HYPERBOLIC PARABOLOIDAL BASED WATER TANK

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ABSTRACT

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The main achievements of this dissertation are the analysis of a hyperbolic paraboloidal shell by the membrane theory and the design of the reinforcement with respect to the analysis. Before the analysis, there is a brief account of the derivation of the membrane theory from the general basic shell theory. Then, the membrane theory is applied to the hyperbolic paraboloidal shell.

The hyperbolic paraboloidal shell concerned is the base of the water tank of a water tower. After the internal stresses and forces have been analyzed, the numerical values are solved by computer. Then the reinforcement of the shell is designed by the working stress design.

There is a brief account on the state of the arts of the design of the water tank.

Finally, there is a discussion on the advantages and disadvantages of the method of analysis and design.

DEDICATION

To my parents

& betrothed

ACKNOWLEDGEMENTS

ACKNOWLEDGEMENTS

The writer would like to express his gratitude to his supervisor, Dr. Z.A. Zielinski, for his encouragement and advice in the course of this dissertation.

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LIST OF NOTATIONS

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x, y, z	Directions of axes
N_x, N_y	Internal normal forces per unit length
N_{xy}, N_{yx}	Internal shearing forces per unit length
$\bar{N}_x, \bar{N}_y$	Projections of internal normal forces on the x-y plane
$\bar{N}_{xy}, \bar{N}_{yx}$	Projections of internal shearing forces on the x-y plane
Q_x, Q_y	Internal transverse forces per unit length
M_x, M_y	Internal normal moments per unit length
M_{xy}, M_{yx}	Internal twisting moments per unit length
r_x, r_y	Radii of curvatures
σ_x, σ_y	Internal normal stresses
σ_1, σ_2	Principal stresses
τ_x, τ_y	Internal shear stresses
u, v, w	Displacements in x, y, z-directions, respectively
E, G	Modulii of elasticity and rigidity
ν	Poisson's ratio
ϵ_1, ϵ_2	Strain of middle surface in x, y-directions, respectively
η_x, η_y	Change of curvatures before and after the deformation of the middle surface
η_{xy}	The change of twist of the middle surface
γ	Shearing strain at the middle surface
X, Y, Z	External loading in x, y, z-directions, respectively
ρ	Density of Water
p_c	Pressure due to concrete weight in z-direction

f'_c	Concrete strength
f_{ta}	Allowable tensile stress of concrete
f_{ca}	Allowable compressive stress of concrete
f_s	Allowable stress of steel
f_c, f_t	Internal concrete and steel stresses
t	Thickness of shell

CHAPTER 1

INTRODUCTION AND STATE OF THE ARTS ON THE DESIGN OF WATER TOWERS

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INTRODUCTION AND STATE OF THE ARTS ON THE DESIGN OF WATER TOWERS

Elevated water tanks are essential to water supply systems. In the early twentieth century, water towers were built for supplying water to a certain factory or a certain urban area. Water towers are especially important for emergencies, e.g., sudden shortage of regular water supply. Sometimes, they are also built for balancing the water flow rate at peak demand of small inflow. More recently, water towers were built with only economical considerations in view while the architecture was not taken into account. Nowadays, water towers are built according to modern architectural design trends, [1], therefore as well as functioning in urban usage, they also provide a pleasant view in the city.

Although every water tower is unique, they have quite a lot of characteristics in common. Most of them are circular in order to minimize stresses and bending moments. They usually have a main shaft at the centre supporting the tank on top.

There are two ways in which concrete water towers can be constructed. The first method is to build it with ordinary formwork. The other way is to construct the central shaft, and then build the tank at ground level. After this, the tank is lifted up to the top by jacking.

Water towers must be protected from water leakage, therefore, no cracks must be allowed. To achieve this, the tank has to be in working stress design. In cold areas, water should be prevented from freezing in order to prevent shortage of supply, as well as cracking.

Water towers can be built in either steel or concrete but, for economic reasons, most towers are built of concrete. In more recent years water towers have been built of reinforced concrete, but the modern ones are usually built of pre-stressed concrete. One of the advantages of pre-stressed concrete is that it can help to reduce cracks if designed properly.

The tank of a water tower is usually a shell structure. Shell structures can be analyzed either by analytical method or by a numerical technique called finite element method. In this project, the main emphasis will be on the practice of the analysis of shell structure. The method of analysis used will be membrane theory derived from the general shell theory. The material used will be reinforced concrete. The loading considered will be symmetrically acting gravity loads.

The main subject of this project is to analyze and design the hyperbolic-paraboloidal shell, which is the base of the water tank. The design of the beams and the shaft will not be included.

CHAPTER 2

THEORY OF SHELLS

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THEORY OF SHELLS

2.1 GENERAL CONCEPT AND NOTATIONS

A shell is a curved surface structure. This definition implies that the thickness "t" is small compared with its other dimensions. The examples of shell structures are: the soap bubble, the body of an aeroplane, the dome roof of a building, the cylindrical water tank, etc.

Consider a small element with dimensions dy x dx, which is cut out from a shell as shown in Fig. 1.

The notations are as follows:

x, y, z are mutually orthogonal.

u, v, w are components of displacement.

Nx and Ny are normal forces.

Nyx and Nxy are shear forces.

Qx and Qy are transverse forces.

Mx and My are normal moments.

Myx and Mxy are twisting moments.

All N, Q and M values are called stress resultants and are forces and couples per unit length of the middle surface of the shell.

r_x and r_y are radii of curvatures.

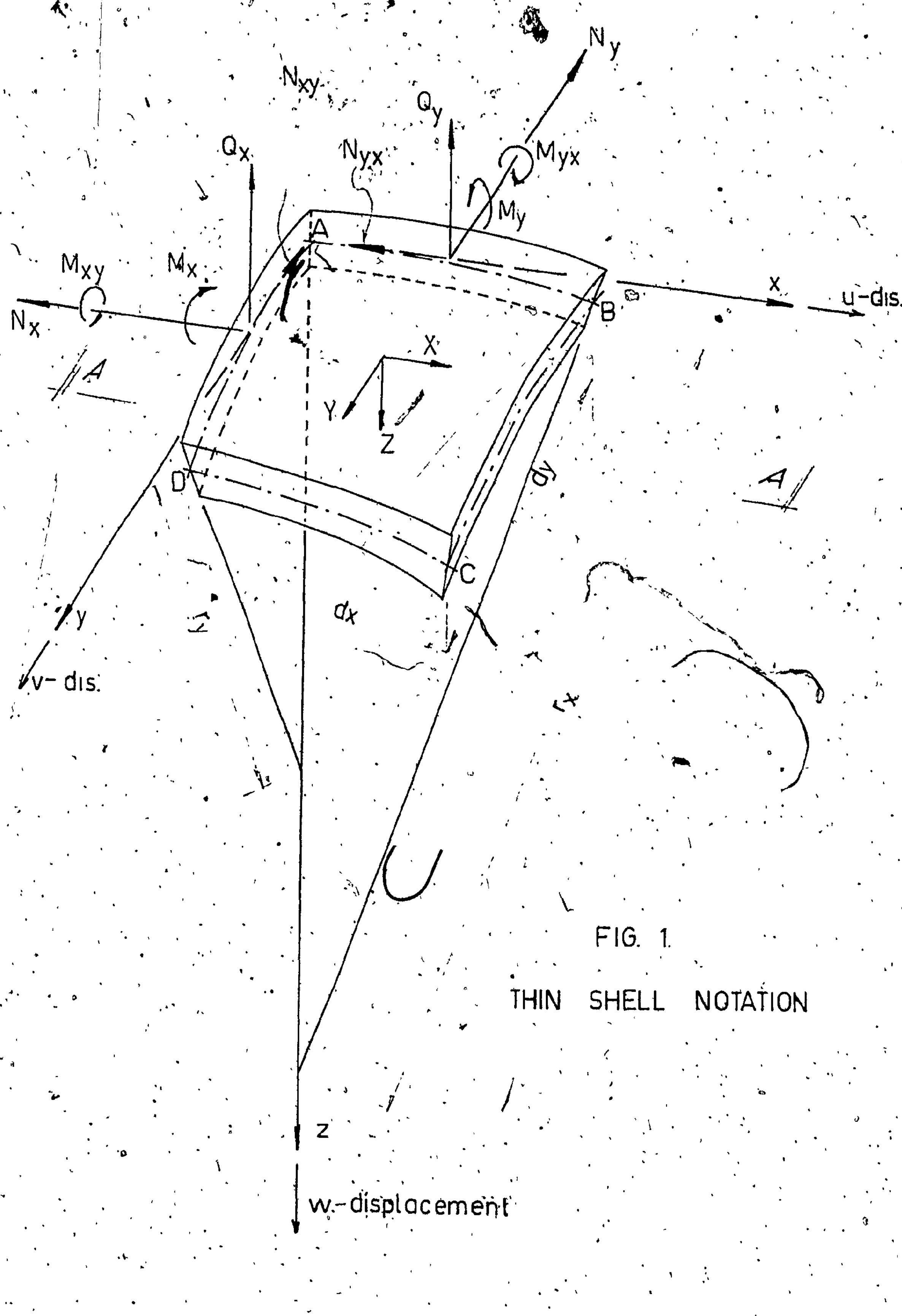


FIG. 1.

THIN SHELL NOTATION

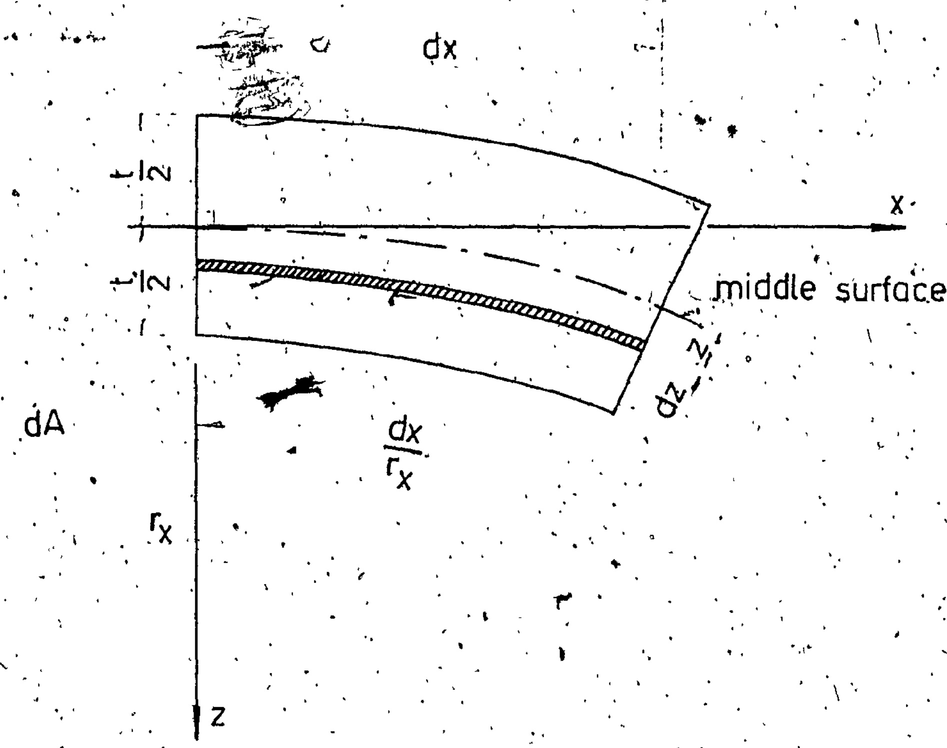


FIG. 2. SECTION A-A OF FIG. 1

2.2 THE RELATIONSHIP BETWEEN THE STRESS RESULTANTS AND STRESSES

Consider Figure 2, $dA = (r_x - z)\frac{dx}{r_x}dz$ (shaded part)

$$= (1 - \frac{z}{r_x})\,dx\,dz$$

$$Ny = \frac{1}{dx}\int_{-t/2}^{t/2} \sigma_y \, dA$$

where σ_y is the normal stress coming out of the paper.

$$Ny = \frac{1}{dx}\int_{-t/2}^{t/2} \sigma_y (1 - \frac{z}{r_x})\,dx\,dz$$

$$Ny = \int_{-t/2}^{t/2} \sigma_y (1 - \frac{z}{r_x})\,dz \qquad \ldots 1(a)$$

Similarly,

$$Nx = \int_{-t/2}^{t/2} \sigma_x (1 - \frac{z}{r_y})\,dz \qquad \ldots 1(b)$$

$$Nxy = \int_{-t/2}^{t/2} \tau_{xy} (1 - \frac{z}{r_y})\,dz \qquad \ldots 1(c)$$

$$Nyx = \int_{-t/2}^{t/2} \tau_{yx} (1 - \frac{z}{r_x})\,dz \qquad \ldots 1(d)$$

Note: - τyx and τxy shown in Fig. 3 are shear stresses.

These shear stresses are equal:

$$\tau_{yx} = \tau_{xy}$$

$$Qx = \int_{-t/2}^{t/2} \tau_{xz} (1 - \frac{z}{r_y})\,dz \qquad \ldots 1(e)$$

$$Qy = \int_{-t/2}^{t/2} \tau_{yz} (1 - \frac{z}{r_x})\,dz \qquad \ldots 1(f)$$

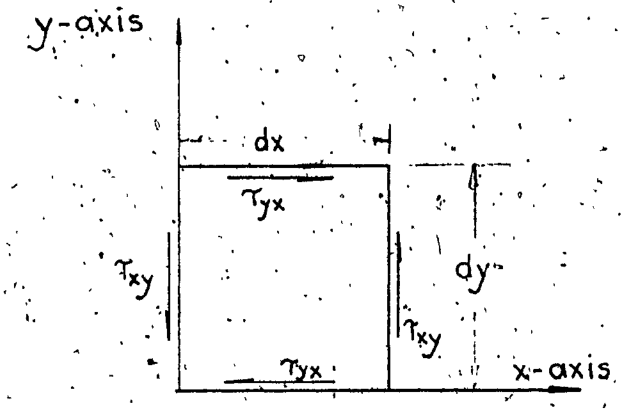


FIG. 3. SHEAR STRESS OF AN ELEMENT

$$Mx = \int_{-t/2}^{t/2} \sigma_x z(1-\frac{z}{r_y})dz \qquad \ldots 1(g)$$

$$My = \int_{-t/2}^{t/2} \sigma_y z(1-\frac{z}{r_x})dz \qquad \ldots 1(h)$$

$$Mxy = -\int_{-t/2}^{t/2} \tau_{xy} z(1-\frac{z}{r_y})dz \qquad \ldots 1(i)$$

$$Myx = \int_{-t/2}^{t/2} \tau_{yx} z(1-\frac{z}{r_x})dz \qquad \ldots 1(j)$$

There are altogether ten stress resultants that were derived as above. There is one interesting point to be noticed; even though $\tau xy = \tau yx$, in general, Nxy is not equal to Nyx, and Mxy is not equal to Myx because r_x is not equal to r_y. However, since "t" is much less than r_x and r_y, $\frac{z}{r_x}$ and $\frac{z}{r_y}$ can be neglected if compared to unity; then we have

$$N_{xy} = N_{yx} \quad \text{and} \quad M_{xy} = - M_{yx}$$

We therefore have eight instead of ten stress resultants to be determined in a general case.

From the theory of elasticity [8]

$$\sigma_x = \frac{E}{1-\nu^2}\left[\epsilon_1 + \nu\epsilon_2 - z(\eta_x + \nu\eta_y)\right] \quad \ldots 2(a)$$

$$\sigma_y = \frac{E}{1-\nu^2}\left[\epsilon_2 + \nu\epsilon_1 - z(\eta_y + \nu\eta_x)\right] \quad \ldots 2(b)$$

$$\tau_{xy} = G(\gamma - 2z\eta_{xy}) \quad \ldots 2(c)$$

where η_x and η_y are change of curvatures before and after the deformation of the middle surface

ϵ_1 and ϵ_2 are strains of the middle surface in the x and y directions, respectively.

Other variations in the middle surface include:

γ is the shearing strain of the middle surface

η_{xy} is the change of twist of the middle surface

If we substitute Eq. (2) into Eq. (1)(a),(b),(c),(g),(h),(i), we get

$$Nx = \frac{Et}{1-\nu^2}(\epsilon_1 + \nu\epsilon_2)$$

$$Ny = \frac{Et}{1-\nu^2}(\epsilon_2 + \nu\epsilon_1)$$

$$Nxy = Nyx = \frac{\gamma \cdot t E}{2(1+\nu)}$$

$$Mx = -D(\eta_x + \nu\eta_y)$$

$$My = -D(\eta_y + \nu\eta_x)$$

$$Mxy = -Myx = D(1-\nu)\eta_{xy}$$

where $D = \frac{Et^3}{12(1-\nu^2)}$

All values of ϵ_1, ϵ_2, η_x, η_y, η_{xy}, and γ are in function of the displacements u, v, and w, therefore, we can write

$$\left.\begin{array}{l} N_x = N_x(u,v,w);\ N_y = N_y(u,v,w);\ N_{xy} = N_{xy}(u,v,w); \\ M_x = M_x(u,v,w);\ M_y = M_y(u,v,w);\ M_{xy} = M_{xy}(u,v,w). \end{array}\right\} \quad \ldots 3$$

see [6] and [10].

2.3 STATIC EQUILIBRIUM EQUATION OF A SHELL ELEMENT

Figures 4 and 5 show the middle surface of a differential element. $d\alpha_x$ and $d\alpha_y$ are the central angle of the element.

Five equations of equilibrium can be set as follows [6]:

$$\sum F_x = 0$$

$$\frac{\partial}{\partial \alpha_x}(N_x r_y) + \frac{\partial}{\partial \alpha_y}(N_{yx} r_x) + N_{xy}\frac{\partial r_x}{\partial \alpha_y} - N_y \frac{\partial r_y}{\partial \alpha_x} - Q_x r_y + X r_x r_y = 0 \qquad \ldots 4(a)$$

$$\sum F_y = 0$$

$$\frac{\partial}{\partial \alpha_y}(N_y r_x) + \frac{\partial}{\partial \alpha_x}(N_{xy} r_y) + N_{yx}\frac{\partial r_y}{\partial \alpha_x} - N_x \frac{\partial r_x}{\partial \alpha_y} - Q_y r_x + Y r_y r_x = 0 \qquad \ldots 4(b)$$

$$\sum F_z = 0$$

$$\frac{\partial}{\partial \alpha_x}(Q_x r_y) + \frac{\partial}{\partial \alpha_y}(Q_y r_x) + N_x r_y + N_y r_x + Z r_x r_y = 0 \qquad \ldots 4(c)$$

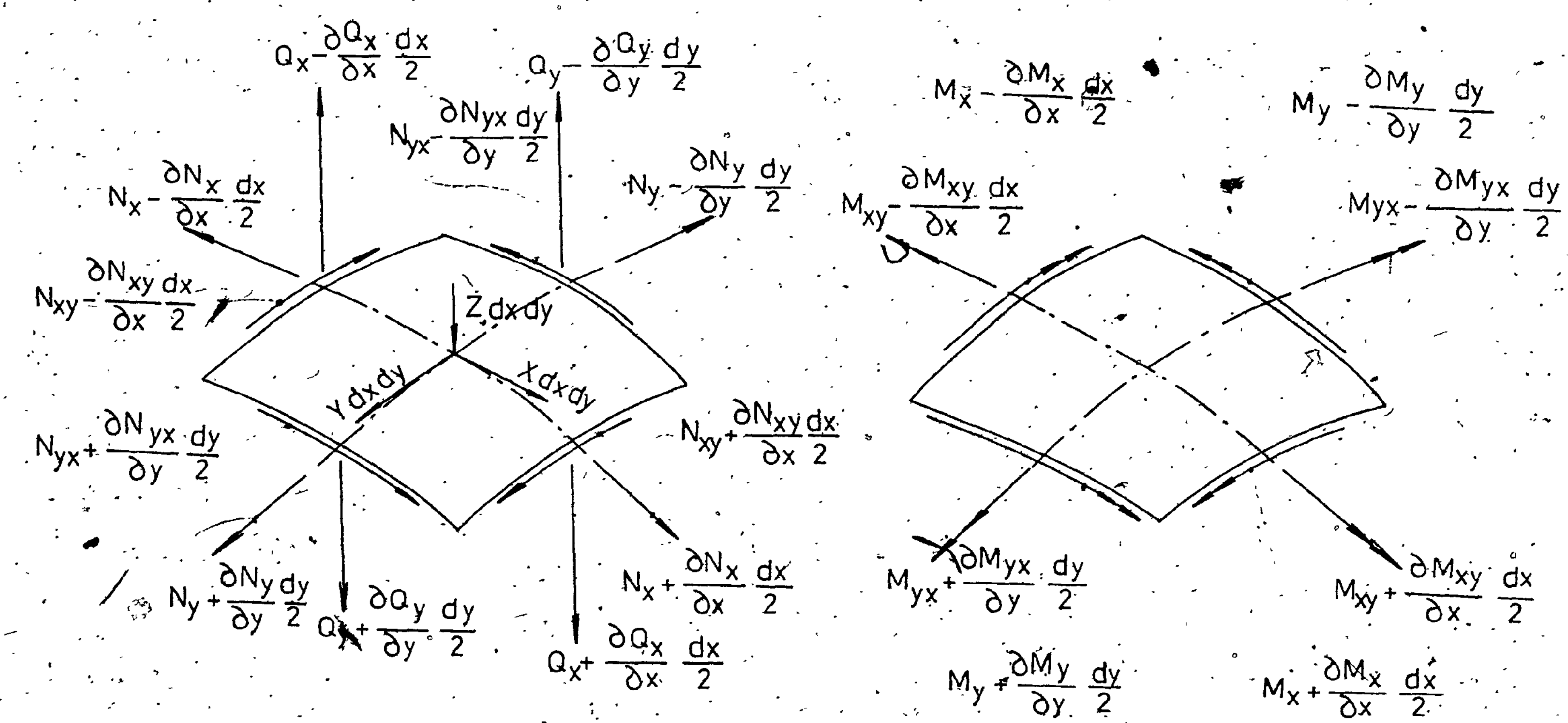
$$\sum M_x = 0$$

$$\frac{\partial}{\partial \alpha_x}(M_{xy} r_y) - \frac{\partial}{\partial \alpha_y}(M_y r_x) - M_{yx}\frac{\partial r_y}{\partial \alpha_x} + M_x \frac{\partial r_x}{\partial \alpha_y} + Q_y r_x r_y = 0 \qquad \ldots 4(d)$$

$$\sum M_y = 0$$

$$-\frac{\partial}{\partial \alpha_y}(M_{yx} r_x) - \frac{\partial}{\partial \alpha_x}(M_x r_y) + M_{xy}\frac{\partial r_x}{\partial \alpha_y} + M_y \frac{\partial r_y}{\partial \alpha_x} + Q_x r_y r_x = 0 \qquad \ldots 4(e)$$

From Equation 3 and Equation 4, altogether we have 11 equations and 11 unknowns, so we can solve the equations. The eleven unknowns include the eight stress resultants and the three displacements in the three directions.



(a) Stress resultants (b) Stress couples

FIG. 4. DIFFERENTIAL ELEMENT

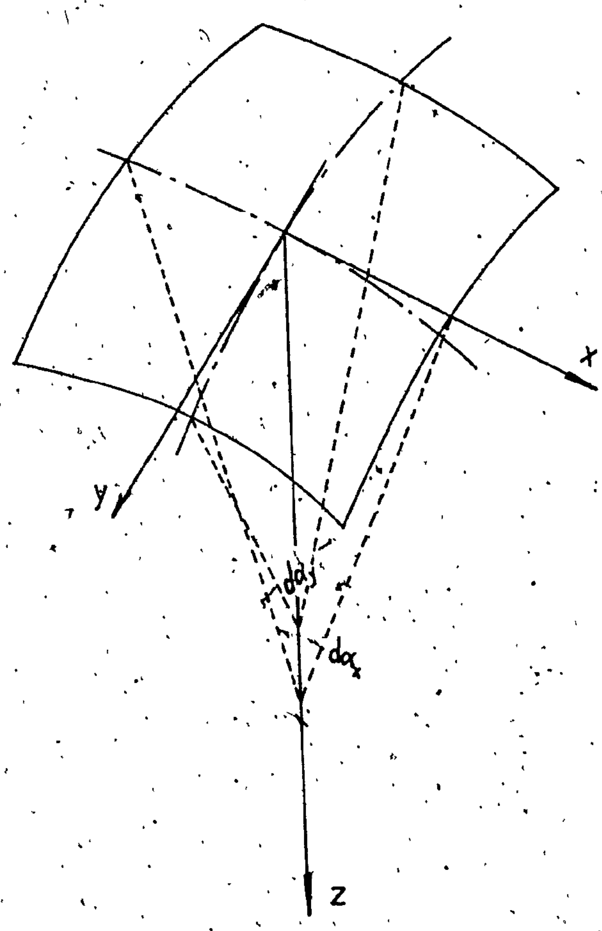


FIG. 5. GEOMETRY OF DIFFERENTIAL ELEMENT

CHAPTER 3

THEORY OF SHELL SIMPLIFIED TO MEMBRANE THEORY

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THEORY OF SHELL SIMPLIFIED TO MEMBRANE THEORY

In most engineering shells, there are substantially no bending and twisting stress couples and no radial shear stress resultants. Therefore, the load is carried substantially by the membrane forces: Nx, Ny and Nxy. All other stress resultants: Mx, My, Mxy, Qx and Qy are neglected and assumed to be zero. Bending and twisting couples and radial shears are only significant in the vicinity of the boundaries or near discontinuities in the loading or in geometry of the shell structure.

So, if we consider Fig. 4(a), neglecting transverse forces Q and considering the x and y-axis intersect at any angle of ω instead of 90°, we get the following equations of static equilibrium in the x and y- direction, respectively[7]:

$$\left(\bar{N}_x + \frac{\partial \bar{N}_x}{\partial x}\frac{dx}{2}\right)dy - \left(\bar{N}_x - \frac{\partial \bar{N}_x}{\partial x}\frac{dx}{2}\right)dy + \left(\bar{N}_{yx} + \frac{\partial \bar{N}_{yx}}{\partial y}\frac{dy}{2}\right)dx$$

$$-\left(\bar{N}_{yx} - \frac{\partial \bar{N}_{yx}}{\partial y}\frac{dy}{2}\right)dx + X(x,y)\,dx\,dy\,\sin\omega = 0 \quad \ldots \text{i(a)}$$

$$\left(\bar{N}_y + \frac{\partial \bar{N}_y}{\partial y}\frac{dy}{2}\right)dx - \left(\bar{N}_y - \frac{\partial \bar{N}_y}{\partial}\frac{dy}{2}\right)dx + \left(\bar{N}_{xy} + \frac{\partial \bar{N}_{xy}}{\partial x}\frac{dx}{2}\right)dy$$

$$-\left(\bar{N}_{xy} - \frac{\partial \bar{N}_{xy}}{\partial y}\frac{dx}{2}\right)dy + Y(x,y)\,dx\,dy\,\sin\omega = 0 \quad \ldots \text{i(b)}$$

By simplifying:

$$\frac{\partial \bar{N}_x}{\partial x} + \frac{\partial \bar{N}_{xy}}{\partial y} = - X \sin\omega \qquad \ldots 5(a)$$

$$\frac{\partial \bar{N}_y}{\partial y} + \frac{\partial \bar{N}_{yx}}{\partial x} = - Y \sin\omega \qquad \ldots 5(b)$$

Considering the z-direction, and by modifying the equation at the bottom of page 167 [7]

$$\frac{\partial}{\partial x}(\bar{N}_x \frac{\partial z}{\partial x}) + \frac{\partial}{\partial y}(\bar{N}_{yx} \frac{\partial z}{\partial x}) + \frac{\partial}{\partial x}(\bar{N}_{xy} \frac{\partial z}{\partial y}) + \frac{\partial}{\partial y}(\bar{N}_y \frac{\partial z}{\partial y}) + Z(x,y) \sin\omega = 0$$

Differentiating the products in the last equation

$$\bar{N}_x \frac{\partial^2 z}{\partial x^2} + 2\bar{N}_{xy} \frac{\partial^2 z}{\partial x \partial y} + \bar{N}_y \frac{\partial^2 z}{\partial y^2} = - Z \sin\omega$$

$$-\left(\frac{\partial \bar{N}_x}{\partial x} + \frac{\partial \bar{N}_{xy}}{\partial y}\right)\frac{\partial z}{\partial x} - \left(\frac{\partial \bar{N}_{yx}}{\partial x} + \frac{\partial \bar{N}_y}{\partial y}\right)\frac{\partial z}{\partial y} \qquad \ldots i(c)$$

Making use of Eq. 5(a) and (b)

$$\bar{N}_x \frac{\partial^2 z}{\partial x^2} + 2\bar{N}_{xy} \frac{\partial^2 z}{\partial x \partial y} + \bar{N}_y \frac{\partial^2 z}{\partial y^2} = \left(X\frac{\partial z}{\partial x} + Y\frac{\partial z}{\partial y} - Z\right)\sin\omega \qquad \ldots 5(c)$$

It is important to note that the membrane theory solution is a statically determinate one, defined simply by the above three equilibrium equations and the boundary conditions of the structure.

CHAPTER 4

THE GEOMETRY OF THE HYPERBOLIC PARABOLOID

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THE GEOMETRY OF THE HYPERBOLIC PARABOLOID

The hyperbolic paraboloid has a doubly curved surface and the surface can be defined in two ways, either as a surface of translation or as a warped parallelogram. In the first case, the surface can be defined by translating or moving a vertical parabola having upward curvature over another parabola with downward curvature. This is shown in Figure 6, where the saddle-shaped surface is formed by moving parabola ABC over parabola BOF.

The hyperbolic paraboloid surface may also be generated along y-axis, a straight line that remains parallel to the xz plane at all times but pivots while sliding along the straight line ABC, (see Figure 7). The resulting surface is represented in Figure 7 by the grid of straight lines h_n and i_n, and every point on it may be considered to be the intersection of two such lines contained in the surface. This surface can be visualized by considering the horizontal plane A'C'E'G' to be warped by vertically depressing corners A' and E' to new positions A and E.

The shape of the base of the water tank in this project will be generated from the second way.

Let us consider a quadrant in Figure 8:

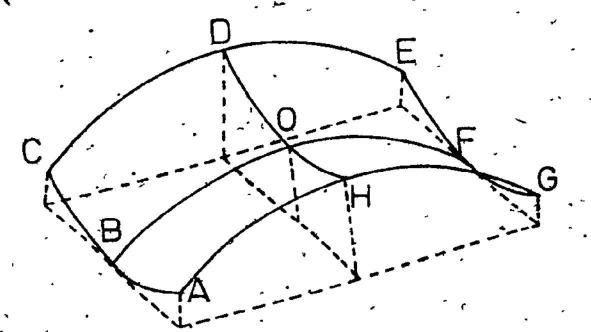


FIG. 6.
SADDLE SHAPED
HYPERBOLIC PARABOLOID

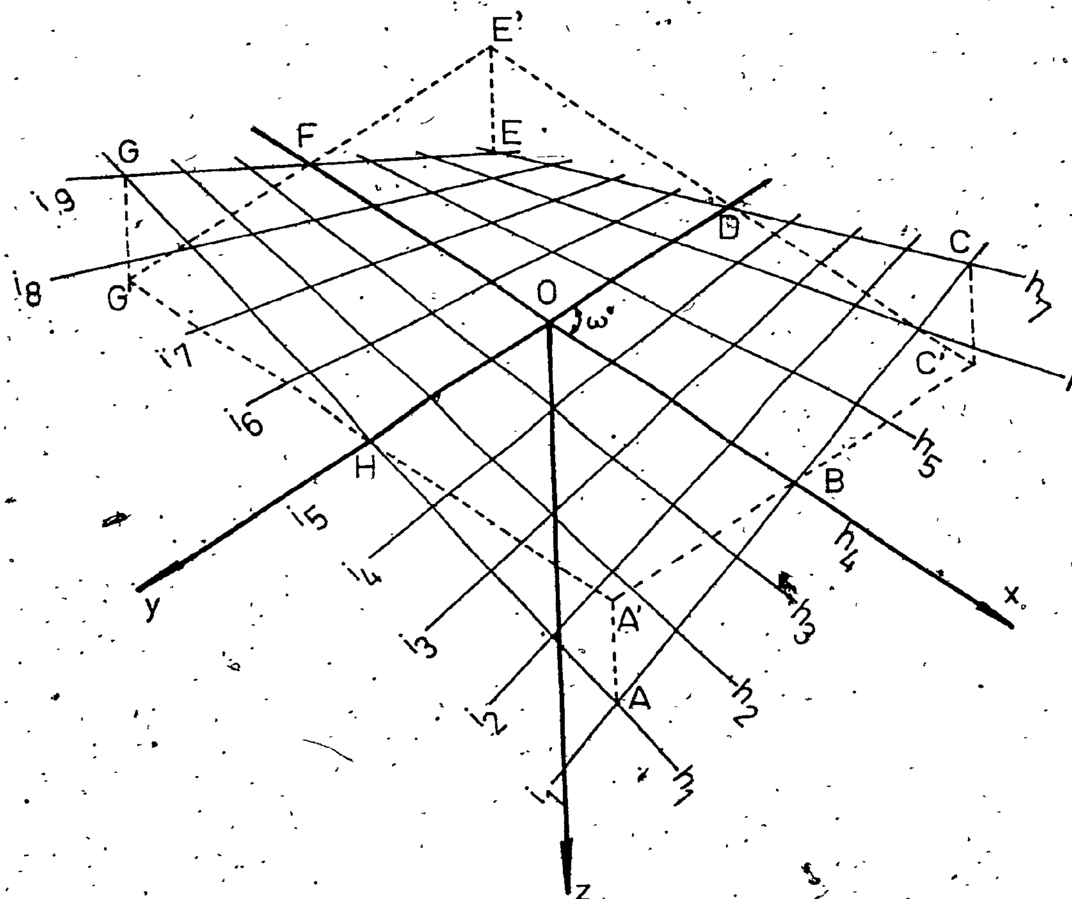


FIG. 7 WARPED PARALLELOGRAM SHAPED
HYPERBOLIC PARABOLOID

$$z = \frac{c}{b} y \qquad \ldots 6$$

$$c = \frac{h}{a} x \qquad \ldots 7$$

Substitute 7 into 6.

$$z = \frac{h}{ab} xy, \quad \text{If } a = b, \text{ then } z = \frac{h}{a^2} xy \qquad \ldots 8$$

In Figure 9:

$$x' = y \cos\frac{\omega}{2} + x \cos\frac{\omega}{2} \qquad \ldots 9(a)$$

$$y' = y \sin\frac{\omega}{2} - x \sin\frac{\omega}{2} \qquad \ldots 9(b)$$

$9(a) \cdot \sin\frac{\omega}{2}$ $$x' \sin\frac{\omega}{2} = y \cos\frac{\omega}{2} \sin\frac{\omega}{2} + x \cos\frac{\omega}{2} \sin\frac{\omega}{2} \qquad \ldots 10(a)$$

$9(b) \cdot \cos\frac{\omega}{2}$ $$y' \cos\frac{\omega}{2} = y \cos\frac{\omega}{2} \sin\frac{\omega}{2} - x \cos\frac{\omega}{2} \sin\frac{\omega}{2} \qquad \ldots 10(b)$$

$10(b) - 10(a)$ $$x' \sin\frac{\omega}{2} - y' \cos\frac{\omega}{2} = 2x \cos\frac{\omega}{2} \sin\frac{\omega}{2}$$

$10(b) + 10(a)$ $$x' \sin\frac{\omega}{2} + y' \cos\frac{\omega}{2} = 2y \cos\frac{\omega}{2} \sin\frac{\omega}{2}$$

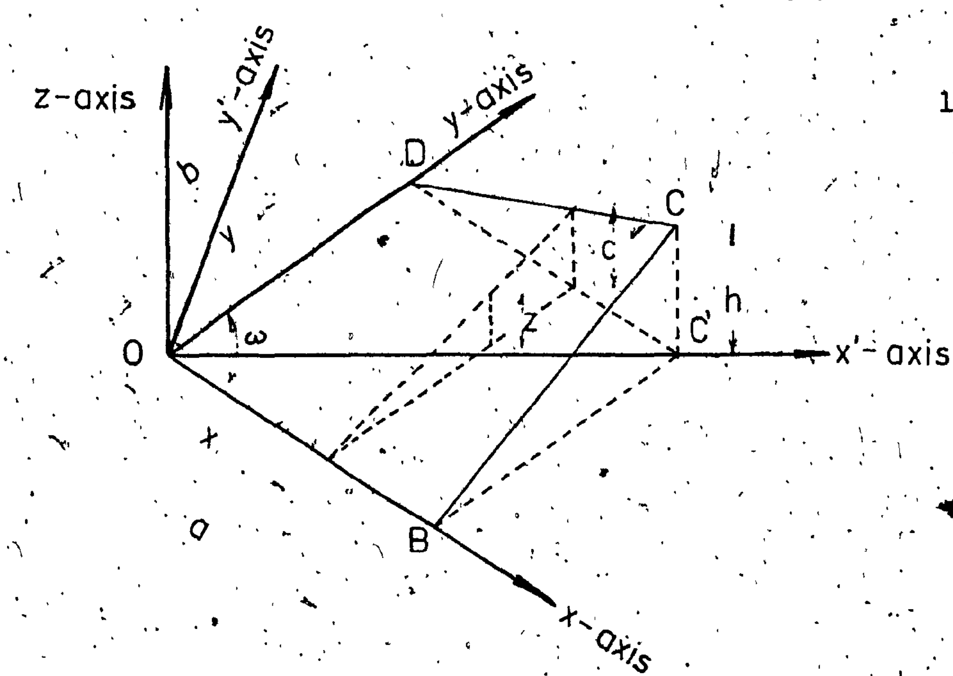


FIG. 8. A SKEW HYPERBOLIC PARABOLOID.

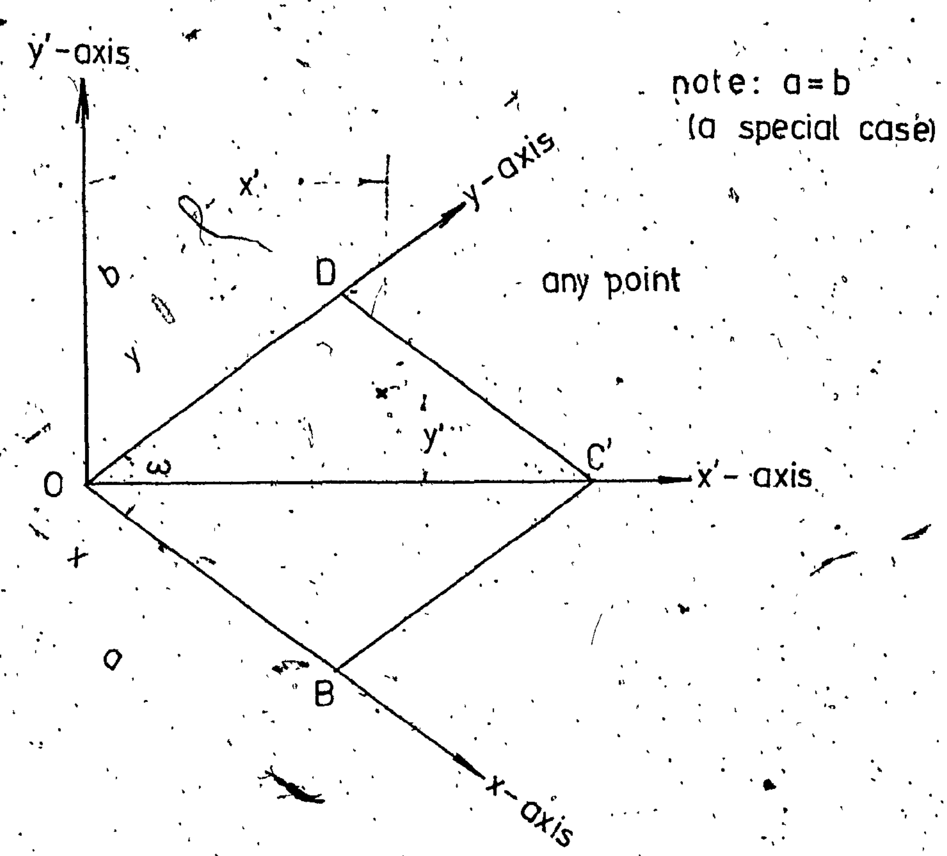


FIG. 9. NORMAL VIEW OF PLANE OBC'D OF FIG. 8.

$$\therefore x = \left(x' \sin\frac{\omega}{2} - y' \cos\frac{\omega}{2}\right) \Big/ 2\cos\frac{\omega}{2}\sin\frac{\omega}{2}$$

$$y = \left(x' \sin\frac{\omega}{2} + y' \cos\frac{\omega}{2}\right) \Big/ 2\cos\frac{\omega}{2}\sin\frac{\omega}{2}$$

Substitute into Equation 8:

$$z = \frac{h}{a^2}\frac{1}{\sin\omega}\left(x'\sin\frac{\omega}{2} - y'\cos\frac{\omega}{2}\right)\left(x'\sin\frac{\omega}{2} + y'\cos\frac{\omega}{2}\right)$$

$$z = \frac{h}{a^2}\frac{1}{\sin\omega}\left(x'^2\sin^2\frac{\omega}{2} - y'^2\cos^2\frac{\omega}{2}\right) \qquad \ldots 11$$

If x' is constant, we have:

$$z = \frac{h}{a^2}\frac{1}{\sin\omega}\left(\text{constant} - y'^2\cos^2\frac{\omega}{2}\right)$$

This formula is an expression of a parabola.

If y' is constant, then the formula also becomes an expression of a parabola.

If z is constant, then the formula becomes an expression of a hyperbola. That is how this structural shell gets its name.

CHAPTER 5

MEMBRANE THEORY APPLIED TO HYPERBOLIC PARABOLOID [11,12]

CHAPTER 5

MEMBRANE THEORY APPLIED TO HYPERBOLIC PARABOLOID [11,12]

In Chapter 3, it is shown that how shell theory is simplified to membrane theory. In the last Chapter 4, the geometry of the hyperbolic-paraboloid is expressed as:

$$z = kxy\sin\omega \qquad \text{where} \quad k = \frac{h}{a^2}\frac{1}{\sin\omega}$$

Note: In this dissertation, h = 14 ft. a = 70 ft.

$$\frac{\partial z}{\partial x} = p = ky\sin\omega$$

$$\frac{\partial^2 z}{\partial x^2} = 0$$

$$\frac{\partial z}{\partial y} = q = kx\sin\omega$$

$$\frac{\partial^2 z}{\partial y^2} = 0$$

$$\frac{\partial^2 z}{\partial x\,\partial y} = k\sin\omega$$

Substitute into Equation 5

$$\frac{\partial \bar{N}_x}{\partial x} + \frac{\partial \bar{N}_{xy}}{\partial y} = -X\sin\omega \qquad \ldots 12(a)$$

$$\frac{\partial \bar{N}_{xy}}{\partial x} + \frac{\partial \bar{N}_y}{\partial y} = -Y\sin\omega \qquad \ldots 12(b)$$

$$2k\sin\omega\,\bar{N}_{xy} = (ky\sin\omega X + kx\sin\omega Y - Z)\sin\omega \qquad \ldots 12(c)$$

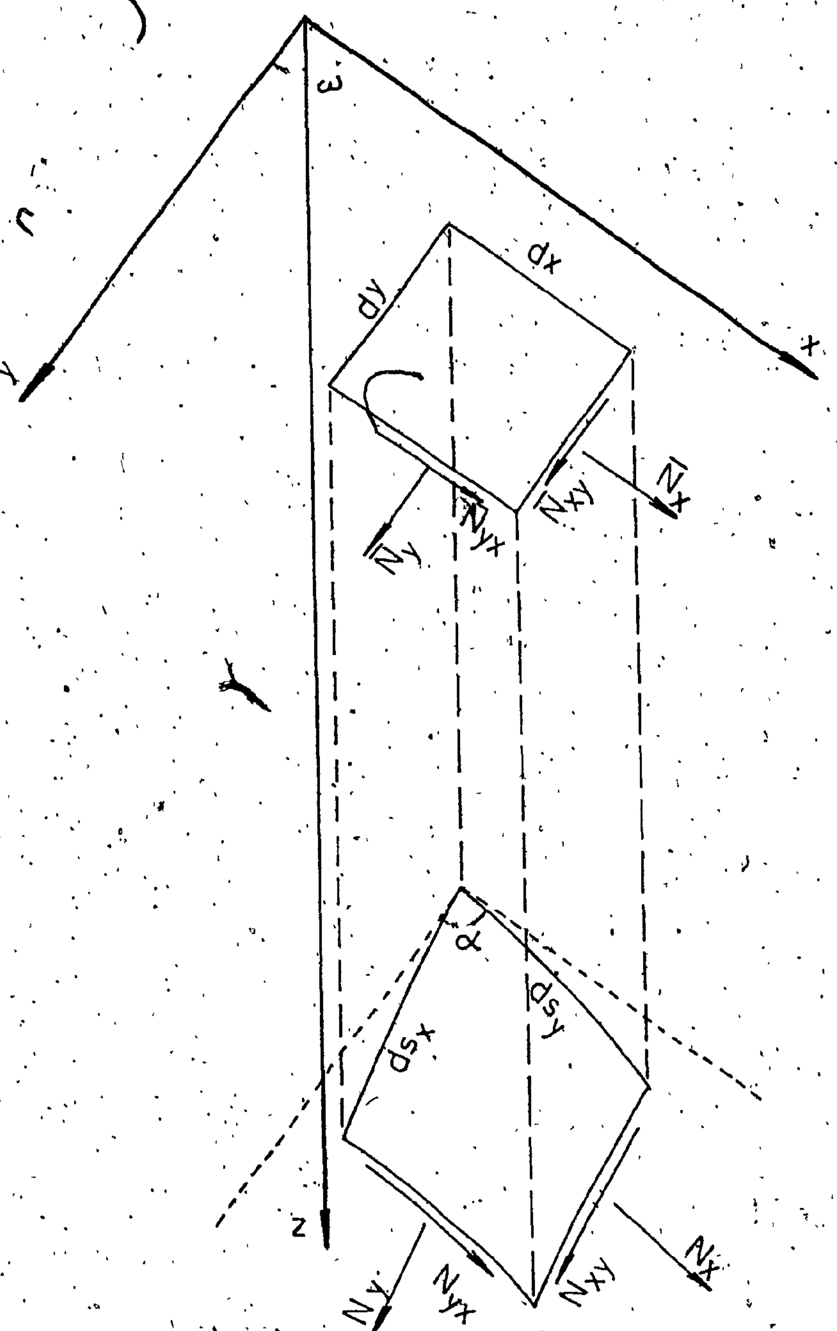


FIG. 10 A SHELL ELEMENT WITH PROJECTION ON THE x-y PLANE

$$\left.\begin{aligned} \bar{N}_x &= N_x\sqrt{\frac{1+q^2}{1+p^2}} \\ \text{and} \quad \bar{N}_y &= N_y\sqrt{\frac{1+p^2}{1+q^2}} \\ \text{also} \quad \bar{N}_{xy} &= N_{xy} \end{aligned}\right\} \qquad \ldots 12(d)$$

where N_x and N_y are stress resultants of the shell and $\bar{N}_x$ and $\bar{N}_y$ are the projections of the stress resultants on the x-y plane.

CHAPTER 6

THE SHAPE OF THE WATER TOWER

CHAPTER 6

THE SHAPE OF THE WATER TOWER

In this project, the water tower being designed will be composed of a central shaft of one hundred feet in height, and ten feet in radius. The thickness of the wall of the shaft will be two feet. A tank will be supported on the top of the shaft. The base of the tank will be slanting outward at about thirty-five degrees from a height of about fifty feet to the top, (for exact dimensions, please refer to Figures 11 to 15). The base of the tank will be divided into six equal sectors radially. Between two sectors, there will be a beam slanting radially outward at a slope of $\frac{4}{3}$ from the shaft to the edge. Each sector will be a hyperbolic paraboloid. The shape of this water tower will be approximately the same as the Lahti Reservoir in Finland, (see [1], page 1441).

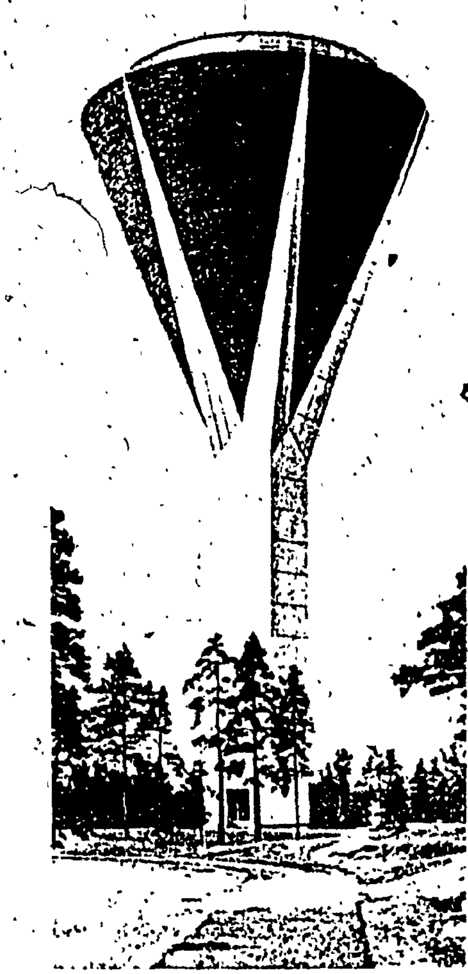
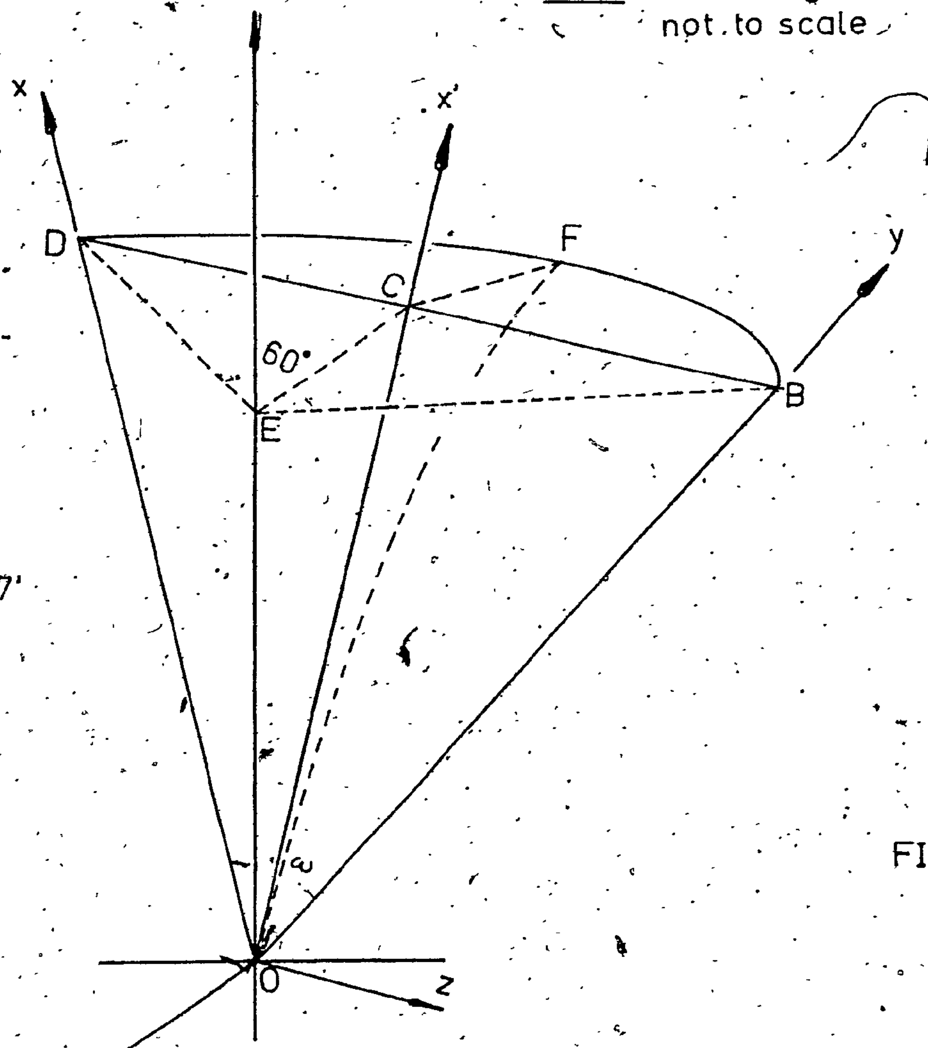


FIG. 10.a. THE LAHTI RESERVOIR IN FINLAND.

note: both diagrams are not to scale



Dimensions:-

OE = 56'-0"
OB = 70'-0"
EB = 42'-0"
CB = 21'-0"
OC = 66.7757'
CF = 3'-6"

FIG. 11. SKETCH OF ONE SECTOR OF THE WATER TANK

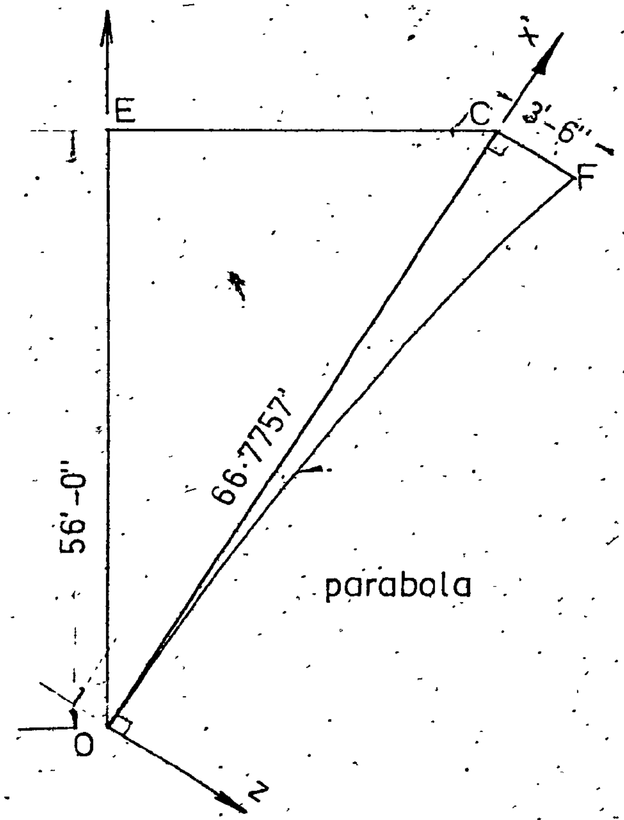
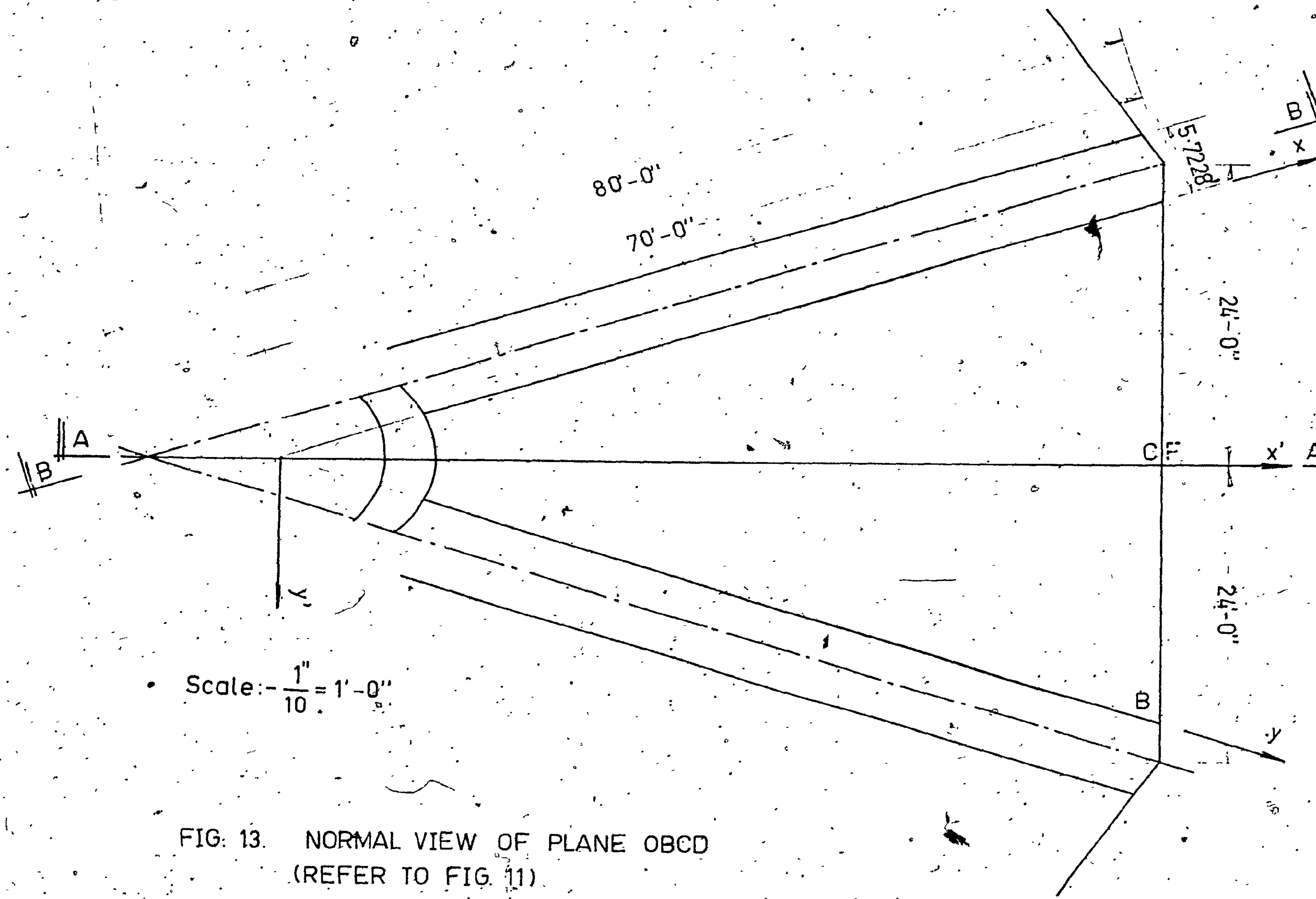


FIG. 12 PLANE OFCE
(REFER TO FIG. 11)



Scale:- $\frac{1''}{10} = 1'-0''$

FIG. 13. NORMAL VIEW OF PLANE OBCD
(REFER TO FIG. 11)

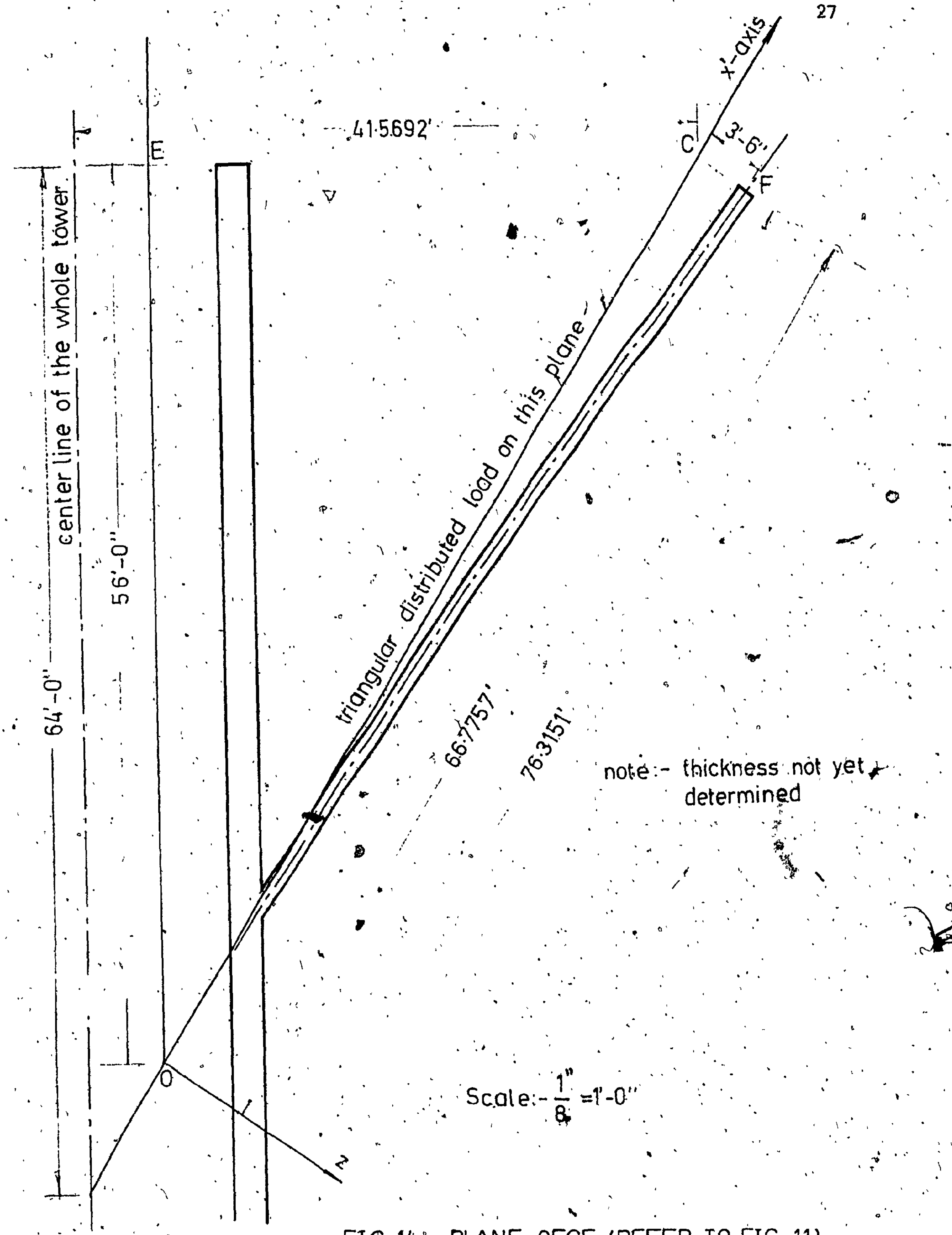


FIG. 14. PLANE OECF (REFER TO FIG. 11.)
SECTION A-A (REFER TO FIG 13)

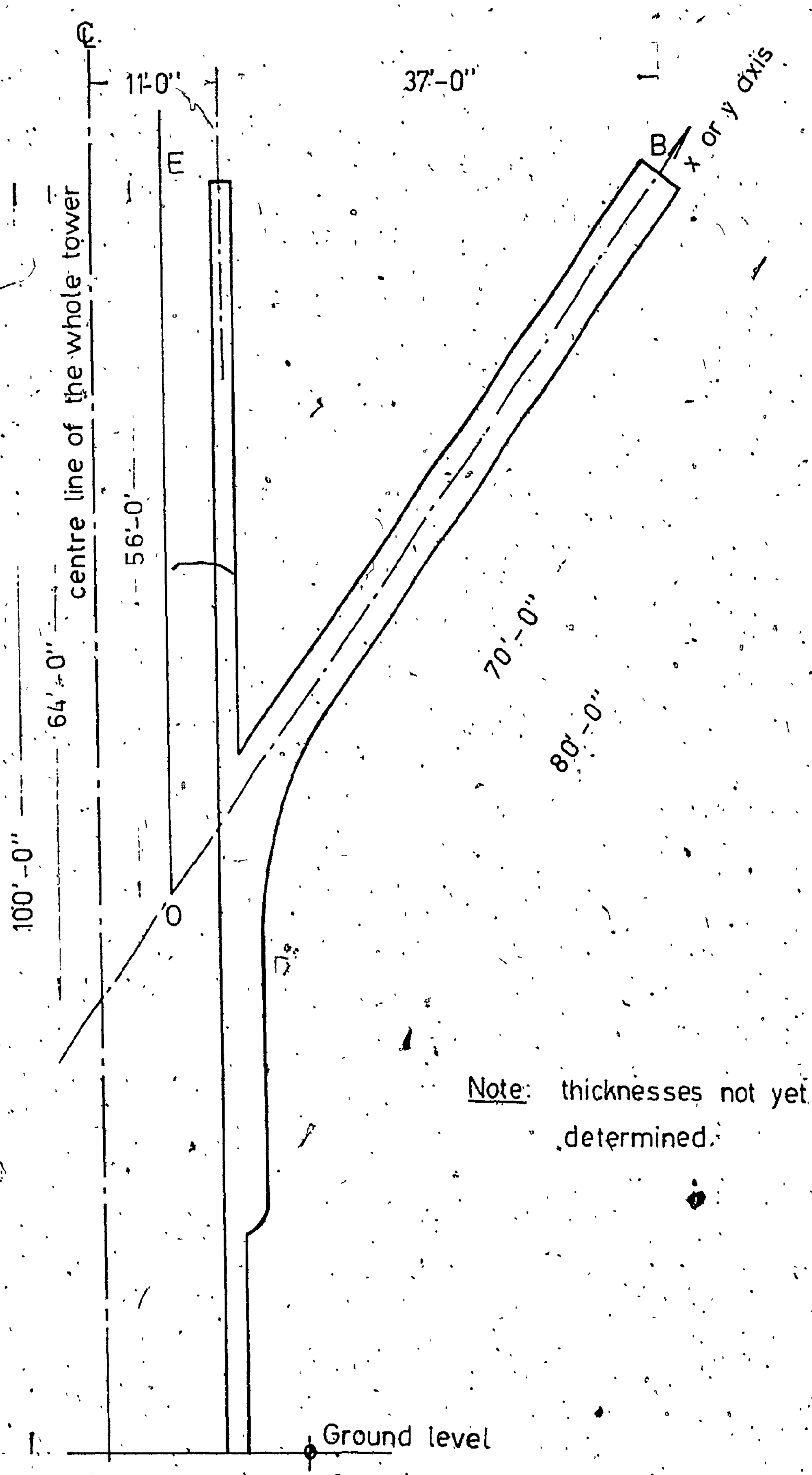


FIG. 15. SECTION OF A BEAM Scale:- $\frac{1"}{12} = 1'-0"$

SECTION B-B (REFER TO FIG. 13)

CHAPTER 7

ANALYSIS OF THE BASE OF THE TANK BY MEMBRANE THEORY

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ANALYSIS OF THE BASE OF THE TANK BY MEMBRANE THEORY

Before going into the analysis, a few assumptions have to be made in order to simplify the problem.

(1) A triangular water pressure is formed normal to plane ABD.

(2) The beams are flexible to prevent secondary moments in the shells.

(3) No moment exists in the shell, or say, the moment can be neglected, so that the membrane theory can be applied. All moments will be sustained by the beam at the connection with the shaft.

Recalling the formulii for hyperbolic paraboloid:

$$\frac{\partial \bar{N}_x}{\partial x} + \frac{\partial \bar{N}_{xy}}{\partial y} = -X \sin\omega \qquad \ldots 12\,(a)$$

$$\frac{\partial \bar{N}_{xy}}{\partial x} + \frac{\partial \bar{N}_y}{\partial y} = -Y \sin\omega \qquad \ldots 12\,(b)$$

$$2k \sin\omega\, \bar{N}_{xy} = (k y \sin\omega X + k x \sin\omega Y - Z) \sin\omega \qquad \ldots 12\,(c)$$

First, consider the water load:

$$X = Y = 0$$

Consider Figures 13, and 14:

$$Z = \rho(56 - \frac{64}{76 \cdot 3151} x')$$ where ρ = density of the water,

as proven before:

$$x' = x \cos\frac{\omega}{2} + y \cos\frac{\omega}{2}$$

$$\therefore Z = \rho(56 - \frac{64}{76 \cdot 3151} x \cos\frac{\omega}{2} - \frac{64}{76 \cdot 3151} y \cos\frac{\omega}{2})$$

Therefore $$\frac{\partial \bar{N}_x}{\partial x} + \frac{\partial \bar{N}_{xy}}{\partial y} = 0 \qquad \text{...13(a)}$$

$$\frac{\partial \bar{N}_{xy}}{\partial x} + \frac{\partial \bar{N}_y}{\partial y} = 0 \qquad \text{...13(b)}$$

$$\bar{N}_{xy} = \frac{-Z \sin\omega}{2k \sin\omega}$$

$$\bar{N}_{xy} = -\frac{\rho\left[56 - \frac{64x}{76 \cdot 3151} \cos\frac{\omega}{2} - \frac{64y}{76 \cdot 3151} \cos\frac{\omega}{2}\right]}{2k}$$

$$\bar{N}_{xy} = -\frac{\rho\left[56 - (x+y)\frac{64}{76 \cdot 3151} \cos\frac{\omega}{2}\right]}{2k} \qquad \text{...13(c)}$$

Substitute $\bar{N}_{xy}$ into Eq. 13(a) and (b).

$$\frac{\partial \bar{N}_x}{\partial x} - \frac{\partial\left[\frac{\rho\left[56 - \frac{64}{76\cdot 3151}(x+y)\cos\frac{\omega}{2}\right]}{2k}\right]}{\partial y} = 0$$

$$\frac{\partial \bar{N}_y}{\partial y} - \frac{\partial\left[\frac{\rho\left[56 - \frac{64}{76\cdot 3151}(x+y)\cos\frac{\omega}{2}\right]}{2k}\right]}{\partial x} = 0$$

by transferring and integrating:

$$\bar{N}_x = -\frac{\rho\, 32}{76\cdot 3151}\cdot\frac{\cos\frac{\omega}{2}}{k}\, x + f_1(y) \qquad \text{...14 (a)}$$

$$\bar{N}_y = -\frac{\rho\, 32}{76\cdot 3151}\cdot\frac{\cos\frac{\omega}{2}}{k}\, y + f_2(x) \qquad \text{...14 (b)}$$

Referring to Fig. 16 (a) on page 33, the boundary condition is determined as follows.

At the free edge, the shear force $\bar{N}x'y'$ and the force ($\bar{N}x'$) normal to the edge are equal to zero. When these forces are resolved in the x'-direction, we get an equation of static equilibrium, as follows:

$$\bar{N}_x \frac{\cos\frac{\omega}{2}}{2\sin\frac{\omega}{2}} + \bar{N}_y \frac{\cos\frac{\omega}{2}}{2\sin\frac{\omega}{2}} + \bar{N}_{xy} \frac{\cos\frac{\omega}{2}}{2\sin\frac{\omega}{2}} + \bar{N}_{yx} \frac{\cos\frac{\omega}{2}}{2\sin\frac{\omega}{2}} = \bar{N}_{x'}\cdot 1$$

$$(\bar{N}_x + \bar{N}_y + \bar{N}_{xy} + \bar{N}_{yx})\cos\frac{\omega}{2} = \bar{N}_{x'}\cdot 2\cdot\sin\frac{\omega}{2}$$

But $\bar{N}_{x'} = 0$ and $\bar{N}_{xy} = \bar{N}_{yx}$

$$\therefore \bar{N}_x + \bar{N}_y + 2\bar{N}_{xy} = 0 \qquad \ldots\text{(i)}$$

In the y'-direction, the equation of static equilibrium is as follows:

$$(\bar{N}_x + \bar{N}_{yx}) \frac{1}{2\sin\frac{\omega}{2}} \sin\frac{\omega}{2} - (\bar{N}_y + \bar{N}_{xy}) \frac{1}{2\sin\frac{\omega}{2}} \sin\frac{\omega}{2} = \bar{N}_{x'y'} \cdot 1$$

But $\bar{N}_{x'y'} = 0$, $\bar{N}_{xy} = \bar{N}_{yx}$

which gives

$$\bar{N}_x = \bar{N}_y \qquad \ldots\text{(ii)}$$

Substitute (ii) into (i), and we get

$$\bar{N}_x = -\bar{N}_{xy}$$

or $\bar{N}_y = -\bar{N}_{xy}$

Therefore, the boundary condition of the problem is:

when $x + y = 70$ ft. $\bar{N}_x$ or $\bar{N}_y = -\bar{N}_{xy}$ ...(iii)

substitute (iii) into Eq. (14).

$$\frac{\rho\left[56 - 70\frac{64}{76.3151}\cos\frac{\omega}{2}\right]}{2K} = -\left(\frac{\rho\, 32}{76.3151}\,\frac{\cos\frac{\omega}{2}}{k}\right)(70 - y) + f_1(y)$$

$$\frac{\rho\left[56 - 70\frac{64}{76.3151}\cos\frac{\omega}{2}\right]}{2K} = -\left(\frac{\rho\, 32}{76.3151}\,\frac{\cos\frac{\omega}{2}}{k}\right)(70 - x) + f_2(x)$$

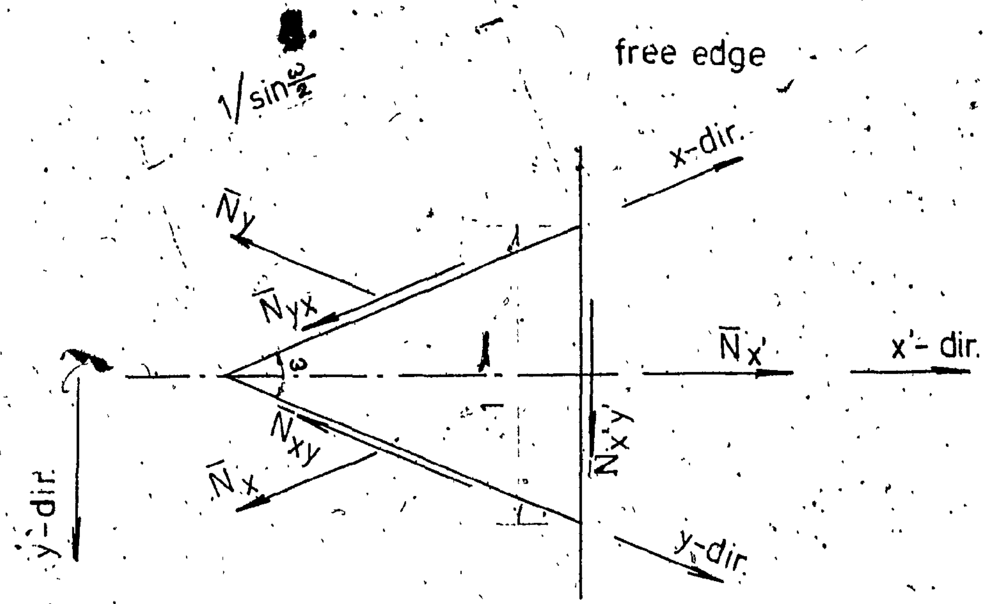


FIG. 16a. AN ELEMENT AT THE BOUNDARY.

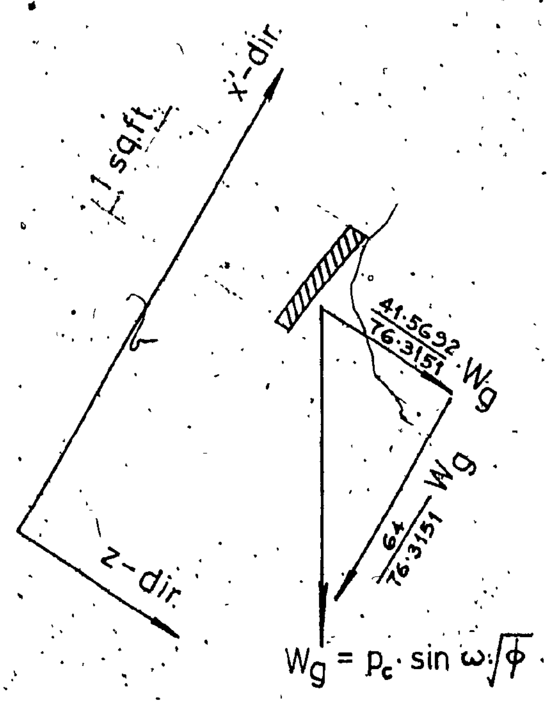


FIG. 16b. DEAD LOAD PER UNIT AREA.

$$\therefore f_1(y) = \frac{\rho\left(56 - 70\frac{64}{76\cdot3151}\cos\frac{\omega}{2}\right)}{2k} + \left(\frac{\rho\, 32}{76\cdot3151}\,\frac{\cos\frac{\omega}{2}}{k}\right)(70-y)$$

$$\text{and } f_2(x) = \frac{\rho\left(56 - 70\frac{64}{76\cdot3151}\cos\frac{\omega}{2}\right)}{2k} + \left(\frac{\rho\, 32}{76\cdot3151}\,\frac{\cos\frac{\omega}{2}}{k}\right)(70-x)$$

$$\therefore \overline{N}_x = \frac{\rho\left(56 - 70\frac{64}{76\cdot3151}\cos\frac{\omega}{2}\right)}{2k} + \left(\frac{\rho\, 32}{76\cdot3151}\,\frac{\cos\frac{\omega}{2}}{k}\right)(70-y-x) \qquad \ldots 15(a)$$

$$\text{and } \overline{N}_y = \frac{\rho\left(56 - 70\frac{64}{76\cdot3151}\cos\frac{\omega}{2}\right)}{2k} + \left(\frac{\rho\, 32}{76\cdot3151}\,\frac{\cos\frac{\omega}{2}}{k}\right)(70-x-y) \qquad \ldots 15(b)$$

7.1 DETERMINATION OF X, Y AND Z (CONCRETE'S OWN WEIGHT)

Consider Fig. 16 (b)

Wg = weight of 1 sq.ft. of concrete projected on the x-y plane.

$$Wg = p_c \sin\omega\sqrt{\phi} \qquad \text{(see [11])}$$

where p_c is the pressure due to concrete weight in the z-direction.

$$\phi = 1 + k^2x^2 + k^2y^2 - 2k^2xy\cos\omega$$

$$Z = \frac{41\cdot5692}{76\cdot3151}\, p_c \sin\omega\sqrt{\phi}$$

$$X' = -\frac{64}{76 \cdot 3151} p_c \sin\omega\sqrt{\phi}$$

$$X = Y = -\frac{64}{76 \cdot 3151} p_c \sin\omega\sqrt{\phi}\,\frac{\sin\frac{\omega}{2}}{\sin(180^\circ - \omega)}$$

(see Fig.16(c))

Fig. 16(c) COMPONENTS OF X'

Substitute X, Y, and Z into Eq. 12(c):

$$\bar{N}_{xy} = \left(\frac{y}{2}X + \frac{x}{2}Y - \frac{Z}{2k\sin\omega}\right)\sin\omega$$

$$= -\frac{y}{2}\,\frac{64}{76 \cdot 3151} p_c \sin\omega\sqrt{\phi}\,\frac{\sin\frac{\omega}{2}}{\sin(180^\circ - \omega)}\sin\omega$$

$$-\frac{x}{2}\,\frac{64}{76 \cdot 3151} p_c \sin\omega\sqrt{\phi}\,\frac{\sin\frac{\omega}{2}}{\sin(180^\circ - \omega)}\sin\omega$$

$$-\frac{1}{2k\sin\omega}\,\frac{41 \cdot 5692}{76 \cdot 3151} p_c \sin\omega\sqrt{\phi}\sin\omega \qquad \ldots 16$$

Let $C = \frac{\sin\frac{\omega}{2}}{\sin(180^\circ - \omega)}$

$$\frac{\partial \bar{N}_{xy}}{\partial x} = -\frac{32}{76\cdot3151}\, p_c \sin^2\omega\, C\, y\left[-\frac{1}{\sqrt{\phi}}k(kx-ky\cos\omega)\right]$$

$$-\frac{32}{76\cdot3151}\, p_c \sin^2\omega\, C\left[\sqrt{\phi}-x\frac{1}{\sqrt{\phi}}k(kx-ky\cos\omega)\right]$$

$$-\frac{41\cdot5692}{76\cdot3151}\frac{p_c}{2}\left[\frac{(kx-ky\cos\omega)}{\sqrt{\phi}}\right]\sin\omega$$

$$\frac{\partial \bar{N}_y}{\partial y} = -\gamma\sin\omega - \frac{\partial \bar{N}_{xy}}{\partial x}$$

$$= +\frac{64}{76\cdot3151}\, p_c \sin^2\omega\sqrt{\phi}\, C$$

$$-\frac{32}{76\cdot3151}\, p_c \sin^2\omega\, C\, y\left[\frac{1}{\sqrt{\phi}}k(kx-ky\cos\omega)\right]$$

$$+\frac{32}{76\cdot3151}\, p_c \sin^2\omega\, C\left[\sqrt{\phi}-x\frac{1}{\sqrt{\phi}}k(kx-ky\cos\omega)\right]$$

$$-\frac{41\cdot5692}{76\cdot3151}\frac{p_c}{2}\left[\frac{(kx-ky\cos\omega)}{\sqrt{\phi}}\right]\sin\omega \qquad \ldots 17$$

If we let $m = kx$, $n = ky$; $dx = \frac{dm}{k}$; $dy = \frac{dn}{k}$;

then $\phi = 1 + m^2 + n^2 - 2mn\cos\omega$

Substitute ϕ into Eq. (17).

$$\frac{\partial \bar{N}_y}{\partial n} = \frac{1}{k}\Big(\frac{64}{76 \cdot 3151} p_c \sin^2\omega\, C\sqrt{\phi} - \frac{32}{76 \cdot 3151} p_c \sin^2\omega\, C\, \frac{nm}{\sqrt{\phi}}$$

$$+ \frac{32}{76 \cdot 3151} p_c \sin^2\omega\, C \cos\omega \frac{n^2}{\sqrt{\phi}} + \frac{32}{76 \cdot 3151} p_c \sin^2\omega\, C\sqrt{\phi}$$

$$- \frac{32}{76 \cdot 3151} p_c \sin^2\omega\, C\, \frac{m^2}{\sqrt{\phi}} + \frac{32}{76 \cdot 3151} p_c \sin^2\omega\, C \cos\omega\, \frac{mn}{\sqrt{\phi}}$$

$$- \frac{41 \cdot 5692}{76 \cdot 3151} \frac{p_c}{2} \sin\omega \frac{m}{\sqrt{\phi}} + \frac{41 \cdot 5692}{76 \cdot 3151} \frac{p_c}{2} \cos\omega \sin\omega \frac{n}{\sqrt{\phi}}\Big)$$

$$\bar{N}_y = \frac{1}{k}\frac{64}{76 \cdot 3151} p_c \sin^2\omega\, C\left[\frac{(n - m\cos\omega)}{2}\sqrt{\phi} + \frac{1 + m^2\sin^2\omega}{2} \sinh^{-1}\left(\frac{n - m\cos\omega}{\sqrt{1 + m^2\sin^2\omega}}\right)\right]$$

$$- \frac{1}{k}\frac{32}{76 \cdot 3151} p_c \sin^2\omega\, C\, m\left[\sqrt{\phi} + m\cos\omega \sinh^{-1}\left(\frac{n - m\cos\omega}{\sqrt{1 + m^2\sin^2\omega}}\right)\right]$$

$$+ \frac{1}{k}\frac{32}{76 \cdot 3151} p_c \sin^2\omega\, C \cos\omega\left[\left(\frac{n}{2} + \frac{3m\cos\omega}{2}\right)\sqrt{\phi}\right.$$

$$\left. + \frac{3m^2\cos^2\omega - 1 - m^2}{2} \sinh^{-1}\left(\frac{n - m\cos\omega}{\sqrt{1 + m^2\sin^2\omega}}\right)\right]$$

$$+ \frac{1}{k}\frac{32}{76 \cdot 3151} p_c \sin^2\omega\, C\left[\frac{(n - m\cos\omega)}{2}\sqrt{\phi} + \frac{1 + m^2\sin^2\omega}{2} \sinh^{-1}\left(\frac{n - m\cos\omega}{\sqrt{1 + m^2\sin^2\omega}}\right)\right]$$

$$- \frac{1}{k}\frac{32}{76 \cdot 3151} p_c \sin^2\omega\, C\, m^2\left[\sinh^{-1}\left(\frac{n - m\cos\omega}{\sqrt{1 + m^2\sin^2\omega}}\right)\right]$$

$$+ \frac{1}{k}\frac{32}{76 \cdot 3151} p_c \sin^2\omega\, C \cos\omega\, m\left[\sqrt{\phi} + m\cos\omega \sinh^{-1}\left(\frac{n - m\cos\omega}{\sqrt{1 + m^2\sin^2\omega}}\right)\right]$$

$$- \frac{1}{k}\frac{41 \cdot 5692}{76 \cdot 3151} \frac{p_c}{2} \sin\omega\, m \sinh^{-1}\left(\frac{n - m\cos\omega}{\sqrt{1 + m^2\sin^2\omega}}\right)$$

$$+ \frac{1}{k}\frac{41 \cdot 5692}{76 \cdot 3151} \frac{p_c}{2} \sin\omega \cos\omega\left[\sqrt{\phi} + m\cos\omega \sinh^{-1}\left(\frac{n - m\cos\omega}{\sqrt{1 + m^2\sin^2\omega}}\right)\right]$$

$$+ f(m) \qquad \ldots 18$$

From the boundary condition,

when $x + y = 70$, $\bar{N}_y = -\bar{N}_{xy}$

we get $f(m) = -\bar{N}_y(m, 70k - m) - \bar{N}_{xy}(x, 70-x) + f(m)$

where the terms $\bar{N}y(m,70k-m)$ and $\bar{N}xy(x,70-x)$ are obtained by substituting 70k-m into n in Eqs. 18, and 70-x into y in Eq. 16, respectively.

Similarly, $\bar{N}x$ can be analyzed in the same way. In the computer program, stress resultants Nx, Ny and Nxy are computed by Eq. 12(d).

7.2 DETERMINATION OF PRINCIPAL STRESSES [11]

Angle α in Figure 10 formed by two intersecting generatrix is obtained by the following formula:

$$\cos\alpha = \frac{pq + \cos\omega}{\sqrt{(1+p^2)(1+q^2)}}$$

Consider Figure 17, normal and tangential stresses σ_β, τ_β on any section whose normal forms an angle β with the normal to the x-axis, are obtained by consideration of the equilibrium in σ_β and τ_β directions, respectively (see Fig. 17).

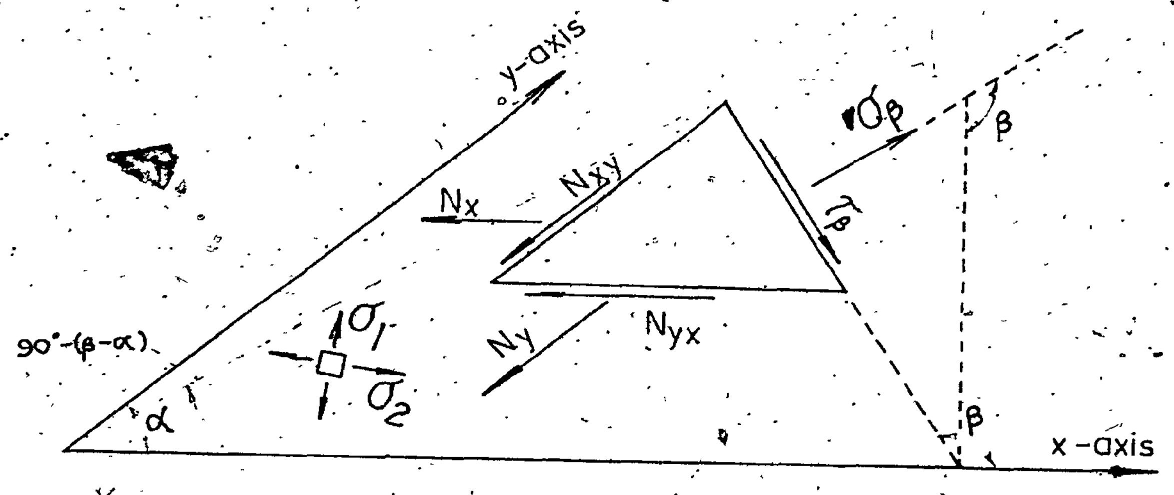


FIG. 17 PRINCIPAL STRESSES

$$\sigma_\beta = N_x \frac{\sin^2\beta}{\sin\alpha} + 2N_{xy}\frac{\sin\beta\sin(\beta-\alpha)}{\sin\alpha} + N_y\frac{\sin^2(\beta-\alpha)}{\sin\alpha} \qquad \ldots 19\,(a)$$

$$\tau_\beta = -N_x \frac{\sin\beta\cos\beta}{\sin\alpha} - N_{xy}\frac{\sin(2\beta-\alpha)}{\sin\alpha} - N_y\frac{\sin(\beta-\alpha)\cos(\beta-\alpha)}{\sin\alpha} \qquad \ldots 19\,(b)$$

Directions of principal stresses are determined by setting τ_β equal to zero as in Eq. 19 (b).

$$\tan 2\theta = \frac{2N_{xy}\sin\alpha + N_y\sin 2\alpha}{N_x + 2N_{xy}\cos\alpha + N_y\cos 2\alpha} \qquad \ldots 20$$

where θ and $90^\circ - \theta$ are the angles of principle stresses with the normal to the x-axis.

Values of principal stresses are:

$$\sigma_1 = N_x \frac{\sin^2\theta}{\sin\alpha} + 2N_{xy}\frac{\sin\theta\sin(\theta-\alpha)}{\sin\alpha} + N_y\frac{\sin^2(\theta-\alpha)}{\sin\alpha} \qquad \ldots 21\,(a)$$

$$\sigma_2 = N_x \frac{\cos^2\theta}{\sin\alpha} + 2N_{xy}\frac{\cos\theta\cos(\theta-\alpha)}{\sin\alpha} + N_y\frac{\cos^2(\theta-\alpha)}{\sin\alpha} \qquad \ldots 21\,(b)$$

Values are calculated in the program, as shown in the Appendix.

The projection of angle θ onto the x-y plane can be found by the following formula [12]:

$$\tan \bar{\theta} = \frac{\xi \sin\theta \sin\omega}{\cos\theta \sin\alpha + \sin\theta(\xi \cos\omega - \cos\alpha)}$$

where $\xi = \sqrt{\frac{1+p^2}{1+q^2}}$

Values of $\bar{\theta}$ are computed in the computer program and the output is shown in Appendix 3.

CHAPTER 8

DESIGN OF THE HYPAR-SHELL

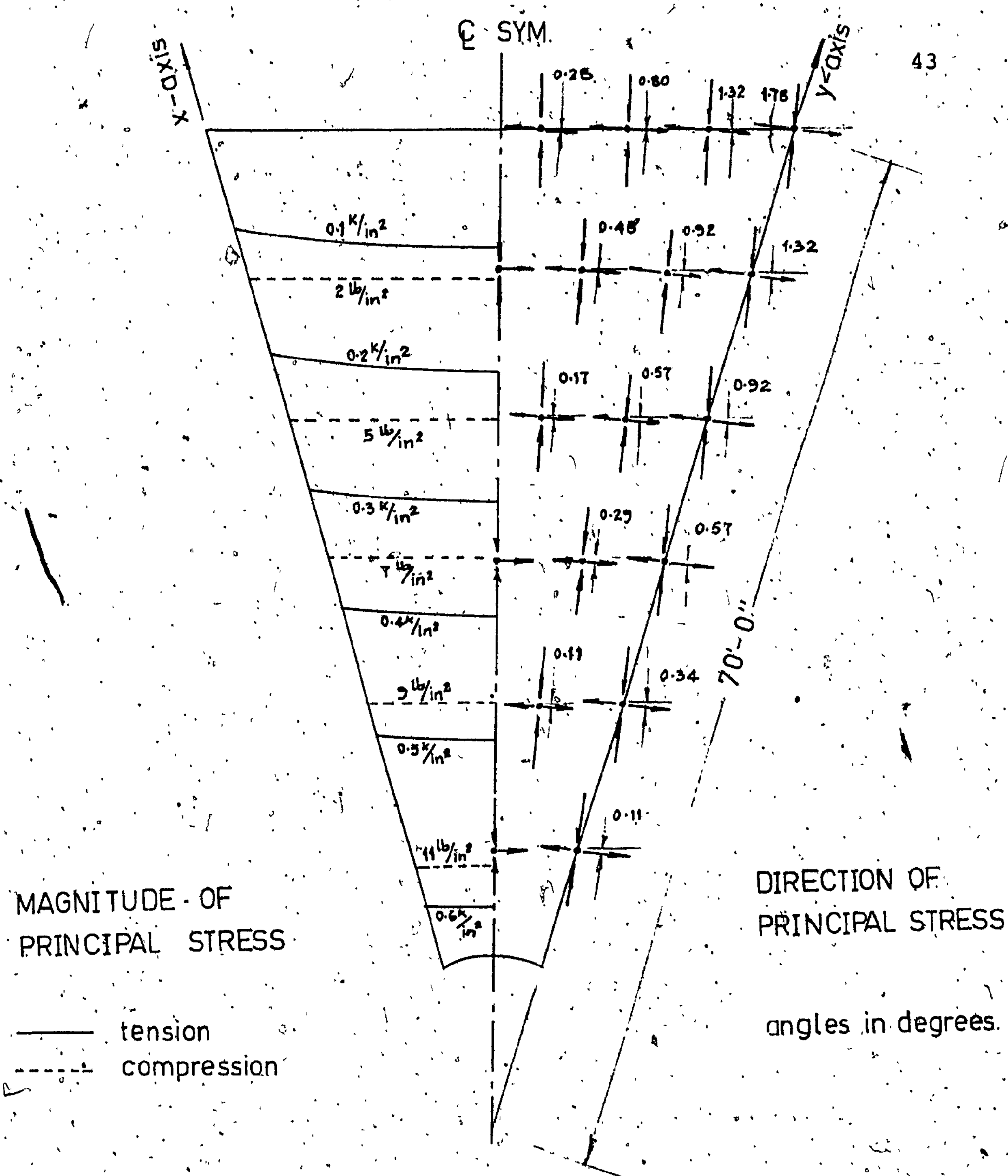
CHAPTER 8

DESIGN OF THE HYPAR-SHELL

Although the thickness of the whole shell is assumed to be two feet, yet in order to save material, the thickness and the amount of steel can be reduced from the shaft to the edge of the tank.

Say, the thickness of the shell starts from 18 inches at the shaft and reduces towards the edge, as shown in Figure 19. From the output of the computer program, values of the principal stresses PS1 and PS2 are calculated. Since the loading and the shell is symmetrical along the x'-axis, consequently, the principal stresses and their directions are also symmetrical on both sides. The directions of the principal stresses are approximately shown in Figure 18.

The layout of reinforcement is arranged in the direction of the x and y-axis. Since the directions of the principal stresses deviate minutely from the x'-axis, they are all assumed to be parallel or perpendicular to the x'-axis. The reinforcements will be designed with respect to the components of the principal stresses resolved into the x and y-direction. The principal compressive stresses (PS2) are all so small, the design of reinforcement will be governed by the principal tensile stresses (PS1).



℄ SYM.
x-axis
y-axis
0.28
0.80
1.32
1.78
0.1 K/in²
2 lb/in²
0.48
0.92
1.32
0.2 K/in²
5 lb/in²
0.17
0.57
0.92
0.3 K/in²
7 lb/in²
0.29
0.57
0.4 K/in²
9 lb/in²
0.11
0.34
0.5 K/in²
11 lb/in²
0.11
0.6 K/in²
70'-0"
MAGNITUDE OF PRINCIPAL STRESS
tension
compression
DIRECTION OF PRINCIPAL STRESS
angles in degrees.

FIG. 18.

Say $f'c$ = 4,000 psi

fta = allowable tensile stress of concrete

= 20% of $f'c$ = 0.2 x 4,000 = 800 psi [15].

fcd = allowable compressive stress of concrete

= 0.45 $f'c$ = 1.8 psi

fs = allowable stress of steel = 20 ksi

Consider x = o, y = 10 ft, t = 18"

PS1 = 0.677 ksi

Internal tensile force per foot = 0.677 x 12 x 24 = 194.98 kips

Force resisted by concrete = 12 x 18 x 0.8 = 172.8 kips/ft

Force resisted by steel = 194.98 - 172.8 = 21.68 kips/ft

Area of steel required = $\frac{21.68}{2} \times \frac{1}{\sin^2(\frac{\omega}{2})} \times \frac{1}{20}$ = 6.02 sq.in/ft.

Use #8 bars at 6 in c/c (4 layers).

Consider x = o, y = 20 ft, t = 15"

PS1 = 0.570 ksi

Internal tensile foroe = 0.570 x 12 x 24 = 166.18 kips/ft

Force resisted by concrete = 12 x 15 x 0.8 = 144 kips/ft

Force resisted by steel = 166.18 - 144 = 22.18 kips/ft

Area of steel required = $\frac{22.18}{2} = \frac{1}{\sin^2(\frac{\omega}{2})} \times \frac{1}{20}$ = 6.16 sq.in/ft.

Use #8 bars at 6 in c/c (4 layers).

Similarly, at x = o, y = 30 ft, t = 12"

PSl = 0.463 ksi

Internal tensile force = 133.34 kips/ft

Force resisted by concrete = 115.2 kips/ft

Force resisted by steel = 18.14 kips/ft

Area of steel required = 5.04 sq.in/ft

Use #8 bars at 5½ in c/c (3 layers).

At x = o, y = 40 ft, t = 12"

PSl = 0.356 ksi

Internal tensile force = 102.5 kips/ft

force resisted by concrete = 115.2 kips/ft

Therefore, no steel is required.

So, to the edge of the tank, 2 layers of #6 bars at 12" c/c are used. Also, in order to be safe, four #6 bars are added along the edge, (see Figure 20).

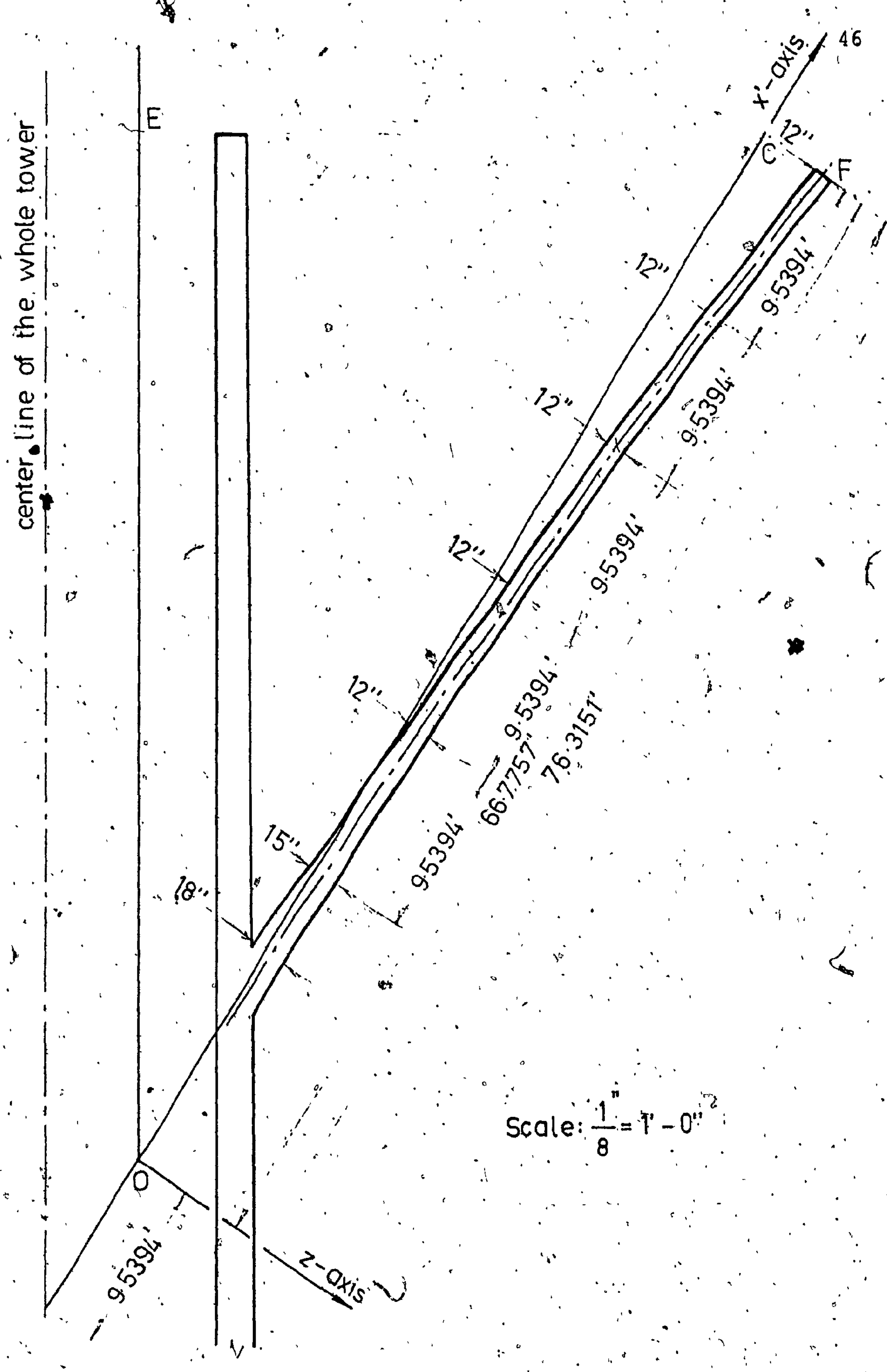
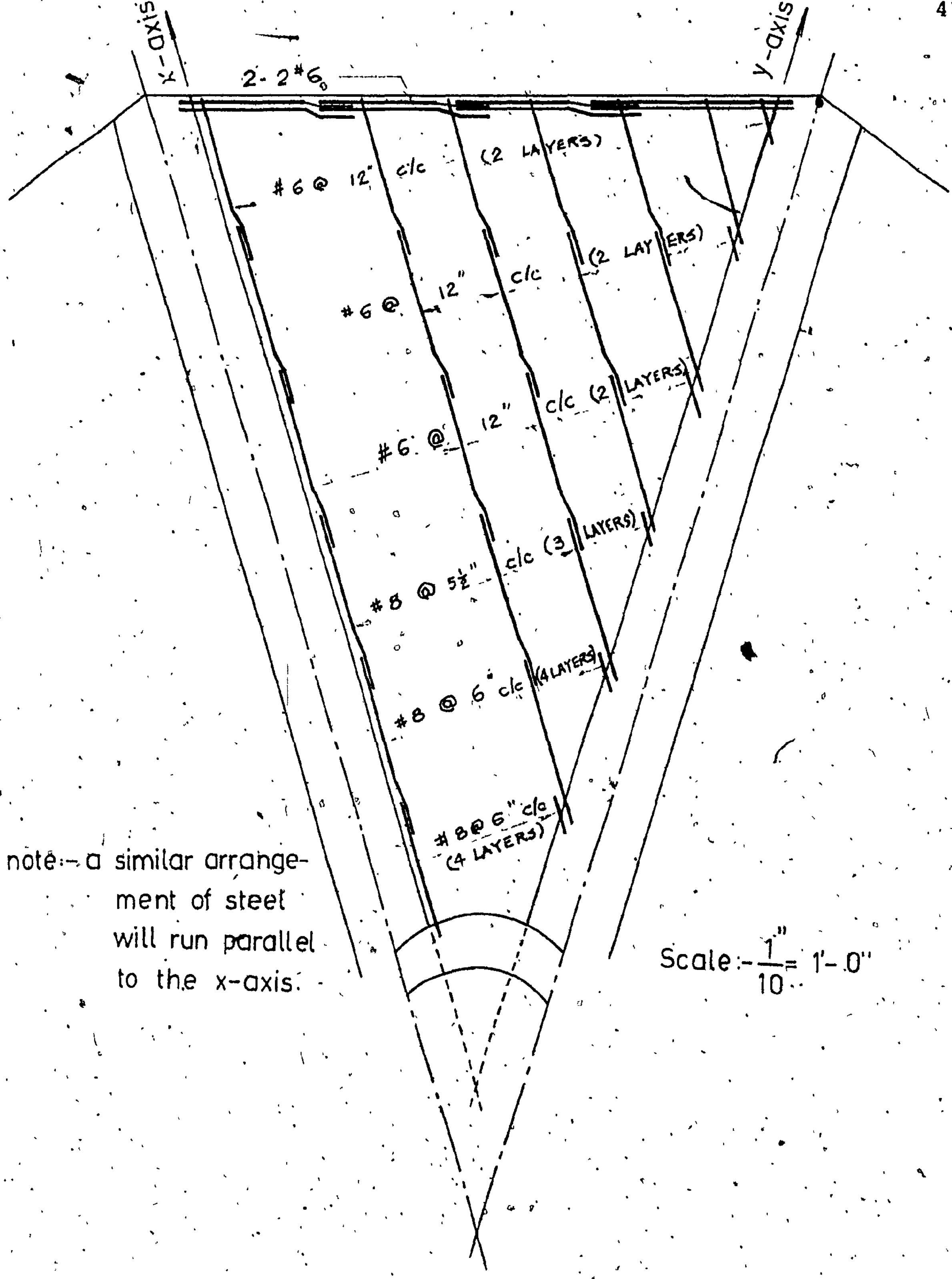


FIG. 19. THICKNESS OF THE SHELL. SECTION A-A (REFER TO FIG. 13)



note:- a similar arrangement of steel will run parallel to the x-axis.

Scale:- $\frac{1"}{10}$ = 1'-0"

FIG. 20. REINFORCEMENT ARRANGEMENT OF THE SHELL.

CHAPTER 9

DISCUSSION

CHAPTER 9

DISCUSSION

Having seen the great advantages of hyperbolic paraboloidal shells, the writer ventured to apply this shell to the water tank being designed and to estimate the significance in structural and economical points-of-view.

Many hyperbolic paraboloidal shell roofs, having a square shape on the x-y plane, and having a uniformly distributed load in the z-direction, if analyzed by the membrane theory, consist only of shear forces (Nxy). In contrast, the base of the water tank being designed in this dissertation, is a skew hyperbolic paraboloidal shell, having an isosceles triangle (half rhombic) on the x-y plane, and having a triangular distributed live load in the z-direction and dead load in the x and y-direction. After being analyzed by the membrane theory, it consists of direct forces (Nx, Ny) as well as shear forces (Nxy).

The other reason which the writer has applied a hyperbolic paraboloidal shell to the base of the water tank is: having noticed most of the tanks of water towers are in inverted cone shape. Thus, writer would like to investigate whether a hyperbolic paraboloidal shell in the tank could help in the saving of materials which imply to a hyperbolic paraboloidal shell roof, such as that described in the previous paragraph. The result of analysis and design shows, in this particular case, a saving of a lot of material. The thickness of this shell is one-and-a half feet at the shaft which is much less than the thickness of many other water tanks which are of inverted cone shape. The amount of steel used in this water tank is also less.

The membrane theory is not an ideal method used for analyzing this particular shell structure. Cracks which must be free from in-water tanks may occur in the x'-direction on the outer face and y'-direction in the inner face of the tank due to the moments which are neglected in the membrane theory. Therefore, the writer suggests that at the present stage, finite element method, which moments can be included, is the best tool to analyze this shell structure. Due to its simplicity, the membrane theory is best used as a preliminary analysis and design, so as to estimate the amount of materials being used.

The generation of hyperbolic paraboloids by straight lines will provide an advantage on the construction of the shell. If formwork and reinforcements are placed in these directions of straight lines, no bending of wooden plates and steel bars are required.

If the meshes of reinforcement are designed to run along the straight lines which are in the direction of the x and y-axes, a lot of steel will be required. Reinforcement can be reduced by placing bars in the direction of the principal stresses, which are in the x' and y'-directions. Furthermore, prestressed steel can be applied to the direction of the principal tensile stresses to reduce requirements of steel.

In conclusion, a hyperbolic paraboloidal shell does furnish some advantage to the construction, design and economy of this water tank, as well as providing a graceful shape to the structure.

BIBLIOGRAPHY

BIBLIOGRAPHY

References for the General Information of Water Towers

[1] B.E. Nagler, Elevated Water Reservoir in Finland, AWWA J., V 58, n11, Nov.1969, p.1429.

[2] S.H. Medwadowski, World Conference on Shell Structures, San Francisco, 1962. Proceedings- p.329-338.

[3] Anonymous, Water Tower at Felsted Essex, India Concrete J., V 44, n8, Aug.1970, p.352.

[4] R.C.Parkinson, Water Tower at Columbus Serves Dual Purpose, Water and Waste Eng. V 6, July 1969, p.34-35.

[5] K. Eriksson, Prestressed Concrete Water Tower in Orebro Sweden, Civil Eng., 28:750, 1958.

References for the Theory of Shell and Membrane Theory

[6] Norris and Wilbur, Elementary Structural Analysis, Chapter 17, McGraw-Hill, 1948.

[7] W. Flügge, Stresses in Shells, Berlin, New York, Springer-Verlag, 1973.

[8] Timoshenko and Woinowsky-Krieger, Theory of Shells, New York, McGraw-Hill, 1959.

[9] A. Pflüger, Elementary Statics of Shells, New York, F.W.Dodge Corp., 1961.

[10] D.P. Billington, Thin Shell Concrete Structures, McGraw-Hill, 1965.

References for The Analysis of Hypobolic Paraboloid By Membrane Theory

[11] F. Candella, Structural Application of Hyperbolic Paraboloidal Shell, ACI Proceedings, Vol. 57, Oct.1960, p.399.

[12] H.D. Ramirez, A.C. Scordelis and D. Ngo, Membrane Stresses in Hyperbolic Paraboloid Shells Having a Parallelogram Shape in Plan, ACI J., Dec. 1969, Vol. 66, p. 994.

[13] A.C. Scordelis, H.D. Ramirez and D. Ngo, Membrane Stresses in Hyperbolic Paraboloid Shells Having an Arbitrary Quadrilateral Shape in Plan, ACI, J., Jan.1970, p. 36.

References for the Design of the Shell

[14] L. Fischer, Theory and Practice of Shell Structures, Wilhelm Ernst and Sohn, Berlin, Munich, 1968.

[15] P.M. Furguson, Reinforced Concrete Fundamentals, John Wiley & Son, Inc., New York, 1958.

APPENDIX 1

COMPUTER PROGRAM - NOTATIONS

APPENDIX 1

LIST OF NOTATIONS IN THE COMPUTER PROGRAM

T	Thickness of shell in feet
DC	Pressure due to concrete weight in z-direction
DW	Density of water
SOMD2	$\sin\frac{\omega}{2}$
COMD2	$\cos\frac{\omega}{2}$
COSOM	$\cos\omega$
SINOM	$\sin\omega$
TH	Angle θ
PS1, PS2	Principal stresses
XK	Constant k
SINAF, COAF	$\sin\alpha$ $\cos\alpha$
SNXBD, SNYBD	Direct stresses due to dead load
SNXBL, SNYBL	Direct stresses due to live load (water)
SNXBT, SNYBT	Total direct stresses
SNXYD	Shear stress due to dead load
SNXYL	Shear stress due to live load
SNXYT	Total shear stress

APPENDIX 2

COMPUTER PROGRAM

COMPUTER PROGRAM

```
      PROGRAM MEMTHE (INPUT,OUTPUT)
      DIMENSION COSA(15,15)
      DIMENSION SNYBD(16,16), SNYBL(16,16), SNYBT(16,16), SNXYD(16,16)
      DIMENSION SNXYT(16,16), SNXYL(16,16), TH(16,16),  PS1(16,16)
      DIMENSION PS2(16,16),SNXBT(16,16) ,TB(16,16)
      DIMENSION SQ(15,15), FF(15,15)
C T IS THE THICKNESS OF THE SHELL
      T=2.
      DC=150.*T
      DW=62.5
C  CALCULATION OF COEFFICIENTS
      SOMD2=.3
      COMD2=SQRT(1.-.3**2)
      COSOM=1.-2.*SOMD2**2
      SINOM=2.*SOMD2*COMD2
      XK=14.0/70.0**2/SINOM
      C1  = SQRT(80.**2-24.**2)
      CK1 = 32.*DC*SINOM*SOMD2/XK/C1
      CK2 = CK1/2.
      CK3= CK2* COSOM
      CK4 = CK1 * COSOM
      CK5 = 41.5692*DC*SINOM/XK/C1/2.
      CK6 = CK5 * COSOM
      CK7 = CK1+CK2
      CK8 = (CK1*XK*70.+CK5)
      CK9 = 64./C1*COMD2
      CK10= 2.*XK
      CK11= DW*32.*COMD2/C1/XK
C CLEAR ARRAY
      DO 9 I=1,15
      DO 9 J=1,15
      SNYBD(I,J)=0.0
      SNYBL(I,J)=0.0
      SNYBT(I,J)=0.0
      SNXBT(I,J)=0.0
      SNXYD(I,J)=0.0
      SNXYL(I,J)=0.0
      SNXYT(I,J)=0.0
      TH(I,J)=0.0
      TB(I,J) = 0.0
      PS1(I,J)=0.0
    9 PS2(I,J)=0.0
C  CALCULATION OF STRESSES
      DO 10 I=1,15
      X = I*5-5
      XM=XK*X
      N=16-I
      DO 10 J=1,N
      Y = J*5-5
      XN=XK*Y
      GNMC = XN -XM*COSOM
      SQPH = SQRT(1.0+XM**2+XN**2-2.*XM*XN*COSOM)
      SQ1M = SQRT(1.0+XM**2*SINOM**2)
      GDSQ = GNMC/SQ1M
      LOGD = ALOG(GDSQ+SQRT(GDSQ**2+1.0))
C  SNXYD IS THE SHEAR STRESS DUE TO DEAD LOAD
      SNXYD(I,J) =-(CK1*XK*(X+Y)+CK5)*SQPH/1000.
```

```
      V1 =   CK7*(GNMC*SQPH+SQ1M**2*LOGD)
      V2 =  (CK4*XM-CK1*XM+CK6)*(SQPH+XM*COSOM*LOGD)
      V3 =   CK3*((XN+3.*XM*COSOM)*SQPH+(3.*(XM*COSOM)**2-1.-XM**2)*LOGD
     1)
      V4 =  -(CK1*XM**2+CK5*XM)*LOGD
      XN = 70.0 * XK - XM
      GNMC = XN -XM*COSOM
      SQPH = SQRT(1.0+XM**2+XN**2-2.*XM*XN*COSOM)
      GDSQ = GNMC/SQ1M
      LOGD = ALOG(GDSQ+SQRT(GDSQ**2+1.0))
      V5 = -CK7*(GNMC*SQPH+SQ1M**2*LOGD)
      V6 = -(CK4*XM-CK1*XM+CK6)*(SQPH+XM*COSOM*LOGD)
      V7 = - CK3*((XN+3.*XM*COSOM)*SQPH+(3.*(XM*COSOM)**2-1.-XM**2)*LOGD
     1)
      V8 =   (CK1*XM**2+CK5*XM)*LOGD
      V9 = CK8*SQPH
      VYD=(V1+V2+V3+V4+V5+V6+V7+V8+V9)
      P=XK*Y*SINOM
      Q=XK*X*SINOM
      QS=Q**2
      PS=P**2
C  SNYBD IS THE DIRECT STRESS DUE TO DEAD LOAD   Y DIR
      SNYBD(I,J) = VYD*SQRT((1.+QS)/(1.+PS))/1000.
      VYL = CK11*(70.-X-Y)
C  SNYBL IS THE DIRECT STRESS DUE TO LIVE LOAD   Y DIR
      SNYBL(I,J) = VYL*SQRT((1.+QS)/(1.+PS))/1000.
C  SNYBT IS THE TOTAL DIRECT STRESS DUE TO DEAD PLUS LIVE LOAD   Y DIR
      SNYBT(I,J) =SNYBD(I,J)+SNYBL(I,J)
C  SNXYL IS THE SHEAR STRESS DUE TO LIVE LOAD
      SNXYL(I,J) =-DW*(56.-CK9*(X+Y))/CK10/1000.
C  SNXYT IS THE SHEAR STRESS DUE TO LIVE PLUS DEAD LOAD
   10 SNXYT(I,J) = SNXYD(I,J)+SNXYL(I,J)
      DO 20 I =1,15
      DO 20 J=1,15
   20 SNXBT(J,I)=SNYBT(I,J)
      DO 21 I=1,15
      X=I*5-5
      N=16-I
      DO 21 J=1,N
      Y =J*5-5
      P=XK*Y*SINOM
      Q=XK*X*SINOM
      QS=Q**2
      PS=P**2
      FF(I,J) =P*Q+COSOM
      SQ(I,J)=SQRT((1.+PS)*(1.+QS))
      COSAF=FF(I,J)/SQ(I,J)
      COSA(I,J)=COSAF
      SINAF = SQRT(1.-COSAF**2)
      SIN2A = 2.*COSAF*SINAF
      COS2A = 1.-2.*SINAF**2
      XX = 2.*SNXYT(I,J)*SINAF+SNYBT(I,J)*SIN2A
      YY = SNXBT(I,J)+2.*SNXYT(I,J)*COSAF+SNYBT(I,J)*COS2A
      TAN2T=XX/YY
      THT2 = ATAN(TAN2T)
      TH(I,J) = THT2/2.
      TE=TH(I,J)
```

```
      SINTH=SIN(TE)
      PHI = SQRT((1.+PS)/(1.+QS))
      TANTB=PHI*SIN(TE)*SINOM/(COS(TE)*SINAF+SIN(TE)*(PHI*COSOM-COSAF))
      TB(I,J)=ATAN(TANTB)
      STHMA =-SINAF*COS(TE) + SIN(TE)*COSAF
      CTHMA = COS(TE)*COSAF + SINAF*SIN(TE)
C   PS1 AND PS2 ARE THE PRINCIPAL STRESSES DUE TO DEAD PLUS LIVE LOAD
      PP        = SNXBT(I,J)*SIN(TE)**2
      QQ        = 2.*SNXYT(I,J)*SIN(TE)*STHMA
      RR        =SNYBT(I,J)*STHMA**2
      SS        = PP+QQ+RR
      PS1(I,J)=SS/SINAF/144./T
   21 PS2(I,J) =(SNXBT(I,J)*COS(TE)**2+2.*SNXYT(I,J)*COS(TE)*CTHMA+
     1SNYBT(I,J)*CTHMA**2)/SINAF/144./T
C   PRINT OUT OF RESULTS
      PRINT 888
      PRINT 998,
      DO 11 I = 1,15
      PRINT 999, (SNYBD(I,J),J=1,15)
  999 FORMAT (6X, 15F7.1)
      II=I*5-5
   11 PRINT 997,II
      PRINT 887
      PRINT 998
      DO 12 I = 1,15
      PRINT 999, (SNYBL(I,J),J=1,15)
      II = I*5-5
   12 PRINT 997,II
      PRINT 886
      PRINT 998
      DO 13 I = 1,15
      PRINT 999, (SNYBT(I,J),J=1,15)
      II = I*5-5
   13 PRINT 997,II
      PRINT 885
      PRINT 998
      DO 14 I = 1,15
      PRINT 999, (SNXYD(I,J),J=1,15)
      II = I*5-5
   14 PRINT 997,II
      PRINT 884
      PRINT 998
      DO 15 I = 1,15
      PRINT 999, (SNXYL(I,J),J=1,15)
      II = I*5-5
   15 PRINT 997,II
      PRINT 883
      PRINT 998
      DO 16 I =,1,15
      PRINT 999, (SNXYT(I,J),J=1,15)
      II = I*5-5
   16 PRINT 997,II
  888 FORMAT ("1", " DIRECT STRESS DUE TO DEAD LOAD    KIPS PER FT")
  887 FORMAT ("1", " DIRECT STRESS DUE TO LIVE LOAD    KIPS PER FT")
  886 FORMAT ("1", " TOTAL DIRECT STRESS    KIPS PER FT")
  885 FORMAT ("1", " SHEAR STRESS DUE TO DEAD LOAD    KIPS PER FT")
  884 FORMAT ("1", " SHEAR STRESS DUE TO LIVE LOAD    KIPS PER FT")
```

```
883 FORMAT ("1", " TOTAL SHEAR STRESS   KIPS PER FT")
996 FORMAT (6X,16F7.3)
998 FORMAT("0","           Y= 0    Y= 5    Y=10    Y=15    Y=20    Y=25    Y=30
   1  Y=35    Y=40    Y=45    Y=50    Y=55    Y=60    Y=65    Y=70")
997 FORMAT ("+","X=", I2)
    STOP
    END
```

APPENDIX 3

COMPUTER OUTPUT

COMPUTER OUTPUT

DIRECT STRESS DUE TO DEAD LOAD, KIPS PER FT. (1)

DIRECT STRESS DUE TO LIVE LOAD, KIPS PER FT. (2)

DIRECT STRESS DUE TO DEAD LOAD KIPS PER FT

	Y= 0	Y= 5	Y=10	Y=15	Y=20	Y=25	Y=30	Y=35	Y=40	Y=45	Y=50	Y=55	Y=60	Y=65	Y=70
X= 0	8.0	6.2	5.4	8.6	8.9	9.1	9.3	9.5	9.3	10.0	10.3	10.5	10.8	11.0	11.3
X= 5	8.2	6.4	8.6	8.6	9.1	9.3	9.5	9.7	10.0	10.2	10.4	10.7	10.9	11.2	0.0
X=10	8.4	6.6	8.8	9.1	9.3	9.5	9.7	9.9	10.1	10.4	10.6	10.8	11.1	0.0	0.0
X=15	8.7	6.9	9.1	9.3	9.5	9.7	9.9	10.1	10.3	10.5	10.6	11.0	0.0	0.0	0.0
X=20	6.9	9.1	9.3	9.5	9.7	9.9	10.1	10.3	10.5	10.7	11.0	0.0	0.0	0.0	0.0
X=25	9.1	9.3	9.5	9.7	9.9	10.1	10.3	10.5	10.7	10.9	0.0	0.0	0.0	0.0	0.0
X=30	9.4	9.5	9.7	9.9	10.1	10.3	10.5	10.7	10.0	0.0	0.0	0.0	0.0	0.0	0.0
X=35	9.6	9.8	10.0	10.2	10.4	10.5	10.7	10.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=40	9.9	10.0	10.2	10.4	10.6	10.8	11.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=45	10.1	10.3	10.5	10.7	10.9	11.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=50	10.4	10.6	10.8	11.0	11.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=55	10.7	10.9	11.1	11.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=60	11.1	11.2	11.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=65	11.4	11.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=70	11.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

(1)

DIRECT STRESS DUE TO LIVE LOAD KIPS PER FT

	Y= 0	Y= 5	Y=10	Y=15	Y=20	Y=25	Y=30	Y=35	Y=40	Y=45	Y=50	Y=55	Y=60	Y=65	Y=70
X= 0	350.6	325.5	300.4	275.2	250.0	224.8	199.6	174.4	149.3	124.2	99.2	74.2	49.4	24.6	0.0
X= 5	325.6	300.5	275.4	250.2	225.0	199.8	174.7	149.5	124.4	99.4	74.4	49.5	24.7	0.0	0.0
X=10	300.6	275.5	250.4	225.3	200.1	174.9	149.8	124.6	99.6	74.5	49.6	24.7	0.0	0.0	0.0
X=15	275.7	250.6	225.5	200.3	175.2	150.0	124.9	99.8	74.7	49.7	24.8	0.0	0.0	0.0	0.0
X=20	250.8	225.7	200.6	175.4	150.2	125.1	100.0	74.9	49.5	24.9	0.0	0.0	0.0	0.0	0.0
X=25	225.9	200.8	175.7	150.5	125.3	100.2	75.0	50.0	24.9	0.0	0.0	0.0	0.0	0.0	0.0
X=30	201.1	175.9	150.7	125.5	100.4	75.2	50.1	25.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=35	176.2	151.0	125.8	100.6	75.4	50.2	25.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=40	151.2	126.0	100.8	75.5	50.3	25.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=45	126.2	101.0	75.7	50.4	25.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=50	101.2	75.9	50.6	25.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=55	76.0	50.7	25.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=60	50.8	25.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=65	25.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

(2)

COMPUTER OUTPUT

SHEAR STRESS DUE TO DEAD LOAD, KIPS PER FT. (1A)

SHEAR STRESS DUE TO DEAD LOAD, KIPS PER FT. (2A)

SHEAR STRESS DUE TO DEAD LOAD KIPS PER FT

	Y= 0	Y= 5	Y=10	Y=15	Y=20	Y=25	Y=30	Y=35	Y=40	Y=45	Y=50	Y=55	Y=60	Y=65	Y=70
X= 0	-9.4	-9.5	-9.6	-9.7	-9.8	-10.0	-10.1	-10.3	-10.4	-10.6	-10.8	-10.9	-11.1	-11.3	-11.5
X= 5	-9.5	-9.6	-9.7	-9.8	-9.9	-10.1	-10.2	-10.4	-10.5	-10.7	-10.8	-11.0	-11.2	-11.4	0.0
X=10	-9.6	-9.7	-9.8	-9.9	-10.0	-10.2	-10.3	-10.4	-10.6	-10.7	-10.9	-11.1	-11.2	0.0	0.0
X=15	-9.7	-9.8	-9.9	-10.0	-10.1	-10.3	-10.4	-10.5	-10.7	-10.8	-11.0	-11.1	0.0	0.0	0.0
X=20	-9.8	-9.9	-10.0	-10.1	-10.3	-10.4	-10.5	-10.6	-10.8	-10.9	-11.0	0.0	0.0	0.0	0.0
X=25	-10.0	-10.1	-10.2	-10.3	-10.4	-10.5	-10.6	-10.7	-10.9	-11.0	0.0	0.0	0.0	0.0	0.0
X=30	-10.1	-10.2	-10.3	-10.4	-10.5	-10.6	-10.7	-10.8	-11.0	0.0	0.0	0.0	0.0	0.0	0.0
X=35	-10.3	-10.4	-10.4	-10.5	-10.6	-10.7	-10.8	-10.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=40	-10.4	-10.5	-10.6	-10.7	-10.8	-10.9	-11.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=45	-10.6	-10.7	-10.7	-10.8	-10.9	-11.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=50	-10.8	-10.8	-10.9	-11.0	-11.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=55	-10.9	-11.0	-11.1	-11.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=60	-11.1	-11.2	-11.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=65	-11.3	-11.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=70	-11.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

(1A)

SHEAR STRESS DUE TO LIVE LOAD KIPS PER FT

	Y= 0	Y= 5	Y=10	Y=15	Y=20	Y=25	Y=30	Y=35	Y=40	Y=45	Y=50	Y=55	Y=60	Y=65	Y=70
X= 0	-350.6	-325.5	-300.5	-275.4	-250.4	-275.4	-200.3	-175.3	-150.2	-125.2	-100.2	-75.1	-50.1	-25.0	-.0
X= 5	-325.5	-300.5	-275.4	-250.4	-225.4	-200.3	-175.3	-150.2	-125.2	-100.2	-75.1	-50.1	-25.0	-.0	0.0
X=10	-300.5	-275.4	-250.4	-225.4	-200.3	-175.3	-150.2	-125.2	-100.2	-75.1	-50.1	-25.0	-.0	0.0	0.0
X=15	-275.4	-250.4	-225.4	-200.3	-175.3	-150.2	-125.2	-100.2	-75.1	-50.1	-25.0	-.0	0.0	0.0	0.0
X=20	-250.4	-225.4	-200.3	-175.3	-150.2	-125.2	-100.2	-75.1	-50.1	-25.0	-.0	0.0	0.0	0.0	0.0
X=25	-225.4	-200.3	-175.3	-150.2	-125.2	-100.2	-75.1	-50.1	-25.0	-.0	0.0	0.0	0.0	0.0	0.0
X=30	-200.3	-175.3	-150.2	-125.2	-100.2	-75.1	-50.1	-25.0	-.0	0.0	0.0	0.0	0.0	0.0	0.0
X=35	-175.3	-150.2	-125.2	-100.2	-75.1	-50.1	-25.0	-.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=40	-150.2	-125.2	-100.2	-75.1	-50.1	-25.0	-.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=45	-125.2	-100.2	-75.1	-50.1	-25.0	-.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=50	-100.2	-75.1	-50.1	-25.0	-.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=55	-75.1	-50.1	-25.0	-.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=60	-50.1	-25.0	-.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=65	-25.0	-.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=70	-.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

(2A)

PS1 TOTAL PRINCIPAL STRESS, KIPS PER SQ. IN. (3)
PS2 TOTAL PRINCIPAL STRESS, KIPS PER SQ. IN. (4)

PS1 TOTAL PRINCIPAL STRESS KIPS PER SQ. IN.

	Y= 0	Y= 5	Y=10	Y=15	Y=20	Y=25	Y=30	Y=35	Y=40	Y=45	Y=50	Y=55	Y=60	Y=65	Y=70
X= 0	.785	.730	.677	.623	.570	.516	.463	.410	.356	.302	.248	.193	.138	.083	.026
X= 5	.730	.676	.622	.568	.514	.460	.407	.353	.299	.245	.191	.136	.081	.026	0.000
X=10	.677	.622	.567	.513	.459	.405	.351	.297	.243	.189	.135	.080	.025	0.000	0.000
X=15	.623	.568	.513	.458	.404	.350	.296	.242	.188	.134	.079	.025	0.000	0.000	0.000
X=20	.570	.514	.459	.404	.349	.295	.241	.187	.133	.079	.024	0.000	0.000	0.000	0.000
X=25	.516	.460	.405	.350	.295	.241	.186	.132	.078	.024	0.000	0.000	0.000	0.000	0.000
X=30	.463	.407	.351	.296	.241	.186	.132	.078	.024	0.000	0.000	0.000	0.000	0.000	0.000
X=35	.410	.353	.297	.242	.187	.132	.078	.024	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=40	.356	.299	.243	.188	.133	.078	.024	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=45	.302	.245	.189	.134	.079	.024	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=50	.248	.191	.135	.079	.024	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=55	.193	.136	.080	.025	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=60	.138	.081	.025	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=65	.083	.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=70	.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

(3)

PS2 TOTAL PRINCIPAL STRESS KIPS PER SQ. IN.

	Y= 0	Y= 5	Y=10	Y=15	Y=20	Y=25	Y=30	Y=35	Y=40	Y=45	Y=50	Y=55	Y=60	Y=65	Y=70
X= 0	-.015	-.014	-.013	-.012	-.011	-.010	-.009	-.008	-.007	-.006	-.005	-.003	-.002	-.001	-.000
X= 5	-.014	-.013	-.012	-.011	-.010	-.009	-.008	-.007	-.005	-.004	-.003	-.002	-.001	-.000	0.000
X=10	-.013	-.012	-.011	-.010	-.008	-.007	-.006	-.005	-.004	-.003	-.002	-.001	-.000	0.000	0.000
X=15	-.012	-.011	-.010	-.008	-.007	-.006	-.005	-.004	-.003	-.002	-.001	-.000	0.000	0.000	0.000
X=20	-.011	-.010	-.008	-.007	-.006	-.005	-.004	-.003	-.002	-.001	-.000	0.000	0.000	0.000	0.000
X=25	-.010	-.009	-.007	-.006	-.005	-.004	-.003	-.002	-.001	-.000	0.000	0.000	0.000	0.000	0.000
X=30	-.009	-.008	-.006	-.005	-.004	-.003	-.002	-.001	-.000	0.000	0.000	0.000	0.000	0.000	0.000
X=35	-.008	-.007	-.005	-.004	-.003	-.002	-.001	-.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=40	-.007	-.005	-.004	-.003	-.002	-.001	-.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=45	-.006	-.004	-.003	-.002	-.001	-.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=50	-.005	-.003	-.002	-.001	-.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=55	-.003	-.002	-.001	-.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=60	-.002	-.001	-.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=65	-.001	-.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=70	-.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

(4)

TOTAL DIRECT STRESS, KIPS PER FT. (5)

TOTAL SHEAR STRESS, KIPS PER FT. (6)

TOTAL DIRECT STRESS KIPS PER FT

	Y= 0	Y= 5	Y=10	Y=15	Y=20	Y=25	Y=30	Y=35	Y=40	Y=45	Y=50	Y=55	Y=60	Y=65	Y=70
X= 0	358.6	333.7	308.8	283.8	258.9	233.9	208.9	184.0	159.1	134.2	109.4	84.7	60.1	35.7	11.3
X= 5	333.8	308.9	284.0	259.1	234.1	209.1	184.2	159.2	134.4	109.6	84.8	60.2	35.6	11.2	0.0
X=10	309.1	284.2	259.3	234.3	209.3	184.4	159.5	134.5	109.7	84.9	60.2	35.6	11.1	0.0	0.0
X=15	284.4	259.5	234.5	209.6	184.6	159.7	134.8	109.9	85.0	60.3	35.6	11.0	0.0	0.0	0.0
X=20	259.7	234.8	209.9	184.9	159.9	135.0	110.0	85.2	60.4	35.6	11.0	0.0	0.0	0.0	0.0
X=25	235.1	210.1	185.2	160.2	135.2	110.3	85.3	60.5	35.7	10.9	0.0	0.0	0.0	0.0	0.0
X=30	210.4	185.5	160.5	135.5	110.5	85.5	60.6	35.7	10.9	0.0	0.0	0.0	0.0	0.0	0.0
X=35	185.8	160.8	135.8	110.7	85.7	60.8	35.8	10.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=40	161.1	136.1	111.0	86.0	60.9	35.9	11.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=45	136.4	111.3	86.2	61.1	36.1	11.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=50	111.6	86.5	61.4	36.2	11.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=55	86.8	61.6	36.4	11.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=60	61.9	36.6	11.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=65	36.9	11.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=70	11.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

(5)

TOTAL SHEAR STRESS KIPS PER FT

	Y= 0	Y= 5	Y=10	Y=15	Y=20	Y=25	Y=30	Y=35	Y=40	Y=45	Y=50	Y=55	Y=60	Y=65	Y=70
X= 0	-359.9	-335.0	-310.1	-285.2	-260.3	-235.4	-210.5	-185.6	-160.7	-135.8	-110.9	-86.1	-61.2	-36.4	-11.5
X= 5	-335.0	-310.1	-285.1	-260.2	-235.3	-210.4	-185.5	-160.6	-135.7	-110.8	-86.0	-61.1	-36.2	-11.4	0.0
X=10	-310.1	-285.1	-260.2	-235.3	-210.4	-185.5	-160.5	-135.6	-110.7	-85.9	-61.0	-36.1	-11.2	0.0	0.0
X=15	-285.2	-260.2	-235.3	-210.4	-185.4	-160.5	-135.6	-110.7	-85.8	-60.9	-36.0	-11.1	0.0	0.0	0.0
X=20	-260.3	-235.3	-210.4	-185.4	-160.5	-135.6	-110.7	-85.7	-60.8	-35.9	-11.0	0.0	0.0	0.0	0.0
X=25	-235.4	-210.4	-185.5	-160.5	-135.6	-110.6	-85.7	-60.8	-35.9	-11.0	0.0	0.0	0.0	0.0	0.0
X=30	-210.5	-185.5	-160.5	-135.6	-110.7	-85.7	-60.8	-35.9	-11.0	0.0	0.0	0.0	0.0	0.0	0.0
X=35	-185.6	-160.6	-135.6	-110.7	-85.7	-60.8	-35.9	-10.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=40	-160.7	-135.7	-110.7	-85.8	-60.8	-35.9	-11.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=45	-135.8	-110.8	-85.9	-60.9	-35.9	-11.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=50	-110.9	-86.0	-61.0	-36.0	-11.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=55	-86.1	-61.1	-36.1	-11.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=60	-61.2	-36.2	-11.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=65	-36.4	-11.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
X=70	-11.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

(6)

PROJECTION OF THE ANGLE THETA ON THE X-Y PLANE (7)
(RADIANS)

PROJECTION OF ANGLE THETA ON THE X-Y PLANE   RADIANS

	Y= 0	Y= 5	Y=10	Y=15	Y=20	Y=25	Y=30	Y=35	Y=40	Y=45	Y=50	Y=55	Y=60	Y=65	Y=70
X= 0	.305	.305	.305	.306	.307	.309	.311	.313	.315	.318	.321	.324	.329	.331	.336
X= 5	.305	.305	.305	.306	.307	.309	.311	.313	.315	.318	.321	.324	.328	.332	0.000
X=10	.304	.304	.305	.306	.307	.308	.310	.312	.315	.318	.321	.324	.328	0.000	0.000
X=15	.303	.303	.304	.305	.306	.307	.309	.311	.314	.317	.320	.324	0.000	0.000	0.000
X=20	.302	.302	.303	.303	.305	.306	.308	.310	.313	.316	.319	0.000	0.000	0.000	0.000
X=25	.300	.301	.301	.302	.303	.305	.307	.309	.311	.315	0.000	0.000	0.000	0.000	0.000
X=30	.299	.299	.299	.300	.301	.303	.305	.307	.310	0.000	0.000	0.000	0.000	0.000	0.000
X=35	.296	.297	.297	.298	.299	.301	.302	.305	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=40	.294	.294	.295	.295	.297	.298	.300	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=45	.291	.291	.292	.293	.294	.295	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=50	.288	.288	.289	.289	.290	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=55	.285	.285	.285	.285	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=60	.282	.282	.281	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=65	.278	.277	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X=70	.273	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

(7)