DISEQUILIBRIUM MODELS AND SWITCHING REGRESSIONS: A SURVEY AND A CASE STUDY

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ABSTRACT

DISEQUILIBRIUM MODELS AND SWITCHING REGRESSIONS: A SURVEY AND A CASE STUDY

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This thesis analyzes several econometric techniques for dealing with switching regressions and disequilibrium models. Formulations designed to produce maximum likelihood estimates as well as other approaches are examined. The particularities and similarities among these techniques are pointed out and the underlying assumptions are discussed. The applications and results related to these methods are compared and evaluated. The estimation of an inter-city travel demand for passenger as a specific application of the disequilibrium techniques is then proposed.

RESUME

Plusieurs techniques économétriques pour estimer les régressions à changement de régimes et les modèles de déséquilibre sont analysées dans cette thèse. Les formulations destinées à produire des estimations par la méthode du maximum de vraissemblance ainsi que d'autres approches sont examinées. Nous prenons soin de relever les particularités et similitudes parmi ces techniques et d'en discuter les hypothèses sous-jacentes. Nous donnons aussi une comparaison et une évaluation des applications et résultats reliés à ces méthodes. Nous proposons ensuite une estimation de la demande inter-ville de transport de passager en tant qu'application particulière des techniques d'estimation des modèles de déséquilibre.

To my friend, Dr. Constantin Stamos

ACKNOWLEDGEMENTS

A thesis involves the efforts and thinking of several individuals. Some of the criticisms are incorporated while others are not. It is not possible to acknowledge every single person's contribution.

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This thesis is dedicated to my friend, philosopher and guide, Dr. Constantin Stamos, Director, Technical Education, Ministry of Education and Religious Affairs, Greece. He has been more than a friend or an inspiration.

I am truly indebted to the many others whom it has not been possible to acknowledge by name.

Responsibility for the general interpretations offered in this thesis and whatever errors and inconsistencies it contains, is mine alone.

TABLE OF CONTENTS

LIST OF SYMBOLS

α

β

γ

δ

Δ

ε

θ

Θ

λ

μ

ξ

π

Π

ρ

σ

τ

ψ

The above symbols are explicitly defined within the text whenever they occur.

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

The idea that markets may not clear, that is, may persist in a state of disequilibrium, has recently been considered as a central part of one reinterpretation of Keynes' theory. This suggests an inquiry on the possibilities of testing empirically these "new" theories, and whether it is possible to derive econometric methods which will be compatible with the new disequilibrium concepts. Until recently, most of the estimation of market demand and supply models has been carried out on the assumption that the market clears in each time period. The market model is thus usually formulated as

(1.1) Demand Equation: $D_t = D(P_t, x_{1t}) + u_t^D, t = 1,\ldots,T$

(1.2) Supply Equation: $S_t = S(P_t, x_{2t}) + u_t^S, t = 1,\ldots,T$

(1.3) Market Equilibrium Equation: $Y_t = D_t = S_t \quad t=1,\ldots,T$

where D stands for demand, S stands for supply and Y is the quantity transacted in the market. Disequilibrium models assume that (1.3) does not hold for some t $(t = 1,\ldots,T)$. The crucial assumption of the equilibrium models that prices adjust each period to equate demand and supply is therefore relaxed. A disequilibrium model of a market is more general in that it assumes every observation on the quantity traded in the market is generated either by the demand regime or the supply regime or both. Any specification or estimation techniques developed for models of markets in disequilibrium should also allow the testing of whether the system is in fact out of equilibrium and in that respect such methods are more general than existing equilibrium approaches, which do not allow this possibility. When demand and supply are not in equilibrium it is assumed that the "short side" of the market dominates (see Figure 1), i.e.

(1.4) $y_t = \min(D_t, S_t)$

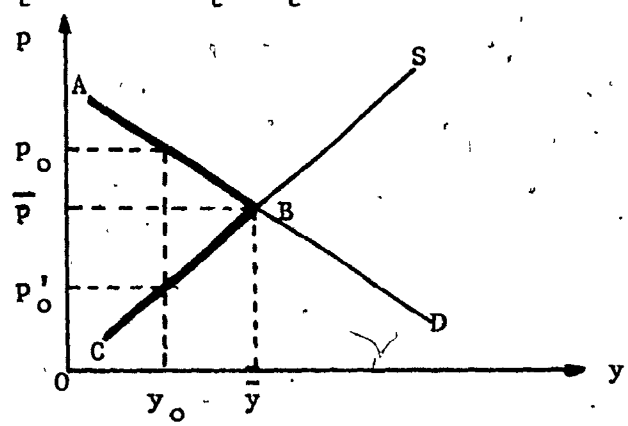


Figure 1

Although equation (1.4) has certain shortcomings (see Solow and Stiglitz (1968)) it is considered a reasonable description of the functioning of actual markets. A closed disequilibrium model is therefore defined as consisting of equations (1.1), (1.2) and (1.4). The equilibrium point B is denoted $(\bar{P}, \bar{y})$. If price is currently P_o the quantity transacted y_o is the resulting transacted quantity, determined by the quantity people wish to buy at this high price, rather than the larger quantity producers wish to sell. The shaded locus ABC is the "short side" of the market, the locus of transacted quantities corresponding to the range of prices. The whole idea behind this model is that consumers cannot be sold more than they want, and that not more can be sold than is produced. As illustrated in Figure 1, unlike the one-to-one relationships, the inverse mapping from y to p does not have a single value, y_o for instance may be associated with either p_o or p'_o and that causes estimation difficulties which are extensively discussed in Chapter 3.

If data were available on both the quantity demanded and the quantity supplied (and the exogenous variable, of course), then the estimation of the parameters of the demand and supply schedules within a disequilibrium framework would present no new problems; both equations can be estimated separately using familiar estimation techniques. It is normally the case that data is available only on the quantity traded so that the use of conventional techniques will induce inconsistent parameter estimates. Thus, the quantities traded and observed may either belong to the demand regime or the supply regime and it is usually unknown, a priori, under which regime the data is generated. A readily available indicator of the applicable regime might be the change in prices. Standard estimation techniques appropriate for equilibrium models in estimating disequilibrium models yield inconsistent parameter estimates.

Any realistic specification for quantities in disequilibrium may have to be highly nonlinear and in that respect a topic closely related to disequilibrium models is that of switching regressions. It may sometimes be reasonable to assume that the data on the dependent variable is generated by two separate regimes. Thus, instead of working with the familiar linear model of the form

$$(1.5) \quad y_t = \sum_{h=1}^{k} \beta_h x_{ht} + u_t, \; t = 1,\ldots,T$$

it may be plausible to assume that a subset I_1 of the T observations on y_t came from a regime

$$(1.6) \quad y_t = \sum_{h=1}^{k} \beta_{1h} x_{ht} + u_{1t}, \; t \in I_1$$

and the complementary subset I_2 of the observations came from a regime

$$(1.7) \quad y_t = \sum_{h=1}^{k} \beta_{2h} x_{ht} + u_{2t}, \; t \in I_2$$

If observations on y_t are thought to have been generated in this way, then it is an example of switching regressions. Note that (1.6) and (1.7) are very general formulations in which variables can be made specific to either regime by setting their coefficients in the other regime equal to zero. We may also consider regression models in which the parameters do not have fixed values. Such parameter variations could arise if the econometric relationships we estimate are derived from the maximization (minimization) behavior of economic agents involving some policy variables. In that case, the economic agents would be taking these policy variables into account in their decisions and hence the variables would be entering the model as determinants of the parameters. In general, parameter variations might change the standard model by allowing for an infinite number of possible parameter values. But it is also possible that the number of parameter changes is finite in which case we may call each possible state of the parameter vector regime. In time series applications these regimes may be associated with the state of the business cycle. In cross-section studies different regimes may be said to hold for behavior units with different characteristics (e.g., income, sex). In each case the switch among regimes may occur as the result of either deterministic phenomenon or some random mechanisms. A variety of techniques may be applied to such switching regressions either to estimate the different sets of parameters or to test the hypothesis that no switch took place.

The problem of switching regressions was first studied by Quandt (1958) who considered the case of a single switch between two regimes. It was only recently that the more general problem of multiple switching between regimes received attention. The extension in this direction is primarily due to Quandt (1972, 1975) and Goldfeld and Quandt (1972, 1973, 1976), but it was influenced by a paper by Fair and Jaffee (1972), in which it was pointed out perhaps for the first time, that estimation of demand and supply schedules in disequilibrium was an extension of a single switch to a multi-switch regression problem. The importance of the Fair and Jaffee paper, however, lies in its seminal contribution to the estimation of disequilibrium models. It has inspired an interest in the subject which was very long overdue. Many important contributions have now been made in this area, notable among them is a paper by Maddala and Nelson (1974). These improved estimation techniques have been applied to actual or experimental data by Fair and Jaffee (1972), Maddala and Nelson (1974), Goldfeld and Quandt (1975), Rosen and Quandt (1978), Laffont and Garcia (1977), and Portes and Winter (1980).

The purpose of this study is first to survey the estimation techniques for switching regressions and disequilibrium models and discuss some applied work with disequilibrium models. Secondly, we propose to use the disequilibrium approach to estimate an inter-city travel demand for passenger under the fixed supply assumption. Study of the fixed supply case is of interest because despite its inherent simplicity, it suffices to capture most of the important features of the disequilibrium estimation problems.

In Chapter 2 we outline the techniques for estimating switching regressions. In Chapter 3 disequilibrium models and their estimation are discussed. Chapter 4 is on the applications of various disequilibrium estimation techniques to actual or experimental data. Chapter 5 deals with a disequilibrium demand for passenger transport. General conclusions are drawn in Chapter 6.

CHAPTER 2

TECHNIQUES FOR ESTIMATING SWITCHING REGRESSIONS

One of the earliest contributions on the subject of switching regressions is by Quandt (1958). He considers the case of a once-and-for-all shift from regime 1 to regime 2 and proposes a maximum likelihood method for the estimation of the parameters in the two regimes. Fair and Jaffee (1972) and Goldfeld and Quandt (1973) discuss further modification of this switching regression model to cases where there is a continuous switching back and forth. Goldfeld and Quandt also consider deterministic and stochastic switching models and a model which is a mixture of two normal distributions. They suggest estimating this model under the assumption that the residuals have a common variance. But problems arise when no a priori information is available concerning the variance of a normal mixture. Quandt and Ramsey (1978) have proposed a method known as the moment generating function (mgf) method for estimating such switching regression models. We start first by outlining Quandt's method and then go on to discuss the more recent contributions.

The Switch Among Regimes

Consider relationships of the form (1.6) and (1.7) which for notational convenience are rewritten as:

(2.1) $y_i = x_i'\beta_1 + u_{1i}$ $\quad i \varepsilon I_1$

(2.2) $y_i = x_i'\beta_2 + u_{2i}$ $\quad i \varepsilon I_2$

where $x_i' = (x_{1i} \ldots x_{ki})$ is the vector of observations at time i, I_1 and I_2 are the sets of indices for which the two different regression equations hold and $\beta_h = (\beta_{h1} \ldots \beta_{hk})$ h = 1,2 is the vector of unknown coefficients. It is assumed that u_{1i} and u_{2i} are distributed normally with mean zero and variances σ_1^2 and σ_2^2 respectively; the error terms are also assumed to be distributed independently of each other and their own lagged values.

A single switch problem is considered where the first t observations are generated by equation (2.1) and the last T-t by equation (2.2). The problem is to estimate β_1, β_2 and t, where t is the point at which the system switches from one regime to another.

Quandt suggests a maximum likelihood method for estimation of parameters and the switching point. The density of the observable random variable y is obtained from the density of the error terms. The first t observations on y are distributed normally with mean $x_i'\beta_1$ and variance σ_1^2 and the last T-t observations are normally distributed with mean $x_i'\beta_2$ and variance σ_2^2. The likelihood function for the entire sample is therefore

(2.3) $$L = \left(\frac{1}{\sqrt{2\pi}\,\sigma_1}\right)^t \left(\frac{1}{\sqrt{2\pi}\,\sigma_2}\right)^{T-t} \operatorname{Exp}\left[\frac{-1}{2\sigma_1^2}\sum_{i=1}^{t}(y_i - x_i'\beta_1)^2 - \frac{1}{2\sigma_2^2}\sum_{i=t+1}^{T}(y_i - x_i'\beta_2)^2\right]$$

Estimates $\hat{\beta}_1$ and $\hat{\beta}_2$ are obtained by maximizing the likelihood function (or the log likelihood function) with respect to the parameters. To obtain estimators $\hat{\sigma}_1$ and $\hat{\sigma}_2$ the likelihood function is first maximized with respect to σ_1 and σ_2 and $\hat{\sigma}_1$ $\hat{\sigma}_2$ are substituted out and the function is maximized with respect to β_1 & β_2. Then in the resulting expressions $\hat{\beta}_1$ and $\hat{\beta}_2$ are substituted for β_1 and β_2. Substituting $\hat{\sigma}_1$ and $\hat{\sigma}_2$ into the log likelihood function provides the following expression

(2.4) $$\log L = -T \log \sqrt{2\pi} - t \log \hat{\sigma}_1 - (T-t) \log \hat{\sigma}_2 - \frac{T}{2}$$

where

$$\hat{\sigma}_1 = \frac{\sum_{i=1}^{t}(y_i - x_i'\beta_1)^2}{t} \quad \text{and} \quad \hat{\sigma}_2 = \frac{\sum_{i=t+1}^{T}(y_i - x_i'\beta_2)^2}{T - t}$$

To obtain maximum likelihood estimate of t, differentiation techniques are not useful since t is not continuous. Quandt suggests trying all possible values of t and selecting the value of t for which (2.4) is a maximum maximoram.

Fair and Jaffee (1972) have extended the above problem to the case where the system switches back and forth a number of times from one regime to another; the exact number of switches being unknown. In total there are m observations generated by the first régime and n observations by the second. Both m and n are unknown and m + n = T. The demand and supply equations (1.1) and (1.2) can be considered as an example of this multi-switch problem. For this problem an expression analogous to (2.4) is given by:

$$(2.5) \qquad \log L = -(m + n) \log\sqrt{2\pi} - m \log \hat{\sigma}_1 - n \log \hat{\sigma}_2 - \frac{m + n}{2}$$

The problem is to divide observations into regimes 1 and 2 and obtain corresponding estimates $\hat{\beta}_1$ and $\hat{\beta}_2$ such that the log likelihood function (2.5) is a maximum. To obtain a maximum of (2.5) all possible sample separations into regimes 1 and 2 have to be tried. There are 2^T possible ways of doing this and therefore for any reasonable value of T it will be impossible to carry out all the calculations. In their study of a disequilibrium model of the U.S. Housing market, Fair and Jaffee (1972) tried a number of algorithms to implement the likelihood method. Their attempts were not very fruitful for none of their algorithms seemed to lead to a global maximum.

They conjecture that it may be due to a not well behaved likelihood function and conclude, "Unless better algorithms than those considered in this study can be found or unless the likelihood function is better behaved for other examples, the maximum likelihood techniques does not appear to be of much practical use". We may note in passing that the authors have claimed that the procedure outlined above provides consistent estimates. The proof to which they refer assumes that the likelihood function is derived from independently and identically distributed random variables. It is now well known that since $(Y_1, Y_2 \ldots Y_T)$ are not identically distributed, the usual properties of maximum likelihood estimators cannot be invoked for estimators obtained by maximizing a likelihood function of the form (2.3). As far as we are aware the proof of consistency of estimators proposed by Fair and Jaffee has not been rigorously derived. A possible proof may follow along the lines of the consistency proof suggested by Amemiya (1973) for MLE for regression models when the dependent variable is normal but truncated to the left of zero. Another proof is given by Kiefer (1978b). We feel compelled to give here Amemiya's proof since it is one of the few attempts to derive consistency properties for MLE for regression models.

Amemiya considers the regression model defined by:

(2.6) $y_t = x_t'\beta_1 + u_{1t}$ if the right hand side (RHS) is > 0

(2.7) $\quad = 0$ if RHS ≤ 0 with $t = 1, 2, \ldots T$.

and u_{1t} is independent with distribution $N(0, \sigma_1^2)$. The problem here is how to estimate β_1, and σ_1^2 on the basis of observation $(y_1, y_2, \ldots, y_T)$. It should be noted here that the model was first studied by Tobin (1958) who defined the maximum likelihood estimator, or more precisely a root of the normal equations and proposed an iterative procedure starting from a certain initial estimator. But Tobin did not prove the consistency and the asymptotic normality of the maximum likelihood estimator he proposed and it appeared that the initial estimator he proposed is not consistent. Amemiya assumes that first $\theta = (\beta, \sigma^2)$ and $\theta_1 = (\beta_1, \sigma_1^2)$. The parameter space θ is compact and does not contain the region $\sigma^2 \leq 0$ and contains an open neighborhood of θ. Second, x_t is unbounded and the empirical distribution function G_n defined by $G_n(x) = j/n$, where j is the number of points $x_1, x_2 \ldots, x_n$ less than or equal to x converges to a distribution function (G). Third, $\lim_{T\to\infty} (\frac{1}{T}) \sum_{t=1}^{T} x_t x_t'$ is positive definite.

We can now define the likelihood function as

$$(2.8)\quad L = \prod_{i\varepsilon I_1} \left[1 - F(\beta' x_t, \sigma^2)\right] \prod_{i\varepsilon I_2} \frac{1}{\sqrt{2\pi}\sigma} \operatorname{Exp}\left\{\frac{-1}{2\sigma^2}(y_t - \beta' x_t)^2\right\}$$

defined over θ where

$$F(\beta' x_t, \sigma^2) = \int_{-\infty}^{\beta' x_t} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\lambda/\sigma^2)} d\lambda$$

and I_1 is the subset (1, 2, ..., T) such that $y_t = 0$ for $t\varepsilon I_1$, and I_2 is its complement. Taking the logarithm of the above likelihood function we can easily get the first and second derivatives. Furthermore, let u^*_{1t} be the random variable with the density

$$(2.9)\quad h(\lambda) = \frac{1}{F_{1t}} \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{1}{2}(\lambda/\sigma_1)^2} \quad \text{where } -\beta_1' x_t < \lambda < \infty$$

$$= 0 \qquad \text{elsewhere}$$

with $F_{1t} = F(x'_t \beta_1, \sigma_1^2)$. Then we have

(2.10) $y_t = x'_t \beta_1 + u^*_{1t}$ for $t \in I_2$

Therefore, the conditional moments of y_t, given $t \epsilon I_2$ can be easily calculated from the moments of u^*_{1t} and since F_{1t} can never be nil by the second assumption, all the moments of u^*_{1t} are bounded uniformly in t. Then, by Amemiya's four lemmas, it follows that the normal equations $\frac{\partial \text{ Log } L^{(1)}}{\partial \Theta} = 0$ where $\Theta = (\beta\ \sigma'^2)$ have a strong consistent root. $\hat{\Theta}$ is a root of the normal equations if it is a solution of the normal equations corresponding to a local maximum of the logarithmic function. Thus $\frac{\partial^2 \log L(\hat{\Theta})}{\partial \Theta\ \partial \Theta'}$ is a negative definite matrix. It is argued that the second and third assumption will practically ensure the asymptotic normality of the estimator. However, generally, the normal equations have multiple roots and it is difficult in practice to know which is consistent. What if there is more than one consistent root it will be impossible to attain the convergence of the iteration of the global maximum. Yet although we may attain convergence of the iteration algorithm it is not guaranteed that a root for the initial estimate is consistent.

We may also mention at this stage that the maximum

(1) Note that $\frac{\partial \text{Log} L}{\partial \theta}$ are non-linear and a root of the normal equations can be found only by an iterative procedure.

likelihood technique assumes that any observation on the dependent variable can be classified into regime 1 or regime 2 but not in both. Sometimes prior information may be available which may suggest that certain observations are equilibrium values. In this case the appropriate model may have to be reformulated to incorporate this information and a different likelihood function derived. The problem of classification of equilibrium observations will be taken up in Chapter 4.

2 The Use of Extraneous Information

The method we have discussed above does not use extraneous information to classify observations as demand points or supply points. Such information is frequently available and it would be wasteful not to utilize such information. Goldfeld and Quandt (1972) have suggested a method of estimating switching regressions using information on an extraneous variable Z. It is assumed that there is a cut-off point z_o such that if $Z \leq z_o$ the observations belong to regime 1 and if $Z > z_o$ the observations belong to regime 2. The cut-off point may be known a priori in which case estimation is straightforward or it may be unknown, in which case it is a parameter to be estimated from the data. As an example, consider relating industrial profits to structural characteristics within the industry like barriers to entry, advertising, the extent of import competition etc.[1] There are reasons to believe that for a cross section of industries this relationship is not linear; that profits are generated by two separate regimes and that industries can be classified into regime 1 or 2 depending upon whether they fall below or above a critical level of an appropriately defined concentration ratio z_o. Industries which fall below z_o realize normal competitive profits and those above it enjoy oligopolist profits. The reason why two regimes are posited is the presumption that firms display qualitatively different behavior as they realize the advantages of cooperation as against competition. Such cooperative behavior, however, isfeasible only after a critical concentration level is reached. The problem is to estimate the two regimes and the switching point z_o. Many writers on the subject have worked with an arbitrary value of z_o but it can be treated as a parameter and estimated from the data using the procedure developed below.

Define a variable D where D = o if $Z \leq z_o$ and D = 1 if $Z > z_o$. Then multiplying (2.1) by $(1 - D_i)$ and (2.2) by D_i and adding we obtain an alternative expression for equation (2.1) and (2.2),

[1]This example is taken from L. J. White: "Searching for the Critical Industrial Concentration Ratio: An Application of the 'Switching of Regimes' Technique", in Goldfeld and Quandt (1976) Chapter 3.

$$(2.11) \quad y_i = x_i'\left[(1 - D_i)\ \beta_1 + D_i\beta_2\right] + (1 - D_i)\ u_{1i} + D_i u_{2i}.$$

A likelihood function for the sample can be derived making precise assumptions on the error terms but since values of D_i are not known (because z_o is not known a priori), any observation can take one of the two values, zero or one. There are 2^T possible ways of dividing T observations in this way. It follows that maximization of the likelihood function is beset with the same problems as mentioned in the discussion following (2.5). Fortunately in the present case the problem can be resolved by approximating the step function $D(z_i)$ by the cumulative normal integral

$$(2.12) \quad D(z_i) = \int_{-\infty}^{z_i} \frac{1}{\sqrt{2\pi}\ \sigma} \text{Exp}\left[-\frac{1}{2}\left(\frac{\zeta - \mu}{\sigma}\right)^2\right] d\zeta$$

The step function and its approximation are shown in Figure 2. If we make the same assumptions on the error terms that we have made previously then the variance of the error term in equation (2.11) is $\sigma_1^2 (1 - D_i)^2 + \sigma_2^2 D_i$. The appropriate likelihood function is

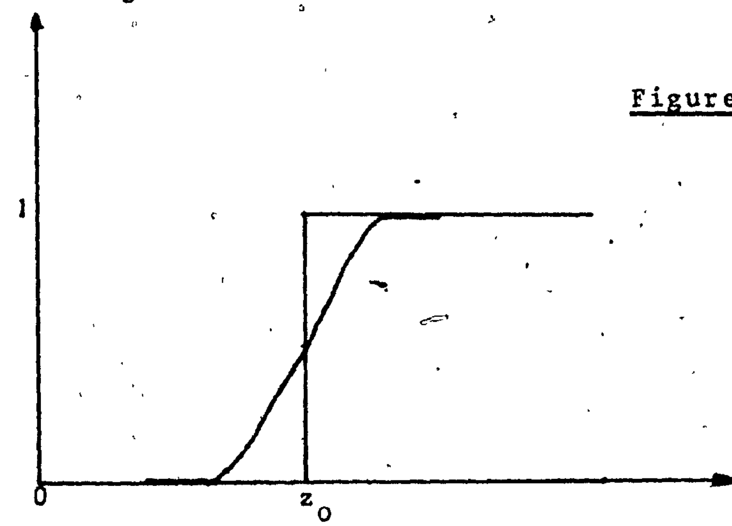
$$(2.13) \quad \log L = -\frac{T}{2} \log 2\pi - \frac{1}{2} \sum_{i=1}^{T} \log\left[\sigma_1^2 (1-D_i)^2 + \sigma_2^2 D_i^2\right]$$

$$- \frac{1}{2} \sum_{i=1}^{T} \frac{\{y_i - x_i'\left[(1 - D_i)\ \beta_1 + D_i\ \beta_2\right]\}^2}{\sigma_1^2 (1 - D_i)^2 + \sigma_2^2 D_i^2}$$

Substituting $D(z_i)$ from (2.12) for D_i in (2.13) and maximizing with respect to β_1, β_2, σ_1^2, σ_2^2, μ and σ provides the maximum likelihood estimates. The estimate μ would in this case be an estimate of the cut-off point z_o and would indicate the closeness of the approximation to the step function.

It may be mentioned that the above method which we will refer to as the D-method can be extended so that information on a set of variables $(Z_1 \ldots Z_S)$ may be used to obtain sample separation into regime 1 or 2. The method can also be extended to the case where switching takes place between more than two regimes.

Figure 2



We may note that the maximum likelihood method suggested by Quandt (1958) is a special case of the D-method where the extraneous variable is the time index i and the cut-off point is t which is estimated from the data.[1]

[1] This point is made in Goldfeld and Quandt (1973b).

The D-method relies on outside information to provide parameter estimates of demand and supply schedules. Such information may not always be available and even if available it may sometimes not be appropriate to use the D-method; (we shall come back to this latter point soon). If it can be assumed that nature selects regime 1 with probability λ and regime 2 with probability $1 - \lambda$, then it is possible to obtain parameter estimates without using such extraneous information. Quandt (1972) has proposed a maximum likelihood method when such an assumption can be made.

The same assumptions which allowed the derivation of (2.3) are made about the error terms to derive the conditional density of the i^{th} observation of y conditional on the values of the independent variables. Let I_1 and I_2 be the set of indices for which the observations belong to regime 1 and 2 respectively. Let $f_1(y_i | x_i)$ be the conditional density when $i \varepsilon I_1$ and $f_2(y_i | x_i)$ be the conditional density when $i \varepsilon I_2$; then the conditional density for the i^{th} observation is

$$(2.14) \quad h(y_i | x_i) = f_1(y_i | x_i) \Pr(i \varepsilon I_1) + f_2(y_i | x_i) \Pr(i \varepsilon I_2)$$

$$= \frac{\lambda}{\sqrt{2\pi}\sigma_1} \operatorname{Exp}\left[-\frac{1}{2\sigma_1^2}(y_i - x_i'\beta_1)^2\right]$$

$$+ \frac{(1-\lambda)}{\sqrt{2\pi}\sigma_2} \operatorname{Exp}\left[-\frac{1}{2\sigma_2^2}(y_i - x_i'\beta_2)^2\right]$$

The likelihood function for the entire sample is

$$(2.15) \qquad L = \prod_{i=1}^{T} h(y_i | x_i)$$

This function has to be maximized with respect to β_1, β_2, σ_1, σ_2, and λ. We will call this λ-method. As Quandt points out, the advantage of the λ-method over the D-method is that it does not require information on extraneous variables. Its weakness is that it does not identify a particular observation as a demand observation or a supply observation but computes the probability that it belongs to regime 1 or 2.

Sometimes the variable on which data segregation is based may itself be endogenous, in which case the D-method will provide inconsistent estimates.[1] Such an endogenous extraneous variable is discussed on page 42. The λ-method which does not depend on extraneous information is therefore preferable in this case.

The λ-method and the D-method can be combined by making probability λ depend on an extraneous variable Z. $D(z_i)$ as defined in (2.12) can now replace λ in the likelihood function (2.15). The latter is now maximized with respect to

[1] This remark may suggest that when the extraneous variable is exogenous the D-method yields consistent estimates. Although assertions to this effect are made in Goldfeld and Quandt (1973 a) there is no reference to an appropriate proof.

the previous parameters as well as the parameters appearing in the expression for $D(z_i)$.

Some experimental work done by Quandt suggests that the D-method performs slightly better than the λ-method as judged by the mean bias and the mean square error performance - the mean being taken over the number of successful replications[1] of an experiment - but the λ-method does better in terms of the ratio of the mean asymptotic variance to the mean square error. This ratio is close to unity for the method as the sample size is increased indefinitely but the convergence to unity for the D-method is slow. However, one disadvantage of the λ-method is that it does not allow individual observations to be identified with particular regimes which might explain its relative good performance. Although the appropriate but unknown criterion is small sample performance.

In the λ-method observations are treated as a set of independent trials. This assumption is now relaxed [(Goldfeld and Quandt (1973 a)], to make every trial depend on and only on the previous trial. A matrix T is defined whose rs^{th} $(r,s = 1, 2)$ element denotes the probability that the system will switch from regime r to regimes s. Let $\lambda_0' = (\lambda_{10}, \lambda_{20}) = (\lambda_{10}, 1 - \lambda_{10})$ be a vector whose first and

[1]An unsuccessful replication was defined as one for which the maximization algorithm failed to converge 'satisfactorily'.

second element represent the probability that the system is initially in regime 1 and 2 respectively and $\lambda_i' = (\lambda_{1i}, \lambda_{2i})$ be a vector defined similarly for the i^{th} trial. Then it is possible to write

$$\lambda_i' = \lambda_{i-1}' T$$

and using this relation recursively we obtain

$$(2.16) \qquad \lambda_i' = \lambda_0' T^i$$

The conditional density of the i^{th} observation on y is as in (2.14) except that λ is now replaced by λ_{1i}. Using the relation (2.16) the likelihood function can be written as a function of β_1, β_2, σ_1^2, σ_2^2, λ_{10} and elements of the matrix T (say) τ_{11} and τ_{22}. (Note the rs^{th} $r \neq s$ elements in T are simply $1 - \tau_{ii}$ $i = 1,2$). This method will be called the τ-method.

The τ-method can be extended by making probabilities τ_{ij} depend on some variable Z. If, for example, high values of Z were thought to be associated with high values of τ_{11} then $\tau_{11}(z_i)$ and $\tau_{22}(z_i)$ could be defined as

$$\tau_{11}(z_i) = \frac{1}{\sqrt{2\pi}\sigma_{\tau_{11}}} \int_{-\infty}^{z_i} \text{Exp}\left[-\frac{1}{2} \frac{(\zeta - Z_{11})^2}{\sigma_{\tau_{11}}^2}\right] d\zeta$$

and

$$\tau_{22}(z_i) = \frac{1}{\sqrt{2\pi}\sigma_{\tau_{22}}} \int_{Z_i}^{\infty} \text{Exp}\left[-\frac{1}{2} \frac{(\zeta - Z_{22})^2}{\sigma_{\tau_{22}}^2}\right] d\zeta$$

The likelihood function will now have to be maximized with respect to the previous parameters and the parameters appearing in the above expression for $\tau_{11}(z_i)$ and $\tau_{22}(z_i)$.

None of the methods discussed above has treated the case of serial correlation of disturbances. If we assume a first order Markov process for the error term

$$u_i = \rho u_{i-1} + \varepsilon_i$$

then more alternatives arise than in the case of the usual regression model, firstly because of the switching mechanism and secondly because we may want to approach the problem either with the D-method or the λ-method.

If the D-method is to be employed then one must assume that

$$u_{1i} = \rho_1 \left[(1 - D_{i-1}) u_{1i-1} + D_{i-1} u_{2i-1}\right] + \varepsilon_{1i}$$
$$u_{2i} = \rho_2 \left[(1 - D_{i-1}) u_{1i-1} + D_{i-1} u_{2i-1}\right] + \varepsilon_{2i}$$

where $\varepsilon_{1i} \sim N(0, \sigma_1^2)$ and $\varepsilon_{2i} \sim N(0, \sigma_2^2)$ and are independent of each other.

The equivalent assumption for the λ-method (with $\sigma_1^2 = \sigma_2^2$) is

$$u_{1i} = \rho_1 u_{1i-1} + \varepsilon_{1i} \qquad \text{with prob } \lambda^2$$
$$u_{1i} = \rho_1 u_{2i-1} + \varepsilon_{1i} \qquad \text{with prob } \lambda(1-\lambda)$$
$$u_{2i} = \rho_2 u_{2i-1} + \varepsilon_{2i} \qquad \text{with prob } \lambda(1-\lambda)$$
$$u_{2i} = \rho_2 u_{2i-1} + \varepsilon_{2i} \qquad \text{with prob } \lambda(1-\lambda)^2$$

The idea behind the assumption is that there are two autocorrelation coefficients, each associated with one of the regimes. Those coefficients are applied to the error term of the previous period, regardless of which regime that error term came from. Then, the appropriate likelihood functions can be derived and parameter estimates can be obtained by maximizing the likelihood function. Presentation of the likelihood functions is omitted here because their relative complexity needlessly burdens the task of analysis.

An alternative specification would be that if in period i regime 1 operates and in period i-1 regime 1 operated as well, the error term follows the usual first order auto-correlation process; if in period i-1 regime 2 operated showing that a switch took place, then a non-autocorrelated error term is generated and the patterns of the error term would be modified according to the method that one wishes to employ. The corresponding likelihood functions can again be derived and parameter estimates can be obtained but are also omitted here.

3 More Recent Developments

Recently, attention has been given to two other methods for estimating switching regressions. These are the Moments method and the Moment Generating Function (mgf) method. Both methods make the same assumptions that underlie equation

(2.14). The problem arises when no prior information is available concerning the variances of a finite mixture of two normal densities. In that particular case the likelihood function corresponding to such distributions is known to be unbounded and maximum likelihood estimation may break down in practice due to this unboundedness and the potential singularity of the matrix of second partials of the likelihood function. Quandt and Ramsey (1978) introduced the "moment generating function estimator" which is defined as the estimator which minimizes the sum of squares of differences between the theoretical and sample moment. Their method is a generalization of the Moments method which can be seen in the following framework borrowed from Kiefer (1978a). Given observations $x = (x_1,\dots,x_T)$ from a distribution with parameters $\gamma = (\gamma_t,\dots,\gamma_k)$, choose functions $g_j(x)$, $j=1,\dots,m$. and solve the system of equations

$$(2.17) \qquad \frac{1}{T} \Sigma\, g(x_t) = E\,(g(x)\,|\,\gamma)$$

For reasonable distributions and choice of function g, a strong law of large numbers will ensure consistency of $\hat{\gamma}_T$ estimated this way. For m >k the sum of squared distances between both sides of (2.17) can be minimized. Here, the "astuteness" in constructing a useful estimator is in choosing the form and number of the g_j.

In the case where x is generated by a normal mixture as

$$(2.18)\quad f(x) = \frac{\lambda}{\sqrt{2\pi}\,\sigma_1} \operatorname{Exp}\left[-\frac{1}{2\sigma_1^2}(x-\mu_1)^2\right] + \frac{1-\lambda}{\sqrt{2\pi}\,\sigma_2} \operatorname{Exp}\left[\frac{-1}{2\sigma_2^2}(x-\mu_2)^2\right]$$

the moments method consists of solving (2.17) with m = 5 and $g_j(x) = x^j$ for j = 1,...,5. The method considered by Quandt and Ramsey consists of choosing $g_j(x) = \operatorname{Exp}\theta_j x$ for j = 1, ...,m with m > k and solving (2.17) by minimizing the sum (over j) of squared deviations numerically. Clearly, many functions of g_j would suffice to produce consistent estimates. More particularly, consider the model

$$(2.19)\quad Y_t \sim N(x'\beta_1, \sigma_1^2) \quad t\in I_1 \quad Pr(t\in I_1) = \lambda$$

$$(2.20)\quad Y_t \sim N(x'\beta_2, \sigma_2^2) \quad t\in I_2, \quad Pr(t\in I_2) = 1-\lambda$$

Note here the absence of the subscript t in x. The pdf of the random variable Y_t is

$$(2.21)\quad h(y_t) = \frac{1}{\sqrt{2\pi}\,\sigma_1} \operatorname{Exp}\left[\frac{-1}{2\sigma_1^2}(y_t - x'\beta_1)^2\right] + \frac{1-\lambda}{\sqrt{2\pi}\,\sigma_2} \operatorname{Exp}\left[-\frac{1}{2\sigma_2^2} \cdot (y_t - x'\beta_2)^2\right]$$

Now, consider the following expression

$$(2.22)\quad S_T = \sum_{j=1}^{m}\left[\frac{1}{T}\sum_{t=1}^{T} e^{\theta_j y_t} - \left(\lambda e^{x'\beta_1\theta_j + \sigma_1^2\theta_j^2/2} + (1-\lambda)\, e^{x'\beta_2\theta_j + \sigma_2^2\theta_j^2/2}\right)\right]^2$$

The term in the parenthesis is the mgf, $E(e^{\theta_j y_t})$ and the first term in the brackets is its consistent estimator. It follows that plim (min S_T) = 0 and by the uniqueness of moments it follows that

$$\text{plim}\ (\hat{\beta}_{1t}, \hat{\beta}_{2t}, \hat{\sigma}^2_{1t}, \hat{\sigma}^2_{2t}, \hat{\lambda}_t) = (\beta_1, \beta_2, \sigma_1^2, \sigma_2^2, \lambda)$$

The mgf method for the model (2.19) - (2.20) entails minimizing S_T in (2.22). The parameters so obtained will be consistent. However, model (2.19) - (2.20) is not an example of a regression model since the independent variable does not have a subscript t. To obtain parameters of a regression model Quandt and Ramsey suggest an approximation[1]; the terms $e^{x'\beta_r \theta_j}$ r = 1, 2 in (2.22) should be replaced by $\frac{1}{T} \sum_{t=1}^{T} e^{x'_t \beta_r \theta_j}$

Equation (2.22) may sometimes be difficult to minimize in which case the terms in the parenthesis may be replaced by their Taylor Series approximation.

Sampling experiments were carried out by Quandt and Ramsey employing both methods. Each experiment was replicated as many times as was necessary to produce 50 success-

[1] The estimator so derived seems to be inefficient in that it does not adequately take into account variation in the y_t due to variation in the corresponding x_t and it is not clear if the consistency property will carry through with the approximation.

ful matched replications of the 2 methods.[1] The mgf method performed much better than the Moments method as judged by (1) the causes of failures in computing estimates for constant parameter mixture, (2) the Shapiro-Wilk Statistic values for the normality of the coefficient estimates, (3) the mean square error. (1), (2), (3) taken over the number of successful replications of the experiment; the superiority of the mgf method was shown as well by the percentage (over coefficient and replications) by which the mgf method produces estimates closer to the true value than the Moments method. As for the switching regression case, Quandt and Ramsey re-examined Hamermesh (1970) model of wage bargains determination from a pooled cross-section time series sample of 180 observations on wage changes W, changes in the consumer price index CPI and unemployment U [ΔW=f (CPI, U)] and found that changes in the cost of living begin to matter at a lower value of CPI than the value employed by Hamermesh

[1] An unsuccessful replication was considered as one for which the numerical minimization of (2.22) failed either because the estimates stayed in prohibited regions in the parameter space or because of each of positive definiteness of the matrix of second partials of (2.22) at the stationary point of the minimization algorithm. The one for which the computations of the estimates of the Moments method failed because the nonic equation had more than one negative root and no admissible solution or it had more than one negative root with more than one admissible solution or it had no negative root at all was also considered unsuccessful.

and that when the cost of living matters wage bargains came close to compensating for it.

Hence, Quandt and Ramsey have shown that the mgf method yields consistent and asymptotically normal estimates. The appeal of their functions is that it is a natural generalization of the Moments method. It appears to work well in the case of mixtures components with constant means as well as regression mixtures in which the mean value of the random variable depends on the regressor. Disadvantages are that computational difficulties may be encountered in achieving satisfactory convergence. The other problem concerns the choice of θ. Quandt and Ramsey point out that the choice of θ is important but do not give it any more attention than to point out that small values of θ ($\leq .04$) made the minimand flat and large values ($>.75$) created computational difficulties.

Some authors have attempted to resurrect the use of the maximum likelihood estimation for mixture problems. In particular Kiefer (1978b) showed that a root of the likelihood equations corresponding to a local maximum is consistent, asymptotically normal and efficient. Let $\theta = (\lambda, \beta_1, \beta_2, \sigma_1, \sigma_2)$ and let the parameters space given by $0 < \lambda < 1, -\infty < \beta_1 < \infty, \quad -\infty < \beta_{2j} < \infty \; j = 1, \ldots, p, \; 0 < \sigma_1 < \infty \quad 0 < \sigma_2 < \infty$

Let θ_0, the true parameter value be contained in some closed region which does not contain $\lambda = 0$, $\lambda = 1$, $\sigma_1 = 0$

or $\sigma_2 = 0$ [$\lambda = 0$ or 1 is not admissible here since when λ is at one of these points the parameters of one of the regimes are not identified. Tests of the equality of the coefficients in each regime can be made using the estimates of the asymptotic variance-covariance matrix]. If the random variable y_t is distributed according to the probability density as in (2.18) then for large enough T it can be shown that there exists a unique consistent root θ_t of the likelihood equation and $\sqrt{T}\ (\theta_T - \theta_0)$ is asymptotically normally distributed with mean zero and variance $I(\theta_0)^{-1}$ where $I(\theta_0)$ is the Fisher information matrix. But Kiefer does not provide any information about which root, in the event there is more than one, is consistent. He argues that an initial consistent estimate is needed as a starting point for the maximization. Furthermore, the estimates calculated using the mgf technique may be used as the starting point for a Newton step toward the (local) maximum of the likelihood function. The Newton iterative scheme is defined by

$$\theta^{i+1} = \theta^i - \left[\frac{\partial^2 \ln L(\theta^i)}{\partial\theta\ \partial\theta'}\right]^{-1} \left[\frac{\partial \ln L(\theta^i)}{\partial\theta}\right]$$

One step provides asymptotically efficient estimates. The negative of the inverse of the matrix of second derivatives of the L is a consistent estimate of the asymptotic variance-covariance matrix of the parameters.

Hartley (1977, 1978) made some limited Monte Carlo experiments with the EM algorithm [(Dempster, Laird and Rubin (1977)] which indicate that convergence to a solution of the likelihood equations corresponding to a local maximum is always obtained and point estimates are very close to the true parameter value, which means that Quandt and Ramsey may have exagerated the importance of computational obstacles to maximum likelihood estimation. Hence, it appears that the maximum likelihood estimation dominates the mgf on asymptotic grounds and has the further advantage that its asymptotic distribution is independent of the θ's which must be selected a priori to implement the latter. Unfortunately, finite sample properties of such estimates are unknown and the attainment of a maximum may be difficult in practice.

A number of other authors have suggested several ways of generalizing Quandt and Ramsey estimates. A characteristic function may be used in the place of the moment generating function [Binder (1978), Clarke and Heathcote (1978), Kumar Nicklin and Paulson (1979)]. Instead of picking discrete θ's, a weighting function continuous in θ may be introduced and θ integrated out [Fowlkes (1978) and Kumar et al. (1979)]. Different objective functions may be selected for optimization [Binder (1978), Clarke and Heathcote (1978)] Bayesian approaches may be considered (Binder). Some of the generalizations suggested above may be even more effective but they, in their turn, encounter other computational and convergence problems; more work is required before drawing any conclusions on the validity of the method. More particularly, a deeper comparison between the maximum likelihood estimation and the mgf estimator with a minimum distance is needed, as well as, a theoretical comparison of the covariance matrix of the mgf estimator and the inverse of the information matrix.

Very recently, the switching regression problem has been connected to the problem of discrimination by Kiefer (1980b). He bears upon Behboodian's (1970) representation of the so-called likelihood equations in terms of weighted averages of sample moments (where the weights depend on unknown parameters), and suggests an iterative procedure of reweighting the moments in order to obtain parameter estimates. Rewriting (2.1) and (2.2) in notation familiar in weighted regression we have

$$(2.23) \quad \sqrt{d_i}\, y_i = \sqrt{d_i}\, x_i'\beta_1 + \sqrt{d_i}\, u_{1i}$$

$$(2.24) \quad \sqrt{1-d_i}\, y_i = \sqrt{1-d_i}\, x_i'\beta_2 + \sqrt{1-d_i}\, u_{2i}$$

where d_i, $i = 1,\ldots,T$ is a series of zero-one variable with $d_i = 1$ if observation i belongs to regime one and $d_i = 0$ otherwise. From (2.23) and (2.24) we can derive the usual parameter estimators β_1, β_2, σ^2 and the population proportion can be estimated by the sample proportion $\hat{\lambda} = \frac{1}{T}$ tr D where D is a diagonal matrix with i^{th} element d_i.

The usefulness of Kiefer's way of looking at the switching model is that it suggests an instrumental variable estimator. He proposes to use predicted probabilities in place of the d_i. That can be done using the fact that the distributional assumption made on the errors in (2.23) and (2.24) imply a distribution for the d_i. It follows, by Bayes Theorem,

$$(2.25) \quad p(d_i=1|y_i) = \frac{\lambda h \quad (y_i|d_i=1)}{\lambda h \quad (y_i|d_i=1) + (1-\lambda)h(y_i|d_i=0)}$$

which is the marginal probability that $d_i = 1$ times the conditional density of y_i given that $d_i = 1$, divided by the marginal density of y_i. Using the normality assumptions that $h(y_i|d_i=1)$ is a normal density with mean $x_i\beta_1$ and variance σ^2 and that $h(y_i|d_i=0)$ is a normal density with mean $x_i \ \beta_2$ and same variance, (2.25) can now be transformed into a logit probability

$$(2.26) \quad p(d_i=1|y_i) = \{1 + \text{Exp}\left[\ln\left(\frac{1-\lambda}{\lambda}\right) + \frac{1}{2\sigma^2}\left[(x_i'\beta_1)^2 - (x_i'\beta_2)^2\right] + \frac{1}{\sigma^2}(x_i'\beta_1 - x_i'\beta_2)y_i\right]\}^{-1} = w_i$$

Then, replacing d_i with the probabilities w_i in the usual parameter estimators derived from (2.23) and (2.24) solves the likelihood equations obtained from the maximization of the likelihood function $L = \prod_{i=1}^{T} f(y_i|\Theta, x_i)$ where $\Theta = (\beta_1, \beta_2$

σ^2,λ). Hence, Kiefer gives an interpretation of the maximum likelihood estimator of the switching regression model as a weighted regression estimator with logit probabilities as weights. The main weakness of his approach is that a non-iterative solution is not available since the conditional probabilities coming from each regime are functions of unknown parameters. He suggests to start with an initial

Θ, calculate probabilities w_i and use these in place of the d_i to obtain a new value of ($\Theta=\Theta^*$) re-evaluate w_i with this new value, Θ^*, to obtain another value for Θ^* and iterate the process until convergence is reached.

4 Summary

1. Estimation techniques discussed in this section normally require maximization of functions which are non-linear. Obtaining expressions for derivatives can be quite tedious. Maximization can be carried out using numerical methods of optimization discussed in Goldfeld and Quandt (1972, Chp. 1). Although the problem may not be the tediousness of the estimates of normal equations but that non-linear estimates have "odd" properties like nonconvergence.

2. The likelihood functions which appear in Chapter 2 are unbounded unless it can be assumed that $\sigma_1^2 = k\ \sigma_2^2$ where k is known. If this assumption cannot be made global maximum lead to inconsistent estimates. Although estimates corresponding to a local maximum of the likelihood function in the interior of the parameter space are consistent (Kiefer 1978b). the attainment of such a maximum may be difficult in practice and the finite samples properties of such estimates are unknown. The mgf method used by Quandt and Ramsey yields consistent and asymptotically normal estimates, but a numerical problem in achieving convergence remains and the choice of Θ is arbitrary and affects the asymptotic covariance matrix which suggests that further research is needed in order to find optimal values of Θ. A better strategy seems as suggested by Kiefer, to use mgf technique in order to

obtain an initial value and then use this point as the starting value for a Newton step towards the solution of the likelihood equations. But here again the finite sample properties of this method remain unknown and computational difficulties are likely to be encountered.

3. The methods we have discussed are frequently justified in the literature by assertions about the consistency properties of the estimators. Such assertions are not well substantiated and more work is required in that direction.

CHAPTER 3

TECHNIQUES FOR ESTIMATING DISEQUILIBRIUM MODELS

The problem of estimation of disequilibrium markets was first studied by Fair and Jaffee (1972). The basic disequilibrium model we defined in the introduction uses no extraneous information for classifying observations into demand and supply regimes. In their paper Fair and Jaffee use information on prices in order to identify observations as demand points or supply points. By making alternative assumptions on price adjustments they considered different disequilibrium models and proposed methods for estimating these models. The authors themselves acknowledged that these methods were lacking in some respects and improvements or alternative approaches have been suggested by Fair and Kelejian (1974), Amemiya (1974) and Maddala and Nelson (1974). Some other disequilibrium models were also considered in these later contributions and appropriate estimation techniques investigated. The paper by Fair and Jaffee is extremely important for setting the direction of research in the area of disequilibrium econometrics. Of the others, the paper by Maddala and Nelson (1974) is important. It improves on certain methods proposed by previous writers and removes ambiguities in others. It also proposes a maximum likelihood method for estimation of a model which does not rely on any extraneous information. We start this chapter by considering this basic model and its estimation. We will then outline two disequilibrium models from the Fair and Jaffee paper and discuss estimation techniques. We will finally consider a disequilibrium model with a very general price adjustment and we will show how a disequilibrium model can be dynamically specified.

The Basic Disequilibrium Model

The basic disequilibrium model is the following set of equations

(3.1) $D_t = x_t'\beta_1 + u_{1t} \qquad t = 1,\ldots,T$

(3.2) $S_t = x_t'\beta_2 + u_{2t} \qquad t = 1,\ldots,T$

(3.3) $y_t = \min(D_t, S_t) \qquad t = 1,\ldots,T$

Observations are available on the exogenous variables and y_t.

Once equation (3.3) is added to (3.1) and (3.2) the estimation techniques discussed in Chapter 2 become inappropriate. These techniques, however, continue to be relevant in cases where the assumption of short side dominance seems untenable.

A maximum likelihood method to estimate the above model is provided by Maddala and Nelson (1974). Price is assumed to be exogenous.

This in effect implies that price enters with a lag or it is regulated by a government agency. It will be assumed as before that u_{1t} and u_{2t} are normally and independently distributed and are also serially uncorrelated. Unless otherwise stated this assumption will be made throughout this chapter.

Define a variable Z_t where $Z_t = 1$ if $t \varepsilon I_1$, and $Z = 0$ if $t \varepsilon I_2$ where I_1 and I_2 are set of indices for which the observations belong to the demand and supply regimes respectively. The endogenous variables are Y_t and Z_t. Since any observation can belong to regime 1 or 2, it follows that

$$h(y_1 \ldots y_T z_1 \ldots z_T) = \prod_{t=1}^{T} \left[h_1(y_t | Z_t = 1) \Pr(Z_t = 1) + h_2(y_t | Z_t = 0) \Pr(Z_t = 0) \right]$$

if we let $g\ (D_t\ S_t)$ be the joint density of D_t and S_t and $g\ (D_t\ S_t | D_t < S_t)$ be the joint density of D_t and S_t conditional on the event that $D_t < S_t$ then we can write

$$(3.4)\quad h_1(y_t | Z_t = 1) = h_1(y_t | D_t < S_t) = \int_{y_t}^{\infty} g(y_t\ S_t | D_t < S_t)\, dS_t$$

$$= \frac{1}{\Pr(D_t < S_t)} \int_{y_t}^{\infty} g(y_t\ S_t)\, dS_t$$

A similar expression can be derived for $h_2\ (y_t | Z_t = 0)$. It then follows that

$$(3.5)\quad h(y_1 \ldots y_T z_1 \ldots z_T) = \prod_{t=1}^{T} \int_{y_t}^{\infty} g(y_t\ S_t)\, dS_t + \int_{y_t}^{\infty} g(D_t\ y_t)\, dD_t$$

For simplicity we have avoided conditioning the densities on the exogenous variables but it is implicit all along. From the independence of the error terms the first integral in (3.5) can be written as:

$$\int_{y_t}^{\infty} g(y_t\ S_t)\ dS_t = f_1\ (y_t) \int_{y_t}^{\infty} f_2\ (S_t)\ dS_t$$

$$= f_1\ (y_t)\ F_2\ (y_t)$$

Similarly the second integral in (3.5) can be written as

$$\int_{y_t}^{\infty} g(D_t\ y_t)\ dD_t = f_2\ (y_t)\ F_1\ (y_t)$$

where

$$f_1(y_t) = \frac{1}{\sqrt{2\pi}\sigma_1} \text{Exp}\left[\frac{-1}{2\sigma_1^2} (y_t - x_t{}'\ \beta_1)^2\right]$$

$$f_2(y_t) = \frac{1}{\sqrt{2\pi}\sigma_2} \text{Exp}\left[\frac{-1}{2\sigma_2^2} (y_t - x_t{}'\ \beta_2)^2\right]$$

(3.6)

$$F_1(y_t) = \frac{1}{\sqrt{2\pi}\sigma_1} \int_{y_t}^{\infty} \text{Exp}\left[\frac{-1}{2\sigma_1^2} (D_t - x_t{}'\ \beta_1)^2\right] dD_t$$

$$F_2(y_t) = \frac{1}{\sqrt{2\pi}\sigma_2} \int_{y_t}^{\infty} \text{Exp}\left[\frac{-1}{2\sigma_2^2} (S_t - x_t{}'\ \beta_2)^2\right] dS_t$$

The likelihood function is then

$$\text{(3.7)} \quad L = \prod_{t=1}^{T} \left[f_1(y_t)\ F_2(y_t) + f_2(y_t)\ F_1(y_t)\right]$$

Expressions for the first and second derivatives for the likelihood function (3.7) have been derived by Maddala and Nelson

(1974 pp. 1016-1018). Once estimates for β_1, β_2, σ_1^2, σ_2^2 are obtained, an estimate of the probabilities that an observation belongs to regime 1 or 2 can then be obtained. The probability of an observation belonging to the demand regime is:

(3.8) $$\Pr(D_t < S_t) = \Pr(x_t' \beta_1 + u_{1t} < x_t' \beta_2 + u_{2t})$$

$$= \Pr(u_{1t} - u_{2t} < x_t' \beta_2 - x_t' \beta_1)$$

$$= \int_{-\infty}^{\alpha} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$

where $\alpha = (x'_t\beta_2 - x'_t\beta_1)/\sigma$ and where $\sigma^2 = \sigma_1^2 + \sigma_2^2 = \text{var}(u_1 - u_2)$. The probability of an observation belonging to the supply regime is $\Pr(D_t > S_t) = 1 - \Pr(D_t < S_t)$. It is clear, therefore, that once equation (3.3) is added to (3.1) and (3.2) the data itself provides estimates of the probability of an observation belonging to the demand regime or the supply regime. The λ-method described in Chapter 2 or other methods which ignore this information are therefore not appropriate for disequilibrium models.

The derivation of the likelihood function and its maximization is easily accomplished even when the independence of u_{1t} and u_{2t} is relaxed. The problem is computationally unfeasible when serial correlation is introduced.

2 Extension of the Basic Model

The basic model uses no extraneous information. If such information is available it allows observations to be classified into demand and supply regimes. For example, Fair and Jaffee (1972) in one of their models assume that rising prices are associated with excess demand and falling prices with excess supply. With this assumption and the additional information on prices the disequilibrium model can be rewritten as

$$(3.9) \qquad D_t = x_t'\beta_1 + u_{1t} \qquad t = 1, \ldots, T$$

$$(3.10) \qquad S_t = x_t'\beta_2 + u_{2t} \qquad t = 1, \ldots, T$$

$$(3.11) \qquad \Delta P_t \gtrless 0 \text{ as } D_t - S_t \gtrless 0 \qquad \Delta P_t^{(1)} = P_t - P_{t-1}$$

$$(3.12) \qquad y_t = \min (D_t, S_t) \qquad t = 1, \ldots, T$$

Equation (3.11) suggests that if in period t prices are higher than in period t − 1 then the t^{th} period observation on y is associated with excess demand and from (3.12) it belongs to the supply regime. Similarly during periods of falling prices observations can be classified into the demand regime. We note that the procedure is equivalent to defining an extraneous variable ΔP and its cut-off value zero and obtaining sample separation from information on whether the extraneous variable is greater or less than its

(1) In Chapter 4 ΔP_t is sometimes defined as $P_{t+1} - P_t$

cut-off value. As in the D-method the step function can be approximated by a cumulative normal integral (2.11) with μ explicity assumed to be zero.[1] The variable ΔP is an example of an extraneous variable which is endogenous.

Once sample separation is obtained the demand and supply parameters in the disequilibrium model (3.9) to (3.12) can be estimated easily by applying estimation techniques to the segregated data in a piecemeal way. This is one of the methods considered by Fair and Jaffee and they call it Directional method I. They point out that if data are segregated in this way then the mean of u_{1t} will not be independent of x_t over those points for which demand is observed and the mean of u_{2t} will not be independent of x_t over those points for which supply is observed. It follows therefore that parameter estimates may be inconsistent. The equilibrium observations under Directional method I are treated as both demand points and supply points. Fair and Jaffee consider an alternative method of estimating model (3.9) to (3.12), which they call Directional method II, differing from Directional method I crucially in the treatment of equilibrium values. Further discussion of these methods is delayed till Chapter 4.

Another method of estimating model (3.9) to (3.12) is a maximum likelihood method proposed by Maddala and Nelson

[1]This point is made in Goldfeld and Quandt (1972)

(1974). The method assumes that the price variables does not enter contemporaneously in the demand and supply equations. This assumption is necessary because P_t is endogenous in the model and there are not enough equations to determine the joint density of y_t and P_t. As in the basic model we define a joint density of the entire sample for the endogenous variables Z_t and y_t

(3.13)

$$h(y_1 \ldots y_T z_1 \ldots z_T) = \prod_{t \in I_1} h_1(y_t | Z_t = 1) Pr(Z_t = 1) .$$

$$\prod_{t \in I_2} h_2(y_t | Z_t = o) Pr(Z_t = o)$$

As in the previous model expressing $h_1(y_t | Z_t = 1)$ as in (3.4) and finding a similar expression for $h_2(y_t | Z_t = 0)$, it is possible to write the likelihood function as

(3.14)
$$L = \prod_{t \in I_1} f_1(y_t) F_2(y_t) \prod_{t \in I_2} f_2(y_t) F_1(y_t)$$

$$= \left[\prod_{t \in I_1} f_1(y_t) \prod_{t \in I_2} F_1(y_t) \right] \cdot \left[\prod_{t \in I_2} f_2(y_t) \prod_{t \in I_1} F_2(y_t) \right] ,$$

$$= L_1 \; L_2 .$$

Since L_1 and L_2 have no parameter in common, maximizing L is equivalent to maximizing L_1 and L_2 separately, provided the separation is known or estimated before.

Fair and Kelejian (1974) have also derived a likelihood function for this model but as Maddala and Nelson (1974) point out their method amounts to obtaining the joint density of $y_1 \ldots y_T$ dependent on the values of $Z_1 \ldots Z_T$. Since the model does give information on the joint probability of $(Y_1 \ldots Y_T Z_1 \ldots Z_T)$ the Fair Kelejian method does not use all the information whereas the Maddala and Nelson method utilizes this information.

3The Price Adjustment Equation and the Dynamic Specification of the Disequilibrium Model

The next variant of the disequilibrium model we consider makes a more specific assumption about the price response in event of a disequilibrium, namely, that a change in price is directly proportional to the amount of excess demand. The disequilibrium model (which we will sometimes refer to as the Quantitative model) is

$$D_t = x_t'\beta_1 + u_{1t} \qquad t = 1, \ldots, T \tag{3.15}$$

$$S_t = x_t'\beta_2 + u_{2t} \qquad t = 1, \ldots, T \tag{3.16}$$

$$\Delta P_t^{(1)} = \gamma(D_t - S_t) \qquad t = 1, \ldots, T \tag{3.17}$$

$$y_t = \min(D_t, S_t) \qquad t = 1, \ldots, T \tag{3.18}$$

In equation (3.17) the coefficient γ is the adjustment

(1) The definition ΔP_t is left unspecified for the time being. It will be later defined as either $\Delta P_t = P_t - P_{t-1}$, or as $\Delta P_t = P_{t+1} - P_t$. In the former case we will refer to it as the F.J. Hyp. and in the latter case as the L.G. Hyp. For examples of estimation under these alternative hypotheses, see Chapter 4.

factor. A disequilibrium estimation is carried out when there is reason to believe that γ is 'small', i.e. there is sluggishness in the adjustment process. Equation (3.17) can be rewritten as

$$(3.19) \qquad D_t - S_t = \frac{1}{\gamma} \Delta P_t \qquad \Delta P_t = P_t - P_{t-1}$$

In periods of rising prices equations (3.19) and (3.18) allow classification of observations in the supply regime and the following supply equation can be estimated

$$(3.20) \qquad y_t = S_t = x_t'\beta_2 + u_{2t} \qquad \Delta P_t \geq 0$$

It is also possible to estimate parameters of the demand equation by noting that (3.19) can be written as

$$y_t = S_t = D_t - \frac{1}{\gamma} \Delta P_t$$

$$(3.21) \qquad = x_t' \beta_1 - \frac{1}{\gamma} \Delta P_t + u_{1t}$$

Observations on y_t, x_t and ΔP_t then allow parameters of the demand equation to be estimated.

Similarly during falling prices the following two equations can be estimated

$$(3.22) \qquad y_t = D_t = x_t' \beta_1 + u_{1t} \qquad \Delta P_t \leq 0$$

and

$$(3.23) \qquad y_t = D_t = x_t'\beta_2 - \frac{1}{\gamma} \Delta P_t + u_{2t}$$

Equations (3.21) and (3.23) can together be written as

$$(3.24) \qquad y_t = x_t'\beta_1 - \frac{1}{\gamma} g_t + u_{1t}$$

$$g_t = \begin{cases} \Delta P_t & \text{if } \Delta P_t \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

and (3.20)and (3.23) can together be written as

$$(3.25) \qquad y_t = x_t' \ \beta_2 - \frac{1}{\gamma} h_t + u_{2t}$$

$$h_t = \begin{cases} \Delta P_t & \text{if } \Delta P_t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Parameter estimates are obtained by applying estimation techniques to equations (3.24) and (3.25). This method will be referred to as the Quantitative method.

The disequilibrium model (3.15) - (3.18) and the estimation procedure just outlined was suggested by Fair and Jaffee (1972). They also pointed out the problems in estimating the parameters of equation (3.24) and (3.25). Firstly, the coefficient $\frac{1}{\gamma}$ appears in both equations and this constraint has to be imposed in the estimation. Secondly, the variables P_t, g_t and h_t are endogenous and therefore estimation techniques which avoid simultaneous equation bias have to be used. A two stage least squares method avoids the second problem. Amemiya (1974) suggests a procedure along the following lines. To obtain consistent estimators of parameters in equation (3.24) regress g_t on all the exogenous variables in the equation and obtain its estimator $\hat{g}_t$. Also, if one of the elements in the vector x_t is the price variable then P could be regressed on the exogenous variables and its estimator $\hat{P}$ obtained. Replacing g_t and P_t by $\hat{g}_t$ and $\hat{P}_t$ in (3.24) and then applying OLS to the equation would provide consistent estimators. Amemiya says that this two stage least squares procedure will not provide asymptotically efficient estimators because it does not take into account the constraint and also because g_t and h_t are not linear functions of P_t. He has proposed a maximum likelihood method which he says is expected to possess desirable large sample properties. We may note that these properties have not been rigorously investigated. We discuss this method below.[1]

Let $f_1(y_tP_t)$ be the joint density of D_t and P_t when $y_t = D_t$ and $f_2(y_tP_t)$ be the joint density of D_t and P_t when $y_t = S_t$. Define an indicator variable Z_t as before. Then it is obvious that

(3.26 $\quad f_1(y_tP_t) = h_1(y_tP_t|Z_t = 1)\ Pr(Z_t = 1)$

and

(3.27) $\quad f_2(y_tP_t) = h_2(y_tP_t|Z_t = 0)\ Pr(Z_t = 0)$

To obtain $f_1(.)$ note that when Z_t = 1

[1] The approach used here is the one given in Maddala and Nelson (1974).

$$D_t = y_t = x_t'\beta_1 + u_{1t}$$
$$\Delta P_t = \gamma(D_t - S_t) = \gamma(y_t - x_t'\beta_2 - u_{2t})$$

and the joint density of y_t and P_t is obtained from the joint density of u_{1t} and u_{2t}. Similarly when $Z_t = 0$

$$S_t = y_t = x_t'\beta_2 + u_{2t}$$
$$\Delta P_t = \gamma(D_t - S_t) = \gamma(x_t'\beta_1 + u_{1t} - y_t)$$

and it is possible to obtain $f_2(.)$ from the joint density of the error terms. The joint density for the endogenous variables y_t, P_t and Z_t for the entire sample is

$$(3.28)\quad h(y_1 \ldots y_T p_1 \ldots p_T z_1 \ldots z_T) = \prod_{t\in I_1} h_1(y_t P_t | Z_t = 1)\, Pr(Z_t = 1)$$
$$= \prod_{t\in I_2} h_2(y_t P_t | Z_t = 0)\, Pr(Z_t = 0)$$

Now using relations (3.26) and (3.27) it is possible to write the likelihood function as

$$(3.29)\quad L = \prod_{t\in I_1} f_1(y_t\ P_t) \cdot \prod_{t\in I_2} f_2(y_t\ P_t)$$

We will now consider a very general form of the price adjustment equation making it both multivariate and stochastic at the same time. Prices change due to variables other than excess demand some of which can be included as regressors, but we pretend either not to know or not be able to quantify all the variables that may cause this change. The effect of these variables is captured by the disturbance term.

As usual we assume the effect of these various factors to be mutually offsetting on average and the error term satisfies the standard properties. The disequilibrium model now takes the generalized form

$$(3.30)\quad D_t = x_t'\beta_1 + u_{1t} \qquad t = 1, \ldots, T$$

$$(3.31)\quad S_t = x_t'\beta_2 + u_{2t} \qquad t = 1, \ldots, T$$

$$(3.32)\quad \Delta P_t = x_t'\beta_3 + u_{3t} \qquad t = 1, \ldots, T$$

$$(3.33)\quad y_t = \min (D_t\ S_t) \qquad t = 1, \ldots, T$$

Note the formulation (3.32) is very general. One of the explanatory variables is $D_t - S_t$; some of the variables may be common with equations (3.30) and (3.31) but there may be others which are not. By appropriately fixing the coefficients at zero, variables can be excluded from any of the equations. The disturbance term u_{3t} is assumed to be distributed normally and independently with mean zero and variance σ_3^2; it is also serially uncorrelated. This disequilibrium model was first considered by Fair and Kelejian (1974) who also suggested a maximum likelihood procedure. A more straightforward presentation is due to Maddala and Nelson (1974). We outline below this full information maximum likelihood procedure as proposed by Maddala and Nelson.

The full information maximum likelihood procedure requires determining the joint density of the endogenous variables P_t and y_t. To obtain this first note that the joint density $g(D_t\ S_t\ P_t)$ of D_t, S_t and P_t is obtained from equations (3.30) to (3.34). In fact it is simply the joint density of u_{1t}, u_{2t} and u_{3t} multiplied by the jacobian of the transformation from $(D_t\ S_t\ P_t)$ to $(u_{1t}\ u_{2t}\ u_{3t})$. Let $f_1(y_t\ P_t)$ be the joint density of y_t and P_t when $y_t = D_t$ and $f_2(y_t\ P_t)$ be the joint density when $y_t=S_t$. We can write

$$(3.34) \qquad f_1(y_t\ P_t) = \int_{y_t}^{\infty} g_1\ (y_t\ S_t\ P_t | D_t < S_t)\ dS_t$$

$$= \frac{1}{Pr(D_t < S_t)} \int_{y_t}^{\infty} g(y_t\ S_t\ P_t) dS_t$$

similarly

$$(3.35) \qquad f_2(y_t P_t) = \frac{1}{Pr(D_t > S_t)} \int_{y_t}^{\infty} g(D_t\ y_t\ P_t) dD_t$$

The joint density $f(y_t\ P_t)$ can then be obtained using (3.34) and (3.35). Thus

$$(3.36) \qquad f(y_t\ P_t) = f_1(y_t\ P_t)\ Pr\ (D_t < S_t) + f_2(y_t\ P_t)\ Pr(D_t > S_t)$$

$$= \int_{y_t}^{\infty} g(y_t\ S_t\ P_t)\ dS_t + \int_{y_t}^{\infty} g(D_t\ y_t\ P_t) dD_t$$

The maximum likelihood estimators are then obtained by maximizing with respect to the parameters the likelihood function

$$(3.37) \qquad L = \prod_{t=1}^{T} f(y_t\ P_t).$$

We finally consider a dynamic specification of a disequilibrium model. A dynamic extension of disequilibrium models would occur in situations where unsatisfied demand or supply in any given period carries over, at least in part, into the next period. We can assume, for example, that suppliers keep stocks but that demanders do not, such as in the case of bank loan markets in which suppliers' stocks correspond to first and secondary bank reserves or in new residential market where suppliers' stocks correspond to unsold new houses. Then, following Dagenais (1980), specification of the model (3.15) - (3.18) can be modified to take account of the fact that previously unsatisfied demand or supply affects the present situation in the following way:

(3.38) $D_t = \beta_o + x'_t\beta_1 + x'_{1t}\ \beta_2 + \beta_3 P_t + \beta_4\ (D_{t-1} - Q_{t-1}) + u_{1t}$

(3.39) $S_t = \lambda_o + x'_t\lambda_1 + x'_{2t}\lambda_2 + \lambda_3 P_t + \lambda_4\ (I_{t-1} - I^d_t) + u_{2t}$

(3.40) $\Delta P_t = \gamma(D_{t-1} - S_{t-1}) + u_{3t}$

(3.41) $y_t = \min\ (D_t,\ S_t)$

where I = end of period inventories; I^d = desired inventories and $\Delta P_t = P_t - P_{t-1}$.

I^d_t may be in turn assumed to be a linear function of the quantity supplied when no stock adjustment is required:

(3.42) $I^d_t = \theta_o + \theta_1\ (\ \lambda_o + x'_t\ \lambda_1 + x'_{2t}\ \lambda_2 + \lambda_3 P_t + u_{2t}) + u_{4t}$

and replacing I^d_t in (3.39) yields:

(3.43) $S_t = \psi_0 + x'_t \psi_1 + x'_{2t}\psi_2 + \psi_3 P_t + \psi_4 I_{t-1} + u_{5t}$ (1)

where $\psi_0 = \lambda_0 - \lambda_4 (\theta_0 + \theta_1 \lambda_0)$, $\psi_1 = \lambda_1(1-\lambda_4\theta_1)$, $\psi_2 = \lambda_2(1-\lambda_2\theta_1)$,

$$\psi_3 = \lambda_3(1-\lambda_4\theta_1),\ \psi_4 = \lambda_4 \text{ and } u_{5t} = u_{2t}(1-\lambda_4\theta_1) - \lambda_4 u_{4t}$$

The problem is now to estimate the parameter vector: $(\beta_0, \beta_1', \beta_2', \beta_3, \beta_4, \psi_0, \psi_1', \psi_2', \psi_3, \psi_4, \gamma)$ from a sample of T observations (t = 1,...,T) on the variables x, y, I and P. Now, we can rewrite equation (3.38) with D_{t-1} and S_{t-1} replaced by their values in (3.40) and (3.43) respectively, we get:

$$\begin{aligned}(3.44)\quad D_t = {} & \beta_0 + \beta_4\psi_0 + x_t'\beta_1 + x_{1t}'\beta_2 + x_{t-1}'\psi_1 + x_{1t-1}'\psi_2\beta_4 + \\ & (\beta_4/\gamma + \beta_3)P_t - \beta_4(1/\gamma - \psi_3)P_{t-1} + \psi_4\beta_4 I_{t-2} - \beta_4 Y_{t-1} \\ & -(\beta_4/\gamma)u_{3t} + u_{1t} + \beta_4 u_{5t-1}\end{aligned}$$

A simple examination of the system composed by equations (3.44), (3.43), (3.41), (3.40), shows that a full information solution would be intractable in practice. A limited information solution (LIML) can be obtained instead. Unfortunately, the consistency of the LIML estimator has not been proved yet but is simply "expected" to be consistent. Nevertheless, despite its "incompleteness" the model described

(1) Note that, according to Bowden (1978) equation (3.40) can be written as: $P_t = \mu_t P_{t-1} + (1-\mu)P^*_{t-1} + \mu u_{6t}$ where P^*_{t-1} is the equilibrium price that would clear the market in period t - 1, $\mu = 1 + \gamma(\beta_3 - \psi_3)$ and u_{6t} is a residual error. As $\mu \to 0$ the model becomes a dynamic equilibrium model and in case of applications it would be interesting to estimate and test the null hypothesis $\mu = 0$.

above shows clearly that situations in which unsatisfied demand or supply in any given period carried over into the next period can be adequately taken into account in a disequilibrium framework. Unfortunately, as far as we are aware no actual application has been carried out yet.

Finally, it should be noted that an important question is that of testing the hypothesis that the market is in disequilibrium against the alternative that it is in equilibrium. We will not discuss here the various test procedures available because it appears that whenever such tests are carried out they usually depend heavily on the precise specification of both the equilibrium and disequilbrium versions of the model. The question is very important and the reader interested in that topic may find fruitful discussion in Goldfeld and Quandt (1976) and Quandt (1978).

4 Summary

1. As in switching regressions maximization of the likelihood functions by obtaining derivatives can be a formidable task. It is almost invariably necessary to adopt numerical methods of optimization. A discussion of which is found in Goldfeld and Quandt [(1972) ch. 1].

2. The likelihood function (3.7) for the basic model is unbounded and therefore a global maximum does not exist. A priori information on the ratio of error variances $k = \frac{\sigma_1^2}{\sigma_2^2}$ if available avoids this problem.
However, if a price adjustment equation is added to model (3.1) - (3.3) the resulting likelihood function satisfies the regularity conditions for a global maximum to exist. (See Quandt 1976 a).

3. Quandt (1976 a) points out that disequilibrium models can be internally inconsistent. The problem of inconsistency is present even in equilibrium models but it is not considered serious since the set of points for which there is no solution is a set of measure zero. For disequilibrium models Quandt recommends checking for internal inconsistency before proceeding to estimate the model. It is possible that the non-linearity of disequilibrium models makes internal inconsistency a more serious problem.

4. For the disequilibrium model (3.1) – (3.3) Hartley and Mallela (1976) have under very general conditions proved that maximum likelihood estimators are strongly consistent and asymptotically normal. One of the assumptions they make is that the parameter space is compact and does not contain the region $\sigma_j^2 \leq 0 (j = 1,2)$ and has as its interior point the true parameter vector. Other assumptions are the familiar assumptions on the error terms, about the boundedness of the regressors, the identifiability of the equations, and about the limiting behaviour of the matrices $M_{ij} = \frac{1}{T} \sum_{t=1}^{T} x_t^i x_t^j$ $i,j=1,2$ where x_t^1 and x_t^2 are regressors specific to the demand and supply equations. The proof is extremely complex. Statistical properties for other disequilibrium models discussed in chapter 3 have not so far been investigated.

CHAPTER 4

APPLICATIONS

In this chapter we will look at applications of methods discussed in Chapter 3 to actual or experimental data. We will not always spell out the specifications of demand and supply relationships employed in the various studies but merely point out the experience of researchers in estimating different disequilibrium models.

1 Applications Under Various Price Adjustment Mechanisms

Fair and Jaffee (1972) estimated the U.S. housing market under alternative assumptions about the price adjustment behaviour. They estimated the disequilibrium model (3.9) - (3.12) by what they call Directional Method I and Directional Method II. Directional Method I consists in data segregation into demand and supply regimes by utilizing information on price change and then estimating demand and supply schedules from the segregated data. Observations during periods when price does not change are equilibrium values and treated as both demand observations and supply observations. As was noted on page 43 estimates under this method would be inconsistent because of the lack of independence of the error term and the explanatory variables in the segregated data. If OLS is applied to the segregated data then there is another source of inconsistency. This stems from the endogeneity of p in the model. A 2SLS method avoids this problem although it seems that Fair and Jaffee used OLS in their estimation under Directional Method I. Laffont and Garcia (1977) in their estimation of the Canadian business loan market have also tried a number of disequilibrium models and one of their formulations is model (3.9) - (3.12) which they have estimated under alternative price change hypotheses. One of their hypotheses is the same as that Fair and Jaffee (F. J. Hyp.) namely that price adjusts in the same period although the adjustment is not always completed and the other hypothesis (called the L. G. Hyp.) is that supply and demand depend on price prevailing at the beginning of the period and prices are revised in the following period on the basis of excess demand (i.e. $\Delta P_t = P_{t+1} - P_t$ in equation 3.11). Under the L. G. Hypothesis P_t is exogenous at time t and hence there is no problem of simultaneous equation bias in estimating demand and supply schedules. However, the use of OLS may still suffer from inconsistency on account of the lack of independence of the error terms and the explanatory variables (see Laffont and Garcia 1977).

The other method tried by Fair and Jaffee for estimating disequilibrium model (3.9) - (3.12) is Directional Method II. An important difference from Directional Method I is the treatment of equilibrium values i.e. how to separate observations into demand and supply regimes when $\Delta P = 0$. Under Directional Method I such observations are treated as both demand points and supply points. To implement Directional Method II for the housing model the periods during which price did not change were left unspecified initially except for the assumption that in each period either all observations were demand observations or supply observations. There were four such periods during which price did not change. All 16 ways of separating these periods into demand and supply regimes were obtained. For each sample separation likelihood function (2.5) was computed and the sample separation for which (2.5) was highest was chosen.[1] As Maddala and Nelson (1974) have pointed out the log likelihood function (2.5) is derived for a model given by equations (2.1) and (2.2) and is not appropriate for the disequilibrium model under consideration. The probabilities with which observations belong to the demand regime or the supply regime are determined within the model; Directional Method II overlooks this information.

Of the two methods, Directional Method I performed better for the Housing model. Although the signs of the coefficients were as expected under both methods the predictive

[1] Fair and Jaffee actually tried a slight variation on this method.

performance of Directional Method II was poor. It seems that Directional Method II is affected by extreme behavior of data. It also uses less information than Directional Method I since it classifies equilibrium values into one or the other of the two regimes and not in both, which perhaps explains its poor relative performance.

In the banking loan model of Laffont and Garcia the results from Directional Method I are roughly the same as those obtained by treating the loan market as an equilibrium model. They did not try Directional Method II.

Fair and Jaffee also used model (3.15) - (3.18) for the Housing market and used the Quantitative model first by imposing the constraint (see page 47) and ignoring the endogeneity of the variables and then by taking into account the endogeneity and ignoring the constraint.[1] The results are very close in both cases. Since the sample used was a large one and the results quite close it is difficult to say which method is preferable for small samples.

Laffont and Garcia also used the Quantitative model but assumed different adjustment speeds for periods of excess demand and excess supply. This is an important variation on the Fair and Jaffee model and results seem to justify this

[1]Fair and Jaffee also estimated the model by ignoring both the constraint and the endogeneity of variables.

assumption. In estimation therefore we do not have to impose the constraint that the coefficients of g_t and h_t are the same. But the two stage least squares method will not be asymptotically efficient because g_t and h_t are not linear functions of P_t.

Maximum likelihood methods have also been employed to estimate various disequilibrium models and we now briefly summarize the experience with these methods.

For the basic model (3.1) - (3.3) there seems to be a consensus that if the estimated probabilities suggest that most of the observations belong to only one of the regimes then the parameters of that regime are reasonably estimated but the parameters estimates of the other schedule are quite poor. Estimated probabilities can according to Maddala and Nelson (1974) show a heavy bias in favour of one of the two regimes because of poor specification of the model or due to the data not being informative. When these deficiencies are removed as was tried in some experimental work by Maddala and Nelson, the maximum likelihood method for the basic model does fairly well. The maximum likelihood method for model (3.9) - (3.12) also performs poorly when data is not informative but has done well in Monte Carlo experiments.

Laffont and Garcia have also used maximum likelihood methods to estimate models (3.15) - (3.18) of the business loan market. They estimate the following variants of the model. (i) L. G. hypothesis with no difference in the speed of price adjustment (ii) L. G. hypothesis with different speeds of price adjustment (iii) F. J. hypothesis with different speeds of price adjustment. For each of these cases the appropriate likelihood function is derived in the Laffont and Garcia paper.[1]

Estimates of the demand equation in the Laffont and Garcia paper do not differ very much under alternative estimation techniques. The authors think that this was to be expected since most of the observations in the period belonged to the demand regime. But they point out that there is a sensitivity of the estimates with respect to alternative price adjustment hypotheses and the different estimation techniques. In the estimation of the supply equation the results from different methods did not always agree. In particular the maximum likelihood methods picked up the influence of a certain variable and pointed out the lack of significance of another, which was in contrast to the results obtained under other methods and under the equilibrium hypothesis. Laffont and Garcia's conclusions is that the "weakness of the (disequilibrium) approach remains the sensibility

[1] Laffont and Garcia (1977) suggest that the likelihood function derived by Amemiya (1974) and Maddala and Nelson (1974) for the model (3.15) - (3.18) is correct if no endogenous variables appear in the demand or supply equation (page 1193). We think that Maddala and Nelson have derived the likelihood function when the endogenous variable (P) appears in the demand and supply equations.

of numerical results to alternative price adjustment behavior

Rosen and Quandt (1978) have estimated a model of the U.S. labour market. As the authors point out the study is perhaps not sophisticated enough to be taken as a guide for policy but it is one of the first proper approaches to the disequilibrium treatment of the labour market. The model they consider is of the form given by equations (3.30) - (3.33). The general form of the price adjustment equation they use is

$$\log W_t - \log W_{t-1} = \gamma\,(\log L_t^D - \log L_t^S) + \gamma_2 U_t + \varepsilon_t$$

where W is gross wage, L^D and L^S are the quantities of labour demanded and supplied respectively and U is the percentage of labour unionized.

The estimation technique used is the maximum likelihood method outlined on page 51. The results obtained are very encouraging, they accord with a priori expectations and are also in line with results from previous studies in this area. The authors also estimated an equilibrium version of the model but it gave poor results as some of the coefficients estimates were very large and some others had the wrong sign.

2 The Suits Model

An interesting example of a disequilibrium market is Suits' study of the Watermelon market. The model was first studied by Suits (1955) and has recently been reformulated as a disequilibrium model by Goldfeld and Quandt (1975). We briefly outline this model and then report on some results obtained by Goldfeld and Quandt in their empirical and Monte Carlo estimation of the reformulated Suits model.

Let q_t be the crop of watermelons, P_t the price and x_t the ex ante or intended harvest of watermelons, then the modified Suits model can be written as

(4.1) $$q_t = b_1 z_{1t} + b_2 + u_{1t}$$

(4.2) $$x_t = b_3 P_t + b_4 q_t + b_5 z_{2t} + b_6 + u_{2t}$$

(4.3) $$P_t = b_7 z_{3t} + b_8 y_t + b_9 + u_{3t}$$

(4.4) $$y_t = \min (q_t, x_t)$$

The z_{1t}, z_{2t} and z_{3t} are sets of exogenous variables that affect equations (4.1), (4.2) and (4.3) respectively and y_t is the actual harvest. Equation (4.1) describes how the crop is determined and equation (4.3) is a demand equation. Equation (4.2) expresses the fact that planned harvest and crop may not be equal. Planned harvest depends on the price of the crop and other variables z_{2t}. If price of the crop is low and (say) farm wages are high then farmers may plan not to harvest the entire crop in which case $y_t = x_t$ from (4.4). But it is also possible that intended harvest given by (4.2) exceeds the actual crop given by (4.1) in which case actual harvest equals actual crop i.e. $y_t = q_t$. More particularly, we can assume that p^*_t is such that $q_t = x_t$, i.e.

(4.5) $q_t = b_3 p^*_t + b_4 q_t + b_5 z_{2t} + b_6 + u_{2t}$

Thus,

(4.6) $$p^*_t = \frac{(1-b_4)\ b_5 - b_6}{b_3} + \frac{b_1\ (1-b_4)}{b_3} z_{1t} - \frac{b_5}{b_3} z_{1t} + \frac{1-b_4}{b_3} u_{1t} - \frac{1}{b_3} u_{2t}$$

Note here that

$$x_t = q_t + b_3\ (p_t - p^*_t)$$

and since that, apparently, there is no reason to believe that, a priori, b_3 will not be positive, it follows that $p_t \gtrless p^*_t$ according to whether $x_t \gtrless q_t$ which means that the sign of $p_t - p^*_t$ can be considered in order to discriminate between regimes. In cases where $p_t > p^*_t$ we have

$$y_t = b_1\ z_{1t} + b_2 + u_{1t}$$

$$p_t = b_7\ z_{3t} = b_8\ y_t + b_9 + u_{3t}$$

$$p^*_t = \frac{(1 - b_4) b_2 - b_6}{b_3} + \frac{b_1 (1-b_4)}{b_3} z_{1t} - \frac{b_5}{b_3} z_{2t} + \frac{1-b_4}{b_3} u_{1t} - \frac{1}{b_3} u_{2t}$$

and where $p_t < p^*_t$ we have

$$y_t = b_1 z_{1t} + b_2 + b_3\ (p_t - p^*_t) + u_{1t}$$

$$p_t = b_7 z_{3t} + b_8\ y_t + b_9 + u_{3t}$$

$$p^*_t = \frac{(1 - b_4) b_2 - b_6}{b_3} + \frac{b_1 (1-b_4)}{b_3} z_{1t} - \frac{b_5}{b_3} z_{2t} + \frac{1-b_4}{b_3} u_{1t} - \frac{1}{b_3} u_{2t}$$

The appropriate likelihood functions were obtained under each of the following specifications (1) q_t unobserved and sample separation unknown (1.A), q_t unobserved but sample separation known (2) q_t observed. Maximum likelihood estimates were then obtained for these specifications.

Results obtained for specifications (1.A) and (2) were "in general good". The estimated standard errors for specification (2) were smaller than specification (1.A.) which was to be expected since specification (2) uses more information. Estimation under specification (1) caused considerable computational problems which suggests that the likelihood function may have been "flat over extensive ranges". The authors succeeded in obtaining sub-optimal estimates and the values were not plausible. They think that the poor performance of specification (1) may be due to, "the simultaneous presence of specification error, inadequate data and excessive denial of information to the model".

Monte Carlo experiments were tried with specifications (1) and (2) and the results are very encouraging. Of the 50 replications of the experiment there was not a single computational failure. The value of using q data is also obvious in these experiments. Increasing the sample size reduces mean biases and the root means square error (RMSE) for both specifications. The RMSE for specification 2 declines faster relative to specification 1 which again reflects the importance of information on q_t.

Since specification (1) uses less information than specification (2) some variation with b_7 in (4.2) and the variance of the error term in (4.1) was tried to see the "effect of these variations on the value of additional information". Changes in b_4 confirmed the a priori expectation that the greater the value of b_4 the more valuable the information on q_t for the estimation of equation (4.2) and also (4.1). Changes in the variance of the error term σ_1^2 also confirmed the prior expectations that parameter estimates of specification (2) will deteriorate. Higher values of σ_1^2 were associated with higher RMSE for coefficients of equation (4.1) both for specification (1) and (2) but the effect was more pronounced for specification (2). A comparison of the ratio of the RMSE [of specification (2) to specification (1)] for the parameters of equation (4.2) indicates that the higher the value of σ_1^2 the more important the value of information on q_t for the estimation of equation (4.2).

In the experiments for specification (1), "σ_1^2 was constrained by $\sigma_1^2 = \frac{1}{k}\sigma_2^2$ "where k was taken to be the true value of σ_1^2/σ_2^2. Values of k equal to twice or one half of the true values were tried. "The estimation procedure seems relatively insensitive to the actual k value assumed".

A non-parametric test of the hypothesis that the maximum likelihood estimates are normally distributed was also carried out and the conclusion seems to be that the normally hypothesis cannot be rejected.

3 Consumption Goods Markets in Centrally Planned Economies

Recently, one of the most interesting studies in disequilibrium estimates is the paper by Portes and Winter (1980) on consumption goods markets in centrally planned economies (CPE's). Its originality lies in the fact that they consider the sustained repressed inflation situation of the CPE's (defined as excess demand in the consumption goods market) as a hypothesis to be tested rather than an admitted proposition. This allows the authors to deal with macro-economic relationships in a disequilibrium framework.

Assuming that prices do not adjust enough to equate demand for and supply of consumption goods and services in period t and that the minimum condition holds, we have

(4.7) $c^d_t = c^d (x^d_t) + u_{1t}$

(4.8) $c^s_t = c^s (x^s_t) + u_{2t}$

(4.9) $c_t = \min (c^d_t, c^s_t)$

Equations (4.7) is specified according to the Houthakker-Taylor saving function for households and equation (4.8) according to a theory of planner's behaviour. Equation (4.9) specifies the quantity traded at the micro-level making c_t

to be the sum of consumption demand in those markets in excess supply and of consumption supply in the remaining markets. It is clear that in this case c_t will be less than c^d_t or c^s_t unless either all markets are in excess demand or all in excess supply. A fall in income will lead to a fall in excess demand on each market and more and more markets will switch to excess supply and vice versa in case of a rise in income. When $c_t = c^s_t < c^d_t$ we have repressed inflation and buyers are rationed in their purchases. When $c_t = c^d_t < c^s_t$ planners would be rationed in sales. One might expect that such rationing will affect the labour market as it does in reality but Portes and Winter do not postulate market interaction in their model and estimation is not undertaken although it is argued that this may be soon possible.[1]

Appropriate likelihood functions of the observation (c_t = t = 1,...,T) conditional on (x^d_t, x^s_t; t=1,...,T) and on $c^d_t > c^s_t$ in excess demand regimes and $c^d_t < c^s_t$ in excess supply regimes are constructed and maximized. Ad hoc search procedures were adopted for the determination of a "well-defined" maximum for each country since the common algorithms failed to converge. It is argued that was possible

[1]An excellent study of multi-market estimation can be found in I to (1980). See also Gourieroux et al (1980).

because of the small number of parameters being estimated (9 or 10), the absence of constant terms in both equations and the large amount of a priori information on the value of the parameters. It is claimed also that the aggregate procedure used allows to estimate the probability π that a given point data is generated by an excess demand regime. The probability π_t is given by $\pi_t = H(x^d_t|\partial)$ where H (.) is the c.d.f. of $(u_{2t} - u_{1t})$, $x^d_t = \hat{c}^d_t - \hat{c}^s_t$ and $\partial = \sqrt{\sigma_1^2 + \sigma_2^2}$.[(1)]

One of the most important problems with which Portes and Winter were faced was to test their model against an alternative hypothesis (equilibrium model or they could ask for each observation what kind of regime it generates). Both methods have been tried in the past, the former by the authors (1978) and the latter by Howard (1976) and both did not give "satisfactory" and reliable results. That forced Portes and Winter to choose a different method, more particularly they used the results of the disequilibrium estimation as a sample separation technique and then applied usual OLS to the separated sub-samples providing hence an

[(1)]Note that these probabilities are marginal probabilities in the sense that they do not depend on the corresponding observed quantities as has been pointed out by Kiefer (1980a) but this does not immune us from the computational burden that might be ensued from such a task as Kiefer pretends.

alternative hypothesis testing and a check on the disequilibrium estimates. One might ask the usefulness of the method because of the small size of the samples (the overall sample consists approximately of 20 observations depending upon the country) but it is nevertheless argued that it provides a fairly reliable alternative hypothesis testing.

The results obtained show that excess supply was the dominant regime in 3 out of 4 countries. Comparing those results to OLS estimates for each country it turned out that the variance of the equation which appears in disequilibrium estimation to be the dominant regime is always lower than the variance of the other equation. It is argued that one explanation may lie on the fact that disequilibrium estimation picks up as the dominant regime the relationship which has the stronger specification but the same reasoning can be applied for OLS estimate making difficult to give any complete answer. The alternative hypothesis test was carried out using the method discussed above and the results were that "If there is disequilibrium and one uses equilibrium estimation, not only will the coefficients be affected but also the residuals may not at all indicate when one observation is far off the true schedule or even in which direction".

Carefully the authors do not draw any strong conclusions about their estimates and point out that many problems remain unsolved such as improvement on sample separation technique, aggregation in macroeconomics and in general all the problems involved in disequilibrium econometrics for testing hypothesis. Nevertheless they do believe that one may easily reject the hypothesis of sustained repressed inflation in the market for consumption goods and services in CPE's which seems to contradict all the previous beliefs on the matter.

4 Summary

1. It seems that the problem of multiple maxima is encountered very often in the applications of maximum likelihood methods as the computational experiences have shown. Likelihood functions which are the product of sums are more likely to have multiple maxima since they are high order polynomials. If the demand and supply equations do not have a constant term then the problem is less likely to arise. Because there is this uncertainty caused by the existence of multiple maxima it is necessary to check the reasonabless of the estimates by trying alternative techniques. It seems that numerical methods of optimization are being made in this direction.

2. All applications reported in this section look at disequilibrium in single market in isolation. Laffont and Garcia (1977), Rosen and Quandt (1978) and Portes and Winter (1980) point out that transactors may be constrained in other markets and these effects would spill over into the

market under consideration. The treatment called for is a general multi-market disequilibrium estimation of market which has already begun [see Ito (1980) and Gourieroux et al. (1980)], but empirical studies have not so far been attempted.

CHAPTER 5

THE DEMAND FOR PASSENGER TRANSPORT

Transport facilities have a powerful influence upon the distribution of population, industry and most other activities in the city. Thus, we have an apparent paradox: the use of transport facilities depends wholly upon the location of activities which in turn depends largely upon the transport facilities. "This seems to be a classical case of supply creating its own demand. We must, however, look closely at the meaning of demand: no other word is so misused in the literature of transport" (Thomson, 1974). Hence, in the following pages we are going to present some of the distinguishing characteristics of this demand and then go on in studying the construction and structure of the various models formulated in the literature. In the remainder of this chapter two travel demand models which have been estimated for Canada are discussed and their weakness as a tool of forecasting travel demand is pointed out. Finally, it is presented how this type of model can be reformulated and estimated in a totally different disequilibrium approach.

The Characteristics of the Demand for Passenger Transport

One of the distinguishing features of transport, in contrast with most industries, is that quality is also a function of demand (indeed we know already that cost is a function of quality and demand a function of price and quality). Even though cost and quality are in theory interchangeable, in practice there are limits to the costs that can be accepted in order to maintain quality. More specifically, high levels of demand create congestion. This overcrowding phenomenon, with all associated inconvenience and discomfort is common to all forms of transport: it is an inevitable result of short sharp peaks in demand for which it would be unreasonably costly to provide extra capacity. On the other hand, low levels of demand also affect quality with all forms of public transport, except for continuous belt systems. Service frequency is an important quality aspect which, again for reasons of cost, is likely to suffer from low levels of demand. Thus, transport can be, and is, provided by most modes even when demand is quite small, but inevitably the quality is minimal in many aspects. A consequence of this type of interaction between quality and demand is that "it makes quality differentials more difficult both during peak periods, if shortage of capacity militates against the provision of de luxe option and during off-peak periods if lack of demand makes it impossible to provide more than one quality service without severe loss of frequency" (Thomson, 1974).

Another important characteristic of demand for transport which differentiates it from other products (e.g. food, clothing, etc.) is that although transport plays an important and essential role in the standard of living of a developed economy "yet the demand for transport is not fundamental in human nature" (Bonavia, 1954). To some extent, therefore, transport is analogous to other services for which demand is very strong although direct demand is weak or non-existent, for example, justice, police, national defence. These cases all exhibit the same characteristic: that is, the individual has a low conscious demand but "the collective derived demand is however strong enough to ensure the supply at the expense of the community" (Thomson, 1974). The analogy between transport and justice or defence is, nevertheless, only partial since the latter services are cases in which there is no response to individual demand through competitive supply. The system of prices breaks down when it is a question of providing an elementary school, or a police station: supply is not directly related to individual demand, even though the public authority should be making the best use of the public's money. As far as transport is concerned, in some cases (the construction of roads from the proceeds of taxation, legal provision of workmen, (transit or railway etc.) the community similarly bridges the gap between demand and supply by special action which ignores the criterion of "effective demand price". More often than not, a large part of the demand for transport can be and is effectively expressed as a money price, especially in a highly organized industrial community.

A last point should be emphasized at this stage of the analysis: this is that transport is by essence an intermediate good, scarce are the cases where an individual consumes transport as such. There is then, a big variety of transport consumption and consequently as many different demand functions. The first consequence is that demand studies at a high geographical level have little economic significance. The same natural aggregate values (traveller per mile or ton per mile) could correspond at radically different situations. A second consequence of the variety of transport is the necessity to distinguish finely the class of economic agents involved and purposes of transport. There is the same relationship between travel to work and personal travel as there is between butter and tanks.

Given the characteristics described above, should one explore the underlying theory of demand as it relates to urban transport demand decisions?

The answer is given by T.A. Domencich and D. McFadden (1975), when they write: "There is a need to develop models of urban travel demand which can satisfy the end objective of transportation planning (1) the fine running of costing public transports and transit related tax policy by adjusting fares, headways, feeder service, toll, etc., within given budget constraints to maximize social benefit; (2) the estimation of benefit for alternative designs of new transit systems; (3) simulation of the urban economy projections of long term transit needs. These objectives require demand models that are sensitive to transportation policy and that depend implicitly on policy variables so that the effect of policy alternatives can be forecast". Consequently, if travel demand models are to forecast constructively the effects of policy changes they ought to be causal, establishing the behavioural link between the attributes of the transportation system and the desire of the individuals. According to the authors, in order to be a productive tool for policy making, the model should also be flexible, transferable and efficient.

One should, however, insist on the importance of the errors the economist is willing to make when the information available to him about economic characteristics of the demand are insufficient. More precisely, a source of errors may be a bad appreciation of the economic variables involved such as underestimation or overestimation of collective units In addition, errors can be related to the apprehension of the structure of the demand function. Broadly speaking, given the actual level of knowledge on models of demand, rich information on individual consumption demands are required in order to derive significant conclusions on the process of decision making.

2 The Construction and Structure of Models

The discussion here is altered at this stage of the analysis since the structure is determined by the hypotheses (explicit or implicit) which are made in connection with the behavior of the uses of transport.

A first problem one faces is what data to use. Thomson considers that most models are inefficient and useless because they use cross-sectional rather than time-series analysis. M. Rousselot and P. Gluntz (1976) are less affirmative. Even though the classical experimental base for the analysis is the time-series, they admit that there are some problems associated with these data: first, those data are not always available since they deal with the complex motion of transport economics and even if they are available, their use is very delicate because of the specific characteristic of the transport market and, in particular, the important role played by the factor quality which, very often, makes it difficult to isolate a "price" effect or a "time" effect. On the other hand, the use of cross-section data creates the opposite problem, that is, one should determine very carefully the influences attributed to the supply. Every error on the former will have an amplified effect on the latter.

We will now attempt to derive the hypotheses most commonly used, to critize them and to suggest possible research areas. These hypotheses cover essentially three elements: (1) the nature of choices available to the individual, (2) the order in which those choices are made, (3) the variables which will allow us to explain them.

1. For the nature of choices explained by the models, one should notice that their number in every model is limited (usually depending on the data available). The limitations of choice is fundamental: this means that other alternatives are not considered by the individual and that a change in the relevant parameters will not influence the analysis. One should accept this hypothesis as far as the purpose of travel is concerned. Usually one can distinguish the travel from home to work, for business, for shopping, for leisure (friends' visit, sport, etc.). Even though some trips may have mixed purposes, there is usually quasi-independence among the demands corresponding to different purposes.

A more fundamental limitation in the models analysing the demand is that they concentrate only on a particular mode. So that, more often than not, road traffic for long distance trips is treated independently of railway or plane which are substitutes for this type of trip. Even if there are not direct substitutes, it would have been more interesting to treat globally the trip by all modes. In addition, causal models should be able to explain causal arbitration among destinations.

2. The traffic models dealing with the process of individual choice are usually composed of submodels, chained up in the following order: determination of traffic from one zone, received by another, known as generation models; distribution of this traffic among different destinations and origins (O-D studies); distribution among modes (modal split models); distribution over network.

The authors of those models assume that the individuals are ordering their choices by following the above scheme, something that has never been verified. As R.W. Schmenner (1976) writes: "contrary to the implicit logic of these conventional techniques, the tripmaker does not first decide to take a trip, then decide where to go and only then decide how to get there. Rather the tripmaker's decisions on where, how and when are made simultaneously. Demand analysis should not force a rigid and artificial analysis". It is easy to find counter examples to the above scheme: we may think of certain car owners who never consider moving by any other mode than their car.

T. Domencich and D. McFadden try, in their model, to avoid this criticism by introducing the notion of "choice situation". Each of these situations corresponds to a precise order in which the decisions are supposed to be taken and to a particular domain of substitutability at each level of choice. They are able to do that by defining probability functions corresponding to a particular choice situation. Those probability functions depend upon x vector of observed attributes for each alternative. The utility function of an individual is then composed of two elements: a non-stochastic one which reflects the representative taste of the population and a stochastic that "reflects the effect of individual idiosyncrasies in tastes or unobserved variation in attributes for each observed attribute vector" (McFadden, 1974). Some examples of choices at given levels are:

- a choice of mode of transport, given the destination and having decided to travel;
- a choice of destination given the mode and the decision to travel;
- a choice between the trip and non-trip situation, given the destination and the mode.

This idea is a very interesting one but one should bear in mind the difficulty of defining the universe of possible choices which must be finite in order to be used in the model. One can assume that the results of such a model will be much different from those of the classical model since, if nothing else, they are closer to the reality.[1] An additional difficulty is that if such a model is to be operational it must be the case that the probability that an individual makes a trip in certain conditions is based on objective variables, either on the individual or on the type of trip. For Domencich and McFadden this role is fulfilled by the "attribute" of the trip which enters the utility function. Furthermore, this method is based on the assumption that the individual faces only binary choices (e.g. choice between auto and bus transit modes, choice between trip and non-trip situation). Finally and most importantly, even though the authors recognize the intermediable nature of the demand for transport and hence the fact that this demand is a derived one, they nevertheless specify the travel demand and the demand for other consumption goods simultaneously.

3. The nature of the explanatory variables: the big variety of transport requires that one should characterize a trip not only by its price but also by factors of quality which must, in principle, define perfectly every possible substitute, even those with different destinations at different periods.

[1]For more details, see the work by Domencich and Mc Fadden (1975), and for an application of their theory, see McCarthy, P. (1977).

We will not analyse here extensively the general socio-economic variables which are fundamental to the explanation of traffic, for reasons given above.[1] One point should be made, however, about the role played by the variable "car ownership". Most of the models treat this variable as exogenous, yet W. Y. Oi and P.W. Shuldiner (1962) believe that "theoretical consideration and available empirical evidence suggest that vehicle ownership is clearly not an exogenous variable, especially over time. Indeed, the future urban transportation system may itself influence the level of car ownership to the extent that this variable will be affected by such factors as (a) familiar travel demand, (b) quality of transport service, (c) adequacy of parking facilities and street capacity, (d) the development of pattern of the city. An analysis which treats car ownership as the variable to be explained, in our opinion, sheds considerable light on the phenomenon of urban travel". In addition, some models consider in their analysis some behavioural variables (Domencich and McFadden, McCarthy) defined by the answer or combination of answers in motivation studies.

The most fundamental variables, which are introduced in all models, are time and price. In the most simple ones,

[1] For an extensive and complete analysis of those variables one can refer to W.Y. Oi and P. Shuldiner (1962), or Martin et al (1961).

those variables are replaced by the distance.[1] The evaluation of those on these apparently simple variables is in fact quite complicated. Since (a) individuals consider in their choice total price and time of trip for a given O-D. But the specific part of those total attributes for transport itself is usually very small. (For time, we have travel time, terminal time, waiting time to and in the station, walking time, etc.) so that the evaluation of those elements becomes delicate; they require a more detailed analysis. (b) the non-existence of a unique market for transport prevents us from using "objective prices", imposed on the individual and which will guide their decision. The subjective prices on which are based the economic agents in order to make the choices, are sometimes, different from prices that one could calculate. An example of the differences between the perceived and actual prices is given by A. Baum (1973), who presents the total cost per passenger item of journeys by public and private transport at three different dates, 1964, 1966 and 1968. The study establishes that private transport is at least 60% more expensive than public transport. The studies carried out in the U.S. established that 72% of motorists travelling to work have never made an estimate

[1] The simplification is somehow abusive since the coefficient of proportionality with the distance varies according to the mode and infrastructure.

of their trip cost. (c) The same phenomenon is observed as far as time is concerned, subjective time is probably different from the objective one, calculated as the sum of elements enumerated in (a). (d) In several studies the variables used is not the absolute value of time or price but rather the difference of the ratio between their values for alternative solution.

As emphasized in the beginning of the paragraph, time and price are not enough to explore the choices of the individuals, one must find some other subjective factors influencing the demand. For passengers transport, comfort is an important element in motivation studies. In a sense, safety is the most important factor of all, "but this does not mean that it is the most important in affecting the choice of transport method by passengers as the difference in the level of danger between, for example, train and coach does not appear significant to most people"[1], (Sharp, 1973). Beesley (1965), in order to compare the public transport mode with the private one, considers that the value of time is smaller for modes with low comfort (generally public transport), he assumes implicitly that the perception of discomfort is proportional to the time one undergoes it.

All those factors comprise what is called the generalized cost. The immediate problem one is facing then is to

[1] This view is also shared by Baum.

translate those variables into monetary terms. This is an important problem since it will be useless to improve the mathematical formulation of the models without defining precisely the variables which influence the choice of economic agents.

In concluding, it should be pointed out what the topics which speed further development in this area are:

(1) We have noticed in the previous pages that however sophisticated the mathematical and statistical tool used, the model, in order to be a useful tool for prediction and forecasting, should be based on realistic hypotheses concerning economic agents. Consequently, research in this field of psycho-sociology applied to transport is more likely to open new paths in this area as would the study of a typology of economic agents so that the behavior of the agent belonging to a particular category is more homogeneous. This typology should be based on objective variables such as income, size of household, etc. and lead to a definition of typical behavior. One of the most important aspects of this behavior which has been completely neglected is the problem of distribution over time which is translated by the notion of elasticity of relation among trips at different moments.

(2) The research would not be successful if it were not based on real data; thus the need for improved transport statistics will go on growing. These improvements could

be directed towards a more detailed classification of transport. The data actually collected treat only of macro-economics values on a national level and are useful as indices of global growth on the market. On the other hand, O-D studies are limited to some zone and some modes. The heterogeneity of such studies creates great problems of aggregation and of adjustment. A great progress is to be expected from the collection of homogeneous time-series data. In addition, carrying out full-scale experiments to vary fares, speed, grand comfort provoked deliberately on the base of concrete facts, could be very useful. Such experiments should allow the observation of marginal effects of the variables under study, other things given.

3. The S.L.A.G. Travel Demand Model

Until now, we have focused on the characteristics of the demand for passenger transport and contrasted it to the demand analysis in general. Furthermore, we have analyzed and critized the construction and structure of models dealing with the demand for passenger transport. In summary, we reached the conclusion that all models encountered display deficiencies on both theoretical and practical grounds. It is not our task here to propose alternative models that will palliate the shortcomings exposed above, but rather to draw from the literature a representative model of the dominant stream of thought for the purpose of applying disequilibrium econometrics. The reader should bear in mind then that the criticisms previously raised apply to the model considered hereafter.

The Spatial Linkage Analysis Group (S.L.A.G.) in the Urban Program Evaluation Directory of the Ministry of State for Urban Affairs developed a travel demand model in 1976 used to predict total passenger travel and passenger shifts between modes. This model is based on another model which was first formulated by S.C. Monsod (1956) for estimating the demand for rail passenger services in the North-East corridor of the U.S.

In its more general form the S.L.A.G. model can be written as follows:

(5.1)

$$V_{ti} = \left[e^{\alpha_1} P_t^{\beta_1} L_t^{\gamma_1}\right] \cdot \left[\sum_{i=1}^{m} C_{ti}^{\alpha_2} H_{ti}^{\beta_2} F_{ti}^{\gamma_2}\right]^{\delta} \cdot \left[e^{K_2} C_{ti}^{\alpha_2} H_{ti}^{\beta_2} F_{ti}^{\gamma_2} \Big/ \sum_{i=1}^{m} e^{K_2} C_{ti}^{\alpha_2} H_{ti}^{\beta_2} F_{ti}^{\gamma_2}\right] \cdot (A_{ti})$$

where:

V_{ti}: Forecast travel demand for city-pair t on mode i

P_t : Population cross-products, city-pair t.
(e.g. product of the population of the city modes)

L_t : Linguistic pairing index, city-pair t.

C_{ti}: Cost of fare of mode i for city-pair t (one way)

H_{ti}: Travel time (hours) of mode i for city-pair t (one way)

F_{ti}: Departure frequency (per week) of mode i for city-pair t (one way)

K_i : Modal constants which may be interpreted as modal acceptability factors representing the unmeasured convenience variables involved in intercity travel.

A_{ti}: A city-pair modal specific adjustment factor.

The equation is composed of three terms:

(1) Potential for travel. This term calculates the potential to interact from demographic and cultural attributes of the modes.

(2) Impedence against travel. This term calculates to what extent the potential for travel is achieved. The more attractive the physical quality attributes and fares of the various modes, the more likely are passengers to use them, and

(3) Modal split. This term calculates the proportion of travellers who choose each mode. Again, the physical quality attributes and fares are the variables; for a particular mode, the more attractive they are relative to those of the other modes, the more likely is a person to select that mode.

The model presents several difficulties regarding its calibration[1] especially because of deficiencies in the

[1] Before estimates of passenger movements could be made it is necessary to adjust the model's parameters so that the simulation of existing travel demand reasonably replicate base year conditions. The accuracy sought depends to a considerable extent on the policy issue under discussion. For more details on the matter see J.R. Meyer (1971).

calibration data set, the dominance of the automobile as a mode of passenger travel, the mathematical structure of the modal-split term which uses the automobile attributes as base, the exclusion of an income factor, the exclusion of supplementary changes and the sensitivity of calibration to change.

An earlier version of this model was presented and estimated in a study by the Canadian Transport Commission (C.T.C) in 1970 which focused on the demand for common carriers only (rail, bus, air) and using the automobile as the competing mode. In that case one can regroup the common carrier in one category and the automobile in the other:

$$(5.2) \qquad V_{tj} = K.P_{t_A}^{\alpha_1}.P_{t_B}^{\alpha_2}.L_t^{\beta_1}.e^{-\beta_2/r_{t_A}}.e^{\gamma(H_1-H_2)}.(C_2-C_1)^{\delta}.$$

$$(\sum_j K_j H_{tj}^{\alpha'} C_{tj}^{\beta'} e^{-\gamma'/F_{tj}})^{\theta}$$

where:

V_{tj}: total annual trips generated from t_A to t_B by one of the j common carriers.

t_A, t_B: the origin A and destination B of a trip in city-pair t.

r_{t_A}: fraction of families with annual income above a certain level in t_A.

H_1: highway driving time, center to center (hours)

H_2: average total trip time by common carrier, weighted according to the modal split (hours)

C_1: perceived cost of automobile

C_2: average total trip cost by common carrier, weighted according to the modal split (dollars)

Those two models are very similar in their structure since they propose to estimate the demand for passenger travel between city pair on the basis of a similar modal split. Although the second model differs from the previous one by the introduction of socio-economic characteristics of the cities involved.

Even though the S.L.A.G. model, appeared to perform better statistically than the one estimated in 1970 by the same commission we deliberately chose not to investigate it for several reasons. As admitted by the authors the S.L.A.G. model suffered from severe limitations and biases given its highly aggregated form, the lack of reliable data and the choice of explanatory variables (mode attributes). It is more tractable empirically to concentrate on the common carriers given the numerous common characteristics. Albeit our acceptance of model (5.2) in opposition to the S.L.A.G. model the results obtained were not satisfactory. Indeed, regarding the modal split estimation unusually low confidence limits had to be taken (80%) for the results to have any meaning. Furthermore, no mention is made about the significance level of the estimation as well as about the behavior of the residuals. Regarding the estimation of the total carrier demand function we can observe that the study does not give any explanation about the specification adopted or about the exclusion of such explanatory variables like the comparison of frequency of services between modes. It should be pointed out also that the statistical properties of the estimates are extremely poor (relatively high degree of correlation between the exogenous variables, low confidence limits and non-significance of the parameter estimates as given by the t-statistics results).

It is our contention that, besides the improvements that could be made regarding the specification of the model, the results would improve dramatically if the model is estimated in a disequilibrium framework.

4 A Disequilibrium Travel Demand Model

In the previous paragraph we have presented the S.L.A.G. model, in its equilibrium formulation, and have discussed the drawbacks of the underlying assumption.

Here, the discussion will be centered on the modal split demand since this constitutes the core of the S.L.A.G.'s model estimates and it is where the equilibrium hypothesis is most needed. Indeed, if the statistical properties of the parameter estimates for this demand could be improved then the entire S.L.A.G. model would be a more reliable analytical and forecasting instrument.

It should be stressed here that assumptions underlying the structure of the model are that the cost and time values are such that the respective markets, in which those prices prevail, are in equilibrium and that the time and cost preferences for all modes are the same which implies that the parameters appearing in the modal split equation are common to all modes. This second hypothesis seems to be reasonable since we do not have reason to believe that people's time-cost preferences should be affected by the choice of mode, neither by the distance involved in each trip. However, the first assumption regarding the state of equilibrium in which the various markets are supposed to be, is more difficult to endorse. Indeed, this assumption overlooks important characteristics of the markets under consideration. In particular, it is commonly accepted, at least for some modes and for relatively short periods of time, that the supply of services is fairly fixed (air and rail carriers) since any increase of availability of services involves intensive investment outlays, the realization of which usually requires large periods of time. It is precisely why we observe such phenomenon as either an excess supply, for some modes, and/or during given periods of time, excess demand or congestion. We believe that those phenomena point towards a disequilibrium situation and consequently the econometric techniques used are inappropriate. Furthermore, our argument is substantiated by the fact the fares practiced, the entry in the industry and the level and quality of services offered are very closely regulated, hence preventing the mechanism to operate and clear the markets. Finally, it could be argued, that, in fact, users of transport facilities tend to internalize congestion into their demand by adapting their choices according to the behavior of the market. This implies that the decision of the individual is a dynamic one and his decision-making process is inter-temporal where the individual might decide to forego a trip in the present period and postpone it for later on. As it has been already mentioned (see page 52), Dagenais (1980) attempted to deal with this problem and has shown that a priori nothing can ensure that the future markets will be in equilibrium. In fact, the author contends that they are more likely to be in disequilibrium.

Following then, the 1970 version of the S.L.A.G. model, we will consider the modal split demand only for common carrier. Let

(5.3) $$d_{tj} = x'_t \beta + u_t$$

(5.4) $$\bar{S}_t = \sum_{j=1}^{m} S_{tj}$$

(5.5) $$y_{tj} = \min (d_{tj}, \bar{S}_t)$$

with:

$j = 1,2,\ldots,m$ (modes available for common carriers)

$t = 1,2,\ldots,T$ (city-pairs)

and,

$x'_t = (H_{tj}; C_{tj}; F_{tj}; K_j)$ (vector of exogenous variables as defined in (5.2)

$\beta = (\alpha', \beta', \gamma'; 1)$ (vector of unknown coefficients as defined in (5.2))

d_{tj} = quantity demanded on city-pair t for mode j.

S_{tj} = the volume of the supplied services on city-pair t for mode j.

$\overline{S}_t$ = total capacity of the system, assumed here fixed.

y_{tj} = observed volume of passengers carried.

This presentation is deliberately fairly general and can be specified in various ways to fit different assumptions on the nature of the disequilibrium in the market. Indeed, the model implies that the entire system might be in disequilibrium (for all modes and for all city-pairs). But we know that some city-pairs are more attracted to each other than others (Montreal-Toronto, Ottawa-Montreal, Toronto-Ottawa) and that some modes have a more flexible supply than others, hence the possibility of disaggregating the model in 2 directions:

(a) take a given city-pair (t=1) and study the modal split on this particular route using time series data, then (5-3) (5-4) (5-5) can be rewritten as:

$$d_{1j} = x'_1\beta + u_1$$

$$\overline{S}_1 = \sum_{j=1}^{m} S_{1j}$$

$$y_{1j} = \min (d_{1j}, \overline{S}_1)$$

(b) take a given mode and study the behavior of the parameters in two different ways, either using time series, study a particular city-pair system or, using cross-section data, investigate the market for a particular mode. In the first case, assuming t = 1 and j = 1, the model becomes:

$d_{11} = x'_1 \beta + u_1$

$\overline{S}_1 = S_{11}$

$y_{11} = \min (d_{11}, \overline{S}_1)$

The formulation of the second case presents no additional difficulty. One should note that the particular functional form those equations may take need not concern us here, since they depend as much upon theoretical as empirical considerations. Indeed, it is common practice among the authors we have encountered in this study to advance no justifications regarding the choice of particular specifications adopted. It seems that this choice is a question of practical convenience rather than theoretical rationalization.

In order to estimate the coefficients of the demand one can observe, following Hartley's (1976) suggestion, that the censored normal regression is applicable. Two methods of estimation are available: the maximum-likelihood estimation (M.L.E.) and the Non-linear least squares and weighted least squares. We first discuss the full-information maximum likelihood estimator and then present certain limited-information methods.

Let $f_t(d_t|x'_t)$ denote the (conditional) density of D_t, given $X_t = x'_t$. Let $F_t(\cdot)$ denote the (conditional) c.d.f. of D_t and let $g_t(y_t|x'_t)$ denote the density of the observed quantity Y_t given $X_t = x'_t$. Hence,

$$g_t(y_t \mid x'_t) = \begin{cases} f_t(y_t \mid x'_t) & y_t < \bar{S}_t \\ 1 - F_t(\bar{S}_t \mid x'_t), & y_t = \bar{S}_t \end{cases}$$

In the present case $f_t(\cdot)$ is normal with mean $x'_t\beta$ and variance σ^2. Then let $\tau = \{1, 2, \ldots, T, \ldots mT\}$ denote an index set for a sample of mT observation on y_t, $\bar{s}_t$ and x'_t. Let τ_D denote the subset of observations where d_t constraints y_t i.e., $\tau_D = \{ t: y_t < \bar{s}_t, t \in \tau\}$ and let $\tau_s = \tau - \tau_D$, the complement of τ_D, denote the observation where the supply side is binding, i.e., $y_t = \bar{s}_t$. Then, from the density of the observed quantity we have the likelihood function,

$$L_\tau(\beta, \sigma^2 \mid y, \bar{s}, X) = \prod_{t=\tau_D} f_t(y_t \mid x_t) \cdot \prod_{t=\tau_s} [1 - F_t(\bar{s}_t \mid x_t)]$$

The MLE's for the parameter vector β and σ^2, $\hat{\beta}$ and $\hat{\sigma}^2$ should maximize L_τ or $\log L_\tau$, i.e.

$$\log_\tau(\hat{\beta}, \sigma^2) = \sup_\theta \{\log L_\tau(\beta, \sigma^2)\}$$

where $\theta = [\beta', \sigma^2]'$

As for the nonlinear least squares (N.L.S.) method, Hartley suggests to use the nonlinear weighted least squares (N.W.L.S.) because the N.L.S. estimator even though strongly consistent and asymptotically normal, have heteroskedastic residuals. But then again, the N.W.L.S. estimator is inconsistent. To remove the inconsistency Hartley suggests an iterative procedure (I.N.W.L.S.) which will give a more efficient and asymptotically normal estimator than the N.L.S. It is his conjecture, however, that the M.L.E. may be shown to be asymptotically more efficient than the I.N.W.L.S. and hence the N.L.S. estimator, although the finite sample properties remain unknown. This question is still open and only an empirical investigation could settle this problem.

CHAPTER 6

CONCLUSION

In this dissertation, our analysis was threefold. First, we have reviewed the literature on the methods of estimation for switching regressions and the estimation for markets in disequilibrium where care was taken to point out the particularities and similarities among those methods and to discuss critically the assumptions underlying those techniques. Secondly, the applications and results related to these techniques are compared and evaluated and thirdly, the estimation of the demand for inter-city passenger as a particular application of the disequilibrium techniques is proposed.

In particular, the analysis on the switching regressions led us to the fact that the estimation techniques require maximization of non-linear functions and hence manipulation of very tedious expressions requiring almost always numerical methods of optimization. Furthermore, in cases where the proportionality factor between the variances of the 2 regimes is unknown, the likelihood function is unbounded and consequently global maximization will lead to inconsistent estimates. Important headway has been made in estimation techniques of switching regressions although the empirical performance of some of these methods has not always been satisfactory given the computational difficulties encountered and the unknown finite sample properties of the estimates. Much depends on the form of the likelihood functions and the information content of the data.

The same problems persist with the estimation of disequilibrium models. Besides the more complex and expensive estimation techniques and the problems of existence of a global maximum, the problem of internal consistency of the models has to be reckoned with. Although statistical properties (consistency and asymptotic normality) of the likelihood estimators for the basic disequilibrium model have been established, it was not the case for other disequilibrium models. Of the little empirical evidence we have, multiple maxima are often encountered in the application of maximum likelihood methods. This problem is more likely to arise when a constant term appears in the demand and supply equations and is aggravated when the likelihood functions are the product of sum; for this reason it is recommended to try alternative techniques.

Finally, the analysis of the characteristics of the demand for inter-city passenger transport and the discussion of the structure and results of the existing models have led us to the conclusion that disequilibrium techniques are appropriate for estimating this type of demand. We have suggested a particular specification of a model where the supply was fixed and proposed an estimation technique. The formulation of this model was deliberately general to allow particular specification according to the problem at hand. However, an estimation of this model will have to tackle impediments associated with the aggregate nature of the formulation and the paucity of data. An important but difficult area in which empirical research needs to be directed is models of multi-market disequilibrium. Unfortunately, these models tend to be either inconsistent or have multiple solutions.

BIBLIOGRAPHY

Amemiya, Takeshi. "Regression Analysis When the Dependent Variable is Truncated Normal". Econometrica. 1973, (41), pp. 997-1016.

Amemiya, Takeshi. "Multivariate Regression and Simultaneous Equation Models When the Dependent Variables are Truncated Normal". Econometrica. 1974, (42), pp. 999-1012.

Amemiya, Takeshi. "A Note on a Fair and Jaffee Model", Econometrica. 1974, (48), pp. 759-762.

Baum, A.J. "Free Public Transport", Journal of Transport Economics and Policy. 1973, (7), pp. 3-19.

Beesley, M.E. "The Value of Time Spent in Travelling: Some New Evidence", Economica. 1965, (32), pp. 174-185.

Beesley, M.E. & Foster, C.D. "Estimating the Social Benefits of Constructing an Underground Railway in London", Journal of Royal Statistical Society. 1963, (126 series A), pp. 46-93.

Beesley, M.E. & Foster, C.D. "Victoria Line, Social Benefit and Finance", Journal of Royal Statistical Society. 1965, (128 series A), pp. 67-88.

Bonavia, M.R. The Economics of Transport. Cambridge University Press, 1954.

Bowden, R.J. The Econometrics of Disequilibrium. North-Holland Publishing Co., Amsterdam, 1978.

Binder, D.A. "Comment on Quandt and Ramsey". Journal of the American Stastical Association. 1978, (73), pp. 746-747.

Bryant, P. "Comment on Quandt and Ramsey". Journal of the American Statistical Association. 1978, (73), pp. 748-749.

Behboodian, J. "On a Mixture of Normal Distribution". Biometrika. 1970, pp. 215-217.

Clarke, B.R. & Heathcothe, C.R. "Comment on Quandt and Ramsey". Journal of the American Statistical Association. 1978, (73), pp. 749-750.

Culley, E. "Demand Model". Intercity Passenger Transport Study, Canadian Transport Commission, Research Branch, pp. 49-57.

Dagenais, M.G. "Specification and Estimation of a Dynamic Disequilibrium Model", cahier 8046, Université de Montréal.

Davies, G.N. "The Effect of Subway on the Spatial Distribution of Population:, Journal of Transport Economics and Policy. 1976, (10), pp. 126-136.

Domencich, T.A. & McFadden, D. Urban Travel Demand. North-Holland Publishing Co., Amsterdam, 1975.

Drazen, A. "Recent Developments in Macroeconimic Disequilibrium Theory". Econometrica. 1980, (48), pp. 283-306.

Fair, R.C. & Jaffee, D.M. "Methods of Estimation for Markets in Disequilibrium". Econometrica. 1972, (40), pp. 497-514.

Fair, R.C. & Kelejian, H.H. "Methods of Estimation for Markets in Disequilibrium". Econometrica. 1974, (40), pp. 177-190.

Fitch, L.C. et at. Urban Transportation and Public Policy. San Francisco, 1964.

Foster, C.D. The Transport Problem. London, 1975.

Fowlkes, E.B. "Comment on Quandt and Ramsey". Journal of the American Statistical Association. 1978, (73), pp. 747-748.

Gerondeau, C. Les Transports Urbains. Presses Universitaires de France, Paris, 1969.

Goldfeld, S.M., Jaffee, D.M. & Quandt, R.E. "A Model of FHLBB Advances: Rationing or Market Clearing". Review of Economics and Statistics. 1980 (LXII), pp. 339-347.

Goldfeld, S.M. & Trotter, H.F. "Maximization by Quadratic Hill climbing". Econometrica. 1966, (34), pp. 541-555.

Goldfeld, S.M. & Quandt, R.E. Non-linear Methods in Econometrics. North-Holland Publishing Co., Amsterdam, 1972.

Goldfeld, S.M. & Quandt, R.E. "A Markov Model for Switching Regressions". Journal of Econometrics. 1973 (a), (1), pp. 3 - 15.

Goldfeld, S.M. & Quandt, R.E. "The Estimation of Structural Shifts by Switching Regressions". Annals of Economic and Social Measurement. 1973 (b), (2), pp. 475 - 485.

Goldfeld, S.M. & Quandt, R.E. "Estimation in a Disequilibrium Model and the Value of Information". Journal of Econometrics. (1975, (3), pp. 325 - 348.

Goldfeld, S.M. & Quandt, R.E. Studies in Non-linear Estimation. Ballinger Publishing Company, 1976.

Goldfeld, S.M. & Trotter, H.F. "Maximization by Quadratic Hill climbing". Econometrica. 1966, (34), pp. 541 - 555.

Gourieroux, C. Laffont, J.J. & Monfort, A. "Coherency Conditions in Simultaneous Linear Equation Models with Endogenous Switching Regimes". Econometrica. 1980, (48), pp. 675 - 695.

Gourieroux, C., Laffont, J.J. & Monfort, A. "Disequilibrium Enconometrics in Simultaneous Equation Systems", Econometrica. 1980, (48), pp. 75 - 95.

Grossman, H.I. "The Nature of Quantities in Market Disequilibrium". American Economic Review. 1974, (64), pp. 509 - 514.

Gwilliam, K.M. & Mackie, P.J. Economics and Transport Policy. George Allen and Unwin Ltd., London, 1975.

Harrison, A.J. & Quarmly, D.A. "The Values of Time in Transport Planning". Sixth Round Table on Transport Economics. Paris, 1969.

Harrison, A.J. The Economics of Transport Appraisal, London, 1974.

Hartley, M.J. "The Estimation of Markets in Disequilibrium: the Fixed Supply Case". International Economic Review. 1976, (17), pp. 687 - 699.

Hartley, M.J. "Comment on Quandt and Ramsey". Journal of the American Statistical Association. 1978, (73), pp. 738 - 741.

Hamermesh, D.S. "Wage Bargains, Threshold Effect, and the Phillips Curve". Quarterly Journal of Economics. 1970, (84), pp. 501-517.

Hartley, M.J. & Mallela, P. "The Asymptotic Properties of a Maximum Likelihood Estimator for a Model of Markets in Disequilibrium". Econometrica. 1977, (45), pp. 1205-1220.

Henscher, D.A. "The Value of Commuter Travel Time Savings". Journal of Transport Economics and Policy. 1976, (10), pp. 167 - 176.

Hosmer, D.W. "Comment on Quandt and Ramsey". Journal of the American Statistical Association. 1978, (73), pp. 741-744.

Howard, D. "The Disequilibrium Model in a Controlled Economy: An Empirical Test of the Barro-Grossman Model". American Economic Review. 1976, (66), pp. 871 - 879.

Ito, T. "Methods of Estimation for Multi-Market Disequilibrium Models". Econometrica. 1980, (48), pp. 97 - 125.

Johnson, N.L. "Comment on Quandt and Ramsey". Journal of the American Statistical Association. 1978 , (73), p. 750.

Kiefer, N.M. "Discrete Parameter Variation: Efficient Estimation of a Switching Regression Model". Econometrica. 1978 b, (46), pp. 417 - 434.

Kiefer, N.M. "Comment on Quandt and Ramsey". Journal of the American Statistical Association. 1978 a, (73), pp. 744-745.

Kiefer, N.M. "A Note on Regime Classification in Disequilibrium Models". Review of Economic Studies. 1980 a, (XLVII). pp. 637 - 639.

Kiefer, N.M. "A Note on Switching Regressions and Logistic Discrimination". Econometrica. 1980 b, (48), pp. 1065 - 1069.

Kumar, K.D., Nicklin, E.H. & Paulson, A.S. "Comment on Estimating Mixtures of Normal Distributions and Switching Regressions". Journal of the American Statistical Association. 1979, (74), pp. 52 - 55.

Laffont, J.J. & Garcia, R. "Disequilibrium Econometrics for Business Loans". Econometrica. 1977 (45), pp. 1187-1204.

Lewis, D. "Estimating the Influence of Public Policy on Road Traffic Levels in Greater London". Journal of Transport Economics and Policy. 1977, (11), pp. 155-168.

Maddala, G.S. Econometrics. McGraw-Hill Book Company. 1977.

Maddala, G.S. & Nelson, F.D. "Maximum Likelihood Methods for Markets in Disequilibrium". Econometrica. 1974, (42), pp. 1013 - 1030.

Martin, B.V., Memmot, F.W., & Bone, A.J. "Principles and Techniques of Predicting Future Demand for Urban Area Transportation". M.I.T. Report No. 3, 1961.

Meyer, J.R. "Techniques of Transport Planning: System Analysis and Simulation Models". The Brookings Institution Transport Research Program. 1971, (II), Washington, D.C.

Meyer, J.R., Vain, J.F., Wohl, M. The Urban Transportation Problem. Harvard University Press, Cambridge, 1968.

Monsod, S. "Further Developments in the Cross-sectional Rail Passenger Demand Model". Studies in Travel Demand, (III), Mathematica. Springfield, Virginia. Clearinghouse for Federal Scientific and Technical Information, 1967.

Monsod, S. "Relative Shares Model". Studies in Travel Demand, (V), Mathematica. Springfield, Virginia. Clearinghouse for Federal Scientific and Technical Information, 1969.

Moses, L.N. & Williamson, H.F. "Value of Time, Choice of Mode and Subsidy Issues in Urban Transportation". Journal of Political Economy. 1963, (71), pp. 247 - 264.

Muellbauer, J. & Portes, R. "Macroeconomic Models with Quantity Rationing". Economic Journal. 1978, (88), pp. 788-821.

McCarthy, P. "Residential Location and the Journey to Work". Journal of Transport Economics and Policy. 1977, (11), pp. 169 - 184.

McDonald, J.F. & Moffit, R.A. "The Uses of Tobit Analysis". Review of Economics and Statistics. 1980, (LXII), pp. 318 - 321.

McFadden, D. "The Measurement of Urban Travel Demand". Journal of Public Economics. 1974, (3), pp. 303 - 328.

MacKinnon, J. & Olewiler, N.D. "Disequilibrium Estimation of the Demand for Copper". Bell Journal of Economics. 1980, (11), pp. 197 - 211.

Oi, W.Y. & Shuldiner, P.W. An Analysis of Urban Travel Demands. Northwestern University Press, Eraston, 1962.

Portes, R. & Winter, D. "Disequilibrium Estimates for Consumption Goods Markets in Centrally Planned Economies". Review of Economic Studies. 1980, (XLVII), pp. 137 - 159.

Quandt, R.E. "The Estimation of Parameters of a Linear Regression System Obeying Two Separate Regimes". Journal of the American Statistical Association. 1958, (53), pp. 873 - 880.

Quandt, R.E. "A New Approach to Estimating Switching Regressions". Journal of the American Statistical Association. 1972, (67), pp. 306 - 310.

Quandt, R.E. "Tests of the Equilibrium vs. Disequilibrium Hypotheses". International Economic Review. 1978, (19), pp. 435 - 452.

Quandt, R.E. & Baumol, W.J. "The Demand for Abstract Transport Modes: Theory and Measurement". Journal of Regional Science. 1966, (6), pp. 13 - 26.

Quandt, R.E. & Ramsey, J.B. "Estimating Mixtures of Normal Distribution and Switching Regressions". Journal of the American Statistical Association. 1978, (73), pp. 730-738.

Rea, J.C. "Intercity Passenger Transport in Canada". Analysis of the Consequences of Alternative Pricing and Network Strategies. Canadian Transport Commission, Research Branch. Report No. 254, pp. 1 - 65., 1976.

Richard, J.F. "Models with Several Regimes and Changes in Exogeneity". Review of Economic Studies. 1980, (XLVII), pp. 1 - 20.

Rosen, H.S. & Quandt, R.E. "Estimation of a Disequilibrium Aggregate Labor Market". Review of Economics and Statistics. 1978, (LX), pp. 371 - 379.

Rousselot, M. & Gluitz, P. "Theory and Practice". International Symposium on Theory and Practice in Transport Economics. Symposium International. 1976.

Schmenner, R. "The Demand for Urban Bus Transit". Journal of Transport Economics and Policy. 1976, (10), pp. 68 - 86.

Schmidt, P. & Strauss, R.P. "The Effect of Unions on Earnings and Earnings on Unions: A Mixed Logit Approach". International Economic Review. 1976, (17), pp. 204 - 212.

Sharp, C.H. The Problem of Transport. Pergamon Press, Oxford, 1965.

Sharp, C.H. The Problem of Urban Passenger Transport. Leicester University Press, 1967.

Sharp, C.H. Transport Economics. Macmillan Studies in Economics. 1973.

Solow, R.M. & Stiglitz, J.E. "Output Employment and Wages in the Short Run". Quarterly Journal of Economics. 1968, (LXXXII), pp. 537 - 560.

Suits, D. "An Econometric Model of the Watermelons Market". Journal of Farm Economics. 1955, (37), pp. 237 - 259.

Thomson, J.M. "Some Aspects of Evaluating Road Improvement in Congested Areas". Econometrica. 1970, (38), pp. 298-310.

Thomson, J.M. Modern Transport Economics. Penguin Education. 1974.

Tobin, J. "Estimation of Relationships for Limited Dependent Variables". Econometrica. 1958, (26), pp. 24 - 36.

Van Doorn, J. Disequilibrium Economics. London, 1975.

Veendorp, E.C. "The Nature of Quantities in Market Disequilibrium". American Economic Review. 1974, (64), pp. 515 - 517.