DEVELOPMENT OF A MODEL FOR OPTIMIZING WATER STORAGE

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A Thesis

in

The School for

Building

Civil Engineering Program

Presented in Partial Fulfillment of the Requirements

for the Degree of Doctor of Philosophy at

Concordia University

Montreal, Quebec, Canada

September 1997

0-612-39783-1

ABSTRACT

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Concordia University, 1997

The purpose of this study is to develop an efficient optimization model to determine storage strategies for water supply purpose in multi-reservoir systems. The specific goals in designing a multi-reservoir system for water supply purpose is to optimize the reservoir sizes and configurations to satisfy demands for water supply at the minimum cost.

Four models are developed to optimize the configuration of multi-reservoir systems for water supply purposes. These models apply optimal control theory (OCT) and penalty successive linear programming (PSLP) as the most promising techniques to optimize large and complex water resources systems. Three of these models are based on the OCT. They have, however, different approaches to join the cost function to other objectives. The fourth model employs a new composite optimization algorithm, which is introduced in this study. This is called PSLP-OCT model and consists of two OCT and PSLP modules. These two modules interactively share their results during the optimization iterations. Multi-objective programming methods are implemented in the four models in order to consider the two non-commensurate objectives of minimizing cost and water deficit. The weighting and epsilon constraint methods are used as the most suitable generating techniques to incorporate the problem objectives.

The comparative performances of the design models on several case studies showed that the design models based on the OCT algorithm fail to design the multi-reservoir system optimally. However, The PSLP-OCT performance indicated that it is a very promising optimization method to design multi-reservoir systems regardless of their size. The PSLP-OCT model is the first model of its type that applies the proposed composite algorithm and incorporates multi-objective programming into the multi-reservoir design problems. Due to the inherent characteristics of the optimal control theory, the control variables in the OCT, module is not sensitive to the initial solution. The model structure is adapted such that the PSLP module is independent of the design period length. Therefore, using large hydrologic data does not affect its problem dimension.

ACKNOWLEDGEMENT

I take this opportunity to express my sincere appreciation to my thesis advisors, Prof. H.B. Poorooshasb and Prof. A.S. Ramamurthy for their indispensable support and helpful guidance.

Thanks are also due to Dr. F. Albuquerque for introducing the Optimal Control Theory to me. I am grateful to my committee members, Dr. W.G. Habashi, Dr. S.P. Simonovic, Dr. V. Ramachandran, Dr. A.M. Hanna, and Dr. J. Campanelli for their suggestions and constructive criticisms.

The author would like to thank the International Development Research Center of Canada (IDRC) for partial funding issued through Concordia University during the first year of his study. The financial support from the Ministry of Culture and Higher Education of Iran is gratefully acknowledged. Further thanks go to Prof. Poorooshasb for his financial help at the final stages of this work.

Finally, I wish to give my deepest gratitude to my wife and little children for their endurance and sacrifice over the years.

DEDICATION

I would like to dedicate this thesis to my wife, for her patience, support, understanding, and love.

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NOMENCLATURE

$C_i(X)$:	Nonlinear constraints in a PSLP general formulation.
C_n^e :	Coefficient in the *n*th reservoir area-storage relationships.
$D\#j$:	The *jth* demand area.
D^A :	Total annual water demand in the hydrosystem (L^3).
D_j^A :	Annual water demand at demand area *j* (L^3).
D_j^t :	Monthly water demand at demand area *j* (L^3).
$D_j^{t,max}$:	Maximum predicted water requirement at demand area *j* (L^3).
D_o :	Degree of optimality.
E_n^t:	Losses (e.g., evaporation) from the *nth* reservoir during time *t*.
$F(.)$:	Objective function value.
N_ε :	Number of selected non-inferior solutions.
N_d :	Total number of municipal, industrial, and irrigation demand areas.
N_r :	Number of state and control variables (reservoir capacities, yields and releases).
P_n^e:	Exponent in the *n*th reservoir area-storage relationships.
$Pl(X)$	Linear approximation of the exact penalty in the PSLP formulation.
Q_n^t :	Unregulated local inflow into reservoir *n* during month *t* (L^3).
R_n^t:	Control variable; a variable showing the release/yield from the *nth* reservoirs during time *t*.
R_n^{max}, R_n^{min}:	The upper and lower bounds on the *nth* control variable.

$RES\#n$:	The *nth* candidate reservoir.
W_n^{br} :	A weight coefficient applied to the bang-bang control on reservoir release.
W_n^{by} :	A weight coefficient applied to the bang-bang control on reservoir yield.
W_n^{fs} :	A weight coefficient applied to the terminal storage function corresponding to the *nth* reservoir.
W_i^p:	Penalty weights used in the PSLP module.
W_n^s :	A weight coefficient applied to the cost function corresponding to the *nth* reservoir.
W_j^y :	A weight coefficient applied to the water supply objective.
X :	Decision vector in the PSLP module with X=[x_1, x_2, .., x_N] elements.
X_0:	A vector [$x_{1,0}$, $x_{2,0}$,.. $x_{N,0}$] showing the initial/trial solution.
d :	A vector [d_1, d_2,.. d_N] showing the step size for N decision variables.
$d_{n,j}^t$:	Water supply target, corresponding to demand area j, for reservoir n during month t.
e_n^t:	Evaporation per unit area (L^3/L^2) from reservoir n during month t.
f_n^c :	The cost function of the nth reservoir.
N_i :	A *deviation variable* corresponding to the ith nonlinear constraint in the PSLP module.
n_k^e :	Denote the smallest value of the objective function $f_k(X)$ in the epsilon constraint algorithm.
m_k^e :	Denote the largest value of the objective function $f_k(X)$) in the epsilon constraint algorithm.
p_c :	Cost penalty weight in the OCT-I model.
P_i :	A *deviation variable* corresponding to the ith nonlinear constraint in the PSLP module.
p_s :	Storage penalty weight in the OCT module/model.
r_n^{max} :	The maximum permissible release from reservoir n (L^3).

r_n^{min} :	The minimum permissible release from reservoir n (L^3).
r_n^t :	The release from the reservoir n during month t (L^3).
$s_n^{initial}$:	Initial storage capacity of the reservoir n (initial condition of the system).
s_n^{max} :	The upper bound on reservoir n (L^3).
s_n^{min} :	The lower bound on reservoir n (L^3).
s_n^t :	State variable; a variable showing the storage capacities of the nth reservoirs at the beginning of the time period t (L^3).
$y_{n,j,o}^t$:	Reservoir yield obtained at previous PSLP iteration (L^3).
$y_{n,j}^t$:	Yield from reservoir n to the demand area j during month t (L^3).
$y_{n,j}^{t,max}$:	The maximum permissible yield of reservoir n to demand area j during month t (L^3).
$y_{n,j}^{t,min}$:	The minimum permissible yield of reservoir n to demand area j during month t (L^3).
Γ_n^j :	The element of the return flow matrix.
Λ_n^k :	The element of the layout configuration matrix.
Ω_n :	A pre-specified ratio of the dead storage to capacity of the nth reservoir.
Ψ_n :	The cost penalty function corresponding to the reservoir n.
α_j^t :	A monthly fraction of the annual water deficit.
β_j :	A ratio, showing the spatial distribution of water demands and/or water deficits in the hydrosystem.
ε_k :	The upper bound to the kth objective function value specified by the epsilon constraint method.
ε^A :	Total annual water deficit in the hydrosystem (L^3).
ε_j^A :	Annual water deficit at demand area j (L^3).
ε_j^t :	Monthly water deficit at the Demand area j (L^3).
$\phi(s_n^{T+1})$:	Terminal function representing the storage deviation from terminal storage target.

η_n^{t+1} : Storage penalty function of the *n*th reservoir at the beginning of time *t+1*.

π_n^{max} : Maximum permissible dead storage bound (L^3).

π_n^{min} : Minimum permissible dead storage bound (L^3).

ρ_j^m: A coefficient for return flow from the demand area *j* during month *m*.

ω : Upper bound (step size) on the change in the nonlinear variables.

ξ_n^j : The element of the demand area matrix

ζ_n^k : The element of the upstream reservoir matrix.

CHAPTER 1

BACKGROUND

1.1 Definition of the Problem:

Allocating and managing water resources are among the most important problems that planners and water resources engineers are facing. This is because water is often available at times, locations, in quantities and with qualities that may not be desired. To satisfy the growing demand for water, reservoir storage is required to control the uneven temporal and spatial distribution of water and provide enough water to consistently meet the demand at specified locations. However, due to financial and environmental reasons only a limited number of reservoirs can be constructed in a river basin. Therefore, an optimal policy is needed to design a multi-reservoir system to accomplish the problem objectives (e.g., supplying water demand) at the minimum cost.

The limited water supply must satisfy the demand of a rapidly increasing population for water quality control, fish and wild life maintenance, recreation, navigation, irrigation, domestic and industrial use. Therefore, the limited water resources must be developed and managed in the most efficient way. Social interests should also be considered as the maximum benefit to society, which are not always obtained by producing the maximum profit. For instance, in some cases, under certain social or environmental conditions, water may be attributed to some users, even though it could be attributed to other users in a more optimal fashion.

To have a fully beneficial and optimal multi-reservoir system, first the design of the system configuration of the reservoirs and then, the operating policy of the system should be optimized. In the design phase, the layout and sizing of reservoirs are the major concerns. However, with the existing reservoirs, in the operation phase, the primary concern is to decide, when to release or store water to achieve the prescribed goals. Optimizing the operating rule of a multi-reservoir system without optimizing the reservoir sites and sizes is beneficial. However, in an ideal situation, configurations of reservoirs and operating rules should be optimized together with the same objectives. In this case, the hydrosystem will work with the highest possible efficiency. Neglecting optimal design of a system for certain objectives will result in a poor or less optimal operating policy.

Optimal design of a large multi-reservoir system usually results in a nonlinear, non-convex optimization problem with a high dimension. Therefore, achieving the global optimality cannot be guaranteed. It is difficult to solve such a problem by any of the existing optimization technique. Therefore, an optimization technique with a high degree of accuracy, convergence, and low computer time and memory is required. The algorithm should be capable of handling different hydrological characteristics of the hydrosystem such as losses, reservoirs layout, tributaries, and routing (if applicable). In some optimization techniques such as discrete dynamic programming, these factors cannot be implemented directly in the optimization algorithm. As a result, a simulation model is required to verify the feasibility of the final result. Inclusion of these factors will remove the necessity of applying a simulation model to verify the feasibility of the final solution of the optimization model.

1.2. Importance of the Problem:

Construction of a reservoir has become increasingly difficult for a variety of political, financial, and environmental reasons. The number of future reservoir projects may be limited. Therefore, those reservoirs that are constructed should represent an optimal configuration that will meet the water demands at the minimum cost. Constructing a reservoir on a particularly advantageous site not only may result in a foreclosure of future opportunities, but also lead to a high construction cost.

A water resources planner needs to evaluate the different scenarios to find the best configuration of reservoirs that can supply water at the minimum cost. Optimizing the layout of a multi-reservoir system may help the decision-makers to save millions of dollars in construction cost. It can also avoid any inappropriate social and environmental impact in the region. A review of available design models indicates that most of these models lead to a large number of control variables and constraints in practical situations. Consequently, the development of an efficient algorithm, to optimize the configuration of reservoirs in multi-reservoir systems is still called for.

1.3 Research Objectives:

The main objective of this study is to develop a more compact and computationally efficient screening model to select between potential reservoir sites and optimize their sizes and the associated yields in a large multi-purpose reservoir system. The criteria to achieve this objective are:

1- The model should be accurate and fast.

2- The formulation of the model should be as general as possible in a way that minor changes in computer program codes are required, if it is applied to different hydrosystems.

3- The model operator does not have to assume relative values for each variable by not using a single monetary objective function in the optimization formulation.

4- The model should be capable of introducing tradeoffs between different possible water demand in the future and the optimized reservoir configurations. This feature lets decision makers evaluate the sensitivity of each candidate reservoir to different water demand levels. The less sensitive reservoir has the priority in being built.

5- Finding the best initial solutions of the optimization variables, especially for large multi-reservoir systems, is a very cumbersome task. To avoid this difficulty, the design model should preferably be insensitive to initial conditions. This feature decreases noticeably the necessary time to design a multi-reservoir system.

The developed design model should screen the sizes and locations of a series of potential reservoirs sites in a multi-reservoir system by applying the fastest and most promising optimization technique(s) applicable to large-scale multi-reservoir systems.

Based on the study performed by Hiew (1987), optimal control theory (OCT) and the penalty successive linear programming (PSLP) would be the candidate techniques. To alleviate the dimension of the optimization problem in PSLP and consequently decrease its required computer time, a methodology will be introduced in this study to reduce the number of decision variables and constraints in PSLP. Then, screening models based on these two optimization techniques will be developed and the performance of each model will be analyzed. Finally, the model which results in a better optimal solution and gives a more suitable performance will be selected as the proposed screening model.

The selected reservoirs are going to be used to store water to supply for domestic, industrial, or irrigation demands at any time and location later. One possible strategy is to find the optimal design of such a system to satisfy water demand at the minimum cost subject to a set of system constraints that incorporate all the other objectives such as controlling minimum flow requirements and maximum flows in channels. To determine the optimal reservoirs configurations, it is necessary to develop a computer model to optimize the layout of several reservoirs in a river basin based on two non-commensurate objectives of supplying water and minimizing the cost. The related problem, is a nonlinear, constrained optimization problem. The multi-objective programming approach (generating method) is being used to achieve these two objectives to design the number of reservoirs and their locations. This will maintain the classical role of the analyst in designing the multi-reservoir system.

1.4 Anticipated Benefit:

Once the model is developed, it can be used by planners to determine the best strategies for utilizing water resources in large multi-reservoir systems. The model will optimize the configuration of reservoir system at minimum cost. The optimization model is formulated to supply water for different water demand areas while considering other objectives such as flood control and minimum channel flow requirement. The model will be a general model, capable of considering any possible reservoir configuration to supply water. Therefore, it can be used easily for local situations and adapting the program codes to the physical networking of the candidate reservoirs would be the only change required of the model.

1.5 Contributions:

This research provides a contribution to the systems analysis in general and water resources planning and management area in particular, by introducing a new composite optimization algorithm. The proposed algorithm is a general-purpose optimization technique that can be used in any optimization problem consisting of both static and dynamic (time dependent) control variables. This study also contributes to the design of multi-reservoir systems by adapting the proposed composite optimization technique that can incorporate the different water resources planning objectives (such as flood control and minimum flow requirements) more accurately. Using the fastest and the most promising techniques in analyzing the large-scale multi-reservoir systems provides the ability to include system characteristics in more detail in the optimization problem. In all previously developed screening models, a single objective function was used and some restrictive assumptions were made to simplify the design procedures. The assumptions of neglecting evaporation losses, concentrating all the water demand areas at the downstream of the last reservoir, or neglecting the channel flow capacity render some of these models to be less interesting in applying to real situations. Unlike previously developed models, the current design model considers the target demands and minimal design cost as two equal objectives in a multi-objective decision making analysis. The detail contributions of this research can be described as follows:

1- The new composite optimization algorithm introduced in this study employs the optimal control theory and the penalty successive linear programming as the two most promising optimization techniques in optimizing large and complex systems. This algorithm helps engineers to get benefit from the advantages of the optimal control theory in problems consisting of both static and dynamic control variables.

2- A design model based on the new composite algorithm is proposed. This model optimizes the layout and sizes of a series of candidate reservoirs to supply water for irrigation, municipal, and industrial needs at different demand locations.

3- The proposed design model requires less variables than all previously developed models of the same type, by not using the monetary objective function.

4- A strategy is used in the design model that reduces the number of optimization constraints dramatically. This strategy optimizes the capacities of the mutli-reservoir system simultaneously while optimizes reservoir yields by analyzing one reservoir at a time.

5- Finding the best initial solution to run the model is not a major task. This is because the major part of the optimization process is assigned to the OCT and hence is insensitive to the initial solution. The model may need an initial guess for reservoir capacities, if the reservoir cost functions are not convex.

6- More realistic system constraints are included in the optimization formulation as: (*i*) minimum channel flow requirement for recreation, navigation, wild life, and dilution of wastewater, *(ii)* maximum channel capacity for flood control, *(iii)* reservoir evaporation, *(iv)* reservoir dead storage, and (*v*) return flows from upstream demand areas considered in the screening model. All these components have not been generally considered together in any of the previously developed models.

7- The model can result in designing a system with less operating policy requirement. This can be done by minimizing the rapid variations of each reservoir operating policy over the time by considering the bang-bang control as one of the objectives of the problem. This feature results in an easier reservoir gate operation.

8- Using generating multi-objective programming techniques, the two objectives of (1) minimizing the cost and (2) supplying water are considered equally important to represent a non-inferior set that shows the tradeoff between two objectives. This approach will not only show the different scenarios for different possible future demands, but will enable water resources planners to perform the sensitivity analysis tests. This feature is very important one and can help the policy makers to find out the chronological priority in building the reservoirs. The reservoir, most insensitive to different demand levels, should be built first.

9- The present study enhanced the understanding of the OCT capabilities and its application in the water resources engineering area by demonstrating some of its shortfall in designing multi-reservoir systems.

CHAPTER 2

LITERATURE REVIEW

The study on simulation and optimization models is an important topic in water resources planning and management. Simulation models are descriptive models that represent the physical characteristics of a system (Yeh 1985). The main disadvantage of simulation models is that for a complex-large system, many efforts are needed to find the best strategy that although may meet the objectives, may be far from the optimal solution. CEQUEAU and SSARR (WMO 646, 1986), HEC-3, and HEC-5 (Simonovic 1992) are examples of simulation models. Simulation model with some capability of self-optimization is preferable to alleviate the number of trials needed to obtain an optimum or near-optimum policy (Yeh 1985). ACRES Reservoir Simulation Program (ACRES 1988), and Jacoby and Loucks (1972) applied a combination of simulation and optimization models to a river basin planning.

Optimization models consist of a set of mathematical equations developed to explain the response of a system. They use mathematical programming techniques to find the best possible solution based on a specified performance function and some physical constraints. Mathematical programming includes several techniques such as dynamic programming (DP), nonlinear programming (NLP), linear programming (LP), optimal control theory (OCT), and genetic algorithms (GAs). Each of these techniques has its advantages and disadvantages. Based on the characteristics of each hydrosystem and the objective used, one of the above techniques may perform better than the others. In the following sections, each method is described briefly.

2.1 Dynamic Programming Models (DP):

Although DP can be formulated either in continuous or discrete form, the later is more popular for its simplicity (Hiew 1987). Bellman (1962) developed the discrete dynamic programming and defined it as "the theory of multi-stage decision processes". DP has been widely used in water resources engineering problems (Hiew 1987). The advantages of discrete DP over other methods such as LP are that the problem is solved one stage at a time and there is no restriction of any kind on the type and form of the objective function. In discrete DP, the computational burden is dependent on the number and discretization of the state variables. For a system with one control variable, n state variables (e.g., reservoir storage), and M levels of discretization, There are M^n combinations that should be considered at each stage of analysis (Mays 1992). Therefore, computational efforts increase exponentially in DP, which creates the so-called "curse of dimensionality" and is considered as the major disadvantage of DP (Bellman 1962). The large increase in computational time and computer memory requirements, constitutes one of the major difficulties in using DP (Hiew 1987). Different techniques have been introduced in the literature to reduce the computational burden and dimensionality problem of DP (e.g., Heidari et. al. 1971; Becker et. al. 1976; Labadie et.

al. 1980; Turgeon 1980). Yakowitz (1982) has reviewed the application of DP models in water resources problems.

2.2 Nonlinear Programming Models (NLP):

Nonlinear programming is the most generalized mathematical programming method (Hiew 1987). Analytical solution of the NLP may not be computationally feasible for the most large-scale nonlinear problems. Therefore "direct" methods, which are iterative search procedures, will be used to find the optimal solution. NLP method has been used in some studies in the field of water resources planning and management (e.g., Lee and Wazirrudin 1971; Hanscom et. al. 1980; Rosental 1981; Lansey et. al. 1989). However, NLP methods are less popular than DP and LP methods in water resources systems analyses. Slow convergence, large computer memory requirement, and the complicated theory of the method are the reasons for its unpopularity (Yeh 1985).

2.3 Linear Programming Models (LP):

LP has been used by many researchers such as Bechard et. al. (1981), Loucks et. al. (1981), and Draper and Adamowski (1976) in planning and management of water resources systems. The main limitation of LP is that the objective function and all the constraints have to be linear. However in water resources systems (W.R.S.), objective functions or system constraints are mostly nonlinear. In order to use LP in this type of problems, a linearization scheme has to be applied.

There are two methods to problem linearization. The first method is called the *piecewise linearization* that converts the original nonlinear function to series of linear functions (Windsor 1973). The second method solves nonlinear optimization problem via a sequence of linear programs. The linearization scheme in the second method is based on the first-order Taylor series expansion of a nonlinear function about a given initial solution. This method has different names in the literature as the method of approximation programming (Palacios-Gomez et. al. 1982), iterative LP (Grygier 1983), and conventional successive linear programming (SLP) (Hiew 1987).

SLP method performs better, if the nonlinearity is related to the objective function only. In this case, the existance of convergence can be proved theoretically (Palacios-Gomez et. al. 1982). Another algorithm known as *SLP Reject* (SLPR) has been proposed by Palacios-Gomez et. al. at 1982 that can cope better with nonlinear constraints. SLPR algorithm is attractive because it is fairly easy to implement, if an efficient and flexible LP code is available. However, there is no theoretical proof of convergence for it (Hiew 1987). The last generation of the SLP is called *penalty SLP* (PSLP). Baker and Lasdon (1985) have developed a simplified version of PSLP. Later, Zhang et. al. (1985) improved the PSLP algorithm and gave a convergence proof of it. In both SLP and SLPR, the values of two successive objective functions were used to accept or reject the step bound and the new point at each iteration. This criterion may yield an optimal solution with a non-vertex optimum point i.e., the optimum point lies in the interior of the feasible solution space (Hiew 1987 p-52). In this case, SLP and SLPR may face "Oscillations" and "Zigzagging" in the vicinity of the optimum solution Therefore, choosing an appropriate step size at each iteration is crucial. To fix this problem, a new criteria for varying the Taylor series step bounds for accepting or rejecting new iterates are introduced in PSLP (Zhang et. al. 1985).

Grygier (1983), Hiew (1987), and Reznicek and Simonovic (1992) applied SLP to optimize reservoir operations. In SLP, the computer time is a function of the product of number of variables and number of constraints (Grygier 1985). Based on his work, it may be concluded that computer time would increase as the square of the number of reservoirs and time stages.

To solve the LP-based problems, separate constraints should be considered for each time stage. Therefore, many variables and constraints are required in an LP model to analyze a large hydrosystem. For example to optimize the operation of 17 reservoirs over 24 months, 872 constraints and 433 variables are needed (Draper and Adamowski, 1976). "Cycling" is another problem in LP-based models i.e., The LP method may go through an endless sequence of iterations without ever finding an optimal solution (Chvatal 1983). Cycling may happen, if large numbers of equality constraints exist in the problem formulation (Roef and Bodin 1970). However, cycling is a rare phenomenon (Chvatal 1983).

2.4 Optimal Control Theory (OCT):

OCT is a method that applies the minimum principle of Pontryagin to optimize dynamic system over time and space (Bryson and Ho 1975). In this sense, it is like DP. However, it has many similarities with NLP in terms of computational procedures. Pontryagin's minimum principle is based on the theory of calculus of variation, which was limited by the requirement that the control variable must be the time rate of change of the state variable. The Pontryagin's minimum principle originally explains the set of optimality condition for continuous-time optimal control problems. However, the discretized form of minimum principle was developed later (Hiew 1987).

Hiew (1987) compared five deterministic optimization algorithms and evaluated their performances on a hypothetical five-reservoir hydropower system. The algorithms were incremental DP (IDP), successive linear programming (SLP), feasible direction method (FDM), objective-space dynamic programming (OSDP), and optimal control theory (OCT). The criteria to compare were: accuracy of results, rate of convergence, smoothness of resulting storage and release trajectories, computer time and memory requirements, robustness of the algorithm, and sensitivity to the initial solution. In his test, the optimal objective values reached were within 1% for all methods except objective space dynamic programming (OSDP). IDP obtained the highest objective value, while OCT and SLP had values very close to IDP. In terms of computer time, the differences among the different methods were considerable. OCT and SLP are the fastest among the five mentioned algorithms and needed only 10 and 20 seconds respectively while IDP used 2000 seconds to run on the Cyber 205 "super" computer at Colorado State University. Heiw's study showed that the optimum state and release trajectories determined by OCT are also insensitive to the initial solution. This feature of OCT is an important one especially in large-scale systems where efforts in determining an initial solution can be significant. OCT can generally lead to smoother policies in comparison to other methods. These characteristics come from the gradient-based and non-extreme point technique of OCT. Hiew (1987) concluded that considering all the criteria mentioned before, OCT and SLP are the most efficient algorithms for optimizing the nonlinear, non-convex objective functions of large hydrosystems.

The augmented objective function is obtained by adjoining the system dynamic equations to the original objective function using Lagrange multipliers (co-state or adjoint variables). The physical bound will also be added to the original objective function by Lagrange multiplier or penalty functions, where the later is a simpler approach. According to Pontryagin's minimum principle, an optimal trajectory can be found by minimizing the augmented objective function over the set of all possible control values (Bryson and Ho 1975).

Obtaining an analytical solution of the OCT Problem is computationally burdened and sometimes impossible. Therefore, like NLP, the "direct" method will be used to find the optimal solution. The OCT has been used by electrical engineers during the last three decades. However, it still is not popular in water resources system analysis. Dillon and Morsztyn (1971, 1972) applied OCT using a first order gradient method in a hydrothermal network optimization. To improve the convergence, Tun & Dillon (1978) used a second order gradient method. Turgeon (1981) applied discrete OCT to optimize the daily operation of 3 reservoirs. In all these studies, quadratic penalty functions were applied to bring the constraints on state variables to the augmented objective function. Papageorgio (1985, 1986) applied the penalty function technique with the variable metric method in the OCT algorithm to optimize the hydroelectric power generation of a multi-reservoir system. Grygier (1983) used "hard" constraints on state variables' lower and upper bounds, i.e., the state constraints were included in the augmented objective function by using the Lagrange multiplier. He reported that his optimal control algorithm was efficient for small systems. When the optimal trajectory is tangential to storage constraints, i.e., when storage and release constraints are active at their maximum or minimum values simultaneously, a significant computational difficulty will be encountered. This is traced to the fact that the trajectories of Lagrange multipliers make a jump by some unknown magnitude, when state variables are tangent to storage constraints (Grygier 1983, Bensalem 1988). Under these conditions, Grygier (1983) used an incremental algorithm to adjust those multipliers. Hence, the convergence was quite slow. Grygier (1983) proposed that for an efficient algorithm, penalty functions should be applied to state constraints and a fast gradient technique with second order convergence should be used.

In penalty based OCT like any other constrained optimization algorithm, the optimum can be achieved, if the penalty weight theoretically approaches infinity. However, it has been found that large penalties can lead to solution divergence (Bryson and Ho 1975) and make gradient search techniques slow and unstable. On the other hand, if medium penalty is used, although divergence will not occur, the final objective value cannot be improved toward the optimal one. An iterative penalty scheme is the key to the success of OCT and affects the rate of convergence and convergence itself. The choice of gradient search technique may have some secondary influence on the rate of convergence (Hiew 1987). The iterative penalty technique applies quadratic penalty weights, which are increased in a series of iterations. Choosing the initial penalty weight is crucial and would affect the optimal objective function value, total number of iterations and computer time. Hiew (1987) showed that any initial penalty larger than 1.00 would lead to sub-optimal result for any problem. High initial penalty weight causes the estimation of Lagrange multipliers to be dominated by penalty terms on state variables constraints. Hence, it would affect the calculation of new state variables over time in the system equation. Very small penalty values will change the original problem to an unconstrained optimization problem and hence the solution may be infeasible. As the penalty weights start to increase, the objective function values will improve at each iteration until it reaches a limiting value corresponding to a certain penalty weight. In this solution, some state variables obtained may still violate the feasibility constraints and hence fail to yield the final solution. With the larger penalty weight in the next iterations, the magnitude of violation of constraints decreases, although the magnitude of objective function does not change significantly. At the final stage of the solution with a large penalty weight, none of the constraints are violated and hence feasible and optimal solution will be obtained. Considering a penalty weight as being small or large depends on the order of magnitude of the state variables. Choosing different sets of increasing penalty weights would lead to slightly different results. However these differences are usually small and insignificant.

Hiew (1987) applied the iterative penalty weight technique in his OCT algorithm. He used an initial penalty value on the order of 0.001 and at each successive iteration increased this value 5-10 fold. He showed that if a quadratic penalty function approach is used, the OCT can be implemented on both large and small systems. Albuquerque (1997) applied the same iterative penalty weights in optimizing the hydraulic control of irrigation canals. Ouarda (1991) compared the performances of the exterior quadratic penalty functions and the barrier function methods in OCT and concluded that the barrier function methods show no improvement over exterior quadratic penalty function.

Theoretically, the computer time required in OCT depends on the number of control variables, number of time stages, and number of iterations required in a general N-dimensional optimization algorithm to converge to a stationary point for each penalty weight chosen. For an objective function with N control variables and T time stages, the computer time requirement for a conjugate-gradient algorithm would be proportional to $N*T$. Therefore, the total computer time requirement for OCT would be a function of $(N*T)^2$. However, studies done by Papageorgio (1985) and Hiew (1987) showed that if penalty-based OCT with varying penalty weight is used, even for a highly nonlinear objective function with N variables, only a small number of iterations (10 to 15% of $N*T$) is required for OCT to converge. Although a more conclusive assessment in this regard was proposed by Hiew (1987), with increase in system dimensions, the computer time requirement in OCT is slightly higher than linear (power of 1.2 for his case). In other words, in OCT, the computer time requirement would be equal to $(N*T)^p$, where $2>p>1$. This feature of penalty based OCT makes it attractive in spite of the requirement that the algorithm has to be run repeatedly at increasing levels of penalty.

2.5 Genetic Algorithms:

The theory behind genetic algorithms (GAs) were proposed by Holland (1975) and further developed by Goldberg (1989). There are many variations of GAs. They are adaptive methods, which may be used to solve search and optimization problems. GAs are based on the genetic processes of biological organisms and principles of natural selection. The analogy with nature is established by the creation within a computer of a set of random initial solutions called a population. Each set consists of the problem variables. These variables are joined together to form a string of values and are encoded into chromosomes, which are sets of character strings analogous to the chromosomes found in DNA (Holland 1975). Each variable is represented by a set of binary numbers (known as genes). The initial population is allowed to evolve over a number of generations. At each generation, a measure of how good each chromosome is with respect to the objective function is calculated. Then, based on their fitness values, individuals are selected from population and recombined to produce the next generation. This process is referred to as "crossover" operation. GAs are also referred as stochastic optimization techniques in that the next generating candidate solutions are obtained with the help of a pseudo-random number generator. Due to its stochastic nature, there is no guarantee that the global optimum will be found. However, the number of applications suggests a good rate of success in identifying good solutions

GA applications in the water resources engineering area have demonstrated their capabilities to yield good approximate solutions especially in the cases of discontinuous, non-differentiable functions (Savic and Walters 1997).

2.6 Existing Design Models:

Optimization models have been used greatly in planning and management of multi-reservoir systems. The subject of optimization in reservoir systems in literature is mainly focused on reservoir operation. However, a few attempts have been made to design the optimal configuration of reservoirs in a multi-reservoir system. To find the best configuration of reservoirs, based on the watershed topography, several preliminary proposed reservoir locations should be specified. These potential reservoirs are called the *candidate reservoirs*. Then, an optimization model will determine the optimal configuration of reservoirs based on some pre-specified objectives. In literature, a model of this type is called a screening model, because it acts the same way as a screen through which all of the possible combinations of planning alternatives are passed (Cohon 1978). Loucks et. al. (1981), presented the yield, chance-constrained, and stochastic LP to design a multi-reservoir system. The piecewise linearization technique was used in their models to deal with the nonlinearity of the problem. The objective functions of their models were based on maximizing the expected net benefit. Fontane (1982) used the discrete DP to develop the methodologies for determining water storage strategies in a multi-reservoir system. The deterministic approach was used to maximize the water supply at a minimum cost. Supangat (1987) modified Fontane's model and extended it to design a multi-reservoir system for hydroelectric power productions. Mays and Bedient (1982) applied DP to determine the size, location and minimum cost of detention basins for the flood control problem in urban areas. Their model was a simplistic one and was applied to a hypothetical watershed. Later, Bennet and Mays (1985), and Taur et. al. (1987) improved the model developed by Mays and Bedient in 1982. They included the sizes and types of outlet structures (i.e., overflow weirs and outlet pipes), and downstream channel modifications in their models. Lall and Miller (1988) developed a screening model to provide electricity and water to meet different demands at certain reliability levels. Recreation and flood control at each reservoir site were also considered in their formulation. They integrated a simulation model into the PSLP to reduce the number of constraints in the optimization formulation. The modified sequent peak algorithm (MSPA) was implemented in their simulation model to determine the optimal reservoir storage capacities while the PSLP maximized the total annual net benefit. Khaliquzzaman and Chander (1997) applied network linear programming to determine optimum sizes of a multi-reservoir system at the minimum cost. A piecewise linearization model was used to incorporate the reservoir cost functions into their objective function. They used HEC-5 program as a simulation model to consider the detail of the hydrosystem and corrected the performance of their optimization model.

A review of these screening models shows that each has some limited applicabilities in practical situation due to the selected optimization methods and some assumptions made in the models. The yield model developed by Loucks et. al. (1981) leads to a linear programming model with a large number of constraints, as the physical problem size increases (Lall and Miller 1988). A trial and error procedure is required by the yield model to find those years that permissible failure will reduce the reservoir capacities. The stochastic LP model developed by Loucks et. al. (1981) is only appropriate for analyzing relatively small multi-reservoir basin systems where streamflows are highly cross-correlated (Loucks et. al. 1981). There is also a dimensionality problem associated with their model in real situations, which can easily exceed several thousands of constraints (Yeh 1985). Stedinger et. al. (1983) reviewed and compared the performance of deterministic LP, yield, and chance-constrained screening models developed by Loucks et. al. in 1981. These models were tested on a small three-parallel- reservoir system. They concluded that:

1- The deterministic LP design model based on historical mean monthly flows does not provide sufficient reservoir capacity to supply the water demand in dry years. On the other hand, the use of the most critical flows in the record leads to larger reservoir capacity than what is needed.

2- Yield model produces reasonable reservoir system design.

3- Chance-constrained formulation of Loucks (1970) leads to a conservative system design.

All the proposed screening models that use DP suffer from the "curse of dimensionality". To cope with the dimensionality problem of DP, Fontane (1982) and Supangat (1987) used a decomposition procedure in which only two reservoirs at a time were considered for analysis. Their models were originally developed for multi-reservoir systems with demand areas located downstream of the last reservoir. The main disadvantage of their models is due to a restricting assumption that is required to apply their model to systems with demand areas distributed all over the watershed. Based on this assumption, the water demands in all areas follow the same seasonal distribution.

The deterministic models developed by Bennet and Mays (1985) and Taur et. al. (1987) had the same dimensionality problem of DP. Selecting a few discretized values for the state variable restricted their effort to achieve the optimal policy. The model developed by Lall and Miller (1988) may be considered as the best screening model developed so far. Their model was not fully successful, compared with the yield model proposed by Loucks et. al. in 1981, to reduce the required number of variables and constraints. This was due to the nature of the objective of the model formulated by them, which maximized the total net benefit. This approach required introducing additional variables for expected costs and benefits related to each control and state variable. The uncertainty inherent in their assumed values for cost and benefit of each variable is another disadvantage of their model.

Lall and Miller (1988) applied a simplistic assumption to estimate flood control benefit and constraints on channel flow capacity were not considered. Another disadvantage of their model is related to the initial solution requirement by the screening model. To run their model, some effort is needed to choose the best initial values of variables. To find the best initial solution, they restricted their efforts by comparing the results of different runs of their screening model by setting: (1) all variables at their lower bounds, (2) all variables at their upper bounds, (3) all variables halfway between their bounds, and (4) some variables at their upper and others at their lower bounds. In their application, Lall and Miller (1988) assumed that all reservoirs are full at the start of the operation period. However, their MSPA failed to provide full reservoirs on the last month of the operating period. Therefore in the applications performed, the maximum storage capacity in the last 36 months, instead of the full capacity, of operation was used for the last operating period. Another limitation of their model, as Lall and Miller (1988) mentioned, is the model requirement to pre-specify yield reliabilities over a critical period that may not be a common period, if the region of application is not hydrologically homogeneous. The model developed by Khaliquzzaman and Chander (1997), though has some advantages over other LP-based models, requires piecewise linearization technique and suffers from the same problem of dimensionality as other LP-based models.

2.7 Multi-Objective Programming (MOP):

In most water resources problems, there are several non-commensurate objectives that have to be achieved simultaneously. Using a single-objective approach on such problems is unrealistic and restrictive. In such cases, improvement of some objectives will result in sacrificing the other objectives. It is not possible to say which objective is more important without making judgment about the relative importance of the objectives. The decision-makers will usually evaluate the relative importance of the objectives differently. Therefore, the resolution of the conflicts among the objectives will usually require a political process. It would be a mistake for the planners to select only one of these objectives or to assume relative values for the objectives (Cohon 1978). The traditional economic approach to optimize a water resources system will try to make all objectives commensurable and optimizes the problem with respect to the "economic-efficiency" aspects (Changchit and Terrell 1989). In other words, one finds a policy in which benefits will exceed the costs and the policy with the highest benefit/cost ratio will be selected.

There are some considerations that make it difficult to put all objectives in a water resources system optimization problem into a single monetary dimension (Cohon 1978). First, there are questions of distribution of project impact in terms of classical upstream-downstream conflict that the economic efficiency objectives cannot address. In order to maximize the benefit/cost ratio, it may be decided to attribute more water to a certain region than others located in the water resources system. A pure efficiency criterion tends to favor further development in the developed regions since some infrastructure costs may be avoided. This decision may create some discomfort among the different users that cannot be converted into the monetary equivalence. Second, the development of river basins, especially in developed countries, may create environmental impacts that are considered by many to be undesirable and yet difficult to transfer into monetary value. Third, There are some uncertainties in terms of predicting the interest rate, inflation rate, labor cost, and the benefit obtained by selling water to different users during the project life. Finally, there is always uncertainty in evaluating the social values into a single monetary equivalence.

Multi-objective programming is an approach that can analyze the tradeoff between objectives (Wurps 1993). Application of multi-objective Programming has a number of advantages over conventional single-objective techniques as (Simonovic 1992): (1) more realistic problems can be addressed since the requirement of a single-objective function is eliminated, (2) non-commensurable objectives can be incorporated in the analysis, (3) tradeoff functions are available explicitly so that the decision makers can formulate more appropriate decisions.

The future water demand forecasting in a complex multi-reservoir system is not an easy task. There is a rather fast increase in water demand and it is very difficult to make any reliable forecast. Therefore in planning a multi-reservoir system, instead of assuming only one solution, it is wise to evaluate a range of solutions for different water demand levels within two boundary conditions of minimum and maximum water demand (Miloradov 1992). Multi-objective approach can introduce a range of options larger than the one "optimal" solution identified by single-objective methods. Therefore, the term of the "optimal solution" in the single-objective context is no longer appropriate and is replaced by the concept of "non-inferiority" which is also called as "Pareto Optimality" in the multi-objective analysis (Mays and Tung 1992). In other words, in multi-objective optimization problems, there is no single optimal solution. A solution, which minimizes one objective, may not, in general, minimize any of the other objectives. Hence, information about preferences is needed to compare the alternative solutions (Loganathan and Bhattacharya 1990).

The multi-objective techniques can be divided into three categories of preference-oriented techniques, generating techniques, and multiple-decision-maker methods. The latter refers to a case where there are several decision makers with conflicting viewpoints and the methods are directed at the resolution of conflict among many decision makers (Cohon 1978). In preference-oriented techniques, the decision-makers articulate their preference in advance to the analyst and the best compromise solution is defined without introducing a non-inferior set. Generating techniques end up with a range of solution and the tradeoff among the different objectives, which is called as the "non-inferior set". By definition, any non-inferior set is a feasible solution to a MOP. Hence, an improvement in one objective will cause a degradation in at least one other objective. The idea of non-inferiority in multi-objective approaches will indicate a range of choice rather than a single optimal solution. Among the obtained non-inferior solutions, the preferred alternative is selected by the decision-maker, which is called the "best compromise solution". Generating methods do not need input of preference and therefore are compatible with a wide range of decision context and are generally more suitable. Consequently some researchers are biased toward generating methods. The rationale for this preference is based on the implied roles for the analyst and decision-makers. In generating techniques, the analyst plays the classical role as a scientist and information provider while decision makers maintain complete control over the decision without transferring any of that responsibility to the analyst. The only disadvantage of generating methods is their sensitivity to the number of objectives. Considering more than three objectives will cause two problems of high computational burden and the difficulty to display the results. Furthermore, analyzing the tradeoff among more than three or four objectives is difficult and doubtful (Cohon 1978).

There are relatively few published papers in engineering of experiments involving multi-objective Programming. However, a notable number of papers has been published in other areas (Goicoechea et. al. 1992). Shafike et. al. (1992) used MOP to analyze a groundwater contamination management problem. Ko et. al. (1992) compared the performances of generating and preference-oriented techniques in dealing with large-scale multi-reservoir system operations. After extensive comparative evaluation of alternate methods, they concluded that the epsilon constraint method is the most suitable generating technique for problems with different dynamic characteristics of objective functions. This method provides the decision-maker with sufficient information to select the most satisfactory solution among the generated non-inferior set (Ko et. al. 1992).

CHAPTER 3

THE PROBLEM FORMULATION

The formulation of the water supply problem to meet the different objectives is described in this chapter. The proposed formulation is discretized over time. The concept of the problem formulation and its related variables are explained by applying them to a hypothetical watershed with a variety of demand types and reservoir configurations.

The duration of time periods of a model depends on the availability of data, the particular objectives, computer capacity, and the purpose for which the model is to be used (Loucks et. al. 1981). The objective in designing a multi-reservoir system is to find reservoir sites and related storage capacities. In designing stage, on the contrary to optimizing the operating policy, it is not intended to find system responses to real time demands of different areas. The shortest time period considered in such analyses is usually no less than the time it takes water to travel from the upper end of a river basin to the lower end of the basin (Loucks et. al. 1981). If a time base less than the river flow travel time is selected, reservoir and channel routing procedures must be performed in the optimization algorithm (Chow et. al. 1988). Including routing procedures in the optimization model would be computationally very intensive with only minor effect, if any, in the reservoir system design (Mays and Bedient 1982). Therefore, based on the watershed size and hydraulic characteristics, e.g., the channel geometry and the flow velocity, any time base greater than the flow travel time can be selected. This will reduce the computer time and memory requirement of the screening model. Depending on the watershed size, the time base can be daily, weekly, or monthly. It is clear that monthly time base is related to larger watersheds, where the reservoirs may be located far from each other and the flow travel time from one reservoir to other(s) takes longer. Monthly period is usually selected as the proper time base in water supply design models. Loucks et. al. (1981), Fontane (1982), Supangat (1987), Lall and Miller (1988), and Khaliquzzaman and Chander (1997) developed monthly design models to design a multi-reservoir system. Once the volume and configuration of reservoirs are specified, the real time based operating policy of the selected reservoirs can be optimized.

The objective function formulated in this study can accept any time base greater than the flow travel time. The selected time base is referred as T_b. The entire potential reservoir sites (candidate reservoirs) and the demand areas in the hydrosystem should be numbered sequentially from upstream to downstream. The reservoir storage capacities at the beginning of the time T_b are considered as the state variables. The control variables are divided into two groups of reservoir releases (spills) and yields. The yield definitions correspond to those used by Loucks et. al. (1981) and are the volume of water delivered to demand areas during the time base T_b. The reservoir release is the volume of the *nth* release from a reservoir in the hydrosystem spilling directly to the next reservoir at the downstream during T_b. The unregulated local inflow(s) and the upstream reservoir release(s) are the total inflows to the next downstream reservoir. The infiltration loss is assumed to be negligible due to sedimentation and deposition of fine soil (e.g., clay) on the reservoir bed area. Evaporation from the reservoir has been assumed to be the only source of system loss.

A hypothetical multi-reservoir system is shown in Fig. (3.1). This system consists of six reservoirs where one of them (RES#5) is a mass balance node. Mass balance nodes are dummy storage nodes with the yield, lower and upper storage bounds of zero. They are used where river diversions or tributaries exist to check the conservation of mass at locations. Four demand areas are shown in the system, where two of them are for industrial (IN), one for agrarian (IR), and one for municipal (M) use. This multi-reservoir system will be used to explain the concept of some variables in the objective function later.

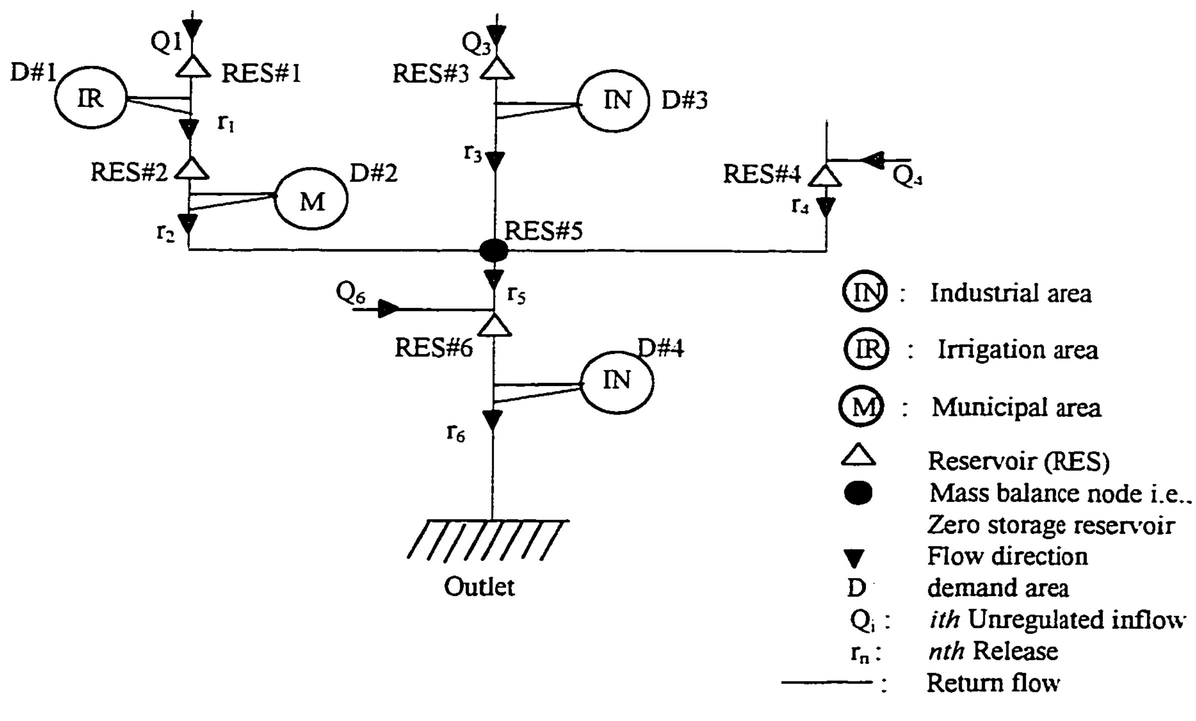


Figure (3.1): A multi-reservoir system and its related demand areas.

3.1 The Problem Objectives:

The primary objectives of the screening model formulated are to determine a set of optimal reservoir capacities that supplies water for different demand areas at the minimum cost. Minimizing the rapid variations on reservoir yield and minimizing the storage differences at the beginning and end of the optimization period are among the secondary objectives that have been included in the problem formulation. In practice some water supply objectives have to be met at any cost. These objectives usually appear in the problem constraints and are mandated by law for political, ecological, or environmental reasons. Hard constraints in the water supply problem may cause an infeasible problem and may not be met especially during dry periods. Therefore, most of the formulated objectives are *"soft"* in that they can be violated at a price. For example, in the water supply problem, some demand areas may receive less water than planned.

These objectives are:

$$Min\ f_1 = \sum_{j=1}^{Nd} W_j^y \left[\sum_{t=1}^{T} \left(\sum_{n=1}^{Nr} y_{n,j}^t - D_j^t \right)^2 \right] + \left\{ \begin{array}{l} \sum_{n=1}^{Nr} W_n^{by} \left[\sum_{t=1}^{T-1} \sum_{j=1}^{Nd} \left(y_{n,j}^{t-1} - y_{n,j}^t \right)^2 \right] + \\ W_n^{br} \sum_{t=1}^{T-1} \left(r_n^{t-1} - r_n^t \right)^2 + W_n^{fs} \left(s_n^{T-1} - s_n^1 \right)^2 \end{array} \right\} \qquad (3.1)$$

$$Min\quad f_2 = \sum_{n=1}^{N_r} f_n^c \qquad (3.2)$$

where

s_n^t : State variable; a variable showing the storage capacities of the *nth* reservoirs at the beginning of the time period *t*.

$y_{n,j}^t$: Yield (L^3) from reservoir *n* to the demand area *j* during the time *t*.

N_d : Total number of municipal, industrial, and irrigation demand areas.

N_r : Number of candidate reservoirs in the system.

D_j^t : Water requirement (L^3) at the demand area j during the time t.

W_j^y : A weight coefficient applied to the first term in equation (3.1).

W_n^{by} : A weight coefficient applied to the second term in equation (3.1).

W_n^{br} : A weight coefficient applied to the third term in equation (3.1).

r_n^t : The release (L^3) from the reservoir n during the time t.

W_n^{fs} : A weight coefficient applied to the terminal storage function.

f_n^c : Total cost for reservoir n as a function of reservoir storage/capacity.

Reservoir yield and release are control variables and reservoir storage is the state variable in the formulation presented. The first objective in Eq. (3.1) explains the desired criteria related to the reservoir yields and releases as the control variables. It consists of four terms. The first term in Eq. (3.1) tries to minimize the water supply shortage for each demand area in the multi-reservoir system. The square term is used to minimize the difference between water supply and demand in either way.

The second and third terms are intended to avoid rapid variations on control variables (y_j^t and r_n^t). This is referred as the bang-bang control in the literature (Albuquerque 1993). Controlling the rapid variation of reservoir yields and releases over the time makes the reservoir gate operation smoother and easier. The fourth term is the terminal function, which controls the final state of the system and is intended to provide storage volumes in the reservoirs, which are needed for the next operation period. The square terms in the bang-bang control and terminal function have the advantage of penalizing larger deviations (in either direction) from their targets.

Based on the order of magnitude and importance of each criterion in Eq. (3.1) to the designer, different values can be assigned to the weight coefficients (W_j^y, W_n^{by}, and W_n^{br}). The most important term in Eq. (3.1) should use the highest value. For the design purposes, usually the first term is the most important term. Quadratic forms were selected for the last three terms. The advantage of using a quadratic form is that most search directions algorithms can take advantage of this form by increasing their convergence characteristics and speed.

The objective in Eq. (3.2) is related to the total cost of the multi-reservoir system. It tries to minimize total reservoir costs as a function of their storage capacities. The total cost depends on some factors such as the type, dimension of the dam, and the selected spill out structure. These factors can be related to the reservoir storage capacity. Therefore, for any candidate reservoir location the total cost can be easily related to the reservoir capacity by a mathematical function. The formulation presented in Eqs. (3.1) and (3.2) show that objectives will conflict with each other. That is, the improved achievement with one objective (e.g., storing enough water to supply demands) can only be accomplished at the expense of increasing total reservoir costs.

3.2 Constraint Equations:

The related system constraints applied to the objectives described in the previous section are as:

1. Continuity equation;

$$
\begin{aligned}
&for\ n = 1,2,\ldots,N_r \\
&s_n^1 = x_n \\
&for\ t = 1,2,\ldots,T \qquad\qquad (3.3)\\
&s_n^{t+1} = s_n^t + Q_n^t + \sum_{k=1}^{Nr}\left(\Lambda_n^k r_k^t\right) - \sum_{j=1}^{N_d} y_{n,j}^t + \sum_{\substack{J=1\\J\neq j}}^{N_d}\left[\Gamma_n^J \sum_{m=1}^{t}\left(\rho_J^{t-m+1}\sum_{\substack{i=1\\i=n}}^{N_r} y_{i,J}^m\right)\right] - \\
&e_n^t C_n^e \left(\frac{s_n^t + s_n^{t+1}}{2}\right)^{p_n^e}
\end{aligned}
$$

where the new variables are as:

Q_n^t: Inflow into reservoir *n* during time interval *t*.

Λ_n^k: The element of the layout configuration matrix of the multi-reservoir system with N_r rows and columns (Λ_{Nr*Nr}). Each row of this matrix shows the reservoir number and each column shows the release number. The concept of the matrix Λ_n^k is explained in Fig. (3.2) for the multi-reservoir system shown in Fig. (3.1). As it is shown, for the system with six reservoirs, Λ has six rows and columns The state of any element at the *nth* row and the *kth* column is defined as follows:

1: if the reservoir *n* receives the *kth* release

-1: if reservoir *n* delivers the *kth* release

0: otherwise

Γ_n^J: The element of the return flow matrix with N_r rows and N_d columns (ΓN_r*N_d) that shows demand areas with return flow influent to reservoir *n*.

Each row of this matrix shows the reservoir number and each column shows the upstream demand area that its return flow discharges into the reservoir n. The concept of the matrix Γ for the hydrosystem in Fig. (3.1) is explained in Fig. (3.2). As it is shown, for the system with six reservoirs and four demand areas, Γ has six rows and four columns. The state of any element at the *nth* row and the *jth* column is defined as follows:

1: if the return flow from demand area j is flowing into reservoir n

0: otherwise

ρ_J^m: A coefficient for return flow from the demand area J during time m.

e_n^t: Evaporation per unit area (L^3/L^2) from reservoir n during time t.

C_n^e and P_n^e: The coefficient and exponent in the surface area-storage relationships of the reservoir n.

$$\Lambda = \begin{vmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{vmatrix} \quad \Gamma = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \quad \xi = \begin{vmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad \varsigma = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{vmatrix}$$

Fig. (3.2): The concept of the system matrices for the multi-reservoir system defined in Fig. (3.1).

Eq. (3.3) is the discrete form of the continuity equation for a reservoir n over T time periods, where s_n^t is the storage (L^3) of the reservoir n at the beginning of time t; Q_n^t

is the volume of the unregulated local inflow (L^3) into reservoir n during time t. The third term ($\Sigma A_n^k * r_k^t$) specifies both the release(s) from reservoir n and all the inflows resulting from upstream reservoir releases to reservoir n.

The fourth term ($\Sigma y_{n,j}^t$) shows the total yield supplied by the reservoir n to all demand areas during time t. N_d is the number of demand areas in the hydrosystem. The fifth term in Eq. (3.3) determines the summation of all return flows that the reservoir n receives during the time t from influent areas. In the last term, the evaporation during time t is related to the average reservoir capacities through the reservoir's area-storage relationship.

2. Constraints on release based on the downstream minimum flow needs and flood control requirements;

$$0 \le r_n^t \qquad\qquad for\ k = 1,2,\ldots,N_r\ ;\ for\ t = 1,2,\ldots,T$$
$$r_n^{\min} \le r_n^t + \sum_{j=1}^{N_d} \xi_n^j \left\{ \sum_{k=1}^{N_r} \left(\zeta_n^k (y_{k,j}^t) \right) \right\} \le r_n^{\max} \tag{3.4}$$

where the new variables are as:

ξ_n^j: The element of the demand area matrix with N_r rows and N_d columns ($\xi N_r * N_d$). This matrix is used to specify the demand areas located in the downstream of a release. Each row of this matrix shows the reservoir release number n and each column shows demand area located in the downstream of a reservoir release n. An element of ξ at column j and row n is equal to 1, if the demand area j is located in the downstream of the release n. The elements of ξ are equal to 0 otherwise. The concept of the matrix ξ_n^j, for the multi-reservoir system shown in Fig. (3.1), is explained in Fig. (3.2). As it is shown, for the system with six releases and four demand areas, ξ has six rows and four columns.

ζ_n^k: The element of the upstream reservoir matrix with N_r rows and columns ($\zeta N_r * N_r$). This matrix is used to specify the reservoirs located in the upstream of a release. Each row of this matrix shows the reservoir release number n and each column shows whether a reservoir is located in the upstream of a reservoir release n. An element of ζ at column k and row n is equal to 1, if the related reservoir k is located in the upstream of the release n. The elements of ζ are equal to 0 otherwise. The concept of the matrix ζ_n^k, for the multi-reservoir system shown in Fig. (3.1), is explained in Fig. (3.2). As it is shown, for the system with six reservoirs, ζ has six rows and columns.

The constraint in Eq. (3.4) controls the minimum and the maximum flow downstream of each reservoir. Eq. (3.4) determines the total streamflows i.e., the summation of *nth* release and all upstream reservoir yields flowing toward downstream. The minimum flow constraint (r_n^{min}) can be other than zero to consider instream recreation, navigation, and water quality control. The maximum flow constraint r_n^{max} will prevent the system from being flooded due to excess reservoir release and the upstream reservoir yields attributed to downstream demand areas.

3. Reservoir yield constraints for water supply;

$$\begin{aligned} &(a)\ \sum_{n=1}^{N_r} y_{n,j}^{t} \le D_j^{t} && for\ j = 1,2,...,N_d\ ;\ for\ t = 1,2,...,T \\ &(b)\ y_{n,j}^{t,\min} \le y_{n,j}^{t} \le y_{n,j}^{t,\max} && for\ n = 1,2,...,N_r \end{aligned} \qquad (3.5)$$

The first reservoir yield constraint in Eq. (3.5.a) is to guarantee that the total water supplied to any demand area j is within a desirable range. The second constraint in Eq. (3.5.b) specifies the lower and upper bounds on the yield supplied by reservoir n for the demand area j. The upper bound $y_{n,j}^{t,max}$ can have the same value as D_j^t. In this case the reservoir n will have the potential to supply the entire water requirement at the demand area j. The lower bound $y_{n,j}^{t,min}$ is the minimum permissible yield that reservoir n can supply for the demand area j. If the reservoir n is located at the downstream of the demand area j or due to any reason is not supposed to supply water for it, the $y_{n,j}^{t,min}$ and $y_{n,j}^{t,max}$ will be set equal to zero

4. Storage constraints based on physical limits;

$$\begin{aligned} &for\ n = 1,2,...,N_r\ ;\ for\ t = 1,2,...,T \\ &\quad s_n^{\min} \le s_n^{t+1} \le s_n^{\max} \end{aligned} \qquad (3.6)$$

In Eq. (3.6), the upper and lower bounds on the reservoir storage are defined. s_n^{min} and s_n^{max} are the lower and upper bounds on reservoir n respectively. Storage upper bound (s_n^{max}) of each candidate reservoir can be selected by using the topographic map of the reservoir site. In Eq. (3.6), the lower bound s_n^{min} can be considered as the conservation (dead) storage of reservoir n to provide a minimum storage for recreation or the reservoir sedimentation. A constraint to maintain prescribed ratios of minimum storage to reservoir storage at each site has been considered:

$$\begin{aligned} &(a)\ s_n^{\min} = \Omega_n * x_n \\ &(b)\ \pi_n^{\min} \le s_n^{\min} \le \pi_n^{\max} \end{aligned} \qquad (3.7)$$

where Ω_n is a pre-specified ratio of the dead storage to reservoir capacity x_n. Therefore, the dead storage is a function of the reservoir storage capacity. If a candidate reservoir is not selected in the optimization process, the related capacity and consequently its dead storage (s_n^{min}) will be equal to zero. π_n^{min} and π_n^{max} are minimum and maximum permissible dead storage bounds. Eq. (3.7.b) ensures that the dead storage will not fall beyond the maximum and minimum permissible dead storage for the reservoir n.

The formulation presented in this chapter (Eqs. 3.1 through 3.7) is a general formulation that describes the system configuration and constraints in detail. By introducing the Four Λ, Γ, ξ, and ζ system matrices, the defined problem formulation can be easily applied to any multi-reservoir system with any possible configuration.

CHAPTER 4

TECHNICAL APPROACH

Considering the literature review in chapter (2), OCT and PSLP are selected as the most promising optimization techniques that can be applied to any complex large-scale multi-reservoir system with a nonlinear objective function. The OCT that is being used in this research is based on the discrete minimum principle of Pontryagin and the PSLP technique is based on the algorithm proposed by Zhang et. al. (1985). To consider different objectives in optimizing multi-reservoir systems, the multi-objective approach is selected to avoid the monetary quantification of the social and political impacts resulting from choosing a certain policy. Considering section (2.6), the generating method (weighting and epsilon constraint) is selected to analyze the tradeoff between the different problem objectives.

In this chapter, the formulation of general minimization problems in OCT and PSLP are described separately. These problems resemble the typical optimization problems in multi-reservoir systems. The theoretical approaches in the OCT and PSLP methods and the related numerical algorithms to solve optimization problems are explained. These algorithms are implemented in the developed computer programs.

Finally, the application of two generating methods (weighting and epsilon constraint) to optimization problems and their algorithms are reviewed.

4.1 General Objective Functions in OCT:

A general minimization problem in a multi-reservoir system is considered in this section. The problem is a multi-stage system with state variables specified at the beginning of the stages. The minimization is over a time span of T periods with (1) known inflows and (2) initial reservoir capacities. The general problem has N_r reservoirs with any arbitrary layout configuration, whether in series or parallel.

The objective function for this problem is typically multi-dimensional, nonlinear, non-convex, and separable in time. A generalized objective function J in the minimization problem can be stated in order to minimize the function $F(.)$ over time t=1,2, ..,T and the deviation from the target $\phi(.)$ at the end of period $T+1$. The function $F(.)$ in the minimization problem is usually a cost function or a function that shows the deviation from any target. A general minimization formulation is as follows:

$$Minimize\ J = \sum_{n=1}^{N_r}\sum_{t=1}^{T} F(s_n^t, R_n^t) + \phi(s_n^{T+1}) \tag{4.1}$$

Subject to:

for $n=1, 2, ..., N$; *for* $t=1, 2, ..., T$

$$s_n^1 = s_n^{initial} \tag{4.2}$$

$$s_n^{t+1} = s_n^t + Q_n^t - R_n^t - E_n^t \qquad (4.3)$$

$$\begin{aligned} &a) \quad s_n^{\min} \le s_n^{t+1} \le s_n^{\max} \\ &b) \quad R_n^{\min} \le R_n^t \le R_n^{\max} \end{aligned} \qquad (4.4)$$

where

R_n^t: Control variable; a variable showing the release from the *nth* reservoirs during time interval t.

$F(s_n^t, R_n^t)$: Objective function value at time t.

$\phi(s_n^{T-1})$: Terminal function representing the deviation from ending target (if there is any) at the end of the final time period T.

$s_n^{initial}$: Initial storage capacity of the reservoir n (initial condition of the system).

N_r : Number of state and control variables (reservoir capacities, releases, and yields).

E_n^t: Losses (e.g., evaporation) from the *nth* reservoir during time t.

s_n^{max}, s_n^{min}: The upper and lower bounds on the capacity of the *nth* reservoir.

R_n^{max}, R_n^{min}: The upper and lower bounds on the release/yield from the *nth* reservoir.

The system dynamic equation (4.3) adjoins the objective function J by using a set of Lagrange multipliers λ. The state-space inequality constraints (4.4.a) are included by using a quadratic penalty function η and a penalty weight p_s to account for the violation of constraints on state variables. The augmented objective function is called the Lagrangian function L:

$$\begin{aligned} &Minimize \quad L = \sum_{n=1}^{N}\sum_{t=1}^{T} F\left(s_n^t, R_n^t\right) + \phi\left(s_n^{T+1}\right) + \\ &\sum_{n=1}^{N_r}\sum_{t=1}^{T} \lambda_n^t\left(s_n^t - s_n^{t+1} + Q_n^t - R_n^t - E_n^t\right) + \sum_{n=1}^{N_r}\sum_{t=1}^{T} p_s * \left(\eta_n^{t-1}\right)^2 \end{aligned} \tag{4.5}$$

where:

$$\begin{aligned} &\eta_n^{t+1} = \gamma_1\left(s_n^{t-1} - s_n^{\min}\right) + \gamma_2\left(s_n^{t-1} - s_n^{\max}\right) \\ &\gamma_1 = \begin{cases} 1 & if \quad s_n^{t-1} - s_n^{\min} > 0 \\ 0 & otherwise \end{cases} \quad ; \quad \gamma_2 = \begin{cases} 1 & if \quad s_n^{t-1} - s_n^{\max} > 0 \\ 0 & otherwise \end{cases} \end{aligned} \tag{4.6}$$

By the minimum principle of Pontryagin, the necessary condition for L to be the minimum value (or stationary in general) is that the differential changes in L due to differential changes in control variables must be zero. It can be shown that this happens only if (Bryson and Ho 1975):

$for\ n = 1,2,\ldots,N$

$$\frac{\partial L}{\partial s_n^t} = 0 \qquad For \quad t = 2,3,\ldots,T \tag{4.7}$$

$$\frac{\partial L}{\partial s_n^{T-1}} = 0 \tag{4.8}$$

$$\frac{\partial L}{\partial R_n^t} = 0 \qquad For \quad t = 1,2,\ldots,T \tag{4.9}$$

Eq. (4.7) is called the "adjoint equation" and Eq. (4.8) is the terminal condition for it and is also called the "transversality equation". Eq. (4.9) represents the "stationary condition". To find the optimal solution using the initial values for state variables, Eqs.

(4.5) to (4.9) should be solved simultaneously. That is, the continuity, transversality, and adjoint conditions together with the stationary condition should be solved to obtain the optimum values for the Lagrange multipliers λ^* and R^*. Practically speaking, solving these nonlinear equations is not an easy task and in some cases it may be possible to solve them only numerically. Hence, direct solution methods of mathematical programming are used instead. In this approach, an initial guess of control trajectories is made. By applying transversality and then adjoint equations, the related Lagrange multipliers are determined. Using stationary conditions and the Lagrange multiplier obtained before, a new search direction is determined to reduce the value of the objective function. The new search direction shows the next move of the control variable toward the minimum. Then, the transversality and adjoint conditions are applied again using the new control variables and the whole procedure is repeated iteratively until the control trajectories converge to the optimum values.

4.2 Multi-Dimensional Optimization:

In the OCT, the minimization procedure uses nonlinear programming methods. In these methods to minimize a function $f(\boldsymbol{X})$ of $\boldsymbol{X}=[x_1, x_2, .., x_N]$ variables, a multi-dimensional optimization method is used to direct the vector $\boldsymbol{X}$ in N-dimensional space in some vector direction $\boldsymbol{n}$ at each step. Then, the $f(\boldsymbol{X})$ can be minimized along the $\boldsymbol{n}$ by a one-dimensional method. This procedure continues until the minimum point is reached. There are various multi-dimensional optimization methods, each differs only by the way it chooses the direction $\boldsymbol{n}$ for the next calculation step. In all these multi-dimensional optimization methods, a line minimization procedure is needed. That is a one-dimensional optimization technique which is used to find the scalar θ in order to minimize the function $f(X+\theta n)$. The value of θ shows the magnitude of the step and n indicates the search direction toward the next iteration point, which finally goes toward the optimal point. The application of optimal control theory to optimization problems is summarized in the flow chart shown in Appendix B-1.

4.2.1 Selecting an Multi-Dimensional Optimization Algorithm:

Generally speaking, in the case of involving non-convex functions, none of the optimization algorithms can guarantee that the global minimum can be found. However, they differ by their convergence rate and memory requirement. Compared to linear convergence of the steepest descent method, all the conjugate gradient and quasi-Newton methods have quadratic convergence. Therefore, these methods should always be preferred over the steepest descent method. Fletcher-Reeves, Polak-Ribiere, and variable metric methods are among popular methods that use the gradient information. N line minimization is required in any of these methods to find the minimum of a quadratic function. However, in the variable metric method, a matrix of size $N*N$ to approximate the inverse of the Hessian matrix H^{-1} should be stored at each iteration while the Fletcher-Reeves method requires storage of the order of N. To choose a proper algorithm in optimizing a large hydrosystem with N-dimensional function, a method that requires storage of order N is preferable. Fletcher-Reeves and Polak-Ribiere algorithms are among the most important conjugate gradient methods that are of this type, where the latter is probably a superior method (Press et. al. 1990). The Polak-Ribiere method is a variant of the Fletcher-Reeves method and there is only a small difference between them, which vanishes when optimizing a quadratic function. In general, objective functions are not quadratic and more than N iterations are needed to reach the minimum point. Experimental evidence seems to favor the Polak-Ribiere method over the Fletcher-Reeves in optimizing non-quadratic functions (Luenberger 1984).

4.2.2 Line Minimization Algorithm:

Every multi-dimensional optimization algorithm requires a line minimization procedure to minimize the objective function in the search direction specified by the algorithm. The line minimization algorithm requires bracketing to ensure that a minimum is in the specified search domain. Therefore, initial bracketing is an essential part of any one-dimensional minimization technique. Using initial extended boundaries for certainty can significantly affect the speed and accuracy of the algorithm. Press et. al. (1990) proposed an algorithm to define the initial bracketing in the line minimization technique. In this algorithm, an initial guess of two close points a and b (e.g., $a=0$, $b=1$) is made. If $f(a)$ is not greater than $f(b)$, the role of the two points a and b should be switched. Then, the third point c is defined using point b and the golden section interval. To find out whether the interval bracketed by a and c includes the minimum point or whether the minimum point is outside this interval, a parabola is fitted through three points a, b, and c. Then by finding the minimum abscissa of the parabola, the function values at this point and the points b, and c are compared. Depending on the function values at these points, the points a, b, and c are updated and the search continued until the downhill trend stops.

Once the bracketing of the interval containing the minimum point is specified, the line minimization begins in which the function $f(X+\theta n)$ will be optimized with respect to θ. In this case it would be a one-dimensional optimization procedure where any one-dimensional optimization algorithm such as Dichotomous, Golden Section Search, Fibonacci, or Brent's method may be used to minimize $f(X+\theta n)$ at any multi-dimensional optimization iteration step. The new point i.e., $X_{new}=X_{old}+\theta_{min}n$ will be stored for the next iteration required in the multidimensional optimization search technique. Albuquerque (1993) showed that depending on the interval determined in the Dichotomous method, Brent's method is 3.5 to 4.6 times faster and results in a smaller minimum objective value. Brent's method is a combination of the Golden Section Search method and the inverse parabolic interpolation (Press et. al. 1990). In this algorithm, using the step interval a and b, the third point will be specified by Golden Section method. Then, an inverse parabolic interpolation would be done to reduce the step interval. If it was unsuccessful i.e., the parabolic step for example fell outside the bounding interval a and b, the method will switch to the Golden Section Search method which is a "sure-but-slow technique" (Press et. al. 1990). Regardless of the type of the function. near the minimum point, the function generally has a convex shape. Hence, using a parabolic interpolation catches the minimum or at least very near to it and can lower the computational time. Brent's method is a one dimensional search method, which does not calculate the derivative. Since finding abscissa rather than an ordinate is important, the procedure is technically called "inverse parabolic interpolation".

4.3 OCT Algorithm:

The algorithm used in this study follows closely the work done by Labadie et. al. (1988) and Albuquerque (1993) as follows:

1. Set the iteration number $i=1$.

2. Make an initial guess of the control variables i.e.,

$$R_n^t \quad for\ n=1,\ 2,\ ..,\ N_r \qquad ; \quad for \quad t=1,\ 2,\ ...\ T$$

   This guess needs not to be a feasible solution. However, a good guess would cause a faster convergence.

3. Use the system dynamic equation (4.3) to find the corresponding state trajectory. Then, evaluate the Lagrangian function (Eq. 4.5) for the existing state and control trajectories; $(s_n^t)_i$ and $(R_n^t)_i$.

4. Test for termination if $i>1$. The termination criteria are based on the: either (1) the difference in values of the Lagrangian functions obtained at two successive iterations, or (2) the gradient of the Lagrangian with respect to the control variables.

5. Calculate Lagrange multipliers in two steps:

5a. Solve the transversality equation (4.7) to find:

$(\lambda_n^{T-1})_i \quad for \quad n=1,2,.., N_r$

5b. Solve the adjoint equation (4.6) to find:

$(\lambda_n^t)_i \quad for \quad n=1,2,.., N_r \quad ; \quad for \quad t=T, T-1,..., 2$

6. Determine the gradient of the Lagrangian function with respect to control variables R_n^t ($\partial L/\partial R_n^t$), using the Lagrange multiplier, control and state trajectories obtained at previous steps (Eq. 4.9).

7. Determine the search directions (S_d) using a multidimensional optimization method.

8. Use a one-dimensional constrained search algorithm to determine a scalar step size θ that minimizes the Lagrangian function with respect to the scalar step size θ. That is,

$$Minimize_{\theta}\ L\left[s_n^t,\left(R_n^t+\theta s_{d,n}^t\right),\lambda_n^t\right] \ for\ n=1,2,...,N_r\ ;for\ t=1,2,..,T$$

9. Set the new estimate of control variables:

$$\begin{aligned}&set\ i=i+1\\&\left(R_n^t\right)_i=\left(R_n^t\right)_{i-1}+\theta\left(s_{d,n}^t\right)_{i-1} \quad for\ n=1,2,...,N_r\ ;for\ t=1,2,..,T\end{aligned}$$

If new control variable values violate the control constraints upper and lower bounds in Eq. (4.4.b), their values should be set equal to their respective upper and lower bounds. In this case, Eq. (4.4.b) is called "saturation function" which refers to the procedure that simply truncates control values, which are violating their respective lower and upper bounds.

10. Go to step 3.

The above mentioned optimal control algorithm is called the *"double sweep"* algorithm in the sense that it uses backward sweep to calculate the Lagrange multipliers and forward sweep to update the control variables. The whole steps 1-10 will be repeated for a sequence of increasing penalty weights starting from a very small value.

4.4 General Objective Functions in PSLP:

The basic concept of PSLP is to approximate all the nonlinear terms in the objective function and constraints, using first-order Taylor series expansion about an initial or trial solution. This results in an approximated problem, which is linear in the decision variables. The PSLP, like all LP-based programming techniques, is a static optimization technique. That is, control and state variables are treated alike and will be determined simultaneously from the solution of the PSLP problem. In this algorithm, the original problem with nonlinear/linear constraints is converted into an *exact penalty function* P(**X**). P(**X**) is formed by keeping the linear constraints and adding all the nonlinear constraints to the original objective function using some penalty weights. The result is a linearly constrained penalty problem:

$$P(\mathbf{X}) = F(\mathbf{X}) + \sum_{i=1}^{k} W_i^p \left|C_i(\mathbf{X})\right| + \sum_{i=k+1}^{h} W_i^p Max[0, C_i(\mathbf{X})] \tag{4.10}$$

where:

X : Decision vector with $X=[x_1, x_2, \ldots, x_N]$ elements. In the multi-reservoir system optimization, X consists of reservoir yields, releases, and reservoir capacities.

$F(\mathbf{X})$: Objective function value.

$C_i(\mathbf{X})$: Nonlinear constraints. The problem has h nonlinear constraints where k of them are in the form of equality and h-k of them are inequality constraints.

W_i^P: Positive scalars used as the penalty weights.

Eq. (4.10) is written for a general minimization problem. It should be noted that in maximization problems, the penalty terms are subtracted from the original objective function. In the next step, the first-order Taylor series is applied to the exact penalty function (4.10) to define *the Approximating Function Pl(X)* as:

$$Pl(\mathbf{X}) = F(\mathbf{X}_0) + \nabla F(\mathbf{X}_0)d + \left(\sum_{i=1}^{k} W_i^p \left|C_i + \nabla C_i d\right| + \sum_{i=k+1}^{h} W_i^p Max(0, C_i + \nabla C_i d)\right) \quad (4.11)$$

where X_0 is the initial/trial solution and $d=[d_1 \ d_2 \ \dots \ d_N]$ is a vector showing the step size for N decision variables. *Pl(X)* is a good approximation to *P(X)* if the step size d is not large. Thus the nonlinear problem of *P(X)* can be minimized by a sequence of minimization of *Pl(X)* with an upper bound on the step size d. This leads to the *Approximating Problem*:

$$Minimize \quad Pl(X) \quad (4.12)$$

subject to the linear constraints and a new deviation constraint as:

$$-\omega \le d_i \le \omega \quad For\ i = 1,2,\dots,N \quad (4.13)$$

where ω is the upper bound on the Taylor series step size. The deviation constraint is to maintain a solution in the neighborhood of the current solution.

To solve the optimization problem (Eqs. 4.12 and 4.13), a Linear Program equivalent to the Approximating problem will be defined:

$$Minimze \quad \nabla F(X_0)d + \sum_{i=1}^{k} W_i^P(p_i + n_i) + \sum_{i=k+1}^{h} W_i^P(p_i) \tag{4.14}$$

subject to all linear constraints, constraint (4.13) and:

$for\ i = 1,2,\ldots,h$

$$C_i(X_0) + \nabla C_i(X_0)d - p_i + n_i = 0 \tag{4.15}$$

$$p_i \geq 0 \quad ; \quad n_i \geq 0 \tag{4.16}$$

In the above, p_i and n_i are *deviation variables*, which allow us to represent the piecewise linear terms of *Pl* linearly. Eqs. (4.14) to (4.16) constitute a linear problem that can be solved by the LP algorithm.

PSLP can be viewed as a steepest descent procedure applied to the exact penalty function associated with the original nonlinear problem. The search direction is determined by solving a linear program, and the distance moved along that direction is determined by the size of the step bounds defined for the Taylor series expansion.

SLP-based algorithms have at least linear convergence (Grygier 1983). PSLP is significantly more robust than SLP and SLPR and at least as efficient. For problems with vertex optima (at least as many active constraints as variables), PSLP is quadratically convergent (Zhang et. al. 1985). PSLP usually converges to a local optimum (Zhang et. al. 1985). The global convergence can be guaranteed, only if the objective function of the minimization (maximization) problem is known to be convex (concave) over the feasible region (Lall and Miller 1988).

4.5 PSLP Algorithm:

The PSLP algorithm is of the *trust region* or *restricted step* type which is defined by the constraint (4.13). This algorithm is based on Zhang et. al. (1985) as follows:

1- Set the iteration number $k=0$

2- Select an admissible initial values $X^k=[x_1^k\ x_2^k\ .\ x_N^k]$ for N decision variables that satisfy all the linear constraints. Also choose W_i^p; the penalty weights in Eq. (4.14).

3- Select ω^k; the step bound at *kth* iteration in Eq. (4.13).

4- Use the Simplex method and solve the equivalent LP problem (Eqs.4.13 to 4.16). This will result in new values of decision variables $X^{k+1}=[x_1^{k-1}\ x_2^{k-1}\ \ x_N^{k-1}]$.

5- Test for stopping criteria. The algorithm terminates if any one of the four separate criteria is satisfied to the pre-specified tolerances:

5-a Step sizes have been reduced to below a tolerance τ_1,

$$\left|x_n^{k-1} - x_n^k\right| < \tau_1\left(1 + \left|x_n^k\right|\right) \quad ; \quad \tau_1 = 10^{-4} \quad for \qquad n = 1,2,\ldots Nr$$

5-b No significant change in the exact penalty function value is observed for three consecutive iterations,

$$\left|P\left(X^{k+1}\right) - P\left(X^k\right)\right| < \tau_2\left(1 + \left|P(X^k)\right|\right) \quad ; \quad \tau_2 = 10^{-4}$$

5-c No significant improvement in the objective function value is observed for three consecutive iterations,

$$\left|F(X^{k+1}) - F(X^k)\right| < \tau_2\left(1 + \left|F(X^k)\right|\right)$$

5-d Kuhn-Tucker condition for optimality is satisfied in Eq. (4.10) within the tolerance τ_1.

6- Update the step bound ω. The expansion and reduction of the step size depends on the comparative values of the exact penalty functions *P(X)* and their approximating functions *Pl(X)* in two successive iterations as:

6-a Compute the actual change in the exact penalty function:

$$\Delta P_k = P(X^{k+1}) - P(X^k)$$

and the change "predicted" by its piecewise linear approximation $Pl(X^k)$, noting that $Pl(X^k)=P(X^k)$:

$$\Delta Pl_k = Pl(X^{k-1}) - P(X^k)$$

6-b: Compute the ratio of actual to predicted change: $r^k = \dfrac{\Delta P_k}{\Delta Pl_k}$

6-c: If $r^k < 0$, then $\omega^k = \dfrac{\omega^k}{2}$, go to step 4;

otherwise, update ω^k based on the criteria given below:

6-c-1 If $\left|1 - r^k\right| < \rho_1$ *then* $\omega^{k+1} = 2 * \omega^k$

6-c-2 If $\left|1 - r^k\right| > \rho_2$ *then* $\omega^{k+1} = \dfrac{\omega^k}{2}$

6-c-3 If $|x_n^{k+1}-x_n^k| = \omega^k$ for *3* consecutive iterations, *then* $\omega^{k+1} = 2 * \omega^k$

where $0<\rho_1<\rho_2<0$. Following Zhang et. al. (1985), the ρ_1 and ρ_2 are set equal to 0.25 and 0.75 respectively.

7- Set $k=k+1$ and go to step 4

Selecting small penalty weights (W_i^p) may result in an infeasible solution. However, if large penalty weights are selected, the decision variables X will be forced to stay too close to the feasible region for the majority of PSLP iterations. Consequently, it may cause slow convergence in some problems (Baker & Lasdon 1985). Therefore, based on the order of magnitude of the objective function and related nonlinear constraints, one can select reasonable small penalty weights to start the problem. If the PSLP iterations terminate with an infeasible solution in the original nonlinear problem, increase the weights and start again.

4.6 Application of the Generating Methods:

A general multi-objective formulation with p-objective problems can be written as:

$$\begin{aligned} &\textit{Minimize} \quad F(X) = \left[f_1(X), f_2(X), \ldots, f_p(X)\right] \\ &\textit{subject to:} \\ &X = \left[x_1, x_2, \ldots, x_N\right] \in F_d \end{aligned} \tag{4.17}$$

Where, F_d is the feasible domain in decision variable space. Eq. (4.17) is a vector optimization problem. The p objectives f_1 to f_p conflict with each other and consequently, the minimum of p objectives cannot be obtained simultaneously. Considering the literature review in chapter two, generating methods were selected to be used in designing multi-reservoir systems. In the subsequent sections, the algorithms of two selected generating methods to solve vector optimization problems are reviewed briefly.

4.7 Weighting Method:

In the weighting method, a weight is applied to each objective that identifies a desirable tradeoff between p objectives. The related weighting method problem is formed by articulating the p weighted objectives as:

$$\begin{aligned}&Min \quad w_1 f_1(X) + w_2 f_2(X) + \ldots w_p f_p(X)\\&subject\ to:\\&X \in F_d\end{aligned} \tag{4.18}$$

The solution to this problem would be the best compromise solution for the user who articulates the values of weights. The purpose of the weight coefficients in each objective function is to set the priority or the order of importance of each of the functions to the user. For example if the first objective function f_1 is a more important factor than the second one, the w_1 should be selected such that the term $w_1 f_1$ has a higher value than the $w_2 f_2$ term. In optimizing multi-reservoir systems, the determination of the weight coefficients is a trial and error process of utmost importance (Albuquerque 1993).

One strategy to determine appropriate weight coefficients is (1) to make a hand calculation to determine the order of magnitude of all objective functions; (2) to determine a set of weight coefficients to give priority to the more important terms; (3) to solve the optimization problem and see if the objectives are being accomplished. If a desired compromise solution is not obtained, repeat steps (2) and (3).

4.8 Epsilon Constraint Method:

In this section, the development of objective functions under epsilon constraint method is considered. The corresponding epsilon constraint problem to the optimization problem (4.17) is:

$$Min \quad f_h(X) \tag{4.19}$$

$$\begin{aligned} &Subject\ to: \\ &f_k(X) \le \varepsilon_k \quad for \quad k = 1,2,\dots,h-1,h+1,\dots,p \\ &X \in F_d \end{aligned} \tag{4.20}$$

Where ε_k is a set of upper bounds for the objective function values specified by the epsilon constraint method. The problem formulated in Eqs. (4.19) and (4.20) is a single objective problem and can be solved by an appropriate optimization method. For any fixed ε_k value, the optimal solution of the optimization method results in a non-inferior (also called un-dominated) solution of the original multi-objective problem of Eq. (4.17). By changing the values of ε_k within the feasible range of objective functions, a set of solutions i.e., a non-inferior set will be obtained.

In the epsilon constraint algorithm, different ε_k values will be considered during successive solutions of the problem (4.19) in order to examine tradeoffs between the objectives. The values for the ε_k in Eq. (4.20) are selected so that (Cohon 1978): (1) feasible solutions to the single-objective problem in Eq. (4.19) exist, and (2) all the constraints on objectives are binding at the optimal solution to the optimization problem. The epsilon constraint algorithm for the problem introduced in Eqs. (4.19) and (4.20) can be described as follows:

1- Solve the p individual minimization problems ($f_k(X)$, $k=1,..p$). Call the optimal solution to the objective k, $X^k=[x^k_1, x^k_2, \ldots x^k_N]$.

2- Compute the values of each objective function at each of the p optimal solutions i.e., $f_1(X^k)$, $f_2(X^k)$, $f_p(X^k)$, $k=1, 2,\ldots, p$. This gives p values for each of the p objective functions.

3- Denote the smallest value of the objective function $f_k(X)$, as n_k^e and the largest one as m_k^e. The n_k^e and m_k^e are the lower and upper bounds of the kth objective function respectively i.e., $n_k^e \leq f_k \leq m_k^e$.

4- Convert the multi-objective programming problem (4.17) into its corresponding epsilon constraint problem as in Eqs. (4.19) and (4.20).

5- Choose the number of different ε_k, in Eq. (4.20), that are going to be used in the generation of non-inferior solution. Call it N_ε.

6- Solve the constrained problem of (4.19) and (4.20) for all values for the ε_k obtained by:

$$\begin{gathered} for\ i = 0,1,2,\ldots,N_\varepsilon - 1 \ ; \quad for\, k = 1,2,\ldots h-1, h+1,\ldots,p \\ \varepsilon_k = n_k^e + \left(\frac{i}{N_\varepsilon - 1}\right)\left(m_k^e - n_k^e\right) \end{gathered} \tag{4.21}$$

Each objective function in constraint (4.20) is used N_ε times in the constrained problem. Therefore, there are N_ε^{N-1} constrained problems that result in N_ε^{N-1} non-inferior solutions to the problem (4.17).

CHAPTER 5

NUMERICAL STUDIES OF ALGORITHMS

The problem formulation to optimize multi-reservoir systems to supply water with a minimum cost was presented in chapter 3. The theories of two optimization techniques, OCT and PSLP, were reviewed briefly in chapter 4. In this chapter, four optimization models based on the OCT algorithm alone or in conjunction with the PSLP algorithm will be introduced. Then, by numerical experimentation, the performances of these optimization models in designing multi-reservoir systems will be compared. This includes development of computer programs for two OCT and PSLP optimization methods, verification of computer program codes by applying test examples, proposing different design models based on these two optimization algorithms, and comparing the performance of each design model on a design test example. Results obtained by each design model are presented and discussed. Finally, the results of these comparative studies are used to select the most suitable design model for use in the case study of CE-646 project in the next chapter.

5.1 Developing Computer Programs:

Two computer programs were developed based on the OCT and PSLP algorithms described in chapter 4. FORTRAN 77 has been chosen as the programming language. Based on section (4.2), Polak-Ribiere conjugate gradient and Brent's methods were selected as the most suitable minimization techniques. The computer code developed by Press et. al. (1990) for these two techniques have been modified and updated for the constrained optimization problems and implemented in the OCT program code. The line bracketing algorithm developed by Press et. al. (1990) was used to define the boundaries in the line minimization technique. A PSLP program code was also developed based on the algorithm explained in section (4.5). The Simplex method program code of Press et. al. (1990) was extensively modified and used in the PSLP computer program to solve *the equivalent linear problem* of Eq. (4.19).

The OCT and PSLP programs constitute the body of the design models that are being developed later in this chapter. Prior to this, the program codes have to be verified. The verification of the computer program codes can only be ascertained by numerical experiments requiring the implementation of the test examples to the program codes. Finding suitable numerical problems may not be easy since most papers on optimization of multi-reservoir systems do not provide full details of the data used.

5.2 Testing the Program Codes:

From literature, two optimization problems were selected as the test problems to verify the OCT and PSLP computer program codes. The performances of the developed programs on these test problems were compared with their known solutions. The selected test problems are relatively small problems but resemble the objective functions that are going to be implemented in the design models. In the following sections, the test problems are described. Then, numerical solutions obtained by two OCT and PSLP programs are compared to their exact solutions.

5.2.1 Testing OCT Code:

Optimal control theory is a dynamic optimization model and consequently the control variable must be the time rate change of the state variable. Considering this characteristic, a test problem was selected from section (6.2) of the textbook written by Fryer and Greenman (1987). This problem has an analytical solution and resembles the water supply formulation assigned to the OCT. The selected problem has an objective function with nonlinear terms with respect to state (x) and control variables (u) as:

$$Minimize \quad J = \frac{1}{2}\sum_{k=0}^{3}(x_k^2 + u_k^2) \tag{5.1}$$

Subject to:

$$for \quad k = 0,1,2,3 \tag{5.2}$$

$$x_{k+1} = x_k - u_k + 1$$

$$\begin{aligned} &a)\ 0 \le x_k \le 3 \\ &b)\ x_0 = 3 \\ &c)\ x_4 = 0 \end{aligned} \tag{5.3}$$

The OCT program was used to optimize the above test problem. Constraint (5.3.a) was joined to the objective function by using an external penalty function. The terminal condition (5.3.c) is treated as a soft constraint and consequently the Lagrange multiplier would not be required to adjoin the constraint (5.3.c) to the objective function. This is to avoid any possible jump in the value of the related Lagrange multiplier that has been recorded in the literature for large hydrosystems. Therefore, the terminal condition (5.3.c) was converted into a quadratic form to make the terminal state value as close to zero as possible. The weighting method is used to consider the order of importance of the original objective function and the terminal condition. The system dynamic equation (5.2) adjoins the objective function with a set of Lagrange multipliers. The Lagrangian function and its corresponding stationary, adjoint, and transversality conditions are:

$$L = \frac{w_1}{2}\sum_{k=0}^{3}(x_k^2 + u_k^2) + \sum_{k=0}^{3}\left[\lambda_k(x_k - x_{k-1} - u_k + 1) + p(\eta_{k-1})^2\right] + w_2 x_4^2 \tag{5.4}$$

$$\frac{\partial L}{\partial u_k} = w_1 u_k - \lambda_k \tag{5.5}$$

$$\begin{aligned} \lambda_3 &= 2w_2 x_4 + 2p\eta_4 \\ \lambda_k &= x_{k-1} + \lambda_{k+1} + 2p\eta_{k-1} \qquad \textit{For} \quad k = 2,1,0 \end{aligned} \tag{5.6}$$

The weight coefficients of 1 and 2000 were found to be appropriate for w_1 and w_2 respectively. The starting penalty weight of .0001 was used to solve the problem and the maximum penalty weight of 10 was found to be large enough to meet the constraint (5.3.a). The analytical solution to the problem and the related OCT solution are given in table (5.1). In this table, the optimal objective function $J(X^*)$, the corresponding optimal points $\mathbf{X}^*$ and $\mathbf{U}^*$, and the error E are shown, where E is the difference between the analytical and OCT solution.

Table (5.1): A comparison of OCT and analytical solutions.

	Analytical	**OCT program**	**Error**
$J(\mathbf{x}^*,\mathbf{u}^*)$	12.286	12.286	0.0
$\mathbf{u}_0^*$	2.857	2.855	0.002
$\mathbf{u}_1^*$	1.714	1.717	-0.003
$\mathbf{u}_2^*$	1.286	1.283	0.003
$\mathbf{u}_3^*$	1.143	1.144	-0.001
$\mathbf{x}_1^*$	1.143	1.145	-0.002
$\mathbf{x}_2^*$	0.429	0.428	0.001
$\mathbf{x}_3^*$	0.143	0.145	-0.002
$\mathbf{x}_4^*$	0.0	0.001	0.001

Table (5.1) shows no significant difference between the analytical and the OCT solutions, which verifies the developed OCT program code. It can be noticed from table (5.1) that the maximum absolute round off error of OCT is $3*10^{-3}$. That could be due to either the approximations in the optimization procedures or the computing round off error in the OCT.

5.2.2 Testing PSLP Code:

A comprehensive list of nonlinear test problems and their solutions is given in Hock and Schittkowski (1981). Among the different optimization problems presented, problem number 13 was selected. This problem resembles the optimization formulation assigned to the PSLP model. The objective function of the selected problem has a quadratic form with a nonlinear constraint and two variables as:

$$\begin{aligned} &Minimize \ \ f(x) = (x_1 - 2)^2 + x_2^2 \\ &subject to: \\ &\quad (1 - x_1)^3 - x_2 \geq 0 \\ &\quad x_i \geq 0 \ \ For \ i = 1,2 \end{aligned} \tag{5.7}$$

The PSLP program was used to optimize the above test problem. A penalty weight of 1.0 was found to be large enough to keep the solution in the feasible region. The analytical solution to the problem and the related PSLP solution are given in table (5.2). In this table, the optimal objective function $f(X^*)$, the corresponding optimal points X^*, and the error E is shown.

Table (5.2): A comparison of PSLP and analytical solutions.

	Analytical	**PSLP program**	**Error**
$F(X^*)$	1.0	1.0	0.0
x_1^*	1.0	1.0	0.0
x_2^*	0.0	0.0	0.0

Table (5.2) shows that the PSLP solution matches perfectly with the exact solution and consequently this verifies the developed PSLP computer program code.

5.3 Design Models:

Using the OCT and PSLP computer programs, four design models are developed to screen the best possible reservoir configuration. Three of these models are based on the OCT algorithm and hence are referred to as the OCT-based models in this study. The fourth model is a composite model based on a combination of OCT and PSLP algorithms. Since one of the primary objectives of this research is to develop a fast optimization model, developing a design model based on only PSLP is avoided. This is because (1) OCT is reported in the literature to be faster than PSLP, (2) too many constraints and variables are required in the PSLP algorithm and that increases computer run time, and (3) OCT is insensitive to the initial solutions. All of these 4 models share the same subroutines for reading the multi-reservoir system information. They perform a screening of a series of potential reservoirs for water supply to identify reservoir capacities and sites. In the subsequent sections, the formulation of each design model is explained. Prior to this, the design problem and its site characteristics must be introduced. This is because the derivation of the transversality and adjoin conditions in all design models (Eqs. 3.7, 3.8, and 3.9) depend on the type of the selected cost function in each problem.

5.4 Selection of Design Test Problem:

A test problem to evaluate the performance of design models has been selected. Based on the fact that this study is oriented towards applications, the experimental approach is adopted to evaluate the performances of different design models based on the PSLP and OCT techniques. A problem from Supangat (1985) was selected as a test problem to compare the performances of the developed design models. This problem is based on the project of a graduate course CE-646 in Colorado State University (at Fort Collins) to develop water storage strategies for water supply (Supangat 1985). The CE-646 was selected as the test case because (1) the required data and its best-known solution are available in Supangat (1985) and (2) the numerical problem fairly represents a large-scale multi-reservoir system. The CE-646 used in this study consists of six reservoirs with both serial and parallel reservoirs. The schematic diagram of CE-646 problem is shown in Fig. (5.1):

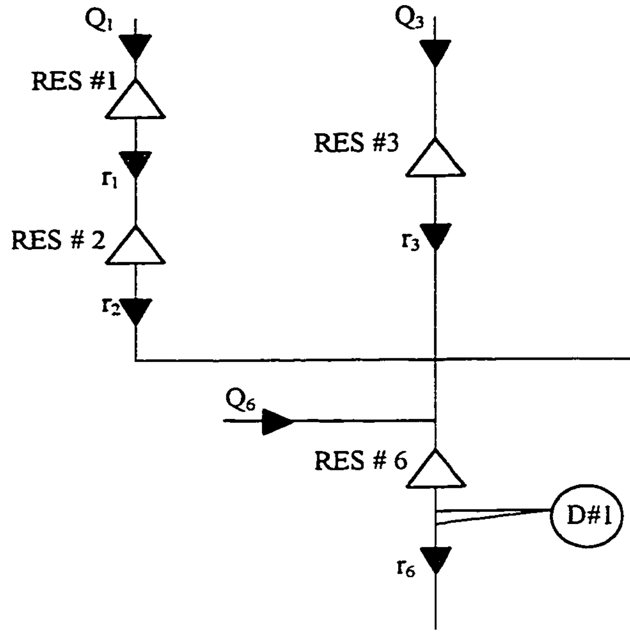


Fig. (5.1): The layout of the CE-646 test problem. All symbols are defined in the Fig (3.1)

The site characteristics given in Supangat (1985) have been used to develop mathematical relationships to predict construction costs as a function of reservoir capacities. Unlike Supangat (1985), nonlinear equations based on the best-fit curve (regression) were selected to estimate the reservoir cost. This approach is believed to represent more accurately the cost estimation. The correlation coefficients (r^2) in all the fitted cost equations were higher than 0.992. Other site information such as inflows to each reservoir, monthly evaporation rates, and physical constraints of the system (S_{max}, S_{min}) are taken from Supangat (1985). The data file for the CE-646 problem is presented in Appendix A and will be used as the input file to the developed design models. The specific goal in applying the design models is to determine the best storage strategies among these six reservoirs to satisfy demands for water supply. Following the CE-646 test problem, the hydrological data of two consecutive dry years is used to design the system. However, it should be emphasized that in a real system we need more data with large number of years to design the system. For the test problem, all the different demand areas are assumed to be located at the outlet of the most downstream reservoir. This is called *case 1 CE-646* problem in this study. In the sections to follow, a hydrosystem is designed using the case 1 CE-646 problem for water supply purposes and the capabilities of design models in meeting the optimization objectives are evaluated. The results obtained by each model are compared to the best-known solution of that numerical problem. The criteria to evaluate the performance of each model are based on the total water supply and the construction cost of the designed system.

5.5 OCT-based Design Model Formulation:

Three different design models, referred as OCT-I, OCT-II, and OCT-III, are introduced in this section. These models are based on the problem formulation explained in chapter (3). The sweep method, as described in section (4.3), is applied to these models to find the Lagrange multipliers and the optimal trajectories. Each of these models however, has a different approach to incorporate the cost function (3.2) with other objectives in Eq. (3.1). The OCT-I applies the epsilon constraint method and OCT-II and OCT-III models are based on the weighting method. All of the three models include the same terms with respect to control variables and are constrained to the availability of inflow and reservoir storage.

In the following sections, the Lagrangian function, transversality, adjoint and stationary conditions of each OCT-based design models are derived. To avoid repeating the derivation of formulation, first the common parts are used to develop the Lagrangian function to the objectives in Eq. (3.1). Then, the common stationary conditions are derived for all three models. Finally based on each approach, the related transversality and adjoint conditions are derived separately.

To solve the optimization problem for designing the multi-reservoir system, first the *Lagrangian function* (L_l) from the objective function (3.1) is formed. The L_l is obtained by adjoining the continuity equation and state-space constraints to the first objective function. The Lagrange multipliers are used to adjoin the continuity equation (3.3) to the objective function (3.1):

$$
\begin{aligned}
L_1 = & \sum_{j=1}^{Nd}\left[W_j^{y}\sum_{t=1}^{T}\left(\sum_{n=1}^{Nr} y_{n,j}^{t} - D_j^{t}\right)^2\right] + \sum_{n=1}^{Nr}\left\{W_n^{by}\sum_{t=1}^{T-1}\left[\sum_{j=1}^{Nd}\left(y_{n,j}^{t+1} - y_{n,j}^{t}\right)^2\right] + \right. \\
& \left. W_n^{br}\left[\sum_{t=1}^{T-1}\left(r_n^{t-1} - r_n^{t}\right)^2\right] + W_n^{fs}\left(s_n^{T-1} - s_n^{1}\right)^2 + \sum_{t=1}^{T} p_s\left(\eta_n^{t-1}\right)^2\right\} + \\
& \sum_{t=1}^{T}\sum_{n=1}^{Nr}\lambda_n^{t}\left\{\begin{array}{l} s_n^{t} - s_n^{t-1} + Q_n^{t} + \sum_{k=1}^{M}\left(\Lambda_{n,k} r_k^{t}\right) - \sum_{j=1}^{Nd} y_{n,j}^{t} + \\ \sum_{\substack{J=1 \\ J \neq j}}^{Nd}\left[\Gamma_{n,J}\sum_{m=1}^{t}\left(\rho_J^{t-m+1}\sum_{\substack{i=1 \\ i \neq n}}^{Nr} y_{i,J}^{m}\right)\right] - e_n^{t} C_n^{e}\left(\dfrac{s_n^{t} + s_n^{t+1}}{2}\right)^{p_n^{e}} \end{array}\right\}
\end{aligned}
\tag{5.8}
$$

where p_s in Eq. (5.8) is the storage penalty weight for violating constraints on the state-space and η_n^{t-1} is the exterior penalty function of the reservoir storage constraints as given by Eq. (4.6). The stationary condition is obtained by using Eq. (4.9) and setting it equal to zero. In the present optimization problem, the control variables for each reservoir consist of reservoir yield and release. Consequently, the stationary condition should be applied to both control variables. For reservoir yields, the stationary conditions are as:

$$\text{for } n = 1,2,\ldots,Nr \quad ; \quad \text{for } j = 1,2,\ldots,Nd$$

$$\vartheta_{n,j}^{t} = 2W_{j}^{y}\left(\sum_{n=1}^{Nr} y_{n,j}^{t} - D_{j}^{t}\right) + \sum_{m=t}^{T}\sum_{i=1}^{Nr}\left(\lambda_{i}^{m}\Gamma_{i,j}\rho_{j}^{m-t+1}\right) - \lambda_{n}^{t}$$

$$\frac{\partial L}{\partial y_{n,j}^{t}} = \begin{cases} \vartheta_{n,j}^{t} + 2W_{n}^{by}\left(y_{n,j}^{t} - y_{n,j}^{t+1}\right) & \text{for } t = 1 \\ \vartheta_{n,j}^{t} + 2W_{n}^{by}\left(-y_{n,j}^{t-1} + 2y_{n,j}^{t} - y_{n,j}^{t+1}\right) & \text{for } t = 2,3,\ldots,T-1 \\ \vartheta_{n,j}^{t} + 2W_{n}^{by}\left(-y_{n,j}^{t-1} + y_{n,j}^{t}\right) & \text{for } t = T \end{cases} \tag{5.9}$$

and for any reservoir release, the stationary condition would be:

$$\text{for} \quad n = 1,2,\ldots,N_r$$

$$\frac{\partial L}{\partial r_{n}^{t}} = \begin{cases} 2W_{n}^{br}\left(r_{n}^{t} - r_{n}^{t+1}\right) + \sum_{k=1}^{Nr}\left(\lambda_{n}^{t}\Lambda_{k}^{n}\right) & \text{for } t = 1 \\ 2W_{n}^{br}\left(-r_{n}^{t-1} + 2r_{n}^{t} - r_{n}^{t-1}\right) + \sum_{k=1}^{Nr}\left(\lambda_{n}^{t}\Lambda_{k}^{n}\right) & \text{for } t = 2,3,\ldots,T-1 \\ 2W_{n}^{br}\left(-r_{n}^{t-1} + r_{n}^{t}\right) + \sum_{k=1}^{Nr}\left(\lambda_{n}^{t}\Lambda_{k}^{n}\right) & \text{for } t = T \end{cases} \tag{5.10}$$

The OCT algorithm implemented at all OCT-based models is based on the iterative penalty weight method. The iterative approach is used to avoid any divergence or instability that often occurs when a high penalty weight is used. Therefore, a series of penalty weights is used to apply the penalty functions to all these models. These weights start with a small initial value and increase at each iteration up to a pre-specified final penalty weight.

5.5.1 OCT-I Model:

The OCT-I model is formulated to practice the epsilon constraint method in the OCT as the most suitable generating method. In this model the cost of the reservoir *n* is assumed to be a continuous function of the reservoir capacity (x_n). The logic behind the cost function in the OCT-I model is based on the simple fact that the required capacity for each reservoir *n* is equal to the largest reservoir storage occurring in time t_s (s_n^{ts}). The cost function f_n^c in the case 1 CE-646 problem has the following quadratic form.

$$\begin{aligned} &f_n^c = A_n x_n + B_n x_n^2 \\ &where,\ x_n = s_n^{ts} = Max_t(s_n^t) \quad for \quad t = 2,\ldots,T+1 \end{aligned} \tag{5.11}$$

The OCT-I applies the epsilon constraint method to minimize the cost of constructing multi-reservoir systems as:

$$\sum_{n=1}^{Nr} \left(A_n x_n + B_n x_n^2 \right) \leq \varepsilon_k \tag{5.12}$$

The quadratic exterior penalty function method was used to incorporate Eq. (5.12) into the rest of the objectives in the Eq. (5.8). For the same reason mentioned in section (5.2.1), the "hard" constraint approach is not used to avoid jumps in the values of Lagrange multipliers related to the epsilon constraint. Therefore, the Lagrangian function for the whole problem in the OCT-I model is as:

$$L = L_1 + p_c \psi^2 \tag{5.13}$$

where p_c is the cost penalty weight for violating the constraints on the second (cost) objective function. The iterative penalty weights p_s and p_c are specified at each loop of the optimization algorithm. The following is the cost penalty function ψ in Eq. (5.13):

$$\psi = \gamma_3\left(\sum_{n=1}^{Nr} f_n^c(x_n) - \varepsilon_k\right) \quad ; \quad \gamma_3 = \begin{cases} 1 & \text{if} \quad \sum_{n=1}^{Nr} f_n^c(x_n) - \varepsilon_k > 0 \\ 0 & \text{otherwise} \end{cases} \tag{5.14}$$

The related transversality and adjoint conditions obtained by using Eqs. (4.7) and (4.8) and setting them equal to zero:

for $n = 1,2,\ldots,Nr$

$$\lambda_n^t = \begin{cases} \dfrac{2\left[W_n^{fs}\left(s_n^{t-1} - s_n^1\right) + p_s\eta_n^{t-1}\right] + \mu_n^{t-1}}{\left(1+\delta_n^t\right)} & \text{for} \quad t = T \\ \dfrac{\lambda_n^{t-1}\left(1-\delta_n^{t-1}\right) + 2p_s\eta_n^{t-1} + \mu_n^{t-1}}{\left(1+\delta_n^t\right)} & \text{for} \quad t = T-1, T-2,\ldots,1 \end{cases} \tag{5.15}$$

where μ and δ are dummy variables. These variables represent the derivative of the cost function and the evaporation term (in the mass balance equation) with respect to the reservoir storage respectively as:

$$\mu_n^{t+1} = \begin{cases} 2p_c\psi\left(A_n + 2B_n s_n^{t-1}\right) & \text{if} \quad t+1 = ts \\ 0 & \text{otherwise} \end{cases} \tag{5.16}$$

$$\delta_n^t = 0.5 e_n^t C_n^e P_n^e \left(\frac{s_n^t + s_n^{t-1}}{2}\right)^{(p_n^e - 1)} \tag{5.17}$$

In the iterative OCT algorithm as mentioned by Hiew (1987), at low storage penalty weights (p_s), the solution is infeasible and violates many feasibility constraints;

only the final solution with the highest selected penalty weight meets the optimality and feasibility conditions. Therefore, the penalty function on the cost violation is not implementable until the system is feasible with respect to systems physical constraints. Consequently, two iterative penalty weights were implemented in the OCT-I model to consider the storage and cost violations. The iterative storage penalty weights (p_s) were used in the inner loop and the iterative cost penalty weights (p_c) were implemented in the outer loop. In other words for each p_c, a complete OCT algorithm given in section (4.3) was being solved. That is, the OCT-I started with a small p_c and the whole system was designed using an iterative penalty approach on storage constraints. The result of the model at this stage would be a system that is feasible on storage constraints. Such a system however, violates the cost constraint because of applying a low cost penalty weight. In the next iteration, a slightly higher p_c was selected and the whole problem was solved using an iterative penalty method on the storage constraint. The flow chart of the OCT-I model is presented in Appendix B-2.

5.5.2 OCT-II Model:

In the OCT-I model, the state variable term in the cost function does not play a role in all optimization iterations unless the total cost is violated. In other words, the control variable trajectories do not receive any information about the system cost until the cost upper bound ε_c is violated. This violation may impose sudden changes in multiplier values, which ultimately affect the control trajectories. To give the control gradients more information about the cost throughout the whole optimization iterations, the OCT-II model was developed.

The weighting method has been used in this model. That is, weighting coefficients (W_n^s) are used to add the second objective (Eq. 4.2) to the rest of the objectives in Eq. (5.8). The OCT-II uses the same cost functions as the OCT-I model. The square term is used to penalize higher costs. The following is the Lagrangian for the whole problem.

$$L = L_1 + \sum_{n=1}^{Nr} W_n^s \left(f_n^c\right)^2 \tag{5.18}$$

consequently, the related transversality and adjoint conditions for the Lagrangian function (Eq. 5.18) are:

$for\ n = 1,2,\ldots,Nr$

$$\lambda_n^t = \begin{cases} \dfrac{2\left[W_n^{fs}\left(s_n^{t-1} - s_n^1\right) + p_s \eta_n^{t-1}\right] + \mu_n^{t-1}}{\left(1 + \delta_n^t\right)} & for\ t = T \\ \dfrac{\lambda_n^{t+1}\left(1 - \delta_n^{t+1}\right) + 2 p_s \eta_n^{t-1} + \mu_n^{t-1}}{\left(1 + \delta_n^t\right)} & for\ t = T-1, T-2,\ldots,1 \end{cases} \tag{5.19}$$

$$\mu_n^{t-1} = \begin{cases} 2W_n^s f_n^c\left(A_n + 2B_n s_n^{t-1}\right) & if\ \ t+1 = ts \\ 0 & otherwise \end{cases} \tag{5.20}$$

where μ_n^{t+1} is a dummy variable that represents the derivative of the cost function with respect to the reservoir storage. The flow chart of the OCT-II model is shown in the Appendix B-3.

5.5.3 OCT-III Model:

Like the OCT-II, the OCT-III model uses the weighting method to add the cost function to the rest of the objectives. However, a different approach to cost function has been considered. In the OCT-III model, cost is a function of reservoir storage and applies to all operating times *t=2,.., T+1*. This approach is based on the Bellman's *principle of optimality* that states (Loucks et. al. 1981): "*no matter in what state of what stage one may be, in order for a policy to be optimal, one had to get that state and stage in an optimal manner.*" Hence, for the current design test, for each reservoir *n* at any time *t*, the cost functions $f_{n,t}^{c}$ and the Lagrangian function for the whole problem in the OCT-III model are:

$$f_{n,t}^{c} = A_n s_n^t + B_n \left(s_n^t\right)^2 \quad for \quad t = 2,\ldots,T+1 \tag{5.21}$$

$$L = L_1 + \sum_{n=1}^{Nr} \left[W_n^s \sum_{t=2}^{T+1} \left(f_{n,t}^{c}\right)^2 \right] \tag{5.22}$$

consequently, the transversality and adjoint conditions are:

$$for\ n = 1,2,\ldots,Nr \quad ; \quad for\ t = T, T-1,\ldots,1$$

$$\lambda_n^t = \frac{2\left[W_n^s f_{n,t+1}^{c}\left(A_n + 2B_n s_n^{t+1}\right)\right] + p_s \eta_n^{t+1} + \mu_n^t}{\left(1+\delta_n^t\right)} \tag{5.23}$$

$$\mu_n^t = \begin{cases} 2W_n^{fs}\left(s_n^{t-1} - s_n^1\right) & if \quad t = T \\ \lambda_n^{t+1}\left(1 - \delta_n^{t+1}\right) & otherwise \end{cases} \tag{5.24}$$

The optimization procedure in this model is the same as the OCT-II model presented in Appendix B-3.

5.6 PSLP-OCT Composite Algorithm:

A new methodology is introduced in this section which uses a composite optimization strategy. This composite algorithm constitutes the PSLP-OCT technique, which employs OCT and PSLP algorithms as the most promising optimization techniques. The PSLP-OCT is a general-purpose optimization technique that can be used in any *mixed type* optimization problem consisting of both static and dynamic (time dependent) control variables. In the conventional approach to optimize such problems, only static optimization techniques (e.g., NLP or PSLP) have to be used. The major drawback in these methods is their initial solution requirement. In the non-convex problems, this requirement could greatly affect the final solution proposed by these methods. Therefore, the related computer programs should be run several times to achieve the best possible local optimum solution. The introduced PSLP-OCT method however, recognizes the dynamism of the problem and differentiates the dynamic variables from the static ones. Therefore, based on the nature of the decision variables, the mixed type problem can be divided into two parts. The OCT optimizes the dynamic part and the PSLP optimizes the static part of the problem. This approach not only reduces the computer execution time, but it alleviates the necessity of those programs to be run for several times. That feature is due to insensitivity of the OCT method to initial solutions and could be extremely helpful in optimizing large systems with non-convex, mixed type optimization problems. Another benefit of the introduced PSLP-OCT method is due to its capability in optimizing problems with linear (dynamic or static) and dynamic nonlinear variables. The objectives with nonlinear dynamic variables can be assigned to the OCT part of the composite algorithm and the PSLP part optimizes the objectives with (non)linear variables. The application of this new technique in designing the multi-reservoir system is described in the following sections. That is, the problem is divided into two parts and each part is assigned to the corresponding component in the PSLP-OCT method. The explained procedure can be used as a guidance to apply the PSLP-OCT method to other mixed type optimization problems.

5.6.1 PSLP-OCT Model:

A computer model based on the PSLP-OCT method was developed. To adapt the objectives described in chapter four to the composite optimization model (PSLP-OCT), the objectives and system constraints are split into two parts in order to obtain maximum benefit from the capabilities of the two PSLP and OCT methods. The epsilon constraint method is also implemented into the composite model as the most suitable generating method to perform the sensitivity analysis. Based on the literature review in chapter two, dimension/complexity of the problem in the PSLP will significantly affect the computational time and the efficiency of the algorithm. Previous studies in the literature showed that OCT is the most promising technique to optimize the operation of large hydrosystems. Therefore, the design problem in chapter four is split into two parts. A small part of the optimization problem (minimizing the cost) is assigned to the PSLP and the OCT handles the major computational part of the optimization problem addressed in Eq. (5.8).

The essence of the composite model is that the reservoir yield and release over the operation period are dependent directly on the reservoir capacity and streamflow sequence. This dependence may be functionalized and evaluated independently by using OCT with respect to target reservoir capacities in the multi-reservoir system. The introduced composite model consists of outer and inner optimization modules. The PSLP is used as the outer module. The reservoir capacities needed are evaluated in the PSLP module. The PSLP applies the epsilon constraint algorithm to the water supply objective. This module minimizes the total reservoir costs to supply water at a pre-specified demand level. Then, the OCT is used as the inner module. The OCT module uses the reservoir capacities obtained by the PSLP at each iteration and optimizes the corresponding reservoir releases and yields for different purposes. These two most promising optimization methods constitute the structure of the new screening PSLP-OCT model that is presented in this study and is generically illustrated in Appendix B-4. Based on this flow chart, the PSLP module at each iteration specifies a set of reservoir capacities. Then, the OCT module minimizes the water deficit for such a system. The optimized yield variables are transferred to the PSLP module through water supply constraints. Based on these results, the PSLP module selects the next move to propose a new reservoir configuration. The process in the PSLP module terminates when optimal reservoir capacities are found or when a series of successive iterations fails to improve the solution.

5.6.2 PSLP Module Formulation:

The objectives of the PSLP module are to minimize (1) total reservoir costs, and (2) monthly water deficits in all demand areas. The PSLP module optimizes these objectives subject to the reservoir storage constraints:

$$\text{Minimize}\left\{\sum_{n=1}^{Nr}\left(A_n x_n + B_n x_n^2\right), \sum_{j=1}^{Nd}\sum_{t=1}^{T}\left(D_{jMAX}^t - \sum_{n=1}^{Nr} y_{n,j}^t\right)\right\} \tag{5.25}$$

$$x_n^{\min} \le x_n \le x_n^{\max} \quad for \quad n = 1,2,\ldots,N_r \tag{5.26}$$

where D_{jMAX}^t is the maximum predicted water demand during month t at area j. Based on the epsilon constraint algorithm, the PSLP module keeps the cost minimization in the objective function and transfers the water deficit minimization objectives to the equivalent constraints:

$$\text{Minimize} \quad \sum_{n=1}^{Nr}\left(A_n x_n + B_n x_n^2\right) \tag{5.27}$$

subject to Eq. (5.26) and the *water deficit constraints*:

$$for\ j = 1,2,\ldots,N_d \ ; \quad for \ \ t = 1,2,\ldots.,T$$
$$D_{jMAX}^t - \sum_{n=1}^{Nr} y_{n,j}^t \le \varepsilon_j^t \tag{5.28}$$

where ε_j^t is the monthly water deficit at the D#j. The constraint (5.28) can be rearranged as:

$$for\ j = 1,2,...,Nd\ ; \quad for\ \ t = 1,2,.....,T$$

$$\sum_{n=1}^{Nr} y_{n,j}^{t} \geq \left(D_{jMAX}^{t} - \varepsilon_{j}^{t}\right) = D_{j}^{t} \tag{5.29}$$

Eq. (5.29) represents the *water supply constraints* and specifies a set of lower bounds on the reservoir yields to force the PSLP module to meet water demands in different demand areas. The lower bounds are equivalent to ε_k values explained in section (4.8) and are determined by the epsilon constraint algorithm. Reservoir capacities $X=[x_1\ x_2\ .\ x_{Nr}]$ are decision variables in the PSLP module. This module has a nonlinear objective function and a set of water supply constraints in terms of the decision variables of the PSLP module. The reservoir yield ($y^t_{n,j}$) in Eq. (5.29) is a nonlinear implicit function of the reservoir capacity x_n and can be obtained through the OCT module.

5.6.3 OCT Module Formulation:

OCT is used as the inner optimization module in the PSLP-OCT model. The OCT module includes other objectives in the design problem, the mass balance equation, and other system constraints. This strategy reduces the computational burden of the PSLP module and assigns the major part of the optimization problem to the OCT as the most promising optimization technique. Therefore, the OCT module optimizes objectives in Eq. (3.1) subject to constraints (3.3), (3.5), (3.6), and (3.7). Once the reservoir capacities are specified by PSLP at each iteration, the OCT module is applied to determine the optimal reservoir yields corresponding to the candidate reservoir capacities.

5.6.4 PSLP-OCT Model Solution Procedure:

The Penalty Successive LP algorithm of Zhang et. al. (1985) is used to solve the nonlinear optimization problem formulated in section (5.6.1). The procedure to solve nonlinear optimization problems is given in Zhang et. al. (1985). However, the application of their algorithm to the problem described in Eqs. (5.26) to (5.29) is explained here as follows:

1. Define the *exact penalty function* by adding the nonlinear constraints to the objective function using pre-specified penalty weights. The penalty weights in the exact penalty function are positive scalars that have to be in excess of the largest Lagrange multiplier (dual variable) value expected. The exact penalty function for the formulation proposed in the PSLP model would be as:

$$P(X)=\sum_{n=1}^{Nr}\left(A_n x_n + B_n x_n^2\right)+\sum_{j=1}^{Nd} W_j^p \sum_{t=1}^{T} Max\left[0,\left(D_j^t - \sum_{n=1}^{Nr} y_{n,j}^t\right)\right] \quad (5.30)$$

N_d penalty weights (W^p_j), corresponding to N_d demand areas, have been used for the nonlinear constraints to consider the scaling of each water supply constraint. The resulting optimization problem has a nonlinear objective function subject to the linear constraints (Eq. 5.26). The exact penalty function together with the linear constraints constitutes the *linearly constrained penalty* (*LCP*) problem.

2. Define the *Approximating function Pl(X)* by replacing all nonlinear parts in the exact penalty function (Eq. 5.30) by its first order Taylor series approximation about a base point $X_0 = [x_{1,0}, x_{2,0}, \ldots x_{Nr,0}]$:

$$PI(X)=\sum_{n=1}^{Nr}\left[\left(A_n x_{n,0}+B_n x_{n,0}^2\right)+\left(A_n+2B_n x_{n,0}\right)\left(x_n-x_{n,0}\right)\right]+$$
$$\sum_{j=1}^{Nd}\left\{W_j^p\sum_{t=1}^{T}Max\left[0,D_j^t-\sum_{n=1}^{Nr}\left(y_{n,j,0}^t+\left(x_n-x_{n,0}\right)\frac{\partial y_{n,j}^t}{\partial x_n}\right)\right]\right\} \qquad (5.31)$$

PI(X) is a good approximation to *P(X)* if the step size $(x_n - x_{n,0})$ is not large. Thus the *P(X)* can be minimized by a sequence of minimization of *PI(X)* with an upper bound on the step size. This leads to the *Approximating Problem*:

$$\textit{Minimize} \quad pl(X) \qquad (5.32)$$

subject to the linear constraint (5.26) and the new *deviation constraint* as:

$$-\omega \le \left(x_n - x_{n,0}\right) \le \omega \quad \textit{for } n=1,2,..,Nr \qquad (5.33)$$

where ω is the upper bound on the Taylor series step size. The deviation constraint is to maintain a solution in the neighborhood of the current solution.

3. Apply the first order Taylor series expansion to linearize the nonlinear part of water supply constraints (5.29) about initial solutions $(y_{n,j,0}^t)$. According to the continuity equation (3.3), each yield is a function of its reservoir storage and all incoming return flows:

$$y_{n,j}^t = f\left((s_n^t),\sum_{\substack{i=1\\i\neq n}}^{Nr}\sum_{\substack{J=1\\J\neq j}}^{Nd}\sum_{m=1}^{t} y_{i,J}^m\right) \qquad (5.34)$$

Considering the fact that reservoir storage is a function of the reservoir capacity, it can be stated that every yield is a function of all reservoir capacities as:

$$y_{n,j}^{t} = H(x_i) \ \ for\ i = 1,2,\ldots,N_r \qquad (5.35)$$

Therefore, the Taylor series expansion of the constraint (5.29) is as:

$$for\ j = 1,2,..,N_d\ ;\ for\ t = 1,2,..,T$$
$$\sum_{n=1}^{Nr} y_{n,j}^{t} \cong \sum_{n=1}^{Nr}\left(y_{n,j,0}^{t} + \sum_{i=1}^{Nr}(x_i - x_{i,0})\frac{\partial y_{n,j}^{t}}{\partial x_i}\right) \geq D_j^t \qquad (5.36)$$

By applying the chain rule for differentiation, a direct relation between reservoir yield ($y_{n,j}^{t}$) and capacities ($x_i = s_i^{\prime}$, $i=1,2, N_r$) can be established. Ignoring minor changes in evaporation losses in the continuity equation, the $\frac{\partial y_{n,j}^{t}}{\partial x_i}$ can be directly evaluated as:

$$\frac{\partial y_{n,j}^{t}}{\partial x_i} = \begin{cases} Z_{i,n} & if\ y_{n,j}^{t,\max} \geq 0 \\ 0 & othewise \end{cases};\ Z_{i,n} = \begin{cases} 1 & if\ i = n \\ \sum_{\substack{J=1 \\ J \neq j}}^{Nd} \Gamma_{n,J} \sum_{m=1}^{t} \rho_J^{t-m-1} & otherwise \end{cases} \qquad (5.37)$$

Eq. (5.37) implies that if a reservoir cannot supply yield to a demand area (maximum yield is zero), its corresponding yield derivative with respect to x is zero. The percent error in excluding the evaporation term in Eq. (5.37) is usually less than 0.1% (Lall 1995) and therefore negligible.

4. Define a Linear Program equivalent to problem defined in step 2 as:

$$Minimize \qquad \sum_{n=1}^{Nr}\left[(A_n + 2B_n x_{n,0})x_n\right] + \sum_{j=1}^{Nd}\left(W_j^p \sum_{k=(j-1)T+1}^{j^*T} p_k \right) \qquad (5.38)$$

subject to the linear constraints (5-26), (5-33) and a new linearized water supply constraint as:

$$for \quad j = 1,2,\ldots,N_d \; ; \; for \; t = 1,2,\ldots,T$$

$$\sum_{n=1}^{Nr}\sum_{i=1}^{Nr}\left(Z_{i,n}x_i\right) + p_k - n_k = \sum_{n=1}^{Nr}\left(\sum_{i=1}^{Nr} Z_{i,n}x_{i,0} - y^t_{n,j,0}\right) + D^t_j \tag{5.39}$$

$$where \; k = j * t$$

In the above, p_k and n_k are *deviation variables*, which allow us to represent the water supply constraints (5.29) linearly in an LP algorithm.

Applying small penalty weights (W^P_j) may result in an infeasible solution. If, however, large penalty weights are selected, the decision variables X are forced to stay too close to the feasible region for the majority of PSLP iterations. Consequently, it may cause slow convergence in some problems (Baker & Lasdon 1985). Therefore, based on the order of magnitude of the objective function (Eq. 5.27) and related nonlinear constraints (Eq. 5.29), one can select reasonable small penalty weights to start the problem. If the PSLP iterations have terminated with an infeasible solution in the original nonlinear problem i.e., the demand is not satisfied fully, increase the weights and start again.

5.7 Numerical Studies of Design Models:

In this section, case 1 CE-646 test problem is applied to the design models introduced in sections (5.5) and (5.6). The optimal solution to the case 1 CE-646 test problem is used as a benchmark to evaluate the performance of each design model. These models have two main objectives: to minimize the cost and water deficit. Therefore, the success of each model depends on the cost of the designed system and the corresponding level of water supply. The other objectives are considered as minor (secondary) objectives and are considered in the assessment only if all models succeed in meeting the two main objectives equally.

A series of optimized solutions to the case1 CE-646 problem with different water supply levels and corresponding reservoir layouts is given in Supangat (1985). These solutions have been obtained by applying the combination of DP and simulation models. It is believed that providing the highest possible water supply in dry periods (as in the CE-646 case) is the most difficult task for every optimization algorithm. Considering the series of solutions given at different supply levels, the reservoir layout with the highest water supply of 51900 MCM and the total cost of $\$182.8*10^6$ is selected. This solution is called the *benchmark solution* of the case 1 CE-646 problem for the rest of this text and serves as an assessment of the performances of the developed models. To compare the model performance with the benchmark solution, the same assumptions of zero minimum storage and return flows are applied to the system. To minimize the water deficit in the system, proper ε_k/weight coefficients were assigned in the related models. Very small/zero weight coefficients were selected for those minor objectives that were not considered in the benchmark solution. The model with the lowest cost and/or highest water supply would be the most successful model and is selected as the final design model proposed in this study.

5.8 Model Performances:

The case 1 CE-646 problem was used to evaluate the performances of the four design models formulated in sections (5.5) and (5.6). The best cost/storage penalty scheme for the case 1 CE-646 problem was one with a maximum value of 100, initial value of .0001, and a 5 to 10 fold increase after each round of iteration. The ε_k in the OCT-I model was set equal to \$182.8*10^6. Several adjustments, based on the procedure mentioned in section (4.7), were made to find the proper final set of weight coefficients. The best compromise solution obtained by each design model is presented in the subsequent sections.

5.9 OCT-based Model Solutions:

In all the OCT-based models, initial storage is a function of the reservoir capacity and is an initial condition of the algorithm. Zero initial storage was assumed for each candidate reservoir to allow the OCT algorithm to reject/select any candidate reservoir in the system. However, a full initial storage assumption was required to make the results of the OCT-based models comparable to the benchmark solution. Therefore, each model started with zero initial storage assumption. Then, the final layout of each model was used in an additional run to specify the reservoir yields with the full initial reservoir storage assumption. Table (5.3) shows the selected weight coefficients for all objectives in each OCT-based model. These weights are selected such that the models obtain the highest yield with the minimum possible construction costs:

Table (5.3): Weight coefficients in the design models.

Model	Weight Coefficients									
	W_v^t	W_n^{by} $\forall n$	W_n^{br} $\forall n$	W_1^s	W_2^s	W_3^s	W_4^s	W_5^s	W_6^s	W_n^{fs} $\forall n$
OCT-I	1	0	.01	-	-	-	-	-	-	.01
OCT-II	1	0	.01	700	300	600	1	500	1	.01
OCT-III	1	0	.01	350	200	250	10	280	10	.01

The outputs of each OCT-based model, including their proposed monthly reservoir storage and the corresponding total yields are presented in Appendices C-1 to C-3 respectively. A study of these results shows that, not only did all the OCT-based models result in sub-optimal solutions, but also all these models are very sensitive to the selected weight coefficients. This is because the appropriate weight coefficients, as mentioned in section (4.7), depend on the relative magnitude of different objectives. Therefore, selecting the constant weights conflicts with the fact that in design problems the magnitudes of cost functions depend on their reservoir capacities that are changing at each iteration. Regardless of taking enormous weight coefficients in Eqs. (5.8), (5.18), and (5.22), sub-optimal solutions were always obtained and all the candidate reservoirs were selected regardless of their cost expenses. Based on many experiments undertaken in these models, it was found that the mixed effects of the Lagrange multipliers of each reservoir (λ_n^t) and its downstream ones on the release gradients were mainly responsible for the sub-optimal solutions. The following paragraphs elaborate the impact of multipliers on each release gradient and consequently its reservoir capacity.

In optimizing the configuration of a multi-reservoir system, the most expensive candidate reservoirs have to be excluded from the optimal reservoir layout. That means the design model has to assign zero storage to the most expensive reservoirs at all time periods. Considering the inflows to each reservoir, inappropriate reservoir releases at different periods will assign non-zero reservoir storage in the continuity equation (3.3). To survey the reasons for obtaining sub-optimal solutions in all the OCT-based models, the effects of different components in Eq. (5.10) on the release gradients have to be elaborated.

The objective function formulated in chapter four is a typical formulation for water supply problems. Unlike the objective functions in Hydropower optimization problems, the current objective function does not consist of any multiplicative term with respect to control and state variables. Consequently, stationary conditions do not include any direct term with respect to state variables and only Lagrange multipliers at each iteration will transfer state variable conditions to the control gradients.

Considering the weight coefficients on controlling the bang-bang (W_n^{br}), two main situations can be taken into account for the release gradients of each reservoir. If large weights are assigned to smooth the release trajectory, the model will try to have a uniform release over time. This may not accord with fluctuations of yield and natural inflow in each time period. Consequently, all the candidate reservoirs will need storage in some periods to store some part of incoming water to keep the outgoing release as smooth as possible. That means, regardless of their costs, all the candidate reservoirs have to be selected in the final layout proposed by the OCT-based models. Comparing monthly inflows and water demands in the case 1 CE-646 problem, there are water deficits in the beginning months and excess inflows at the final months. Having excess incoming water requires more releases in the corresponding months. However, large W_n^{br} weights will push the model to keep the releases low at the final months, resulting in non-zero storage at those periods.

If very small/zero W_n^{br} weights are assigned to control the bang-bang problem, only Lagrange multipliers (λ) of each reservoir and its subsequent reservoir(s) will mainly affect the release gradients. Based on the Eq. (5.10), the release gradient of each reservoir

n consists of additive terms with respect to λ of the current and the linked downstream reservoir(s). The difficulty occurs if the cost constraint or both storage and cost constraints are binding at the optimum. That is, the minimum/zero water deficit happens when the minimum storage/cost requirements are equal to their lower/upper bounds.

In the subsequent sections, it is shown that regardless of the initial solution selected for the control variables, the OCT-based models end up with a system with cost/storage infeasibilities which result in non-zero Lagrange multipliers (λ). Once it happens, the multipliers affect release gradients, which in turn will keep the non-zero values of multipliers and result in sub-optimal solutions.

5.9.1 Non-Zero Multipliers:

The terminal storage function results in non-zero final multipliers (λ_n^T) in all the OCT-based models at Eqs. (5.15), (5.19), and (5.23). This value will propagate backward in time and gives non-zero values to other multipliers at all periods. The cost functions in the OCT-based models are other sources of non-zero multipliers and their effects depend on whether the weighting or epsilon constraint method is applied to the cost function.

It goes without saying that if the OCT-I model starts from an initial reservoir storage that violates the cost, multipliers have non-zero values from the beginning. Let us assume the system starts with feasible (e.g., small/zero) reservoir storage. Therefore, the cost and storage constraints are not violated. The algorithm in the OCT-I model starts with a small penalty on storage violation. This lets the algorithm meet the water demand at the expense of pushing the storage to stay in the negative infeasible side, which will activate the storage penalty function (p_s) to bring back the storage into the feasible region. Therefore, at the end of the first outer (cost penalty function) loop, none of the storage constraints is violated. This solution will not necessarily assign proper combination of reservoir capacities and hence violates the system's total cost constraint. Consequently, the cost penalty function will assign a positive value to the multipliers.

The behavior of the multipliers in the OCT-II and OCT-III models is different, however. In these models, due to the nature of the weighting method used in the formulation, the system cost term is present in the multiplier formulations. In the OCT-II model, the reservoir capacities at each p_s iteration are selected by choosing the maximum reservoir storage over the whole time period of analysis. This will happens at time t_s (see Eq. 5.20). Therefore, the related multiplier has a positive value which propagates backward in time. In the OCT-III model, the system cost is a function of monthly reservoir storage and consequently, a positive value is added to all multipliers at all periods.

5.9.2 Downstream Multiplier Effect:

The reservoir capacity is a function of its monthly storage, which, in turn, depends on the unregulated inflow, release from the upstream reservoir(s), water demand and supply, and the reservoir cost. Therefore, at each iteration the candidate reservoirs may have different storage and hence different multipliers from each other. When the release of the *nth* reservoir increases within the feasible range of state space at each iteration, the corresponding storage and consequently its λ_n^t, will decrease/cease. This is because a permanent tendency to supply the demand tries to keep the yield as close as possible to the water demand. If the downstream reservoir has a greater/non-zero multiplier (λ^t_{n+1}), the $\lambda_n^t \Lambda_n^k$ term in Eq. (5.10) pushes the release gradient of the upstream reservoir to decrease. The decrease in the release of the *nth* reservoir increases the corresponding reservoir storage. In other words, if a candidate reservoir has a positive multiplier, all its upstream candidate reservoirs, regardless of their costs, are selected in the final solution. This is called the *downstream multiplier problem* in this study and applies to all candidate reservoirs starting from the upstream part of the watershed except for the most downstream reservoir, where the release gradient in that reservoir depends only on its Lagrange multiplier.

The situation for the most downstream reservoir is different, however. This is traced to the fact that its existence depends on its cost and the hydrology of the watershed. In the case 1 CE646 problem, the last reservoir (RES#6) is one of the two most inexpensive reservoirs. Consequently, it will stay as one of the selected reservoirs in the system. The presence of this reservoir pushes all the upstream reservoirs to stay in the proposed layout of the model due to the downstream multiplier problem.

In the next sections, the performances of the OCT-based models are presented. Then, their specific problems, in addition to the downstream multiplier effect, are analyzed.

5.9.3 OCT-I Solution:

The proposed storage trajectories by the OCT-I model represent a sub-optimal solution due to the problems addressed earlier in this chapter. Fig. (5.2) shows the monthly storage as determined by the OCT-I model. The detail of the OCT-I solution to the case 1 CE-646 problem is presented in Appendix C-1.

In spite of the great number of trials and the use of different combinations of weight coefficients in Eq. (5.8), the OCT-I model failed to design the layout at the pre-specified cost of $182.8*10^6$. This was due to the instability of the algorithm caused by the jumps in the Lagrange multipliers i.e., their values changed in different directions. The experimental results showed that the two storage and cost penalty terms ($p_s*\eta$ and $p_c*\psi$) are mainly responsible for that and this is explained in the next paragraphs.

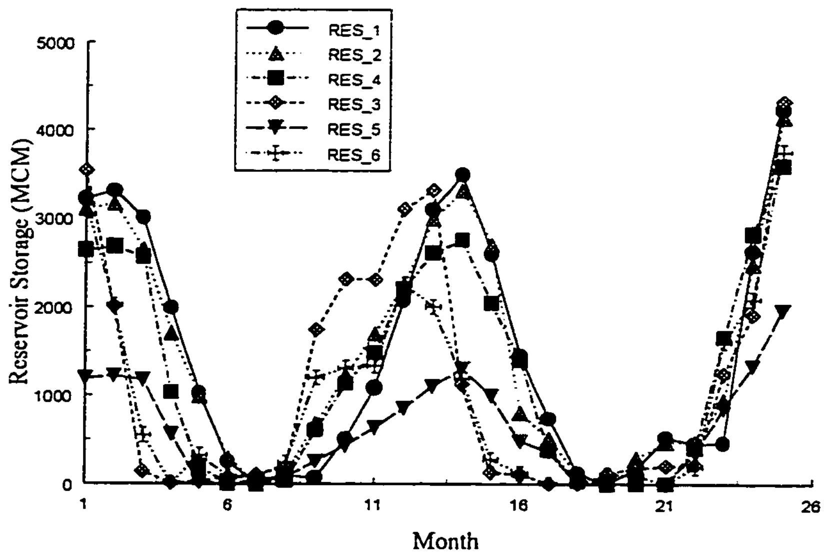


Fig. (5.2): OCT-I solution of storage trajectories for the case 1 CE-646 problem.

With an initial low penalty weight, the OCT algorithm rapidly supplies the full demand at the expense of assigning infeasible negative storage values. Although this solution does not violate cost constraint, it is not an acceptable one. As the storage penalty weight increases in subsequent iterations, the magnitudes of storage violations decrease. These decreases are achieved by increasing the storage and reconfiguring the control variable trajectories in order to supply the water demands.

The iterative cost penalty function implemented in the OCT-I model is supposed to avoid any divergence or instability in the storage trajectories. However, due to the downstream multiplier problem, the designed layout at the end of each inner loop fails to get closer to the optimal solution. Consequently, even with increasing the cost penalty weight in the next outer loops, a large cost violation still exists. Therefore, at each inner loop, the cost function imposes a large positive multiplier and hence, the storage trajectories jump back to negative/small values. As soon as this happens, the cost penalty function is inactivated and the storage penalty function has to bring the storage trajectories back to the feasible region. By building up storage and bringing them into feasible region, the same scenario of cost violation happens and the jumping of the system back and forth continues. At the final OCT-I solution, when the effect of the storage penalty function is dominant over the cost function, the $p_s*\eta$ term forces the storage trajectories to stay in the feasible region and increase the reservoir storage at the expense of violating the cost constraint. Due to the downstream multiplier problem, although this system violates the cost constraint, the total reservoir capacities are not large enough to fully supply the demand.

To reduce the instability of the OCT-I model, the penalty function approach was replaced by the barrier terms. The results showed that, despite the reduction in the model instability, the OCT-I failed to reach closer to the optimum solution. This is obviously due to the permanent presence of non-zero multipliers in Eq. (5.15) and consequently, the downstream multiplier problem. The barrier function applied to Eq. (5.12) could not keep the cost upper bound inside the feasible region either. This is because, as the upper cost boundary is approached, and since search techniques use discrete steps, a step leading outside the region may indicate a false success by showing a decrease in the value of the Lagrangian function L. An explicit check of the cost constraint value in Eq. (5.12) can prevent such a false success. However, this approach does not eliminate the downstream multiplier problem and hence was not applied.

5.9.4 OCT-II Solution:

The OCT-II model was applied to the case 1 CE-646 problem. The result is presented in detail in Appendix C-2. The proposed monthly storage of each reservoir is also shown in Fig. (5.3).

The experimental investigation showed that, like the OCT-I model, the OCT-II model suffers from the downstream multiplier problem and the multipliers jump. However, the instability problem observed in the OCT-II model was slightly less than in the OCT-I model. This may be due to the permanent effect of the cost function on control gradients at periods less than t_s that is felt through multipliers by the algorithm.

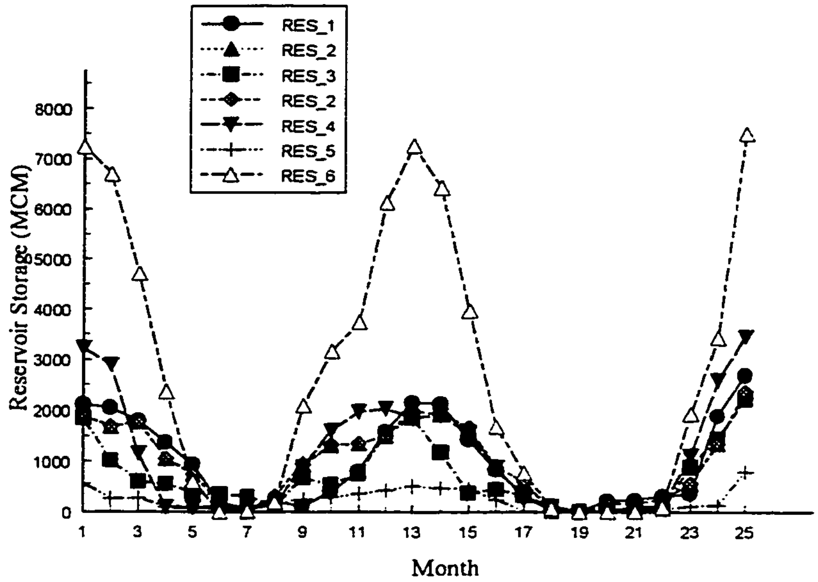


Fig. (5.3): OCT-II solution of storage trajectories for the case 1 CE-646 problem.

When p_s is small in the beginning, the effect of the system cost through multipliers will increase the control variables (r and y), resulting in negative storage values in the next iteration. These infeasibilities stay until p_s is large enough, compared to the cost weight coefficient, to bring back the storage into a feasible region. Unlike the OCT-I model, control gradients receive the effect of the system cost through multipliers even inside the feasible region. In other words, the system cost adds a positive value to the multipliers at time t_s which propagates backward in time. This positive value reduces the effect of the negative storage penalty function on multipliers. Consequently, control variables will take smaller steps to bring back the storage into the storage feasible region. This results in smoother changes in state and control trajectories and hence, the OCT-II is less unstable than the OCT-I model.

5.9.5 OCT-III Solution:

Like two previous OCT-based models, the downstream multiplier problem resulted in a sub-optimal solution. Appendix C-3 presents the detail of the proposed solution and Fig. (5.4) shows the monthly storage as determined by the OCT-III model.

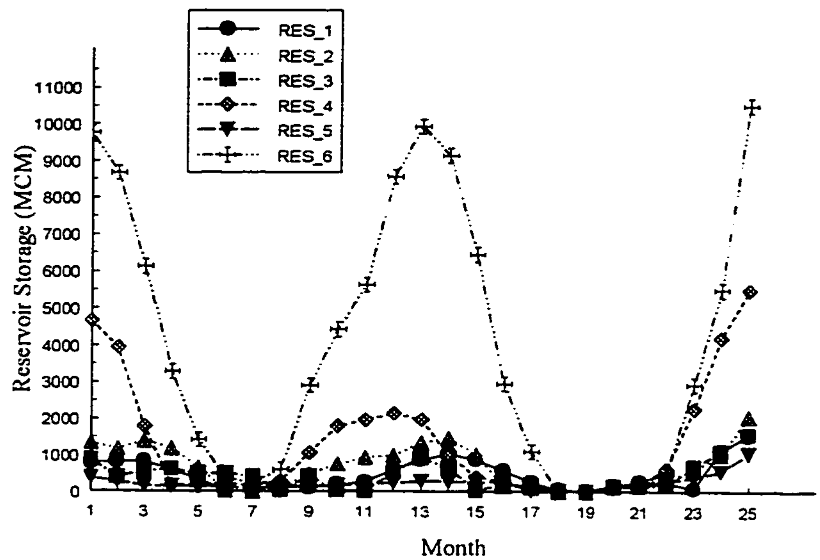


Fig. (5.4): OCT-III solution of storage trajectories for the case 1 CE-646 problem.

The formulation and the behavior of the OCT-III model are quite similar to the OCT-II model, except the cost function affects the multipliers at entire optimization periods. This might be the reason for its better performance compared to the OCT-II model. Compared to other OCT-based models, the OCT-III model was able to supply a higher demand at a lower cost.

5.10 PSLP-OCT Solution:

The PSLP-OCT model was applied to the CE-646 problem. To meet the water demand fully, zero deficits (ε_j^t) were considered in the water supply constraints. In spite of applying different combinations of weight coefficients to the objective components in Eq. (5.8), the PSLP-OCT model, though close to the benchmark solution, was never able to fully supply the monthly water demands.

Several model experiments showed that two factors are mainly important in selecting the proper yield weight coefficients in the OCT module. These factors are: (1) the relative magnitudes of candidate reservoir capacities with respect to each other and (2) the ratio of the system's hydrology (flow variability) to the demand levels. To survey the effect of reservoir sizes and their inflow on the OCT module performance, a closer look into the OCT algorithm is necessary. As was mentioned earlier in section (5.9.3), with an initial low penalty weight, the OCT algorithm supplies the demand fully at the expense of violating the storage constraints. As the storage penalty weights increase in subsequent iterations, the magnitudes of these feasibility violations decrease. At the final solution with the largest penalty weight implemented, the multi-reservoir is supposed to supply the demand without any storage constraint infeasibilities. In the problem formulation (Eq. 4.1), all the reservoirs are contributing to supply a certain demand and hence share the same yield gradients. The yield gradients at each OCT iteration depend on the total water supplied at the previous iteration.

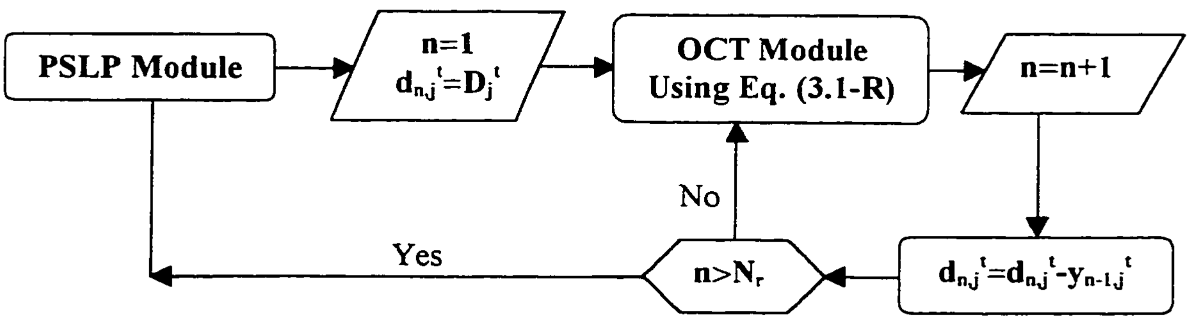
A multi-reservoir system like the case 1 CE-646 problem can be characterized as a system with high water demand levels and large differences in the selected reservoir capacities. In this system, small candidate reservoirs are incorporated with low inflows and large reservoirs with large ones. High demand levels impose large yield gradients in the OCT module. As mentioned in the previous paragraph, large yield gradients in small reservoirs push the algorithm to supply yield at the expense of infeasible storage trajectories (i.e., negative storage) that stay in the infeasible region for most of the penalty iterations of the OCT algorithm. Therefore, even with moderate storage penalty weights (p_s), the small reservoir yields are more than what they can supply in reality and other (large) reservoirs in the system supply the remaining unsatisfied demand. When the p_s are large enough, the yields of small reservoirs decrease and hence their storage goes back to the feasible region. This may change the flow pattern (upstream reservoir releases) in the system and increase the unsatisfied demand (yield gradient) which has to be supplied by other reservoirs in the system. The new situation for the gradient search techniques is like a new problem starting with a high penalty weight. This situation, as was mentioned in the literature review, leads to solution divergence and makes gradient search techniques slow and unstable.

To remove the effect of a small reservoir on large reservoir yield trajectories, a slightly different approach is required. In this approach, the OCT module can optimize the water supply problem using one reservoir at a time. To accelerate the OCT convergence, different yield weight coefficients, based on the ratio of flow availability to demand level, can be assigned to candidate reservoirs. Using smaller yield weight coefficients for the small reservoirs with low inflow reduces their yield gradients and hence increases the OCT convergence. In this approach, the OCT module optimizes the first objective (Eq. 4.1) by using one reservoir at a time.

To do this, reservoirs are numbered sequentially from upstream to downstream. Once the PSLP module determines the reservoir capacities at each outer (PSLP) iteration, the OCT module starts from the most upstream reservoir and considers one reservoir at a time to optimize its yield trajectories in order to minimize the water deficit. Then, the remaining water deficit is calculated and the OCT module uses the next downstream reservoir to supply the water deficit. To accelerate the OCT convergence, the original yield weight coefficients (W_j^y) can be changed into new coefficients ($W^y_{n,j}$) that can be different for each candidate reservoir *n*. Based on the new approach, for each reservoir *n*, the Eq. (3.1) is rewritten as:

$$Min \quad f_1 = \sum_{j=1}^{Nd} W_{n,j}^{y}\left[\sum_{t=1}^{T}\left(y_{n,j}^{t} - d_{n,j}^{t}\right)^2\right] + W_n^{by}\left[\sum_{t=1}^{T-1}\sum_{j=1}^{Nd}\left(y_{n,j}^{t-1} - y_{n,j}^{t}\right)^2\right] +$$
$$W_n^{br}\sum_{t=1}^{T-1}\left(r_n^{t+1} - r_n^{t}\right)^2 + W_n^{fs}\left(s_n^{T-1} - s_n^{1}\right)^2 \qquad (3.1\text{-R})$$

Equation (3.1-R) is the revised version of the original water supply objectives. In this equation, $d_{n,j}^t$ is the water demand assigned to the *nth* reservoir and is determined in the sequential water supplying procedure in the OCT module as:



The new approach was adapted into the PSLP-OCT model and was applied to the case 1 CE-646 problem. The model was able to optimize the system successfully at

$\$160.83*10^6$. The penalty weight W_l^p of 80 was large enough to meet the monthly demand levels in Eq. (5.39) and there was no difficulty in assigning proper weights in the OCT module. Table (5.4) shows the assigned weight coefficients for only the selected reservoirs in the OCT module. The weight coefficients related to non-selected reservoirs are not reported, because they were just mass balance nodes (inflow=outflow) to the OCT module and could not affect the optimal result.

Table (5.4): Selected weight coefficients in the OCT module.

Selected Reservoirs	Weight Coefficients			
	$W_{n,1}^y$	W_n^{by}	W_n^{br}	W_n^{fs}
RES#4	1.0e-7	0.0	1.0e-3	1.0e-5
RES#6	1.0	0.0	1.0e-4	1.0e-2

The PSLP-OCT model was able to supply water demand successfully at a proposed construction cost lower than the benchmark solution. The monthly reservoir storage of the selected reservoirs and the corresponding total water supply are shown in Figs. (5.5) and (5.6). The detail of the PSLP-OCT model computations regarding the monthly yield and release of all reservoirs is shown in Appendix C-4. Zero values for monthly yields and releases, and reservoir capacities are assumed as the initial solution for the PSLP and OCT modules. Using these initial solutions, the PSLP-OCT model required 68 seconds of execution time on a Pentium-Pro 180 Personal Computer to solve the case 1 CE-646 problem. It goes without saying that a better initial solution can further reduce the execution time.

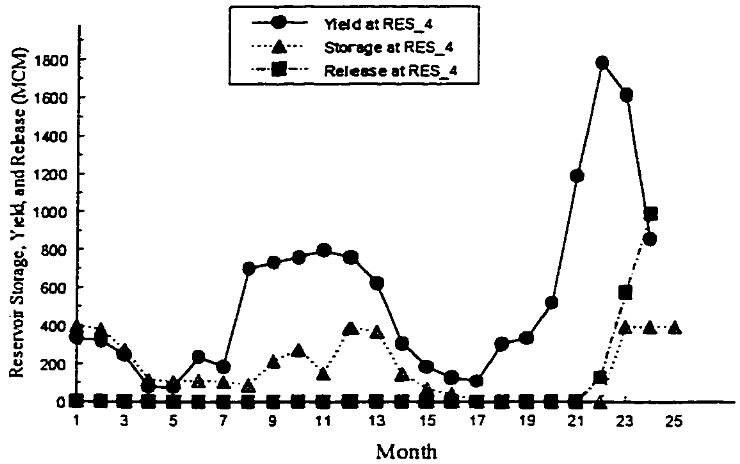


Fig. (5.5): PSLP-OCT solution of optimum state and control trajectories of RES#4 for the case 1 CE-646 problem

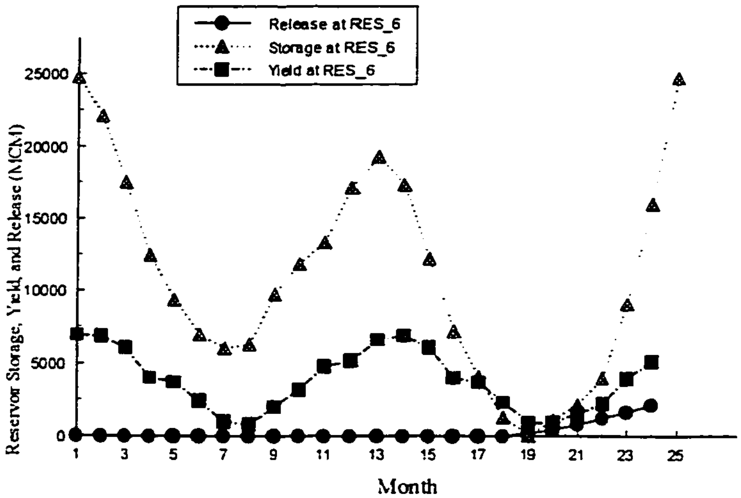


Fig. (5.6): PSLP-OCT solution of optimum state and control trajectories of RES#6 for the case 1 CE-646 problem

It took only a few (3-4) iterations to construct a good approximation to the optimal solution. However, the gradient algorithm in PSLP converged slowly near the optimum. Consequently, a relatively large number of iterations (16 iterations in this example) were required to find the true optimum, which resulted in only a small improvement in the objective function value. At the optimal result, the terminal storage objective ($S_n^{T+1}=S_n^1$) though very close, was not fully met. Increasing the terminal storage weight coefficient could be a remedy. However, this will affect the optimum yield trajectories as is discussed in the weight coefficient sensitivity analysis. A simpler remedy is to adjust corresponding reservoir releases manually. These adjustments are unlikely to affect the optimality of the final results since they are relatively small in magnitude.

5.11 Comparison of Results:

The layouts designed and the total water supplied by the models together with the benchmark solution are shown in Table (5.5). These results show that all three OCT-based models give sub-optimal solutions and the PSLP-OCT model is the most successful model.

To compare the relative performance and achievement of each design model in minimizing the water deficit at the minimum cost, a *degree of optimality* (D_o) is introduced. The D_o is indicated by the ratio of the yield unit cost (Y^{uc}) of each design model to the benchmark solution:

$$(D_o)_m = \frac{(Y^{uc})_m}{(Y^{uc})_b} \quad ; \quad Y^{uc} = \left(\frac{\sum_{j=1}^{Nd}\sum_{t=1}^{T}\sum_{n=1}^{Nr} y_{n,j}^t}{\sum_{n=1}^{Nr} R_n^c} \right) \tag{5-40}$$

where the subscripts *m* and *b* denote the design model and benchmark solutions and R_n^c is the construction cost of the *nth* reservoir. Table (5.6) shows the degree of optimality of each design model with respect to the benchmark solution obtained by the combination of DP and simulation models.

Table (5.5): Comparison of design model performances.

Model	Required Storage Size in Reservoir No. (MCM)*10^-2 1	2	3	4	5	6	Design Cost $*10^-6	Annual Yield (MCM)*10^-2
OCT-I	32.14	31.00	35.35	26.38	11.92	31.51	254.7	474.4
OCT-II	21.11	18.84	18.43	32.20	5.648	72.28	201.4	477.2
OCT-III	7.685	13.21	8.659	46.51	3.646	97.66	186.6	482.1
PSLP-OCT	-	-	-	3.954	-	247.5	160.8	519.0
Bench Mark	-	-	-	4.000	-	288.0	182.8	519.0

Table (5.6): Degree of optimality of the different design models.

Degree of Optimality	Design Model OCT-I	OCT-II	OCT-III	PSLP-OCT
$(D_o)_m$	0.656	0.834	0.910	1.137

The degree of optimality of the design models shows that the proposed PSLP-OCT is the most successful model, able to design the multi-reservoir system at a lower cost than the benchmark solution. All the objectives in the OCT module are formulated in a way to have a quadratic form. This assists the gradient search techniques to achieve the minimum for the related inner problem of water supply. The objective function of the PSLP module in this problem is convex and its nonlinear constraint is concave. Therefore, it can be ascertained that the global optimum was achieved in this case study.

Compared to the benchmark solution, the proposed PSLP-OCT model requires smaller reservoir capacities to supply water demand. There is a small difference between the proposed capacity of RES#4 in the two models that obviously is because of the discretization procedure used in the DP model. However, the difference in proposed capacity for RES#6 is quite significant. In the original reference (Supangat 1985), there is no discussion on the result obtained and no details are given on the monthly reservoir storage and releases. In his result discussion, Supangat (1985) focuses only on the sensitivity of reservoir capacities to different demand levels.

According to Supangat (1985), the simplifications made in his DP model might have resulted in a system, which may be under-designed or over-designed. The simplifications were due to the decomposition technique and variable discretizations in the DP model. According to Supangat (1985), the storage discretization was equal to 1% of the mean annual critical flow to each reservoir (i.e., 200 to 800 MCM) and yield discretization is determined such that the maximum number of yield levels and their associated storage values is 30 (Supangat 1985, p-121). The degree of optimality of the PSLP model shows that the DP model developed by Supangat over-designs the system by a bit less than 14 percent.

5.12 Sensitivity of the Weight Coefficients:

As was mentioned earlier in section (4.7), the appropriate selections of the weight coefficients depend on the order of magnitude and the importance of all objectives. In this section, the sensitivity of four weight coefficients in the OCT module is analyzed. Then the effect of different penalty weights in the PSLP module is evaluated. Among the four weight coefficients ($W_{n,j}^{y}$, W_n^{br}, W_n^{by} and W_n^{fs}) the yield coefficient is the most important one. One strategy to select the proper weight could be to choose yield weight coefficients for each reservoir and then find the appropriate weights for other objectives. Based on this strategy and considering the order of magnitude of the hydrologic data, yield weight coefficient equal to one and zero weights for secondary objectives were selected in the beginning and the performance of the OCT module was examined. Then based on the obtained result, appropriate weights for all objectives were determined. For the case 1 in CE-646 problem, 4-5 adjustments were made for each weight coefficient to find the most appropriate coefficients.

As was mentioned in section (5.10), larger terminal weight coefficients (W_n^{fs}) were not used to meet the terminal storage conditions. This is due to the fact that larger weights (W_n^{fs}) increase the effect of the Lagrange multiplier on the yield gradient and may eventually reduce the reservoir yield. Small W_n^{fs} on the other hand, increase the deviation of final storage from its target storage. The same situation applies to the bang-bang weight coefficients. Increasing W_n^{br} and W_n^{by} will smooth the control trajectories over time. However, they generally lower the total water supply and hence lead to a higher system cost. Therefore, if in designing a multi-reservoir system, having smooth control trajectories is not as important as the system cost, the related weights should be kept as small as possible. While bang-bang control on yield W_n^{by} will definitely reduce the total system yield all the time, assigning reasonable bang-bang weights to the release may help the system to supply water more efficiently. However, large W_n^{br} causes the bang-bang term dominate the multiplier term in the release gradient. This may result in inappropriate (too large/small) releases and consequently, pushes the storage to stay in the infeasible region.

The penalty weights in the PSLP module have to be at least greater than the related decision variables in the dual problem. The PSLP-OCT model performance showed that the model is insensitive to the penalty weight W_j^p as long as its value is greater than the decision variables in the dual problem. However, large W_j^p may keep the deviation variables in the basis for the majority of PSLP iterations which usually slow convergence. Therefore, the best strategy to find the proper W_j^p, as Zhang et. al. (1985) mentioned, is to start the problem with a small W_j^p. If the current solution is infeasible in the PSLP module (monthly demands are not satisfied), increase the weight and start again.

CHAPTER 6

MODEL EXTENSION TO MULTI-DEMAND AREA PROBLEMS

In chapter 5, the performances of four screening models in designing the case 1 CE-646 multi-reservoir system were compared to the benchmark solution obtained from an available DP model. It was shown that the proposed PSLP-OCT model offers the best compromise solution and is a promising optimization model to design multi-reservoir systems regardless of their sizes. In this chapter, a more complex and realistic design problem with a multi-demand area distributed over the whole watershed is considered. Then, the capability of the PSLP-OCT model in designing multi-reservoir systems to supply water for those demand areas is investigated and the necessary model adaptation is introduced. By implementing this adaptation, the finalized form of the proposed model in this study is established. This model can be used to design any multi-reservoir system with any number and types of demand area.

6.1 Case 2 Study with Multi-Demand Areas:

Case 2 study in this research is based on the CE-646 problem, which is generalized to a situation with demand areas distributed over the whole watershed. The new scenario shown in Fig. (6.1) illustrates the different demand areas of different types. Each demand area can be supplied by its upstream candidate reservoir(s). This case study is based on Supangat (1985) and annual demand levels of 2500, 5000, 5000, 2500, and 36900 MCM are required by demand areas #1 to #5. Due to inherent limitations of the DP model in Supangat (1985), all the demand areas in the case 2 CE-646 problem share the same seasonal distributions of water demands. This feature, however, does not affect the PSLP-OCT model performance since it is indifferent to demand distributions and is capable of handling any seasonal arrangement.

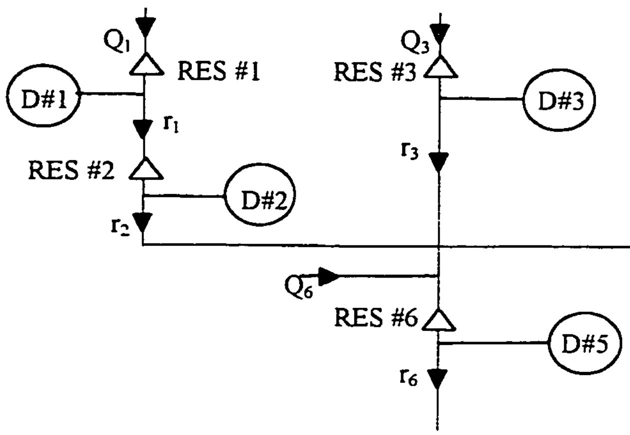


Fig. (6.1): The hydrosystem layout of CE-646 multi-demand area. All symbols are defined in Fig (3.1).

The PSLP-OCT model was applied to the case 2 CE-646 problem to design the multi-reservoir system considering all the demand areas at the same time. The PSLP-OCT solution showed a considerable sensitivity to the selected weight coefficients, which makes it inconvenient for any model user. The case 2 CE-646 problem can be characterized as a hydrosystem with high differences between demand levels in different areas. It goes without saying that large demand levels (such as D#5) result in larger yield gradients. This causes a problem in the model performance and is named here as *domination of large gradients* over small ones. During several model experiments, it was found that the OCT module gives priority to satisfy large demand levels by increasing the corresponding reservoir yields. Consequently, in spite of assigning reservoir capacities in the PSLP module to meet water requirement for a certain demand area, the OCT module may use them to satisfy large demand levels at other areas. This trend in the OCT continues until yield gradients of the large and small demand areas fall into the same order of magnitude. At that iteration, the OCT module can use the assigned capacities to provide yield for the same demand area. This causes the PSLP module to assign extra capacities for some reservoirs. Consequently, up to that iteration, a more expensive system is designed, if those reservoirs happen to be the expensive ones in the hydrosystem. Once all demands are satisfied in an expensive selected layout, the PSLP module has to reconfigure the whole system by reducing the size of those expensive reservoirs that supply water for the large demand areas and equivalently increase the size of other inexpensive reservoirs. Therefore, the domination of large gradients increases the computational burden of both the OCT and the PSLP modules and increases computer execution time. Moreover, this calls for an efficient, large LP solver which, as Hiew (1987) and Grygier (1983) pointed out, is crucial to the success and satisfactory performance of PSLP.

For instance, in the case 2 problem, the first reservoir is among the most expensive candidate reservoirs and it should not be selected unless it is the only reservoir capable of supplying water to a certain demand area. In the current case study, the first demand area can only be supplied by RES#1. Therefore, the model is expected to increase the capacity of the RES#1 only to that level to supply water for the D#1. To design the whole system at the same time, the PSLP-OCT model can be used directly without requiring any modification to the original code. In this case, the PSLP module increases the capacity of the first reservoir to satisfy the D#1. However, the OCT module will use this capacity to provide water mainly for higher demand levels at areas #2 and #5. Consequently, in the next PSLP iteration, the module increases the capacity of the RES #1 again to meet the water requirement of D#1. This causes the RES#1 to have a capacity larger than it should. The same scenario happens to other reservoirs upstream of D#5 and consequently, the system will be designed at a higher cost and hence the result will be sub-optimal at this point. Therefore, the PSLP module tries to reconfigure the system. That is, it reduces the size of RES#1 to a value to supply water for the first demand area and increases the size(s) of some other inexpensive reservoirs such as RES#4 or RES#6 to provide the demands in the D#5.

Step size reduction is another potential problem that is created by the domination of large gradients. As was explained in section (3.5), a certain relative improvement (25%) in the PSLP objective values is required to keep the same step size in the next PSLP iterations. Supplying water for small demand levels compared to the effects of supplying large demand levels at other areas results in a small improvement in the objective value. Consequently, the step size in the PSLP module is reduced by half in the next iteration and eventually, the step size tends quickly to zero and terminates the PSLP iteration. In other words, the step size at each PSLP iteration depends mainly on the improvement of that part of the objective function that supplies the large demand levels. To cope with the rapid reduction of step sizes, a larger initial step size is required in order to provide the necessary number of iterations to reach the optimal solution. Hence, a higher computer execution time is required to design the entire system.

One strategy to cope with these problems is to use lower weight coefficients $W_{n,j}^{y}$ for large demand levels. In this case, the yield gradients in Eq. (3.1-R) would have the same order of magnitude. During the experiments, it was found that this strategy, though effective, calls for careful $W_{n,j}^{y}$ selections. Selection has to be based on the combining effects of the system hydrology and the scaling of the demands, and consequently, may not be an easy task in large, complex hydrosystems.

To cope with the problems of rapid step size reduction, domination of large gradients and sensitivity of the weight coefficients to the yield gradients, an alternative approach was implemented in the PSLP-OCT model. In this approach, the design model considers one demand area at a time and tries to design the multi-reservoir system for that single demand area. To do this, demand areas are numbered sequentially from upstream to downstream. This allows the algorithm to proceed sequentially from upstream to downstream demand areas, specifying storage strategies to supply each demand area in the hydrosystem. The model formulated thus leads to a sequential reservoir screening algorithm with tradeoffs between the OCT and PSLP modules. The PSLP algorithm then begins with the first demand area and proceeds downstream. For each demand area, the OCT module considers one reservoir at a time and optimizes the corresponding yield. This approach will require running the OCT module as many times as the number of demand areas multiplied by the number of reservoirs (N_d*N_r). That is, for case 2 CE-646 problem, first the OCT module is used for a small system of one reservoir (RES#1) with one demand area (D#1) and the model optimizes the capacity of the RES#1 to supply D#1. Then, in the next step, a system with two reservoirs and one demand area (RES#1, RES#2, and D#2) will be considered. These two reservoirs are the candidate reservoirs that potentially can supply water to D#2. Once the PSLP module specifies the related reservoir capacities (x_1 and x_2), the OCT module optimizes the yield of RES#1 ($y_{1,1}^t$, t=1,..,T). Then, the yield trajectories of RES#2 are optimized in a way to supply the water deficit that could not be satisfied by RES#1. In other words, at each step, the PSLP module considers one demand area with all the related candidate reservoirs. Then, the OCT module considers one reservoir at a time to optimize the corresponding yield. At first glance, this approach may seem to increase computer time due to its sequential procedure. However, by the sequential policy and considering one demand area at a time a great simplification in the PSLP formulation can be done that dramatically reduces the number of yield constraints.

The computer execution time in the PSLP-OCT model depends on several factors such as the problem size, the shape of objective functions, and the initial solution in each module. In the next paragraphs, it is shown that the sequential procedure reduces the problem dimensions in both the OCT and the PSLP modules and eventually, lowers the computer time. Solving the problem for one reservoir and one demand area at a time needs N_r*N_d times running the OCT module sequentially. This is a N_d increase in number of times that the OCT module is called. However, the problem size of the OCT module in the sequential strategy is much smaller than before. According to literature, for a problem with N variables and T time stages, the rate of change in the computer time in the OCT varies as $(N*T)^p$, where $1<p<2$. By applying the sequential strategy, the change in computer execution time for the OCT module varies from $N_r*\{T*(N_d+1)\}^p$ in the original model to $N_r*N_d*(2*T)^p$. Several experiments with a different number of variables were performed on the OCT module. It was found that the power p for the water supply problem of the OCT module is equal to 1.23. This result shows a close agreement with the value ($p\approx1.2$) that Hiew (1987) has recorded. That is, for the case 2 CE-646 problem with 24 months hydrologic data and 5 demand areas, the sequential policy increases the total OCT execution time by 1.3.

The problem size in the PSLP module depends on the number of variables and the problem constraints. For a problem with N variables and T time stages, the changes in execution time in the PSLP modules are proportional to $N_r*(N_d*T+2N_r)$. Unlike the case for the OCT module, the sequential approach reduces the PSLP execution time. This is achieved by decreasing the number of time periods of yield constraints T fold and is discussed in the next paragraph. Consequently, for case 2 CE-646 problem, the sequential policy decreases the PSLP execution time by 7.8.

It should be noted that in most practical cases, the hydrologic data length (T) is much larger than the number of candidate reservoirs or demand areas. Considering the slow rate of increase in the OCT execution time and rapid rate of decrease in the PSLP execution time, one can expect an overall reduction in the execution time of the sequential PSLP-OCT model. It should be added that due to the sequential policy, a smaller initial step size would be sufficient which can further lower computer time. The approximated water supply constraint (5.36) can be rearranged as:

$$for\ j=1,2,..,N_d\ ;\ for\ t=1,2,..,T$$

$$\sum_{n=1}^{N_r}\sum_{i=1}^{Nr}\left(Z_{i,n}x_i\right)\geq D_j^t-\sum_{n=1}^{N_r}\left(y_{n,j,0}^t-\sum_{i=1}^{Nr}(Z_{i,n}x_{i,0})\right) \tag{6.1}$$

Due to the sequential policy of the PSLP-OCT model, the return flows from the upstream demand areas are treated as constants and consequently, cease during yield differentiation. Therefore, the Eq. (5.37) reduces to:

$$\frac{\partial y_{n,j}^t}{\partial x_i}=Z_{i,n}=\begin{cases}1 & if\ y_{n,j}^{t,\max}\geq 0\ ;\ if\ i=n\\ 0 & othewise\end{cases} \tag{6.2}$$

Eq. (6.2) shows that the right hand side of Eq. (6.1) is a constant and its value depends on the reservoir yields obtained at the previous iteration ($y_{n,j,o}^t$). The left-hand side of Eq. (6.1) is the summation of the capacities of reservoirs that are physically capable of supplying water for demand area j. Therefore, Eq. (6.1) states that for every demand area, the summation of reservoir capacities should at least be equal to the largest right hand side value over the design time period. This concept can be used to reduce the number of constraints in the PSLP module and consequently alleviate the optimization problem dimension dramatically. That is, for every demand area *j*, there will be only one constraint to reflect the water supply constraint as:

$$for\ j=1,2,..,N_d$$

$$\sum_{n=1}^{N_r}\sum_{i=1}^{Nr}\left(Z_{i,n}x_i\right)\geq Max_t\left[D_j^t-\sum_{n=1}^{N_r}\left(y_{n,j,0}^t-\sum_{i=1}^{Nr}(Z_{i,n}x_{i,0})\right)\right] \tag{6.3}$$

Therefore, the *equivalent Linear Problem* in the sequential model has N_r decision variables and N_d+2N_r constraints and consequently, is independent of the design period length. This feature enables the PSLP module to use longer periods of hydrologic data to design multi-reservoir systems without affecting its dimension.

The objective function (4.1-R) and system constraints (Eq. 4.3 to 4.7) in the OCT module are similar to those specified for single demand area formulation in section (5.10). The algorithm, however, is slightly different, in that the PSLP module needs to account for the physical networking of both the reservoir system and the demand areas. Therefore, the candidate reservoirs and demand areas should be numbered sequentially from upstream to downstream. The PSLP algorithm then starts with the first demand area and proceeds downstream. For each demand area, the PSLP module may assign/increase the capacity of some reservoirs. These reservoirs are contributing to supply water to that demand area. Therefore, the OCT module considers only the contributing reservoir(s) and optimizes their yield. If a reservoir is not contributing to a demand area, the OCT module assigns zero values to the corresponding yield variables. This feature not only reduces the computational burden of the OCT, but also limits the number of yield weight coefficients that have to be selected. The new approach was implemented into the PSLP-OCT model. To evaluate its performance and speed, the PSLP-OCT model was applied to the case 1 CE-646 problem. The result showed that the new approach achieved the same result while the execution time reduced from 68 to 46 seconds on the Pentium Pro 180 personal computer.

6.2 Solution to the Multi-Demand Area Case Study:

The PSLP-OCT model was applied to the case 2 CE-646 problem. Unlike the case 1 problem, all the decision variables (reservoir capacities) do not appear in the yield constraint of the PSLP module i.e., for each demand area, only those reservoirs capable of supplying water are considered. Following the assumptions made in Supangat (1985), no return flows from demand areas into the stream system were considered. The optimal configuration designed by the PSLP-OCT model and the benchmark solution obtained by the DP model are shown in table (6.1).

Table (6.1): Model performances comparison between the DP and PSLP-OCT models.

Model	Required Storage Size in Reservoir No. (MCM)*10^{-2}						Design Cost \$*10^{-6}	Annual Yield (MCM)*10^{-2}
	1	2	3	4	5	6		
PSLP-OCT	0.7285	19.65	2.534	6.156	-	225.7	187.6	519.0
Bench Mark	3.160	19.86	5.970	8.780	-	255.5	218.2	519.0

The monthly storage of the selected candidate reservoirs and their corresponding yield and release trajectories for each demand area are plotted in Figs. (6.2) to (6.6). As expected, the required construction costs are higher than in the previous case due to the additional yield constraints.

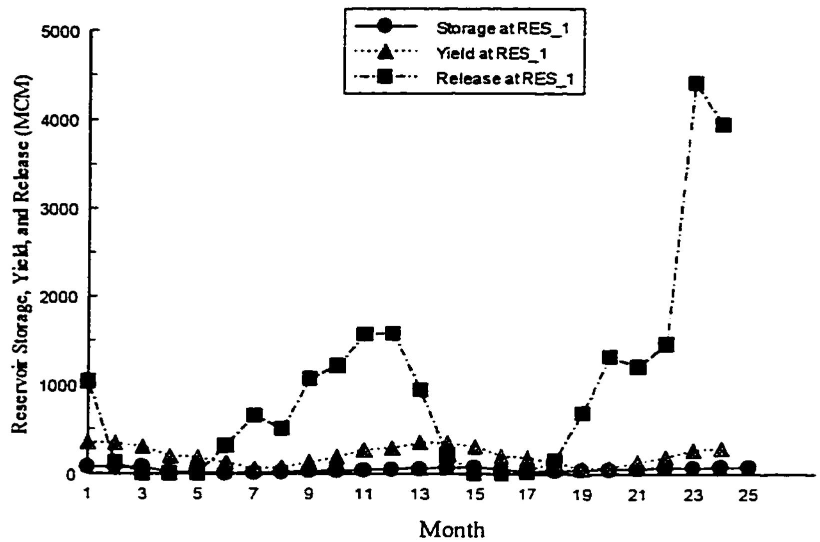


Fig. (6.2): Optimum state and control trajectories of RES#1 by the PSLP-OCT model for case 2 CE-646 problem.

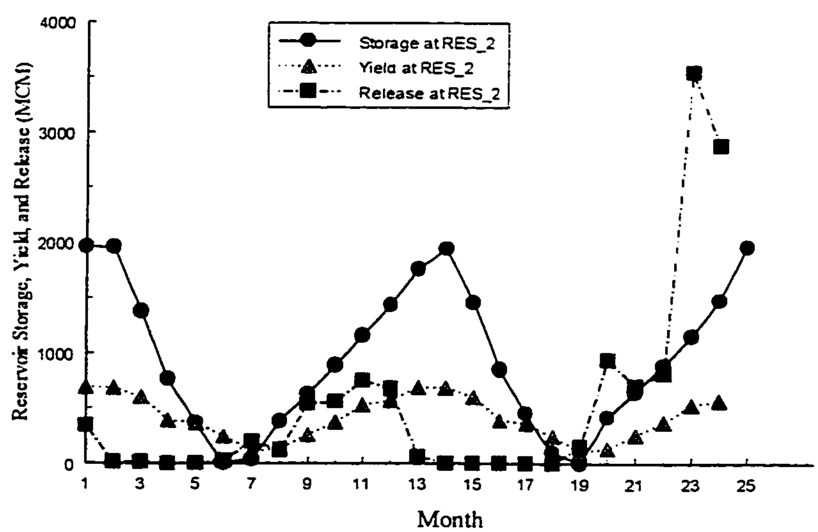


Fig. (6.3): Optimum state and control trajectories of RES#2 by the PSLP-OCT model for case 2 CE-646 problem.

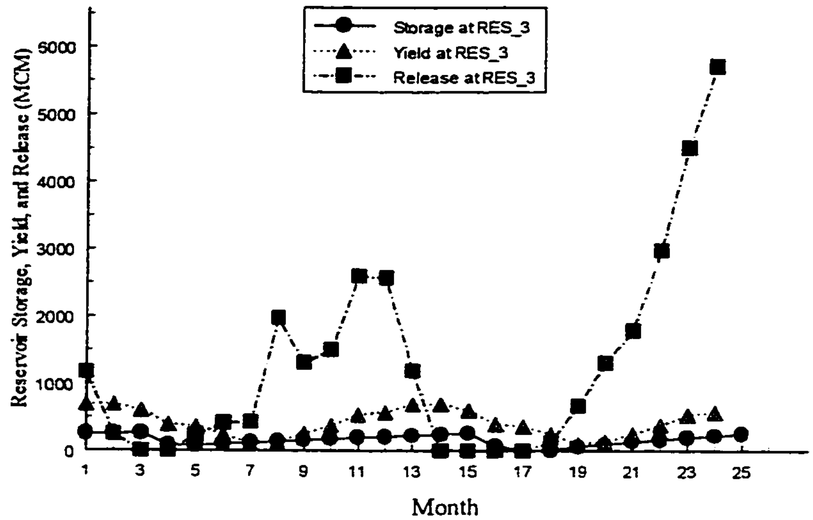


Fig. (6.4): Optimum state and control trajectories of RES#3 by the PSLP-OCT model for case 2 CE-646 problem.

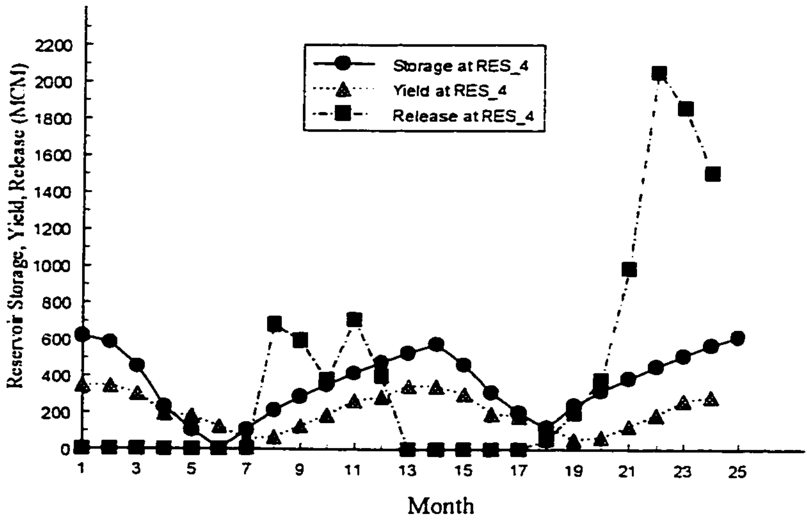


Fig. (6.5): Optimum state and control trajectories of RES#4 by the PSLP-OCT model for case 2 CE-646 problem.

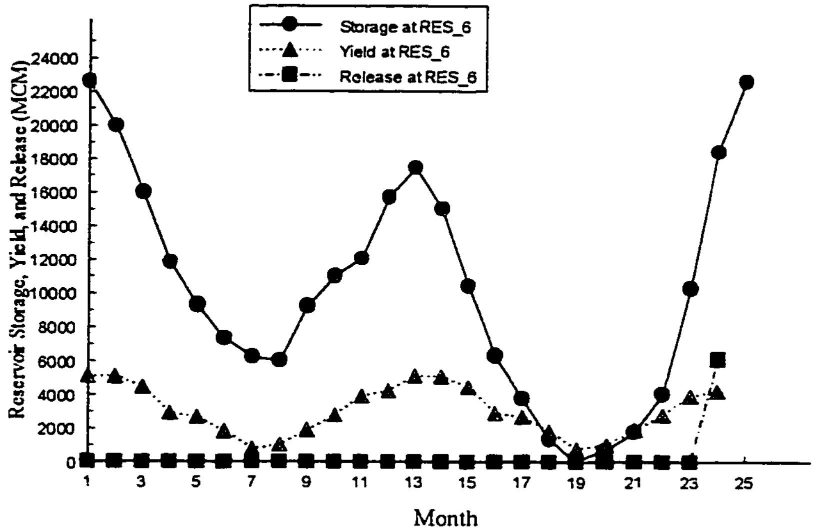


Fig. (6.6): Optimum state and control trajectories of RES#6 by the PSLP-OCT model for case 2 CE-646 problem.

The configurations obtained from the above water supply screening procedure represent the best compromise solution. The computer execution time for this case on a PC (Pentium-Pro 180) was 83 seconds. The degree of optimality of the PSLP-OCT model for the case 2 CE-646 problem was 1.163. The result shows that, like the case 1 study, the PSLP-OCT was capable of designing a system at around 16% lower cost. As was mentioned earlier, the main reason for proposing an over-designed system by the DP model has to be related to 1) the decomposition technique implemented in the DP model: and 2) the discretization levels considered for the storage and decision variables. Due to lack of information on the monthly storage and releases proposed by the DP model at Supangat (1985), it is not possible to investigate its proposed storage/release trajectories and to analyze further the reason(s) for over-designing the system. Regardless of the reasons that the DP model over-designed the system, the lower cost multi-reservoir system designed by the PSLP-OCT model indeed proves the success of the PSLP-OCT model.

As in the case 1 CE-646 problem, the terminal storage objective was not fully met and the minor deviation (from target storage) had to be adjusted by changing the corresponding release. The assigned weight coefficients for the contributing reservoirs are presented in Table (6.2). Following the strategy explained in section (5.12) and considering the order of magnitude of the hydrologic data, yield weight coefficients of *1* were selected a priori for all demand areas. Then with the zero weights for the secondary objectives, the performance of the OCT-PSLP model was examined. Based on the obtained result, appropriate weights for the first and secondary objectives were determined such that the system could supply water at the minimum cost.

Table (6.2): Selected weight coefficients in the OCT module for case 2 CE-646 problem.

Weight Coefficients	**Selected Reservoirs**				
	RES#1	**RES#2**	**RES#3**	**RES#4**	**RES#6**
$\mathbf{W}_{n,1}^{y}$	1.0	-	-	-	-
$\mathbf{W}_{n,2}^{y}$	-	1.0	-	-	-
$\mathbf{W}_{n,3}^{y}$	-	-	2.0	-	-
$\mathbf{W}_{n,4}^{y}$	-	-	-	1.0	-
$\mathbf{W}_{n,5}^{y}$	-	-	-	-	1.0
$\mathbf{W}_{n}^{by}$	0.0	0.0	0.0	0.0	0.0
$\mathbf{W}_{n}^{br}$	1.e-5	0.0	1.e-5	0.0	0.0
$\mathbf{W}_{n}^{fs}$	.01	.01	.01	.01	1.e-8

Table (6.3) shows the selected penalty weights in the PSLP module. These weights have to be greater than the related decision variables in the dual problem. Several experiments on the PSLP-OCT model performances showed that besides the above-mentioned restriction, any penalty weight W_j^p can be selected and the PSLP module is not sensitive to penalty weights at all. Therefore, one can simplify the model formulation to one penalty weight for the whole system without any effect on the final result. The unique penalty weight in this case has to be greater than the largest decision variable in the dual problem. However, in problems where scaling of yield constraints varies widely, it may be desirable to have separate weights for each constraint. This is because selecting a large unique penalty weight may keep the deviation variables in the basis for the majority of PSLP iterations which usually slow convergence. Based on Table (6.2), the proper unique penalty weight for the above case study hence would be 200. The unique penalty weigh approach was applied to the case 2 problem and the OCT-PSLP model resulted in the same solution with no significant difference in execution time.

Table (6.3): Selected penalty weight coefficients in the PSLP module for case 2 CE-646 problem.

Weight Coefficients	**Demand Area**				
	D#1	**D#2**	**D#3**	**D#4**	**D#5**
W_j^P	190	180	200	150	150

6.3 Sensitivity of the Candidate Reservoirs to Demand Levels:

The epsilon constraint method provides a range of solutions for various levels of water deficit. It should be noted that despite having N_d*T yield constraints in the original PSLP formulation (5.39), only N_d upper bounds would be sufficient to investigate the effect of water demand levels on designing the reservoir sizes and configurations. This can be done by assuming that the seasonal water demand at each area (D_j^t) is a monthly fraction of its annual water demand (D_j^A). Consequently, the monthly water deficit at each demand area (ε_j^t) can be defined as a monthly fraction of the annual water deficit (ε_j^A) i.e.,

$$\frac{D_j^t}{D_j^A} = \frac{\varepsilon_j^t}{\varepsilon_j^A} = \alpha_j^t \qquad ; \qquad \sum_{t=1}^{12} \alpha_j^t = 1 \tag{6.4}$$

Hence, the epsilon constraint method will consist of N_d+1 components corresponding to the objectives of minimizing the total cost (f_1) and N_d water supply constraints (f_j , $j=2,N_d+1$). The procedure explained in section (3.8), is used to find the proper upper and lower bounds on the f_j objectives. Due to the independence of these objectives from each other, their upper and lower bounds would simply be equal to the maximum and minimum annual water deficit at each demand area ($0.\leq f_j \leq D_j^A$). Therefore, one can use the procedure explained in section (3.8) to specify a set of annual water deficits at each demand area and investigate the sensitivity of reservoirs to different water deficit levels.

Taking the whole hydrosystem as a unit can result in a further simplification in reservoir sensitivity analysis. This will call for considering an allowable water deficit for the entire multi-reservoir system (ε^A) and analyzing reservoir patterns and sizes under different total water deficits imposed on the system. Based on this concept, the annual water deficit (ε^A) follows the same spatial distribution as the corresponding total annual demand level (D^A). Therefore, the annual water deficit at each demand area can be defined as a fraction of ε^A i.e.,

$$\frac{D_j^A}{D^A} = \frac{\varepsilon_j^A}{\varepsilon^A} = \beta_j \qquad ; \qquad \sum_{j=1}^{Nd} \beta_j = 1 \tag{6.5}$$

where β_j represents the spatial distribution of the total annual water demands and deficits in the hydrosystem. This feature not only shows the relationships between different levels of water supply and their associated reservoir costs, but also clarifies the sensitivity of an individual reservoir size to the hydrosystem's demand levels. Considering the uncertainty of future demands for water, this sensitivity information can be extremely useful in water resources planning and greatly facilitates the decision making process. The epsilon constraint approach applied to the case 2 CE-646 problem combines the effects of N_d water deficit constraints (f_j, $j=2,N_d+1$) and considers them as one deficit constraint. Therefore, the epsilon constraint method should consider the objective of minimizing the total cost (f_1) and the total allowable water deficit constraint (f_2). The procedure explained in section (3.8), is used to find the proper upper and lower bounds on the f_2 objective as follows:

1- Optimize the f_1 objective function individually. This would mean that one should solve an optimization problem that tries to minimize the cost without any restriction on the supplying demand. The result of this optimization problem, without requiring solving it, is a system with zero cost (no reservoirs) i.e., a system whose water supply depends on the river yield capabilities. Therefore, the PSLP-OCT model was adapted to minimize the water deficit for a zero storage reservoir system. That is, for each demand area, the OCT module considers only the most upstream zero storage reservoir to supply water for that demand area and all other zero storage reservoirs are considered as mass balance nodes. This is called the case 3 CE-646 problem and the corresponding model results are presented in Appendix C-5. The same penalty weights as the case2 CE-646 were used in this case. However, based on the flow hydrology and the demand levels, slightly different weight coefficients are selected for the OCT module and are shown in table (6.4). The water deficit obtained in this problem is equal to 19301.86 MCM. This is the maximum possible deficit and is denoted as m_2^e.

2- Optimize the objective function f_2 without considering the system cost. That would mean solving an optimization problem that tries to supply fully the total demand. The result of this problem is obviously a system with zero deficit and is known in advance without any computational requirement. The zero deficit in this situation is denoted as n_2^e.

3- Set the upper and lower bounds on the water deficit objective: $0. \leq f_2 \leq 19301.86$

4- Choose $N\varepsilon$, the number of different ε_k in Eq. (3.25). The number $N\varepsilon$, depends on the decision-makers' preferences and their certainty about the future water demand scenarios. The larger the $N\varepsilon$, the more information is provided about the sensitivity of each candidate reservoir to the possible future demand levels. For illustrative purpose, four levels of water deficit are considered in this section. Consequently, Eq. (3.26) can be rewritten as:

$$\varepsilon_2 = n_2^\varepsilon + \left(\frac{i}{N_\varepsilon - 1}\right)\left(m_2^\varepsilon - n_2^\varepsilon\right) \qquad \textit{for } i = 0,1,\ldots,3 \tag{6.6}$$

The upper bound ε_2 is equivalent to the annual water deficits (ε^A) in the CE-646 water supply problem. By inserting the corresponding parameters in the Eq. (6.6), a set of annual deficit levels was calculated and rounded up. Then, the demand levels were calculated by subtracting the annual deficits from the maximum annual demand (51900 MCM). Table (6.5) shows the selected annual deficits and their corresponding water demand levels.

Table (6.4): Selected weight coefficients in the OCT module for case 3 CE-646 problem.

Weight Coefficients	Selected Reservoirs RES#1	RES#2	RES#3	RES#5	RES#6
$W_{n,1}^{y}$	1.0	-	-	-	-
$W_{n,2}^{y}$	-	1.0	-	-	-
$W_{n,3}^{y}$	-	-	1.0	-	-
$W_{n,4}^{y}$	-	-	-	1.0	-
$W_{n,5}^{y}$	-	-	-	-	1.0
W_{n}^{by}	0.0	0.0	0.0	0.0	0.0
W_{n}^{br}	1.e-4	0.0	0.0	0.0	0.0
W_{n}^{fs}	1.e-1	1.e-1	1.e-1	1.0	1.e-2

Table (6.5): Selected annual water deficit and their corresponding demand levels (MCM).

Parameters	Deficit Counter (*i*) in Eq. (6.6) 0	1	2	3
Annual Deficit	0.00	6450.00	12900.00	19301.86
Annual Demand	51900.00	45450.00	39000.00	32598.14

Based on Table (6.5), two additional cases were required to investigate the multi-reservoir system's response to the selected demand levels. The results of the case 4 with 45450 MCM and case 5 with 39000 MCM annual demand levels are shown in Appendices C-6 and C-7 respectively. The same penalty weights as in Table (6.3) were used for these cases. The selected weight coefficients for the OCT module in each case are also presented in Tables (6.6) and (6.7).

Table (6.6): Selected weight coefficients in the OCT module for case 4 CE-646 problem.

Weight Coefficients	Selected Reservoirs RES#1	RES#2	RES#3	RES#4	RES#6
$W_{n,1}^{y}$	1.0	-	-	-	-
$W_{n,2}^{y}$	-	1.0	-	-	-
$W_{n,3}^{y}$	-	-	2.0	-	-
$W_{n,4}^{y}$	-	-	-	1.0	-
$W_{n,5}^{y}$	-	-	-	-	2.0
W_{n}^{by}	0.0	0.0	0.0	0.0	0.0
W_{n}^{br}	1.e-3	1.e-3	0.0	0.0	1.e-5
W_{n}^{fs}	1.e-2	1.e-3	1.e-3	1.e-2	1.e-2

Table (6.7): Selected weight coefficients in the OCT module for case 5 CE-646 problem.

Weight Coefficients	Selected Reservoirs				
	RES#1	RES#2	RES#3	RES#4	RES#6
$W_{n,1}^{y}$	1.0	-	-	-	-
$W_{n,2}^{y}$	-	1.0	-	-	-
$W_{n,3}^{y}$	-	-	2.0	-	-
$W_{n,4}^{y}$	-	-	-	1.0	-
$W_{n,5}^{y}$	-	-	-	-	2.0
W_{n}^{by}	0.0	0.0	0.0	0.0	0.0
W_{n}^{br}	1.e-5	0.0	0.0	0.0	1.e-5
W_{n}^{fs}	1.e-3	1.e-5	1.e-3	1e-3	1.e-2

6.4 Discussion of Results:

Based on the results obtained from the epsilon constraint method, the effects of different water deficit levels on the selected reservoir storage strategies can be analyzed. The responses of the hydrosystem to different water supply levels are shown in Fig. (6.7). Two axes with different scales are used in Fig. (6.7) to consider the scaling of the reservoir capacities.

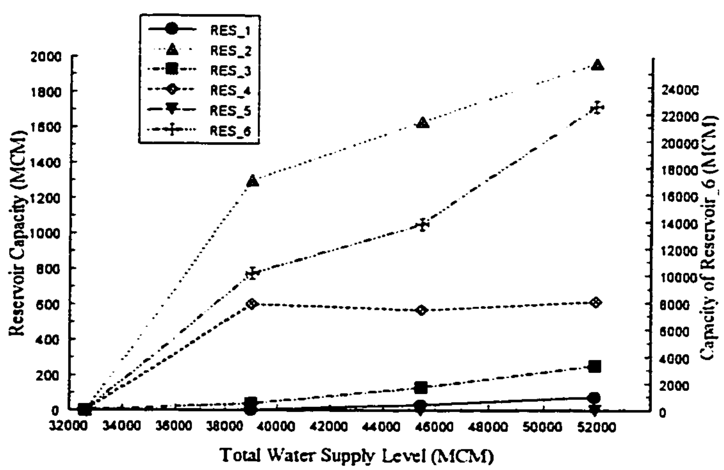


Fig. (6.7): Water storage strategies for CE-646 problem at different supply levels

Fig. (6.7) indicates that RES#1, RES#2, RES#3, and RES#6 are sensitive to the levels of total water demand. Their sizes increase monotonically with increasing supply levels and therefore, should be carefully sized. Among these reservoirs, RES#2 and RES#6 are more sensitive in that their sizes change rapidly at different supply levels. Therefore, caution should be used to avoid foreclosure of future opportunity. The results also show that RES#5 should not be built regardless of the demand levels. Candidate reservoirs #1, #2, and #3 are also insensitive to the levels of demand at the area D#5 and they should be built only to satisfy the demand areas located at their downstream limit. Candidate reservoir #4 is exceptional. The results of CE-646 problems (cases 1 to 5) show that RES#4 should be built at all demand levels greater than the yield capability of the river (Case3). The size of RES#4 can be decided easily, as it shows a mild sensitivity to the change of demand levels. The required capacity at this site ranges between 570 to 615 MCM when supply levels change from 39000 to 51900 MCM. The selected capacity of RES#4 in the case 5 is slightly larger than the case 4 problem. This is due to the relatively higher cost of RES#6 at that size and consequently, some part of the assigned capacity in the RES#4 is reserved to supply demands at D#5. As the capacity of RES#6 increases, it becomes a less expensive reservoir than RES#4 as in the case 2 problem, where the PSLP module will not increase RES#4 capacity to supply water for D#5.

The sensitivity of the individual reservoir sizes can be useful in developing guidance for construction and investment timing in the river basin. The less sensitive reservoir has a chronological priority over others and should be built first. Fig. (6.7) shows that if, at that moment, the available total budget is less than the required total cost, decision-makers should definitely give priority to build RES#4. Then, depending on some other factors such as the remaining budget, social aspects, and the possibility of expanding the capacity of each reservoir in the future, the decision-makers can give their preferences for building other candidate reservoirs.

Similar analyses can be done in multi-reservoir design problems by using different ε^A levels to specify a set of water storage strategies. The slope of each curve in graphs such as Fig. (6.7) show the sensitivity of a candidate reservoir to different water deficit levels. High slope curves show that extra caution should be used in estimating the future water demand levels in the related demand area(s). This information can help the decision-makers to select the best reservoir configuration and to avoid building these reservoirs either too small or too large.

CHAPTER 7

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

7.1 Summary and Conclusions:

Four optimization models are developed to optimize the configuration of multi-reservoir systems for water supply purposes. These models apply optimal control theory (OCT) and penalty successive linear programming (PSLP) as the most promising techniques to optimize large and complex water resources systems. Three of these models are based on optimal control theory. However, they differ from each other by taking different approaches to join the cost function to other objectives. The fourth model employs a new composite optimization algorithm, which is introduced in this study. This model consists of two modules. The first module uses the PSLP to minimize the reservoir costs subject to the water supply constraint. The second module employs the OCT to minimize the water deficit of the system. These two modules interactively share their results during the optimization iterations. The final solution of the PSLP-OCT model proposes a multi-reservoir system that meets the objectives in the two OCT and PSLP modules.

The developed models are design models that determine water storage strategies in a multi-reservoir system. These models are written in FORTRAN 77 to optimize the reservoir sizes and select from given configurations for water supply purposes at a minimum cost. The multi-objective programming methods are implemented in the models in order to consider the two non-commensurate objectives of minimizing cost and water supply. The weighting method and epsilon constraint method are used as the most suitable generating techniques to incorporate the problem objectives.

To develop the design models, first the computer codes based on the OCT and PSLP algorithms were developed and tested for general nonlinear problems. The performance of the OCT program was compared to the analytical solution of a test problem. The results were consistent between them with a maximum absolute difference of $4*10^{-3}$. The performance of the PSLP program was compared to another test problem and the PSLP result exactly matched the problem's best known solution. The four design models were developed based on these two computer codes.

A multi-reservoir system was selected as a case study to evaluate the performances of the developed models in designing the reservoir configuration. This case study consists of six candidate reservoirs with serial/parallel layout that fairly represent a large hydrosystem. The best known solution to the case study is proposed by the combination of dynamic programming and simulation models and was used as the benchmark to evaluate the success of each design model. The results of the design model performances were illustrated in chapter 5. Based on these results, all the design models based on the OCT algorithm fail to design the multi-reservoir system optimally. However, the PSLP-OCT model provided the design for the system successfully. Compared to the benchmark solution, the PSLP-OCT model supplied the same level of water demand at a lower cost. The PSLP-OCT performance showed that it is a very promising optimization method to design multi-reservoir systems regardless of their size.

The newly proposed PSLP-OCT model incorporates multi-objective programming into the multi-reservoir design problems. The water supply objectives in the OCT module have quadratic forms. Therefore, not only does the OCT achieve the non-inferior solution, but also yields a result that is not sensitive to the initial solution.

The PSLP-OCT model applies a sequential strategy. That is, the demand areas and candidate reservoirs are numbered sequentially. The PSLP module starts from the most upstream demand area. Considering each demand area at a time, the proposed model designs the multi-reservoir system by considering one reservoir at a time. Designing the system for one demand area at a time has the advantage of enabling the user to interrupt the analysis after supplying water for that demand area. For example, the user may feel that a particular weight coefficient should be changed to provide a higher yield or meet a specific objective. The screening procedure can be interrupted at that point and the related weight coefficients can be modified. If the user is satisfied with the result (water supply level vs. the system cost), the program then proceeds to the next demand area. The entire procedure of considering one demand area at a time is repeated until all demand areas in the system have been analyzed. Due to the sequential strategy, the PSLP module is adapted such that it is independent of the design period length. Therefore, using large hydrologic data does not affect its problem dimension.

The proposed PSLP-OCT, when applied to different case studies, demonstrates the application of the epsilon constraint in designing multi-reservoir systems. The sensitivity analysis performed to show the response of the system configuration to different water supply levels. The sensitivity of the individual reservoir size to different water supply levels can be a useful information in developing guidance for construction and investment timing in the river basin. The most insensitive reservoir to the demand levels has priority and must be built.

7.2 Recommendations:

The present study has been limited to dealing with problems of deterministic approach: That is, the stream flows are assumed to be known with certainty. A stochastic formulation is generally a more realistic representation of a hydrosystem since stream flows have randomness and are stochastic in nature. However, in stochastic optimization models, the reliability of the designed system is not considered. Therefore, a probabilistic approach (e.g., chance-constrained/yield formulations) is recommended to be incorporated to the PSLP-OCT model. The probabilistic approach though adds to the complexity of the problem, will help the water resources engineer to design the multi-reservoir system with a desired reliability.

The second recommendation is to extend the developed methodology to include the objective of hydropower production in addition to the objective of water supply. The objective of hydropower optimization can be considered by adding the related constraints and objective(s) in the PSLP and OCT modules respectively. This requires some modifications in the OCT and PSLP computer codes. The objectives of supplying water and generating hydropower in a multi-reservoir system may be two non-commensurate objectives and oppose each other. That is, the best strategy to supply water is not necessarily the best one to generate hydropower. Depending on the order of importance of water supply and hydropower objectives, proper weight coefficients must be assigned to the objectives in the OCT module to meet the design objectives.

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APPENDIX A

INPUT DATA FILE

<u>**Input Data File of the Case 1 CE-646 Problem:**</u>

* (Total months) T (Total reservoirs) Nr (Total demand areas) ND

24 6 1

* Ratio of minimum storage Smin(n) to reservoir storage i.e., OMEG(n).

6*.0

* Upper bounds on minimum storage TAUU(n)

6*0.

* Lower bounds on minimum storage TAUL(n)

6*0.

* Upper bounds on reservoir storage/capacity Xmax(n)

10780 15360 16030 1000 9500 48700

* Lower bound on reservoir storage Xmin(n)

6*0.

* Maximum water demand at area j during month t [D(j,t)]

7214.1 7162.2 6279.9 4100.1 3788.7 2595.0 1141.8 1453.2 2698.8 3944.4 5553.3 5968.5 7214.1 7162.2 6279.9 4100.1 3788.7 2595.0 1141.8 1453.2 2698.8 3944.4 5553.3 5968.5

* Upper & lower bounds on reservoir yields Ymax(n,j,t),Ymin(n,j,t)

* RES#1	Ymax(n,j,t)	Ymin(n,j,t)
	24*50000.	24*0.
* RES#2	Ymax(n,j,t)	Ymin(n,j,t)
	24*50000.	24*0.
* RES#3	Ymax(n,j,t)	Ymin(n,j,t)
	24*50000.	24*0.
* RES#4	Ymax(n,j,t)	Ymin(n,j,t)
	24*50000.	24*0.
* RES#5	Ymax(n,j,t)	Ymin(n,j,t)
	24*50000.	24*0.
* RES#6	Ymax(n,j,t)	Ymin(n,j,t)
	24*50000.	24*0.

* Upper and lower bounds on reservoir releases Rmax(n) and Rmin(n)

6*99999. 6*0.

* WY(n,j)

1.e-7 1. 1. 1. 1. 1.

* Wby(n)

6*0.

* Wbr(n)

1.e-3 0. 0. 0. 0. 1.e-4

* Wfs(n)

1.e-5 0. 0. 0. 0. 1.e-2

* Coefficients of Evaporation function

* RES#1 Ce(n) Pe(n)

0.410 0.7

* RES#1 monthly evaporation rate m/month EVAP(n,t)

0.2103 0.3737 0.412 0.2798 0.141 0.0717 0.0312 0.025 0.0247 0.026 0.0388
0.1030 0.2103 0.3737 0.412 0.2798 0.141 0.0717 0.0312 0.025 0.0247 0.026
0.0388 0.1030

* RES#2 Ce(n) Pe(n)

1 .64

* RES#2 monthly evaporation rate m/month EVAP(n,t)

0.2103 0.3737 0.412 0.2798 0.141 0.0717 0.0312 0.025 0.0247 0.026 0.0388
0.1030 0.2103 0.3737 0.412 0.2798 0.141 0.0717 0.0312 0.025 0.0247 0.026
0.0388 0.1030

* RES#3 Ce(n) Pe(n)

0.33 0.67

* RES#3 monthly evaporation rate m/month EVAP(n,t)

0.2103 0.3737 0.412 0.2798 0.141 0.0717 0.0312 0.025 0.0247 0.026 0.0388
0.1030 0.2103 0.3737 0.412 0.2798 0.141 0.0717 0.0312 0.025 0.0247 0.026
0.0388 0.1030

* RES#4 Ce(n) Pe(n)

1.730 0.51

* RES#4 monthly evaporation rate m/month EVAP(n,t)

0.2103 0.3737 0.412 0.2798 0.141 0.0717 0.0312 0.025 0.0247 0.026 0.0388 0.1030 0.2103 0.3737 0.412 0.2798 0.141 0.0717 0.0312 0.025 0.0247 0.026 0.0388 0.1030

* RES#5 Ce(n) Pe(n)

0.475 0.72

* RES#5 monthly evaporation rate m/month EVAP(n,t)

0.2103 0.3737 0.412 0.2798 0.141 0.0717 0.0312 0.025 0.0247 0.026 0.0388 0.1030 0.2103 0.3737 0.412 0.2798 0.141 0.0717 0.0312 0.025 0.0247 0.026 0.0388 0.1030

* RES#6 Ce(n) Pe(n)

0.74 0.74

* RES#6 monthly evaporation rate m/month EVAP(n,t)

0.2103 0.3737 0.412 0.2798 0.141 0.0717 0.0312 0.025 0.0247 0.026 0.0388 0.1030 0.2103 0.3737 0.412 0.2798 0.141 0.0717 0.0312 0.025 0.0247 0.026 0.0388 0.1030

* Coefficients of reservoir cost functions cost(n)=A(n)*X(n)+ B(n)*X(n)^2

* RES#1 A B

0.01875305419 -1.1624993e-07

* RES#2 A B

.01565203799 2.302339e-7

* RES#3 A B

.01963009329 2.78487678e-07

* RES#4 A B

.00427404517 1.818244446e-6

* RES#5 A B

0.02175264227 -3.96866e-9

* RES#6 A B

0.00712305011 -2.844271e-8

* Inflows to reservoirs Q(n,t)

* RES#1

1379.7 470.412 239.148 191.844 178.704 444.132 717.444 575.532 1200.996 1411.236 1847.484 1876.392 1295.604 559.764 278.568 176.076 207.612 281.196 743.724 1395.468 1340.28 1663.524 4685.724 4231.08

*RES#2

24*0.

*RES#3

1865.880 940.824 438.876 375.804 617.580 680.652 551.880 2128.68- 1584.684 1902.672 3148.344 3161.484 1905.300 706.932 417.852 325.872 362.664 430.992 814.680 1479.564 2084.004 3390.120 5072.04 6312.456

*RES#4

312.732 212.868 84.096 70.956 78.840 228.636 165.564 825.192 791.028 633.348 1032.804 738.468 396.828 228.636 155.052 84.096 102.492 299.592 331.128 517.716 1185.228 2304.756 2183.868 1844.856

*RES#5

24*0.

*RES#6

1011.780 851.472 317.988 365.292 520.344 312.732 -44.676 1500.588 1274.58 1400.724 3524.148 2341.548 1450.656 512.46 339.012 354.78 302.22 312.732 473.040 -512.460 696.420 3211.416 2147.076 4420.296

* Return flow fraction RHO(j,t)

24* 0.

* Maximum penalty weight Pmax

100

APPENDIX B

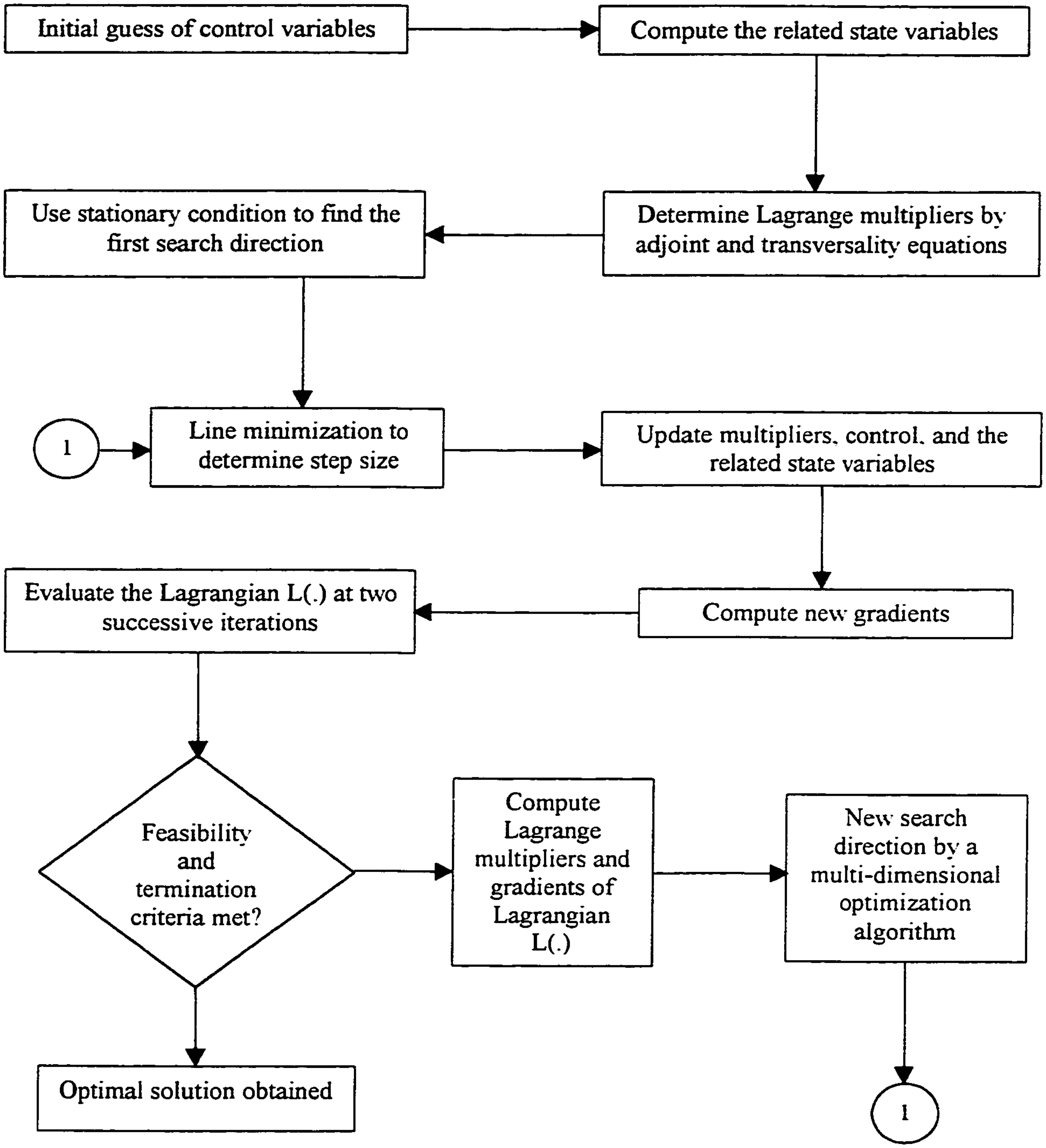
FLOW CHARTS OF COMPUTER PROGRAMS

B-1 Flow Chart of the Optimal Control Theory Algorithm

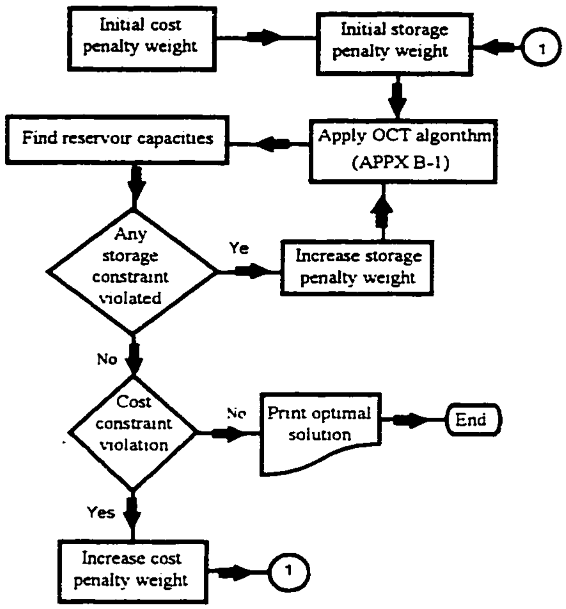
B-2 Flow Chart of the OCT-I Model

B-3 Flow Chart of the OCT-II and OCT-III Models

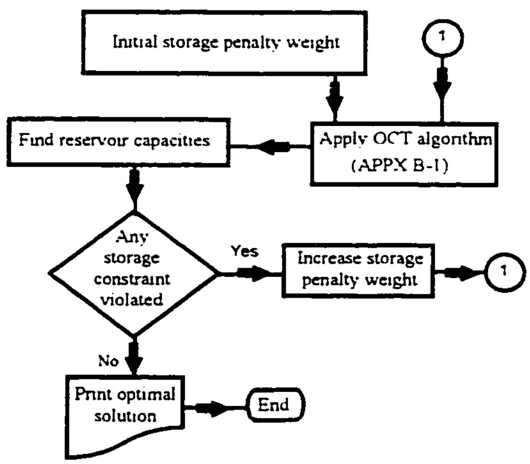
APPENDIX B-1: Flow Chart of the Optimal Control Theory



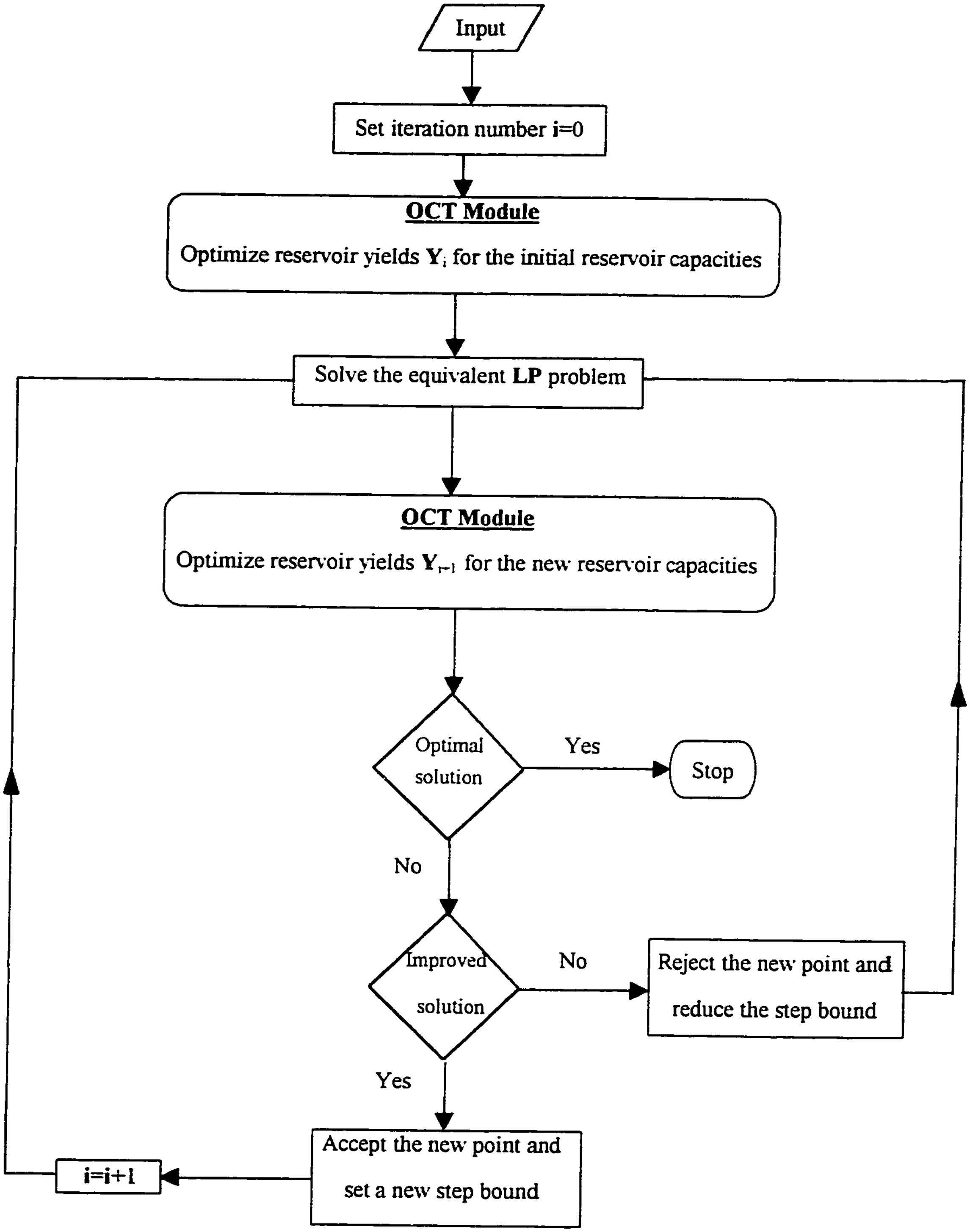
APPENDIX B-2: Flow Chart of the OCT-I Model



APPENDIX B-3: Flow Chart of the OCT-II and OCT-III Models



APPENDIX B-4: Flow Chart of the PSLP-OCT Model



APPENDIX C

C-1 OCT-I Model Solution to the Case 1 CE-646 Problem

C-2 OCT-II Model Solution to the Case 1 CE-646 Problem

C-3 OCT-III Model Solution to the Case 1 CE-646 Problem

C-4 PSLP-OCT Model Solution to the Case 1 CE-646 Problem

C-5 PSLP-OCT Model Solution to the Case 3 CE-646 Problem

C-6 PSLP-OCT Model Solution to the Case 4 CE-646 Problem

C-7 PSLP-OCT Model Solution to the Case 5 CE-646 Problem

C-1: OCT-I Model Solution to the Case 1 CE-646 Problem

Reservoir Yields y(n,j,t), n=1 to Nr, j=1 to Nd. t=1 to T

y(1,j,t)

250.608 2.135 497.270 384.606 163.137 0.000 23.554 11.063 31.101 31.124 31.124 31.124 35.338 500.468 493.541 0.000 0.000 0.000 0.000 189.801 225.673 120.224 669.758 641.707

y(2,j,t)

934.010 1284.934 1703.776 1486.401 1482.691 987.666 466.089 14.569 170.429 309.283 244.749 72.788 529.128 1569.877 2836.380 1174.329 1240.500 499.505 279.504 254.411 987.242 770.265 0.088 0.088

y(3,j,T)

3012.509 2421.943 150.190 2.737 70.545 146.138 43.145 218.368 717.537 1650.948 2128.951 2741.347 3908.696 1552.148 250.849 365.378 362.664 105.467 39.252 401.426 1204.431 1485.258 3508.121 2993.674

y(4,j,T)

104.850 201.742 1472.236 764.613 162.554 57.709 42.765 38.490 47.167 47.057 47.057 47.057 11.992 638.599 467.121 725.821 23.261 0.000 0.000 0.000 149.377 184.378 181.027 189.256

y(5,j,T)

20.253 132.192 739.394 686.407 132.974 81.780 72.613 13.945 15.862 15.886 15.886 15.886 21.366 607.173 848.349 495.229 794.147 331.000 331.000 517.000 93.331 95.563 92.144 48.501

y(6,j,T)

2665.865 2750.652 1301.859 399.001 1256.256 836.911 253.695 922.448 1497.204 1670.607 2866.038 2840.803 2475.014 1591.025 680.194 546.251 302.220 84.137 487.410 0.000 38.954 1288.712 1102.716 2095.828

Reservoir Releases r(n,t), n=1 to Nr t=1 to T

r(1,T)

1034.000 776.000 756.000 774.000 783.000 694.000 599.000 596.000 732.000 798.000 819.000 827.000 864.000 957.000 935.000 886.000 829.000 395.000 587.000 841.000 1186.000 1532.000 1847.000 1978.000

r(2,T)

36.000 0.000 0.000 0.000 0.000 0.000 15.000 16.000 16.000 16.000 16.000 15.000 14.000 0.000 0.000 0.000 0.000 0.000 14.000 406.000 198.000 283.000 307.000 313.000

r(3,T)

379.000 391.000 413.000 358.000 494.000 507.000 431.000 349.000 296.000 258.000 228.000 204.000 185.000 159.000 183.000 71.000 0.000 210.000 718.000 1049.000 863.000 877.000 892.000 900.000

r(4,T)

163.000 129.000 141.000 147.000 111.000 133.000 126.000 212.000 220.000 237.000 259.000 285.000 243.000 300.000 336.000 378.000 427.000 331.000 331.000 517.000 632.000 863.000 832.000 882.000

r(5,T)

113.000 41.000 16.000 0.000 0.000 8.000 19.000 22.000 22.000 22.000 22.000 22.000 19.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 188.000 262.000 262.000 200.000

r(6,T)

0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 337.000 721.000 1040.000 1712.000 1924.000 2024.000 2066.000

Reservoir storage s(n,t), n=1 to Nr t=1 to T+1

S(1,T)

3213.689 3308.387 2999.981 1985.220 1018.154 250.638 0.756 95.643 64.109 501.996 1084.090 2081.406 3099.511 3495.380 2597.010 1446.485 736.318 114.867 1.055 157.774 522.432 451.027 462.315 2631.238 4242.410

S(2,T)

3099.802 3163.541 2654.181 1706.015 993.419 293.666 0.000 117.911 683.334 1228.892 1701.590 2259.806 2998.908 3319.528 2706.215 804.495 516.043 104.505 0.000 293.496 474.078 474.827 953.550 2493.430 4158.213

S(3,T)

3535.205 2009.428 137.171 12.831 27.890 80.918 108.426 186.158 1747.462 2318.594 2312.302 3103.668 3319.724 1131.202 126.888 110.857 0.337 0.336 115.858 173.283 202.418 218.989 1246.843 1918.743 4337.447

S(4,T)

2637.984 2682.843 2564.928 1035.751 195.079 0.363 38.289 35.088 609.789 1133.648 1482.937 2209.681 2616.081 2757.894 2047.891 1399.786 380.043 32.270 0.862 0.989 1.705 405.556 1662.932 2833.769 3607.356

S(5,T)

1191.875 1221.276 1176.474 561.546 21.974 0.000 43.220 77.601 253.646 435.768 634.858 855.927 1102.895 1305.184 997.418 484.593 367.147 0.000 0.000 0.000 0.000 350.669 856.080 1333.876 1967.163

S(6,T)

3151.100 2023.577 554.871 0.000 324.291 82.232 73.016 239.619 1204.693 1315.970 1341.979 2265.885 2007.019 1199.653 279.077 120.471 0.000 0.000 101.595 98.206 0.737 195.188 1615.809 2096.961 3767.657

C-2: OCT-II Model Solution to the Case 1 CE-646 Problem

Reservoir Yields y(n,j,t), n=1 to Nr, j=1 to Nd, t=1 to T

y(1,j,T)
287.345 3.004 2.819 2.693 323.076 4.840 1.835 1.044 11.685 11.435 11.435
255.271 397.355 364.677 64.259 0.000 0.000 0.000 0.000 92.852 56.145 159.459
1449.792 1653.685
y(2,j,T)
1370.887 619.863 1392.007 944.928 1155.235 732.480 275.744 32.748 513.398
959.273 889.732 551.720 959.623 1142.217 1524.451 687.564 807.917 292.516
32.992 933.028 577.540 503.661 248.029 98.085
y(3,j,T)
2281.853 914.499 4.441 4.285 5.099 7.999 3.248 430.125 502.890 489.071
1199.378 1885.790 2184.900 1191.958 3.419 2.884 8.193 0.000 2.756 188.329
270.869 502.139 2262.851 3240.984
y(4,j,T)
599.511 1901.003 1102.653 8.224 9.038 110.489 14.792 5.804 6.099 200.601
900.108 833.317 251.218 403.033 640.938 490.663 55.890 129.411 71.648 38.976
681.413 740.072 183.964 205.231
y(5,j,T)
351.982 34.324 216.255 28.337 0.000 71.469 0.000 0.000 0.000 0.000 0.000 0.000
72.190 129.370 419.054 435.461 109.224 274.010 239.059 192.964 113.216
104.177 104.171 0.000
y(6,j,T)
1972.093 3271.289 3157.479 2716.874 1843.530 1069.461 528.514 751.098
1413.772 2023.236 2291.864 2181.968 2781.262 3334.985 3087.165 1977.002
1973.766 888.476 683.825 7.050 999.617 1934.892 1304.493 770.954
Reservoir Releases r(n,t), n=1 to Nr t=1 to T
r(1,t)
1168.000 719.000 657.000 631.000 606.000 551.000 588.000 665.000 899.000
998.000 1043.000 1049.000 925.000 896.000 818.000 664.000 538.000 281.000
551.000 1275.000 1213.000 1453.000 1694.000 1784.000
r(2,t)
0.000 0.000 0.000 0.000 0.000 0.000 5.000 5.000 5.000 5.000 5.000 6.000 28.000
17.000 92.000 252.000 306.000 0.000 366.000 338.000 557.000 628.000 657.000
664.000
r(3,t)
420.000 439.000 481.000 546.000 646.000 724.000 728.000 1149.000 1207.000
1208.000 1147.000 962.000 402.000 314.000 348.000 451.000 575.000 522.000
716.000 1387.000 1760.000 2070.000 2234.000 2291.000
r(4,t)
47.000 48.000 54.000 73.000 86.000 114.000 78.000 71.000 60.000 63.000 71.000
82.000 103.000 158.000 210.000 223.000 183.000 274.000 259.000 394.000
485.000 568.000 513.000 779.000
r(5,t)

3.000 5.000 13.000 43.000 83.000 55.000 0.000 0.000 0.000 0.000 0.000 0.000
73.000 42.000 12.000 12.000 78.000 0.000 10.000 211.000 335.000 392.000
392.000 116.000
r(6,t)
0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
0.000 0.000 0.000 0.000 0.000 881.000 1416.705 2261.000 2524.000 2624.000
2657.000
Reservoir storage s(n,t), n=1 to Nr t=1 to T+1
S(1,t)
2110.887 2034.955 1782.882 1361.748 919.648 169.200 57.480 185.082 94.565
384.869 786.655 1579.668 2151.658 2124.614 1423.245 819.188 331.109 0.688
0.884 193.603 221.212 292.339 343.394 1885.292 2678.537
S(2,t)
1884.376 1681.314 1780.145 1044.843 730.765 181.480 0.000 307.256 934.497
1315.083 1348.792 1497.033 1988.228 1925.420 1661.891 863.165 587.470 11.516
0.000 152.008 155.976 234.431 555.762 1344.712 2366.539
S(3,t)
1843.316 1007.249 594.460 547.795 373.254 339.710 288.351 108.980 658.530
533.318 738.912 1540.864 1854.506 1172.808 373.671 440.024 311.960 91.414
0.403 96.325 0.559 53.694 871.670 1446.844 2227.261
S(4,t)
3219.786 2885.982 1149.811 77.232 66.960 50.760 54.905 127.676 876.063
1600.990 1970.735 2032.427 1855.569 1898.159 1565.729 869.812 240.231
103.829 0.009 0.489 85.228 104.043 1100.726 2587.626 3448.240
S(5,t)
564.825 256.684 265.156 89.731 91.323 94.286 81.800 159.790 230.779 290.766
353.749 424.721 506.636 464.267 450.591 229.265 4.718 0.492 0.481 10.421
0.455 37.237 109.054 125.871 788.787
S(6,T)
7227.767 6687.465 4707.068 2358.030 594.186 0.000 22.271 182.062 2085.459
3158.097 3748.365 6132.220 7258.378 6427.820 3974.013 1674.856 766.537
53.744 0.000 0.215 0.000 87.802 1930.238 3431.548 7493.666

C-3: OCT-III Model Solution to the Case 1 CE-646 Problem

Reservoir Yields y(n,j,t), n=1 to Nr, j=1 to Nd, t=1 to T

y(1,j,t)

549.571 42.511 0.000 0.000 0.000 0.000 0.906 0.000 43.399 165.913 235.803 418.530 240.305 0.539 0.425 0.429 0.000 0.000 0.000 198.277 0.000 0.000 1220.282 1742.662

y(2,j,t)

814.277 88.924 647.041 978.007 1136.078 479.447 306.376 235.526 691.901 935.221 1089.717 755.599 776.564 1138.129 1431.470 511.939 353.493 133.370 0.000 117.060 480.451 829.310 1236.745 225.568

y(3,j,t)

2046.235 435.321 0.078 0.061 0.767 2.185 0.000 392.025 515.549 522.515 1247.960 1602.964 1719.553 1050.139 0.278 0.284 0.464 0.000 0.000 0.000 0.000 311.303 1795.931 3218.708

y(4,j,t)

1015.958 2354.221 1685.371 7.643 33.728 279.265 0.000 0.000 0.000 396.117 784.118 837.591 1255.322 692.382 89.277 69.030 165.954 272.399 104.959 133.214 564.139 419.138 0.584 56.625

y(5,j,t)

101.619 106.657 25.366 10.930 21.162 13.797 0.000 0.000 0.000 0.000 0.000 0.000 126.144 146.639 96.378 342.601 51.958 27.000 0.000 0.000 22.747 16.691 0.066 0.066

y(6,j,t)

2535.206 3871.366 3556.839 2738.355 2227.720 1221.582 639.592 670.067 1318.661 1729.025 2090.831 2249.668 2720.079 3489.900 4158.177 2666.766 2100.240 931.693 1036.375 1004.740 1631.630 2368.082 1300.246 725.384

Reservoir Releases r(n,t), n=1 to Nr t=1 to T

r(1,t)

790.000 423.000 451.000 463.000 508.000 444.000 520.000 684.000 1058.000 1207.000 1239.000 1196.000 913.000 700.000 575.000 484.000 438.000 306.000 619.000 1078.000 1349.000 1831.000 2434.000 2059.000

r(2,t)

129.000 126.000 0.000 0.000 0.000 0.000 93.000 93.000 93.000 93.000 93.000 93.000 19.000 4.000 4.000 4.000 198.000 172.961 493.000 834.000 773.000 769.000 778.000 771.000

r(3,t)

298.000 331.000 396.000 493.000 618.000 762.000 928.000 1359.000 1444.000 1409.000 1380.000 1165.000 462.000 262.000 267.000 350.000 505.000 446.000 693.000 1478.000 2034.000 2594.000 2852.000 2616.000

r(4,t)

13.000 14.000 20.000 25.000 21.000 20.000 39.000 54.000 59.000 62.000 65.000 76.000 118.000 135.000 180.000 208.000 17.000 27.000 151.000 242.000 234.000 239.000 251.000 497.000

r(5,t)

11.000 12.000 13.000 17.000 28.000 34.000 19.000 23.000 28.000 30.000 31.000 24.000 4.000 13.000 52.000 107.000 0.000 0.000 51.000 199.000 96.000 97.000 90.000 0.000

r(6,t)

0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 512.000 941.000 1656.000 1906.000 2009.000 2043.000

Reservoir storage s(n,t), n=1 to Nr t=1 to T+1

S(1,t)

768.480 808.464 813.101 600.984 329.694 0.365 0.497 197.030 88.556 188.149 226.465 599.129 860.923 1003.058 861.992 564.869 256.393 25.976 1.169 125.889 245.074 236.347 68.865 1100.285 1529.601

S(2,t)

1320.634 1167.218 1375.043 1178.725 663.566 35.447 0.000 120.624 476.092 749.181 927.947 984.208 1331.544 1448.831 1006.456 145.820 113.837 0.331 0.000 126.000 252.935 348.477 581.158 1000.394 2062.748

S(3,t)

865.894 387.485 561.908 604.604 487.281 486.064 402.516 26.392 404.043 29.175 0.332 520.711 914.201 637.886 32.615 183.156 158.714 15.904 0.895 122.574 124.136 174.137 658.949 1083.045 1560.750

S(4,t)

4650.805 3934.549 1779.153 157.851 196.156 220.264 149.633 276.196 1047.386 1779.413 1954.641 2138.323 1963.190 986.679 387.912 273.671 80.730 0.266 0.459 75.628 218.129 605.218 2251.833 4184.112 5475.327

S(5,t)

364.632 264.882 160.049 141.532 138.505 110.297 82.481 102.473 133.465 164.456 196.445 230.427 282.372 270.108 245.267 276.664 34.958 0.000 0.000 100.000 142.992 258.234 383.526 544.428 1041.238

S(6,t)

9765.957 8676.850 6120.397 3286.163 1421.317 359.504 246.554 602.229 2907.623 4428.322 5631.730 8568.489 9940.562 9152.361 6448.142 2947.599 1095.020 0.000 0.000 161.665 214.441 526.191 2923.393 5480.843 10518.132

C-4: PSLP-OCT Model Solution to the Case 1 CE-646 Problem

Reservoir Yields y(n,j,t), n=1 to Nr, j=1 to Nd, t=1 to T

y(1, 1,t)

24*0.000

y(2, 1,t)

24*0.000

y(3, 1,t)

24*0.000

y(4, 1,t)

328.602 319.673 243.803 76.302 75.245 233.783 182.874 697.918 732.738 757.005
795.902 756.884 621.448 303.648 182.594 122.462 106.113 299.592 331.128
517.716 1185.228 1780.563 1611.122 854.713

y(5, 1,t)

24*0.000

y(6, 1,t)

6885.499 6842.527 6036.097 4023.798 3713.455 2361.217 958.926 755.282
1966.062 3187.396 4757.398 5211.616 6592.653 6858.552 6097.306 3977.638
3682.586 2295.408 810.672 935.484 1513.572 2163.836 3942.177 5113.787

Reservoir Releases r(n,t), n=1 to Nr t=1 to T

r(1,t)

1379.700 470.412 239.148 191.844 178.704 444.132 717.444 575.532 1200.996
1411.236 1847.484 1876.392 1295.604 559.764 278.568 176.076 207.612 281.196
743.724 1395.468 1340.280 1663.524 4685.724 4231.080

r(2,t)

1379.700 470.412 239.148 191.844 178.704 444.132 717.444 575.532 1200.996
1411.236 1847.484 1876.392 1295.604 559.764 278.568 176.076 207.612 281.196
743.724 1395.468 1340.280 1663.524 4685.724 4231.080

r(3,t)

1865.880 940.824 438.876 375.804 617.580 680.652 551.880 2128.680 1584.684
1902.672 3148.344 3161.484 1905.300 706.932 417.852 325.872 362.664 430.992
814.680 1479.564 2084.004 3390.120 5072.040 6312.456

r(4,t)

0.000 128.756 572.744 990.138

r(5,t)

0.000 128.756 572.744 990.138

r(6,t)

0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 137.000 379.000 729.000 1162.000 1590.000 2113.000

Reservoir storage s(n,t), n=1 to Nr t=1 to T+1

s(1,t)

25*0.000

s(2,t)

25*0.000

s(3,t)

25*0.000

s(4,t)

395.436 379.558 272.738 113.019 107.667 111.259 106.111 88.800 216.074 274.363 150.706 387.607 369.186 144.559 69.538 41.990 3.622 0.000 0.000 0.000 0.000 0.000 395.436 395.436 395.436

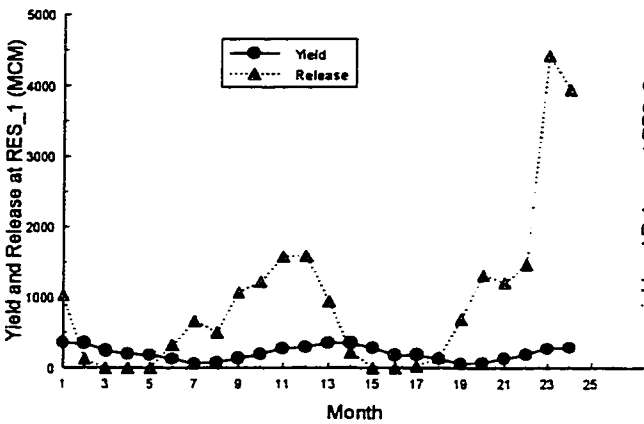
s(5,t)

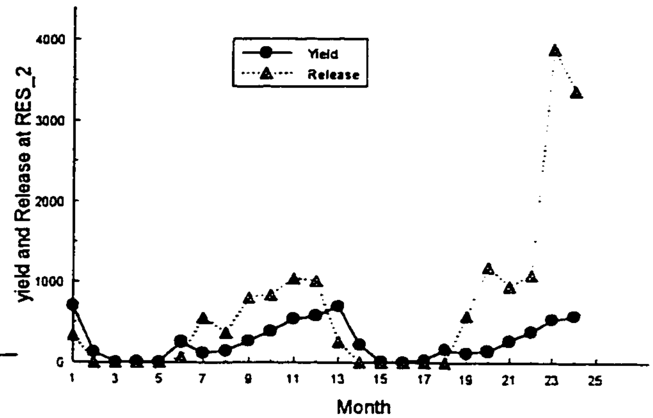
25*0.000

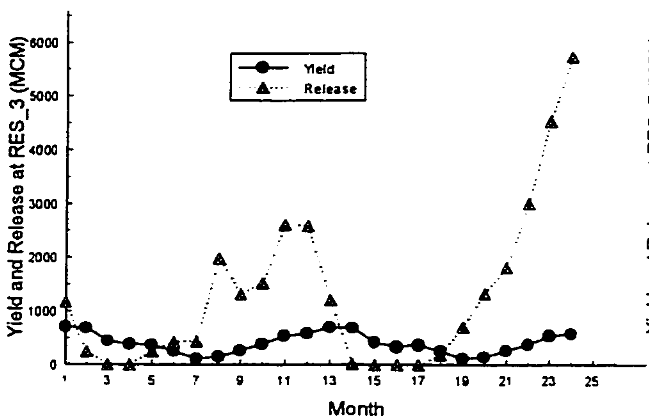
s(6,t)

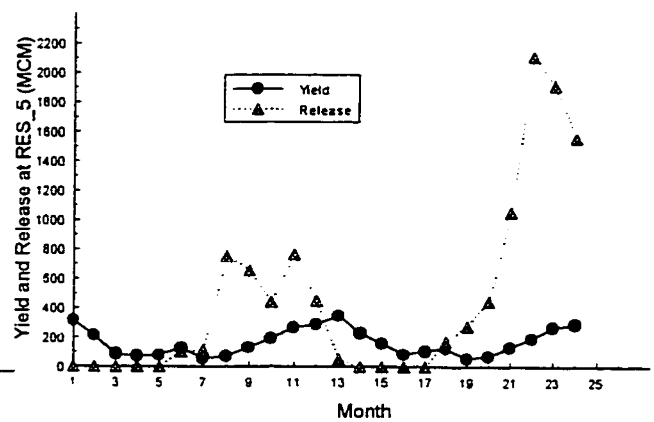
24747.750 22112.271 17520.930 12470.509 9374.100 6975.015 6050.343 6315.658 9764.780 11858.492 13385.155 17146.748 19311.570 17364.355 12275.666 7206.281 4081.965 1270.886 0.225 1083.930 2131.898 4009.838 9077.465 16022.018 24745.959

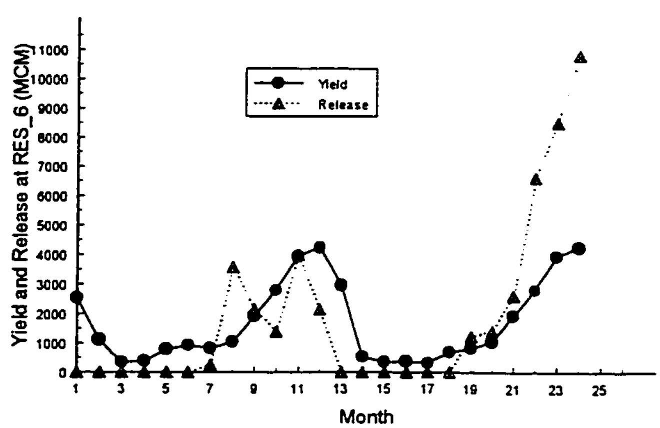
C-5: PSLP-OCT Model Solution to the Case 3 CE-646 Problem



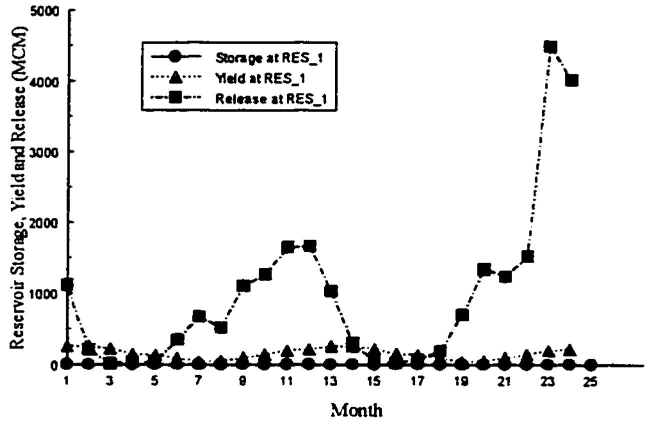


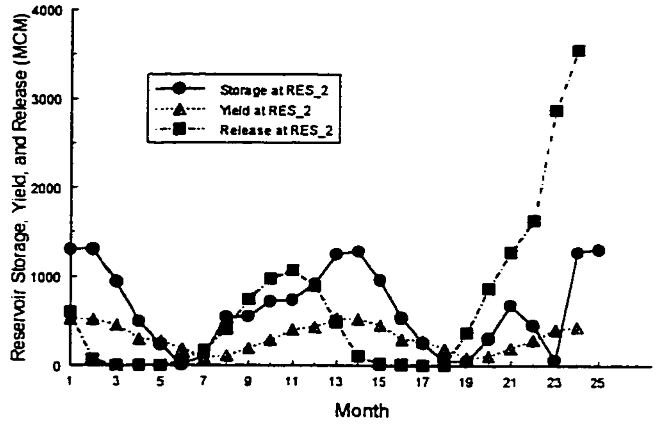


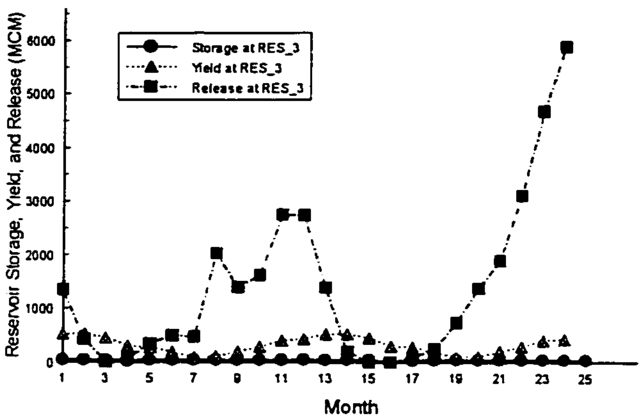


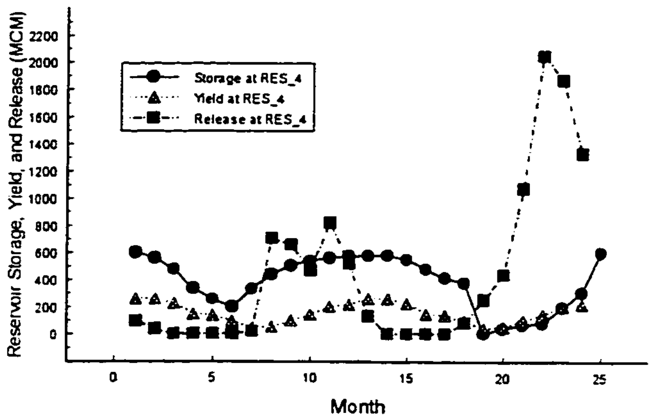


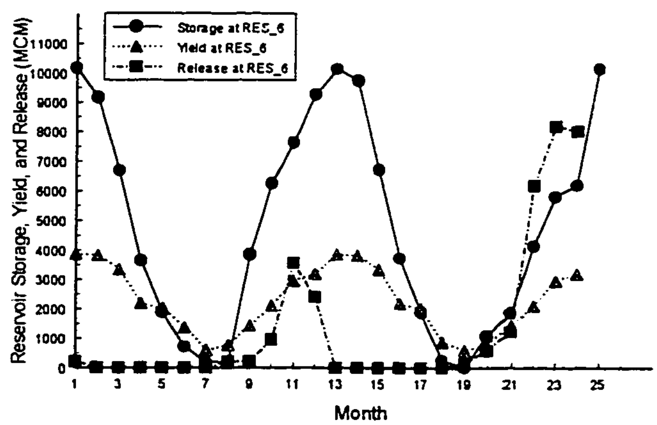
C-6: PSLP-OCT Model Solution to the Case 4 CE-646 Problem











C-7: PSLP-OCT Model Solution to the Case 5 CE-646 Problem