

COOPERATION BETWEEN ADOLESCENTS
IN COMPUTER-ASSISTED
ALGEBRAIC PROBLEM SOLVING

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ABSTRACT

COOPERATION BETWEEN ADOLESCENTS IN COMPUTER-ASSISTED ALGEBRAIC PROBLEM SOLVING

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The purpose of this study was to compare the achievement of individuals and two types of cooperating pairs on a computer-assisted problem-solving task. The first set of pairs worked together but were evaluated separately. The second set of pairs was subjected to a low-performance contingency; they were told beforehand that their final performance would be judged according to the lowest mark obtained by either member on the post-test.

Subjects consisted of 51 English speaking grade 10 mathematics students at Lemoyne D'Iberville High School in Longueuil, Quebec. Three treatment groups consisting of 12 to 18 subjects were used. All three groups were given one hour to solve a problem using a specially designed computer program as an aid. This was followed by two post-tests, one administered immediately and another two weeks later. An analysis of variance for repeated measures revealed no significant differences across treatment groups or over time. It was concluded that cooperating pairs were to be recommended from a cost-effectiveness standpoint and that a follow-up study, relieved of the constraints and

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resource limitations affecting the original, might possibly
detect an advantage in terms of performance as well.

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Chapter 1

Introduction

There is no doubt that the computer can play an important role in student learning.

Numerous studies have shown that CAI can be tremendously effective, especially with the slower groups. (Doerr, 1979; p. 122)

The computer, therefore, offers a dynamic new tool for developing a range of cognitive skills and for helping learners develop useful learning strategies. (Caldwell, 1980a; p. 142)

The potential of educational computing is, of course, far greater than just the automation of programmed instruction ... (Rushby 1979; p. 19)

Indeed, given the potential of the computer as an educational resource, we might wonder why computer assisted

learning (CAL) is taking so long to infiltrate our educational institutions. As late as April 1981, the use of CAL in the public schools of the Province of Quebec was virtually non-existent (DICOS, Note 1).

A number of factors account for this state of affairs. Most striking for those who have computers is the shortage of high quality software (Caldwell, 1980b; Shaw, 1981; Rushby, 1979). The production of such software is difficult and time consuming and, unfortunately, those who have the required skills often work in institutions which do not value this activity as much as other, usually more intellectual, ones (Rushby, 1979). Furthermore, those who do produce educational software often limit themselves to 'drill and practice' or tutorial programs. Though useful, these instructional forms of CAL, often referred to as CAI, are limited. For example, the student cannot ask questions or request explanations unless they have been anticipated by the author (Hartley and Lovell, 1977). Luehrmann (1980) believes that CAI is not likely to succeed within the school system because it offers to do what is already being done by teachers. What is needed is software which will involve teachers rather than bypass them (Olds, 1981).

More powerful forms of CAL do exist; Shaw (1981) divides these less common forms into three categories: 1) simulations, 2)

construction and/or exploration, evaluation and modification of models, and 3) academic games. These fall roughly into Boyd's (1982) category of 'auto-elaborative' CAL, a prime example of which is Papert's (1980) Turtle/Logo computer system. The single most important characteristic of these kinds of CAL is the relatively high degree of control which they offer the learner. Caldwell (1980a) states that 'drill and practice' as well as tutorial programs are useful but that they attend to only one form of learning strategy, rote memorization. He suggests that the computer should be used to encourage individual thought. It is interesting to compare this with a statement made by Polya (1973).

Thus, a teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students in routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking. (p. V)

Caldwell might have us apply this statement to the teaching of other subjects as well. In any case, it is evident that the application of such a philosophy to CAL implies increased learner control. Mager (1961) suggests that motivation is a function of the amount of control the learner is allowed to exercise over the learning experience. Thus, the element of control might provide the student with a more interesting as well as a more potent learning experience.

The increased availability of valuable software would certainly result in the greater use of computers as educational tools. Apart from the scarcity of quality software, however, at least two factors have influenced the introduction of CAL into our schools. These are cost and the introduction of new courses to the high school curriculum. Hardware and software-related costs are obvious and unavoidable and, in most cases, cannot be compensated for by higher student/teacher ratios as suggested by Doerr (1979) since these are generally written into union contracts (Young, 1981). Hardware costs remain considerable even though they have decreased dramatically over the last decade. The cost of furnishing one room with a sufficient number of machines to accommodate a class of students can easily run over \$100 000 not including the cost of maintenance or the purchase of software. In spite of the cost, many schools are buying small numbers of relatively inexpensive microcomputers to be used

primarily by students taking courses in computer science and data processing (Chevalier, 1976; DICOS, Note 1; Bip Bip, Note 2). Whether these represent a more valuable use of the computer than CAL is debatable; the fact remains that in the foreseeable future CAL will be a secondary use of computers in the high schools of this province at least.

Given this situation and assuming that educators are committed to some form of CAL, it is important that they make effective use of the resources available to them. One interesting and partial solution to this problem is small-group CAL; the use of a single program at a single terminal by more than one learner. Careful consideration of this solution raises some important questions (Cartwright, 1973; Okey and Major, 1975):

- 1) Will student learning suffer as a consequence?
- 2) Will group work have any positive effects on the learning experience?
- 3) Are some kinds of CAL better suited to group use than others?
- 4) Are special provisions required to encourage cooperation within the group?
- 5) How large can such groups be?

6) Should groups be formed according to specific learner characteristics?

The purpose of this study was to seek answers to some of these questions in order to help determine whether group CAL is a desirable alternative to individual CAL.

Chapter 2

Review of Literature

Most introductory literature on CAL points to the following advantages: instruction can be tailored to the individual, instruction can take place at the individual's own pace, the individual can be protected from criticism or ridicule by peers. It is implied that these factors influence the quality and the quantity of learning in a positive way. However, in a number of studies (Love, 1969; Karweit and Livingston, 1969; Cartwright, 1972; Okey and Major, 1975) the researchers compared small group computer-assisted instruction (CAI) with individual CAI and found little, if any, evidence to show that individual CAI is preferable to group CAI in any way. On the contrary, the great gains in cost-effectiveness seem to favor group CAI.

Love (1969) investigated pairs as a possible technique for improving instructional achievement as well as efficiency. Senior high school students took five CAI lessons in Boolean algebra. Some students worked in pairs while others worked individually. The results on a final examination taken individually by all the students showed no significant difference between those who had worked in pairs and those who had worked

alone.

Karweit and Livingston (1969) had high-ability grade six students play a computer game individually and in teams of two or three segregated by sex. A test designed to measure learning from the game failed to detect a significant difference.

Okey and Major (1975) studied college students working individually, in pairs, and in groups of three or four on a Plato IV instructional module dealing with Bloom's mastery learning strategy. Following instruction, all students completed a criterion test and a questionnaire designed to test their attitudes toward the content of the instructional module. No significant differences were found in either achievement or attitude. There were, however, important differences in time of study with larger groups taking the least amount of time and pairs taking the most. It was observed that pairs had more frequent discussions thus accounting for their higher study time.

Cartwright (1972) reported similar findings for college students working individually, in pairs, and in groups of three or four. During a summer course 282 college students took three CAI lessons (each one week apart) in general psychology. During the fourth week students responded individually to a criterion test; no significant differences in achievement were found. The

author later took certain personality measures into consideration and found that they were unrelated to performance on the test (Cartwright, 1973).

One explanation for this pattern of no significant differences is offered by Cartwright (1973). He argues that CAI provides individual rather than individualized instruction, the latter implying that personal characteristics such as age, intelligence, personality, verbal ability, socio-economic status, and experiential background have been taken into consideration. Secondly, he suggests that even individualized instruction need not be presented individually and that a certain degree of individualization is maintained in group CAI through differential interpretation of feedback (Cartwright, 1976). Even though a group response is given to the computer, the student will personalize the feedback by comparing it to his/her own response whether it differs from the group response or not.

Even though researchers interested in group CAI did not find the increased cognitive gains they were looking for, their other findings have important implications in terms of cost-effectiveness. All of the authors report reductions in cost ranging from 50% (Love, 1969) to 75% (Cartwright, 1972). This implies that, given limited computer facilities, more students can be accommodated without any detrimental effects on

achievement and, possibly, with some beneficial effects. Cartwright suggests that group CAI may be less dehumanizing than conventional CAI, a point of view which is shared by Boyd (1982).

Unfortunately, past research on group CAL is of little help when cost-effectiveness is not a problem and we are faced with a situation such as having three computers for three students. Do we have them work alone or should we encourage them to work together? A number of studies and much of the literature on cooperation suggests that groups may be preferable especially where problem solving is involved.

A study by Dick (1963) suggests that group PI may result in better long term retention than individual PI. Dick had 70 college students work through a 3500-frame algebra program. Half the students worked individually and the others worked in pairs. Results on the final examination showed ~~no~~ significant difference. However, retesting of 80% of the subjects, one year later, indicated significantly better retention by the paired group. Dick postulated that the difference may have been a consequence of the discussion which took place between the students who worked together.

The opportunity to communicate, even in a limited way, may explain the effectiveness of groups in dealing with certain

situations. Durling and Schick (1976) became interested in studies which indicated that group work might be preferable when problem solving was involved. They succeeded in demonstrating that vocalization, and not the number of persons, was the more important factor influencing success in problem-solving. They found that problem-solving efficiency was greatest for individuals vocalizing with a partner or to a person supposedly learning the task than for individuals working with a partner when vocalization was denied.

Another potential advantage of group work may lie in the opportunity to cooperate. Johnson and Johnson (1975) have written an extensive review of the literature pertaining to cooperative, competitive, and individual goal structures. They state that whenever problem solving is desired a cooperative goal structure should be used. Of course putting students to work in groups does not necessarily imply cooperation; given their previous experience, North American students are just as likely to compete especially if they are to be evaluated on an individual basis. Cooperation can only be ensured by carefully structuring the learning experience.

Johnson and Johnson state that "a cooperative goal structure exists when students perceive that they can obtain their goal if, and only if, the other students with whom they are linked obtain

their goal", (p. 7). One method of promoting such a mutuality of goals, and therefore cooperation, is the use of a group contingency. For example, reinforcement in the form of a reward might be given on the basis of the average performance of the group or it may be based on the lowest or highest scores of part of the group. These are called low, average and high-performance group contingencies. Research indicates that the most powerful of these is the low performance group contingency (Johnson and Johnson, 1975) because it motivates the more gifted students to help the slower students in their group.

Such a strategy was employed by Wodarski, Hamblin, Buckholdt and Ferritor (1973) in a study comparing the effects of individual consequences with different shared consequences on three grade five arithmetic classes. They found that as the proportion of shared consequences increased, so did the incidence of peer tutoring and the amount of learning. Better students benefited less than poorer students although they were not hurt by the experience while the slower students were helped substantially.

Johnson and Johnson fail to discuss the disadvantages of group work. A more balanced view of group problem-solving is described by Freedman, Carlsmith, and Sears (1970).

To sum up, it seems that under most circumstances, groups are less efficient than individuals working alone. Group members distract, inhibit, and generally tend to interfere with one another. Groups do provide a means of catching errors, and on certain types of problems, this might overcome their relative inefficiency. Also, when differing skills are needed for a solution, groups have a big advantage. Finally, as in minimal social situations, group members tend to motivate each other to work harder, and they probably do this to an even greater extent than in the minimal situations. If other incentives have not already produced a sufficiently high motivational level, this would be an advantage of working in groups. (p. 193)

Thus, for certain tasks, the advantages of working in groups may outweigh the disadvantages. This, however, may only become apparent once individuals have learned to work together. The authors, like Johnson and Johnson, recommend heterogeneous groups: "Overall results indicated that heterogeneous groups are superior to homogeneous groups on a wide variety of problems" (p.

190). As for group size, Johnson and Johnson make the following statement:

In problem-solving activities, a group size that maximizes each member's participation is preferable, and this may mean using pairs or triads for tasks that require high involvement and are not easily broken down into a division of labor. On the other hand, bringing the whole class together to share ideas and results is sometimes necessary. No rule of thumb is possible in deciding what size group to utilize; the teacher simply has to experiment to determine what size works best for a particular purpose. (p. 90)

If (as suggested by Johnson and Johnson) a 'cooperative goal structure' is more effective for problem-solving, one might expect this to be reflected in the application of a problem-solving strategy. Success in problem-solving depends very much on the strategy which is used and the efficiency with which it is applied. When the problem consists of the attainment of a conjunctive concept and the problem solver controls the situation a selection strategy is in order. Bruner, Goodnow, and Austin (1956) describe four ideal selection strategies along with

their advantages and disadvantages. Given a problem, the application of any of these strategies will produce a solution albeit at the expense of varying amounts of time and cognitive strain. For instance, the application of 'simultaneous scanning' guarantees maximum informativeness yet makes such cognitive demands as to be totally impractical. On the other hand, 'successive scanning' results in very little cognitive strain but is terribly inefficient. The 'conservative focussing' strategy offers a compromise; it consists of the selection of one example to be used as a focus and is followed by a series of choices each of which alters a single attribute of the example. The result is a strategy which guarantees the informativeness of each instance (not maximum informativeness) while making reasonable cognitive demands. "It is always possible, given the use of conservative focussing, to complete the job with only as many tests as there are attributes to be tested" (p. 89). Whether groups are more successful than individuals in the application of such a strategy is still an open question.

Chapter 3Goals and Objectives

All of the research on group CAL, except for the study by Karweit and Livingston, has been limited to tutorial CAI. The question remains whether similar results can be expected for non-instructional forms of CAL. The principal goal of this study was to determine whether pairs perform better than individuals on a computer-assisted problem-solving task involving the exploration of a mathematical model. Even though no such differences in performance had been found by previous studies it is also true that none of these studies except Cartwright's had looked for effects on long term retention, but even more important is the fact that this study would be using a non-instructional type of CAL in the form of a problem-solving exercise. Gagne (1977) states that "the evidence strongly suggests that achieving a higher-order rule by means of problem solving produces a highly effective capability that is well retained over time" (p. 164). This, in combination with the studies by Durling and Schick (1976) and Wodarski et al. (1973), led the author to believe that pairs (especially those under a low-performance contingency) might outperform individuals on a criterion test following a problem-solving exercise. This belief

was further supported by the literature on cooperation (Johnson and Johnson, 1975; Yelon and Weinstein, 1977; Freedman et al, 1970).

The secondary goal of this study was to determine the extent to which the 'conservative focussing' strategy was used by students and how this was related to success on the problem-solving task. Part of the problem was the definition of a method of quantification which would allow such comparisons to be made. It was hoped that the resulting information might be useful in determining what kind of guidance to offer less successful students in order to improve their performance.

Hypotheses

- 1) Achievement on a criterion test following a computer-assisted problem-solving exercise will be greater for students working in pairs than for students working individually.
- 2) Achievement on a criterion test following a computer-assisted problem-solving exercise will be greatest for students working in pairs under a low performance contingency.

3) The retention of concepts learned during a computer-assisted problem-solving exercise will be greater for students working in pairs than for students working individually.

4) The retention of concepts learned during a computer-assisted problem-solving exercise will be greatest for students working in pairs under a low performance contingency.

Operational Definitions

Achievement - the score obtained on a criterion test immediately following the exercise.

Retention - the score obtained on a criterion test one week after the exercise.

Low performance contingency - a system of evaluation in which paired students both receive the lowest mark obtained by either student.

Variables

The independent variable for this study consisted of three levels of cooperation on a problem-solving exercise:

- 1) Individuals - were denied any form of cooperation.
- 2) Pairs - worked together at a single terminal and were free to cooperate as they wished. They were told that evaluation would take place on an individual basis at the end of the exercise.
- 3) Pairs under a low performance contingency - worked together at a single terminal. They were told that testing would take place on an individual basis but that each member would receive a mark equal to the lowest obtained by either member of the pair.

There was one dependent variable which was measured on two occasions: the score obtained on a criterion test administered immediately after the exercise (achievement) and that obtained on a different but equivalent test administered two weeks after the exercise (retention).

Chapter 4MethodSubjects

The subjects for this study consisted of fifty-one English-speaking students (2 classes) from Lemoyne D'Iberville High School in Longueuil, Quebec. The students (ages 15-17) were enrolled in the regular Secondary IV (grade 10) mathematics course.

Research Design

This experiment used a two-way repeated measures design as illustrated below.

| | | | |
|---|----------------|----------------|----------------|
| R | X ₁ | O ₁ | O ₄ |
| R | X ₂ | O ₂ | O ₅ |
| R | X ₃ | O ₃ | O ₆ |

Subjects were randomly assigned (R) to one of three

experimental groups. Each group interacted with the same CAL module (a model of the general quadratic function) for one hour; however, this interaction took place on an individual basis (X_1) for the first experimental group, in pairs (X_2) for the second group, and also in pairs but under a low-performance contingency (X_3) for the third group. Immediately following this activity, a post-test (O_1, O_2, O_3) was administered to each of the subjects in order to measure their understanding of, as well as their ability to generalize, the concepts which were to be learned as a result of the exercise. A parallel version of the same test (O_4, O_5, O_6) was administered two weeks later in order to detect possible differences in retention by the three experimental groups. The immediate and delayed post-tests were counterbalanced in order to increase their reliability by eliminating differences which might have occurred had the two forms been used respectively as post-test and delayed post-test. This had the added advantage of making it more difficult for students to pass answers to their peers. Rejection of the null hypotheses would be based on the 0.05 level of statistical significance.

In order to ensure that subjects concentrate on the problem solving activity and not on the novelty of the situation, each student was made to interact with a CAL module on linear functions a few days before the experiment was conducted.

Materials

An important task in this study was the selection of a problem and the design of a program to assist in its solution. Because of the author's background, a problem in mathematics was favoured however, it was important that the problem meet certain criteria: it should be relevant to the students' course of studies, and the computer should offer some advantages over other methods of solution. The intention was to produce a program which would be useful to students and teachers of high school mathematics as well as to this study.

The problem consists of the exploration of a mathematical model. It is at the senior high school level that students are introduced to the notion of a general function. Thus, functions such as $y = x^2 + 2$ and $y = 3(x + 1)^2 - 5$ become special cases of the general quadratic function $y = a(x - b)^2 + c$ where a , b , and c are the parameters. The advantage of the general quadratic function is that it represents every one of the infinity of quadratic functions which exist, and any knowledge concerning it also applies to each of the special cases which it represents. Furthermore, the knowledge gained here can easily be applied to other general functions where the parameters will play similar, if not identical, roles.

The empirical method is often used when studying a general function for the first time. After having graphed a number of functions of the type $y = a(x - b)^2 + c$, the student quickly recognises the characteristic shape of the parabola. It is immediately obvious that these functions have much in common and that their prime distinguishing feature is position in the Cartesian plane. At this point attention may be drawn to the parameters and the fact that they are undoubtedly responsible for this variation in position. The student may then be presented with the problem of determining how each of these parameters affects the graph of the function. This may be done by conducting simple experiments such as graphing the three functions $y = 1x^2$, $y = 2x^2$, and $y = 3x^2$ on the same set of axes. Since parameters 'b' and 'c' have been set to zero for all three functions it is easy to identify the effect of variations in the value of 'a'. This result might be confirmed by further experimentation and finally generalized. Similar experiments will reveal the role which is played by 'b' and 'c'.

The shortcoming of this method is that it requires the drawing of many graphs.

In secondary schools examination of models is often limited to qualitative discussion or a

simple quantitative treatment involving a few sample calculations. This is understandable because the calculations are often complex, and to perform sufficient of them to explore the model in any detail would be tedious and time-consuming. A computer program can enable the quantitative exploration of such models to be carried out rapidly and in considerable detail. Furthermore, the student is released from the chore of carrying out a large number of complicated calculations, an activity which can easily obscure the topic under investigation. (Shaw, 1981; p. 182)

Such a program was designed to assist students in their study of the general quadratic function.

The Program

The program (appendix G), which was written in AppleSoft Basic for the Apple II microcomputer, is a tool which can be used by students to explore the general quadratic function. Its

purpose was to release students from the time-consuming task of drawing a large numbers of graphs by hand thus allowing them to concentrate on the problem. Using this program, many students were able to study 50 to 80 different functions in just one hour, something which no student could ever do, nor would want to do, by hand.

The original program was evaluated and revised twice over a period of a few months. The result is a product which responds well to the needs of the user. It is composed of two main parts: part one allows the student to define up to three functions by specifying the value of each parameter in each function (fig. 1), part two illustrates the functions graphically on a single set of axes and identifies them (fig. 2). Both of these procedures are set in a loop which may be repeated as often as desired by the user (fig. 3). A few suggestions are made concerning strategy, these being introduced periodically as the student uses the program (refer to lines 710 - 714 in appendix G).

Figure 1

Definition of Functions to be Graphed

DEFINE FUNCTIONS

$$Y = A(X - B)^2 + C$$

CHOOSE ANY INTEGRAL VALUE FOR 'C'
FROM -48 ... TO +48 ?

FUNCTION 1 $Y = 2(X - 9)^2 + 5$

FUNCTION 2 $Y = 2X^2 + 5$

FUNCTION 3 $Y = 2(X + 9)^2$

Figure 2

Final Graphs

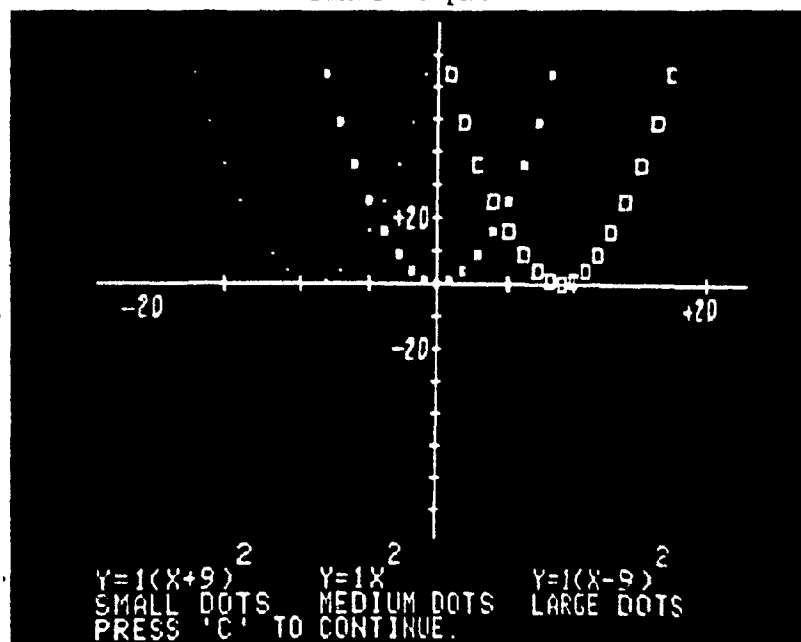
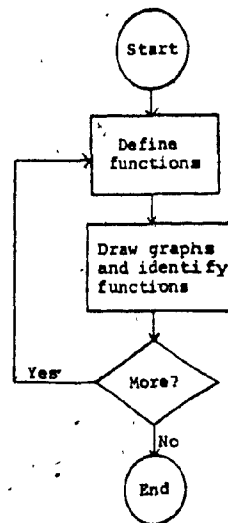


Figure 3
Basic Program Structure



The Lesson

In order for the problem to make sense it had to be put into context. The first step consisted of an assignment (appendix A) requiring students to draw the graphs of three quadratic functions. These graphs (appendix B) served as the basis for a class discussion in which the general quadratic function was introduced. During this discussion students recognized that each function was represented by a parabola, that this was probably due to the fact that in each case y was expressed as a second degree expression in x , and that any differences were likely accounted for by the various constants which appeared in each

function. It was at this point that the concept of a general function was introduced. Students were shown how each of these functions could be considered to be a special case of the general quadratic function. They easily identified the value of each parameter in all three functions. This introduction was followed by an exercise (appendix C) which required students to state the value of each parameter in various functions and to write the equation of a function given the value of its parameters. Upon completion and correction of this exercise students were ready to be introduced to the problem.

The problem was presented to students in the form of a questionnaire (appendix D). The purpose of the questionnaire was to reduce of the original problem to a series of smaller problems in accordance with Polya's (1973) model for problem solving; the formative evaluation had revealed that most students could not do without such guidance. The problem required that the effect of each of the three parameters on the graph of the function be discovered. Under normal circumstances a classroom discussion would have followed. However, because of the nature of this study, the problem-solving exercise was succeeded by a test designed to measure the extent to which students had been able to solve the problem and whether or not they could apply this knowledge to non-quadratic functions.

Instrumentation

The test (appendix E) consisted of fourteen multiple choice questions divided into two parts. The first seven questions tried to determine how well students understood the individual and combined effects of the three parameters on the graph of the quadratic function, while the purpose of the second set of questions was to find out if students could apply their knowledge to non-quadratic functions. The test was developed concurrently with the program and was modified twice before arriving at the final version. A second form of the test (appendix F) was also prepared in order to reduce the effect of learning (as a result of taking the first test) on the delayed post-test. A final analysis was performed on both tests (tables 1 and 2) using the experimental data. The analysis included the calculation of indices of difficulty and discriminability for each item on both forms of the test as well as the Kuder-Richardson index of reliability for each of these (Tuckman, 1972). A t-test revealed no significant differences ($\alpha = .05$) between the means of the tests.

Table 1
Item Analysis of Form A

| Item | Index of difficulty | Index of discriminability |
|------|------------------------|------------------------------|
| 1 | 0.46 | 0.89 |
| 2 | 0.42 | 0.75 |
| 3 | 0.29 | 0.73 |
| 4 | 0.46 | 0.75 |
| 5 | 0.54 | 0.88 |
| 6 | 0.63 | 0.88 |
| 7 | 0.67 | 1.00 |
| 8 | 0.67 | 0.71 |
| 9 | 0.79 | 1.00 |
| 10 | 0.79 | 0.75 |
| 11 | 0.87 | 0.67 |
| 12 | 0.58 | 0.56 |
| 13 | 0.87 | 1.00 |
| 14 | 0.75 | 0.75 |
| Mean | 0.63 | 0.81 |

Mean score = 5.21

Standard deviation = 3.01

Kuder - Richardson reliability = 0.74

Table 2
Item Analysis of Form B

| Item | Index of difficulty | Index of discriminability |
|------|------------------------|------------------------------|
| 1 | 0.65 | 0.86 |
| 2 | 0.70 | 1.00 |
| 3 | 0.26 | 0.64 |
| 4 | 0.78 | 1.00 |
| 5 | 0.57 | 0.71 |
| 6 | 0.57 | 0.86 |
| 7 | 0.48 | 0.75 |
| 8 | 0.70 | 0.83 |
| 9 | 0.35 | 0.45 |
| 10 | 0.65 | 1.00 |
| 11 | 0.65 | 1.00 |
| 12 | 0.70 | 0.83 |
| 13 | 0.61 | 0.80 |
| 14 | 0.43 | 0.63 |
| Mean | 0.58 | 0.81 |

Mean score = 5.91

Standard deviation = 3.42

Kuder - Richardson reliability = 0.79

Procedure

Each of the two classes was treated separately but in identical fashion. Students within a single class were assigned to one of three treatment groups using a table of random numbers; 17 students were assigned to the first group, 16 to the second, and 18 to the third. Students in the first group worked alone on a problem while students in the second and third groups worked on the same problem in pairs. Formation of the dyads within each of the latter two groups was done in random fashion.

Because of the limited number of microcomputers (4), the experiment was conducted over a period of three consecutive days. Each day was divided into three periods of one and one half hours: 9:30 - 11:00 a.m., 11:10 a.m. - 12:40 p.m., and 1:45 - 3:15 p.m. Students were given one hour to solve the problem after which they were tested. So that no group would be favoured by the time or the day on which they were treated, scheduling was achieved by assigning students from each group across all nine periods in such a way that during each period there would be two individuals from the first experimental group, a pair of students from the second, and another pair from the third. Also, students from each class were scheduled together so that each class would be processed in the least amount of time (approximately one and one half days for each class).

The day before the experiment was begun students were introduced to the general quadratic function as described on page 28. This was necessary since the problem they solved dealt with some new concepts. Students were then given their appointment cards and began reporting to the lab the following morning. Here they were greeted by the lab assistant who proceeded as follows: First, students were asked to read and sign one of the three statements (each corresponding to a different treatment) describing the conditions under which they would work and the method of evaluation (appendix H). Secondly, students who were to work in pairs were introduced to their partners. Thirdly, each individual or pair was given a statement of the problem in the form of a questionnaire (appendix D). Finally, the lab assistant placed each pair or individual at a computer and started the program for them.

Students were allowed to work on the problem for a maximum of one hour. During this time the program collected all inputs and eventually stored them in a sequential file on the same diskette as the program. This information was later used to 'reconstruct' the work done by each individual or pair. Finally, students were tested; half of them (one from each pair and one individual) took form A of the criterion test (appendix E) while the other half took form B (appendix F). Students were given thirty minutes in which to complete the test.

Treatment was limited to one hour for several reasons. First, this allowed a reasonable amount of time in which to solve the problem. Secondly, it controlled the amount of time so that different levels of achievement would not simply reflect differences in time spent working on the problem. Finally, limited resources would have made it impractical to run the experiment any other way.

By the end of the third day all students had been processed with no major difficulties. Only four students escaped treatment due to absenteeism while a pair of students from the second group were mistakenly placed in the third group by the lab assistant. Two weeks later students were re-tested using form B of the test with those who had originally taken form A and vice versa. No subjects were lost during this final phase.

Chapter 5

Results

Mean scores and standard deviations for achievement on both post-tests are shown in Table 4 (page 40). A one-way analysis of variance (Table 5, page 41) for repeated measures was used to compare the mean scores on both post-tests across the three experimental groups; the same analysis was used to compare mean scores on part 1, and then part 2 of the tests. No significant differences ($p > .05$) were found, thus the rejection of the hypotheses of this study is called for.

The BMDP2V program (Dixon & Brown, 1979) was used to conduct the above analysis. All other analysis used SPSS (Nie, Hull, Jenkins, Steinbrenner, & Bent, 1975) programs.

The relationship between the scores obtained on parts 1 and 2 of the immediate post-test was studied in order to determine whether success on part 2 of the test was related to achievement on the first part. Subjects were divided into thirds according to their score on part 1 of the immediate post-test. A T-test comparing the means of the low and high thirds on part 2 of the test revealed a significant difference (Table 3).

Table 3

Comparison of Low and High Thirds on Part 2 of the Immediate Post-test

| Test | Number | Mean | Standard | Standard | T | 2-tail |
|------------|----------|------|-----------|----------|-------|--------|
| Part 1 | of Cases | | Deviation | Error | Value | Prob |
| Low third | 16 | 1.69 | .87 | .22 | -2.29 | .033 |
| High third | 17 | 3.06 | 2.30 | .99 | | |

A complete list of raw data can be found in appendix I. Apart from scores on the criterion tests, the list contains four measures of performance during the problem-solving exercise: the total number of experiments conducted by each individual or pair, the total number of functions which they looked at, a measure of the extent to which they applied the 'conservative focussing strategy', and the total time spent on the exercise. Means and standard deviations for each of these are shown in Table 6 (page 42). A one way analysis of variance of each of these variables failed to reveal any significant differences between the means.

A measure of focussing strategy was arrived at by 'reconstructing' (see page 34) the experiments performed by students during the problem-solving exercise. Experiments were assigned a value according to the following scheme: when an experiment consisted of three functions in which one parameter

was varied while the others were kept constant, one-and-one-half points were assigned; if the same treatment was applied to only two functions, one point was assigned; finally, when the strategy was applied to two functions in separate, but consecutive, experiments, one-half point was given. It is the sum of these values which constituted the final measure. This scheme was designed to reward deliberate and forceful applications of 'conservative focussing'. The Pearson correlation coefficient was .40 ($p=.003$) between this measure and the number of experiments and .58 ($p=.001$) between this same measure and the number of functions. The correlation coefficient between the number of experiments and the number of functions was much higher ($r=.80$, $p=.001$).

Pearson correlation coefficients were calculated in order to determine how: a) the number of experiments, b) the number of functions, c) the 'conservative focussing' strategy, and d) the ratio of the number of functions to the number of experiments, varied with the scores obtained on either post-tests. These correlation coefficients are listed in Table 7 (page 43) together with significance-test results. In an effort to clarify the situation, first and second order partials were also calculated for each of the variables. Of interest was the increase in the correlation coefficient (for pairs) between the measure of 'conservative focussing' and post-test scores to .68 ($p=.016$).

when the number of experiments and the number of functions were controlled for. Due to the small size of the cells, however, many of the results were not significant and those which were, generally supported the results already evident in the table.

Finally, it is interesting to note that students, even though they were assigned to each other on a random basis, were sufficiently taken by the problem to put aside personal differences. No conflicts of any sort were observed by the lab assistant or the author and most pairs seemed to work together very well.

Table 4

Test Means and Standard Deviations for the Three Treatment Groups

| Dependent Measures | | Individuals N = 17 | Pairs N = 12 | Contingency Pairs N = 18 |
|------------------------|------|-----------------------|-----------------|-----------------------------|
| Post-test | | | | |
| -Part I (out of 7) | Mean | 3.17 | 3.50 | 3.22 |
| | S.D. | 2.19 | 2.15 | 2.07 |
| -Part II (out of 7) | Mean | 2.29 | 2.25 | 2.28 |
| | S.D. | 1.96 | 1.48 | 1.64 |
| -Total (out of 14) | Mean | 5.47 | 5.75 | 5.50 |
| | S.D. | 3.57 | 3.08 | 3.07 |
| Delayed post-test | | | | |
| -Part I (out of 7) | Mean | 3.23 | 2.50 | 2.72 |
| | S.D. | 2.19 | 1.83 | 1.96 |
| -Part II (out of 7) | Mean | 2.18 | 1.58 | 2.39 |
| | S.D. | 1.67 | 0.90 | 1.97 |
| -Total (out of 14) | Mean | 5.41 | 4.08 | 5.11 |
| | S.D. | 3.37 | 2.39 | 3.48 |

Table 5

Results of Analysis of Variance for Repeated MeasuresImmediate and Delayed Post-tests

| ANALYSIS OF VARIANCE FOR 1-ST DEPENDENT VARIABLE - POST | | | | | | DPOST |
|---|--------|----------------|--------------------|--------------|--------|------------------|
| | SOURCE | SUM OF SQUARES | DEGREES OF FREEDOM | MEAN SQUARES | F | TAIL PROBABILITY |
| 1 | MEAN | 2481.80727 | 1 | 2481.80727 | 139.67 | .0000 |
| | GROUP | 4.01777 | 2 | 2.00888 | .11 | .8934 |
| | ERROR | 781.85458 | 44 | 17.76942 | | |
| 2 | R | 11.30581 | 1 | 11.30581 | 3.61 | .0641 |
| | RGROUP | 9.71676 | 2 | 4.85836 | 1.55 | .2237 |
| | ERROR | 137.94281 | 44 | 3.13506 | | |

Part 1 of Immediate and Delayed Post-tests

| ANALYSIS OF VARIANCE FOR 1-ST DEPENDENT VARIABLE - POST1 | | | | | | DPOST1 |
|--|--------|----------------|--------------------|-------------|--------|------------------|
| | SOURCE | SUM OF SQUARES | DEGREES OF FREEDOM | MEAN SQUARE | F | TAIL PROBABILITY |
| 1 | MEAN | 852.12254 | 1 | 852.12254 | 125.17 | .0000 |
| | GROUP | 1.08598 | 2 | .54299 | .08 | .9235 |
| | ERROR | 299.53105 | 44 | 6.80752 | | |
| 2 | R | 5.25255 | 1 | 5.25255 | 2.86 | .0977 |
| | RGROUP | 4.02409 | 2 | 2.01205 | 1.10 | .3429 |
| | ERROR | 80.72059 | 44 | 1.83456 | | |

Part 2 of Immediate and Delayed Post-tests

| ANALYSIS OF VARIANCE FOR 1-ST DEPENDENT VARIABLE - POST2 | | | | | | DPOST2 |
|--|--------|----------------|--------------------|-------------|--------|------------------|
| | SOURCE | SUM OF SQUARES | DEGREES OF FREEDOM | MEAN SQUARE | F | TAIL PROBABILITY |
| 1 | MEAN | 425.45673 | 1 | 425.45673 | 107.00 | .0000 |
| | GROUP | 2.60221 | 2 | 1.30111 | .33 | .7227 |
| | ERROR | 174.95098 | 44 | 3.97616 | | |
| 2 | R | 1.14611 | 1 | 1.14611 | .66 | .4200 |
| | RGROUP | 2.21457 | 2 | 1.10729 | .64 | .5320 |
| | ERROR | 76.10458 | 44 | 1.72965 | | |

Table 6
Means and Standard Deviations for Four Performance Variables

| | | Individuals | Pairs | Contingency Pairs | Overall |
|---|------|-------------|--------|----------------------|---------|
| | | N = 17 | N = 12 | N = 18 | N = 47 |
| Number of experiments | Mean | 22.5 | 25.5 | 24.8 | 24.1 |
| | S.D. | 8.2 | 6.3 | 9.5 | 8.2 |
| Number of functions | Mean | 49.4 | 52.0 | 50.6 | 50.5 |
| | S.D. | 17.9 | 7.5 | 19.4 | 16.3 |
| Time (minutes) | Mean | 54.3 | 57.3 | 56.3 | 55.9 |
| | S.D. | 10.1 | 6.2 | 7.7 | 8.3 |
| Measure of conservative focussing strategy | Mean | 8.8 | 11.1 | 12.8 | 10.9 |
| | S.D. | 5.4 | 4.6 | 9.0 | 6.9 |

Table 7

Pearson Correlation Between Performance Variables and Test Scores

| | Individuals | Pairs | Contingency Pairs | Overall |
|---|--------------|--------------|----------------------|---------------|
| | N = 17 | N = 12 | N = 18 | N = 47 |
| Number of experiments | | | | |
| -Post-test | .37 (p=.07) | .24 (p=.22) | .42 (p=.04) | .36 (p=.01) |
| -Delayed | .20 (p=.21) | .33 (p=.15) | .15 (p=.28) | .17 (p=.13) |
| posttest | | | | |
| Number of functions | | | | |
| -Post-test | .24 (p=.18) | .47 (p=.06) | -.04 (p=.44) | .14 (p=.18) |
| -Delayed | .07 (p=.39) | .52 (p=.04) | -.18 (p=.23) | -.03 (p=.43) |
| post-test | | | | |
| Ratio of no. of functions to no. of experiments | | | | |
| -Post-test | -.11 (p=.33) | -.10 (p=.38) | -.70 (p=.001) | -.37 (p=.005) |
| -Delayed | -.13 (p=.31) | -.18 (p=.29) | -.47 (p=.03) | -.28 (p=.03) |
| post-test | | | | |
| Measure of conservative focussing strategy | | | | |
| -Post-test | .66 (p=.002) | .48 (p=.06) | -.14 (p=.29) | .21 (.08) |
| -Delayed | .43 (p=.04) | .41 (p=.09) | -.21 (p=.20) | .05 (p=.37) |
| Post-test | | | | |

Note: p is the result of a one-tailed test of significance.

Discussion

Interpretation of the results of the bivariate analysis between test scores and the performance variables is limited by the large proportion of coefficients which are non-significant. Even so, a few observations can be made.

There exists a consistent and significant positive correlation between achievement on the post-test and the number of experiments which were conducted but, even though the number of functions is highly correlated to the number of experiments ($r=.80$, $p=.001$), no such statement can be made concerning the variation of test scores with the number of functions which were inspected. This peculiar state of affairs may be explained by the negative correlation which exists between post-test scores and the ratio of the number of functions to the number of experiments. This suggests that subjects who conducted experiments with single functions tended to obtain higher scores on the post-test than those who studied two or three functions simultaneously. Thus, for a given number of experiments, a lower ratio (and therefore a smaller number of functions) seems to have been more effective. This appears to indicate that the simultaneous comparison of functions might result in a certain amount of confusion unless some other condition is met; this

condition might well be the presence of a strategy.

There is a significant and fairly strong positive correlation between conservative focussing and immediate post-test scores. For pairs the coefficient increased to .68 ($p=.016$) from .48 ($p=.06$) when the number of experiments and functions were controlled for. The results for the contingency pairs were not as clear and greatly affected the size and significance of the overall coefficient. It is noteworthy that where a strong relationship exists between post-test scores and conservative focussing, the relationship with the ratio is weak and non-significant whereas when the first relationship is weak, the second is strong. This observation tends to support the idea that the simultaneous inspection of larger numbers of functions is not a problem when 'conservative focussing' is successfully applied.

The above observation raises the question of validity for the measure of 'conservative focussing'. Pearson correlation coefficients revealed that the number of experiments could account for 16% ($r=.40$, $p=.003$) of the variance of this measure whereas the number of functions might account for as much as 34% ($r=.58$, $p=.001$). These results are acceptable (considering that a strategy cannot be applied unless experiments are conducted and functions are studied) since they leave a large part of the

measure unexplained. Hence the measure appears to be a valid one.

Finally, the criterion test used in this study was divided into two parts, one designed to measure the acquisition of concepts as they apply to the general quadratic function and the second to measure the ability to generalize the newly acquired concepts to other functions. The fact that subjects who did better on part 1 of the test also did better on part 2 (Table 3) indicates that this kind of generalization did take place even though subjects were asked to apply the concepts to functions with which they were totally unfamiliar.

Chapter 6

Conclusions and Recommendations

The results of this study are consistent with previous studies; pairs were found to be as successful on the problem-solving task as individuals thus supporting the use of pairs as a means of increasing the cost-effectiveness of CAL. Unlike previous studies, however, this study placed a constraint on time and it is conceivable that this may have suppressed a major effect. A study of the means in Table 5, reveals consistently higher values for 'pairs' over 'individuals' and, even though none of these is statistically significant, there is an intimation that the removal of the constraint on time might produce such differences with a corresponding effect on achievement. A follow up study in which the constraint on time is removed is certainly called for. Will pairs work longer than individuals? Will they be more successful as a result? How will this affect cost-effectiveness? These questions should be answered.

Any follow up study should take into consideration the following points. It is possible that exposure to the various treatments was insufficient to produce a measurable effect; a

series of problems should be considered. If each problem were followed by a post-test and the final score were the cumulative mark on all of these, the result would be a more sensitive instrument than the 14-item test used in this study. Such extended treatment might also give students the time to get to know each other, get to know the medium, and sharpen their cooperative and problem-solving skills thus improving their overall effectiveness. Finally, the use of a larger sample and a control group would allow more satisfactory statistical analysis as well as the ability to perform a summative evaluation of the materials and exercise.

The success rate for the solution of the problem used in this experiment was low with roughly fifty percent of students receiving a mark of less than thirty percent on the immediate post-test. This lack of success can only be attributed to poor problem-solving strategies. The suggestions included in the program were clearly insufficient and the one recommending the graphing of two or three functions per experiment (as opposed to a single function) may even have been harmful. It could be that students require formal instruction in the use of 'conservative focussing'. How successful such instruction would be is worthy of investigation.

Even though the hypotheses for this study were rejected, it may yet be found that computer-assisted problem solving and other types of non-instructional CAL are better served by a 'cooperative goal structure' than solo study arrangements. Meanwhile, the use of pairs is recommended since it makes limited resources available to a greater number of students while reducing the cost per student by fifty percent.

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QUADRATIC FUNCTIONS

1. Complete the table of ordered pairs for each function.
2. Draw the graphs of all three functions on a single set of axes.

a) $y = x^2$

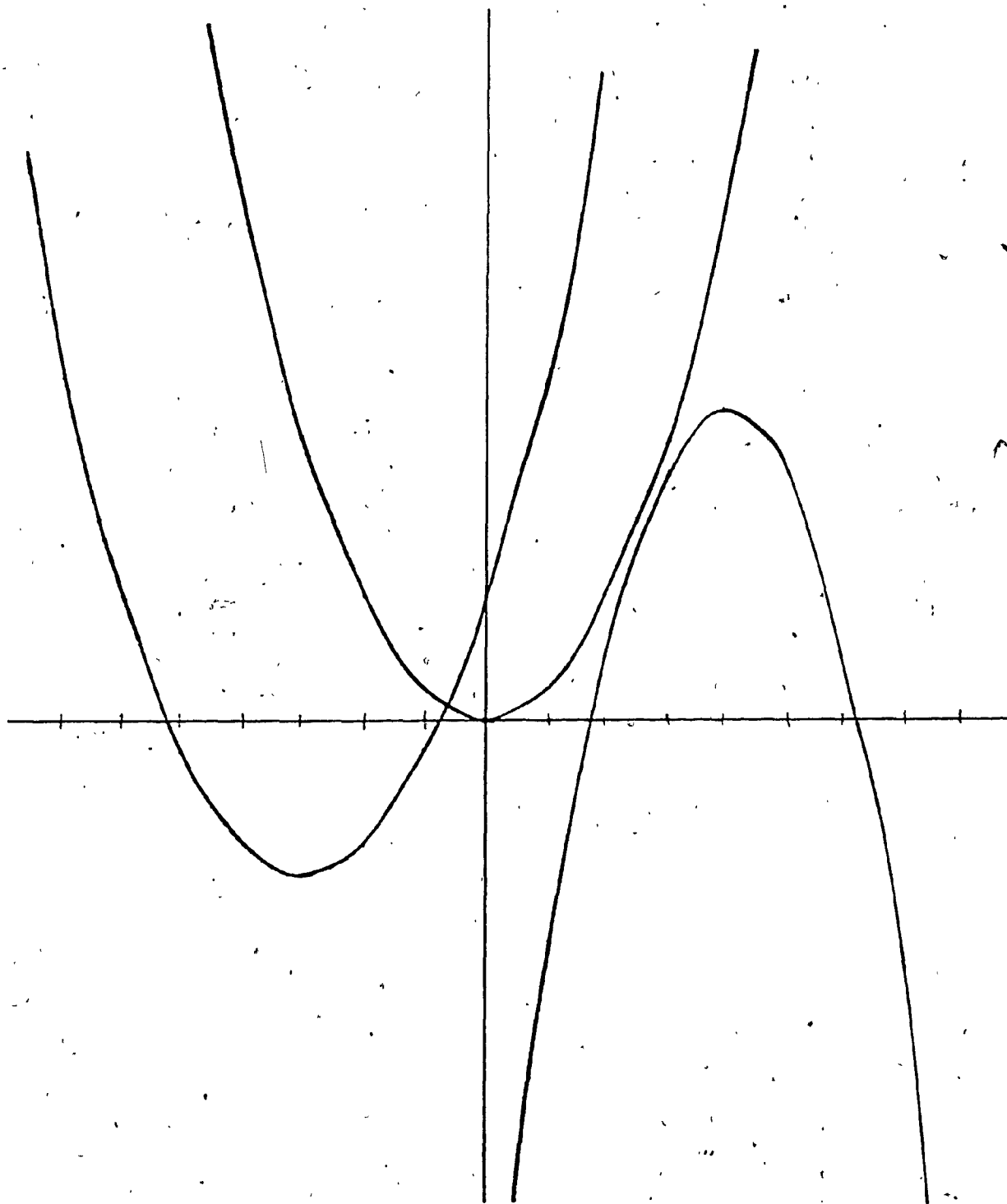
| x | y |
|----|---|
| -4 | |
| -3 | |
| -2 | |
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |

b) $y = (x + 3)^2 - 5$

| x | y |
|----|---|
| -8 | |
| -7 | |
| -6 | |
| -5 | |
| -4 | |
| -3 | |
| -2 | |
| -1 | |
| 0 | |
| 1 | |
| 2 | |

c) $y = -2(x - 4)^2 + 10$

| x | y |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |



Appendix C . Second ExerciseTHE GENERAL QUADRATIC FUNCTION

$$y = A(x - B)^2 + C$$

1. For each of the following functions, state the value of the parameters A, B, and C.

a) $y = -2x^2 + 1$

b) $y = 3(x - 1)^2 + 5$

c) $y = -(x + 3)^2 - 4$

d) $y = 0,5x^2 - 7$

e) $y = (x + 9)^2$

2. Write the equation of the quadratic function defined by each of the following.

a) $A = 1, B = 1, C = 1$

b) $A = -1, B = 0, C = 2$

c) $A = 3, B = -2, C = 0$

d) $A = -0,8, B = -3, C = -10$

e) $A = -1, B = -1, C = 0$

Appendix D Questionnaire

DO NOT TOUCH THE RESET BUTTON

By comparing all functions to $y = x^2$, where $A = 1$, $B = 0$ and $C = 0$, answer the following questions.

For the general quadratic function

$$y = A(x - B)^2 + C$$

what happens to the graph of the function

- when A is negative? positive?
- as A increases?
- when B is negative? zero? positive?
- when C is negative? zero? positive?

If you have answered the above questions you should be able to predict what the graphs of the following functions will look like.

Can you? If not, go back to the questions.

$$y = x^2$$

$$y = x^2 + 10$$

$$y = (x - 5)^2$$

$$y = -5x^2$$

$$y = -x^2 - 20$$

$$y = -3(x + 5)^2$$

$$y = (x - 5)^2 - 10$$

$$y = 0.5x^2$$

TEST

THE GENERAL QUADRATIC FUNCTION

INSTRUCTIONS:

1. Please do not write on this question-naire.
2. You may do rough work on the back of the answer sheet.
3. Choose the best answer for each question and circle your choice on the answer sheet.
4. You have 30 minutes to complete this test.

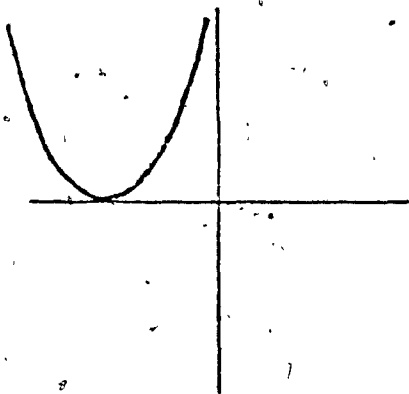
Which graph below best illustrates each of the following functions?

1. $y = 3(x - 2)^2$

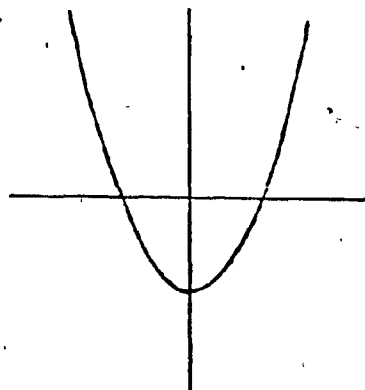
2. $y = 3x^2 - 2$

3. $y = -3x^2 - 2$

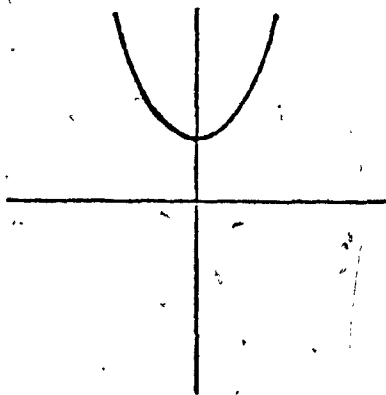
A)



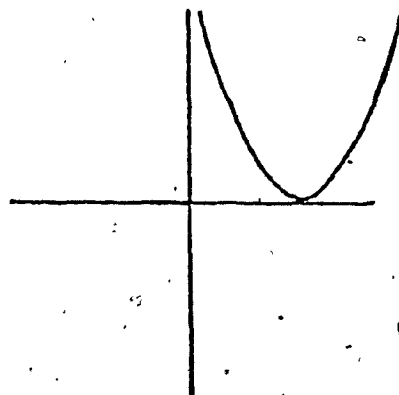
B)



C)



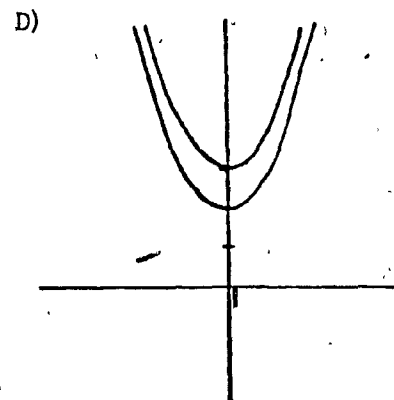
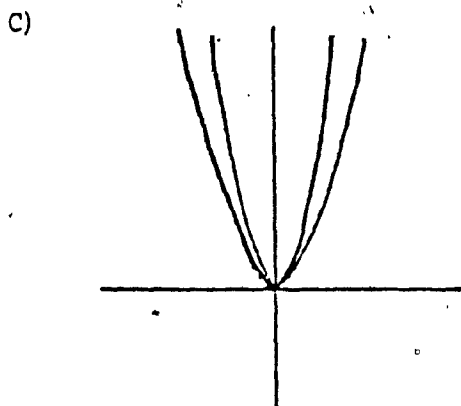
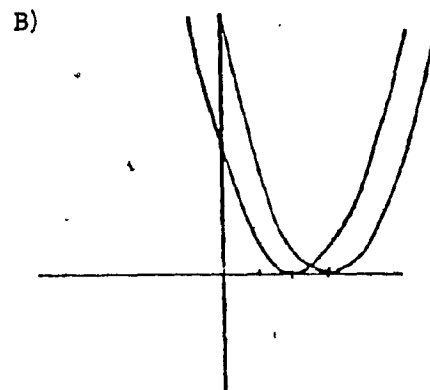
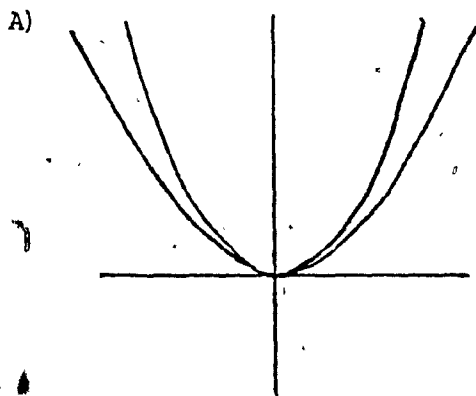
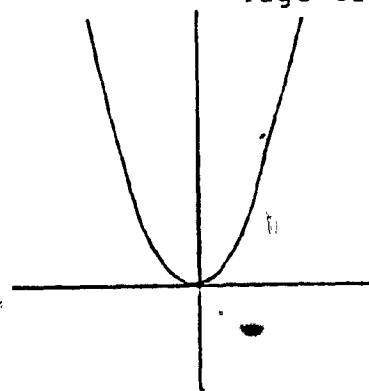
D)



E) None of these

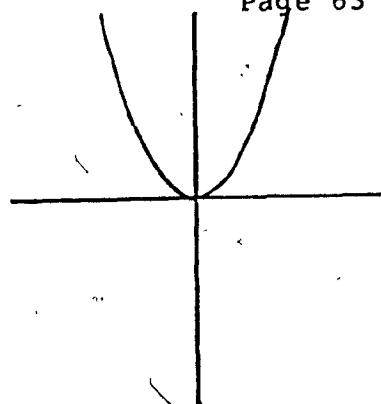
2

4. If the graph on the right represents $y = x^2$, then which of the graphs below best represents the functions $y = 2x^2$ and $y = 3x^2$?

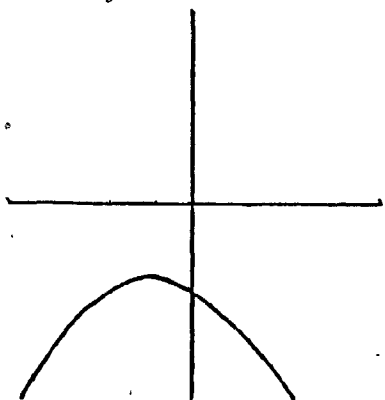


- E) None of these

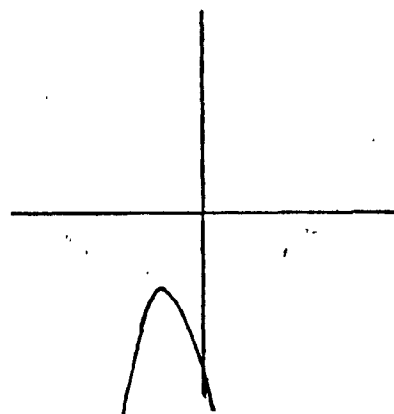
5. If the graph on the right represents $y = x^2$, then which of the graphs below best represents the function $y = -2(x + 5)^2 - 10$?



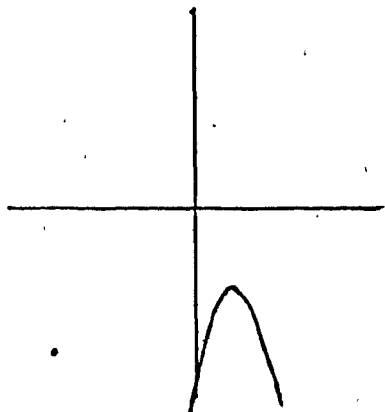
A)



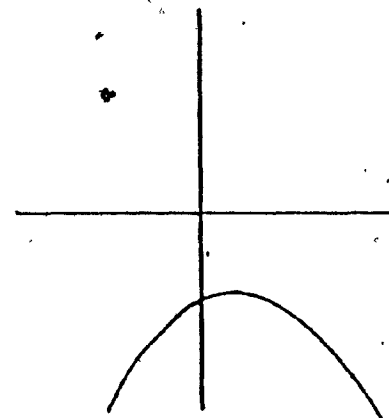
B)



C)

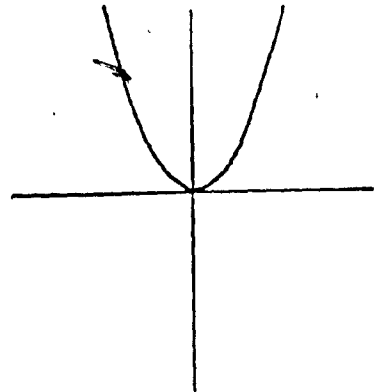


D)

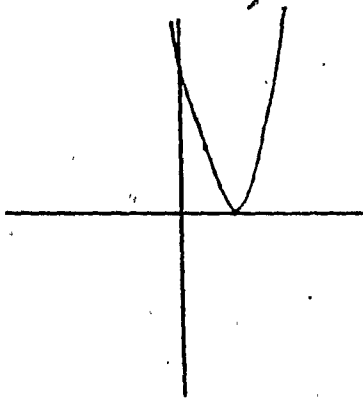


E) None of these.

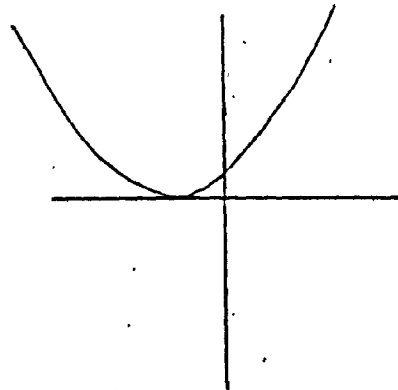
6. If the graph on the right represents the function $y = x^2$, then which of the graphs below best represents the function $y = 0,25(x - 3)^2$?



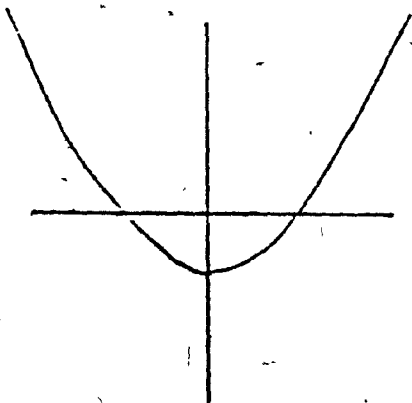
A)



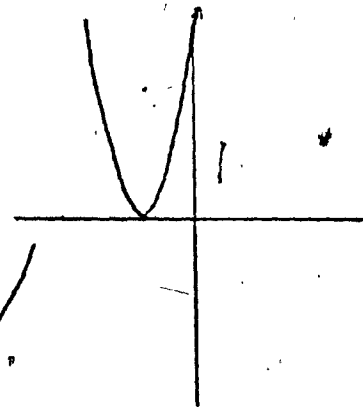
B)



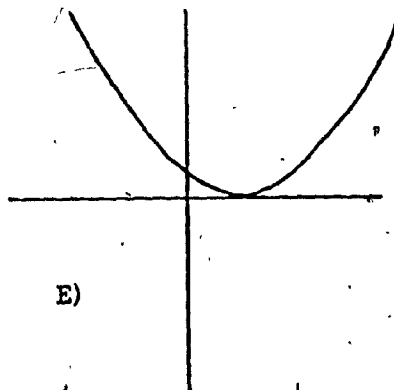
C)



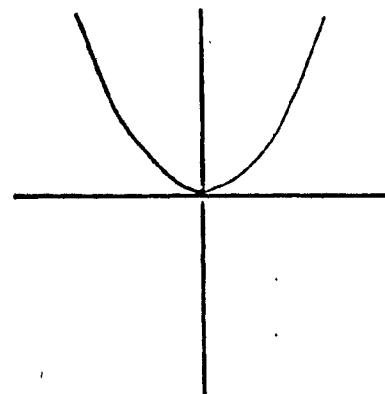
D)



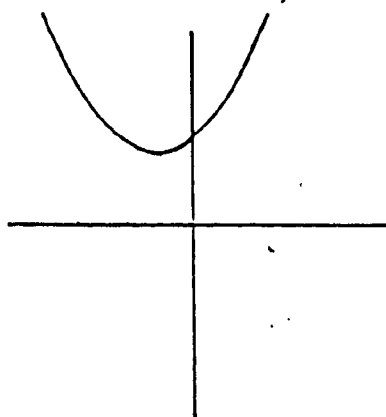
E)



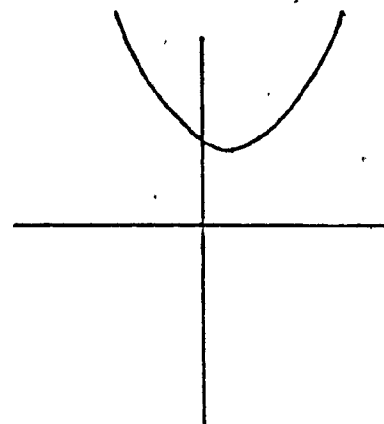
7. If the graph on the right represents the function $y = x^2$, then which of the graphs below best represents the function $y = (x + 2)^2 - 6$?



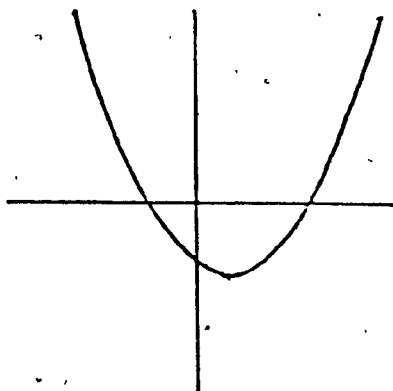
A)



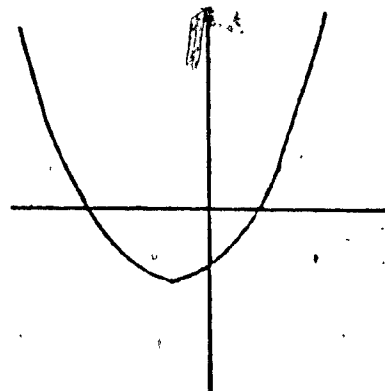
B)



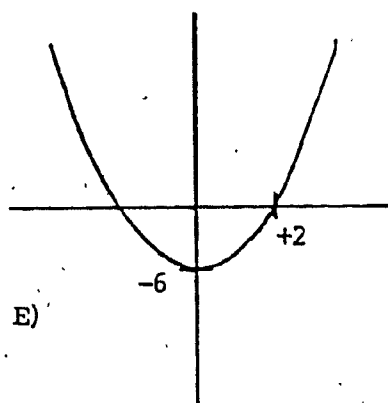
C)



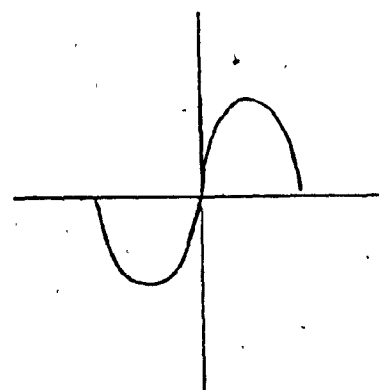
D)



E)



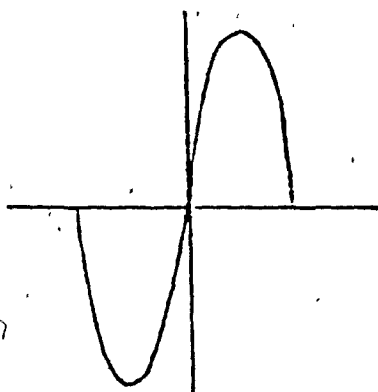
If the graph on the right represents some unknown function $y = f(x)$, then which of the graphs below best represents each of the following functions?



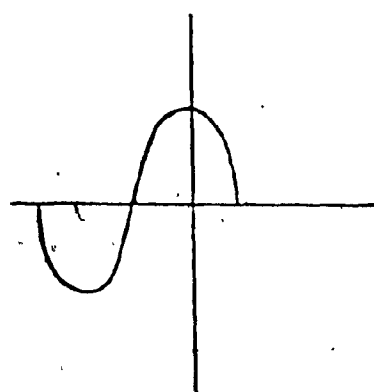
8. $y = f(x) + 2$

9. $y = f(x - 2)$

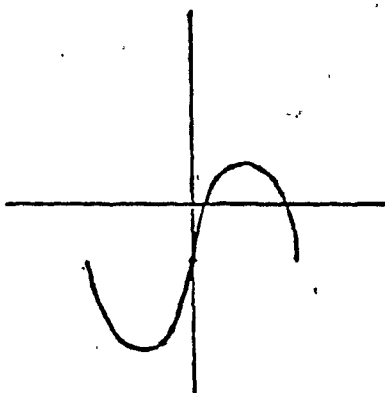
A)



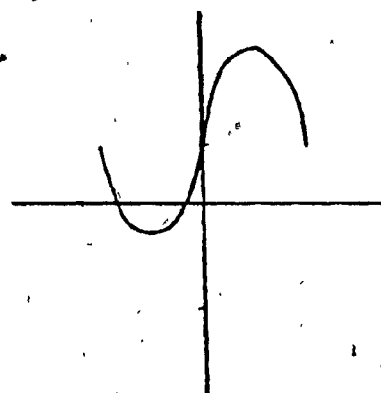
B)



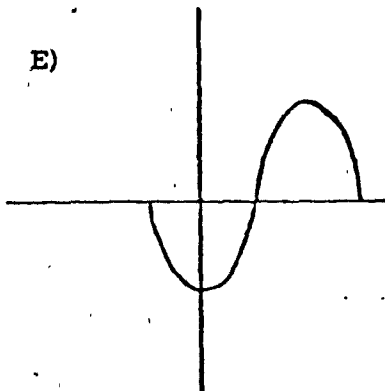
C)



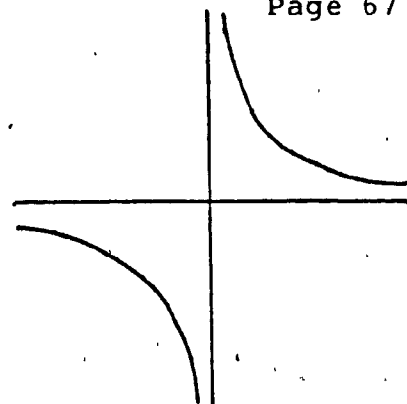
D)



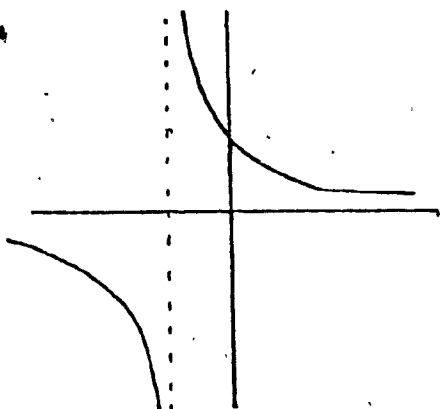
E)



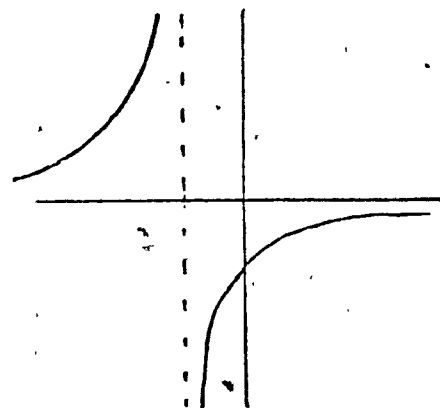
10. If the graph on the right represents some unknown function $y = g(x)$, then which of the graphs below best represents the function $y = -g(x) - 7$?



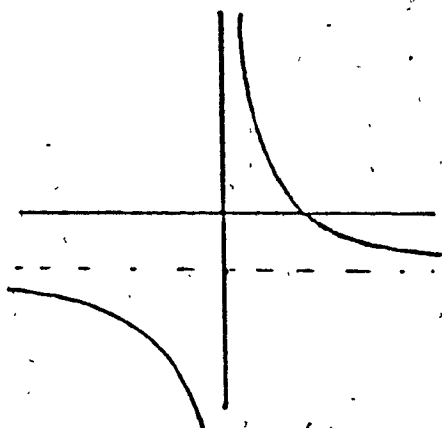
A)



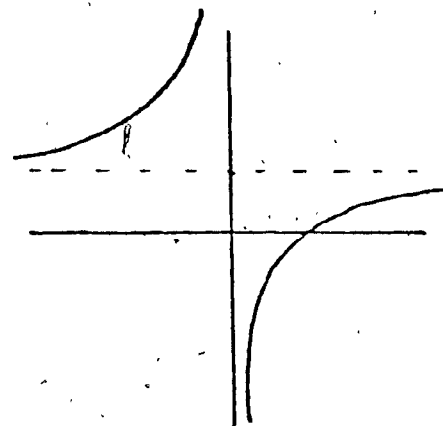
B)



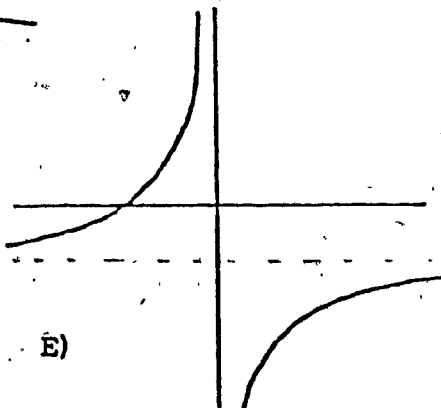
C)



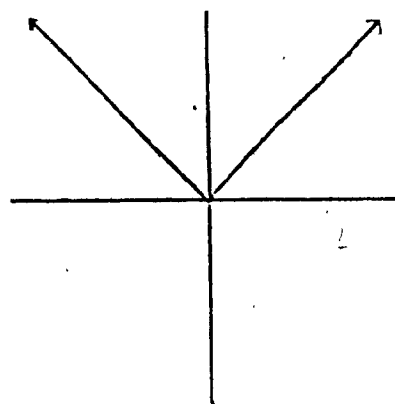
D)



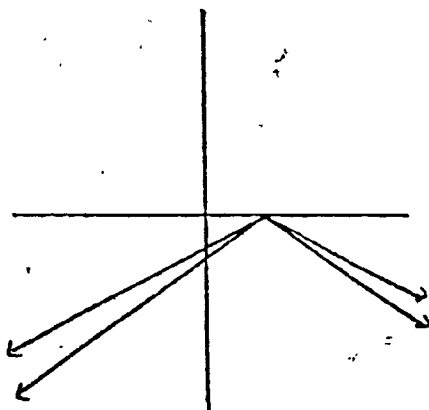
E)



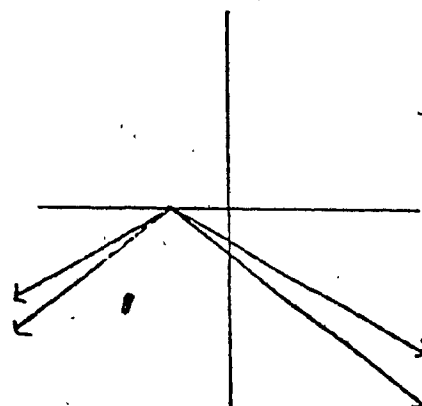
11. If the graph on the right represents the function $y = h(x)$, then which of the graphs below best represents the functions $y = -3h(x - 4)$ and $y = -5h(x - 4)$?



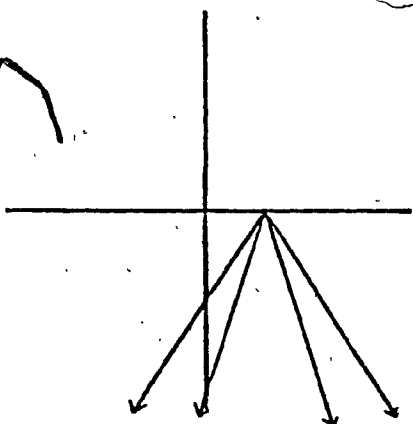
A)



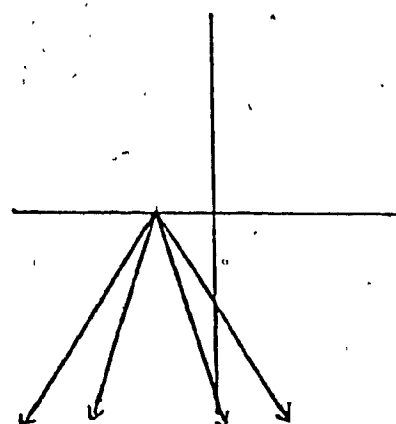
B)



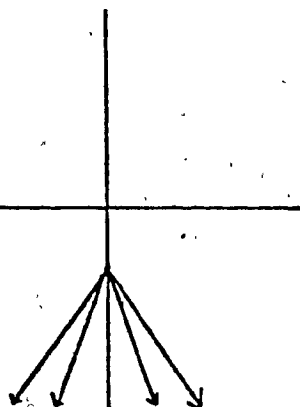
C)



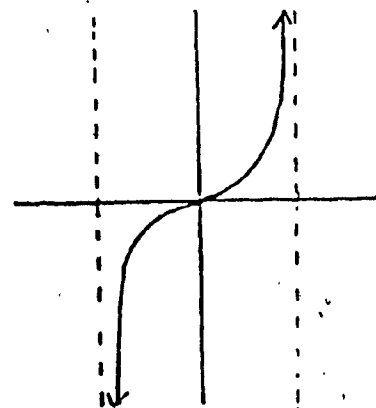
D)



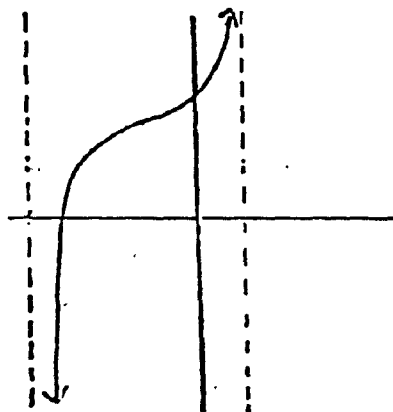
E)



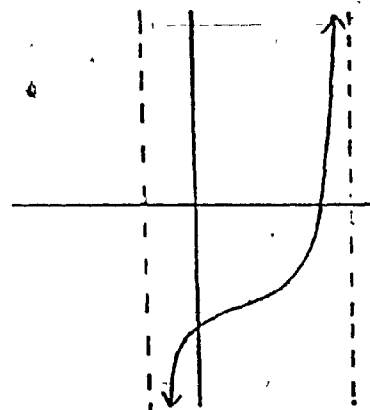
12. If the graph on the right represents the function $y = \tan x$, then which of the graphs below best represents the function $y = \tan (x - 3) + 5$?



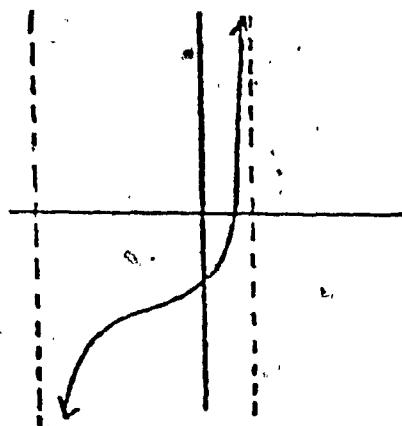
A)



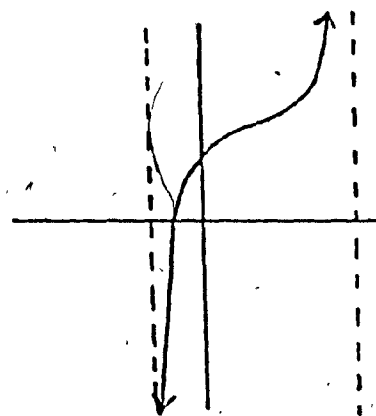
B)



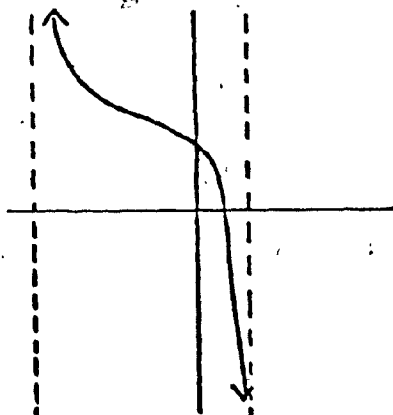
C)



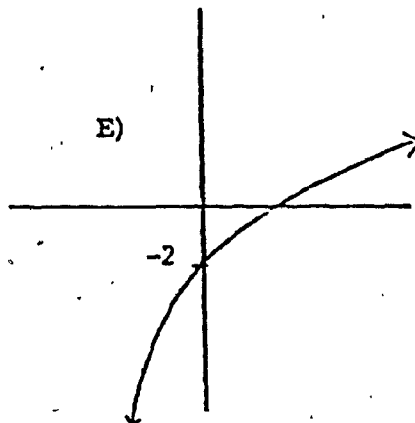
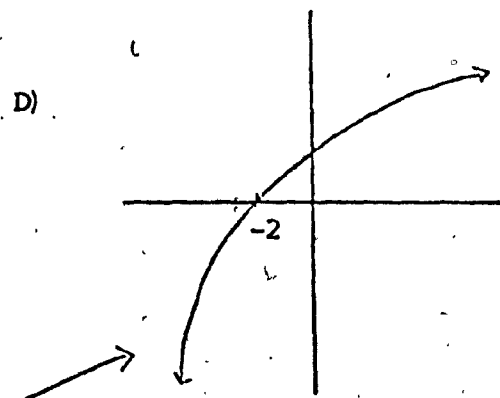
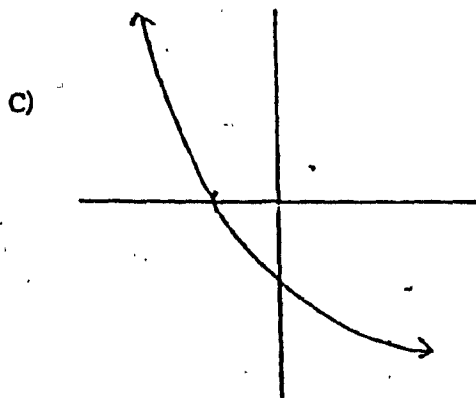
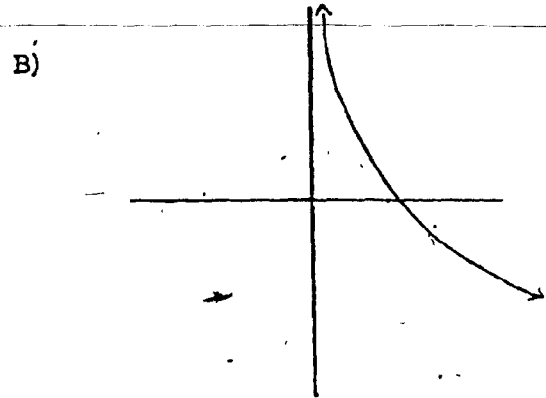
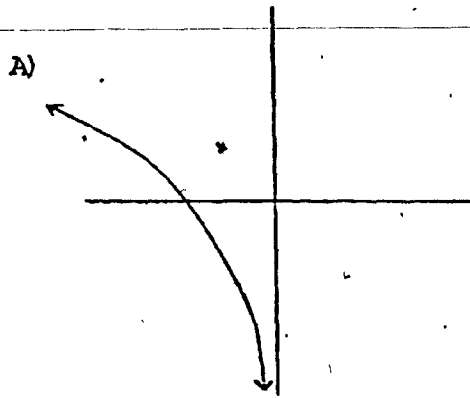
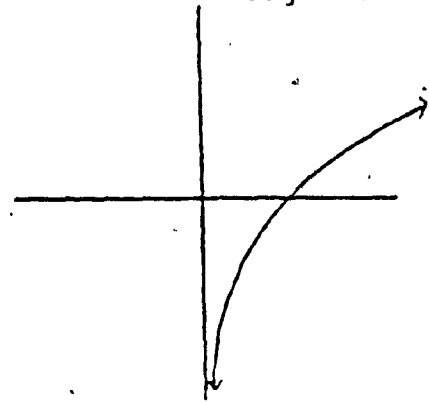
D)



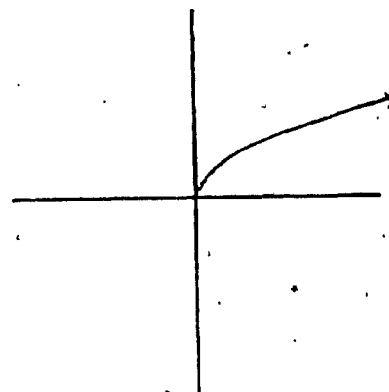
E)



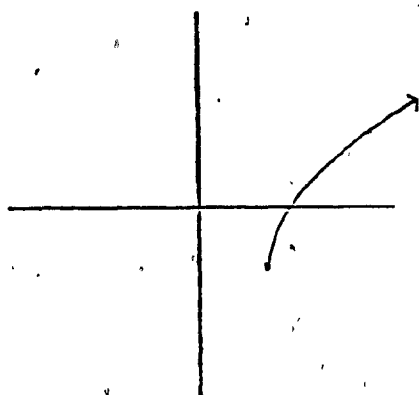
13. If the graph on the right represents the function $y = 2 \log x$, then which of the graphs below best represents the function $y = -2 \log x$?



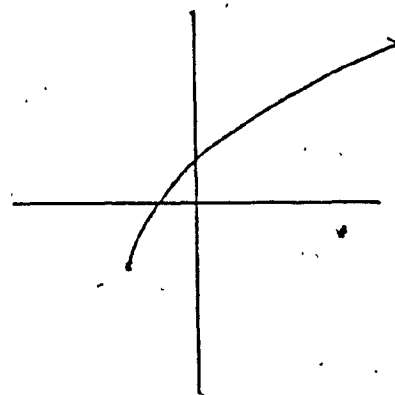
14. If the graph on the right represents the function $y = q(x)$, then which of the graphs below best represents the function $y = 2q(x - 3) - 3$?



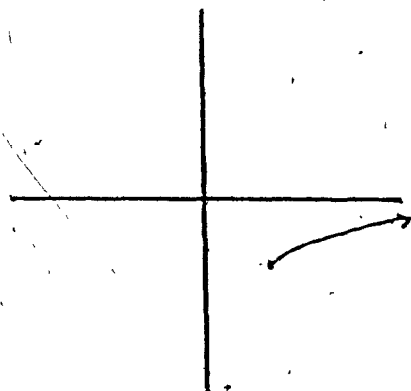
A)



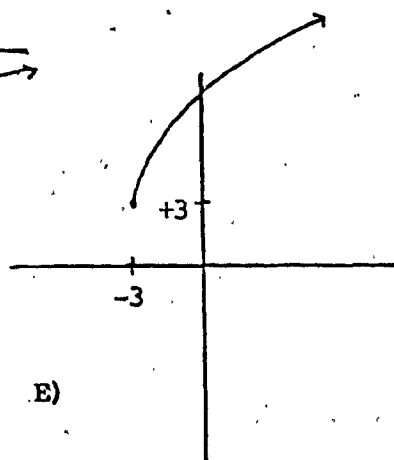
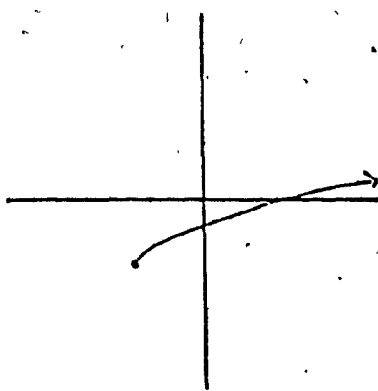
B)



C)



D)



E)

Appendix F Test Form B

TEST

THE GENERAL QUADRATIC FUNCTION

INSTRUCTIONS:

1. Please do not write on this question-naire.
2. You may do rough work on the back of the answer sheet.
3. Choose the best answer for each question and circle your choice on the answer sheet.
4. You have 30 minutes to complete this test.

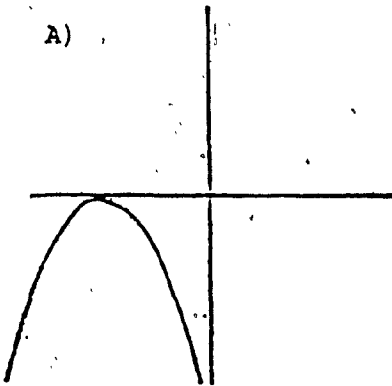
Which graph below best illustrates each of the following functions?

1. $y = -6(x + 5)^2$

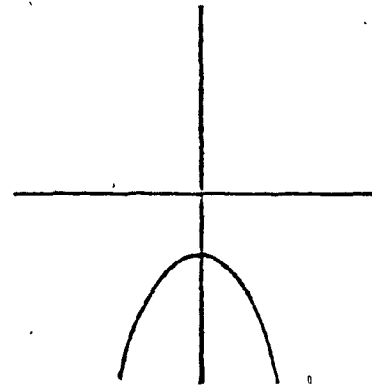
2. $y = -6x^2 + 5$

3. $y = 6x^2 + 5$

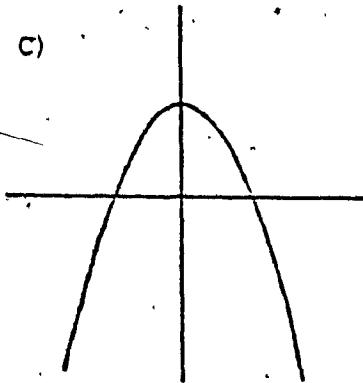
A)



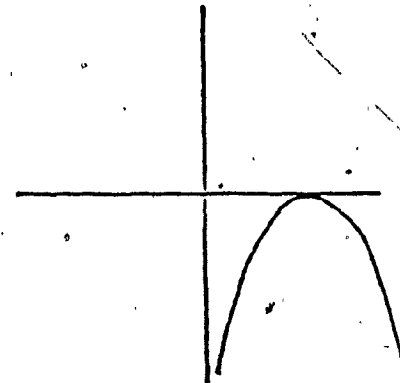
B)



C)

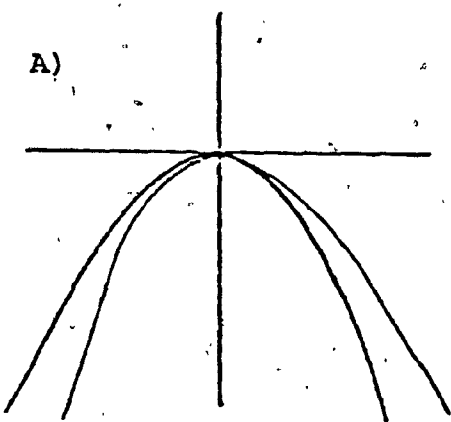
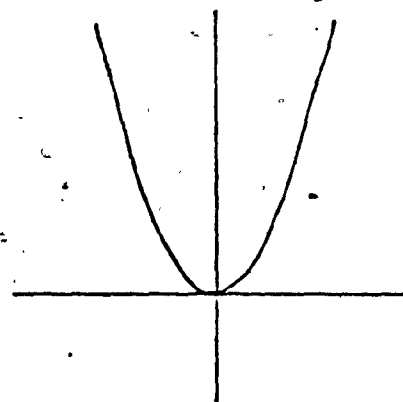


D)

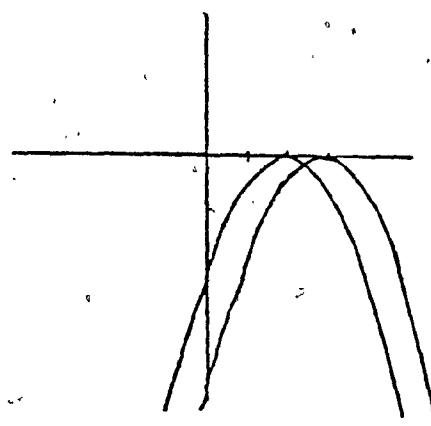


E) None of these

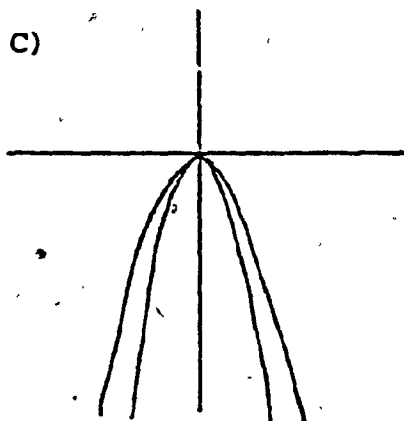
4. If the graph on the right represents $y = 4x^2$, then which of the graphs below best represents the functions $y = -2x^2$ and $y = -3x^2$?



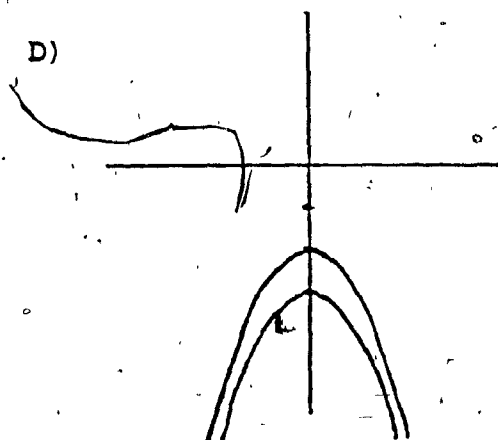
B)



C)

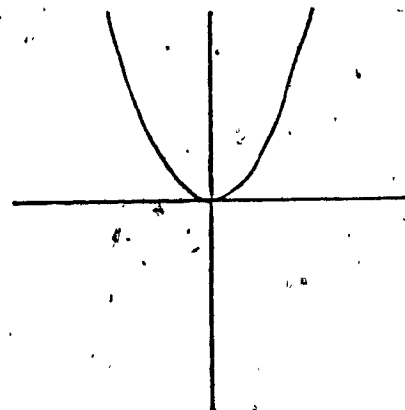


D)

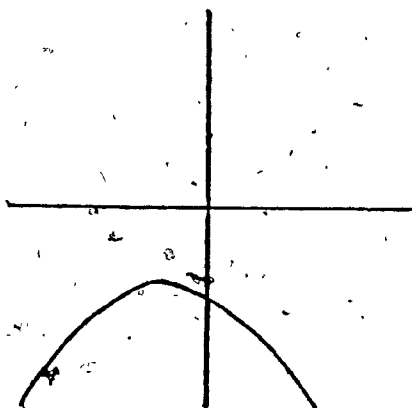


E) None of these

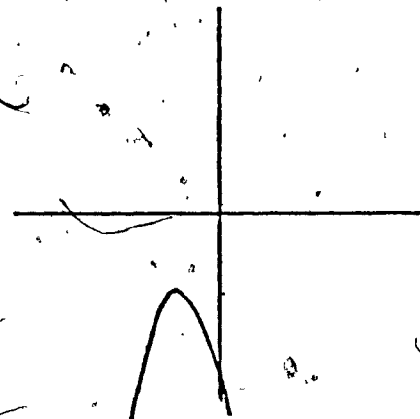
5. If the graph on the right represents $y = x^2$, then which of the graphs below best represents the function $y = -0,5(x - 2)^2 - 7$?



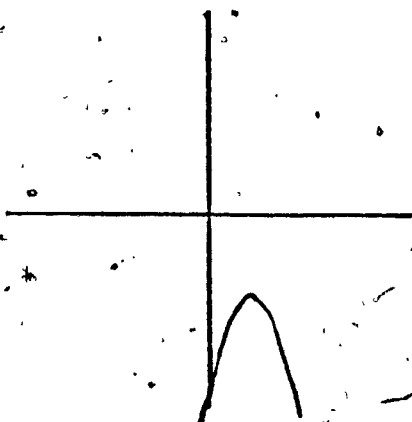
A)



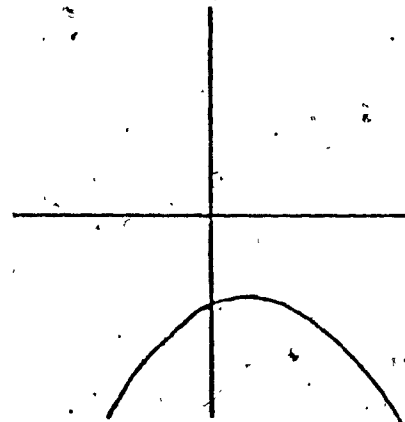
B)



C)

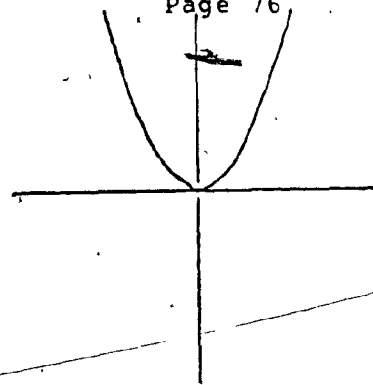


D)

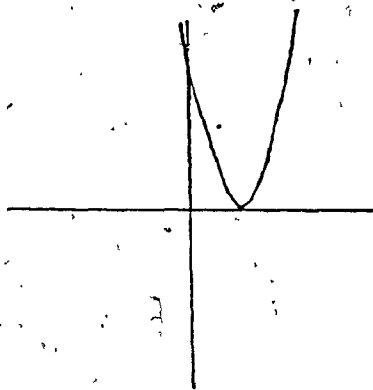


E) None of these.

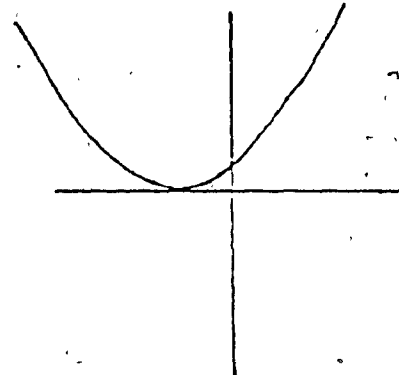
6. If the graph on the right represents the function $y = x^2$, then which of the graphs below best represents the function $y = 2(x + 4)^2$?



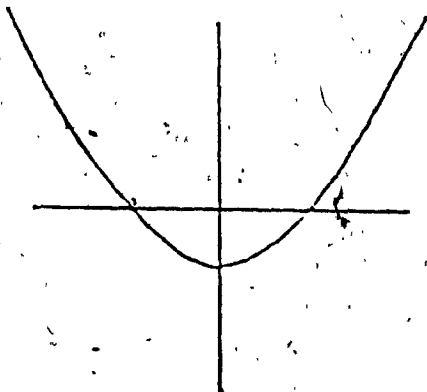
A)



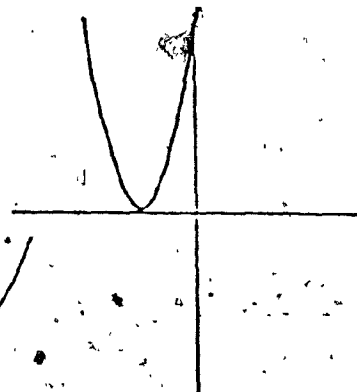
B)



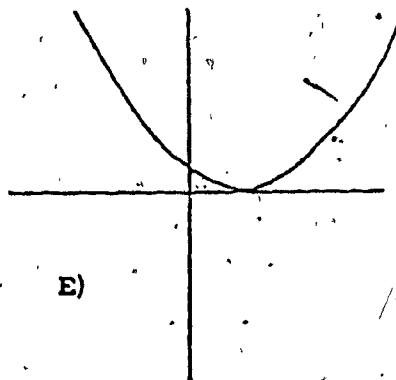
C)



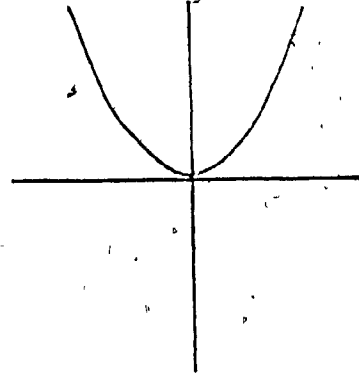
D)



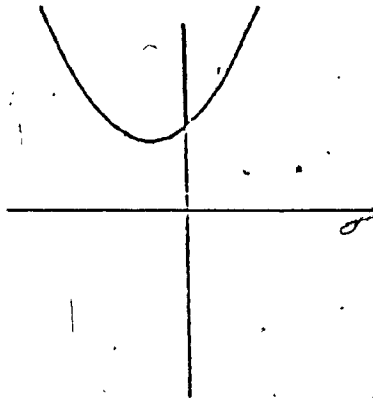
E)



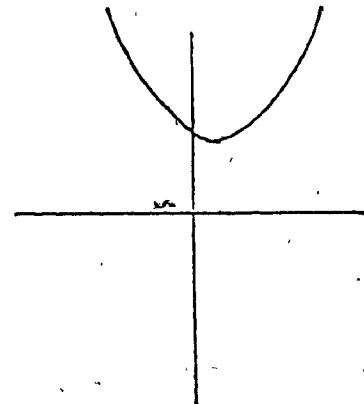
7. If the graph on the right represents the function $y = x^2$, then which of the graphs below best represents the function $y = (x - 3)^2 + 5$?



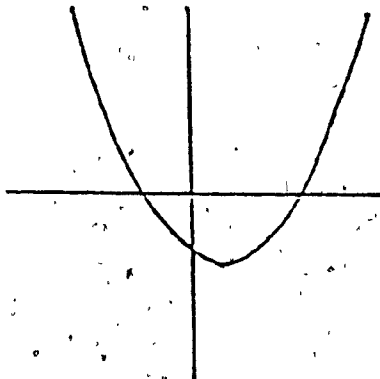
A)



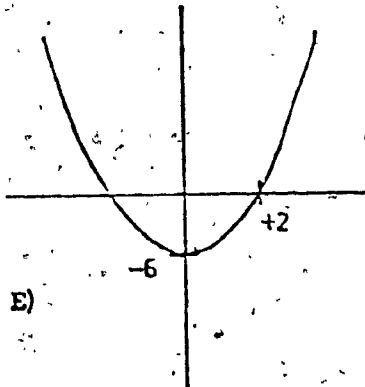
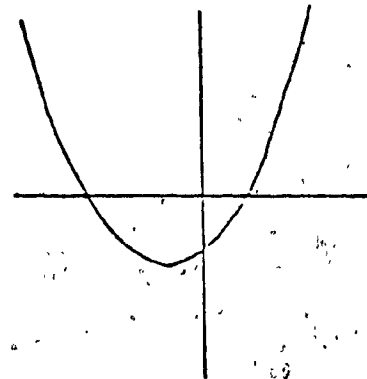
B)



C)

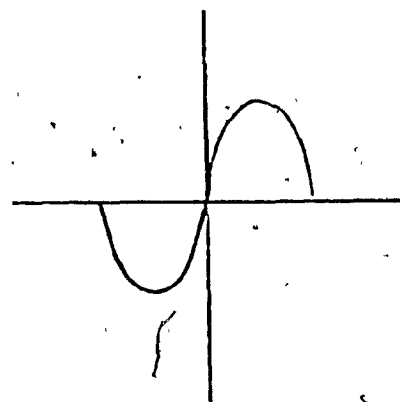


D)



E)

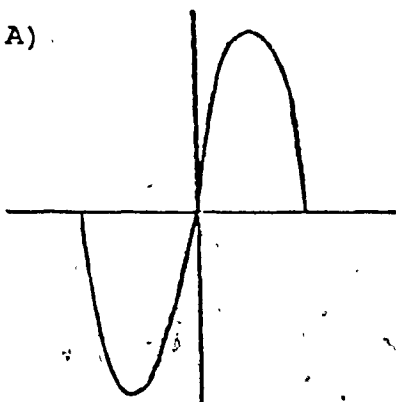
If the graph on the right represents some unknown function $y = g(x)$, then which of the graphs below best represents each of the following functions?



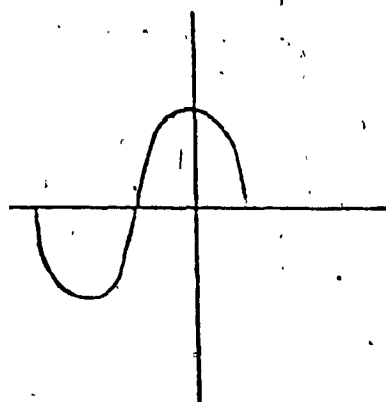
8. $y = g(x + 4)$

9. $y = g(x) - 6$

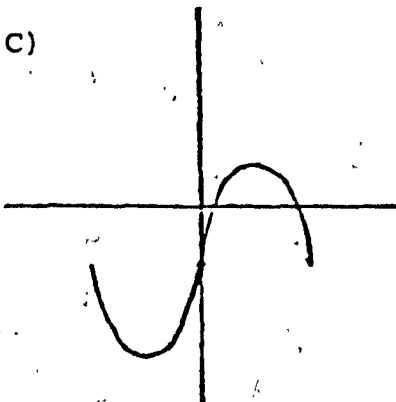
A)



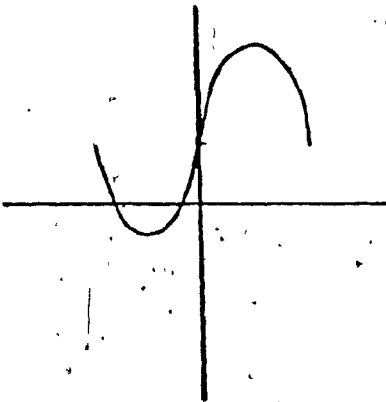
B)



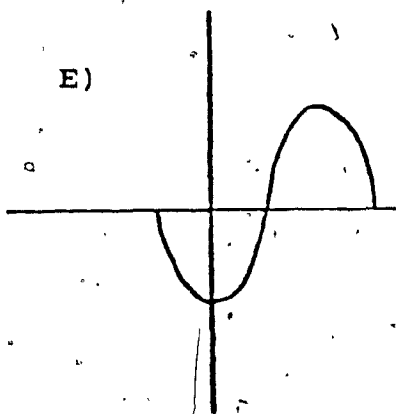
C)



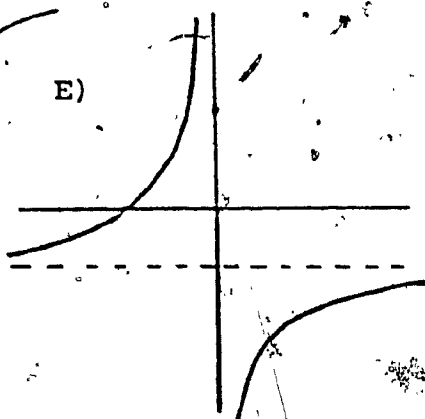
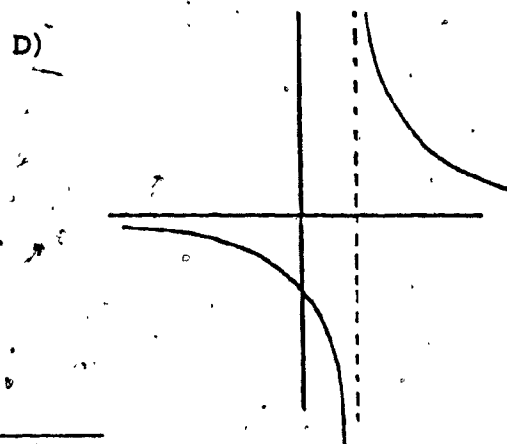
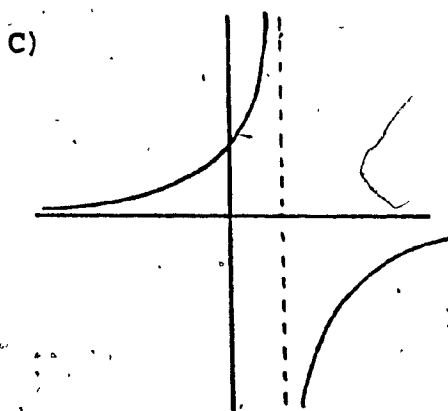
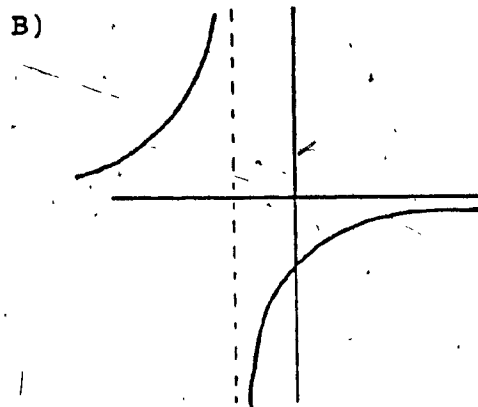
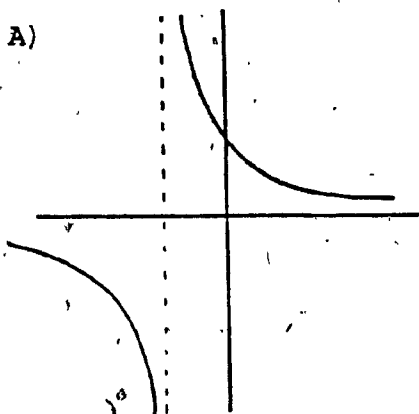
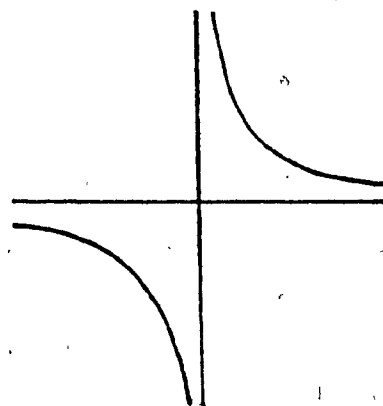
D)



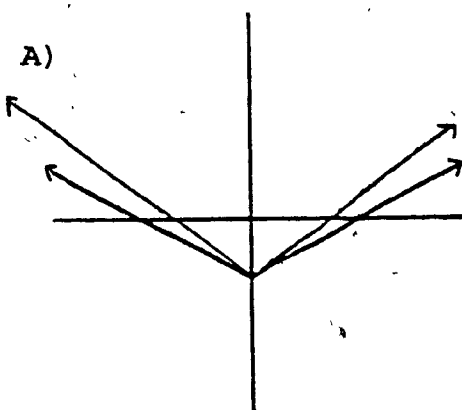
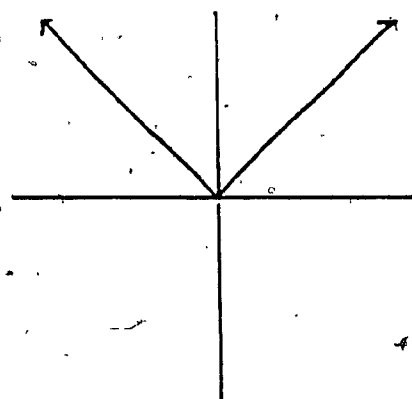
E)



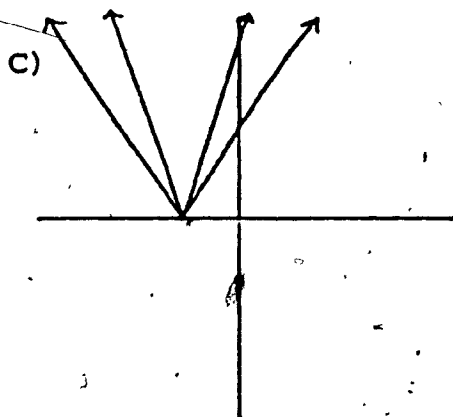
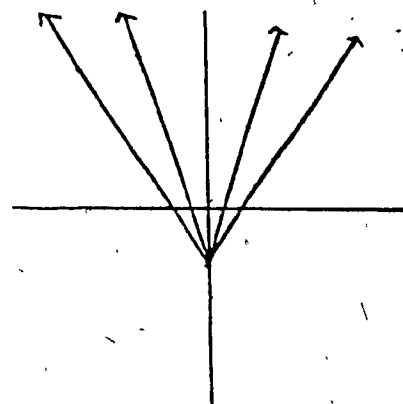
10. If the graph on the right represents some unknown function $y = g(x)$, then which of the graphs below best represents the function $y = -g(x - 9)$?



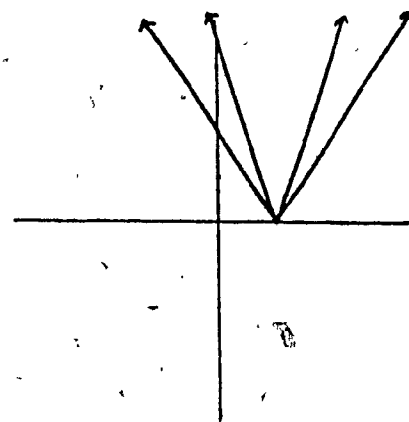
11. If the graph on the right represents the function $y = h(x)$, then which of the graphs below best represents the functions $y = 3h(x) - 4$ and $y = 5h(x) - 4$?



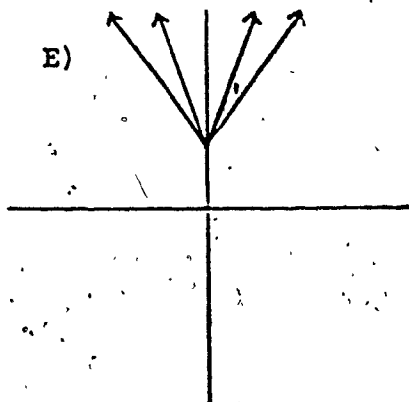
B)



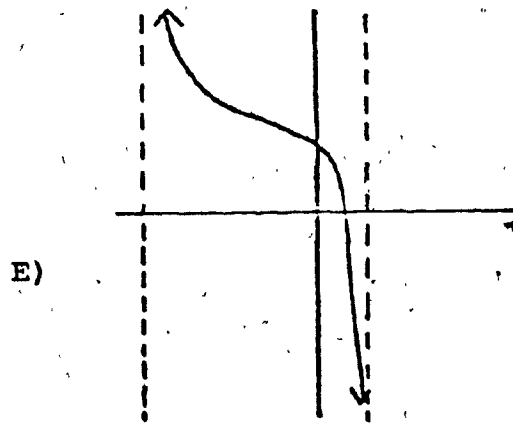
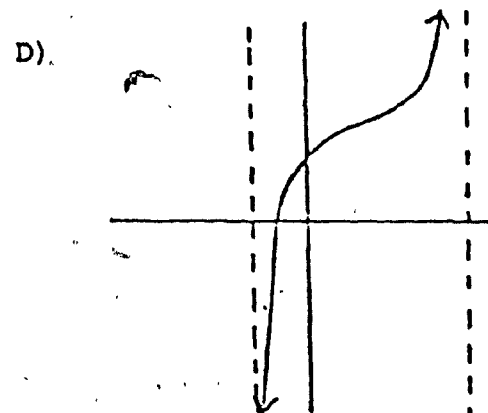
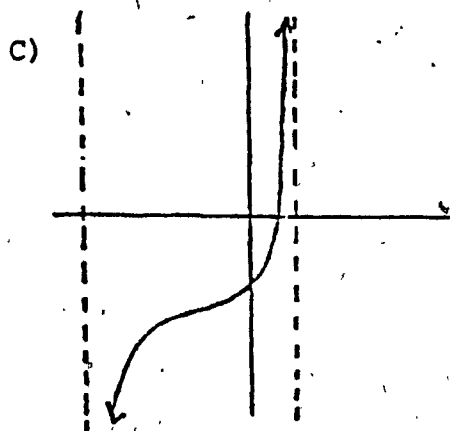
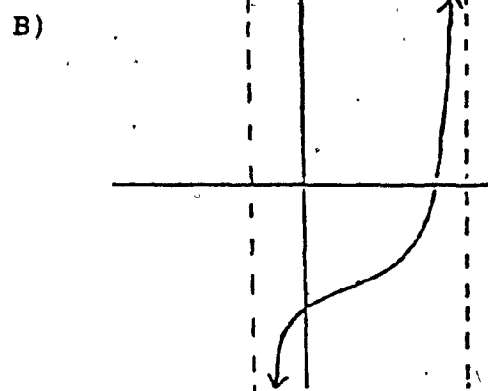
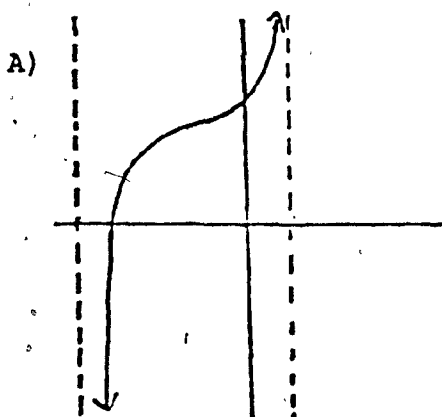
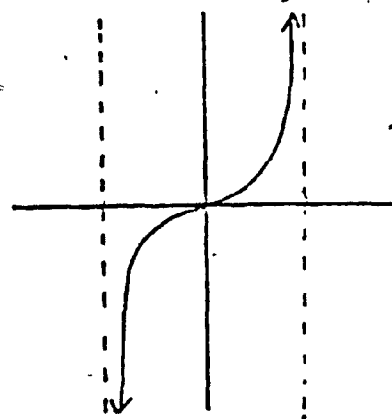
D)



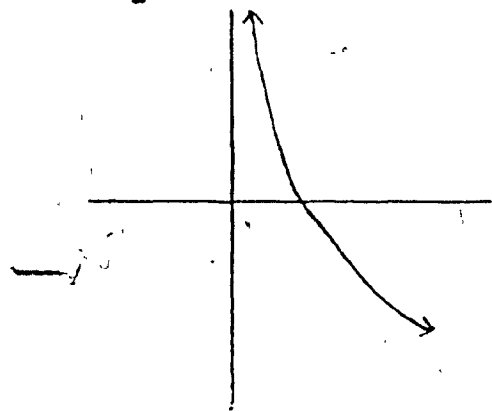
E)



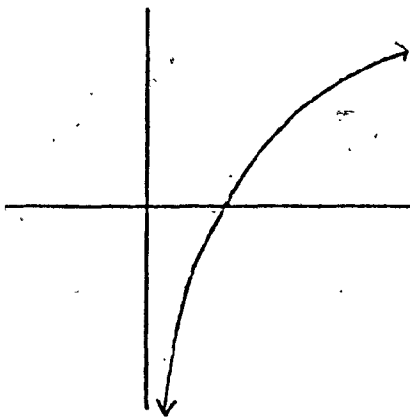
12. If the graph on the right represents the function $y = \tan x$, then which of the graphs below best represents the function $y = \tan(x + 4) - 7$?



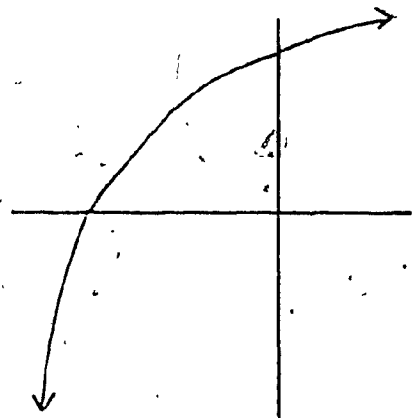
13. If the graph on the right represents the function $y = -3 \log x$, then which of the graphs below best represents the function $y = 3 \log x$?



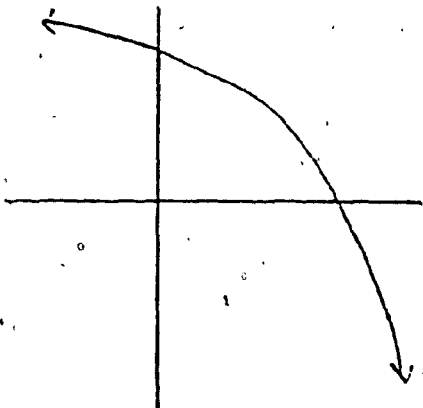
A)



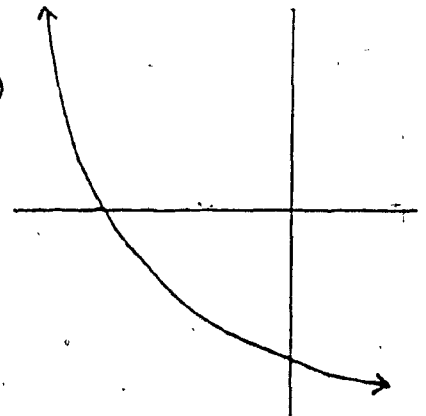
B)



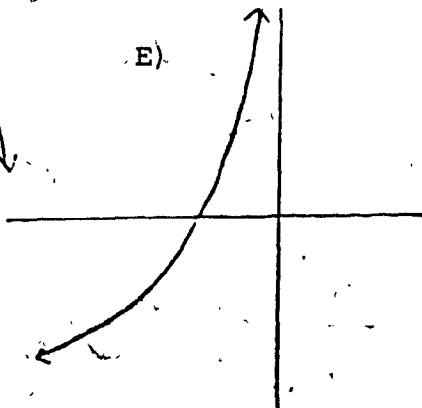
C)



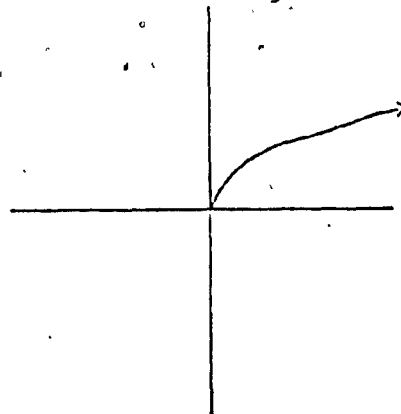
D)



E)

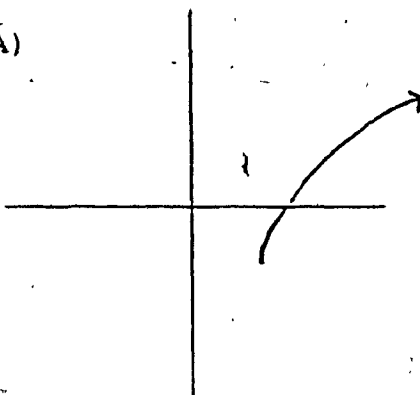


14. If the graph on the right represents the function $y = q(x)$, then which of the graphs below best represents the function $y = 0,5q(x + 3) - 3$?

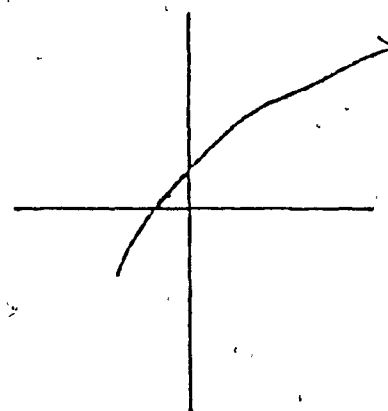


8

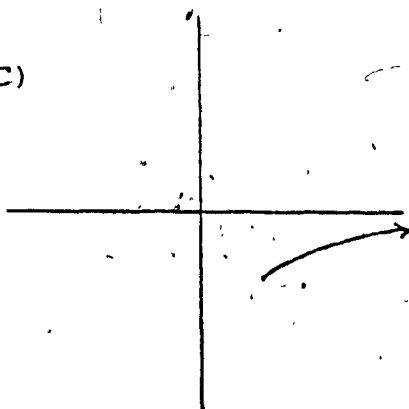
A)



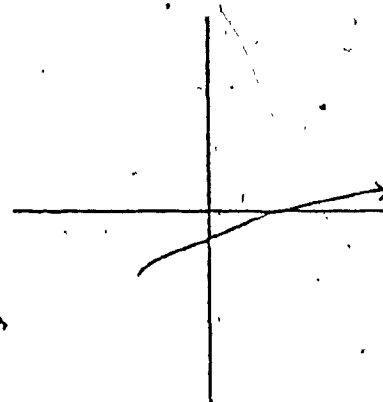
B)



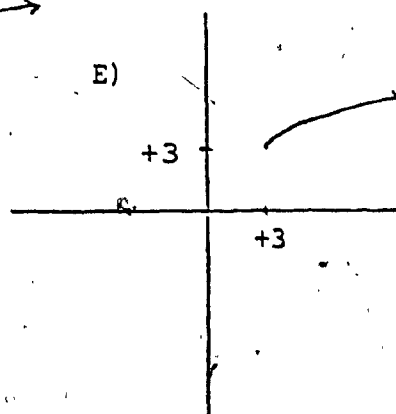
C)



D)



E)



Appendix G Program Listing

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140 IF K = 1 THEN HPlot C1,C2
500 REM MAIN PROGRAM
510 GOSUB 10000: REM TITLE
520 TEXT : HOME
530 VTAH 10: INPUT "ARE YOU WORKING ALONE?(Y/N)";R$
550 IF LEFT$(R$,1) = "N" OR LEFT$(R$,1) = "Y" GOTO 570
560 PRINT "ANSWER YES OR NO.": GOTO 530
570 IF LEFT$(R$,1) = "Y" GOTO 620
580 PRINT : INPUT "HOW MANY ARE YOU?";AZ
590 PRINT : PRINT "PLEASE TYPE IN YOUR FULL NAMES"
600 PRINT : PRINT "AND PRESS RETURN AFTER EACH ONE."
610 GOTO 640
620 PRINT : PRINT "PLEASE TYPE IN YOUR FULL NAME."
630 AZ = 1
640 FOR I = 1 TO AZ
650 PRINT : INPUT N$(I)
660 NEXT
670 PRINT : PRINT "THANK YOU"
680 FOR K = 1 TO 1500: NEXT K
690 GOSUB 5000: REM EXPLANATION
700 ENOZ = 0: REM NO. OF EXPERIMENTS
705 DIM NFZ(50),PA(50,3),PBZ(50,3),PCZ(50,3)
710 IF ENOZ = 5 THEN GOSUB 20000
711 IF ENOZ = 7 THEN GOSUB 21000
712 IF ENOZ = 9 THEN GOSUB 22000
713 IF ENOZ = 11 THEN GOSUB 23000
714 IF ENOZ = 13 THEN GOSUB 24000
715 ENOZ = ENOZ + 1
720 GOSUB 6000: REM COLLECT PARAMETERS
730 GOSUB 9000: REM DRAW AXES
740 GOSUB 8000: REM DRAW GRAPHS
750 GOSUB 7000: REM IDENTIFY FUNCTIONS
755 GOSUB 1000: REM STORE PARAMETERS IN ARRAYS
760 HGR : HOME : TEXT
770 VTAH 8
780 PRINT "DO YOU WANT MORE GRAPHS?(Y/N)";: GET R$
785 IF LEFT$(R$,1) = "N" OR LEFT$(R$,1) = "Y" GOTO 790
787 PRINT "ANSWER YES OR NO.": GOTO 780
790 IF LEFT$(R$,1) = "Y" GOTO 710
792 PRINT : PRINT : INPUT "ARE YOU READY TO TAKE THE TEST?(Y/N)";R$
794 IF LEFT$(R$,1) = "Y" OR LEFT$(R$,1) = "N" GOTO 796
795 PRINT : PRINT "PLEASE ANSWER YES OR NO.": GOTO 792
796 IF LEFT$(R$,1) = "Y" GOTO 800
798 PRINT : PRINT "IN THAT CASE YOU HAD BETTER STUDY": PRINT : PRINT "A F
    EW MORE GRAPHS."
799 FOR I = 1 TO 3000: NEXT I: GOTO 710
800 PRINT : PRINT "END OF PROGRAM"
810 GOSUB 2000: REM STORE ARRAYS IN TEXTFILE QUADATA
820 END
1000 REM *****
1005 REM STORE PARAMETERS IN ARRAYS
1010 NFZ(ENOZ) = N
1020 FOR I = 1 TO N
1030 PA(ENOZ,I) = A(I):PBZ(ENOZ,I) = BZ(I):PCZ(ENOZ,I) = CZ(I)
1040 NEXT
1050 RETURN

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2000 REM *****
2010 REM SUBROUTINE WHICH STORES ALL USER RELATED DATA INTO TEXT FILE.
2020 IS = " ": REM IS IS CTRL-D
2030 PRINT IS;"OPEN QUADATA"
2040 PRINT IS;"APPEND QUADATA"
2050 PRINT IS;"WRITE QUADATA"
2060 PRINT AZ: REM NUMBER OF STUDENTS AT TERMINAL
2070 FOR I = 1 TO AZ
2080 PRINT N$(I): REM THE NAMES OF EACH STUDENT
2090 NEXT
2100 PRINT ENOZ: REM NO. OF EXPERIMENTS
2110 FOR I = 1 TO ENOZ
2120 PRINT NFZ(I): REM NO. OF FUNCTIONS IN EXP. NO. 1.
2130 FOR J = 1 TO NFZ(I)
2140 PRINT PA(I,J): PRINT PB(I,J): PRINT PC(I,J): REM THE VALUES OF TH
E PARAMETERS IN THE J'TH FUNCTION OF THE I'TH EXPERIMENT.
2150 NEXT : NEXT
2160 PRINT IS;"CLOSE QUADATA"
2170 RETURN
5000 REM *****
5010 REM EXPLAIN PROGRAM
5020 TEXT : HOME
5040 PRINT "THIS PROGRAM IS DESIGNED TO HELP YOU"
5050 PRINT : PRINT "STUDY THE GENERAL QUADRATIC FUNCTION"
5060 VTAB 6: HTAB 11
5070 PRINT "Y = A(X - B)";: VTAB 5: PRINT "2";: VTAB 6: PRINT " + C"
5075 VTAB 9: PRINT "YOU WILL BE REQUIRED TO CHOOSE VALUES"
5080 PRINT : PRINT "FOR A, B, AND C IN THE FUNCTION ABOVE"
5090 VTAB 14: PRINT "IN THIS WAY, YOU MAY DESCRIBE UP TO '3'"
5100 PRINT : PRINT "DIFFERENT FUNCTIONS."
5110 PRINT : PRINT "THE COMPUTER WILL THEN GRAPH ALL"
5120 PRINT : PRINT "OF THESE FUNCTIONS FOR YOU ON A"
5130 PRINT : PRINT "SINGLE SET OF AXES."
5140 VTAB 24: INPUT "PRESS 'RETURN' TO BEGIN."#R$
5160 RETURN
6000 REM *****
6010 REM COLLECT PARAMETERS AND STORE IN ARRAYS A,BZ AND CZ(I)
6015 ONERR GOTO 6600
6020 TEXT : HOME : HTAB 12
6040 PRINT "DEFINE FUNCTIONS"
6050 HTAB (12): PRINT "-----"
6060 T = 7: N = 1
6070 VTAB 6: HTAB 11
6080 PRINT "Y = A(X - B)";: VTAB 5: PRINT "2";: VTAB 6: PRINT " + C"
6090 VTAB 11: PRINT "CHOOSE ANY REAL VALUE FOR 'A'"
6110 PRINT " FROM -9 ... TO ... +9, EXCEPT 0";
6120 INPUT A(N)
6122 IF A(N) = 0 OR A(N) < -9 OR A(N) > 9 GOTO 6126
6124 GOTO 6130
6126 VTAB 13: PRINT "ILLEGAL VALUE, TRY AGAIN.";: GOTO 6120
6130 VTAB T + 9: PRINT "FUNCTION ";N;" Y = ";A(N);
6135 CF = POS(0) + 1: REM HORZ. POSITION OF CURSOR
6140 GOSUB 6500
6145 VTAB 11: HTAB 1
6150 PRINT "CHOOSE ANY INTEGRAL VALUE FOR 'B'"
6160 PRINT " FROM -9 ... TO ... +9 ";
6170 INPUT BZ(N)
6175 IF BZ(N) > 9 OR BZ(N) < -9 GOTO 6172
6176 GOTO 6178
6177 VTAB 13: PRINT "ILLEGAL VALUE, TRY AGAIN.";: GOTO 6170
6178 VTAB T + 9: HTAB CF
6180 IF BZ(N) = 0 THEN PRINT "X";
6190 IF BZ(N) > 0 THEN PRINT "(X - ";BZ(N);)";
6200 IF BZ(N) < 0 THEN PRINT "(X + "; - BZ(N);)";

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6210 UTAB T + 8: PRINT "2";CP = POS (0) + 1
6220 GOSUB 6500
6225 UTAB 11: HTAB 1: PRINT "CHOOSE ANY INTEGRAL VALUE FOR 'C'"
6240 PRINT "FROM -40 .... TO +40 ";
6250 INPUT CX(N)
6255 IF CX(N) > 40 OR CX(N) < -40 GOTO 6265
6260 GOTO 6270
6265 UTAB 13: PRINT "ILLEGAL VALUE, TRY AGAIN.": GOTO 6250
6270 UTAB T + 9: HTAB CP
6280 IF CX(N) = 0 THEN PRINT ;
6290 IF CX(N) > 0 THEN PRINT " + ";CX(N);
6300 IF CX(N) < 0 THEN PRINT " - "; - CX(N);
6310 IF N = 3 GOTO 6390
6320 GOSUB 6500
6340 UTAB 11: HTAB 1
6350 PRINT "DO YOU WISH TO DEFINE"
6360 PRINT "ANOTHER FUNCTION?(Y/N)"; GET R$
6365 IF LEFT$(R$,1) = "N" OR LEFT$(R$,1) = "Y" GOTO 6370
6367 HTAB 1: UTAB 13: PRINT "ANSWER 'Y' OR 'N'. TRY AGAIN.": GOTO 6340
6370 IF LEFT$(R$,1) = "N" GOTO 6390
6380 N = N + 1: T = T + 4: GOSUB 6500: GOTO 6090
6390 GOSUB 6500: UTAB 11: HTAB 1
6395 INVERSE: PRINT "PRESS ANY KEY TO OBTAIN THE GRAPHS"; NORMAL
6400 GET R$: HOME
6405 POKE 216,0: REM TURN OFF ON ERROR
6410 RETURN
6500 REM *****
6510 REM ERASE LINES 11 TO 13
6520 UTAB 11: HTAB 1: FOR I = 1 TO 4: FOR K = 0 TO 39: PRINT " "; NEXT I
6530 RETURN
6600 REM *****
6610 REM INPUT ERROR HANDLING ROUTINE FOR PARAMETERS
6620 ET = PEEK (222)
6630 IF ET = 254 OR ET = 255 THEN 6640
6635 GOTO 6645
6640 UTAB 13: PRINT "ILLEGAL VALUE, TRY AGAIN.";
6645 ONERR GOTO 6600
6650 RESUME
7000 REM *****
7010 REM IDENTIFY GRAPHS
7040 REM PRINT EQUATIONS AND CORRESPONDING DOT SIZE
7050 HOME
7060 M(1) = 1:M(2) = 14:M(3) = 27: REM THESE ARE THE LEFTMOST STARTING POS
7070 ITIONS FOR PRINTING FUNCTIONS AND THEIR DOT SIZE
7080 FOR I = 1 TO N
7090 UTAB 22: HTAB M(I): PRINT "Y=";A(I);
7100 IF BZ(I) = 0 THEN PRINT "X";
7110 IF BZ(I) > 0 THEN PRINT "(X-";BZ(I);")";
7120 IF BZ(I) < 0 THEN PRINT "(X+"; - BZ(I);")";
7130 UTAB 21: PRINT "2";: UTAB 22
7140 IF CX(I) = 0 THEN GOTO 7160
7150 IF CX(I) < 0 THEN PRINT CX(I)
7160 IF CX(I) > 0 THEN PRINT "+"CX(I)
7170 NEXT
7180 REM NEXT LOOP PRINTS DOT SIZE BELOW EACH FUNCTION
7190 D$(1) = "SMALL DOTS":D$(2) = "MEDIUM DOTS":D$(3) = "LARGE DOTS"
7200 FOR I = 1 TO N
7210 UTAB 23: HTAB M(I): PRINT D$(I): NEXT
7220 GOSUB 11000
7230 RETURN
8000 REM *****
8010 REM GRAPH FUNCTIONS (UP TO THREE IN BLACK AND WHITE)
8050 K = 1: REM COUNT FUNCTIONS
8055 J = 0: REM FLAG TO SHORT CIRCUIT FOR X=-20 TO 20

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8060 IF A(K) > = 5 OR A(K) < = - 5 THEN S = .4: GOTO 8080
8065 IF A(K) < 1 OR A(K) > - 1 THEN S = 1: GOTO 8080
8070 S = .5: REM STEP SIZE
8080 FOR X = - 20 TO 20 STEP S
8090 YZ = A(K) * (X - BZ(K)) + 2 + CZ(K)
8100 IF YZ > 78 OR YZ < - 78 THEN 8155
8105 J = 1: REM EXIT WHEN YZ OUT OF RANGE
8110 REM CALCULATE C1 AND C2, VERT. AND HORZ. COORDINATES
8120 C1 = 6 * X + 140: C2 = - YZ + 80
8130 IF K = 1 THEN HPLLOT C1, C2
8140 IF K = 2 THEN HPLLOT C1 - 1, C2 - 1 TO C1 + 1, C2 - 1 TO C1 + 1, C2 + 1
      TO C1 - 1, C2 + 1 TO C1 - 1, C2 - 1
8150 IF K = 3 THEN HPLLOT C1 - 2, C2 - 2 TO C1 + 2, C2 - 2 TO C1 + 2, C2 + 2
      TO C1 - 2, C2 + 2 TO C1 - 2, C2 - 2
8153 GOTO 8160
8155 IF J = 1 THEN 8170
8160 NEXT
8170 K = K + 1: IF K < = N GOTO 8055
8180 RETURN
9000 REM *****
9010 REM AXES
9040 HGR : HCOLOR= 7: HOME
9050 HPLLOT 0,80 TO 279,80: HPLLOT 140,0 TO 140,159: REM HORZ. AND VERT. A
      XES
9060 REM DRAW SCALE
9070 FOR I = 1 TO 15: HPLLOT 138,10 * I TO 142,10 * I: NEXT : REM VERTICA
      L SCALE
9075 FOR I = 0 TO 8: HPLLOT 30 * I + 20,78 TO 30 * I + 20,82: NEXT : REM
      HORIZONTAL SCALE
9080 REM WRITE COORDINATES +20 AND -20 ON BOTH AXES
9090 H = 255: V = 85: GOSUB 9500: HPLLOT 249,88 TO 253,88: HPLLOT 251,86 TO 2
      51,90
9100 H = 15: GOSUB 9500: HPLLOT 9,88 TO 13,88
9110 H = 126: V = 58: GOSUB 9500: HPLLOT 120,60 TO 124,60: HPLLOT 122,58 TO 1
      22,62
9120 V = 98: GOSUB 9500: HPLLOT 120,100 TO 124,100
9130 RETURN
9500 REM *****
9530 REM '20' IN HIRES
9540 REM REM H AND V SPECIFY THE POSITION OF 20 IN THE PLANE
9550 HPLLOT H,V TO H + 1,V - 1 TO H + 2,V - 1 TO H + 3,V TO H + 3,V + 1 TO
      H,V + 4 TO H,V + 5 TO H + 3,V + 5
9560 HPLLOT H + 7,V - 1 TO H + 8,V - 1 TO H + 9,V TO H + 9,V + 4 TO H + 8,
      V + 5 TO H + 7,V + 5 TO H + 6,V + 4 TO H + 6,V
9570 RETURN
10000 REM *****
10010 REM TITLE 'QUAD'
10040 HOME : GR : COLOR= 14
10050 REM LETTER 'Q'
10060 HLIN 6,8 AT 15: HLIN 6,8 AT 22: VLIN 16,21 AT 5: VLIN 16,22 AT 9: PLOT
      7,20: PLOT 8,21: PLOT 10,23
10090 REM LETTER 'U'
10100 VLIN 15,22 AT 13: VLIN 15,22 AT 17: HLIN 14,16 AT 22
10120 REM LETTER 'A'
10130 VLIN 16,22 AT 21: VLIN 16,22 AT 25: HLIN 22,24 AT 15: HLIN 22,24 AT
      19
10150 REM LETTER 'D'
10160 VLIN 15,22 AT 29: VLIN 17,20 AT 33
10170 PLOT 30,15: PLOT 31,15: PLOT 32,16: PLOT 30,22: PLOT 31,22: PLOT 32
      21
10190 PRINT TAB( 19 ); "BY"
10200 PRINT : PRINT TAB( 14 ); "CLAUDE LEBEL"
10210 FOR I = 1 TO 5000: NEXT I
10230 RETURN

```

```

11000 REM *****
11010 REM PAUSE
11020 VTA 24: HTAB 1
11025 PRINT "PRESS 'C' TO CONTINUE.": GET C:
11030 IF LEFT$(C$,1) < > "C" THEN 11020
11040 RETURN
20000 REM *****
20100 HOME: VTA 10
20200 PRINT "REMEMBER"
20300 PRINT: PRINT "IT IS MORE EFFICIENT TO DRAW"
20400 PRINT: PRINT "2 OR 3 FUNCTIONS AT A TIME."
20500 GOSUB 11000: REM PAUSE
20600 RETURN
21000 REM *****
21100 HOME: VTA 10
21200 PRINT "REMEMBER"
21300 PRINT: PRINT "AT FIRST IT IS BETTER TO CHANGE ONLY"
21400 PRINT: PRINT "ONE PARAMETER AT A TIME WHILE"
21500 PRINT: PRINT "KEEPING THE OTHERS CONSTANT."
21600 GOSUB 11000: REM PAUSE
21700 RETURN
22000 REM *****
22100 HOME
22200 PRINT "IT IS A GOOD IDEA TO CHOOSE VALUES"
22300 PRINT: PRINT "WHICH ARE FAR APART IN ORDER"
22400 PRINT: PRINT "TO MAKE THE EFFECTS MORE VISIBLE."
22500 PRINT: PRINT: PRINT "CHOOSING VALUES OF 1, 2 AND 3"
22600 PRINT: PRINT: PRINT "WILL HAVE LITTLE EFFECT"
22700 PRINT: PRINT "WHILE CHOOSING VALUES OF 1, 5 AND 9"
22800 PRINT: PRINT "OR OF +10 AND -10"
22900 PRINT: PRINT "WILL HAVE A MUCH GREATER EFFECT."
22910 GOSUB 11000: REM PAUSE
22920 RETURN
23000 REM *****
23100 HOME: VTA 10
23200 PRINT "TRY A VALUE OF 'A' BETWEEN 0 AND 1."
23400 GOSUB 11000: REM PAUSE
23500 RETURN
24000 REM *****
24100 HOME: VTA 10
24200 PRINT "DON'T JUMP TO CONCLUSIONS."
24300 PRINT: PRINT "TEST YOUR IDEAS."
24400 GOSUB 11000: REM PAUSE
24500 RETURN

```

Appendix H Instructions to Subjects - Treatment

GROUP I

INSTRUCTIONS

1. You will work alone.
2. You are not to talk to anyone except the lab assistant.
3. You will have a maximum of one hour to solve the problem.
4. At the end of this time you will be given a multiple choice test.
You will have 30 minutes to answer 14 questions.
5. The mark you obtain on this test will count towards your
final grade in mathematics.

Sign below after you have read the instructions.

GROUP II

INSTRUCTIONS

1. You will work with a partner.
2. You are not to talk to anyone except your partner or the lab assistant.
3. You will have a maximum of one hour to solve the problem.
4. At the end of this time you will be tested individually.
You will have 30 minutes to answer 14 multiple choice questions.
5. The mark you obtain on this test will count towards your final grade in mathematics.

Sign below after you have read the instructions.

GROUP III

INSTRUCTIONS

1. You will work with a partner.
2. You are not to talk to anyone, except your partner or the lab assistant.
3. You will have a maximum of one hour to solve the problem.
4. At the end of this time you will be tested individually.
You will have 30 minutes to answer 14 multiple choice questions.
5. The mark you obtain will depend on how well your partner succeeds.
Each of you will receive the lowest mark obtained by either you or your partner. This mark will count towards your final grade in mathematics.

Sign below after you have read the instructions.

Appendix 1 Raw Data

| Subject Number | Exp. Group | Posttest | | | Delayed posttest. | | | No. Exp. | No. Func. | Focusing Strategy | Time (Min.) |
|-------------------|---------------|----------|-------|-----|-------------------|-------|-----|-------------|--------------|----------------------|----------------|
| | | Prt 1 | Prt 2 | Tot | Prt 1 | Prt 2 | Tot | | | | |
| 1 | 1 | 1 | 1 | 2 | 1 | 2 | 3 | 20 | 38 | 7.5 | 60 |
| 2 | " | 3 | 1 | 4 | 0 | 2 | 2 | 30 | 64 | 18.5 | 60 |
| 3 | " | 1 | 1 | 2 | 1 | 3 | 4 | 36 | 84 | 4.0 | 60 |
| 4 | " | 2 | 2 | 4 | 4 | 3 | 7 | 18 | 50 | 9.0 | 53 |
| 5 | " | 6 | 5 | 11 | 7 | 5 | 12 | 31 | 65 | 19.5 | 60 |
| 6 | " | 3 | 3 | 6 | 3 | 0 | 3 | 21 | 41 | 9.5 | 40 |
| 7 | " | 3 | 1 | 4 | 3 | 3 | 6 | 29 | 44 | 2.5 | 60 |
| 8 | " | 7 | 0 | 7 | 2 | 3 | 5 | 29 | 73 | 4.0 | 60 |
| 9 | " | 5 | 6 | 11 | 7 | 5 | 12 | 19 | 44 | 13.5 | 60 |
| 10 | " | 7 | 7 | 14 | 6 | 4 | 10 | 36 | 60 | 15.5 | 60 |
| 11 | " | 5 | 2 | 7 | 4 | 2 | 6 | 11 | 30 | 9.0 | 55 |
| 12 | " | 3 | 3 | 6 | 1 | 3 | 4 | 19 | 52 | 9.5 | 60 |
| 13 | " | 0 | 2 | 2 | 4 | 3 | 7 | 12 | 29 | 3.5 | 25 |
| 14 | " | 3 | 1 | 4 | 2 | 1 | 3 | 26 | 73 | 9.5 | 60 |
| 15 | " | 2 | 1 | 3 | 2 | 1 | 3 | 16 | 34 | 2.0 | 50 |
| 16 | " | 3 | 1 | 4 | 6 | 0 | 6 | 18 | 38 | 8.5 | 60 |
| 17 | " | 0 | 2 | 2 | 2 | 0 | 2 | 11 | 21 | 4.0 | 40 |
| 18 | 2 | 5 | 5 | 10 | 3 | 2 | 5 | 28 | 60 | 15.0 | 60 |
| 19 | " | 7 | 4 | 11 | 7 | 3 | 10 | " | " | " | " |
| 20 | " | 2 | 3 | 5 | 3 | 2 | 5 | 28 | 55 | 5.0 | 60 |
| 21 | " | 4 | 0 | 4 | 1 | 1 | 2 | " | " | " | " |
| 22 | " | 2 | 3 | 5 | 1 | 2 | 3 | 15 | 38 | 9.0 | 44 |
| 23 | " | 1 | 0 | 1 | 0 | 1 | 1 | " | " | " | " |
| 24 | " | 1 | 1 | 2 | 4 | 0 | 4 | 34 | 58 | 13.0 | 60 |
| 25 | " | 3 | 3 | 6 | 2 | 1 | 3 | " | " | " | " |
| 26 | " | 1 | 2 | 3 | 1 | 1 | 2 | 21 | 51 | 7.0 | 60 |
| 27 | " | 6 | 2 | 8 | 3 | 2 | 5 | " | " | " | " |
| 28 | " | 4 | 2 | 6 | 2 | 1 | 3 | 27 | 50 | 17.5 | 60 |
| 29 | " | 6 | 2 | 8 | 3 | 3 | 6 | " | " | " | " |
| 30 | 3 | 4 | 0 | 4 | 1 | 0 | 1 | 29 | 49 | 14.0 | 50 |
| 31 | " | 3 | 4 | 7 | 2 | 2 | 4 | " | " | " | " |
| 32 | " | 3 | 1 | 4 | 1 | 3 | 4 | 18 | 37 | 3.5 | 60 |
| 33 | " | 1 | 3 | 4 | 2 | 0 | 2 | " | " | " | " |
| 34 | " | 0 | 1 | 1 | 1 | 4 | 5 | 11 | 28 | 7.0 | 37 |
| 35 | " | 4 | 3 | 7 | 2 | 4 | 6 | " | " | " | " |
| 36 | " | 5 | 1 | 6 | 4 | 2 | 6 | 26 | 38 | 10.0 | 60 |
| 37 | " | 7 | 3 | 10 | 7 | 6 | 13 | " | " | " | " |
| 38 | " | 7 | 3 | 10 | 6 | 7 | 13 | 30 | 45 | 9.0 | 60 |
| 39 | " | 6 | 7 | 13 | 4 | 2 | 6 | " | " | " | " |
| 40 | " | 0 | 2 | 2 | 1 | 2 | 3 | 19 | 53 | 16.0 | 60 |
| 41 | " | 3 | 1 | 4 | 2 | 1 | 3 | " | " | " | " |
| 42 | " | 3 | 2 | 5 | 2 | 1 | 3 | 41 | 74 | 10.0 | 60 |
| 43 | " | 3 | 4 | 7 | 4 | 4 | 8 | " | " | " | " |
| 44 | " | 3 | 2 | 5 | 5 | 1 | 6 | 34 | 91 | 35.5 | 60 |
| 45 | " | 3 | 1 | 4 | 1 | 0 | 1 | " | " | " | " |
| 46 | " | 2 | 1 | 3 | 0 | 2 | 2 | 15 | 40 | 10.5 | 60 |
| 47 | " | 1 | 2 | 3 | 4 | 2 | 6 | " | " | " | " |