HEDGING EFFECTIVENESS OF MORTGAGE BACKED SECURITIES USING EMPIRICAL AND DEALER ESTIMATES OF DURATION

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ABSTRACT

Hedging Effectiveness of Mortgage Backed Securities Using Empirical and Dealer Estimates of Duration

Robert Scott

Mortgage duration estimation and the hedging of interest rate risk are topics of fundamental importance for holders of mortgage backed securities. In this study, we estimate several measures of empirical duration, (market-implied duration), and compare them with dealer estimates derived from analytical interest rate and prepayment models. These duration measures are used to calculate hedge ratios with the intention of hedging the interest rate risk of a wide selection of mortgage backed securities. Hedged returns indicate that the empirical measures are more effective and that the single factor estimates provide the best measure of interest rate risk. Hedging with two futures contracts is generally more effective than one when the risk exposure to changes in the slope of the yield curve are immunized.

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I - Introduction

The market for mortgage-backed securities (MBS) is the largest fixed-income market in the world[1]. MBS are also among the most difficult to understand. An embedded option, combined with departures from apparent rational behaviour by those with the option[2] make the price performance relatively unpredictable. Unlike mortgages, other callable bonds will be called when it becomes profitable to do so. Mortgages are often not called even when the mortgagor has a financial incentive to exercise the option.

An MBS is called when the homeowner, or mortgagor, prepays the mortgage, usually by refinancing at a lower rate. Similarly, a mortgagor can choose to make a prepayment of less than the full amount of the mortgage, but more than a normally scheduled principal payment. There is also the issue of mobility, requiring some homeowners to sell their house and prepay their mortgage, regardless of rates, in order to relocate. Financing rates lower than the MBS coupon rate lead to an incentive for the mortgagor to refinance. Higher rates, on the other hand, will cause the mortgagor to prolong or lock in the mortgage.

The duration risk of an MBS (the dollar weighted average maturity of a security) can be illustrated as follows. As interest rates fall, the likelihood of early pre-payment rises, and so the weighted average time to maturity shortens. The opposite holds true for increasing rates, namely, the number of payments in the future will increase, since the mortgagor will maintain the mortgage for a longer period.

These unique features of mortgages present quite a problem for the investors who hold mortgage-backed securities. Many fund managers who hold MBS have a targeted duration, the dollar weighted average duration of all securities in the portfolio. In certain cases, fund managers who hold MBS are required to hedge duration risk. Under normal circumstances, as interest rates fall, the value of a portfolio increases in proportion to the duration of the portfolio. If there are MBS in the portfolio, however, the duration can shorten just as rates are falling, causing the portfolio to under-perform the targeted duration. The opposite holds true for increasing rates. Under both circumstances, the portfolio loses relative to a benchmark. When rates are stable, however, the higher yield over equivalent credit risk Treasuries causes mortgages to generally outperform the target duration.

MBS holders try to correctly measure the duration of their securities, but these estimates are highly dependent upon the model used in the calculations. If the MBS holder incorrectly measures duration, there is an unknown risk that could seriously harm the value of a portfolio when interest rates change. Once a measure of duration is obtained, the investor can use Treasury futures, or other derivative instruments, to target or eliminate a duration exposure.

Some problems exist with the use of duration to measure the interest rate risk of these securities. First, duration can capture risk for parallel shifts in interest rates. In other words, short, intermediate and long rates all move by the same amount at the same time. While this may account for many interest rate movements, shifts can vary by as much as 200 basis points between maturities. Another problem is the rate of change in duration as rates change, known as convexity. This usually only takes on importance for large changes in interest rates. Premium MBS are generally negatively convex, as compared to the positive convexity of most non callable risk free securities. This can be an important factor when there are large changes in the level of interest rates. While duration can be approximated by the equation[3]:

$$D = \frac{\sum_{t=1}^{T} PVCF \times t}{PVB} \tag{1}$$

where D is the duration in years, PVCF is the present value of each cash flow at time t=1, and PVB represents the total value of the bond. Convexity is estimated as the second moment of price changes with respect to interest rates, formally written as[4]:

$$C = \frac{1}{2} \times \frac{\sum_{t=1}^{T} PVCF \times t \times (t+1)}{PVB} \tag{2}$$

Negative convexity suggests that as interest rates rise, the duration of a security lengthens, and conversely, as the level of interest rates falls, so does the duration.

To estimate duration, an investor may turn to analytical model or 'street' estimates which are based on elaborate prepayment models. The option used in some models is valued with an interest rate lattice framework using economic variables to forecast the amount of prepayments. These models will give a probability weighted estimate of duration. These estimates are based on several factors, namely, the future path of short rates, burnout[5], coupon, age, geographic location, historical prepayments, volatility, and other economic variables (see Fabozzi, 1992).

A competing way of estimating interest rate risk is by the use of empirical measures. A market as large as the MBS market should efficiently price the embedded prepayment option. Use of an empirical measure can allow for innovations which are not captured by a prepayment model such as a shift in mortgagor behaviour, or a shift in refinancing costs which may change the value of the option. In addition, there is a cost involved in estimation of mortgage durations. Development of a model may be expensive in data collection, formulation, and maintenance costs, whereas an empirical measure is more parsimonious in data collection costs and can be updated and re-estimated with relative ease.

While many studies have examined various mortgage models and empirical measures of duration, there are several obvious areas for extending this body of knowledge to incorporate new theoretical advances and estimation techniques. It is the intention of this thesis to examine the efficacy of hedging using different duration measures and to develop an empirical model for optimal estimation of the duration of mortgage-backed securities.

This thesis is organized as follows: In the next section, previous studies of empirical duration measures are examined. Section III will discuss the time interval and data sources used in this study. Section IV will present a methodology for testing several duration measures discussed in Section II, with the objective of testing these measures in hedged portfolios. The results of these tests are presented in section V. Conclusions and implications for further study are presented in section VI.

II - Literature review

The price change of a fixed income security can be approximated by the equation:

$$\Delta P = -D\frac{\Delta r}{(1+r)} + \frac{1}{2}C^{*}\Delta r^{2} \tag{3}$$

where ΔP is the change in price, r is the interest rate, C is the convexity, and D is the duration of the security. For small changes in r, the convexity, C, can be ignored, resulting in the shorter equation:

$$\Delta P = -D\frac{\Delta r}{(1+r)} \tag{4}$$

Batlin(1989) describes the hedge ratio which minimizes the variance of the gain or loss as

$$H = {\Delta P_m}/{\Delta P_t} \tag{5}$$

where ΔP_m is the change in price on the mortgage, and ΔP_t is the change in price of the Treasury futures price. Substituting equation 4 into equation 5 for the mortgage and Treasury prices, at time t, H_t, the hedge ratio becomes:

$$H_t = {}^{D_m}\!/_{D_t} \tag{6}$$

In other words, an MBS hedge involves a short position relative to the ratio of durations of the MBS and the underlying asset of the futures contract. As D_m and D_t can vary through time, so will the hedge ratio H_t. A second method involves the Ederington (1979) approach where the hedge ratio is empirically estimated from the least-squares coefficient of the regression of the security price on its corresponding hedging instrument. A drawback to this procedure is that the implied hedge ratios are constant through time which is inconsistent with recent studies (see e.g. Langowski, Park and Switzer, 1996). In this paper, we will use the first method so as to provide a way of accounting for time variation in duration risk of the MBS. In addition, we will incorporate recent developments in the estimation of duration.

Duration measures are sensitive to the choice of interest rates used and to the yield dynamics assumptions. Price changes relative to changes in the 2-year rate have a different elasticity to those of the 10-year rate. If we are to take yield curve dynamics into account, it is important to examine several different elements of risk embedded in the yield curve. Litterman, Scheinkman, and Weiss(1991), (henceforth LSW), present a useful theoretical look at the shape of the curve. They point out that given certain volatility assumptions, the discount function is downward sloping and convex as rates increase. Under a multiplicative process, yields are biased higher in the short run, but as maturity extends, the convexity of the discount function causes yields to decrease. The result is an increasing, then decreasing discount function. When plotted in maturity space, the yield curve will usually be upward sloping, but will be humped, decreasing at the long end.

LSW point to three factors present in the yield curve. The first being the level of short rates. The second, is the slope of the yield curve, described as the eventual yield for the short rate, or the long yield. The third is a representation of volatility (which embodies mean reversion), approximated by a butterfly spread, or the yield difference between a duration matched portfolio of a short and a long bond and the yield on an intermediate security. In the LSW model, volatility can cause the shorter and intermediate sections of the curve to surge upwards, while the longer end moves downwards. This can be explained in another way. Since a portfolio consisting of a very short and a very long security has more convexity than an intermediate security (assume matched duration), it must have a lower yield to account for the benefit of providing a volatility hedge. As a result, increases in volatility can be manifested as a bowing of the intermediate section of the curve relative to the ends to account for the increased value of convexity. LSW show a very consistent relationship between the butterfly spread and the implied volatility of interest rates as calculated from bond futures options. The implication from this research is that a protection from curve reshaping due to volatility, and flattening/steepening could provide a more effective hedge.

Similarly, Waldman (1992) proposes taking yield curve dynamics into account when measuring the risk of MBS. He suggests that the correct measure of interest rate risk is a series of 'partial' durations which measure the risk of the security to changes in yields of particular sections of the yield curve. In other words, this measures sensitivity to any type of reshaping of the yield curve as opposed to strictly parallel shifts. In effect, there can be a duration measure for every cash flow of a security. This type of measurement involves many assumptions about prepayments on a par with those used in OAS models. Nevertheless, it does show the outer extreme method which can be used to measure duration to provide the perfect hedge. It could be usefully extended to an empirical measure, but would still be of limited use to portfolio managers who would have to hedge as many as 5 maturity areas (i.e. 5 futures contracts to insulate against changes in any part of the yield curve).

DeRosa, Goodman and Zazarino(1993) (DGZ) provide the first model for empirically estimating the durations of MBS. DGZ estimate empirical duration with one sample regression, estimating the coefficients for the following equation:

$$\Delta P/P = c + b_1 \Delta r + b_2 (P - 100) \Delta r + b_3 (P - 100)^2 \Delta r K_t + \varepsilon_t \qquad (7)$$

Where K_t is a dummy variable, which is equal to 1 when P is greater than 100 and 0, otherwise. The resulting coefficient b_2 will be the per dollar increment value of duration. Equation 7 transformed can indicate the duration based on the price of the mortgage:

$$D = b_1 + b_2(P-100) + b_3\left[(P-100)^2\right]K_t \tag{8}$$

The third term, b3, is added to allow for a change in pricing behaviour when the price is at a premium i.e. the coupon is higher than the current mortgage coupon rate. A normal fixed income security, with a convexity of zero, has a fixed duration irrespective of price. Under this scenario, all coefficients in equation 8 become zero except for b1. When convexity is added, the relationship between price and duration becomes roughly linear. In the case of the MBS, as price increases, duration shortens, causing b2 to be negative. This is explained by prepayment behaviour. As mortgage rates rise, and MBS prices fall, the mortgagor prolongs the mortgage. When the mortgage is prolonged, the duration of the MBS increases. Conversely, as rates fall and the price exceeds 100, the mortgagor prepays the mortgage principal at a considerably faster rate. Since the mortgagor is allowed to prepay more principal than what is agreed upon in the mortgage, but is not allowed to reduce principal payments to below these levels, there is a non-symmetric relationship between duration and price over and under 100. It is for this reason that b3 is included in the equation which allows for non-linear shifts in duration for prices above 100.

As a comparison, an analytical model, creating a lattice tree of interest rates will give a per basis point shift in price for a corresponding shift in interest rates. This measure is known as effective duration. DGZ compare the empirical measures with analytical model estimates and found them lower in most cases. The empirical measures are compared using the 10- and 7-year Treasury rates showing no difference. This is somewhat problematic, as it should have been obvious to compare measures with those from a maturity further from the 10-year yield.

Breeden(1994) compares duration measures of FNMA, GNMA, and IO and PO STRIPs[6], for use in hedging with Treasury note and bond futures. He provides two main findings: First, interest rate elasticity is best described in a two factor model, i.e., 1 and 10-year rates. Secondly, he finds that averages of dealer estimates of duration based on prepayment models are more effective than empirical measures. His measurement criterion is based on root mean squared error of the hedged total returns. He explains much of the reasoning behind the need to hedge mortgage securities. His estimates of the price elasticities given changes in interest rates are obtained as least squares estimate of D in equation 4. He uses daily data from 1986 to 1994, but hedges only based on quarterly duration estimates. Given the rapid shifts in empirical duration evident in chart I, this quarterly rebalancing is unrealistic. His "dealer" duration estimate used is the median of all broker dealer estimates. This makes no assumption of the efficacy of any one model, and treats all prepayment models as equal. Nevertheless, the median estimate is still more accurate than the empirical measure.

Breeden finds that estimates of price elasticities overestimate realized values especially in high coupon areas. In addition, a random walk model showed empirical durations provided better estimates of next period elasticities than dealer estimates. The best model, defined as the one with the lowest mean square forecast error, however, was a combination of the random walk with the dealer estimates of price elasticities. Futures hedging, however, provided the best return using the dealer estimates.

A practitioner's approach is a simple estimation of price elasticity relative to a given interest rate as shown in Sobti(1995). This method estimates duration directly as a function of recent price behaviour(i.e. 40 day). A ratio of price changes of mortgages vs. 10-year Treasuries yields is calculated to get the duration of the MBS. For example:

$$D = \frac{\sum_{t=1}^{40} \Delta P_t / \Delta r_t}{40} \qquad (9)$$

where ΔP_t is the change in price of the mortgage, Δr is the change in the 10-year yield, and D is the duration of the MBS. The D is the time series estimate of D in equation 4.

Often, MBS hedgers will turn to duration measures calculated from an elaborate option valuation model. This model involves a basic assumption about the behaviour of interest rates. There are myriad of varying models currently being used. Nevertheless, they all operate using roughly the same procedure. First, a lattice framework of the possible future paths of interest rates is created. Some more popular models include Cox, Ingersoll and Ross (1985), and Vasicek (1977). Once the possible future paths of short rates has been laid out, the expected level of prepayments are estimated for each node. This step involves a prepayments model which can accurately estimate the level of refinancing and prepayments for each level of rates. Once this has been accomplished, a probability weighted estimate of present value can be calculated, along with duration and convexity. This process leaves room for errors at every step. Assumptions must be made about volatility, the diffusion process of interest rates, and the appropriate factors affecting the level of prepayments for each MBS. While the body of knowledge in this area is large, and the number of possible models enormous, the series selected for this study should be considered among the best and most often used.

No direct comparison has been done between empirical measures of duration, or for that matter with more sophisticated multi-factor models. A logical extension of these studies is a comparison of several duration measures for use in hedging. In this paper, we will attempt to contribute to the literature by examining the hedging benefits of several of the alternative empirical duration models. The following measurements of interest rate risk will be tested and compared for use in alternative hedging portfolios:

1. Empirical measure as calculated by DGZ on a stretching regression basis[7]
2. Empirical measure as calculated by DGZ on a rolling regression basis
3. 30 Day empirical elasticity
4. 40 day empirical elasticity
5. 2-factor model with short and long rate elasticities similar to Breeden[1994]
6. A single factor model using 30 day elasticity, but hedging for parallel and non-parallel shifts
7. Analytical model estimate from Salomon Brothers
8. Analytical estimate from JP Morgan

These results will be useful in that while some of these variables have been looked at individually in the past, no study has yet looked at this wide a selection of alternatives.

In addition, no study has looked at the benefits of accounting for both parallel and non-parallel shifts simultaneously.

III - Data:

Mortgage prices for Federal National Mortgage Association 6.5, 7, 7.5, 8, 8.5, 9, 9.5, and 10% coupons were obtained from Bloomberg. Pricing observations are daily between October 1990 and March, 1996. Option adjusted duration measures were obtained from Salomon Brothers[8], and JP Morgan[9]. Treasury futures settle prices were obtained for the 2-, 5-, 10- year, and long bond contracts. The cheapest to deliver bond was calculated as per Hull and White[10] from Treasury security prices obtained from Bank Credit Analyst. Interest rates used are constant maturity yields from the U.S. Federal Reserve H.15 release. All empirical measures of duration are calculated using daily data, but hedging performance and duration adjustment is estimated on a weekly basis between 1990(1) and 1996.

IV - Methodology:

A portfolio of MBS can be hedged for one of two reasons: to lock in a risk free return by going short in the futures market on a duration neutral basis, and to go long or short in the futures market in order to target a particular interest rate exposure or duration. While there are several other ways of hedging, namely with interest rate floors and caps, a duration hedge in the futures market is easier to monitor as it is a market settled security as opposed to a harder to value OTC security. For the purpose of analyzing a particular hedge strategy, it is more practical to use a duration neutral strategy. The optimal hedge (in the sense of risk minimization)[11] is the one which produces the smallest mean squared error. A hedge which produces a zero average return, but a high mean squared error is of little use to a portfolio manager who may need to liquidate a position at any time. In addition, since futures are settled daily, there should be no surprise payments at settlement time due to a poor hedge.

For the simple one variable duration measures, the choice of hedging vehicle is important. For example, if a 2 year duration mortgage is hedged with a duration equivalent bond futures contract, the hedger is being exposed to curve reshaping risk. A flattening or steepening curve can affect the hedged position significantly. A multi-variable measure of duration will require hedging with more than one contract (e.g. a short and a long maturity contract). Increasing volatility can affect the intermediate section of the curve by increasing the (absolute) value of the butterfly spread. In this case, a combined 2-30 futures position would outperform a 10- or 5- year bulleted hedge because yields on the intermediate section would increase relative to the short and long end.[12]

The general procedure followed for evaluation of each strategy is summarized in the following four steps:

1. Estimate Duration
2. Calculate the hedge ratio
3. Calculate the holding period return for the hedged portfolio
4. Rank duration measures based on the lowest mean squared error of holding period returns

IV-1 DGZ Estimates

Equation 7 is rearranged by dividing through by Δr to give the duration estimate:

$$D = b_1 + b_2(P-100) + b3\left[(P-100)^2\right]K_t \qquad (10)$$

Where K_t is a dummy variable which is equal to 1 when the price is greater than 100. This measure will be estimated on daily data on a rolling basis so as to provide as long a hedging period as there is for the elasticity measures. This measure will be estimated in two ways. The first is to use all data available from t=1 to t, while the second is to only use the previous 120 trading days worth of data. This will allow us to control for the possibility of stale data influencing later estimates of duration.

IV-2 Single Factor Elasticity

Elasticity measures are the most simple of the duration estimates provided here. Equation 4 will be estimated by a time series approach. Since the assumption has been made that duration shifts in MBS are frequent, these estimates must only use the most recent data. Based on interviews at two major dealers, it was decided to' use two different measures of elasticities. Both a 30-day and 40-day estimate will be made of D in the following equation:

$$\Delta P_t = \alpha + \beta \frac{\Delta r_t}{(1+r_t)} + \varepsilon_t \tag{11}$$

Where e_t is a random error term. The β estimate is our proxy for -D (duration).

IV-3 2-Factor Duration Model

A two factor model will use one short rate, and one long rate for an estimate of partial durations. This will be of the form:

$$\Delta P / P = -D_s \Delta r_s + -D_L \Delta r_L \tag{12}$$

Where D_S is the partial duration with respect to short rates and D_L is the partial duration with respect to long rates. Because of the embedded option, these partial durations are not additive to provide a whole duration. They must simply be interpreted as two factor sensitivities. This will be estimated on a rolling basis with only the most recent observations so as to allow for changes in duration measures through time. Each partial duration estimate will then be hedged using both 2-year and 10-year futures contracts.

Hedging positions for this portfolio were calculated in the following way: Since the assumption is made that the percentage change in price with respect to changes to the 2-year yield is independent of changes with respect to changes in the 10-year yield, the dollar duration exposure of the 2-year and 10- year positions are calculated and hedged separately. The resulting hedge ratio is:

$$\frac{DM_2 \times P_m}{DF_2 \times F_2} + \frac{DM_{10} \times P_m}{DF_{10} \times F_{10}} \tag{13}$$

Where DM is the partial duration of the mortgage with respect to the 2-year yield, P_m is the price of the mortgage, DF is the duration of the futures contract, and F is the futures settle price.

IV-4 Slope Hedge

The Litterman, Sheinkman and Weiss paper will be extended to create a two factor hedge which incorporates changes in levels, as well as slope. The daily 10-year rate will be used for changes in the level of rates. The relationship will be estimated as follows:

$$\Delta P / P = -D_L \Delta Y_{10} \tag{14}$$

where, D_L is the elasticity with respect to parallel changes in the yield curve. The difference, however, will be in the hedging. A portfolio of 2- and 10- year futures contracts will be created which minimizes the exposure to changes in the slope of the yield curve. (See Appendix 1). Since this portfolio has duration risk, it will be levered so as to match the duration of the MBS. The result should be a duration neutral hedge which is insensitive to linear changes in the slope of the yield curve. The hedge ratio involves a short position in a 2-year and 10- year futures contract with a weighting (W) in the 2-year contract determined as follows:

$$W = \frac{D_L^2 - D_M^2}{D_L^2 - D_S^2} \tag{15}$$

where D_L D_M and D_S is the duration of the 10-year futures contract, the mortgage and the 2-year futures contract. This combined futures position itself has duration. The duration of the combined portfolio is offset by the duration of the MBS to create a portfolio which should minimize exposure to parallel changes and changes in the slope of the yield curve. (See Appendix 1 for a proof of the futures weighting procedure)

IV-5 Analytical Estimates

Analytical estimates of duration will be used 'as is' to conform with previous studies using these measures. These are from Salomon Brothers, and JP Morgan. Both are calculated using a proprietary prepayments model.

IV-6 Hedge Ratios

Hedge ratios will be calculated using each duration measure and equation 6. These ratios will be adjusted each week based on new estimates of duration.

An alternative method for controlling duration risk, which eliminates inefficiencies or idiosyncrasies within the futures market, is the short hedging with actual Treasury securities. To hedge in this manner, the MBS holder would short an equivalent duration Treasury security. The advantage to this hedge is that the duration of the underlying hedge is not dependent on an uncertain basket of deliverable securities. For example, when rates are below 8% and the yield curve is upwards sloping, the cheapest to deliver Treasury bond is the 11¼% of 2/15/2015 , whereas when rates are above 8%, the cheapest to deliver bond can have a maturity of between 15 and 30 years. This translates to a relatively large shift in duration which may skew the results of a duration neutral hedge. The downside, of course, is that this method is rarely used because of the transaction and financing costs involved, and short sale limits. Nevertheless, for the purpose of measuring MBS duration, it will help confirm the best measure. The hedge ratio will be the same as that of the futures contract:

$$HR = {D_M}/{D_T} \tag{17}$$

Where D_M is the estimated duration of the MBS and D_T is the duration of the treasury security, or treasure futures contract.

IV-7 Holding Period Return

Holding period return will be calculated with the standard equation:

$$\text{Holding Period Return} = ({P_{Mt}}/{P_{Mt-1}} - 1) - H \times ({P_{ft}}/{P_{ft-1}} - 1) \tag{18}$$

where, P_M and P_f are the prices of the mortgage and the futures contracts, and H is the hedge ratio. Each strategy will then be ranked according to Mean Squared Error, or the variance of the returns. The duration measure which provides the lower MSE will be considered superior. For one factor hedges, both the 30-year and 10-year futures contracts are tested for hedging.

A Kruskal-Wallis means test was employed to determine if there was a significant difference between each of the mean squared errors of hedged returns. This is similar to an F-test, and employs a chi-squared distribution. T-tests would not be appropriate because of the non-normal distribution of these means This is due to the fact that the mean squared error cannot be less than zero.

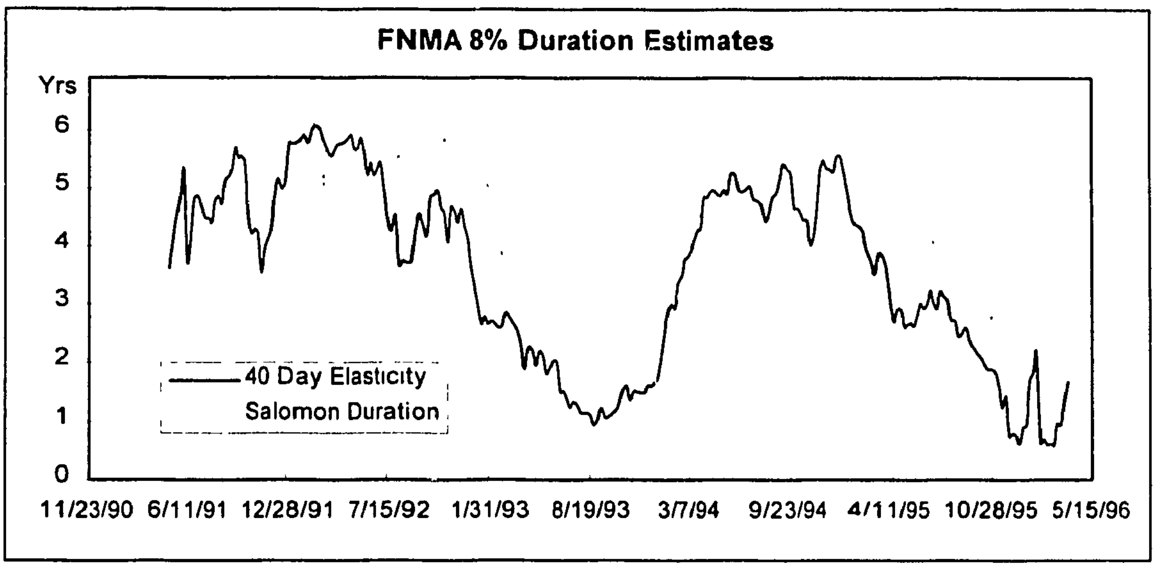
All empirical duration measures will be calculated out of sample. This means that any duration measure at time t will be a function of price behaviour from $t=1$, to t. These hedge ratios are used to hedge from period t to $t+1$ (one week). In the case of the elasticity measures, only the previous 30 or 40 days are used. No hedges are performed with in sample data. DGZ estimates are made on a rolling and a fixed interval basis. The fixed interval basis, all data points between $t=1$, and t will be used, whereas the rolling method uses the previous 180 observations.

Transaction costs were not considered in this study for several reasons. The main reason is due to the fact that these costs will have to be undertaken under each hedging strategy and will result in adding a constant fixed amount to the total return of each hedged portfolio, not affecting the results. The exception to this is with the hedged portfolios that use two futures contracts. It is arguable, however, that a practical application of these hedging strategies can legitimately be applied on a marginal basis. What this means is that the net effect of switching from one duration hedge to another is merely an incremental adjustment in the quantity and mix of hedging vehicles. It is a reasonable assumption to believe that a holder of a large portfolio of securities uses several instruments to hedge the duration. A shift in methodology to the use of a 2 contract hedge merely implies purchasing more of one type of futures contract and less of another. Because a large balance of MBS are hedged in aggregate portfolios, it is considered as arbitrary to add specific costs as it is top exclude them. In addition, almost all previous studies which are reviewed in this thesis do not incorporate transaction costs.

V - Results:

Attempts to account for empirical negative convexity provided insignificant estimates in approximately 75% of the observations. In addition, estimates of sensitivity to changes in the butterfly spreads showed insignificant results in roughly 80% of the estimation periods.

Chart I - Duration Estimates



Based on hedging performance, it is evident that the empirical measures provide better hedge ratios than those produced from analytical models. For every coupon, the Kruskal-Wallis means test indicated that the mean squared errors were significantly different even at the 97½% confidence interval. With a few exceptions, the 2-contract slope and duration neutral hedge provided the lowest mean squared error. There was no case in which the analytical durations provided better hedge ratios than the empirical measures.

This is consistent with previous research,(e.g. Breeden, 1994) that showed that empirical measures were the best predictor of future elasticities. Unfortunately, the only assertion that conclusively can be made regarding dealer estimates of duration, is that these empirical duration measures are better than analytical ones from two of the major Wall Street investment dealers. There are, theoretically, as many analytical measures as there are analysts and econometricians. Nevertheless, Salomon and JP Morgan are among the most reputable for their expertise in the field of fixed income, so these measures should be considered among the best. The most successful hedging vehicle is the combined hedge of 2- and 10-year futures contracts weighted to hedge against both parallel shifts and changes in the slope between 2- and 10- years. These hedges are found to be most successful for the low coupon mortgages, whereas the simple one contract hedge for elasticities proved the most successful for coupons over 9%. The slope of the yield curve has had the most dramatic shifts between 1993 and 1994 since the late 1970's and the early 1980's. This adds intuitive evidence as to why slope hedging should add to the efficacy of shielding MBS from interest rate risk. Since high coupon mortgages have very low durations, it is likely that they are sensitive only to parallel changes in the yield curve. Changes in the slope at the very short end of the yield curve (less than 2 years) do not have a substantial effect on the value of the MBS. Likewise, relative changes between the 2-year and 10-year yields will likely have almost no effect on the value of the security since relatively few cash flows are expected to be received later than 2 years in the future.

For one factor hedges, the most successful was the 10-year US Treasury futures contract, followed by the 30-year contract, and lastly by the short sale of a 10-year constant maturity Treasury. This is consistent with anecdotal evidence which suggests that the 10-year contract is the most frequented hedging vehicle for mortgage securities. It stands to reason that the more effective hedge should generally be the one which matches duration closest. Although all MBS used in this thesis were 30 year mortgages, the duration is almost always below 10 years. As a result, it makes good intuitive sense that the price of the mortgage, and the hedged returns, should be more responsive to changes in the 10-year interest rate than the 30-year interest rate. The other frequent hedging vehicle is the interest rate floor and collar, but these are much more expensive and should be considered out of reach for the majority of MBS holders.[13]

Chart II - Slope of US Treasury Yield Curve

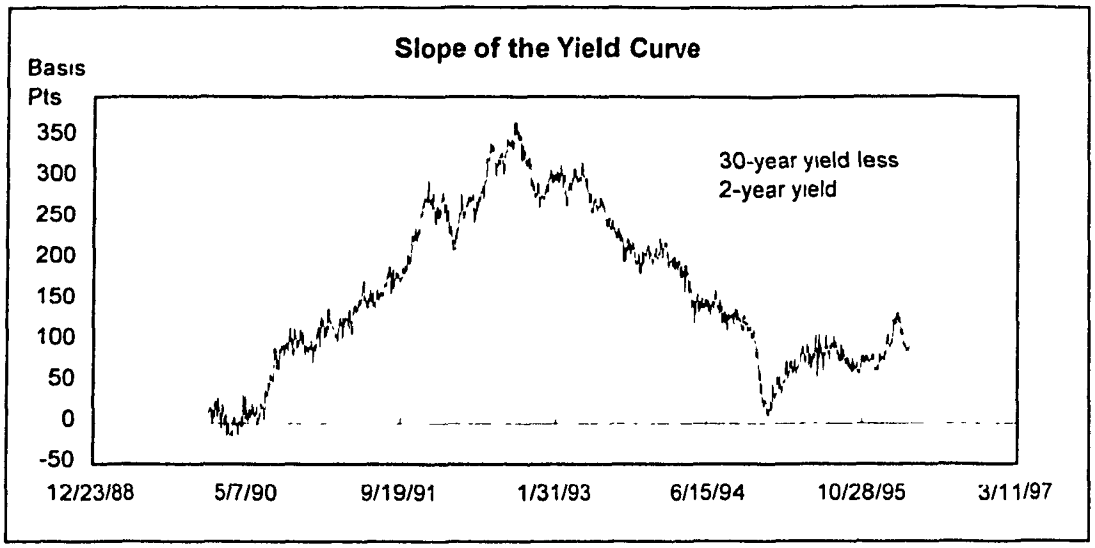


Chart II shows the large change in the slope of the yield curve during the sample period. Clearly, hedging effectiveness will be affected by changes in the slope of the curve. Unfortunately, the 2-factor partial duration model did not outperform the one factor elasticities, but did generally outperform the dealer estimates of duration. This may be due to several factors. First, partial durations for mortgage securities are not additive, so an additive hedge may not be appropriate against re-shaping of the yield curve. Secondly, the 2-year futures contract may not be an appropriate hedging vehicle, because of the lack of significant volume and open interest. Thirdly, the 2-year yield is the most volatile in the money market which, again, provides poor functionality for hedging purposes. Speculation about Fed policy is generally done with 2-year Treasuries and futures. This again, adds to the volatility of the hedged position. When the one factor measures of duration are used, with a 2 contract hedge, however, significant improvement in hedging performance is observed as was mentioned above. These 2-contract hedges used the 40-day elasticity measures. This type of hedge seems to be the most promising for future research on duration hedging.

VI - Conclusion:

For the time period analyzed, it is apparent that the information conveyed in the durations of reputable investment houses do not outperform those generated using market data. Empirical durations are gaining popularity now among many fund managers, who had underestimated the excessive prepayments in 1993, and had not forecasted the consequences to the detriment of their portfolios. Had they rebalanced their portfolios using empirical measures of duration, instead of analytical estimates, it is arguable that there would have been less significant under-performance of mortgage securities in these two years. It is noticeable that in both 1993 and 1995, the analytical model estimates of duration are considerably higher than the 30-day elasticity estimate. There is a practical explanation for this. Since the analytical models are backwards looking, they must try to explain future mortgage prepayments based on historical data. In 1993, and 1995, the costs of refinancing a mortgage decreased dramatically[14]. Because of this, the option for the mortgagor to refinance becomes worth more since the strike price of the option is effectively decreased. For example, even though a mortgagor could refinance when rates are lower than the agreed upon financing rate, the mortgagor will not refinance because there is an additional cost of prepaying the mortgage early. It is only when the benefit of refinancing exceeds the extra cost of refinancing that the mortgagor will consider renewing the mortgage at a lower rate. Since these costs have declined, interest rates have to move by a smaller amount than previously in order for the mortgagor to prepay the mortgage. This increase in the value of the option will decrease the duration of the MBS. It is likely that the dealer analytical models did not anticipate this change, whereas the decreased costs were immediately reflected in the market estimates. (See chart I)

There are many avenues along which this research can and should be expanded. This study was limited to the use of duration hedging, and not convexity. From a partial equilibrium perspective. this negative convexity can be hedged, but from a full equilibrium perspective, there is a net amount of negative convexity in the market, since the mortgagors are the possessors of the positive convexity. This means that all negative convexity of mortgages cannot be hedged in the marketplace. The result, is an increase in the cost of convexity, or the cost of options until those who wish to hedge are satisfied. Avoiding extreme losses due to negative convexity can be minimized by the use of frequent duration hedging.

The addition of more derivatives for hedging, i.e. caps, collars, floors, CMT futures, OTC futures and options, all make it possible to hedge more effectively. Curve reshaping will be an issue during specific periods within the business cycle (i.e. shifts and reversals in Fed policy), and can be appropriately hedged using these derivatives. The lack of a centralized market and standardized contracts makes it very difficult to conduct econometric research on these instruments. Changes in the mortgage industry microstructure and investor fears about prepayments have increased the perceptive riskiness of these instruments. Traditional analytic models are failing to anticipate these changes, and as a result, investors are becoming more dependent on market aggregate estimates of durations. These issues remain topics for future research.

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Endnotes

[1] Fabozzi, 1992

[2] In some cases, prepayment may occur in a manner that does not minimize financing costs. Changing domiciles may impose further uncertainty to the exercise of the option.

[3] See Elton and Gruber, P. 539

[4] Ibid

[5] Burnout is defined as the degree to which principal amounts have already been prepaid in a pool. Older pools generally have a higher degree of burnout, and are subsequently expected to have lower levels of prepayments in the future. Conversely, pools with low burnout are much more likely to have prepayments.

[6] IO and PO stand for interest only and principal only. These mortgage derivatives consist of payment streams from either interest payments or principal payments, but not both. They are highly sensitive to changes in interest rates and are considered very risky.

[7] This is explained further in the methodology section

[8] Bond Market Roundup: Abstract 1989-1996

[9] Mortgage-Backed Sector Report 1992 1996

[10] Hull and White, *Derivatives*, Cheapest to deliver calculations.

[11] Langowski, Park and Switzer (1996) develop an alternative means of optimizing the hedge. Their optimal hedge combines derivative securities to minimize the transactions costs of hedging a portfolio that is duration immunized that allows for some positive overalll convexity.

[12] Litterman, Sheinken and Weiss,1991

[13] A Wall Street senior trader suggested in a personal interview that no dealers on 'the street' hedge with collars and floors because they realize how expensive they are relative to futures and options. They are more so a money making scheme than an effective hedging vehicle.

[14] Source: Federal Reserve Data on aggregate point costs for taking out mortgages

Appendix 1: Proof for Yield Curve Slope Immunization

Hedge ratio for changes in the slope of the yield curve

A duration neutral butterfly consists of a long(short) position in a medium maturity security and a short(long) position in a combination of a very short and a very long maturity security. The weighting is a linear averaging of the short and long maturities totaling the duration of the medium term security. The weight for the short term security is determined as follows:

$$W * D_s + (1 - W) * D_L = D_M$$

Where, D is the duration of the short, medium and long term security (s,m and l). Solving for W yields the following:

$$W = \frac{D_L - D_M}{D_L - D_S}$$

A slope neutral position must proportionately match returns of the barbell and the bullet position, not durations. W then must satisfy the following:

$$W * D_s \Delta r_s + (1 - W) * D_L \Delta r_L = D_M \Delta r_m$$

If we define the slope of the curve as the partial derivative of the yield curve with respect to duration as S, then the total return for any security given a change in the slope of the curve is,

$$D * (DS_{new} - DS_{old})$$

or

$$D^2 (S_{new} - S_{old})$$

since DS gives the nominal yield for any point in the curve. Substituting this into equation 2 and dividing both sides by (S_{new}-S_{old}), the return minimizing hedge for changes in the slope of the curve then becomes:

$$W = \frac{D_L^2 - D_M^2}{D_L^2 - D_S^2}$$

where W is the weight on the short duration security, resulting in a weight of (1-W) for the longer security.

Appendix 2: Tables of Hedging Results

Table II - 8: FNMA 6.5% Coupon Hedged Returns*

Coupon:	FNMA65							
	DGZ	Rolling DGZ	30-Day	40-Day	2-Factor*	Slope Hedge	Salomon	JP Morgan
CMT Hedge								
Average Hedged Weekly Return	0 026%	-0 007%	0 036%	0 038%	0 040%	0 012%	0.051%	0 057%
Mean Squared Error	0 769%	0 642%	0.904%	0 934%	1 066%	0 467%	0 994%	1 000%
10-Year Futures Hedge								
Average Hedged Weekly Return	0.034%	-0 033%	0.035%	0 017%	NA	0 037%	0 059%	0 055%
Mean Squared Error	0 746%	0.799%	0.781%	0.830%	NA	0.877%	0.906%	0 896%
30-Year Futures Hedge								
Average Hedged Weekly Return	0.025%	-0 043%	0.015%	0.005%	0 028%	NA	0 031%	0 031%
Mean Squared Error	0 780%	0.630%	0.806%	0.845%	0 940%	NA	0 923%	0.916%
Unhedged MBS Return	0.045%							
Mean Squared Error	1 081%							

* Significant Difference in MSE at 97 5% significance level

** Hedge involves short 2- and 30-year securities

Table II - 8: FNMA 7% Coupon Hedged Returns*

Coupon:	FNMA70							
	DGZ	Rolling DGZ	30-Day	40-Day	2-Factor**	Slope Hedge	Salomon	JP Morgan
CMT Hedge								
Average Hedged Weekly Return	-0.007%	0.017%	0.003%	0.003%	-0.006%	0.002%	0.015%	0.001%
Mean Squared Error	0.669%	0.625%	0.637%	0.644%	0.747%	0.413%	0.600%	0.627%
10-Year Futures Hedge								
Average Hedged Weekly Return	-0.004%	0.025%	-0.004%	0.000%	NA	0.009%	-0.009%	-0.015%
Mean Squared Error	0.522%	0.484%	0.485%	0.512%	NA	0.481%	0.554%	0.499%
30-Year Futures Hedge								
Average Hedged Weekly Return	-0.023%	0.006%	-0.019%	-0.021%	-0.023%	NA	-0.034%	-0.028%
Mean Squared Error	0.575%	0.498%	0.507%	0.523%	0.560%	NA	0.549%	0.521%
Unhedged MBS Return	-0.015%							
Mean Squared Error	0.728%							

* Significant Difference in MSE at 97.5% significance level

** Hedge involves short 2- and 30-year securities

Table II - 8: FNMA 7.5% Coupon Hedged Returns*

Coupon:	FNMA75 DGZ	Rolling DGZ	30-Day	40-Day	2-Factor*	Slope Hedge	Salomon	JP Morgan
CMT Hedge								
Average Hedged Weekly Return	0.046%	0 043%	0 027%	0 035%	0 025%	-0 018%	0 041%	0.034%
Mean Squared Error	0 571%	0.575%	0.580%	0 585%	0.651%	0.376%	0 588%	0 597%
10-Year Futures Hedge								
Average Hedged Weekly Return	0.027%	0.014%	-0.005%	0 012%	NA	0.015%	0 018%	0 025%
Mean Squared Error	0.437%	0.430%	0 441%	0.445%	NA	0.416%	0.493%	0.539%
30-Year Futures Hedge								
Average Hedged Weekly Return	0.022%	0 008%	-0.007%	0.007%	-0.008%	NA	0.007%	0 018%
Mean Squared Error	0.467%	0.454%	0.462%	0.462%	0.490%	NA	0.510%	0.552%
Unhedged MBS Return	0.025%							
Mean Squared Error	0.643%							

* Significant Difference in MSE at 97.5% significance level

** Hedge involves short 2- and 30-year securities

Table II - 8: FNMA 8% Coupon Hedged Returns*

Coupon:	FNMA80							
	DGZ	Rolling DGZ	30-Day	40-Day	2-Factor*	Slope Hedge	Salomon	JP Morgan
CMT Hedge								
Average Hedged Weekly Return	0.036%	0.033%	0.026%	0.029%	0.022%	-0.021%	0.030%	0.029%
Mean Squared Error	0 508%	0.513%	0.510%	0 518%	0.571%	0 390%	0.483%	0 526%
10-Year Futures Hedge								
Average Hedged Weekly Return	0.018%	0.011%	-0.008%	0.011%	NA	0 014%	-0 010%	0.020%
Mean Squared Error	0.395%	0 394%	0.400%	0 400%	NA	0 376%	0.482%	0.488%
30-Year Futures Hedge								
Average Hedged Weekly Return	0 015%	0 007%	-0.008%	0.008%	-0.008%	NA	-0.008%	0.016%
Mean Squared Error	0 420%	0 413%	0.418%	0.414%	0 439%	NA	0.495%	0 494%
Unhedged MBS Return	0 030%							
Mean Squared Error	0 558%							

* Significant Difference in MSE at 97.5% significance level

** Hedge involves short 2- and 30-year securities

Table II - 8: FNMA 8.5% Coupon Hedged Returns*

Coupon:	FNMA85							
	DGZ	Rolling DGZ	30-Day	40-Day	2-Factor**	Slope Hedge	Salomon	JP Morgan
CMT Hedge								
Average Hedged Weekly Return	0.025%	0.029%	0 017%	0 025%	0.021%	-0.022%	0.025%	0.026%
Mean Squared Error	0 436%	0 436%	0 439%	0.434%	0.477%	0 410%	0.409%	0 451%
10-Year Futures Hedge								
Average Hedged Weekly Return	0 008%	0 011%	-0.013%	0 013%	NA	0.026%	-0.008%	0 019%
Mean Squared Error	0 364%	0.352%	0 354%	0.343%	NA	0 338%	0.459%	0.434%
30-Year Futures Hedge								
Average Hedged Weekly Return	0.006%	0.007%	-0.011%	0.010%	-0.009%	NA	-0 006%	0.016%
Mean Squared Error	0 388%	0 369%	0 372%	0.354%	0.385%	NA	0 471%	0 436%
Unhedged MBS Return	0.029%							
Mean Squared Error	0 468%							

* Significant Difference in MSE at 97.5% significance level

** Hedge involves short 2- and 30-year securities

Table II - 8: FNMA 9% Coupon Hedged Returns*

Coupon:	FNMA90							
	DGZ	Rolling DGZ	30-Day	40-Day	2-Factor**	Slope Hedge	Salomon	JP Morgan
CMT Hedge								
Average Hedged Weekly Return	0.019%	0.022%	0 019%	0.020%	0 020%	-0.022%	0 020%	0 026%
Mean Squared Error	0.368%	0.365%	0.355%	0.363%	0.394%	0 426%	0 352%	0.371%
10-Year Futures Hedge								
Average Hedged Weekly Return	0.006%	0.008%	-0.011%	0.012%	NA	0.023%	-0.005%	0.022%
Mean Squared Error	0.316%	0.304%	0 305%	0 299%	NA	0 307%	0 425%	0 357%
30-Year Futures Hedge								
Average Hedged Weekly Return	0 003%	0 004%	-0.008%	0 009%	-0 010%	NA	-0.003%	0 019%
Mean Squared Error	0.333%	0 317%	0.321%	0 308%	0.338%	NA	0.436%	0 360%
Unhedged MBS Return	0.016%							
Mean Squared Error	0.385%							

* Significant Difference in MSE at 97 5% significance level

** Hedge involves short 2- and 30-year securities

Table II - 8: FNMA 9.5% Coupon Hedged Returns*

Coupon:	FNMA95							
	DGZ	Rolling DGZ	30-Day	40-Day	2-Factor*	Slope Hedge	Salomon	JP Morgan
CMT Hedge								
Average Hedged Weekly Return	0.020%	0 024%	0 019%	0 023%	0.029%	-0 022%	0.023%	0 027%
Mean Squared Error	0.252%	0 249%	0 251%	0.250%	0.260%	0.449%	0 250%	0 253%
10-Year Futures Hedge								
Average Hedged Weekly Return	0.012%	0 016%	0.007%	0.019%	NA	0.027%	0.007%	0 021%
Mean Squared Error	0.253%	0 247%	0.244%	0 238%	NA	0.255%	0 354%	0.284%
30-Year Futures Hedge								
Average Hedged Weekly Return	0.012%	0.015%	0.007%	0.017%	0 007%	NA	0.008%	0.020%
Mean Squared Error	0.255%	0.247%	0.247%	0.240%	0.260%	NA	0 358%	0.282%
Unhedged MBS Return	0.012%							
Mean Squared Error	0.251%							

* Significant Difference in MSE at 97.5% significance level

** Hedge involves short 2- and 30-year securities

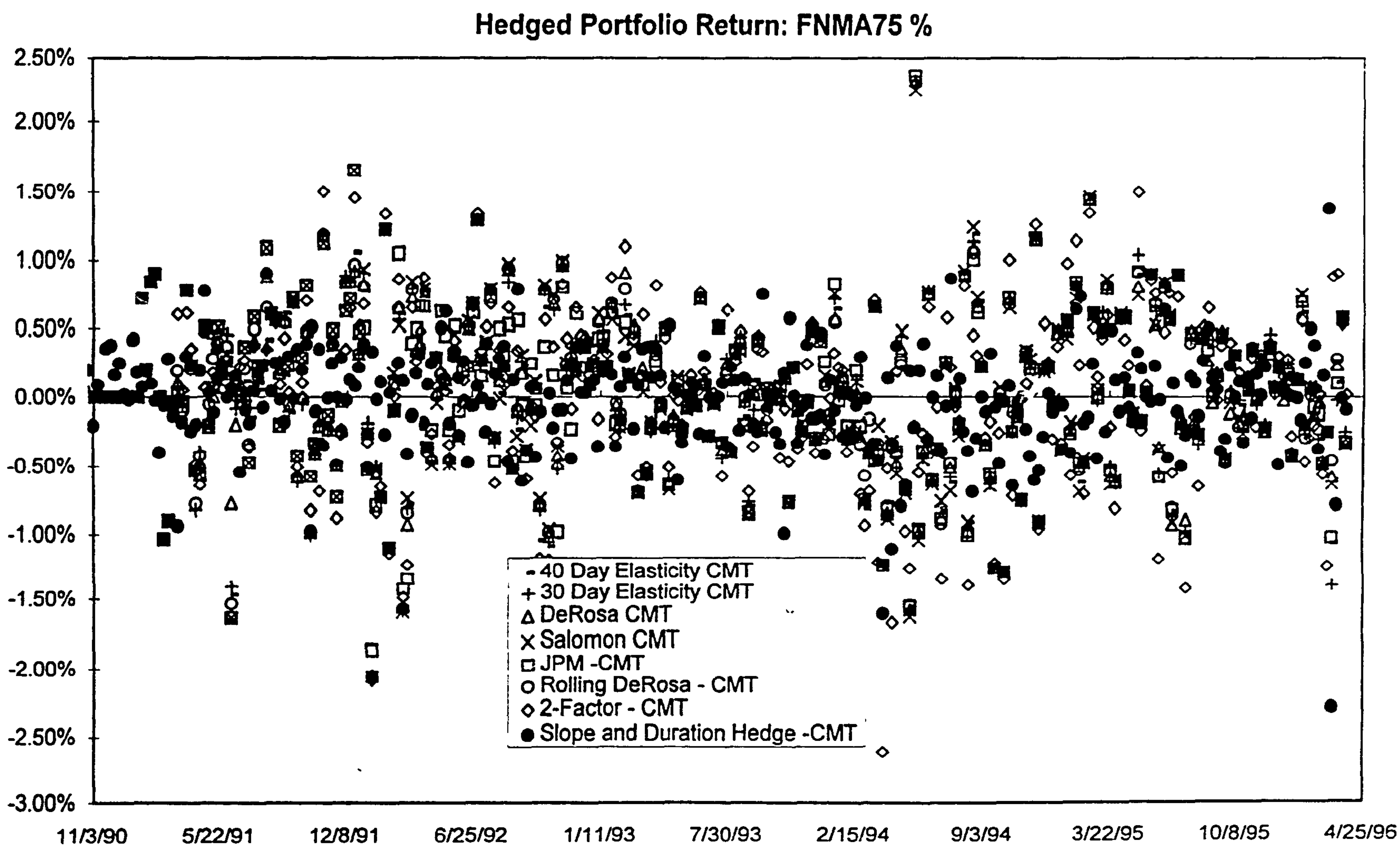
Table II - 8: FNMA 10% Coupon Hedged Returns*

Coupon:	FNMA10							
	DGZ	Rolling DGZ	30-Day	40-Day	2-Factor**	Slope Hedge	Salomon	JP Morgan
CMT Hedge								
Average Hedged Weekly Return	0.020%	0.024%	0.019%	0.023%	0.029%	-0.022%	0.023%	0.027%
Mean Squared Error	0.252%	0.249%	0.251%	0.250%	0.260%	0.449%	0.250%	0.253%
10-Year Futures Hedge								
Average Hedged Weekly Return	0.012%	0.016%	0.007%	0.019%	NA	0.027%	0.007%	0.021%
Mean Squared Error	0.253%	0.247%	0.244%	0.238%	NA	0.255%	0.354%	0.284%
30-Year Futures Hedge								
Average Hedged Weekly Return	0.012%	0.015%	0.007%	0.017%	0.007%	NA	0.008%	0.020%
Mean Squared Error	0.255%	0.247%	0.247%	0.240%	0.260%	NA	0.358%	0.282%
Unhedged MBS Return	0.012%							
Mean Squared Error	0.251%							

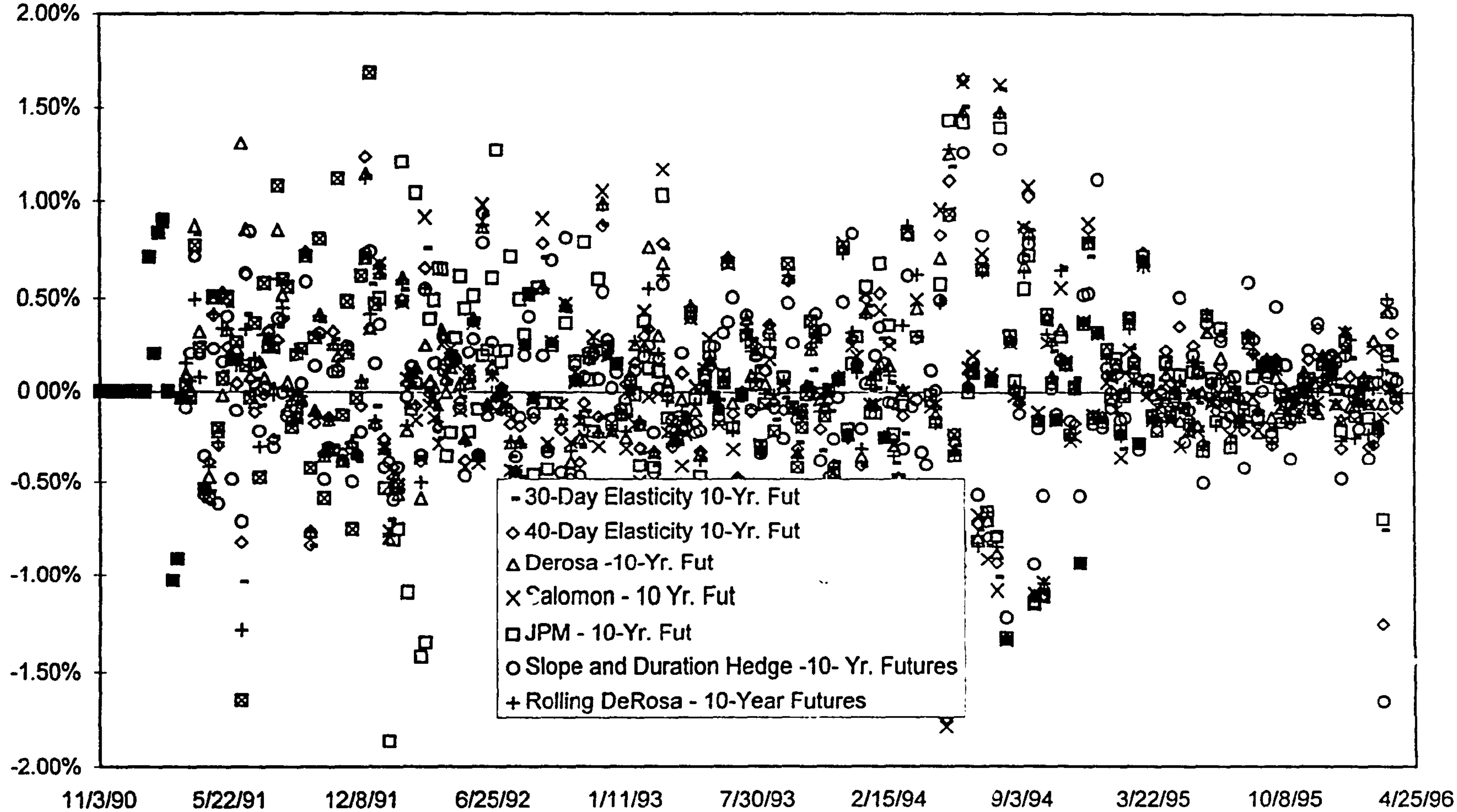
* Significant Difference in MSE at 97.5% significance level

** Hedge involves short 2- and 30-year securities

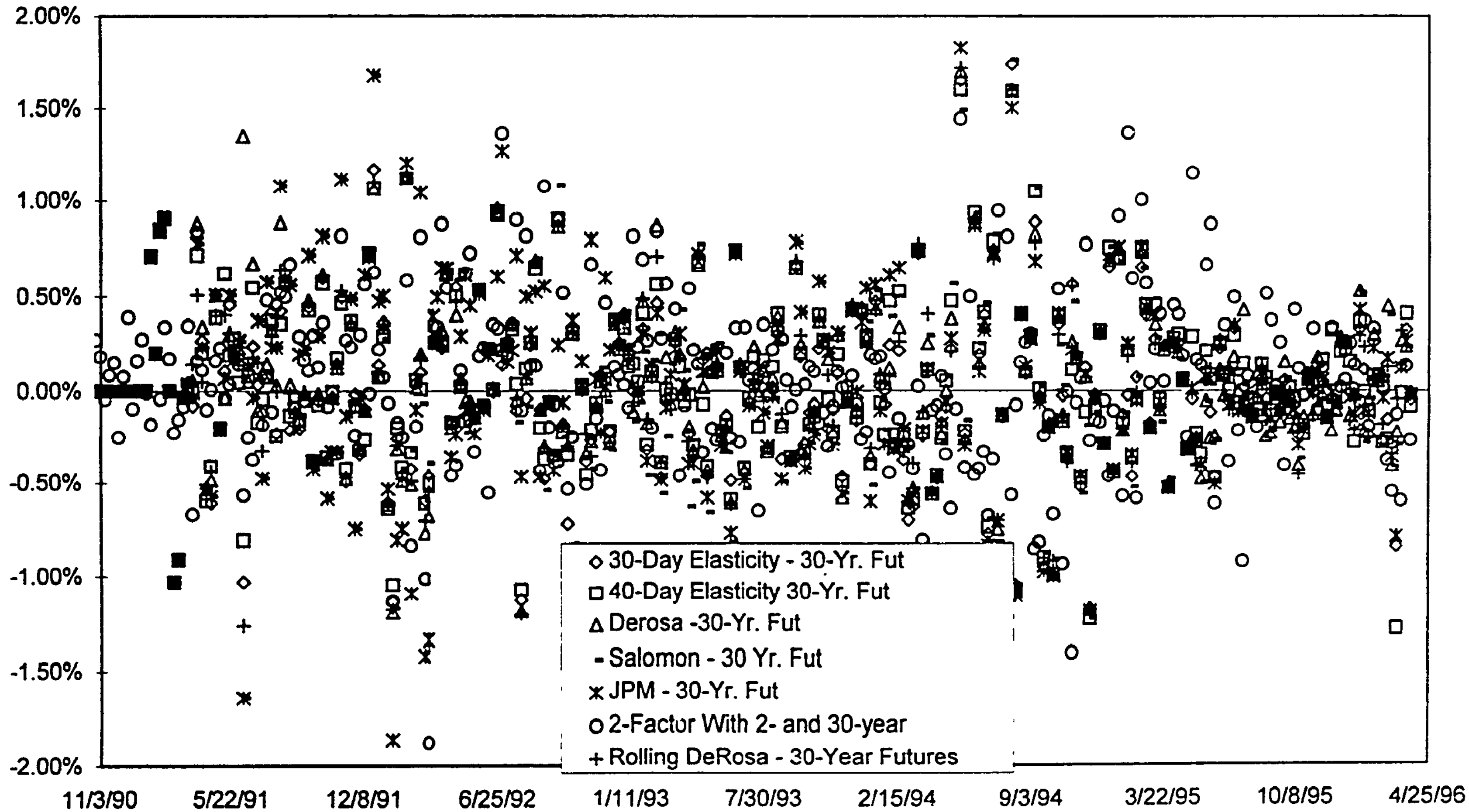
Appendix 3: Charts of Hedging Performance


Hedged Portfolio Return: FNMA75 %
2.50%
2.00%
1.50%
1.00%
0.50%
0.00%
-0.50%
-1.00%
-1.50%
-2.00%
-2.50%
-3.00%
11/3/90
5/22/91
12/8/91
6/25/92
1/11/93
7/30/93
2/15/94
9/3/94
3/22/95
10/8/95
4/25/96
40 Day Elasticity CMT
30 Day Elasticity CMT
DeRosa CMT
Salomon CMT
JPM -CMT
Rolling DeRosa - CMT
2-Factor - CMT
Slope and Duration Hedge -CMT

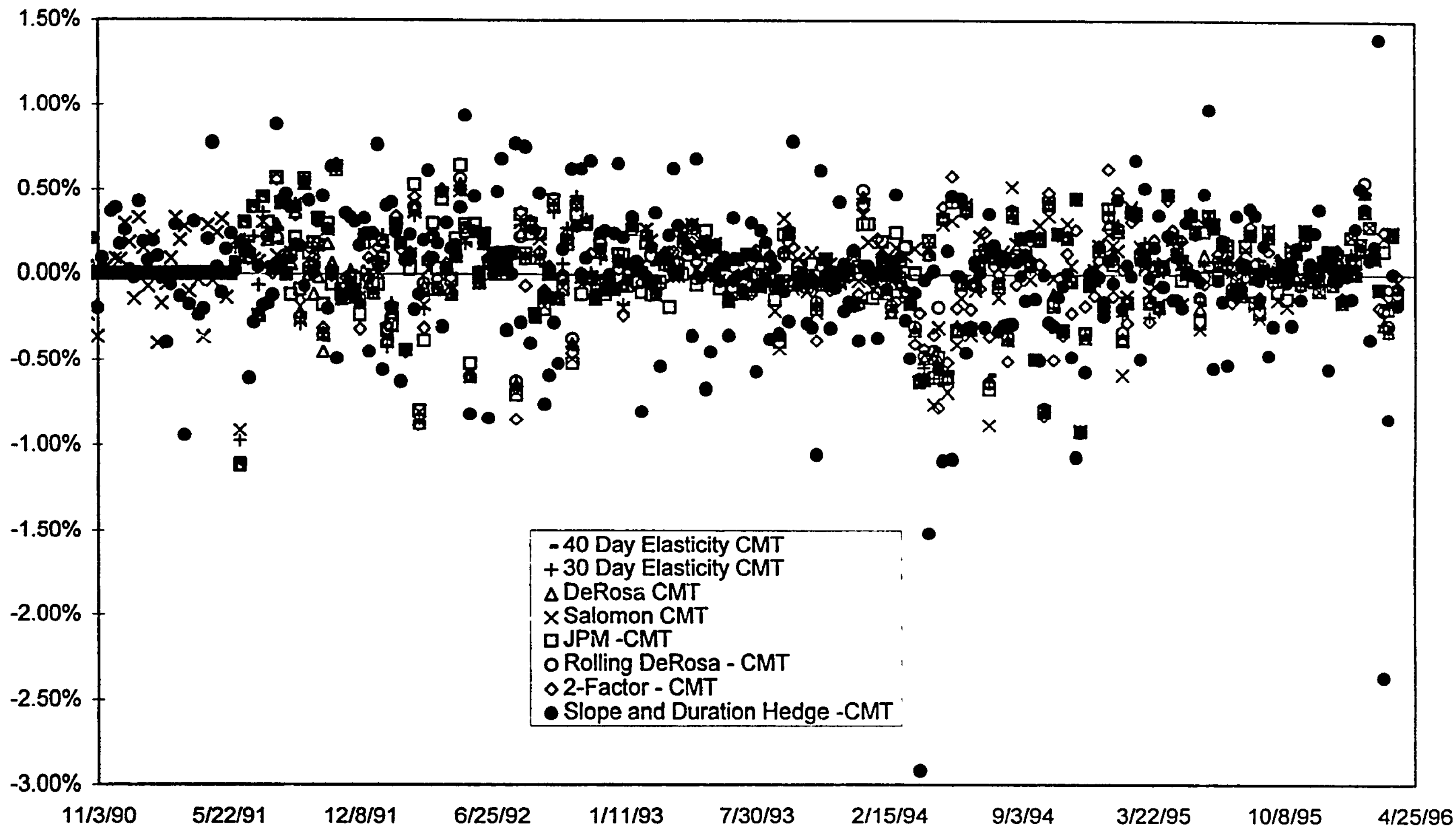
Hedged Portfolio Return: FNMA75 %

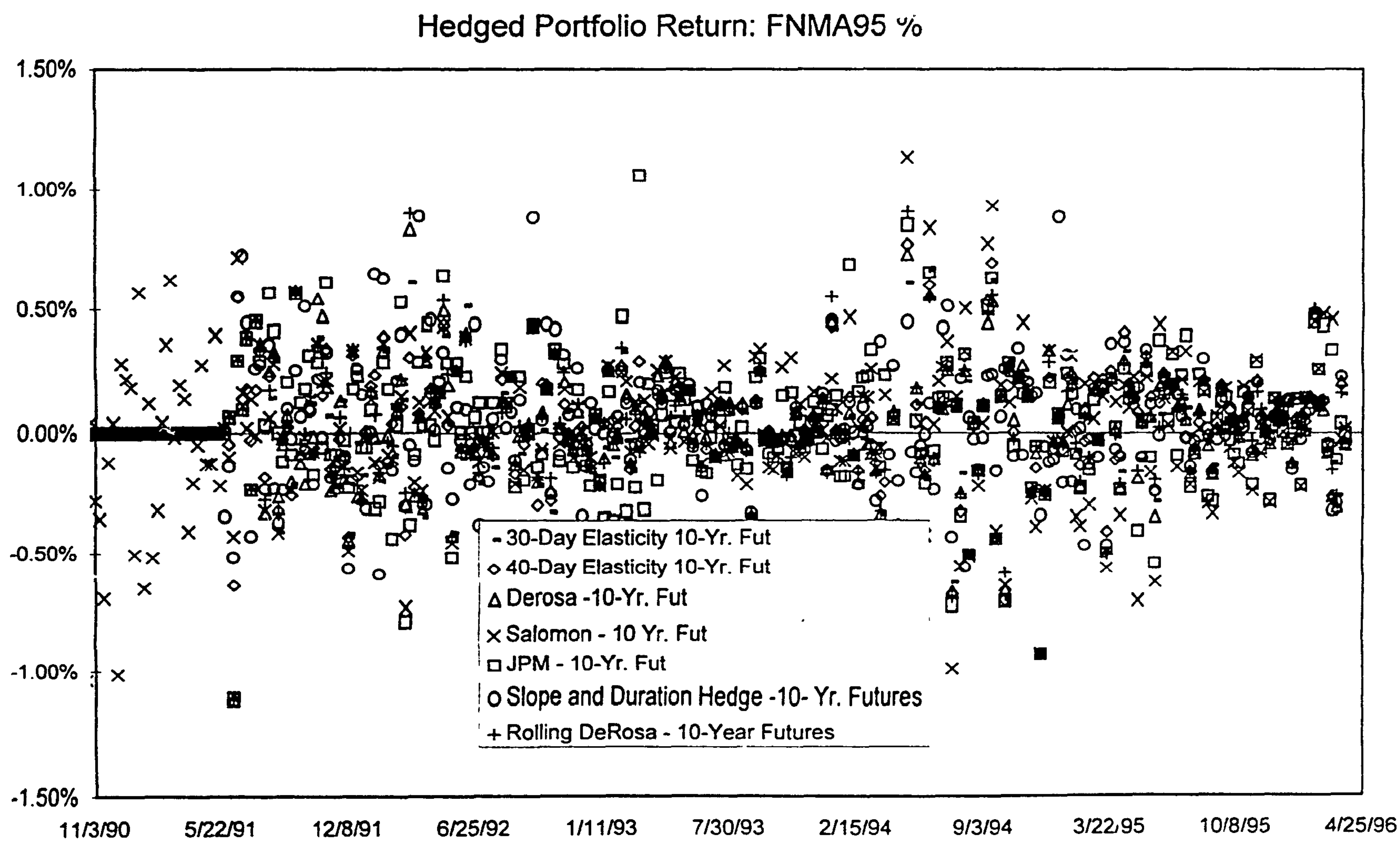


Hedged Portfolio Return: FNMA75 %



Hedged Portfolio Return: FNMA95 %





Hedged Portfolio Return: FNMA95 %
1.50%
1.00%
0.50%
0.00%
-0.50%
-1.00%
-1.50%
11/3/90
5/22/91
12/8/91
6/25/92
1/11/93
7/30/93
2/15/94
9/3/94
3/22/95
10/8/95
4/25/96
- 30-Day Elasticity 10-Yr. Fut
◇ 40-Day Elasticity 10-Yr. Fut
△ Derosa -10-Yr. Fut
× Salomon - 10 Yr. Fut
□ JPM - 10-Yr. Fut
○ Slope and Duration Hedge -10- Yr. Futures
+ Rolling DeRosa - 10-Year Futures

Hedged Portfolio Return: FNMA95 %