LATERAL INSTABILITY

OF

SINGLE ANGLE BEAM-COLUMNS

By

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A TECHNICAL REPORT

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ABSTRACT

The problem of lateral-torsional buckling of angles subjected to axial load with moments about both principal axes is studied as a special case of the general solution given by Timoshenko.

In this solution, two geometric terms, β_x and β_y, require evaluation. A computer program to perform this, together with simplified approximate expression, is developed and the values for standard sections are listed.

A computer program is also developed to solve the general cubic equation for the critical eccentric compression load, representing the desired combination of axial load and moments.

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FOREWORD

This technical report was prepared to satisfy the dissertation requirement of the Master of Engineering Degree at Sir George Williams University.

Laterally unsupported single angle steel members subject to bending, or axial compression plus bending, occur frequently in towers and in bracing systems. At present, major specifications do not provide adequate design information for these situations.

In September, 1970 the Engineering and Research Committee of the Canadian Steel Industries Construction Council invited proposals for research in this area. It was hoped that this research would suggest a practical design formula and/or rules to assist engineers.

Professor Cedric Marsh of the Department of Engineering, Sir George Williams University, submitted a proposal to the Committee. This proposal was accepted and the requested funds were allocated.

This paper presents the theoretical determination of buckling loads and moments. This includes the computer programs developed for this solution.

The author wishes to acknowledge the assistance of Professor Cedric Marsh in all aspects of the preparation of this paper.

INTRODUCTION

Structural steel angles, often referred to as "angle irons", have long been a standard structural element. This shape has existed for more than 150 years: first in wrought iron, then in steel. It is commonly used in tension and compression members, both singly and combined.

Yet the design of a laterally unsupported beam-column comprised of a single angle is not specified in any standard. Design manuals (8, 9, 10) refer to the problem, but only offer incomplete solutions.

The theoretical aspects of the problem have been understood for the last twenty-five years (1, 2, 4, 5, 7). The phenomena involved concerns primarily the elastic stability of a member which may buckle in flexure or in a combined flexural-torsional mode.

The research described in this report proceeded to determine the predictability of actual buckling loads by the existing theory. From this, an attempt can be made to arrange this information in a form conveniently useful to practising designers.

I. SOURCES

The theoretical background for this study is presented fairly thoroughly in the book "Theory of Elastic Stability" (1) by S. P. Timoshenko and J. M. Gere. This book also documents other references for this material.

The symbols and development presented in this report will follow the above text closely. Some improvements were adopted from the book "Thin-Walled Elastic Beams" (2) by V. Z. Vlasov.

II. GENERAL CASE

In the general case of a column of thin-walled open cross section, buckling failure usually occurs by a combination of torsion and buckling. Consider the unsymmetrical cross section shown in Figure 1(a). The x- and y-axes are the principal centroidal axes of the cross section and x_o, y_o are the coordinates of the shear centre, O. (Figure 1 (b) and (c) show the same orientation for the angle shapes dealt with specifically later in this report.) During buckling the cross section will undergo translation and rotation.

The most general loading case is of a bar subjected to the action of bending couples M_x, M_y at the ends in addition to a central thrust P, as shown in Figure 2. It is assumed in the analysis that the effect of P on the bending stresses can be neglected. The initial deflection of the bar due to the couples M_x, M_y will be considered as very small.

From the three equations of equilibrium for the buckled form of the bar, the critical values of the external forces can be calculated for any end condition.

The most useful assumption of end conditions is that the bar has simple supports, so that the ends are free to warp and to rotate about the x- and y-axes, but cannot rotate about the z-axis or deflect in the x- and y-directions. These conditions are satisfied if solutions to the resulting

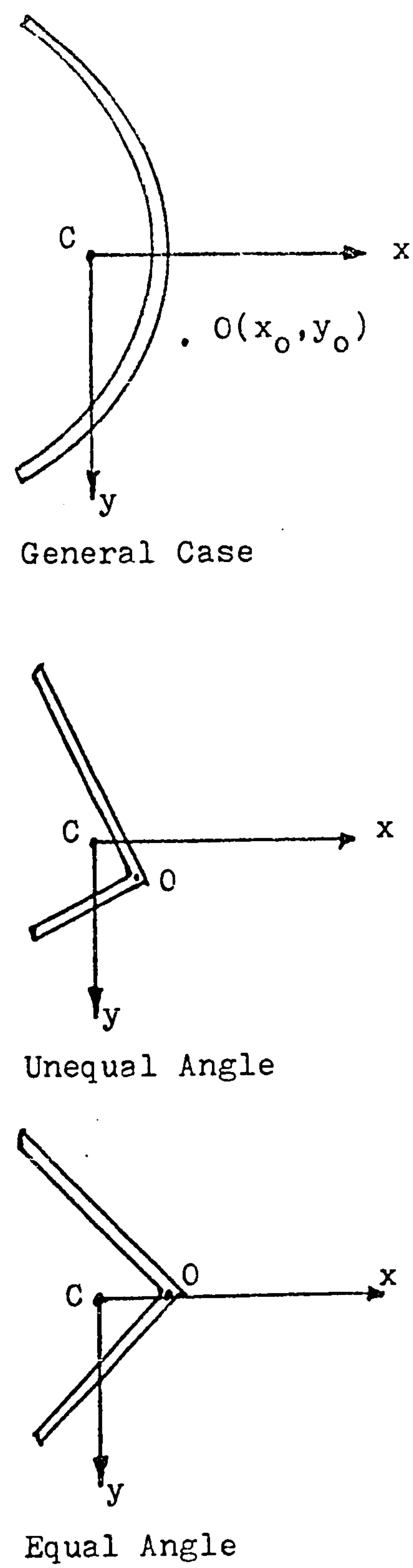


FIGURE 1: CROSS SECTIONS OF OPEN THIN-WALLED SHAPES

Principal axes indicated.

C = Centroid; O = Shear Centre

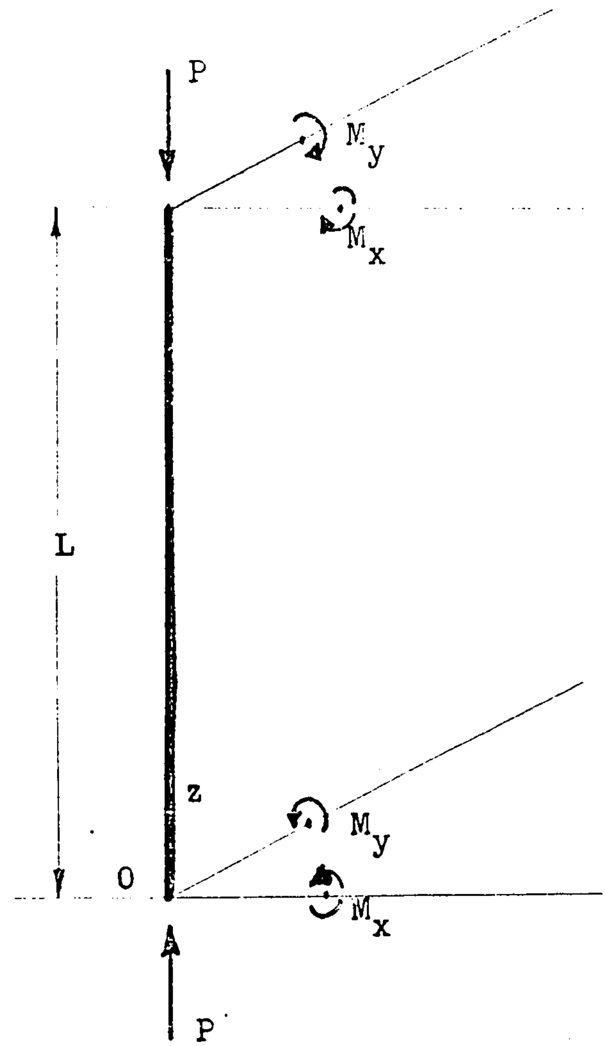


FIGURE 2: CENTRAL THRUST AND END MOMENTS

differential equations of bending and twisting are taken in the form:

$$\text{Deflection (or twist)} = A_i \sin \frac{\pi z}{L} \quad \ldots\ldots\ldots\ldots \quad (1)$$

where A_i = constants,

i = 1 for x-direction,

= 2 for y-direction,

= 3 for z-direction (twist)

Substituting these solutions into the differential equations leads to the following determinant:

$$\begin{vmatrix} (P_y - P) & 0 & -(Py_o - M_x) \\ 0 & (P_x - P) & (Px_o - M_y) \\ -(Py_o - M_x) & (Px_o - M_y) & \frac{I_o}{A}\left[P_t - \frac{A}{I_o}(M_x\beta_y + M_y\beta_x - P)\right] \end{vmatrix} = 0$$

$$\ldots\ldots\ldots\ldots \quad (2)$$

where: $P_x = \dfrac{\pi^2 EI_x}{L^2}$, Euler critical load for buckling about the x-axis, (kips),

$P_y = \dfrac{\pi^2 EI_y}{L^2}$, Euler critical load for buckling about the y-axis, (kips),

$P_t = \dfrac{A}{I_o}\left[GJ + \dfrac{\pi^2 EH}{L^2}\right]$, the critical load for pure torsional buckling, (kips),

$\beta_y = \dfrac{1}{I_x}\left[\int_A y^3 dA + \int_A x^2 y dA\right] - 2y_o$, (inches),

$\beta_x = \dfrac{1}{I_y}\left[\int_A x^3 dA + \int_A xy^2 dA\right] - 2x_o$, (inches), E = Modulus of elasticity (29,000 ksi. for steel),

G = Shear modulus of elasticity (11,200 ksi. for steel),

L = Length of member, (inches),

I_x = Moment of inertia of the cross section area about the x-axis (major principal axis), (inches^4),

I_y = Moment of inertia of the cross section area about the y-axis (minor principal axis), (inches^4),

$I_o = I_x + I_y + A(x_o^2 + y_o^2)$,

= Polar moment of inertia with reference to an axis through the shear centre, (inches^4),

A = Cross section area of the member, (inches^2).

H = Total warping constant of the cross section, (inches^6),

J = Torsional constant of the cross section, (inches^4).

While most of these properties are familiar quantities, their exact determination requires some attention. The calculation of the total warping constant, H, and the torsional constant, J, are described in detail later in the paper. (See Equations (22) and (25).)

The parameters β_x and β_y correspond to those which Timoshenko (1) describes by the notation β_2 and β_1, respectively. (i.e. $\beta_1 = \beta_y$ and $\beta_2 = \beta_x$.) The use of the notation in this paper is taken from Vlasov (2).

III. ECCENTRIC THRUST

a) Development of Cubic Equation

Under the particular loading of eccentric thrust, the end moments are applied by means of an eccentric load P. Denoting the coordinates of the point of application of this load by eccentricities e_x and e_y, we have:

$$M_x = Pe_y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$M_y = Pe_x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Substituting these expressions into the determinant of Equation (2) gives:

$$\begin{vmatrix} (P_y - P) & 0 & -P(y_o - e_y) \\ 0 & (P_x - P) & P(x_o - e_x) \\ -P(y_o - e_y) & P(x_o - e_x) & \frac{I_o}{A}\left[P_t - \frac{AP}{I_o}(e_y\beta_y + e_x\beta_x) - P\right] \end{vmatrix} = 0 \quad \ldots\ldots\ldots\ldots\ldots(5)$$

The determinant can be expanded into the form of a cubic equation.

$$(P_y - P)\left\{(P_x - P)\frac{I_o}{A}\left[P_t - \frac{AP}{I_o}(e_y\beta_y + e_x\beta_x) - P\right] - P^2(x_o - e_x)^2\right\}$$

$$- P(y_o - e_y)\left[0 + (P_x - P)P(y_o - e_y)\right] = 0 \quad \ldots\ldots \quad (6)$$

For convenience we define a new term, K, as

$$K = \frac{A}{I_o}(e_y\beta_y + e_x\beta_x) + 1 \quad \ldots\ldots\ldots\ldots \quad (7)$$

Substituting Equation (7) and rearranging Equation (6) in terms of P gives:

$$f(P) = k_1P^3 + k_2P^2 + k_3P + k_4 = 0 \quad \dots\dots\dots\dots\dots \quad (8)$$

where:

$$k_1 = (x_o - e_x)^2 + (y_o - e_y)^2 - \frac{I}{A}_o K$$

$$k_2 = \frac{I}{A}_o P_t + \frac{I}{A}_o K(P_x + P_y) - P_y(x_o - e_x)^2 - P_x(y_o - e_y)^2$$

$$k_3 = - \frac{I}{A}_o P_t(P_x + P_y) - \frac{I}{A}_o K P_x P_y$$

$$k_4 = \frac{I}{A}_o P_t P_x P_y$$

This is the cubic equation for the buckling load, P, which forms the basis for most of the remainder of the thesis.

b) Shape of the Cubic Function

Studying Equation (8) leads to some conclusions about the solutions to the buckling load.

For small values of the critical load, P, f(P) assumes the value of the last term, which is positive. This determines the point at which the curve cuts the vertical axis in Figure 3.

For the particular buckling load, $P = P_x$,

$$f(P) = f(P_x) = P_x^2(x_o - e_x)^2(P_x - P_y) \quad \dots\dots\dots \quad (9)$$

and for the case of $P = P_y$,

$$f(P) = f(P_y) = P_y^2(y_o - e_y)^2(P_y - P_x) \quad \dots\dots\dots \quad (10)$$

For the angle sections dealt with in this research, it is clear that P_x is greater than P_y, (i.e. that the

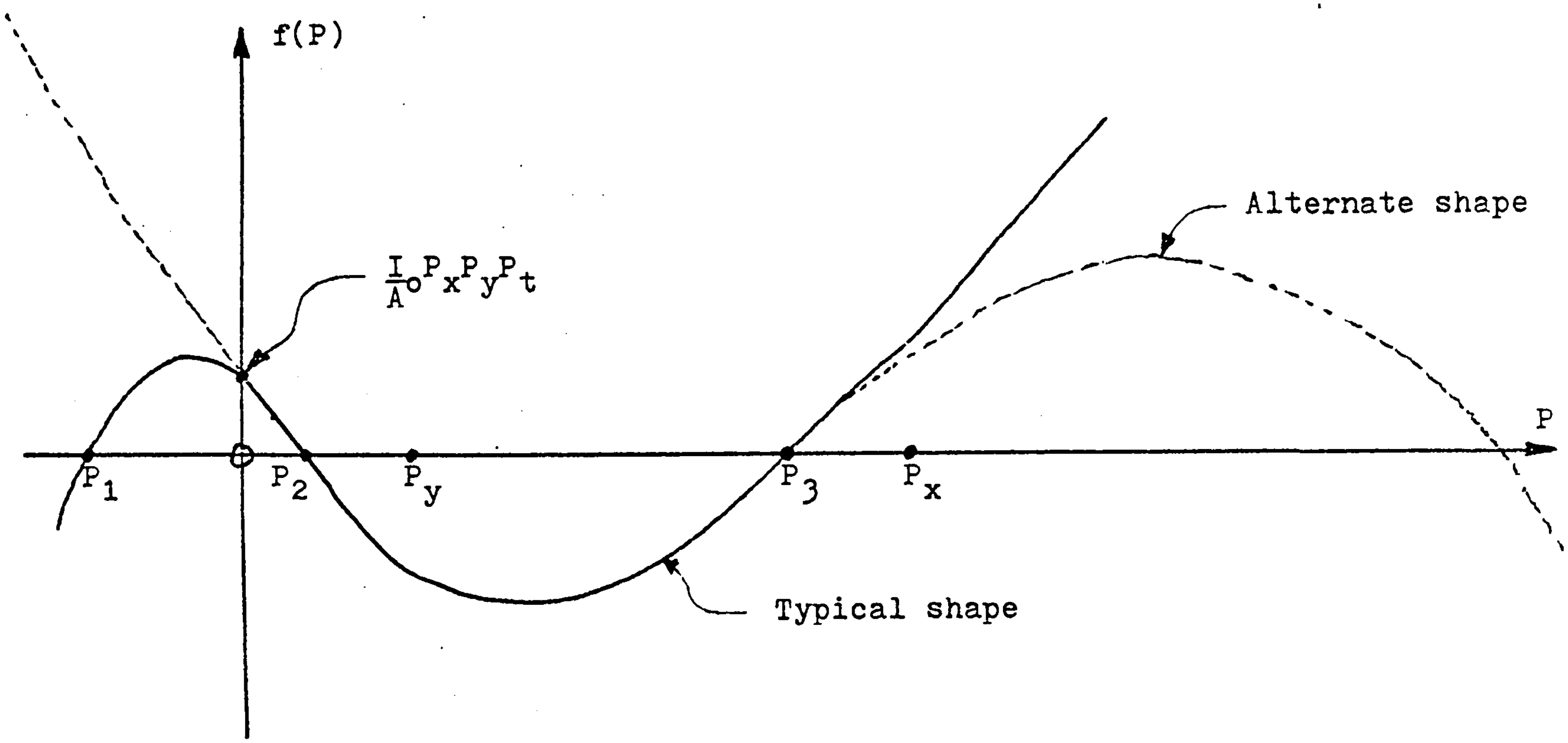


FIGURE 3: SHAPE OF THE CUBIC FUNCTION

Euler buckling load is greater about the x-axis than about the y-axis).

Therefore $f(P_x)$ is of opposite sign to $f(P_y)$, and one root of the equation lies between P_x and P_y. Each root of the equation of course represents a critical buckling load. A root with a negative sign represents a tension force applied to the member to cause buckling. This last failure is more easily imagined as a negative moment applied at each end to cause buckling.

c) Thrust Applied at the Shear Centre

One special case of interest occurs if the thrust, P, is applied along the shear-centre axis. This is described by:

$$e_x = x_o$$
$$e_y = y_o$$

Substituting this into the determinant of Equation (5) gives:

$$\begin{vmatrix} (P_y - P) & 0 & 0 \\ 0 & (P_x - P) & 0 \\ 0 & 0 & \frac{I_o}{A}(P_t - PK) \end{vmatrix} = 0 \quad \ldots\ldots \quad (11)$$

In this case lateral buckling in the two principal planes and torsional buckling may occur independently. This may be observed mathematically from Equation 11, or in the form of the expanded equation:

$$(P_y - P)(P_x - P)\frac{I_o}{A}(P_t - PK) = 0 \;\ldots\ldots\ldots \quad (12)$$

The three roots of Equation (12) are:

$$P = P_y$$

$$P = P_x$$

$$P = \frac{P_t}{K}$$

and the critical buckling load is the smallest of these three roots.

d) Thrust Applied at the Centroid

Another special case occurs if the thrust, P, is applied along the centroidal axis of the member. This is described by:

$$e_x = e_y = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

From Equation (7) $K = 1$,

and the determinant in Equation (5) becomes:

$$\begin{vmatrix} (P_y - P) & 0 & -Py_o \\ 0 & (P_x - P) & Px_o \\ -Py_o & Px_o & \frac{I_o}{A} (P_t - P) \end{vmatrix} = 0 \quad \ldots\ldots \quad (14)$$

This may be expanded as:

$$(P_y - P) \left[(P_x - P) \frac{I_o}{A} (P_t - P) - (Px_o)^2 \right]$$
$$- (Py_o)^2 (P_x - P) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

This can be further simplified if we let :

$$I_c = I_x + I_y$$

and rearrange Equation (15) as :

$$\frac{I_c}{I_o} P^3 + \left[\frac{A}{I_o} (x_o{}^2 P_y + y_o{}^2 P_x) - (P_x + P_y + P_t) \right] P^2 + (P_x P_y + P_x P_t + P_y P_t) P - P_x P_y P_t = 0 \quad \ldots\ldots \quad (16)$$

The curve for this function is graphed in Figure 4. It is a particular case of the curve in Figure 3 (and with the function graphed negatively to the earlier one).

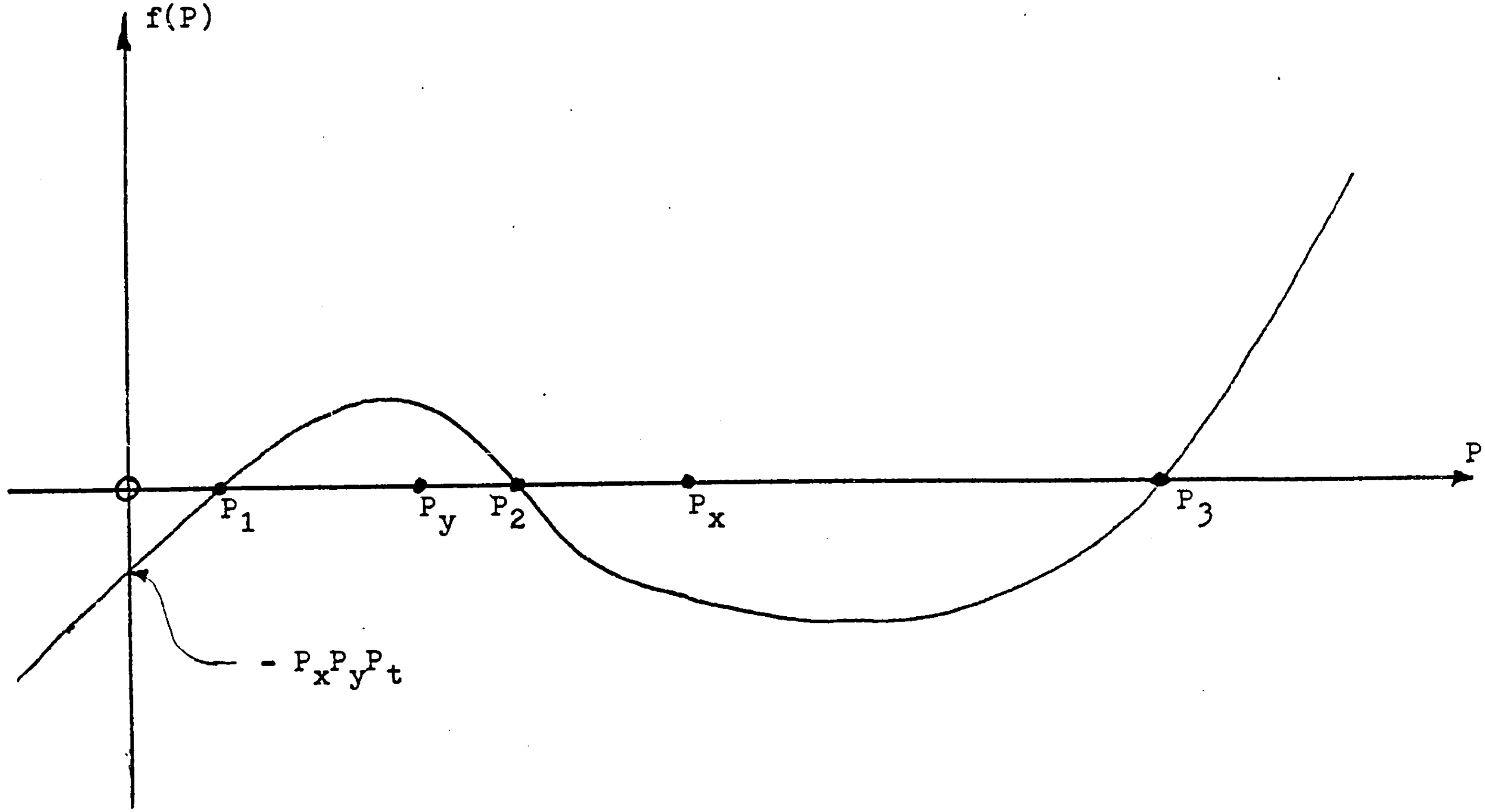


FIGURE 4: CUBIC FUNCTION FOR THRUST AT CENTROID

IV. END MOMENTS

The case of pure bending of the angles is also of interest. This is mathematically equivalent to making the axial force, P, very small (approaching zero) and the eccentricities very large. In this situation, the determinant of Equation (2) becomes:

$$\begin{vmatrix} P_y & 0 & M_x \\ 0 & P_x & -M_y \\ M_x & -M_y & \frac{I_o}{A}\left[P_t - \frac{A}{I_o}(M_x\beta_y + M_y\beta_x)\right] \end{vmatrix} = 0 \qquad \text{.....................(17)}$$

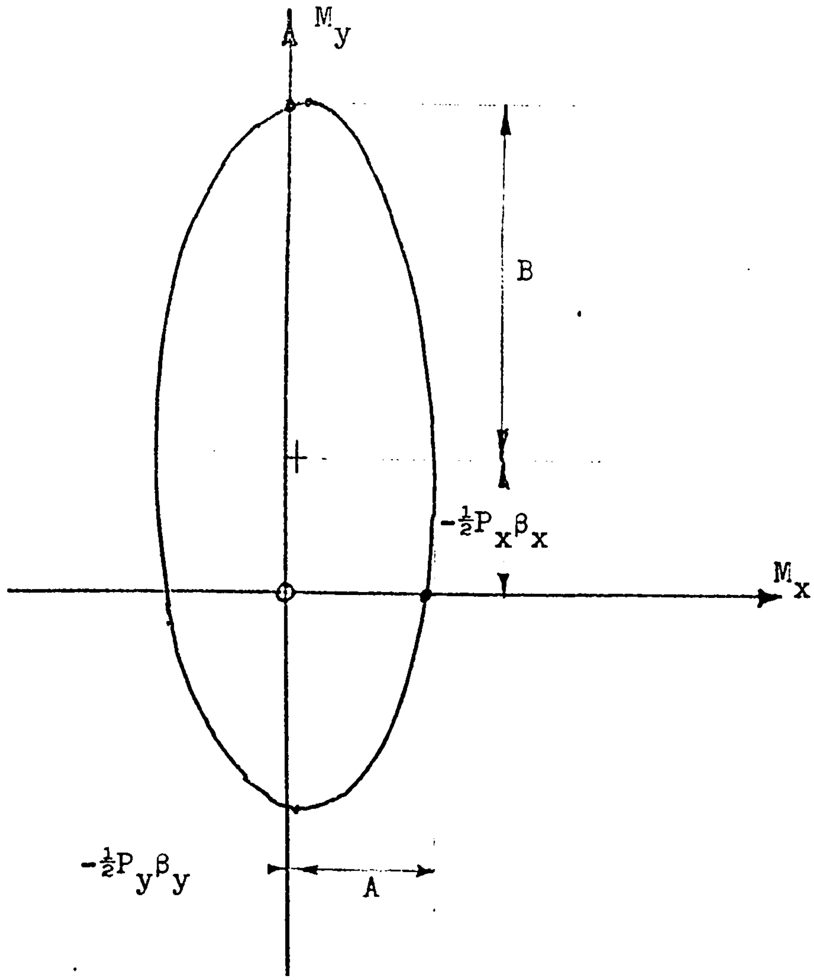
Expanding the determinant gives:

$$P_y\left\{P_x\frac{I_o}{A}\left[P_t - \frac{A}{I_o}(M_x\beta_y + M_y\beta_x)\right] - P_yM_y^2\right\} - P_xM_x^2 = 0 \qquad \text{.................. (18)}$$

And expressing this in terms of M_x and M_y, the critical buckling moments, gives:

$$P_yM_y^2 + (P_xP_y\beta_x)M_y + P_xM_x^2 + (P_xP_y\beta_y)M_x - P_xP_yP_t\frac{I_o}{A} = 0 \qquad \text{...................... (19)}$$

This equation describes an ellipse, as shown in Figure 5. The intercepts are determined in the usual manner. By completing the squares for the M_x and M_y terms and rearranging Equation (19)



$$A = \sqrt{P_x P_y(\tfrac{1}{2}\beta_x)^2 + (\tfrac{1}{2}P_y\beta_y)^2 + \frac{P_y P_t I_o}{A}}$$

$$B = \sqrt{(\tfrac{1}{2}P_x\beta_x)^2 + \frac{P_x P_t I_o}{A} + P_x P_y(\tfrac{1}{2}\beta_y)^2}$$

FIGURE 5 : FUNCTION FOR CRITICAL END MOMENTS

it assumes the standard form for an ellipse whose centre is not at the origin of the axes:

$$\frac{(M_x + \frac{1}{2}P_y\beta_y)^2}{P_y\ D} + \frac{(M_y + \frac{1}{2}P_x\beta_x)^2}{P_x\ D} = 1 \quad \ldots\ldots\ldots \quad (20)$$

where:

$$D = P_x\ (\tfrac{1}{2}\beta_x)^2 + P_y(\tfrac{1}{2}\beta_y)^2 + P_t\ \frac{I}{A}o \quad \ldots\ldots\ldots\ldots \quad (21)$$

Since P_x is greater than P_y for Euler buckling of angles, the critical ellipse will always have its major axis (M_y-axis) vertical. This corresponds to the intuitive expectations that the angle is stronger about its principal y-axis than about its principal x-axis.

V. PROGRAMMING FOR SOLUTION

Computer programs were developed to calculate the geometrical parameters (e.g. principal moments of inertia, total warping constants, torsional constants, β_x, β_y) and to solve the cubic equations for critical thrusts. Critical moments for pure bending were determined for a few points of the standard ellipse shown in Figure 5.

The length and difficulty of making these calculations necessitated a computer solution. Each particular case of loading eccentricity and member length would otherwise have to be performed individually. With the aid of the program, several combinations of interest could be compared easily.

In addition, one objective was to develop some general formulas which could be used instead of long and difficult calculations. This was also aided by use of computer facilities. In particular, regression analysis of the β_x and β_y parameters led to some simplified formulas. (Appendix C.)

The background for the β_x and β_y parameters is described in Appendix A. A computer program for their calculation is listed, in addition to the values calculated for each angle shape. The results of regression analysis of these properties is also presented. A comparison is made between the approximate and the exact calculations. This computer progam was used in the larger calculations for critical buckling.

The values input to the main buckling program can be divided into mathematical and material constants (π, E, G), angle properties ($\angle$ b x a x t, Area, I_{xx}, $\bar{y}$, I_{yy}, $\bar{x}$, r_{zz}, $\tan\alpha$, root radius r_1), and program variables (L, e_x, e_y).

From these inputs, the shear centre coordinates (x_o, y_o), principal axes properties (I_x, I_y, I_o), and geometrical properties (J, G, β_x, β_y) can be calculated. The calculation of some of these will be mentioned here and the remainder described in detail in Appendix A.

The principal axes properties are obtained from the input values (copied from Steel Handbooks 8, 9, 10) directly:

$$I_y = \text{Area}\ (r_{zz})^2$$

$$I_x = I_c - I_y$$
$$= (I_{xx} + I_{yy}) - I_y$$

From F. Bleich (6), the total warping constant is calculated as:

$$H = \frac{t^3}{36}\left[(a - \tfrac{1}{2}t)^3 + (b - \tfrac{1}{2}t)^3\right] \quad \ldots\ldots\ldots\ldots (22)$$

One simplification which reduced the requirement of inputting the root radius for each angle is based on the graphs of Figures 6 and 7. The resulting formula for root radius is:

$$r_1 = 0.12 + 0.03\ (b + a) \quad \ldots\ldots\ldots\ldots (23)$$

From the Aluminum Design Handbook (11), the formulas for the torsional constant are:

$$n = 0.85 + 0.24\ \frac{r_1}{t} \quad \ldots\ldots\ldots\ldots (24)$$

$$J = \left[(a - r_1 - t) + (b - r_1 - t)\right]\frac{t^3}{3} + (nt)^4 \ldots (25)$$

FIGURE 6 : ANGLE RADII VS. SUM OF LEG LENGTHS

UNEQUAL ANGLES

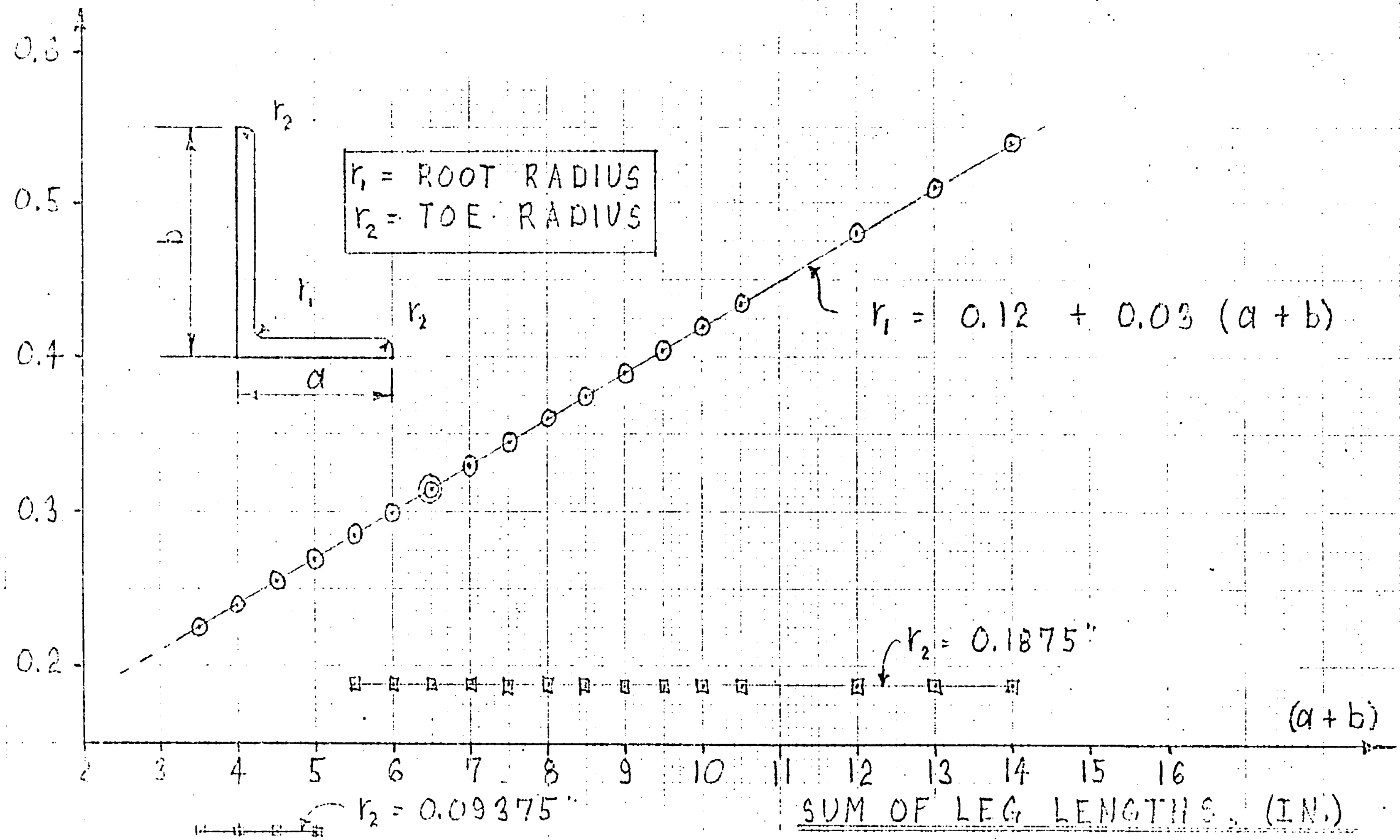
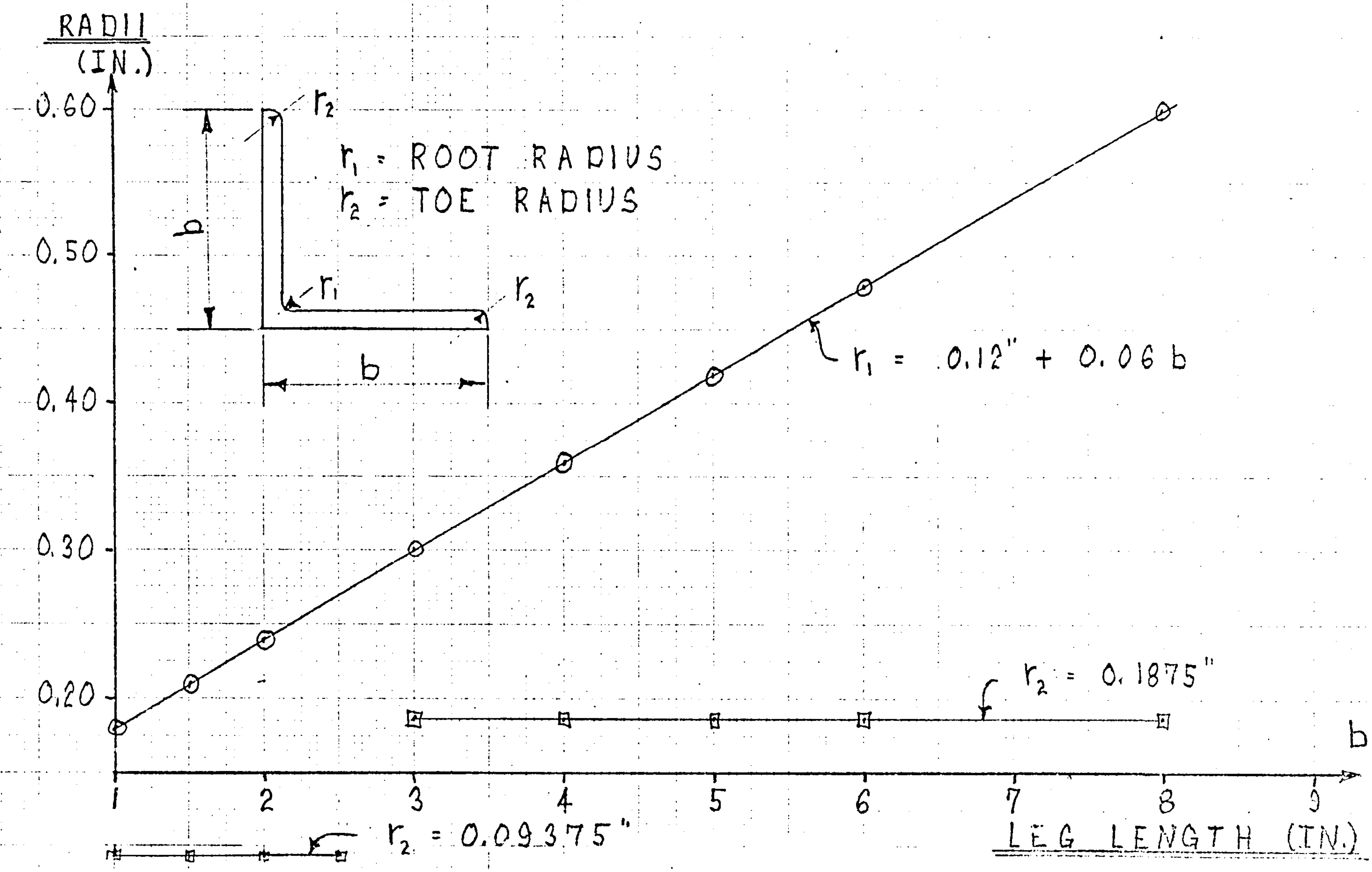


FIGURE 7: EQUAL ANGLE RADII VS. LEG LENGTH



After the various parameters and coefficients have been calculated for a particular angle under a specified loading, the critical buckling loads and moments must be calculated. The numerical technique used to solve the cubic equation (Eq. 8) for three critical roots is called the Newton-Raphson Formula. It uses a second order approximation to converge on the roots. (See Figure 8 for a graphical illustration of this method.)

For the cubic equation:

$$f(P) = k_1P^3 + k_2P^2 + k_3P + k_4 = 0 \quad \ldots\ldots\ldots \quad (8)$$

the first derivative (with respect to P) is:

$$f'(P) = 3k_1P^2 + 2k_2P + k_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (26)$$

and the second derivative (again with respect to P) is:

$$f''(P) = 6k_1P + 2k_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (27)$$

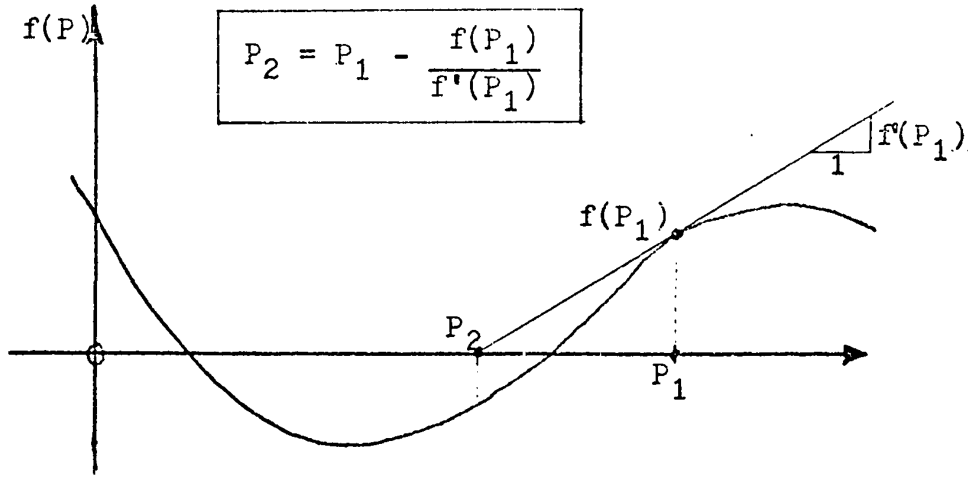
The Newton-Raphson Formula is based on the improvement in an approximation to the root of Equation (8). Thus, if

$$P = P_i$$

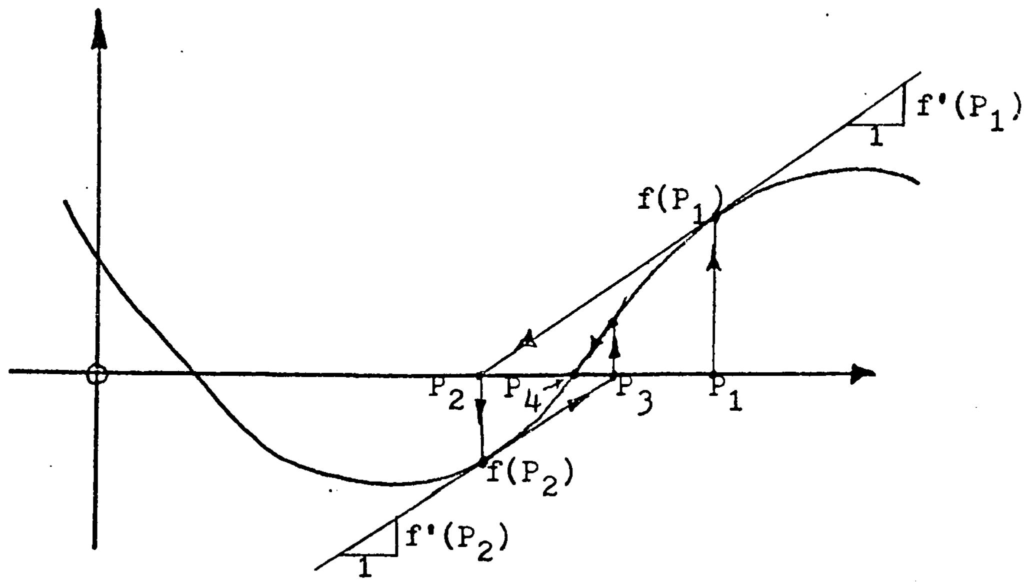
is an approximation to a root of a polynomial, then,

$$P = P_{i+1} = P_i - \frac{f'(P_i)\ f(P_i)}{(\ f'(P_i)\ )^2 - f(P_i)\ f''(P_i)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (28)$$

is a better approximation to the root.



(a) First Iteration



(b) Cycle of Iterations

FIGURE 8: NEWTON-RAPHSON TECHNIQUE

(First-order variation shown.)

To obtain the initial approximation to use as starting values for the iterations, note that maximum and minimum values for the function f(P) occur when Equation (26) assumes the value of zero. (The slope of the curve is zero.) Since this is a quadratic equation, the two roots are obtained directly as:

$$P_a = -\frac{k_2}{3k_1} + \frac{\sqrt{k_2^2 - 3k_1k_3}}{3k_1} \quad \ldots\ldots\ldots\ldots \quad (29)$$

$$P_b = -\frac{k_2}{3k_1} + \frac{\sqrt{k_2^2 - 3k_1k_3}}{3k_1} \quad \ldots\ldots\ldots\ldots \quad (30)$$

The iteration is then started below the lower value of P_a and P_b to determine the first root, P_1, of Equation (8). The program performs this calculation by selecting the minimum of the two values and then subtracting from it one-half of its absolute magnitude. This generalizes the starting value for positive and negative values of P_a or P_b.

A second starting value is calculated as the average of P_a and P_b. This is intended to search for the second root, P_2, which should be larger than P_1.

The third starting value is chosen above the maximum value of P_a and P_b. It is selected by a similar procedure to that used for the first root.

This approach was applied with satisfactory results in the computer program. A tabulation of symbols used in the program is included with a sample listing of the program and its typical output in Appendix B.

LIST OF REFERENCES

1. Timoshenko and Gere, "Theory of Elastic Stability", McGraw-Hill, 1961, 2nd Edition.
2. V. Z. Vlasov, "Thin-Walled Elastic Bars", Moscow, 1940 (Translated for U.S. Dept. of Commerce)
3. C. Marsh, "Single-Angle Members in Tension and Compression", ASCE, Journal of the Structural Division, Vol. 95, No. ST5, February, 1969.
4. S. P. Timoshenko, "Theory of Bending, Torsion and Buckling", Journal of the Franklin Institute, Vol. 239, Nos. 3, 4, and 5 , April and May, 1945.
5. A. Chajes and G. Winter, "Torsional-Flexural Buckling of Thin-Walled Members", ASCE, Journal of the Structural Division, Vol. 91, No. ST4, August, 1965.
6. F. Bleich, "Buckling Strength of Metal Structures", McGraw-Hill, 1952.
7. J. N. Goodier, "Flexural-Torsional Buckling of Bars of Open Section, Under Bending, Eccentric Thrust, or Torsional Loads", Cornell Univ. Eng. Expt. Sta. Bull. 27, Dec. 1941.
8. Canadian Institute of Steel Construction, "Steel Construction Handbook", 1970.

9. American Institute of Steel Construction,
"Steel Construction Handbook", 1968.

10. British Steel Producer's Conference,
"Steel Designer's Manual", 1969.

11. Aluminum Company of Canada Limited,
"Strength of Aluminum".

APPENDIX A

CALCULATION OF PARAMETERS β_x AND β_y

The general equations for critical buckling loads in this report include the parameters β_x and β_y.

$$\beta_y = \frac{1}{I_x}\left[\int_A y^3\, dA + \int_A x^2 y\, dA\right] - 2y_o \quad \ldots\ldots\ldots \quad (A\text{-}1)$$

$$\beta_x = \frac{1}{I_y}\left[\int_A x^3\, dA + \int_A x y^2\, dA\right] - 2x_o \quad \ldots\ldots\ldots \quad (A\text{-}2)$$

In these equations, x and y refer to the principal axes, x_o and y_o are the coordinates of the shear centre, and I_x and I_y are the moments of inertia about the x and y principal axes respectively.

The determination of these parameters is most difficult for an unequal angle, This is because of its lack of symmetry. (A cross section which is symmetric about both principal axes, such as a wide flange beam, has $\beta_x = \beta_y = 0$.)

An exact determination will first be considered. The properties available in the Canadian Institute of Steel Construction Design Handbook will be used. These are identical to the properties in the American Institute of Steel Construction's manual and somewhat less complete than those in the British "Steel Designer's Manual". The properties used are based on the sketch reproduced in Figure A-1.

The information read from the handbooks mentioned includes the properties: A, I_{xx}, $\bar{y}$, I_{yy}, $\bar{x}$, r_{zz}, $\tan\alpha$ in approximately that order. (The nomenclature used here is for our purposes and differs slightly from the handbook's. A glance at Figures A-1 and A-2 will help explain the distinction.)

For the principal axes,

$$I_y = I_{zz} = A\,(r_{zz})^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (A\text{-}3)$$

$$I_x = (I_{xx} + I_{yy}) - I_{zz} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (A\text{-}4)$$

The angle is sketched again in Figure A-2 in a position rotated from that in Figure A-1 in order to make the principal axes vertical and horizontal.

In order to perform the required integrations, the intersection points of the leg centre-lines with the principal axes must be determined. These points (0, 1, 2, 3, 4, 5 in Figure A-2) are located as follows:

$$\alpha = \arctan(\,\tan\alpha)$$

$$y_3 = \frac{\bar{y} - \frac{1}{2}t}{\cos\alpha}$$

$$x_5 = \frac{\bar{x} - \frac{1}{2}t}{\cos\alpha}$$

$$y_4 = -\frac{x_5}{\tan\alpha}$$

From these coordinates, the equations for the centre-lines of the angle legs can be written as:

Line 1 (Short Leg): $y_S = y_3 - (\tan\alpha)\,x$

Line 2 (Long Leg): $y_L = y_4 + \dfrac{x}{\tan\alpha}$

By setting these equations equal for the point of intersection the coordinates of the shear centre are determined:

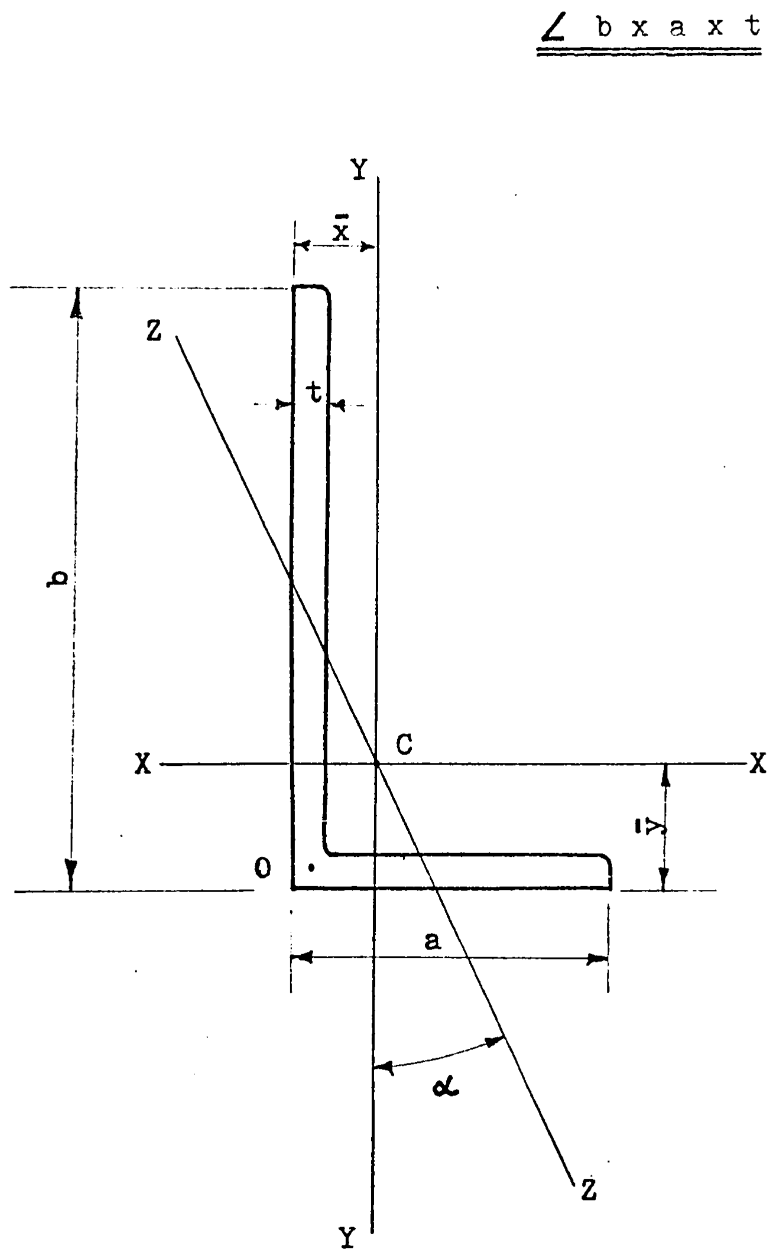


FIGURE A-1: UNEQUAL ANGLE

Handbook notation indicated.

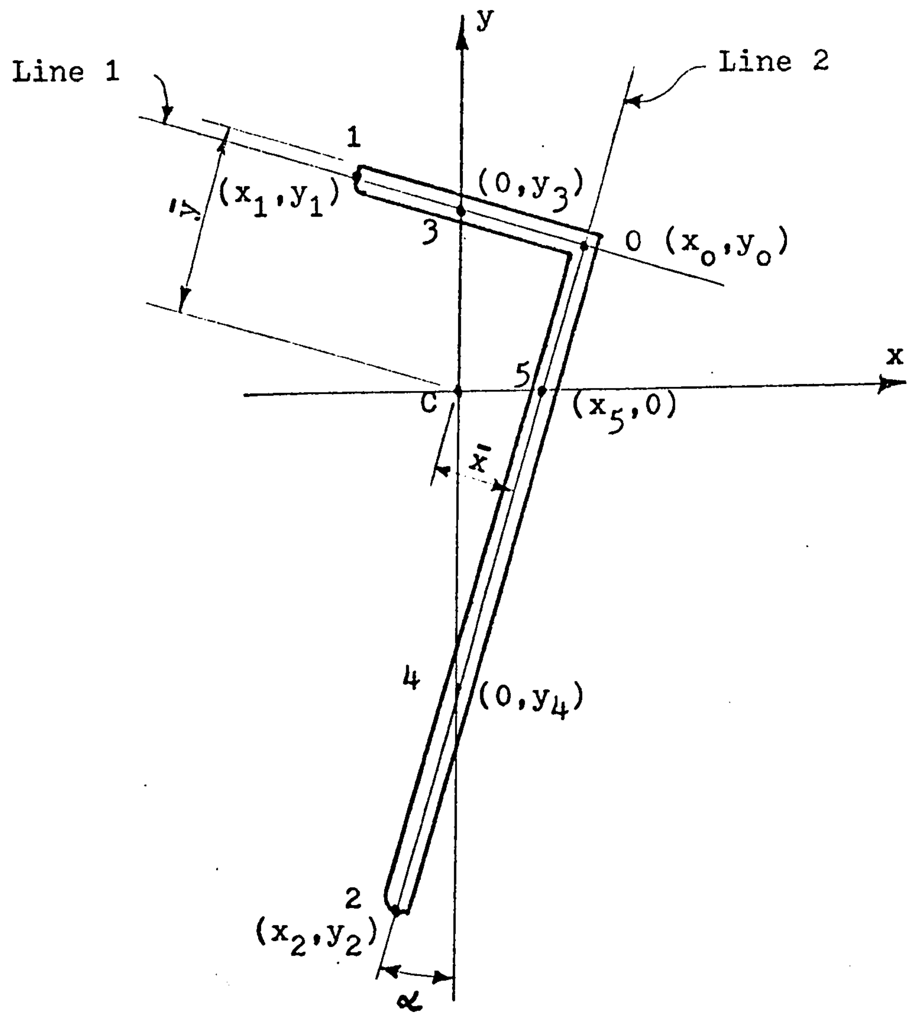


FIGURE A-2: UNEQUAL ANGLE

Intersection points noted.

$$x_o = (y_3 - y_4) \sin\alpha \cos\alpha$$

$$y_o = y_3 \cos^2\alpha + y_4 \sin^2\alpha$$

Using the equations for the leg centre-lines, the leg lengths a(short leg) and b(long leg), and the above coordinates gives the following points of intersection:

$$x_1 = x_o - (a - \tfrac{1}{2}t) \cos\alpha$$

$$y_1 = y_o + (a - \tfrac{1}{2}t) \sin\alpha$$

$$x_2 = x_o - (b - \tfrac{1}{2}t) \sin\alpha$$

$$y_2 = y_o - (b - \tfrac{1}{2}t) \cos\alpha$$

Having determined these intersection points and the equations of the leg centre-lines, the integrations required can be performed. Each integral is calculated separately along the long and short legs, since their equations are different. Development of the integrals is indicated in Figures A-3 and A-4. Figure A-3 illustrates vertical elements and their distance from the y-axis. Figure A-4 shows horizontal elements and their distance from the x-axis.

The integrals of Equations A-1 and A-2 are dealt with individually. Thus:

$$\int_A x^3 \, dA = \int_{A_1} x^3 \, dA_1 + \int_{A_2} x^3 \, dA_2$$

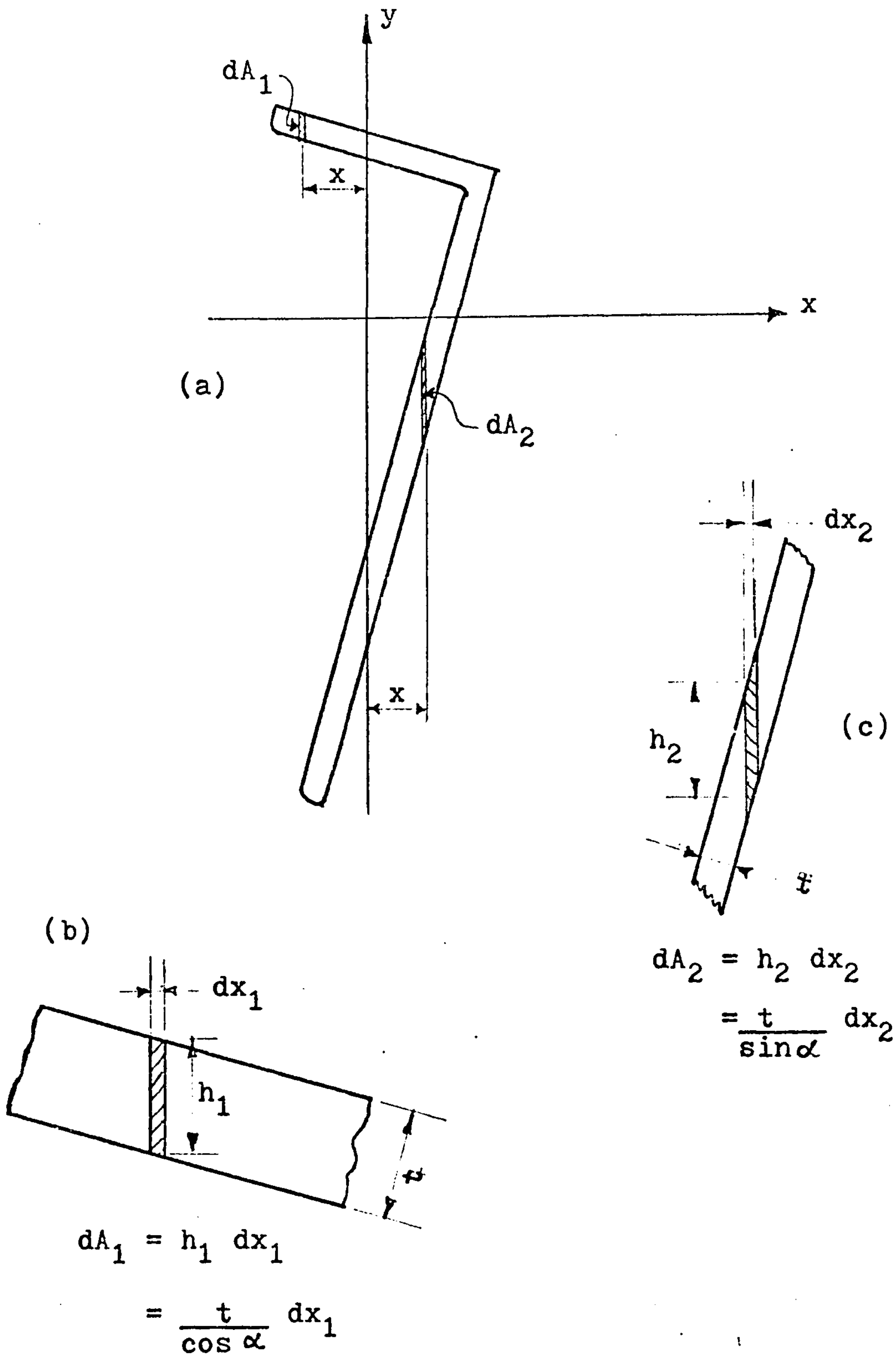


FIGURE A-3: VERTICAL ELEMENTS FOR INTEGRATION

$$\int_A x^3\, dA = \int_{x_1}^{x_o} x^3 \frac{t}{\cos\alpha}\, dx + \int_{x_2}^{x_o} x^3 \frac{t}{\sin\alpha}\, dx$$

$$= \frac{t}{4}\left[\frac{1}{\cos\alpha}(x_o^4 - x_1^4) + \frac{1}{\sin\alpha}(x_o^4 - x_2^4)\right]$$

.... (A-5)

The other integral which is more conveniently performed along the x-axis (i.e. with respect to x), is for the integral:

$$\int_A x^2 y\, dA = \int_{A_1} x^2 y_s\, dA_1 + \int_{A_2} x^2 y_L\, dA_2$$

where: $y_s = y_3 - (\tan\alpha)\, x$ = short leg equation

$y_L = y_4 + \dfrac{x}{\tan\alpha}$ = long leg equation

Therefore, substituting these equations into the integration gives:

$$\int_A x^2 y\, dA = \int_{x_1}^{x_o} x^2 (y_3 - x \tan\alpha) \frac{t}{\cos\alpha}\, dx + \int_{x_2}^{x_o} x^2 \left(y_4 + \frac{x}{\tan\alpha}\right) \frac{t}{\sin\alpha}\, dx$$

$$= \frac{t}{12} \frac{1}{\sin\alpha \cos\alpha} \left\{ \sin\alpha \left[4y_3(x_o^3 - x_1^3) - 3\tan\alpha (x_o^4 - x_1^4)\right] + \cos\alpha \left[4y_4(x_o^3 - x_2^3) + \frac{3}{\tan\alpha}(x_o^4 - x_2^4)\right] \right\}$$

.... (A-6)

For the remaining two integrals, the integration is conveniently performed along the y-axis with the elements indicated in Figure A-4.

$$\int_A y^3 \, dA = \int_{A_1} y^3 \, dA_1 + \int_{A_2} y^3 \, dA_2$$

$$= \int_{y_o}^{y_1} y^3 \frac{t}{\sin\alpha} \, dy + \int_{y_2}^{y_o} y^3 \frac{t}{\cos\alpha} \, dy$$

$$= \frac{t}{4} \frac{1}{\sin\alpha \cos\alpha} \left[\cos\alpha \, (y_1^4 - y_o^4) + \sin\alpha \, (y_o^4 - y_2^4) \right]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots \quad (A\text{-}7)$$

The last integral requires that the centre-line equations be used to substitute for x in terms of y. From the short leg equation,

$$x_s = \frac{y_3 - y}{\tan\alpha}$$

and from the long leg equation:

$$x_L = (y_L - y_4) \tan\alpha$$

So that the last integral becomes:

$$\int_A xy^2 \, dA = \int_{A_1} xy^2 \, dA_1 + \int_{A_2} xy^2 \, dA_2$$

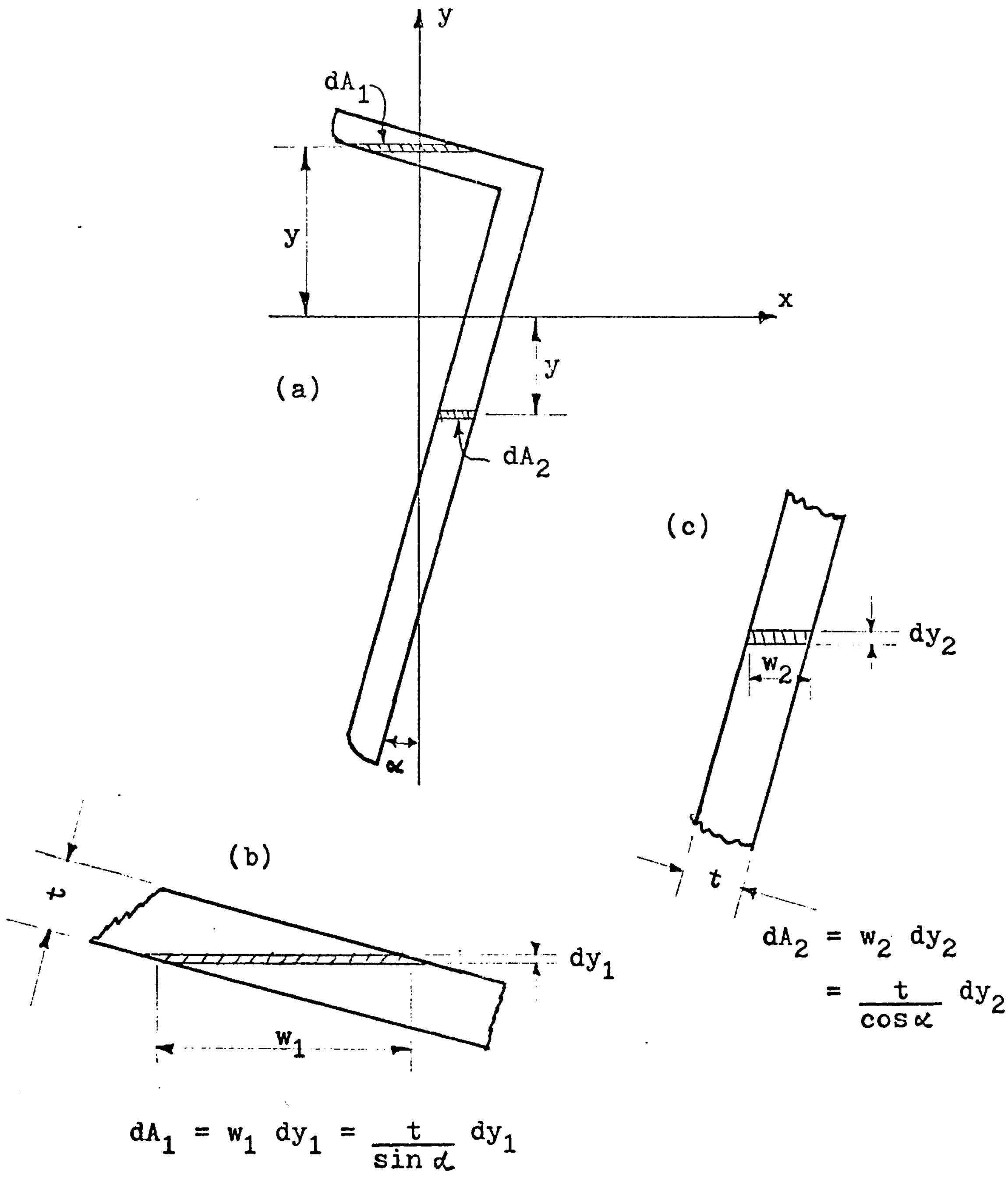


FIGURE A-4: HORIZONTAL ELEMENTS FOR INTEGRATION

$$\int_A xy^2\, dA = \int_{y_o}^{y_1} \frac{1}{\tan\alpha} (y_3 - y)\, y^2 \frac{t}{\sin\alpha}\, dy$$

$$+ \int_{y_2}^{y_o} (\tan\alpha)(y - y_4) y^2 \frac{t}{\cos\alpha}\, dy$$

$$= \frac{t}{12} \frac{1}{\sin^2\alpha \cos^2\alpha} \left\{ \cos^3\alpha \left[4y_3(y_1{}^3 - y_o{}^3) - 3(y_1{}^4 - y_o{}^4) \right] \right.$$

$$\left. + \sin^3\alpha \left[3(y_o{}^4 - y_2{}^4) - 4y_4(y_o{}^3 - y_2{}^3) \right] \right\} \qquad \text{.................. (A-8)}$$

These last four equations (A-5, -6, -7, -8) represent the exact evaluation of the integrals forming the parameters β_x and β_y. Their results were used directly in the computer program written to evaluate the parameters. Slide rule calculations and extra output checks were used in developing the program.

The next step in determining these parameters was to examine the simplifications possible for equal leg angles. Although the same calculations would be adequate for this special case, some saving in input time and effort can be effected by developing distinct equations for the equal angles.

By symmetry,

$\alpha = 45^o$

$\tan\alpha = 1$

$\cos\alpha = \sin\alpha = \sqrt{\tfrac{1}{2}}$

Refering to Figure A-5 for the position of points of intersection and the angle's orientation with respect to the principal axes, we continue with:

$$y_3 = - y_4$$

$$x_1 = x_2$$

$$y_1 = - y_2$$

$$\bar{x} = \bar{y}$$

$$x_o = \frac{\bar{x} - \frac{1}{2}t}{\cos\alpha} = \sqrt{2}\ (\bar{x} - \tfrac{1}{2}t)$$

The equations for the two legs will still use the designations short and long, refering to the upper and lower respectively.

Line 1 (Short): $y_s = y_3 - x\ (\tan\alpha)$

$$= \sqrt{2}\ (\bar{y} - \tfrac{1}{2}t) - x$$

Line 2 (Long): $y_L = y_4 + \dfrac{x}{\tan\alpha}$

$$= -\sqrt{2}\ (\bar{y} - \tfrac{1}{2}t) + x$$

From the intersection of the two lines, it is proven that:

$$y_o = 0$$

$$x_1 = x_o - (b - \tfrac{1}{2}t)\cos\alpha$$

$$= \sqrt{2}\ (\bar{x} - \tfrac{1}{2}t) - \frac{1}{\sqrt{2}}\ (b - \tfrac{1}{2}t)$$

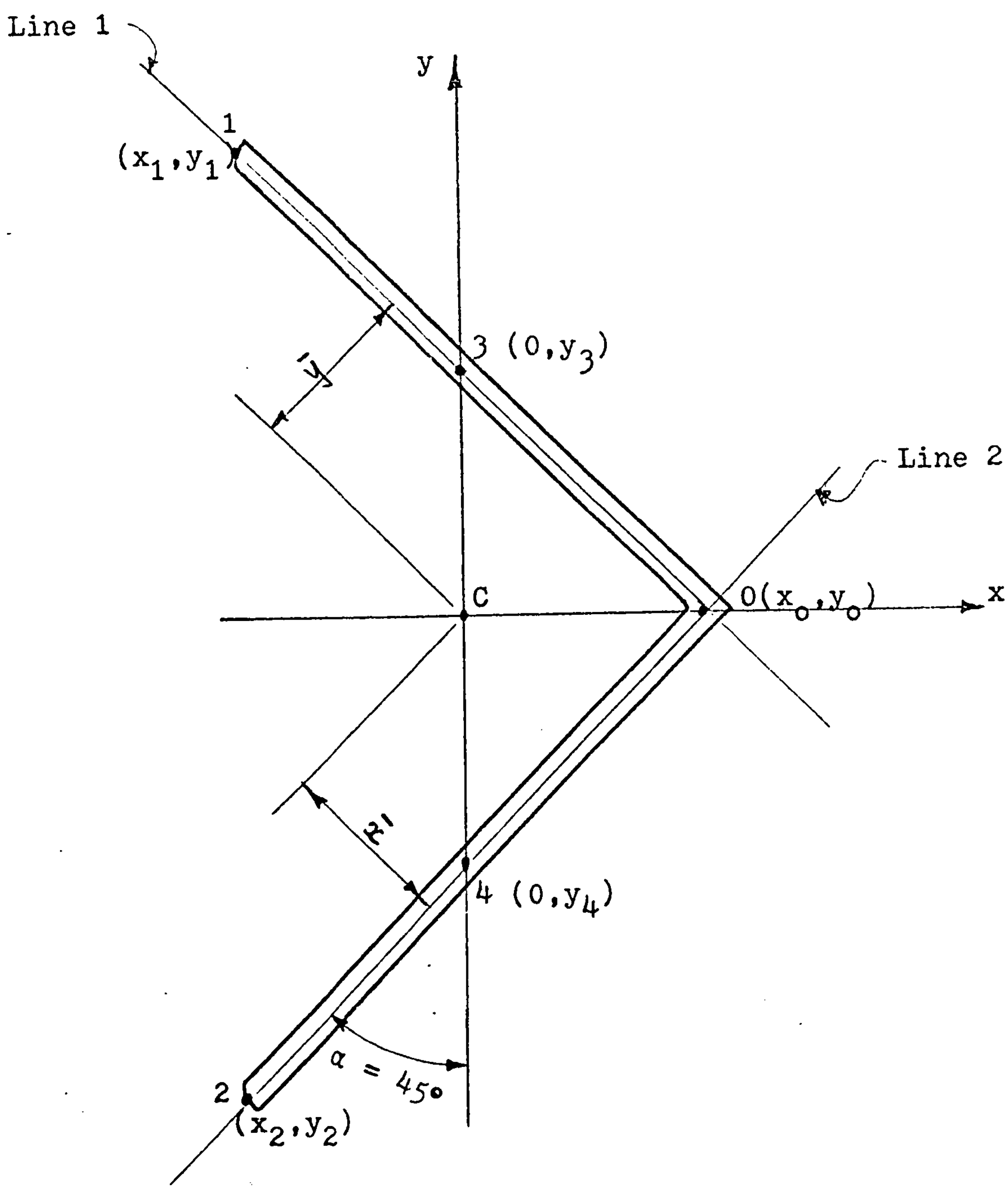


FIGURE A-5: EQUAL ANGLE

Intersection points noted.

$$y_1 = y_o + (b - \tfrac{1}{2}t) \sin\alpha$$

$$= \frac{1}{\sqrt{2}} (b - \tfrac{1}{2}t)$$

The number of integrations for equal angle parameters is considerably reduced by the symmetry of the angle.

$$\int_A x^3 \, dA = \int_A y^3 \, dA = \int_A x^2 y \, dA = 0$$

The only integral with nonzero value is handled as before:

$$\int_A xy^2 \, dA = \int_{A_1} xy^2 dA_1 + \int_{A_2} xy^2 \, dA_2$$

Proceeding as for the unequal angles,

$$x_s = y_3 - y_s \quad ; \qquad x_L = y_L - y_4$$

$$dA_1 = \sqrt{2} \; t \; dy; \qquad dA_2 = \sqrt{2} \; t \; dy$$

$$\int_A xy^2 \, dA = \int_0^{y_1} (y_3 - y) \, y^2 \, (\sqrt{2} \; t \; dy)$$

$$+ \int_{y_2}^{0} (y - y_4) \, y^2 \, (\sqrt{2} \; t \; dy)$$

$$= \frac{\sqrt{2} \; t}{12} \left(4y_3 \, y_1^{\,2} - 3y_1^{\,4} - 3y_2^{\,4} + 4y_4 y_2^{\,3} \right)$$

With $y_2 = - y_1$, and $y_4 = -y_3$, the above reduces to:

$$\int_A xy^2 \, dA = \frac{\sqrt{2} t}{6} \left(4 \, y_3 \, y_1^{\,3} - 3 \, y_1^{\,4} \right) \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(A-9)}$$

From this last integral and the values of x_o and I_y, the value of β_x for the equal leg angle can be calculated. This was the approach used in the computer program.

It is also evident from these calculations that the two integrals and y_o which make up β_y are all zero. Therefore this parameter has a zero value for equal angles.

The following pages present the symbols used in this appendix, their program equivalents, a listing of the computer program for unequal angles, and the results for all shapes listed in the CISC Manual and the British "Steel Designer's Manual". Only one program is listed, although the equal angles were run with a slightly different program as just described.

Appendix B contains the computer program for solving the cubic buckling equation. This program incorporates the calculations of the β_x and β_y parameters. It was developed after these properties were determined.

Appendix C presents later work directed towards determining a simple formula for calculating these parameters without recourse to the long calculations of this appendix. The results were not used in the programming for critical buckling loads, but were intended to simplify any further work with these coefficients.

	Report Symbol	Computer Program Equivalent
Input of Angle Properties:	b	B
	a	A
	t	T
	A	AREA
	I_{xx}	IXX
	$\bar{y}$	YBAR
	I_{yy}	IYY
	$\bar{x}$	XBAR
	r_{zz}	RZZ
	$\tan\alpha$	TA
Calculated Properties:	I_x	IX
	I_y	IY
	$\cos\alpha$	CA
	$\sin\alpha$	SA
	y_3	Y3
	x_5	X5
	y_4	Y4
	x_o	XO
	y_o	YO
	x_1	X1
	y_1	Y1
	x_2	X2
	y_2	Y2
	$\int_A x^3 \, dA$	SX3
	$\int_A x^2 y \, dA$	SX2Y
	$\int_A y^3 \, dA$	SY3
	$\int_A x y^2 \, dA$	SXY2
	β_x	BETAX
	β_y	BETAY

```
FORTRAN (3.2)/MASTER

      PROGRAM DALTON
C  ROBERT ERROL DALTON ..737-0632..... 482-1695....
C  PROGRAM FOR ..LATERAL STABILITY OF SINGLE-ANGLE BEAMS
C  PROFESSOR CEDRIC MARSH .... SUPERVISOR..... 879-7365
      REAL IXX, IYY, IX, IY
      WRITE(61,450)
  450 FORMAT( 1H1, 5X , 32HCALCULATION OF BETA-X AND BETA-Y     ///  )
      WRITE(61,453)
  453 FORMAT(1H- , 10X, 41HPROPERTIES FROM...STEEL DESIGNERS MANUAL  )
      WRITE(61,463)
  463 FORMAT(1H ,10X,44HBY THE...BRITISH STEEL PRODUCERS CONFERENCE    )
      WRITE(61,452)
  452 FORMAT(1H0, 5X, 1HB, 11X, 1HA, 9X, 1HT, 8X, 6HBETA-X , 9X,
     1  6HBETA-Y , 12X, 2HXO, 13X, 2HYO, 12X, 4H IX  , 10X, 4H IY   / )
      READ(60,1000) NANGL
 1000 FORMAT(I5 )
      NNNN = 0
   50 CONTINUE
      NNNN = 1 + NNNN
      READ(60,1) B, A, T, AREA, IXX, YBAR, IYY, XBAR, RZZ, TA
    1 FORMAT( 11F5.0  )
C*************************************************************************
C  CALCULATION OF COORDINATES AND PRINCIPAL MOMENTS OF INERTIA
C*************************************************************************
      IY = (RZZ**2) * AREA
      IX = (IXX + IYY) - IY
      TTA = ATAN(TA)
      CA = COS(TTA)
      SA = SIN(TTA)
      Y3 = (YBAR - T/2.) / CA
      X5 = (XBAR - T/2.) / CA
      Y4 = - X5 / TA
      XO = (Y3 - Y4) * SA * CA
      YO = Y3 * (CA**2)  +  Y4 * (SA**2)
      X1 = XO - (A - T/2.) * CA
      Y1 = YO + (A - T/2.) * SA
      X2 = XO - (B - T/2.) * SA
      Y2 = YO - (B - T/2.) * CA
C*************************************************************************
C  CALCULATION OF BETA-X AND BETA-Y BY EXACT INTEGRATION
C*************************************************************************
      DD1 = XO**4 - X1**4
      DD2 = XO**4 - X2**4
      SX3 = (T/4.) * (DD1/CA + DD2/SA)
      DD3 = XO**3 - X1**3
      DD4 = XO**3 - X2**3
      DD5 = 4.* Y3 * DD3 - 3. * TA * DD1
      DD6 = 4. * Y4 * DD4  +  (3./TA) * DD2
      SX2Y = (T/ (12. * SA * CA) ) * ( SA*DD5 + CA * DD6)
      DD7 = Y1**4  -  YO**4
      DD8 = YO**4 - Y2**4
      SY3 = (T/ (4. * SA * CA) ) * (CA * DD7 + SA * DD8)
```

```
      DD9 = Y1**3 - Y0**3
      DD10 = Y0**3 - Y2**3

      DD11 = 4. * Y3 * DD9 - 3. * DD7
      DD12 = 3. * DD8 - 4. * Y4 * DD10
      SXY2 = (T/ (12.* SA**2 * CA**2) ) * (CA**3 * DD11 + SA**3 * DD12)
      BETAX = (SX3 + SXY2) / IY - 2. * XO
      BETAY = (SY3 + SX2Y) / IX - 2. * YO
         WRITE(61,112) B, A, T, BETAX, BETAY, XO, YO, IX, IY
  112 FORMAT(1H , F8.2, 4X, F8.2, 2X, F8.4, 4X, 6( F8.3, 7X )   /    )
      IF( NNNN . LT . NANGL )  GO TO 50
      STOP
      END
```

CALCULATION OF RETA-X AND RETA-Y

PROPERTIES FROM...CISC HANDBOOK OF STEEL CONSTRUCTION

B	A	T	RETA-X	BETA-Y	XO	YO	IX	IY
9.00	4.00	1.0000	-7.449	-6.500	1.087	2.841.	100.733	8.267
9.00	4.00	.8750	-7.614	-6.524	1.115	2.845	90.114	7.486
9.00	4.00	.7500	-7.742	-6.538	1.153	2.858	79.216	6.484
9.00	4.00	.6250	-7.901	-6.562	1.179	2.863	67.615	5.585
9.00	4.00	.5625	-8.115	-6.577	1.186	2.862	61.643	5.057
9.00	4.00	.5000	-8.186	-6.582	1.204	2.868	55.584	4.516
8.00	6.00	1.0000	-8.631	-3.310	2.037	1.341	98.301	21.299
8.00	6.00	.8750	-8.770	-3.307	2.071	1.343	88.391	18.809
8.00	6.00	.7500	-8.908	-3.310	2.092	1.342	77.559	16.541
8.00	6.00	.6250	-9.044	-3.310	2.126	1.346	66.488	13.912
8.00	6.00	.5625	-9.038	-3.309	2.143	1.347	60.524	12.776
8.00	6.00	.5000	-9.186	-3.311	2.147	1.344	54.592	11.407
8.00	6.00	.4375	-9.184	-3.310	2.164	1.345	48.324	10.176
8.00	4.00	1.0000	-6.575	-5.448	1.145	2.344	73.252	7.948
8.00	4.00	.8750	-6.848	-5.464	1.174	2.346	65.970	7.030
8.00	4.00	.7500	-7.113	-5.481	1.200	2.350	58.202	6.098
8.00	4.00	.6250	-7.138	-5.496	1.236	2.361	49.741	5.259
8.00	4.00	.5625	-7.346	-5.506	1.244	2.359	45.444	4.756
8.00	4.00	.5000	-7.412	-5.512	1.263	2.364	40.947	4.253
8.00	4.00	.4375	-7.494	-5.527	1.269	2.363	36.270	3.830
7.00	4.00	.8750	-6.276	-4.354	1.224	1.828	46.547	6.553
7.00	4.00	.7500	-6.424	-4.359	1.262	1.835	41.212	5.688
7.00	4.00	.6250	-6.673	-4.374	1.286	1.838	35.407	4.793

B	A	T	β_x	β_y	x_o	y_o	IX	IY
7.00	4.00	.5625	-6.653	-4.376	1.305	1.041	32.357	4.443
7.00	4.00	.5000	-6.728	-4.378	1.325	1.845	29.226	3.974
7.00	4.00	.4375	-6.807	-4.391	1.329	1.843	25.922	3.578
7.00	4.00	.3750	-6.879	-4.394	1.347	1.848	22.618	3.082
6.00	4.00	.8750	-5.766	-3.130	1.282	1.286	31.598	5.902
6.00	4.00	.7500	-5.909	-3.129	1.319	1.290	28.067	5.133
6.00	4.00	.6250	-6.140	-3.133	1.343	1.289	24.266	4.334
6.00	4.00	.5625	-6.251	-3.131	1.361	1.291	22.317	3.883
6.00	4.00	.5000	-6.199	-3.137	1.378	1.295	20.105	3.595
6.00	4.00	.4375	-6.361	-3.142	1.383	1.292	17.936	3.164
6.00	4.00	.3750	-6.348	-3.143	1.401	1.294	15.604	2.796
6.00	4.00	.3125	-6.418	-3.142	1.419	1.296	13.254	2.346
6.00	3.50	.5000	-5.789	-3.681	1.144	1.542	18.301	2.599
6.00	3.50	.3750	-5.845	-3.689	1.181	1.549	14.172	2.028
6.00	3.50	.3125	-6.012	-3.697	1.185	1.548	12.098	1.702
6.00	3.50	.2500	-5.995	-3.703	1.203	1.552	9.795	1.405
5.00	3.50	.7500	-4.917	-2.400	1.146	.984	16.232	3.268
5.00	3.50	.6250	-5.135	-2.406	1.169	.983	14.032	2.767
5.00	3.50	.5000	-5.275	-2.404	1.204	.987	11.750	2.250
5.00	3.50	.4375	-5.347	-2.412	1.208	.984	10.461	2.039
5.00	3.50	.3750	-5.419	-2.406	1.227	.985	9.238	1.762
5.00	3.50	.3125	-5.490	-2.409	1.244	.988	7.821	1.479
5.00	3.50	.2500	-5.651	-2.413	1.248	.985	6.410	1.190
5.00	3.00	.5000	-4.755	-2.976	.975	1.245	10.516	1.584
5.00	3.00	.4375	-4.827	-2.983	.994	1.248	9.302	1.398

B	A	T	BETA-X	BETA-y	XO	YO	IX	IY
5.00	3.00	.3750	-4.995	-2.990	.999	1.246	8.192	1.208
5.00	3.00	.3125	-4.983	-2.993	1.018	1.249	6.955	1.045
5.00	3.00	.2500	-5.047	-2.999	1.036	1.253	5.655	.845
4.00	3.50	.6250	-4.687	-0.877	1.167	.349	8.671	2.229
4.00	3.50	.5000	-4.818	-0.874	1.200	.350	7.286	1.814
4.00	3.50	.4375	-4.884	-0.870	1.216	.350	6.598	1.602
4.00	3.50	.3750	-4.882	-0.870	1.233	.351	5.777	1.423
4.00	3.50	.3125	-5.010	-0.869	1.235	.349	5.001	1.199
4.00	3.50	.2500	-5.084	-0.871	1.251	.350	4.035	.965
4.00	3.00	.6250	-4.144	-1.657	.990	.670	7.270	1.630
4.00	3.00	.5000	-4.279	-1.656	1.025	.672	6.069	1.331
4.00	3.00	.4375	-4.423	-1.656	1.029	.670	5.524	1.176
4.00	3.00	.3750	-4.496	-1.653	1.046	.671	4.884	1.016
4.00	3.00	.3125	-4.485	-1.658	1.063	.673	4.117	.883
4.00	3.00	.2500	-4.551	-1.647	1.080	.674	3.486	.714
3.50	3.00	.5000	-4.049	-0.869	1.024	.350	4.647	1.153
3.50	3.00	.4375	-4.185	-0.869	1.027	.348	4.181	1.019
3.50	3.00	.3750	-4.246	-0.871	1.043	.348	3.616	.884
3.50	3.00	.3125	-4.249	-0.868	1.060	.349	3.134	.766
3.50	3.00	.2500	-4.314	-0.866	1.076	.349	2.581	.619
3.50	2.50	.5000	-3.621	-1.620	.820	.658	3.828	.772
3.50	2.50	.4375	-3.621	-1.625	.838	.660	3.391	.709
3.50	2.50	.3750	-3.688	-1.615	.855	.661	3.085	.615

B	A	T	BETA-X	BETA-Y	XO	YO	IX	IY
3.50	2.50	.3125	-3.808	-1.612	.864	.667	2.621	.519
3.50	2.50	.2500	-3.906	-1.621	.877	.660	2.160	.420
3.00	2.50	.5000	-3.342	-0.863	.832	.346	2.724	.676
3.00	2.50	.4375	-3.412	-0.858	.849	.347	2.502	.598
3.00	2.50	.3750	-3.475	-0.860	.866	.347	2.181	.519
3.00	2.50	.3125	-3.542	-0.865	.868	.345	1.845	.455
3.00	2.50	.2500	-3.610	-0.858	.885	.346	1.572	.368
3.00	2.50	.1875	-3.666	-0.856	.902	.346	1.209	.281
3.00	2.00	.5000	-2.809	-1.568	.622	.641	2.154	.416
3.00	2.00	.4375	-2.879	-1.570	.641	.643	1.940	.370
3.00	2.00	.3750	-2.959	-1.570	.660	.645	1.720	.320
3.00	2.00	.3125	-3.022	-1.567	.678	.647	1.498	.272
3.00	2.00	.2500	-3.180	-1.568	.682	.645	1.270	.220
3.00	2.00	.1875	-3.179	-1.571	.701	.647	.976	.174
2.50	2.00	.3750	-2.038	-1.094	.523	.412	1.219	.521
2.50	2.00	.3125	-2.826	-0.848	.688	.343	1.009	.231
2.50	2.00	.2500	-2.898	-0.848	.705	.343	.833	.187
2.50	2.00	.1875	-2.961	-0.851	.708	.341	.650	.150
2.50	2.00	.1250	-3.020	-0.854	.724	.344	.448	.102
2.50	1.50	.3125	-2.303	-1.484	.475	.622	.782	.118
2.50	1.50	.2500	-2.369	-1.486	.494	.625	.654	.096
2.50	1.50	.1875	-2.442	-1.495	.499	.623	.512	.078
2.00	1.50	.3125	-2.033	-0.824	.502	.337	.458	.102
2.00	1.50	.2500	-2.180	-0.827	.506	.334	.387	.083
2.00	1.50	.1875	-2.248	-0.826	.523	.335	.307	.063
2.00	1.50	.1250	-2.246	-0.830	.540	.337	.209	.046
1.75	1.25	.2500	-1.771	-0.813	.410	.329	.235	.050
1.75	1.25	.1875	-1.839	-0.810	.428	.331	.189	.039
1.75	1.25	.1250	-1.912	-0.810	.445	.332	.133	.026

CALCULATION OF BETA-X AND BETA-Y

PROPERTIES FROM...CISC HANDBOOK OF STEEL CONSTRUCTION

B	A	T	BETA-X	BETA-Y	XO	YO	IX	IY
8.00	8.00	1.1250	-10.319	0	2.613	0	155.286	40.714
8.00	8.00	1.0000	-10.444	0	2.645	0	141.496	36.504
8.00	8.00	.8750	-10.574	0	2.662	0	126.589	32.611
8.00	8.00	.7500	-10.698	0	2.694	0	111.202	28.198
8.00	8.00	.6250	-10.828	0	2.712	0	94.810	23.990
8.00	8.00	.5625	-10.894	0	2.728	0	86.531	21.669
8.00	8.00	.5000	-10.887	0	2.744	0	77.607	19.593
6.00	6.00	1.0000	-7.564	0	1.923	0	55.942	15.058
6.00	6.00	.8750	-7.692	0	1.955	0	50.481	13.319
6.00	6.00	.7500	-7.815	0	1.987	0	44.846	11.554
6.00	6.00	.6250	-7.944	0	2.005	0	38.500	9.900
6.00	6.00	.5625	-8.010	0	2.021	0	35.247	8.953
6.00	6.00	.5000	-8.141	0	2.022	0	31.794	8.006
6.00	6.00	.4375	-8.135	0	2.038	0	28.235	7.165
6.00	6.00	.3750	-8.200	0	2.054	0	24.626	6.174
6.00	6.00	.3125	-8.326	0	2.056	0	20.817	5.183
5.00	5.00	.8750	-6.253	0	1.602	0	28.092	7.508
5.00	5.00	.7500	-6.441	0	1.619	0	24.870	6.530
5.00	5.00	.6250	-6.503	0	1.651	0	21.572	5.628
5.00	5.00	.5000	-6.698	0	1.669	0	18.038	4.562
5.00	5.00	.4375	-6.765	0	1.685	0	15.986	4.014
5.00	5.00	.3750	-6.757	0	1.701	0	13.862	3.538
5.00	5.00	.3125	-6.819	0	1.717	0	11.830	2.970

B	A	T	BETA-X	BETA-Y	XO	YO	IX	IY
4.00	4.00	.7500	-4.937	0	1.266	0	12.090	3.310
4.00	4.00	.6250	-5.063	0	1.298	0	10.595	2.805
4.00	4.00	.5625	-5.128	0	1.313	0	9.657	2.543
4.00	4.00	.5000	-5.256	0	1.315	0	8.918	2.281
4.00	4.00	.4375	-5.319	0	1.331	0	7.986	2.014
4.00	4.00	.3750	-5.316	0	1.347	0	7.015	1.785
4.00	4.00	.3125	-5.382	0	1.363	0	5.902	1.498
4.00	4.00	.2500	-5.441	0	1.365	0	4.758	1.242
3.50	3.50	.5000	-4.502	0	1.146	0	5.697	1.503
3.50	3.50	.4375	-4.566	0	1.161	0	5.273	1.327
3.50	3.50	.3750	-4.633	0	1.163	0	4.619	1.181
3.50	3.50	.3125	-4.693	0	1.179	0	3.805	.995
3.50	3.50	.2500	-4.753	0	1.195	0	3.195	.805
3.00	3.00	.5000	-3.814	0	.962	0	3.475	.925
3.00	3.00	.4375	-3.881	0	.978	0	3.183	.817
3.00	3.00	.3750	-3.941	0	.993	0	2.890	.710
3.00	3.00	.3125	-3.935	0	1.009	0	2.380	.620
3.00	3.00	.2500	-4.067	0	1.011	0	1.899	.501
3.00	3.00	.1875	-4.135	0	1.027	0	1.541	.379
2.50	2.50	.5000	-3.003	0	.792	0	1.860	.540
2.50	2.50	.3750	-3.192	0	.810	0	1.545	.415
2.50	2.50	.3125	-3.246	0	.826	0	1.347	.353
2.50	2.50	.2500	-3.311	0	.841	0	1.114	.286
2.50	2.50	.1875	-3.454	0	.843	0	.884	.216
2.50	2.50	.1250	-3.442	0	.859	0	.607	.153

B	A	T	BETA-X	BETA-Y	XO	YQ	IX	IY
2.00	2.00	.3750	-2.438	0	.640	0	.753	.207
2.00	2.00	.3125	-2.566	0	.642	0	.665	.175
2.00	2.00	.2500	-2.634	0	.658	0	.557	.143
2.00	2.00	.1875	-2.701	0	.674	0	.432	.108
2.00	2.00	.1250	-2.700	0	.689	0	.303	.077
1.75	1.75	.3125	-2.184	0	.557	0	.424	.116
1.75	1.75	.2500	-2.254	0	.573	0	.366	.094
1.75	1.75	.1875	-2.316	0	.589	0	.288	.072
1.75	1.75	.1250	-2.384	0	.590	0	.209	.051
1.50	1.50	.3750	-1.817	0	.456	0	.303	.077
1.50	1.50	.3125	-1.813	0	.472	0	.249	.071
1.50	1.50	.1875	-2.000	0	.490	0	.175	.045
1.50	1.50	.1250	-2.000	0	.506	0	.124	.032
1.25	1.25	.3125	-1.502	0	.373	0	.141	.039
1.25	1.25	.2500	-1.566	0	.389	0	.122	.032
1.25	1.25	.1875	-1.634	0	.405	0	.097	.025
1.25	1.25	.1250	-1.617	0	.421	0	.069	.019
1.13	1.13	.1875	-1.401	0	.362	0	.069	.019
1.13	1.13	.1250	-1.457	0	.378	0	.051	.013
1.00	1.00	.2500	-1.128	0	.304	0	.056	.018
1.00	1.00	.1875	-1.250	0	.320	0	.048	.012
1.00	1.00	.1250	-1.261	0	.336	0	.035	.009

CALCULATION OF BETA-X AND BETA-Y

PROPERTIES FROM...STEEL DESIGNERS MANUAL
BY THE...BRITISH STEEL PRODUCERS CONFERENCE

B	A	T	BETA-X	BETA-Y	XO	YO	IX	IY
9.00	4.00	.8720	-7.734	-6.532	1.115	2.836	90.064	7.316
9.00	4.00	.8100	-7.831	-6.550	1.131	2.833	84.723	6.827
9.00	4.00	.7450	-7.877	-6.556	1.142	2.842	78.810	6.470
9.00	4.00	.6840	-7.978	-6.573	1.158	2.839	73.291	5.969
9.00	4.00	.6220	-8.171	-6.578	1.166	2.847	67.509	5.461
9.00	4.00	.5590	-8.121	-6.597	1.183	2.844	61.353	5.057
9.00	4.00	.4950	-8.339	-6.611	1.191	2.843	55.079	4.501
8.00	6.00	.8700	-8.817	-3.313	2.061	1.340	87.668	18.792
8.00	6.00	.8080	-8.916	-3.318	2.074	1.331	82.533	17.547
8.00	6.00	.7460	-8.906	-3.326	2.089	1.335	77.005	16.525
8.00	6.00	.6830	-9.039	-3.337	2.091	1.336	71.490	15.210
8.00	6.00	.6210	-9.095	-3.343	2.106	1.340	65.898	13.912
8.00	6.00	.5590	-9.132	-3.345	2.112	1.350	59.964	12.776
8.00	6.00	.4950	-9.251	-3.329	2.132	1.333	53.866	11.374
8.00	4.00	.7470	-7.134	-5.488	1.204	2.339	57.852	6.098
8.00	4.00	.6830	-7.300	-5.507	1.206	2.340	53.683	5.607
8.00	4.00	.6230	-7.351	-5.519	1.218	2.347	49.646	5.144
8.00	4.00	.5600	-7.303	-5.543	1.230	2.346	45.167	4.763
8.00	4.00	.4960	-7.466	-5.562	1.232	2.347	40.595	4.245
7.00	3.50	.6230	-6.326	-4.810	1.058	2.048	32.586	3.384
7.00	3.50	.5590	-6.380	-4.832	1.060	2.050	29.641	3.139

B	A	T	BETA-X	Beta-Y	XO	YO	IX	IY
7.00	3.50	.4970	-6.446	-4.854	1.071	2.048	26.777	2.813
7.00	3.50	.4360	-6.602	-4.863	1.074	2.058	23.869	2.481
7.00	3.50	.3720	-6.654	-4.861	1.106	2.047	20.661	2.189
6.00	4.00	.7480	-5.988	-3.133	1.309	1.283	27.857	5.133
6.00	4.00	.6860	-6.069	-3.152	1.320	1.280	26.009	4.741
6.00	4.00	.6220	-6.120	-3.161	1.336	1.286	24.016	4.334
6.00	4.00	.5580	-6.262	-3.176	1.338	1.287	21.948	3.912
6.00	4.00	.4960	-6.284	-3.158	1.364	1.281	19.792	3.588
6.00	4.00	.4340	-6.407	-3.162	1.369	1.291	17.634	3.156
6.00	4.00	.3730	-6.493	-3.181	1.379	1.287	15.428	2.732
6.00	3.50	.6200	-5.721	-3.674	1.113	1.533	21.779	3.111
6.00	3.50	.5600	-5.805	-3.695	1.122	1.530	20.021	2.829
6.00	3.50	.4970	-5.761	-3.708	1.137	1.537	18.031	2.599
6.00	3.50	.4360	-5.922	-3.725	1.138	1.538	16.107	2.293
6.00	3.50	.3710	-6.057	-3.720	1.164	1.526	13.970	1.970
6.00	3.50	.3080	-6.090	-3.729	1.169	1.536	11.770	1.690
6.00	3.00	.6230	-5.340	-4.129	.869	1.750	19.970	2.080
6.00	3.00	.5590	-5.468	-4.121	.900	1.750	18.259	1.881
6.00	3.00	.4960	-5.420	-4.146	.911	1.749	16.443	1.737
6.00	3.00	.4340	-5.565	-4.154	.915	1.759	14.668	1.532
6.00	3.00	.3730	-5.639	-4.177	.925	1.757	12.833	1.327
6.00	3.00	.3090	-5.696	-4.199	.928	1.759	10.799	1.141
5.00	3.50	.6210	-5.136	-2.423	1.166	.976	13.908	2.762
5.00	3.50	.5610	-5.252	-2.426	1.170	.986	12.816	2.514
5.00	3.50	.4980	-5.372	-2.421	1.189	.972	11.630	2.250

B	A	T	BETA-X	BETA-Y	XO	YO	IX	IY
5.00	3.50	.4360	-5.432	-2.429	1.203	.977	10.404	1.986
5.00	3.50	.3710	-5.488	-2.444	1.207	.979	9.010	1.750
5.00	3.50	.3100	-5.553	-2.452	1.221	.984	7.677	1.473
5.00	3.00	.5600	-4.762	-2.996	.951	1.235	11.408	1.712
5.00	3.00	.4960	-4.889	-3.002	.957	1.246	10.342	1.528
5.00	3.00	.4340	-4.923	-3.005	.979	1.234	9.186	1.394
5.00	3.00	.3720	-4.981	-3.015	.993	1.241	8.056	1.204
5.00	3.00	.3080	-5.131	-3.032	.996	1.242	6.814	1.006
4.00	3.50	.6230	-4.673	-0.888	1.168	.353	8.541	2.229
4.00	3.50	.5580	-4.813	-0.875	1.173	.346	7.859	2.011
4.00	3.50	.4980	-4.875	-0.878	1.187	.348	7.186	1.814
4.00	3.50	.4350	-4.942	-0.882	1.203	.351	6.433	1.597
4.00	3.50	.3740	-5.071	-0.887	1.204	.351	5.671	1.389
4.00	3.50	.3090	-5.065	-0.892	1.222	.353	4.772	1.188
4.00	3.00	.5600	-4.352	-1.658	.996	.667	6.667	1.433
4.00	3.00	.4960	-4.411	-1.665	1.013	.671	6.058	1.282
4.00	3.00	.4330	-4.421	-1.684	1.023	.667	5.403	1.167
4.00	3.00	.3710	-4.550	-1.666	1.035	.669	4.738	1.012
4.00	3.00	.3110	-4.644	-1.684	1.044	.664	4.064	.856
4.00	2.50	.4350	-3.981	-2.297	.808	.937	4.686	.744
4.00	2.50	.3730	-4.023	-2.306	.813	.947	4.092	.668
4.00	2.50	.3080	-4.142	-2.309	.833	.937	3.473	.557
4.00	2.50	.2480	-4.289	-2.328	.833	.937	2.855	.455
3.50	3.00	.5580	-4.042	-0.877	.997	.348	4.954	1.276
3.50	3.00	.4990	-4.107	-0.882	1.010	.351	4.547	1.153

B	A	T	BETA-X	BETA-Y	XO	YO	IX	IY
3.50	3.00	.4350	-4.241	-0.872	1.015	.344	4.081	1.019
3.50	3.00	.3730	-4.311	-0.875	1.030	.347	3.600	.880
3.50	3.00	.3080	-4.447	-0.882	1.034	.347	3.056	.734
3.50	3.00	.2480	-4.430	-0.888	1.048	.350	2.501	.619
3.50	2.50	.4320	-3.756	-1.631	.827	.657	3.363	.677
3.50	2.50	.3700	-3.815	-1.639	.842	.662	2.963	.587
3.50	2.50	.3090	-3.936	-1.641	.857	.651	2.546	.494
3.50	2.50	.2450	-3.997	-1.658	.860	.652	2.059	.411
3.00	2.50	.4350	-3.465	-0.871	.836	.345	2.392	.598
3.00	2.50	.3720	-3.559	-0.887	.847	.340	2.114	.516
3.00	2.50	.3110	-3.673	-0.870	.855	.343	1.822	.438
3.00	2.50	.2460	-3.762	-0.885	.867	.338	1.488	.352
3.00	2.00	.4350	-2.889	-1.578	.639	.634	1.950	.370
3.00	2.00	.3720	-2.966	-1.576	.658	.637	1.740	.320
3.00	2.00	.3110	-3.096	-1.572	.666	.642	1.498	.272
3.00	2.00	.2450	-3.178	-1.582	.682	.637	1.232	.218
3.00	2.00	.1870	-3.239	-1.580	.688	.642	.964	.176
2.50	2.00	.3670	-2.760	-0.869	.670	.337	1.120	.270
2.50	2.00	.3060	-2.883	-0.854	.678	.341	.982	.228
2.50	2.00	.2460	-2.937	-0.861	.692	.344	.813	.187
2.50	2.00	.1810	-3.000	-0.870	.696	.345	.624	.146
2.50	1.50	.3080	-2.370	-1.490	.466	.617	.772	.118
2.50	1.50	.2490	-2.440	-1.499	.478	.623	.654	.096
2.50	1.50	.1830	-2.525	-1.512	.497	.617	.497	.073
2.00	1.50	.3110	-2.090	-0.836	.488	.335	.448	.102
2.00	1.50	.2410	-2.179	-0.843	.506	.328	.369	.081
2.00	1.50	.1840	-2.285	-0.844	.509	.337	.297	.063

CALCULATION OF BETA-X AND BETA-Y

PROPERTIES FROM...STEEL DESIGNERS MANUAL
BY THE...BRITISH STEEL PRODUCERS CONFERENCE

B	A	T	BETA-X	BETA-Y	XO	YO	IX	IY
8.00	8.00	.9960	-10.484	0	2.633	0	140.592	36.528
8.00	8.00	.9330	-10.547	0	2.635	0	132.936	34.804
8.00	8.00	.8710	-10.610	0	2.651	0	125.665	32.635
8.00	8.00	.8090	-10.674	0	2.666	0	118.198	30.442
8.00	8.00	.7460	-10.735	0	2.669	0	110.041	28.559
8.00	8.00	.6830	-10.797	0	2.685	0	102.058	26.262
8.00	8.00	.6210	-10.927	0	2.686	0	93.990	23.990
6.00	6.00	.8720	-7.738	0	1.943	0	50.127	13.333
6.00	6.00	.8090	-7.800	0	1.959	0	47.277	12.443
6.00	6.00	.7470	-7.864	0	1.975	0	44.366	11.554
6.00	6.00	.6830	-7.995	0	1.978	0	41.277	10.623
6.00	6.00	.6230	-7.991	0	1.992	0	38.086	9.914
6.00	6.00	.5600	-8.052	0	2.008	0	34.813	8.967
6.00	6.00	.4960	-8.181	0	2.011	0	31.368	7.992
6.00	6.00	.4340	-8.176	0	2.027	0	27.789	7.151
6.00	6.00	.3710	-8.301	0	2.029	0	24.140	6.160
5.00	5.00	.7480	-6.431	0	1.621	0	24.730	6.530
5.00	5.00	.6860	-6.560	0	1.622	0	23.129	6.031
5.00	5.00	.6220	-6.551	0	1.639	0	21.272	5.628
5.00	5.00	.5580	-6.615	0	1.656	0	19.479	5.081
5.00	5.00	.4960	-6.743	0	1.657	0	17.648	4.552
5.00	5.00	.4340	-6.813	0	1.673	0	15.755	4.005

B	A	T	BETA-X	BETA-Y	XO	YO	IX	IY
5.00	5.00	.3730	-6.875	0	1.674	0	13.722	3.538
4.00	4.00	.7490	-4.996	0	1.266	0	11.995	3.225
4.00	4.00	.6860	-5.121	0	1.269	0	11.238	2.982
4.00	4.00	.6240	-5.117	0	1.284	0	10.389	2.811
4.00	4.00	.5600	-5.181	0	1.301	0	9.557	2.543
4.00	4.00	.4960	-5.313	0	1.304	0	8.671	2.269
4.00	4.00	.4340	-5.371	0	1.319	0	7.772	2.008
4.00	4.00	.3720	-5.438	0	1.335	0	6.826	1.734
4.00	4.00	.3080	-5.500	0	1.338	0	5.735	1.485
3.50	3.50	.6210	-4.430	0	1.102	0	6.704	1.836
3.50	3.50	.5600	-4.498	0	1.117	0	6.211	1.669
3.50	3.50	.4960	-4.561	0	1.134	0	5.646	1.494
3.50	3.50	.4330	-4.690	0	1.136	0	5.062	1.318
3.50	3.50	.3710	-4.752	0	1.152	0	4.458	1.142
3.50	3.50	.3110	-4.749	0	1.166	0	3.805	.995
3.50	3.50	.2470	-4.876	0	1.169	0	3.100	.800
3.00	3.00	.5620	-3.748	0	.946	0	3.751	1.029
3.00	3.00	.4960	-3.875	0	.950	0	3.422	.918
3.00	3.00	.4320	-3.935	0	.967	0	3.069	.811
3.00	3.00	.3700	-3.998	0	.983	0	2.717	.703
3.00	3.00	.3090	-4.141	0	.984	0	2.328	.592
3.00	3.00	.2450	-4.134	0	1.001	0	1.889	.491
2.50	2.50	.4930	-3.113	0	.783	0	1.906	.514
2.50	2.50	.4350	-3.176	0	.795	0	1.739	.461
2.50	2.50	.3720	-3.178	0	.812	0	1.525	.415

B	A	T	BETA-X	BETA-Y	XO	YO	IX	IY
2.50	2.50	.3110	-3.303	0	.812	0	1.327	.353
2.50	2.50	.2450	-3.356	0	.831	0	1.097	.283
2.25	2.25	.3670	-2.855	0	.716	0	1.084	.296
2.25	2.25	.3060	-2.926	0	.731	0	.930	.250
2.25	2.25	.2460	-2.982	0	.745	0	.775	.205
2.25	2.25	.1810	-3.116	0	.749	0	.607	.153
2.00	2.00	.3700	-2.489	0	.629	0	.735	.205
2.00	2.00	.3080	-2.545	0	.645	0	.645	.175
2.00	2.00	.2490	-2.618	0	.658	0	.537	.143
2.00	2.00	.1830	-2.734	0	.663	0	.412	.108
1.75	1.75	.3110	-2.178	0	.558	0	.424	.116
1.75	1.75	.2410	-2.301	0	.565	0	.349	.091
1.75	1.75	.1840	-2.360	0	.577	0	.268	.072
1.50	1.50	.3090	-1.861	0	.460	0	.249	.071
1.50	1.50	.2490	-1.931	0	.474	0	.222	.058
1.50	1.50	.1860	-1.991	0	.491	0	.175	.045
1.25	1.25	.2460	-1.551	0	.392	0	.108	.032
1.25	1.25	.1820	-1.672	0	.395	0	.095	.025
1.25	1.25	.1220	-1.761	0	.409	0	.063	.017
1.00	1.00	.2500	-1.184	0	.304	0	.064	.016
1.00	1.00	.1850	-1.305	0	.308	0	.048	.012
1.00	1.00	.1240	-1.361	0	.322	0	.031	.009

APPENDIX B

COMPUTER PROGRAM FOR CALCULATION OF CRITICAL BUCKLING LOADS

The general background of the program has been discussed in the body of the thesis. Therefore this appendix will do little more than suggest a few corrections or possible extensions and present the meaning of symbols used in the program.

One obvious improvement which would reduce input work was mentioned in the main part of the dissertation. The present program requires an input of the root radius for each angle. Since this data was already prepared in card deck form, it would not be worthwhile to change it. However, if a change were desired in the future , Equation (23) could be input as:

```
R1 = 0.12 + 0.03 * (B + A)   ..........   (B-1)
```

(Computer programming symbols have been used in the above version of Equation (23).)

A small correction to the calculation of the torsional constant, J, could be made by using the exact form of Equation (25):

```
J = ( (A - R1 - T) + (B - R1 - T) ) * T**3 /3 + XXXX
                                   ...................   (B-2)
```

One possible extension to the program would be an option to run the routine for a series of eccentricities along the legs. This would be fairly easy to implement.

Along the short leg, where <u>da</u> is some increment along the leg,

$$e_x = x_o - (da) \cos \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(B-3)}$$

$$e_y = y_3 - e_x \tan \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(B-4)}$$

Or, in programming symbols, with <u>da</u> = DELTA,

```
EX = X0 - DELTA * CA

EY = Y3 - EX * TA
```

Similarly, along the long leg, with increment <u>db</u>,

$$e_x = x_o - (db) \sin \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(B-5)}$$

$$e_y = y_4 + \frac{e_x}{\tan \alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(B-6)}$$

Again, in programming nomenclature, with <u>db</u> = DELTB,

```
EX = X0 - DELTB * SA

EY = Y4 + EX / TA
```

The whole of these calculations would probably be located within a DO LOOP to have them performed the number of times required.

The calculation of critical bending moments in the absence of axial load is currently quite limited. The points where the critical ellipse cuts the M_x and M_y axes are found. This analysis could be extended as required.

The remainder of this Appendix is made up of a listing of the symbols used in the programming and a copy of the program and sample output. Some of the symbols listed have already been explained in Appendix A. They are repeated here for the sake of completeness.

	Report Symbol	Computer Program Equivalent
Constants:	E	E
	G	G
	π	PI
Variables:	L	L
	e_x	EX
	e_y	EY
Input of Angle Properties:	b	B
	a	A
	t	T
	A	AREA
	I_{xx}	IXX
	$\bar{y}$	YBAR
	I_{yy}	IYY
	$\bar{x}$	XBAR
	r_{zz}	RZZ
	$\tan \alpha$	TA
Calculated Properties:	x_o	XO
	y_o	YO
	I_x	IX
	I_y	IY

	Report Symbol	Computer Program Equivalent
Calculated Properties:	I_o	IO
	J	J
	H	H
	β_x	BETAX
	β_y	BETAY
Calculated Constants and Functions:	P_x	PX
	P_y	PY
	P_t	PT
	k_1	C1
	k_2	C2
	k_3	C3
	k_4	C4
	K	K
	f(P)	FP
	f'(P)	DFP
	f"(P)	D2FP
	$\cos\alpha$	CA
	$\sin\alpha$	SA
	P_i	PP1
	P_{i+1}	PP2
Program Dummy Variables:		XXXX
		TWIST
		WARP
		OUT1
		OUT2

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```
      PROGRAM ROBERT
C     ROBERT ERROL DALTON ..737-0632..... 482-1695....
C     PROGRAM FOR ..LATERAL STABILITY OF SINGLE-ANGLE BEAMS
C     PROFESSOR CEDRIC MARSH .... SUPERVISOR..... 879-7365
      REAL IXX, IYY, IX, IY, IO, J, K ,L
      NNNN = 0
      READ(60,1000) NANGL
 1000 FORMAT( I5 )
   50 CONTINUE
      NNNN = 1 + NNNN
C*************************************************************************
C     INPUT OF ANGLE PROPERTIES
C*************************************************************************
      READ(60,1)     B, A, T, AREA, IXX, YBAR, IYY, XBAR, RZZ, TA, R1
    1 FORMAT(11F5.0)
      WRITE(61,460)
  460 FORMAT( 1H  ,  5X,  16HANGLE PROPERTIES      )
      WRITE(61,462)
  462 FORMAT( 1H0 , 8X , 1HB , 10X, 1HA, 10X, 1HT, 10X, 4HAREA, 9X,
     1  3HIXX, 7X, 4HYBAR, 6X, 3HIYY, 9X, 4HXBAR, 6X, 3HRZZ, 9X,
     2   3HTAN  , 7X  , 1X,  2HR1        )
      WRITE(61,102)  B, A, T, AREA, IXX, YBAR, IYY, XBAR, RZZ, TA, R1
  102 FORMAT(1H , 4X, F6.2, 5X, F6.2, 5X, F6.4, 6X, 8(F7.2 , 4X)  )
C*************************************************************************
C     CALCULATION OF COORDINATES AND PRINCIPAL MOMENTS OF INERTIA
C*************************************************************************
      IY = (RZZ**2) * AREA
      IX = (IXX + IYY) - IY
      TTA = ATAN(TA)
      CA = COS(TTA)
      SA = SIN(TTA)
      Y3 = (YBAR - T/2.) / CA
      X5 = (XBAR - T/2.) / CA
      Y4 = - X5 / TA
      XO = (Y3 - Y4) * SA * CA
      YO = Y3 * (CA**2)  +  Y4 * (SA**2)
      X1 = XO - (A - T/2.) * CA
      Y1 = YO + (A - T/2.) * SA
      X2 = XO - (B - T/2.) * SA
      Y2 = YO - (B - T/2.) * CA
      WRITE(61,104) IX, IY, CA, SA
  104 FORMAT( 1H-,15X, 5HIX =   , F8.2 ,10X, 5HIY =  ,F8.2,10X,
     1   13HCOS(ALPHA) =    , F6.4,10X , 13HSIN(ALPHA) =   ,F6.4 )
      WRITE(61,470)
  470 FORMAT( 1H-   , 5X , 29HCOORDINATES IN PRINCIPAL AXES    )
      WRITE(61,472)
  472 FORMAT( 1H , 13X, 2HY3, 10X, 2HX5, 10X, 2HY4, 10X, 2HXO, 10X,
     1  2HYO, 10X, 2HX1, 10X, 2HY1, 10X, 2HX2, 10X, 2HY2    )
      WRITE(61,106) Y3, X5, Y4, XO, YO, X1, Y1,X2, Y2
  106 FORMAT( 1H ,  4X,  9( 4X, F8.3 )    )
```

```
C*****************************************************************************
C  CALCULATION OF BETA-X AND BETA-Y BY EXACT INTEGRATION
C*****************************************************************************
      DD1 = X0**4 - X1**4


      DD2 = X0**4 - X2**4
      SX3 = (T/4.) * (DD1/CA + DD2/SA)
      DD3 = X0**3 - X1**3
      DD4 = X0**3 - X2**3
      DD5 = 4.* Y3 * DD3 - 3. * TA * DD1
      DD6 = 4. * Y4 * DD4  +  (3./TA) * DD2
      SX2Y = (T/ (12. * SA * CA) ) * ( SA*DD5 + CA * DD6)
      DD7 = Y1**4  -  Y0**4
      DD8 = Y0**4 - Y2**4
      SY3 = (T/ (4. * SA * CA) ) * (CA * DD7 + SA * DD8)
      DD9 = Y1**3 - Y0**3
      DD10 = Y0**3 - Y2**3
      DD11 = 4. * Y3 * DD9 - 3. * DD7
      DD12 = 3. * DD8 - 4. * Y4 * DD10
      SXY2 = (T/ (12.* SA**2 * CA**2) ) * (CA**3 * DD11 + SA**3 * DD12)
      WRITE(61,474)
  474 FORMAT( 1H- , 5X,30HINTEGRALS IN BETA CALCULATIONS     )
      WRITE(61,476)
  476 FORMAT( 1H , 15X, 14HINT(X**3 * DA)  , 5X,18HINT(X**2 * Y * DA)  ,
     1  5X, 14HINT(Y*** * DA)  , 5X, 19HINT(X * Y**2 * DA )    )
      WRITE(61,108) SX3, SX2Y, SY3, SXY2
  108 FORMAT( 1H  , 15X,  4(F8.3 ,12X) /.)
      BETAX = (SX3 + SXY2) / IY - 2. * X0
      BETAY = (SY3 + SX2Y) / IX - 2. * Y0
      X02 = -2. * X0
      Y02 = - 2. * Y0
      WRITE(61,452)
  452 FORMAT(1H0 , 5X, 1HB, 10X, 1HA, 10X, 1HT, 7X, 6HBETA-X , 10X,
     1  6HBETA-Y , 13X, 2HX0, 13X, 2HY0  )
      WRITE(61,112) B, A, T, BETAX, BETAY, X0, Y0
  112 FORMAT(1H ,        F8.2,  4X, F8.2,   2X, F8.5,  4X, 4(F8.3, 7X)
     1   ////      )
C*****************************************************************************
C   CALCULATION OF BUCKLING LOADS AND CUBIC EQUATION COEFFICIENTS
C*****************************************************************************
      E = 29.0E+03
      G = 11.2E+03
      PI = 3.141593
      IO = IX + IY + AREA * (X0**2 + Y0**2)
      CAPN = PI / T
      SCRPN = 0.85 + 0.24 * CAPN
      XXXX =  (SCRPN * T)**4
      J = (A - T + B - T) * (T**3)/ 3. + XXXX
      H = (T**3/36.)*( (A - T/2.)**3 +     (B - T/2.)**3  )
      WRITE(61,116) IO, XXXX, J, H
  116 FORMAT(1H0, 5X, 5HIO =  , F10.4, 5X, 11H(N*T)**4 =  , F12.6, 5X,
     1   4HJ =  , F12.6,  5X,  4HH =   , F12.4  )
```

```
      EX = -2.0
      EY = 5.0
      L = 20.0
      DO 200 II = 1, 2
      L = L + 20.0
      PX = PI**2 * E * IX / L**2
      PY = PI**2 * E * IY / L**2

      TWIST = G * J
      WARP = PI**2 * E * H / L**2
      PT = (AREA/IO) * (TWIST + WARP)
      WRITE(61,120) PX, PY, TWIST, WARP, PT
  120 FORMAT(1H0, 5X, 5HPX = , F9.2, 8X,5HPY = , F9.1, 8X, 5HG*J = ,
     1  F9.1, 8X, 23HPI**2 * E * H / L**2 = ,F9.1 , 8X,5HPT = , F9.1 )
      K = (AREA / IO) * (EY * BETAY + EX * BETAX  +  1.)
      C1 = (XO-EX)**2 + (YO - EY)**2 - IO * K / AREA
      C2 = IO*PT/AREA + IO*K*(PX+PY)/AREA -PY*(XO-EX)**2 -PX*(YO-EY)**2
      C3 = -(IO/AREA) * (PT*PX + PT*PY + K*PX*PY)
      C4 = IO * PT * PX * PY / AREA
      WRITE(61,122) K, C1, C2, C3, C4
  122 FORMAT(1H-, 5X, 4HK = ,F12.3, 5X,5HC1 = , F12.1, 5X, 5HC2 = ,
     1 E12.2, 5X, 5HC3 =  , E12.2, 5X, 5HC4 =  , E12.2  /  )
C**********************************************************************
C   SOLUTION OF CUBIC EQUATION FOR CRITICAL BUCKLING LOADS
C**********************************************************************
      WRITE(61,150)
  150 FORMAT(1H-, 2X, 26HSOLUTION OF CUBIC EQUATION  )
      PA = ( -C2 + SQRT(C2**2 - 3. * C1 * C3) )      / (3. * C1)
      PB = ( -C2 - SQRT(C2**2 - 3. * C1 * C3)  )  /  (3. * C1)
      CR1 = -.50 * PY * BETAY
      CR2 = -.50 * PX * BETAX
      CR3 = PT * IO / AREA
      CRMX = CR1   +   SQRT( CR1**2 + PY * CR3)
      CRMY = CR2 + SQRT(CR2**2   +   FX * CR3)
      WRITE(61,124) PA, PB
  124 FORMAT(1H-,15X, 5HPA = , F10.2,10X, 5HPB = , F10.2  // )
      WRITE(51,151)
  151 FORMAT(1H , 5X,4HROOT,8X, 5HTRIAL,12X,9HP(FIRST) ,5X,
     113HF( P(FIRST) )   , 7X, 9HD( F(P) )  ,10X, 9HP(SECOND) , 4X,
     2  14HF( P(SECOND) ) ,5X,10HD2( F(P) )   )
      DO 300 NN = 1, 3
      KOUNT = 0
      GO TO (301, 302, 303), NN
  301 CONTINUE
      PP1 = AMIN1(PA,PB)
      PP1 = PP1 - .50 * ABS(PP1)
      GO TO 360
  302 PP1 =  .50 * (PA + PB)
      GO TO 360
  303 CONTINUE
      PP1 = AMAX1(PA,PB)
      PP1 = PP1 + .50 * ABS(PP1)
  360  P = PP1
```

```
      KOUNT = KOUNT + 1
      FP   = C1 * P**3 + C2 * P**2 + C3 * P + C4
      DFP  = 3. * C1 * P**2 + 2. * C2 * P + C3
      D2FP = 6. * C1 * P + 2. * C2
      PP2 = PP1 - FP * DFP / ( DFP**2 - FP * D2FP)
      OUT1 = FP
      P = PP2
      FP   = C1 * P**3 + C2 * P**2 + C3 * P + C4
      OUT2 = FP


      WRITE(6,126) NN, KOUNT, PP1, OUT1, DFP, PP2, OUT2, D2FP
 126  FORMAT(1H , 2X, I5, 8X, I5, 8X, F12.2, 6X, 2(E12.2, 6X), F12.2,
     1    6X, E12.2,6X, E12.2   )
      IF( ABS(OUT2) . LT . 100. )       GO TO 351
 350  PP1 = PP2
      IF(KOUNT.GE. 10) GO TO 351
      GO TO 360
 351  CONTINUE
      GO TO (361, 362, 363), NN
 361  P1 = PP2
      GO TO 300
 362  P2 = PP2
      GO TO 300
 363  P3 = PP2
 300  CONTINUE
      WRITE(6,170)
 170  FORMAT(1H-,  5X, 1HB , 10X, 1HA, 10X, 1HT,  8X, 6HLENGTH , 7X,
     1 2HEX , 9X, 2HEY,  9X, 2HP1, 11X, 2HP2, 10X, 2HP3, 13X, 2HMX,
     2   10X, 2HMY   )
      WRITE(6,128) B, A, T, L, EX, EY, P1, P2, P3, CRMX, CRMY
 128  FORMAT(1H , 1X, F7.2, 3X, F7.2, 5X, F6.3, 5X, F7.2, 4X,F7.2,4X,
     1   F7.2,       1X, 3(F12.2, 1X), 2X, 2(F12.2, 1X) ///////)
 200  CONTINUE
      IF(NNNN.LT.NANGL ) GO TO 50
      STOP
      END
```

ANGLE PROPERTIES

B	A	T	AREA	IXX	YBAR	IYY	XBAR	RZZ	TAN	R1
2.50	1.50	.3080	1.15	.70	.89	.19	.39	.32	.35	.24

IX = .77 IY = .12 COS(ALPHA) = .9439 SIN(ALPHA) = .3304

COORDINATES IN PRINCIPAL AXES

Y3	X5	Y4	X0	Y0	X1	Y1	X2	Y2
.780	.250	-0.714	.466	.617	-0.805	1.061	-0.309	-1.598

INTEGRALS IN BETA CALCULATIONS

INT(X**3 * DA)	INT(X**2 * Y * DA)	INT(Y*** * DA)	INT(X * Y**2 * DA)
-0.021	.060	-0.258	-0.148

B	A	T	BETA-X	BETA-Y	X0	Y0
2.50	1.50	.30800	-2.370	-1.490	.466	.617

IO = 1.5770 (N*T)**4 = .010407 J = .043365 H = .0125

PX = 138.14 PY = 21.1 G*J = 485.7 PI**2 * E * H / L**2 = 2.2 PT = 355.8

K = -1.245 C1 = 27.0 C2 = -2.57E 03 C3 = -7.27E 04 C4 = 1.42E 06

SOLUTION OF CUBIC EQUATION

PA = 75.28 PB = -11.92

ROOT	TRIAL	P(FIRST)	F(P(FIRST))	D(F(P))	P(SECOND)	F(P(SECOND))	D2(F(P))
1	1	-17.88	1.74E 06	4.50E 04	-22.78	1.43E 06	-8.03E 03
1	2	-22.78	1.43E 06	8.62E 04	-28.92	7.23E 05	-8.82E 03
1	3	-28.92	7.23E 05	1.43E 05	-32.67	1.15E 05	-9.82E 03
1	4	-32.67	1.15E 05	1.81E 05	-33.28	2.09E 03	-1.04E 04
1	5	-33.28	2.09E 03	1.88E 05	-33.29	6.49E-01	-1.05E 04
2	1	31.68	-2.60E 06	-1.54E 05	14.79	-1.30E 05	0
2	2	14.79	-1.30E 05	-1.31E 05	13.78	1.32E 03	-2.74E 03
2	3	13.78	1.32E 03	-1.28E 05	13.79	1.54E-01	-2.90E 03
3	1	112.93	-6.33E 05	3.81E 05	114.50	-1.80E 04	1.32E 04
3	2	114.50	-1.80E 04	4.02E 05	114.54	-1.35E 01	1.34E 04

B	A	T	LENGTH	EX	EY	P1	P2	P3	MX	MY
2.50	1.50	.308	40.00	-2.00	5.00	-33.29	13.79	114.54	118.28	470.65

PX = 61.40 PY = 9.4 OBJ = 485.7 PI**2 * E * H / L**2 = 1.0 PT = 354.9

K = -1.245 C1 = 27.0 C2 = -8.71E 02 C3 = -3.35E 04 C4 = 2.80E 05

SOLUTION OF CUBIC EQUATION

PA = 33.74 PB = -12.24

ROOT	TRIAL	P(FIRST)	F(P(FIRST))	D(F(P))	P(SECOND)	F(P(SECOND))	D2(F(P))
1	1	-18.36	4.33E 05	2.58E 04	-22.49	2.85E 05	-4.72E 03
1	2	-22.49	2.85E 05	4.67E 04	-26.07	8.18E 04	-5.39E 03
1	3	-26.07	8.18E 04	6.70E 04	-27.17	4.36E 03	-5.97E 03
1	4	-27.17	4.36E 03	7.37E 04	-27.23	1.08E 01	-6.14E 03
2	1	10.75	-1.47E 05	-4.28E 04	7.32	-1.09E 03	-1.19E-07
2	2	7.32	-1.09E 03	-4.19E 04	7.29	1.88E-01	-5.56E 02
3	1	50.61	-1.44E 05	8.59E 04	52.09	-8.82E 03	6.46E 03
3	2	52.09	-8.82E 03	9.57E 04	52.19	-2.84E 01	6.70E 03

B	A	T	LENGTH	EX	EY	P1	P2	P3	MX	MY
2.50	1.50	.308	60.00	-2.00	5.00	-27.23	7.29	52.19	74.84	260.32

APPENDIX C

SIMPLIFIED BETA FORMULAS

As an extension of the work presented in Appendix A, the β_x and β_y values calculated exactly were studied in the hope of finding a more convenient method of calculation. The approach to this problem, the results, and an analysis of their accuracy are the subject of this Appendix.

It was expected that the values of the parameters would be in some proportion to the leg lengths and thicknesses, especially since they all share the common dimension of length. Therefore the work began by preparing input data files with the two parameters and the leg properties.

Standard package programs for regression analysis are available on most computer systems. The one used for this work was the AQD (Analysis of Quantitative Data) package at Harvard Business School. The program performed the usual least squares linear regression for the two data files (Equal and Unequal Angles). Different independent variable combinations were checked to balance accuracy and ease of calculation.

For Equal Angles, the regression analysis was used for β_x calculations only, since:

$$\beta_y = 0 \quad \text{..............................} \quad \text{(C-1)}$$

Two formulas were developed from the analysis:

$$\beta_x = 0.09134 - 1.344\ b \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(C-2)}$$

$$\beta_x = 0.002858 - 1.433\ b + 1.017\ t \quad \ldots\ldots\ldots \quad \text{(C-3)}$$

The "adjusted R-squared" term, a measure of the accuracy of the regression in predicting the input values, was 0.997 for Equation (C-2) and a perfect 1.000 for the second formula.

In order to confirm the accuracy of these formulas, a small program was prepared to compare predicted with calculated values. A listing of the program and some partial output is displayed in Figure C-1. Either formula (Equation (C-2) or (C-3)) gave very small errors in the calculations. Therefore the simpler of the two, Equation (C-2) is selected as a means of determining β_x values for an Equal Angle.

This conclusion is supported by Figures C-2 and C-3. The correspondence of a linear relationship with leg length is indicated by the first graph, and the accuracy of the prediction verified by the second one.

The calculations for Unequal Angles proceeded in the same manner as for Equal Angles, but the results were less accurate. The equations from the linear regression analysis are:

$$\beta_x = -0.6529 - 0.7411\ b - 0.5509\ a + 2.474\ t$$

$$\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(C-4)}$$

```
2 A$="←.  ←.←←←←  ←←←.←←      ←←←.←←   ←←.←      ←←←.←←
10 DIM B(66),T(66),E(66),P(66),D(66),Q(66),F(66)
15 N = 66
20 FILES EQUAL
30 FOR X = 1 TO N
40 READ #1, B(X),T(X),E(X)
60 P(X)= 2.858E-03 -1.433*E(X) + 1.017 * T(X)
70 D(X) = ( E(X) - P(X) ) / E(X)
75 D(X) = 100. * D(X)
80 Q(X) = 9.134E-02 - 1.344 * B(X)
90 F(X) = ( E(X) - Q(X) ) / E(X)
95 F(X) = 100. * F(X)
100 NEXT X
110 PRINT "COMPARISON OF PREDICTED WITH CALCULATED BETA-X"
115 PRINT"**********************************************"
1  16 PRINT" "
125 PRINT"B         T      BETA-X          P1        D1        P2
1  30 FOR Y = 1 TO N
145 PRINT USING A$:B(Y),T(Y),E(Y),P(Y),D(Y),Q(Y),F(Y)
150 NEXT Y
160 END
```

COMPARISON OF PREDICTED WITH CALCULATED BETA-X

****

B	T	BETA-X	$\beta_x(B,T)$ P1	% D1	$\beta_x(B)$ P2	% D2
8.	1.1250	-10.32	-10.32	0.0	-10.66	-3.3
8.	1.0000	-10.44	-10.44	-0.0	-10.66	-2.1
8.	0.8750	-10.57	-10.57	0.0	-10.66	-0.8
8.	0.7500	-10.70	-10.70	-0.0	-10.66	0.3
8.	0.5625	-10.89	-10.89	0.0	-10.66	2.1
8.	0.5000	-10.89	-10.95	-0.6	-10.66	2.1
6.	1.0000	-7.56	-7.58	-0.2	-7.97	-5.4
6.	0.8750	-7.69	-7.71	-0.2	-7.97	-3.6
6.	0.7500	-7.82	-7.83	-0.2	-7.97	-2.0
6.	0.6250	-7.94	-7.96	-0.2	-7.97	-0.4
6.	0.5625	-8.01	-8.02	-0.2	-7.97	0.5
6.	0.5000	-8.14	-8.09	0.7	-7.97	2.1
6.	0.4375	-8.14	-8.15	-0.2	-7.97	2.0
6.	0.3750	-8.20	-8.21	-0.2	-7.97	2.8
6.	0.3125	-8.33	-8.28	0.6	-7.97	4.2
5.	0.7500	-6.44	-6.40	0.6	-6.63	-2.9
5.	0.6250	-6.50	-6.53	-0.4	-6.63	-1.9
5.	0.5000	-6.70	-6.65	0.7	-6.63	1.0

FIGURE C-1: COMPARISON PROGRAM AND SAMPLE OUTPUT

β_x FOR EQUAL ANGLES

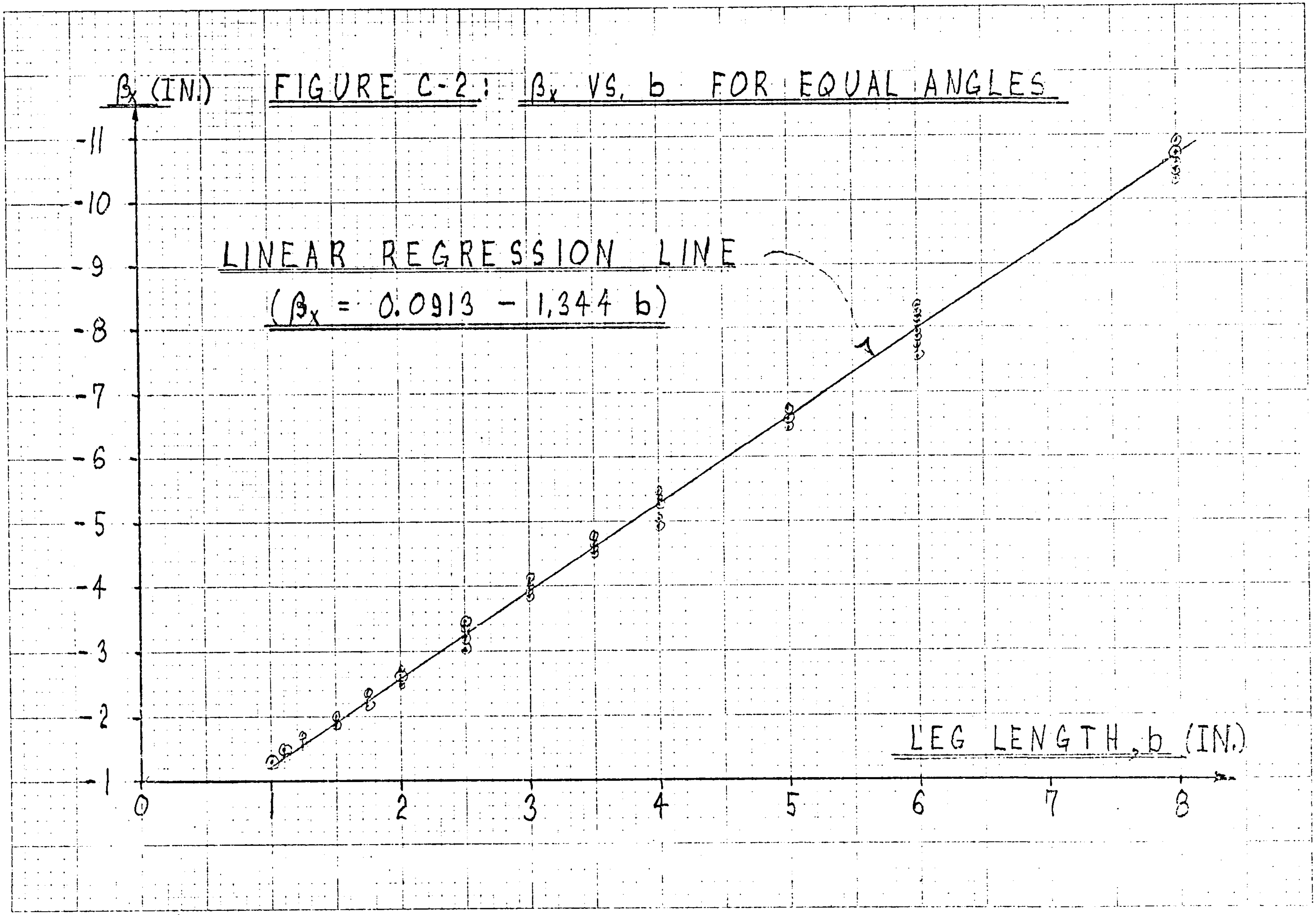
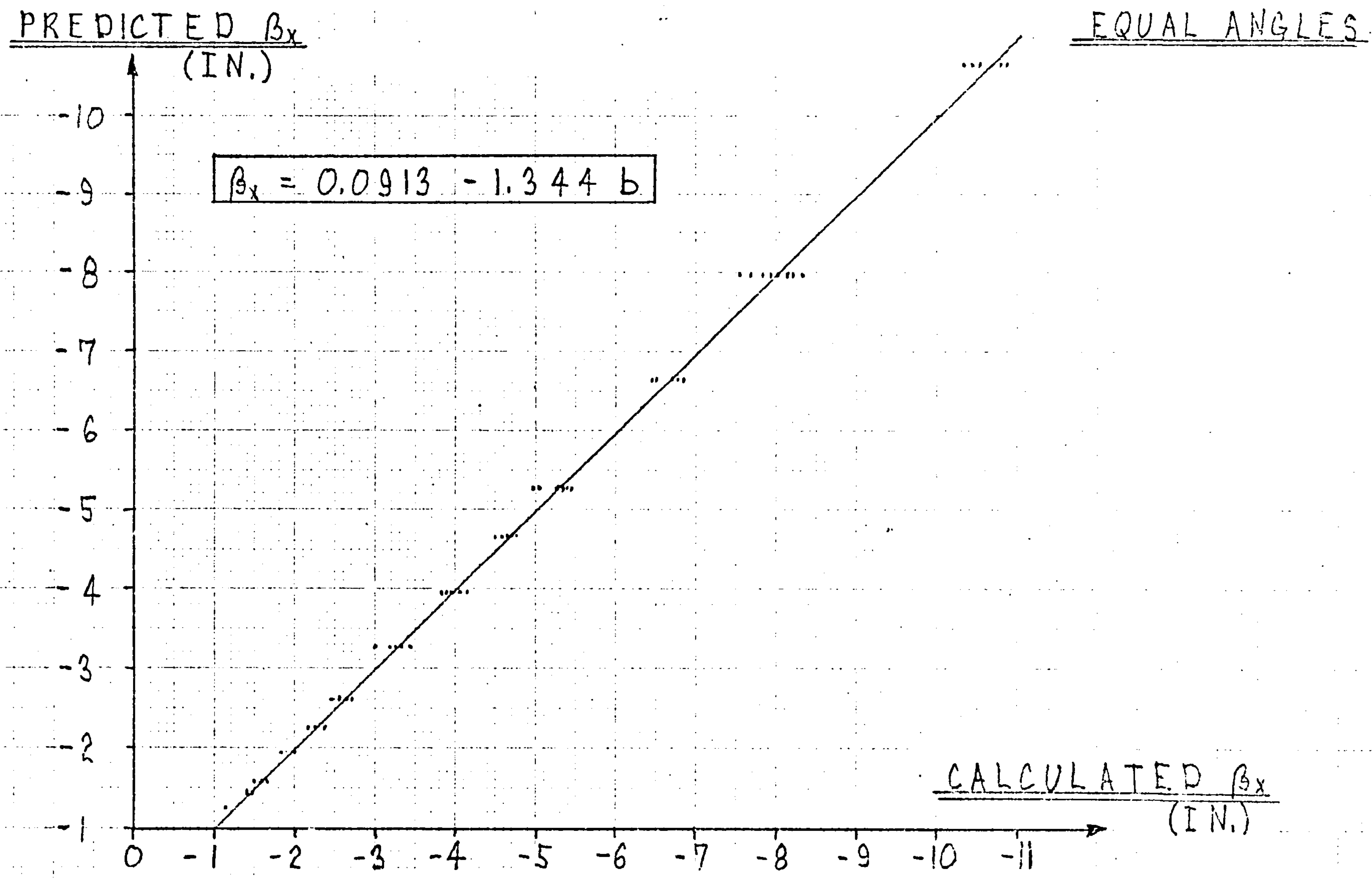


FIGURE C-2: β_x VS. b FOR EQUAL ANGLES
β_x (IN.)
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
LINEAR REGRESSION LINE
(β_x = 0.0913 − 1.344 b)
LEG LENGTH, b (IN.)

FIGURE C-3 : PREDICTED β_x VS. CALCULATED β_x
EQUAL ANGLES



$$\beta_x = -0.4638 - 0.5999\ b - 0.4806\ a \quad \ldots\ldots \quad (C\text{-}5)$$

$$\beta_x = -0.9321 - 0.8187\ b \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (C\text{-}6)$$

The "adjusted R-squared" terms for these equations were 0.829, 0.798, and 0.781 in that order. These numbers are not as favorable as those for the Equal Angles were. Some indication of this is found in Figure C-4, where a program listing and output is displayed to compare the predicted with the calculated values.

This lack of accuracy is also apparent in the graphs of Figures C-5, C-6, and C-7. From analyzing this data, I would suggest that Equation (C-5) is the least accurate approximation which can be used, and Equation (C-4) should be employed if very close calculations are required. Figures C-6 and C-7 show the difference in moving from Equation (C-6) to (C-5).

The β_y results of the regression analysis were:

$$\beta_y = 0.4048 - 1.131\ b + 0.8902\ a - 0.6773\ t \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (C\text{-}7)$$

$$\beta_y = 0.3530 - 1.170\ b + 0.8709\ a \quad \ldots\ldots \quad (C\text{-}8)$$

$$\beta_y = 1.202 - 0.7732\ b \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (C\text{-}9)$$

```
10 B$="←.←←    ←.←←    ←.←←←←    ←←.←←     ←←←.←←"
20 C$="  ←←←.←      ←←.←←     ←←←.←"
30 DIM B(97), A(97), T(97), E(97), F(97)
40 DIM P(97), Q(97), R(97), S(97)
50 N = 97
60 FILES EOB
70 FOR X = 1 TO N
80 READ #1, B(X), A(X), T(X), E(X), F(X)
90 P(X)= -.6529 -.7411*B(X) -.5509*A(X) + 2.474*T(X)
100 Q(X) = ( E(X) - P(X) ) / E(X)
110 Q(X) = 100.0 * Q(X)
120 R(X) = -.9321 - .8187 * B(X)
130 S(X) = ( E(X) - R(X) ) / E(X)
140 S(X) = 100.0 * S(X)
150 NEXT X
160 PRINT "COMPARISON OF PREDICTED WITH CALCULATED BETA-X"
170 PRINT "**********************************************"
180 PRINT "      "
190 PRINT"                                          ";
200  PRINT"                 %                    %   "
210 PRINT"  B       A       T      BETA-X      PRED.     DIFF.";
220 PRINT"  PRED.    DIFF.   "
230 PRINT"  "
240 FOR Y = 1 TO N
250 PRINT USING B$: B(Y), A(Y), T(Y), E(Y),P(Y);
260 PRINT USING C$: Q(Y), R(Y), S(Y)
270 NEXT Y
280 END
```

COMPARISON OF PREDICTED WITH CALCULATED BETA-X
****

B	A	T	BETA-X	$\beta_X(B,A,T)$ PRED.	% DIFF.
9.00	4.00	1.0000	-7.45	-7.05	5.3
9.00	4.00	0.8750	-7.61	-7.36	3.3
9.00	4.00	0.7500	-7.74	-7.67	0.9
9.00	4.00	0.6250	-7.90	-7.98	-1.0
9.00	4.00	0.5625	-8.12	-8.13	-0.2
9.00	4.00	0.5000	-8.19	-8.29	-1.3
8.00	6.00	1.0000	-8.63	-7.41	14.1
8.00	6.00	0.8750	-8.77	-7.72	11.9
8.00	6.00	0.7500	-8.91	-8.03	9.8
8.00	6.00	0.6250	-9.04	-8.34	7.8
8.00	6.00	0.5625	-9.04	-8.50	6.0
8.00	6.00	0.5000	-9.19	-8.65	5.8

FIGURE C-4: COMPARISON PROGRAM AND SAMPLE OUTPUT

β_X FOR UNEQUAL ANGLES

FIGURE C-5: β_x VS. b FOR UNEQUAL ANGLES

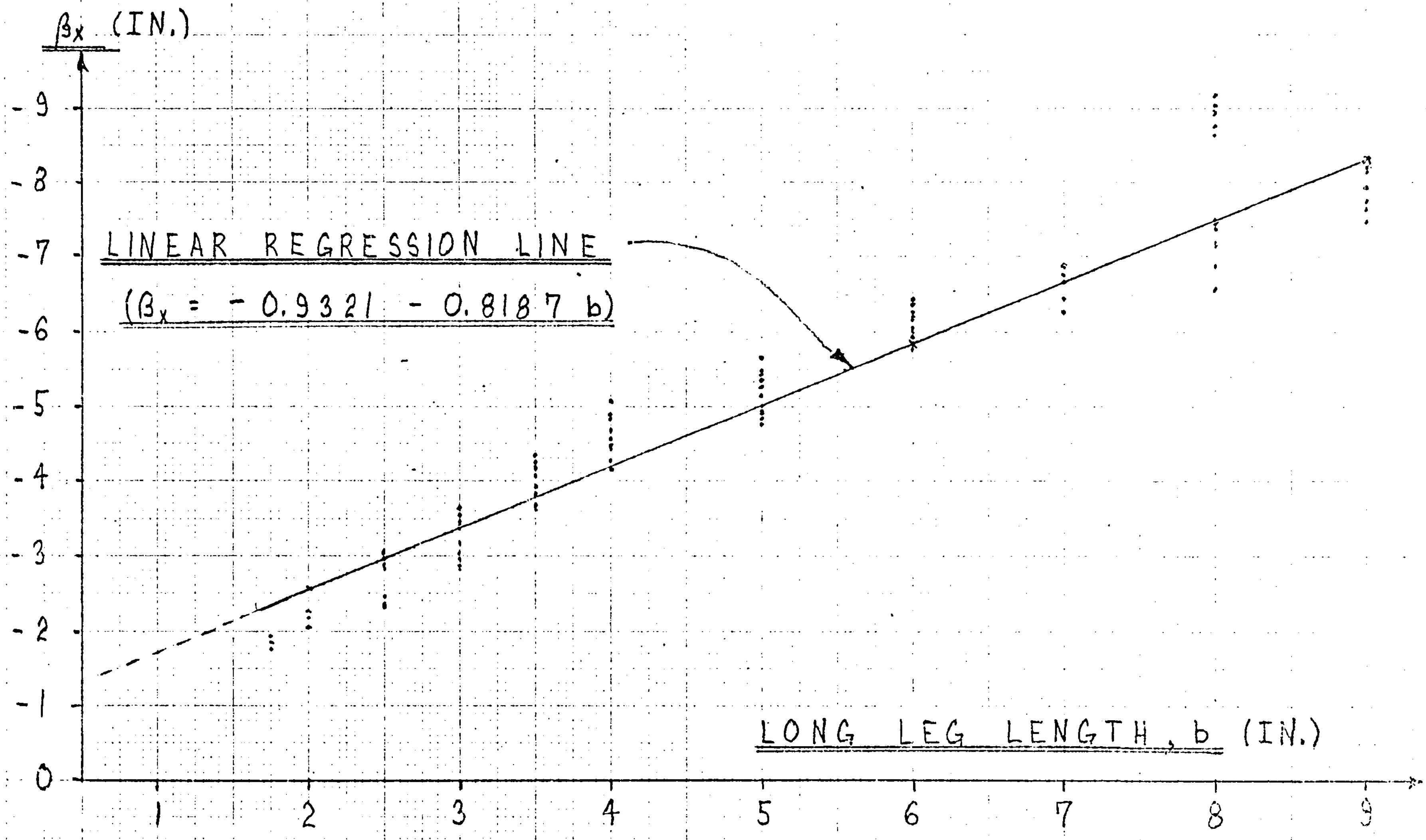
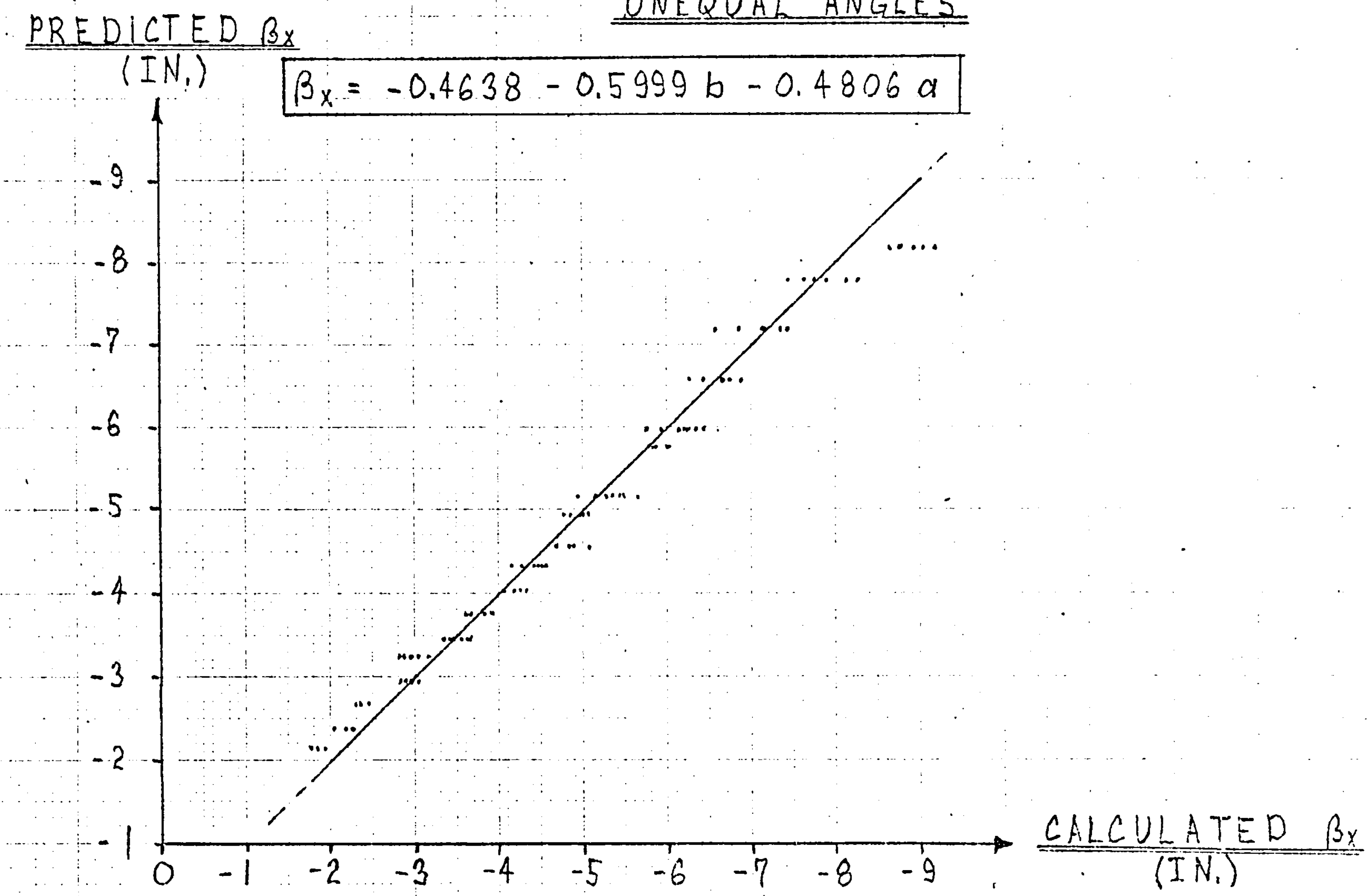


FIGURE C-6: PREDICTED β_x VS. CALCULATED β_x

UNEQUAL ANGLES

$$\beta_x = -0.4638 - 0.5999\,b - 0.4806\,a$$



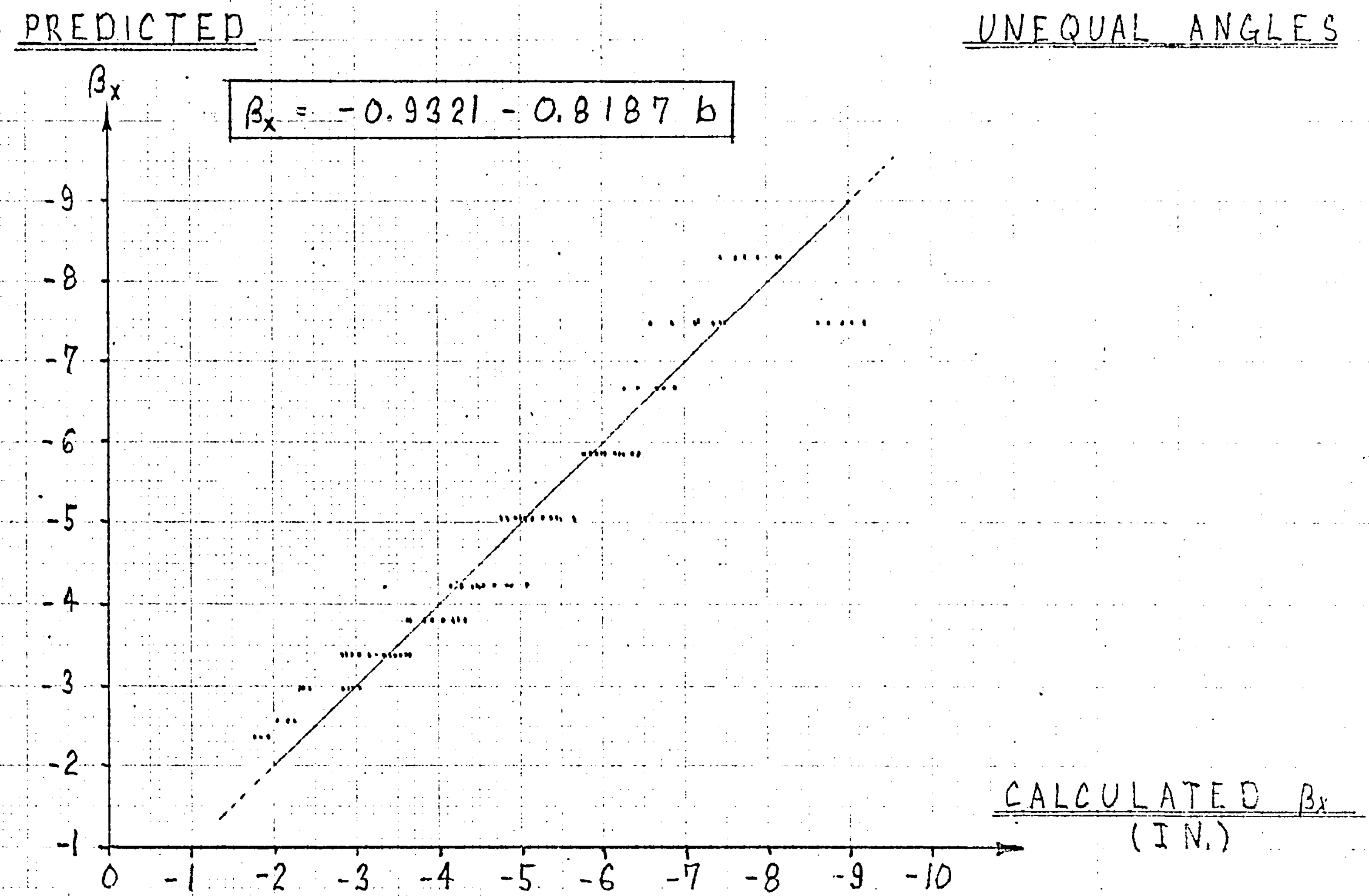


FIGURE C-7 : PREDICTED β_x VS. CALCULATED β_x

UNEQUAL ANGLES

The "adjusted R-squared" terms for these three equations are 0.916, 0.914, and 0.839. It is interesting to notice that the addition of a coefficient for the leg thickness, t, does not improve the accuracy very much. Therefore, Equation (C-8) would certainly be used instead of Equation (C-7), if these calculations are done by hand.

The regression equations were again compared with the exact calculations by a computer program (Figure C-8) and by graphs (Figures C-9, C-10, C-11). It would seem advisable to propose a two-variable approach with the more conservative Equation (C- 8) unless the errors from Equation (C-9) can be tolerated.

In summary, the results of this analysis are that the parameters β_x and β_y can be determined by much shorter calculations that those outlined in Appendix A. The actual approximating equation used will depend on the accuracy requirements of the calculations balanced against the means available for making the calculations.

```
10 B$="←.←←     ←.←←     ←.←←←←     ←←.←←        ←←←.←←"
20 C$="  ←←←.←        ←←.←←     ←←←.←"
30 DIM B(97), A(97), T(97), E(97), F(97)
40 DIM P(97), Q(97), R(97), S(97)
50 N = 97
60 FILES FOR
70 FOR X = 1 TO N
80 READ #1, B(X), A(X), T(X), E(X), F(X)
90 P(X)= .4046 - 1.131*B(X) +.8902*A(X) -.6773*T(X)
100 Q(X) = (F(X) - P(X)) / F(X)
110 Q(X) = 100.0 * Q(X)
120 R(X) = 1.202 - .7732*B(X)
130  S(X) = (F(X) - R(X)) / F(X)
140 S(X) = 100.0 * S(X)
150 NEXT X
160 PRINT "COMPARISON OF PREDICTED WITH CALCULATED BETA-Y"
170 PRINT "**********************************************"
180 PRINT "     "
190 PRINT"                                    ";
200  PRINT"                  %                   %  "
210 PRINT"  B       A       T      BETA-Y      PRED.     DIFF.";
220 PRINT"  PRED.   DIFF.  "
230 PRINT"  "
240 FOR Y = 1 TO N
250 PRINT USING B$: B(Y), A(Y), T(Y), F(Y), P(Y);
260 PRINT USING C$: Q(Y), R(Y), S(Y)
270 NEXT Y
280 END
COMPARISON OF PREDICTED WITH CALCULATED BETA-Y
**********************************************
```

B	A	T	BETA-Y	β(B,A,T) PRED.	% DIFF.
9.00	4.00	1.0000	-6.50	-6.89	-6.0
9.00	4.00	0.8750	-6.52	-6.81	-4.3
9.00	4.00	0.7500	-6.54	-6.72	-2.8
9.00	4.00	0.6250	-6.56	-6.64	-1.1
9.00	4.00	0.5625	-6.58	-6.59	-0.3
9.00	4.00	0.5000	-6.58	-6.55	0.5
8.00	6.00	1.0000	-3.31	-3.98	-20.2
8.00	6.00	0.8750	-3.31	-3.89	-17.8
8.00	6.00	0.7500	-3.31	-3.81	-15.1
8.00	6.00	0.6250	-3.31	-3.73	-12.5
8.00	6.00	0.5625	-3.31	-3.68	-11.3
8.00	6.00	0.5000	-3.31	-3.64	-10.0
8.00	6.00	0.4375	-3.31	-3.60	-8.7

FIGURE C-8: COMPARISON PROGRAM AND SAMPLE OUTPUT

β_y FOR UNEQUAL ANGLES

FIGURE C-9 : β_y VS. b FOR UNEQUAL ANGLES

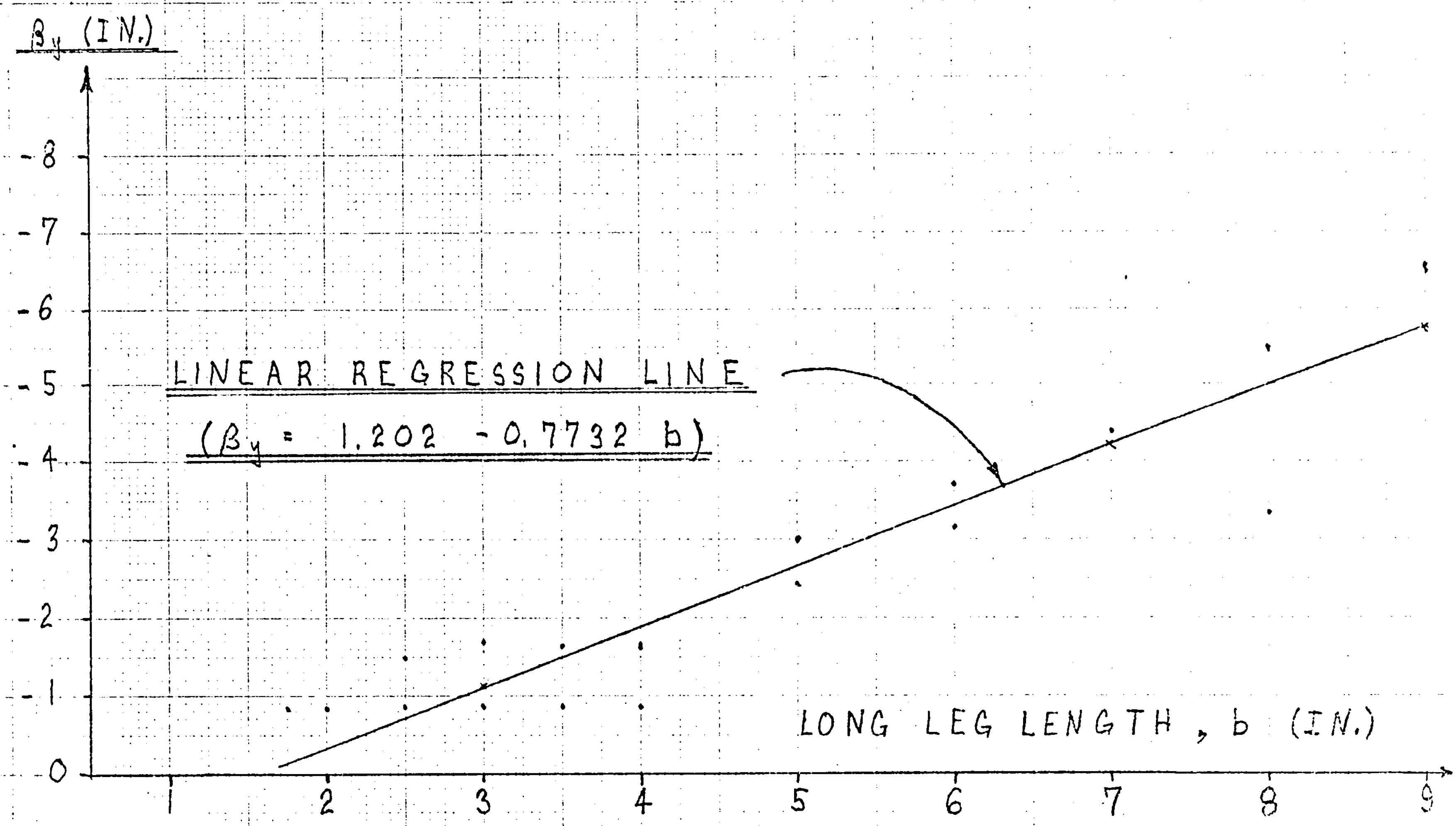


FIGURE C-10: PREDICTED β_y VS. CALCULATED β_y UNEQUAL ANGLES

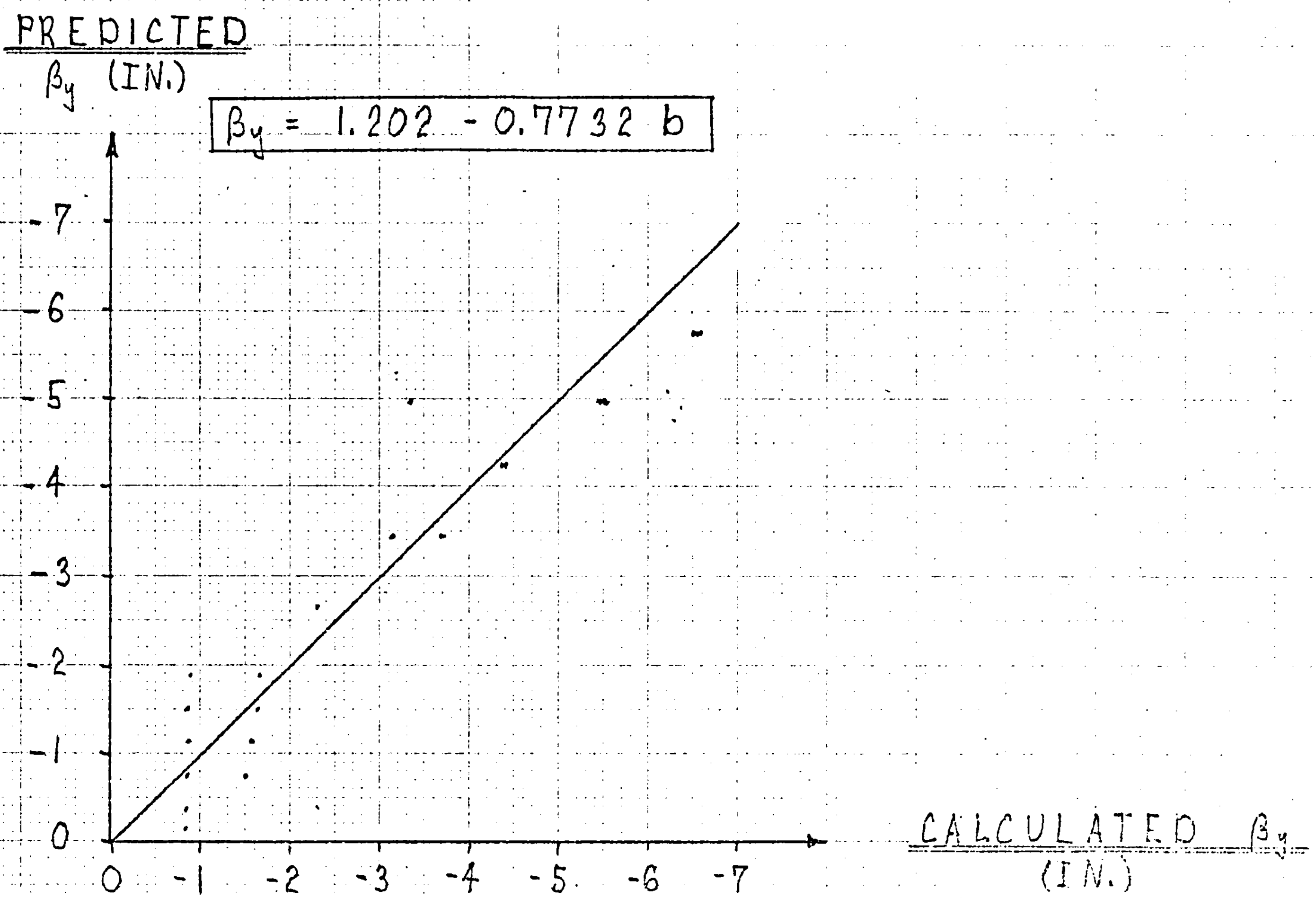


FIGURE C-11: PREDICTED β_y VS. CALCULATED β_y UNEQUAL ANGLES