

DETAILING OF REINFORCED CONCRETE STRUCTURES  
FOR EARTHQUAKE RESISTANCE

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D. Stavropoulos

ABSTRACT

Earthquakes are one of nature's greatest hazards to life on this planet. Throughout historic time they have caused the destruction of countless cities and villages on nearly every continent. This earthquake hazard has been the main driving force behind the development of analytical methods for predicting the response of structures to earthquakes. This report presents a brief overview of the dynamic analysis procedures and the seismic response of structures. The main emphasis of the report is on the detailing of reinforcing steel for the seismic response of concrete structures.

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## NOTATION

A	Constant
$A_c$	Area of concrete
$A_g$	Gross sectional area of concrete
$A_s$	Area of tension reinforcement
$A'_s$	Area of compression reinforcement
a	Depth of equivalent rectangular stress block, amplitude, acceleration
B	Constant
c	Distance from extreme compression fibre to neutral axis at ultimate conditions, damping coefficient
C	Damping coefficient matrix
d	Distance of extreme compression fibre to centroid of tension reinforcement
D, $D_n$	Plan dimension
e	Base of natural logarithms
$e_x$	Eccentricity
$E_s$	Young's Modulus for steel

$F$	Force
$F_t$	Force at top of structure
$F(t)$	Force varying with time
$F_I$	Inertia force
$F_D$	Damping force
$F_S$	Elastic force
$F_I$	Inertia force vector
$F_D$	Damping force vector
$F_S$	Elastic force vector
$f'_c$	Compressive strength of concrete
$f_{su}$	Ultimate strength of steel
$f_y$	Yield strength of steel
$h$	Height
$J$	Overturning moment reduction coefficient
$k$	Stiffness
$K$	Stiffness matrix
$l$	Length of span



$m$	Mass
$M$	Mass matrix
$M$	Moment
$M_u$	Ultimate moment
$n$	Modular ratio
$R$	Amplitude
$t$	time
$T$	Natural period of vibration
$u$	Displacement in x direction
$\dot{u}$	Velocity
$\ddot{u}$	Acceleration
$u_t$	Total displacement
$u_g$	Ground displacement
$\ddot{u}_t$	Total acceleration
$\ddot{u}_g$	Ground acceleration
$u_0$	Initial displacement
$\dot{u}_0$	Initial velocity

$u$  Displacement vector  
 $\dot{u}$  Velocity vector  
 $\ddot{u}$  Acceleration vector  
 $V$  Total horizontal force, shear force  
 $W$  Weight  
 $\alpha$  Angle  
 $\Delta$  Increment  
 $\Delta y$  Deformation  
 $\epsilon_{su}$  Steel strain at ultimate  
 $\epsilon_{sy}$  Steel strain at yield  
 $\epsilon_{cu}$  Concrete strain at ultimate  
 $\epsilon_{cy}$  Concrete strain at yield  
 $\zeta$  Damping ratio  
 $\mu$  Ductility factor  
 $\rho$  Ratio of tension reinforcement  
 $\rho'$  Ratio of compression reinforcement  
 $\tau$  Time

- $\phi$  Mode shape vector
- $\omega$  Frequency
- $\omega_n$  Natural circular frequency
- $\omega_D$  Damped natural circular frequency

## INTRODUCTION

Concrete has certain intrinsic advantages over other construction materials. It is highly versatile, can be cast into a variety of shapes and offers the possibility of a wide range of architectural finishes. Concrete is also known to have good durability and fire resistance. When properly reinforced, concrete structures having built-in continuity provide a rigid-frame action which prevents the development of undesirable story drifts or vibrations. However, because of its relatively heavy weight and low ductility, reinforced concrete structures have some disadvantages when built in regions susceptible to severe earthquake ground shaking. Based on these characteristics, and on the observed damage induced by moderate and strong earthquakes, it has been suggested that concrete structures are particularly vulnerable to earthquakes. However, as Bresler<sup>1\*</sup> points out, many concrete structures have withstood severe earthquakes without significant damage, suggesting that there is nothing inherent in concrete structures which makes them particularly vulnerable to earthquakes. Regardless of the material used, properly designed and detailed structures will perform well. Therefore, the design and construction of seismic resistant concrete structures require special attention.

\*Numbers refer to references at the end of this report.

In designing a building against severe earthquakes, economic considerations usually require that the large seismic energy input to the building be absorbed and dissipated through large but controllable inelastic deformations of its structure. However, these deformations should be limited to values which would avoid inducing severe damage to either structural and nonstructural elements. To achieve a large energy absorption and dissipation capacity, the sources for potential types of structural brittle failure should be eliminated. Therefore, it is necessary to prevent premature crushing and shear of concrete; sudden cracks and simultaneous fractures of steel which occurs in the case of members with extremely low reinforcement ratios; sudden loss of bond and anchorage; premature crushing and/or splitting of the concrete cover accompanied by local buckling of the main reinforcement; and the possibility of dynamic instability resulting from large lateral drifts.

Among these types of failure, two, which have been the cause of severe local damage and even collapse of many structures during recent strong earthquakes<sup>2</sup>, that are most undesirable, are local shear and bond (anchorage) failures. One remedy for these types of failure is proper detailing of the reinforcement, or, according to some designers, provide more material. However, a unique feature of the earthquake excitation provides the key to the solution of this

problem. In contrast to the other loads considered in structural design - wind, gravity, hydrostatic, etc. - the intensity of the earthquake loading depends on the properties of the structure. Thus, adequate earthquake resistance may be provided by either increasing strength or by the unique seismic design concept of reducing stiffness and thereby reducing the forces to be resisted. This additional approach to earthquake design imposes a greater need for understanding of structural behavior in earthquake engineering than in any other field of civil engineering design. Seemingly minor changes in the framing system or in design details may have an overwhelming influence on the seismic performance of the structure, and merely adding more material will not guarantee satisfactory performance.

It is because details are part of the key to good earthquake design that this report was undertaken. It is hoped that the report will be of value to the writer and to those who believe that, principles rather than rules are more important in engineering design.

# Chapter 1

## SEISMIC RESPONSE OF STRUCTURAL MATERIALS

### INTRODUCTION

Structure response to seismic motion is complex. The response is interdependently affected by the characteristics of the input motion (frequency, amplitude, phase relationships, and duration), the stiffness characteristics of the foundation materials, and the dynamic response characteristics of the structure. The input motion pulse frequencies and their distribution can have a major effect on earthquake response. In general, however, as the amplitude of the input motion increases, the response of the structure increases.

This chapter is principally concerned in presenting a basic understanding of the dynamic response characteristics of structures, in order to obtain the maximum benefit from even the simplest method of seismic analysis.

## ELASTIC SEISMIC RESPONSE OF STRUCTURES

The way in which a structure responds to a given dynamic excitation depends on the nature of the excitation and the dynamic characteristics of the structure, i.e. on the manner in which it stores and dissipates vibrational energy. Seismic excitation may be described in terms of displacement, velocity or acceleration varying with the time. When this excitation is applied to the base of a structure, it produces a time-dependent response in each element of the structure which may be described in terms of motions or forces.

Consider a simple structure, such as that shown in Figure 1.1; the dynamic characteristics of such a system are simply described by its natural period of vibration  $T$ , (or frequency  $\omega$ ) and its damping  $\zeta$ . If subjected to a harmonic base motion described by  $u_g = a \sin \omega t$ , the response of the mass at top of the spring is fully described in Figure 1.2. The ratios of response amplitude to input amplitude are shown for displacement  $R_d$ , velocity  $R_v$ , and accel-



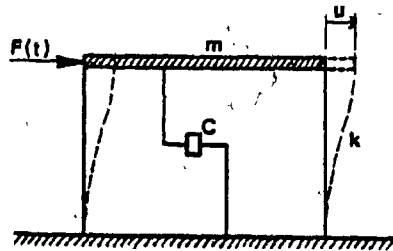


Figure 1.1 Idealized single-degree-of-freedom system

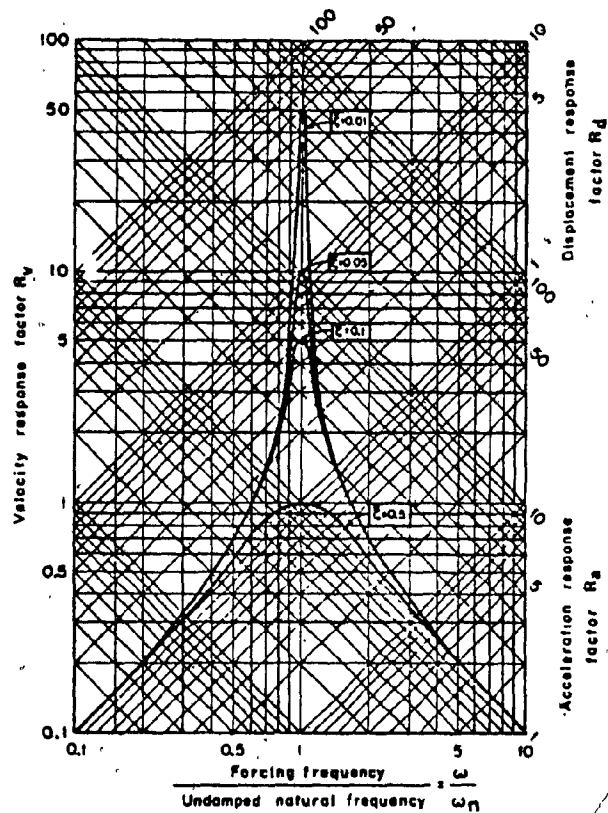


Figure 1.2 Response of linear elastic single-degree-of-freedom system to a harmonic forcing function

eration  $R_a$ , in terms of the ratio between the frequency of the forcing function  $\omega$  and the natural frequency<sup>3</sup> of the system  $\omega_n$ .

The significance of the natural period or frequency of the structure is demonstrated by the large amplifications of the input motion at or near the resonance conditions, i.e. when  $\omega/\omega_n = 1$ . Figure 1.2 also shows the importance of damping particularly near resonance.

If the structure of Figure 1.1 is subjected to a ground motion of the type given in Figure 1.3, it will be excited into motion of acceleration response according to its natural period and damping. Figure 1.4 is a plot of the motion of a series of single-degree-of-freedom structures with different natural periods and damping.

Most structures are more complex dynamically than the single-degree-of-freedom system discussed above. Multi-story buildings, for example, are better represented as multi-degree-of-freedom structures, with one degree of freedom for each story, and one natural mode and period of vibration for each story (figure 1.5). The response history of any element of such a structure is a function of all the modes of vibration, as well as of its position within the overall structural configuration.

Further discussion of both single and multi-degree-of-freedom systems follows in Chapter 3.

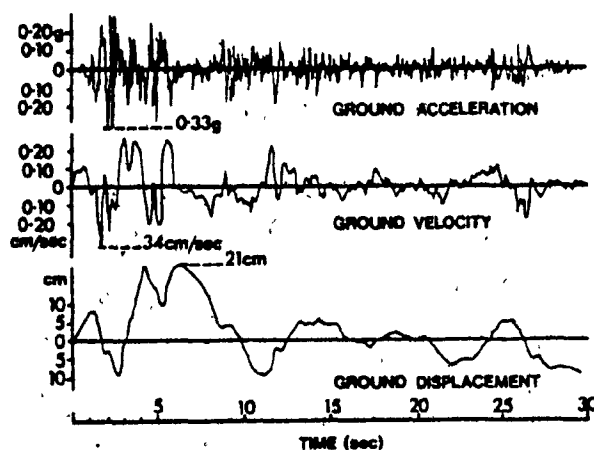


Figure 1.3 1940 El Centro earthquake ground motion: acceleration, velocity and displacement in north-south direction

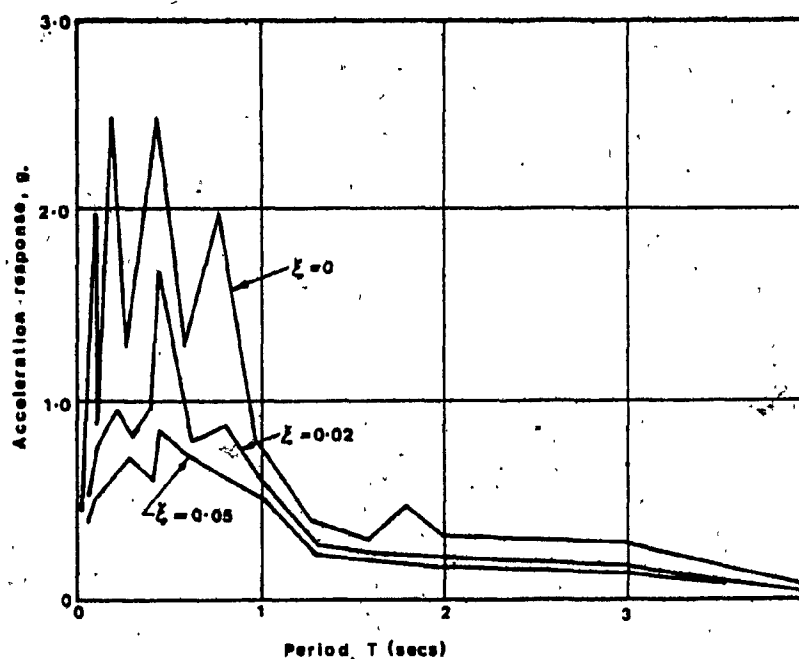


Figure 1.4 Elastic acceleration response spectra of north-south component of the 1940 El Centro E.Q. for a series of single-degree-of-freedom structures with different natural periods and damping (from reference 4)

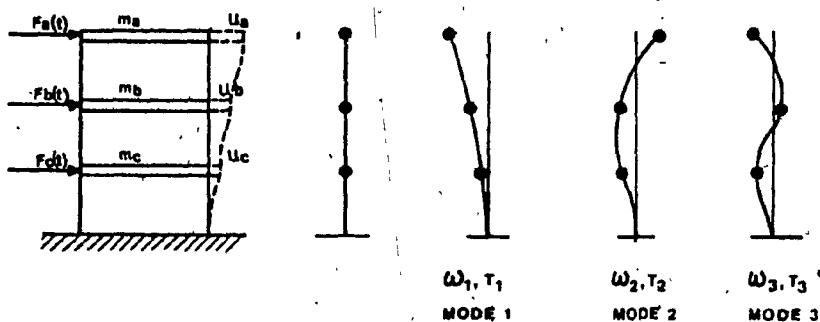
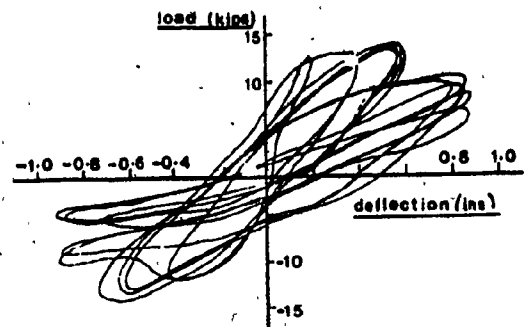
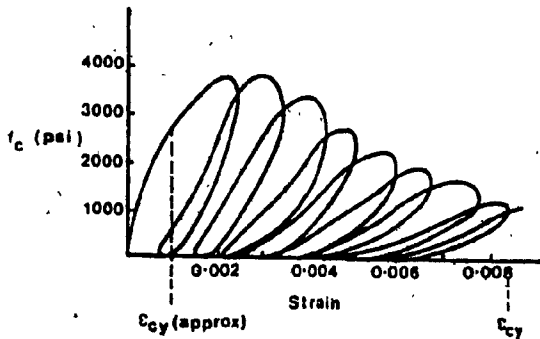
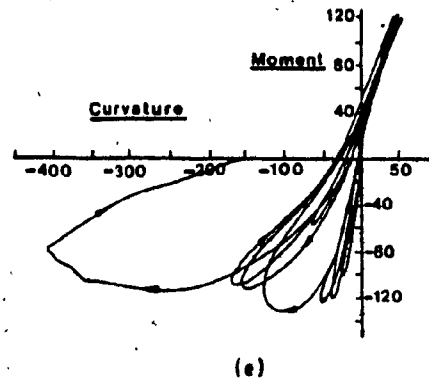
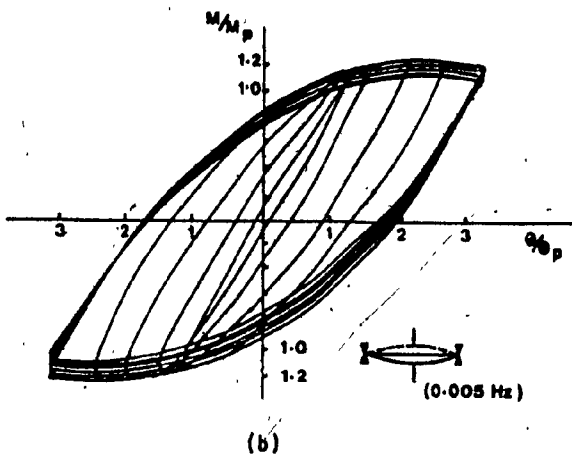
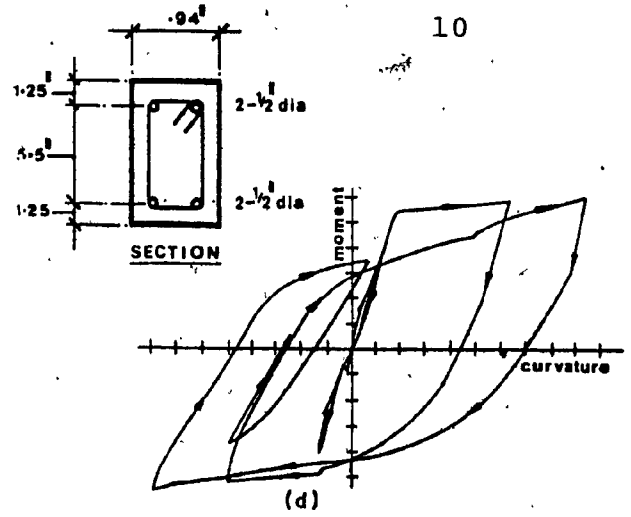
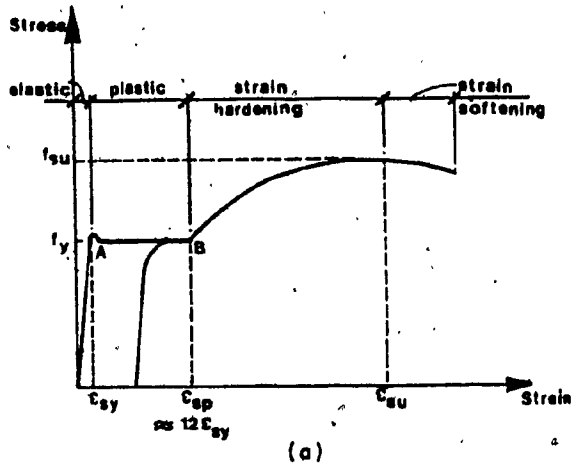


Figure 1.5 Multi-degree-of-freedom system subjected to dynamic loading

## INELASTIC SEISMIC RESPONSE OF STRUCTURES

The typical stress-strain curves for various materials under repeated and reversed direct loading shown in Figure 1.6 illustrate the chief characteristics of inelastic dynamic behavior, namely: plasticity, strain hardening, strain softening, stiffness degradation, ductility, and energy absorption.

Plasticity, as exhibited by mild steel (Figure 1.6 a), is a desirable property in that it is easy to simulate mathematically and provides a convenient control on the load developed by a member. Unfortunately, the higher the grade of steel, the shorter the plastic plateau, and the sooner the strain hardening effect sets in. Strain softening is the opposite of strain hardening, involving a loss of stress or strength with increasing strain (Fig. 1.6 a and c).



(c)

(f)

Figure 1.6 Elastic and inelastic stress-strain behavior of various materials under repeated and reversed loading.

- (a) Mild steel, monotonic (or repeated axial) loading.
- (b) Structural steel under cyclic bending.
- (c) Unconfined concrete, repeated loading.
- (d) Doubly reinforced concrete beam, cyclic loading.
- (e) Prestressed concrete column, cyclic bending.
- (f) Masonry wall, cyclic lateral loading.

Stiffness degradation is an important feature of inelastic cyclic loading of concrete. The stiffness as measured by the overall stress/strain ratio of each hysteresis loop of figures 1.6 (c) to (f) is clearly reducing with each successive loading cycle.<sup>5</sup>

The ductility of a member or structure may be defined in general by the ratio:

$$\frac{\text{Deformation at failure}}{\text{Deformation at yield}}$$

In various uses of this definition, "deformation" may be measured in terms of deflection, rotation or curvature. The numerical value of ductility will be difficult to calculate depending on the exact combination of applied forces and moments. Deformation at yield, also presents difficulty when the load or moment-deformation curve is not elastoplastic. In such a case, Park<sup>6</sup> suggests that the deformation at yield be taken as the deformation calculated for the structure assuming elastic behaviour up to the strength of the structure in the first load application to yield (Figure 1.7). Ductility is generally desirable in structures because of the gentler and less explosive onset of failure than that occurring in brittle materials. The favourable ductility of mild steel may be seen from Figure 1.6 (a) by the large value of ductility in direct tension measured by the ratio  $\epsilon_{su}/\epsilon_{sy}$ .

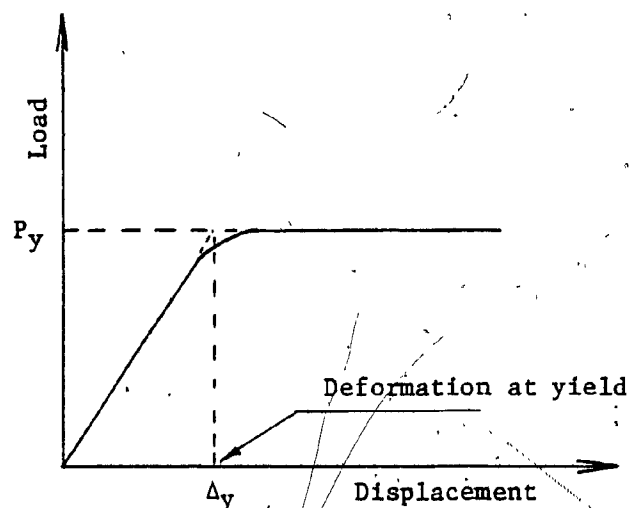


Figure 1.7 Possible definition for deformation at yield when load-displacement relationship is curved.

This ductility is particularly useful in seismic problems because it is accompanied by an increase in strength in the inelastic range. By comparison, the high value of compressive ductility for plain concrete, expressed by the ratio  $\epsilon_{cu}/\epsilon_{cy}$  in Figure 1.6 (c) is far less useful because of the inelastic loss of strength. Steel has the best ductility properties of normal building materials, while concrete can be made moderately ductile with appropriate reinforcement.

A high energy absorption capacity is a desirable property of earthquake-resistant construction insofar as a substantial part of the energy is temporarily stored by the structure in elastic strain energy and kinetic energy. As an elucidation of the seismic energy of structures, consider

Figure 1.8 derived from the inelastic seismic energy of the Bank of New Zealand Building, Wellington.<sup>7</sup> It is observed that after three seconds the earthquake motion is so strong that the yield point is exceeded in parts of the structure and permanent energy dissipation in the form of inelastic strain energy begins. A brittle building with the same yield strength, but with no inelastic behavior, would have begun to fail after three seconds of the earthquake.

#### MATHEMATICAL MODELS OF SEISMIC BEHAVIOR

When examining the range and complexity of hysteretic behavior shown in Figure 1.6, the problems involved in establishing usable mathematical stress-strain models for realistic seismic analysis are obvious. The simplest models of hysteresis that are shown in Figure 1.9 are reasonable for design purposes when building in steelwork. For concrete, the problems of adequately predicting inelastic seismic response are daunting at the present time.

A mathematical model, however, has been formulated, based upon the mechanical characteristics of concrete under this type of strain reversal.<sup>8</sup> A somewhat crude cyclic stress-strain diagram was adopted and the experimentally obtained results are indicated by dashed lines (Figure 1.10); the analytical results, by solid lines. From the figure, it can



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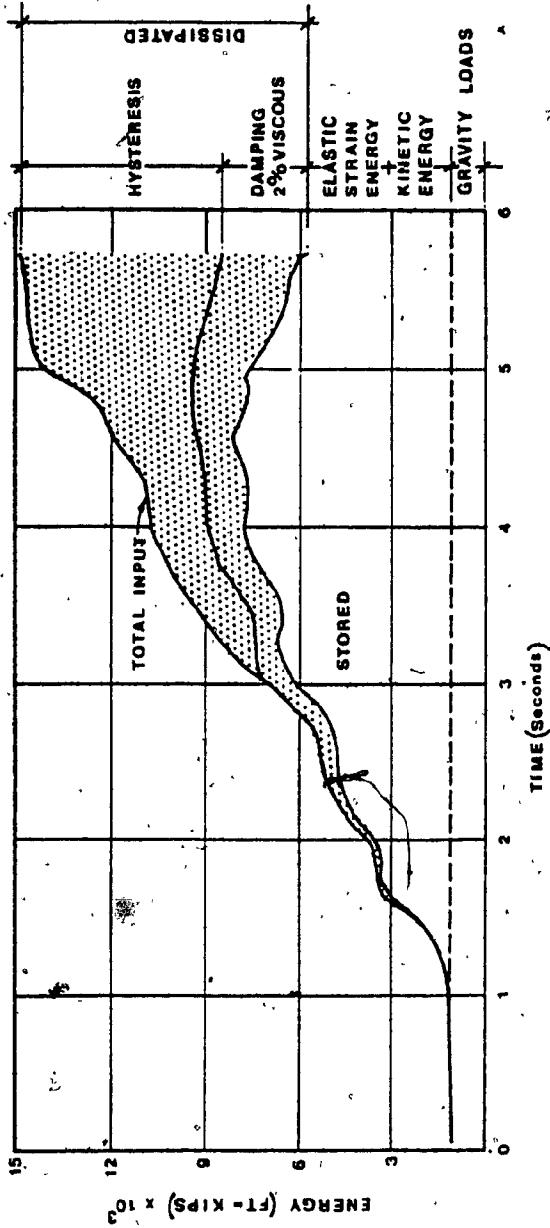


Figure 1.8 Energy expenditure in Bank of New Zealand Building, Wellington computed for first part of an earthquake equal to 1.5 times El Centro 1940, north-south component

be seen that the concrete mathematical model does not compare favorably with the experimental curve. However, it is believed that for cyclic loading, this relatively simple concrete model may be sufficiently accurate, since the inelastic behavior of ductile reinforced concrete is controlled by the steel.

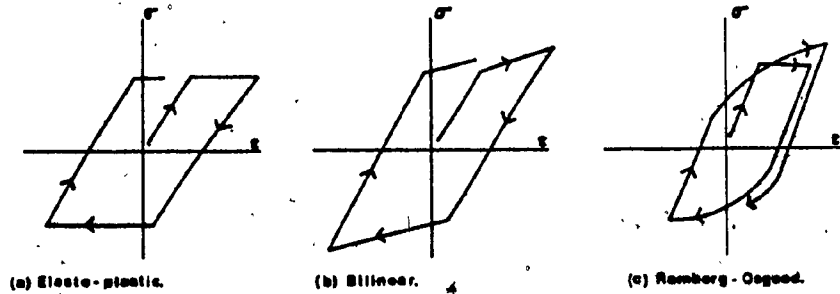


Figure 1.9 Idealized hysteresis loops for cyclic behavior of steel

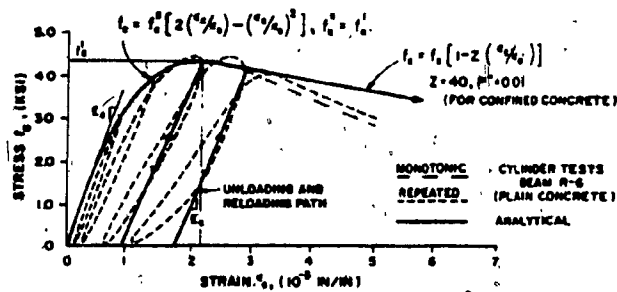
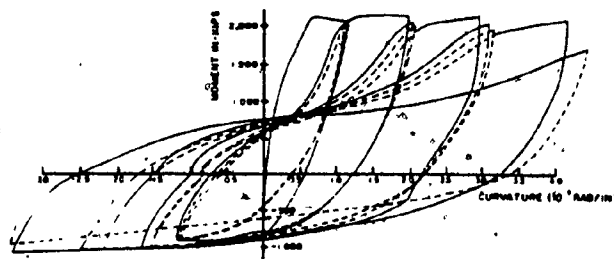
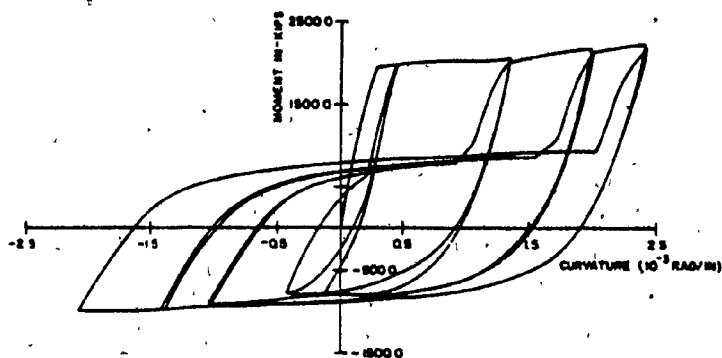


Figure 1.10 Concrete model under repeated loading

A comparison of the experimental and analytical results (Figure 1.11) is very encouraging.



(a) TEST RESULTS



(b) ANALYTICAL RESULTS

Figure 1.11 Comparison of experimental and analytical moment-average curvature relationship

#### LEVEL OF DAMPING IN DIFFERENT STRUCTURES

Damping varies with the materials used, the form of the structure, the nature of the subsoil and the nature of the vibration. Large amplitude post-elastic vibration is more heavily damped than small amplitude vibration, while buildings with heavy shear walls and heavy cladding and partitions have

greater damping than lightly clad skeletal structures. There are many reports of differences in damping in different modes of vibration, but no underlying pattern has as yet been established. The overall damping of a structure is clearly related to the damping characteristics of the subsoil, and some allowance may be made for this in more sophisticated analysis involving soil-structure interaction\* Table 1.1 indicates representative values of damping for a range of construction. These values are suitable for normal response spectrum or modal analysis in which viscous damping, equal in all modes, is assumed.

\*It is beyond the scope of this report to discuss soil-structure interaction. The interested reader is referred to: Dowrick, D.J., "Earthquake Resistant Design", Sections 5.5.2.3 and 5.5.3, and the references given therein.

Table 1.1 Typical damping ratios for structures

Type of construction	Damping percent of critical
Steel frame, welded, with all walls of flexible construction	2
Steel frame, welded or bolted, with stiff cladding, and all internal walls flexible	5
Steel frame, welded or bolted, with concrete shear walls	7
Concrete frame, with all walls of flexible construction	5
Concrete frame, with stiff cladding and all internal walls flexible	7
Concrete frame, with concrete or masonry shear walls	10
Concrete and/or masonry shear wall buildings	10
Timber shear wall construction	15

## Note

(1) The term "frame" indicates beam and column bending structures as distinct from shear structures.

(2) The term "concrete" includes both reinforced and prestressed concrete in buildings. For isolated prestressed concrete members such as in bridge decks damping values less than 5 percent be appropriate, e.g. 1-2 percent if the structure remains substantially uncracked.

# Chapter 2

7

## METHODS OF SEISMIC ANALYSIS

### INTRODUCTION

There are many methods of determining seismic forces in structures. However, they may all be classified into two distinct categories: equivalent static force analysis, and dynamic analysis.

This chapter is concerned with the brief presentation of these two types of analyses.

## EQUIVALENT STATIC FORCE ANALYSIS

The static force methods are approximate methods which have been evolved because of the difficulties involved in carrying out realistic dynamic analysis. Codes of practice world-wide rely mainly on the simpler static force approach, and incorporate varying degrees of refinement in an attempt to simulate the real behavior of the structure.

Basically the static force approach is a means of determining the total horizontal force  $V$  on the structure, more commonly called base shear:

$$V = ma \quad (2.1)$$

where  $m$  is the mass of the structure and  $a$  is the seismic horizontal acceleration.  $V$  is then applied to the structure by a simple rule describing its vertical distribution. In buildings, this generally consists of horizontal point loads at each concentration of mass (most typically the floor levels) as indicated in Figure 2.1. The seismic forces and moments in the structure are then determined by any suitable

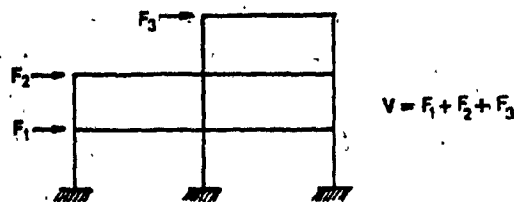


Figure 2.1 Example of frame with equivalent static forces applied at floor levels

statical analysis and the results added to those for the normal gravity load cases according to the code combinations.

In the 1975 National Building Code of Canada, the minimum lateral seismic force  $V$  is specified as:

$$V = A.S.K.I.F.W. \quad (2.2)$$

where the terms of this expression are defined as follows:

$A$  is the assigned horizontal design ground acceleration for the zone in question. The values of  $A$  are 0.00, 0.02, 0.04, 0.08, for seismic zones 0, 1, 2 and 3 respectively (Figure 2.2).

$S$  is a seismic response factor taken as  $0.5/\sqrt[3]{T}$  but not greater than 1.0.

$T$  is the fundamental period of the structure, taken as  $0.05h_n/\sqrt{D}$  or  $0.1N$  where  $h_n$  is the height above the base,  $D$  is the dimension of the building in a direction parallel to the applied forces.

$K$  is a numerical coefficient reflecting the influence of the type of construction on the damping, ductility and/or energy-absorption capacity of the structure. Values of  $K$  are given in Table 2.1.



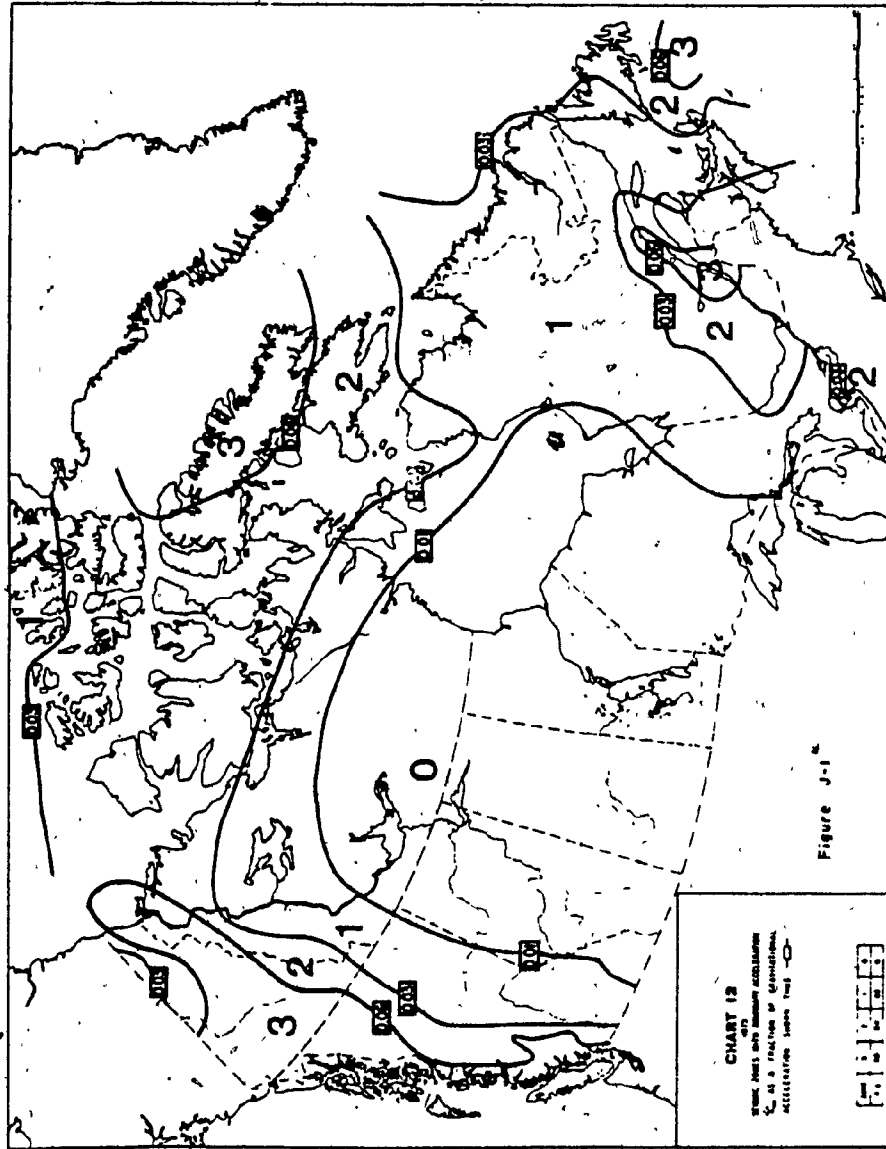


Figure 2.2 Seismic zone map of Canada

Table 2.1

Case <sup>(1)</sup>	Type or Arrangement of Resisting Elements	Value of K
1	<i>Buildings with a ductile moment-resisting space frame<sup>(2)(3)</sup> with the capacity to resist the total required force.</i>	0.7
2	<i>Buildings with a dual structural system consisting of a complete ductile moment-resisting space frame and ductile flexural walls<sup>(4)</sup> designed in accordance with the following criteria: The frames and ductile flexural walls shall resist the total lateral force in accordance with their relative rigidities considering the interaction of the flexural walls and frames. In this analysis the maximum shear in the frame must be at least 25 per cent of the total base shear.</i>	0.7
3	<i>Buildings with a dual structural system consisting of a complete ductile moment-resisting space frame and shear walls<sup>(5)</sup> or steel bracing designed in accordance with the following criteria: (a) The shear walls or steel bracing acting independently of the ductile moment-resisting space frame shall resist the total required lateral force. (b) The ductile moment-resisting space frame shall have the capacity to resist not less than 25 per cent of the required lateral force, but in no case shall the ductile moment-resisting space frame have a lower capacity than that required in accordance with the relative rigidities.</i>	0.8
4	<i>Buildings with ductile flexural walls<sup>(4)</sup> and buildings with ductile framing systems not otherwise classified in this Table as Cases 1, 2, 3 or 5.</i>	1.0
5	<i>Buildings with a dual structural system consisting of a complete ductile moment-resisting space frame with masonry infilling designed in accordance with the following criteria: (a) The wall system comprising the infilling and the confining elements acting independently of the ductile moment-resisting space frame shall resist the total required lateral force. (b) The ductile moment-resisting space frame shall have the capacity to resist not less than 25 per cent of the required lateral force.</i>	1.3
6	<i>Buildings (other than Cases 1, 2, 3, 4 and 5) of (a) continuously reinforced concrete, (b) structural steel, and (c) reinforced masonry shear walls.</i>	1.3 <sup>(6)</sup>
7	<i>Buildings of unreinforced masonry and all other structural systems except Cases 1 to 6 inclusive and those set forth in Table 4.1.9.C.</i>	2.0
8	<i>Elevated tanks plus full contents, on 4 or more cross-braced legs and not supported by a building, designed in accordance with the following criteria: (a) The minimum and maximum value of the product SKI shall be taken as 1.2 and 2.5 respectively. (b) For overturning, the factor J as set forth in Sentence 4.1.9.1.(14) shall be 1.0. (c) The torsional requirements of Sentence 4.1.9.1.(15) shall apply.</i>	3.0
Column 1	2	3

I is an importance factor taken as 1.3 for schools and buildings designed for post-disaster services and as 1.0 for all other buildings.

F is a foundation factor accounting for the influence of soil conditions. Values of F are given in Table 2.2.

Table 2.2

Type and Depth of Soil <sup>(1)</sup>	F
<i>Rock, dense and very dense coarse-grained soils, very stiff and hard fine-grained soils; compact coarse-grained soils and firm and stiff fine-grained soils from 0 to 50 ft deep</i>	1.0
<i>Compact coarse-grained soils, firm and stiff fine-grained soils with a depth greater than 50 ft; very loose and loose coarse-grained soils and very soft and soft fine-grained soils from 0 to 50 ft deep</i>	1.3
<i>Very loose and loose coarse-grained soils, and very soft and soft fine-grained soils with depths greater than 50 ft</i>	1.5 <sup>(2)(3)</sup>
Column I	.2

W is the dead load of the building plus 25% of the design snow load.

Figure 2.3 indicates how the lateral seismic force V is distributed through the structure as equivalent lateral static forces. If the height-to-width ratio of the structure is greater than 3, then part of the total force V, up to 15% maximum, is arbitrarily applied to the top of the structure. The purpose of this is to increase the shear in the upper parts of the structure to account

for the increasing participation of the higher modes of vibration in the response of taller, more flexible structures.

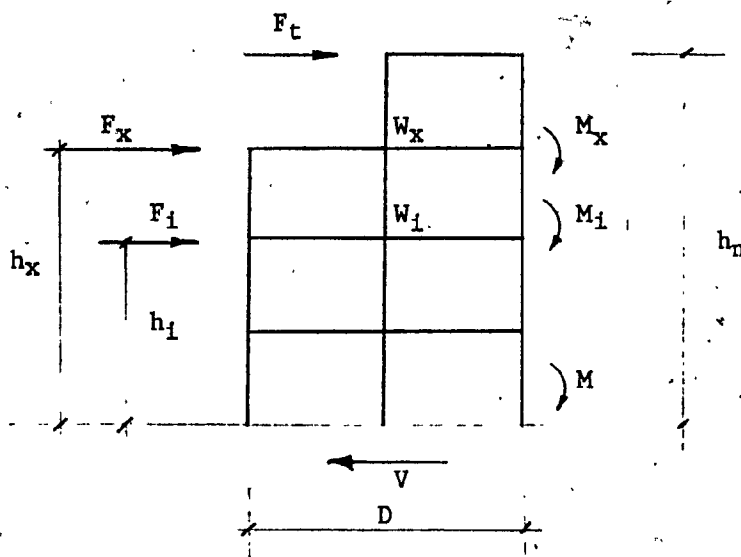


Figure 2.3 Distribution of the lateral seismic force V

It is also recognized that the fundamental mode deflection curve departs from the assumed straight line as the structure deflection is due more to bending action rather than shear deflection. This tends to cause greater shears in the top of the structure than would result from the so-called "triangular distribution"

$$F_t = 0.004 V \left( \frac{h_n}{D} \right)^2 \quad (2.3)$$

$$F_t = 0 \text{ for } \left( \frac{h_n}{D} \right) \leq 3 \quad (2.4)$$

$$F_t = 0.15 V \text{ for } \left( \frac{h_n}{D} \right) > 3 \quad (2.5)$$

(max.)

$$F_x = (V - F_t) \frac{W_x h_x}{\sum_{i=1}^n W_i h_i} \quad (2.6)$$

Having determined the distribution of the lateral seismic force  $V$  by the use of the formula (2.6), the remaining basic design criteria relate to the overturning moment on the structure as a whole and at any horizontal plane (Figure 2.3).

The moment at any section is given by:

$$M = M_x J_x \quad (2.7)$$

where

$$M_x = F_t (h_n - h_x) + \sum_{i=x}^n F_i (h_i - h_x) \quad (2.8)$$

and

$$J = 1.0 \quad \text{if } T < 0.5 \text{ sec.}$$

$$J = 1.1 - 0.2T \quad \text{if } 0.5 \leq T \leq 1.5 \text{ sec.}$$

$$J = 0.8 \quad \text{if } T > 1.5 \text{ sec.}$$

The torsional moments are computed by requiring that the design eccentricity at each floor be computed by

whichever of the following equations produces the greater effects:

$$e_x = 1.5e + 0.05 D_n \quad (2.9)$$

or 
$$e_x = 0.5e - 0.05 D_n$$

where  $e$  is the computed eccentricity between the centre of mass and the centre of rigidity and  $D_n$  is the plan dimension of the building in the direction of the computed eccentricity.

#### DYNAMIC ANALYSIS

For large or complex structures, static methods of seismic analysis are no longer accurate enough and a dynamic analysis is required. Various methods have been developed for the dynamic seismic analysis of structures. They all have in common the solution of the equations of motion, as well as the usual statical relationships of equilibrium and stiffness.

The three main techniques currently used for dynamic analysis are:

- i. Direct integration of the equations of motion by step-by-step procedures;

- ii. Normal mode analysis;
- iii. Response spectrum analysis.

The direct integration technique provides the most powerful and informative analysis for any given earthquake motion. A time-dependent forcing function (earthquake accelogram such as Figure 1.3) is applied and the corresponding response-history of the structure during the earthquake is computed. That is, the moment and force diagrams at each of a series of prescribed intervals throughout the applied motion can be found. Computer programs have been written for both linear elastic and nonlinear inelastic material behavior, using step-by-step integration procedures. However, linear behavior is seldom analyzed by direct integration as normal mode techniques are easier and nearly as accurate.

The normal mode analysis is a more limited technique than direct integration, as it depends on artificially separating the normal modes of vibration and combining the forces and displacements associated with a chosen number of them by superposition. Because of the use of superposition,

\*It is beyond the scope of this report to present this method of dynamic analysis in detail. The interested reader however, is referred to references 9 and 10 for a complete treatment of the subject.

the technique is limited to linear material behavior. Although modal analysis can provide any desired order of accuracy for linear behavior by incorporating all the modal responses, some approximation is usually made by using only the first few modes in an attempt to save computation time.

The response spectrum technique is really a simplified special case of modal analysis. The modes of vibration are determined in period and shape and the maximum response magnitudes corresponding to each mode are found by reference to a response spectrum (such as the one shown in Figure 1.4). An arbitrary rule is then used for superposition of the responses in the various modes (Figure 2.4).

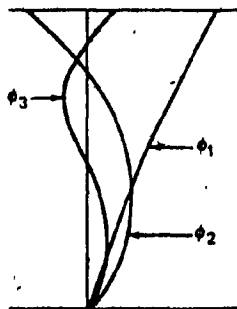


Figure 2.4 Natural modes of vibration for a three-degree-of-freedom system

The resultant moments and forces in the structure correspond to the envelopes of maximum values, rather than a set of simultaneously existing values. Further discussion of the theory of dynamic analysis is presented in Chapter 3.



## SELECTING A METHOD OF ANALYSIS

In the past there has generally been little choice in the method of analysis, mainly because suitable and economical computer programs have not been readily available. To date, most earthquake-resistant structures have been analyzed with an equivalent static load derived from a code of practice. However, with an increasing number of efficient and economical dynamic analysis programs that have been written for fast computers, many design offices are changing to the use of dynamic analyses.

It is difficult to give clear general advice on selecting the means of analysis, as each structure will have its own requirements. Table 2.3, however, gives a very simple indication of the applicability of the main methods of analysis.

Table 2.3  
Simple indication on selecting a means of analysis  
for different structures

Type of structure	Method of analysis
Small simple structures	(1) Equivalent static forces (appropriate code)
↓	(2) Response spectra (appropriate spectrum)
Progressively more demanding structures	(3) Modal analysis (appropriate dynamic input)
↓	(4) Non-linear plane frame (appropriate dynamic input)
Large complex structures	(5) Non-linear 3-D frame (appropriate dynamic input)

# Chapter 3

## THEORY OF DYNAMICS AND SEISMIC RESPONSE\*

### INTRODUCTION

The purpose of this chapter is to discuss the problem of formulating a mathematical model which satisfactorily describes the behavior of a structure subjected to damaging earthquake motions.

Having given a general description in Chapter 2 of the main types of dynamic analysis used in earthquake

\*The basis of this chapter is a course on Structural Dynamics, taught by Dr. Oscar A. Pekau at Concordia University, which the writer was privileged to take during the course of his graduate study.

engineering, a summary of the mathematical processes involved now follows. First the formulation of the dynamic response equations employed in typical computer programs will be discussed briefly, and the usual approach to the numerical solution of the equations will be outlined for single and multi-degree-of-freedom systems.

Fuller treatments of the following theory have been given by Biggs<sup>9</sup>, Clough and Penzien<sup>10</sup>, and Newmark and Rosenblueth<sup>11</sup>.

#### SINGLE-DEGREE-OF-FREEDOM SYSTEMS

To find the displacement history of a structure, it is necessary to solve the equations of motion of the system. There is one such equation of dynamic equilibrium for each degree of freedom based on the fact that

$$\sum F \neq 0 \quad (3.1)$$

rather

$$\sum F = ma \quad (3.2)$$

(i.e. the summation of forces is no longer zero, as in statics, but rather equal to the mass times the acceleration of the system).

A common representation of a single-degree-of-freedom system is shown in Figure 1.1, where  $F(t)$  is the forcing function varying with time. If the system of Figure 1.1 is idealized as shown in Figure 3.1(a) and considering the equilibrium of its free body (Figure 3.1(b)),

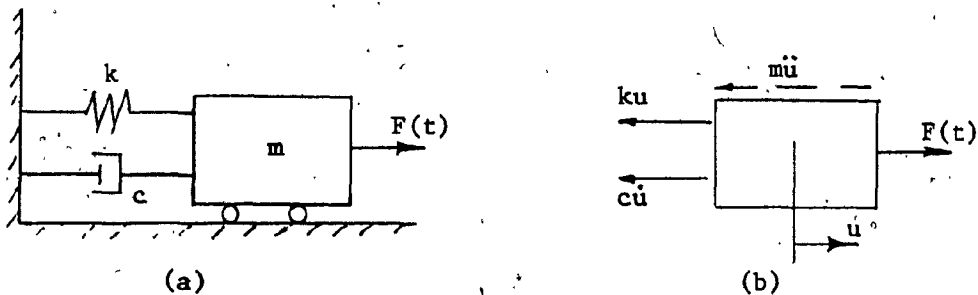


Figure 3.1 Idealized fixed base single-degree-of-freedom system subjected to a time-varying force

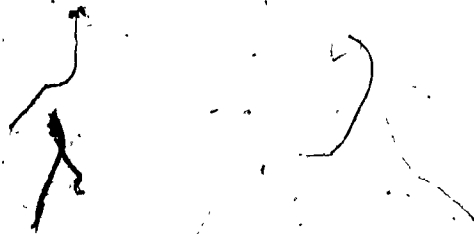
the equation of motion is given by

$$m\ddot{u} + c\dot{u} + ku = F(t) \quad (3.3)$$

where the dot ( $\dot{\phantom{u}}$ ) indicates differentiation with respect to time.

For the case of earthquake excitation (Figure 3.2), the only external loading is in the form of an applied motion at ground level. The total acceleration of the mass is

$$\ddot{u}_t = \ddot{u} + \ddot{u}_g \quad (3.4)$$



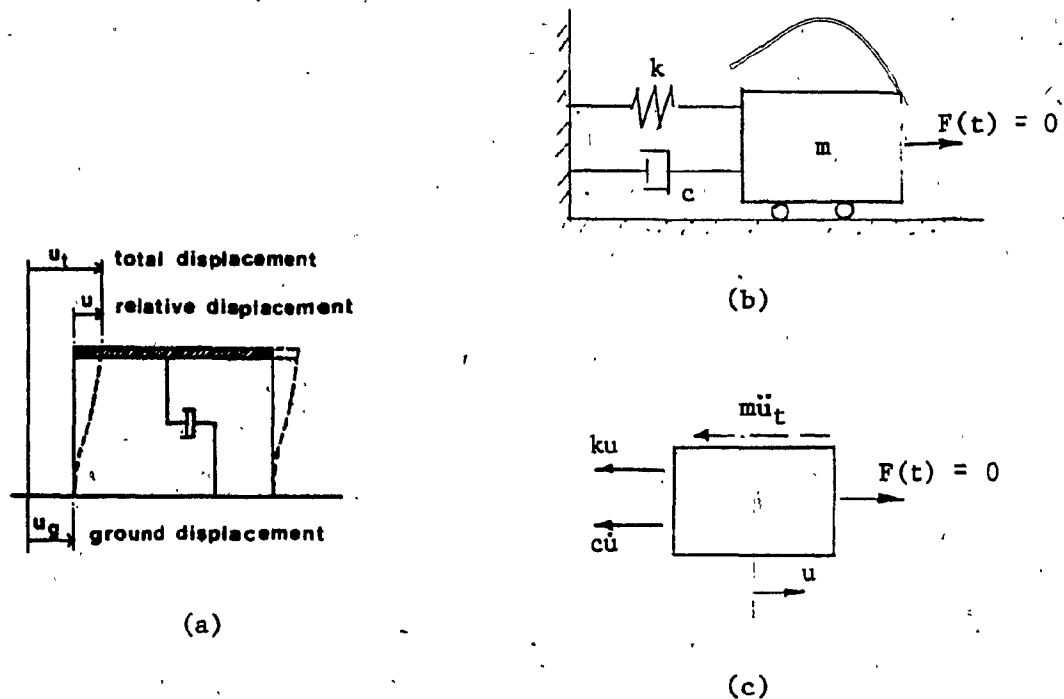


Figure 3.2 Single-degree-of-freedom system subjected to ground motion

Therefore, the equation of motion becomes

$$m\ddot{u} + m\ddot{u}_g + c\dot{u} + ku = F(t) = 0 \quad (3.5)$$

or

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g \quad (3.6)$$

A comparison of equations (3.3) and (3.6) indicates that the relative displacement,  $u$  (Figure 3.2), of the mass in a system subjected to a time-varying base acceleration,  $\ddot{u}_g$ , is equal to the absolute displacement,  $u$  (Figure 1.1), of the mass in a fixed-base system in which the mass is subjected to a time-varying force equal to  $-m\ddot{u}_g$ . The quantity  $m\ddot{u}_g$  is called the effective load resulting from the base motion.

In order to solve the equation of motion (equation 3.6), first consider the case of free vibration with zero damping. The equation of motion becomes

$$m\ddot{u} + ku = 0 \quad (3.7)$$

$$\ddot{u} + \frac{k}{m}u = 0$$

or

$$\ddot{u} + \omega^2 u = 0 \quad (3.8)$$

where

$$\omega^2 = k/m$$

or

$$\omega = \sqrt{k/m} \quad (3.9)$$

is the circular frequency of this free vibration system.

The solution of equation (3.8) is

$$u = A \sin \omega t + B \cos \omega t$$

solving for A and B,

$$u = \frac{\dot{u}_0}{\omega} \sin \omega t + u_0 \cos \omega t \quad (3.10)$$

The resulting simple harmonic motion is shown in Figure 3.3.

The period of the motion is

$$T = 2\pi/\omega$$

and the amplitude is

$$R = \sqrt{\left(\frac{\dot{u}_0}{\omega}\right)^2 + u_0^2}$$

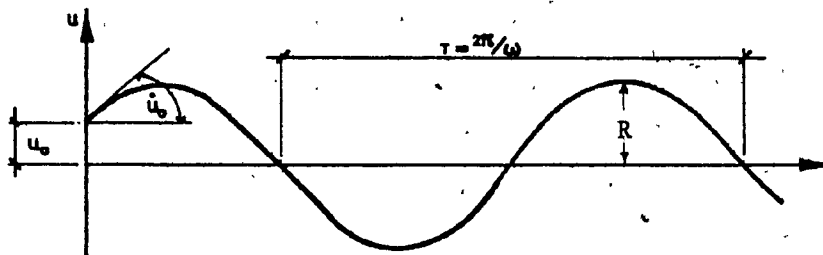


Figure 3.3 Undamped simple harmonic motion of single-degree-of-freedom system given initial displacement and velocity

For the damped system, the equation of motion may be written

$$\ddot{u} + 2\zeta\omega\dot{u} + \omega^2u = 0 \quad (3.11)$$

from which (for moderate damping)

$$u = e^{-\zeta\omega t} \left( \frac{\dot{u}_0 + \zeta\omega u_0}{\omega_D} \sin\omega_D t + u_0 \cos\omega_D t \right) \quad (3.12)$$

where  $\zeta = \frac{c}{2m\omega}$  is the damping ratio,

and  $\omega_D = \omega\sqrt{1 - \zeta^2}$  is the damped circular frequency.

To find the response of a single-degree-of-freedom system to an impulsive loading (Figure 3.4), the free vibration results may be used.

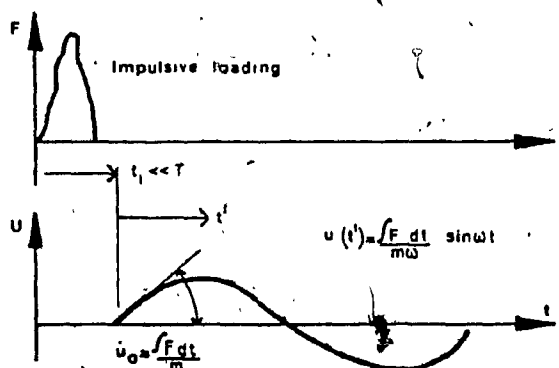


Figure 3.4 Response of single-degree-of-freedom system to impulsive undamped loading

If the length of the impulse  $t_1 \ll T$ , the period of vibration, it may be assumed that  $u_0 = 0$  and from impulse-momentum

$$m\Delta\dot{u} = \int F dt$$

therefore

$$\dot{u}_0 = \frac{\int F dt}{m}$$

Using these values of  $u_0$  and  $\dot{u}_0$  in equation (3.10)

$$u = \frac{\int F dt}{m\omega} \sin \omega t$$

For arbitrary loading (Figure 3.5), a series of impulses may be assumed. The displacement response due to any individual increment of loading ending at time  $t$  and of



duration  $d\tau$ , can be written as

$$du = \frac{F(\tau)}{m\omega} \sin\omega(t - \tau) d\tau$$

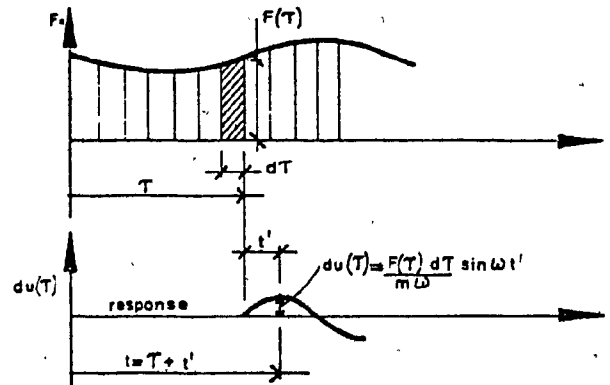


Figure 3.5 Response of single-degree-of-freedom system to arbitrary undamped loading.

The total response to the arbitrary loading is the sum of all the impulses of duration  $d\tau$

$$u(t) = \int_0^t \frac{F(\tau)}{m\omega} \sin\omega(t - \tau) d\tau$$

This is an exact expression called the Duhamel Integral. Because it depends on the principle of superposition, it is applicable to linear structures only.

The response of damped single-degree-of-freedom structures to earthquake motion may be derived from the Duhamel Integral as follows.

$$u(t) = \int_0^t \frac{F(\tau)}{m\omega_D} e^{-\zeta\omega(t-\tau)} \sin\omega_D(t-\tau) d\tau \quad (3.13)$$

is the damped form of the Duhamel Integral which can be rewritten in terms of the ground acceleration  $\ddot{u}_g(\tau) = F(\tau)/m$ , taking  $\omega = \omega_D$ , which is reasonable for small damping. Thus,

$$u(t) = \frac{1}{\omega} \int_0^t \ddot{u}_g(\tau) e^{-\zeta\omega(t-\tau)} \sin\omega(t-\tau) d\tau \quad (3.14)$$

Letting  $V(t)$  equal the part under the integral of equation (3.14), the earthquake deflection response of a lumped mass system becomes

$$u(t) = \frac{1}{\omega} V(t) \quad (3.15)$$

The forces generated in the structure are found in terms of the effective acceleration

$$\ddot{u}_e(t) = \omega^2 u(t) \quad (3.16)$$

The effective earthquake force on the structure is then

$$\begin{aligned} Q(t) &= m\ddot{u}_e(t) \\ &= m\omega^2 u(t) \end{aligned}$$

Therefore,

$$Q(t) = m\omega V(t) \quad (3.17)$$

Thus the effective earthquake force (or base shear) is found in terms of the mass of the structure, its circular frequency, and the response function  $V(t)$ .

#### MULTI-DEGREE-OF-FREEDOM SYSTEMS

In the dynamic analysis of most structures, it is necessary to assume that the mass is distributed in more than one discrete lump. For most buildings, the mass is assumed to be concentrated at the floor levels, and to be subjected to lateral displacements only.

The number of degrees of freedom of a system is defined as the least number of independent displacement coordinates necessary to completely determine the configuration of the system at any instant.

The use of lumped-mass models to represent the actual distributed mass structure is a convenient device for reducing the infinite number of degrees of freedom of the structure to a manageable few. This makes possible the formulation of the force equilibrium of the system in terms of a set of ordinary differential equations instead of the

partial differential equations which would be required for the continuous system.

To illustrate the multi-degree-of-freedom analysis consider the three-story building shown in Figure 1.5. Each story mass represents one degree-of-freedom, each with an equation of dynamic equilibrium

$$\begin{aligned} F_{Ia} + F_{Da} + F_{Sa} &= F_a(t) \\ F_{Ib} + F_{Db} + F_{Sb} &= F_b(t) \\ F_{Ic} + F_{Dc} + F_{Sc} &= F_c(t) \end{aligned} \quad (3.18)$$

where  $F_I$ ,  $F_D$  and  $F_S$  are the inertia, damping and elastic force respectively.

The inertia forces in equation (3.18) are simply

$$\begin{aligned} F_{Ia} &= m_a \ddot{u}_a \\ F_{Ib} &= m_b \ddot{u}_b \\ F_{Ic} &= m_c \ddot{u}_c \end{aligned} \quad (3.19)$$

or more generally in matrix form

$$F_I = M\ddot{u} \quad (3.20)$$

Similarly for the damping and elastic forces

$$F_D = C\dot{u} \quad (3.21)$$

and

$$F_S = Ku \quad (3.22)$$

It should be noted that the mass matrix is of diagonal form for a lumped mass system, giving no coupling between the masses. In more generalized shape coordinate systems, coupling generally exists between the coordinates, complicating the solution.

Using equations (3.20), (3.21) and (3.22), the equations of dynamic equilibrium may be written generally as

$$M\ddot{u} + C\dot{u} + Ku = F(t) \quad (3.23)$$

This equation is identical to equation (3.3), for a single-degree-of-freedom system.

As the dynamic response of a structure is dependent upon the frequency and the displaced shape, the first step in the analysis of a multi-degree-of-freedom system is to find its vibration frequencies and mode shapes. For free vibration, there is no external force and no damping.

Equations (3.23) then become

$$M\ddot{u} + Ku = 0 \quad (3.24)$$

but in free vibration, the motion is simple harmonic

$$u = \hat{u}\sin\omega t$$

Therefore

$$\ddot{u} = -\omega^2\hat{u}\sin\omega t \quad (3.25)$$

where  $\hat{u}$  represents the amplitude of vibration.

Substituting in equation (3.24)

$$K\hat{u} - \omega^2 M\hat{u} = 0 \quad (3.26)$$

equation (3.26) is an eigenvalue equation and is readily solved for  $\omega$  by standard computer programs. Its solution for a system of  $N$  degrees of freedom yields a vibration frequency  $\omega_n$  and a mode shape vector  $\phi_n$  for each of its  $N$  modes.  $\phi_n$  represents the relative amplitudes of motion (Figure 3.6) for each of the displacement components in

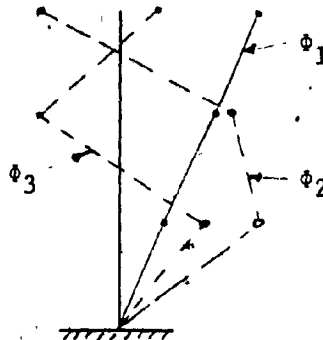


Figure 3.6 Natural modes of vibration of a three-degree-of-freedom system

mode  $n$ . Equation (3.26) cannot be solved for absolute values of  $\phi$ , as the amplitudes are arbitrary in free vibration.

An important simplification can be made in the equations of motion because of the fact that each mode has an independent equation of exactly equivalent form to that for a single-degree-of-freedom system. Because of the orthogonality properties of mode shapes, equation (3.23) can be written

$$\ddot{Y}_n + 2\zeta_n\omega_n\dot{Y}_n + \omega_n^2 Y_n = \frac{\phi_n^T F(t)}{\phi_n^T M \phi_n} \quad (3.27)$$

where  $Y_n$  is a generalized displacement in mode  $n$ , leading to the actual displacement.

In terms of excitation by earthquake ground motion  $u_g(t)$ , equation (3.27) becomes

$$\ddot{Y}_n + 2\zeta_n\omega_n\dot{Y}_n + \omega_n^2 Y_n = \frac{L_n}{\phi_n^T M \phi_n} \ddot{u}_g(t) \quad (3.28)$$

where  $L_n$  is the earthquake participation factor and is given by

$$L_n = \frac{\phi_n^T \hat{M} \mathbf{1}}{\phi_n^T M \phi_n}$$

in which  $\mathbf{1}$  is a unit column vector of dimension  $N$ .

The solution of equation (3.28) may be done by evaluating the Duhamel Integral

$$Y_n(t) = \frac{L_n}{\phi_n^T M \phi_n} \cdot \frac{1}{\omega_n} \int_0^t \ddot{u}_g(\tau) e^{-\zeta_n \omega_n (t-\tau)} \sin \omega_n (t-\tau) d\tau \quad (3.29)$$

The displacement of floor (or mass)  $i$  at time  $t$  is then obtained by superimposing the response of all modes evaluated at this time  $t$

$$u_i = \sum_{n=1}^N \phi_{in} Y_n(t) \quad (3.30)$$

where  $\phi_{in}$  is the relative amplitude of displacement of mass  $i$  in mode  $n$ .

The earthquake forces in the structure may then be expressed in terms of the effective accelerations

$$\ddot{Y}_n \text{ eff.}(t) = \omega_n^2 Y_n(t) \quad (3.31)$$

from which the acceleration at any floor  $i$  is

$$\ddot{u}_{in \text{ eff.}}(t) = \omega_n^2 \phi_{in} Y_n(t) \quad (3.32)$$

and the earthquake force at any floor  $i$  at time  $t$  is

$$Q_{in}(t) = m_i \omega_n^2 \phi_{in} Y_n(t) \quad (3.33)$$



Superimposing all the modal contributions, the earthquake forces in the whole structure are given by

$$Q(t) = M\phi\omega^2 Y_n(t) \quad (3.34)$$

where  $\phi$  is the square matrix of relative amplitude distributions in each mode and  $\omega^2$  is the diagonal matrix of  $\omega^2$  for each of the  $n$  modes.

Having determined the modal response amplitudes of equation (3.30), the entire history of displacement and force response can be defined from equation (3.34).

# Chapter 4

## DETAILING OF CAST IN PLACE REINFORCED CONCRETE

### INTRODUCTION

There is more information available about the seismic performance of reinforced concrete than any other material. No doubt this is because of its widespread use and because of the difficulties involved in ensuring its adequate ductility.

Reinforced concrete is generally desirable because of its availability and economy, and its stiffness can be used to advantage to minimize seismic deformations and hence reduce the damage to non-structure.

The purpose of this chapter is to define the strength and deformation characteristics of reinforced concrete members and to provide notes on the detailing of these members for earthquake resistance.

## PRINCIPLES OF EARTHQUAKE-RESISTANT DESIGN

The seismic response of structural materials has been presented generally in Chapter 1, where some stress-strain diagrams were presented. The hysteresis loops of Figure 1.6(d) indicate that considerable ductility without strength loss can be achieved in doubly reinforced beams having adequate confinement reinforcing. This is in distinct contrast to the loss of strength and stiffness degradation exhibited by plain unconfined concrete under repeated loading as shown in Figure 1.6(c). However, even in well reinforced concrete members, the root cause of failure under earthquake loading is usually concrete cracking. Degradation occurs in the cracked zone under cyclic loading. Cracks do not close up properly when the tensile stress drops because of permanent elongation of reinforcement in the crack, and aggregate interlock is destroyed in a few cycles. In hinge and joint zones reversed diagonal cracking breaks down the concrete between the cracks completely, and sliding shear failure occurs (Figure 4.1).

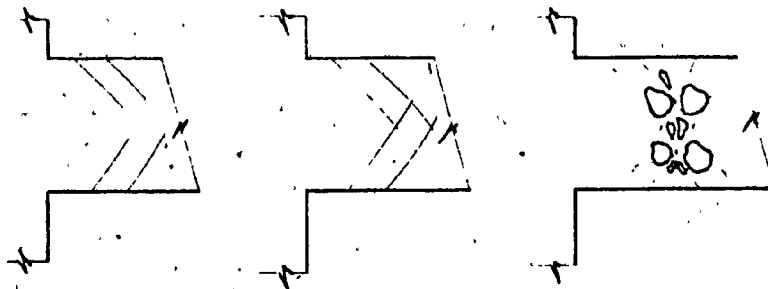


Figure 4.1 Progressive failure of reinforced concrete hinge zone under seismic loading

In reinforced concrete structures, the essential features of earthquake resistance are embodied in ensuring the following:

- (i) Beams should fail before columns;
- (ii) Failure should be in flexure rather than shear;
- (iii) Premature failure of joints between members should be prevented;
- (iv) Ductile rather than brittle failure should be obtained.

The ACI Code<sup>12</sup> specifies strength factors which try to ensure that beams fail before columns, but this situation can be further facilitated by the use of mild steel for the longitudinal reinforcement of beams, and higher strength steels for columns.

To prevent shear failure occurring before bending failure, it is good practice to design so that the flexural steel in a member yields while the shear reinforcement is working at a stress less than yield. In beams a conservative approach to safety in shear is to make the shear strength equal to the maximum shear demands which can be made on the beam in terms of its bending capacity.

Referring to Figure 4.2, the shear strength of

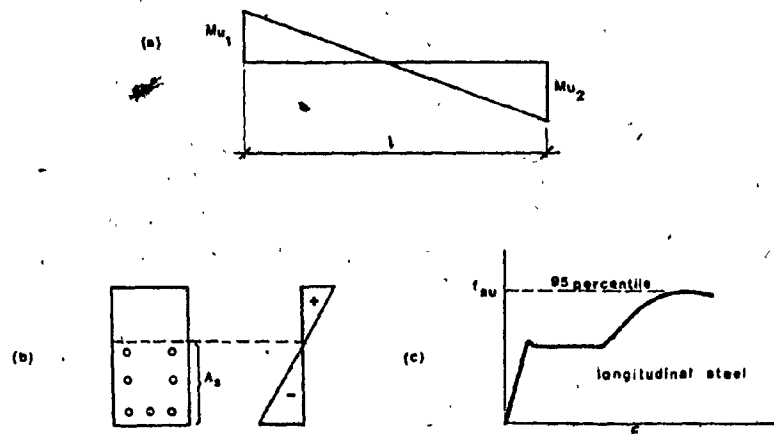


Figure 4.2 Shear strength considerations for reinforced concrete beams

the beam should correspond to

$$V_{\max} = \frac{M_{u1} - M_{u2}}{l} + V_D \quad (4.1)$$

where  $V_D$  is the dead load shear force and

$$M_u = A_s f_{su} d$$

Recently Paulay<sup>13</sup> suggested that the shear can be carried through the broken concrete zone by sloping the main bars (Figure 4.3) through the hinge zone towards a point of contraflexure at the centre of the beam.

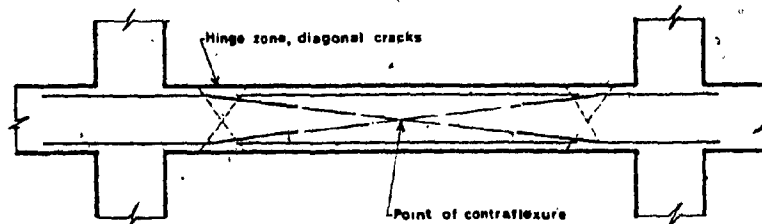


Figure 4.3 Prevention of sliding shear in plastic hinge zones using sloping main steel to carry shear after heavy concrete cracking

#### DUCTILITY OF REINFORCED CONCRETE MEMBERS

The most practical way of determining the ductility demand for a reinforced concrete structure is by the ultimate stress method. Simplified methods of determining the hinge rotations in reinforced concrete frames have been suggested by Hollings<sup>14</sup> and Park<sup>15</sup>. These techniques involve the assumption of hinge mechanisms for the frame, such as shown in Figure 4.4, and the imposition of an arbitrary lateral deflection ductility factor  $\mu$  on the frame. These simple mechanisms are only guesses at the actual rotational demands due to an earthquake. However, they are likely to produce better aseismic structures than if no rotation assessment is made.

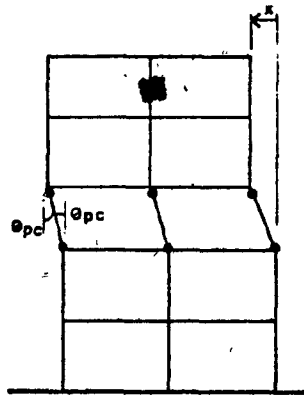
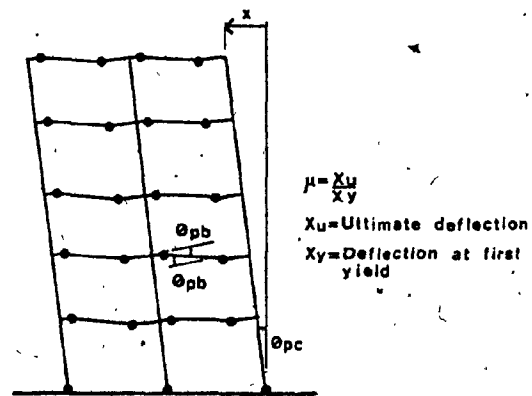
(a) Column Sidesway(b) Beam Sidesway

Figure 4.4 Alternative plastic hinge mechanisms for a typical multi-storey frame

The available section ductility of a concrete member is most conveniently expressed as the ratio of its curvature at ultimate moment  $\phi_u$  to its curvature at first yield  $\phi_y$ . The expression  $\phi_u/\phi_y$  may be evaluated from first principles, depending on the geometry of the section, the reinforcement arrangement, the loading, and the stress-strain relationships of the steel and the concrete.

Consider conditions at first yield and ultimate moment as shown in Figure 4.5.

For an under-reinforced section with no compression steel, first yield will occur in the steel, and the curvature is given by

$$\phi_y = \frac{\epsilon_{sy}}{(1-k)d} = \frac{f_y}{E_s(1-k)d} \quad (4.2)$$

where

$$k = \frac{\rho n}{\sqrt{(\rho n)^2 + 2\rho n}} - \rho n \quad (4.3)$$

provided

$$f_c = \frac{2\rho f_y}{k} \leq .7f_c'$$



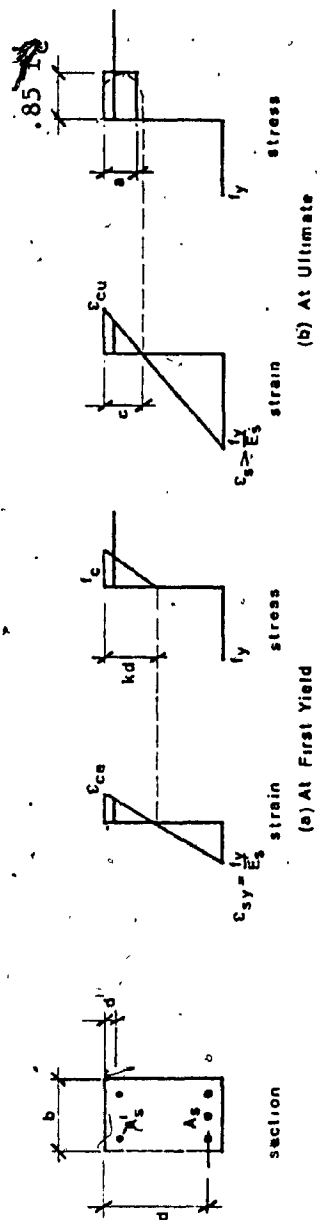


Figure 4.5 Reinforced concrete section in flexure

Referring to Figure 4.6, it can be shown that the ultimate curvature is

$$\phi_u = \frac{\epsilon_{cu}}{c} = \frac{\beta_1 \epsilon_{cu}}{a} \quad (4.4)$$

where

$$a = \frac{A_s f_y}{.85 f'_c b} \quad (4.5)$$

and  $\beta_1$  may be taken according to the ACI Code<sup>12</sup>. The available section ductility is then

$$\frac{\phi_u}{\phi_y} = \frac{\epsilon_{cu} d (1-k) E_s}{c f_y} \quad (4.6)$$

For estimating the ductility available from reinforced concrete in a strong earthquake, a value of 0.004 may be taken as representing the limit of useful concrete strain<sup>16</sup>.

For a doubly reinforced section, the ductility may be determined in the same way as for singly reinforced sections but with allowances for the effect of compression steel ratio  $\rho'$ . The expressions for  $c$  and  $k$  become

$$\begin{aligned} c &= \frac{a}{\beta_1} \\ &= \frac{(\rho - \rho') f_y d}{.85 f'_c \beta_1} \end{aligned} \quad (4.7)$$

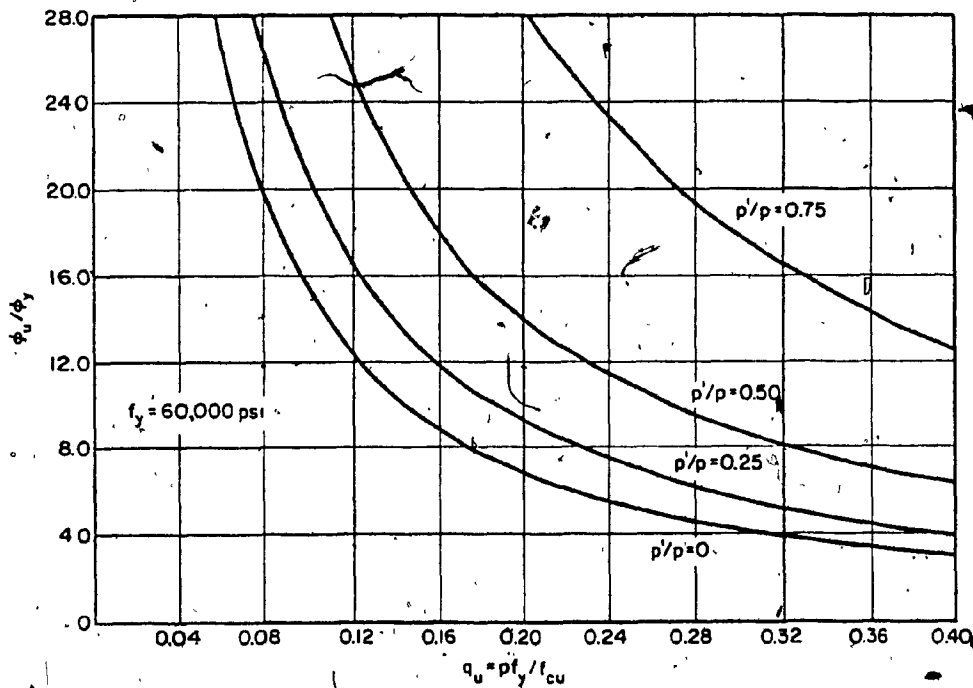
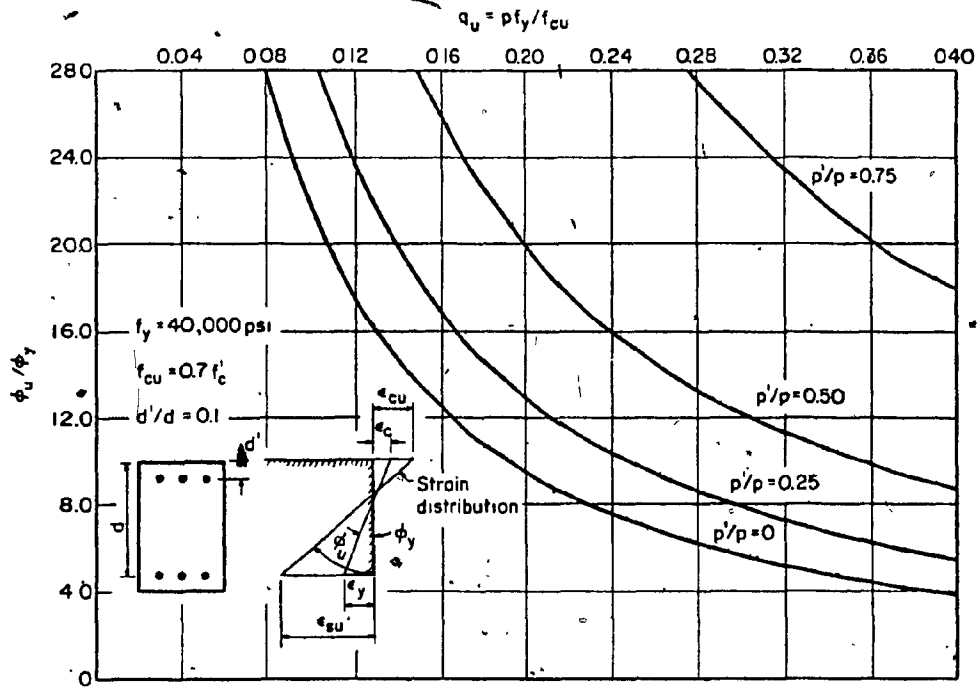


Figure 4.6 Variation of  $\phi_u / \phi_y$  for beams of unconfined concrete in respect to  $q_u$

and

$$k = \sqrt{(\rho + \rho')^2 n^2 + 2 \left[ \rho + (\rho' d'/d) \right] n} - (\rho + \rho') n \quad (4.8)$$

The above equations assume that compression steel yields. If this is not so, the actual value of the steel stress should be substituted for  $f_y$ .

### BEAM DETAILS

The significant provisions of the ACI Code<sup>12</sup> relating to flexural members will be discussed in the following.

Referring to the details of Figures 4.7 and 4.8,

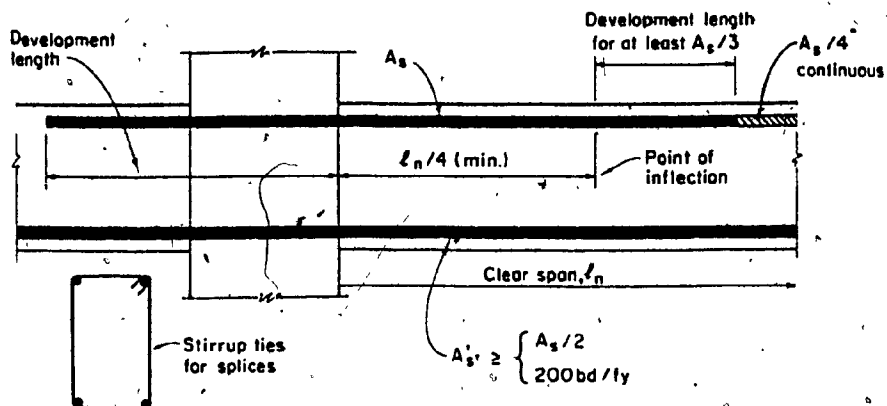


Figure 4.7 Anchorage of beam reinforcement in beam on opposite side of column

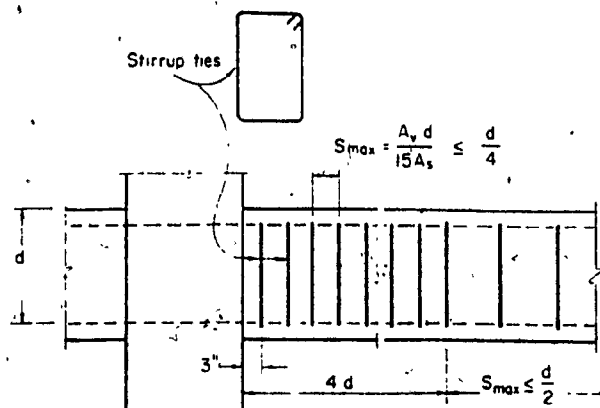


Figure 4.8 Beam web reinforcement limitations,  
min. bar size - #3

the limitations on flexural reinforcement ratio are

$$\rho_{min} = \frac{200}{f_y}$$

$$\rho_{max} = 0.50 \rho_{balanced}$$

Because the ductility of a flexural member decreases with increasing values of the reinforcement ratio,  $\rho$ , the code limits the maximum value of this ratio to 0.50 of the ratio corresponding to the balanced condition.

In order to allow for the possibility of the positive moment at one end of a beam due to earthquake-induced lateral displacement (Figure 4.9) exceeding the negative moment capacity due to gravity loads, the code requires a minimum positive moment capacity at beam ends

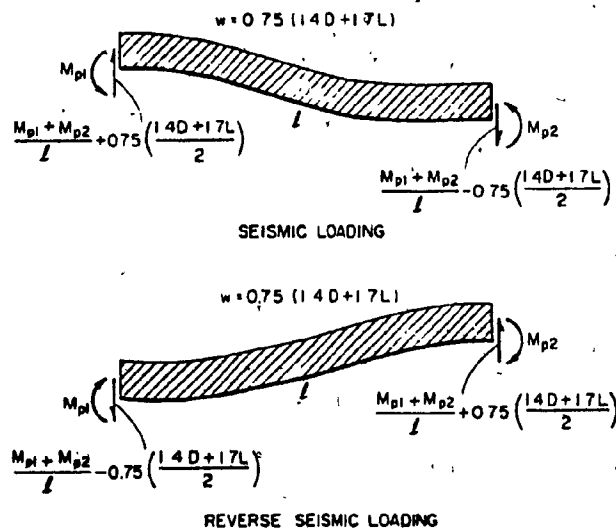


Figure 4.9 Loadings for design of transverse reinforcement in beams

equal to 50% of the negative moment capacity, i.e at the end

$$M_{(\text{pos})} \geq 0.50 M_{(\text{neg})}$$

It is usually not possible to develop the yield strength of a reinforcing bar from a framing beam within the width of a column (except in very large columns). Where beam reinforcement can extend through a column, its capacity is developed within the compression zone of the beam on the far side of the connection. For exterior columns, the flexural reinforcement in a framing beam has to be developed within the confined regions of the column. For such cases, standard 90° hooks are used plus whatever extension is necessary to develop the bar, the development

length being measured from the face of the column (Figure 4.10).

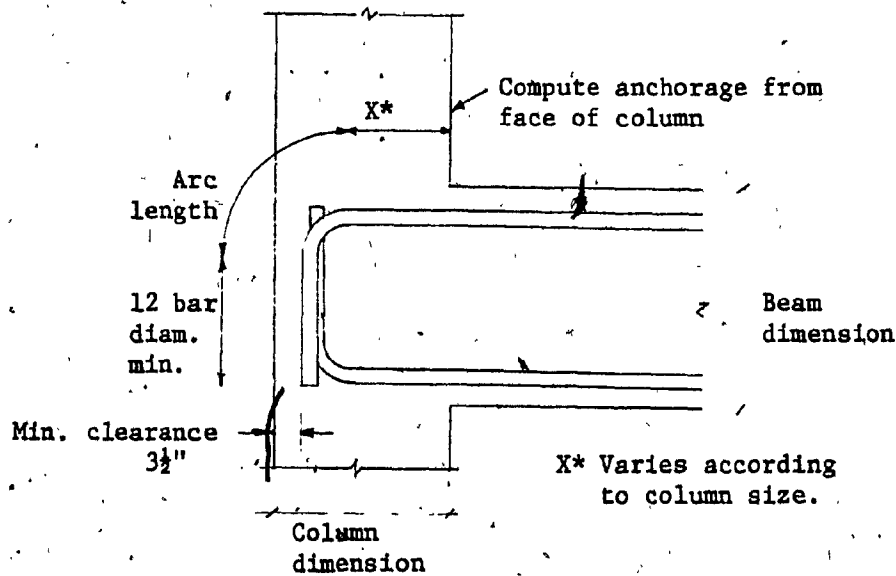
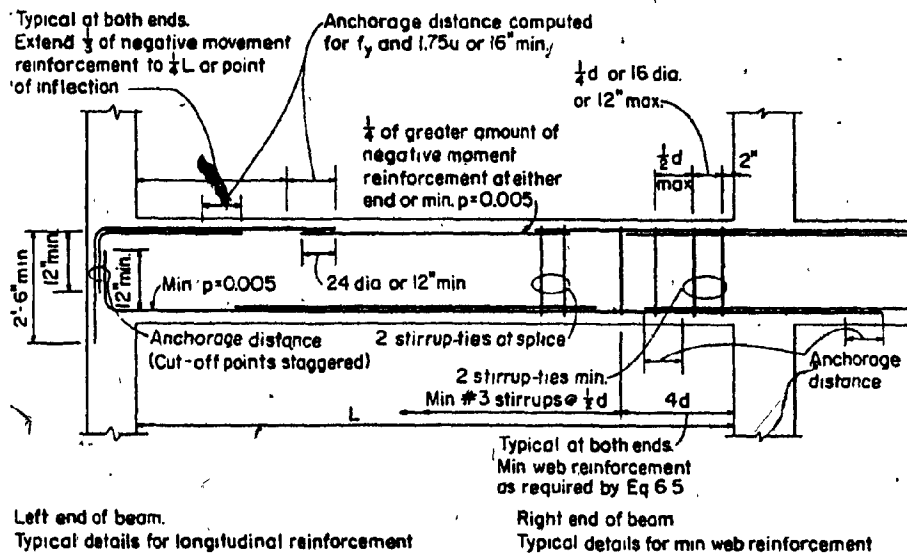


Figure 4.10 Typical longitudinal and web reinforcement details for girders and beams on column lines

FRAME-COLUMN DETAILING

The limitations on reinforcement ratio of columns are (Figures 4.11 to 4.14):

$$\rho_{min} = 1\%$$

$$\rho_{max} = 6\%$$

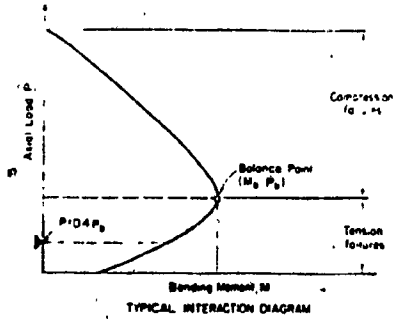
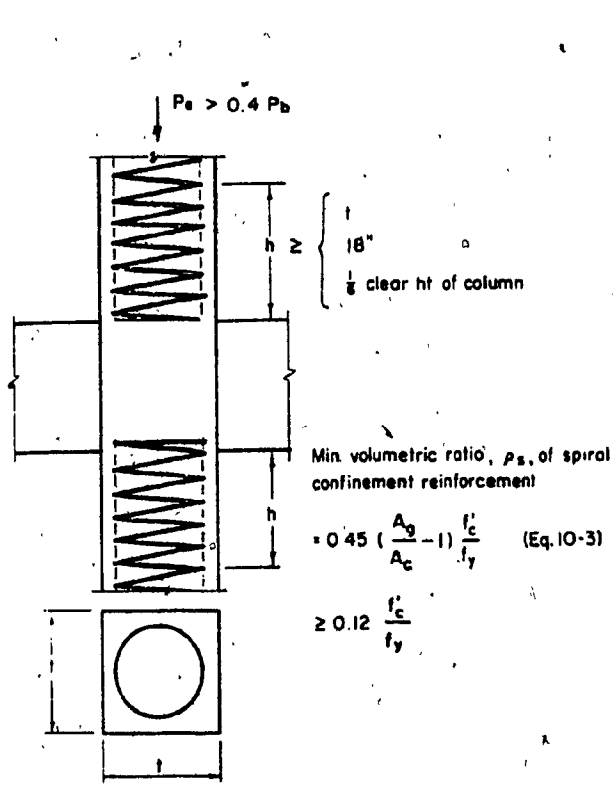


Figure 4.11 Confinement reinforcement at column ends - spirals



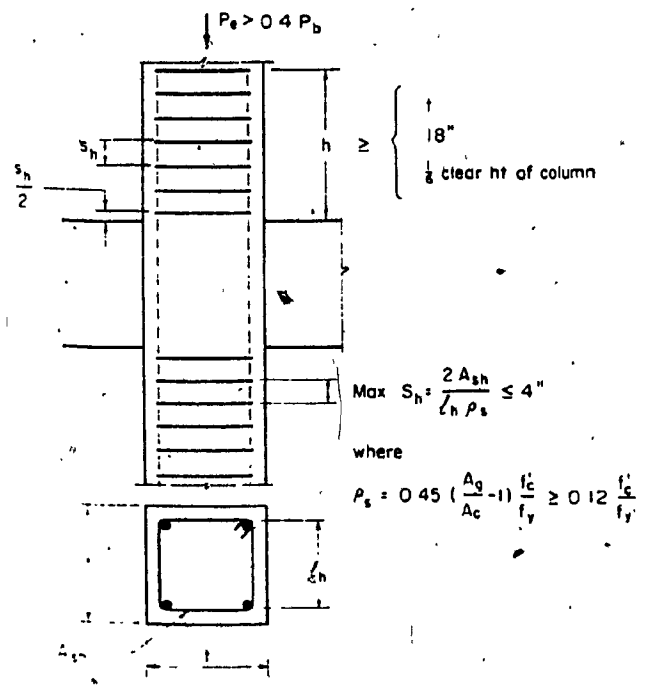
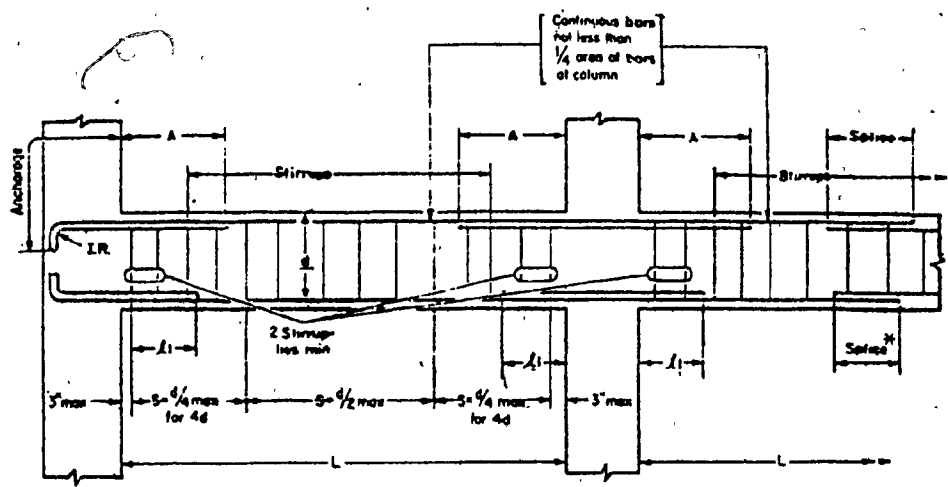


Figure 4.12 Confinement reinforcement at column ends - rectangular



Designer should provide dimension A, s, d, L, anchorage length, cutoff points of discontinuous bars, etc.

A = distance to point of inflection plus anchorage length but not less than L/4. Designer may cut some bars shorter than this but at least one-third the area of bars of the column must extend this distance

d = beam design depth

I.R. = 40 minimum, 60 preferable

L = distance required by design for moment plus anchorage length

\* Provide not less than two stirrup ties throughout splice length, spaced no further apart than 16 bar diameters or 12 in.

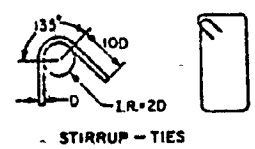


Figure 4.13 Example of typical bearing bar details for special ductile moment resisting frames

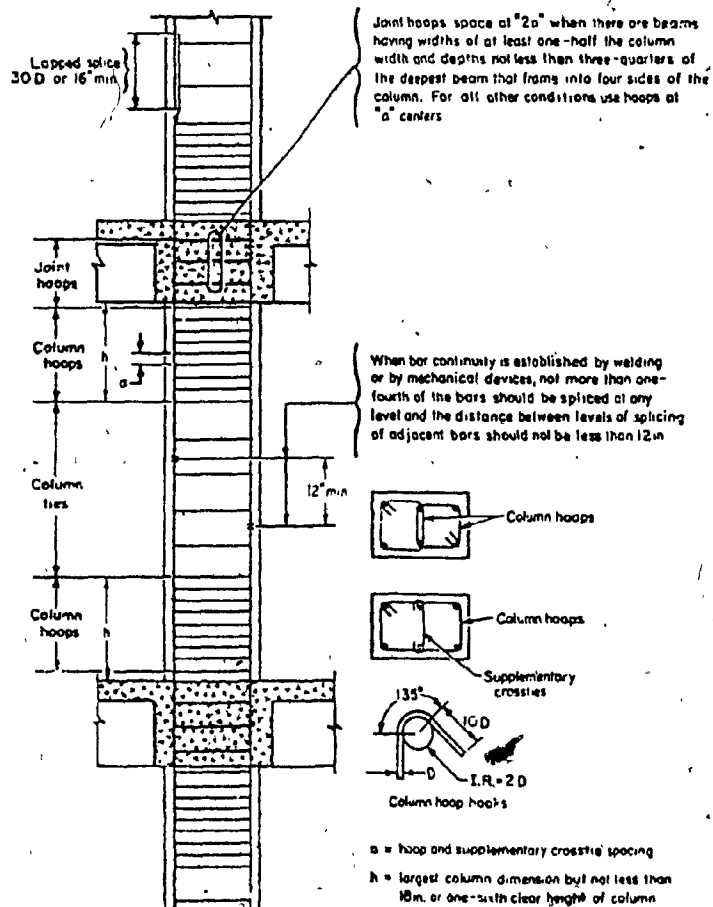


Figure 4.14 Typical rectangular tied column bar details for special ductile moment-resisting frames

However, the practical upper limit for the reinforcement ratio is, 4% due to construction considerations and convenience in detailing and placing reinforcement in beam-column connections.

In order to ensure that beams will fail before columns, the sum of the flexural strengths of the columns meeting at a joint should be greater than the sum of the

moment strengths of the framing beams in the same plane (Figure 4.15).

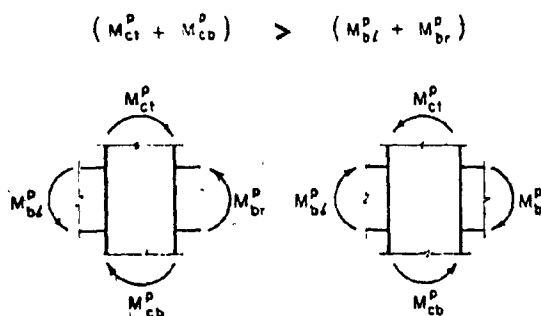


Figure 4.15 "Strong column - weak beam" frame

When the axial load of a column exceeds  $0.4 P_b$ , special transverse reinforcement has to be provided near the ends to ensure adequate strength after loss of the shell, should hinging take place in the columns. A minimum volumetric ratio of spiral reinforcement,  $\rho_s$ , equal to  $0.12 f'_c / f_y$  is specified. This value will usually govern in the case of large columns. This confinement reinforcement is to be used throughout the entire length of columns supporting discontinuous shear walls (Figure 4.16).

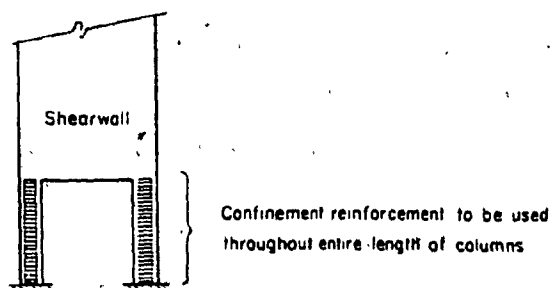


Figure 4.16, Columns supporting discontinued shearwall

It should be noted from the above that

$$\rho_s = 0.45 \left( \frac{A_g}{A_c} - 1 \right) \frac{f'_c}{f_y}$$

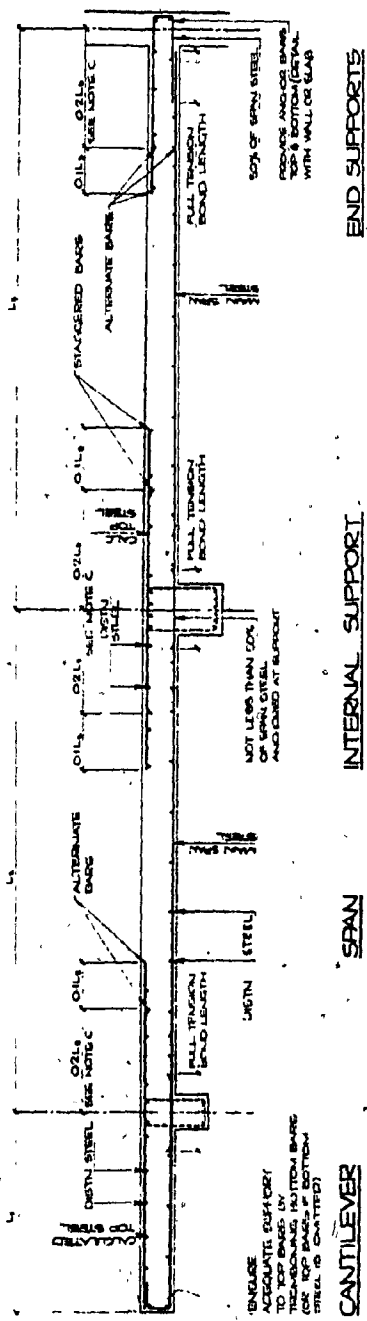
as given by the Code<sup>12</sup> provides insufficient confinement<sup>17</sup>.

### SLAB DETAILS

Reinforcement designed for gravity loads in the slabs of beam-and-slab type floors spanning in one or more than one direction is usually adequate to ensure good performance of the slabs both as a flexural member and as a horizontal diaphragm. In view of the anticipated diaphragm action, additional reinforcement should be provided at the sides and corners of openings in slabs as shown in Figure 4.17.

### SHEAR WALLS

Shear walls are normally much stiffer than regular frame elements and therefore can be expected to be subjected to correspondingly greater lateral forces during response to earthquake motions. Because of their relatively greater depth, their lateral-deformation capacity is



**END SUPPORTS & SINGLE SPAN SLABS**

- NOTES**
- A EXCEPT WHERE SHOWN SLAB DETAILS ARE TO BE AS BALANCED DRAWING OFFICE PRACTICE
  - B  $\phi$  IS THE DIAMETER OF THE BAR CONCERNED
  - C BARS TO BE CURTAILED AS CALCULATED BUT NOT SHORTER THAN SHOWN

**INTERNAL SUPPORT**

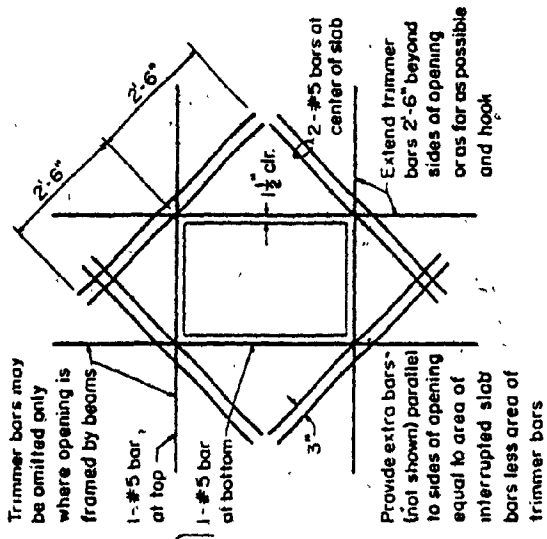


Figure 4.17 Typical slab details

limited, so that, for a given amount of lateral-displacement, shear walls will tend to exhibit greater apparent distress than frames. Because of this, and the fact that such elements possess a lower degree of structural redundancy than frames of comparable stiffness, codes have tended to penalize shear walls. However, the satisfactory performance of many shear wall and frame-shear wall structures<sup>18</sup> subjected to actual earthquakes attests to the reliability of structures which rely on properly designed and well founded shear walls for their lateral support.

The principal provisions of the Code<sup>12</sup> relating to shear walls are shown in Figure 4.18.

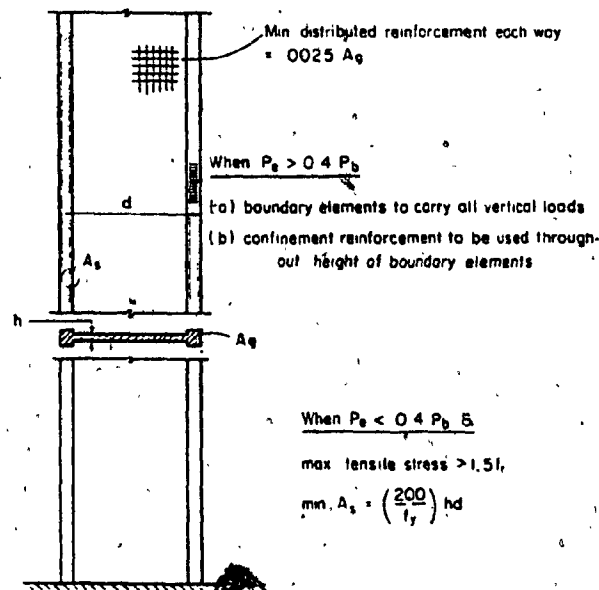


Figure 4.18 Special shear walls



arrangement of reinforcement as shown in Figure 4.20.

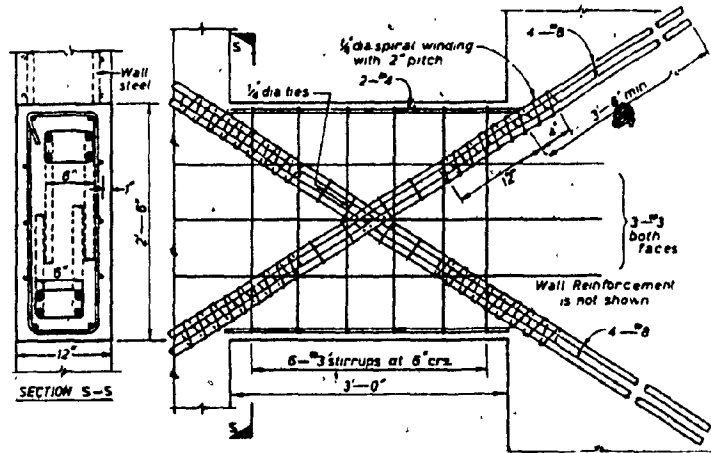


Figure 4.20 A prototype diagonally reinforced coupling beam

The disposition of internal forces that balance the external actions, consistent with seismic loading, are presented in Figure 4.21. Using the notation given in the figure  $A_s$

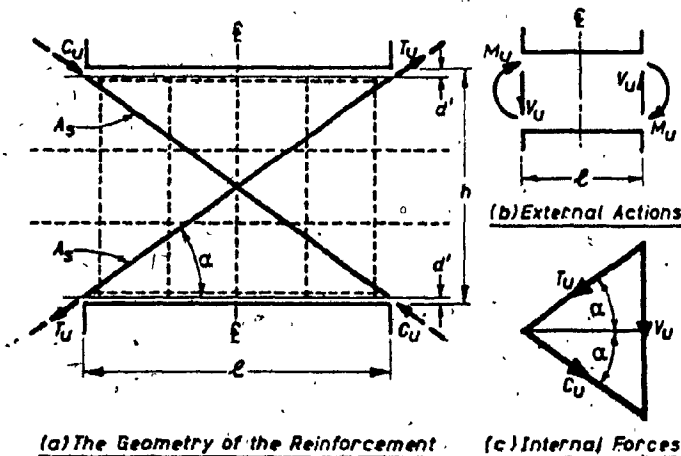


Figure 4.21 External actions and internal forces in diagonally reinforced coupling beams



required to resist the external moment  $M_u$  or shear force

$V_u = \frac{M_u}{l}$ , is obtained by

$$A_s = \frac{V_u}{2f_y \sin \alpha} \quad (4.9)$$

where

$$\tan \alpha = (h - 2d')/l$$

A comparison of the failure pattern shown in Figure 4.22 reveals the superior performance of the diagonally reinforced coupling beams.

Figure 4.23 suggests a typical detail for coupling beams.

#### WALL, STAIRCASE AND PARAPET DETAILING

Figures 4.24 to 4.27 suggest typical details for walls, staircases and parapets based on recommendations according to California codes.

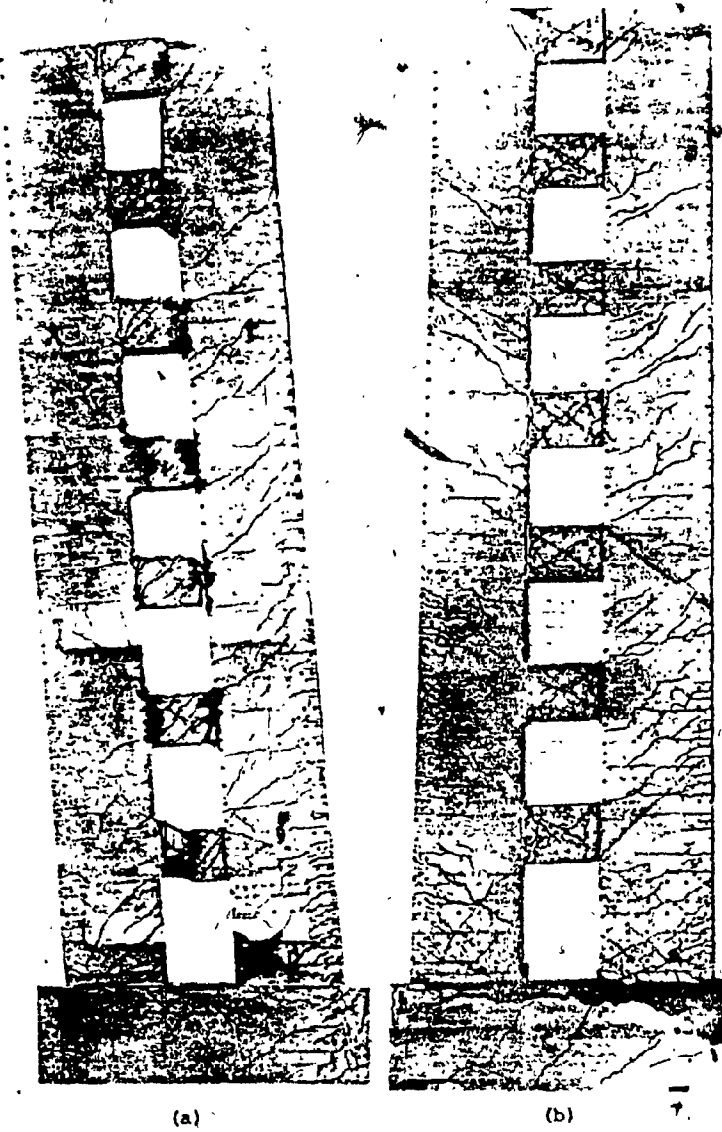
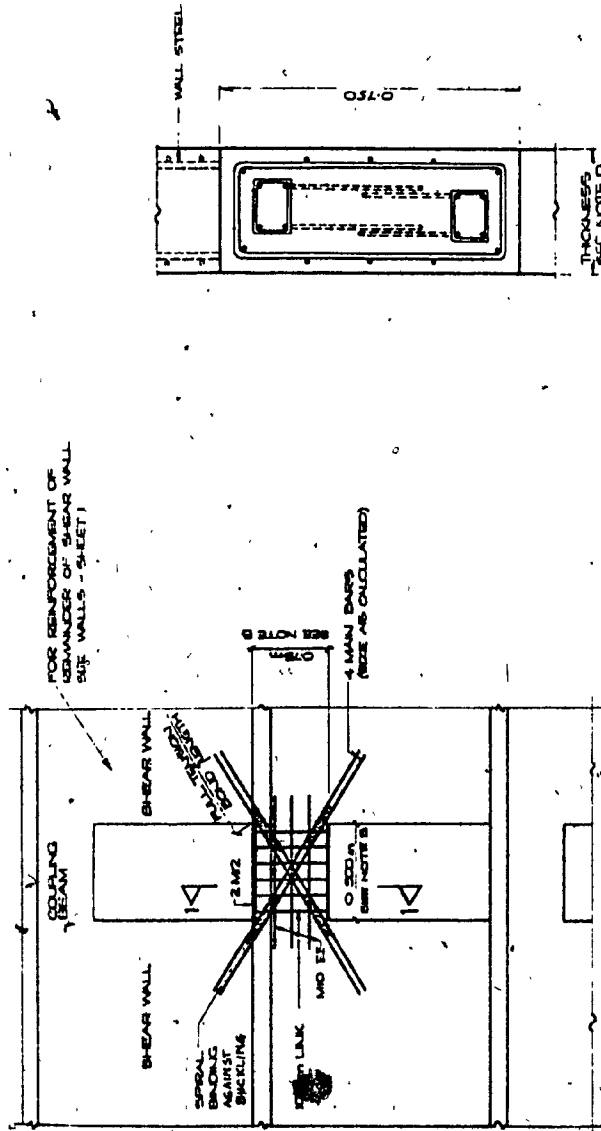


Figure 4.22 The crack pattern after failure of reinforced coupled shear wall models with (a) conventionally, (b) diagonally reinforced coupling beams



**HIGH-STRENGTH COUPLING BEAM**

**NOTES**

- A TO BE USED WHERE COUPLED SHEAR WALL ACTION IS ESSENTIAL
- B THIS DETAIL IS APPROPRIATE FOR DEEP COUPLING BEAMS OF SHEAR SPAN/DEPTH RATIO TO THAT SHOWN
- C CARE IS NECESSARY IN DETAILING TO AVOID THE STEEL FRUITS AND CONCRETING PROBLEMS
- D USE WITH WALLS THINNER THAN 0.30M IS NOT RECOMMENDED

Figure 4.23 Typical details for coupling beams



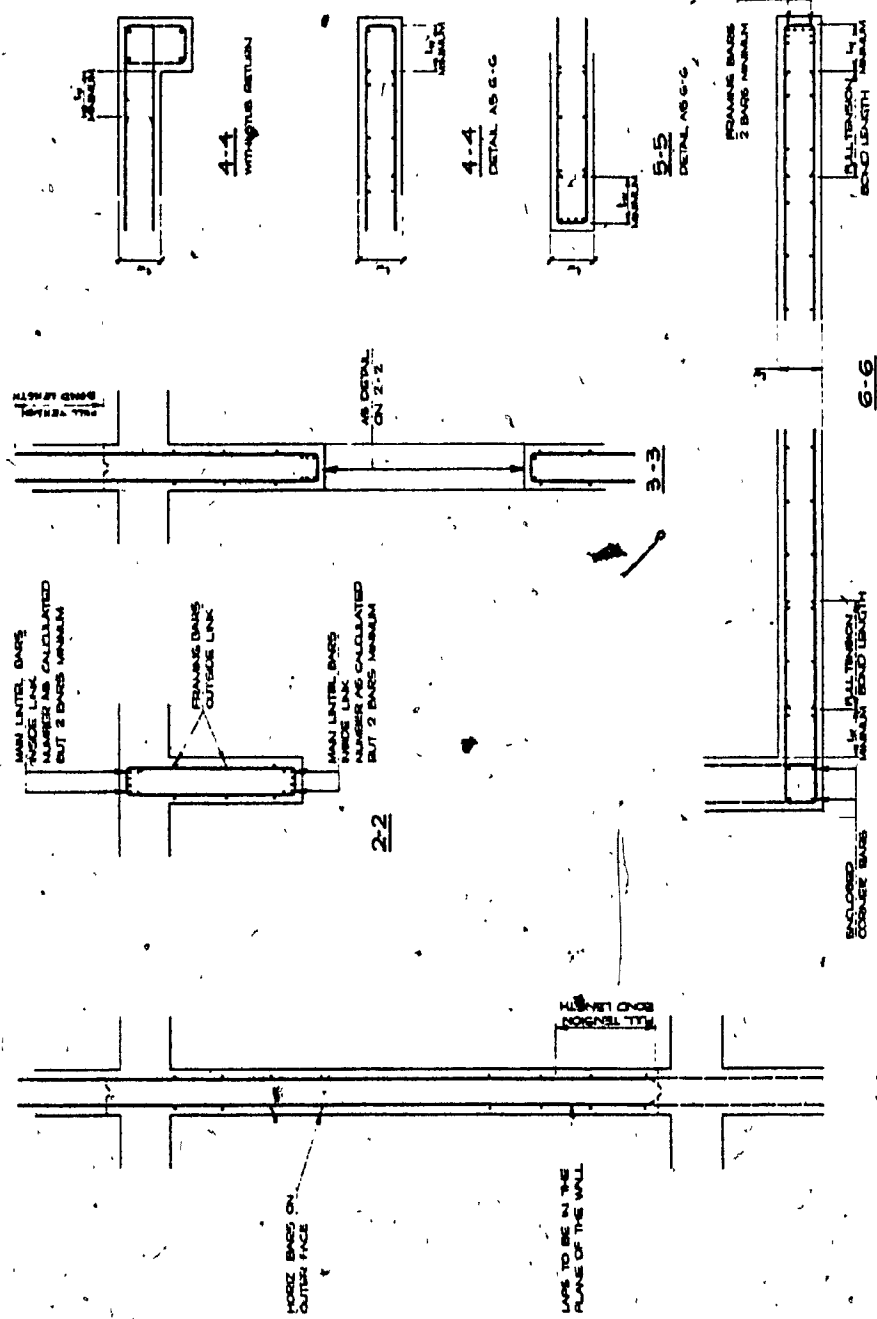
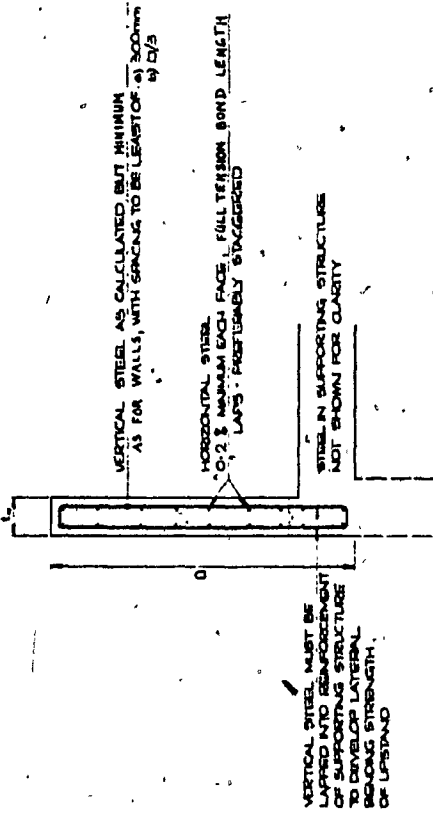


Figure 4.25 Typical details for walls



- NOTES**
- A.  $\phi$  IS THE DIAMETER OF THE BAR CONCERNED
  - B. IF UPSTAND OR PARAPETS BEIN LONGITUDINALLY THEN DETAIL AS FOR BEAMS
  - C. VERTICAL STEEL PREFERABLY IN FORM OF LANKS BUT IF HEIGHT OF LEG IS EXCESSIVE USE TWIN U BARS



**UPSTANDS AND PARAPETS**

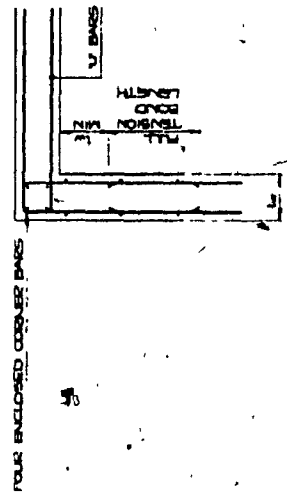


Figure 4.27 Typical details for upstands and parapets

# Chapter 5

## DETAILS FOR PRESTRESSED AND PRECAST CONCRETE

### INTRODUCTION

Prestressed and precast concrete structures have given mixed performance in earthquakes, difficulties mainly being experienced at connections between members. Most prestressed concrete elements, when designed for loading reversals, perform well in earthquakes. The failures that have occurred have been due mainly to failures of the supporting structures or connections<sup>22</sup>.



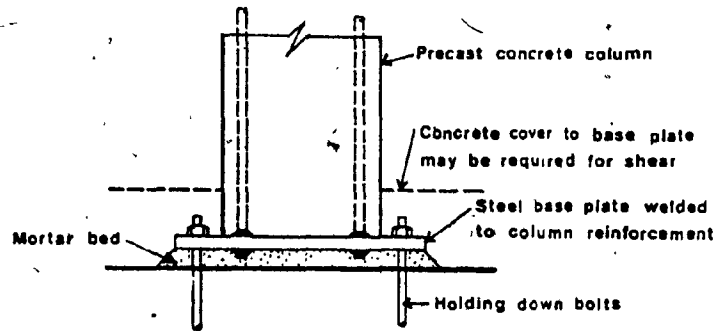
Basic principles in the design and detail of precast and prestressed concrete structures are the same as those for cast-in-place concrete structures. However, for precast structures, the methods of connection and ductility present special problems not always encountered in cast-in-place concrete. In order to overcome the connection problem, partial precasting is often done. For example, precast beams may be used with cast-in-place columns, or precast walls may be used with cast-in-place floors, or vice versa.

As the basic detailing of reinforced concrete has been discussed earlier in this report, only the essential problem of connection is considered in this chapter\*.

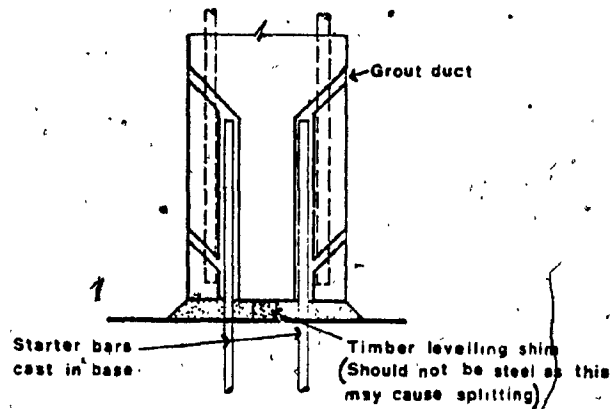
The following details must be individually designed for the forces acting on the joint. Variations may be made to suit the circumstances. Member reinforcement not shown for reasons of clarity.

\*It is not the intention of this chapter to present a complete review of of prestressed and precast concrete structures. Only some essential connection details, based on the California Code, will be presented. For a more complete treatment of the subject, the interested reader is referred to References 7 and 23.

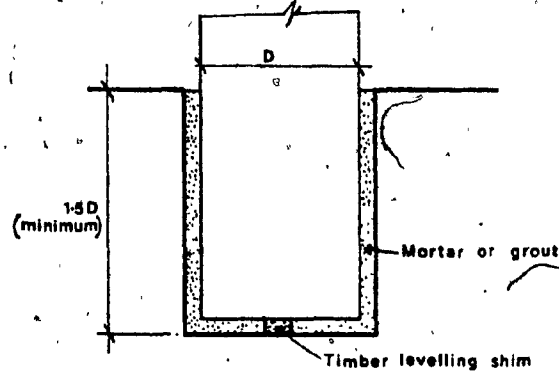
## CONNECTIONS BETWEEN BASES AND PRECAST COLUMNS



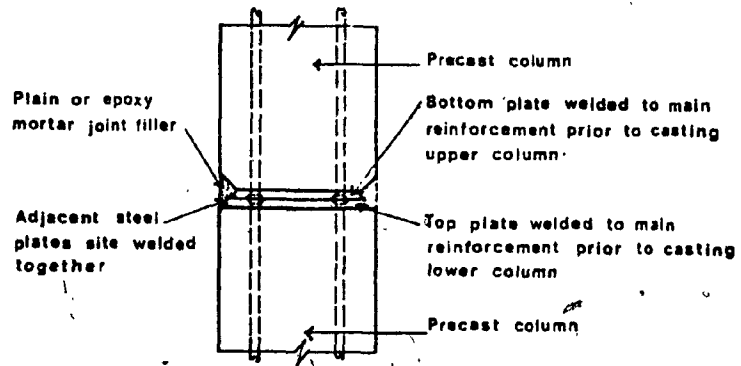
Detail 5.1 Site bolted. Moment transfer controlled by base plate



Detail 5.2 Site grouted. Effectiveness depends on grouting

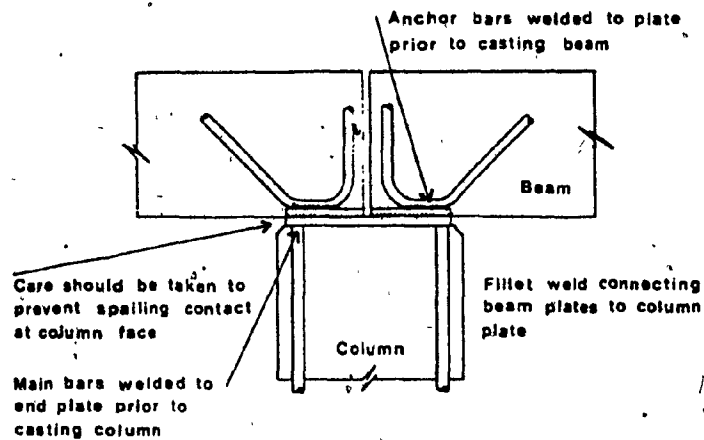


Detail 5.3 Site grouted. Best all-round joint of this type. Method of transfer of vertical load to base must be checked.

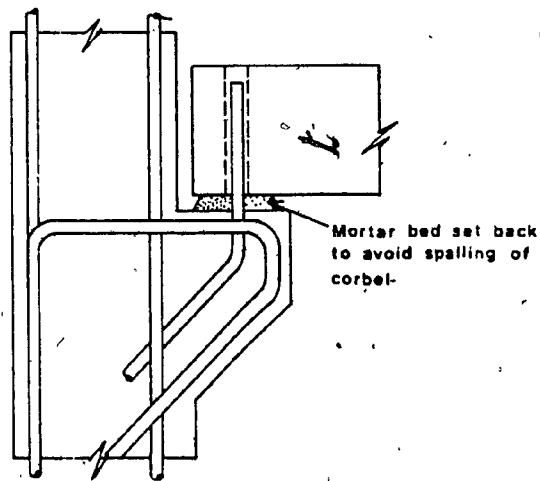


Detail 5.4 Site welded

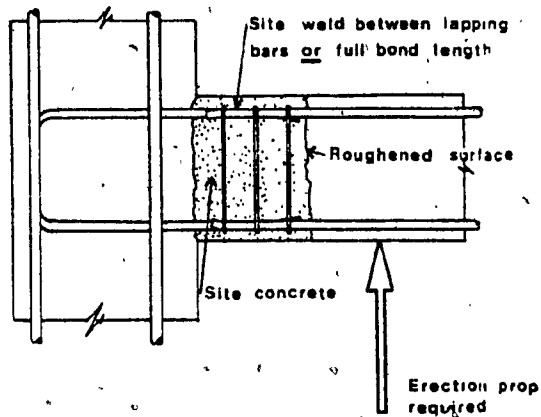
CONNECTIONS BETWEEN PRECAST COLUMNS AND BEAMS



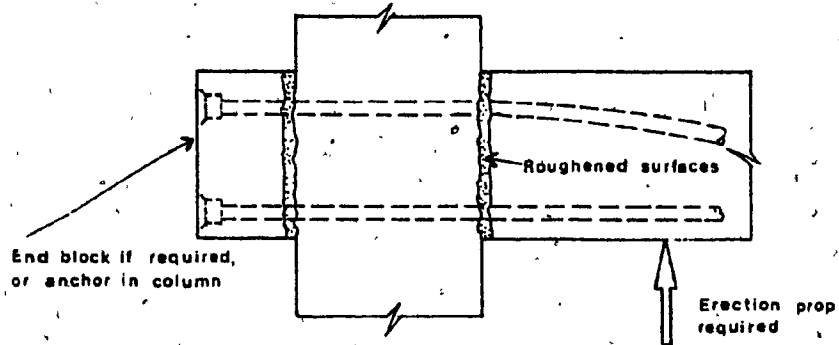
Detail 5.5 Site welded. Low moment capacity



Detail 5.6 Site grouted. Low moment capacity, poor in horizontal shear

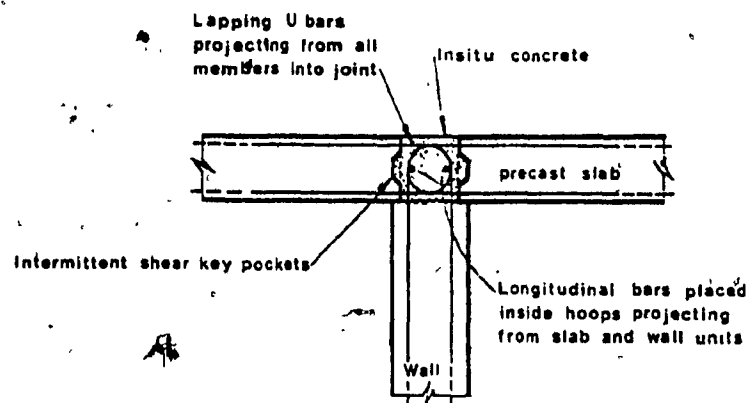


Detail 5.7 Site concreted and welded and links fixed

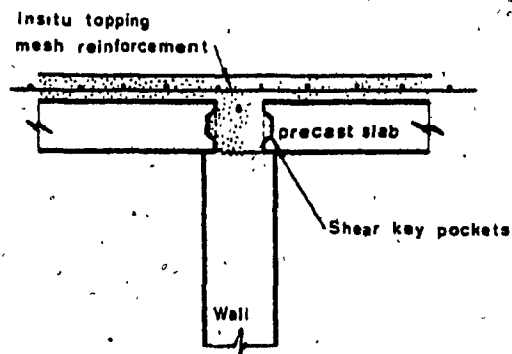


Detail 5.8 Site mortared and post-tensioned

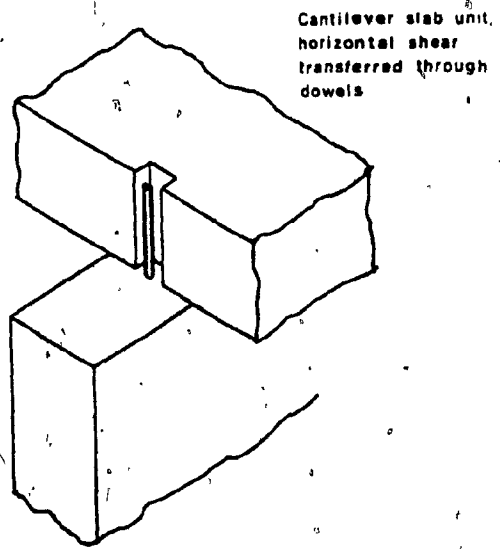
## CONNECTIONS BETWEEN PRECAST FLOORS AND WALLS



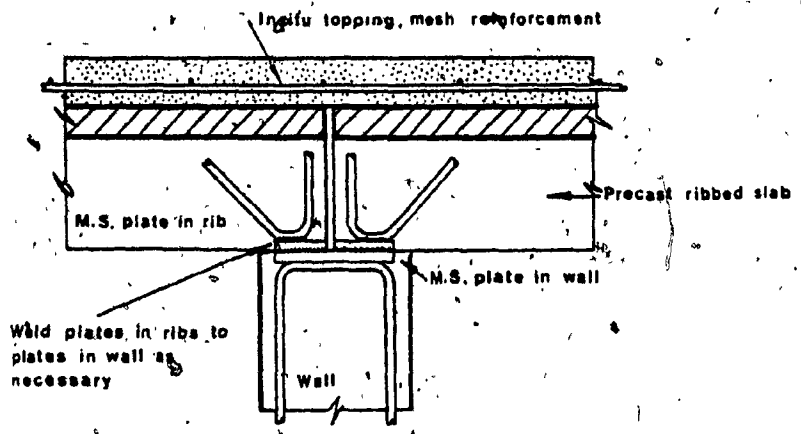
Detail 5.9 Site concrete and reinforcement



Detail 5.10 Site concrete and reinforcement

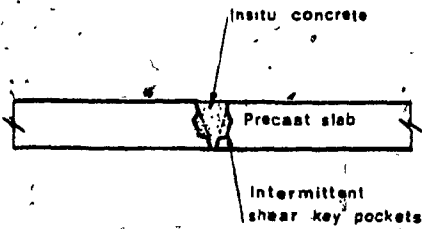


Detail 5.11 Site grouting

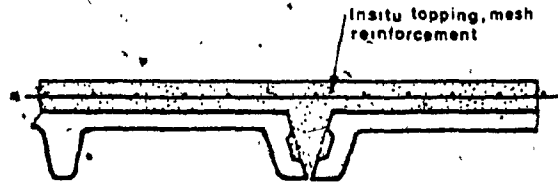


Detail 5.12 Site concrete, reinforcement and welding

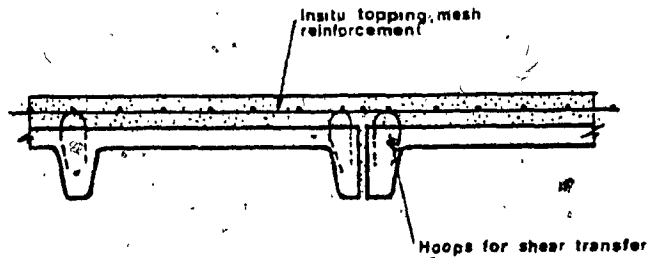
## CONNECTIONS BETWEEN ADJACENT PRECAST FLOOR AND ROOF UNITS



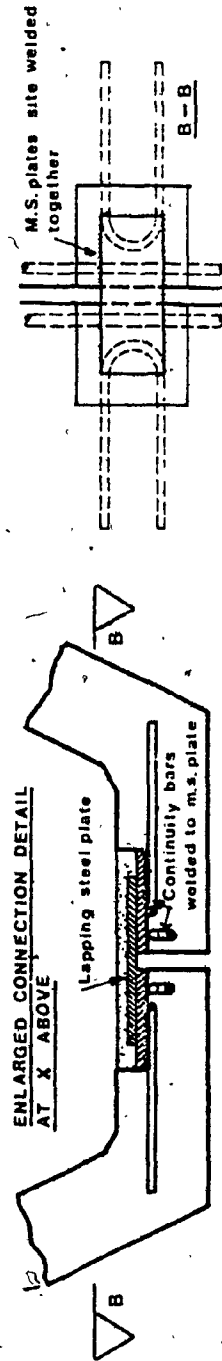
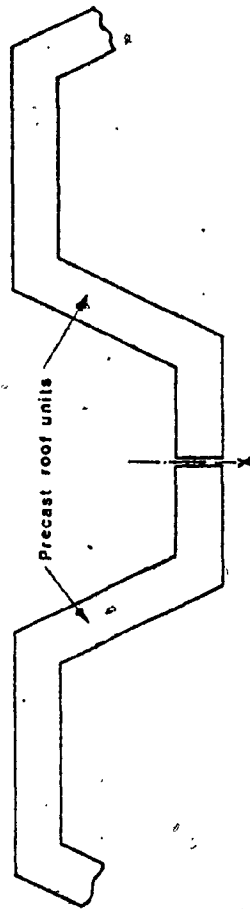
Detail 5.13 Site concreting. This joint depends on perimeter reinforcement to complete shear transfer system



Detail 5.14 Site concreting and reinforcing



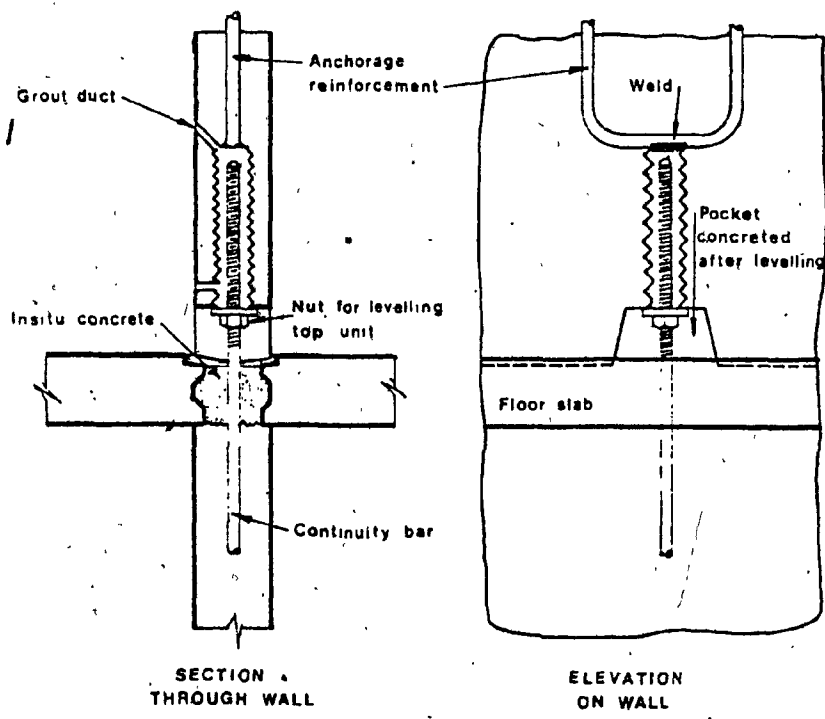
Detail 5.15 Site concreting and reinforcing



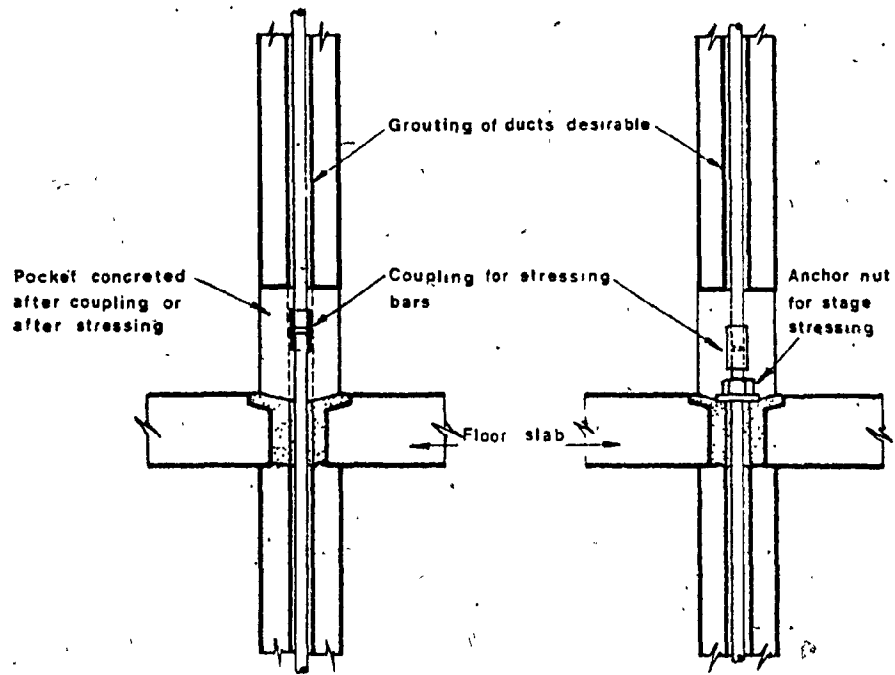
Detail 5.16 Site welding and mortaring. Lapping steel plate bent on site to suit differential camber of adjacent precast units



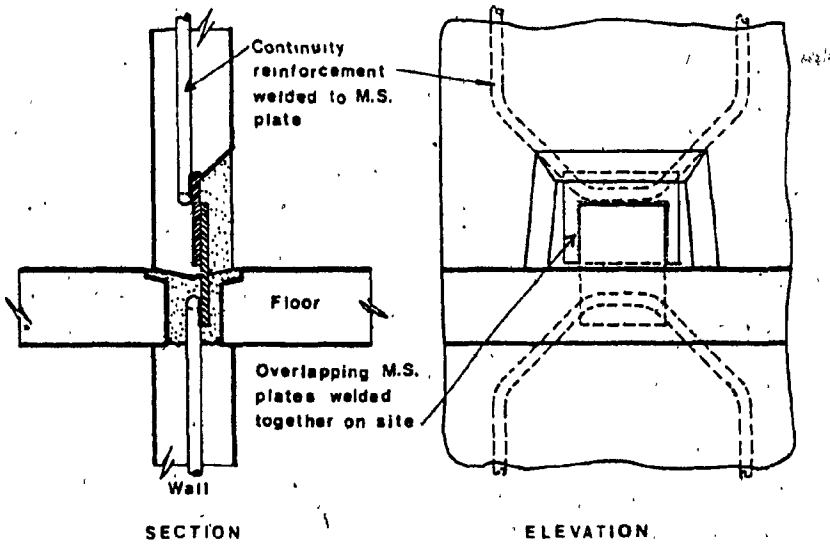
### CONNECTIONS BETWEEN ADJACENT PRECAST WALL UNITS



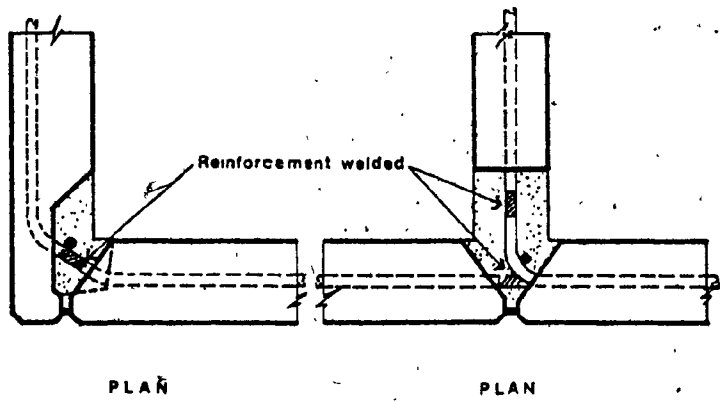
Detail 5.17 Site concreting and grouting



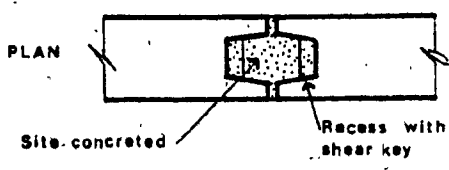
Detail 5.18 Site concreting and post-tensioning and grouting of ducts



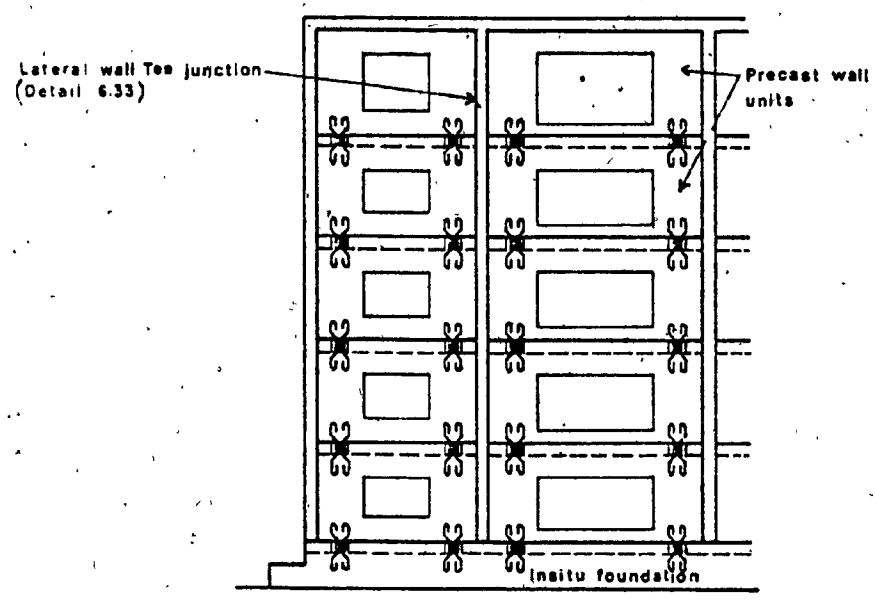
Detail 5.19 Site welded and concreted



Detail 5.20 Site welded and concreted



Detail 5.21 Site concreted



Detail 5.22 Layout of joints in wall elevation

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