

**COMPUTER-AIDED OPTIMIZATION OF A RC - ACTIVE BAND-PASS FILTER**

**Paraskevas Nikolaou Prokopoulos**

**A Major Technical Report  
in  
The Department  
of  
Electrical Engineering**

**Presented in Partial Fulfillment of the Requirement  
for the degree of Master of Engineering at  
Concordia University  
Montréal, Québec  
Canada**

**October, 1983**

**© Paraskevas Nikolaou Prokopoulos, 1983**

ABSTRACTCOMPUTER - AIDED OPTIMIZATION  
OF A RC-ACTIVE BAND-PASS FILTER

Paraskevas Nikolaou Prokopoulos

An ever increasing demand for high circuit density to decrease the size and costs of electronic systems, such as space electronic circuits, has become a major concern of the active filter designers. Several RC-active filter designs have been proposed in the literature in an attempt to meet this demand.

A new RC-active filter design using unity gain-amplifiers is presented in this report. The design realizes a second-order band-pass filter with good stability and sensitivity properties.

The total capacitance and total resistance are minimized in the design through computer optimization procedures. Therefore, the required substrate area for integrated circuit-fabrication is also minimized. Non-linear programming techniques for the minimization algorithms are employed.

Extensive simulation results have been provided to illustrate the validity of the optimization techniques.

## ACKNOWLEDGMENTS

I would like to thank Dr. B.B. Bhattacharyya for his warm interest in this study. His guidance and valuable assistance in order to accomplish this work have been of great value.

I would also like to thank Dr. B.B. Bhattacharyya for suggesting this interesting topic and for encouraging me with his enthusiasm and constructive consultation to alleviate all the difficulties faced during the course of the computer simulation work.

My thanks are due also to Professor Chris Charalambou for his assistance in one area of the minimization algorithms. I am grateful to Concordia University Computer Centre for allotting me ample computer time when intensive work was involved.

Finally, I dedicate this work to the memory of my beloved late brother, Gerasimos, and to my parents.

TABLE OF CONTENTS

	Page
ABSTRACT.....	iii
ACKNOWLEDGMENTS.....	iv
LIST OF ABBREVIATIONS.....	xviii
LIST OF TABLES.....	viii
LIST OF FIGURES.....	xi

Chapter

1. INTRODUCTION.....	1
2. ACTIVE FILTERS.....	5
Brief background.....	5
Synthesis methods for RC-active filters.....	7
Active network elements.....	9
3. DESIGN OF A RC-ACTIVE FILTER (BPF).....	19
The proposed band-pass RC-active filter.....	19
Voltage transfer function.....	20
Design-equations.....	23
Sensitivity.....	25
Stability.....	26

4. MINIMIZATION OF $C_T$ AND $R_T$ .....	29
General concepts of optimization.....	29
$C_T$ and $R_T$ .....	30
5. SOME OPTIMAL DESIGNS FOR THE PROPOSED FILTER.....	33
Formulation of the problem.....	33
The optimal design objective function.....	34
The optimal design constraints.....	47
$v_f(\bar{x})$ .....	49
Determination of $A_c$ and $A_r$ .....	50
Thin-film components.....	50
Capacitors.....	50
Resistors.....	52
Range of passive component values.....	53
6. PROBLEM SOLUTION WITH COMPUTER PROGRAMMING...	54
Computer program considerations.....	54
Minimization programs.....	54
Amplitude and phase response programs.....	56
7. CONCLUSIONS AND COMMENTS.....	57

REFERENCES.....	92
APPENDICES.....	93
A. DERIVATION OF THE VOLTAGE TRANSFER FUNCTION OF THE PROPOSED FILTER.....	93
B. FILTER DESIGN EQUATIONS.....	96
C. MINIMIZATION METHODS.....	100
D. $G(\bar{x})$ .....	111
E. $A_c$ AND $A_r$ .....	114
F. COMPUTER-AIDED SOLVING OF THE PROBLEM.....	133

## LIST OF TABLES

## Table

1. Linear active elements used in the active filter design..... 8
2. Filter elements and time constants in terms of the specifications and any two of the other elements.. 24
3.  $Q_0$  and  $\omega_0$  -sensitivities with respect to the passive and active elements..... 27
4. Index of minimization and "plot" computer programs.. 36
5. Expressions of elements and parameters (general)... 39
6. Expressions of elements and parameters used in the minimization programs (Newton's method).... 40
7. Expressions of elements and parameters used in the minimization programs (Fletcher-Powell's method)..... 41
8. Calculated gradients of Fletcher-Powell's minimization algorithm..... 43

9. Optimized component values using Newton's method.....	64
10. Optimized component values using Fletcher-Powell's method.....	65
11. Minimization effects on $H_{sp}$ - Newton's method...	68
12. Minimization effects on $H_{sp}$ - Fletcher-Powell's method.....	69
13. Processes compatible with various types of thin-film materials.....	72
14. Characteristics of some important thin-film materials for resistors.....	73



E1. Some dielectric materials and typical characteristics that can be achieved..... 117

E2. Properties of thin-film materials..... 118

E3. Tradeoff involved in reduction of width to increase resistance..... 122

E4. Specific resistivity of various "Ta" films..... 122

## LIST OF FIGURES

## Figure

- 2.1 Controlled-source model of a differential-input OA..... 10
- 2.2 A typical ideal OA in a non-inverting gain configuration: (a) Actual circuit, (b) Equivalent circuit..... 11
- 2.3 A typical ideal OA in an inverting gain configuration: (a) Actual circuit, (b) Equivalent circuit..... 12
- 2.4 Open-loop gain response versus frequency of a practical OA..... 14
- 2.5 Transfer function characteristics of the practical frequency-dependent OA..... 15

Figure

Page

3.1	Proposed band-pass RC-active filter using unity-gain amplifier.....	19
3.2	Ideal models of UGA,s: (a) a non-inverting GVUGA, (b) construction of UGA in (a).....	20
3.3	Proposed band-pass RC-active filter of figure 3.1 using non-unity-gain amplifiers $K_1$ and $K_2$ .....	21
4.1	Classification of minimization cases in terms of filter parameters and constraints.....	32

## Figure

## Page

6.1	Amplitude v.s. frequency response assuming ideal UGAs and general component values.....	74
6.2	Phase v.s. frequency response assuming ideal UGAs and general component values.....	75
6.3	Amplitude v.s. frequency response assuming non- ideal UGAs and general component values ( $f_0 = 500\text{HZ}$ ).....	76
6.4	Phase v.s. frequency response assuming non-ideal UGAs and general component values.....	77
6.5	Amplitude v.s. frequency response assuming non- ideal UGAs and general component values ( $f_0 = 1000\text{HZ}$ ).....	78
6.6	Phase v.s. frequency response assuming non- ideal UGAs and general component values ( $f_0 = 1000\text{HZ}$ ).....	79

6.7	Amplitude v.s. frequency response assuming non-ideal non-unity gain amplifiers and general component values.....	80
6.8	Phase v.s. frequency response assuming non-ideal non-unity gain amplifiers and general component values ( $A_0 = 150000$ , $K_0 = 1000$ ).....	81
6.9	Phase v.s. frequency response assuming non-ideal non-unity gain amplifiers and general component values.....	82
6.10	Amplitude v.s. frequency response assuming ideal UGAs and "optimized" element values (Newton's minimization algorithm; case: $C_1 = C_2$ )....	83
6.11	Amplitude v.s. frequency response assuming ideal UGAs and "optimized" element values (Newton's minimization algorithm; case: all passive elements are floating, $f_0 = 1000\text{HZ}$ ).....	84
6.12	Amplitude v.s. frequency response assuming ideal UGAs and "optimized element values (Newton's minimization algorithm; case: all passive elements are floating, $f_0 = 1000\text{HZ}$ ).....	85

Figure

Page

6.13 Phase v.s. frequency response assuming ideal UGAs and "optimized" element values (Newton's minimization algorithm; case: all element values are floating (graph I and III),  $C_1 = C_2$  (graph II)).....86

6.14 Phase v.s. frequency response assuming ideal UGAs and "optimized" element values (Newton's minimization algorithm; case: all passive elements are floating).....87

6.15 Phase v.s. frequency response assuming ideal UGAs and "optimized" element values Fletcher- Powell's minimization algorithm; case: all element values are floating (graph I and III),  $C_1 = C_2$ .....88 (graph II)).

6.16 Amplitude v.s. frequency response assuming ideal UGAs and "optimized" passive element values (Fletcher-Powell's minimization algorithm; case: all element values are floating graph I and II),  $C_1 = C_2$  (graph III)).....89

6.17 Amplitude v.s. frequency response assuming ideal unity-gain OAs and "optimized" element values (Fletcher-Powell's minimization algorithm; case: all passive elements are floating).....90

6.18 Phase v.s. frequency response assuming ideal unity-gain OAs and "optimized" element values (Fletcher-Powell's algorithm; case: all passive elements are floating).....91

A<sub>1</sub> Band-pass RC-active filter with non-unity-gain amplifiers (proposed filter, figure 3.3).....93

Figure	Page
E <sub>1</sub> Thin-film capacitor pattern.....	115
E <sub>2</sub> A thin-film resistor pattern (a) Top view, (b) cross-section A-A' .....	119
F Flow charts.....	127
F <sub>1</sub> Main program of minimization using Quasi- Newton's algorithm.....	127
F <sub>2</sub> Function subroutine FUNCT <sub>1</sub> (N,X,F,).....	133
F <sub>3</sub> Function subroutine ZXMIN(FUNCT <sub>1</sub> ,N...,X,G,F).....	135
F <sub>4</sub> Main program of minimization using Fletcher- Powell's algorithm.....	136
F <sub>5</sub> Subroutine program TRANF(X,F,CONT (I),I,...3)....	137
F <sub>6</sub> Function subroutine FUNCT (N,X,F,G).....	138
F <sub>7</sub> Function subroutine FMFP (FUNCT, N,X,F,G,...)....	139
F <sub>8</sub> Organization flow-chart of "PLOT" programs (Comp. programs no. 49-105, per Table-4).....	153



LIST OF SYMBOLS AND ABBREVIATIONS

Symbol or Abbreviation	Description	Page
A or A <sub>0</sub>	Open-loop d.c. gain of the voltage controlled amplifier using ideal OA .....	9
A <sub>i</sub>	Unit-square area of the capacitive or resistive thin-film in integrated-circuit fabrication .....	51
Å	Armstrong, length unit (1Å = 10 <sup>-8</sup> cm) .....	118
A(s)	Open-loop frequency-dependent gain of a practical OA .....	13
A <sub>1</sub>	Aluminum chemical element .....	50
Al <sub>2</sub> O <sub>3</sub>	Aluminum-trioxide .....	118
Au	gold chemical element .....	50
Ac	Total area of the thin-film occupied by the total capacitance .....	32
A <sub>r</sub>	Total area of the thin-film occupied by the total resistance .....	32
A <sub>p</sub> A <sub>2</sub>	constant-multipliers defined in Table-7....	42
B	Zero-dB gain-bandwidth (GB) of the practical OA .....	14
BPF	Band-pass filter .....	19
C or C <sub>i</sub> (i = 1, 2)	Capacitors of the proposed RC-active filter	19
C <sub>r</sub>	Total capacitance .....	25
CCCS	Current-controlled current source .....	8
CCVS	Current-controlled voltage source .....	8
d	distance between electrodes in a capacitive film or thickness of a resistive thin-film of the integrated circuit. ....	51
dB	decibel(s) $10 \log \left( \frac{P_2}{P_1} \right)$ for power ratio and $20 \log \left( \frac{V_2}{V_1} \right)$ for voltage ratio .....	13
d.c.	direct current	13

ABBREVIATIONS AND SYMBOLS CONTINUED

Symbol or Abbreviation	Description	Page
$\epsilon_0$	permittivity of free space.....	51
$\epsilon_r$	Relative dielectric constant.....	51
$\mathcal{E}$	Electric field strength developed between two conducting capacitor plates.....	114
$f_c$ or $f_H$	3-dB cut-off frequency of the practical OA ( $f_c = \frac{\omega_0}{2\pi}$ ).....	13
$f_0$	center frequency of the BPF ( $f_0 = \frac{\omega_0}{2\pi}$ ).....	20
$f$	frequency (Hertz or cycles/sec).....	14
$f(x)$	objective function with respect to the single variable.....	100
$f(\bar{x})$	constrained objective function with respect to the variable-vector.....	29
$f(\bar{x}_i)$	constrained objective function of of $i$ th iteration.....	102
$g(\bar{x})$ or $G(\bar{X})$	gradient-vector of the objective function.....	49
$g_i(\bar{x})$ ( $i=1,2,..m$ )	$i$ th equality-constraint function related to the objective function.....	29
GB	gain-bandwidth of a practical OA.....	13
GVUA	Grounded-voltage unity-gain amplifier.....	19
H	Hessian matrix of a twice differentiable objective function.....	100
$H_i$ or $H^{-1}$	Inverse hessian matrix.....	49
$h_j(\bar{x})$	$j$ th inequality-constraint of $f(\bar{x})$ .....	29
$H_z$	Hertz-units of frequency (prefixed k for kilo and M for megs).....	14
$H_{BP}$	Gain or insertion loss of the second-order transfer function at d.c. (for low-pass) or $f_0$ (for band-pass) frequency of a RC-active filter.....	20

ABBREVIATIONS AND SYMBOLS CONTINUED

Symbol or Abbreviation	Description	Page
$i$	a subscript.....	11
IC	Integrated-circuit(s).....	50
$i_i$	$i$ th branch current of an electric circuit.....	93
$j$	a subscript (or prefix of the angular frequency $\omega$ ).....	15
$\bar{k}$	a penalty-function multiplier vector.....	107
$k_i$	penalty-function multiplier.....	107
K	Closed-loop d.c. gain of an ideal OA.....	20
$K_1, K_2$	Closed-loop d.c. gain of the ideal OA no. 1 and 2 respectively.....	10
KCL	Kirchoff's current loop.....	93
KVL	Kirchoff's voltage loop.....	93
K(s)	Closed-loop frequency-dependent gain of a practical OA.....	16
$K_o$	Closed-loop d.c. gain of an ideal OA.....	17
$L(\bar{x}; \lambda_1)$	Unconstrained objective function as a function of the variable-vector and Lagrangian multiplier $\lambda$ .....	102
LHP	Left-hand plane of the complex frequency "s".....	26
$l_i$	unit-length of a resistive thin-film in IC fabrication.....	115
m	an integer.....	30
mil	one thousandth of an inch.....	122
MnO <sub>2</sub>	Manganese dioxide.....	50
n	an integer.....	30
NIC	Negative-impedance converter.....	8
OA	operational amplifier.....	8

Symbol or Abbreviation	Description	Page
PIV	Positive impedance inverter (also called gyrator).....	8
$P(\bar{x}; k_i)$	Unconstrained objective function in quadratic form, as a function of the variable-vector $\bar{x}$ and the penalty-function multiplier $k_i$ .....	47
Q or $Q_0$	Figure of quality of the transfer function of the proposed BPF at $f_0$ ( $Q_0 = \frac{f_0}{\Delta f}$ ).....	20
r	an integer.....	30
RC	resistor-capacitor (combination).....	21
RHP	Right-half plane of the complex frequency "s".....	26
$R_s$	sheet-resistance of a resistive thin-film in IC fabrication.....	52
R or $k_i$ (i=1,2,3)	Resistors of the proposed RC-active filter.....	19
$R_T$	Total resistance of the proposed RC-active filter .....	25
rad/sec	radians per second-units of angular frequency.....	15
S	complex frequency ( $s = \sigma + j\omega$ ).....	14
sec	seconds .....	15
SiO	Silicon-monoxide.....	50
SiO <sub>2</sub>	Silicon-dioxide.....	118
Ta	Tantalum.....	30
TaO <sub>5</sub>	Tantalum-pentoxide .....	50
TiO <sub>2</sub>	Titanium-dioxide.....	118
TCC	Temperature coefficient of a capacitor.....	118
TCR	Temperature coefficient of a resistor.....	52
TM	Tantalum-metal (capacitors).....	53
TMM	Tantalum-Manganese - Dioxide-Metal (capacitors).....	53

Symbol or Abbreviation	Description	Page
UGA	Unity-gain amplifier.....	19
V <sub>a</sub>	Anodizing voltage during thin-film deposition process.....	51
V <sub>BR</sub>	Break-down voltage between the electrodes of a capacitor during the thin-film deposition process.....	51
VCCS	Voltage-controlled current source.....	8
VCVS	Voltage-controlled voltage source.....	8
V <sub>i</sub> (i=1,2,3)	Branch voltages of the proposed filter.....	93
V <sub>in</sub>	applied input voltage of the proposed filter or OAs.....	11
V <sub>o</sub>	output voltage of the proposed filter or OAs.....	8
$\omega$	angular frequency.....	15
$\omega_H$	angular cut-off frequency of the open-loop gain response of the practical OA.....	13
$\omega_o$	angular center frequency of the proposed BPF.....	20
w <sub>i</sub>	unit-width of the resistive thin-film in IC fabrication.....	116
$\omega_{-3dB}$	angular cut-off frequency of the closed- loop gain response using the practical OA.....	17
x	single variable of the constrained objective function.....	100
$\bar{x}$	independent variable-vector of the unconstrained objective function and constrained objective functions.....	30
x <sub>i</sub> or x <sup>i</sup>	variable-element of the objective function at the ith iteration of the minimization algorithm.....	102
$\bar{x}^*$	independent variable-vector at the "minimum".....	34

Symbol or Abbreviation	Description	Page
$\nabla_{\mathbf{x}}f(\bar{\mathbf{x}})$	Gradient vector of $f(\bar{\mathbf{x}})$ with respect to $x_i$ .....	101
$\nabla_{\mathbf{xx}}L$	Gradient vector of $L(\mathbf{x};\lambda)$ with respect to $x_i$ .....	103
$\nabla_{\lambda\lambda}L$	Gradient vector of $L(\mathbf{x};\lambda_i)$ with respect to $\lambda_i$ .....	103
$\nabla\nabla_{\mathbf{x}}f(\bar{\mathbf{x}})$	First-derivative of $\nabla_{\mathbf{x}}f(\bar{\mathbf{x}})$ .....	101
$\Delta f$	3-dB bandwidth of the proposed BPF .....	21
$\lambda_i$	Langrangian multiplier of the unconstrained objective function $L(\bar{\mathbf{x}}; \lambda_i)$ or step-size of the optimizing algorithm .....	102
$\frac{\partial f}{\partial x}$	Partial derivative of $f$ with respect to the independent variable $x$ .....	49
$\rho$	Specific resistivity of the resistive thin-film in IC fabrication .....	52
$\sigma$	real part (dumping factor) of the complex frequency "s" .....	14
$S_{\omega_0, x}$ , $S_{Q_0, x}$	Sensitivity of $\omega_0$ and $Q_0$ , respectively, with respect to the element $x$ (i.e., $x=C_i, R_i, K_i$ ) ..	26
$\sum_n$ or $\sum_{i=1}^k$	Summation over all $n$ over $i=1, 2, \dots, K$ .....	47
$\begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}$	Matrix representation of a set of multivariable linear equations .....	103
$\begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}^T$	Transposed matrix .....	110

## Chapter One

### INTRODUCTION

#### 1.1 General

Filter design consists of the realization of a specific transfer function that represents a gain and phase response by means of a synthesis procedure and implementation of the realization in the form of an electric two-port network.

A wide range of filter types exist, such as passive RLC, ~~RC~~-active, crystal, digital, and lately, switched-capacitor (SC) filters.

RC-active filter design is one of the areas of electronic circuit technology that has evolved very fast and has accumulated a vast amount of literature in a very short time.

One of the most significant impacts on the design of the active filters has been provided in recent years by the introduction of inexpensive monolithic OAs as active elements. These elements are available as off-the-shelf components in large quantities and low cost. Needless to say, the abundance of these active elements made has possible large volume production of high performance active

inductorless filters, in particular filters using hybrid IC design and fabrication techniques.

A major concern of the active filter designers, as related to the hybrid IC implementation, is the following.

To minimize the cost of a circuit, more components must be contained in a given substrate area; thus, the size of a circuit must be minimized. Basically, in an integrated circuit, a capacitor requires more space than a resistor, which in turn uses more space than a transistor.

As a result, for economic production of an integrated circuit active filter, and in order to increase the system capacity, such as in satellite communication systems where microelectronics play an important role in the satellite payload, the circuit must be designed with a minimum number of components, particularly capacitors.

In addition, an attempt must be made to minimize the total capacitance and resistance in the circuit.



## 1.2 Scope of the Report

In this report, procedures are described to minimize the total capacitance and total resistance of a low frequency low Q second-order band-pass active filter by using computer-aided minimization algorithms. Towards this end, a brief discussion of active filters, their related synthesis methods and the associated active elements with their characteristics are briefly discussed in chapter two.

The proposed active filter is described and analyzed in chapter three.

The total capacitance and total resistance of this filter are briefly outlined, and a discussion on the principles of minimization is given in chapter four.

The minimization procedures and several relevant aspects of the problem, such as the constraints, gradients, etc, and appropriate concepts of thin-film component designs are described in chapter five. An important part of this report is the solution of the optimization procedures formulated by means of appropriate computer programs. Certain steps must be taken in order to make the computer programs effective. These steps are important considerations in order to guarantee a feasible solution. These considerations, along with the comments on the computer programs results and

conclusions are given in chapter six.

A list of references and appendices conclude the context of this report. The appendices contain the calculations, theory on optimization, concepts of thin-film fabrication and computer programs with description and output results.

## Chapter Two

### ACTIVE FILTERS

#### 2.1. Brief Background

Spurred by technological advancements and requirements, modern filter theory has produced a class of networks called RC-active networks. These networks include only resistors, capacitors, and active elements.

The active elements have evolved from the vacuum tubes, through the transistors to the integrated circuit OAs in present use. This evolution stems from the need to operate the filter in reduced size, cost, and power consumption, as well as in increased system reliability and functional performance.

RC active filters are now available in hybrid integrated circuit forms and are used in many areas. Some applications are: telephony, where they are used as pilot, voice-channel and some channel-bank filters; telemetry and tracking for satellite systems; phase-lock loop (PLL) filters; instrumentation, used as spring-dumping filters; medical instrumentation and equipment, due to very low frequency

signal processing; digital data communications, used as carrier and time reference-recovery filters, sync-pulse recovery and clock generation filters, etc...

RC-active networks offer some attractive advantages over the RLC networks, such as:

- 1) Increased operating speed, since they are confined in compact, solid state form; propagation delay and parasitics are minimized.
- 2) The input impedance is high, compared to the source impedance; this reduces the need for signal power.
- 3) The output impedance is low; this feature keeps the input impedance of the next filter-stage unloaded, thus providing isolation between stages.
- 4) They can realize functions not realizable by RLC networks since the restrictions of passivity and reciprocity of the RLC networks are inoperative in RC-active networks.

Nevertheless, design of RC-active networks should be exercised carefully, otherwise, oscillations will cause unstable performance. Last, but not least, sensitivity analysis of the resulting networks should be done before implementing the networks.

## 2.2 Synthesis Methods for RC-Active Filters

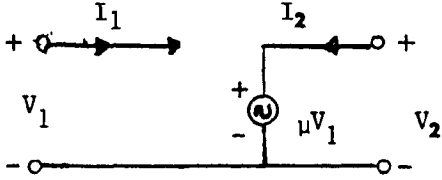
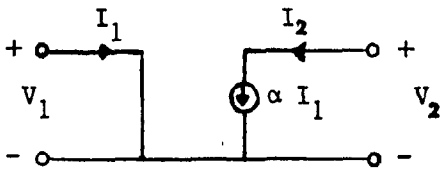
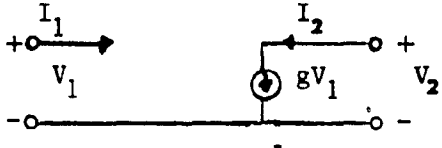
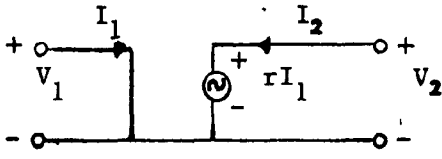
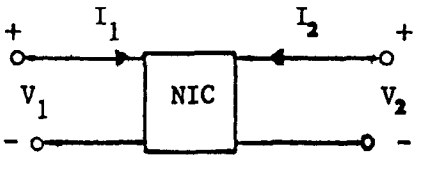
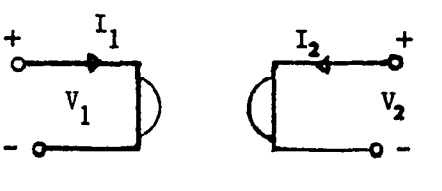
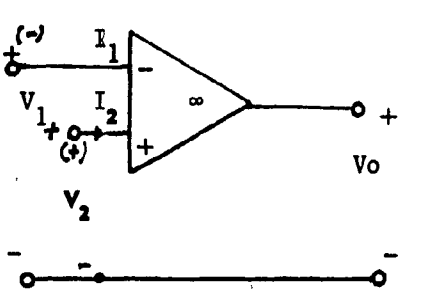
### 2.2.1 Methods

Considering recent synthesis methods, there are several classifications available. One such classification is based on the feedback representation of the active network configuration.<sup>(1)</sup> Another classification is based on the way by which the passive and active elements are identified. Still, another classification divides the RC-active filter literature into: 1) the conventional active-synthesis approach 2) the coefficient-matching approach and 3) the simulated-inductor approach.

The design presented in this work can be described as an application of the coefficient-matching technique. The technique is briefly described below.

Coefficient-matching technique: A RC-active network, employing a given type of active elements, is selected and its network function is analyzed. The analyzed network function is compared with a specific function. The element values are determined by equating coefficients of the same power in the complex frequency " $s$ ".<sup>(2)</sup>

Table 1 - Linear active elements used in the active-filter design:

Active-Network Element	Input-Output Relations	Network Symbol
VCVS	$V_2 = \mu V_1$ $I_1 = 0$	
CCCS	$I_2 = \alpha I_1$ $V_1 = 0$	
VCCS	$I_2 = gV_1$ $I_1 = 0$	
CCVS	$V_2 = rI_1$ $V_1 = 0$	
NIC	$V_1 = -K_1 V_2$ $I_2 = K_2 I_1$	
PIV or active gyrator	$I_1 = G_2 V_2$ $I = -G_1 V_1$	
OA	$V_o = A(V_2 - V_1)$ $A \rightarrow \infty$ $I_1 = I_2 = 0$	

### 2.2.2 Active Network Elements (3)

Numerous papers have been written on active-filter synthesis using a variety of active elements.

Table-I shows some of these active elements with their characteristics.

One of the major contributions to the active-filter designs has come from the use of a particular type of active elements, namely, OAs.

#### The OA

Considering the ideal OA, and from the input-output relationship of OA in Table-1, as the open-loop d.c gain "A" approaches infinity,  $V_2 - V_1$  approaches zero, where  $V_1$  and  $V_2$  are the inverting and non-inverting input-terminal voltages, respectively. Hence, the output voltage " $V_o$ " approaches zero. As is shown in Table-1, the ideal OA is an ideal two-input one-output VCVS with A approaching infinity. A is independent of frequency, temperature and input-voltage levels. Figure 2.1 shows, a typical representation of the equivalent circuit of an ideal OA.

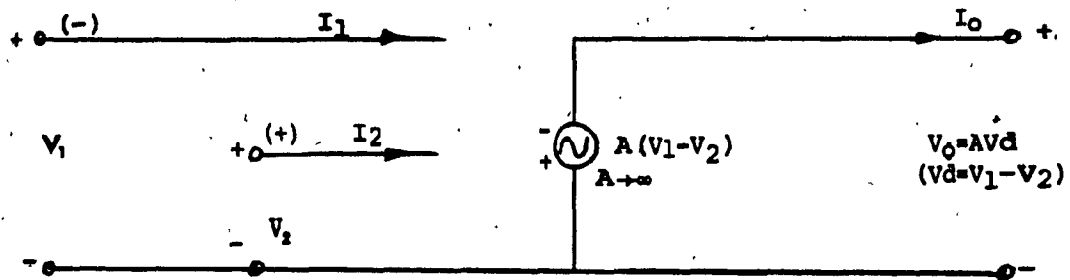


Figure 2.1 - Controlled-source model of an ideal differential input OA.

The output voltage  $V_o$  is always of the same polarity with the input voltage, when the latter is applied at the non-inverting input terminal, and of opposite polarity when it is applied at the inverting terminal.

The corresponding closed-loop d.c. gain ( $K_1$  or  $K_2$ ) follows the input-terminal sign; the closed-loop connected OA is referred to as non-inverting or inverting OA, respectively.

The non-inverting OA and its equivalent circuit are shown in Figure 2.2, connected in a closed-loop gain configuration.



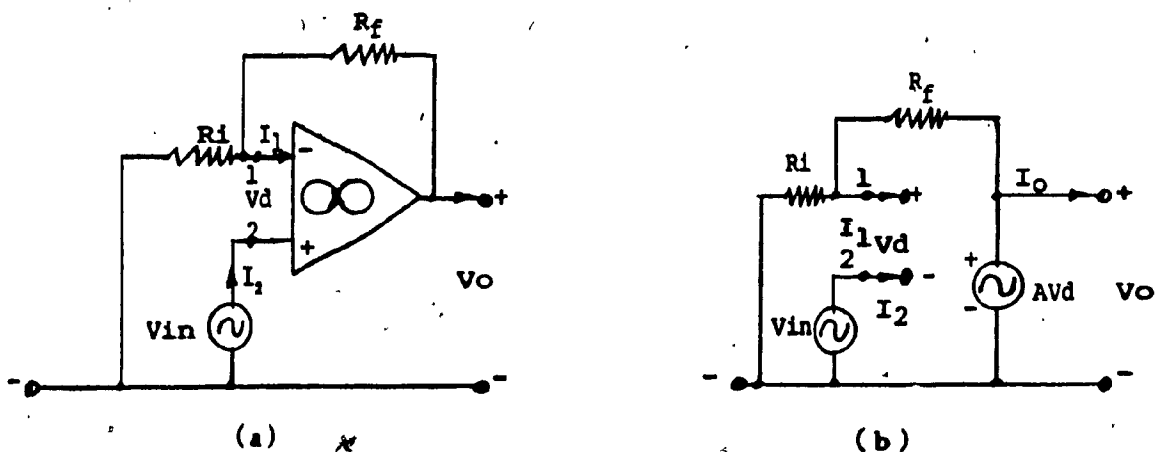


Figure 2.2 - A typical ideal OA in non-inverting gain configuration: (a) Actual circuit, (b) equivalent circuit.

The closed loop d.c. gain<sup>(4)</sup> is given by:

$$\frac{V_o}{V_i} = 1 + \frac{R_f}{R_i} = G + 1 \dots\dots\dots(2.1)$$

From eq. (2.1)  $\frac{V_o}{V_i} \geq 1$ , for real positive value, in  $R_f$  and  $R_i$ . By adjusting  $R_f$  and  $R_i$ , the d.c. gain of the OA can attain a large range of values.

The d.c. gain is independent of any source-impedance variations, since the non-inverting terminal draws zero current.

The inverting OA and its, equivalent circuit are shown in Figure 2.3. The closed-loop d.c. gain is given by:

$$\frac{V_o}{V_i} = - \frac{R_f}{R_i} = - G \dots\dots\dots(2.2)$$

From eq. (2.2), any real positive value (limited by design-constraints) of  $\frac{V_o}{V_i}$  can be obtained by adjusting  $R_f$  and  $R_i$ .

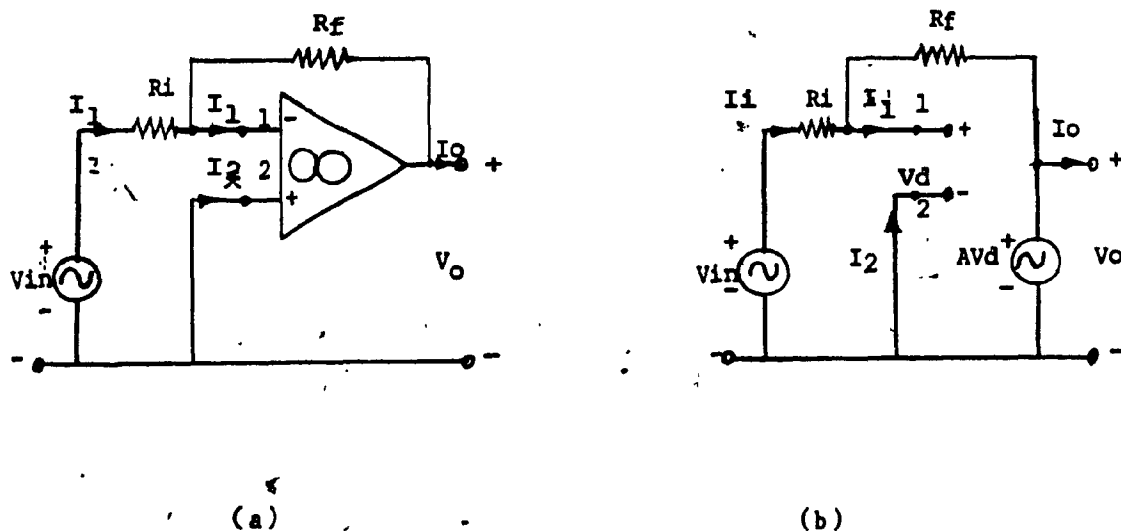


Figure 2.3 - A typical ideal OA in inverting gain configuration: (a) Actual circuit, (b) equivalent circuit.

From the equivalent circuit in figure 2.3 it is obvious that if the source-voltage  $V_i$  exhibits an impedance  $R_i$  it may be incorporated in series with  $R_i$ . Hence, the source-voltage  $V_i$  must be able to deliver a current  $\frac{V_i}{R_i}$ .

The output-impedance approaches zero, consequently the load-resistance  $R_L$  does not enter the expression for the d.c.-gain. However, the OA must be able to deliver output-current, required by the feedback current  $\frac{V_o}{R_f}$  and the load current  $\frac{V_o}{R_L}$ , where  $R_L$  is the load resistance.

Additional performance characteristics of the OAs, (5) when manufactured in silicon monolithic form, are: i) Close matching and tracking of the active and passive components over a wide range of temperature variations; ii) limited constraint on the number and geometry of the active devices, and iii) excellent thermal coupling through the circuit.

However, the integrated OAs are complex circuits, composed of versatile differential-amplifier cascaded stages with appropriate output stages.

The practical OA is a non-ideal device characterized by a frequency and temperature-dependent finite gain  $A(s)$ . This open-loop gain decreases monotonically with frequency at frequencies higher than its limited corner-frequency (or 3 dB cut-off frequency) value  $\omega_H$ . The input and output impedances are finite and the input and output signal levels are limited by the dynamic characteristics of the OA. Other non-ideal characteristics of the practical OA are: input offset-voltage and offset-current, common-mode rejection error, and finite unequal common-mode impedances of the two input-terminals. The Bode-plot of a typical practical OA with internal compensation is shown in Figure 2.4. Near d.c., the open-loop gain  $A_0$  is in the range of 200,000 or 106 dB, but at  $\omega_H$ , the voltage gain starts decreasing with a steady slope of 6dB/octave, or 20dB/decade.

The value of  $B$  usually depends on the model. The best OAs attain values of  $B$  in the range 1 to 65 MHz. At frequencies above  $B$ , the roll-off of the gain-slope is steeper than 12dB/octave. The OA then begins to appear as an attenuator which is useful in some cases, when stability is needed.

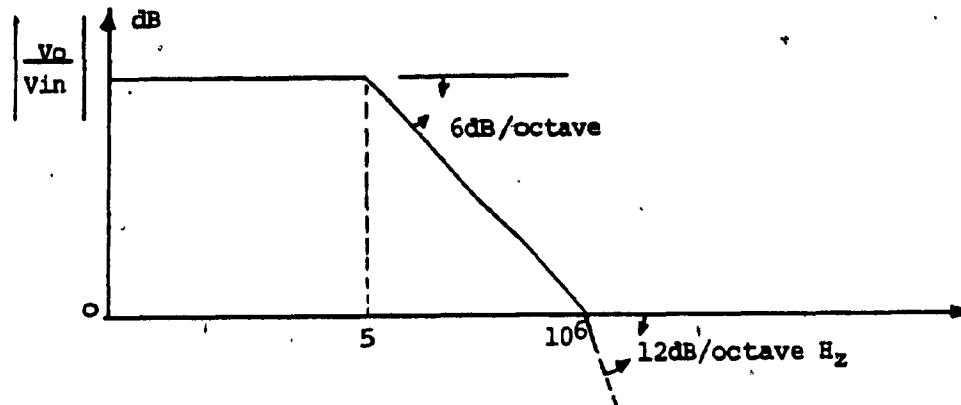


Figure 2.4 - Open-loop gain response versus frequency of a practical OA.

Most OAs are designed so that, for small signal operation,  $A(s)$  is represented by a first-order transfer function (ignoring the higher-order terms) that exhibits low-pass characteristics. The transfer-function of the practical OA, in open-loop configuration, is given by:  $A(s) = \frac{A_0 \omega_H}{s + \omega_H} = \frac{B}{s + \omega_H}$

... (2.3)

The gain and phase characteristics are shown in Figure 2.5

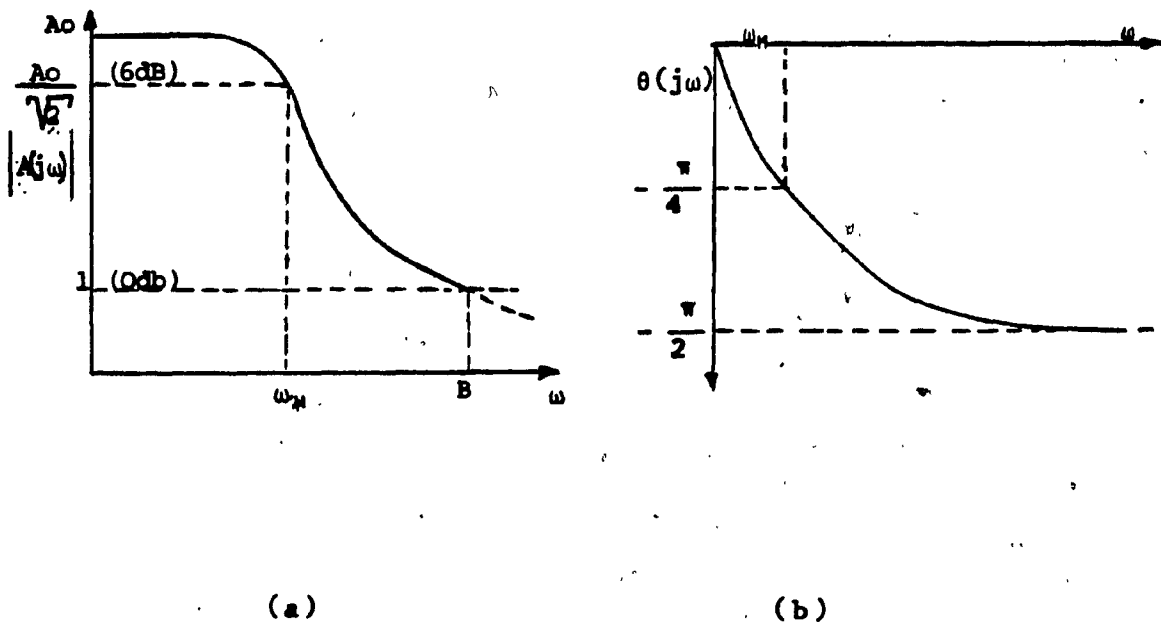


Figure 2.5 - Transfer function characteristics of the practical, frequency-dependent OA.

(a) Gain characteristics (b) Phase characteristics

Electronic-circuit designers must operate the OAs in the linear region of their transfer characteristics, in closed-loop modes, and with small input signal, otherwise the above small signal model is not valid. Furthermore, temperature variations may cause fluctuations on  $A_0$  characteristics, even with temperature-compensation circuits. For these reasons, closed-loop gain configurations should be used, by connecting external resistors, as in Figure 2.2 and 2.3 for the non-inverting and inverting-input OA configurations, respectively.

For the ideal OA, K is:

$$1 + \frac{R_f}{R_i} = G + 1 \dots \dots \dots (2.4)$$

$$\text{and } -\frac{R_f}{R_i} = -G \dots \dots \dots (2.5)$$

for the non-inverting and inverting OA configurations, respectively.

When the dominant-pole model of the practical OA is considered,  $K(s)$ , for the non-inverting configuration is:

$$K(s) = \frac{\frac{1}{\beta}}{\frac{s}{\beta B} + \frac{\omega_H}{\beta B} + 1} = \frac{B}{s + \omega_H + \beta B} = \frac{B}{s + \omega_H + \frac{B}{G}} \dots \dots \dots (2.6)$$

$$\beta = \frac{R_i}{R_i + R_f}$$

Similarly, for the inverting-input configuration,

$$K(s) = -\gamma \frac{\frac{1}{\beta}}{\frac{s}{\beta B} + \frac{\omega_H}{\beta B} + 1} = \frac{-G}{G+1} \cdot \frac{B}{s + \omega_H + \frac{B}{G+1}} \dots \dots \dots (2.7)$$

$$\text{where } \gamma = \frac{R_f}{R_i + R_f}$$

For  $\frac{B}{G} \gg \omega_H$ , the corresponding gains are:

$$K(s) = \frac{B}{s + \frac{B}{G+1}} \dots \dots \dots (2.8)$$

and

$$K(s) = \frac{-G}{G+1} \cdot \frac{B}{s + \frac{B}{G+1}} \dots \dots \dots (2.9)$$

respectively.

At zero frequency, the gains in equations (2.8) and (2.9) reduce to:  $K(s)|_{s=0} = 1 + G$  and  $K(s)|_{s=0} = -G$ , respectively, which are identical to the d.c. gains of equations (2.1) and (2.2) respectively. From equations (2.6) and (2.7) the 3-dB bandwidths ( $\omega_{-3dB}$ ) of the closed-loop practical OA are:

$$\omega_{-3dB} = \omega_H + \frac{B}{G+1} = \frac{B}{G+1} \left(1 + \frac{G+1}{A_o}\right) \dots \dots \dots (2.10)$$

and

$$\omega_{-3dB} = \omega_H + \frac{B}{G+2} = \frac{B}{G+2} \left(1 + \frac{G+2}{A_o}\right) \dots \dots \dots (2.11)$$

for the non-inverting and inverting-input OAs, respectively.

It can be observed that, for  $A_o \gg G+1$  and  $A_o \gg G+2$ , the corresponding 3-dB bandwidths are:

$$\omega_{-3dB} = \frac{B}{G+1} \dots \dots \dots (2.12)$$

and

$$\omega_{-3dB} = \frac{B}{G+2} \dots \dots \dots (2.13)$$

Comparing equations (2.12) and (2.13),  $\omega_{-3dB}$  for the non-inverting practical OA is greater than that of inverting OA. It can be seen, also, that the smaller the term G the wider the 3-dB bandwidth.

The unity-gain of the non-inverting OA is given by

$$K_o = \frac{R_f}{R_i} + 1 = G + 1 = 1 \dots \dots \dots (2.14)$$

since  $R_f$  is zero. Therefore, equation (2.12) becomes:

$$\omega_{-3dB} = B \dots \dots \dots (2.15)$$

since  $A_o \gg 1$  and  $G = 0$ .

From equations (2.12) and (2.15), it is obvious that the

cut-off frequency  $\omega_H$  is at its maximum value B when the OA is configured for unity-gain. The phase response varies between zero and  $-\frac{\pi}{2}$  radians at the maximum cut-off frequency B.

Other imperfections of the practical OA are the closed-loop input and output impedances  $Z_{in}$  and  $Z_{out}$ , respectively, which are represented by the expressions:

$$Z_{in} = (1 + A(s)\beta) Z_i \dots\dots\dots(2.16)$$

and 
$$Z_{out} = \frac{Z_o}{1 + A(s)\beta} \dots\dots\dots(2.17)$$

Here,  $Z_i$  and  $Z_o$  are the input and output impedances of the open-loop OA, respectively. When  $Z_i$  is in the order of 50,000 ohms  $Z_{in}$  is in the range of 50 to 100 megohms.  $Z_o$  decreases to  $Z_{out} = \frac{Z_o}{1 + A\beta}$ . When  $Z_o$  is in the order of 200 ohms, the corresponding impedance  $Z_{out}$  is in the order of 0.1 ohms.

From equations (2.16) and (2.17) it is clear that a unity-gain OA would offer the largest value of input impedance and the smallest value of output impedance.

Some of the other advantages of operating an OA in the unity-gain mode are simplicity, low cost and high stability.



## Chapter Three

### DESIGN OF A RC-ACTIVE BAND-PASS FILTER (BPF)

#### 3.1 The Proposed Band-Pass Filter

It is quite common to use second-order transfer-function sections in cascade, in order to realize higher order active filter, due to low-sensitivity and post-design adjustment considerations. (6)

In this report a new band-pass second-order RC-active filter using unity-gain amplifiers is discussed. As described in the previous section, UGAs can be realized conveniently by OAs. The filter is shown in Figure 3.1.

The required OA circuit in closed-loop configuration, to form the positive unity-gain amplifier, is shown in Figure 3.2

Note that ideally  $Z_{in} = \infty$  and  $Z_{out} = 0$ .

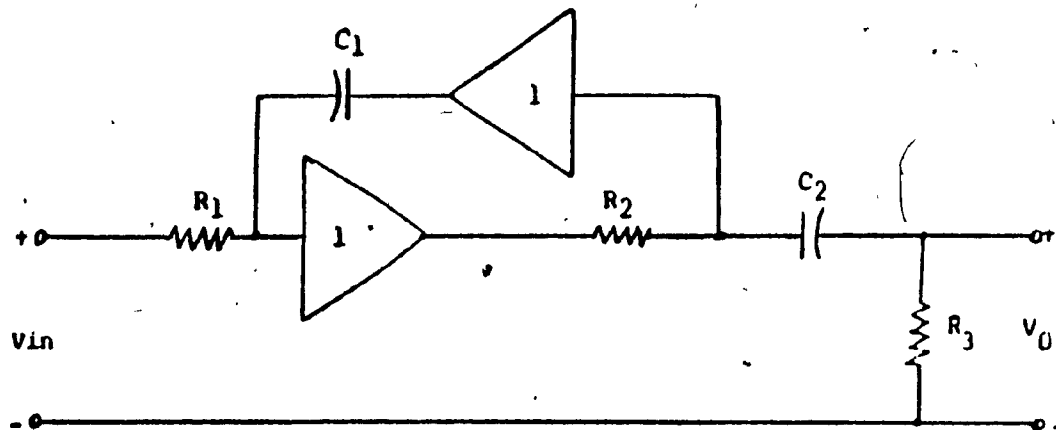


Figure 3.1 - Proposed band-pass RC-Active filter using unity-gain amplifiers.

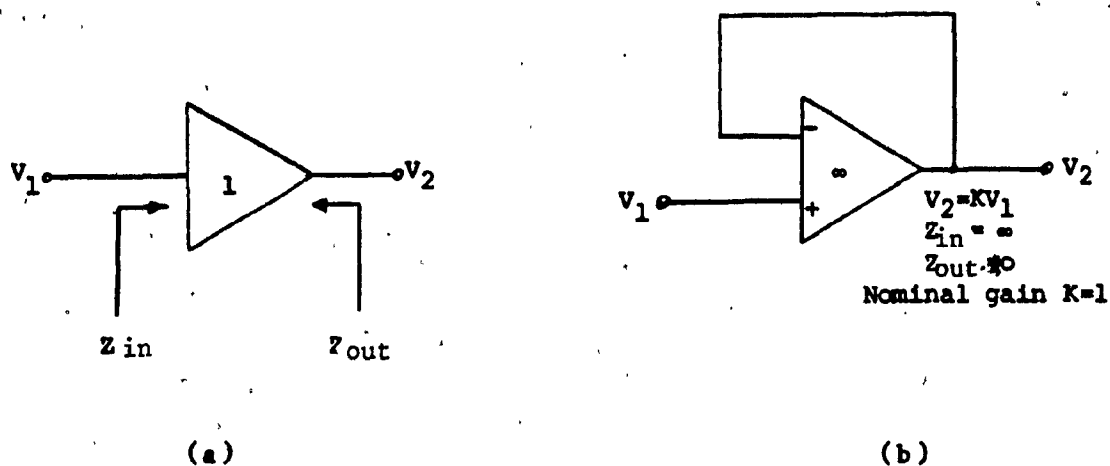


Figure 3.2 - Ideal models of UGAs:

(a) The symbol of the UGA

(b) Construction of UGA

### 3.2 Voltage Transfer Function

The voltage transfer function for a second-order band-pass filter, in the complex frequency domain "s" is:

$$\frac{V_o}{V_i}(s) = \frac{H_{BP} \frac{\omega_0}{Q_0} s}{s^2 + \frac{\omega_0}{Q_0} s + \omega_0^2} \dots \dots \dots (3.1)$$

The band-pass filter of figure 3.1 will be analyzed assuming ideal, then practical OAs, unity-gain or non-unity-gain amplifier configuration. The unity-gain amplifier configuration will be used later in the minimization algorithms.

The filter transfer function, Figure 3.1, is given by:

$$\frac{V_o}{V_i}(s) = \frac{\frac{R_3}{R_1 R_2 C_1} s}{s^2 + \frac{R_2 + R_3}{R_1 R_2 C_1} s + \frac{1}{R_1 R_2 C_1 C_2}} \dots\dots\dots (3.2)$$

and is derived in Appendix A.

Assuming non-unity gain configuration ( $K_1, K_2 \neq 1$ ), figure 3.3, the transfer function of the same filter is given by:

$$\frac{V_o}{V_i}(s) = \frac{\frac{K_1 R_3}{R_1 C_1 [R_2 + R_3 (1 - K_1 K_2)]} s}{s^2 + \frac{R_1 C_1 + R_2 C_2 + R_3 C_2 - K_1 K_2 R_1 C_1}{R_1 C_1 C_2 [R_2 + R_3 (1 - K_1 K_2)]} s + \frac{1}{R_1 C_1 C_2 [R_2 + R_3 (1 - K_1 K_2)]}} \dots\dots (3.3)$$

and is derived in Appendix A

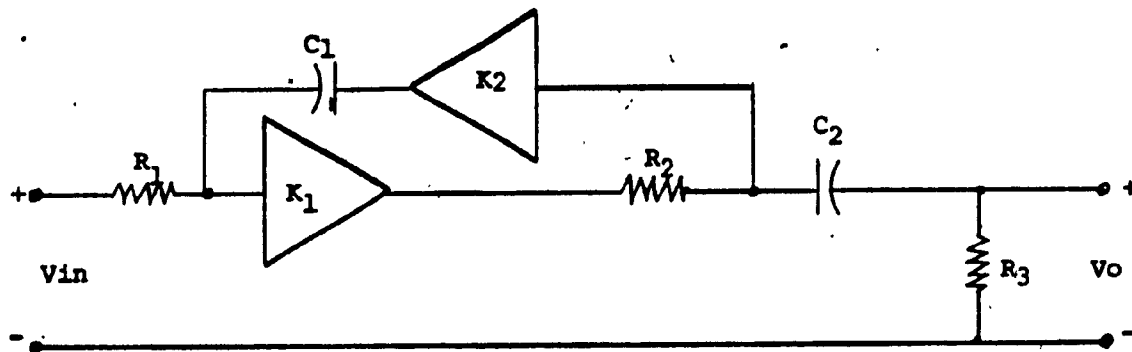


Figure 3.3 - Proposed band-pass RC-active filter of figure 3.1, using non-unity gain amplifiers  $K_1$  and  $K_2$ .

Let  $K_1 = K_2 = K$

In this case, the transfer functions of eq. (3.3) becomes:

$$\frac{V_o}{V_i}(s) = \frac{\frac{KR_3}{R_1 C_1 [R_2 + R_3 (1 - K^2)]} s}{s^2 + \frac{R_1 C_1 + R_2 C_2 + R_3 C_2 - K^2 R_1 C_1}{R_1 C_1 C_2 [R_2 + R_3 (1 - K^2)]} s + \frac{1}{R_1 C_1 C_2 [R_2 + R_3 (1 - K^2)]}} \dots\dots\dots (3.4)$$

By letting  $K = 1$ , the transfer function of equation (3.4), reduces to the transfer function of eq. (3.2). When the

dominant-pole model is used with unity-gain amplifiers (K=1)

$$\text{then } K(s) = \frac{B}{s + B} \dots\dots\dots (3.5)$$

Substituting eq. (3.5) in eq. (3.2), then:

$$\frac{V_o}{V_i}(s) = \frac{\frac{R_2 C_2 B}{R_1 R_2 C_1 C_2 + R_1 R_3 C_1 C_2} s(s + B)}{s^4 + \frac{2R_1 R_2 C_1 C_2 B + 2R_1 R_3 C_1 C_2 B + R_1 C_1 + R_2 C_2 + R_3 C_3}{R_1 R_2 C_1 C_2 + R_1 R_3 C_1 C_2} s^3 + \frac{R_1 R_2 C_1 C_2 B^2 + 2R_1 C_1 B + 2R_2 C_2 B + R_3 C_3 B}{R_1 R_2 C_1 C_2 + R_1 R_3 C_1 C_2}}$$

$$\frac{2R_1 C_2 B + 1}{s^2} + \frac{R_2 C_2 B^2 + R_3 C_3 B^2 + 2B}{R_1 R_2 C_1 C_2 + R_1 R_3 C_1 C_2} s + \frac{B^2}{R_1 R_2 C_1 C_2 + R_1 R_3 C_1 C_2} \dots\dots\dots (3.6)$$

Again, in an ideal situation, where B approaches infinity, the transfer functions in eq. (3.6) is simplified to:

$$\frac{V_o}{V_i}(s) = \frac{\frac{R_2}{R_1 R_2 C_1} s}{s^2 + \frac{R_2 + R_3}{R_1 R_2 C_1} s + \frac{1}{R_1 R_2 C_1 C_2}} \dots\dots\dots (3.7)$$

which is identical to eq. (3.2) derived directly with the assumption of unity-gain ideal amplifiers employing ideal OAs.

## 3.3

Design - Equations

Due to the complexity of operations involved in solving the fourth-order transfer function to derive the filter parameters  $\omega_0$ ,  $Q_0$  and  $H_{BP}$ , as well as the element-equations, only the ideal-OA second-order transfer function is analyzed. This implies that the design will be useful only for low frequency applications, that is for  $\omega_0 \ll B$ . Comparing eq. (3.1) and (3.2) the following equations are derived:

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \dots \dots \dots (3.8)$$

$$Q_0 = \frac{1}{R_2 + R_3} \sqrt{\frac{C_1 R_1 R_3}{C_2}} \dots \dots \dots (3.9)$$

$$H_{BP} = \frac{R_3}{R_2 + R_3} \dots \dots \dots (3.10)$$

From eq. (3.8),  $\omega_0$  can be adjusted by trimming the values  $C_1$ ,  $C_2$  and  $R_1$  or  $R_2$ . Once  $\omega_0$  is adjusted,  $H_{BP}$  or  $Q_0$  (but not both) can be adjusted by varying  $R_3$ , without affecting  $\omega_0$ . The parameters  $\omega_0$ ,  $Q_0$  and  $H_{BP}$  are usually specified by the filter designer. The corresponding element-values are calculated in Appendix B. All element-value expressions are presented in Table 2-5-6- and -7

The following component-equations will be useful in establishing the initial variable-locations in the minimization algorithms, when minimizing  $C_T$  and  $R_T$

TABLE - 2 FILTER ELEMENTS AND TIME CONSTANTS IN TERMS OF THE SPECIFICATIONS AND ANY TWO OF THE OTHER ELEMENTS

Given:  $\omega_0 \sqrt{C_1 C_2 R_1 R_2} \cdot Q_0 = \frac{R_1}{R_2 + R_3} \sqrt{\frac{GIR_1}{C_1 R_2}} \cdot H_{sp} = \frac{R_1}{R_2 + R_3}$

FLOATING ELEMENTS	$C_1$	$C_2$	$R_1$	$R_2$	$R_3$	$R_1 C_1$	$R_2 C_2$	$R_3 C_2$	COMMENTS
$C_1$ and $C_2$	FLOATING	FLOATING	$\frac{Q_0}{\omega_0 C_1 (1-H_{sp})}$	$\frac{1-H_{sp}}{\omega_0 Q_0 C_1}$	$\frac{H_{sp}}{\omega_0 Q_0} \cdot \frac{1}{C_1}$	-	-	-	UNIQUE SOLUTION
$C_1$ and $R_1$	FLOATING	$\frac{H_{sp}}{\omega_0 Q_0} \cdot \frac{1}{R_3}$	FLOATING	$\frac{1-H_{sp}}{C_2 \omega_0 Q_0}$	$\frac{H_{sp}}{1-H_{sp}} \cdot R_2$	$\frac{Q_0}{\omega_0} \cdot \frac{1}{1-H_{sp}}$	$\frac{1-H_{sp}}{\omega_0 Q_0}$	-	$R_1 C_1$ and $R_2 C_2$ are fixed; for solution $R_2$ or $C_2$ is required
$C_1$ and $R_2$	FLOATING	$\frac{H_{sp}}{\omega_0 Q_0} \cdot \frac{1}{R_3}$	$\frac{Q_0 R_3}{\omega_0 H_{sp} R_2 C_1}$	FLOATING	$\frac{H_{sp} R_2}{1-H_{sp}}$	-	-	-	UNIQUE SOLUTION
$C_1$ and $R_3$	FLOATING	$\frac{H_{sp}}{\omega_0 Q_0} \cdot \frac{1}{R_3}$	$\frac{1}{\omega_0 C_1 R_2 C_2}$	$\frac{1-H_{sp}}{H_{sp}} \cdot R_3$	FLOATING	-	-	-	UNIQUE SOLUTION
$C_2$ and $R_1$	$\frac{1}{\omega_0 R_1 R_2 C_2}$	FLOATING	FLOATING	$\frac{1-H_{sp}}{H_{sp}} \cdot R_3$	$\frac{H_{sp}}{\omega_0 Q_0} \cdot \frac{1}{C_2}$	-	-	-	UNIQUE SOLUTION
$C_2$ and $R_2$	$\frac{1}{\omega_0 R_1 R_2 C_2}$	FLOATING	$\frac{Q_0}{\omega_0 C_1 (1-H_{sp})}$	FLOATING	$\frac{H_{sp} R_2}{1-H_{sp}}$	$\frac{Q_0}{\omega_0} \cdot \frac{1}{1-H_{sp}}$	$\frac{1-H_{sp}}{\omega_0 Q_0}$	-	$R_1 C_1$ and $R_2 C_2$ are fixed; for solution, $R_1$ and $C_1$ is required.
$C_2$ and $R_3$	$\frac{1}{\omega_0 R_1 R_2 C_2}$	FLOATING	$\frac{1}{\omega_0 R_2 C_1 C_2}$	$\frac{1-H_{sp}}{H_{sp}} \cdot R_3$	FLOATING	$\frac{1}{\omega_0 R_2 C_2}$	-	$\frac{H_{sp}}{\omega_0 Q_0}$	$R_3 C_2$ and $R_1 C_1$ are fixed; for solution $R_1$ or $C_1$ is required.
$R_1$ and $R_2$	$\frac{1}{\omega_0 R_1 R_2 C_2}$	$\frac{H_{sp}}{\omega_0 Q_0} \cdot \frac{1}{R_3}$	FLOATING	FLOATING	$\frac{H_{sp}}{1-H_{sp}} \cdot R_2$	-	-	-	UNIQUE SOLUTION
$R_1$ and $R_3$	$\frac{1}{\omega_0 R_1 R_2 C_2}$	$\frac{H_{sp}}{\omega_0 Q_0} \cdot \frac{1}{R_3}$	FLOATING	$\frac{1-H_{sp}}{H_{sp}} \cdot R_3$	FLOATING	-	-	-	UNIQUE SOLUTION
$R_2$ and $R_3$	$\frac{1}{\omega_0 R_1 R_2 C_2}$	$\frac{H_{sp}}{\omega_0 Q_0} \cdot \frac{1}{R_3}$	$\frac{1}{\omega_0 R_1 R_2 C_2}$	FLOATING	FLOATING	$\frac{1}{\omega_0 R_2 C_2}$	-	-	$R_2$ and $R_3$ are interdependent

of the proposed filter. It is, therefore, convenient to calculate all the element-values classified in two groups; the resistor-equations, with capacitor-values as variables, and the capacitor-equations, with resistor-values as variables. These calculated values will be used, initially, as arbitrary locations of the variables ( $X_i$ ) in the minimization algorithms, assuming that the parameters  $\omega_o$ ,  $Q_o$ , and  $H_{BP}$  are specified. The element-equations are:

$$R_3 = \frac{1}{C_2} \cdot \frac{H_{BP}}{\omega_o Q_o} \dots\dots\dots(3.11)$$

$$R_2 = \frac{1}{C_2} \cdot \frac{1-H_{BP}}{\omega_o Q_o} \dots\dots\dots(3.12)$$

$$R_1 = \frac{1}{C_2} \cdot \frac{Q_o}{\omega_o (1-H_{BP})} \dots\dots\dots(3.13)$$

Hence,  $H_{BP} < 1$  for eq. (3.12) and (3.13) to take always positive values.

$$C_2 = \frac{1}{R_2+R_3} \cdot \frac{1}{\omega_o Q_o} \dots\dots\dots(3.14)$$

$$C_1 = \frac{R_2+R_3}{R_1 R_2} \cdot \frac{Q_o}{\omega_o} \dots\dots\dots(3.15)$$

$$C_T = C_1 + C_2 \dots\dots\dots(3.16)$$

$$R_T = R_1 + R_2 + R_3 \dots\dots\dots(3.17)$$

3.4

### Sensitivity

The RC-active filter designer is concerned with the sensitivities of the resulting transfer function. The sensitivity performance of any active filter is best described in terms of its sensitivities with respect to variations of its active and passive elements. (7)

The sensitivities  $S_x^{\omega_0}$  and  $S_x^{Q_0}$ , where x is any passive or active element are shown in Table - 3.

## 3.5

Stability

The second-order transfer function of the proposed band-pass filter can be written as:

$$\frac{V_o}{V_i}(s) = \frac{N(s)}{D(s)} = \frac{H_{BP} \frac{\omega_0}{Q_0} s}{s^2 + \frac{\omega_0}{Q_0} s + \omega_0^2} \dots \dots \dots (3.18)$$

or:

$$\frac{V_o}{V_i}(s) = \frac{a_1 s}{b_2 s^2 + b_1 s + b_0} \dots \dots \dots (3.19)$$

The bi-coefficients of the  $i^{\text{th}}$  power in "S" determine whether the poles ( $S_p$ ) are located in the LHP, in the RHP, or on the imaginary axis ( $j\omega$ ). The last two pole-locations define the "critical stability" or the "oscillatory" condition of the filter. For stability, the  $b_i$ -coefficients for the transfer function of (3.19) must be real positive constants.

The denominator of the transfer function, eq. (3.4), will be examined in order to observe the importance of K of the amplifiers at the instant of power activation, and after activation. K is given by:  $K = \frac{A_0}{A_0 + 1}$ . As  $A_0$  increases its values from zero, during activation, to infinity, during the steady-state period of the power (post activation), the gain K increases from zero to one, where it remains constant.



$X$	$S_x^{\omega_0}$	$S_x^{\omega_0}$	Comments
$R_1$	$\frac{1}{2}$	$-\frac{1}{2}$	
$R_2$	$\frac{1}{2} \frac{R_1 - R_2}{R_2 + R_3}$	$-\frac{1}{2}$	$S_{R_2}^{\omega_0} = \frac{1}{2}$
$R_3$	$-\frac{R_1}{R_2 + R_3}$	0	$S_{R_3}^{\omega_0} = 1$
$C_1$	$\frac{1}{2}$	$-\frac{1}{2}$	
$C_2$	$-\frac{1}{2}$	$-\frac{1}{2}$	
$K_1$	$\frac{R_1 C_1}{(R_2 + R_3) C_2} \frac{1}{2K_2} \begin{matrix} K_1 = K_2 \\ -K \\ -1 \end{matrix}$	$\frac{1}{2} K_1 K_2 R_1 C_1 C_2$	$S_{K_1}^{\omega_0} \approx \frac{R_1 C_1}{(R_2 + R_3) C_2}$ , where $K_1 = K_2 = K = 1$ $S_{K_1}^{\omega_0} = \frac{1}{2} R_1 C_1 C_2$ , where $K_1 = K_2 = K = 1$
$K_2$	$\frac{R_1 C_1}{(R_2 + R_3) C_2} \frac{1}{2K_2} \begin{matrix} K_1 = K_2 \\ -K \\ -1 \end{matrix}$	$\frac{1}{2} K_1 K_2 R_1 C_1 C_2$	$S_{K_2}^{\omega_0} \approx \frac{R_1 C_1}{(R_2 + R_3) C_2}$ , where $K_1 = K_2 = 1$ $S_{K_2}^{\omega_0} \approx \frac{1}{2} R_1 C_1 C_2$ , where $K_1 = K_2 = 1$

Table - 3 -  $Q$  and  $\omega_0$  sensitivities with respect to the passive and active elements

### 3.5.1 Stability During Activation

For stability during activation, this is, right after power is switched on, the  $b_1$ -coefficient of the denominator should remain real and positive. For this circuit, the following conditions should hold:

$$1) \ b_0 = \omega_0^2 > 0 \qquad \text{where } \omega_0^2 = \frac{1}{R_1 C_1 C_2 (R_2 + R_3 (1 - K^2))}$$

Since

$$0 < K < 1, \ \omega_0^2 > 0$$

2)  $b_2 > 0$ . In this case,  $b_2 = 1$ . Therefore,  $b_2$ , the second-order coefficient, is always real positive.

$$3) \ b_1 > 0. \ \text{For this filter, } b_1 = \frac{\omega_0}{Q_0}$$

$$\text{or } \frac{\omega_0}{Q_0} = \frac{R_1 C_1 + R_2 C_2 + R_3 C_2 - K_1 K_2 R_1 C_1}{R_1 C_1 C_2 [R_2 + R_3 (1 - K_1 K_2)]} = \frac{R_1 C_1 (1 - K^2) + C_2 (R_2 + R_3)}{R_1 C_1 C_2 [R_2 + R_3 (1 - K^2)]}$$

It is obvious that  $\frac{\omega_0}{Q_0}$  or  $b_1$  is real positive, since  $K^2 < 1$ . From the above stability analysis, it can be concluded that the proposed configuration yields a stable filter.

## Chapter Four

### MINIMIZATION OF TOTAL CAPACITANCE ( $C_T$ )

### AND TOTAL RESISTANCE ( $R_T$ )

#### 4.1 General Concepts of Optimization (Non-linear Programming)<sup>(9)</sup>

In any optimization problem, there is a function  $f(\bar{x})$  to be minimized, which satisfies some rules called constraints  $g_i(\bar{x})$ . The function to be minimized is called "objective function". The optimization process can be stated as:  $\min f(\bar{x})$ , such that  $g_i(\bar{x})=0$ ,  $i=1,2,\dots,m \leq n$ ,  $h_j(\bar{x}) \geq 0$ ,  $j=1,2,\dots,r$  where  $\bar{x} = [x_1, x_2, \dots, x_n]^T$ , and all functions  $f(\bar{x})$ ,  $g_i(\bar{x})$  and  $h_j(\bar{x})$  are differentiable.

With the given objective function, the  $m$  equality-constraints, and  $r$  inequality-constraints, it is possible to find a set (or sets, since the solution might not be unique)  $[x_1^*, x_2^*, \dots, x_n^*]$  which yields a minimum. The function  $f(\bar{x})$  will be feasible for a solution if the dimension  $m$  in  $g_i(\bar{x})$  does not exceed the number of variables in  $f(\bar{x})$ . Therefore, the inequality  $m \leq n$  should hold. If  $m \geq n$ , the problem is termed "overconstrained." When  $m < n$ , the number of degrees of freedom, for optimizing,

is  $n - m$ . The equality-constraints can be divided into two inequality-constraints:

$$g_i(\bar{x}) \geq 0, \quad i=1,2,\dots,m$$

$$g_i(\bar{x}) \leq 0, \quad i=1,2,\dots,m$$

From the above definitions, the non-linear programming problem can be written as:

$$\min_{\bar{x}} f(\bar{x}), \quad \text{such that} \quad \begin{cases} g_i(\bar{x}) \\ -g_i(\bar{x}) \\ h_j(\bar{x}) \end{cases} \geq 0, \quad \begin{matrix} i=1,2,\dots,m < n \\ j=1,2,\dots,r \end{matrix}$$

There are many minimization methods (or algorithms) available in literature; some of these are briefly described in Appendix C.

#### 4.2 Total Capacitance ( $C_T$ ) and Total Resistance ( $R_T$ )

For high volume production of active filters, hybrid integrated circuit realizations using thin-film RC-networks and silicon monolithic operational amplifiers have been a state of the art in recent filter technology and have very efficient production yields. To this end, however, due consideration must be given to the inherent properties of hybrid integrated circuits.

In a thin-film realization, a capacitor uses more space than a resistor, and the resistor uses more space than a transistor. It is estimated that if the area occupied by a transistor on an IC chip is taken to be one unit, then a resistor requires an area of two units and a capacitor occupies an area of three units.

Consequently, the aim of the designer should not only be to minimize the number of the passive components, but also to minimize  $C_T$  and  $R_T$

In the context of the minimization procedure, the terms "total capacitance" or "total resistance" is interpreted in this report to mean a quantity that is directly proportioned to the sum of the individual areas of each capacitor or resistor, as the case may be. For the proposed active filter, minimization of the areas corresponding to  $C_T$  and/or  $R_T$  is the major topic of this report. Analysis deals with forty eight cases of minimizing the capacitance and/or resistance-areas of a thin-film substrate based on the proposed filter component requirements and parameter ( $\omega_0$ ,  $Q_0$  and  $H_{sp}$ ) specifications. In some cases, minimization procedures have been given directly for the quantities  $C_T$  or  $R_T$  without considering corresponding areas.

The different optimization cases considered in this report are shown in Fig. 4.1. The expressions for  $C_T$  and  $R_T$  are defined in section 3.3 and are given in Table 5, 6 and 7.

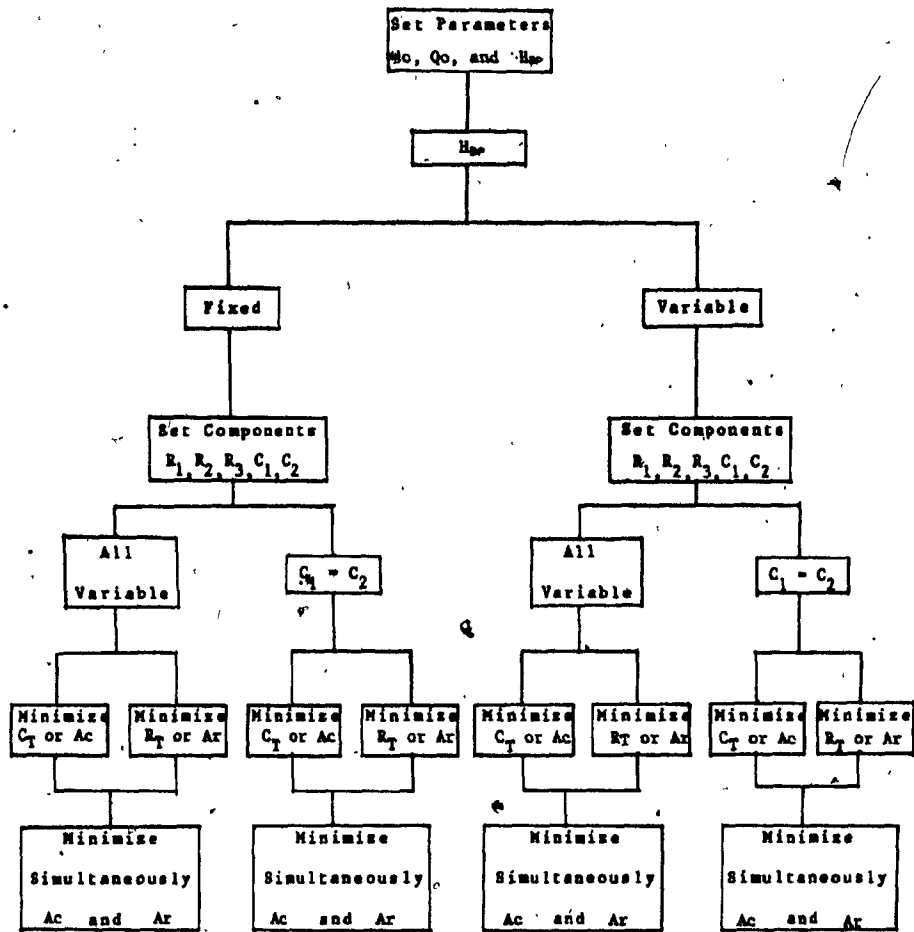


Figure 4.1 - Classification of minimization cases in terms of filter parameters and constraints

## Chapter Five

### SOME OPTIMAL DESIGNS FOR THE PROPOSED FILTER

#### 5.1 Formulation of the problem

The filter design problem is viewed here as the determination of a set of "optimum" element values for a given network structure.

These values yield the minimum value of the objective function (ie.  $C_T$  ,  $B_T$  ,  $A_c$  ,  $A_r$  ) for the given filter specifications.

In applying the computer-aided analysis, proper formulation of the above objective function, selection of the constraints to be satisfied, together with the choice of suitable minimization algorithms, are the essential aspects of optimization problem.

The filter element-values are bounded in a feasible region, specified by the solid-state circuit manufacturer. These element-value bounds can be introduced in the computer program, either as variables in a range defined by "hard" limits (stop commands) or as minimization constraints. The constraints of the 48 objective functions are the filter parameters  $W_0$  and  $Q_0$  and the component-requirement  $C_1 = C_2$  .

However,  $W_0$  and  $Q_0$  are functions of the filter elements; in view of this dependence, their values in each iteration of the minimization algorithm, change throughout the algorithm until the "optimum" (minimum set  $x^* = [x_1^*, x_2^*, \dots, x_n^*]$ ) is achieved, in which instance, these constraints are satisfied, or until the parameters obtain their desired values.

## 5.2 The Optimal-Design Objective Function

The minimization problem can be stated as follows:

Min<sub>x</sub>  $f(\bar{x})$ , such that the equality-constraints  $g_i(\bar{x}) = 0$ ,  
 $i=1,2,\dots,r$ , hold, where  $\bar{x} = [x_1, x_2, \dots, x_m]^T$ .

The objective function  $f(\bar{x})$ , in this study, is determined by the specific requirements of the filter design to deliver the desired filter parameters at the end of the minimization process, where the physical size of  $C_T$  and  $R_T$  representing the corresponding areas  $A_c$  and  $A_r$  respectively, are minimized. This function is the "constrained" objective function to be minimized, without considering the penalty paid by the presence of the constraints; this penalty is the inability of the algorithm to reach the "global" (or sometimes "local") point when constraints ( $g_i(\bar{x})$ ) are satisfied.

Computer-programs are compiled, in Fortran IV, utilizing two of the Computer-Library subroutines which minimize the



specific five-dimensional twice-differentiable functions with some constraints, Figure 4.1 and Table - 4.

Quasi-Newton's algorithm is used to minimize  $f(\bar{x})=C_T$  for four cases, each case pertaining to three sets of values in  $W_0$  and  $Q_0$ , per Table -5, -6, -9, and -11.

Fletcher-Powell's algorithm solves the more feasible functions  $f(\bar{x})$  given by:  $Ax$ ,  $2Ax + 3Ac$ , where  $Ac=A_1C_T$ ,  $Ax=A_2R_T$ , and  $A_1$  and  $A_2$  are constants defined by the electric and physical properties of the thin-film material used in this analysis. There are twelve cases for all three objective functions ( $f(x)$ ), each case pertaining to three sets of values in  $W_0$  and  $Q_0$ . Table -4, -7, -8, -10, and -12 show the various objective functions of this algorithm.

$F(\bar{x})$  is a two-dimensional constrained objective function since  $\bar{x}=[x_1, x_2]^T$ , where  $x_1 = C_1$ ,  $x_2 = C_2$ , when  $f(\bar{x})=C_T=C_1+C_2$  and three-dimensional ( $\bar{x}=[x_1, x_2, x_3]^T$ ), where  $x_1=R_1$ ,  $x_2=R_2$ ,  $x_3=R_3$ , when  $f(\bar{x}) = R_T = R_1 + R_2 + R_3$ .

In a real situation technical problem, where the filter design has to meet certain parameter-specifications, besides minimizing the physical size of its components, constraint-functions  $g_i(\bar{x})$  are present.

Table - 4 INDEX OF MINIMIZATION AND "PLOT" PROGRAMS

a) fo=500 HZ, Qo=50

b) fo=500 HZ, Qo = 10

c) fo=1000 HZ, Qo = 10

Program No	Program Name	Case	Description
1	MINCT <sub>1</sub>	1a	Minimize C <sub>T</sub> using Newton's method
2		1b	when H <sub>BP</sub> is floating and all
3		1c	elements are floating ( $H_{BP} = \frac{R_3}{R_2 + R_3}$ )
4	MINCT <sub>1</sub>	2a	Minimize C <sub>T</sub> using Newton's method
5		2b	when H <sub>BP</sub> is floating and C <sub>1</sub> =C <sub>2</sub> =C
6		2c	
7	MINCT <sub>2</sub>	3a	Minimize C <sub>T</sub> using Newton's method
8		3b	when H <sub>BP</sub> is constant ( $R_3 = KR_2; H_{BP} = \frac{K}{K+1}$ )
9		3c	and C <sub>1</sub> , C <sub>2</sub> , R <sub>1</sub> , R <sub>2</sub> are floating
10	MINCT <sub>3</sub>	4a	Minimize C <sub>T</sub> using Newton's method
11		4b	when H <sub>BP</sub> is constant and C <sub>1</sub> = C <sub>2</sub> = C
12		4c	
13	C <sub>1</sub> MIN	1a	Minimize area "Ac" of C <sub>T</sub> using
14		1b	Fletcher's method when H <sub>BP</sub> is float
15		1c	-ing and all elements are floating
16	C <sub>2</sub> MIN	2a	Minimize area "Ac" of C <sub>T</sub> using
17		2b	Fletcher's method when H <sub>BP</sub> is
18		2c	floating and C <sub>1</sub> = C <sub>2</sub> = C
19	C <sub>3</sub> MIN	3a	Minimize area "Ac" of C <sub>T</sub> using
20		3b	Fletcher's method when H <sub>BP</sub> is
21		3c	constant ( $H_{BP} = \frac{K}{K+1}; R_3 = KR_2$ )
22	C <sub>4</sub> MIN	4a	Minimize area "Ac" of C <sub>T</sub> using
23		4b	Fletcher's method when H <sub>BP</sub> is
24		4c	constant and C <sub>1</sub> = C <sub>2</sub> = C

## INDEX CONTINUED

Program No	Program Name	Case	Description
25	R <sub>1</sub> MIN	5a	Minimize area "Ar" of R <sub>T</sub> using Fletcher's method when H <sub>BP</sub> and all elements are floating .
26		5b	
27		5c	
28	R <sub>2</sub> MIN	6a	Minimize area "Ar" of R <sub>T</sub> using Fletcher's method when H <sub>BP</sub> is floating and C <sub>1</sub> =C <sub>2</sub> = C
29		6b	
30		6c	
31	R <sub>3</sub> MIN	7a	Minimize area "Ar" of R <sub>T</sub> using Fletcher's method when H <sub>BP</sub> is constant ( $H_{BP} = \frac{K}{K+1}$ ; R <sub>3</sub> =KR <sub>2</sub> )
32		7b	
33		7c	
34	R <sub>4</sub> MIN	8a	Minimize area "Ar" of R <sub>T</sub> using Fletcher's method when H <sub>BP</sub> is constant and C <sub>1</sub> = C <sub>2</sub>
35		8b	
36		8c	
37	RCMIN <sub>1</sub>	9a	Minimize area of 3Ac+2Ar using Fletcher's method when H <sub>BP</sub> is floating and all elements are floating
38		9b	
39		9c	
40	RCMIN <sub>2</sub>	10a	Minimize area of 3Ac + 2Ar using Fletcher's method when H <sub>BP</sub> is floating and C <sub>1</sub> = C <sub>2</sub>
41		10b	
42		10c	
43	RCMIN <sub>3</sub>	11a	Minimize area of 3Ac + 2Ar using Fletcher's method when H <sub>BP</sub> is constant ( $H_{BP} = \frac{K}{K+1}$ ; R <sub>3</sub> = KR <sub>2</sub> )
44		11b	
45		11c	
46	RCMIN <sub>4</sub>	12a	Minimize area of 3Ac + 2Ar using Fletcher's method when H <sub>BP</sub> is constant and C <sub>1</sub> = C <sub>2</sub>
47		12b	
48		12c	

## INDEX CONTINUED

Program No	Program Name	Case	Description
49	Paris 2	fo=500HZ, Qo=50, H <sub>BP</sub> =.909	Plot the Amplitude and Phase Response with Ideal OAs in the Unity-Gain Mode
50	"	fo=500HZ, Qo=50, H <sub>BP</sub> =.75	
51	"	fo=500HZ, Qo=10, H <sub>BP</sub> =.75	
52	Paris	fo=500HZ, Qo=50, H <sub>BP</sub> =.909	Plot the Amplitude and Phase Response with Non-Ideal OAs in the Unity-Gain Mode. B=100MHZ
53	"	fo=500HZ, Qo=10, H <sub>BP</sub> =.909	
54	"	fo=1000HZ, Qo=10, H <sub>BP</sub> =.909	
55	Paris 1	fo=500HZ, Qo=50, H <sub>BP</sub> =.909	Plot the Amplitude and Phase Response with Non-Ideal OAs in the Non Unity-Gain Mode
56	"	Ao=150000, Ko=1000 fo=500HZ, Qo=50, H <sub>BP</sub> =.909	
57	"	Ao=150000, Ko=10 fo=500HZ, Qo=50, H <sub>BP</sub> =.909	
58 through 69	Paris	Optimal-element values of Quasi-Newton minim. programs (Program nos: 1 - 12)	Plot the Amplitude and Phase Response with Non-Ideal OAs in the Unity-Gain Mode. B=100MHZ
70 through 105	Paris	Optimal-element values of Fletcher-Powell's minim. programs (program nos: 13 - 48)	"

TABLE - 5 EXPRESSIONS OF ELEMENTS AND PARAMETERS - GENERAL

CASE	DESCRIPTION	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	C <sub>1</sub>	C <sub>2</sub>	H	W <sub>0</sub>	Q <sub>0</sub>
1a	minimize C <sub>t</sub> when H <sub>sp</sub> and all elements are floating	FLOATING	FLOATING	FLOATING	$\frac{R_2+R_3}{R_1} \frac{Q_0}{W_0}$	$\frac{1}{Q_0 H_0} \frac{1}{R_2+R_3}$	$\frac{R_3}{R_2+R_3}$	$\frac{1}{\sqrt{C_1 C_2 R_1 R_2}}$	$\frac{R_2}{R_2+R_3} \sqrt{\frac{C_1 R_1}{C_2 R_2}}$
1b	minimize C <sub>t</sub> when H <sub>sp</sub> is floating and C <sub>1</sub> -C <sub>2</sub> =C	FLOATING	FLOATING	FLOATING	$\frac{R_2+R_3}{R_1} \frac{Q_0}{W_0}$	C <sub>1</sub>	$\frac{R_3}{R_2+R_3}$	$\frac{1}{C \sqrt{R_1 R_2}}$	$\frac{R_2}{R_2+R_3} \sqrt{\frac{R_1}{R_2}}$
1c	minimize C <sub>t</sub> when H <sub>sp</sub> is constant (H <sub>y</sub> =R <sub>2</sub> ) and C <sub>1</sub> , C <sub>2</sub> , R <sub>1</sub> , R <sub>2</sub> are floating	FLOATING	FLOATING	K R <sub>2</sub>	$\frac{K+1}{R_1} \frac{Q_0}{W_0}$	$\frac{1}{(K+1) Q_0 H_0} \frac{1}{R_2}$	$\frac{K}{K+1}$	$\frac{1}{\sqrt{C_1 C_2 R_1 R_2}}$	$\frac{1}{K+1} \sqrt{\frac{C_1 R_1}{C_2 R_2}}$
1d	minimize C <sub>t</sub> when H <sub>sp</sub> is constant and C <sub>1</sub> -C <sub>2</sub>	FLOATING	FLOATING	K R <sub>2</sub>	$\frac{K+1}{R_1} \frac{Q_0}{W_0}$	C <sub>1</sub>	$\frac{K}{K+1}$	$\frac{1}{C \sqrt{R_1 R_2}}$	$\frac{1}{K+1} \sqrt{\frac{R_1}{R_2}}$
2a	minimize R <sub>t</sub> when H <sub>sp</sub> and all elements are floating	$\frac{1}{C_1 (1-H_{sp})} \frac{Q_0}{W_0}$	$\frac{1-H_{sp}}{C_2 W_0 Q_0}$	$\frac{H_{sp}}{C_2 W_0 Q_0}$	FLOATING	FLOATING	$\frac{R_3}{R_2+R_3}$	$\frac{1}{\sqrt{C_1 C_2 R_1}}$	$\frac{R_2}{R_2+R_3} \sqrt{\frac{C_1 R_1}{C_2 R_2}}$
2b	minimize R <sub>t</sub> when H <sub>sp</sub> is floating and C <sub>1</sub> -C <sub>2</sub> =C	$\frac{1}{C (1-H_{sp})} \frac{Q_0}{W_0}$	$\frac{1-H_{sp}}{C W_0 Q_0}$	$\frac{H_{sp}}{C W_0 Q_0}$	FLOATING	C <sub>1</sub>	$\frac{R_3}{R_2+R_3}$	$\frac{1}{C \sqrt{R_1 R_2}}$	$\frac{R_2}{R_2+R_3} \sqrt{\frac{R_1}{R_2}}$
2c	minimize R <sub>t</sub> when H <sub>sp</sub> is constant (H <sub>y</sub> =K R <sub>2</sub> ) and C <sub>1</sub> , C <sub>2</sub> , R <sub>1</sub> , R <sub>2</sub> are floating	$\frac{1}{C_1 (1-H_{sp})} \frac{Q_0}{W_0}$	$\frac{1-H_{sp}}{C_2 W_0 Q_0}$	K R <sub>2</sub>	$\frac{K+1}{R_1} \frac{Q_0}{W_0}$	$\frac{1}{(K+1) Q_0 H_0} \frac{1}{R_2}$	$\frac{K}{K+1}$	$\frac{1}{\sqrt{C_1 C_2 R_1 R_2}}$	$\frac{1}{K+1} \sqrt{\frac{C_1 R_1}{C_2 R_2}}$
2d	minimize R <sub>t</sub> when H <sub>sp</sub> is constant and C <sub>1</sub> -C <sub>2</sub>	$\frac{1}{C_1 (1-H_{sp})} \frac{Q_0}{W_0}$	$\frac{1-H_{sp}}{C W_0 Q_0}$	K R <sub>2</sub> ≠	FLOATING	C <sub>1</sub>	$\frac{K}{K+1}$	$\frac{1}{C \sqrt{R_1 R_2}}$	$\frac{1}{K+1} \sqrt{\frac{R_1}{R_2}}$

6

TABLE - 6 EXPRESSIONS OF ELEMENTS AND PARAMETERS USED IN THE MINIMIZATION PROGRAMS (NEWTON'S METHOD)

Program Name	$R_1$	$B_2$	$B_3$	$C_1$	$C_2$	Constrain 1	Constrain 2	Constrain 3	$C_T$	$R_T$
MIN C1 (1003)	FLOATING	FLOATING	FLOATING	$\frac{B_2+B_3}{B_1 \cdot B_2} \frac{Q_0}{W_0}$	$\frac{1}{W_0 Q_0} \frac{1}{B_2+B_3}$	$W_0^2 B_1 C_1 B_2 C_2 - 1 = 0$	$Q_0^2 (B_2+B_3)^2 \frac{C_2}{C_1 B_1 B_2} - 1 = 0$	N/A	$C_1 + C_2$	$B_1 + B_2 + B_3$
MIN C1 (1000)	FLOATING	FLOATING	FLOATING	$\frac{B_2+B_3}{B_1 \cdot B_2} \frac{Q_0}{W_0}$	$\frac{1}{W_0 Q_0} \frac{1}{B_2+B_3}$	$W_0^2 B_1 C_1 B_2 C_2 - 1 = 0$	$Q_0^2 (B_2+B_3)^2 \frac{C_2}{C_1 B_1 B_2} - 1 = 0$	$\frac{C_1}{C_2} - 1 = 0$	$C_1 + C_2$	$B_1 + B_2 + B_3$
MIN C1 (7009)	FLOATING	FLOATING	KB <sub>2</sub>	$\frac{Q_0}{W_0} \frac{K+1}{B_1}$	$\frac{1}{W_0 Q_0} \frac{1}{B_2 (K+1)}$	$W_0^2 B_1 C_1 B_2 C_2 - 1 = 0$	$Q_0^2 (K+1)^2 \frac{C_2 B_2}{C_1 B_1} - 1 = 0$	N/A	$C_1 + C_2$	$B_1 + B_2 + B_3$
MIN C1 (10001)	FLOATING	FLOATING	KB <sub>2</sub>	$\frac{Q_0}{W_0} \frac{K+1}{B_1}$	$\frac{1}{W_0 Q_0} \frac{1}{B_2 (K+1)}$	$W_0^2 B_1 C_1 B_2 C_2 - 1 = 0$	$Q_0^2 (K+1)^2 \frac{C_2 B_2}{C_1 B_1} - 1 = 0$	$\frac{C_1}{C_2} - 1 = 0$	$C_1 + C_2$	$B_1 + B_2 + B_3$

TABLE - 7 EXPRESSIONS OF ELEMENTS AND PARAMETERS USED IN THE MINIMIZATION PROGRAMS (FLETCHER-POWELL'S METHOD)

PROGRAM NAME AND NUMBER	ELEMENT CALCULATIONS				CONSTRAINTS				OBJECTIVE FUNCTION			
	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	C <sub>1</sub>	C <sub>2</sub>	Constrain 1	Constrain 2	Constrain 3	C <sub>1</sub>	R <sub>1</sub>	A <sub>1</sub> A <sub>2</sub> A <sub>3</sub>	PAR +
C <sub>1</sub> MIN (13to15)	FLOATING	FLOATING	FLOATING	$\frac{R_2+R_3}{R_1 \cdot R_2 \cdot W_0}$	$\frac{1}{R_2+R_3} \frac{1}{Q_0 W_0}$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$	$Q_0^2 (R_2+R_3)^2 \frac{C_2}{C_1 R_1 R_2} - 1 = 0$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$	$C_1 + C_2$	-	-	-
C <sub>2</sub> MIN (16to18)	FLOATING	FLOATING	FLOATING	$\frac{R_2+R_3}{R_1 \cdot R_2 \cdot W_0}$	$\frac{1}{R_2+R_3} \frac{1}{Q_0 W_0}$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$	$Q_0^2 (R_2+R_3)^2 \frac{C_2}{C_1 R_1 R_2} - 1 = 0$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$	$C_1 + C_2$	-	-	-
C <sub>3</sub> MIN (19to21)	FLOATING	FLOATING	KE <sub>2</sub>	$\frac{Q_0}{W_0} \frac{R_2+R_3}{R_1}$	$\frac{1}{W_0 Q_0} \frac{1}{R_2 (k+1)}$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$	$Q_0^2 (k+1)^2 \frac{C_2 R_2}{C_1 R_1} - 1 = 0$	NONE	$C_1 + C_2$	-	-	-
C <sub>4</sub> MIN (22to24)	FLOATING	FLOATING	KE <sub>2</sub>	$\frac{Q_0}{W_0} \frac{R_2+R_3}{R_1}$	$\frac{1}{W_0 Q_0} \frac{1}{R_2 (k+1)}$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$	$Q_0^2 (k+1)^2 \frac{C_2 R_2}{C_1 R_1} - 1 = 0$	$\frac{C_1}{C_2} - 1 = 0$	$C_1 + C_2$	-	-	-
R <sub>1</sub> MIN (25to27)	FLOATING	FLOATING	FLOATING	$\frac{R_2+R_3}{R_1 \cdot R_2 \cdot W_0}$	$\frac{1}{R_2+R_3} \frac{1}{Q_0 W_0}$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$	$Q_0^2 (R_2+R_3)^2 \frac{C_2}{C_1 R_1 R_2} - 1 = 0$	NONE	-	$R_1 + R_2 + R_3$	-	-
R <sub>2</sub> MIN (28to30)	FLOATING	FLOATING	FLOATING	$\frac{R_2+R_3}{R_1 \cdot R_2 \cdot W_0}$	$\frac{1}{R_2+R_3} \frac{1}{Q_0 W_0}$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$	$Q_0^2 (R_2+R_3)^2 \frac{C_2}{C_1 R_1 R_2} - 1 = 0$	$\frac{C_1}{C_2} - 1 = 0$	-	$R_1 + R_2 + R_3$	-	-
R <sub>3</sub> MIN (31to33)	FLOATING	FLOATING	KE <sub>2</sub>	$\frac{Q_0}{W_0} \frac{R_2+R_3}{R_1}$	$\frac{1}{W_0 Q_0} \frac{1}{R_2 (k+1)}$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$	$Q_0^2 (k+1)^2 \frac{C_2 R_2}{C_1 R_1} - 1 = 0$	NONE	-	$R_1 + (k+1) R_2$	-	-
R <sub>4</sub> MIN (34to36)	FLOATING	FLOATING	KE <sub>2</sub>	$\frac{Q_0}{W_0} \frac{R_2+R_3}{R_1}$	$\frac{1}{W_0 Q_0} \frac{1}{R_2 (k+1)}$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$	$Q_0^2 (k+1)^2 \frac{C_2 R_2}{C_1 R_1} - 1 = 0$	$\frac{C_1}{C_2} - 1 = 0$	-	$R_1 + (k+1) R_2$	-	-

TABLE - 7 EXPRESSIONS OF ELEMENTS AND PARAMETERS USED IN THE MINIMIZATION PROGRAMS (FLITCHER-POWELL'S METHOD) CONT.

PROGRAM NAME AND NUMBER	ELEMENT CALCULATIONS					CONSTRAINTS			OBJECTIVE FUNCTION		
	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	C <sub>1</sub>	C <sub>2</sub>	Constraint 1	Constraint 2	Constraint 3	C <sub>1</sub>	R <sub>T</sub>	$\frac{A_1^2 + 2A_2^2}{C_1^2 + A_2^2}$
SCHEM 1 (3710039)	FLOATING	FLOATING	FLOATING	$\frac{R_2 + R_3}{R_1} \frac{Q_1}{R_2} \frac{Q_2}{R_3}$	$\frac{1}{R_2 + R_3} \frac{1}{Q_1 Q_2}$	$Q_1^2 R_1 C_1 R_2 C_2 - 1 = 0$	$Q_1^2 (R_2 + R_3)^2 \frac{C_2}{C_1 R_1 R_2} - 1 = 0$	NONE	$C_1 + C_2$	$R_1 + R_2 + R_3$	X X X
SCHEM 2 (4010042)	FLOATING	FLOATING	FLOATING	$\frac{R_2 + R_3}{R_1} \frac{Q_1}{R_2} \frac{Q_2}{R_3}$	$\frac{1}{R_2 + R_3} \frac{1}{Q_1 Q_2}$	$Q_1^2 R_1 C_1 R_2 C_2 - 1 = 0$	$Q_1^2 (R_2 + R_3)^2 \frac{C_2}{C_1 R_1 R_2} - 1 = 0$	$\frac{C_2}{C_1} - 1 = 0$	$C_1 + C_2$	$R_1 + R_2 + R_3$	X X X
SCHEM 3 (4310045)	FLOATING	FLOATING	KB <sub>2</sub>	$\frac{Q_1}{R_1} \frac{R_2 + R_3}{R_1}$	$\frac{1}{R_2 C_2} \frac{1}{R_3 C_1}$	$Q_1^2 R_1 C_1 R_2 C_2 - 1 = 0$	$Q_1^2 (R_2 + R_3)^2 \frac{C_2}{C_1 R_1 R_2} - 1 = 0$	NONE	$C_1 + C_2$	$R_1 + (R_2 + R_3)$	X X X
SCHEM 4 (4610048)	FLOATING	FLOATING	KB <sub>2</sub>	$\frac{Q_1}{R_1} \frac{R_2 + R_3}{R_1}$	$\frac{1}{R_2 C_2} \frac{1}{R_3 C_1}$	$Q_1^2 R_1 C_1 R_2 C_2 - 1 = 0$	$Q_1^2 (R_2 + R_3)^2 \frac{C_2}{C_1 R_1 R_2} - 1 = 0$	$\frac{C_2}{C_1} - 1 = 0$	$C_1 + C_2$	$R_1 + (R_2 + R_3)$	X X X

$A_1^2 = A_1 C_1$ , where  $A_1 = \frac{V}{E_0 k r g}$ , and  $V$  = Breaking Voltage of the dielectric material  
 $E_0 = 8.85 \times 10^{-12}$ , is the permittivity of the free space ( $\frac{\text{Farads}}{\text{cm}}$ )

$A_2^2 = A_2 R_1$ , where  $A_2 = \frac{V}{E_0}$ , and  $V$  = width of the thin-film resistive strip (cm)  
 $R_1$  = sheet resistance of the resistive strip (ohms)

$R_2 = \frac{Q_1}{R_1} \frac{Q_2}{R_3}$ , and  $V$  = width of the thin-film resistive strip (cm)  
 $R_3$  = sheet resistance of the resistive strip (ohms)



TABLE - 8 CALCULATED GRADIENTS OF FLETCHER-POWELL'S MINIMIZATION ALGORITHM

	C <sub>1</sub> MIN	C <sub>2</sub> MIN	C <sub>3</sub> MIN
<b>PENALTY MULTIPLIER</b>			
K <sub>1</sub>	1000	500	
K <sub>2</sub>	1000	100000000	
K <sub>3</sub>		10000	
<b>CONSTRAINT:</b>			
b <sub>1</sub>	W <sub>0</sub> <sup>2</sup> R <sub>1</sub> C <sub>1</sub> R <sub>2</sub> C <sub>2</sub> -1=0	W <sub>0</sub> <sup>2</sup> R <sub>1</sub> C <sub>1</sub> R <sub>2</sub> C <sub>2</sub> -1=0	
b <sub>2</sub>	C <sub>2</sub> <sup>2</sup> (R <sub>2</sub> <sup>2</sup> +R <sub>3</sub> <sup>2</sup> )+C <sub>2</sub> (C <sub>1</sub> R <sub>1</sub> R <sub>2</sub> )-1=0	C <sub>2</sub> <sup>2</sup> (R <sub>2</sub> <sup>2</sup> +R <sub>3</sub> <sup>2</sup> )+C <sub>2</sub> (C <sub>1</sub> R <sub>1</sub> R <sub>2</sub> )-1=0	
b <sub>3</sub>	C <sub>1</sub> /C <sub>2</sub> -1=0	C <sub>1</sub> /C <sub>2</sub> -1=0	
<b>VARIABLE : x<sub>i</sub></b>			
<b>(NORMALIZED)</b>			
x <sub>1</sub>	A <sub>1</sub> C <sub>1</sub>	A <sub>1</sub> C <sub>1</sub>	A <sub>1</sub> C <sub>1</sub>
x <sub>2</sub>	A <sub>1</sub> C <sub>2</sub>	A <sub>1</sub> C <sub>2</sub>	A <sub>1</sub> C <sub>2</sub>
x <sub>3</sub>	A <sub>2</sub> R <sub>1</sub>	A <sub>2</sub> R <sub>1</sub>	A <sub>2</sub> R <sub>1</sub>
x <sub>4</sub>	A <sub>2</sub> R <sub>2</sub>	A <sub>2</sub> R <sub>2</sub>	A <sub>2</sub> R <sub>2</sub>
x <sub>5</sub>	A <sub>2</sub> R <sub>3</sub>	A <sub>2</sub> R <sub>3</sub>	A <sub>2</sub> R <sub>3</sub>
<b>UNCONSTRAINED</b>			
FUNCTION: F(x)	A <sub>1</sub> (C <sub>1</sub> +C <sub>2</sub> )	A <sub>1</sub> (C <sub>1</sub> +C <sub>2</sub> )	A <sub>1</sub> (C <sub>1</sub> +C <sub>2</sub> )
CONSTRAINED			
FUNCTION: P(x)	A <sub>1</sub> (C <sub>1</sub> +C <sub>2</sub> )+k <sub>1</sub> b <sub>1</sub> <sup>2</sup> +k <sub>2</sub> b <sub>2</sub> <sup>2</sup>	A <sub>1</sub> (C <sub>1</sub> +C <sub>2</sub> )+k <sub>1</sub> b <sub>1</sub> <sup>2</sup> +k <sub>2</sub> b <sub>2</sub> <sup>2</sup> +k <sub>3</sub> b <sub>3</sub> <sup>2</sup>	A <sub>1</sub> (C <sub>1</sub> +C <sub>2</sub> )+k <sub>1</sub> b <sub>1</sub> <sup>2</sup> +k <sub>2</sub> b <sub>2</sub> <sup>2</sup>
<b>GRADIENT OF</b>			
<b>G(i) = ∂F/∂x<sub>i</sub></b>			
G(1)	1 + $\frac{2}{C_1} (k_1 b_1 (b_1+1) - k_2 b_2 (b_2+1))$	1 + $\frac{2}{C_1} (k_1 b_1 (b_1+1) - k_2 b_2 (b_2+1)) + k_3 b_3 (b_3+1)$	1 + $\frac{2}{C_1} (k_1 b_1 (b_1+1) - k_2 b_2 (b_2+1))$
G(2)	1 + $\frac{2}{C_2} (k_1 b_1 (b_1+1) + k_2 b_2 (b_2+1))$	1 + $\frac{2}{C_2} (k_1 b_1 (b_1+1) + k_2 b_2 (b_2+1) - k_3 b_3 (b_3+1))$	1 + $\frac{2}{C_2} (k_1 b_1 (b_1+1) + k_2 b_2 (b_2+1))$
G(3)	$\frac{2}{R_1} (k_1 b_1 (b_1+1) - k_2 b_2 (b_2+1))$	$\frac{2}{R_1} (k_1 b_1 (b_1+1) - k_2 b_2 (b_2+1))$	$\frac{2}{R_1} (k_1 b_1 (b_1+1) - k_2 b_2 (b_2+1))$
G(4)	$\frac{2}{R_2} (k_1 b_1 (b_1+1) + \frac{R_2 - R_3}{R_2 + R_3} k_2 b_2 (b_2+1))$	$\frac{2}{R_2} (k_1 b_1 (b_1+1) + \frac{R_2 - R_3}{R_2 + R_3} k_2 b_2 (b_2+1))$	$\frac{2}{R_2} (k_1 b_1 (b_1+1) + k_2 b_2 (b_2+1))$
G(5)	$\frac{4}{(R_2 + R_3)} k_2 b_2 (b_2+1)$	$\frac{4}{(R_2 + R_3)} k_2 b_2 (b_2+1)$	$\frac{4}{R_2 + R_3} k_2 b_2 (b_2+1)$

$\frac{4}{R_2 + R_3} A_1 = \frac{2}{3} \frac{V}{E_0 E_1 E_2} \text{ AND } A_2 = \frac{V^2}{E_0 E_1} \text{ (SEE TABLE - 7)}$

TABLE - 8 CONTINUED

PENALTY MULTIPLIERS	$C_1$ MIN	$R_1$ MIN	$G$	$P_2$ MIN
$K_1$ $K_2$ $K_3$	100000 1000 10000	500 500		1000 100000 50000
CONSTRAINT :				
$h_1$ $h_2$ $h_3$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$ $Q_0^2 (1+K) C_2 R_2 / (C_1 R_1) - 1 = 0$ $C_1 / C_2 - 1 = 0$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$ $Q_0^2 (R_2 + R_3)^2 / (C_1 R_1 R_2) - 1 = 0$		$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$ $Q_0^2 (R_2 + R_3)^2 C_2 / (C_1 R_1 R_2) - 1 = 0$
VARIABLE : $x_i$ (normalised)				
$x_1$ $x_2$ $x_3$ $x_4$ $x_5$	$A_1 C_1$ $A_1 C_2$ $A_2 R_1$ $A_2 R_2$ $A_2 R_3$	$A_1 C_1$ $A_1 C_2$ $A_2 R_1$ $A_2 R_2$ $A_2 R_3$		$A_1 C_1$ $A_1 C_2$ $A_2 R_1$ $A_2 R_2$ $A_2 R_3$
UNCONSTRAINED FUNCTION: $P(x_i)$	$A_1 (C_1 + C_2)$	$A_2 (R_1 + R_2 + R_3)$		$A_2 (R_1 + R_2 + R_3)$
CONSTRAINED FUNCTION: $F(x_i)$	$A_1 (C_1 + C_2) + K_1 h_1^2 + K_2 h_2^2 + K_3 h_3^2$	$A_2 (R_1 + R_2 + R_3) + K_1 h_1^2 + K_2 h_2^2$		$R_2 (R_1 + R_2 + R_3) + K_1 h_1^2 + K_2 h_2^2 + K_3 h_3^2$
GRADIENT $G(1) = \frac{\partial P}{\partial x_1}$	$1 + \frac{2}{C_1} (K_1 h_1 (h_1 + 1) - K_2 h_2 (h_2 + 1) + K_3 h_3 (h_3 + 1))$ $1 + \frac{2}{C_2} (K_1 h_1 (h_1 + 1) + K_2 h_2 (h_2 + 1) - K_3 h_3 (h_3 + 1))$ $1 + \frac{2}{R_1} (K_1 h_1 (h_1 + 1) - K_2 h_2 (h_2 + 1))$ $1 + \frac{2}{R_2} (K_1 h_1 (h_1 + 1) + K_2 h_2 (h_2 + 1))$ $1 + \frac{4}{R_2 + R_3} K_2 h_2 (h_2 + 1)$	$1 + \frac{2}{C_1} (K_1 h_1 (h_1 + 1) - K_2 h_2 (h_2 + 1))$ $1 + \frac{2}{C_2} (K_1 h_1 (h_1 + 1) + K_2 h_2 (h_2 + 1))$ $1 + \frac{2}{R_1} (K_1 h_1 (h_1 + 1) - K_2 h_2 (h_2 + 1))$ $1 + \frac{2}{R_2} (K_1 h_1 (h_1 + 1) + K_2 h_2 (h_2 + 1))$ $1 + \frac{4}{R_2 + R_3} K_2 h_2 (h_2 + 1)$		$1 + \frac{2}{C_1} (K_1 h_1 (h_1 + 1) - K_2 h_2 (h_2 + 1) - K_3 h_3 (h_3 + 1))$ $1 + \frac{2}{C_2} (K_1 h_1 (h_1 + 1) + K_2 h_2 (h_2 + 1) + K_3 h_3 (h_3 + 1))$ $1 + \frac{2}{R_1} (K_1 h_1 (h_1 + 1) - K_2 h_2 (h_2 + 1))$ $1 + \frac{2}{R_2} (K_1 h_1 (h_1 + 1) + K_2 h_2 (h_2 + 1))$ $1 + \frac{4}{R_2 + R_3} K_2 h_2 (h_2 + 1)$

TABLE - 8 CONTINUED

FENALTY MULTIPLIER	R <sub>3</sub> MIN	R <sub>4</sub> MIN	R <sub>5</sub> MIN	KCHIM <sub>1</sub>
K <sub>1</sub> K <sub>2</sub> K <sub>3</sub>	1200 500	5000 1000 5000		100 500
CONSTRAINT:				
b <sub>1</sub> b <sub>2</sub> b <sub>3</sub>	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$ $Q_0^2 (K+1) R_2 C_2 / (R_1 C_1) - 1 = 0$ $C_1 / C_2 - 1 = 0$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$ $Q_0^2 (K+1) R_2 C_2 / (R_1 C_1) - 1 = 0$ $C_1 / C_2 - 1 = 0$		$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$ $Q_0^2 (K+1) R_2 C_2 / (C_1 R_1 R_2) - 1 = 0$
VARIABLE: x <sub>i</sub> (normalised)				
x <sub>1</sub> x <sub>2</sub> x <sub>3</sub> x <sub>4</sub> x <sub>5</sub>	A <sub>1</sub> C <sub>1</sub> A <sub>1</sub> C <sub>2</sub> A <sub>2</sub> R <sub>1</sub> A <sub>2</sub> R <sub>2</sub> A <sub>2</sub> R <sub>3</sub> -KA <sub>2</sub> R <sub>2</sub>	A <sub>1</sub> C <sub>1</sub> A <sub>1</sub> C <sub>2</sub> A <sub>2</sub> R <sub>1</sub> A <sub>2</sub> R <sub>2</sub> A <sub>2</sub> R <sub>3</sub> -KA <sub>2</sub> R <sub>2</sub>		A <sub>1</sub> C <sub>1</sub> A <sub>1</sub> C <sub>2</sub> A <sub>2</sub> R <sub>1</sub> A <sub>2</sub> R <sub>2</sub> A <sub>2</sub> R <sub>3</sub>
UNCONSTRAINED				
FUNCTION: F(x <sub>i</sub> )	A <sub>2</sub> , R <sub>1</sub> +(K+1)R <sub>2</sub>	A <sub>2</sub> (R <sub>1</sub> +(K+1)R <sub>2</sub> )		3(A <sub>1</sub> (C <sub>1</sub> +C <sub>2</sub> ))+2(A <sub>2</sub> (R <sub>1</sub> +R <sub>2</sub> +R <sub>3</sub> ))A <sub>2</sub> C
CONSTRAINED				
FUNCTION: F(x <sub>i</sub> )	A <sub>2</sub> R <sub>1</sub> +(K+1)R <sub>2</sub> + K <sub>1</sub> b <sub>1</sub> <sup>2</sup> +K <sub>2</sub> b <sub>2</sub> <sup>2</sup>	A <sub>2</sub> (R <sub>1</sub> +(K+1)R <sub>2</sub> )+K <sub>1</sub> b <sub>1</sub> <sup>2</sup> +K <sub>2</sub> b <sub>2</sub> <sup>2</sup> +K <sub>3</sub> b <sub>3</sub> <sup>2</sup>		A <sub>2</sub> C+K <sub>1</sub> b <sub>1</sub> <sup>2</sup> +K <sub>2</sub> b <sub>2</sub> <sup>2</sup>
GRADIENT				
G(1) = $\frac{\partial F}{\partial x_1}$	$\frac{2}{C_1} (K_1 b_1 (b_1+1) - K_2 b_2 (b_2+1))$ $\frac{2}{C_2} (K_1 b_1 (b_1+1) + K_2 b_2 (b_2+1))$ $1 + \frac{2}{K_1} K_1 b_1 (b_1+1) - K_2 b_2 (b_2+1)$ $K+1 - \frac{2}{K_2} (K_1 b_1 (b_1+1) + K_2 b_2 (b_2+1))$ $(\frac{4}{K_2+K_3}) K_2 b_2 (b_2+1)$	$\frac{2}{C_1} (K_1 b_1 (b_1+1) - K_2 b_2 (b_2+1))$ $\frac{2}{C_2} (K_1 b_1 (b_1+1) + K_2 b_2 (b_2+1))$ $1 + \frac{2}{K_1} K_1 b_1 (b_1+1) - K_2 b_2 (b_2+1)$ $K+1 - \frac{2}{K_2} (K_1 b_1 (b_1+1) + K_2 b_2 (b_2+1))$ $(\frac{4}{K_2+K_3}) K_2 b_2 (b_2+1)$	$\frac{2}{C_1} (K_1 b_1 (b_1+1) - K_2 b_2 (b_2+1))$ $\frac{2}{C_2} (K_1 b_1 (b_1+1) + K_2 b_2 (b_2+1))$ $1 + \frac{2}{K_1} K_1 b_1 (b_1+1) - K_2 b_2 (b_2+1)$ $K+1 - \frac{2}{K_2} (K_1 b_1 (b_1+1) + K_2 b_2 (b_2+1))$ $(\frac{4}{K_2+K_3}) K_2 b_2 (b_2+1)$	$3 - \frac{2}{C_1} (K_1 b_1 (b_1+1) - K_2 b_2 (b_2+1))$ $3 - \frac{2}{C_2} (K_1 b_1 (b_1+1) + K_2 b_2 (b_2+1))$ $2 + \frac{2}{K_1} (K_1 b_1 (b_1+1) - K_2 b_2 (b_2+1))$ $2 + \frac{2}{K_2} (K_1 b_1 (b_1+1) + K_2 b_2 (b_2+1))$ $2 + (\frac{4}{K_2+K_3}) K_2 b_2 (b_2+1)$

TABLE - 8 CONTINUED

PRIORITY MULTIPLIER	RCHLN2	RCHLN3	RCHLN4
$K_1$ $K_2$ $K_3$	100 500 1000	2000 2000	3200 6400 3200
CONSTRAINT:			
$h_1$ $h_2$ $h_3$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$ $Q_0^2 (K_2 + H_3) C_2 / (C_1 R_1 R_2) - 1 = 0$ $C_2 / C_1 - 1 = 0$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$ $Q_0^2 (K_2 + H_3) C_2 / (C_1 R_1 R_2) - 1 = 0$	$W_0^2 R_1 C_1 R_2 C_2 - 1 = 0$ $Q_0^2 (K_2 + H_3) C_2 / (C_1 R_1 R_2) - 1 = 0$ $C_2 / C_1 - 1 = 0$
VARIABLE: $x_i$ (normalized)			
$x_1$ $x_2$ $x_3$ $x_4$ $x_5$	$A_1 C_1$ $A_1 C_2$ $A_2 R_1$ $A_2 R_2$ $A_2 R_3$	$A_1 C_1$ $A_1 C_2$ $A_2 R_1$ $A_2 R_2$ $A_2 R_3 = K_4 R_2$	$A_1 C_1$ $A_1 C_2$ $A_2 R_1$ $A_2 R_2$ $A_2 R_3 = K_4 R_2$
UNCONSTRAINED			
FUNCTION: $F(x_i)$	$3(A_1(C_1+C_2)) + 2(A_2(R_1+R_2+R_3)) = A_3 C$	$3(A_1(C_1+C_2)) + 2(A_2(R_1+(K_2+1)R_2)) = A_3 C$	$3(A_1(C_1+C_2)) + 2(A_2(R_1+(K_2+1)R_2)) = A_3 C$
CONSTRAINT			
FUNCTION: $F(x_i)$	$A_3 C + K_1 h_1^2 + K_2 h_2^2 + K_3 h_3^2$	$A_3 C + K_1 h_1^2 + K_2 h_2^2$	$A_3 C + K_1 h_1^2 + K_2 h_2^2 + K_3 h_3^2$
GRADIENT $\frac{\partial F}{\partial x_i}$			
G(1)	$3 + \frac{2}{C_1} (K_1 h_1 (h_1+1) - K_2 h_2 (h_2+1) - K_3 h_3 (h_3+1))$	$3 + \frac{2}{C_1} (K_1 h_1 (h_1+1) - K_2 h_2 (h_2+1))$	$3 + \frac{2}{C_1} (K_1 h_1 (h_1+1) - K_2 h_2 (h_2+1) - K_3 h_3 (h_3+1))$
G(2)	$3 + \frac{2}{C_2} (K_1 h_1 (h_1+1) + K_2 h_2 (h_2+1) + K_3 h_3 (h_3+1))$	$3 + \frac{2}{C_2} (K_1 h_1 (h_1+1) + K_2 h_2 (h_2+1))$	$3 + \frac{2}{C_2} (K_1 h_1 (h_1+1) + K_2 h_2 (h_2+1) + K_3 h_3 (h_3+1))$
G(3)	$2 + \frac{2}{R_1} (K_1 h_1 (h_1+1) - K_2 h_2 (h_2+1))$	$2 + \frac{2}{R_1} (K_1 h_1 (h_1+1) - K_2 h_2 (h_2+1))$	$2 + \frac{2}{R_1} (K_1 h_1 (h_1+1) - K_2 h_2 (h_2+1))$
G(4)	$2 + \frac{2}{R_2} (K_1 h_1 (h_1+1) + \frac{R_2 - R_3}{R_2 R_3} K_2 h_2 (h_2+1))$	$2(K_2+1) + \frac{2}{R_2} K_1 h_1 (h_1+1) + K_2 h_2 (h_2+1)$	$2 + \frac{2}{R_2} (K_1 h_1 (h_1+1) + \frac{R_2 - R_3}{R_2 R_3} K_2 h_2 (h_2+1))$
G(5)	$2 + (\frac{4}{R_2 + R_3} + R_3) K_2 h_2 (h_2+1)$	$(\frac{4}{R_2 + R_3} + R_3) K_2 h_2 (h_2+1)$	$2 + (\frac{4}{R_2 + R_3} + R_3) K_2 h_2 (h_2+1)$

To take into account the presence of these constraints, the penalty-function (9,10) is introduced, where the constrained function (i.e.  $\min_{\bar{x}} f(\bar{x})$ , such that  $g_i(\bar{x})=0, i=1,2,\dots,r$ ) is converted into the new unconstrained function  $P(\bar{x};k_i)$  where  $P(\bar{x};k_i) = f(\bar{x}) + \sum_{i=1}^r k_i g_i(\bar{x})^2 \dots\dots\dots(5.1)$  Here,  $k_i$  is the penalty-function multiplier, which is arbitrary, but a very important real positive constant.

The dimension of  $f(\bar{x})$  is  $m$ , but that of  $P(\bar{x};k_i)$  is  $n$ ;  $n>m$ . So  $\bar{x} = [x_1, x_2, \dots, x_n]^T$ , for all the unconstrained objective functions  $P(\bar{x};k_i)$  and for both algorithms.

### 5.3 The Optimal-Design Constraints

In order to maintain the specific parameters  $\omega_0$  and  $Q_0$  within close limits of their desired values, constraint-functions  $g_1$  and  $g_2$ , respectively, must be specified in the main programs. For example, the parameters  $\omega_0$  and  $Q_0$  can retain their values, at 10 and 50 KHZ, respectively, within .1 percent accuracy. The parameter  $H_{BP}$  can also be assigned a value within a range of real positive values. For example,  $0 < H_{BP} < 1$  where  $H_{BP} = \frac{R_3}{R_2 + R_3}$ , and  $R_2$  and  $R_3$  take real positive values.

In order to achieve inequality condition in  $H_{BP}$ , two constraint-functions  $g_3$  and  $g_4$  should be specified, where  $g_3 = H_{BP}$ , when  $H_{BP} > 0$ , and  $g_4 = H_{BP}$ , when  $H_{BP} < 1$ . Note that  $H_{BP}$  is not allowed to take any of the two limit values in the inequality-constraints  $g_3$  and  $g_4$ .

If it did, both values would violate the feasible region of values in the transfer function  $T(s)$ , which is a function of  $C_1$ ,  $C_2$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .

The equality  $H_{BP} = \frac{R_3}{R_2 + R_3}$  can be specified as another constraint-function,  $g_5$ , where  $g_5 = H_{BP} - \frac{R_3}{R_2 + R_3} = 0$ .

The variables  $x_i$ ,  $i=1,2,\dots,5$ , must be real positive ( $0 < x_i < \epsilon$ ), where " $\epsilon$ " is the maximum component-value achieved by solid-state techniques. Ten inequality-constraint functions

$g_6$  through  $g_{15}$  are required for the five variables  $x_i$ . Finally, the constraint-function  $g_{16}$  will determine the required case:  $C_1 = C_2$ . Here,  $g_{16} = \frac{C_1}{C_2} - 1 = 0$ . A more careful observation, for this technical problem to be feasible, shows that constraints  $g_2$  and  $g_5$  are inter-dependent, thus, they cannot be satisfied independently, and hence simultaneously. In one case,  $H_{BP}$  is required to be constant. This can be achieved by letting  $R_3 = KR_2$ , where  $K$  is a constant. Constraints  $g_3$ ,  $g_4$  and  $g_5$  can be eliminated, allowing constraint  $g_2$  to be satisfied. Constraints  $g_6$  through  $g_{15}$  can be eliminated, being redundant since all elements  $x_i$  are real positive and bounded by upper and lower limits:

The remaining three constraint-functions are  $g_1$ ,  $g_2$ , and  $g_{16}$  which will be called  $g_1$ ,  $g_2$  and  $g_3$ , respectively. Given

the three specifications  $\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$ ,  $Q_0 = \frac{1}{R_2 + R_3} \sqrt{\frac{C_1 R_1 R_2}{C_2}}$  and

$C_1 = C_2$ , the corresponding constraints are:

$$g_1 = \omega_0^2 R_1 R_2 C_1 C_2 - 1 = 0 \dots \dots \dots (5.2)$$

$$g_2 = Q_0^2 (R_2 + R_3)^2 \frac{C_2}{C_1 R_1 R_2} - 1 = 0 \dots \dots \dots (5.3a)$$

when  $R_2$  is independent of  $R_3$ , or

$$g_2 = Q\sigma^2 (k+1)^2 \frac{C_2 R_2}{C_1 R_1} - 1 = 0 \dots \dots \dots (5.3b)$$

where,  $R_3 = KR_2 \dots \dots \dots (5.3c)$

$$g_3 = \frac{C_1}{C_2} - 1 = 0 \dots \dots \dots (5.4)$$

5.4 The Objective-Function Gradients

Quasi-Newton's and Fletcher-Powell's optimization algorithms compute the gradients  $\nabla f(\bar{x})$  of the objective function  $f(\bar{x})$  which are required in the respective recurrence formulae:

$$\bar{x}_{i+1} = \bar{x}_i - H^{-1}(\bar{x}_i) \nabla f(\bar{x}_i) \dots \dots \dots (9) \dots \dots \dots (5.5)$$

$$\text{and } \bar{x}_{i+1} = \bar{x}_i - \lambda_n H_i \nabla f(\bar{x}_i) \dots \dots \dots (5.6)$$

Here,  $\lambda_n$  is the step-size of the gradient change, during the "descent" from one iteration to the next, and  $H^{-1}$  and  $H_i$  are the inverse Hessian matrices of the objective function  $f(\bar{x})$ . Subscript "i" denotes the iteration number. In this study, given the above constraints, the objective function  $f(\bar{x})$  is

replaced by the new objective function  $P(\bar{x}; K_i)$  that includes the penalty function. Similarly  $\nabla f(\bar{x})$  is replaced by  $\nabla P(\bar{x}, K_i)$ ,

where:  $P(\bar{x}; K_i) = f(\bar{x}) + \sum_{i=1}^r K_i f(\bar{x})^2 \dots \dots \dots (5.7)$

$$\text{and } \nabla P(\bar{x}; K_i) = \begin{bmatrix} \frac{\partial P}{\partial x_1} \\ \frac{\partial P}{\partial x_2} \\ \frac{\partial P}{\partial x_3} \\ \frac{\partial P}{\partial x_4} \\ \frac{\partial P}{\partial x_5} \end{bmatrix} = \begin{bmatrix} G(1) \\ G(2) \\ G(3) \\ G(4) \\ G(5) \end{bmatrix} \dots \dots \dots (5.8)$$

For local minimum,  $\nabla P(\bar{x}; K_1) = 0$ .....(5.9)

The computer-subroutines corresponding to the Quasi-Newton's and Fletcher-Powell's algorithms are "ZXMIN" and FMFM", respectively. Both subroutines use the gradient-vector  $\nabla P(\bar{x}; K_i)$ . For illustration purposes, the gradient vector of the objective function  $C_T$  in case 1a (Table - 4) is derived in Appendix D. The gradient-vectors of all cases in the Fletcher-Powell algorithm are presented in Table-8.

## 5.5 Determination of Ac and Ar

### 5.5.1 Thin-Film Components (11,12,13)

#### 5.5.1.1 Capacitors

In the IC fabrication, the two basic types of capacitors are: "thin-film" and "junction" capacitors. Both types consist of two (or more) low-resistance layers of electrodes; the bottom electrode is called "base electrode" and the top electrode is called "counterelectrode".

The plates (electrodes) of thin-film capacitors are formed by depositing a true metal (Al, Au and Ta), and the carrier-free region is formed by a dielectric material ( $S_iO$ ,  $Ta_2O_5$ , and  $MnO_2$ ). A typical group of these metals and dielectrics is demonstrated in Table - 13.



In the case of "junction" capacitors, both plates are formed by diffused low-resistance layers of opposite dopant types, and the space-charge layer results from the depletion of charges at the pn-junction.

The capacitance is given by:

$$C = \frac{\epsilon_r \epsilon_0 A}{d} \text{ Farads} \dots\dots\dots (5.10)$$

The capacitance-density is given by:  $\frac{C}{A} = \frac{\epsilon_r \epsilon_0}{d} \dots\dots\dots (5.11)$

If  $A_c$  is the total thin-film area occupied by the total capacitance  $C_T$ , then, it can be shown that:  $\frac{C_T}{A_c} = K \dots\dots (5.12)$

where,  $K = \frac{3}{2} \epsilon_0 (\epsilon_r / V_{bi}) \dots\dots\dots (5.13)$

It is estimated that  $V_{bi} \approx \frac{2}{3} V_a$ . Eq. (5.12) can be written as:

$$A_c = \frac{1}{K} C_T = nA_i = nw^2 \dots\dots\dots (5.14)$$

where the length "l" of the unit-square and the width "w" of the thin-film are equal ( $l = w$ ) and n is the number of unit-squares required. From eq. (5.14), it can be seen that the total area  $A_c$  is directly related to the total capacitance  $C_T$ . Therefore, minimization of  $A_c$  implies corresponding reduction of  $C_T$ , since K is a constant multiplier.

Detailed analysis for calculating  $A_c$  is presented in Appendix E.

## 5.5.1.2

Resistors

Integrated resistors are of two types: "diffused" and "deposition" (thin-or thick-film) resistors.

Even though diffused resistors can be found simultaneously with transistors, diodes, and diffused capacitors, they exhibit large TCR and their range of resistor values is limited.

Thin-and thick-film resistors require additional steps of processing, but they offer much wider range of resistance, together with reduced TCR and narrower tolerances. Table - 14 identifies some thin-film materials suitable for integrated resistors. From Table-14 it is clear that tantalum is a preferred material.

In the integrated-circuit technology, a thin-film exhibits total resistance given by:

$$R_T = \frac{\rho [\text{ohm-cm}]}{d [\text{cm}]} \cdot \frac{l [\text{cm}]}{w [\text{cm}]} = \frac{\rho}{d} \cdot \frac{l}{w} \text{ ohms} \dots\dots\dots(5.15)$$

The ratio  $(\frac{l}{w})$  is called "aspect ratio" and the ratio  $(\frac{\rho}{d})$  is called "sheet resistance"  $R_S$ . When  $l = w$ , then eq.

(5.15) becomes:

$$R_T = \frac{\rho}{d} = R_S \text{ ohms / } \square \dots\dots\dots(5.16)$$

When  $l = nw$ , where  $n$  is an integer number,

$$R_T = n \frac{\rho}{d} = nR_S \text{ ohms} \dots\dots\dots(5.17)$$

The resistance-density is given by

$$\text{From eq. (5.15)} \quad \frac{R_T}{A_R} = \frac{R_S}{l^2} \dots\dots\dots(5.18)$$

$$\text{and} \quad A_R = \frac{w^2}{R_S} \cdot R_T, \text{ cm}^2 \dots\dots\dots(5.19)$$

Note that eq. (5.18) was derived by assuming:

$$nA_i = n(l \times w) = nw^2 \dots\dots\dots(5.20)$$

Derivation of eq. (5.19) is presented in Appendix E.

From eq. (5.19 and (5.20),  $A_T$  can be minimized (i.e. the number of resistance squares can be reduced) by minimizing the total resistance  $R_T$  through minimization algorithms.

#### 5.5.2 Range of the Passive Component Values

The range of resistance per substrate, using tantalum thin-film material, is limited at the high and low ends, the upper limit being  $10,000R_s$ , where  $R_s$  is in the range of 10 - 100 ohms/ $\square$ . This limit is due to the maximum number of squares attained in a given resistor pattern, compatible with an acceptable yield in production. The lower limit is  $\frac{1}{2} R_s$ , due to the tolerance of the minimum film-square and the specific resistance ( $\rho$ ) of the material.

Recently, tantalum thin-film resistor values are available in the range of 1 to 5 megohms per substrate. The range of capacitor values, available with TM capacitors, is determined at the upper-end by the acceptable yield, and at the low-end by the acceptable capacitor tolerances. It is not the number of capacitors, nor the value of any one capacitor, that is important, but rather the total capacitance per substrate, since a defect in the area of any one capacitor will cause a defective device.

The range of values of capacitors is 100pF to  $.08\mu F$  for TM capacitors and 10pF to  $5\mu F$  for TMM capacitors.

## Chapter Six

### PROBLEM SOLVING WITH COMPUTER PROGRAMMING

#### 6.1 Computer-Program Considerations

##### 6.1.1 Minimization Programs

Computer programming is a powerful tool to handle the minimization problems in this study. It is time-effective for: solving the sets of simultaneous equations, manipulating matrices, and computing the feasible points of the "descending" path on which the "minimum" point is located. (14)

The amount of work involved in minimizing the given non-linear functions is dictated by the size of the variable-vector  $\bar{x}_i$ , the complexity of the functions in the minimization subroutines ZXMIN and FMFP, and the choice of the initial values  $\bar{x}_0$ , as well as the penalty-function coefficients  $K_i$ .

The accuracy of computations is determined by the number of decimal-point digits.

If the accuracy is not adequate, "round-off" errors will

result. These errors, when compared with the magnitude of any of the variables, make it difficult for the algorithm to determine the next vector  $\bar{x}_{i+1}$  in each iteration, because the least-significance decimal digits will be truncated, and the vector  $\bar{x}_i$  will remain unchanged. Therefore, the algorithm will be "zig-zagging" (or oscillating) in a region far from the "feasible" region which contains the "minimum" point-vector  $\bar{x}^*$ .

Consequently, scaling of the variables (usually normalized to unity) before entering the subroutine ZXMIN or FMFP is a safe approach. In general, when correct input data are specified, fast computations will occur, which otherwise would be lengthy. The computer time must be taken into consideration when dealing with a complex problem. In view of the above considerations, fast convergence, and hence computer-time and costs, will be minimized, if proper algorithms are employed.

Since high accuracy of the final values of the constraints specified (  $w_0$ ,  $Q_{10}$ , and  $H_{BP}$  ) causes slower convergence, and adversely more computer time, compromise between high accuracy and expensive computer time must be accepted.

Finally, small step-size (high penalty-function multipliers) is a must, in order to achieve convergence along the descending path of the function to be minimized. This requirement will cause relatively slow convergence but will

ensure satisfactory parameter-values. In order to minimize the volume of the output "printout" paper, a command is necessary to print only one in every desired number of iterations.

#### 6.1.2 Amplitude and Phase Response Programs

The "optimized" filter components and parameters can be further used in a second class of programs, the "plot-programs", which depict, in tabular and graphic form, the amplitude and phase versus frequency responses.

These programs verify the fact that the minimization algorithms not only achieve minimization of the total capacitance and the total resistance of the filter, but also they preserve the amplitude and phase responses, as specified. In order to show the steep slope of the graphs around the center frequency, the plotted points in this region must be increased.

## Chapter Seven

### CONCLUSIONS AND COMMENTS

In this report, a new RC-active band-pass filter using unity-gain amplifiers has been proposed. The circuit has been shown to be stable and to possess low sensitivity properties. Initial design equations have also been given. Computer algorithms have been developed to yield a variety of design procedures yielding minimum total resistance or minimum total capacitance while meeting the required specifications.

Optimization procedure is also described that simultaneously minimizes the total resistance and the total capacitance in the circuit.

Assuming that all the necessary considerations have been taken into account, in applying computer programming to solve the minimization problems, together with proper input data, the expected program-results were retrieved in "hard copy" form and were properly tabulated.

The volume of computer-programs is divided into three major groups. Each group is also divided into four cases; each case is further divided into three sets of data on  $\omega_0$  and  $Q_0$ .

In the first group, the total capacitance is minimized, and in the second group, the total areas  $A_C$ ,  $A_T$ , and  $(2A_T + 3A_C)$  are minimized, as they were described in sections 5.2 and 5.5

In the third group, the minimized passive components of group one and two, in addition to a group with general component values, are fed in the "plot" programs.

In the first group, the computer programs are summarized in Table - 4 (programs 1 through 12) and the other relevant data required for the programs are shown in Table - 5 and - 6. Table - 9 shows the original values of the components, as input, and the minimized values as output. Table - 11 shows the effect of the minimized components on  $H_{sp}$ .

The second group minimizes the area  $A_r$  or  $A_c$  which corresponds to the thin-film total areas occupied by the total resistance and by the total capacitance, respectively, and their combined area  $2A_r + 3A_c$ . Since the purpose of the problem in this report is to minimize the physical size of the passive components of the proposed RC-active filter, the above areas are minimized with the Fletcher-Powell's algorithm. This algorithm is more effective than the Quasi-Newton's algorithm. (9) The solutions of the above minimization problems are obtained by executing several programs divided in 12 cases, each case being solved for 3 sets of values in  $f_0$  and  $Q_0$ . There are 36 programs (program no. 13 through 48), per Table - 4, and the relevant data required for the programs are shown in Table - 7 and - 8. Table - 10 shows the input original values and the output minimized values of the passive components. Table - 12 shows the effect of the optimized passive elements on the insertion loss ( $H_{sp}$ ) of the transfer function.

The third group of 57 programs plots the magnitude and phase responses of the proposed filter versus frequency divided into three subgroups, per Table - 4. The first subgroup, consisting of nine programs (program no. 49 through 57) utilizes either one of the OA mode (unity or non-unity amplifier and the condition of the OA (ideal or practical OA)).



The second subgroup, consisting of 12 programs (program no. 58 through 69), per Table - 4, uses the optimal results of the Quasi-Newton's algorithm, with unity-gain, amplifiers (non-ideal OAs).

The third subgroup consists of 36 programs (program no. 70 through 105), per Table - 4, that utilizes the Fletcher-Powell's optimal results, with unity-gain amplifiers (non-ideal OAs).

The summary results of both minimization algorithms, extracted from the "computer-printouts", are shown in Table-9 and - 10 which demonstrate and verify the expected minimized total capacitance and total areas ( $A_c$  and  $A_r$ ) of both algorithms. Table - 11 and - 12 show the percentage-reduction of the objective functions and its effects over the filter insertion loss ( $H_{sp}$ ).

Due to the excessive volume of the computer-program package, only sample-printouts with corresponding sample flow-charts are presented in this report, Appendix F.

Study of Table - 9 and - 10 shows the original and the final values of the passive elements of the filter. Indeed, the total capacitance, total resistance, and the corresponding thin-film areas are reduced. Furthermore, these values satisfy the desired values of the parameters  $f_0$ ,  $Q_0$  and  $H_{sp}$  (i.e.  $H_{sp}$  can be specified as "fixed" at the beginning of the algorithm or it can be left floating).

An important feature of the minimization algorithms is that not only do they minimize the passive element values, but also, they solve the filter-design equations for the required component value, simultaneously. In the conventional case, for example, given five elements to determine their values, and three known parameters (i.e.  $\omega_0$ ,  $Q_0$ , and  $H_{BP}$ ), at least two element values have to be specified, before any computation. In the minimization case, however, there is no need for arbitrary element-values; all five elements can be floating. The element-values are determined through the algorithm operations to determine the minimum point of the objective function, especially whenever the constraints are satisfied. As it was mentioned in previous chapters, the Fletcher-Powell's algorithm is faster than the Quasi-Newton's algorithm. Table - 11 and - 12 show the number of iterations in each algorithm, for the same minimization case. It can be seen that Fletcher-Powell's algorithm requires fewer iterations.

The speed of convergence depends not only on the penalty function multiplier, but also, on how close to the local minimum point the original point is located.

The percentage-reduction of the objective functions in both algorithms, shown in Table - 11 and - 12, shows that the more reduction attained the more insertion loss (i.e. the smaller  $H_{BP}$ ) is allowed in the filter amplitude response,

especially for low center-frequency. It is worth noticing that there is no difference in the amount of reduction produced when comparing both algorithms, assuming identical input data. The percentage reduction of an objective function, regardless of the algorithm, depends, mainly, on the starting point (i.e. how far it is from the final minimum point) of the convergence path and on the number of the constraints.

These constraints force the algorithm to follow a lesser steep path along the boundary of the constraint functions, so that the "forced" minimum point is higher than that achieved without constraint-functions. This argument can be verified by observing the difference in "minimum" points obtained between heavily constrained functions (i.e. in the case where  $C_1=C_2$ ,  $H_{sp} = -G = \frac{K}{K+1}$ ) and less constrained functions (i.e., in cases where  $C_1$ ,  $C_2$  and  $H_{sp}$  are floating.)

Both algorithms, in their process to reduce  $C_T$ ,  $R_T$ ,  $A_c$ , and  $A_r$  show that, while  $C_T$  or  $A_c$  is reduced (i.e.  $C_1$  and  $C_2$  are minimized), the resistors  $R_1$  through  $R_3$  increase in a complex manner, dictated by different factors, such as location of the variable-descending point, constraints, etc. For example, when the constraint-function that is determined by the specification  $\omega_0^2 = \frac{1}{(C_1 R_1)(C_2 R_2)}$  is forced to be kept constant, along the iteration process, the resistors  $R_1$  and  $R_2$  have to increase disproportionally, while  $C_1$  and  $C_2$  decrease,

which is the case of minimizing  $C_T$  or  $A_C$ . The reverse process takes place when  $R_T$  or  $A_{Rr}$  is minimized. These arguments can be verified by observing the results in Table - 9

The above observations explain the reason why the objective function ( $3A_C + 2A_r$ ) is very "shallow", in the sense that its gradient-slopes are not steep and the final (minimum) point is not far from the initial point. This can be considered as a "conflict" between two senses of optimization; the capacitor decreases while the resistor increases, the net effect being a small decrease (if not at all). Table - 4, and Table - 12 (program no. 37 through 48) verify these arguments.

In both minimization algorithms, the computer-programs have a "stopping" command which is used whenever any of the five variables reaches its design-limit, imposed by the manufacturers, regardless of whether the minimum point was reached.

Finally, the plots of the amplitude and phase responses show that this filter configuration is feasible only with unity-gain amplifiers. Comparison of the amplitude and phase responses, where non-optimal versus optimal components were used, shows that they are identical, for the same values of the parameters  $\omega_0$ ,  $Q_0$  and  $H_{sp}$ .

This, once again, proves that the transfer-function is independent of the minimization process, provided that the parameters  $\omega_0$ ,  $Q_0$  and  $H_{sp}$  remain unchanged.

Fig 6.1 and 6.2 show the amplitude and phase responses of the Band-Pass Filter configured with unity-gain amplifiers. The three graphs verify the dependence of the insertion loss on  $H_{sp}$  and the shape (or bandwidth) on  $Q_0$  of its transfer function.

The transfer function of the present filter is unaffected by the non-idealness of the unity-gain model of OAs, provided that  $f_0$  is significantly smaller than  $B$ , as it is demonstrated in fig. 6.3, 6.4, 6.5 and 6.6.

When the d.c. gain of the amplifiers is non-unity ( $K_{1,2} \neq 1$ ) the transfer function is non-feasible, as it can be seen from the amplitude and phase responses, fig 6.7, 6.8, and 6.9.

Provided that the unity-gain model of the OAs is employed, the amplitude and phase responses of the transfer function, using "optimized" passive components (from Newton's and Fletcher-Powell's minimization algorithms) are identical to those using non-optimal passive components, fig 6.10 to 6.18.

TABLE - 9 OPTIMIZED COMPONENT VALUES USING NEWTON'S METHOD

Program No.	ORIGINAL VALUES						FINAL VALUES					
	$R_1(K\Omega)$	$R_2(K\Omega)$	$R_3(K\Omega)$	$C_1(mF)$	$C_2(mF)$	$C_3(mF)$	$R_1(K\Omega)$	$R_2(K\Omega)$	$R_3(K\Omega)$	$C_1(mF)$	$C_2(mF)$	$C_3(mF)$
1	0.1	1	1	318.31	3.1831	321.493	4.90394	4.85175	.335591	3.22246	.132088	3.35634
2	0.1	1	1	63.662	15.9155	79.5775	4.86033	5.01807	2.81786	.617930	.671953	1.2899
3	0.1	1	1	31.831	7.95775	39.7887	4.86033	5.01807	2.81786	.308979	.335976	.644955
4	0.1	0.1	0.1	318.31	31.831	350.141	4.90095	.727431	4.66471	5.3302	5.33068	10.6609
5	1	0.1	1	35.0141	28.9373	63.9514	4.8242	.226882	3.08095	9.61617	9.61678	19.223
6	1	0.1	1	17.507	14.4686	31.9757	4.8242	.226882	3.08095	4.80809	4.80839	9.61648
7	1	1	3	63.662	1.59155	65.253	5.0119	1.51847	4.5554	12.6962	1.04808	13.7443
8	1	1	3	12.7324	7.95775	20.6901	4.9404	1.54738	4.64216	2.57677	5.14263	7.7194
9	1	1	3	6.3662	3.97887	10.3457	5.00159	1.49508	4.48747	1.27245	2.65987	3.93232
10	1	1	3	636.62	1.59155	638.211	4.92382	0.12312	0.369359	12.9267	12.9269	25.8535
11	0.1	1	3	127.324	7.95775	135.282	4.98984	3.11848	9.35563	2.55161	2.55164	5.10325
12	0.1	1	3	63.662	3.97887	67.6409	4.93091	3.08182	9.24545	1.29106	1.29107	2.58213

TABLE -10 OPTIMIZED COMPONENT VALUES USING FLETCHER'S METHOD

Program No	ORIGINAL VALUES						FINAL VALUES					
	R <sub>1</sub> (Mg)	R <sub>2</sub> (Mg)	R <sub>3</sub> (Mg)	C <sub>1</sub> (Mg)	C <sub>2</sub> (Mg)	%A (cm <sup>2</sup> )	R <sub>1</sub> (Mg)	R <sub>2</sub> (Mg)	R <sub>3</sub> (Mg)	C <sub>1</sub> (Mg)	C <sub>2</sub> (Mg)	%A (cm <sup>2</sup> )
13	1	1	1	31.831	3.1831	.26376	4.9385	5.1082	4.5252	6.0727	.66043	.50572
14	1	1	1	6.3662	15.915	.16785	4.8755	4.9933	4.4037	1.2287	3.3868	.034754
15	1	1	1	3.1831	7.9577	.083924	4.8755	4.9933	4.4037	.61437	1.6924	.017377
16	1	1	1	31.831	3.1831	.26376	4.7135	1.5236	0.17375	3.7486	3.7378	.056345
17	1	1	1	6.3662	15.915	.16785	4.749	7.6806	1.1425	1.6677	1.6664	.023116
18	1	1	1	3.1831	7.9577	.083924	4.749	7.6806	1.1425	.8387	.8321	.012558
19	0.5	5	15	126.32	.31831	.96152	1.4833	1.5149	4.5447	42.84	.10491	.32350
20	0.5	5	15	35.465	1.5195	.20381	5.0347	5.1093	153.28	2.5274	.15571	.020212
21	0.5	5	15	12.732	.79577	.10191	3.7935	38.519	11.556	1.6767	.10324	.01348
22	0.05	100	300	1273.2	.15915	9.5925	5.2603	0.13114	0.39354	12.102	12.101	.18232
23	0.05	10	30	254.65	.79577	1.9242	5.0963	3.185	9.5551	2.4987	2.4984	.037643
24	0.05	10	30	127.32	.39789	.96212	5.0963	3.185	9.5551	1.2494	1.2494	0.01882

\* A : AREA OF "AC" OR "AR" OR "ZAC + ZAR" ASSOCIATED WITH THE EQUIVALENT PROGRAM

TABLE -10 CONTINUED

Program No	ORIGINAL VALUES										FINAL VALUES									
	R <sub>1</sub> (MO)	R <sub>2</sub> (MO)	R <sub>3</sub> (MO)	C <sub>1</sub> (MF)	C <sub>2</sub> (MF)	A(cm <sup>2</sup> )	R <sub>1</sub> (MO)	R <sub>2</sub> (MO)	R <sub>3</sub> (MO)	C <sub>1</sub> (MF)	C <sub>2</sub> (MF)	A(cm <sup>2</sup> )	R <sub>1</sub> (MO)	R <sub>2</sub> (MO)	R <sub>3</sub> (MO)	C <sub>1</sub> (MF)	C <sub>2</sub> (MF)	A(cm <sup>2</sup> )		
25	40	0.5	1	11.937	4.2441	2.0008	0.30999	0.073388	0.038723	78.447	58.737	0.15505								
26	4	0.5	1	2.3873	2.1221	2.0008	0.96595	0.23055	0.017138	5.5087	75.304	0.485								
27	4	0.5	1	1.1937	10.610	2.008	0.54388	0.13191	0.077223	4.8224	75.908	0.27306								
28	1	1	10	175.07	0.57875	0.50550	206.6	0.079054	0.001773	78.702	78.702	.10334								
29	0.5	0.5	1	19.099	21.221	0.25075	0.11056	0.13691	0.2623	79.790	79.799	0.058477								
30	0.5	0.5	1	9.5493	10.610	0.25075	0.073362	0.053968	0.14772	78.909	78.895	.037782								
31	4	1	3	15.915	1.5915	2.0022	0.83149	0.07806	0.23418	76.510	20.380	0.41590								
32	4	1	3	3.183	7.9577	2.002	1.0056	0.10098	0.30294	12.615	78.907	0.50349								
33	4	1	3	1.5195	3.9789	2.002	0.59478	0.050552	0.15166	10.674	78.814	0.29747								
34	1	1	3	6.3662	1.5915	0.50200	0.81130	0.020282	0.060847	78.444	78.467	0.40569								
35	1	1	3	12.732	7.9577	0.50200	0.16256	0.10159	0.30476	78.292	78.294	0.081483								
36	1	1	3	6.3662	3.9789	0.50200	0.080437	0.050264	0.15079	79.155	79.15819	0.040319								
37	4	1	10	43.768	0.57875	5.0132	4.0894	1.1359	7.5744	29.830	73.052	4.7887								



TABLE -10 CONTINUED

Program No	ORIGINAL VALUES						FINAL VALUES					
	R <sub>1</sub> (MO)	R <sub>2</sub> (KO)	R <sub>3</sub> (KO)	C <sub>1</sub> (nF)	C <sub>2</sub> (nF)	A (cm <sup>2</sup> )	R <sub>1</sub> (MO)	R <sub>2</sub> (KO)	R <sub>3</sub> (KO)	C <sub>1</sub> (nF)	C <sub>2</sub> (nF)	A (cm <sup>2</sup> )
38	4	1	10	8.7535	2.8937	4.2742	4.0894	1.1359	7.5744	5.9661	3.6526	4.3155
39	4	1	10	4.3768	1.4469	4.1426	4.0894	1.1359	7.5744	2.9830	1.8263	4.2068
40	10	0.01	0.1	175.07	57.875	6.2644	1.111	0.012345	0.061723	85.912	85.910	4.9941
41	10	0.01	0.1	35.014	289.37	8.3309	0.92695	0.010299	0.29862	102.93	102.95	5.5799
42	10	0.01	0.1	17.507	144.69	4.6655	0.92695	0.010299	0.29862	51.464	51.475	3.2336
43	1	1	3	63.662	1.5915	2.4787	1.2247	0.61236	1.8371	51.979	2.599	2.4606
44	1	1	3	12.732	7.9577	1.4716	1.2247	0.61236	1.8371	10.396	12.995	1.7588
45	1	1	3	6.3662	3.9789	1.2378	1.2247	0.61236	1.8371	5.1979	6.4974	1.4915
46	1	1	3	6.3662	1.5915	2.4787	7.4764	0.18694	0.56083	8.5145	8.5133	7.862
47	1	1	3	12.732	7.9577	1.4716	1.0556	0.65974	1.9792	12.062	12.062	1.6034
48	1	1	3	6.3662	3.9789	1.2378	1.0556	0.65974	1.9792	6.0308	6.0309	1.3308

TABLE -11 MINIMIZATION EFFECTS ON FILTER INSERTION LOSS ( $\epsilon = \frac{E_1}{E_1 + E_3}$ ) NEWTON'S METHOD

Program No	Initial Values		Final Values		% of Capacitance Reduction	Number of Iterations	Minimum Obtained	Comments
	$E_p$	Loss(dB)	$E_p$	Loss(dB)				
1	0.500	6.02	$.686938 \times 10^{-1}$	43.2616	99	2380	NO	$E_1$ reached max. value
2	0.500	6.02	$.531685 \times 10^{-1}$	25.487	98	2200	NO	"
3	0.500	6.02	$.531685 \times 10^{-1}$	25.487	98	2200	NO	"
4	0.500	6.02	.390705	8.163	97	3120	NO	"
5	0.90907	0.828	.931411	0.617	70	830	NO	"
6	0.90907	0.828	.931411	0.617	70	830	NO	"
7	0.750	2.498	.75	2.498	79	1160	NO	"
8	0.750	2.498	.75	2.498	63	1120	NO	"
9	0.750	2.498	.75	2.498	62	1350	NO	"
10	0.750	2.498	.75	2.498	96	1080	NO	"
11	0.750	2.498	.75	2.498	96	3600	NO	"
12	0.750	2.498	.75	2.498	96	5000	NO	"

TABLE -12 MINIMIZATION EFFECTS ON FILTER INSERTION LOSS ( $R_p = \frac{R_1}{R_2 + R_3}$ ) FLETCHER'S METHOD

Program No	Initial Values		Final Values		Loss (dB)	% of Area (Ac, A <sub>c</sub> ) Reduction	Number of Iterations	Minimum Obtained	Comments
	$R_p$	Loss (dB)	$R_p$	Loss (dB)					
13	0.500	6.028	0.469741	6.5636	81	454	NO	R1 reached min. value	
14	0.500	6.028	0.468630	6.583	79	494	NO	"	
15	0.500	6.028	0.468630	6.583	79	494	NO	"	
16	0.500	6.028	0.102366	19.79688	79	214	NO	"	
17	0.500	6.028	0.598001	4.466	85	514	NO	"	
18	0.500	6.028	0.598001	4.466	85	514	NO	"	
19	0.75	6.028	0.75	2.498	66	274	NO	"	
20	0.75	2.498	0.75	2.498	90	894	NO	"	
21	0.75	2.498	0.75	2.498	87	734	NO	"	
22	0.75	2.498	0.75	2.498	98	274	NO	"	
23	0.75	2.498	0.75	2.498	98	634	NO	"	
24	0.75	2.498	0.75	2.498	98	634	NO	"	
25	0.6666	3.522	3.654	9.23355	92	674	NO	"	

TABLE - 12 CONTINUED

Program No	Initial Values		Final Values		% of Area (Ac. Ar) Reduction	Number of Iterations	Minimum Obtained	Comments
	Loss (dB)	Loss (dB)	Loss (dB)	Loss (dB)				
26	0.6666	3.522	0.4062	7.8256	75	214	NO	C <sub>2</sub> reached its max. value
27	0.6666	3.522	0.36925	8.6536	86	394	NO	C <sub>2</sub> reached its max. value
28	0.9090	0.8278	0.02193	33.18	80	1574	NO	C <sub>1</sub> and C <sub>2</sub> reached their max. values
29	0.6666	3.522	0.65788	3.637	77	714	NO	" " " "
30	0.6666	3.522	0.7324	2.7048	85	1354	NO	" " " "
31	0.75	2.498	0.75	2.498	79	514	NO	C <sub>1</sub> reached its max. value
32	0.75	2.498	0.75	2.498	75	434	NO	C <sub>2</sub> reached its max. value
33	0.75	2.498	0.75	2.498	85	674	NO	C <sub>2</sub> reached its max. value
34	0.75	2.498	0.75	2.498	19	1234	NO	C <sub>2</sub> reached its max. value
35	0.75	2.498	0.75	2.498	84	1594	NO	C <sub>1</sub> and C <sub>2</sub> reached their max. values
36	0.75	2.498	0.75	2.498	92	1854	NO	" " " "
37	0.909	0.8278	0.8695	1.213	4	185	YES	No component limits
38	0.909	0.8278	0.8695	1.213	1(increase)	185	YES	No component limits

TABLE - 12 CONTINUED

Program No	Initial Values		Final Values		Loss (dB)	% of Area (A <sub>c</sub> /A <sub>r</sub> ) Reduction	Number of Iterations	Minimum Obtained	Comments
	R <sub>sp</sub>	Loss (dB)	R <sub>sp</sub>	Loss (dB)					
39	0.909	0.8278	0.8695	1.213		2 (increase)	182	YES	No component limit
40	0.909	0.8278	0.8333	1.583	20		295	YES	C <sub>1</sub> and C <sub>2</sub> reached their limits
41	0.909	0.8278	0.9666	0.2945	33		473	YES	" "
42	0.909	0.8278	0.9666	0.2945	30		473	YES	" "
43	0.75	2.498	0.75	2.498	1		501	YES	" "
44	0.75	2.498	0.75	2.498	19 (increase)		501	YES	" "
45	0.75	2.498	0.75	2.498	20 (increase)		501	YES	" "
46	0.75	2.498	0.75	2.498	300 (increase)		1290	YES	" "
47	0.75	2.498	0.75	2.498	9 (increase)		226	YES	" "
48	0.75	2.498	0.75	2.498	7 (increase)		226	YES	" "

Table 13 - Process compatible with various types of thin-film materials

	DEPOSITION PROCESS	MATERIAL
METALS	Vacuum Evaporation Cathode Sputtering Vapor Plating	Nichrome, aluminum chromium, gold, nickel Tantalum Copper, gold, nickel tin, oxide
DIELECTRICS	Vacuum Evaporation Anodization Vapor Plating	Silicon monoxide silicon dioxide Tantalum oxide Silica, alumina, glass

Table 14 - Characteristics of some important thin-film materials for resistors

MATERIALS	SUITABLE PROCESS	ohms/sq. range	TEMPERATURE COEFFIC	CHARACTERISTICS
Nickel-chromium (Nichrome)	Vacuum Evaporation	50-400	+ 50-100	Good temperature coefficient, good adhesion
Tin-Oxide	Vapor-plating	100-5000	+ 100-300	High sheet resistance, good adhesion.
Tantalum	Sputtering	50-500	+ 100-200	Good process control and temperature coefficient, good adhesion, high stability
Cermets	Silk Screen	10-100000	+ 100-300	Wide range of resistivity process not suitable for close tolerances

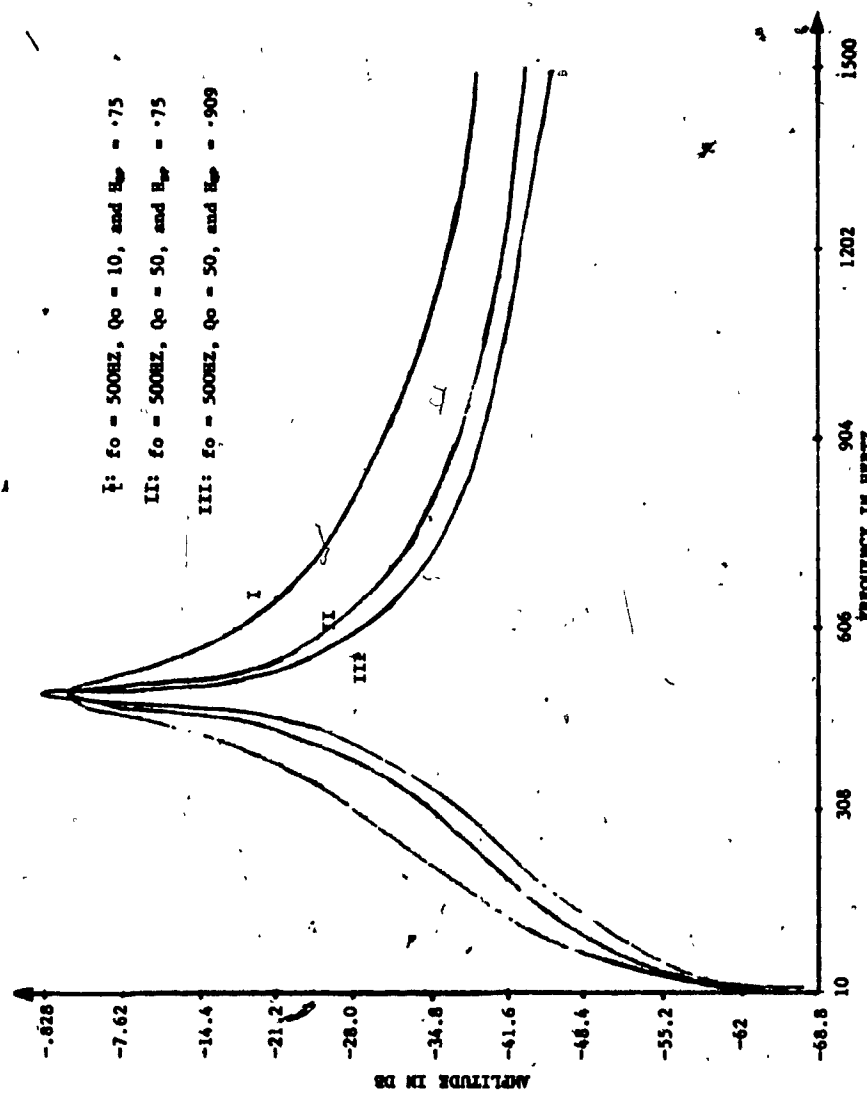


Fig. 6.1 - Amplitude v.s. frequency response of the proposed RC-active filter assuming ideal OAs in unity-gain mode and general component values.



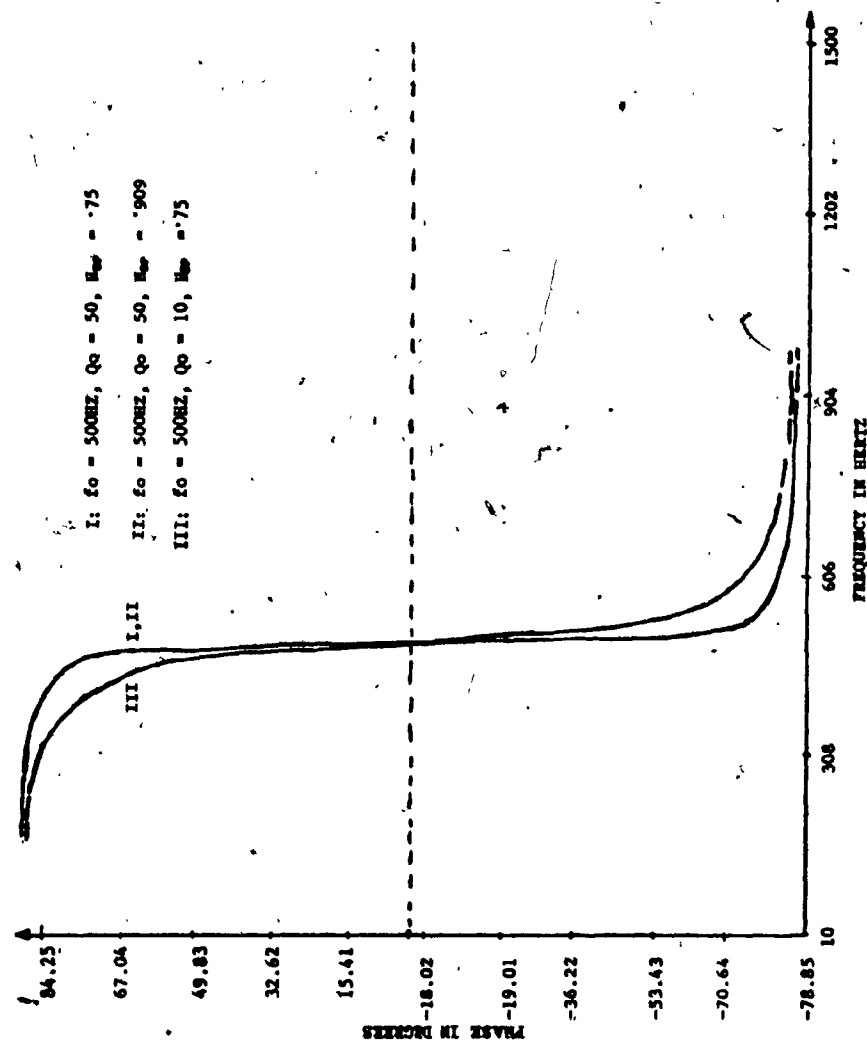


Fig. 6.2 - Phase v.s. frequency response of the proposed RC-active filter assuming ideal OAs in unity-gain mode and general component values.

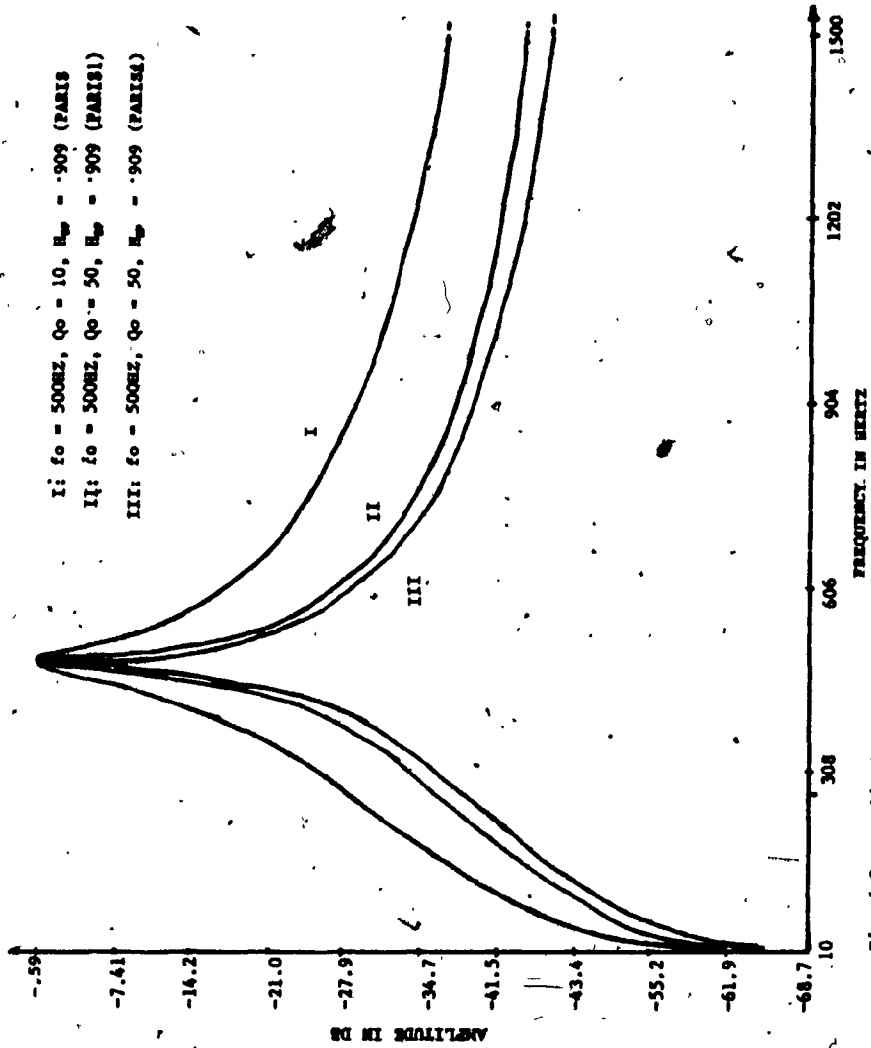


Fig. 6.3 - Amplitude v.s. frequency response of the RC-active filter assuming non-ideal OAs in unity-gain mode and general component values.

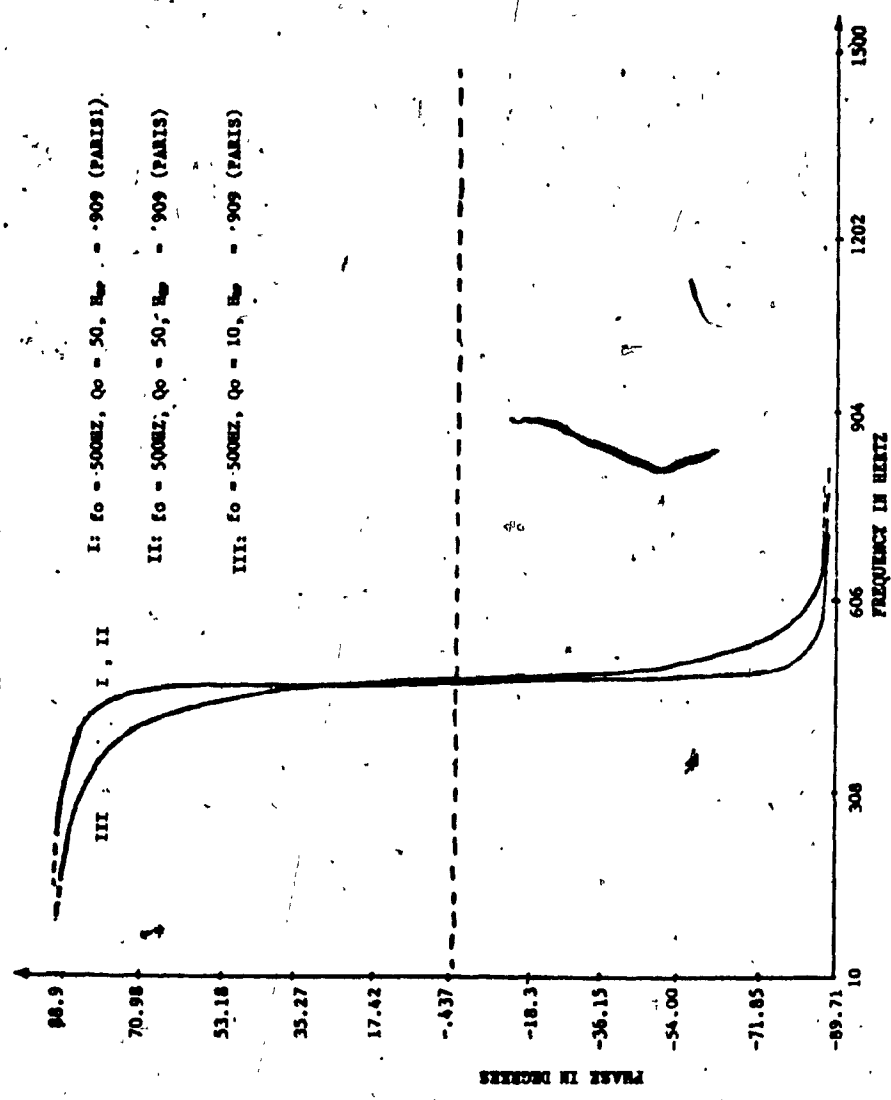


Fig. 6.4 - Phase v.s. frequency response of the proposed RC-active filter assuming non-ideal OAs in unity-gain mode and general component values.

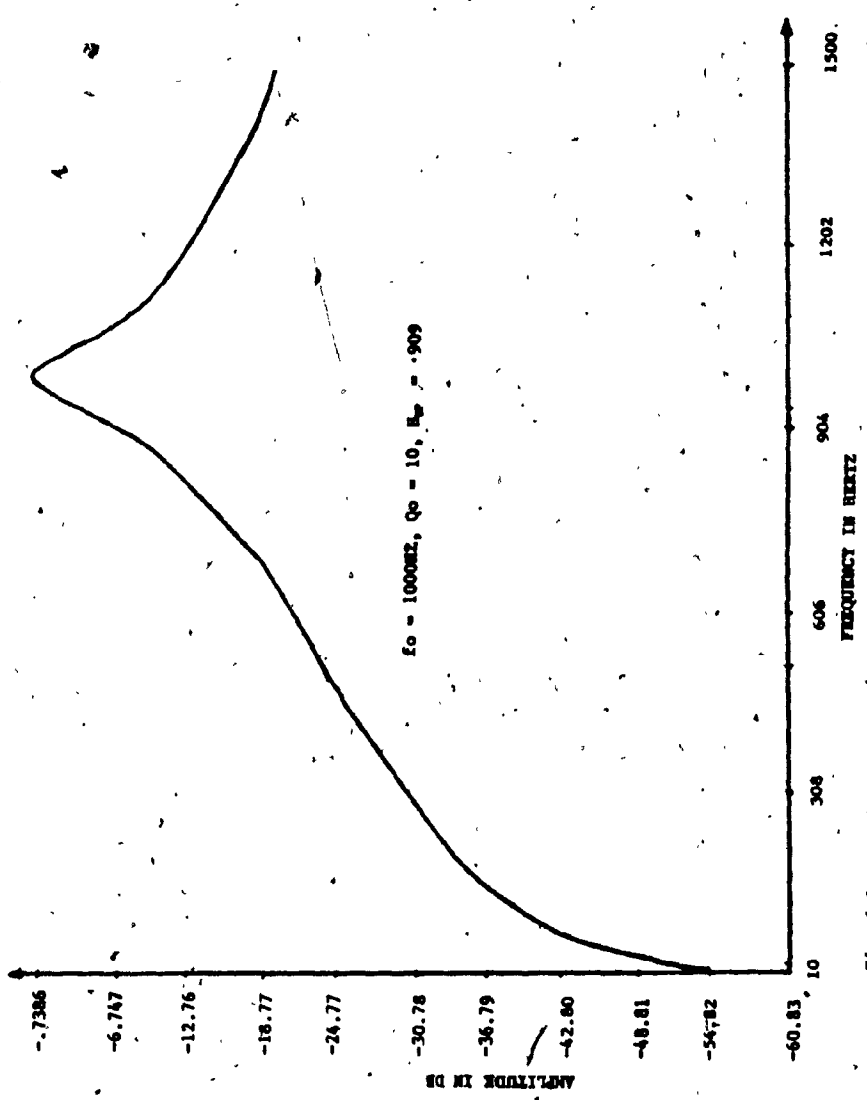


Fig. 6.5 - Amplitude v. f. frequency response of the proposed MC-active filter assuming non-ideal OAs in unity-gain mode and general component values.

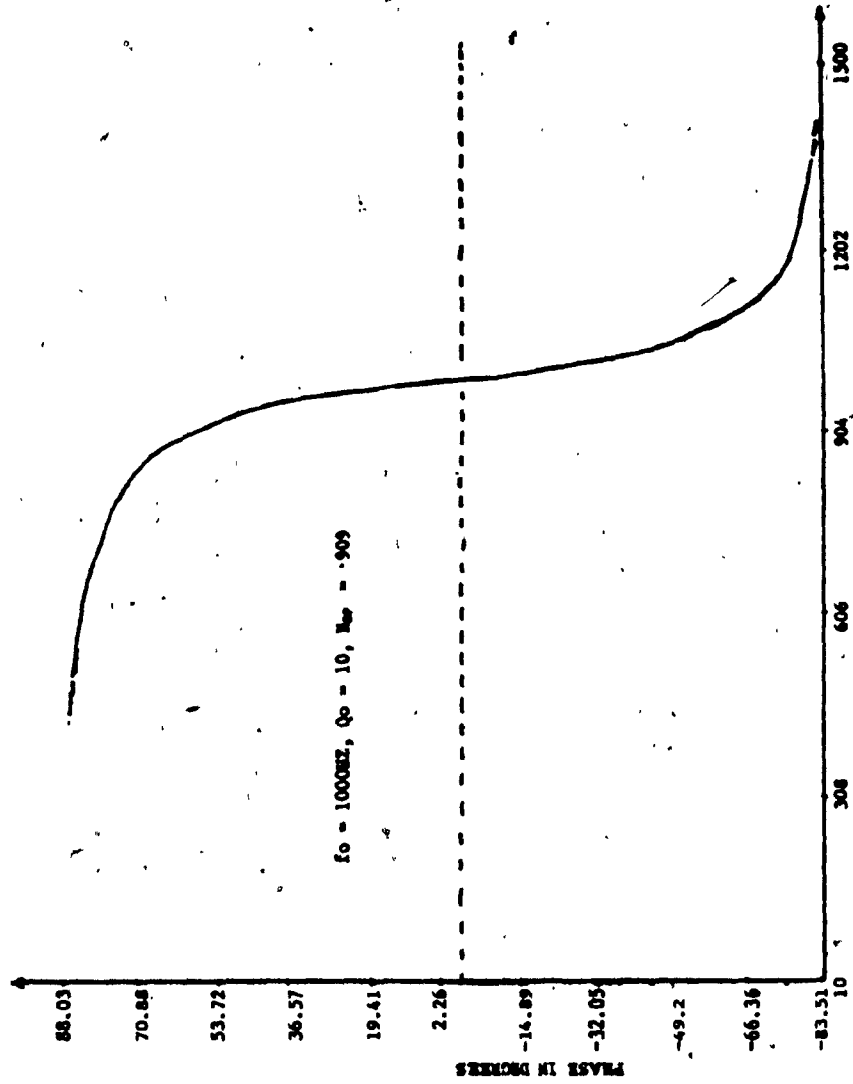


Fig. 6.6 - Phase v.s. frequency response of the proposed MC-active filter assuming non-ideal OAs in unity-gain mode and general component values.

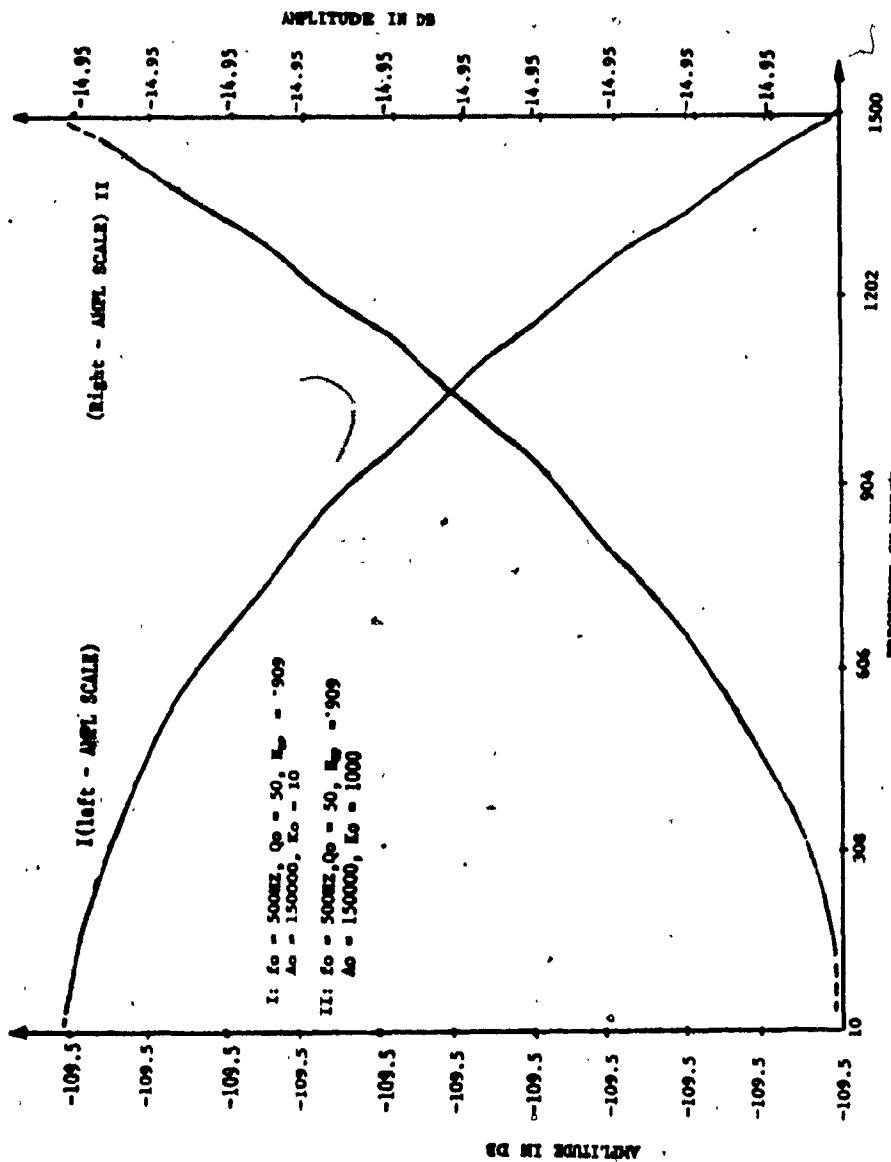


Fig. 6.7 - Amplitude v.s. frequency response of the proposed EC-active filter assuming non-ideal Qs in non-unity-gain mode and general component values.

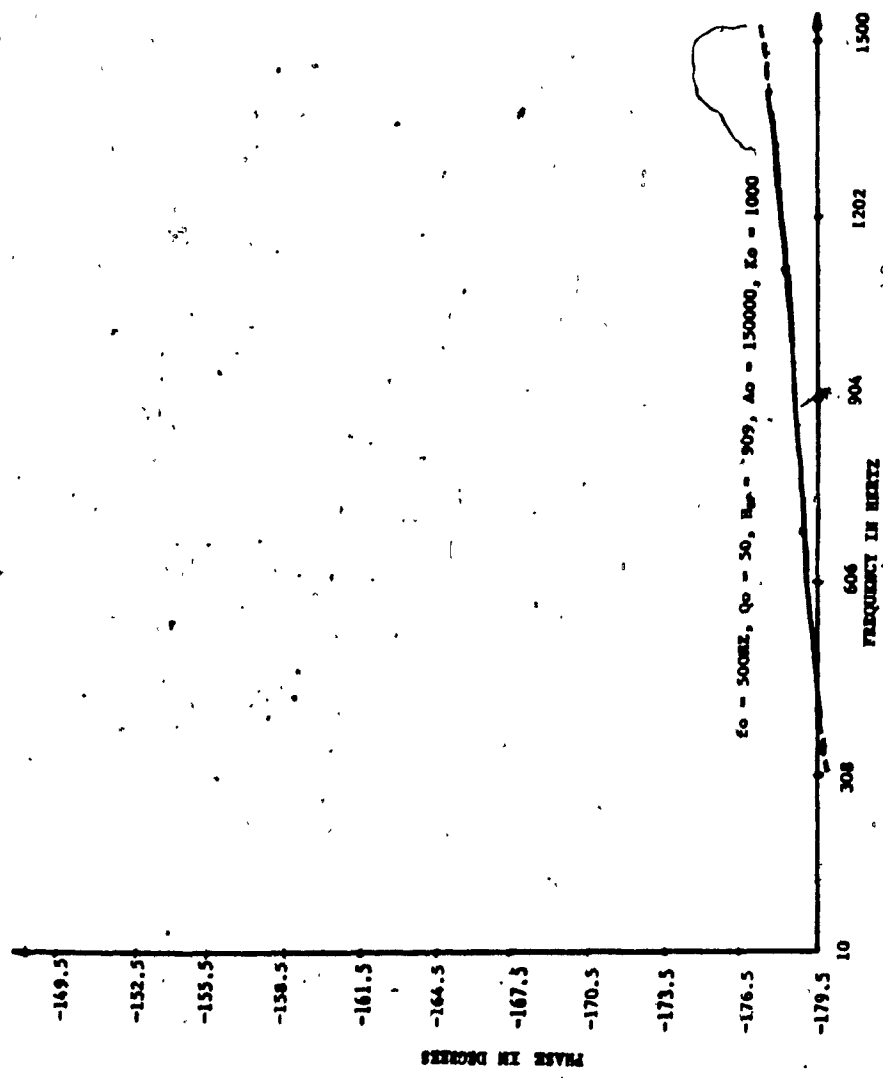


Fig. 6.8 - Phase v.s. frequency response of the proposed RC-active filter assuming non-ideal OAs in non-unity-gain mode and general component values.

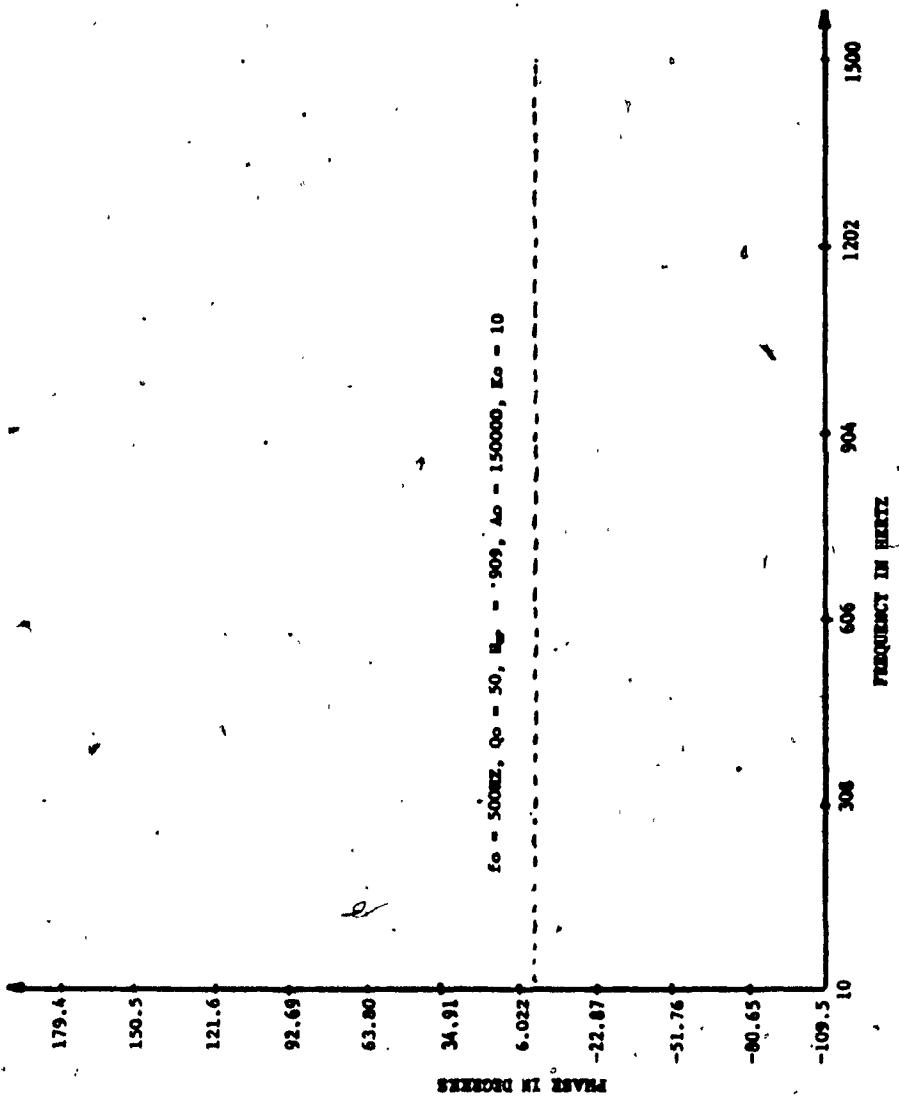


Fig. 6.9 - Phase v.s. frequency response of the proposed RC-active filter assuming non-ideal OAs in non-unity gain mode and general component values.



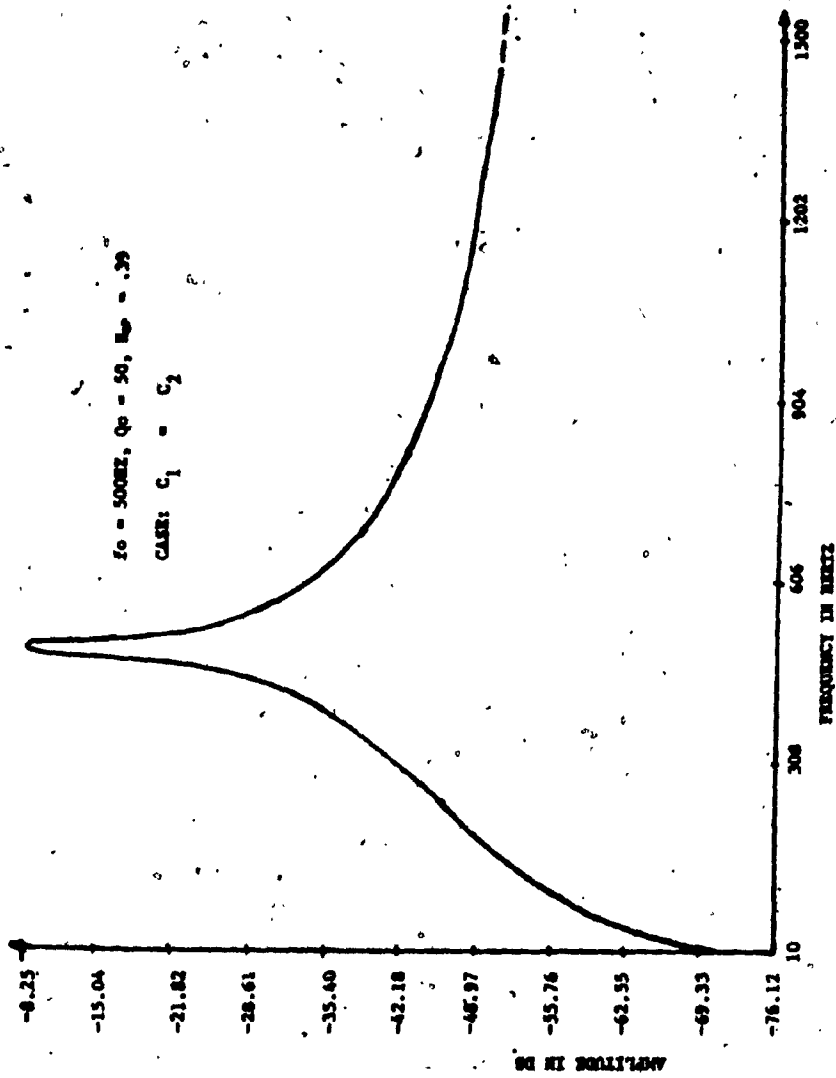


Fig. 6.10 - Amplitude v.s. frequency response of the proposed RC-active filter assuming ideal OAs in unity-gain mode and 'optimized' passive element values using Newton's minimization algorithm.

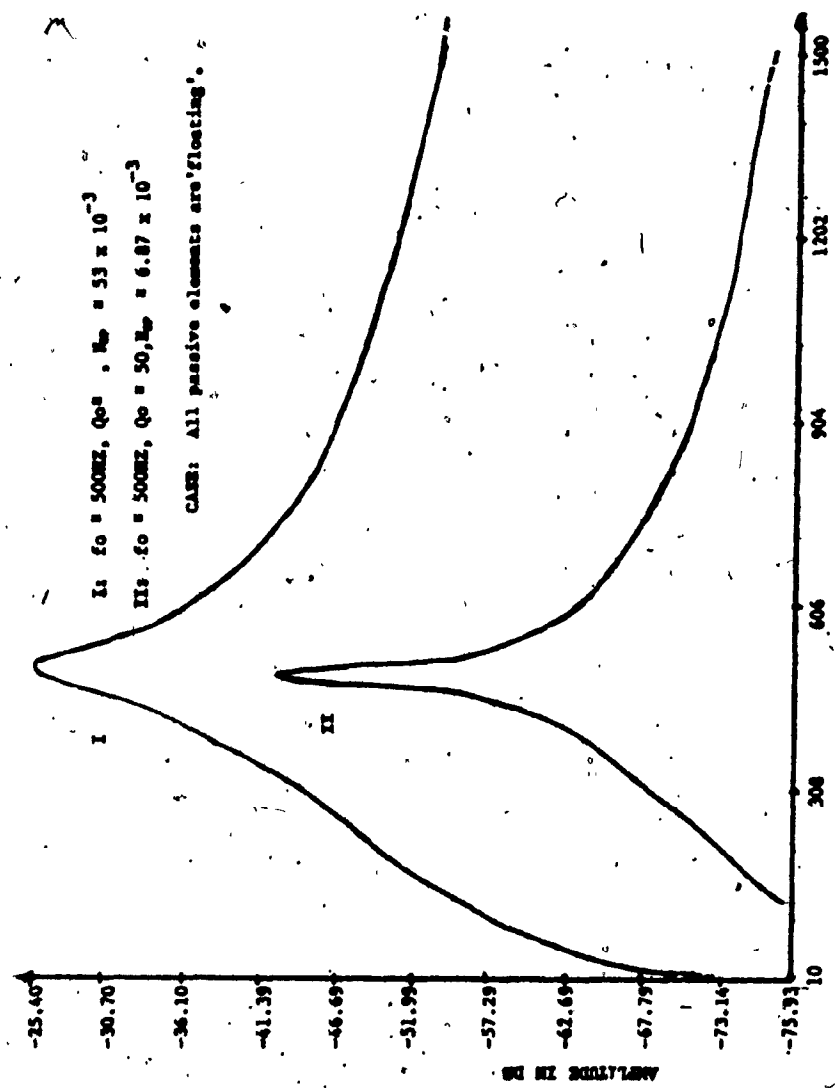


Fig. 6.11 - Amplitude v.s. frequency response of the proposed NC-active filter assuming ideal OAs in unity-gain mode and 'optimized' passive element values using Newton's minimization algorithm.

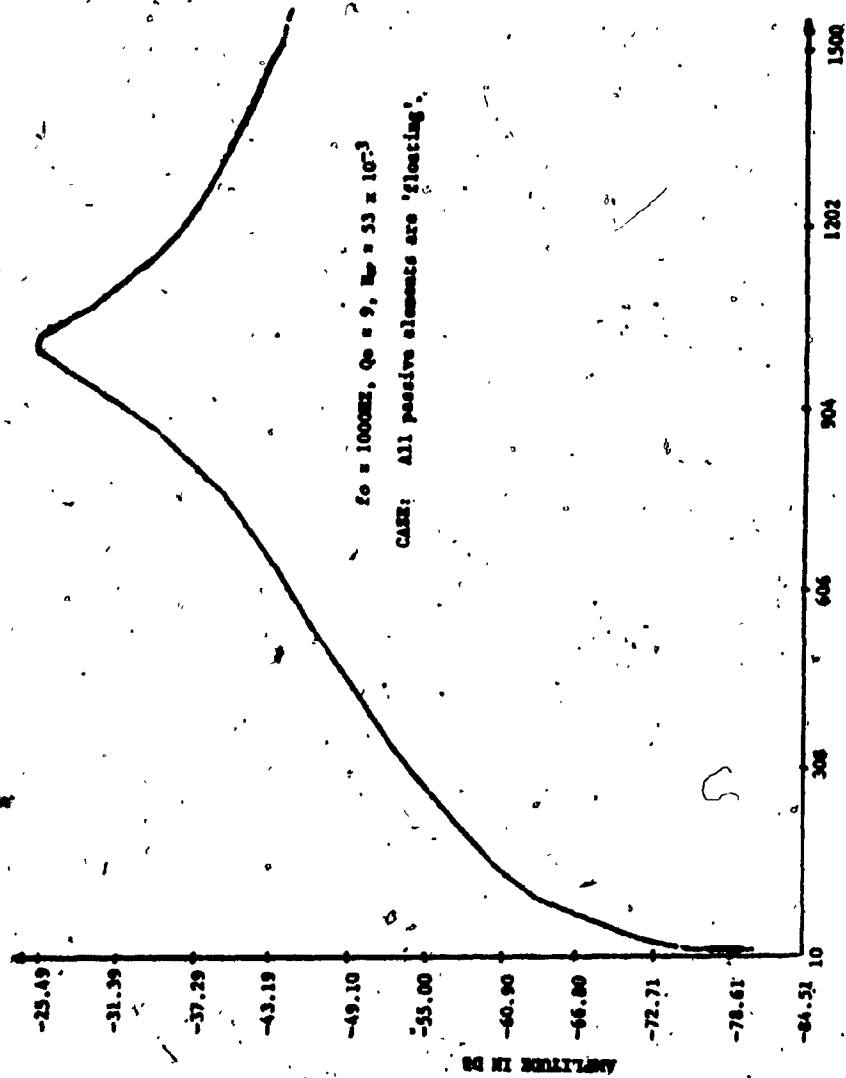


Fig. 6.12 - Amplitude v.s. frequency response of the proposed active filter assuming ideal OAs in unity-gain mode and 'optimized' passive element values using Newton's minimization algorithm.

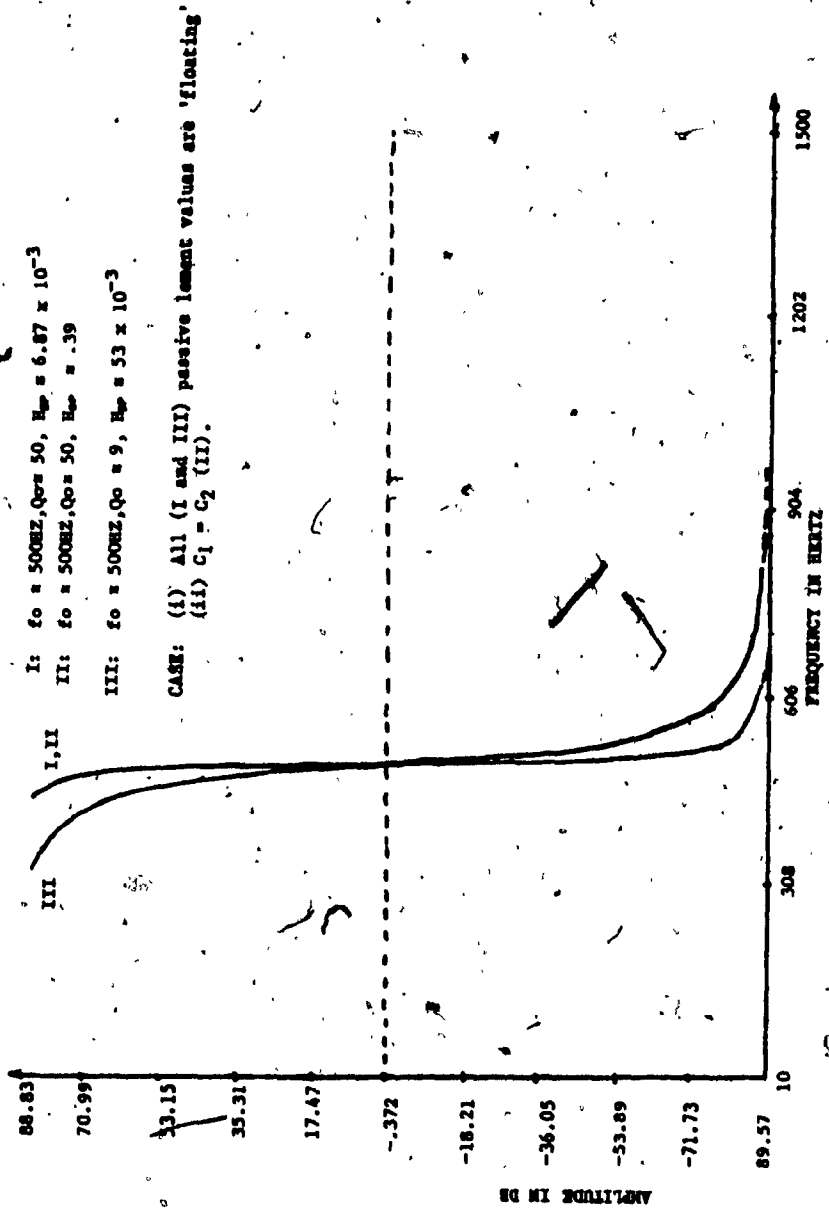


Fig. 6.13 - Phase v.s. frequency response of the proposed R-active filter assuming ideal OAs in unity-gain mode and 'optimized' passive element values using Newton's optimization algorithm.

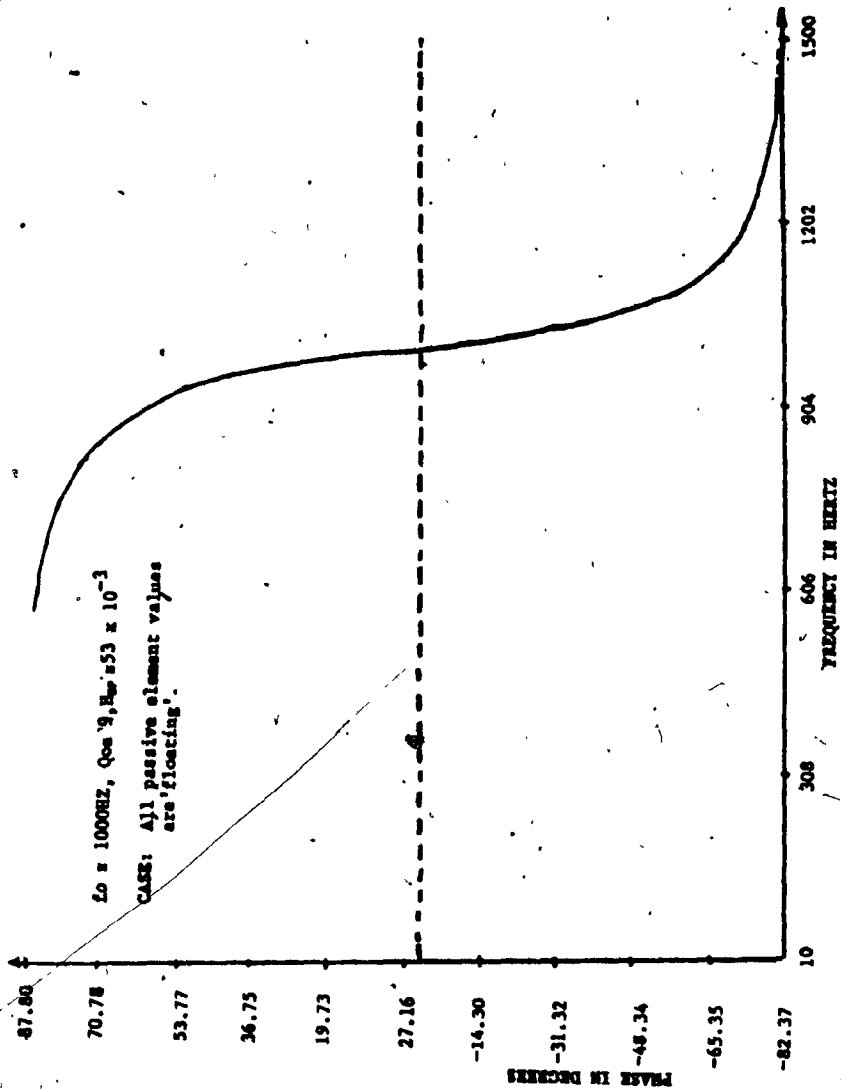


Fig. 6.14 - Phase v.s. frequency response of the proposed EC-active filter assuming ideal OAs in unity-gain mode and 'optimized' passive element values using Newton's minimization algorithm.

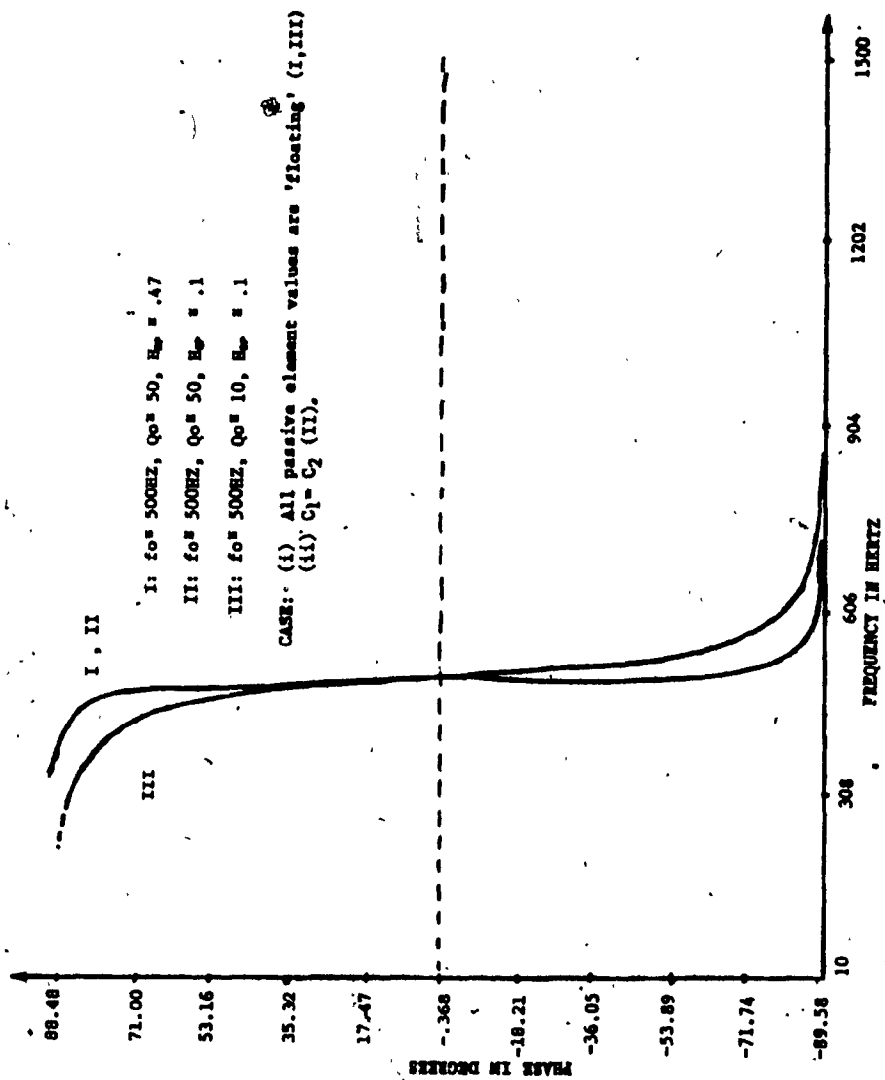


Fig. 6.15 - Phase v.s. frequency response of the proposed RC-active filter assuming ideal OAs in unity-gain mode and 'optimized' passive element values using Fletcher-Powell's optimization algorithm.

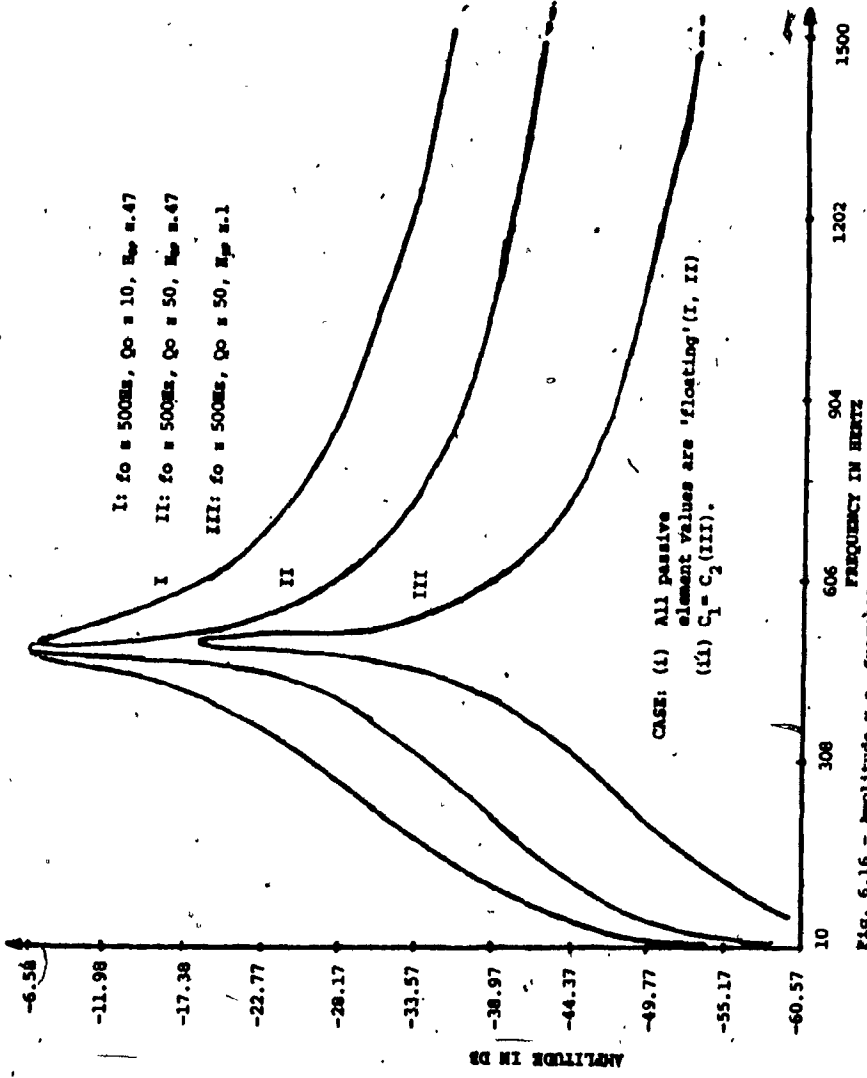


Fig. 6.16 - Amplitude v.s. frequency response of the proposed RC-active filter assuming ideal OAs in unity-gain mode and 'optimized' passive element values using Fletcher-Powell's optimization algorithm.

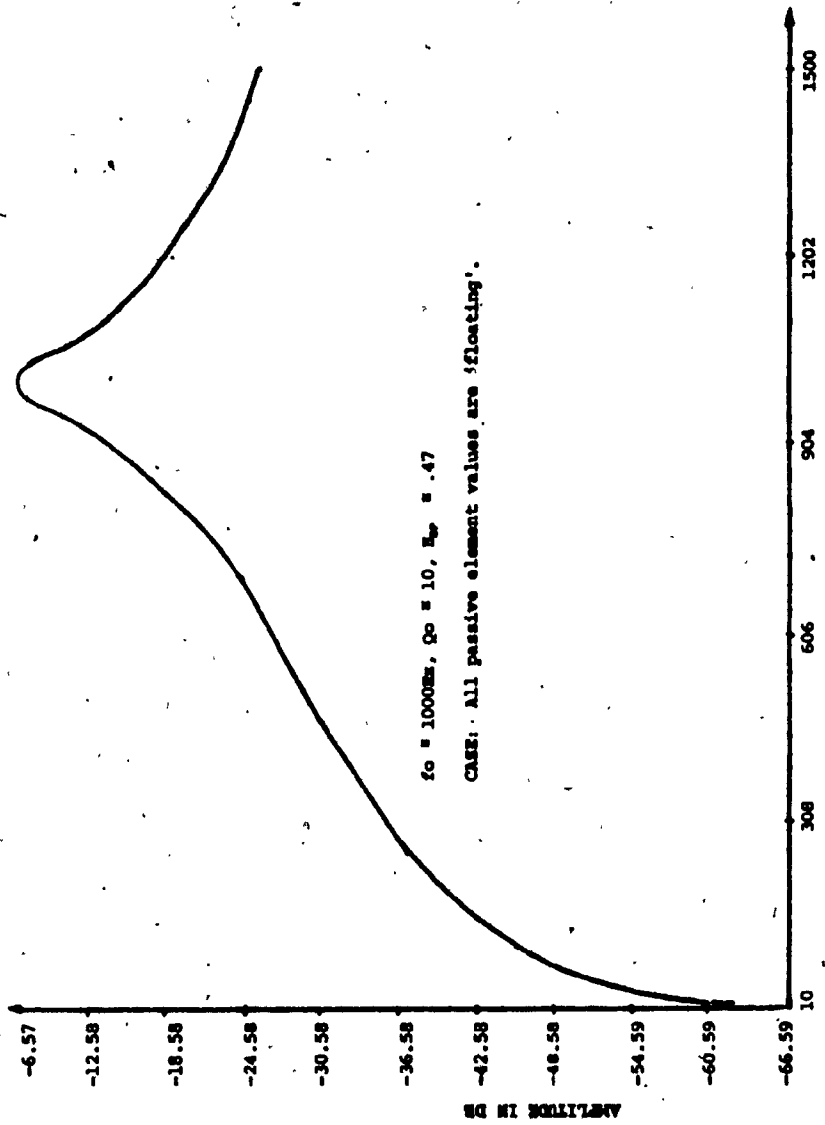


Fig. 6.17 - Amplitude v.s. frequency response of the proposed MC-active filter assuming ideal OAs in unity-gain mode and 'optimized' passive element values using Fletcher-Powell's minimization algorithm.



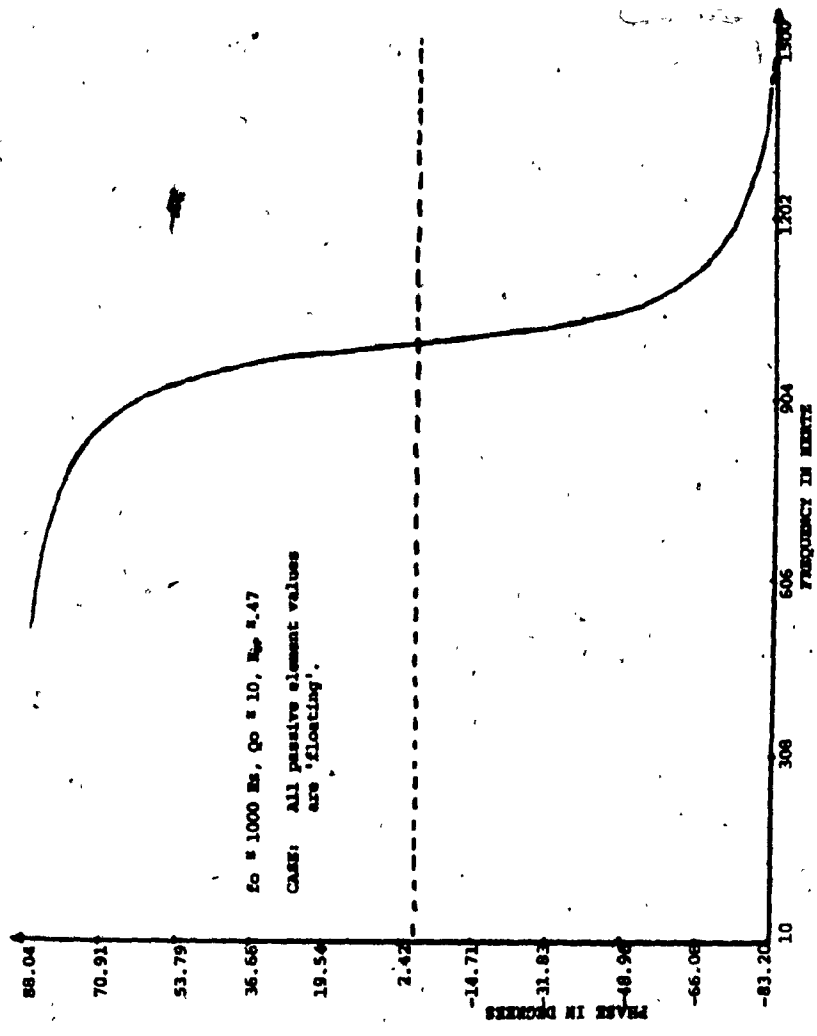


Figure 6.18 - Phase v.s. frequency responses of the proposed MC-active filter assuming ideal OAs in unity-gain mode and 'optimised' passive element values using Fletcher-Powell's minimisation algorithm.

REFERENCES

1. Horowitz, I.M. and Branner, G.R., "A Unified Survey of Active RC Synthesis Techniques", Proc. Nat'l Electron, Conf. 23, 1967
2. Sallen, R.P. and Key, E.L. "A Practical Method of Designing RC Active Filters", IRE Trans. Circuit Theory, Vol CT-2, pp. 74-85. Mar. 1955.
3. Keen, A.W. "Active and non-reciprocal networks", Aeton Polytech (Prague), Vol. 14, no. 1, pp. 47-76, 1966
4. Bhattachanyya, B.B. "Active filter design", class notes, Concordia University, 1975.
5. Leeds, M.B. "Linear IC's: Part 4 - Inside the O's" Electronics pp. 86-91, Oct. 16, 1967.
6. Mitra, S.K., "Analysis and Synthesis of Linear Active Networks," John Wiley and Sons, N.Y. 1969, pp 1-20.
7. Maschytz, G.S. "The Operational Amplifier in Linear Active Networks," Bell Telephone Inc. (Reprinted from IEEE Spectrum, Vol, 7, pp. 42-50, Jan. 1969.
8. Polak, E. "Computational Methods in Optimization" Mathematics in Science and Engineering, University at Southern California, Vol. 77, 1971.
9. Lasdon, L.S, "Optimization Theory for Large Systems", MacMillan Co., Callier - MacMillan Limited. London, 1970
10. Camenzond, H.R. "Circuit Design for Integrated Electronics", Addison - Wesley Publishing Co., 1968
11. Meiksin, Z.H. "Thin and Thick Films for Hybrid Microelectronics", Lexington Cooks, D.C. Heath Co 1976.
12. Allison, J, "Electronic Integrated Circuits" McGraw Hill 1975.
13. Temes, G.C. and Calahan, D.A. "Computer-aided Network Optimization", IEEE Vol 53, pp. 1832 - 1863, Nov. 1967

APPENDIX A

DERIVATION OF THE TRANSFER FUNCTIONS OF EQUATION 3.2 AND 3.2

In this section, the voltage transfer functions of the proposed filter are derived. For convenience, the filter configuration of figure -3.3 will be repeated.

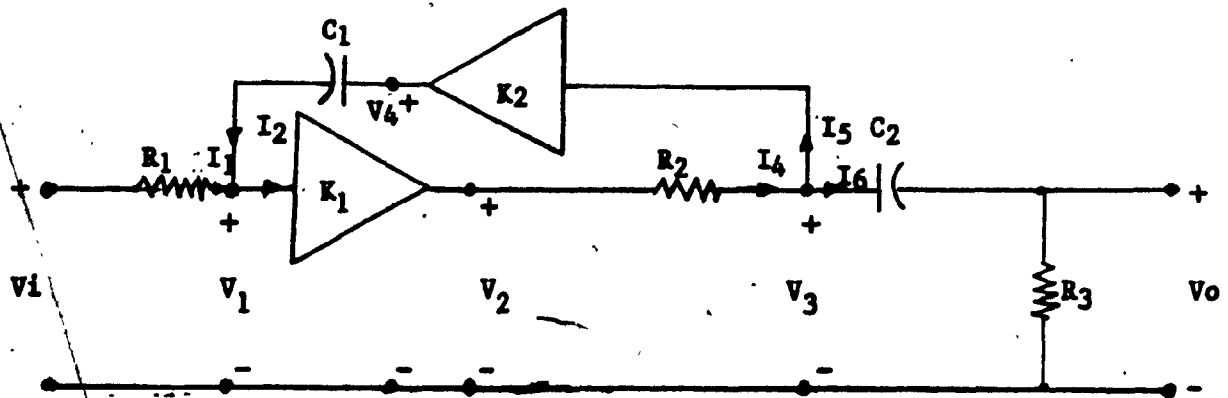


Figure A1- Band-Pass RC-active filter with OAs of non-unity-gain mode (Proposed filter, figure 3,3)

Observing the branch-voltages and branch-currents of figure A , the KCL and KVL analysis follows:

$$I_3 = I_1 + I_2 = 0 \dots \dots \dots (A -1)$$

$$I_1 = -I_2 \dots \dots \dots (A -2)$$

$$V_1 = \frac{V_2}{K_1} \dots \dots \dots (A_1-3)$$

$$V_4 = K_2 V_3 \dots \dots \dots (A_1-4)$$

$$V_i = I_1 R_1 + V_1 \dots \dots \dots (A_1-5)$$

Substituting eq. (A<sub>1</sub>-2) and eq. (A<sub>1</sub>-3) into eq. (A<sub>1</sub>-5), then

$$V_i = -I_2 R_1 + \frac{V_2}{K_1} \dots \dots \dots (A_1-6)$$

$$I_2 = (V_4 - V_1) C_1 S \dots \dots \dots (A_1-7)$$

Substituting eq. (A<sub>1</sub>-3) and (A<sub>1</sub>-4) into eq. (A<sub>1</sub>-7), then

$$I_2 = K_2 V_3 C_1 S - \frac{V_2}{K_1} C_1 S \dots \dots \dots (A_1-8)$$

Substituting eq. (A<sub>1</sub>-7) into eq. (A<sub>1</sub>-6), the latter becomes

$$V_i = -K V_3 R_1 C_1 S + \frac{V_2}{K_1} R_1 C_1 S + \frac{V_2}{K_1} \dots \dots \dots (A_1-9)$$

Also, from the same configuration,

$$V_2 = I_4 R_2 + V_3 \dots \dots \dots (A_1-10)$$

$$\text{and } I_4 = I_5 + I_6 = I_6 \dots \dots \dots (A_1-11)$$

where  $I_5 = 0$

$$\text{Eq. (A}_1\text{-10) becomes } V_2 = I_6 R_2 + V_3 \dots \dots \dots (A_1-12)$$

Also, by observing the filter configuration,

$$I_6 = (V_3 - V_0) C_2 S \dots \dots \dots (A_1-13)$$

Substituting eq. (A<sub>1</sub>-13) into eq. (A<sub>1</sub>-12), then

$$V_2 = V_3 (1 + R_2 C_2 S) - V_0 R_2 C_2 S \dots \dots \dots (A_1-14)$$

Substituting eq. (A<sub>1</sub>-14) into eq. (A<sub>1</sub>-9), the latter becomes:

$$V_i = V_3 \left[ \frac{1}{K_1} R_1 C_1 (R_2 C_2 s + 1) s + \frac{1}{K_1} (R_2 C_2 s + 1) - K_2 R_1 C_1 s - \frac{V_o}{K_1} [R_1 R_2 C_1 C_2 s^2 + R_2 C_2 s] \right] \dots (A_1 - 15)$$

By observation,  $V_o = \frac{V_3 R_3 C_2 s}{R_3 C_2 s + 1} \dots \dots \dots (A_1 - 16)$

From eq. (A<sub>1</sub>-16),  $V_3 = V_o \frac{(R_3 C_2 s + 1)}{R_3 C_2 s} \dots \dots \dots (A_1 - 17)$

Substituting eq. (A<sub>1</sub>-17) into eq. (A<sub>1</sub>-15) and solving for  $\frac{V_o}{V_i}$  after some calculations, then

$$\frac{V_o}{V_i} = \frac{\frac{K_1 R_3}{R_1 C_1 [R_2 + R_3 (1 - K_1 K_2)]}}{s^2 + \frac{R_1 C_1 + R_2 C_2 + R_3 C_2 - K_1 K_2 R_1 C_1}{R_1 C_1 C_2 [R_2 + R_3 (1 - K_1 K_2)]} s + \frac{1}{R_1 C_1 C_2 [R_2 + R_3 (1 - K_1 K_2)]}} \dots \dots (A_1 - 18)$$

Letting  $K_1 = K_2 = K$ , then

$$\frac{V_o}{V_i} = \frac{\frac{K_1 R_3}{R_1 C_1 [R_2 + R_3 (1 - K^2)]}}{s^2 + \frac{R_1 C_1 + R_2 C_2 + R_3 C_2 - K^2 R_1 C_1}{R_1 C_1 C_2 [R_2 + R_3 (1 - K^2)]} s + \frac{1}{R_1 C_1 C_2 [R_2 + R_3 (1 - K^2)]}} \dots \dots (A_1 - 19)$$

Finally, setting  $K=1$ , then

$$\frac{V_o}{V_i} = \frac{\frac{R_3}{R_1 R_2 C_1}}{s^2 + \frac{R_2 + R_3}{R_1 R_2 C_1} s + \frac{1}{R_1 R_2 C_1 C_2}} \dots \dots \dots (A_1 - 20)$$

APPENDIX B

FILTER DESIGN EQUATIONS

The filter design-equations are analyzed in this section. In this analysis, the transfer-functions, eq. (3.2) and (3.3), are considered, which are repeated for convenience.

Case 1. Ideal OA of unity-gain mode (see figure 3.1)

(A) Parameter-equations ( $\omega_0$ ,  $Q_0$  and  $H_{sp}$ )

For convenience, eq. (3.2) is repeated here:

$$\frac{V_o}{V_i} = \frac{\frac{R_3}{R_1 R_2 C_1} s}{s^2 + \frac{R_2 + R_3}{R_1 R_2 C_1} s + \frac{1}{R_1 R_2 C_1 C_2}} \dots \dots \dots (B1-1)$$

$$\text{or } \frac{V_o}{V_i} = \frac{H_{sp} \frac{\omega_0}{Q_0} s}{s^2 + \frac{\omega_0}{Q_0} s + \omega_0^2} \dots \dots \dots (B1-2)$$

$$\text{From eq. (B1-1) and (B1-2) } \frac{\omega_0}{Q_0} = \frac{R_2 + R_3}{R_1 R_2 C_1} = \frac{1}{R_1 C_1} \left(1 + \frac{R_3}{R_2}\right) \dots \dots \dots (B1-3)$$

$$\omega_0^2 = \frac{1}{R_1 R_2 C_1 C_2} \dots \dots \dots (B1-4)$$

$$\text{and } H_{sp} \cdot \frac{\omega_0}{Q_0} = \frac{R_3}{C_1 R_1 R_2} \dots \dots \dots (B1-5)$$

$$\text{From eq. (B1-4), } \omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \dots \dots \dots (B1-6)$$

Substituting eq. (B1-6) into eq. (B1-3) and solving for  $Q_0$ ,

then,  $Q_0 = \frac{1}{R_2+R_3} \cdot \sqrt{\frac{R_1 R_2 \epsilon_1}{C_2}} \dots\dots\dots (B1-7)$

Substituting eq. (B1-3) into eq. (B1-5) and solving for  $H_{sp}$

then  $H_{sp} = \frac{R_3}{R_2+R_3} \dots\dots\dots (B1-8)$

(B) Element-Equations (Resistors)

Multiplying eq. (B1-6) by eq. (B1-7) and replacing the term  $(R_2 + R_3)$  in eq. (B1-8) by  $R_2 + R_3 = \frac{R_3}{H_{sp}} \dots\dots\dots (B2-1)$

then  $\omega_0 Q_0 = \frac{H_{sp}}{R_3} \cdot \frac{1}{C_2} \dots\dots\dots (B2-1)$

Solving eq. (B2-1) for  $R_3$ , then  $R_3 = \frac{1}{C_2} \cdot \frac{H_{sp}}{\omega_0 Q_0} \dots\dots\dots (B1-3)$

Solving eq. (B1-8) for  $R_2$  and replacing  $R_3$ , from eq. (B1-3),

then  $R_2 = \frac{1}{C_2} \cdot \frac{1 - H_{sp}}{\omega_0 Q_0} \dots\dots\dots (B1-4)$

Solving eq. (B1-6) for  $R_1$  and replacing  $R_2$ , from eq. (B1-4),

then  $R_1 = \frac{1}{C_1} \cdot \frac{Q_0}{\omega_0 (1 - H_{sp})} \dots\dots\dots (B1-5)$

(C) Element-equations (Capacitors)

Solving eq. (B1-3) for  $C_2$ , then  $C_2 = \frac{1}{R_3} \cdot \frac{H_{sp}}{\omega_0 Q_0} \dots\dots\dots (B2-1a)$

or multiplying eq. (B1-6) by eq (B1-7) and solving for  $C_2$ ,

then  $C_2 = \frac{1}{R_2+R_3} \cdot \frac{1}{\omega_0 Q_0} \dots\dots\dots (B2-1b)$

Solving eq. (B1-3) for  $C_1$ , then  $C_1 = \frac{R_2+R_3}{R_1R_2} \cdot \frac{Q_0}{\omega_0} \dots (B2-1c)$

Case 2 Ideal amplifiers (non-unity-gain mode)

In this case, only the parameter-equations will be analysed which are used in the sensitivity and stability sections, 3.4 and 3.5 respectively.

Eq. (3.3) is repeated for convenience:

$$\frac{V_o}{V_i}(s) = \frac{\frac{K_1 R_3}{R_1 C_1 [R_2 + R_3 (1 - K_1 K_2)] s}}{s^2 + \frac{R_1 C_1 + R_2 C_2 + R_3 C_2 - K_1 K_2 R_1 C_1}{R_1 C_1 C_2 [R_2 + R_3 (1 - K_1 K_2)]} s + \frac{1}{R_1 C_1 C_2 [R_2 + R_3 (1 - K_1 K_2)]}} \dots (B3-1)$$

From eq. (B1-2) and (B3-1),

$$\omega_0^2 = \frac{1}{R_1 C_1 C_2 [R_2 + R_3 (1 - K_1 K_2)]} \dots (B3-2)$$

$$\frac{\omega_0}{Q_0} = \frac{R_1 C_1 + R_2 C_2 + R_3 C_2 - K_1 K_2 R_1 C_1}{R_1 C_1 C_2 [R_2 + R_3 (1 - K_1 K_2)]} \dots (B3-3)$$

$$\text{and } H_{sp} \frac{\omega_0}{Q_0} = \frac{K_1 R_3}{R_1 C_1 [R_2 + R_3 (1 - K_1 K_2)]} \dots (B3-4)$$

$$\text{From eq. (B3-2), } \omega_0 = \frac{1}{\sqrt{R_1 C_1 C_2 [R_2 + R_3 (1 - K_1 K_2)]}} \dots (B3-5)$$



Replacing  $W_0$  from eq. (B3-5) in eq. (B3-3) and solving for  $Q_0$ , then

$$Q_0 = \frac{\sqrt{R_1 C_1 C_2 [R_2 + R_3 (1 - K_1 K_2)]}}{R_1 C_1 (1 - K_1 K_2) + C_2 (R_2 + R_3)} \dots\dots\dots (B3-6)$$

Substituting eq. (B3-3) in eq. (B3-4) and solving for  $H_{sp}$ ,

$$\text{then } H_{sp} = \frac{K_1 R_3 C_2}{R_1 C_1 (1 - K_1 K_2) + C_2 (R_2 + R_3)} \dots\dots\dots (B3-7)$$

APPENDIX C

MINIMIZATION METHODS

C1 Unconstrained Problems

In this class of minimization problems, the function  $f(\mathbf{x})$  is twice differentiable with  $\bar{\mathbf{x}} = [x, y]^T$ . Letting  $x = x_0 + h$  and  $y = y_0 + k$ , then  $\min_{\bar{\mathbf{x}}} f(\bar{\mathbf{x}})$  is found by Taylor series expansion and a strong local point " $x_0, y_0$ " is sought, so that  $f(x_0 + h, y_0 + k) \gg f(x_0, y_0)$  for all  $h$  and  $k$ .

For minimization,

$$hf_x(x_0, y_0) + kf_y(x_0, y_0) + \frac{1}{2} \begin{bmatrix} h, k \end{bmatrix} H(x_0, y_0) \begin{bmatrix} h \\ k \end{bmatrix} + o(h^3, k^3) > 0,$$

where  $H(x_0, y_0)$  is the Hessian matrix evaluated at " $x_0, y_0$ " and  $f_x(x_0, y_0) = 0, f_y(x_0, y_0) = 0$

C2 Newton's method for minimization

For one-dimensional cases, the roots of a function  $f(x) = 0$  are sought by, first, approximating it with its tangent at some point " $x_0$ ", and, then by solving for its next value at " $x_1$ " when the tangent crosses the  $x$ -axis. The process is repeated at subsequent points  $x_2, x_3, \dots, x_n$ , by the recurrence formula:

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)} \dots \dots \dots (C2-1)$$

until the algorithm converges. The closer the initial point,

is to the actual solution " $x^*$ " the faster the convergence.

For multi-dimensional problem, where " $x$ " is replaced by the vector  $\bar{x} = [x_1, x_2, \dots, x_n]^T$ , the corresponding recurrence formula in eq. (C2-1) is replaced by:

$$\nabla f(x_{n+1}) = \nabla f(x_n) + \nabla \nabla^T f(x_n) \cdot (x_{n+1} - x_n) = 0 \dots\dots\dots (C2-2)$$

Substituting  $\nabla \nabla^T f(x_n)$  with the hessian matrix  $H(x_n)$  and solving for  $x_{n+1}$ , then:  $\bar{x}_{n+1} = \bar{x}_n - H^{-1}(\bar{x}_n) \nabla f(\bar{x}_n) \dots\dots\dots (C2-3)$

This is one of a few algorithms under the Quasi-Newton's method, and it converges fast if a suitable value " $\bar{x}_0$ " is chosen. The only drawback of this algorithm is that it requires second partial derivative of  $f(\bar{x})$  at each iteration. In view of this,  $H^{-1}$  will assume new values at each iteration.

In order to alleviate this problem, this algorithm is replaced by other algorithms wherever the hessian matrix  $H$  is required. However, when the function is of the quadratic form,  $f(\bar{x}) = \frac{1}{2} \bar{x}^T A \bar{x} + b^T \bar{x} + c$ , then  $H^{-1}$  is constant. For this reason, non-linear functions are often approximated by quadratic forms which can be conveniently used in the Penalty-function minimization algorithms.

One recommended type of the Quasi-Newton's algorithms is the following:

- Step 0. Select  $\bar{x}^0$ ; also, select a real positive constant  $\beta$  ( $.5 < \beta < .8$ ) and set  $i = 0$ .
- Step 1. Compute  $\nabla f(\bar{x}^i)$ .
- Step 2. If  $\nabla f(\bar{x}^i) = 0$ , stop; else, go to step 3.
- Step 3. Compute  $H(\bar{x}^i)^{-1}$  if it exists, and go to step 5; else, set  $h(\bar{x}^i) = -\nabla f(\bar{x}^i)$  and go to step 5.
- Step 4. Select step-size  $k_i$  (i.e.  $k_i = k=1$ , or  $k_i = k\beta$ ) from known algorithms.
- Step 5. Set  $\bar{x}^{i+1} = \bar{x}^i + k_i h(\bar{x}^i)$ , set  $i = i+1$  and go the step 1.

### C3 Constrained problems

These problems may be stated as follows:  $\min_{\bar{x}} f(\bar{x})$ , such that  $g_i(\bar{x}) = 0$ ,  $i=1,2,\dots,m$ ..... (C3-1) where  $m < n$ . Using the "Lagrangian multipliers"  $\lambda$  these

constrained problems can be converted into unconstrained ones by the relation:  $L(\bar{x}; \lambda_i) = f(\bar{x}) + \sum_{i=1}^m \lambda_i g_i(\bar{x})$ ..... (C3-2) where  $\lambda_1 = \lambda_2, \lambda_3, \dots, \lambda_m$  are the Lagrangian multipliers.

Necessary conditions for "stationary" (maximum or minimum) point are:

$$\frac{\partial L}{\partial x_j} = \frac{\partial f(\bar{x})}{\partial x_j} + \sum_{i=1}^m \lambda_i \frac{\partial g_i}{\partial x_j} = 0, \quad \begin{matrix} j=1,2,\dots,n \\ i=1,2,\dots,m \end{matrix} \dots\dots\dots (C3-3)$$

Using the gradient notation " $\nabla$ " the expression in eq. (C3-3) can be re-written in the compact form:

$$\nabla_{\mathbf{x}} f(\bar{\mathbf{x}}) + \left[ \frac{\partial g_i}{\partial \mathbf{x}} \right] = 0 \dots\dots\dots (C3-4)$$

### C3.1 Computational methods

The Lagrangian computational problem leads to  $m+n$  simultaneous equations in  $m+n$  unknown variables by using the necessary convergence conditions at " $\mathbf{x}^*$ " and " $\lambda^*$ ", namely:

$$\nabla_{\mathbf{x}} L(\mathbf{x}^*, \lambda^*) = 0 \dots\dots\dots (C3.5)$$

$$\text{and } \nabla_{\lambda} L(\mathbf{x}^*, \lambda^*) = 0 \dots\dots\dots (C3.6)$$

Newton-Raphson's method is numerically straight-forward, but often inadequate, especially when  $(m+n)$  increases. Necessary convergence condition at " $\mathbf{z}^*$ " is:

$$\nabla_{\mathbf{z}} L(\mathbf{z}^*) = 0 \dots\dots\dots (C3.7)$$

$$\text{where } \mathbf{z} = \begin{bmatrix} \bar{\mathbf{x}} \\ \lambda \end{bmatrix} \dots\dots\dots (C3.8)$$

The recurrence formula is:

$$\mathbf{z}_{n+1} = \mathbf{z}_n - \mathbf{H}^{-1}(\mathbf{z}_n) \nabla_{\mathbf{z}} L(\mathbf{z}_n) \dots\dots\dots (C3.9)$$

where  $\mathbf{H}(\mathbf{z})$  is the hessian matrix of  $f(\bar{\mathbf{x}}, \lambda)$  given by:

$$\mathbf{H}(\mathbf{z}_n) = \begin{bmatrix} \nabla_{\mathbf{x}\mathbf{x}} L & \nabla_{\mathbf{x}} \lambda L \\ \nabla_{\lambda \mathbf{x}} L & \nabla_{\lambda \lambda} L \end{bmatrix} \quad \text{and } \nabla_{\mathbf{z}} L(\mathbf{z}_n) = \begin{bmatrix} \nabla_{\mathbf{x}} L \\ \nabla_{\lambda} L \end{bmatrix} \dots\dots\dots (C3.10)$$

### C3.2 Kuhn and Tucker convergence conditions

Kuhn-Tucker theorem refers to Lagrangian-type functions using inequality constraints and converting to unconstrained functions with the stationary convergence conditions:

$$\begin{aligned}
 \nabla_{\bar{x}} L(\bar{x}^*, \bar{\lambda}^*) &= 0 \\
 \nabla_{\bar{\lambda}} L(\bar{x}^*, \bar{\lambda}^*) &\leq 0 \quad \dots\dots\dots(C3.11) \\
 (\bar{\lambda}^*)^T \bar{g}(\bar{x}^*) &= 0 \\
 \bar{\lambda}^* &\geq 0
 \end{aligned}$$

### C3.3 One-dimensional search techniques

If  $f(x)$  is a real-valued one-dimensional function then  $x^*$  is its "minimum" in an interval  $[a^0, b^0]$  after the  $n$ th iteration, and after this interval is reduced to a smaller one  $[a^n, b^n]$  by using different search techniques.

Some search techniques are: dichotomous search, equal-interval search, Fibonacci search, Golden-section search, and quadratic interpolation search, each being exhibiting merits and (or) drawbacks from the other. A distinct feature of these techniques is that there is no need for derivatives in  $f(x)$ .

## C4 Gradient techniques

### C4.1 Unconstrained algorithms

In this class of gradient techniques, multi-dimensional problems are solved using "direct methods". The "gradient-direction" vector,  $\nabla f(\bar{x}) = \left[ \frac{\partial f(\bar{x})}{\partial x_1}, \frac{\partial f(\bar{x})}{\partial x_2}, \dots, \frac{\partial f(\bar{x})}{\partial x_n} \right]^T$ . (C4.1)

is used in the algorithms.

This vector is also called "direction of steepest descent" for minimizing  $f(\bar{x})$  and "direction of steepest ascent" for maximizing  $f(\bar{x})$ .

For example, for steepest descent, the difference relation:  $f(\bar{x}_0) - f(\bar{x}_0 + d\bar{x})$  is maximized, where the vector  $d\bar{x}$  describes an incremental distance  $ds$  along the path of steepest descent:

$$ds^2 = \sum_{i=1}^n (dx_i)^2 \quad \text{or} \quad 1 - \sum_{i=1}^n \left( \frac{dx_i}{ds} \right)^2 = 0 \quad \dots \dots \dots (C4.2)$$

where  $\frac{dx_i}{ds}$  are the direction cosines.

The Lagrangian-multiplier function  $L\left(\frac{dx_i}{ds}, \lambda\right)$  and its necessary convergence conditions are used. The recurrence relation (discrete algorithm) is  $x^{p+1} = x^p + k \nabla f(x^p)$ .....(C4.3)

where  $k$  is the step-size of the gradient change, on the descent-path (i.e.,  $k < 0$  for convergence). With small  $k$ , the discrete algorithm will follow the gradient path very closely

but the convergence rate will be slow. On the other hand, with large  $k$ , the convergence will be fast, but the optimum point  $x^*$  might be missed, causing "oscillations" around  $\bar{x}^*$ . One way to avoid this situation is to reduce the number of steps by increasing the number of computations at each step.

#### C4.2 Constrained algorithms

In this class of gradient techniques, "boundary-following" and "penalty-function" problems are solved. In the "boundary-following" problems, linear quadratic and non-linear multi-dimensional problems can be optimized using linear or non-linear programming algorithms.

In the non-linear function case, gradient-type (such as feasible direction and projection methods) algorithms are employed.

The gradient techniques using equality-constraints ( $g_1(\bar{x}) = 0$ ) are similar to those using inequality-constraints, except that the iterative path must lie on the boundary at all times.



### C4.3 Penalty-function method

Direct and Lagrangian multiplier-methods are used to solve these converted (from constrained into unconstrained) objective functions.

#### C4.3.1 Equality-constrained problems

In this category, a problem can be formulated as follows:  $\text{Min}_{\bar{x}} f(\bar{x})$ , such that  $g_i(\bar{x}) = 0, i=1,2,\dots,m$ .....(C4.4)

Here,  $f(\bar{x})$  is the constrained objective function. The above objective function is converted into the new unconstrained function:

$$P(\bar{x}; k_i) = f(\bar{x}) + \sum_{i=1}^m k_i (g_i(\bar{x}))^2 \dots \dots \dots (C4.5)$$

$$\text{where } \bar{k} = [k_1, k_2, \dots, k_m]^T \dots \dots \dots (C4.6)$$

is a positive real-valued vector, which is a specified weighting factor, depending on how much the constraint-functions is to be satisfied. As  $k_i$  increases from zero-value to infinity, the constraint-functions are satisfied more closely to their desired values. Thus, these added "weighting" terms (i.e. the second term of the right hand side of eq. (C4.5) of the objective function represent "penalties" for closeness to the constraint boundaries. This penalty, implies more effort and longer process in order to reach optimum solution.

#### C4.3.2. Inequality-constraints

In this category, the problem is stated as follows:

$$\min_{\bar{x}} f(\bar{x}), \text{ such that } g_i(\bar{x}) \leq 0, i=1,2,\dots,r \dots\dots\dots (C4.7)$$

The new unconstrained function is:

$$P(\bar{x};k) = f(\bar{x}) + \sum_{i=1}^r k_i [g_i(\bar{x})]^2 U_i(g_i) \dots\dots\dots (C4.8)$$

$$\text{where } U_i(g_i) = \begin{cases} 0 & \text{if } g_i(\bar{x}) \leq 0 \\ 1 & \text{if } g_i(\bar{x}) > 0 \end{cases} \dots\dots\dots (C4.9)$$

and  $k_i > 0$ .  $U_i(g_i)$  is a step-function served to ignore the constraint, whenever " $\bar{x}$ " is inside the feasible region and allows the existence of the equality-constraint,  $g_i(\bar{x}) = 0$  whenever " $\bar{x}$ " is outside the feasible region.

#### C4.4 Variable-metric algorithms

The variable-metric algorithms, as part of the quadratically convergent algorithms, optimize the convergence direction near the optimum " $\bar{x}^*$ ", by determining the vector difference ( $\bar{x}^* - \bar{x}^0$ ) from  $\nabla f(\bar{x}^0)$  where  $f(\bar{x}^0)$  is approximated by a sequence of quadratic functions near " $\bar{x}^*$ ".

Using first-order Taylor series expansion of  $\nabla f(\bar{x})$  and assuming that  $f(\bar{x})$  is twice-differentiable, that " $\bar{x}^*$ " and " $\bar{x}^0$ "

are its minimum and close-to-minimum points respectively, and  $H(\bar{x})$  is its Hessian matrix at  $\bar{x}^*$ , then  $\nabla f(\bar{x}^*) = 0$  and  $H(\bar{x}^*)$  is positive definite.

The respective recurrence formulae for a general function  $f(\bar{x})$  described above (and for a quadratic function

$$f(\bar{x}) = c + [\bar{b}]^T \bar{x} + \frac{1}{2} [\bar{x}]^T A \bar{x}) \text{ are:}$$

$$\bar{x}^* = \bar{x}^0 - H[\bar{x}^*]^{-1} \cdot \nabla f(\bar{x}^0) \dots \dots \dots (C4.10)$$

$$\text{and } \bar{x}^* = \bar{x}^0 - A^{-1} \cdot \nabla f(\bar{x}^0) \dots \dots \dots (C4.11)$$

$$\text{where } \nabla f(\bar{x}^0) = A\bar{x}^0 + \bar{b} \dots \dots \dots (C4.12)$$

$$\text{and } \bar{x}^* = -A^{-1}\bar{b} \dots \dots \dots (C4.13)$$

#### C4.5 Fletcher-Powell variable-metric algorithm

Let  $\bar{x}^i$  and  $H_i$  denote the approximations to  $\bar{x}^*$  and the inverse of the Hessian matrix, respectively. Let  $\bar{g}_i = \nabla f(\bar{x}^i)$ . The Fletcher-Powell algorithm consists of the following four basic steps:

1. Choose  $\bar{x}^0$  and  $H_0$

2. For  $i = 0, 1, \dots, n-1$ , define:

$$\bar{x}^{i+1} = \bar{x}^i + \lambda_i \bar{v}^i \dots \dots \dots (C4.14)$$

$$\text{where } \bar{v}^i = -H_i \bar{g}_i \dots \dots \dots (C4.15)$$

and  $\lambda_i$  minimizes  $f(\bar{x}^i + \lambda \bar{v}^i)$  with respect to  $\lambda$

3. For  $i=0,1,\dots,n-1$ , let  $\bar{u}^i = \lambda_i \bar{v}^i \dots\dots\dots(C4.16)$

and  $\bar{y}^i = \bar{g}^{(i+1)} - \bar{g}^i \dots\dots\dots(C4.17)$

Define  $H_{i+1} = H_i + A_i + B_i \dots\dots\dots(C4.18)$

where  $A_i = \frac{\bar{u}^i(\bar{u}^i)^T}{(\bar{u}^i)^T(\bar{y}^i)}$  and  $B_i = \frac{H_i\bar{y}^i(H_i\bar{y}^i)^T}{(\bar{y}^i)^T(H_i\bar{y}^i)}$  (C4.19)

4. If the "stopping" criteria are not satisfied, then let  $\bar{x}^0 = \bar{x}^n$  and go to step 2.

The starting point  $\bar{x}^0$  is arbitrary but can be suitably chosen by methods available in the literature.

$H_0$  is any symmetric positive-definite matrix, as initial approximation to the inverse of the Hessian matrix; the simplest  $H_i$  is the diagonal matrix with positive scalars along the main diagonal.

Stopping rules are available in the literature. In many computer-program minimization techniques, such as the variable-metric algorithm, the convergence point " $\bar{x}^*$ " may be missed when "round-off" errors occur in the matrix and gradient computations. In order to minimize this effect, a good approach is to discard  $H_n$  after  $n$  iterations and start with new approximation.

APPENDIX D

GRADIENT-VECTOR G( $\bar{x}$ )

In this section, the elements  $G(i)$ ,  $i=1,2,\dots,5$  are calculated for a five-dimensional gradient-vector,  $G(\bar{x}) = \nabla P(\bar{x}; k_i)$ , per equation (5.9). As an example, the gradient-vector  $\nabla P(\bar{x}; k_i)$ , will be derived (minimization case 1a), where  $P(\bar{x}; k_i) = C_T + K_1 g_1^2 + K_2 g_2^2 \dots \dots \dots (D-1)$

$$P(\bar{x}; k_i) = C_T + K_1 g_1^2 + K_2 g_2^2 \dots \dots \dots (D-1)$$

$$\text{and } C_T = C_1 + C_2 \dots \dots \dots (D-2)$$

$$g_1 = \omega_0^2 R_1 R_2 C_1 C_2 - 1 = 0 \dots \dots \dots (D-3)$$

$$g_2 = \omega_0^2 (R_2 + R_3)^2 \frac{C_2}{R_1 R_2 C_1} - 1 = 0 \dots \dots \dots (D-4)$$

$$\text{Also } x_1 = C_1, x_2 = C_2, x_3 = R_1, x_4 = R_2, x_5 = R_3 \dots \dots \dots (D-5)$$

$$G(1) = G(x_1) = \frac{\partial P(\bar{x}; K_i)}{\partial x_1} = \frac{\partial P(\bar{x}; K_i)}{\partial C_1} \dots \dots \dots (D-6)$$

$$\text{or } G(1) = \frac{\partial C_T}{\partial C_1} + K_1 \frac{\partial}{\partial C_1} (g_1^2) + K_2 \frac{\partial (g_2^2)}{\partial C_1} \dots \dots \dots (D-7)$$

$$\frac{\partial C_T}{\partial C_1} = 1 \dots \dots \dots (D-8)$$

$$k_1 \frac{\partial (g_1^2)}{\partial C_1} = 2k_1 (\omega_0^2 R_1 R_2 C_1 C_2 - 1) \omega_0^2 R_1 R_2 C_2$$

$$= \frac{2k_1}{C_1} (\omega_0^2 R_1 R_2 C_1 C_2 - 1) \cdot (\omega_0^2 R_1 R_2 C_1 C_2) \dots \dots \dots (D-9)$$

$$= \frac{2k_1}{C_1} g_1 (g_1 + 1) \dots \dots \dots (D-10)$$

where eq. (C-3) was used in eq. (D-8).

$$K_2 \frac{\partial (g_2^2)}{\partial C_1} = - \frac{2K_2}{C_1} \left[ Q_0^2 (R_2 + R_3)^2 \frac{C_2}{R_1 R_2 C_1} - 1 \right] \cdot \left[ Q_0^2 (R_2 + R_3)^2 \frac{C_2}{R_1 R_2 C_1} \right] \quad (D-11)$$

$$= - \frac{2k_2}{C_1} g_2 (g_2 + 1) \dots \dots \dots (D-12)$$

Substituting eq. (D-8), (D-10), and (D-12) in eq. (D-7),

$$\text{then } G(1) = 1 + \frac{2}{C_1} \left[ K_1 g_1 (g_1 + 1) - K_2 g_2 (g_2 + 1) \right] \dots \dots \dots (D-13)$$

$$\text{Similarly, } G(2) = \frac{\partial C_T}{\partial C_2} + K_1 \frac{\partial}{\partial C_2} (g_1^2) + K_2 \frac{\partial (g_2^2)}{\partial C_2} \dots \dots \dots (D-14)$$

$$\text{where, } \frac{\partial C_T}{\partial C_2} = 1 \dots \dots \dots (D-15)$$

$$K_1 \frac{\partial}{\partial C_2} (g_1^2) = \frac{2k_1}{C_2} (\omega_0^2 R_1 R_2 C_1 C_2 - 1) (\omega_0^2 R_1 R_2 C_1 C_2) \dots \dots \dots (D-16)$$

$$= \frac{2k_1}{C_2} g_1 (g_1 + 1) \dots \dots \dots (D-17)$$

$$K_2 \frac{\partial}{\partial C_2} (g_2^2) = \frac{2k_2}{C_2} \left[ Q_0^2 (R_2 + R_3)^2 \frac{C_2}{R_1 R_2 C_1} - 1 \right] \cdot \left[ Q_0^2 (R_2 + R_3)^2 \frac{C_2}{R_1 R_2 C_1} \right] \dots (D-18)$$

$$= \frac{2k_2}{C_2} g_2 (g_2 + 1) \dots \dots \dots (D-19)$$

Substituting eq. (D-15), (D-17), and (D-19) in eq. (D-14),

$$G(2) = 1 + \frac{2}{C_2} \left[ K_1 g_1 (g_1 + 1) + K_2 g_2 (g_2 + 1) \right] \dots \dots \dots (D-20)$$

$$G(3) = \frac{\partial C_T}{\partial R_1} + K_1 \frac{\partial (g_1^2)}{\partial R_1} + K_2 \frac{\partial (g_2^2)}{\partial R_1} \dots \dots \dots (D-21)$$

$$\text{where } \frac{\partial C_T}{\partial R_1} = 0 \dots \dots \dots (D-22)$$

$$K_1 \frac{\partial (g_1^2)}{\partial R_1} = \frac{2K_1}{R_1} (R_1 R_2 C_1 C_2 - 1) (R_1 R_2 C_1 C_2) \dots \dots \dots (D-23)$$

$$= \frac{2K_1}{R_1} g_1 (g_1 + 1) \dots \dots \dots (D-24)$$

$$K_2 \frac{\partial (g_2^2)}{\partial R_1} = - \frac{2k_2}{R_1} \left[ Q_0^2 (R_2 + R_3)^2 \frac{C_2}{R_1 R_2 C_1} - 1 \right] \cdot \left[ Q_0^2 (R_2 + R_3)^2 \frac{C_2}{R_1 R_2 C_1} \right] \dots (D-25)$$

$$= -\frac{2k_2}{R_1} g_2 (g_2 + 1) \dots \dots \dots (D-26)$$

Substituting eq. (D-22), (D-24), and (D-26) in eq. (D-21)

then,  $G(3) = \frac{2}{R_1} [K_1 g_1 (g_1 + 1) - K_2 g_2 (g_2 + 1)] \dots \dots \dots (D-27)$

$$G(4) = \frac{\partial C_T}{\partial R_2} + k_1 \frac{\partial (g_1^2)}{\partial R_2} + K_2 \frac{\partial (g_2^2)}{\partial R_2} \dots \dots \dots (D-28)$$

where  $\frac{\partial C_T}{\partial R_2} = 0 \dots \dots \dots (D-29)$

$$K_1 \frac{\partial (g_1^2)}{\partial R_2} = \frac{2}{R_2} K_1 (R_1 R_2 C_1 C_2 - 1) (R_1 R_2 C_1 C_2) \dots \dots \dots (D-30)$$

$$= \frac{2K_1}{R_2} g_1 (g_1 + 1) \dots \dots \dots (D-31)$$

$$K_2 \frac{\partial (g_2^2)}{\partial R_2} = \frac{2(R_2 - R_3)}{R_2(R_2 + R_3)} K_2 \left[ Q_o^2 (R_2 + R_3)^2 \frac{C_2}{R_1 R_2 C_1} - 1 \right] \cdot \left[ Q_o^2 (R_2 + R_3)^2 \frac{C_2}{R_1 R_2 C_1} \right] \dots (D-32)$$

$$= \frac{2(R_2 - R_3)}{R_2(R_2 + R_3)} k_2 g_2 (g_2 + 1) \dots \dots \dots (D-33)$$

Substituting eq. (D-29), (D-31), and (D-33) in eq. (D-28),

then  $G(4) = \frac{2}{R_2} \left[ K_1 g_1 (g_1 + 1) + \frac{R_2 - R_3}{R_2 + R_3} K_2 g_2 (g_2 + 1) \right] \dots \dots \dots (D-34)$

Finally,  $G(5) = \frac{\partial C_T}{\partial R_3} + K_1 \frac{\partial (g_1^2)}{\partial R_3} + k_2 \frac{\partial (g_2^2)}{\partial R_3} \dots \dots \dots (D-35)$

where,  $\frac{\partial C_T}{\partial R_3} = 0 \dots \dots \dots (D-36)$

$$K_1 \frac{\partial (g_1^2)}{\partial R_3} = K_1 \frac{\partial}{\partial R_3} (R_1 R_2 C_1 C_2 - 1) = 0 \dots \dots \dots (D-37)$$

$$K_2 \frac{\partial (g_2^2)}{\partial R_3} = \frac{4}{R_2 + R_3} K_2 \left[ Q_o^2 (R_2 + R_3)^2 \frac{C_2}{R_1 R_2 C_1} - 1 \right] \cdot \left[ Q_o^2 (R_2 + R_3)^2 \frac{C_2}{R_1 R_2 C_1} \right] \dots (D-38)$$

$$= \frac{4}{R_2 + R_3} K_2 g_2 (g_2 + 1) \dots \dots \dots (D-39)$$

Substituting eq. (D-36), (D-37), and (D-39) in eq. (D-35),

the latter becomes:  $G(5) = \frac{4}{R_2 + R_3} k_2 g_2 (g_2 + 1) \dots \dots \dots (D-40)$

APPENDIX E

Thin-film area: Ac and Ar

I) Calculation of Ac

As an example, the tantalum metal (TM) capacitor is considered, which consists of a tantalum pentoxide dielectric, a tantalum base electrode, and a counterelectrode of evaporated gold, aluminum, or other metal, as shown in figure E.1

Electric field  $\mathcal{E}$  (volts/cm) is developed between the plates, separated by distance  $d$  (cm) and sustaining potential  $V$ , which represents the anodizing voltage  $V_a$  during deposition of the dielectric film. It is estimated, by general acceptance, that the relation between the anodization voltage  $V_a$  and the breakdown voltage  $V_{BR}$  is:

$$V_{BR} \approx \left(\frac{2}{3}\right) V_a \dots\dots\dots (E 1-1)$$

Since thickness  $d$  is a function of the applied voltage and the electric field  $\mathcal{E}$ , then  $d = f(\mathcal{E}, V_a)$  or  $\mathcal{E} = \frac{V_a}{d}$ , and  $d = \frac{V_a}{\mathcal{E}}$  (cm)  $\dots\dots\dots (E 1-2)$

The capacitance density  $\frac{C}{A}$  (Farads/cm<sup>2</sup>) is given by:  $\frac{C}{A} = \frac{\epsilon_r \epsilon_0}{d} \dots\dots (E 1-3)$

where  $\epsilon_r$  and  $\epsilon_0$  were defined in section 5.5.1.1



After substituting eq. (E1-2) in eq. (E1-3),

$$\frac{C}{A} = \epsilon_r \epsilon_0 \frac{E}{V_a} = \epsilon_0 (\epsilon_r E/V_a) \dots \dots \dots (E1-4)$$

Substituting eq. (E1-1) in eq. (E1-4), then the latter becomes:

$$\frac{C}{A} = \frac{3\epsilon_0}{2} (\epsilon_r E V_{BR}) \dots \dots \dots (E1-5)$$

Eq. (E1-5) can be written as:  $\frac{C}{A} = k \dots \dots \dots (E1-6)$

where  $k = \frac{3}{2} \epsilon_0 (\epsilon_r E/V_{BR}) \dots \dots \dots (E1-7)$

So,  $k$  is a constant number; it defines the thin-film material and the anodization characteristics, during deposition process.

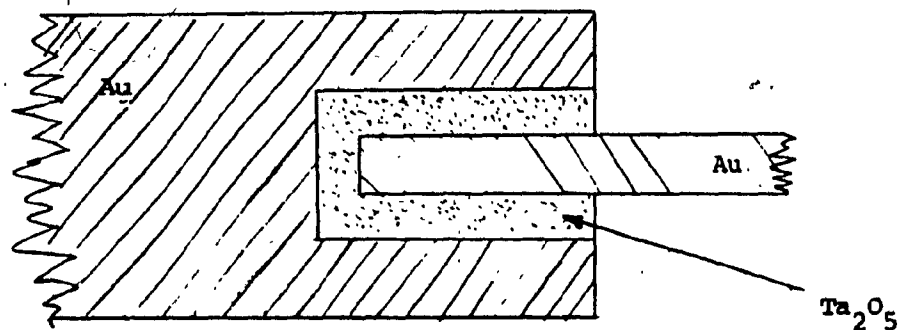
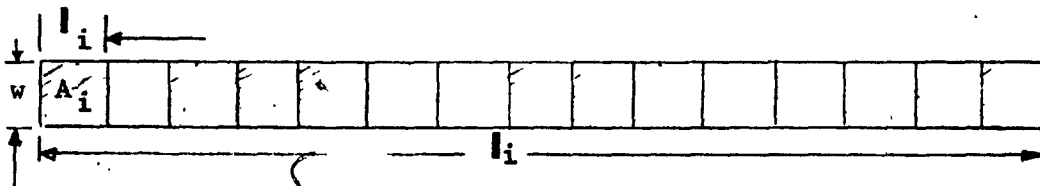


Figure E.1 Thin-film capacitor pattern

The capacitance area  $A$  can be thought of the summation of small unit-squares  $A_i$ , such that  $A_i = w \times l_i$ , as shown in a simple geometry of the capacitance pattern below.



The width "w" is uniform, and the unit length "l<sub>i</sub>" can be estimated to be equal to the width "w". Therefore, l<sub>i</sub> = w, and the unit-square is A<sub>i</sub> = w x l<sub>i</sub> = w<sup>2</sup>

Table E.1 shows typical values of capacitance per unit-area (F/□), for different dielectric films, and some characteristics, such as quality factor Q, TCC and accuracy before trimming.

The total area A<sub>T</sub> (or A<sub>c</sub>) is  $A_c = \sum_{i=1}^n A_i = \sum_{i=1}^n w_i^2 = nw^2, \dots \dots (E1-8)$   
since w<sub>1</sub> = w<sub>2</sub> = ... = w<sub>n</sub> = w

The total capacitance density  $\frac{C}{A}$  is given by:  $\frac{C_T}{A_c} = k.$

Conversely, the total area required for the total capacitance C<sub>T</sub> is:  $A_c = \frac{1}{k} C_T \dots \dots \dots (E1-9)$

or  $A_c = \frac{2}{3} \cdot \frac{V}{\epsilon_0 \epsilon_r \epsilon} \cdot C_T \dots \dots \dots (E1-10)$

where  $A_c = nA_i = nw^2$

For example, from Table E.2, for dielectric film Ta<sub>2</sub>O<sub>5</sub>  $\epsilon = 25$ ,  $\epsilon_0 = 8.85 \times 10^{-14}$  [F/cm],  $\epsilon = 4 \times 10^6$  [Volts/cm]

and V<sub>rr</sub> = 100 [volts],

$A_c = \frac{2}{3} \cdot \frac{100}{8.85 \times 10^{-14} \times 25 \times 4 \times 10^6} = (7.533 \times 10^7) C_T$  [cm<sup>2</sup>]

Table 2.1 - Some Dielectric materials and typical characteristics that can be achieved

Dielectric	Deposition process	Capacitance per $\square$	$Q_d$ at 10 MHz	Tempet coeff/°C	Accuracy before trimming
Silicon	vacuum evaporation	0.013 pf/ $\mu\text{m}^2$ at 6 v	200	200 - 250	$\pm 15\%$
Alumina	vapor plating	0.3 pf/ $\mu\text{m}^2$ at 30 v	10 - 100	150 - 400	$\pm 10\%$
Tantalum	Anodic oxidation	1.55 pf/ $\mu\text{m}^2$ at 12 v	good	150 - 350	$\pm 20\%$
Silicon Dioxide	vapor oxidation	0.25-0.4 pf/ $\mu\text{m}^2$ at 12 v	10 - 100	100	$\pm 15\%$

Table E.2 - Properties of typical thin-film materials

Dielectric film Material	Dielectric constant ( $\epsilon_r$ )	Dissipation factor ( $\tan\delta$ )	TCC (ppm/°c)	Breaking voltage $V_{90}$ (v)	Electric field $\mathcal{E}$ (v/cm $\times 10^6$ )	Capacitance Density C/A ( $\mu\text{F}/\text{cm}^2$ )	Dielectric film Thickness ( $\text{\AA}$ )
SiO	5-7	.01-.03	150-400	5-100	1 - 2	.001-.015	3000-40000
SiO <sub>2</sub>	3-4	.004-.04	100	50-200	3	.002-.02	800-10000
Ta <sub>2</sub> O <sub>5</sub>	20-27	.002-.006	180-220	30-150	3 - 4	.03-.2	1000-6000
TiO <sub>2</sub>	30-100	.01-1.0	200-800	25-90	.3 - 1	.1-1.0	1000-2000
Al <sub>2</sub> O <sub>3</sub>	8-100	.2-.24	200-300	25-120	2 - 4	.03-.25	400-2500

II) Calculation of Ar

A thin-film resistor is considered. A simplified top and cross-sectional view of a thin-film resistor is shown in figure E.2.

The resistance of a complete film, with resistivity , uniform width 'w', length 'l', and thickness 'd' is given by:

$$R = \frac{\rho \text{ [ohms-cm]} l \text{ [cm]}}{d \text{ [cm]} w \text{ [cm]}} , \text{ or } R = \frac{\rho}{d} \cdot \frac{l}{w} \text{ ohms} \dots\dots\dots(E2-1)$$

The ratio  $(\frac{l}{w})$  is called "aspect ratio" and the ratio  $(\frac{\rho}{d})$  is called the "sheet resistance"  $R_s$ .

when  $l = w$ , then  $R = \frac{\rho}{d} = R_s \text{ ohms}/\square \dots\dots\dots(E2-2)$

when  $l = n \cdot w$ , then  $R = n \frac{\rho}{d} = nR_s \dots\dots\dots(E2-3)$

where n is the number of square-units or the number of sheet squares.

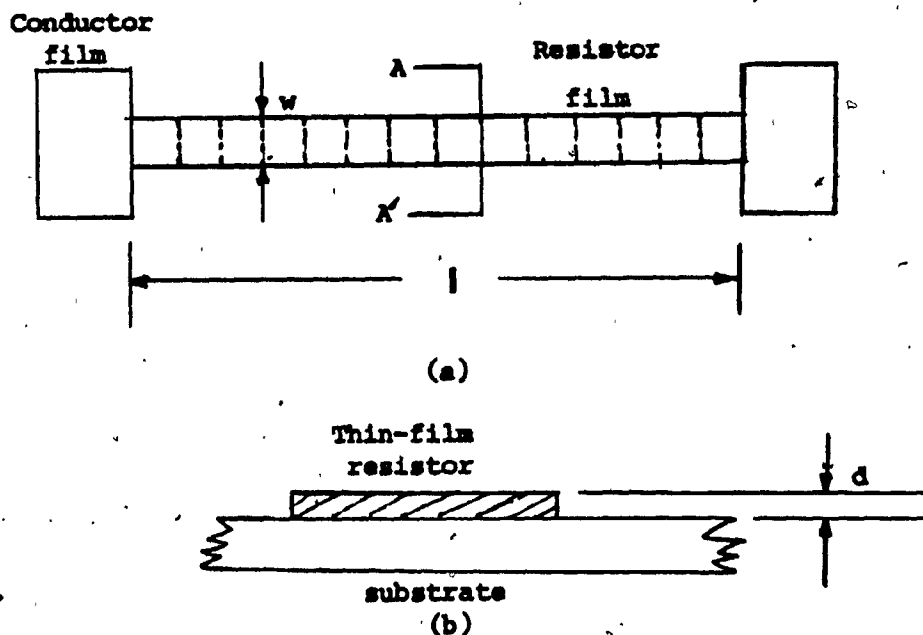


Fig. E.2 - A thin-film resistor pattern.  
 (a) Top view, (b) cross-section A-A'

The sheet resistance "Rs" characterizes the specific resistivity of a given resistor film whose thickness "d" is constant. It also corresponds to the resistance of a single square of material. The amount of resistivity depends on the impurity-type of dopant used. The thickness of the film "d" depends on the impurity-carrier diffusion and the diffusion coefficient D, by the relation:

$$N(x,t) = N_0 \left[ \operatorname{Erfc} \left( \frac{x}{2 \sqrt{Dt}} \right) \right] \dots \dots \dots (E2-4)$$

So, by defining the amount  $N(x,t)$ , (which in turn determines the amount of resistivity  $\rho$ ) the coefficient "D", and the time duration "t", the thickness "d" is determined from error function (erfc) tables or graphs.

Table E.3 illustrates the relation between tolerances (and yields) and the geometry of resistor films, while Table E.4 illustrates the specific resistivity " $\rho$ " obtained for different tantalum films.

Multiplying both numerator and denominator of the expression in eq. (E2-1) by w, then  $R = \frac{\rho}{d} \cdot \frac{l w}{w^2} \dots \dots \dots (E2-5)$

$$\text{or } R = \frac{\rho}{d} \frac{A}{w^2} \dots \dots \dots (E2-6)$$

and the resistance density is given by:

$$\frac{R}{A} = \frac{\rho}{d} \frac{1}{w^2} \dots \dots \dots (E2-7)$$

$$\text{or } \frac{R}{A} = R_s \frac{1}{w^2} \dots \dots \dots (E2-8)$$

The total area  $A_T$  (or  $A_r$ ) is the summation of unit-squares  $A_i$  or the product  $n \times A_1$ , where  $n$  is the total number of

unit-squares. Hence,  $A_r = \sum_{i=1}^n A_i = nA_1$  where  $A_1$  is the first unit-square. Also,  $A_r = nA_1 = nw^2$  ..... (E2-9)

and  $R_T = \frac{R_s}{w^2} \cdot A_r = \frac{R_s}{w^2} (A_1 + A_2 + \dots + A_n)$  ..... (E2-10)

or  $R_T = \frac{nR_s}{w^2} A_1$  ..... (E2-11)

since,  $A_1 = A_2 = \dots, A_n$

Finally, from eq. (E2-10).

$A_r = \left( \frac{w^2}{R_s} \right) R_T$  mils<sup>2</sup> ..... (E2-12)

where  $A_T$  is given by eq. (E2-9).

For example, from Table E.3, for  $w = .5$  mils

$R_s = 4000$  ohms/ $\square$ , and  $R_T = 40,000$  ohms,  $A_r = 50$  (mils)<sup>2</sup>.

Table E.3 - Tradeoffs involved in reduction of width to increase resistance. The following are assumed:

$$\rho_s = 4000 \text{ ohms / sq.}, \quad A = 50 \text{ mils}^2$$

w [mils]	l [mils]	R [ohms]	Resistor tolerance %	Representative overall yield %
2.5	10.0	1600	9	98
2.0	12.5	2500	10	95
1.0	25.0	10000	15	90
0.5	50.0	40000	25	80
0.25	100.0	160000	45	60

Table E.4 - Specific Resistivity of various Tantalum (Ta) films

"Ta" film	bcc-Ta	$\beta$ - Ta	Ta <sub>2</sub> N	Ta+ $\theta$	Ta + O + Ta <sub>2</sub> O <sub>5</sub> <sup>1</sup>
$\rho$ (micro-ohm-cm)	24 - 50	180-220	240-300	40-300	250 - 2000

1. Reactive sputtering with oxygen.



## APPENDIX F

### COMPUTER-AIDED SOLVING OF THE PROBLEM

#### F1. Programs and Subroutines-General

Formulation of the minimization problem, as it is described in chapter four, is a simple task that requires simple mathematical manipulations. However, the execution of the minimization algorithm is, by itself, a highly complex and tedious task.

Forty-eight computer programs are written to solve the minimization problem and fifty-seven programs to plot the amplitude and phase responses of the transfer functions of this filter.

Since both minimization algorithms, used as function subroutines ZXMIN and FMFP (called in the main programs), require external data to perform computations and iterations, these data are supplied by additional subroutines, such as FUNCT, FUNCTI and TRANF.

The "plot" programs also use function subroutine, such as USPLH to depict the proper graphs that represent the filter responses.

The computer programs were compiled in FORTRAN IV; they were run in BATCH and edited in NTEX D text editor. PERMANENT-FILE "PARIS" was made which consisted of DATA and WORK-FILE, PERMANENT PROGRAMS, listed in proper DUMP FILES, stored on DISC BASA XXX of the NOS. 1.3, 472 Computer System.

The input data and the repetition (loops) of the program "runs", which were designed so that different input parameters (i.e.  $W_0$ ,  $Q_0$ , penalty-multipliers  $K_i$ ) were selected, were kept to a minimum. The algorithms require only initial values of variables, once, then they iterate different values by themselves, until minimum (or forced-logic termination) is reached.

## F2 Description of minimization programs

Forty-eight minimization programs minimize the objective function  $F$ , per Table-4 (program no. 1 to 48). Program no. 1 to 12 use the Quasi-Newton's algorithm. Program no. 13 to 48 use the Fletcher-Powell's algorithm.

All the minimization programs, regardless of the algorithm used, follow similar procedure; gathering of data and computing the "minimum" by means of subroutines which employ the minimization algorithms.

F2.1 Description of main programs that use Quasi-Newton's algorithm

All twelve programs that minimize the total capacitance  $C_T$  are practically identical; the only difference is the type and the number of the constraint-functions  $g_i$ . This distinction, in terms of the specific constraints, identifies the four basic cases of minimization per Table-4 (programs no. 1 to 12).

The functional structure of these programs is represented by the flow chart of figure F1.

These programs share their variable "xi", the filter parameters ( $W_0$  and  $Q_0$ ) and penalty-function multipliers " $K_1$ ", as common elements, with their function sub-routines. To identify the problem as a whole and the different cases explored, proper description is printed. The minimization sub-routine ZXMIN will not recognize the requirements of the problem unless its predescribed arguments (such as size  $N$  of the variable  $\bar{x}$  and gradient  $\bar{g}$  vector, accuracy of computations, etc.) are determined.

Due to the large difference of magnitude between capacitance and resistance elements, truncation of the least significant decimal digits would occur during the computations so that the algorithm would not be able to derive the new variables for the next iteration (cycle of computations).

Therefore, initial scaling is required to bring these variables in the same order of magnitude.

Provided that the above requirements are met, the minimization algorithm (subroutine ZXMIN) can be called to perform the necessary steps and computations, iteration by iteration, until the objective is reached, while printing of the output variables in each iteration takes place. Upon completion of the required iterations for minimum, the main-program algorithm repeats the whole process with new filter parameters (unless zero frequency is encountered for which the program is terminated) and new multipliers  $K_i$ . Listing of two samples of the main programs, and subroutine FUNCT1 also demonstrates all the steps of the problem solution process.

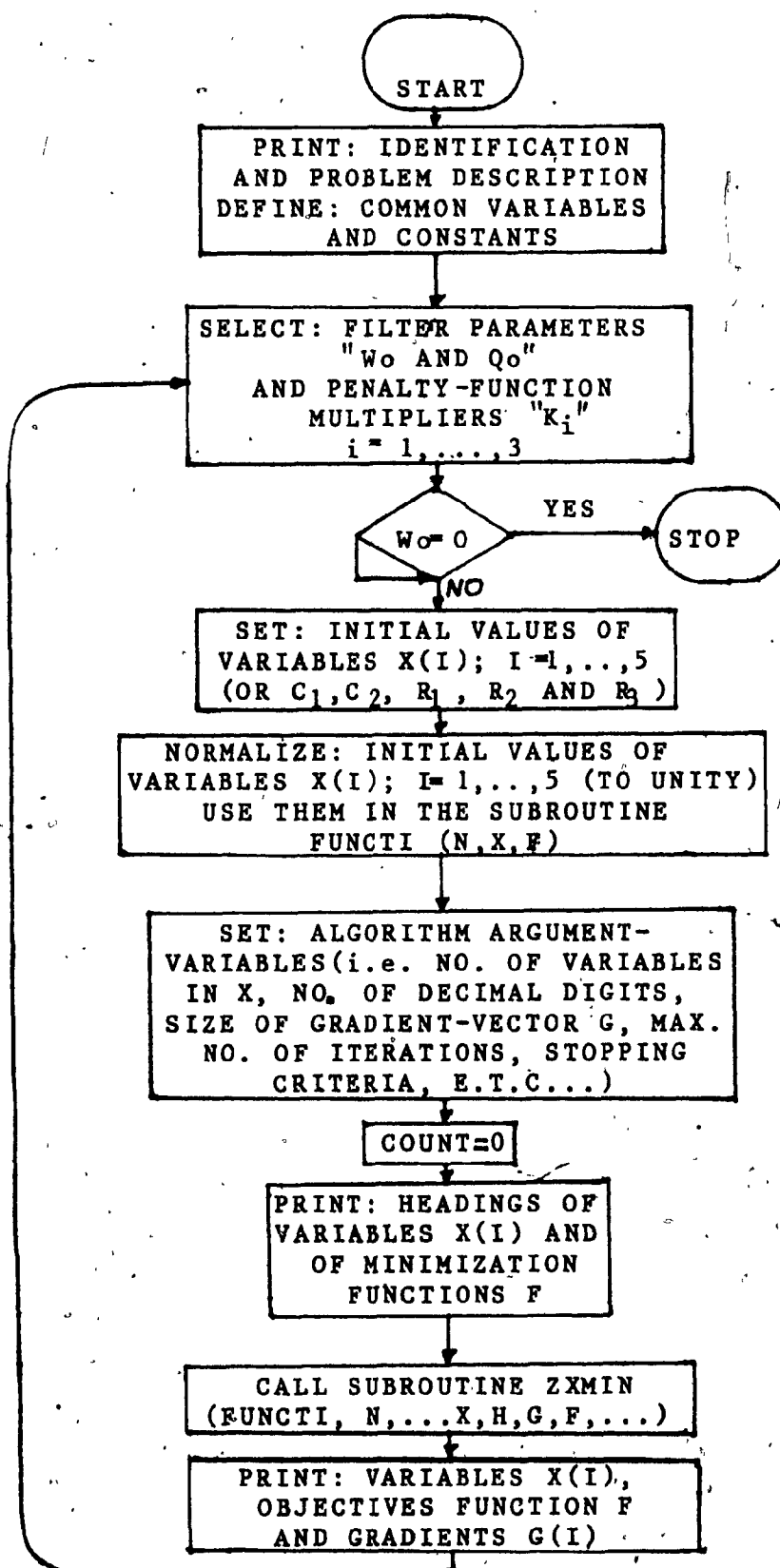


Fig. F.1 - Main Program of minimization by Quasi-Newton's algorithm.

```

PROGRAM NENCT 73/174 OPT-1          FTM 4.0000          00706708. 14-23-73          PAGE 1
1      PROGRAM NENCT(INPUT,OUTPUT)
      CINTERNAL FUNCT1
      COMMON F0,F1,OP,K1,K2,FR,OPP,X1N,X2N,X3N,X4N,X5N,F,IN,OPN
      REAL K1,K2
      DIMENSION N1(2),K1(5),N1(5),C1(5)
      PRINT 1
      FORMAT(///,5X,'CONCORDIA UNIVERSITY-SCMU CAMPUS.          MONTREAL
      * 1980.01
      PRINT 3
18      3 FORMAT(//,5X,'FOR A TECHNICAL REPORT IN EL. ENGINEERING.//
      *SH,OPTIMIZ. GAIN BAND PASS ACTIVE FILTER.//
      *SH,OPTIMIZATION OF FILTER ELEMENTS.//
      *SH,.....BY PARASKEVAS PRONOPIIS,I.D. NO. 0001056.....//
      *SH,.....ADVISOR DR. S. BHATTACHARYYA, EL. ENG.....//
19      *SH,THIS PROGRAM MINIMIZEZ THE TOTAL CAPACITANCE "CT" OF THE//
      *SH,FILTER GIVEN CONSTRAINTS "G1" AND "G2" FOR SPECIFIED//
      *SH,PERFORMANCE FREQUENCY "F0" AND POLE "OP" RESP. AND INITIAL//
      *SH,COMPONENT VALUES R1,R2,R3,C1,AND C2(ALL INDEPENDENT FROM//
      *SH, EACH OTHER. THE BAND PASS INSERTION LOSS "MOP" IS//
20      *SH,DEPENDENT ON FILTER PASSIVE COMPONENTS AND IS GIVEN BY //
      *SH,0          MOP=R3/R2+R310.//
      *SH,.....IN THIS ANALYSIS GAUSS-NEWTON ALGORITHM IS USED.....//
105     READ *,F0,OP,K1,K2
      IF(F0.EQ.0.) GO TO 100
      PRINT 5,F0,OP,K1,K2
      5 FORMAT(5X,LOWFREQUENCY=F0,1.500HERTZ,7X,3MOP,F0,1.7X,10
      *PENALTY FACTOR K1=F10,2.7X,LOWPENALTY FACTOR K2=
      *F10,2.//)
      PRINT 2
      2 FORMAT(5X,000.....MINIMIZE "CT" WHEN MOP IS FLOATING AND C1,C2=R1
      *R2,R3 ARE DIFFERENT .....//)
35     F1=2.9,1019026000
      K1=1.E+9
      K2=1.E+3
      N1(1)=1.E+3
      N1(2)=1.E+3
      N1(3)=1.E+3
      N1(4)=1.E+3
      N1(5)=1.E+3
      N1(1)=1./N1(1)
      N1(2)=1./N1(2)
      N1(3)=1./N1(3)
      N1(4)=1./N1(4)
      N1(5)=1./N1(5)
      F1N=F1/F1
      OPN=OP/OP
      N=5
      NSIG=24
      N1XFN=10000
      IOPT=0
      ICDUNT=0.
      PRINT 6
      6 FORMAT(5X,0MOP,0X,0R1(0MHS),0X,0R2(0MHS),
      *0X,0R3(0MHS),0X,0C1(FARADS),0X,0C2(FARADS),3X,0P(K),7X,0CT(E
      *ARADS),1X,0MOP,3X,0F0,0X,0OP,0X,0K1,3X,0K2,//)
      CALL ZMINIFUNCT1,N,NSIG,MAXFN,IOPT,X,N,G,F,d,IER)
      PRINT 15
15     FORMAT(//,15X,0K1(1),7X,0K1(2),10X,0K1(3),12X,0K1(4),15X,0K1(5)
      PRINT 10,(K1(I),I=1,5)
10     FORMAT(//,5X,0C1(5.6)
      PRINT 20,F,C1,C2
20     FORMAT(//,10X,1MOP,10X,2MOP,10X,2MOP/1E,3G15.6)
      PRINT 25,FR,OPP,IER
25     FORMAT(//,15X,2F0,15X,2MOP,7,0X,2C15.6,7,5X,0MOP,0.15,//)
      GO TO 105
100     STOP
      END

```

```

SUBROUTINE FUNCT1 73/174 OPT-1          FTM 4.0000          00706708. 14-23-73          PAGE 1
1      SUBROUTINE FUNCT1(N,F)
      DIMENSION K(N)
      CMINIMIZE TOTAL CAPACITANCE CT GIVEN CONSTRAINTS
      CIMPUSED BY CENTER FREQUENCY F0 AND OP OF THE BAND PASS FILTER
      C CASE 1-MOP FLOATING,R1,R2,R3,AND C1,C2 ARE DIFFERENT-
      COMMON F0,FT,OP,K1,K2,FR,OPP,X1N,X2N,X3N,X4N,X5N,F,IN,OPN
      REAL K1,K2
20     CCONSTRAINT ON CT IMPUSED BY THE EQUALITY-F0*FT*(K1*K2*(C1+C2)-
      *G1-(F1*02)*(K1(1)*K1(2)*K1(3)*K1(4)*K1(5)*K1(6)*K1(7)*K1(8)*K1(9)*K1(10))-1.
      CCONSTRAINT ON CT IMPUSED BY THE EQUALITY-OP*(R1,R2,R3),C1,C2)-
      *G2-(OP*02)*(K2(1)*K2(2)*K2(3)*K2(4)*K2(5)*K2(6)*K2(7)*K2(8)*K2(9)*K2(10))-1.
      CUNCONSTRAINED TOTAL CAPACITANCE "CT"
15     CCT=(K1(1)*K1(2)*K1(3)*K1(4)*K1(5)*K1(6)*K1(7)*K1(8)*K1(9)*K1(10)*
      *K2(1)*K2(2)*K2(3)*K2(4)*K2(5)*K2(6)*K2(7)*K2(8)*K2(9)*K2(10))
      CCONSTRAINED TOTAL CAPACITANCE "CT" TO BE MINIMIZED GIVEN
      C ABOVE CONSTRAINTS
      F=C*F
      R1=K1(1)*K1(2)
      R2=K1(3)*K1(4)
      R3=K1(5)*K1(6)
      C1=K1(7)*K1(8)
      C2=K1(9)*K1(10)
      R1=ABS(R1)
      R2=ABS(R2)
      R3=ABS(R3)
      C1=ABS(C1)
      C2=ABS(C2)
30

```

POOR COPY  
COPIE DE QUALITEE INFERIEURE

```

MOP=17/1R2+R3
CTO=C1+C2
FR=(1./G-203105101./SQR(C1C2010R2))
35
MOP=17/1R2+R3
ICOUNT=201020
IF(17.E0.ICOUNT) PRINT 17,MOP,R1,R2,R3,C1,C2,F,CTO,ICOUNT,FR,OPF.
P01,02
ICOUNT=ICOUNT+1
17
FORMAT(1X,0G12.5,1X,14,0P10.2,F7.2,F7.4,F7.4)
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100

```

COCONCHA UNIVERSITY-TOON CAMPUS. MONTREAL 1988.

RAJON TECHNICAL REPORT IN EL. ENGINEERING.  
UNITY\_GAIN BAND PASS ACTIVE FILTER

OPTIMIZATION OF FILTER ELEMENTS

.....BY PARASRAT PRASAD,S.D. NO. 0001930.....

.....ADVISOR DR. S. BHATTACHARYA, EL. ENG.....

THIS PROGRAM MINIMIZE THE TOTAL CAPACITANCE "CT" OF THE FILTER,GIVEN CONSTRAINTS "P01" AND "G2" FOR SPECIFIED RESONANCE FREQUENCY "FO" AND Q-FACTOR "Q" RESP. AND INITIAL COMPONENT VALUES R1,R2,R3,C1,AND C2(ALL INDEPENDENT FROM EACH OTHER). THE BAND PASS INSERTION LOSS "MOP" IS DEPENDENT ON FILTER PASSIVE COMPONENTS AND IS GIVEN BY MOP=17/1R2+R3

....IN THIS ANALYSIS GUST\_MBYOIN ALGORITHM IS USED....

FREQUENCY= 500.0HERTZ OP= 50.0 PENALTY FACTOR K1= 1000.00 PENALTY FACTOR K2= 500.00

.....MINIMIZE "CT" WHEN MOP IS FLOATING AND C1,C2,R1,R2,R3 ARE DIFFERENT .....

MOP	R1(OHMS)	R2(OHMS)	R3(OHMS)	C1(FARADS)	C2(FARADS)	PER.K1	CT(FARADS)	COUNT	FO	Q	G1	G2
500000	100000.	1000.00	1000.00	.318310E-06	.318310E-08	2.50000	.321493E-06	0	500.00	50.00	-.0000	0.0000
499945	100102.	1000.02	999.992	.317830E-06	.318110E-08	2.49657	.321020E-06	20	499.01	50.00	-.0000	-.0017
499203	101308.	1100.07	879.923	.274730E-06	.328705E-08	2.57700	.282942E-06	70	497.04	49.01	-.0001	-.0010
498734	100401.	1070.01	921.794	.282680E-06	.318707E-08	2.52130	.285870E-06	60	499.01	49.97	-.0001	-.0012
492322	100700.	1080.77	812.000	.254770E-06	.334071E-08	2.10093	.258119E-06	80	500.35	49.99	-.0001	-.0004
392000	113923.	1223.02	701.041	.220330E-06	.315356E-08	2.01041	.231490E-06	100	501.31	49.79	-.0000	-.0000
380113	121140.	1265.04	600.167	.214100E-06	.304061E-08	1.89300	.217070E-06	120	501.00	49.96	-.0001	-.0010
353945	127403.	1310.03	718.101	.192720E-06	.315362E-08	1.71797	.199073E-06	140	500.22	49.99	-.0000	-.0005
292173	139025.	1444.90	590.107	.159030E-06	.311292E-08	1.74912	.162942E-06	160	503.54	49.78	-.0010	-.0009
262010	137700.	1410.02	500.433	.157020E-06	.331032E-08	1.51104	.160341E-06	180	499.52	50.00	-.0010	-.0023
225723	144325.	1490.03	435.991	.142110E-06	.330023E-08	1.42545	.145022E-06	200	499.07	49.88	-.0005	-.0009
175070	140395.	1540.41	350.005	.129520E-06	.338302E-08	1.34779	.132000E-06	220	498.00	50.00	-.0000	-.0025
164464	140011.	1460.00	331.435	.110930E-06	.315001E-08	1.25334	.122000E-06	240	500.70	50.03	-.0010	-.0012
160000	175000.	1012.02	367.001	.109500E-06	.291137E-08	1.17577	.112437E-06	260	499.50	50.19	-.0017	-.0010
137000	180701.	1090.52	345.957	.119430E-06	.290000E-08	1.12291	.107130E-06	280	500.74	49.99	-.0010	-.0010
161014	204050.	2022.93	300.332	.123200E-07	.244256E-08	1.03016	.104033E-07	300	500.79	49.83	-.0031	-.0000
160000	200390.	2002.00	000.570	.102470E-07	.250417E-08	0.93401	.105052E-07	320	500.49	50.01	-.0020	-.0005
164007	277007.	2211.62	437.304	.130100E-07	.240010E-08	0.93020	.104033E-07	340	501.30	49.93	-.0009	-.0009
160937	220702.	2231.76	043.124	.103513E-07	.237579E-08	0.90144	.105001E-07	360	499.04	50.07	-.0007	-.0027
160000	230279.	2331.75	049.020	.790050E-07	.229300E-08	0.802103	.103597E-07	380	500.32	49.94	-.0013	-.0023
135070	231000.	2494.90	033.300	.747000E-07	.210000E-08	0.81000	.700000E-07	400	500.23	49.90	-.0010	-.0007
150010	204034.	2300.03	437.504	.700000E-07	.200000E-08	.775000	.720700E-07	420	500.27	50.00	-.0011	-.0001
164236	275000.	2715.00	057.005	.675013E-07	.200732E-08	0.73000	.695000E-07	440	499.03	50.00	-.0007	-.0000
137570	200424.	2077.00	458.701	.635200E-07	.190470E-08	.700000	.654350E-07	460	500.27	50.02	-.0011	-.0007
134501	200420.	2047.24	450.011	.619700E-07	.180422E-08	0.603161	.630400E-07	480	499.93	50.01	-.0003	-.0000
127202	314207.	3130.02	457.131	.970000E-07	.170070E-08	0.600170	.930370E-07	500	501.03	49.97	-.0001	-.0011
121500	329201.	3290.00	490.113	.940000E-07	.180000E-08	0.610001	.900000E-07	520	500.92	49.99	-.0001	-.0022
110100	302273.	3031.10	010.754	.920400E-07	.163402E-08	.504000	.942007E-07	540	500.63	49.98	-.0025	-.0007
111034	300404.	3071.09	000.407	.400030E-07	.154000E-08	.550201	.500442E-07	560	500.47	49.94	-.0010	-.0024
107003	303420.	3032.00	000.270	.444070E-07	.147145E-08	.533303	.470300E-07	580	500.77	50.01	-.0031	-.0003
100001	000073.	0000.00	000.577	.430020E-07	.130701E-08	.503210	.450000E-07	600	500.07	49.97	-.0030	-.0012
940502E-01	033972.	0350.10	077.001	.302050E-07	.129410E-08	0.400120	.295190E-07	620	505.05	49.90	-.0100	-.0012
905102E-01	044723.	0444.23	076.320	.295050E-07	.120775E-08	.450000	.280770E-07	640	500.34	49.90	-.0027	-.0009
920000E-01	072175.	0732.00	002.373	.370010E-07	.121950E-08	.430320	.303100E-07	660	500.57	49.99	-.0023	-.0005
900033E-01	000290.	0000.00	000.341	.257030E-07	.110257E-08	.417502	.260702E-07	680	500.63	50.00	-.0025	-.0002
070330E-01	000102.	0001.02	000.300	.342367E-07	.111045E-08	.300114	.253701E-07	700	500.74	50.00	-.0019	-.0002
050001E-01	025335.	0240.00	003.500	.331327E-07	.110023E-08	.302000	.342000E-07	720	500.17	50.00	-.0007	-.0000
032050E-01	031343.	0300.00	009.170	.315120E-07	.100000E-08	.305050	.325730E-07	740	499.04	50.03	-.0007	-.0010
000011E-01	074020.	0724.00	035.912	.301190E-07	.102000E-08	.319350	.311342E-07	760	500.30	50.02	-.0012	-.0000
793090E-01	000240.	0000.00	000.000	.293070E-07	.100002E-08	.342032	.303090E-07	780	499.00	50.00	-.0000	-.0002
777372E-01	000790.	0002.11	007.930	.284930E-07	.103525E-08	.335019	.294292E-07	800	500.47	49.99	-.0010	-.0003
750700E-01	027032.	0230.00	011.013	.274250E-07	.943092E-08	.320000	.281070E-07	820	500.02	50.00	-.0001	-.0001
720000E-01	000000.	0000.00	015.350	.253020E-07	.893000E-08	.303210	.260440E-07	840	500.21	49.99	-.0007	-.0000
607070E-01	071100.	0704.23	021.000	.239015E-07	.837791E-08	.280151	.247000E-07	860	500.00	50.00	-.0024	-.0016
070101E-01	072700.	0724.00	021.071	.232000E-07	.810771E-08	.275271	.240070E-07	880	500.17	49.99	-.0007	-.0005
055932E-01	075100.	0739.50	022.240	.226010E-07	.794900E-08	.260000	.234090E-07	900	500.10	49.99	-.0000	-.0004
029772E-01	070500.	0692.00	024.215	.213530E-07	.759400E-08	.250000	.221120E-07	920	500.35	50.00	-.0014	-.0001
001012E-01	030300.	0201.17	014.410	.203050E-07	.729000E-08	.242000	.211140E-07	940	500.00	49.99	-.0003	-.0005
074530E-01	072700.	0613.07	019.055	.193100E-07	.695000E-08	.230745	.200130E-07	960	500.70	50.00	-.0031	-.0000
070100E-01	070301.	0670.35	014.000	.191970E-07	.691600E-08	.229300	.199000E-07	980	000.00	50.01	-.0001	-.0003
334000E-01	070100.	0717.75	017.000	.180700E-07	.650700E-08	.213050	.187100E-07	1000	500.77	50.00	-.0011	-.0005
320700E-01	000030.	0100.00	012.000	.177230E-07	.645000E-08	.213003	.180000E-07	1020	417.71	49.99	-.0012	-.0004
307000E-01	007375.	0100.00	010.000	.170000E-07	.620000E-08	.200000	.170000E-07	1040	500.00	50.00	-.0000	-.0000

POOR COPY  
COPIE DE QUALITEE INFIERIEURE

PROGRAM FUNCT1 73/174 OPT-1 YIN 4.0498 06/06/06 14.24.34 PAGE 1

```

1      PROGRAM FUNCT1(INPUT,OUTPUT)
      EXTERNAL FUNCT1
      COMMON /F/,PI,OP,K1,K2,K3,FR,OPP,NIN,X2N,X3N,X4N,X5N,FIN,OPN
      REAL K1,K2,K3
      DIMENSION N(20),N(1),N(2),N(3),G(1)
      PRINT 1
      1      FORMAT(///,9X,'CONCORDIA UNIVERSITY-5000 CAMPUS',          MONTREAL
      2      0 1900.0)
      PRINT 2
      3      FORMAT(//,9X,'TECHNICAL REPORT IN EL. ENGINEERING,/'
      4      05X,'UNITY_GAIN BAND PASS ACTIVE FILTER',//
      5      05X,'OPTIMIZATION OF FILTER ELEMENTS',//
      6      05X,'.....BY PARASKEVAS PNEKOPIS,I.O. NO. 0901954.....',//
      7      05X,'.....ADVISOR DR. S. BHATTACHARYYA, EL. ENG.....',//
      8      05X,'THIS PROGRAM MINIMIZES THE TOTAL CAPACITANCE "CT" OF THE,/'
      9      05X,'FILTER, GIVEN CONSTRAINTS "C1", "C2", AND "C3" FOR SPECIFIED,/'
      10     05X,'RESONANCE FREQUENCY "F0" AND POLE_0 "OP" RESP. AND INITIAL,/'
      11     05X,'COMPONENT VALUES K1,K2,K3,C1, AND C2(ALL INDEPENDENT FROM,/'
      12     05X,'EACH OTHER.) THE BAND PASS INSERTION LOSS "MIP" IS,/'
      13     05X,'DEPENDENT ON FILTER PASSIVE COMPONENTS AND IS GIVEN BY,/'
      14     05X,'MIP=2/(1+2*G1),/'
      15     05X,'.....IN THIS ANALYSIS QUASY_NEWTON ALGORITHM IS USED.....',//
      16     100 READ /F/,OP,K1,K2,K3
      17     IF(FR.EQ.0) GO TO 100
      18     PRINT 5,F0,OP,K1,K2,K3
      19     5      FORMAT(5X,'FREQUENCY=F0,1,000HRTZ,3X,2MOP,5,1,3X,'PERALTY COE
      20     6      05X,'FFICIENTS =,2X,2M1,0,F10,2,2X,2M2,0,F10,2,2X,2M3,0,F10,2,2//)
      21     PRINT 2
      22     2      FORMAT(5X,'G1N.....MINIMIZE "CT" WHEN MIP IS FLOATING ,C1-C2,AND R
      23     3      05X,'E1,R2,R3 ARE DIFFERENT .....//)
      24     3      F1=2.03,1415916*PI
      25     K1=1.E+9
      26     K2=1.E+2
      27     N(1)=1.E+9
      28     N(2)=1.E+2
      29     N(3)=1.E+9
      30     N(4)=1.E+2
      31     N(5)=1.E+9
      32     N(6)=1.E+2
      33     N(7)=1.E+9
      34     N(8)=1.E+2
      35     N(9)=1.E+9
      36     N(10)=1.E+2
      37     N(11)=1.E+9
      38     N(12)=1.E+2
      39     N(13)=1.E+9
      40     N(14)=1.E+2
      41     N(15)=1.E+9
      42     N(16)=1.E+2
      43     N(17)=1.E+9
      44     N(18)=1.E+2
      45     N(19)=1.E+9
      46     N(20)=1.E+2
      47     N(21)=1.E+9
      48     N(22)=1.E+2
      49     N(23)=1.E+9
      50     N(24)=1.E+2
      51     N(25)=1.E+9
      52     N(26)=1.E+2
      53     N(27)=1.E+9
      54     N(28)=1.E+2
      55     N(29)=1.E+9
      56     N(30)=1.E+2
      57     N(31)=1.E+9
      58     N(32)=1.E+2
      59     N(33)=1.E+9
      60     N(34)=1.E+2
      61     N(35)=1.E+9
      62     N(36)=1.E+2
      63     N(37)=1.E+9
      64     N(38)=1.E+2
      65     N(39)=1.E+9
      66     N(40)=1.E+2
      67     N(41)=1.E+9
      68     N(42)=1.E+2
      69     N(43)=1.E+9
      70     N(44)=1.E+2
      71     N(45)=1.E+9
      72     N(46)=1.E+2
      73     N(47)=1.E+9
      74     N(48)=1.E+2
      75     N(49)=1.E+9
      76     N(50)=1.E+2
      77     N(51)=1.E+9
      78     N(52)=1.E+2
      79     N(53)=1.E+9
      80     N(54)=1.E+2
      81     N(55)=1.E+9
      82     N(56)=1.E+2
      83     N(57)=1.E+9
      84     N(58)=1.E+2
      85     N(59)=1.E+9
      86     N(60)=1.E+2
      87     N(61)=1.E+9
      88     N(62)=1.E+2
      89     N(63)=1.E+9
      90     N(64)=1.E+2
      91     N(65)=1.E+9
      92     N(66)=1.E+2
      93     N(67)=1.E+9
      94     N(68)=1.E+2
      95     N(69)=1.E+9
      96     N(70)=1.E+2
      97     N(71)=1.E+9
      98     N(72)=1.E+2
      99     N(73)=1.E+9
      100    N(74)=1.E+2
      101    N(75)=1.E+9
      102    N(76)=1.E+2
      103    N(77)=1.E+9
      104    N(78)=1.E+2
      105    N(79)=1.E+9
      106    N(80)=1.E+2
      107    N(81)=1.E+9
      108    N(82)=1.E+2
      109    N(83)=1.E+9
      110    N(84)=1.E+2
      111    N(85)=1.E+9
      112    N(86)=1.E+2
      113    N(87)=1.E+9
      114    N(88)=1.E+2
      115    N(89)=1.E+9
      116    N(90)=1.E+2
      117    N(91)=1.E+9
      118    N(92)=1.E+2
      119    N(93)=1.E+9
      120    N(94)=1.E+2
      121    N(95)=1.E+9
      122    N(96)=1.E+2
      123    N(97)=1.E+9
      124    N(98)=1.E+2
      125    N(99)=1.E+9
      126    N(100)=1.E+2
      127    N(101)=1.E+9
      128    N(102)=1.E+2
      129    N(103)=1.E+9
      130    N(104)=1.E+2
      131    N(105)=1.E+9
      132    N(106)=1.E+2
      133    N(107)=1.E+9
      134    N(108)=1.E+2
      135    N(109)=1.E+9
      136    N(110)=1.E+2
      137    N(111)=1.E+9
      138    N(112)=1.E+2
      139    N(113)=1.E+9
      140    N(114)=1.E+2
      141    N(115)=1.E+9
      142    N(116)=1.E+2
      143    N(117)=1.E+9
      144    N(118)=1.E+2
      145    N(119)=1.E+9
      146    N(120)=1.E+2
      147    N(121)=1.E+9
      148    N(122)=1.E+2
      149    N(123)=1.E+9
      150    N(124)=1.E+2
      151    N(125)=1.E+9
      152    N(126)=1.E+2
      153    N(127)=1.E+9
      154    N(128)=1.E+2
      155    N(129)=1.E+9
      156    N(130)=1.E+2
      157    N(131)=1.E+9
      158    N(132)=1.E+2
      159    N(133)=1.E+9
      160    N(134)=1.E+2
      161    N(135)=1.E+9
      162    N(136)=1.E+2
      163    N(137)=1.E+9
      164    N(138)=1.E+2
      165    N(139)=1.E+9
      166    N(140)=1.E+2
      167    N(141)=1.E+9
      168    N(142)=1.E+2
      169    N(143)=1.E+9
      170    N(144)=1.E+2
      171    N(145)=1.E+9
      172    N(146)=1.E+2
      173    N(147)=1.E+9
      174    N(148)=1.E+2
      175    N(149)=1.E+9
      176    N(150)=1.E+2
      177    N(151)=1.E+9
      178    N(152)=1.E+2
      179    N(153)=1.E+9
      180    N(154)=1.E+2
      181    N(155)=1.E+9
      182    N(156)=1.E+2
      183    N(157)=1.E+9
      184    N(158)=1.E+2
      185    N(159)=1.E+9
      186    N(160)=1.E+2
      187    N(161)=1.E+9
      188    N(162)=1.E+2
      189    N(163)=1.E+9
      190    N(164)=1.E+2
      191    N(165)=1.E+9
      192    N(166)=1.E+2
      193    N(167)=1.E+9
      194    N(168)=1.E+2
      195    N(169)=1.E+9
      196    N(170)=1.E+2
      197    N(171)=1.E+9
      198    N(172)=1.E+2
      199    N(173)=1.E+9
      200    N(174)=1.E+2
      201    N(175)=1.E+9
      202    N(176)=1.E+2
      203    N(177)=1.E+9
      204    N(178)=1.E+2
      205    N(179)=1.E+9
      206    N(180)=1.E+2
      207    N(181)=1.E+9
      208    N(182)=1.E+2
      209    N(183)=1.E+9
      210    N(184)=1.E+2
      211    N(185)=1.E+9
      212    N(186)=1.E+2
      213    N(187)=1.E+9
      214    N(188)=1.E+2
      215    N(189)=1.E+9
      216    N(190)=1.E+2
      217    N(191)=1.E+9
      218    N(192)=1.E+2
      219    N(193)=1.E+9
      220    N(194)=1.E+2
      221    N(195)=1.E+9
      222    N(196)=1.E+2
      223    N(197)=1.E+9
      224    N(198)=1.E+2
      225    N(199)=1.E+9
      226    N(200)=1.E+2
      227    N(201)=1.E+9
      228    N(202)=1.E+2
      229    N(203)=1.E+9
      230    N(204)=1.E+2
      231    N(205)=1.E+9
      232    N(206)=1.E+2
      233    N(207)=1.E+9
      234    N(208)=1.E+2
      235    N(209)=1.E+9
      236    N(210)=1.E+2
      237    N(211)=1.E+9
      238    N(212)=1.E+2
      239    N(213)=1.E+9
      240    N(214)=1.E+2
      241    N(215)=1.E+9
      242    N(216)=1.E+2
      243    N(217)=1.E+9
      244    N(218)=1.E+2
      245    N(219)=1.E+9
      246    N(220)=1.E+2
      247    N(221)=1.E+9
      248    N(222)=1.E+2
      249    N(223)=1.E+9
      250    N(224)=1.E+2
      251    N(225)=1.E+9
      252    N(226)=1.E+2
      253    N(227)=1.E+9
      254    N(228)=1.E+2
      255    N(229)=1.E+9
      256    N(230)=1.E+2
      257    N(231)=1.E+9
      258    N(232)=1.E+2
      259    N(233)=1.E+9
      260    N(234)=1.E+2
      261    N(235)=1.E+9
      262    N(236)=1.E+2
      263    N(237)=1.E+9
      264    N(238)=1.E+2
      265    N(239)=1.E+9
      266    N(240)=1.E+2
      267    N(241)=1.E+9
      268    N(242)=1.E+2
      269    N(243)=1.E+9
      270    N(244)=1.E+2
      271    N(245)=1.E+9
      272    N(246)=1.E+2
      273    N(247)=1.E+9
      274    N(248)=1.E+2
      275    N(249)=1.E+9
      276    N(250)=1.E+2
      277    N(251)=1.E+9
      278    N(252)=1.E+2
      279    N(253)=1.E+9
      280    N(254)=1.E+2
      281    N(255)=1.E+9
      282    N(256)=1.E+2
      283    N(257)=1.E+9
      284    N(258)=1.E+2
      285    N(259)=1.E+9
      286    N(260)=1.E+2
      287    N(261)=1.E+9
      288    N(262)=1.E+2
      289    N(263)=1.E+9
      290    N(264)=1.E+2
      291    N(265)=1.E+9
      292    N(266)=1.E+2
      293    N(267)=1.E+9
      294    N(268)=1.E+2
      295    N(269)=1.E+9
      296    N(270)=1.E+2
      297    N(271)=1.E+9
      298    N(272)=1.E+2
      299    N(273)=1.E+9
      300    N(274)=1.E+2
      301    N(275)=1.E+9
      302    N(276)=1.E+2
      303    N(277)=1.E+9
      304    N(278)=1.E+2
      305    N(279)=1.E+9
      306    N(280)=1.E+2
      307    N(281)=1.E+9
      308    N(282)=1.E+2
      309    N(283)=1.E+9
      310    N(284)=1.E+2
      311    N(285)=1.E+9
      312    N(286)=1.E+2
      313    N(287)=1.E+9
      314    N(288)=1.E+2
      315    N(289)=1.E+9
      316    N(290)=1.E+2
      317    N(291)=1.E+9
      318    N(292)=1.E+2
      319    N(293)=1.E+9
      320    N(294)=1.E+2
      321    N(295)=1.E+9
      322    N(296)=1.E+2
      323    N(297)=1.E+9
      324    N(298)=1.E+2
      325    N(299)=1.E+9
      326    N(300)=1.E+2
      327    N(301)=1.E+9
      328    N(302)=1.E+2
      329    N(303)=1.E+9
      330    N(304)=1.E+2
      331    N(305)=1.E+9
      332    N(306)=1.E+2
      333    N(307)=1.E+9
      334    N(308)=1.E+2
      335    N(309)=1.E+9
      336    N(310)=1.E+2
      337    N(311)=1.E+9
      338    N(312)=1.E+2
      339    N(313)=1.E+9
      340    N(314)=1.E+2
      341    N(315)=1.E+9
      342    N(316)=1.E+2
      343    N(317)=1.E+9
      344    N(318)=1.E+2
      345    N(319)=1.E+9
      346    N(320)=1.E+2
      347    N(321)=1.E+9
      348    N(322)=1.E+2
      349    N(323)=1.E+9
      350    N(324)=1.E+2
      351    N(325)=1.E+9
      352    N(326)=1.E+2
      353    N(327)=1.E+9
      354    N(328)=1.E+2
      355    N(329)=1.E+9
      356    N(330)=1.E+2
      357    N(331)=1.E+9
      358    N(332)=1.E+2
      359    N(333)=1.E+9
      360    N(334)=1.E+2
      361    N(335)=1.E+9
      362    N(336)=1.E+2
      363    N(337)=1.E+9
      364    N(338)=1.E+2
      365    N(339)=1.E+9
      366    N(340)=1.E+2
      367    N(341)=1.E+9
      368    N(342)=1.E+2
      369    N(343)=1.E+9
      370    N(344)=1.E+2
      371    N(345)=1.E+9
      372    N(346)=1.E+2
      373    N(347)=1.E+9
      374    N(348)=1.E+2
      375    N(349)=1.E+9
      376    N(350)=1.E+2
      377    N(351)=1.E+9
      378    N(352)=1.E+2
      379    N(353)=1.E+9
      380    N(354)=1.E+2
      381    N(355)=1.E+9
      382    N(356)=1.E+2
      383    N(357)=1.E+9
      384    N(358)=1.E+2
      385    N(359)=1.E+9
      386    N(360)=1.E+2
      387    N(361)=1.E+9
      388    N(362)=1.E+2
      389    N(363)=1.E+9
      390    N(364)=1.E+2
      391    N(365)=1.E+9
      392    N(366)=1.E+2
      393    N(367)=1.E+9
      394    N(368)=1.E+2
      395    N(369)=1.E+9
      396    N(370)=1.E+2
      397    N(371)=1.E+9
      398    N(372)=1.E+2
      399    N(373)=1.E+9
      400    N(374)=1.E+2
      401    N(375)=1.E+9
      402    N(376)=1.E+2
      403    N(377)=1.E+9
      404    N(378)=1.E+2
      405    N(379)=1.E+9
      406    N(380)=1.E+2
      407    N(381)=1.E+9
      408    N(382)=1.E+2
      409    N(383)=1.E+9
      410    N(384)=1.E+2
      411    N(385)=1.E+9
      412    N(386)=1.E+2
      413    N(387)=1.E+9
      414    N(388)=1.E+2
      415    N(389)=1.E+9
      416    N(390)=1.E+2
      417    N(391)=1.E+9
      418    N(392)=1.E+2
      419    N(393)=1.E+9
      420    N(394)=1.E+2
      421    N(395)=1.E+9
      422    N(396)=1.E+2
      423    N(397)=1.E+9
      424    N(398)=1.E+2
      425    N(399)=1.E+9
      426    N(400)=1.E+2
      427    N(401)=1.E+9
      428    N(402)=1.E+2
      429    N(403)=1.E+9
      430    N(404)=1.E+2
      431    N(405)=1.E+9
      432    N(406)=1.E+2
      433    N(407)=1.E+9
      434    N(408)=1.E+2
      435    N(409)=1.E+9
      436    N(410)=1.E+2
      437    N(411)=1.E+9
      438    N(412)=1.E+2
      439    N(413)=1.E+9
      440    N(414)=1.E+2
      441    N(415)=1.E+9
      442    N(416)=1.E+2
      443    N(417)=1.E+9
      444    N(418)=1.E+2
      445    N(419)=1.E+9
      446    N(420)=1.E+2
      447    N(421)=1.E+9
      448    N(422)=1.E+2
      449    N(423)=1.E+9
      450    N(424)=1.E+2
      451    N(425)=1.E+9
      452    N(426)=1.E+2
      453    N(427)=1.E+9
      454    N(428)=1.E+2
      455    N(429)=1.E+9
      456    N(430)=1.E+2
      457    N(431)=1.E+9
      458    N(432)=1.E+2
      459    N(433)=1.E+9
      460    N(434)=1.E+2
      461    N(435)=1.E+9
      462    N(436)=1.E+2
      463    N(437)=1.E+9
      464    N(438)=1.E+2
      465    N(439)=1.E+9
      466    N(440)=1.E+2
      467    N(441)=1.E+9
      468    N(442)=1.E+2
      469    N(443)=1.E+9
      470    N(444)=1.E+2
      471    N(445)=1.E+9
      472    N(446)=1.E+2
      473    N(447)=1.E+9
      474    N(448)=1.E+2
      475    N(449)=1.E+9
      476    N(450)=1.E+2
      477    N(451)=1.E+9
      478    N(452)=1.E+2
      479    N(453)=1.E+9
      480    N(454)=1.E+2
      481    N(455)=1.E+9
      482    N(456)=1.E+2
      483    N(457)=1.E+9
      484    N(458)=1.E+2
      485    N(459)=1.E+9
      486    N(460)=1.E+2
      487    N(461)=1.E+9
      488    N(462)=1.E+2
      489    N(463)=1.E+9
      490    N(464)=1.E+2
      491    N(465)=1.E+9
      492    N(466)=1.E+2
      493    N(467)=1.E+9
      494    N(468)=1.E+2
      495    N(469)=1.E+9
      496    N(470)=1.E+2
      497    N(471)=1.E+9
      498    N(472)=1.E+2
      499    N(473)=1.E+9
      500    N(474)=1.E+2
      501    N(475)=1.E+9
      502    N(476)=1.E+2
      503    N(477)=1.E+9
      504    N(478)=1.E+2
      505    N(479)=1.E+9
      506    N(480)=1.E+2
      507    N(481)=1.E+9
      508    N(482)=1.E+2
      509    N(483)=1.E+9
      510    N(484)=1.E+2
      511    N(485)=1.E+9
      512    N(486)=1.E+2
      513    N(487)=1.E+9
      514    N(488)=1.E+2
      515    N(489)=1.E+9
      516    N(490)=1.E+2
      517    N(491)=1.E+9
      518    N(492)=1.E+2
      519    N(493)=1.E+9
      520    N(494)=1.E+2
      521    N(495)=1.E+9
      522    N(496)=1.E+2
      523    N(497)=1.E+9
      524    N(498)=1.E+2
      525    N(499)=1.E+9
      526    N(500)=1.E+2
      527    N(501)=1.E+9
      528    N(502)=1.E+2
      529    N(503)=1.E+9
      530    N(504)=1.E+2
      531    N(505)=1.E+9
      532    N(506)=1.E+2
      533    N(507)=1.E+9
      534    N(508)=1.E+2
      535    N(509)=1.E+9
      536    N(510)=1.E+2
      537    N(511)=1.E+9
      538    N(512)=1.E+2
      539    N(513)=1.E+9
      540    N(514)=1.E+2
      541    N(515)=1.E+9
      542    N(516)=1.E+2
      543    N(517)=1.E+9
      544    N(518)=1.E+2
      545    N(519)=1.E+9
      546    N(520)=1.E+2
      547    N(521)=1.E+9
      548    N(522)=1.E+2
      549    N(523)=1.E+9
      550    N(524)=1.E+2
      551    N(525)=1.E+9
      552    N(526)=1.E+2
      553    N(527)=1.E+9
      554    N(528)=1.E+2
      555    N(529)=1.E+9
      556    N(530)=1.E+2
      557    N(531)=1.E+9
      558    N(532)=1.E+2
      559    N(533)=1.E+9
      560    N(534)=1.E+2
      561    N(535)=1.E+9
      562    N(536)=1.E+2
      563    N(537)=1.E+9
      564    N(538)=1.E+2
      565    N(539)=1.E+9
      566    N(540)=1.E+2
      567    N(541)=1.E+9
      568    N(542)=1.E+2
      569    N(543)=1.E+9
      570    N(544)=1.E+2
      571    N(545)=1.E+9
      572    N(546)=1.E+2
      573    N(547)=1.E+9
      574    N(548)=1.E+2
      575    N(549)=1.E+9
      576    N(550)=1.E+2
      577    N(551)=1.E+9
      578    N(552)=1.E+2
      579    N(553)=1.E+9
      580    N(554)=1.E+2
      581    N(555)=1.E+9
      582    N(556)=1.E+2
      583    N(557)=1.E+9
      584    N(558)=1.E+2
      585    N(559)=1.E+9
      586    N(560)=1.E+2
      587    N(561)=1.E+9
      588    N(562)=1.E+2
      589    N(563)=1.E+9
      590    N(564)=1.E+2
      591    N(565)=1.E+9
      592    N(566)=1.E+2
      593    N(567)=1.E+9
      594    N(568)=1.E+2
      595    N(569)=1.E+9
      596    N(570)=1.E+2
      597    N(571)=1.E+9
      598    N(572)=1.E+2
      599    N(573)=1.E+9
      600    N(574)=1.E+2
      601    N(575)=1.E+9
      602    N(576)=1.E+2
      603    N(577)=1.E+9
      604    N(578)=1.E+2
      605    N(579)=1.E+9
      606    N(580)=1.E+2
      607    N(581)=1.E+9
      608    N(582)=1.E+2
      609    N(583)=1.E+9
      610    N(584)=1.E+2
      611    N(585)=1.E+9
      612    N(586)=1.E+2
      613    N(587)=1.E+9
      614    N(588)=1.E+2
      615    N(589)=1.E+9
      616    N(590)=1.E+2
      617    N(591)=1.E+9
      618    N(592)=1.E+2
      619    N(593)=1.E+9
      620    N(594)=1.E+2
      621    N(595)=1.E+9
      622    N(596)=1.E+2
      623    N(597)=1.E+9
      624    N(598)=1.E+2
      625    N(599)=1.E+9
      626    N(600)=1.E+2
      627    N(601)=1.E+9
      628    N(602)=1.E+2
      629    N(603)=1.E+9
      630    N(604)=1.E+2
      631    N(605)=1.E+9
      632    N(606)=1.E+2
      633    N(607)=1.E+9
      634    N(608)=1.E+2
      635    N(609)=1.E+9
      636    N(610)=1.E+2
      637    N(611)=1.E+9
      638    N(612)=1.E+2
      639    N(613)=1.E+9
      640    N(614)=1.E+2
      641    N(615)=1.E+9
      642    N(616)=1.E+2
      643    N(617)=1.E+9
      644    N(618)=1.E+2
      645    N(619)=1.E+9
      646    N(620)=1.E+2
      647    N(621)=1.E+9
      648    N(622)=1.E+2
      649    N(623)=1.E+9
      650    N(624)=1.E+2
      651    N(625)=1.E+9
      652    N(626)=1.E+2
      653    N(627)=1.E+9
      654    N(628)=1.E+2
      655    N(629)=1.E+9
      656    N(630)=1.E+2
      657    N(631)=1.E+9
      658    N(632)=1.E+2
      659    N(633)=1.E+9
      660    N(634)=1.E+2
      661    N(635)=1.E+9
      662    N(636)=1.E+2
      663    N(637)=1.E+9
      664    N(638)=1.E+2
      665    N(639)=1.E+9
      666    N(640)=1.E+2
      667    N(641)=1.E+9
      668    N(642)=1.E+2
      669    N(643)=1.E+9
      670    N(644)=1.E+2
      671    N(645)=1.E+9
      672    N(646)=1.E+2
      673    N(647)=1.E+9
      674    N(648)=1.E+2
      675    N(649)=1.E+9
      676    N(650)=1.E+2
      677    N(651)=1.E+9
      678    N(652)=1.E+2
      679    N(653)=1.E+9
      680    N(654)=1.E+2
      681    N(655)=1.E+9
      682    N(656)=1.E+2
      683    N(657)=1.E+9
      684    N(658)=1.E+2
      685    N(659)=1.E+9
      686    N(660)=1.E+2
      687    N(661)=1.E+9
      688    N(662)=1.E+2
      689    N(663)=1.E+9
      690    N(664)=1.E+2
      691    N(665)=1.E+9
      692    N(666)=1.E+2
      693    N(667)=1.E+9
      694    N(668)=1.E+2
      695    N(669)=1.E+9
      696    N(670)=1.E+2
      697    N(671)=1.E+9
      698    N(672)=1.E+2
      699    N(673)=1.E+9
      700    N(674)=1.E+2
      701    N(675)=1.E+9
      702    N(676)=1.E+2
      703    N(677)=1.E+9
      704    N(678)=1.E+2
      705    N(679)=1.E+9
      706    N(680)=1.E+2
      707    N(681)=1.E+9
      708    N(682)=1.E+2
      709    N(683)=1.E+9
      710   
```



25	<pre> C1=C1*2 R1=R1*2 R2=R2*2 R3=R3*2 C1=C1*2 C2=C2*2 </pre>
30	<pre> R1=ABS(R1) R2=ABS(R2) R3=ABS(R3) C1=ABS(C1) C2=ABS(C2) MOP=R3/R2+R3 </pre>
35	<pre> C1=C1/C2 PR=1./0.20109*11./507(C1+C2+R1+R2) OPP=1./12+R3*507(R1+C1+R2/C2) IP=1/COUNT/201+20 IF(IP.E0.1/COUNT) PRINT 17,MOP,R1,R2,R3,C1,C2,C1/C2,COUNT,PR,OPP,61 0.62-63 </pre>
40	<pre> 17 FORMAT(1X,F612.0,1X,10,F10.2,F7.2,F7.4,F7.4,F7.4) IF(C1.DR.C2).LT.1.E-10) STOP IF(R1.E7.0.E+6) STOP RETURN END </pre>
45	<pre> END </pre>
SYMBOLIC REFERENCE MAP (R=1)	
ENTRY POINTS	
3 FUNCT1	

CONCORDIA UNIVERSITY

COMPUTER CENTRE

POOR COPY  
 COPIE DE QUALITEE INFERIEURE

## F2.2 Description of Function Subroutine FUNCTI

This subroutine shares the "common" variables and constants with the main program and the second subroutine ZXMIN. These "common" elements feed the minimization algorithm ZXMIN.

The constraints and the final unconstrained objective function are calculated in this subroutine. The computed variables (filter components) and parameters are denormalized and printed every 20 iterations. Then, if the minimum point, or the end-values of any of the five components is reached a "stop" command terminates the main program. Figure F.2 describes the function of FUNCTI in more detail

## F2.3 Function subroutine ZXMIN

Again, the set of variables (COMMON) and constants, as well as the function subroutine FUNCTI, initialization of counter ( $i=0, \bar{x} = \bar{x}^i$ ), and initialization of H (i.e.  $H = H_0 = I$ ) are called to prepare the algorithm for the process of minimization. Next, the loop is starting where all the minimization steps are included. In this loop, the gradients  $G_i^j(I)$  are computed, and are tested if they reached the minimum value " $\epsilon$ ", whereby minimum point is assumed and the algorithm terminates. Subsequently, the hessian matrix is computed; if it exists a step size " $k$ " is properly estimated

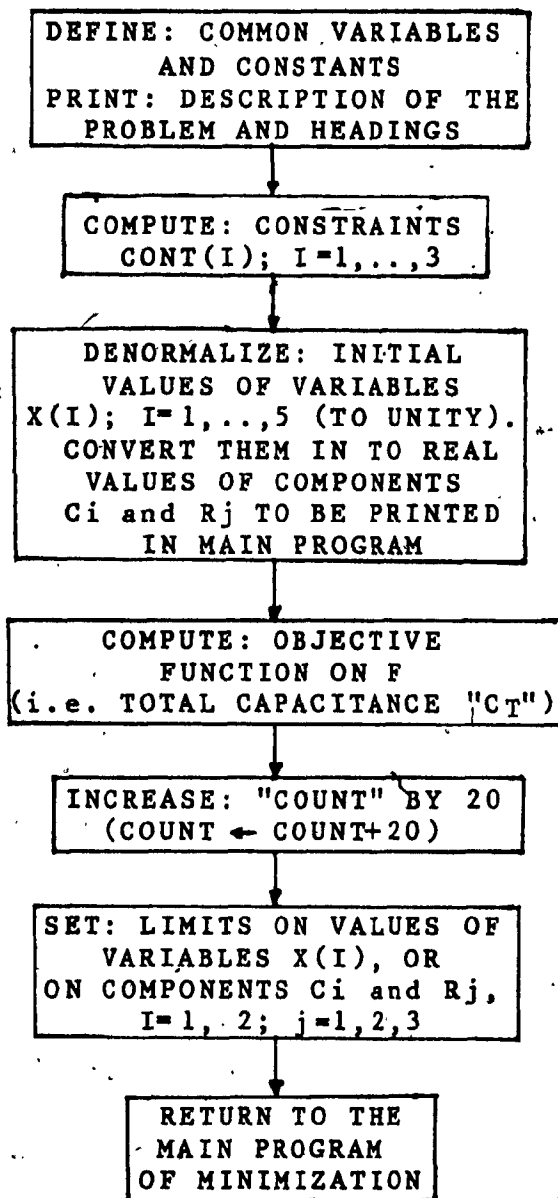


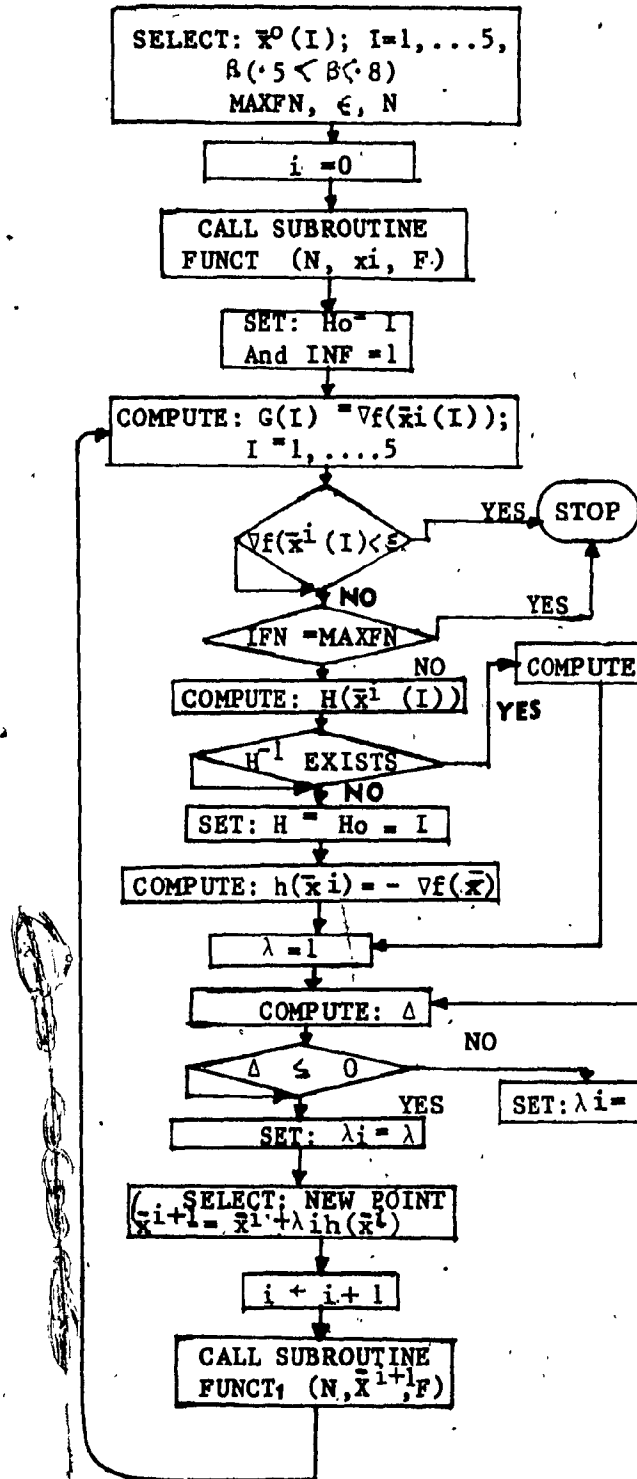
Fig E2 - Subroutine Function FUNCTI (N,X,F)

and the new point  $\bar{x}^{i+1}$  is calculated according to prescribed formula. Then, the counter is increased ( $i \leftarrow i + 1$ ) and the process is repeated by assigning new value in  $F$  and gradients  $G^{i+1}(I)$ . Detailed description is found in the flow chart, Fig. F.3.

#### F2.4 Description of main programs and subroutines of the Fletcher-Powell's algorithm

This group of programs, which is the most important part of the minimization analysis in this report, minimizes the total capacitance, the total resistance area, and their combination, of the filter configuration under study. Each of the above three objective functions to be minimized consists of four cases, which describe the different constraints imposed, per Table - 4 (program No. 13 to 48). As in Quasi-Newton's algorithm, all programs are similar in formulation, the only difference being the constraints and consequently the objective functions.

The functional description of the main programs and subroutines are detailed in the flow charts, fig. F.4, F.5, F.6 and F.7 respectively and in their corresponding listings.



**FUNCT** - Function sub-routine (calculates function F at point  $\bar{x}^i$ )

$\bar{x}^0(I)$  = initial guess of  $\bar{x}$

MAXFN = max. no. of function evaluations

$\epsilon$  = tolerance no. (used in stopping criteria)

$$\nabla f(\bar{x}^i(I)) = \begin{bmatrix} \frac{\partial f(\bar{x}^i)}{\partial x_1} \\ \vdots \\ \frac{\partial f(\bar{x}^i)}{\partial x_n} \end{bmatrix}$$

$$H(\bar{x}^i) = \begin{bmatrix} \frac{\partial^2 f(\bar{x}^i)}{\partial x_1^2} & \dots & \frac{\partial^2 f(\bar{x}^i)}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f(\bar{x}^i)}{\partial x_n \partial x_1} & \dots & \frac{\partial^2 f(\bar{x}^i)}{\partial x_n^2} \end{bmatrix}$$

$$= \nabla \nabla f(\bar{x}^i)$$

$$h(\bar{x}^i) = - H(\bar{x}^i)^{-1} \nabla f(\bar{x}^i)$$

$$\Delta = f(\bar{x}^i + \lambda h(\bar{x}^i)) - f(\bar{x}^i)$$

$$= \frac{\lambda}{2} \langle \nabla f(\bar{x}^i), h(\bar{x}^i) \rangle$$

$$\text{where, } \langle \bar{x}, \bar{y} \rangle = \sum_{i=1}^n x_i y_i$$

Figure F3 - Function subroutine ZMIN (FUNCT<sub>1</sub>, N, ..., X, H, G, F, ...)

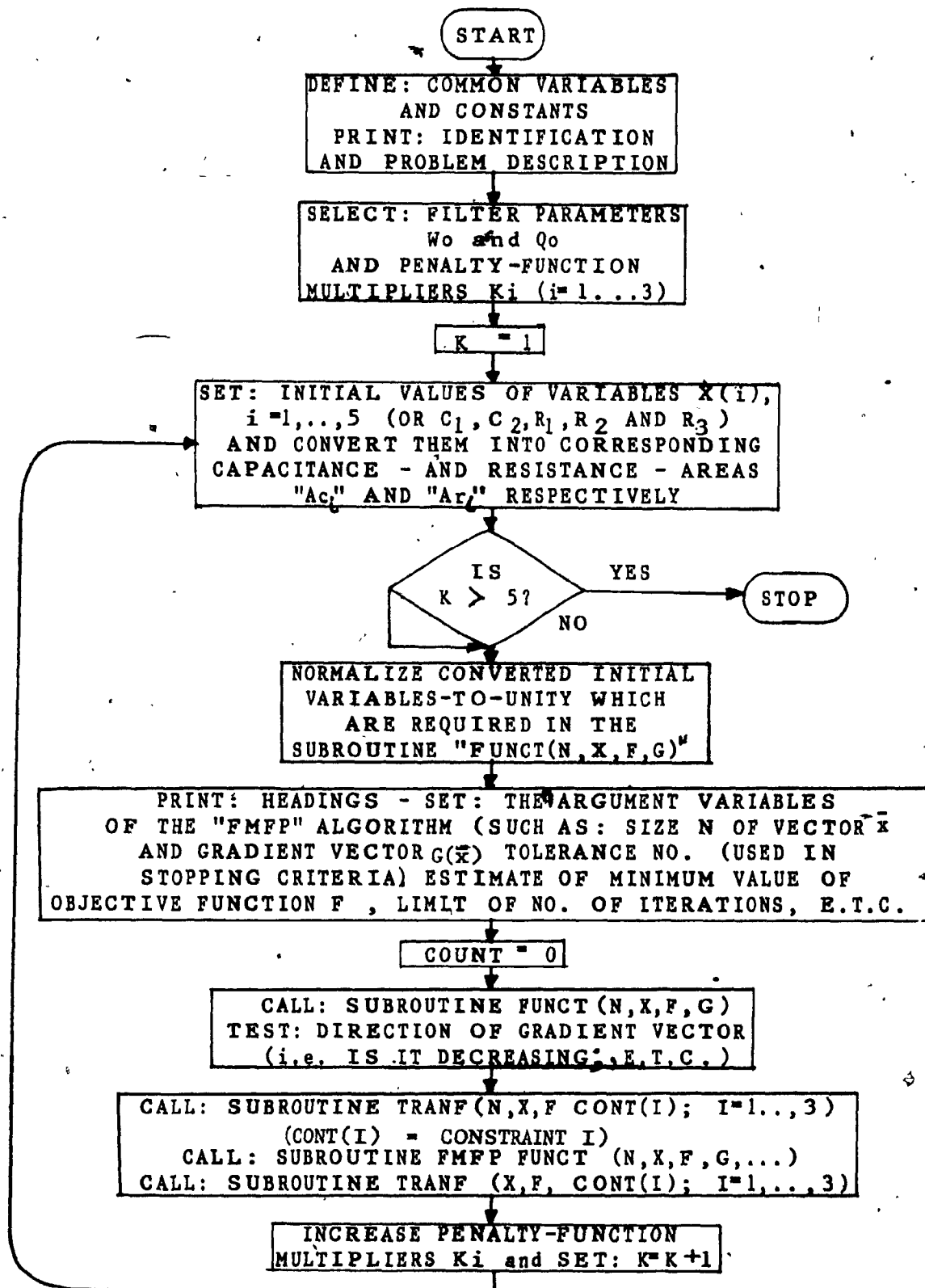


Fig R4 - Main program of minimization by Fletcher-Powell's algorithm

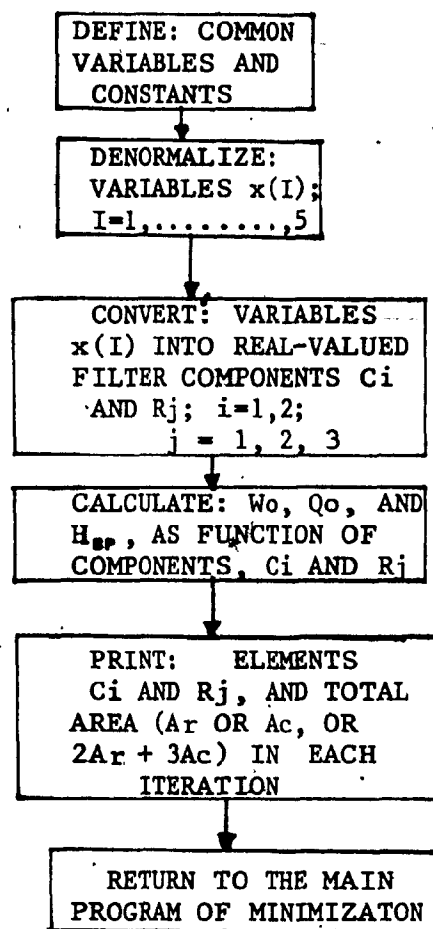


Fig. F5 - Subroutine TRANF. ( X, F, CONT (I); I = 1, ...3)

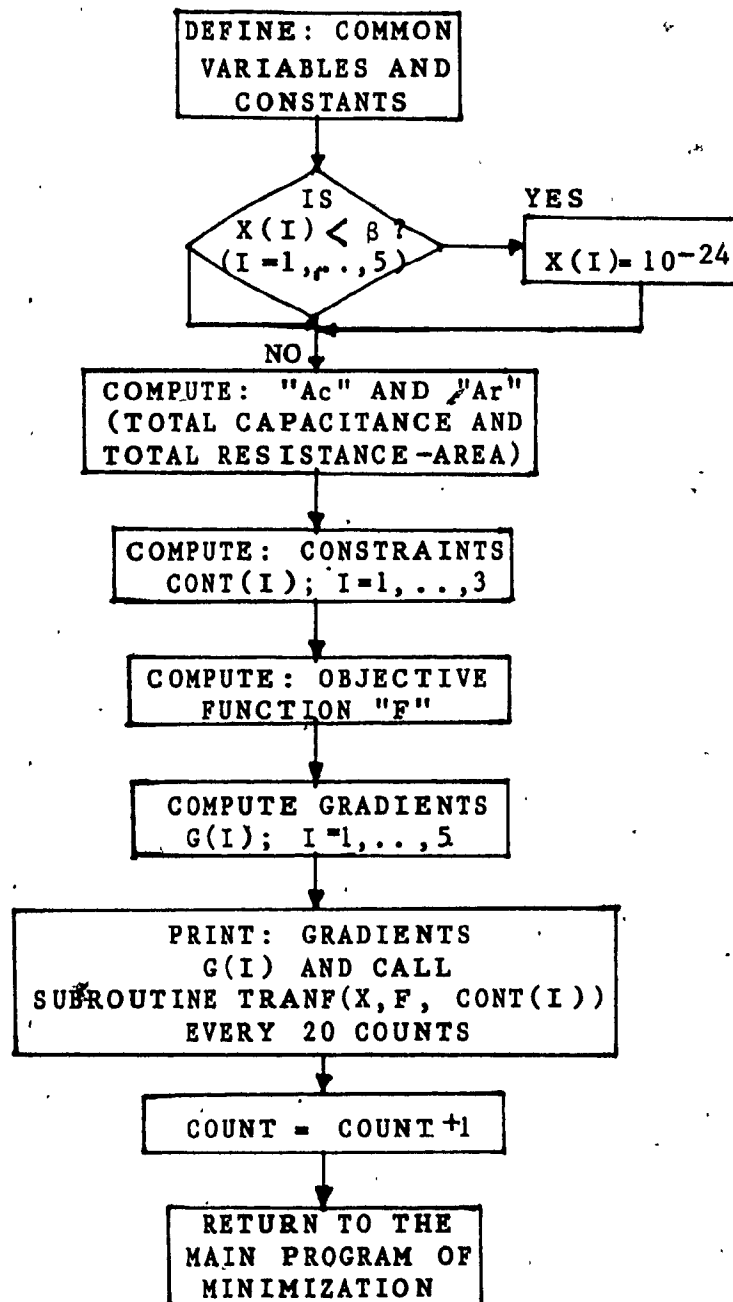
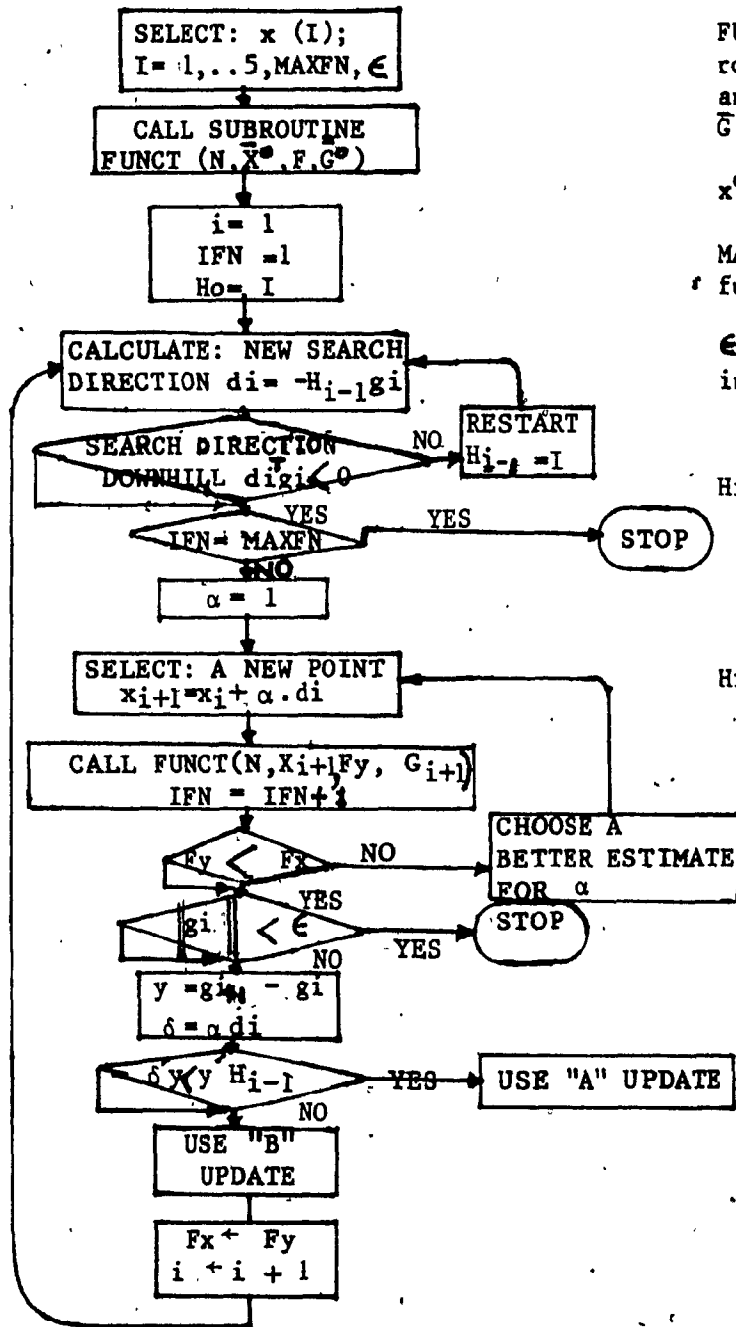


Fig F.6 - Subroutine FUNCT (N,X,F,G)





FUNCT Function sub-  
routine (Calculates F  
and gradient vector  
Ḡ at point x̄i)

x<sup>0</sup>(I) = initial guess

MAXFN = max. no. of  
function evaluations

ε = tolerance no. (used  
in stopping criteria)

"A" UPDATE

$$H_i = H_{i-1} + \frac{\delta \delta'}{\delta'^2 y} - \frac{H_{i-1} y y' H_{i-1}}{y' H_{i-1} y}$$

"B" UPDATE

$$H_i = \left( I - \frac{y \delta'}{y' \delta} \right) H_{i-1} \left( I - \frac{\delta y'}{y \delta} \right) + \frac{y y'}{\delta' y}$$

Figure F.7 - Subroutine FMFP (FUNCT, N, X, F, G, ...)

```

1 PROGRAM C17H(INPUT,3UPP3I)
INTERNAL FUNCT
COMMON PI,OP,HI,MZ,ICOUNT,A1H,A2H,A3H,A4H,A5H,A1,A2,X1,X2,X3,X4,X5
SYNOPSIS (X1),Y1,Y2,C1,C2,HC1,HC2,C1
PRINT 25
25 FORMAT(10H,42HMAJOR TECHNICAL REPORT IN EL. ENGINEERING.)
PRINT 26
26 FORMAT(10H,60HMINORITY GAIN BAND PASS ACTIVE FILTER OPTIMIZATION OF PI
ETER ELEMENTS,/)
PRINT 27
27 FORMAT(10H,37HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 28
28 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 29
29 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 30
30 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 31
31 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 32
32 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 33
33 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 34
34 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 35
35 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 36
36 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 37
37 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 38
38 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 39
39 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 40
40 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 41
41 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 42
42 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 43
43 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 44
44 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 45
45 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 46
46 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 47
47 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 48
48 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 49
49 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 50
50 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 51
51 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 52
52 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 53
53 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 54
54 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 55
55 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 56
56 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 57
57 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 58
58 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 59
59 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 60
60 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 61
61 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 62
62 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 63
63 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 64
64 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 65
65 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 66
66 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 67
67 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 68
68 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 69
69 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 70
70 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 71
71 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 72
72 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 73
73 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 74
74 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 75
75 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 76
76 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 77
77 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 78
78 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 79
79 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 80
80 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 81
81 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 82
82 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 83
83 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 84
84 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 85
85 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 86
86 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 87
87 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 88
88 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 89
89 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 90
90 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 91
91 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 92
92 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 93
93 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 94
94 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 95
95 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 96
96 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 97
97 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 98
98 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 99
99 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)
PRINT 100
100 FORMAT(10H,31HBY PARASKEVAS PROKOPIE I.G. 6901996,/)

```

POOR COPY  
COPIE DE QUALITEE INFERIEURE



MAJOR TECHNICAL REPORT IN EL. ENGINEERING.  
OPTV\_GAIN BAND PASS ACTIVE FILTER OPTIMIZATION OF FILTER ELEMENTS

BY PARAMANIV PANDRUPIS I.S. 6001906

ADVISED DR. S.S. SUNITACHARYYA

THIS PROGRAM MINIMIZES THE AREA "AC" CORRESPONDING TO THE TOTAL CAPACITANCE "CT" OF THE RC ACTIVE FILTER USING PROPER DEPOSITION THIN FILM ON PROPER SUBSTRATE (I.E. THE SUBSTRATE COEFFICIENTS THE TOTAL AREA INCLUDING THAT OF THE ACTIVE COMPONENTS) GIVEN CONSTRAINTS FOR SPECIFIED CENTER FREQUENCY "F0", POLE Q, "Q0" AND FLATTEN BAND PASS MAX. GAIN "GAIN" AS WELL AS ALL FIVE FILTER ELEMENTS (C1,C2,R1, R2, AND R3)

IN THIS ANALYSIS PLATTNER POWELL MINIMIZATION METHOD IS USED

MINIMIZATION FUNCTION F="AC".....HERE  
AC=(2/3)\*(VDR/(EO\*ER))CT\*AL\*CT (CM^2)  
AL=(1/2)\*L\*(1/RS)\*RY\*AL\*RY (CM^2)  
VDR=DRIVING VOLTAGE (VOLTS)  
EO=0.99910E+12 IS THE PERMISSIVITY OF THE FREE SPACE.  
(PARADIS/CM)  
ER=DIELECTRIC CONSTANT RELATIVE TO THAT OF FREE SPACE.  
E=DIELECTRIC STRENGTH OF THE THIN FILM (VOLTS/CM)  
G=WIDTH OF THE THIN FILM RESISTIVE LINE (CM)  
RS=Sheet RESISTANCE OF THE RESIST. LINE (OHMS/SQCM)  
CT=TOTAL CAPACITANCE=C1+C2 OF THE FILTER (PARADS)  
RY=TOTAL RESISTANCE=R1+R2+R3 OF THE FILTER (OHMS)

NOTE THE ABOVE OBJECTIVE FUNCTION IS A CONSTRAINED ONE.

INPUT PR.OP

VDR 10 E1 E2 E3 R1 R2 R3  
100.0 .000E+00 10.0 .000E+00 .000E+00 .000E+00 .000E+00

A1=(2/3)\*(VDR/(EO\*ER)) A2=(1/2)\*L\*(1/RS)\*RY

.753294E+07 .500000E-04  
F0= 900.0 Q0= 30.0

INPUT X VECTOR

.100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01

N1 1000.0000000000000000 N2 1000.0000000000000000

1.000000 1.000000 COMPUTED GRADIENTS G(1) .1421005E-10 -.1421005E-10 -.1021171E-10

C1=.31031E-07 C2=.31031E-07 R1=.10000E+07 OHMS R2= 1000.0 OHMS R3= 1000.0 OHMS AC=.500100 (CM^2)

TOTAL CAPACITANCE SURFACE AREA "AC"=.26376 (CM^2) CONSTRAINTS -.710543E-10 -.142100E-10 F= 900.0000

F0= 900.0000000000000000 Q0= 30.000000 GAIN=.500000

1.00002 1.00002 VERIFIED VALUE OF G(1) .140022E-09 .999760E-09 .999760E-09

COUNT= 0  
C1=.31031E-07 C2=.31031E-07 R1=.10000E+07 OHMS R2= 1000.0 OHMS R3= 1000.0 OHMS AC=.500100 (CM^2)

TOTAL CAPACITANCE SURFACE AREA "AC"=.26376 (CM^2) CONSTRAINTS 0. 0. F= 900.0000

F0= 900.0000000000000000 Q0= 30.000000 GAIN=.500000

MINIMIZATION ALGORITHM STARTS AT THIS POINT.

CIRCUIT PARAMETERS

-.946392 -.10.04075 COMPUTED GRADIENTS G(1) -.946392 -.4057310

COUNT= 14  
C1=.30273E-07 C2=.333333E-07 R1=.10000E+07 OHMS R2= 1000.0 OHMS R3= 1000.0 OHMS AC=.500100 (CM^2)

POOR COPY  
COPIE DE QUALITEE INFERIEURE

TOTAL CAPACITANCE_SURFACE AREA "AC"= .2500 (CN_100) CONSTRAINTS -.000000E-02 -.243779E-03 P= 1.0295				
FB=	531.266442MERTZ	OP=	50.000000	GAIN= .000000
COMPUTED GRADIENTS G(1)				
	.0291073	1.491003	-.3304007	.9930202E-01 .3090570
COUNT= 70				
C1= .30250E-07F C2= .33390E-00F R1= .10495E+07DMS R2= 1049.9 DMS R3= 1049.2 DMS AC= .32500 (CN_500)				
TOTAL CAPACITANCE_SURFACE AREA "AC"= .2500 (CN_100) CONSTRAINTS -.300729E-04 .100002E-03 P= 1.0000				
FB=	199.92232MERTZ	OP=	50.000000	GAIN= .000000
COMPUTED GRADIENTS G(1)				
	-3.253060	3.712950	-4.244492	-1.110017 1.672070
COUNT= 90				
C1= .25070E-07F C2= .25640E-00F R1= .10160E+07DMS R2= 1270.0 DMS R3= 1177.9 DMS AC= .26700 (CN_500)				
TOTAL CAPACITANCE_SURFACE AREA "AC"= .21571 (CN_100) CONSTRAINTS -.710940E-03 .1030700E-02 P= 1.0000				
FB=	500.17773MERTZ	OP=	49.990000	GAIN= .001213
COMPUTED GRADIENTS G(1)				
	-12.44019	-0.407104	-7.700493	-0.700772 1.022172
COUNT= 74				
C1= .23070E-07F C2= .23070E-00F R1= .10160E+07DMS R2= 1270.0 DMS R3= 1200.7 DMS AC= .27170 (CN_500)				
TOTAL CAPACITANCE_SURFACE AREA "AC"= .19170 (CN_100) CONSTRAINTS -.002129E-02 .122012E-02 P= 1.0071				
FB=	501.00035MERTZ	OP=	49.999120	GAIN= .007700
COMPUTED GRADIENTS G(1)				
	-10.61100	-12.51036	-9.100301	-0.050503 -0.3207270
COUNT= 90				
C1= .21020E-07F C2= .21650E-00F R1= .10010E+07DMS R2= 1400.0 DMS R3= 1400.0 DMS AC= .70320 (CN_500)				
TOTAL CAPACITANCE_SURFACE AREA "AC"= .17600 (CN_100) CONSTRAINTS -.029030E-02 -.301900E-03 P= 1.0000				
FB=	501.00193MERTZ	OP=	50.000000	GAIN= .000000
COMPUTED GRADIENTS G(1)				
	1.075010	-0.353950	.770050	-1.770100 -2.777310
COUNT= 110				
C1= .10000E-07F C2= .33270E-00F R1= .15700E+07DMS R2= 1010.0 DMS R3= 1020.0 DMS AC= .70070 (CN_500)				
TOTAL CAPACITANCE_SURFACE AREA "AC"= .10000 (CN_100) CONSTRAINTS -.110000E-02 -.170270E-02 P= 1.0000				
FB=	500.29607MERTZ	OP=	50.000000	GAIN= .001222
COMPUTED GRADIENTS G(1)				
	-0.000070	-0.449054	-2.279790	-2.076017 -0.400053
COUNT= 130				
C1= .17000E-07F C2= .10000E-00F R1= .17230E+07DMS R2= 1770.0 DMS R3= 1691.7 DMS AC= .00200 (CN_500)				
TOTAL CAPACITANCE_SURFACE AREA "AC"= .10000 (CN_100) CONSTRAINTS -.230001E-02 -.306120E-03 P= 1.0000				
FB=	500.07225MERTZ	OP=	50.000000	GAIN= .002201
COMPUTED GRADIENTS G(1)				
	-7.011010	-11.00906	-2.430540	-2.002001 -0.330000
COUNT= 150				
C1= .10010E-07F C2= .27100E-00F R1= .10010E+07DMS R2= 1910.0 DMS R3= 1700.0 DMS AC= .03000 (CN_500)				
TOTAL CAPACITANCE_SURFACE AREA "AC"= .17000 (CN_100) CONSTRAINTS -.277010E-02 -.300000E-03 P= 1.0000				
FB=	500.01000MERTZ	OP=	50.012372	GAIN= .002007

POOR COPY  
COPIE DE QUALITEE INFERIEURE

COMPUTED GRADIENTS G(1)				
-21.18164	-1.702942	-9.171299	-2.762974	2.292113
COUNT= 174				
C1= .13004E-07 C2= .13004E-06 R1= .20674E+07 DMS R2= 2075.7 DMS R3= 1924.5 DMS AC= 1.0154 (CR_500)				
TOTAL CAPACITANCE SURFACE AREA "AC"= .12540 (CR_500) CONSTRAINTS -.206400E-02 .227712E-02 P= .00797				
FB= 500.74335HERTZ OP= 49.943169 SAIR= .001100				
COMPUTED GRADIENTS G(1)				
-13.61670	-9.699770	-2.950047	-2.561007	.342667

COUNT= 174				
C1= .14634E-07 C2= .14730E-06 R1= .20674E+07 DMS R2= 2075.7 DMS R3= 1924.5 DMS AC= 1.0154 (CR_500)				
TOTAL CAPACITANCE SURFACE AREA "AC"= .11606 (CR_500) CONSTRAINTS -.206221E-02 .267994E-03 P= .01405				
FB= 500.71709HERTZ OP= 49.900003 SAIR= .000110				

COMPUTED GRADIENTS G(1)				
-11.15640	-11.16715	-2.169123	-2.177006	-.632275E-01
COUNT= 216				
C1= .13160E-07 C2= .13013E-06 R1= .23104E+07 DMS R2= 2370.1 DMS R3= 2100.3 DMS AC= 1.1615 (CR_500)				
TOTAL CAPACITANCE SURFACE AREA "AC"= .10900 (CR_500) CONSTRAINTS -.209359E-02 -.723403E-04 P= .05752				
FB= 500.64363HERTZ OP= 50.001009 SAIR= .079330				

COMPUTED GRADIENTS G(1)				
-9.600193	-16.07009	-1.204032	-2.273611	-1.106036
COUNT= 234				

C1= .12878E-07 C2= .12802E-06 R1= .25051E+07 DMS R2= 2572.3 DMS R3= 2399.3 DMS AC= 1.2950 (CR_500)				
TOTAL CAPACITANCE SURFACE AREA "AC"= .10139 (CR_500) CONSTRAINTS -.207309E-02 -.137020E-02 P= .70717				
FB= 500.72001HERTZ OP= 50.034010 SAIR= .070405				

COMPUTED GRADIENTS G(1)				
-11.92223	-12.32227	-1.001009	-1.022272	-.002010E-01
COUNT= 254				

C1= .11050E-07 C2= .12161E-06 R1= .20687E+07 DMS R2= 1720.3 DMS R3= 2070.3 DMS AC= 1.3329 (CR_500)				
TOTAL CAPACITANCE SURFACE AREA "AC"= .09394E-01 (CR_500) CONSTRAINTS -.206210E-02 -.116700E-03 P= .74443				
FB= 500.65761HERTZ OP= 50.002015 SAIR= .077593				

COMPUTED GRADIENTS G(1)				
-17.40009	-6.003491	-2.713750	-1.402776	.6074961
COUNT= 274				

C1= .13756E-07 C2= .11405E-06 R1= .20207E+07 DMS R2= 2094.7 DMS R3= 2044.0 DMS AC= 1.4151 (CR_500)				
TOTAL CAPACITANCE SURFACE AREA "AC"= .09740E-01 (CR_500) CONSTRAINTS -.220711E-02 .092349E-03 P= .70475				
FB= 500.22209HERTZ OP= 49.970000 SAIR= .077052				

COMPUTED GRADIENTS G(1)				
1.631425	-7.702937	.2902113	-.3472620	-.3006760
COUNT= 294				

C1= .07530E-07 C2= .13420E-06 R1= .31126E+07 DMS R2= 3106.5 DMS R3= 2903.5 DMS AC= 1.9904 (CR_500)				
TOTAL CAPACITANCE SURFACE AREA "AC"= .01376E-01 (CR_500) CONSTRAINTS -.311302E-03 -.013254E-03 P= .03939				
FB= 500.12700HERTZ OP= 50.022900 SAIR= .075994				

COMPUTED GRADIENTS G(1)				
1.030230	-12.00071	.3032007	-1.310009	-1.044007

POOR COPY  
COPIE DE QUALITEE INFERIEURE

COUNT= 314				
C1= .0014E-00F	C2= .9210E-00F	E1= .3501E+07DMS	E2= 3013.1	DMS E3= 1255.0 DMS AC= 1.7567 (CR_501)
TOTAL CAPACITANCE SURFACE AREA "AC"= .7190E-01(CR_501) CONSTRAINTS --.22919E-02 --.20001E-02 P= .97449				
FO=	100.55973HERTZ	OP=	50.87113	GAIN= .47399
-23.76009	-0.272525	COMPUTED GRADIENTS G111	-1.109173	.6070002
COUNT= 324				
C1= .0173E-00F	C2= .1000E-00F	E1= .3610E+07DMS	E2= 3003.4	DMS E3= 3418.0 DMS AC= 1.0409 (CR_501)
TOTAL CAPACITANCE SURFACE AREA "AC"= .6820E-01(CR_501) CONSTRAINTS --.23097E-02 --.87095E-03 P= .97099				
FO=	100.97740HERTZ	OP=	40.970051	GAIN= .47301
-25.31939	-3.849272	COMPUTED GRADIENTS G111	-1.750078	-.0669529
COUNT= 334				
C1= .7107E-00F	C2= .0121E-00F	E1= .4065E+07DMS	E2= 4138.4	DMS E3= 3070.2 DMS AC= 2.0072 (CR_501)
TOTAL CAPACITANCE SURFACE AREA "AC"= .6209E-01(CR_501) CONSTRAINTS --.10093E-02 --.12017E-02 P= .49041				
FO=	100.46690HERTZ	OP=	40.960906	GAIN= .471917
-20.02145	-14.97972	COMPUTED GRADIENTS G111	-1.440471	-1.121509
COUNT= 374				
C1= .7100E-00F	C2= .7100E-00F	E1= .4065E+07DMS	E2= 4138.4	DMS E3= 3070.2 DMS AC= 2.0072 (CR_501)
TOTAL CAPACITANCE SURFACE AREA "AC"= .5077E-01(CR_501) CONSTRAINTS --.24737E-02 --.57130E-03 P= .97009				
FO=	100.61997HERTZ	OP=	40.909721	GAIN= .471009
-1.700025	2.172001	COMPUTED GRADIENTS G111	-1.140114	-.2910300E-02
COUNT= 394				
C1= .6001E-00F	C2= .7101E-00F	E1= .4378E+07DMS	E2= 4534.4	DMS E3= 4028.2 DMS AC= 2.1934 (CR_501)
TOTAL CAPACITANCE SURFACE AREA "AC"= .5750E-01(CR_501) CONSTRAINTS --.22700E-02 --.27000E-03 P= .44090				
FO=	100.00370HERTZ	OP=	40.991004	GAIN= .470000
-13.09962	29.34164	COMPUTED GRADIENTS G111	-1.957911	1.590412
COUNT= 414				
C1= .6101E-00F	C2= .7101E-00F	E1= .4400E+07DMS	E2= 4763.3	DMS E3= 4224.8 DMS AC= 2.3045 (CR_501)
TOTAL CAPACITANCE SURFACE AREA "AC"= .5502E-01(CR_501) CONSTRAINTS --.43007E-03 --.21210E-02 P= .43790				
FO=	100.10700HERTZ	OP=	40.921079	GAIN= .469997
-20.07235	-24.24130	COMPUTED GRADIENTS G111	-.902376	-.9921300
COUNT= 434				
C1= .6200E-00F	C2= .6112E-00F	E1= .4771E+07DMS	E2= 4937.4	DMS E3= 4377.5 DMS AC= 2.3906 (CR_501)
TOTAL CAPACITANCE SURFACE AREA "AC"= .5240E-01(CR_501) CONSTRAINTS --.24390E-02 --.27300E-03 P= .61792				
FO=	100.61060HERTZ	OP=	50.660220	GAIN= .469994
-19.02794	-11.22730	COMPUTED GRADIENTS G111	-.602900	.7297990E-01
COUNT= 454				
C1= .6072E-00F	C2= .6603E-00F	E1= .4910E+07DMS	E2= 5100.2	DMS E3= 4929.2 DMS AC= 2.4701 (CR_501)
TOTAL CAPACITANCE SURFACE AREA "AC"= .5572E-01(CR_501) CONSTRAINTS --.14434E-02 --.17313E-03 P= .40037				
FO=	100.36120HERTZ	OP=	40.979000	GAIN= .469791

CONCORDIA UNIVERSITY

COMPUTER CENTRE

POOR COPY  
COPIE DE QUALITEE INFERIEURE







DESIGN TECHNICAL REPORT IN EL. ENGINEERING.  
 UNITV\_GAIN BAND PASS ACTIVE FILTER OPTIMIZATION OF POLYMER ELEMENTS  
 BY VERASHEVAI PERUMPTI J.S. COULVIS  
 ADVISEE DR. D.S. BHATTACHARYYA

THIS PROGRAM MINIMIZES THE AREA "A" CORRESPONDING  
 TO THE TOTAL RESISTANCE "RT" OF THE DC ACTIVE FILM  
 UNDER PROPER DEPOSITION THIN FILM ON PROPER SUBSTRATE  
 I.E. THE SUBSTRATE COMPIRES THE TOTAL AREA INCLUDING  
 THAT OF THE ACTIVE COMPONENTS, GIVEN CONSTRAINTS FOR  
 SPECIFIED CUTOFF FREQUENCY "FC", POLE "P",  
 "Q" AND FINED BAND PASS GAIN "G" AS WELL AS,  
 ALL FIVE FACTOR RESISTIVITY "CI, C2, R1, R2, R3" IS

IN THIS ANALYSIS FLATCHER POWELL MINIMIZATION METHOD IS USED

MINIMIZATION FUNCTION F="A".....HERE  
 R1=12/31.1VDR/150.0R.011-C7=01.C7 (CM.300)  
 R2=110.0R7/751.0V-02.RY (CM.300)  
 VDR=DRIVING VOLTAGE (VOLTS)  
 C7=0.00010(-14) IS THE PERMITTIVITY OF THE PORE SPACE.  
 IFARADS/CM)  
 R3=0-DIELECTRIC CONSTANT RELATIVE TO THAT OF PORE SPACE.  
 C=0-DIELECTRIC STRENGTH OF THE THIN FILM (VOLTS/CM)  
 R=0-RESISTIVITY OF THE THIN FILM RESISTIVE LINE (OHMS/CM)  
 R1=0-RESISTANCE OF THE RESIST. LINE (OHMS/CM)  
 C1=TOTAL CAPACITANCE=C1+C2 OF THE FILTER (FARADS)  
 RY=TOTAL RESISTANCE=R1+R2+R3 OF THE FILTER (OHMS)

NOTE THE ABOVE OBJECTIVE FUNCTION IS A CONSTRAINED ONE.

INPUT P0.0P  
 VDR 00 00 00 00 00 00  
 100.0 0.00010 15.0 0.00007 0.00001 10.0

CONCORDIA UNIVERSITY

COMPUTER CENTRE

POOR COPY  
COPIE DE QUALITEE INFERIEURE







F3 Description of "PLOT" programs

In all, fifty-seven programs plot the transfer function  $T(s)$  of the proposed filter. Brief descriptions of the function of these programs are outlined in chapter six and in Table - 4 (program no. 49 to 105). Forty-eight of these programs use data which are the "optimized" filter passive components, per Table - 9 and -10. The rest use arbitrary data, but the ideal and practical operation of the OAs, in either unity-or non-unity-gain mode, are verified in the present filter configuration.

The operation of this group of programs is demonstrated by the aid of the flow chart, as shown in fig. F.8, which serves the same purpose for all three subgroups of the "plot" computer programs, depending on the mode of operation of the connected OAs, in the same filter configuration.

This purpose is, as outlined previously, to gather required information from the filter designer, as input data, to process this data in order to provide the required variables for calculation of the amplitude and phase responses, and finally to feed these variables and responses into the subroutine USPLH, condition the plot points, and have these manipulated data be printed as tables, and as curves.

The listing of some of these programs further describes each step of the computer-aided presentation of the filter transfer function.

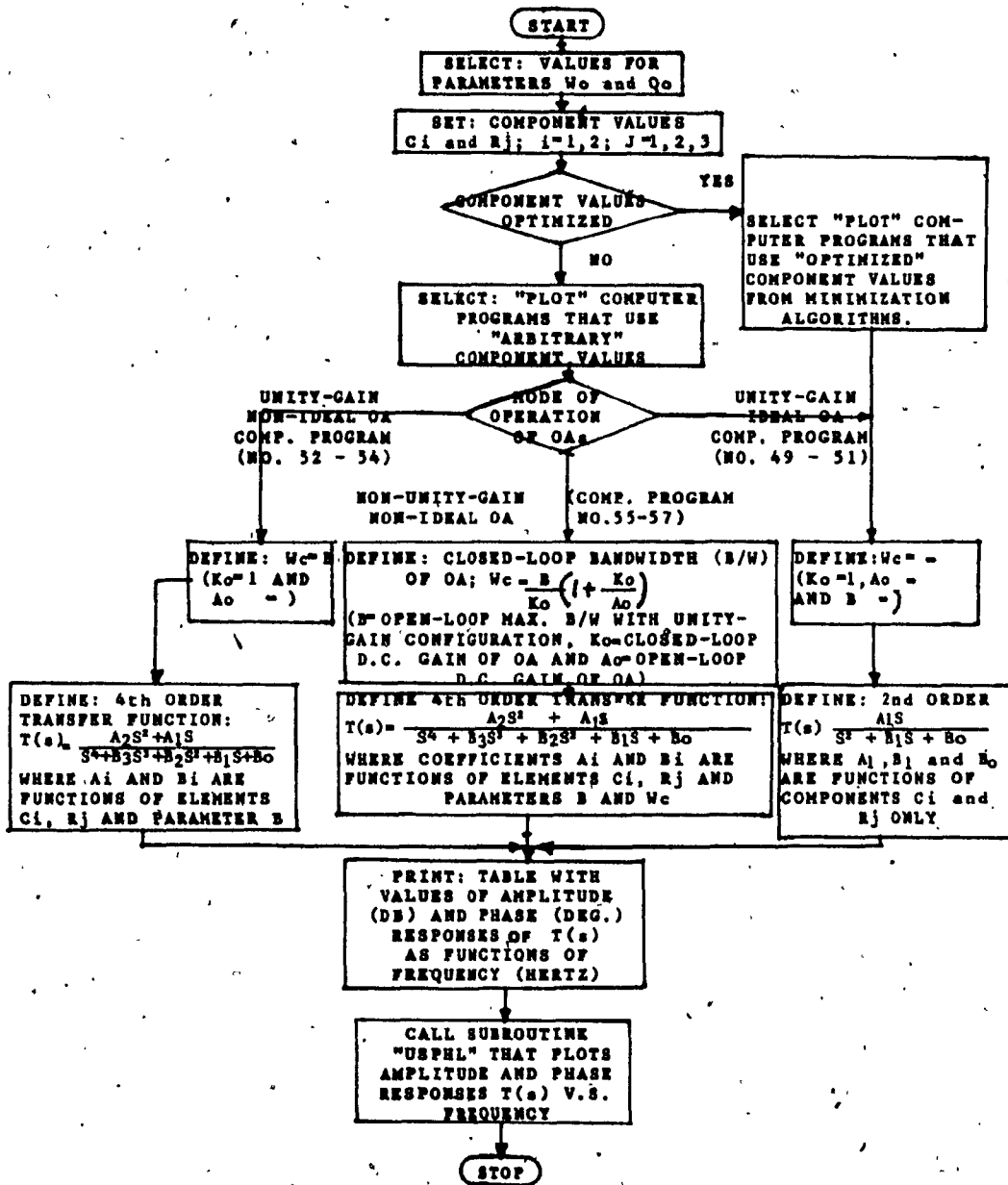


Fig P.8 - Organisational flow-chart of "PLOT" programs (computer program no. 49 - 105, per Table - 4)









PRG-	1316.000	RAC- IN RATIO-	.81226444-02	RAC- IN 00-	-.41884577-02	PRGSE IN 00-	-.80000000-02
PRG-	1320.000	RAC- IN RATIO-	.80402472-02	RAC- IN 00-	-.41894204-02	PRGSE IN 00-	-.80000000-02
PRG-	1324.000	RAC- IN RATIO-	.79600074-02	RAC- IN 00-	-.41901730-02	PRGSE IN 00-	-.80000000-02
PRG-	1328.000	RAC- IN RATIO-	.78813144-02	RAC- IN 00-	-.42000077-02	PRGSE IN 00-	-.80000000-02
PRG-	1332.000	RAC- IN RATIO-	.78043070-02	RAC- IN 00-	-.42133120-02	PRGSE IN 00-	-.80000000-02
PRG-	1336.000	RAC- IN RATIO-	.77299772-02	RAC- IN 00-	-.42270400-02	PRGSE IN 00-	-.80000000-02
PRG-	1340.000	RAC- IN RATIO-	.76581700-02	RAC- IN 00-	-.42370000-02	PRGSE IN 00-	-.80000000-02
PRG-	1344.000	RAC- IN RATIO-	.75880190-02	RAC- IN 00-	-.42433000-02	PRGSE IN 00-	-.80000000-02
PRG-	1348.000	RAC- IN RATIO-	.75199700-02	RAC- IN 00-	-.42460011-02	PRGSE IN 00-	-.80000000-02
PRG-	1352.000	RAC- IN RATIO-	.74535960-02	RAC- IN 00-	-.42455071-02	PRGSE IN 00-	-.80000000-02
PRG-	1356.000	RAC- IN RATIO-	.73885100-02	RAC- IN 00-	-.42415000-02	PRGSE IN 00-	-.80000000-02
PRG-	1360.000	RAC- IN RATIO-	.73245100-02	RAC- IN 00-	-.42340000-02	PRGSE IN 00-	-.80000000-02
PRG-	1364.000	RAC- IN RATIO-	.72615100-02	RAC- IN 00-	-.42230000-02	PRGSE IN 00-	-.80000000-02
PRG-	1368.000	RAC- IN RATIO-	.72000000-02	RAC- IN 00-	-.42090000-02	PRGSE IN 00-	-.80000000-02
PRG-	1372.000	RAC- IN RATIO-	.71395000-02	RAC- IN 00-	-.41920000-02	PRGSE IN 00-	-.80000000-02
PRG-	1376.000	RAC- IN RATIO-	.70800000-02	RAC- IN 00-	-.41720000-02	PRGSE IN 00-	-.80000000-02
PRG-	1380.000	RAC- IN RATIO-	.70215000-02	RAC- IN 00-	-.41500000-02	PRGSE IN 00-	-.80000000-02
PRG-	1384.000	RAC- IN RATIO-	.69640000-02	RAC- IN 00-	-.41260000-02	PRGSE IN 00-	-.80000000-02
PRG-	1388.000	RAC- IN RATIO-	.69075000-02	RAC- IN 00-	-.41000000-02	PRGSE IN 00-	-.80000000-02
PRG-	1392.000	RAC- IN RATIO-	.68520000-02	RAC- IN 00-	-.40720000-02	PRGSE IN 00-	-.80000000-02
PRG-	1396.000	RAC- IN RATIO-	.67975000-02	RAC- IN 00-	-.40420000-02	PRGSE IN 00-	-.80000000-02
PRG-	1400.000	RAC- IN RATIO-	.67440000-02	RAC- IN 00-	-.40100000-02	PRGSE IN 00-	-.80000000-02

CONCORDIA UNIVERSITY

COMPUTER CENTRE

PROGRAM PARTS	7/17/74	00701	PTN 4.0000	00/00/11-17-22-00	PAGE 1
1	PROGRAM PARTS (INPUT,OUTPUT) DIMENSION A(100),P(100),X(100),B(100),C(100),I(MAG(151)) DIMENSION V(120),I(1),V(120,1)				
5	COMPLEX TYPE PRINT 25 25 FORMAT(1X,10X)MAJOR TECHNICAL REPORT IN EL. ENGINEERING.1				
10	PRINT 26 26 FORMAT(1X,10X)COMMUNITY GAIN BAND PASS ACTIVE FILTER OPTIMIZATION OF PI ELEMENTS.//				
15	PRINT 27 27 FORMAT(1X,10X)PARASKEVAS PROPOSIS I.D. 000196.// PRINT 28 28 FORMAT(1X,10X)PROPOSIS D.G. 000196.// READ 3,1F0-10P 3 FORMAT(15,12)				
20	PRINT 5,1F0-10P 5 FORMAT(//,5X)THIS PROGRAM COMPUTES DEFLECTION TABLES AND PLOTS// ON X,Y COORDINATES THE AMPLITUDE AND// PHASE RESPONSE OF THE PRACTICAL BAND PASS ACTIVE FILTER WITH// ORDER AND TRANSFER CHARACTERISTIC IS 0P// ORDER SECOND DEGREE BUT THAT OF THE NON IDEAL OP AMP IS OF THE FOURTH ORDER// USING APPROXIMATED COMPONENT VALUES// ORDER CLOSED TO THE OPTIMIZED ONES,AS WERE CALCULATED BY THE PLETCHER ORDER OR PLETCHER MINIMIZATION ALGORITHM.// ORDER CASE PD=0,TS,IDENTITY AND 0P=0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,97,98,99,100,101,102,103,104,105,106,107,108,109,110,111,112,113,114,115,116,117,118,119,120,121,122,123,124,125,126,127,128,129,130,131,132,133,134,135,136,137,138,139,140,141,142,143,144,145,146,147,148,149,150,151,152,153,154,155,156,157,158,159,160,161,162,163,164,165,166,167,168,169,170,171,172,173,174,175,176,177,178,179,180,181,182,183,184,185,186,187,188,189,190,191,192,193,194,195,196,197,198,199,200,201,202,203,204,205,206,207,208,209,210,211,212,213,214,215,216,217,218,219,220,221,222,223,224,225,226,227,228,229,230,231,232,233,234,235,236,237,238,239,240,241,242,243,244,245,246,247,248,249,250,251,252,253,254,255,256,257,258,259,260,261,262,263,264,265,266,267,268,269,270,271,272,273,274,275,276,277,278,279,280,281,282,283,284,285,286,287,288,289,290,291,292,293,294,295,296,297,298,299,300,301,302,303,304,305,306,307,308,309,310,311,312,313,314,315,316,317,318,319,320,321,322,323,324,325,326,327,328,329,330,331,332,333,334,335,336,337,338,339,340,341,342,343,344,345,346,347,348,349,350,351,352,353,354,355,356,357,358,359,360,361,362,363,364,365,366,367,368,369,370,371,372,373,374,375,376,377,378,379,380,381,382,383,384,385,386,387,388,389,390,391,392,393,394,395,396,397,398,399,400,401,402,403,404,405,406,407,408,409,410,411,412,413,414,415,416,417,418,419,420,421,422,423,424,425,426,427,428,429,430,431,432,433,434,435,436,437,438,439,440,441,442,443,444,445,446,447,448,449,450,451,452,453,454,455,456,457,458,459,460,461,462,463,464,465,466,467,468,469,470,471,472,473,474,475,476,477,478,479,480,481,482,483,484,485,486,487,488,489,490,491,492,493,494,495,496,497,498,499,500,501,502,503,504,505,506,507,508,509,510,511,512,513,514,515,516,517,518,519,520,521,522,523,524,525,526,527,528,529,530,531,532,533,534,535,536,537,538,539,540,541,542,543,544,545,546,547,548,549,550,551,552,553,554,555,556,557,558,559,560,561,562,563,564,565,566,567,568,569,570,571,572,573,574,575,576,577,578,579,580,581,582,583,584,585,586,587,588,589,590,591,592,593,594,595,596,597,598,599,600,601,602,603,604,605,606,607,608,609,610,611,612,613,614,615,616,617,618,619,620,621,622,623,624,625,626,627,628,629,630,631,632,633,634,635,636,637,638,639,640,641,642,643,644,645,646,647,648,649,650,651,652,653,654,655,656,657,658,659,660,661,662,663,664,665,666,667,668,669,670,671,672,673,674,675,676,677,678,679,680,681,682,683,684,685,686,687,688,689,690,691,692,693,694,695,696,697,698,699,700,701,702,703,704,705,706,707,708,709,710,711,712,713,714,715,716,717,718,719,720,721,722,723,724,725,726,727,728,729,730,731,732,733,734,735,736,737,738,739,740,741,742,743,744,745,746,747,748,749,750,751,752,753,754,755,756,757,758,759,760,761,762,763,764,765,766,767,768,769,770,771,772,773,774,775,776,777,778,779,780,781,782,783,784,785,786,787,788,789,790,791,792,793,794,795,796,797,798,799,800,801,802,803,804,805,806,807,808,809,810,811,812,813,814,815,816,817,818,819,820,821,822,823,824,825,826,827,828,829,830,831,832,833,834,835,836,837,838,839,840,841,842,843,844,845,846,847,848,849,850,851,852,853,854,855,856,857,858,859,860,861,862,863,864,865,866,867,868,869,870,871,872,873,874,875,876,877,878,879,880,881,882,883,884,885,886,887,888,889,890,891,892,893,894,895,896,897,898,899,900,901,902,903,904,905,906,907,908,909,910,911,912,913,914,915,916,917,918,919,920,921,922,923,924,925,926,927,928,929,930,931,932,933,934,935,936,937,938,939,940,941,942,943,944,945,946,947,948,949,950,951,952,953,954,955,956,957,958,959,960,961,962,963,964,965,966,967,968,969,970,971,972,973,974,975,976,977,978,979,980,981,982,983,984,985,986,987,988,989,990,991,992,993,994,995,996,997,998,999,1000,1001,1002,1003,1004,1005,1006,1007,1008,1009,1010,1011,1012,1013,1014,1015,1016,1017,1018,1019,1020,1021,1022,1023,1024,1025,1026,1027,1028,1029,1030,1031,1032,1033,1034,1035,1036,1037,1038,1039,1040,1041,1042,1043,1044,1045,1046,1047,1048,1049,1050,1051,1052,1053,1054,1055,1056,1057,1058,1059,1060,1061,1062,1063,1064,1065,1066,1067,1068,1069,1070,1071,1072,1073,1074,1075,1076,1077,1078,1079,1080,1081,1082,1083,1084,1085,1086,1087,1088,1089,1090,1091,1092,1093,1094,1095,1096,1097,1098,1099,1100,1101,1102,1103,1104,1105,1106,1107,1108,1109,1110,1111,1112,1113,1114,1115,1116,1117,1118,1119,1120,1121,1122,1123,1124,1125,1126,1127,1128,1129,1130,1131,1132,1133,1134,1135,1136,1137,1138,1139,1140,1141,1142,1143,1144,1145,1146,1147,1148,1149,1150,1151,1152,1153,1154,1155,1156,1157,1158,1159,1160,1161,1162,1163,1164,1165,1166,1167,1168,1169,1170,1171,1172,1173,1174,1175,1176,1177,1178,1179,1180,1181,1182,1183,1184,1185,1186,1187,1188,1189,1190,1191,1192,1193,1194,1195,1196,1197,1198,1199,1200,1201,1202,1203,1204,1205,1206,1207,1208,1209,1210,1211,1212,1213,1214,1215,1216,1217,1218,1219,1220,1221,1222,1223,1224,1225,1226,1227,1228,1229,1230,1231,1232,1233,1234,1235,1236,1237,1238,1239,1240,1241,1242,1243,1244,1245,1246,1247,1248,1249,1250,1251,1252,1253,1254,1255,1256,1257,1258,1259,1260,1261,1262,1263,1264,1265,1266,1267,1268,1269,1270,1271,1272,1273,1274,1275,1276,1277,1278,1279,1280,1281,1282,1283,1284,1285,1286,1287,1288,1289,1290,1291,1292,1293,1294,1295,1296,1297,1298,1299,1300,1301,1302,1303,1304,1305,1306,1307,1308,1309,1310,1311,1312,1313,1314,1315,1316,1317,1318,1319,1320,1321,1322,1323,1324,1325,1326,1327,1328,1329,1330,1331,1332,1333,1334,1335,1336,1337,1338,1339,1340,1341,1342,1343,1344,1345,1346,1347,1348,1349,1350,1351,1352,1353,1354,1355,1356,1357,1358,1359,1360,1361,1362,1363,1364,1365,1366,1367,1368,1369,1370,1371,1372,1373,1374,1375,1376,1377,1378,1379,1380,1381,1382,1383,1384,1385,1386,1387,1388,1389,1390,1391,1392,1393,1394,1395,1396,1397,1398,1399,1400,1401,1402,1403,1404,1405,1406,1407,1408,1409,1410,1411,1412,1413,1414,1415,1416,1417,1418,1419,1420,1421,1422,1423,1424,1425,1426,1427,1428,1429,1430,1431,1432,1433,1434,1435,1436,1437,1438,1439,1440,1441,1442,1443,1444,1445,1446,1447,1448,1449,1450,1451,1452,1453,1454,1455,1456,1457,1458,1459,1460,1461,1462,1463,1464,1465,1466,1467,1468,1469,1470,1471,1472,1473,1474,1475,1476,1477,1478,1479,1480,1481,1482,1483,1484,1485,1486,1487,1488,1489,1490,1491,1492,1493,1494,1495,1496,1497,1498,1499,1500,1501,1502,1503,1504,1505,1506,1507,1508,1509,1510,1511,1512,1513,1514,1515,1516,1517,1518,1519,1520,1521,1522,1523,1524,1525,1526,1527,1528,1529,1530,1531,1532,1533,1534,1535,1536,1537,1538,1539,1540,1541,1542,1543,1544,1545,1546,1547,1548,1549,1550,1551,1552,1553,1554,1555,1556,1557,1558,1559,1560,1561,1562,1563,1564,1565,1566,1567,1568,1569,1570,1571,1572,1573,1574,1575,1576,1577,1578,1579,1580,1581,1582,1583,1584,1585,1586,1587,1588,1589,1590,1591,1592,1593,1594,1595,1596,1597,1598,1599,1600,1601,1602,1603,1604,1605,1606,1607,1608,1609,1610,1611,1612,1613,1614,1615,1616,1617,1618,1619,1620,1621,1622,1623,1624,1625,1626,1627,1628,1629,1630,1631,1632,1633,1634,1635,1636,1637,1638,1639,1640,1641,1642,1643,1644,1645,1646,1647,1648,1649,1650,1651,1652,1653,1654,1655,1656,1657,1658,1659,1660,1661,1662,1663,1664,1665,1666,1667,1668,1669,1670,1671,1672,1673,1674,1675,1676,1677,1678,1679,1680,1681,1682,1683,1684,1685,1686,1687,1688,1689,1690,1691,1692,1693,1694,1695,1696,1697,1698,1699,1700,1701,1702,1703,1704,1705,1706,1707,1708,1709,1710,1711,1712,1713,1714,1715,1716,1717,1718,1719,1720,1721,1722,1723,1724,1725,1726,1727,1728,1729,1730,1731,1732,1733,1734,1735,1736,1737,1738,1739,1740,1741,1742,1743,1744,1745,1746,1747,1748,1749,1750,1751,1752,1753,1754,1755,1756,1757,1758,1759,1760,1761,1762,1763,1764,1765,1766,1767,1768,1769,1770,1771,1772,1773,1774,1775,1776,1777,1778,1779,1780,1781,1782,1783,1784,1785,1786,1787,1788,1789,1790,1791,1792,1793,1794,1795,1796,1797,1798,1799,1800,1801,1802,1803,1804,1805,1806,1807,1808,1809,1810,1811,1812,1813,1814,1815,1816,1817,1818,1819,1820,1821,1822,1823,1824,1825,1826,1827,1828,1829,1830,1831,1832,1833,1834,1835,1836,1837,1838,1839,1840,1841,1842,1843,1844,1845,1846,1847,1848,1849,1850,1851,1852,1853,1854,1855,1856,1857,1858,1859,1860,1861,1862,1863,1864,1865,1866,1867,1868,1869,1870,1871,1872,1873,1874,1875,1876,1877,1878,1879,1880,1881,1882,1883,1884,1885,1886,1887,1888,1889,1890,1891,1892,1893,1894,1895,1896,1897,1898,1899,1900,1901,1902,1903,1904,1905,1906,1907,1908,1909,1910,1911,1912,1913,1914,1915,1916,1917,1918,1919,1920,1921,1922,1923,1924,1925,1926,1927,1928,1929,1930,1931,1932,1933,1934,1935,1936,1937,1938,1939,1940,1941,1942,1943,1944,1945,1946,1947,1948,1949,1950,1951,1952,1953,1954,1955,1956,1957,1958,1959,1960,1961,1962,1963,1964,1965,1966,1967,1968,1969,1970,1971,1972,1973,1974,1975,1976,1977,1978,1979,1980,1981,1982,1983,1984,1985,1986,1987,1988,1989,1990,1991,1992,1993,1994,1995,1996,1997,1998,1999,2000,2001,2002,2003,2004,2005,2006,2007,2008,2009,2010,2011,2012,2013,2014,2015,2016,2017,2018,2019,2020,2021,2022,2023,2024,2025,2026,2027,2028,2029,2030,2031,2032,2033,2034,2035,2036,2037,2038,2039,2040,2041,2042,2043,2044,2045,2046,2047,2048,2049,2050,2051,2052,2053,2054,2055,2056,2057,2058,2059,2060,2061,2062,2063,2064,2065,2066,2067,2068,2069,2070,2071,2072,2073,2074,2075,2076,2077,2078,2079,2080,2081,2082,2083,2084,2085,2086,2087,2088,2089,2090,2091,2092,2093,2094,2095,2096,2097,2098,2099,2100,2101,2102,2103,2104,2105,2106,2107,2108,2109,2110,2111,2112,2113,2114,2115,2116,2117,2118,2119,2120,2121,2122,2123,2124,2125,2126,2127,2128,2129,2130,2131,2132,2133,2134,2135,2136,2137,2138,2139,2140,2141,2142,2143,2144,2145,2146,2147,2148,2149,2150,2151,2152,2153,2154,2155,2156,2157,2158,2159,2160,2161,2162,2163,2164,2165,2166,2167,2168,2169,2170,2171,2172,2173,2174,2175,2176,2177,2178,2179,2180,2181,2182,2183,2184,2185,2186,2187,2188,2189,2190,2191,2192,2193,2194,2195,2196,2197,2198,2199,2200,2201,2202,2203,2204,2205,2206,2207,2208,2209,2210,2211,2212,2213,2214,2215,2216,2217,2218,2219,2220,2221,2222,2223,2224,2225,2226,2227,2228,2229,2230,2231,2232,2233,2234,2235,2236,2237,2238,2239,2240,2241,2242,2243,2244,2245,2246,2247,2248,2249,2250,2251,2252,2253,2254,2255,2256,2257,2258,2259,2260,2261,2262,2263,2264,2265,2266,2267,2268,2269,2270,2271,2272,2273,2274,2275,2276,2277,2278,2279,2280,2281,2282,2283,2284,2285,2286,2287,2288,2289,2290,2291,2292,2293,2294,2295,2296,2297,2298,2299,2300,2301,2302,2303,2304,2305,2306,2307,2308,2309,2310,2311,2312,2313,2314,2315,2316,2317,2318,2319,2320,2321,2322,2323,2324,2325,2326,2327,2328,2329,2330,2331,2332,2333,2334,2335,2336,2337,2338,2339,2340,2341,2342,2343,2344,2345,2346,2347,2348,2349,2350,2351,2352,2353,2354,2355,2356,2357,2358,2359,2360,2361,2362,2363,2364,2365,2366,2367,2368,2369,2370,2371,2372,2373,2374,2375,2376,2377,2378,2379,2380,2381,2382,2383,2384,2385,2386,2387,2388,2389,2390,2391,2392,2393,2394,2395,2396,2397,2398,2399,2400,2401,2402,2403,2404,2405,2406,2407,2408,2409,2410,2411,2412,2413,2414,2415,2416,2417,2418,2419,2420,2421,2422,2423,2424,2425,2426,2427,2428,2429,2430,2431,2432,2433,2434,2435,2436,2437,2438,2439,2440,2441,2442,2443,2444,2445,2446,2447,2448,2449,2450,2451,2452,2453,2454,2455,2456,2457,2458,2459,2460,2461,2462,2463,2464,2465,2466,2467,2468,2469,2470,2471,2472,2473,2474,2475,2476,2477,2478,2479,2480,2481,2482,2483,2484,2485,2486,2487,2488,2489,2490,2491,2492,2493,2494,2495,2496,2497,2498,2499,2500,2501,2502,2503,2504,2505,2506,2507,2508,2509,2510,2511,2512,2513,2514,2515,2516,2517,2518,2519,2520,2521,2522,2523,2524,2525,2526,2527,2528,2529,2530,2531,2532,2533,2534,2535,2536,2537,2538,2539,2540,2541,2542,2543,2544,2545,2546,2547,2548,2549,2550,2551,2552,2553,2554,2555,2556,2557,2558,2559,2560,2				



