CALENDER BARRING,

NORMAL MODES OF VIBRATION OF A CALENDER STACK

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ABSTRACT

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The barring phenomenon Which occurs in paper machine calenders is investigated. The available information on the subject is reviewed, summarized and presented in a condensed form. From the literature survey the concensus of opinion emerged that barring is caused by the resonant vibration of the calender stack with the paper acting as both an elastic component of the system and a source of excitation. Based on the various field observations reported in the literature a two-dimensional physical model of an closed-frame calender stack is constructed. A mathematical model is developed for the physical model to determine the undamped natural frequencies and modes of vibration of such a system. The analysis is programed for computer solution and tested by applying it to an operational calender stack. The results of the computations are in good agreement with field observations of calender behaviour in general.

ACKNOWLEDGEMENTS

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NOMENCLATURE

$[a]_n$	flexibility matrix for the nth roll
$[A]$	flexibility matrix for the calender stack
$[c]_n$	constraint matrix for the nth free-free roll
$[C]$	constraint matrix for the calender stack
d_i	flexural displacement of ith point mass of a roll
$(d)_n$	flexural displacement vector of the nth roll
(d)	flexural displacement vector for the system
D_i	absolute displacement of ith point mass of a roll
$(D)_n$	absolute displacement vector of the nth roll
(D)	absolute displacement vector for the system
$[I]$	identity matrix
$[kf]_n$	flexural stiffness matrix for the nth roll
$[KF]$	flexural stiffness matrix for the calender stack
$[KS]$	stiffness matrix for calender stack due to paper
$[m]_n$	inertia matrix for nth roll
$[M]$	inertia matrix for the calender stack
N	number of point masses per upper roll
NR	number of rolls in calender stack
T	kinetic energy of calender stack in free vibration
vf_n	flexural strain energy of the nth roll
VF	flexural potential energy of the calender stack
VS	potential energy in compressed paper
w_j	the jth natural circular frequency of the system

CHAPTER 1

INTRODUCTION AND LITERATURE SURVEY

1.1 INTRODUCTION

Calendering is the last stage of paper making where the final finish is given to the paper. The finish is produced in the calender stack by passing the sheet between cast iron rolls and subjecting it to high speed compressive stress cycles. In the case of certain types of paper, the calender applies chemical finish to the paper in addition to a mechanical finish. However, this study is mainly concerned with the calenders of newsprint machines whose sole function is to impart the final surface finish and caliper to the paper.

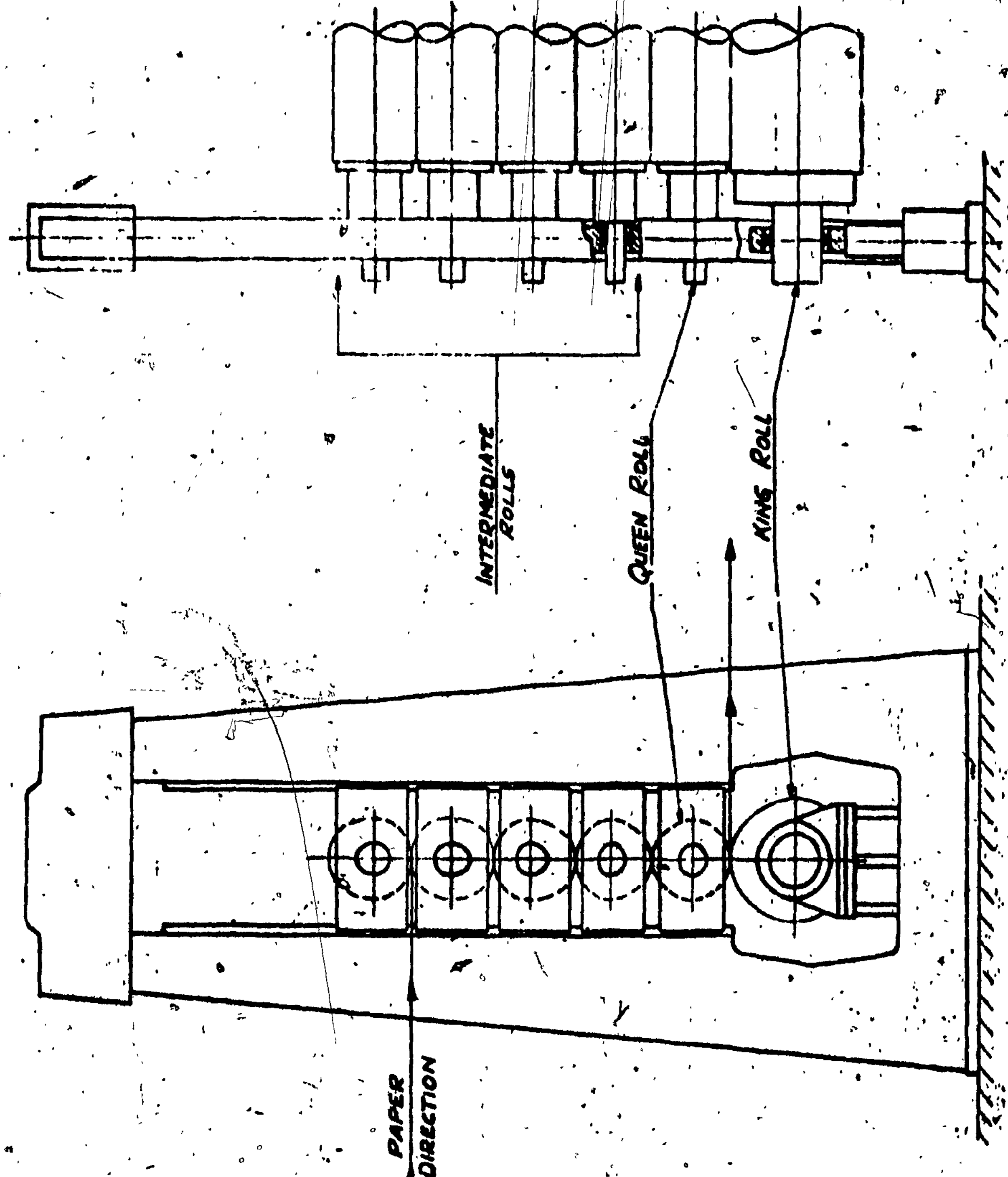
The calender stack consists of a frame and a number of calender rolls stacked on one another with their axis usually in the same vertical plane. The number of rolls thus assembled may vary from four to eight rolls to a calender stack. In rare cases there may be ten rolls in a stack. The upper rolls are of equal diameter and are supported on antifriction bearings. The two lower rolls, sometimes referred to as the queen and king rolls may in some cases be running on sleeve bearings. On modern machines the queen roll is usually the same size as the intermediate ones but on older machines it is larger than the intermediate rolls. The bottom roll, the king roll is always larger than the rest of the rolls. It is supporting the weight of all the rolls above it and tends to deflect under the load. In order to compensate for the deflection and to maintain a straight

1.1 Introduction (cont'd)

line of contact (called the nip) between rolls the king roll is crowned, that is it is ground in such a way that its diameter gradually decreases towards the ends. On more modern calenders a shell type king roll and pressure applied over a segment of the inside of the shell provide means for compensating for roll deflection. This has the advantage of allowing variation of the loading conditions or nip pressures. Most calender stacks contain one hollow roll or more which can be heated by passing steam through the bore along the center of the roll. The surface temperature of such rolls is an important operating factor.

The calender stack may be open or closed depending on whether the bearing housings of the rolls are set in a slot in the frame or supported on a cantilever. In the latter arrangement, means of counterbalancing the weight of the roll not in contact with the paper is usually provided. Figures 1, 2 and 3 schematically illustrate the two types of calender stacks. The power input to the calender stack is most often through the king roll but in some cases the queen roll is driven. To the rest of the rolls the necessary driving power is transmitted through the paper being calendered.

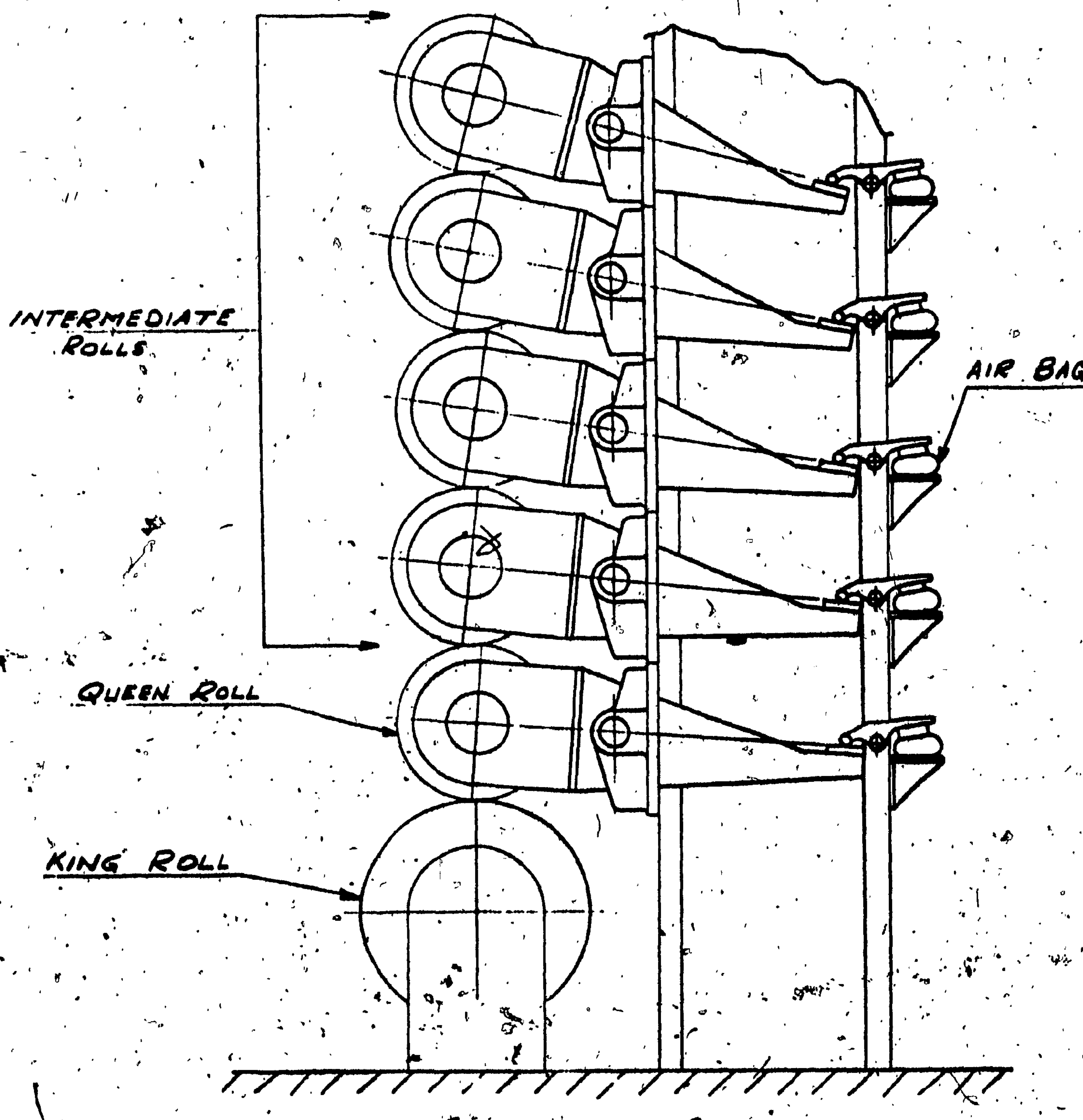
The mechanism by which calendering imparts the final finish and caliper to the paper is a complex one and not all aspects of it are well understood. The desired end result of



CONVENTIONAL CLOSED CALENDER STACK

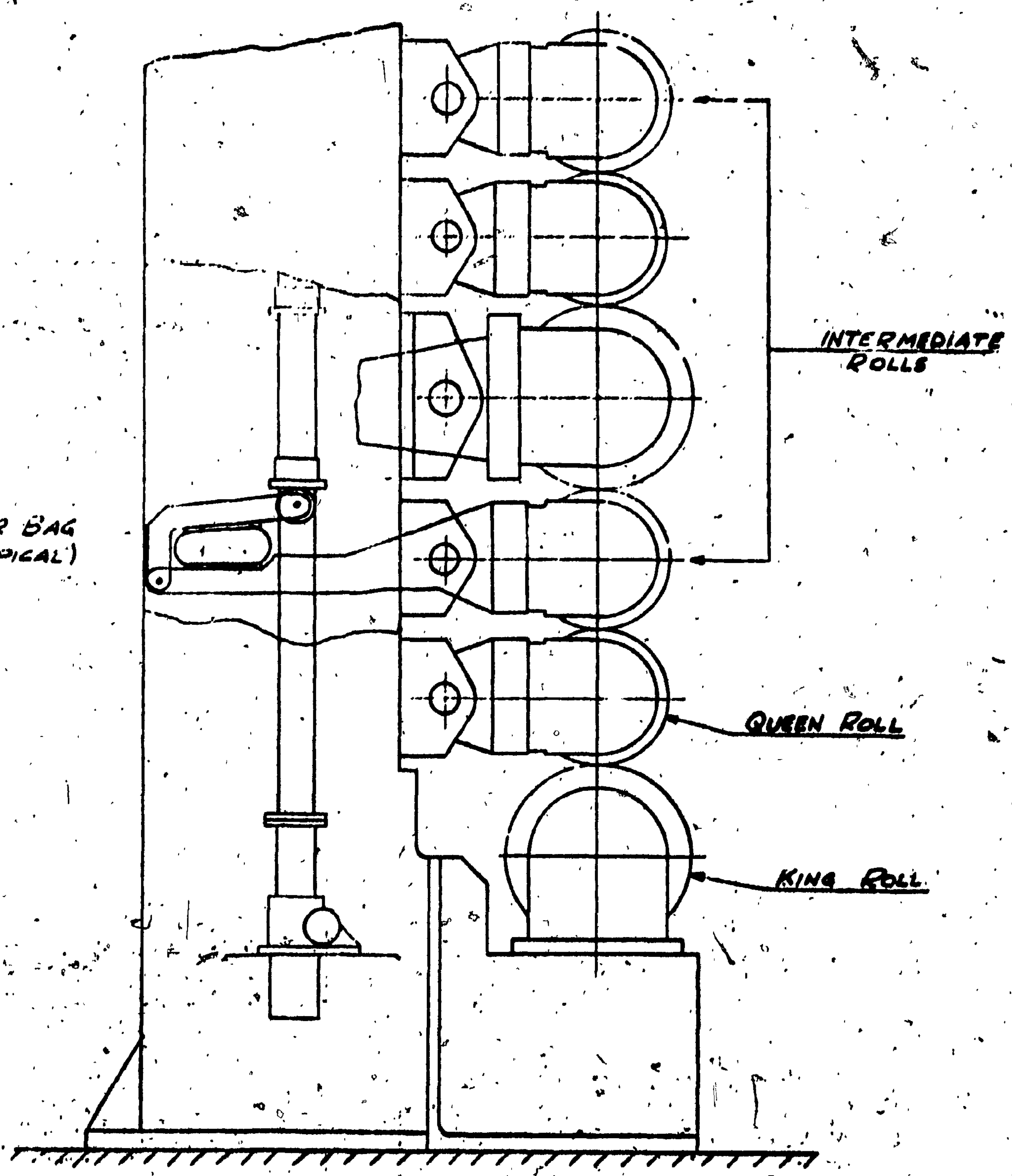
FIGURE N°1

1.1 Introduction (cont'd)



CANTILEVERED CALENDER

FIGURE Nº 2



"TANDOM" CANTILEVERED CALENDER

FIGURE Nº 8

1.1 Introduction (cont'd)

uniform finish qualities and caliper in the machine direction is obtained through the visco-elastic compression and shear stress cycles the paper is subjected to as it passes through the nips. The parameters and factors affecting the performance of the calender stack and therefore the quality of the paper produced are numerous. The number, size, weight, finish, and speed of the rolls are the most important design parameters. Roll condition, roll temperature and variables which affect nip pressures are the most important operating factors.

Newsprint machines operating at speeds above 1000 fpm are very often plagued with operating problems commonly referred to as barring. The visual manifestation of barring is the appearance of shiny stripes, called bars across the width of the paper and axial markings on the calender rolls. The two phenomena appear to be related and usually, but not always occur together. In the paper, barring is a quality defect manifested by periodic variation in the surface finish and caliper of the paper. It is a serious problem that may cause operating problems further down the paper machine, making winding of the paper into rolls difficult or it may effect the printability of the paper causing operating problems in the printing presses. Furthermore, the optical effect of shiny streaks across the paper gives an undesirable appearance to the paper and is in itself enough to make

1.1 Introduction (cont'd)

marketing of such paper difficult. On the calender rolls, barring is tangential corrugation running along the axial direction necessitating the frequent regrinding of rolls resulting in lost production and increased maintenance costs.

Although barring is a serious operating problem that has been with the industry for a long time, there is relatively little published literature on the problem. Most of the available literature deals with physical observations, measurements and descriptions of the numerous complex phenomena associated with barring. There are several theories based upon the findings of these investigations to explain the mechanism of barring but none of them can account for all the observed phenomena. There is a general agreement amongst the investigators that the direct cause of barring is the bouncing and banging together of the calender rolls. The cause and the mechanism of calender roll vibration however, are not well understood. Consequently, systematic methods of avoiding or remedying calender vibrations are not available. Rather, paper mills must depend on the experience and intuition of mill personnel and on trial and error procedures to deal with the barring problem when it is encountered. This can become quite frustrating and expensive as one means that has been successfully applied on one machine may have no effect or

1.1 Introduction (cont'd)

may even have an adverse effect when tried on another machine. There have been a few attempts to develop theoretical models to simulate calender stack behaviour. Because of the complexity of the problem the models used were highly simplified but did achieve various degrees of success in simulating observed phenomena. On the basis of such analyses, two investigators (8,16) proposed means of eliminating barring. One of these (16) is reported to have been successfully applied on a paper machine. The following is a review of the published literature related to the problem of calender barring. The available knowledge has been organized into categories in order to facilitate the use of this report as a quick source of information. Each category is relevant to certain aspects of the phenomenon of calender barring. The literature reviewed is listed at the end of this dissertation in chronological order and a brief description of the contents of each publication is also given.

1.2 THE NATURE OF CALENDER BARRING

Calender stack barring has long been an important operating problem of paper making. It has become especially common and troublesome with the advent of the high speed newsprint machine. The term barring describes the optical effect produced by periodic variation in the surface texture of both the calendered paper and the calender rolls. The two phenomena appear to be related but differ in several aspects.

Barring of Calender Rolls

Calender rolls are made from cast iron of a special composition (3). They are chilled iron rolls with a chill depth of 5/8 of an inch to 1 inch depending on the size of the roll. Roll size usually varies from 10 inches to 30 inches diameter for intermediate rolls. The queen and king rolls on some older machines are larger than intermediate rolls. The surface hardness of the chilled iron rolls is quite high measuring 68-72 on the scleroscope scale. It has been found (3) that this hardness further increases in service through work hardening reaching a hardness of 84 on the above scale after four to six weeks of service.

Bars on calender rolls are broad lines on the roll surface along which the surface texture has been altered. They are believed to be the result of differential work hardening caused by impact between adjacent rolls. The visual

1.2 The Nature of Calender Barring (cont'd)

appearance of the bars is difficult to describe and will in general depend on the angle of incidence of light and the position and distance of the viewer from the roll. The bars are usually straight and run parallel to the roll axis. However, wavy bars and bars running at a considerable angle to the roll axis are not uncommon. Bars may extend over the entire length of the roll or more often only over a portion of it. A bar may also be interrupted at several places along the face of the roll or it may be interrupted and then continue parallel to the roll axis but slightly shifted in the circumferential direction. The spacing of the bars around the circumference of the roll is usually irregular and adjacent rolls show very different bar spacing. Reference (7) contains the most detailed description of the characteristics of calender roll bars.

Close examination of the barred rolls using a microscope reveals lines of severe pitting alternating with slightly pitted regions. There are also signs of circumferential scratching and on some slower machines bars were found to consist of a series of closely spaced, fine, circumferential scratches (7). The size, shape and depth of the pits at the bars and between bars are not significantly different but the intensity of the pitting is greater at the bars. Whether the strongly pitted bar appears darker or lighter than the surrounding area depends on the angle of illumination and

1.2 The Nature of Calender Barring (cont'd)

viewing. One investigator (7) attempted to measure the hardness variation around the roll and to correlate it with the presence of bar markings. The results obtained showed no obvious correlation and it was concluded that any variation in hardness associated with the bars must have been less than about 4 points on the scleroscope scale.

Barring is also accompanied by measurable corrugation of the rolls. At the location of the bars the surface is slightly indented. On a severely barred roll the depth of the corrugation was found to be about 0.0002 inches (7). One investigator (13) using a specially constructed curvature gage measured corrugation depths up to 0.0003 inches. He also found that the severely pitted line, which gives rise to the optical effect of bars was not located on the bottom of the corrugation but rather on the slope off-set in the direction opposite to the direction of roll rotation. The off-set was found to vary within the range of 40 to 70 degrees. He also found that the depth of the corrugations was the greatest near the centre of the machine.

In calenders where barring is a problem, rolls begin to show barring shortly, sometimes immediately after their installation in the stack. The development of the barring pattern can be gradual, starting with one roll and then spreading to other rolls or all rolls may appear to start barring at the same time (4) which is probably just a more

1.2 The Nature of Calender Barring (cont'd)

rapid development of the former phenomenon. Light barring does not always get worse with time and strong bars have been observed to fade completely (7). Most investigators believe that the barring pattern on the roll changes continually in an apparently unpredictable manner. However, evidence is claimed by one investigator (13) that the barring intensity grows exponentially with time and that the roll corrugation migrates in a direction opposite to the direction of rotation. One piece of evidence supporting this is the offset position of the strongly pitted area on the slope of the corrugation.

Barring of Newsprint

The barring of paper in the calender is usually evident from a visual inspection of the paper and is manifested by glossy streaks running in the cross machine direction. For the more extreme cases local blackening along the bars may also be found. The bars usually run across the entire sheet and are 1/2 inch to 1 inch wide. In the machine direction the streaks may be spaced in more or less regularly repeated patterns. In most instances the spacing is very regular. In reference (7) based on a survey of barring patterns on nine newsprint machines, the following categories were established.

(a) bars spaced regularly in machine direction

1.2 The Nature of Calender Barring (cont'd)

(b) bars spaced regularly but severity modulated

(c) bars spaced irregularly, perhaps two or more patterns superimposed

(d) bars spaced irregularly within a group but groups spaced regularly.

The most common categories were found to be (a) and (b).

Barring is also associated with caliper changes in the paper and can be detected by taking machine direction caliper profiles. The thickness of the paper is reduced along the bars and the reduction is more or less in proportion to the visual severity of the bars. In fact, caliper profile measurements will detect barring that is not visually apparent (10,11). Also, caliper measurements make the quantitative description of barring intensity possible. A barring intensity scale of 1 to 10 corresponding to "total caliper variation" of 0.0001 inches to 0.0010 inches in a 0.0032 inches thick newsprint sheet was proposed in (10). It is not quite clear whether the word "total" stands for "peak" or "peak to peak" values. Most likely it refers to "peak to peak" values. This was the first attempt to define and standardize barring severity quantitatively. However, there is no reference in any of the subsequent literature to this scale. Most investigators describe newsprint barring intensity in percentage peak to peak machine direction caliper variation. Barring intensities of up to 12.5%, 14.0%

1.2 The Nature of Calender Barring (cont'd)

and 20.0% peak to peak variations are reported in (6,4,8) respectively. A newsprint committee survey reported barring intensities of up to 28% caliper variation (10). Caliper variation becomes visible barring in the range of 9% to 12% peak to peak thickness variation (10). The paper entering the calender would normally have machine direction caliper variation in the order of 9% peak to peak (10). These variations however, are usually completely random both in magnitude and in machine direction spacing. However, on some occasions periodic caliper variation of this magnitude caused by certain forming conditions at the wet end have been observed (6,12).

In a comprehensive observational investigation of barring in newsprint one investigator (11) found that the caliper reduction along a bar was as much as three times greater at the centre of the machine than near the ends of the calender rolls. Intermediate measurements indicated a nearly linear decrease of paper caliper along the bar with the distance from the edge of the paper. Another investigator (7) made measurement of basis weight variation in the machine direction in barred paper. Paper disks of 3/4 inch diameter were punched out at half-inch intervals along two closely spaced rows in the machine direction. He found no obvious correlation between the observed basis weight and caliper variations.

1.2 The Nature of Calender Barring (cont'd)

With certain types of continuous caliper profilers a continuous analog recording of the sheet thickness in the machine direction can be obtained. Samples of such records representing newsprint barring on many paper machines spread over two continents are presented in references (7,8,10-13). Some of the caliper variations are almost sinusoidal in appearance with fairly constant amplitude or with a beat. Others show random variation of amplitude and the presence of low amplitude harmonics. The range of the fundamental frequency of the caliper variations, taking the paper speed into consideration, reported by the above investigators is surprisingly narrow, being 65-90 Hz. The higher frequency component is about four to five times the fundamental. In all cases the fundamental component is of considerably greater amplitude than the high frequency one and it is the visible bar across the sheet.

Summary

Bars on calender rolls are axial streaks where the surface texture of the roll has been altered. Material along the bar has been removed resulting in corrugation of the roll down to depths of 0.0003 inches. The depth of the corrugations is greatest at the mid-section of the roll. The circumferential spacing of the bars is usually quite irregular. Differential work hardening due to impact between adjacent rolls is believed to be the cause. Bars in newsprint are cross

1.2 The Nature of Calender Barring (cont'd)

machine direction, glossy, sometimes blackened streaks along which the paper caliper has been considerably reduced. The caliper reduction is most severe at the center portion of the sheet. In most cases the bars are spaced very regularly in the machine direction. Barring of rolls and barring of paper do not always occur together.

1.3 SURVEYS RELATED TO CALENDER BARRING

There is every indication in the literature that barring is a widespread problem in the newsprint industry. High speed machines are generally more prone to barring than slower ones. However, barring problems at speeds as low as 1000 fpm have been reported. Two comprehensive surveys of the barring problem have been conducted and reported.

The first survey involved thirty newsprint machines in Canada (5). It was found that all high speed conventional calender stacks experienced barring of various intensities. From the information gathered in the survey it was concluded that high speed and high nip pressure were conducive to barring. One outstanding example of the effect of lower nip pressures was the case of two identical 2400 fpm newsprint machines in the same mill. One was equipped with a breaker stack and a four-nip calender while the other was operating with a conventional six-nip calender. The machine with the breaker stack had no barring problem while on the other machine with the six-nip conventional stack severe barring occurred.

The results of a second survey were reported in (10). Seventeen low speed machines (1100-1600 fpm) and twenty-three high speed machines (1800-2450 fpm) in forty newsprint mills across the North American continent were surveyed. All the calender characteristics with the exception of roll length and diameters were recorded. No

1.3 Surveys Related to Calender Barring (dont'd)

definite relationship between the occurance or the absence of barring and the calender characteristics such as speed, number of rolls in stack, number of steam rolls, steam pressure, nip pressure, roll crowning or roll off-set could be established. However, the following observations were made. Of the seventeen low speed machines surveyed five had no barring problem. The steam rolls in the calenders of these machines had much lower steam temperatures than the ones in the other machines. The three different newsprint mills operating these barring free machines were quite aware of a connection between steam roll temperatures and barring. A higher degree of deformation of the roll geometry and associated imbalance caused by the higher steam temperatures was suspected. Of the twenty-three high speed machines only three experienced no barring problem at all. Another four machines were considered as having a minor problem and for eleven machines the problem was serious enough to produce press room complaints. Two of the three machines without barring problem had non-conventional calender stacks. One had a breaker stack and a four nip calender while the other operated with two three-nip calender stacks. The third barring free machine employed a conventional six-nip calender stack with off-set rolls. The mills that could pin point where barring started in the stack indicated that the second roll from the top was the first to show evidence of barring. In all those cases, the second roll from the top

1.3 Surveys Related to Calender Barring (cont'd)

was either a steam roll or a solid roll adjacent to a steam roll. However, this may only be an indication of the practice of locating steam rolls in the stack and it is the two top rolls that have the tendency to bar first because of their physical location.

Summary

Most of the newsprint machines with conventional calender stacks were found to experience barring problems. There was no clear evidence that any one design or operating parameter or combination of parameters was responsible for barring on the conventional calender stack. The few machines with breaker stack arrangement or with two calender stacks and fewer nips in each were free of barring. On low speed machines calenders with higher temperature steam rolls were found more likely to have barring problems.

1.4 CAUSE OF BARRING

There is general agreement in the literature that the direct cause of the barring of calender rolls and paper is the vertical bouncing of the calender rolls. There is ample observational evidence to support this hypothesis. Extensive noise and vibration measurements on and in the vicinity of barring calender stacks (6,7,8,11) indicated the presence of strong vertical vibrations of the rolls and accompanying noise with a fundamental frequency very near or identical to the frequency of the bars on the paper. In the case of calender stacks where barring was intermittent the vibration and noise amplitudes were observed to increase greatly as barring set in but remained unchanged in their spectra. Yet, the same calender stacks when driven without paper in the nips vibrated and produced noise of considerably lower amplitude and at much higher frequencies than those observed during the calendering process. It appears therefore that the individual rolls in the calender stack and the paper being calendered form a vibratory system which is capable of large amplitude vibrations when excited near one of its natural frequencies. Overcalendering, the blackening of newsprint in the calender stack due to large nip pressures or high moisture content which cause excessive compression of the paper is a familiar phenomenon in paper mills. Calender rolls bouncing and colliding with one another as the stack vibrates can create very high instantaneous nip pressures and it is generally assumed that the bars on the

1.4 Cause of Barring (cont'd)

paper are caused by such local overcalendering of the paper. Paper samples taken from each nip of calender stacks during operation (7,10) revealed that practically all the caliper variation is put into the paper as it passes through the first nip. This and observations of the relative amplitudes of the motion of the calender rolls led some investigators (7,18) to conclude that the vibration of the top roll is the cause of barring of the paper.

There is considerably less agreement amongst the various investigators regarding the cause of the calender stack vibration and why some calenders bar and others do not or why the same calender stack will bar some of the time and will operate free of problems other times. The various opinions may be regarded broadly speaking to represent two schools of thought.

According to one theory the major cause of calender vibration is either the mechanical condition of the calender stack itself or some external excitation transmitted through the floor. The poor balancing or poor alignment of rolls, the poor condition of bearings and chatter marks left on the calender rolls after grinding are the most often quoted causes. References (1,2,4,5,8-10,13) emphasize the importance of these mechanical conditions of the calender stack as the primary causes of stack vibration and barring and provide field evidence to support their claim. The most

1.4 Cause of Barring (cont'd)

widely accepted hypothesis of roll barring is that small amplitude vibrations of the calender rolls due to any of the above sources of excitation will cause local work hardening along the rolls resulting in uneven wear and longitudinal corrugation of the rolls which will then lead to greater vibration amplitudes, more corrugation and barring of both the rolls and the paper. As evidence to support this hypothesis instances where upon regrinding of rolls the barring problems disappeared are cited. In (13) the spacing of the corrugations on rolls were found to fit the bar spacing on the paper. Removal of the corrugation from the rolls eliminated the paper barring altogether. The fact that some calender stacks will develop barring very shortly after regrinding of the rolls while others will be barring free for a long time is explained by emphasising the importance of a careful start-up procedure (10) to avoid damage to the rolls due to thermal shock and associated deformations or due to the jamming of the paper and the resulting bouncing of the rolls during the start-up of the calender stack.

While the importance of the mechanical condition of the stack is recognized by all investigators, according to the second school of thought the condition of the incoming paper to the calender is the most important factor in the excitation of stack vibration. Reference (7) quotes instances when barring of the paper occurred without any

1.4 Cause of Barring (cont'd)

visible barring of the rolls and claims that slight barring of rolls will not cause barring of the paper. He considers cyclic or random machine direction variation in basis weight and moisture content as important excitation sources for calender stack vibration. In reference (11) incoming sheet non-uniformity is recognized as a vibration exciter and examples of several calender stacks are quoted where all the corrugation was removed from the calender rolls using non-rotating stone type super-finisher without significantly affecting the severity of barring. In reference (6) an extensive experimental program on an experimental paper machine to determine the effect of wet-end sheet forming conditions on calender operation is described. It was found that table roll vibration caused the stock on the Fourdrinier wire to jump resulting in significant cyclic machine direction basis weight variation. Subsequently barring improvement was achieved on three paper machines and on one machine barring was virtually eliminated as a result of balancing of table rolls. Another case is mentioned in (5) where the stiffening of table rolls, forming board and headbox lip eliminated barring by eliminating "chop" on the Fourdrinier wire. Reference (12) also traces barring to sheet non-uniformity due to sheet forming conditions on the wire or at the headbox slice. In (14) an interesting theory is proposed linking the somewhat thicker edges of the incoming sheet to the phenomenon of calender barring.

1.4 Cause of Barring (cont'd)

According to this theory the excess material at the edges is not able to pass through the nip but flows in opposite direction to the travel of the sheet until the accumulated bulk is caught and pulled through the nip imparting a large bounce to the stack. The cyclic repetition of the process results in barring of the rolls and leads to the barring of the paper. Severe barring on several machines (other than newsprint) was eliminated by either trimming the incoming sheet or by slightly relieving the ends of the top roll in the stack.

Summary

There is a great deal of evidence that the barring of calender rolls and the paper is caused by calender stack vibration. The vibration of the top roll appears to be responsible for most of the caliper variation associated with barred paper. There appear to be many sources of excitation to cause and maintain the vibration of a calender stack. The most significant ones are roll unbalance, roll misalignment, roll surface corrugation and machine direction variation in the properties of the incoming sheet. The latter do not necessarily have to be cyclic to cause barring as the calender stack will respond at its natural frequencies if the random excitation contains sufficient energy at those frequencies in its spectrum. Most likely a combination of several sources of excitation is involved in

1.4 Cause of Barring (cont'd)

every barring problem making each barring case unique. Regardless of the sources of excitation involved, the concensus of opinion of the investigators is that calender stack vibration is the major cause of the barring phenomenon.

1.5 MECHANISM OF CALENDER BARRING

Despite of the fact that calender barring is a serious and common problem that has been with the paper industry for a long time there have been relatively few published investigations of the mechanism of calender vibration. Experimental data from controlled experiments to determine the effect of the various operating factors believed to be causing calender vibration are virtually non-existent. References (1,7,8,10,11) contain some experimental data on the nature of calender stack vibration with (7) and (8) being the most detailed investigations.

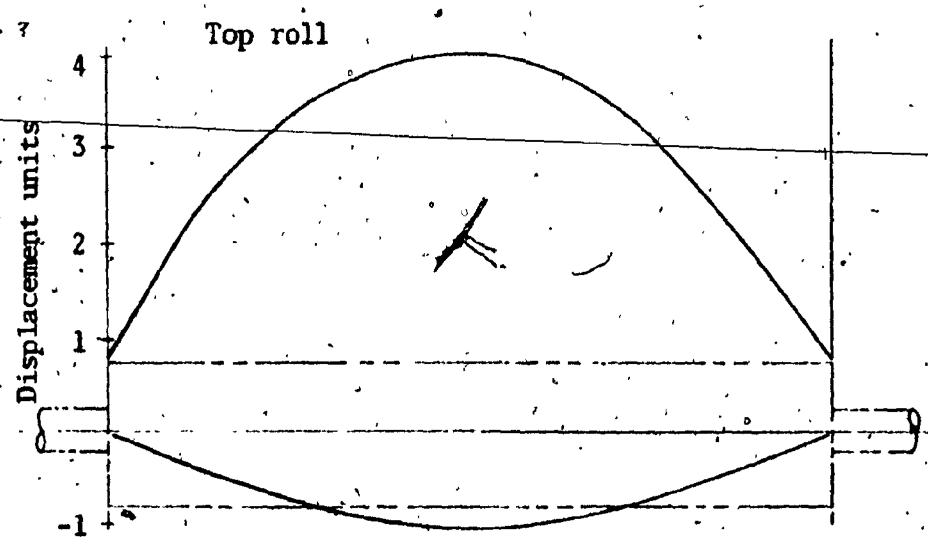


Fig. 4 Displacements of Top and Bottom Rolls

Several investigators demonstrated that the fundamental and most significant frequency of the vibration and noise spectra around a barring calender stack is the same as the frequency at which the bars in the paper are produced. The

1.5 Mechanism of Calender Vibration (cont'd)

most extensive experimental investigation of the mechanism of calender stack vibration is reported in (8). The motions of the top roll and the bottom roll of a 264" wide six-roll calender stack were recorded during barring. The diameters of the four top rolls, the queen and the king rolls were 18", 24" and 40" respectively. The following observations were made. Both rolls oscillated at the same frequency. The vibration amplitude in the vertical plane in the case of the top roll was about four times greater than in the horizontal plane and consequently all other reported measurements were made in the vertical plane. The vertical movement of the top roll was the greatest in the stack and about four times greater than the movement of the bottom roll. The middle and the two ends of the top roll moved in phase but the displacement at the middle of the roll was again about five times as great as the movement of the ends. This is in agreement with (11) where the paper caliper variation along the center of a barred sheet was found to be about three times as great as along either edge. In the case of the bottom roll there was practically no displacement at the ends. It appears therefore that the bottom roll vibrates mainly in a flexural mode whereas the top roll moves in translation as well as in the flexural mode. Figure 4 illustrates and compares the motions of the two rolls which move 180 degrees out of phase with each other. There is no mention in the above paper (11) of the intermediate rolls

1.5 Mechanism of Calender Barring (cont'd)

other than a comparison of the movements of the ends of the two top rolls. The end of the second roll from the top moves about twice as much as that of the top roll and somewhat out of phase. It should be mentioned here that the vibration signals obtained from the journals and the bearing housings were not representative of the vibration of the rolls. Reference (7) after a somewhat less detailed experimental investigation reported that all rolls oscillated at the same frequency and that the movement of the top roll was the most significant. The fact that the top roll goes through the greatest displacement correlates well with the findings of (7) and (10) where it is reported that paper samples taken between nips indicated that practically all of the barring damage was already present after the first nip and the successive nips tended to smoothen it out.

The frequencies at which various newsprint calenders vibrate and cause barring fall within a remarkably narrow band. In the case of one mill (7) all nine machines barred in the frequency range of 70-85 bars per second. In a continent wide survey thirty seven machines were reported to bar in the range of 64-84 bars per second and 78% of them barred within the much narrower range of 76-84 bars per second. As most calender stacks are quite similar in construction the fundamental or first two natural frequencies of the vibratory system formed by the rolls and the paper between them would be expected to be quite similar from one machine to another. For this reason it is generally assumed that barring is the result of the calender stack vibrating at one of its natural frequencies near the fundamental one. It is a reasonable assumption even though there is no report in the literature of a resonance test of a calender stack to verify the assumption.

The frequency at which any one calender stack bars has been found by several investigators (1,7,8 and 11) to vary with the speed of the paper running through the calender. They observed that when the machine speed was increased the vibration frequency increased with it in a linear fashion but only over a relatively small range. Further increase in the machine speed resulted in a sudden drop in the vibration (or barring) frequency. The most detailed investigation of this phenomenon is reported in (11). The speed range of about 1820-2180 fpm was covered in the experiments. At 1820 fpm the vibration frequency of 76 cps was observed. With increasing machine speed this frequency increased linearly with calender speed up to about 2060 fpm when it suddenly dropped to 73 cps and from there it increased again linearly with machine speed. At the same time it was observed that the spacing between adjacent calender bars in the paper remained relatively constant. The recording of the dimensions of the spacings between bars on two machines over

1.5 Mechanism of Calender Barring (cont'd)

a period of several months revealed only three or four different sizes of bar spacing for each machine. From this and from similar observations made by other investigators it appears that a calender stack will bar only at certain discrete bar spacings and in a small band of frequencies probably centered around some resonant frequency of the calender rolls-paper system. There is some uncertainty regarding the relationship between the size of the spacing between bars and the linear dimension of the wrap around the calender roll. It has not been established with certainty whether the number of spacings between two nips is an integer or not. It does appear however, that if the machine speed is varied this number will change discontinuously at increments or decrements of one (11).

Based on these observations the following mechanism for calender stack vibration has been postulated by several investigators (7,8,11). The calender stack once excited near one of its natural frequencies will vibrate at significant amplitudes. The resulting large forces generated by the motion of the top roll will create cyclic caliper variations in the paper as it passes through the first nip. The cyclic caliper non-uniformities (bars) in passing through successive nips will further excite the calender stack and help to sustain the vibration of the uppermost roll if they arrive at those nips (especially the second and third nip

1.5 Mechanism of Calender Barring (cont'd)

from the top) with a suitable phase relationship to the motion of the top roll. Thus the calender stack acts as a mechanical oscillator with the paper providing the feedback and the drive supplying the energy to maintain the oscillations. When the calender speed is increased the vibration frequency increases to keep the required number of bar spacings (and phase relationship) between two nips constant. When the vibration frequency has increased to the point where one fewer spaces per wrap would result in a vibration frequency closer to the natural frequency of the system then a discontinuous change in the number of spaces per wrap and in the vibration frequency will occur. A similar process would keep the system tuned if the calender speed were to decrease. The mechanism can be further illustrated with the following equations.

If F is the frequency of vibration, L is the spacing between successive bars and S is the surface speed of the calender rolls then

$$S=(F)(L)$$

If (n + c) is the number of spacings between two nips, where n is an integer variable and c is a phase related constant which is either zero or some positive fraction then

$$W=(n + c)(L)$$

1.5 Mechanism of Calender Barring (cont'd)

where W is the wrap around a roll (some fraction of the circumference, usually one half). From these two equations W and F can be expressed as follows

$$W=(n + c)(S/F)$$

$$F=(n + c)(S/W)$$

The last equation with c and W constant, F and S as variables and n as a parameter has been plotted below.

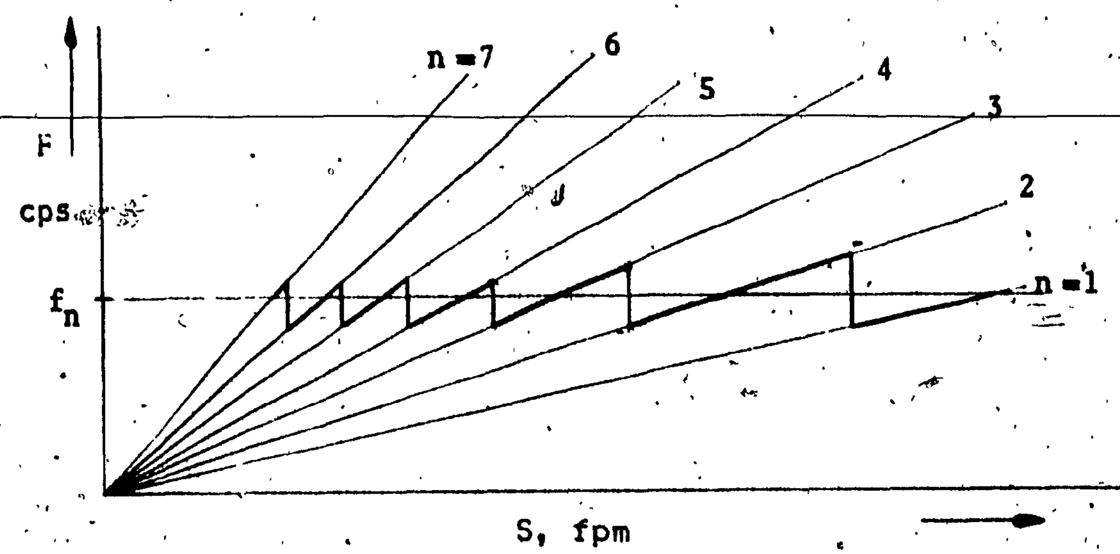


Fig. 5 Barring Frequency vs Calender Speed

The oscillation of the vibration frequency about the natural frequency of the system and the discrete jumps in the value of n are illustrated in the diagram.

1.5 Mechanism of Calender Barring (cont'd)

Summary

Experimental observations of the vibration of barring calender stacks indicate that all rolls move at the same frequency and mainly in the vertical plane. The frequency of the vibration is the same as that at which bars are produced in the paper. Both flexural and translational motions have been observed. The motion of the top roll seems to be the most significant. The displacements at the center of the top and bottom rolls are much greater than at their ends indicating that for those rolls flexural displacements are more significant than translational ones. Since the frequency of vibration is relatively uniform amongst calender stacks of similar construction it is assumed that it represents one of the natural frequencies of the calender stack with paper running through the nips. The observation of discontinuous changes in bar spacing and vibration frequencies when the calender speed is varied suggests that a feedback mechanism in the form of bars travelling through successive nips is present which keeps the vibration frequency close to the natural frequency of the system allowing only discrete spacings of the bars. It is conceivable that only the top two or possibly three rolls play a significant role in the barring process.

As none of the investigators mentions it, it may be worthwhile to point out here that non-linear oscillatory systems are known to exhibit discontinuities in their behaviour, such as the jump phenomenon for example. The non-linear elastic properties of the paper certainly make the calender stack a non-linear oscillatory system and could account for some of the observed discontinuous behaviour.

1.6 THEORETICAL STUDIES OF CALENDER BARRING

There are very few theoretical studies of the problem of calender barring in the published literature. The first such paper was published in 1963 by three investigators (8) who also performed the most detailed experimental investigation of the mechanism of calender stack vibration. They analyzed a simple lumped mass-spring model of a six-roll five-nip calender stack with all six masses restricted to move in one plane and in one direction. The spring constants were computed from compression-pressure curves obtained in laboratory experiments on uncalendered newsprint for compression cycles of about 0.0005 seconds duration believed to be representative of compression cycles encountered when calendering at 1800 fpm. In this manner spring constants were assigned at the various nips depending on the nip pressure there. The bottom roll was assumed to rest on springs representing the stiffness of its journals. The natural modes and frequencies of this oscillatory system were computed but only the mode shapes were published while the frequencies are only mentioned as being in fair agreement with experimental observations. The shape of the computed second mode is in agreement with the observations made on the top and bottom rolls of the full size stack. There is mention in the paper of a digital computer analysis of the dynamic behaviour of the calender stack including the effect of calender speed change on bar spacing. The results of the analysis are said to be in satisfactory agreement with observations. No details of this analysis are given. As an outcome of the study a barring suppression device based on keeping the nip pressure at the top roll constant was developed, patented and used successfully.

The dynamic behaviour of the same calender stack was studied three years later by another investigator (14) using an analog computer. Essentially the same mathematical model was used except for the addition of damping. As another improvement a new improved set of spring coefficients based on the work published in (21) was calculated. Either by error or on purpose the bottom roll was assumed to rest on the ground instead of on springs. The model was independent of stack and roll geometry and without a feedback mechanism. Sinusoidal, random and sinusoidal on random excitations were applied to the top roll of the model. Practically none of the phenomena observed in (8) were reproduced. On the whole the investigation was rather limited as essentially a one dimensional, multi-degree of freedom system was excited in various ways with predictable results. No more information was gained from it than would have been obtained from a solution of the undamped system for its natural modes and frequencies. The investigator realized that the lack of a feedback mechanism rendered the model essentially a stationary one but as he pointed out the delay times required would have been within the inaccuracy of the analog computer.

The same calender stack was again studied three years later by another group (16) on a hybrid computer. The simulation model was the same as that in (14) except that the bottom roll was placed on springs again as in (8). The spring constants used were computed by the method described in (14). The damping constants were obtained experimentally by an indirect method. Using a pendulum the coefficient of restitution of the paper resting on a suitable backing was determined and then the coefficient of damping was calculated from a relationship of the two properties. The mathematical model incorporated a feedback mechanism in the form of forcing functions acting at the various nips. A forcing function for any one nip was a train of impulses simulating the effect of bars created at the first nip arriving some time later at the particular nip. To do this the force disturbance expected at lower nips because of a bar put in the paper at the uppermost nip was stored in the digital part of the hybrid computer and delayed for the length of time required for it to travel to a particular nip before sending it back to the analog computer to create an impulse at the right time. In this way machine speeds and roll geometry were represented by delay time length. The model was excited at the top roll by step and random excitation. The response of the system was in good agreement with the observations of other investigators on several points. The amplitudes and phase relationships of the various rolls in the stack during simulated barring were in good agreement with the observed and computed results in (8). However the frequencies of calender vibration at 2020 fpm are reported as 58 cycles/minute and 333 cycles/minute. This is very different from the 96 cps frequency reported in (8) and there is no mention of any scaling factor to be applied. This may be an omission in the report. The effect of calender speed on barring frequency was demonstrated even though in a somewhat limited way. At speeds above 2000 fpm the frequency of vibration was 58 cycles/minute. At speeds below 2000 fpm the frequency was 62 cycles/minute. Near the transition point strong beats were evident which is also in agreement with observation (11). The idea that the motion of the two massive lower rolls can be used to oppose the vibration of the upper ones was tested succesfully with the model and the use of a roll movable in the horizontal direction to vary the travel time (essentially varying the diameter of the queen roll) between the last two nips was proposed as a means to eliminate calender vibration.

A fourth investigator (13) analyzed a single degree of freedom system with base excitation to study the mechanism of the corrugation of calender rolls. His theory predicts the growth rate of corrugations and the rate of their migration in a direction opposite to the rotation of the roll. Computations based on the theory are claimed to be in good agreement with observations made on the top roll of a calender.

Summary

Three theoretical analyses of the phenomenon of calender stack vibration have been published. The same six-roll calender stack was analyzed using very similar but progressively more refined mathematical models. The models were simplified lumped mass, one dimensional, six-degree of freedom systems with the refinements of damping and feedback effect added by the successive investigators. The flexural vibrations of the rolls observed by some of the investigators were not considered. The analyses succeeded to various degrees to reproduce several of the observed phenomena. In the case of some of the phenomena only qualitative agreement between real and simulated behaviour was obtained.

It is interesting to note that even though experimental evidence indicates that at least the top and bottom rolls undergo very significant flexural vibrations all the investigators modeled the calender stack with lumped masses assuming translational motion of the rolls only.

1.7 CORRECTIVE MEASURES AGAINST CALENDER BARRING

There is universal agreement in the literature that barring is caused by the vibration of the calender stack. The elastic system formed by the calender stack and the paper running through it is a complex vibratory system with elastic and damping properties, external and internal sources of excitation all dependent on and affected by a large number of operating factors. The behaviour of such a system is difficult to predict or even understand especially since the factors affecting the behaviour change continuously and often without the operating personnel being aware of the changes. As a result barring is observed to come and go without any apparent cause for it and some measures used successfully to eliminate barring on one machine will worsen the problem on another surrounding the whole problem of barring with an air of mysticism.

There are many corrective measures mentioned in the literature some of them simple and often applied or practiced successfully others more involved and less frequently tried or only postulated as possible remedies. As the problem is essentially a vibration problem all measures to remedy it have to do with reducing or eliminating the excitation or damping the response. These measures can be classified into four major groups having to do with

a. Mechanical maintenance of the calender stack

b. Operation of the calender stack

1.7 Corrective Measures Against Calender Barring (cont'd)

c. Controlling external excitation sources

d. Design of calender stack

The most involved and least proven remedies belong in the last of the four groups.

a) Mechanical Condition

The importance of the mechanical condition of the calender stack is well recognized throughout the industry. Roll balancing, roll alignment, roll corrugation and bearing condition are the most widely believed sources of excitation. These are also the most easily remedied and therefore the first corrective measures taken when a barring problem is encountered. Roll corrugation is particularly closely associated with barring and roll grinding is a very frequently applied remedial action. Some investigators regard it as the sole cause of calender vibration (13). However, others report serious vibrations without any corrugation in the calender rolls (7). In a survey (10) covering most of the North American newsprint industry most mills claimed that roll grinding helped barring but one mill reported that barring was most severe immediately after regrinding the rolls and then decreased until roll corrugation developed. References (2,10,13) contain information about roll grinding and the factors that may affect the condition of the ground roll.

1.7 Corrective Measures Against Calender Barring (cont'd)

b) Calender Operation

The mode of starting up a calender stack is emphasized by two investigators (10) as a means of controlling barring. In particular they strongly recommend the gradual heating up of the steam rolls and the careful starting of the calendering process in order to avoid upsetting the calender stack through a thermal shock or physical jamming of the rolls during calender start-up. The use of steam rolls is a controversial point as many mills have found that shutting off one or two steam rolls in the stack reduced the barring intensity significantly. The mere presence of steam rolls in a stack is believed to be conducive to barring because steam rolls are less well balanced in the first place and condensate inside them further increases their imbalance. Non-uniform moisture profile in the cross machine direction is mentioned by one investigator (11) as a possible cause of barring and he recommends that cross machine profiles of sheet properties be checked when barring occurs. The addition or removal of a roll is sometimes tried as a means of eliminating barring. This should be expected to affect the natural frequencies of the calender stack and the resulting effect on barring intensity is only predictable if the nature of the excitation and the vibratory characteristics of the calender stack are reasonably well known. This is illustrated by the case of two machines, reported in (11) where adding of a roll to the stacks produced opposite effects on the barring intensities. One investigator (15) claimed that in many cases thicker caliper along the two edges of the sheet was causing barring and suggested that trimming of the sheet before the calender may help in some cases to eliminate barring. The mechanism postulated for the edge effect is the accumulation and the sudden passing of excess bulk through a nip. According to the paper edge trimming has been successfuly used to eliminate barring in several cases.

c) External Excitation

Sources of excitation external to the components of the calender stack are also important. Floor vibration caused by some vibrating machinery in the vicinity of the calender stack has been known to induce barring (6). The investigation and elimination of floor vibrations is recommended whenever barring occurs. However, perhaps the most important and most common external source of excitation is the paper being calendered. This excitation is in the form of machine direction basis weight, sheet caliper or moisture content variation. Several investigators believe the paper to be one of the major sources of excitation (6,7,10-12). One claims that the barring frequency can be often found in the stock deposited on the wire and emphasizes the importance of checking wet-end components for vibration when barring is encountered (6). He mentions several examples of wet-end component vibration such as severe machine direction headbox vibration whose removal did not reduce the barring intensity but also reports of cases where relatively slight vibration of a table roll was found to be the cause of calender barring. It is widely believed that machines that are equipped with a breaker stack are free of barring and their use is highly recommended. The effectiveness of the breaker stack in preventing barring seems to confirm the idea that random or cyclic sheet caliper or basis weight variation is the major source of external excitation of calender vibrations. The breaker stack precalenders and smoothens the paper for the calender stack thereby reducing the paper borne vibration excitation.

d) Calender Design

There are several recommendations in the literature concerning certain calender stack configurations proven or believed to be potentially usefull in preventing or eliminating calender barring. The most common and least expensive of these is the off-setting of the calender rolls relative to one another. Usually every other intermediate roll is off-set horizontally about one inch creating a staggered roll arrangement. This is a well tried and usually successful means of reducing barring. Off-setting may be effective because it affects the regenerative feedback

1.7 Corrective Measures Against Calender Barring (cont'd)

mechanism by altering the wrap around some of the rolls. If that is the case its effectiveness may be limited to a certain calender speed range (11). Another hypothesis is that it acts as a damper by allowing some of the vibrational energy to be absorbed as the tangential component of the impact forces acts on the off-set rolls to change their rotational kinetic energy (17). Another effective means of eliminating calender barring is the use of two or three calender stacks with fewer nips (5,9,10). The North American survey mentioned earlier (5,10) seemed to confirm this as machines with such non-conventional stacks were amongst the few that had no barring problem. There are three calender features recommended in the literature which are based on theoretical considerations with little or no field testing to prove their effectiveness in controlling calender vibration. One is a device designed to make the nip pressure in the first nip independent of the movement of the top roll (8). This device has been patented and is claimed to have been successfully tested in operation. The top roll was replaced by a light tube roll which is pressed down with air cushions to provide a nip pressure equivalent to the dead weight of a normal roll. Another investigator (18) suggested that a top roll with a soft cover would distribute the deformation in the first nip between the sheet and the soft cover thereby reducing the severeness of the barring. Based on a computer simulation of the dynamic behaviour of a six-roll, five-nip calender stack another group (16) suggested that the motion of the two heavy bottom rolls might be used to counteract the vibration of the upper rolls if a control device adjusted the path of the sheet between the two last nips in such a way as to obtain the correct phase relationship between the motions of the upper and bottom rolls. This would in fact amount to having a queen roll with a readily variable diameter. The computer simulation indicated that dramatic reduction of the barring intensity is possible by such adjustment.

Summary

The most frequently tried remedies are the rebalancing, realigning and regrinding of rolls. These measures are often but not always effective in reducing barring intensity. Many operating factors from steam temperatures to cross machine moisture profile are believed to have an effect on barring but the manipulation of these factors as a means of controlling or eliminating barring intensity does not produce easily predictable results. The off-setting of rolls and the reduction of the number of nips in a stack are highly recommended remedies to barring. Several investigators have proposed means of controlling the vibration of the top roll in order to eliminate barring. One such scheme has been successfully tested in a paper mill.

1.8 CONCLUSIONS AND RECOMMENDATIONS

There is general agreement in the literature that calender barring of paper is caused by calender stack vibration. A great deal of convincing experimental evidence to support this theory is presented. Most of the vibration energy involved in the barring process is expended in the vibratory motion of the top roll as almost all of the barring damage to the paper occurs as the sheet passes through the first nip. For a mechanically well maintained calender stack the main source of excitation is believed to be the machine direction caliper and basis weight variation. Both cyclic and random non-uniformities are capable of causing calender stack vibrations. The existence of some regenerative feedback mechanism that helps to sustain vibration of the calender stack has been experimentally demonstrated. This feedback effect also appears to maintain the frequency of the calender stack vibration within a relatively narrow band. The vibration is believed to be centered at one of the natural frequencies of the calender stack. However, this has not been experimentally demonstrated. The published theoretical analyses of the barring problem used a somewhat simplified model ignoring the flexural vibrations of the rolls which have been experimentally observed to be very significant. The publications claim some success in simulating calender behaviour. Owing to the lack of sufficient detail a critical evaluation of the success is difficult.

1.8 Conclusions and Recommendations (cont'd)

In order to be able to deal with the calender vibration problem effectively a good understanding of calender behaviour and some form of mathematical modeling of the observed phenomena are important. The following program to achieve this is recommended. If, as it appears to be the case, the vibration that causes barring occurs near one of the natural frequencies of the calender stack then it is important to be able to compute these frequencies and to know how to affect them through design modifications. For this purpose a natural mode and frequency analysis of the undamped calender stack-paper system vibrating in the vertical plane should be performed. In this analysis both a lumped mass system and one where the flexural vibrations of at least the top roll are taken into consideration should be investigated and the results compared. The validity of the analysis should then be checked by resonance tests on full scale calender stacks. If this is done on calender stacks where barring occurs then the hypothesis that barring takes place at a resonant frequency of the calender stack can be tested. It would be desirable to analyze both the closed frame and cantilever type calender stacks. However, the availability of the calender for field testing should be the deciding factor. Once the undamped elastic system representing the calender stack with paper running through it is modeled successfully as a static system then a digital or hybrid computer simulation of the dynamic behaviour of the model can be attempted. In this simulation some form of damping should be introduced and roll geometry as well as the feedback effect of the bars travelling through the stack should be incorporated. As there is much evidence that the vibratory motion of the top roll of the calender stack is largely responsible for barring damage to the sheet most of the efforts should be directed towards finding practical ways of controlling the vibration of that roll. Some of the ideas proposed in the literature should be more closely examined and investigated.

CHAPTER 2

MATHEMATICAL ANALYSIS

2.1 INTRODUCTION

The calendering process and the nature of the calender barring problem as seen by numerous investigators has been discussed in detail in the literature survey. There have been many investigations of experimental or observational nature into the problem and it appears that most investigators regard barring as a result of the vibration of the calender rolls. However, only a few attempts have been made at the mathematical treatment of the underlying vibration problem. In all cases the physical model analysed was a one dimensional vibrating system consisting of a series of point masses representing the rolls and a series of springs between the point masses representing the paper passing between adjacent rolls. Experimental observations of roll displacement and barring intensity along the width of calenders indicate that the rolls undergo flexural motion during vibration. The available experimental evidence is described in detail in the literature survey. In view of the possibility of flexural vibrations of the calender rolls a one dimensional model appears to be inadaquate and a model capable of flexural vibrations becomes necessary for a good approximation of the vibrational characteristics of the calender stack. Several investigators observed flexural vibrations of the rolls in both the horizontal and vertical planes. They all agreed however, that the flexural vibration taking place in the vertical plane was considerably more significant than the one in the horizontal plane. It is likely that flexural vibrations of some magnitude may occur in any axial plane of a calender roll. A model that posesses the necessary degrees of freedom to describe such a complex motion would be rather unwieldy and very difficult to treat mathematically. The fact that flexural motions in the vertical plane have been observed to be the most significant suggests that a two dimensional model of the calender stack which permits flexural vibrations in the vertical plane may be a sufficiently good approximation of the calender stack as a vibrating system. In this report such a two dimensional model is presented and analysed. The mass of each roll is lumped into a number of point masses connected to one another by massless elastic rods of uniform elastic and cross sectional properties. The lumped masses of adjacent rolls are connected by ideal springs representing the elasticity of the paper being calendered. The resulting mathematical model is solved for the normal modes and the associated natural frequencies of the vibrating system.

2.2 MATHEMATICAL MODEL OF CALENDER STACK

Experimental observations of many investigators indicate that the rolls of a barring calender stack undergo flexural vibrations in at least the vertical and horizontal planes. The motion taking place in the vertical plane is considered the most significant both in magnitude and in its contribution to barring. Neglecting the flexural vibrations outside the vertical plane the calender stack as a vibrating system may be represented by a two dimensional physical model shown below.

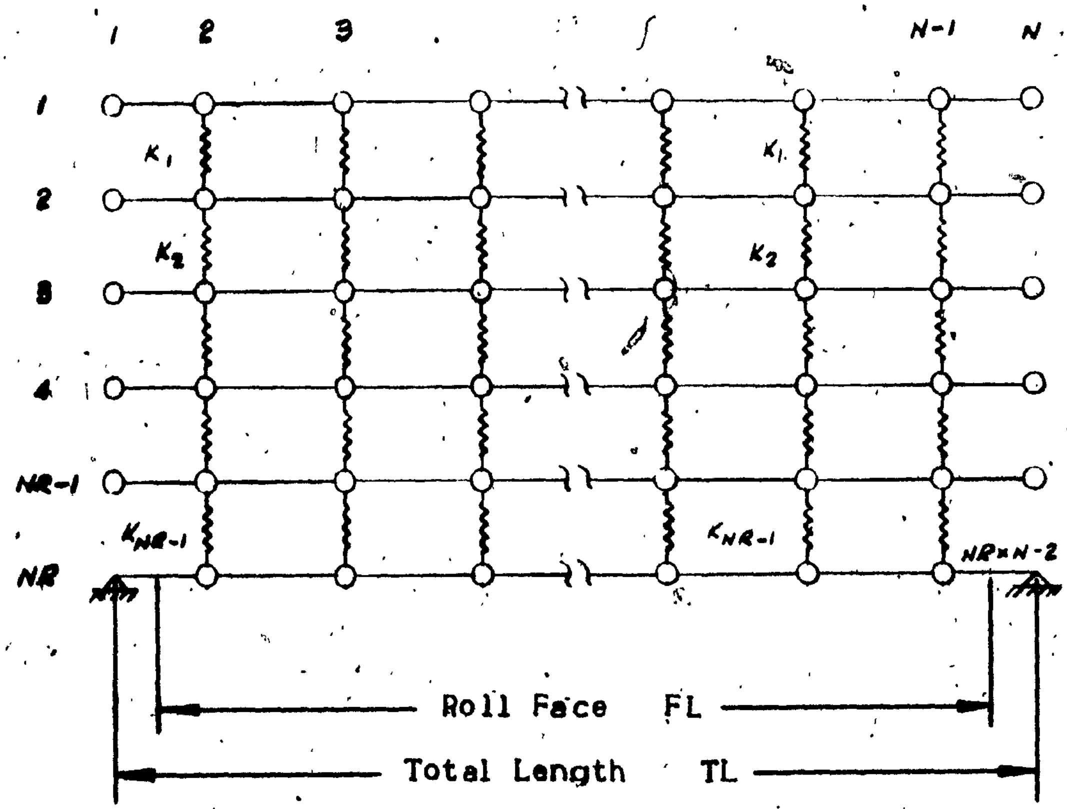


Fig. 6 Physical Model of Calender Stack

2.2 Mathematical Model of Calender Stack (cont'd)

Assumptions

Vibratory motion outside the vertical plane containing the axis of the calender roll is negligible.

Total mass of each roll is lumped into N point masses of equal magnitude. There is no mass point at C.G. of a roll. The point masses are located at C.G. of roll sections. Point masses at the ends of upper rolls represent the combined journal and bearing masses. The rotation of the lumped masses is neglected.

There are massless roll sections of uniform stiffness between point masses.

The upper rolls are unconstrained at their ends. The lowermost roll is simply supported. The potential energy stored in the rolls due to flexure can be satisfactorily approximated by treating them as free-free and simply supported rolls respectively.

Ideal springs represent the paper being calendered. Coupling between the point masses of adjacent rolls is through the ideal springs. The springs can resist both tension and compression. The stiffness of the springs is constant along the space between two adjacent rolls.

The system is conservative.

2.2 Mathematical Model of Calender Stack (cont'd)

In this model the hybrid system of the calender stack which contains both discrete and continuous components is represented by a system of discrete masses coupled to one another. The mass of each roll has been lumped into N point masses which are joined by massless rods with properties corresponding to the flexural stiffness of the roll. The corresponding point masses of adjacent rolls are coupled by springs which model the elastic resilience of the paper being calendered. The two end masses of the upper rolls represent the journal and bearing housing masses connected to the adjacent roll mass by a massless rod whose stiffness corresponds to the effective stiffness of the journal. The ends of the upper rolls are assumed to be unconstrained while the lowermost roll is assumed to be simply supported. This model represents a complex system with a large number of degrees of freedom which is quite representative of the calender stack yet whose mathematical treatment can be made relatively straightforward with the help of some simplifying assumptions. The following is an outline of the proposed procedure to set up the mathematical model from the physical one and to obtain the desired solutions from it.

Assuming that the system is conservative and that the springs between the rolls are ideal linear springs and using potential and kinetic energy expressions in the Lagrange's equations the equations of motion for the system can be set

2.2 Mathematical Model of Calender Stack (cont'd)

up in terms of mass and stiffness matrices. Assuming oscillatory motions for the components of the freely vibrating model the characteristic equation of the system can be obtained in the following form

$$\det(\mu[I] - [S]) = 0$$

where [I] is the unit matrix and [S] is the dynamic matrix $[M][K]^{-1}$ or the inverse dynamic matrix $[M]^{-1}[K]$. The parameter μ is related to the circular frequency associated with the free vibrations of the system. The characteristic equation is then solved to determine its n roots $\mu_1, \mu_2, \ldots, \mu_n$ and the n sets of relative vibration amplitudes which will satisfy the characteristic equation for the various roots. The roots and the sets of relative vibration amplitudes are also known as the eigenvalues and eigenvectors of the matrix [S]. Together they describe the natural or normal modes of vibration of the undamped calender stack and thus provide the most essential information about the vibratory behaviour of the system.

Although it is outside the intended scope of this report to go beyond the free vibration analysis of the calender stack system it should be pointed out here that a slight extension of the mathematical model and the results of the analysis will allow the forced vibration analysis of the system. The method will be described here briefly.

2.2 Mathematical Model of Calender Stack (cont'd).

Define a set of new variables called the normal or principal co-ordinates which are related to the original generalised co-ordinates q. The desired relationship is described by the equation

$$(q) = [A](y)$$

where [A] is the modal matrix consisting of the n eigenvectors obtained from the static analysis. Substituting (y) for (q) in the equation of motion

$$[M](\ddot{q}) + [K](q) = (Q)$$

we obtain

$$[M][A](\ddot{y}) + [K][A](y) = (Q).$$

where (Q) are the generalised forces acting on the system. Premultiplying by $[A]^T$ a set of n de-coupled differential equations of motion in terms of the normal co-ordinates y are obtained by virtue of the fact that

$$[A]^T[M][A]$$

and

$$[A]^T[K][A]$$

are diagonal because of the orthogonality properties of the normal modes, namely

$$(A)_j^T [M] (A)_i = 0$$

and

$$(A)_j^T [K] (A)_i = 0 \; .$$

These de-coupled differential equations can then be solved readily and independently. The results obtained in terms of the normal co-ordinates y can be transformed into the desired generalized co-ordinates by the relationship

$$(y) = [A]^{-1} (q) \; .$$

2.3 ENERGY EXPRESSIONS FOR THE CALENDER SYSTEM

The potential energy stored in the vibrating calender is created by the flexing of the rolls and by the compression of the paper between two adjacent rolls. It is possible to compute the two energy terms separately and then to combine the two scalar quantities. Both energy terms are associated with displacements of the point masses. However, the flexural strain energy depends only on the flexural displacements of the roll, while the potential energy in the compressed paper is related to the absolute displacements of the point masses. For this reason a distinction must be made between displacements caused by flexure and absolute displacement of a point mass as the latter may contain rigid body motion. The flexural and absolute displacement vectors will be denoted by (d) and (D) respectively.

Flexural Strain Energy in Rolls

The total flexural potential energy VF of the system is made up of the energy stored in the free-free upper rolls and that stored in the simply supported lowermost roll. The potential energy of each free-free roll can be expressed as

$$vf = 0.5(d)_n^T [kf]_n (d)_n \qquad (1)$$

where (d) is the displacement vector and $[kf]_n$ is the (NxN) stiffness matrix of the nth free-free roll. The column matrix $(d)_n$ contains the displacements of the point masses relative to the rigid body motion of the roll when it undergoes vibratory motion. In the case of the simply supported lowermost roll no rigid body motion is possible and therefore (d) represents the absolute displacements of the point masses of that roll

$$(d)_1 = \begin{pmatrix} d_1 \\ \cdot \\ \cdot \\ \cdot \\ d_N \end{pmatrix} \quad (d)_2 = \begin{pmatrix} d_{N+1} \\ \cdot \\ \cdot \\ \cdot \\ d_{2N} \end{pmatrix} \quad (d)_{NR} = \begin{pmatrix} d_{(NR-1)(N)+1} \\ \cdot \\ \cdot \\ \cdot \\ d_{(NR)(N)-2} \end{pmatrix}$$

The total flexural potential energy in the system can therefore be expressed as

$$VF = 0.5(d)^T[KF](d) \tag{2}$$

Where [KF] is a (NRxN-2)(NRxN-2) partitioned flexural stiffness matrix for the system which is made up of diagonal terms representing the flexural stiffness matrices of the individual calender rolls

$$[KF] = \begin{bmatrix} [kf]_1 & & & \\ & \cdot & & \\ & & \cdot & \\ & & & [kf]_{NR} \end{bmatrix}$$

and (d) is the flexural displacement vector such that

$$(d) = \begin{pmatrix} (d)_1 \\ \cdot \\ \cdot \\ \cdot \\ (d)_{NR} \end{pmatrix}$$

2.3 Energy Expressions for the Calender System (cont'd)

Potential Energy in the Compressed Paper

The potential energy in the springs which represent the paper between the calender rolls is proportional to the absolute displacements of the point masses and can be expressed as

$$VS = 0.5(D)^T[KS](D) \tag{3}$$

where (D) is the absolute displacement vector of the point masses such that

$$(D) = \begin{bmatrix} (D)_1 \\ \cdot \\ \cdot \\ \cdot \\ (D)_{NR} \end{bmatrix}$$

As in the case of the flexural displacements

$$(D)_1 = \begin{bmatrix} D_1 \\ \cdot \\ \cdot \\ \cdot \\ D_N \end{bmatrix} \quad (D)_2 = \begin{bmatrix} D_{N+1} \\ \cdot \\ \cdot \\ \cdot \\ D_{2N} \end{bmatrix} \quad (D)_{NR} = \begin{bmatrix} D_{(NR-1)(N)+1} \\ \cdot \\ \cdot \\ \cdot \\ D_{(NR)(N)-2} \end{bmatrix}$$

The matrix [KS] is the combined stiffness matrix of the springs containing (NRxN-2)(NRxN-2) elements some of which are zero. It can be formed by expressing the potential energy stored in the springs in terms of the absolute displacements of the point masses and the spring constants between them and then using the relationship

$$(\partial VS/\partial D_i) = [KS](D) \tag{4}$$

The procedure is illustrated in Appendix A.

Total Potential Energy of the System

In the preceeding two sections it was shown that the flexural potential energy is related to the flexural displacements and the potential energy in the compressed paper depends on the absolute displacements. In order to sum the two potential energy terms VF and VS the flexural and absolute displacements of the free-free rolls have to be related. Using a procedure described in Appendix C the absolute displacements of the point masses of the free-free rolls can be related to the flexural displacements through the expression

$$(D)_n = [c]_n (d)_n \tag{5}$$

Matrix $[c]_n$ is an (NxN) constraint matrix for the nth roll which is obtained through the use of the orthagonality relationships of the two rigid body modes of a free-free roll to the flexural natural modes. Since the flexural and absolute displacements of a simply supported roll are the same, for the simply supported bottom roll [c] is a unit matrix. The nature of the constraint matrix $[c]_n$ for a free-free roll is such that its determinant is zero and therefore its inverse does not exist. The potential energy term corresponding to the springs can now be expressed in terms of the flexural displacements by substituting equation (5) in equation (3)

$$VS = 0.5 \begin{bmatrix} [c]_1 (d)_1 \\ \vdots \\ [c]_{NR} (d)_{NR} \end{bmatrix}^T \begin{bmatrix} \\ KS \\ \\ \end{bmatrix} \begin{bmatrix} [c]_1 (d)_1 \\ \vdots \\ [c]_{NR} (d)_{NR} \end{bmatrix}$$

$$VS = 0.5([C](d))^T [KS][C](d) \quad (6)$$

$$VS = 0.5(d)^T [C]^T [KS][C](d) \quad (7)$$

Where [C] is an (NRxN-2)(NRxN-2) partitioned matrix

$$[C] = \begin{bmatrix} [c]_1 & & \\ & \ddots & \\ & & [c]_{NR} \end{bmatrix}$$

From an extension of equation (5) it follows that

$$[D] = [C](d) \quad (8)$$

In the general case where the end masses of the upper rolls are different from the point masses representing the roll [C] is not symmetric. As in the case of $[c]_n$ the combined constraint matrix [C] is singular. By adding equations (2) and (7) the total potential energy of the system can now be expressed as follows

$$V = VF + VS = 0.5(d)^T ([KF] + [C]^T [KS][C])(d) \quad (9)$$

2.3 Energy Expressions for the Calender System (cont'd)

Kinetic Energy of the System

The kinetic energy of the system depends on velocities alone and can be written for the nth roll in matrix form as

$$T_n = 0.5(\dot{D})_n^T [m]_n (\dot{D})_n \qquad (10)$$

The matrix $[m]_n$ is an (NxN) inertia matrix. Since $(D)_n$ are generalized co-ordinates $[m]_n$ is diagonal. The total kinetic energy of the system is

$$T = T_1 + \ldots\ldots + T_{NR} = 0.5(\dot{D})^T [M](\dot{D}) \qquad (11)$$

where [M] is a (NRxN-2)(NRxN-2) diagonal matrix such that

$$[M] = \begin{bmatrix} [m]_1 & & \\ & \ddots & \\ & & [m]_{NR} \end{bmatrix}$$

Differentiating equation (8) and substituting in equation (11) an expression is obtained for the total kinetic energy of the vibrating calender system in terms of the flexural displacement vectors, the inertia matrix and the constraint matrix

$$T = 0.5(\dot{d})^T [C]^T [M][C](\dot{d}) \qquad (12)$$

2.4 EQUATIONS OF MOTION

Having established the potential energy and the kinetic energy of the vibrating system, one can proceed to obtain the equations of motion using Lagrange's equations. Lagrange's equations can be written in matrix form as

$$d/dt[\partial T/\partial \dot{q}_i] - [\partial T/\partial q_i] + [\partial V/\partial q_i] = (Q) \quad . \qquad (13)$$

Where q are generalized co-ordinates and (Q) is a column matrix representing non-conservative generalized forces. For a conservative system in which the kinetic energy depends on velocities alone Lagrange's equations reduce to

$$d/dt[\partial T/\partial \dot{q}_i] + [\partial V/\partial \dot{q}_i] = 0 \qquad (14) \; .$$

The energy terms T and V are given by equations (12) and (9) respectively. After differentiating these matrix expressions and substituting into equation (14) the equations of motion in matrix form are obtained.

$$[C]^T[M][C](\ddot{d}) + ([KF] + [C]^T[KS][C])(d) = 0 \qquad (15) \; .$$

Assuming a sinusoidal solution of the type

$$d_i = B_i \cos(wt) \qquad (16)$$

leads to the eigenvalue problem

$$w^2[C]^T[M][C](d) = ([KF] + [C]^T[KS][C])(d) \qquad (17) \; .$$

Introducing the flexibility matrix $[a]_n$ of the nth roll in

2.4 Equations of Motion (cont'd)

the flexural potential energy expression gives

$$vf_n = 0.5(d)_n^T [kf]_n (d)_n = 0.5(d)_n^T ([a]_n)^{-1} (d)_n \qquad (18)$$

and therefore [KF] can be written as

$$[KF] = ([A])^{-1} = [A]^{-1}$$

where [A] is a (RNxN-2)(RNxN-2) partitioned flexibility matrix such that

$$[A] = \begin{bmatrix} [a]_n & & \\ & \ddots & \\ & & [a]_{NR} \end{bmatrix}$$

Substituting for [KF] in equation (17) gives

$$w^2[C]^T[M][C](d) = ([A]^{-1} + [C]^T[KS][C])(d) \qquad (19)$$

and since

$$[KF]^{-1} = [A]$$

premultiplying by $[KF]^{-1}$ leads to the eigenvalue problem

$$w^2[A][C]^T[M][C](d) = ([I] + [A][C]^T[KS][C])(d) \qquad (20)$$

where [I] is the identity matrix. Premultiplying equation (20) by [C] gives

$$w^2[C][A][C]^T[M][C](d) = ([C] + [C][A][C]^T[KS][C])(d) \qquad (21)$$

Making use of the relationship

2.4 Equations of Motion (cont'd)

$$[D] = [C](d) \qquad (8)$$

leads to the expression

$$w^2[C][A][C]^T[M](D) = ([I] + [C][A][C]^T[KS])(D) \qquad (22)$$

The final eigenvalue problem to be solved is

$$[PST]^{-1}[PIN](D) = L(D) \qquad (23)$$

Where [PIN] is a diagonal partitioned matrix whose diagonal terms are

$$([c][a][c]^T[m])_n$$

$$L = 1/w^2 \qquad (24)$$

and

$$[PST] = ([I] + [C][A][C]^T[KS]) \qquad (25)$$

In general [PIN], [PST] and $[PST]^{-1}$ are not symmetric matrices.

2.5 COMPUTATIONAL PROCEDURES

The generation of the various matrix expressions leading up to the eigenvalue problem represented by equation (23) and the solution of the eigenvalue problem itself have been programmed in the Fortran IV language for digital computer solution. The program is described in some detail in Appendix D. Only a brief summary of the procedures will be given here.

The number of rolls in the calender stack, the number of lumped masses to be used to represent each roll, the overhang mass values, roll dimensions, roll material properties and spring constants representing paper stiffness constitute the input for the program. The first segment of the program computes [KS] the spring stiffness matrix. The procedure used both in establishing paper stiffness and in generating the stiffness matrix [KS] are described in detail in Appendix A.

The second segment generates the matrices [m],[kf] and [c] roll by roll and generates the partitioned matrices [PIN] and [CACT] as it goes along, by forming the diagonal members $[c][a][c]^T[m]$ and $[C][A][C]^T$ respectively. The (NxN) matrix [m] is generated by computing the mass of the nth roll and lumping that into (N-2) point masses, each of which is considered concentrated at the center of gravity of the segment it represents and adding the overhang mass for each end. The flexibility matrix [a] is generated by a routine using the moment-area method. The method used is described in Appendix C. The nature of the constraint matrix [c] and the method of generating it are also discussed in Appendix C.

The third segment of the program computes the system stiffness matrix by forming

$$[PST] = ([I] + [CACT][KS])$$

then after inverting [PST] computes the matrix

$$[PST]^{-1}[PIN]$$

which is the matrix whose eigen solution is sought. This matrix is normally a (NxNR-2)(NxNR-2) general matrix.

The fourth segment calls the subroutine EIG which is one of the subroutines in a mathematical software package called EISPACK. This software package is the creation of Argonne National Laboratory at the University of Chicago and is available at the time of writing this report from the software library of the CDC Computer at the Sir George Williams Campus of Concordia University. This is the only software package in the Montreal area, known to the writer which is capable of solving the eigenvalue problem for a general matrix. Having obtained the (NxNR-2) eigenvalues and eigenvectors which form the (NxNR-2) columns of the modal

2.5 Computational Procedures (cont'd)

matrix the program converts numerically the eigenvalues into natural frequencies and normalizes each modal vector to its largest element. The various output features of the program are described in Appendix D.

CHAPTER 3

NUMERICAL SOLUTION OF AN OPEN-FRAME CALENDER STACK

3.1 CALENDER STACK DATA

The analysis has been applied to an open frame, seven-roll, six-nip calender stack which has been in operation for a number of years and had encountered barring problems from time to time. The barring frequency of the calender stack however, is not known with any degree of reliability. The following input data were used to represent the calender stack.

TL = 203.0 inches (bearing center distance, bottom roll)
FL = 168.0 (face width of rolls)

Roll Diameters

DR (1) = 18.0 inches (uppermost roll)
(2) = 18.0
(3) = 16.0 (with a 2.5" diameter bore)
(4) = 18.0
(5) = 18.0
(6) = 18.0
(7) = 30.0

Effective Journal Diameters

DJ (1) = 9.0 inches
(2) = 9.0
(3) = 9.0
(4) = 9.0
(5) = 9.0
(6) = 9.0
(7) = 20.0

Combined bearing and journal weights, one end only

BMASS (1) = 824.0 lbs.
(2) = 824.0
(3) = 824.0
(4) = 824.0
(5) = 824.0
(6) = 824.0

Paper Stiffness Values

x(1) = 21.6 lbs/sqin (millions)
x(2) = 33.5
x(3) = 45.8

3.1 Calender Stack Data (cont'd)

x(4) = 69.1 lbs/sqin (millions)
x(5) = 81.0
x(6) = 99.8

The following data were used in computing the above values:

Sheet width = 162 inches

Nip pressures

Nip 1	81.0	lbs per linear inch of sheet width
2	162.0	
3	227.8	
4	308.8	
5	389.8	
6	470.8	

Roll Material Properties

DS = 0.268 lbs/cuin density
YM = 20,000,000 lbs/sqin modulus of elasticity

Number of lumped masses per roll

Upper rolls 6
Bottom rolls 4

COMPUTER OUTPUT

EIGENVALUE	1	58.63	EIGENVALUE	2	34.37
FREQ.	1	72.00	FREQ.	2	94.05

VECTOR 1	VECTOR 2
1.000000	1.000000
.131923	.092562
-.419601	-.340240
-.419601	-.340240
.131923	.092562
1.000000	1.000000
.903923	.356275
.114962	.034456
-.375001	-.136949
-.375001	-.136949
.114962	.034456
.903923	.356275
.678270	-.182592
.078633	-.011143
-.325586	.077624
-.325586	.077624
.078633	-.011143
.678270	-.182592
.607463	-.566623
.073108	-.045784
-.247862	.208789
-.247862	.208789
.073108	-.045784
.607463	-.566623
.480284	-.729793
.053880	-.047190
-.192047	.257136
-.192047	.257136
.053880	-.047190
.480284	-.729793
.425690	-.005931
.021105	-.007669
-.143567	.262532
-.143567	.262532
.021105	-.007669
.425690	-.005931
-.049660	.122697
-.114631	.254271
-.114631	.254271
-.049660	.122697

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 3	20.76	EIGENVALUE 4	14.30
FREQ. 3	121.00	FREQ. 4	145.78

VECTOR 3	VECTOR 4
-1.000000	.456140
-.037291	-.006591
.324970	-.124631
.324970	-.124631
-.037291	-.006591
-1.000000	.456140
.695632	-1.000000
.017955	.013586
-.218073	.274093
-.218073	.274093
.017955	.013586
.695632	-1.000000
.918819	.105069
.002499	-.004554
-.337034	-.033781
-.337034	-.033781
.002499	-.004554
.918819	.105069
.553395	.773684
.017990	-.007413
-.177189	-.215159
-.177189	-.215159
.017990	-.007413
.553395	.773684
-.122327	.396423
.014491	.006969
.020700	-.121012
.020700	-.121012
.014491	.006969
-.122327	.396423
-.776426	-.351150
.039245	.039980
.184116	.061038
.184116	.061038
.039245	.039980
-.776426	-.351150
.143943	.099262
.261359	.163431
.261359	.163431
.143943	.099262

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 5	13.48	EIGENVALUE 6	11.01
FREQ. 5	150.16	FREQ. 6	166.19

VECTOR 5	VECTOR 6
1.000000	1.000000
-.349489	-.353130
-.341977	-.331055
.341977	.331055
.349489	.353130
-1.000000	-1.000000
.966480	.355741
-.338140	-.125518
-.329417	-.118084
.329417	.118084
.338140	.125518
-.966480	-.355741
.733852	-.164566
-.327103	.074201
-.310110	.066996
.310110	-.066996
.327103	-.074201
-.733852	.164566
.705177	-.535534
-.247138	.189679
-.239096	.175594
.239096	-.175594
.247138	-.189679
-.705177	.535534
.553801	-.648421
-.194632	.230663
-.186135	.289606
.186135	-.289606
.194632	-.230663
-.553801	.648421
.425577	-.642491
-.152265	.232541
-.134947	.195727
.134947	-.195727
.152265	-.232541
-.425577	.642491
-.127657	.219447
-.081259	.138272
.081259	-.138272
.127657	-.219447

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 7 9.99
FREQ. 7 174.43

EIGENVALUE 8 8.23
FREQ. 8 192.13

VECTOR 7	VECTOR 8
-.121903	1.000000
.010544	-.359068
.024524	-.313240
.024524	.313240
.010544	.359068
-.121903	-1.000000
.519547	-.778695
-.019030	.280032
-.130432	.242636
-.130432	-.242636
-.019030	-.280032
.519547	.778695
-1.000000	-.820014
.114108	.374829
.249985	.318560
.249985	-.318560
.114108	-.374829
-1.000000	.820014
.201207	-.411896
.003333	.147738
-.061216	.129504
-.061216	-.129504
.003833	-.147738
.201207	.411896
.673814	.233161
-.027351	-.005794
-.166491	-.066814
-.166491	.066814
-.027351	.005794
.673814	-.233161
-.003607	.604048
.025084	-.252687
-.001032	-.193070
-.001032	.193070
.025084	.252687
-.003607	-.604048
.079662	-.314063
.122929	-.194078
.122929	.194078
.079662	.314063

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 9 7.51 EIGENVALUE 10 6.20
FREQ. 9 201.24 FREQ. 10 221.38

VECTOR 9	VECTOR 10
-.023956	.438800
.005179	-.160427
.001713	-.128846
.001713	.128846
.005179	.160427
-.023956	-.438800
.124795	-1.000000
-.003338	.365554
-.032563	.293783
-.032563	-.293783
-.003338	-.365554
.124795	1.000000
-.535337	.146992
.076208	-.068198
.118704	-.054081
.118704	.054081
.076208	.068198
-.535337	-.146992
1.000000	.014336
-.083398	-.297261
-.204281	-.240508
-.204281	.240508
-.083398	.297261
1.000000	-.014336
-.913307	.394163
.075523	-.142469
.187215	-.120656
.187215	.120656
.075523	.142469
-.913307	-.394163
-.183422	-.317387
.004162	.121001
.048605	.078307
.048605	-.078307
.004162	-.121001
-.183422	.317387
-.096285	.266212
-.149995	.162835
-.149995	-.162835
-.096285	-.266212

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 11	5.35	EIGENVALUE 12	5.25
FREQ. 11	238.32	FREQ. 12	240.61

VECTOR 11	VECTOR 12
-.014011	-1.000000
.009930	.736830
-.005899	-.449152
-.005899	-.449152
.009930	.736830
-.014011	-1.000000
-.009323	-.928312
.009367	.672572
-.006685	-.405517
-.006685	-.405517
.009367	.672572
-.009323	-.928312
-.040201	-.510617
.013784	.572394
.000853	-.386482
.000853	-.386482
.013784	.572394
-.040201	-.510617
.136584	-.577784
-.009155	.403332
-.030137	-.237116
-.030137	-.237116
-.009155	.403332
.136584	-.577784
-.470547	-.336146
.054620	.260258
.000746	-.163556
.000746	-.163556
.054620	.260258
-.470547	-.336146
1.000000	-.211394
-.185746	.141725
-.181932	-.080911
-.181932	-.080911
-.185746	.141725
1.000000	-.211394
.140791	.034217
.231536	-.026340
.231536	-.026340
.140791	.034217

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 13 4.35	EIGENVALUE 14 4.33
FREQ. 13 264.25	FREQ. 14 264.94
VECTOR 13	VECTOR 14
-1.000000	.123194
.900101	-.046206
-.612422	-.032677
-.612422	.032677
.900101	.046206
-1.000000	-.123194
-.057380	-.609344
.056828	.227561
-.040321	.164577
-.040321	-.164577
.056828	-.227561
-.057380	.609344
.392893	1.000000
-.510501	-.475714
.367451	-.332641
.367451	.332641
-.510501	.475714
.392893	-1.000000
.759599	-.090010
-.678248	.033063
.459728	.027010
.459728	-.027010
-.678248	-.033063
.759599	.090010
.665231	-.015757
-.585834	.304019
.394461	.222200
.394461	-.222200
-.585834	-.304019
.665231	.015757
.327347	-.131329
-.350259	.046150
.256088	.044155
.256088	-.044155
-.350259	-.046150
.327347	.131329
-.124862	-.233976
.038138	-.141320
.038138	.141320
-.124862	.233976

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 15 3.30	EIGENVALUE 16 3.26
FREQ. 15 303.36	FREQ. 16 305.57
VECTOR 15	VECTOR 16
-.611483	.025512
.740958	-.009787
-.565048	-.006112
-.565048	.006112
.740958	.009787
-.611483	-.025512
.827564	-.201589
-1.000000	.076595
.761928	.050513
.761928	-.050513
-1.000000	-.076595
.827564	.201589
.357818	.646102
-.602057	-.312378
.471778	-.199865
.471778	.199865
-.602057	.312378
.357818	-.646102
-.234032	-1.000000
.289092	.301110
-.222566	.247114
-.222566	-.247114
.289092	-.301110
-.234032	1.000000
-.569255	.590540
.675231	-.224577
-.511469	-.147304
-.511469	.147304
.675231	.224577
-.569255	-.590540
-.396658	.536000
.568580	-.203068
-.454470	-.136107
-.454470	.136107
.568580	.203068
-.396658	-.536000
.255223	.212576
-.096177	.128518
-.096177	-.128518
.255223	-.212576

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 17 2.45
FREQ. 17 352.50

EIGENVALUE 18 2.39
FREQ. 18 356.80

VECTOR 17	VECTOR 18
-.184276	-.000988
.307781	.000189
-.254768	.001048
-.254768	-.001048
.307781	-.000189
-.184276	.000988
.597534	.017799
-1.000000	-.007359
.828103	-.002672
.828103	.002672
-1.000000	.007359
.597534	-.017799
-.393737	-.093052
.890342	.044950
-.746985	.028900
-.746985	-.028900
.890342	-.044950
-.393737	.093052
-.459584	.327960
.762981	-.128092
-.630769	-.071734
-.630769	.071734
.762981	.128092
-.459584	-.327960
.212978	-.743016
-.339045	.287314
.277776	.171182
.277776	-.171182
-.339045	-.287314
.212978	.743016
.429526	1.000000
-.880060	-.389457
.756495	-.222075
.756495	.222075
-.880060	.389457
.429526	-1.000000
-.630660	.182364
.304834	.112997
.304834	-.112997
-.630660	-.182364

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 19	2.21	EIGENVALUE 20	1.97
FREQ. 19	370.51	FREQ. 20	392.35

VECTOR 19	VECTOR 20
.435318	.462086
-.487001	-.547502
.850116	1.000000
-.850116	-1.000000
.487001	.547502
-.435318	-.462086
.465967	.056027
-.522354	-.066205
.919160	.120713
-.919160	-.120713
.522354	.066205
-.465967	-.056027
.402621	-.189305
-.569509	.202239
1.000000	-.513500
-1.000000	.513500
.569509	-.202239
-.402621	.189305
.309465	-.261909
-.349357	.311693
.617776	-.570910
-.617776	.570910
.349357	-.311693
-.309465	.261909
.199840	-.213484
-.217405	.249041
.374340	-.450285
-.374340	.450285
.217405	-.249041
-.199840	.213484
.074924	-.101160
-.095250	.129032
.101595	-.246426
-.101595	.246426
.095250	-.129032
-.074924	.101160
-.016042	.022022
.017037	-.025566
-.017037	.025566
.016042	-.022022

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 21	1.89	EIGENVALUE 22	1.68
FREQ. 21	400.98	FREQ. 22	425.05

VECTOR 21	VECTOR 22
-.037265	-.290615
.081720	.374480
-.071000	-.719379
-.071000	.719379
.081720	-.374480
-.037265	.290615
.197693	.403756
-.434817	-.520467
.377945	1.000000
.377945	-1.000000
-.434817	.520467
.197693	-.403756
-.341505	.183318
1.000000	-.296684
-.875660	.567451
-.875660	-.567451
1.000000	.296684
-.341505	-.183318
.186285	-.207161
-.409892	.267661
.356302	-.514936
.356302	.514936
-.409892	-.267661
.186285	.207161
.240962	-.340372
-.537391	.443992
.468071	-.047583
.468071	.047583
-.537391	-.443992
.240962	.340372
-.054555	-.220549
.250446	.301239
-.234752	-.597056
-.234752	.597056
.250446	-.301239
-.054555	.220549
.046574	.059174
-.370140	-.069949
-.370140	.069949
.046574	-.059174

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 23	1.48	EIGENVALUE 24	1.40
FREQ. 23	452.56	FREQ. 24	466.11

VECTOR 23	VECTOR 24
-.008039	.097607
.022708	-.139712
-.020395	.283420
-.020395	-.283420
.022708	.139712
-.008039	-.097607
.062278	-.339898
-.176466	.400466
.158550	-.974349
.158550	.974349
-.176466	-.400466
.062278	.339898
-.164659	.276689
.691010	-.495638
-.623704	1.000000
-.623704	-1.000000
.691010	.495638
-.164659	-.276689
.353763	.115269
-1.000000	-.165060
.898230	.334905
.898230	-.334905
-1.000000	.165060
.353763	-.115269
-.216656	-.200046
.617045	.282913
-.554718	-.570585
-.554718	.570585
.617045	-.282913
-.216656	.200046
-.233773	-.213021
.559251	.323083
-.491999	-.671942
-.491999	.671942
.559251	-.323083
-.233773	.213021
-.625993	.075249
.405392	-.094921
.405392	.094921
-.625993	-.075249

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 25	1.13	EIGENVALUE 26	1.09
FREQ. 25	517.52	FREQ. 26	528.67

VECTOR 25	VECTOR 26
.013446	.000425
-.021911	-.001656
.047037	.001534
-.047037	.001534
.021911	-.001656
-.013446	.000425
-.084158	-.005031
.136898	.019669
-.293675	-.018222
.293675	-.018222
-.136898	.019669
.084158	-.005031
.197309	.025339
-.401258	-.129646
.856553	.120420
-.856553	.120420
.401258	-.129646
-.197309	.025339
-.286418	-.101344
.466083	.394921
-1.000000	-.365767
1.000000	-.365767
-.466083	.394921
.286418	-.101344
.011542	.217463
-.015401	-.845940
.030155	.783380
-.030155	.783380
.015401	-.845940
-.011542	.217463
.225017	-.266396
-.388984	1.000000
.854077	-.923364
-.854077	-.923364
.388984	1.000000
-.225017	-.266396
-.121737	-.316544
.166995	.230867
-.166995	.230867
.121737	-.316544

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE	27	.88	EIGENVALUE	28	.71
FREQ.	27	587.19	FREQ.	28	655.56

VECTOR 27	VECTOR 28
-.000785	.000035
.001517	-.000079
-.003460	.000189
.003460	-.000189
-.001517	.000079
.000785	-.000035
.008260	-.000531
-.015918	.001198
.036269	-.002857
-.036269	.002857
.015918	-.001198
-.008260	.000531
-.037772	.003722
.090924	-.010473
-.206301	.024869
.206301	-.024869
-.090924	.010473
.037772	-.003722
.133776	-.021483
-.258969	.048404
.590899	-.115341
-.590899	.115341
.258969	-.048404
-.133776	.021483
-.224450	.071913
.437362	-.157556
-1.000000	.372676
1.000000	-.372676
-.437362	.157556
.224450	-.071913
.139606	-.100807
-.299891	.356914
.705559	-.808216
-.705559	.808216
.299891	-.356914
-.139606	.100807
-.206329	-.598819
.314774	1.000000
-.314774	-1.000000
.206329	.598819

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 29 .00
FREQ. 29 0.00

EIGENVALUE 30 .00
FREQ. 30 0.00

VECTOR 29	VECTOR 30
.604317	1.000000
.482339	.786556
.349272	.553707
.216206	.320659
.083139	.088010
-.038839	-.125434
-.034718	.180719
.005286	.158998
.048927	.135303
.092567	.111608
.136208	.087913
.176212	.066192
.042032	-.026600
.102339	.015120
.168129	.060631
.233918	.106143
.299708	.151655
.360015	.193374
-.664036	-.234956
-.421385	-.115939
-.156675	.013097
.108036	.143734
.372746	.273570
.615397	.392587
-.951309	-.266761
-.725127	-.108545
-.478384	.064055
-.231641	.236654
.015103	.409254
.241284	.567470
1.000000	.167949
.718718	.109808
.411864	.046381
.105010	-.017046
-.201843	-.080473
-.483126	-.130614
.000000	-.000000
-.000000	-.000000
-.000000	.000000
-.000000	-.000000

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 31	.00	EIGENVALUE 32	.00
FREQ. 31	0.00	FREQ. 32	0.00

VECTOR 31	VECTOR 32
-.340024	.480403
-.275680	.359563
-.205486	.227738
-.135292	.095913
-.069099	-.035912
-.004754	-.156752
-.372468	-.017472
-.305038	.014905
-.231478	.050225
-.157918	.085545
-.084358	.120865
-.016928	.153242
.031380	.003659
-.001321	.030635
-.036995	.060062
-.072669	.089489
-.108343	.118917
-.141044	.145892
.115890	-.481019
.081880	-.364536
.044779	-.237464
.007678	-.110392
-.029424	.016680
-.063434	.133163
.072777	.951315
.248630	.727823
.440469	.484014
.632308	.240205
.824147	-.003605
1.000000	-.227097
.059225	1.000000
.131329	.729175
.209987	.433729
.288645	.138284
.367303	-.157162
[illegible]9406	-.427987
.000000	-.000000
-.000000	-.000000
.000000	.000000
-.000000	-.000000

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE	33	-.00	
FREQ.	33	0.00	
EIGENVALUE	34	-.00	
FREQ.	34	0.00	

VECTOR 33	VECTOR 34
-.132154	.379851
-.080547	.276932
-.024249	.164657
.032049	.052382
.086347	-.059893
.139953	-.162812
-.145074	-1.000000
-.142907	-.826592
-.140542	-.637421
-.138178	-.448249
-.135814	-.259077
-.133647	-.065670
-.078295	-.357377
-.090526	-.403487
-.103869	-.453789
-.117211	-.504092
-.130554	-.554394
-.142785	-.600504
-.407092	-.434350
-.361374	-.494456
-.224227	-.560026
-.087080	-.625595
.050067	-.691165
.175706	-.751270
-.387015	-.100039
-.304793	-.192671
-.215097	-.293725
-.125400	-.394779
-.035703	-.495833
.046510	-.500465
-1.000000	-.421394
-.799822	-.336146
-.579772	-.243149
-.360523	-.150151
-.141274	-.057153
.059705	.020095
.000000	.000000
.000000	.000000
.000000	.000000
.000000	.000000

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 35 -.00 EIGENVALUE 36 -.00
FREQ. 35 0.00 FREQ. 36 0.00

VECTOR 35	VECTOR 36
.147830	-.302520
.144924	-.203602
.141754	-.175050
.138584	-.060010
.135413	.039615
.132508	.130661
-.709414	-1.000000
-.607382	-.969366
-.496074	-.935948
-.384766	-.902529
-.273458	-.869110
-.171426	-.830476
-.382221	-.792269
-.336262	-.777563
-.286125	-.762146
-.235989	-.746430
-.185852	-.730714
-.139893	-.716308
1.000000	-.800686
.880693	-.778934
.750540	-.755283
.620387	-.731471
.490234	-.707739
.370927	-.685985
.923360	-.362775
.730485	-.331290
.520076	-.296944
.309667	-.262597
.099258	-.228250
-.093617	-.196765
.114789	-.292121
-.004504	-.333590
-.134642	-.378829
-.264779	-.424068
-.394917	-.469307
-.514210	-.510777
-.000000	.000000
-.000000	.000000
-.000000	.000000
.000000	.000000

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 37 -.00
FREQ. 37 0.00

EIGENVALUE 38 -.00
FREQ. 38 0.00

VECTOR 37	VECTOR 38
.374631	-.268896
.253950	-.206161
.122081	-.141904
-.009788	-.075647
-.141657	-.009391
-.262537	.051345
.093446	1.000000
.079887	.798274
.065095	.578209
.050303	.358144
.035512	.138080
.021953	-.063646
-.059237	.384477
-.036607	.495613
-.011920	.616852
.012767	.738091
.037455	.859330
.060085	.970466
.391928	.263463
.127941	.057072
-.160844	-.166082
-.446029	-.393236
-.736014	-.618390
-1.000000	-.824781
-.233391	-.242082
-.139594	-.188137
-.037269	-.129288
.065055	-.070440
.167379	-.011591
.261176	.042354
-.150917	.213594
-.255051	.204223
-.368652	.194000
-.482252	.183778
-.595853	.173555
-.699987	.164165
-.000000	.000000
.000000	.000000
.000000	.000000
.000000	-.000000

COMPUTER OUTPUT (CONTINUED)

EIGENVALUE 39	-.00	EIGENVALUE 40	-.00
FREQ. 39	0.00	FREQ. 40	0.00

VECTOR 39	VECTOR 40
.265509	-1.000000
.182968	-.683028
.092923	-.337224
.002878	.008573
-.087167	.354369
-.169789	.671349
.704666	.021598
.587957	.017624
.460638	.013288
.333319	.008952
.206000	.004617
.089291	.000642
1.000000	-.022948
.795985	-.024855
.573424	-.026937
.350862	-.029018
.128300	-.031099
-.075714	-.033007
.041865	.000387
-.097342	.036819
-.249204	.076562
-.481066	.116305
-.552920	.156048
-.692134	.192480
-.178977	.042640
-.158561	.048400
-.136289	.054851
-.114018	.061222
-.091746	.067592
-.071330	.073432
-.028567	.100872
-.000221	.063852
.030781	.023466
.061623	-.016919
.092545	-.057305
.120890	-.094324
-.000000	.000000
-.000000	-.000000
.000000	-.000000
-.000000	-.000000

MODE SHAPE DIAGRAMS

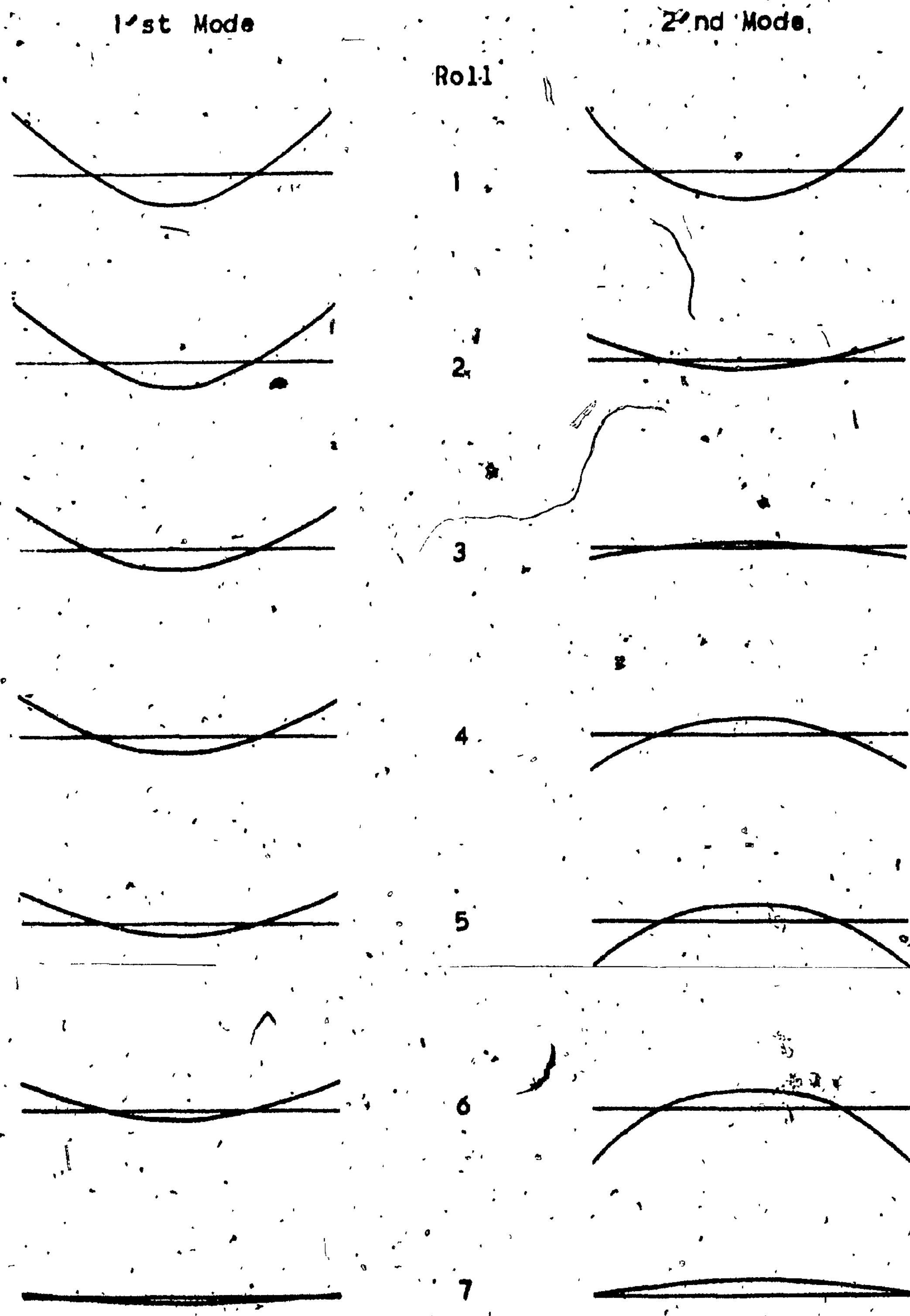


Figure no. 7

Mode Shape Diagrams (cont'd)

Figure no. 8

Mode Shape Diagrams (cont'd)

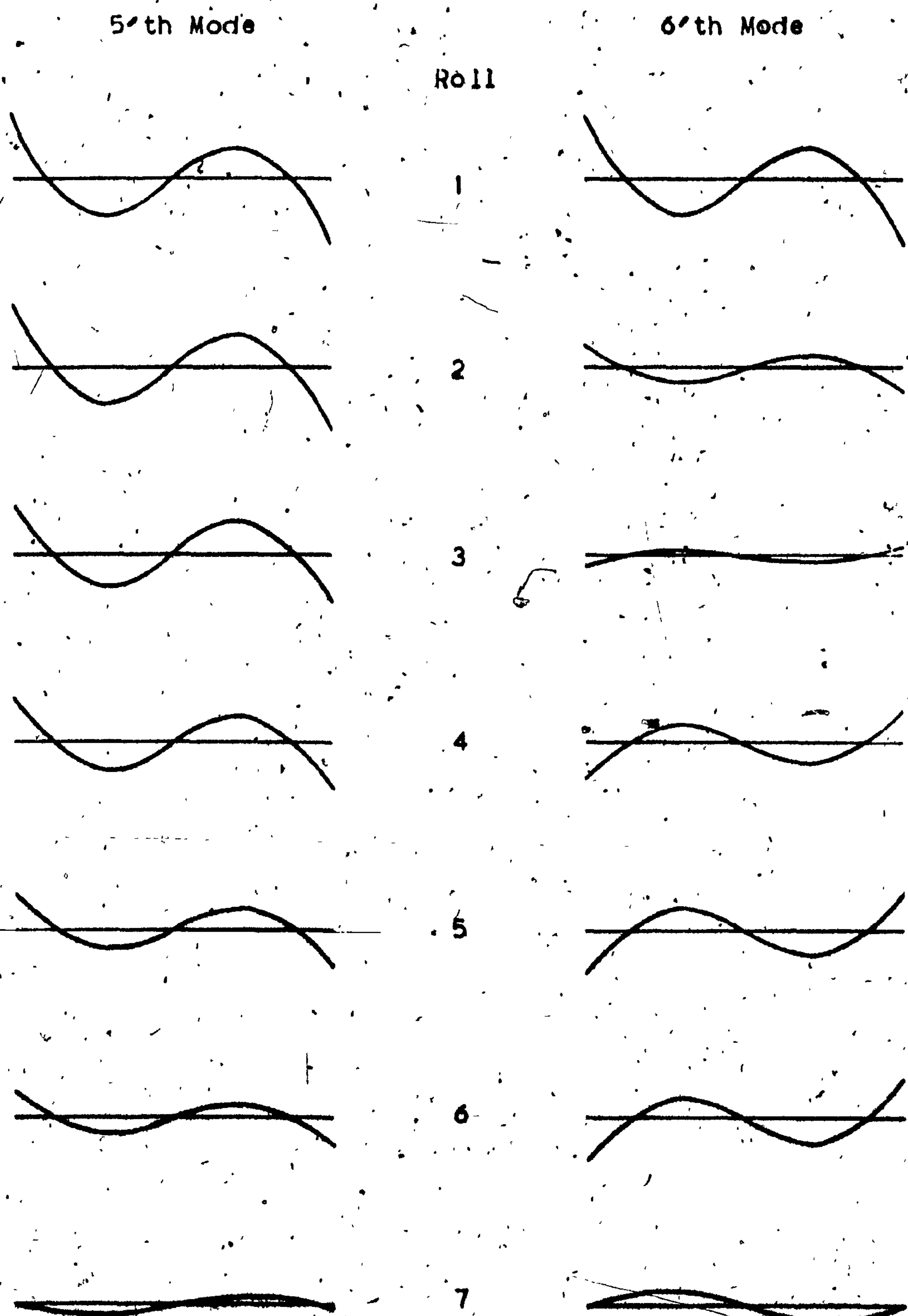


Figure no. 9

Mode Shape Diagrams (cont'd)

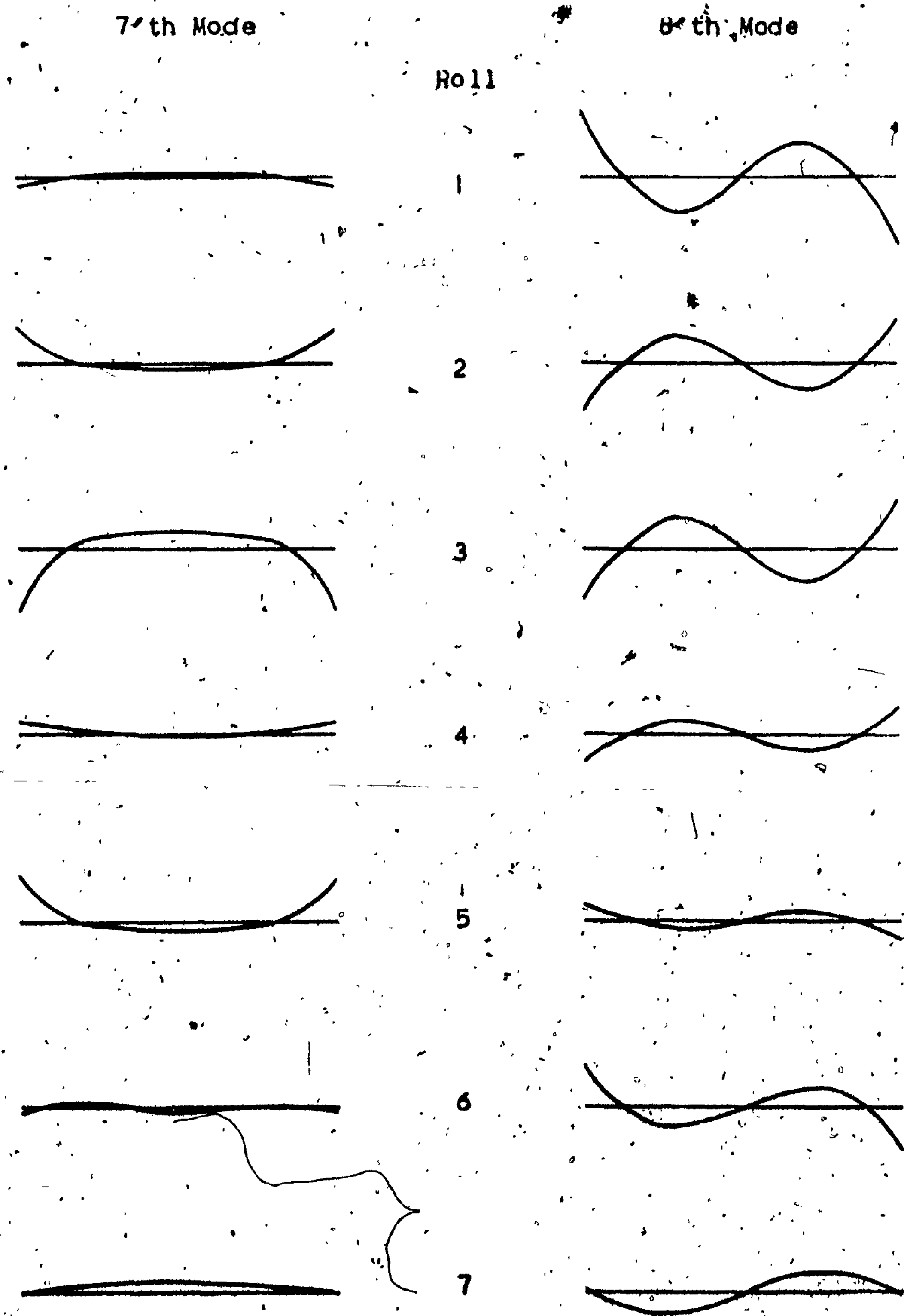


Figure no. 10

Mode Shape Diagrams (cont'd)

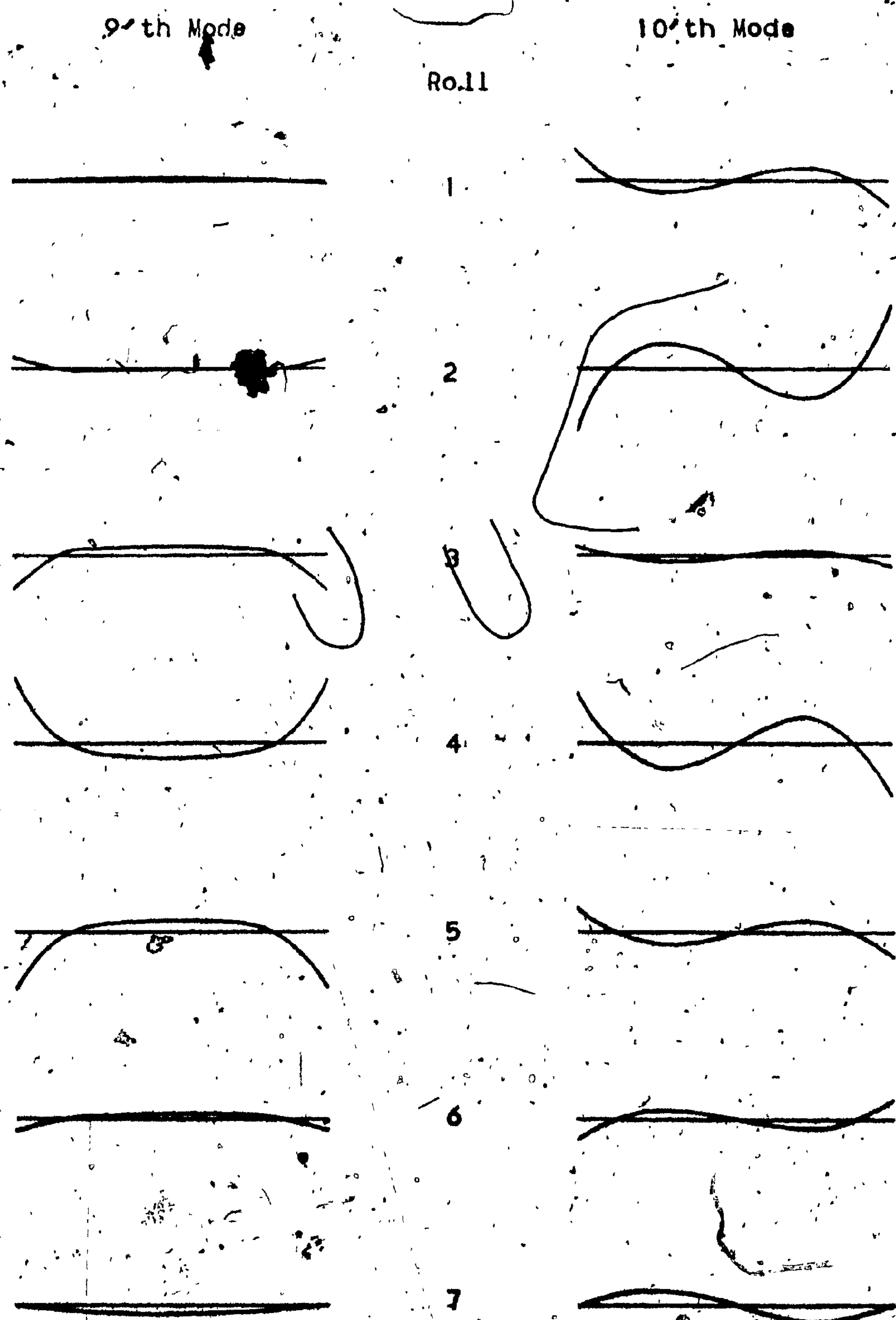


Figure no. 11

Mode Shape Diagrams (cont'd)

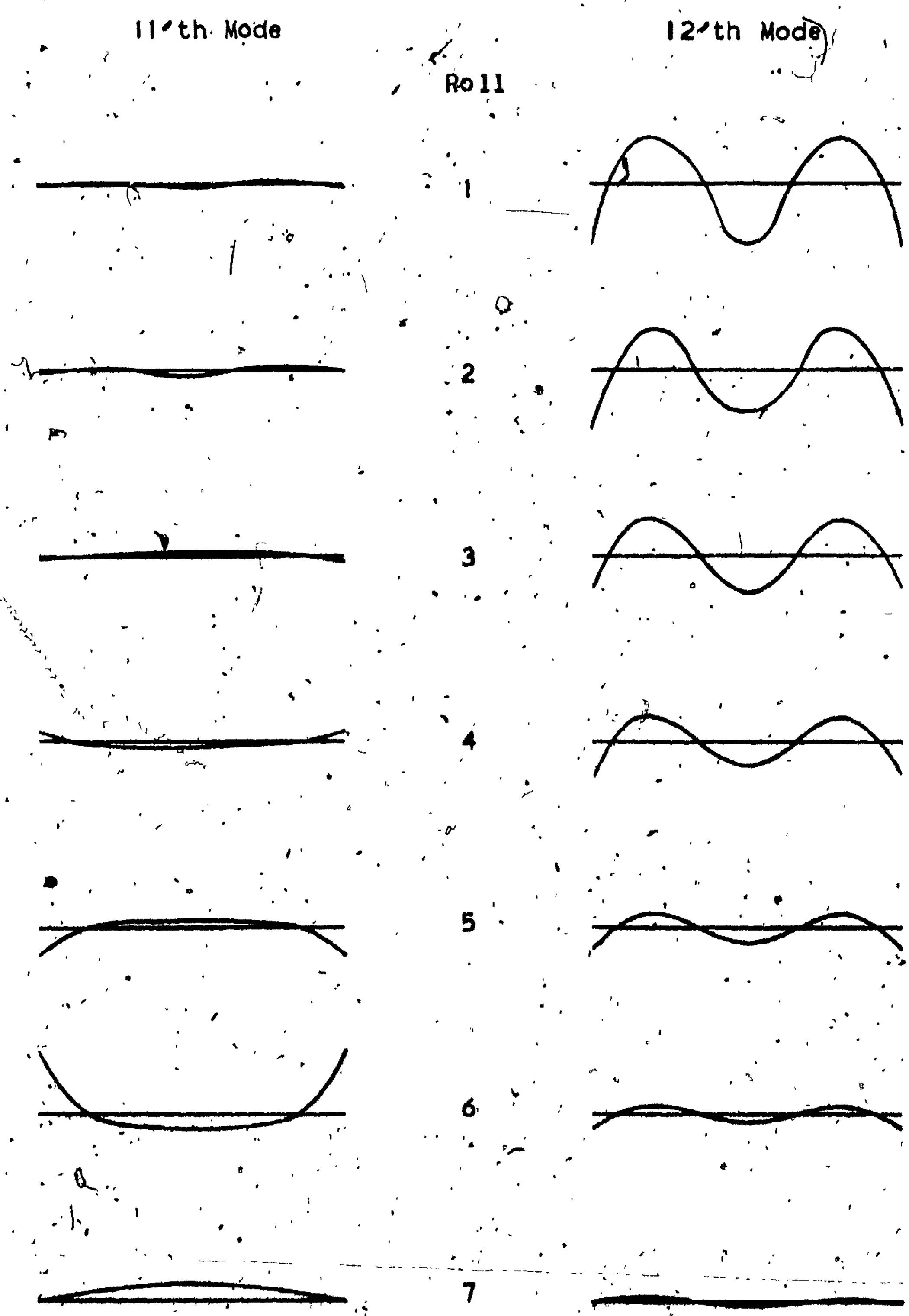


Figure no: 12

Mode Shape Diagrams (cont'd)

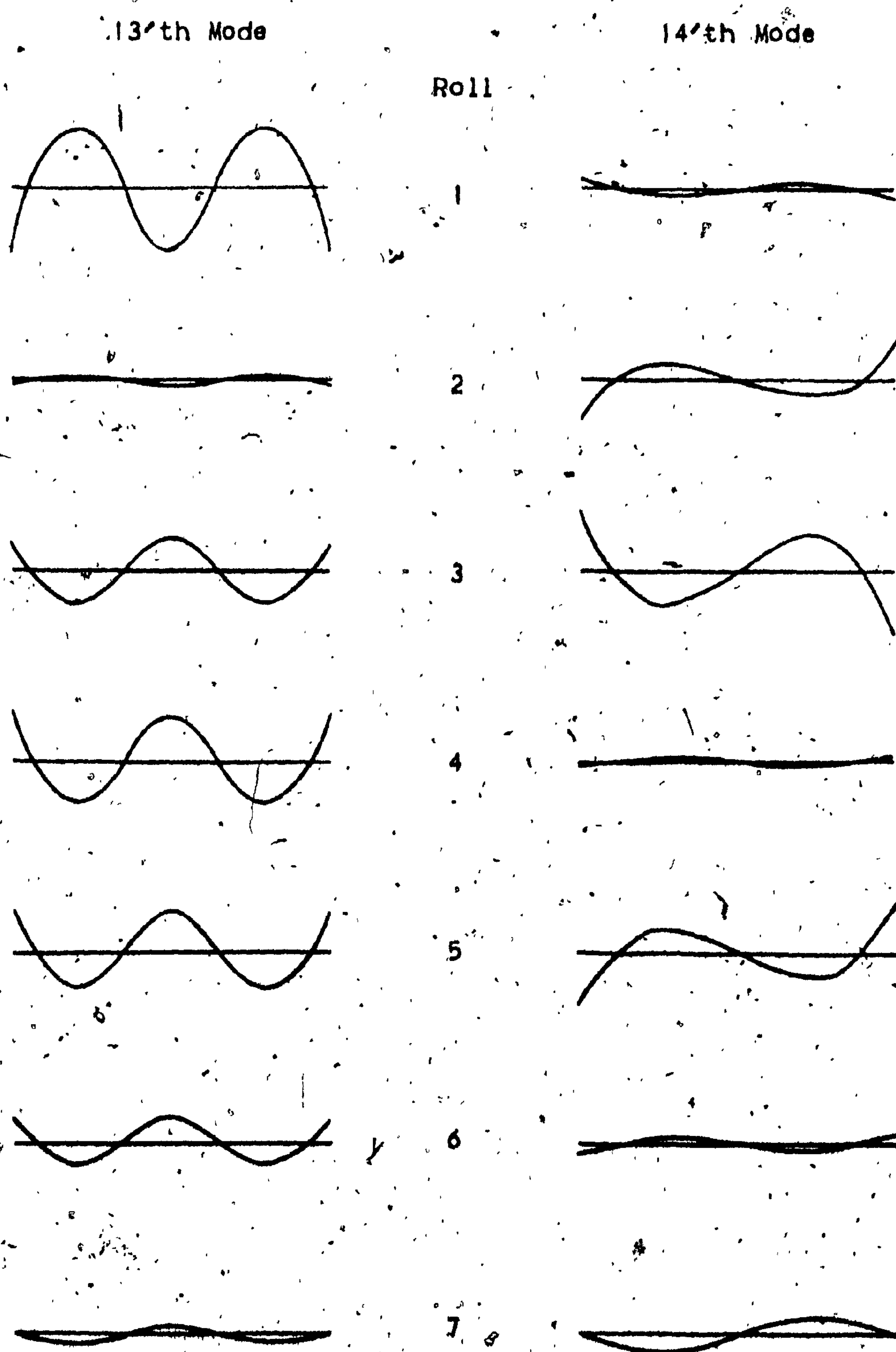


Figure no. 13

Mode Shape Diagrams (cont'd)

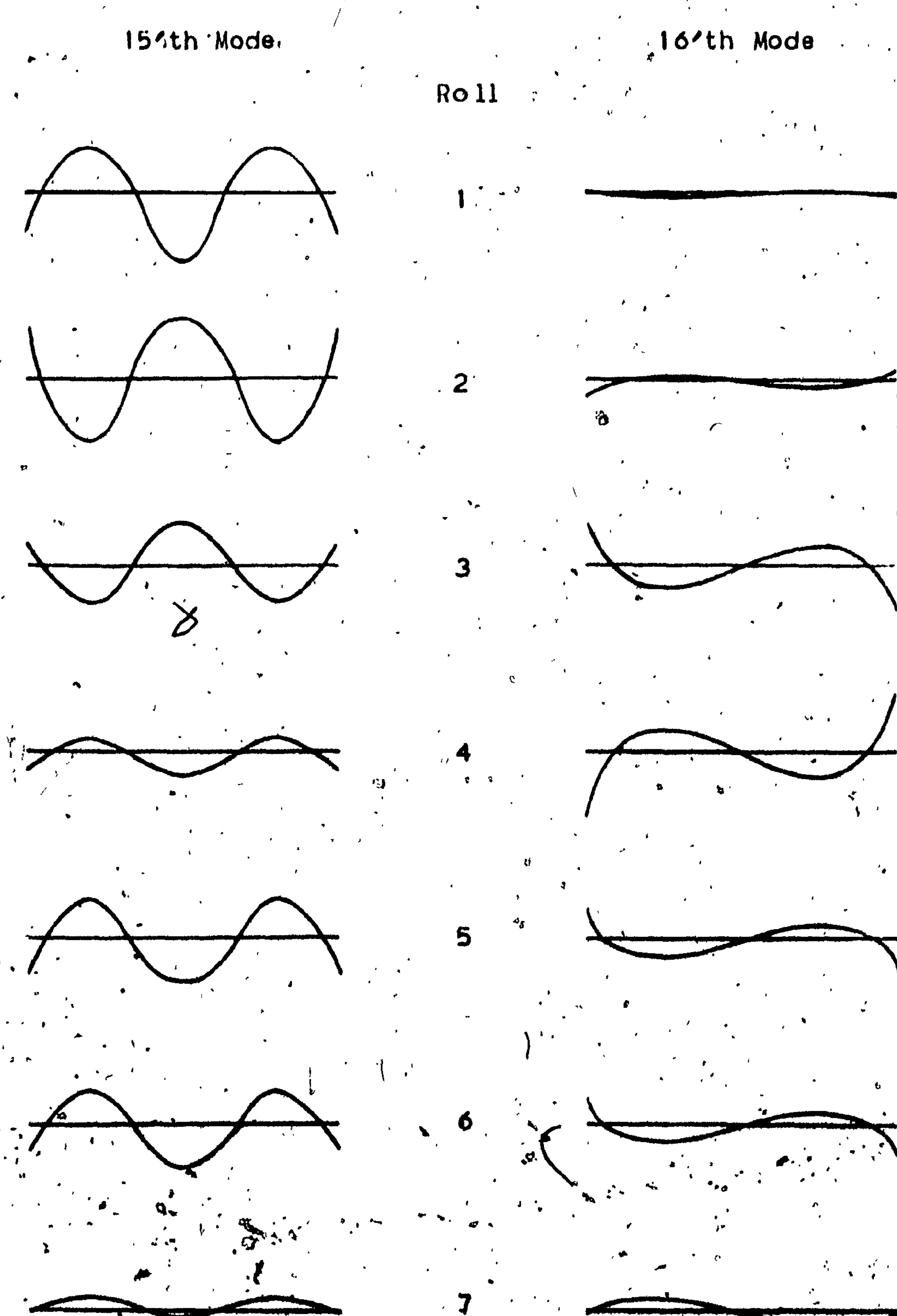


Figure no. 14

Mode Shape Diagrams (cont'd)

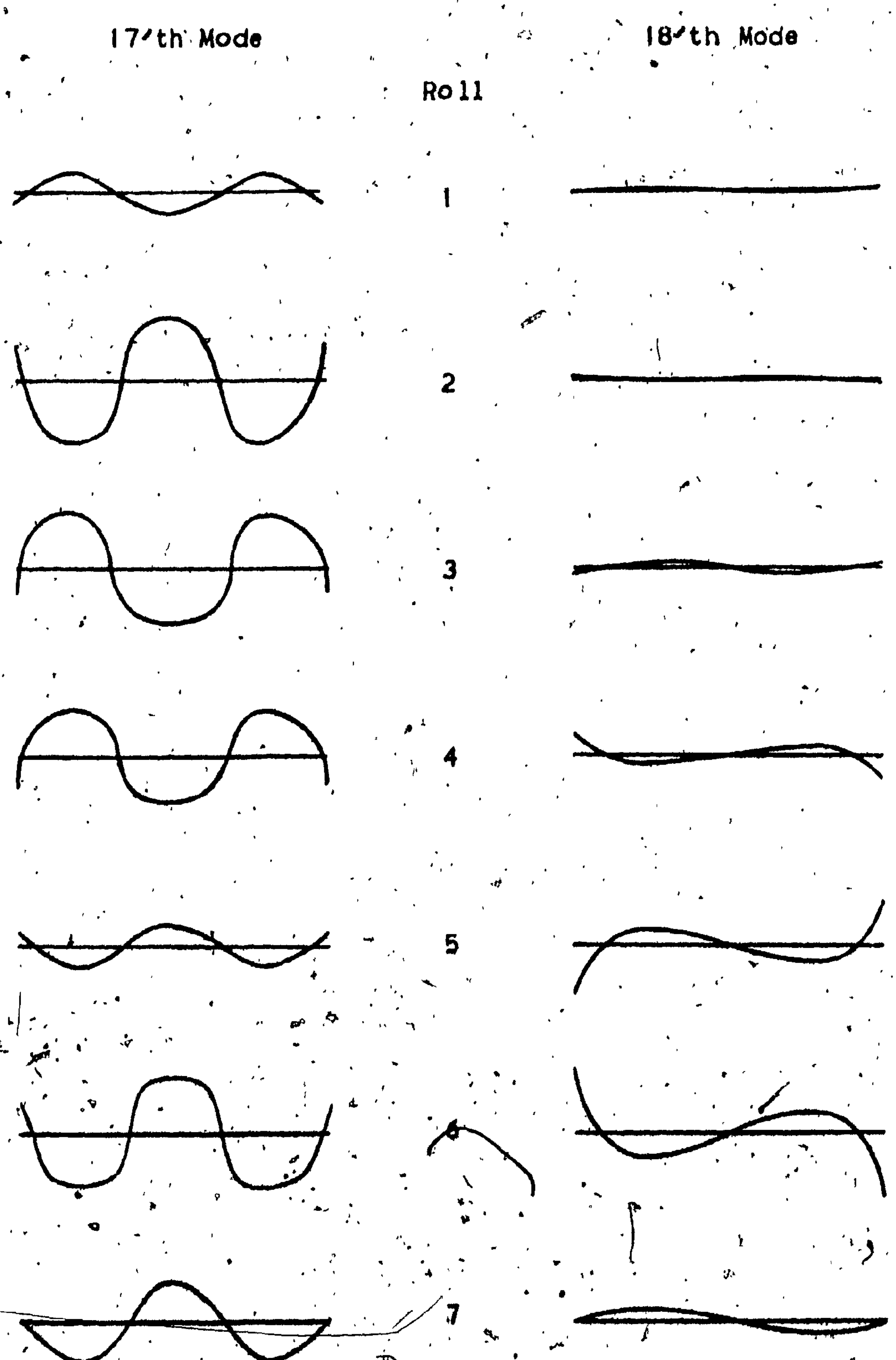


Figure no. 15

Mode Shape Diagrams (cont'd)

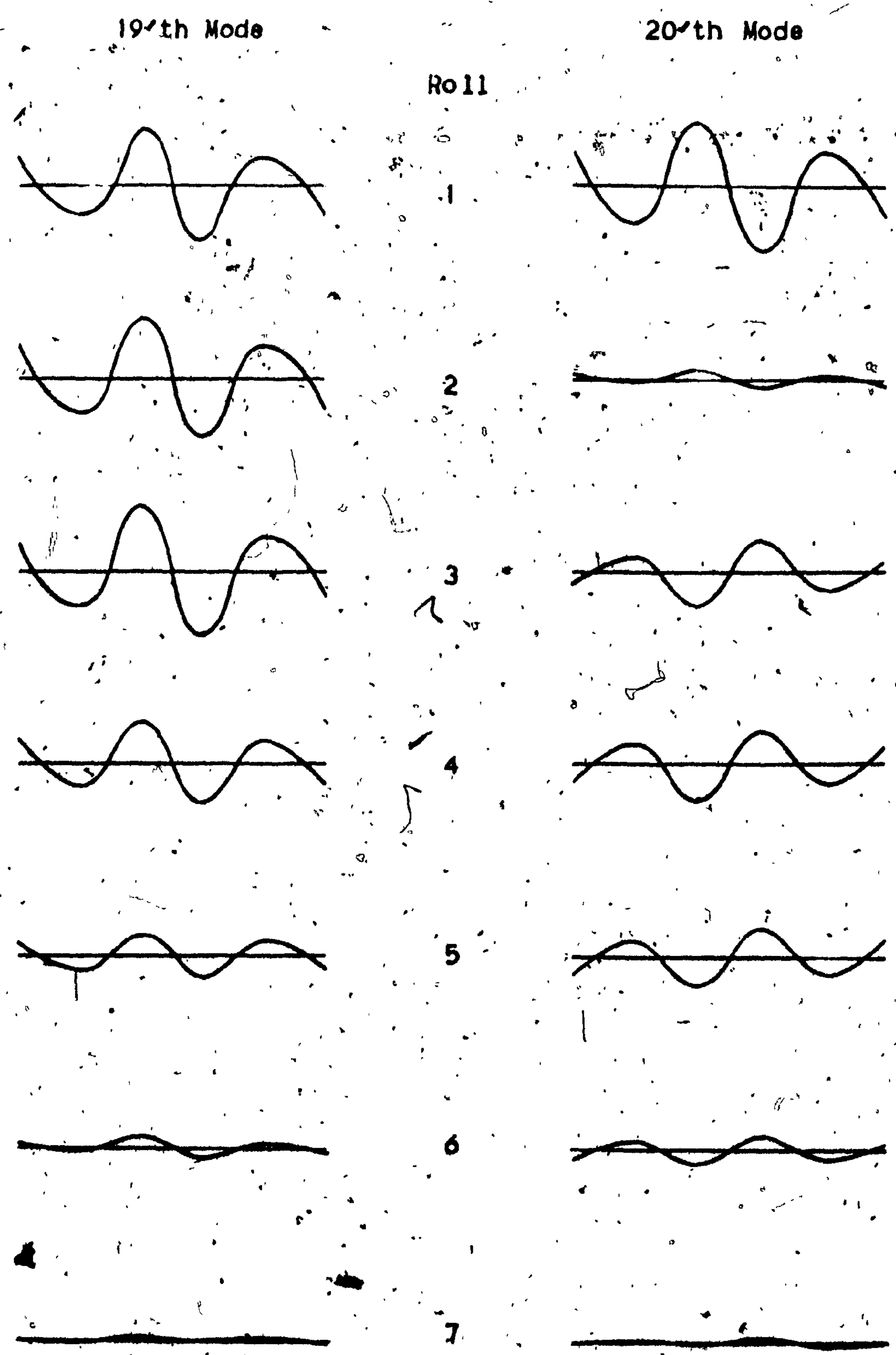


Figure no. 16

Mode Shape Diagrams (cont'd)

21'st Mode 22'nd Mode

Roll

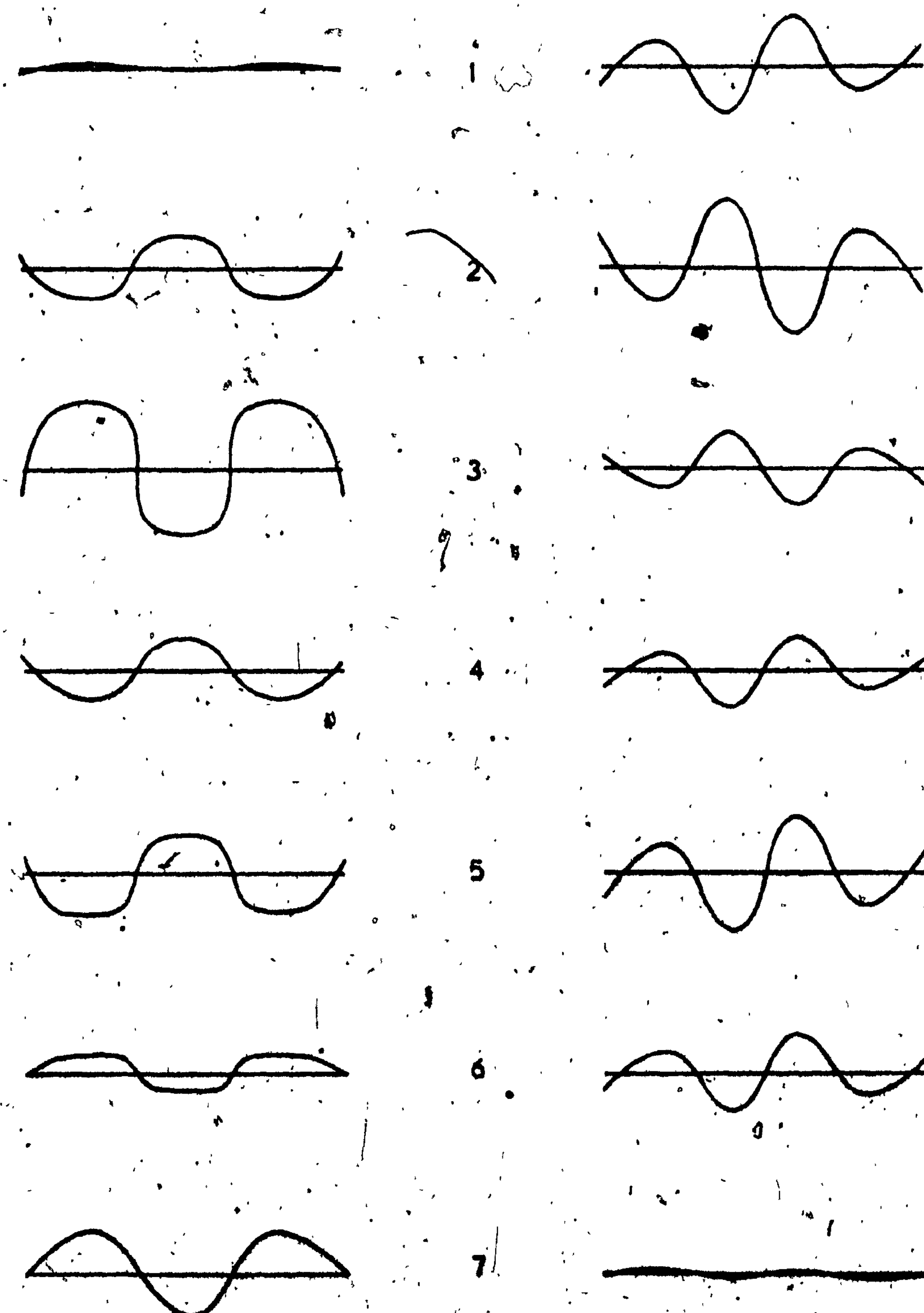


Figure no. 17

Mode Shape Diagrams (cont'd)

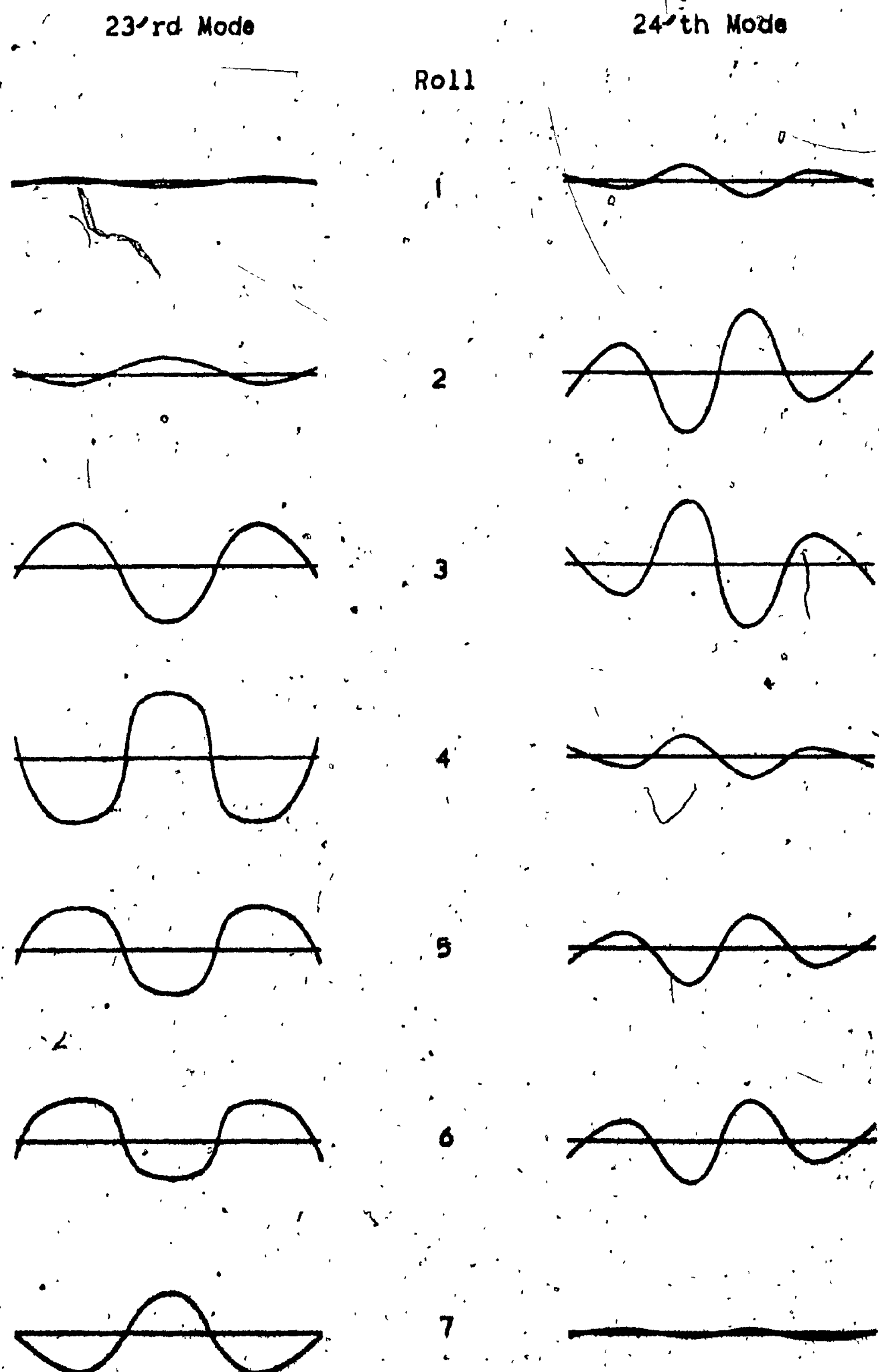


Figure no. 18

Mode Shape Diagrams (cont'd)

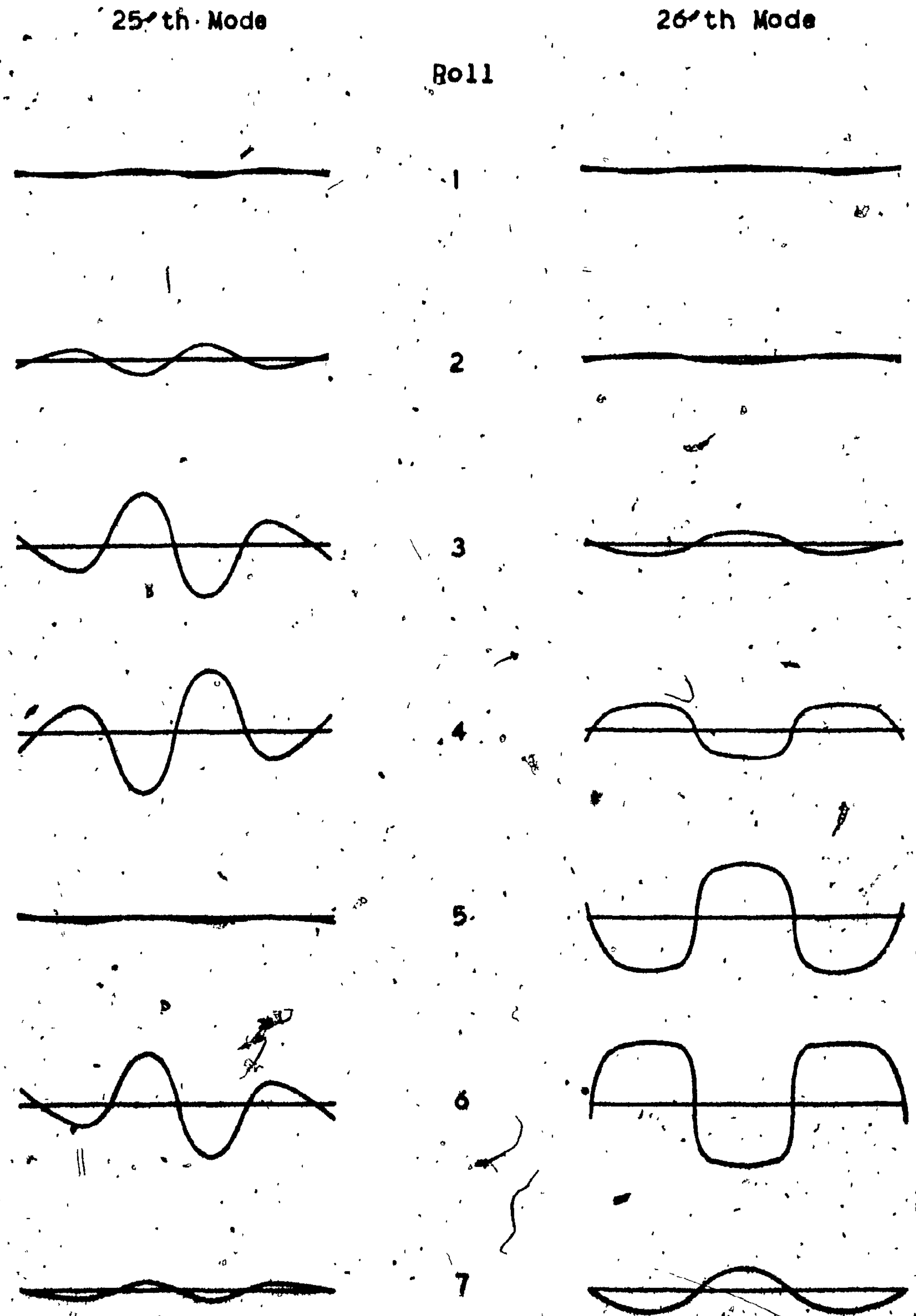


Figure no. 19

Mode Shape Diagrams (cont'd)

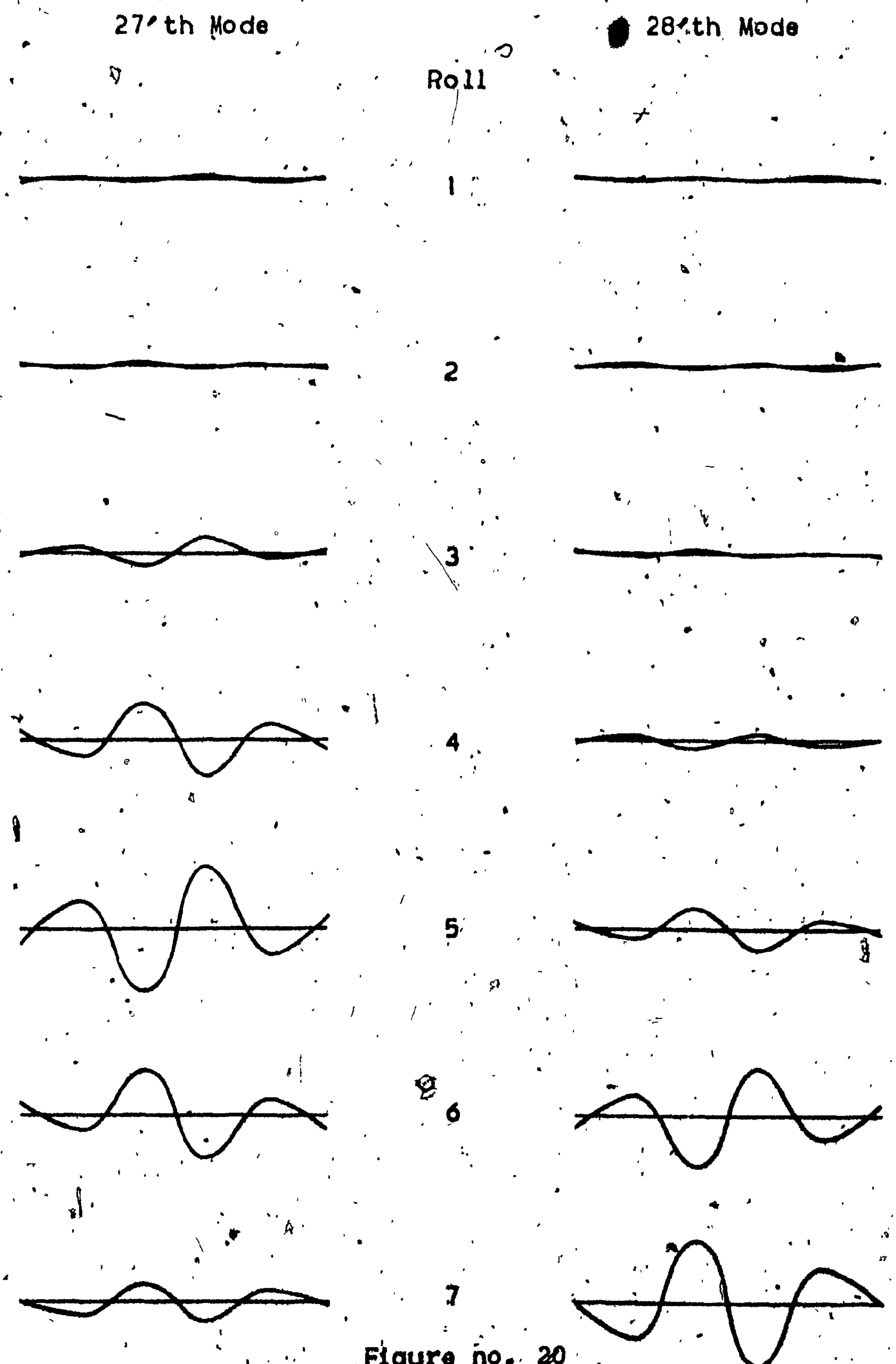


Figure no. 20

Mode Shape Diagrams (cont'd)

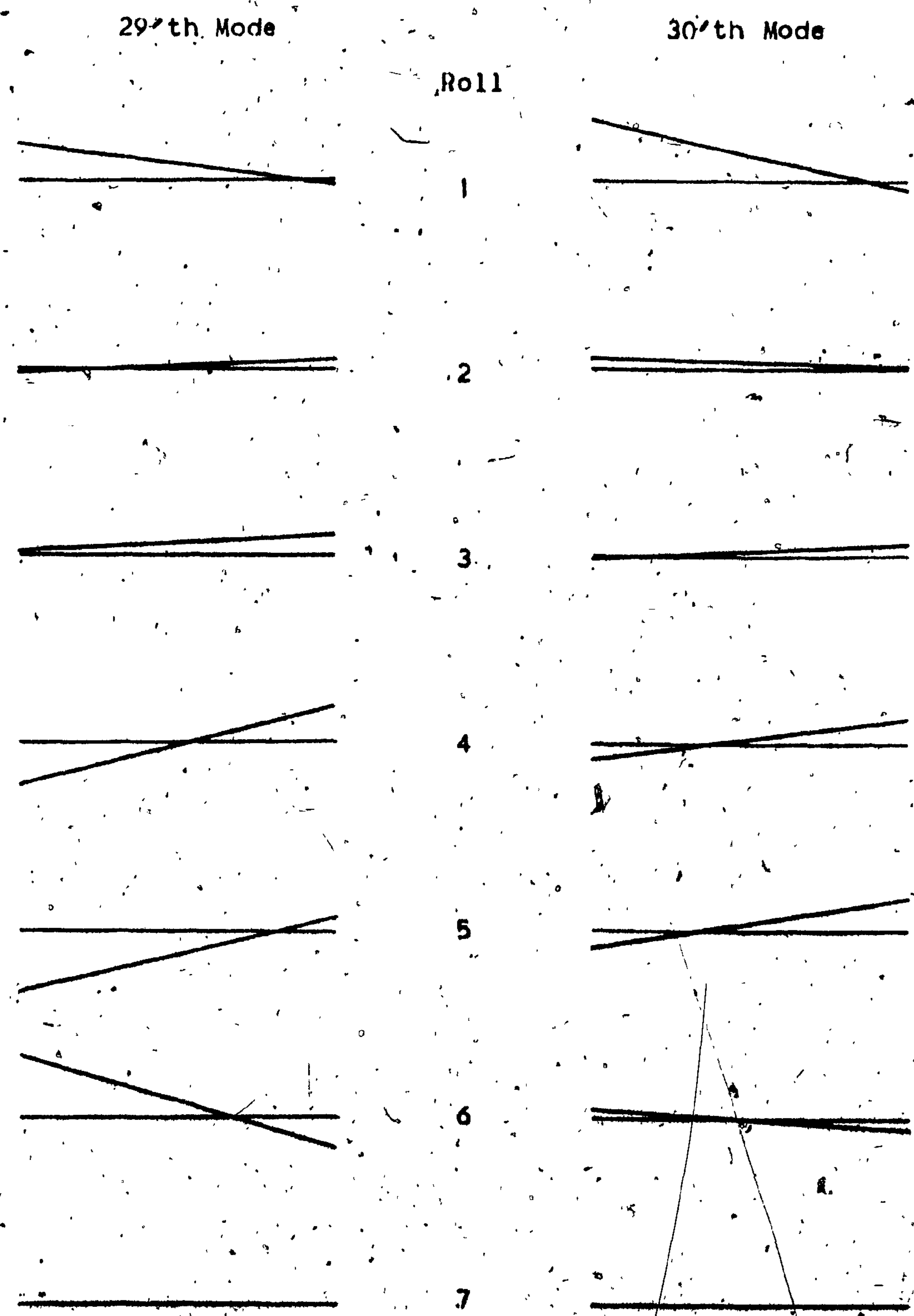


Figure no. 21

CHAPTER 4

EVALUATION OF THE MATHEMATICAL MODEL AND CONCLUSIONS

4.1 EVALUATION OF THE MATHEMATICAL MODEL AND CONCLUSIONS

In order to evaluate the validity of the mathematical model and the computations performed in the computer program the analysis should, ideally be tested against a calender stack whose natural frequencies and natural modes of vibration are known with a reasonable degree of accuracy. Unfortunately such information is not readily available. A less rigorous and less quantitative assessment of the analysis may be made by applying it to an actual calender stack and judging how realistic the computed natural frequencies and modes are and whether the effect of varying certain system parameters is in agreement with expectations based on basic physical principles which govern the behaviour of vibrating systems. Owing to the lack of reliable field data the latter approach has been taken to evaluate the analysis presented in Chapter 2. The numerical results of the analysis of an operational calender stack were presented in Chapter 3. Based on those results the following observations can be made.

The first two natural frequencies computed from the corresponding eigenvalues of the solution were 72.0 and 94.0 cps respectively. In a continent wide survey described in Chapter 1 all newsprint machines with a barring problem were reported to bar at the rate of 64-84 bars per second. The first natural frequency computed in the analysis falls right in the center of this range while the second one is just outside it. This is a very encouraging fact as although the barring frequency of the calender stack involved is not known, on the basis of the above statistics it is quite unlikely to be very far, if at all outside the 64-84 cps range. The first two or three natural frequencies, therefore appear to be quite realistic. The natural frequencies of the higher modes are more difficult to judge. One can only say that relative to the first two or three natural frequencies they, too appear to be realistic. It might be noted here that both because of the high frequencies and the nature of the mode shapes involved the natural modes higher than about the fifth or sixth mode are not likely to have practical importance and are probably not reliable for a model with only six lumped masses per roll.

The eigenvectors plotted in Chapter 3 indicate that the first four mode shapes are various combinations of the first bending mode of the rolls which make up the calender stack. This is in qualitative agreement with the field reports discussed in Chapter 1 according to which the uppermost and bottom rolls were observed to vibrate in their first bending mode during the barring phenomenon. Also, as it was pointed out in that report the barring pattern produced by several other barring calenders indicated that the rolls were vibrating in that mode. It is therefore very likely that most or perhaps all calenders bar in that manner. The fact that the first four natural modes computed by the analysis

4.1 Evaluation of the Mathematical Model (cont'd)

involve the first bending mode of the rolls makes the probability of the occurrence of that mode large and further enhances the credibility of the results.

The diagram below illustrates the effect of varying some of the geometric parameters and the paper stiffness.

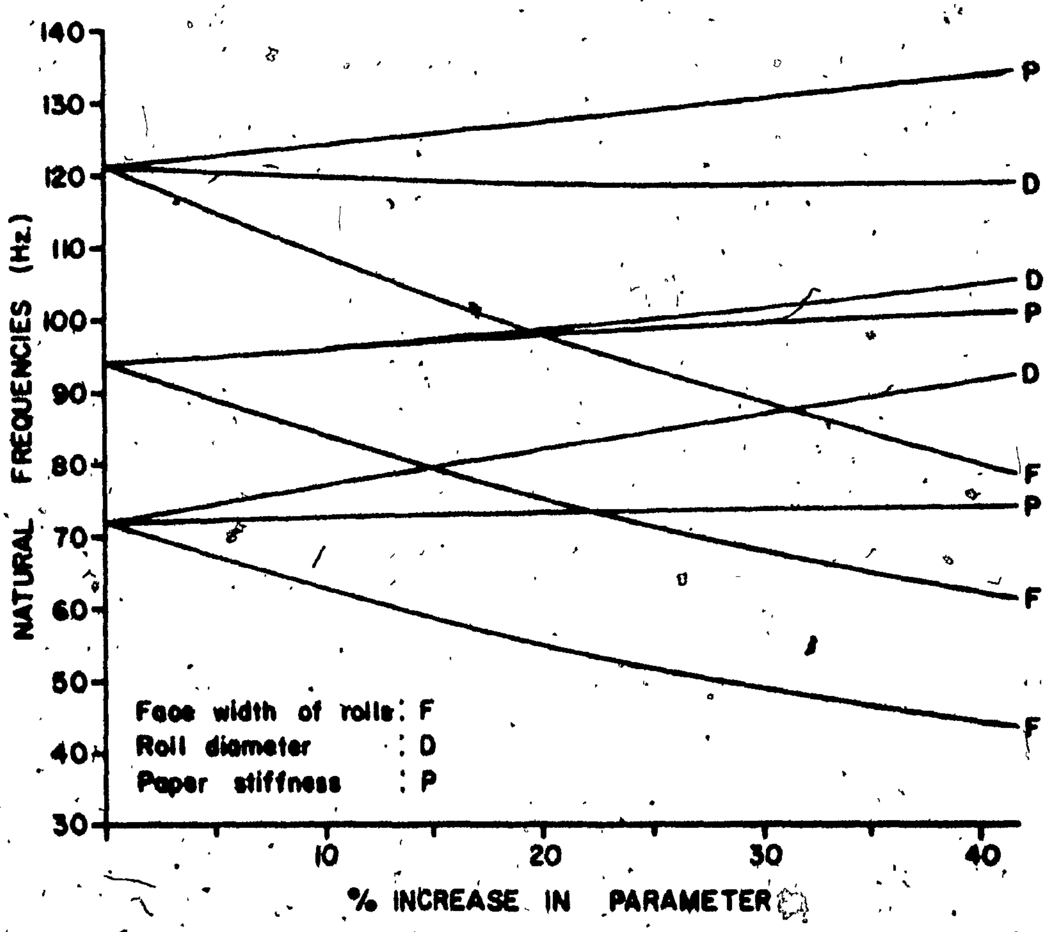


Figure 22

The effect on the behaviour of the calender stack is in qualitative agreement with expectations based on fundamental

4.1 Evaluation of the Mathematical Model (cont'd)

considerations and appear to be realistic quantitatively. Increasing the width of the calender stack lowers the natural frequencies, and enlarging the roll diameters increases them. The stiffening of the springs representing the paper between adjacent rolls increases the natural frequencies.

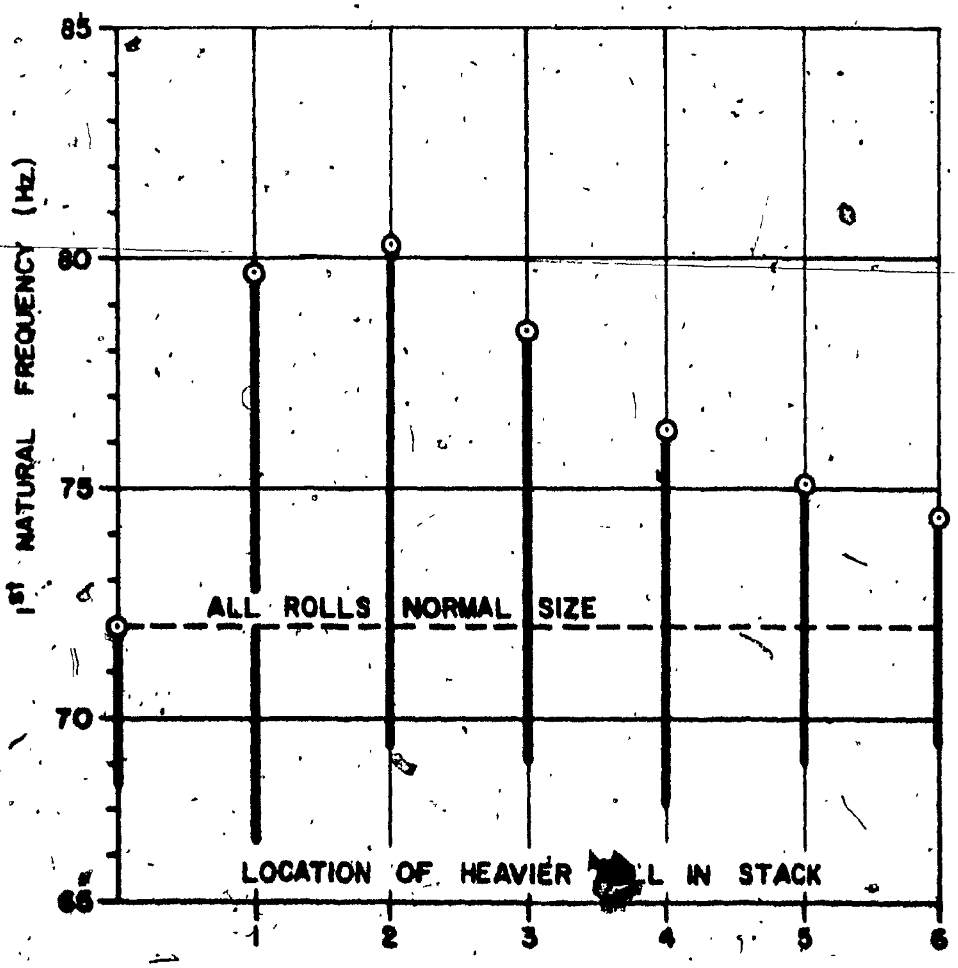


Figure 23

The effect on the first three natural frequencies is small but it increases with the order of the modes. The effect of

4.1 Evaluation of the Mathematical Model (cont'd).

changing the diameter of a roll on the first natural frequency of the calender stack depends on the location of the roll in the stack, as illustrated in the diagram above. The degree of the influence decreases as the roll is placed deeper and deeper in the stack. This is what would be expected intuitively since the deeper the roll is in the stack the more constrained it is and the less influence it can exert on the stack. While the results produced by the solution of the proposed mathematical model of the calender stack on the whole seem to be realistic and reliable they do exhibit one anomaly which, although it is not considered significant is of interest and as such deserves attention. It will be discussed next.

It appears that the mathematical solution will produce two zero frequency modes per each upper roll whose ends are not constrained. The corresponding mode shapes are those representing rigid body motion. In each case the bottom roll whose ends are constrained remains stationary. While it is not intended to assess formally the reason for this portion of the results, from the nature of the anomaly it would appear that the approximation introduced in evaluating the flexural potential energy in the upper rolls by regarding them as free-free rolls is the reason for the phenomenon. In the process leading to the constraint matrix which relates the absolute displacements of the elements of a free-free roll to their displacement relative to the rigid body motion two rigid body modes are assumed and imposed. The eigenvalue problem resulting from the mathematical model carries in it the information about the possibility of the rigid body motion and the algorithm used to solve the eigenvalue problem searches for and forces those zero frequency solutions. That the solutions are fictitious can be seen from the mode shape diagrams by inspection. Two of the zero frequency modes have been plotted in Figure 21 in Chapter 3. In those and in all the other zero frequency modes all of the upper rolls undergo rigid body motion simultaneously while the bottom roll remains stationary. From a qualitative consideration of the spring forces involved in the physical model of the calender stack it will be apparent that the bottom roll cannot remain motionless while one of the upper rolls moves in any manner and that all the upper rolls cannot undergo rigid body motion simultaneously. In fact two adjacent rolls cannot have rigid body motion simultaneously. As it was pointed out earlier the above anomaly is a result of expressing the flexural potential energy in the upper rolls as that of flexed free-free rolls. This approach greatly simplified the computation of the flexibility influence coefficients for the system but introduced an approximation and resulted in the fictitious modes discussed. In view of the nature of the coupling between adjacent rolls (rolls with free ends stacked on one another, 4.1 Evaluation of the Mathematical Model (cont'd).

non-linear springs between them) a more involved approach is not considered justified or likely to be less approximate.

In summary the modal analysis of the calender stack when applied to an operational prototype produced results which are in good agreement with available observational data of calender stack behaviour in general. The effect of the various geometric and other stack parameters as computed by the modal analysis program appear to be correct when judged on the basis of basic physical principles which govern the behaviour of vibrating systems. A more rigorous quantitative assessment of the analysis will be possible when it is applied to a calender stack of accurately known characteristics.

APPENDIX A

COMBINED PAPER STIFFNESS MATRIX.

The diagram below illustrates the physical model of a two dimensional, six-roll, five-nip calender stack. As an example the mass of each roll has been lumped into four point masses. Ideal springs are assumed to represent the resilience of the paper passing through the nips.

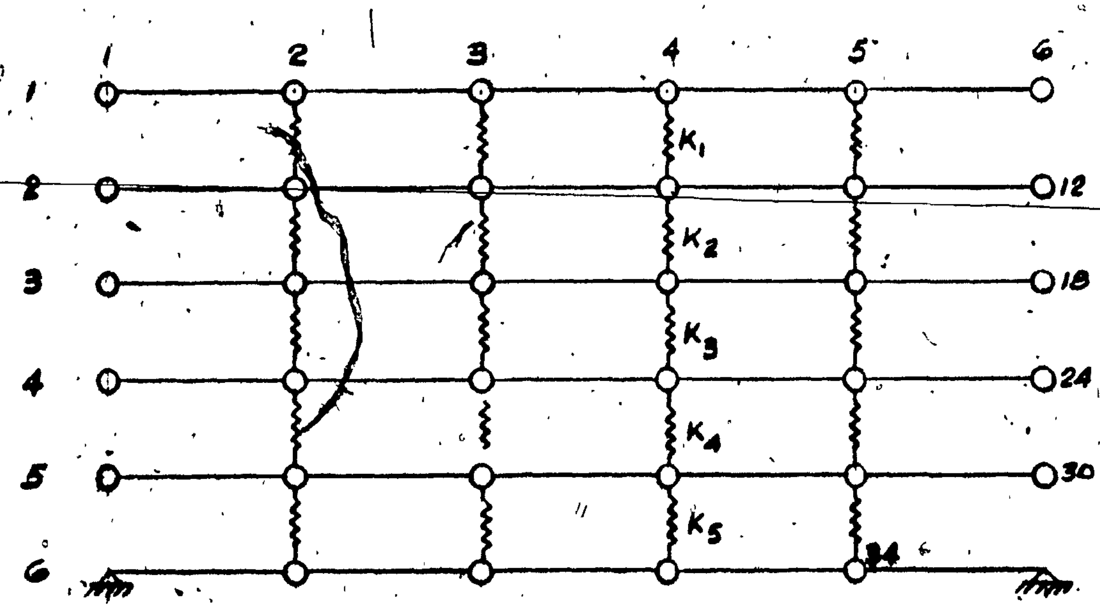


Figure 24

Each lumped mass of a roll, except the end ones which represent the journal and bearing housing masses and are not normally in contact with the paper, is coupled through ideal springs to its counterparts of the adjacent rolls. The potential energy of the above system due to the deflection

Paper Stiffness Matrix (cont'd)

of the ideal springs alone can be expressed as follows

$$VS = 0.5(KS[(D_2 - D_8)^2 + (D_3 - D_9)^2 + (D_4 - D_{10})^2 + (D_5 - D_{11})^2] +$$
$$+ KS[(D_8 - D_{14})^2 + (D_9 - D_{15})^2 + (D_{10} - D_{16})^2 + (D_{11} - D_{17})^2] +$$
$$+ KS[(D_{14} - D_{20})^2 + (D_{15} - D_{21})^2 + (D_{16} - D_{22})^2 + (D_{17} - D_{23})^2] +$$
$$+ KS[(D_{20} - D_{26})^2 + (D_{21} - D_{27})^2 + (D_{22} - D_{28})^2 + (D_{23} - D_{29})^2] +$$
$$+ KS[(D_{26} - D_{31})^2 + (D_{27} - D_{32})^2 + (D_{28} - D_{33})^2 + (D_{29} - D_{34})^2])$$

It can be shown that in matrix form

$$VS = 0.5(D)^T[KS](D)$$

and

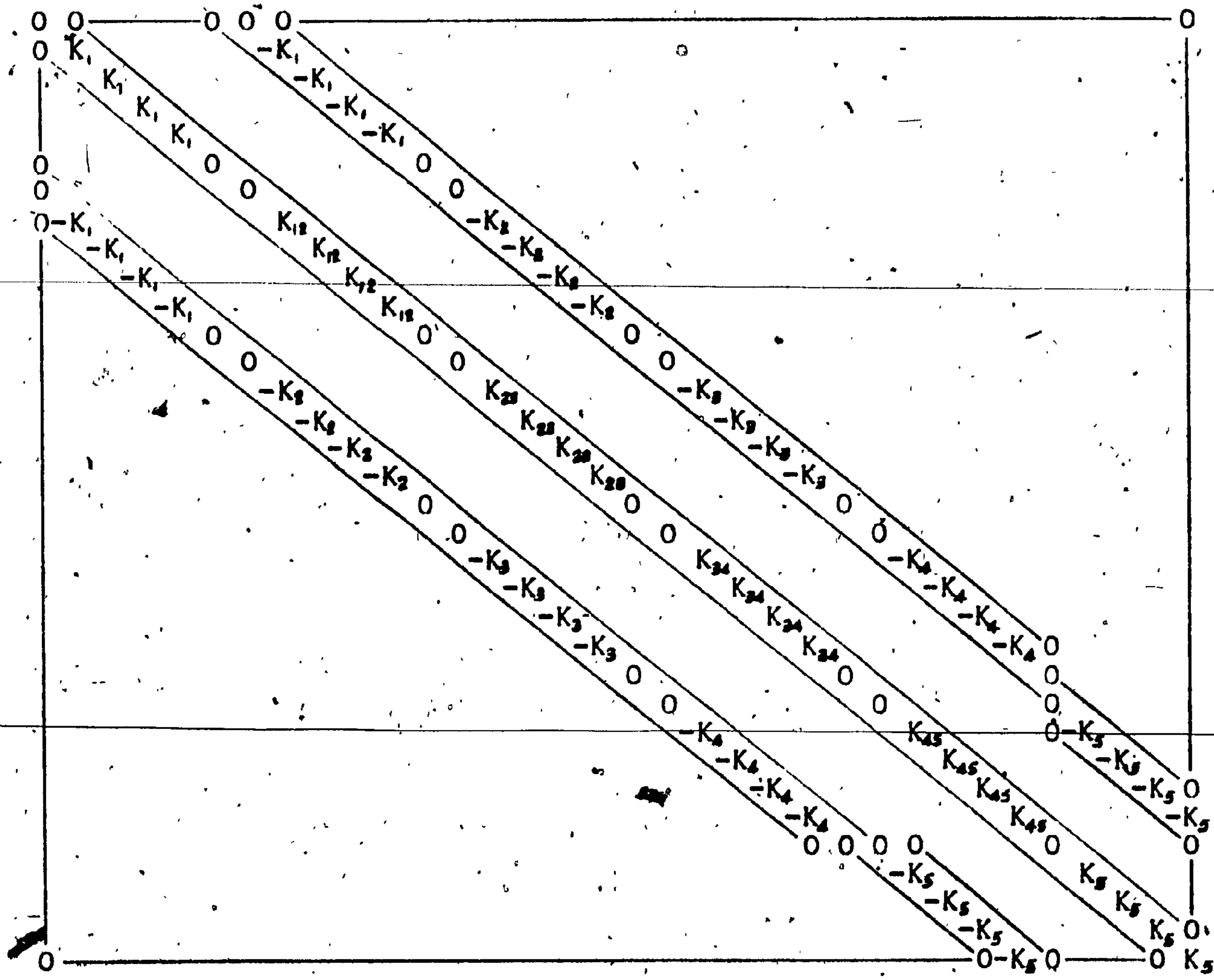
$$(\partial VS/\partial D_i) = [KS](D)$$

Performing the differentiation leads to

$$\begin{pmatrix} K_1D_2 - K_1D_8 \\ K_1D_3 - K_1D_9 \\ K_1D_4 - K_1D_{10} \\ K_1D_5 - K_1D_{11} \\ 0 \\ 0 \\ -K_1D_2 + (K_1 + K_2)D_8 - K_2D_{14} \\ \cdot \\ \cdot \\ \cdot \\ -K_4D_{23} + (K_4 + K_5)D_{29} - K_5D_{34} \\ 0 \\ 0 \\ -K_5D_{26} + K_5D_{31} \\ -K_5D_{27} + K_5D_{32} \\ -K_5D_{28} + K_5D_{33} \\ -K_5D_{29} + K_5D_{34} \end{pmatrix} = [KS](D)$$

from which [KS] may be constructed as follows.

Paper Stiffness Matrix (cont'd)



The diagonal terms K_{12}, K_{23}, K_{34} and K_{45} correspond to $(K_1 + K_2)$, $(K_2 + K_3)$, $(K_3 + K_4)$ and $(K_4 + K_5)$ respectively. The paper stiffness matrix [KS] is generated in the first segment of the computer program (described in appendix D) from the input data, namely the number of rolls NR, the number of lumped point masses per roll NN and the paper stiffness constants $K_1, K_2, \ldots, K_n$ corresponding to the (NR-1) nips in the calender stack.

APPENDIX B

DETERMINATION OF PAPER STIFFNESS

Several investigators of the calender barring problem on newsprint machines represented the paper being calendered as an ideal, linear spring and attempted to determine the best value for its stiffness [Ref. (8),(14),(23) and (24) in Chapter 1]. In particular Wahlstrom, Crouse and Davidson obtained numerical values for the spring constant as a function of nip loading. Their results have been plotted in Figure 25. The nip load and the paper stiffness are expressed on the graph as quantities per unit length of the nip and sheet width respectively. In order to obtain the total spring constant attributable to the entire sheet the value read from the ordinate has to be multiplied by the width of the paper sheet. Crouse assigned a single value to the constant assuming it independent of nip load. Wahlstrom used experimental means of determining the values of the spring constant over a wide range of nip loads. Davidson used an empirical method, based on work done by Mardon et al. (21) to obtain values for the spring constant. The method involved the assumption that a good approximation of the spring constant of the paper as it passes through the nip can be obtained by taking 25% of the paper caliper after a nip as the magnitude of the elastic deformation which the paper underwent as it passed through that nip. The ratio of the nip load to this elastic deformation is then taken as the effective spring constant at that nip. The results of

Wahlstrom and Davidson agree reasonably well in the higher nip load range but differ considerably at lighter nip loads.

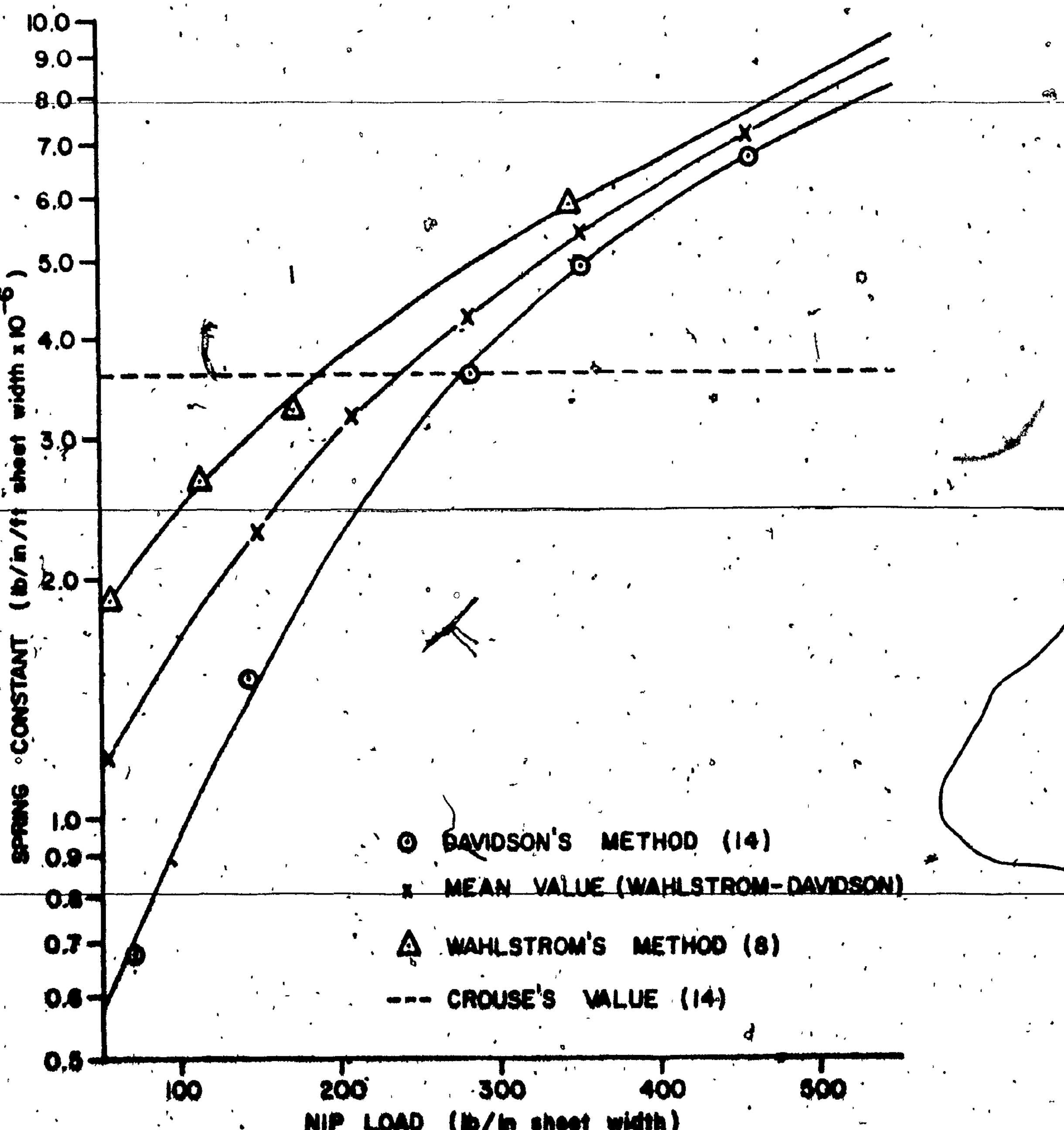


Figure 25

Determination of Paper Stiffness (cont'd)

This is quite unfortunate since the loads at the two uppermost nips where most of the barring deformation takes place fall in the range where the two investigators differ the most.)

In the computations presented here the numerical values used to represent the paper stiffness were obtained by taking the average of the results of Wahlstrom and Davidson. There was no justification for this choice other than that there was no reason to consider the findings of one investigator more reliable than those of the other and it was outside the scope of this work to determine newsprint paper stiffness independently. For each nip the value of the paper spring constant is obtained by computing the total weight of the rolls resting on the nip and determining from Figure 25 the spring constant corresponding to that nip load. The obtained value represents the total spring effect of the paper passing through the nip and is expressed in lbs/in per foot of sheet width. This figure is then multiplied by the face width in feet and the resulting total spring constant expressed in millions of lbs/in is used as input to the computer program. In the program this value is converted into lbs/in and is divided by the number of point masses that represent each roll in order to lump the paper stiffness.

APPENDIX C

CONSTRAINT MATRIX FOR THE UPPER ROLLS

The upper rolls are assumed to be capable of rigid body motions such as translation and rotation as well as elastic deformation. Since the kinetic energy of such a system depends on the absolute motion whereas its potential energy is a function of elastic deformations alone, it is necessary to relate the absolute motion of the roll to the motion of its particles relative to the rigid body motions. Using an approach described in Ref. (27) the two motions can be related for a lumped system, such as the one below as follows.

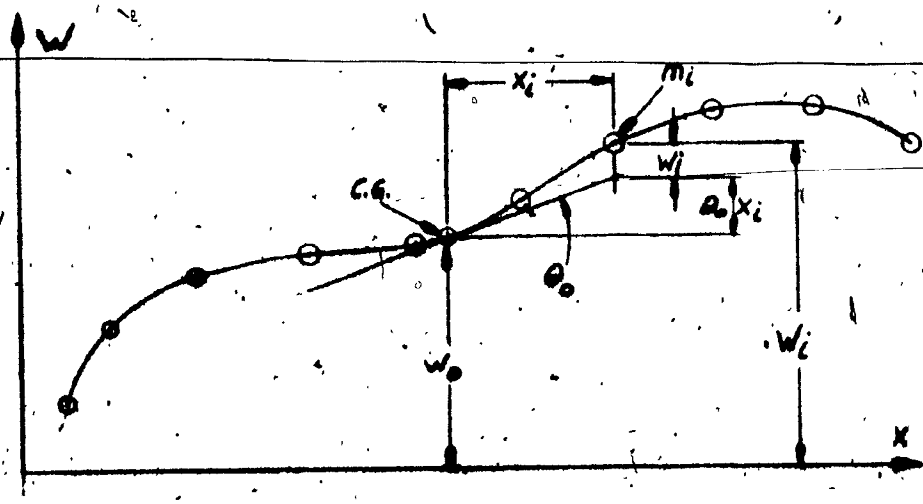


Figure 26

Assume that all displacements are sufficiently small so that the center of mass does not change relative to the beam and no longitudinal motion of the masses is caused by rigid body

Constraint Matrix, Upper Rolls (cont'd)

rotation. The absolute displacement of a point can be written as

$$W_i = w_i + \theta_o x_i + w_o$$

where w_o and θ_o are rigid body motions and w_i is the elastic displacement of point i when regarding the beam clamped at the center of mass. W_i represents the absolute displacement of point i. In matrix form

$$(W) = w_o \; (1) + \theta_o \, (x) + (w) \qquad (1)$$

If the two rigid body modes are

$$(W)_1 = w_o \, (1)$$

and $\qquad$ (2)

$$(W)_2 = \theta_o \, (x)$$

then from the orthagonality relationship of natural modes

$$(W)_1^T \, [m] \; (W)_2 = 0$$

and $\qquad$ (3)

$$(W)_2^T \, [m] \; (W)_2 = 0$$

Substituting (1) into equations (2) and (3) gives

$$w_o (1)^T [m](1) + \theta_o (1)^T [m](x) + (1)^T [m](w) = 0$$

$$(4)$$

$$w_o (x)^T [m](1) + \theta_o (x)^T [m](x) + (x)^T [m](w) = 0$$

Constraint Matrix, Upper Rolls (cont'd)

where

$$(1)^T[m](1) = \mathrm{sum}(m) = M$$

$$(1)^T[m](x) = (x)^T[m](1) = 0 \qquad (5)$$

$$(x)^T[m](x) = \mathrm{sum}(m_i\, x_i^2) = I$$

then from equations (4) and (5)

$$w_o = -\ (1/M)\ (1)^T[m](w)$$

and (6)

$$\theta_o = -\ (1/I)\ (x)^T[M](w)$$

Equation (1) can then be rewritten as

$$(W) = -\ 1/M(1)(1)^T[m](w) - 1/I\ (x)(x)^T[m](w) + (w)$$

or

$$(W) = ([I] - 1/M(1)(1)^T[m] - 1/I\ (x)(x)^T[m])(w) \qquad (7)$$

or

$$(W) = [c](w) \qquad (8)$$

Where [c] is the constraint matrix relating the absolute motion to the motion relative to the rigid body motion

$$[c] = ([I] - 1/M(1)(1)^T[m] - 1/I\ (x)(x)^T[m]) \qquad (9)$$

FLEXIBILITY INFLUENCE COEFFICIENTS FOR UPPER ROLLS

For the computation of the flexural potential energy the upper rolls are assumed to be unconstrained. This essentially amounts to assuming that the mode shape of the vibrating roll will not be significantly different from that of a free-free roll. The flexibility influence coefficients are not defined for such a roll. The determination of the stiffness influence coefficients on the other hand is a cumbersome process. For small displacements where the position of the center of mass of the roll does not change relative to the roll the elastic displacements of the roll particles and therefore the flexibility of the influence coefficients can be derived by considering the free-free roll clamped at the center of gravity as shown below.

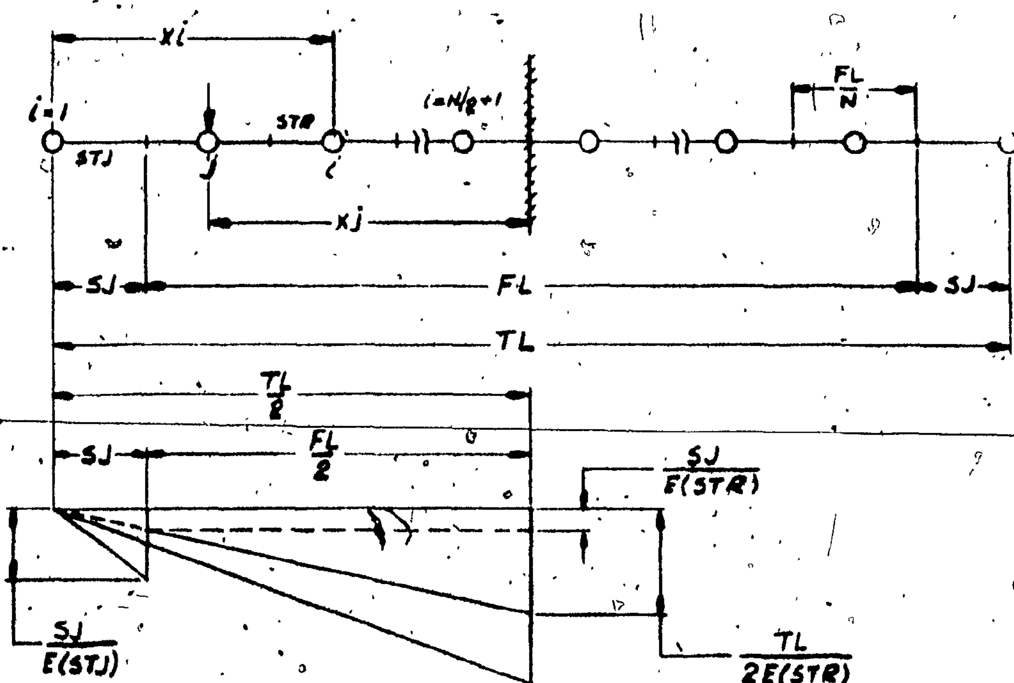


Figure 27

Flexibility Coefficients, Upper Rolls (cont'd)

The parameters not identified in the diagram but used in the following discussion can be defined as follows.

N = number of lumped roll masses

SGL = FL/N

STR = moment of inertia of roll cross sections

STJ = effective moment of inertia of journal sections

E = modulus of elasticity of roll material

[SIF] = flexibility matrix for roll

Applying unity load at the lumped mass at $i = 1$ and using the moment-area method the flexibility influence coefficient SIF(1,1) is obtained as follows.

$$SIF(1,1) = (1/E)((SJ/STJ)(SJ/2)(2SJ/3) + (1/STR)[(TL/2 - SJ)(FL/4)(2FL/6 + SJ) + (SJ)(FL/2)(FL/4 + SJ)])$$

$$= (1/E)((SJ^3/3STJ) + (FL/STR)[(FL/24)(TL + SJ) + SJ/8)(TL + 2SJ)])$$

Using the equation of the elastic curve for a cantilever beam the flexibility influence coefficients can be obtained for the left half of the roll as follows.

For $j = 1$ and $1 \le i < (N/2)$, load at j, deflection at i

$$SIF(i,1) = (1/E)([1/(6STR)][xi^3 - 3(TL/2)^2 (xi) + TL^3/4])$$

$$= (1/E)([1/(24STR)][4xi^3 - 3(TL)^2 (xi) + TL^3])$$

Flexibility Coefficients, Upper Rolls (cont'd)

where

$$xi = SJ + SGL/2 + (i - 2)(SGL)$$

$$= SJ + (SGL)(2i - 3)/2 .$$

For $j>2$ and $2<i<(N/2)$, load at j, deflection at i

$$SIF(i,j) = (1/E)([1/(6STR)][xi^3 - 3(xj)^2 (xi) + 2(xj)^3])$$

where

$$xi = (i - j)(SGL)$$

$$xj = (1/2)[FL - (SGL)(2J - 3)] .$$

In this manner the diagonal and one half of the off-diagonal elements of the flexibility matrix for the left half of the roll are computed. The remaining off-diagonal elements are obtained from the Maxwell reciprocity relationship. The flexibility matrix of the right hand half of the roll can be obtained from symmetry since

$$SIF(1,1) = SIF(6,6)$$

$$SIF(1,2) = SIF(6,5) \quad \text{etc.}$$

The resulting flexibility matrix for the roll with eight lumped masses takes the form shown below

A	b	c	d	0			0
B	E	f	g				
C	F	H	i				
D	G	I	J	0			0
0			0	J	i	g	d
				i	h	f	c
				g	f	e	b
0			0	d	c	b	a

Flexibility Coefficients, Upper Rolls (cont'd)

The elements in the upper case notation are the only ones computed from first principles.

FLEXIBILITY INFLUENCE COEFFICIENTS FOR BOTTOM ROLL

The bottom roll is assumed to be simply supported. The absolute and elastic displacements of such a roll are identical and the flexibility coefficients are computed readily using the moment-area method. The diagram below illustrates the model and identifies some of the variables used in the computations.

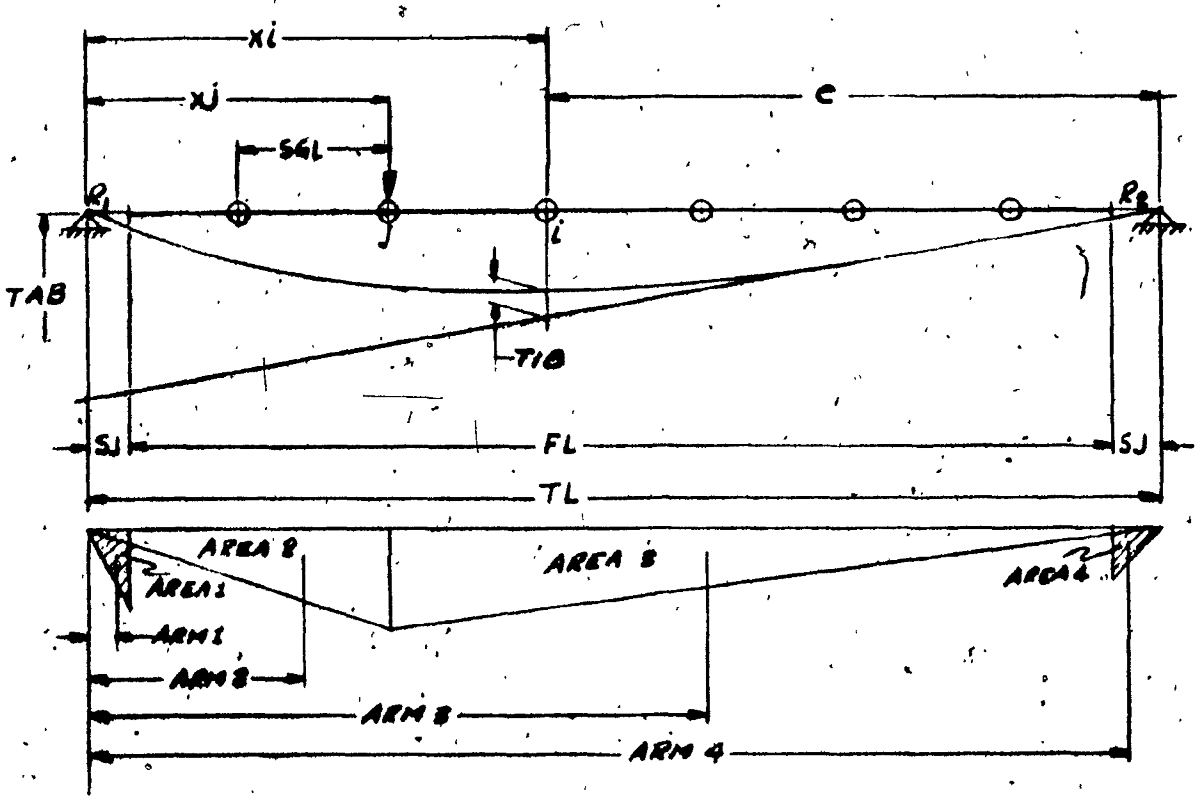


Figure 28

From the diagram the deflection at point i due to the load at point j is given by

$$y = (e/TL)(TAB) - TIB$$

Flexibility Coefficients, Bottom Roll (cont'd)

where

e = TL - xi

Assuming a unit load at j the reaction forces at the supports can be computed. To obtain TAB, area-moments are taken about R1. TAB is then given by

TAB = (area1)(arm1) + ... + (area4)(arm4) .

The following are the intermediate computations.

R1 = 1 - xj/TL

R2 = xj/TL

area1 = (1/E)[(SJ)(R1)/(STJ)][SJ/2]

= (1/E)[(R1)(SJ)^2/(2STJ)]

arm1 = 2SJ/3

area2 = (1/E)[(R1)(xj + SJ)(xj - SJ)]/(2STR)

arm2 = [(SJ + 2xj)(xj - SJ)]/[3(xj + SJ)] + SJ

area3 = (1/E)[(R1)(xj) + (R2)(SJ)][TL - xj - SJ]/(2STR)

arm3 = [2(R2)(SJ) + (R1)(xj)][TL - xj - SJ]/((3)[(R1)(xj) + (R2)(SJ)]) + xj

area4 = (1/E)[(R2)(SJ)^2]/(2STJ)

arm4 = TL - SJ/3

To obtain TIB area-moments are taken about i as shown in Figure 29.

Flexibility Coefficients, Bottom Roll (cont'd)

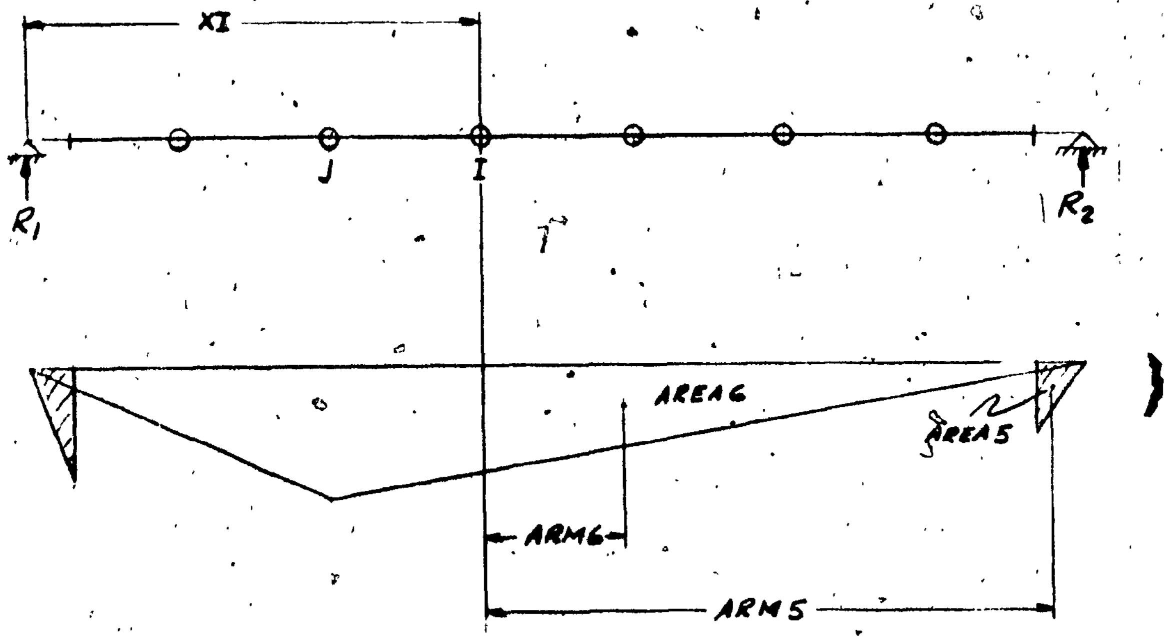


Figure 29

TIB is then given by the equation

TIB = (area5)(arm5) + (area6)(arm6) . . .

The following intermediate steps are involved

area5 = area4

arm5 = [TL - xi - 2SJ/3]

area6 = (1/E)[R2(TL + SJ - xi)][TL - xi - SJ]/(2STR)

arm6 = (TL + 2SJ - xi)(TL - xi - SJ)/[3(TL + SJ - xi)]

The desired flexibility influence coefficient for point i is

SIF(i,j) = (TL - xi)(TAB)/TL - TIB .

As in the case of the upper rolls the presence of symmetry

Flexibility Coefficients, Bottom Roll (cont'd)

is used to reduce the number of computations. Half of the diagonal elements and part of the off-diagonal elements of the flexibility matrix are computed. The rest of the elements are obtained from symmetry and from the Maxwell reciprocity relationship. Using a simply supported roll with six lumped masses as an example the following flexibility matrix is obtained.

A	b	c	d	e	f
B	G	h	i	j	e
C	H	K	l	i	d
D	I	L	k	h	c
E	J	i	h	g	b
F	e	d	c	b	a

The upper case characters represent the elements computed from first principles.

APPENDIX D

COMPUTER PROGRAM

General Description

The program has been written in the Fortran IV language. With the storage requirement for 23 arrays and with the associated subroutines the program requires 26K words of memory for loading. It requires a software package called by the acronym EISPACK which can solve the eigenvalue problem for six classes of matrices. In particular it solves the class of real general matrices which is the requirement in this application. There is no size limitation to the matrix that EISPACK can solve. However, the present dimension statements in the program limit the size of calender system matrices to 50x50. The dimension statements also limit the inertia, stiffness and constraint matrices to a maximum size of 12x12. These limitations are only for the purpose of memory economy and can be altered readily if the need arises. The total computing time of a seven-roll calender stack with six lumped masses per roll leading to the eigen solution of a 40x40 matrix is 36 seconds on the CDC 6000 computer. There are two computing options built into the program. Depending on the value of an index (LCH) which is one of the input data to the program it will compute the eigen solution of either the entire calender stack system or that of the Nth roll from the top treating the upper rolls as free-free and the lowermost one as simply supported. There are three output options available depending on how many intermediate results are to be printed.

Input-Output Features

The input data required for calender stack sizes of up to eight rolls are contained on seven data cards. The first card contains five integer data, namely NN, NR, LCH, IP, and NP. These define the number of lumped masses per roll, the number of rolls in the stack, the computation option, the output option and the number of the first page of the eigen solution output respectively. The use of the latter two are defined in comment statements in the source listing of the program. The first four data are read from I2 fields. The last one is contained in an I4 field.

The next three data cards contain the journal, outside and inside diameters (inches) of the rolls in the calender stack respectively. If a roll is not bored, as most rolls are not, its inside diameter is entered as zero. On each card the data are contained in F10.3 fields and read in the sequence of the corresponding rolls beginning with the uppermost roll. Data for up to eight rolls can be entered on each card.

From the fifth data card the combined weight (lbs) of one journal and one bearing housing for each roll is read. The data are again contained in F10.3 fields and start with the uppermost roll.

Computer Program (cont'd)

The sixth data card contains the estimated total spring constants (in millions of lbs/in) each of which represents the elastic behaviour of the paper passing through a nip. The values on the data card are entered in F10.3 fields in the sequence of the nips to which they correspond starting with the uppermost nip.

The last data card contains four real variables which are the total width of the stack measured between bearing centers (inches), the face width of the calender rolls (inches), the density of the roll material (lbs/cuin) and finally the value of Young's modulus for the roll material (millions of lbs/sqin). These data are entered in fields of F6.1, F6.1, F6.3 and F5.1 respectively.

The amount of output from the program is governed by the value assigned to the index IP. There are three output options for the printing of various amounts of intermediate results. The final results representing the eigen solution of the calender stack are printed in ascending order of the natural modes. The eigenvalue, the natural frequency (cps) and the normalized eigenvector representing the modal shape of the calender stack are printed out for each mode.

APPENDIX E

BIBLIOGRAPHY
Calender Barring

(1) A.T. EDWARDS — E.B. EDDY CO. VIBRATION PROBLEM
Private Communication, 1950

Experimental investigation of calender stack vibration at the E.B. Eddy Plant, a manufacturer of high grade paper. Vibration levels and frequencies were measured on and around a 146" wide calender stack. The effect of calender speed on vibration frequencies was also investigated.

(2) G.R. EWAN — BARRING ON CALENDER ROLLS
Private Communication, 1953

A brief summary of observations and opinions regarding the barring, regrinding and checking of calender rolls is presented. Usefull hints for roll grinding and checking procedures are given.

(3) G. EWING TAIT — METALLURGICAL FACTORS AFFECTING CALENDER ROLL PERFORMANCE
Pulp & Paper Mag. Canada, May 1959

The nature and the method of manufacture of chilled iron rolls is described. Some data relating to the properties of chilled iron rolls and the work hardening of calender rolls in service are presented.

(4) B.I. HOWE, J.E. LAMBERT — AN ANALYSIS OF THE THEORY AND OPERATION OF HIGH SPEED CALENDER STACKS
C.P.P.A. Tech. Sect. Proc. 305-6, 1961

A comprehensive analysis of the calendering process and its effects on paper quality, including a brief review of the phenomenon of barring. The effects of many operating factors are discussed.

(5) NEWSPRINT COMMITTEE C.P.P.A. — BREAKER STACKS AS RELATED TO BARRING
Summary of questionnaire issued by Newsprint Committee - C.P.P.A., 1962

Thirty newsprint machines in Canada were surveyed to investigate the nature

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and causes of the calender barring problem.

(6) W.H. CUFFEY, G. INGRAM — CYCLIC MACHINE DIRECTION BASIS WEIGHT VARIATION. Pulp & Paper Mag. Canada, April 1962

Experimental investigation of wet-end causes of basis weight variation and barring is reported. The effect of the vibration of wet-end components such as headbox, lead roll, table rolls and Fourdrinier wire on basis weight uniformity was investigated. On experimental and production paper machines wet-end disturbances such as pressure waves in the headbox were introduced artificially.

(7) I.T. PYE — CALENDER BARRING IN NEWSPRINT. Pulp & Paper Mag. Canada, 1963, T194

Extensive investigation of the phenomenon of barring of calender rolls and newsprint on the nine Powell River machines of the MacMillan, Bloedel and Powell River Company. Spectrum analyses of basis weight and caliper profiles and calender vibrations were carried out. Records of roll history were kept over a long period. Many details of the barring phenomenon are presented.

(8) P.B. WAHLSTROM, K.O. LARSSON, C.A. ASKLOF — CALENDER BARRING, ITS MECHANISM AND POSSIBLE ELIMINATION. Pulp & Paper Mag. Can. 1963, T205-12

Experimental investigation of calender stack vibration on a 264 inch wide 2000 fpm. newsprint machine. Vibration amplitudes and frequencies were measured at several points along three of the six calender rolls in the machine speed range of 1800-2100 fpm. In a theoretical analysis the calender stack was modeled as a one dimensional spring-mass system to obtain the frequencies and amplitudes of the normal modes of vibration. A device successfully applied to eliminate barring is described.

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(9) T. MATOMAKI — BAR MARKING CAUSED BY CALENDER STACKS OF NEWSPRINT PAPER MACHINES
In Finnish, Paperi Puu 45, No. 7
Jan. 1963

A short review and qualitative explanation of the barring problem based on observation of the problem on various newsprint machines. Division of the stack into smaller units is suggested as a possible means of avoiding barring.

(10) B.I. HOWE, J.C. COSGROVE — CALENDER STACK BARRING ON NEWSPRINT MACHINES
Pulp & Paper Mag. Can. 1963, T259-74

A very comprehensive examination of all aspects of newsprint and calender roll barring. Barring terminology and a barring intensity scale are defined. The operating conditions in twenty seven newsprint mills were surveyed. Based on experimental data a mechanism for both paper and calender roll barring is postulated. Methods to avoid or to eliminate barring are proposed.

(11) W.H. CUFFEY — NEWSPRINT CALENDER VIBRATION AS IT AFFECTS MACHINE-DIRECTION CALIPER UNIFORMITY
Pulp & Paper Mag. Can. 1963, T379

An experimental investigation of the barring phenomenon was carried out on a newsprint machine in the machine speed range of 1900-2200 fpm. Bar wavelength, sheet caliper, stack vibration and noise measurements were made. Empirical correlation between machine speed and barring wavelength and frequency was established. Remedial actions and their effect are described. A qualitative hypothesis for the mechanism of barring is proposed.

(12) W. MULLER-RID, M. SCHADLER, A. STARK, H. FAISS — A CONTRIBUTION TO EXPLAIN THE APPEARANCE OF BARRING ON HIGH SPEED PAPER MACHINES
In German, Papier 17, No. 5, May 1963

Experimental data are analysed to

determine the cause of paper barring. A qualitative explanation of the underlying mechanism is given tracing the problem to incoming sheet thickness variations caused by sheet forming conditions. The mechanism of sheet thickness variation produced by headbox lip vibration is quantitatively analysed.

(13) J.R. PARKER — CORRUGATION OF CALENDER ROLLS AND THE BARRING OF NEWSPRINT
Paper Technology 6 No. 1, Feb. 1965

The barring of calender rolls was measured with a precision curvature gage. Empirical correlation between roll bars and paper bars was established. The calender stack was modeled as a one dimensional mass-spring vibrating system. The analysis produces the frequencies and amplitudes of the normal modes of vibration. The vibrating motion of a single mass and spring system with base excitation is analysed to show how roll barring occurs, increases and migrates around the circumference of the roll.

(14) R.W. DAVIDSON — A STUDY OF CALENDER BARRING THROUGH ANALOG SIMULATION
Thesis, University of Maine, 1966

A mathematical model of a five-nip calender stack was constructed and studied on an analog computer. The model consisted of masses, springs and dampers. Both sinusoidal and random excitations of the model were employed. The mathematical model, the analog computer program and the results of the simulation are described in detail. The calender stack studied was the same as that in (8).

(15) W.C. NOTBOHM — EDGE EFFECT SEEMS TO BE TRIGGERING MECHANISM IN CALENDER BARRING
Pulp & Paper, November 1967

Operating evidence is presented to support the hypothesis that the thick edges of the incoming sheet cause

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periodic roll bouncing and thus paper and roll barring. Remedies are proposed.

(16) K.I. MUMMIE
T.L. TUTTLE

CALENDER VIBRATION - A SIMULATION STUDY AND A CURE
Tappi, ol. 52, No. 7, July 1969

A one dimensional model of masses, springs, and dampers was studied on a hybrid computer. The simulation allowed for the feedback effect of the bars travelling down the calender stack. The calender modeled was the same as that studied in (8). A tunable calender stack using an adjustable nip-out roll is recommended to eliminate barring.

(17) J. SAKURAI

STUDY OF MACHINE CALENDER FOR BETTER PERFORMANCE
Japan Pulp & Paper, No. 3, Oct. 1970

A review of various aspects of calender operation including the problem of calender vibrations is presented. Several remedies are discussed. Off-setting of rolls is discussed in most details.

(18) B.F. VALEEV

VIBRATIONS OF CALENDER ROLLS AND MEANS FOR THEIR PREVENTION
In Russian, Bumazh-Prom. No. 8, 19-21 Aug. 1971

A review of the various existing theories about calender vibration is presented. Concludes that none explain all aspects of the problem satisfactorily. Offers possible solution through damping the motion of the upper roll.

(19) E.J. JUSTUS

TODAY'S NEW CALENDER STACKS
Tappi, June 1972

The relationship between paper quality and calender characteristics is discussed. The nature of calender vibration is briefly discussed.

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Compressibility of Paper

(20) G.E. MACKIN, E.L. KELLER, P.K. BAIRD — EFFECT OF CALENDERING PRESSURE ON SHEET PROPERTIES
Technical Assoc. Papers 1941 Series 24

Extensive tests on book, bond and kraft paper grades were carried out in order to determine the effect of calendering pressure on the physical properties of the paper produced.

(21) R.L. BLANCHARD, J. MARDON, R.E. MONOHAN, R.J. QUINT, J.E. WILDER — THE CHANGE OF PAPER PROPERTIES THROUGH MACHINE CALENDER STACKS PARTS 1 & 2
Pulp & Paper Mag. Can. 1964, 65 (11)

Extensive measurements of the physical properties of newsprint from four paper machines were made to establish the change in these properties from nip to nip in the calender stack. In particular the compressibility of the paper at various stages of calendering was determined.

(22) J. MARDON, R.E. MONOHAN, R.A. CARTER, J.E. WILDER — DYNAMIC CONSOLIDATION OF PAPER DURING CALENDERING
Transactions of the symposium on the dynamic compressibility of paper
Cambridge, Sept. 1965

Two dynamic compressibility tests are described. The relationship between static and dynamic compressibility through the calender stack and the dwell time/pressure relationship for caliper reduction is given for three different grades of paper.

(23) D.L.T. CHAPMAN, J.D. PEEL — CALENDERING PROCESSES AND THE COMPRESSIBILITY OF PAPER, PART 1
Paper Technology Vol. 10, No. 2, 1969

Measurements of change in paper properties due to compression were made. The experiments involving the compression of single sheets are described. Empirical relationships described. Empirical relationships between applied load, dwell time, initial, under load and recovered thicknesses are given.

(24) V. COLLEY, J.D. PEEL — CALENDERING PROCESSES AND THE COMPRESSIBILITY OF PAPER, PART 2
Paper Technology, October 1972

The effects of moisture content, temperature, duration and magnitude of pressure on the under-load and recovered thicknesses and on the roughness and optical properties of the sheet were investigated.

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(26) W.T. THOMSON — Vibration Theory and Applications
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(27) L. MEIROVITCH — Analytical Methods in Vibrations
The MacMillan Company (1967)

(28) I. SOKOLNIKOFF, R.M. REDHEFFER — Mathematics of Physics and Modern Engineering
McGraw Hill Book Co. (1958)

(29) C.M. HARRIS, C.E. CREDE — Shock and Vibration Handbook
McGraw Hill Book Co. (1961)