	$n_3 = 200$	$n_4 = 800$	$n_5 = 100$	$n_6 = 250$
		Prod	$n_1 = 200$	$n_2 = 720$	$n_3 = 200$	$n_4 = 800$	$n_5 = 100$	$n_6 = 250$
	l_3	*Dem*	$n_1 = 500$	$n_2 = 200$	$n_3 = 300$	$n_4 = 100$	$n_5 = 200$	$n_6 = 650$
		Prod	$n_1 = 500$	$n_2 = 200$	$n_3 = 300$	$n_4 = 100$	$n_5 = 200$	$n_6 = 650$

Table A20: Demand vs. Model 1: Production by $f = 2$ in the Time Horizon

f_2t_1	l_1	*Dem*	$n_1 = 300$	$n_2 = 250$	$n_3 = 350$	$n_4 = 300$	$n_5 = 500$	$n_6 = 300$
		Prod	$n_1 = 300$	$n_2 = 450$*	$n_3 = 350$	$n_4 = 300$	$n_5 = 500$	$n_6 = 300$
	l_2	*Dem*	$n_1 = 300$	$n_2 = 250$	$n_3 = 400$	$n_4 = 150$	$n_5 = 350$	$n_6 = 300$
		Prod	$n_1 = 300$	$n_2 = 370$*	$n_3 = 400$	$n_4 = 150$	$n_5 = 350$	$n_6 = 300$
	l_3	*Dem*	$n_1 = 300$	$n_2 = 200$	$n_3 = 250$	$n_4 = 300$	$n_5 = 350$	$n_6 = 150$
		Prod	$n_1 = 300$	$n_2 = 300$*	$n_3 = 250$	$n_4 = 300$	$n_5 = 350$	$n_6 = 150$
f_2t_2	l_1	*Dem*	$n_1 = 300$	$n_2 = 200$	$n_3 = 250$	$n_4 = 350$	$n_5 = 450$	$n_6 = 850$
		Prod	$n_1 = 300$	$n_2 = 0$	$n_3 = 250$	$n_4 = 350$	$n_5 = 450$	$n_6 = 850$
	l_2	*Dem*	$n_1 = 500$	$n_2 = 120$	$n_3 = 100$	$n_4 = 500$	$n_5 = 200$	$n_6 = 350$
		Prod	$n_1 = 500$	$n_2 = 0$	$n_3 = 100$	$n_4 = 500$	$n_5 = 200$	$n_6 = 350$
	l_3	*Dem*	$n_1 = 500$	$n_2 = 100$	$n_3 = 200$	$n_4 = 300$	$n_5 = 500$	$n_6 = 600$
		Prod	$n_1 = 500$	$n_2 = 0$	$n_3 = 200$	$n_4 = 300$	$n_5 = 500$	$n_6 = 600$
f_2t_3	l_1	*Dem*	$n_1 = 100$	$n_2 = 250$	$n_3 = 150$	$n_4 = 300$	$n_5 = 400$	$n_6 = 550$
		Prod	$n_1 = 100$	$n_2 = 250$	$n_3 = 150$	$n_4 = 300$	$n_5 = 400$	$n_6 = 550$
	l_2	*Dem*	$n_1 = 300$	$n_2 = 100$	$n_3 = 300$	$n_4 = 550$	$n_5 = 300$	$n_6 = 250$
		Prod	$n_1 = 300$	$n_2 = 100$	$n_3 = 300$	$n_4 = 550$	$n_5 = 300$	$n_6 = 250$
	l_3	*Dem*	$n_1 = 550$	$n_2 = 150$	$n_3 = 250$	$n_4 = 500$	$n_5 = 550$	$n_6 = 300$
		Prod	$n_1 = 550$	$n_2 = 150$	$n_3 = 250$	$n_4 = 500$	$n_5 = 550$	$n_6 = 300$

Table A21: Demand vs. Model 1: Production by $f = 3$ in the Time Horizon

f_3t_1	l_1	*Dem*	n_1 =500	n_2 = 200	n_3 = 250	n_4 = 200	n_5 = 250	n_6 = 300
		Prod	n_1 = 500	n_2 = 450*	n_3 = 250	n_4 = 200	n_5 = 250	n_6 = 300
	l_2	*Dem*	n_1 = 250	n_2 = 500	n_3 = 350	n_4 = 250	n_5 = 200	n_6 = 250
		Prod	n_1 = 250	n_2 = 750*	n_3 = 350	n_4 = 250	n_5 = 200	n_6 = 250
	l_3	*Dem*	n_1 = 250	n_2 = 300	n_3 = 450	n_4 = 250	n_5 = 200	n_6 = 400
		Prod	n_1 = 250	n_2 = 500*	n_3 = 450	n_4 = 250	n_5 = 200	n_6 = 400
f_3t_2	l_1	*Dem*	n_1 = 300	n_2 = 250	n_3 = 350	n_4 = 300	n_5 = 500	n_6 = 300
		Prod	n_1 = 300	n_2 = 0	n_3 = 350	n_4 = 300	n_5 = 500	n_6 = 300
	l_2	*Dem*	n_1 =300	n_2 = 250	n_3 = 400	n_4 =150	n_5 = 350	n_6 = 300
		Prod	n_1 = 300	n_2 = 0	n_3 = 400	n_4 = 150	n_5 = 350	n_6 = 300
	l_3	*Dem*	n_1 =300	n_2 = 200	n_3 = 250	n_4 =300	n_5 = 350	n_6 = 150
		Prod	n_1 = 300	n_2 = 0	n_3 = 250	n_4 = 300	n_5 = 350	n_6 = 150
f_3t_3	l_1	*Dem*	n_1 = 300	n_2 = 250	n_3 = 350	n_4 = 300	n_5 = 500	n_6 = 300
		Prod	n_1 = 300	n_2 = 250	n_3 = 350	n_4 = 300	n_5 = 500	n_6 = 300
	l_2	*Dem*	n_1 =300	n_2 = 250	n_3 = 400	n_4 =150	n_5 = 350	n_6 = 300
		Prod	n_1 = 300	n_2 = 250	n_3 = 400	n_4 = 150	n_5 = 350	n_6 = 300
	l_3	*Dem*	n_1 =300	n_2 = 200	n_3 = 250	n_4 =300	n_5 = 350	n_6 = 150
		Prod	n_1 = 300	n_2 = 200	n_3 = 250	n_4 = 300	n_5 = 350	n_6 = 150

Table A22: Demand vs. Model 2: Production by f = 1 in the Time Horizon

f_1t_1	l_1	*Dem*	n_1 =300	n_2 = 200	n_3 = 250	n_4 = 350	n_5 = 450	n_6 = 850
		Prod	n_1 =300	n_2 = 400*	n_3 = 250	n_4 = 350	n_5 = 700*	n_6= 1060*
	l_2	*Dem*	n_1 = 500	n_2 = 120	n_3 = 100	n_4 = 500	n_5 = 200	n_6 = 350
		Prod	n_1 =500	n_2 = 620*	n_3 = 100	n_4 = 500	n_5 = 400*	n_6 = 350
	l_3	*Dem*	n_1 = 500	n_2 = 100	n_3 = 200	n_4 = 300	n_5 = 500	n_6 = 600
		Prod	n_1 =500	n_2 = 400*	n_3 = 200	n_4 = 300	n_5 = 700*	n_6 = 600
f_1t_2	l_1	*Dem*	n_1 = 500	n_2 = 200	n_3 = 250	n_4 = 200	n_5 = 250	n_6 = 300
		Prod	n_1 =500	n_2 = 0	n_3 = 250	n_4 = 200	n_5 = 0	n_6 = 90
	l_2	*Dem*	n_1 =250	n_2 = 500	n_3 = 350	n_4 =250	n_5 = 200	n_6 = 250
		Prod	n_1 =250	n_2 = 0	n_3 = 350	n_4 = 250	n_5 = 0	n_6 = 250
	l_3	*Dem*	n_1 =250	n_2 = 300	n_3 = 450	n_4 =250	n_5 = 200	n_6 = 400
		Prod	n_1 =250	n_2 = 0	n_3 = 520*	n_4 = 250	n_5 = 0	n_6 = 400
f_1t_3	l_1	*Dem*	n_1 = 350	n_2 = 100	n_3 = 350	n_4 = 300	n_5 = 250	n_6 = 500
		Prod	n_1 =350	n_2 = 100	n_3 = 350	n_4 = 300	n_5 = 250	n_6 = 500
	l_2	*Dem*	n_1 =200	n_2 = 720	n_3 = 200	n_4 =800	n_5 = 100	n_6 = 250
		Prod	n_1 =200	n_2 = 720	n_3 = 200	n_4 = 800	n_5 = 100	n_6 = 250
	l_3	*Dem*	n_1 =500	n_2 = 200	n_3 = 300	n_4 =100	n_5 = 200	n_6 = 650
		Prod	n_1 =500	n_2 = 200	n_3 = 230	n_4 = 100	n_5 = 200	n_6 = 650

Table A23: Demand vs. Model 2: Production by $f = 2$ in the Time Horizon

f_2t_1	l_1	*Dem*	n_1 =300	n_2 = 250	n_3 = 350	n_4 = 300	n_5 = 500	n_6 = 300
		Prod	n_1 = 300	n_2 = 450	n_3 = 350	n_4 = 300	n_5 = 500	n_6 = 300
	l_2	*Dem*	n_1 = 300	n_2 = 250	n_3 = 400	n_4 = 150	n_5 = 350	n_6 = 300
		Prod	n_1 = 300	n_2 = 370	n_3 = 400	n_4 = 150	n_5 = 350	n_6 = 300
	l_3	*Dem*	n_1 = 300	n_2 = 200	n_3 = 250	n_4 = 300	n_5 = 350	n_6 = 150
		Prod	n_1 = 300	n_2 = 300	n_3 = 250	n_4 = 300	n_5 = 350	n_6 = 150
f_2t_2	l_1	*Dem*	n_1 = 300	n_2 = 200	n_3 = 250	n_4 = 350	n_5 = 450	n_6 = 850
		Prod	n_1 = 300	n_2 = 0	n_3 = 250	n_4 = 350	n_5 = 450	n_6 = 850
	l_2	*Dem*	n_1 =500	n_2 = 120	n_3 = 100	n_4 =500	n_5 = 200	n_6 = 350
		Prod	n_1 = 500	n_2 = 0	n_3 = 100	n_4 = 500	n_5 = 200	n_6 = 350
	l_3	*Dem*	n_1 =500	n_2 = 100	n_3 = 200	n_4 =300	n_5 = 500	n_6 = 600
		Prod	n_1 = 500	n_2 = 0	n_3 = 200	n_4 = 300	n_5 = 500	n_6 = 600
f_2t_3	l_1	*Dem*	n_1 = 100	n_2 = 250	n_3 = 150	n_4 = 300	n_5 = 400	n_6 = 550
		Prod	n_1 = 100	n_2 = 250	n_3 = 150	n_4 = 300	n_5 = 400	n_6 = 550
	l_2	*Dem*	n_1 =300	n_2 = 100	n_3 = 300	n_4 =550	n_5 = 300	n_6 = 250
		Prod	n_1 = 300	n_2 = 100	n_3 = 300	n_4 = 550	n_5 = 300	n_6 = 250
	l_3	*Dem*	n_1 =550	n_2 = 150	n_3 = 250	n_4 =500	n_5 = 550	n_6 = 300
		Prod	n_1 = 550	n_2 = 150	n_3 = 250	n_4 = 500	n_5 = 550	n_6 = 300

Table A24: Demand vs. Model 2: Production by $f = 3$ in the Time Horizon

f_3t_1	l_1	*Dem*	n_1 =500	n_2 = 200	n_3 = 250	n_4 = 200	n_5 = 250	n_6 = 300
		Prod	n_1 = 500	n_2 = 450*	n_3 = 250	n_4 = 200	n_5 = 250	n_6 = 300
	l_2	*Dem*	n_1 = 250	n_2 = 500	n_3 = 350	n_4 = 250	n_5 = 200	n_6 = 250
		Prod	n_1 = 250	n_2 = 750*	n_3 = 350	n_4 = 250	n_5 = 200	n_6 = 250
	l_3	*Dem*	n_1 = 250	n_2 = 300	n_3 = 450	n_4 = 250	n_5 = 200	n_6 = 400
		Prod	n_1 = 250	n_2 = 500*	n_3 = 450	n_4 = 250	n_5 = 200	n_6 = 400
f_3t_2	l_1	*Dem*	n_1 = 300	n_2 = 250	n_3 = 350	n_4 = 300	n_5 = 500	n_6 = 300
		Prod	n_1 = 300	n_2 = 0	n_3 = 350	n_4 = 300	n_5 = 500	n_6 = 300
	l_2	*Dem*	n_1 =300	n_2 = 250	n_3 = 400	n_4 =150	n_5 = 350	n_6 = 300
		Prod	n_1 = 300	n_2 = 0	n_3 = 400	n_4 = 150	n_5 = 350	n_6 = 300
	l_3	*Dem*	n_1 =300	n_2 = 200	n_3 = 250	n_4 =300	n_5 = 350	n_6 = 150
		Prod	n_1 = 300	n_2 = 0	n_3 = 250	n_4 = 300	n_5 = 350	n_6 = 150
f_3t_3	l_1	*Dem*	n_1 = 300	n_2 = 250	n_3 = 350	n_4 = 300	n_5 = 500	n_6 = 300
		Prod	n_1 = 300	n_2 = 250	n_3 = 350	n_4 = 300	n_5 = 500	n_6 = 300
	l_2	*Dem*	n_1 =300	n_2 = 250	n_3 = 400	n_4 =150	n_5 = 350	n_6 = 300
		Prod	n_1 = 300	n_2 = 250	n_3 = 400	n_4 = 150	n_5 = 350	n_6 = 300
	l_3	*Dem*	n_1 =300	n_2 = 200	n_3 = 250	n_4 =300	n_5 = 350	n_6 = 150
		Prod	n_1 = 300	n_2 = 200	n_3 = 250	n_4 = 300	n_5 = 350	n_6 = 150

Table A25: Time Availability for the RMPs

RMPs	1			2		
Time Periods	$t = 1$	$t = 2$	$t = 3$	$t = 1$	$t = 2$	$t = 3$
Available Production Time	1000	700	700	1000	700	700

Table A26: Time Availability for the RMPs

FPs	$f = 1$			$f = 2$			$f = 3$		
Time Periods	$t = 1$	$t = 2$	$t = 3$	$t = 1$	$t = 2$	$t = 3$	$t = 1$	$t = 2$	$t = 3$
Available Production Time	800	800	700	800	800	700	800	800	700

Table A27: Raw Material Capacities for the Suppliers

Suppliers	1			2			3		
Time Periods	$t = 1$	$t = 2$	$t = 3$	$t = 1$	$t = 2$	$t = 3$	$t = 1$	$t = 2$	$t = 3$
Raw Material Capacity	5000	5000	5000	10000	5000	5000	12000	5000	5000

Table A28: Raw Material Purchase Plan

Model 1

RMPs	Time Periods	Suppliers	Quantity Supplied
c= 1	*t = 1*	j_1	5000
		j_2	4500
		j_3	0
	t = 2	j_1	5000
		j_2	3000
		j_3	0
	t = 3	j_1	5000
		j_2	1000
		j_3	0
c= 2	*t = 1*	j_1	0
		j_2	5000
		j_3	12000
	t = 2	j_1	0
		j_2	2000
		j_3	5000
	t = 3	j_1	0
		j_2	4000
		j_3	5000

Model 2

RMPs	Time Periods	Suppliers	Quantity Supplied
c= 1	*t = 1*	j_1	5000
		j_2	4500
		j_3	0
	t = 2	j_1	5000
		j_2	3000
		j_3	0
	t = 3	j_1	5000
		j_2	1000
		j_3	0
c= 2	*t = 1*	j_1	0
		j_2	5000
		j_3	12000
	t = 2	j_1	0
		j_2	2000
		j_3	5000
	t = 3	j_1	0
		j_2	4000
		j_3	5000

Table A29: Model 1: Production of Product Families

$c_1t_1 = 9500$	$f=1$	$m_1 = 9000$
		$m_2 = 0$
	$f=2$	$m_1 = 500$
		$m_2 = 0$
	$f=3$	$m_1 = 0$
		$m_2 = 0$
$c_1t_2 = 8000$	$f=1$	$m_1 = 0$
		$m_2 = 0$
	$f=2$	$m_1 = 0$
		$m_2 = 8000$
	$f=3$	$m_1 = 0$
		$m_2 = 0$
$c_1t_3 = 6000$	$f=1$	$m_1 = 0$
		$m_2 = 6000$
	$f=2$	$m_1 = 0$
		$m_2 = 0$
	$f=3$	$m_1 = 0$
		$m_2 = 0$

$c_2t_1 = 17000$	$f=1$	$m_1 = 0$
		$m_2 = 0$
	$f=2$	$m_1 = 0$
		$m_2 = 8000$
	$f=3$	$m_1 = 0$
		$m_2 = 9000$
$c_2t_2 = 7000$	$f=1$	$m_1 = 0$
		$m_2 = 0$
	$f=2$	$m_1 = 0$
		$m_2 = 0$
	$f=3$	$m_1 = 7000$
		$m_2 = 0$
$c_2t_3 = 9000$	$f=1$	$m_1 = 0$
		$m_2 = 0$
	$f=2$	$m_1 = 4000$
		$m_2 = 0$
	$f=3$	$m_1 = 5000$
		$m_2 = 0$

Table A30: Model 2: Production of Product Families

c_1t_1 (9500 units)	j_1	m_1	$f_1 = 4500$	$f_2 = 500$	$f_3 = 0$
		m_2	$f_1 = 0$	$f_2 = 0$	$f_3 = 0$
	j_2	m_1	$f_1 = 4500$	$f_2 = 0$	$f_3 = 0$
		m_2	$f_1 = 0$	$f_2 = 0$	$f_3 = 0$
	j_3	m_1	$f_1 = 0$	$f_2 = 0$	$f_3 = 0$
		m_2	$f_1 = 0$	$f_2 = 0$	$f_3 = 0$
c_1t_2 (8000 units)	j_1	m_1	$f_1 = 0$	$f_2 = 0$	$f_3 = 0$
		m_2	$f_1 = 5000$	$f_2 = 0$	$f_3 = 0$
	j_2	m_1	$f_1 = 0$	$f_2 = 0$	$f_3 = 0$
		m_2	$f_1 = 3000$	$f_2 = 0$	$f_3 = 0$
	j_3	m_1	$f_1 = 0$	$f_2 = 0$	$f_3 = 0$
		m_2	$f_1 = 0$	$f_2 = 0$	$f_3 = 0$
c_1t_3 (6000 units)	j_1	m_1	$f_1 = 0$	$f_2 = 0$	$f_3 = 0$
		m_2	$f_1 = 5000$	$f_2 = 0$	$f_3 = 0$
	j_2	m_1	$f_1 = 0$	$f_2 = 0$	$f_3 = 0$
		m_2	$f_1 = 1000$	$f_2 = 0$	$f_3 = 0$
	j_3	m_1	$f_1 = 0$	$f_2 = 0$	$f_3 = 0$
		m_2	$f_1 = 0$	$f_2 = 0$	$f_3 = 0$

Table A31:Model 1: Production Correlation of Product Families and Products

		n_1	n_2	n_3	n_4
f_1t_1	Available	m_1 = 9000		m_2= 0	
	Production	n_1 = 6000	n_2 = 3000	n_3 = 0	n_4 = 0
f_2t_1	Available	m_1 = 500		m_2= 8000	
	Production	n_1 = 500	n_2 = 0	n_3 = 0	n_4 = 8000
f_3t_1	Available	m_1 = 0		m_2= 9000	
	Production	n_1 = 0	n_2 = 0	6000	3000
f_1t_2	Available	m_1 = 0		m_2= 0	
	Production	n_1 = 0	n_2 = 0	n_3 = 0	n_4 = 0
f_2t_2	Available	m_1 = 0		m_2= 8000	
	Production	n_1 = 0	n_2 = 0	n_3 = 8000	n_4 = 0
f_3t_2	Available	m_1 = 7000		m_2= 0	
	Production	n_1 = 4000	n_2 = 3000	n_3 = 0	n_4 = 0
f_1t_3	Available	m_1 = 0		m_2= 6000	
	Production	n_1 = 0	n_2 = 0	n_3 = 1000	n_4 = 5000
f_2t_3	Available	m_1 = 4000		m_2= 0	
	Production	n_1 = 0	n_2 = 4000	n_3 = 0	n_4 = 0
f_3t_3	Available	m_1 = 5000		m_2= 0	
	Production	n_1 = 5000	n_2 = 0	n_3 = 0	n_4 = 1500

Table A32: Demand vs. Shipment for Model 1

		n_1			n_2			n_3			n_4		
$l_1 t_1$	Demand	2000			1500			500			1000		
	Shipment f_1, f_2, f_3	2000	0	0	1500	0	0	0	0	500	0	0	1000
$l_2 t_1$	Demand	2000			1000			1500			500		
	Shipment f_1, f_2, f_3	1500	500	0	1000	0	0	0	0	1500	0	0	500
$l_3 t_1$	Demand	2500			500			1000			1500		
	Shipment f_1, f_2, f_3	2500	0	0	500	0	0	0	0	1000	0	0	1500
Production	f_1, f_2, f_3	6000	500	0	3000	0	0	0	0	6000	0	8000	3000
$l_1 t_2$	Demand	1000			1500			500			2000		
	Shipment f_1, f_2, f_3	0	0	1000	0	0	1500	0	0	500	0	2000	0
$l_2 t_2$	Demand	1500			1000			1500			2500		
	Shipment f_1, f_2, f_3	0	0	1500	0	0	1000	0	0	1500	0	2500	0
$l_3 t_2$	Demand	1500			500			1000			3500		
	Shipment f_1, f_2, f_3	0	0	1500	0	0	500	0	0	1000	0	3500	0
Production	f_1, f_2, f_3	0	0	4000	0	0	3000	0	8000	0	0	0	0
$l_1 t_3$	Demand	2000			2500			3500			2000		
	Shipment f_1, f_2, f_3	0	0	2000	0	2500	0	1000	2500	0	2000	0	0
$l_2 t_3$	Demand	1500			1000			2500			1500		
	Shipment f_1, f_2, f_3	0	0	1500	0	1000	0	0	2500	0	1500	0	0
$l_3 t_3$	Demand	1500			500			3000			1500		
	Shipment f_1, f_2, f_3	0	0	1500	0	500	0	0	3000	0	1500	0	0
Production	f_1, f_2, f_3	0	0	5000	0	4000	0	1000	0	0	5000	0	0

Table A33: Demand vs. Shipment for Model 2

		n_1			n_2			n_3			n_4		
$l_1 t_1$	Demand	2000			1500			500			1000		
	Shipment f_1, f_2, f_3	1500	500	0	1500	0	0	0	0	500	0	0	1000
$l_2 t_1$	Demand	2000			1000			1500			500		
	Shipment	2000	0	0	1000	0	0	0	0	1500	0	0	500
$l_3 t_1$	Demand	2500			500			1000			1500		
	Shipment f_1, f_2, f_3	2500	0	0	500	0	0	0	0	1000	0	0	1500
Production	f_1, f_2, f_3	6000	500	0	3000	0	0	0	0	6000	0	8000	3000
$l_1 t_2$	Demand	1000			1500			500			2000		
	Shipment f_1, f_2, f_3	0	0	1000	0	0	1500	0	0	500	0	2000	0
$l_2 t_2$	Demand	1500			1000			1500			2500		
	Shipment f_1, f_2, f_3	0	0	1500	0	0	1000	0	0	1500	0	2500	0
$l_3 t_2$	Demand	1500			500			1000			3500		
	Shipment f_1, f_2, f_3	0	0	1500	0	0	500	0	0	1000	0	3500	0
Production	f_1, f_2, f_3	0	0	4000	0	0	3000	8000	0	0	0	0	0
$l_1 t_3$	Demand	2000			2500			3500			2000		
	Shipment f_1, f_2, f_3	0	2000	0	0	0	2500	3500	0	0	2000	0	0
$l_2 t_3$	Demand	1500			1000			2500			1500		
	Shipment f_1, f_2, f_3	0	1500	0	0	0	1000	2500	0	0	1500	0	0
$l_3 t_3$	Demand	1500			500			3000			1500		
	Shipment f_1, f_2, f_3	0	1500	0	0	0	500	3000	0	0	1500	0	0
Production	f_1, f_2, f_3	0	5000	0	0	0	4000	1000	0	0	5000	0	0

Table A34: Sensitivity Analysis on Inventory Cost at RMP

Incremental Percentage	Revenue	Total Cost	Profit	Integrated Plan
0%	4100800	623480	3477320	Basic Plan
+10%	4100800	623480	3477320	Integrated Plan Changed
-10%	4100800	621370	3479430	Integrated Plan Changed

Table A35: Sensitivity Analysis on Inventory Cost at FP

Incremental Percentage	Revenue	Total Cost	Profit	Integrated Plan Changed or Not Changed
0%	4100800	699500	3401300	Basic Plan
+10%	4100800	700150	3400650	Integrated Plan Changed
-10%	4100800	698070	3402730	Integrated Plan Changed

Table A36: Sensitivity Analysis Setup Cost at RMP

Incremental Percentage	Revenue	Total Cost	Profit	Integrated Plan Changed or Not Changed
0%	4100800	1662420	2438380	Basic Plan
+20%	4100800	1662420	2438380	Integrated Plan Changed
-20%	4100800	1662370	2438430	Integrated Plan Changed

Table A37: Setup Cost at FP, Sensitivity Analysis

Incremental Percentage	Revenue	Total Cost	Profit	Integrated Plan Changed or Not Changed
0%	4100800	1662420	2438380	Basic Plan
+20%	4100800	1666825	2433975	Integrated Plan Changed
-20%	4100800	1666550	2434250	Integrated Plan Changed

Table A38: Raw Material Purchase Price Sensitivity

Incremental Percentage	Revenue	Total Cost	Profit	Integrated Plan Changed or Not Changed
0%	4100800	1666550	2434250	Basic Plan
+10%	4100800	1683700	2417100	Integrated Plan Changed
-10%	4100800	1662550	2438250	Integrated Plan Changed

Table A39: Raw Material Transportation Price Sensitivity

Incremental Percentage	Revenue	Total Cost	Profit	Integrated Plan Changed or Not Changed
0%	4100800	1666550	2434250	Basic Plan
+50%	4100800	1676550	2424250	Integrated Plan Changed
-50%	4100800	1662440	2438360	Integrated Plan Changed

Table A40: Finished Products Transportation Sensitivity

Incremental Percentage	Revenue	Total Cost	Profit	Integrated Plan Changed or Not Changed
0%	4100800	1666550	2434250	Basic Plan
+50%	4100800	1666970	2433830	Integrated Plan Changed
-50%	4100800	1665380	2435420	Integrated Plan Not Changed