FATIGUE LIFE PREDICTION UNDER COMPLEX LOADING

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ABSTRACT

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A cumulative damage procedure for predicting the fatigue failure of engineering metals subjected to complicated stress-strain histories is described.

The rain-flow cycle counting method which counts all closed stress-strain loops as cycles is employed in the damage assessment procedure.

Different counting methods are described and compared to the rain-flow counting method.

ACKNOWLEDGEMENTS

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NOMENCLATURE

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D	Damage
n	Number of cycles at given amplitude
$2N_f$	Reversals to failure
N_f	Fatigue life in cycles
$\Delta\varepsilon$	Total strain range
εa	Strain amplitude
εea	Elastic strain amplitude
ε_p	Plastic strain amplitude
$\Delta\sigma$	Stress range
σa	Stress amplitude
E	Elastic modulus
ε'_f	Cyclic fatigue ductility coefficient
σ'_f	Cyclic fatigue strength coefficient
σ_f	True fracture strength
S'_y	Cyclic 0.2% offset yield strength
b	Cyclic fatigue strength exponent
c	Cyclic fatigue ductility coefficient
n'	Cyclic strain hardening exponent
K'	Cyclic strength coefficient
σ_{cr}	Equivalent completely reversed stress amplitude
σ_o	Mean stress

CHAPTER I

INTRODUCTION

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INTRODUCTION

Fatigue design of machine components is one of the more difficult tasks that engineers face. Many factors are involved and the relationships between these factors are only partially established and largely empirical. Fatigue failure is caused by repeated loading with the number of cycles to failure varying with the load range. "Damage" accrues gradually over the life span of cyclical loading, with small changes taking place during each cycle. Crack initiation occurs at varying percentages of the final life and crack propagation continues until final fracture takes place.

Fatigue design requires a knowledge of the following [1]*:

1) The expected load-time history (or better, the local stress-time and strain-time history at the most critical locations.)

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*Numbers in parentheses refer to the list of references at the end of the report.

2) The nature of the environment in which the component is operated (wet, dry, corrosive, temperature, etc.)

3) The properties of the material as it exists in the finished component at the most critically stressed locations ("inherent" fatigue properties, residual stress effects, surface effects, sensitivity to corrosion, "cleanliness", etc.)

4) The geometry of the component and its notches (stress concentration, surface finish, manufacturing variability, etc.)

In the decades that have followed the first recorded observation of the fatigue phenomenon, well over 100 years ago, [1]. much has been done to enable better fatigue life predictions. The concept of the fatigue limit - the stress level below which fatigue failure is highly improbable - has been used extensively, probably because it is simple to apply and because it has an empirical relationship to the ultimate tensile strength.

This concept requires the knowledge of the fatigue limit of the material in question, fatigue strength diagram and the applied stresses.

The fatigue limit can be determined by testing a number of similar test pieces of the same metal at stress ranges f_1, f_2, f_3, ..., etc.; and the number of cycles of stress which each specimen can endure to fracture is noted. The stress range applied to any test piece is kept constant throughout the test. It is usual to test about eight or ten specimens each at a different load, so chosen that at least two test pieces remain unbroken after testing.

Generally, the determination is made by subjecting successive test pieces to decreasing stress ranges, and noting the increasing fatigue lives. If the test piece breaks before the fatigue lives attain the specified number of cycles for which the determination is being conducted, the applied stress is higher than the fatigue limit; if, on the other hand, the test piece is not broken at the specified number of cycles, the applied stress is either less than, or equal to, the fatigue limit. A considerable amount of time is needed to conduct these tests in order to find the fatigue limit of the material and to construct the S-N curve, as shown in Figure 1.1 [2].

The fatigue limit is determined for the case of the completely reversed loading, where the mean stress is zero. The effect of the mean stress on the fatigue limit should also be determined experimentally, so that a fatigue strength diagram can be established.

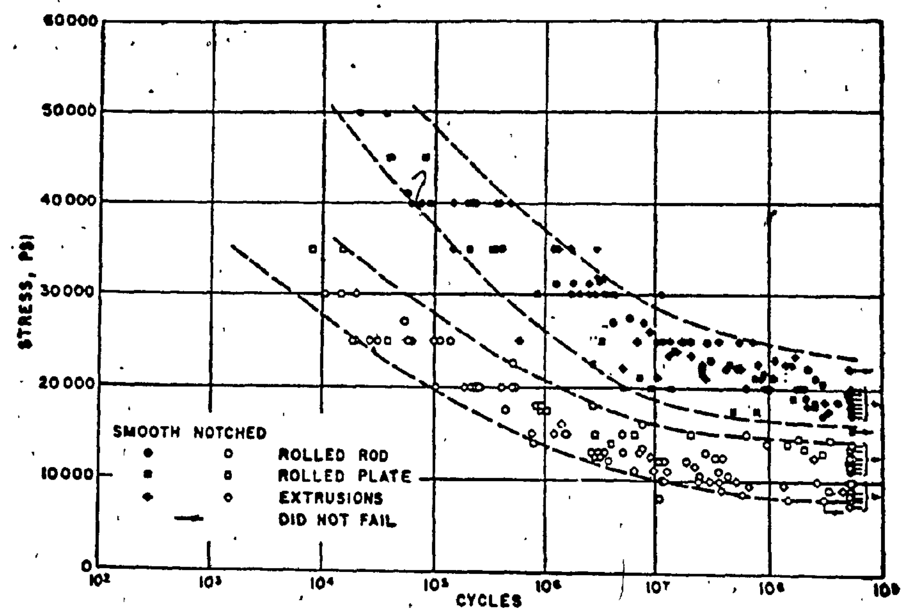

FIG. 1.1 Rotating Beam Fatigue Data for 2024-T_4 Aluminum [2]

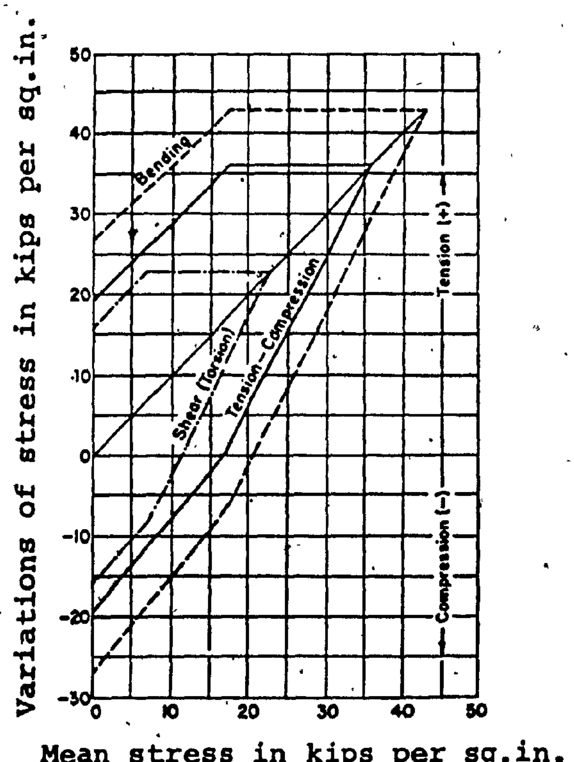

FIG. 1.2 Fatigue Strength Diagram For Polished Specimens of Structural Steel [2]

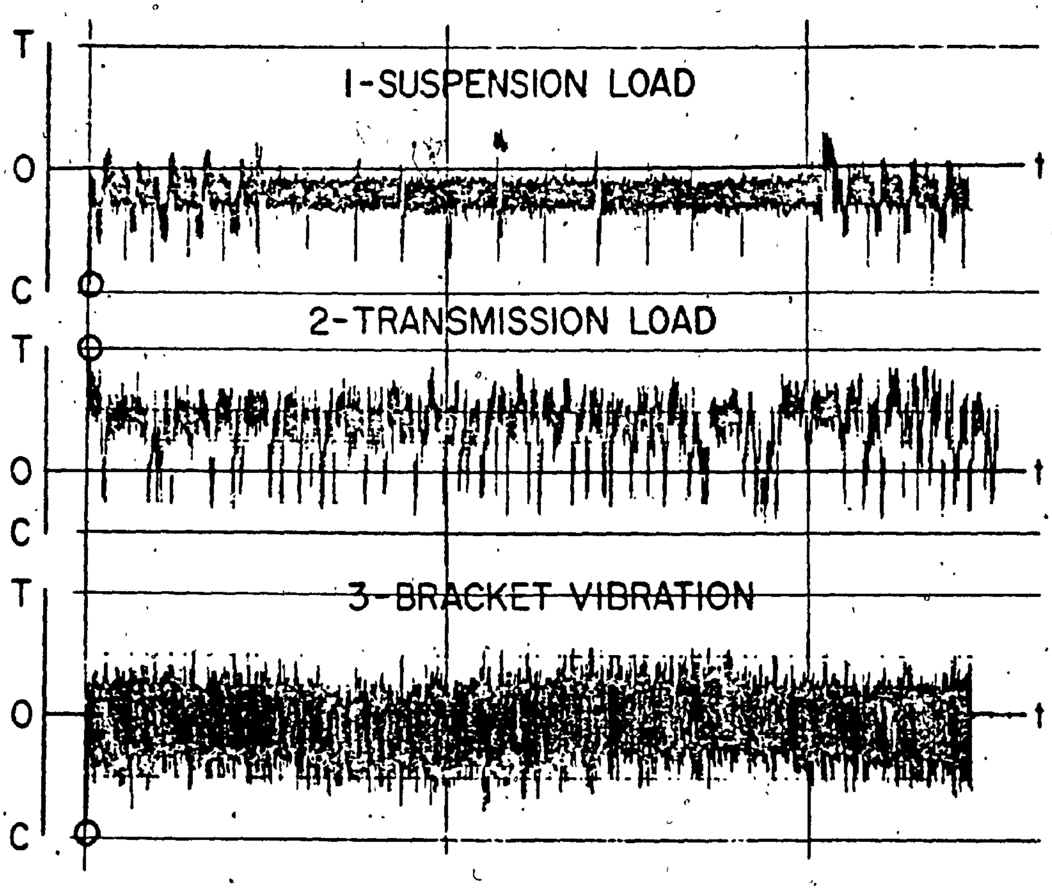

FIG. 1.3 Amplitude-Time Display of Real Service Histories [9]

As shown in Figure 1.2 [2], the fatigue strength diagram is constructed for bending, tensile and torsion loading, so that machine components subjected to different modes of loading can be designed.

After obtaining the fatigue limit and the fatigue strength diagram, the design of the machine components require the knowledge of stresses resulting from their operating conditions, and for the design to be accepted, the operating stress should be below the allowable fatigue strength of the material.

Even though the fatigue limit concept has been used widely in design, it is too simple to predict fatigue lives in components subjected to real complex load histories such as those shown in Figure 1.3. The fatigue limit concept is based upon sinusoidal repetitive loading of constant amplitude where real load histories may have consecutive excitations of different amplitudes. The actual fatigue damage is an accumulation of damages done during these non-uniform cycles. Therefore, during the past twenty years, considerable effort has been spent in improving fatigue life prediction. Many cumulative theories have been proposed: the three frequently used fatigue life predication routines include load life prediction, nominal stress life prediction, and nominal strain life prediction [12]. All three methods of prediction are based on the Palmgren [13] and Miner [14] linear damage rule where the fatigue damage D is calculated by linearly summing cycle ratios for the history, as indicated by

$$D = \Sigma \frac{n}{Nf} \qquad (1.1)$$

where

n = Number of cycles at a given amplitude

Nf = Fatigue life in cycles from constant amplitude load-life curve at the given amplitude.

The procedures for these fatigue life prediction routines are as follows.

1.1 LOAD-LIFE PREDICTION (Fig.1.4)

1) Input

a) - Load - time history.

b) - Load-vs-cycles to failure curve.

2) Load time history rain flow counted for load range.

3) Linear damage summation and life prediction.

1.2 NOMINAL STRESS-LIFE PREDICTION (Fig. 1.5)

1) Input

a) - Load-time history.

b) - Smooth specimen monotonic stress-strain and fatigue properties.

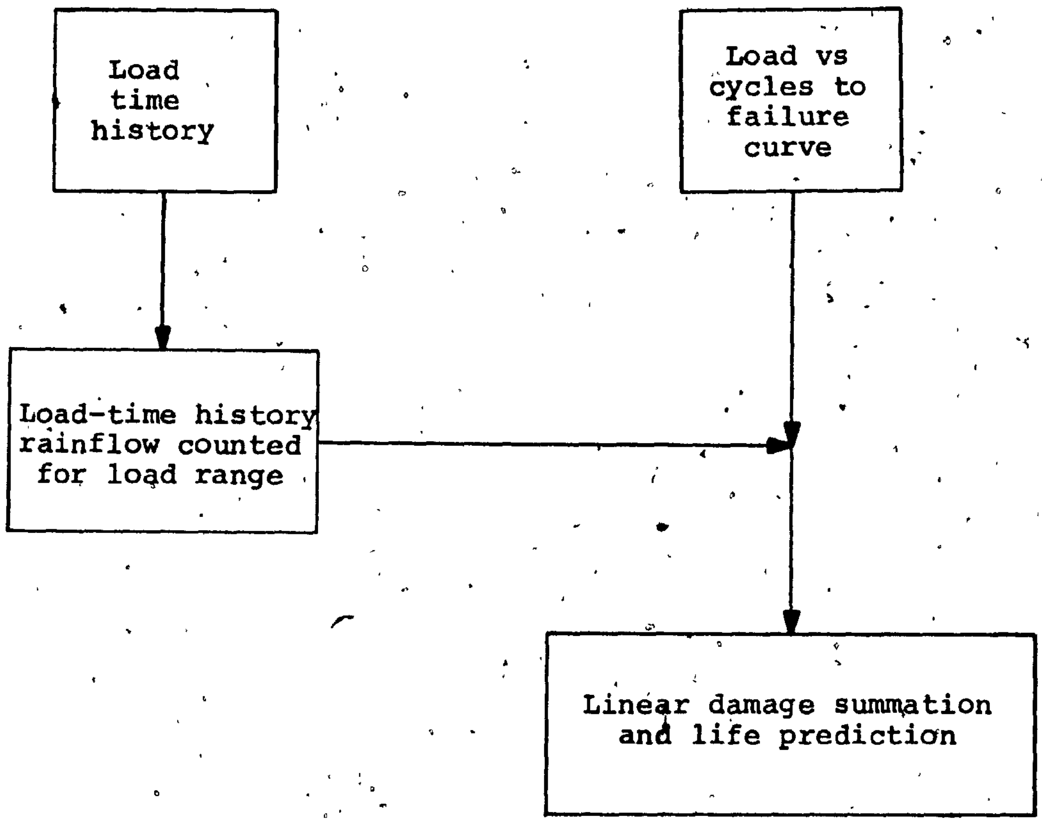

FIG. 1.4 Flow Chart Diagram For Load-Life Prediction Technique

c) Fatigue notch factor.

2) "Nominal" strain calculated from load or measured with strain gage.

3) "Nominal" stress-time found by following stress strain response for the material (as determined by uniaxial testing of small polished specimen [9, Appendix C].

4) "Nominal" stress-time rainflow counted.

5) Notched S-N curve estimated.

a) Smooth specimen fatigue strength at 1,000,000 cycles divided by the fatigue notch factor.

b) Notched S-N curve drawn between fatigue strength coefficient and reduced strength at 1,000,000 cycles.

c) Linear damage summation and life prediction.

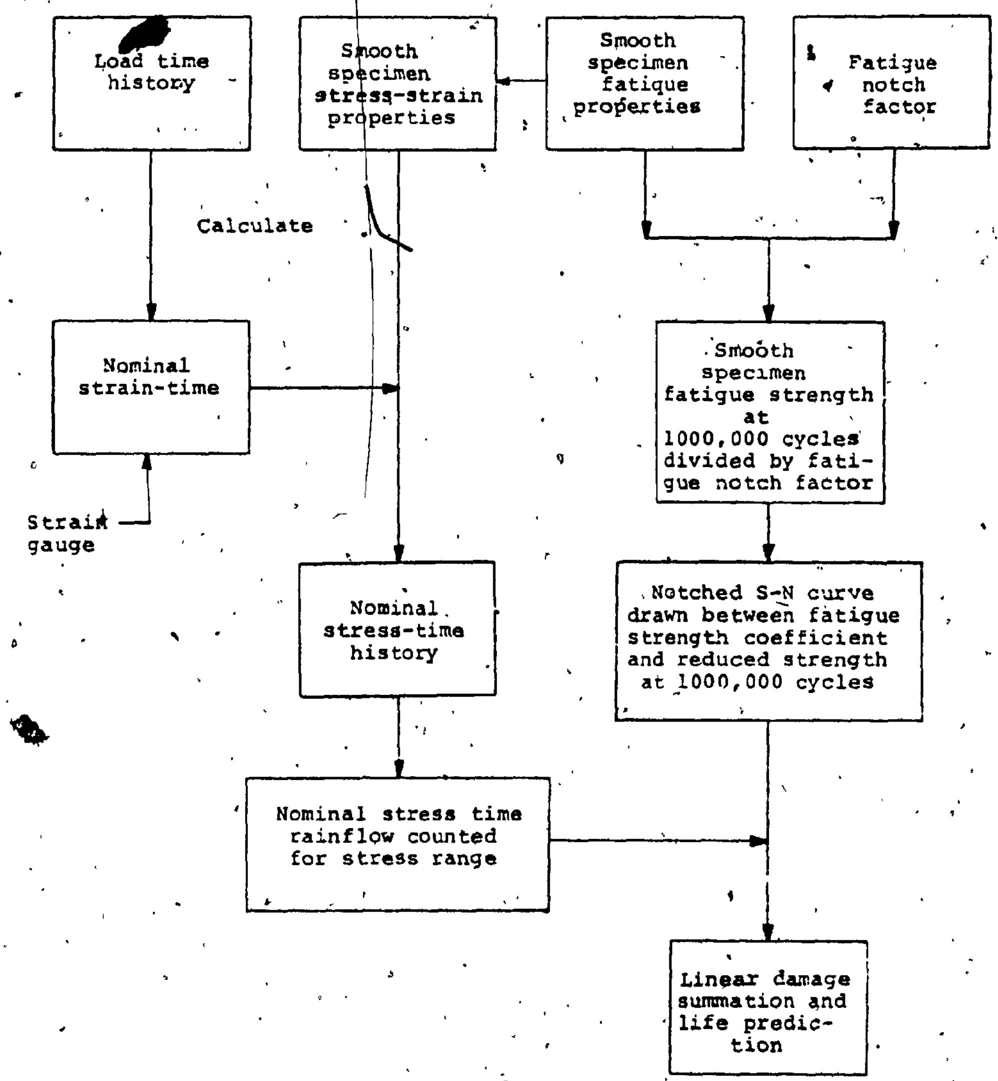

FIG. 1.5 Flow Chart Diagram for the Nominal Stress Life Prediction

1.3 ABBREVIATED "NOMINAL" STRAIN-LIFE PREDICTION (Fig.1.6)

1) Input

   a) Load-time history.

   b) Smooth specimen monotonic stress-strain and fatigue properties.

   c) Theoretical stress concentration factor.

2) History abbreviation

   a) Find maximum peak and minimum valley and use history which lies between those two points.

   b) Screen out all ranges less than 40 percent of the maximum range.

3) Load converted to notch root strain

   a) Load converted to "nominal" strain by assuming linear elastic condition.

   b) "Nominal" strain converted to notch root strain by multiplying the "nominal" strain by stress concentration factor.

4) Strain time history is rainflow counted.

5) Linear damage summation and life prediction.

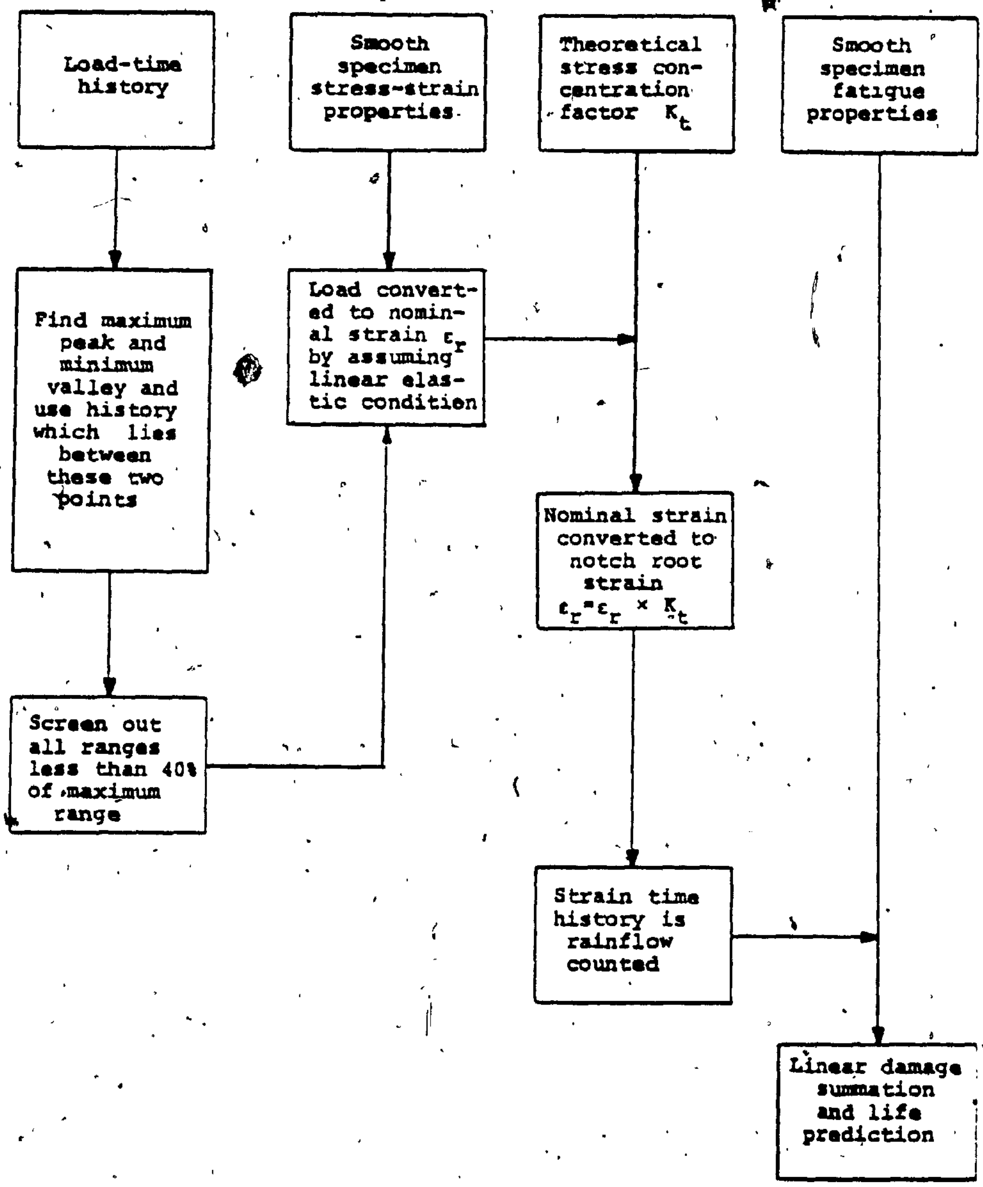

FIG. 1.6 Flow Chart Diagram for the Abbreviated Nominal Strain Life Prediction

In the load life prediction routine, the only improvement over the conventional construction load vs. cycles to failure method is that the load history is rainflow counted for load range and linear damage summation is done for damage prediction. The nominal stress life prediction routine adds the precautious feature of reducing the allowable fatigue lives by the fatigue notch factor. The abbreviated nominal strain life prediction technique uses the theoretical stress concentration factor K_t instead of the fatigue notch factor as in the nominal stress life prediction technique. One additional simplification in the abbreviated nominal strain life prediction technique is that the load history is screened out to eliminate ranges that are less than 40% of the maximum range.

Reference [12] evaluates the accuracy that might be expected in using these procedures in design analysis.

Predictions given by the load-life method were always longer than the actual life measured by test (conservative prediction) "Nominal" stress-life analysis yielded estimates of life which were generally within the scatter of the test data. Even though a majority of the service load history was not considered, the abbreviated nominal strain life prediction gave reasonable accuracy.

Furthermore, emphasis has recently been shifted from using nominal stresses, elastic analysis, and the concept of safe stress to the analysis using the elastic-plastic behavior at the most highly strained region. This is called the local stress-strain approach [4,5] and is used to make a prediction of life to the formation of an engineering size crack.

Briefly, the local stress-strain approach involves the following: (See Fig. 1.7)

1) A model of uniaxial stress-strain behavior is employed to simulate the deformation history of an imaginary filament at the critical location, such as notch root.

2) Using the applied loads or nominal stresses in conjunction with cyclic load versus notch strain curve [16,17] or Neuber's rule [15] to generate the local strain-time history.

3) A counting procedure is used to identify events or cycles, which can be related to increments of damage or life exhaustion.

4) Smooth specimen fatigue test data are needed to establish cyclic stress-strain curves, as well as fatigue resistance properties used in damage equations.

5) Fatigue life prediction.

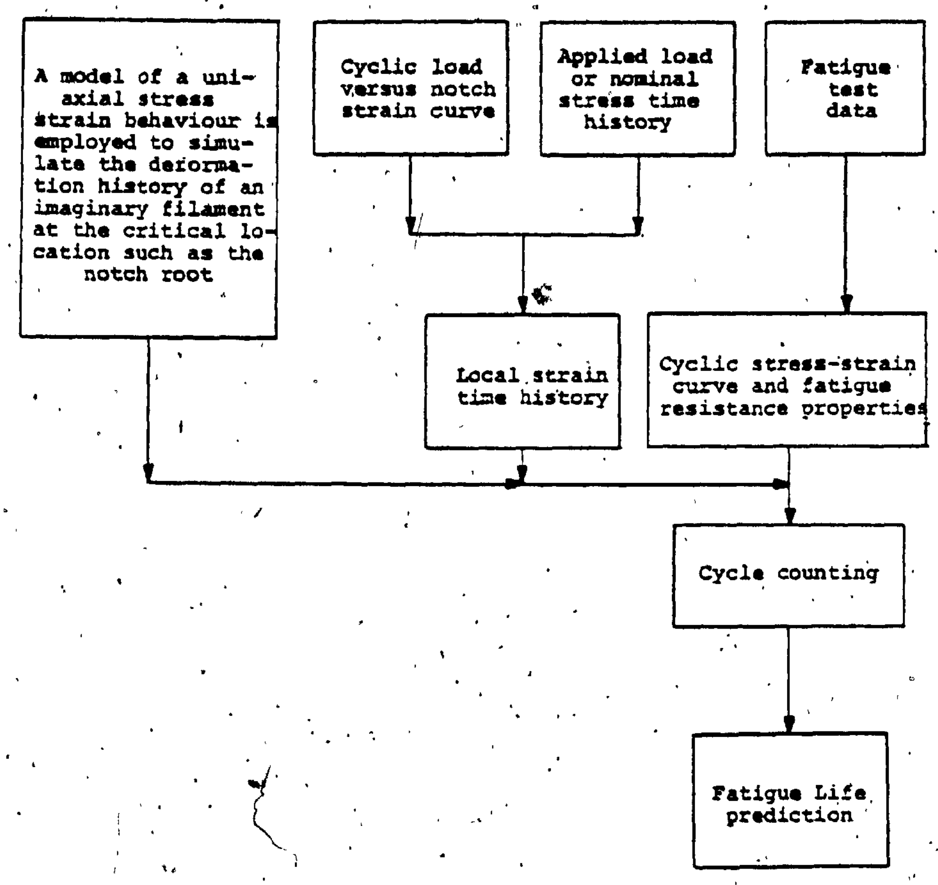

FIG. 1.7 Flow Chart Diagram for Fatigue Life Prediction by Local Strain Approach

As can be seen in the previous presentation, all fatigue life prediction techniques utilize the cycle counting technique to obtain an adequate number of load cycles that do significant damage to the structure without having to include all cycles, most of which are unimportant.

The cycle-counting technique therefore plays an important role in the determination of the fatigue life prediction. For that reason, many different counting techniques have been developed and improved upon. Among the different techniques, the rainflow counting method has proven to be the most efficient and accurate.

In the following Chapter, various counting techniques will be presented, along with the rainflow counting method.

Utilizing the rainflow counting method, the local strain approach has in recent years been developed to a high degree of sophistication and has received favorable acceptance. This method utilizes the cyclic stress-strain properties and the low cycle fatigue resistance properties which are experienced by the elements of materials at the most highly stressed local regions. These low cycle fatigue resistance properties are also presented in a subsequent Chapter.

CHAPTER II

COUNTING PROCEDURES

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COUNTING PROCEDURES

2.1 OVERVIEW

The specific procedure used to identify cycles can have a significant effect on the resulting fatigue life estimate. Some of the various "counting methods" currently used are: peak-crossing-peak, range mean, order overall method, range pair and rain flow. This Chapter discusses the above-mentioned counting procedures. Also, the comparison of the various cycle counting methods is considered.

2.2 PEAK-CROSSING-PEAK COUNTING METHOD

As illustrated in Figure 2.1, this method [3] counts the maximum peaks (and valleys) between zero crossings. The strain history shown in Figure 2.1, is composed of 9 reversals, the reversals ab, bc, cd, ef are above zero, the reversal fh is crossing the time axis at point g, indicating zero strain and also reversals hI, IJ and JK are crossing the time axis at points m, n and K, points d and I indicate the maximum peaks between zero crossing and points h and J indicate the minimum valleys between zero crossings. According to this method, the history can be reduced to five reversals: ad, dh, hI, IJ and JK. Each reversal is counted as a half-cycle for damage calculation.

As we can see, the method is neglecting the small range reversal and is detecting the large range reversal. It can be applied to any complex strain history resulting in a higher fatigue life than actual, due to the error introduced by omitting the small range reversals.

2.3 RANGE MEAN COUNTING METHOD

This method [3] counts the ranges and means between the reversals. As illustrated in Figure 2.2, the strain history is composed of 9 reversals with ranges $r_1, r_2, \ldots, r_9$. The range and the mean of each reversal is counted to form cycles in the following manner. If the load sequence is random (the probability density curve of the truly random signal should be similar to the probability density curve of the gaussian random signal [19]), positive (r_3) and negative (r_4) ranges can be paired, and each pair is considered a cycle. If the sequence is not exactly random, one may count a range (from valley-to-peak) as a half-cycle. This method tends to divide a history into a large number of small range reversals, with mean offsets and neglects any large-range reversals.

This method can be easily applied to any complex history resulting in higher fatigue life due to the error introduced by omitting the large reversals which contribute most of the damage.

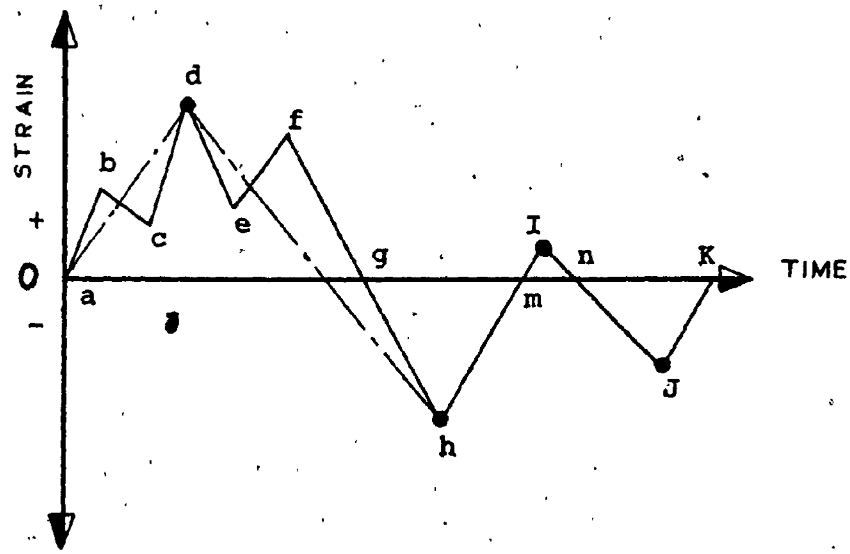


FIG. 2.1 Peak Crossing Peak [3]

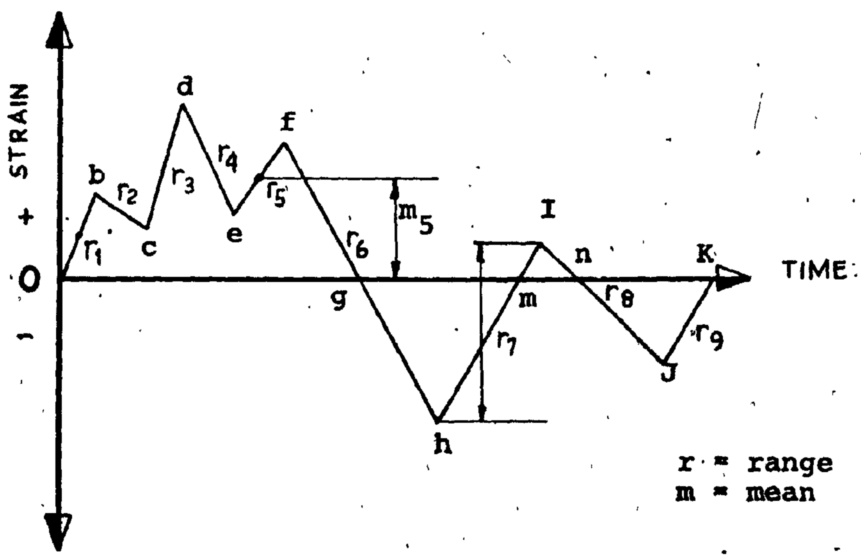


FIG. 2.2 Range Mean [3]

2.4 ORDER OVERALL COUNTING METHOD

2.4.1 Overview

This method [7] starts by looking for the highest peak P and the lowest valley v of the sequence. The difference between their values is assigned 100%. The method then proceeds to select those peaks and valleys whose levels are more than a certain specified level (for example, 50% of the maximum range.) The neighboring peak and valley will then be connected to form cycles.

Many smaller ranges may intervene between the selected peaks and valleys. They are screened out and neglected. As an example, Figure 2.4 shows how this process is applied to the sequence of Figure 2.3. When "screening level" is as low as the smallest range of recorded history, all reversals of the history are included in their proper sequence among the order overall ranges. Higher levels of screening lead to greater abbreviation of the history.

This method permits the selection of a small number of reversals (say, 10% of the total number) which account for a large fraction (say 90%) of the total damage. It also retains the sequence of important events so that the residual stress effects in notches can be evaluated. This method can be applied where more confident design of accelerated life tests is required. If a large fraction of reversals can be eliminated

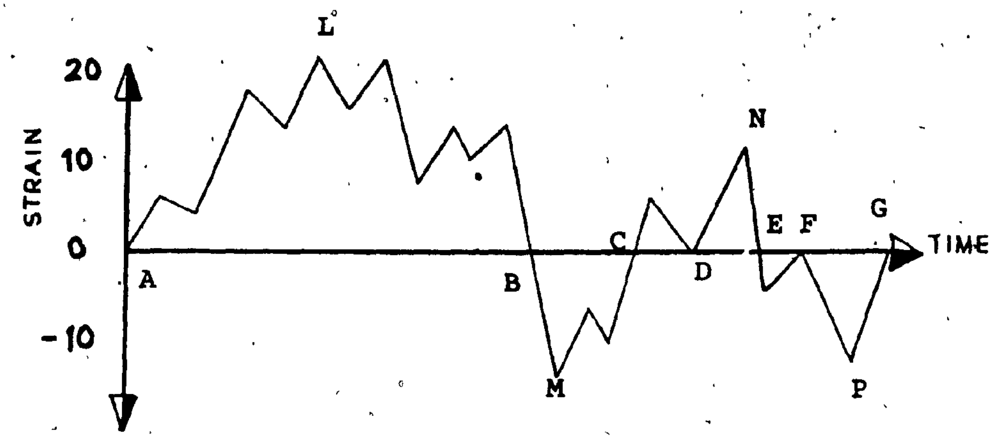


FIG. 2.3 Complex Strain History [7]

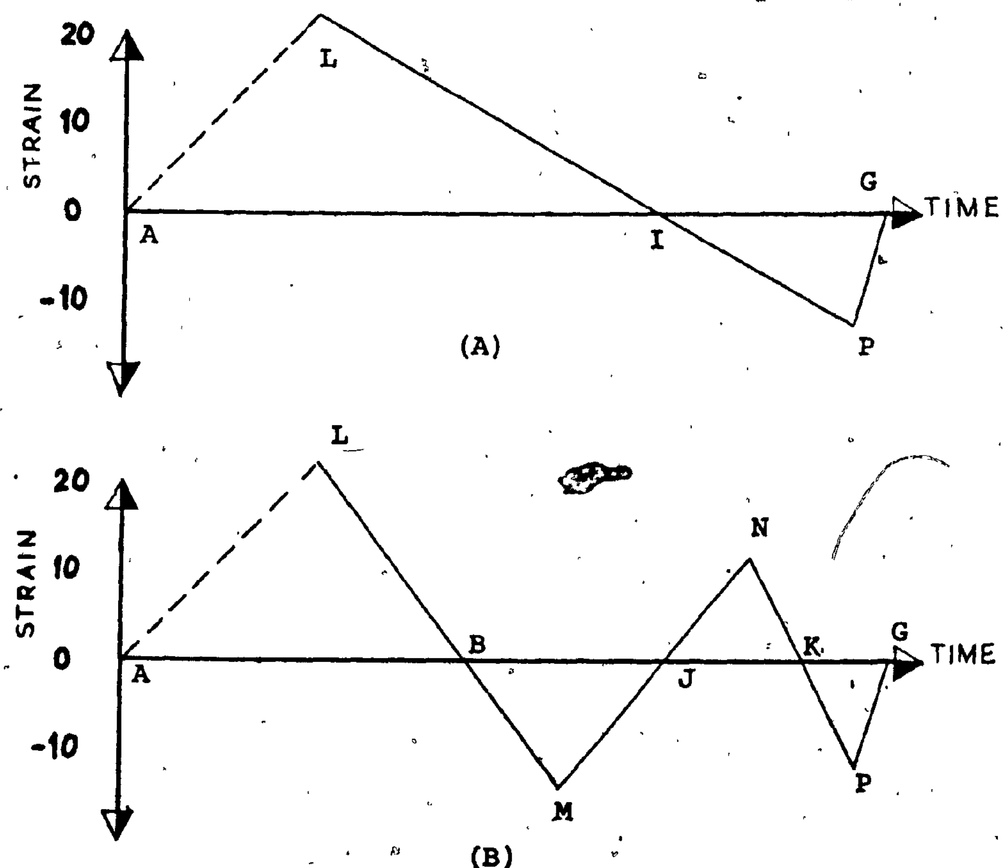


FIG. 2.4 Order Overall Counting Method Applied To The Sequence of Fig. 2.3 [7]

without major effect on the severity of the test, then the results can be obtained at less expense and in a shorter time.

2.4.2 Order Overall Counting Procedures

The range of stress or of strain is the main factor in all damage laws. The S-N curve depends upon these factors to a very high degree. If the fatigue life is proportional to the sixth power of the stress range, which is a reasonably low guess [7], then half of the stress range will produce only 1/64 of the damage of the full stress range, and one cycle at full range will produce as much damage as 4,000 cycles at 25% of the full range. The range is therefore chosen as the primary variable that shall determine whether the reverals are counted or neglected.

At the beginning, in the counting process, two reversals that include the largest range between them are selected. This will almost always be an "overall" range in the sense that smaller ranges intervene between them.

The next step can be approached in several equally valid ways.

2.4.2.1 First method

This method looks for those two reversals that include the next largest range between them and which form a series of four reversals with the previous pair or, in other words, which form an alternating series of peaks and valleys with the previously selected reversals. Figure 2.5 illustrates the meaning of this clause. One can then proceed to the next largest, and so on, until all reversals have been used.

2.4.2.2 Second method

This method subdivides each of the two or three ranges defined by the first pair into subranges, and so on, until all ranges that are larger than a selected minimum have been found.

2.4.2.3 Third method

The third method starts from an extreme reversal (say, the highest peak.) The next valley that differs by more than the screening level from the peak becomes the first tentative candidate. Succeeding reversals are then examined. The peaks are checked to see whether they differ from the candidate by more than the screening level (event "X".) The valleys are checked to see whether they are lower than the candidate (event "Y".) If event "Y" occurs first (before event "X"), the candidate is rejected, and the new valley becomes a candidate. If event "X" occurs first (before event "Y"), the candidate is validated and the newly found peak becomes the next candidate.

This method can be easily applied to the complex strain history shown in Figure 2.5, as follows:

1) Screening level is to be fixed (say 50% of the overall range.)

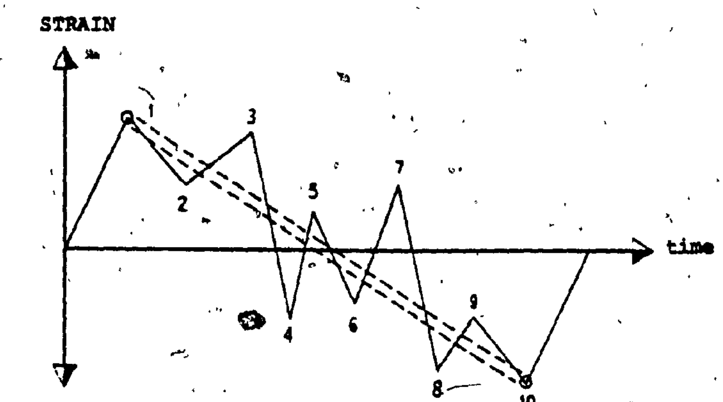
2) The highest peak (point 1) is to be taken as the starting point.

3) The next valley (point 4) that differs by more than the screening level from point 1, becomes the first tentative candidate.

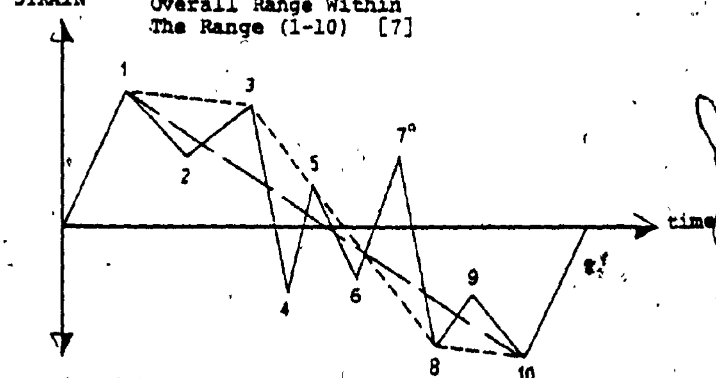
4) Since the next peak (point 5) does not differ from the candidate (point 4) by more than the screening level, the next valley (point 6) is to be examined.

5) Point (6) is not lower than point 4 and the next peak (point 7) differs from point 4 by more than the screening level; therefore, point 7 will be the new candidate.

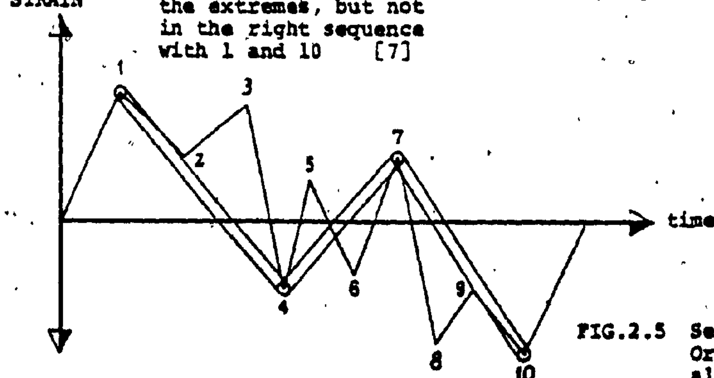
6) The process is to be continued until the reversal number 10 is reached.



(A) Finding the Largest Overall Range Within The Range (1-10) [7]



(B) Reversals 3 and 8 are the extremes, but not in the right sequence with 1 and 10 [7]



(C) Reversals 7 and 4 are the largest that form an alternating sequence of peaks and valleys with 1 and 10 [7]

FIG.2.5 Sequence of Order Overall Counting Method [7]

2.5 RANGE PAIR COUNTING TECHNIQUE

In fatigue life prediction, the complex strain history obtained from component tests should be primarily reduced into discrete cycles so that the damage calculation can be easily made. The range pair counting method reduces the actual strain history into discrete cycles. This technique [8] treats a strain history as the collection of n peaks and valleys designated by x_i, i=1,...,2n, such that if x_j is peak x_{j+1} is a valley, $1 \leq j \leq 2n - 1$. This method considers four points (x_1,x_2,x_3,x_4) at a time and the conditions for counting a cycle (x_2,x_3) are as follows:

If $x_2 > x_1$, then the cycle is counted, provided that $x_2 \leq x_4$ and $x_3 \geq x_1$ (Figure 2.6a).

Conversely, if $x_2 < x_1$, then a cycle is counted provided that $x_2 \geq x_4$ and $x_3 \leq x_1$ (Figure 2.6b).

Thus, starting at the beginning of the history, the first four points x_1,x_2,x_3 and x_4 are considered. If x_2 and x_3 meet the above conditions, a cycle is defined and these two points are deleted from the history. Consequently, x_4 becomes x_2 and the next two points are added to again yield four points. Counting continues until the four points considered do not define a cycle. Then x_1 is omitted from consideration and becomes an element of a residue history. The three remaining points are updated, i.e., x_2 becomes x_1,

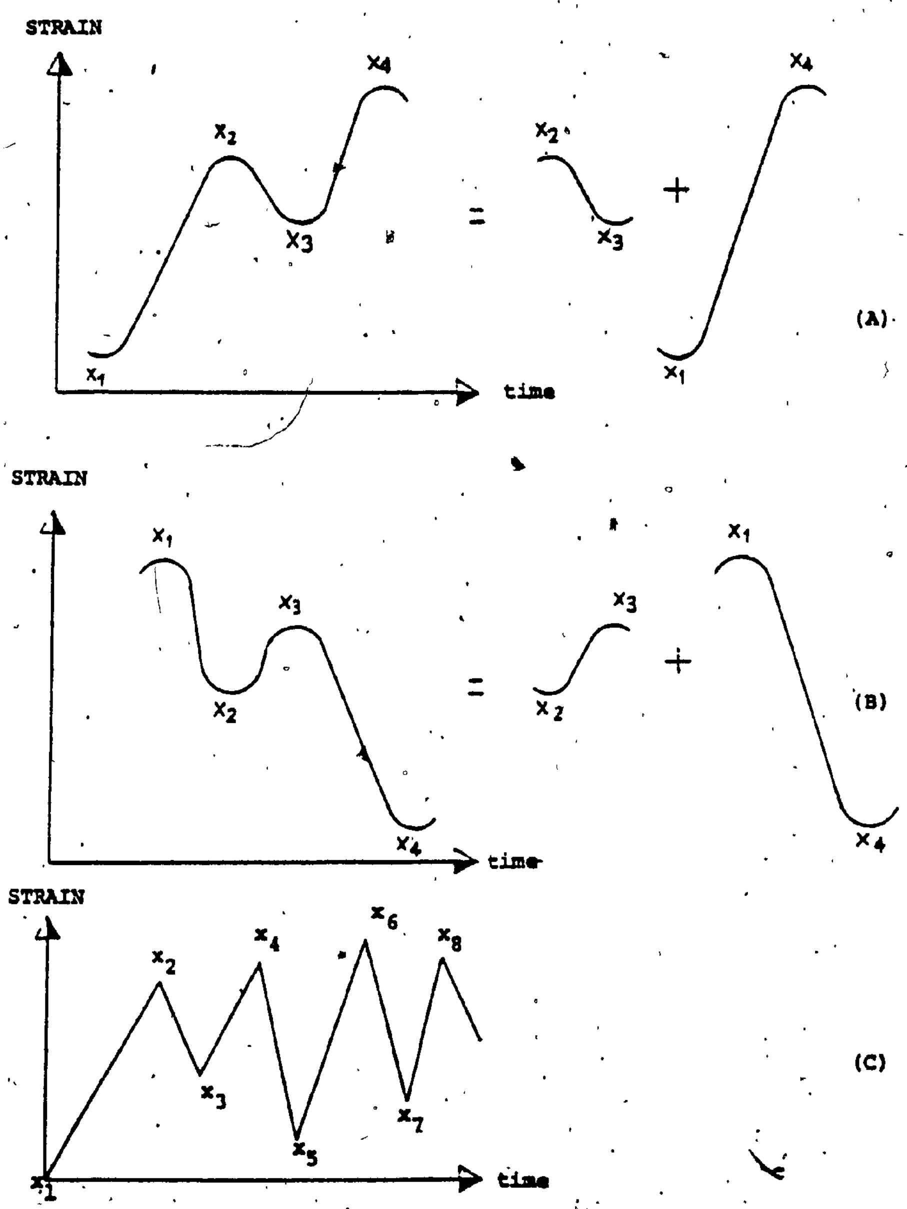


FIG. 2.6 Range Pair Counting Applied to Strain History [8]

x_3 becomes x_2, x_4 becomes x_3, and x_4 is added sequentially from the original history. This process continues until there are only two or three points remaining. These points are added to the residue spectrum, which is then analyzed in the same manner as the original history. Continuing in this manner, a residue spectrum is finally generated which will yield no cycles that can be counted by the range pair cycle counting method. This residue spectrum diverges to a maximum range and then converges as shown in Figure 2.7. Cycles are generated from the final residue history as follows: pair the highest peak D with the lowest valley E, to form a cycle. Then, moving away from this cycle in both directions, each successive peak and valley are paired together, i.e., pair peak B with valley C to form a cycle and peak F with valley G to form another cycle. If there is an extra peak or valley left on either side, it is omitted, i.e., peak H is to be omitted.

2.6 RAIN FLOW COUNTING TECHNIQUE

2.6.1 Overview

The name of the rain flow counting technique is derived from the process of rain dripping down a series of pagoda roofs, as described by the strain-time history shown in Figure 2.8.

The rain flow counting method has been shown by extensive experimental tests to be superior [3] to those counting methods

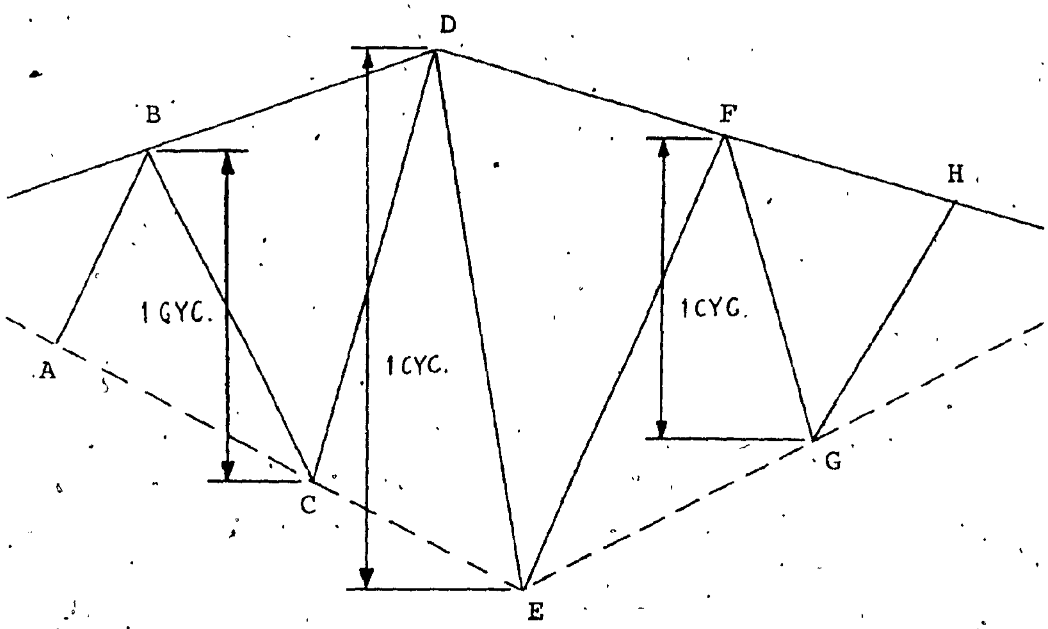


FIG. 2.7 Residue Spectrum Generated by Range Pair Counting Method [8]

mentioned above. The apparent reason for this is that the rain-flow method defines cycles as closed stress-strain hysteresis loops. To exemplify this point, a brief complex strain-time history and its associated stress-strain response are shown in Figure 2.8. The cycles that would be determined by the rain flow counting method are identified by a-d-g, b-c-b^1, e-f-e^1. Each of these cycles is a closed stress-strain hysteresis loop of the type obtained from the constant amplitude test.

2.6.2 Description of Method

The purpose of this section is to describe in detail the means by which the corresponding stable stress-strain response, Figure 2.8, is generated and how the completion of a hysteresis loop is detected mathematically, [4].

As shown in Figure 2.8, the strain range between the maximum and minimum strain reversal points is divided into a number of strain "bands". Ten bands are used in the example. It is helpful to think of graphically reconstructing the strain history by using one-dimensional strain elements, each of which can traverse one strain band.

Since it will never be required to traverse more than a distance of ten bands, a total of ten elements are needed.

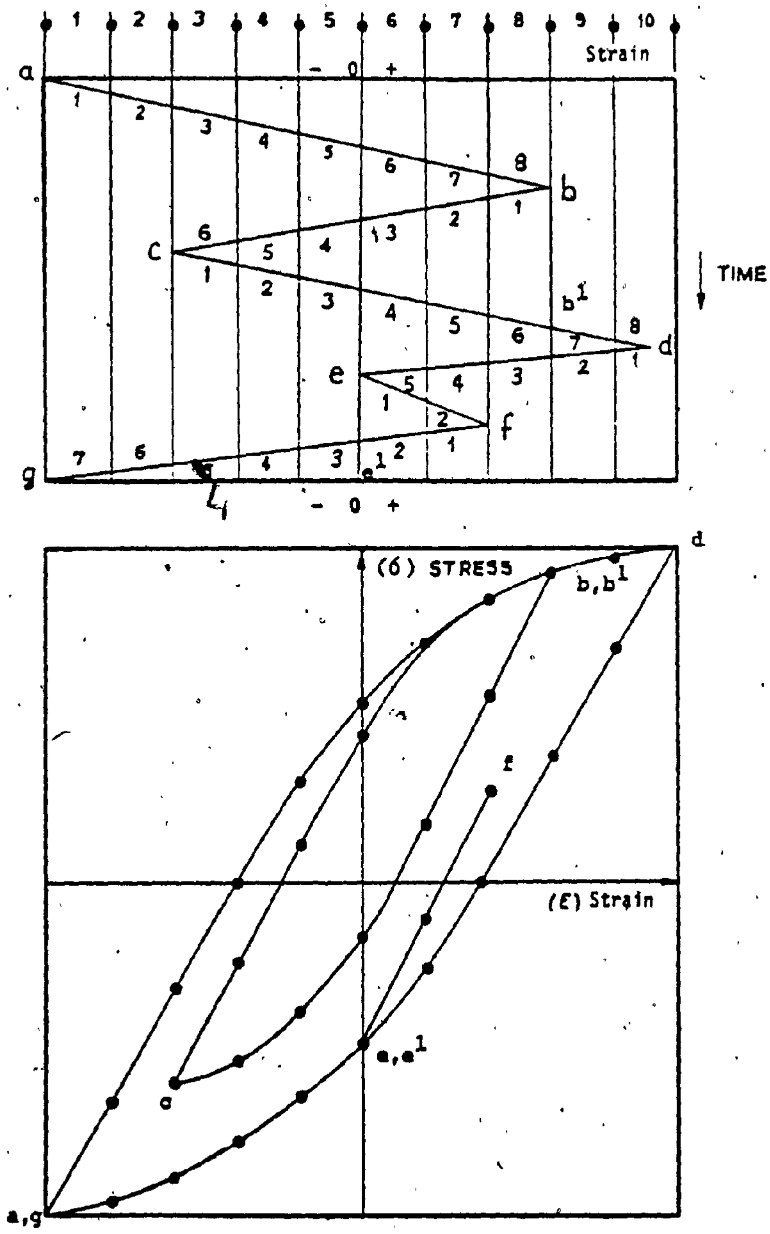


FIG.2.8 Block of Strain History and Its Stress-Strain Response [4]

An orderly set of rules specify the use of these elements. Each element is assigned what will be called an "availability sign" or "availability". At point a, all ten elements are assigned an availability of -1, because it is a negative strain. This indicates that each element can now be used only in the tensile direction. At point a, and after every reversal point, the elements are used in order, starting with the first. An element is used for tensile loading only if its availability is -1, or for compressive loading, +1. When an element is used, its availability is changed. If an element is not available to be used in its turn, it is passed over and its availability remains unchanged. If a strain reversal point lies within a strain band and not at the edge of one, its position is approximated by being placed at the far edge of the strain band. This is done because an element can only be used as a whole.

A reversal-by-reversal record of the element availabilities is shown in Figure 2.9. Note again, that at point a, the most negative strain in the history, all element availabilities are initiatized at -1. At point b, the eight elements representing the reversal a→b were used in the tensile direction and their availabilities changed from -1 to +1; elements nine and ten are not used and their availability signs remain unchanged. At point a, the availability of every element is +1. This indicates that the last time each element was used, it was used in the tensile direction. Upon reaching point b, on the reversal between c and d, the availability numbers of elements seven and eight indicate that they cannot be used in the tensile direction. Thus, these elements are "skipped", and elements nine and ten are used to reach point d. However, the skipping of an element is the key to the means of cycle counting. The fact that element seven is unavailable after the use of element six indicates that the strain cycle has been completed with element six. Completion of a cycle is indicated in the same way when point e is reached during the reversal between points f and g.

The most important cycle is the largest one which is completed at point g. If the example problem here were actually set up for computer use, eleven strain bands, and thus eleven elements, would have been established; and the eleventh element's availability would also have been initiatized at -1. Element ten is used to reach the last peak in the strain history at point g. But the unavailability of the eleventh element, regardless of the fact that it is not used, signals that a cycle whose strain range equals ten strain bands, has been completed.

Extension of the above concepts to simultaneously simulate stress-strain response and count cycles is a simple step. The one-dimensional strain elements which were used as a tool for explaining cycle counting are replaced by the two-

REVERSAL

ELEMENTS	a	→b	→c	→d	→e	→f	→g
1	−	+	−	+	−	+	−
2	−	+	−	+	−	+	−*
3	−	+	−	+	−	−	−
4	−	+	−	+	−	−	−
5	−	+	−	+	−	−	−
6	−	+	−	+*	+	+	−
7	−	+	+	+	+	+	−
8	−	+	+	+	+	+	−
9	−	−	−	+	+	+	−
10	−	−	−	+	+	+	−*

* Loop Closed.

FIG. 2.9 Record of Element Availabilities [4] dimensional stress-strain elements which represent a cyclic loop shape. Skipping an element then provides the memory effect seen at points b and e (Figure 2.8), where the stress-strain path of the large loop is rejoined.

When counting starts at any point in the history other than the maximum peak or minimum valley, the method is counting half cycles, as well as full cycles, as shown in Figure 2.10 [3]. The counting method corresponds to the stable cyclic stress-strain behavior of a metal in that strain ranges counted as cycles will form closed stress-strain hysteresis loops, and those counted as half cycles will not, other than that the method can be applied to any complex history, considering the effect of each stress-strain hysteresis loop on damage calculation.

2.7 COMPARISON OF THE CYCLE COUNTING METHODS

If various cycle-counting methods are compared on the basis of their applicability to complicated strain histories, it is easily seen that for most of them there are situations where unreasonable results are obtained. In the sequence shown in Figure 2.11a, the small reversals do some fatigue damage that may or may not be significant, compared to the damage done by the large cycle on which they are superimposed. Peak-crossing-peak counting gives the result that is equivalent to that of Figure 2.11c, which is nonconservative in the

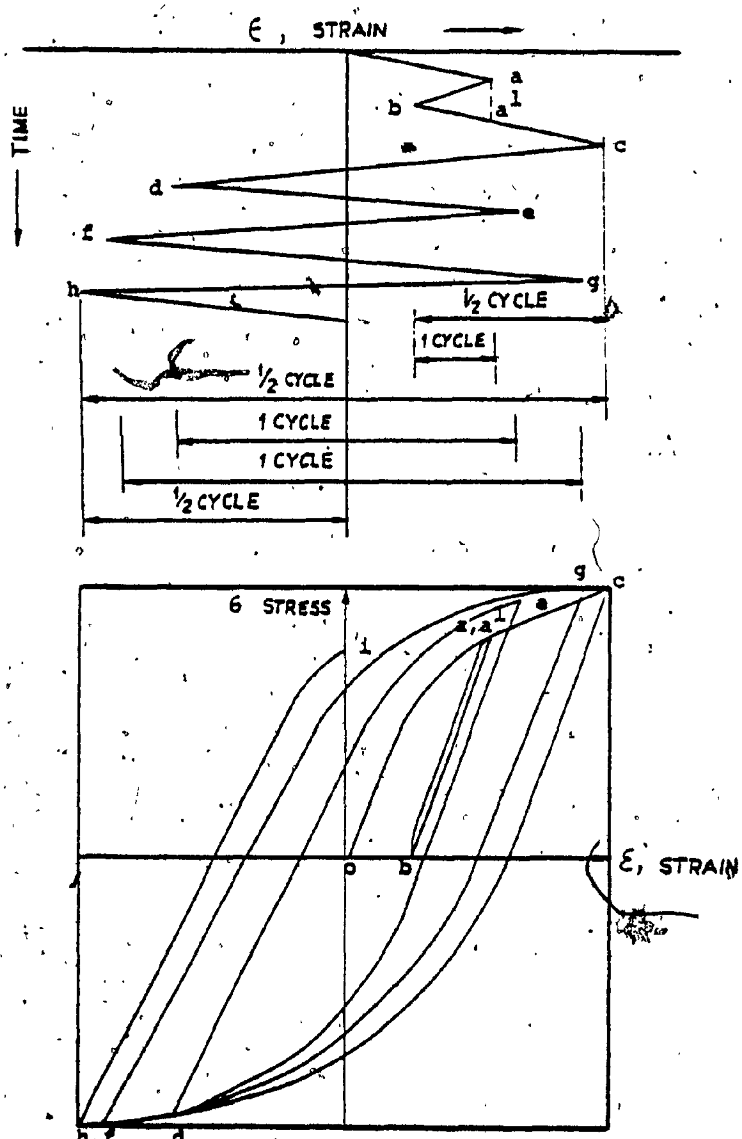


FIG. 2.10 Full and Half Cycles Shown on Complex Strain History and Its Stress-Strain Response [3]

case where the small reversals do significant damage.

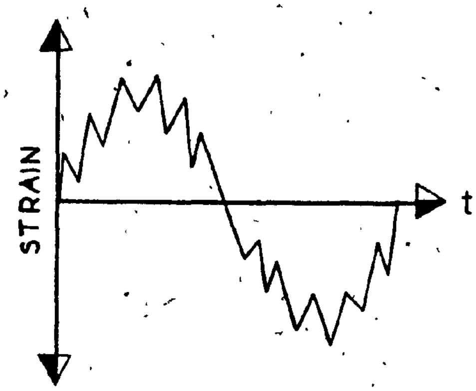
The range mean counting method has the characteristic that, if small reversals are counted, the large range is broken up and counted as several smaller ones. This gives the unrealistic results that small excursions do negative damage, as the calculated damage can be decreased by including them. For example, in Figure 2.11a, the large cycle on which the smaller ones are superimposed is not recognized by range counting; therefore, the calculated damage could easily be less than for Figure 2.11c.

The order overall range (OOR) method abbreviates the strain history to a certain screening level, but counts all possible ranges above the screening level.

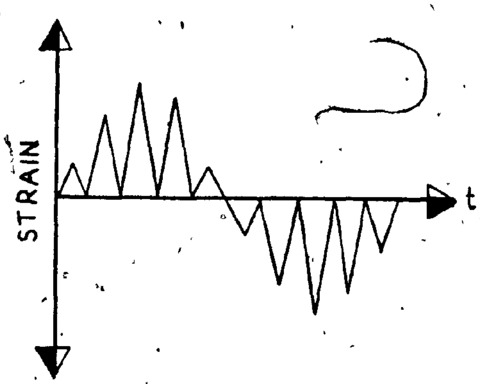
The range pair and rain flow counting results are much more reasonable, the small reversals being treated as interruptions of large strain ranges, and the damages for the large and small strain ranges are simply added.

Except when half cycles are being counted, the rain flow counting method reduces to the range pair method. All of the cycles counted by the rain flow method are therefore counted as cycles by the range pair method. But the half cycles counted by the rain flow method (Figure 2.10) are handled differently by the range pair method, resulting in no damage being calculated for some parts of the strain time history if the range

(A)



(B)



(C)

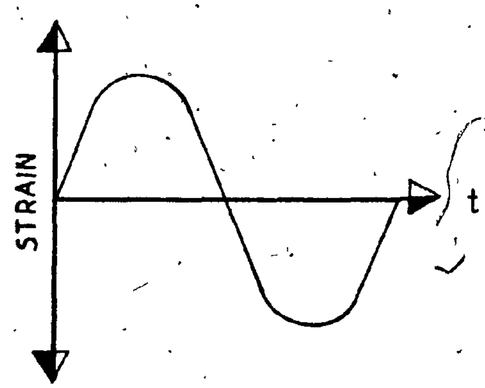


FIG. 2.11 Sequences Which Cause Problems For Several Cycle Counting Methods [3]

pair method is used. This difference is significant only in situations where the damage due to individual half cycles is important, namely where there are only a few reversals to failure, or where there are insignificant minor reversals and most of the damage is done by a few major reversals.

CHAPTER III

EVALUATION OF CYCLIC STRESS-STRAIN AND FATIGUE RESISTANCE MATERIAL PROPERTIES

CHAPTER III

EVALUATION OF CYCLIC STRESS-STRAIN AND FATIGUE RESISTANCE MATERIAL PROPERTIES

3.1 OVER VIEW.

The purpose of this Chapter is to outline a method for evaluating the cyclic stress-strain and fatigue resistance material properties. As mentioned by D.T. Rask and Jo Dean Morrow in [6] the fatigue testing in the low cycle regime will be employed for this purpose. Consequently, it is necessary to distinguish between low cycle fatigue testing and fatigue testing more generally. By definition, failure in low cycle fatigue occurs in fewer than 50,000 cycles. It is further characterized by the existence of stress-strain hysteresis loop and by measurement of plastic strain range in the test specimen. To meet these requirements, certain special testing techniques not generally considered in high cycle fatigue testing assume importance. To produce failure in a few cycles, strain rather than stress must be controlled. This places special emphasis on strain-detecting devices and accompanying instrumentation for measurement and control. Specimen configuration becomes important, since buckling or bending must be avoided and reliable strain measurements must be made.

3.2 THE SPECIMENS

The overall size and type of specimen used for low cycle fatigue testing is often dictated by the form of the available metal stock. Even when this is not a consideration, there are other factors which may influence the design. Obviously, the specimens must be "sized" to the load capacity of the testing system employed. Also, the enlarged threaded ends which are generally used to fasten the specimen to the testing system must be designed to ensure failure in the reduced section. A ratio of between 4 to 6 for the area of the threaded end and the reduced section is usually satisfactory.

Several successful low cycle fatigue specimen configurations are given in Figure 3.1, along with comments on their limitations and applicability.

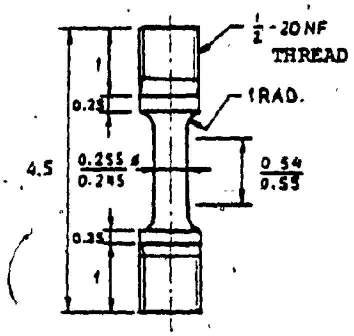
The correct or optimum choice of specimen size and shape also depends upon the strength and ductility of the metal being investigated. Consequently, one should plan to measure the tensile properties of the metal before extensive fatigue testing is performed.

While it is important to machine the samples carefully so that they are straight and axial, it is normally not necessary to be as careful about the surface finish in the reduced section as it is for long-life fatigue specimens.

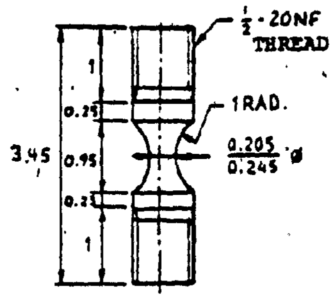
Since plastic strain is usually present in low cycle fatigue testing, the scatter in the results is usually not as great as in tests where longer lives result. One reason is that the plastic action "washes out" many of the scatter-producing factors. This is particularly true for reasonably ductile metals in the low cycle fatigue region because the large amount of cyclic plastic strain imposed during the fatigue test eliminates the initial residual stresses and greatly reduces the influence of small scratches and other stress raisers. When longer lives are expected and when metals have low ductility, the same care should be taken to obtain a smooth surface which is as free of residual stresses as is generally taken with long-life fatigue samples.

After the machining has been completed, the diameter and hardness of the specimens are measured. Normally, the specimen diameter is measured at three locations, the diameter is measured at least twice (90° apart) and the average of the two values is used to compute the area. If these areas differ greatly, the specimen should be rejected or used only for preliminary tests.

Hardness measurements should be made on each specimen to ensure uniformity. It is usually satisfactory to use the average of three hardness values from each shoulder adjacent to the gauge section.

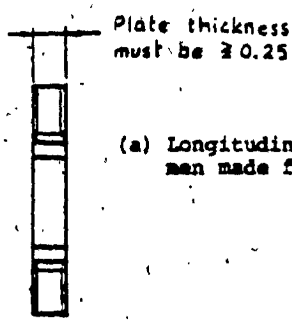


(A) Longitudinal Specimen - 0.5 in. gauge length used for strain amplitudes up to ± 2.0%, L/D ≧ 2. Test section diameter uniform to ± 0.0005 in gauge section. Surface finish 16 r.m.s. max.



(B) Hourglass Specimen - Used for strain amplitudes greater than ± 2.0%. Requires diameteral strain measurement. Surface finish 16 r.m.s.max.

(C) Special Longitudinal Specimens - (All dimentions the same as those above.



(a) Longitudinal specimen made from plate.

(b) Longitudinal specimen made from tubing.

FIG. 3.1 Fatigue Specimens [6]

Once these preliminary steps have been completed, it is desirable to store the specimens in an evacuated bell jar or individually in test tubes containing noncorrosive anhydrous salts.

3.3 THE CYCLIC STRESS-STRAIN CURVE

The initial cyclic rate of change in properties is greatly influenced by the cyclic strain range. However, the amount of change rapidly diminishes with repeated cycling during the first few percent of fatigue life. As long as the control conditions are not altered, a metal will quickly adjust to a nearly stable condition which is reflected by a nearly stable mechanical hysteresis loop.

By connecting the tips of several stable loops at different strain ranges, a smooth curve is formed which is called the cyclic stress-strain curve. This curve can be mathematically expressed as

$$\left(\frac{\Delta\varepsilon}{2}\right) = \left(\frac{\Delta\varepsilon_e}{2}\right) + \left(\frac{\Delta\varepsilon_p}{2}\right) = \left(\frac{\sigma_a}{E}\right) + \varepsilon'_f\left(\frac{\sigma a}{\sigma'_f}\right)^{\frac{1}{n'}} \quad (3.1)$$

where

σ_f, ε_f, σ_a and n' are as defined in Figures 3.7, 3.8, 4.2 and 3.9, respectively.

Since most metals stabilize rather quickly after changes in the cyclic strain amplitude, it is possible to determine the cyclic stress-strain curve from one specimen. The following describes two techniques to determine this curve.

3.3.1 Multiple Step Tests

Determining the cyclic stress strain curve from multiple step testing consists of cycling one specimen at several levels of strain amplitude. The number of cycles at each level should be sufficient to achieve stability but small enough to avoid serious fatigue damage. With the stable hysteresis loops superimposed, the cyclic stress-strain curve is obtained by drawing a smooth curve through the tips of the loops.

3.3.2 Incremental Step Tests

Another means of producing the cyclic stress-strain curve consists of subjecting a specimen to blocks of gradually decreasing and then increasing strain amplitudes as shown in Figure 3.2. A maximum strain amplitude of ± 1.5 to 2.0 percent is usually sufficient to cyclically stabilize the metal quickly without the danger of causing the specimens to neck, fail, or buckle before a stable state is achieved. The cyclic stress-strain curve is then determined by the locus of superimposed hysteresis loop tips.

One means of producing these curves is to use a Data-Trak (MTS Systems Corp., Minneapolis, Minn.) function generator to control the strain level of an MTS electro-hydraulic closed-loop testing system. With this equipment, the maximum strain amplitude is set on the testing system and the Data-Trak is used to provide a uniformly decreasing and increasing envelope for the strain amplitude. Figure 3.2 depicts a typical strip chart record obtained by this technique. Usually, the Data-Trak and testing system are co-ordinated so that the zero strain amplitude is reached after 40 cycles and the maximum strain amplitudes after another 40 cycles. In this way, a cyclic stress-strain curve is generated after three or four blocks of decreasing and increasing strain amplitude, as shown in Figure 3.3.

3.4 STRAIN-CONTROLLED LOW CYCLE FATIGUE TESTING

3.4.1 Testing Sequence

Usually, from seven to ten specimens are required to determine the low cycle fatigue resistance of a metal. For most metals, this can be adequately described by testing at strain amplitudes between ± 2.0 and ± 0.2 percent. The result is the strain-life amplitude similar to that shown in Figure 3.4.

The testing should be conducted so that the data points will be uniformly distributed along the log-life axis. To accomplish this, the strain life-curve is approximated so that

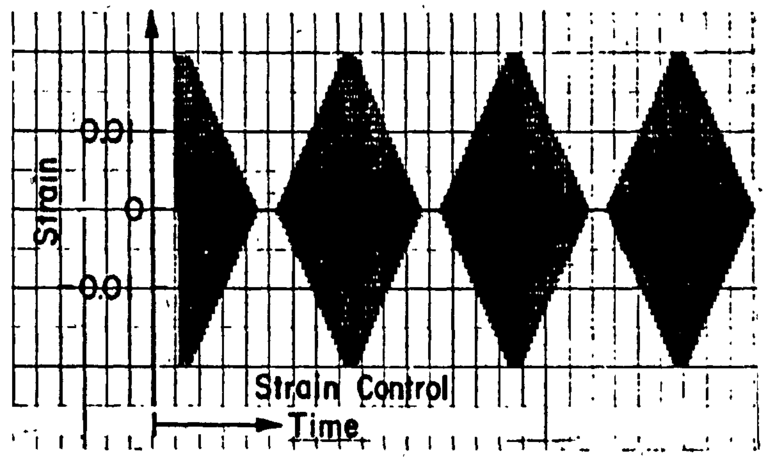


FIG. 3.2 Strain Time Record For Incremental Step Test [6]

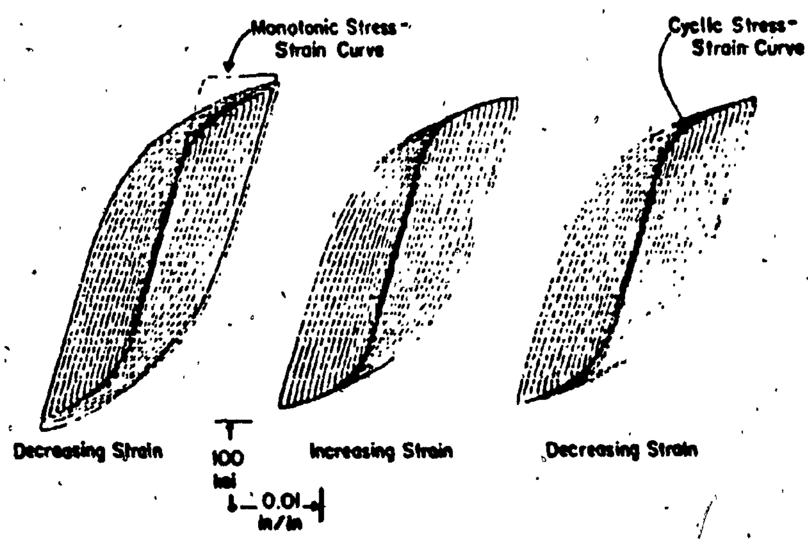


FIG. 3.3 Stress-Strain Record of Incremental Step Test on Quenched and Tempered (SAE. 4142) [6]

fatigue lives can be predicted. The initial approximation is made from the results of the tension test, the cyclic stress-strain curve, and one strain-controlled fatigue test. The strain amplitude for this fatigue test must be selected,for those plastic strains larger than 10^{-3} and failures at about 10^3, reversals result. It has been empirically found that a strain amplitude of ± 1 percent fulfills these requirements for all steels regardless of strength and for most ductile metals. Thus, testing is usually begun at this strain amplitude. Figure 3.6 shows typical hysteresis loops and strain life curves for metals of different strengths and ductilities at this strain amplitude.

Before proceeding with this approximation, it is necessary to develop the equations which relate strain to fatigue life. As shown in Figure 3.4, the total strain amplitude can be separated into its elastic and plastic components. In low cycle fatigue testing, the plastic component of the strain amplitude is characterized by the fatigue ductility properties and elastic component by the fatigue strength properties of metals.

3.4.2 Fatigue Ductility Properties

For a series of completely reversed tests at different strain ranges, a log-log plot of the fatigue life versus the stable plastic strain amplitude usually gives a straight line.

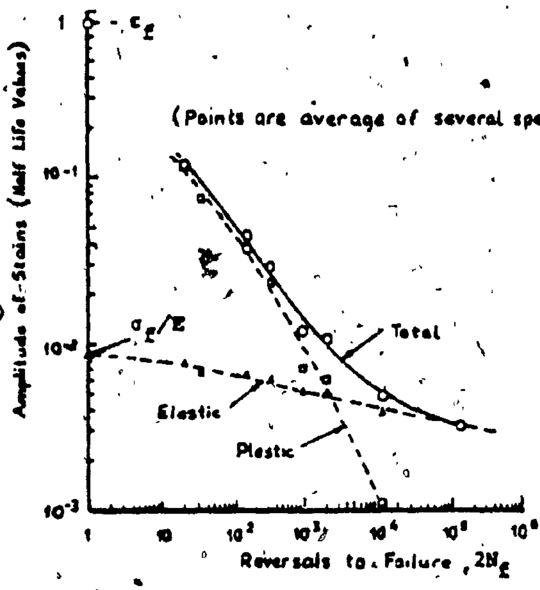


FIG. 3.4 Life as a Function of Elastic, Plastic, and Total Strain - SAE 4340 steel [6]

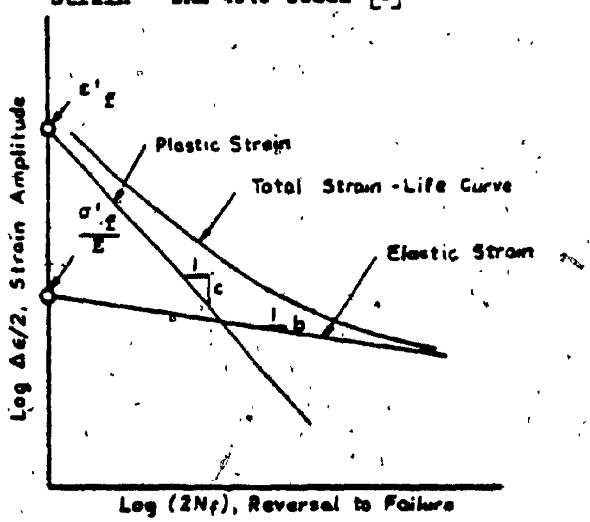
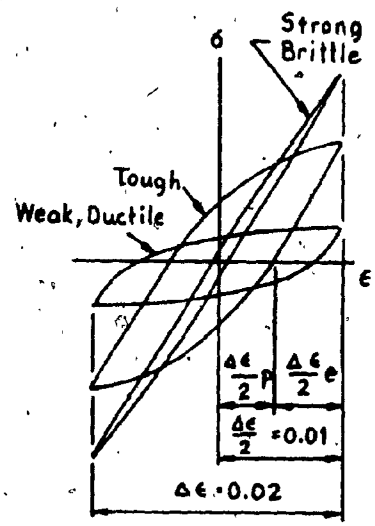


FIG. 3.5 Life as a Function of elastic, Plastic and Total Strain Amplitude [6]



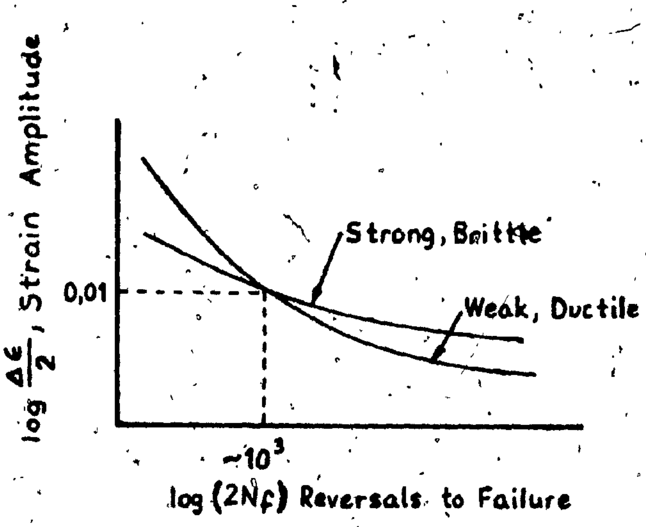


FIG. 3.6 Schematic Representation of the Manner in Which Ductile and Strong Metals Respond to Cyclic Strains of ± 1 Percent [6]

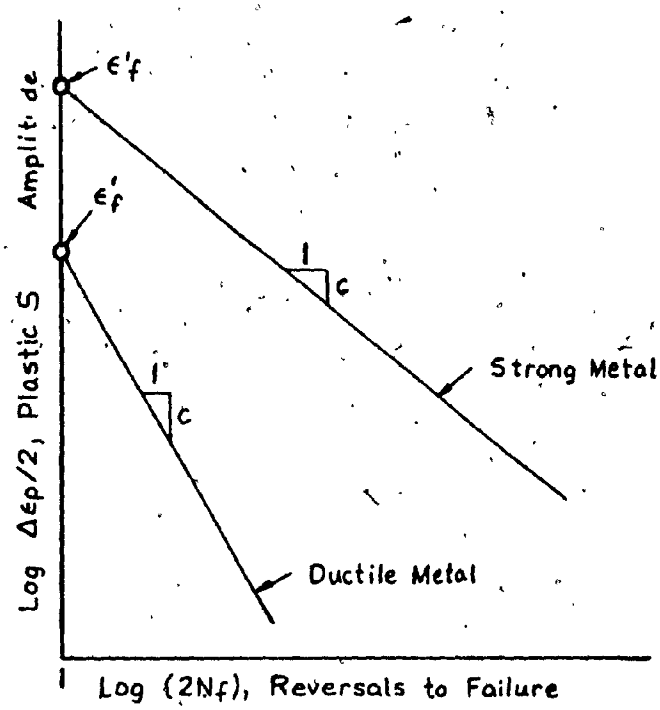


FIG. 3.7 Schematic Plastic Strain-Life Curves Showing the Influence of Strength and Ductility [6]

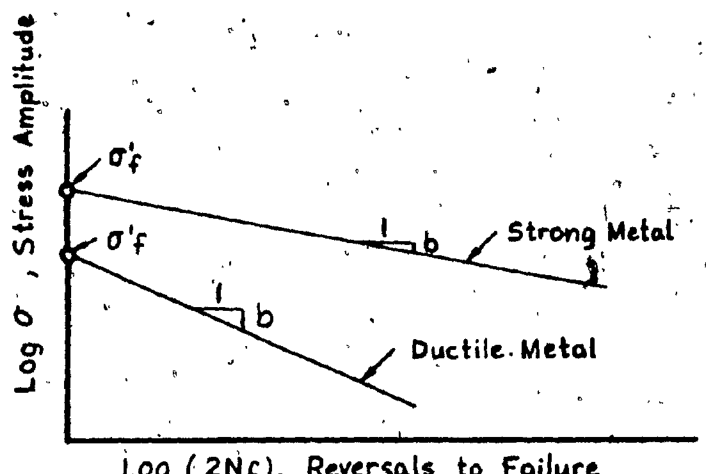


FIG. 3.8 Schematic Stress-Life Curves Showing the Influence of Strength and Ductility [6]

As illustrated in Figure 3.7, the plastic strain intercept at one reversal is defined as the fatigue ductility coefficient [6] and it can be approximated by

$$\varepsilon'_f = 0.002 \left(\frac{\sigma'_f}{S'y_s}\right)^{\frac{1}{n'}} \tag{3.2}$$

where

$\sigma'_f \cong \sigma_f$ = True tensile stress to cause fracture, $\frac{P_f}{A_f}$

S'_y = cyclic 0.2 percent offset yield strength

The slope of the curve in Figure 3.7 is also a fatigue ductility property. This is defined by the fatigue ductility exponent, C.

Thus, the plastic strain amplitude can be written as

$$\left(\frac{\Delta\varepsilon_p}{2}\right) = \varepsilon'_f (2N_f)^C \tag{3.3}$$

3.4.3 Fatigue Strength Properties

As illustrated in Figure 3.8, the stress amplitude intercept at one reversal is defined as the fatigue strength coefficient. The fatigue strength exponent, b, is defined as the slope of the curve. Hence, a relationship similar to that of Equation (3.3) can be written for the stress amplitude.

$$\sigma_a = \sigma'_f \; (2N_f)^b \tag{3.4}$$

or

$$\left(\frac{\Delta\varepsilon_e}{2}\right) = \frac{\sigma_a}{E} = \left(\frac{\sigma'_f}{E}\right)(2N_f)^b \tag{3.5}$$

where

σ_a is taken as the half-life value of the stress amplitude.

The strain life curve can now be approximated as follows: The first reversal intercepts are determined by the fatigue ductility coefficient computed from Equation (3.2) and the fatigue strength coefficient approximated by the true fracture strength. The half-life plastic strain amplitude from ± 1 percent test is determined from the hysteresis loops recorded during the test. This results in a datum point on the strain-life curve. A straight line drawn through this point and the fatigue ductility coefficient results in the plastic strain-life curve. Similarly, the elastic strain-life curve is the line drawn through the half-life elastic strain amplitude and the fatigue strength coefficient.

The curves represented by Equations (3.3) and (3.5), are those summed to give the total strain-life curve, as shown in Figure 3.4. The equation of this curve is written as

$$\left(\frac{\Delta\varepsilon}{2}\right) = \left(\frac{\Delta\varepsilon_e}{2}\right) + \left(\frac{\Delta\varepsilon_p}{2}\right) \qquad (3.6)$$

or

$$\left(\frac{\Delta\varepsilon}{2}\right) = \left(\frac{\sigma'_f}{E}\right)(2N_p)^b + \varepsilon'_f(2N_f)^c \qquad (3.7)$$

The results of subsequent tests can then be used to improve the approximation of the strain-life curve. An accurate strain-life curve is especially important in long lives where a small change in the strain amplitude can change the life of the specimen by several orders of magnitude. Therefore, it is best to test the larger strain amplitude first. Also, since the scatter is usually less at higher strain amplitudes, the fatigue lives of any subsequent tests can be predicted with more confidence.

3.5 CYCLIC STRESS-STRAIN PROPERTIES

3.5.1 Cyclic Strain Hardening Exponent (n')

For each metal there is a range of potential strength or hardness which can be achieved by cold working, annealing, heat-treating, etc. If a metal is initially soft, it will cyclically harden; if it is initially hard, it will probably soften. Some intermediate state will approach which represents a stable condition for the particular metal under the imposed conditions. The initial state is reflected by the monotonic strain-hardening exponent, n. For most metals subjected to

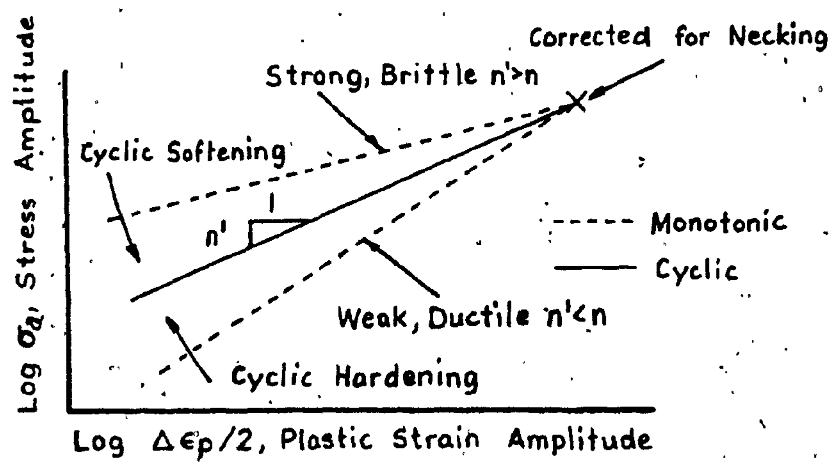


FIG. 3.9 Comparison of Monotonic and Cyclic Strain Hardening Exponents [6]

cyclic loadings, the value of the cyclic strain-hardening exponent n', lies between 0.1 and 0.2. As illustrated in Figure 3.9, we can use the cyclic stress-strain curve to find the cyclic strain-hardening exponent.

3.5.2 Cyclic Strength Coefficient (K')

As illustrated in Figure 3.9, the relationship between the stress amplitude and the plastic strain amplitude can be written in the following form:

$$\sigma_a = K' (\varepsilon_p)^{n'} \tag{3.8}$$

The cyclic strength coefficient (K') can be calculated from Equation (3.8) as follows:

$$K' = \frac{\sigma_a}{(\varepsilon_p)^{n'}}$$

$$= \frac{\sigma'_f}{(\varepsilon_p)^{n'}}.$$

CHAPTER IV

DAMAGE ASSESSMENT

CHAPTER IV

DAMAGE ASSESSMENT

4.1 OVERVIEW

In this Chapter, the basic requirements of the damage calculation are considered separately. The following areas are covered: stable stress-strain hysteresis loop, strain-life curves and cyclic stress-strain curve. This information will be combined into a specific method of damage assignment.

4.2 STABLE STRESS-STRAIN HYSTERESIS LOOP AND CYCLIC STRESS-STRAIN CURVE

During a constant amplitude controlled strain test, after the initial rapid hardening or softening is complete so that the stress-strain behavior is approximately stable, a stress-strain hysteresis loop, as is shown in Figure 4.1a, is formed. Usually, hysteresis loops taken at half the fatigue life may be used to approximate the behavior during most of the life. If half-life hysteresis loops from tests at several different strain levels are plotted on the same set of axes as that illustrated in Figure 4.1b, a cyclic stress-strain curve is defined by the locus of the loop tips, Curve OABC in Fig. 4.1b.

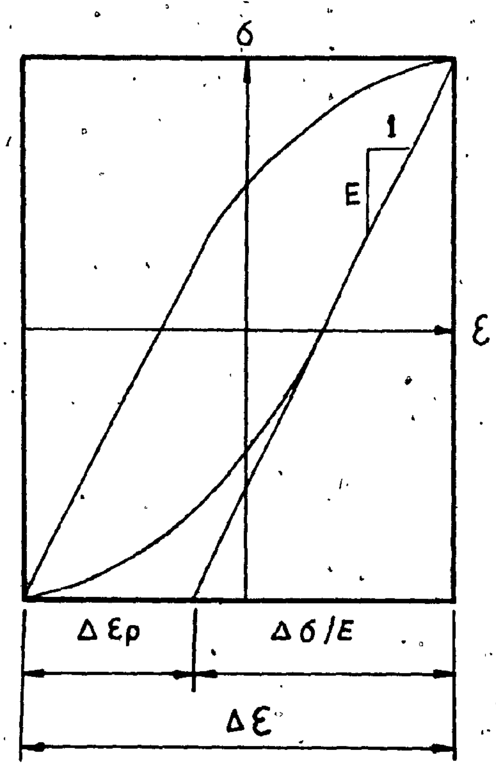


FIG. 4.1(a) Stress-Strain Hysteresis Loop [5]

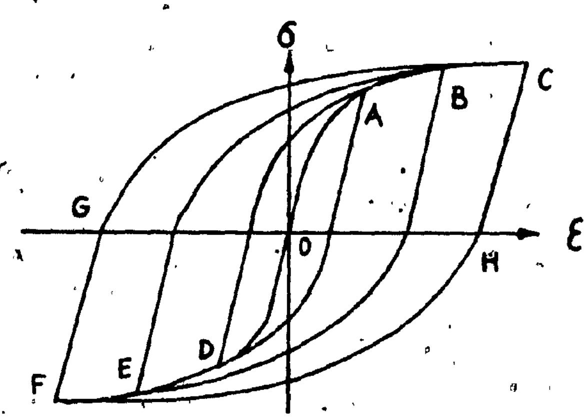


FIG.4.1(b) Cyclic Stress-Strain Curve Determined From a Set of Stable Hysteresis Loop [5]

4.3 STRAIN-LIFE FATIGUE CURVE

A strain versus cycles to fatigue crack initiation curve is needed. This is usually obtained in the uniaxial state of stress by testing un-notched axial test specimens.

Constant amplitude, completely reversed, controlled strain tests on such specimens provide both of the needed curves. Strain-life curves so obtained are shown in Figure 3.5 [6].

4.4 DAMAGE ASSIGNMENT DEVELOPMENT

To obtain mathematical relationships realistically describing the cyclic stress-strain and strain-life curves, the total strain range, $\Delta\varepsilon$ in Figure 4.1a, is divided into elastic and plastic components:

$$\Delta\varepsilon = \left(\frac{\Delta\sigma}{E}\right) + \Delta\varepsilon_p \qquad (4.1)$$

where

$\left(\frac{\Delta\sigma}{E}\right)$ is the elastic strain range determined from the stress range, $\Delta\sigma$, and the elastic modulus, E, and where

$\Delta\varepsilon_p$ is the plastic strain range.

Amplitudes, or half-ranges, of these quantities are usually employed:

$$\varepsilon_a = \frac{\Delta\varepsilon}{2}, \quad \sigma_a = \frac{\Delta\sigma}{2}$$

and

$$\varepsilon_p = \frac{\Delta\varepsilon_p}{2}.$$

If a log-log plot is made of stress versus plastic strain, a straight line usually results, implying a mathematical relationship of the form

$$\sigma_a = K'(\varepsilon_p)^{n'} \qquad (4.2)$$

where

K' = cyclic strength coefficient,

n' = cyclic strain-hardening exponent.

To mathematically represent the strain-life curve, elastic and plastic strains are plotted separately versus cycles on log-log coordinates, as in Figure 3.5. Straight lines usually result, giving equations of the forms:

$$\frac{\sigma_a}{E} = \sigma'_f \,(2N_f)^b \qquad (4.3)$$

$$\varepsilon_p = \varepsilon'_f \,(2N_f)^c \qquad (4.4)$$

where

σ'_f = cyclic fatigue strength coefficient

ε'_f = cyclic fatigue ductility coefficient

b = cyclic fatigue strength exponent

c = cyclic fatigue ductility exponent

$2N_f$ = reversals to failure.

The above damage equations have been developed for completely reversed stress-strain cycles with zero mean stress, however, most of the stress-strain cycles to be evaluated have a tensile or compressive mean stress associated with them. It is generally recognized that tensile mean stresses reduce fatigue life and compressive mean stresses increase it. For the purpose of damage assessment, an equivalent stress-strain cycle of zero mean stress is calculated. Most approximations involve an equivalent completely reversed stress cycle. The following form was suggested by Morrow [11]:

$$\sigma_{cr} = \frac{\sigma_a}{1 - \sigma_o/\sigma'_f} \qquad (4.5)$$

where

σ_{cr} = equivalent completely reversed stress amplitude

σ_a = stress amplitude of the cycle with mean stress σ_o

4.5 METHOD OF DAMAGE ASSIGNMENT

To evaluate cycles on the basis of strain amplitude directly requires iteration because life cannot be explicitly expressed in the strain-life equation. As stated by W.R. Brose

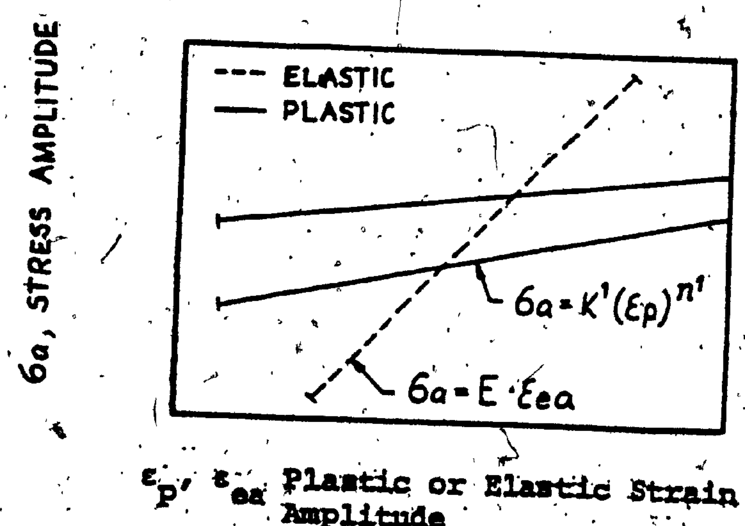


FIG. 4.2 Plastic and Elastic Components of Cyclic Strain Plotted on Log-Log Coordinates [5]

in [4], the stress-life equation will be used at long lives and the inelastic strain-life equation at short lives. Division is made on the basis of a somewhat arbitrarily chosen "transition strain amplitude", ε_{tr}. The cyclic stress-plastic strain relation, representing the results of an incremental step cyclic stress-strain test, is also used. Very small cycles below a specified "computational limit strain amplitude", $\varepsilon_{c\ell}$, will be ignored to reduce computational time. Miner's rule [14] is being used for linearly summing cycle ratios for the strain history. The method of calculating the damage for stress-strain cycles is summarized:

1) $\varepsilon_a \leq \varepsilon_{c\ell}$:

$$\frac{1}{N_f} = 0 \qquad (4.6)$$

2) $\varepsilon_{c\ell} < \varepsilon_a \leq \varepsilon_{tr}$:

$$\sigma_{cr} = \frac{\sigma_a}{1 - (\sigma_o/\sigma'_f)} \qquad (4.7)$$

$$\frac{1}{N_f} = 2\left(\frac{\sigma'_f}{\sigma_{cr}}\right)^{1/b} \qquad (4.8)$$

3) $\varepsilon_a > \varepsilon_{tr}$:

$$\sigma_{cr} = \frac{\sigma_a}{1 - (\sigma_o/\sigma'_f)} \qquad (4.9)$$

$$\varepsilon_p = \left(\frac{\sigma_{cr}}{K'}\right)^{1/n'} \qquad (4.10)$$

$$\frac{1}{N_f} = 2\left(\frac{\varepsilon'_f}{\varepsilon_p}\right)^{1/c} \qquad (4.11)$$

where

$\frac{1}{N_f}$ = damage per cycle.

Following Miner's role, the damage per cycle should be added up to find the total damage due to the entire strain history.

CHAPTER V

ILLUSTRATIONS

CHAPTER V

ILLUSTRATIONS

5.1 OVERVIEW

This Chapter illustrates the procedure required in predicting the fatigue life of structures under complex loading. It will discuss the use of the computer in counting the complex cycles, using rain flow counting technique, the low cycle fatigue resistance properties and the damage assessment.

5.2 STRAIN HISTORY

5.2.1 Nominal Strain History

In fatigue analysis of components in real structures, it is often impractical to obtain strain gauge data at critical locations such as those at the notch root. Access to those locations is often difficult, and gauge life is uncertain when large cyclic strains are present, strain gauge recordings at essentially elastically stressed locations away from notches are easier to obtain, or elastic analysis may provide accurate stresses at small distances from plastically strained areas. Consequently, the strain measurements were intended to be nominal strain measurements and the strain history generated in this Chapter is, in fact, nominal strain history.

5.2.2 The Experiment

The strain history is generated throughout an experimental laboratory work, in which three strain gauges were installed (see Figure 5.1) on the diesel engine mounting bracket, located in the laboratory of the Mechanical Engineering Department, Concordia University, to produce the complex strain history, while a random loading was applied to the running engine by the hydraulic brake. Figure 5.1 illustrates the experiment set-up, as well as the strain gauge locations, while Figure 5.2 illustrates the experimental connection diagram.

Each strain gauge was calibrated by shaking the engine manually and reading the value of the output signal from the strain indicator. The main output signal, as well as the calibration signal, were recorded separately for each strain gauge on a magnetic tape, using a tape-recorder, as shown in Figure 5.1.

The firing frequency of the engine was considered to be the main exciting frequency, hence the maximum frequency content was set to 200 HZ(4-cycle diesel engine at 3,000 r.p.m.) and the analog signal was sampled at 5 KHZ to eliminate the bias errors. Each signal was then converted into digital series of peaks and valleys, while the original sequence of events was preserved. Valleys and peaks were then defined as being separated by a range of at least 10% of the maximum range, and the smaller ranges were filtered out from the original analog data. The output signals of the strain gauges were compared and the largest one was considered.

The calibration signal was then introduced through a special computer program to generate the strain history. The calibration and strain signa ls, as displayed on the FFT (Fast Fourier Transform Analyzer) are shown in Figures 5.3a and 5.3b, respectively.

5.2.3 Local Strain History

In order for an accurate fatigue life to be made the nominal strain history must be converted to local strain history, i.e., the local strain approach, as described in Chapter I, should be followed.

Utilizing the technique developed by W.R. Brose in [4] the local strain history can be calculated from loads or nominal strain in conjunction with load-notch strain curves. The load-notch strain curve can be obtained either experimentally [16] or analytically [17]. Experimentally, the cyclic load-notch strain curve can be obtained in one test, by subjecting the component to blocks of incrementally increasing and decreasing reversed loading which allows the metal at the notch to cyclically stabilize. Analytically, an elastic-plastic finite element analysis can produce the curve if the cyclic stress-strain curve is employed in place of the static stress-strain curve.

Also, the Neuber Rule can be used to calculate the local strains from nominal stresses, if nominal elasticity is assumed. The Neuber rule [15] states that, for a notched member subjected to a nominal stress excursion, Δs, the notch root will deform until the product of the local stress strain excursions is equal to the following constant:

$$\Delta\sigma \times \Delta\varepsilon = \frac{(K_f \ \Delta s)^2}{E} \qquad (5.1)$$

where

K_f = the fatigue notch factor for any particular material and geometry, and

E = the elastic modulus.

Equation (5.1) is of the form of rectangular hyperbola and defines the control condition at the notch root. Now, the material will be allowed to deform along the cyclic stress-strain curve until it intersects the control hyperbola, thus satisfying the required equality.

In order to obtain the cyclic load-notch strain curve or fatigue notch factor, an additional experimental work is required which is not considered throughout this report. Consequently, the nominal strain history produced in this Chapter, will be considhred as local strain history, for the purpose of illustrations only.

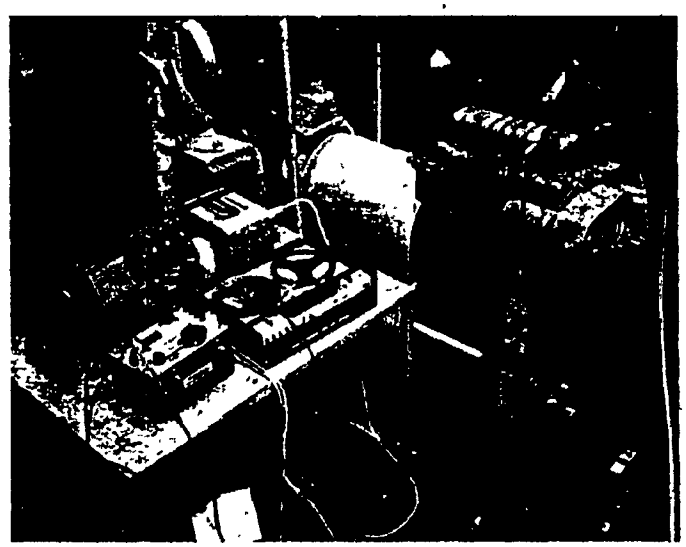


FIG. 5.1 Instrumentation Used for Generating the Strain History in the Lab

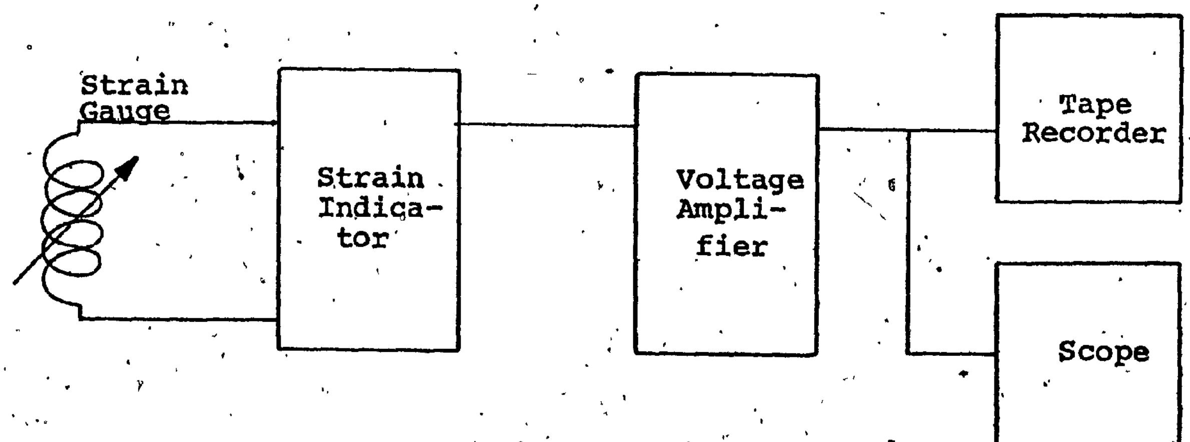


FIG. 5.2 Connection Diagram

5.3 FORTRAN PROGRAM FOR GENERATING THE STRAIN HISTORY

A special sorting program has been designed to re-arrange the digitized signal for the fatigue program. The program is designed to locate the maximum peak and minimum valley in the array, so that the fatigue analysis can begin with the peak of the maximum value. This guarantees proper orientation of the stress-strain response since either peaks or valleys must lie on the cyclic stress-strain curve, as shown in Figure 4.1b. All other points lie on the outer loop curve. Starting with the maximum peak or minimum valley insures only the closed loop full cycles for evaluation of damage.

For the purpose of this Chapter, the program is creating a magnified strain history (300x), rearranged so that the history ends with the maximum peak or minimum valley.

The Computer Program is shown in Figure 5.4, and the produced strain history is presented in Appendix A.

5.4 FORTRAN PROGRAM FOR SIMULATING STRESS-STRAIN RESPONSE AND CALCULATING DAMAGE

5.4.1 FORTRAN Listing

Figure 5.5 contains a listing of FORTRAN program which predicts the initiation life for a given strain history and material. The body of the program is shown in flow chart form, in Figure 5.6, and the FORTRAN notation is described in Table 5.1.

5.4.2 Comments on the Structure of the Program

In the form shown, the program accepts one strain history, arranged so that the maximum magnitude strain is last. The stress and strain values which represent the point where the stress-strain path begins are input as initial values of the running stress and running strain. While the program uses 50 elements, ten straight-line segments adequately represent the stress-strain loop shape curve. Thus, the 50 elements are actually ten sets of five. Only the first element of each set is input, and the program is duplicating the rest.

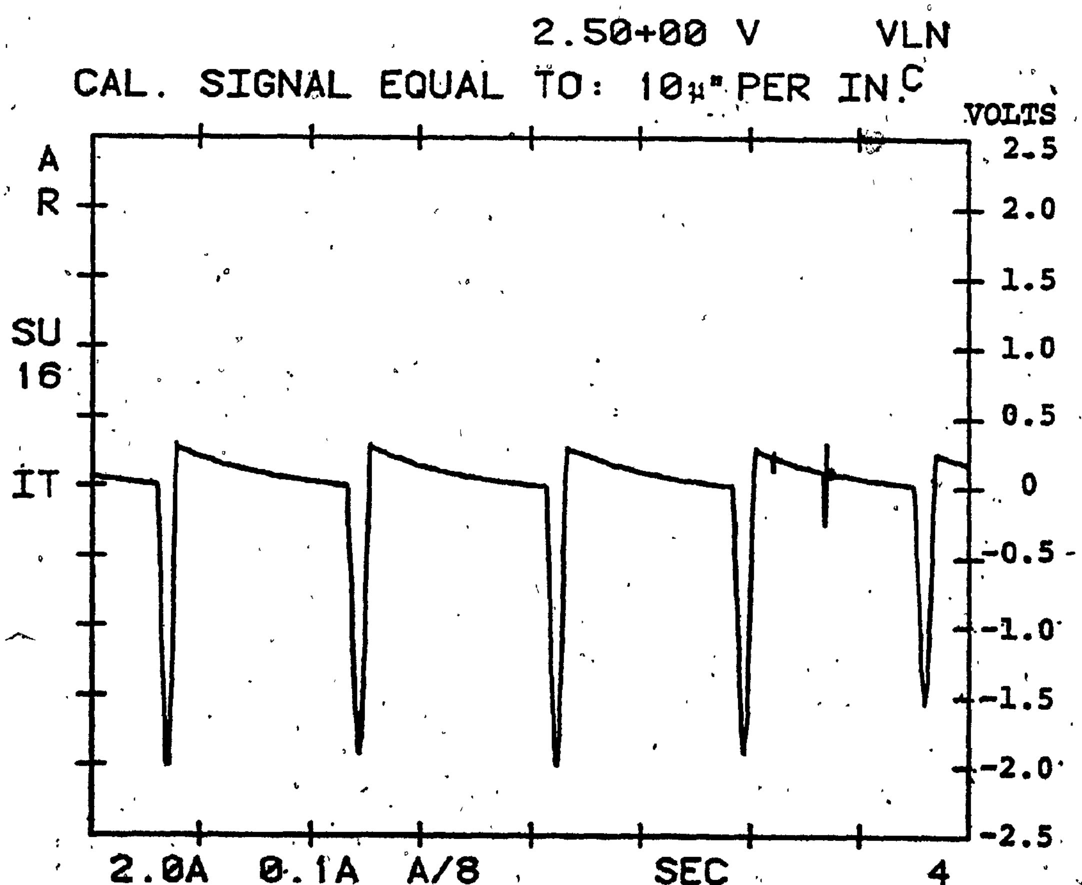


FIG. 5.3 (A) Calibration Signal

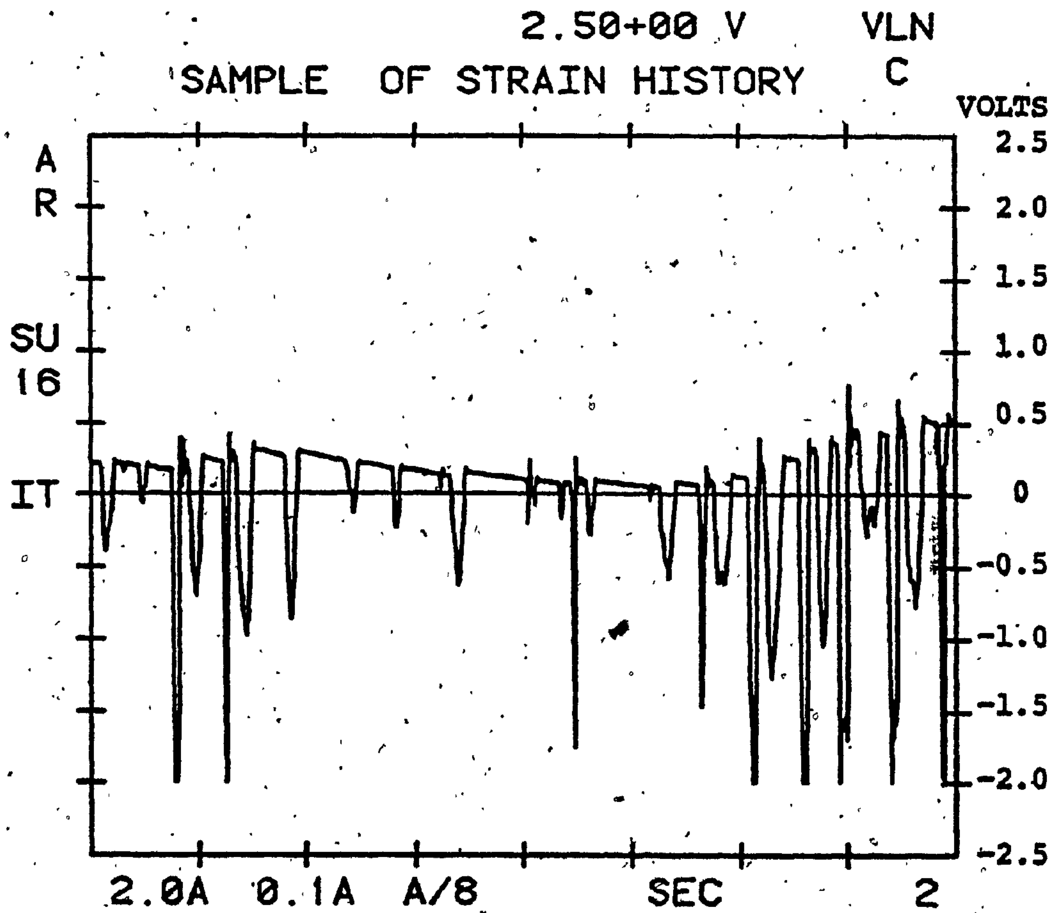


FIG. 5.3 (B). Sample of the Strain History

```
      PROGRAM  ATTIA(VOLT,OUTPUT,TAPE1=VOLT)
      DIMENSION   A(2210),K(2210)
      NS=1
      NF=10
      DO  10  KL=1,220
      READ(1,20),NI,(A(I),I=NS,NF)
      NS=NS+10
      NF=NF+10
20    FORMAT(I6,10F6.2)
10    CONTINUE
      I=2
      PMAX=A(I)
      N=2200
      DO  30  J=2,N
      IF(A(J).LE.0)  GO  TO  30
      IF(PMAX.GE.A(J))  GO  TO  30
      PMAX=A(J)
      JMAX=J
30    CONTINUE
      PRINT  40,PMAX,JMAX
40    FORMAT(7X,F6.2,I6)
      I=1
      VMIN=A(I)
      DO  50  J=2,N
      IF(A(J).GE.0)  GO  TO  50
      IF(VMIN.LE.A(J))  GO  TO  50
      VMIN=A(J)
      JMIN=J
50    CONTINUE
      PRINT  40,VMIN,JMIN
      IF(IABS(PMAX).LE.IABS(VMIN))  GO  TO  60
```

FIG. 5.4 Listing of FORTRAN Program for Generating the Strain History From Digitized Strain Signal

```
      I=JMAX+1
      GO TO 70
60    I=JMIN+1
70    DO  80  L=1,2200
      K(L)=1500*A(I)
      I=I+1
      IF(I.EQ.2201)  GO  TO  90
      GO  TO  80
90    I=1
80    CONTINUE
      PRINT  120
120   FORMAT(////)
      KH=1
      KF=1
      KZ=1
      NR=1
      NZ=10
      DO  100  IL=1,220
      PRINT  110,KH,(K(L),L=NR,NZ)
      NR=NR+10
      NZ=NZ+10
      KH=KH+1
      IF(KH/KF.EQ.5)  GO  TO  107
      GO  TO  100
107   PRINT  108
108   FORMAT(/)
      KF=KF+1
100   CONTINUE
110   FORMAT(I3,7X,10I7)
      STOP
      END
```

FIG. 5.4 Listing of FORTRAN Program for Generating the Strain History From Digitized Strain Signal

(continued)

TABLE 5.1 NOTATION USED IN FORTRAN PROGRAM FOR STRESS-STRAIN SIMULATION AND DAMAGE CALCULATION

J	Strain peak counter
I	Element counter
KEPK(I)	Strain peak j
JMAX	Number of strain peaks in history
KAE(I)	Availability of element i
KRE	Running value of strain
KRS	Running value of stress
KEB	Strain band; strain range of element
KSEL(I)	Stress range of element i
KSS(I)	Save stress for element i; when a loop is closed on element i, the save stress is the stress reached the last time element i was used, and thus is the opposite stress peak
EA	Strain amplitude
SA	Stress amplitude
SM	Mean stress
PE	Plastic strain amplitude
FSC	Fatigue strength coefficient
FSE	Fatigue strength exponent
FDC	Fatigue ductility coefficient
FDE	Fatigue ductility exponent

TABLE 5.1 NOTATION USED IN FORTRAN PROGRAM FOR STRESS-STRAIN SIMULATION AND DAMAGE CALCULATION (Continued)

CSC	Cyclic strength coefficient
CSE	Cyclic strain-hardening exponent
ET	Transition strain amplitude; used to divide small cycles from big cycles for damage calculation
EFL	Computational limit strain amplitude, for computational efficiency, damage is not assessed for cycles below this level (it is much less than the conventional fatigue limit)
DAM	Damage assessed for one cycle
DAMSUM	Damage sum
BTF	Predicted blocks to failure (initiation)

```
      PROGRAM  HALA(HIS,STRAIN,OUTPUT,TAPE1=HIS)
      DIMENSION  KEPK(2210),KAE(52),KSEL(50),KSS(50)
C
C  FILL  ARRAY  WITH  STRAINS  FROM  FILE
C
C
      NS=1
      NF=10
      DO  10  I=1,220
      READ  20, (KEPK(J),J=NS,NF)
      NS=NS+10
      NF=NF+10
   10 CONTINUE
   20 FORMAT  (10I7)
      READ  21,(KEPK(J),J=2201)
   21 FORMAT (I7)
C
C
C  READ  IN  CONSTANTS
C
      READ  35,JMAX,KRE,KRS,KEB
      READ  36,FSC,FSE,FDC,FDE,CSC,CSE,ET,EFL
   35 FORMAT (4I7)
   36 FORMAT(F9.0,1X,3F6.3,F9.0,F5.3,2F5.0)
C
C  INITALIZE   AVAILABILITIES
C
      IF(KRE.GT.0)  GO  TO  40
      L=-1
      GO  TO   50
   40 L=1
   50 DO  60   I=1,51
      KAE(I)=L
   60 CONTINUE
      KAE(52)=-L
C
C  READ  IN  STRESS   STRAIN  ELEMENTS
C
      NS=1
      NF=5
      DO   70    M=1,10
      READ   75,K
      DO  74  I=NS,NF
      KSEL(I)=K
   74 CONTINUE
      NS=NS+5
      NF=NF+5
   70 CONTINUE
   75 FORMAT(I5)
```

FIG. 5.5 Listing of FORTRAN Program for Simulating the Stress-Strain Response and Calculating Damage [4]

```
C
C  BODY  OF  THE  PROGRAM
C
        DAMSUM=0.0
        DO  195  J=1,JMAX
        I=1
   80   KAE(I)=-KAE(I)
        KRE=KRE+KAE(I)*KEB
        KRS=KRS+KAE(I)*KSEL(I)
C
C  LOOP  CLOSURE  TEST  (CHECK  AVAILABILITY  OF  NEXT  ELEMENT
C
        IT=I+1
   90   IF(KAE(I).GT.0) GO  TO   100
        GO  TO   110
  100   IF(KAE(IT).GT.0)  GO  TO  160
        GO  TO   120
  110   IF(KAE(IT).LT.0)  GO  TO  160
  120   N=0
C
C  STRAIN  LIMIT   TEST
C
        IF(KAE(I).GT.0)  GO  TO  130
        GO  TO  140
  130   IF((KEPK(J)-KRE).LE.0)  GO  TO  190
        GO  TO   150
  140   IF((KEPK(J)-KRE).GE.0) GO  TO  190
  150   I=IT
        GO  TO   80
  160   IT=IT+1
        IF(N.GT.0)  GO  TO  90
C
C  DAMAGE   CALCULATIONS
C
        EA=I*KEB/2
        IF(EA.LE.EFL)   GO   TO   90
        SB=IABS(KRS-KSS(I))/2
        SM=(KRS+KSS(I))/2
        SA=SB/(1-(SM/FSC))
        IF(EA.GT.ET)  GO  TO  170
        DAM=2.0*(FSC/SA)**(1.0/FSE)
        GO  TO   180
  170   PE=(SA/CSC)**(1.0/CSE)
        DAM=2*(FDC/PE)**(1.0/FDE)
  180   DAMSUM=DAMSUM+DAM
        N=1
        GO  TO  90
  190   KSS(I)=KRS
  195   CONTINUE
        BTF=1.0/DAMSUM
C  OUTPUT   BLOCKS   TO   FAILURE
C
        WRITE  200,BTF
  200   FORMAT(E10.4)
        STOP
        END
```

FIG.5.5 **Listing of FORTRAN Program for Simulating the Stress-Strain Response and Calculating Damage [4]**

(continued)

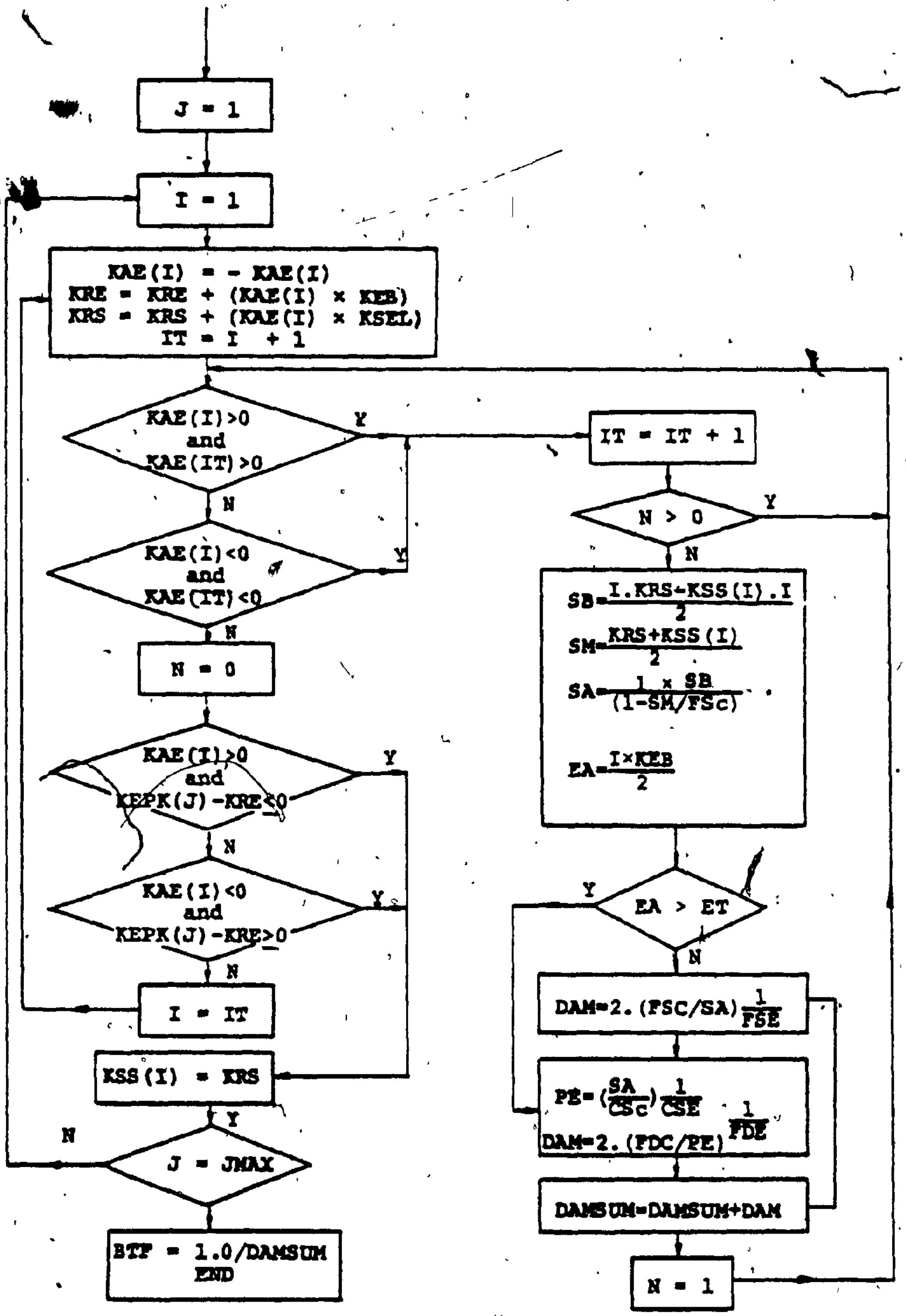


FIG. 5.6 Block Diagram for the Program Shown in FIG. 5.5 [4]

For computational efficiency, part of the program uses integer arithmetic. Thus, variables of both types, integer and real, are used in the program. The necessary mixed mode statements were written so as not to result in inaccurate calculations. Integer arithmetic also requires that the units of input data representing stress and strain quantities are psi and micro inches per inch. The one exception to this is FDC (Fatigue ductility coefficient) which is input in inch per inch.

5.4.3 Development of Stress-Strain Elements

The cyclic stress-strain curve of RQC-100 Steel, shown in Figure 5.7, is used to develop the stress-strain elements.

The total strain range was calculated to cover the maximum peak to minimum valley (maximum range) in the strain history, the total strain range, as described by the cyclic stress strain curve, was divided into ten equal strain bands, creating ten straight-line segments on the cyclic stress-strain curve. Each of these elements were used to generate the corresponding element on the outer loop curve, the length of the element on the outer loop curve is twice the length of the element on the cyclic stress-strain curve while retaining the same slope [18]. The outer loop curve generated this way, is shown in Figure 5.8), while the values of stress and strain are shown in Table (5.2).

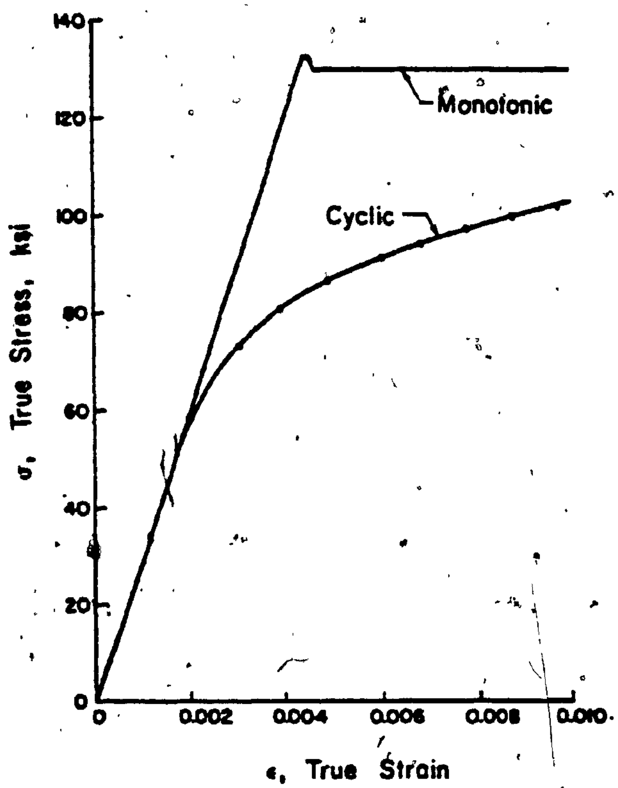


FIG. 5.7 Stress-Strain Curves for RQC-100 Steel, 298 BHN [4]

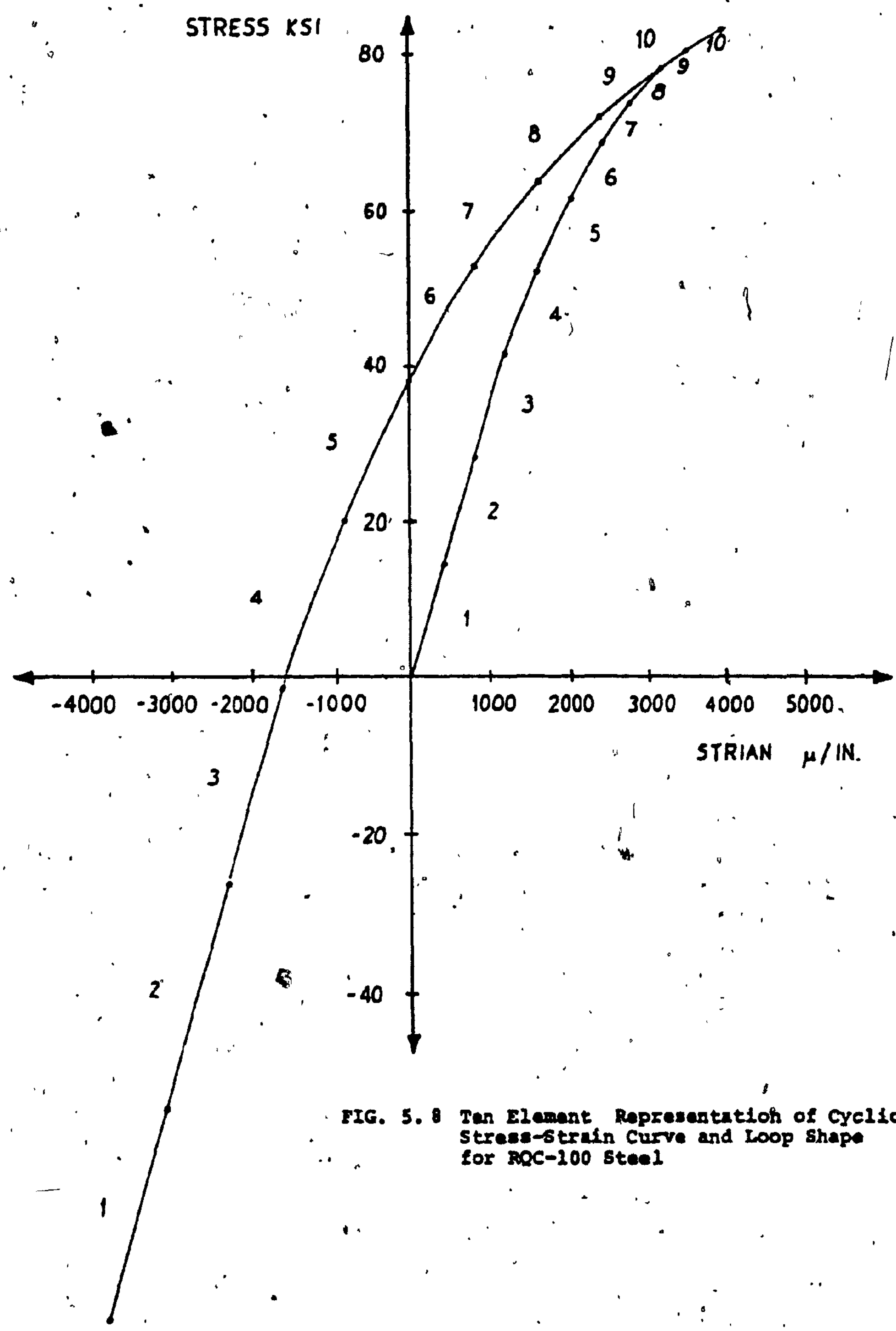


FIG. 5.8 Ten Element Representation of Cyclic Stress-Strain Curve and Loop Shape for RQC-100 Steel

5.4.4 Material Constants

RQC-100 Steel was taken to be the material of the bracket. The cyclic stress-strain and fatigue resistance material properties of the RQC-100 Steel have been determined [4] in accordance with the technique given in Chapter III. These constants are given in Table 5.3. Also, the chemical composition and processing are given in Table 5.4..

5.5 RESULTS

Utilizing the Computer Program listed above in conjunction with the available data in RQC-100 Steel and the strain history produced in this Chapter, the fatigue life is predicted to be 730 blocks, i.e., in order to initiate a fatigue crack in this specific material, the strain history produced in this Chapter should be applied on this type of steel 730 times.

Using this Computer Program with a small amount of materials property information, the life prediction may be made for a wide variety of situations. The needed material properties may be obtained by testing small laboratory specimens, and such information is becoming increasingly available through the published literature.

TABLE 5.2 OUTER LOOP STRESS-STRAIN ELEMENTS

Element	Strain ε $\frac{\mu''}{inch}$	Stress (psi) σ	$\frac{\sigma}{5}$ *
1	800	28468	5694
2	800	28468	5694
3	800	25766	5153
4	800	21659	4332
5	800	18865	3773
6	800	14527	2905
7	800	8032	1606
8	800	8032	1606
9	800	3352	670
10	800	3352	670

* Stress element as read by the Computer Program.

TABLE 5.3 CYCLIC STRESS-STRAIN AND FATIGUE RESISTANCE MATERIAL PROPERTIES FOR RQC-100 STEEL

σ'_f	200	KSI
b	-0.094	
ε_f'	1.0	
c	-0.75	
K'	208	KSI
n'	0.14	

TABLE 5.4 COMPOSITION AND PROCESSING OF RQC-100 STEEL

COMPOSITION	WT. %
Carbon	0.19
Manganese	0.79
Phosphorus	0.005
Sulphur	0.021
Silicon	0.24
Boron	0.0028
PROCESSING	
RQC-100 (Bethlehem Steel Corp.) - 9.5 mm (3/8 in.) plate, roller quenched from approximately 1,650 F, tempered at 900 F or higher to 690 MPa (100 KSI) minimum yield strength.	

CHAPTER VI

CONCLUSION AND RECOMMENDATIONS

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CONCLUSION AND RECOMMENDATIONS

In order for accurate fatigue life prediction to be made, the local strain approach described in Chapter I must be followed.

The use of any method of cycle counting other than range pair or rain flow methods can result in inconsistencies and gross differences between the predicted and actual fatigue life.

Measuring local strain at the critically stressed locations is often impractical. Access to those locations is often difficult and gauge life is uncertain when large cyclic strains are present. Elastic strains measured at small distances from plastically strained areas can be used for calculating nominal stresses and subsequent local strains.

The cyclic stress-strain curve is the most important concept in predicting the fatigue life by estimating local strains. Aside from the tensile properties, it is the single piece of experimental information essential for making life estimates. Consequently, the technique of using the cyclic stress-strain curve for simulating the stress-strain response from an input strain history can be used for all steels with the exception of annealed austenitic stainless steels, as they appear to have hysteresis loops that change shape significantly with strain level. [5]

Additional development of the local strain approach is needed in certain areas. Approximate methods of obtaining load versus notch strain calibrations, such as Neuber's Rule, need to be verified for additional geometries and modes of loading. Procedures are needed for defining crack initiation in terms of crack size, which is small, relative to the notch root radius.

This would remove the difficulty now encountered with the size effects in sharply notched members, and it would allow the local strain analysis to be linked with a fracture mechanics' type analysis for determining the total fatigue life for both initiation and propagation.

Additional experimental work is needed that results in improved procedures for using uniaxial data to estimate material properties for other states of stress.

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REFERENCES

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APPENDIX A

BRACKET VIBRATION STRAIN HISTORY

TABLE A

LINE NO.	STRAIN (MICROINCH PER INCH)									
1	480	-809	705	-960	269	-495	495	-585	735	-809
2	570	-585	554	-795	914	-1140	1470	-884	1410	-809
3	944	-809	960	-600	509	-419	690	-629	615	-539
4	254	-884	914	-839	1214	-930	1125	-1410	884	-450
5	674	-690	1065	-1335	960	-720	735	-480	419	-434
6	869	-570	795	-389	615	-779	615	-674	570	-779
7	570	-735	389	-585	615	-300	539	-570	465	-360
8	450	-615	1319	-720	825	-765	855	-539	629	-855
9	644	-509	1500	-1410	900	-1244	990	-1035	1065	-465
10	1214	-1440	990	-1424	1125	-629	705	-720	524	-705
11	434	-480	480	-450	960	-509	1109	-855	750	-539
12	914	-825	1065	-554	1065	-765	434	-900	750	-1005
13	600	-930	629	-750	809	-554	690	-900	465	-690
14	779	-914	1574	-1275	825	-750	795	-1049	779	-600
15	1109	-1650	1484	-1244	809	-930	1349	-1170	1244	-705
16	855	-314	659	-434	674	-750	509	-539	615	-674
17	570	-629	825	-375	674	-300	690	-300	674	-450
18	779	-314	659	-570	600	-554	434	-495	419	-554
19	434	-434	705	-600	450	-329	779	-450	629	-419
20	570	-434	629	-465	765	-389	674	-404	585	-269
21	629	-539	600	-480	509	-720	509	-944	524	-465
22	629	-314	585	-434	600	-539	539	-539	750	-524
23	1454	-2054	1724	-1454	735	-1200	1005	-1170	855	-1380
24	930	-1365	1035	-1154	825	-1125	750	-765	855	-795
25	1440	-1170	1679	-1230	825	-450	524	-375	914	-720
26	509	-524	554	-765	495	-615	480	-839	539	-570
27	600	-495	629	-524	779	-434	674	-509	1005	-735
28	1559	-1410	2190	-629	1184	-1065	809	-1349	1140	-509
29	914	-629	674	-809	825	-735	855	-855	705	-884
30	674	-495	809	-1019	1349	-1019	1470	-884	1500	-765
31	809	-1065	825	-285	659	-1095	944	-104	690	-1305
32	884	-1140	795	-450	570	-795	765	-1230	990	-1454
33	1275	-1484	1605	-690	825	-1065	1019	-539	570	-509
34	509	-720	360	-539	450	-495	389	-554	674	-690
35	839	-600	705	-434	809	-434	750	-644	554	-375
36	644	-300	554	-570	539	-495	509	-629	554	-480
37	674	-705	795	-600	1065	-1710	944	-1289	990	-1514
38	1664	-825	1365	-585	1095	-795	990	-960	570	-480
39	659	-720	615	-884	944	-750	869	-1109	914	-720

40	779	-1410	1049	-1319	944	-1019	1035	-1289	750	-480
41	1005	-884	944	-779	1184	-1695	2144	-1170	1755	-1200
42	1530	-1200	1154	-1275	1125	-855	495	-509	930	-1365
43	1154	-1125	690	-674	509	-765	855	-1109	1035	-1140
44	809	-1170	674	-1184	690	-465	465	-585	809	-434
45	615	-554	495	-524	795	-779	839	-809	705	-345
46	539	-570	855	-300	539	-720	495	-434	1109	-615
47	254	-659	1214	-375	419	-705	884	-615	990	-659
48	750	-585	990	-480	585	-644	960	-974	1230	-750
49	884	-644	1109	-644	615	-600	1275	-839	930	-720
50	1184	-765	930	-795	1230	-450	809	-434	720	-554
51	690	-659	524	-404	750	-300	1005	-1049	1095	-720
52	914	-765	855	-269	615	-524	404	-585	345	-524
53	389	-615	600	-839	600	-750	389	-720	570	-329
54	450	-450	735	-1065	1095	-629	615	-779	629	-779
55	735	-659	690	-375	600	-974	629	-765	419	-705
56	884	-1500	1109	-1440	825	-960	705	-554	1230	-1170
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59	524	-615	674	-735	585	-659	659	-539	659	-765
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63	1484	-1440	1065	-1619	1619	-1019	1365	-1184	1319	-1125
64	720	-644	345	-585	509	-629	509	-254	615	-434
65	705	-944	765	-884	1095	-1200	1484	-1605	1934	-2190
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69	1619	-1005	1170	-1949	2610	-2054	1889	-1980	1394	0
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76	750	-404	539	-659	750	-570	450	-629	539	-434
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78	1484	-1275	1289	-1619	2340	-1125	1065	-1275	2099	-1035
79	90	-944	2280	-1559	1125	-1305	930	-419	1170	-480

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86	495	-404	720	-389	570	-450	300	-539	389	-779
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