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DECISION BY EXCLUSION:

ON THE DESIGN OF THE BUILDING ENCLOSURE

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In memory of my late Father,

Joseph Naoum Mattar

ABSTRACT

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Design alternatives are usually chosen on the basis of cost; this has been the only currency by which "trade-offs" between disparate building quantities can be effected. The problem with these economic comparisons as a means of selecting design alternatives is that cost -- whether capital cost, annual cost, life cycle cost or some other kind -- is a very unreliable indicator. Complications are caused by the prevalent unstable prices for building work, variations among several cost indices and also variations in costs from one contractor to another.

In building enclosure design, decisions are made on the basis of physical units (fire resistances, sound transmission losses, risk of condensation and so on) as well as such variable quantities as economic units. Such design decisions can be made by a process of exclusion. In this process, the performance of design alternatives, for each objective considered, is assessed and normalized by means of utility transformations. Each alternative is then compared with the others generated in the design.

An alternative is dominated by the others if the sum of

its weighted attributes is less than that of at least one of the others for every possible set of positive weights. Initially-dominated alternatives are eliminated. If the performance objectives are ranked in priority, these constraints on the weights can be used to exclude more alternatives. The final selection by the designer is made from a reduced set of "*best*" or "*at least as good as*" alternatives obtained by progressively adding constraints attaching to the performance objectives.

This decision-making process is one of many stages in systematic design. A framework of a systematic design procedure is presented and illustrated by means of an example involving the design of external walls for single-family houses in the Montreal area. The sensitivity of decisions to changes in the designer's input is also considered.

In implementing the Decision by Exclusion rule, the design method has been objectified. The application of the Decision Rule to design will encourage the use of a wider range of alternatives. It will also encourage the use of research findings. Because of its versatility in making use of the limited information available in ranking of performance objectives' priorities, the Decision by Exclusion rule is applicable to a wide range of problems.

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No research is completely isolated from other areas, and, in this study, interaction with a number of people from widely-different disciplines has provided me with valuable insights. I wish to thank both Concordia University and Nova Scotia Technical College for the resources which were put at my disposal while I conducted the research.

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NOTATIONS

Because of the wide scope of the thesis it has been necessary to divide the notations into three parts:

N-1 NOTATION FOR THESIS

A,B,K	Coefficients used in utility function
C	Initial cost
$\bar{e}_j$	The j^{th} unit vector
F	Factor used in parametric analysis
f_r, f_s	Positive real numbers
FR	Fire resistance
J_o	Parametrized variable
n	Number of performance objectives
m	Number of alternative constructional system
$\bar{o}$	Zero vector
R	Risk of condensation
R_y	A weak ordering relationship
r_i, r_s	Any indices between 1 and n
s	Sound transmission loss
T	Thermal resistance
u_{ij}	The utility of an alternative i with respect to performance objective j
v_i	Sum of weighted utilities of alternative
$\bar{x}$	m-dimensional row vector
Y_i, Y_j, Y_k	Performance variables in the set {Y} of variables considered in design

N-2 NOTATION FOR APPENDIX III

A	Fraction of "g" assigned as horizontal ground acceleration
D, L	Specified dead and live loads, respectively
h	Height of wall
R	Resistance of wall or thermal insulation depending on context (Limit state design)
STL	Sound transmission loss
S_p	Horizontal force factor for earthquake
Q	Air infiltration rate
W	Lateral load
W_p	Weight of fixture
ϕ	Performance factor
γ	Importance factor
$\alpha_D, \alpha_L, \alpha_W$	Variability factors for dead, live and wind loads
Δ	Deflection of wall
δ	Structure vibration
Y_{ij}	Level of performance of an alternative i with respect to an objective j
Y_{mj}	Most desirable limit on performance objective j
Y_{Tj}	The least acceptable limit on performance objective j
Z	Objective function in linear programming formulation
ω_j	The weight attaching to performance variable j
$[C]_{t \times n}$	An $(t \times n)$ matrix where t is the number of linear constraints on weight attaching to performance objectives
$[U]_{m \times n}$	The matrix of attributes
$\{C\}$	The convex set of non-dominated alternatives

N-2 (continued)

$\{\tilde{C}\}$	The convex set of non-dominated alternatives with priorities on objectives
$\{i\}$	A set of candidate solution or alternatives
$\{W_i\}$	The set of weights which will result in alternative i being selected
$\{Y_j\}$	Set of performance variables or performance objectives established by the CIE master list of properties
λ	Mathematical weight used in development of decision rule
μ	Scalar quantity

N-3 NOTATION FOR APPENDIX IV

A	Area normal to flow of heat
b	Spacing of line studs
c	Velocity of sound in air
d	Separation of panels in a double panel construction
f	Frequency
f_B	Bridging frequency
f_c	Critical frequency for single panel
f_ℓ	Limiting frequency for double panel
f_o	Fundamental double panel resonance
f_i, f_o	Surface coefficients of the air flow inside and outside, respectively
h	Thermal conductivity
M	Total mass of multiple panel per unit area
ℓ	Length of flow path (thickness)
m	Mass of panel per unit area
m_1, m_2	Mass of panel 1,2 per unit area
m'	Effective mass of double panel for determining f_o
P_i, P_o	Vapour pressure inside and outside
q	Heat flow rate
R_T	Total thermal resistance
$R_1, R_2, \ldots R_n$	Individual resistances
$TL(f)$	Transmission Loss of Construction at frequency (f)
$TL_1(f)$ $TL_2(f)$	Transmission Loss for panels 1 and 2 at a frequency (f)
$TL_I(f)$	Transmission loss at frequency (f) of a multiple panel with no inter-panel connections

$TL_M(f)$	Transmission loss at a frequency (f) according to mass law
ΔTL_M	Increase in transmission loss over that calculated according to the mass law
t_i, t_o	Inside and outside temperatures, respectively
V_T	Overall vapour resistance
W	Weight of water vapour transmitted through a unit area in unit time
μ	Permeability of the material
$\bar{\mu}$	Average permeability

CHAPTER 1

DECISION IN BUILDING DESIGN

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1.1 INTRODUCTION

In pre-industrial societies, the materials and methods of construction changed slowly. Buildings were erected by craftsmen familiar with the materials available in their locality, the kind of building wanted, current techniques of construction and local climatic conditions. Consequently, the enclosure of a building tended to be reasonably well-adapted to its functions and to the climate. Generally-accepted vocabularies of design and material usage enabled ordinary builders to provide environments which ministered to the needs and requirements of their building owners and users.

With the technological advances since the industrial revolution, there has been a proliferation of new building materials, constructional systems and methods. Building designers* -- architects, building engineers and others -- are aware that:

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* Throughout this study, the words "*building designer*" and "*designer*" have been used instead of "*architect*". Traditionally, it has been architects who have been responsible for designing the enclosure of buildings; the use of the word "*designer*" in place of "*architect*" now reflects the increasing participation of other professions such as engineers -- building-, materials-, and so on -- and developers, in addition to architects, in the design of the building enclosure.

> *"Their conventional methods of satisfying new requirements and making use of new constructional possibilities are no longer adequate."* (1)

The attitudes and practices of the past -- the consensus of opinions on how buildings ought to be built -- whose consequence was to convert requirements into successful buildings, have to give way to new approaches more suited to changing conditions.[2] The changes are not only in the means of construction but also in people's continually-varying requirements and use of buildings as well as their increasing standards of expectations.

Although there seems to be no consensus among building designers on what the new approaches to design should be, it is clear that they must adopt a disciplined (systematic, even) framework* for design activities if they hope to make

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* Design performed by the congenitally unmethodical has resulted in much wasteful and inefficient expenditure of effort and creative skill exploring routine problems. Consequently, second-best results have often been accepted as design solutions when better ones already exist. It would be reasonable to expect that method in building enclosure design would be beneficial to designers. However, the effectiveness of methodical approaches to design (a goal-directed problem-solving activity) is to a great extent determined by the tools available and the skill and willingness with which they are employed. Systematic method used by designers combining the art and science of building offers a framework for making greater use of the designer's imagination and for the conscious application of knowledge.

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1 P.N. Manning, "Appraisals of Building Performance and Their Use in Design", Vol. 1, Synthesis, Ph.D. Thesis, University of Liverpool, 1967.

2 A.J. Elder, AJ Handbook of Building Enclosure. The Architectural Press, London, 1974.

effective use of developments in the many other disciplines that bear upon design. For instance, system analysis techniques can be used for developing performance objectives while methods in operations research (e.g., Linear Programming) and decision analysis can be adopted in the evaluative phase of the systematic design process. Such developments, however, have only become practicable with the advent of computers.

1.2 SOME ASPECTS OF BUILDING ENCLOSURE DESIGN

It is the nature of studies of building design problems to be complex because they involve the simultaneous consideration of many interesting factors. Though much design decision-making is done "*intuitively*", on a subjective assessment of whatever facts are available, a theoretical model of the structure of the design process necessarily must rely on many techniques embodied in the structure of such subjects as philosophy, value theory, decision theory, psychology, mathematics, economics, building engineering, theory of measurement, and so on, for only in this way can the large variety of activities and aspects of mind and personality involved in design be reflected in a formal way.

One very simple view of the design process is that a designer first identifies the performance requirements and then selects from among all the possible solutions that solution which meets the objectives most nearly optimally. This is the way in which, in practice, a designer might design (say) the enclosure of buildings. However, concealed in this simple concept is a large collection of uncertainties, conflicts and ambiguities: For example, the building enclosure is concerned, *inter alia*, with physical and economic needs, yet it is seldom possible to determine with any certainty what priorities may be ascribed to these needs. Or, it is possible that at least some of the performance objectives are in conflict: that, while making improvements in one aspect of a constructional system's performance, another aspect is concurrently worsening. The formulations of design programs are often vague and ambiguous, so even the definition of a set of objectives is liable to be fraught with difficulties.

Basic to the design of the building enclosure is the definition of performance objectives. The specification of performance objectives involves the reconciliation of two complementary types of information:

(i) The owner's requirements (i.e., what is wanted or needed), and

(ii) The constraints acting upon the design (i.e., factors which control the fulfillment of the requirements such as codes and by-laws).

It is therefore essential to identify the requirements and constraints as well as general background information about the context of the design problem (e.g., site and climatic conditions). The performance objectives specify the desired level(s) of performance in terms of criteria or "*least acceptable*" limits and, in some cases, "*most desirable*" limits as well. Thus, the objectives are considered as performance variables in which some degree of achievement is desirable.

If several solutions are proposed to meet the performance objectives, a decision must be made to adopt one. The alternative selected should be better than or at least as good as the other solutions from the standpoint of overall performance. In order to compare the effectiveness of the alternative solutions it is necessary to measure their performance even if the measurements are crude and approximate. A set of analytical models is needed to predict the performance of alternatives without favouring any candidate solutions. These analytical models enable the designer to quantify the performance of any or all of the alternatives. The degree to which a candidate solution satisfies the desired level of achievement of an objective is ascertained by comparing the performance score against the limits specified on the objectives.

Different features of the building enclosure are measured in different ways using different types of measuring scales. And, each feature is liable to be appraised to a different level of precision, depending on the way in which the objective has been defined and on the nature of the prediction obtained from the analytical models. The most useful measurements -- and the ones with which designers are most familiar -- are those made on interval scales. In the design of external walls, for example, the limits of desirability for thermal resistance provide the basis for an interval scale against which the predicted performance of a wall can be measured. For thermal resistance measurements, the alternative's performance can be predicted reasonably accurately and the intervals on the scale are regular. In many cases, however, comparative relationships rather than precise relationships between the alternatives, although not so adequate, have to suffice. In estimating the risk of condensation in the enclosure, for example, the designer has to make informed judgements about the chances of condensation occurring in the constructional system. In such instances the designer can only rank the performance of alternatives, and the risk-of-condensation scale is thus an ordinal scale reflecting both the approximate nature of the prediction and the definition of the objective. When the performance objective is specified only in terms of a criterion, the performance of an alternative is compared with that criterion, and the alternative meets or does not meet the desired level of performance. This criterion (and its nominal scale) provides a basis for appraising whether minimum standards have been met and cannot be used to differentiate between candidate solutions.

In the design of the building enclosure, the main difficulty facing designers choosing among alternatives is that constructional systems have many attributes. For example, a designer may want a solution that provides good thermal insulation, high fire resistance, low risk of condensation, adequate sound transmission loss and low cost. But, solutions generated for the design are likely to possess widely different characteristics on each of the performance variables -- good on some, poor on others, indifferent on many -- so that there is no readily-evident "*best*" alternative. The difficulty is enhanced by uncertainty about the way in which requirements may interact one with another. Arbitrating between interacting requirements and keeping track of all the relevant variables during the course of the design process imposes a heavy cognitive load on the designer, particularly if the problem is complex with a large number of variables[3,4] -- which is certainly the case with the process of designing a building's enclosure. How, then, are alternatives to be compared when they are likely to have these different properties?

1.3 DECISIONS IN BUILDING DESIGN

Three methods of decision-making in building design can

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[3] G.A. Miller, "The Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Processing Information", Psychological Review, Vol. 63 (1956), pp. 81-97.

[4] D.B. Yntema and G.E. Mueser, "Remembering the Present States of a Number of Variables", Journal of Experimental Psychology, Vol. 60 (July 1960), pp. 18-22.

be discerned[5]. In these methods, the decisions are either:

(i) of a rather rough-and-ready kind, made in a not-very-systematic way, but using understanding derived from practical experience;

(ii) made by assigning a common unit (e.g., dollars) to performance variables and evaluating alternatives in terms of these common (e.g., monetary) units; or

(iii) based on some manner of aggregating the performance attributes after they have been valued independently.

Perhaps the most generalized model of the decision process is represented by the evaluation matrix (Figure 1.1). The matrix provides a simple tabulation of the level of achievement (represented by a quantity) afforded by each alternative on each of the objectives. Besides showing each alternative's degree of achievement on the objectives, the numbers in the evaluation matrix may reflect the relative importance to the designer of each performance objective. A crude overall measure of the effectiveness of an alternative is then found by summing the attributes in

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5 D.P. Grant ["The Problem of Weighting", DMG-DRS Journal, Vol. 8, No. 3, (Jul-Sept 1974), pp. 136-141] hints in this article that building design decisions can be classified into these three methods.

		OBJECTIVES					
		1	2	....,	i	.....	n
ALTERNATIVES	1						
	2						
	⋮						
	j						
	⋮						
	m						

FIGURE 1.1 DECISION EVALUATION MATRIX

the relevant row of the matrix. However, in the application of this theoretically elegant and simple concept, difficulties of a fundamental nature are encountered. In arriving at a single numerical value to enter into the cell of the matrix, many diverse considerations have to be combined. This, though, may not be possible in the design of the building enclosure for the reasons that follow.

Firstly, raw performance measurements have to be converted into levels of achievement. This would be done by normalizing the raw performance scores (which are based on diverse units of measurement) to a percentile scale by means of transformation functions. These transformation functions can reflect the utility the designer attaches to each performance objective. For example, transformation functions based on the law of diminishing marginal utility may be used when greater sensitivity to achievements at low levels of performance is desired. Thus, a designer would probably consider a small increase in (say) fire resistance at levels close to the minimum acceptable limit more beneficial than a similar increase further along the scale.

Secondly, it is conceivable that raw performance scores are obtained from other-than-interval scales. The transformation to percentile scale of measurements ~~from a scale~~ lower than the interval -- say, from an ordinal scale -- could be done via step-functions. Though permissible, this would be limiting: it is not meaningful to perform arithmetic operations (such as addition) on numbers which are, at best, crude expressions of judgements.

Finally, there is the problem of measuring the relative importance the designer attaches to the various performance objectives. The most common and most reliable method for expressing judgements concerning the relative importance of objectives is to rank them.[6] Ranking of performance objectives by order of importance results in ordinal measurements. However, it is not possible to derive a set of weights (i.e., interval scale measurements) from the ranking method nor to make use of the rank measurements in arithmetic operations.

Limitations on the use of normal additive decision models in design can be overcome by using the *Decision by Exclusion* Rule which it has been the purpose of this thesis to establish. The Decision by Exclusion Rule can be likened to a version of the evaluation matrix at Figure 1.1. Each alternative's utility or performance attributes obtained as a result of the transformations of the raw scores are entered into the appropriate cells of the matrix. These attributes,

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6 R.T. Eckenrode ["Weighting Multiple Criteria", Management Science, Vol. 12, No. 3, 1964, pp. 180-192] in reporting this finding, adds that ranking needs no training and requires the least time to use of all the methods compared. The explicit ordering of the priorities on the objectives is an approximate rationalization of a normally subjective process. Thus, the qualities associated with ranking tend to support the assumption that designers are more likely to rank preferences than (say) rate them or use the method of successive comparison.

however, are not weighted. Initially, the exclusion process is based on the comparison of the attributes -- pure dominance* being an obvious and special case. Further exclusion of alternatives may be achieved by taking into account the priorities among the objectives. This usually results in the constriction of the decision space. When there are several "*optimal*" solutions, a more exacting specification of priorities reduces the number of "*optimal*" solutions. Unless the priorities are quantified, it is unlikely that one "*best*" solution will emerge.

1.4 AIM OF THE RESEARCH

The purpose of the study is to develop a rational and realistic method for making decision in design. Owing to their complexity, the numerous and varied steps of the design decision-making process are delineated in a systematic manner. Design methods pertinent to the determination of a building's enclosure are reviewed critically and so the design model proposed in this thesis is put in context. The operational model is then described in some detail with considerable emphasis on the basis for and theoretical development of the decision rule. The attendant postulates and analyses are also discussed. An appreciation of the

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* The dominance model assumes that within a variable, the decision-maker can compare any pair of alternatives with regard to value, for all variables taken singly. However, the model does not consider comparisons between objectives.

design decision-making process in the context of other decision-making procedures is also presented.

In order to demonstrate the application of the decision model that has been developed in this particular study, the scope has been restricted to the design of the enclosure of single-family houses -- and external walls of such houses specifically. Nevertheless, the use of the decision model can readily be extended to the design of other building sub-systems, the evaluation of building alternatives at the planning stage, and even to the assessment of development plans.

CHAPTER 2

DECISION BY EXCLUSION : ON THE DESIGN OF THE BUILDING ENCLOSURE

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2.1 INTRODUCTION

It is possible to identify two approaches to the systematic design of the building enclosure.[1] In one approach, the unknown variables are in the constituent material properties, component configuration and constructional factors which are to be selected to fulfill a set of design requirements while simultaneously optimising an objective function. In contrast, the other approach treats feasible and admissible constructional systems whose properties can be determined by analysis or experiment. The design best-suited to the use of this second approach is the one that optimises an objective function such as cost or weight. Until recently, the objective function in both approaches had invariably been in terms of one variable (structural or monetary) while the remaining requirements were considered as constraints. But this method of decision-making -- on the basis of a single objective function -- imposes severe limitations on the designer. It is, for example, difficult for him either to trade-off between various performance attributes or to express preferences between performance requirements.

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[1] C.C. Chamis, "Closing Materials Research Structural Design Cycle", Proc. ASCE, Journal of Engineering Mechanics Division, EM5 (October 1969), pp. 1255-68.

2.2 A REVIEW OF SYSTEMATIC METHODS OF BUILDING ENCLOSURE DESIGN

In an attempt to formulate an optimal multi-factor design procedure for constructional systems, Krokosky[2,3] devised a method which accommodates the designer's preferences within each performance requirement. Each performance objective (in this case structural, thermal and acoustic) is specified in quantitative terms of most- and least-desirable values, together with different levels of desirability of the performance attributes in the form of a ranking matrix reflecting the attribute's worth to the designer. An ideal multifunctional material which meets the design requirements optimally is then sought, using a random search technique. For ease of computation, Krokosky expresses various material properties relevant to the design problem in terms of a "*prime unifying material parameter*".[4] Though he works in this way, it is not always possible to correlate all of the performance variables in terms of a prime parameter (e.g.,

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[2] E.M. Krokosky, "The Ideal Multifunctional Constructional Material", Proc. ASCE, Journal of Structural Division, Vol. 94, ST4, Proc. Paper 5896 (April 1968), pp. 959 - 981.

[3] E.M. Krokosky, "Optimal Multifunctional Material Systems", Proc. ASCE, Journal of Engineering Mechanics Division, Vol. 37, EM2 (April 1971), pp. 559 - 575.

[4] E.M. Krokosky, *op. cit.* (2).

density).* Much more seriously, it is also doubtful whether, in practice, any solution obtained from this design procedure would actually be constructed, for it is unlikely that any materials available would possess the assumed ideal multi-functional properties.

The procedure developed by Krokosky contains an implicit assumption of indifference between performance requirements and the design solution is considered to be as desirable as its least desirable attribute on the ranking vectors.[5] But the relative importance of the various performance requirements cannot be taken into account in this design process. This is a serious weakness because designers often wish to express their priorities between requirements. These priorities have a significant effect on the solution selected. Rao, *et al*,[6] have discussed at length some further merits and shortcomings of Krokosky's scheme.

The Building Research Group in Japan has tackled the problem of optimal material choice more directly -- in a

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* It should be noted that Krokosky's method belongs to the first approach (performance characteristics unknown) described above.

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5 *Ibid.*

6 J.K.S. Rao, V.K. Kapur and C.V.S.K. Rao, "Discussion of Paper by E.M. Krokosky", *op. cit.* p. 7, Proc. ASCE, Journal of Engineering Mechanics Division, EM6 (December, 1971) pp. 1750 - 1753.

systemic and systematic way.[7,8,9,10] The successful application of this method depends on the existence, in a readily-accessible form, of an extensive information system which takes into account both design context and individual material properties in their multitudinous variations. This procedure, though, offers no shortcut from systematic computation and evaluation of alternative materials. It virtually disregards the problem of preferences between performance variables and -- a crucial omission -- does not attempt to reassemble the alternatives into the overall design.

The design decision-making process presented in this thesis attempts to avoid some of the shortcomings noted in both Krokosky's and the Japanese Building Research Group's methods. Its use is restricted to constructional systems

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7 Building Research Institute, Ministry of Construction, Building Research Group, Japan (March 1965), Forty-Fourth Report: On the Systematic Method for Selecting Building Materials.

8 Building Research Institute, Ministry of Construction, Building Research Group, Japan (March 1968), Fifty-First Report (second report): On the Systematic Method for Selecting Building Materials.

9 Building Research Institute, Ministry of Construction, Building Research Group, Japan (November 1968), Thirty-Sixth BRI Research Paper: On the Systematic Method for Selecting Building Materials.

10 Building Research Institute, Ministry of Construction, Building Research Group, Japan (January 1970), Fifty-Sixth Report (third report): On the Systematic Method for Selecting Building Materials.

available (or possible within the constraints of professional practice) whose constituent material properties are known.* Moreover, in this method, the design process is undertaken at a higher level in the hierarchy of design decisions -- specifically, at the level of sub-components rather than at the atomistic (or very detailed) level to which the Japanese procedure is confined (Figure 2.1). In addition to the variations due to the use of different materials, a designer using the method expounded in this work develops and identifies a large number of feasible and admissible design solutions that result from changes in configuration and dimension. Where it is feasible ~~to do so~~, the performance requirements are specified in terms of "*least acceptable*" and "*most desirable*" values, and preference between requirements can be taken into account in the decision rule. While the application of the decision rule may not necessarily result in a unique design solution, its use results in a reduced set of "*optimal*" alternatives reflecting both the values of the designer and the limitation imposed by the nature of value measurement, i.e., ranking or ordinal measurement.

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* This approach corresponds to the second (feasible and admissible alternatives) one outlined at the beginning of this section.

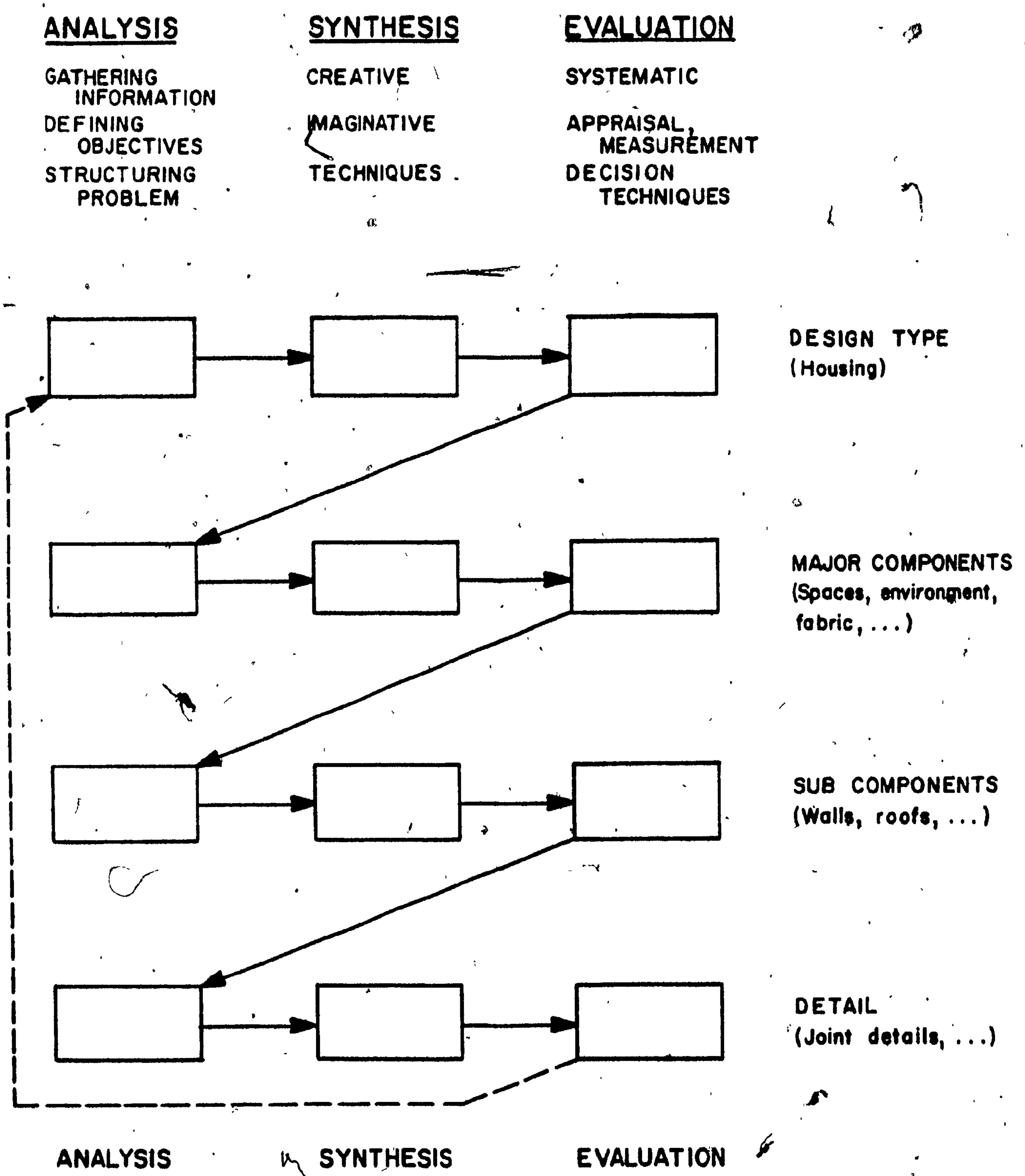

FIGURE 2.1. GENERAL STRUCTURE OF DESIGN (After Manning, op. cit.(13).)

2.3 OPERATIONAL STRUCTURE FOR THE DESIGN OF THE BUILDING ENCLOSURE*

The modelling of design-decision-making processes is fraught with difficulties. Notwithstanding these, most descriptions of the activities involved in design recognize a basic pattern.[11] Page,[12] in summing up the Conference on Design Methods, describes the pattern thus:

> *". . .there only seems to be one common point of agreement, and that is that systematic design is a three-stage process demanding analysis, synthesis and evaluation. . ."*

The differences between methods reflect the variations in design problems and variations in the operating mode of individual designers.

Manning,[13] in proposing a design framework (Figure 2.1) suggests that design comprises a sequence of decisions leading from relatively abstract terms to the final specification of hardware. A need (say, for housing) is recognized and the designer attempts to satisfy this need by the design and construction of new buildings. The first task is the

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11 See, for instance, the numerous descriptions in J.C. Jones, Design Methods: Seeds of Human Futures, Wiley-Interscience (1970).

12 J.K. Page, A Review of the Papers Presented at the Conference, in J.C. Jones and D.G. Thornley (eds.) Conference on Design Methods, (London, 1962) Pergamon Press (1963).

13 P.N. Manning, "Appraisals of Building Performance and their Use in Design", Vol. 1, Synthesis, Ph.D. Thesis, (Liverpool University, 1967).

understanding, structuring and statement of the design problem. This corresponds to the design method phase of analysis and comprises the gathering of relevant information, the definition of constraints and objectives and, where feasible, the definition of relationships between objectives. The writing of the design program (as the activity of specifying the design requirements of a building is termed in North America)

> ". . .*is probably the most crucial aspect of the entire design process, a clear statement being essential to a successful solution. . .*" (14)

Programming is fraught with substantial difficulties

> ". . .*arising from the fact that the different parties to a building project have different interest in and different expectations of it: their requirements will differ according to the viewpoint from which they see the building.*" (15)

The design context, (i.e., the set of environments: physical, economic, aesthetic, technological, social and so on) affects the requirements embodied in the program. But, during the early phases, at least, of a design program, it may not be obvious how and to what extent the context determines the performance requirements. Furthermore, the party responsible for drawing up the program may not be the designer and may not know what information is needed or

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14 *Ibid.*

15 *Ibid.*

available. The uncertainty in defining the performance requirements is reflected in the steps involved in stating the problem (Figure 2.2).

2.3.1 Performance Objectives

The precise specification of limits of a performance requirement results in a performance objective. The <u>CIB Master List of Properties</u>[16] has been used to establish a detailed set of performance variables or performance objectives $\{Y_i\}$. When appropriate, a performance criterion, corresponding to the least acceptable value of the variable, is defined for each design context. In some instances, a performance in excess of the least acceptable limit is desirable. In such cases, an upper limit corresponding to the most desirable or useful value of the variable, beyond which no advantage will be gained, is defined. The program or technical limitations may afford guidance for the definition of the upper bound while codes and other legal documents delimit the least acceptable value of the performance variable. Thus the performance objective on each variable is defined either by a criterion (e.g., acceptable/unacceptable) or by a range of desirable limits (least acceptable to most desirable).

The range of each performance objective (i.e., the

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16 <u>CIB Master List of Properties for Structuring Documents Relating to Buildings, Building Elements, Building Components, Materials and Services</u>, Report No. 18, International Council for Building Research Studies and Documentation, Rotterdam, 1972.

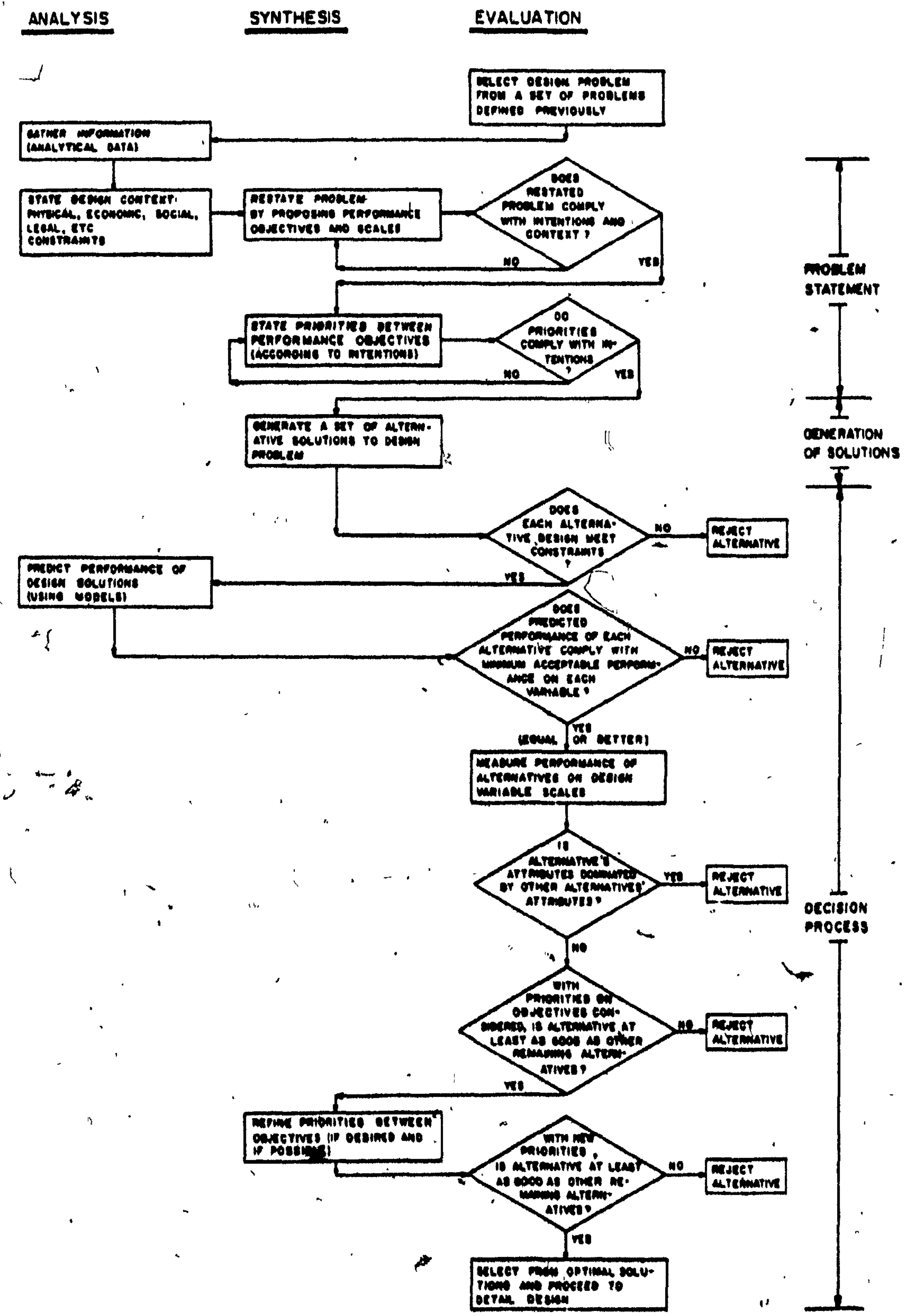

FIGURE 2.2 OPERATIONAL FRAMEWORK OF DESIGN SEQUENCE FOR COMPONENTS OF BUILDING FABRIC

[DEVELOPED FROM R MANNING, op. cit.(1).]

difference between upper and lower bounds) is normalized to a percentile scale so that any distortions in the relative importance of the variables in the design due to variations in the magnitude of actual measurements are eliminated. In the case of ordinal scales, the rankings are prescribed, fixed values on the percentile scale. For the interval scales, however, the transformations to the percentile scale may be linear or on the basis of other utility functions (such as the law of diminishing marginal utility).[17]

2.3.2 Priorities Among Performance Objectives

Wherever a decision exists, preferences based on a system of values may be exercised. Whether a designer will value one performance objective over another will vary from person to person, from circumstance to circumstance and from time to time. Furthermore, some individual designers may make their choices according to well-formulated systems of values while others may be capricious. A design decision-making model can do no more than represent and predict the consequences of the designer's preferences. Thus there is no methodology which will substitute for the designer's identifying the performance variables, defining limits of acceptability and the utility of the objectives, and expressing

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[17] The limitations on the type of mathematical transformation possible with each scale type is discussed at some length by S.S. Stevens, "Measurement, Psychophysics and Utility", in C.W. Churchman and P. Ratoosh (eds.), Measurement: Definitions and Theories (Wiley, 1959).

preferences between objectives. But, by making the values in the decision process explicit, two advantages are achieved:

(i) the systematic and conscious exercise of judgement is assisted, and

(ii) the consequences for the decision of changes in values can be studied.

2.3.3 Generation of Alternatives

In order to achieve his objectives, a designer generates a number of feasible and admissible alternatives from which to choose. This is known as the "*synthesis*" stage. It is largely imaginative -- or, at least, dependent on "*lateral*"[18] rather than "*vertical*" modes of thought -- and it is widely believed that it must, to a large extent, be personal to each individual designer. A variety of processes -- rational, intuitive, ordered or random -- that may be appropriate to aid the synthesis of different solutions by different personalities can be found in the literature.[19,20] Hall[21] also discusses a number of different schemes (e.g., functional

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[18] E. de Bono, The Use of Lateral Thinking, Penguin Books, 1967.

[19] J.C. Jones, Design Methods: Seeds of Human Futures (Wiley-Interscience, 1970).

[20] G. Broadbent, Design in Architecture (John Wiley, 1973).

[21] A.D. Hall, A Methodology for Systems Engineering (Van Nostrand, 1962).

analysis of "*ideation*") which are at the designer's disposal. The morphological box[22] which can be used to strengthen the synthesis stage, provides means both of ordering and visualizing many different ways of tackling a design problem. The quality of solution obtained from such generating models depends on the type and number of parameters chosen as well as the types and combinations of sub-solutions and the ability of the designer using them.

In the method for the design of external walls (that is applicable to other components of the building fabric) proposed here, a set of candidate solutions or alternatives {i} is generated as a result of the process of synthesis. In the case of walls, an alternative is defined by the materials, geometry and configuration of the component's constituents. In practice, most construction materials and products are available in discrete sizes (e.g., bricks, blocks, batt insulation, gypsum board, and so on). So the approach to the design of a building's enclosure in which only feasible and admissible[23] alternatives are considered as solutions to the problem is adopted. Any change in the materials specified or in the configuration or in the geometry is treated as another altnernative to be considered

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22 H.H. Marbach, Remarks on the Use of the Morphological Box, The Design Activity Conference, Mimeographed Proceedings (Strathclyde, 1973).

23 See P.C. Fishburn [Decision and Value Theory, Wiley, 1964] for distinction between the terms "*feasible*" and "*admissible*".

in the evaluation.

2.3.4 Evaluation of Alternatives

Depending on the synthesis technique used, the number of alternatives postulated as candidate solutions to a design problem is likely to be large. The process of reducing all the possible solutions to a set of admissible alternatives is the first step in the evaluation phase and the time and cost to do this could be prohibitive. It is, therefore, essential to eliminate some alternatives -- the least useful ones, of course -- quickly, while still leaving a reasonable number, which contain acceptable solutions, for more detailed comparisons. Some alternatives might be rejected because their materials are not available, while others could fail to satisfy some principles of building. It is also possible to use the owners' requirements (say, aesthetic preference for a facing material) to reduce the decision field considerably. The exclusion for such reasons of certain constructional systems could eliminate some solutions that, on the basis of the performance objectives, might be considered optimal; on the other hand, the solutions finally arrived at will not be unacceptable to the owner (e.g., on subjective grounds of appearance).

The next step in the evaluation procedure is to model the performance of the alternatives in any suitable way -- verbal, mathematical, visual or experimental. The alternatives' predicted behaviour is then measured and normalized with respect to the range of the performance objective defined earlier. These transformed performance measures together form the matrix of attributes $[u]_{m \times n}$, where u_{ij} refers to the attribute of alternative i with respect to the performance objective j (Table 2.1). Each row vector in the matrix of attributes describes quantitatively the predicted behaviour of an alternative. Thus, the matrix of attributes forms the basis for decision-making.

2.3.5 Additive Models in Decision-Making

The many attributes which characterize constructional systems pose difficulties for designers choosing between alternatives. Several decision-making rules have been developed in such other disciplines as management science and economics.[24] However, in multi-attribute decision-making, one idea -- the additive composition notion -- dominates the literature.[25] This asserts that the utility of a multi-attributed alternative compound equals the sum of the weighted utilities of its components. In terms of building enclosure design, it is assumed that, in an additive model, the various component attributes contribute additively but independently to the alternative's overall worth. A precise

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24 For a succinct description of the different approaches to multiple-attribute decision-making, see K.R. MacCrimmon, Decision-Making Among Multiple-Attribute Alternatives: A Survey and Consolidated Approach, Rand Memorandum RM-4823-ARPA (December, 1968).

25 W. Edwards and A. Tversky, Decision-Making, Hammondsworth (Penguin, 1967).

TABLE 2.1

MATRIX OF ATTRIBUTES $[u]_{m \times n}$

ALTERNATIVE CONSTRUCTIONAL SYSTEM	PERFORMANCE VARIABLES								
	Y_1	Y_2			Y_j				Y_n
1	u_{11}	u_{12}			u_{1j}				u_{1n}
2	u_{21}	u_{22}			u_{2j}				u_{2n}
i	u_{i1}	u_{i2}			u_{ij}				u_{in}
m	u_{m1}	u_{m2}			u_{mj}				u_{mn}

definition of priorities among objectives -- in terms of weights -- is required, and the total value or overall utility, v_i, of an alternative i is equal to the sum of the weighted component attributes. In mathematical form:

$$v_i = \omega_1 u_{i_1} + \omega_2 u_{i_2} + \ldots + \omega_j u_{ij} + \ldots + \ldots \omega_n u_{in} = \sum_{j=1}^{n} \omega_j u_{ij} \qquad (2.1)$$

where

ω_j = the weight attaching to performance variable j, j = 1, ... , n.

The optimal solution is then found by choosing the alternative which has the highest total utility. Stated formally, the additive decision rule is:

(i) Calculate the overall utility of each alternative constructional system, and

(ii) Select that alternative with the highest utility.

The additive model is a conceptually simple model which makes it attractive to the designer faced with a complex choice and also provides a means of explaining decisions. Moreover, as yet, no other tractable formulations that describe non-independent structures have been satisfactorily constructed.[26]

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[26] *Ibid.*

In the design of the building enclosure, two problems make the application of this decision rule difficult or -- perhaps -- meaningless. The first problem concerns the measurement of the alternatives' performance. The predictive models[27] vary significantly in precision of predicted performance. Consequently, the measurement of performance would be on nominal, ordinal, interval or ratio scales, depending on the precision of the predictive model and the statement of the performance objective. However, arithmetic operations, which are implied by the additive decision rule, can be performed only on measurements on interval or ratio scales.[28]

The second problem encountered with the use of the additive model for designing building enclosures stems from designers' inability to express with any precision what sometimes must be vague preferences concerning the performance requirements. For example, this drawback may be caused because a designer, in his everyday practice, would never think of "*decomposing*" the design program and its labyrinth of inter-relationships into its component requirements. The kinds of minds that write research reports (such as this)

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27 These could be either iconic, analogue or symbolic. For more information on model types, see C.W. Churchman, R.L. Ackoff and E.L. Arnoff, Introduction to Operations Research (Wiley, 1964).

28 S.S. Stevens, "Mathematics, Measurement and Psychophysics", in S.S. Stevens (Ed.) Handbook of Experimental Psychology, Wiley, (New York, 1951).

are usually entirely different from the minds of the more pragmatic, intuitive, "*action-rather-than-thought*" individuals who constitute the bulk of professional designers.

Yet another problem that might arise could be due to variations in the design context. Variations in climatic conditions could, for instance, make the task of assigning weights to performance requirements very difficult. It is reasonable, therefore, to expect that a designer would express the weights in terms of a range (or an interval) of numbers. So far, however, there is no generally-acceptable technique for dealing with the effect of large tolerances in weightings used in making decisions among multi-attribute alternatives.[29]

2.4 DECISION BY EXCLUSION

For the theoretical development of the Decision by Exclusion Rule and its computer application, it has been expedient to use the notion of "*weights*" even though weights are never specified. An alternative is said to be dominated by the others if the sum of its weighted attributes, v_i, is less than that of at least one of the others for every possible set of non-negative weights. Thus, in the first phase of the Decision by Exclusion Rule, many attributes can be eliminated by dominance (with implicit additivity).

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[29] C.W. Churchman, *et al*, *op. cit.* (27)

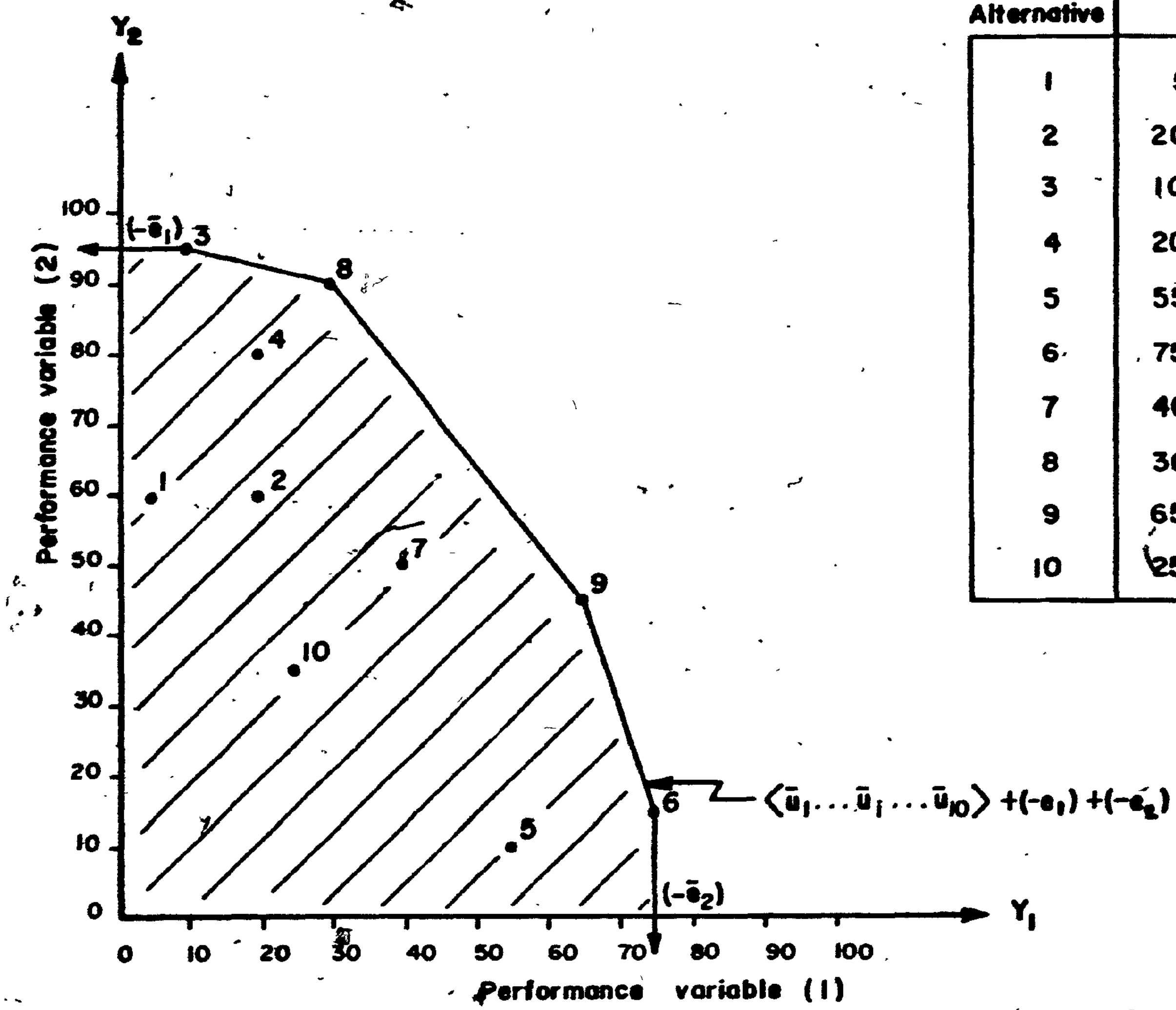
Graphically, an alternative can be represented by a point in n-dimensional space, where each coordinate corresponds to one attribute. The convex hull formed by connecting the extreme points in this attribute space encloses these joints which represent the dominated alternatives. For the 2-dimensional attribute space (in the sample shown in Figure 2.3) the non-dominated alternatives which form the convex hull are:

3, 6, 8 and 9

and the interior points are excluded from further consideration. If a priority between the performance objectives is not supposed, then, any of the four attributes may be selected because it would be at least as good as any other.

The relative positions of points in the attribute space are unique up to and including a linear transformation. Thus, adding or multiplying either y_1 or y_2 attributes, or both, by a positive constant does not change the dominance structure, i.e., the resulting convex hull comprises the same extreme point as those prior to transformation.

The inequality $\omega_1 \geq \omega_2$ (corresponding to performance objective 1 being more important than objective 2) restricts consideration in the attribute space approximately to the lower side of the line OE (Figure 2.4). The constricted attribute space can be determined more precisely by moving the vector $-\vec{e}_1 + \vec{e}_2$ (which is perpendicular to OE) outward



Alternative	Attributes 1	Attributes 2
1	5	60
2	20	60
3	10	95
4	20	80
5	55	10
6	75	15
7	40	50
8	30	90
9	65	45
10	25	35

Figure 2.3 Representation of exclusion procedure for two variables

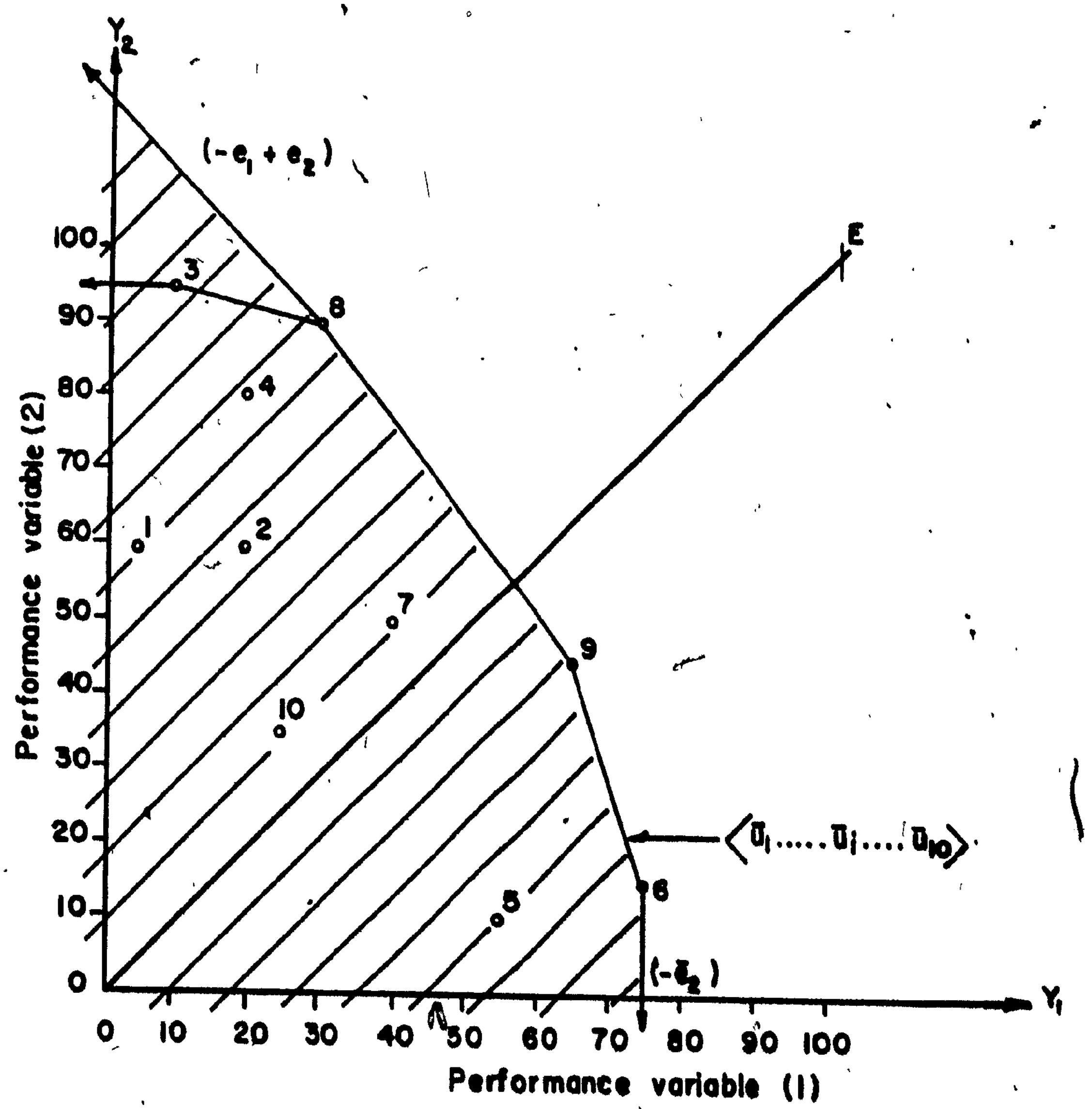


Figure 2.4 Representation of exclusion procedure for two variables with priorities between variables ($\omega_1 \geq \omega_2$)

from the origin until it passes through the outermost extreme point. The resulting convex hull obtained by adding this vector to that defined by the extreme point (shown shaded in Figure 2.4) changes the dominance structure, for, alternative 3 now becomes an interior point and so it is excluded.

As the preference for performance objective 1 increases, i.e., $\omega_1 \geq k\omega_2$, $k \geq 1$, the direction of the vector $(-e_1 + ke_2)$ approaches the line parallel to y_2-axis passing through the appropriate extreme point in the clockwise direction. For the case of $k = 2$, i.e., objective 1 being more than twice as important as objective 2, only alternatives 6 and 9 remain on the convex hull (Figure 2.5). In the limit as k becomes infinitely large, i.e., only objective 1 matters, the vector $(-e_1 + ke_2)$ becomes parallel to the y_2-axis and only alternative 6 remains for further consideration in the design.

It is conceptually easier to consider the weighting space instead of the attribute space. If the weights are allowed to take on any value (positive or negative), it is possible to determine a range of weights for which only one alternative is dominant. For alternatives with two attributes (e.g., matrix in Figure 2.6), this range of weights can be represented by a region in the plane defined by the ω_1-, ω_2-axes. All these regions, defined for all alternatives, constitute the whole ω-space (Figure 2.7).

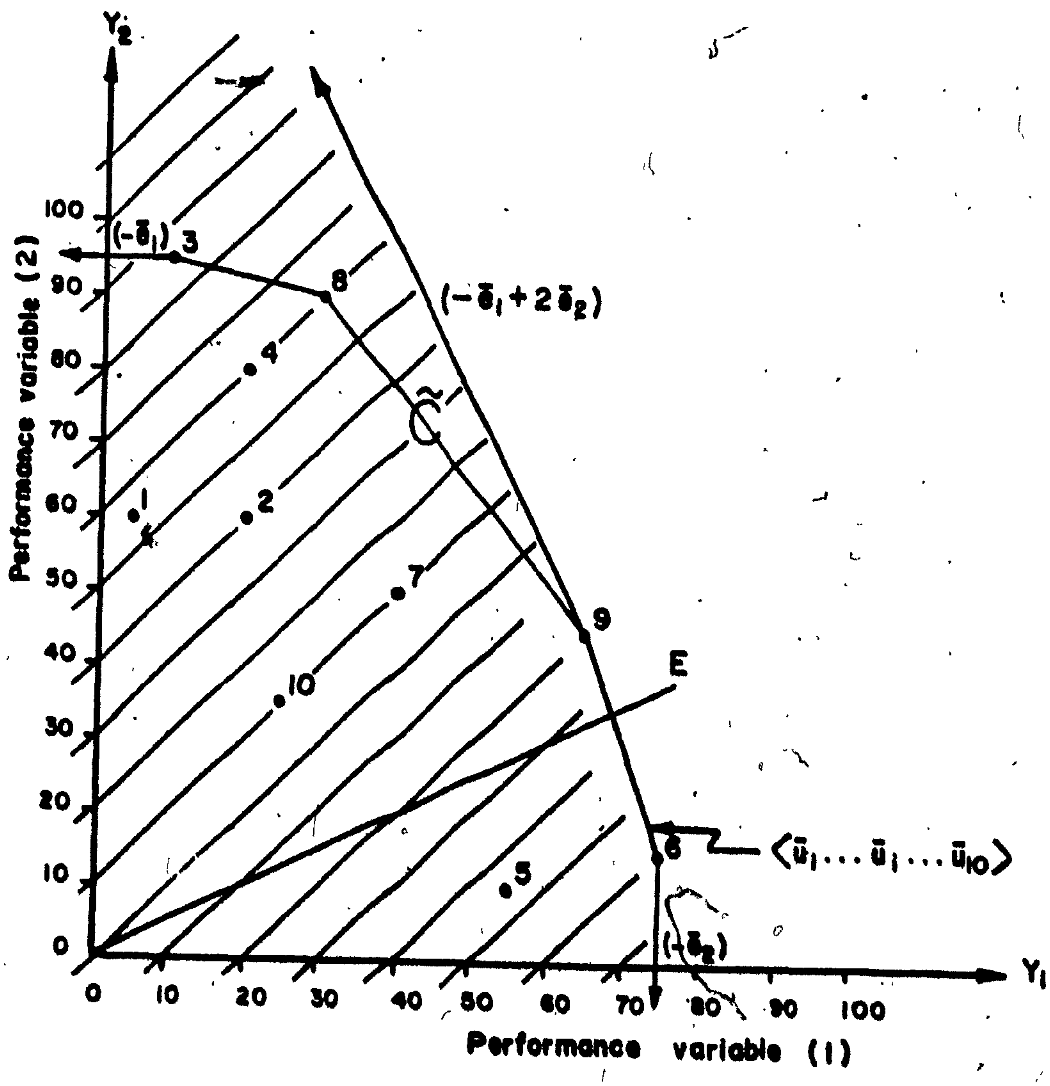


FIGURE 2.5 Representation of exclusion procedure for two variables with priorities between variables ($\omega_1 \geq 2\omega_2$)

	Attributes	
Alternatives	1	2
1	28	40
2	36	50
3	44	28
4	36	48
5	20	80

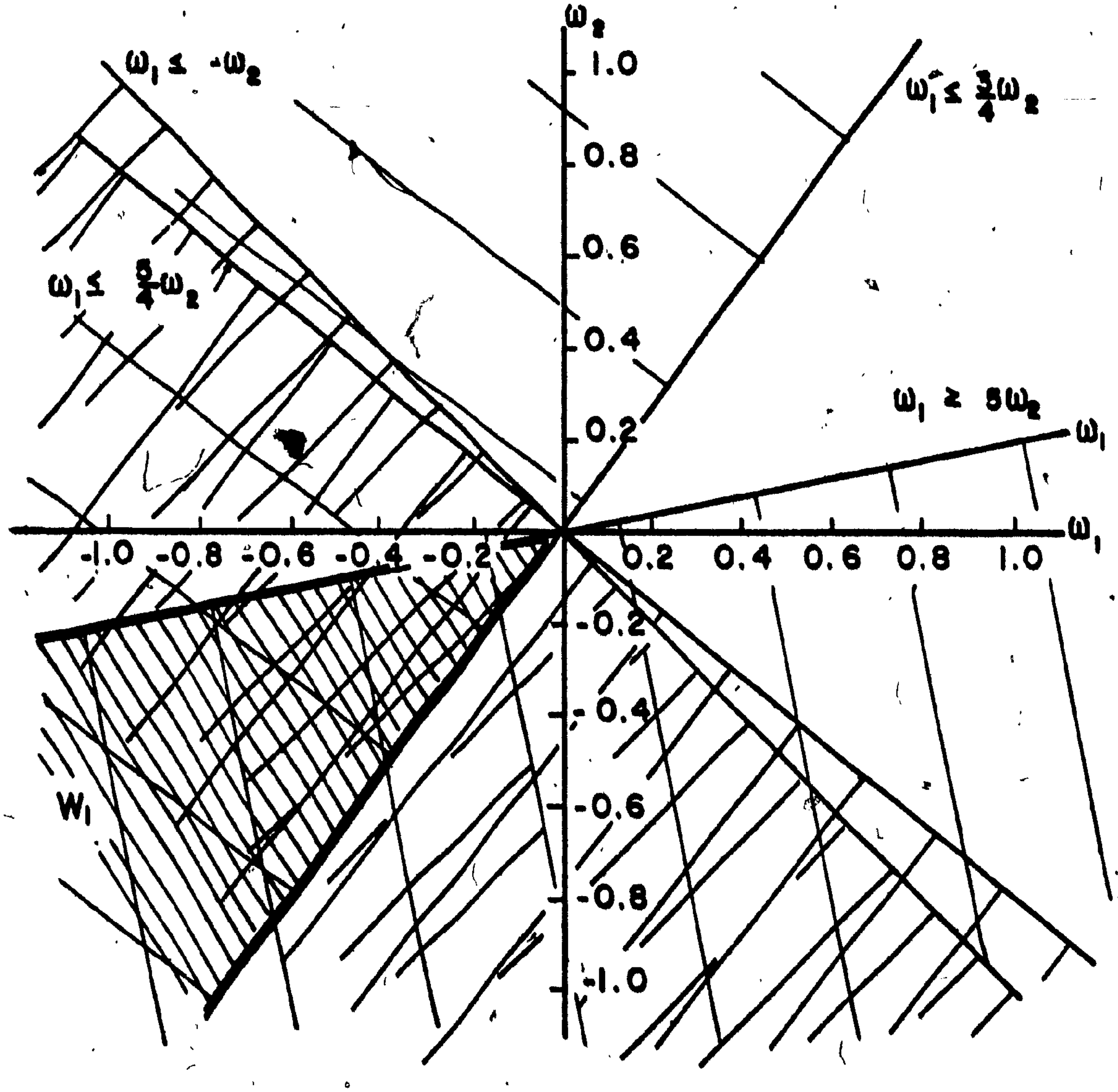


FIGURE 2.6 Region in ω-space in which alternative 1 is selected (intersection of regions of inequality)

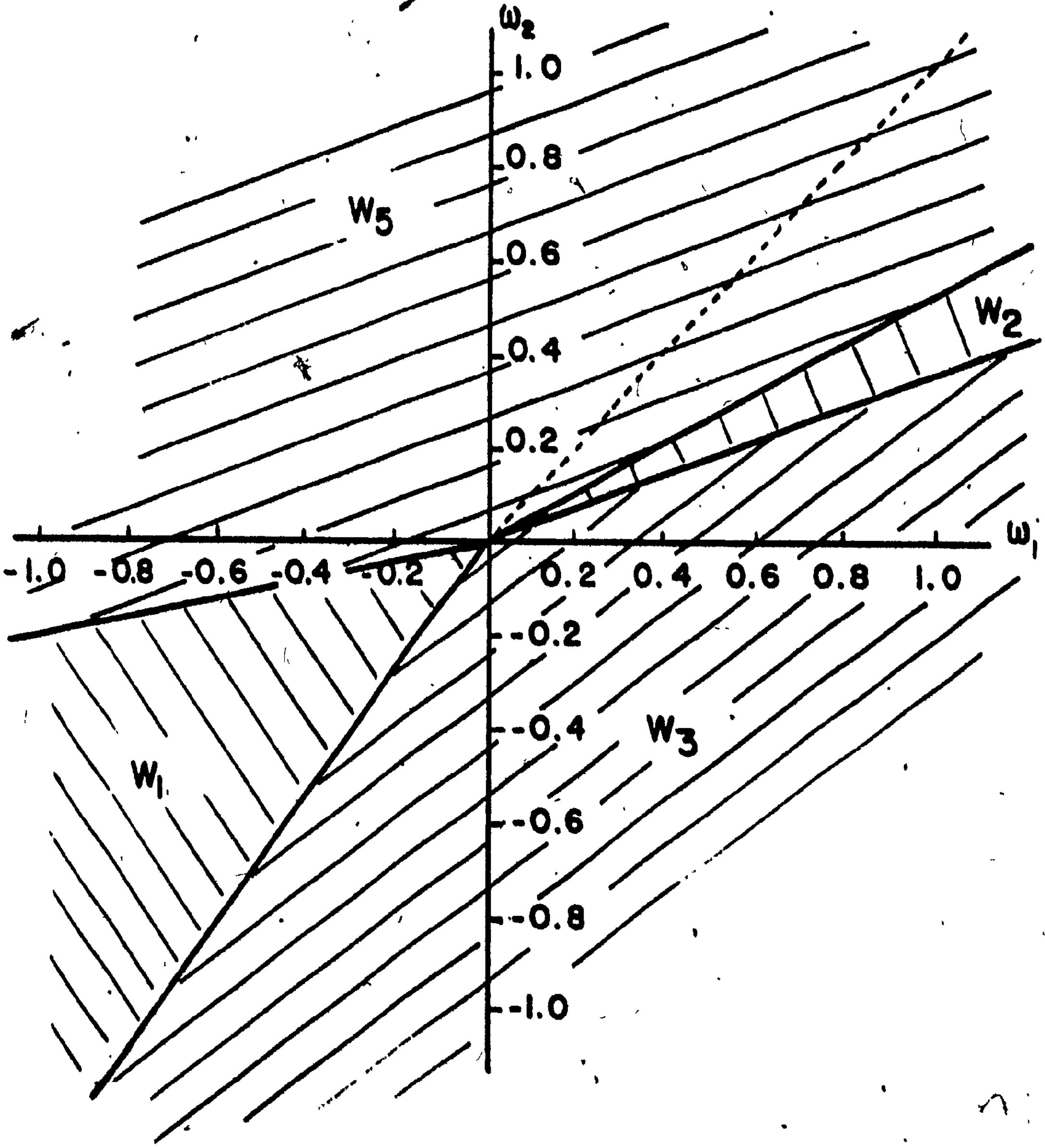


FIGURE 2.7 Union of regions in ω-space where alternatives may be selected.
(Note: W_i's span ω-space)

When the restriction $\omega \geq 0$ is imposed, the set of feasible solutions is restricted to the first quadrant. In this particular example, alternatives 1 and 4 will be excluded because alternative 1 is entirely in the negative ω-space, while alternative 4 corresponds to a point at the origin.

The restriction $\omega_1 \geq \omega_2$ can be represented by a region bounded by the lines $\omega_1 = \omega_2$ and the positive ω_1-axis, i.e., alternatives 2, 3 and 5 remain for further consideration (Figure 2.7). As the importance of objective 1 relative to objective 2 increases, i.e., $\omega_1 \geq k\omega_2$, $k \geq 1$, and increasing, the decision space diminishes since the line $\omega_1 = k\omega_2$ approaches the positive ω_1 axis. In the limit, only alternative 3 can be selected.

2.4.1 Assumptions

A more rigorous presentation of the Decision by Exclusion Rule involving multi-attribute alternatives is given below. Its mathematical development is facilitated by the use of linear algebra and the theory of linear inequalities. In this section, some of the assumptions, which form the bases of the decision rule, are made explicit.

(i) Assumption of Ordinality -- The design decision-maker can weakly order the preferences between performance objectives.

If R_y is a weak ordering relationship (such as preference or indifference) then R_y should satisfy the conditions:

(a) for all Y_i and Y_j in Y, either $Y_i R_y Y_j$ or $Y_j R_y Y_i$, and

(b) for all Y_i, Y_j and Y_k in Y, if $Y_i R_y Y_j$ and $Y_j R_y Y_k$, then $Y_i R_y Y_k$.*

where Y_i, Y_j and Y_k are performance variables in the set Y of variables considered in the design context.

(ii) Assumptions of Independence -- The performance variables are assumed not to be redundant.

This assumption assures the consistency of the order of preference between performance attributes. Easton[31] proposes a geometric analogy in order to establish the independence of the performance variables. In building enclosure design, Easton's independence criterion is not likely to be satisfied, so some redundancy between performance variables may exist. Thus the independence assumption can be reformulated to imply that no attribute should be redundant with respect to other attributes: in

* Note: Condition (b) is known as the transivity axiom, which in decision theory is considered to be a rationality axiom. (30)

30 P. Suppes, Studies in the Methodology and Foundations of Science, D. Reidel Publishing Company, Dordrecht, Holland (1969).

31 A. Easton, Complex Managerial Decisions Involving Multiple Objectives, John Wiley and Sons (1973).

other words, logically or empirically it should not be possible to imply an attribute value by some combination of other attributes.

(iii) Implicit Additivity -- It is assumed that the decision will be influenced by the contribution of all the attributes as if these were additive even though the addition of the component attributes is not explicitly performed.

The assumption of implicit additivity is necessary because the nature of performance objectives is such that preferences between them can be measured only on an ordinal scale. Furthermore, the nature of some objectives is such that not all attributes can be measured on interval scales.

(iv) Taken collectively, all attributes are assumed to be sufficient for choosing an alternative constructional system.

(v) Each performance variable considered in the design decision-making process is assumed to have some weight or preference value.

2.4.2 Theoretical Development of the Decision Rule

It has been argued that the weights attaching to the performance variables can, at best, be measured on an ordinal scale. When considering weights to be put on the attributes, it may be possible to state that performance variable Y_i is preferred to or more important than performance variable Y_j.

Thus,

$$\omega_i \geq \omega_j \tag{2.2}$$

or the weight (or rank) on Y_i is greater than that on Y_j, but the magnitude of the difference between the weights is unknown. If a weighting scheme $\overline{\omega}$ were obtainable, then the objective would be to find the maximum utility, v_k.

Thus,

$$v_k = \overline{u}_k \, \overline{\omega} = \max_i \, \overline{u}_i \, \overline{\omega} \tag{2.3}$$

where $i = 1, \ldots , m$ and $\overline{u}_i$ is the i^{th} row of the attribute matrix u, and k denotes the best alternative.

Consider a set of weights $(\omega_1, \ldots , \omega_n)$ as a column vector $\overline{\omega}$ in the Euclidean space E^n.*

The equation

$$\overline{v} = u\overline{\omega} \tag{2.4}$$

can be interpreted as a linear transformation from E^n into E^m, (u being a (m x n) matrix and $\overline{v}$ a column vector in E^m).

---

* A Euclidean space E^n is the space of n-tuples $(x_1, \ldots , x_n)$ of real numbers.

For a given weighting vector $\overline{\omega} \geq 0$, and a given attribute matrix u, alternative i is selected, if $v_i \geq v_j$ for all j.

Defining

$$V_i = \{\overline{v} | v_i \geq v_j \text{ for all } j\}, \quad (i = 1, \ldots , m)$$

alternative i will be chosen if $u\overline{\omega} \in V_i$.

The V_i divide the m-dimensional $\overline{v}$-space into m convex sets whose boundaries are the hyperplanes* $v_i = v_j$, i and j = 1, ... , m.

All sets have, in common, the half line defined by

$$v_1 = v_2 = v_3 = \ldots = v_m$$

If it were possible to state that $u \geq 0$, then it would be in order to restrict considerations to the positive orthant of the $\overline{v}$-space.

The cone V_i can be expressed in matrix notation, thus:

$$V_i = \{\overline{v} | P^{(i)} \overline{v} \leqq 0\} \tag{2.5}$$

where the "*preference*" matrix $P^{(i)}$ is defined by:

---

* <u>Hyperplane</u>: The set of points whose coordinates satisfy $\overline{a}\,\overline{x} = k$ (where a is a fixed vector and k is constant) is called a hyperplane. The set of points satisfying a linear inequality such as $\overline{a}\,\overline{x} \leq k$ is called a half-space. (32)

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[32] B. Noble, <u>Applied Linear Algebra</u>, Prentice-Hall, 1969.

$$P^{(i)} = \begin{bmatrix} +1 & 0 & 0 & \cdots & -1 & \cdots & 0 \\ 0 & +1 & 0 & \cdots & -1 & \cdots & 0 \\ 0 & 0 & +1 & \cdots & -1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & 0 & & -1 & +1 \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots & & \\ 0 & 0 & 0 & \cdots & -1 & 0 \quad 0 & +1 \end{bmatrix}$$

$P^{(i)}$ has m rows and m columns. For convenience of notation, the trivial inequality $(v_i - v_i \geq 0)$ is not omitted. Hence, $P^{(i)}$ contains a zero row.

Inserting Equation (2.4) into (2.5) results in

$$V_i = \{\bar{v} | \; P^{(i)} \; u \; \bar{\omega} \; \leqq 0\}$$

Instead of considering the cones V_i which are defined in the v-space, it is possible to investigate the corresponding sets in the $\bar{\omega}$-space:

$$W_i = \{\bar{\omega} \geq 0 | P^{(i)} \; u \; \bar{\omega} \leqq 0\} \qquad (2.6)$$

where W_i is the set of weights which will result in alternative i being selected as the "*best*" choice.

Defining $Q^{(i)} = P^{(i)}u$, Equation (2.6) becomes

$$W_i = \{\bar{\omega} \geq 0 | Q^{(i)} \cdot \bar{\omega} \leqq 0\}, \; i = 1, \ldots, m \qquad (2.7)$$

where $Q^{(i)}$ is a (m x n) matrix of the form

$$Q^{(i)} = \begin{bmatrix} u_{11} - u_{i1} & u_{12} - u_{i2} & \cdots & u_{1n} - u_{in} \\ u_{21} - u_{i1} & u_{22} - u_{i2} & \cdots & u_{2n} - u_{in} \\ \vdots & \vdots & & \vdots \\ u_{m1} - u_{i1} & u_{m2} - u_{i2} & & u_{mn} - u_{in} \end{bmatrix}$$

Thus the $k^{\underline{th}}$ row of $Q^{(i)}$ equals $\bar{u}_k - \bar{u}_i$, where $\bar{u}_k$ and $\bar{u}_i$ are the $k^{\underline{th}}$ and $i^{\underline{th}}$ rows respectively. Therefore, W_i can be described in terms of the rows of U.

$$W_i = \{\bar{\omega} \geq 0 \,|\, (\bar{u}_k - \bar{u}_i)\bar{\omega} \leq 0, \; k = 1, \ldots , m\}$$

If the weights are to be normalized then the linear constraint $\Sigma\omega_i = 1$ should be stipulated; however, this is not essential to the development of the theory.

Theorem I:

The sets W_i $(i = 1, \ldots , m)$ have no common interior points, i.e., only a boundary point can be an element of two or more such sets.

An interior point of W_i is a vector $\bar{\omega} \varepsilon W_i$ which satisfies the inequality $Q^{(i)} \bar{\omega} < 0$. Hence, it will be shown that there does not exist an $\bar{\omega}$ such that

$$Q^{(k)} \bar{\omega} < 0 \text{ and } Q^{(\ell)} \bar{\omega} < 0 \text{ for any } k \neq \ell.$$

Or, stated differently, the sets W_i are non-overlapping, and any given weighting vector $\bar{\omega}$ will be an element of one set W_i only, unless it falls on the boundary of W_i.

Proof (by contradiction):

From the definition (Equation (2.7)):

$$W_i = \{\overline{\omega} \geq 0 | Q^{(i)} \overline{\omega} \leq 0\} = \{\overline{\omega} | (\overline{u}_k\overline{\omega} - \overline{u}_i\overline{\omega}) \leq 0 \text{ for } k = 1, \ldots, m\}$$

Without loss of generality, assume that there exists an $\overline{\omega}$ which is an interior point of W_1 and W_2. Therefore,

$$Q^{(1)} \overline{\omega} < 0 \text{ and } Q^{(2)} \overline{\omega} < 0$$

or,

$$(\overline{u}_k\overline{\omega} - \overline{u}_1\overline{\omega}) < 0 \quad \text{for } k = 1, \ldots, m \tag{2.8}$$

and

$$(\overline{u}_k\overline{\omega} - \overline{u}_2\overline{\omega}) < 0 \quad \text{for } k = 1, \ldots, m \tag{2.9}$$

Choosing k = 2 and 1, respectively, Equation (2.8) becomes

$$(\overline{u}_2\overline{\omega} - \overline{u}_1\overline{\omega}) < 0$$

and Equation (2.9) becomes

$$(\overline{u}_1\overline{\omega} - \overline{u}_2\overline{\omega}) < 0$$

i.e., a contradiction exists.

Therefore, the theorem holds.

Thus, W_i is the set in the $\overline{\omega}$-space where alternative i will be at least as good as any other alternative, and the interior of W_i is the set of $\overline{\omega}$ for which alternative i will be better than any other alternative.

W_i could be visualized as the intersection of (m - 1) half spaces which are described by the inequalities:

$$(\overline{u}_k - \overline{u}_i)\ \overline{\omega} \leq 0 \quad k = 1, \ldots, m, k \neq i$$

Since $\overline{\omega} = 0$ is on the boundary of each half space, each W_i is a convex* cone originating at $\overline{\omega} = 0$.

Theorem II:

The W_i span the $\overline{\omega}$-space.

Proof:

Given any $\overline{\omega}$, let $\overline{u}_i\overline{\omega} = \max_k (\overline{u}_k\overline{\omega})$.

It follows that $(\overline{u}_k - \overline{u}_i)\ \overline{\omega} \leq 0$ for all k. Therefore,

$\overline{\omega} \in W_i$

Q.E.D.

Example:

Considering five alternatives and two attributes, a graphical representation of the theorem is given in Figure

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* A set in E is said to be a convex set if it contains the line-segment joining any points x_1 and x_2 of the set. Thus, a hyperplane and a half-space can be shown to be convex sets. (33)

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33 *Ibid.*

2.7.

Considering the effect of the restriction

$$\bar{\omega} \geq 0, \text{ (i.e., } \bar{\omega} \geqq 0 \text{ and } \bar{\omega} \neq 0)$$

$\bar{\omega}$ is said to be semi-positive, i.e., only the positive orthant of the $\bar{\omega}$ is considered. This restriction will remove from further consideration all W_i's which do not have a point in the positive orthant.

Thus, if $\bar{u}_i \leq \bar{u}_k$ for some $k \neq i$, W_i can be eliminated. In the example in Figure 2.7, W_1 and W_4 can be eliminated from further consideration.

Consider W_i, the set of weights $\bar{\omega}$ for which alternative i is selected:

$$\begin{aligned} W_i &= \{\bar{\omega} \geq 0 \mid \text{alternative } i \text{ is selected}\} \\ &= \{\bar{\omega} > 0 \mid (\bar{u}_k - \bar{u}_i)\ \bar{\omega} \leq 0 \text{ for all } k\} \\ &\quad \{\bar{\omega} \geq 0 \mid Q^{(i)}\ \bar{\omega} \leqq 0\} \end{aligned}$$

In other words, alternative i will be chosen for all semi-positive $\bar{\omega}$ which solve the linear inequalities,

$$Q^{(i)}\bar{\omega} \leqq 0, \quad \bar{\omega} \geq 0 \qquad (2.10)$$

If this system of linear inequalities has no solution, then alternative i will never be selected.

Gale's theorem of the alternative[34] states that either the above system has a solution or the system

$$\bar{x}\, Q^{(i)} > 0, \quad \bar{x} \geq 0 \qquad (2.11)$$

has a solution $\bar{x}$, but never both systems.

Note that $\bar{x}$ is an m-dimensional row vector, m being the number of alternatives.

Gale's theorem of the alternative makes it possible to examine Equation (2.11) rather than Equation (2.10). Equation (2.10) has no solution $\bar{\omega}$ if Equation (2.11) has a solution $\bar{x}$. Equation (2.11) has a solution $\bar{x}$ only if

$$\sum_{k=1}^{m} x_k\, \bar{q}_k^{(i)} > 0, \; \bar{x} \geq 0$$

has a solution. Where $\bar{q}_k^{(i)}$ is the k^{th} row of $Q^{(i)}$, or making use of the definition of $Q^{(i)}$:

$$\sum_{k=1}^{m} x_k\, (\bar{u}_k - \bar{u}_i) > 0, \; \bar{x} \geq 0$$

This is equivalent to

$$\bar{u}_i \sum_{k=1}^{m} x_k < \sum_{k=1}^{m} x_k\, \bar{u}_k, \quad \bar{x} \geq 0 \qquad (2.12)$$

Since

$$\sum_{k=1}^{m} x_k > 0 .$$

[34] See *Theorem* 2.10, p. 49, in D. Gale, The Theory of Linear Economic Models (McGraw-Hill Book Co. Inc., 1960).

($\bar{x}$ is semi-positive), it is possible to divide Equation (2.12) by $\sum_{k=1}^{m} x_k$:

$$\bar{u}_i < \frac{\sum_{k=1}^{m} x_k \bar{u}_k}{\sum_{k=1}^{m} x_k}, \quad \bar{x} \geq 0$$

Defining

$$\lambda_k = \frac{x_k}{\sum_{\ell=1}^{m} x_\ell}$$

then

$$\lambda_k \geq 0 \text{ for all } k \text{ and } \sum_{k=1}^{m} \lambda_k = 1.$$

Hence,

$$\bar{u}_i < \sum_{k=1}^{m} \lambda_k \bar{u}_k, \text{ with } \lambda_k \geq 0 \text{ for all } k \text{ and}$$

$$\sum_{k=1}^{m} \lambda_k = 1 \qquad (2.13)$$

It is now possible to state the problem in terms of Equation (2.13). Thus, alternative i will not be chosen if there exists λ_k ($k = 1, \ldots, m$) such that

$$\bar{u}_i < \sum_{k=1}^{m} \lambda_k \bar{u}_k, \text{ with } \lambda_k \geq 0 \text{ for all } k$$

and

$$\sum_{k=1}^{m} \lambda_k = 1$$

holds.

Using the theory of convex sets, this result can be expressed in a different manner. Thus, there exists λ_k ($k = 1, \ldots, m$) and μ_j ($j = 1, \ldots, n$) such that

$$\bar{u}_i = \sum_{k=1}^{m} \lambda_k \bar{u}_k - \sum_{j=1}^{n} \mu_j \bar{e}_j$$

with $\mu_j > 0$ ($j = 1, \ldots, n$), $\lambda_k \geq 0$ ($k = 1, \ldots, m$), $\Sigma\lambda_k = 1$, where $\bar{e}_j$ is the j^{th} unit vector of E^n, i.e., a vector consisting of the coefficient 1 in position j and zeroes elsewhere. $\bar{u}_i$ is therefore expressed as an interior point of a convex set which is the sum of the polytope generated by the $\bar{u}_k$ ($k = 1, \ldots, m$) and the negative orthant generated by $(-\bar{e}_j)$*, ($j = 1, \ldots, n$).

Theorem III:

Alternative i will never be selected if $\bar{u}_i$ is an interior point of the convex set generated by the sum of the polytope $< \bar{u}_1, \ldots, \bar{u}_m >$ and the negative orthant.

---

* The notation used in the theoretical development is consistent with Gale's notation. (35) Hence, $(\bar{x})$ connotes the half-line generated by $\bar{x}$ and $< \bar{x}_1, \ldots, \bar{x}_m >$ is the convex hull of $\bar{x}_1, \ldots, \bar{x}_m$ (also called convex polytope).

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35 *Ibid.*

If $\bar{u}_i$ is on the boundary of that convex set and not a corner point, then there will always be an alternative "*at least as good as*" alternative i.

Example:

Consider ten alternatives of two attributes each: $m = 10$, $n = 2$ (See Figure 2.3).

The ten rows $\bar{u}_i$ of u are plotted in the performance variable space. The alternatives 1, 2, 4, 5, 7, 10, which are not extreme points of the convex set, can be eliminated.

Thus, if it were possible to determine all the extreme points $\bar{u}_k$ of the convex set $C = < \bar{u}_1, \ldots, \bar{u}_m > + \sum_{j=1}^{n} (-\bar{e}_j)$, then, for any given set of weights $\bar{\omega}$, a "*best*" or "*at least as good as*" solution could be found from the extreme points, i.e., all $\bar{u}_k$ which are not extreme points can be eliminated from further consideration.

If it were possible to stipulate that $u \geq 0$, then the convex set C can be restricted to the positive orthant, thus becoming a polytope.

Consider, now, linear constraints on the weights $(\omega_1 \ldots \omega_n)$ of the type $\omega_r \leq \omega_s$, $r \neq s$. This inequality can be represented as

$$(\bar{e}_r - \bar{e}_s)\ \bar{\omega} \leq 0$$

Adding $(\bar{e}_r - \bar{e}_s)$ as a row vector to the matrix $Q^{(i)}$, an

[(m + 1) x n] matrix denoted by $\tilde{Q}^{(i)}$ results.

Applying Gale's theorem of the alternative (as previously), either the system $\tilde{Q}^{(i)} \overline{\omega} \leq 0$, $\overline{\omega} \geq 0$ or the system $\overline{x} \tilde{Q}^{(i)} > 0$, $\overline{x} \geq 0$ has a solution but never both systems. (Note that $\overline{x}$ now has (m + 1) coefficients.)

Therefore, alternative i will not be selected, if the first system has no solution, or, equivalently, if the second system has a solution, in which case, there exists an $\overline{x}$ such that

$$\sum_{k=1}^{m} x_k \; (\overline{u}_k - \overline{u}_i) + x_{m+1}(\overline{e}_r - \overline{e}_s) > 0, \; \overline{x} \geq 0$$

or,

$$\overline{u}_i \sum_{k=1}^{m} x_k \; < \sum_{k=1}^{m} x_k \; \overline{u}_k + x_{m+1}(\overline{e}_r - \overline{e}_s).$$

The first m coefficients of $\overline{x}$ cannot all be zero, since in this instance, the resulting inequality would be

$$x_{m+1}(\overline{e}_r - \overline{e}_s) > 0, \; x_{m+1} > 0$$

from which it follows that $\overline{e}_r > \overline{e}_s$, which is not true.

Therefore,

$$\sum_{k=1}^{m} x_k > 0$$

and it is possible to divide by it.

$$\bar{u}_i < \frac{\sum_{k=1}^{m} x_k \bar{u}_k}{\sum_{k=1}^{m} x_k} + \frac{x_{m+1}}{\sum_{k=1}^{m} x_k} (\bar{e}_r - \bar{e}_s)$$

Defining

$$\lambda_k = \frac{x_k}{\sum_{\ell=1}^{m} x_\ell}, \; k = 1, \ldots, m+1,$$

then

$$\sum_{k=1}^{m} \lambda_k = 1 \quad \text{and} \quad \lambda_k \geq 0 \;\; \text{for } k = 1, \ldots, m+1$$

(Note that there is no upper limit on λ_{m+1}, since λ_{m+1} is not part of the normalization.)

Alternative i will never be chosen, if there exist λ_k ($k = 1, \ldots, m + 1$), such that

$$\bar{u}_i < \sum_{k=1}^{m} \lambda_k \bar{u}_k + \lambda_{m+1}(\bar{e}_r - \bar{e}_s)$$

$$0 \leq \lambda_k \;\; (k = 1, \ldots, m + 1) \;\text{and}\; \sum_{k=1}^{m} \lambda_k = 1.$$

Equivalently: Alternative i will never be chosen, if there exist $(m + 1)$ λ_k and n μ_j such that

$$\bar{u}_i = \sum_{k=1}^{m} \lambda_k \bar{u}_k + \lambda_{m+1}(\bar{e}_r - \bar{e}_s) - \sum_{j=1}^{n} \mu_j \bar{e}_j,$$

$$0 \leq \lambda_k \ (k = 1, \ldots, m + 1), \ \sum_{k=1}^{n} \lambda_k = 1$$

and

$$\mu_j > 0 \ (j = 1, \ldots, n).$$

This result is similar to the previous finding but the restriction $\omega_r \leq \omega_s$ causes the half-line generated by $\bar{e}_r - \bar{e}_s$ to be added to the convex set.

Theorem IV:

Under the restriction $\omega_r \leq \omega_s$, alternative i will never be chosen if $\bar{u}_i$ is an interior point of the convex set $\tilde{C}$ generated by the polytope $< \bar{u}_1, \ldots, \bar{u}_m >$, the negative orthant $\sum_{j=1}^{n} (-\bar{e}_j)$, and the half-line $(\bar{e}_r - \bar{e}_s)$.

2.4.3 Remarks

(i) Since $(-\bar{e}_s) = (\bar{e}_r - \bar{e}_s) + (-\bar{e}_r)$, the half-line $(-\bar{e}_s)$ is in the convex set generated by the other half lines and can, therefore, be omitted as a generator. It is thus possible to write:

$$\tilde{C} = < \bar{u}_1, \ldots, \bar{u}_m > + \sum_{j \neq s} (-\bar{e}_j) + (\bar{e}_r - \bar{e}_s).$$

(ii) If the constant $\omega_r \leq \omega_s$ is replaced by the more general linear constraint

$$f_r \, \omega_r \leq f_s \, \omega_s$$

where f_r and f_s are any positive real numbers then $\tilde{C}$ can be modified to

$$\tau = < \overline{u}_1, \ldots, \overline{u}_m > + \sum_{j \neq s} (-\overline{e}_j) + (f_r\overline{e}_r - f_s\overline{e}_s)$$

(The proof follows that of the simpler case almost line for line, and is therefore omitted.)

Example:

Using the same attribute matrix as in the previous example, the constraint $\omega_1 \geq 2\omega_2$ has been added. $(-\overline{e}_1)$ is not needed as a generator of the convex set $\tilde{C}$. (Figure 2.5.)

(iii) Another important generalization can be made by allowing t (where $t \geq 1$) linear constraints to be applied simultaneously on the set of weights $\overline{\omega}$:

$$f_{r_i} \omega_{r_i} - f_{s_i} \omega_{s_i}, \; i = 1, \ldots, t$$

where r_i and s_i can be any indices between 1 and n. In this case, τ becomes

$$\tau = < \overline{u}_1, \ldots, \overline{u}_m > + \sum_{j=1}^{n}(-\overline{e}_j) + \sum_{i=1}^{t}(f_{r_i}\overline{e}_{r_i} - f_{s_i}\overline{e}_{s_i})$$

Once more, the half-lines $(-\overline{e}_j)$ can be omitted for all j's which are contained in the index set $\{s_i | i = 1, \ldots, t\}$.

It should be noted that, by adding several constraints on the weights simultaneously, more (never less) alternatives might be eliminated than by applying the elimination process successively, considering one constraint at a time.

The following example will illustrate the difference to be noted:

$$u = \begin{bmatrix} 4 & 4 & 6 \\ 6 & 0 & 6 \\ 2 & 6 & 9 \\ 3 & 2 & 9 \\ 4 & 3 & 4 \end{bmatrix}$$

Alternative 5 is excluded before adding any constraints. Constraint $\omega_1 \leq \omega_3$ excludes alternative 4; constraint $\omega_3 \leq \omega_2$ excludes alternative 4. However, applying both constraints simultaneously, alternatives 1, 2 and 4 can be excluded.

2.4.4 Computational Aspects of the Elimination of Alternatives

At present, an algorithm is being developed* to determine the extreme points of a convex set which is generated by a polytope and a set of half-lines, the polytope being described as the convex hull of a set of points (not as an intersection of half-spaces, as is the case in linear programming). Such an algorithm should eliminate (in a computationally efficient manner) all alternatives which will

* Jointly by W. Bitterlich and the author.

not be selected (for they correspond to non-extreme points in the convex set).

Since at present there is no direct suitable algorithm, a "*brute force*" approach using the equations for the sets

$$W_i = \{\bar{\omega} \geq 0 \,|\, (\bar{u}_k - \bar{u}_i)\ \bar{\omega} \leq 0 \text{ for } k = 1, \ldots, m\}$$

$i = 1, \ldots, m$ has been developed. Alternative i will be eliminated if W_i is empty. Since computer algorithms for linear programming (LP) are readily available, it is reasonable to reformulate the elimination process as an LP problem.

Formulation I

Maximize

$$Z = \sum_{i=1}^{m} \omega_i$$

Subject to

$$(\bar{u}_k - \bar{u}_i)\ \bar{\omega}' \leq 0, \quad k = 1, \ldots, m$$

$$\sum_{i=1}^{m} \omega_i \leq 1$$

$$\bar{\omega} \geq 0$$

This problem always has a feasible solution $\bar{\omega} = 0$. (A feasible solution to an LP problem is a solution which satisfies all the constraints but does not necessarily optimize the objective function z.) The existence of an initial feasible solution simplifies the computation.

If the LP problem thus formulated only has the solution $\overline{\omega} = 0$, then the objective function Z remains at zero and the set W_i is empty since W_i contains only semi-positive weights $\overline{\omega}$.

If, on the other hand, the LP problem has any non-zero solution, it can easily be seen that the value of the objective function will be 1. This yields a simple criterion in deciding whether or not to eliminate alternative i.

The formulation of the LP problem above is not the only way of expressing the elimination process in terms of linear programming.

Consider, for example, the following modified LP problem.

Formulation II

Maximize

$$z = \overline{o}\ \overline{\omega}$$

Subject to

$$(\overline{u}_k - \overline{u}_i)\ \overline{\omega} \leq 0,\ k = 1, \ldots, m$$

$$\sum_{i=1}^{m} \omega_i = 1$$

$$\overline{\omega} \geq 0$$

$\bar{o}$ is a zero vector. The objective function in this case is artificial and meaningless. The criterion for elimination of alternatives becomes one of the existence of a feasible solution. Thus, any objective function would have been acceptable, the simplest one being the choice made. The non-existence of a feasible solution to this LP problem is equivalent to W_i being empty, hence alternative i is eliminated.

This approach is computationally more complex since special attention has to be paid to the setting-up of an artificial initial solution and to applying the Phase I -- Phase II procedure of linear programming.

Both linear programming formulations have their dual LP formulations which look slightly different but do not seem to have any computational advantage and, as such, are not formulated here.

Linear constraints on the weighting scheme of the form

$$f_{r_j} \omega_{r_j} \leq f_{s_j} \omega_{s_j} , \quad (r_j, s_j = 1, \ldots , m)$$
$$(j = 1, \ldots , t)$$

do not pose a serious burden on the LP formulations previously described. The inequalities $(f_{r_j} \bar{e}_{r_j} - f_{s_j} \bar{e}_{s_j}) \bar{\omega} \leq 0$, $(j = 1, \ldots , t)$ are added to the existing set of constraints. All these approaches have the disadvantage that they make it necessary to apply an LP program m times

-- once for each alternative, hence the term "*brute force*".

A flow-chart for a computer algorithm that uses the LP Formulation I is given below.

2.4.5 Parametric Analysis

To determine the robustness of a decision, the following set of constraints $(1 - F)\omega_{j_0} \geq F\,\omega_j$ is used. This is of the same type as $F_r\,\omega_r \leq F_s\,\omega_s$ but here, F is allowed to vary continuously and the value of F for which alternative i is eliminated is determined for each alternative. The formulation chosen assumes a value between 0 and 1. The algorithm to find the critical F for each i is presented in the flow-chart. The class of equations $F\omega_{j_0} \leq (1 - F)\omega_j$ for all j, $j \neq j_0$ was treated similarly.

2.5 SUMMARY

Many systematic methods have been developed to assist in the process of designing building enclosures.[36] However, very few methods (but notably those of Krokosky[37] and the Japanese Building Research Group[38]) have considered the multi-attribute nature of the problem. Lewis *et al*[39], in

[36] See J.C. Jones, *Op. Cit.* (11).

[37] See E.M. Krokosky, *Op. Cit.* (2) and (3).

[38] See Building Research Group, *Op. Cit.* (9), (10), (11) and (12).

[39] W.P. Lewis, A.E. Samuel and W.B. Field, "An Example of the Application of a Systematic Method to Design", Operations Research Quarterly, Vol. 24, No. 2, pp. 217-223.

FLOW DIAGRAM FOR THE BASIC EXCLUSION PROCESS

Enter

m (number of alternatives)

n (number of variables)

m x n matrix U (attribute matrix)

For all alternatives (rows) k

Solve the LP problem

Maximize $Z = \sum_{j=1}^{n} \omega_j$

Subject to:

$$\sum_{j=1}^{n} \omega_j \le 1$$

$$(\bar{u}_i - \bar{u}_k)\,\bar{\omega} \le 0 \quad \text{for} \quad i = 1, \ldots, m$$

$$\bar{\omega} \ge 0$$

If $Z = 0$

Exclude alternative k and the corresponding row of U

FLOW DIAGRAM FOR ADDING LINEAR CONSTRAINTS

$$C\,\overline{\omega} \leq 0$$

Enter

m (number of alternatives remaining after the Basic Exclusion Process)

n (number of variables)

m x n matrix U (attribute matrix)

Constraint matrix C†

↓

For all (remaining) alternatives k

Solve the LP problem

Maximize

$$z = \sum_{j=1}^{n} \omega_j$$

Subject to:

$$\sum_{j=1}^{n} \omega_j \leq 1$$

$$(\overline{u}_i - \overline{u}_k)\,\overline{\omega} \leq 0 \quad \text{for } i = 1, \ldots, m$$

$$C\,\overline{\omega} \leq 0$$

$$\overline{\omega} \geq 0$$

↓

If $z = 0$
Exclude alternative k.

---

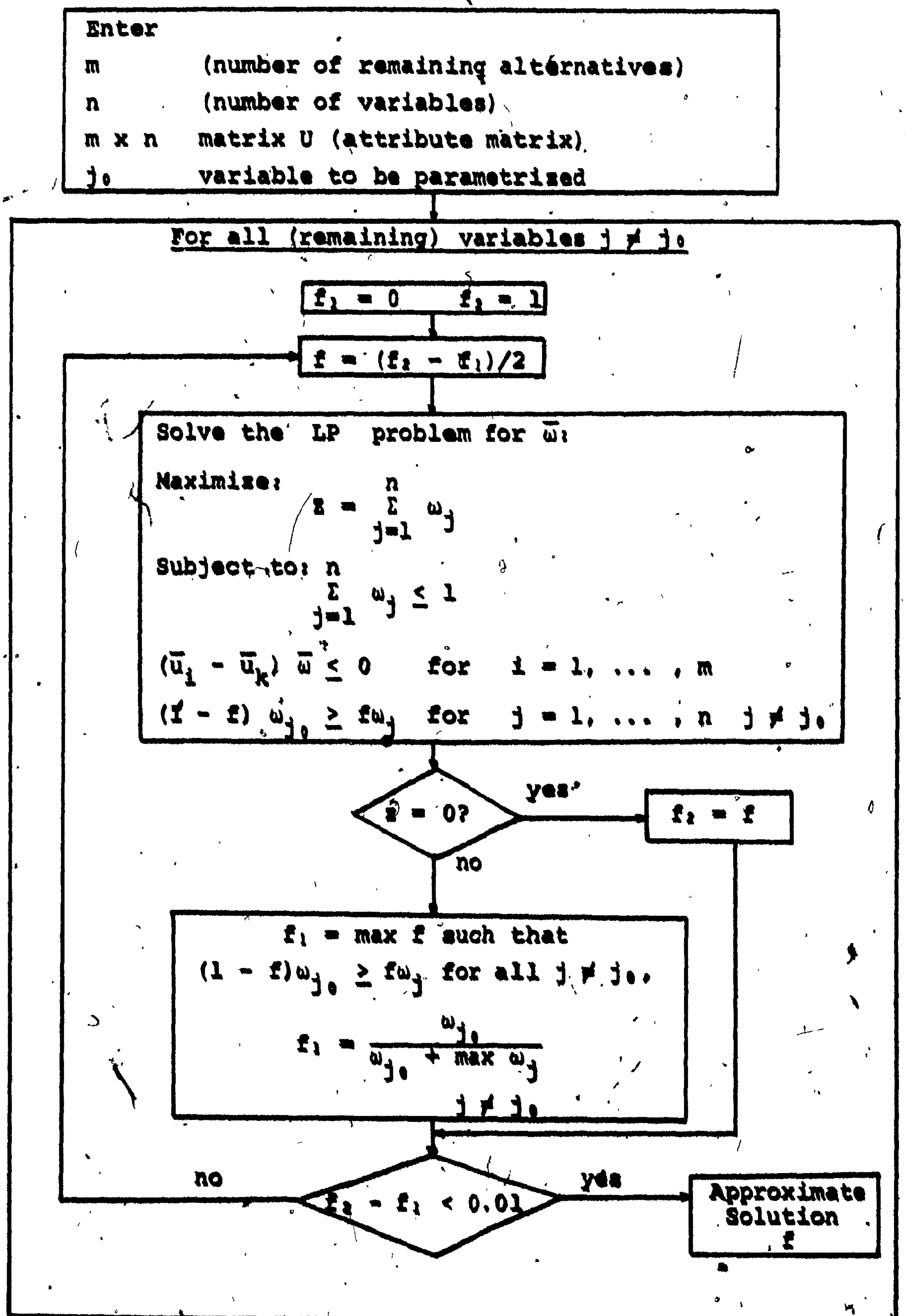
† C is the matrix which describes linear constraints on $\overline{\omega}$. For instance, consider the case there n = 3 and the weights are the following: $3\omega_1 \leq \omega_2$ and $2\omega_2 \leq \omega_3$, then:

$$C = \begin{pmatrix} 3 & -1 & 0 \\ 0 & 2 & -1 \end{pmatrix}$$

C is an (t x n) matrix where t is the number of constraints.

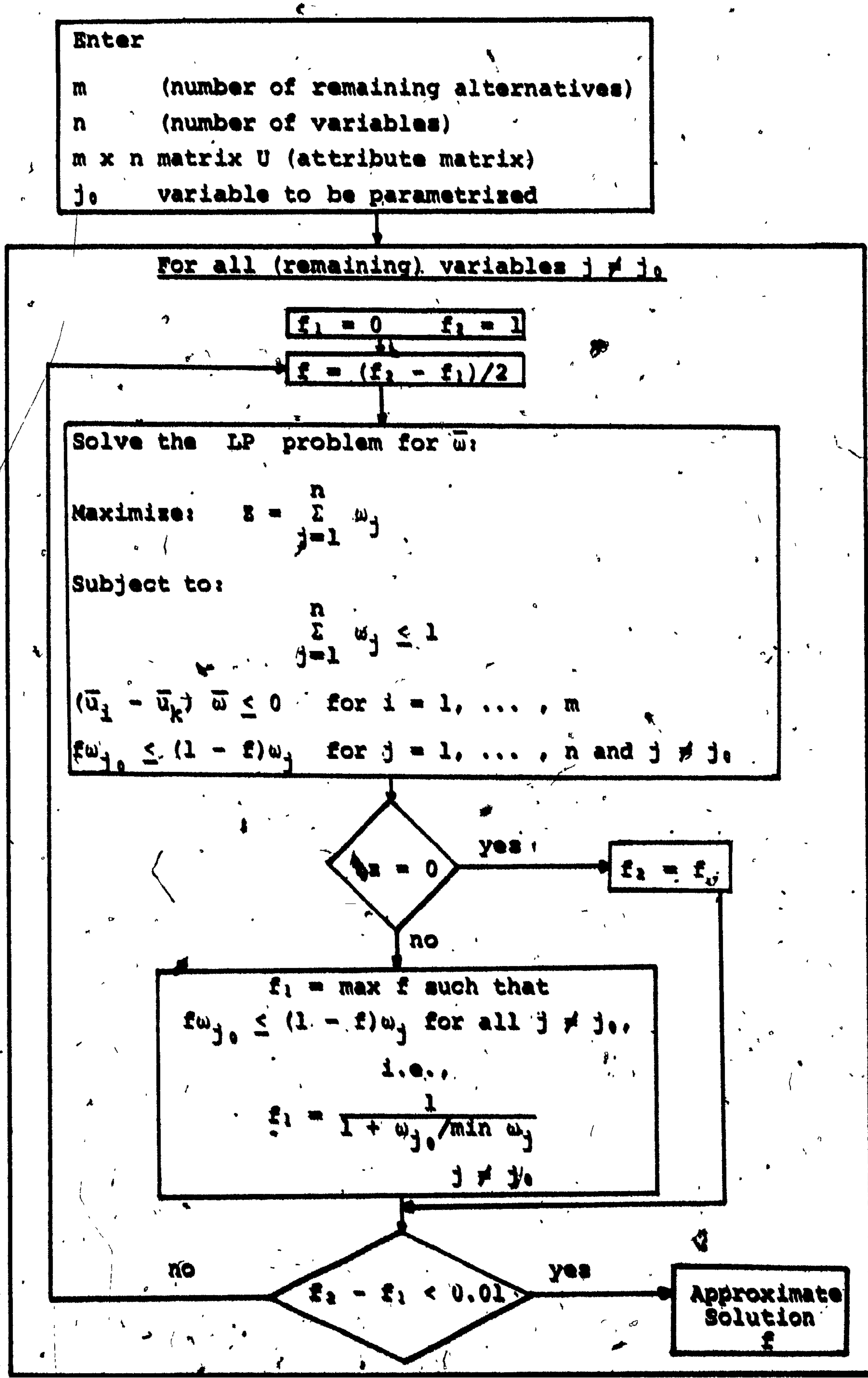
FLOW DIAGRAM FOR PARAMETRIC LINEAR CONSTRAINTS

$$(1 - f)\,\omega_{j_0} \geq f\,\omega_j$$



FLOW DIAGRAM FOR PARAMETRIC LINEAR CONSTRAINTS

$$f\omega_{j_0} \leq (1-f)\ \omega_j$$



discussing the application of systematic methods to design, noted that there is a considerable gap between research and application. In the design decision-making scheme proposed here an attempt has been made to bridge the gap -- partly by simulating practice and partly by reducing the cognitive load on the designer and, in so doing, improving the quality of design.

The design program for a building is used to define the limits of acceptability, utility functions and preferences among performance objectives; any changes being easily accommodated in the design procedure, which remains virtually invariant. Thus, having ensured that the requirements of design are responsibly cared for, more effort can be expended on the more demanding, and imaginative, synthesis phase of design, adapting existing solutions to new situations or proposing entirely new constructional systems. The predictive and measurement models used in the evaluation phase are invariant -- improvements in the models are easily incorporated into the design scheme.

On applying the decision rule, a number of alternatives are excluded at each step in the procedure:

(i) By checking them against constraints such as building science principles, (e.g., "*two-stage weather tightening*"[40]), transportation limitations, etc.

[40] G.K. Garden, "Rain and Air Leakage at Joints", Industrialization Forum, Vol. 2, No. 4 (July 1971), pp. 7-12.

(ii) By comparing their predicted performance with the least acceptable limit on each objective.

(iii) By comparing an alternative's attributes with those of the other alternatives in the decision field to determine whether the alternative is dominated or not.

(iv) By taking preferences among objectives and checking whether remaining alternatives become dominated or not.

The reduced set of alternatives which results from the application of the decision rule comprises the available set of optimal solutions. Given the limitations of the preference measurements, any alternative chosen from the reduced set would be at least as good as any other remaining alternative.

In the past, two methods have been available for resolving the multi-attribute decision problem encountered in the design of a building enclosure. One of them is the "*one-dimensional comparison method*", which defines a utility function (usually additive) over the attributes considered in the design and then finds the optimum alternative. The other is the "*multi-dimensional comparison method*", in which the alternatives are considered on the basis of each attribute separately. Neither method has been accepted universally. The decision by exclusion rule attempts to bridge the gap between those prevalent approaches which are two extreme cases in the domain of domination structures.[41] By modifying the multi-dimensional comparison method to account for the assumption that the constituent attributes contribute to the overall worth of the alternative, it is possible to identify a domination structure in which an optimal solution is not dominated by the other alternatives in the decision space. The assumption of implicit additivity makes possible the use of linear programming to identify the extreme points in the attribute space corresponding to the non-dominated alternatives. Preferences between objectives are treated as additional constraints in the LP formulation and these, in turn, are reflected by a constriction of the decision space resulting in the exclusion of alternatives. The more explicitly the preferences are stated, the greater the constriction of the decision space and thus the fewer would be the number of alternatives remaining. If weighting were possible, these would result in a *"one-dimensional comparison method"*, in which it is likely that only one alternative, the optimal, would remain in the decision space.

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41 L.P. Yu, "Cone Convexity, Cone Extreme Points, and Non-dominated Solutions in Decision Problems with Multi-objectives", Journal of Optimisational Theory and Applications, Vol. 14, No. 3 (1974).

CHAPTER 3

APPLICATION OF THE DECISION BY EXCLUSION RULE TO THE DESIGN OF EXTERNAL WALLS

CHAPTER 3

APPLICATION OF THE DECISION BY EXCLUSION RULE TO THE DESIGN OF EXTERNAL WALLS

3.1 STATEMENT OF THE PROBLEM

In this Chapter, it is proposed to examine the design process for a component of the building fabric, i.e., the external wall of single-family houses in Montreal. The design process comprises many stages (as shown in Figure 2.2) of which the principal ones are:

(i) Establishing the design context

(ii) Specification of performance objectives

(iii) Generation of alternatives

(iv) Prediction of alternatives' performance

(v) Evaluation and selection of optimum alternative(s) using the Decision by Exclusion rule.

Each of these stages will be elaborated further. However, to illustrate the application of the Decision by Exclusion rule, the emphasis will be on the evaluation and selection processes. This in no way diminishes the importance of the other stages, which are essential in providing a context for the application.

3.2 DESIGN CONTEXT

The external environment, the availability of resources and design, and statutory constraints assumed in this example are typical of conditions found in suburban Montreal. In practice, the designer would make measurements or judgements on the site to determine conditions, establish data from such published sources as "The Climate of Canada"[1] or "Supplements to the National Building Code"[2], and so on. A number of design guides[3] and exhaustive checklists[4] are also available to help the designer in gathering information. For the purpose of this example, the AJ building enclosure design guide[5] has been used to outline kinds of data appropriate and the environmental conditions in Montreal have been assumed (Appendix II).

3.3 PERFORMANCE OBJECTIVES

The specification of performance objectives makes explicit the purposes to be served by the constructional systems without restricting the designer in the solutions he puts forward. It is the designer's responsibility to specify objectives as precisely as he can, but there are wide variations in the extent to which this is possible. The set of

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1 The Climate of Canada, Meterological Branch, Dept. of Transport. Information Canada, Ottawa, 1962.

2 Associate Committee of the National Building Code, National Building Code of Canada 1975, National Research Council of of Canada, Ottawa, 1975, NRCC 13992.

3 See, for example, A.J. Elder (Ed.), AJ Handbook of Building Enclosure. The Architectural Press, London, 1974.

4 See, for example, Jaeggin, K.W., and Brass, A.E., A Study of the Performance of Buildings. National Research Council of Canada, Ottawa, 1967, NRC 9352.

5 A.J. Elder, *Op. Cit.* (3).

performance objectives shown in Appendix III developed for this study provides examples that range from the precisely-definable to the broadly general. But performance objectives of these kinds are presently the most specific that can be devised and they provide a suitable basis for evaluation of whatever alternatives are generated during the design process. The summary of the performance objectives, which is based on the CIB Master List of Properties[6] is presented in Table 3.1; but only five objectives will be considered further in this design example, to wit:

(i) Fire resistance (FR)

(ii) Thermal resistance (T)

(iii) Risk of condensation (R)

(iv) Sound transmission loss (S)

(v) Initial cost (C)

The remaining objectives in Table 3.1 are considered as constraints.

3.4 ALTERNATIVE DESIGN SOLUTIONS

In most cases, there will be a large number of constructional systems constituting possible alternative answers to the statement of performance objectives. The trade and

6 CIB Master Lists for Structuring Documents Relating to Buildings, Building Elements, Components, Materials and Services, International Council for Building Research, Studies and Documentation, CIB Report No. 18, Rotterdam, 1972.

TABLE 3.1

SUMMARY OF PERFORMANCE OBJECTIVES FOR EXTERNAL WALLS OF SINGLE FAMILY HOUSES IN MONTREAL

PERFORMANCE VARIABLES (based on CIB Master List of Properties)	PERFORMANCE OBJECTIVES	
	Least Acceptable	Most Desirable
1 Compressive Strength	$\phi R = \gamma[\alpha_D D + \alpha_L L]$	---
2 Bending Strength	$\phi R = \gamma[\alpha_W W]$	---
3 Bending Stiffness	$\Delta = h/50$ in(h/50 mm)	---
4 Impact Strength Safety	R = 750 ft-lb(1000 J)	---
5 Impact Strength Service	R = 90 ft-lb(120 J)	---
6 Strength relating to holding power of fixing	$R = 0.4\ W_p$	---
7 Vibration	$\delta \leq 0.2\ \delta_t$ in 0.5 sec.	---
8 Fire Resistance	Fire Resistance Rating = 0.5h	Fire Resistance Rating = 2.0h
9 Ignitability and Flame Spread	Flame Spread Rating = 150	Flame Spread Rating = 0
10 Smoke Development	Smoke Development Rating = 150	Smoke Development Rating = 0
11 Air Tightness	Q = 11.8 ft/hr under 0.30 in Q = 3.6 in/hr under 7.5 mm of water	---
12 Prevention of Condensation	Condensation Risk - High	Condensation Risk - None
13 Dimensional Control	---	---
14 Prevention of Water Penetration	No water penetration	---
15 Thermal Insulation	R = 9.09°F.ft²h/Btu (1.6°C m²/W)	R = 20.00°F. ft h/Btu (3.5°C m²/W)
16 Transmission of Sound	STL = 30 dB	STL = 70 dB
17 Service Life	Service Life = 20 yrs	Service Life = 100 yrs
18 Initial Cost	$6.00/ft² ($64.57/m²)	$2.00/ft² ($21.52/m²)

<u>N.B:</u> For key to symbols see Appendix III

technical literature[7,8] abound with feasible solutions,* and modifications of these constructional systems may result in yet more alternatives. Considerations of different configurations, variations of thicknesses and different materials can also be used to generate alternatives.

It is possible to identify a wall by describing the materials and configuration of the laminae which constitute the constructional system. The set of alternative solutions shown in Figure 3.1 began in this way with a number of commonplace systems. By changing the materials as well as the number and configuration of laminae, the original number of alternative solutions was enlarged. The description of wall sections (Figure 3.1) is systematic -- starting with the outermost lamina (i.e., the surface facing the weather) to the innermost. Each design alternative is designated with a number which serves as an identification tab in the

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* The wall constructions presented as alternative solutions to the design problem may not include "*the global optimum*" system, but it is seldom possible to know whether the designer's efforts have uncovered the best alternative(s). The search for good prospects was stopped, as often happens in practice, because of time limitations. However, the set of alternatives is large enough that comparisons against performance criteria and among the proposed solutions can be significant.

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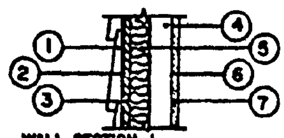
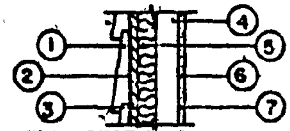
7 For example, that collection of trade catalogues available in most design offices: <u>Sweets' Catalogue Services Canadian Construction Catalogue File</u>, McGraw-Hill, Scarborough, Ontario, 1974.

8 Another universal standy is: C.G. Ramsey and H.R. Sleeper, <u>Architectural Graphic Standards</u>, John Wiley and Sons, New York, 1974.

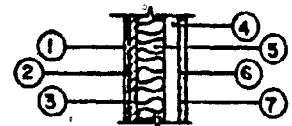
FIGURE 3.1 EXAMPLE OF ALTERNATIVE DESIGN SOLUTIONS.



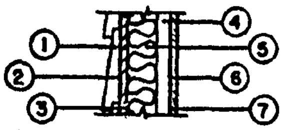
WALL SECTION 1
1 Wood Bevel Siding (3/4" x 8" Cedar)
2 Building Paper - (15# Felt)
3 Wood Sheathing - (3/4")
4 Wood Stud at 16" o.c - (2" x 4")
5 Batt Insulation - (2")
6 Polyethylene Vapour Barrier - (4 mil)
7 Gypsum Board - (1/2")



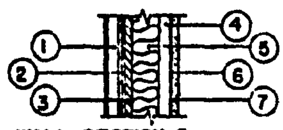
WALL SECTION 2
1 Wood Bevel Siding - (3/4" x 8" Cedar)
2 Building Paper - (15# Felt)
3 Wood Sheathing - (3/4")
4 Wood Stud at 16" o.c - (2" x 4")
5 Batt Insulation - (3")
6 Polyethylene Vapour Barrier - (4 mil)
7 Gypsum Board - (1/2")



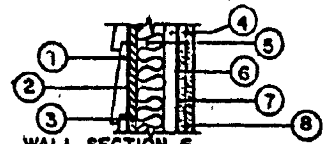
WALL SECTION 3
1 22 Gge. stl, light zinc coated flat siding
2 Building Paper (15# Felt)
3 Wood Sheathing (3/4)
4 Wood Stud at 16" o.c (2" x 4")
5 Batt Insulation (3")
6 Polyethylene Vapour Barrier (4 mil)
7 Gypsum Board (1/2")



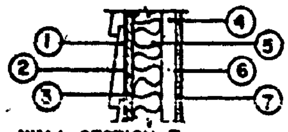
WALL SECTION 4
1 .025" Aluminum Siding + Baked Enamel
2 Building Paper - (15# Felt)
3 Wood Sheathing (3/4")
4 Wood Stud at 16" o.c (2" x 4")
5 Batt Insulation (3")
6 Polyethylene Vapour Barrier (4 mil)
7 Gypsum Board (1/2")



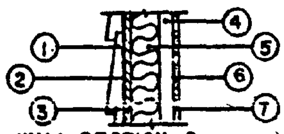
WALL SECTION 5
1 Corrug. Asb. Cement Siding (1/4" x 4" x 1 3/8")
2 Building Paper - (15# Felt)
3 Wood Sheathing (3/4")
4 Wood Stud at 16" o.c (2" x 4")
5 Batt Insulation (3")
6 Polyethylene Vapour Barrier (4 mil)
7 Gypsum Board (1/2")



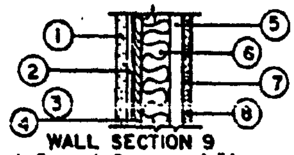
WALL SECTION 6
1 Wood Bevel Siding (3/4" x 8" Cedar)
2 Building Paper (15# Felt)
3 Wood Sheathing (3/4")
4 Wood Stud at 16" o.c (2" x 4")
5 Batt Insulation (3")
6 Polyethylene Vapour Barrier (4 mil)
7 Gypsum Board (1/2")
8 Gypsum Board (1/2")



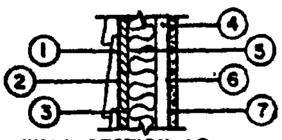
WALL SECTION 7
1 Cedar Shingles (Double Coursing)
2 Building Paper (15# Felt)
3 Wood Sheathing (3/4")
4 Wood Stud at 16" o.c (2" x 4")
5 Batt Insulation (3")
6 Polyethylene Vapour Barrier (4 mil)
7 Gypsum Board (1/2")



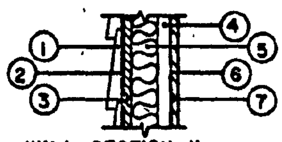
WALL SECTION 8
1 Wood Bevel Siding (3/4" Cedar)
2 Building Paper - (15# Felt)
3 Wood Sheathing - (1/2")
4 Wood Stud at 16" o.c (2" x 4")
5 Batt Insulation - (3")
6 Polyethylene Vapour Barrier (4 mil)
7 Gypsum Board (1/2")



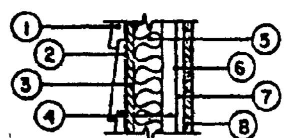
WALL SECTION 9
1 Cement Stucco (1")
2 Self Furring Steel Lath
3 Building Paper (15# Felt)
4 Wood Sheathing (3/4")
5 Wood Stud 16" o.c (2" x 4")
6 Batt Insulation (3")
7 Polyethylene Vapour Barrier (4 mil)
8 Gypsum Board (1/2")



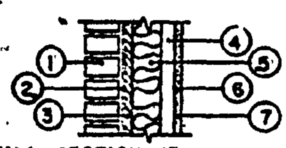
WALL SECTION 10
1 Wood Bevel Siding (3/4" x 8" Cedar)
2 Building Paper (15# Felt)
3 Wood Sheathing (3/4")
4 Wood Stud at 16" o.c (2" x 4")
5 Expanded Polystyrene Type I Class B (2")
6 Polyethylene Vapour Barrier (4 mil)
7 Gypsum Board (1/2")



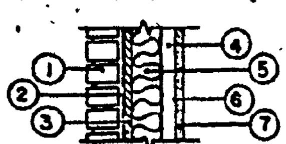
WALL SECTION 11
1 Wood Bevel Siding (3/4" x 8" Cedar)
2 Building Paper (15# Felt)
3 Wood Sheathing (3/4")
4 Wood Stud at 16" o.c (2" x 4")
5 Batt Insulation (3")
6 Polyethylene Vapour Barrier (4 mil)
7 Fir Plywood (3/8")



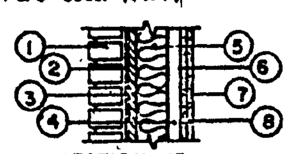
WALL SECTION 12
1 Wood Bevel Siding (3/4" x 8" Cedar)
2 Building Paper (15# Felt)
3 Wood Sheathing (3/4")
4 Wood Studs at 16" o.c (2" x 4")
5 Batt Insulation (3")
6 Polyethylene Vapour Barrier (4 mil)
7 Wood Furring (1" x 2")
8 T&G Cedar (1" x 4")



WALL SECTION 13
1 Face Brick (Running Bond)
2 Building Paper (15# Felt)
3 Wood Sheathing (3/4")
4 Wood Stud at 16" o.c (2" x 4")
5 Batt Insulation (2")
6 Polyethylene Vapour Barrier (4 mil)
7 Gypsum Board (1/2")

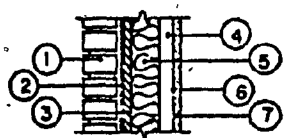


WALL SECTION 14
1 Face Brick (Running Bond)
2 Building Paper (15# Felt)
3 Wood Sheathing (3/4")
4 Wood Stud at 16" o.c (2" x 4")
5 Batt Insulation (3")
6 Polyethylene Vapour Barrier (4 mil)
7 Gypsum Board (1/2")



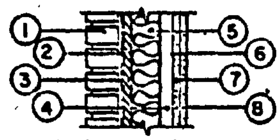
WALL SECTION 15
1 Face Brick (Running Bond)
2 Building Paper (15# Felt)
3 Wood Sheathing (3/4")
4 Wood Stud at 16" o.c (2" x 4")
5 Batt Insulation (3")
6 Polyethylene Vapour Barrier (4 mil)
7 Gypsum Board (1/2")
8 Gypsum Board (1/2")

FIGURE 3.1 EXAMPLE OF ALTERNATIVE DESIGN SOLUTIONS. (Cont.)



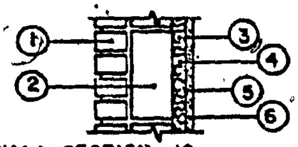
WALL SECTION 16

1 Face Brick (Common Bond)
2 Building Paper (15# Felt)
3 Wood Sheathing (3/4")
4 Wood Stud at 16" o.c (2"x4")
5 Batt Insulation (3")
6 Polyethylene Vapour Barrier (4 mil)
7 Gypsum Board (1/2")



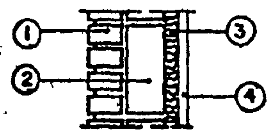
WALL SECTION 17

1 Face Brick (Common Bond)
2 Building Paper (15# Felt)
3 Wood Shedthing (3/4")
4 Wood Studs at 16" o.c (2"x4")
5 Batt Insulation (3")
6 Polyethylene Vapour Barrier (4 mil)
7 Gypsum Board (1/2")
8 Gypsum Board (1/2")



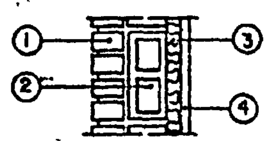
WALL SECTION 18

1 Face Brick (Running Bond)
2 Concrete Block (Solid - 4")
3 Wood Furring
4 Batt Insulation (3")
5 Polyethylene Vapour Barrier (4 mil)
6 Gypsum Board (1/2")



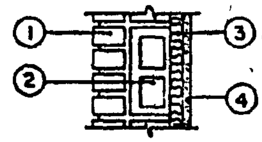
WALL SECTION 19

1 Face Brick (Running Bond)
2 Concrete Block (Solid - 4")
3 Expanded Polystyrene (Type 2 Class B-2)
4 Gypsum Board (1/2")



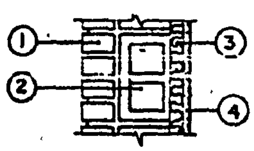
WALL SECTION 20

1 Face Brick (Running Bond)
2 Concrete Block (Hollow - 4")
3 Expanded Polystyrene (Type 2 Class B-1½)
4 Gypsum Board (1/2")



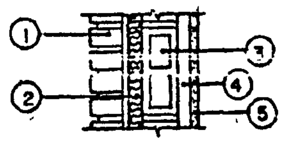
WALL SECTION 21

1 Face Brick (Running Bond)
2 Concrete Block (Hollow - 4")
3 Expanded Polystyrene (Type 1 Class B-2)
4 Gypsum Board (1/2")



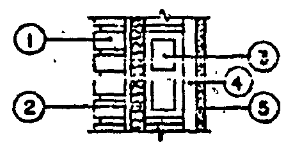
WALL SECTION 22

1 Face Brick (Running Bond)
2 Concrete Block (Hollow - 6")
3 Expanded Polystyrene (Type 2 Class B-1½)
4 Gypsum Board (1/2")



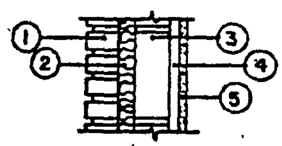
WALL SECTION 23

1 Face Brick (Running Bond)
2 Expanded Polystyrene (Type 1 Class B-1½")
3 Concrete Block (Hollow - 4")
4 Wood Furring (1"x 2")
5 Gypsum Board (1/2").



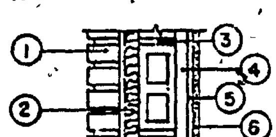
WALL SECTION 24

1 Face Brick (Running Bond)
2 Expanded Polystyrene (Type 1 Class B-2")
3 Concrete Block (Hollow - 4")
4 Wood Furring (1"x 2")
5 Gypsum Board (1/2")



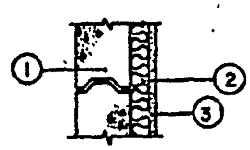
WALL SECTION 25

1 Face Brick (Running Bond)
2 Expanded Polystyrene (Type 1 Class B-2")
3 Concrete Block (Solid - 4")
4 Wood Furring (1"x 2")
5 Gypsum Board (1/2")



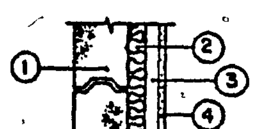
WALL SECTION 26

1 Face Brick (Running Bond)
2 Expanded Polystyrene (Type 2 Class B 1½
3 Concrete Block (Hollow - 4")
4 Wood Furring (1"x 2")
5 Gypsum Lath (3/8")
6 Gypsum Plaster (1/2")



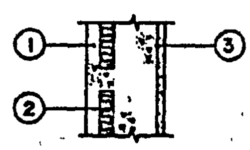
WALL SECTION 27

1 Precast Concrete Panel (6")
2 Expanded Polystyrene (Type 2 Class B-2)
3 Gypsum Board (1/2")



WALL SECTION 28

1 Precast Concrete Panel (6")
2 Expanded Polystyrene (Type 2 Class B 1½
3 Wood Furring (2"x 2")
4 Gypsum Board (1/2")



WALL SECTION 29

1 Precast Concrete Panel (8")
2 Expanded Polystyrene (Type 2 Class B- 2")
3 Gypsum Board (1/2")

NOTE:

The design alternatives are made up of products and materials commonly available in Montreal, and as such they are desribed in units currently used in practice.

subsequent computation.

The final set of 29 admissible alternatives (Figure 3.1) is an example of constructional systems assumed to have complied with the least acceptable criteria of those performance objectives established for a particular situation and with other constraints, one of which might be the building owner's aesthetic tastes. For this example, the performance objectives are elaborated in Appendix III.

3.5 ANALYTICAL MODELS

It is possible to estimate the likely behaviour of the constructional systems by representing them by mathematical models, by physical models or by prototypes. In the mathematical models (e.g., for the calculation of thermal resistance, sound transmission loss and rate of condensation) reliance is placed on previous scientific knowledge of the behaviour of similar constructional systems with respect to the performance variables. In many other cases (e.g., fire resistance rating), tests simulating the conditions under which the systems may operate are used to assess their probable performance. Evaluation of the reduced performance likely to result from future use of a construction subject to wear and deterioration provides an illustration of a model that must contain an element of interpretation based, in this case, on the suitability of the test methods and estimates of the performance predicted from tests, and modified by any relevant further information, experience or other determining factors that may be available to the designer. Initial costs are estimated on the basis of surveys of materials and labour costs published periodically.

The decision rule proposed here is capable of operating with any number of performance objectives, subject only to computer limitations. For the purpose of the illustrations presented in this chapter to explain the use of the decision rule, only the five performance objectives specified previously (FR, T, R, S, C) have been considered.

These performance objectives were chosen because they represent:

(i) designers' major considerations respecting the building enclosure; and,

(ii) varying degrees of precision in predicting performance of alternatives.

It is possible, using either mathematical models or codes of practice, to predict the behaviour of walls on each of these five performance objectives (Appendix IV). However, in all the calculations, there is an element of interpretation which varies greatly from one model to the next, depending on their representational validity; some may be excessively precise by comparison with the crudity of others. Ideally, all should possess a similar degree of accuracy but, practically, this is an impossibility.

3.6 EVALUATION OF ALTERNATIVES

Using the approach described above, the performance of alternatives is predicted with respect to each objective. For the application given in this Chapter, the models are detailed in Appendix IV. The predicted performance scores are then entered into the appropriate cell in a matrix of performance characteristics (Table 3.2).

It is evident from the most cursory examination of the rows of the matrix that the disparate units of measurement and the orders of magnitude of the scores would make it difficult to assess the overall performance of one alternative by comparison with the others. Furthermore, it is not possible, from the raw (predicted) performance scores alone, to obtain an indication of an alternative's effectiveness in meeting every performance objective.

Alternatives with predicted performance values less than the least acceptable limits are rejected. In the external wall design problem, this has been done for all the performance objectives except the cost objective. To facilitate the comparison of objectives, the predicted performance data should be expressed on a common basis, i.e., in terms of its corresponding utility. The utility scale is arbitrarily defined such that 0 and 100 correspond to the least acceptable and the most desirable limits, respectively. The predicted performance score is transformed into the appropriate utility attribute by means of utility functions.

TABLE 3.2

EXAMPLE OF MATRIX OF PREDICTED PERFORMANCE SCORES

PERFORMANCE CHARACTERISTICS

Wall Section/ Alternative #	Fire Resistance Rating (Hr)	Risk of Condensation	Thermal Resistance (°F/Btu/hr ft²) (°C/W/m²)	Sound Transmission (dB)	Initial Cost ($/ft²) ($/m²)
1	0.83	None	11.90/2.10	43.0	2.62/28.20
2	0.83	None	14.79/2.61	43.0	2.66/28.63
3	0.83	None	14.37/2.53	42.0	3.18/34.23
4	0.83	None	14.37/2.53	39.5	3.48/37.46
5	0.83	Negligible	14.00/2.47	39.6	2.83/30.46
6	1.25	None	14.57/2.57	43.0	3.03/32.26
7	0.83	None	14.75/2.60	46.2	3.06/33.15
8	0.83	Negligible	13.45/2.37	41.5	2.48/26.70
9	0.83	None	13.35/2.35	50.3	3.11/33.48
10	0.58	None	12.04/2.12	43.0	2.83/30.46
11	0.67	None	14.14/2.49	40.0	3.61/38.86
12	0.92	None	15.75/2.77	40.0	6.04/65.01
13	0.83	None	10.84/1.91	65.0	4.34/46.72
14	0.83	None	13.84/2.43	65.0	4.38/47.15
15	1.25	None	14.30/2.52	64.8	4.75/51.13
16	0.83	None	13.89/2.45	65.0	4.77/51.34
17	1.25	None	14.30/2.52	64.8	5.14/55.33
18	4.00	None	13.42/2.36	70.0	4.84/52.10
19	4.00	High	12.94/2.28	70.0	5.02/54.68
20	3.50	High	10.70/1.88	68.2	4.69/50.48
21	3.50	High	9.95/1.75	68.2	4.50/48.44
22	4.00	High	10.90/1.92	69.8	4.83/51.99
23	3.50	Low	10.11/1.78	64.0	4.63/49.84
24	3.50	Low	11.89/2.09	64.0	4.67/49.84
25	3.50	Low	11.50/2.03	63.8	4.87/52.42
26	3.50	Low	15.14/2.67	63.8	5.21/56.08
27	3.00	High	12.30/2.17	68.4	6.53/70.29
28	3.00	High	12.64/2.23	69.0	6.68/71.90
29	3.00	Low	12.30/2.17	63.0	7.36/79.22

3.7 TRANSFORMATION OR UTILITY FUNCTIONS

The overall worth of an alternative is determined by its constituent utility attributes, which, in turn, depend on the level of achievement on a performance objective and on the shape of the utility function. It is evident that, given a level of achievement, the shape of the utility function is important in determining the magnitude of the attribute. Since design decisions are likely to be sensitive to changes in attribute values, the effect of changes in the utility function on decisions will be investigated (later in the Chapter).

The relationship between a level of performance of an alternative i with respect to an objective j, y_{ij}, and the corresponding utility u_{ij} is given by the transformation function $u_{ij} = f(y_{ij})$. The utility may be considered in terms of benefits, comfort, annoyance, and so on, that an individual derives from a level of performance on that objective.

The shapes of the utility function depend to a certain extent on the scale on which the performance is measured and also on the individual's judgement of utility. Some utility functions appropriate to the design situation are described below.

(i) Threshold Function

Consider, for instance, the case of the owner whose wish is only to comply with the least acceptable limit on the sound transmission loss objective. In such a case, the satisfactory performance of the wall, y_{ij}, is defined as

$$y_{ij} \geq y_{Tj} \qquad (3.1)$$

where y_{Tj} is the least acceptable limit.

If the above inequality is satisfied, then the utility corresponding to Y_{ij} is 100, otherwise it is zero. Thus the form of the utility function is of a threshold function type (Figure 3.2). It should be noted, however, that this type of utility function is more conveniently treated as a constraint.

(ii) Step-function

If there is some concern over acoustical comfort, the owner's utility may differ from the above case. In the Residential Standards[9] only three ratings of sound transmission class* are specified. Each class corresponds to a 5dB range of airborne sound transmission loss. The performance level at a particular rating (5dB) would correspond to one utility level. Thus, the sound transmission loss

* It should be noted that these ratings are more suitable for party walls since the ratings are based on Sound Transmission Class.

[9] Associate Committee on The National Building Code, Residential Standards, 1975, National Research Council of Canada, Ottawa, NRCC No. 13991.

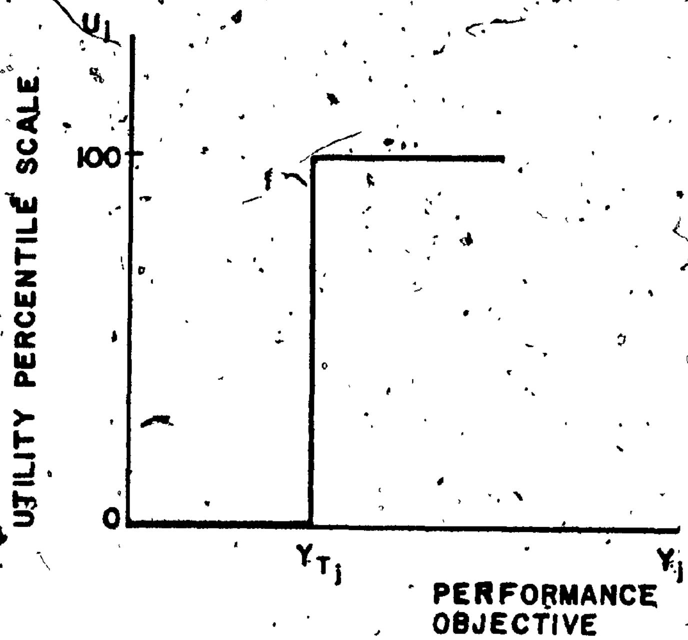


FIGURE 3.2 UTILITY TRANSFORMATION
THRESHOLD FUNCTION TYPE

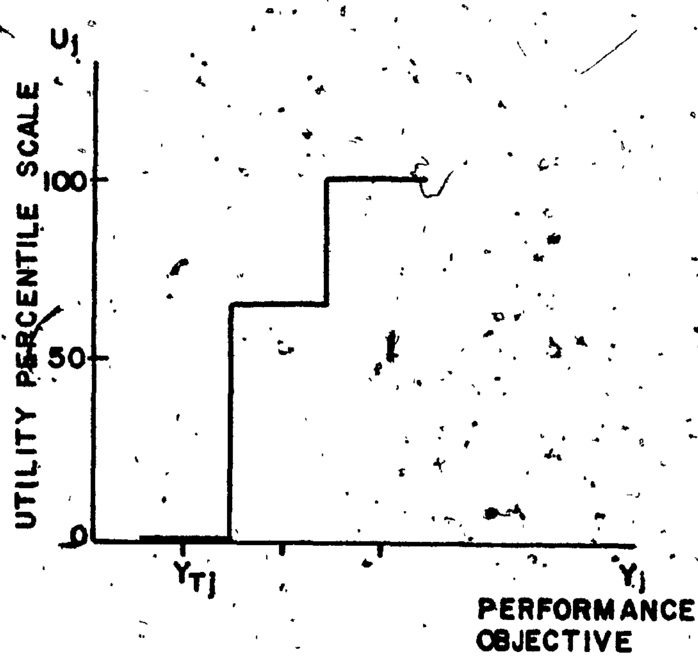


FIGURE 3.3 UTILITY TRANSFORMATION—
STEP-FUNCTION TYPE

ratings correspond to three utility levels -- hence the utility function can be represented by a step-function akin to that in Figure 3.3.

(iii) Continuous functions

If the performance can be measured on an interval scale and the utility varies accordingly, then the utility function is continuous. Three different shapes of utility functions can be discerned:

(a) Concave (upwards)

(b) linear, and

(c) convex (upwards).

The first type is usually based on the principle of diminishing marginal utility[10] which is expressed in the following inequalities:

$$\frac{du_j}{dy_j} > 0 \quad \text{and} \quad \frac{d^2u_j}{dy_j^{\,2}} < 0 \qquad (3.2)$$

A utility function which fulfills these inequalities is of the form $u_{ij} = A\,B^{y_{ij}} + K$. This indicates that each additional increment in performance contributes a decreasing amount of utility as the performance level increases (Figure 3.4). The principle is compatible with risk-averse behaviour, i.e., a gamble will always be sold for less than its expected

10 M.W. Lifson, Decision and Risk Analysis for Practicing Engineers, Cashner Books, 1972.

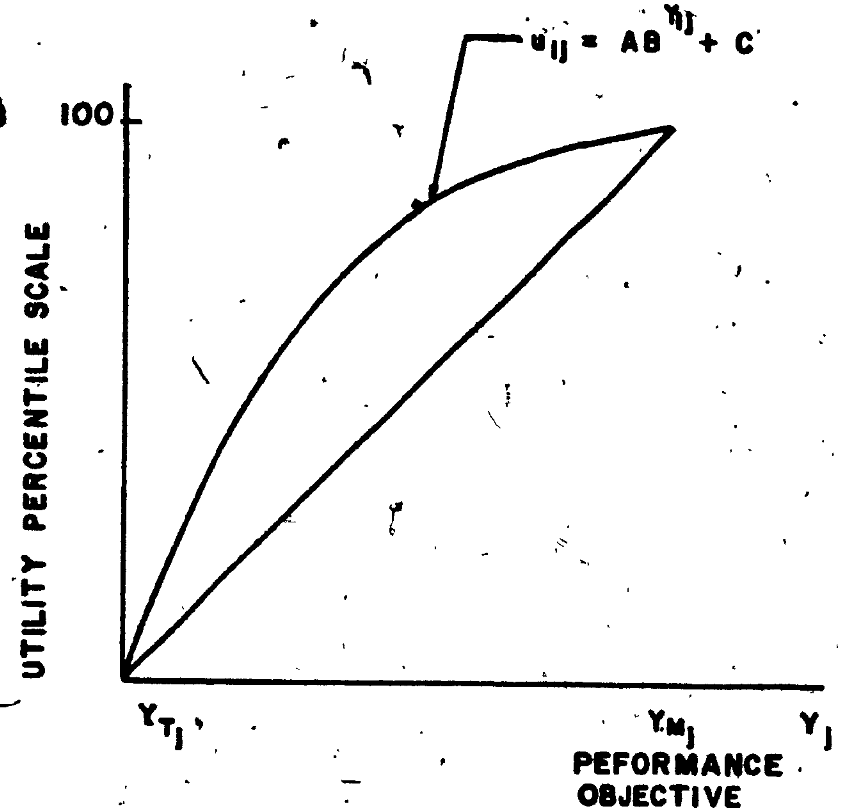


FIGURE 3.4 CONTINUOUS UTILITY FUNCTIONS — LINEAR AND DIMINISHING MARGINAL UTILITY

value. It is obvious that the risk aversion increases with increasing upward concavity of the utility curve.

The linear type of transformation produces an increase in utility magnitude proportional to an increase in performance at any level of performance (Figure 3.4). This type of utility transformation is also known as a risk-neutral utility function. A convex (upwards) utility function of Type (c) is the so-called "gambler's" utility curve -- i.e., it is risk-attractive -- and not of interest in a design situation.

(iv) Special Design Utility Functions

Utility functions can sometimes be derived on the basis of theoretical and empirical considerations. The sound transmission loss and thermal resistance objectives are amenable to this approach. The AJ Handbook of Building Enclosure[11] provides a relationship between design internal 10% noise climate in dBA and the external walls' average sound insulation in dB over a range of external 10% noise climate in dBA. This relationship is plotted as shown in the first quadrant of Figure 3.5 for an external noise climate of 85 dBA. Using Waller's[12] empirical correlation between noise level in dBA and the loss of the value of the

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11 A.J. Elder, *Op. Cit.* (3)

12 R.A. Waller, "Environmental Quality, Its Measurement and Control", Regional Studies, Vol. 4, 1970, pp. 177-191.

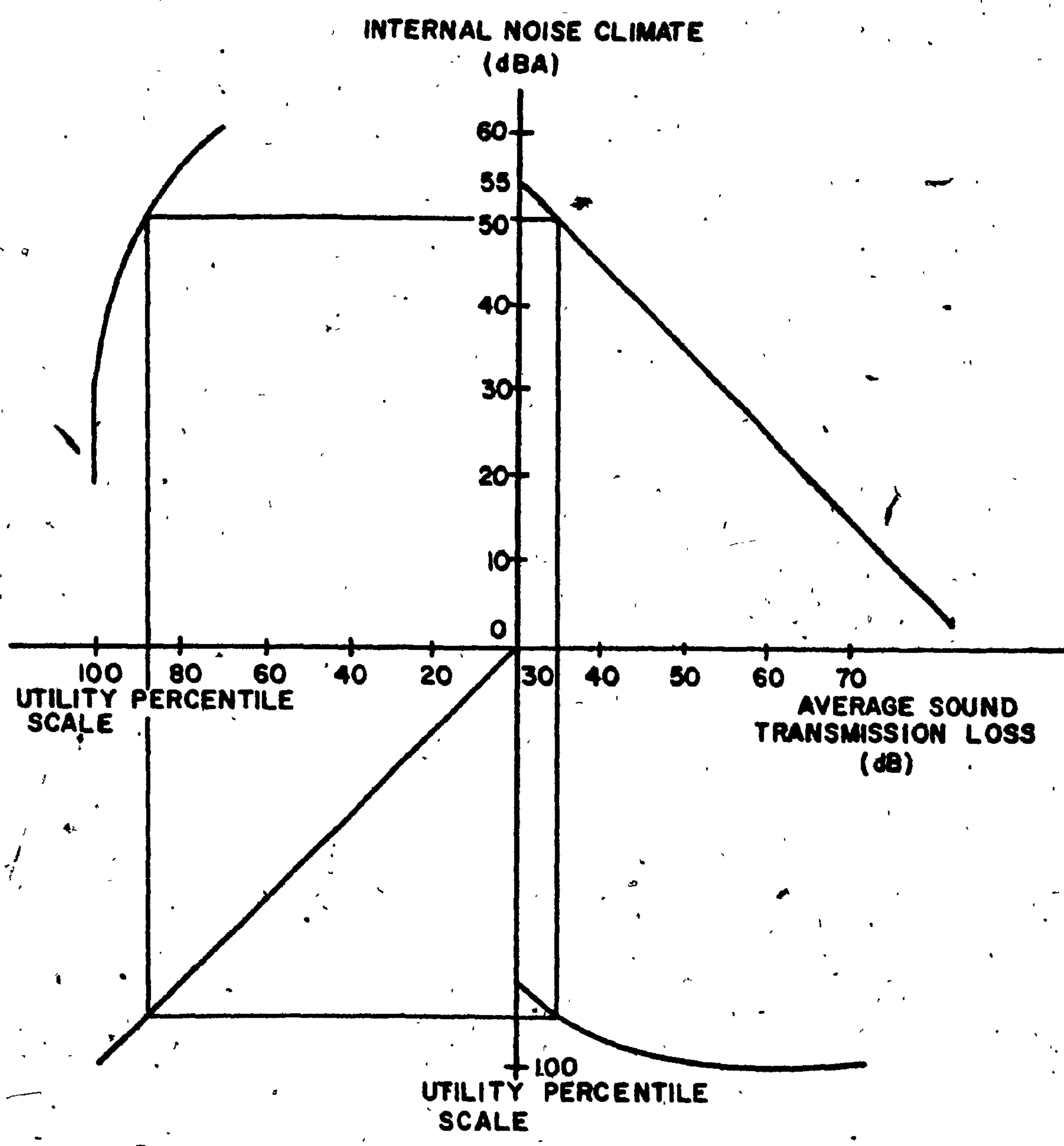


FIGURE 3.5 UTILITY FUNCTION BASED ON THEORETICAL AND EMPIRICAL RELATIONSHIP

— AVERAGE SOUND TRANSMISSION LOSS TRANSFORMATION TO UTILITY amenity in houses, a relationship between internal noise level and utility can be assumed as shown in the second quadrant of Figure 3.5. Combining these two sets of data, the utility function for average sound transmission loss is plotted as shown in the fourth quadrant.

The utility function for thermal resistance is derived in a similar manner and shown in Figure 3.6. It should be noted that the heat loss is inversely proportional to the thermal resistance. Two utility functions for resistance are, however, developed -- one on the basis of linear relationships between heat loss and utility and the other by using the principle of diminishing marginal utility. The resulting utility functions are both risk-averse; the function based on the linear transformation being less risk-averse than the other.

In the example design problem under consideration, a linear utility transformation has, for simplicity, been assumed to be applicable for all performance objectives except the risk of condensation objective. In this objective, a step-function is used (Figure 3.7) because the risk of condensation is measured on an ordinal scale.

Transformation of performance into utility by means of linear functions is given by the equation

$$u_{ij} = \left(\frac{y_{ij} - y_{Tj}}{y_{mj} - y_{Tj}}\right) \times 100 \qquad (3.3)$$

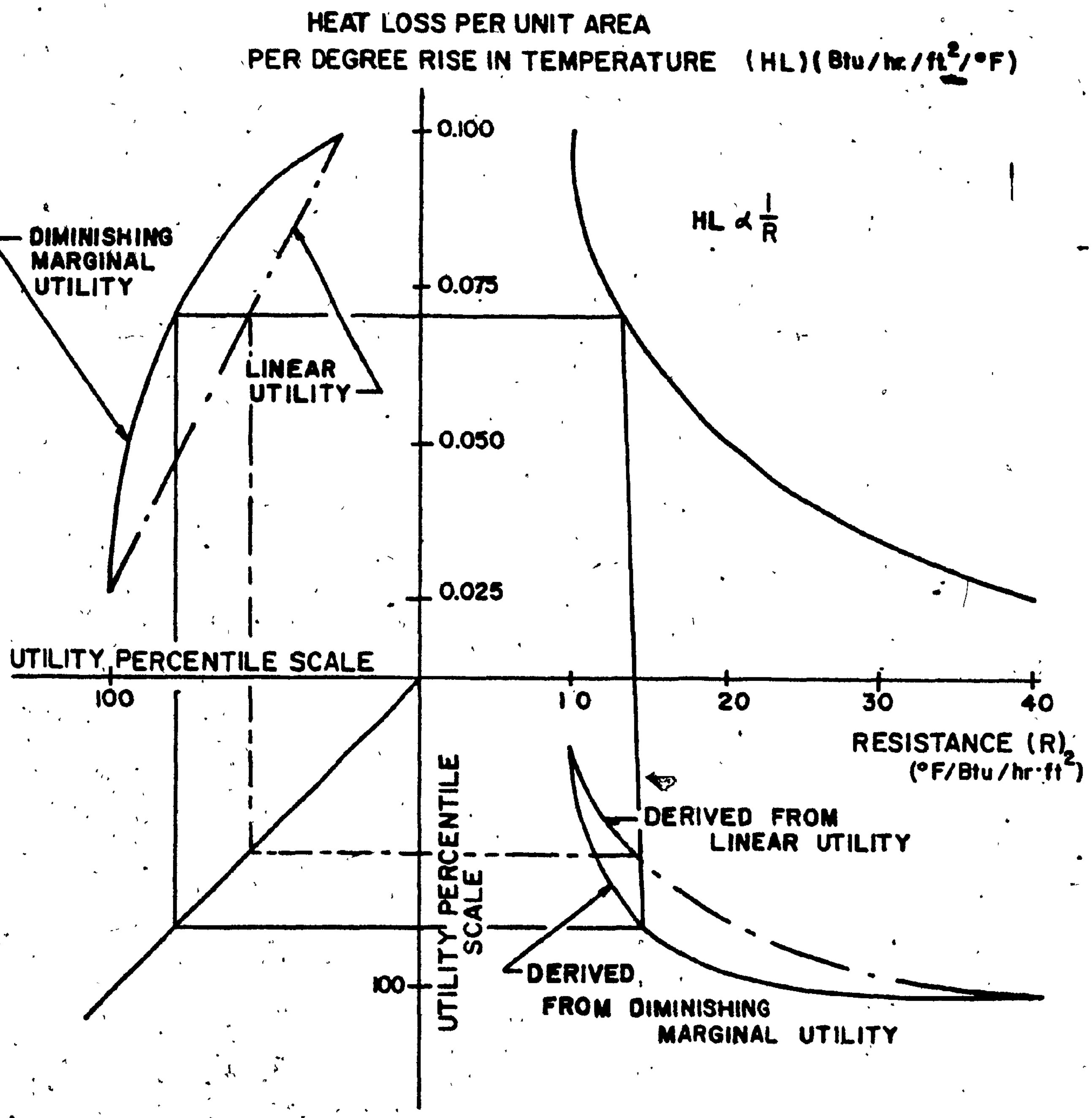


FIGURE 3.6 UTILITY BASED ON THEORETICAL AND JUDGEMENTAL RELATIONSHIP

— THERMAL RESISTANCE TRANSFORMATION TO UTILITY

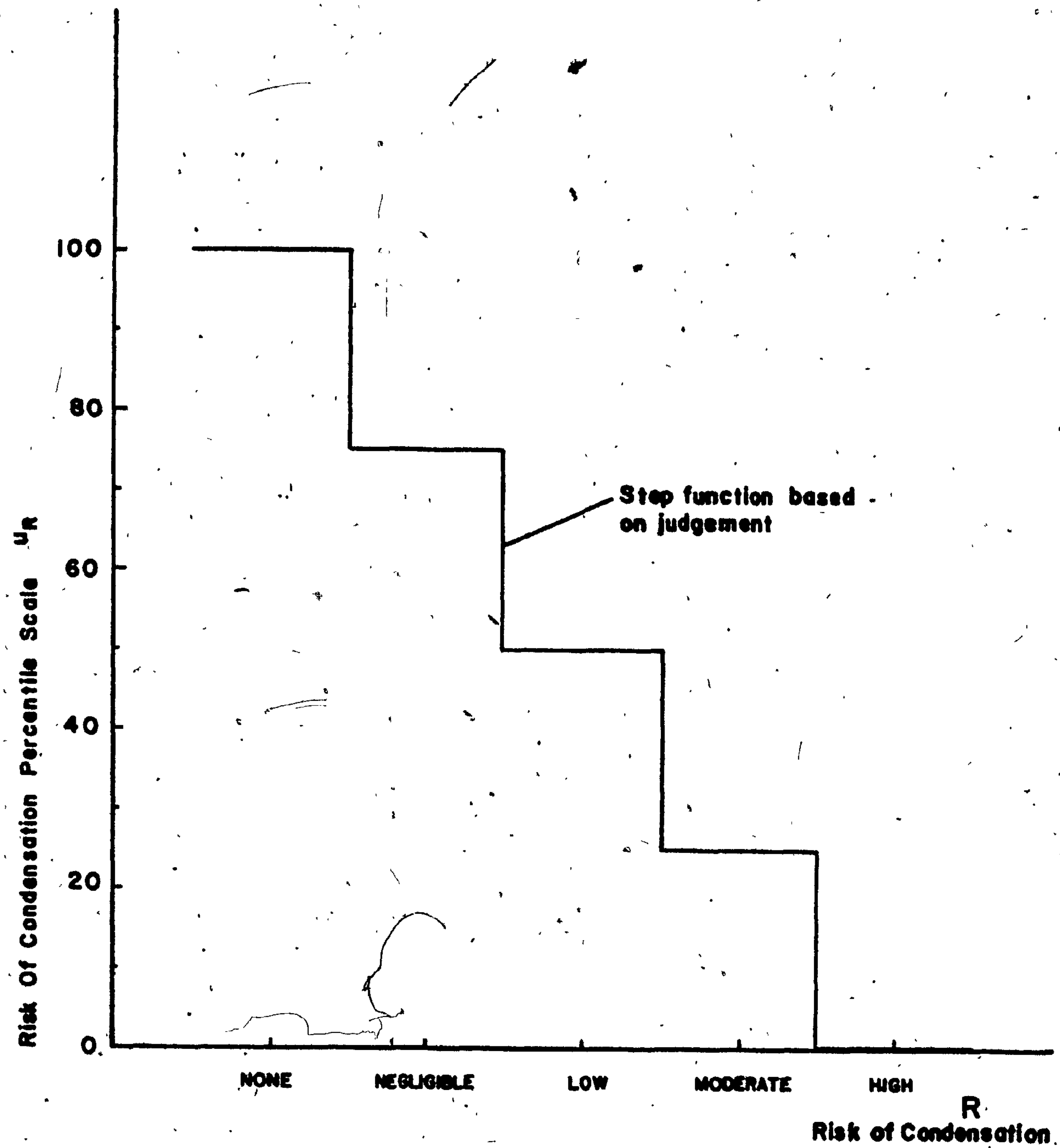


Fig. 3-7 Graph of step-function utility transformation for risk of condensation.

where u_{ij} is the utility corresponding to the performance value y_{ij}, and y_{mj} and y_{Tj} are the most desirable and least acceptable limits respectively. Using the limits of the five performance objectives considered in this example (given in Table 3.1), the predicted performance scores in Table 3.2 are transformed into utility attributes and presented in Table 3.3. It should be noted that, in this Table, the two columns for fire resistance attributes correspond to two different sets of limits. Originally, the limits were 0.5 to 4.0 hr. They were later revised to 0.5 to 2.0 hrs. in order to be more realistic. This change of objective limits provides data for studying the effect on decisions of changes in limits (this is done later in this chapter). Another point which is worthy of note concerns the cost transformation function -- a high cost corresponds to a low utility, and so the function is of an inverse nature.

3.8 DECISION BY EXCLUSION

Thus far, the performance objectives for external walls have been defined, a number of alternative solutions to the objectives have been proposed and their predicted behaviour calculated and measured on normalized (percentile) scales. The evaluation matrix in Table 3.3 presents the data in a form suitable for application of the decision rule. Because of the large number of alternatives and performance objectives considered in this example, it is necessary to use a computerized version of the Decision by Exclusion rule

TABLE 3.3

EXAMPLE OF MATRIX OF DESIGN ALTERNATIVES AND THEIR PERFORMANCE ATTRIBUTES

Performance Objective	1	2	3	4	5	1 Revised Scale
Design Alternative (#)	Fire Resistance	Risk of Condensation	Thermal Resistance	Sound Transmission loss	Initial Cost	Fire Resistance ***
1	9.5	100	25.8	32.5	84.5	22.2
2	9.5	100	52.2	32.5	83.5	22.2
3	9.5	100	48.4	30.0	70.5	22.2
4	9.5	100	48.4	23.8	63.0	22.2
5	9.5	75	45.0	24.0	79.3	22.2
6	21.4	100	50.0	32.5	74.3	50.0
7	9.5	100	51.9	40.5	74.3	22.2
8	9.5	75	40.0	28.8	88.0	22.2
9	9.5	100	39.0	50.8	72.3	22.2
10	2.4	100	27.0	32.5	79.3	5.6
11	4.8	100	46.3	25.0	59.8	11.1
12	11.9	100	60.9	25.0	-1.0	27.8
13	9.5	100	16.0	87.5	41.5	22.2
14	9.5	100	44.0	87.5	40.5	22.2
15	21.4	100	47.8	87.0	31.3	50.0
16	9.5	100	44.0	87.5	30.7	22.2
17	21.4	100	47.8	87.0	21.5	50.0
18	100.0	100	39.7	100.0	29.0	100.0
19	100.0	0	35.3	100.0	24.5	100.0
20	85.7	0	14.8	95.5	32.8	100.0
21	85.7	0	7.9	39.5	37.5	100.0
22	100.0	0	16.6	99.5	29.5	100.0
23	85.7	50	9.3	85.0	34.3	100.0
24	85.7	50	25.7	85.0	33.3	100.0
25	85.7	50	22.1	84.5	28.3	100.0
26	95.2	50	55.5	84.5	19.8	100.0
27	71.4	0	29.4	96.0	-13.3	100.0
28	71.4	0	38.3	97.5	-17.0	100.0
29	71.4	50	29.4	82.5	-34.0	100.0

*** The attributes in this column are derived from the linear transformation of Fire Resistance measurements based on the Objective scale with 0.50 to 2.00 h as limits. A "revised" matrix is obtained by replacing the resistance attributes in the "original" matrix (which are based on 0.50 to 4.00 h scale limits) with the corresponding amended values in this column vector.

described in Chapter II (the computer program is given in Appendix I). The process involved in reaching a decision can be considered in three distinct steps:

(i) Exclusion of dominated alternatives -- without consideration of priorities among objectives.

(ii) Further exclusion of alternatives by specifying priorities among objectives.

(iii) Robustness analysis.

3.8.1 Exclusion of Dominated Alternatives

Two analyses are undertaken using the original matrix of attributes (fire resistance limits -- 0.5 to 4.0 hr.) and the revised matrix (fire resistance limits -- 0.5 to 2.0 hr.). The results are presented and compared below.

(i) Original Matrix

17 out of the 29 alternatives are excluded from further consideration, since, no matter what the priorities assigned to the objectives, these alternatives are inferior to those remaining. This is based on the rule of dominance with implicit additivity.

The remaining alternatives, corresponding to the extreme points of the convex polygon (the non-dominated alternatives), are the wall assemblies (with the identification numbers in Figure 3.1):

1, 2, 7, 8, 9, 12, 13, 14, 15, 18, 22, 26

(ii) Revised Matrix

With the revised set of fire resistance attributes, only 12 of the 29 alternatives are excluded, the remaining alternatives being:

1, 2, 6, 7, 8, 9, 12, 13, 14, 15, 18, 20, 21, 22, 23, 24, 26

The revision in the fire resistance performance limits had the effect of boosting most alternatives' score on that objective's percentile scale so that five fewer alternatives were excluded. These are underlined in the above list.

3.8.2 Further Exclusion of Alternatives -- by Specifying Priorities Among Objectives

Where a designer is able to express preferences among objectives, he is likely to use one or the other of two ways of expressing priorities:

(i) Simple priorities (i.e., between any two objectives)

(ii) Partially-ordered sets of priorities among objectives.

(i) Simple Priorities Between Objectives

The purpose of this section is to show the effect of specifying simple priorities between objectives. A simple priority consists of expressing one objective as being more important than another (say, thermal resistance more important than fire resistance rating, or $\omega_T \geq \omega_{FR}$). The effect of these simple inequalities is to reduce the decision field further by excluding more alternatives from further consideration in the design. The five performance objectives considered in this design example result in 20 possible simple priority combinations expressed mathematically as $\omega_i \geq \omega_j$, $i \neq j$, $i = 1, \ldots, m$; $j = 1, \ldots, n$. The excluded alternatives are presented in Tables 3.4(a) and 3.4(b). Using the original matrix, if $\omega_T \geq \omega_{FR}$, the wall assembly 22 will be excluded from the non-dominated set of alternatives. This is plausible because the fire resistance attribute of wall 22 (of magnitude equal to 100) contributes to the alternative's overall utility significantly more than the thermal resistance attribute (of magnitude equal to 16.6); its overall worth is now diminished by the priority. In general, however, it would be difficult in advance of the use of the computer program to predict which alternative(s), if any, would be excluded: the interactions among the attributes are usually too complex.

(ii) <u>Partially Ordered Sets of Priorities Among Objectives</u>

Often a designer may express a number of preferences simultaneously. In this section, a number of such "*partially ordered sets of priorities (POSET)*" will be used in the decision rule. In the following, various applications of the Decision by Exclusion rule with partially-ordered sets will be examined.

TABLE 3.4(a)

SIMPLE PREFERENCE ANALYSIS -- ORIGINAL MATRIX

Preference Between Objectives ω_i	$\geq$ ω_j	Additional Alternatives Excluded from Design									
T	FR									22	
T	R	1									
T	S				9		13				
T	C	1		8	9		13			22	
FR	T		7			12		14	15		
FR	R		7						15		
FR	S		7		9		13	14	15		
FR	C	1	7	8	9		13	14			
R	T										26
R	FR									22	
R	S										
R	C			8						22	
S	T		7			12			15		
S	FR										
S	R										
S	C	1		8	9		13			22	
C	T		7			12			15		
C	FR										
C	R										
C	S		7						15		

TABLE 3.4(b)

SIMPLE PREFERENCE ANALYSIS -- REVISED MATRIX

Preference Between Objectives		Additional Alternatives Excluded from Design													
ω_i	$\geq$ ω_j														
T	FR									20	21	22	23	24	
T	R	1													
T	S				9		13			20		22			
T	C	1		8	9		13			20	21	22	23	24	
FR	T		7			12		14	15						
FR	R		7						15						
FR	S		7		9		13	14							
FR	C	1	7	8	9		13	14							
R	T														26
R	FR									20	21	22	23	24	
R	S									20	21				
R	C			8						20	21	22	23	24	
S	T		7			12			15						
S	FR										21		23	24	
S	R														
S	C	1		8	9		13			20	21	22	23	24	
C	T		7			12			15						
C	FR														
C	R														
C	S		7						15						

POSET 1

A designer considers intiial cost (C) and fire resistance (FR) more important than sound transmission loss (S), while having no preference between the cost and fire resistance objectives. No priorities are stated for the other objectives. What is the effect of this partially ordered set of priorities on the selection of an alternative solution?

Expressed in terms of inequalities, the designer's preferences are:

$$C \geq S$$

$$FR \geq S$$

(Only the original performance attributes are considered.)

Using Table 3.4(a) of the simple preference analysis, two separate applications of the above simple priorities result in the exclusion of the following alternatives:

7, 9, 13, 14, 15

Alternatively, the above partially ordered set of priorities may be considered simultaneously in the computer program as constraints. This results in the exclusion of the same set of alternatives. Alternatives remaining are:

1, 2, 8, 12, 18, 22, 26

In cases where the designer's priorities are more complex than POSET 1, the use of the simple preference results (Tables 3.4(a) and 3.4(b)) soon becomes too cumbersome. Furthermore, it can be shown by set theory analysis, that the number of alternatives excluded from the design when simple preferences are added may be fewer than the actual number eliminated when preferences are applied simultaneously. Numerical evidence will be given in POSET 4.

POSET 2

In a particular instance, a designer's priorities among the performance objectives could be specified in a manner something like the following:

Initial Cost (C) is the most important objective while risk of condensation (R) is the least; fire resistance (FR), though less important than cost, is more important than either thermal resistance (T) or sound transmission loss (S). Lastly, thermal resistance is more important than sound insulation. How would this ordering of the objectives affect the decision field?

In order to use it in the decision rule, the statement of priorities has first to be expressed mathematically in terms of inequalities:

$$C \geq FR \geq T \geq S \geq R$$

Simultaneous application of these contraints results in the elimination of the following alternatives in addition to those excluded in the first stage of the decision analysis (when no priorities between objectives were considered):

1, 7, 9, 12, 13, 14, 15

Thus, the alternatives which remain for further consideration in the design are

2, 8, 18, 22, 26

POSET 3

What is the effect on the decision field if the priorities on the objectives as expressed in POSET 2 are refined as the statement below?

$$1.0C \geq 2.0FR \geq 2.0T \geq 2.0S \geq 2.0R$$

i.e., cost is at least twice as important as fire resistance whereas fire resistance is more important than thermal resistance, which in turn is more important than sound transmission loss; finally, sound transmission loss is more important than risk of condensation.

The alternatives 22 and 26 in addition to those in the previous POSET are excluded by the simultaneous application of these constraints. The set of alternatives remaining is, then, reduced to

2, 8, 18

It is reasonable to infer that as priorities between objectives are specified more precisely, the number of alternatives remaining for design consideration diminishes. When the priorities are defined exactly, the designer is usually left with only one alternative.*

POSET 4

A designer considers that the fire resistance objective is the most important objective in the design but he has no preferences among the other objectives. What is the effect of this partially ordered set of priorities on the decision field?

The designer's preference can be stated in terms of inequalities, thus:

$$FR \geq C$$
$$FR \geq T$$
$$FR \geq R$$
$$FR \geq S$$

Applying these constraints simultaneously in the decision rule, the following alternatives are eliminated in addition to those already excluded in the first stage:

1, 2, 7, 8, 9, 12, 13, 14, 15

and so the alternatives remaining are:

---

* This is the case when weights are specified, i.e., priorities are measured on interval scales.

18, 22, 26

If the simple preference analyses are used to exclude alternatives (Table 3.4(a)), wall assembly 2 would not have been excluded. This observation corroborates the statement made in POSET 2 that the separate applications of simple priorities may result in fewer alternatives being excluded than is the case with simultaneous application of the constraints.

3.8.3 Robustness Analysis

In the usual additive utility models, the assumption that precise weighting of objectives is possible facilitates the choice of an optimum alternative, but restricts the practical use of the model. When significant variations in weighting of objectives occur, robustness analyses are undertaken to determine the extent to which certain weights may vary before an optimal solution becomes non-optimal. A systematic variation of the weights is termed a parametric analysis. On the other hand, uncertainties in the attribute values can be more directly studied by assessing the sensitivity of the decisions to changes in the magnitude of the attributes. Both techniques are treated in this section.

(i) Parametric Analysis on Objective Priorities

In the parametric analysis, the degree of importance of one objective relative to the others is changed systematically and the effect of such changes in priority on the alternatives remaining in the decision field is examined. Variations in objective priorities can range from the point where an objective is the most important factor in the decision (in fact, the only factor) through an indifference towards all objectives, on to where the objective is the least important factor and, finally, not included in design considerations.

In the following, the effect of variations in degree of importance of initial cost objective relative to the other objectives is examined. It should be noted that preferences are not specified among the other objectives (fire resistance (FR), thermal resistance (T), risk of condensation (R) and sound transmission loss (S)), i.e., they remain indifferent to each other. A similar analysis is presented for the fire resistance objective with respect to the others (the original matrix of attributes is used in both cases).

Let F be a parameter varying from 0 to 1. An increase in degree of importance of an objective (say, initial cost) over all the other objectives is effected by increasing F in the inequality

$$(1 - F)\omega_c \geq F\ \omega_j \qquad (3.4)$$

where j represents all the other objectives. A decrease in the degree of importance of cost with respect to all other objectives is affected by increasing F in the inequality:

$$F\omega_c \leq (1 - F)\omega_j \qquad (3.5)$$

For clarity of presentation, the left-hand side of Figure 3.8 is used to depict the increase in the degree of importance of the cost objective over all the other objectives. There are three specific values of F which are of special interest.

(a) Point A of Figure 3.8, $F = 1.0$

Inequality (3.4) gives

$$0\ \omega_c \geq \omega_j \quad \text{for all } j,\ j \neq c$$

This means that the cost objective is infinitely more important than the others.

(b) Point B of Figure 3.8, $F \to 0$

Inequality (3.4) gives

$$\omega_c \geq 0\ \omega_j \quad \text{for all } j,\ j \neq c$$

and Inequality (3.5)

$$0\ \omega_c \leq \omega_j \quad \text{for all } j,\ j \neq c$$

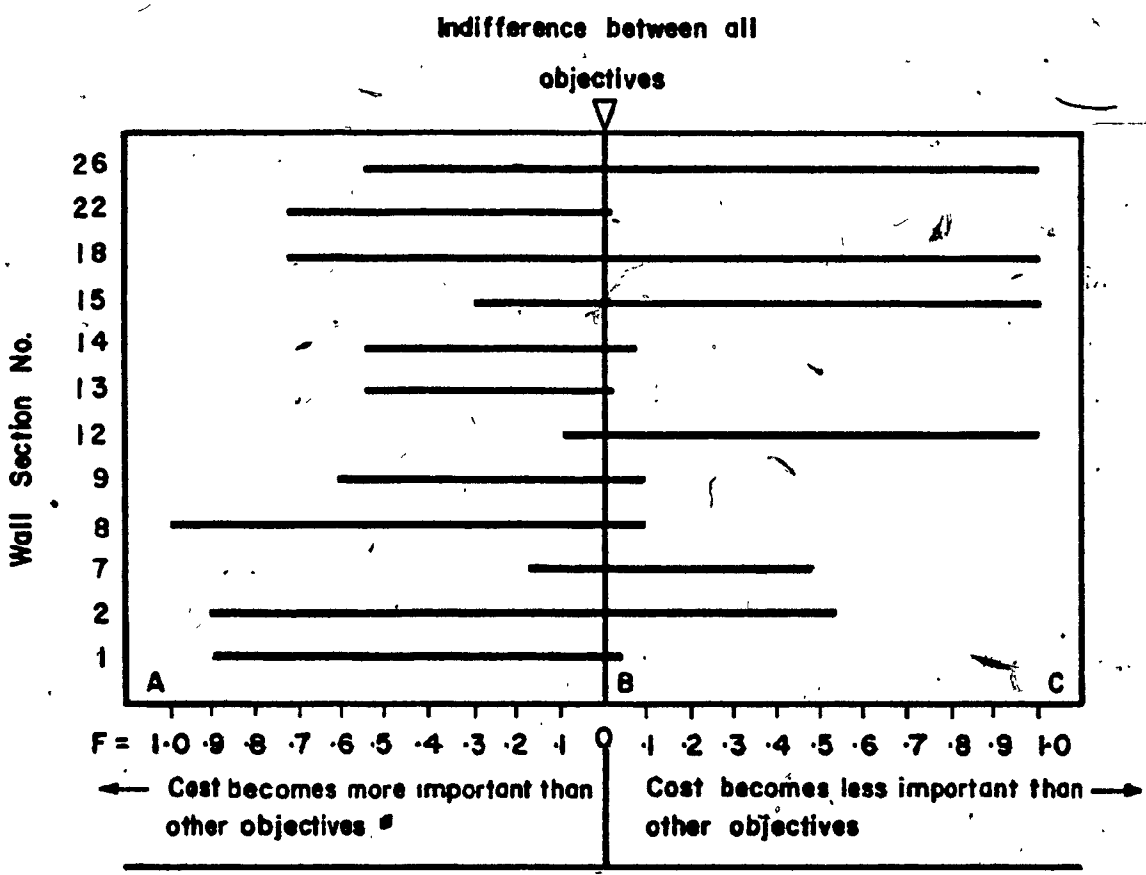
Both inequalities represent an indifference among all objectives.

(c) Point C of Figure 3.8 similarly represents the situation where cost is no longer considered as an objective.

(a) <u>Importance of Cost Objective</u>

A horizontal solid line in Figure 3.8 represents the

FIGURE 3.8 PARAMETRIC ANALYSIS FOR INITIAL COST



$(1-F)\times\omega_c \geq F\times\omega_j$ $\qquad$ $F\times\omega_c \leq (1-F)\times\omega_j$

(The inequalities hold for all j, where $j \neq c$)

<u>Robustness Analysis</u> — Effect of change in importance ranking of initial Cost objective with respect to the other performance objectives.

(Data from original matrix)

range of F over which the corresponding alternative remains in the non-dominated set. When no priorities are specified among the objectives, Point B, the 12 non-dominated alternatives are the same as those remaining after exclusion by dominance. The number of remaining alternatives is reduced as the importance of the cost factor is increased or decreased. At the point where the cost objective is infinitely more important than the other objectives, wall assembly 8 is the optimum alternative. This can be verified by examining the predicted performance scores (Table 3.2) where wall section 8 can be seen to be the least costly of the candidate solutions.

At F = 0.5 on the left-hand side of Figure 3.8, Inequality (3.4) becomes

$$\omega_C \geq \omega_{FR}$$
$$\omega_C \geq \omega_T$$
$$\omega_C \geq \omega_R$$
$$\omega_C \geq \omega_S$$

The 9 remaining alternatives are

1, 2, 8, 9, 13, 14, 18, 22, 26

At F = 0.8 on the left-hand side of Figure 3.8 the cost objective is considered to be more important than four times each of the other variables. The three remaining alternatives are:

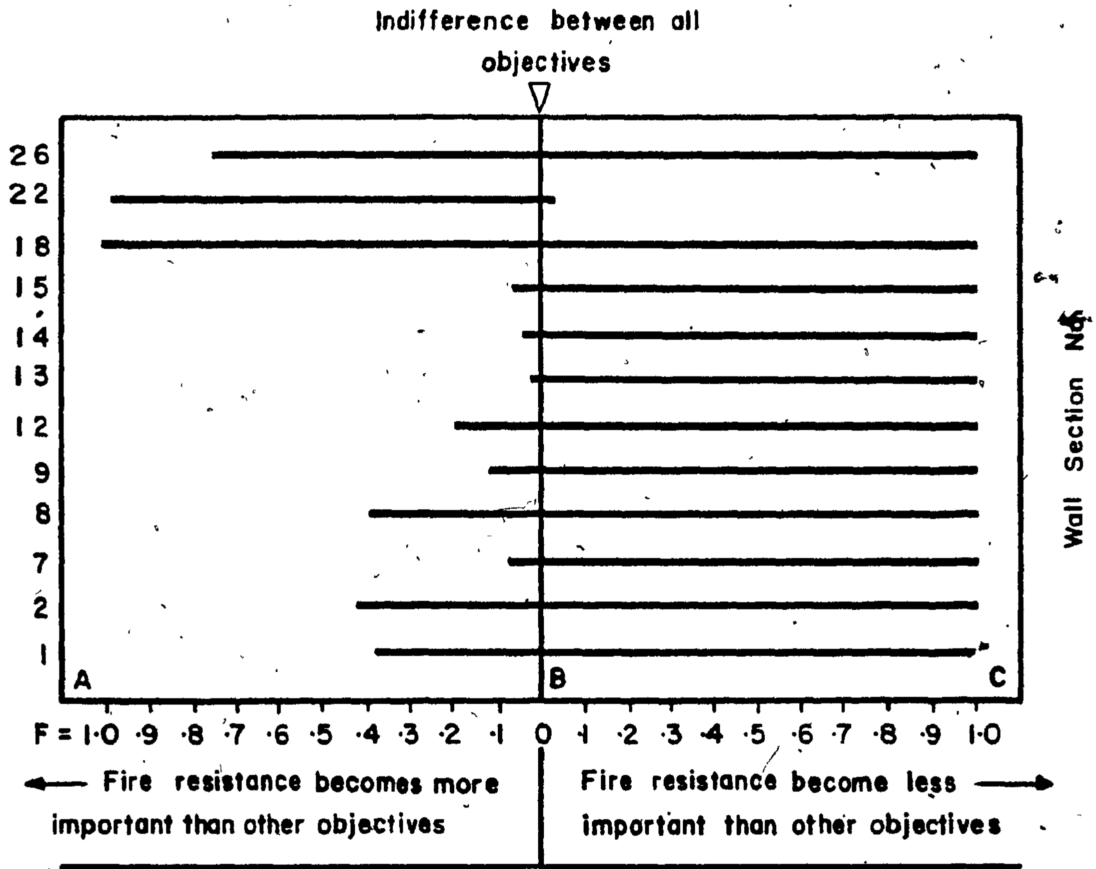
1, 2, 8

At the point C where cost is no longer considered in the decision, and no preferences are ascribed to the other objectives, there are only 4 alternatives remaining. It is interesting to note the large number of alternatives at B compared with the number remaining at C. It is easy to explain the reason for this observation in physical terms. A wall which performs poorly on the four objectives (fire resistance, thermal resistance, risk of condensation and sound transmission loss) may compensate for this deficiency by scoring highly on the fifth (cost) objective, i.e., it may be relatively inexpensive.

(b) Importance of Fire Resistance

The results of varying the degree of importance of fire resistance objective with respect to all the other objectives are shown in Figure 3.9. The three alternatives 18, 22 and 26 remain non-dominated over a relatively large range of variation in importance of the fire resistance objective. At F = 0.67 on the left-hand side, the fire resistance objective is more important than twice the other objectives and at F = 0.5 it is only just more important. Nevertheless, the same three alternatives remain. Thus, a designer selecting any of the three wall sections, 18, 22 or 26 on the basis of the preference expressed in POSET 4 is making a reasonably certain choice.

FIGURE 3.9 PARAMETRIC ANALYSIS FOR FIRE RESISTANCE



$(1-F) \times u_F \geqslant F \times u_j$ | $F \times u_F \leqslant (1-F) \times u_j$

(The inequalities hold for all j, where $j \neq F$)

Robustness Analysis — Effect of change in importance ranking of Fire Resistance objective with respect to the other performance objectives.

(Data from original matrix)

(ii) Effect of Changes in the Attribute Values on the Decision

The robustness analysis described so far has dealt only with the parametric analysis of changes in preferences among objectives. It is possible to alter the partial ordering of alternatives by means of changes in the values of the attributes. The change of the fire resistance scale resulted in an increase in value of some fire resistance attributes (these increased values were shown in the revised matrix) and these, in turn, increased the number of alternatives remaining for further consideration in the design. To clarify the effect of those increases in value of fire resistance attributes upon the decision, a partial plot of initial cost (C) versus fire resistance (FR) is presented in Figure 3.10. This plot demonstrates how wall assembly 6, for example, which is an interior (hence, inferior) point in the original plot becomes an extreme (hence, non-dominated) point in the revised plot.

The effects of uncertainty in the definition of utility functions can be studied by varying their shape. These functions have so far been assumed to be linear, except for the risk-of-condensation transformation, which is a step-function. If, however, the designer were risk-averse, the equation $u_{ij} = AB^{y_{ij}} + K$ may be used to represent the utility function. For convenience of computation, y_{ij} is measured on a linear percentile scale such that $y_{ij} = 0$ at the least acceptable

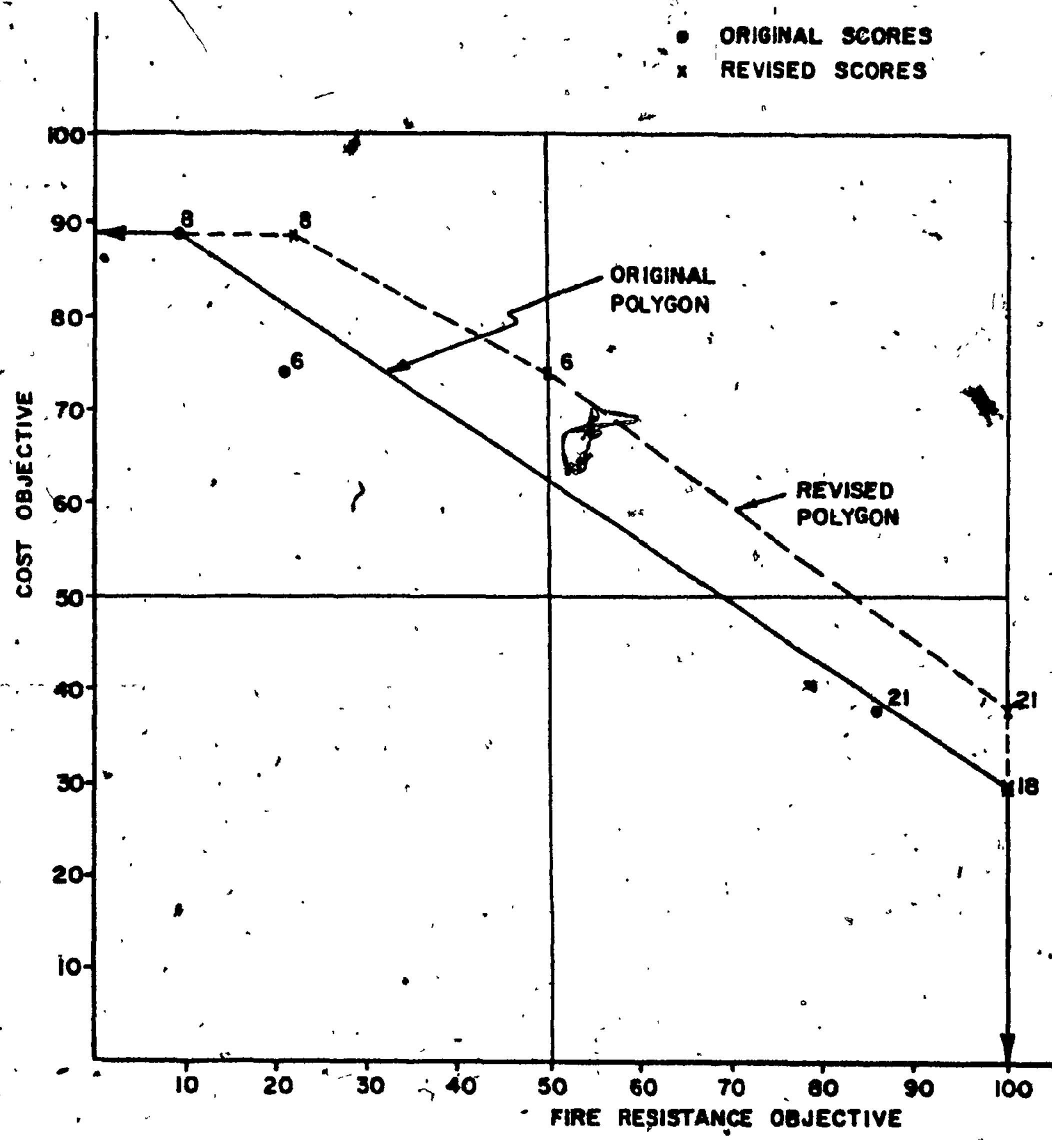


Figure 3-10 Plot of cost attributes vs fire resistance attributes (original and revised)

(Note: To improve clarity of presentation, all the inferior alternatives which do not become extreme points on the revised plot are not shown.)

limit and $y_{ij} = 100$ at the most desirable limit. The coefficients A, B and K can be determined by specifying three points on the utility curve, e.g., at $u_{ij} = y_{ij} = 0$, $u_{ij} = y_{ij} = 100$ and at the mid-point of the performance range, $y_{ij} = 50$. (Figure 3.11). For the purposes of this exercise, the values of utility at this mid-point are taken to be 55, 60, 65, ... , 95.

Taking the cost objective as an example, the remaining alternatives corresponding to the various shapes of cost utility function are shown in Table 3.5. It can be observed from the Table that, the more pronounced the risk aversion, i.e., the higher the values of utility at $y_{ij} = 50$, the larger is the number of remaining alternatives. A risk-averse utility transformation causes an increase in the attribute values, which, in turn, are likely to become extreme points in the decision field. Nevertheless, the overall effect is marginal. An increase of utility from 60 to 80 at $y_{ij} = 50$ produces only one additional non-dominated alternative, i.e., wall assembly 20. Over the whole range from 50 to 95, only 4 additional alternatives are added to the non-dominated set.

With the limited numerical evidence in this section, it can be observed that the change that has the most impact on the decision is a variation of the priorities among objectives. The effect of changing an objective's limits is akin to changing the priority attaching to the objective. This,

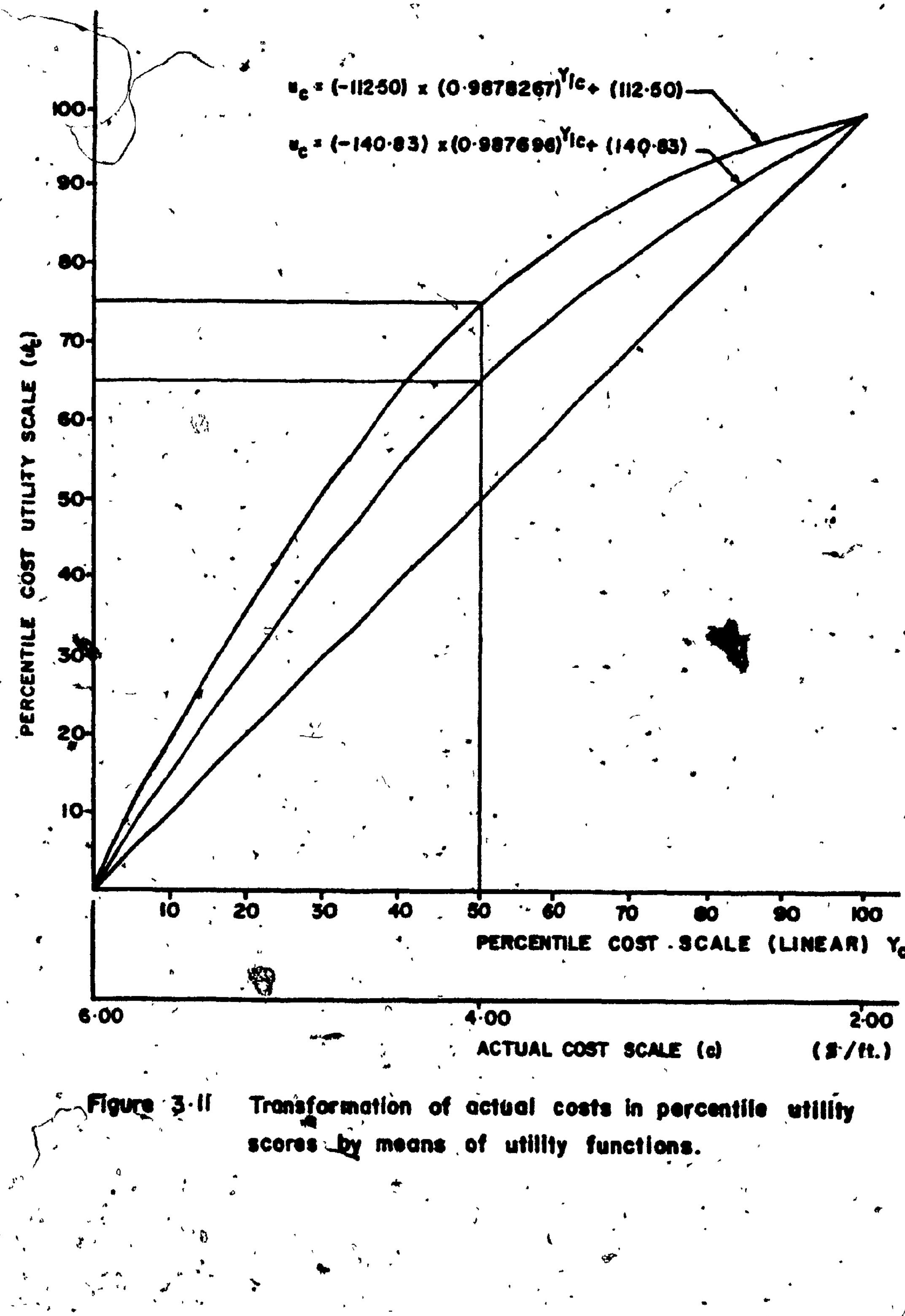


Figure 3·11 Transformation of actual costs in percentile utility scores by means of utility functions.

TABLE 3.5 EFFECT OF CHANGE OF UTILITY COST OBJECTIVE ON DECISION FIELD
— NO PREFERENCE BETWEEN OBJECTIVES

VALUE OF U CORRESPONDING TO $Y_{lc} = 50$	UTILITY FUNCTION FOR COST OBJECTIVE $U_{lc} = AB^{Y_{lc}} + K$	ALTERNATIVES REMAINING IN DESIGN																
50	———	1	2		7	8	9	12	13	14	15	18			22			26
55	$U = (-302.5) \times (0.995995)^{Y_{lc}} + 302.5$	1	2		7	8	9	12	13	14	15	18			22			26
60	$U = (-180.0) \times (0.991923)^{Y_{lc}} + 180.0$	1	2		7	8	9	12	13	14	15	18		21	22			26
65	$U = (-140.83) \times (0.987698)^{Y_{lc}} + 140.83$	1	2	6	7	8	9	12	13	14	15	18	20	21	22			26
70	$U = (-122.5) \times (0.983197)^{Y_{lc}} + 122.5$	1	2	6	7	8	9	12	13	14	15	18	20	21	22			26
75	$U = (-112.5) \times (0.978267)^{Y_{lc}} + 112.5$	1	2	6	7	8	9	12	13	14	15	18	20	21	22			26
80	$U = (-106.67) \times (0.972655)^{Y_{lc}} + 106.67$	1	2	6	7	8	9	12	13	14	15	18	20	21	22			26
85	$U = (-103.21) \times (0.965903)^{Y_{lc}} + 103.21$	1	2	6	7	8	9	12	13	14	15	18	20	21	22	23		26
90	$U = (-101.25) \times (0.957007)^{Y_{lc}} + 101.25$	1	2	6	7	8	9	12	13	14	15	18	20	21	22	23		26
95	$U = (-100.28) \times (0.942812)^{Y_{lc}} + 100.28$	1	2	6	7	8	9	12	13	14	15	18	20	21	22	23	24	26

in turn, is equivalent to a percentage error in the performance prediction model for that objective. This can be seen by realising that the column of attribute values pertaining to that objective is multiplied by a constant. (One exception is the case where performance limits are contracted; here, utility attributes of magnitude 100 remain the same by definition.) The implication of this for design is that effort in determining performance objective priorities and performance limits and efforts in refining predictive models would be highly beneficial to the success of a project.

CHAPTER 4

IN CONCLUSION: DECISIONS IN BUILDING DESIGN

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In the process of developing a building design, decisions at various levels of responsibility are made by a variety of people ranging from the project designer at one end to the draftsman preparing detailed drawings at the other. These decisions are not independent of one another, but in making a particular decision, the designer evaluates each alternative in the light of the objectives and his personal experience and judgement, and compares alternatives against each other. Ability to make decisions is, therefore, fundamental to the design process, and the Decision by Exclusion rule has been developed in response to the need for more systematic and comprehensive appraisals in design decision-making.

The purpose of this final Chapter is to indicate the decision rule's potential value in design practice, to extend understanding and use of the rule and to outline the kind of supporting research that will be needed.

4.1 <u>DECISIONS IN DESIGN</u>

To recapitulate, the major assumptions on which the Decision by Exclusion rule is based are that:

(a) the performance objectives are independent of each other (this is the assumption of independence);

(b) the designer can, at best, only rank the priorities among the performance objectives (the assumption of ordinality), and

(c) when appraising alternatives, an alternative's component attributes contribute additively -- albeit implicitly -- to its overall worth (the assumption of implicit additivity).

Because of Assumption (c), more alternatives are eliminated as inferior to others than would have been the case when only pure dominance obtains. By attaching priorities to the performance objectives and progressively refining these priorities, it is possible for the designer to make a final selection from a reduced set of "*best*" or "*at least as good as*" solutions. The flexibility of the decision rule in allowing for preferences among objectives -- as implied by Assumption (b) -- extends the applicability of the Decision by Exclusion rule over a wide range of problems. This is especially so in the instances in multi-dimensional decision-making where, because of the conflicting nature of some objectives, the decision-maker is often hindered from weighting the objectives.

The decision rule may also be applied by planning authorities to the evaluation of development plans for communities[1], or by institutions or government agencies to the evaluation of specific building design proposals. Making optimal choices from among the hundreds of SEF[2] or Operation Breakthrough[3] proposals would have been examples of the important uses to which the rule could have been applied. In the application of the Decision by Exclusion rule to the design of external walls it has been necessary to confine the application to simple examples but more general implications can be inferred from this study.

There are two caveats to be borne in mind when interpreting the results:

(i) The caveat of incompleteness

In a comprehensive design project, the designer would have to consider all the objectives deemed important and also the alternatives generated while taking into account the local context.

(ii) The caveat of the cyclical nature of the design process

---

1 See, for instance, the Coventry-Solihull-Warwickshire subregional study prepared by P. Smith and K. Barnes for the Local Government Operational Research Unit, Development Plan Evaluation and Robustness, (Department of the Environment, Research Report 5, HMSO, London, 1976).

2 See, the Metropolitan Toronto School Board Study of Educational Facilities, Introduction to the First SEF Building System, Toronto, 1969.

3 See O.E. Pfrang, Guide Criteria for the Evaluation of Operation Breakthrough Housing Systems, National Bureau of Standards, Washington, 1970.

Since the design process is cyclic in nature, the evaluation and decision stages are not necessarily the final steps in the design sequence. At the end of a design decision sequence, it is quite possible for there to be no unique solutions, only optimal solutions that are compatible with the requirements and assumptions which have been made at the time and in the circumstances of the project. Having cycled once through the design procedure the designer acquires an appreciation of the relative strengths and weaknesses of both the objectives and proposed alternatives. Depending on the designer's experience and knowledge and the needs and emphases of the project, the design sequence may be recycled with a revised set of performance objectives and changes in priorities among objectives.

As a consequence of using the Decision by Exclusion rule. that part of the design process that involves deliberating over alternatives and deciding which one to adopt becomes a matter of routine and so a large number of design proposals can be analysed. This is beneficial on three counts:

(a) The reliability of the designer's judgement is enhanced because his decision is based upon a wider selection of alternatives than usually considered in traditional design methods.

(b) The process of preparing the data prior to the application of the decision rule is in itself a valuable activity that helps to define ends, and, as a consequence, means.

(c) The time saved in the arduous task of weighting and comparing alternatives can be used to produce better predictive models, improved performance objectives and utility functions, more alternatives to choose between, and so on.

The parametric analysis for determining robustness also serves some useful purposes. By examining the results of the robustness analysis, a designer may be able to determine the key issues to which an alternative is particularly sensitive. These may then be investigated in more detail, and the refinements used in the evaluation process.

The sensitivity study undertaken in Chapter III has revealed that changing the priorities among objectives has a significant impact upon the decisions. Changes in the performance limits or percentage errors in the predictive model have a similar effect on the decision. In contrast, variations in utility functions have less impact on the decisions.

4.2 RECOMMENDATION FOR FURTHER RESEARCH

In developing the performance objectives for external walls, a number of problems which also apply to other aspects of building enclosure were discerned. These problems are formulated below in general terms.

There are many performance objectives which cannot yet be formulated precisely. Such lack of precision can lead to ambiguities in interpretation. There is a need to develop more and better performance objectives and models (mathematical or other) to predict the performance of constructional systems proposed as answers to the performance objectives. For instance, durability of constructions, functional life, and associated requirements for building maintenance comprise one set of problems of this kind.

The CIB Master List[4] provides a useful checklist for establishing performance objectives, but it is not complete. For instance, racking strength of components is not included in the checklist even though it is of considerable importance in wall design. Moreover, the Master List is not suitable as a basis for specification of performance objectives because the form in which it was compiled is not consistent with the way in which functional requirements are perceived.[5] Consequently, the reorganization and refinement of performance objectives for externals walls to resolve these deficiencies may prove to be beneficial to designers.

It is also necessary to develop performance objectives

---

4 CIB Master List for Structuring Documents Relating to Buildings, Building Elements, Components, Materials and Services, International Council for Building Research Studies and Documentation, Report No. 18, Rotterdam, 1972.

5 See, for example, N.B. Hutcheon, "Fundamental Considerations in the Design of External Walls for Buildings", 67th Annual and Professional Meeting of the Engineering Institute of Canada, Halifax, 1953.

for such other components of the building enclosure as roofs and floors, along the lines of those developed here for external walls. A compilation of performance objectives for buildings, building elements, and components, is essential for subsequent utilization of systematic design and decision-methods.

For the performance objectives to be realistic, it is necessary to examine the service conditions in the locality of their use (e.g., Montreal) and also the performance of materials and constructional systems *in situ*. This type of research, apart from providing an understanding of the relationships between material, design and environment, serves as a check upon the performance levels defined in performance objectives.

Standardization and optimization of constructional systems for single-family housing (and certain other building types) are undertaken mainly on the basis of fashion and practical experimentation over a period of time and many buidings. A research approach to this task would involve the design generation of alternative constructional systems and evaluation of the proposed alternatives by means of the Decision by Exclusion rule.

Some features of the Decision by Exclusion rule which could be improved to facilitate access and use include the normalization of the raw performance scores and the development of a more efficient algorithm for establishing extreme points (which correspond to non-dominated solutions). With these refinements to the computer program, more complex applications of the Decision Rule can be tackled economically.

A computerized design system, of which the Decision by Exclusion rule would be a component, could be developed to place emphasis on storing, manipulating and retrieving design alternatives. Though such a comprehensive system would appear to many to leave designers only the tasks of exercising judgement in selecting performance objectives, their priorities and interpretation of results, these would still be important responsibilities which, today, are performed substantially less than optimally.

APPENDIX I

COMPUTER PROGRAM OF DECISION BY EXCLUSION RULE

```
      PROGRAM UTILITY (INPUT=/80,OUTPUT=65,TAPE1=INPUT,TAPE2=65,
     + TAPE3)
      DIMENSION UT(29,10),IELIM(30),IND(29),ILEFT(29),IEQ(11),FEQ(11)
      DIMENSION W(29),FLAM1(10,29),FLAM2(10,29),CPA1(10,29)
      DIMENSION CPA2(10,29)
      DIMENSION C(30,10),COPI(30,30),WA(30),X(30),ICOLMS(30),IDES(30)
      DIMENSION ROW(30),D(30)
C
C        DIMENSION AND DATA REQUIREMENTS
C
C        UT(M,N),IELIM(M+1),IND(M),ILEFT(M),IEQ(N+1),FEQ(N+1),W(M)
C
C        C(M1,N),COPI(M1,M1),WA(M1),X(M1),ICOLMS(M1),IDES(M1)
C        ROW(M1),D(M1)
C
C        DATA IR,IC/M1,N/
C
C        M1 MUST BE AT LEAST M+1. IT ALSO MUST BE AT LEAST EQUAL TO
C           1+(NO. OF REMAINING ALTERNATIVES AFTER PHASE 1)+
C            (NO. OF INEQUALITIES ADDED IN PHASE 3)
C
C          FOR PHASE 4 (ONE ATTRIBUTE - FULL RANGE)
C              M1 >= 1 + 2*(REMAINING ALTERNATIVES)
C
      DATA D/0.0,1.0,28*0.0/
      DATA CPA1,CPA2,FLAM1,FLAM2/290*0.0,290*0.0,290*0.0,290*0.0/
      DATA TLIMIT/80.0/
      DATA ROW/1.0,29*0.0/
      DATA IR,IC/30,10/
      DATA DFMAX/0.001/
      DO 200 J=1,IC
      C(1,J)=-1.0
      C(2,J)=1.0
  200 CONTINUE
      DO 202 I=1,IR
      ICOLMS(I)=I
  202 CONTINUE
C
C
C     CARD 1     FREE FORMAT
C
C           N = THE NUMBER OF VARIABLES
C           M = THE NUMBER OF ALTERNATIVES
C           NDIGIT = NUMBER OF DIGITS TO BE PRINTED AFTER PERIOD
C                    (APPLIES ONLY TO UTILITY MATRIX).
C           RAND = RANDOM NUMBER SEED. IF RAND = 0. THEN M (THE NUMBER
C                OF ALTERNATIVES) CARDS FOR UTILITY MATRIX ARE TO FOLLOW.
C           VALMIN = MIN VALUE IN UTILITY MATRIX (IF RANDOMLY GENERATED)
C           VALMAX = MAX VALUE IN UTILITY MATRIX (IF RANDOMLY GENERATED)
C                    (ALL VALUES WILL BE LESS THAN VALMAX AND GREATER
C                     OR EQUAL TO VALMIN)
C           JUMP1  = 1 IF SECTION 1 IS TO BE JUMPED OVER.
C                    IN THIS CASE THE DATA ARE READ FROM TAPE 3.
C
C     CARD 2 TO M+1      OMITTED IF RAND IS NON-ZERO.
C
C              ROW OF UTILITY MATRIX. IT CONTAINS N VALUES IN FREE FORMAT.
```

```
C
C     SPECIAL INEQUALITIES ARE INTRODUCED AFTER THE FIRST CARD AND
C     (IF RAND=0) THE UTILITY MATRIX CARDS ARE READ.
C
C        THESE ARE EXPRESSED AS CHAINS OF INEQUALITIES.
C        A CHAIN CONSISTS OF AT MOST N GROUPS OF
C               MULTIPLIER BLANK INDEX"
C        SEPARATED BY COMMAS AND ENDED BY A SLASH.
C
C        A SET OF INEQUALITIES CONSISTS OF ONE OR MORE SUCH CHAINS.
C        THE SET IS TERMINATED BY THREE SLASHES.
C        EACH SET COULD BE ON ONE OR SEVERAL CARDS.
C
C        EXAMPLE
C                1 5,1 2,1 3/2.5 5,2 4///
C
C        REPRESENTS THE SET OF INEQUALITIES
C                      1*W5 >= 1*W2 >= 1*W3
C             AND      2.5*W5 >= 2*W4
C
C     THERE CAN BE ZERO, ONE OR MORE SUCH SETS OF INEQUALITIES FOR THE
C     SAME UTILITY MATRIX, BUT EACH UTILITY MATRIX REQUIRES A SEPARATE
C     RUN.
C     THE MULTIPLIERS SHOULD NOT HAVE MORE THAN 2 DIGITS AFTER
C     THE POINT, SINCE ONLY 2 DIGITS ARE PRINTED.
C
C
      READ *,N,M,NDIGIT,RAND,VALMIN,VALMAX,JUMP1
      IF (EOF(1).EQ.1.0) STOP
      EPSILON=0.1*0.1**NDIGIT
      N1=N+1
      WRITE (2,2)
    2 FORMAT (5(/))
      IF (RAND.EQ.0.0) GO TO 8
      CALL RANSET (RAND)
      WRITE (2,16) RAND
   16 FORMAT (19H THE RANDOM SEED IS,F10.3//)
      DIGITS=10.**NDIGIT
      RANGE=VALMAX-VALMIN
      IF (RANGE.LE.0.0) RANGE=10.0
      DO 3 I=1,M
      DO 3 J=1,N
      UT(I,J)=IFIX(DIGITS*(VALMIN+RANF(0)*RANGE))/DIGITS
    3 CONTINUE
      GO TO 14
    8 DO 12 I=1,M
      READ (1,*) (UT(I,J),J=1,N)
      IF (EOF(1).EQ.1.0) STOP
   12 CONTINUE
   14 NSPACE=NDIGIT+6
      WRITE (2,7) N,NSPACE,(J,J=1,N)
    7 FORMAT (//16H UTILITY MATRIX,//,1X,=I=)
      DO 10 I=1,M
   10 WRITE (2,11) I,N,NSPACE,NDIGIT,(UT(I,J),J=1,N)
   11 FORMAT (I3,=F=,=)
C
      IF (JUMP1.EQ.1) GO TO 39
```

```
C
   13 DO 4 I=1,M
    4 IND(I)=I
      NIND=M
      M1=M
      NELIM=0
      NLEFT=0
      IELIM(1)=0
      K=0
      DO 20 L=1,M
      K=K+1
      DO 15 I=2,NIND
      I1=I
      IF (I.LE.K) I1=I-1
      DO 15 J=1,N
      C(I+1,J)=UT(I1,J)-UT(K,J)+EPSILON
   15 CONTINUE
      LIC=NIND+1
      DO 206 I=1,LIC
      DO 204 J=1,LIC
      COPI(I,J)=0.0
  204 CONTINUE
      COPI(I,I)=1.0
  206 IDES(I)=N+I
      CALL ZX2LP (C,D,ICOLMS,ROW,1,LIC-1,N,100,LIC,IR,COPI,IDES,X,
     + WA,IER)

      IF (X(1).GT.0.9) GO TO 21
      NELIM=NELIM+1
      IELIM(NELIM)=IND(K)
      NIND=NIND-1
      DO 18 KK=K,NIND
      IND(KK)=IND(KK+1)
      DO 19 J=1,N
   19 UT(KK,J)=UT(KK+1,J)
   18 CONTINUE
      K=K-1
      GO TO 20
   21 NLEFT=NLEFT+1
      ILEFT(NLEFT)=IND(K)
   20 CONTINUE
   39 IF (JUMP1.EQ.1) READ (3) NIND,ILEFT,NLEFT,IELIM,NELIM,UT,
     + FLAM1,FLAM2,CPA1,CPA2
      WRITE (2,40) (IELIM(K),K=1,NELIM)
   40 FORMAT (//24H ELIMINATED ALTERNATIVES/20I3)
      WRITE (2,42) (ILEFT(K),K=1,NLEFT)
   42 FORMAT (//23H REMAINING ALTERNATIVES/20I3)
      IF (JUMP1.NE.1) WRITE (3) NIND,ILEFT,NLEFT,IELIM,NELIM,UT,
     + FLAM1,FLAM2,CPA1,CPA2
C
C       L2=0    STOP
C          1    SIMPLE INEQUALITIES
C          2    SPECIAL INEQUALITIES
C          3    ONE ATTRIBUTE - FULL RANGE
C
  300 READ *, L2
      IF (EOF(1).EQ.1.0 .OR. L2.EQ.0) STOP
```

```
      IF (L2.EQ.1) GO TO 301
      IF (L2.EQ.2) GO TO 64
      IF (L2.EQ.3) GO TO 304
      WRITE (2,302)
  302 FORMAT (/19H WRONG L2-INDICATOR)
      STOP
C
C        SIMPLE INEQUALITIES     L2=1
C
  301 WRITE (2,43)
   43 FORMAT (///45H FOR WI>=WJ ELIMINATE FURTHER THE ALTERNATIVES./)
      LIC=NIND+2
      DO 60 J1=1,N
      DO 58 J2=1,N
      IF (J1.EQ.J2) GO TO 58
      DO 45 J=1,N
   45 C(LIC,J)=0.0
      C(LIC,J1)=-1.0
      C(LIC,J2)=1.0
      NELIM=0
      IELIM(1)=0
      DO 56 K=1,NIND
      DO 52 I=2,NIND
      I1=I
      IF (I.LE.K) I1=I-1
      DO 52 J=1,N
      C(I+1,J)=UT(I1,J)-UT(K,J)+EPSILON
   52 CONTINUE
      DO 226 I=1,LIC
      DO 224 J=1,LIC
      COPI(I,J)=0.0
  224 CONTINUE
      COPI(I,I)=1.0
  226 IDES(I)=N+I
      CALL ZX2LP(C,D,ICOLMS,ROW,1,LIC-1,N,100,LIC,IR,COPI,IDES,X,WA,IER)
      IF (X(1).GT..95) GO TO 56
      NELIM=NELIM+1
      IELIM(NELIM)=IND(K)
   56 CONTINUE
      WRITE (2,57) J1,J2,NELIM,(IELIM(K),K=1,NELIM)
   57 FORMAT (I7,I4,5X,=I3.0)
   58 CONTINUE
   60 CONTINUE
      GO TO 300
C
C        SPECIAL INEQUALITIES     L2=2
C
   64 LIC=NIND+1
      WRITE (2,63)
   63 FORMAT (3(/))
   65 DO 66 I=1,N1
      IEQ(I)=0
   66 FEQ(I)=1.0
      READ (1,*) (FEQ(I),IEQ(I),I=1,N)
      IF (EOF(1).EQ.1.0) STOP
      IF (IEQ(1).EQ.0) GO TO 76
      IF (LIC.GT.NIND+1) GO TO 70
```

```
      WRITE (2,68)
   68 FORMAT (44H THE INEQUALITIES    FI*W   >=  FJ*WJ  >=  ...)
   70 DO 73 I=1,N
      IF (IEQ(I+1).EQ.0) GO TO 74
      LIC=LIC+1
      DO 72 J=1,N
   72 C(LIC,J)=0.0
      C(LIC,IEQ(I))=-FEQ(I)
      C(LIC,IEQ(I+1))=FEQ(I+1)
   73 CONTINUE
   74 WRITE (2,75) (FEQ(J),IEQ(J),J=1,I)
   75 FORMAT (15X,10(F8.2,I3))
      GO TO 65
   76 NELIM=0
      IELIM(1)=0
      NLEFT=0
      ILEFT(1)=0
      DO 86 K=1,NIND
      DO 82 I=2,NIND
      I1=I
      IF (I.LE.K) I1=I-1
      DO 82 J=1,N
      C(I+1,J)=UT(I1,J)-UT(K,J)+EPSILON
   82 CONTINUE
      DO 286 I=1,LIC
      DO 284 J=1,LIC
      COPI(I,J)=0.0
  284 CONTINUE
      COPI(I,I)=1.0
  286 IDES(I)=N+I
      CALL ZX2LP(C,D,ICOLMS,ROW,1,LIC-1,N,100,LIC,IR,COPI,IDES,X,WA,IER)
      IF (X(1).GT..95) GO TO 84
      NELIM=NELIM+1
      IELIM(NELIM)=IND(K)
      GO TO 86
   84 NLEFT=NLEFT+1
      ILEFT(NLEFT)=IND(K)
   86 CONTINUE
      WRITE (2,87) (IELIM(K),K=1,NELIM)
   87 FORMAT (/27H ELIMINATE THE ALTERNATIVES,T30,16I3)
      WRITE (2,88) (ILEFT(I),I=1,NLEFT)
   88 FORMAT (23H REMAINING ALTERNATIVES,T30,16I3)
      GO TO 300
C
C        F*W(J0) <= (1-F)*W(J)  AND  (1-F)*W(J0) >= F*W(J)     L2=3
C
  308 READ *, J0
      WRITE (2,309) J0,J0,J0
  309 FORMAT(///10H ATTRIBUTE,I3//12H ALTERNATIVE,5X,4HF*W(,
     +I1,15H) <= (1-F)*W(J),5X,8H(1-F)*W(,I1,11H) >= F*W(J)/)
      DO 380 K=1,NIND
         IF (FLAM1(J0,K).GT.0.0 .AND. FLAM2(J0,K).GT.0.0) GO TO 380
         DO 384 I=2,NIND
            I1=I
            IF (I.LE.K) I1=I-1
            DO 382 J=1,N
  382       C(I+1,J)=UT(I1,J)-UT(K,J)+EPSILON
```

```
384     CONTINUE
        DO 316 I=2,N
           DO 314 J=1,N
314        C(NIND+I,J)=0.0
316     CONTINUE
C
C          F*W(J0) <= (1-F)*W(J)
C
        IF (FLAM1(J0,K).GT.0.0) GO TO 331
        CALL SECOND (CP)
        IF (CP.GT.TLIMIT) GO TO 400
        F1=0.0
        F2=1.0
        NN1=0
312     DF=F2-F1
313     NN1=NN1+1
        F=F1+0.5*DF
        IF (DF.LT.DFMAX) GO TO 330
        LIC=NIND+1
        DO 320 I=1,N
           IF (I.EQ.J0) GO TO 320
           LIC=LIC+1
           C(LIC,J0)=F
           C(LIC,I)=F-1.0
320     CONTINUE
        DO 324 I=1,LIC
           DO 322 J=1,LIC
322        COPI(I,J)=0.0
           COPI(I,I)=1.0
           IDES(I)=N+I
324     CONTINUE
        CALL ZX2LP(C,D,ICOLMS,ROW,1,LIC-1,N,50,LIC,IR,COPI,
        IDES,X,WA,IER)
        CALL ERROR (NN1,IER,J0,ILEFT(K),DF,F),RETURNS(330,313)
        IF (X(I).GT.0.5) GO TO 323
        F2=F
        GO TO 312
323     DO 325 I=1,N
325     W(I)=0.0
        DO 327 I=1,LIC
        II=IDES(I)
        IF (II.LE.N) W(II)=X(I)
327     CONTINUE
        PRINT 500,NN1,F,F1,F2,DF,X(1),(W(I),I=1,N)
500 FORMAT (/I3,4F6.3,F6.1/5F7.4)
        IF (W(J0).EQ.0) GO TO 329
        F1=1.0
        DO 326 J=1,N
           IF (J.EQ.J0) GO TO 326
           F1=AMIN1(F1,W(J))
326     CONTINUE
        F1=F1/(F1+W(J0))
        F1=AMAX1(F1,F)
        GO TO 312
329     F=1.0
330     FLAM1(J0,K)=F
        CALL SECOND (CP1)
```

```
            CPA1(J0,K)=CP1-CP
C
C            (1-F)*W(J0) >= F*W(J)
C
      331   IF (FLAM2(J0,K).GT.0.0) GO TO 380
            CALL SECOND (CP1)
            IF (CP1.GT.TLIMIT) GO TO 400
            F1=0.0
            F2=1.0
            NN2=0
      332   DF=F2-F1
      333   NN2=NN2+1
            FF=F1+0.5*DF
            IF (DF.LT.DFMAX) GO TO 370
            LIC=NIND+1
            DO 346 I=1,N
                IF (I.EQ.J0) GO TO 346
                LIC=LIC+1
                C(LIC,I)=FF
                C(LIC,J0)=FF-1.0
      346   CONTINUE
            DO 354 I=1,LIC
                DO 352 J=1,LIC
      352       COPI(I,J)=0.0
                COPI(I,I)=1.0
                IDES(I)=N+I
      354   CONTINUE
            CALL ZXPLP (C,D,ICOLMS,ROW,1,LIC-1,N,50,LIC,IR,COPI,
           + IDES,X,YA,IER)
            CALL ERROR (NN2,IER,J0,ILEFT(K),DF,FF),RETURNS(370,333)
            IF (X(1).GT.0.5) GO TO 353
            F2=FF
            GO TO 332
      353   DO 355 I=1,N
      355   W(I)=0.0
            DO 356 I=1,LIC
                II=IDES(I)
                IF (II.LE.N) W(II)=X(I)
      356   CONTINUE
            F1=0.0
            DO 366 J=1,N
                IF (J.EQ.J0) GO TO 366
                F1=AMAX1(F1,W(J))
      366   CONTINUE
            F1=W(J0)/(W(J0)+F1)
            GO TO 332
      370   CALL SECOND (CP)
            FLAM2(J0,K)=FF
            CPA2(J0,K)=CP-CP1
            IF (CP.GT.TLIMIT) GO TO 400
      380 CONTINUE
      400 DO 410 K=1,NIND
            WRITE (2,408) ILEFT(K),FLAM1(J0,K),FLAM2(J0,K),CPA1(J0,K),
           + CPA2(J0,K)
      408   FORMAT (I7,F22.3,F25.3,8X,2F5.1)
      410 CONTINUE
          WRITE (2,411)
```

```
  411 FORMAT (5(/))
      REWIND 3
      WRITE (3) NIND,ILEFT,NLEFT,IELIM,NELIM,DT,FLAM1,FLAM2,CPA1,
     + CPA2
      IF (SECOND(CP).LT.TLIMIT) GO TO 300
      STOP
      END
```

APPENDIX II

DESIGN CONTEXT - EXTERNAL ENVIRONMENT IN MONTREAL SUBURB

APPENDIX II

DESIGN CONTEXT -- EXTERNAL ENVIRONMENT IN MONTREAL SUBURB

The data presented below relates to the design of the building enclosure. In the absence of an actual site, data from the local meterological station* can serve to describe the climate.

A2.1 CLIMATIC DESIGN DATA FOR MONTREAL

(i) Design Temperature:

January (2½%):	-10° F.	(-23° C.)
July (2½%):		
Dry bulb:	86° F.	(30° C.)
Wet bulb:	74° F.	(23° C.)

(ii) Heating Factor:

Degree days below 65° F. = 8,130

(Degree days below 18° C. = 4,437)

(iii) Annual Total Precipitation:

42 in. (1,067 mm)

(iv) Ground Snow Load:

54 psf (2,585 N/m^2)

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* Montreal International Airport, Dorval.

(v) Hourly Wind Pressures:

1/30: 7.8 psf (374 N/m^2)

(vi) Seismic Code:

(as per National Building Code of Canada) = 2

A2.2 ACOUSTIC ENVIRONMENT

(i) Location:

Near highway, 3000 vehicles per hour.

(ii) External Noise Level:

85 dBA (10% Level)

APPENDIX III

PERFORMANCE OBJECTIVES FOR EXTERNAL WALLS OF SINGLE FAMILY HOUSES IN MONTREAL

APPENDIX III

PERFORMANCE OBJECTIVES FOR EXTERNAL WALLS OF SINGLE-FAMILY HOUSES IN MONTREAL

A3.1 INTRODUCTION

The specification of performance objectives makes explicit the purposes to be served by the constructional systems without restricting the designer in the solutions he puts forward. When defining a performance objective, the following items should be clear:

(i) by which element it should be fulfilled;

(ii) to which category of objective it belongs;

(iii) what the performance objective is;

(iv) what units of measurement are used;

(v) what the limits of desirability are (e.g., least acceptable, most desirable)

(vi) how fulfullment of the objective will be assessed;

(vii) on what kind of scale the measurements are made;

(viii) how the level of achievement of an alternative will be evaluated (i.e., type of transformation function).

An example of a performance objective definition

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1 K. Büchin, T. Hagenbrock, I. Hess, H. Küsgen and P. Sulzer, in "Performance Specifications for Housing", CIB 6th Congress, The Impact of Research on the Built Environment, International Council for Building Research Studies and Documentation, Budapest, 1974, recommend the definition of performance requirements according to a procedure of which this approach is a modification.

appropriate to external walls of single-family houses in Montreal is shown in this appendix. These performance objectives are compatible with the classification in the CIB Master List[2] for building elements. These categories, though not always identical with performance requirements for external walls as outlined either by Fitzmaurice[3] or Hutcheon[4], treat most objectives by subdividing them into their constituent parts. The thermal objectives of walls, to give one example, are considered in terms of thermal resistance, thermal capacity, thermal expansion and physical or chemical changes arising from temperature variations.

Sometimes it is not possible to formulate performance objectives for all walls in simple terms. The lack of adequate models to predict the service life of constructional systems also makes the definition of durability requirements difficult to achieve. Notwithstanding these difficulties, the set of performance objectives established during this study (that is shown in this appendix) provides a suitable basis for evaluation of the alternatives generated during

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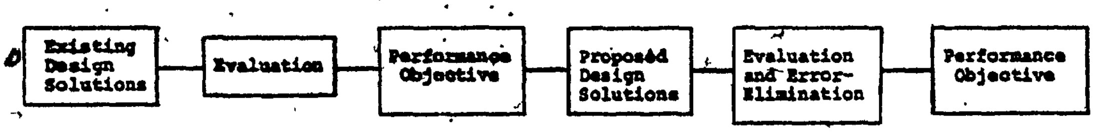
2 CIB Master List for Structuring Documents Relating to Buildings, Building Elements, Components, Materials and Services, International Council for Building Research Studies and Documentation, Report No. 18, 1972.

3 R. Fitzmaurice, Principles of Modern Building, HMSO (1st edition), 1939.

4 N.B. Hutcheon, "Fundamental Consideration in the Design of External Walls for Buildings", 67th Annual General and Professional Meeting of the Engineering Institute of Canada, Halifax, May, 1953.

the design process -- and also for the subsequent refinement of the objectives.

The development of performance objectives is evolutionary. It is akin to the task of stating the problem in problem-solving procedures:



This sequence is one of creative evolution. Even though the statement of desired performance is made without regard to the specific means to be used in achieving the results, it is evident that, if they are to be realistic and useful, performance objectives have to be based on preceeding design solutions. The common expectation that there should be an "*original*" approach to unoriginal, recurring problems is wasteful and inefficient in its demands upon creative skills.

a)	Building Element	:	External Walls
b)	Requirement Category	:	Structural Safety
c)	Performance Requirement	:	Compressive Strength
d)	General Statement of Requirement	:	Walls should resist imposed gravity loads so that the probability of failure is small
e)	Unit of Measurement	:	Kips (kN)
f)	Limits of Desirability		
	(i) least acceptable	:	$\phi R = \gamma[\alpha_D D + \alpha_L L]$
	(ii) most desirable	:	--------------
g)	Assurance	:	By calculation or laboratory test
h)	Type of Scale	:	Nominal
i)	Measurement of Alternative's Performance (Transformation function) : u_{ij}		--------------

j) Remarks: Compliance with criterion is in accordance with limit states design provisions in the National Building Code of Canada, 1975.

R = Resistance of Wall; D and L = specified dead and live loads

ϕ = Performance factor; γ = Importance factor

α_D, α_L = Variability factors for dead and live loads

a)	Building Element	:	External Walls
b)	Requirement Category	:	Structural Safety
c)	Performance Requirement	:	Bending Strength
d)	General Statement of Requirement	:	Walls should support horizontal loads arising from wind, occupancy, etc., so that the probability of failure is sufficiently small
e)	Unit of Measurement	:	Kip-ft (kN.m)
f)	Limits of Desirability		
	(i) least acceptable	:	$\phi R = \gamma[\alpha_W W]$
	(ii) most desirable	:	---------------
g)	Assurance	:	By calculation or laboratory test
h)	Type of Scale	:	Nominal
i)	Measurement of Alternative's Performance (Transformation function) : u_{ij}		---------------

j) <u>Remarks</u>: Compliance with criterion is in accordance with limit states design provisions in the National Building Code of Canada, 1975.

R = Resistance of wall; W = lateral load

ϕ = performance factor; γ = importance factor

α_W = load factor

a)	Building Element	:	External Wall
b)	Requirement Category	:	Structural Serviceability
c)	Performance Requirement	:	Bending Stiffness
d)	General Statement of Requirement	:	Walls should resist lateral loads to minimize damage to finishes and components that may be included in the wall
e)	Unit of Measurement	:	in. (mm.)
f)	Limits of Desirability		
	(i) least acceptable	:	Δ = h/50 in (h/50 mm)
	(ii) most desirable	:	----------------
g)	Assurance	:	By calculation or laboratory tests
h)	Type of Scale	:	Nominal
i)	Measurement of Alternative's Performance (Transformation function) : u_{ij}		----------------

j) **Remarks:** Compliance with criterion is in accordance with limit states design provisions in the National Building Code of Canada, 1975.

Δ = deflection of wall; h = height of wall (in. or mm.)

Lateral loads : (a) 150 lbs (~ 700 N.) on area 5 in^2 (30 cm^2) at centre of wall

(b) 10 lb/ft^2 (~ 500 N/m^2) on entire surface of wall

a)	Building Element	:	External Wall
b)	Requirement Category	:	Structural Safety
c)	Performance Requirement	:	Impact Strength
d)	General Statement of Requirement	:	Walls should withstand accidental impacts without becoming dislodged. Fractures resulting from such impacts should not produce falling debris which may be a safety hazard
e)	Unit of Measurement	:	ft-lb (Joules)
f)	Limits of Desirability		
	(i) least acceptable	:	R = 750 ft-lb (1000 J)
	(ii) most desirable	:	------------------
g)	Assurance	:	By laboratory test
h)	Type of Scale	:	Nominal
i)	Measurement of Alternative's Performance (Transformation function) : u_{ij}		------------------

j) **Remarks**: This criterion corresponds to the impact of a person (e.g, cyclist), hitting the wall with his full weight.

ASTM E 72-68 "*Conducting Strength Tests of Panels for Building Construction*" is the standard test.

R = Impact resistance of wall.

a)	Building Element	:	External Wall
b)	Requirement Category	:	Structural Serviceability
c)	Performance Requirement	:	Impact strength
d)	General Statement of Requirement	:	Walls should be able to withstand impact loads without suffering visually unacceptable deformations
e)	Unit of Measurement	:	ft-lb (Joules)
f)	Limits of Desirability		
	(i) least acceptable	:	R = 90 ft-lb (120 J.)
	(ii) most desirable	:	------------------
g)	Assurance	:	By laboratory test
h)	Type of Scale	:	Nominal
i)	Measurement of Alternative's Performance (Transformation function) : u_{ij}		------------------

j) <u>Remarks:</u> It is possible to relax the serviceability criterion if replacement of parts is easy and inexpensive.

R = impact resistance of wall

Test performance according to ASTM E-72-68 "*Conducting Strength Tests of Panels for Building Construction*".

a)	Building Elements	:	External Walls
b)	Requirement Category	:	Structural Serviceability
c)	Performance Requirement	:	Strength relating to holding power of fixings
d)	General Statement of Requirement	:	Fixings should be capable of carrying their intended loads without loosening or causing damage to the face of a wall
e)	Unit of Measurement	:	Pound-force (Newton)
f)	Limits of Desirability		
	(i) least acceptable	:	$R = 0.4\ W_p$
	(ii) most desirable	:	----------------
g)	Assurance	:	By laboratory test
h)	Type of Scale	:	Nominal
i)	Measurement of Alternative's Performance (Transformation function) : u_{ij}		----------------

j) **Remarks:** Under earthquake conditions (Zone 2 in Montreal), the National Building Code of Canada defines the pull force caused by fixtures in terms of ground acceleration and mass.

Thus $R \geq A\ S_p\ W_p$

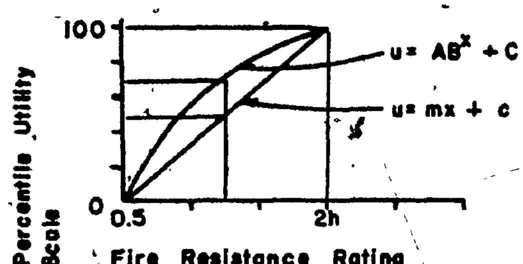
where

R = resistance capacity of fixing.
A = fraction of "g" assigned as horizontal ground acceleration
S_p = horizontal force factor
W_p = weight of fixture

For Montreal - $R \geq 0.04 \times 10\ W_p$

a)	Building Element :	External Walls
b)	Requirement Category :	Structural Serviceability
c)	Performance Requirement :	Vibration
d)	General Statement of Requirement :	Walls should withstand structural vibrations from gusts of wind, etc., in such a way that safety is not jeopardised nor are the occupants made uncomfortable
e)	Unit of Measurement :	in/sec. , (mm/sec)
f)	Limits of Desirability	
	(i) least acceptable:	$\delta = 0.2\ \delta_0$ in 0.5 sec.
	(ii) most desirable :	----------------
g)	Assurance :	By laboratory test
h)	Type of Scale :	Nominal
i)	Measurement of Alternative's Performance (Transformation function) : u_{ij}	----------------

j) **Remarks:** This criterion is based on perception of and tolerance to transient vibrations. It might be possible to propose vibration criteria similar to floor vibration requirements where deflection limitations have been correlated with damping and floor spans.

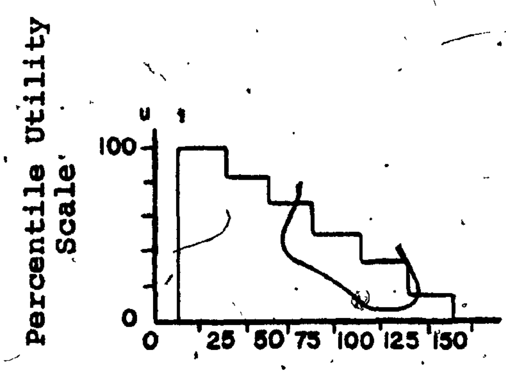
a)	Building Element :	External Walls
b)	Requirement Category :	Fire Safety
c)	Performance Requirement :	Fire Resistance
d)	General Statement of Requirement :	Walls should support imposed loads at elevated temperatures to provide time for people to escape and for the fire brigade to arrive. A higher fire resistance to provide time to the fire brigade to control the fire is also desirable
e)	Unit of Measurement :	Hours (h)
f)	Limits of Desirability	
	(i) least acceptable:	Fire Resistance Rating = 0.5h
	(ii) most desirable :	Fire Resistance Rating = 2.0h
g)	Assurance :	Code Calculations or Laboratory tests
h)	Type of Scale :	Interval - linear or utility function transformation
i)	Measurement of Alternative's Performance (Transformation function) : u_{ij}	

J) <u>Remarks:</u> Although the Residential Code* does not provide for a minimum fire resistance rating, 0.5-hour is thought necessary if the performance requirements are to be fulfilled. The 2-hour rating is deemed adequate for salvaging belongings and possibly the building.

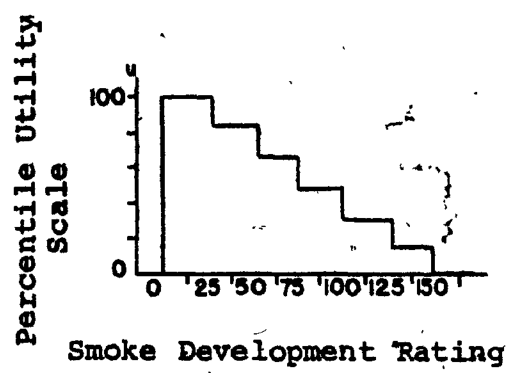
Supplement No.2, of the National Building Code provides a useful basis for calculating the fire resistance ratings of walls.

**Associate Committee on the National Building Code, <u>Canadian Code for Residential Construction, 1970, Residential Standards</u>, National Research Council of Canada, Ottawa, NRCC No. 11562.*

a)	Building Elements	:	External Walls
b)	Requirement Category	:	Fire Safety
c)	Performance Requirement	:	Ignitability and flame spread
d)	General Statement of Requirement	:	External walls should minimize the danger resulting from a rapid spread of flame on the surface
e)	Unit of Measurement	:	----------------------
f)	Limits of Desirability		
	(i) least acceptable	:	Flame Spread Rating = 150
	(ii) most desirable	:	Flame Spread Rating = 0
g)	Assurance	:	Code or laboratory test
h)	Type of Scale	:	Ordinal (step-function transformation)
i)	Measurement of Alternative's Performance (Transformation function) : u_{ij}		



j) <u>Remarks</u>: The flame spread rating is an accepted method for classified surface burning characteristics of building materials. The least acceptable rating was obtained from the Residential Code, the most desirable is feasible.

a)	Building Element :	External Walls
b)	Requirement Category :	Fire Safety
c)	Performance Requirement :	Smoke Development
d)	General Statement of Requirement :	Walls should not contain substances which on combustion will emit smoke
e)	Unit of Measurement :	------------------
f)	Limits of Desirability	
	(i) least acceptable:	Smoke Development Rating = 150
	(ii) most desirable :	Smoke Development Rating = 0
g)	Assurance	Code or laboratory test
h)	Type of Scale	Ordinal - (step function transformation)
i)	Measurement of Alternative's Performance (Transformation function) : u_{ij}	

j) **Remarks**: This feature of fire safety, although relatively new in the fire field, is used to stress the safety of life as opposed to "safety of materials".

a)	Building Element	:	External Walls
b)	Requirement Category	:	Air Penetration
c)	Performance Requirement	:	Air Tightness
d)	General Statement of Requirement	:	Air infiltration through walls should be minimized to avoid unacceptably high rate of air change and deterioration in thermal performance
e)	Unit of Measurement	:	ft/hr (m/hr)
f)	Limits of Desirability		
	(i) least acceptable	:	Q = 11.8 ft/hr under 0.30 in Q = 3.6 m/hr under 7.5 mm of water
	(ii) most desirable	:	------------------
g)	Assurance		By testing
h)	Type of Scale		Nominal
i)	Measurement of Alternative's Performance (Transformation function) : u_{ij}		

j) <u>Remarks:</u> Although Nordic Building Regulations propose an acceptance curve for external walls based on inflation rate versus pressure difference, it is thought that the criterion for air infiltration rate, Q, is justified. The pressure difference corresponds to the stagnation pressure resulting from a wind velocity of 25 mph (11 m/s). Wind speeds in excess of this are usually not significant in air leakage problems.

a) Building Element : External Walls

b) Requirement Category : Air/Moisture Movement

c) Performance Requirement : Prevention of Condensation

d) General Statement of Requirement : Walls should experience limited and preferably no condensation within them. If condensation occurs, the construction should accomodate it with no undesirable effects

e) Unit of Measurement : ------------------

f) Limits of Desirability

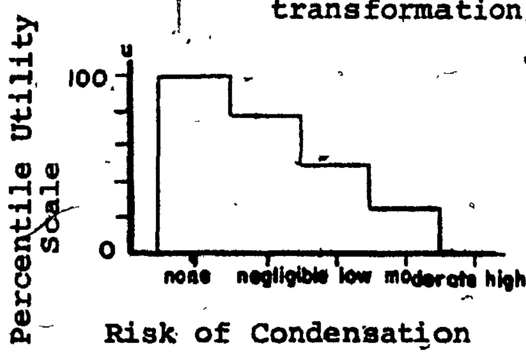
(i) least acceptable: Condensation Risk - High

(ii) most desirable : Condensation Risk - None

g) Assurance : By calculation or laboratory test

h) Type of Scale : Ordinal (Step-function transformation)

i) Measurement of Alternative's Performance (Transformation function) : u_{ij}



j) <u>Remarks:</u> Calculations for condensation are based on internal conditions of 70°F(21°C) and 50% R.H. and external conditions of 0°F(-18°C) and 100% R.H.

a)	Building Elements :	External Walls
b)	Requirement Category :	Hygrothermal Requirements
c)	Performance Requirement :	Dimensional Control
d)	General Statement of Requirement :	Changes in dimension due to variations in both temperature and humidity should be compensated for in the design and fabrication of joints to permit movement of components and adequate sealing of joint
e)	Unit of Measurement :	----------------
f)	Limits of Desirability	
	(i) least acceptable:	----------------
	(ii) most desirable :	----------------
g)	Assurance :	----------------
h)	Type of Scale :	Nominal
i)	Measurement of Alternative's Performance (Transformation function) : u_{ij}	

j) **Remarks:** Variation in climatic conditions:

(i) Within buildings

Air Temperature: 60 to 80°F (15.5 to 26.7°C)
Relative Humidity: 20 to 80%

(ii) Outside buildings

Air Temperature: -40 to 100°F (-40 to 38°C)
Relative Humidity: 30 to 100%

(iii) Outside surfaces

Surface Temperature range: 200°F (93°C)

a)	Building Element	:	External Walls
b)	Requirement Category	:	Water Tightness
c)	Performance Requirement	:	Prevention of water penetration
d)	General Statement of Requirement	:	Walls should be able to prevent rain from reaching the internal face. If objective cannot be met then provision should be made for adequate drainage.
e)	Unit of Measurement	:	----------------------
f)	Limits of Desirability		
	(i) least acceptable	:	No water penetration
	(ii) most desirable	:	----------------------
g)	Assurance	:	By laboratory test
h)	Type of Scale	:	Nominal
i)	Measurement of Alternative's Performance (Transformation function) : u_{ij}		
j)	Remarks: The laboratory tests simulate wind driven rain conditions of 5 gal/hr/ft^2 (0.24 m/hr) of water under a pressure of 3 p.s.f. (144 Pa) or 20% of structural wind load.		

a) Building Element : Exterior Walls

b) Requirement Category : Thermal Requirements

c) Performance Requirement : Thermal Insulation

d) General Statement of Requirement : Walls should have sufficiently high thermal resistance

e) Unit of Measurement : $^{\circ}F\ ft^2h/Btu$ ($^{\circ}C.m^2/W$)

f) Limits of Desirability

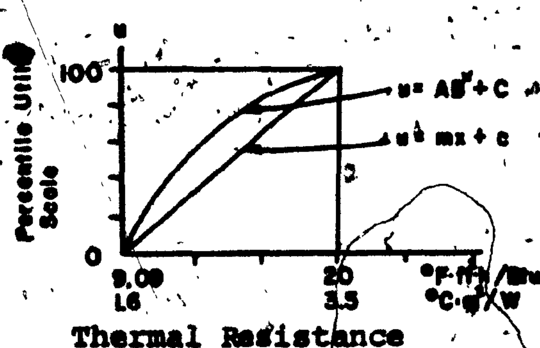
(i) least acceptable : $R = 9.09^{\circ}F.\ ft^2h/Btu$ ($1.6^{\circ}C.m^2/W$)

(ii) most desirable : $R = 20.00^{\circ}F.\ ft^2h/Btu$ ($3.5^{\circ}C\ m^2/W$)

g) Assurance : By calculation or laboratory test

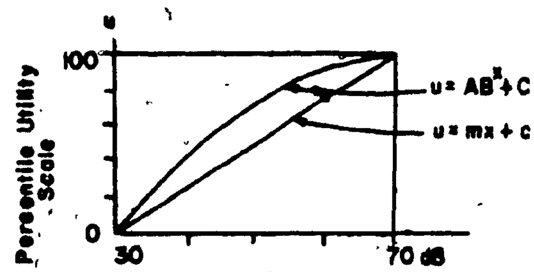
h) Type of Scale : Interval - (linear or utility function transformation)

i) Measurement of Alternative's Performance (Transformation function) : u_{ij}



j) **Remarks:** The least acceptable criterion corresponds to the Residential Code Specification. The most desirable value is both feasible and realistic. A criterion of minimum surface temperature of inside face ($50^{\circ}F$ ($10^{\circ}C$)) reduces the risk of condensation and pattern staining.

a)	Building Element	:	External Walls
b)	Requirement Category	:	Acoustic Requirements
c)	Performance Requirement	:	Transmission of Sound
d)	General Statement of Requirement	:	Walls should provide a sound insulation against external noise (e.g., traffic)
e)	Unit of Measurement	:	dB
f)	Limits of Desirability		
	(i) least acceptable	:	STL = 30 dB
	(ii) most desirable	:	STL = 70 dB
g)	Assurance	:	By calculation or tests
h)	Type of Scale	:	Interval - (linear or utility function transformation)
i)	Measurement of Alternative's Performance (Transformation function) : u_{ij}		



Sound Transmission Loss

j) Remarks: Sound Transmission Loss measurements are an average of losses on 16 frequencies. The Residential Code does not specify a minimum requirement. Limits of desirability were established for this immediate purpose.

a) Building Element : External Walls

b) Requirement Category : Durability Requirements

c) Performance Requirement : Service Life

d) General Statement of Requirement : Walls should maintain a satisfactory life in which only moderate expenditure on maintenance or repair is required

e) Unit of Measurement : Years

f) Limits of Desirability

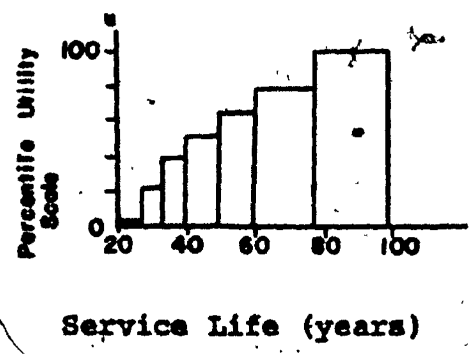
(i) least acceptable: Service life = 20 years

(ii) most desirable : Service life = 100 years

g) Assurance : -------------

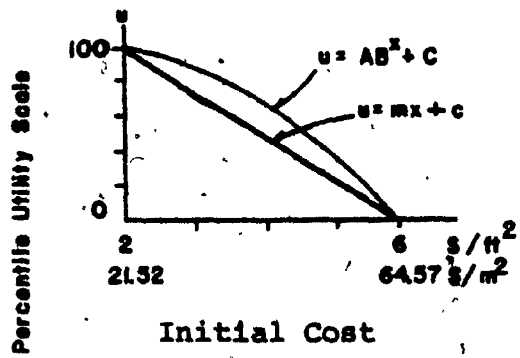
h) Type of Scale : Ordinal - Step-function transformation

i) Measurement of Alternative's Performance (Transformation function) : u_{ij}



j) Remarks: The performance objective limits were based on the British Standard Code of Practice (p. 3, Chapt. IX on Durability.) The step-function transformation is made necessary because of the amount of judgement necessary in assessing the large number of factors involved estimating the service life of a component.

a)	Building Element	:	External Walls
b)	Requirement Category	:	Economic Requirement
c)	Performance Requirement	:	Initial Cost
d)	General Statement of Requirement	:	The initial cost of walls should be low
e)	Unit of Measurement	:	$/ft^2 ($/m^2)
f)	Limits of Desirability		
	(i) least acceptable	:	$6.00/ft^2 ($64.57/m^2)
	(ii) most desirable	:	$2.00/ft^2 ($21.52/m^2)
g)	Assurance	:	---------------
h)	Type of Scale	:	Interval - linear or utility function transformation
i)	Measurement of Alternative's Performance (Transformation function) : u_{ij}		



j) Remarks: The initial cost limits were established in consultation with CMHC officials and based on rough estimates (10 to 30%) of proportion of house costs. Because of these approximations, it was thought that the least acceptable limit for this objective may be exceeded but a negative number would be assigned to the performance measurement.

APPENDIX IV

MODELS FOR PREDICTING THE PERFORMANCE OF EXTERNAL WALLS WITH RESPECT TO FIRE RESISTANCE, THERMAL RESISTANCE, RISK OF CONDENSATION, SOUND TRANSMISSION LOSS AND INITIAL COST

APPENDIX IV

MODELS FOR PREDICTING THE PERFORMANCE OF EXTERNAL WALLS WITH RESPECT TO FIRE RESISTANCE, THERMAL RESISTANCE, RISK OF CONDENSATION, SOUND TRANSMISSION LOSS AND INITIAL COST

A4.1 FIRE RESISTANCE RATING

The fire resistance ratings are based on the standard method described in the National Building Code of Canada[1] and Supplement No. 2 to the National Building Code[2] sets out ratings based on results of tests reported and analyzed by the Division of Building Research, National Research Council of Canada. In the Supplement, provision is made for estimating the fire endurance ratings of constructional systems for which test results are not available.

A4.2 THERMAL RESISTANCE

The fundamental equation for steady-state heat flow in one dimension is:

$$q = -k A \frac{dt}{dx} \qquad \text{(A4.1)}$$

where:

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1 Associate Committee on the National Building Code, "National Building Code of Canada", 1975, National Research Council of Canada, Ottawa, NRCC 13982.

2 Associate Committee on the National Building Code, "Fire Performance Ratings", Supplement No. 2 to the National Building Code of Canada, 1975, National Research Council of Canada, Ottawa, NRCC 13987.

q = the heat flow rate

h = thermal conductivity

A = area normal to flow

$\frac{dt}{dx}$ = temperature gradient

Integrating along a path of constant heat flow, the following relationship obtains:

$$q = -k \frac{A_m}{L_m} \Delta t = - \frac{\Delta t}{R} \quad \text{(A4.2)}$$

It has been shown that Equation (A.4.2) is analogous to Ohm's Law for electrical circuits[3]. Based on this analogy, it is possible to show that the total resistance to heat flow through a composite system is numerically equal to the sum of the individual resistances in series.

Thus,

$$R_T = R_1 + R_2 \quad . \; . \; . + R_n \quad \text{(A4.3)}$$

where:

R_1, R_2 and R_n = the individual resistances

R_T = the total resistance.

Consider a cavity wall construction comprising more than one (for the purposes of illustration, say, two) homogeneous

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3 ASHRAE Handbook of Fundamentals, published by The American Society of Heating, Refrigerating, and Air-Conditioning Engineers, New York, 1972.

materials of conductivities k_1 and k_2, thickness x_1 and x_2, and the conductance of the air space separating them equal to C.

$$R_T = \frac{1}{f_i} + \frac{x_1}{k_1} + \frac{1}{C} + \frac{x_2}{k_2} + \frac{1}{f_0} \qquad \text{(A4.4)}$$

where:

f_i and f_0 = the surface coefficients of the air flow inside and outside, respectively

At any interface, the temperature can be calculated since the drop in temperature through any component of the wall is proportional to its resistance. Thus, the drop in temperature, Δt_i, through the inside air film, for example, is:

$$\Delta t_i = \frac{R_i}{R_T}(t_i - t_0) \qquad \text{(A4.5)}$$

where:

t_i and t_0 = the inside and outside temperatures, respectively.

Hence, the temperature, t_1, at the inner face of component 1 is:

$$t_1 = t_i - \Delta t_i \qquad \text{(A4.6)}$$

A correction for the effect of framing in insulated wall sections can be made using charts already developed in

ASHRAE's Handbook of Fundamentals[4]. Although the most exact method of determining the heat resistance coefficient for a given constructional system is to evaluate the heat loss through a prototype wall by means of heat meters, the series model results in adequate predictions. The model does not represent the actual heat flow through walls exactly, but the resistance values predicted by the series model compare favorably with average values obtained from tests[5].

A4.3 RISK OF CONDENSATION

The fundamental equation for estimating water vapour transmission through materials is:

$$w = -\mu \frac{dp}{dx} \qquad \text{(A4.7)}$$

where:

w = the weight of water vapour transmitted through a unit area in unit time

$\frac{dp}{dx}$ = vapour pressure gradient

μ = permeability of the material

The close parallel with Equation (A4.1) for rate of heat flow suggests that a series model would provide an

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[4] *Ibid.*

[5] N.B. Hutcheon, W.H. Ball and E.E. Brooks, "A Test Hut Study of Two Types of Insulation", University of Saskatchewan Report PRH-4, Saskatoon, 1953.

adequate basis for predicting vapour flow and vapour pressures through composite walls. The permeability, μ, of a material is a function of relative humidity and temperature, hence there is a need to simplify by treating the average permeability, $\bar{\mu}$:

$$\bar{\mu} = \left(\frac{\int_{p_1}^{p_2} \mu \, dp}{p_1 - p_2} \right) \qquad \text{(A4.8)}$$

Integrating Equation (A.7) from 0 to ℓ (i.e., across the thickness of the material) and from p_1 to p_2, the following equation can be obtained[6]:

$$w = \bar{\mu} \frac{(p_1 - p_2)}{\ell} = \frac{\Delta p}{(\ell / \bar{\mu})} \qquad \text{(A.4.9)}$$

Thus, by analogy, it can be shown that the resistance to vapour flow provided by a sheet of material is the reciprocal of the permeance ($\bar{\mu}/\ell$) and the overall vapour resistance (V_T) of an assembly is the sum of the resistances of its components in series. The drop in vapour pressure Δp across a lamina in an assembly is proportional to its resistance. Thus, the drop in vapour pressure Δp_1 across a vapour resistance $v_1 = (\frac{\bar{\mu}_1}{\ell_1})$ is:

$$\Delta p_1 = \frac{V}{V_T} (p_i - p_o) \qquad \text{(A4.10)}$$

where:

p_i and p_o = the vapour pressures inside and outside, respectively.

[6] ASHRAE Handbook of Fundamentals, *Op. Cit.* (3)

To check for interstitial condensation, then the following steps are followed:

(i) Determine internal and external air temperatures and vapour pressure*;

(ii) On a scale diagram, plot calculated drop in temperature (from thermal resistance series model) across the wall assembly;

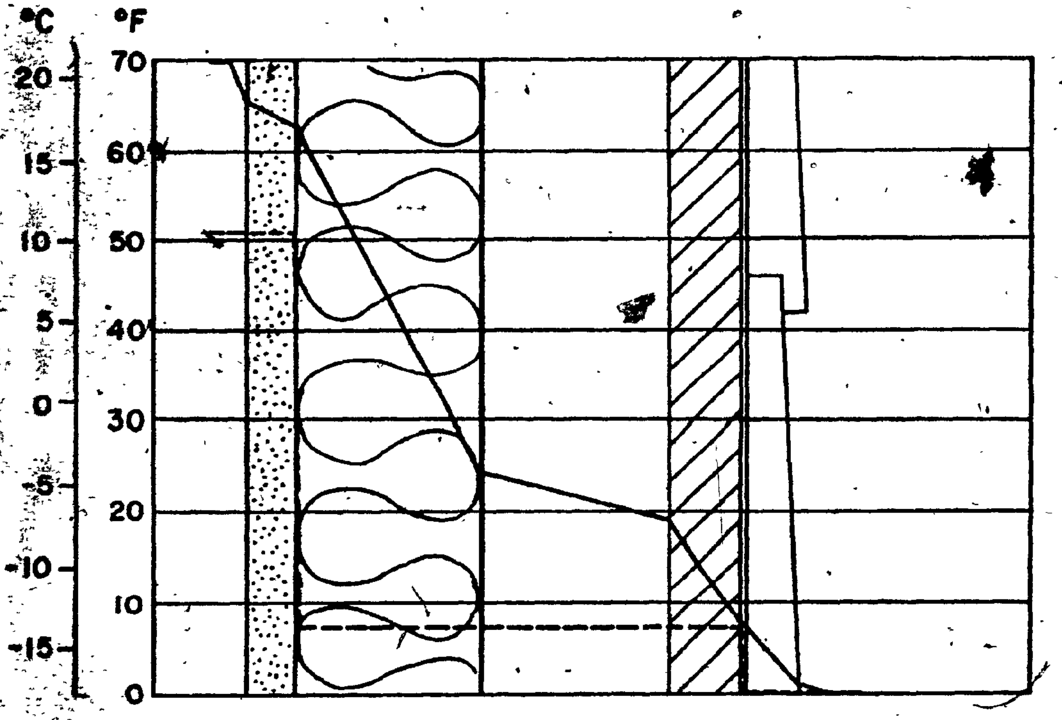
(iii) On the same diagram, plot calculated dew point temperature across the wall assembly (conversions are made by means of psychrometric charts);

(iv) Comparison of plots from (ii) and (iii) above, shows whether the wall assembly is below dew point.

A graphical representation of the calculations of risk of condensation for wall assembly 1 is given in Figure A4.1. The lamina in which condensation occurs indicates to the designer the risk of condensation. If the actual and dew point temperature plots do not intersect, there is no risk of condensation due to vapour migration through the materials. If, however, the temperature plots intersect, condensation is probable at the intersection of the plots. If the intersection of the temperature plots is at the innermost lamina, the condensation risk is judged to be high, whereas, for intersection of the plots at the outermost lamina, the

* The conditions assumed are:

Internal conditions: 70° F. (21° C.) and 50% R.H.
External conditions: 0° F. (-18° C.) and 100% R.H.



WALL SECTION I

Wood Bevel Siding (3/4" x 8" Cedar)

Building Paper - (15# Felt)

Wood Sheathing - (3/4")

Wood Stud at 16" o.c - (2" x 4")

Batt Insulation - (2")

Polyethylene Vapour Barrier - (4 mil)

Gypsum Board - (1/2")

Figure A 4.1 Graphical Representation of Risk of Condensation Calculations (Example)

- - - Dew. Point Temperture
—— Actual Temperture

RISK OF CONDENSATION —— NONE

risk is negligible. Any condensation occurring within the intermediate laminae is judged as either low or moderate, depending on whether the intersection of the temperature plots is closer to the inside or outside of the wall, respectively.

Even though air leakage through gaps and cracks accounts for six to seven times more condensation than the amount that results from vapour diffusion through materials[7], it is still necessary to control water vapour diffusion through materials in the wall (especially porous material exposed to freeze-thaw conditions). It is not yet possible to predict the condensation rate due to air leakage without assuming cracking conditions which, in turn, depend on design and workmanship as well as materials.

A4.4 SOUND TRANSMISSION LOSS

The effectiveness of a wall in providing acoustical insulation is described in terms of sound transmission loss. This depends on frequency of the incident sound and the mass per unit area, as well as the separation and bridging of panels in the wall assembly. The mathematical model used here for determining the transmission loss for 16 frequencies ($f \approx$ 125 to 4000 Hz for every third octave) is based on the Wyle Laboratories' study on ways to improve the sound

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7 J.K. Latta, "Vapour Barriers: What Are They? Are They Effective?" CBD 175, Division of Building REsearch, National Research Council of Canada, March, 1976.

insulation of building elements[8].

The transmission loss (TL) of a finite double construction, with absorption in the cavity at frequencies lower than the critical frequency of either panel, is given by the equation*:

$$TL_I = 10 \log \{1 + \left| \frac{\omega M}{3.6\rho c} - \frac{\omega^2 m_1 m_2}{(3.6\ \rho c)^2} (1 - e^{-2ikd}) \right|^2\} \quad \text{(A4.11)}$$

where:

$$M = m_1 + m_2 \text{ and } k = 2\pi f/c$$

The subscript I in the expression TL_I (Equation A.11) signifies that the expression is valid for ideal multiple panels.

From Equation (A4.11), it can be shown that at low frequencies where the wavelength is much greater than the dimension of the panel separation, the transmission at the fundamental resonance frequency, f_0, becomes 0 where:

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* The definitions of the symbols are given at the end of this section in order to simplify the task of presentation of the model.

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8 Wyle Laboratories, "A Study of Techniques to Increase the Sound Insulation of Building Elements", prepared for the Department of Housing and Urban Development, National Technical Information Service, PB-222-829.

$$f_0 = \frac{1}{2}\pi\sqrt{\frac{3.6\rho c^2}{m'd}} \quad \text{(A4.12)}$$

where:

$$m' = \left(\frac{2m_1\, m_2}{m_1 + m_2}\right) \quad \text{(A4.13)}$$

At $f < f_0$,

$$TL_I = 10 \log \{1 + (\frac{\omega M}{3.6\rho c})^2\} \quad \text{(A4.14)}$$

Thus, if $\omega M >> 3.6\rho c$,

$$TL_I \simeq 20 \log(\frac{\omega M}{3.6\rho c}) = 20 \log Mf - 33.5 \text{ db} \quad \text{(A4.15)}$$

Equation (A4.15) is a statement of the "*mass law*" transmission loss for double panel assemblies.

At $f_0 < f < f_\ell$,

$$TL_I \simeq 20 \log [\frac{\omega^2\, m_1 m_2}{(3.6\, \rho c)^2} \cdot 2\, kd]$$

$$= TL_1 + TL_2 + 20 \log 2kd \quad \text{(A4.16)}$$

where:

$$f_\ell = c/2\pi d$$

characterises the cavity resonance. This can be avoided by absorption in the cavity.

TL_1 and TL_2 are the transmission loss of the two panels calculated according to the mass law equation:

$$TL_m \approx 20 \log(1 + \frac{m}{3.6\ c})$$

$$\approx 20 \log(mf) - 33.5 \text{ dB} \qquad (A4.17)$$

At $f > f_\ell$,

$$TL_I = TL_1(f) + TL_2(f) + 6 \text{ dB} \qquad (A4.18)$$

These equations are adequate for the case where there is no bridging[9]. Assuming that the inter-panel connections are line connections (studs), the transmission loss calculations are performed on the following basis.

The overall transmission loss of a bridged double panel is given by:

$$TL = TL_I - TL_B \qquad (A4.19)$$

The curve of the bridged transmission loss is parallel to the mass law line and a convenient way of specifying the transmission loss of the bridged double panel is in terms of an increase, ΔTL_M, in transmission loss over and above that predicted by the mass law for the whole assembly.

$$\Delta TL_M = 10 \log(bf_c) + 20 \log (\frac{m_1}{m_1 + m_2}) - 28 \text{ dB} \qquad (A4.20)$$

The bridging frequency, f_B, is given by the equation:

$$f_B = f_0 \text{ antilog } \frac{\Delta TL_M}{40} \qquad (A4.21)$$

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9 *Ibid.*

Thus, at $f < f_B$,

$$TL(f) = TL_I(f)$$

$$= \{TL_1(f) + TL_2(f) + 20 \log fd - 39\} dB \quad (A4.22)$$

and, at $f > f_B$,

$$TL(f) = TL_M(f) + \Delta TL_M \quad (A4.23)$$

The effects of coincidence are neglected in these calculations even though they can be significant and even though the Wyle model is capable of taking them into account. All constructional systems are assumed to be double panels even though some would be more accurately described as triple panels (the Wyle model can treat triple panels only when symmetrical, which, in practice, is rarely achieved in external walls).

A plot of sound transmission loss predictions against frequency is shown in Figure (A4.2). The Sound Transmission Class ratings were created for internal partitions and are not applicable to external walls. Thus, the transmission loss of a wall assembly is calculated as the average of the 16 frequencies. This transmission loss value can be used as an indication of the acoustical insulation performance of external walls.

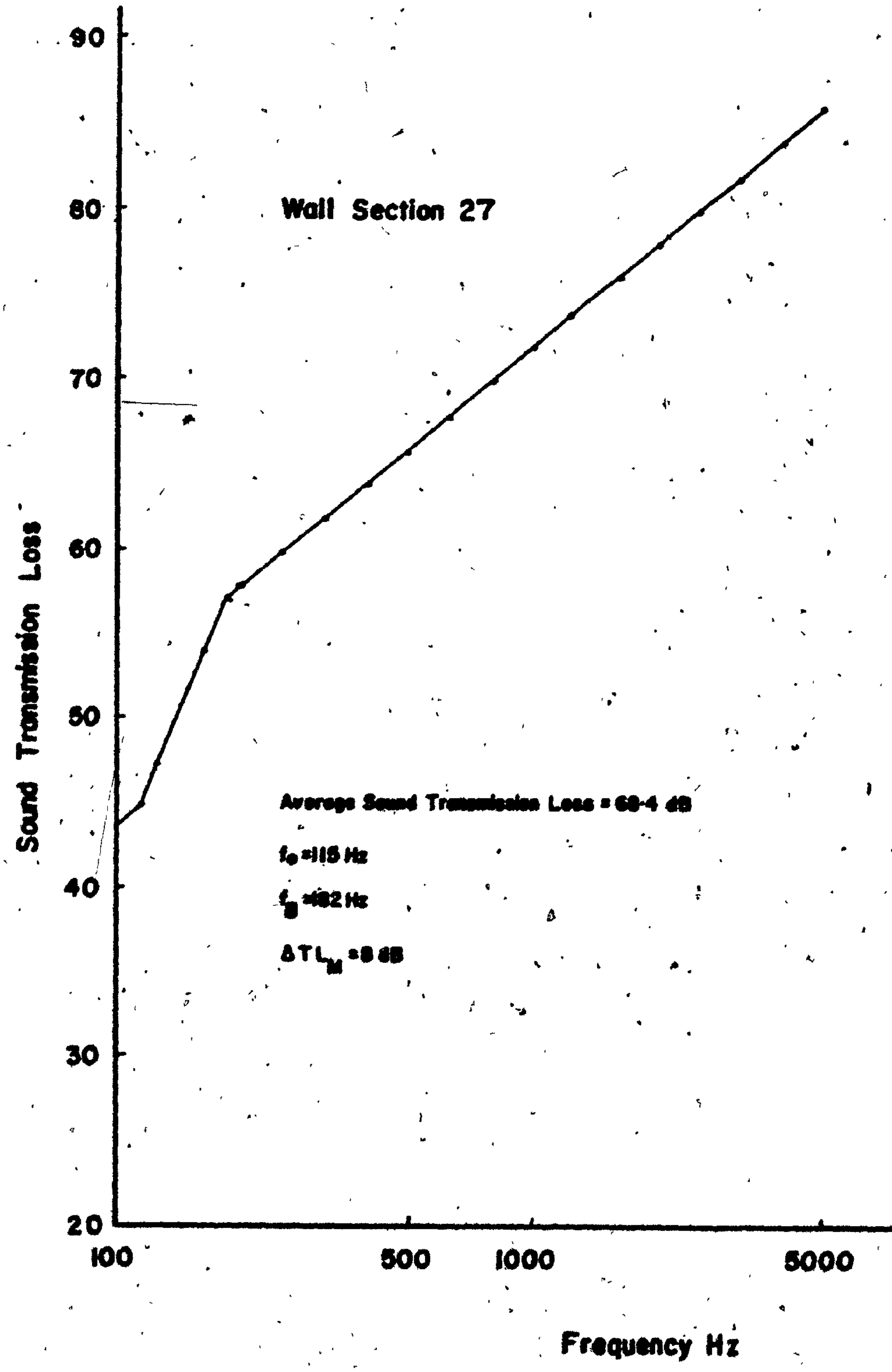

FIGURE A 4.2 PLOT OF SOUND TRANSMISSION LOSS VS. FREQUENCY

A4.5 KEY TO SYMBOLS USED IN SOUND TRANSMISSION LOSS MODEL

b = spacing of line studs

c = velocity of sound in air

d = separation of panels in a double panel construction

f = frequency

f_B = bridging frequency

f_c = critical frequency for single panel

f_ℓ = limiting frequency for double panel

f_0 = fundamental double panel resonance

M = total mass of multiple panel per unit area

m = mass of panel per unit area

m_1, m_2 = mass of panel 1, 2 per unit area

m' = effective mass of double panel for determining f_0

$TL(f)$ = transmission loss of construction at frequency (f)

$TL_1(f)$, $TL_2(f)$ = transmission loss for panels 1 and 2 at a frequency (f)

$TL_I(f)$ = transmission loss at frequency f of a multiple panel with no interpanel connections

$TL_M(f)$ = transmission loss at a frequency (f) according to mass law

ΔTL_M = increase in transmission loss over that calculated according to the mass law

ρ = density of air

ω = angular frequency

A4.6 INITIAL COST

Calculations of initial (capital) costs are based on Lansdownes' Construction Cost Handbook[10]. The costs for building work are derived from detailed unit rates of the wall's constituent components. These unit rates comprise base, overtime, etc., labor rates plus benefits, material costs plus handling, federal and provincial taxes and small plant and hand tools costs. The initial cost-in-place of a wall assembly is the sum of the unit costs of the constituents which comprise the wall.

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10 D.K. Lansdownes and Partners, Limited, Lansdownes' Construction Cost Handbook, McGraww-Hill Ryerson Ltd., Toronto, 1974.