AN ITERATIVE METHOD FOR KINEMATIC ANALYSIS OF FOUR-BAR SPATIAL LINKAGES

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DISSERTATION

IN THE

FACULTY OF ENGINEERING

Presented in partial fulfilment of the requirements for the Degree of MASTER OF ENGINEERING

at

Sir George Williams University

Montreal, Canada

April 1972

ACKNOWLEDGEMENTS

The author wishes to express his gratitude and deep appreciation to his supervisor Dr. M.O.M. Osman for initiating the project and the helpful assistance given throughout the investigation.

The investigation forms a part of a research project supported by the research grant No. A-5181 from the National Research Council of Canada.

The author's gratitude goes also to the computing facilities at Sir George Williams University which were extensively used, and to Mr. G. Boast who helped in setting the Calcomp plotting program.

The author expresses his special gratitude to his wife Yardena for her encouragement and patience during his course of studies.

Finally, thanks are also due to Canadian Marconi Co. for their continuing education assistance.

ABSTRACT

An iterative proximity perturbation technique based on Newton-Raphson method is introduced to solve a set of non-linear equations which represent the displacement equations of a four-bar spatial linkage with a coupler point. For small increments of the input motion the roots of the equations are iterated in the proximity of their old values. The starting values for each iteration are taken to be those previously available.

A Fortran program based on the mathematical model of a spherical RRRR - mechanism is written to demonstrate the feasibility of the method. The displacements of the coupler point and output link are computed. Convergence proved to be rapid and consistent for each cycle.

The method is applied to determine the kinematic quantities of a RGCR- spatial linkage. Displacements, velocities and accelerations of the coupler point and the output member are obtained, and Calcomp - plots are generated from the numerical results. The displacement curves compare very well with those obtained from a model built for this purpose (see Fig. 6).

The method proved to be suitable for detecting locking regions for most mechanisms, and it is best suited for computed-aided design of complex planer as well as spatial linkages.

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NOMENCLATURE

A,B,C,D;	refer to linkage joints
f;	number of freedoms of linkage
P;	refers to coupler point
R_1,R_2,R_3,R_4;	lengths of links
U_p,V_p,W_p;	coordinates of coupler point P with respect to UVW - system
X Y Z;	principal coordinate system
U V W;	moving coordinate system
δX, δY, δZ;	incremental corrections for X, Y and Z
$\alpha, \beta, \gamma, \theta, \varphi$;	angles between links and coordinate axis (to be defined explicitly in the text)

Regarding types of linkage-joints

C;	Cylindric joint
G;	Globular (spherical) joint
R;	Revolute joint

Subscripts

a,b,c,d,p;	as in X_p, Y_p, Z_p denote coordinates of point P.
j;	as in $(X_p)_j$, $(Y_p)_j$, $(Z_p)_j$ denote coordinates of joint P, for the j-th increment.

Superscripts

(o), (i), (n);	as in $X_c^{(n)}$, $Y_c^{(n)}$, $Z_c^{(n)}$ denote the n-th iteration for the coordinates of joint C.

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CHAPTER 1

PURPOSE AND SCOPE OF INVESTIGATION

1.1 Introduction

When a linkage is synthesized to serve as a motion converter or function generator, the methods of synthesis can only at best produce a mechanism that approximates the desired function. Therefore, a necessary step in the synthesis procedure is to perform a motion analysis of a complete cycle of the mechanism to determine its accuracy and range of compliance with motion specifications. Such an analysis is not only an evaluation of the kinematic properties of the mechanism, but can be extended to give a complete picture of the dynamic characteristics of the system required for the machine design phase. Thus, there are specific properties of the mechanism that are desired at any instantaneous position in its cycle. These include the displacement, the linear and angular velocities and accelerations of the output member, the displacement and linear velocity and acceleration of the coupler point, and for dynamic analysis also the acceleration of the centers of gravity of each of the moving links. The analysis is usually made for a number of positions of the mechanism throughout its cycle, and the obtained information is ploted to show the variation of each parameter. When the input crank has a constant angular velocity and acceleration the various parameters can then be ploted as a function of the angular position (θ) of the input crank.

1.2 Survey of the Existing Methods for Displacement Analysis

1.2.1 Formulation of the Position-Vector Equation

The basic and also most difficult part of the kinematic ananlysis of mechanisms in general, and that of spatial linkages in particular, is the formulation of the displacement equation. An expression for the displacement of the output member as a function of the input, has to be solved before proceeding with the analysis.

Mechanisms with complicated geometric configuration involve rather sophisticated approaches to determine the corresponding displacement of the driven link for a given displacement of the driving link. However, once the displacement of the output is determined, the corresponding velocities and accelerations can be obtained by straightforward differentiation with respect to the time.

The position of a spatial four-bar linkage is defined by the vector equation:

$$\bar{R}_1 + \bar{R}_2 + \bar{R}_3 + \bar{R}_4 = 0 \qquad (1.1)$$

and basically all the methods available for displacement analysis of four-link mechanisms are in one way or another solving this fundamental equation. These methods for analysis fall into three main classes; graphical, analytical and analog-simulation methods.

1.2.2 The Graphical Method

This method (1)* is of limited scope. It involves a precise point by point projection of the linkage on each of three mutual perpendicular planes. The path of the coupler point and output link are then traced as the input varies by small increments.

The graphical determination of velocity and acceleration is also elaborate. It involves the construction of a vector polygon for velocity and acceleration. These vector polygons present the solutions of the relative motion equations for the pairs of related points on the mechanism. Also, this is a repetative procedure depending on the number of input crank increments required for the analysis.

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*Numbers in brackets () designate references at end of text.

For the rapid determination of the kinematics of a mechanism with different link proportions or different coupler points the graphical method seems to be impractical.

1.2.3 Analytical Methods

Several analytical methods for obtaining the displacement equation of the linkage are available. The solution of the mathematical model is usually obtained in a numerical form. Solutions for simple spatial linkages may be obtained, however, by the classical approach of closed form solution.

The classical approach makes basically use of the application of analytic geometry to the mechanism configuration. An example of this procedure is given in nearly every book on analysis of mechanisms. The application of the method on a more complex RCGR mechanism can be found in (2).

Other mathematical tools are also used. Vector algebra is used by Chace (3) to formulate the displacement equation, while complex variables is introduced by Raven (4).

The main disadvantage of the closed form solution is that it requires elaborate algebraic manipulations to bring a set of equations to an explicit form. The final equation which relates the output displacement of the mechanism to the input one in its explicit form is usually of high order, and there are as many configurations in which the mechanism may be closed, as there are real roots to the equation. This may pose the dilemma: "Which of the solutions is the next point on the analysed path?"

The matrix method of analysis has a systematic approach. It is based on the use of four parameters needed for the geometric description of a connection between links. The use of only four parameters was first suggested by Denavit and Hartenberg (5) and is based on the use of a unique common perpendicular between successive-pair axes. Since a mechanism may be viewed as being a sequence of connections, it may be described by means of a symbolic equation whose terms contain the parameters defining the successive connections. On replacing each term by a matrix also formed from parameters, an algebraic manipulative procedure suited to analysis becomes possible.

Applying an iterative technique the matrix method was further developed by Uicker (6) to be used on a digital computer.

As pointed out by the authors, the method has one main disadvantage. Since the matrix is singular at the deadpoint of the mechanism the iteration process fails to converge and this point cannot be calculated. Another form of the displacement matrix was suggested by suh (7). In this method, a three dimensional rigid body is displaced in space from $\varepsilon 1$ to $\varepsilon 2$ by first rotating the body through an angle φ about an axis U which has directional cosines Ux, Uy, Uz and passes through point $P_1(x_1, y_1, z_1)$, followed by a translation to the position ε_2 that $P_1(x_1, y_1, z_1)$ moves to $P_2(x_2, y_2, z_2)$. The path from position 1 to position 2 is chosen such that point P_1 coincides with the origin of the coordinate system, the translation is done along U while the rotation is about U which is passing through the origin.

1.2.4 Analog Simulation Method

An electronic analog computer technique for spatial mechanisms analysis has been introduced by Keller (8).

The general approach used in his method is similar to the digital iteration technique. The mechanism is represented as a set mathematical constraints for the linkage configuration. These constraints are not satisfied by solving for one variable in a given relationship; instead, the variables are varied simultaneously by an automatic control which minimizes the error to satisfy the given constraint.

The analog computer offers a few advantages over other methods. It is relatively easy to operate. All calculations are performed simultaneously in real time. The results are given in graphical form. Optimization of a mechanism is easily achieved, since many different proportions of links can be computed and ploted by just changing the setting of control potentiometers. The main disadvantage of the analog computer is that the accuracy of the results depends on the built-in accuracy of the computing elements, and at best can compare with a hand drawn graphical solution.

Complex mechanisms require elaborate circuitry. Handling of bulk data requires all together the use of an hybrid computer where the digital part is used to control the input data to the analog part.

CHAPTER 2

THE ITERATIVE METHOD

The methods developed so far for displacement analysis of linkages can be subdivided into two main groups: a) graphical and analytical close form methods which are most suited for relatively simple mechanisms, and b) highly sophisticated methods including matrix and tensor techniques.

Although these techniques are powerful tools for solving linkage-kinematics, many require a specialized knowledge in mathematical analysis which is beyond the scope of study of most practicing engineers. Since computer-aided design methods are gaining momentum in all fields of engineering there exists the need for a relatively simple numerical technique by which a designer with a common knowledge in analytical geometry and computer programming will be able to solve complicated mechanism problems and get satisfactory results leaving the bulk of the mathematical manipulation and computational work to the machine.

This need for an algorithm for the analysis of mechanisms, that utilizes the full advantages offered by high speed digital computers was recognized by Osman and

Mansour (9, 10) as well as Chi-Yeh (11). The present work is an extension of these efforts to the solution of spatial linkages.

2.1 Formulation of the Problem

2.1.1 The Displacement Equation

The formulation of the displacement equation for any arbitrary point on the mechanism is the basis for a complete kinematic analysis of a linkage.

In the method presented here the mechanism is described as a set of mathematical constraints between the successive links of the mechanism. In general it can be expressed as;

$$R_m = f_m(\theta, k, x, y, z)$$

where; $m = 1, 2, 3, 4$ refers to the link-number of the mechanism; R is the link length, θ is a given displacement of the driving member, k specifies the geometric configuration of the mechanism, and (x, y, z) are the unknown coordinates of joint. This set of simultaneous non linear algebraic equations takes the form of

$$f_m\ (\theta, k, x, y, z, R) = 0 \quad m = 1, 2, 3, 4$$

This set is solved iteratively with the help of a fast converging numerical method to obtain the coordinates x, y, z of the joints. Once the displacements of all the joints are known, coupler curves can easily be obtained.

Successive differentiation of the displacement equations yields independent equations for the velocity and acceleration of the linkage as a function of the input angular velocity and angular acceleration. This procedure is illustrated in the next section for the case of a Spherical four-bar linkage.

2.1.2 The Spherical four-bar RRRR-linkage

Fig. 1 shows a schematic of a spherical four-bar RRRR-linkage with a coupler point. Let R_1, R_2, R_3, R_4 denote the length of the links DA, AB, BC and CD respectively. R_1 is the fixed member. R_2 is the driver and is rotating about the X-axis. Θ is the input to the system and is assumed known. The output link R_4 is free to rotate about the Y-axis. φ is the output angle and is considered unknown. The coordinates of the coupler point P with respect to the XYZ-system are X_p, Y_p and Z_p. U_p, V_p, W_p define the position of P in the moving coordinate system of UVW which is originated at B.

The position of each of the joints at any input angle Θ of the driving link is given by the joints' coordinates in a cartesian system XYZ. Let it be desired to find the linear displacement of the coupler point P as well as the angular displacement of the output member φ .

The coordinates of each of the joints are written in terms of the known or unknown parameters. The geometric relationship between the joints is then used to establish the system of equations governing the motion of the linkage.

2.1.3. The Mathematical Model

From Fig. 1 the following relationships can be written:

$$\begin{aligned} X_a &= R_1 \sin\gamma \\ Y_a &= 0 \\ Z_a &= 0 \end{aligned} \qquad (2.1)$$

$$\begin{aligned} X_b &= X_a \\ Y_b &= R_2 \cos\theta \\ Z_b &= R_2 \sin\theta \end{aligned} \qquad (2.2)$$

$$\begin{aligned} X_c &= - R_4 \cos\varphi \\ Y_c &= R_1 \cos\gamma \\ Z_c &= R_4 \sin\varphi \end{aligned} \qquad (2.3)$$

$$\begin{aligned} X_d &= 0 \\ Y_d &= Y_c \\ Z_d &= 0 \end{aligned} \qquad (2.4)$$

There are two unknowns (X_c, Z_c) to completely define this linkage. Considering the geometrical constraints of this mechanism, two additional equations can be written; see Fig. 1:

$$(X_c - X_b)^2 + (Y_c - Y_b)^2 + (Z_c - Z_b)^2 = R_3^2 \quad (2.5)$$

$$(X_d - X_c)^2 + (Y_d - Y_c)^2 + (Z_d - Z_c)^2 = R_4^2 \quad (2.6)$$

Substituting the known values from equations (2.1) to (2.4) we obtain:

$$F(x, z) = (X_c - R_1 \sin\gamma)^2 + (R_1 \cos\gamma - R_2 \cos\theta)^2 + (Z_c - R_2 \sin\theta)^2 - R_3^2 = 0 \quad (2.7)$$

$$G(x, z) = X_c^2 + Z_c^2 - R_4^2 = 0 \quad (2.8)$$

Equations (2.7) and (2.8) are solved iteratively for the values X_c and Z_c as a function of the input angle θ.

It is again important to note that once we solve for the unknown coordinates X_c and Z_c the displacement of any point on the mechanism can be determined.

The angular displacement φ of the output is computed from eq. (2.3).

$$\varphi = \tan^{-1} (- Z_c / X_c) \tag{2.9}$$

Using the results of Appendix II the coordinates of the coupler point P are given by the following:

$$\begin{bmatrix} X_p - X_b \\ Y_p - Y_b \\ Z_p - Z_b \end{bmatrix} = \begin{bmatrix} \cos(xu) & \cos(xv) & \cos(xw) \\ \cos(yu) & \cos(yv) & \cos(yw) \\ \cos(zu) & \cos(zv) & \cos(zw) \end{bmatrix} \begin{bmatrix} U_p \\ V_p \\ W_p \end{bmatrix} \tag{2.10}$$

hence;

$$\begin{aligned} X_p &= U_p \cos(xu) + V_p \cos(xv) + W_p \cos(xw) + X_b \\ Y_p &= U_p \cos(yu) + V_p \cos(yv) + W_p \cos(yw) + Y_b \\ Z_p &= U_p \cos(zu) + V_p \cos(zv) + W_p \cos(zw) + Z_b \end{aligned} \tag{2.11}$$

where U_p, V_p and W_p are the coordinates of the coupler point P referred to the UVW-moving system.

2.1.4 The Newton-Raphson Method

Two iteration techniques;

a. The method of approximation (Newton-Raphson)

b. The method of successive substitution, are considered by Milne (14) as the most suitable techniques for solving a set of non-linear algebraic equations. In this investigation we choose the first method because of its second order convergence. To demonstrate this method let us consider the case of two equations in two unknowns. Let equations (2.7) and (2.8) be written as

$$\begin{aligned} F(x,z) &= 0 \\ G(x,z) &= 0 \end{aligned} \qquad (2.12)$$

Now if x_o, z_o be values of a pair of roots and δx, δz be corrections, so that,

$$\begin{aligned} x &= x_o + \delta x \\ z &= z_o + \delta z \end{aligned} \qquad (2.13)$$

then (2.12) becomes

$$\begin{aligned} F(x_o + \delta x,\ z_o + \delta z) &= 0 \\ G(x_o + \delta x,\ z_o + \delta z) &= 0 \end{aligned} \qquad (2.14)$$

expanding (2.14) by Taylor's theorem, we get;

$$\begin{aligned} F(x_o + \delta x,\ z_o + \delta z) &= F(x_o,\ z_o) + \delta x\left(\frac{\partial F}{\partial x}\right)_o + \delta z\left(\frac{\partial F}{\partial z}\right)_o = 0 \\ G(x_o + \delta x,\ z_o + \delta z) &= G(x_o,\ z_o) + \delta x\left(\frac{\partial G}{\partial x}\right)_o + \delta z\left(\frac{\partial G}{\partial z}\right)_o = 0 \end{aligned} \qquad (2.15)$$

solving eq. (2.15) for δx and δz, we have,

$$\delta x = \frac{\begin{vmatrix} -F(x_o,\ z_o) & \left(\frac{\partial F}{\partial z}\right)_o \\ -G(x_o,\ z_o) & \left(\frac{\partial G}{\partial z}\right)_o \end{vmatrix}}{D} \qquad (2.16)$$

$$\delta z = \frac{\begin{vmatrix} \left(\frac{\partial F}{\partial z}\right)_o & -F(x_o,\ z_o) \\ \left(\frac{\partial F}{\partial x}\right)_o & -G(x_o,\ z_o) \end{vmatrix}}{D}$$

where

$$D = \begin{vmatrix} \left(\frac{\partial F}{\partial x}\right)_o & \left(\frac{\partial F}{\partial z}\right)_o \\ \left(\frac{\partial G}{\partial x}\right)_o & \left(\frac{\partial G}{\partial z}\right)_o \end{vmatrix}$$

The case of only two equations with two unknowns is illustrated here since for a one degree of freedom four-bar linkage of any configuration, only two equations similar to equations (2.5) and (2.6) can be written. Differentiating equations (2.7) and (2.8) once with respect to x and once in respect to z we get,

$$\frac{\partial F}{\partial x} = 2(x_c - R_1 \sin\gamma)$$

$$\frac{\partial F}{\partial z} = 2(z_c - R_2 \sin\theta) \qquad (2.17)$$

$$\frac{\partial G}{\partial x} = 2x_c$$

$$\frac{\partial G}{\partial z} = 2z_c$$

2.1.5 The Starting Values for the Iteration Process

The Newton-Raphson iterative procedure, similar to all functional iteration methods does not, in general, converge to a solution from an arbitrary starting point. The initial estimate must be within some small region around the actual solution. In general, the number and order of the nonlinear algebraic equations are inversely proportional to the size of the region for estimation. Freudenstien and Roth (12) pointed out that for two simultaneous second order equations, almost any initial estimate will lead to one of the roots. But since for the same input angle θ, there are usually two possible configurations a four-bar linkage can assume in space, we are generally faced with the problem of estimating correctly the starting values for each iteration cycle, so that a continuous path for the joint under consideration is obtained.

Fig. 2 shows the two possible starting configurations for the spherical RRRR-linkage for $\theta = 0^{\circ}$. The mechanism motion is started when $\theta = 0^{\circ}$ and incremented equally in the clockwise direction. For $\theta = 0^{\circ}$ we have to compute the starting values for the first iteration. Several methods exist for this purpose. Those are:

a. <u>The algebraic method</u>

From equations (2.7) and (2.8) we have;

$$x_c = \left[-R_3^2 + R_4^2 + (R_1 \cos\gamma - R_2)^2 + R_1 \sin\gamma\right] / 2R_1 \sin\gamma \tag{2.18}$$

$$Z_c = \pm \sqrt{R_4^{\,2} - x_c^{\,2}}$$

Thus, two sets of coordinates are obtained. Each set corresponding to a particular linkage with a particular coupler curve.

b. The method of intelligent guess

A reasonably intelligent guess can be obtained by drawing a sketch of the mechanism to scale. This method is used in this investigation. Our initial estimations for X_c, Y_c and Z_c were;

X_c = - 0.300 in. Y_c = 0.500 in. Z_c = 0.800 in.

while the actual values obtained were:

X_c = - 0.289 in. Y_c = 0.559 in. Z_c = 0.957 in.

c. The parameter perturbation method

The parameter-perturbation procedure developed by Freudenstien and Roth (12) is an algorith for determining a root of a set of simultaneous non-linear equations and is particularly suitable to approximate "hard to find" initial estimations. The pre-requisite with this method is a known solution to a linkage of the same type. Now, instead of working with the equations of the particular linkage we wish to solve, we start with a different mechanism of the same type. This mechanism has the same displacement equation but with different parameters, and the solution for these equations is known.

That is to say, we start with a RRRR mechanism with different length of links for which there is a known solution.

Then we begin to increment both the unknown coordinates X_c, Y_c, Z_c and the parameters until we obtain the set of desired equations and their solution.

2.1.6. The Proximity Perturbation Method

Once the starting point for the first iteration is found and the process converges to a certain value, the input angle θ is incremented by a small amount. To solve the displacement equation for this new position the last known coordinates of the previous iteration are used. $(X_c)_j^{(n)}$, $(Y_c)_j^{(n)}$, $(Z_c)_j^{(n)}$ are taken as the initial estimates $(X_c)_{j+1}^{(o)}$, $(Y_c)_{j+1}^{(o)}$, $(Z_c)_{j+1}^{(o)}$. After n iterations the initial estimates converge to the correct solution $(X_c)_{j+1}^{(n)}$, $(Y_c)_{j+1}^{(n)}$, $(Z_c)_{j+1}^{(n)}$ (within computing tolerance) on the path. This assumes that the increments of θ are taken sufficiently small, otherwise the procedure might diverge or even converge to a solution corresponding to the other possible configuration of the linkage.

The use of this proximity perturbation method assures that the new set of assumed coordinates is always in the proximity of the actual results. There are no further decisions required concerning the positive or negative signs of particular variables and therefore the only possible path for a particular configuration is obtained.

2.2 Convergence Criteria

Convergence may be assumed to be obtained when the corrections δx and δz of equation (2.16) are negligibly small. In a mathematical sense, the criteria of smallness would be related to the capabilities of a particular digital computer being utilized. However, in a kinematic sense, it is recognized that the desired solution is a function of the linkage parameters used for the particular design. Hence taking the correction in the order of ± 0.001 is compatible with the mechanical tolerance of link lengths, which is sufficient for practical design problems. Thus, each iteration loop examines δx and δz and compares the magnitude of the largest against a programed limiting value. If this value is not exceeded, convergence is indicated and the iteration procedure is completed for this particular point.

In addition, convergence may also be defined on the basis of the structural error between the generated and prescribed functions as measured by the Fi and Gi residuals of equation (2.14). Thus, after each iteration loop, the residuals are compared against a programed convergence limit. If satisfied, convergence is indicated and the iteration procedure is completed for this particular point.

2.3 Computational Procedure and Results

The flow-chart for displacement analysis of four-bar spatial linkages with coupler point is shown in Fig. 3. The following points are worth mentioning:

a. The input starting point for the analysis is taken at $\theta = 0$, and is incremented in 10° steps. However, any other θ for which an approximation is available can be chosen. If the mechanism is known to lock for a certain range θ_1 to θ_2 of input motion θ, the input motion can vary from $\theta = 0^{\circ}$ to $\theta = \theta_1$ in equal increments of $\Delta\theta$ and then from $\theta = 0^{\circ}$ to $\theta = \theta_2$ in steps of $-\Delta\theta$.

b. The number of iterations of each cycle is controlled. After ten iterations the already stored values of $F(x,z)$ of equation (2.7) are checked to determine whether the iteration process is converging. In case of the opposite or when looping occurs, Θ is incremented by 10° and a new iteration process starts using as its starting point the last known values $(X_c)^{(n)}_{j-1}$, $(Y_c)^{(n)}_{j-1}$, $(Z_c)^{(n)}_{j-1}$ on the path.

In some cases where the mechanism is locked for a large range of the input angle Θ, this procedure might produce misleading results, since the iteration process might converge to a solution which is not on the path of the particular mechanism in question.

c. In those cases where convergence is progressing slowly the process stops after thirty iterations and Θ is incremented to the next position. Again the last known values of X_c, Y_c, Z_c are used as initial approximations for the iteration of a consecutive point.

d. The corrections δx and δy are taken as .001 in. This value proved to be economical as far as computing time is concerned as well as it provides sufficient accuracy for design purposes.

e. The output angle φ of the mechanism is computed by means of the special sub-routine ATAN 2 [SP,CP] where $SP = \sin\varphi$ and $CP = \cos\varphi$. This sub-routine is specially prepared to avoid ambiguity regarding the sign of the trigonometric function. It chooses the correct quadrant for the angle by checking both the signs and values of $\sin\varphi$ and $\cos\varphi$.

A computer program for the computation of the coupler and output link displacements is written using Fortran IV language, and entered by card deck to a CDC 6600 machine. Complete descriptions of computer programs and results are given in Appendix I. As an example, the coupler curve for a spherical four-bar RRRR linkage with the following parameters have been obtained.

$R_1 = 1.000$ in. $\qquad R_2 = 0.270$ in.

$R_3 = 1.500$ in. $\qquad R_4 = 1.000$ in.

$\gamma = 56^{\circ}$

$U_p = 1.000$ in. $\qquad V_p = 1.000$ in. $\qquad W_p = 1.000$ in.

Estimated Starting point:

$(X_c)^{(o)} = -0.300$ in. $\qquad (Y_c)^{(o)} = 0.500$ in. $\qquad (Z_c)^{(o)} = 0.800$ in.

Convergence proved to be very rapid, with only two to three iterations for each input increment. In addition to numerical listings of results, coupler curves and other kinematic quantities were obtained using a Calcomp-plotter. The computer listing and Calcomp-plots are given in Appendix I.

CHAPTER 3

KINEMATIC ANALYSIS OF FOUR BAR RGCR SPATIAL LINKAGE

From the nine, one degree of freedom spatial linkages reported by Harrisberger (13) as having potential engineering applications, the RGCR type shown in Fig. 4 is chosen for a kinematic study by the iterative method.

3.1 The Mobility Equation

The number of freedoms of a kinematic chain is obtained by multipying the number of links by the freedoms of each link, and then substructing those freedoms which are destroyed by each kinematic pair. A space link has six degrees of freedom, three translation and three rotation.

The number of freedoms of motion "f" in a mechanism of n links is given by the Kutzbach-Grübler criterion

$$f = 6(n-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - 1P_5$$

where P_i is the number of pairs with i degrees of freedom.

Thus, for the RGCR mechanism

$$f = 6(4-1) - 5 \times 2 - 4 \times 1 - 3 \times 1$$
$$= 18 - 10 - 4 - 3 = 1$$

The designation RGCR identifies the kinematic pairs, beginning with the input link and proceeding through the coupler and ending with the frame-link.

Thus, the input-crank is pivoted to the frame by revolute (R) joint of one degree of freedom. The coupler link is pivoted to input crank by a globular (G) joint with three freedoms and is paired to the output member by a cylindric (C) joint with two freedoms. The motion of the output member is then constrained by the revolute (R) joint.

This is a type two linkage i.e. having two single freedom pairs, one double freedom and one triple freedom pair. Thus, it is $2P_1 + 1P_2 + 1P_3$ type mechanism.

3.2 Description of the RGCR-Linkage

The input link R_2 as shown in Fig. 4 rotates about the Z axis, with θ being the input angle. R_3 is the coupler link. Joint B is the origin of the moving co-ordinate system UVW, and P is the coupler point. φ is the output of link 4 about an axis parallel to the Y axis. R_1 is the fixed link. α and γ are constant angles determining the orientation of R_1 and R_2 respectively.

The parameters θ, $\dot{\theta}$ and $\ddot{\theta}$ are the input angular displacement, velocity and acceleration respectively and are assumed known.

3.3 The Mathematical Model

The displacement analysis follows the procedure established in section 2.1.3 of chapter 2.

The following relationships can be written from Fig. 4:

$$X_b = R_2 \cos\alpha \sin\theta$$
$$Y_b = R_2 \cos\alpha \cos\theta \qquad (3.1)$$
$$Z_b = R_2 \sin\alpha$$

$$X_c = X_d + R_4 \sin\varphi$$
$$Y_c = Y_d \qquad (3.2)$$
$$Z_c = R_4 \cos\varphi$$

$$X_d = R_1 \sin\gamma$$
$$Y_d = R_1 \cos\gamma \qquad (3.3)$$
$$Z_d = 0$$

There are two unknown coordinates (X_c, Z_c) to completely define this mechanism. Since it is a closed loop mechanism with one degree of freedom, two mathematical constraints govern its motion. These are:

$$(X_c - X_b)^2 + (Y_c - Y_b)^2 + (Z_c - Z_b)^2 = R_3^3 \tag{3.4}$$

$$(X_d - X_c)^2 + Z_c^2 = R_4^2 \tag{3.5}$$

Substituting the known coordinates into the equations (3.4) and (3.5) we have,

$$Y_c = R_2 \cos\alpha\cos\Theta \tag{3.6}$$

$$(X_c - R_2 \cos\alpha\sin\Theta)^2 + (Z_c - R_2 \sin\alpha)^2 = R_3^2 \tag{3.7}$$

$$(R_1 \sin\gamma - X_c)^2 + Z_c^2 = R_4^2 \tag{3.8}$$

Equations (3.7) and (3.8) are solved iteratively for different values of Θ. Equation (3.6) is computed independently.

Let equations (3.7) and (3.8) be given by:

$$F(x,z) = (X_c - R_2 \cos\alpha\sin\Theta)^2 + (Z_c - R_2 \sin\alpha)^2 - R_3^2 = 0 \tag{3.9}$$

$$G(x, z) = (X_c - R_1 \sin\gamma)^2 + Z_c^2 - R_4^2 = 0$$

following the Newton-Raphson method, the necessary partial derivatives are given as;

$$\frac{\partial F}{\partial x} = 2(X_c - R_2 \cos\alpha \sin\theta)$$

$$\frac{\partial F}{\partial z} = 2(Z_c - R_2 \sin\alpha)$$

(3.10)

$$\frac{\partial G}{\partial x} = 2(X_c - R_1 \sin\gamma)$$

$$\frac{\partial G}{\partial z} = 2\, Z_c$$

In addition, the following quantities are derived:

$$\delta x = \frac{\begin{vmatrix} -F & \frac{\partial F}{\partial z} \\ -G & \frac{\partial F}{\partial z} \end{vmatrix}}{D}$$

$$\delta z = \frac{\begin{vmatrix} \frac{\partial F}{\partial x} & -F \\ \frac{\partial G}{\partial x} & -G \end{vmatrix}}{D} \qquad (3.11)$$

$$D = \begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial z} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial z} \end{vmatrix}$$

$$D = 4Z_c\ (X_c - R_2 \cos\alpha \sin\theta) -$$

$$4(Z_c - R_2 \sin\alpha)\ (X_c - R_1 \sin\gamma)$$

$$\delta x = \{-2Z_c \left[(X_c - R_2 \cos\alpha \sin\theta)^2 + \right.$$

$$\left.(Z_c - R_2 \sin\alpha)^2 - R_3^2 \right] +$$

$$2(Z_c - R_2 \sin\alpha) \left[(X_c - R_1 \sin\gamma)^2 + Z_c^2 - R_4^2 \right] \} / D \qquad (3.12)$$

$$\delta z = \{-2(X_c - R_2 \cos\alpha \sin\theta) \left[(X_c - R_1 \sin\gamma)^2 + Z_c^2 - R_4^2 \right]$$

$$+ 2(X_c - R_1 \sin\gamma) \left[(X_c - R_2 \cos\alpha \sin\theta)^2 \right.$$

$$\left. - (Z_c - R_2 \sin\alpha)^2 - R_3^2 \right] \} / D$$

Equations (3.7) through (3.12) are used to derive X_c and Z_c by the iterative proximity perturbation method.

Once the quantities X_c, Y_c, Z_c are known, the displacement equation of the coupler point P and the output φ can be written:

The output angular displacement φ is computed from eq. (3.2)

$$\varphi = \tan^{-1} (X_c - R_1 \sin\gamma)/Z_c \tag{3.13}$$

Using the computer subroutine ATAN2(SP,CP).

Applying the results obtainted in Appendix II, the coordinates of the coupler point P are given by the following:

$$\begin{bmatrix} X_p - X_b \\ Y_p - Y_b \\ Z_p - Z_b \end{bmatrix} = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} U_p \\ V_p \\ W_p \end{bmatrix} \tag{3.14}$$

where;

$$\begin{aligned} X_p &= U_p \cos\beta - W_p \sin\beta + X_b \\ Y_p &= V_p + Y_b \\ Z_p &= U_p \sin\beta + W_p \cos\beta + Z_b \end{aligned} \tag{3.15}$$

The values of $\sin\beta$ and $\cos\beta$ are determined from the following relationships:

$$\begin{aligned} \sin\beta &= (Z_c - R_2 \sin\alpha)/R_3 \\ \cos\beta &= (X_c - R_2 \cos\alpha \sin\theta)/R_3 \end{aligned} \tag{3.16}$$

Making use of equations (3.1) to (3.16), the final form of the coordinates of point P are:

$$X_p = (U_p / R_3)\ (X_c - R_2 \cos\alpha \sin\theta) - (W_p / R_3)(Z_c - R_2 \sin\alpha)$$

$$+ R_2 \cos\alpha \sin\theta$$

$$Y_p = V_p + R_2 \cos\alpha \cos\theta \qquad (3.17)$$

$$Z_p = (U_p / R_3)(Z_c - R_2 \sin\alpha) + (W_p / R_3)(X_c - R_2 \cos\alpha \sin\theta)$$

$$+ R_2 \sin\alpha$$

3.3.1 <u>Velocities</u>

Differentiating (3.9) and rearranging, we obtain:

$$\frac{dF}{dt} = 2(X_c - R_2 \cos\alpha \sin\theta)(\dot{X}_c - R_2 \cos\alpha \cos\theta\dot{\theta}) +$$

$$2(Z_c - R_2 \sin\alpha)\ \dot{Z}_c = 0$$

$$(3.18)$$

$$\frac{dG}{dt} = 2(X_c - R_1 \sin\gamma)\ \dot{X}_c - 2Z_c\ \dot{Z}_c = 0$$

From equations (3.18) and (3.6), we have,

$$\dot{X}_c = Z_c\ (X_c - R_2\ \cos\alpha\ \sin\theta)\ R_2\ \cos\alpha\cos\theta\dot{\theta} /$$

$$\left[Z_c\ (X_c - R_2\ \cos\alpha\sin\theta) - (X_c - R_1\ \sin\gamma)(Z_c - R_2\ \sin\alpha)\right]$$

$$\dot{Y}_c = -R_2\ \cos\alpha\ \sin\theta\dot{\theta} \tag{3.19}$$

$$\dot{Z}_c = -\dot{X}_c\ (X_c - R_1\ \sin\gamma)\ / Z_c$$

Differentiating (3.15) in respect to time we obtain the linear velocity of the coupler point

$$\dot{X}_p = (U_p / R_3)(\dot{X}_c - R_2\ \cos\alpha\cos\theta\dot{\theta}) - (W_p / R_3)\ \dot{Z}_c + R_2\ \cos\alpha\cos\theta\dot{\theta}$$

$$\dot{Y}_p = -R_2\ \cos\alpha\ \sin\theta\dot{\theta} = \dot{Y}_c \tag{3.20}$$

$$\dot{Z}_p = (U_p / R_3)\ \dot{Z}_c + (W_p / R_3)(\dot{X}_c - R_2\ \cos\alpha\ \cos\theta\dot{\theta})$$

Using eq. (3.2) the angular velocity of the output link is derived analytically as,

$$\dot{\varphi} = \dot{X}_c / R_4\ \cos\varphi \tag{3.31}$$

3.3.2 Accelerations

Differentiating equation (3.19) we obtain the linear acceleration of joint C,

$$\ddot{X}_c = [\dot{X}_c(\dot{Z}_c R_2 \cos\alpha \sin\theta - 2Z_c R_2 \cos\alpha \cos\theta\dot{\theta} - \dot{X}_c R_2 \sin\alpha$$

$$- \dot{Z}_c R_1 \sin\gamma) - Z_c R_2^2 \cos^2\alpha\, \dot{\theta}^2 (-\sin^2\theta + \cos^2\theta) +$$

$$X_c R_2 \cos\alpha \cos\theta (Z_c\ddot{\theta} + \dot{Z}_c\dot{\theta}) - X_c Z_c R_2 \cos\alpha \sin\theta\dot{\theta}^2 -$$

$$R_2^2 \cos^2\alpha \sin\theta \cos\theta (Z_c\ddot{\theta} + \dot{Z}_c\dot{\theta})] / (-Z_c R_2 \cos\alpha \sin\theta$$

$$+ X_c R_2 \sin\alpha + Z_c R_1 \sin\gamma - R_1 R_2 \sin\gamma \sin\alpha)$$

$$\ddot{Y}_c = -(R_2 \cos\alpha \sin\theta\ddot{\theta} + R_2 \cos\alpha \cos\theta\dot{\theta}^2) \tag{3.22}$$

$$\ddot{Z}_c = [\ddot{X}_c (R_1 \sin\gamma - X_c) - \dot{X}_c^2 - \dot{Z}_c^2] / Z_c$$

Differentiating eq. (3.20) we obtain the linear acceleration of the coupler point P.

$$\ddot{X}_p = (U_p / R_3)[(\ddot{X}_c + R_2 \cos\alpha(\sin\theta\dot{\theta}^2 - \cos\theta\ddot{\theta})] - (W_p / R_3)\ \ddot{Z}_c$$

$$- R_2 \cos\alpha(\sin\theta\dot{\theta}^2 - \cos\theta\ddot{\theta})$$

$$\ddot{Y}_p = \ddot{Y}_c \tag{3.23}$$

$$\ddot{Z}_p = (U_p / R_3)\ddot{Z}_c - (W_p / R_3)[\ddot{X}_c + R_2 \cos\alpha(\sin\theta\dot{\theta}^2 - \cos\theta\ddot{\theta})]$$

output member angular acceleration is found by differentiating eq. (3.21)

$$\ddot{\varphi} = (\ddot{X}_c / R_4 + \dot{\varphi}^2 \sin\varphi) / \cos\varphi \qquad (3.24)$$

3.4 Computational Procedure and Results

A computer program for the computation of the displacement, velocities, and accelerations of the coupler point as well as the output link is written.

As an example, the coupler curve for a linkage with the following dimensions has been obtained.

R_1 = 1.480 in. R_2 = 1.050 in.

R_3 = 1.000 in. R_4 = 1.050 in.

$\alpha = 50^\circ$ $\gamma = 35^\circ$

$\dot{\theta}$ = 1.00 rad/sec $\ddot{\theta}$ = 0 rad/sec^2

U_p = 0.500 in. V_p = 1.000 in. W_p = 1.000 in.

Estimated starting point

$(X_c)^{(o)}$ = 1.000 in. $(Y_c)^{(o)}$ = 1.000 in. $(Z_c)^{(o)}$ = 1.000 in.

As an additional example, the following RGCR-linkage is considered:

$R_1 = 2.800$ in. $\qquad R_2 = 1.000$ in.

$R_3 = 2.400$ in. $\qquad R_4 = 1.000$ in.

$\alpha = 45^\circ \qquad \gamma = 45^\circ$

$\dot{\theta} = 1.00$ rad/sec $\qquad \ddot{\theta} = 0$ rad/sec^2

$U_p = 1.200$ in. $\qquad V_p = 1.000$ in. $\qquad W_p = 1.000$ in.

In this case an estimate for the starting point is obtained by the parameter perturbation method. We start with a similar linkage that has different parameters, but with a known solution:

$R_1 = 1.410$ in. $\qquad R_2 = 1.000$ in.

$R_3 = 1.000$ in. $\qquad R_4 = 1.000$ in.

$\alpha = 45^\circ \qquad \gamma = 45^\circ$

For which

$X_c = 1.000$ in. $\qquad Z_c = 1.000$ in.

Both R_1 and R_2 are incremented simultaneously in four steps of .45 in. and .35 in. respectively, obtaining each time new values for X_c and Z_c

$(X_c)^{(0)} = 2.500$ in. $(Y_c)^{(0)} = 1.000$ in. $(Z_c)^{(0)} = .900$ in.

Results of these two examples are given in Appendix I.

3.5 Additional Examples

Another configuration of the RGCR-linkage is studied. In this case, the input link R_2 rotates with the variable angle α being the input angle and angle θ fixed. α, $\dot{\alpha}$ and $\ddot{\alpha}$ are therefore the angular input motion and are assumed to be known.

The displacement equations are the same as equations (3.1 - 3.17). Velocities and accelerations are computed as a function of time considering α as the input variable. Starting with (3.9) and differentiating with respect to time

$$\frac{dF}{dt} = X_c\dot{X}_c + X_cR_2 \sin\alpha\sin\theta\dot{\alpha} - \dot{X}_cR_2 \cos\alpha\sin\theta - R_2^2 \sin\alpha\cos\alpha\sin^2\theta\dot{\alpha} + Z_c\dot{Z}_c - Z_cR_2 \cos\alpha\dot{\alpha} - \dot{Z}_cR_2 \sin\alpha + R_2^2 \sin\alpha\cos\alpha\dot{\alpha} = 0 \tag{3.25}$$

$$\frac{dF}{dt} = X_c\dot{X}_c + X_cR_2 \sin\alpha\sin\theta\dot{\alpha} - \dot{X}_cR_2 \cos\alpha\sin\theta - R_2^2 \sin\alpha\cos\alpha\sin^2\theta\dot{\alpha} + \dot{X}_c(R_1\sin\gamma - X_c) - Z_cR_2 \cos\alpha\dot{\alpha} - R_2 \sin\alpha(R_1 \sin\gamma - X_c)\, \dot{X}_c/Z_c + R_2^2 \sin\alpha\cos\alpha\dot{\alpha} = 0$$

rearranging (3.25), we have,

$$\dot{X}_c = (- X_cR_2 \sin\alpha \sin\theta\dot{\alpha} + R_2^2 \sin\alpha\cos\alpha\sin^2\theta\dot{\alpha} + Z_cR_2 \cos\alpha\dot{\alpha} - R_2^2 \sin\alpha\cos\alpha\dot{\alpha}) / [- R_2 \cos\alpha\sin\theta + R_1 \sin\gamma - R_2 \sin\alpha(R_1 \sin\gamma - X_c) / Z_c] \tag{3.26}$$

$$\dot{Z}_c = \dot{X}_c (R_1 \sin\gamma - X_c) / Z_c$$

differentiating Y in eq. (3.1), we have,

$$\dot{Y}_c = - R_2 \cos\theta \sin\alpha\dot{\alpha}$$

Differentiating equations (3.26) we obtain the linear acceleration of joint C.

$$\ddot{X}_c = \{- \dot{X}_c \{2R_2 \sin\alpha\sin\theta\dot{\alpha} - [Z_c (R_2 \cos\alpha\dot{\alpha}(R_1 \sin\gamma - X_c) - \dot{X}_cR_2 \sin\alpha) - \dot{Z}_cR_2 \sin\alpha(R_1 \sin\gamma - X_c)] / Z_c^2\} + R_2^2 \sin^2\theta\dot{\alpha}^2 (\cos^2\alpha - \sin^2\alpha) - X_cR_2 \sin\theta (\sin\alpha\ddot{\alpha} + \cos\alpha\dot{\alpha}^2) + R_2^2 \sin\alpha\cos\alpha\sin^2\theta\ddot{\alpha} + \dot{Z}_cR_2 \cos\alpha\dot{\alpha} - Z_cR_2 \sin\alpha\dot{\alpha}^2 + Z_cR_2 \cos\alpha\ddot{\alpha} - R_2^2 \cos^2\alpha\dot{\alpha}^2 + R_2^2 \sin^2\alpha\dot{\alpha}^2 - R_2^2 \sin\alpha\cos\alpha\ddot{\alpha}\} / [R_1 \sin\gamma - R_2 \cos\alpha\sin\theta - R_2 \sin\alpha(R_1 \sin\gamma - X_c) / Z_c] \tag{3.27}$$

$$\ddot{Y}_c = -R_2 \cos\theta \,(\cos\alpha\dot{\alpha}^2 + \sin\alpha\ddot{\alpha})$$

$$\ddot{Z}_c = \{Z_c\,[\ddot{X}_c\,(R_1 \sin\gamma - X_c) - \dot{X}_c^{\,2}] - \dot{Z}_c\dot{X}_c\,(R_1 \sin\gamma - X_c)\} / Z_c^2$$

differentiating now the displacement equation for the coupler point P (3.17)

$$\dot{X}_p = (U_p/R_3)(\dot{X}_c - R_2 \sin\theta \sin\alpha\dot{\alpha}) - (W_p/R_3)(\dot{Z}_c - R_2 \cos\alpha\dot{\alpha})$$
$$- R_2 \sin\theta \sin\alpha\dot{\alpha}$$

$$\dot{Y}_p = \dot{Y}_c \qquad (3.28)$$

$$\dot{Z}_p = (U_p/R_3)(\dot{Z}_c - R_2 \cos\alpha\dot{\alpha}) + (W_p/R_3)(\dot{X}_c + R_2 \sin\theta \sin\alpha\dot{\alpha})$$
$$+ R_2 \cos\alpha\dot{\alpha}$$

$$\ddot{X}_p = (U_p/R_3)\,[\ddot{X}_c + R_2 \sin\theta\,(\cos\alpha\dot{\alpha}^2 + \sin\alpha\ddot{\alpha})] -$$
$$(W_p/R_3)\,[\ddot{Z}_c - R_2(\cos\alpha\ddot{\alpha} - \sin\alpha\dot{\alpha}^2)] -$$
$$R_2 \sin\theta\,(\cos\alpha\dot{\alpha}^2 + \sin\alpha\ddot{\alpha})$$

$$\ddot{Y}_p = \ddot{Y}_c \qquad (3.29)$$

$$\ddot{Z}_p = (U_p/R_3)\,[\ddot{Z}_c - R_2\,(-\sin\alpha\dot{\alpha}^2 + \cos\alpha\ddot{\alpha})] +$$
$$(W_p/R_3)\,[\ddot{X}_c + R_2 \sin\theta\,(\cos\alpha\dot{\alpha}^2 + \sin\alpha\ddot{\alpha})]$$
$$+ R_2\,(-\sin\alpha\dot{\alpha}^2 + \cos\alpha\ddot{\alpha})$$

The angular displacement, velocity and acceleration are computed from equations (3.13), (3.21) and (3.24).

The following examples are considered:

<u>Example 1</u>

$R_1 = 1.480$ in. $\quad$ $R_2 = 1.050$ in.

$R_3 = 1.000$ in. $\quad$ $R_4 = 1.050$ in.

$\theta = 70^\circ$ $\quad$ $\gamma = 35^\circ$

$\dot{\alpha} = 1.000$ rad/sec $\quad$ $\ddot{\alpha} = 0$ rad/sec^2

$U_p = 0.500$ in. $\quad$ $V_p = 1.000$ in. $\quad$ $W_p = 1.000$ in.

$(X_c)^{(0)} = 1.000$ in. $\quad$ $(Y_c)^{(0)} = 1.000$ in. $\quad$ $(Z_c)^{(0)} = 1.000$ in.

<u>Example 2</u>

$R_1 = 1.480$ in. $\quad$ $R_2 = 1.050$ in.

$R_3 = 1.000$ in. $\quad$ $R_4 = 1.050$ in.

$\theta = 0^\circ$ $\quad$ $\gamma = 35^\circ$

$\dot{\alpha} = 1.000$ rad/sec $\quad$ $\ddot{\alpha} = 0$ rad/sec^2

$U_p = 0.500$ in. $\quad$ $V_p = 1.000$ in. $\quad$ $W_p = 1.000$ in.

$(X_c)^{(0)} = 1.000$ in. $\quad$ $(Y_c)^{(0)} = 1.000$ in. $\quad$ $(Z_c)^{(0)} = 1.000$ in.

Example 3

$R_1 = 2.000$ in. $R_2 = 12.000$ in.

$R_3 = 7.000$ in. $R_4 = 12.000$ in.

$\theta = 90^\circ$ $\gamma = 90^\circ$

$\dot{\alpha} = 1.000$ rad/sec $\ddot{\alpha} = 0$ rad/sec^2

$U_p = 3.000$ in. $U_p = 0.000$ in. $W_p = 3.000$ in.

By choosing $\theta = 90^\circ$ and $\gamma = 90^\circ$ this type of spatial linkage is a special case of a plane mechanism

$(X_c)^{(0)} = 12.000$ in. $(Y_c)^{(0)} = 0.000$ in. $(Z_c)^{(0)} = 7.000$ in.

Results are given in Appendix I.

3.6 Sources of Divergence and Locking Regions

The Newton-Raphson iteration procedure fails to converge when the Jacobian matrix D, (eq. 2.16) is singular in the neighbourhood of the solution or the initial approximation. (14)

Mathematically speaking, the vanishing of D indicates either of the three cases:

a. No solutions.

b. Multiple solutions.

c. Two or more solutions close together.

Unfortunately, from kinematic point of view, the case of no mathematical solution does not necessarily mean a locked linkage, but rather the case of an "ill conditioned" mechanism, with parameters that yeild a zero or close to zero Jacobian Matrix for a particular input angle Θ. To illustrate this, let us reqrite the Matrix D using eq. (2.16) and (2.17).

$$D = 4Z_c(X_c - R_1 \sin\gamma) - 4X_c(Z_c - R_2 \sin\Theta) \qquad (3.30)$$

let us consider the spherical RRRR-linkage with the following parameters:

R_1 = 1.414 in. R_2 = 1.000 in.

R_3 = 2.000 in. R_4 = 1.000 in.

For $\Theta = 0^o$ the starting points are:

X_c = - 1.000 in. Y_c = 1.000 in. Z_c = 0.000 in.

These parameters will make the Jacobian matrix vanish and terminate the computer program.

Since it is evident from eq. (3.30) that for $\theta = 0^o$ Z_c should not be zero we can choose $(Z_c)^{(0)} = .001$in. this will not terminate the computer program but the iteration process fails to converge. This problem is not encountered at $\theta = 180^o$, since, because of tranquilations in the computer calculations Z_c does not converge to zero.

Another special case is the RSCR linkage with α variable.

For the following parameters

$R_1 = 1.000$ in. $\qquad R_2 = 1.000$ in.

$R_3 = 1.000$ in. $\qquad R_4 = 1.000$ in.

$\theta = 90^o \qquad \gamma = 90^o$

which is a planer linkage.

For $\alpha = 0$ the starting points are chosen as:

$(X_c)^{(0)} = 1.980$ in. $\qquad (Y_c)^{(0)} = .000$ in. $(Z_c)^{(0)} = .020$ in.

instead of the exact solution

$X_c = 2.000$ in. $\qquad Y_c = 0.000$ in. $\quad Z_c = 0.000$ in.

that will zero the Jacobian. But in this special case the iteration converged to

$(X_c)^{(n)} = 1.074$ in. $\quad (Y_c)^{(n)} = .000$ in. $\quad (Z_c)^{(n)} = .907$ in.

which is the other mechanical configuration this linkage can assume in space for $\alpha = 0^o$.

However, this problem can be eliminated in both cases by starting the iteration process with $\alpha = 5^o$ and incrementing every 10^o, thus bypassing the singularity point at $\alpha = 0^o$.

Excluding those cases of "ill conditioned" mechanisms locking of the linkage can be assumed each time the iteration process fails to converge for a non-zero Jacobian.

CONCLUSIONS

The problem of solving the displacement equation of spatial four-bar linkage is considered and the iterative method is employed.

To the best of the author's knowledge, no coupler curves were computed by any other method on the RSCR linkage, but in the special case where the spatial linkage becomes a plane linkage the procedure is the same as the one devised by Osman and Mansour (8).

The method is best suited for computer aided design. Unlike the existing methods for displacement analysis of complicated geometrical configuration it does not require any knowledge of advanced mathematical analysis such as is required when using screw matrices, dual number quaternion algebra or tensors.

In the determination of the displacement of the driven members, we usually encounter the problem of solving a set of non-linear algebraic equations which in general do not have an explicit solution. With the help of an electronic digital computer, Newton-Raphson iteration method proved to yield fast convergence.

In the case where a reasonable initial guess of the starting point for the iteration is not available a parameter perturbation method is suggested. Once the first point is solved, the iteration process for the consecutive point can be initiated by proximity perturbation of the previous results.

The technique of kinematic analysis employed herein is very effective for complicated mechanisms. However, the problem of poor convergence of iteration associated with the Newton-Raphson method may arise in some particular cases for which the Jacobian Matrix D (eq.2.16) happens to be zero. The vanishing of D sometimes causes the iteration process to diverge or the computer program is terminated all together.

The technique is also well suited to detect the locking regions of most linkages.

The following areas are recommended for further investigation.

a. Extention of the technique to the analysis of five bars and more spatial linkages.

b. Extension of the technique to the synthesis of spatial mechanism.

c. Further investigation of the mathematical aspects of convergence, locking regions, and their relationship to the Jacobian Matrix D.

d. Further investigation to determine the necessary and sufficient domain of perturbation of the initial approximation that will still assure convergence.

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ILLUSTRATIONS AND COMPUTING FLOW-CHARTS

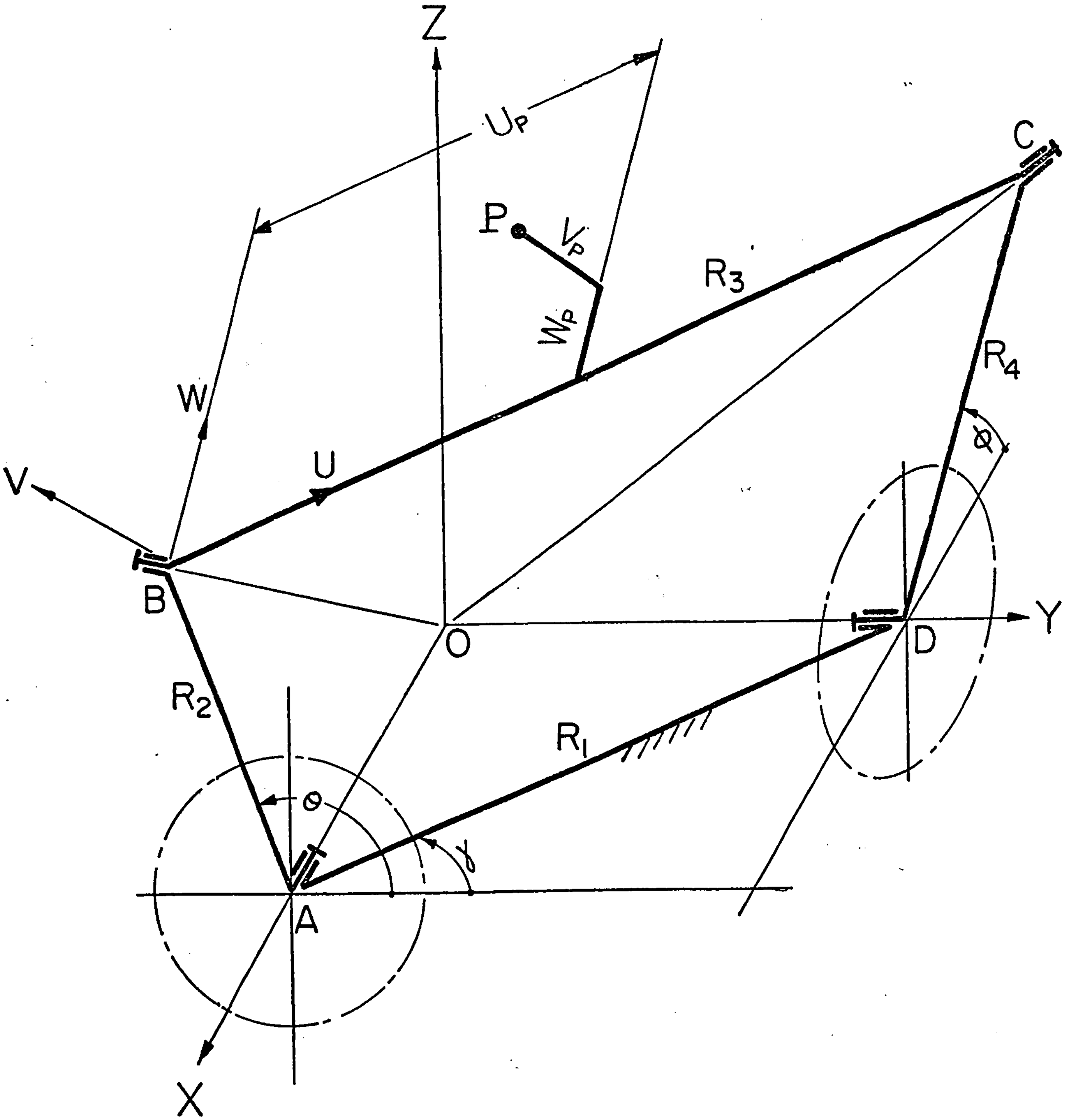


Fig. 1: The spherical four-bar RRRR-linkage with coupler point.

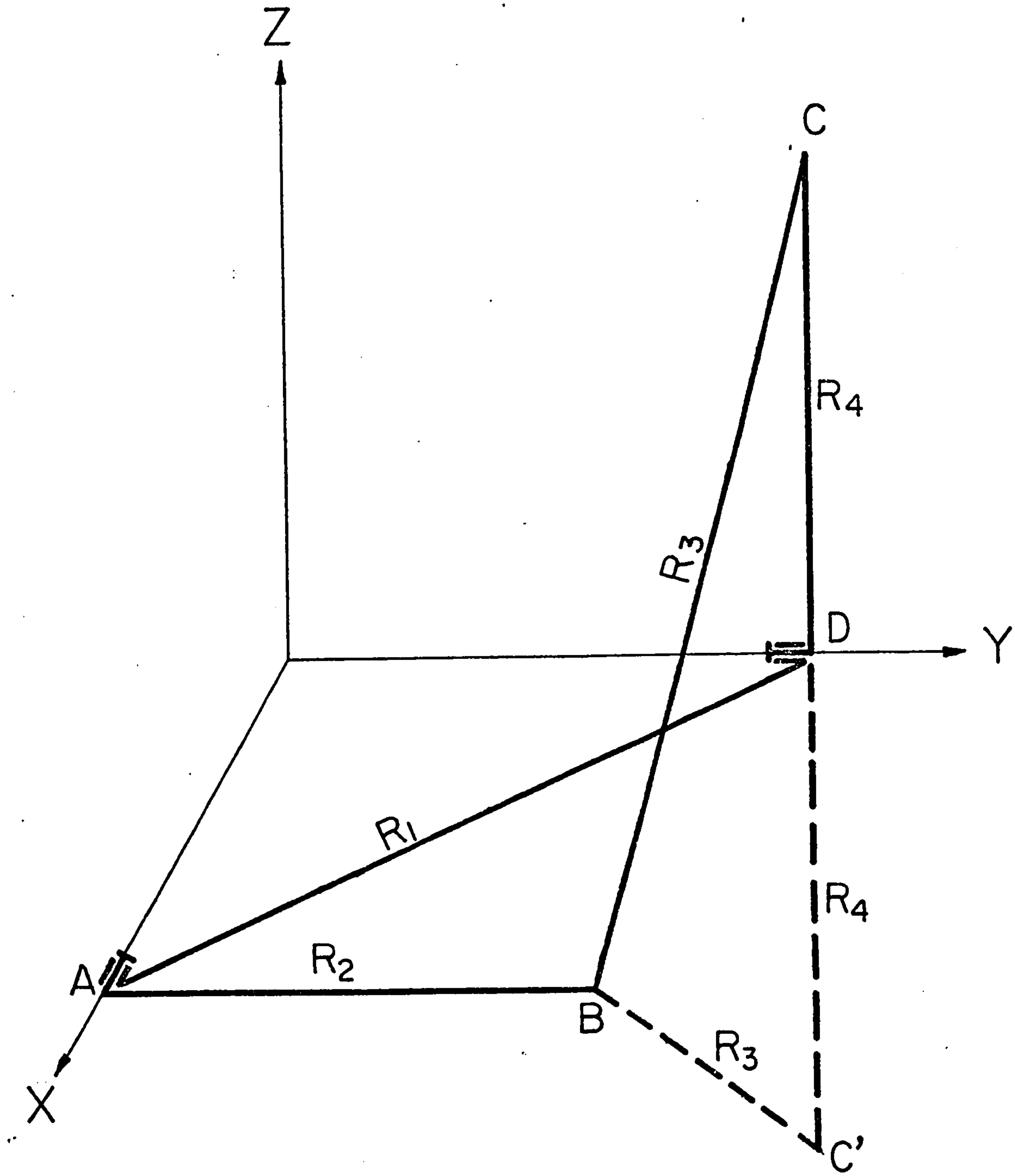
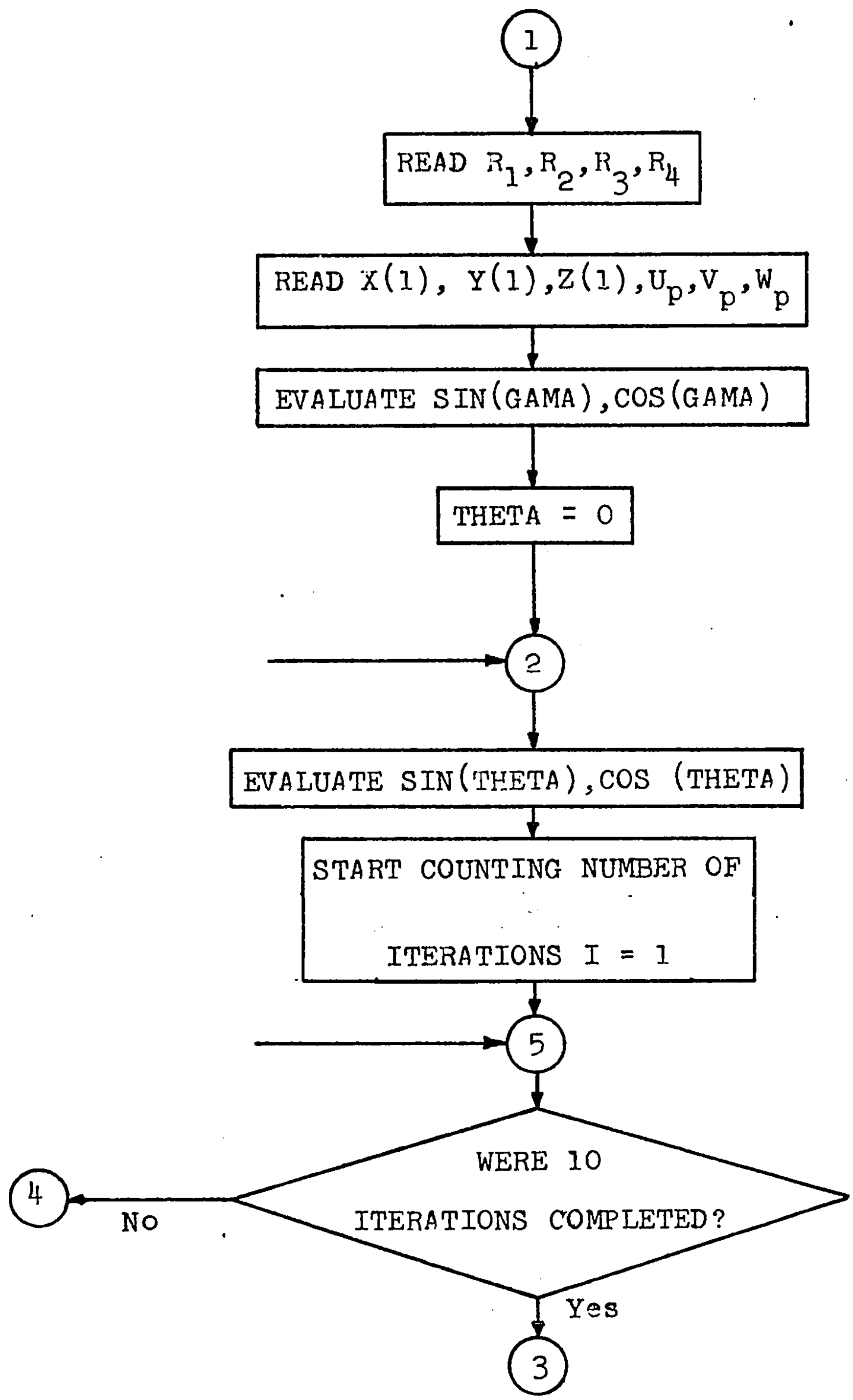
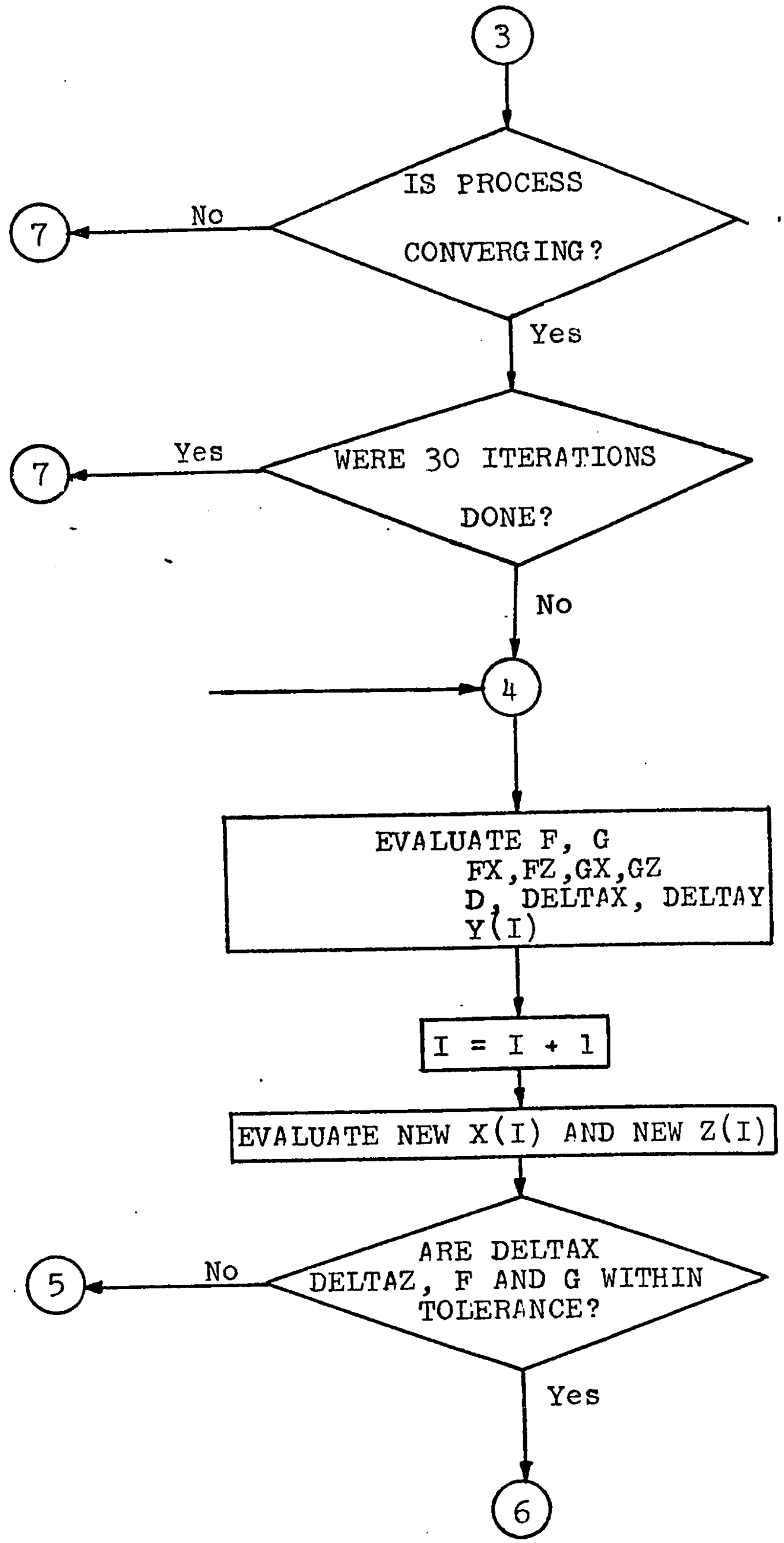
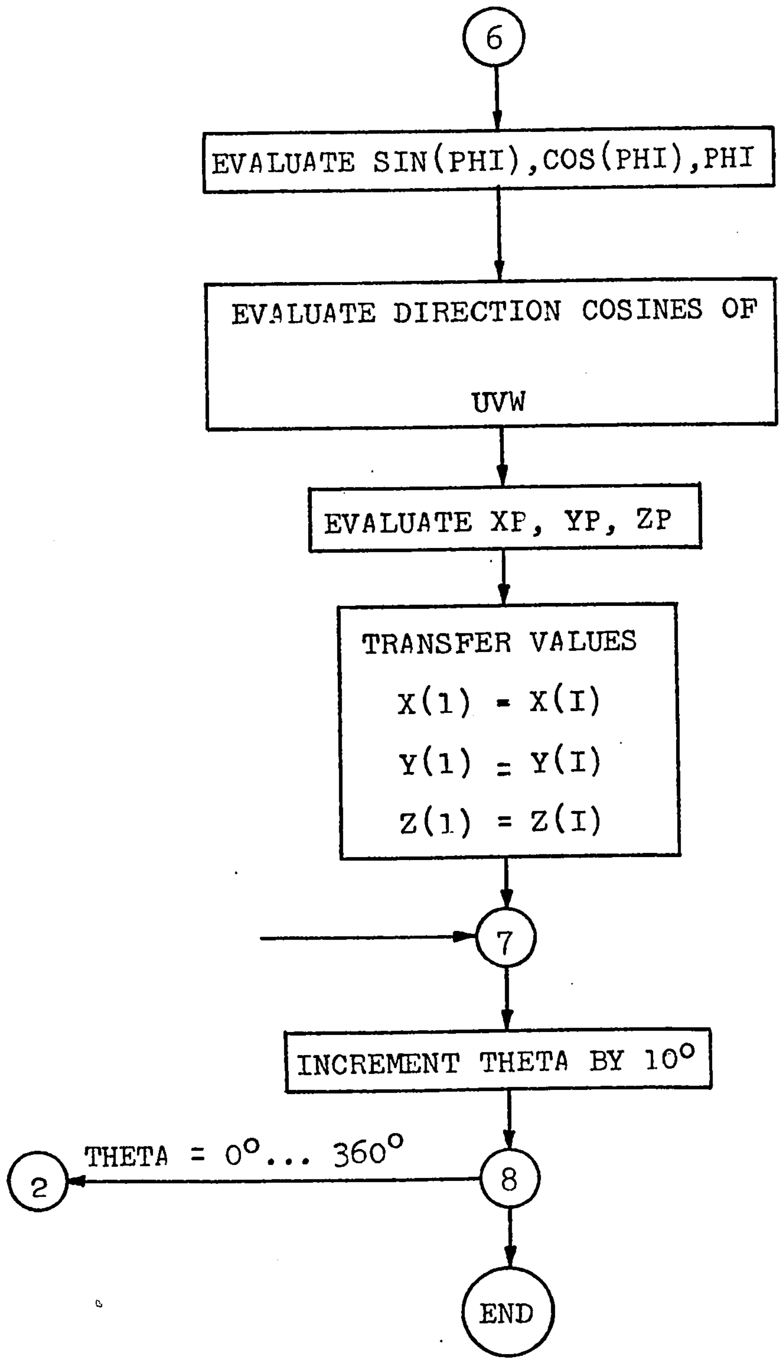


Fig. 2: The two possible starting configurations for the RRRR-linkage.

Fig. 3: Flow-chart for displacement analysis of the spherical RRRR-linkage.




3
IS PROCESS CONVERGING?
No
7
Yes
WERE 30 ITERATIONS DONE?
Yes
7
No
4
EVALUATE F, G
FX,FZ,GX,GZ
D, DELTAX, DELTAY
Y(I)
I = I + 1
EVALUATE NEW X(I) AND NEW Z(I)
ARE DELTAX DELTAZ, F AND G WITHIN TOLERANCE?
No
5
Yes
6


6
EVALUATE SIN(PHI),COS(PHI),PHI
EVALUATE DIRECTION COSINES OF UVW
EVALUATE XP, YP, ZP
TRANSFER VALUES
X(1) = X(I)
Y(1) = Y(I)
Z(1) = Z(I)
7
INCREMENT THETA BY 10°
THETA = 0°... 360°
2
8
END

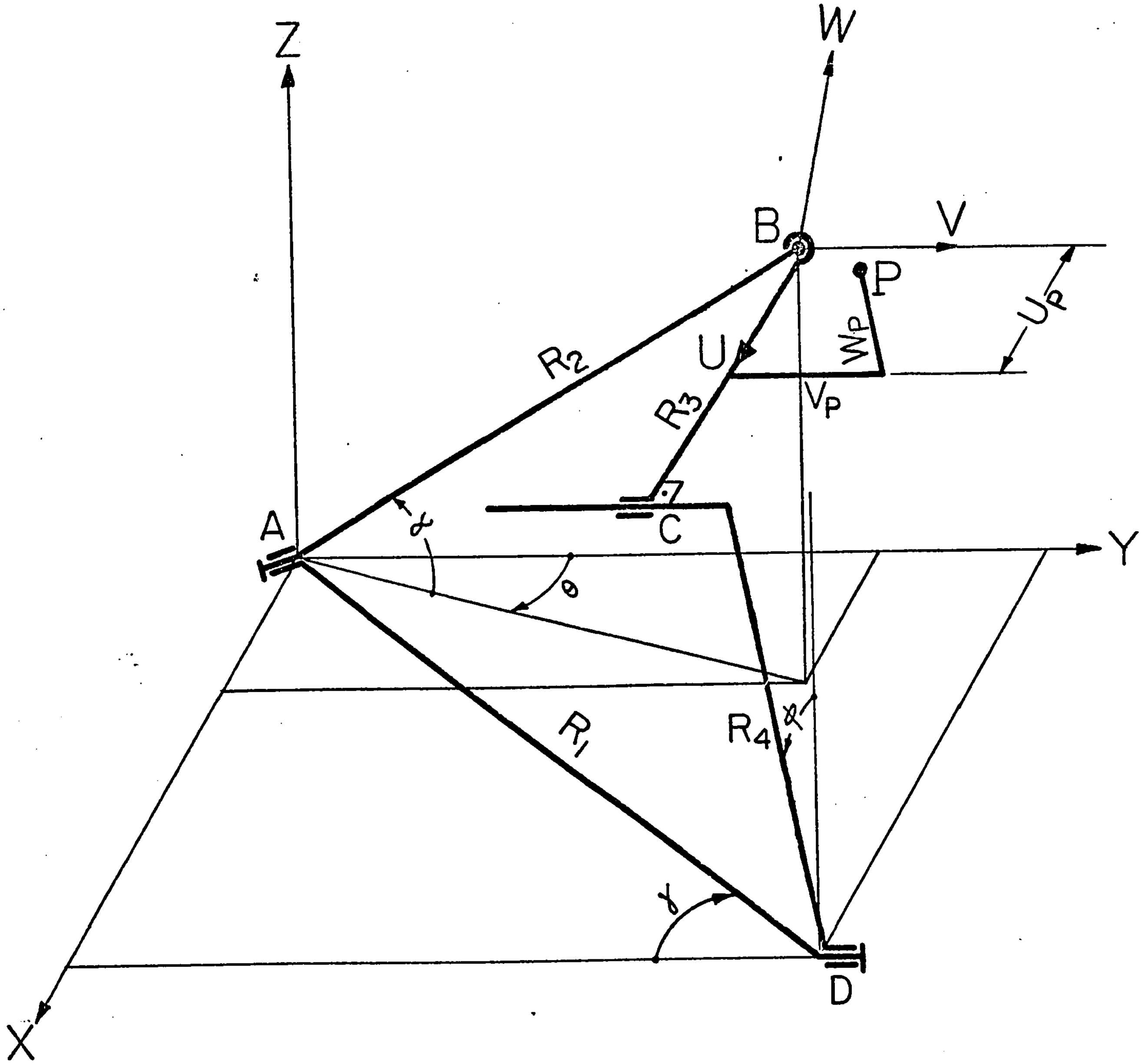


Fig. 4: The RSCR linkage

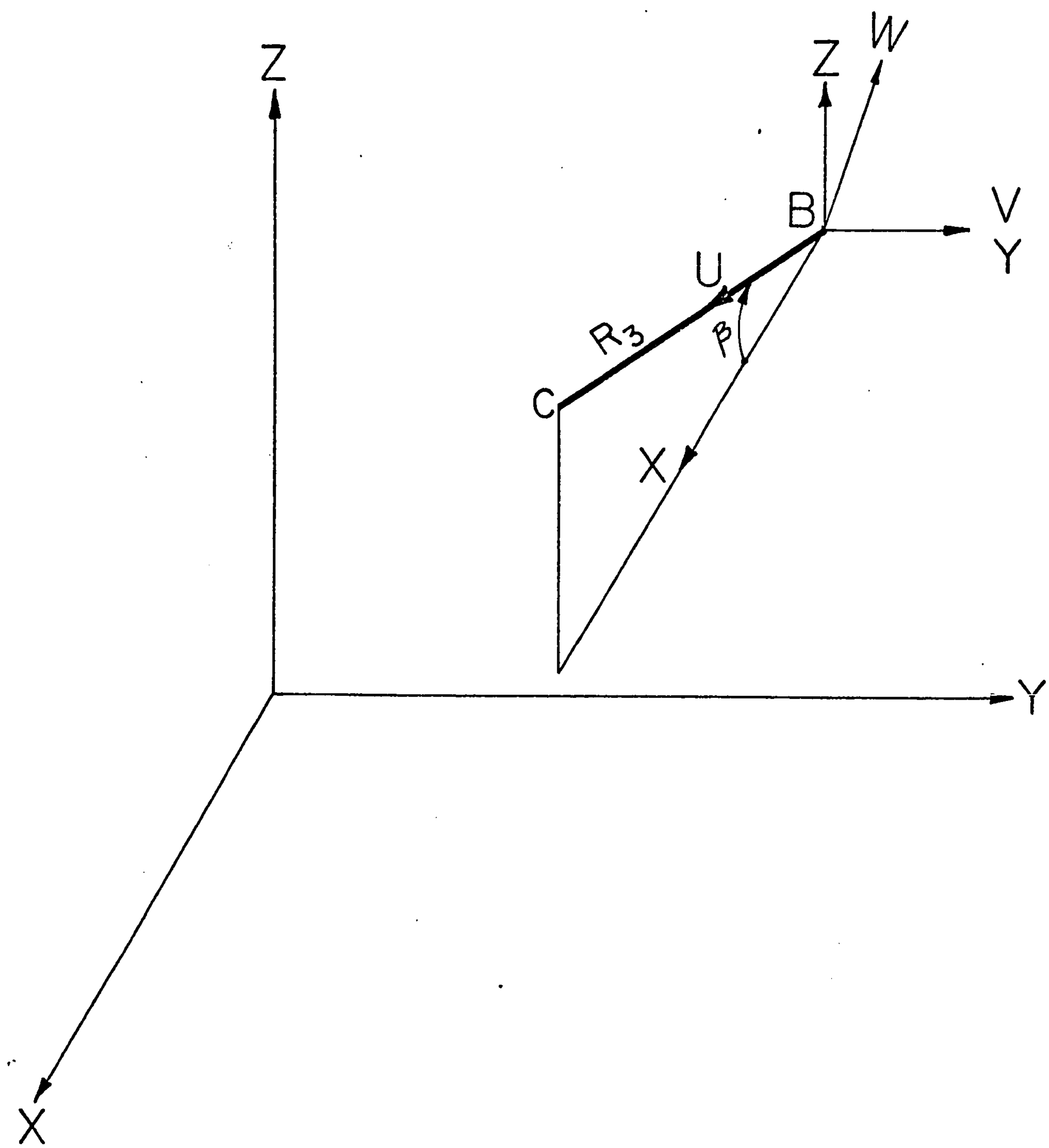


Fig. 5: The Euler angle β of the UVW-coordinate system.

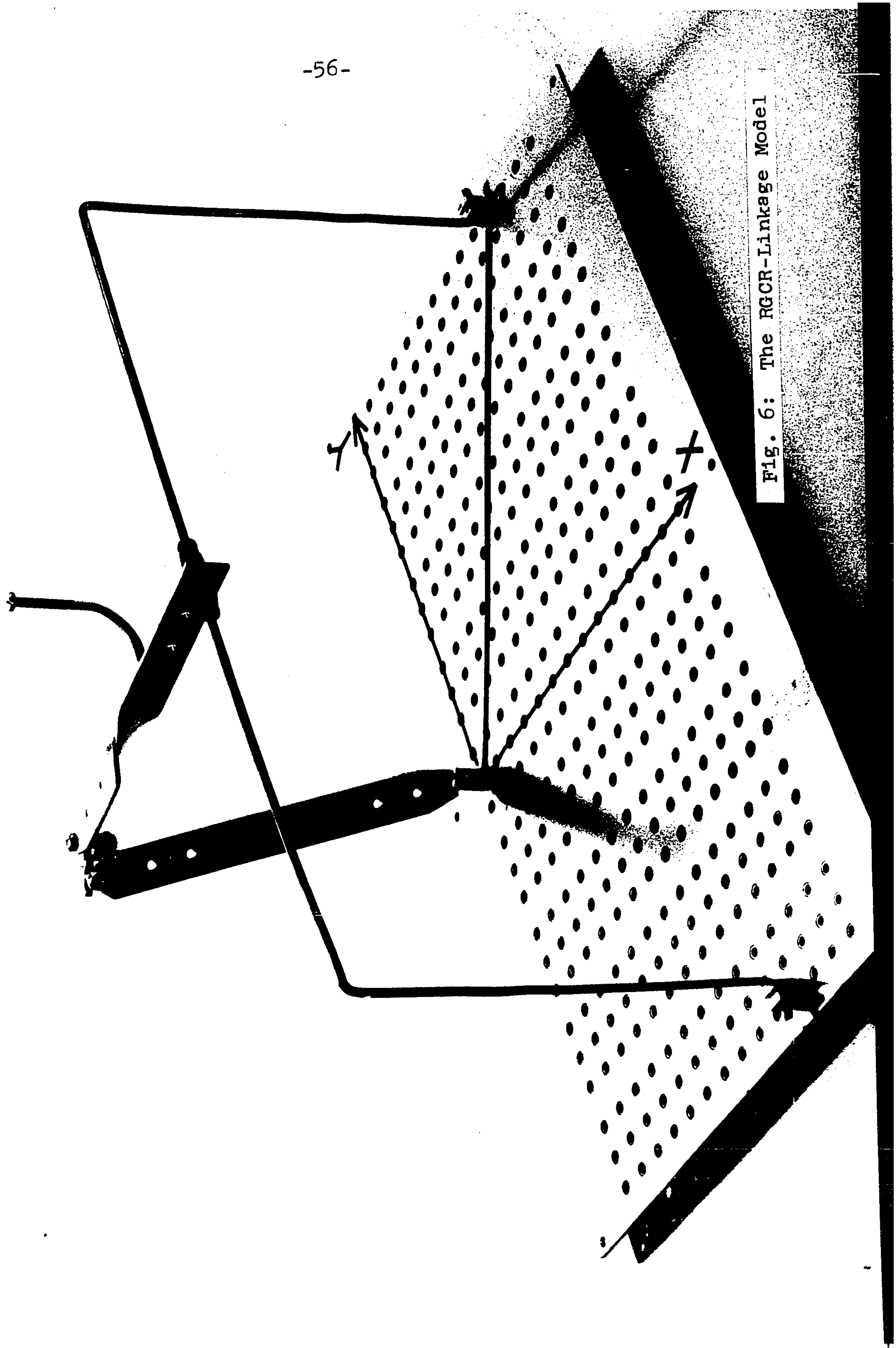


Fig. 6: The RGCR-Linkage Model

APPENDIX I

COMPUTER PROGRAMS AND RESULTS

1. The spherical four-bar RRRR-linkage

```
      PROGRAMLINKAGE(INPUT,OUTPUT)
C
C           ITERATIVE  METHOD  FOR  DISPLACMENT
C              ANALYSIS  OF  SPATIAL  LINKAGES
C
C                     RRRR  SPHERICAL  LINKAGE
C
      DIMENSION F(30),X(30),Y(30),Z(30)
    1 READ101,R1,R2,R3,R4,GAMA
      READ101,X(1),Y(1),Z(1),UP,VP,WP
  101 FORMAT(6F10.3)
      PRINT 102
  102 FORMAT(1H1,21X,≠R1≠,8X,≠R2≠,8X,≠R3≠,8X,≠R4≠,7X,≠GAMA≠,7X,≠UP≠,8X,
     1≠VP≠,8X,≠WP≠)
      PRINT103,R1,R2,R3,R4,GAMA,UP,VP,WP
  103 FORMAT(1H0,15X,8F10.3)
C     T=ANGLE THETA IN RADIANS
C     G=ANGLE GAMA  IN RADIANS
      G=GAMA*ATAN(1.0)/45.0
      CG=COS(G)
      SG=SIN(G)
      THETA=0.0
      DO 7J=1,37
      T=THETA*ATAN(1.0)/45.0
      CT=COS(T)
      ST=SIN(T)
      PRINT104
  104 FORMAT(1H0,19X,≠THETA≠,7X,≠I≠,10X,≠X≠,12X,≠Y≠,12X,≠Z≠,12X,≠F≠,12X,
     1≠G≠,12X,≠D≠)
      I=1
   12 CONTINUE
      IF(I-10)15,15,32
   32 IF((ABS(F(9)-F(10))).LT.(ABS(F(7)-F(8))).AND.
     1(ABS(F(7)-F(8))).LT.(ABS(F(5)-F(6))).AND.
     2(ABS(F(5)-F(6))).LT.(ABS(F(3)-F(4))).AND.
     3(ABS(F(3)-F(4))).LT.(ABS(F(1)-F(2))))17,7
   17 IF(I-30)15,15,7
C     ITERATION  PROCESS  STARTS
   15 F(I)=(X(I)-R1*SG)**2+(R1*CG-R2*CT)**2+(Z(I)-R2*ST)**2-R3**2
      G=(X(I))**2+(Z(I))**2-R4**2
      FX=2.0*(X(I)-R1*SG)
      FZ=2.0*(Z(I)-R2*ST)
      GX=2.0*X(I)
      GZ=2.0*Z(I)
      D=FX*GZ-FZ*GX
      DELTAX=(-F(I)*GZ+G*FZ)/D
      DELTAZ=(-G*FX+F(I)*GX)/D
      Y(I)=R1*CG
      PRINT105,THETA,I,X(I),Y(I),Z(I),F(I),G,D
  105 FORMAT(1H ,15X,F8.1,I9,5F13.3,F17.7)
      I=I+1
C     COORDINATES  OF  JOINT  C
      X(I)=X(I-1)+DELTAX
      Z(I)=Z(I-1)+DELTAZ
      Y(I)=Y(I-1)
```

```
      IF(ABS(DELTAX)-0.001)10,12,12
   10 IF(ABS(DELTAZ)-0.001)11,12,12
   11 IF(ABS(F(I-1))-0.001)13,12,12
   13 IF(ABS(G)-0.001)14,12,12
C     ITERATION  PROCESS  COMPLETED
   14 CONTINUE
      SP=Z(I)/R4
      CP=-X(I)/R4
      P=ATAN2(SP,CP)
C     ANGULAR  DISPLCEMENT  OF  OUTPUT  LINK
      PHI=P*45.0/ATAN(1.0)
C     DIRECTION PARAMETERS OF U
      XU=X(I)-R1*SG
      YU=Y(I)-R2*CT
      ZU=Z(I)-R2*ST
C     DIRECTION COSINES OF U
      BB=SQRT(XU**2+YU**2+ZU**2)
      CXU=XU/BB
      CYU=YU/BB
      CZU=ZU/BB
C     DIRECTION PARAMETERS OF W
      XW=R2*CT*Z(I)-R2*ST*R1*CG
      YW=R2*ST*X(I)-R1*SG*Z(I)
      ZW=R1*SG*R1*CG-R2*CT*X(I)
C     DIRECTION COSINES OF W
      AA=SQRT(XW**2+YW**2+ZW**2)
      CXW=XW/AA
      CYW=YW/AA
      CZW=ZW/AA
C     DIRECTION PARAMETERS OF V
      XV=YU*ZW-ZU*YW
      YV=ZU*XW-XU*ZW
      ZV=XU*YW-YU*XW
C     DIRECTION COSINES OF V
      CC=SQRT(XV**2+YV**2+ZV**2)
      CXV=XV/CC
      CYV=YV/CC
      CZV=ZV/CC
C     COORDINATES  OF  COUPLER  POINT  P
      XP=UP*CXU+VP*CXV+WP*CXW+R1*SG
      YP=UP*CYU+VP*CYV+WP*CYW+R2*CT
      ZP=UP*CZU+VP*CZV+WP*CZW+R2*ST
      PRINT106
  106 FORMAT(1H0,21X,≠XP≠,8X,≠YP≠,8X,≠ZP≠,8X,≠PHI≠)
      PRINT107,XP,YP,ZP,PHI
  107 FORMAT(1H ,15X,4F10.3)
      X(1)=X(I)
      Y(1)=Y(I)
      Z(1)=Z(I)
    7 THETA=THETA+10.0
      GO TO 1
      END
```

1.2: NUMERICAL RESULTS

R1	R2	R3	R4	GAMA	UP	VP	WP
1.000	.270	1.500	1.000	56.000	1.000	1.000	1.000

THETA	I	X	Y	Z	F	G	D
0.0	1	-.300	.559	.800	-.252	-.270	-2.6529202
0.0	2	-.289	.559	.973	.030	.030	-3.2262894
0.0	3	-.289	.559	.957	.000	.000	-3.1751315

XP	YP	ZP	PHI
.958	.237	1.727	73.206

THETA	I	X	Y	Z	F	G	D
10.0	1	-.289	.559	.957	-.085	.000	-3.2289049
10.0	2	-.339	.559	.942	.003	.003	-3.1878243
10.0	3	-.339	.559	.941	.000	.000	-3.1830072

XP	YP	ZP	PHI
.860	.229	1.778	70.162

THETA	I	X	Y	Z	F	G	D
20.0	1	-.339	.559	.941	-.072	.000	-3.2447131
20.0	2	-.381	.559	.926	.002	.002	-3.2102368
20.0	3	-.381	.559	.925	.000	.000	-3.2068307

XP	YP	ZP	PHI
.768	.208	1.823	67.608

THETA	I	X	Y	Z	F	G	D
30.0	1	-.381	.559	.925	-.057	.000	-3.2718234
30.0	2	-.413	.559	.911	.001	.001	-3.2453768
30.0	3	-.413	.559	.911	.000	.000	-3.2432926

XP	YP	ZP	PHI
.683	.173	1.860	65.616

THETA	I	X	Y	Z	F	G	D
40.0	1	-.413	.559	.911	-.046	.000	-3.3069568
40.0	2	-.435	.559	.901	.001	.001	-3.2891169

XP	YP	ZP	PHI
.607	.128	1.890	64.225

THETA	I	X	Y	Z	F	G	D
50.0	1	-.435	.559	.901	-.023	.000	-3.3459780
50.0	2	-.447	.559	.895	.000	.000	-3.3365138

XP	YP	ZP	PHI
.542	.073	1.912	63.446

THETA	I	X	Y	Z	F	G	D
60.0	1	-.447	.559	.895	-.005	.000	-3.3844622
60.0	2	-.450	.559	.893	.000	.000	-3.3824861

XP	YP	ZP	PHI
.489	.013	1.928	63.271

THETA	I	X	Y	Z	F	G	D
70.0	1	-.450	.559	.893	.012	.000	-3.4182542
70.0	2	-.443	.559	.896	.000	.000	-3.4224221

XP	YP	ZP	PHI
.448	-.052	1.937	63.679

THETA	I	X	Y	Z	F	G	D
80.0	1	-.443	.559	.896	.029	.000	-3.4439377
80.0	2	-.428	.559	.904	.000	.000	-3.4526478

XP	YP	ZP	PHI
.419	-.117	1.941	64.643

THETA	I	X	Y	Z	F	G	D
90.0	1	-.428	.559	.904	.045	.000	-3.4591762
90.0	2	-.405	.559	.915	.001	.001	-3.4707401

XP	YP	ZP	PHI
.403	-.181	1.939	66.132

THETA	I	X	Y	Z	F	G	D
100.0	1	-.405	.559	.914	.060	.000	-3.4629137
100.0	2	-.373	.559	.928	.001	.001	-3.4756942
100.0	3	-.373	.559	.928	.000	.000	-3.4736035

XP	YP	ZP	PHI
.401	-.242	1.933	68.112

THETA	I	X	Y	Z	F	G	D
110.0	1	-.373	.559	.928	.073	.000	-3.4554385
110.0	2	-.333	.559	.944	.002	.002	-3.4679600
110.0	3	-.333	.559	.943	.000	.000	-3.4648262

XP	YP	ZP	PHI
.413	-.297	1.923	70.544

THETA	I	X	Y	Z	F	G	D
120.0	1	-.333	.559	.943	.085	.000	-3.4383244
120.0	2	-.286	.559	.959	.002	.002	-3.4493610
120.0	3	-.286	.559	.958	.000	.000	-3.4451331

XP	YP	ZP	PHI
.440	-.347	1.908	73.380

THETA	I	X	Y	Z	F	G	D
130.0	1	-.286	.559	.958	.095	.000	-3.4142467
130.0	2	-.233	.559	.974	.003	.003	-3.4228979
130.0	3	-.232	.559	.973	.000	.000	-3.4176302

XP	YP	ZP	PHI
.482	-.389	1.890	76.562

THETA	I	X	Y	Z	F	G	D
140.0	1	-.232	.559	.973	.102	.000	-3.3866895
140.0	2	-.174	.559	.987	.004	.004	-3.3924380
140.0	3	-.173	.559	.985	.000	.000	-3.3863048

XP	YP	ZP	PHI
.538	-.423	1.867	80.010

THETA	I	X	Y	Z	F	G	D
150.0	1	-.173	.559	.985	.106	.000	-3.3595485

THETA	I	X	Y	Z	F	G	D
150.0	2	-.111	.559	.996	.004	.004	-3.3622928
150.0	3	-.111	.559	.994	.000	.000	-3.3555921

XP	YP	ZP	PHI
.608	-.450	1.839	83.631

THETA	I	X	Y	Z	F	G	D
160.0	1	-.111	.559	.994	.107	.000	-3.3366593
160.0	2	-.047	.559	1.001	.004	.004	-3.3367073
160.0	3	-.047	.559	.999	.000	.000	-3.3298413

XP	YP	ZP	PHI
.690	-.469	1.805	87.309

THETA	I	X	Y	Z	F	G	D
170.0	1	-.047	.559	.999	.104	.000	-3.3212981
170.0	2	.016	.559	1.002	.004	.004	-3.3193066
170.0	3	.016	.559	1.000	.000	.000	-3.3127332

XP	YP	ZP	PHI
.781	-.480	1.765	90.916

THETA	I	X	Y	Z	F	G	D
180.0	1	.016	.559	1.000	.098	.000	-3.3157261
180.0	2	.075	.559	.999	.004	.004	-3.3125799
180.0	3	.075	.559	.997	.000	.000	-3.3067390

XP	YP	ZP	PHI
.877	-.484	1.718	94.319

THETA	I	X	Y	Z	F	G	D
190.0	1	.075	.559	.997	.089	.000	-3.3208568
190.0	2	.129	.559	.993	.003	.003	-3.3174981
190.0	3	.129	.559	.992	.000	.000	-3.3127301

XP	YP	ZP	PHI
.975	-.480	1.666	97.390

THETA	I	X	Y	Z	F	G	D
200.0	1	.129	.559	.992	.077	.000	-3.3361158
200.0	2	.174	.559	.986	.002	.002	-3.3333524
200.0	3	.174	.559	.985	.000	.000	-3.3298360

XP	YP	ZP	PHI
1.070	-.469	1.609	100.021

THETA	I	X	Y	Z	F	G	D
210.0	1	.174	.559	.985	.062	.000	-3.3595240
210.0	2	.210	.559	.978	.001	.001	-3.3578587
210.0	3	.210	.559	.978	.000	.000	-3.3555862

XP	YP	ZP	PHI
1.160	-.450	1.551	102.129

THETA	I	X	Y	Z	F	G	D
220.0	1	.210	.559	.978	.045	.000	-3.3879869
220.0	2	.236	.559	.972	.001	.001	-3.3875054

XP	YP	ZP	PHI
1.242	-.423	1.495	103.658

THETA	I	X	Y	Z	F	G	D
230.0	1	.236	.559	.972	.027	.000	-3.4177315
230.0	2	.252	.559	.968	.000	.000	-3.4180673

XP	YP	ZP	PHI
1.313	-.389	1.442	104.578

THETA	I	X	Y	Z	F	G	D
240.0	1	.252	.559	.968	.009	.000	-3.4448051
240.0	2	.257	.559	.966	.000	.000	-3.4451790

XP	YP	ZP	PHI
1.372	-.347	1.397	104.882

THETA	I	X	Y	Z	F	G	D
250.0	1	.257	.559	.966	-.009	.000	-3.4655643
250.0	2	.252	.559	.968	.000	.000	-3.4648744

XP	YP	ZP	PHI
1.418	-.297	1.362	104.576

THETA	I	X	Y	Z	F	G	D
260.0	1	.252	.559	.968	-.027	.000	-3.4770870
260.0	2	.236	.559	.972	.000	.000	-3.4740321

XP	YP	ZP	PHI
1.450	-.242	1.339	103.677

THETA	I	X	Y	Z	F	G	D
270.0	1	.236	.559	.972	-.044	.000	-3.4774824
270.0	2	.212	.559	.978	.001	.001	-3.4706987

XP	YP	ZP	PHI
1.468	-.181	1.330	102.210

THETA	I	X	Y	Z	F	G	D
280.0	1	.212	.559	.977	-.060	.000	-3.4660825
280.0	2	.177	.559	.985	.001	.001	-3.4542862
280.0	3	.177	.559	.984	.000	.000	-3.4521500

XP	YP	ZP	PHI
1.469	-.117	1.335	100.208

THETA	I	X	Y	Z	F	G	D
290.0	1	.177	.559	.984	-.075	.000	-3.4435145
290.0	2	.134	.559	.992	.002	.002	-3.4256494
290.0	3	.134	.559	.991	.000	.000	-3.4223346

XP	YP	ZP	PHI
1.455	-.052	1.355	97.711

THETA	I	X	Y	Z	F	G	D
300.0	1	.134	.559	.991	-.087	.000	-3.4116581
300.0	2	.084	.559	.998	.003	.003	-3.3870445
300.0	3	.083	.559	.997	.000	.000	-3.3824728

XP	YP	ZP	PHI
1.424	.013	1.388	94.772

THETA	I	X	Y	Z	F	G	D
310.0	1	.083	.559	.997	-.097	.000	-3.3734859
310.0	2	.026	.559	1.001	.003	.003	-3.3419660
310.0	3	.026	.559	1.000	.000	.000	-3.3361959

XP	YP	ZP	PHI
1.377	.073	1.433	91.463

THETA	I	X	Y	Z	F	G	D
320.0	1	.026	.559	1.000	-.104	.000	-3.3327925
320.0	2	-.037	.559	1.001	.004	.004	-3.2948537
320.0	3	-.037	.559	.999	.000	.000	-3.2881006

XP	YP	ZP	PHI
1.314	.128	1.487	87.873

THETA	I	X	Y	Z	F	G	D
330.0	1	-.037	.559	.999	-.107	.000	-3.2938184
330.0	2	-.102	.559	.997	.004	.004	-3.2506629
330.0	3	-.103	.559	.995	.000	.000	-3.2433003

XP	YP	ZP	PHI
1.238	.173	1.547	84.114

THETA	I	X	Y	Z	F	G	D
340.0	1	-.103	.559	.995	-.107	.000	-3.2607885
340.0	2	-.168	.559	.988	.004	.004	-3.2143113
340.0	3	-.168	.559	.986	.000	.000	-3.2068376

XP	YP	ZP	PHI
1.150	.208	1.609	80.320

THETA	I	X	Y	Z	F	G	D
350.0	1	-.168	.559	.986	-.103	.000	-3.2374042
350.0	2	-.231	.559	.975	.004	.004	-3.1900464
350.0	3	-.231	.559	.973	.000	.000	-3.1830113

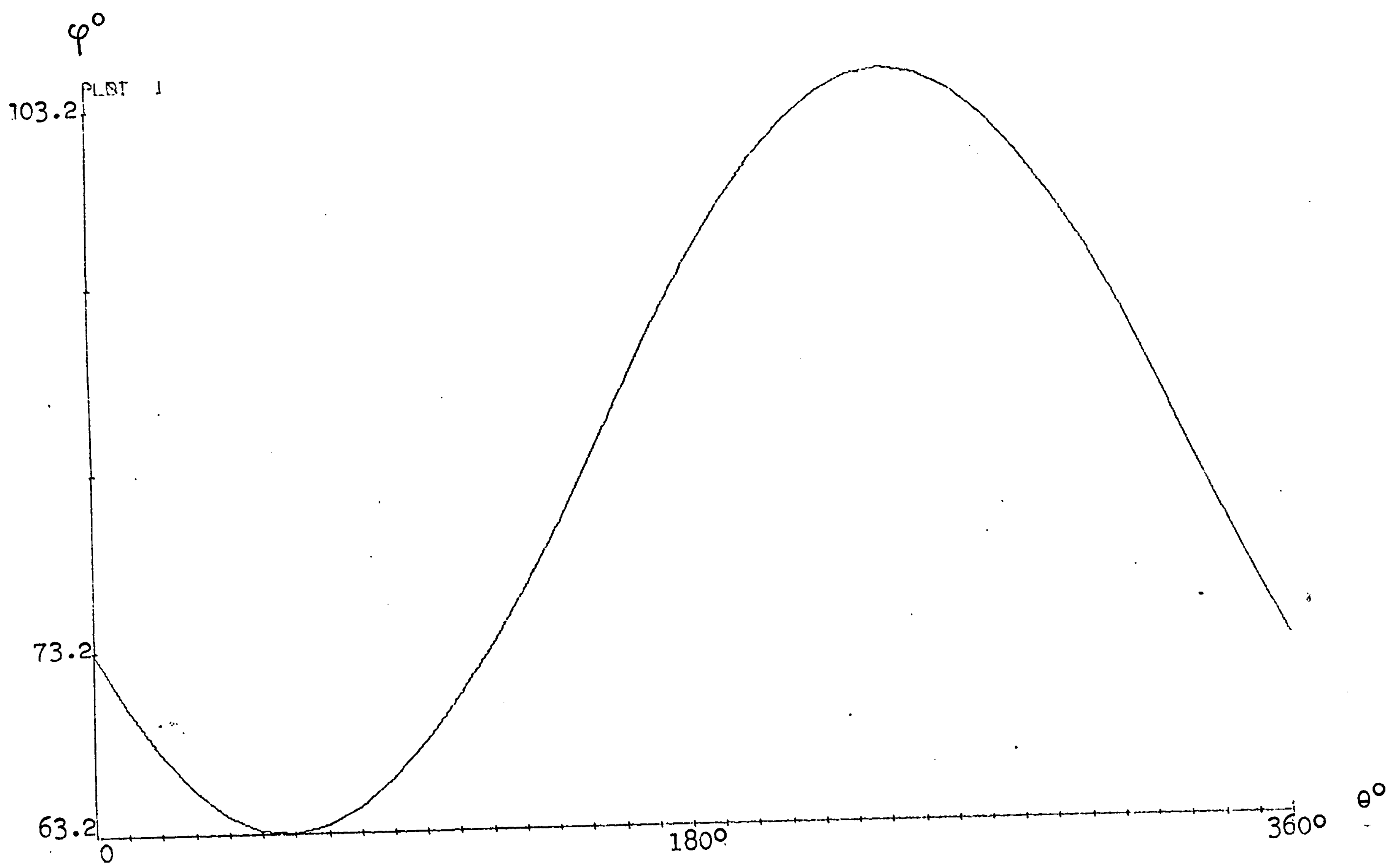
XP	YP	ZP	PHI
1.055	.229	1.670	76.636

THETA	I	X	Y	Z	F	G	D
360.0	1	-.231	.559	.973	-.096	.000	-3.2263514
360.0	2	-.289	.559	.959	.004	.004	-3.1808238
360.0	3	-.289	.559	.957	.000	.000	-3.1747252

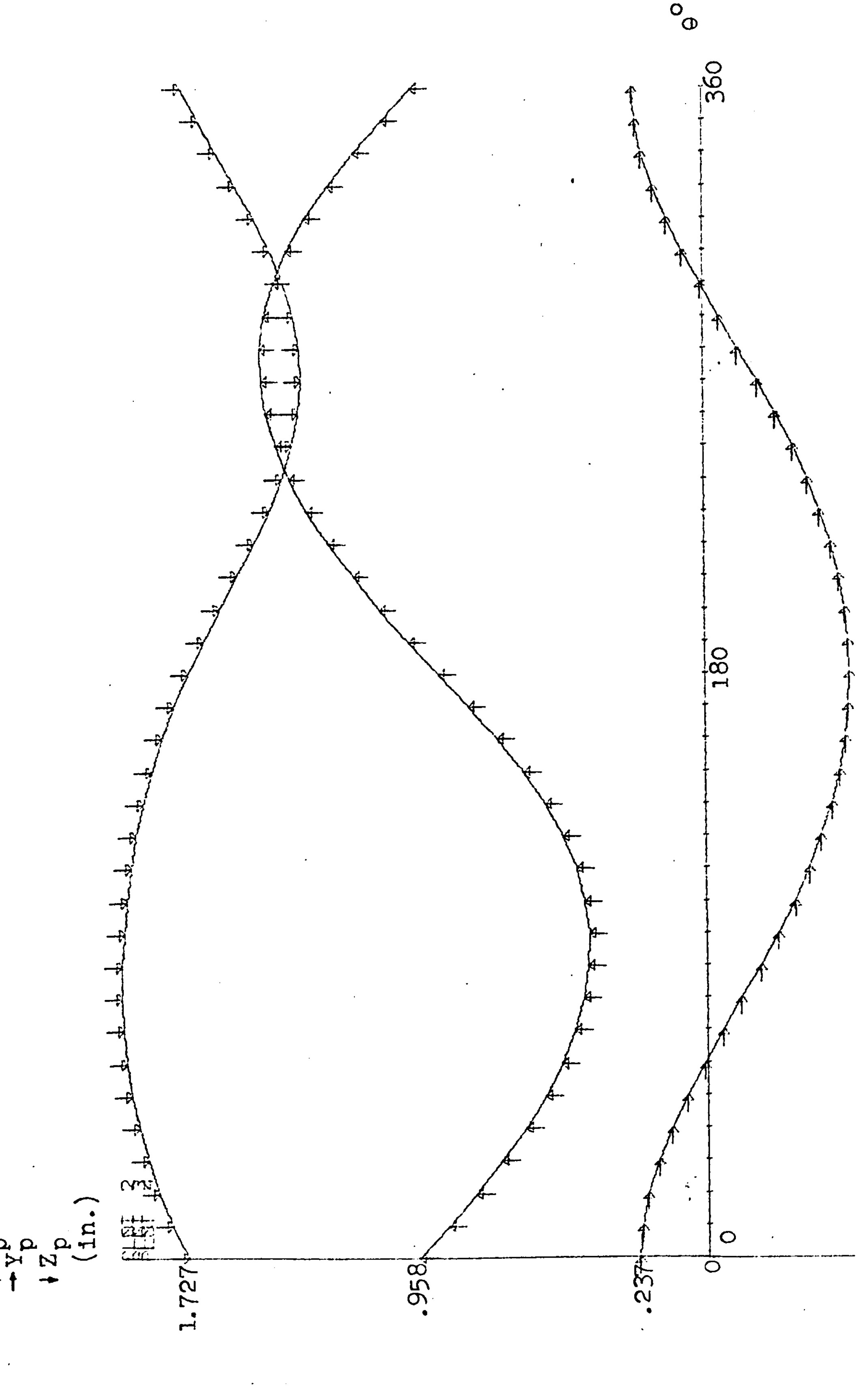
XP	YP	ZP	PHI
.958	.237	1.727	73.206

1.3: Calcomp plots

Angular Displacement of Output Link



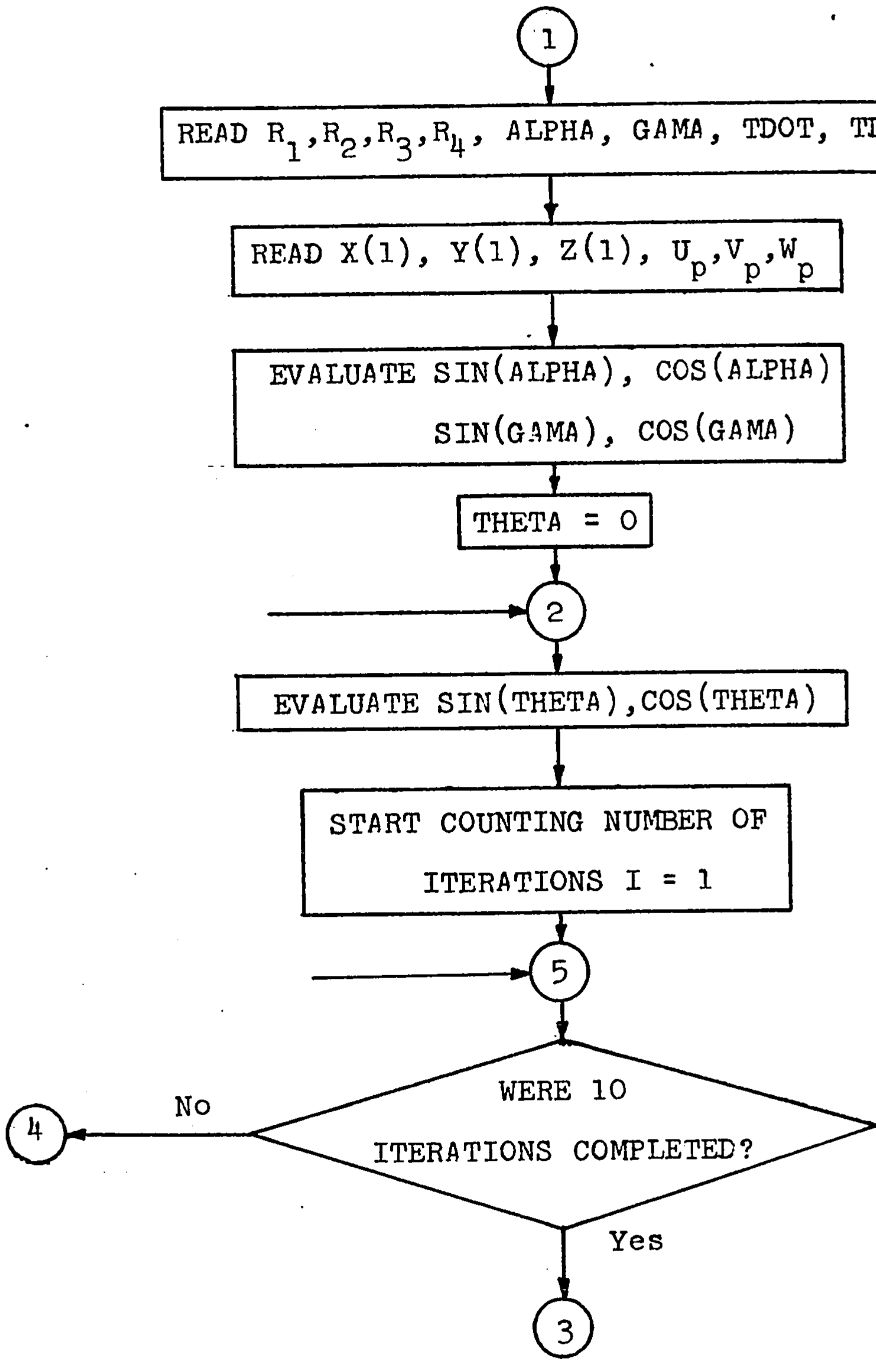
Linear Displacement of Coupler Point

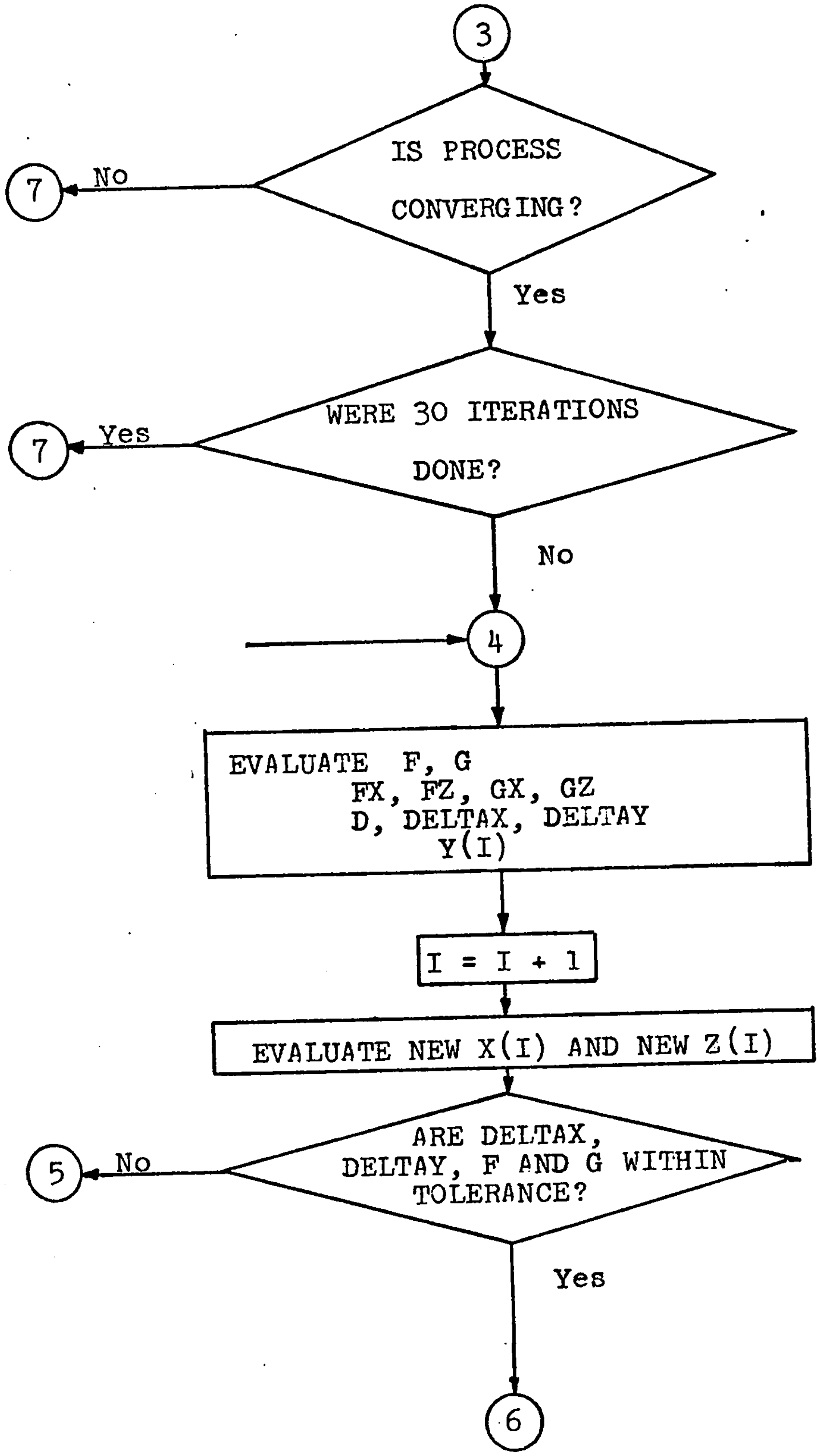


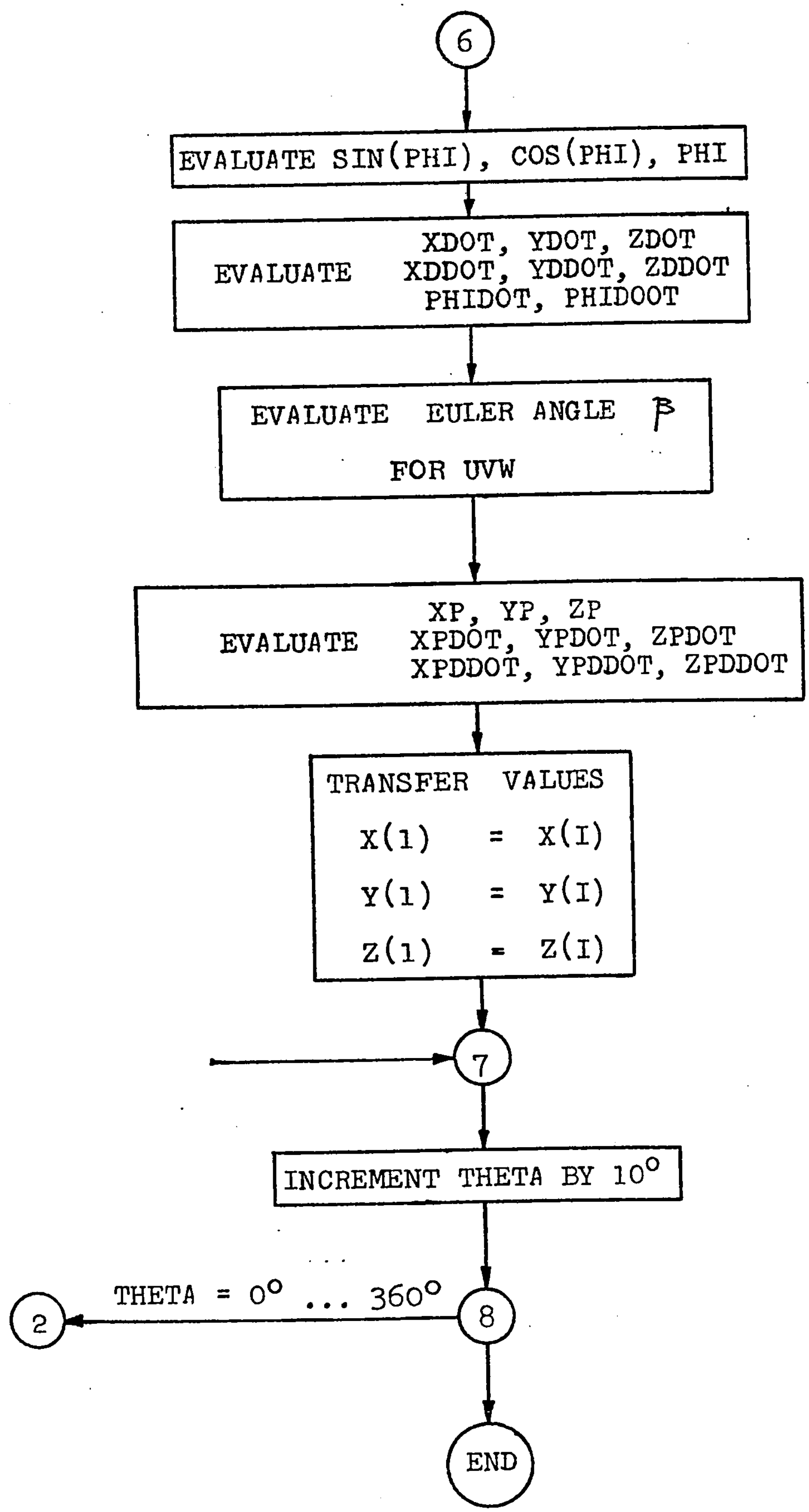
2. RGCṘ linkage (Θ variable)

2.1 FLOW-CHART

Iterative method for kinematic Analysis of Four-Bar spatial linkages.




3
IS PROCESS CONVERGING?
No
7
Yes
WERE 30 ITERATIONS DONE?
Yes
7
No
4
EVALUATE F, G
FX, FZ, GX, GZ
D, DELTAX, DELTAY
Y(I)
I = I + 1
EVALUATE NEW X(I) AND NEW Z(I)
ARE DELTAX, DELTAY, F AND G WITHIN TOLERANCE?
No
5
Yes
6


6
EVALUATE SIN(PHI), COS(PHI), PHI
EVALUATE XDOT, YDOT, ZDOT XDDOT, YDDOT, ZDDOT PHIDOT, PHIDDOT
EVALUATE EULER ANGLE β FOR UVW
EVALUATE XP, YP, ZP XPDOT, YPDOT, ZPDOT XPDDOT, YPDDOT, ZPDDOT
TRANSFER VALUES
X(1) = X(I)
Y(1) = Y(I)
Z(1) = Z(I)
7
INCREMENT THETA BY 10°
THETA = 0° ... 360°
8
2
END

```
      PROGRAMLINKAGE(INPUT,OUTPUT,TAPE10,TAPE60=INPUT)
C
C                ITERATIVE  METHOD  FOR  KINEMATICE
C                 ANALYSIS  OF  SPATIAL  LINKAGES
C
C                 RGCR  LINKAGE  (THETA  VARIABLE)
C
      DIMENSION F(30),X(30),Y(30),Z(30)
    1 READ101,R1,R2,R3,R4,ALPHA,GAMA,TDOT,TDDOT
      READ101,X(1),Y(1),Z(1),UP,VP,WP
  101 FORMAT(7F10.3)
      PRINT 102
  102 FORMAT(1H1,21X,≠R1≠,8X,≠R2≠,8X,≠R3≠,8X,≠R4≠,6X,≠ALPHA≠,
     16X,≠GAMA≠,7X,≠TDOT≠,5X,≠TDDOT≠)
      PRINT103,R1,R2,R3,R4,ALPHA,GAMA,TDOT,TDDOT
  103 FORMAT(1H0,15X, 8F10.3)
      PRINT108
  108 FORMAT(1H0,22X,≠UP≠,9X,≠VP≠,9X,≠WP≠)
      PRINT109,UP,VP,WP
  109 FORMAT(1H0,15X,3F10.3)
C     T=ANGLE THETA IN RADIANS
C     P=ANGLE PHI   IN RADIANS
C     A=ANGLE ALPHA IN RADIANS
C     B=ANGLE BETA  IN RADIANS
C     G=ANGLE GAMA  IN RADIANS
      G=GAMA*ATAN(1.0)/45.0
      A=ALPHA*ATAN(1.0)/45.0
      CA=COS(A)
      SA=SIN(A)
      CG=COS(G)
      SG=SIN(G)
      THETA=0.0
      DO 7J=1,37
      T=THETA*ATAN(1.0)/45.0
      CT=COS(T)
      ST=SIN(T)
      PRINT104
  104 FORMAT(1H0,19X,≠THETA≠,7X,≠I≠,10X,≠X≠,12X,≠Y≠,12X,≠Z≠,12X,≠F≠,12X,
     1≠G≠,12X,≠D≠)
      I=1
   12 CONTINUE
      IF(I-10)15,15,32
   32 IF((ABS(F(9)-F(10))).LT.(ABS(F(7)-F(8))).AND.
     1(ABS(F(7)-F(8))).LT.(ABS(F(5)-F(6))).AND.
     2(ABS(F(5)-F(6))).LT.(ABS(F(3)-F(4))).AND.
     3(ABS(F(3)-F(4))).LT.(ABS(F(1)-F(2))))17,7
   17 IF(I-30)15,15,7
C     ITERATION PROCESS STARTS
   15 F(I)=(X(I)-R2*CA*ST)**2+(Z(I)-R2*SA)**2-R3**2
      G=(X(I)-R1*SG)**2+Z(I)**2-R4**2
      FX=2.0*(X(I)-R2*CA*ST)
      FZ=2.0*(Z(I)-R2*SA)
      GX=2.0*(X(I)-R1*SG)
      GZ=2.0*Z(I)
      D=FX*GZ-FZ*GX
```

```
      DELTAX=(-F(I)*GZ+G*FZ)/D
      DELTAZ=(-G*FX+F(I)*GX)/D
      Y(I)=R2*CA*CT
      PRINT105,THETA,I,X(I),Y(I),Z(I),F(I),G,D
  105 FORMAT(1H ,15X,F8.1,I9,5F13.3,F17.7)
      I=I+1
C     COORDINATES  OF  JOINT  C
      X(I)=X(I-1)+DELTAX
      Z(I)=Z(I-1)+DELTAZ
      Y(I)=Y(I-1)
      IF(ABS(DELTAX)-0.001)10,12,12
   10 IF(ABS(DELTAZ)-0.001)11,12,12
   11 IF(ABS(F(I-1))-0.001)13,12,12
   13 IF(ABS(G)-0.001)14,12,12
C     ITERATION  PROCESS  COMPLETED
   14 CONTINUE
      CP=Z(I)/R4
      SP=(X(I)-R1*SG)/R4
      P=ATAN2(SP,CP)
C     ANGULAR  DISPLCEMENT  OF  OUTPUT  LINK
      PHI=P*45.0/ATAN(1.0)
C     LINEAR  VELOCITY  OF  JOINT  C
      XDOT=(Z(I)*(X(I)-R2*CA*ST)*(R2*CA*CT*TDOT))/(Z(I)*(X(I)-R2*CA*ST)-
     1(X(I)-R1*SG)*(Z(I)-R2*SA))
      YDOT=-R2*CA*ST*TDOT
      ZDOT=-(X(I)-R1*SG)*XDOT/Z(I)
C     LINEAR  ACCELERATION  OF  JOINT  C
      XDDOT=(XDOT*(ZDOT*R2*CA*ST+2.0*Z(I)*R2*CA*CT*TDOT-XDOT*R2*SA-
     1ZDOT*R1*SG)-Z(I)*R2**2*CA**2*TDOT**2*(CT**2-ST**2)
     2+R2*CA*CT*X(I)*(ZDOT*TDOT+Z(I)*TDDOT)-Z(I)*X(I)*R2*CA*ST*TDOT**2
     3-R2**2*CA**2*ST*CT*(Z(I)*TDDOT+ZDOT*TDOT))/(X(I)*R2*SA-Z(I)
     4*R2*CA*ST+Z(I)*R1*SG-R1*R2*SG*SA)
      YDDOT=-R2*CA*(ST*TDDOT+CT*TDOT**2)
      ZDDOT=(-XDDOT*(X(I)-R1*SG)-XDOT**2-ZDOT**2)/Z(I)
C     ANGULAR  VELOCITY  OF  OUTPUT  LINK
      PHIDOT=XDOT/(R4*CP)
C     ANGULAR  ACCELERATION  OF  OUTPUT  LINK
      PHIDDOT=(XDDOT/R4+PHIDOT**2*SP)/CP
      PRINT110
  110 FORMAT(1H0,25X,≠PHI≠,12X,≠PHIDOT≠,9X,≠PHIDDOT≠)
      PRINT111,PHI,PHIDOT,PHIDDOT
  111 FORMAT(1H ,15X,3F15.3)
C     SB=SIN(B)
C     CB=COS(B)
      SB=(Z(I)-R2*SA)/R3
      CB=(X(I)-R2*CA*ST)/R3
C     COORDINATES  OF  COUPLER  POINT  P
      XP=UP*CB-WP*SB+R2*CA*ST
      YP=VP+Y(I)
      ZP=UP*SB+WP*CB+R2*SA
C     LINEAR  VELOCITY  OF  COUPLER  POINT  P
      XPDOT=(UP/R3)*(XDOT-R2*CA*CT*TDOT)+R2*CA*CT*TDOT-(WP/R3)*ZDOT
      YPDOT=YDOT
      ZPDOT= (UP*ZDOT)/R3+(WP/R3)*(XDOT-R2*CA*CT*TDOT)
C     LINEAR  ACCELERATION  OF  COUPLER  POINT  P
```

```
      XPDDOT=(UP/R3)*(XDDOT+R2*CA*(ST*TDOT**2-CT*TDDOT))-R2*CA*
     1(ST*TDOT**2-CT*TDDOT)-(WP/R3)*ZDDOT
      YPDDOT=YDDOT
      ZPDDOT= (UP*ZDDOT)/R3+(WP/R3)*(XDDOT+R2*CA*(ST*TDOT**2-CT*TDDOT))
      PRINT106
  106 FORMAT(1H0,22X,≠XP≠,8X,≠YP≠,8X,≠ZP≠,6X,≠XPDOT≠
     1,5X,≠YPDOT≠,5X,≠ZPDOT≠,4X,≠XPDDOT≠,4X,≠YPDDOT≠,4X,≠ZPDDOT≠)
      PRINT107,XP,YP,ZP,XPDOT,YPDOT,ZPDOT,XPDDOT,YPDDOT,ZPDDOT
  107 FORMAT(1H ,15X, 9F10.3)
      X(1)=X(I)
      Y(1)=Y(I)
      Z(1)=Z(I)
    7 THETA=THETA+10.0
      GO TO 1
      END
```

2.3: NUMERICAL RESULTS

R1	R2	R3	R4	ALPHA	GAMA	TDOT	TDDOT
1.480	1.050	1.000	1.050	50.000	35.000	1.000	0.000

UP	VP	WP
.500	1.000	1.000

THETA	I	X	Y	Z	F	G	D
0.0	1	1.000	.675	1.000	.038	-.080	3.8817417
0.0	2	.972	.675	1.044	.003	.003	3.9419434
0.0	3	.971	.675	1.043	.000	.000	3.9344255

PHI	PHIDOT	PHIDDOT
6.686	.666	.159

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.247	1.675	1.895	.766	-0.000	-.021	.538	-.675	-.130

THETA	I	X	Y	Z	F	G	D
10.0	1	.971	.665	1.043	-.214	.000	3.4455264
10.0	2	1.101	.665	1.028	.017	.017	3.8176966
10.0	3	1.093	.665	1.021	.000	.000	3.7755872

PHI	PHIDOT	PHIDDOT
13.467	.687	.078

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.389	1.665	1.889	.851	-.117	-.047	.425	-.665	-.170

THETA	I	X	Y	Z	F	G	D
20.0	1	1.093	.634	1.021	-.209	.000	3.3111924
20.0	2	1.222	.634	.990	.018	.018	3.6495524
20.0	3	1.215	.634	.984	.000	.000	3.6099389

PHI	PHIDOT	PHIDDOT
20.378	.691	-.041

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.543	1.634	1.878	.910	-.231	-.080	.243	-.634	-.212

THETA	I	X	Y	Z	F	G	D
30.0	1	1.215	.585	.984	-.198	.000	3.1899239
30.0	2	1.337	.585	.939	.017	.017	3.4908112
30.0	3	1.329	.585	.934	.000	.000	3.4552917

PHI	PHIDOT	PHIDDOT
27.213	.671	-.198

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.704	1.585	1.861	.928	-.337	-.119	-.044	-.585	-.227

THETA	I	X	Y	Z	F	G	D
40.0	1	1.329	.517	.934	-.185	.000	3.0951505
40.0	2	1.439	.517	.877	.015	.015	3.3544828
40.0	3	1.431	.517	.874	.000	.000	3.3247454

PHI	PHIDOT	PHIDDOT
33.697	.621	-.381

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.863	1.517	1.837	.891	-.434	-.156	-.381	-.517	-.181

THETA	I	X	Y	Z	F	G	D
50.0	1	1.431	.434	.874	-.159	.000	3.0339216
50.0	2	1.523	.434	.812	.012	.012	3.2475442
50.0	3	1.517	.434	.810	.000	.000	3.2249373

PHI	PHIDOT	PHIDDOT
39.516	.538	-.560

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.011	1.434	1.807	.794	-.517	-.178	-.722	-.434	-.060

THETA	I	X	Y	Z	F	G	D
60.0	1	1.517	.337	.810	-.130	.000	3.0062196
60.0	2	1.587	.337	.752	.008	.008	3.1710180
60.0	3	1.583	.337	.751	.000	.000	3.1560103

PHI	PHIDOT	PHIDDOT
44.364	.427	-.705

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.137	1.337	1.776	.643	-.585	-.174	-1.004	-.337	.112

THETA	I	X	Y	Z	F	G	D
70.0	1	1.583	.231	.751	-.097	.000	3.0066826
70.0	2	1.631	.231	.703	.005	.005	3.1216324
70.0	3	1.629	.231	.703	.000	.000	3.1135095

PHI	PHIDOT	PHIDDOT
47.988	.295	-.801

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.233	1.231	1.748	.449	-.634	-.139	-1.106	-.231	.285

THETA	I	X	Y	Z	F	G	D
80.0	1	1.629	.117	.703	-.060	.000	3.0279057
80.0	2	1.657	.117	.672	.002	.002	3.0943470
80.0	3	1.656	.117	.672	.000	.000	3.0913334

PHI	PHIDOT	PHIDDOT
50.222	.150	-.851

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.293	1.117	1.729	.230	-.665	-.077	-1.209	-.117	.410

THETA	I	X	Y	Z	F	G	D
90.0	1	1.656	-.000	.672	-.020	.000	3.0637782
90.0	2	1.665	-.000	.661	.000	.000	3.0848974

PHI	PHIDOT	PHIDDOT
50.976	-.000	-.866

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.313	1.000	1.722	-.000	-.675	.000	-1.330	.000	.456

THETA	I	X	Y	Z	F	G	D
100.0	1	1.665	-.117	.661	.020	.000	3.1116760
100.0	2	1.656	-.117	.672	.000	.000	3.0916659

PHI	PHIDOT	PHIDDOT
50.222	-.150	-.851

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.293	.883	1.729	-.230	-.665	.077	-1.209	.117	.410

THETA	I	X	Y	Z	F	G	D
110.0	1	1.656	-.231	.672	.061	.000	3.1731560
110.0	2	1.630	-.231	.703	.002	.002	3.1164272
110.0	3	1.629	-.231	.703	.000	.000	3.1135003

PHI	PHIDOT	PHIDDOT
47.988	-.295	-.801

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.233	.769	1.748	-.449	-.634	.139	-1.196	.231	.285

THETA	I	X	Y	Z	F	G	D
120.0	1	1.629	-.337	.703	.101	.000	3.2532616
120.0	2	1.585	-.337	.751	.004	.004	3.1637500
120.0	3	1.583	-.337	.751	.000	.000	3.1559842

PHI	PHIDOT	PHIDDOT
44.364	-.427	-.705

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.137	.663	1.776	-.643	-.585	.174	-1.084	.337	.112

THETA	I	X	Y	Z	F	G	D
130.0	1	1.583	-.434	.751	.139	.000	3.3585933
130.0	2	1.521	-.434	.812	.008	.008	3.2390789
130.0	3	1.517	-.434	.810	.000	.000	3.2248893

PHI	PHIDOT	PHIDDOT
39.516	-.538	-.560

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.011	.566	1.807	-.794	-.517	.178	-.722	.434	-.060

THETA	I	X	Y	Z	F	G	D
140.0	1	1.517	-.517	.810	.173	.000	3.4943963
140.0	2	1.437	-.517	.876	.011	.011	3.3459255
140.0	3	1.431	-.517	.874	.000	.000	3.3246803

PHI	PHIDOT	PHIDDOT
33.697	-.621	-.381

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.863	.483	1.837	-.891	-.434	.156	-.381	.517	-.181

THETA	I	X	Y	Z	F	G	D
150.0	1	1.431	-.585	.874	.202	.000	3.6613649
150.0	2	1.335	-.585	.938	.013	.013	3.4831122
150.0	3	1.329	-.585	.934	.000	.000	3.4552217

PHI	PHIDOT	PHIDDOT
27.213	-.671	-.198

XP YP ZP XPDOT YPDOT ZPDOT XPDDOT YPDDOT ZPDDOT

.704	.415	1.861	-.928	-.337	.119	-.044	.585	-.227

THETA	I	X	Y	Z	F	G	D
160.0	1	1.329	-.634	.934	.223	.000	3.8533665
160.0	2	1.221	-.634	.989	.015	.015	3.6431944
160.0	3	1.215	-.634	.984	.006	.000	3.6096753

PHI	PHIDOT	PHIDDOT
20.378	-.691	-.041

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.543	.366	1.878	-.910	-.231	.080	.233	.634	-.212

THETA	I	X	Y	Z	F	G	D
170.0	1	1.215	-.665	.984	.236	.000	4.0571340
170.0	2	1.100	-.665	1.027	.015	.015	3.8127058
170.0	3	1.093	-.665	1.021	.006	.000	3.7755354

PHI	PHIDOT	PHIDDOT
13.467	-.687	.078

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.389	.335	1.889	-.851	-.117	.047	.425	.665	-.170

THETA	I	X	Y	Z	F	G	D
180.0	1	1.093	-.675	1.021	.243	.000	4.2540576
180.0	2	.977	-.675	1.049	.014	.014	3.9740879
180.0	3	.971	-.675	1.043	.006	.000	3.9346163

PHI	PHIDOT	PHIDDOT
6.686	-.666	.159

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.247	.325	1.895	-.766	.000	.021	.538	.675	-.130

THETA	I	X	Y	Z	F	G	D
190.0	1	.971	-.665	1.043	.241	.000	4.4233102
190.0	2	.857	-.665	1.056	.013	.013	4.1086853
190.0	3	.852	-.665	1.050	.006	.000	4.0682797

PHI	PHIDOT	PHIDDOT
.178	-.634	.215

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.122	.335	1.897	-.667	.117	.001	.593	.665	-.104

THETA	I	X	Y	Z	F	G	D
200.0	1	.852	-.634	1.050	.233	.000	4.5453596
200.0	2	.744	-.634	1.050	.015	.012	4.2001249
200.0	3	.740	-.634	1.044	.006	.000	4.1600348

PHI	PHIDOT	PHIDDOT
-5.956	-.592	.260

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.015	.366	1.895	-.562	.231	-.016	.608	.634	-.090

THETA	I	X	Y	Z	F	G	D
210.0	1	.740	-.585	1.044	.218	.000	4.6052486
210.0	2	.641	-.585	[illegible]	.010	.010	4.2375386

210.0	3	.637	-.585	1.028	.00n	.000	4.1990750

PHI	PHIDOT	PHIDDOT
-11.636	-.543	.307

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.074	.415	1.891	-.456	.337	-.031	.596	.585	-.079

THETA	I	X	Y	Z	F	G	D
220.0	1	.637	-.517	1.028	.197	.000	4.5953391
220.0	2	.549	-.517	1.010	.008	.008	4.2182292
220.0	3	.546	-.517	1.005	.00n	.000	4.1829477

PHI	PHIDOT	PHIDDOT
-16.780	-.484	.364

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.145	.483	1.884	-.355	.434	-.043	.561	.517	-.060

THETA	I	X	Y	Z	F	G	D
230.0	1	.546	-.434	1.005	.17n	.000	4.5173177
230.0	2	.470	-.434	.982	.00n	.006	4.1494274
230.0	3	.468	-.434	.978	.00n	.000	4.1192046

PHI	PHIDOT	PHIDDOT
-21.286	-.415	.435

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.199	.566	1.876	-.262	.517	-.051	.5n7	.434	-.027

THETA	I	X	Y	Z	F	G	D
240.0	1	.468	-.337	.978	.137	.000	4.3831744
240.0	2	.406	-.337	.954	.004	.004	4.0486760
240.0	3	.405	-.337	.951	.00n	.000	4.0255248

PHI	PHIDOT	PHIDDOT
-25.031	-.332	.520

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.237	.663	1.867	-.179	.585	-.052	.438	.337	.022

THETA	I	X	Y	Z	F	G	D
250.0	1	.405	-.231	.951	.101	.000	4.2146690
250.0	2	.359	-.231	.930	.003	.003	3.9421015
250.0	3	.358	-.231	.928	.00n	.000	3.9275203

PHI	PHIDOT	PHIDDOT
-27.869	-.233	.607

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.262	.769	1.859	-.109	.634	-.043	.3n5	.231	.079

THETA	I	X	Y	Z	F	G	D
260.0	1	.358	-.117	.928	.061	.000	4.0405485
260.0	2	.330	-.117	.913	.001	.001	3.8596005
260.0	3	.329	-.117	.913	.00n	.000	3.8534080

PHI	PHIDOT	PHIDDOT
-29.650	-.121	.675

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.276	.883	1.853	-.051	.665	-.025	.387	.117	.128

THETA	I	X	Y	Z	F	G	D
270.0	1	.329	-.000	.913	.028	.000	3.8908296
270.0	2	.320	-.000	.907	.008	.000	3.8265537

PHI	PHIDOT	PHIDDOT
-30.259	-.000	.702

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.280	1.000	1.850	-.000	.675	-.000	.284	.000	.147

THETA	I	X	Y	Z	F	G	D
280.0	1	.320	.117	.907	-.028	.000	3.7885962
280.0	2	.330	.117	.913	.008	.000	3.8541747

PHI	PHIDOT	PHIDDOT
-29.650	.121	.675

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.276	1.117	1.853	.051	.665	.025	.387	-.117	.128

THETA	I	X	Y	Z	F	G	D
290.0	1	.329	.231	.913	-.068	.000	3.7422606
290.0	2	.359	.231	.929	.007	.001	3.9339768
290.0	3	.358	.231	.928	.008	.000	3.9274986

PHI	PHIDOT	PHIDDOT
-27.869	.233	.607

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.262	1.231	1.859	.109	.634	.043	.365	-.231	.079

THETA	I	X	Y	Z	F	G	D
300.0	1	.358	.337	.928	-.096	.000	3.7428884
300.0	2	.406	.337	.953	.003	.003	4.0410698
300.0	3	.405	.337	.951	.008	.000	4.0254883

PHI	PHIDOT	PHIDDOT
-25.031	.332	.520

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.237	1.337	1.867	.179	.585	.052	.448	-.337	.022

THETA	I	X	Y	Z	F	G	D
310.0	1	.405	.434	.951	-.129	.000	3.7686617
310.0	2	.470	.434	.982	.005	.005	4.1442092
310.0	3	.468	.434	.978	.008	.000	4.1191698

PHI	PHIDOT	PHIDDOT
-21.286	.415	.435

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.199	1.434	1.876	.262	.517	.051	.587	-.434	-.027

THETA	I	X	Y	Z	F	G	D
320.0	1	.468	.517	.978	-.157	.000	3.7935342
320.0	2	.549	.517	1.010	.008	.008	4.2157526
320.0	3	.546	.517	[illegible]	.008	.000	4.1829277

PHI	PHIDOT	PHIDDOT
-16.780	.484	.364

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.145	1.517	1.884	.355	.434	.043	.541	-.517	-.060

THETA	I	X	Y	Z	F	G	D
330.0	1	.546	.585	1.005	-.180	.000	3.7952735
330.0	2	.641	.585	1.034	.010	.010	4.2373451
330.0	3	.637	.585	1.028	.000	.000	4.1990733

PHI	PHIDOT	PHIDDOT
-11.636	.543	.307

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.074	1.585	1.891	.456	.337	.031	.596	-.585	-.079

THETA	I	X	Y	Z	F	G	D
340.0	1	.637	.634	1.028	-.196	.000	3.7602783
340.0	2	.745	.634	1.051	.012	.012	4.2015984
340.0	3	.740	.634	1.044	.000	.000	4.1600491

PHI	PHIDOT	PHIDDOT
-5.956	.592	.260

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.015	1.634	1.895	.562	.231	.016	.608	-.634	-.090

THETA	I	X	Y	Z	F	G	D
350.0	1	.740	.665	1.044	-.208	.000	3.6851353
350.0	2	.858	.665	1.057	.014	.014	4.1113935
350.0	3	.852	.665	1.050	.000	.000	4.0683072

PHI	PHIDOT	PHIDDOT
.178	.634	.215

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.122	1.665	1.897	.667	.117	-.001	.593	-.665	-.104

THETA	I	X	Y	Z	F	G	D
360.0	1	.852	.675	1.050	-.213	.000	3.5758421
360.0	2	.978	.675	1.050	.016	.016	3.9778870
360.0	3	.971	.675	1.043	.000	.000	3.9346558

PHI	PHIDOT	PHIDDOT
6.686	.666	.159

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.247	1.675	1.895	.766	-.000	-.021	.538	-.675	-.130

R1	R2	R3	R4	ALPHA	GAMA	TDOT	TDDOT
2.800	1.000	2.400	1.000	45.000	45.000	1.000	0.000

UP	VP	WP
1.200	1.000	1.000

THETA	I	X	Y	Z	F	G	D
0.0	1	2.500	.707	.900	.527	.081	8.5987042
0.0	2	2.393	.707	.917	.015	.012	8.4311243
0.0	3	2.391	.707	.911	.000	.000	8.3821239

PHI	PHIDOT	PHIDDOT
24.292	.807	.311

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.110	1.707	1.806	.860	-0.000	-.154	.308	-.707	-.354

THETA	I	X	Y	Z	F	G	D
10.0	1	2.391	.696	.911	-.575	.000	7.9343152
10.0	2	2.523	.696	.852	.021	.021	7.8653830
10.0	3	2.519	.696	.842	.000	.000	7.7818373

PHI	PHIDOT	PHIDDOT
32.621	.858	.270

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.265	1.696	1.773	.902	-.123	-.220	.173	-.696	-.402

THETA	I	X	Y	Z	F	G	D
20.0	1	2.519	.664	.842	-.556	.000	7.3802839
20.0	2	2.646	.664	.761	.023	.023	7.1743889
20.0	3	2.641	.664	.750	.000	.000	7.0851353

PHI	PHIDOT	PHIDDOT
41.419	.900	.213

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.424	1.664	1.728	.918	-.242	-.293	.003	-.664	-.430

THETA	I	X	Y	Z	F	G	D
30.0	1	2.641	.612	.750	-.524	.000	6.7494971
30.0	2	2.758	.612	.647	.024	.024	6.4108180
30.0	3	2.753	.612	.635	.000	.000	6.3163303

PHI	PHIDOT	PHIDDOT
50.585	.930	.126

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.583	1.612	1.671	.901	-.354	-.368	-.201	-.612	-.422

THETA	I	X	Y	Z	F	G	D
40.0	1	2.752	.542	.635	-.474	.000	6.0591975
40.0	2	2.852	.542	.514	.025	.025	5.6024443
40.0	3	2.846	.542	.501	.000	.000	5.5035431

PHI	PHIDOT	PHIDDOT
59.964	.942	-.009

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.736	1.542	1.600	.846	-.455	-.437	-.4[illegible]2	-.542	-.350

THETA	I	X	Y	Z	F	G	D
50.0	1	2.846	.455	.501	-.409	.000	5.3281552
50.0	2	2.922	.455	.368	.024	.024	4.7806329
50.0	3	2.916	.455	.353	.006	.000	4.6792284

PHI	PHIDOT	PHIDDOT
69.316	.923	-.225

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.876	1.455	1.519	.750	-.542	-.485	-.671	-.455	-.184

THETA	I	X	Y	Z	F	G	D
60.0	1	2.915	.354	.353	-.331	.000	4.5782439
60.0	2	2.966	.354	.218	.021	.021	3.9833758
60.0	3	2.959	.354	.204	.006	.000	3.8840685

PHI	PHIDOT	PHIDDOT
78.254	.855	-.583

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.995	1.354	1.433	.612	-.612	-.493	-.809	-.354	.119

THETA	I	X	Y	Z	F	G	D
70.0	1	2.959	.242	.204	-.242	.000	3.8403805
70.0	2	2.985	.242	.080	.016	.016	3.2642833
70.0	3	2.978	.242	.068	.006	.000	3.1776375

PHI	PHIDOT	PHIDDOT
86.137	.704	-1.190

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
2.088	1.242	1.351	.438	-.664	-.432	-1.101	-.242	.614

THETA	I	X	Y	Z	F	G	D
80.0	1	2.978	.123	.067	-.147	.000	3.1678602
80.0	2	2.984	.123	-.025	.009	.009	2.7114863
80.0	3	2.979	.123	-.033	.006	.000	2.6572346

PHI	PHIDOT	PHIDDOT
91.900	.422	-2.071

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
2.146	1.123	1.288	.230	-.696	-.268	-1.248	-.123	1.283

THETA	I	X	Y	Z	F	G	D
90.0	1	2.979	-.000	-.033	-.049	.000	2.6581052
90.0	2	2.978	-.000	-.070	.001	.001	2.4673030
90.0	3	2.977	-.000	-.071	.006	.000	2.4579843

PHI	PHIDOT	PHIDDOT
94.091	-.000	-2.612

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
2.167	1.000	1.264	-.000	-.707	.000	-1.346	.000	1.675

THETA	I	X	Y	Z	F	G	D
100.0	1	2.977	-.123	[illegible]71	.049	.000	2.4549007

100.0	2	2.980	-.123	-.032	.002	.002	2.6669208
100.0	3	2.979	-.123	-.033	.000	.000	2.6567004

PHI	PHIDOT	PHIDDOT
91.900	-.422	-2.071

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
2.146	.877	1.288	-.230	-.696	.268	-1.268	.123	1.283

THETA	I	X	Y	Z	F	G	D
110.0	1	2.979	-.242	-.033	.147	.000	2.6524507
110.0	2	2.983	-.242	.077	.012	.012	3.2444382
110.0	3	2.978	-.242	.067	.000	.000	3.1771682

PHI	PHIDOT	PHIDDOT
86.137	-.704	-1.190

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
2.088	.758	1.351	-.438	-.664	.432	-1.101	.242	.614

THETA	I	X	Y	Z	F	G	D
120.0	1	2.978	-.354	.067	.244	.000	3.1904922
120.0	2	2.967	-.354	.220	.023	.023	3.9951403
120.0	3	2.959	-.354	.204	.000	.000	3.8843782

PHI	PHIDOT	PHIDDOT
78.254	-.855	-.583

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.995	.646	1.433	-.612	-.612	.493	-.809	.354	.119

THETA	I	X	Y	Z	F	G	D
130.0	1	2.959	-.455	.204	.337	.000	3.9403692
130.0	2	2.924	-.455	.371	.029	.029	4.8047681
130.0	3	2.916	-.455	.353	.000	.000	4.6797984

PHI	PHIDOT	PHIDDOT
69.316	-.923	-.225

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.876	.545	1.519	-.750	-.542	.485	-.671	.455	-.184

THETA	I	X	Y	Z	F	G	D
140.0	1	2.915	-.542	.353	.421	.000	4.8012692
140.0	2	2.853	-.542	.517	.031	.031	5.6278457
140.0	3	2.846	-.542	.501	.000	.000	5.5040468

PHI	PHIDOT	PHIDDOT
59.964	-.942	-.009

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.736	.458	1.600	-.846	-.455	.437	-.432	.542	-.350

THETA	I	X	Y	Z	F	G	D
150.0	1	2.846	-.612	.501	.493	.000	5.7048042
150.0	2	2.759	-.612	.650	.030	.030	6.4335439
150.0	3	2.753	-.612	.635	.000	.000	6.3167042

PHI	PHIDOT	PHIDDOT
50.585	-.930	.126

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.583	.388	1.671	-.901	-.354	.368	-.201	.612	-.422

THETA	I	X	Y	Z	F	G	D
160.0	1	2.752	-.664	.635	.548	.000	6.5993327
160.0	2	2.647	-.664	.763	.028	.028	7.1935685
160.0	3	2.642	-.664	.750	.000	.000	7.0853988

PHI	PHIDOT	PHIDDOT
41.419	-.900	.213

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.424	.336	1.728	-.918	-.242	.293	.003	.664	-.430

THETA	I	X	Y	Z	F	G	D
170.0	1	2.641	-.696	.750	.586	.000	7.4416908
170.0	2	2.523	-.696	.854	.025	.025	7.8812924
170.0	3	2.519	-.696	.842	.000	.000	7.7820220

PHI	PHIDOT	PHIDDOT
32.621	-.858	.270

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.265	.304	1.773	-.902	-.123	.220	.173	.696	-.402

THETA	I	X	Y	Z	F	G	D
180.0	1	2.519	-.707	.842	.604	.000	8.1950630
180.0	2	2.395	-.707	.922	.022	.022	8.4730049
180.0	3	2.391	-.707	.912	.000	.000	8.3824696

PHI	PHIDOT	PHIDDOT
24.292	-.807	.311

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.110	.293	1.806	-.860	.000	.154	.308	.707	-.354

THETA	I	X	Y	Z	F	G	D
190.0	1	2.391	-.696	.911	.602	.000	8.8296462
190.0	2	2.267	-.696	.968	.019	.019	8.9499853
190.0	3	2.264	-.696	.959	.000	.000	8.8681064

PHI	PHIDOT	PHIDDOT
16.504	-.750	.345

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.966	.304	1.827	-.796	.123	.097	.412	.696	-.298

THETA	I	X	Y	Z	F	G	D
200.0	1	2.264	-.664	.959	.582	.000	9.3243361
200.0	2	2.144	-.664	.994	.016	.016	9.3009350
200.0	3	2.142	-.664	.987	.000	.000	9.2279446

PHI	PHIDOT	PHIDDOT
9.317	-.687	.380

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.833	.336	1.840	-.717	.242	.050	.490	.664	-.240

THETA	I	X	Y	Z	F	G	D

THETA	I	X	Y	Z	F	G	D
210.0	1	2.142	-.612	.987	.545	.000	9.6685953
210.0	2	2.031	-.612	1.005	.013	.013	9.5242920
210.0	3	2.029	-.612	.999	.000	.000	9.4608207

PHI	PHIDOT	PHIDDOT
2.795	-.617	.421

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.716	.388	1.846	-.627	.354	.014	.545	.612	-.180

THETA	I	X	Y	Z	F	G	D
220.0	1	2.029	-.542	.999	.491	.000	9.8639919
220.0	2	1.929	-.542	1.004	.010	.010	9.6298955
220.0	3	1.928	-.542	.999	.002	.000	9.5769322

PHI	PHIDOT	PHIDDOT
-2.992	-.539	.472

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.615	.458	1.845	-.528	.455	-.013	.581	.542	-.119

THETA	I	X	Y	Z	F	G	D
230.0	1	1.928	-.455	.999	.423	.000	9.9249336
230.0	2	1.843	-.455	.994	.007	.007	9.6394006
230.0	3	1.842	-.455	.990	.000	.000	9.5980742

PHI	PHIDOT	PHIDDOT
-7.952	-.451	.532

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.532	.545	1.842	-.425	.542	-.028	.602	.455	-.058

THETA	I	X	Y	Z	F	G	D
240.0	1	1.842	-.354	.990	.342	.000	9.8780546
240.0	2	1.773	-.354	.981	.005	.005	9.5847728
240.0	3	1.772	-.354	.978	.000	.000	9.5558835

PHI	PHIDOT	PHIDDOT
-11.984	-.353	.597

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.467	.646	1.836	-.319	.612	-.033	.610	.354	.000

THETA	I	X	Y	Z	F	G	D
250.0	1	1.772	-.242	.978	.251	.000	9.7596613
250.0	2	1.722	-.242	.968	.003	.003	9.5042327
250.0	3	1.722	-.242	.966	.000	.000	9.4875272

PHI	PHIDOT	PHIDDOT
-14.973	-.243	.658

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.421	.758	1.831	-.212	.664	-.029	.611	.242	.051

THETA	I	X	Y	Z	F	G	D
260.0	1	1.722	-.123	.966	.153	.000	9.6107846
260.0	2	1.691	-.123	.958	.001	.001	9.4354382
260.0	3	1.691	-.123	.957	.000	.000	9.4288308

PHI PHIDOT PHIDDOT

PHI	PHIDOT	PHIDDOT
-16.818	-.124	.702

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.393	.877	1.827	-.106	.696	-.016	.608	.123	.086

THETA	I	X	Y	Z	F	G	D
270.0	1	1.691	-.000	.957	.051	.000	9.4699607
270.0	2	1.680	-.000	.954	.000	.000	9.4068143

PHI	PHIDOT	PHIDDOT
-17.442	-.000	.718

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.384	1.000	1.825	-.000	.707	-.000	.606	.000	.098

THETA	I	X	Y	Z	F	G	D
280.0	1	1.680	.123	.954	-.051	.000	9.3650462
280.0	2	1.691	.123	.957	.000	.000	9.4296050

PHI	PHIDOT	PHIDDOT
-16.818	.124	.702

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.393	1.123	1.827	.106	.696	.016	.608	-.123	.086

THETA	I	X	Y	Z	F	G	D
290.0	1	1.691	.242	.957	-.151	.000	9.3066815
290.0	2	1.722	.242	.967	.001	.001	9.4941870
290.0	3	1.722	.242	.966	.000	.000	9.4875148

PHI	PHIDOT	PHIDDOT
-14.973	.243	.658

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.421	1.242	1.831	.212	.664	.029	.611	-.242	.051

THETA	I	X	Y	Z	F	G	D
300.0	1	1.722	.354	.966	-.246	.000	9.2862249
300.0	2	1.773	.354	.980	.003	.003	9.5727726
300.0	3	1.772	.354	.978	.000	.000	9.5558548

PHI	PHIDOT	PHIDDOT
-11.984	.353	.597

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.467	1.354	1.836	.319	.612	.033	.610	-.354	.000

THETA	I	X	Y	Z	F	G	D
310.0	1	1.772	.455	.978	-.332	.000	9.2792138
310.0	2	1.842	.455	.993	.005	.005	9.6272431
310.0	3	1.842	.455	.990	.000	.000	9.5980297

PHI	PHIDOT	PHIDDOT
-7.952	.451	.532

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.532	1.455	1.842	.425	.542	.028	.602	-.455	-.058

THETA	I	X	Y	Z	F	G	D
320.0	1	1.842	.542	.990	-.408	.000	9.2527148

THETA	I	X	Y	Z	F	G	D
320.0	2	1.929	.542	1.003	.008	.008	9.6184122
320.0	3	1.928	.542	.999	.000	.000	9.5768759

PHI	PHIDOT	PHIDDOT
-2.992	.539	.472

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.615	1.542	1.845	.528	.455	.013	.581	-.542	-.119

THETA	I	X	Y	Z	F	G	D
330.0	1	1.928	.612	.999	-.471	.000	9.1734719
330.0	2	2.030	.612	1.004	.011	.011	9.5134374
330.0	3	2.029	.612	.999	.000	.000	9.4607545

PHI	PHIDOT	PHIDDOT
2.795	.617	.421

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.716	1.612	1.846	.627	.354	-.014	.545	-.612	-.180

THETA	I	X	Y	Z	F	G	D
340.0	1	2.029	.664	.999	-.520	.000	9.0143047
340.0	2	2.144	.664	.993	.013	.013	9.2901160
340.0	3	2.142	.664	.987	.000	.000	9.2278659

PHI	PHIDOT	PHIDDOT
9.317	.687	.380

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.833	1.664	1.840	.717	.242	-.050	.490	-.664	-.240

THETA	I	X	Y	Z	F	G	D
350.0	1	2.142	.696	.987	-.559	.000	8.7577111
350.0	2	2.267	.696	.966	.016	.016	8.9383594
350.0	3	2.264	.696	.959	.000	.000	8.8680075

PHI	PHIDOT	PHIDDOT
16.504	.750	.345

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.966	1.696	1.827	.796	.123	-.097	.412	-.696	-.298

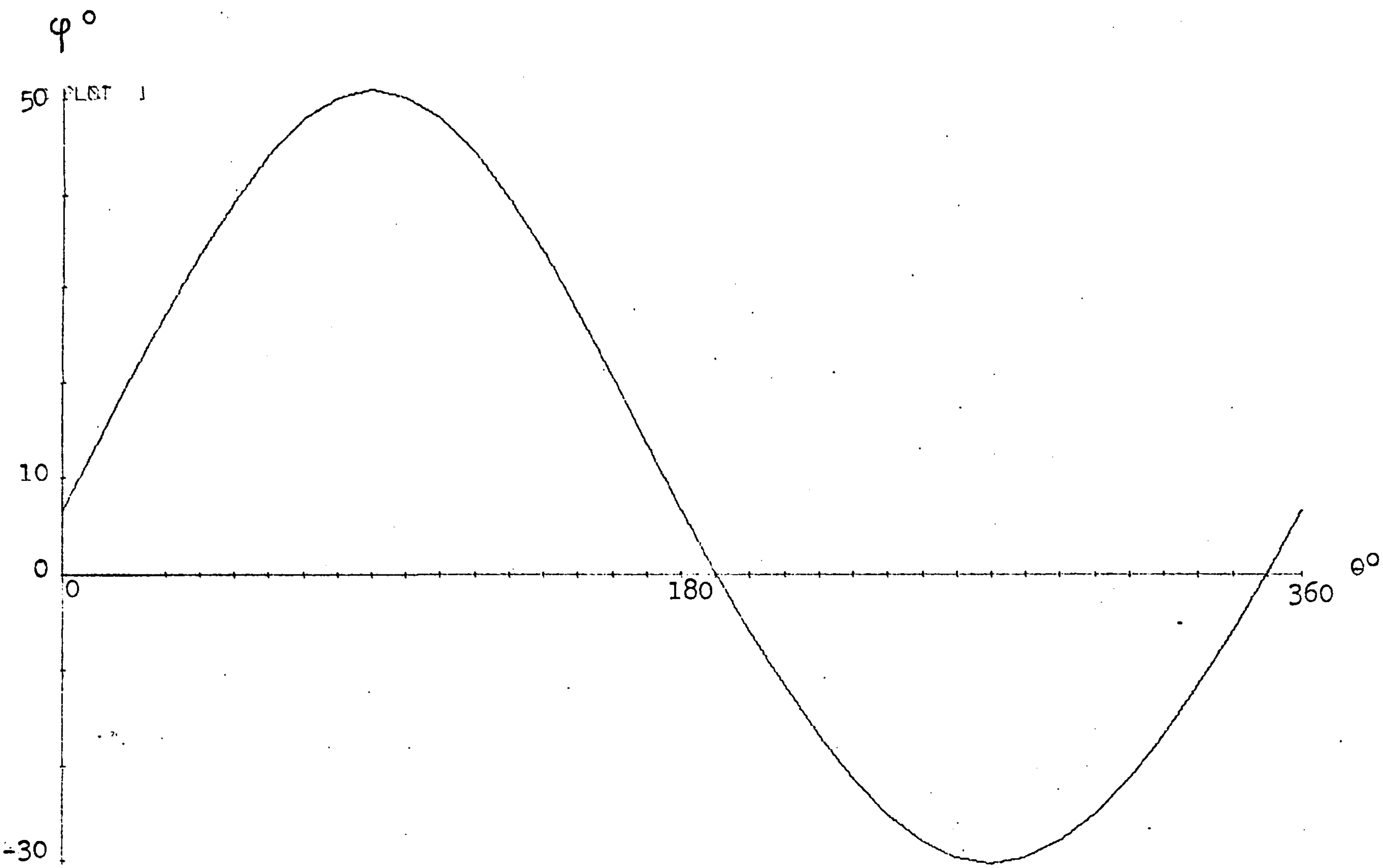
THETA	I	X	Y	Z	F	G	D
360.0	1	2.264	.707	.959	-.571	.000	8.3968122
360.0	2	2.394	.707	.920	.019	.019	8.4596669
360.0	3	2.391	.707	.912	.000	.000	8.3823374

PHI	PHIDOT	PHIDDOT
24.292	.807	.311

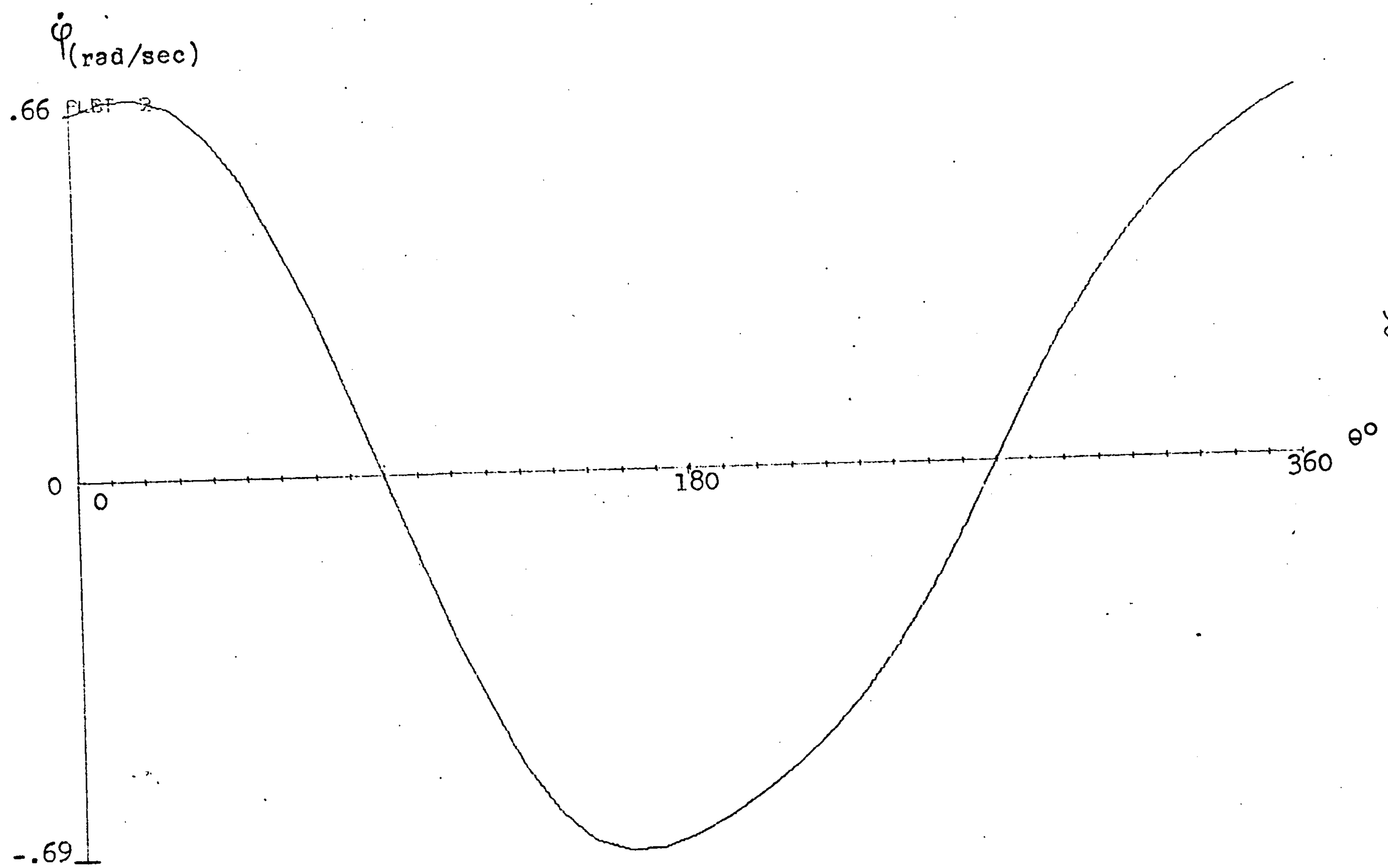
XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.110	1.707	1.806	.860	-.000	-.154	.348	-.707	-.354

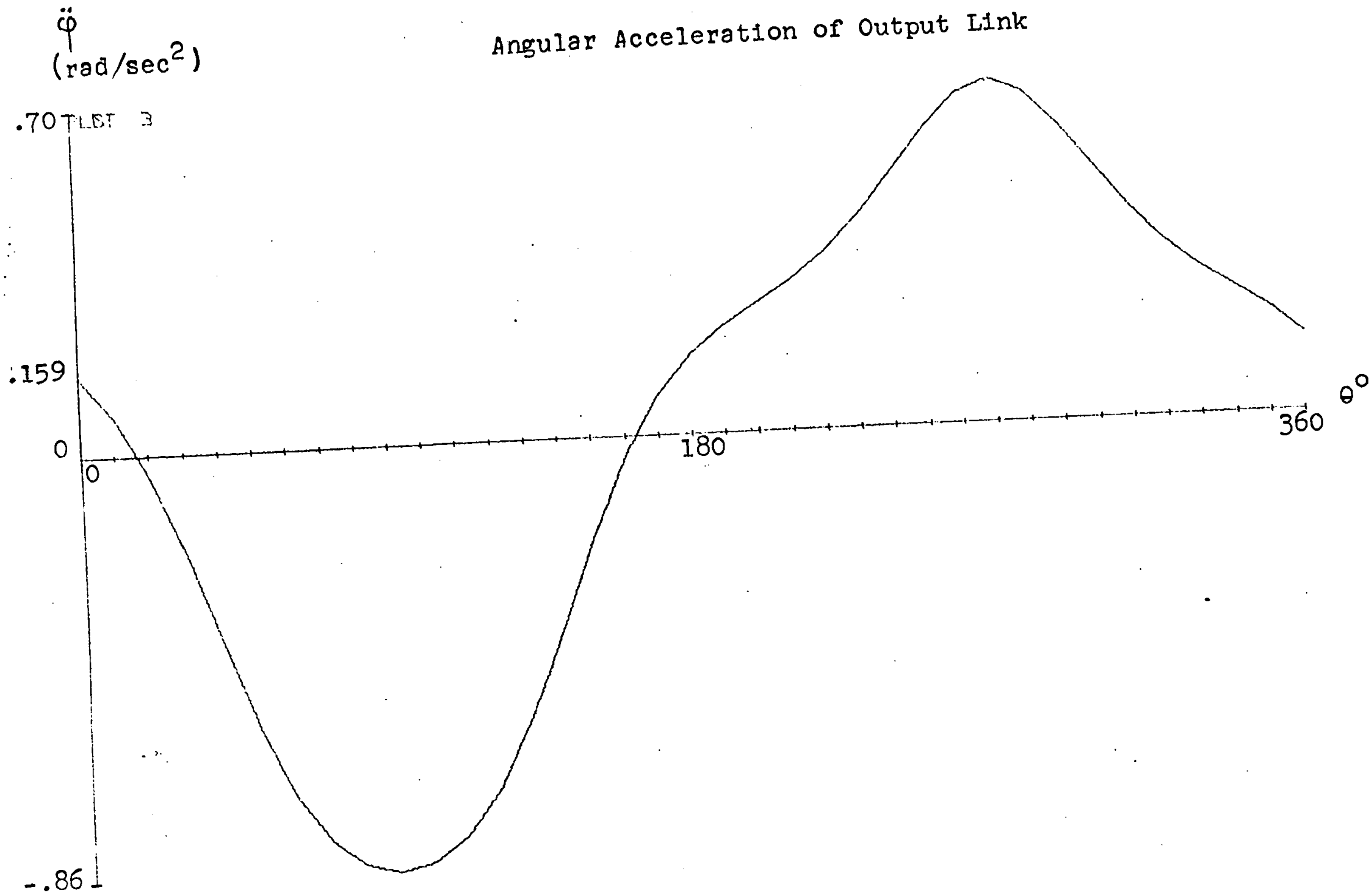
2.4: Calcomp plots

Angular Displacement of Output Link

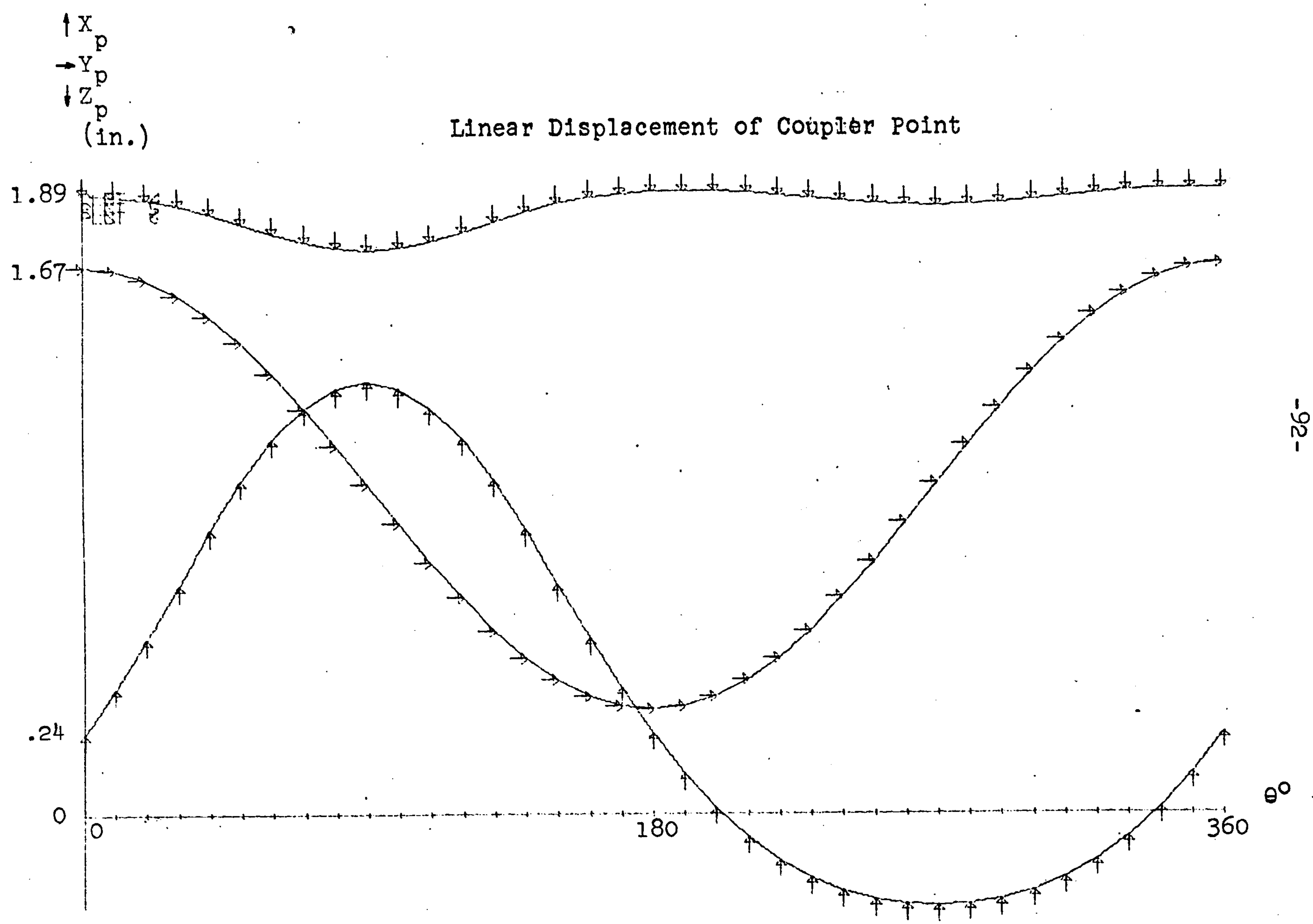


Angular Velocity of Output Link

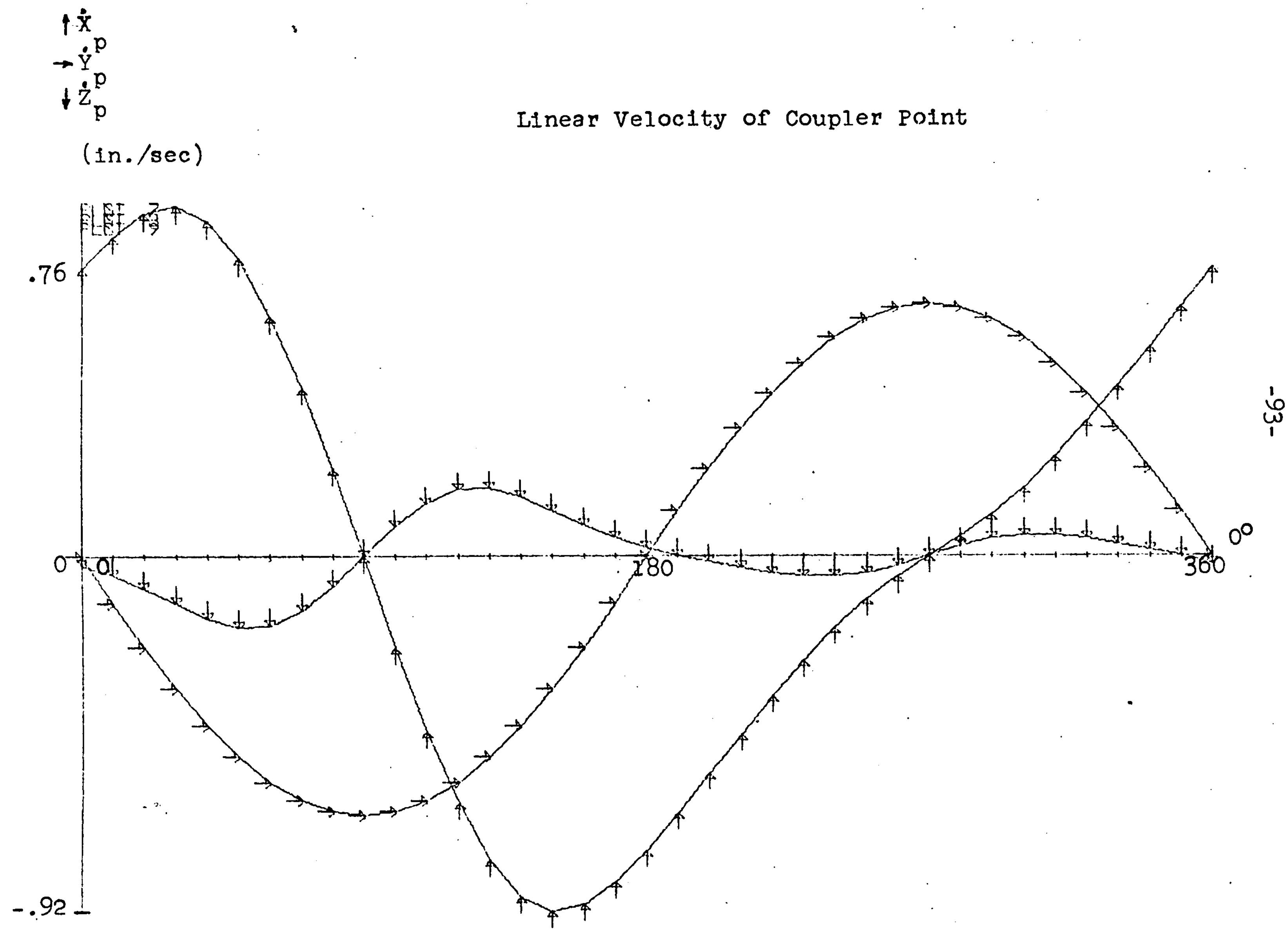



Angular Acceleration of Output Link
$\ddot{\varphi}$ (rad/sec^2)
.70
.159
0
-.86
0
180
360
θ°

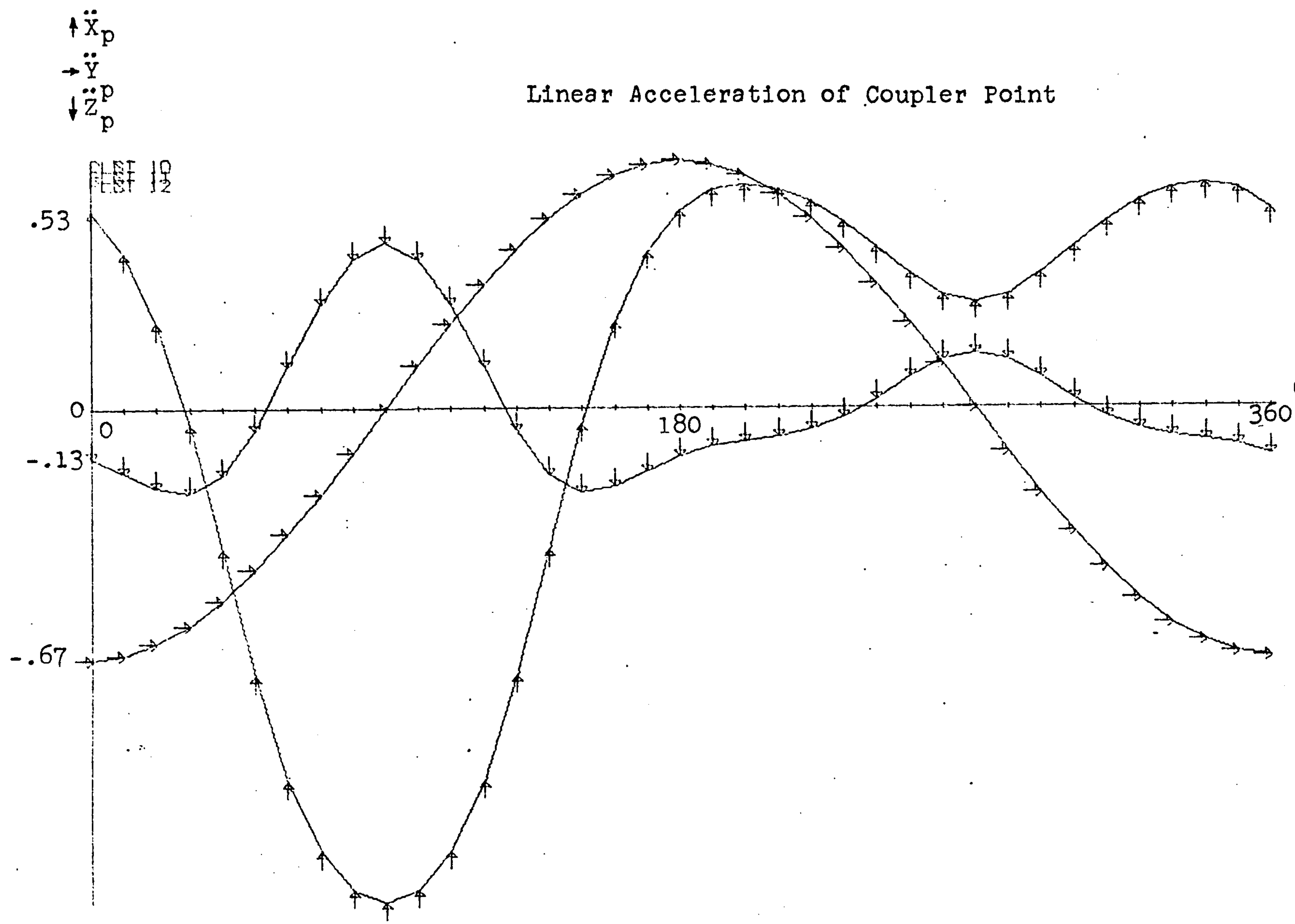
Linear Displacement of Coupler Point



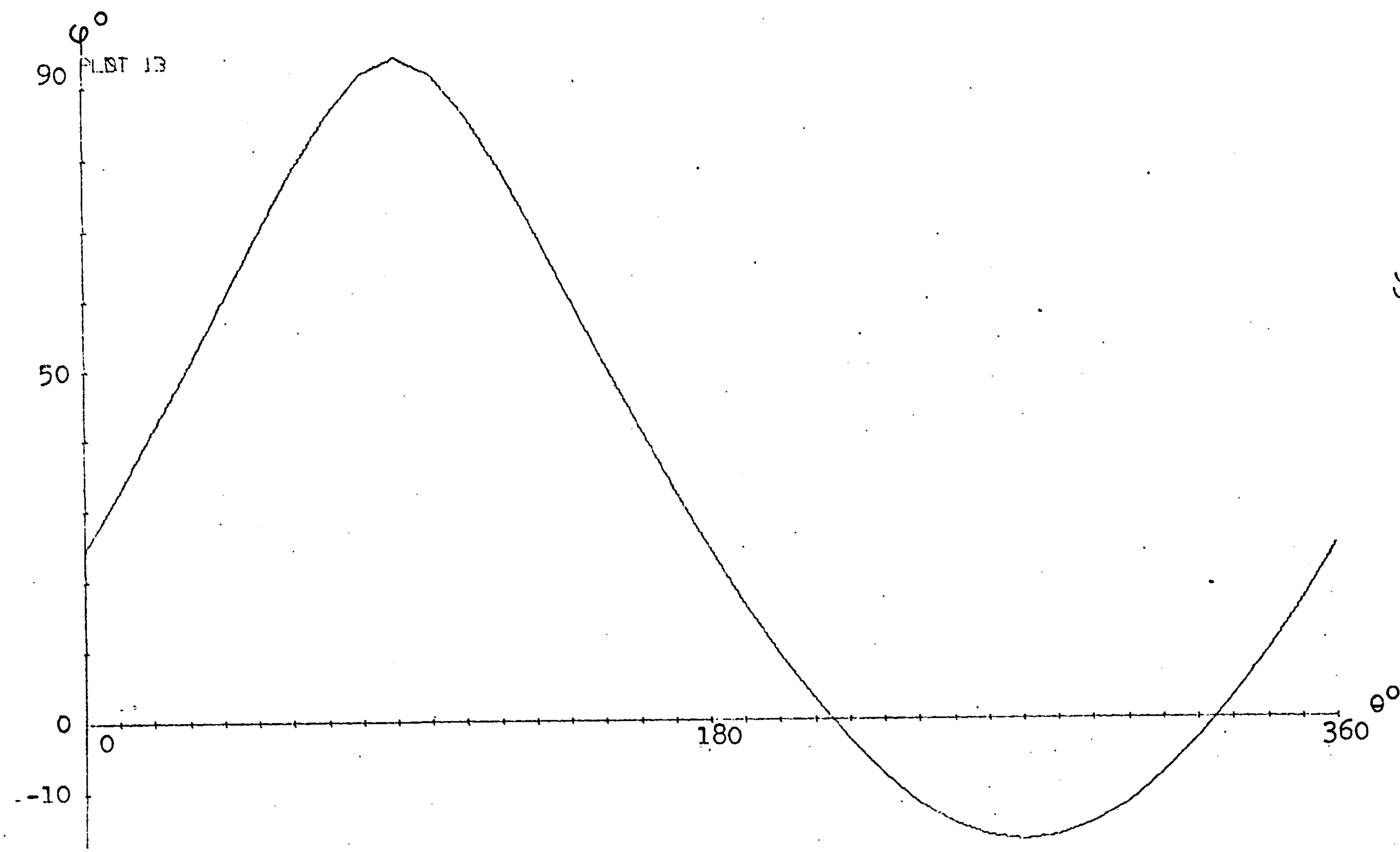
Linear Velocity of Coupler Point



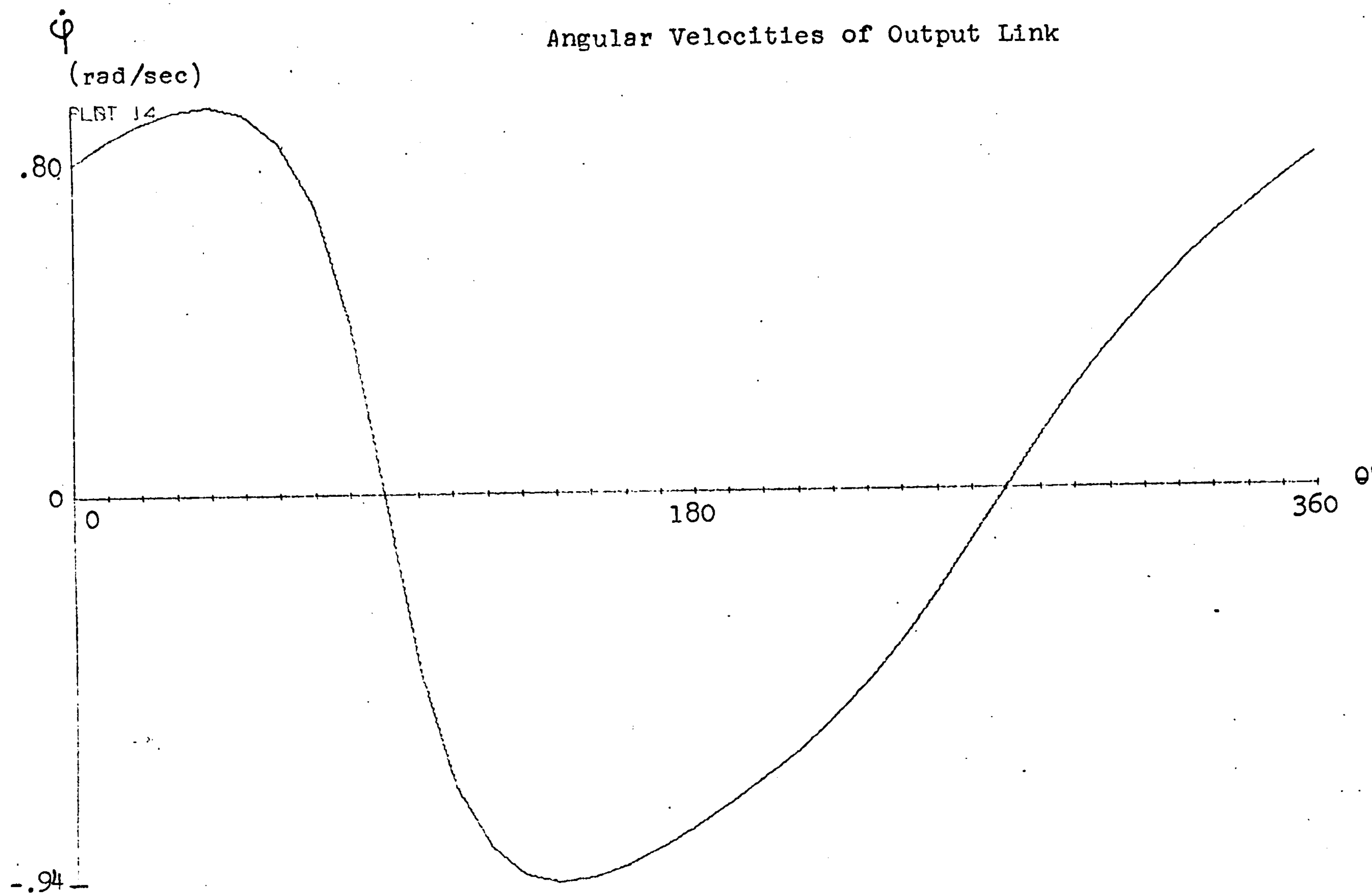
Linear Acceleration of Coupler Point



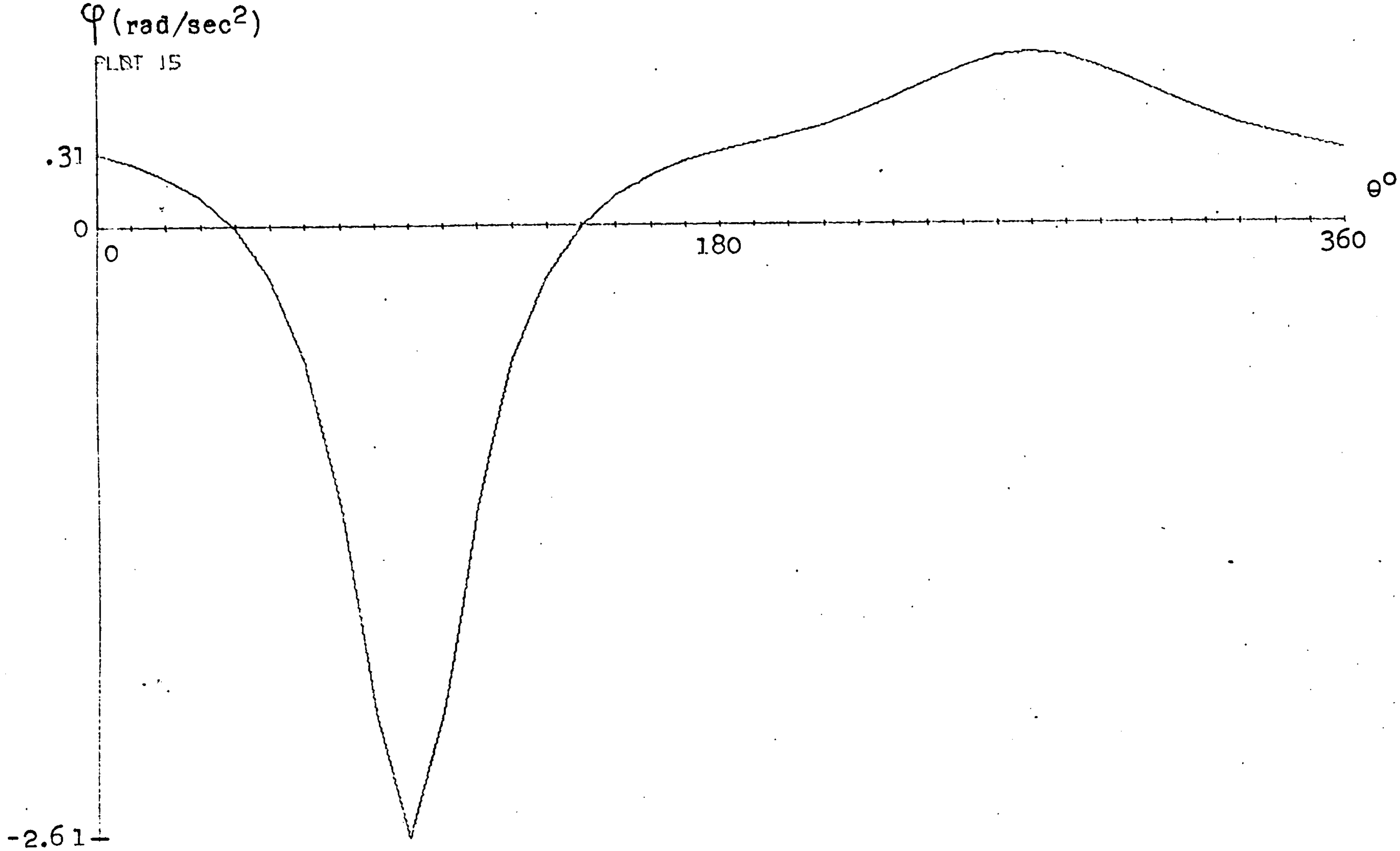
Angular Displacement of Output Link

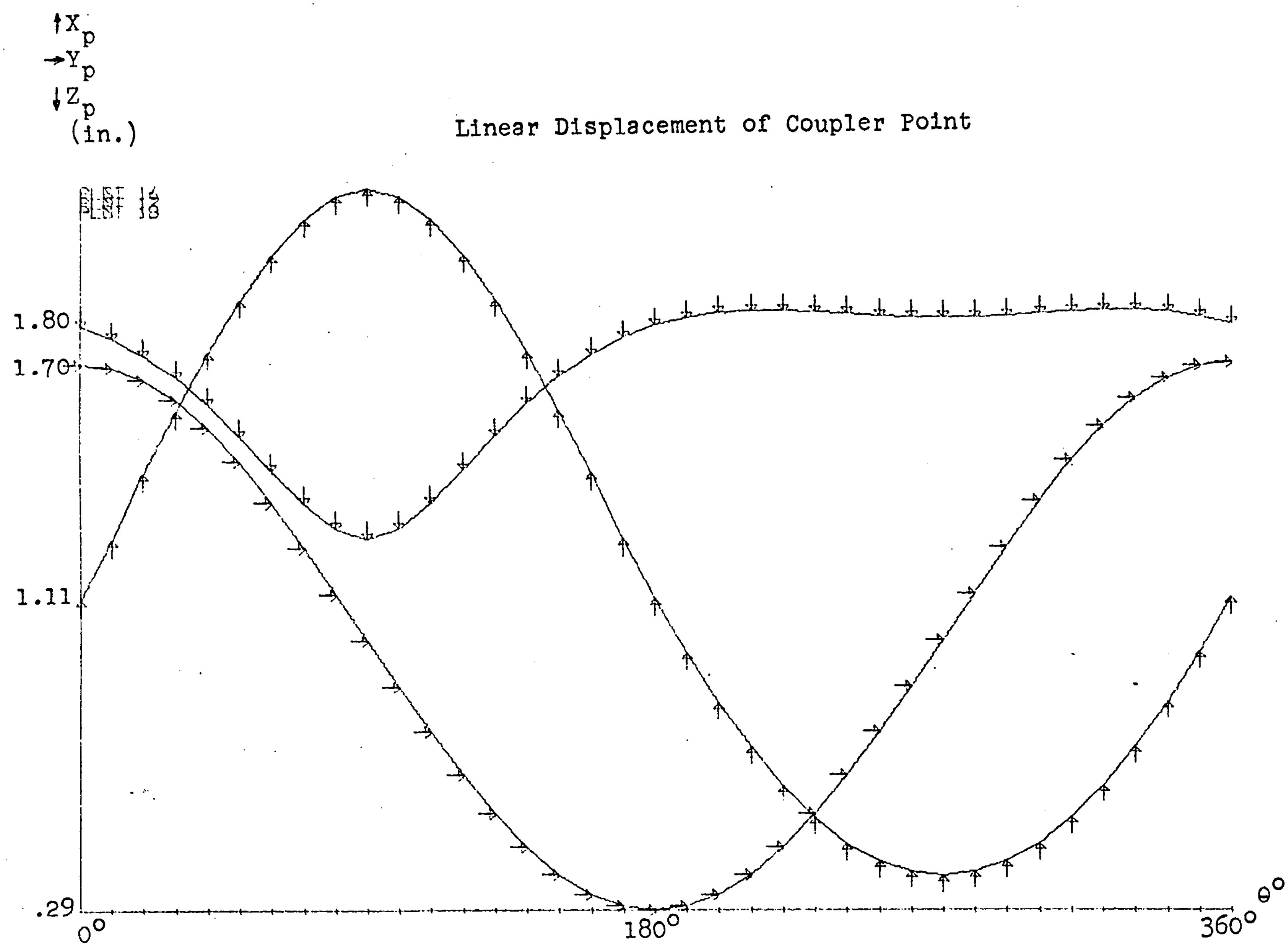


Angular Velocities of Output Link

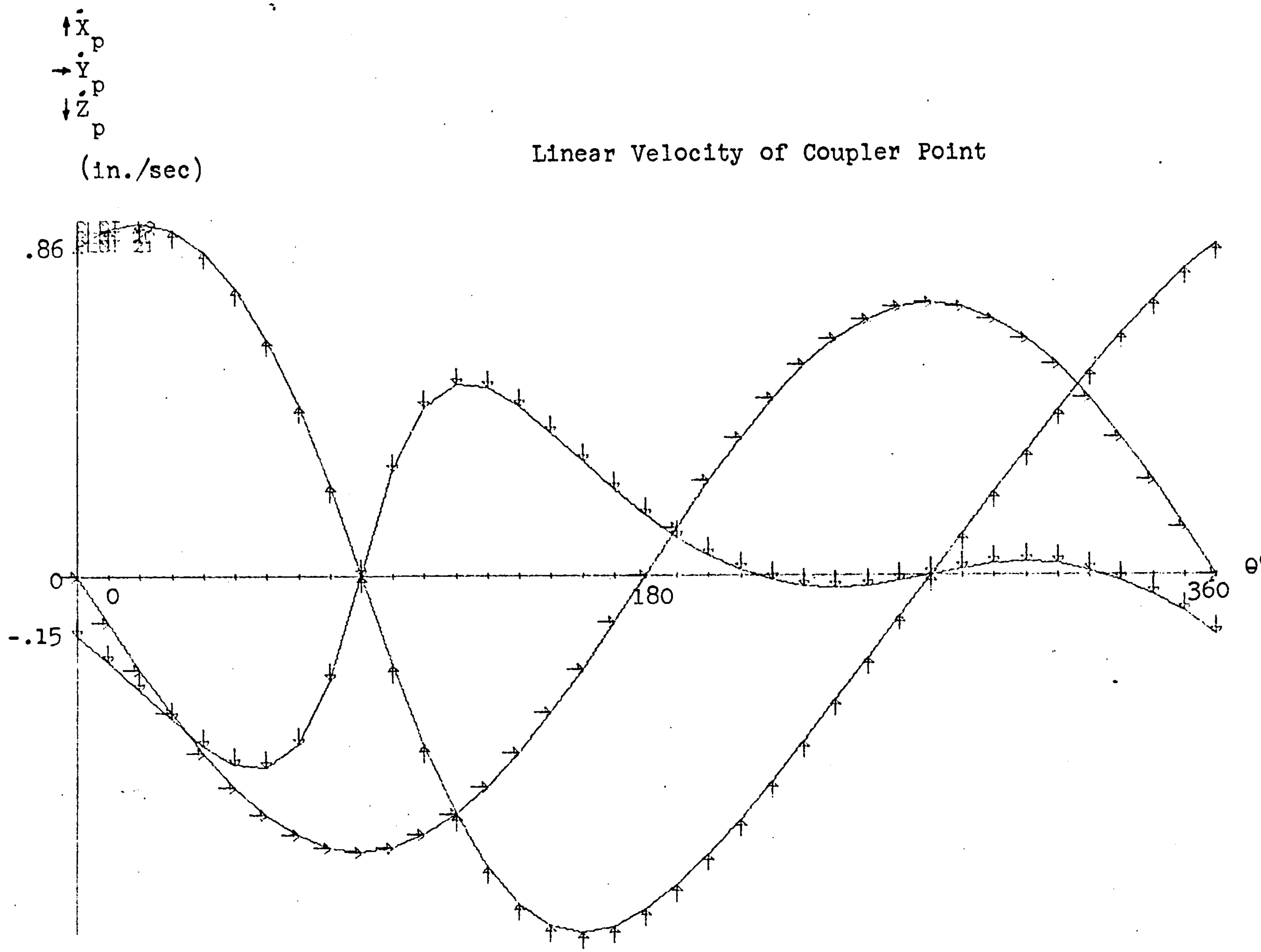


Angular Acceleration of Output Link

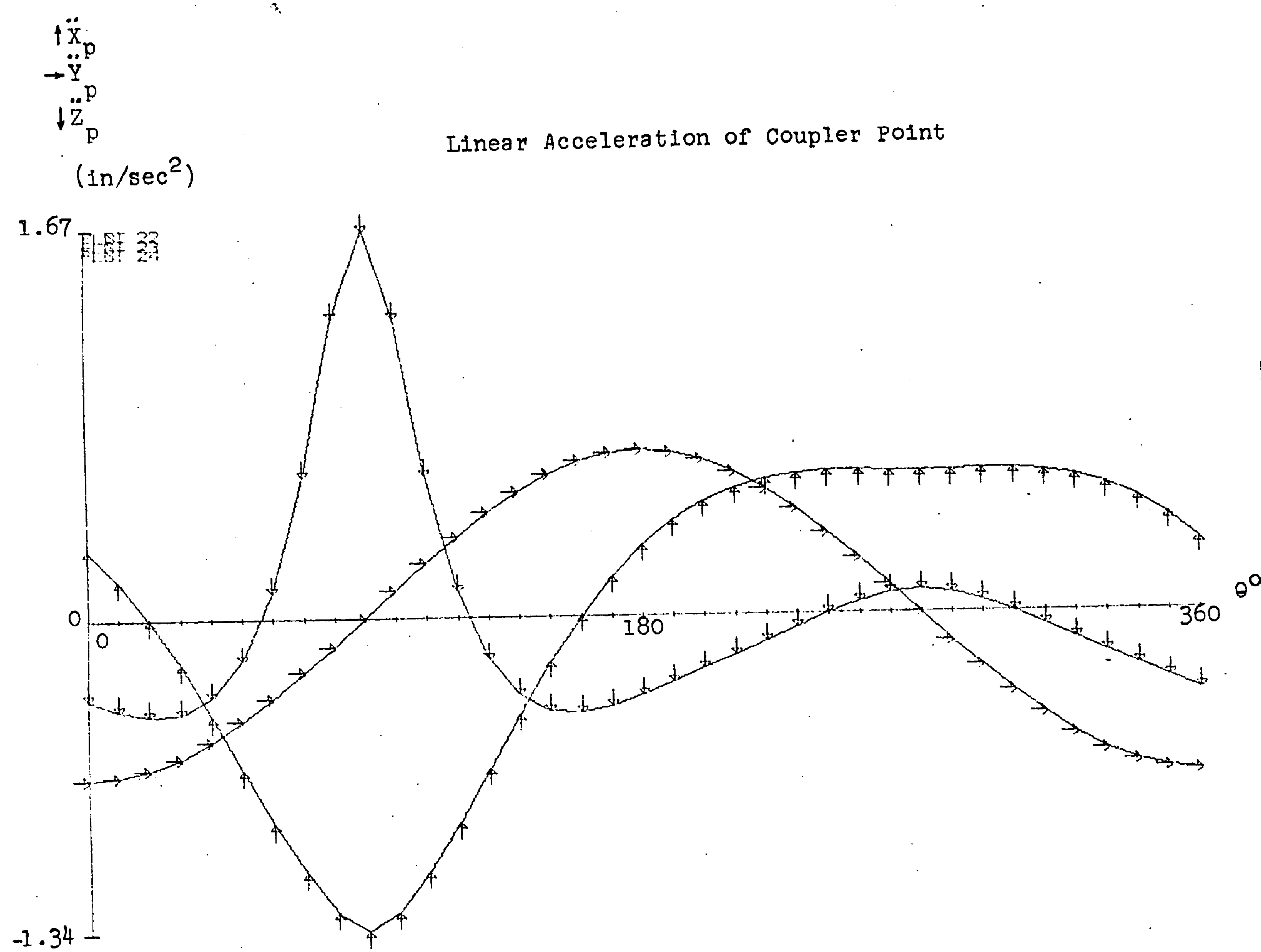



Linear Displacement of Coupler Point
↑X_p
→Y_p
↓Z_p
(in.)
1.80
1.70
1.11
.29
0°
180°
360°
θ°

Linear Velocity of Coupler Point



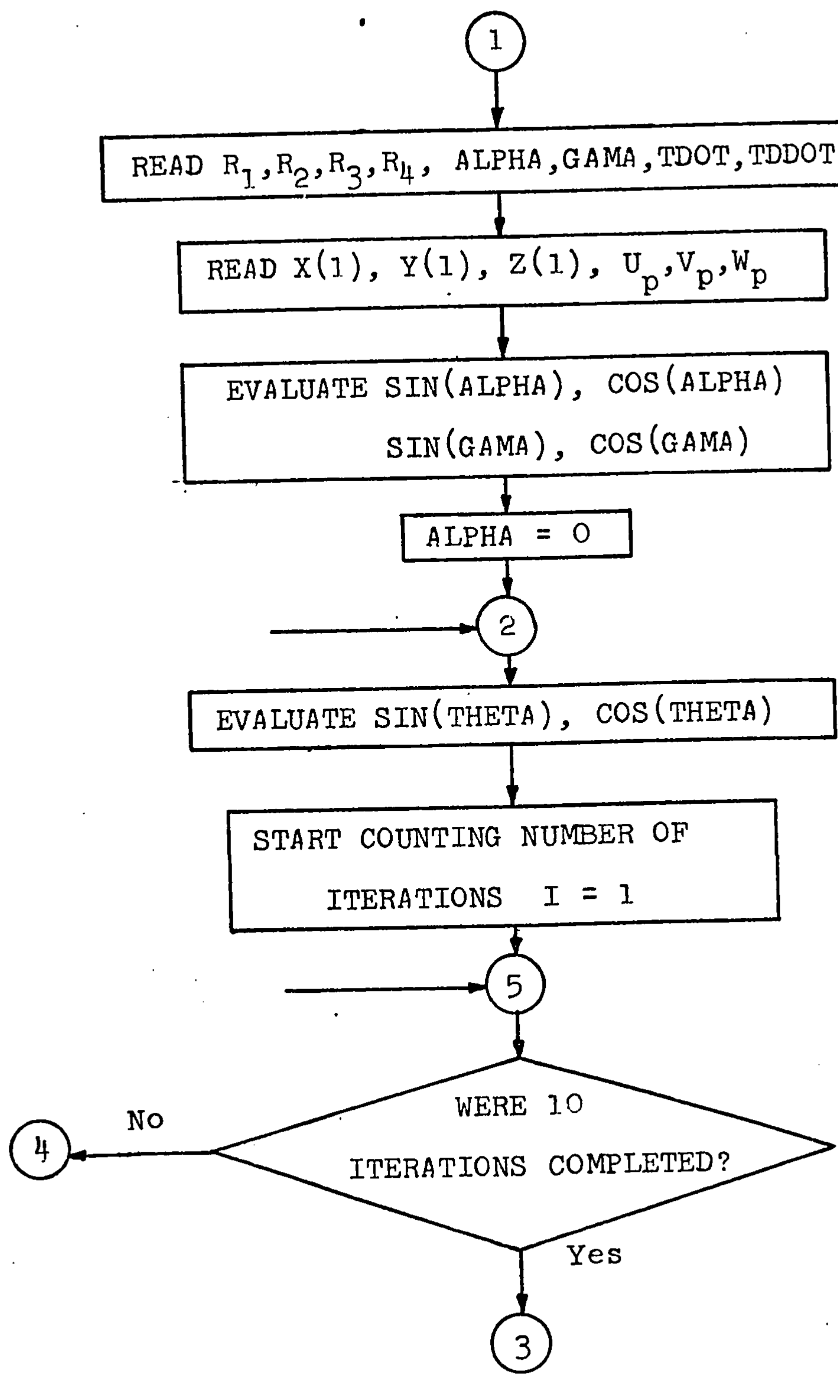
Linear Acceleration of Coupler Point

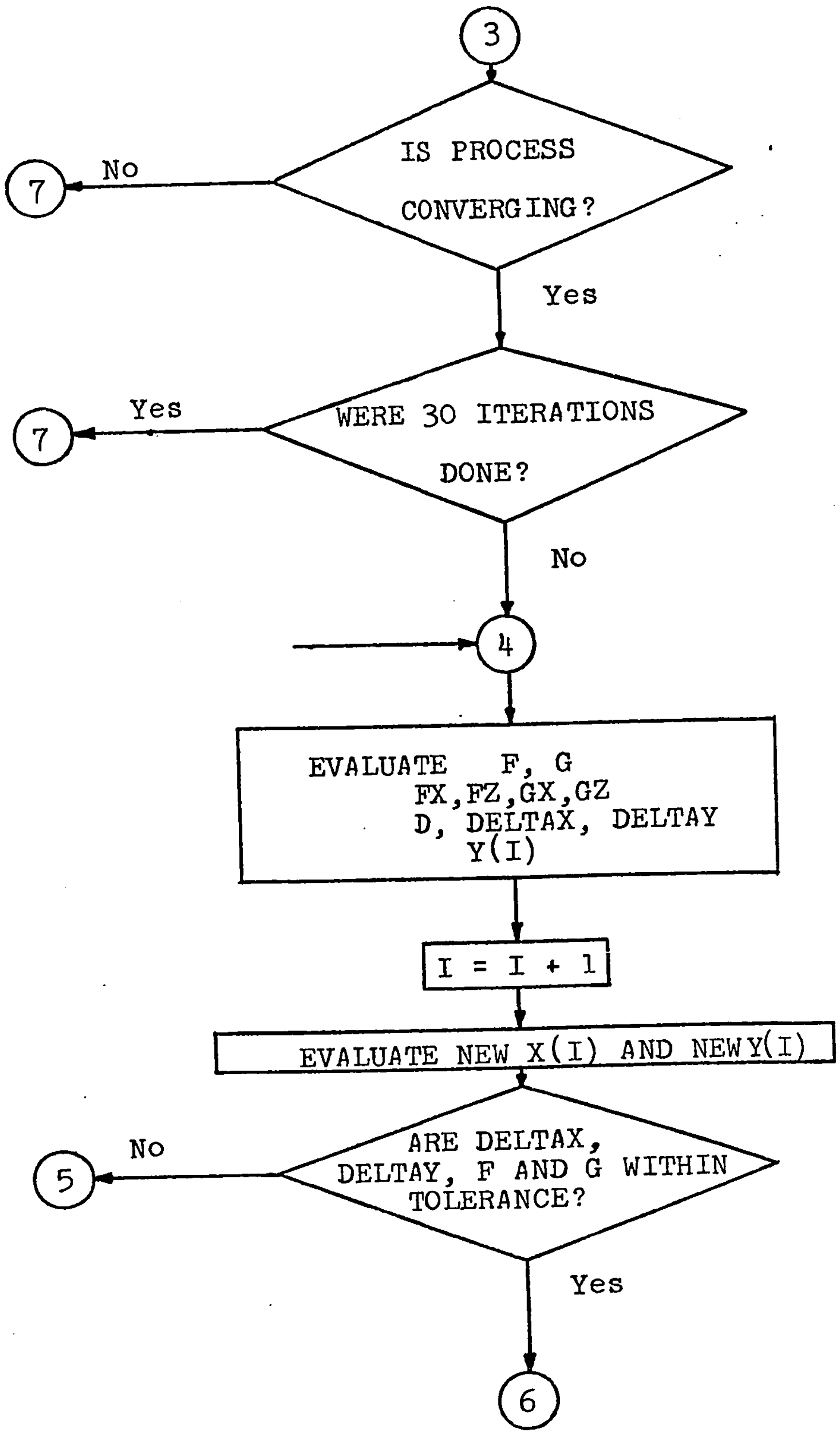


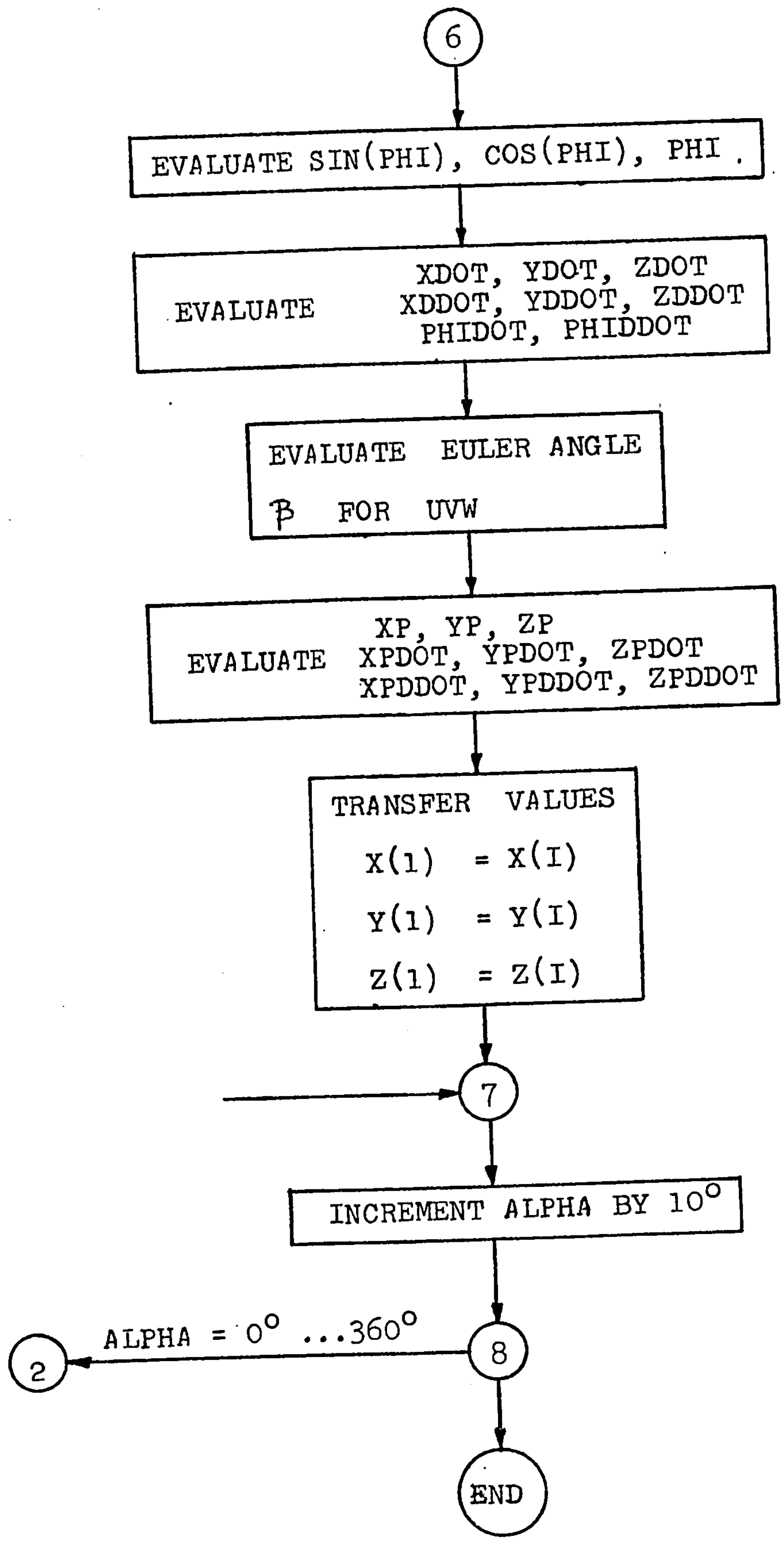
3. RGCR - linkage (α variable)

3.1 FLOW-CHART

Iterative method for kinematic Analysis of Four-Bar spatial linkages.




3
IS PROCESS CONVERGING?
No
7
Yes
WERE 30 ITERATIONS DONE?
Yes
7
No
4
EVALUATE F, G
FX,FZ,GX,GZ
D, DELTAX, DELTAY
Y(I)
I = I + 1
EVALUATE NEW X(I) AND NEW Y(I)
ARE DELTAX, DELTAY, F AND G WITHIN TOLERANCE?
No
5
Yes
6


6
EVALUATE SIN(PHI), COS(PHI), PHI
EVALUATE XDOT, YDOT, ZDOT XDDOT, YDDOT, ZDDOT PHIDOT, PHIDDOT
EVALUATE EULER ANGLE β FOR UVW
EVALUATE XP, YP, ZP XPDOT, YPDOT, ZPDOT XPDDOT, YPDDOT, ZPDDOT
TRANSFER VALUES
X(1) = X(I)
Y(1) = Y(I)
Z(1) = Z(I)
7
INCREMENT ALPHA BY 10°
ALPHA = 0° ...360°
8
2
END

3.2: PROGRAM LINKAGE CDC 6600 FTN V3.0-P213 OPT=1 72/

```
      PROGRAMLINKAGE(INPUT,OUTPUT,TAPE10,TAPE60=INPUT)
C
C                 ITERATIVE  METHOD  FOR  KINEMATICE
C                  ANALYSIS  OF  SPATIAL  LINKAGES
C
C                  RGCR  LINKAGE  (ALPHA VARIABLE)
C
      DIMENSION F(30),X(30),Y(30),Z(30)
    1 READ101,R1,R2,R3,R4,THETA,GAMA,ADOT,ADDOT
      READ101,X(1),Y(1),Z(1),UP,VP,WP
  101 FORMAT(7F10.3)
      PRINT 102
  102 FORMAT(1H1,21X,≠R1≠,8X,≠R2≠,8X,≠R3≠,8X,≠Z4≠,6X,≠THETA≠,
     16X,≠GAMA≠,7X,≠ADOT≠,6X,≠ADDOT≠)
      PRINT103,R1,R2,R3,R4,THETA,GAMA,ADOT,ADDOT
  103 FORMAT(1H0,15X, 8F10.3)
      PRINT108
  108 FORMAT(1H0,22X,≠UP≠,9X,≠VP≠,9X,≠WP≠)
      PRINT109,UP,VP,WP
  109 FORMAT(1H0,15X,3F10.3)
C     T=ANGLE THETA IN RADIANS
C     P=ANGLE BETA  IN RADIANS
C     A=ANGLE ALPHA IN RADIANS
C     B=ANGLE BETA  IN RADIANS
C     G=ANGLE GAMA  IN RADIANS
      G=GAMA*ATAN(1.0)/45.0
      T=THETA*ATAN(1.0)/45.0
      CT=COS(T)
      ST=SIN(T)
      CG=COS(G)
      SG=SIN(G)
      ALPHA=0.0
      DO 7J=1,37
      A=ALPHA*ATAN(1.0)/45.0
      CA=COS(A)
      SA=SIN(A)
      PRINT104
  104 FORMAT(1H0,19X,≠ALPHA≠,7X,≠I≠,10X,≠X≠,12X,≠Y≠,12X,≠Z≠,12X,≠F≠,12X,
     1≠G≠,12X,≠D≠)
      I=1
   12 CONTINUE
      IF(I-10)15,15,32
   32 IF((ABS(F(9)-F(10))).LT.(ABS(F(7)-F(8))).AND.
     1(ABS(F(7)-F(8))).LT.(ABS(F(5)-F(6))).AND.
     2(ABS(F(5)-F(6))).LT.(ABS(F(3)-F(4))).AND.
     3(ABS(F(3)-F(4))).LT.(ABS(F(1)-F(2))))17,7
   17 IF(I-30)15,15,7
C     ITERATION  PROCESS  STARTS
   15 F(I)=(X(I)-R2*CA*ST)**2+(Z(I)-R2*SA)**2-R3**2
      G=(X(I)-R1*SG)**2+Z(I)**2-R4**2
      FX=2.0*(X(I)-R2*CA*ST)
      FZ=2.0*(Z(I)-R2*SA)
      GX=2.0*(X(I)-R1*SG)
      GZ=2.0*Z(I)
      D=FX*GZ-FZ*GX
```

```
      DELTAX=(-F(I)*GZ+G*FZ)/D
      DELTAZ=(-G*FX+F(I)*GX)/D
      Y(I)=R2*CA*CT
      PRINT105,ALPHA,I,X(I),Y(I),Z(I),F(I),G,D
  105 FORMAT(1H ,15X,F8.1,I9,5F13.3,F17.7)
      I=I+1
C     COORDINATES OF JOINT C
      X(I)=X(I-1)+DELTAX
      Z(I)=Z(I-1)+DELTAZ
      Y(I)=Y(I-1)
      IF(ABS(DELTAX)-0.001)10,12,12
   10 IF(ABS(DELTAZ)-0.001)11,12,12
   11 IF(ABS(F(I-1))-0.001)13,12,12
   13 IF(ABS(G)-0.001)14,12,12
C     ITERATION PROCESS COMPLETED
   14 CONTINUE
      CP=Z(I)/R4
      SP=(X(I)-R1*SG)/R4
      P=ATAN2(SP,CP)
C     ANGULAR DESPLCEMENT OF OUTPUT LINK
      PHI=P*45.0/ATAN(1.0)
C     LINEAR VELOCITY OF JOINT C
      XDOT=(-X(I)*R2*SA*ST*ADOT+R2**2*SA*CA*ST**2*ADOT+R2*CA*Z(I)*ADOT
     1-R2**2*SA*CA*ADOT)/(-R2*CA*ST+R1*SG-R2*SA*(R1*SG-X(I))/Z(I))
      YDOT=-R2*CT*SA*ADOT
      ZDOT=(R1*SG-X(I))*XDOT/Z(I)
C     LINEAR ACCELERATION OF JOINT C
      XDDOT=(-XDOT*(2.0*R2*SA*ST*ADOT-(Z(I)*(R2*CA*ADOT*(R1*SG-X(I))-
     1R2*SA*XDOT)-ZDOT*R2*SA*(R1*SG-X(I)))/Z(I)**2)+R2**2*ST**2*
     2ADOT**2*(CA**2-SA**2)-X(I)*R2*ST*(CA*ADOT**2+SA*ADDOT)+R2**2*SA
     3*CA*ST**2*ADDOT-R2*SA*Z(I)*ADOT**2+R2*ZDOT*CA*ADOT+R2*CA*Z(I)*
     4ADDOT-R2**2*CA**2*ADOT**2+R2**2*SA**2*ADOT**2-R2**2*SA*CA*ADDOT)
     5/(R1*SG-R2*CA*ST-R2*SA*(R1*SG-X(I))/Z(I))
      YDDOT=-(R2*CA*CT*ADOT**2+R2*CT*SA*ADDOT)
      ZDDOT=((-XDOT**2+(R1*SG-X(I))*XDDOT)*Z(I)-ZDOT*XDOT*(R1*SG-X(I)))
     1/Z(I)**2
C     ANGULAR VELOCITY OF OUTPUT LINK
      PHIDOT=(ZDOT*SP-XDOT*CP)/R4
C     ANGULAR ACCELERATION OF OUTPUT LINK
      PHIDDOT=(ZDDOT*SP-XDDOT*CP)/R4
      PRINT110
  110 FORMAT(1H0,25X,≠PHI≠,12X,≠PHIDOT≠,8X,≠PHIDDOT≠)
      PRINT111,PHI,PHIDOT,PHIDDOT
  111 FORMAT(1H ,15X,3F15.3)
C     SB=SIN(B)
C     CB=COS(B)
      SB=(Z(I)-R2*SA)/R3
      CB=(X(I)-R2*CA*ST)/R3
C     COORDINATES OF COUPLER POINT P
      XP=UP*CB-WP*SB+R2*CA*ST
      YP=VP+Y(I)
      ZP=UP*SB+WP*CB+R2*SA
C     LINEAR VELOCITY OF COUPLER POINT P
      XPDOT=(UP/R3)*(XDOT+R2*ST*SA*ADOT)-(WP/R3)*(ZDOT-R2*CA*ADOT)-
     1R2*ST*SA*ADOT
```

```
      YPDOT=YDOT
      ZPDOT= (UP/R3)*(ZDOT-R2*CA*ADOT)+(WP/R3)*(XDOT+R2*ST*SA*ADOT)+
     1R2*CA*ADOT
C     LINEAR ACCELERATION OF COUPLER POINT P
      XPDDOT=(UP/R3)*(XDDOT+R2*ST*(CA*ADOT**2+SA*ADDOT))-(WP/R3)*(ZDDOT-
     1R2*(-SA*ADOT**2+CA*ADDOT))-(R2*ST*(CA*ADOT**2+SA*ADDOT))
      YPDDOT=YDDOT
      ZPDDOT= (UP/R3)*(ZDDOT-R2*(-SA*ADOT**2+CA*ADDOT))+(WP/R3)*(XDDOT+
     1R2*ST*(CA*ADOT**2+SA*ADDOT))+R2*(-SA*ADOT**2+CA*ADDOT)
      PRINT106
  106 FORMAT(1H0,22X,≠XP≠,8X,≠YP≠,8X,≠ZP≠,6X,≠XPDOT≠
     1,5X,≠YPDOT≠,5X,≠ZPDOT≠,4X,≠XPDDOT≠,4X,≠YPDDOT≠,4X,≠ZPDDOT≠)
      PRINT107,XP,YP,ZP,XPDOT,YPDOT,ZPDOT,XPDDOT,YPDDOT,ZPDDOT
  107 FORMAT(1H ,15X, 9F10.3)
      X(1)=X(1)
      Y(1)=Y(1)
      Z(1)=Z(1)
    7 ALPHA=ALPHA+10.0
      GO TO 1
      END
```

3.3: NUMERICAL RESULTS

(Example 1)

R1	R2	R3	R4	THETA	GAMA	ADOT	ADDOT
1.480	1.050	1.000	1.050	70.000	35.000	1.000	0.000

UP	VP	WP
.500	1.000	1.000

ALPHA	I	X	Y	Z	F	G	D
0.0	1	1.000	.359	1.000	.000	-.080	-.5511365
0.0	2	1.290	.359	.996	.084	.084	-.5489601
0.0	3	1.290	.359	.954	.002	.002	-.5257297
0.0	4	1.290	.359	.953	.000	.000	-.5252165

PHI	PHIDOT	PHIDDOT
24.826	7.621	16.191

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.185	1.359	.780	-5.941	-0.000	-5.057	27.195	-.359	-64.147

ALPHA	I	X	Y	Z	F	G	D
10.0	1	1.290	.354	.953	-.305	.000	-.1465545
10.0	2	-2.676	.354	2.788	19.095	19.095	-3.9401601
10.0	3	-.909	.354	1.597	4.540	4.540	-2.0667037
10.0	4	-.108	.354	1.058	.933	.933	-1.2175644
10.0	5	.171	.354	.870	.113	.113	-.9214660
10.0	6	.216	.354	.840	.003	.003	-.8738928
10.0	7	.217	.354	.839	.000	.000	-.8725979

PHI	PHIDOT	PHIDDOT
-36.971	3.705	-24.265

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.062	1.354	-.244	1.734	-.062	-3.589	10.032	-.354	31.808

ALPHA	I	X	Y	Z	F	G	D
20.0	1	.217	.337	.839	-.266	.000	-1.1697909
20.0	2	-.164	.337	.552	.228	.228	-1.6280031
20.0	3	-.064	.337	.530	.011	.011	-1.4765564
20.0	4	-.058	.337	.529	.000	.000	-1.4687897

PHI	PHIDOT	PHIDDOT
-59.775	1.361	-6.552

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.265	1.337	-.542	1.693	-.123	-.506	-3.216	-.337	8.374

ALPHA	I	X	Y	Z	F	G	D
30.0	1	-.058	.311	.529	-.167	.000	-1.9170738
30.0	2	-.150	.311	.371	.033	.033	-2.1065634
30.0	3	-.134	.311	.371	.000	.000	-2.0716627

PHI	PHIDOT	PHIDDOT
-69.331	.670	-2.283

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.515	1.311	-.540	1.197	-.180	.370	-2.385	-.311	2.917

ALPHA	I	X	Y	Z	F	G	D
40.0	1	-.134	.275	[illegible]	-.116	.000	-2.5142878

ALPHA	I	X	Y	Z	F	G	D
						.009	-2.6408216
40.0	2	-.168	.275	.280	.009	.000	-2.6275236
40.0	3	-.163	.275	.280	.000		

PHI	PHIDOT	PHIDDOT
-74.517	.404	-.987

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.691	1.275	-.441	.839	-.231	.719	-1.785	-.275	1.337

ALPHA	I	X	Y	Z	F	G	D
50.0	1	-.163	.231	.280	-.090	.000	-3.0149728
50.0	2	-.180	.231	.220	.004	.004	-3.1204586
50.0	3	-.178	.231	.221	.000	.000	-3.1135066

PHI	PHIDOT	PHIDDOT
-77.874	.282	-.477

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.812	1.231	-.299	.556	-.275	.886	-1.560	-.231	.655

ALPHA	I	X	Y	Z	F	G	D
60.0	1	-.178	.180	.221	-.075	.000	-3.4202850
60.0	2	-.187	.180	.175	.002	.002	-3.5199332
60.0	3	-.186	.180	.176	.000	.000	-3.5152985

PHI	PHIDOT	PHIDDOT
-80.363	.223	-.233

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.887	1.180	-.137	.309	-.311	.963	-1.342	-.180	.247

ALPHA	I	X	Y	Z	F	G	D
70.0	1	-.186	.123	.176	-.069	.000	-3.7259906
70.0	2	-.193	.123	.138	.001	.001	-3.8287776
70.0	3	-.192	.123	.138	.000	.000	-3.8249745

PHI	PHIDOT	PHIDDOT
-82.430	.195	-.098

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.921	1.123	.033	.085	-.337	.978	-1.256	-.123	-.054

ALPHA	I	X	Y	Z	F	G	D
80.0	1	-.192	.062	.138	-.064	.000	-3.9302824
80.0	2	-.197	.062	.104	.001	.001	-4.0436972
80.0	3	-.196	.062	.104	.000	.000	-4.0399727

PHI	PHIDOT	PHIDDOT
-84.319	.185	-.014

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.918	1.062	.202	-.119	-.354	.947	-1.172	-.062	-.303

ALPHA	I	X	Y	Z	F	G	D
90.0	1	-.196	-.000	.104	-.067	.000	-4.0354060
90.0	2	-.199	-.000	.069	.001	.001	-4.1668689
90.0	3	-.199	-.000	.070	.000	.000	-4.1626679

PHI	PHIDOT	PHIDDOT
-86.180	.188	.045

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.881	1.000	.361	-.302	-.359	.875	-.983	.000	-.516

ALPHA	I	X	Y	Z	F	G	D
100.0	1	-.199	-.062	.070	-.070	.000	-4.0478742
100.0	2	-.201	-.062	.034	.001	.001	-4.2052351
100.0	3	-.201	-.062	.034	.000	.000	-4.1999756

PHI	PHIDOT	PHIDDOT
-88.118	.201	.092

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.814	.938	.505	-.461	-.354	.768	-.821	.062	-.698

ALPHA	I	X	Y	Z	F	G	D
110.0	1	-.201	-.123	.034	-.075	.000	-3.9782129
110.0	2	-.202	-.123	-.005	.002	.002	-4.1699911
110.0	3	-.201	-.123	-.004	.000	.000	-4.1629147

PHI	PHIDOT	PHIDDOT
-90.217	.220	.134

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.721	.877	.628	-.591	-.337	.633	-.652	.123	-.849

ALPHA	I	X	Y	Z	F	G	D
120.0	1	-.201	-.180	-.004	-.080	.000	-3.8405211
120.0	2	-.201	-.180	-.048	.002	.002	-4.0761344
120.0	3	-.200	-.180	-.047	.000	.000	-4.0661480

PHI	PHIDOT	PHIDDOT
-92.549	.247	.174

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.609	.820	.725	-.687	-.311	.474	-.446	.180	-.966

ALPHA	I	X	Y	Z	F	G	D
130.0	1	-.200	-.231	-.047	-.087	.000	-3.6519750
130.0	2	-.198	-.231	-.097	.003	.003	-3.9420527
130.0	3	-.197	-.231	-.095	.000	.000	-3.9275201

PHI	PHIDOT	PHIDDOT
-95.186	.281	.218

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.484	.769	.792	-.745	-.275	.298	-.215	.231	-1.043

ALPHA	I	X	Y	Z	F	G	D
140.0	1	-.197	-.275	-.095	-.095	.000	-3.4323270
140.0	2	-.192	-.275	-.153	.003	.003	-3.7890753
140.0	3	-.190	-.275	-.150	.000	.000	-3.7675342

PHI	PHIDOT	PHIDDOT
-98.204	.324	.266

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.352	.725	.828	-.761	-.231	.112	.038	.275	-1.075

ALPHA I X Y F G D

150.0	1	-.190	-.311	-.150	-.104	.000	-3.2033486
150.0	2	-.181	-.311	-.217	.005	.005	-3.6407202
150.0	3	-.179	-.311	-.213	.000	.000	-3.6085605

PHI	PHIDOT	PHIDDOT
-101.686	.374	.317

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.221	.689	.831	-.731	-.180	-.074	.364	.311	-1.053

ALPHA	I	X	Y	Z	F	G	D
160.0	1	-.179	-.337	-.213	-.114	.000	-2.9880043
160.0	2	-.163	-.337	-.291	.006	.006	-3.5210873
160.0	3	-.162	-.337	-.285	.006	.000	-3.4733795

PHI	PHIDOT	PHIDDOT
-105.721	.434	.365

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.099	.663	.803	-.655	-.123	-.252	.567	.337	-.970

ALPHA	I	X	Y	Z	F	G	D
170.0	1	-.162	-.354	-.285	-.126	.000	-2.8089642
170.0	2	-.136	-.354	-.375	.009	.009	-3.4516843
170.0	3	-.135	-.354	-.366	.000	.000	-3.3826175

PHI	PHIDOT	PHIDDOT
-110.390	.501	.400

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.005	.646	.745	-.535	-.062	-.409	.862	.354	-.820

ALPHA	I	X	Y	Z	F	G	D
180.0	1	-.135	-.359	-.366	-.141	.000	-2.6860420
180.0	2	-.097	-.359	-.469	.012	.012	-3.4465998
180.0	3	-.097	-.359	-.456	.006	.000	-3.3510901

PHI	PHIDOT	PHIDDOT
-115.753	.572	.410

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.086	.641	.662	-.379	.000	-.535	.980	.359	-.609

ALPHA	I	X	Y	Z	F	G	D
190.0	1	-.097	-.354	-.456	-.160	.000	-2.6326498
190.0	2	-.042	-.354	-.571	.016	.016	-3.5078790
190.0	3	-.043	-.354	-.554	.006	.000	-3.3841740

PHI	PHIDOT	PHIDDOT
-121.828	.642	.392

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.136	.646	.560	-.197	.062	-.619	1.081	.354	-.356

ALPHA	I	X	Y	Z	F	G	D
200.0	1	-.043	-.337	-.554	-.181	.000	-2.6524463
200.0	2	.032	-.337	-.675	.020	.020	-3.6246239
200.0	3	.028	-.337	-.655	.000	.000	-3.4762210

PHI	PHIDOT	PHIDDOT

-128.583	.707	.349

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.154	.663	.448	-.006	.123	-.658	1.182	.337	-.088

ALPHA	I	X	Y	Z	F	G	D
210.0	1	.028	-.311	-.655	-.204	.000	-2.7380591
210.0	2	.126	-.311	-.777	.025	.025	-3.7771999
210.0	3	.119	-.311	-.755	.001	.001	-3.6121882

PHI	PHIDOT	PHIDDOT
-135.946	.764	.296

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.138	.689	.333	.183	.180	-.650	1.053	.311	.169

ALPHA	I	X	Y	Z	F	G	D
220.0	1	.119	-.275	-.755	-.229	.000	-2.8728017
220.0	2	.239	-.275	-.871	.028	.028	-3.9431887
220.0	3	.229	-.275	-.848	.001	.001	-3.7713878

PHI	PHIDOT	PHIDDOT
-143.826	.811	.243

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.091	.725	.224	.359	.231	-.600	.953	.275	.398

ALPHA	I	X	Y	Z	F	G	D
230.0	1	.229	-.231	-.848	-.253	.000	-3.0343251
230.0	2	.370	-.231	-.951	.031	.031	-4.1010469
230.0	3	.358	-.231	-.929	.001	.001	-3.9311656

PHI	PHIDOT	PHIDDOT
-152.131	.849	.197

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.014	.769	.126	.514	.275	-.514	.818	.231	.591

ALPHA	I	X	Y	Z	F	G	D
240.0	1	.358	-.180	-.928	-.275	.000	-3.1982986
240.0	2	.518	-.180	-1.013	.033	.033	-4.2310780
240.0	3	.504	-.180	-.992	.001	.001	-4.0693507

PHI	PHIDOT	PHIDDOT
-160.783	.880	.161

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.087	.820	.046	.643	.311	-.396	.658	.180	.745

ALPHA	I	X	Y	Z	F	G	D
250.0	1	.503	-.123	-.991	-.293	.000	-3.3410092
250.0	2	.677	-.123	-1.052	.034	.034	-4.3154393
250.0	3	.662	-.123	-1.033	.001	.001	-4.1656043

PHI	PHIDOT	PHIDDOT
-169.717	.906	.136

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.208	.877	-.011	.742	.337	-.256	.480	.123	.861

ALPHA	I	X	Y	Z	F	G	D
260.0	1	.661	-.062	-1.033	-.306	.000	-3.4408596
260.0	2	.845	-.062	-1.067	.035	.035	-4.3382978
260.0	3	.829	-.062	-1.050	.007	.001	-4.2021776

PHI	PHIDOT	PHIDDOT
-178.889	.928	.119

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.344	.938	-.042	.810	.354	-.098	.289	.062	.939

ALPHA	I	X	Y	Z	F	G	D
270.0	1	.829	-.000	-1.050	-.314	.000	-3.4792037
270.0	2	1.018	-.000	-1.053	.036	.036	-4.2863001
270.0	3	1.000	-.000	-1.039	.007	.001	-4.1644488

PHI	PHIDOT	PHIDDOT
171.729	.948	.112

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.489	1.000	-.045	.843	.359	.070	.089	.000	.981

ALPHA	I	X	Y	Z	F	G	D
280.0	1	1.000	.062	-1.039	-.313	.000	-3.4409071
280.0	2	1.189	.062	-1.012	.037	.037	-4.1492044
280.0	3	1.171	.062	-1.000	.000	.000	-4.0414089

PHI	PHIDOT	PHIDDOT
162.151	.968	.116

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.636	1.062	-.017	.840	.354	.242	-.118	-.062	.989

ALPHA	I	X	Y	Z	F	G	D
290.0	1	1.171	.123	-.999	-.305	.000	-3.3148320
290.0	2	1.355	.123	-.940	.037	.037	-3.9205351
290.0	3	1.336	.123	-.930	.006	.000	-3.8261372

PHI	PHIDOT	PHIDDOT
152.368	.989	.134

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.780	1.123	.040	.801	.337	.414	-.334	-.123	.967

ALPHA	I	X	Y	Z	F	G	D
300.0	1	1.336	.180	-.930	-.296	.000	-3.0942935
300.0	2	1.510	.180	-.839	.039	.039	-3.5981828
300.0	3	1.491	.180	-.831	.006	.000	-3.5162502

PHI	PHIDOT	PHIDDOT
142.349	1.016	.178

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.914	1.180	.127	.723	.311	.578	-.566	-.180	.920

ALPHA	I	X	Y	Z	F	G	D
310.0	1	1.490	.231	-.831	-.266	.000	-2.7774714
310.0	2	1.650	.231	-.708	.041	.041	-3.1849364
310.0	3	1.629	.231	-.703	.006	.000	-3.1143000

PHI	PHIDOT	PHIDDOT
132.012	1.054	.272

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.030	1.231	.241	.601	.275	.734	-.844	-.231	.857

ALPHA	I	X	Y	Z	F	G	D
320.0	1	1.629	.275	-.703	-.237	.000	-2.3677702
320.0	2	1.770	.275	-.547	.044	.044	-2.6890703
320.0	3	1.747	.275	-.544	.000	.000	-2.6281951

PHI	PHIDOT	PHIDDOT
121.183	1.118	.491

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.120	1.275	.382	.421	.231	.878	-1.251	-.275	.798

ALPHA	I	X	Y	Z	F	G	D
330.0	1	1.747	.311	-.544	-.203	.000	-1.8742494
330.0	2	1.865	.311	-.349	.052	.052	-2.1255962
330.0	3	1.839	.311	-.350	.001	.001	-2.0720604

PHI	PHIDOT	PHIDDOT
109.448	1.246	1.082

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.171	1.311	.547	.140	.180	1.014	-2.006	-.311	.765

ALPHA	I	X	Y	Z	F	G	D
340.0	1	1.839	.337	-.350	-.169	.000	-1.3127846
340.0	2	1.929	.337	-.095	.073	.073	-1.5212961
340.0	3	1.894	.337	-.103	.001	.001	-1.4696759
340.0	4	1.894	.337	-.103	.000	.000	-1.4687694

PHI	PHIDOT	PHIDDOT
95.631	1.577	3.196

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.154	1.337	.736	-.411	.123	1.142	-4.834	-.337	.637

ALPHA	I	X	Y	Z	F	G	D
350.0	1	1.894	.354	-.103	-.143	.000	-.7114868
350.0	2	1.935	.354	.318	.179	.179	-.9486197
350.0	3	1.867	.354	.272	.007	.007	-.8756432
350.0	4	1.864	.354	.270	.000	.000	-.8726022

PHI	PHIDOT	PHIDDOT
75.111	2.844	14.725

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.966	1.354	.936	-2.150	.062	1.021	-19.153	-.354	-4.736

ALPHA	I	X	Y	Z	F	G	D
360.0	1	1.864	.359	.270	-.158	.000	-.1486984
360.0	2	1.290	.359	2.428	4.988	4.988	-1.3382964
360.0	3	1.290	.359	1.401	1.055	1.055	-.7722091
360.0	4	1.290	.359	1.025	.142	.142	-.5647168
360.0	5	1.290	.359	.955	.005	.005	-.5265977
360.0	6	1.290	.359	.953	.000	.000	-.5252180

PHI	PHIDOT	PHIDDOT
24.826	7.621	16.191

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.185	1.359	.780	-5.941	-.000	-5.057	27.105	-.359	-64.147

(Example 2)

R1	R2	R3	R4	THETA	GAMA	ADOT	ADDOT
1.480	1.050	1.000	1.050	0.000	35.000	1.000	0.000

UP	VP	WP
.500	1.000	1.000

ALPHA	I	X	Y	Z	F	G	D
0.0	1	1.000	1.050	1.000	1.000	-.080	3.3955725
0.0	2	.364	1.050	1.136	.423	.423	3.8571197
0.0	3	.364	1.050	.950	.035	.035	3.2250746
0.0	4	.364	1.050	.932	.000	.000	3.1631411

PHI	PHIDOT	PHIDDOT
-27.499	-1.237	.598

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.749	2.050	.830	1.026	-0.000	1.977	1.807	-1.050	-.673

ALPHA	I	X	Y	Z	F	G	D
10.0	1	.364	1.034	.931	-.306	.000	2.8089452
10.0	2	.567	1.034	1.037	.052	.052	3.3162620
10.0	3	.561	1.034	1.010	.001	.001	3.2208428

PHI	PHIDOT	PHIDDOT
-15.891	-1.063	1.332

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.547	2.034	1.158	1.265	-.182	1.744	.832	-1.034	-1.874

ALPHA	I	X	Y	Z	F	G	D
20.0	1	.561	.987	1.010	-.261	.000	3.0161252
20.0	2	.736	.987	1.060	.039	.033	3.4367052
20.0	3	.729	.987	1.043	.000	.000	3.3712208

PHI	PHIDOT	PHIDDOT
-6.533	-.801	1.585

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.319	1.987	1.431	1.309	-.359	1.377	-.289	-.987	-2.186

ALPHA	I	X	Y	Z	F	G	D
30.0	1	.729	.909	1.043	-.199	.000	3.2913162
30.0	2	.856	.909	1.058	.016	.016	3.6059497
30.0	3	.851	.909	1.050	.000	.000	3.5701647

PHI	PHIDOT	PHIDDOT
.121	-.535	1.414

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.099	1.909	1.639	1.191	-.525	1.016	-.970	-.909	-1.896

ALPHA	I	X	Y	Z	F	G	D
40.0	1	.851	.804	1.050	-.135	.000	3.5713110
40.0	2	.930	.804	1.050	.006	.006	3.7849651
40.0	3	.928	.804	1.047	.003	.000	3.7693127

PHI	PHIDOT	PHIDDOT
4.332	-.318	1.065

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.092	1.804	1.789	.996	-.675	.722	-1.235	-.804	-1.471

ALPHA	I	X	Y	Z	F	G	D
50.0	1	.928	.675	1.047	-.080	.000	3.8103373
50.0	2	.972	.675	1.044	.003	.002	3.9397609
50.0	3	.971	.675	1.043	.000	.000	3.9344219

PHI	PHIDOT	PHIDDOT
6.686	-.164	.706

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.247	1.675	1.895	.780	-.804	.498	-1.232	-.675	-1.112

ALPHA	I	X	Y	Z	F	G	D
60.0	1	.971	.525	1.043	-.039	.000	3.9857519
60.0	2	.992	.525	1.040	.000	.000	4.0519577

PHI	PHIDOT	PHIDDOT
7.800	-.068	.404

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.365	1.525	1.966	.570	-.909	.328	-1.172	-.525	-.849

ALPHA	I	X	Y	Z	F	G	D
70.0	1	.991	.359	1.040	-.014	.000	4.0947520
70.0	2	.999	.359	1.039	.000	.000	4.1200072

PHI	PHIDOT	PHIDDOT
8.198	-.018	.178

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.447	1.359	2.012	.371	-.987	.197	-1.105	-.359	-.665

ALPHA	I	X	Y	Z	F	G	D
80.0	1	.999	.182	1.039	-.003	.000	4.1482009
80.0	2	1.000	.182	1.039	.000	.000	4.1532057

PHI	PHIDOT	PHIDDOT
8.274	-.001	.037

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.495	1.182	2.037	.183	-1.034	.092	-1.059	-.182	-.553

ALPHA	I	X	Y	Z	F	G	D
90.0	1	1.000	-.000	1.039	.000	.000	4.1628400

PHI	PHIDOT	PHIDDOT
8.271	-.000	-.011

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.511	1.000	2.044	-.000	-1.050	-.000	-1.043	.000	-.514

ALPHA	I	X	Y	Z	F	G	D
100.0	1	1.000	-.182	1.039	-.000	.000	4.1530278

PHI	PHIDOT	PHIDDOT
8.274	.001	.037

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.495	.818	2.037	-.183	-1.034	-.092	-1.069	.182	-.553

ALPHA	I	X	Y	Z	F	G	D
110.0	1	1.000	-.359	1.039	.003	.000	4.1245692
110.0	2	.999	-.359	1.039	.00[illegible]	.000	4.1198371

PHI	PHIDOT	PHIDDOT
8.198	.018	.178

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.447	.641	2.012	-.371	-.987	-.197	-1.145	.359	-.665

ALPHA	I	X	Y	Z	F	G	D
120.0	1	.999	-.525	1.039	.014	.000	4.0735062
120.0	2	.991	-.525	1.040	.00[illegible]	.000	4.0508255

PHI	PHIDOT	PHIDDOT
7.800	.068	.404

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.365	.475	1.966	-.570	-.909	-.328	-1.172	.525	-.849

ALPHA	I	X	Y	Z	F	G	D
130.0	1	.991	-.675	1.040	.039	.000	3.9908273
130.0	2	.971	-.675	1.043	.00[illegible]	.000	3.9355604

PHI	PHIDOT	PHIDDOT
6.686	.164	.706

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.247	.325	1.895	-.780	-.804	-.498	-1.232	.675	-1.112

ALPHA	I	X	Y	Z	F	G	D
140.0	1	.971	-.804	1.043	.078	.000	3.8711342
140.0	2	.929	-.804	1.048	.005	.002	3.7738027
140.0	3	.928	-.804	1.047	.00[illegible]	.000	3.7692829

PHI	PHIDOT	PHIDDOT
4.332	.318	1.065

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.092	.196	1.789	-.996	-.675	-.722	-1.215	.804	-1.471

ALPHA	I	X	Y	Z	F	G	D
150.0	1	.928	-.909	1.047	.134	.000	3.7217172
150.0	2	.853	-.909	1.053	.006	.006	3.5827343
150.0	3	.851	-.909	1.050	.00[illegible]	.000	3.5700080

PHI	PHIDOT	PHIDDOT
.121	.535	1.414

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.099	.091	1.639	-1.191	-.525	-1.016	-.970	.909	-1.896

ALPHA	I	X	Y	Z	F	G	D
160.0	1	.851	-.987	1.050	.202	.000	3.5685186
160.0	2	.732	-.987	1.050	.014	.014	3.3988723
160.0	3	.729	-.987	1.043	.00[illegible]	.000	3.3707025

PHI	PHIDOT	PHIDDOT
-6.533	.801	1.585

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.319	.013	1.431	-1.309	-.359	-1.377	-.289	.987	-2.186

ALPHA	I	X	Y	Z	F	G	D
170.0	1	.729	-1.034	1.043	.273	.000	3.4550677
170.0	2	.564	-1.034	1.024	.028	.028	3.2706460
170.0	3	.561	-1.034	1.010	.000	.000	3.2198301

PHI	PHIDOT	PHIDDOT
-15.891	1.063	1.332

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.547	-.034	1.158	-1.265	-.182	-1.744	.832	1.034	-1.874

ALPHA	I	X	Y	Z	F	G	D
180.0	1	.561	-1.050	1.010	.335	.000	3.4291042
180.0	2	.364	-1.050	.954	.045	.042	3.2383573
180.0	3	.364	-1.050	.932	.000	.000	3.1634223

PHI	PHIDOT	PHIDDOT
-27.499	1.237	.598

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.749	-.050	.830	-1.026	.000	-1.977	1.807	1.050	-.673

ALPHA	I	X	Y	Z	F	G	D
190.0	1	.364	-1.034	.931	.373	.000	3.5161243
190.0	2	.167	-1.034	.829	.050	.050	3.3110332
190.0	3	.172	-1.034	.803	.001	.001	3.2206962

PHI	PHIDOT	PHIDDOT
-40.135	1.266	-.244

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.899	-.034	.482	-.685	.182	-1.961	1.943	1.034	.810

ALPHA	I	X	Y	Z	F	G	D
200.0	1	.172	-.987	.803	.380	.000	3.6980515
200.0	2	.007	-.987	.664	.046	.046	3.4629988
200.0	3	.017	-.987	.641	.001	.001	3.3718178

PHI	PHIDOT	PHIDDOT
-52.394	1.171	-.777

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.991	.013	.158	-.388	.359	-1.730	1.410	.987	1.702

ALPHA	I	X	Y	Z	F	G	D
210.0	1	.017	-.909	.641	.359	.000	3.9225248
210.0	2	-.100	-.909	.488	.037	.037	3.6516235
210.0	3	-.090	-.909	.471	.000	.000	3.5708979

PHI	PHIDOT	PHIDDOT
-63.349	1.015	-.966

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-1.041	.091	-.117	-.196	.525	1.409	.814	.909	1.895

ALPHA	I	X	Y	Z	F	G	D
220.0	1	-.090	-.804	.471	.321	.000	4.1327773
220.0	2	-.163	-.804	.325	.027	.027	3.8351706
220.0	3	-.153	-.804	.313	.000	.000	3.7698462

PHI	PHIDOT	PHIDDOT
-72.642	.843	-.982

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-1.065	.196	-.334	-.091	.675	-1.089	.424	.804	1.738

ALPHA	I	X	Y	Z	F	G	D
230.0	1	-.153	-.675	.313	.273	.000	4.2880893
230.0	2	-.193	-.675	.186	.018	.018	3.9836445
230.0	3	-.186	-.675	.178	.000	.000	3.9347224

PHI	PHIDOT	PHIDDOT
-80.227	.674	-.958

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-1.076	.325	-.499	-.037	.804	-.806	.215	.675	1.499

ALPHA	I	X	Y	Z	F	G	D
240.0	1	-.186	-.525	.178	.217	.000	4.3689361
240.0	2	-.204	-.525	.075	.011	.011	4.0838769
240.0	3	-.199	-.525	.071	.000	.000	4.0507987

PHI	PHIDOT	PHIDDOT
-86.138	.508	-.949

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-1.079	.475	-.618	-.011	.909	-.565	.182	.525	1.280

ALPHA	I	X	Y	Z	F	G	D
250.0	1	-.199	-.359	.071	.159	.000	4.3747982
250.0	2	-.204	-.359	-.005	.006	.006	4.1386155
250.0	3	-.201	-.359	-.007	.000	.000	4.1198735

PHI	PHIDOT	PHIDDOT
-90.388	.342	-.962

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-1.080	.641	-.698	.001	.987	-.356	.034	.359	1.114

ALPHA	I	X	Y	Z	F	G	D
260.0	1	-.201	-.182	-.007	.095	.000	4.3187840
260.0	2	-.201	-.182	-.053	.002	.002	4.1605573
260.0	3	-.200	-.182	-.054	.000	.000	4.1532056

PHI	PHIDOT	PHIDDOT
-92.958	.172	-.980

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-1.080	.818	-.744	.003	1.034	-.172	-.066	.182	1.010

ALPHA	I	X	Y	Z	F	G	D
270.0	1	-.200	-.000	-.054	.032	.000	4.2201076
270.0	2	-.199	-.000	-.070	.000	.000	4.1635274

PHI	PHIDOT	PHIDDOT
-93.820	.000	-.989

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-1.079	1.000	-.759	.000	1.050	-.000	-.021	.000	.974

ALPHA	I	X	Y	Z	F	G	D
280.0	1	-.199	.182	-.070	-.031	.000	4.0958169
280.0	2	-.200	.182	-.054	.006	.000	4.1540707

PHI	PHIDOT	PHIDDOT
-92.958	-.172	-.980

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-1.080	1.182	-.744	-.003	1.034	.172	-.066	-.182	1.010

ALPHA	I	X	Y	Z	F	G	D
290.0	1	-.200	.359	-.054	-.091	.000	3.9545066
290.0	2	-.202	.359	-.006	.002	.002	4.1274371
290.0	3	-.201	.359	-.007	.006	.000	4.1198378

PHI	PHIDOT	PHIDDOT
-90.388	-.342	-.962

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-1.080	1.359	-.698	-.001	.987	.356	.034	-.359	1.114

ALPHA	I	X	Y	Z	F	G	D
300.0	1	-.201	.525	-.007	-.146	.000	3.7949658
300.0	2	-.202	.525	.073	.006	.006	4.0704438
300.0	3	-.199	.525	.071	.006	.000	4.0507117

PHI	PHIDOT	PHIDDOT
-86.138	-.508	-.949

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-1.079	1.525	-.618	.011	.909	.565	.162	-.525	1.280

ALPHA	I	X	Y	Z	F	G	D
310.0	1	-.199	.675	.071	-.195	.000	3.6107491
310.0	2	-.191	.675	.184	.019	.013	3.9699338
310.0	3	-.186	.675	.178	.006	.000	3.9345772

PHI	PHIDOT	PHIDDOT
-80.227	-.674	-.958

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-1.076	1.675	-.499	.037	.804	.806	.215	-.675	1.499

ALPHA	I	X	Y	Z	F	G	D
320.0	1	-.186	.804	.178	-.238	.000	3.3987434
320.0	2	-.161	.804	.323	.025	.022	3.8226931
320.0	3	-.153	.804	.313	.006	.000	3.7696533

PHI	PHIDOT	PHIDDOT
-72.642	-.843	-.982

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-1.065	1.804	-.334	.091	.675	1.089	.424	-.804	1.738

ALPHA	I	X	Y	Z	F	G	D
330.0	1	-.153	.909	.313	-.274	.000	3.1682627
330.0	2	-.099	.909	.486	.033	.033	3.6427882
330.0	3	-.090	.909	.471	.000	.000	3.5707129

PHI	PHIDOT	PHIDDOT
-63.349	-1.015	-.966

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-1.041	1.909	-.117	.196	.525	1.409	.874	-.909	1.895

ALPHA	I	X	Y	Z	F	G	D
340.0	1	-.090	.987	.471	-.303	.000	2.9473139
340.0	2	.007	.987	.664	.047	.047	3.4632409
340.0	3	.017	.987	.641	.001	.001	3.3718242

PHI	PHIDOT	PHIDDOT
-52.394	-1.171	-.777

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.991	1.987	.158	.388	.359	1.730	1.410	-.987	1.702

ALPHA	I	X	Y	Z	F	G	D
350.0	1	.017	1.034	.641	-.322	.000	2.7823379
350.0	2	.165	1.034	.833	.059	.059	3.3284189
350.0	3	.172	1.034	.803	.001	.001	3.2212135

PHI	PHIDOT	PHIDDOT
-40.135	-1.266	-.244

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.899	2.034	.482	.685	.182	1.961	1.943	-1.034	.810

ALPHA	I	X	Y	Z	F	G	D
360.0	1	.172	1.050	.803	-.326	.000	2.7258090
360.0	2	.364	1.050	.965	.063	.063	3.2754874
360.0	3	.364	1.050	.932	.001	.001	3.1644822
360.0	4	.364	1.050	.931	.000	.000	3.1625353

PHI	PHIDOT	PHIDDOT
-27.499	-1.237	.598

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.749	2.050	.830	1.026	-.000	1.977	1.847	-1.050	-.673

(Example 3)

R1	R2	R3	R4	THETA	GAMA	ADOT	ADDOT
2.000	12.000	7.000	12.000	90.000	90.000	1.000	0.000

UP	VP	WP
3.000	0.000	3.000

ALPHA	I	X	Y	Z	F	G	D
0.0	1	12.000	-.000	7.000	0.000	5.000	-280.0000000
0.0	2	11.750	-.000	7.000	.062	.062	-280.0000000
0.0	3	11.750	-.000	6.996	.006	.000	-279.8214286

PHI	PHIDOT	PHIDDOT
54.341	1.200	-.009

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
8.895	-.000	2.891	-3.469	-0.000	8.274	-8.495	.000	-5.202

ALPHA	I	X	Y	Z	F	G	D
10.0	1	11.750	-.000	6.996	-24.876	.000	-193.4526860
10.0	2	9.951	-.000	9.502	9.520	9.520	-306.8927187
10.0	3	10.081	-.000	8.893	.389	.388	-281.8958364
10.0	4	10.086	-.000	8.866	.001	.001	-280.7875473

PHI	PHIDOT	PHIDDOT
42.366	1.193	-.069

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
8.169	-.000	4.249	-4.795	.000	7.247	-6.646	.000	-6.448

ALPHA	I	X	Y	Z	F	G	D
20.0	1	10.086	-.000	8.866	-24.907	.000	-196.2325309
20.0	2	7.836	-.000	10.919	9.279	9.279	-309.3493186
20.0	3	8.082	-.000	10.362	.370	.370	-284.6574553
20.0	4	8.093	-.000	10.338	.001	.001	-283.5865398

PHI	PHIDOT	PHIDDOT
30.512	1.176	-.129

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
7.240	-.000	5.412	-5.793	.000	6.063	-4.779	.000	-7.014

ALPHA	I	X	Y	Z	F	G	D
30.0	1	8.093	-.000	10.338	-24.890	.000	-200.8279198
30.0	2	5.530	-.000	11.849	8.849	8.848	-313.0267295
30.0	3	5.869	-.000	11.374	.340	.340	-288.9606499
30.0	4	5.883	-.000	11.354	.001	.001	-287.9584786

PHI	PHIDOT	PHIDDOT
18.881	1.149	-.180

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
6.165	-.000	6.362	-6.477	.000	4.832	-3.111	.000	-7.017

ALPHA	I	X	Y	Z	F	G	D
40.0	1	5.883	-.000	11.354	-24.793	.000	-206.8512915
40.0	2	3.161	-.000	12.285	8.275	8.275	-317.6117426
40.0	3	3.563	-.000	11.910	.302	.302	-294.4283463
40.0	4	3.579	-.000	[illegible]	.006	.000	-293.5156117

PHI	PHIDOT	PHIDDOT
7.562	1.114	-.216

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
4.994	-.000	7.100	-6.901	.000	3.633	-1.825	.000	-6.680

ALPHA	I	X	Y	Z	F	G	D
50.0	1	3.579	-.000	11.896	-24.601	.000	-213.7925795
50.0	2	.842	-.000	12.259	7.627	7.627	-322.7637050
50.0	3	1.276	-.000	11.989	.262	.262	-300.6180794
50.0	4	1.292	-.000	11.979	.000	.000	-299.8023787

PHI	PHIDOT	PHIDDOT
-3.382	1.074	-.235

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
3.767	-.000	7.635	-7.137	.000	2.506	-.934	.000	-6.243

ALPHA	I	X	Y	Z	F	G	D
60.0	1	1.292	-.000	11.979	-24.317	.000	-221.0966457
60.0	2	-1.343	-.000	11.823	6.967	6.967	-328.1392049
60.0	3	-.902	-.000	11.653	.224	.224	-307.0759830
60.0	4	-.887	-.000	11.648	.000	.000	-306.3535896

PHI	PHIDOT	PHIDDOT
-13.919	1.033	-.239

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
2.511	-.000	7.979	-7.245	.000	1.449	-.343	.000	-5.886

ALPHA	I	X	Y	Z	F	G	D
70.0	1	-.887	-.000	11.648	-23.954	.000	-228.2349197
70.0	2	-3.331	-.000	11.042	6.345	6.345	-333.4158220
70.0	3	-2.902	-.000	10.962	.191	.191	-313.3833509
70.0	4	-2.889	-.000	10.959	.000	.000	-312.7430809

PHI	PHIDOT	PHIDDOT
-24.040	.992	-.228

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
1.243	-.000	8.143	-7.265	.000	.441	.098	.000	-5.695

ALPHA	I	X	Y	Z	F	G	D
80.0	1	-2.889	-.000	10.959	-23.539	.000	-234.7588831
80.0	2	-5.086	-.000	9.979	5.791	5.791	-338.3161848
80.0	3	-4.682	-.000	9.976	.164	.164	-319.1915044
80.0	4	-4.670	-.000	9.976	.000	.000	-318.6185672

PHI	PHIDOT	PHIDDOT
-33.766	.954	-.207

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.021	-.000	8.134	-7.211	.000	-.548	.558	.000	-5.669

ALPHA	I	X	Y	Z	F	G	D
90.0	1	-4.670	.000	9.976	-23.098	.000	-240.3320755
90.0	2	-6.587	.000	8.694	5.320	5.320	-342.6291910
90.0	3	-6.214	.000	8.756	.143	.143	-324.2437719
90.0	4	-6.204	.000	8.758	.000	.000	-323.7225227

PHI	PHIDOT	PHIDDOT
-43.130	.920	-.180

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-1.269	.000	7.952	-7.075	.000	-1.544	1.051	-.000	-5.750

ALPHA	I	X	Y	Z	F	G	D
100.0	1	-6.204	.000	8.758	-22.662	.000	-244.7432660
100.0	2	-7.826	.000	7.238	4.938	4.938	-346.2244515
100.0	3	-7.489	.000	7.355	.127	.127	-328.3854071
100.0	4	-7.479	.000	7.358	.006	.000	-327.9008675

PHI	PHIDOT	PHIDDOT
-52.180	.891	-.148

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-2.485	.000	7.594	-6.836	.000	-2.557	1.722	-.000	-5.854

ALPHA	I	X	Y	Z	F	G	D
110.0	1	-7.479	.000	7.358	-22.256	.000	-247.9054799
110.0	2	-8.801	.000	5.656	4.643	4.643	-349.0575502
110.0	3	-8.501	.000	5.818	.116	.116	-331.5631106
110.0	4	-8.493	.000	5.823	.006	.000	-331.1015764

PHI	PHIDOT	PHIDDOT
-60.973	.868	-.115

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-3.648	.000	7.058	-6.465	.000	-3.584	2.564	-.000	-5.891

ALPHA	I	X	Y	Z	F	G	D
120.0	1	-8.493	.000	5.823	-21.905	.000	-249.8450921
120.0	2	-9.514	.000	3.983	4.428	4.428	-351.1655223
120.0	3	-9.252	.000	4.185	.109	.109	-333.8158349
120.0	4	-9.245	.000	4.190	.006	.000	-333.3649701

PHI	PHIDOT	PHIDDOT
-69.565	.851	-.081

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-4.732	.000	6.343	-5.936	.000	-4.605	3.522	-.000	-5.783

ALPHA	I	X	Y	Z	F	G	D
130.0	1	-9.245	.000	4.190	-21.627	.000	-250.6838837
130.0	2	-9.968	.000	2.250	4.287	4.287	-352.6531882
130.0	3	-9.744	.000	2.486	.106	.106	-335.2582931
130.0	4	-9.738	.000	2.492	.006	.000	-334.8070255

PHI	PHIDOT	PHIDDOT
-78.015	.840	-.048

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-5.710	.000	5.453	-5.230	.000	-5.590	4.579	-.000	-5.469

ALPHA	I	X	Y	Z	F	G	D
140.0	1	-9.738	.000	2.492	-21.437	.000	-250.6158339
140.0	2	-10.165	.000	.484	4.214	4.214	-353.6717722
140.0	3	-9.981	.000	.750	.105	.105	-336.0584846
140.0	4	-9.976	.000	.757	.006	.000	-335.5969163

PHI	PHIDOT	PHIDDOT
-86.381	.834	-.018

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-6.547	.000	4.397	-4.337	.000	-6.499	5.640	-.000	-4.906

ALPHA	I	X	Y	Z	F	G	D
150.0	1	-9.976	.000	.757	-21.342	.000	-249.8799104
150.0	2	-10.105	.000	-1.288	4.202	4.202	-354.3919738
150.0	3	-9.963	.000	-.994	.107	.107	-336.4108215
150.0	4	-9.959	.000	-.987	.000	.000	-335.9302754

PHI	PHIDOT	PHIDDOT
-94.716	.834	.011

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-7.213	.000	3.191	-3.257	.000	-7.287	6.694	-.000	-4.076

ALPHA	I	X	Y	Z	F	G	D
160.0	1	-9.959	.000	-.987	-21.349	.000	-248.7302454
160.0	2	-9.790	.000	-3.040	4.243	4.243	-354.9745462
160.0	3	-9.692	.000	-2.722	.110	.110	-336.5070633
160.0	4	-9.689	.000	-2.713	.000	.000	-336.0003162

PHI	PHIDOT	PHIDDOT
-103.069	.838	.035

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-7.674	.000	1.863	-2.007	.000	-7.907	7.602	-.000	-2.983

ALPHA	I	X	Y	Z	F	G	D
170.0	1	-9.689	.000	-2.713	-21.454	.000	-247.4056776
170.0	2	-9.219	.000	-4.741	4.332	4.332	-355.5423281
170.0	3	-9.168	.000	-4.404	.116	.116	-336.5080646
170.0	4	-9.166	.000	-4.395	.000	.000	-335.9697370

PHI	PHIDOT	PHIDDOT
-111.483	.846	.056

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-7.905	.000	.444	-.615	.000	-8.314	8.312	-.000	-1.649

ALPHA	I	X	Y	Z	F	G	D
180.0	1	-9.166	.000	-4.395	-21.657	.000	-246.1014156
180.0	2	-8.393	.000	-6.360	4.461	4.461	-356.1581375
180.0	3	-8.393	.000	-6.009	.123	.123	-336.5199243
180.0	4	-8.393	.000	-5.999	.000	.000	-335.9469130

PHI	PHIDOT	PHIDDOT
-119.995	.857	.074

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-7.883	.000	-1.025	.879	-.000	-8.471	8.768	-.000	-.120

ALPHA	I	X	Y	Z	F	G	D
190.0	1	-8.393	.000	-5.999	-21.947	.000	-244.9461211
190.0	2	-7.318	.000	-7.861	4.622	4.622	-356.8124298
190.0	3	-7.372	.000	-7.503	.131	.131	-336.5780791
190.0	4	-7.374	.000	-7.492	.000	.000	-335.9698563

PHI	PHIDOT	PHIDDOT
-128.635	.871	.087

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-7.595	.000	-2.497	2.423	-.000	-8.349	8.887	-.000	1.542

ALPHA	I	X	Y	Z	F	G	D
200.0	1	-7.374	.000	-7.492	-22.291	.000	-243.9876286
200.0	2	-6.005	.000	-9.205	4.808	4.808	-357.4229582
200.0	3	-6.115	.000	-8.848	.140	.140	-336.6419549
200.0	4	-6.119	.000	-8.837	.000	.000	-336.0005463

PHI	PHIDOT	PHIDDOT
-137.426	.887	.097

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-7.038	.000	-3.922	3.960	-.000	-7.929	8.665	-.000	3.264

ALPHA	I	X	Y	Z	F	G	D
210.0	1	-6.119	.000	-8.837	-22.687	.000	-243.1895950
210.0	2	-4.470	.000	-10.352	5.013	5.013	-357.8462196
210.0	3	-4.638	.000	-10.004	.149	.149	-336.6010631
210.0	4	-4.643	.000	-9.993	.000	.000	-335.9306007

PHI	PHIDOT	PHIDDOT
-146.386	.905	.104

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-6.217	.000	-5.248	5.427	-.000	-7.211	8.076	-.000	4.962

ALPHA	I	X	Y	Z	F	G	D
220.0	1	-4.643	.000	-9.993	-23.105	.000	-242.4397142
220.0	2	-2.738	.000	-11.260	5.232	5.232	-357.8981600
220.0	3	-2.964	.000	-10.932	.158	.158	-336.2914329
220.0	4	-2.971	.000	-10.922	.000	.000	-335.5973167

PHI	PHIDOT	PHIDDOT
-155.527	.924	.109

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-5.151	.000	-6.422	6.758	-.000	-6.203	7.126	-.000	6.555

ALPHA	I	X	Y	Z	F	G	D
230.0	1	-2.971	.000	-10.922	-23.520	.000	-241.5682309
230.0	2	-.844	.000	-11.890	5.460	5.460	-357.3802968
230.0	3	-1.125	.000	-11.593	.167	.167	-335.5196403
230.0	4	-1.134	.000	-11.583	.000	.000	-334.8074788

PHI	PHIDOT	PHIDDOT
-164.859	.943	.113

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-3.869	.000	-7.398	7.894	-.000	-4.933	5.839	-.000	7.962

ALPHA	I	X	Y	Z	F	G	D
240.0	1	-1.134	.000	-11.583	-23.907	.000	-240.3739367
240.0	2	1.170	.000	-12.207	5.698	5.698	-356.1074038
240.0	3	.837	.000	-11.951	.176	.176	-334.0908958
240.0	4	.826	.000	-11.942	.000	.000	-333.3654545

PHI	PHIDOT	PHIDDOT
-174.387	.963	.116

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-2.410	.000	-8.131	8.779	-.000	-3.439	4.255	-.000	9.112

ALPHA	I	X	Y	Z	F	G	D
250.0	1	.826	.000	-11.942	-24.247	.000	-238.6540642
250.0	2	3.253	.000	-12.181	5.946	5.946	-353.9337726
250.0	3	2.874	.000	-11.976	.186	.186	-331.8377256
250.0	4	2.861	.000	-11.969	.000	.000	-331.1020720

PHI	PHIDOT	PHIDDOT
175.884	.983	.119

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
-.822	.000	-8.588	9.365	-.000	-1.771	2.428	-.000	9.941

ALPHA	I	X	Y	Z	F	G	D
260.0	1	2.861	.000	-11.969	-24.522	.000	-236.2345687
260.0	2	5.346	.000	-11.790	6.207	6.207	-350.7761297
260.0	3	4.928	.000	-11.646	.196	.196	-328.6464183
260.0	4	4.914	.000	-11.641	.000	.000	-327.9013559

PHI	PHIDOT	PHIDDOT
165.946	1.004	.123

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
.839	.000	-8.743	9.616	-.000	.009	.422	-.000	10.397

ALPHA	I	X	Y	Z	F	G	D
270.0	1	4.914	.000	-11.641	-24.724	.000	-232.9979289
270.0	2	7.384	.000	-11.022	6.486	6.486	-346.6320180
270.0	3	6.935	.000	-10.948	.207	.207	-324.4791944
270.0	4	6.920	.000	-10.945	.000	.000	-323.7229863

PHI	PHIDOT	PHIDDOT
155.795	1.026	.127

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
2.514	.000	-8.582	9.507	-.000	1.834	-1.698	-.000	10.439

ALPHA	I	X	Y	Z	F	G	D
280.0	1	6.920	-.000	-10.945	-24.849	.000	-228.9064051
280.0	2	9.296	-.000	-9.877	6.789	6.788	-341.5926469
280.0	3	8.827	-.000	-9.880	.221	.221	-319.3906565
280.0	4	8.810	-.000	-9.880	.000	.000	-318.6189865

PHI	PHIDOT	PHIDDOT
145.422	1.049	.132

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
4.136	-.000	-8.105	9.026	-.000	3.627	-3.819	.000	10.036

ALPHA	I	X	Y	Z	F	G	D
290.0	1	8.810	-.000	-9.880	-24.904	.000	-224.0192984
290.0	2	11.007	-.000	-8.366	7.118	7.118	-335.8489292
290.0	3	10.529	-.000	-8.455	.236	.236	-313.5372926
290.0	4	10.512	-.000	-8.458	.000	.000	-312.7434331

PHI	PHIDOT	PHIDDOT
134.819	1.072	.137

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
5.643	-.000	-7.322	8.178	-.000	5.310	-5.875	.000	9.170

ALPHA	I	X	Y	Z	F	G	D
300.0	1	10.512	-.000	-8.458	-24.902	.000	-218.5029470
300.0	2	12.440	-.000	-6.518	7.481	7.481	-329.6887974
300.0	3	11.968	-.000	-6.700	.255	.255	-307.1786253
300.0	4	11.951	-.000	-6.706	.000	.000	-306.3536481

PHI	PHIDOT	PHIDDOT
123.978	1.096	.140

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
6.971	-.000	-6.262	6.986	-.000	6.801	-7.749	.000	7.838

ALPHA	I	X	Y	Z	F	G	D
310.0	1	11.951	-.000	-6.706	-24.863	.000	-212.6319164
310.0	2	13.519	-.000	-4.379	7.875	7.875	-323.4832106
310.0	3	13.072	-.000	-4.658	.278	.278	-300.6681368
310.0	4	13.055	-.000	-4.668	.000	.000	-299.8025187

PHI	PHIDOT	PHIDDOT
112.893	1.121	.140

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
8.064	-.000	-4.964	5.491	-.000	8.020	-9.314	.000	6.062

ALPHA	I	X	Y	Z	F	G	D
320.0	1	13.055	-.000	-4.668	-24.809	.000	-206.7792155
320.0	2	14.175	-.000	-2.015	8.291	8.291	-317.6581010
320.0	3	13.772	-.000	-2.391	.303	.303	-294.4317330
320.0	4	13.756	-.000	-2.406	.000	.000	-293.5156222

PHI	PHIDOT	PHIDDOT
101.565	1.145	.133

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
8.874	-.000	-3.483	3.760	-.000	8.895	-10.433	.000	3.905

ALPHA	I	X	Y	Z	F	G	D
330.0	1	13.756	-.000	-2.406	-24.765	.000	-201.3927871
330.0	2	14.348	-.000	.486	8.710	8.710	-312.6509297
330.0	3	14.014	-.000	.018	.330	.330	-288.9319489
330.0	4	14.000	-.000	-.001	.001	.001	-287.9583804

PHI	PHIDOT	PHIDDOT
90.006	1.167	.117

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
9.368	-.000	-1.883	1.883	-.000	9.368	-10.949	.000	1.485

ALPHA	I	X	Y	Z	F	G	D
340.0	1	14.000	-.000	-.001	-24.747	.000	-196.9557805
340.0	2	14.000	-.000	3.014	9.094	9.094	-308.8547921
340.0	3	13.759	-.000	2.468	.357	.357	-284.6183155
340.0	4	13.748	-.000	[illegible]	.001	.001	-283.5863953

PHI	PHIDOT	PHIDDOT
78.245	1.185	.088

XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
9.529	-.000	-.238	-.029	-.000	9.409	-10.817	.000	-1.008

ALPHA	I	X	Y	Z	F	G	D
350.0	1	13.748	-.000	2.445	-24.765	.000	-193.9303675
350.0	2	13.124	-.000	5.445	9.393	9.393	-306.5601999
350.0	3	12.996	-.000	4.844	.378	.378	-281.8689032
350.0	4	12.991	-.000	4.817	.001	.001	-280.7874427

PHI	PHIDOT	PHIDDOT
66.332	1.197	.046

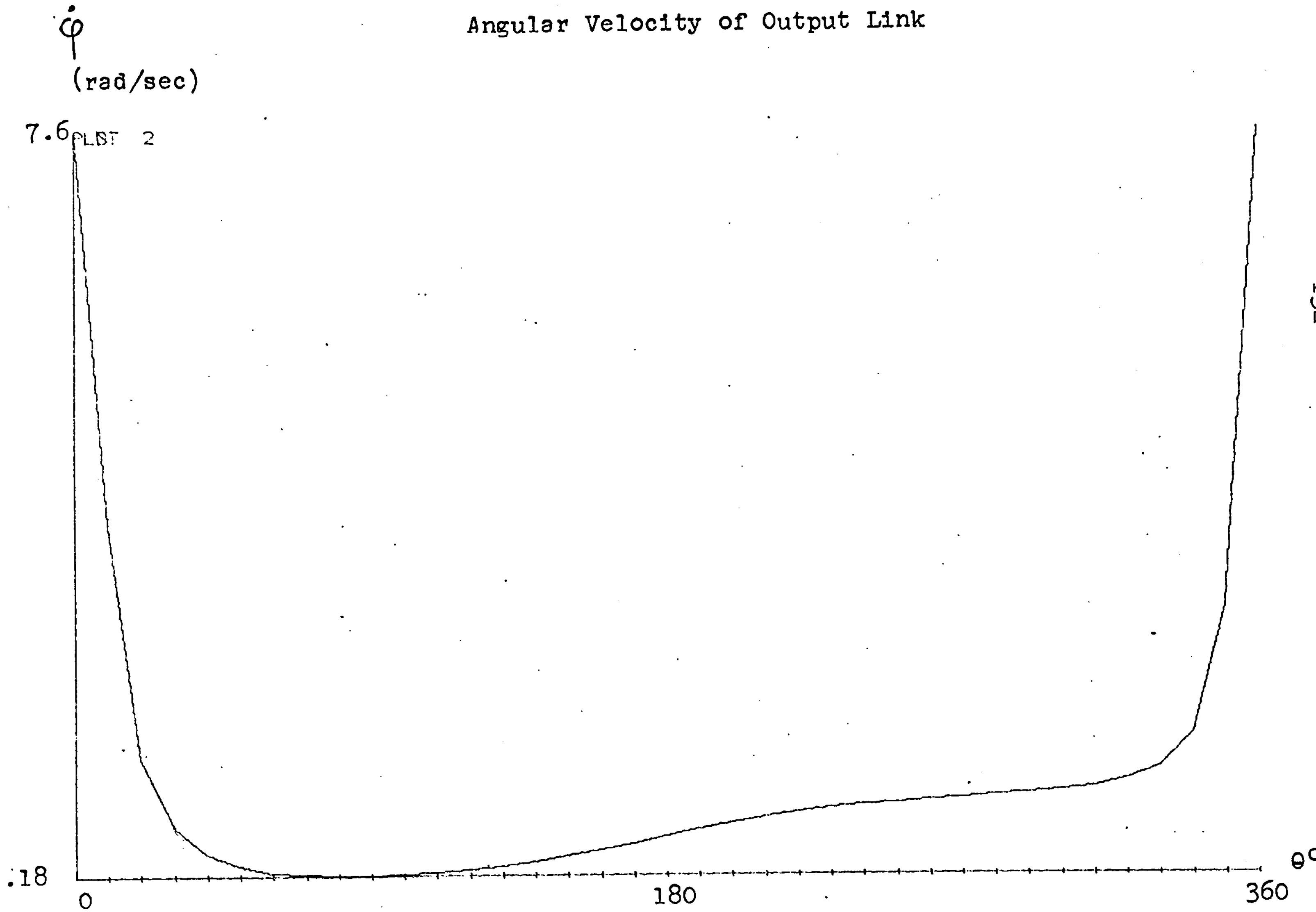
XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
9.363	-.000	1.376	-1.851	-.000	9.026	-9.958	.000	-3.325

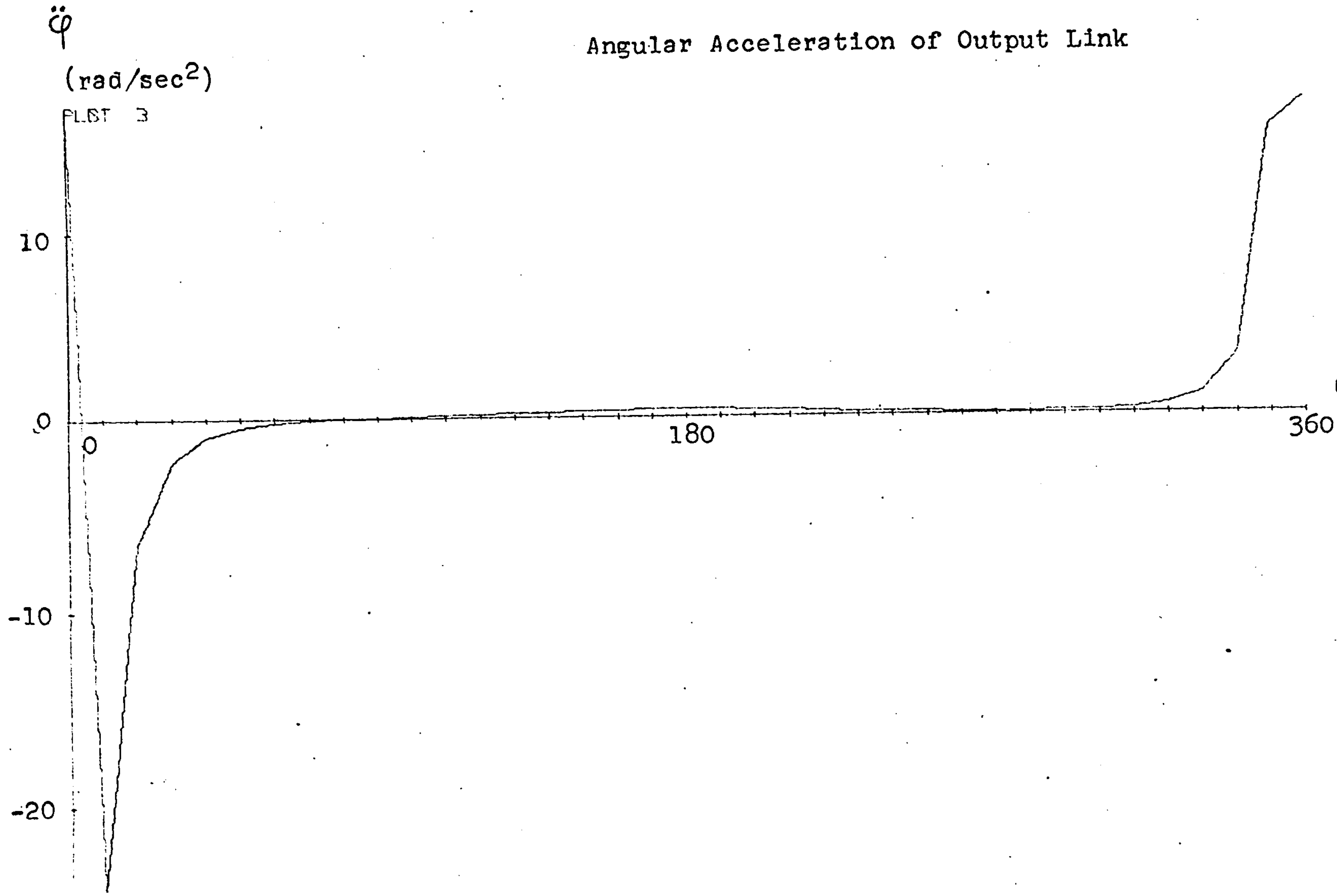
ALPHA	I	X	Y	Z	F	G	D
360.0	1	12.991	-.000	4.817	-24.813	.000	-192.6900410
360.0	2	11.750	-.000	7.648	9.551	9.551	-305.9113432
360.0	3	11.750	-.000	7.023	.390	.390	-280.9339270
360.0	4	11.750	-.000	6.996	.001	.001	-279.8235746

PHI	PHIDOT	PHIDDOT
54.341	1.200	-.009

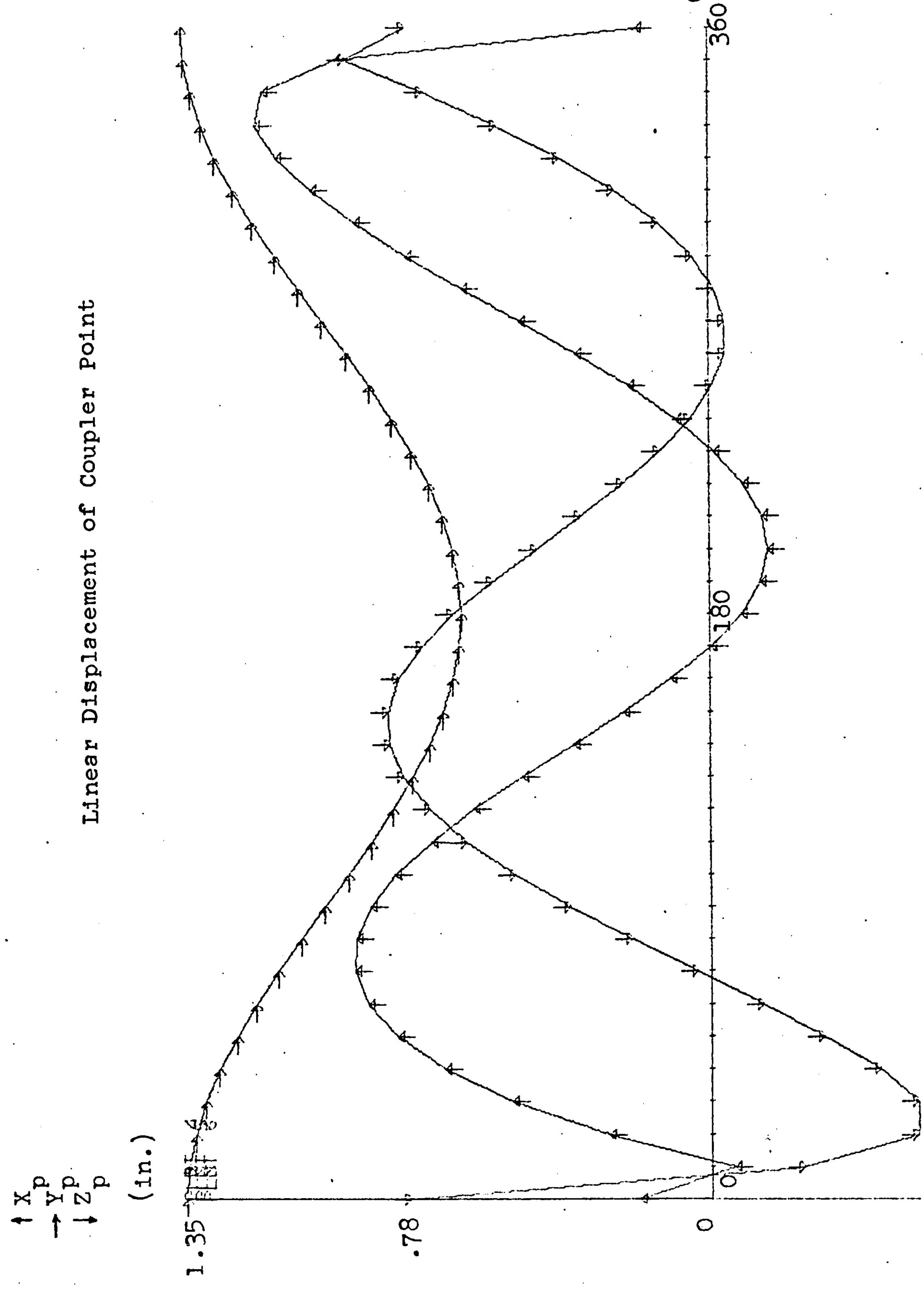
XP	YP	ZP	XPDOT	YPDOT	ZPDOT	XPDDOT	YPDDOT	ZPDDOT
8.895	-.000	2.891	-3.469	.000	8.274	-8.405	.000	-5.202

Angular Velocity of Output Link

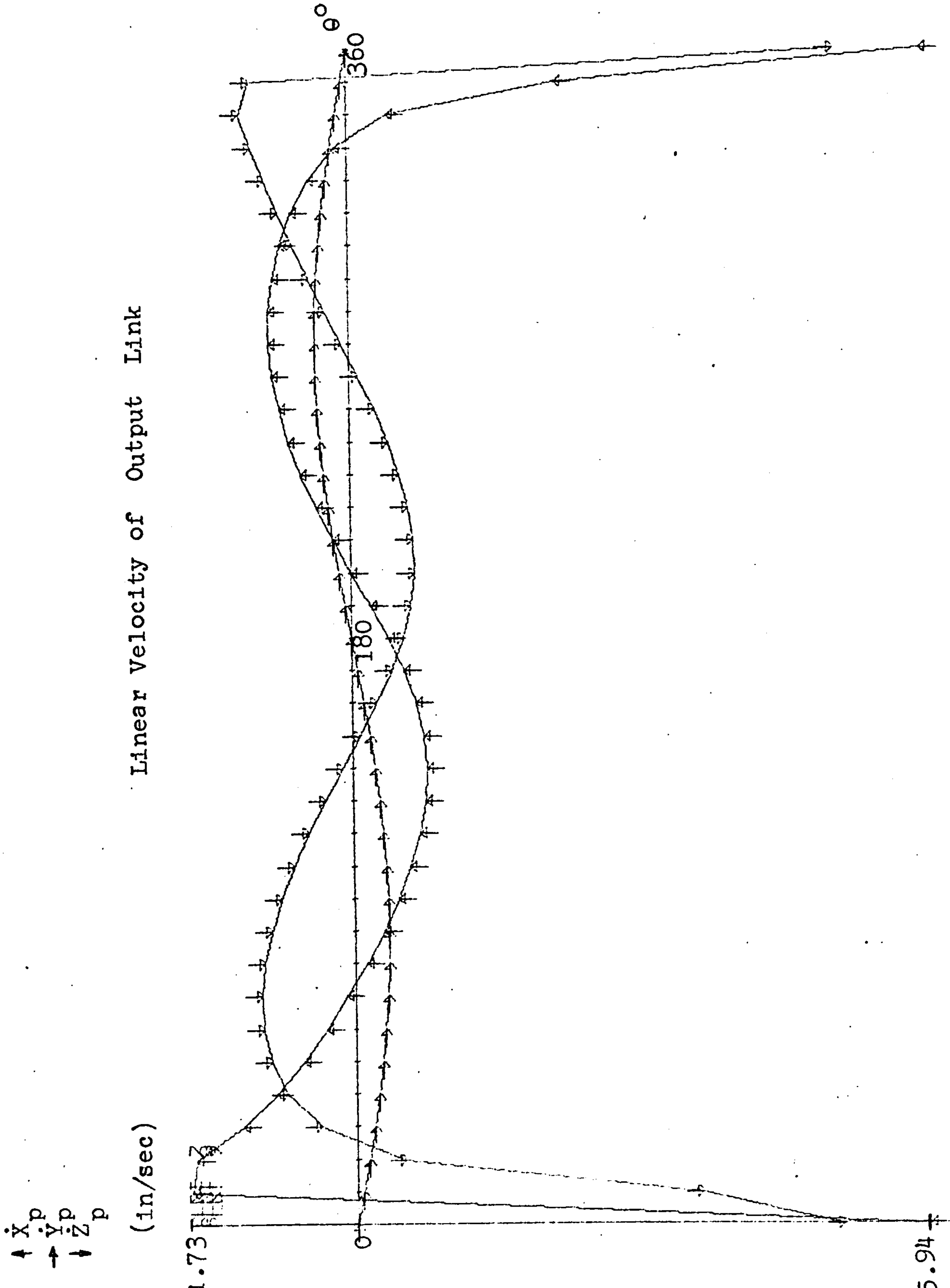




$\ddot{\varphi}$
(rad/sec^2)
PLOT 3
Angular Acceleration of Output Link
10
0
-10
-20
0
180
360
θ°

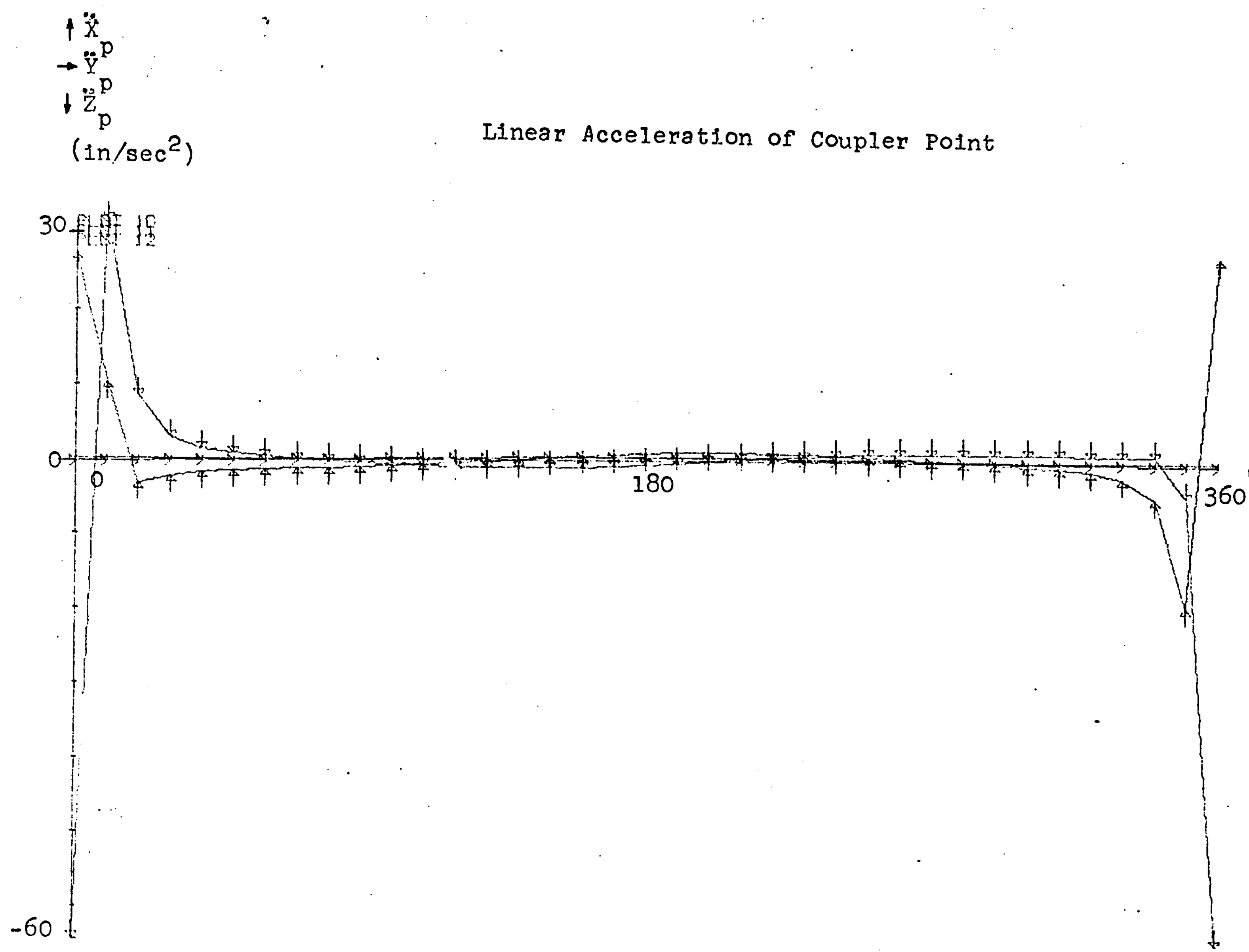


Linear Displacement of Coupler Point



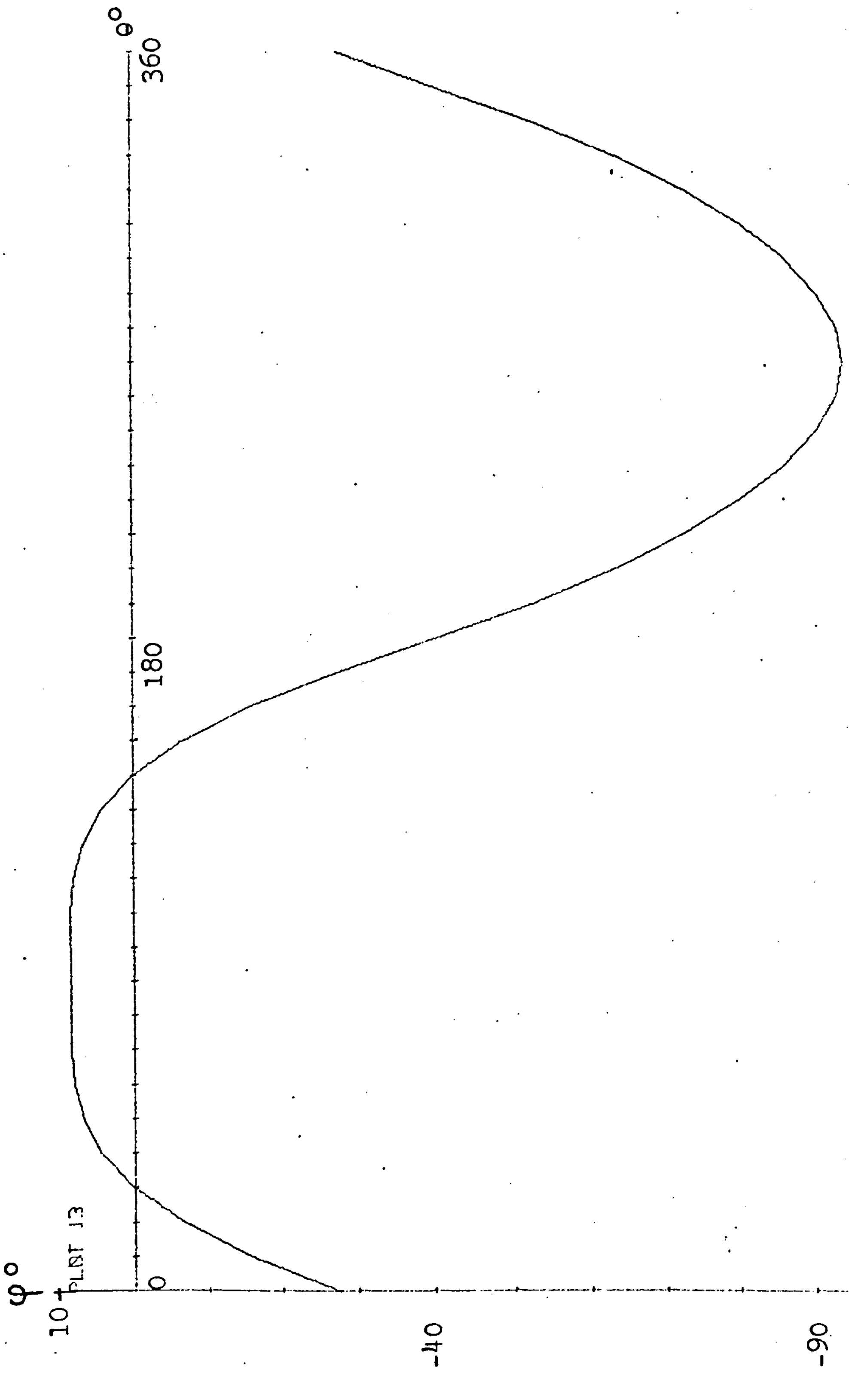
Linear Velocity of Output Link

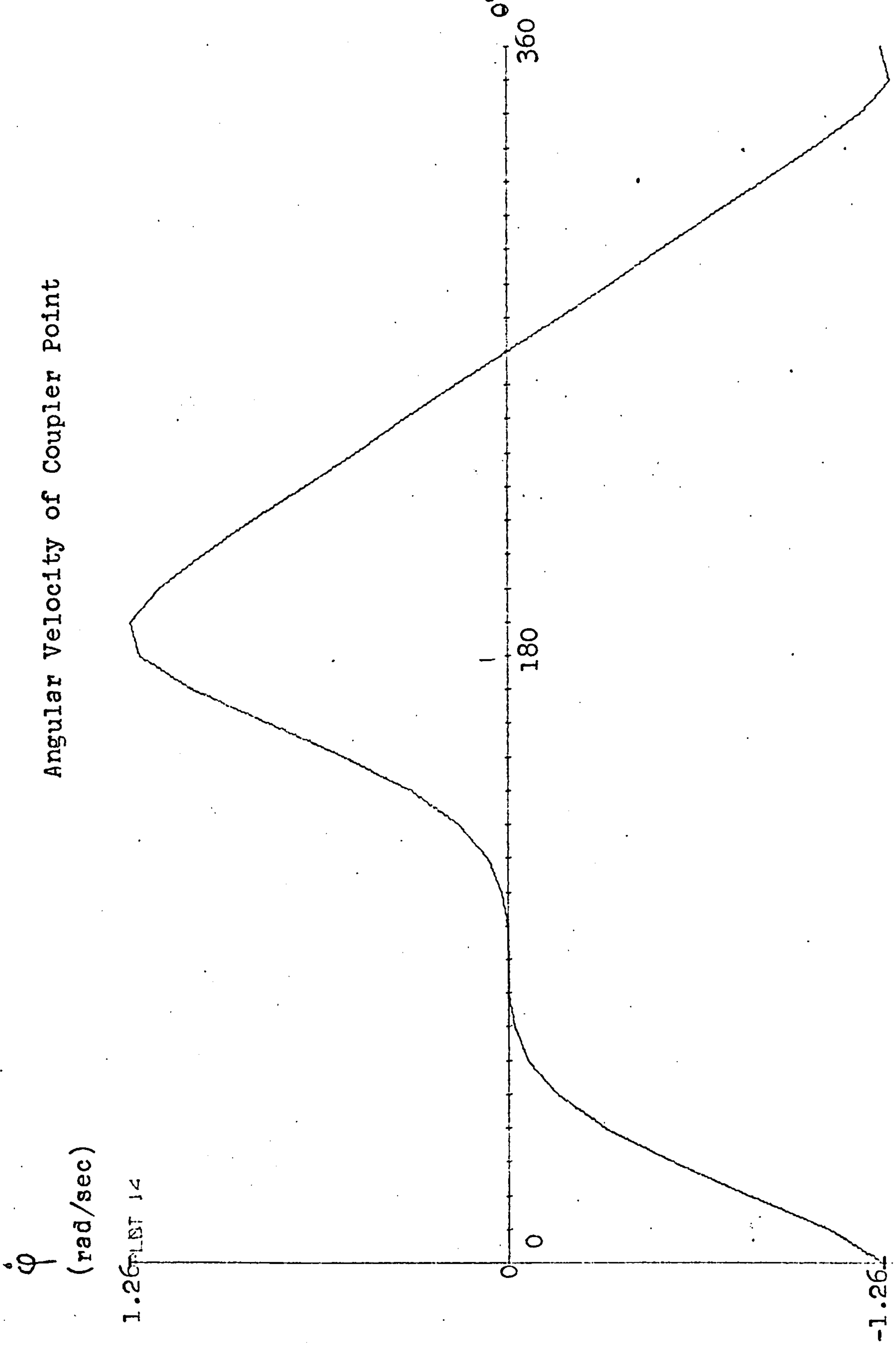
Linear Acceleration of Coupler Point



(Example 2)

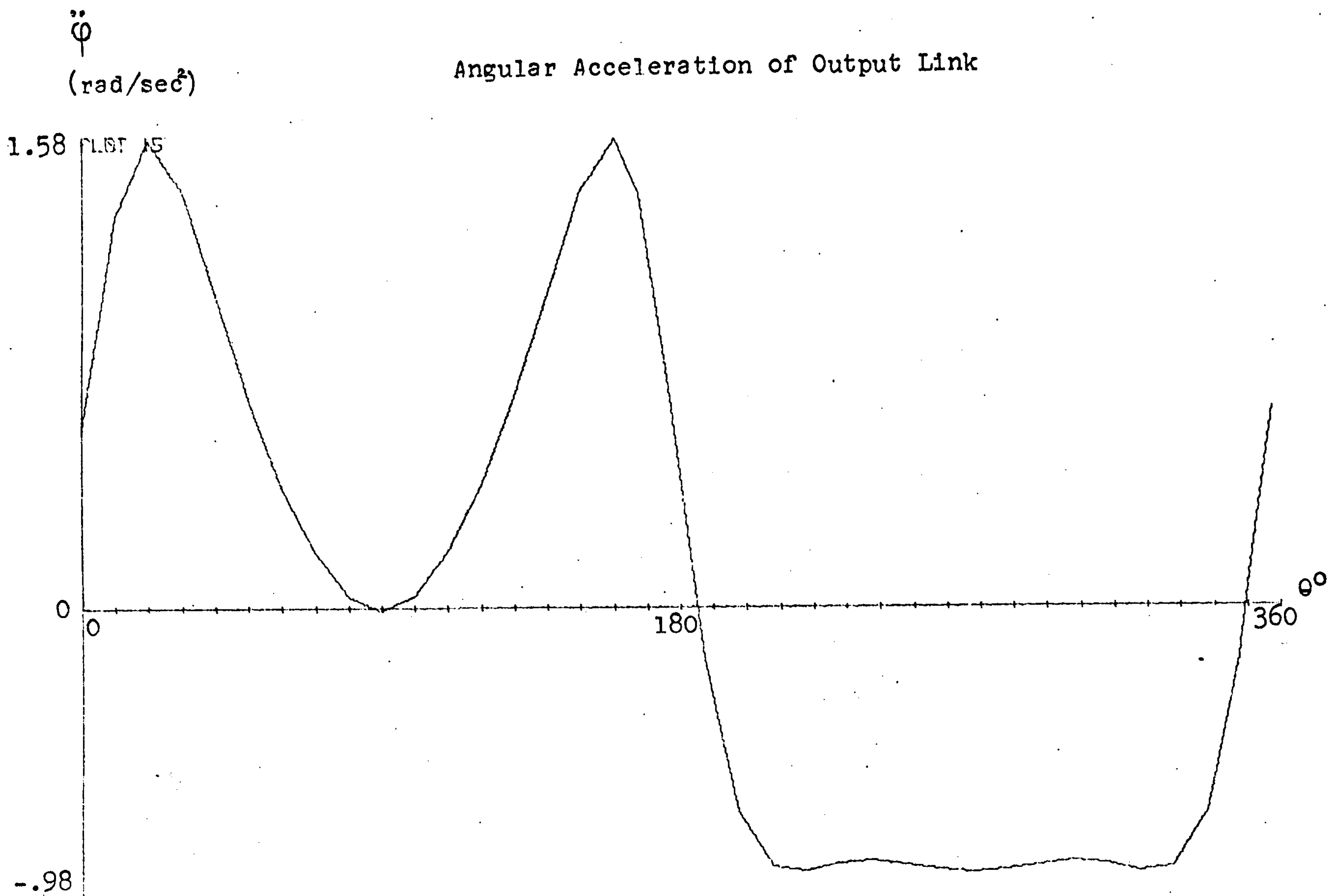
Angular Displacement of Coupler Point



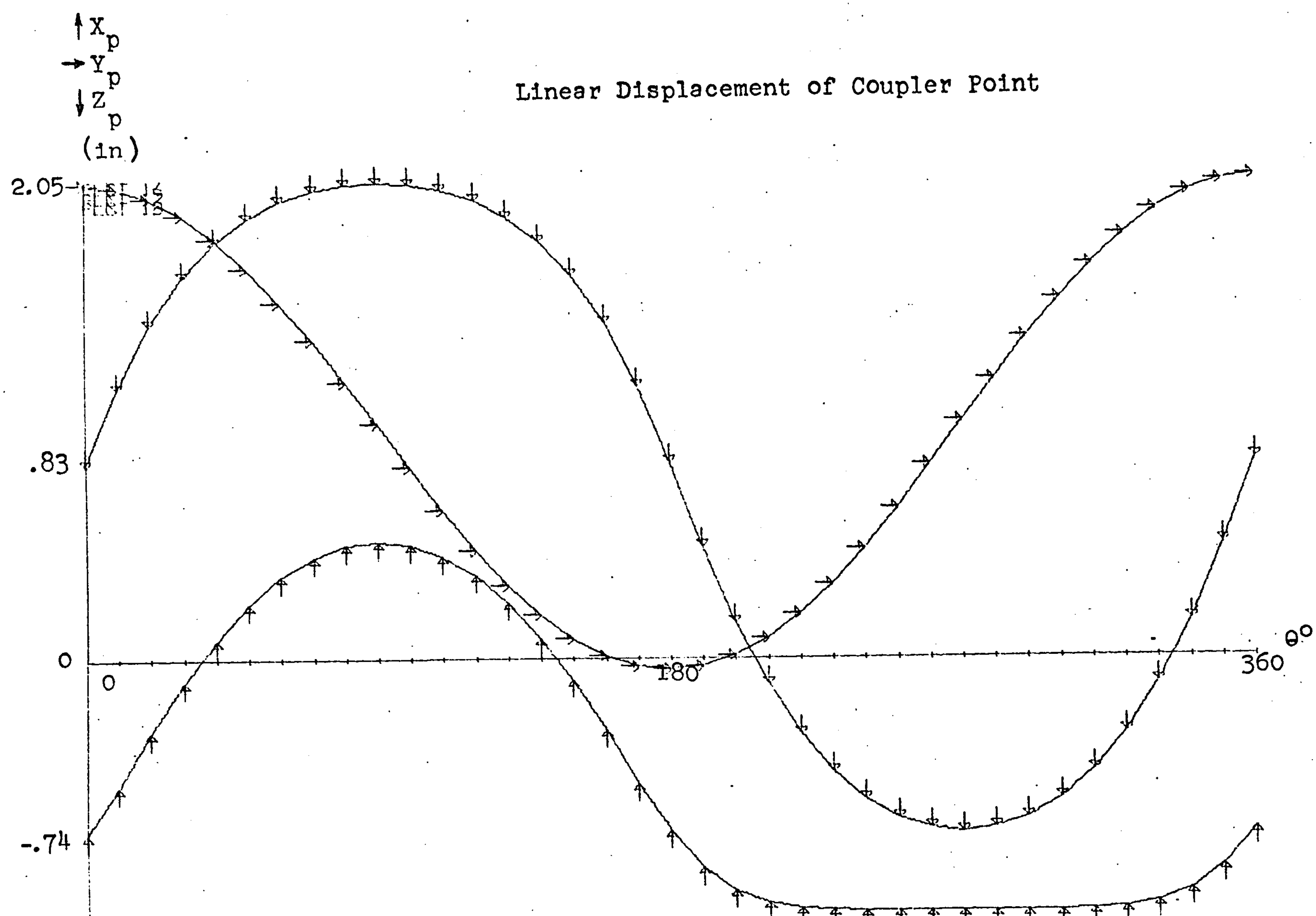


Angular Velocity of Coupler Point

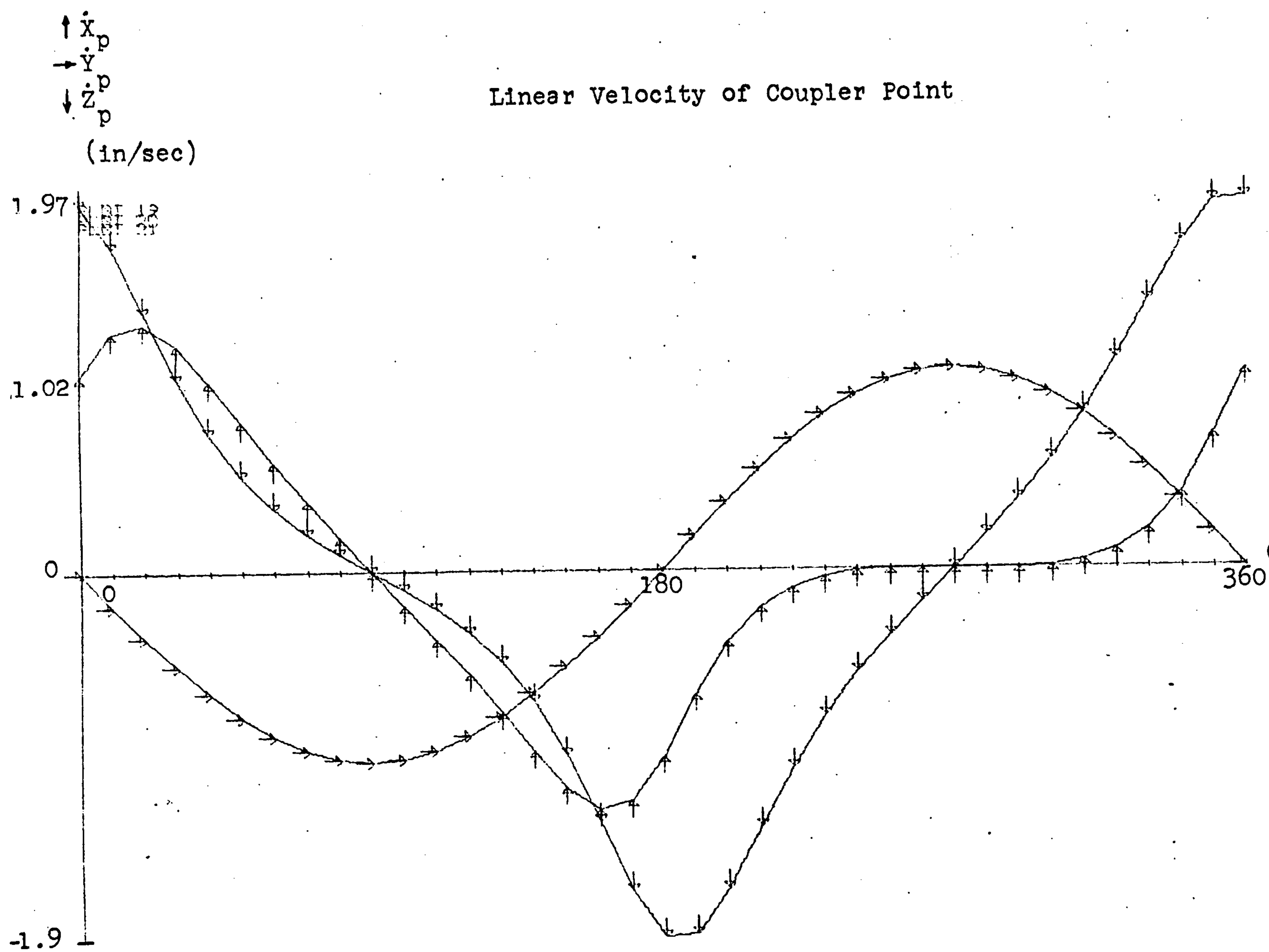
Angular Acceleration of Output Link



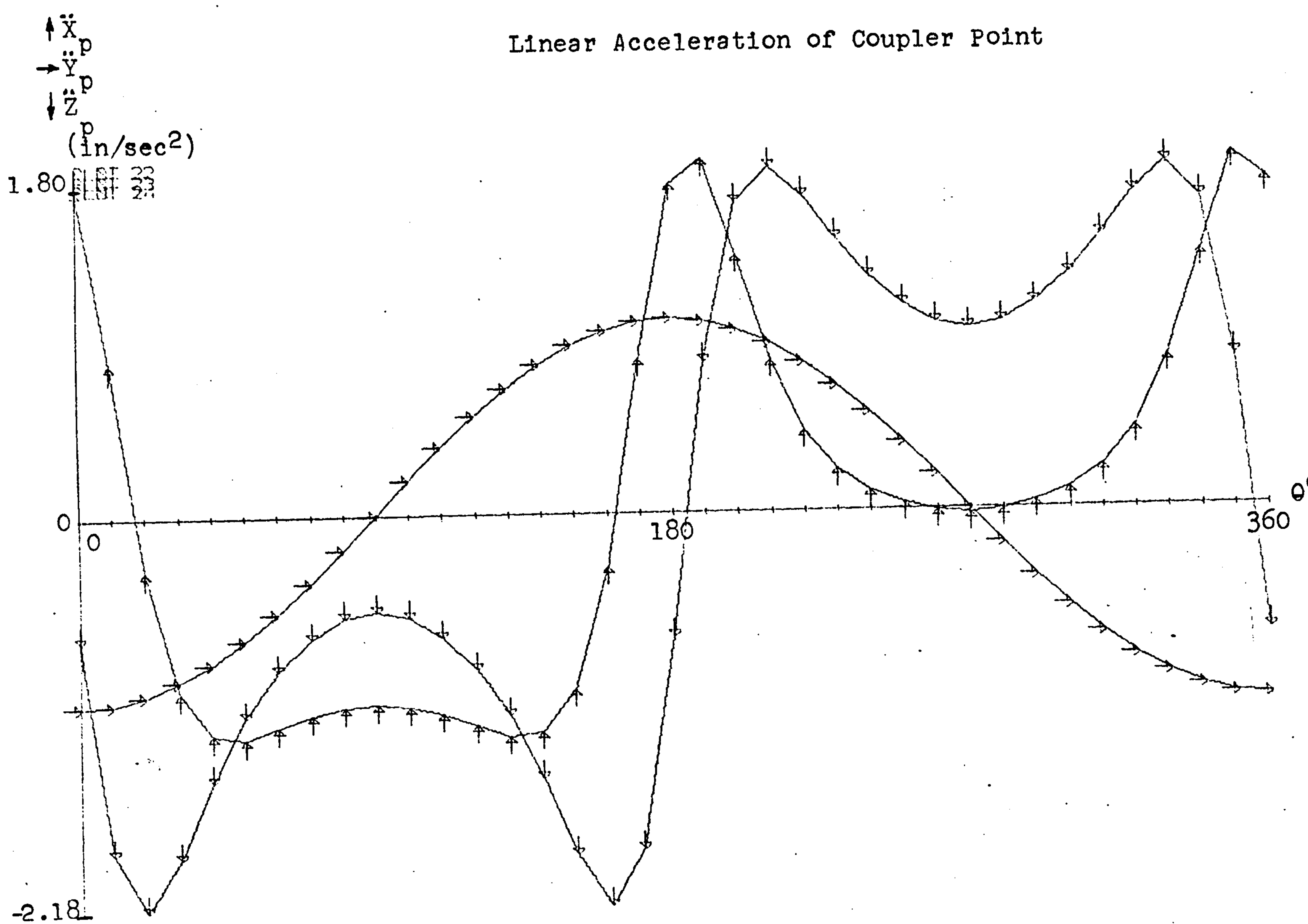
Linear Displacement of Coupler Point



Linear Velocity of Coupler Point

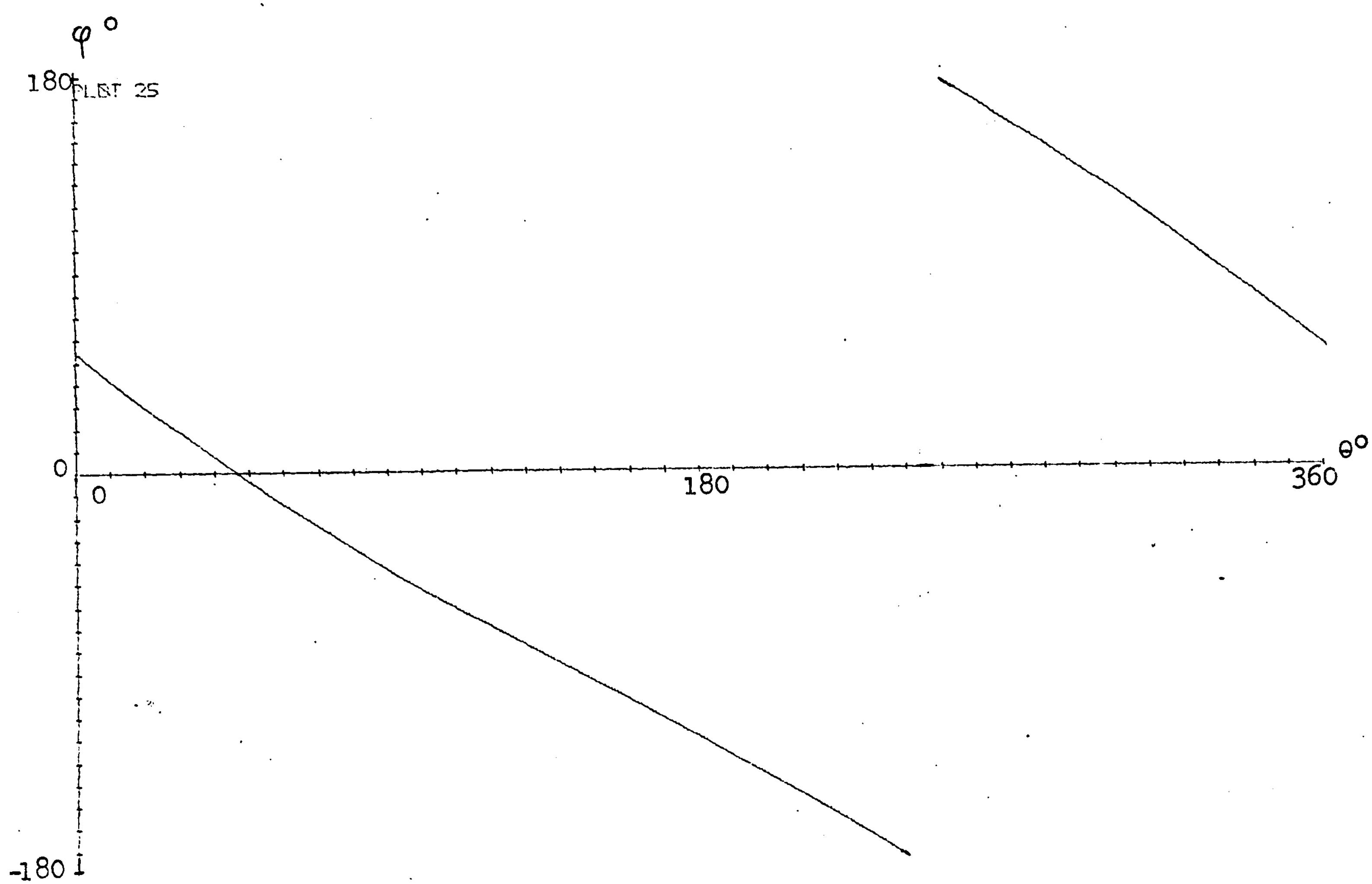


Linear Acceleration of Coupler Point

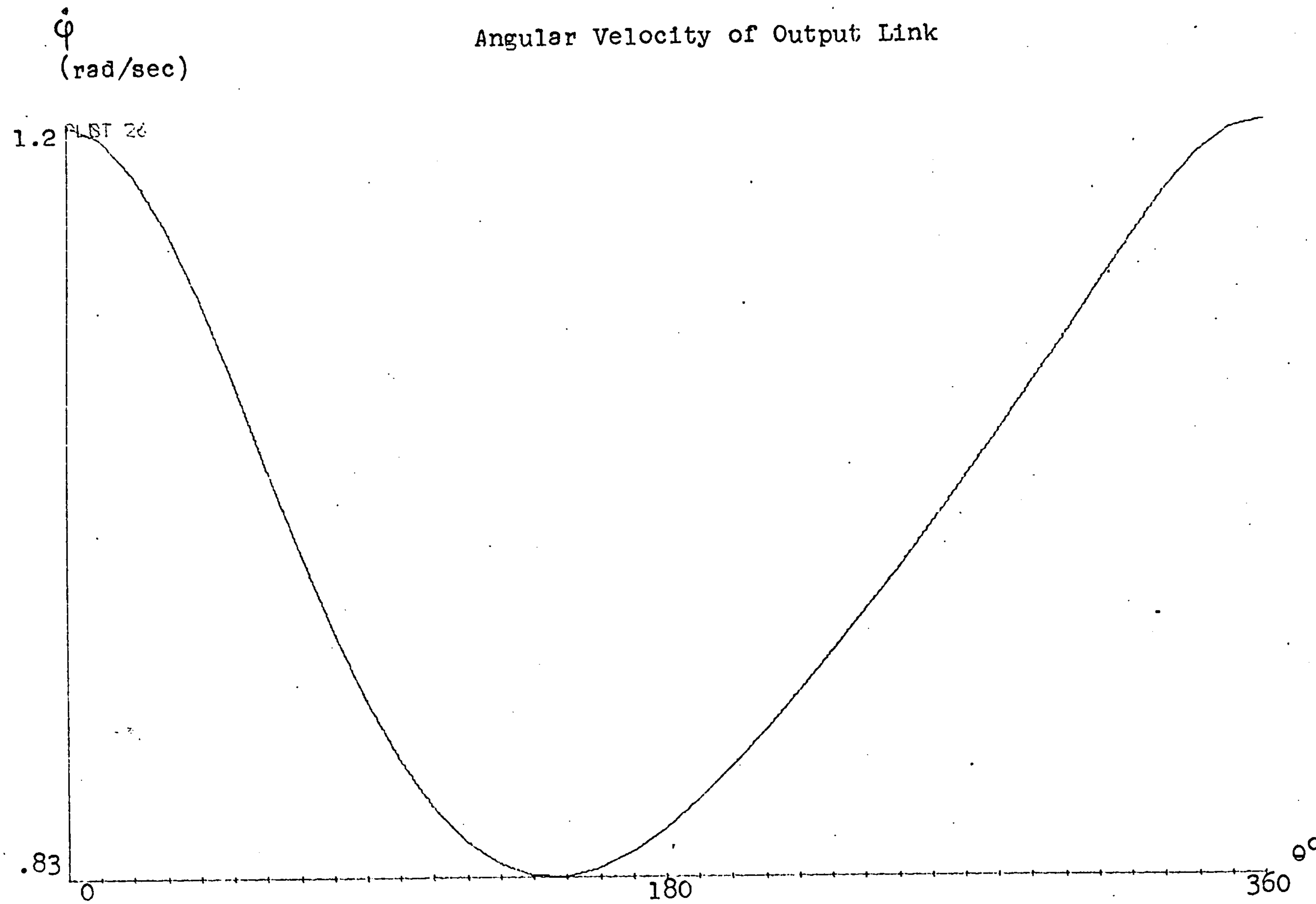


(Example 3)

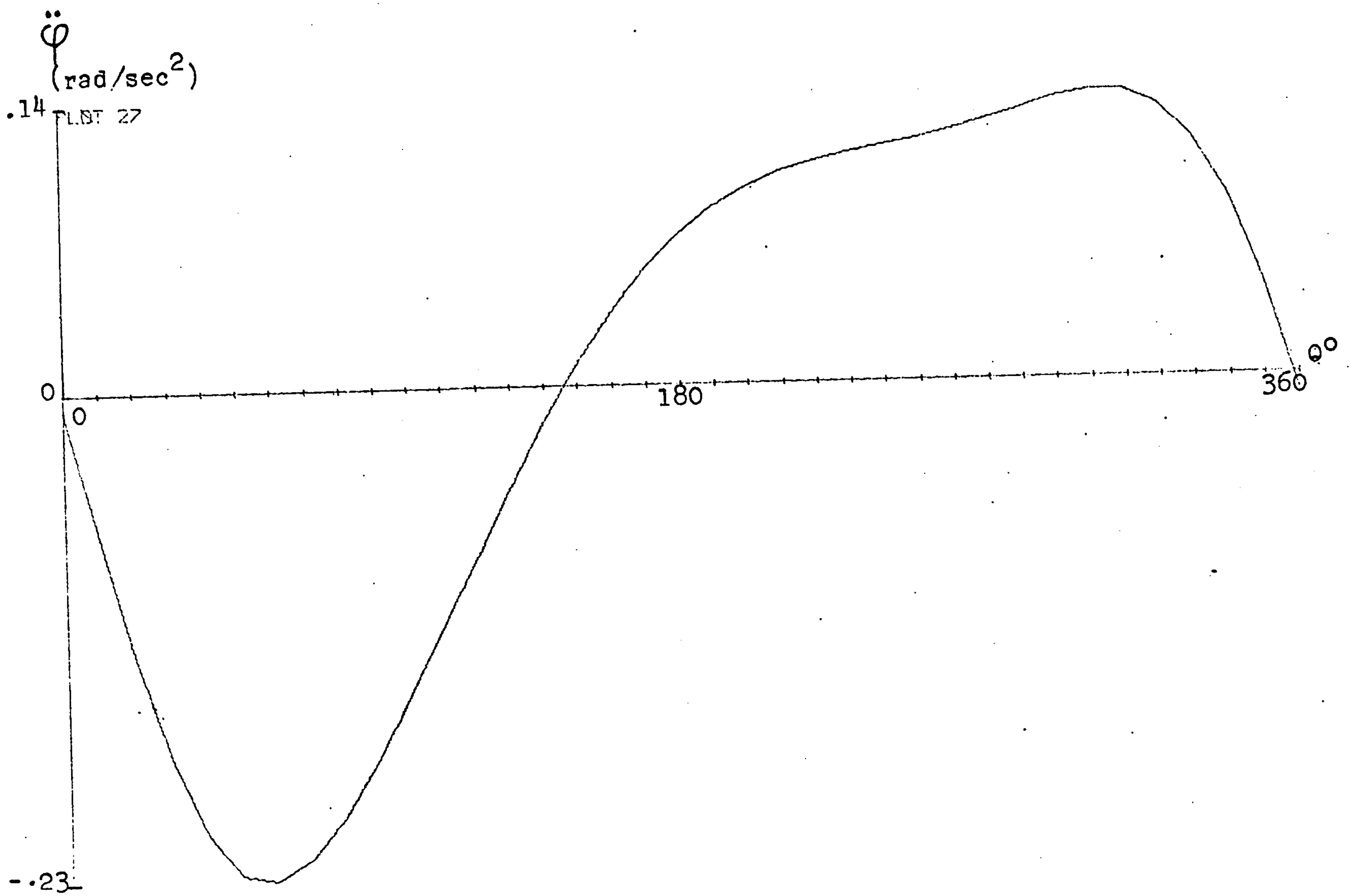
Angular Displacement of Output Link

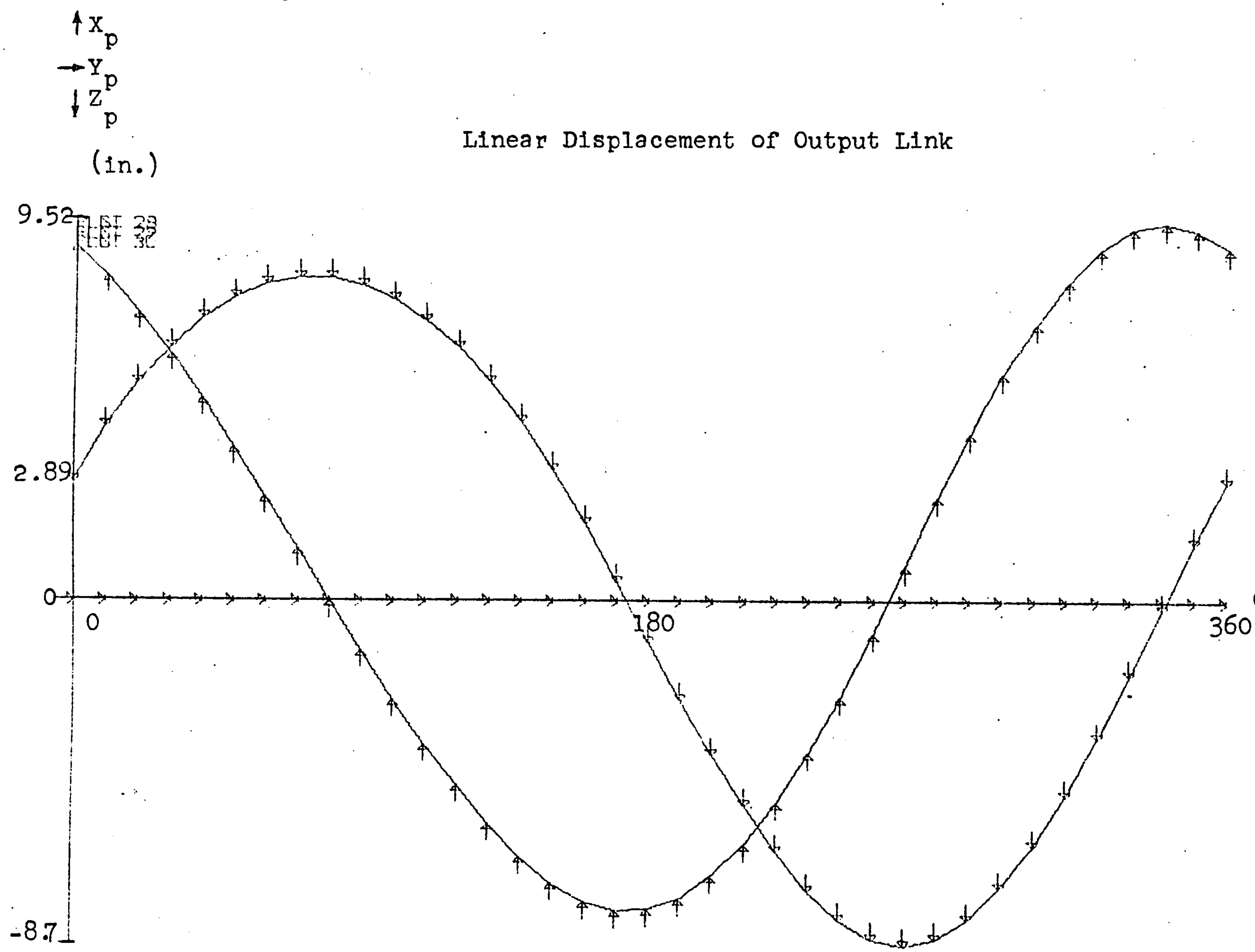


Angular Velocity of Output Link

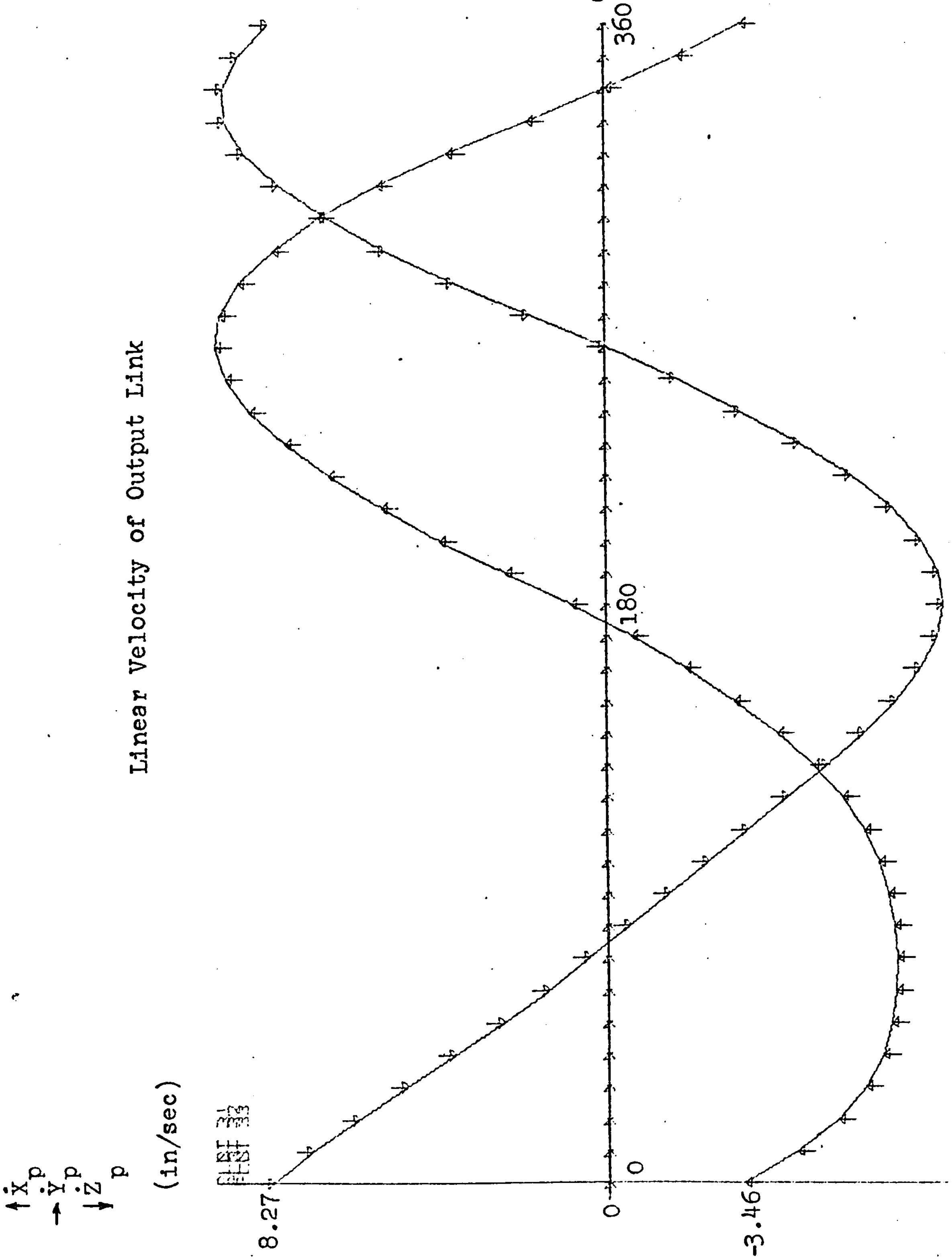


Angular Acceleration of Output Link

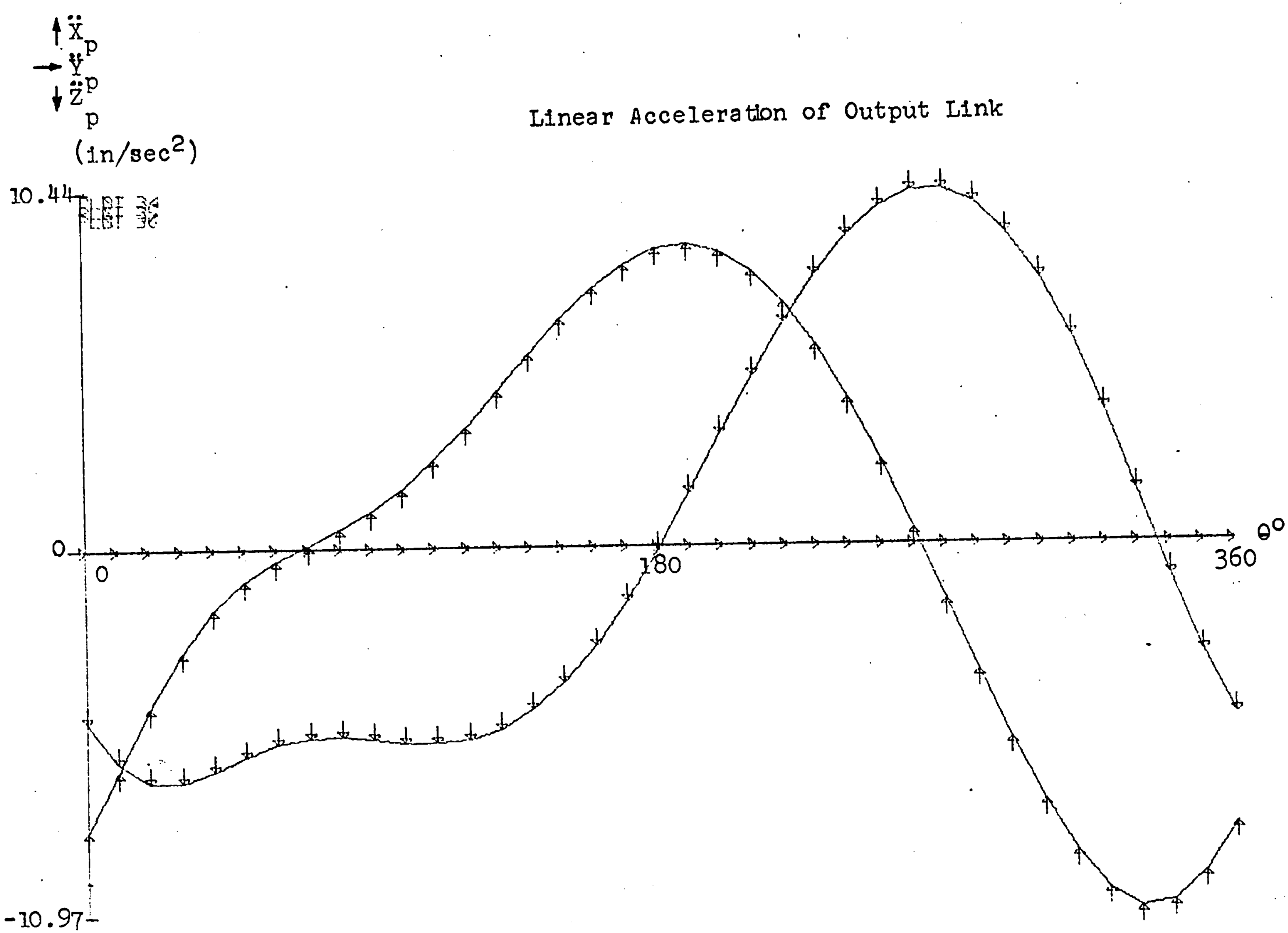




Linear Displacement of Output Link


Linear Velocity of Output Link
$\dot{X}_p$
$\dot{Y}_p$
$\dot{Z}_p$
(in/sec)
8.27
0
-3.46
0
180
360
θ°

Linear Acceleration of Output Link



APPENDIX II

Transformation Matrix for the moving reference system UVW by means of the direction cosines and Euler angles.

The moving system UVW is established in the following way.

U is chosen along the coupler link R_3.
W is perpendicular to the plane OBC (Fig. 1).
V is perpendicular to U and W to establish a right hand system.

The direction parameters of U are:

$$XU = X_c - X_b$$

$$YU = Y_c - Y_b$$

$$ZU = Z_c - Z_b$$

and the direction cosines are:

$$\cos(XU) = \frac{XU}{\sqrt{XU^2 + YU^2 + ZU^2}}$$

$$\cos(YU) = \frac{YU}{\sqrt{XU^2 + YU^2 + ZU^2}}$$

$$\cos(ZU) = \frac{ZU}{\sqrt{XU^2 + YU^2 + ZU^2}}$$

The lines OB and OC establish the plane OBC to which W is perpendicular. Thus, if the direction parameters of OB are X_b, Y_b, Z_b and that of OC are X_c, Y_c, Z_c the direction parameters of W are:

$$XW = \begin{vmatrix} Y_b & Z_b \\ Y_d & Z_c \end{vmatrix} \qquad YW = \begin{vmatrix} Z_b & X_b \\ Z_d & X_c \end{vmatrix}$$

$$ZW = \begin{vmatrix} X_b & Y_b \\ X_d & Y_c \end{vmatrix}$$

and the direction cosines of W are:

$$\cos(XW) = \frac{XW}{\sqrt{XW^2 + YW^2 + ZW^2}}$$

$$\cos(YW) = \frac{YW}{\sqrt{XW^2 + YW^2 + ZW^2}}$$

$$\cos(ZW) = \frac{ZW}{\sqrt{XW^2 + YW^2 + ZW^2}}$$

The direction parameters of V which is perpendicular to both U and W are:

$$XV = \begin{vmatrix} YU & ZU \\ YW & ZW \end{vmatrix} \qquad YV = \begin{vmatrix} ZU & XU \\ ZW & XW \end{vmatrix}$$

$$ZV = \begin{vmatrix} XU & YU \\ XW & YW \end{vmatrix}$$

and the direction cosines of V are

$$\cos(XV) = \frac{XV}{\sqrt{XV^2 + YV^2 + ZV^2}}$$

$$\cos(YV) = \frac{YV}{\sqrt{XV^2 + YV^2 + ZV^2}}$$

$$\cos(ZV) = \frac{ZV}{\sqrt{XV^2 + YV^2 + ZV^2}}$$

The transformation matrix becomes

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos(XU) & \cos(XV) & \cos(XW) \\ \cos(YU) & \cos(YV) & \cos(YW) \\ \cos(ZU) & \cos(ZV) & \cos(ZW) \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

Using the Euler's angle β defined in Fig. 5 as the angle between link R_3 and the X-axis, the transformation matrix of the moving reference system UVW can be expressed as:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

where β is the Euler Angle defining the rotation of the UVW-system; from Fig. 5,

$$\sin\beta = (Z_c - Z_b) / R_3$$

$$\cos\beta = (X_c - X_b) / R_3$$