Compression of a Thin Layer Overlying Deep Soil Deposit

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ABSTRACT

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Roads constitute the largest and most expensive project governments undertake. The deterioration of the infrastructure of these roads represents a major and outstanding problem in transportation engineering. Construction of roads is usually made by stripping the top soil (600 to 1000 mm), which often contains organic materials, and replacing it with a layer of subgrade material (crushed stones, well-graded sand). A thin layer of asphalt or concrete is usually placed on the top of the subgrade layer to provide a durable surface.

This thesis examines the role of a deep deposit on the compression of the overlain subgrade layer. The object of this study is to provide a practical method of analysis for the design of airport runways. The cross-anisotropic elastic body that is characterized by three independent elastic constant with a plane of isotropy is suggested as an improved mathematical model of natural soil deposit. The theory of stresses and displacements in a two-layer system is presented in accordance with the theory of elasticity. The theory present herein reveals the controlling influence of two important ratios on the load-settlement characteristics of the "two-layer system," namely; the ratio of the thickness of the upper layer to the radius of the bearing area and the ratio of the modulus of the deposit to that of the upper layer. For practical design purposes, the theoretical results of settlement and compression of the upper layer have been evaluated numerically and expressed in basic influence curves, for rough and smooth interfaces at the center and the

edge of the load. These influence curves are made for a various combination of anisotropic and isotropic two-layer system. The influence curves of the compression of a thin layer overlying deep deposit, confirm that the stiffness of the lower layer has a significant influence on the compression of the upper layer.

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LIST OF SYMBOLS

The following symbols are used

a	Radius of uniform circular load
P_0	Uniformly distributed load
P	Point load
E_i	Modulus of elasticity of the layers where i =1, 2
ν_i	Poisson's ratio of the layers where i =1, 2
(r,θ,z)	Cylindrical coordinates.
$J\nu(mr)$	Bessel function of the first kind of order ν, where ν = 0, 1, 2...
m	Dimensionless parameter of Bessel function expansion
$\partial/\partial z$	First partial derivate
$\partial^2/\partial z^2$	Second partial derivate
$\phi(r,z)$	Love's potential strain function
∇^2	Laplace's operator
A,B,C,D	Constants depending on boundary condition
W_i	Settlement (normal displacement) of the surface of the layers where i =1, 2
U_i	Horizontal displacement of the surface of the layers where i =1, 2
σ_{zi}	Normal stress of the surface of the layers where i =1, 2
σ_{ri}	Radial stress of the surface of the layers where i =1, 2
Trz_i	Shearing stress of the surface of the layers where i =1, 2

σ_{zm}	Normal stress in terms of the function $\phi_0(z,m)$
W_m	Settlement in terms of the function $\phi_0(z,m)$
n	Degree of anisotropy
S	Ratio of anisotropy
Γ	Gamma function
K_p	Passive earth pressure coefficient
K_a	Active earth pressure
φ	Angle of internal friction

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CHAPTER 1

INTRODUCTION

1.1 General:

The problem of calculating the stresses and displacement in a layer elastic system is one which arises in engineering analysis and design, especially in the field of soil mechanics. Soils in general follow extremely complicated stress-strain-time laws. This, together with the fact that soil is rarely homogeneous, makes it difficult to predict stresses and displacements accurately. One of the main problems of soil mechanics is to predict settlements, and to accomplish this, it is necessary to accept over-simplified models of soil behavior in order to arrive at an engineering approximation.

The universally accepted mathematical model of the soil is the homogeneous isotropic elastic half-space, the solution to which was provided by Boussinesq (1885). Soil is certainly not a truly elastic material and it is necessary first to investigate the validity of such solutions based on the theory of elasticity.

1.2 Definition:

Engineering structures settle for many reasons, owing to the effect of additional loading of subsoil by a structure, lowering of the groundwater level, diverse forms of ground surface sinking (mining, sliding, subsidence, underground erosion). Several components put together to make up the magnitude of the final settlements such components are: (1) instantaneous (immediate) settlement that occurs immediately after the application of load without a change in moisture content. (2) settlement produced by the effect of primary consolidation. (3) settlement due to secondary consolidation. There are many methods used for calculating the final settlement, they form four qualitatively distinct groups of solution, the first group includes mathematically exact methods of the elasticity theory which satisfy the equilibrium conditions, the constitutive equation and the boundary conditions prescribed

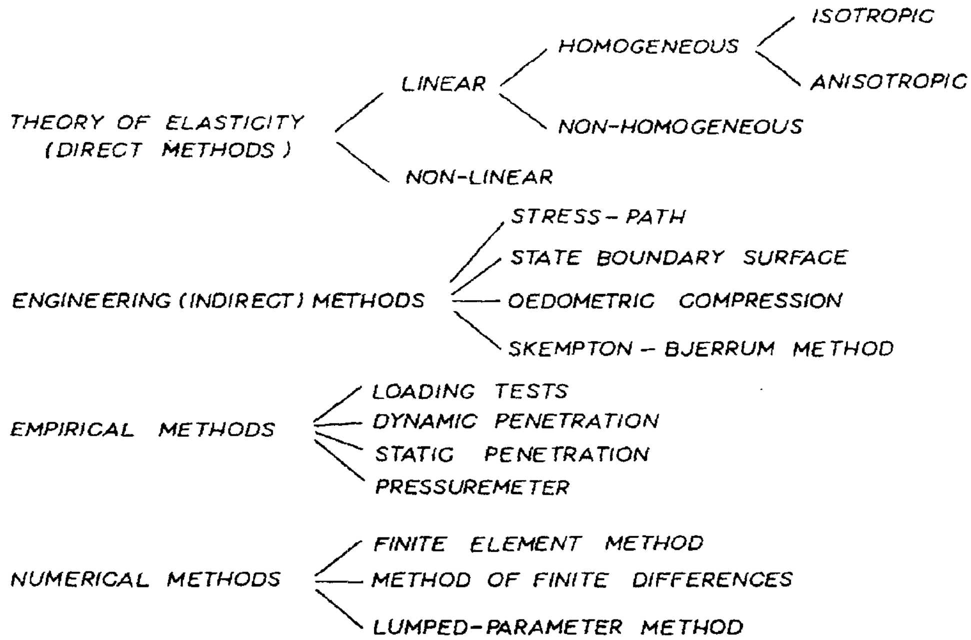

Figure 1-1 Methods of calculating the final settlement

Calculating settlement of soils which more often than not are layered in character is one which often arises in engineering analysis especially in the field of soil mechanics.

if in a two layer system the stiffness of lower layer were infinite that means a layer overlaying a bed rock, the settlement of upper layer will be equal the compression, and the total load would be used to compress the upper layer.

However if hypothetically the stiffness of the lower layer were zero the upper layer will sink downward as an elevator without any compression and the settlement of the system would be from lower layer only. Between these two extremely cases part of the load will be consumed to compress the upper layer and the rest to settle the lower layer.

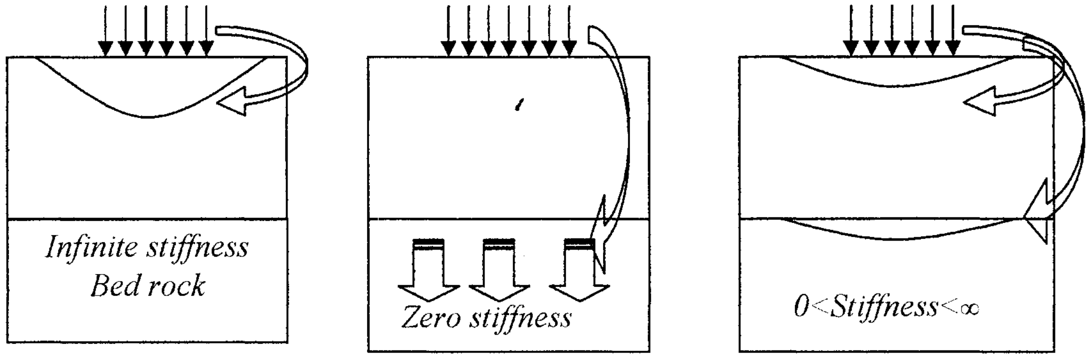

Figure 1-2 Compression of upper layer.

1.3 Anisotropic elastic material:

A comprehensive experimental investigation of the distribution of stresses and displacements for remolded clays, by the U.S. Army Engineers Waterways Experiment Station (1958) show that for clay there is remarkable agreement between the experimentally measured values of $\sigma_x, \sigma_y, \sigma_z$ and τ_{xz}, and the values computed from Boussinesq's equations. These results suggest that, in clay soils at least, computation of stresses by elastic theory is admissible. It is accepted on the basis of the limited experimental evidence that normally consolidated clays may be effectively isotropic. Many undisturbed deposits of over consolidated clay will exhibit marked anisotropy, so that such clays will possess a greater load spreading capacity than indicated by Boussinesq, it is possible that this effect contributes to the fact that observed settlements in heavily over-consolidated clays are usually considerably smaller than the predicted values. Unfortunately no direct measurements of stress distributions have been made in undisturbed clays as the placing of pressure measuring cells require an artificially prepared test bed. Such test beds, consisting of remolded clay that has been compacted rather than consolidated, will result in an over all isotropy. Moreover; experiments show that the vertical stress in sand is greater in the region of the load than indicated by Boussinesq (U. S. Army Engineering 1954), anisotropy is not likely to be the entire explanation, however, as in sand compressibility decrease with depth that this effect can also cause an increase in the intensity of vertical stress near the load. The idea of using the theory of cross –anisotropic for the foundation soil stems from the concept that the soils have been formed by sedimentation. This concept implies that the mechanical properties of soil in vertical direction should differ from those of horizontal direction (all horizontal directions are being equivalent to one another).

1.4 Research objectives:

This study has the following objectives

1- Introduce the cross-anisotropic elastic body that is characterized by three independent elastic constants as an improved mathematical model of natural soil deposit.

2- Examine the role of deep deposit on the compression of the overlain subgrade layer.

3- Reveal some of the fundamental relations existing between the physical factors, which control the load-settlement relations, and provide a practical method of analysis for the design of airport runways.

CHAPTER 2

LITERATURE REVIEW

2.1 GENERAL

In foundation engineering, designers must deal with real soils in natural deposits, which, more often are layered in character. Several solutions for surface loading conditions associated with multilayered elastic solids, of infinite lateral extent, are already available in existing literature. The boundaries between the individual elastic layers are usually assumed to be either perfectly continuous or completely smooth. However, most of the numerical solutions for stresses and displacements have been done for the purpose of airport design, where the upper layer is stiffer than the lower layer. None of the numerical solutions has dealt with the opposite case where the lower layer is stiffer than the upper layer except for Ueshita and Meyerhof (1967). This case could be encountered in field compaction.

Furthermore; all these solutions have dealt with isotropic elastic solids, where experiments show that overconsolidated clay possesses a greater load spreading capacity than indicated by Boussinesq. This may well contribute to the fact that the predicted settlements in such soil usually exceed the observed values. Also the observed stress distributions in sand are partly a result of anisotropy.

2.2 REVIEW OF PREVIOUS WORK

Burmister, (1943)

Burmister developed the theory of stresses and displacements in a two-layer system in accordance with the methods of the mathematical theory of elasticity. The necessary assumptions of the theory of elasticity were made that the soils of each of the two layers are homogeneous, isotropic, elastic materials, for which Hooke's law is valid. Moreover the surface of layer 1 is assumed to be weightless and to be infinite in extent in the horizontal direction, but of finite thickness h, and must be free of normal and shearing stresses outside the limits of the loaded area. Layer 2 is assumed to be infinite in extent both horizontally and vertically downward, and the stresses and displacements must be equal to zero. It is assumed that the two layers are continuously in contact and act together as an elastic medium of composite nature. Continuity requires that the normal and shearing stresses and the vertical and horizontal displacements must be equal in the two layers at the interface. The theory reveals some of the fundamental relations existing between the physical factors, which control the load-settlement relations. The theoretical results are evaluated numerically and expressed in basic influence curves, giving values of the settlement coefficient F, in terms of the basic ratios that control the load-settlement relations, at the center of a circular flexible bearing area, (Figure 2.1)

Ueshita, and Meyerhof, (1967)

Ueshita and Meyerhof evaluated the surface displacement of an elastic layer on a rigid base (a soil-rock system) under uniformly loaded areas of various shapes. They also evaluate the displacement of a two-layer elastic system, where the upper layer is more compressible than the lower layer, under a uniformly loaded circular area. The surface displacement is computed according to a rigorous solution of the theory of elasticity. Concerning a two-layer elastic system, Burmister computed the displacement factor of the system where the upper layer is stiffer than the lower layer.

Ueshita and Meyerhof evaluated the surface displacement factor, for the center and the edge of a loaded circular area on a two-layer system for the cases where E_1/E_2= 0. 01, 0.1, 0. 2 and 0. 5, assuming $\nu_1 = \nu_2 = 0.5$ (Figure 2.2).

Hoskin and Lee, (1959)

Hoskin and Lee explained in their paper that for axially symmetrical problems both in three dimensional elasticity theory and in plate theory, stress components which vary radially in proportion with the Bessel function $J_0(mr)$ where r is the radius and *m* a constant, play a particularly significant role. For elasticity theory in cylindrical coordinates the Bessel function loading distribution produces separation of the axial coordinate z and the radial coordinate r in the basic equation and so generates a simple deformation expression and simple solution. However this load function, introduces certain difficulties. $J_0(mr)$ represents a damped oscillatory load with maximum intensity at the origin. The amplitude of the oscillations decrease, to zero with increasing r in such a way that $J_0(mr)$ can be represented asymptotically for a large values of r by:

$$J_0(mr) \approx \sqrt{\frac{2}{\pi mr}} \cos(mr - \frac{\pi}{4}) \tag{2.1}$$

This ensures that all stress components approach zero as r increases, so. However the total load applied on the surface is not defined.

The total load p within an arbitrary radius $r = b$ is given by

$$P = P_0 \int_0^b \int_0^{2\pi} J_0(mr) r dr d\theta = \frac{2\pi b p_0}{m} J_1(mb) \approx 2 p_0 \sqrt{\frac{2\pi b}{m^3}} \cos(mb - \frac{3\pi}{4}) \tag{2.2}$$

As b increases the value p given by equation (2.2) of oscillates with an amplitude that tend to infinity so that the total load is not defined. This difficulty can be overcome by

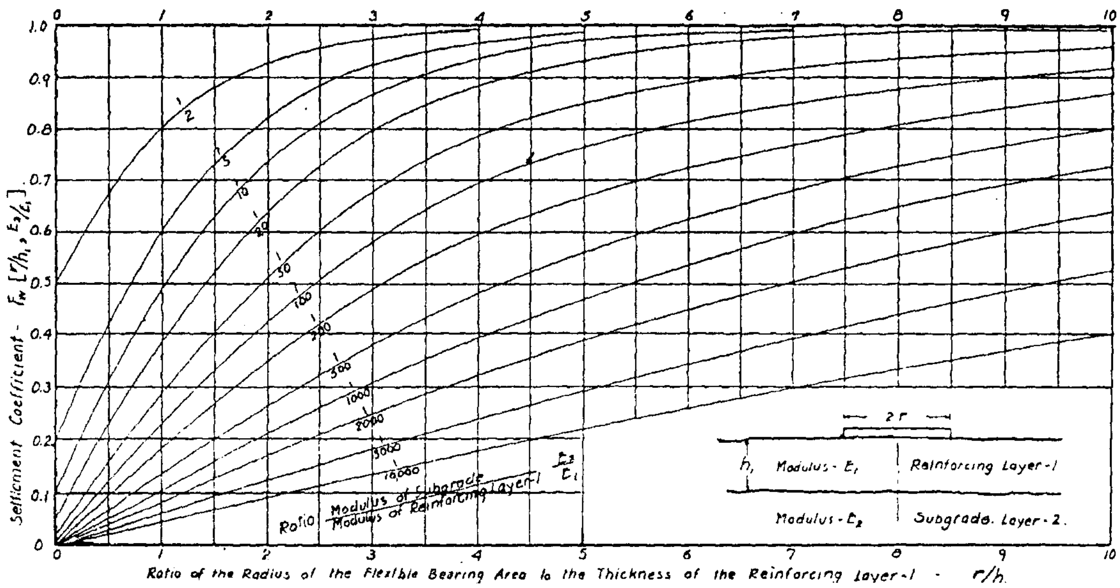

Figure 2-1 Factor of settlement of the upper the layer for Poisson's ratio $\nu_1 = \nu_2 = 0.5$

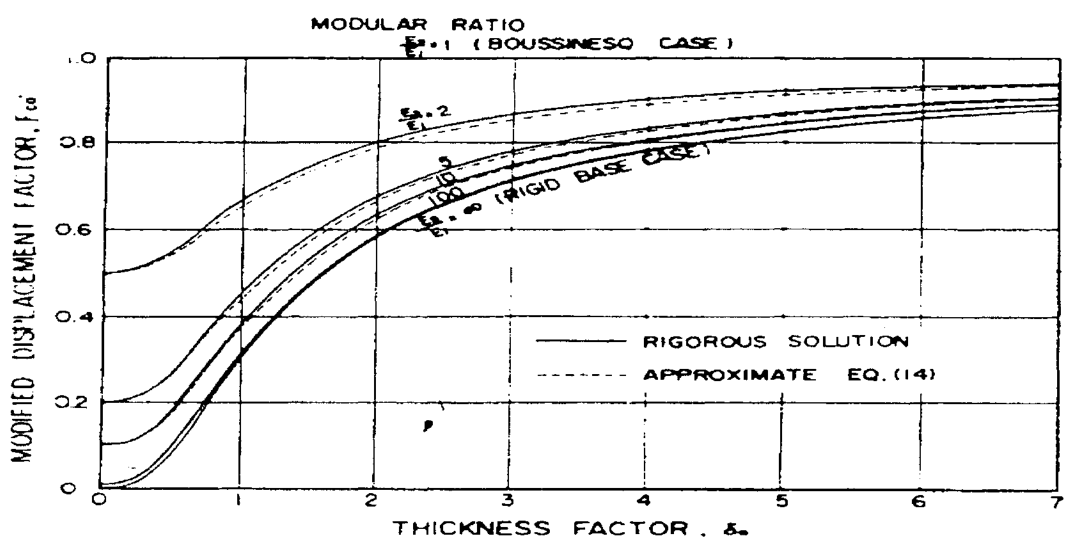

Figure 2-2 Factor of settlement of the upper layer on a stiffer elastic layer, at the center of a flexible circular load, for Poisson's ratio $\nu_1 = \nu_2 = 0.5$

considering an integral of terms of this type over varying m, for example; a distribution of load that is of uniform intensity p_0 over an area of radius a, but which is zero outside this area can be represented in the form

$$P(r) = P_0 a \int_0^{\infty} J_0(mr) J_1(ma)\, dm \tag{2.3}$$

Since r appears only in the term $J_0(mr)$ the analysis detailed above for deflection of a flexible surface supported by certain types of foundation, can be applied directly to the integrand, resulting in an infinite integral over m.

In general an arbitrary load distribution p(r) can be represented in the form of a

Fourier-Bessel integral $$P(r) = \int_0^{\infty} p^*(m) j_0(mr)\, dm \tag{2.4}$$

Where $$p^*(m) = m \int_0^{\infty} n p(n) j_0(mn)\, dn \tag{2.5}$$

Expressions for forms of surface loading can be obtained by considering the general Fourier-Bessel integral, for p(r). Each surface loading is characterized by the expression for $p^*(m)$,

(i) For a uniform circular load of stress intensity p_o and radius a

$$p^*(m) = p_0 a j_1(ma) \tag{2.6}$$

(ii) For a concentrated force P applied at the origin

$$p^*(m) = \frac{pm}{2\pi} \tag{2.7}$$

(iii) For a parabolic loading, with maximum stress p_o, acting over a circular area of radius a

$$p^*(m) = \frac{2p_0}{m} j_2(ma) \tag{2.8}$$

(iv) For a hemispherical loading, with maximum stress p_o acting over a circular area of radius a

$$p^*(m) = p_0 a \left(\frac{\pi}{2ma}\right)^{\frac{1}{2}} j_{\frac{3}{2}}(ma) \tag{2.9}$$

Cauwelaert, (1977)

Cauwelaert shows that the anisotropic materials with a plane of isotropy, characterized by five independent elastic constants, can quite probably be reduced to three fundamental constants as follows, in the case of an anisotropic body with a plane of isotropic the relation given in equation of Hooke's law,

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 \\ a_{12} & a_{22} & a_{13} & 0 & 0 & 0 \\ a_{13} & a_{13} & a_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{pmatrix} * \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{pmatrix} \tag{2.10}$$

may be reduced to contain five independent terms $a_{11}, a_{12}, a_{13}, a_{22}, a_{33}, and a_{44}$. These independent terms may be expressed by way of the following technical constants, also shown in Figure (2.3)

$$a_{11} = a_{22} = \frac{1}{\frac{E}{n}} = \frac{n}{E} \qquad a_{12} = -\frac{\nu}{\frac{E}{n}} = \frac{n\nu}{E} \qquad a_{13} = -\frac{\frac{\mu}{n}}{\frac{E}{n}} = \frac{\mu}{E}$$

$$a_{33} = \frac{1}{E} \qquad a_{44} = a_{55} = \frac{1}{G_{yz}} = \frac{1}{G_{xz}} \qquad a_{66} = 2*(a_{11} - a_{12}) = \frac{2*n*(1+\nu)}{E} \tag{2.11}$$

In which $E_z = E$ = elastic modulus along axis of symmetry; $E_x = E_y = E/n$ elastic modulus in isotropic plane, $E_z / E_x = n$ = degree of anisotropy; μ = Poisson's ratio defining strain induced in isotropic plane by stress applied along the axis of symmetry z, μ/n = Poisson's ratio defining strain induced along axis of symmetry z by a stress applied along one of the axes x or y, ν = Poisson's ratio in isotropic plane (*xy*); and G_{yz} and G_{xz} = shear moduli in anisotropic planes (*yz* and *xz*).

By transformation of the coordinate axes (a rotation about the y-axis in this case), the number of independent technical constants could logically be reduced to three.

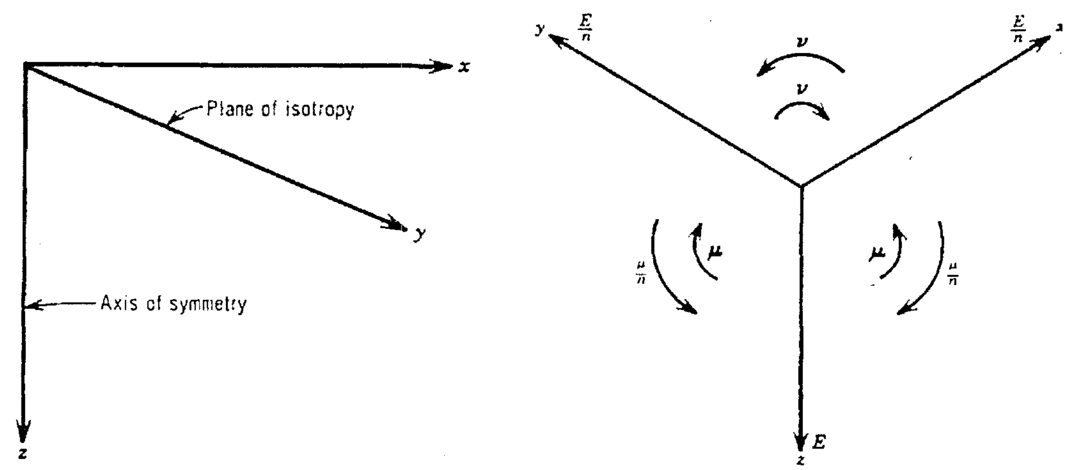

Figure 2-3 Technical constants of axisymmetric anisotropic body with plane of isotropy

The values obtained after transformation are

$$\frac{1}{G_{xz}} = \frac{1+n+2\mu}{E} \qquad \nu = \frac{\mu}{n} \tag{2.12}$$

The coefficients reduced to three independent coefficients namely

1. E : Young's modulus along the axis perpendicular to the plane of isotropy.

2. n: The degree of anisotropy ($n = E_z / E_x = E_z / E_y$)

3. μ : Poisson's ratio in an anisotropic plane.

Generally these three values can easily be determined in a laboratory.

Cauwelaert shows as an application to problems with axial symmetry that the stress equation becomes.

$$\sigma_z = \frac{\partial}{\partial z}\left[\left(n^2 + \mu n + n - \mu^2\right)\nabla^2\Phi - n(1+\mu)\frac{\partial^2\Phi}{\partial z^2}\right] \qquad \tau_{rz} = \frac{\partial}{\partial z}\left[\left(n - \mu^2\right)\nabla^2\Phi - n(1+\mu)\frac{\partial^2\Phi}{\partial z^2}\right],$$

$$\sigma_r = \frac{\partial}{\partial z}\left[\mu(n+\mu)\nabla^2\Phi - n(1+\mu)\frac{\partial^2\Phi}{\partial r^2}\right] \qquad \sigma_\theta = \frac{\partial}{\partial z}\left[\mu(n+\mu)\nabla^2\Phi - n(1+\mu)\frac{1}{r}\frac{\partial\Phi}{\partial r}\right] \tag{2.13}$$

In which the Φ function has to satisfy $\nabla^2\nabla_1^2\Phi = 0$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \quad \text{, and} \qquad \nabla_1^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{s^2}\frac{\partial^2}{\partial z^2} \qquad (2.14)$$

With s (the ratio of isotropy) $= \left(\frac{n - \mu^2}{n^2 - \mu^2}\right)^{\frac{1}{2}}$ (2.15)

The displacements are

$$U = \frac{n(n+\mu)(1+\mu)}{E}\frac{\partial^2\Phi}{\partial r\partial z} \qquad W = \frac{1}{E}\left[(1+n+2\mu)(n-\mu^2)\nabla^2\Phi - n(1+\mu)^2\frac{\partial^2\Phi}{\partial z^2}\right] \qquad (2.16)$$

Obviously; all these relations can be reduced to the relations given by Love (1927) in the case of an isotropic body (n = 1), with the exception of one factor $(1+\mu)$

The compatibility equation, $\nabla^2\nabla_1^2\Phi = 0$, is satisfied for

$$\Phi = J_0(mr)(Ae^{mz} - Be^{-mz} + Ce^{msz} - De^{-msz}) \qquad (2.17)$$

It is interesting to note that n will always be greater than μ, as s must remain real and μ cannot be greater than unit. In the case of anisotropic materials, μ can be greater than 0.5

The stress and displacement expressions then become.

$$\sigma_z = -mJ_0(mr)\left[n(1+\mu)(Am^2e^{mz} + Bm^2e^{-mz}) + ns(n+\mu)(Cm^2e^{msz} + Dm^2e^{-msz})\right] \qquad (2.18)$$

$$\tau_{rz} = mJ_1(mr)\left[n(1+\mu)(Am^2e^{mz} - Bm^2e^{-mz}) + ns^2(n+\mu)(Cm^2e^{msz} - Dm^2e^{-msz})\right] \qquad (2.19)$$

$$U = \frac{1+\mu}{E}n(n+\mu)J_1(mr)\left[Am^2e^{mz} + Bm^2e^{-mz} + sCm^2e^{msz} + sDm^2e^{-msz}\right] \qquad (2.20)$$

$$W = -\frac{1+\mu}{E}J_0(mr)\left[n(1+\mu)(Am^2e^{mz} - Bm^2e^{-mz}) + ns^2\frac{(n+\mu)^2}{(1+\mu)}(Cm^2e^{msz} + Dm^2e^{-msz})\right] \qquad (2.21)$$

CAUWELAERT and CERISIER, (1982)

Cauwelaret and Ceriser studied the relation between mechanical and geotechnical properties of an anisotropic semi-infinite body with a plane of isotropy submitted to a uniform circular load. In the Prandtl failure model, the limit of equilibrium under a continuous footing (Figure 2-4) is reached when the stresses along a slip line (OA or OB) satisfy Coulomb's criterion $\tau = \sigma tg\varphi$, where φ is the angle of internal friction.

The real appearance of the slip lines underneath the footing is debatable. Failure starts however in A and B under a well specified direction.

The mechanical properties for the case of cylindrical coordinate (Figure 2-5) are given by

$$E\theta = Er = \frac{E}{n}, \qquad Ez = E$$

$$\mu_{rz} = \frac{\mu}{n}, \qquad \mu_{zr} = \mu, \qquad \mu_{r\theta} = \mu_{\theta r} = \frac{\mu}{n}, \tag{2.22}$$

$$\frac{1}{Grz} = \frac{1+n+2\mu}{E}$$

The stress function and stresses are

$$\Phi = J_0(mr)(Ae^{mz} - Be^{-mz} + Ce^{msz} - De^{-msz}) \tag{2.23}$$

$$\sigma_z = -mJ_0(mr)\left[n(1+\mu)(Am^2e^{mz} + Bm^2e^{-mz}) + ns(n+\mu)(Cm^2e^{msz} + Dm^2e^{-msz})\right] \tag{2.24}$$

$$\tau_{rz} = mJ_1(mr)\left[n(1+\mu)(Am^2e^{mz} - Bm^2e^{-mz}) + ns^2(n+\mu)(Cm^2e^{msz} - Dm^2e^{-msz})\right] \tag{2.25}$$

$$\sigma_r = mJ_0(mr)\left[n(1+\mu)(Am^2e^{mz} + Bm^2e^{-mz}) + \frac{ns(n-\mu^2)}{(n+\mu)}(Cm^2e^{msz} + Dm^2e^{-msz})\right] - \frac{J_1(mr)}{r}n(1+\mu)\left[Am^2e^{mz} + Bm^2e^{-mz} + sCm^2e^{msz} + sDm^2e^{-msz}\right] \tag{2.26}$$

using Hankel transform

$$F = -P_o a\int_0^\infty \frac{J_1(ma)}{m}F(m)dm \,. \tag{2.27}$$

Where F(m) is one of the aforementioned stresses.

One can obtain for a homogeneous deposit for r=0 and z=0

$$\sigma_r = \frac{P\mu(1+s)}{2n} \qquad \sigma_z = \frac{P}{2}, \qquad \tau_{rz} = 0 \tag{2.28}$$

Transforming the stresses in A from (z, r)'to (z', r'), one obtains

$$\sigma'_r = \frac{P}{2}\left[\mu\frac{(1+s)}{n}\cos^2\alpha + \sin^2\alpha\right], \qquad \sigma'_z = \frac{P}{2}\left[\mu\frac{(1+s)}{n}\sin^2\alpha + \cos^2\alpha\right] \tag{2.29}$$

$$\tau'_{rz} = \frac{P}{2}\left[(1-\mu)\frac{(1+s)}{n}\right]\sin\alpha\cos\alpha$$

for the case of non cohesive soil $\tau'_{rz} = \sigma'_z\, tg\varphi$

$\frac{\mu(1+s)}{n} = \frac{tg(\alpha-\varphi)}{tg\alpha}$ If, like Prandtl, we take $\alpha = \frac{\pi}{4} + \frac{\varphi}{2}$

$$\frac{\mu(1+s)}{n} = \frac{tg(\pi/4-\varphi/2)}{tg(\pi/4+\varphi/2)} = Ka$$

and thus $$\frac{n^2}{\mu^2(1+s)^2} = \frac{tg^2(\pi/4+\varphi/2)}{tg^2(\pi/4-\varphi/2)} = \frac{Kp}{Ka} \tag{2.30}$$

Where Kp is the passive earth pressure coefficient and Ka the active earth pressure coefficient. In the case of cohesive soil

$$\frac{\mu(1+s)}{n} = [\frac{tg(\alpha-\varphi)}{tg\alpha}][1 - \frac{C}{tg\alpha - tg\varphi}] \tag{2.31}$$

Plate bearing tests allow very easily to determine the Young modulus, Poisson's ratio and the degree of anisotropy, as follows. The elastic deflection on top of a semi-infinite anisotropic body is given by

$W = \frac{1+\nu}{E} * \frac{-s(1-k)}{(1-s)} * p_0 a$. Where $K = \frac{n+\nu}{1+\nu}$, therefore

$W = \frac{s(1-n)}{(1-s)} * \frac{p_0 a}{E_1}$, thus $\frac{W}{P_0 a} = \frac{s(1-n)}{E_1(1-s)}$

This equation gives us a relationship between $\frac{W}{P_0 a}, E \; and \; n$

Measurements realized with 2 plates of different diameters allow then to determine by succesive approximations the values of E and n, and thus of ν. A test realized with another plate, or another load, permits to verification of the validity of the results. Then the value of ν can be obtained from equation (2.30)

The geotechnical parameters ϕ and C can be determined by the classic shear test.

Veverka, (1973) has realized a large scale experiment on different granular materials placed in a trial pit on a Winkler foundation (K = 7.8kg/cm^2). He measured the vertical stresses (within and beyond the axis of the load) at the interface and the deflections at the surface (in the axis of a circular load). He determined the modulus E_1 of the granular materials in a "Geonor" type triaxial cell. He recorded the concentration of the vertical stresses in the axis of the load for all the materials. Cauwelaert and Cerisier have treated the problem theoretically as a two-layer with a frictionless interface. The first layer is anisotropic with a vertical modulus E_1, and the second is assimilated with a semi-infinite isotropic body with modulus E_2. Concerning the geotechnical aspect, they have analyzed in greater detail the cases where the thickness of the granular layer was sufficient in comparison with the diameter of the load to allow a full development of Prandtl's failure model in the material.

Using Winkler's relation σ_z = Kw, they have then computed the value of E_1 taking into account a relation, established by Zimpfer, between deflections under rigid and flexible plates : w(rigid) = (1.18/1.50) w(flexible). The results of the computations are given in Table (2-1).

Table1 (2-1) Computation of Young moduli out of geotechnical properties

MATERIAL :	Porosity (%)	Watercontent (%)	Angle of int. friction	Cohesion (Kg/cm²)	Thickness (cm)	Radius of the load (cm)	Applied load (Kg/cm²)	Observed vert. stress (Kg/cm²)	Observed surf. deflection (cm)	Measured E_1 (Kg/cm²)	Computed E_1/E_2	Computed n	Computed E_2 (Kg/cm²)	Computed ν
1	58.8	6.30	30.0	0.07	60	15.5	2	12.9	0.053	1940	4.15	3.15	1640	.64
1	58.8	6.30	30.0	0.07	45	15.5	3	22.7	0.087	1940	2.4	2.5	750	.50
2	83.0	4.75	37.5	0.13	60.	15.5	6	11.9	0.113	2420	8.4	4.6	3370	.75
2	83.0	4.75	37.5	0.13	60	22.0	6	22.2	0.186	2420	7.4	4.8	3170	.79
2	83.0	4.75	37.5	0.13	45	15.5	6	16.9	0.130	2420	5.3	2.8	1850	.41
3	80.4	0.00	44.0	0.09	45	15.5	3	13.4	0.062	4220	7.1	2.4	2750	.37
4	81.5	0.00	39.0	0.11	45	15.5	6	17.4	0.123	5450	16.3	7.1	5465	.50

Material: 1) Natural sand 0/1mm

2) Natural gravel 0/40mm

3) Crushed stone 40/63mm + natural sand

4) Crushed stone 40/63mm + crushed sand.

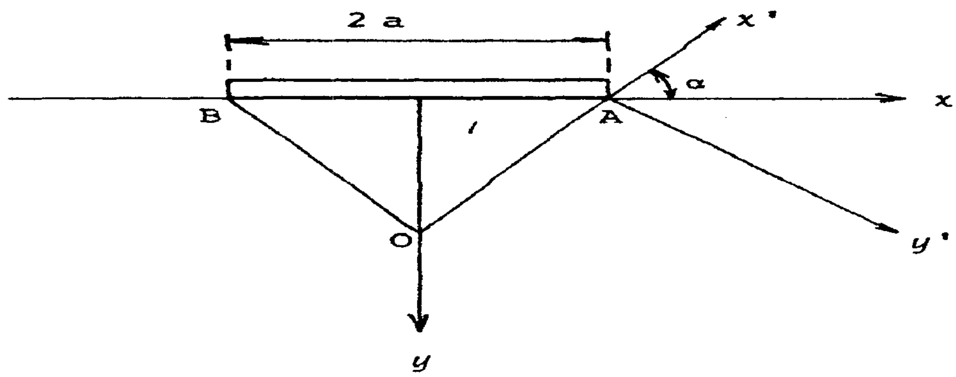

Figure (2-4) Prandtl's failure model

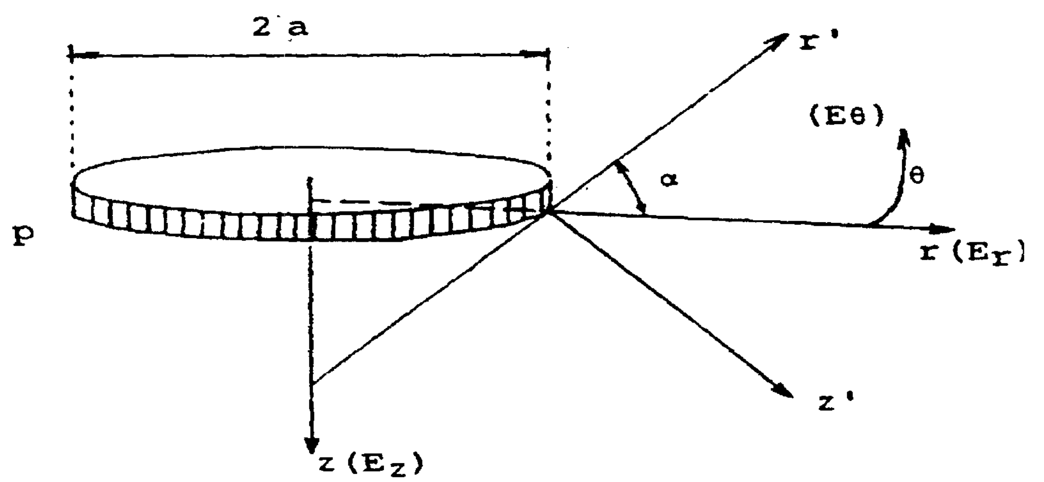

Figure (2-5) Failure model for a circular load

CHAPTER 3

THEORETICAL ANALYSES

3.1 General

The linear theory of elasticity has enjoyed a fairly long and profitable history in the field of foundation engineering. Geotechnical engineers have turned to elasticity for answers to a variety of questions, and despite the sure knowledge that the answers are at best approximate, they will continue to do so for some time. The reasons for this lie in the essential simplicity of the relevant elastic solutions. This can be a significant advantage when compared with the time and effort involved in obtaining numerical solutions that employ one of the multitudes of existing plasticity models for soil.

In geotechnical engineering, particularly in airport design the engineer is dealing with real soils in natural deposits, which, more often are layered in character. Moreover; design practice for flexible pavements follows at least two radically different approaches. One of the most common methods is to base the pavement design on an empirical measure of the performance characteristics of the soil and pavement system. Such a measure can be the CBR test. The design of the pavement system is based upon the use of tables and diagrams, previously prepared on the basis of experience and regional data. Another approach to pavement design is to combine theoretical analyses with field and laboratory experiments. The field and laboratory testing are designed to determine the material properties in the pavement system. These properties make up the "numbers" which go into the theoretical analysis to provide data on predicted stresses and/or displacements.

3.2 Two-layer system of isotropic elastic solid:

The case of two-layer was considered and the theory of elasticity was applied, see Figure (3-1). In this model, the necessary assumption of the theory of elasticity and the essential boundary and continuity conditions were satisfied across the interface between the two layers. Specifically:

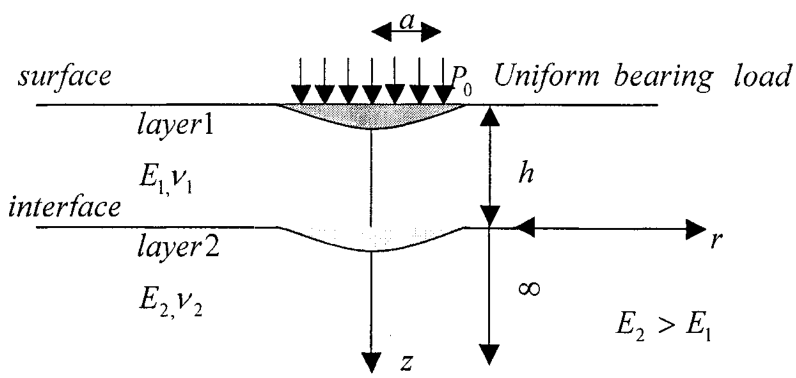

Figure (3-1) Two-Soil layer system

a) The soil of each of the two layers is assumed to be homogeneous, isotropic elastic material for which Hooke's Law is valid.

b) The surface layer no.1 is assumed to be infinite in extent in the horizontal direction, but of finite thickness h. The underlying layer no.2 is assumed to be infinite in extent in the horizontal and vertical direction.

c) The surface of layer no.1 is free of shearing and free of normal stresses outside the limits of the surface loading. Also, the stress and displacement must be equal to zero in layer no.2 at infinite depth. Therefore the boundary conditions are.

$$\sigma_{zm1} = -j_0(mr) \ldots\ldots\ 0 \le r \le a, \ldots\ z = 0$$

$$\sigma_{zm1} = 0, \ldots\ldots\ r > a, \ldots\ldots\ z = 0 \qquad (3.1)$$

$$\tau_{rz} = 0, \ldots\ldots\ 0 \le r \le \infty, \ldots\ldots\ z = 0$$

$$\sigma_{zm2} = 0, \dots\dots\dots \quad \dots\dots\dots \quad \dots\dots\dots \quad z = \infty$$

$$w_{m2} = 0, \dots\dots\dots \quad \dots\dots\dots \quad \dots\dots\dots \quad .. z = \infty$$

Furthermore

Case 1: It is assumed that the two layers are continuously in contact with shearing resistance fully active between them, so that the two layers act together as the elastic medium of composite nature with full continuity of stress and displacement across the interface between the layers. (Rough interface, no slip occurs at the interface) Thus; (referring to Figure 3-2) the continuity conditions for rough interface

$$U_1 = U_2, \quad Trz_1 = Trz_2, \quad W_1 = W_2, \quad \sigma_{z1} = \sigma_{z2} \tag{3.2}$$

Case 2: It is assumed that the two layers of the system are continuously in contact but with a frictionless interface and a continuity of normal stress and normal displacement only. (Smooth interface the layers can slip over each other without any shear stress) Thus; (referring to Figure 3.1-2) continuity conditions for smooth Interface

$$Trz_1 = 0, \quad Trz_2 = 0, \quad W_1 = W_2, \quad \sigma_{z1} = \sigma_{z2} \tag{3.3}$$

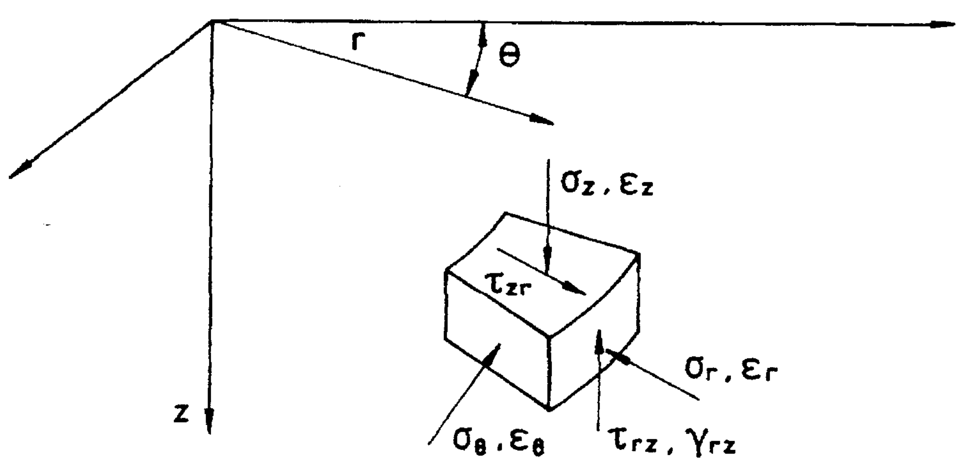

Figure (3-2) Coordinate system

The general solution of the axisymmetric problem in the isotropic theory of elasticity in the absence of body forces can be formulated in terms of Love's potential strain function $\phi(r,z)$, which satisfies the equations of equilibrium and compatibility.

1) Equations of Equilibrium: $$\frac{\partial \sigma_r}{\partial r}+\frac{\partial Trz}{\partial z}+\frac{\partial r-\partial\theta}{r}=0$$
$$\frac{\partial Trz}{\partial r}+\frac{\partial \sigma_z}{\partial z}+\frac{Trz}{r}=0 \tag{3.4}$$

2) Equations of Compatibility (2): $\nabla^4\Phi(r,z)=0$
$$\text{Where}\,\nabla^2=\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}\right] \tag{3.5}$$

is the Laplace operator referred to a system of cylindrical polar coordinates (r,θ,z).

The components of displacement vector in the r and z directions U, W, respectively ($U\theta=0$ for axial symmetry) can be expressed in terms of $\Phi(r,z)$

$$U=\frac{1+\nu}{E}\left[\frac{\partial^2\Phi}{\partial r\partial z}\right] \qquad W=\frac{1+\nu}{E}\left[2(1-\nu)\nabla^2\Phi-\frac{\partial^2\Phi}{\partial z^2}\right] \tag{3.6}$$

Where ν is Poisson's ratio and E modulus of elasticity. The non-zero components of the Cauchy stress tensor σ referred to the cylindrical polar coordinates system can be expressed as

$$\sigma_Z=\frac{\partial}{\partial z}\left[(2-\nu)\nabla^2\Phi-\frac{\partial^2\Phi}{\partial z^2}\right], \qquad \sigma_r=\frac{\partial}{\partial z}\left[\nu\nabla^2\Phi-\frac{\partial^2\Phi}{\partial r^2}\right]$$
$$\sigma_\theta=\frac{\partial}{\partial z}\left[\nu\nabla^2\Phi-\frac{1}{r}\frac{\partial\Phi}{\partial r}\right], \qquad Trz=\frac{\partial}{\partial r}\left[(1-\nu)\nabla^2\Phi-\frac{\partial^2\Phi}{\partial z^2}\right] \tag{3.7}$$

The logical approach to seek a solution to a three dimensional problem is to introduce some functions that satisfy the equilibrium equations and to use the compatibility equation to define these functions. Since the problem is one which possesses axial symmetry, it is convenient to formulate the associated elasticity problem by recourse to the theory of Hankel transforms:

$$F_\nu(m,z)=H_\nu\{f(r,z)\}=\int_0^\infty rf(r,z)J_\nu(mr)dr$$

The basic properties of the Hankel transform depend on the properties of $J\nu(mr)$.

Where $J\nu(mr)$ is the Bessel function of the first kind of order ν which is the bounded solution of the Bessel differential equation,

$$\frac{d^2\Phi}{dr^2}+\frac{1}{r}\frac{d\Phi}{r}-\frac{\nu^2}{r^2}\Phi=0$$

The Hankel transform is compatible with the Bessel differential operators in the sense that it algebraizes the variable coefficient part of the Bessel equation

$$H_\nu\left\{\frac{d^2\Phi}{dr^2}+\frac{1}{r}\frac{d\Phi}{dr}-\frac{\nu^2}{r^2}\Phi\right\}=-m^2\Phi_\nu(m)$$

So that when $\nu=0$, it is clear that the J_0 - Hankel transform is compatible with

$\frac{d^2\Phi}{dr^2}+\frac{1}{r}\frac{d\Phi}{dr}$, this means that $H_0\left\{\frac{d^2\Phi}{dr^2}+\frac{1}{r}\frac{d\Phi}{dr}\right\}=-m^2\Phi_0(m)$.

For the boundary value problem which is associated with the biharmonic equation

$\nabla^2(\nabla^2\Phi)=0$, where $\nabla^2=\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}\right]$,

The Hankel transform of $\Phi(r,z)$ is

$$\Phi_0(m,z)=H_0\{\Phi(r,z)\}=\int_0^\infty r\Phi(r,z)J_0(mr)dr$$

$$H_0=\{\nabla^4\Phi(r,z)\}=\left[\frac{d^2}{dz^2}-m^2\right]^2\Phi_0(m,z)$$

The J_0 - Hankel transform reduces the radial part of the biharmonic operator to an algebraic one. The general solution to this equation is:

$$\Phi_0(m,z)=J_0(mr)\left[Ae^{mz}-Be^{-mz}+Cze^{mz}-Dze^{-mz}\right] \tag{3.8}$$

Where A, B, C, D are arbitrary functions, and m is a dimensionless parameter which are to be determined by satisfying the boundary conditions.

The expression for displacement and stress components can be expressed in terms of the transformed strain potential $\Phi_0(m,z)$

$$W_1 = \frac{1+\nu_1}{E}\left[2(1-\nu_1)\nabla^2\Phi(r,z) - \frac{\partial^2\Phi}{\partial z^2}\right] \qquad \text{In terms of } \Phi(r,z)$$

$$W_{m1} = \frac{1+\nu_1}{E}\left[2(1-\nu_1)\left[\frac{d^2}{dz^2} - m^2\right]\Phi_0 - \frac{d^2\Phi_0}{dz^2}\right] \qquad \text{In terms of } \Phi_0(m,z)$$

$$W_{m1} = \frac{1+\nu_1}{E}\left[2\frac{d^2\Phi_0}{dz^2} - 2\nu_1\frac{d^2\Phi_0}{dz^2} - \frac{d^2\Phi_0}{dz^2} - 2(1-\nu_1)m^2\Phi_0\right]$$

$$W_{m1} = \frac{1+\nu_1}{E}\left[(1-2\nu_1)\frac{d^2\Phi_0}{dz^2} - 2(1-\nu_1)m^2\Phi_0\right]$$

$$\frac{d\Phi_0}{dz} = J_0(mr)[A_1 me^{mz} + B_1 me^{-mz} + C_1 e^{mz} + C_1 zme^{mz} - D_1 e^{-mz} + D_1 zme^{-mz}]$$

$$\frac{d^2\Phi_0}{dz} = J_0(mr)\begin{bmatrix} A_1 m^2 e^{mz} - B_1 m^2 e^{-mz} + C_1 me^{mz} + C_1 me^{mz} \\ + C_1 zm^2 e^{mz} + D_1 me^{-mz} + D_1 me^{-mz} - D_1 zm^2 e^{-mz} \end{bmatrix}$$

$$\frac{d^2\Phi_0}{dz} = J_0(mr)[A_1 m^2 e^{mz} - B_1 m^2 e^{-mz} + C_1 m(2+zm)e^{mz} + D_1 m(2-zm)e^{-mz}],$$

Substitution into W_{m1}

$$W_{m1} = \frac{1+\nu_1}{E_1} J_0(mr)\begin{bmatrix} (1-2\nu_1)A_1 m^2 e^{mz} - (1-2\nu_1)B_1 m^2 e^{-mz} + (1-2\nu_1)C_1 m(2+mz)e^{mz} \\ + (1-2\nu_1)D_1 m(2-mz)e^{-mz} - 2(1-\nu_1)A_1 m^2 e^{mz} + 2(1-\nu_1)B_1 m^2 e^{-mz} \\ - 2(1-\nu_1)C_1 m^2 ze^{mz} + 2(1-\nu_1)D_1 m^2 ze^{-mz} \end{bmatrix}$$

$$W_{m1} = -\frac{1+\nu_1}{E_1} J_0(mr)\left[A_1 m^2 e^{mz} - B_1 m^2 e^{-mz} - C_1 m(2-4\nu_1 - mz)e^{mz} - D_1 m(2-4\nu_1 + mz)e^{-mz}\right] \quad (3.9)$$

This equation is the equation of displacement in terms of strain potential $\Phi_0(m,z)$

Doing the same procedure for the rest of the displacement and stress components with the use of recurrence relation of Bessel function

$$J'_n(x) = \frac{n}{x}J_n(x) - J_{n+1}(x) \qquad J'_n(x) = J_{n-1}(x) - \frac{n}{x}J_n(x)$$

$$\sigma_{zm1} = -mj_0(mr)\left[A_1 m^2 e^{mz} + B_1 m^2 e^{-mz} - C_1 m(1-2\nu_1 - mz)e^{mz} + D_1 m(1-2\nu_1 + mz)e^{-mz}\right] \quad (3.10)$$

$$T_{rzm1} = mJ_1(mr)\left[A_1 m^2 e^{mz} - B_1 m^2 e^{-mz} + C_1 m(2\nu_1 + mz)e^{mz} + D_1 m(2\nu_1 - mz)e^{-mz}\right] \quad (3.11)$$

$$\sigma_{rm1} = mJ_0(mr)\left[A_1 m^2 e^{mz} + B_1 m^2 e^{-mz} + C_1 m(1+2\nu_1 + mz)e^{mz} - D_1 m(1+2\nu_1 - mz)e^{-mz}\right]$$
$$- m\frac{J_1(mr)}{mr}\left[A_1 m^2 e^{mz} + B_1 m^2 e^{-mz} + C_1 m(1+mz)e^{mz} - D_1 m(1-mz)e^{-mz}\right] \quad (3.12)$$

$$U_{m1} = \frac{1+\nu_1}{E_1} J_1(mr)\left[A_1 m^2 e^{mz} + B_1 m^2 e^{-mz} + C_1 m(1+mz)e^{mz} - D_1 m(1-mz)e^{-mz}\right] \qquad (3.13)$$

All these equations are for layer number 1, for layer number 2 a similar set of equation could be obtained with elastic properties E_2, ν_2 and coefficients B_2 and D_2, because the coefficients A_2 and C_2 must be equal to zero to satisfy the boundary conditions that the stress and displacement must be equal to zero at infinite depth.

The boundary and continuity equations which must be satisfied for case one (rough interface), are expressed mathematically in the following equation.

Boundary conditions at the surface of the ground where Z = -h are

$\sigma_{zm1} = -J_0(mr)$ Distribution of surface loading must be equal to the normal stress:

$$-mJ_0(mr)\left[A_1 m^2 e^{-mh} + B_1 m^2 e^{mh} - C_1 m(1-2\nu_1 + mh)e^{-mh} + D_1 m(1-2\nu_1 - mh)e^{mh}\right] = -J_0(mr)$$

Shearing stress at surface $T_{rzm1} = 0$

$$mJ_1(mr)\left[A_1 m^2 e^{-mh} - B_1 m^2 e^{mh} + C_1 m(2\nu_1 - mh)e^{-mh} + D_1 m(2\nu_1 + mh)e^{mh}\right] = 0$$

Continuity conditions at rough interface where Z=0 are

$W_{m1} = W_{m2}$, $U_{m1} = U_{m2}$, $\sigma_{zm1} = \sigma_{zm2}$, $T_{rzm1} = T_{rzm2}$, and the coefficients A_2 and C_2 must be equal to zero at infinite depth

Vertical displacements must be equal to $W_{m1} = W_{m2}$

$$-\frac{1+\nu_1}{E_1}\left[A_1 m^2 - B_1 m^2 - 2(1-2\nu_1)C_1 m - 2(1-2\nu_1)D_1 m\right] = -\frac{1+\nu_2}{E_2}\left[-B_2 m^2 - 2(1-2\nu_2)D_2 m\right]$$

Horizontal displacements must be equal to $U_{m1} = U_{m2}$

$$+\frac{1+\nu_1}{E_1}\left[A_1 m^2 + B_1 m^2 + C_1 m - D_1 m\right] = +\frac{1+\nu_2}{E_2}\left[B_2 m^2 - D_2 m\right]$$

Normal stresses must be equal to $\sigma_{zm1} = \sigma_{zm2}$

$$-\left[A_1 m^2 + B_1 m^2 - (1-2\nu_1)C_1 m + (1-2\nu_1)D_1 m\right] = -\left[B_2 m^2 + (1-2\nu_2)D_2 m\right]$$

Shearing stresses must be equal $T_{rzm1} = T_{rzm2}$

$$+\left[A_1 m^2 - B_1 m^2 + 2\nu_1 C_1 m + 2\nu_1 D_1 m\right] = +\left[-B_2 m^2 + 2\nu_2 D_2 m\right]$$

From the above equations the coefficients A_1, B_1, C_1, D_1 were determined so as to satisfy these boundary conditions

$$2A_1 m^2 = \left[K(1-4\nu_1)(1+2mh)e^{mh} - Le^{mh} + KL(4\nu_1 - 2mh)e^{-mh}\right] * \frac{1}{\Delta}$$

$$2B_1 m^2 = \left[(4\nu_1 + 2mh)e^{mh} - Le^{-mh} + K(1-4\nu_1)(1-2mh)e^{-mh}\right] * \frac{1}{\Delta}$$

$$C_1 m = \left[K(1+2mh)e^{mh} - KLe^{-mh}\right] * \frac{1}{\Delta} \qquad (3.14)$$

$$D_1 m = \left[e^{mh} - K(1-2mh)e^{-mh}\right] * \frac{1}{\Delta}$$

Where Δ is the common denominator.

$$\Delta = m * \left[e^{2mh} - (L + K + 4km^2h^2) + KLe^{-2mh}\right] \qquad (3.15)$$

Where the coefficients of the strength properties of the two layers are:

$$n = \frac{E_2[1+\nu_1]}{E_1[1+\nu_2]} \quad k = \left[\frac{1-n}{1+n(3-4\nu_1)}\right] \quad L = \left[\frac{(3-4\nu_2) - n(3-4\nu_1)}{(3-4\nu_2)+n}\right] \qquad (3.16)$$

Substituting the values of coefficients A_1, B_1, C_1, D_1 into equation (3.9) after replacing Z = -h at the surface, this equation becomes:

$$W_{m1} = -\frac{1+\nu_1}{E_1} j_0(mr)\left[A_1 m^2 e^{-mh} - B_1 m^2 e^{mh} - C_1 m(2-4\nu_1 + mh)e^{-mh} - D_1 m(2-4\nu_1 - mh)e^{+mh}\right]$$

$$W_{m1} = -\frac{1+\nu_1}{E_1} J_0(mr)\left[\begin{array}{l} \frac{K}{2}(1-4\nu_1)(1+2mh) - \frac{L}{2} + \frac{KL}{2}(4\nu_1 - 2mh)e^{-2mh} \\ -\frac{(4\nu 1 + 2mh)}{2}e^{2mh} + \frac{L}{2} - \frac{K}{2}(1-4\nu_1)(1-2mh) \\ -K(1+2mh)(2-4\nu_1 + mh) + KL(2-4\nu_1 + mh)e^{-2mh} \\ -(2-4\nu_1 - mh)e^{2mh} + K(1-2mh)(2-4\nu_1 - mh) \end{array}\right] * \frac{1}{\Delta}$$

$$W_{m1} = -\frac{1+\nu_1}{E_1} j_0(mr)\left[-(2-2\nu_1)e^{2mh} + KL(2-2\nu_1)e^{-2mh} - (2-2\nu_1)4mhK\right] * \frac{1}{\Delta}$$

$$W_{m1} = \frac{2(1-\nu_1^2)}{E_1}\left[\frac{e^{2mh} + 4Kmh - KLe^{-2mh}}{e^{2mh} - (L+K+4Km^2h^2) + KLe^{-2mh}}\right] * \frac{1}{m} * J_0(mr) \quad (3.17)$$

A distribution of a uniform load with intensity P_0 over an area of radius a can be represented in the form $P(r) = p_0 a \int_0^\infty j_0(mr) j_1(ma) dm$

Since the equation of displacements and stresses was obtained for an arbitrary load $\sigma_{zm1} = -j_0(mr)$, the values of the stresses and displacements can be obtained by applying the transformations

$$-P_o a \int_0^\infty J_1(ma) F(m) dm \qquad \text{For a distributed load} \quad (3.18)$$

$$\frac{-p}{2\pi}\int_0^\infty mF(m) dm \qquad \text{For a point load p}$$

in which F(m) represents the aforementioned stress and displacement relations. Hence the equation of settlement can be written in this form

$$W_1 = P_0 a \int_0^\infty W_{m1} J_1(ma) dm$$

$$W_1 = \frac{2(1-\nu_1^2)}{E_1} P_0 a \int_0^\infty \left[\frac{e^{2mh} + 4Kmh - KLe^{-2mh}}{e^{2mh} - (L+K+4Km^2h^2) + KLe^{-2mh}}\right] * \frac{J_1(ma)}{m} * J_0(mr) * dm \quad (3.19)$$

$$W_1 = \frac{2(1-\nu_1^2)}{E_1} P_0 a Fw1_{\substack{isotropic \\ rough\ interface}},$$

where $Fw1_{\substack{isotropic \\ rough\ interface}}$ is the Factor of settlement of isotropic elastic upper layer

$$Fw1_{\substack{isotropic \\ rough\ interface}} = \int_0^\infty \left[\frac{e^{2mh} + 4Kmh - KLe^{-2mh}}{e^{2mh} - (L+K+4Km^2h^2) + KLe^{-2mh}}\right] * \frac{J_1(ma)}{m} * J_0(mr) * dm \quad (3.20)$$

The equation of settlement at the interface, Z = 0 is

$$W_{m2} = -\frac{1+\nu_1}{E_1} j_0(mr)\left[A_1 m^2 - B_1 m^2 - C_1 m(2-4\nu_1) - D_1 m(2-4\nu_1)\right]$$

Substituting the values of coefficients A_1, B_1, C_1, D_1 into this equation:

$$W_{m2} = -\frac{1+\nu_1}{E_1}\begin{bmatrix} \frac{K}{2}(1-4\nu_1)(1+2mh)e^{mh} - \frac{L}{2}e^{mh} + \frac{KL}{2}(4\nu_1 - 2mh)e^{-mh} \\ -\frac{(4\nu_1+2mh)}{2}e^{mh} + \frac{L}{2}e^{-mh} - \frac{K}{2}(1-4\nu_1)(1-2mh)e^{-mh} \\ -K(1+2mh)(2-4\nu_1)e^{mh} + KL(2-4\nu_1)e^{-mh} \\ -(2-4\nu_1)e^{mh} + K(2-4\nu_1)(1-2mh)e^{-mh} \end{bmatrix} * \frac{1}{\Delta} * J_0(mr)$$

$$W_{m2} = -\frac{1+\nu_1}{E_1}\left\{ \begin{array}{l} \begin{bmatrix} \frac{K}{2}(1-4\nu_1)(1+2mh) - \frac{L}{2} - (2\nu_1 + mh) \\ -(2-4\nu_1) - K(2-4\nu_1)(1+2mh) \end{bmatrix} e^{mh} \\ - \begin{bmatrix} \frac{K}{2}(1-4\nu_1)(1-2mh) - K(2-4\nu_1)(1-2mh) \\ -\frac{L}{2} - \frac{KL}{2}(4\nu_1 - 2mh) - KL(2-4\nu_1) \end{bmatrix} e^{-mh} \end{array} \right\} * \frac{1}{\Delta} * J_0(mr)$$

$$W_{m2} = +\frac{1+\nu_1}{E_1}\left\{ \frac{\begin{array}{l}[2-2\nu_1 + mh + 0.5L + 0.5K(3-4\nu_1)(1+2mh)]e^{mh} \\ -[KL(2-2\nu_1 - mh) + 0.5L + 0.5K(3-4\nu_1)(1-2mh)]e^{-mh}\end{array}}{e^{2mh} - (L + K + 4Km^2h^2) + KLe^{-2mh}} \right\} * \frac{1}{m} * J_0(mr)$$

As for the settlement of the surface, the settlement of the interface would be

$$W_2 = +\frac{1+\nu_1}{E_1}P_0 a\int_0^\infty \left\{ \frac{\begin{bmatrix} 2-2\nu_1 + mh + 0.5L \\ +0.5K(3-4\nu_1)(1+2mh) \end{bmatrix} e^{mh} - \begin{bmatrix} KL(2-2\nu_1 - mh) + 0.5L \\ +0.5K(3-4\nu_1)(1-2mh) \end{bmatrix} e^{-mh}}{e^{2mh} - (L + K + 4Km^2h^2) + KLe^{-2mh}} \right\} * \frac{J_1(ma)}{m} * J_0(mr) * dm \quad (3.21)$$

$$W_2 = \frac{(1+\nu_1)}{E_1} P_0 a Fw2_{\substack{isotropic \\ rough\ interface}},$$

Where $Fw2_{\substack{isotropic \\ rough\ interface}}$ is the factor of settlement of the isotropic elastic lower layer

$$Fw2_{\substack{isotropic \\ riugh\ interface}} = \int_0^\infty \left\{ \frac{\begin{bmatrix} 2-2\nu_1+mh+0.5L \\ +0.5K(3-4\nu_1)(1+2mh) \end{bmatrix} e^{mh} - \begin{bmatrix} KL(2-2\nu_1-mh)+0.5L \\ +0.5K(3-4\nu_1)(1-2mh) \end{bmatrix} e^{-mh}}{e^{2mh}-(L+K+4Km^2h^2)+KLe^{-2mh}} \right\} * \frac{J_1(ma)}{m} * J_0(mr) * dm \quad (3.22)$$

To ensure the correctness of these equations the following checks were made.

If the elastic properties E and ν are equal in the two layers, that is, a homogeneous deposit throughout, $E_1 = E_2$ & $\nu_1 = \nu_2 \Rightarrow n = 1$ that means $K = 0$ & $L = 0$, hence the integration of equation (3.19) reduces to

$$W_1 = \frac{2(1-\nu^2)}{E} P_0 a \int_0^\infty \frac{1}{m} * J_0(mr) * J_1(ma) * dm$$

At the center of the load $r = 0 \Rightarrow J_0(mr) = 1$

Where $J_\nu(0) = \begin{cases} 1 & \nu = 0 \\ 0 & \nu > 0 \\ 0 & \nu = -1, -2, -3\ldots \\ \infty & \nu < 0,\ \nu \neq -1, -2\ldots \end{cases}$

Subsequently, the equation reduces to

$$W_1 = \frac{2(1-\nu^2)}{E} P_0 a \int_0^\infty \frac{1}{m} * J_1(ma) * dm$$

Applying the well-known integration

$$\int_0^\infty J_\nu(mr) m^n dm = \frac{2^n \Gamma\left(\frac{1}{2} + \frac{1}{2}n + \frac{1}{2}\nu\right)}{r^{n+1}\Gamma\left(\frac{1}{2} - \frac{1}{2}n + \frac{1}{2}\nu\right)}$$

where Γ is Gamma function, and $n = -1$ & $\nu = 1$

$$\int_0^\infty J_1(mr) m^{-1} dm = \frac{\Gamma\left(\frac{1}{2}\right)}{2\Gamma\left(\frac{3}{2}\right)} = \frac{\sqrt{\pi}}{2\frac{\sqrt{\pi}}{2}} = 1$$

consequently; the two layer system equation reduces to the well-known Boussinesq equation for a distributed circular load

$$W = \frac{2(1-\nu^2)}{E} pa$$

For equation of displacement at the interface, if the modulus E_2 becomes infinite, that means a rough rock surface at the base of upper layer will exist, the coefficients of the strength properties becomes

$$E_2 \to \infty \Rightarrow n = \frac{E_2}{E_1}\frac{1+\nu_1}{1+\nu_2} \to \infty, \quad \text{so } K \to \frac{-1}{(3-4\nu_1)}, \text{ and } L \to -(3-4\nu_1)$$

If Poisson's ratios of both layers are equal to 1/2, the strength coefficients L and K both become equal to minus one (- 1) and the settlement of the interface of the second layer will be equal to zero.

For the case of smooth interface, the boundary and continuity equations are expressed mathematically in the following equation.

Boundary conditions at the surface of the ground where Z = -h are

$\sigma_{zm1} = -J_0(mr)$ Distribution of surface loading must be equal to the normal stress:

$$-mJ_0(mr)\left[A_1m^2e^{-mh} + B_1m^2e^{mh} - C_1m(1-2\nu_1+mh)e^{-mh} + D_1m(1-2\nu_1-mh)e^{mh}\right] = -J_0(mr)$$

Shearing stress at surface $T_{rzm1} = 0$

$$mJ_1(mr)\left[A_1m^2e^{-mh} - B_1m^2e^{mh} + C_1m(2\nu_1-mh)e^{-mh} + D_1m(2\nu_1+mh)e^{mh}\right] = 0$$

Continuity conditions at smooth interface where Z=0 are

$W_{m1} = W_{m2}$, $\sigma_{zm1} = \sigma_{zm2}$, $T_{rzm1} = 0$, $T_{rzm2} = 0$, and the coefficients A_2 and C_2 must be equal to zero at infinite depth

Vertical settlements must be equal to $W_{m1} = W_{m2}$

$$-\frac{1+\nu_1}{E_1}\left[A_1m^2 - B_1m^2 - 2(1-2\nu_1)C_1m - 2(1-2\nu_1)D_1m\right] = -\frac{1+\nu_2}{E_2}\left[-B_2\,m^2 - 2(1-2\nu_2)D_2m\right]$$

Normal stresses must be equal to $\sigma_{zm1} = \sigma_{zm2}$

$$-\left[A_1m^2 + B_1m^2 - (1-2\nu_1)C_1m + (1-2\nu_1)D_1m\right] = -\left[B_2m^2 + (1-2\nu_2)D_2m\right]$$

Shearing stress at the interface for upper layer is equal to zero $T_{rzm1} = 0$

$$+\lfloor A_1 m^2 - B_1 m^2 + 2\nu_1 C_1 m + 2\nu_1 D_1 m \rfloor = 0$$

Shearing stress at the interface for lower layer is equal to zero $T_{rzm2} = 0$

$$\lfloor -B_2 m^2 + 2\nu_2 D_2 m \rfloor = 0$$

From the above equations the coefficients A_1, B_1, C_1, D_1 were determined so as to satisfy, these boundary conditions

$$A_1 m = [C_1(F - 2\nu_1) - D_1(1 - F)],$$

$$B_1 m = [C_1 F - D_1(1 - 2\nu_1 - F)],$$

$$C_1 m = [(1 - 2\nu_1 - F)e^{mh} + (2\nu_1 + mh)e^{mh} - (1 - F)e^{-mh}] * \frac{1}{\Delta}, \tag{3.23}$$

$$D_1 m = [Fe^{mh} - (F - 2\nu_1)e^{-mh} - (2\nu_1 - mh)e^{-mh}] * \frac{1}{\Delta},$$

Where Δ is the common denominator.

$$\Delta = m * \lfloor Fe^{2mh} + (2F - 1)2mh - (1 + 2m^2h^2) + (1 - F)e^{-2mh} \rfloor \tag{3.24}$$

Where the coefficients of the strength properties of the two layers are:

$$n = \frac{E_2[1 + \nu_1]}{E_1[1 + \nu_2]} \quad F = \left[\frac{(1 - \nu_2) + n(1 - \nu_1)}{2(1 - \nu_2)}\right] \tag{3.25}$$

Substituting the values of coefficients A_1, B_1, C_1, D_1 into equation of settlement of layer number1 after replacing Z = -h at the surface, this equation becomes:

$$W_{m1} = -\frac{1 + \nu_1}{E_1} j_0(mr)[A_1 m^2 e^{-mh} - B_1 m^2 e^{mh} - C_1 m(2 - 4\nu_1 + mh)e^{-mh} - D_1 m(2 - 4\nu_1 - mh)e^{+mh}]$$

$$W_{m1} = \frac{2(1 - \nu_1^2)}{E_1}\left[\frac{Fe^{2mh} - [2F - 1 - 2mh] - (1 - F)e^{-2mh}}{Fe^{2mh} + (2F - 1)2mh - (1 + 2m^2h^2) + (1 - F)e^{-2mh}}\right] * \frac{1}{m} * J_0(mr) \tag{3.26}$$

Applying the transformation

$$W_1 = \frac{2(1 - \nu_1^2)}{E_1} P_0 a \int_0^\infty \left[\frac{Fe^{2mh} - [2F - 1 - 2mh] - (1 - F)e^{-2mh}}{Fe^{2mh} + (2F - 1)2mh - (1 + 2m^2h^2) + (1 - F)e^{-2mh}}\right] * \frac{J_1(ma)}{m} * J_0(mr) * dm \tag{3.27}$$

$$W_1 = \frac{2(1-\nu_1^2)}{E_1} P_0 a Fw1_{\substack{isotropic \\ smooth\ interface}},$$

Where $Fw1_{\substack{isotroic \\ smooh\ interface}}$, is the factor of settlement of isotropic elastic upper layer, for smooth interface

$$Fw1_{\substack{isotropic \\ smooth\ interface}} = \int_0^\infty \left[\frac{e^{2mh} + 4Kmh - KLe^{-2mh}}{e^{2mh} - (L + K + 4Km^2h^2) + KLe^{-2mh}} \right] * \frac{J_1(ma)}{m} * J_0(mr) * dm \qquad (3.28)$$

The equation of settlement at the interface, Z = 0 is

$$W_{m2} = -\frac{1+\nu_1}{E_1} j_0(mr)\left[A_1 m^2 - B_1 m^2 - C_1 m(2-4\nu_1) - D_1 m(2-4\nu_1)\right]$$

Substituting the values of coefficients A_1, B_1, C_1, D_1 into this equation:

$$W_{m2} = \frac{2(1-\nu_1^2)}{E_1}\left[\frac{(1+mh)e^{mh} - (1-mh)e^{-mh}}{Fe^{2mh} + (2F-1)2mh - (1+2m^2h^2) + (1-F)e^{-2mh}} \right] * J_0(mr)$$

As for the settlement of the surface, the settlement of the interface would be

$$W_2 = \frac{2(1-\nu_1^2)}{E_1} P_0 a \int_0^\infty \left[\frac{(1+mh)e^{mh} - (1-mh)e^{-mh}}{Fe^{2mh} + (2F-1)2mh - (1+2m^2h^2) + (1-F)e^{-2mh}} \right] * \frac{J_1(ma)}{m} * J_0(mr) * dm \qquad (3.29)$$

$$W_2 = \frac{2(1-\nu_1^2)}{E_1} P_0 a Fw2_{\substack{isotropic \\ smooth\ interface}},$$

Where $Fw2_{\substack{isotroic \\ smooh\ interface}}$, is the factor of settlement of isotropic elastic lower layer, for smooth interface

$$Fw2_{\substack{isotropic \\ smooth\ interface}} = \int_0^\infty \left[\frac{(1+mh)e^{mh} - (1-mh)e^{-mh}}{Fe^{2mh} + (2F-1)2mh - (1+2m^2h^2) + (1-F)e^{-2mh}} \right] * \frac{J_1(ma)}{m} * J_0(mr) * dm \qquad (3.30)$$

3.3 Two-layer system of anisotropic elastic solid:

Cauwelaert, 1977 has shown that the five independent elastic constants that characterize the anisotropic material with a plane of isotropy can probably be reduced to three fundamental constants. Knowing the values of the elastic constant, the stress equation can be easily determined.

$$\sigma_r = \frac{\partial}{\partial z}\left[\nu(n+\nu)\nabla^2\Phi - n(1+\nu)\frac{\partial^2\Phi}{\partial r^2}\right] \quad \sigma_\theta = \frac{\partial}{\partial z}\left[\nu(n+\nu)\nabla^2\Phi - n(1+\nu)\frac{1}{r}\frac{\partial\ \Phi}{\partial r}\right] \tag{3.31}$$

$$\sigma_z = \frac{\partial}{\partial z}\left[\left(n^2+\nu n+n-\nu^2\right)\nabla^2\Phi - n(1+\nu)\frac{\partial^2\Phi}{\partial z^2}\right] \quad \tau_{rz} = \frac{\partial}{\partial z}\left[\left(n-\nu^2\right)\nabla^2\Phi - n(1+\nu)\frac{\partial^2\Phi}{\partial z^2}\right]$$

in which the Φ function has to satisfy $\nabla^2\nabla_1^2\Phi = 0$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\ \text{, and} \qquad \nabla_1^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{s^2}\frac{\partial^2}{\partial z^2} \tag{3.32}$$

with s (the ratio of anisotropy) $= \left(\dfrac{n-\nu^2}{n^2-\nu^2}\right)^{\frac{1}{2}}$ (3.33)

and n the degree of anisotropy ($n = E_z / E_x = E_z / E_y$)

The displacement equations are

$$U = \frac{n(n+\nu)(1+\nu)}{E}\frac{\partial^2\Phi}{\partial r\partial z} \qquad W = \frac{1}{E}\left[(1+n+2\nu)\left(n-\nu^2\right)\nabla^2\Phi - n(1+\nu)^2\frac{\partial^2\Phi}{\partial z^2}\right] \tag{3.34}$$

Where the stress function is

$$\Phi = J_0(mr)(Ae^{mz} - Be^{-mz} + Ce^{msz} - De^{-msz}) \tag{3.35}$$

The stress and displacement expressions become.

$$\sigma_z = -mJ_0(mr)\left[n(1+\nu)(Am^2e^{mz} + Bm^2e^{-mz}) + ns(n+\nu)(Cm^2e^{msz} + Dm^2e^{-msz})\right] \tag{3.36}$$

$$\tau_{rz} = mJ_1(mr)\left[n(1+\nu)(Am^2e^{mz} - Bm^2e^{-mz}) + ns^2(n+\nu)(Cm^2e^{msz} - Dm^2e^{-msz})\right] \tag{3.37}$$

$$U = \frac{1+\nu}{E}n(n+\nu)J_1(mr)\left[Am^2e^{mz} + Bm^2e^{-mz} + sCm^2e^{msz} + sDm^2e^{-msz}\right] \tag{3.38}$$

$$W=-\frac{1+\nu}{E}J_0(mr)\left[n(1+\nu)(Am^2e^{mz}-Bm^2e^{-mz})+ns^2\frac{(n+\nu)^2}{(1+\nu)}(Cm^2e^{msz}-Dm^2e^{-msz})\right] \quad (3.39)$$

For the case of the rough interface the boundary and continuity equations which must be satisfied are expressed mathematically in the following equation.

Boundary conditions at the surface of the ground where Z = -h are

$\sigma_{zm1}=-mJ_0(mr)$ Distribution of surface loading must be equal to the normal stress:

$$-mJ_0(mr)\left[n_1(1+\nu_1)(A_1m^2e^{-mh}+B_1m^2e^{mh})+n_1s_1(n_1+\nu_1)(C_1m^2e^{-msh}+D_1m^2e^{msh})\right]=-mJ_0(mr)$$

Shearing stress at surface $T_{rzm1}=0$

$$mJ_1(mr)\left[n_1(1+\nu_1)(A_1m^2e^{-mh}-B_1m^2e^{mh})+n_1s_1^{\,2}(n_1+\nu_1)(C_1m^2e^{-msh}-D_1m^2e^{msh})\right]=0$$

Continuity conditions at a rough interface where Z=0 are:

$W_{m1}=W_{m2}$, $U_{m1}=U_{m2}$, $\sigma_{zm1}=\sigma_{zm2}$, $T_{rzm1}=T_{rzm2}$, and the coefficients A_2 and C_2 must be equal to zero at infinite depth.

Vertical settlements must be equal to $W_{m1}=W_{m2}$

$$-\frac{1+\nu_1}{E_1}J_0(mr)\begin{bmatrix}n_1(1+\nu_1)(A_1m^2-B_1m^2)\\+n_1s_1^{\,2}\frac{(n_1+\nu_1)^2}{(1+\nu_1)}(C_1m^2-D_1m^2)\end{bmatrix}=-\frac{1+\nu_2}{E_2}J_0(mr)\begin{bmatrix}n_2(1+\nu_2)(-B_2m^2)\\+n_2s_2^2\frac{(n_2+\nu_2)}{(1+\nu_2)}(-D_2m^2)\end{bmatrix}$$

Horizontal displacements must be equal to $U_{m1}=U_{m2}$

$$\frac{1+\nu_1}{E_1}J_1(mr)\begin{bmatrix}n_1(n_1+\nu_1)(A_1m^2+B_1m^2+\\s_1C_1m^2+s_1D_1m^2)\end{bmatrix}=\frac{1+\nu_2}{E_2}J_1(mr)\left[n_2(n_2+\nu_2)(B_2m^2+s_2D_2m^2)\right]$$

Normal stresses are equal to $\sigma_{zm1}=\sigma_{zm2}$

$$-mJ_0(mr)\begin{bmatrix}n_1(1+\nu_1)(A_1m^2+B_1m^2)+\\n_1s_1(n_1+\nu_1)(C_1m^2+D_1m^2)\end{bmatrix}=-mJ_0(mr)[n_2(1+\nu_2)B_2m^2+n_2s_2(n_2+\nu_2)D_2m^2]$$

Shearing stresses are equal to $T_{rzm1}=T_{rzm2}$

$$mJ_1(mr)\begin{bmatrix}n_1(1+\nu_1)(A_1m^2-B_1m^2)\\+n_1s_1^2(n_1+\nu_1)(C_1m^2-D_1m^2)\end{bmatrix}=mJ_1(mr)\left[n_2(1+\nu_2)(-B_2m^2)+n_2s_2^2(n_2+\nu_2)(-D_2m^2)\right]$$

From the above equations the coefficients A_1, B_1, C_1, D_1 were determined so as to satisfy these boundary conditions.

$$n_1(1+\nu_1)A_1m^2 = (Je^{mh} - \beta g e^{mh} + s_1\beta F e^{-s_1 mh} + s_1\lambda J e^{-s_1 mh} + s_1\lambda g e^{s_1 mh} + s_1 F e^{s_1 mh}) * \frac{1}{\Delta}$$

$$n_1(1+\nu_1)B_1m^2 = (Je^{-mh} - \beta g e^{-mh} + s_1\beta e^{-s_1 mh} + s_1 e^{s_1 mh}) * \frac{1}{\Delta}$$

$$n_1 s_1(n_1+\nu_1)C_1m^2 = (\beta e^{mh} - \beta F e^{-mh} - \lambda J e^{-mh} - s_1\lambda e^{s_1 mh}) * \frac{1}{\Delta} \quad (3.40)$$

$$n_1 s_1(n_1+\nu_1)D_1m^2 = (Fe^{-mh} - e^{mh} + \lambda g e^{-mh} - s_1\lambda e^{-s_1 mh}) * \frac{1}{\Delta}$$

Where Δ is the common denominator

$$\begin{aligned}\Delta = 2J - 2\beta g - 2\lambda s_1 - (1-s_1)e^{mh+s_1 mh} + \beta(1+s_1)e^{mh-s_1 mh} \\ + (\lambda g + F)(1+s_1)e^{-mh+s_1 mh} - (\beta F + \lambda J)(1-s_1)e^{-mh-s_1 mh}\end{aligned} \quad (3.41)$$

Where the coefficients of the strength properties of the two layers are

$$N = \frac{E_2[1+\nu_1]}{E_1[1+\nu_2]} \quad K_1 = \frac{n_1+\nu_1}{1+\nu_1} \quad K_2 = \frac{n_2+\nu_2}{1+\nu_2} \quad a = \frac{N-1}{s_2K_2 - s_2} \quad b = \frac{Ns_1K_1 - s_1}{s_2K_2 - s_2}$$

$$F = \frac{a(1-s_2)}{2+a(1-s_2)} \quad g = \frac{(1+s_1)+b(1-s_2)}{2+a(1-s_2)} \quad J = \frac{(1-s_1)-b(1-s_2)}{2+a(1-s_2)}$$

$$\lambda = \frac{(NK_1 - K_2) - a(1-s_2K_2) + F[(NK_1+K_2) + a(1-s_2K_2)}{(N+s_1K_2) + b(1-s_2K_2) - g[(NK_1+K_2) + a(1-s_2K_2)]} \quad (3.42)$$

$$\beta = \frac{(N - s_1K_2) - b(1-s_2K_2) - J[(NK_1+K_2) + a(1-s_2K_2)}{(N+s_1K_2) + b(1-s_2K_2) - g[(NK_1+K_2) + a(1-s_2K_2)]}$$

Substitute the coefficients A_1, B_1, C_1, D_1 and the coefficients of the strength properties of the two layers into the equation of settlement, this equation becomes:

$$W_{m1} = \frac{1+\nu_1}{E_1} J_0(mr) \begin{bmatrix} + s_1(1-K_1)e^{mh+s_1 mh} - s_1(1-K_1)(F+\lambda g)e^{-mh+s_1 mh} \\ - s_1(1-K_1)(\beta F + \lambda J)e^{-mh-s_1 mh} + \beta s_1(1-K_1)e^{mh-s_1 mh} \end{bmatrix} * \frac{1}{\Delta} \quad (3.43)$$

As shown by Burmister, the values of the stresses and displacements beneath a uniform load, P_0 applied to a circular surface (radius a), and for an arbitrary stress $\sigma_{zm1} = -mj_0(mr)$, can be obtained by applying the transformations

$$-P_o a\int_0^\infty \frac{J_1(ma)}{m}F(m)dm \quad \text{For a distributed load} \tag{3.44}$$

$$\frac{-p}{2\pi}\int_0^\infty F(m)dm \qquad \text{For a point load } p$$

in which F(m) represents the aforementioned stress and displacement relations.

The equation of surface settlement becomes

$$W_1=\frac{1+\nu_1}{E_1}P_0 a\int_0^\infty \left[\frac{\begin{matrix}+s_1(1-K_1)e^{mh+s_1mh}-s_1(1-K_1)(F+\lambda g)e^{-mh+s_1mh}\\ -s_1(1-K_1)(\beta F+\lambda J)e^{-mh-s_1mh}+\lambda s_1(1-K_1)e^{mh-s_1mh}\end{matrix}}{\begin{matrix}2J-2\beta g-2\lambda s_1-(1-s_1)e^{mh+s_1mh}+\beta(1+s_1)e^{mh-s_1mh}\\ +(\lambda g+F)(1+s_1)e^{-mh+s_1mh}-(\beta F+\lambda J)(1-s_1)e^{-mh-s_1mh}\end{matrix}}\right]*\frac{J_1(ma)}{m}*J_0(mr)*dm \tag{3.45}$$

$$W_1=\frac{1+\nu_1}{E_1}P_0 aFw1_{\substack{anisotropic\\ rough\ interface}}.$$

where $Fw1_{\substack{anisotropic\\ rough\ interface}}$ is the Factor of settlement for anisotropic elastic upper layer, for rough interface

The equation of settlement at the interface where Z = 0 is

$$W_{m2}=-\frac{1+\nu}{E}J_0(mr)\left[n(1+\nu)(Am^2-Bm^2)+ns^2\frac{(n+\nu)^2}{(1+\nu)}(Cm^2-Dm^2)\right]$$

Substituting the coefficients A_1, B_1, C_1, D_1 and reorganize, the equation becomes

$$W_{m2}=\frac{1+\nu_1}{E_1}J_0(mr)\begin{bmatrix}-(\beta s_1K_1+J+s_1K_1-\beta g)e^{mh}\\ -(\beta g-J-Fs_1K_1-\beta Fs_1k_1-\lambda Js_1K_1-\lambda gs_1K_1)e^{-mh}\\ -(\beta s_1K_1+\lambda Js_1-\beta s_1-\lambda s_1^2K_1)e^{-s_1mh}\\ -(Fs_1+\lambda gs_1-s_1-\lambda s_1^2K_1)e^{s_1mh}\end{bmatrix}*\frac{1}{\Delta} \tag{3.46}$$

Applying the transformation

$$W_2=\frac{1+\nu_1}{E_1}P_0 a\int_0^\infty \begin{bmatrix}-(\beta s_1K_1+J+s_1K_1-\beta g)e^{mh}\\ -(Fs_1+\lambda gs_1-s_1-\lambda s_1^2K_1)e^{smh}\\ -(\beta g-J-Fs_1K_1-\beta Fs_1k_1-\lambda Js_1K_1-\lambda gs_1K_1)e^{-mh}\\ -(\beta s_1K_1+\lambda Js_1-\beta s_1-\lambda s_1^2K_1)e^{-s_1mh}\end{bmatrix}*\frac{1}{\Delta}*\frac{J_1(ma)}{m}*J_0(mr)*dm \tag{3.47}$$

$$W_2=\frac{1+\nu_1}{E_1}P_0 aFw2_{\substack{anisotropic\\ rough\ interface}}$$

where $Fw2_{\substack{anisotropic\\ rough\ interface}}$ is the factor of settlement for lower anisotropic elastic layer, for rough interface

To ensure the correctness of these equations, the following checks were made.

In the case of homogeneous deposit the stress function becomes

$\Phi = J_0(mr)(-Be^{-mz} - De^{-msz})$ The coefficients A_2 and C_2 must be equal to zero to ensure the nullity of stress and displacement at infinite depth.

Applying the same previous procedure, the settlement of homogeneous deposit becomes

$$W = \frac{1+\nu}{E} * \frac{-s(1-k)}{(1-s)} * p_0 a \tag{3.48}$$

where $K = \dfrac{n+\nu}{1+\nu}$, and $s = \left(\dfrac{n-\nu^2}{n^2-\nu^2}\right)^{\frac{1}{2}}$

In the case of two layer system, the settlement of upper layer is

$$W_1 = \frac{1+\nu_1}{E_1} P_0 a \int_0^\infty \begin{bmatrix} +s_1(1-K_1)e^{mh+s_1mh} \\ -s_1(1-K_1)(F+\lambda g)e^{-mh+s_1mh} \\ -s_1(1-K_1)(\beta F+\lambda J)e^{-mh-s_1mh} \\ +\beta s_1(1-K_1)e^{mh-s_1mh} \end{bmatrix} * \frac{1}{\Delta} * \frac{1}{m} * J_0(mr) * J_1(ma) * dm$$

If the Poisson's ratio and modulus of elasticity for the both layers are equal, that means homogeneous deposit throughout, then this equation reduces to the aforementioned equation of homogeneous deposit.

That is when $E_1 = E_2$ & $\nu_1 = \nu_2$ the coefficients of the strength properties N will be equal to 1

$N = \dfrac{E_2[1+\nu_1]}{E_1[1+\nu_2]} = 1$; Consequently the other coefficients $a = F = J = 0$, $g = b = 1$ and

$\lambda = \beta = 0$, then the equation of settlement will reduce to

$$W_1 = \frac{1+\nu}{E} p_0 a \left[\frac{s(1-k)e^{mh+smh}}{-(1-s)e^{mh+smh}}\right] * \int_0^\infty \frac{1}{m} * J_0(mr) * J_1(ma) * dm$$

For the displacement of surface at the center $r = 0 \Rightarrow J_0(mr) = 1$, the equation will reduce to

$$W_1 = \frac{1+\nu}{E} p_0 a \left[\frac{s(1-k)e^{mh+smh}}{-(1-s)e^{mh+smh}} \right] * \int_0^\infty \frac{J_1(ma)}{m} * dm$$

As mentioned previously the integration $\int_0^\infty \frac{J_1(ma)}{m} dm = 1$, hence the equation of two layer system reduces to the equation of homogenous deposit.

Second check on the validity of this equation has been made by comparing between isotropic and anisotropic equation.

Equation of settlement of anisotropic upper layer can be written as follows

$$W_1 = \frac{2(1+\nu_1^2)}{E_1} P_0 a \int_0^\infty \left[\frac{\begin{matrix} +s_1(1-K_1)e^{mh+s_1mh} - s_1(1-K_1)(F+\lambda g)e^{-mh+s_1mh} \\ -s_1(1-K_1)(\beta F+\lambda J)e^{-mh-s_1mh} + \lambda s_1(1-K_1)e^{mh-s_1mh} \end{matrix}}{\begin{matrix} 2J - 2\beta g - 2\lambda s_1 - (1-s_1)e^{mh+s_1mh} + \beta(1+s_1)e^{mh-s_1mh} \\ +(\lambda g+F)(1+s_1)e^{-mh+s_1mh} - (\beta F+\lambda J)(1-s_1)e^{-mh-s_1mh} \end{matrix}} \right] * \frac{1}{2(1-\nu_1)} * \frac{J_1(ma)}{m} * J_0(mr) * dm$$

That means the factor of settlement of anisotropic becomes

$$Fw1_{anisotropic} = \int_0^\infty \left[\frac{\begin{matrix} +s_1(1-K_1)e^{mh+s_1mh} - s_1(1-K_1)(F+\lambda g)e^{-mh+s_1mh} \\ -s_1(1-K_1)(\beta F+\lambda J)e^{-mh-s_1mh} + \lambda s_1(1-K_1)e^{mh-s_1mh} \end{matrix}}{\begin{matrix} 2J - 2\beta g - 2\lambda s_1 - (1-s_1)e^{mh+s_1mh} + \beta(1+s_1)e^{mh-s_1mh} \\ +(\lambda g+F)(1+s_1)e^{-mh+s_1mh} - (\beta F+\lambda J)(1-s_1)e^{-mh-s_1mh} \end{matrix}} \right] * \frac{1}{2(1-\nu_1)} * \frac{J_1(ma)}{m} * J_0(mr) * dm$$

When the degree of anisotropy ($n = E_z / E_x = E_z / E_y$) equal one (1) that means the ratio of isotropy $s = \left(\frac{n - \nu^2}{n^2 - \nu^2} \right)^{\frac{1}{2}}$ equal one too. In this case the factor of settlement of anisotropic should be reduced to the isotropic one

$$W_1 = \frac{2(1-\nu_1^2)}{E_1} P_0 a Fw1_{\substack{isotropic \\ rough\ interface}}$$

$$W_1 = \frac{2(1-\nu_1^2)}{E_1} P_0 a Fw1_{\substack{anisotropic \\ rough\ interface}}$$

A check has been made for the case where n=1 and the Poisson's ratio of lower and upper layer are equal to 0.5. It is found that the settlement of anisotropic upper layer when n=1 is equal to the settlement of isotropic elastic upper layer.

These verifications ensure the correctness of the aforementioned equations of anisotropic elastic solid. The theory of two layer system of anisotropic elastic solid in this sense is more general than the Burmister theory, in the sense that it allows dealing with both cases, isotropic and anisotropic and their combinations in a two-layer system.

For the case of a smooth interface, the boundary and continuity equations which must be satisfied are expressed mathematically in the following equation.

Boundary conditions at the surface of the ground where Z = -h are

$\sigma_{zm1} = -mJ_0(mr)$ Distribution of surface loading must be equal to the normal stress:

$$-mJ_0(mr)\left[n_1(1+\nu_1)(A_1m^2e^{-mh}+B_1m^2e^{mh})+n_1s_1(n_1+\nu_1)(C_1m^2e^{-msh}+D_1m^2e^{msh})\right]=-mJ_0(mr)$$

Shearing stress at surface $T_{rzm1} = 0$

$$mJ_1(mr)\left[n_1(1+\nu_1)(A_1m^2e^{-mh}-B_1m^2e^{mh})+n_1s_1{}^2(n_1+\nu_1)(C_1m^2e^{-msh}-D_1m^2e^{msh})\right]=0$$

Continuity conditions at a smooth interface where Z=0 are

$W_{m1} = W_{m2}$, $\sigma_{zm1} = \sigma_{zm2}$, $T_{rzm1} = 0$, $T_{rzm2} = 0$, and the coefficients A_2 and C_2 must be equal to zero at infinite depth.

Vertical settlements must be equal to $W_{m1} = W_{m2}$

$$-\frac{1+\nu_1}{E_1}J_0(mr)\begin{bmatrix} n_1(1+\nu_1)(A_1m^2-B_1m^2) \\ +n_1s_1{}^2\dfrac{(n_1+\nu_1)^2}{(1+\nu_1)}(C_1m^2-D_1m^2)\end{bmatrix}=-\frac{1+\nu_2}{E_2}J_0(mr)\begin{bmatrix} n_2(1+\nu_2)(-B_2m^2) \\ +n_2s_2^2\dfrac{(n_2+\nu_2)}{(1+\nu_2)}(-D_2m^2)\end{bmatrix}$$

Normal stresses are equal $\sigma_{zm1} = \sigma_{zm2}$

$$-mJ_0(mr)\begin{bmatrix} n_1(1+\nu_1)(A_1m^2+B_1m^2)+ \\ n_1s_1(n_1+\nu_1)(C_1m^2+D_1m^2)\end{bmatrix}=-mJ_0(mr)\left[n_2(1+\nu_2)B_2m^2+n_2s_2(n_2+\nu_2)D_2m^2\right]$$

Shearing stress at the interface for upper layer is equal to zero $T_{rzm1} = 0$

$$mJ_1(mr)\left[n_1(1+\nu_1)(A_1m^2-B_1m^2)+n_1s_1^2(n_1+\nu_1)(C_1m^2-D_1m^2)\right]=0$$

Shearing stress at the interface for lower layer is equal to zero $T_{rzm2} = 0$

$$mJ_1(mr)\lfloor n_2(1+\nu_2)(-B_2m^2) + n_2 s_2^2(n_2+\nu_2)(-D_2m^2)\rfloor = 0$$

From the above equations, the coefficients A_1, B_1, C_1, D_1 were determined so as to satisfy these boundary conditions

$$\begin{aligned} n_1(1+\nu_1)A_1m^2 &= (-s_1(1-U)e^{mh} - s_1(U-s_1V)e^{-s_1mh} + s_1(1-s_1V)e^{s_1mh}) * \frac{1}{\Delta} \\ n_1(1+\nu_1)B_1m^2 &= (s_1Ue^{-mh} - s_1ge^{-mh} - s_1Ue^{-s_1mh} + s_1e^{s_1mh}) * \frac{1}{\Delta} \\ n_1s_1(n_1+\nu_1)C_1m^2 &= (Ue^{-mh} - Ue^{mh} - s_1Ve^{-mh} - s_1Ve^{s_1mh}) * \frac{1}{\Delta} \\ n_1s_1(n_1+\nu_1)D_1m^2 &= (e^{-mh} - e^{mh} - s_1Ve^{-mh} + s_1Ve^{-s_1mh}) * \frac{1}{\Delta} \end{aligned} \tag{3.49}$$

where Δ is the common denominator

$$\begin{aligned} \Delta = {} & +2s_1U + 2s_1V - 2s_1 - (1-s_1)e^{mh+s_1mh} - U(1+s_1)e^{mh-s_1mh} \\ & +(1-s_1V)(1+s_1)e^{-mh+s_1mh} + (U-s_1V)(1-s_1)e^{-mh-s_1mh} \end{aligned} \tag{3.50}$$

Where the coefficients of the strength properties of the two layers are

$$N = \frac{E_2[1+\nu_1]}{E_1[1+\nu_2]} \qquad K_1 = \frac{n_1+\nu_1}{1+\nu_1} \qquad K_2 = \frac{n_2+\nu_2}{1+\nu_2} \qquad c = \frac{s_2 - s_2K_2}{1-s_2}$$

$$V = \frac{2c}{Ns_1K_1 - c - s_1N + s_1c} \qquad U = \frac{Ns_1K_1 + c - s_1N + s_1c}{Ns_1K_1 - c - s_1n + s_1c} \tag{3.51}$$

Substitute the coefficients A_1, B_1, C_1, D_1 and the coefficients of the strength properties of the two layers into the equation of settlement, this equation becomes:

$$W_{m1} = \frac{1+\nu_1}{E_1} J_0(mr) \begin{bmatrix} +s_1(1-K_1)e^{mh+s_1mh} - s_1(1-K_1)(1-s_1V)e^{-mh+s_1mh} \\ +s_1(1-K_1)(U-s_1V)e^{-mh-s_1mh} - s_1U(1-K_1)e^{mh-s_1mh} \end{bmatrix} * \frac{1}{\Delta} \tag{3.52}$$

After applying the transformation, the equation of settlement becomes

$$W_1 = \frac{1+\nu_1}{E_1} P_0 a \int_0^\infty \left[\frac{\begin{matrix} +s_1(1-K_1)e^{mh+s_1mh} - s_1(1-K_1)(1-s_1V)e^{-mh+s_1mh} \\ +s_1(1-K_1)(U-s_1V)e^{-mh-s_1mh} - s_1U(1-K_1)e^{mh-s_1mh} \end{matrix}}{\begin{matrix} 2s_1U + 2s_1V - 2s_1 - (1-s_1)e^{mh+s_1mh} - U(1+s_1)e^{mh-ms_1h} \\ +(1-s_1V)(1+s_1)e^{-mh+ms_1h} + (U-s_1V)(1-s_1)e^{-mh-ms_1h} \end{matrix}} \right] * \frac{J_1(ma)}{m} * J_0(mr) * dm \tag{3.53}$$

$$W_1 = \frac{1+\nu_1}{E_1} P_0 a Fw1_{\substack{anisotropic\\ smooth\ interface}}$$

where $Fw1_{\substack{anisotropic\\ smooth\ interface}}$ is the factor of settlement for anisotropic elastic upper layer, for Smooth interface

The equation of settlement at the interface where Z = 0 is

$$W_{m2} = -\frac{1+\nu}{E} J_0(mr)\left[n(1+\nu)(Am^2 - Bm^2) + ns^2\frac{(n+\nu)^2}{(1+\nu)}(Cm^2 - Dm^2)\right]$$

Substituting the coefficients A_1, B_1, C_1, D_1 and reorganizing, the equation of settlement of the

interface become,

$$W_{m2} = \frac{1+\nu_1}{E_1} J_0(mr)\begin{bmatrix} +s_1(1-K_1)(1-U)e^{mh} - s_1(1-K_1)(1-U)e^{-mh} \\ +s_1^2 V(1-K_1)e^{s_1 mh} - s_1^2 V(1-K_1)e^{-s_1 mh} \end{bmatrix} * \frac{1}{\Delta} \qquad (3.54)$$

After applying the transformation, the equation of settlement becomes

$$W_{m2} = \frac{1+\nu_1}{E_1} P_0 a \int_0^{\infty} \left[\frac{\begin{matrix} +s_1(1-K_1)(1-U)e^{mh} - s_1(1-K_1)(1-U)e^{-mh} \\ +s_1^2 V(1-K_1)e^{s_1 mh} - s_1^2 V(1-K_1)e^{-s_1 mh} \end{matrix}}{\begin{matrix} 2s_1U + 2s_1V - 2s_1 - (1-s_1)e^{mh+ms_1h} - U(1+s_1)e^{mh-ms_1h} \\ +(1-s_1V)(1+s_1)e^{-mh+ms_1h} + (U-s_1V)(1-s_1)e^{-mh-msh} \end{matrix}}\right] * \frac{J_1(ma)}{m} * J_0(mr) * dm \qquad (3.55)$$

$$W_2 = \frac{1+\nu_1}{E_1} P_0 a Fw2_{\substack{anisotropic\\ smooth\ interface}}$$

where $Fw2_{\substack{anisotropic\\ smooth\ interface}}$ is the factor of settlement for anisotropic elastic lower layer, for smooth interface

Furthermore, an attempt has been made to calculate the compression of anisotropic upper layer, the theoretical results have been evaluated numerically and expressed in basic influence curves, for rough and smooth interface at the center of the load, for a various combinations of anisotropic and isotropic elastic materials (Chapter 4).

The compression of upper layer is easily found to be

$$C = W_1 - W_2$$

$$C = \frac{2(1-\nu_1^2)}{E_1} P_0 a \left[\frac{1}{2(1-\nu_1)} Fw1_{\substack{anisotropic \\ rough\ interface}} - \frac{1}{2(1-\nu_1)} Fw2_{\substack{anisotropic \\ rough\ interface}} \right] \quad (3.56)$$

$C = \frac{2(1-\nu_1^2)}{E_1} P_0 a Fc_{\substack{anisotropic \\ rough\ interface}}$. Where $Fc_{\substack{anisotropic \\ rough\ interface}}$

$$Fc_{\substack{anisotropic \\ rough\ interface}} = \left[\frac{1}{2(1-\nu_1)} Fw1_{\substack{anisotropic \\ rough\ interface}} - \frac{1}{2(1-\nu_1)} Fw2_{\substack{anisotropic \\ rough\ interface}} \right]$$

$$C = \frac{2(1-\nu_1^2)}{E_1} P_0 a \left[\frac{1}{2(1-\nu_1)} Fw1_{\substack{anisotropic \\ smooth\ interface}} - \frac{1}{2(1-\nu_1)} Fw2_{\substack{anisotropic \\ smooth\ interface}} \right] \quad (3.57)$$

$C = \frac{2(1-\nu_1^2)}{E_1} P_0 a Fc_{\substack{anisotropic \\ smooth\ interface}}$. Where $Fc_{\substack{anisotropic \\ smooth\ interface}}$

$$Fc_{\substack{anisotropic \\ smooth\ interface}} = \left[\frac{1}{2(1-\nu_1)} Fw1_{\substack{anisotropic \\ smooth\ interface}} - \frac{1}{2(1-\nu_1)} Fw2_{\substack{anisotropic \\ smooth\ interface}} \right]$$

CHAPTER 4

NUMERICAL AND ERROR ANALYSIS

Infinite integrals involving products of Bessel functions are commonly encountered in the analysis of the axisymmetric problem, as the current problem and like the problem of an infinite plate resting on a linearly deformable medium. These integrals can be rerepresented in the generalized form

$$I = \int_0^{\infty} F_1(x)\{F_2(x) + F_3(x)J_1(\alpha x)\}J_0(\beta x)dx \qquad \textbf{(4.1)}$$

The numerical evaluation of this type is carried out by representing the integral as an infinite series bounded by subsequent zeros of $J_0(\beta x)$ and $J_1(\alpha x)J_0(\beta x)$.

Integration which proceeds by one interval at a time is carried out by using a number of Gauss-Legender quadrature points. The summation is terminated when the absolute value of the partial integral is less than 0.01%, or until the following condition is satisfied

$$\frac{\textit{Integration of } n^{th} \textit{ region}}{\sum_{n=1}^{n} (\textit{Integration of } n^{th} \textit{ region})} < 0.001 \qquad \textbf{(4.2)}$$

Another way of evaluating this integral could be as follows. The expressions of the displacements of the surface of upper layer as mentioned before is

$W_1 = \dfrac{2(1-\nu_1^2)}{E_1} P_0 a \displaystyle\int_0^{\infty} \dfrac{Numerator}{Denominator} * \dfrac{1}{m} * J_1(ma)dm$ for the center of a loaded circular area

An analytical evaluation of the integrals is impractical if not impossible. However, in order to obtain numerical solution, Burmister suggested that the reciprocal of the denominator expression can be expressed as a series of exponential terms of the

$\lambda\alpha^{n}e^{-b\alpha}$, where $\lambda, n, b,$ are constants. These terms may be multiplied by the terms in the nominator, so that the expression, $\left(\frac{N}{D}\right)*\frac{1}{m}J_1(ma)dm$ becomes a sum of a series of terms of the form $\lambda_1\alpha^{n_1}e^{-b_1\alpha}J_1(ma)dm$ where $\lambda_1, n_1, b_1,$ are constants, the integrals $\lambda_1\int_0^{\infty}\alpha^{n_1}e^{-b_1\alpha}J_1(ma)dm$ are standard integrals forms involving Bessel functions

$$\int_0^{\infty}\alpha^{n}e^{-bm}J_1(ma)dm = -(-1)^n\left(\frac{1}{a}\right)\frac{\partial^n}{\partial b^n}\left[\frac{b}{\sqrt{b^2+a^2}}\right] \tag{4.3}$$

$$\int_0^{\infty}e^{-bm}J_1(ma)dm = -\left(\frac{1}{a}\right)\left[1-\frac{b}{\sqrt{b^2+a^2}}\right] \tag{4.4}$$

Then the equations may be evaluated numerically for the required values of $\nu_1, \nu_2, \frac{E_1}{E_2}, a$

The accuracy of the values calculated in this manner depends primarily on the accuracy of the approximation of $D'(m)=\frac{1}{D(m)}$

If the error kept to a known value of ε by letting $\sqrt{\int_0^{\infty}\left[\frac{1}{D(m)}-D'(m)\right]^2 dm} < \varepsilon$ then the total error of the approximation can be found

$$\int_0^{\infty}\frac{N(m)}{D(m)}J_1(ma)dm - \int_0^{\infty}N(m)D'(m)J_1(ma)dm \le \varepsilon\sqrt{\int_0^{\infty}N^2(m)J_1^2[ma]dm} \tag{4.5}$$

Since ε can be an arbitrary set, the order of approximation can be as precise as desired. This procedure is a real painful trial and error, even though it has been used by Burmister (1943) in his paper "Theory of stresses and displacements in layered system and application to the design of airport runways", Biot (1935) "Effect of certain discontinuities on the pressure distribution in a loaded soil", and Poulos (1967) "stresses and displacements in an elastic layer underlain by rough rigid base"

This agonizing procedure to solve the integration was used at the time where no efficient computers existed.

For our problem a small program in Cmap (an educational program created by Dr. K. H.

Concordia University) was adopted to evaluate these integrations numerically.

This program has a built in function that facilitates the programming work

Since the Bessel function represent a damped oscillatory load with the maximum intensity at the origin, and since the amplitude of the oscillations decrease to zero with increasing the variable, and because it is impossible to evaluate infinite integral numerically, a finite upper limit has been chosen, the reminder integral will not affect the accuracy of the result. The upper limit has been chosen by trial and error to be (350) then the accuracy will be 0.1%. This program serves for all settlement cases of a layered system, an elastic layer underlain by rough rigid base, two layers, and homogeneous deposits, and for both the center and the edge of the load, for a wide range of ν_1, $\frac{E_2}{E_1}$ and $\frac{h}{a}$, and with a little change, this program could serve for the evaluation of compression of the upper layer.

Program in Cmap(1)
Settlement of isotropic elastic upper layer for the center and the edge of the load

```
main ()
{
v2=0.5;
defmat(v1[8],0.1,0.15,0.2,0.25,0.3,0.35,0.4,0.5);
defmat(ah[15],0.01,0.5,1,1.5,2,2.5,3,3.5,4,4.5,5,5.5,6,6.5,7);
defmat (ee [6], 1, 2, 5, 10, 20,100);
numv1=8;
numah=15;
numee=6;
r =0;
view (v2, numv1, numah, numee, r);
if (numv1! =8) {resizemat (v1 [numv1]) ;}
if (numah! =15) {resizemat (ah [numah]) ;}
if (numee! =6) {resizemat (ee [numee]) ;}
view (v1, ah, ee);
zero (integral [numah, numee]);
//view (integral);
for (i=1; i<=numv1; i=i+1)
{       If(r==0) {print (^^"Center") ;}
        If(r! =0) {print (^^"Edge") ;}
        print (^"for v1=", v1 [i]);
        print (^^"    ","E2/E1  ");
        for (t=1; t<=numee; t=t+1)
        {       print (ee[t]) ;}
        for (j=1; j<=numah; j=j+1)
        {
         print (^"h/a=", ah[j]);
                for (m=1; m<=numee; m=m+1)
                {
//claculation n, k, l
                n=ee[m]*(1+v1 [i])/ (1+v2);
                k= (1-n)/ (1+n*(3-4*v1 [i]));
                l= ((3-4*v2)-n*(3-4*v1 [i]))/ ((3-4*v2) +n);

//integration
        If(r==0) {
integral [j, m] =imteg(x, 0, 354, upf(x)/downf(x)*(1/x)*bessj (1, x/ah[j]));
                }
        If(r! =0) {
integral[j,m]=imteg(x,0,354,upf(x)/downf(x)*(1/x)*bessj(0,x/ah[j]) *bessj(1,x/ah[j]));

                }
                print (integral [j, m]);
```

```
            }
        }
}
//plot(x, 1, 7, integ(y, 0.001, 200, upf(y)/downf(y)/y*bessj (1, y/x)));
}
upf (float x)
{
return exp (2*x) +4*k*x-k*l*exp (-2*x);
}
downf (float x)
{
return exp (2*x)-(l+k+4*k*x*x) +k*l*exp (-2*x);
}
```

The following Figures 4-1 to 4 -16 show the relation between the factor of settlement and the ratios $\frac{h}{a}$ and $\frac{E_2}{E_1}$ for rough interface at the center and the edge of the load, for Poisson's ratio of the lower layer equal to 0.5 and for the upper layer equal to

0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, and 0.5.

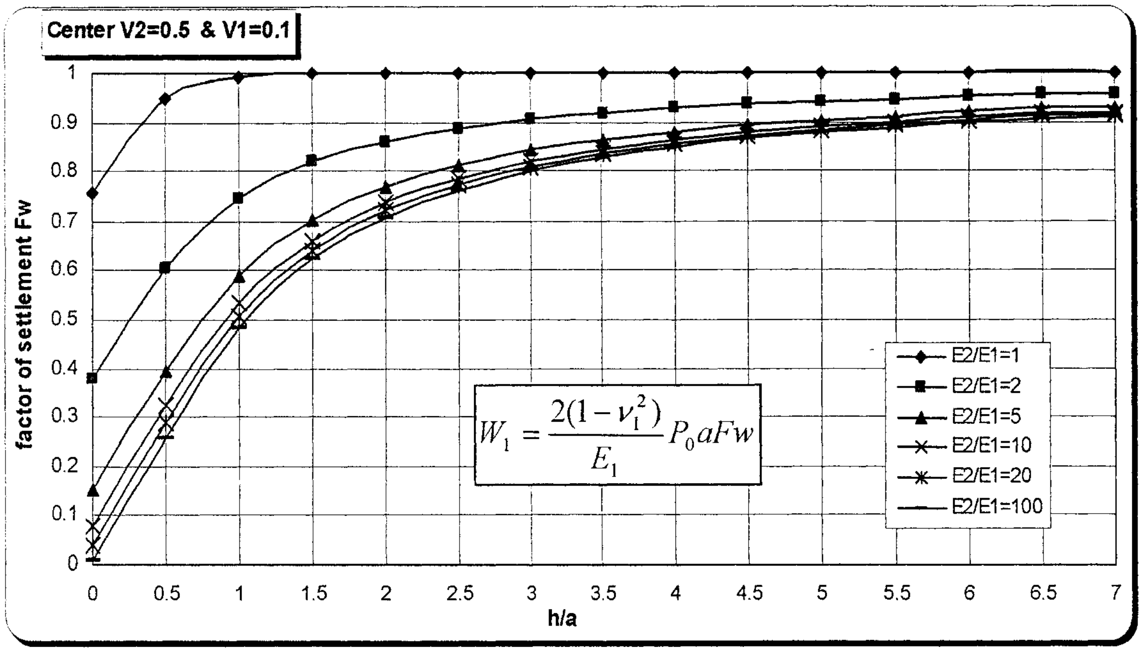

Figure 4-1 Factor of settlement of an isotropic elastic upper layer, at the center of the load, $\nu_1 = 0.1$ & $\nu_2 = 0.5$

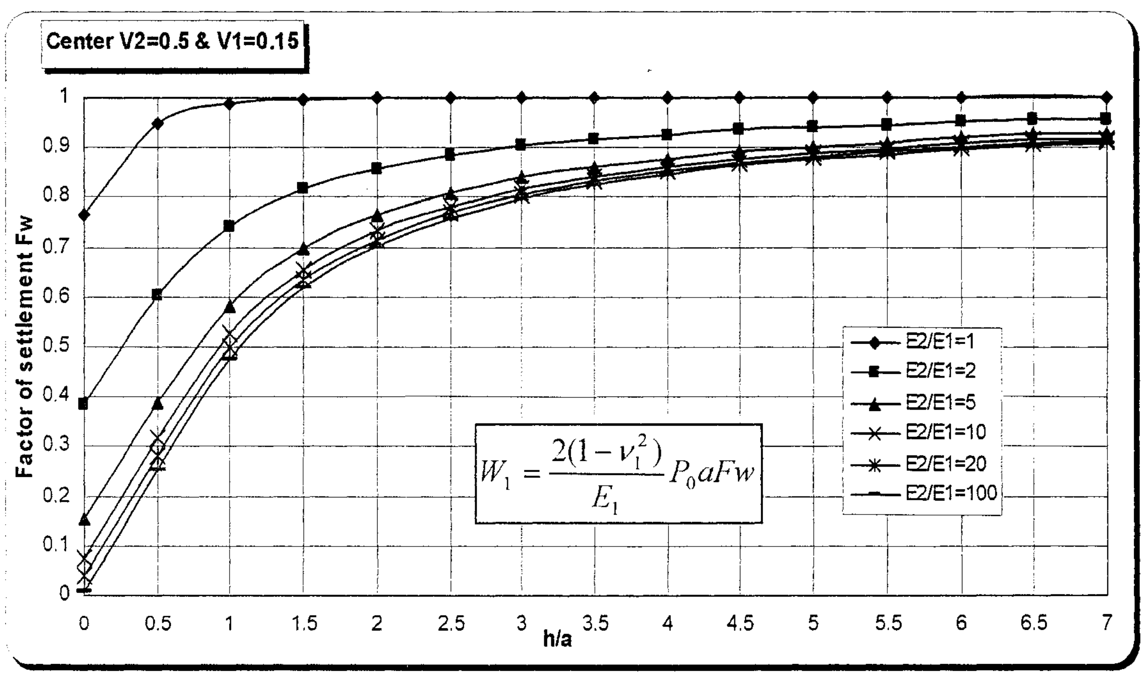

Figure 4-2 Factor of settlement of an ĭsotropic elastic upper layer at the center of the load, $\nu_1 = 0.15$ & $\nu_2 = 0.5$

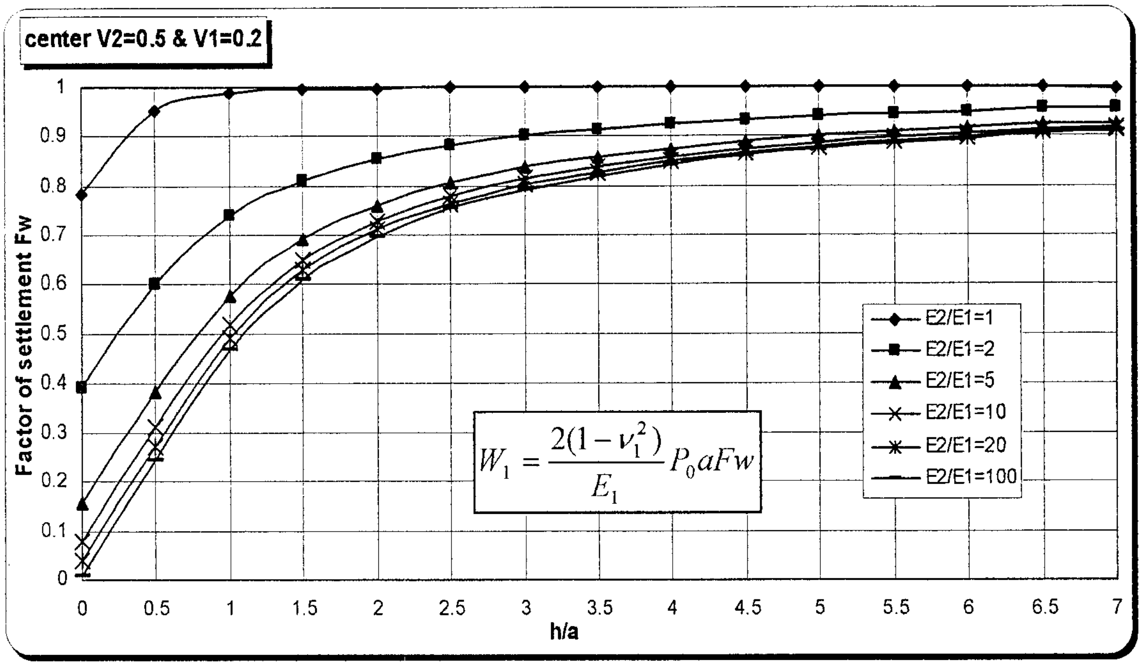

Figure 4-3 Factor of settlement of an isotropic elastic upper layer at the center of the load, ν_1 = 0.2& ν_2 = 0.5

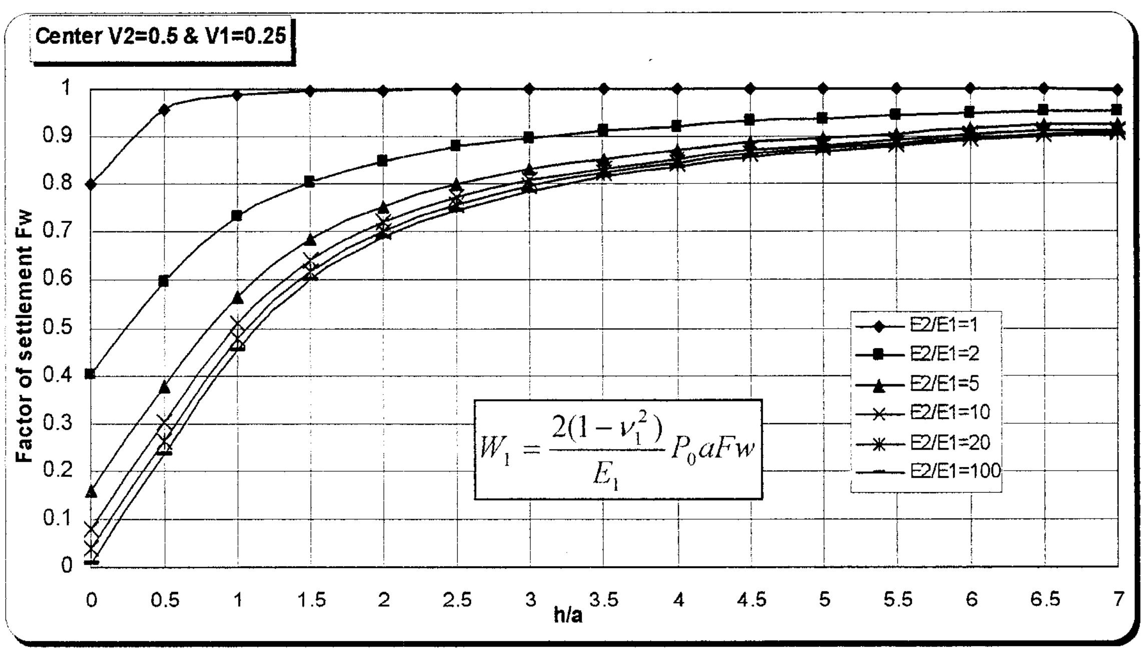

Figure 4-4 Factor of settlement of an isotropic elastic upper layer at the center of the load ν_1 = 0.25 & ν_2 = 0.5

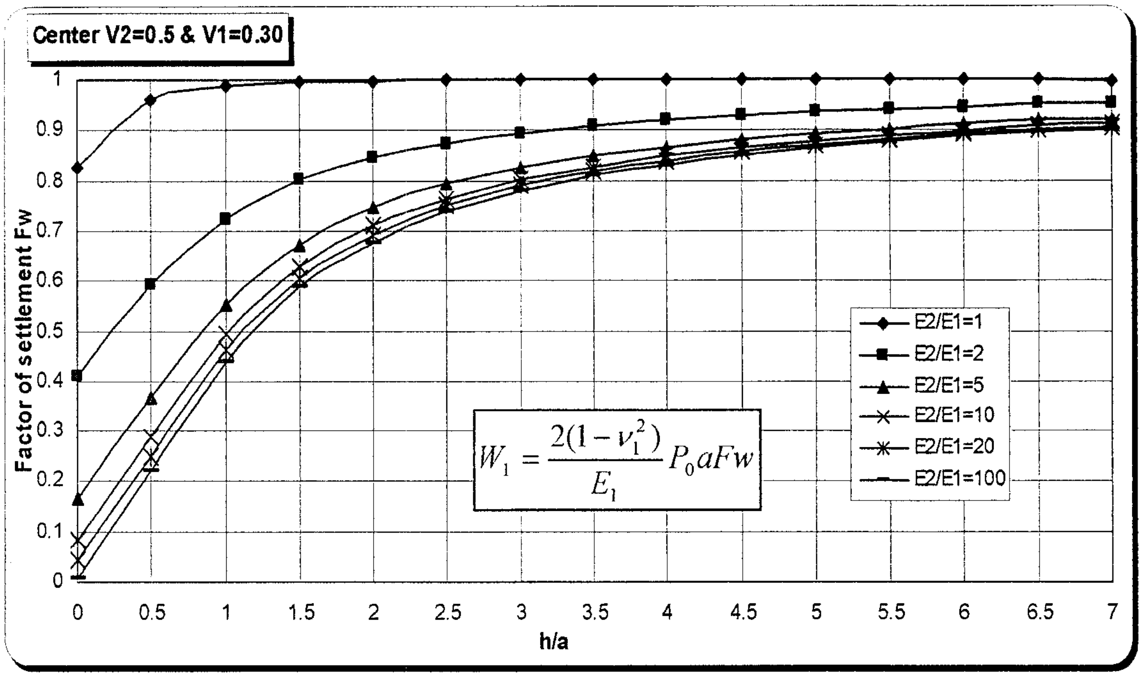

Figure 4-5 Factor of settlement of an isotropic elastic upper layer at the center of the load, $\nu_1 = 0.3$ & $\nu_2 = 0.5$

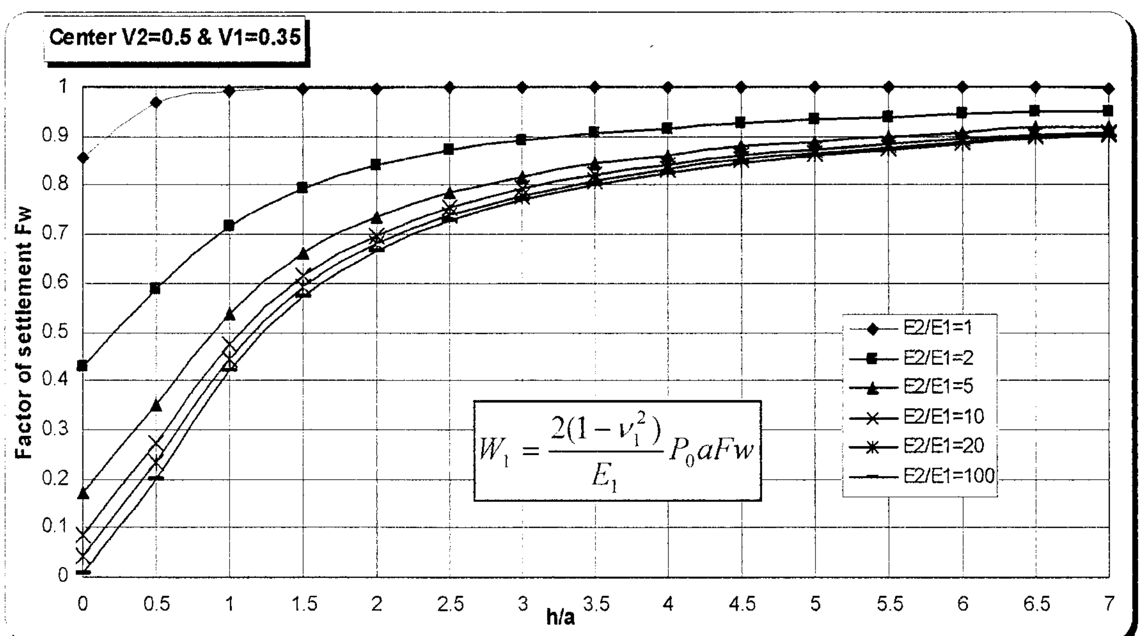

Figure 4-6 Factor of settlement of an isotropic elastic upper layer at the center of the load, $\nu_1 = 0.35$ & $\nu_2 = 0.5$

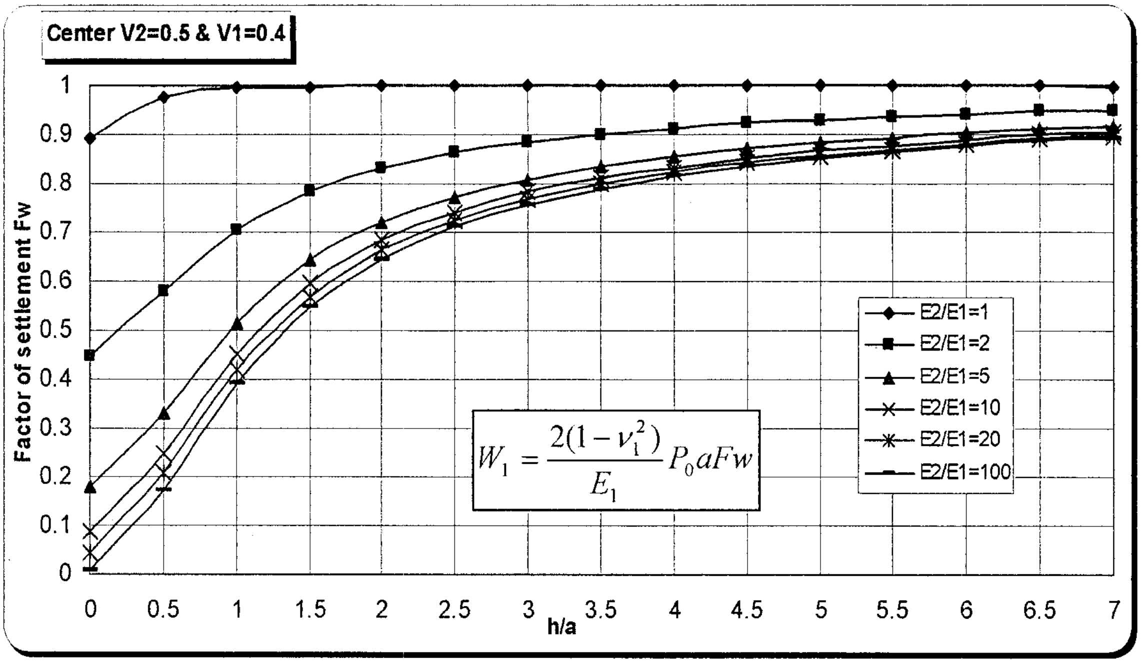

Figure 4-7 Factor of settlement of an isotropic elastic upper layer at the center of the load, $\nu_1 = 0.4$ & $\nu_2 = 0.5$

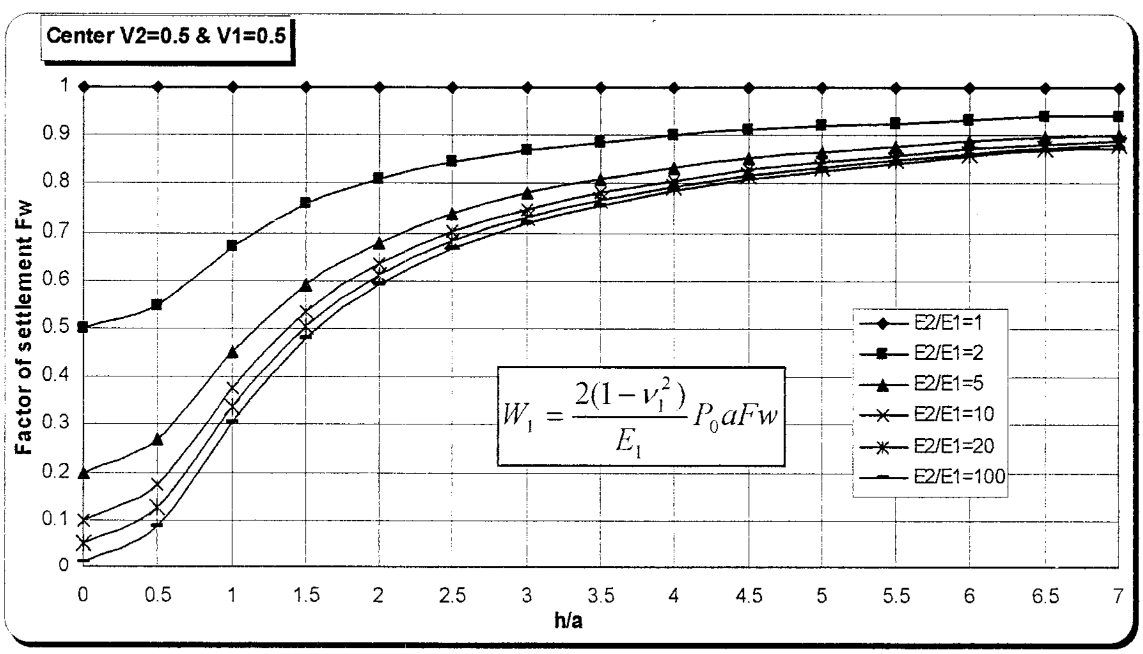

Figure 4-8 Factor of settlement of an isotropic elastic upper layer at the center of the load, $\nu_1 = 0.5$ & $\nu_2 = 0.5$

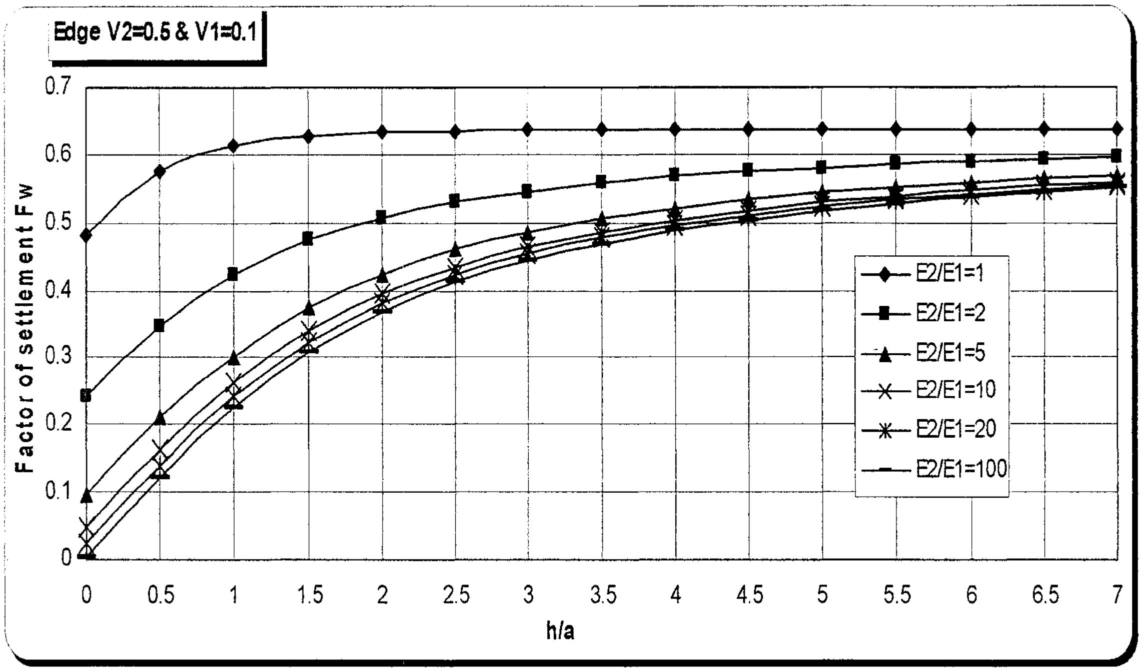

Figure 4-9 Factor of settlement of an isotropic elastic upper layer, at the edge of the load, $\nu_1 = 0.1$ & $\nu_2 = 0.5$

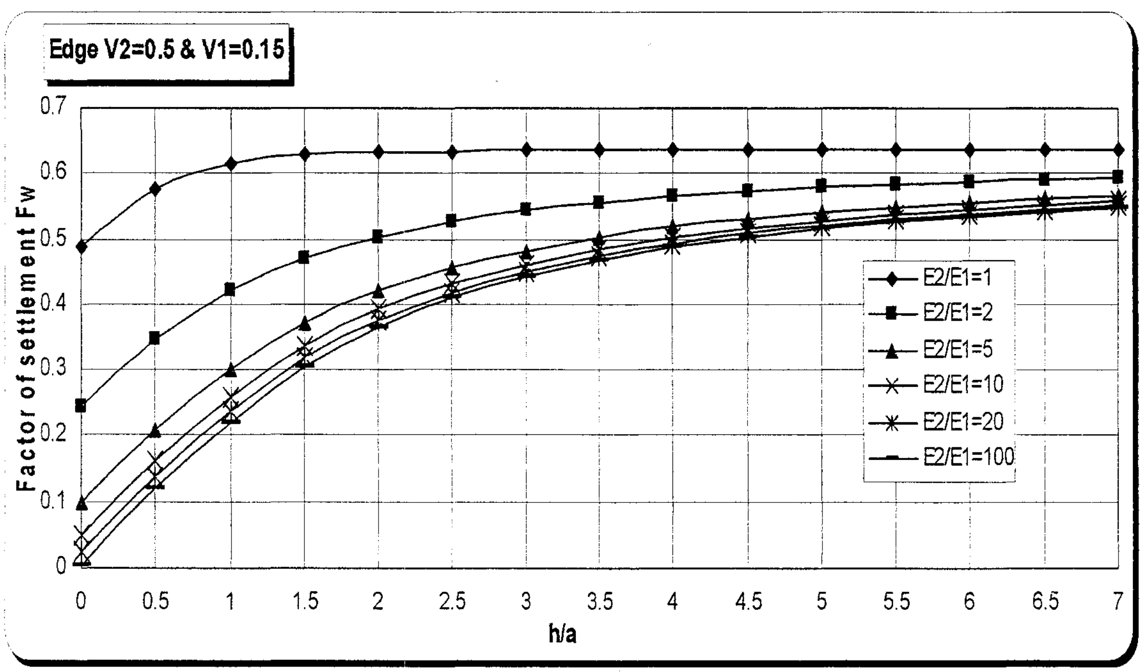

Figure 4-10 Factor of settlement of an isotropic elastic upper layer, at the edge of the load, $\nu_1 = 0.15$ & $\nu_2 = 0.5$

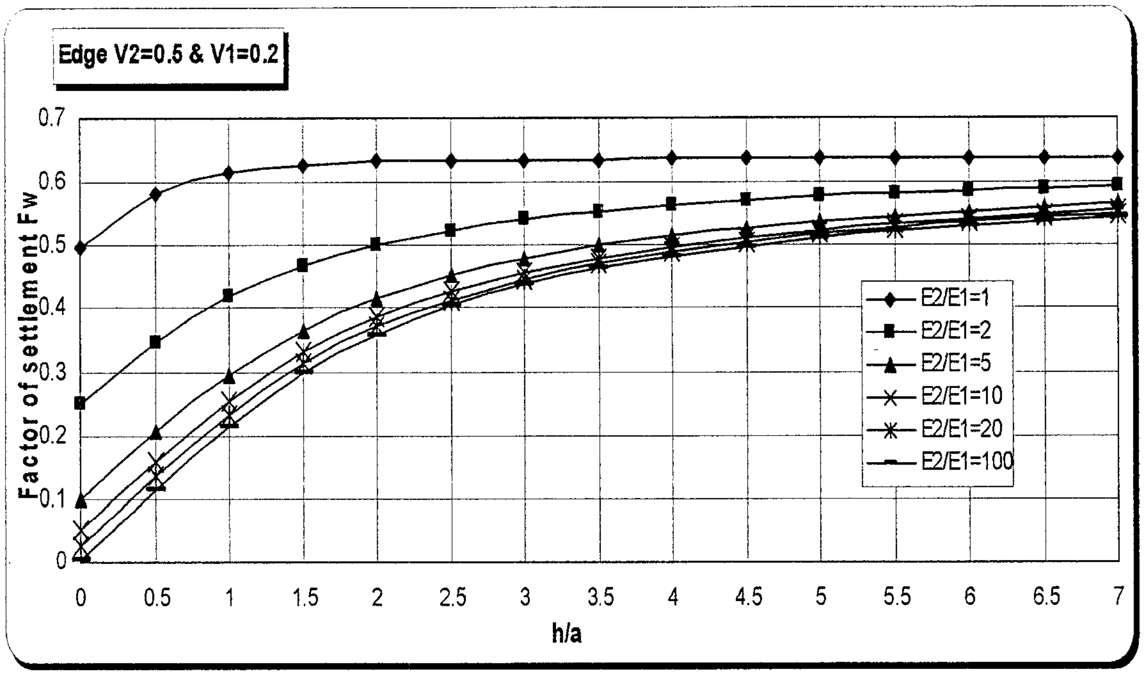

Figure 4-11 Factor of settlement of an isotropic elastic upper layer, at the edge of the load, ν_1 = 0.2 & ν_2 = 0.5

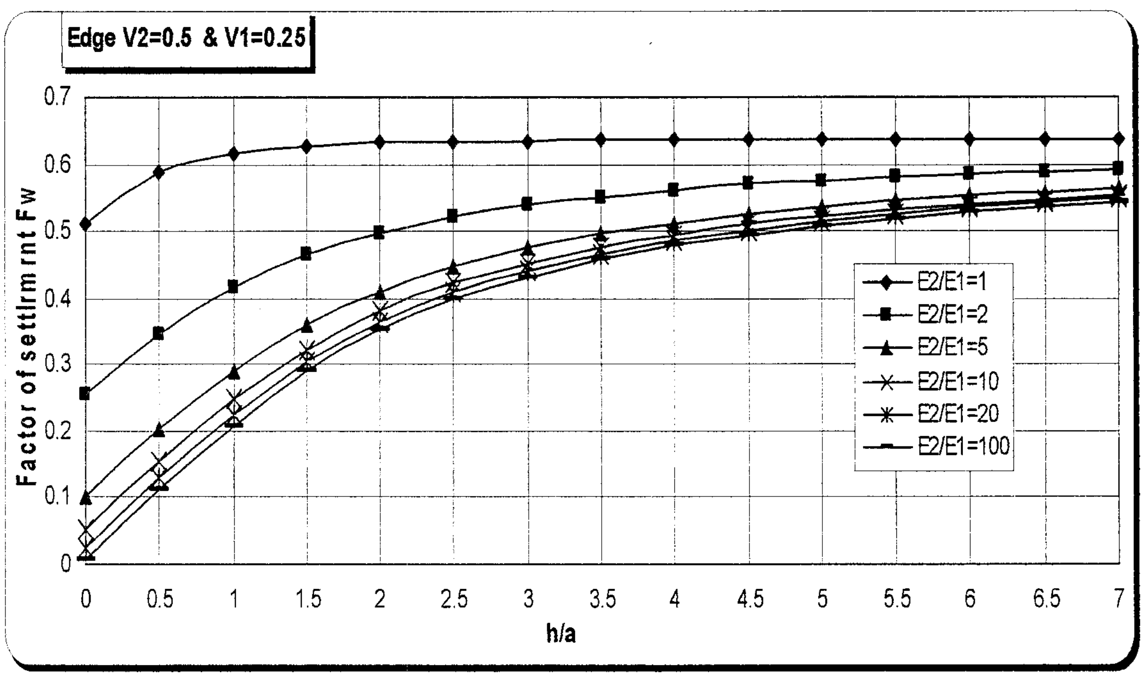

Figure 4-12 Factor of settlement of an isotropic elastic upper layer, at the edge of the load, ν_1 = 0.25 & ν_2 = 0.5

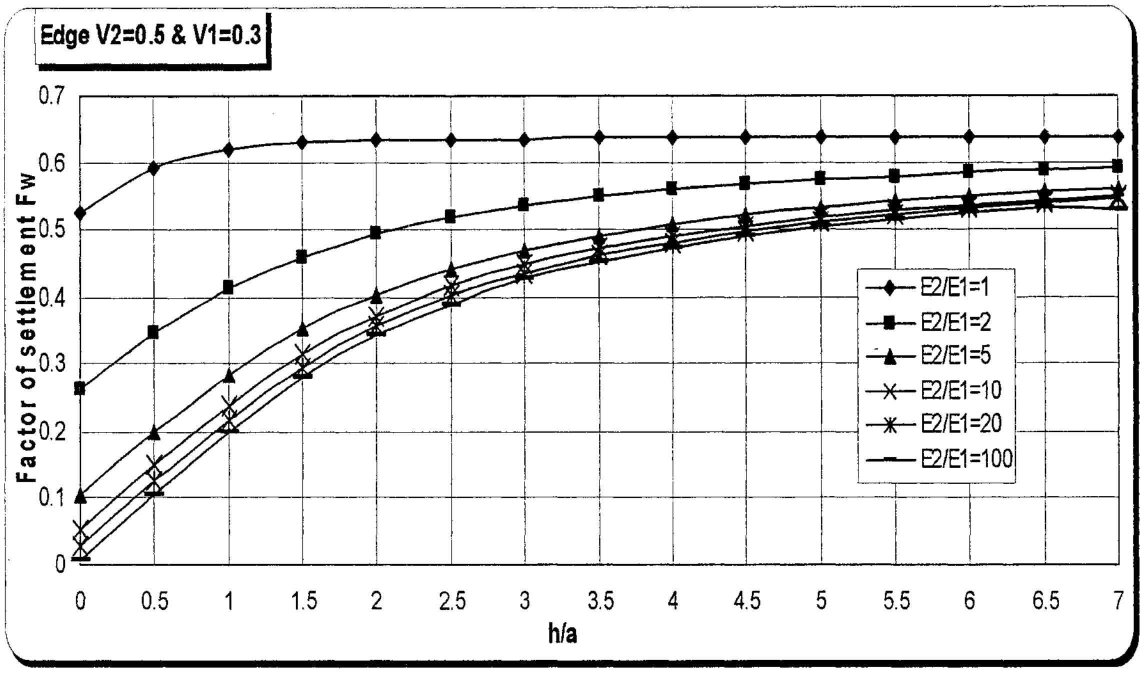

Figure 4-13 Factor of settlement of an isotropic elastic upper layer, at the edge of the load, $\nu_1 = 0.3$ & $\nu_2 = 0.5$

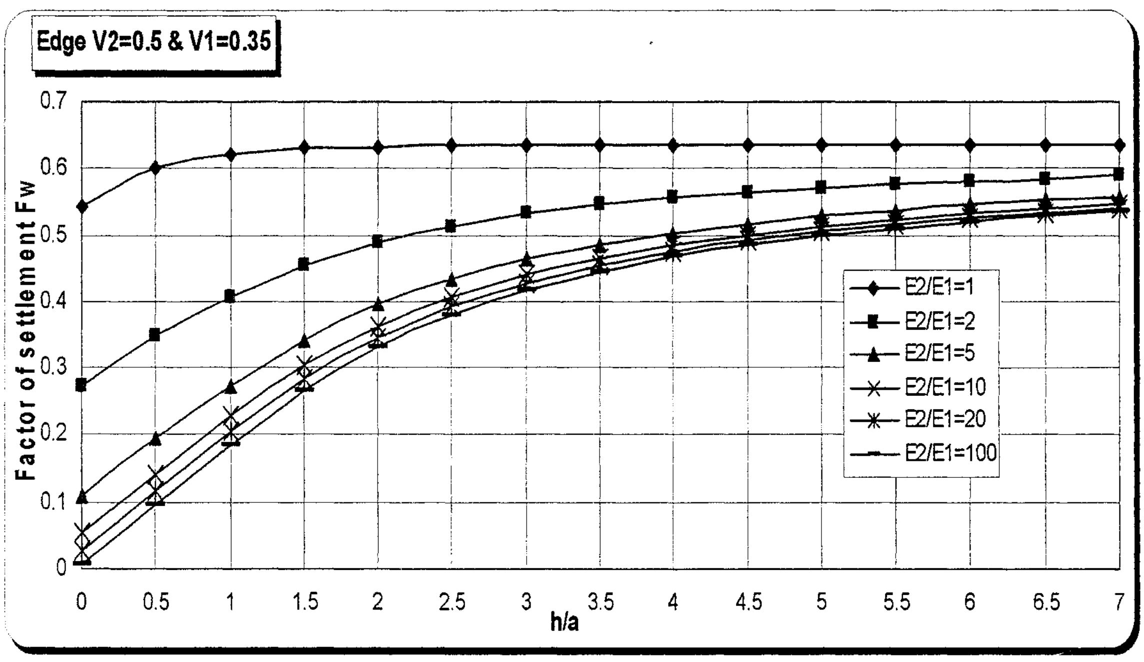

Figure 4-14 Factor of settlement of an isotropic elastic upper layer, at the edge of the load, $\nu_1 = 0.35$ & $\nu_2 = 0.5$

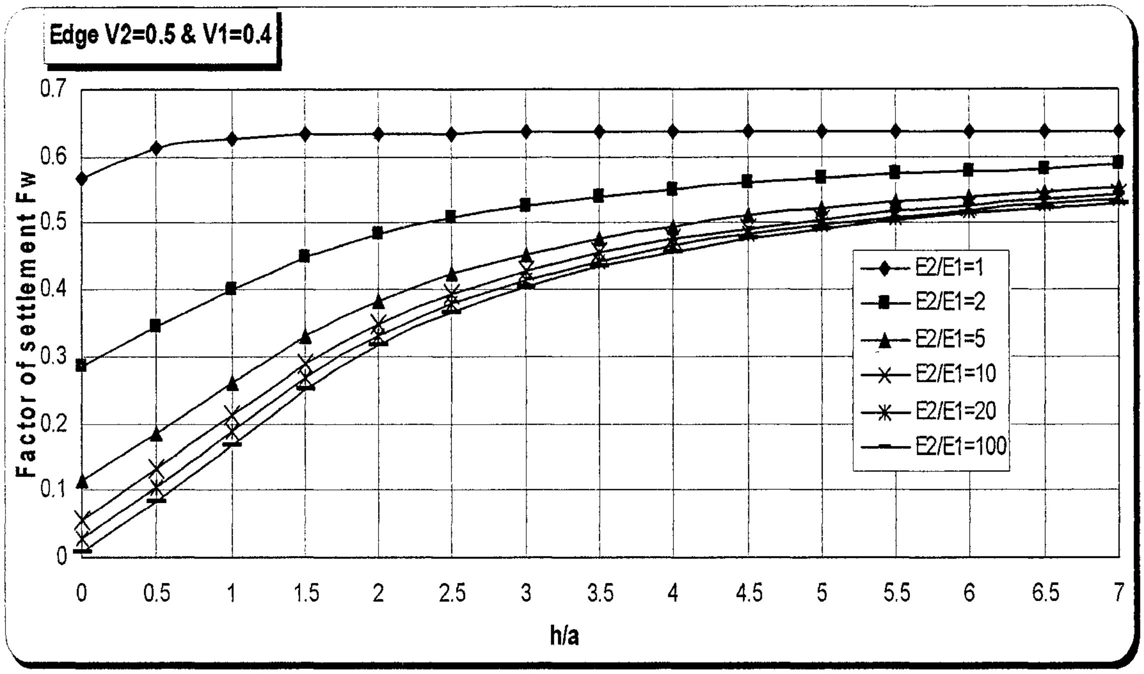

Figure 4-15 Factor of settlement of an isotropic elastic upper layer, at the edge of the load, $\nu_1 = 0.4$ & $\nu_2 = 0.5$

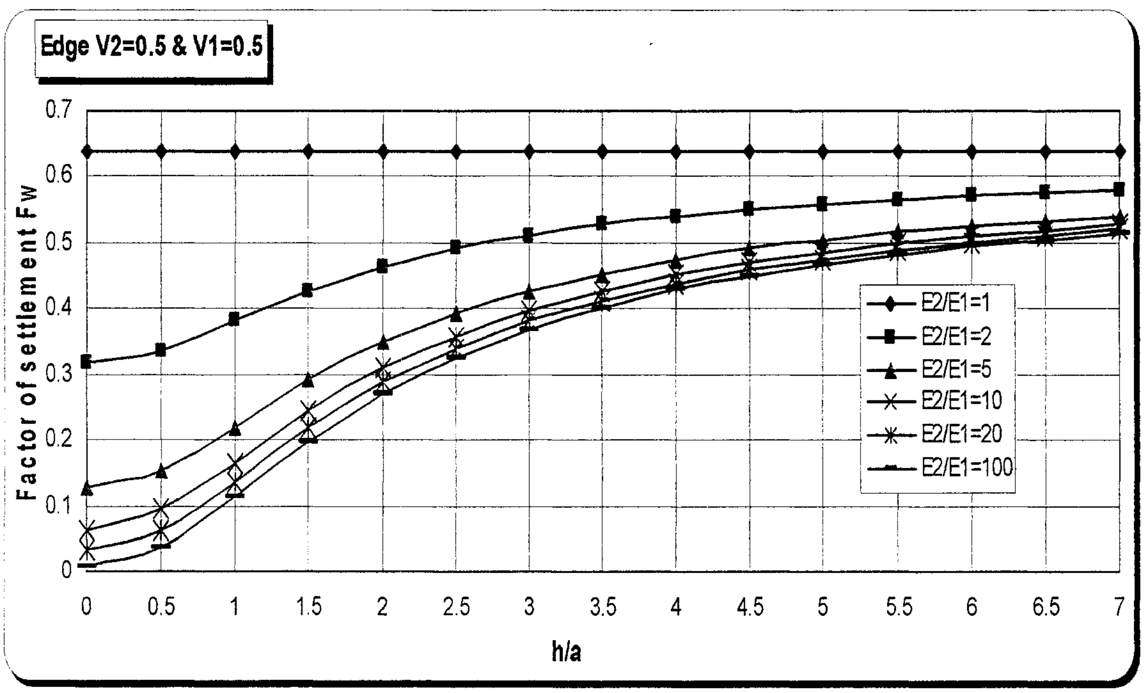

Figure 4-16 Factor of settlement of an isotropic elastic upper layer, at the edge of the load, $\nu_1 = 0.5$ & $\nu_2 = 0.5$

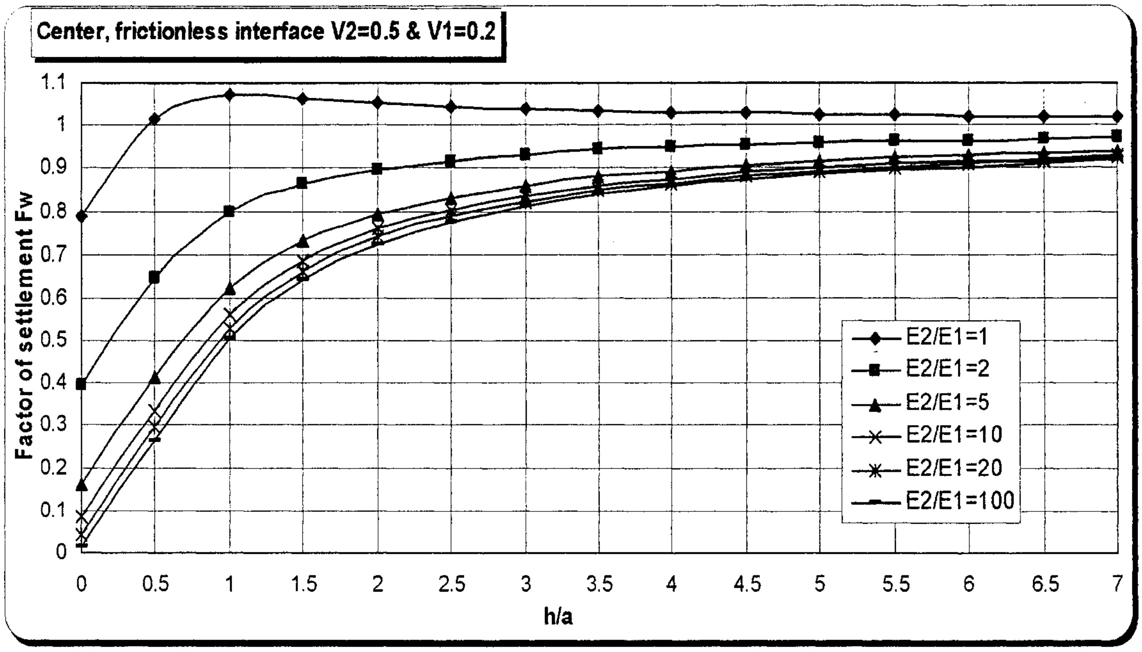

Figure 4-17 Factor of settlement for a frictionless interface, at the center of the load ν_1=0.2 and ν_2=0.5

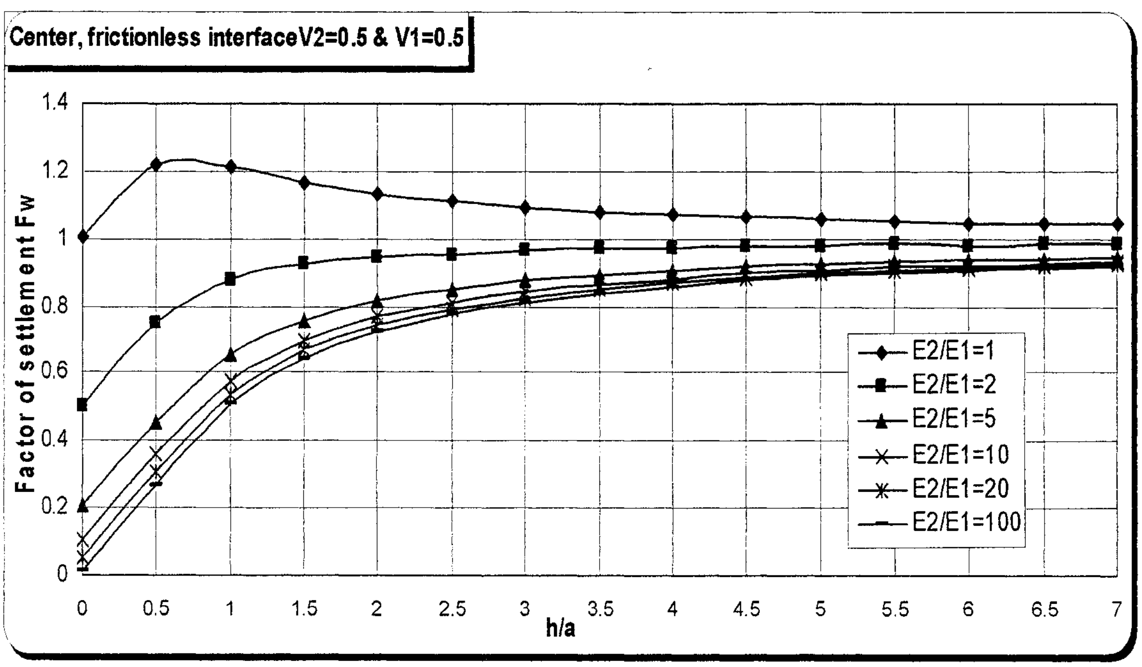

Figure 4-18 Factor of settlement for a frictionless interface, at the center of the load ν_1= 0.5 and ν_2=0.5

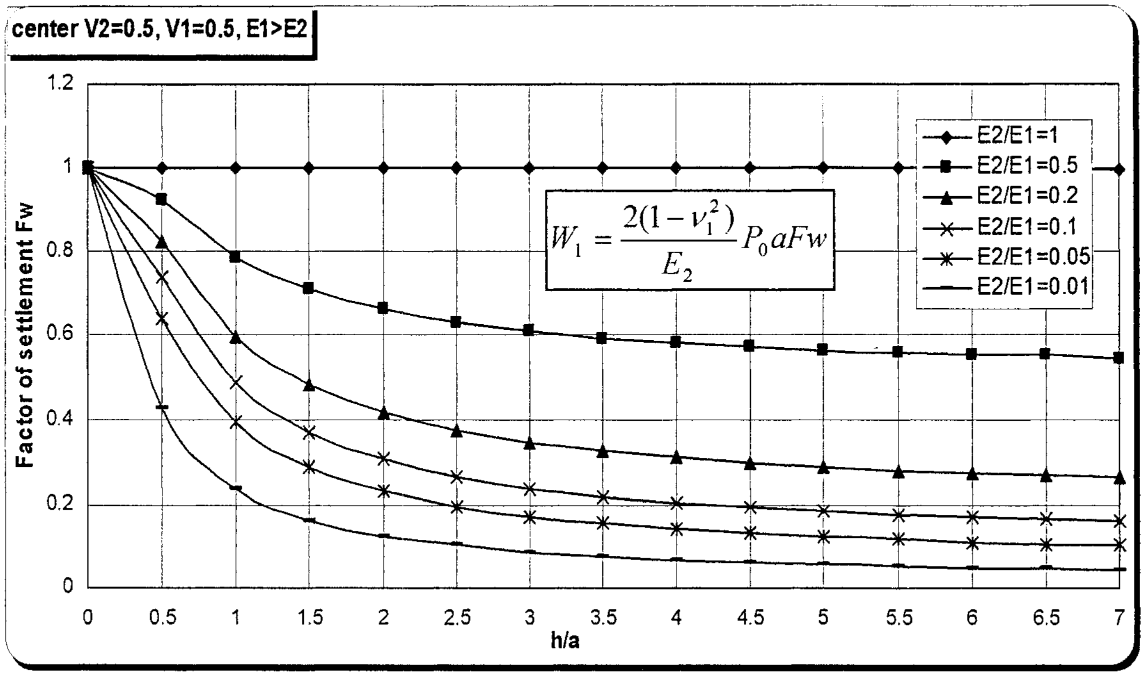

Figure 4-19 Factor of settlement of an isotropic elastic upper layer, at the center of the load, $\nu_1 = 0.5$ & $\nu_2 = 0.5$, for $E_1 > E_2$

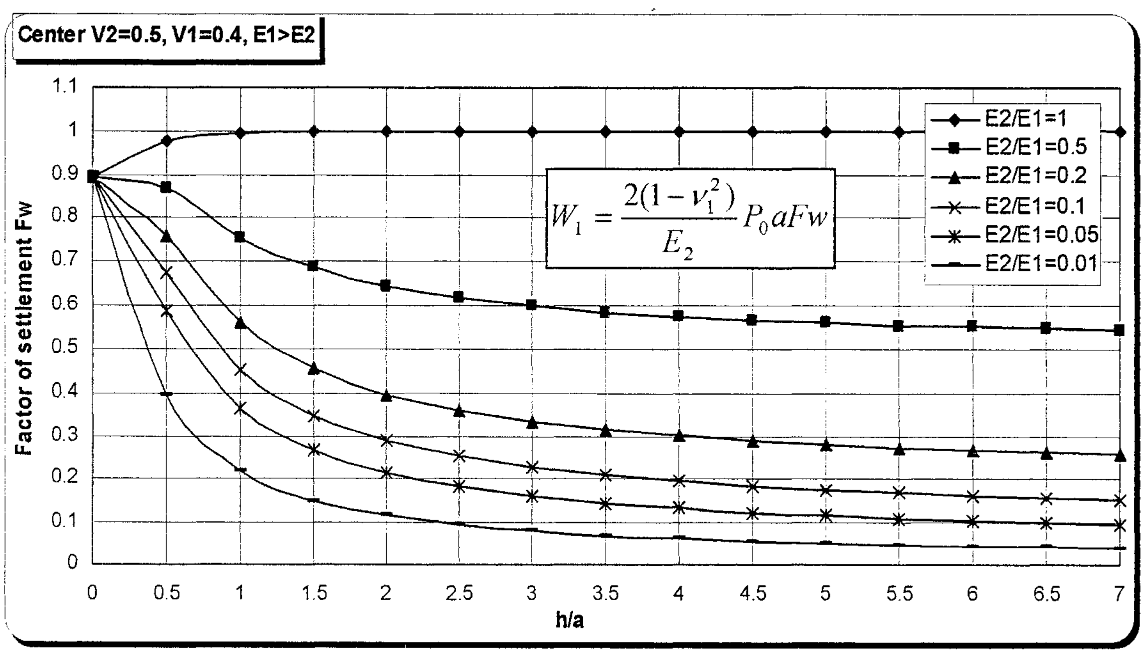

Figure 4-20 Factor of settlement of an isotropic elastic upper layer, at the center of the load, $\nu_1 = 0.4$ & $\nu_2 = 0.5$, for $E_1 > E_2$

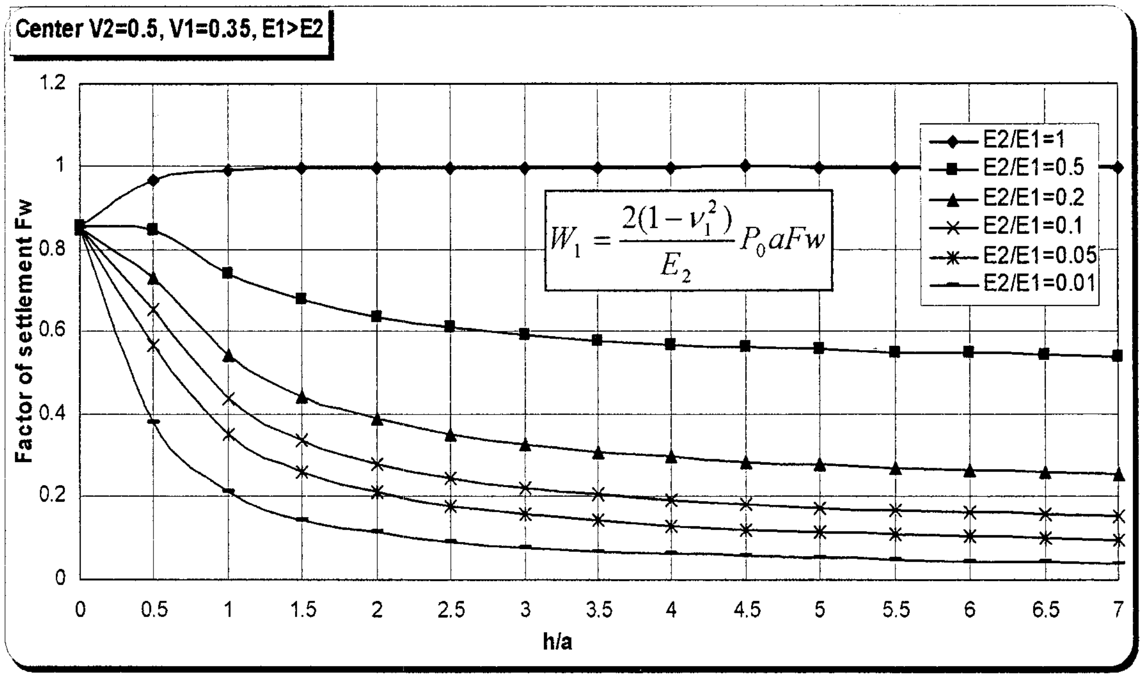

Figure 4-21 Factor of settlement of an isotropic elastic upper layer, at the center of the load, ν_1 = 0.35 & ν_2 = 0.5, for $E_1 > E_2$

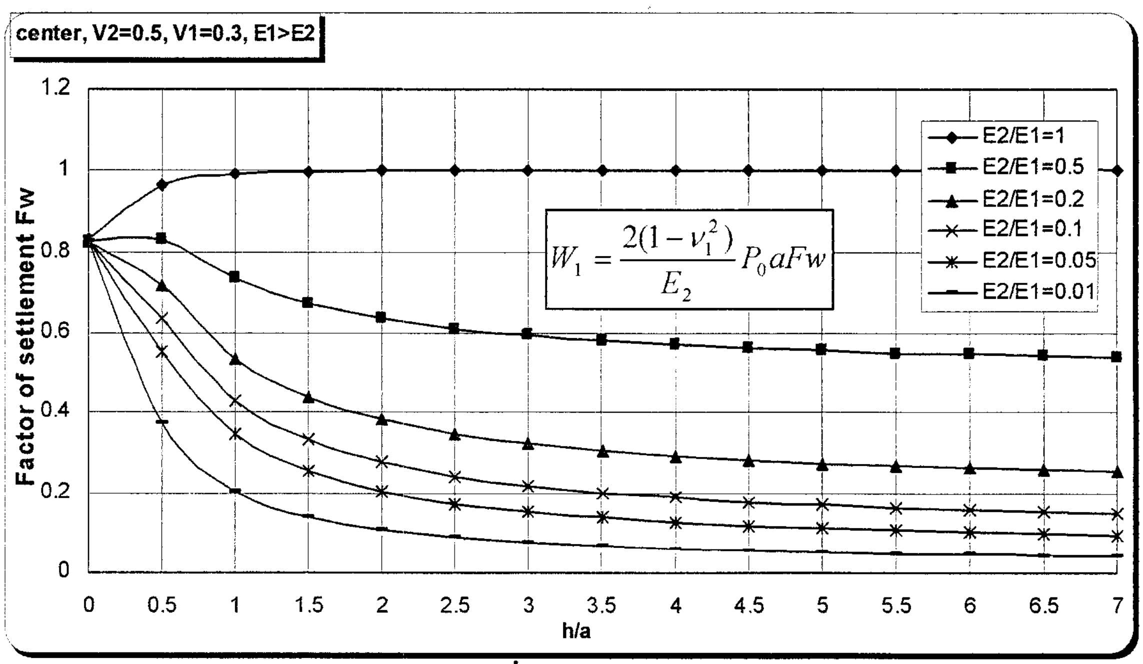

Figure 4-22 Factor of settlement of an isotropic elastic upper layer, at the center of the load, ν_1 = 0.3 & ν_2 = 0.5, for $E_1 > E_2$

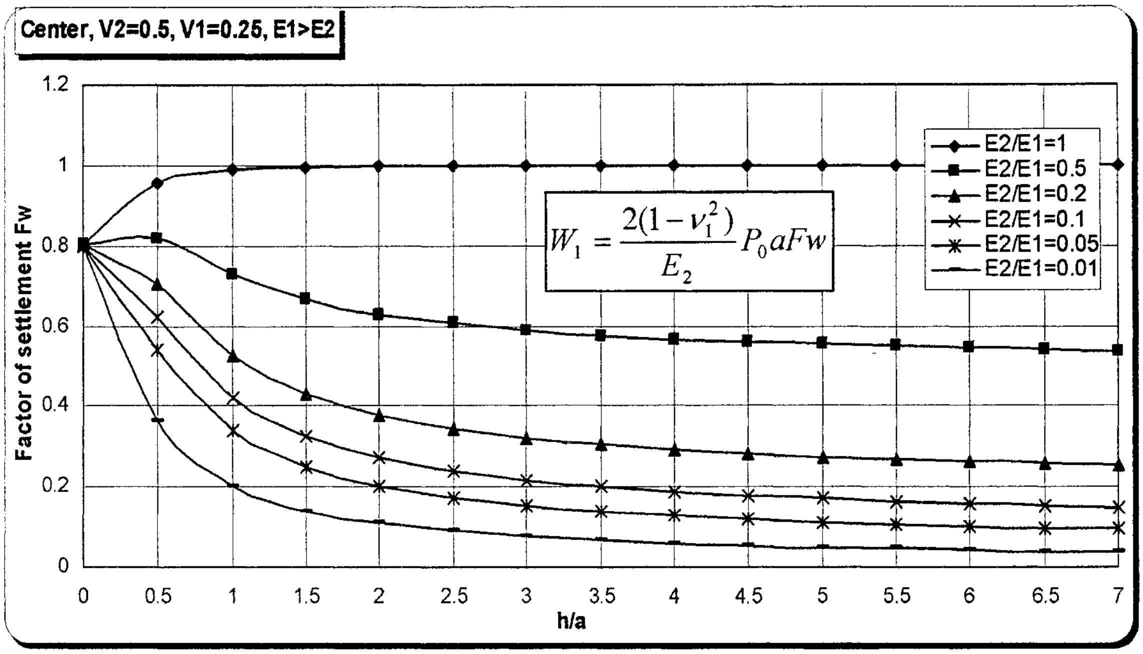

Figure 4-23 Factor of settlement of an isotropic elastic upper layer, at the center of the load, ν_1 = 0.25 & ν_2 = 0.5, for $E_1 > E_2$

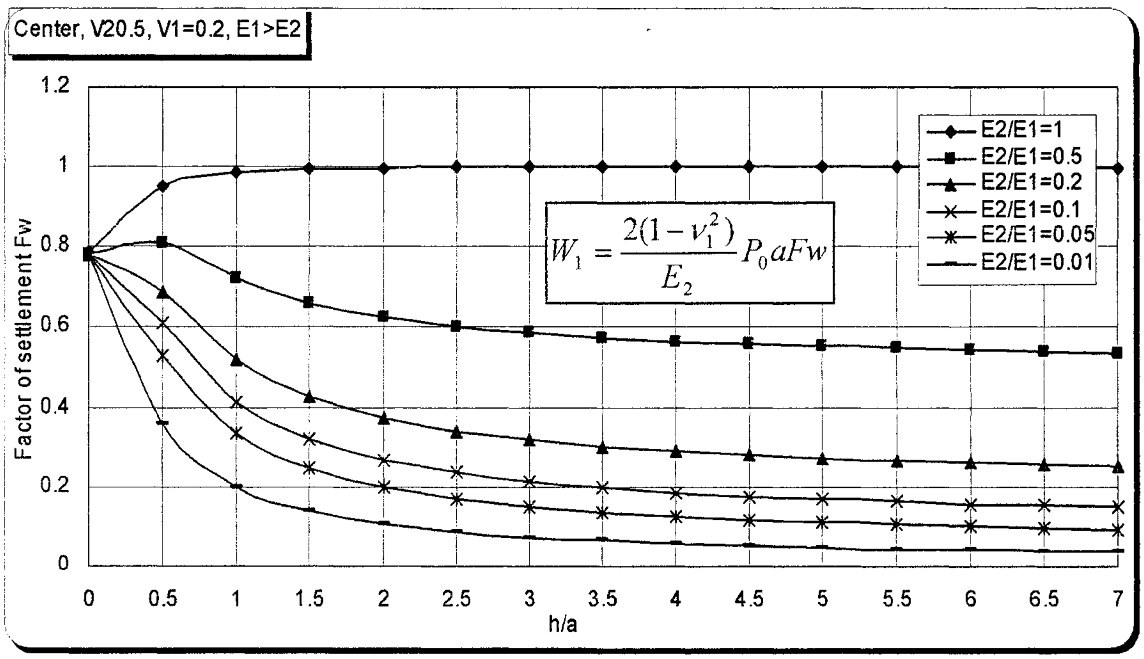

Figure 4-24 Factor of settlement of an isotropic elastic upper layer, at the center of the load, ν_1 = 0.20 & ν_2 = 0.5, for $E_1 > E_2$

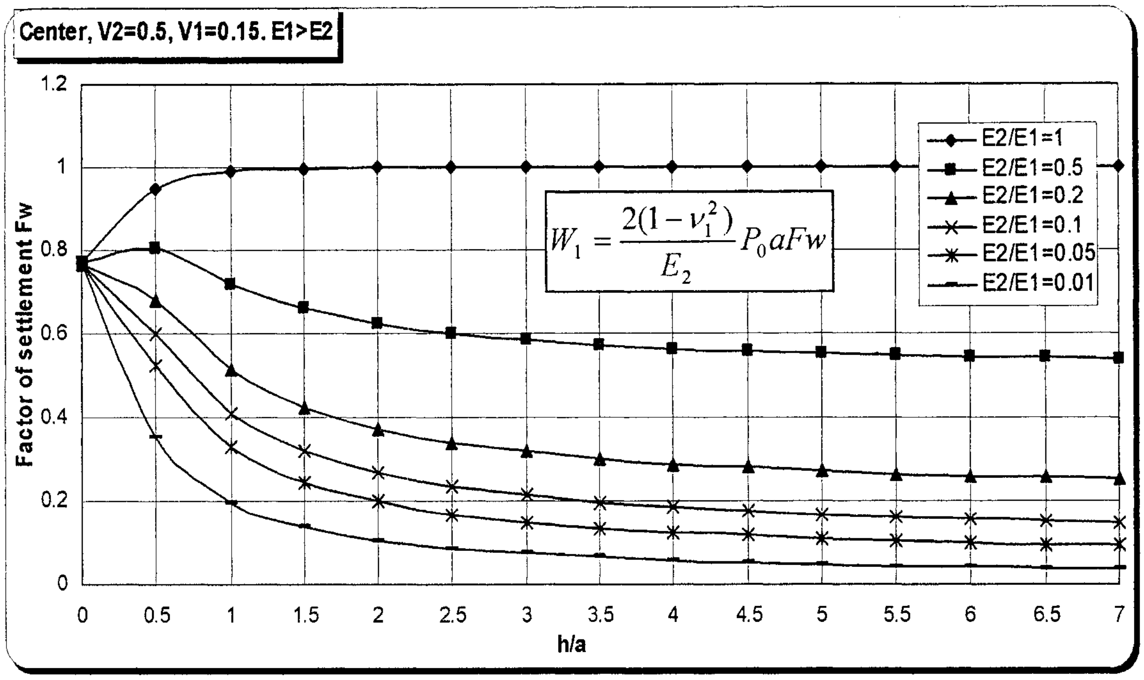

Figure 4-25 Factor of settlement of an isotropic elastic upper layer, at the center of the load, $\nu_1 = 0.15$ & $\nu_2 = 0.5$, for $E_1 > E_2$

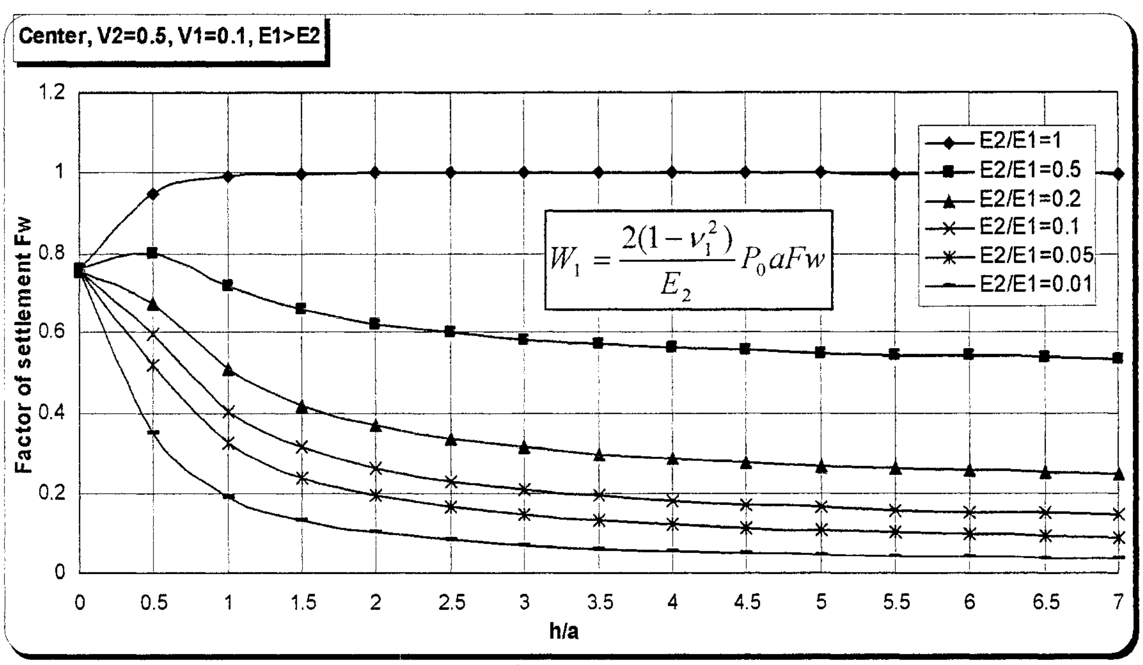

Figure 4-26 Factor of settlement of an isotropic elastic upper layer, at the center of the load, $\nu_1 = 0.1$ & $\nu_2 = 0.5$, for $E_1 > E_2$

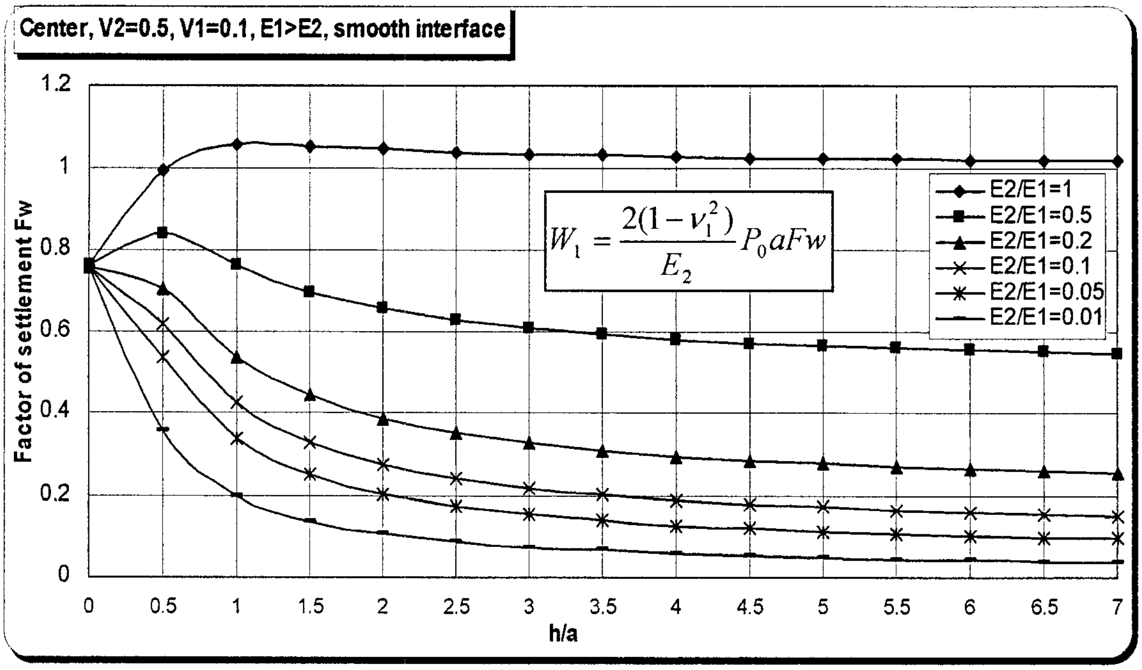

Figure 4-27 Factor of settlement for a frictionless interface, at the center of the load, ν_1 **= 0.1 &** ν_2**= 0.5,** $E_1 > E_2$

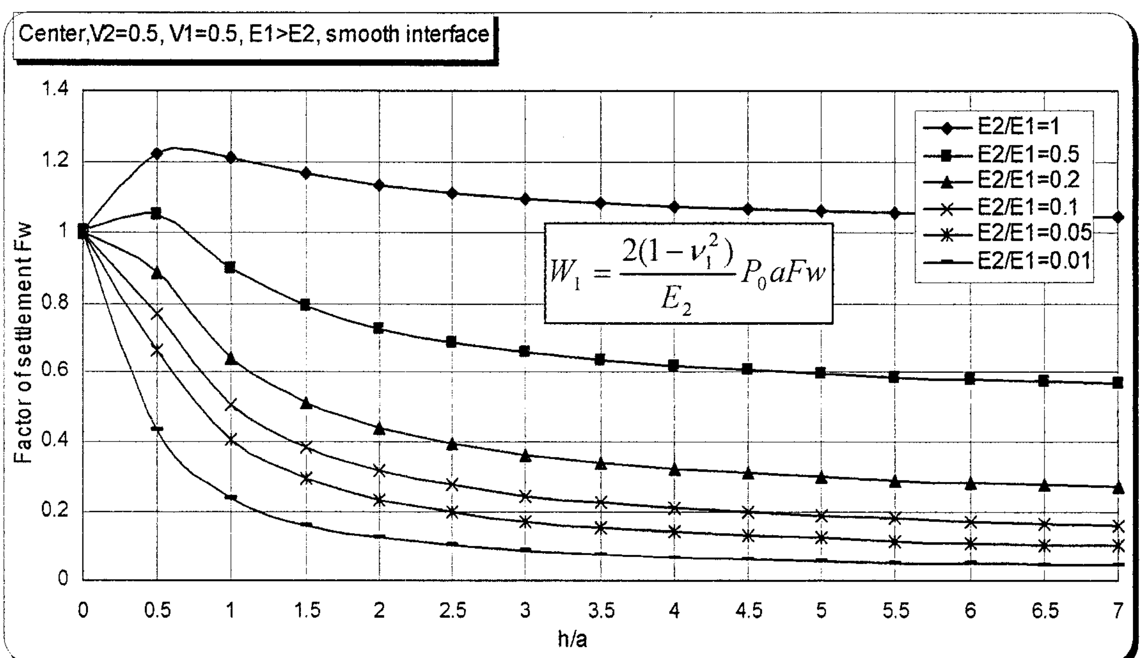

Figure 4-28 Factor of settlement for a frictionless interface, at the center of the load, ν_1 **= 0.5 &** ν_2**= 0.5,** $E_1 > E_2$

Program in Cmap (2)
Settlement of anisotropic elastic upper layer for the center and the edge of the load

```
main()
{
v2=0.5;
n1=1.0000001;
n2=1.0000001;
defmat(v1[8],0.1,0.15,0.2,0.25,0.3,0.35,0.4,0.5);
defmat(ah[15],0.01,0.5,1,1.5,2,2.5,3,3.5,4,4.5,5,5.5,6,6.5,7);
defmat(ee[6],1,2,5,10,20,100);
numv1=8;
numah=15;
numee=6;
r=0;
view(v2,numv1,numah,numee,r);
if(numv1!=8){resizemat(v1[numv1]);}
if(numah!=15){resizemat(ah[numah]);}
if(numee!=6){resizemat(ee[numee]);}
view(v1,ah,ee);
zero(integral[numah,numee]);
//view(integral);
for(i=1;i<=numv1;i=i+1)
{
       if(r==0){print(^^"Center");}
        if(r!=0){print(^^"Edge");}
        print(^"for v1=",v1[i]);
        print(^^"     ","E2/E1  ");
        for(t=1;t<=numee;t=t+1)
        {       print(ee[t]);}
        for(j=1;j<=numah;j=j+1)
        {
         print(^"h/a=",ah[j]);
                for(m=1;m<=numee;m=m+1)
{
//claculation N,k,L,s1,s2,A,B,F,g,J,V,W
        N=ee[m]*(1+v1[1])/(1+v2);
        k=(n1+v1[1])/(1+v1[1]);
        L=(n2+v2)/(1+v2);
        s1=((n1-v1[1]*v1[1])/(n1*n1-v1[1]*v1[1]))^0.5;
        s2=((n2-v2*v2)/(n2*n2-v2*v2))^0.5;
        A=(N-1)/(s2*L-s2);
        B=(N*k*s1-s1)/(s2*L-s2);
        F=(A*(1-s2))/(2+A*(1-s2));
        g=((1+s1)+B*(1-s2))/(2+A*(1-s2));
        J=((1-s1)-B*(1-s2))/(2+A*(1-s2));
```

```
        V=((N*k-L)-A*(1-s2*L)+F*((N*k+L)+A*(1-s2*L)))/((N+s1*L)+B*(1-s2*L)
                -g*((N*k+L)+A*(1-s2*L)));
        W=((N-s1*L)-B*(1-s2*L)-J*((N*k+L)+A*(1-s2*L)))/((N+s1*L)+B*(1-s2*L)-
                g*((N*k+L)+A*(1-s2*L)));
//integration
        if(r==0){
integral[j,m]=imteg(x,0,354,upf(x)/downf(x)*(1/x)*bessj(1,x/ah[j]));
                }
        if(r!=0){
integral[j,m]=imteg(x,0,354,upf(x)/downf(x)*(1/x)*bessj(0,x/ah[j])*bessj(1,x/ah[j]));

                }
                print(integral[j,m]);
                }

        }
}
//plot(x,1,7,integ(y,0.001,200,upf(y)/downf(y)/y*bessj(1,y/x)));
}
upf(float x)
{
return -(-s1*(1-k)*exp(x+s1*x)-W*s1*(1-k)*exp(x-s1*x)+(W*F+V*J)*s1*(1- k)*
exp(-x-s1*x)+(F+V*g)*s1*(1-k)*exp(-x+s1*x));
}
downf(float x)
{
return 2*(1-v1[i])*(-2*W*g-2*V*s1+2*J)-exp(x+s1*x)+W*((1+s1))*exp(x-s1*x)
+(V*g+F)*((1+s1))*exp(-x+s1*x)-(W*F+V*J)*exp(-x-s1*x);
}
}
```

This program allows calculation of the settlement for both isotropic and anisotropic layers. The following figures show the relation between the factor of settlement and the ratios $\frac{h}{a}$ and $\frac{E_2}{E_1}$ for anisotropic elastic upper layer resting on elastic half space, the reverse case, and anisotropic two-layer, for both rough and smooth interfaces at the center of the load. Because of the shortage of time to do some experiments to determine the properties of the anisotropic materials, the results of the Cauwelaert and Cerisier (1982) experiment have been adopted. Therefore an anisotropic layer has been chosen to be material number 1 (natural sand, 0/lmm), and material number 3 (crushed stone + natural sand), see Table (2-1),

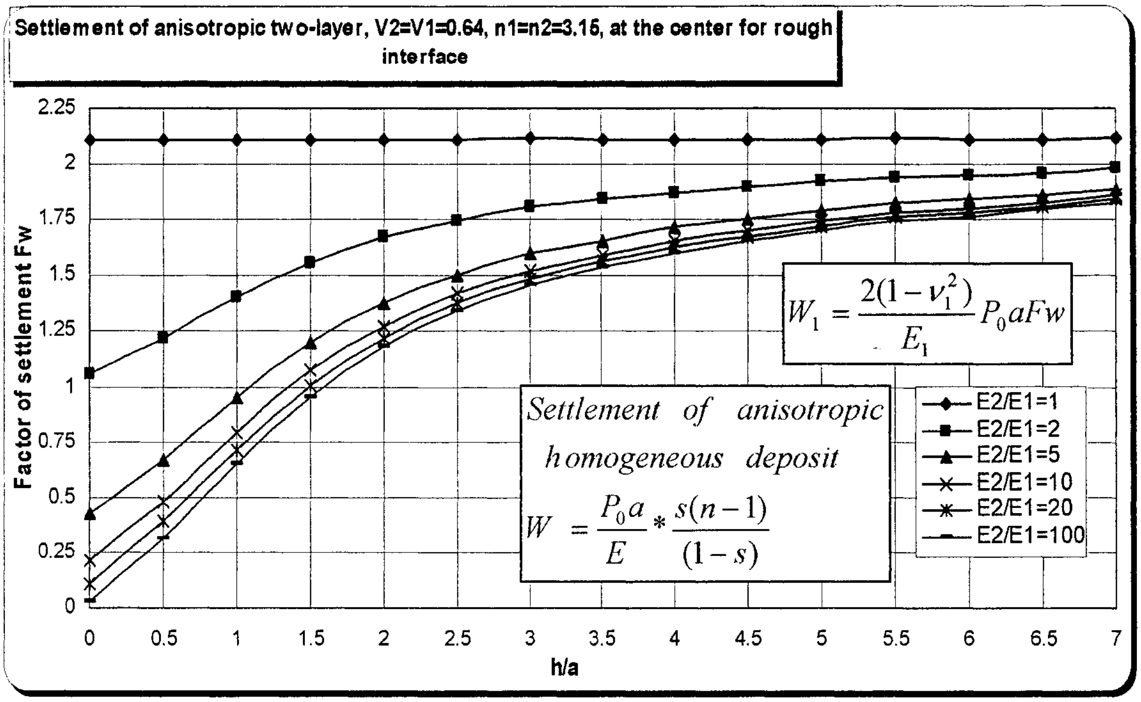

Figure 4-29 Factor of settlement of an anisotropic two-layer, at the center of the load for a rough interface, $\nu_1 = \nu_2$ =0.64, n1=n2=3.15

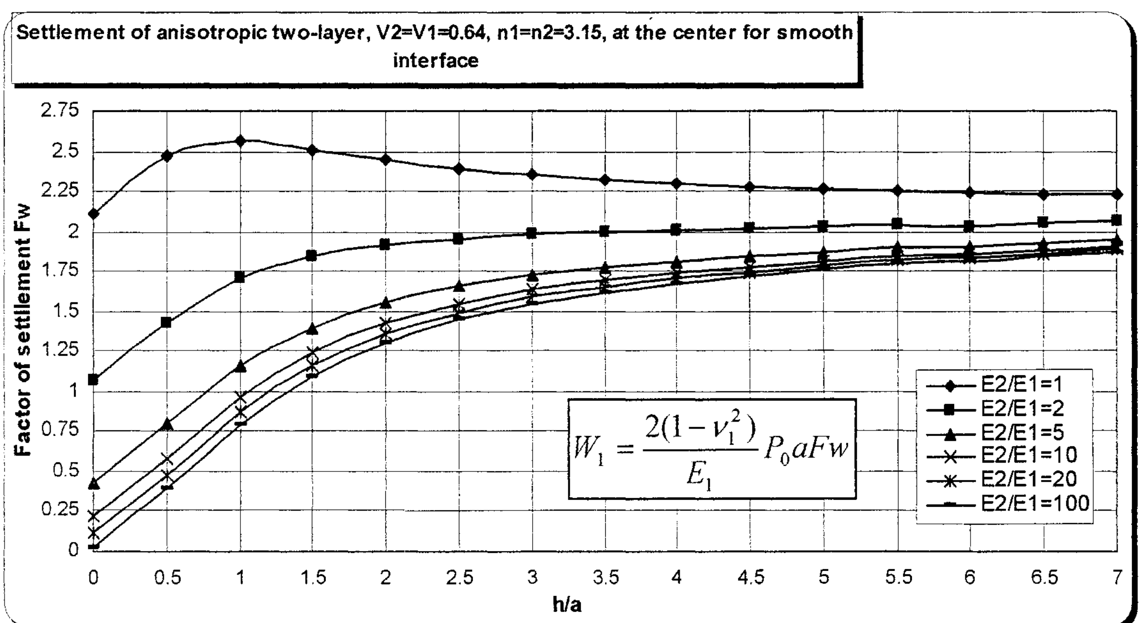

Figure 4-30 Factor of settlement of an anisotropic two-layer, at the center of the load for a smooth interface, $\nu_1 = \nu_2$ =0.64, n1=n2=3.15

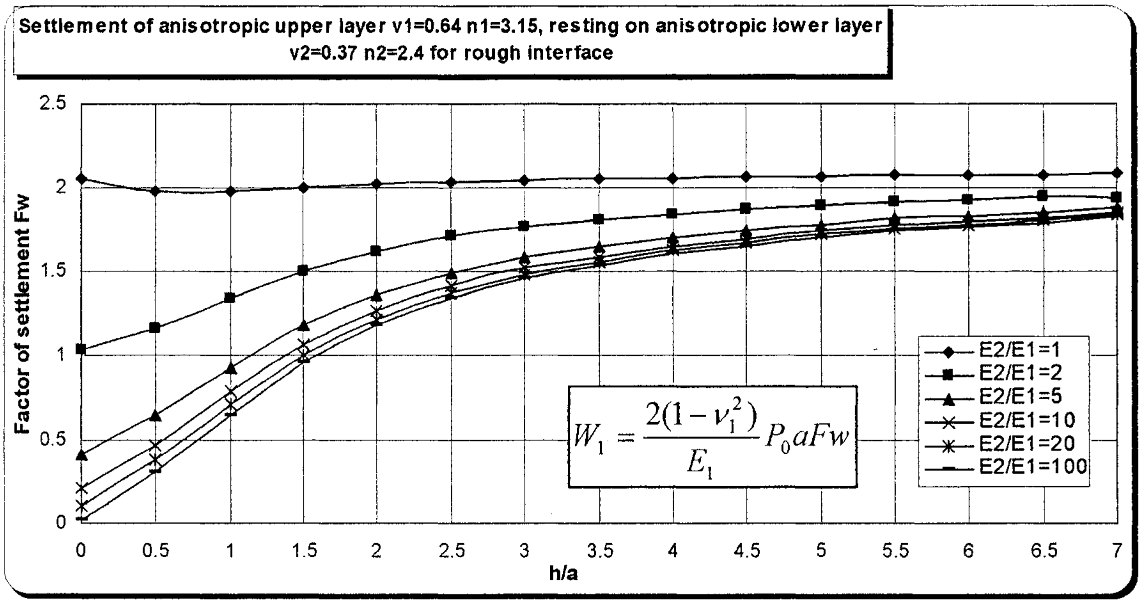

Figure 2-31 Settlement of an anisotropic elastic upper layer ν_1= 0.64, n1=3.15 resting on an anisotropic elastic lower layer ν_2=0.37, n2=2.4 for a rough interface (Natural sand over crushed stone + natural sand)

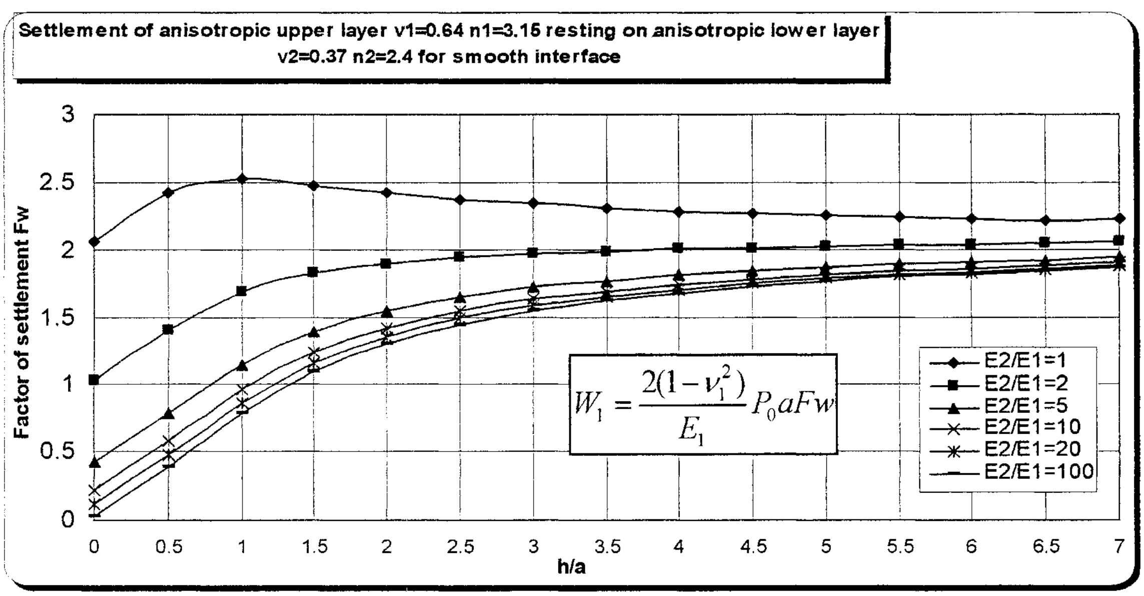

Figure 2-32 Settlement of an anisotropic elastic upper layer ν_1= 0.64, n1=3.15 resting on an anisotropic elastic lower layer ν_2=0.37, n2=2.4 for a smooth interface (Natural sand over crushed stone + natural sand)

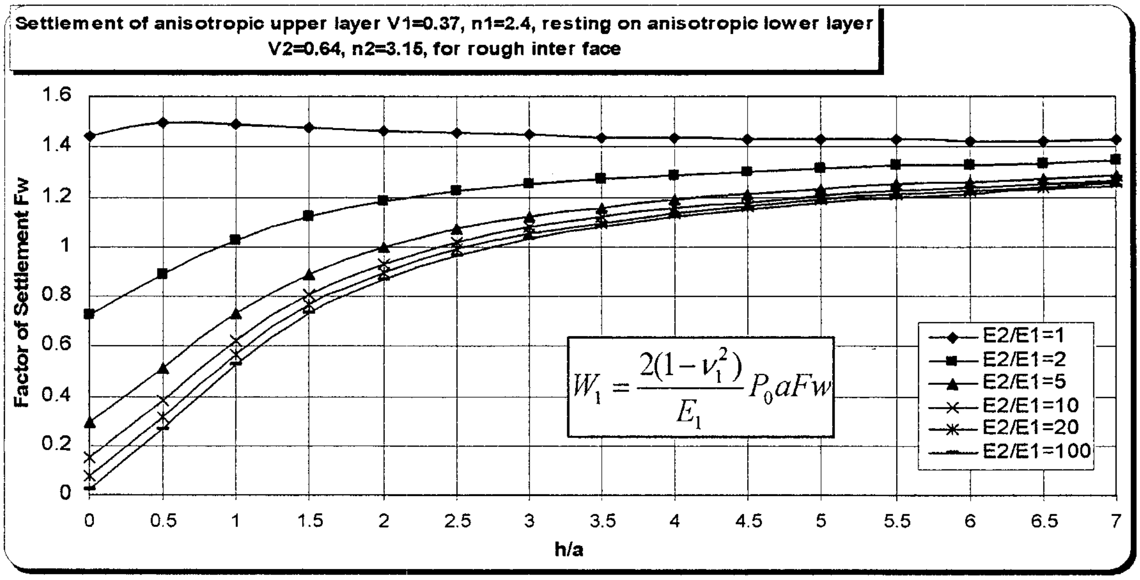

Figure 2-33 Settlement of an anisotropic elastic upper layer $\nu_1 = 0.3$, n1=2.4 resting on an anisotropic elastic lower layer ν_2=0.64, n2=3.15 for a rough interface (crushed stone + natural sand over Natural sand)

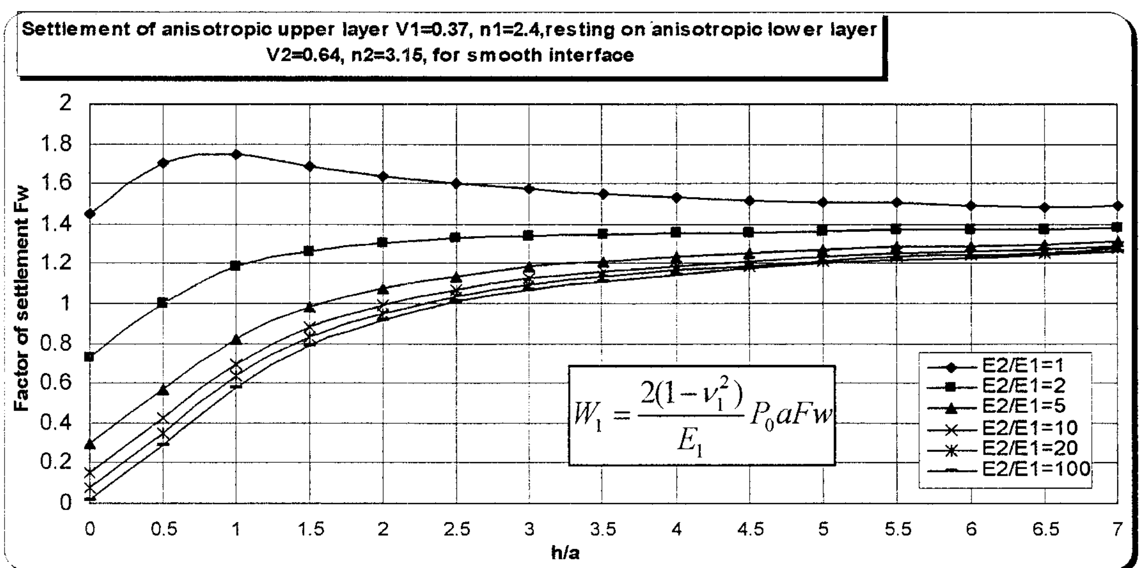

Figure 2-34 Settlement of an anisotropic elastic upper layer $\nu_1 = 0.3$, n1=2.4 resting on an anisotropic elastic lower layer ν_2=0.64, n2=3.15 for a smooth interface (crushed stone + natural sand over Natural sand)

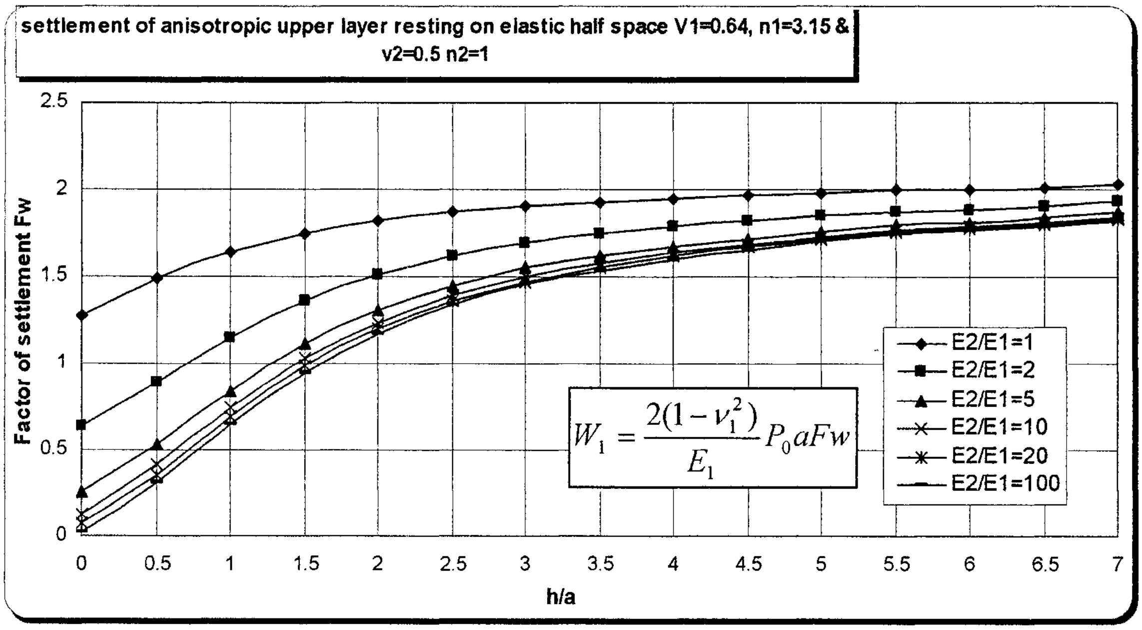

Figure 4-35 Factor of settlement of an anisotropic elastic upper layer resting on an isotropic elastic half space, ν_1 = 0.64 & ν_2 = 0.5, n1=3.15, n2=1, at the center of the load, for a rough interface

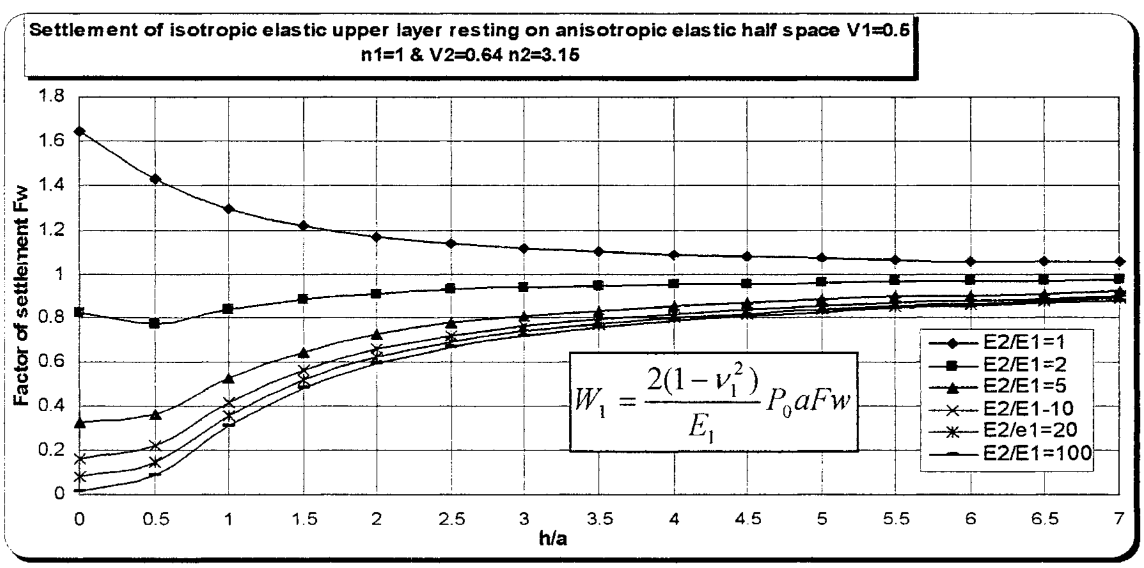

Figure 4-36 Factor of settlement of an isotropic elastic upper layer resting on an anisotropic elastic half space, ν_1 = 0.5 & ν_2 = 0.64, n1=1, n2=3.15, at the center of the load, for a rough interface

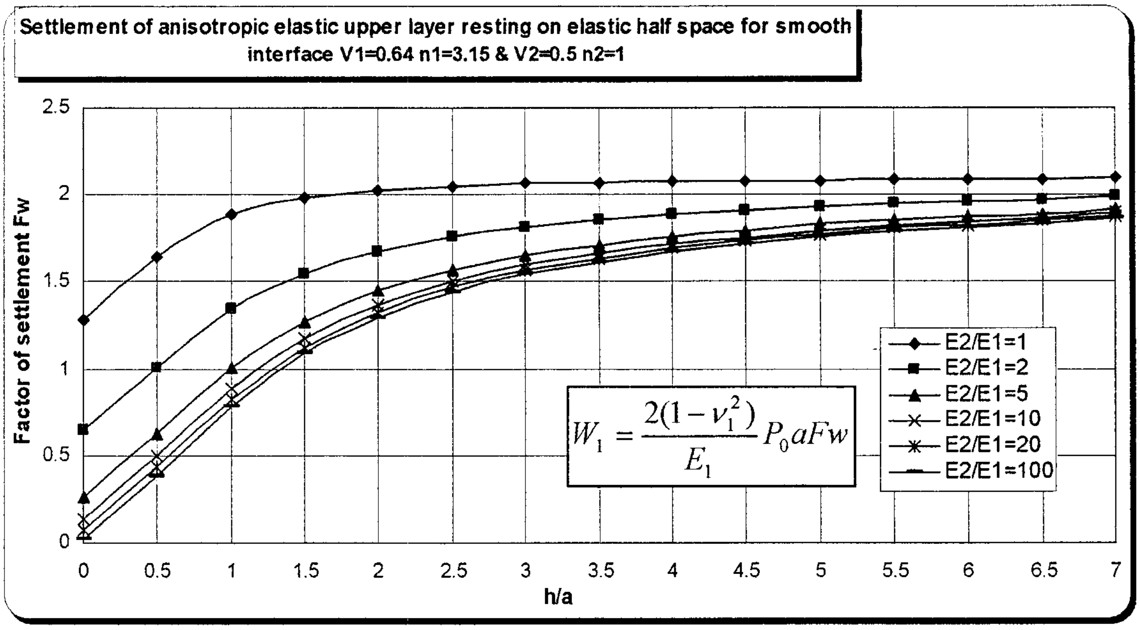

Figure 4-37 Factor of settlement of an anisotropic elastic upper layer resting on An isotropic elastic half space, ν_1 = 0.64 & ν_2 = 0.5, n1=3.15, n2=1, at the center of the load, for a smooth interface

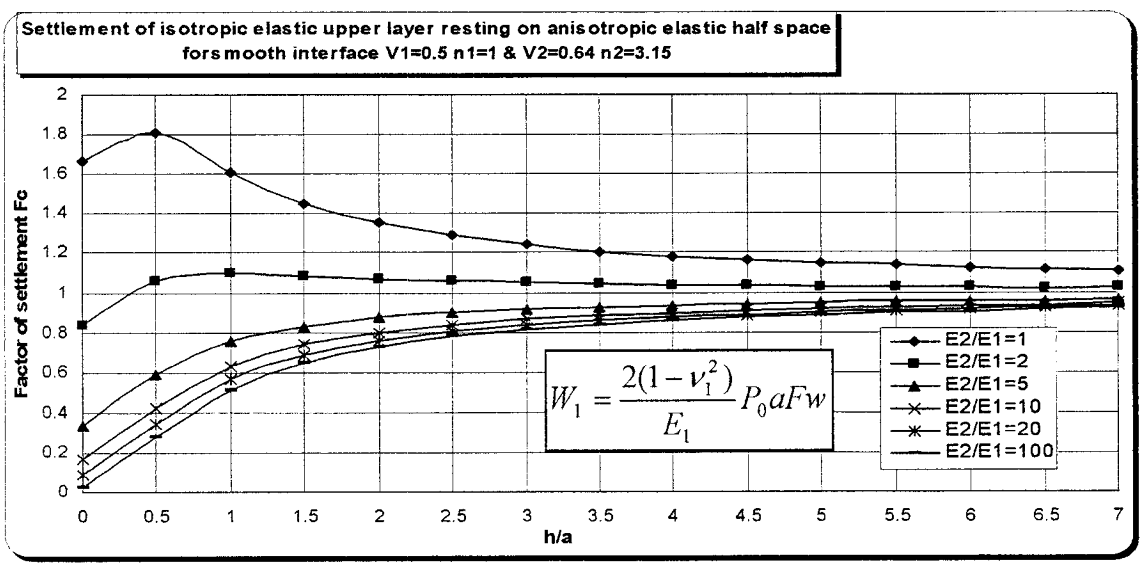

Figure 4-38 Factor of settlement of an isotropic elastic upper layer resting on an anisotropic elastic half space, ν_1 = 0.5 & ν_2 = 0.64, n1=1, n2=3.15, at the center of the load, for a smooth interface

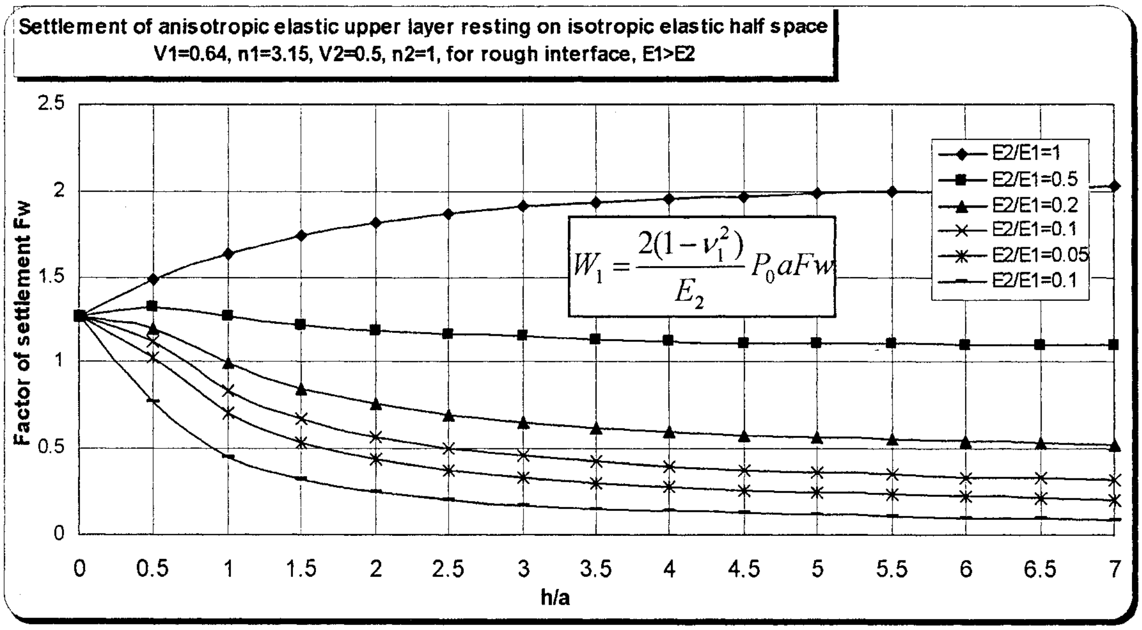

Figure 4-39 Factor of settlement of an anisotropic elastic upper layer resting on an isotropic elastic half space, ν_1 = 0.64 & ν_2 = 0.5, n1=3.15, n2=1, at the center of the load, for a rough interface, $E_1 > E_2$

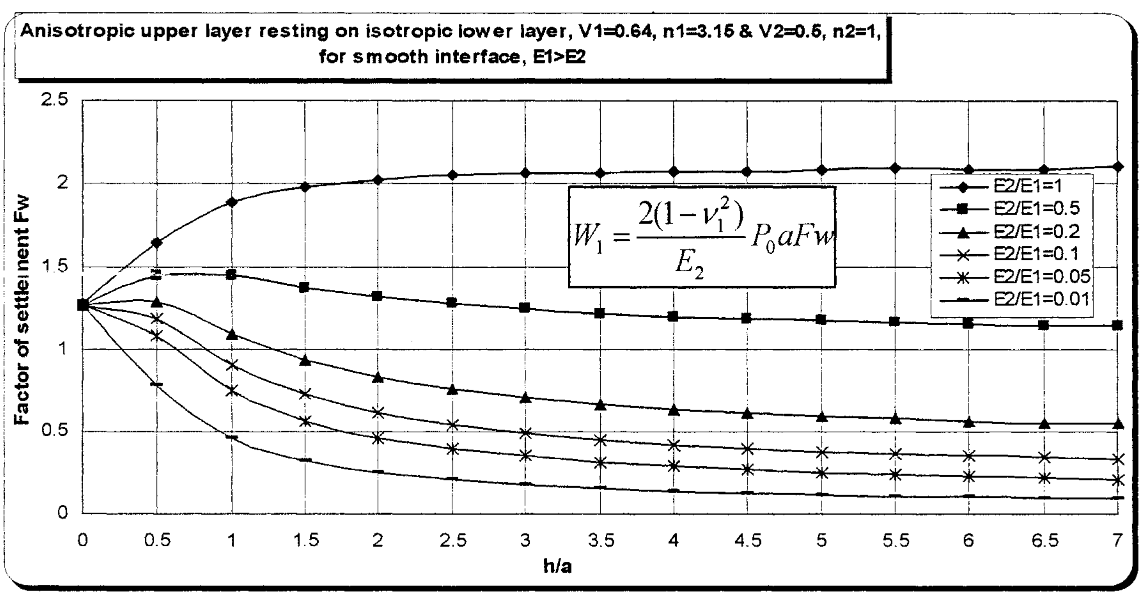

Figure 4-40 Factor of settlement of an anisotropic elastic upper layer resting on an isotropic elastic half space, ν_1 = 0.64 & ν_2 = 0.5, n1=3.15, n2=1, at the center of the load, for a smooth interface, $E_1 > E_2$

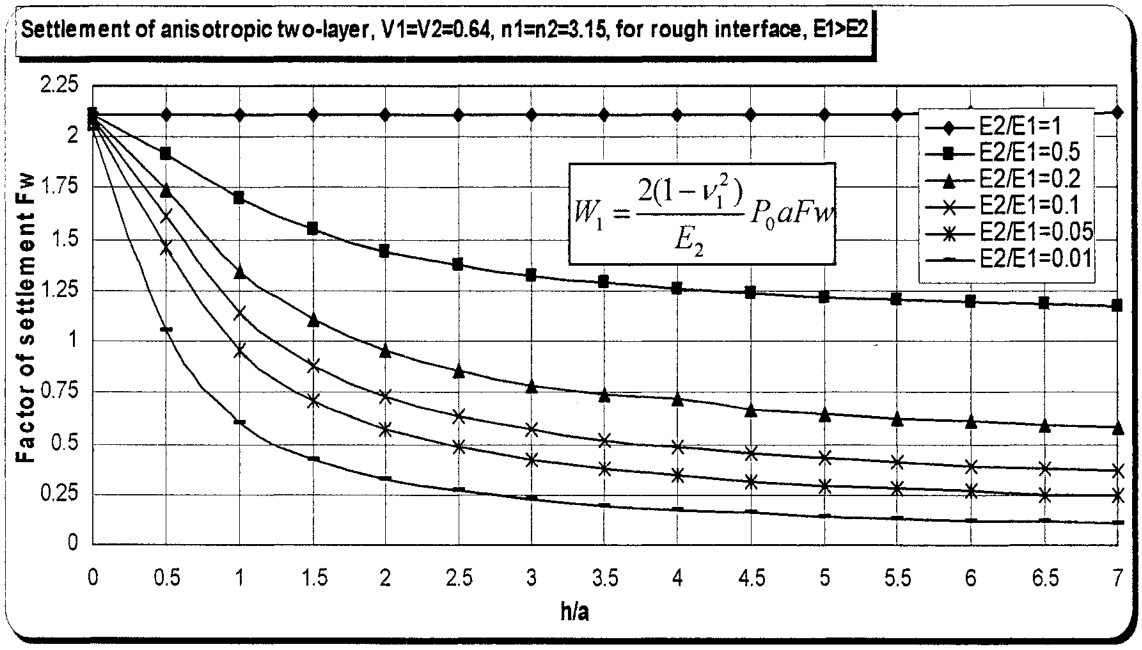

Figure 4-41 Factor of settlement of an anisotropic two-layer, at the center of the load for a rough interface, $\nu_1 = \nu_2$ =0.64, n1=n2=3.15, $E_1 > E_2$

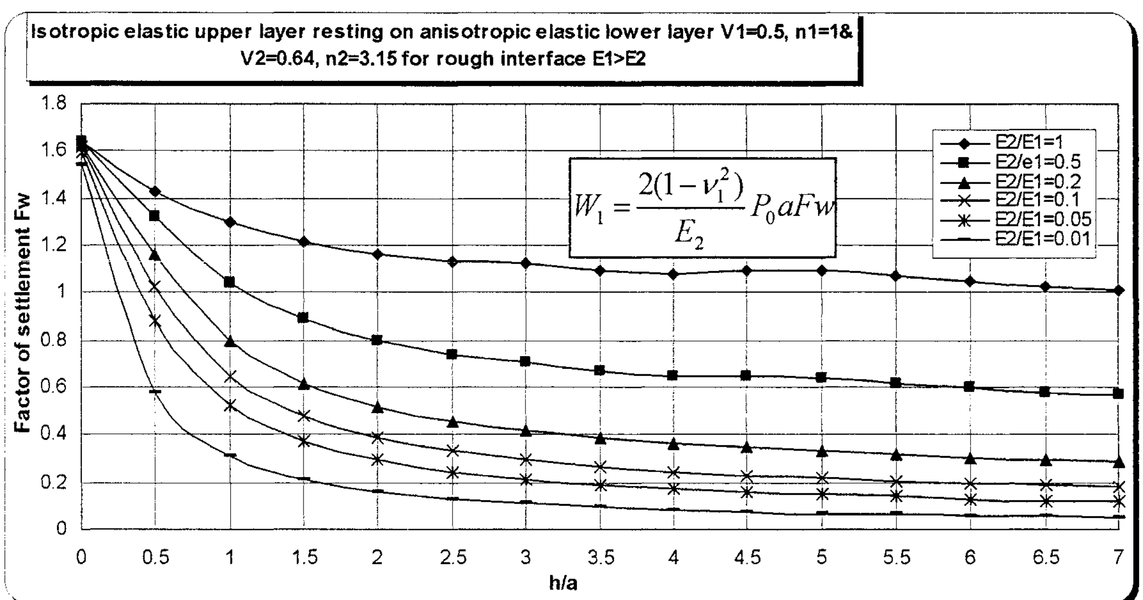

Figure 4-42 Factor of settlement of an isotropic elastic upper layer resting on an anisotropic elastic lower layer, ν_1 = 0.5 & ν_2 = 0.64, n1=3.15, n2=1, at the center of the load, for a rough interface, $E_1 > E_2$

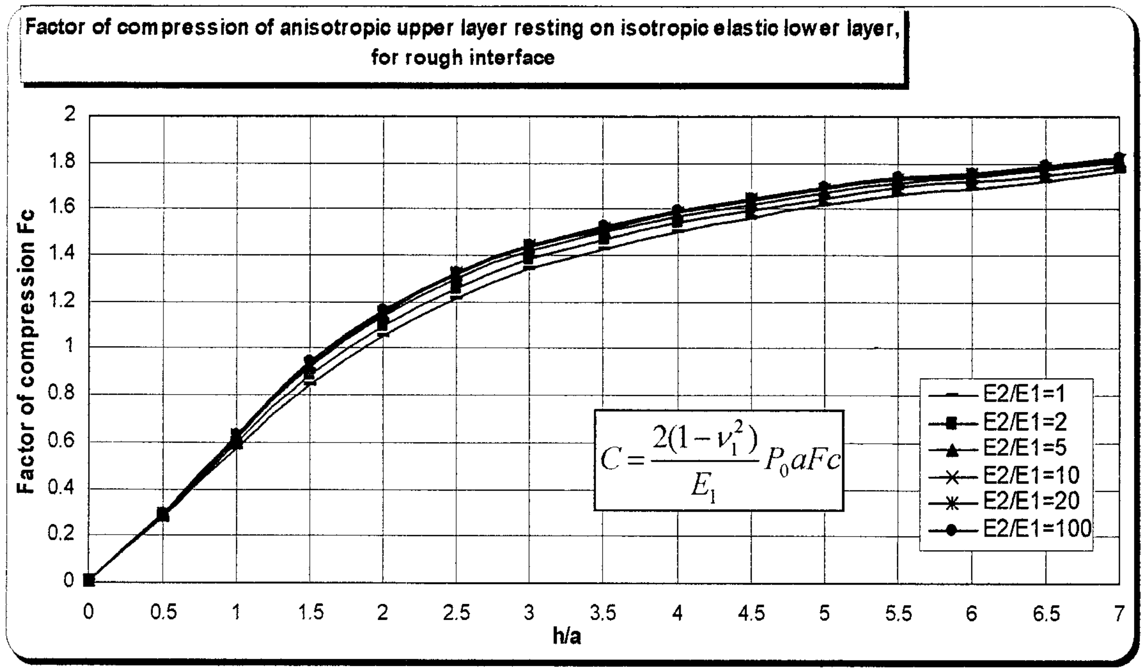

Figure 4-43 Factor of compression for an anisotropic elastic layer ν_1=0.64, n1=3.15 resting on elastic half-space ν_2=0.5, n2=1, for a rough interface.

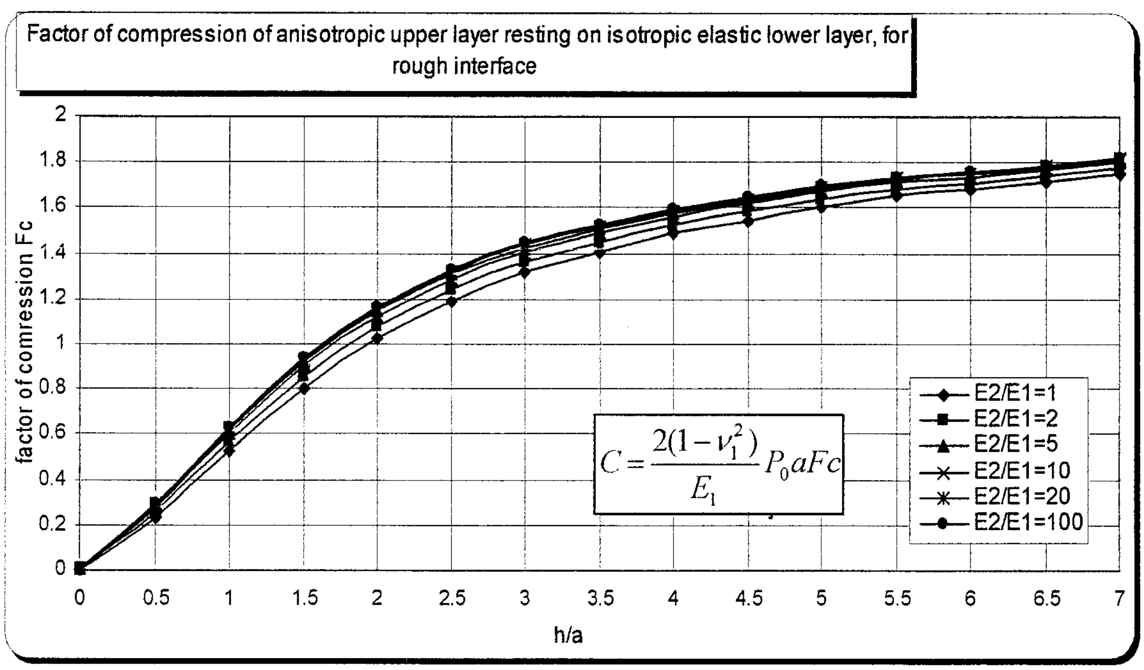

Figure 4-44 Factor of compression for an anisotropic elastic layer ν_1=0.64, n1=3.15 resting on elastic half-space ν_2=0.2, n2=1, for a rough interface.

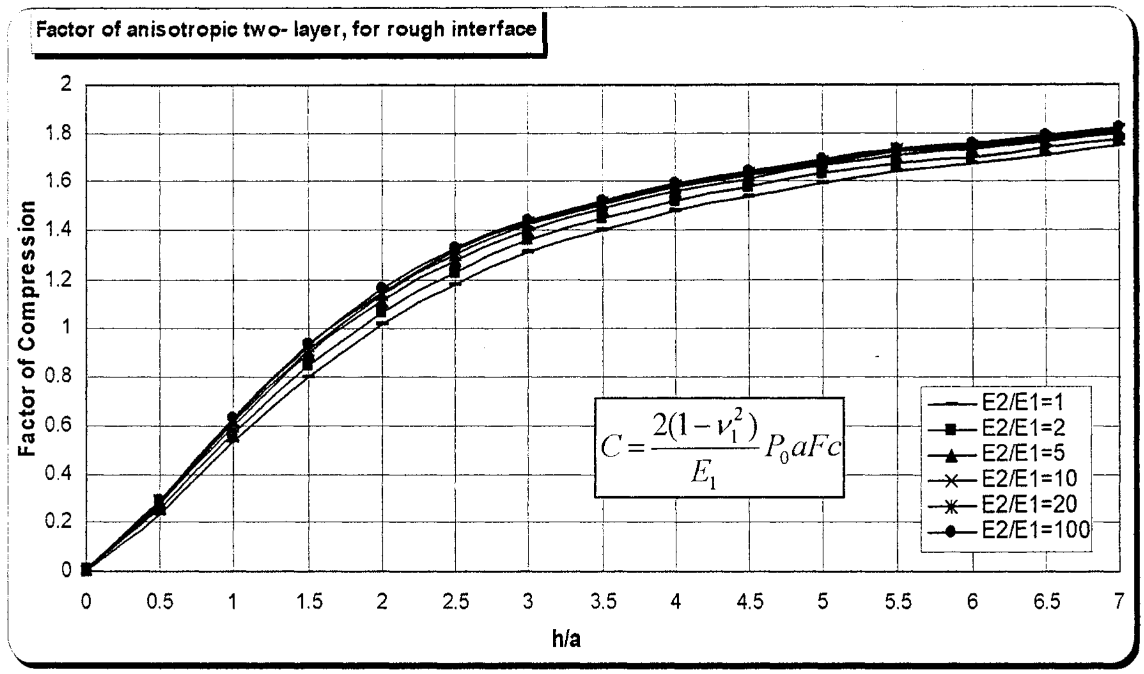

Figure 4-45 Factor of compression for an anisotropic elastic two-layer, rough interface (ν_1=0.64, n1=3.15, ν_2=0.37, n2=2.4

The following checks on the correctness of the influence curves were made

1. for h/a approaching zero that is, with h very small the deposit becomes a homogenous one all of layer (2), (for $\nu_1 = \nu_2 = 0.5$) the settlement coefficient reduces numerically to the value of E_1/E_2 that the equation of settlement reduces to Boussinesq equation for lower layer as would be expected.

$$W = \frac{2(1-\nu_1^2)}{E_1} * \frac{E_1}{E_2} = \frac{2(1-\nu_1^2)}{E_2}$$

2. when the ratio E2/E1equal to infinity in all diagrams, and for the ratio h/a equal zero, (the upper layer does not exist and the load is resting on the strong lower layer), the settlement is equal to zero as would be expected.
3. when the ratio h/a tends to infinity, that is either with (a) very small or h very large,

the deposit becomes a homogeneous one all of layer number (1), the factor of settlement then tends to unity for isotropic elastic diagrams, as it should be.

4. For the case of $E_1 > E_2$, and when E_1 and the ratio h/a are equal to infinity that is with *h* very large, the deposit becomes a homogeneous one all of layer number (1), the factor of settlement tends to zero as would be expected (figure (4-19)).
5. If the contact between the layers is perfectly smooth interface, the settlement increases.
6. Figure (4-38) shows a factor of settlement of isotropic elastic upper layer resting on anisotropic elastic half space, $\nu_1 = 0.5$ & $\nu_2 = 0.64$, n1=1, n2=3.15, when the ratio h/a tend to infinity, that is either with *(a)* very small or *h* very large, the deposit becomes a homogeneous one all of layer number (1), the factor of settlement then tends to unity as would be expected.

CHAPTER 5

CONCLUSIONS

5.1 Conclusion

The fundamental principles of isotropic and anisotropic elastic two layer system is presented, the displacement and the compression of the upper layer is computed where the upper layer is more compressible than the lower layer, under a uniformly loaded circular area, for a wide range of Poisson's ratio ν_1, modulii of elasticity E_2 / E_1 and the thickness of the upper layer over the radius of the loaded circular area h / a and is presented in the form of influence curves.

Based on the results, the following can be concluded:

1) The cross-anisotropy with a plane of isotropy coincides with the plane of the load, which is characterized by three independent elastic constants, suggested as an improved mathematical model of natural soil deposit. The results of the application of the theoty of elasticity on a two-layer system of a cross-anisotropic material has shown that the current application is more general than Burmister theory in the sense that it allows to deal with both materials, isotropic and anisotropic.
2) depending on the result of settlement of two-layer system, compression of upper layer, is computed and is presented in the form of influence curves to examine the role of a deep deposit on the compression of overlain subgrade, these compression's influence curves of anisotropic elastic upper layer show that the compression increases owing to an increase in the stiffness of the lower deep deposit.

5.2 Recommendation for further research

As they say "no one believes the theoretician, except the theoretician himself. Everyone believes the experimentalist, except for the experimentalist himself." Therefore

1) Experimental investigation should be conducted to investigate the correctness and the validity of the current application.
2) Compaction, which consists in a reduction of the void content principally in the vertical direction leads to an increase of the angle of internal friction, Thus to an increase of the ratio K_p / K_a and thus to an increase of the degree of anisotropy n. Further investigation could be conducted to study the relation between compaction and the degree of anisotropy n.

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