Analytical / Computational Techniques in Future Broadband Networks

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ABSTRACT

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In a future broadband ISDN network various types of traffic with a wide range of characteristics are integrated in a common environment. The broadband network should not only be capable of operating efficiently in this vastly heterogenous environment, but also capable of supporting multi-point connections in addition to the more conventional point-to-point connections. The above two features and the anticipated traffic volume make the engineering of a future network a multifaceted challenge. In this context, congestion evaluation and prevention is an issue of crucial importance.

In this thesis we discuss admission control mechanisms that aim at preventing congestion by restricting the access of new service requests when congestion pends. The formulation of the controls is based on analytical results concerning Time Division Multiplexing (TDM) sharing schemes and a hybrid TDM technique of 'movable boundary' type. Thus, two possible scenarios are examined. According to the first, no distinction is made in terms of performance requirements of the various types of traffic, but a 'universal' quality of service is guaranteed, whereas in the second, a distinction is based on the delay/ loss requirements of the supported types of traffic.

TO MY PARENTS

MICHAIL AND ANTIOPI

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INTRODUCTION

The present Ph. D. thesis is motivated by the implementation of Broadband ISDN on optical networks. A broadband end-to-end fiber network is attractive for carrying present and future communications services in a fully integrated manner. Now, the supported services that include variable-rate video and still image transfer in addition to the traditional ones, may have a broad spectrum of traffic characteristics, such as peak rate, average bit rate, call interarrival time and holding time, burst-length statistics etc. Besides operating in a heterogenous environment of unknown characteristics, the network is required to support multi-point connections in addition to the more conventional point-to-point connections [Tur86], [GoAS86].

The integration of these vastly different types of traffic is possible due to the development of high speed packet switching and the reliability of fiber optic transmission. Accordingly, the realization of integration of these types of traffic implies that the network should be capable of handling circuit switched, stream or real time traffic.

In the following we will try to answer questions concerning the above mentioned integration focusing on a particular congestion control mechanism and the analytical expressions associated with it. Although the results we present in this thesis stand by themselves, (for example, fast circuit switching in Chapters I and II, hybrid TDM in Chapter III), it is useful to present in the following paragraphs a more general framework of the broadband integrated environment where they can be understood and then applied to answer questions on the alternatives of its implementation.

With the goal of flexible sharing of the network resources, packet switching seems promising for implementing an integrated access and transport network. To this end, the preferred form of broadband packet switching with CCITT and T1 standards committees is the Asynchronous Transfer Mode (ATM) [GoAS86]. This mode of operation is based on small size, fixed length packets ('cells' in ATM terminology, 53 bytes long) and bandwidth assignment according to the temporal needs of the supported communications services. ATM stands in contrast to the synchronous mode of operation where bandwidth is allocated on a periodic basis, independently of the activity of the sources. In ATM, a call is no longer characterized by the position of the time slot in a frame but by a label designating the logical connection under consideration that may span over several links.

Statistical Multiplexing is a capacity allocation technique that exploits the bursty nature of multiplexed traffic. An increased multiplexing degree is attained by means of this technique, because capacity is allocated not according to the peak rate requirements of a connection, but rather according to an 'average' rate, given that a certain packet-loss probability is acceptable. A great number of bursty types of traffic with low peak-to-link ratios are needed to make the use of statistical multiplexing preferable (because of the attained multiplexing gain) over a simpler non-statistical mode of multiplexing. ATM being targeted to function as a flexible transport mechanism accommodating a number of bursty types of sources, employs statistical multiplexing at some level or at consecutive levels of increased multiplexing capacity.

Congestion in Broadband networks and related parameters

Various aspects of standardization of the ATM concept are still open. They include nodal architectures, statistical multiplexing, signaling, congestion control and usage parameter control, etc. Of these issues, we are mainly concerned with controlling the flow in the network, a function that results in avoiding deterioration of its throughput with satisfaction of the performance requirements of the supported services. More specifically *admission control* strategies that apply directly to a call request are considered here.

Despite the enormous bandwidth of broadband networks, congestion still occurs due to the anticipated traffic volume being further enhanced by the network's features such as broadcasting or multicasting. Control mechanisms should then be applied not on a link per link basis, since throughput maximization is an important issue, but at the points of access to the network transport instead. Two general categories of controls have been suggested, being termed preventive and reactive. We choose to pursue a preventive type of control in this thesis, since, reactive controls may be associated with sensitivity to transient behavior, oscillations and questionable fairness in throttling heterogenous types of traffic when congestion pends [WoRR88].

Implementation of preventive controls is mainly done by means of admission controls. The types of admission we pursue here are associated with the following two prerequisites:

i. the network has a correct knowledge (estimated or measured) of the traffic parameters ('usage parameters') of the supported types of traffic.

ii. the traffic parameters of the supported services remain the same for the duration of each connection.

These functions will be provided by the network by:

i. employment of devices that monitor the stochastic behavior of various users and provide the necessary *traffic descriptors* in terms of peak rate, mean rate, burst length etc. These parameters are subsequently used by the controls to determine the traffic load of the system and to decide whether or not acceptance of an incoming call leads the network to a congested state.

ii. traffic enforcement units that guarantee the agreed upon traffic parameters of the user are not violated. Traffic enforcement schemes such as the 'leaky bucket' [Tur86], [Ra89] appear to implement efficiently the usage parameter control (UPC) by employing a counter that is incremented on each cell transmission and decremented at a constant rate. Cells arriving when the counter has attained its maximum value are discarded or 'tagged' for transmission with no guaranteed grade of service. Thus, the mean rate is enforced to an agreed constant rate of the counter, whose maximum value also restricts the maximum burst length of the source.

Other mechanisms that ensure the usage parameter control have been suggested. They include the 'jumping window', the 'triggered jumping window' and the 'moving window'. A comparison of the above mentioned controls including the 'leaky bucket', in terms of (i) violation probability (ii) sensitivity to static overhead (iii) the dynamic reaction time and (iv) worst-case traffic admitted to the network and implementation complexity, is given in [Rath91].

In [EcDoZo91], a different congestion control strategy is suggested. The controls are partitioned into three domains : (i) network-element internal controls, (ii) network-wide at the ATM level and (iii) call-level controls in an environment in which a limited number of different classes of traffic is suggested. The call-level controls apply during the call set-up negotiations based on the traffic descriptors and the performance needed to be delivered by the network. With the service parameters having been established at the call level, the suggested networkwide mechanisms control the implementation of the service agreement at the cell level. This is done by three central capabilities: namely, the capability for selectively shedding load under congestion conditions, the capability for forward conveyance of encountered congestion conditions and that of backward notification of relevant congestion onset / abatement conditions.

Finally, the third category of controls, those termed network-element internal, are those on which the networkwide ATM-level control depend upon. Their actions include traffic monitoring, selective cell discard, and bandwidth buffer allocation according to the performance requirements of the supported classes of traffic.

A distinction between two classes of traffic is suggested in [GiMaPh91]. The distinction is made in terms of the offered grade of service. Thus, class 1 is given a guaranteed information transfer quality of service per connection, whereas class 2 is offered an average information quality of service over many connections.

A combination of preventive and reactive congestion controls is suggested to ensure the satisfaction of class 1 and 2 requirements. There are two causes of congestion-related cell loss: momentary buffer overflow and sustained overload. The first arises because of coincident arrival of peak-bandwidth bursts from a larger than average number of sources. The second occurs where correlated traffic exists in the network for a sustained period of time. According to their control scheme, reactive mechanisms work to exclude momentary buffer overflows, whereas preventive mechanisms deal with the sustained overload.

In [NoRoSVi91] the cell arrival process in an ATM multiplexer is examined, when loaded with variable-bit-rate sources. Their analysis is based on a result, originally derived by Benes, expressing the distribution of work in a GI/G/1 queue. The result is applied first to a queue where the arrival process is a superposition of periodic sources and then to a variable-input-rate, constant-output fluid system. The latter is used to model the 'burst component' of the considered superposition queueing process. The suggested 'cell component' and 'burst component' of the queueing process represent (i) negative correlation between successive interarrival times due to the periodic nature of cell emissions by active sources and (ii) positive correlation between the average arrival intensities in successive periods of length greater than the inter-cell time of the multiplexed sources, respectively. Their analysis leads to buffer dimensioning and congestion controls that guarantee a given performance threshold (in terms of acceptable cell/burst-scale congestion levels).

In [BMLRW91], the performance of an ATM multiplexer is studied. The input process is considered to be the superposition of a multiplicity of homogeneous (binary) sources modeled by a two-state Markovian process. A two-state Markov-modulated Poisson process (MMPP) is used to approximate the actual superposition of the input traffic. Results concerning the cell loss performance of the system are given via an asymptotic matching procedure, to the 'phase process' [Neu81], which is the resulting superposition of a finite number of homogeneous binary sources. The results of this approximation are verified with simulation data and exhibit the 'sharp knee' behavior between a 'cell scale' congestion and a 'burst scale' congestion that was observed by other researchers [NoRoSVi91].

The work in [Hui88] is closely related to ours. A methodology for congestion evaluation is presented here, by splitting the process into three levels, namely, packet level, burst level and call level. Congestion is subsequently evaluated for each level, in terms of blocking (loss) probability. The given congestion controls are based on the interdependences of the performance characteristics between levels moving from the cell level backwards to the call level. This process results in a call acceptance region that regulates the incoming traffic during call establishment negotiations, by satisfying the predefined performance thresholds at the burst/cell levels, respectively.

A similar approach is followed also in [WoKo90] where admission types of controls are suggested in cooperation with bandwidth enforcement mechanisms to control congestion in ATM networks. Simulation results are provided for the performance of an ATM statistical multiplexer employing an array of input buffers. The considered traffic classes are characterized by the average rate, the peak rate and the average burst length. Statistical versus non-statistical modes of operation of the multiplexers along with several network management and design principles are discussed as well (see also [Wer88]).

In [SaKuYa91], the traffic characteristics in an ATM network are analyzed, focusing on voice and variable-rate video traffic. The superposed traffic is approximated by a 2-state Markov-Modulated Poisson Process (MMPP), whereas the cell processing time is approximated by an Lth order Erlang distribution (E_L). The resulting queuing model is an $MMPP/E_L/1/K$, with K the output buffer capacity. The presented results concern the burstiness of the traffic and the loss probability for a multiplexing link capacity of 156 Mb/s and buffer size of a few tens of cells, for different mixtures of video and voice traffic.

A matrix-analytic method is used in [YaMa91] to analyze the performance of an integrated services TDM switching node. The arrival process consists of two priority classes of packets, those with high priority being generated from a number of binary sources, and those with low priority being Poisson distributed. Two infinite size buffers for the high and low priority traffic are employed as well-as a sharing scheme that allows high-priority packets to experience a minimal nodal delay and low-priority packets to utilize any unused bandwidth.

In [OhMuMi91], the performance of an ATM switch is addressed. The adopted queueing model is that of a discrete-time single-server queue, at which three different kinds of arrival processes are allowed to join together: arrivals of cells with a general interarrival distribution, Bernoulli arrivals of cells in batches and interrupted Poisson processes (IPP, [Kuc73]). An exact and elegant analysis is presented in order to derive the waiting-time distributions and interdeparture-time distribution for arrival cells subject to admission control. It is subsequently extended to approximate end-to-end delay distributions for the designated traffic, and tested via simulations.

The arrival process model.

An issue of crucial importance is the arrival process model we use in Chapters II and III, to describe bursty types of traffic. The Markov correlated model in [MASKR88] is suggested to describe the aggregate arrival process of variable-rate video sources. The aggregate bit rate is quantized into finite discrete levels, with transitions between levels being assumed to occur with exponential transition rates that depend on the current level. Given that there is an infinite number of Markov chains that can fit the steady-state mean and autocovariance of the aggregate video rate, the authors in [MASKR88] suggest a birth-death process to describe the process, since no discontinuities or deep changes occur in observed video traffic samples. This results in a representation of the aggregate process by a number of independent 'binary' sources, that alternate between active and silent periods with exponential rates and, when in the active mode, impose bandwidth demands equal to the quantization step of the aggregate process. The transition from an idle to an active state is called a 'burst' and imposes bandwidth demands for an exponentially-distributed time period.

A model similar to the one described above was initially suggested by Brady [Br69] for voice-traffic modeling, and used again in statistical multiplexing analysis of voice sources with speech activity detection in [SrWh86] (with the active state representing talkspurts and the silent state corresponding to silence intervals).

In this thesis, we assume that the Markov correlated model, or equivalently the 'finite source' model, can be used to describe the bursty nature not only of voice or variable-rate video traffic, but of other sources as well. Then, the aggregate arrival process for various types of traffic is assumed to be composed of groups of binary sources (with parameters that fit the mean rate and covariance of the respective types of traffic) that, when in the active state impose distinctive bandwidth demands to the statistical multiplexers of the network according to the type of traffic they represent.

Another frequently used model is the Markov Modulated Poisson Process (MMPP), which is a special case of the versatile Markovian process [Neu79]. Arrivals are governed by a continuous-time discrete-state Markov chain, with packets arriving according to a Poisson process with parameter λ_m when in state m - called 'phase m'. Thus a K-state MMPP is defined by the state space {1,...,K}, the infinitesimal generator Q (a KxK matrix that denotes the allowable transitions and the respective rates) and the arrival rate matrix $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \cdots \lambda_K)$.

In [HeLu86] a two-state MMPP is used with four parameters $(\lambda_1, \lambda_2, r_1, r_2)$, that match the statistical characteristics of the superposition of a number of voice sources.

The chosen statistical characteristics are: the average arrival rate, the variance-to-mean ratio of the number of arrivals in the time interval $(0, t_1)$, the long-term variance-to-mean ratio of the number of arrivals and the third moment of the number of arrivals in $(0, t_2)$, with t_1, t_2 arbitrarily chosen [Neu79].

An approach similar to ours in traffic characterization is given in [LiMa90], where the binary source model is used to describe traffic types other than voice or variable-rate video. Note that any multistate Markov model can always be approximated by the binary model in [MASKR88], with the correlation of each type of traffic represented in terms of the power spectral density.

Universal grade of service - Admission controls.

It is critical in a multimedia environment that the transport level is guaranteed to meet performance requirements for all the supported applications. That is, from delay sensitive applications such as voice, to less-sensitive applications such as image transfer. This variety could lead to tailoring the network according to the individual needs of the supported services, and consequently to an increased complexity of controls [Wo88]. Thus, in the first two chapters of the thesis, we choose to deal with the heterogenous performance requirements in an integrated manner, by not assigning priorities to different classes of traffic and aiming at simple control mechanisms that apply universally. In the third part, we pursue the topic of service priorities by making a distinction between two classes of traffic that are integrated in a broadband environment (not necessarily ATM).

The distinction is based on delay requirements, the delay sensitive traffic being immediately accepted when bandwidth is available, whereas the delay insensitive second priority traffic is being stored in the buffers of the system waiting to be serviced by any unused portion of the bandwidth dedicated to the first priority traffic.

Then, the basic philosophy of the *admission controls* for the fully-integrated connection-oriented ATM transport is summarized in the following [Hui88]: By suggesting a method for evaluating burst blocking, that is, the calculation of the probability that a source switching to the active mode is blocked, we can draw the admissible region for the combinations of calls of each service type for which the burst-level blocking is acceptable. Given this admissible region of call combinations, we can then calculate the probability of call blocking at the boundary of the region which depends on the call arrival and holding-time statistics. The same basic philosophy applies in the third chapter of the thesis with an additional parameter, the queuing delay (or the packet-loss probability) of the low-priority traffic.

The main volume of our work, that is, the way of evaluating the above-mentioned burst or call-blocking statistics and the queueing delay in the hybrid scheme, is distributed in the following chapters:

Chapter I.

In this first part of the thesis, we suggest a multi-class multi-link generalization of the well-known Erlang B formula that gives the blocking probabilities in telephone networks as a function of the offered load and the number of available channels. Our work is an extension of an earlier work that appeared in [Kau81], [Rob81], and applies to cases in which an incoming call that can be characterized by different bandwidth demands, must access two or several consecutive links in order to reach its destination. Since blocking can occur in any of these links or in any link combination, the blocking probability is calculated exactly by means of a linear recursive formula that takes into account interdependencies of traffic levels on consecutive links. The above analysis is further extended to cover architectural features of the broadband network, such as broadcasting and multicasting.

For practical applications, the Poissonian assumption for the arrival process of the individual traffic streams, is sufficient for traditional types of traffic - for example, telephone calls without silence removal - or cases in which the number of traffic sources having access to the network is assumed to be infinite. Scenarios of traffic integration with a limited number of bursty sources, like still picture graphics stations, are not covered by this analysis.

Chapter II.

The above limitation is considered in the second part of the thesis, where we analyze the situation in which traffic sources are discrete in nature so that the total offered traffic load to the network can never exceed certain limits. In order to take into account the bursty nature of the offered traffic, we introduce the two-state source model as the unit source model for any type of traffic burst [MASKR88]. A basic assumption here, as mentioned before, is that a number of two-state sources can approximate any type of traffic adequately well, by adjusting the statistical parameters of the two-state sources accordingly.

Subsequently, we derive expressions for blocking probabilities - blocking meaning the network's inability to allocate the demanded bandwidth - for two network topologies :

a) a set of multiplexers at the access points of the network where calls can be blocked or bursts can be dropped, as well as, an internally nonblocking routing network.

b) a set of multiplexers at the access points and a blocking TST (Time-Space-Time) $N_i \times N_0$ switch at the routing core.

For these two cases, we calculate burst-loss probabilities as a function of the traffic load and the bandwidth requirements of the supported services. Then, we use these calculations to suggest multidimensional call acceptance regions with the number of dimensions being the number of types of traffic. The point of network operation is defined by the load of each type of traffic present in the network. Acceptance of a new call moves the point of operation closer to the boundary, which, for as long as it is not violated, guarantees the predefined performance threshold.

Chapter III.

In the third part of the thesis, an integration of three types of traffic, typified by voice, data, and video is analyzed. Bandwidth is administrated via a 'movable boundary' technique [Ku74],[Za74], that partitions the available transmission capacity into two portions: one for the video-voice calls, which is never to be exceeded, and another one for the data packets which are allowed to overflow to the video - voice band when unused bandwidth is available. Buffer space for the data traffic is provided in order to maximize bandwidth utilization.

The suggested analysis assumes both a Poissonian arrival process as well as a Markov correlated one for the video-voice traffic and proceeds by construction of a Markovian chain embedded at the start of each frame in the succession of frames in time. This approach is limited by the increased dimensionality of the state transition matrix. The complexity of the problem is effectively reduced when the hierarchical structure of the system is exploited. Sinces the above system is hierarchically structured in terms of arrival rate and service time distribution of the three types of traffic, Courtois' techniques on nearly completely decomposable systems apply naturally [Cou77]. The technique is an approximation that treats *nearly completely decomposable* systems as *completely decomposable* that can be analyzed by treating the subsystems involved independently, consequently an error ε is introduced that depends on the degree of coupling between subsystems. A second order approximation that decreases the error of approximation into an order of ε^2 is also employed, allowing a wide range of input traffic characteristics to be accurately analyzed.

A further extension of the above analysis is suggested for the case of an infinite data buffer assumption. Matrix Geometric techniques [Neu89] are well-known solutions to the problem of solving the equilibrium state distribution of these types of situations. Direct application of the techniques though, is limited by the amount of computations involved in realistic applications (DS3 or higher capacities). Instead, a combination of decomposition, matrix geometric techniques is pursued as an adequate solution.

Based on the above analysis, we calculate blocking probabilities for the first- priority calls and the average delay/ buffer overflow probabilities for the data traffic. The results lead to formulation of admission controls similar to these suggested in the previous two chapters.

Chapter IV.

Finally, in the last chapter of this thesis, we discuss the main results of the above analysis, emphasizing conclusions and practical considerations based on them. We conclude the chapter by mentioning further possible extensions of our work to related subjects. The problem of consecutive levels of statistical multiplexing of increasing capacity is one of them.

CHAPTER I.

Blocking of Circuit Switched Traffic in Tandem ISDN links.

The context of the problem is broadband optical networks which carry a wide range of traffic classes in an integrated fashion. The focus is upon so called stream traffic in which each class is allocated different quantities of bandwidth for the duration of a call on an equal sharing policy. Our main contribution is an exact analysis of the blocking conditions experienced by different classes of traffic demanding transmission through a network with the star or the ring topology is presented. Thus, we calculate blocking probabilities on tandem links by means of a recursive relation that simplifies considerably the complexity of the problem without ignoring the interdependencies among links. Being a solution to a multiple classes - multiple links blocking problem, the present work is a multi-dimensional generalization of a well-known analysis for a single trunk. The efficiency of the solution lies on the linearity of the necessary computations.

The results of the analysis are applied to blocking calculations in an $N_I \times N_o$ star and an N-station ring network. They are further extended to include multicasting and broadcasting switching capabilities.

1.1 Introduction

Realization of ISDN [Wh87] on optical networks, will be required to handle many different kinds of traffic including voice, data, video etc. From the point-of-view of performance, we distinguish two basic types of traffic which we designate as reserved (stream) and unreserved. The reserved type of traffic, typified by voice, can tolerate only a limited amount of delay which constitutes a performance threshold. The same may be said of video signals. For this type, only limited buffering is tolerable. Being also less bursty than many types of data traffic, a reasonable policy is to reserve transmission bandwidth. If the transmission bandwidth is not available, the calls are cleared. An example of such a policy is contained in reference [GeLee89]. In contrast, the unreserved traffic, typified by data signals, does not have a delay threshold. Performance deteriorates with increased delay in a more or less continuous fashion; accordingly, buffering is appropriate.

In this chapter, we concentrate on reserved traffic which is modeled as the Poisson arrival to the system of calls requiring a certain amount of bandwidth for a random time interval. We assume that this traffic may be segmented into different classes each of which has different bandwidth and holding-time requirements.

We are interested in situations where reserved traffic is routed over two or more tandem links. We calculate blocking probabilities over the path formed by these links, with blocked calls cleared. This is a fixed-routing problem and although modern communications networks employ alternate routing - according to which if a call arrives to find insufficient capacity in one of the links along its first attempted path, then a second is tried - usually the complexity of the problem is such, that approximations are given by decomposing the scheme into a series of fixed routing problems. This work is an extension to higher dimensions of analyses by Kaufman [Kau81] and by Roberts [Rob81] who obtained, independently, a generalized Erlang B formula for a single trunk carrying reserved traffic whose calls require a random number of servers for an arbitrarily-long period of time. We find the probability of blocking on one or more of the links in a tandem connection for each traffic class. In presenting the results, we focus upon two configurations, the star and the ring. As we shall see, the methodology is such that the generalization to other configurations is straightforward.

The first topology examined here, the star configuration, is illustrated in Figure 1.1. A space-division switch that might be self-routing [HuAr87] for example, lies at the hub of the star providing routing connections amongst any pairs from the N_I input and N_0 output set of links. A group of sources with the same or different traffic characteristics are statistically multiplexed onto each of the N_I input lines. Demultiplexing respectively occurs at any of the N_0 output lines. The bandwidth of each input or output line, is to be divided among the various traffic classes on a fully-shared basis. Each call occupies for its duration a number of slots in a frame according to its bandwidth requirement. This traffic arrives to multiplexers at the end of the input lines as shown. The switch routes calls from each of the classes between input and output lines.

The multiplexing is effected by Time Division Multiplexing (TDM) in which the flow on the line is organized into frames consisting of a fixed number of slots . Each slot carries a basic unit of information, a cell or a byte, for example. We assume that the duration of frames on input and output lines are F_I and F_o slots, respectively. In the real-time multiplexing scheme input and output frames must have the same duration; hence F_I and F_o denote the relative bandwidth of input and output lines, respectively. We assume the TDM model for the remainder of the chapter; however, other multiplexing techniques can be modeled in a similar manner, as well. For example, the same basic analysis carries through in systems where a peak rate is guaranteed for the duration of a virtual connection in an ATM environment [GeLee89]. An aggregate of ATM sources may be modeled by a large number of independent Markov chains with two states, active and idle [MASKR88]. In the active state, transmission is at a constant rate. When the total guaranteed peak rate reaches a certain threshold, all new connection attempts are rejected.

Under the foregoing model, a call can be blocked on either an input or an output line. In either case a newly arriving call requires more slots that are available in the frame. In this chapter, a method for calculating the probability of blocking is presented.

It is assumed that the switch is capable of rearranging the incoming traffic in order to resolve output contention, that is, signals in different input lines, occupying the same portion of the respective input frames with the same output destination, automatically acquire a phase difference, which allows conflict-free transmission through the switch. If this rearrangement cannot be carried out in practice, the results we present constitute a lower bound on performance.

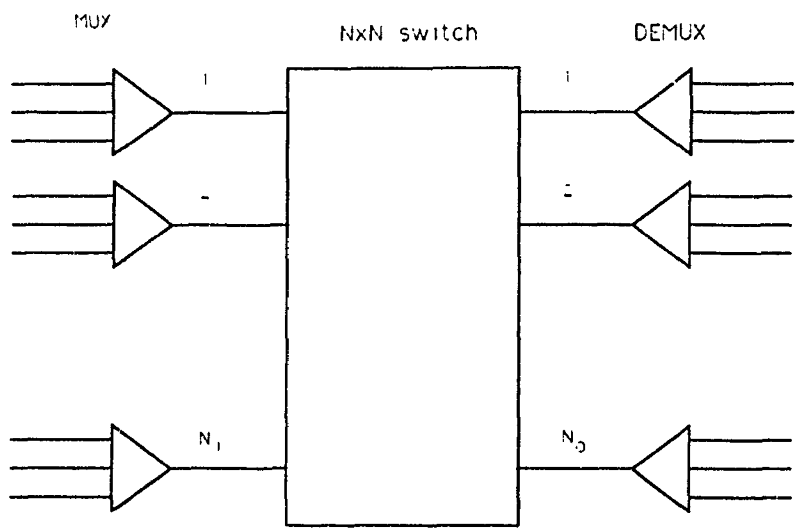


Figure 1.1. Star configuration with $N_I \times N_O$ switch at the hub.

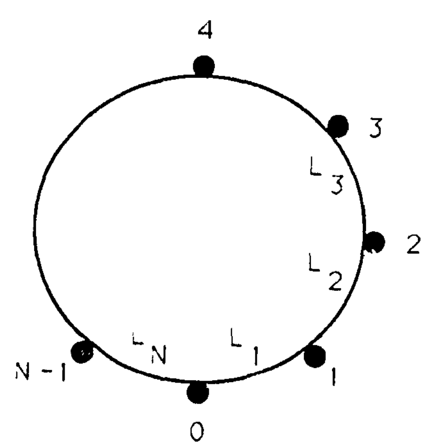


Figure 1.2. Ring configuration with N stations .

The second configuration which is under consideration, is the ring illustrated in Figure 1.2. At each of the N stations in the ring, traffic may enter and leave the system. Again it is assumed that flow on each of the links is organized into the TDM frame structure. A call originating at a particular station occupies a varying number of slots, depending on bandwidth requirements on each of the links in the path to its destination station. If any of the links cannot accommodate the bandwidth requirements of a call, the call is blocked. Notice that the ring topology is a tandem connection of N 2x2 switches. This stands in contrast to the star, a single N_I x N_0 switch. Finally, we conclude this chapter by considering multicasting/broadcasting features in a switch (star) topology. A similar type of analysis applies to a ring topology as well.

Dziong and Roberts [DziRo87] derived a similar result in a somewhat different fashion. They also suggested an approximation based on a multi-variate normal distribution with the same moments as the mean, variance and covariance of the multi-class traffic. Relevant to the present work is also the paper by Kelly [Kel86], where an approximation for the blocking probabilities is introduced in a link independence form. This approximation is valid for networks with an arbitrary topology, under the condition of simultaneous increment of the offered traffic and the capacity of the links. A reduced load approximation is also given in [Wh85].

1.2 Analysis of the N_I x N_0 star.

The analysis of blocking in the N_I x N_0 star is based on the following assumptions :

1). Calls are generated at the multiplexer at a Poisson rate with an average of λ calls/ second.

2). Each call falls into one of K classes of messages, which require b_i frame slots for a random holding time τ_i. The arrival rate of each class i with sourceldestination pair μ, ν is represented by $\lambda_{\mu\nu i}$ calls/sec.

3). The LS transform of the probability density function of the holding time, τ_i, is a rational function with mean value $\frac{1}{\mu_i}$ [Lam77].

The objective is to compute the steady-state blocking probability experienced by a call attempt with bandwidth requirements b_i, presented at an input link μ with destination the output port ν (the pair μ, ν being arbitrarily chosen). Two conditions prohibit the transmission of such a message through the $N_I x N_O$ switch.

I). Lack of available bandwidth in the frame with duration F_I slots, at the input port μ,

II). Lack of available bandwidth in the frame with duration F_o slots, at the output port ν,

By allowing frames on input and output lines to be of different lengths, we allow for concentration of traffic and consideration of the switching speed. Denoting by $P(I_{\mu i})$, the probability that a message of class i is blocked because of condition (I) exclusively, and by $P(I_{\nu i})$, the probability that the same message is blocked because of condition (II), the blocking probability is expressed as :

$$P_{b_{\mu\nu i}} = P(I_{\mu i}) + P(I_{\nu i}) - P(I_{\mu i} \cap I_{\nu i}) \qquad (1.1)$$

The probabilities of the first and the second events are expressed as

$$P(I_{\mu i}) = \sum_{j=0}^{b_i-1} r_\mu(F_I - j)$$

and

$$P(I_{\nu i}) = \sum_{j=0}^{b_i-1} z_\nu(F_o - j)$$

with $r_\mu(x)$,$z_\nu(x)$ the probability of finding x slots full in the frame at the input port μ and the output port ν, respectively. The third event $P(I_{\mu i} \cap I_{\nu i})$ can be expressed by means of the joint probability function $q_{\mu\nu}(j,l)$, which represents the probability of finding j slots of the frame at the input port μ full, and a total number of l slots destined to output ν.

$$P(I_{\mu i} \cap I_{\nu i}) = \sum_{j=0}^{b_i-1} \sum_{l=0}^{b_i-1} q_{\mu\nu}(F_I - j, F_o - l)$$

Noting also that :

$$r_\mu(j) = \sum_{l=0}^{F_o} q_{\mu\nu}(j,l)\,, \quad z_\nu(l) = \sum_{j=0}^{F_I} q_{\mu\nu}(j,l)$$

equation (1.1) yields :

$$P_{b_{\mu\nu i}} = \sum_{k=0}^{b_i-1} \sum_{l=0}^{F_o} q_{\mu\nu}(F_I - k, l) + \sum_{k=0}^{b_i-1} \sum_{j=0}^{F_I} q_{\mu\nu}(j, F_o - k) - \sum_{j=0}^{b_i-1} \sum_{l=0}^{b_i-1} q_{\mu\nu}(F_I - j, F_o - l)$$

An efficient method of computing $q_{\mu\nu}(j,l)$ is presented in the next section.

1.2.1 Local Balance Equation.

Introducing this section, we note that the above-described system can be modeled as a BCMP network with type 3 service, that has both the 'product form state characterization' and the 'insensitivity property', independently of the sharing policy we adopt [Kau81]. The first property, the product form solution, is expressed as:

$$P(\mathbf{n}) = G(\Omega)^{-1} \prod_{i=1}^{N_I} \prod_{j=1}^{N_o} \prod_{k=1}^{K} \frac{a_{ijk}^{n_{ijk}}}{n_{ijk}!}$$

with

$$G(\Omega) = \sum_{\mathbf{n} \in \Omega} \prod_{i=1}^{N_I} \prod_{j=1}^{N_o} \prod_{k=1}^{K} \frac{a_{ijk}^{n_{ijk}}}{n_{ijk}!}$$

the normalization constant and Ω an arbitrary sharing policy. The insensitivity property simply means that the service time distributions of the various classes of traffic are specified only by their mean.

Assume for the moment, that the 'temporal requirements' of the various classes of messages are exponentially distributed. Then the state of the system $\mathbf{n}$ can be written as :

$$\mathbf{n} = (n_{111}, n_{112}, \ldots, n_{11K}, \ldots, n_{N_I N_o K})$$

The nonnegative integer $n_{\mu\nu z}$ is the number of type z calls on input port μ with output destination the port ν. Also, we define the state vectors

$$\mathbf{n}^-_{ijk} = (n_{111}, \ldots, n_{ijk} - 1, \ldots, n_{N_I N_o K}) \quad \text{and} \quad \mathbf{n}^+_{ijk} = (n_{111}, \ldots, n_{ijk} + 1, \ldots, n_{N_I N_o K})$$

The fundamental dynamics of our system are the same as those considered by Kaufman [Kau81]. It is assumed that the slots required by a single call need not be contiguous and the switch is capable of rearranging the incoming traffic in order to resolve spatial conflicts. Thus, the sharing of the available bandwidth among the different classes is considered a *complete sharing* policy, since a call attempt requiring a certain number of slots (say b) of the frame is blocked if and only if fewer than b slots of the respective input and/or output frames are available. Denoting by Ω the set of allowable states determined by the above policy, a state $\mathbf{n} \in \Omega$ satisfies the following conditions :

I. $$\sum_{y=1}^{N_o} \sum_{z=1}^{K} n_{\mu y z} b_z = m_{I,\mu} \leq F_I \qquad \forall \ \mu \in \{1, 2, \cdots N_I\}.$$

II. $$\sum_{x=1}^{N_I} \sum_{z=1}^{K} n_{x \nu z} b_z = m_{o,\nu} \leq F_o \qquad \forall \ \nu \in \{1, 2, \cdots N_o\}.$$

In the next section, we evaluate the occupancy distribution of the system. Thus, we define $R_{\mu\nu}$ to be the collection of the pairs of the input/output occupancies $(m_{I,\mu}, m_{o,\nu})$, for any input/output pair of ports μ, ν, and for every $\mathbf{n}$ in Ω, that is :

$$R_{\mu\nu} = \{(m_{I,\mu}, m_{o,\nu}) \quad \forall \ \mathbf{n} \in \Omega\}$$

We note that Ω belongs to a more general class of policies, the so called *coordinate convex* policies, defined by the following two properties :

i. $\mathbf{n} \in \Omega \rightarrow n_{xyz} \geq 0 \quad x=1, \cdots N_I,\ y=1, \cdots N_O,\ z=1, \cdots K.$

ii. $\mathbf{n} \in \Omega$ and $n_{xyz} > 0 \rightarrow \mathbf{n}^-_{xyz} \in \Omega.$

Condition (ii) here, means that departures from the switch are never blocked, a property which certainly characterizes the complete sharing policy model we adopt. Further, it can be proved that our policy may be viewed as a single node queuing network with population size constraints, characterized by a product-form state distribution. Finally, the "insensitivity property" that concerns the product-form state distribution, is also valid for our model.

Since our policy is coordinate convex, the local balance equation can be written as :

$$a_{\mu\nu i}\gamma_{\mu\nu i}(\mathbf{n})P(\mathbf{n}^-_{\mu\nu i}) = n_{\mu\nu i}P(\mathbf{n}) \qquad \forall\ \mu, \nu, i \quad \text{and } \mathbf{n} \in \Omega. \tag{1.2}$$

with $a_{\mu\nu i} = \lambda_{\mu\nu i} / \mu_i$, and

$$\gamma_{\mu\nu i}(\mathbf{n}) = \begin{cases} 1 & \text{if } n_{\mu\nu i} \geq 1 \\ 0 & \text{if } n_{\mu\nu i} = 0 \end{cases}$$

1.2.2 Calculation of the Blocking Probability.

The function $q_{\mu\nu}(j,l)$ is defined as :

$$q_{\mu\nu}(j,l) = \sum_{\mathbf{n} \in S_{l,j}} P(\mathbf{n}).$$

with $P(\mathbf{n})$ the probability of the state $\mathbf{n}$ and the set $S_{j,l} \subset \Omega$ defined as :

$$S_{j,l} = \{\mathbf{n} : \sum_{y=1}^{N_o} \sum_{z=1}^{K} n_{\mu yz} b_z = j \text{ and } \sum_{x=1}^{N_I} \sum_{z=1}^{K} n_{x\nu z} b_z = l \text{ , } n_{xyz} \geq 0, \quad \forall\ x, y, z \quad \mathbf{n} \in \Omega\}.$$

for an arbitrary pair of input-output ports μ, ν. Note here that j, l in the above expression are not arbitrary integers but represent occupancy values at the respective input-output ports as a result of one or more states in Ω.

Summing (1.2) over $S_{j,l}$ we get:

$$a_{\mu\nu i} \sum_{\mathbf{n} \in S_{j,l}} \gamma_{\mu\nu i}(\mathbf{n}) P(\mathbf{n}^-_{\mu\nu i}) = \sum_{\mathbf{n} \in S_{j,l}} n_{\mu\nu i} P(\mathbf{n}). \tag{1.3}$$

The LHS of this equation is :

$$LHS = a_{\mu\nu i} \sum_{\mathbf{n} \in S_{j,l}} \gamma_{\mu\nu i}(\mathbf{n}) P(\mathbf{n}^-_{\mu\nu i}) = a_{\mu\nu i} \sum_{\mathbf{n} \in S_{j,l} \cap \{n_{\mu\nu i} \geq 1\}} P(\mathbf{n}^-_{\mu\nu i}). \tag{1.3a}$$

But,

$$S_{j,l} \cap \{n_{\mu\nu i} \geq 1\} = \{\mathbf{n} : \sum_{\substack{y=1 \\ y \neq \nu}}^{N_o} \sum_{z=1}^{K} n_{\mu yz} b_z + (n_{\mu\nu i} - 1) b_i = j - b_i \quad \text{and}$$

$$\sum_{\substack{x=1 \\ x \neq \mu}}^{N_I} \sum_{z=1}^{K} n_{x\nu z} b_z + (n_{\mu\nu i} - 1) b_i = l - b_i, \quad n_{\mu\nu i} \geq 1, n_{xyz} \geq 0\} \tag{1.4}$$

substituting in equation (1.4)

$$\hat{n}_{xyz} = \begin{cases} n_{xyz} & x,y,z \neq \mu,\nu,i \\ n_{xyz}-1 & x,y,z = \mu,\nu,i \end{cases}$$

the left part of equation (1.3a) becomes :

$$LHS = a_{\mu\nu i} \sum_{\mathbf{n}\in S_{j,l}\cap\{n_{\mu\nu i}\geq 1\}} P(\mathbf{n}^-_{\mu\nu i}) = a_{\mu\nu i} \sum_{\hat{\mathbf{n}}\in S_{j-b_i,l-b_i}} P(\hat{\mathbf{n}}) = a_{\mu\nu i}\, q_{\mu,\nu}(j-b_i, l-b_i) \quad j,l\geq b_i. \qquad (1.5a)$$

where $q_{\mu,\nu}(j,l)$ is the probability of finding j slots of the frame at the input port μ full, and a total number of l slots destined to output ν.

The right-hand-side of equation (1.3) can be written as :

$$RHS = \sum_{\mathbf{n}\in S_{j,l}} n_{\mu\nu i} P(\mathbf{n}) = \sum_{\mathbf{n}\in S_{j,l}} n_{\mu\nu i} \frac{P(\mathbf{n})}{q_{\mu\nu}(j,l)} q_{\mu\nu}(j,l) \qquad \forall \ (j,l)\in \boldsymbol{R}_{\mu\nu}.$$

Noting that $P(\mathbf{n} \mid j,l)$, the conditional probability of the system being in state $\mathbf{n}$ given j,l the number of full slots in the input and output frames respectively, can be expressed as :

$$P(\mathbf{n} \mid j,l) = \begin{cases} P(\mathbf{n})/q_{\mu\nu}(j,l) & \text{for } \mathbf{n}\in S_{j,l} \\ 0 & \textit{otherwise} \end{cases}$$

we obtain :

$$RHS = \Big(\sum_{\mathbf{n}\in S_{j,l}} n_{\mu\nu i} P(\mathbf{n} \mid j,l)\Big) q_{\mu\nu}(j,l) = E(n_{\mu\nu i} \mid j,l)\, q_{\mu\nu}(j,l) \qquad \forall \ (j,l)\in \boldsymbol{R}_{\mu\nu} (1.5b)$$

with $E(n_{\mu\nu i} \mid j,l)$, the expected number of $n_{\mu\nu i}$ conditioned on the occupancy of the respective frames j,l. Combining equations (1.5a) and (1.5b), we get :

$$a_{\mu\nu i}\, q_{\mu,\nu}(j-b_i, l-b_i) = E(n_{\mu\nu i} \mid j,l)\, q_{\mu\nu}(j,l) \qquad \forall \ (j,l)\in \boldsymbol{R}_{\mu\nu} \text{ and } \textit{arbitrary } \mu,\nu.$$

Note that both the terms involved in the definition of the set $S_{j,l}$ are affected by the intersection of this set with $\{\mathbf{n} : n_{\mu v i} \geq 1\}$, since both the terms in eq.(1.4) contain the element $n_{\mu v i}$ and a unit increment of the term $n_{\mu v i}$ implies an increment of b_i slots to the number of occupied slots in both the frames at the input port μ and the output port ν. This is clearly not the case, when we consider the local balance equation between a state **n** and the state $\mathbf{n}^-_{\mu w i}$, $\forall\ w \neq v$. In this case we have :

$$a_{\mu w i} \sum_{\mathbf{n} \in S_{j,l}} \gamma_{\mu w i}(\mathbf{n}) P(\mathbf{n}^-_{\mu w i}) = \sum_{\mathbf{n} \in S_{j,l}} n_{\mu w i} P(\mathbf{n}). \qquad (1.6a)$$

The LHS of this equation is :

$$LHS = a_{\mu w i} \sum_{\mathbf{n} \in S_{j,l}} \gamma_{\mu w i}(\mathbf{n}) P(\mathbf{n}^-_{\mu w i}) = a_{\mu w i} \sum_{\mathbf{n} \in S_{j,l} \cap \{n_{\mu w i} \geq 1\}} P(\mathbf{n}^-_{\mu w i}).$$

We consider again the intersection of these sets,

$$S_{j,l} \cap \{n_{\mu w i} \geq 1\} = \{\mathbf{n} : \sum_{\substack{y=1 \\ y \neq w}}^{N_o} \sum_{z=1}^{K} n_{\mu y z} b_z + (n_{\mu w i} - 1) b_i = j - b_i \quad \text{and} \quad \sum_{x=1}^{N_I} \sum_{z=1}^{K} n_{x v z} b_z = l, \ \ w \neq v\}.$$

substituting

$$\hat{n}_{xyz} = \begin{cases} n_{xyz} & x,y,z \neq \mu,w,i \\ n_{xyz} - 1 & x,y,z = \mu,w,i \end{cases}$$

the left part of eq.(1.6a) becomes :

$$LHS = a_{\mu w i} \sum_{\mathbf{n} \in S_{j,l} \cap \{n_{\mu w i} \geq 1\}} P(\mathbf{n}^-_{\mu w i}) = a_{\mu w i} \sum_{\hat{\mathbf{n}} \in S_{j-b_i,l}} P(\hat{\mathbf{n}}) = a_{\mu w i} q_{\mu,v}(j - b_i, l) \quad \text{for } j,l \geq b_i. \qquad (1.6b)$$

where $q_{\mu,\nu}(j,l)$ is the probability of finding j slots of the frame at the input port μ full, and a total number of l slots destined to output ν.

The right-hand-side of eq. (1.6a) can be written as :

$$RHS = \sum_{\mathbf{n}\in S_{j,l}} n_{\mu wi} P(\mathbf{n}) = \sum_{\mathbf{n}\in S_{j,l}} n_{\mu wi} \frac{P(\mathbf{n})}{q_{\mu\nu}(j,l)} q_{\mu\nu}(j,l) \qquad \forall\ (j,l) \in \boldsymbol{R}_{\mu}.$$

Noting that :

$$P(\mathbf{n} \mid j,l) = \begin{cases} P(\mathbf{n})/q_{\mu\nu}(j,l) & \text{for } \mathbf{n} \in S_{j,l} \\ 0 & \textit{otherwise} \end{cases}$$

we obtain :

$$RHS = \Big(\sum_{\mathbf{n}\in S_{j,l}} n_{\mu wi} P(\mathbf{n} \mid j,l)\Big) q_{\mu\nu}(j,l) = E(n_{\mu wi} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall\ (j,l) \in \boldsymbol{R}_{\mu\nu} \quad (1.6c)$$

Combining eq. (1.6b) and (1.6c), we get :

$$a_{\mu wi} q_{\mu,\nu}(j-b_i,l) = E(n_{\mu wi} \mid j,l) q_{\mu\nu}(j,l) \quad r \neq \mu \qquad (1.7)$$

The treatment of the local balance equation between the states **n** and $\boldsymbol{n}_{rvi}^{-}$, for any $r \neq \mu$ is exactly the same. Summation over all the states in the set $S_{j,l}$ leads to the following expression:

$$\sum_{\mathbf{n}\in S_{j,l}} a_{r\nu i} \gamma_{r\nu i}(\mathbf{n}) P(\mathbf{n}_{r\nu i}^{-}) = \sum_{\mathbf{n}\in S_{j,l}} n_{r\nu i} P(\mathbf{n}) \quad \forall\ r,\nu,i \quad r \neq \mu \qquad (1.8)$$

Since

$$S_{j,l} \cap \{n_{r\nu i} \geq 1\} = \{\mathbf{n} : \sum_{y=1}^{N_o} \sum_{z=1}^{K} n_{\mu y z} b_z = j \quad \text{and} \quad \sum_{\substack{x=1 \\ x \neq r}}^{N_l} \sum_{z=1}^{K} n_{x\nu z} b_z + (n_{r\nu i} - 1) b_i = l - b_i ,\ r \neq \mu\}.$$

the left-hand-side of (1.8) becomes:

$$LHS = a_{r\nu i} q_{\mu,\nu}(j, l-b_i) \quad \text{for } j, l \geq b_i ,$$

whereas the RHS of eq.(1.8) is again (for exactly the same reasons that yielded eqs.(1.5b),(1.6c):

$$RHS = E(n_{r\nu i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall \ (j,l) \in \boldsymbol{R}_{\mu\nu}$$

Combining the last two expressions we get :

$$a_{r\nu i} q_{\mu,\nu}(j, l-b_i) = E(n_{r\nu i} \mid j,l) q_{\mu\nu}(j,l) \quad r \neq \mu$$

The main results of the previous analysis are shown in the following expressions:

$$a_{\mu\nu i} q_{\mu,\nu}(j-b_i, l-b_i) = E(n_{\mu\nu i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall \ (j,l) \in \boldsymbol{R}_{\mu\nu} \tag{1.9}$$

and

$$a_{\mu y i} q_{\mu,\nu}(j-b_i, l) = E(n_{\mu y i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall \ (j,l) \in \boldsymbol{R}_{\mu\nu} ,\ y \neq \nu . \tag{1.10}$$

and

$$a_{x\nu i} q_{\mu,\nu}(j, l-b_i) = E(n_{x\nu i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall \ (j,l) \in \boldsymbol{R}_{\mu\nu} ,\ x \neq \mu . \tag{1.11}$$

The last three expressions (eqs. (1.9),(1.10),(1.11)) correspond to eq.(A5) in reference

[Kau81]. In order to simplify further the computation of the values of $q_{\mu\nu}(j,l)$, we prove in Appendix A, lemma II that equations (1.9), (1.10) and (1.11) hold for $\forall\ j \le F_{\mu l}\ ,\ l \le F_{\nu o}$ thus full enumeration of the state space is not necessary, and we derive the final two recursive relationships, that yield $q_{\mu\nu}(j,l)$. Multiplying eq. (1.9), (1.10), (1.11) by b_i we get

$$a_{\mu\nu i} b_i q_{\mu,\nu}(j-b_i,l-b_i) = b_i E(n_{\mu\nu i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall\ j \le F_{\mu l}\ ,\ l \le F_{\nu o} \tag{1.12}$$

$$a_{\mu y i} b_i q_{\mu,\nu}(j-b_i,l) = b_i E(n_{\mu y i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall\ j \le F_{\mu l}\ ,\ l \le F_{\nu o}\ \ y \ne \nu . \tag{1.13}$$

and

$$a_{x\nu i} b_i q_{\mu,\nu}(j,l-b_i) = b_i E(n_{x\nu i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall\ j \le F_{\mu l}\ ,\ l \le F_{\nu o}\ \ x \ne \mu . \tag{1.14}$$

Then. summation of equation (1.13) over the range of output ports and the different classes, and use of eq. (1.12) for the ν output yields :

$$\sum_{i=1}^{K}\Big(\sum_{\substack{y=1\\ y\ne\nu}}^{N_o} a_{\mu y i} b_i q_{\mu,\nu}(j-b_i,l) + a_{\mu\nu i} b_i q_{\mu,\nu}(j-b_i,l-b_i)\Big) =$$

$$\sum_{i=1}^{K}\Big(\sum_{\substack{y=1\\ y\ne\nu}}^{N_o} b_i E(n_{\mu y i} \mid j,l) q_{\mu\nu}(j,l) + b_i E(n_{\mu\nu i} \mid j,l) q_{\mu\nu}(j,l)\Big) =$$

$$\sum_{i=1}^{K}\sum_{y=1}^{N_o} b_i E(n_{\mu y i} \mid j,l) q_{\mu\nu}(j,l) = E\Big(\sum_{i=1}^{K}\sum_{y=1}^{N_o} n_{\mu y i} b_i \mid_{j,l}\Big) q_{\mu\nu}(j,l) = j q_{\mu\nu}(j,l) \tag{1.15}$$

Similarly,

$$\sum_{i=1}^{K}\left(\sum_{\substack{x=1\\x\neq\mu}}^{N_l} a_{x\nu i} b_i q_{\mu,\nu}(j,l-b_i) + a_{\mu\nu i} b_i q_{\mu,\nu}(j-b_i,l-b_i)\right) = \sum_{i=1}^{K}\sum_{x=1}^{N_l} b_i E(n_{x\nu i} \mid j,l) q_{\mu\nu}(j,l) =$$

$$E\left(\sum_{i=1}^{K}\sum_{x=1}^{N_l} n_{x\nu i} b_i \mid_{j,l}\right) q_{\mu\nu}(j,l) = l q_{\mu\nu}(j,l). \qquad (1.16)$$

The probability function $q_{\mu\nu}(j,l)$ can be found from the recursive relations above. Note that eq.(1.15) cannot be used in the computation of $q_{\mu\nu}(0,l)$; $l \le F_o$, and eq.(1.16) cannot supply the values of $q_{\mu\nu}(j,0)$; $j \le F_l$. Thus, an additional summation is proposed. Adding eq.(1.15) and (1.16), we get :

$$\sum_{i=1}^{K}\left(\sum_{\substack{y=1\\y\neq\nu}}^{N_o} a_{\mu y i} b_i q_{\mu,\nu}(j-b_i,l) + a_{\mu\nu i} b_i q_{\mu,\nu}(j-b_i,l-b_i)\right) +$$

$$\sum_{i=1}^{K}\left(\sum_{\substack{x=1\\x\neq\mu}}^{N_l} a_{x\nu i} b_i q_{\mu,\nu}(j,l-b_i) + a_{\mu\nu i} b_i q_{\mu,\nu}(j-b_i,l-b_i)\right) =$$

$$= j q_{\mu\nu}(j,l) + l q_{\mu\nu}(j,l) = (j + l) q_{\mu\nu}(j,l). \qquad (1.17)$$

Using also the convention :

$$q_{\mu\nu}(j,l) = 0 \quad (j,l) \notin R_{\mu\nu}$$

and the normalizing condition :

$$\sum_{j=0}^{F_l}\sum_{l=0}^{F_o} q_{\mu\nu}(j,l) = 1 \qquad (1.18)$$

the values of the function $q_{\mu\nu}(j,l)$ can be computed.

1.2.3 Example.

As an example consider a 64x64 symmetric switch loaded at each input link with two classes of traffic having bandwidth requirements $b_1 = 2$, $b_2 = 3$, and load $a_1 = 1/2$, $a_2 = 1/3$ respectively. The capacity of both input and output frames are 5 slots ($F_I = F_o = 5$) , and calls of either the classes arriving at each input port are uniformly destined for all output ports. Solving equations (1.17) and (1.18) for $\mu = 1, \nu = 1$, we get :

$q_{11}(0, 0) = 0.2242$ $\quad q_{11}(0, 1) = 0.0000$

$q_{11}(1, 0) = 0.0000$ $\quad q_{11}(1, 1) = 0.0000$

$q_{11}(2, 0) = 0.1104$ $\quad q_{11}(2, 1) = 0.0000$

$q_{11}(3, 0) = 0.0736$ $\quad q_{11}(3, 1) = 0.0000$

$q_{11}(4, 0) = 0.0272$ $\quad q_{11}(4, 1) = 0.0000$

$q_{11}(5, 0) = 0.0362$ $\quad q_{11}(5, 1) = 0.0000$

$q_{11}(0, 2) = 0.1104$ $\quad q_{11}(0, 3) = 0.0736$

$q_{11}(1, 2) = 0.0000$ $\quad q_{11}(1, 3) = 0.0000$

$q_{11}(2, 2) = 0.0561$ $\quad q_{11}(2, 3) = 0.0362$

$q_{11}(3, 2) = 0.0362$ $\quad q_{11}(3, 3) = 0.0253$

$q_{11}(4, 2) = 0.0142$ $\quad q_{11}(4, 3) = 0.0089$

$q_{11}(5, 2) = 0.0184$ $\quad q_{11}(5, 3) = 0.0125$

$$q_{11}(0,\ 4) = 0.0272 \qquad q_{11}(0,\ 5) = 0.0362$$

$$q_{11}(1,\ 4) = 0.0000 \qquad q_{11}(1,\ 5) = 0.0000$$

$$q_{11}(2,\ 4) = 0.0142 \qquad q_{11}(2,\ 5) = 0.0184$$

$$q_{11}(3,\ 4) = 0.0089 \qquad q_{11}(3,\ 5) = 0.0125$$

$$q_{11}(4,\ 4) = 0.0037 \qquad q_{11}(4,\ 5) = 0.0047$$

$$q_{11}(5,\ 4) = 0.0047 \qquad q_{11}(5,\ 5) = 0.0063$$

Based on these values the blocking probabilities for the two classes of messages are :

$$P_{b_{111}} = 0.2541 \qquad\qquad P_{b_{112}} = 0.4989$$

Since we assumed symmetric load presented to the input ports, we have :

$$P_{b_{111}} = P_{b_{\mu\nu 1}} \quad \forall\ \mu, \nu, \qquad P_{b_{112}} = P_{b_{\mu\nu 2}} \quad \forall\ \mu, \nu.$$

The obvious generalization of the foregoing result is to interconnected star networks, such as Hubnet [LeBou83] shown in Figure 1.3. In this system, a connection is made through one or more intermediate hubs, and can be blocked at any of the connecting links. Further, it can be shown that the same general approach can be used to calculate blocking probabilities for multicast traffic where traffic at an input port is routed to a subset of the output ports. In the next section, we present the results of an analysis of a ring network which would serve to indicate the results in the general case.

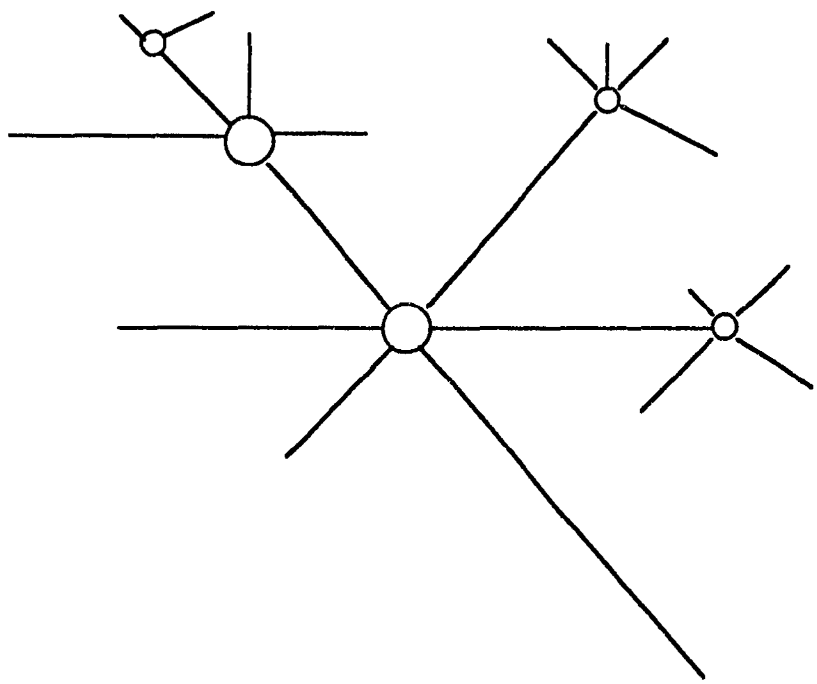


Figure 1.3. Star interconnected network.

1.3 Extension to an N-station ring

The system under study is an optical fiber network in the form of a ring a common topology in Local and Metropolitan Area Networks (LANs and MANs, respectively). A ring in this context is simply a sequence of point-to-point links between stations. Traffic may enter and leave the system at each of these stations.

As in the previous Section, there are different classes of stream traffic. In general, each of these classes will require different amounts of transmission capacity. An easily implementable way of allocating this bandwidth is by means of Time Division Multiplexing (TDM). The flow of a line is blocked into frames which are composed of a fixed number of slots, each slot conveying a certain amount of information. For each call, a specified number of slots in the recurring frame is allocated. Notice that in a ring system, a call will, in general, be established over several tandem links, reserving the same number of slots in a sequence of frames.

The appropriate measure of performance for circuit-switched traffic is the probability of call blocking. Blocking occurs when the number of slots in a frame which are required by a call are not available in a frame because of previously established calls. In a ring system, a call is blocked when the number of slots that it requires are not available on one of the tandem links that it traverses. The analysis of blocking in this case is complicated by the fact that traffic is entering and leaving the system at each of the stations so that each link contains a different mixture of calls.

1.3.1 Performance analysis

We assume N stations distributed in a ring structure shown in Figure 1.2. Propagation of messages is done in a sequential manner, that is, given that there is available bandwidth in the frame, a message asking for transmission from station μ to station ν, is first transmitted to station $\mu+1$, then to $\mu+2$, and so on until its final destination station ν is reached. The links (L_i , i=1,2, ... N) between the stations are assumed to be unidirectional †. The transmission rates at the links connecting the various stations are assumed to be the same. For reasons of simplicity, we assume that the flow on each of the links is organized into frame F slots of duration. The traffic characteristics are the same as indicated in 1) - 3) of the previous section.

By definition a message can be transmitted through the network, if and only if, there is available bandwidth at each one of the successive links that form the virtual path between the entrance node and the destination node. Then given that a call attempt of class i messages demands b_i bandwidth units (slots) in each part of the sequence of links it should traverse, rejection of such an attempt will occur if less than b_i slots are available in any link that constitutes part of its path. Thus, for an arbitrary chosen pair of input output ports μ,ν, the blocking probability of type i messages, is expressed as :

$$P_{b_{\mu\nu i}} = \{P(I_{\mu+1}) + P(I_{\mu+2}) + \ldots + P(I_\nu)\} - \{P(I_{\mu+1} \cap I_{\mu+2}) + \cdots + P(I_{\nu-1} \cap I_\nu)\} + \cdots \pm$$

$$\pm \{P(I_{\mu+1} \cap I_{\mu+2} \cap \cdots \cap I_\nu)\}. \tag{1.19}$$

† A similar analysis can be carried out for bidirectional links.

where $P(I_j)$ is the probability that our call attempt is going to be blocked at link j exclusively. Similarly, $P(I_j \cap I_k)$ is the probability that the same call attempt is blocked in both the links L_j, L_k, etc. All the probabilities in eq.(1.19) can be expressed by means of an N dimensional function of the form, $q(x_1, x_2, x_3, \cdots, x_N)$ that represents the joint probability of finding x_1 slots of the frame L_1 occupied, x_2 in the frame L_2, etc. Thus, the probability that a message is blocked at the link $L_{\mu+1}$, denoted by $P(I_{\mu+1})$ is :

$$P(I_{\mu+1}) = \sum_{j=0}^{b_i-1} q_{\mu+1}(F-j)$$

with $q_{\mu+1}(x)$, the probability of finding x occupied slots in the frame $\mu+1$. Similarly, the probability that a call request is blocked because of lack of bandwidth at both the links $L_{\mu+1}, L_{\mu+2}$ is expressed as:

$$P(I_{\mu+1} \cap I_{\mu+2}) = \sum_{j=0}^{b_i-1} \sum_{l=0}^{b_i-1} q_{\mu+1,\mu+2}(F-j, F-l)$$

and

$$P(I_{\mu+1} \cap \cdots \cap I_{\nu}) = \sum_{j=0}^{b_i-1} \cdots \sum_{l=0}^{b_i-1} q(F-j, \cdots F-l)$$

The probability functions $q_{\mu+1}(.), q_{\mu+1,\mu+2}(.), \cdots$ are expressed by means of the joint probability function $q(x_1, x_2, x_3, \cdots, x_N)$. Thus :

$$q_{\mu+1}(n) = \sum_{x_1=0}^{F} \cdots \sum_{x_N=0}^{F} q(x_1, x_2, \cdots, x_\mu, n, \cdots, x_N) \tag{1.20}$$

Clearly, the calculation of blocking probability depends upon an efficient technique for calculating the joint probability $q(x_1, x_2, x_3, \cdots, x_N)$.

We define the state of the system $\mathbf{n}$ as :

$$\mathbf{n} = (n_{011}, n_{012}, \ldots, n_{01K}, \ldots, n_{N-1,N-2,K})$$

where n_{ijk} is the number of calls in progress which originate at station i, end at station j and are in class k. Also, we define the state vectors

$$\mathbf{n}_{ijk}^{-} = (n_{011}, \ldots, n_{ijk}-1, \ldots, n_{N-1,N-2,K}) \text{ and } \mathbf{n}_{ijk}^{+} = (n_{011}, \ldots, n_{ijk}+1, \ldots, n_{N-1,N-2,K})$$

Again, the sharing of the available bandwidth among the different classes is a *complete sharing* policy, since no priorities, nor bandwidth reservation modes are considered for different types of traffic. As in the previous analysis, it can be shown that the results hold for general service times distributions.

The derivation of the joint probability function $q(.)$ in eq.(1.20), begins with the local balance equation :

$$a_{\mu\nu i}\gamma_{\mu\nu i}(\mathbf{n})P(\mathbf{n}_{\mu\nu i}^{-}) = n_{\mu\nu i}P(\mathbf{n}) \quad \forall\ \mu, \nu, i \quad \text{and } \mathbf{n} \in \Omega. \tag{1.21}$$

with $a_{\mu\nu i}$ representing the load, ($\lambda_{\mu\nu i}/\mu_i$), and $\gamma_{\mu\nu i}(\mathbf{n})$ defined in Section 1.2.1. The remaining steps follow those used to obtain equations (1.1 - 1.7) above. In order to save space these will be omitted. The joint probability function can be derived from the following recursive relationship:

$$\sum_{\mu=0}^{N-1}\sum_{i=1}^{K}(b_i l_{\mu,\mu+1} a_{\mu,\mu+1,i}\, q(x_1, x_2, \ldots, x_{\mu+1}-b_i, \ldots, x_N) +$$

$$b_i l_{\mu,\mu+2} a_{\mu,\mu+2,i}\, q(x_1, x_2, \ldots, x_{\mu+1}-b_i, x_{\mu+2}-b_i, \ldots, x_N) + \cdots +$$

$$b_i l_{\mu,(\mu+N-1)modN}\, a_{\mu,(\mu+N-1)modN,i}\, q(x_1-b_i, \cdots, x_\mu, \cdots, x_N-b_i)\,) =$$

$$(x_1 + x_2 + \cdots + x_N)\, q(x_1, x_2, \cdots, x_N).$$

where the function $l_{i,j} = (j-i)modN$ in the summation takes into account the fact that traffic goes over several consecutive links. Finally, using the normalizing condition :

$$\sum_{x_1=1}^{F}\sum_{x_2=1}^{F}\cdots\sum_{x_N=1}^{F} q(x_1, x_2, \cdots, x_N) = 1$$

the values of $q(.)$ can be computed recursively.

1.3.2 Example

Consider a ring with 5 nodes accessed by two different types of traffic with the following load parameters : $a_1 : 5\cdot10^{-7}$, $a_2 : 3.33\cdot10^{-7}$, and bandwidth requirements b_1 :2 and b_2 :3 slots, respectively. Each node of the ring is accessed by both types of traffic. The destination of the messages in the system, is uniformly distributed among the 5 nodes. The total capacity of the frame at each link is 5 slots. The probabilities of the events $P(I_{\mu+1})$, $P(I_{\mu+1}\cap I_{\mu+2})$, $P(I_{\mu+1}\cap I_{\mu+2}\cap I_{\mu+3})$, and $P(I_{\mu+1}\cap I_{\mu+2}\cap I_{\mu+3}\cap I_{\mu+4})$, in eq.(1.19) concerning type 1, were found to be :

$P(I_1) : 3.3\cdot10^{-6}$ $P(I_1\cap I_2) : 1.9\cdot10^{-6}$ $P(I_1\cap I_2\cap I_3) : 9.9\cdot10^{-7}$ $P(I_1\cap I_2\cap I_3\cap I_4) : 3.3\cdot10^{-7}$

Using these values, the blocking probability of a class 1 call request from node i to nodes i+1 , i+2 , i+3 , i+4 mod N, was found to be:

$$P_{121} = 3.3 \cdot 10^{-6} \quad P_{131} = 4.6 \cdot 10^{-6} \quad P_{141} = 4.9 \cdot 10^{-6} \quad P_{101} = 4.9 \cdot 10^{-6}$$

Similarly for type 2 traffic we have :

$$P(I_1) : 8.3 \cdot 10^{-6} \quad P(I_1 \cap I_2) : 4.9 \cdot 10^{-6} \quad P(I_1 \cap I_2 \cap I_3) : 2.4 \cdot 10^{-6} \quad P(I_1 \cap I_2 \cap I_3 \cap I_4) : 8.3 \cdot 10^{-7}$$

Again, the blocking probability of a class 2 call request from node i to node i+1 , i+2 , i+3 , i+4 mod N, was found to be:

$$P_{122} = 8.3 \cdot 10^{-6} \quad P_{132} = 1.1 \cdot 10^{-5} \quad P_{142} = 1.2 \cdot 10^{-5} \quad P_{102} = 1.2 \cdot 10^{-5}$$

The computations that are involved are fairly simple so that a number of tandem connections can be handled. We recognize, however, that ring networks may have in the order of hundreds of stations, and consequently multidimensional occupancy distributions of this order are extremely difficult to evaluate. Approximate techniques to handle this situation should be considered instead. In the following paragraphs, we introduce an approximation in which the parameters of the previous example are used again to evaluate the significance of the terms $P(I_{\mu+1})$, $P(I_{\mu+1} \cap I_{\mu+2})$, $P(I_{\mu+1} \cap I_{\mu+2} \cap I_{\mu+3})$, and $P(I_{\mu+1} \cap I_{\mu+2} \cap I_{\mu+3} \cap I_{\mu+4})$, in the calculation of the blocking probability. We consider the traffic that traverses the maximum number of links (four in the previous example) in an approximation that progressively discards the terms involving the joint blocking probabilities expressed by the events $P(I_{\mu+1})$, $P(I_{\mu+1} \cap I_{\mu+2})$, $P(I_{\mu+1} \cap I_{\mu+2} \cap I_{\mu+3})$, and $P(I_{\mu+1} \cap I_{\mu+2} \cap I_{\mu+3} \cap I_{\mu+4})$, which are associated with a multidimensional occupancy distribution with dimensions 4,3, and 2 respectively.

The results are shown in Figure 1.4 in terms of type-2 blocking probability as a function of the type -2 traffic load. Curve A represents the exact blocking probability of type-2 traffic from 0 station to station 4, curve B represents the approximation in which the event of blocking because of lack of bandwidth in all four links is neglected, curve C is the approximation in which we neglect both the events (i) blocking in any three and (ii) blocking in all four links, and finally, curve D represents the assumption that all four links are independent. We observe that all the above assumptions are successful for low bandwidth utilization, but only curve B retains its closeness to the exact curve for higher than 10^{-4} traffic load of type-2.

The generalization of the ring topology is Shuffle Net [Aca87] which can be conceived of as a cylinder. The topology consists of k columns each consisting of P^k stations. Each station has P inputs and P outputs connecting to other stations plus connections to locally-generated traffic. Another possible extension of the above results applies to a network with a general topology.

1.4 Broadcasting in an ISDN switch.

A call originating at a particular switching port occupies a varying number of slots, depending on its bandwidth requirements, on each pair of input/output links. Here we consider the case in which broadcasting from any input port to all output ports is also available. We are mainly concerned with broadcasting in the following sections but a similar analysis can be carried through for multicasting switch capabilities.

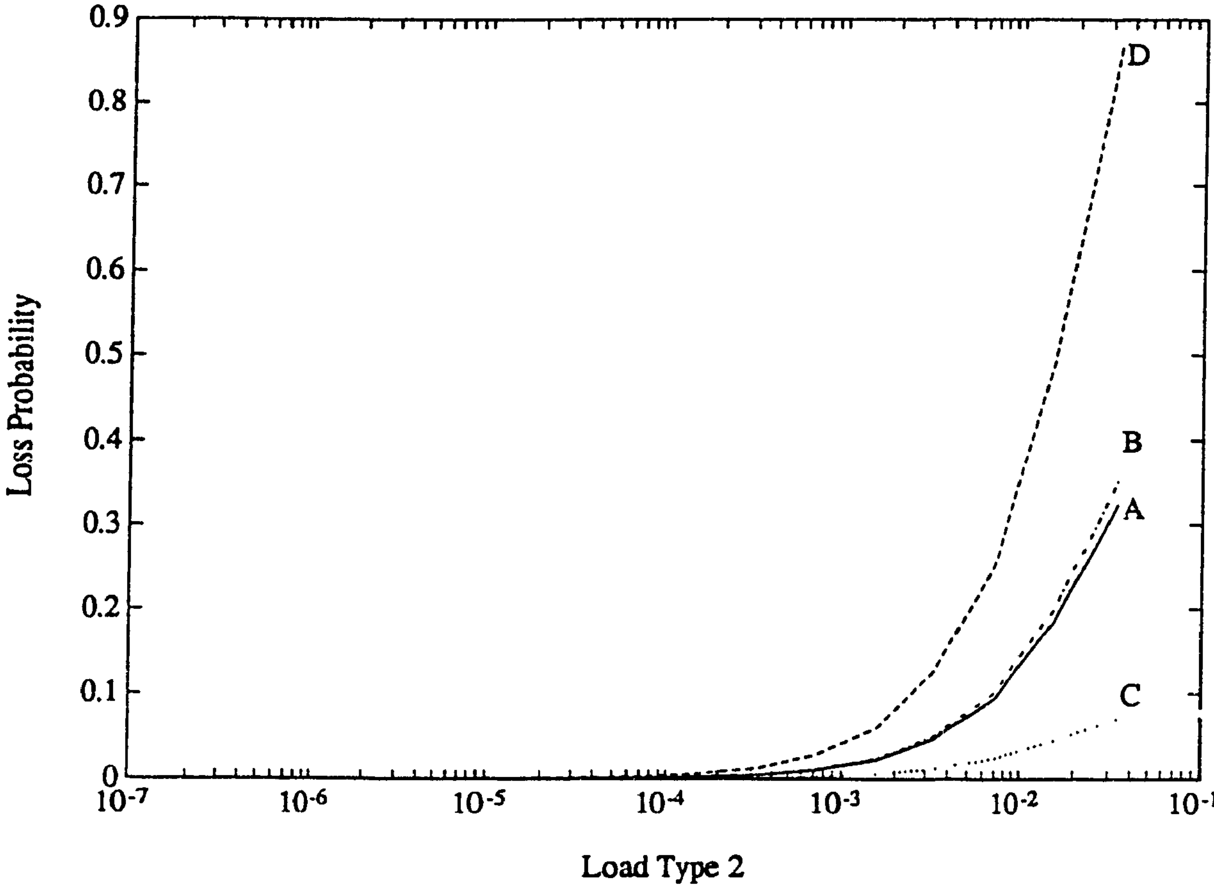


Figure 1.4. Blocking approximations in a 5 nodes' ring.

In the latter case, a particular set of the output ports is chosen as the call destination.

1.4.1 Analysis of an N_I x N_0 switch with broadcasting capabilities.

The analysis is based on the following assumptions :

1). Calls are generated at the multiplexer at a Poisson rate with an average of λ calls/second.

2). Each call falls into one of K classes of messages, which require b_i frame slots (units of bandwidth) for a random holding time τ_i. The arrival rate of each class i with source/destination pair μ, ν is represented by $\lambda_{\mu\nu i}$ calls/sec.

3). The broadcasting services are considered a separate class that uses b_i frame slots in any input frame μ and b_i slots in every output frame. This is denoted by 0 at the destination index. Consequently, the arrival rate f the broadcasting service from, say input μ of type i, is denoted by $\lambda_{\mu 0 i}$.

4). The holding time, τ_i, of any type of a call is generally distributed with mean value $1/\mu_i$.

According to this formulation, the system exhibits the product form solution. Assuming for the moment, that the service requirements of the various classes of messages are exponentially distributed, the state of the system $\mathbf{n}$ is given as:

$$\mathbf{n} = (n_{111}, n_{112}, \ldots, n_{11K}, n_{101}, \cdots, n_{10K}, \ldots, n_{N_I 0K}) \qquad (1.22)$$

The nonnegative variable $n_{\mu\nu z} \in Z_0^+$ is the number of type z calls on input port μ with output destination the port ν. Remember that destination 0 denotes broadcasting and the new state is extended by $N_i K$ elements to include the broadcasting. In the sequel, the analysis proceeds as in the previous sections, resulting in the following expressions:

$$a_{\mu\nu i} q_{\mu,\nu}(j-b_i, l-b_i) = E(n_{\mu\nu i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall \ \{j,l\} \in \boldsymbol{R}_{\mu\nu} \tag{1.23}$$

and

$$a_{\mu y i} q_{\mu,\nu}(j-b_i, l) = E(n_{\mu y i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall \ \{j,l\} \in \boldsymbol{R}_{\mu\nu} \ y \neq \nu . \tag{1.24}$$

and

$$a_{x\nu i} q_{\mu,\nu}(j, l-b_i) = E(n_{x\nu i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall \ \{j,l\} \in \boldsymbol{R}_{\mu\nu} \ x \neq \mu \tag{1.25}$$

There are two more equations involved because of the broadcasting capabilities of the switch. The first represents broadcasting from the input port μ and the second broadcasting from a port other than μ.

$$a_{\mu 0 i} q_{\mu,\nu}(j-b_i, l-b_i) = E(n_{\mu 0 i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall \ \{j,l\} \in \boldsymbol{R}_{\mu\nu} \tag{1.26}$$

$$a_{x 0 i} q_{\mu,\nu}(j, l-b_i) = E(n_{x 0 i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall \ \{j,l\} \in \boldsymbol{R}_{\mu\nu} \ x \neq \mu \tag{1.27}$$

Multiplying the above equations by b_i we get:

$$a_{\mu\nu i} b_i q_{\mu,\nu}(j-b_i, l-b_i) = b_i E(n_{\mu\nu i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall \ j \leq F_{\mu I} \ , \ l \leq F_{\nu o} \tag{1.28}$$

$$a_{\mu\nu i} b_i q_{\mu,\nu}(j-b_i, l) = b_i E(n_{\mu y i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall \ j \leq F_{\mu I} \ , \ l \leq F_{\nu o} \ y \neq \nu . \tag{1.29}$$

$$a_{x\nu i} b_i q_{\mu,\nu}(j,l-b_i) = b_i E(n_{x\nu i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall\ j \le F_{\mu I},\ l \le F_{\nu o}\ \ x \ne \mu . \tag{1.30}$$

$$a_{\mu 0 i} b_i q_{\mu,\nu}(j-b_i,l-b_i) = b_i E(n_{\mu 0 i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall\ j \le F_{\mu I},\ l \le F_{\nu o} \tag{1.31}$$

$$a_{x 0 i} b_i q_{\mu,\nu}(j,l-b_i) = b_i E(n_{x 0 i} \mid j,l) q_{\mu\nu}(j,l) \qquad \forall\ j \le F_{\mu I},\ l \le F_{\nu o} \qquad x \ne \mu \tag{1.32}$$

Then, summation of equation (1.29) over the range of output ports and the different classes, and use of eq. (1.28), (1.31) for the ν output yields :

$$\sum_{i=1}^{K}\Big(\sum_{\substack{y=1\\ y\ne\nu}}^{N_o} a_{\mu y i} b_i q_{\mu,\nu}(j-b_i,l) + a_{\mu\nu i} b_i q_{\mu,\nu}(j-b_i,l-b_i) + a_{\mu 0 i} b_i q_{\mu,\nu}(j-b_i,l-b_i)\Big) =$$

$$\sum_{i=1}^{K}\Big(\sum_{\substack{y=1\\ y\ne\nu}}^{N_o} b_i E(n_{\mu y i} \mid j,l) q_{\mu\nu}(j,l) + b_i E(n_{\mu\nu i} \mid j,l) q_{\mu\nu}(j,l)) + b_i E(n_{\mu 0 i} \mid j,l) q_{\mu\nu}(j,l)\Big) =$$

$$\sum_{i=1}^{K}\sum_{y=0}^{N_o} b_i E(n_{\mu y i} \mid j,l) q_{\mu\nu}(j,l) = E\Big(\sum_{i=1}^{K}\sum_{y=0}^{N_o} n_{\mu y i} b_i \mid_{j,l}\Big) q_{\mu\nu}(j,l) = j q_{\mu\nu}(j,l) \tag{1.40}$$

Similarly, from eq. (1.30), (1.28), (1.32), (1.31),

$$\sum_{i=1}^{K}\Big(\sum_{\substack{x=1\\ x\ne\mu}}^{N_I} a_{x\nu i} b_i q_{\mu,\nu}(j,l-b_i) + a_{\mu\nu i} b_i q_{\mu,\nu}(j-b_i,l-b_i)\Big) +$$

$$\sum_{i=1}^{K}\Big(\sum_{\substack{x=1\\ x\ne\mu}}^{N_I} a_{x 0 i} b_i q_{\mu,\nu}(j,l-b_i) + a_{\mu 0 i} b_i q_{\mu,\nu}(j-b_i,l-b_i)\Big) =$$

$$\sum_{i=1}^{K}\Big(\sum_{\substack{x=1\\ x\ne\mu}}^{N_I} [a_{x\nu i} + a_{x 0 i}]\, b_i q_{\mu,\nu}(j,l-b_i) + [a_{\mu\nu i} + a_{\mu 0 i}] b_i q_{\mu,\nu}(j-b_i,l-b_i)\Big) =$$

$$\sum_{i=1}^{K}\sum_{x=1}^{N_I} b_i E(n_{x\nu i} \mid j,l) q_{\mu\nu}(j,l) + \sum_{i=1}^{K}\sum_{x=1}^{N_I} b_i E(n_{x0i} \mid j,l) q_{\mu\nu}(j,l) =$$

$$E(\sum_{i=1}^{K}\sum_{x=1}^{N_I} [\, n_{x\nu i} + n_{x0i} \mid b_i \mid_{j,l}) q_{\mu\nu}(j,l) = l q_{\mu\nu}(j,l). \tag{1.34}$$

The probability function $q_{\mu\nu}(j,l)$ can be found from the recursive relations above. Adding eq.(1.33) and (1.34) as before we get :

$$\sum_{i=1}^{K}\Big(\sum_{\substack{y=1 \\ y\neq\nu,0}}^{N_o} a_{\mu y i} b_i q_{\mu,\nu}(j-b_i,l) + \sum_{\substack{x=1 \\ x\neq\mu}}^{N_I} [a_{x\nu i} + a_{x0i}]\, b_i q_{\mu,\nu}(j,l-b_i) + 2\cdot[a_{\mu\nu i} + a_{\mu 0 i}]\, b_i q_{\mu,\nu}(j-b_i,l-b_i)\Big) =$$

$$= j q_{\mu\nu}(j,l) + l q_{\mu\nu}(j,l) = (j+l) q_{\mu\nu}(j,l). \tag{1.35}$$

Similarly, as in Section 1.2.2, the following equations serve bounding and normalization purposes. Thus, we define:

$$q_{\mu\nu}(j,l) = 0 \quad i < 0 \vee j < 0$$

$$q_{\mu,\nu}(i,\cdot) = 0 \quad i > F_{\mu I}$$
$$q_{\mu,\nu}(\cdot,j) = 0 \quad j > F_{\nu o}$$

and

$$\sum_{j=0}^{F_I}\sum_{l=0}^{F_o} q_{\mu\nu}(j,l) = 1 \tag{1.36}$$

Having completed the derivation of the occupancy distribution, the loss probability for type-i messages, is given by :

$$P_{b_{\mu\nu i}} = P(I_{\mu i}) + P(I_{\nu i}) - P(I_{\mu i} \cap I_{\nu i}) \tag{1.37}$$

with the individual terms in the summation expressed in terms of the two-dimensional occupancy distribution in equation (1.35).

Note, that the above expression is not applicable when blocking of the broadcasting classes is of interest. In the latter case an (N_o+1)-dimensional occupancy distribution should be computed, that takes into account the input port and all the output port occupancies. We follow the same technique that was used for the computation of the multidimensional occupancy distribution in Section 1.3.1. It is easy to verify that the recursive expression for a broadcasting switch has the form :

$$\begin{aligned}
&\sum_{i=1}^{K}\Big(\sum_{y=1}^{N_o} a_{\mu y i} b_i q(j-b_i, l_1, \cdots l_\nu - b_i, \cdots l_{N_o}) + a_{\mu 0 i} b_i q(j-b_i, l_1-b_i, \cdots l_{N_o}-b_i)\Big) + \\
&\sum_{\nu=1}^{N_o}\Big(\sum_{i=1}^{K}\Big(\sum_{\substack{x=1\\ x\neq\mu}}^{N_I} a_{x\nu i} b_i q(j, l_1, \cdots l_\nu - b_i, \cdots l_{N_o}) + a_{\mu\nu i} b_i q(j-b_i, l_1, \cdots l_\nu - b_i, \cdots l_{N_o})\Big)\Big) + \\
&\sum_{i=1}^{K}\Big(\sum_{\substack{x=1\\ x\neq\mu}}^{N_I} a_{x0i} b_i q(j, l_1-b_i, \cdots l_{N_o}-b_i) + a_{\mu 0 i} b_i q(j-b_i, l_1-b_i, \cdots l_{N_o}-b_i)\Big) = \\
&(j + l_1 + l_2 + \cdots l_{N_o}) \cdot q(j, l_1, \cdots l_\nu, \cdots l_{N_o})
\end{aligned} \tag{1.38}$$

Given the above recursion, exact blocking probabilities for the broadcasting traffic are derived by means of the expression in equation (1.19). Also, the blocking probability of the broadcasting calls can always be upper-bounded by the union bound; that is, the probability that a call is blocked in any of the input/output ports it accesses. The upper bound is directly derivable from the two-dimensional recursion in equation (1.35).

1.4.2 Example.

We consider a 4x4 switch, with $F_I = F_o = 6$ slots/frame and two classes of traffic at each input port. The destination ports are equally probable. Class 1 is point-to-point traffic with bandwidth requirements $b_1 = 2$ slots/frame, and class 2 is broadcasting traffic, again with bandwidth requirements $b_2 = 2$ slots/ frame and a_2=0.005. Figure 1.5 plots the blocking probabilities for both the types of traffic as well as the upper bound for the broadcasting traffic, in terms of the traffic load per input/output port a_1 of the point-to-point traffic. Curve A is the upper bound blocking, B represents the broadcasting blocking and C the blocking experienced by the point-to-point traffic. It is clear, as expected, that the broadcasting traffic has a higher blocking probability than the point-to-point traffic and that the union bound is close to the exact solution over the underload range in particular.

1.5 Conclusions.

We have presented an extension of the work of Kaufman and Roberts who considered blocking of traffic composed of classes with different bandwidth requirements. We considered tandem links and broadcasting/multicasting classes of traffic. We worked out specific cases of an N_I x N_0 switch, an N node ring and an N_I x N_0 switch with broadcasting capabilities. The complexity of the technique proposed above increases exponentially with the dimensions of the occupancy distribution.

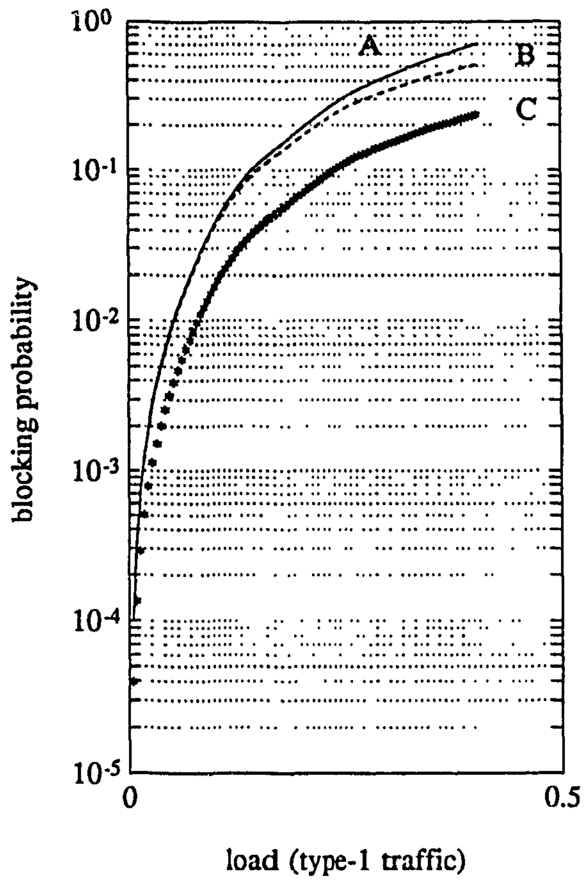


Figure I.5. Broadcasting in a 4x4 switch.

CHAPTER II.

Burst Control in Statistical Multiplexing: Admission Controls

This chapter contains an exact analysis of the blocking conditions at the burst level, experienced by different types of traffic in an ATM network. A basic assumption is that traffic is generated by a finite set of independent two-state sources that share equally the available bandwidth. This is the extension we introduce to the results derived in the previous chapter, in order to make them applicable to a traffic environment that encompasses bursty and variable-rate traffic. When all two-state sources are active, the bandwidth in demand exceeds the capacity of one or more links in the network.

In this chapter, we show, using reversibility arguments, that the product-form solution is valid for our model. Then, we give a recursion that reduces the multidimensional birth-death process of our original model into a one-dimensional birth-death like system. Thus, we calculate blocking probabilities by means of a recursive relation that simplifies considerably the complexity of the problem. In the sequel, the analysis is extended to a two-dimensional case, that models a switch and a similar recursion is suggested. The above results supply the analytical basis for an admission-control mechanism that guarantees a predefined performance threshold for all the classes in the system.

Being a solution to a multiple classes problem the present work is considered a multi-class generalization of an M/M/k/k/N (with k the queue capacity and N the number of sources) finite-source queueing system and its efficiency lies on the linearity of the necessary computations.

2.1 Introduction

A broadband integrated network will be required to provide service to present and future applications. Flexibility is one of the most important features of such a network, since the range of the supported services may have a wide spectrum of traffic characteristics, including average bit rate, peak rate, interarrival time, burst length etc. Traffic may also differ with respect to performance requirements, primarily delay and loss probability.

Broadband packet switching or ATM [GAC86], [Tur86] is most likely to be the transport technique for a broadband network . Its implementation implies handling of a large range of traffic types, including constant rate, bursty and smoothly variable traffic. Thus, bandwidth should be allocated according to the individual needs of the supported services with a guaranteed grade of service. This flexibility is mainly attained through the enormous bandwidth of an end-to-end fiber-optic network and statistical multiplexing, a technique that allocates bandwidth on demand.

This chapter discusses blocking probabilities at the burst level of a finite-source model. The need arises because sources like variable-rate video coders and graphics stations will be integrated in the network and exhibit traffic characteristics that cannot be modeled by a Poisson process. Instead the two-state model which was previously employed in the description of compressed full-motion video and voice sources with silence removal, can adequately describe the burstiness of these types of traffic [MASKR88]. In Section 2.2.1, we give a formal description of the arrival process as a collection of binomial discrete sources that, when in the active mode require a certain amount of bandwidth for a random time interval.

In Section 2.2.2, we establish the product-form solution [KI75] for a mixture of binary sources with distinct bandwidth and service requirements. In Section 2.2.3, we calculate burst-blocking probabilities by means of a recursion based on the occupancy distribution. These results can be used in sizing the multiplexer's capacity for a required grade of service and draw the decision boundaries for acceptance or rejection of a call request for a given mixture of traffic as well. An example of a congestion-control technique based directly on the results in Section 2.2.3 is given in Section 2.2.5. A further generalization to a two-dimensional recursion that covers the case of a broadband network with an $N_I x N_o$ internally-nonblocking switch in its core, is briefly discussed in Section 2.3. In Section 2.4, we consider a Poisson approximation to the finite-source model adopted throughout this Chapter. Finally, in Section 2.5, we introduce an approximation that yields end-to-end loss probabilities in a network employing various levels of statistical multiplexing.

Our main contribution in this chapter is the extension of the analysis in the first chapter to include the finite-source model. Subsequently, we derived call-admission regions for various mixtures of traffic and compared our exact results with those derived by means of an analytical approximation (Chernoff bound in [Hui88]) and simulation [WoKo90].

2.2 Statistical multiplexing and the network topology.

Statistical multiplexing is the appropriate form of multiplexing when dealing with variable-rate and bursty types of traffic, since bandwidth is assigned according to their temporal needs. The sum of the peak rates of the sources exceeds the multiplexer's capacity but the burstiness of the sources allows their integration with a variable degree of burst loss. Because of the communication speeds that are involved, only a limited amount of storage is available; not enough to store a burst until line capacity becomes available. Thus, if the necessary transmission bandwidth is not available, bursts are dropped. This results in either refusal of new connections or deterioration of the quality of calls in progress [MASKR88]. The bandwidth of the multiplexer (F bandwidth units here), is to be divided among the various traffic classes on a fully-shared basis.

Our analysis uses a specific case as an example of an integrated network but can be extended to cover a network of a general structure employing an arbitrary number of multiplexers and switches. An ATM [GAC86] based public network might be end-to-end ATM with statistical multiplexers of increasing capacity at all the stages of the network or, it can have the topology depicted in Figure 2.1, employing statistical multiplexing only at the first level of the user interface [Wo89]. There is always a trade-off between the advantages in bandwidth utilization in statistical multiplexing and the simplicity of circuit switching. This topology implies that the link utilization and, consequently, the mixture of traffic is such that the output of the multiplexer approaches a nearly constant rate stream. We adopt the second alternative (Figure 2.1) as our network model under the assumption that the multiplexers are the only point of contention where a burst can be

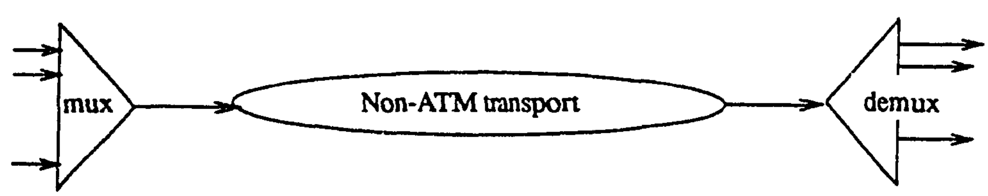


Figure 2.1 ATM access in a broadband network.

lost. Then, we attempt to establish the grade of service for such a network, in terms of burst-loss probability for the various types of traffic in the network.

2.2.1 The traffic model and access characteristics.

Various approaches to modeling the arrival process have been suggested. In [SrWh86] the superposition of arrival processes has been modeled as a renewal process. A more frequently-used approach is to approximate the arrival process by a Markov - modulated Poisson process (MMPP). According to this model, the aggregate arrival rate is governed by the evolution of a discrete-space Markov process; when in state i, calls are generated according to a Poisson process of rate λ_i. The state of the system is then Markovian and the equilibrium equations can be solved algorithmically (e.g. using matrix-geometric techniques, Section 3.3). In [HeLu86], a two-state MMPP is proposed, where the four parameters, that is, the state transition rates and the two arrival intensities, are chosen to match four arrival process characteristics. In an other approach, Ide models the superposition of N binary sources as an N-state MMPP with arrival rate proportional to the number of active sources [Ide88].

In the model, we adopt here for variable-rate types of traffic typified by compressed full-motion video, the actual aggregate rate of $\bar{N}$ variable-rate sources, is quantized into N finite discrete levels and sampled at random Poisson instances [MASKR88] (Figure 2.2). The result is a discrete finite-state, continuous-time Markov process that can be represented by the birth-death Markovian process depicted in Figure 2.3b. Thus, $\bar{N}$ video sources are represented by N binary sources, ($N \geq \bar{N}$), each alternating between an

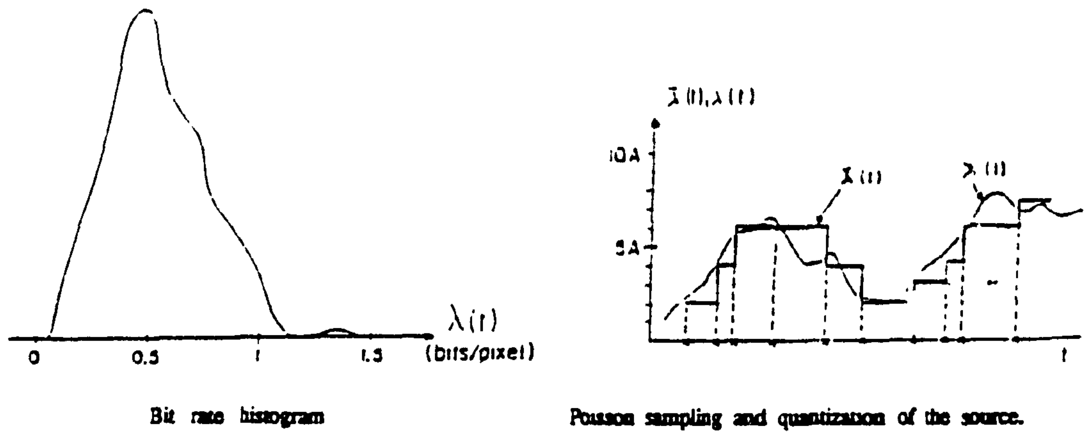


Figure 2.2 Sampling and quantization of the aggregate video traffic (from [MASKR88]).

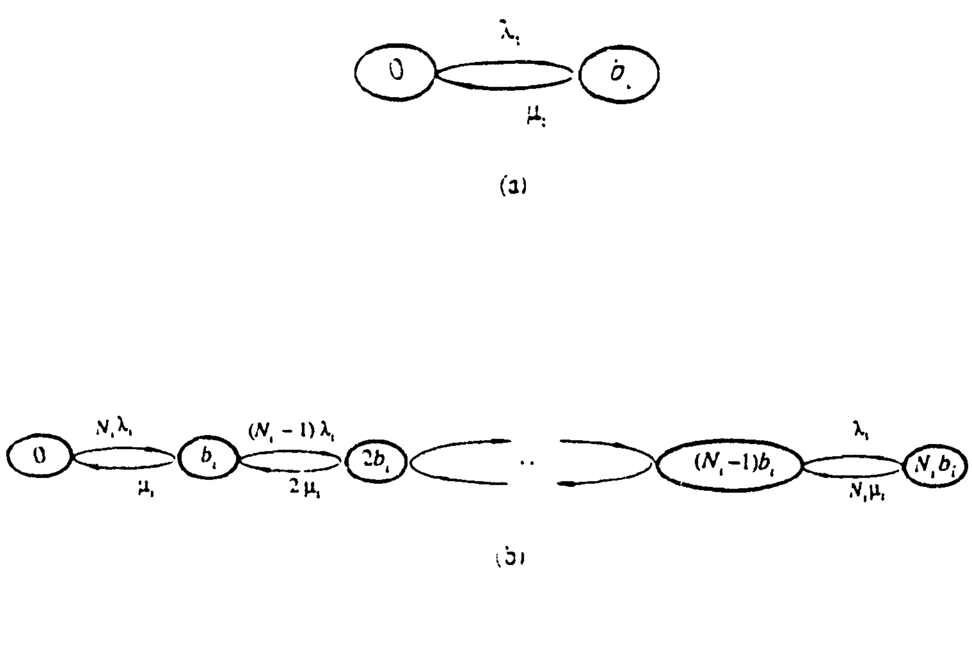


Figure 2.3 Type_i birth-death arrival process.

inactive state in which no bandwidth demands are posed and an active state in which b bits/sec are requested. The transition of a binary source to the active mode constitutes a burst. The model for voice traffic and for uncompressed video is the same as the one above with $N = \bar{N}$. We assume that a number of other types of bursty traffic can be modeled similarly by fitting the birth-death rates in Figure 2.3b to the mean and autocovariance of the aggregate process. The result is a multidimensional birth-death process that characterizes the multi-class arrival process, with transition rates between adjacent states depicted in Figure 2.4.

We outline here the basic traffic assumptions in this chapter:

• Bursts are generated at the multiplexer from a set of discrete independent sources that exhibit the binomial behavior shown in Figure 2.3a.

• Each burst falls into one of K types or classes, and requires b_i bits/sec, for a random holding time τ_i. N_i is the number of sources in class i and λ_i is the rate with which each type i source switches to an active state (Figure 2.5).

• The probability density function of the burst duration τ_i, for every discrete source in class i, is exponentially distributed with mean value $1/\mu_i$.

The multiplexer has a finite capacity of F bandwidth units which are fully shared among all types of bursts on a FCFS basis. Bursts whose bandwidth requirements cannot be satisfied are blocked and depart without further affecting the system. In contrast, bursts in progress have priority over new arrivals and retain their allocated bandwidth for their duration. The objective is to compute the steady-state blocking probability experienced

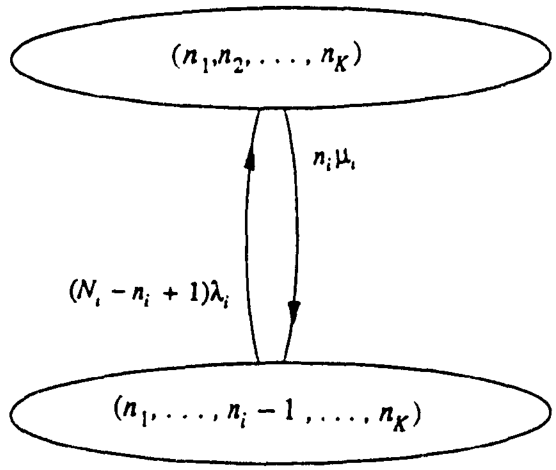


Figure 2.4 State transition rates in the arrival process.

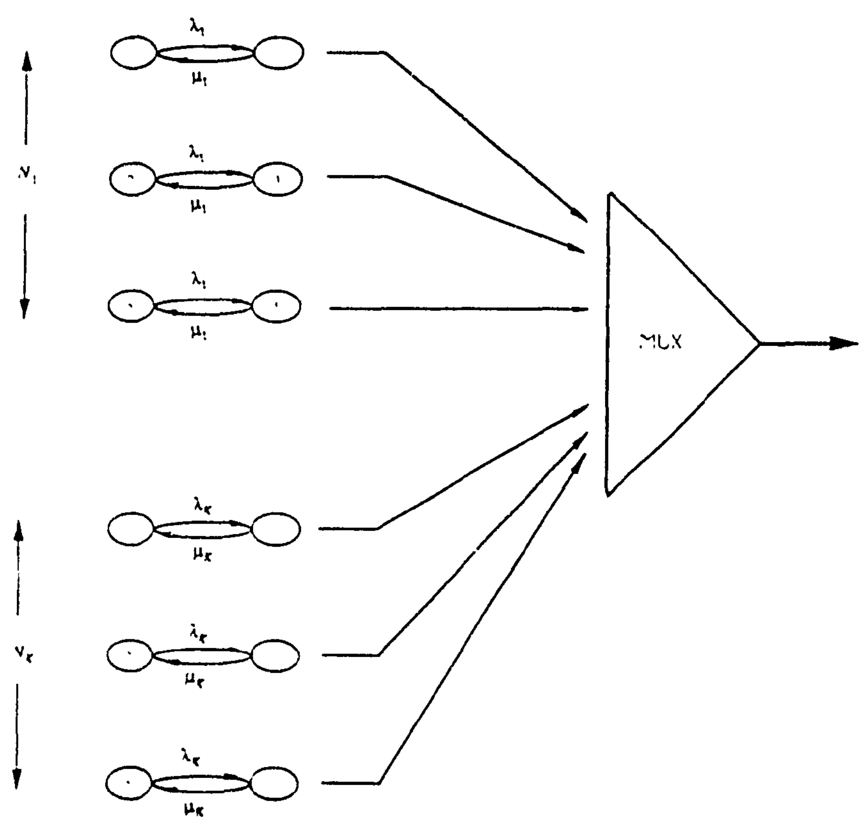


Figure 2.5 The arrival process model.

by a burst with bandwidth requirement b_i.

2.2.2 Local Balance Equation.

Since the durations of the various classes of bursts are exponentially distributed, the state of the system **n** is :

$$\mathbf{n} = (n_1, n_2, \ldots, n_i, \ldots, n_K).$$

The nonnegative integer n_i is the number of type i bursts in the system, that is, the number of type i binary sources in the active mode. Also, we define the state vectors

$$\mathbf{n}_i^- = (n_1, \ldots, n_i - 1, \ldots, n_K) \quad \text{and} \quad \mathbf{n}_i^+ = (n_1, \ldots, n_i + 1, \ldots, n_K).$$

We consider a *complete sharing* policy and denote by Ω the set of allowable states determined by the above policy. Then, a state $\mathbf{n} \in \Omega$ satisfies the following condition :

$$\sum_{i=1}^{K} n_i b_i \leq F \quad \text{with } n_i \leq N_i; \quad \forall\ i \leq K.$$

R is the set of allowable occupancies under the constraint above, that is :

$$R = \{\ j\ ;\ \mathbf{n} \cdot \boldsymbol{b} = j \quad \forall\ \mathbf{n} \in \Omega\} \quad \text{with} \quad \boldsymbol{b} = (b_1, \ldots, b_K)$$

We use the same methodology as in chapter I to establish the local balance equation in the form:

$$(N_i - n_i + 1) a_i \gamma_i(\mathbf{n}) P(\mathbf{n}_i^-) = n_i P(\mathbf{n}) \qquad \forall\ i \quad \text{and } \mathbf{n} \in \Omega. \tag{2.1}$$

with $a_i = \lambda_i / \mu_i$, and

$$\gamma_i(\mathbf{n}) = \begin{cases} 1 & \text{if } n_i \geq 1 \\ 0 & \text{if } n_i = 0 \end{cases}$$

In Appendix B, lemma I we prove that our model, represented from the state diagram in Figure 2.4, has the product-form solution [Lam77],[Kel79]. A solution to the local balance equation in (2.1) is given by:

$$P(\mathbf{n}) = G(\Omega)^{-1} \prod_{i=1}^{K} \begin{bmatrix} N_i \\ n_i \end{bmatrix} a_i^{n_i} \tag{2.2}$$

with $G(\Omega)$ the normalization constant :

$$G(\Omega) = \sum_{\mathbf{n} \in \Omega} \left(\prod_{i=1}^{K} \begin{bmatrix} N_i \\ n_i \end{bmatrix} a_i^{n_i} \right)$$

Notice that the ratio $P(\mathbf{n}) / P(\mathbf{n}_i^-)$ is the local balance equation in (2.1). The solution is unique, since it constitutes a solution to an ergodic Markovian process [Kl75] and it satisfies the global balance equation as well. Now, the blocking probability for each class i is given by (see also [Hui89]):

$$P_{bi} = \sum_{\mathbf{n}:\mathbf{n}\cdot\mathbf{b} > F-b_i} P(\mathbf{n})$$

A more computationally-efficient solution is given in the next section by means of the occupancy distribution $q(j)$, that represents the probability that j out of F bandwidth units are being held.

2.2.3 Calculation of the Blocking Probability.

The function $q(j)$ is defined as :

$$q(j) = \sum_{\mathbf{n} \in S_j} P(\mathbf{n}).$$

with $P(\mathbf{n})$ the probability of the state $\mathbf{n}$ and the set $S_j \subset \Omega$, with $j \in R$, defined as :

$$S_j = \{ \ \mathbf{n} : \sum_{i=1}^{K} n_i b_i = j \qquad \mathbf{n} \in \Omega \ \}.$$

Note here that j in the above expression is not an arbitrary integer but represents a nominal occupancy value as a result of one or more states in Ω. Summing the local balance equation in (2.1) over S_j we get:

$$N_i a_i \sum_{\mathbf{n} \in S_j} \gamma_i(\mathbf{n}) P(\mathbf{n}_i^-) - a_i \sum_{\mathbf{n} \in S_j} (n_i - 1)\gamma_i(\mathbf{n}) P(\mathbf{n}_i^-) = \sum_{\mathbf{n} \in S_j} n_i P(\mathbf{n}). \tag{2.3}$$

The LHS of this equation is :

$$LHS = N_i a_i \sum_{\mathbf{n} \in S_j} \gamma_i(\mathbf{n}) P(\mathbf{n}_i^-) - a_i \sum_{\mathbf{n} \in S_j} (n_i - 1)\gamma_i(\mathbf{n}) P(\mathbf{n}_i^-)$$

$$= N_i a_i \sum_{\mathbf{n} \in S_j \cap \{n_i \geq 1\}} P(\mathbf{n}_i^-) - a_i \sum_{\mathbf{n} \in S_j \cap \{n_i \geq 1\}} (n_i - 1) P(\mathbf{n}_i^-). \tag{2.4}$$

But,

$$S_j \cap \{n_i \geq 1\} = \{\mathbf{n} : \sum_{z \neq i} n_z b_z + (n_i - 1) b_i = j - b_i \quad n_i \geq 1, n_z \geq 0, z \neq i\}$$

Now, we substitute in equation (2.4) the variable

$$\hat{n}_j = \begin{cases} n_j & j \neq i \\ n_j - 1 & j = i \end{cases}$$

and the left part of equation (2.3a) becomes :

$$LHS = N_i a_i \sum_{\mathbf{n} \in S_j \cap \{n_i \geq 1\}} P(\mathbf{n}_i^-) - a_i \sum_{\mathbf{n} \in S_j \cap \{n_i \geq 1\}} (n_i - 1) P(\mathbf{n}_i^-) =$$

$$N_i a_i \sum_{\hat{n} \in S_{j-b}} P(\hat{n}) - a_i \sum_{\hat{n} \in S_{j-b}} \hat{n}_i P(\hat{n}) \qquad (2.5a)$$

where $q(j)$ is the probability of finding j BUs occupied.

The first term in equation (2.5a) is :

$$N_i a_i \sum_{\hat{n} \in S_{j-b}} P(\hat{n}) = N_i a_i q(j - b_i)$$

whereas the second term in the above equation is :

$$a_i \sum_{\hat{n} \in S_{j-b}} \hat{n}_i P(\hat{n}) = a_i \sum_{\hat{n} \in S_{j-b}} \hat{n}_i \frac{P(\hat{n})}{q(j-b_i)} q(j-b_i)$$

Noting that $P(\mathbf{n} \mid j)$, the conditional probability of the system being in state $\mathbf{n}$ given j the number of occupied BUs , can be expressed as :

$$P(\hat{n} \mid j - b_i) = \begin{cases} P(\hat{n})/q(j-b_i) & \text{for } \hat{n} \in S_{j-b_i} \\ 0 & \text{otherwise} \end{cases}$$

we obtain :

$$\sum_{\mathbf{n}\in S_j} \hat{n}_i P(\hat{\mathbf{n}} \mid j-b_i) q(j-b_i) = E(\hat{n}_i \mid j-b_i) q(j-b_i) \qquad \forall \quad j \in \boldsymbol{R}.$$

with $E(\hat{n}_i \mid j-b_i)$, the expected number of $\hat{n}_i$ conditioned on the occupancy j. Substitution in equation (2.5a) yields :

$$LHS = N_i a_i q(j-b_i) - a_i E(\hat{n}_i / j-b_i) q(j-b_i) \quad j \geq b_i, \; j \in R. \tag{2.5b}$$

Similarly, the right-hand side of equation (2.3) is written as :

$$RHS = (\sum_{\mathbf{n}\in S_j} n_i P(\mathbf{n} \mid j)) q(j) = E(n_i \mid j) q(j) \qquad \forall \quad j \subset \boldsymbol{R}. \tag{2.5c}$$

with $E(n_i \mid j)$, the expected number of n_i conditioned on the occupancy j. Combining eq. (2.5b) and (2.5c) we get :

$$[\, N_i \; - \; E(\hat{n}_i \mid j-b_i) \,]\, a_i q(j-b_i) = E(n_i \mid j) q(j) \qquad \forall \quad j \in \boldsymbol{R}$$

Multiplying the above equation by b_i and summing over K, we have:

$$\sum_{i=1}^{K} [\, N_i \; - \; E(\hat{n}_i \mid j-b_i) \,]\, a_i b_i q(j-b_i) = E(\sum_{i=1}^{K} b_i n_i \mid j) q(j) = j q(j). \qquad \forall \quad j \in \boldsymbol{R} \tag{2.6}$$

Note that in the above expression the expectation in the LHS is not known. One way to overcome this difficulty is by means of a simple transformation. In Appendix B, lemma II we prove that two stochastic systems with :

i). the same set of states

ii). the same parameters K, N_i, a_i ; $\quad \forall \; i \leq K$

are equivalent, in the sense that they yield the same blocking probabilities. The bandwidth requirements for the various classes and the transmission bandwidth F enter the calculations for blocking only through the policy Ω. Since we have the freedom to choose the bandwidth requirements and the multiplexer's capacity so that conditions (i) and (ii) are valid, we choose these parameters in such a way that they a yield a stochastic system with states characterized by unique occupancy. This is purely a matter of convenience since the b_i 's can be as close to reality as we please to peak bandwidth demands made by real sources. For example, in the case of two types of traffic, we choose b_i's so that they are prime to one another and their product exceeds the multiplexer's capacity. Thus, every state in Ω has a unique occupancy, therefore the term $E(\hat{n}_i \mid j-b_i)$ reduces to $n_i - 1$, and for such a system the recursion in equation (2.6) is given by :

$$\sum_{i=1}^{K} [\, N_i \; - \; n_i + 1 \,]\, a_i b_i q(j-b_i) = jq(j). \qquad \forall \quad j \leq F \tag{2.7}$$

for any $j \leq F$ (see Appendix B, lemma III). Finally, using the normalizing condition :

$$\sum_{j=1}^{F} q(j) = 1$$

the values of $q(.)$ can be computed recursively. Then the blocking probability for the i^{th} class is:

$$P_{bi} \;=\; \sum_{\mathbf{n} \,\in\, B_i^+} P(\mathbf{n}) \;=\; \sum_{i=0}^{b_i-1} q(F-i) \qquad \forall\, i \leq K. \tag{2.8}$$

Note also that in the case that the number of binomial sources approaches infinity, the recursion in equation (2.7) reduces to :

$$\sum_{i=1}^{K} a_i b_i q(j-b_i) = jq(j). \qquad \forall \quad j \le F \tag{2.9}$$

since the superposition of renewal processes results in a Poisson process [SrWh86] and Kaufman's result applies naturally. Thus, the recursion in equation (2.7) applies for a mixture of Poisson and non-Poisson types of traffic as well. Note though, that the calculation for the finite-source model requires enumeration and processing on the state space, and consequently its complexity is increased relatively to the Kaufman/ Roberts recursion for the infinite (Poisson) model.

2.2.4 Example 1

Consider a multiplexer with bandwidth $F = 7$ Kbits/sec, being accessed by two different types of traffic with the following load parameters : a_1 : 0.10, a_2 : 0.20, and bandwidth requirements b_1 :2 and b_2 :1 Kbits/sec characterizing the binary sources of type 1 and 2, respectively. The total number of type 1 sources is $N_1 = 10$ and the number of type 2 sources is $N_2 = 10$. The equivalent stochastic system we choose has bandwidth requirements $b_1 = 11$, $b_2 = 5$ and capacity $F = 38$. It is easy to see that this system has the same set of states but each state has a distinct occupancy. The state space and the respective occupancies for these two systems are shown in the following table :

n1	n2	j		n1	n2	j
0	0	0		0	0	0
0	1	1		0	1	5
0	2	2		0	2	10
0	3	3		0	3	15
0	4	4		0	4	20
0	5	5		0	5	25
0	6	6		0	6	30
0	7	7		0	7	35
1	0	2		1	0	11
1	1	3		1	1	16
1	2	4		1	2	21
1	3	5		1	3	26
1	4	6		1	4	31
1	5	7		1	5	36
2	0	4		2	0	22
2	1	5		2	1	27
2	2	6		2	2	32
2	3	7		2	3	37
3	0	6		3	0	33
3	1	7		3	1	38

Solving equation (2.7) for these parameters, and putting $q'(0) = 1$, the unnormalized values for the $q(.)$ function were found to be :

$q'(\cdot)$													
1.000	0	0	0	0	2.000	0	0	0	0	1.800	1.000	0	0
0	0.960	2.000	0	0	0	0.336	1.800	0.450	0	0	0.081	0.960	0.900
0	0	0.013	0.336	0.810	0.120	0	0.001	0.081	0.432	0.240			

(the first row contains the values of $q(.)$ in the range 0–13, the second row is the range 14–27, and the third row is the range 28–38). The normalized values of $q(j)$ are given from the expression

$$q(j) = q(0) \cdot q'(j) \qquad j \le F$$

with

$$q(0) = 1 / \sum_{j=0}^{F} q'(j).$$

Thus, $q(j)$ has the following values:

$q(\cdot)$													
0.065	0	0	0	0	0.130	0	0	0	0	0.117	0.065	0	0
0	0.062	0.130	0	0	0	0.022	0.117	0.029	0	0	0.005	0.062	0.058
0	0	0.001	0.022	0.053	0.008	0	0.000	0.005	0.028	0.016			

Then the burst-loss probabilities for classes 1 and 2 are found from equation (2.8)

$$P_{b_1} = 0.133 \qquad P_{b_2} = 0.049$$

2.2.5 A congestion-control technique.

Congestion-control techniques can be based directly on the burst-loss probabilities in eq. (2.8), by drawing multidimensional call-admission regions, where the burst-loss probability is acceptably small, less than a predefined level ε. Then, acceptance of a type i call moves the operation point along the i^{th} axis, closer to the boundary, which for as long as it is not violated, guarantees the predefined grade of service (GOS).

As an example consider two types of traffic, compressed full-motion video terminals and graphics stations, sharing the multiplexer's bandwidth in a case presented in [Hui88]. The peak rate is 10 *Mbits/s* for both types of traffic and their average rates are 5 *Mbits/sec* and 1 *Mbits/s* for the video and graphics calls, respectively. The multiplexer's capacities are 100, 200, 400 *Mbits/s*, respectively, with utilization defined to be the number of calls of each type times their average rate, divided by the total available capacity.

Figure 2.6a plots the admissible region for a 10^{-8} burst loss for both the types of traffic. We observe here the concavity of the boundary as in [Hui88] and [WoKo90], and the fact that bursty traffic benefits less from statistical multiplexing than variable-rate traffic. By choosing the values for the two types of traffic to be the same as those in Hui's example, we also tested the accuracy of the suggested approximation in [Hui88]. The results of our analysis are in excellent agreement with those of Hui, suggesting that the accuracy of the modified Chernoff bound is high, for the examined probability range.

We also notice that the boundary becomes more linear as the peak-to-link ratio of the supported types of traffic decreases. This event has significant implications to the control mechanisms. Given that the peak-to-link ratio is such that the boundary is linear, we can always substitute the same number of calls of one type for the same number of calls of the other type. This means that all we need to do in order to assign certain bandwidth to a connection for a given grade of service (GOS), is to load the multiplexer with one type of traffic only and then divide the multiplexer's capacity by the number of supported calls. The resulting 'virtual bandwidth characterization' of the source is going to be valid for a mixture of traffic as well, as a result of the linearity of the boundary.

The call-admission regions of two bursty types of traffic is shown in Figure 2.6b for three multiplexing capacities. We assume that both have an average rate of 1 *Mbits/s* and peak rates 10 and 20 *Mbits/s*, respectively. The above types of traffic (graphics I and II) access a statistical multiplexer with capacity 100, 200, and 400 *Mbits/s*. The burst-loss

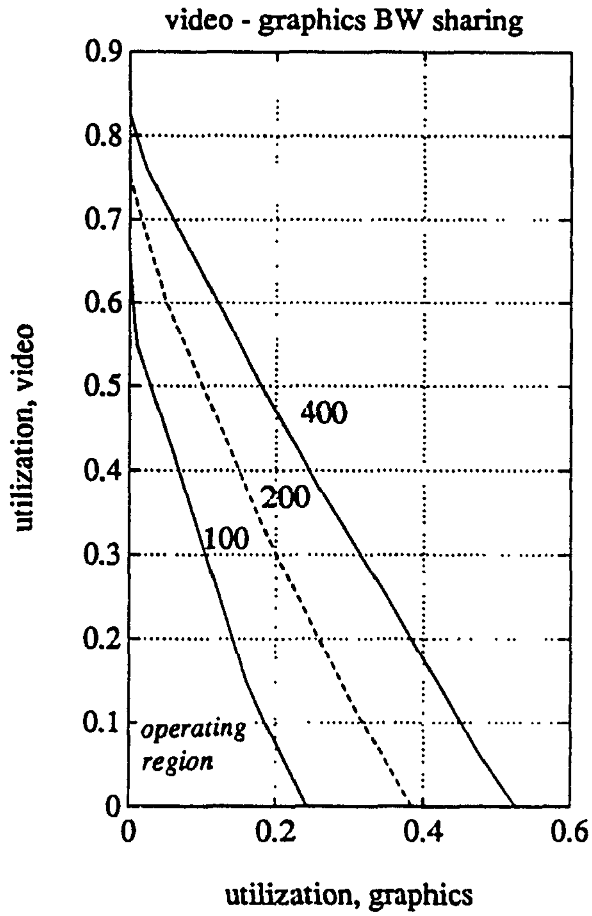


Figure 2.6a Admission call region, variable rate video/graphics traffic.

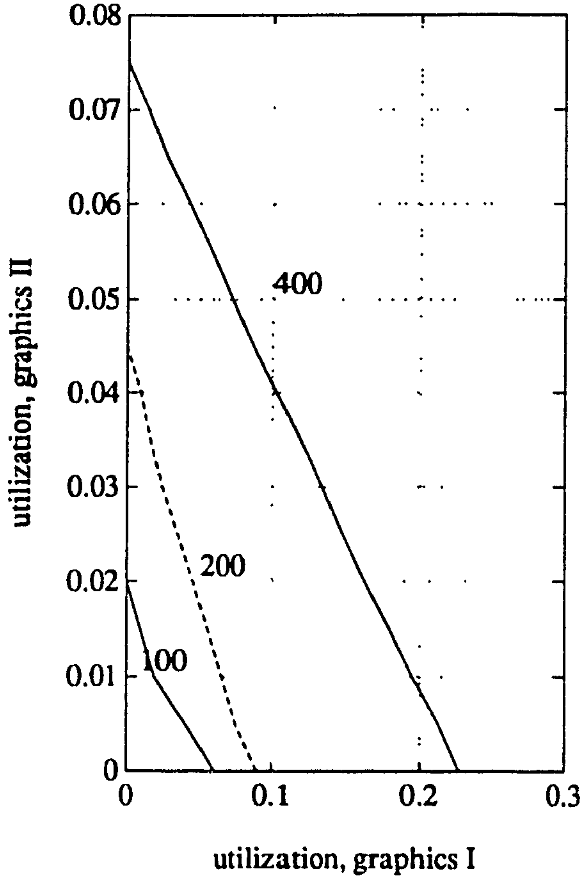


Figure 2.6b Admission call region, two types of bursty traffic.

probability is at most 10^{-6} and the operation region is the area bounded by the axes and the respective boundary that guarantees the predefined performance threshold (10^{-6}).

We note the low-bandwidth utilization levels as the result of the bursty nature of both types of traffic in comparison with the previous example. The linearity of the boundary becomes obvious again as the peak-to-link ratio decreases.

The advantage of statistical multiplexing over a non-statistical mode of operation is increased bandwidth utilization, due to the bursty nature of the carried traffic. However, the disadvantages associated with it are information loss and relatively complex flow-control schemes that are avoided in a non-statistical mode of operation. In the trade-off between complexity of controls and utilization gains, knowledge of the occupancy distribution $q(.)$ can be applied directly in choosing architectural features for the network, for example, how many levels of statistical multiplexing should be present in the network. (see also Section 2.5).

2.3 Burst loss probabilities for an N_I x N_0 switch.

In this section, we present the two-dimensional generalization of the recursion in equation (2.7). A recursive relationship is derived for burst blocking probabilities in a switch. The underlying assumptions are the same as before. In particular, if the bandwidth required by a burst is not immediately available, the burst is blocked. This extension to two dimensions together with the results above, allow the complete modeling of a network multiplexers and switches, with respect to blocking at the burst level.

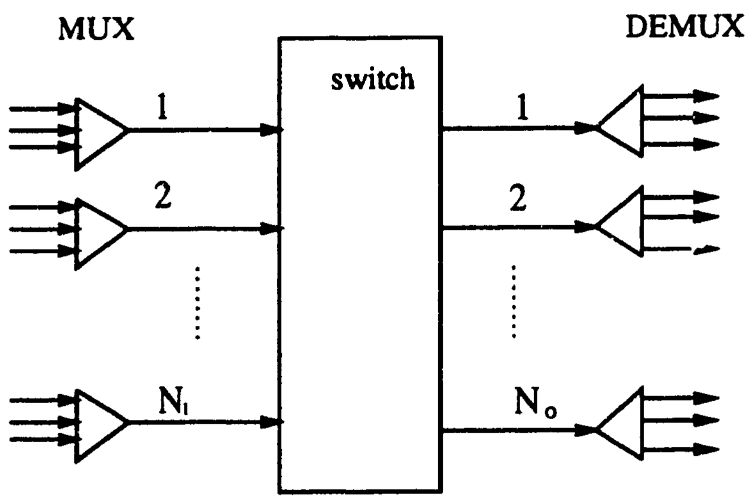


Figure 2.7 Topology of a switched network.

Assume as the network model, the switch presented in Figure 2.7. N_I multiplexers operating in the statistical mode are shown to be connected to N_o output ports via an internally-nonblocking switch. A burst to be transmitted through the switch demands bandwidth at both input and output lines. Different types of traffic have access to various multiplexers at the input ports of the switch with destination any arbitrary chosen subset of the output ports. The statistical assumptions for the traffic characteristics are those presented in Section 2.2.1. The objective is to calculate the type i burst-loss probability (bandwidth requirements b_i), originating at an input link μ with destination the output port ν. All types of traffic share the available transmission bandwidth equally on a FCFS basis. A burst can be lost because of either or both of the following two conditions:

I). Lack of available bandwidth on input line having F_I bandwidth units

II). Lack of available bandwidth on output line having F_o bandwidth units

By allowing input and output lines to have different bandwidth, we allow for consideration of the 'speedup' of the switch. We denote $P(I_{\mu i})$ the probability that a message of class i is blocked because of condition (I) exclusively, and $P(I_{\nu i})$ the probability that the same message is blocked because of condition (II), Then, the burst-loss probability is expressed as :

$$P_{b_{\mu\nu i}} = P(I_{\mu i}) + P(I_{\nu i}) - P(I_{\mu i} \cap I_{\nu i}) \tag{2.9}$$

The probability of the first and the second events is expressed as :

$$P(I_{\mu i}) = \sum_{j=0}^{b_i-1} r_\mu(F_I - j) \qquad P(I_{\nu i}) = \sum_{j=0}^{b_i-1} z_\nu(F_o - j)$$

with $r_\mu(x)$, $z_\nu(x)$ the probability of finding x bandwidth units reserved at the input port μ and the output port ν, respectively. The third event $P(I_{\mu i} \cap I_{\nu i})$ can be expressed by means of the joint probability function $q_{\mu\nu}(j,l)$, which represents the probability of finding j bandwidth units reserved at the input port μ and a total number of l bandwidth units destined to output ν.

$$P(I_{\mu i} \cap I_{\nu i}) = \sum_{j=0}^{b_i-1} \sum_{l=0}^{b_i-1} q_{\mu\nu}(F_I-j, F_o-l) \tag{2.10}$$

Noting also that :

$$r_\mu(j) = \sum_{l=0}^{F_o} q_{\mu\nu}(j,l), \qquad z_\nu(l) = \sum_{j=0}^{F_I} q_{\mu\nu}(j,l)$$

equation (2.9) yields :

$$P_{b_{\mu\nu i}} = \sum_{k=0}^{b_i-1} \sum_{l=0}^{F_o} q_{\mu\nu}(F_I-k,l) + \sum_{k=0}^{b_i-1} \sum_{j=0}^{F_I} q_{\mu\nu}(j,F_o-k) - \sum_{j=0}^{b_i-1} \sum_{l=0}^{b_i-1} q_{\mu\nu}(F_I-j,F_o-l)$$

An efficient method of computing $q_{\mu\nu}(j,l)$ follows along the lines of the derivation of the recursion in Section 2.2.3. The state of the system **n** is :

$$\mathbf{n} = (n_{111}, n_{112}, \ldots, n_{11K}, \ldots, n_{N_I N_o K})$$

The nonnegative integer $n_{\mu\nu z}$ is the number of type z calls on input port μ with destination output port ν. Thus, on top of the bandwidth characterization for each burst, as in the case of a single trunk, another description is added according to the input/output destination pair of each burst. We denote $N_{\mu\nu z}$ to be the maximum number of type z binomial sources at the input port μ with destination output port ν. Also, Ω is the set of allowable states in the system, determined by the 'equal sharing' policy, the multiplexers' bandwidths, and the types of traffic that have access to the switch. Any state $\mathbf{n}$ in Ω satisfies the following two conditions :

$$I. \quad \sum_{y=1}^{N_o} \sum_{z=1}^{K} n_{\mu y z} b_z = m_{I,\mu} \leq F_I \qquad \forall\ \mu \in \{1, 2, \cdots N_I\} \quad \text{and } n_{\mu\nu z} \leq N_{\mu\nu z} \qquad \forall\ \mu, \nu, z.$$

$$II. \quad \sum_{x=1}^{N_I} \sum_{z=1}^{K} n_{x\nu z} b_z = m_{o,\nu} \leq F_o \qquad \forall\ \nu \in \{1, 2, \cdots N_o\} \quad \text{and } n_{x\nu z} \leq N_{x\nu z} \qquad \forall\ x, \nu, z.$$

Next, we evaluate the occupancy distribution of the system. Thus, we define $\boldsymbol{R}_{\mu\nu}$ to be the collection of the pairs of the input/output occupancies $(m_{I,\mu}, m_{o,\nu})$, for any input/output pair of ports μ, ν, and for every $\mathbf{n}$ in Ω, that is :

$$\boldsymbol{R}_{\mu\nu} = \{(m_{I,\mu}, m_{o,\nu}) \qquad \forall\ \mathbf{n} \in \Omega\}$$

The two-dimensional recursion is given in equation (2.14). We omit the detailed derivation of the result here, since it can be easily obtained following the analysis in the previous chapter, under the same traffic assumptions as those in Section 2.2.1. As before, a simple transformation that yields an one-to-one correspondence between any state in Ω and the respective input/output occupancies is also employed.

We begin noting that there are three equations that correspond to equation (2.7) in the single trunk case. This is so, simply because there are three ways $q_{\mu\nu}(j,l)$ can be reached, that is, $q_{\mu\nu}(j-b_i,l)$, $q_{\mu\nu}(j,l-b_i)$, $q_{\mu\nu}(j-b_i,l-b_i)$ - the first denoting an arrival at input port μ with destination a port other than ν, the second an arrival at output port ν with origin an input port other than μ and the third an arrival at the input port μ with destination the output port ν.

$$(N_{\mu\nu i}-n_{\mu\nu i}+1)\,a_{\mu\nu i}q_{\mu\nu}(j-b_i,l-b_i) = E(n_{\mu\nu i}\mid j,l)q_{\mu\nu}(j,l) \qquad \forall\ (j,l)\in \boldsymbol{R}_{\mu\nu} \quad (2.11)$$

$$(N_{\mu yi}-n_{\mu yi}+1)\,a_{\mu yi}q_{\mu\nu}(j-b_i,l) = E(n_{\mu yi}\mid j,l)q_{\mu\nu}(j,l) \qquad \forall\ (j,l)\in \boldsymbol{R}_{\mu\nu}\ \ y\neq\nu \quad (2.12)$$

$$(N_{x\nu i}-n_{x\nu i}+1)\,a_{x\nu i}q_{\mu\nu}(j,l-b_i) = E(n_{x\nu i}\mid j,l)q_{\mu\nu}(j,l) \qquad \forall\ (j,l)\in \boldsymbol{R}_{\mu\nu}\ \ x\neq\mu \quad (2.13)$$

Multiplying equations (2.11), (2.12), (2.13) by b_i and taking the summation over the possible input/output ports and the range of the various types of traffic we have :

$$\sum_{i=1}^{K}\Big(\sum_{\substack{y=1\\y\neq\nu}}^{N_o}(N_{\mu yi}-n_{\mu yi}+1)\,a_{\mu yi}b_iq_{\mu\nu}(j-b_i,l)+(N_{\mu\nu i}-n_{\mu\nu i}+1)\,a_{\mu\nu i}b_iq_{\mu\nu}(j-b_i,l-b_i)\Big)+$$

$$\sum_{i=1}^{K}\Big(\sum_{\substack{x=1\\x\neq\mu}}^{N_I}(N_{x\nu i}-n_{x\nu i}+1)\,a_{x\nu i}b_iq_{\mu\nu}(j,l-b_i)+(N_{\mu\nu i}-n_{\mu\nu i}+1)\,a_{\mu\nu i}b_iq_{\mu\nu}(j-b_i,l-b_i)\Big)=$$

$$= jq_{\mu\nu}(j,l)+lq_{\mu\nu}(j,l) = (j+l)q_{\mu\nu}(j,l). \qquad (2.14)$$

for any pair $(j,l)\in R_{\mu\nu}$ and $n_{\mu yi}, n_{x\nu i}, n_{\mu\nu i}$; $\forall\ x\neq\mu, y\neq\nu, i$, the vector components associated with the input/output occupancy pair (j,l). Using also the convention :

$$q_{\mu\nu}(j,l)=0 \qquad \forall\ (j,l)\notin R_{\mu\nu}$$

and the normalizing condition :

$$\sum_{j=0}^{F_I}\sum_{l=0}^{F_o} q_{\mu\nu}(j,l) = 1$$

the values of the function $q_{\mu\nu}(j,l)$ can be computed recursively as before.

2.3.1 Example 2

As an example consider a 4x2 Time-Space-Time (TST) internally nonblocking switch, loaded with two types of bursty traffic. We assume an input/output link capacity $F_I = F_o = 5\ Mbits/sec$, peak rates for the two types of traffic of 3 and 2 $Mbits/sec$, respectively. Figure 2.8 plots the burst blocking probabilities as a function of the number of type-1 sources that have access to the system for $a_1 = a_2 = 10^{-5}, 10^{-6}$. We observe that, overloading the system with type-1 sources, the type-1 burst-loss probability approaches one quickly. The type-2 traffic exhibits a different behavior. Although we increase the type-1 traffic load in the system, the burst-loss probability for type-2 traffic stabilizes around the values $5 \cdot 10^{-3}$ and $3 \cdot 10^{-4}$ simply because the frame occupancy is 5 slots out of which only three can be occupied from the type-1 traffic. Thus, the remaining two slots are always available to the second class traffic even though the system is saturated by type-1 traffic.

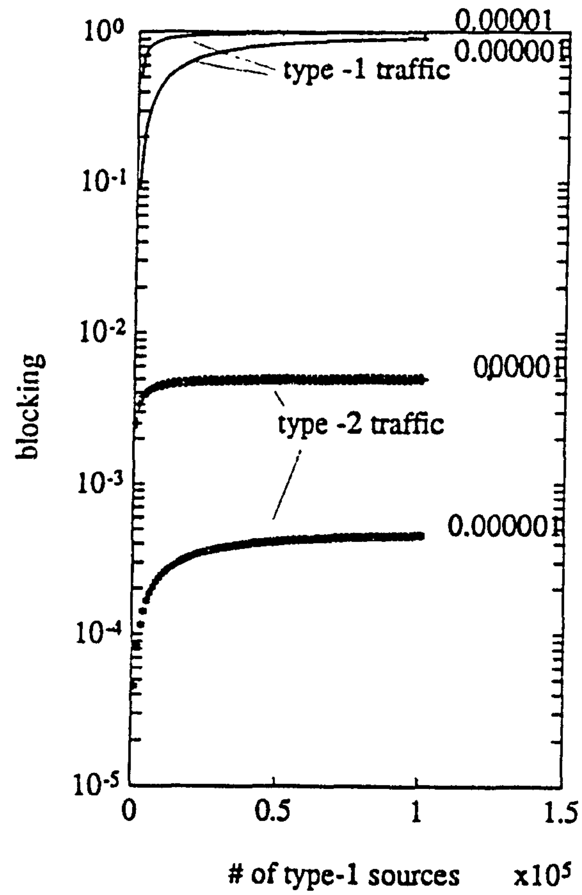


Figure 2.8 Blocking probabilities in a 4x2 switch.

2.3.2 Example 3

Now, consider a 16x16 TST switch loaded with variable-rate video and graphics station traffic with the same parameters as in the exampie of Section 2.2.5. Figure 2.9 plots the utilization of the two types of traffic under the assumption of a state-independent arrival rate and symmetric input traffic. We assume an input/output link capacity $F_I = F_o = 90\ Mbits/s$, and $180 Mbits/s$ and plot the admission region in terms of traffic utilization for a 10^{-6} burst-loss probability. Again, as in the case of a single statistical multiplexer, we observe that the boundary becomes more linear when the peak-to-link ratio of the sources decreases.

2.4 A Poisson approximation to the finite-source model.

Here we consider the question of approximating the finite-source model with one in which the arrival process is Poisson. In particular, we are interested in evaluating the recursions in equations (2.7) and (2.7a) and compare the resulting loss probabilities under the following assumption:

$$\frac{N_i \lambda_i}{\mu_i} = \frac{\lambda_i^P}{\mu_i^P}$$

with N_i, λ_i, μ_i denoting the number of type-i binary sources, the arrival rate, and the service rate, respectively. λ_i^P, μ_i^P the parameters of the approximating Poisson process.

Under this assumption, we plot in Fig. 2.10 the blocking probabilities of two types

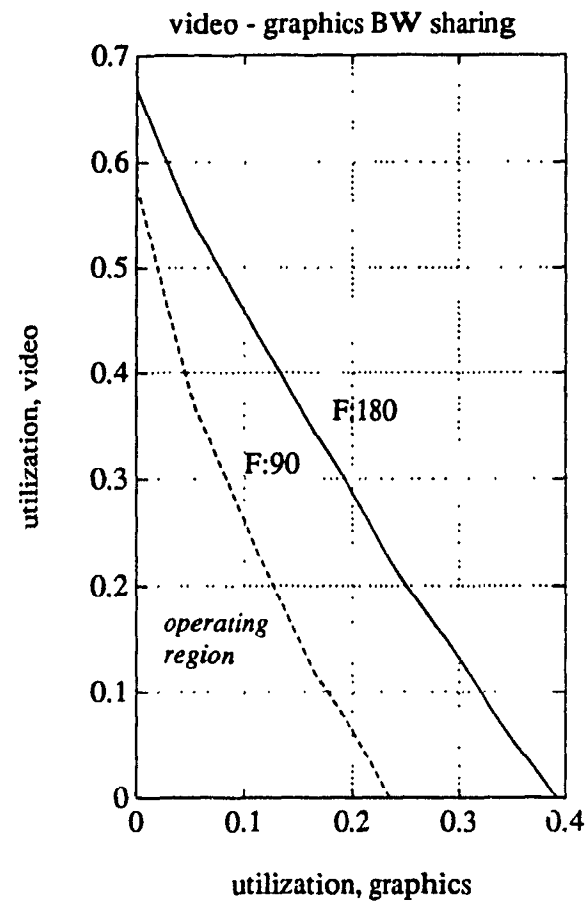
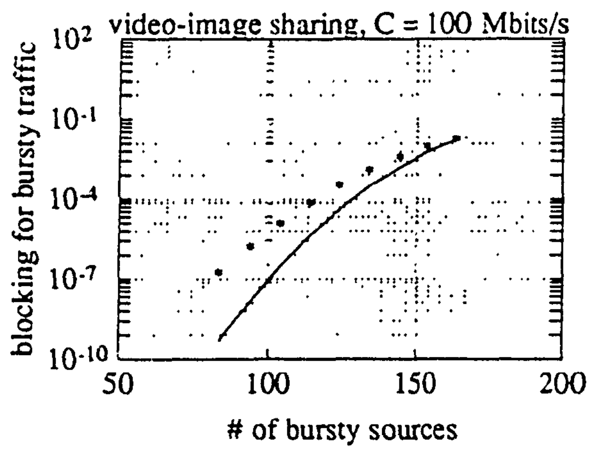


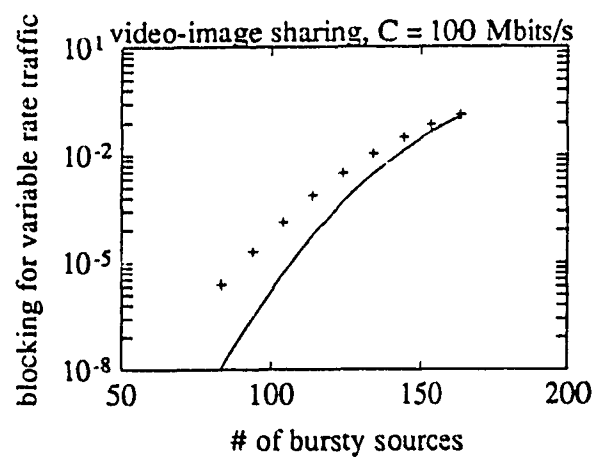
Figure 2.9 Admission call region, 16x16 switch.

of traffic, variable-rate video and image transfer with the same parameters as those in the example of Section 2.2.5, feeding a statistical multiplexer with a capacity of 100 *Mbits/s*. We keep the number of binary video sources fixed, (1 call or 20 binary sources) and vary the number of binary sources of the bursty traffic that have access to the multiplexer. The difference in approximation is shown in Fig. 2.10a for the video traffic and Fig. 2.10b for the bursty traffic, with the solid line denoting the finite-source approximation in both cases. Although the number of binary sources involved is considerable, the significant difference between the two models is due to the fact that only one video call is present in the finite-source model. As a result, the state space of the finite-source model is significantly different than that of the Poisson model, and since blocking is expressed in terms of probabilities of states that lead to congestion, the two sets of states associated with loss are completely different in the two models.

In the second example we present here, we increase the number of video calls that have access to the multiplexer to 10, thus, making the state space in both models the same. We also increase the number sources of each type by decreasing the ratio λ_i / μ_i by a factor of 10. The results on the loss probabilities of the bursty traffic is shown in Fig. 2.11 with the solid line denoting again the finite-source model. The agreement between the two models is significantly improved in this case. Summarizing this Section, we note that the above results indicate that not only a significant number of binary sources is needed in order to approximate successfully the finite source model by a Poisson model in terms of blocking, but also the state space for the two models should be the same.



(a)



(b)

Figure 2.10 Comparison between the two models (* Poisson, - Finite source)

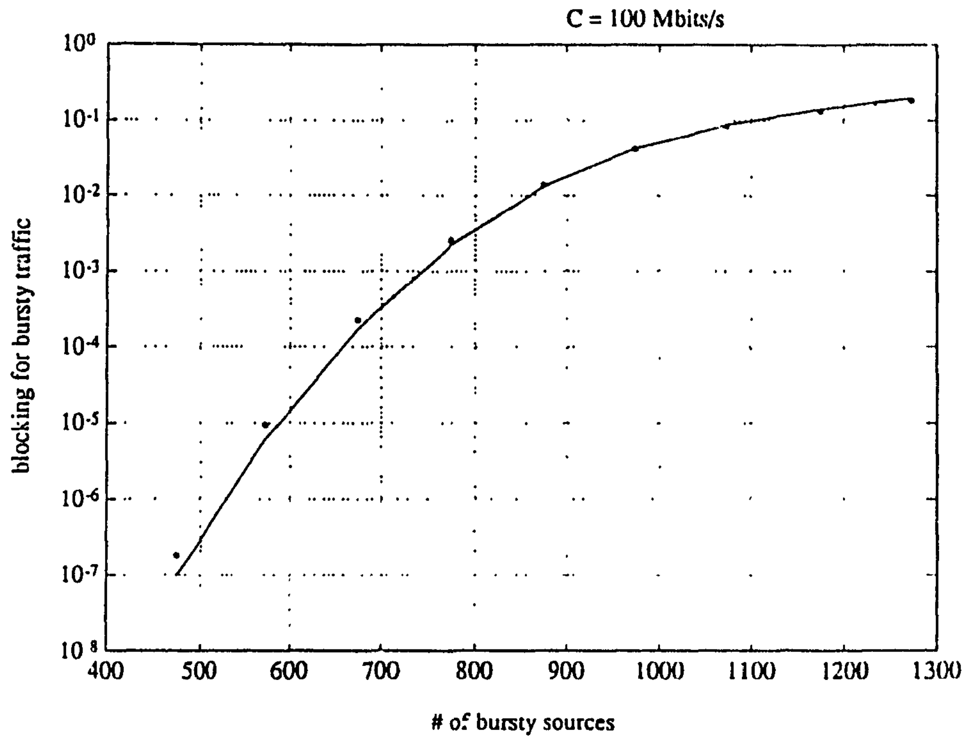


Figure 2.11 Comparison between the two models with the same state space.

As a final comment to this introduction to the subject, we note that the superposition of binary sources has been studied in the context of packetized speech. In [HeLu86], [SrWh86] is shown, in agreement to our results, that the approximation of a large number of independent sources by a Poisson distribution can lead to inaccurate results. Note also that problems in approximating bursty traffic by Poisson distribution is also reported in [NaKuTo91], where a model for packet loss in finite-buffer voice multiplexers is analyzed. The superposition of voice sources is modeled as a two-state MMPP with slightly different matching statistics than that of [HeLu86]. Their results indicate that the Poisson assumption underestimates significantly the packet loss probability.

2.5 Successive levels of statistical multiplexing.

In this section we consider a more realistic topology of a broadband network employing statistical multiplexing at various levels of increased capacity and a switching core. According to this scenario of traffic integration, depicted in Figure 2.12, several incoming links are statistically multiplexed at the first level at a given capacity C_1. The carried traffic is further multiplexed at a higher capacity level C_2, the process being repeated N times, with N depending on the degree of bandwidth utilization we pursue for the given traffic characteristics and the required grade of service.

Here, as a first approach to this problem of estimating the burst-loss probability of the above network, we suggest an approximation based on the results from the previous Sections. The performance of a single statistical multiplexer was analyzed in Section

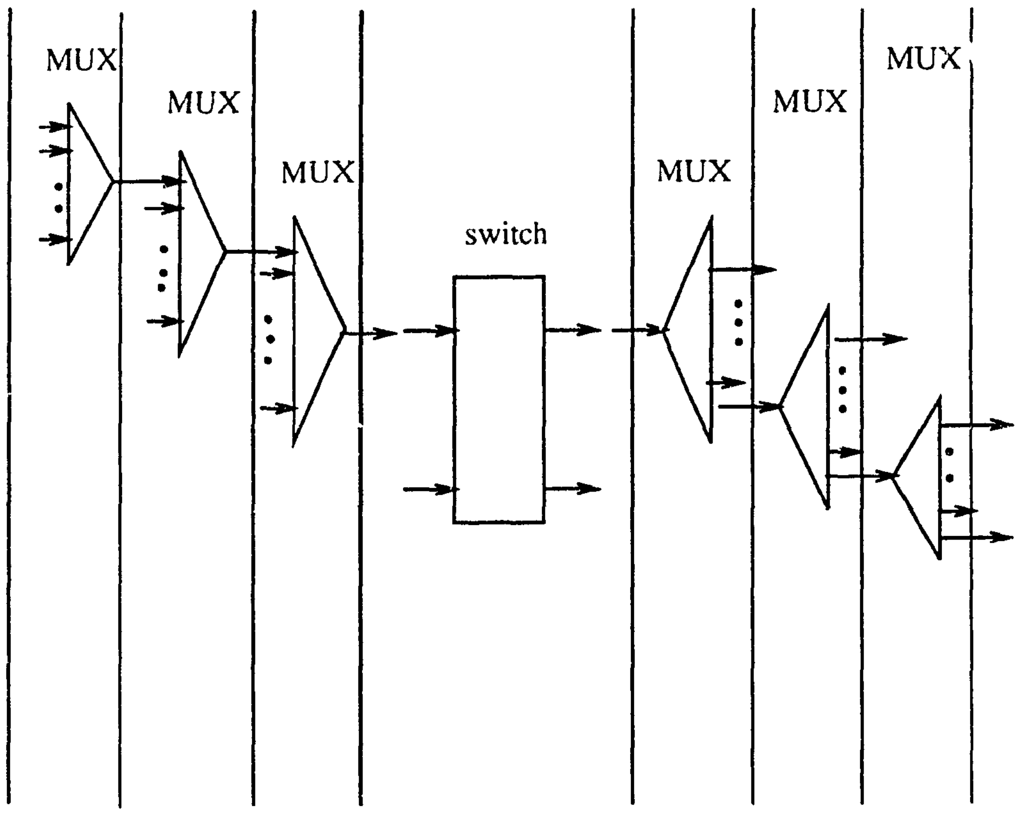


Figure 2.12 Consecutive levels of statistical multiplexing with a switching core.

2.2.3 and the blocking characteristics of the switching core is given in Section 2.3. Starting with the results in 2.2.3, we calculate the burst blocking probability B_i for type i traffic, by means of the recursion in equation (2.7). If λ_i is the transition rate of type i sources, we suggest that the effect of blocking at the first level of statistical multiplexing be included in the overall performance evaluation, by defining the type i transition rate at the second level to be $\lambda_i - B_i$.

At N successive levels of statistical multiplexing the resulting process is approximated by one with binary source rate equal to :

$$\lambda_i - \sum_{i=1}^{N} B_i .$$

Then the analysis in Section 2.3 could be applied directly to the calculation of the blocking probabilities due to the switch operation.

The validity of this approach was tested for a simple case concerning two levels of statistical multiplexing. The approximation was compared with exact results that were derived via our analysis for the switch in Section 2.3 (two levels of statistical multiplexing are equivalent to an Nx1 switch). The before-mentioned approximation was found not to be accurate in many instances and further work is required.

2.6 Conclusions

We analyzed loss conditions in statistical multiplexing under the discrete finite-source model assumption. The results find direct applications in controlling congestion in ATM networks. We showed that the steady-state distribution of our model exhibits the product-form solution and that employment of a simple transformation yields a one-dimensional recursion for the occupancy distribution. The linearity of the suggested recursion makes possible the calculation of blocking probabilities even for large number of sources. The above analysis was extended to an integrated switch and can be further generalized to an arbitrary number of tandem links. A congestion-control technique was suggested as a result of this analysis.

CHAPTER III.

Video, Voice and Data Integration in a 'Movable Boundary' Scheme.

The subject of this chapter is a particular technique for integrating different categories of traffic, typified by data, voice, video in a multimedia environment. In contrast to the previous two chapters where no priorities were present among the various types of traffic, a higher priority is assigned to video and voice traffic and a lower one to the data traffic. The sharing of the transmission bandwidth is accomplished via a 'movable boundary' technique.

An embedded Markov chain analysis is used for the derivation of the blocking probabilities for the first-priority traffic (voice, video) and the overflow probabilities and the queueing delay for the buffered data traffic. The straightforward approach to the problem requires prohibitive computational complexity. The contribution of this chapter is to apply certain mathematical techniques to reduce this complexity. Decomposition and matrix-geometric techniques are combined to supply results for high-capacity applications (up to DS3(T3) level).

3.1 Introduction

The high reliability and the enormous bandwidth in fiber-optic networks allow the integration of services with a wide spectrum of traffic characteristics. The future integrated network should be able to respond adequately to these distinct characteristics and provide the necessary priorities and the required grades of service.

The analysis presented in the previous two chapters aimed at the development of congestion-control techniques that guarantee a universal grade of service for all types of traffic. The above mentioned approach does not ensure optimality in bandwidth utilization. In this chapter, we analyze the performance of a cooperative system that supports three different classes of traffic in an integrated fashion. Two distinct priorities and a 'movable boundary' sharing scheme are used to administer the transmission capacity of the system, allowing the first-priority calls (the video, voice traffic) to go through the network under a given grade of service and the second-priority calls (the packet traffic) to experience a minimum delay and a given buffer-overflow probability.

The 'movable boundary' technique is based on a concept proposed by K. Kummerle [Kum74] and Zafiropoulo [Zaf74] for integration of voice calls and data packets. The problem of modeling and analysis of movable boundary (or SENET [CoVe75]) integration of voice and data traffic has been given considerable attention in the literature. In one of the first approaches to the problem, Fisher and Harris [FiHa76] carried out an analysis for the data delay, assuming the number of voice calls at the beginning of any two different frames to be independent. Later, Weinstein et al. [WeiMF80] showed through simulations the high correlation of voice calls that led Fisher and Harris to underestimate the data delay. Other approaches to the problem include Gaver and Lechoczki's [GaLe82] fluid-flow approximation to estimate the average data delay and Leon-Garcia's model [LeKW82] where a series of approximations are used to suggest upper and lower bounds to the average data delay. A continuation of this work is presented in [WiLe82], where matrix-diagonalization techniques are used to modify some earlier work by Neuts [Neu78] in an attempt to confront the increased dimensionality of the state space. Exact results are given for buffer-overflow probabilities in finite-buffer cases as well as the mean number of packets in a system with an infinite-size buffer.

The concept of movable boundary is applied in this chapter for the integration of video, voice and data calls. The available bandwidth is divided into two portions, one for the first-class traffic that encompasses video and voice, which is never to be exceeded, and a second for the buffered data traffic that varies according to the first-class load. The efficiency of this technique is due to the fact that unused bandwidth dedicated to the service of the first class, is readily available to the second-class traffic, which experiences a variable service rate according to the traffic intensity of the preemptive first-class traffic.

We chose to analyze the above scheme via a Markovian chain embedded at the start of each frame in the succession of frames in time, in a similar manner as [MoTs88], and considered the state of the system to be the number of video calls, voice calls and data packets in the system. For practical applications, such an approach runs into computational difficulties due to the increased dimensionality of the state-transition matrix that describes the dynamic behavior of the system. For example, in the case of a buffer size of one hundred data packets and a total number of video, voice calls in the order of ten, the size of the state-transition matrix is more than a thousand.

The solution to this problem lies on the hierarchical structure with respect to the dynamic behavior of the system under consideration (see also [GhaSc88]). Noting that the service rates of video, voice and data traffic are ordered as $\mu_{vi} \ll \mu_{vo} \ll \mu_d$ and that a similar expression is valid for the respective arrival rates, (to keep load in balance), that is, $\lambda_{vi} \ll \lambda_{vo} \ll \lambda_d$, the system can be decomposed into a three level hierarchy and subsequently analyzed on the basis that interactions within the same level are much stronger than those between different levels.

The technique is an approximation that treats nearly-completely-decomposable systems as completely decomposable. Clearly, in this decomposition an error ε is introduced. The analysis by Courtois in [Cou77] reveals that the error involved (ε) depends essentially on the degree of coupling between subsystems. A second-order approximation that decreases the error of approximation into an order of ε^2 is also employed, allowing a wide range of input traffic characteristics to be accurately analyzed.

Two cases are discussed here. In the first, we assume a finite-buffer capacity, whereas in the second, we examine an infinite-buffer model under a control mechanism that maximizes the bandwidth utilization. Matrix analytic techniques [Neu89] are employed to study the data queue behavior. Finally, in Section 3.4, we examine the integration of four different types of traffic in the framework of the digital multiplexing hierarchy. Again, the presented analysis is based on 'Decomposability' techniques.

3.2 Finite capacity data buffer.

In this section, we consider a multiplexing scheme employing a buffer of finite capacity for the data traffic. Although real systems employ finite-buffers only, infinite-buffer assumptions are usually made in order to improve the analytic tractability of the problem (see Section 3.3).

3.2.1 Description of the system.

The queueing system under consideration is shown in Figure 3.1. Video, voice and data traffic share N channels in a frame of duration τ. The first-class traffic is composed of two streams, carrying voice and video calls, whereas data traffic is segmented into packets that are temporarily stored before transmission in a buffer of size M (packets). s channels are dedicated to the service of the video and voice calls that compete directly on an equal sharing, FCFS basis and the remaining $N-s$ channels for the data traffic, respectively. The bandwidth requirements of each class are distinct and denoted by I_{vo}, I_{vi}, I_d, respectively. That is, I_{vi} slots in a frame are required for the duration of each video call, I_{vo} for each voice call and I_d for the data packets.

We consider two models for the arrival process. In the first, all three classes of traffic have Poisson arrival processes with rates λ_{vi}, λ_{vo}, and λ_d calls/sec, respectively. In the second model the video and voice traffic is generated from a finite-source model and the data traffic is Poisson distributed. In both the models, the service time for both video and voice traffic is assumed to be exponentially distributed (although our model

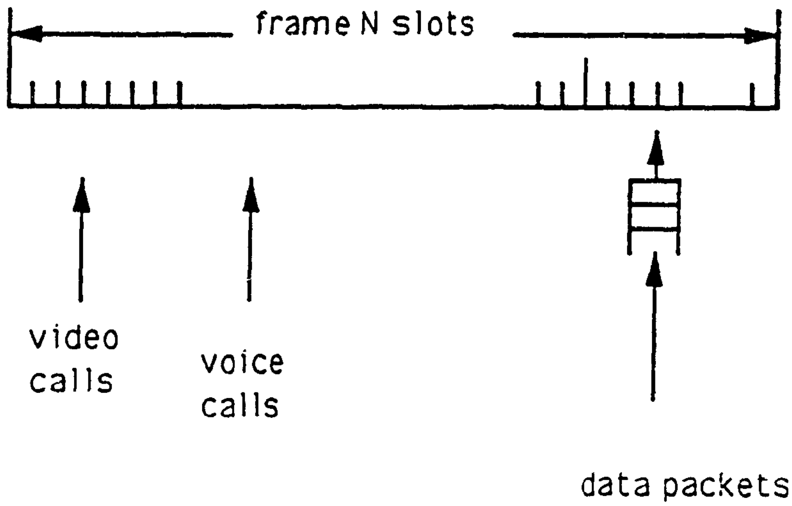


Figure 3.1. Video, voice, data integration in a movable boundary scheme.

allows for a general service-time distribution) and the service time of data calls is constant, of duration τ.

Although calls arrive at any epoch, decisions concerning their transmission are made at the beginning of each frame, when the system knows exactly how many of the existing calls terminate or intend to keep their slots for the duration of the next frame. Video or voice calls are accepted for transmission when the number of available channels exceeds their bandwidth requirements (l_{vi} or l_{vo}). If not, they are blocked and are considered to depart without further affecting the performance of the system. Similarly, acceptance or rejection of data packets depends on the availability of space in the buffer. The number of data packets scheduled for transmission at the beginning of a frame depends on the availability of servers in the first-class portion of the frame on top of the number of servers dedicated exclusively for the data traffic. The queueing model for the scheme above is depicted in Figure 3.2.

3.2.2 The Embedded Markov Chain Analysis.

We choose the embedded Markov chain approach to analyze the cooperative system described above. A finite Markov chain is embedded at the starts of the frames in the succession of frames in time. We assume that each call that completed its service has already departed from the system before the start of the next frame, and new arrivals have already been introduced at this point. The state of the system is then characterized by (i, j, k) with i: the number of video calls, j: the number of video calls, and k: the number of data packets in the system. Thus, the state space is :

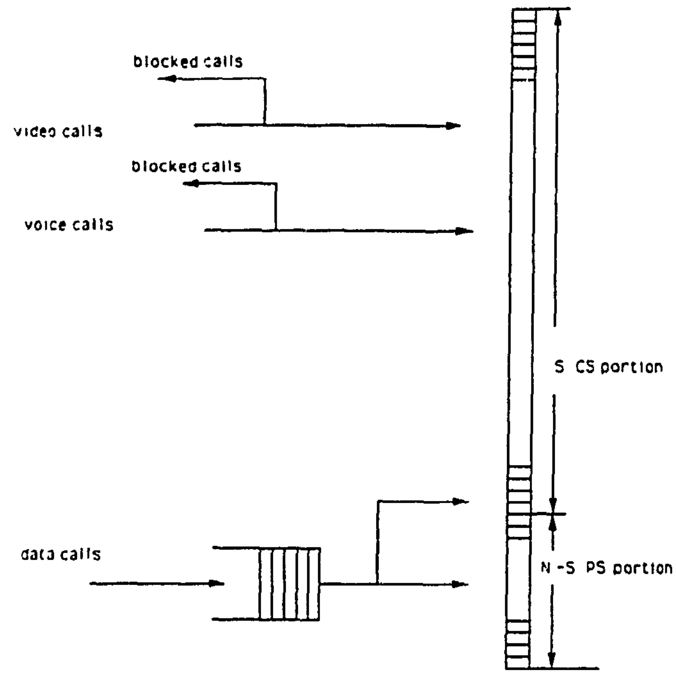


Figure 3.2. Bandwidth sharing of the three types of traffic.

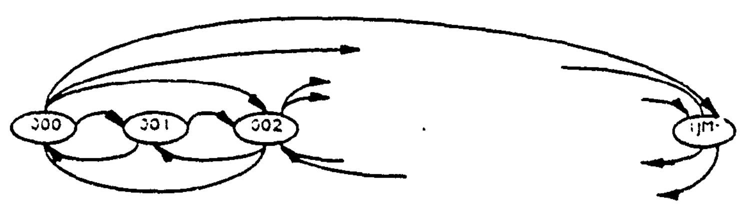


Figure 3.3. The state transition diagram of the embedded Markov chain.

$$S = \{(i, j, k) \quad ; 0 \le i \le s_{VI},\ 0 \le j \le s_{VO},\ 0 \le k \le M, \text{ and } il_{vi} + jl_{vo} \le s\}. \tag{3.1}$$

The variables s_{VI} and s_{VO} represent the maximum number of video and voice calls in the system and are given by the maximum integer which is less or equal to the ratio of the available bandwidth and the bandwidth requirements, that is, $s_{VI} = \lfloor s/l_{vi} \rfloor$ and $s_{VO} = \lfloor s/l_{vo} \rfloor$, respectively. In order to calculate the transition probabilities for the Markov chain depicted in Figure 3.3, we first derive the probability $p_{i_1 j_1}(i_2, j_2)$, that is, the probability of finding i_2 video calls and j_2 voice calls when i_1, j_1 was their distribution in the previous frame. We also need the term $a_d(k, M)$ that represents the probability of having k arrivals in a τ-second interval in a buffer of size M. Thus, the transition probability from state (i_1, j_1, k_1) to (i_2, j_2, k_2) is:

$$q_{i_1 j_1 k_1}(i_2, j_2, k_2) = p_{i_1 j_1}(i_2, j_2)\, a_d(k_2, M); \quad \text{for } k_1 \le U(i_1, j_1), \text{ and } k_2 \le M. \tag{3.2a}$$

or

$$q_{i_1 j_1 k_1}(i_2, j_2, k_2) = p_{i_1 j_1}(i_2, j_2)\, a_d(k_2 - k_1 + U(i_1, j_1), M - k_1 + U(i_1, j_1));$$
$$\text{for } U(i_1, j_1) \le k_1 \le M,\ k_1 - U(i_1, j_1) \le k_2 \le M, \tag{3.2b}$$

where $U(i,j)$ is the temporary transmission capability of the system in packets, given that i video and j voice calls are present. We have

$$U(i,j) = \lfloor (N - il_{vi} - jl_{vo})/l_d \rfloor \text{ and } U(i,j) \ge \lfloor (N - s)/l_d \rfloor \qquad \forall i,j \quad in\ S.$$

Note that in equation (3.2a) the number of the packets in the system k_1 is less than $U(i_1, j_1)$, the transmission capability of the system, and all k_2 packets constitute new arrivals in an empty buffer (capacity M). In equation (3.2b), the number of packets k_1

exceeds the term $U(i_1,j_1)$ by a number $k_1-U(i_1,j_1)$, therefore these packets remain in the queue for transmission during the next frame. Thus k_2 packets during the next frame imply $k_2-(k_1-U(i_1,j_1))$ new arrivals in a buffer of size $M-(k_1-U(i_1,j_1))$.

3.2.2.1 The Poisson arrival process model.

The arrival process of the data traffic is assumed to be Poisson. Then, the probability of k arrivals in a buffer of maximum capacity K is:

$$a_d(k,K) = \begin{cases} e^{\lambda_d \tau}(\lambda_d \tau)^k / k! & k < K \\ 1-\sum_{i=0}^{K-1} e^{\lambda_d \tau}(\lambda_d \tau)^i / i! & k = K \\ 0 & k > K \end{cases} \tag{3.3}$$

When the Poisson arrival model is assumed for the other two types of traffic, the probability of k arrivals in a link of capacity K calls are given by similar expressions and denoted by $a_{vi}(k,K)$, $a_{vo}(k,K)$.

Now, the probability $p_{i_1,j_1}(i_2,j_2)$ is expressed as:

$$p_{i_1,j_1}(i_2,j_2) =$$

$$\sum_{x_1=0}^{\min(i_1,i_2)} \sum_{x_2=0}^{\min(j_1,j_2)} \gamma_{vi}(i_1,x_1)\gamma_{vo}(j_1,x_2) \cdot a_{vi}(i_2-x_1, s-x_1 I_{vi}-j_2 I_{vo}) a_{vo}(j_2-x_2, s-i_2 I_{vi}-x_2 I_{vo}) \tag{3.4}$$

with $\gamma_v(i,j)$ representing the probability that j out of i calls retain their slots during a τ time interval. Then :

$$\gamma_{vi}(i,j) = \binom{i}{j} (1-e^{-\mu_{vi}\tau})^{i-j} e^{-j\mu_{vi}\tau} \tag{3.5}$$

$$\gamma_{vo}(i,j) = \binom{i}{j} (1-e^{-\mu_{vo}\tau})^{i-j} e^{-j\mu_{vo}\tau} \tag{3.6}$$

Using the above expressions, the state-transition probabilities in equation (3.2) can be computed. In the next section, we remove the Poisson assumption for the arrival process model of the first-class traffic and employ a more realistic model that was introduced in Chapter II.

3.2.2.2 The Finite Source Model.

According to this model suggested in [MASKR88] for variable-rate videocoders (see also Section 2.2.1), the aggregate video traffic is modeled by a collection of a binary sources that alternate between active and idle states with probability p and q, respectively. The transition from an idle to an active state constitutes a 'burst' and certain bandwidth demands are imposed. The same model describes the arrival process of voice traffic with bursts imposing bandwidth requirements of $64 kbits/s$. Thus, both the video and voice arrival processes become binomial, with the rest of the expressions in the previous section holding, as well as the state-transition probabilities.

3.2.3 Decomposition Analysis.

Given that the transition probabilities are calculated as above, we choose a lexicographical mapping to enumerate the states of the system as:

$$u = (s_{VO}+1)(M + 1)i + (M + 1)j + k.$$

Then the transition probabilities can be written in a matrix form of dimensionality L, $Q=[q_{ij}]$, $i=0, \cdots L$, $j=0, \cdots L$, with the transition rates q_{ij} given in equations (3.2a), (3.2b). The steady state of the above stochastic system is the row vector π that satisfies

$$\pi = \pi Q \qquad (3.7)$$

The computation of the vector π is increasingly difficult for larger values of S or M. The burden of computation can be effectively reduced by noting that the transition matrix Q, due to its hierarchical structure, is nearly completely decomposable.

The first step in applying decomposition is to consider the 'two-level hierarchy'. According to this scheme a distinction is made between the first-class traffic that encompasses video and voice traffic and the second-class traffic, the data traffic. The above is based on analytical work by Simon and Ando (Cou77). They showed that aggregation of variables in nearly completely-decomposable systems must separate the analysis of the short-run from that of the long-run dynamics. They proved two theorems. The first one states that, provided intergroup dependencies are sufficiently weak as compared to intragroup dependencies, in the short run, the system will behave approximately, and can therefore be analyzed, as if it were completely decomposable. The second theorem states that even in the long run, when neglected intergroup dependencies have had time to influence the system behavior, the values of the variables within any group will remain approximately in the same ratio as if those intergroup dependencies had never existed. The results obtained in the short run will therefore remain approximately valid in the long run, as far as the relative behavior of the variables of the same group is concerned.

3.2.3.1. The first-order approximation.

Given that s_c is the number of states of the video and voice traffic in the lexicographical mapping :

$$y = (s_{VO}+1)i + j \qquad \forall\ i,j\ in\ S, \quad y \le s_c$$

(remember that s_{VO} is the maximum number of voice sources to be accommodated in a frame of capacity s). Then $p_{i_1 j_1}(i_2, j_2)$ in equation (3.4) is written $p_I(J)$ or equivalently $p(I,J)$ and constitutes the I^{th}, J^{th} element of the matrix $P=[p(I,J)]$.

Now, observe that the matrix Q is nearly completely decomposable, since, when partitioned into a set of square $(M+1)\times(M+1)$ submatrices Q_{IJ} $I=0,\cdots,s_c$, $J=0,\cdots,s_c$ the order of the main diagonal elements is significantly larger than that of the peripheral submatrices. This is simply so, because transitions between subsystems are less probable than transitions within the subsystem, given the validity of the initial conditions $\lambda_{vi} \ll \lambda_{vo} \ll \lambda_d$ and $\mu_{vi} \ll \mu_{vo} \ll \mu_d$. Partitioning Q in square $(M+1)\times(M+1)$ submatrices - that correspond to the different modes of the first-class traffic - yields:

$$Q = \begin{bmatrix} Q_{00} & \cdots & Q_{0s_c} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ Q_{s_c 0} & \cdots & Q_{s_c s_c} \end{bmatrix}$$

Q is subsequently expressed as:

$$Q = Q^* + \varepsilon C$$

with C a square matrix of the same order as Q that keeps the r.h.s stochastic, ε a real positive number representing the maximum degree of coupling between subsystems Q_{IJ} and Q^* the block diagonal matrix :

$$Q^* = \begin{bmatrix} Q_0^* & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & & & Q_I^* & & \cdot \\ \cdot & & & & & \cdot \\ 0 & \cdot & \cdot & & \cdot & Q_{s_c}^* \end{bmatrix}$$

Q_I^* are square $(M+1)\times(M+1)$ submatrices, defined by:

$$Q_I^* = \begin{bmatrix} a_d(0,M) & a_d(1,M) & \cdots & a_d(M,M) \\ \cdot & \cdot & \cdots & \cdot \\ a_d(0,M) & a_d(1,M) & \cdots & a_d(M,M) \\ 0 & a_d(0,M-1) & & a_d(M-1,M-1) \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ 0 & \cdot & \cdots & a_d(U(i,j),U(i,j)) \end{bmatrix}$$

The number of identical rows in Q_I^* equals the number of available servers plus one, since any number of packets in the queue less or equal to the number of servers receive service during the same frame. From the definition of the transition probabilities (eq.(3.2)), we have

$$Q_{IJ} = p(I,J)Q_I^*$$

Also,

$$\varepsilon = \max(\varepsilon_0, \varepsilon_1, \cdots \varepsilon_{s_c}) \qquad \varepsilon_k = \sum_{I \neq J} p(I,J) = 1 - p(J,J) \tag{3.8}$$

and

$$C = [c_{kl}]; \quad c_{kk} = -\frac{\varepsilon_k}{\varepsilon} Q_k^* ; \quad c_{kl} = \frac{p(k,l)}{\varepsilon} Q_k^* \quad k \neq l ;$$

According to [Cou77], the steady-state vector π in equation (3.7) can be approximated by the vector x whose elements are defined in the product :

$$x_{il} = X_I \cdot \nu_I^*(i) \quad ; \quad i=0, \ldots, M, \; I=0, \ldots, s_c \tag{3.9}$$

X_I in equation (3.9) represents the steady-state vector of the aggregate matrix P, and $\nu_I^*(i)$ is the steady-state vector of the I^{th} submatrix Q_I^*. The accuracy of this first-order approximation to the invariant vector π is of the order of ε.

3.2.3.2 The second-order approximation.

The second-order approximation, that is the estimation of the invariant vector π with accuracy ε^2, is subsequently obtained. We first calculate the elements of the row vector $[x_{il}] \times \varepsilon C$ and denote it by $\boldsymbol{A}(I,1)$. $[x_{il}]$ represents the L elements of the first-order approximation arranged in (s_c+1) sets of $(M+1)$ elements, the eigenvectors of Q_I^*. Subsequently, the vector $\boldsymbol{A}(I,1)$ is used to solve the system of equations

$$\boldsymbol{B}(I,1)[I - Q_I^*] = \boldsymbol{A}(I,1) \tag{3.10}$$

The matrix $[I - Q_l^*]$ is singular, but a unique solution to the system exists in combination with:

$$\sum_{k \in l} b_k(l,1) = 0$$

where $b_k(l,1)$ are the elements of $\boldsymbol{B}(l,1)$. Subsequently $[b_k(l,1)] \times C$ is computed to yield the vector $\boldsymbol{K}(\varepsilon,1)$ which is used to solve the system

$$[\boldsymbol{\beta} - \boldsymbol{X}][P - I_N] = \varepsilon \boldsymbol{K}(\varepsilon,1) \qquad (3.11)$$

The extra equation we need to resolve the singularity of $[P - I_N]$ comes from the additional condition that the sum of the differences

$$[\beta_l - X_l], \quad l=1, \cdots s_c$$

must be zero. The final equation that yields the differences $\pi_{il} - x_{il}$ with accuracy ε^2 by substitution from equations (3.10),(3.11) is :

$$\pi_{il} - x_{il} = (\beta_l - X_l)\nu_{il}^* + b_k(l,1)$$

3.2.3.3 An alternative approach to the calculation of X_l in equation (3.9).

The nature of the 'movable boundary' model is such that the video and voice traffic is completely independent of the packet traffic. The transition matrix P represents the dynamics of the video/voice traffic, and we are interested in its steady-state solution X_l. Noting that the situation at the first-class portion of the frame can be modeled as a BCMP node with service of type 3 and with population size constraints, the steady-state distribution is analytically described from the product-form solution as:

$$p(i,j) = G(\Omega)^{-1}\frac{\rho_1^i}{i!}\frac{\rho_2^j}{j!}$$

As was seen in the previous chapters, the normalization factor $G(\Omega)^{-1}$ may be efficiently calculated by means of the occupancy distribution in the form:

$$\sum_{i=1}^{2} \rho_i b_i q(j-b_i) = jq(j) \quad \text{and} \quad G(\Omega)^{-1} = q(0). \tag{3.12}$$

with ρ_i the type i load, b_i the bandwidth requirements of type -i traffic and $q(.)$ the occupancy distribution of the link.

A similar expression was derived in Section 2.2.3, under the finite-source model assumption :

$$\sum_{i=1}^{2} [\, N_i \; - \; n_i + 1 \,] \rho_i b_i q(j-b_i) = jq(j). \tag{3.13}$$

for any $j \leq F$. F is the available capacity, N_i the maximum number of binary sources involved in the system, and n_i the number of active sources of type i associated with occupancy j .

Since the long-term variations of the first-class traffic are reflected upon the number of servers seen by the second-class traffic, and more specifically, the steady-state vector X_l of P corresponds to a unique steady-state occupancy distribution, this approach leads to a hybrid solution that combines analytically- and numerically-derived results.

3.2.4 The three-level hierarchy - decomposition of the P matrix.

In the three-level decomposition, the matrix P that describes the dynamics of the first-class traffic, is decomposed into :

$$P = P^* + \varepsilon_1 C_1 \tag{3.14}$$

The subsystems P_I^* ; $I=0, \cdots s_{VI}$ of P^*, represent the variations at the voice level, whereas the stochastic matrix $P^{'}$ represents the long-term variations due to the video traffic at the higher level. The elements of $P^{'}$ are given in the following expression :

$$p_{IJ}^{'} = \sum_{i \in I} \nu_{iI}^* \sum_{j \in J} p_{iIjJ}$$

with I and J the set of entries of aggregates P_I^* P_J^*, the elements p_{iIjJ} of P and ν_{iI}^* the entries of the steady-state vector of each aggregate P_I^*. A similar approach to the one followed in the two-level case is followed here for the decomposition analysis with similar results. The error of the approximation though, is increased and bounded by:

$$\varepsilon_1 / 2 (n_1 + \varepsilon_1)$$

with n_1 the maximum degree of coupling between aggregates of level 1 that belong to the same aggregate of level 2 and ε_1 defined in equation (3.14).

3.2.5 Example 1.

Here, we present results concerning the second-order approximation of the two-level hierarchy under the Poisson assumption for the video/voice traffic. The blocking probabilities for the video/voice traffic can be computed using the recursive relation in equation (3.12). The performance of the data traffic is expressed in terms of the packet-loss probability and the average delay. Given that the steady-state distribution is calculated using the decomposition technique above, the loss probability L_d for the data traffic is given by :

$$L_d = 1 - \tau^{-1} \sum_{i,j} \{ \sum_{k=0}^{U(i,j)} \tilde{\pi}(i,j,k) \sum_{l=0}^{M} la_d(l,M) + \sum_{k=U(i,j)+1}^{M} \tilde{\pi}(i,j,k) \sum_{l=0}^{M-(k-U(i,j))} la_d(l,M-(k-U(i,j))) \}$$

with $\tilde{\pi}(i,j,k)$ the approximated steady-state vector in terms of video, voice sources and data packets, respectively, that we derived in Sections 3.2.3.1 and 3.2.3.2. Then the average packet delay w_p is derived by means of the Little's formula :

$$w_p = \sum_{i,j} \sum_{k=0}^{M} k \tilde{\pi}(i,j,k) / \lambda_d (1 - L_d)$$

A case in which $I_{vi} = 3$, $I_{vo} = 2$, and $I_d = 1$ is presented here, with arrival rates $\lambda_{vi} = 0.001$, $\mu_{vi} = 0.005$ and $\mu_{vo} = 0.02$. Three arrival rates for the voice traffic are used, λ_{vo} : 0.1, 0.05, 0.01 calls/sec and a variable number of packet calls. The buffer-size was assumed to be 20 packets long. The frame duration is 0.1 sec. and contains 8 slots, 6 of which are dedicated to first-class traffic. For the above values, the second-order approximation (Section 3.2.3.2) was found to be high, in the order of 10^{-7}.

The average delay of the packets in the system and the loss probability is plotted in Figure 3.4, as a function of the normalized arrival rate $a_d \tau$, the number of packet arrivals per frame. It is clear that higher arrival rates of the voice traffic result in higher packet-loss probabilities and higher saturation levels of the data delay (given by the ratio of the buffer-size to the steady-state number of servers for the data traffic).

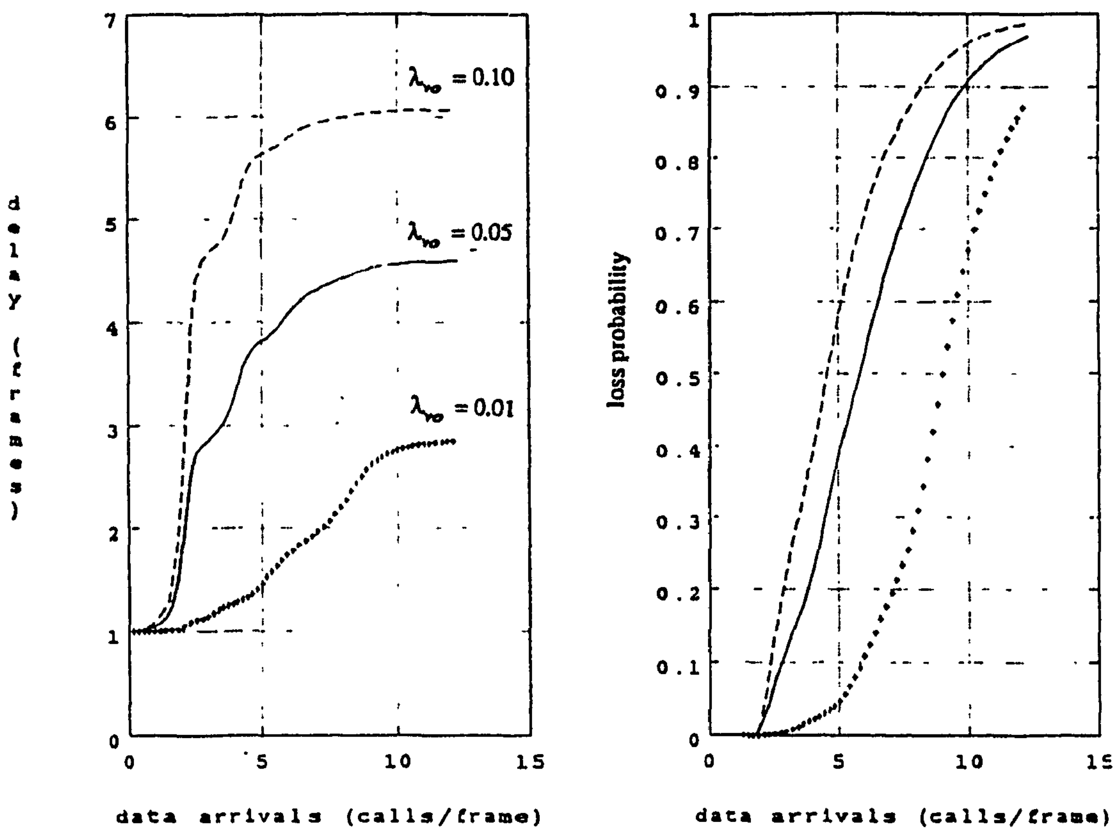


Figure 3.4. Delay/loss characteristics for buffer size: 20 packets,

$\lambda_{vi} = 0.001, \mu_{vi} = 0.005, \mu_{vo} = 0.02.$

3.2.6 Example 2.

Now, we consider a more realistic application involving the finite-source model and a larger capacity, that of a DS3 frame. Exact calculation of the the state-transition probabilities for the video/voice traffic in such scale is clearly intractable. Thus, equation (3.13) was used, instead, to yield analytical results for the occupancy distribution of the first-class portion of the frame. Next, we give an application of this technique.

Assume variable-rate video sources being characterized by a peak rate of 5 *Mbits/s* and average rate of 1 *Mbits/s* share equally 60 % of a DS3 (T3) capacity along with 64*kbits/s* voice sources. The data traffic is composed by ATM packets (53 *bytes*) long, with a dedicated bandwidth of 40% of the DS3 capacity and different buffer sizes. The burst-loss probability for video/voice traffic is 10^{-6}. Figure 3.5 shows the average delay and the buffer-overflow probability for the data traffic when the the buffer size is 20, 50, 100 ATM cells, as a function of the normalized arrival rate in packets per frame. The accuracy of the approximation is of the order of 10^{-3}.

The straight line in both the diagrams corresponds to a buffer capacity of 20 ATM cells, the dotted line to a buffer capacity of 50 cells, and the stars to the buffer capacity of 100 cells. The above results suggest to us that an increment of the buffer size from 20 to 100 improves slightly the cell-loss probability of the system. All three curves increase exponentially with the data traffic load and the lines for the 100 and 50 cells capacity almost coincide. The delay characteristics of the three cases coincide in the underload range of values. They attain a steady value - the ratio of the buffer size to the number of

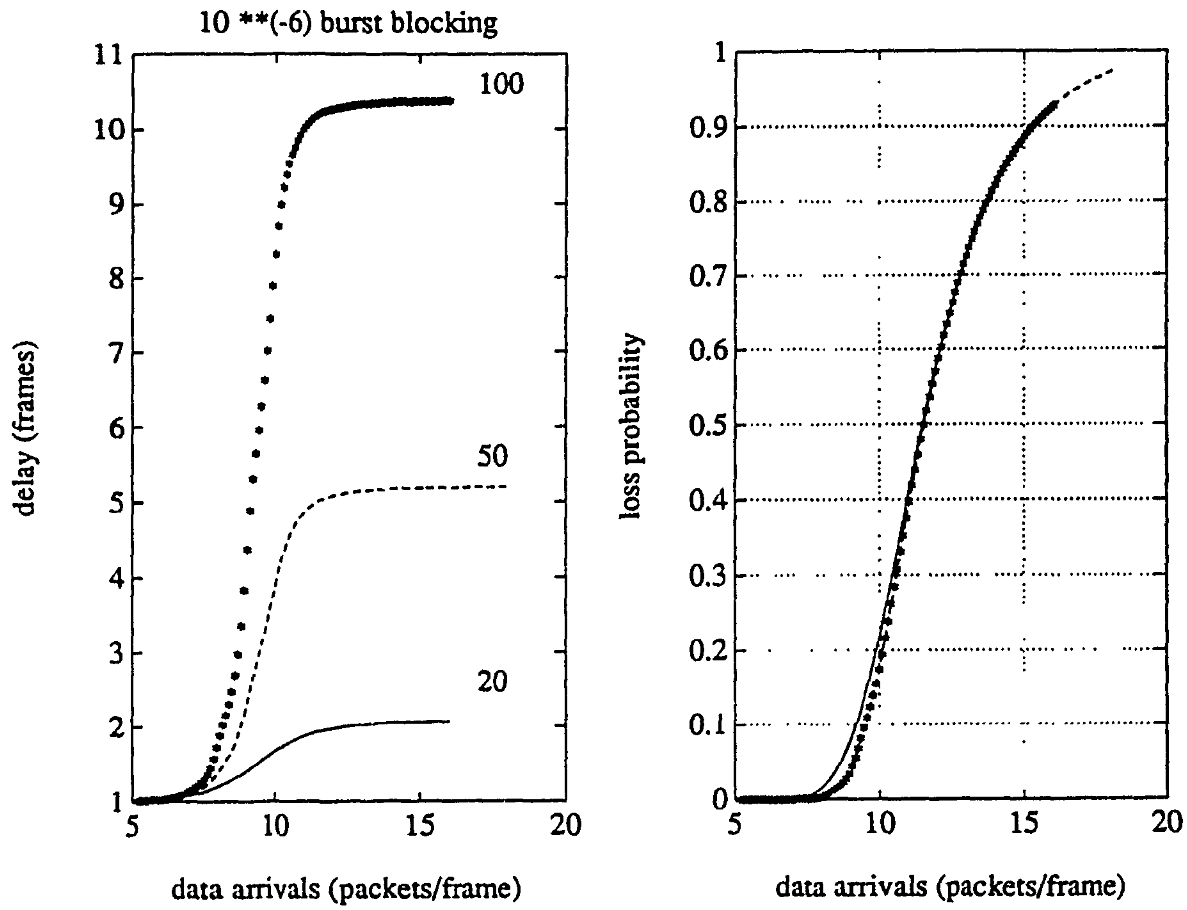


Figure 3.5. DS3 (T3) application, three buffer sizes: 20, 50, 100 ATM cells.

servers - as the offered data traffic exceeds its exclusively-dedicated servers.

The effect of various mixtures of the first-class traffic - in terms of video/voice percentage - is assessed in the following application. Variable-rate video traffic with a peak rate of 5 *Mbits/s* and average rate of 1 *Mbits/s* per call, shares equally 60 % of a DS3 (T3) capacity along with 64*kbits/s* voice sources. Again, the data traffic is composed of ATM cells, with a dedicated bandwidth of 40% The burst-loss probability for the first-class traffic is given in equation (3.13) and was set to be 10^{-6}.

In Figure 3.6, we plot the acceptable mean data rate, for a given cell-loss probability (10^{-3}) and a buffer capacity of 50 ATM cells, as a function of different percentages of video traffic. For the given burst-loss probability, we expect bandwidth utilization of the first-class traffic to be higher if only voice traffic is present, since the bandwidth demands of a voice source is lower than that of a video (binary) source. Their ratio is 64 *kbits/s* / 250 *kbits/s* in this example. Lower bandwidth utilization of the video/voice portion of the frame is translated into more servers available to the data traffic. It is clear that when only voice traffic is present, the first-class portion of the frame is heavily packed, resulting in a lower data rate being acceptable to the system for the given overflow probability.

The plot in Figure 3.6 is an admission region for the data traffic and can be used similarly with those in Figs. 2.6a and 2.6b in deciding the acceptable data-traffic rate for a predefined performance threshold (10^{-3} cell-loss probability in this example). In fact, three-dimensional acceptance regions can be constructed, with the three coordinates representing the load of video, voice and data traffic, respectively.

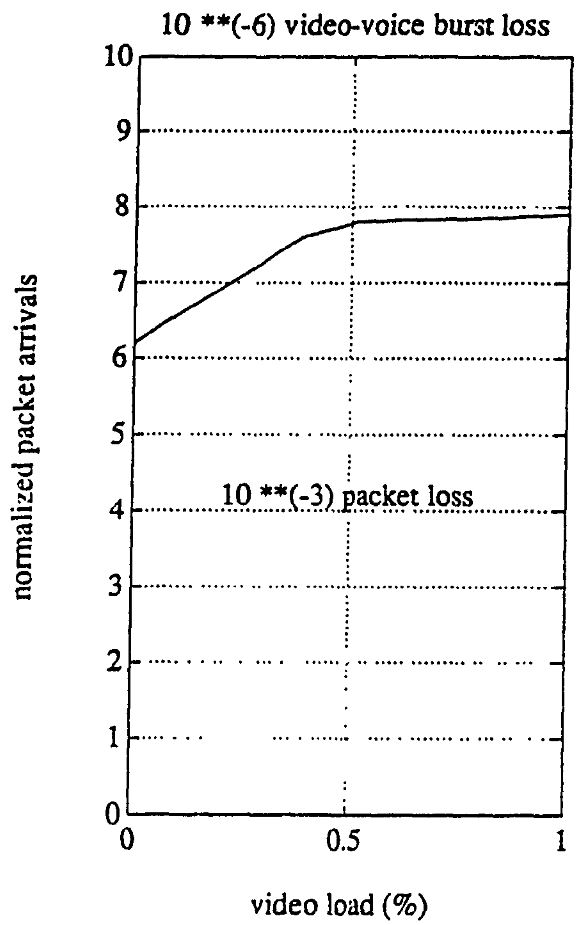


Figure 3.6. Acceptable data rate for buffer size 50 ATM cells.

3.3 Infinite Capacity Data Buffers.

In the second part of this chapter, we analyze the performance of the above-considered system under the 'infinite-buffer capacity' assumption. The matrix-geometric techniques, as suggested in [Neu89], finds direct applications to our problem. 'Exact' solutions of this type though, run into computational difficulties when realistic applications are considered.

We outline the way the technique proceeds in the following paragraphs. Given that the state of the system is (i,j) with i: the number of ATM packets in the system and j : the number of available servers (clusters of 53 voice channels (bytes)), we choose a lexicographical mapping of the state space that yields a state-transition matrix of M/G/1 type that has the following structure:

$$Q = \begin{bmatrix} B_0 & B_1 & B_2 & \cdot & \cdot & \cdot \\ A_0 & A_1 & A_2 & \cdot & \cdot & \cdot \\ 0 & A_0 & A_1 & \cdot & \cdot & \cdot \\ 0 & 0 & A_0 & \cdot & \cdot & \cdot \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \qquad (3.15)$$

The subscripts i of the submatrices B_i, A_i ; $i \geq 0$, represent the levels of the Markov chain, that is, the set of states $\{(i,\cdot)\}$ with the same index i. The steady-state solution of the Q

matrix (if existent) has the form

$$[\,\boldsymbol{x}_0, \boldsymbol{x}_1, \boldsymbol{x}_2, \ldots\ldots\,] \tag{3.16}$$

with the vectors $\{\boldsymbol{x}_i\}$ being derived recursively [Ram88] as :

$$\boldsymbol{x}_i = [\,\boldsymbol{x}_0\bar{B}_i + \sum_{k=1}^{i-1}\boldsymbol{x}_k\bar{A}_{i+1-k}\,](I - \bar{A}_1)^{-1}\ ;\ i \geq 1. \tag{3.17}$$

with

$$\bar{A}_i = \sum_{j=i}^{\infty} A_j G^{j-i} \quad \text{and} \quad \bar{B}_i = \sum_{j=i}^{\infty} B_j G^{j-i}$$

I is the identity matrix and G the minimal nonnegative solution of

$$G = \sum_{i=0}^{\infty} A_i G^i$$

G represents the rate matrix $G(z)$ evaluated at $z=1$. Its entries are generating functions of time distributions which, in turn, govern the rate at which a system would make transitions among the states from level $i+1$ to level i. In general, closed-form expressions for $G(z)$ do not exist, except some special cases, and its computation is done recursively. We follow Neuts' terminology in defining the matrices :

$$A = \sum_{i=0}^{\infty} A_i \qquad B = \sum_{i=0}^{\infty} B_i$$

When the matrix A is irreducible, then the steady-state vector $\boldsymbol{\pi}$ of Q exists and satisfies:

$$\boldsymbol{\pi} = \boldsymbol{\pi} Q$$

Given the evaluation of G, we recursively compute the K matrix in :

$$K = \sum_{i=0}^{\infty} B_i G^i$$

We start the computation of the $\boldsymbol{x}_0$ vector by calculating β.

$$\beta = \sum_{v=1}^{\infty} v A_v \boldsymbol{e}$$

with $\boldsymbol{e}$ a column vector with all entries equal to one. Then $\boldsymbol{x}_0$ is given by

$$\boldsymbol{x}_0 = \boldsymbol{k} / \boldsymbol{k}\boldsymbol{k}_1$$

$\boldsymbol{k}$ is the invariant vector of K and $\boldsymbol{k}_1$ is given in the following expression

$$\boldsymbol{k}_1 = \boldsymbol{e} + \sum_{v=1}^{\infty} \left[B_v \sum_{i=0}^{v-1} G^i \right] \mu$$

with μ defined by:

$$\mu = [I - G + \boldsymbol{e}\boldsymbol{g}]\,[I - A + (\boldsymbol{e} - \beta)\boldsymbol{g}]^{-1}\,\boldsymbol{e}$$

and $\boldsymbol{g}$ the steady-state vector of G. This completes the calculation of $\boldsymbol{x}_0$ that leads to the recursive computation of the vector $[\,\boldsymbol{x}_0, \boldsymbol{x}_1, \boldsymbol{x}_2, ...\,]$ in equation (3.17).

As an example, we examine the applicability of matrix-geometric techniques in a DS3 (T3) application involving ATM cells. Although the size of $\{A_i\}$, $\{B_i\}$ submatrices is considerable, 196x196, direct application of a matrix-geometric technique is limited by the amount of computations involved in calculating the transition probabilities between the states of the first-class traffic. As an alternative, we suggest the following scheme that employs decomposition at a higher level exploiting the hierarchical structure of the system and matrix-geometric techniques at the lower level, that takes advantage of the sparsity of the subsystems under study.

More specifically, we use equation (3.9), that gives the first-order approximation of the steady-state solution in terms of :

i. the equilibrium of the long-term dynamics (first-class traffic) and,

ii. the steady-state solution of the short-term dynamics (data traffic), for the various modes of the first-class traffic.

Equation (3.13) is subsequently used to derive analytical results for (i), that is, the steady-state distribution of the number of available servers as seen from the data traffic, for the various modes of operation of the first-class traffic, and the matrix-geometric technique for the derivation of the steady-state solutions of the subsystems Q_l^* ; $l = 0, 1, \cdots s_r$. Q_l^*, as before, represents state-transition matrices of M/G/m type, similar to those in Section 3.2.3.1, in which the buffer size dependence is dropped since we assumed that it is infinite. Note that the structure of Q_l^* is that of equation (3.15) with A_i, B_i, $m \times m$ matrices with m the number of available servers. Since both the invariant vectors in (i), (ii) are calculated exactly, the error of the approximation is in the order of ε [Cou77] and given in equation (3.8).

In order to demonstrate the flexibility of this technique, we introduce the following control mechanism that maximizes the bandwidth utilization by regulating the data traffic according to the various modes of operation of the first-class traffic as in [WiLe84]. We assume an infinite number of data classes with ordered priorities that can be throttled according to the temporary load (on a frame basis) of video/voice traffic. We use an infinite number of data classes so we can maximize the offered data load, although in practice only a limited number of data classes is available. Thus, we regulate the total offered load of data traffic (that should be strictly less than one), by accepting data traffic of a higher arrival rate when the first-class portion of the frame is underused and lower rate data traffic during overloaded periods of the first-class traffic. Results concerning the average number of ATM cells in the system, are given in the following example.

3.3.1 Example 3.

In this example, we derive the average number of cells by applying the techniques from the previous section. Assume again a DS3 (T3) link (768 VC) and the parameters for the video and voice traffic in example 2. Again, 60 % of the bandwidth is allocated to the first-class traffic. The data traffic is composed of cells 53 bytes long (ATM cells), that are stored in a buffer of infinite capacity. The accuracy of the approximation is again of the order of 10^{-3}. We plot two curves in Figure 3.7 for burst-loss probabilities at least 10^{-6} for both video and voice. Note, that the burst-loss probability of the video/voice traffic is unrelated to the accuracy of the involved approximation, which is used to derive the performance parameters of the data traffic.

Curve A is the average number of ATM cells in the system when fixed boundary is employed, curve B shows the average number of ATM cells when the ideal control mechanism is employed. The before-mentioned control mechanism allows a higher bandwidth utilization for an acceptable cell delay threshold if the required processing

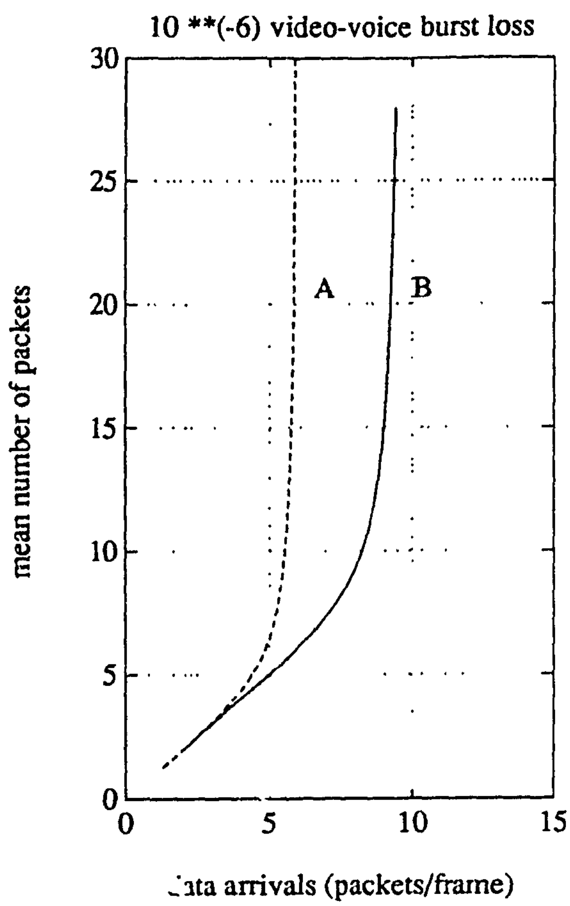


Figure 3.7. DS3 (T3) application, infinite size buffer, A: fixed boundary, B: ideal control.

overhead is acceptable. Curve A is the case in which the two types of traffic (first-class and ATM cells) are completed segregated. Note that the two curves represent the lower and upper bounds to any scheme that involves movable boundary with or without data-flow controls.

It is clear that the implementation of the fixed boundary scheme (curve A) is much simpler than the ideal control mechanism with an infinite number of data classes (curve B) which guarantees the maximum bandwidth utilization. Based on these two bounds one could assess the efficiency of a given sharing scheme (movable boundary with or without data-traffic control) in the tradeoff between complexity of controls and bandwidth-utilization gain.

3.4 Mixed-media integration in the digital multiplexing hierarchy.

In this section, we analyze another case in which decomposition techniques apply in a straightforward manner. According to this scenario of traffic integration, we assume four different types of traffic. Slow-scanning video and voice traffic constitute the low-priority traffic with bandwidth requirements 56 *kbits/s* (DS0) and 1.544 *Mbits/s* (DS1), respectively, that share equally a capacity of 1.544 *Mbits/s*. That is, either the whole bandwidth is dedicated to the slow-scanning video traffic or at most 24 voice calls are supported by the multiplexer on a FCFS basis. We also assume that the above mixture of traffic is multiplexed at higher levels according to the multiplexing hierarchy presented in Fig. 3.8.

The other two types of traffic we consider here are two types of video traffic with bandwidth requirements DS3, DS4, respectively, that is, 44.7 *Mbits/s* and 274.17 *Mbits/s*, sharing equally a DS4 capacity of 274.17 *Mbits/s*. These two types of traffic have preemptive priority over the DS0, DS1 traffic. A number of low-priority video/voice sources is multiplexed up to a DS3 level and uses any unused portion of the DS4 frame that is shared equally by the first-priority traffic. A secondary multiplexing facility that absorbs the overflowing second-priority traffic under overloaded periods of the first-

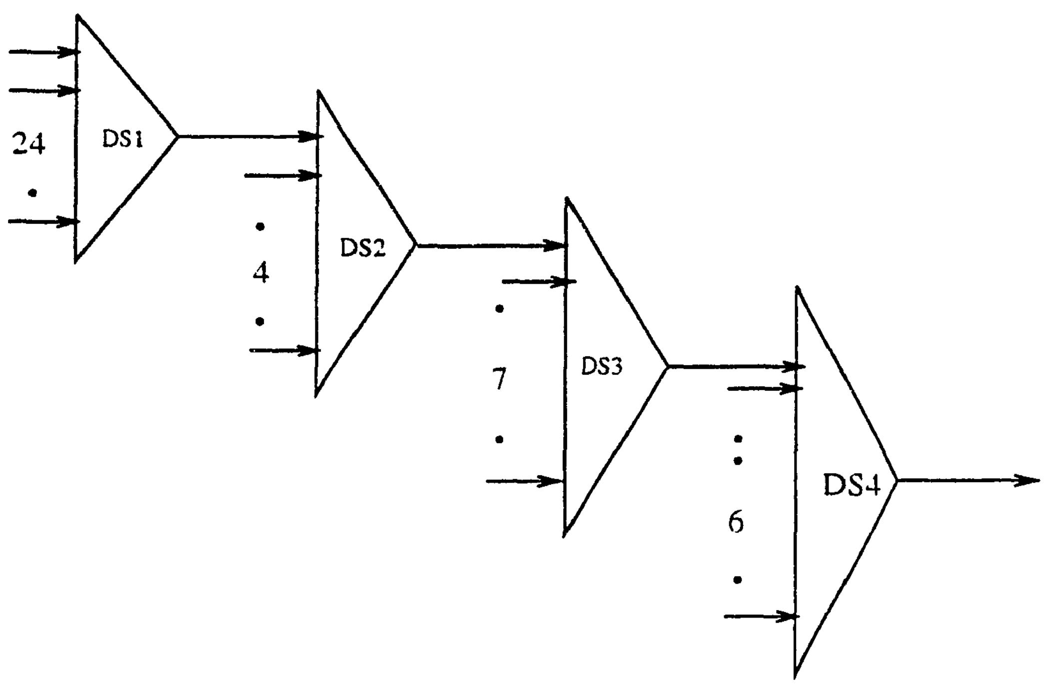


Figure 3.8. Digital multiplexing hierarchy up to DS4 level.

priority traffic is employed as well. Under the above assumptions, the bandwidth sharing scheme is shown in Figure 3.9.

As for the traffic model we assume that the arrival process of the various classes of traffic is given by the birth-death Markov model in [MASKR88], and the service requirements are exponentially distributed. Given that a hierarchical ordering of the arrival rates and the service rates is a reasonable assumption, that is, given that

$$\lambda_{DS4} \ll \lambda_{DS3} \ll \lambda_{DS1} \ll \lambda_{DS0} \quad \text{and} \quad \mu_{DS4} \ll \mu_{DS3} \ll \mu_{DS1} \ll \mu_{DS0}$$

the analysis proceeds as in Section 3.2. The blocking probabilities of the first-class traffic are given by :

$$P_{b_i} = \sum_{i=0}^{b_i-1} q(S-i)$$

in terms of the occupancy distribution of the first-class traffic.

Now, the first-order approximation of the steady-state vector is given in equation (3.9) in the form :

$$x_{il} = X_I \cdot v_I^*(i)$$

By analogy, the occupancy distribution vector f_{il} of the DS4 frame in terms of voice channels is given by :

$$f_{il} = q_I \cdot h_I(i) \quad I = 0, \ldots, 6; \qquad i = 0, \ldots, 4032.$$

with q_I the occupancy distribution vector of the DS3, DS4 traffic at the higher level and $h_I(i)$ the occupancy distribution of the low-priority traffic for the various modes of

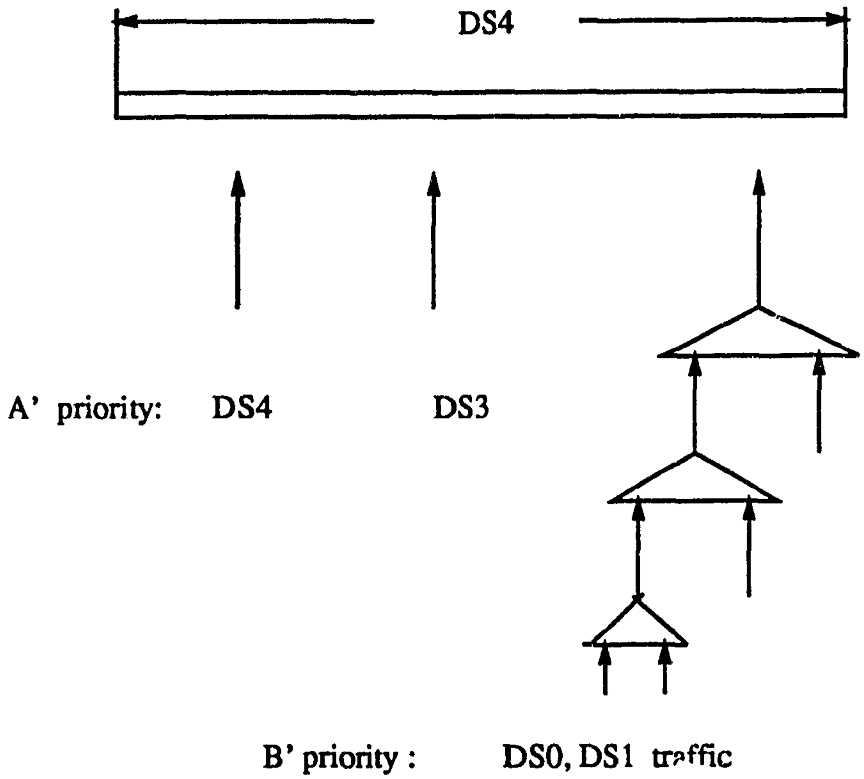


Figure 3.9. Bandwidth sharing scheme.

operation of the first-priority traffic. The latter distribution is given by the convolution of the occupancy distribution of the DS1 frame up to a DS4 level.

Based on the above expressions, we plot in Fig. 3.10a the steady-state occupancy distribution for the following traffic parameters:

- First-priority traffic: 10 DS3 sources with $a_{DS3}: 10^{-4}$ with bandwidth requirements 1 slot per call (in a DS4 frame of 6 DS3 slots) , 2 DS4 sources with $a_{DS4}: 10^{-5}$ (in a DS4 frame of 6 DS3 slots or 4032 voice channels) and bandwidth requirements 6 slots per call. The loss probability is 10^{-5}, 10^{-3} for the DS3, DS4 calls, respectively.

- Second-priority traffic: 100 DS0 sources with $a_{DS0}: 0.0002$ and 4 DS1 sources with $a_{DS1}: 10^{-3}$ and bandwidth requirements 1 and 24 slots, respectively (in a DS1 frame of 24 voice channels). The loss probability is 10^{-3} for both the classes of traffic.

The observed six clusters of the occupancy distribution correspond to the six modes of operation of the DS4, DS3 traffic, whereas the shaded area in each cluster is due to an array of bell-shaped distributions (see the nonlogarithmic version shown in Figure 3.10b, over the range of 0 to 100 voice channels). The straight line located at the 4032 channels (number of voice channels in a DS4 frame) is the probability of a full DS4 frame.

Given the occupancy distribution, it is straightforward to evaluate the performance parameters of the first- and second-priority traffic, as well as the bandwidth-utilization gain associated with this technique.

3.5 Conclusions

Decomposition and matrix-geometric techniques are known to find application in sharing schemes involving voice and data traffic [MoTs88], [GhaSc88], [WiLe84]. In this chapter, we extended the concept to include video traffic and employed the above techniques to analyze two scenarios involving finite and infinite-size buffers, respectively. The second main contribution in this chapter is the suggestion of a hybrid scheme that combines analytical and numerical solutions. This scheme that makes use of an analytical expression derived in the second chapter was shown to be an adequate technique for analyzing high-capacity scenarios with considerable accuracy.

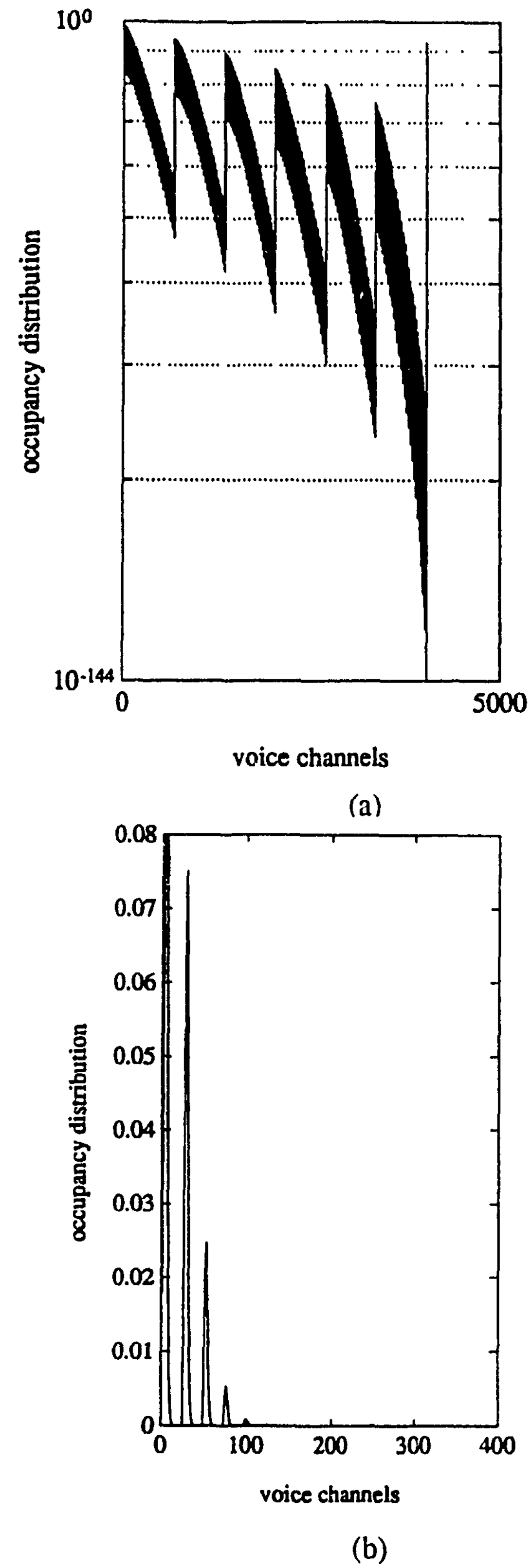


Figure 3.10. The occupancy distribution for the DS4 frame.

CHAPTER IV.

Conclusions and Further Extensions

In this thesis we suggested analytical expressions that constitute the basis for the development of control mechanisms that guarantee the satisfaction of the performance requirements of the various types of traffic with access to the broadband network. The first two chapters concern a scheme of traffic integration in which no distinction is made in terms of the delay/loss requirements of the supported types of traffic but instead, a universal grade of service is guaranteed that satisfies the stringent of the individual performance requirements. In the third chapter though, we study the case in which better bandwidth utilization is attained when taking into account the delay tolerant characteristics of the data traffic.

The results in Chapter I apply to the limiting case in which a practically infinite number of sources are statistically multiplexed in a single or multiple links. As shown in Section 2.4, when these results apply as approximations to situations involving bursty traffic, they should be handled with care. One of the contributions of our analysis is that only a two-dimensional recursion is needed in order to calculate blocking probabilities in a switch that does not provide multicasting services. The suggested two-dimensional recursion for the switch is computationally efficient and we obtained blocking probabilities for large switches (larger than 64×64). The same analysis was extended to derive recursions for broadcasting/multicasting switching capabilities. When increasing the dimensions of the occupancy distribution though, as for example in the ring topology, the computational effort increases as well and soon becomes unmanageable. Thus, reduced-load approximations as suggested in [Kel86], [Wh85] do not only constitute alternatives but a necessity when the number of links is large.

The a priori construction of the call-acceptance regions we suggested in Chapter II is efficient because:

i.) is performed only once, with subsequent recalculations concerning the point of operation only

ii.) is simple as a control mechanism and applies to all the supported classes of traffic

iii.) can be further simplified given that the offered peak-to-link rate ratio of the supported classes of traffic is small, to a 'Circuit Switched' type of operation.

The latter means that, given the linearity of the boundary of the admission region, the 'virtual bandwidth' characterization of a certain type of bursty traffic - and for a given grade of service - is valid when the multiplexer is loaded with a heterogenous mixture of traffic as well. Now, in order for this 'virtual Bandwidth' of the source to be computed for any type of traffic, we feed the multiplexer of a given capacity with one type of traffic only and then divide the multiplexer's capacity by the number of calls that can be supported given that the required GOS is guaranteed. It is easy to see from fig. 2.6 that the smaller the peak-to-link ratio the more linear the boundary becomes, thus, for an output capacity, say F, the calculated Virtual Bandwidth' V_i, and a linear boundary, the admission control is simply:

$$\sum_i n_i V_i \leq F$$

The integers n_i are the number of type -i calls that can be supported in an arbitrary mixture of traffic. In an ATM application, where policing mechanisms, like the 'leaky bucket' [Tur86] are employed, V_i should be reflected at the rate the control mechanism produces the transmission tokens. Similar results are valid in the two-dimensional case, when considering the performance of a switch. In a heterogenous environment of input traffic, 'Virtual Bandwidth' calculations were found to be valid here as well.

Finally, note that in a more realistic scenario of broadband integration, finite-size buffers at the switching nodes are involved. This case is not explicitly analyzed here, but our results apply approximately to cases in which a common input traffic buffer exists, given that the burst length of the supported applications is comparable to (or exceeds) the buffer size. The above observation -if true- is particularly important in the light of traffic integration of video distribution services where the possible delays may considered to be unacceptably long [VePi89].

Our work in Chapters I and II finds direct application in analyzing a reservation protocol that was suggested recently in CCITT. The fast reservation protocol (FPR) [CCITT91] uses in-band signaling to negotiate changes to a connection's information transfer rate. According to this protocol, whenever a connection has a burst of cells to transmit, it will first send a special request cell. Upon receiving this request cell, network elements along the connection path will attempt to reserve bandwidth at the connection's peak rate. If this allocation is successful at all nodes, an acknowledgment will follow and the connection will be allowed to send its burst. Once the burst is completed the reserved bandwidth is automatically released and the connection is assumed to remain idle until the next burst request is received.

Note how the model suggested in Chapters I and II fits the description of the above mentioned protocol. The changes to the connection's information transfer rate are modeled by binary sources with different bandwidth demands (transmission rates) and distinct arrival/service statistics. Burst blocking probabilities are given via the occupancy distribution, of one or more dimensions, depending on the number of links along the connection path.

As for the results in Chapter III, 'movable boundary' techniques constitute an alternative for traffic integration. Two cases were analyzed here, based on 'finite' and 'infinite' buffer assumptions, respectively. 'Near complete decomposability' techniques were shown to be adequate solutions in confronting the computational burden of an embedded Markov chain approach. The adopted queueing model is flexible in analyzing a number of sharing schemes including different priorities for the video-voice traffic as well as a possible employment of buffer space for the voice traffic.

Our results on high-capacity applications indicate that there is not a dramatic improvement in the performance of the data traffic due to the movable boundary scheme. Better results can be obtained by means of priority mechanisms that regulate the data traffic; however, the possible benefits in terms of bandwidth utilization should be well weighted against the increased complexity of controls they imply.

Possible extensions of the above work include the end-to-end performance of a network employing various levels of statistical multiplexing and a switching core (see Section 2.5). Approximations involving the results from Sections 2.2.3 and 2.3 are readily available. An analysis of this type, yields end-to-end loss probabilities for the various types of traffic and subsequent formulation of a single call-acceptance region as in Sections 2.2 and 2.3 regardless of the specific network topology under consideration.

Other unanswered questions concern the linearity of the boundary. For example, what is the necessary peak-to-link ratio that guarantees the linearity of the boundary or how different types of traffic relate to the suggested 'linearity ratio'. Only three specific types of traffic were examined here and further results are required.

Also, the results in Chapter II can be further extended along the lines of the analysis in [Hui88]. Large-deviation techniques and the Chernoff bound are used in this paper to derive tight bounds for the tail distribution of the offered process, and consequently the blocking experienced by different types of traffic feeding a statistical multiplexer. The accuracy of this approximation was examined in Section 2.2.5 and was found to be high. A possible extension could be the derivation of similar bounds for more than one link, starting with the two-dimensional case, the switch, whose analysis in Section 2.3 poses numerical difficulties due to the exhaustive enumeration of the state space.

Finally, as mentioned in Section 3.3, the problem with a direct application of matrix-geometric techniques is the computational burden of the state-transition probabilities of the first class traffic, when high-capacity applications are considered. Thus, we chose to analyze the performance of the data traffic under an infinite-buffer assumption via a combination of decomposability and matrix-geometric techniques. Another alternative that should be examined in the future is an approximation in the calculation of state-transition probabilities that retains their relative orders of magnitude, with subsequent application of matrix-geometric techniques.

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APPENDIX A

LEMMA I.

In the single-dimensional case, (Kaufman's model) with $N_I = N_o = F$ and a set of bandwidth requirements $\boldsymbol{B} = \{b_i \quad i = 1, \cdots, K\}$ we prove that :

if j is not an allowable occupancy, under the constraint $j \le F$, then $j - b_i$ is not an allowable occupancy for any $b_i \in \boldsymbol{B}$.

Proof.

Let $\boldsymbol{R}$ be the set of allowable occupancies under Ω and $\boldsymbol{B}$, and the vector $\boldsymbol{b}$, $\boldsymbol{b} = (b_1, \cdots, b_i, \cdots, b_K)$. Then $j \le F$, and $j \notin \boldsymbol{R}$, implies that there is no $\mathbf{n} \in \Omega$ such that $\mathbf{n}\boldsymbol{b} = j$. If there was an $\mathbf{n}$ in Ω such that $\mathbf{n}\boldsymbol{b} = j - b_i$, then $\mathbf{n}\boldsymbol{b} + b_i = j$ or equivalently $\mathbf{n}_i^+\boldsymbol{b} = j$ and $\mathbf{n}_i^+$ should belong to Ω since $j \le F$.

Therefore, $j - b_i \notin \boldsymbol{R}$ for any $b_i \in \boldsymbol{B}$ and $j \le F$.

LEMMA II.

For any input/output pair μ, ν and an associated set of bandwidth requirements $\boldsymbol{B} = \{b_i \quad i = 1, \cdots, K\}$ we prove that :

if the pair of respective occupancies (j, l) is not allowable under the constraints $j \le F_{\mu I}$, $l \le F_{\nu o}$, then none of the pairs $(j - b_i, l)$ $(j, l - b_i)$ $(j - b_i, l - b_i)$, represents a nominal occupancy pair.

Proof.

$R_{\mu\nu}$ is the set of allowable pairs of occupancies under Ω and B . If the pair $(j, l) \notin R_{\mu\nu}$ then :

a. either j is not an allowable occupancy, $j \notin R$

b. or l is not an allowable occupancy, $l \notin R'$

c. or both j, l are not allowable occupancies for μ, ν respectively.

case a.

If j is not an allowable occupancy then

1. $(j - b_i, l) \notin R_{\mu\nu}$ because $j - b_i$ is not an allowable occupancy from lemma I ,

2. $(j, l - b_i) \notin R_{\mu\nu}$ because $j \notin R$

3. $(j - b_i, l - b_i) \notin R_{\mu\nu}$ because $j - b_i \notin R$ from lemma I .

Exactly the same is the proof in cases (b) and (c) . Using the convention $q_{\mu\nu}(j,l) = 0 \ \forall$ $(j, l) \notin R_{\mu\nu}$ equations (1.9),(1.10) and (1.11) hold for every $j \leq F_{\mu j}$, $l \leq F_{\nu_0}$.

APPENDIX B

LEMMA I.

We prove that the multidimensional birth-death process described in the first section, with the global balance equation :

$$[\sum_{i=1}^{K} (N_i - n_i)\lambda_i \delta_i^+(\mathbf{n}) + \sum_{i=1}^{K} n_i \mu_i \delta_i^-(\mathbf{n})]\, P(\mathbf{n}) =$$

$$\sum_{i=1}^{K} (N_i - n_i + 1)\lambda_i \delta_i^-(\mathbf{n}) P(\mathbf{n}_i^-) + \sum_{i=1}^{K} (n_i + 1)\mu_i \delta_i^+(\mathbf{n})\, P(\mathbf{n}_i^+) \qquad \forall\ \mathbf{n} \in \Omega.$$

and the local balance equation in (1), is characterized by the product-form solution. The functions $\delta_i^+(\mathbf{n})$ and $\delta_i^-(\mathbf{n})$ are defined as :

$$\delta_i^+(\mathbf{n}) = \begin{cases} 1 & \text{if } \mathbf{n}_i^+ \in \Omega \\ 0 & \textit{otherwise} \end{cases}$$

$$\delta_i^-(\mathbf{n}) = \begin{cases} 1 & \text{if } \mathbf{n}_i^- \in \Omega \\ 0 & \textit{otherwise} \end{cases}$$

<u>Proof.</u>

According to Kolmogorov's criteria for reversible stochastic processes ([Kel79], pp. 21), our Markovian process is reversible since for any closed loop in the state-transition diagram, the product of the rates in one direction equals the product of the rates in the reversed direction (for an example, see Figure 2.4). Also, our process is a " spatial process " because ([Kel79], pp. 189)

i). only one component of the state $\mathbf{n}$ can change at a time ii). the transition rates between two states depends exclusively on these two states

iii) any state in Ω can be reached by any other one by a finite sequence of transitions.

The equilibrium distribution of a reversible spatial process is a Markov field that has the product-form solution (Theorem 9.3 , [Kel79]). A different solution is also given by Hui in [Hui89].

LEMMA II.

In a one-to-one mapping between two stochastic systems with

(i) the same set of states Ω and (ii) the same values for the parameters

$N_i, a_i, K \qquad \forall\ i \leq K,$

the blocking probabilities for the different classes are the same in both the systems.

Proof.

The set $B_i^+ = \{\mathbf{n} \in \Omega : \mathbf{n}_i^+ \notin \Omega\}$ is the same for both the systems because of condition (i).

From eq.(2.2)

$$P(\mathbf{n}) = G(\Omega)^{-1} \prod_{i=1}^{K} \binom{N_i}{n_i} a_i^{n_i}$$

with $G(\Omega)$ the normalization constant. Because of condition (ii), $\forall\ \mathbf{n} \in \Omega$, $P(\mathbf{n})$ has the same value in both the stochastic systems. The blocking probability for the i^{th} class is given by:

$$P_{bi} = \sum_{\mathbf{n} \in B_i^+} P(\mathbf{n})$$

Thus, P_{bi} is the same for both the systems.

LEMMA III.

Assuming a multiplexer with capacity F bandwidth units and a set of bandwidth requirements $\boldsymbol{B} = \{b_i \;\; i = 1, \cdots, K\}$ we prove that :

if j is not an allowable occupancy, under the constraint $j \leq F$, then $j - b_i$ is not an allowable occupancy either, for any $b_i \in \boldsymbol{B}$.

Proof.

Let $\boldsymbol{R}$ be the set of allowable occupancies under Ω and $\boldsymbol{B}$, and the vector $\boldsymbol{b}$, $\boldsymbol{b} = (b_1, \cdots, b_i, \cdots, b_K)$. Then $j \leq F$, and $j \notin \boldsymbol{R}$, implies that there is no $\mathbf{n} \in \Omega$ such that $\mathbf{n}\boldsymbol{b} = j$. If there was an $\mathbf{n}$ in Ω such that $\mathbf{n}\boldsymbol{b} = j - b_i$, then $\mathbf{n}\boldsymbol{b} + b_i = j$ or equivalently $\mathbf{n}_i^+\boldsymbol{b} = j$ and $\mathbf{n}_i^+$ should belong to Ω since $j \leq F$.

Therefore, $j - b_i \notin \boldsymbol{R}$ for any $b_i \in \boldsymbol{B}$ and $j \leq F$. With the convention $q(j) = 0$; $\forall \; j \notin \boldsymbol{R}$, then $q(j - b_i) = 0$ as well, and equation (2.7) holds for every $j \leq F$.