MAXIMUM SHEAR STRENGTH OF DAPPED-END OR CORBEL

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ABSTRACT

Maximum Shear Strength of Dapped-end or Corbel

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The behaviour and the ultimate shear strength of a dapped-end with horizontal and vertical reinforement have been studied extensively. This study is devoted to a dapped-end with inclined reinforcement.

The analysis of a free body diagram of a dapped-end showed that the ultimate shear strength of a dapped-end with 45^{o} inclined reinforcement should have twice the strength of a dapped-end with horizontal or vertical reinforcement.

Six models of deep beams resembling twin corbels when seen upside down were tested in the first series. Eight full scale models of a dapped-end with different diameters of reinforcement and varying shapes of a dapped-end were tested in the second series. During the test, crack appearance, type of failure and ultimate capacity of samples were recorded.

The test results were compared with calculated values based on the analysis of the free body diagram.

In order to avoid a secondary collapse, it was proposed to limit yield strength of steel to fy = 40 ksi (280 N/mm^2). A design procedure for the dapped-end has been proposed and a numerical example has also been presented.

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NOTATIONS

a	- Depth of equivalent rectangular block
a_s	- Shear span (measured from center of support to interface of dapped-end)
Ahf	- Area of horizontal web reinforcement
As	- Steel area in general
A's	- Area of main dapped-end reinforcement
Avf	- Area of vertical web reinforcement
Awf	- Total area of main reinforcement (projection area of reinforcement perpendicular to crack)
b	- Width of beam
C	- Compression force
d	- Effective depth of full beam
d_D	- Effective depth of dapped-end, deep beam or corbel
εct	- Limit strain in concrete in tension direction at failure under biaxial tension and compression
Es	- Modulus elasticity for steel
f'c	- Uniaxial compression strength of concrete
fs	- Stress in steel
ftc	- Tension stress in concrete under tension and compression biaxial stress state
fto	- Uniaxial pure tension strength of concrete
fy	- Yield strength of steel
h	- Depth of dapped-end beam (full beam)
h_D	- Depth of deep beam, dapped-end or corbel
N	- Horizontal force applied
P	- Applied load
S	- Factor of safety

T	- Total tension force (flexure)
T_H	- Tension force in horizontal direction due to horizontal reinforcement
Tv	- Tension in vertical direction due to vertical reinforcement
Ty	- Tension force corresponding to the yield of the steel
Twf	- Tension force in direction along the inclined bar
THI	- Projection of Twf in horizontal direction
Vicr, vicr	- Shear force and stress at the time of concrete cracking
vsu	- Ultimate shear strength of concrete subjected to pure shear loading
$vu = \frac{Vu}{\phi b d_D}$	- Nominal shear stress with respect to effective depth of dapped-end
$vuh = \frac{Vu}{b h_D}$	- Nominal shear stress with respect to full depth of dapped-end
Vu	- Ultimate shear strength at failure (full shear load)
Vucr	- Ultimate cracking capacity
vucr	- Ultimate shear stress for reinforced concrete at cracking
V test	- Shear force at failure
V'totest	- Total of V_{test} and N
Vucal	- Calculated shear strength by applying reduction factor, ϕ
Vcal	- Calculated shear strength with ϕ=1.0
σc	- Principal compression stress in concrete under biaxial stress state
σt	- Principal tension stress in concrete under biaxial stress state
$\rho = \frac{As}{bh}$	- Reinforcement ratio

$\rho wf = \frac{Awf\sqrt{2}}{bh_D}$	- Ratio density of web reinforcement in 45° inclined direction
ϕ	- Reduction factor (according to A.C.I. 318-77)
α	- Angle inclination of reinforcement with respect to horizontal axis

CHAPTER - 1

INTRODUCTION

1.1 General

Connections are one of the most important element that should be considered in design of precast concrete structures. The connections may be subjected to high stresses as well as to unforseen forces such as temperature changes.

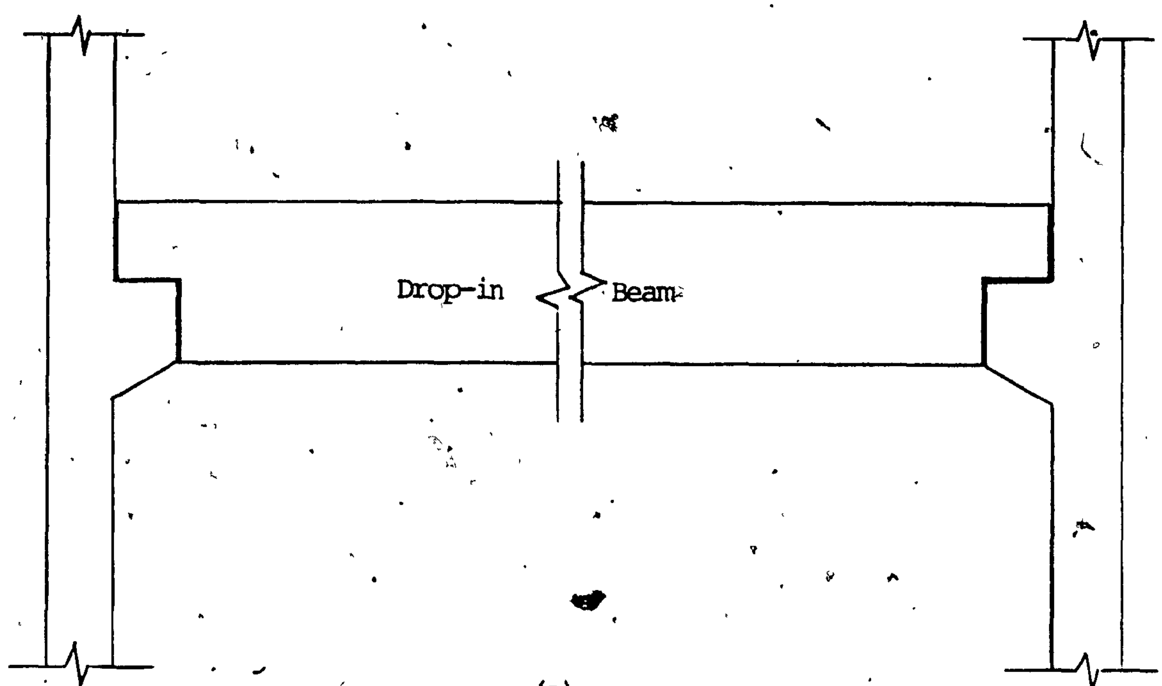
The dapped-end beam provides an efficient and economical connection of precast to precast or precast to cast-in-place concrete components. The connection has several economical advantages such as:

1) The cost of material used for the connection is reduced by eliminating all miscellaneous steel inserts and welding.

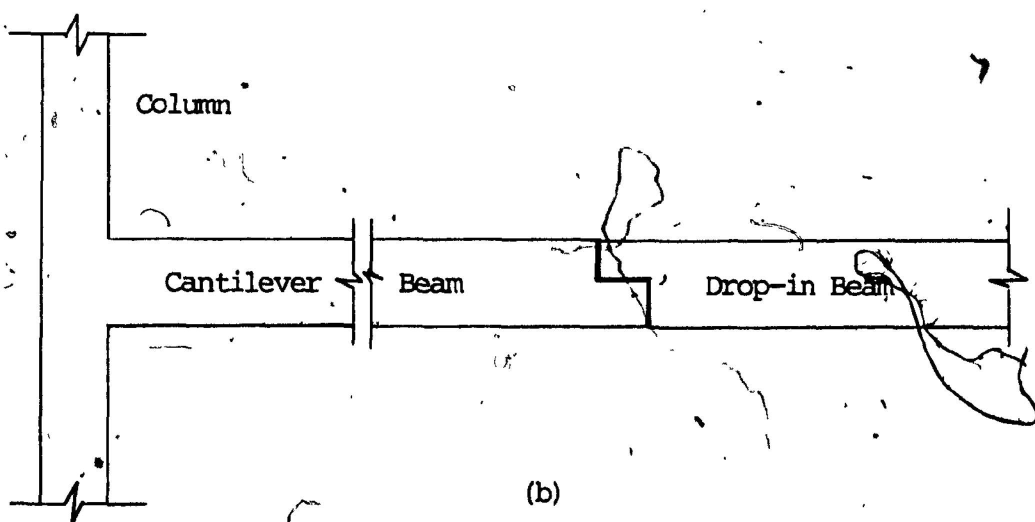
2) The connection is normally located at the point of inflection, thus the moment and corresponding flexural reinforcement are reduced.

3) The cost of construction can be reduced further by providing a simple connection such as drop-in beam.

The dapped-end beam connection may have wide practical applications. Figure 1 illustrates examples of possible applications, such as a drop-in beam between two corbels, a drop-in beam between two cantilever beams, a connection between joist to beam and a connection between beam to column.

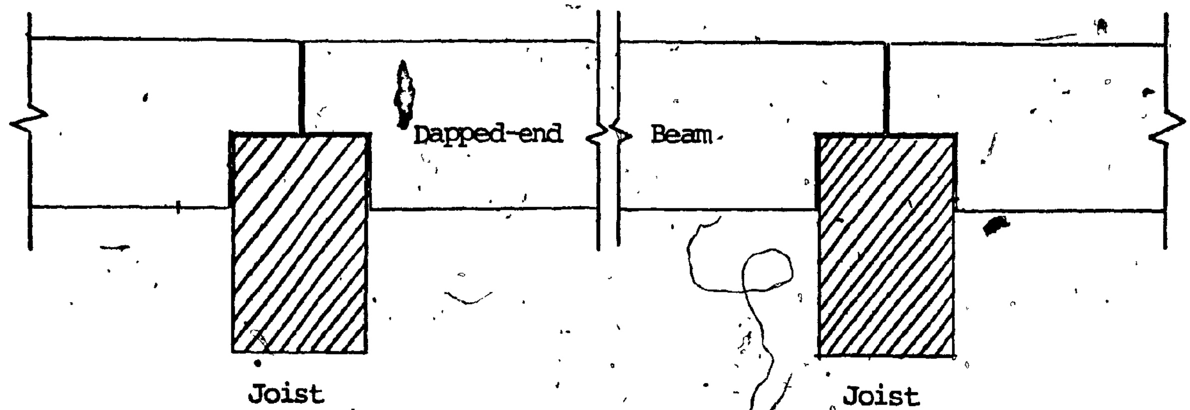


(a)



(b)

Fig.1.1 Some of the applications of dapped-end



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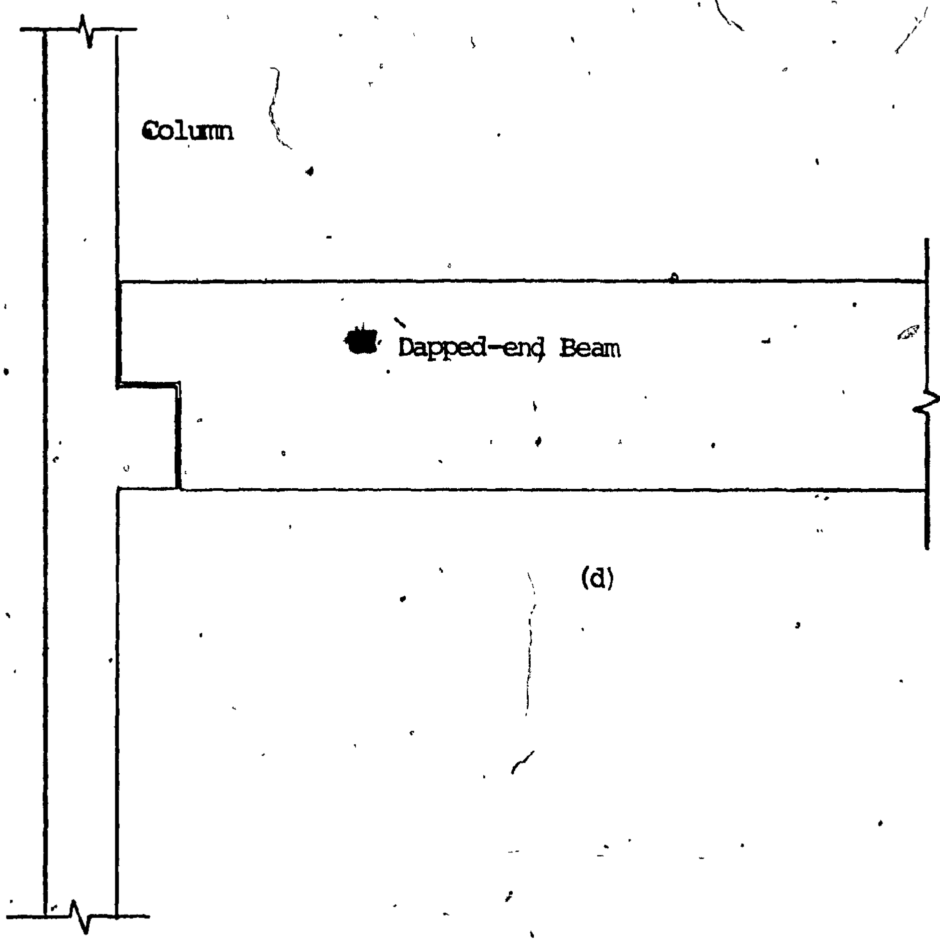

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Fig. 1.1 Some of the applications of dapped-end

1.2. Previous Research Work

In 1970, Sargious and Tadros [1] presented finite element analysis of "Stresses in Prestressed Concrete Stepped Cantilevers Under Concentrated Loads". However their data were not compared with experimental results.

In 1973, Werner and Dilger [2] presented a paper entitled "Shear Design of Prestressed Concrete Stepped Beams", which compared finite element analysis with an experimental program. The results of the experimental program agreed closely with the finite element analysis. The finite element method was used to find the maximum stresses at the re-entrant corner. It was found that the first crack appeared when the tensile stress in concrete reached the approximate value of $6\sqrt{f'c}$. The ultimate shear strength was calculated with the following formula

$$Vu = Vc + Vp + Vs \qquad \text{(for horizontal reinforcement)}$$

$$Vu = Vc + Vp + Vs \sin \alpha \qquad \text{(for inclined reinforcement)}$$

where

Vu = Ultimate shear strength

Vc = Shear force at cracking

Vp = Shear force due to prestressing (vertical component of tendon)

Vs = Shear force in web reinforcement

α = Angle inclination of the reinforcement with respect to horizontal axis

In 1975, Hamoudi, Phang and Bierweiler [3], found that the shear strength of a prestressed concrete beam with dapped-end can be predicted accurately based on elastic analysis. Moreover, prestressing the dapped-end zones with inclined reinforcement provides a satisfactory method for web reinforcement and furthermore prevents shear cracking of a dapped-ends at working loads.

In 1979, Mattock and Chan[4] published results of eight dapped-end tests on four dapped-end beam models. Those beams had horizontal and vertical reinforcement. Four models were subjected to vertical load only and four were subjected to a combination of vertical load and horizontal force corresponding to volume change in structure. The analysis of their design was based on modified shear friction theory recommended by P.C.I.[5] which assumes the shear stresses of corbel or bracket, vu should be less than 0.2 f'c. Based on their research work, Mattock and Chan showed that it is possible to have vu increased to 0.3 f'c, provided the ratio of $^{a}S/d_{D}$ is not more than 1.0. They proposed that the shear span "a_s" should be measured from the line of action of Vu to the center of gravity of the bundled stirrup group.

1.3 Objective of Proposed Research

The purpose of this research work is to study the maximum shear strength of a dapped-end or corbel with inclined reinforcement and compare them to the behaviour of dapped-end with horizontal and vertical reinforcement studied by Mattock's Group[4].

Based on the analysis of the free body diagram, it can be shown that a dapped-end with inclined reinforcement has twice the strength of a dapped-end with horizontal and vertical reinforcement. This research was undertaken in order to prove the validity of this observation. Deep beams resembling twin corbels when seen up-side down and full scale models of dapped-end beams were tested in the first and second series respectively.

CHAPTER - 2

EXPERIMENTAL PROGRAM

2.1 Description of Test Models

2.1.1 First Stage Tests

In the first stage, it was decided to test samples in the form of a deep beam resembling rectangular twin corbels when seen in the up-side down position. Each sample has a dimension of 1.75" x 12" in cross section and 38" in length, as shown in figure 2.1.

Six test samples were made in two series, with series A having a low strength of concrete of f'c = 2000 psi (14 N/mm^2) and series B using a normal strength of concrete of f'c =4000 psi (28 N/mm^2).

All reinforcing bars were of #4 (10M) and fy = 60ksi (420 N/mm). Samples 1, 2, 4 and 5 had 6 #4 (6 x 10M), equivalent to reinforcement ratio ρ = 0.057 and samples 3 and 6 had 4 #4 (4 x 10M) bars and ρ = 0.038. The above ratios of reinforcement were calculated corresponding to full section, i.e. $\rho = {}^{As}/_{bh}$. It should be noted that there were no stirrups provided but only inclined and horizontal bars over the shear transfer area.

The amount of the reinforcement was the same for samples of series A and B, and it was chosen in such manner that series A would fail earlier than sample of series B.

The details of the reinforcement and the concrete strength of sample series A and B are shown in figure 2.1 and Table 2.1.

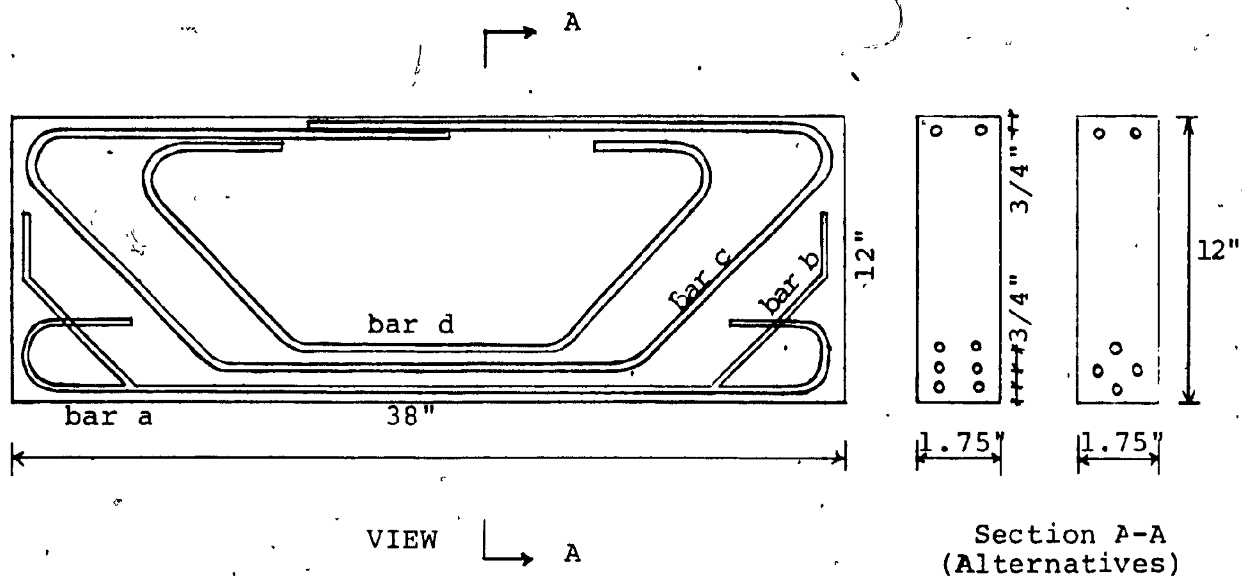


Fig. 2.1. Location of the reinforcement and detail dimensions of the sample.

Table 2.1 Design properties of test samples

SAMPLE		Strength and Type of concrete, psi	Reinforcement					
			No. of Bars				As	ρ
Mark	Series		a	b	c	d	(in^2)	
1	A	Cement mortar of f'c = 2000psi (14mpa)	1	1	2	2	1.2	0.057
2			1	1	2	2	1.2	0.057
3			1	-	2	1	0.8	0.038
4	B	Concrete of f'c = 4000psi (28mpa)	1	1	2	2	1.2	0.057
5			1	1	2	2	1.2	0.057
6			1	-	2	1	0.8	0.038

Note: All bars are of #4 (10M) and fy = 60 ksi ($28N/mm^2$).

2.1.2 Second Stage

In second stage, four rectangular beams having both end dapped were cast. Each beam had a dimension of 5" x 24" in cross section and approximately 10' in length, as shown in figure 2.2.

Four beams were produced by assuming the same strength of concrète, but with different diameters of bars and different shapes of dapped-ends. Only 45^{O} angle inclined bars were used as reinforcement over the dapped-ends.

The samples were chosen to have the same dimensions as those tested by Mattock's[(4)], except that horizontal reinforcement was replaced by inclined bars.

Beams B_1 and B_4 had the same amount of reinforcement made of 20mm bars, fy= 60 ksi (420 mpa) and design concrete strength of f'c = 4000 psi (28 mpa). It should be noted that the location of the reinforcement of B_1 is different from B_4.

Beams B_2 and B_3 had the same number of bars but different diameter of 10mm and 15mm respectively. The same strength of concrete of f'c = 4000 psi (28 mpa) and the yield strength of steel, fy=60 ksi (420 mpa) were assumed for both samples.

The details of the dapped-end beams showing different diameters of reinforcement and different shapes of dapped-ends are presented in figure 2.2. Moreover, the details of those dapped-end beams are shown in table 2.2.

2.2 Testing Arrangement

All samples of first stage were tested as simple supported beams under one load in the center. The loading scheme is shown in figure 2.3. Rollers were used on the supports and under the point load to eliminate the horizontal restraint. The samples were supported on rigid steel plates and intermediate bearing pads made of layers of packing box cardboard, except for sample 1 which was supported and loaded through ½" neoprene pads. An overall view of the testing set up shown in figure 2.4.

In the second stage, all dapped-end beams were tested as simple supprted beams. In this case the load was applied at a distance of 22½" from the end of the dapped-end as in Mattock's test. The loading scheme for the test is shown in figure 2.5. A roller support is provided on one side and pin connection support under the dapped-end. A 6" x 8" x ¼" thick hardwood pad was placed between the concrete and steel plates. Moreover, mortar was used on top of the concrete at the loading location to have an even distribution of load. An overall view of the testing is shown in figure 2.6 (a) and (b).

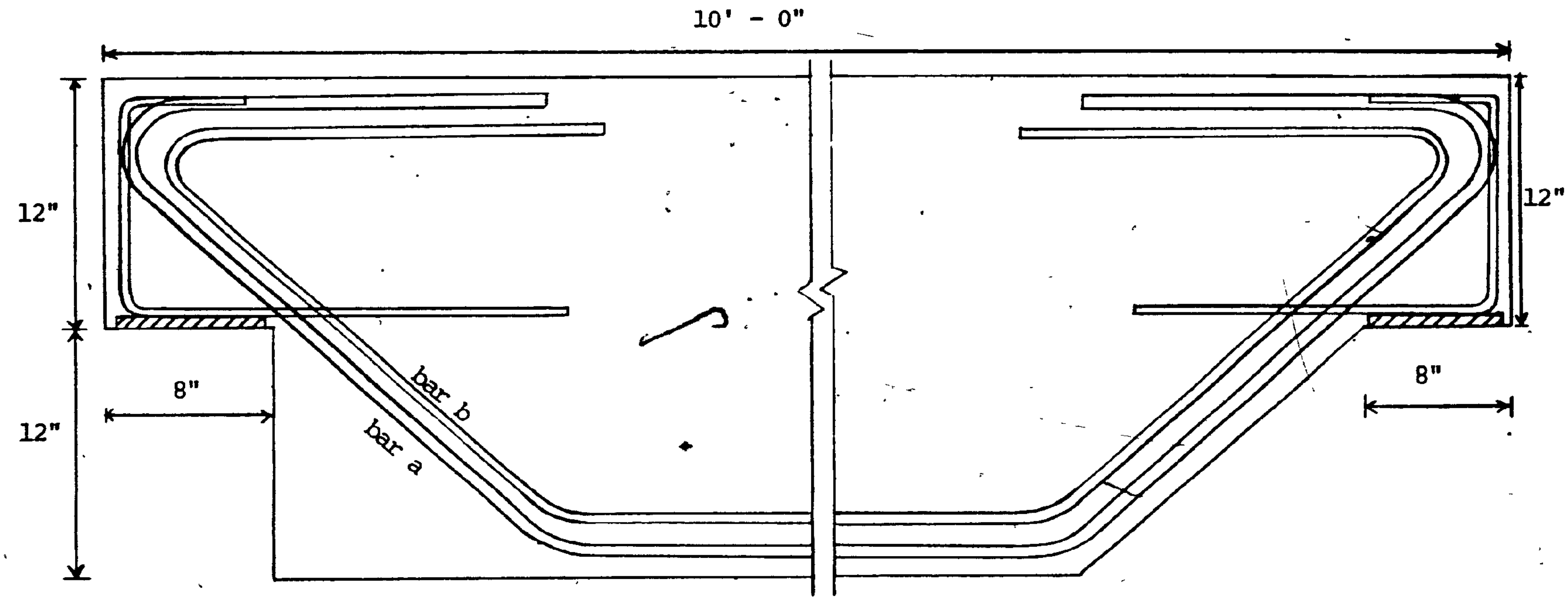


Fig. 2.2 Detail of dapped-end beam having different diameter of reinforcement and different shape of dapped-end.

TABLE 2.2 Detail properties of dapped-end beam

BEAM No.	f'c (psi)	fy (ksi)	Reinforcement				As (in^2)	ρwf
			Diameter of Bar		Number of Bar			
			a (mm)	b (mm)	a	b		
1	4000	60	20	20	2	2	1.95	0.046
2	4000	60	15	10	2	2	0.79	0.018
3	4000	60	15	10	2	2	0.79	0.018
4	4000	60	20	20	2	2	1.95	0.046

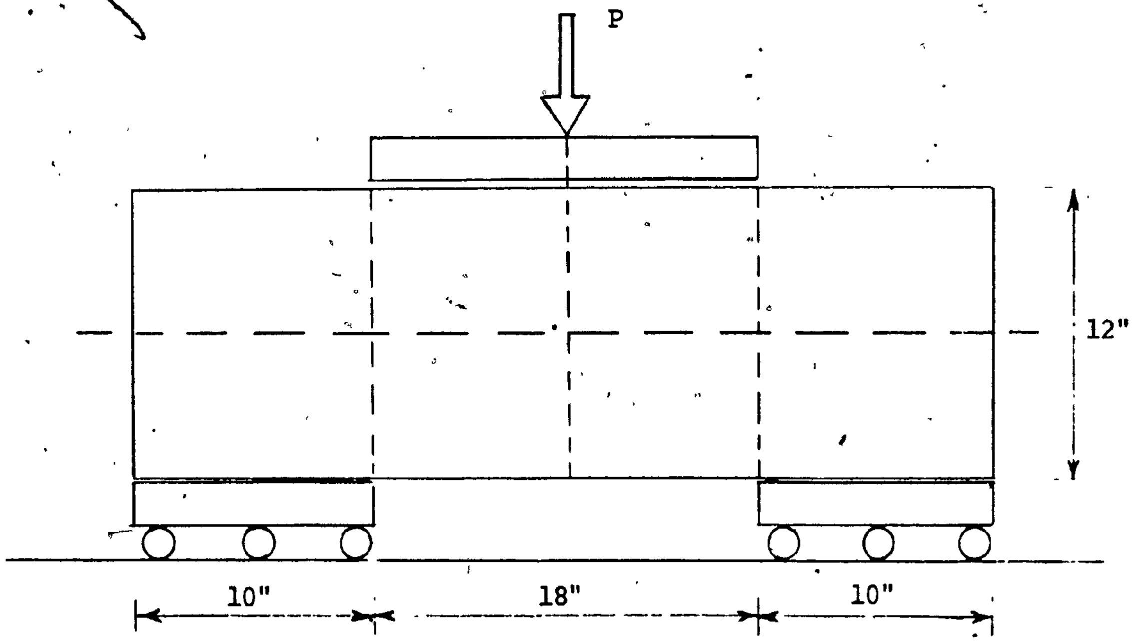


Fig. 2.3 Loading scheme of twin corbels when seen up-side down.

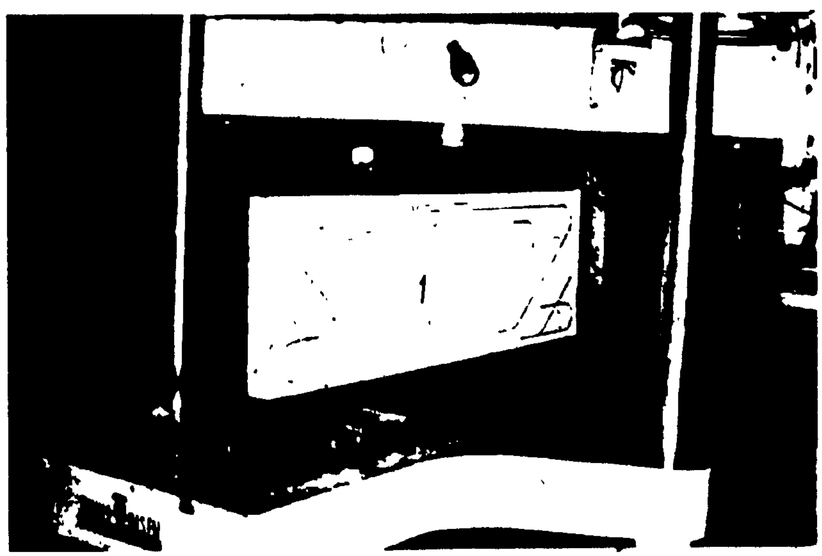


Fig. 2.4 An overall view of the testing.

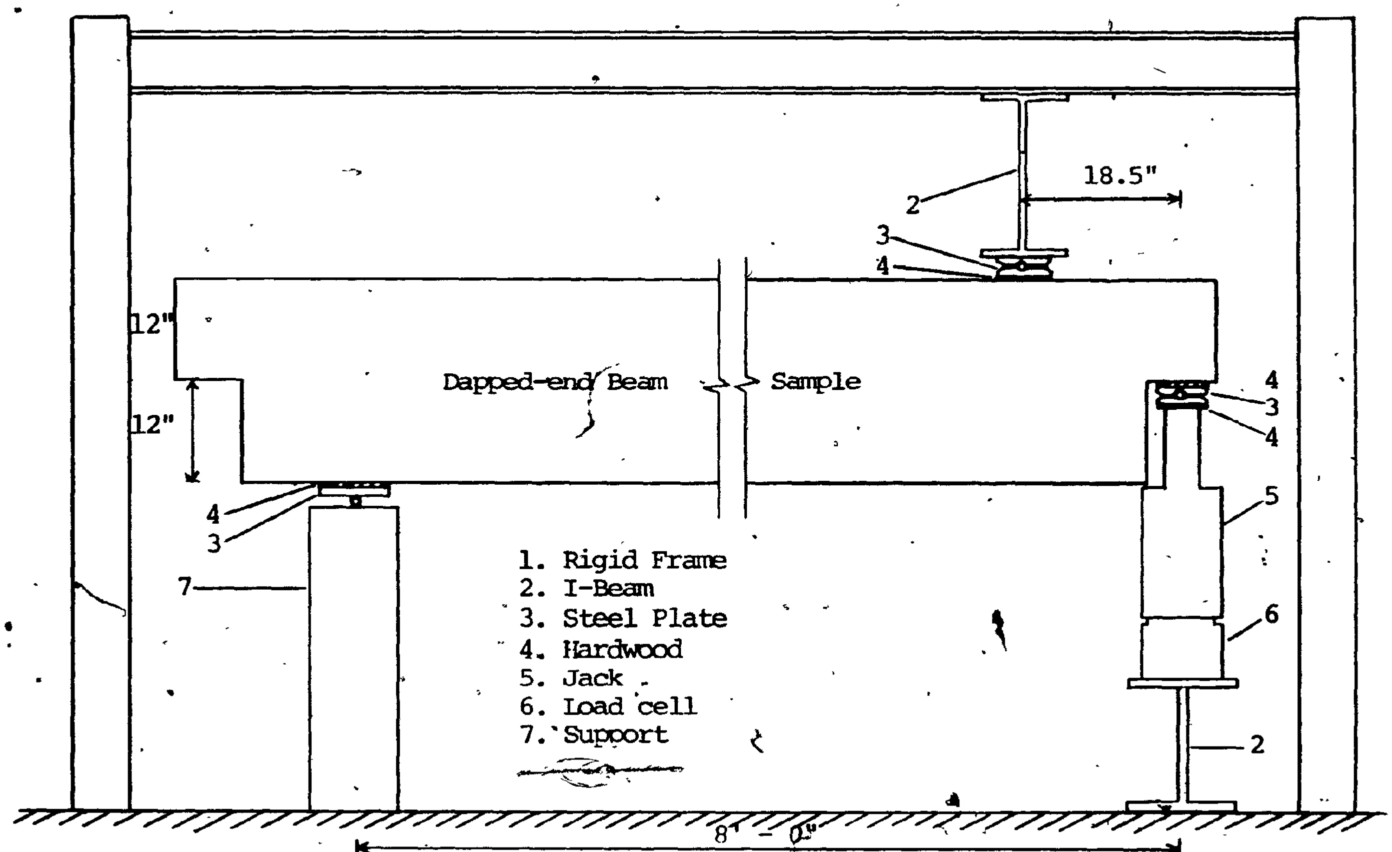
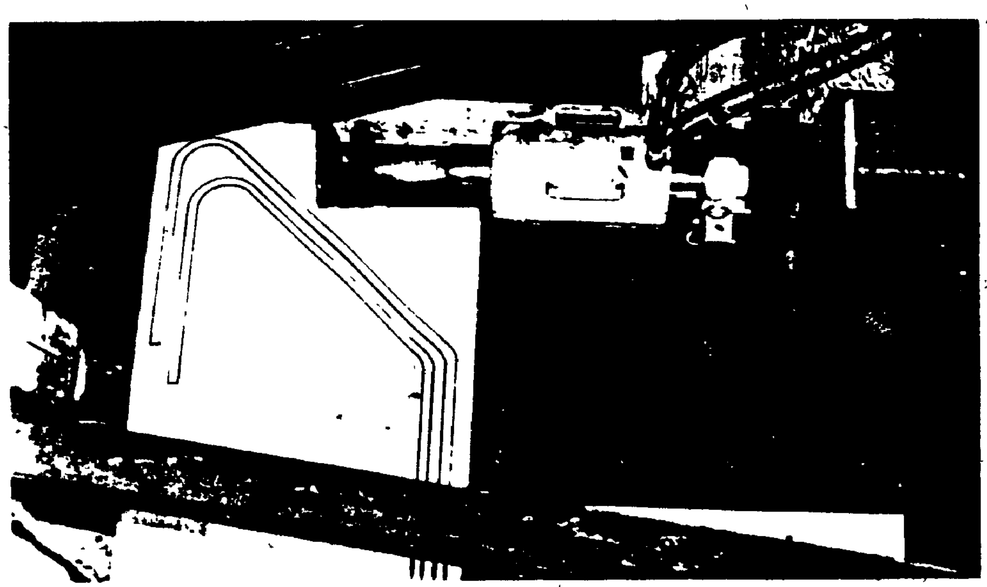


Fig. 2.5 Scheme of the loading on dapped-end

Note: The same beam was tested twice. First right dapped-end was tested as shown in figure, then beam was turned around and left dapped-end was tested in the same frame.

(a)



(b)

2.3 Testing procedure

In order to facilitate the observations, all samples had their main reinforcement drawn on both side faces. In addition, grid lines were drawn to facilitate the location of cracks during testing.

In the first stage of the experimental program, the twin corbels were loaded at the center. The loads were initially increased by increments of 5 kip.

After each load increment, the crack patterns were marked. This procedure was repeated until the corbel reached its ultimate capacity.

The dapped-end sample tested in the second stage of the experimental program were loaded initially by 5 kip increments and later by 10 kip increment.

Specimen B_1R was loaded in increments of 5 kip until the load reached 100 kip and then 10 kip increments were applied. A different procedure was used for B_4R, B_4L, B_1L and B_3L which consist of 5 kip load increments till 30 kip was reached then 10 kip increments were applied. Finally, specimen B_3R, B_2R and B_2L were loaded to failure with increments of 5 kip.

After each load increment, the crack patterns were marked, and the readings of all strain gauges were recorded. This procedure was repeated until the ultimate capacity of the dapped-end was reached.

At failure, the failure load was marked and photographs were taken.

It should be noted that each dapped-end beam was tested twice. Once the test of the right end was done, the beam was turned around and then test of the left end was completed.

2.4 Loading Measurements

First stage tests were done on a Tinus Olsen testing machine. The loading measurements were read directly from machine gauge.

In the case of the dapped-end, the load was applied by means of a hydraulic jack through a calibrated load cell with a strain indicator. From the strain indicator the loads were recorded and the equivalent of loads were obtained from a calibrating table.

Electric strain guages of 350Ω were used on the inclined reinforcement as shown in figure 2.7. After attaching the strain gauges, the wire was soldered and then protected by means of M-bond. The wires were wrapped with tape and brought up to the surface of the beams.

Electric strain gauges of 120Ω were placed on the concrete surface as shown also in figure 2.7.

Corresponding to the load applied, the readings of all strain gauges were recorded. Recorded measurements were plotted as shown in Appendix C.

2.5 Materials

2.5.1 Reinforcement

All the reinforcement bars were tested using a Tinus Olsen machine. The results of the tensile tests are shown in figure 2.8.

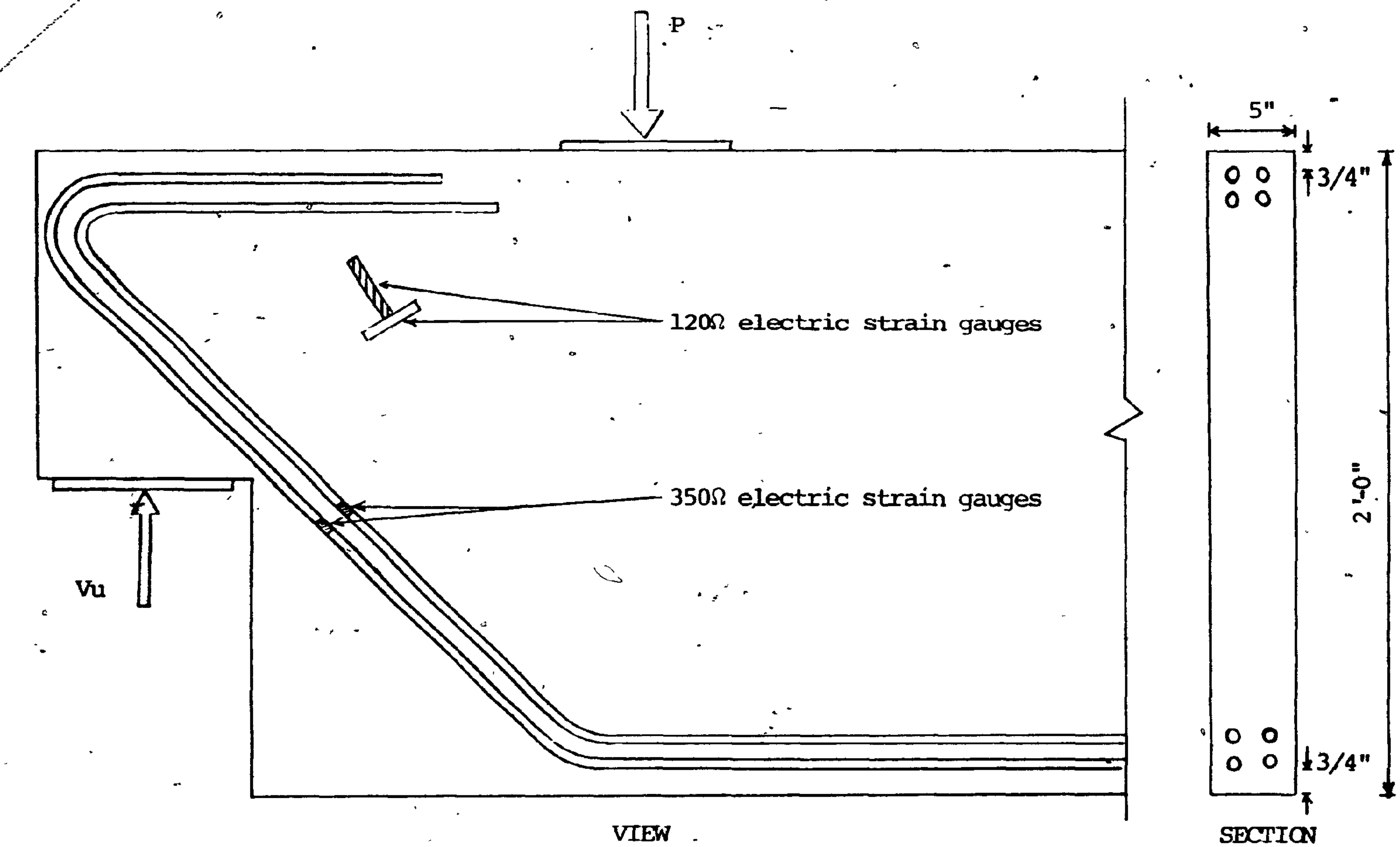


Fig. 2.7 Location of strain gauges

2.5.2. Concrete

Test cylinders were cast together with beams to verify the strength of concrete which was tested at the same time as beams. Six cylinders 6 x 12 in were tested for each beam under compression force. The test results are shown in table 2.3.

Three split-cylinder tests of 6 x 12 in of each beam were carried out in order to check the normal tension strength of the concrete.

Based on splitting tests shown in table 2.4, the normal tension strength was established which was approximately $f't = 0.0734\ f'c$. The normal tension strength of concrete [6] under biaxial stress state, ftc is a linear function as shown in figure 2.9. Based on this figure it can be calculated that the pure tensile strength of concrete, $fto = 1.28\ f't$. The test results were transferred to pure tensile strength as shown in table 2.4.

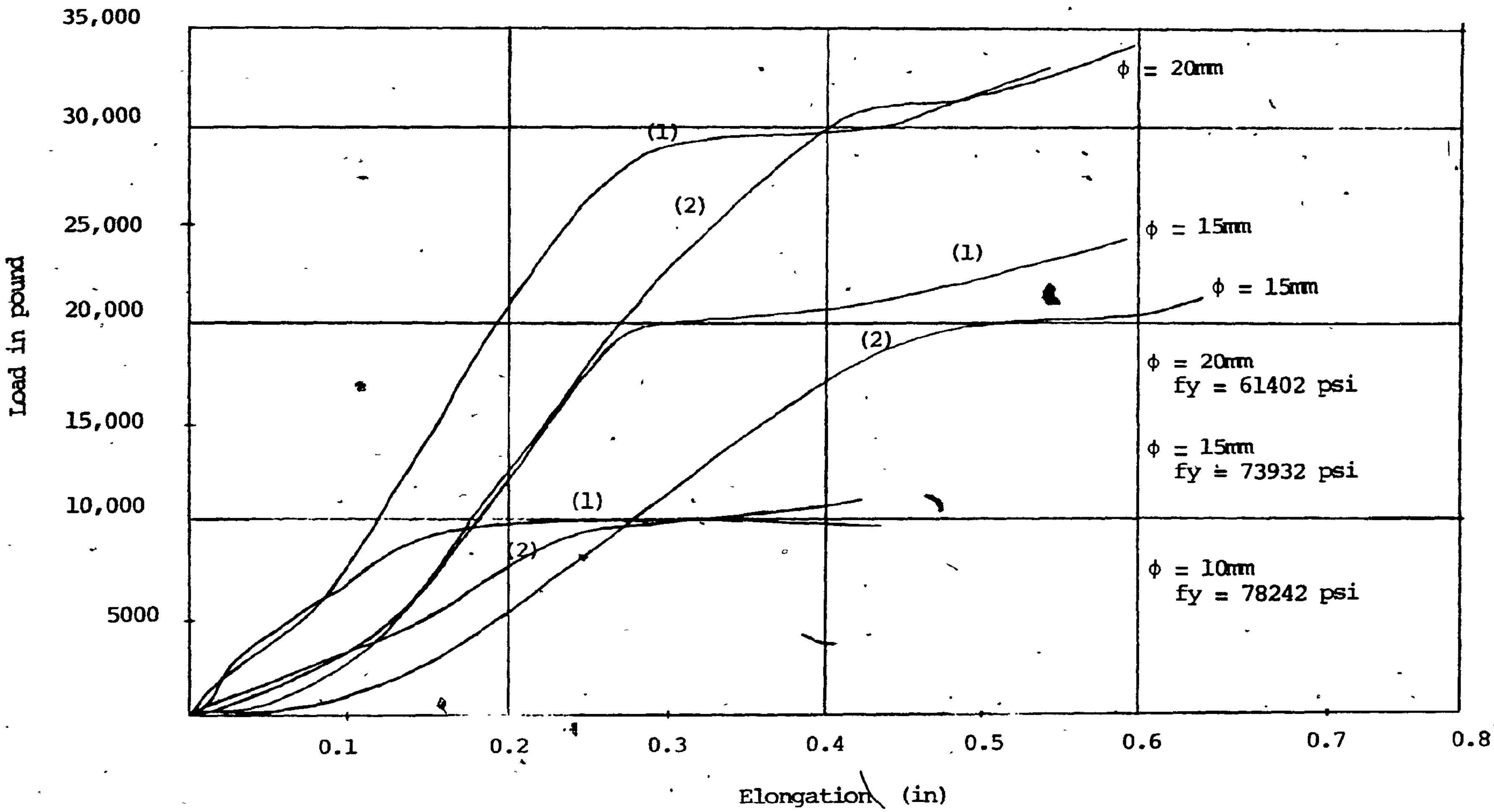


Fig. 2.8 Load-Elongation of the reinforcement

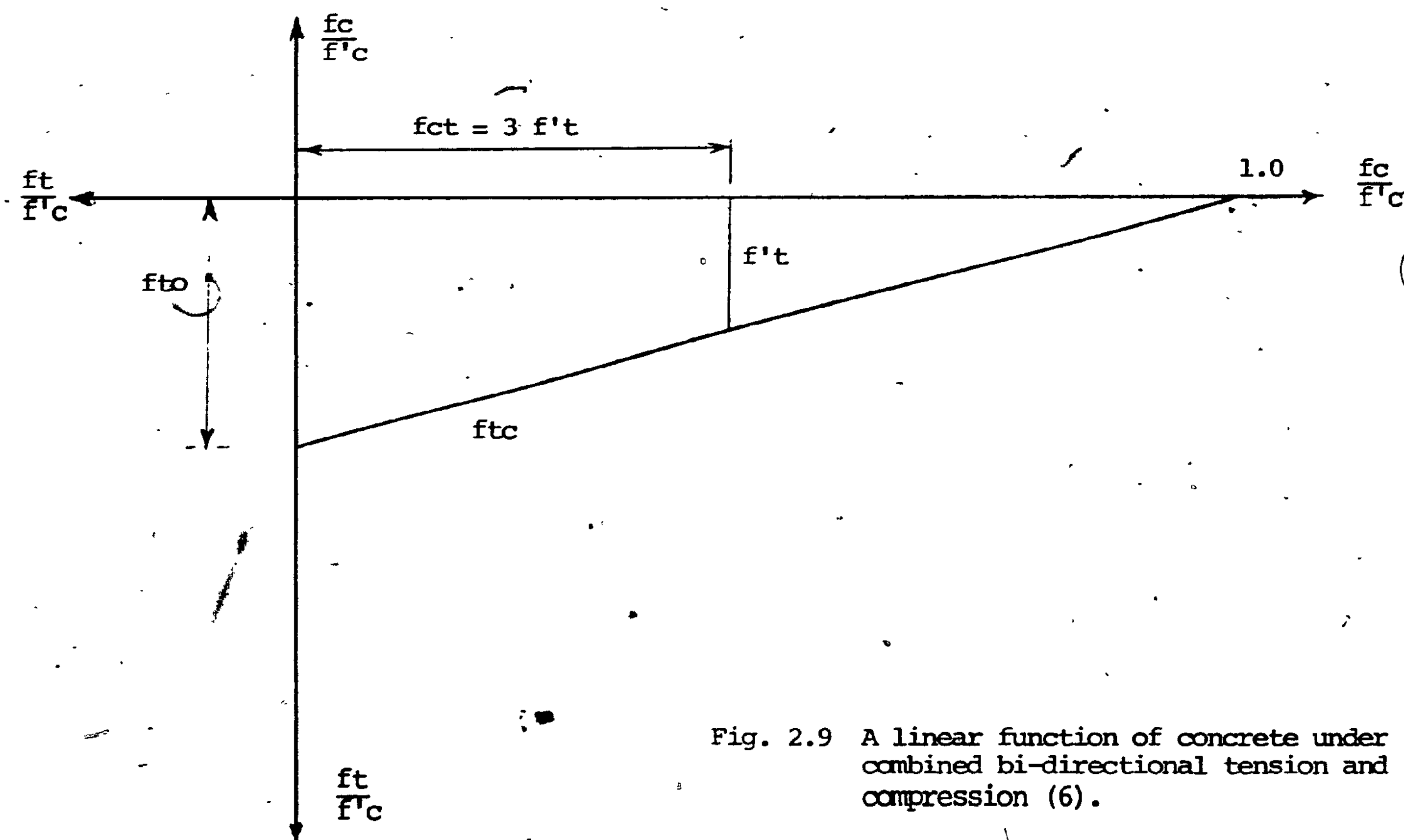


Fig. 2.9 A linear function of concrete under combined bi-directional tension and compression (6).

Table 2.3 Compressive cylinder test results (second stage)

BEAM NO.	CYLINDER NO.	LOAD (lb)	STRESS (psi)	AVERAGE STRESS (psi)
1	1	97,200	3437	4928
	2	145,000	5128	
	3	150,000	5305	
	4	151,000	5340	
	5	143,000	5057	
	6	150,000	5305	
2	1	150,000	5306	4265
	2	142,000	5023	
	3	147,000	5200	
	4	65,000	2299	
	5	139,000	4917	
	6	80,500	2848	
3	1	104,000	3678	4141
	2	105,000	3714	
	3	129,000	4563	
	4	125,000	4421	
	5	120,000	4244	
	6	119,400	4224	
4	1	126,000	4456	3991
	2	91,000	3218	
	3	110,000	3890	
	4	114,000	4032	
	5	128,000	4527	
	6	108,000	3820	

BEAM NO.	CYLINDER NO.	LOAD AT SPLITTING Pu(16)	STRESS AT SPLITTING f't (psi)	fto=1.28f't (psi)	AVERAGE fto, psi
1	1	37,600	332.5	425.6	448
	2	49,300	435.9	557.9	
	3	31,800	281.2	360	
2	1	40,000	353.7	452.7	432
	2	38,000	335.9	430	
	3	36,500	322.7	413	
3	1	36,000	318.3	407.4	377.6
	2	36,700	324.5	415.4	
	3	27,400	242.3	310	
4	1	28,900	355.5	327	370
	2	27,300	241.4	309	
	3	41,900	370.5	474	

Note: $f't = \frac{2Pu}{\pi dL}$

CHAPTER - 3

TEST RESULTS AND OBSERVATIONS

3.1 Test Results

The test results of twin corbels and dapped-end beams are shown in table 3.1 and 3.2 respectively.

Tables also show the actual strength of materials, calculated and recorded shear strength limits for each beam.

Comparison of results for a dapped-end with inclined reinforcement obtained in our tests with results obtained by Mattock's Group[4] is shown in table 3.3. Calculated strength for Mattock's dapped-end beams based on free body analysis are shown in table 3.4.

3.2 Observations

3.2.1 Behaviour of Deep Beam

Beam 1 of series A was loaded as a simply supported beam. The sample was supported on and loaded through ½" neoprene pads. Because of the lateral shape change and expansion of neoprene, concrete spalling occurred over the support. Therefore, this sample was rejected and its result excluded from the comparison. The sample failed at 36.5^k and first spalling of concrete occurred at 20k at the loading point, and then 28^k near the support.

Beam 2 of series A was loaded through rigid steel and intermediate bearing pads made of layer of packing cardboard. The first inclined crack was observed at 32^k, and this crack was extended as the load reached 65^k. Another crack occurred at the right support and fail at 77^k. It should be noted that the failure of the sample was caused by splitting of concrete.

Table 3.1 Test Results of Twin Corbels

Sample	f'c (psi) (mpa)	fy (ksi) (mpa)	Bar No.	As (in^2) (mm^2)	$\rho = \frac{As}{bh}$	Vucal (DC), kips (kN)	Vucal (DS), kips (kN)	VuTest (kips) (kn)	$vu = \frac{Vu}{\phi bd}$ (psi) (kpa)	vu/f'c	Vfcr (kips) (kN)	Vicr (kips) (kN)	$vicr = \frac{Vicr}{\phi bd}$ (psi) (kpa)	vicr/f'c	Observations Mode of Failure
Series A 1	2150 (15)	60 (420)	6#4	1.2 (774)	0.057	26.86 (119.25)	77.9 (346)	18.2 (80.8)	1165 (8026)	0.54	-	-	-	-	Bearing surface failed Test disregarded
2	2150 (15)	60 (420)	6#4	1.2 (774)	0.057	26.86 (119.25)	77.9 (346)	38.5 (171)	2464 (16,976)	1.14	-	32.5 (144)	2080 (14,331)	0.96	Splitting of concrete over support
3	2150 (15)	60 (420)	4#4	0.8 (516)	0.038	26.86 (119.25)	53.3 (237)	34 (151)	2176 (14,992)	1.01	38 (168.7)	29 (128.8)	1857 (12,794)	0.86	- do -
Series B 4	4846 (34)	60 (420)	6#4	1.2 (774)	0.057	60.55 (268.8)	77.9 (346)	45 (199.8)	2881 (19,850)	0.59	80 (355)	25 (111)	1600 (11,024)	0.33	Inplane splitting
5	4846 (34)	60 (420)	6#4	1.2 (774)	0.057	60.55 (268.8)	77.9 (346)	45 (199.8)	2881 (19,850)	0.59	70 (310)	25 (111)	1600 (11,024)	0.33	Splitting around inclined bar
6	4846 (34)	60 (420)	4#4	0.8 (516)	0.038	60.55 (268.8)	53.3 (237)	45 (199.8)	2881 (19,850)	0.59	35 (165)	12.5 (55.5)	1600 (11,024)	0.165	Inplane splitting

Note: P = 2 x Vu

Table 3.2 Test results of dapped-end

Beam No.	f'c (psi) (mpa)	fy (ksi) (mpa)	Bar No.	As (in^2) (mm^2)	$\rho = \frac{As}{Bh}$	Vucal (fl.) kips (kN)	Vucal (DC) kips (kN)	Vucal (DS) kips (kN)	Vu test kips (kN)	$vu = \frac{Vu}{\phi bd}$ psi kpa	vu/f'c	Vfcr (kips) (kN)	Vicr kips (kN)	$vicr = \frac{Vicr}{\phi bd}$ psi Pa	vicr/f'c	$X \sqrt{f'c}$	Observation Mode of Failure
B1R	4928 (34.4)	61.4 (429.8)	4x20M	1.95 (1257)	0.0163	109.4 (485.7)	175.4 (778.7)	155 (688)	110 (488)	2671 (18,403)	0.54	25 (111)	15 (66.6)	364 (2508)	0.074	5.2	Flexural failure
B1L	4928 (34.4)	61.4 (429.8)	4x20M	1.95 (1257)	0.0163	109.4 (485.7)	175.4 (778.7)	155 (688)	110 (488)	2671 (18,403)	0.54	40 (178)	20 (89)	486 (3348)	0.098	6.9	Diagonal splitting as secondary failure phenomena
B2R	4265 (29.8)	74.3 (520.1)	2x15M 2x10M	0.80 (516)	0.0067	58.6 (260.2)	152.3 (676.2)	86.65 (385)	68 (302)	1610 (11,092)	0.38	20 (89)	10 (44)	237 (1633)	0.055	3.6	Flexural failure
B2L	4265 (29.8)	74.3 (520.1)	2x15M 2x10M	0.80 (516)	0.0067	58.6 (260.2)	152.3 (676.2)	86.65 (385)	72.5 (322)	1716 (11,823)	0.40	20 (89)	15 (66.6)	355 (2446)	0.083	5.4	- do -
B3R	4141 (28.9)	74.3 (520.1)	2x15M 2x10M	0.80 (516)	0.0067	58.6 (260.2)	147.8 (656.2)	86.65 (385)	65 (289)	1539 (10,604)	0.37	25 (111)	15 (66.6)	355 (2446)	0.086	5.4	Diagonal splitting of the beam
B3L	4141 (28.9)	74.3 (520.1)	2x15M 2x10M	0.80 (516)	0.0067	58.6 (260.2)	147.8 (656.2)	86.65 (385)	70 (311)	1657 (11,416)	0.40	15 (66.6)	20 (89)	473 (3259)	0.114	7.3	- do -
B4R	3991 (27.8)	61.4 (429.8)	4x20M	1.95 (1257)	0.0163	105.3 (467.5)	142.4 (632.2)	155 (688)	90 (400)	2267 (15,619)	0.57	15 (66.6)	20 (89)	504 (3473)	0.126	7.9	Diagonal splitting as secondary failure phenomena
B4L	3991 (27.8)	61.4 (429.8)	4x20M	1.95 (1257)	0.0163	105.3 (467.5)	142.4 (632.2)	155 (688)	89 (395)	2267 (15,619)	0.57	20 (89)	25 (111)	630 (4340)	0.158	9.9	- do -

Average: 6.45

Table 3.3 Comparison Results of horizontal and inclined reinforcement

Mattock's Dapped-end Test Results (4)										Inclined dapped-end test results										
Specimen No.	Bar No.	A's (in^2)	f'c (psi)	fy (ksi)	N (kips)	Vtest (kips)	Vtotest (kips)	vu (psi)	vu/f'c	Specimen No.	Bar No.	Awf (in^2)	f'c (psi)	fy (ksi)	Vucal(DS) (kips)	Vucal(DC) (kips)	Vucal(f) (kips)	Vtest (kips)	$vu = \frac{Vu}{\phi bd}$ (psi)	vu/f'c
1B	2#6	0.88	4425	59.8	30	42.93	72.93	913	0.2	B2L	2x15M 2x10M	0.79	4265	74.3	86.65	152.3	58.6	72.5	1716	0.4
2B	2#6	0.88	4475	59.8	25	38.10	63.10	810	0.18	B2R	2x15M 2x10M	0.79	4265	74.3	86.65	152.3	58.6	68	1610	0.38
3B	2#6	0.88	4590	63.6	28	39.70	67.70	845	0.18	B3L	2x15M 2x10M	0.79	4141	74.3	86.65	147.8	58.6	70	1657	0.4
4B	2#6	0.88	4260	63.6	28	39.78	67.28	846	0.2	B3R	2x15M 2x10M	0.79	4141	74.3	86.65	147.8	58.6	65	1539	0.37

Note : $vu = \frac{vutest}{\phi bd_D}$

Table 3.4. Calculated strength for Mattock's tests according to free body Analysis.

Specimen No.	Bar No.	A's (in^2)	Ah (in^2)	Atot (in^2)	f'c (psi)	fy (ksi)	N (kip)	Vcal (kip)	Vtot (kip)	Vcal (DS) (kip)	Vcal (DC) (kip)
1A	2#3 1#2	0.22	0.10	0.32	4875	69.1	-	36.12	36.12	22.1	102.3
2A	3#3 2#2	0.33	0.20	0.53	4785	69.4	-	36.98	36.98	36.78	100.5
3A	3#3 2#2	0.33	0.20	0.53	5370	69.1	-	37.04	37.04	36.62	112.7
4A	3#3 2#2	0.33	0.20	0.53	4590	69.1	-	36.74	36.74	36.62	96.4
1B	2#6 2#2	0.88	0.20	1.08	4425	59.8	30	44.13	74.13	64.8	92.9
2B	2#6 2#2	0.88	0.20	1.08	4475	59.9	25	38.84	63.84	64.8	93.9
3B	2#6 2#2	0.88	0.20	1.08	4590	63.6	28	38.91	66.9	68.6	96.4
4B	2#6 2#2	0.88	0.20	1.08	4260	63.6	28	38.67	66.7	68.6	89.4

Beam 3 of series A had the same support conditions as beam 2. The first flexural crack was observed at 38k and it was extended when the load reached 60k. The first inclined cracking was observed at 58k, and it became wider when the load was increased. Furthermore, diagonal splitting occurred at the same loaction. At the area of the left support, there was spalling of the concrete and the sample failed at a load of 68k.

Beam 4 of series B was loaded and failed at 90k. The first inclined cracking appeard on both load-support segments at a load of 50k. As the load was increased, second and third inclined cracks occurred on both load-support segments. The first flexural crack was observed at 80k. In the area of the right support, failure of the sample was observed between the point loading and the right segment of the support.

Beam 5 of series B was loaded and failed at 90k. Failure of the sample occurred around the inclined bar of the left support. The first inclined and flexural cracks were observed at 50k and 70k respectively.

Beam 6 of series B was loaded and failed at approximately 90k. The first crack at 20k was an inclined crack. The crack was extended as the load reached 25k. As the load increased, new cracks appeared around that area. When the load reached 35k, a flexural crack was observed. The location of the failure was identical to beam 4. This sample failed because of steel yielding. Photographs of all crack patterns are shown in Appendix B.

3.2.2 Behaviour of dapped-end beam

BIR : A few flexural cracks were first observed at 25k. Some of them propogated as the load was increased, and some of them had longer crack patterns than the others. When the applied load reached 80k, a wide crack was observed near the previous crack, and it became wider as the load reached 100k. At the re-entrant corner of the dapped-end, cracks were observed at 20k and 55k. Diagonal inclined cracks were observed at 30k and 40k which propagated as the load was increased. The failure load of this beam was 110k. It should be noted that specimen B1R failed in flexure because of steel yielding.

B1L: A number of cracks were observed along the dapped-end area. Those cracks propagaged as the load was increased. More diagonal cracks occurred between the point load and the left segment of the support. The first diagonal and flexural cracks were observed at 20k and 40k respectively. New flexural cracks continued to appear until the load reached 110k.

Diagonal cracks started at 40k in the front face of the beam and at 20k in the back face of the beam. The failure load of the dapped-end was recorded at a load of 110k. Secondary collapse occurred within the load-support segment and at that time failure of the dapped-end occurred. secondary collapse occurred because of excessive strain deformation of steel which resulted a failure comparable to diagonal splitting.

B2R: This dapped-end had identical behaviour with specimen B1L, except the failure load was observed at 68k. At the re-entrant corner, a crack was observed at 10k. Moreover, the first inclined and flexural cracks appeared at a load of 10k and 20k respectively. A number of inclined cracks occurred at the dapped-end area. Even though a lot of crack occurred, the flexural failure and yielding of the steel caused collapse of the specimen.

B2L: The first flexural and inclined cracks were observed at load of 20k and 15k respectively. The flexrual cracks appeared in the middle of the beam and as the load was increased it propagated to the bottom fiber as well as to the top fiber. Similar to specimen B2R, the diagonal cracks of specimen B2L occurred on both load-support segments. Careful observation of the dapped-end zone showed that diagonal cracks were less numerous than the other end (B2R). The failure load of the dapped-end was at a load of 72.5k, and it was flexural failure.

<u>B3R:</u> At the re-entrant corner of the dapped-end, a number of inclined cracks were observed. At that location, cracks occurred at loads of 15k, 35k and 45k in the front face of the beam and occurred at a load of 30k and 45k in the back face of the beam. At the dapped-end zone, a few inclined cracks appeared in the front face of the beam, but there were not any inclined cracks appearing in the back face of the beam. In general, cracks appearing on each end of those beams had similar patterns. The first flexural crack was observed at a load of 25k. Between the point load and the right segment, inclined cracking became wider as the load was increased. In addition, failure by diagonal splitting on the beam occurred at load of 65k.

<u>B3L:</u> The first inclined and flexural cracks were observed at a load of 15k and 20k respectively. A number of inclined cracks occurred perpendicular to the inclined reinforcement of the dapped-end as well as at the dapped-end zone. In this beam, number of flexural cracks appeared and propagated as the load was increased. Flexural failure was observed at a load of 70k. At the re-entrant corner cracks were observed at 50k in the front face of the beam and at 15k in the back face of the beam.

<u>B4R:</u> The first flexural and inclined cracks were observed at loads of

15k and 20k respectively. Flexural crack propagated as the load reached 25k. Similar to specimen B4R and B1L, a number of inclined cracks of the specimen B4R occurred perpendicular to the inclined reinforcement of the dapped-end beam. At the dapped-end zone inclined cracks started at a load of 50k and it became wider as the load reached 80k. Secondary collapse occurred and introduced failure comparable to diagonal splitting at the dapped-end. Because of that, there was crushing of the concrete and there was failure a load of 90k.

<u>B4L:</u> In general, the dapped-end had an identical behaviour with specimen B4R. The first flexural and inclined cracks were observed at loads of 20k and 25k respectively. As usual, cracks propagated and became wider as the load was increased. Secondary collapse occurred and introduced failure comparable to diagonal splitting at the dapped-end. Moreover, failure occurred because of concrete crushing. The failure load was observed at a 89k.

It should be noted that the load mentioned in this section refer to the full shear load, Vu. All crack patterns and photographs taken are shown in Appendix A and B respectively.

CHAPTER - 4

ANALYSIS OF DAPPED-END OR CORBEL

4.1 Introduction

A corbel, deep beams or a dapped-end in the vicinity of support, may fail because of inclined cracking when subjected to pure shear or combined loading. Corresponding to a pattern of inclined cracks, a free body diagram in the form of multiple inclined compression struts of concrete combined with the reinforcing bars which acts as tension member, can be used as working model for strength analysis.

Theoretically, it can be shown that, a dapped-end or corbel may have much higher ultimate shear transfer capacity than is accepted in present design practice, if proper and sufficient reinforcement are used. This has been proven in tests on corbels with horizontal web reinforcement done by Mattock[(4),(7),(8),(9),(10)], and in our tests described in this report.

Analysis based on free body diagram for a model with inclined reinforcement under 45° angle leads to the observation that the ultimate shear strength can be twice as great as that obtainable with a horizontal or vertical web reinforcement.

Based on their research the following definition of strength limits in shear has been proposed by Zielinski[(11)].

4.2. Shear Strength Limit

Shear transfer capacity in reinforced concrete is dependent upon the magnitude of loading, the amount, placement, and strength of reinforcement, and also on the strength of the concrete itself.

The shear capacity limits can be differentiated for two basic failure modes:

1) Strength limit at cracking
2) Ultimate strength at failure

4.2.1 Strength at cracking

Consider a dapped-end or corbel on an interface line A-B as shown in figure 4.1. The bi-axial stress state with orthogonal compression stress σc, and tension stress σt occurs on that interface line, where both stresses act in the 45° inclined direction with respect to horizontal axis. Under pure shear action, both stresses are of equal magnitude, i.e., $\sigma c = \sigma t$.

According to previous work done by Zielinski[12], [13] and lately by Tasuji et al[14], it can be stated that under a bi-axial stress state, a corbel will fail by inclined cracking as shown in figure 4.2, when the limit stress vsu will be reached. The value of vsu is equal to :

$$vsu = ftc \simeq 0.9\ fto \qquad (1)$$

If uniaxial tensile strength of concrete, fto is assumed to have a strength of $6\sqrt{f'c}$, then the ultimate shear strength of unreinforced concrete corresponding to bi-axial stress state when $\sigma c = \sigma t$ will be equal to:

$$vsu = 0.9 \times 6\sqrt{f'c} = 5.4\sqrt{f'c} \qquad (2)$$

$$\text{or approximately} = 5.5\sqrt{f'c}$$

According to figure 4.1, the ultimate shear capacity can then be expressed as follows:

$$Vu = (C + T)\ \phi/\sqrt{2} = C\phi\sqrt{2}\ \text{ OR } = T\phi\sqrt{2}$$

$$= \sigma c\phi bh_D \text{ or } \sigma t\phi bh_D \qquad (3)$$

Based on equation (2), the limiting shear force can be written as follows:

$$Vu = \phi vsu\ bh_D = \phi 5.5\sqrt{f'c}\ bh_D \qquad (4)$$

Comparable values of vsu were observed in Mattock's [10] push-off tests and on deep beams and wall beams tested by Zielinski[12],[13] and Taner [15].

When the above stress is reached, inclined cracks, will occur as shown in figure 4.2.

It is necessary to note that, the cracking of concrete as shown in figure 4.2 will not mean failure of the dapped-end or corbel, when the corbel has reinforcement which is able to take over the whole tension force, T. Only the reinforcement which intersects the cracks may take the tension force. Obviously reinforcement which is placed perpendicular to the cracks will be the most efficient in providing the tension strength. Thus the web reinforcement, Awf on the line perpendicular to **the crack's direction** should be considered as follows:

$$Awf = Ahf \sin \alpha + Avf \cos \alpha \qquad (5)$$

For a dapped-end or corbel having horizontal and vertical reinforcement which make a 45° angle with respect to the line of the crack direction, the web reinforcement will be:

$$Awf = (Ahf + Avf) / \sqrt{2}$$

According to our tests and test results by Tasuji et al [14], cracks appear in concrete when under equal bi-axial compression and tension, the strain in the tension direction reaches the limit of approximately $\varepsilon ct = 0.0001$. Thus at the time of cracking, the reinforcement, if present, is subjected to a stress of

$$fs \cong \varepsilon ct . Es = 3000 \text{ psi}$$

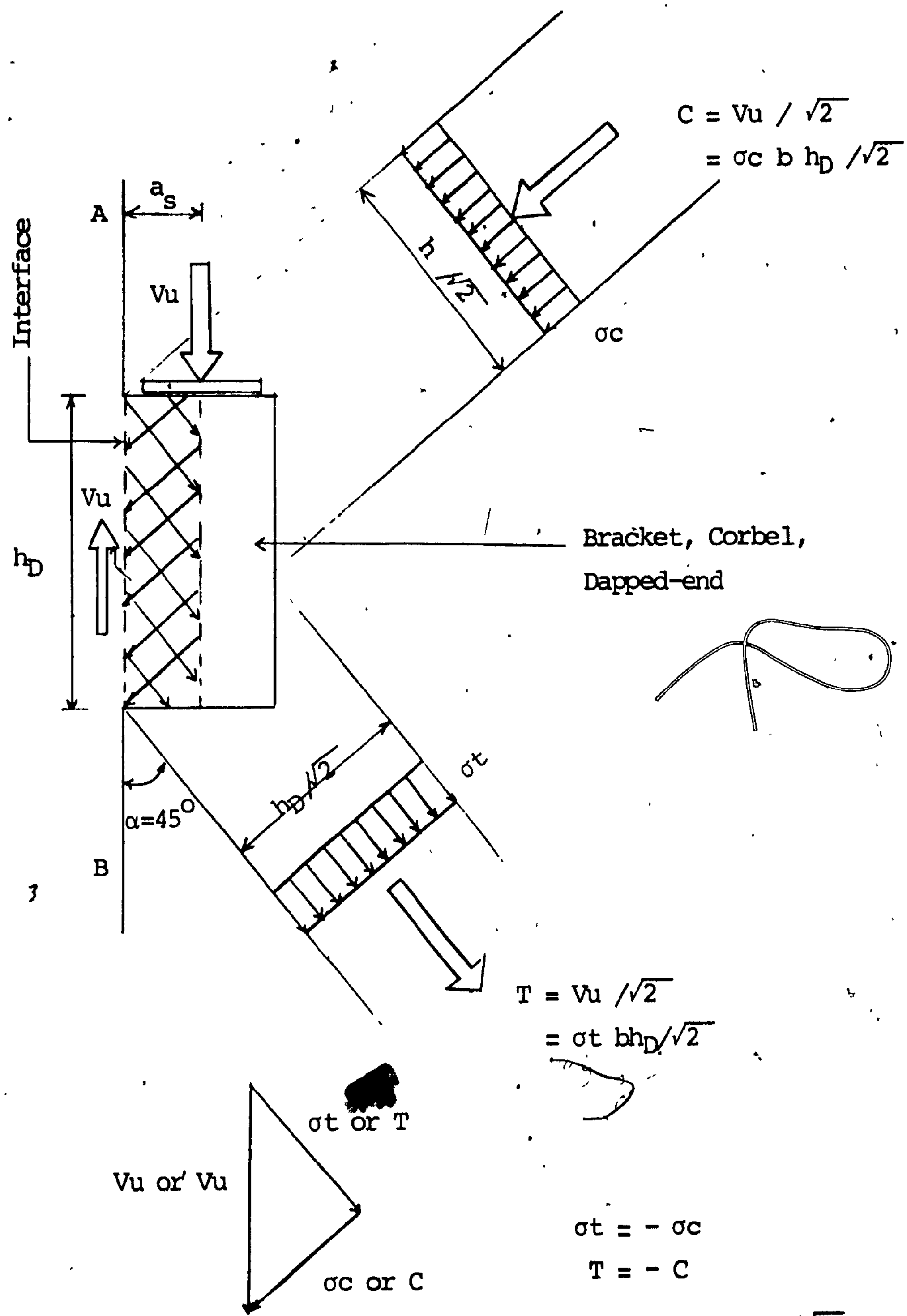


Fig 4.1 Free body diagram of dapped-end or corbel

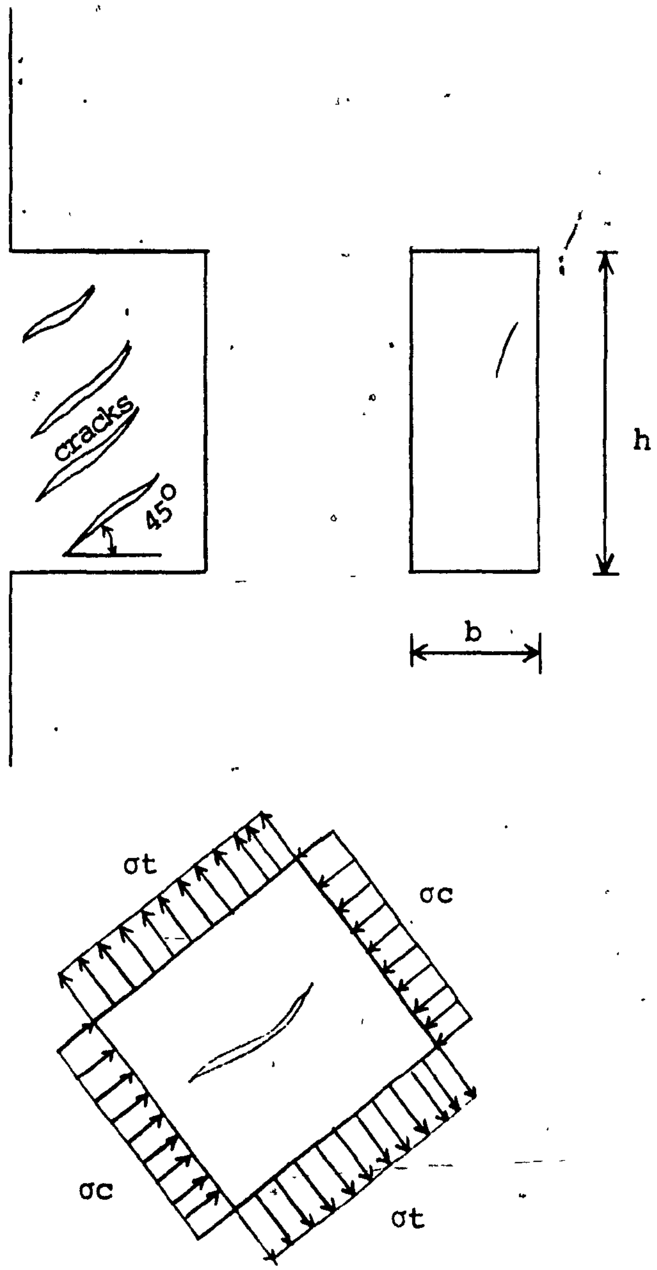


Fig. 4.2 Crack patterns of dapped-end or corbel under bi-axial stress state

As a result, the ultimate cracking capacity, Vucr of a reinforced dapped-end or corbel can be expressed as below:

$$Vucr = vsu \ \phi b h_D \ (1 + \rho wf \ 3000/\sqrt{vsu}) \qquad (6)$$

Thus the limit shear stress at cracking can be expressed as follows:

$$vucr = (5.5\sqrt{f'c} + 3000 \ \rho wf) \ \phi$$

$$= 5.5\sqrt{f'c} \ \phi \ (1 + \rho wf \ 545/\sqrt{f'c}) \qquad (7)$$

where $\rho wf = Awf \sqrt{2}/bh_D$

If the amount of the reinforcement is not sufficient and the shear force exceeds the cracking force Vucr, failure of the dapped-end or corbel is expected. In such a case, failure of the dapped-end will be sudden and without warning.

The minimum amount of reinforcement required to prevent sudden failure of a dapped-end can be derived as follows: based on equation (3) and (6) the total tension force component at the time of cracking can be expressed as:

$$T = vsu \ bh_D \ \phi \ (1 + \rho wf \ 3000/\sqrt{vsu})/\sqrt{2}$$

$$= Vucr \ \ \phi/\sqrt{2}$$

At the moment of cracking, this force must be taken over entirely by the reinforcement, which should not be subjected to yield. Therefore, sudden failure can be prevented if Ty > T. Where

$$Ty = \phi \ Awf \ fy = \phi \ \rho wf \ bh_D \ /\sqrt{2}$$

$$\phi \ \rho wf \ bh_D \ /\sqrt{2} > vsu \ \phi \ bh_D \ (1 + \rho wf \ 3000/\sqrt{vsu}) \ /\sqrt{2}$$

or $\rho wf \ fy > vsu \ (1 + \rho wf \ 3000/\sqrt{vsu})$

As a result, the minimum ratio of web reinforcement, ρwfcr required to prevent rapid failure at the moment of cracking of a dapped-end can be defined as follows:

$$\rho wfcr = \rho wf > vsu/(fy - 3000)$$

$$= 5.5 \cdot \sqrt{f'c}/(fy - 3000) \qquad (8)$$

By applying an additional factor of safety against the sudden failure in shear at cracking, equation (8) can be defined as:

$$S\rho wfcr > 6\sqrt{f'c} \,/\, (fy - 3000) \qquad (9)$$

The above formula demonstrates that "pure shear" cracking strength of reinforced concrete elements varies and depends primarily on the strength of the concrete. The contribution of steel for cracking strength is rather insignificant.

4.2.2 Strength limit after cracking

After inclined cracks appear, further functioning of the reinforced concrete dapped-end is still possible if the amount of reinforcement is sufficient.

Consider a free body diagram which has inclined compression struts defined by cracks and inclined or horizontal web reinforcement as it is shown in figure 4.3 and 4.4 respectively.

The use of the free body diagram requires the definition of "C" the ultimate capacity of concrete in this state.

It is proposed that the strength of the concrete in cracked struts be reduced to $fcs = 0.7\ f'c$.

As a result, the total compression capacity of all concrete struts would be:

$$C = fcs\ bh_D\ \phi\ /\sqrt{2} = 0.7\ f'c\ \phi\ bh_D/\sqrt{2}$$

or

$$C = 0.5\ f'c\ \phi\ bh_D \qquad (10$$

The capacity of the inclined tension member provided by the reinforcement would be :

$$Twf = Awf\ fy \qquad (11)$$

As it is shown in figure 4.3, the ultimate capacity of dapped-end having 45^{o} inclined reinforcement can be expressed as the sum of vertical pro-projections of C and T which leads to the equation below:

$$Vu = (C + T)\ \phi\ /\sqrt{2} \qquad (12)$$

$$= (Cv + Tv)\ \phi$$

Substituting equations (9), (10) and (11) into (12) we will get:

$$Vu = \phi\ (0.5\ f'c\ bh_D + Awf\ fy)\ /\sqrt{2}$$

$$= 0.35\ \phi\ f'c\ bh_D + 0.7\ \phi\ Awf\ fy \qquad (13)$$

where $\phi = 0.85$ (capacity reduction factor for shear strength). From equation (13) the ultimate shear stress for 45^{o} inclined reinforcement can be rewritten as below:

$$vu = (0.35\ f'c + 0.5\ \rho wf\ fy) \qquad (14)$$

Since for equilibrium of internal forces, T = C must be satisfied the equations (13) and (14) can be written in similar form based on the capacity of the steel or concrete only as :

$$Vu = 1.41\ \phi\ Awf\ fy \qquad (15)$$

or $$Vu = 0.7\ \phi\ f'c\ bh_D \qquad (15b)$$

and the corresponding ultimate nominal shear stress as

$$vu = \phi\ \rho wf\ fy \qquad (16a)$$

or $$vu = 0.7\ \phi f'c \qquad (16b)$$

Based on equations (15) and (16), the required amount of inclined web reinforcement can be calculated as follows:

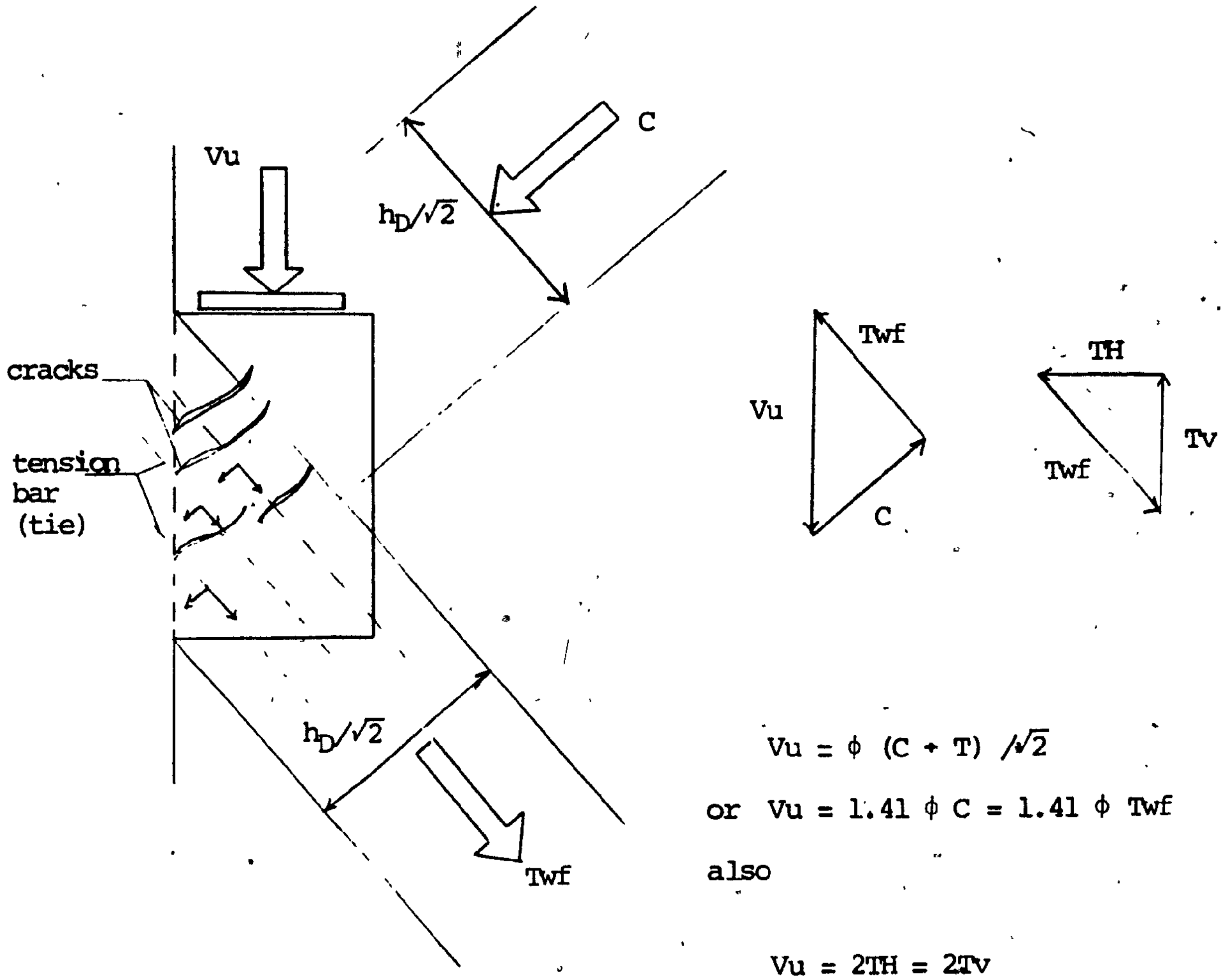
$$A_{wf} = {}^{V_u}/1.41\ \phi\ f_y \qquad (17a)$$

or

$$\rho_{wf} = A_{wf}\ \sqrt{2}\ /\phi b h_D = \frac{V_u}{\phi f_y} \qquad (17b)$$

or

$$\rho_{wf} = 0.7\ \phi\ f'_c/f_y \qquad (17c)$$



$$Vu = \phi\ (C + T)\ /\sqrt{2}$$

or $Vu = 1.41\ \phi\ C = 1.41\ \phi\ Twf$

also

$$Vu = 2TH = 2Tv$$

Fig 4.3 Free body diagram of reinforced dapped-end with inclined reinforcement

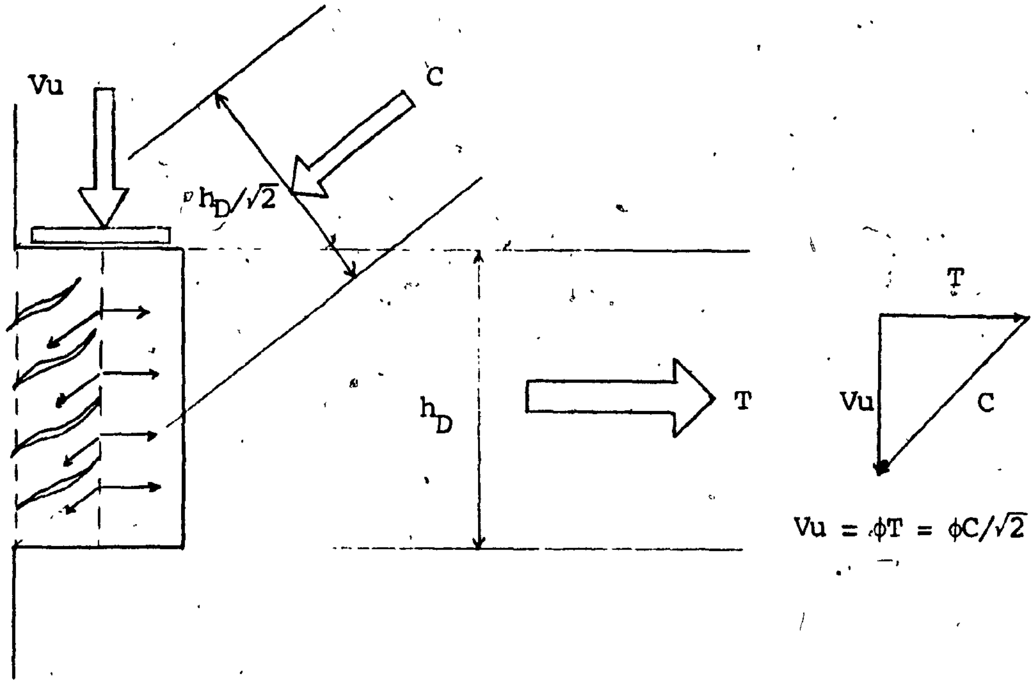


Fig. 4.4 Free body diagram of reinforced dapped-end with horizontal reinforcement

The maximum usable amount and corresponding ratio of inclined web reinforcement can be defined by comparing the ultimate capacity of compression struts with the ultimate capacity of tension (steel) component which must be in equilibrium as follows:

$$0.71\ \phi\ Awf\ fy = 0.35\ \phi\ f'c\ bh_D$$

Therefore the maximum usable amount and ratio of web reinforcement will be respectively equal to:

$$Awf \leq 0.5\ f'c\ bh_D/fy \qquad (18a)$$

and $$\rho wf = 0.7\ \phi f'c/fy \qquad (18b)$$

It should be stressed that the usable amount of web reinforcement is limited to the amount corresponding to the ultimate capacity of concrete (shear balanced condition).

In order to eliminate secondary collapse due to excessive cracking and shape change of free body model, it is proposed that the maximum strain in the tension direction should not exceed a certain limit which is proposed here to be $\varepsilon = 0.00125$. As a result, the maximum usable yield strength of web reinforcement, fy should be taken as :

$$fy = 0.00125 \times Es = 40{,}000 \text{ psi } (280 \text{ Mpa})$$

Following the above procedure, the ultimate shear capacity of a dapped-end with horizontal reinforcement can be easily derived from a free body diagram as shown in figure 4.4. In this case, the maximum usable shear transfer capacities can be expressed in terms of strength of steel or concrete respectively as:

$$Vu = \phi T = \phi\, Ahf\, fy \qquad (19a)$$

or $$Vu = \phi C/\sqrt{2} = \phi\, 0.35\, f'c\, bh_D \qquad (19b)$$

Corresponding to maximum ultimate nominal shear stress, (19) becomes:

$$vuh = \frac{Vu}{bh_D} = \phi\, \rho hf\, fy$$

or $$vuh = 0.35\, \phi f'c \qquad (20)$$

In addition, the maximum usable amount and ratio of horizontal web reinforcement will be:

$$Afh = {}^{Vu}/\phi fy \quad \text{or} \quad \rho hf = Vuh/\phi fy \qquad (21a)$$

or $$\rho hf = 0.35\, \phi\, f'c/fy \qquad (21b)$$

It should be noted here that this ratio web reinforcements represents ½ of the amount for a dapped-end with inclined reinforcement.

The ratio density of horizontal web reinforcement, ρhf should not be less than $\sqrt{2}$ $\rho wcfr$ in order to prevent rapid failure without warning at first cracking. Hence,

$$\rho hf \leq 7.7\, f'c\, / \,(fy - 3000) \qquad (22)$$

Based on equation (19a) it should be noted that, the ultimate shear strength is equal to the whole capacity of the horizontal reinforcement.

As a result, the required amount of horizontal web reinforcement can be calculated by assuming the whole shear force is carried by steel only.

By applying the reduction factor of $\phi = 0.85$ (as is used in practice), equation (20) can be written as follows:

$$v_{uh} = 0.35\ \phi\ f'c$$

$$= 0.35 \times 0.85\ f'c \simeq 0.3\ f'c \qquad (23)$$

This stress limit was recorded in tests done by several authors [8], [9], [10], [12], [16], [17]. Comparison between equation (16b) and (23) shows that the dapped-end having inclined reinforcement had twice the capacity of the dapped-end having horizontal reinforcement.

CHAPTER - 5

CONCLUSION AND DESIGN PROCEDURE

5.1 Conclusion

The following conclusions can be drawn from the results of the concluded tests and analysis.

1. The present A.C.I. design concept based on the nominal shear must be revised. If the nominal shear concept is retained then the maximum allowable nominal shear stress value could be increased to 0.3 f'c for corbels and dapped-ends with horizontal reinforcement and to 0.6 f'c in the case of 45^{o} inclined reinforcement.

2. Free body diagram analysis can be safely applied for predicting the capacity and calculating the required reinforcement for deep beams, corbels, and dapped-ends.

3. After cracking, the contribution of concrete and steel is equal. Thus shear capacity is defined by the capacity of the concrete or steel component whichever is smaller. Increasing the amount of web reinforcement over $\rho wf = 0.7$ f'c/fy which defines the balanced condition will not increase the strength of deep beam, dapped-end or corbel.

4. In order to eliminate early collapse in shear, the yield strength of steel used for shear reinforcement should be limited to 40 ksi (280 mpa).

5. Generally all dapped-ends subjected to high shear fail because of the steel capacity. The capacity of the concrete is usually greater than the steel component. Some of the beams tested had flexural failure which did not reach the capacity in shear.

6. The test results of specimens B1R and B1L as well as B2R and B2L had equivalent strengths for each beam, thus the variations of dapped-end profiles did not change the ultimate shear strength.

7. The reinforcement detailing of specimen B1 resulted in an 18% increase in shear capacity over those obtained from specimen B4.

8. Dapped-end beams with inclined reinforcement are $\sqrt{2}$ times more economical than the horizontal reinforcement with respect to the amount of steel required as shown in equations (17a) and (21a).

5.2. Summary on proposed design procedure

1. For a given load, calculate the full shear load, Vu and the maximum moment of the beam.

2. Choose appropriate section for a given load or moment and check whether $a_s/h_D < 1$.

3. Calculate the area of steel required, Awf or Ahf needed to carry the full shear load such that:

$$Awf = \frac{Vu}{\phi 1.41\ fy} \quad \text{for inclined reinforcement}$$

or

$$Ahf = \frac{Vu}{\phi fy} \quad \text{for horizontal reinforcement}$$

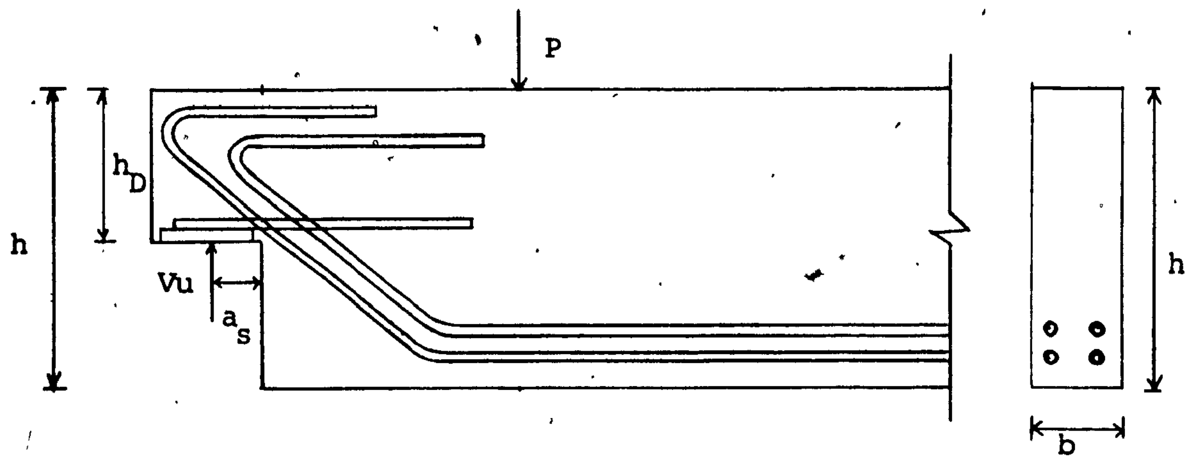
4. Check approximate concrete strength, f'c based on balanced condition:

$$Vu = 1.41\ \phi\ Awf\ fy \qquad (15a)$$

$$Vu = 0.7\ \phi f'c\ b\ h_D \qquad (15b)$$

$$f'c = \frac{1.41\ Awf\ fy}{0.7\ b\ h_D}$$

5. Check capacity of dapped-end reinforcement



$$T = THI + TH$$

$$THI = Twf \cos 45^{\circ}$$

$$M = T\,(d_D - a/2)$$

Moment, M should be greater than moment at A, otherwise repeat procedure 3 and 4.

6. Check capacity of main reinforcement

$$T = As\ fy$$

$$M = T\,(d - a/2)$$

Moment M should be greater than moment at B, otherwise repeat procedure 3, 4 and 5.

7. Vertical and horizontal stirrups are provided for structural purpose.

In applying the above procedures, a numerical example of designing a dapped-end is presented in appendix D. Calculation of the actual shear strength of all the samples corresponding to actual strength of the materials are presented in appendix E.

5.3 Recommendation

This paper shows the improved behaviour of dapped-end beams with inclined reinforcement. Since all the beams of the second series failed in flexure, it is therefore recommended that the next samples should fail in shear. The following figure demonstrates this.

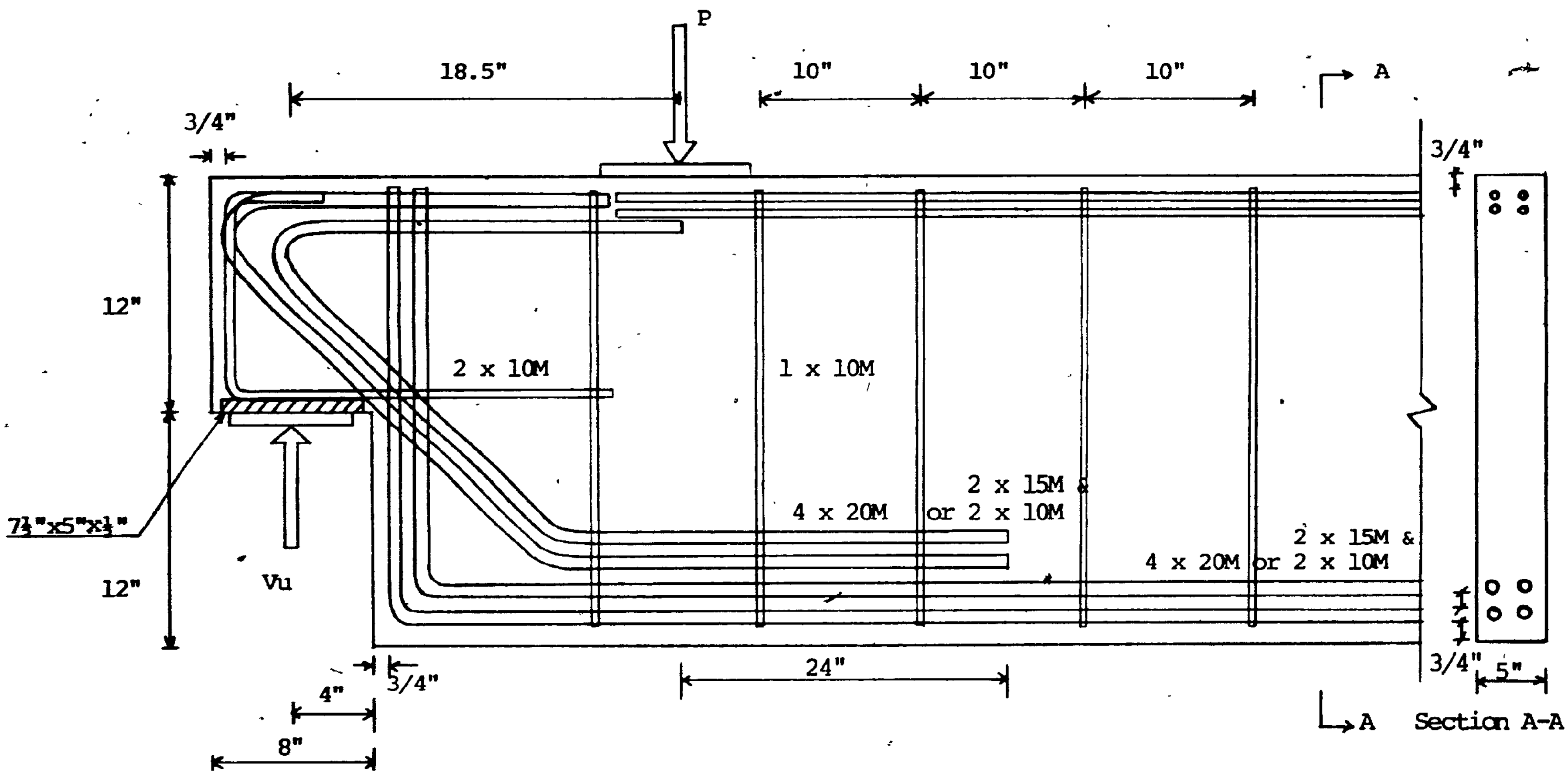


Fig 5.1 Recommended beam to be tested

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APPENDIX A

CRACK PATTERNS OF THE TEST

SPECIMENS

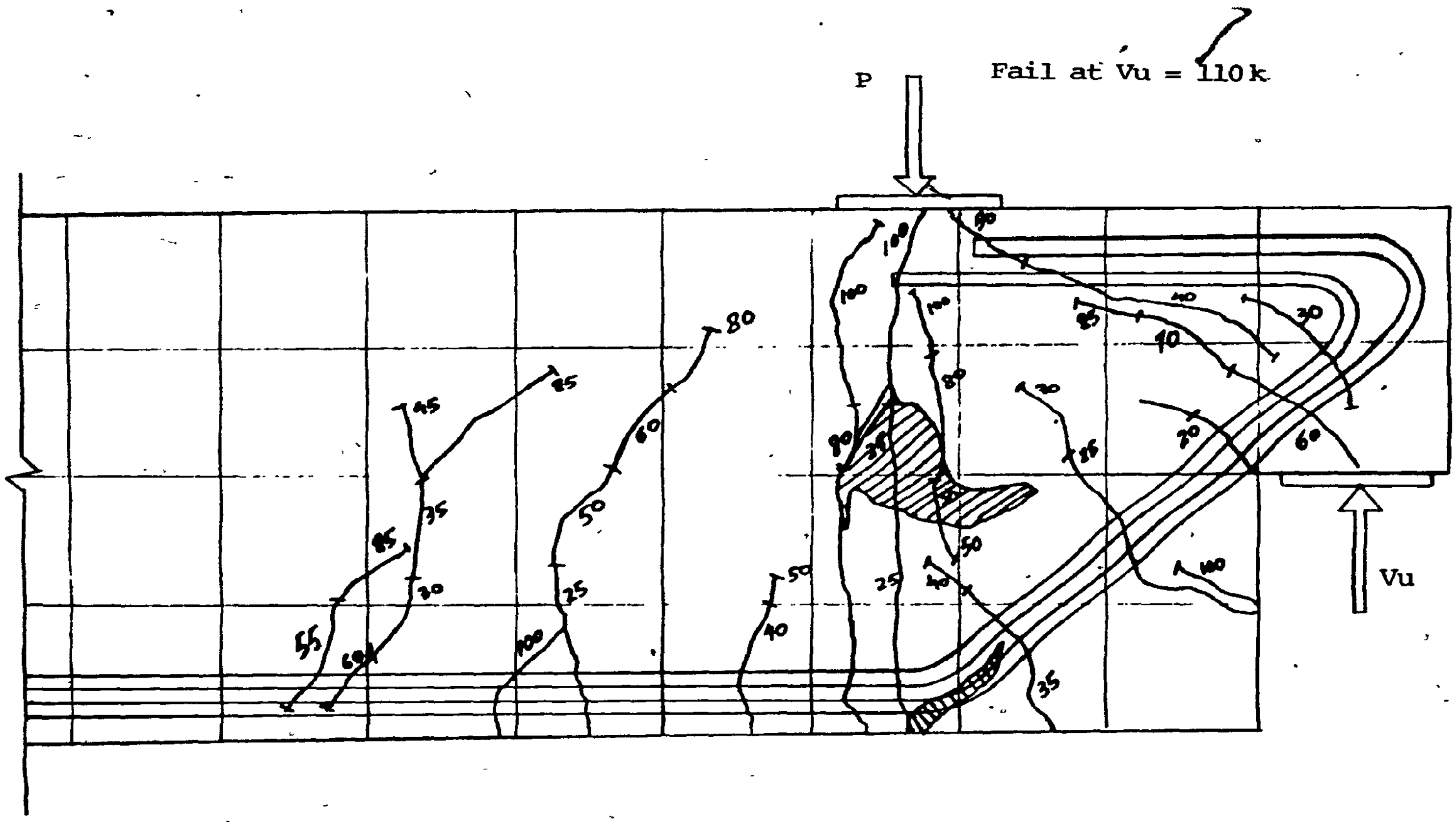


Fig. A1 Crack Patterns of Specimen B1R (Front)

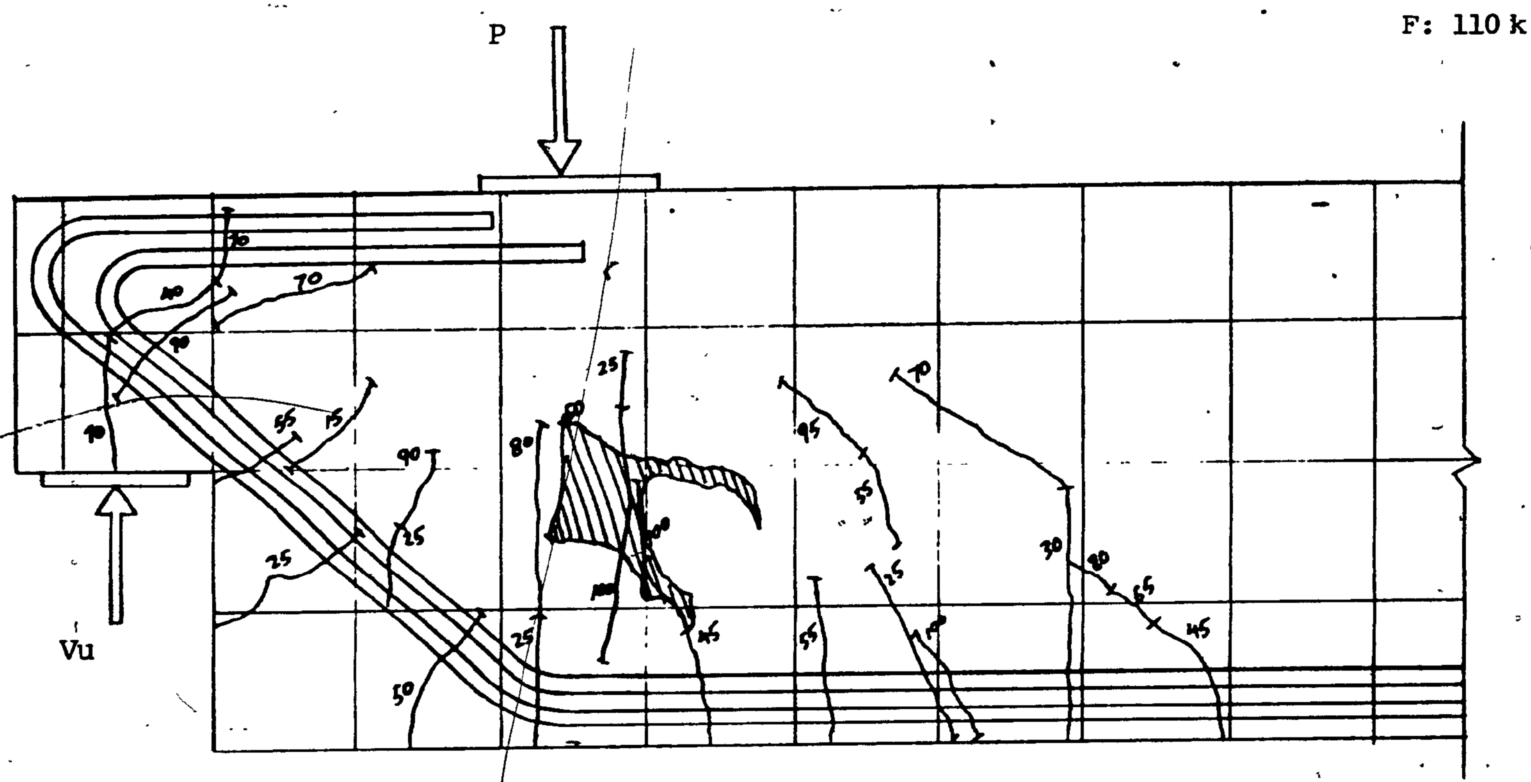


Fig. A2. Crack Patterns of Specimen B1R (Back)

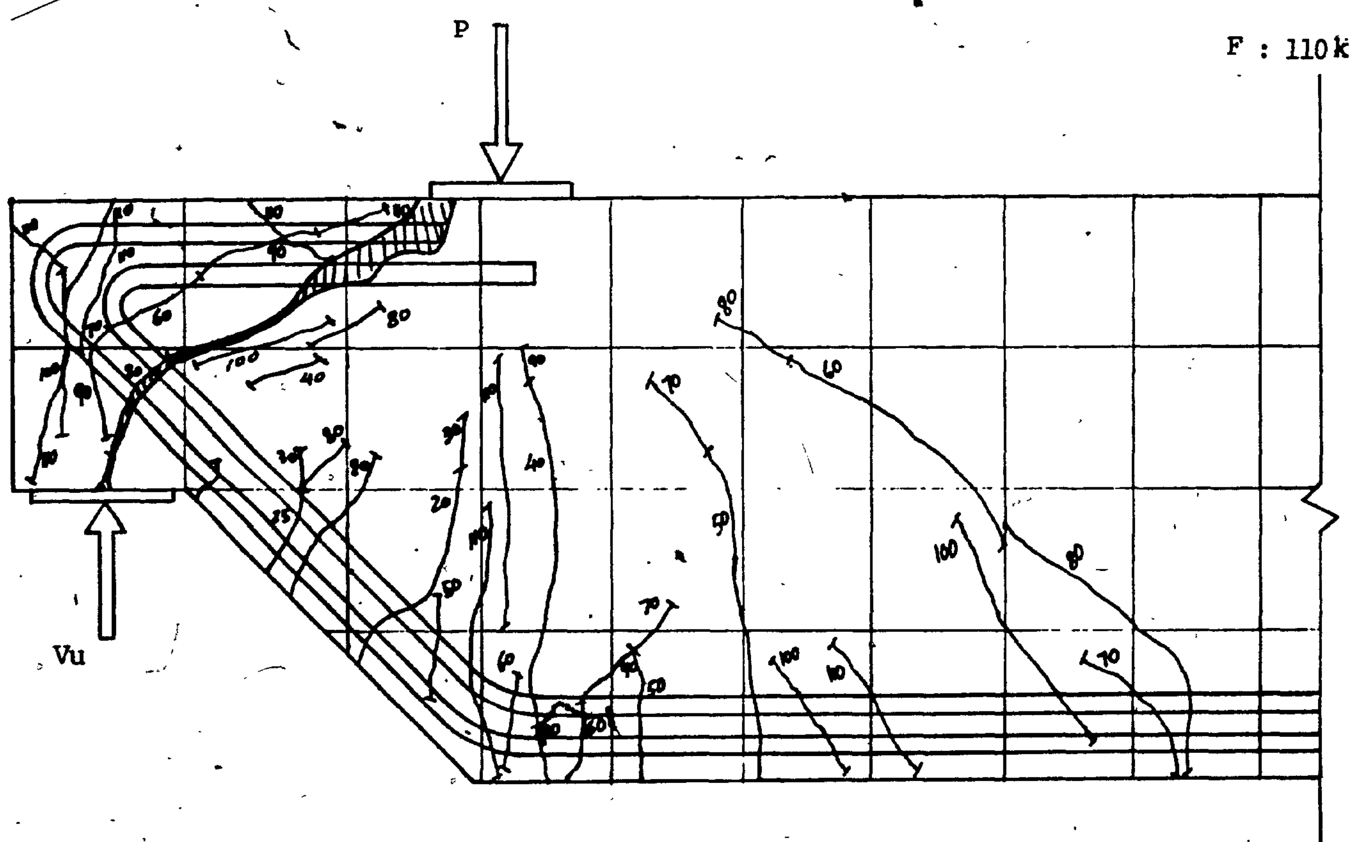


Fig. A3. Crack Patterns of Specimen B1L (Front)

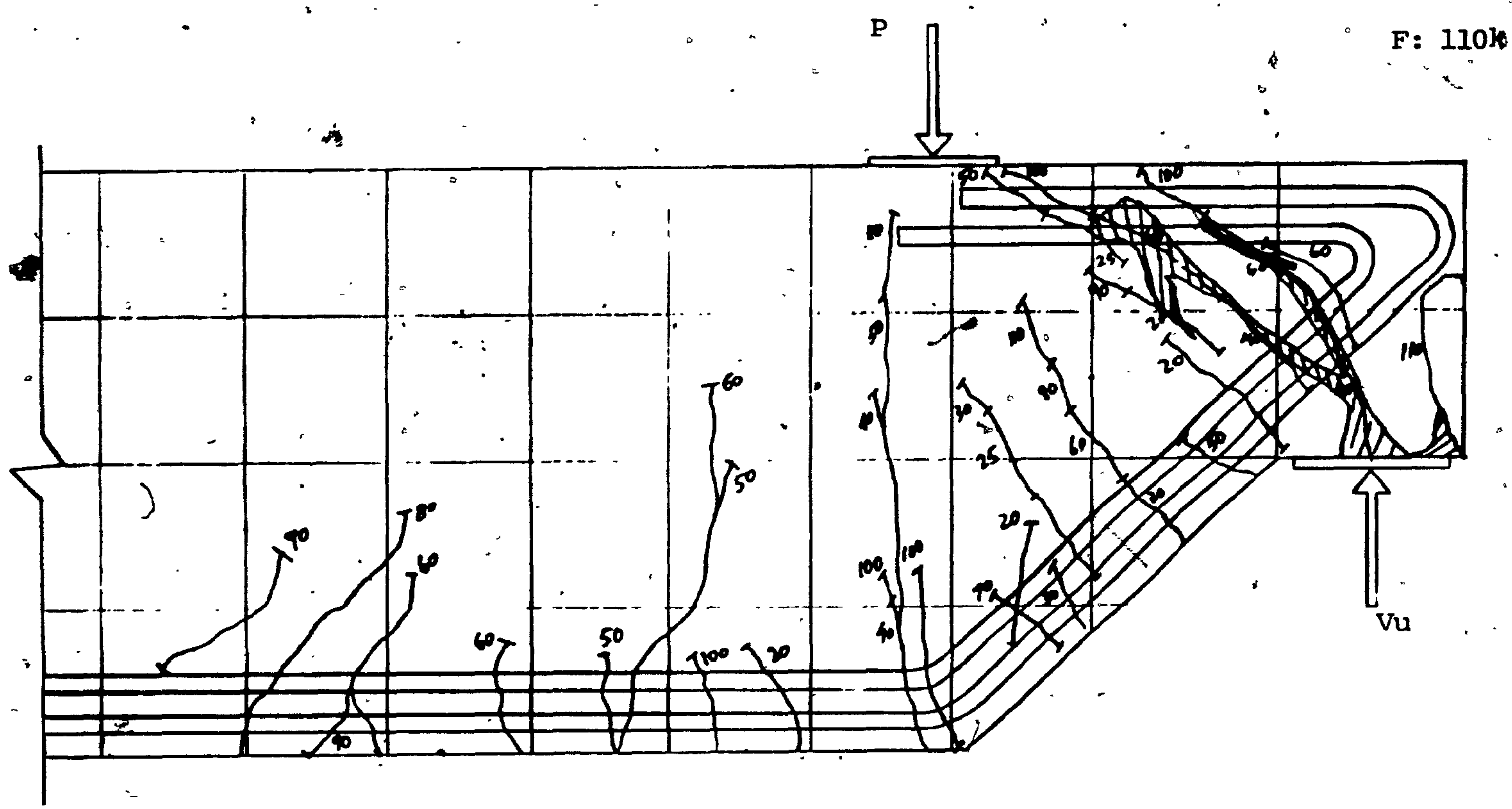


Fig. A4. Crack Patterns of Specimen B1L (Back)

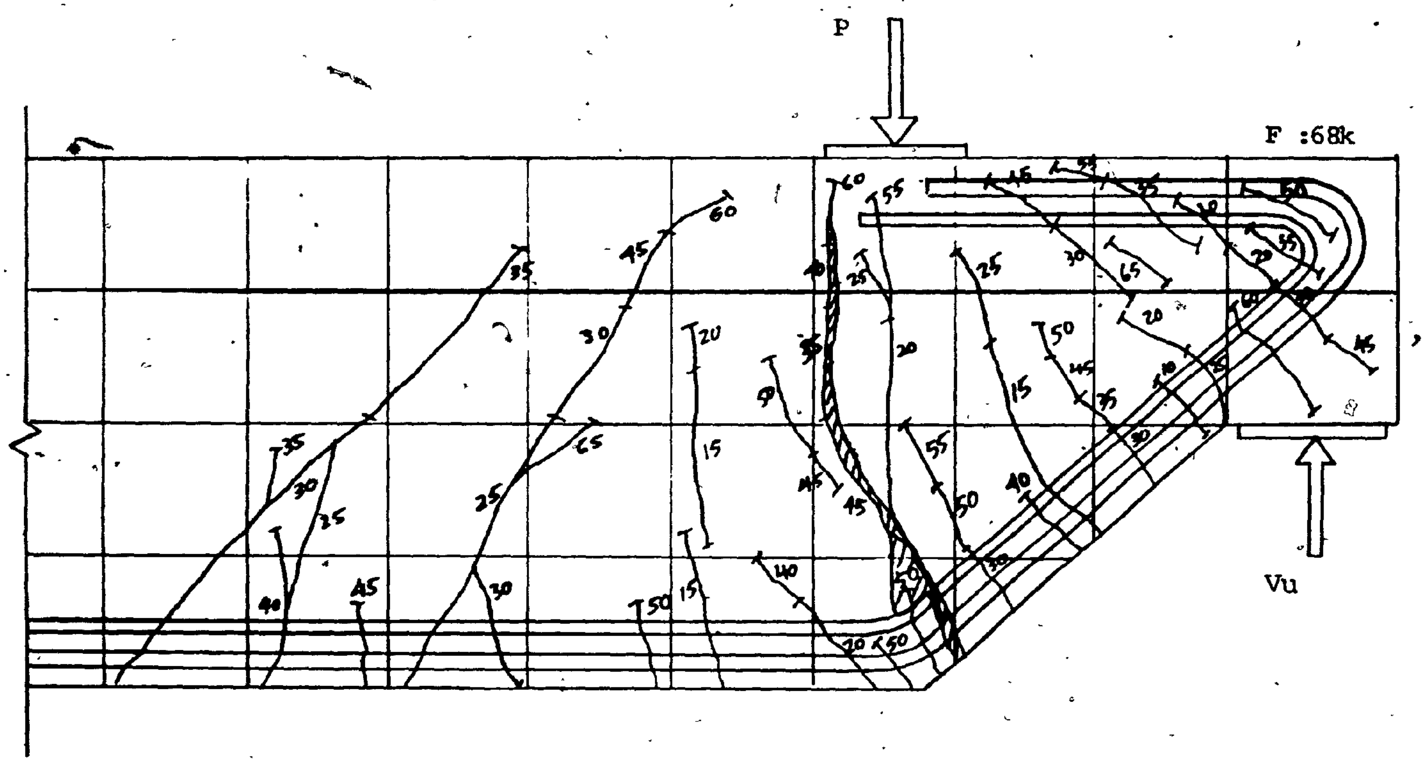


Fig. A5. Crack Patterns of Specimen B2R (Front)

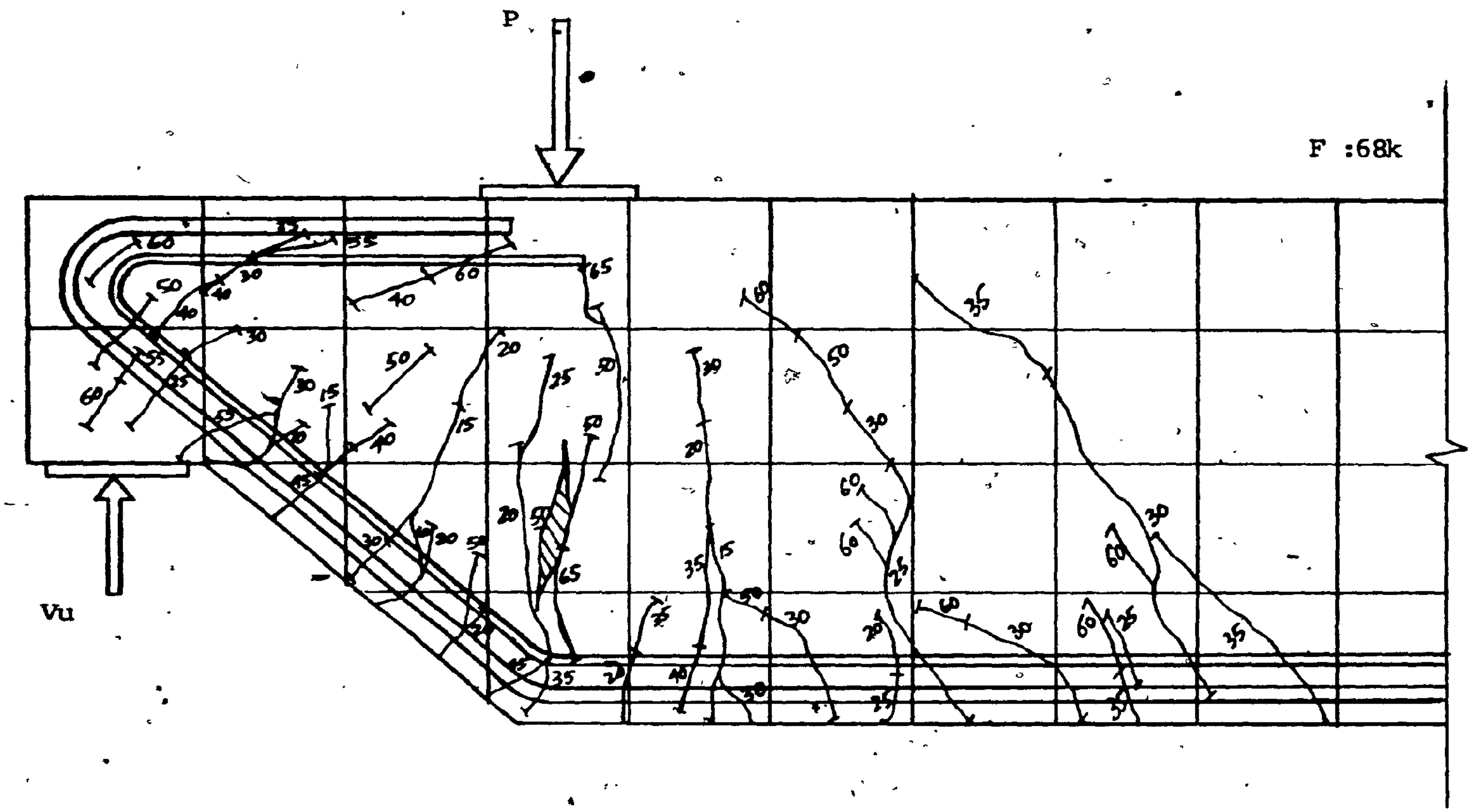


Fig. A6: Crack Patterns of Specimen B2R (Back)

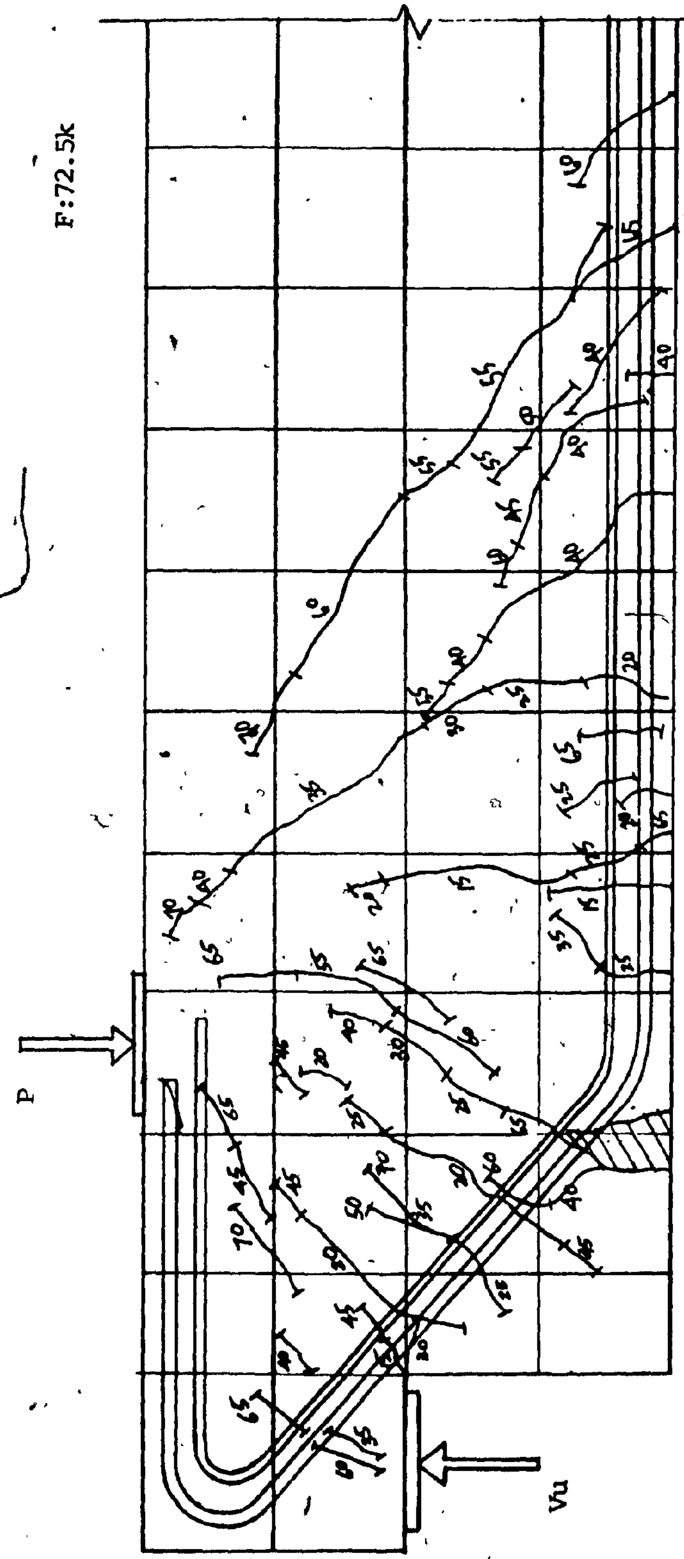


Fig. A7. Crack Patterns of Specimen B2L (Front)

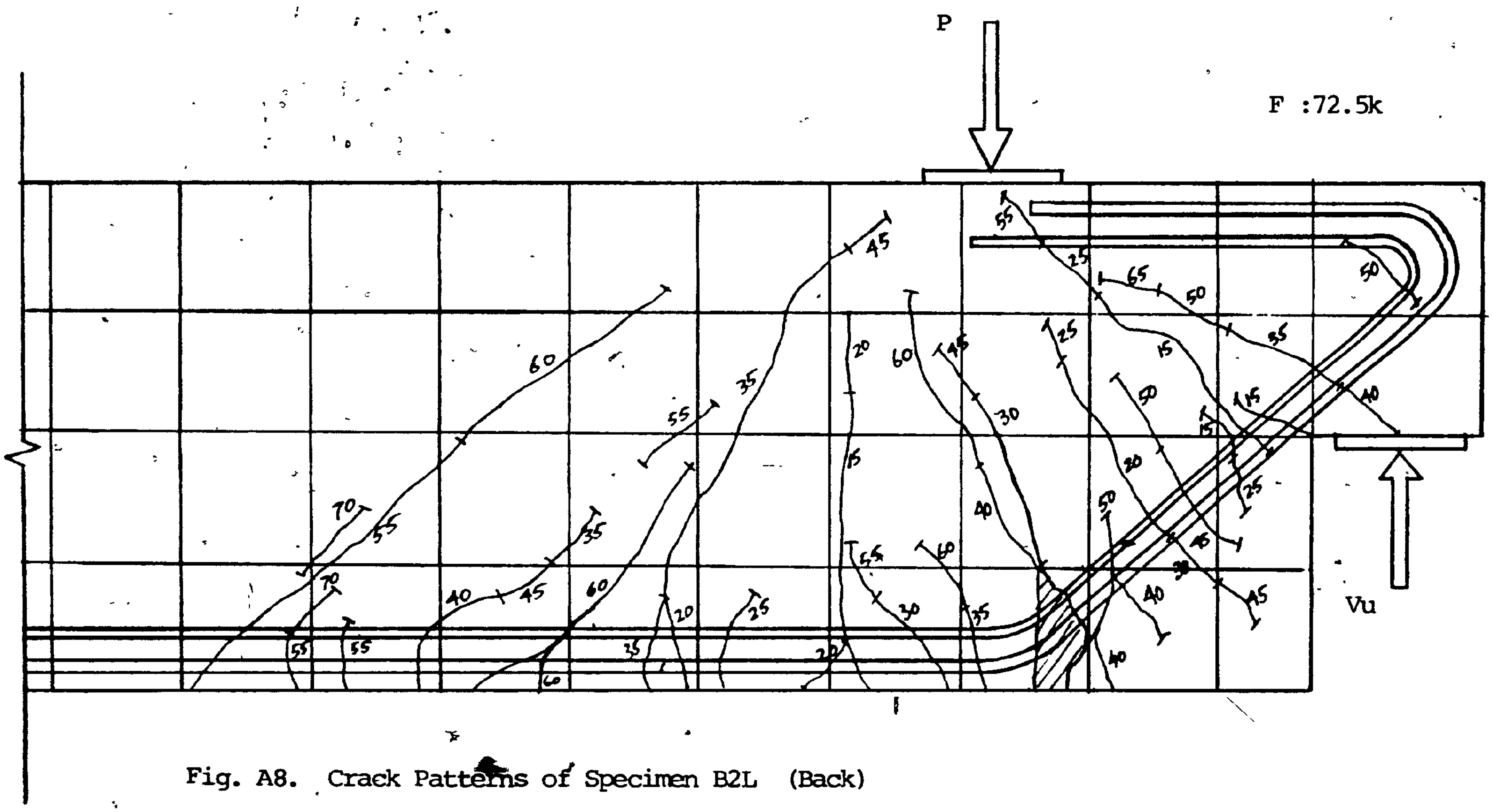


Fig. A8. Crack Patterns of Specimen B2L (Back)

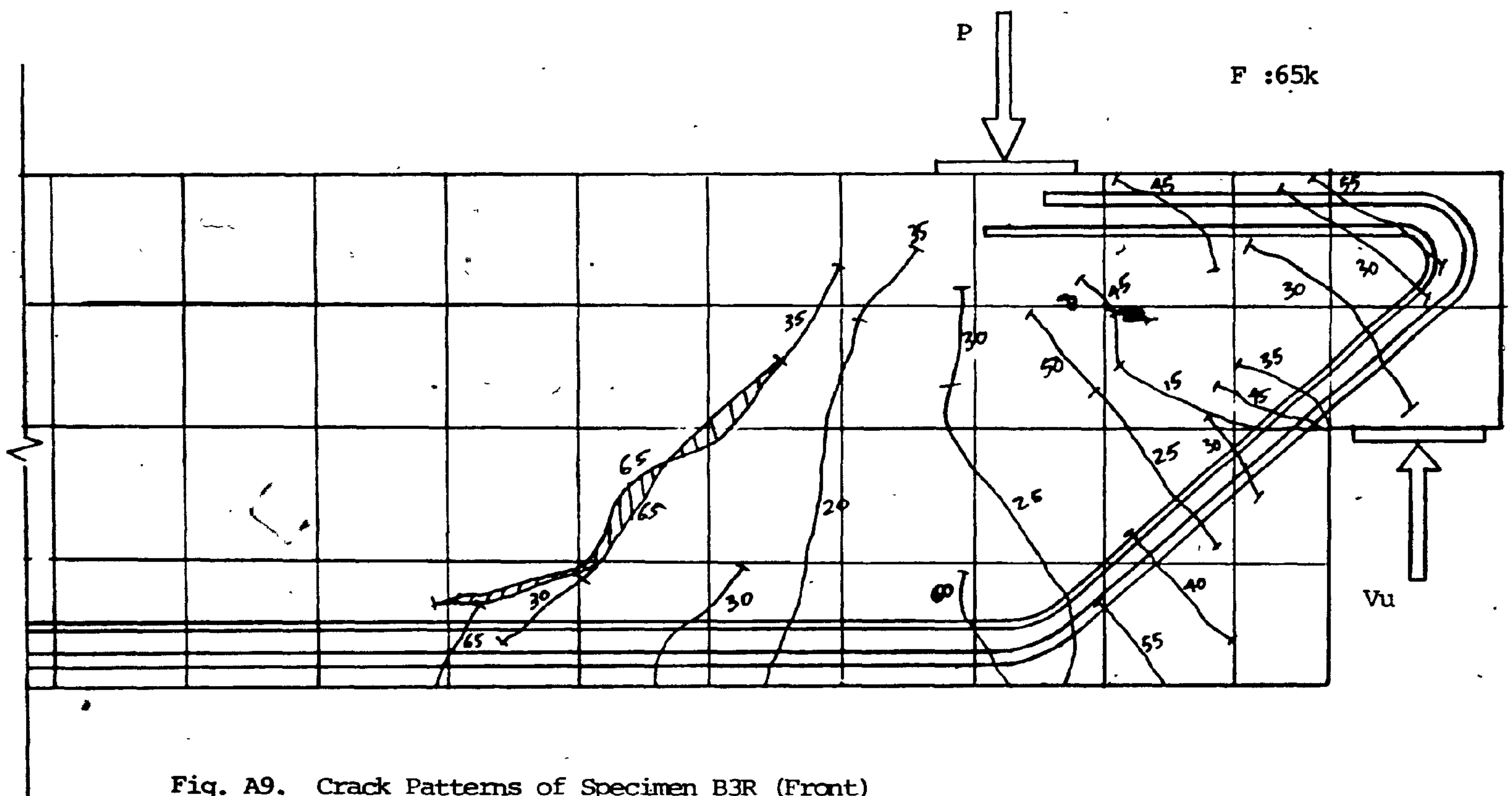


Fig. A9. Crack Patterns of Specimen B3R (Front)

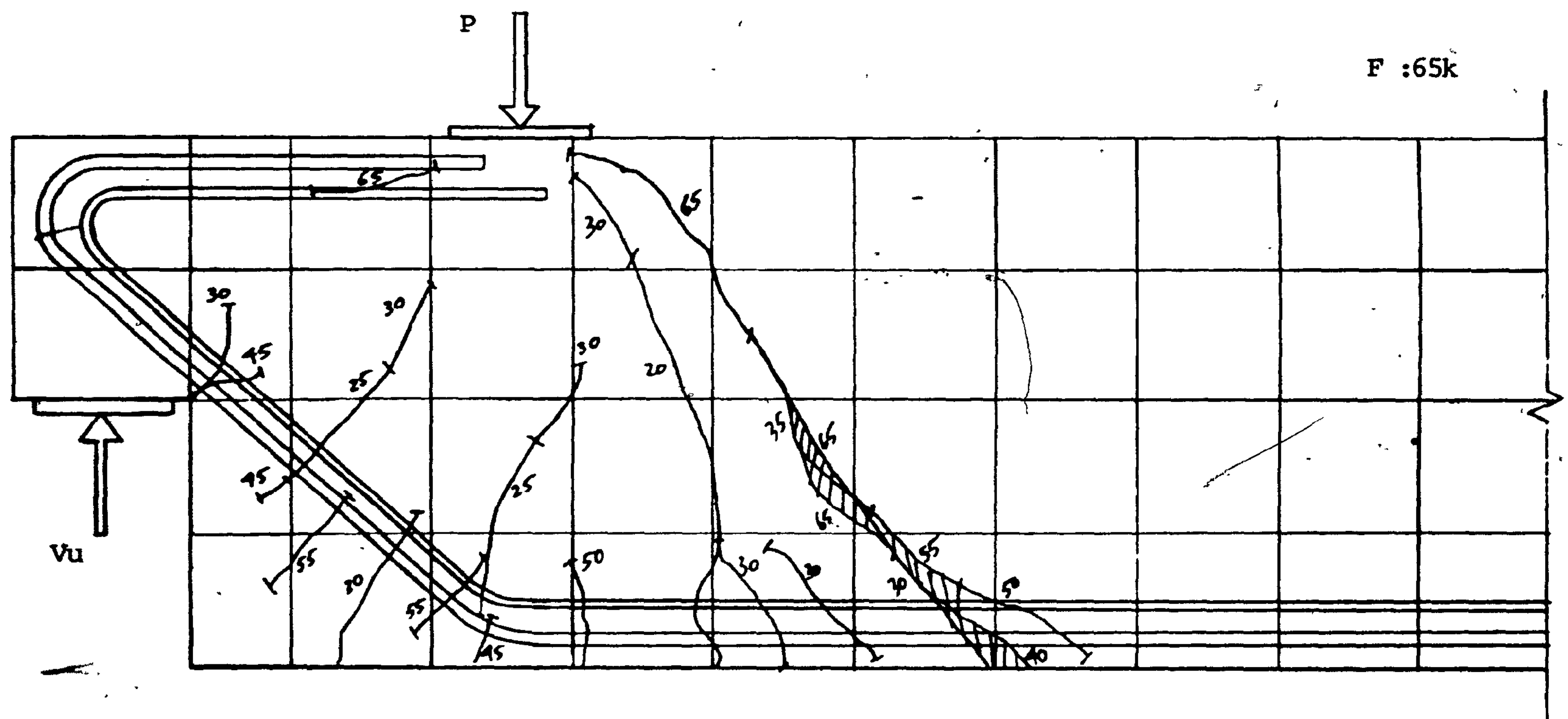


Fig. A10. Crack Patterns of Specimen B3R (Back)

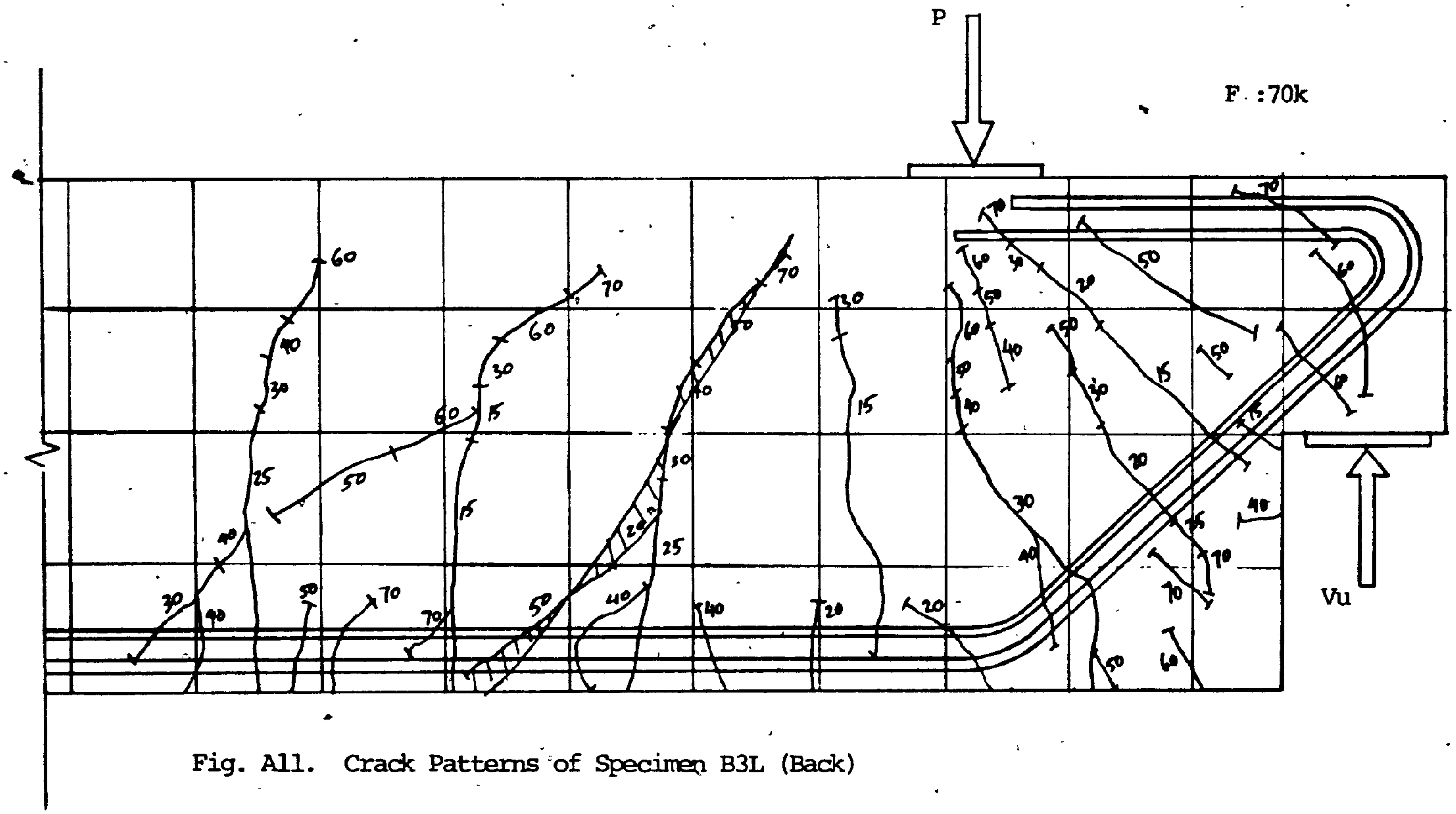


Fig. All. Crack Patterns of Specimen B3L (Back)

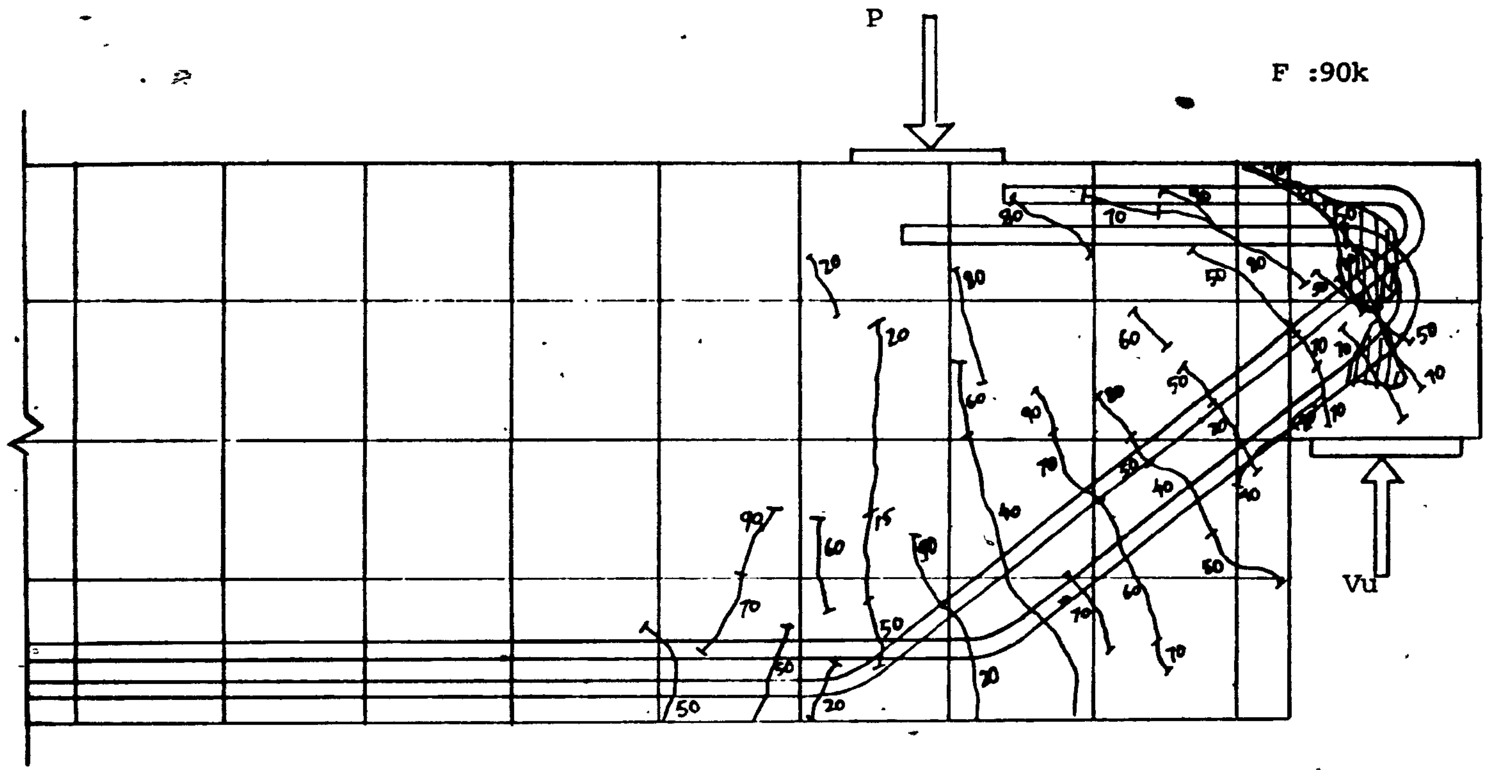


Fig. A12. Crack Patterns of Specimen B4R (Front)

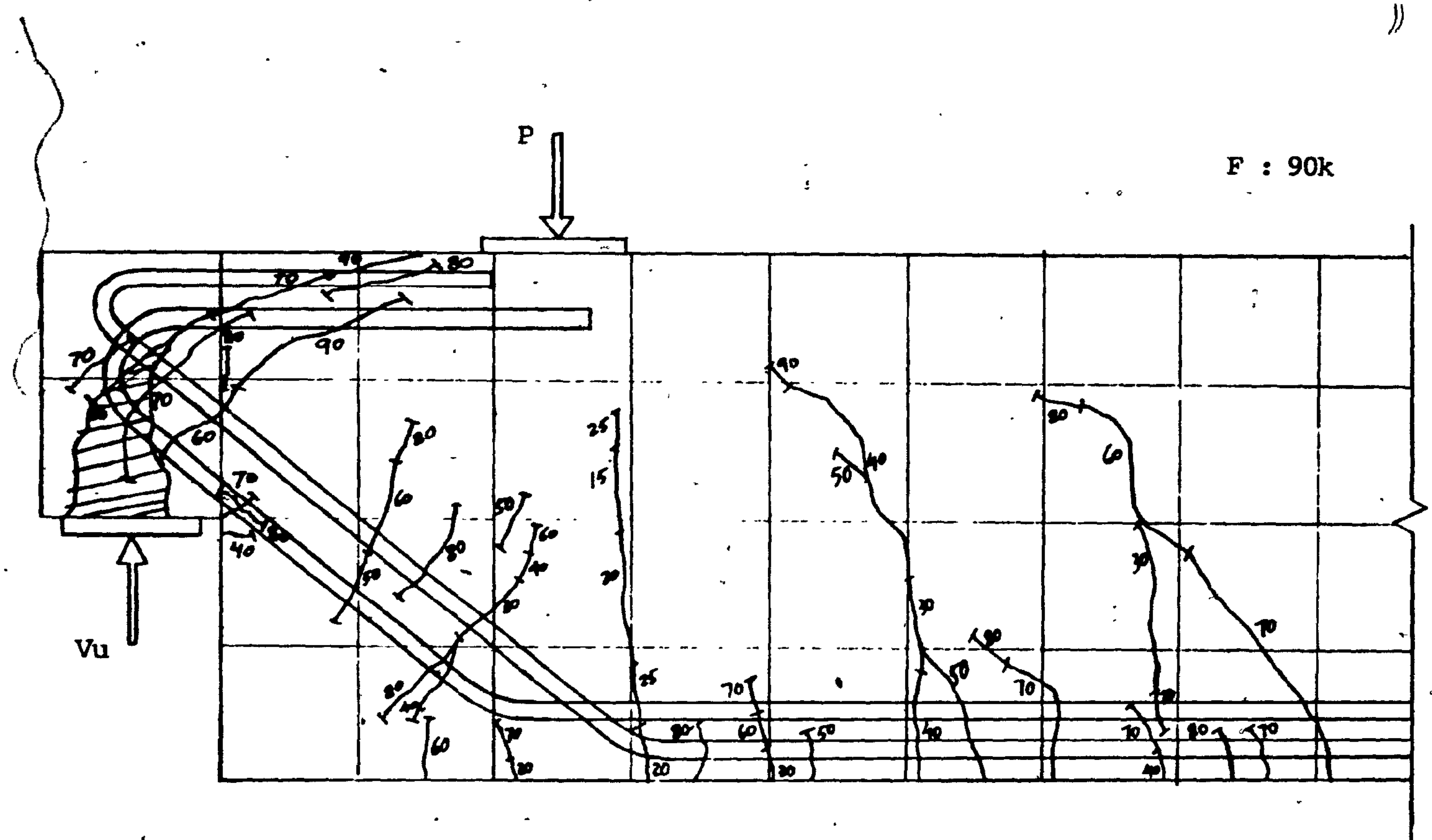


Fig. A13. Crack Patterns of Specimen B4R (Back)

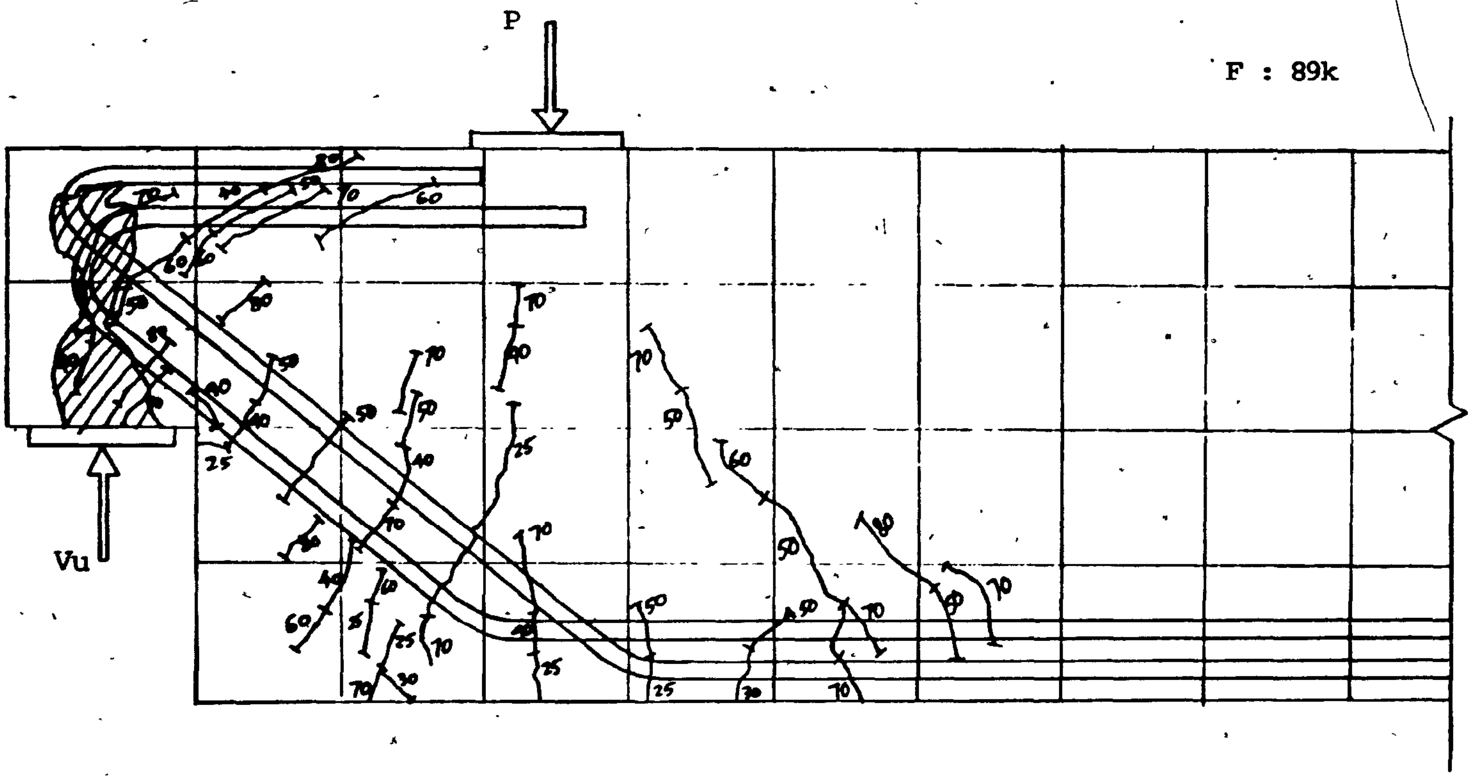


Fig. A14. Crack Patterns of Specimen B4L (Front)

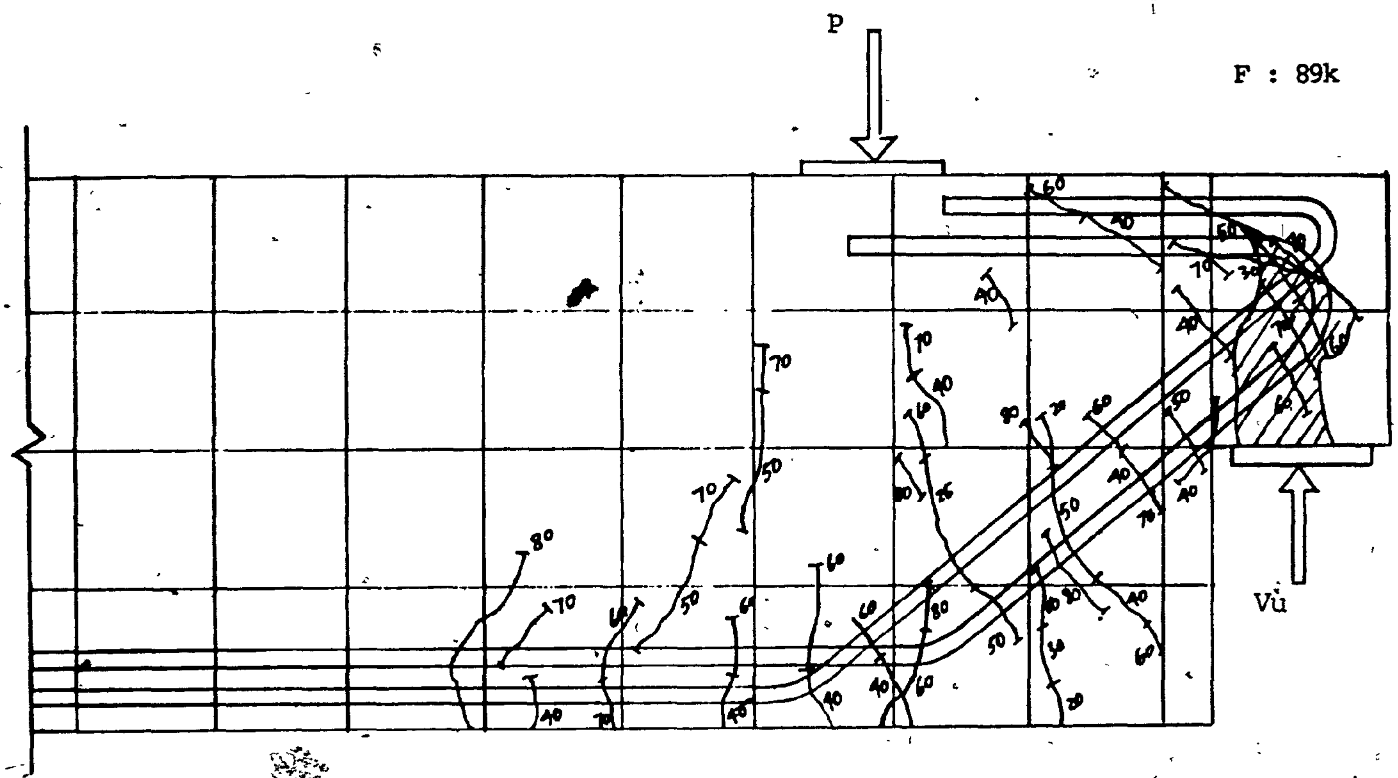
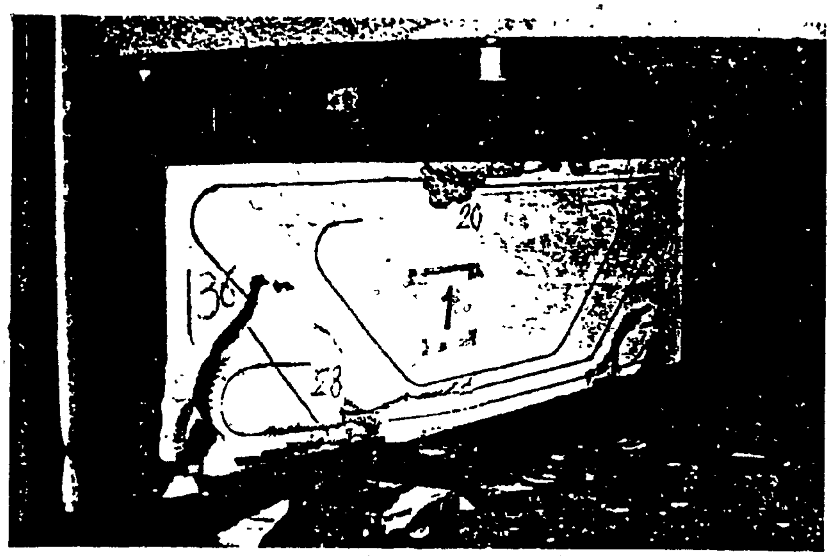


Fig. A15. Crack Patterns of Specimen B4L (Back)

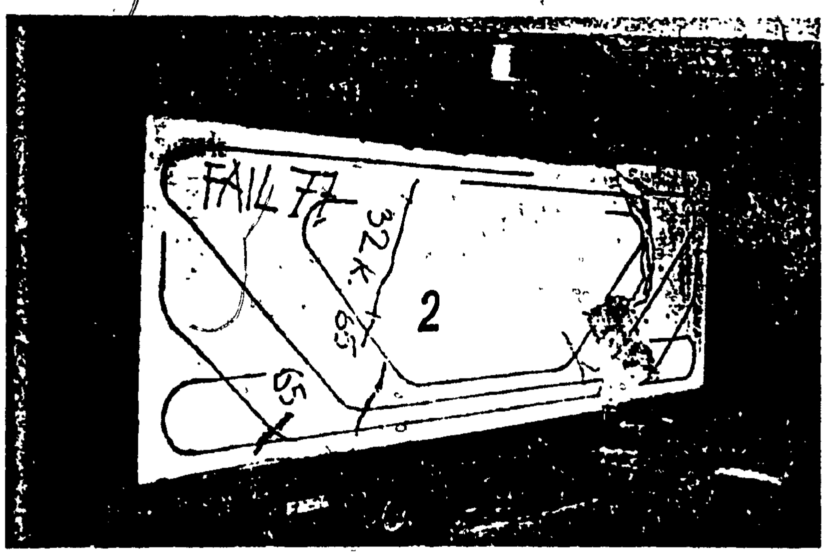
APPENDIX B

CRACK PATTERNS

(Photographs)

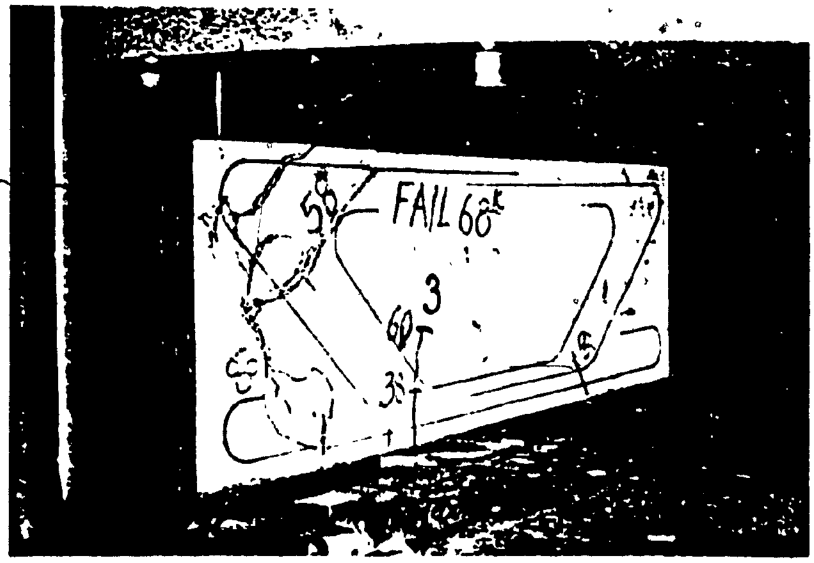


F = 36.4k

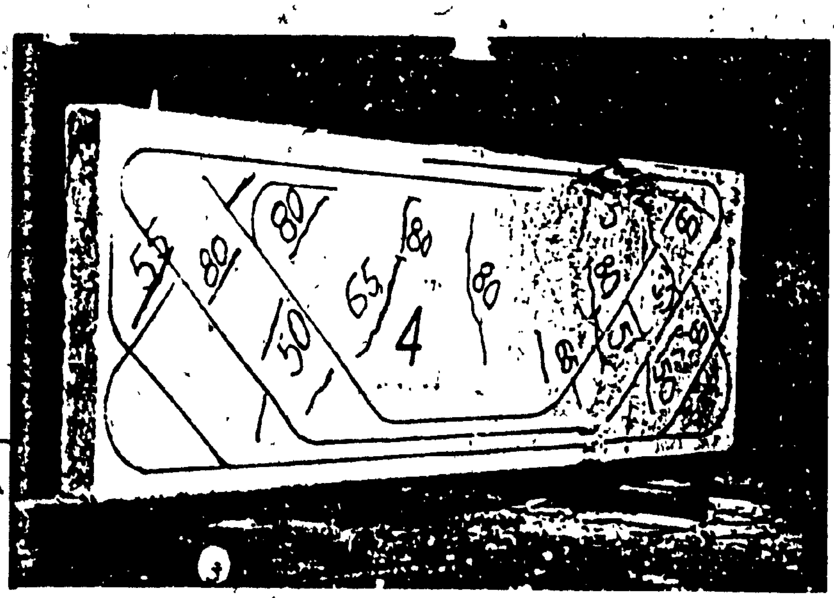


F = 77k

Fig. B1 Crack Pattern of A1 and A2

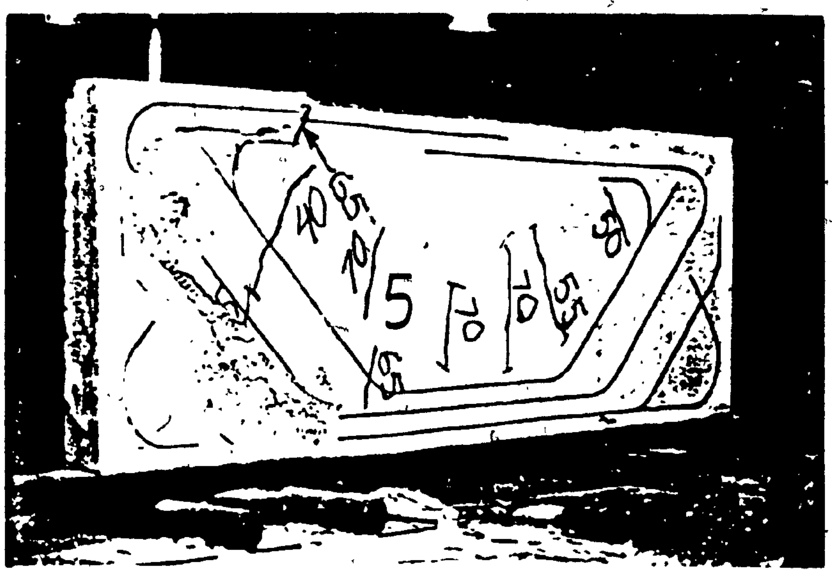


F = 68k

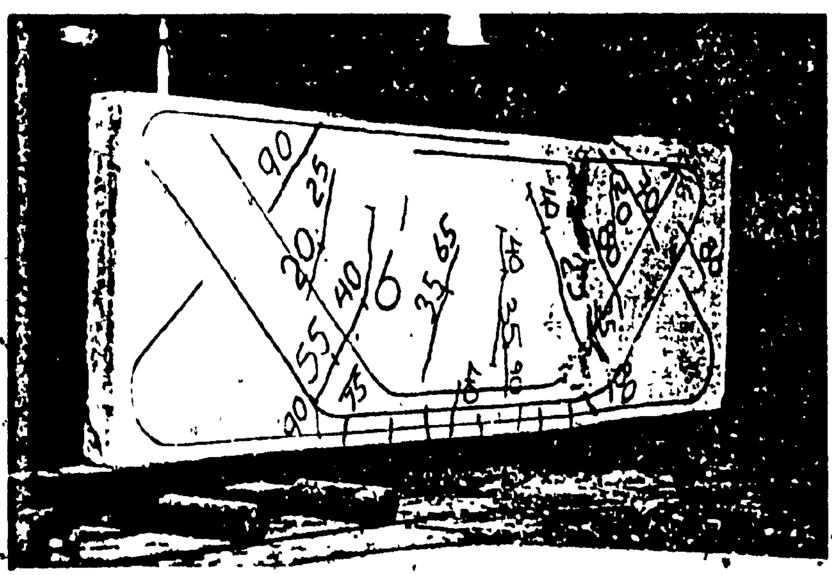


F = 90k

Fig. B2 Crack Patterns of A3 and B4



F = 90k



F = 90k

Fig. B3 Crack Patterns of B5 and B6

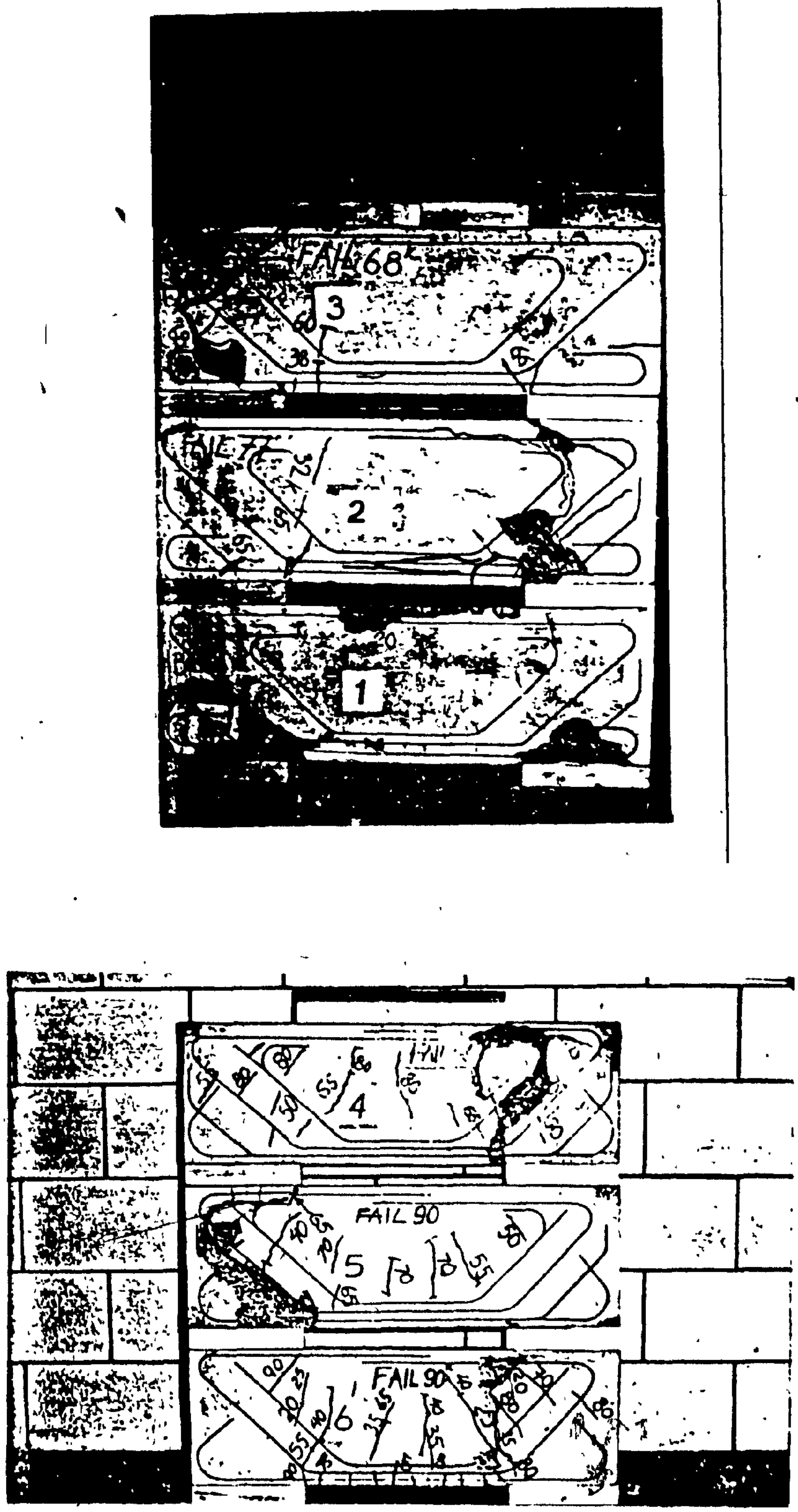
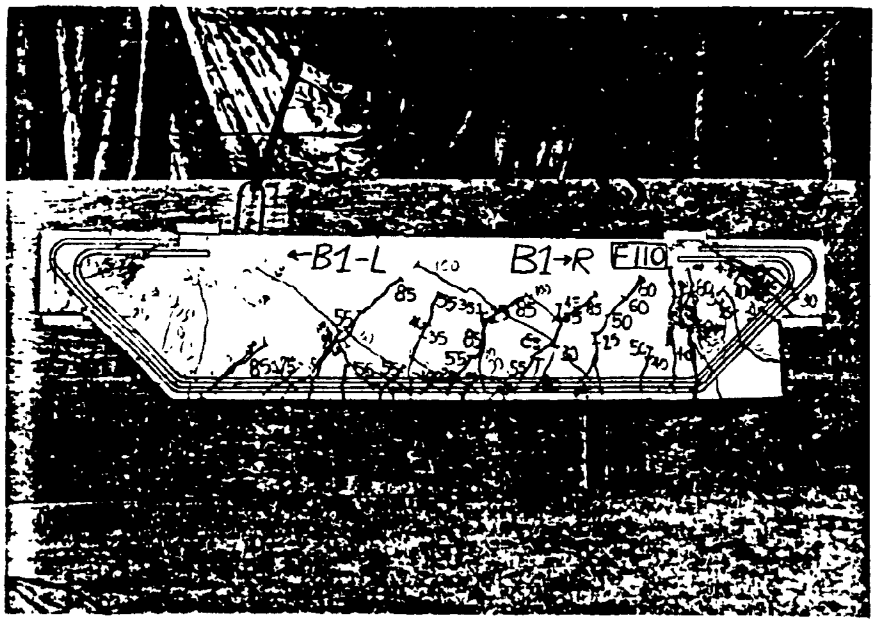
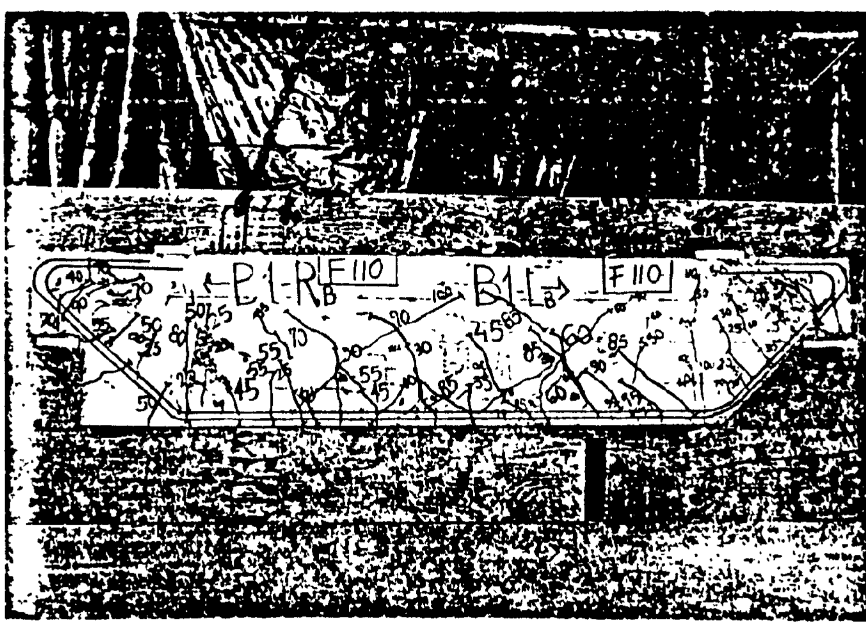


Fig. B4 Cracks pattern of all deep beam tested

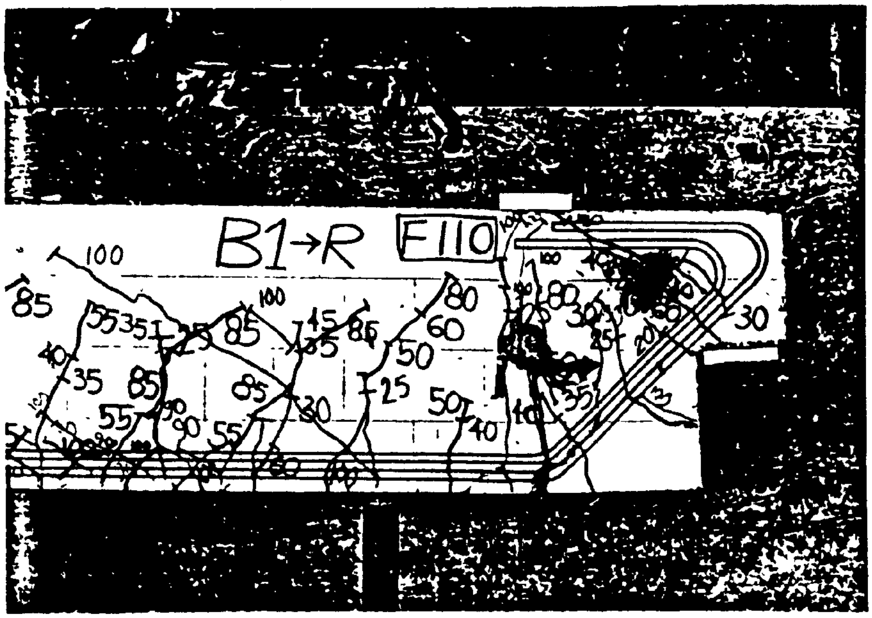


(front)



(back)

Fig. B5 Cracks pattern on overall beam (front and back face of the beam, B1)



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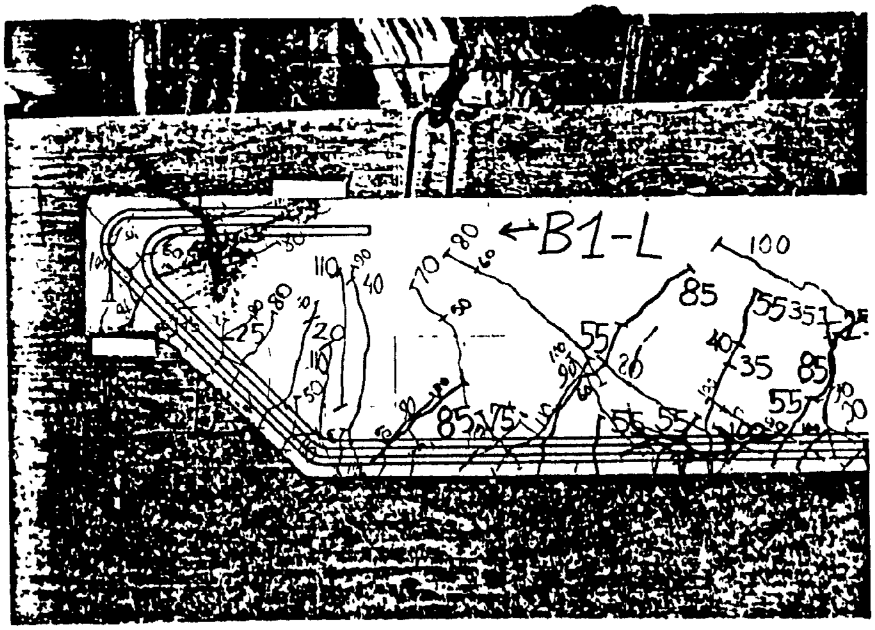

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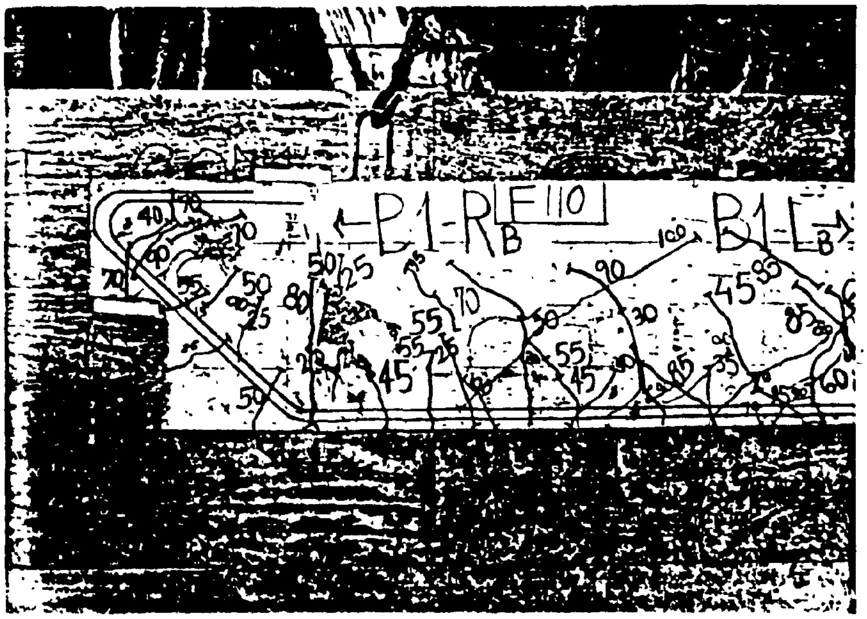
Fig. B6 Cracks pattern on right and left side of the beam, B1



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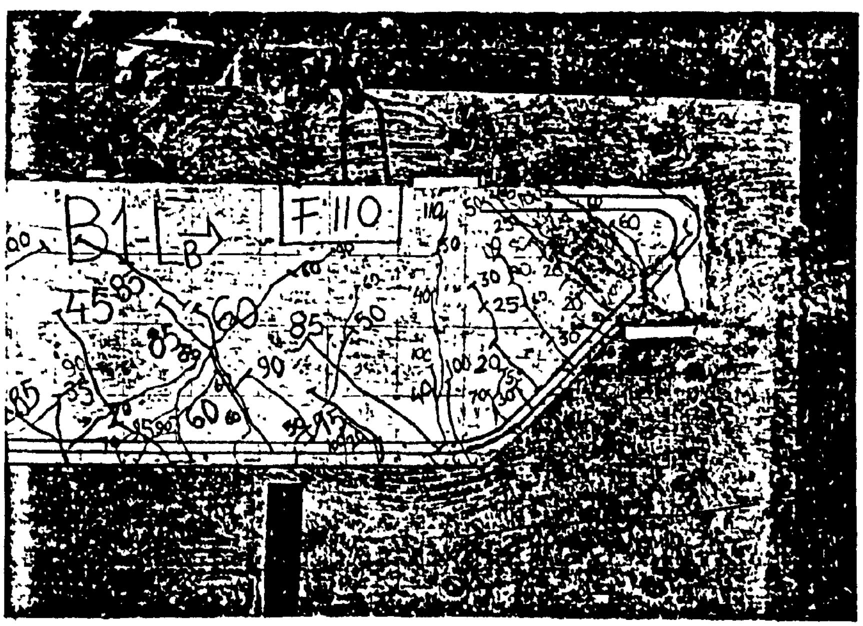

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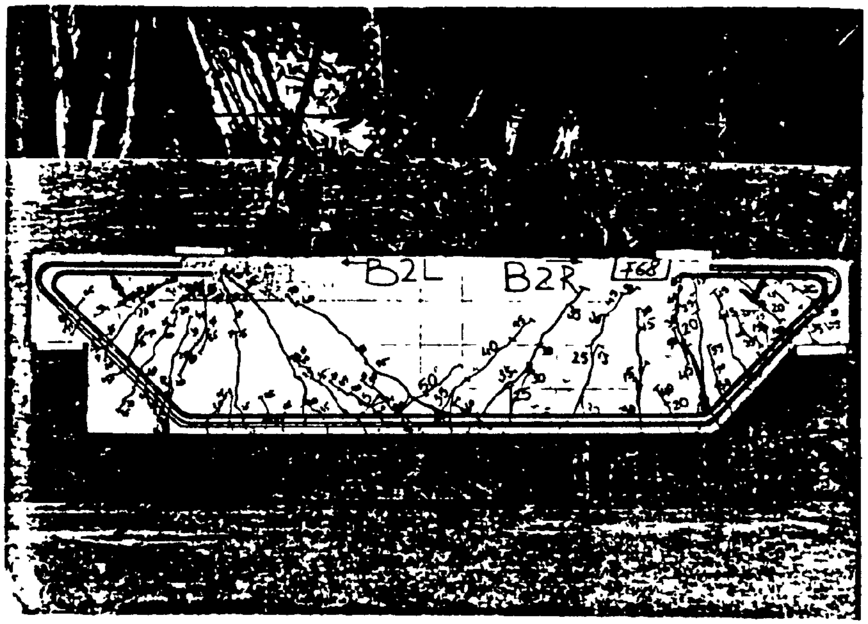
Fig. B7 Cracks pattern on left and right side of the beam, B1



(front)

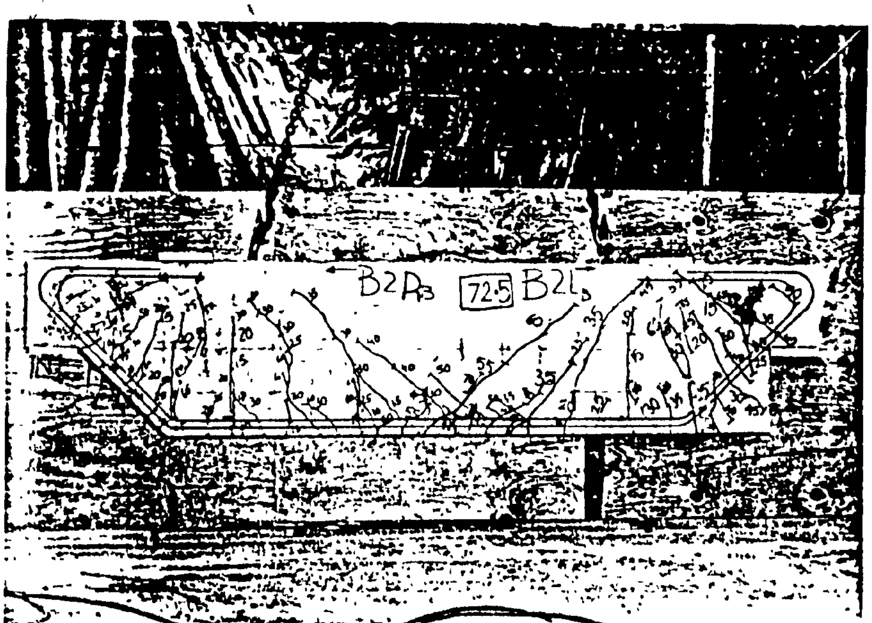


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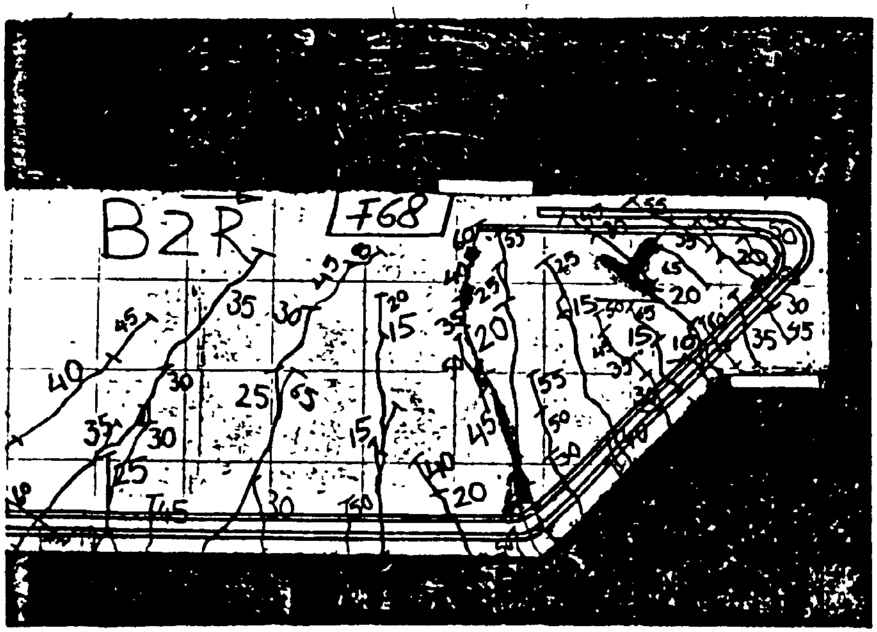
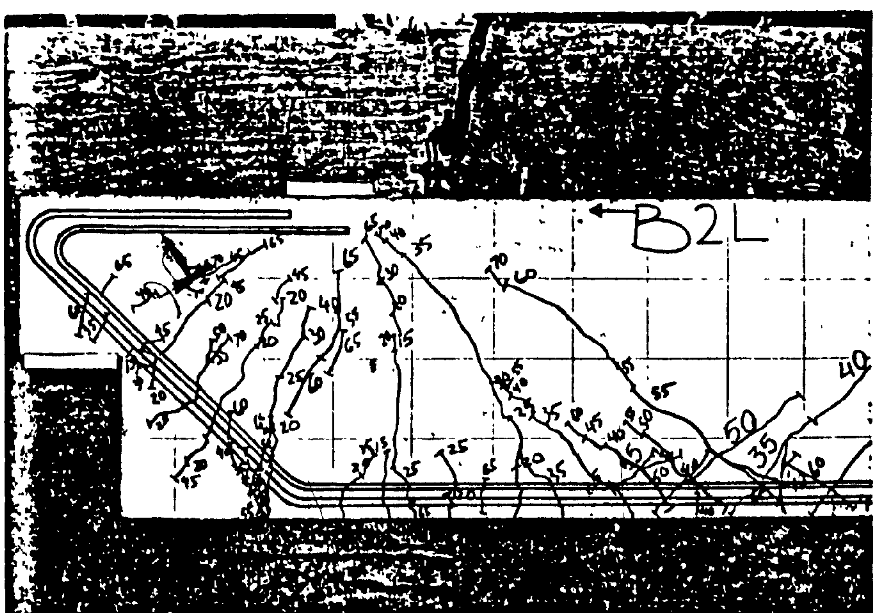
Fig. B8 Cracks pattern on overall beam (front and back face of the beam, B2)

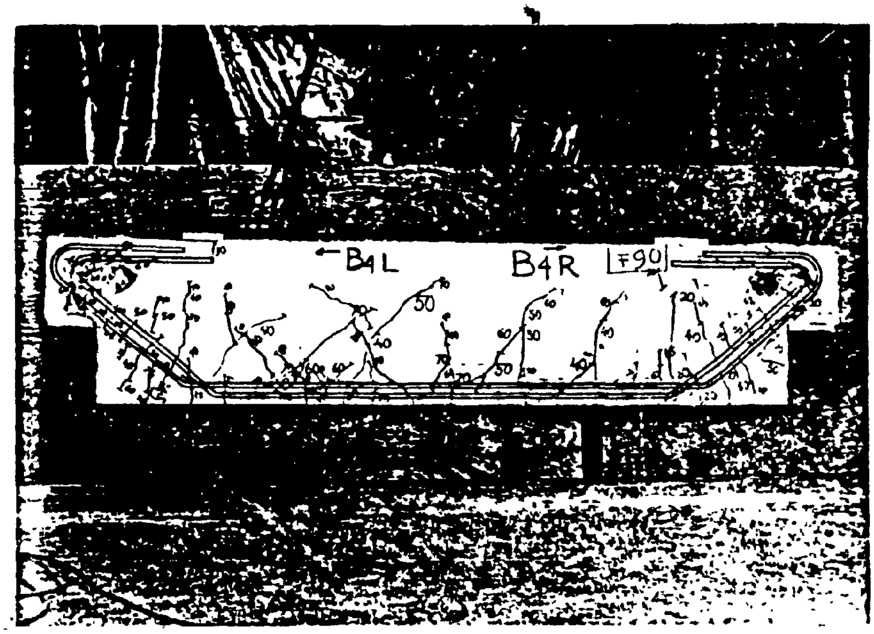


(front)



(front)

Fig. B9 Cracks pattern on right and left side of the beam, B2



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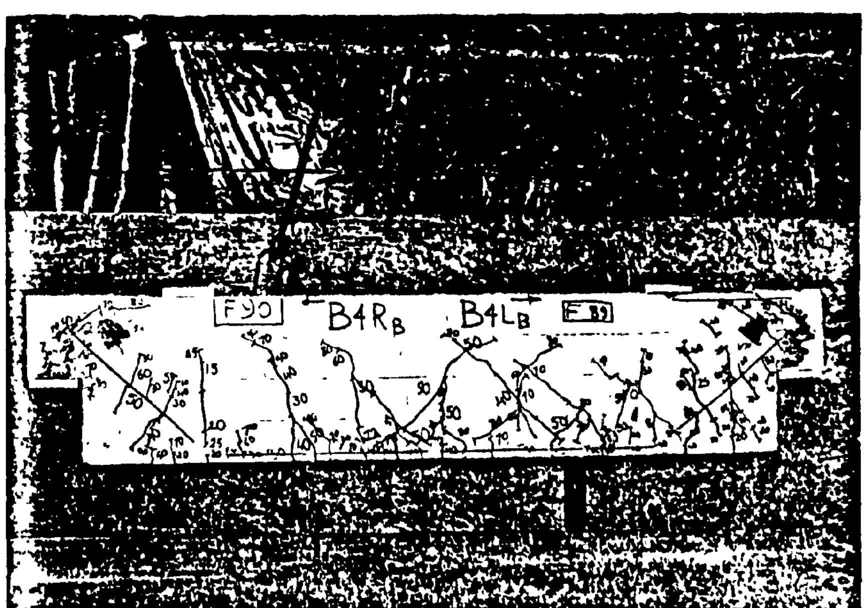
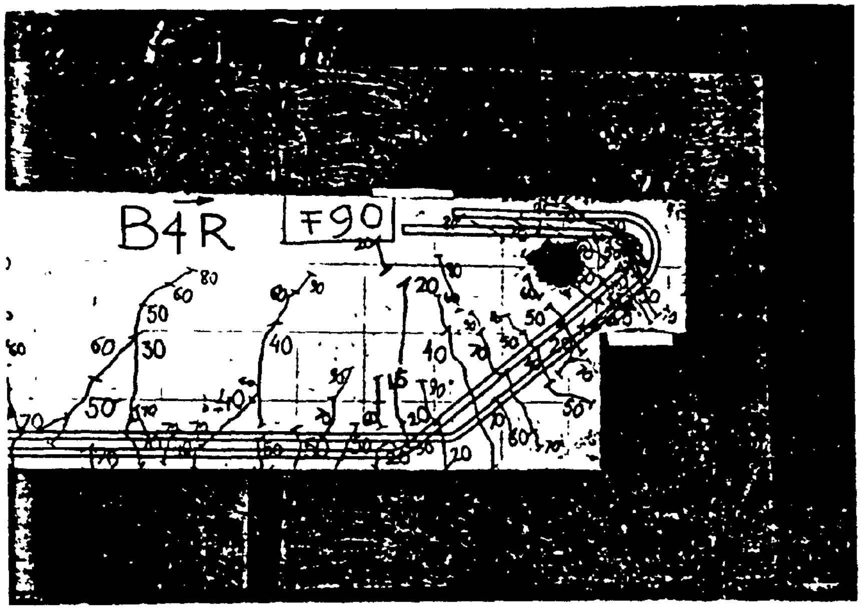
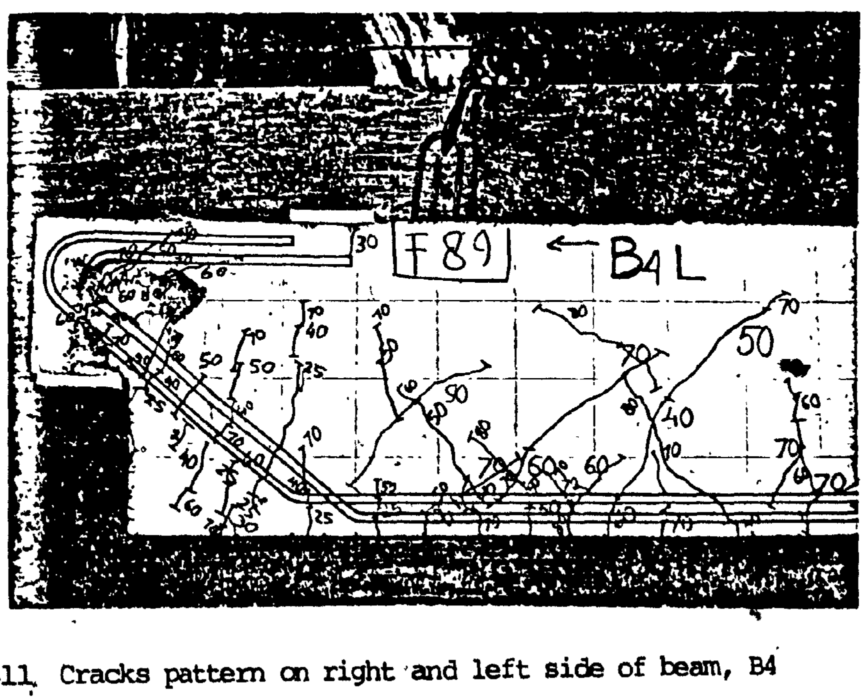


(front)

Fig. B10 Cracks pattern on overall beam (front and back faces of the beam, B4)

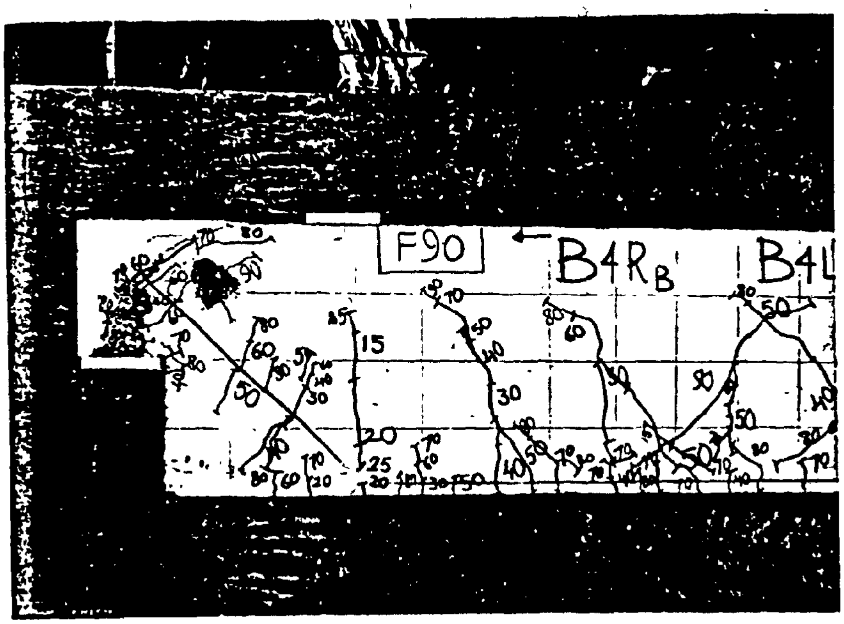


(front)



(front)

Fig. B11. Cracks pattern on right and left side of beam, B4



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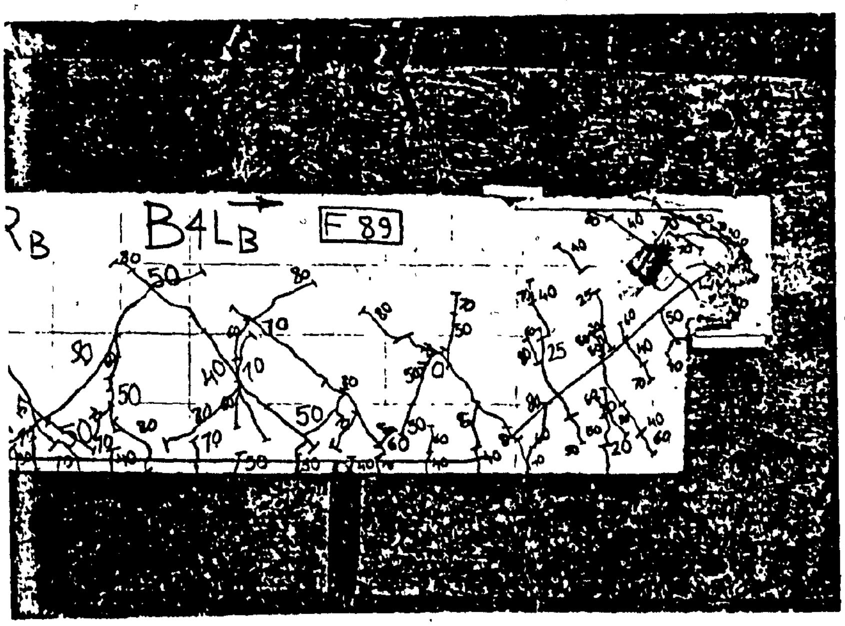


(back)

Fig. B12 Cracks pattern on left and right side of beam, B4

APPENDIX C

LOAD-STRAIN RELATIONSHIP OF

DAPPED-END BEAMS

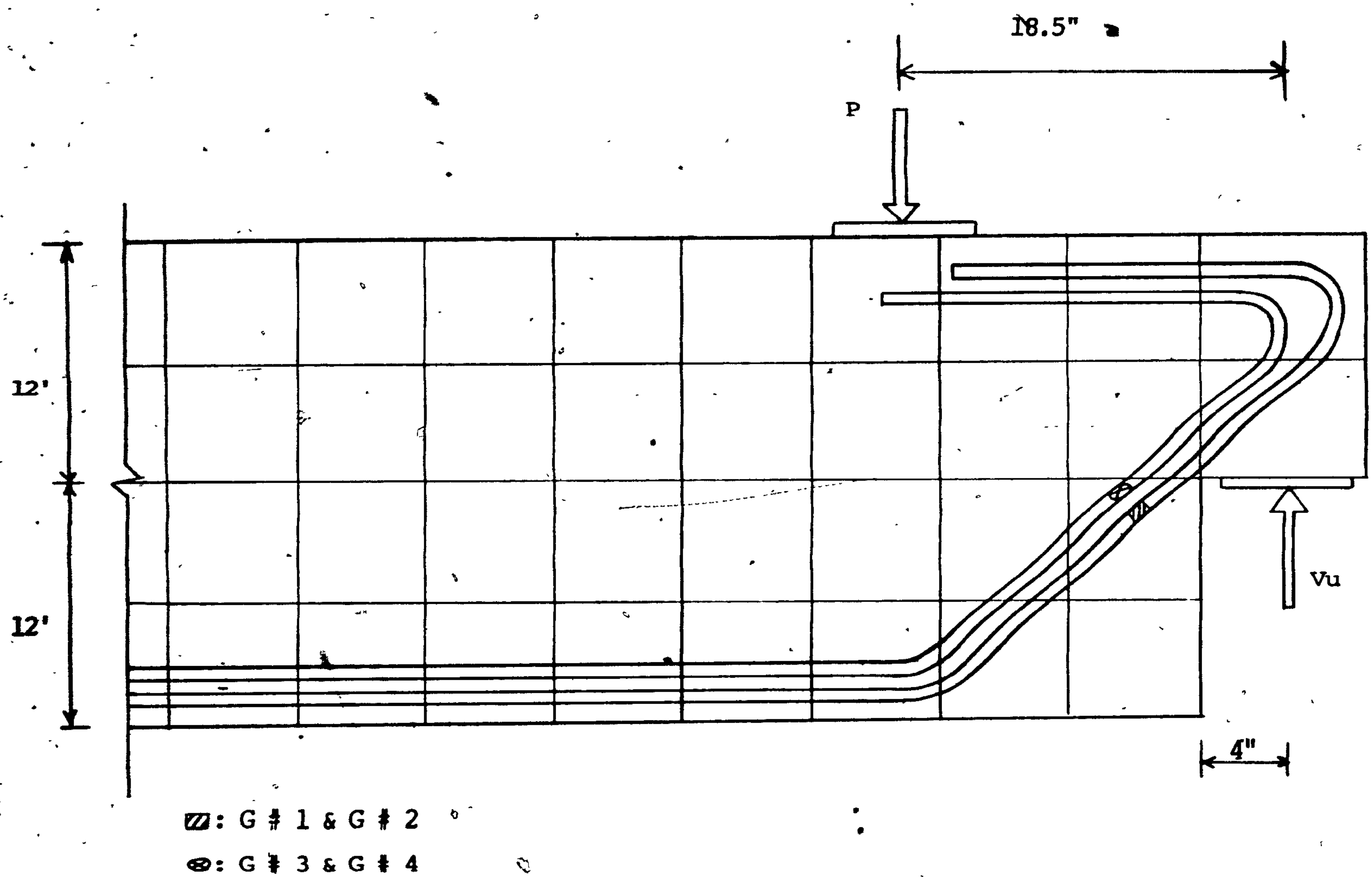
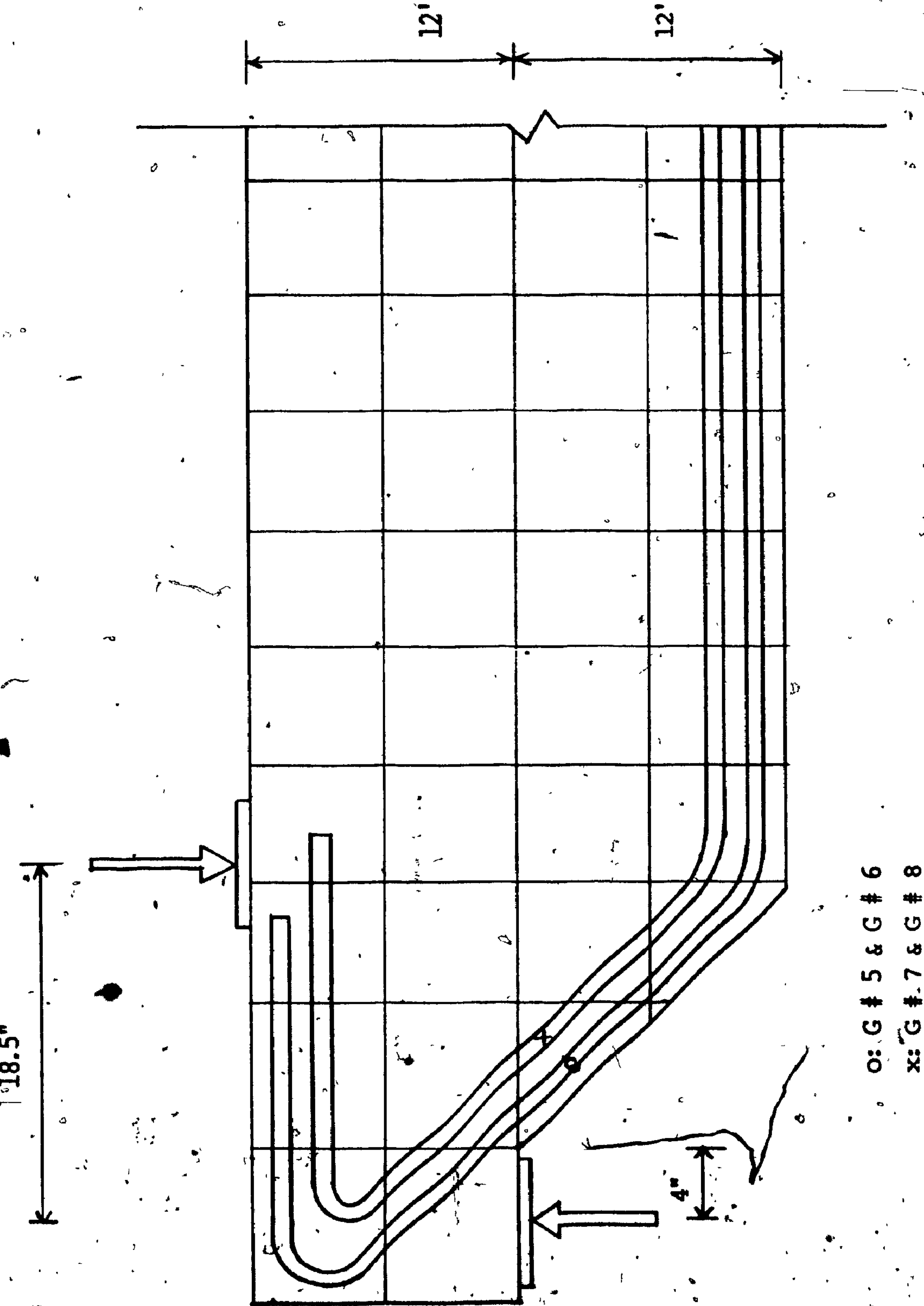


Fig. C1. Locations of the Gauges on Reinforcements


12'
12'
18.5"
4"
o: G # 5 & G # 6
x: G # 7 & G # 8

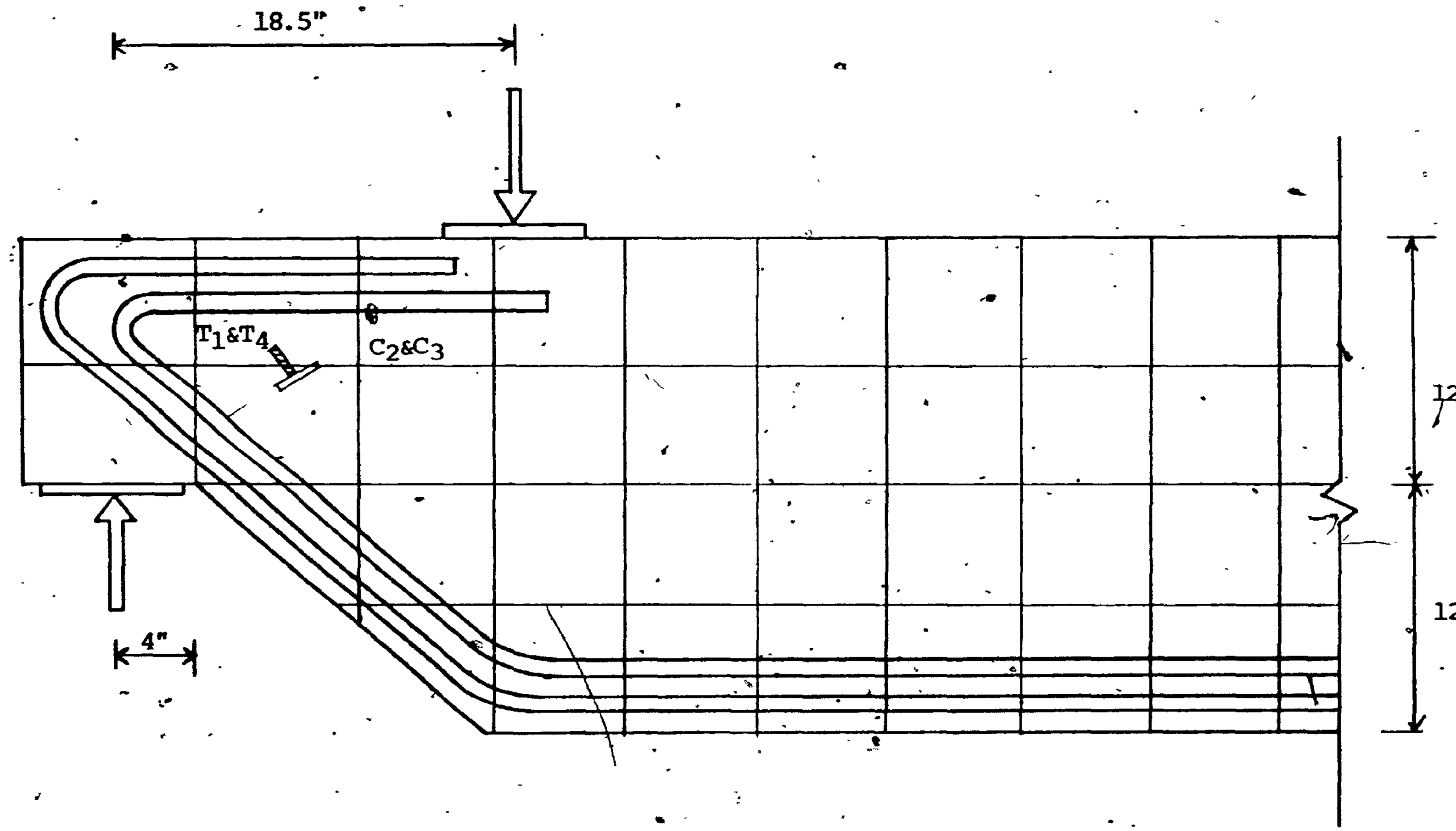


Fig. C3 Gauge Locations on Concrete

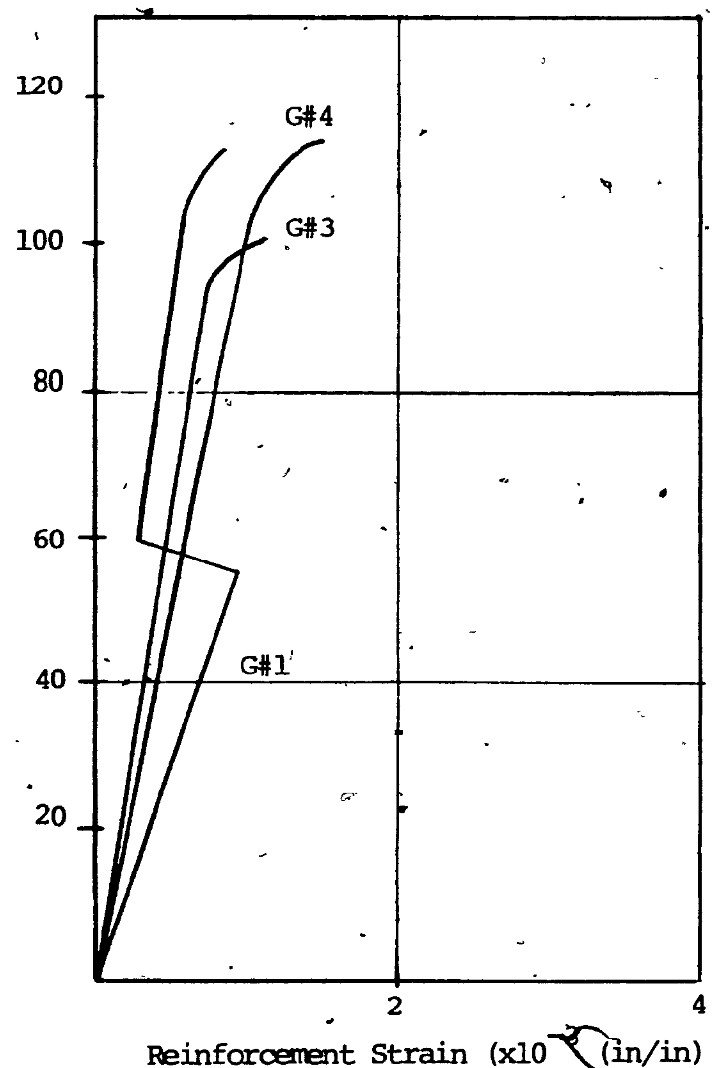


Fig. C4 Load Strain Relationship for Specimen B1R

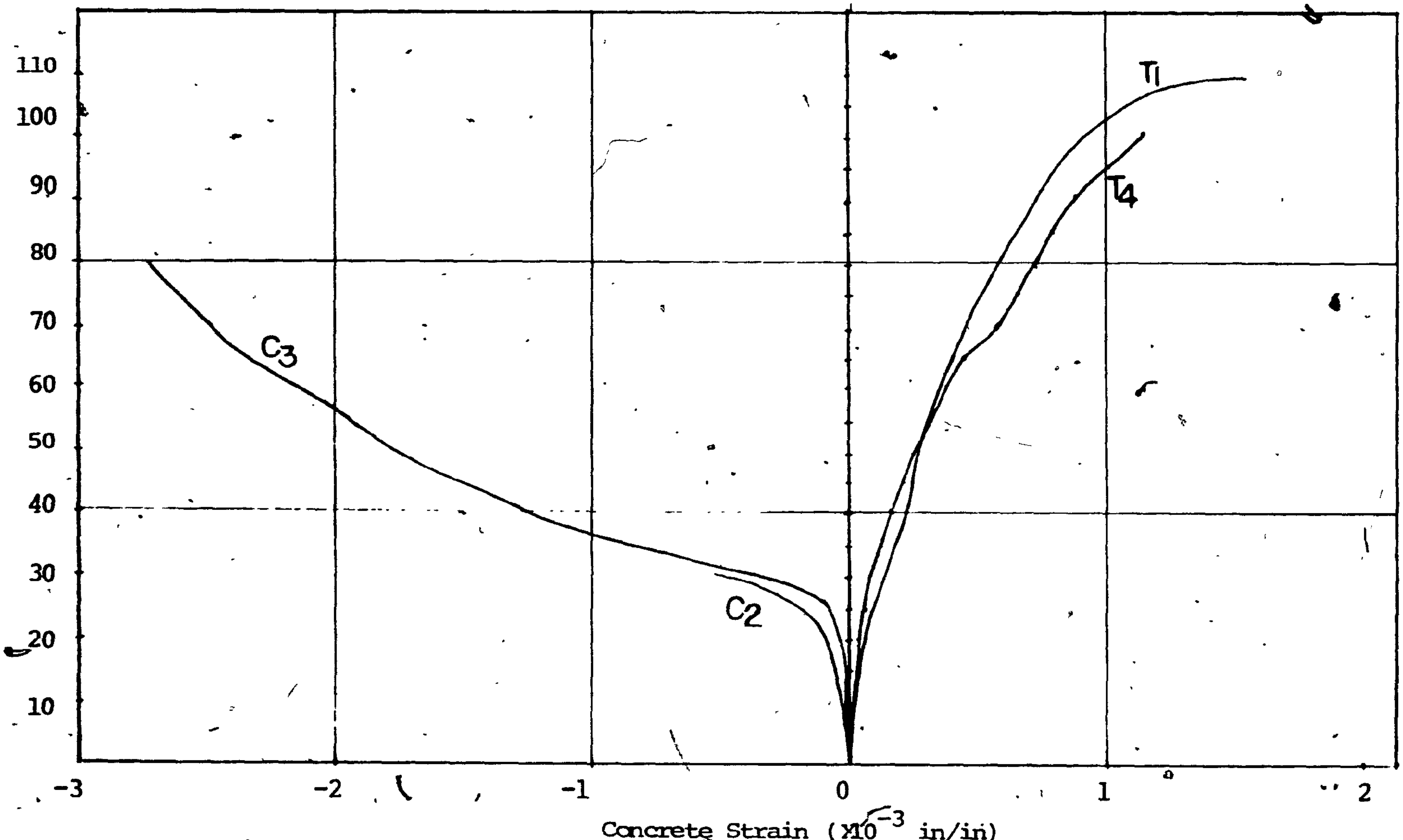


Fig. C5 Load-Strain Relationship for Specimen B1R

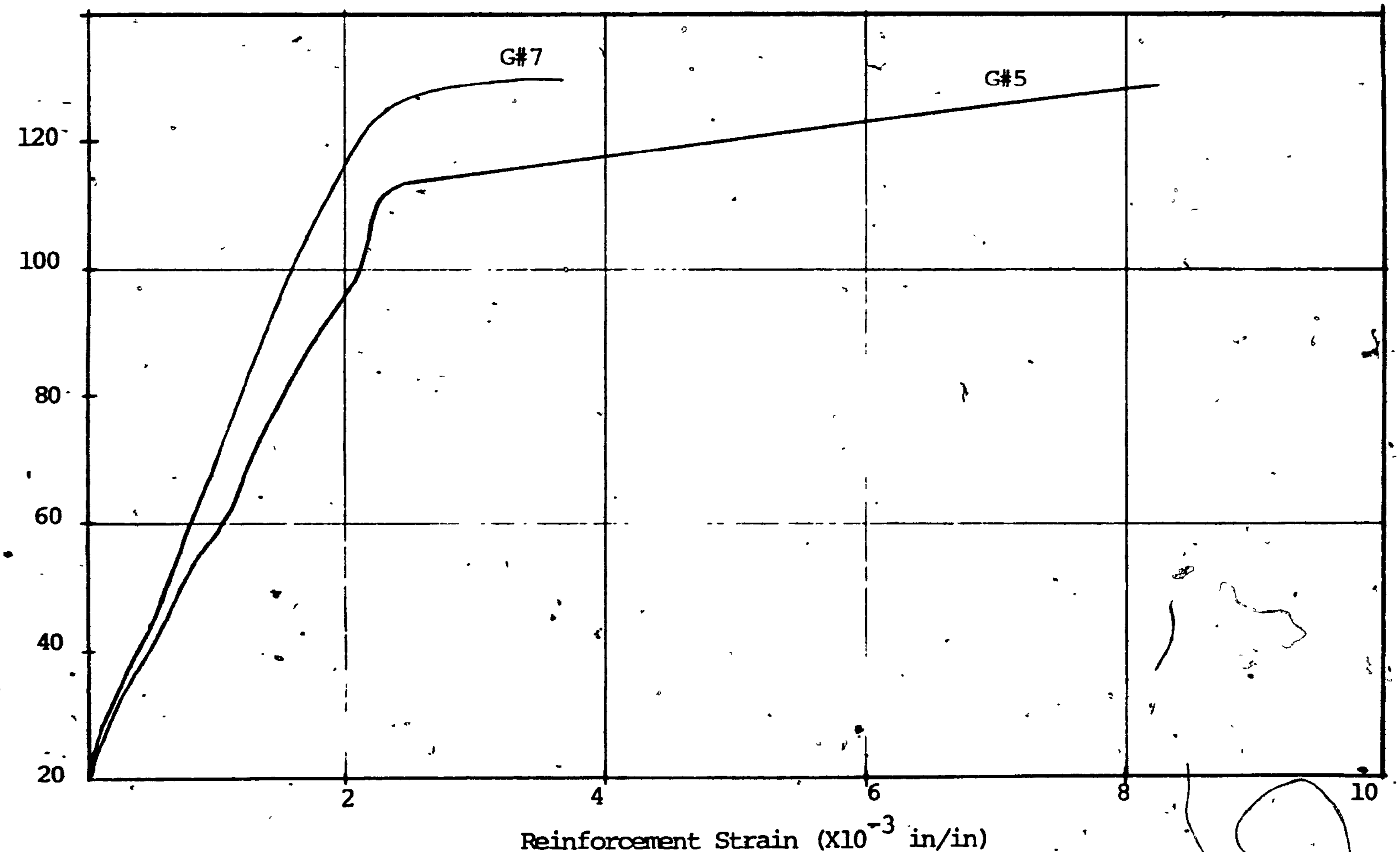


Fig. C6 Load-Strain Relationship for Specimen B1L

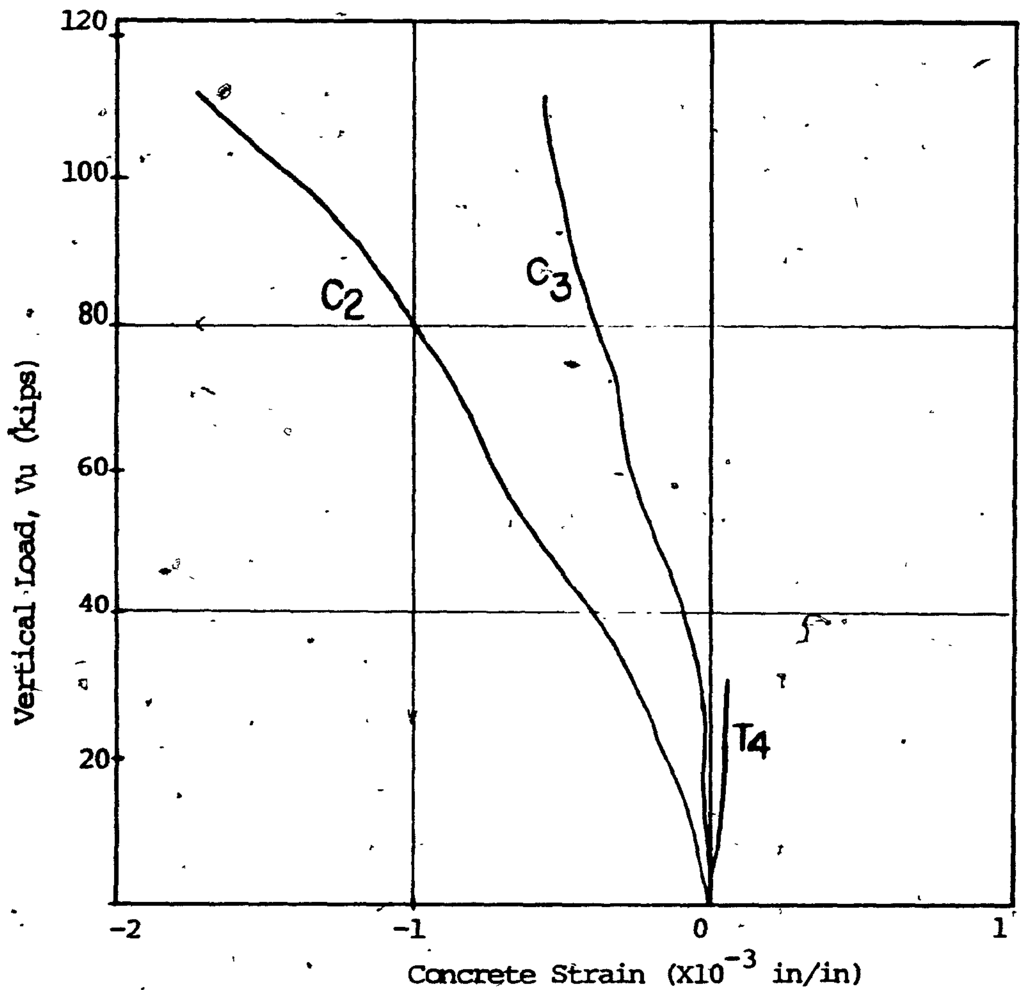


Fig. C7. Load-Strain Relationship for Specimen B1L

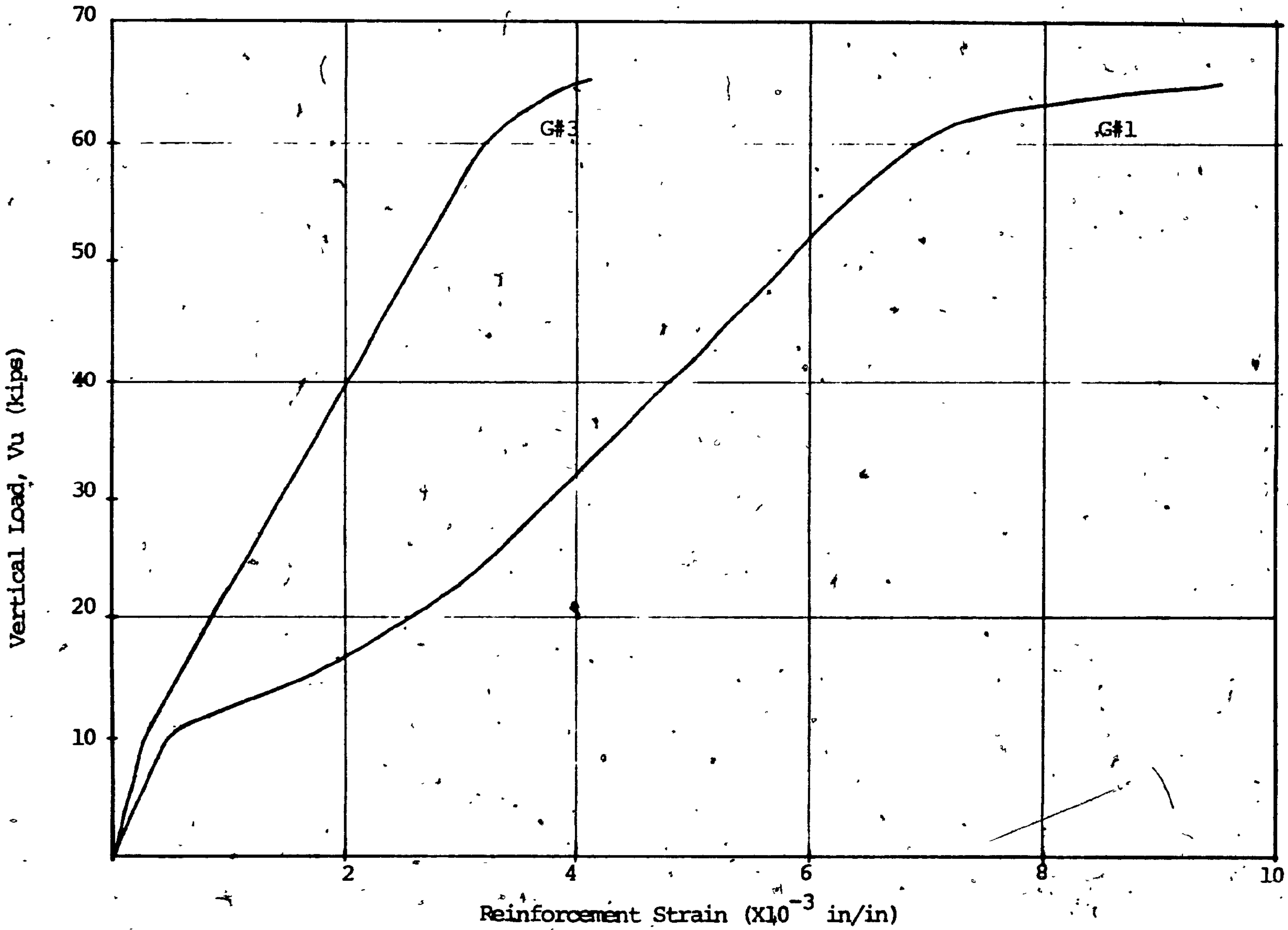


Fig. C8 Load-Strain Relationship for Specimen B2R

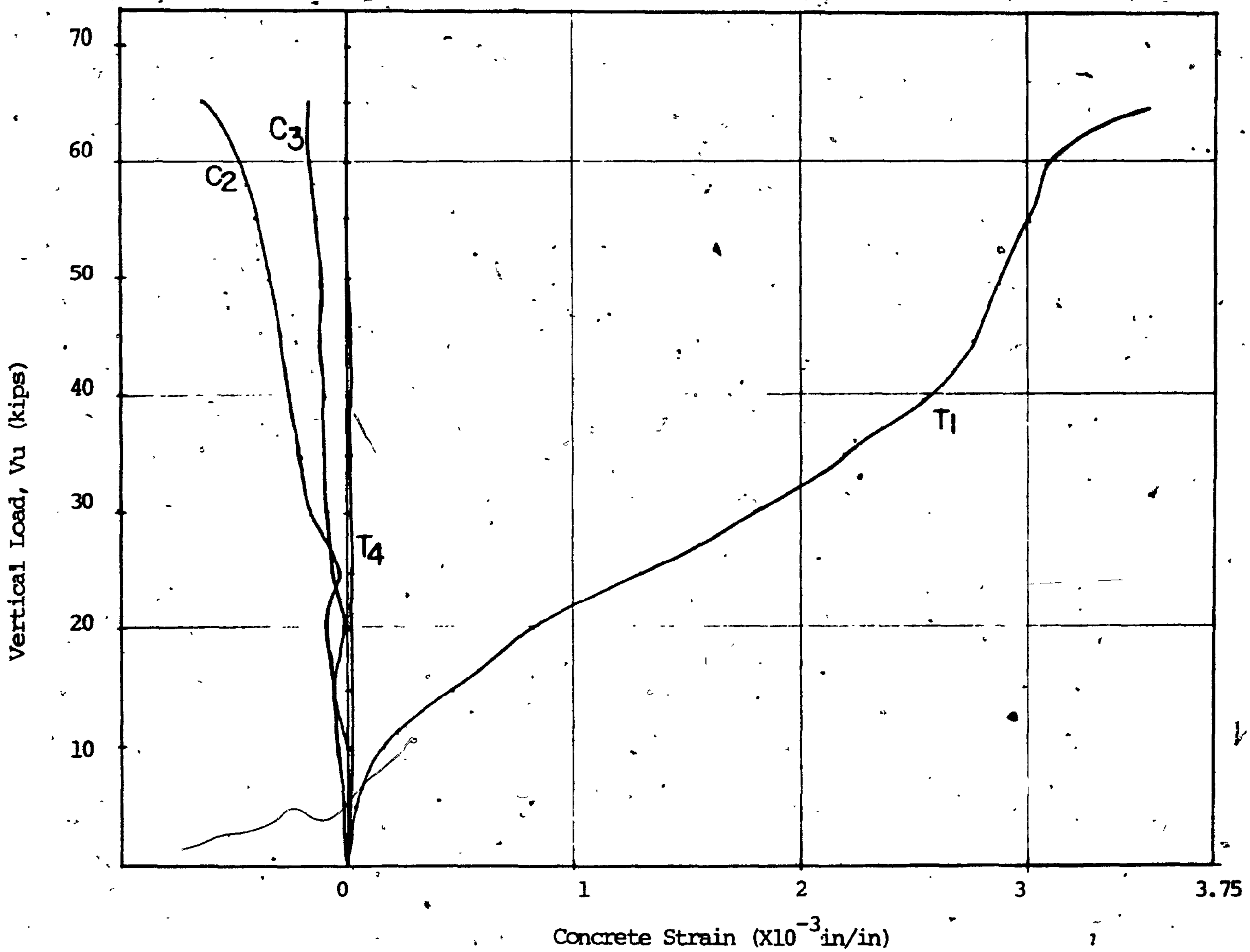


Fig. C9 Load-Strain Relationship for Specimen B2R

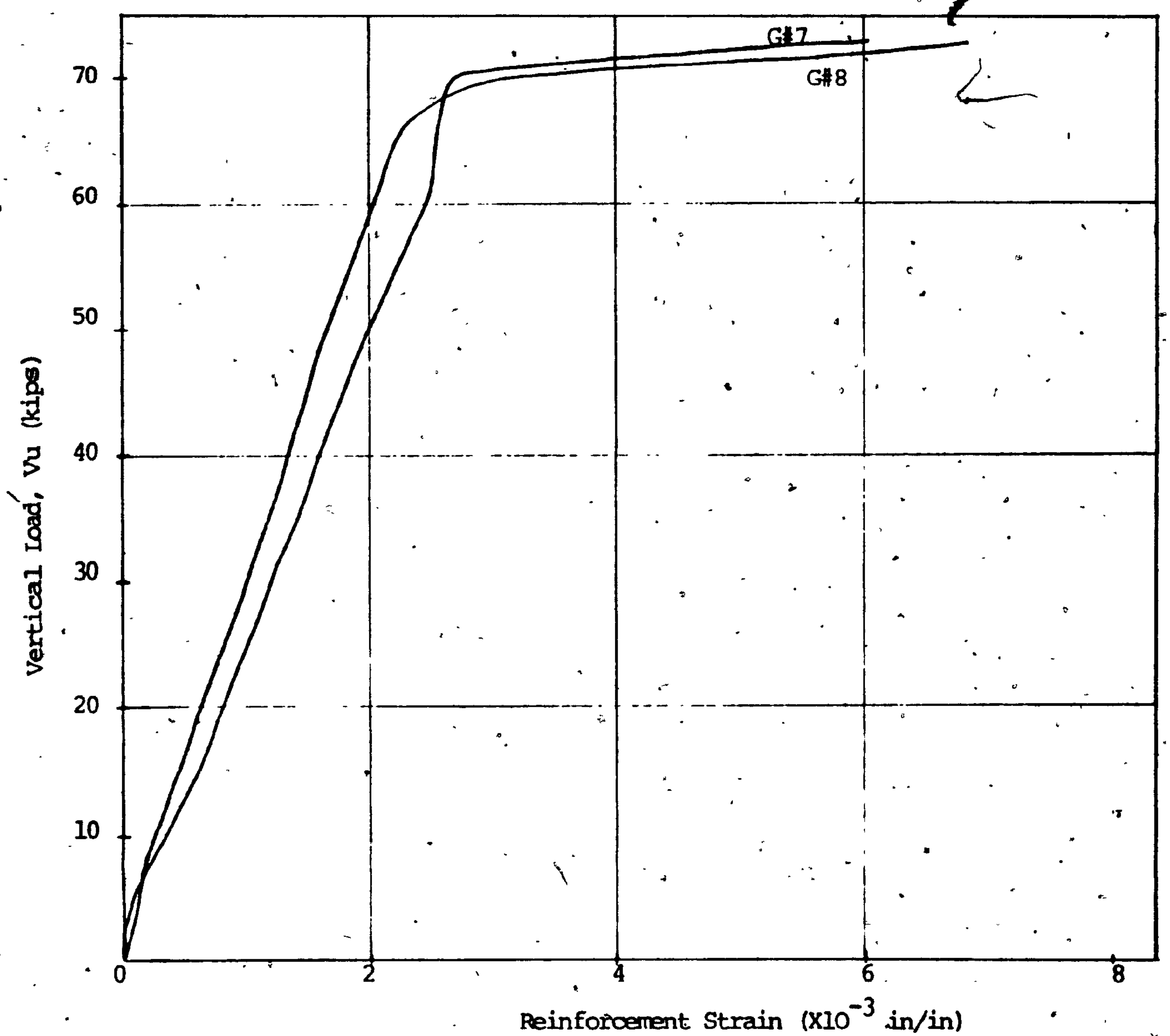


Fig. C10 Load-Strain Relationship for Specimen B2L

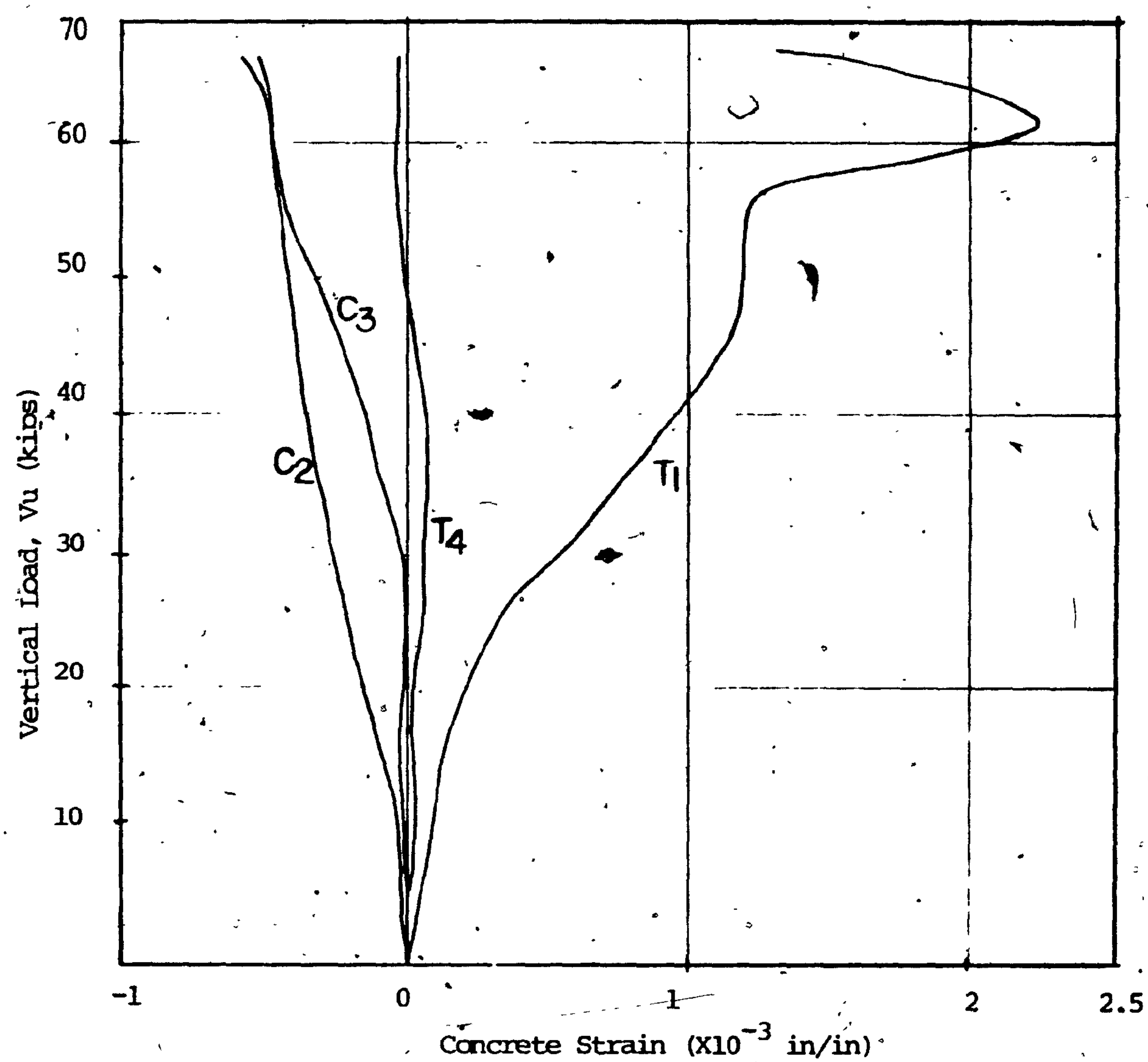


Fig. C11 Load-Strain Relationship for Specimen B2L

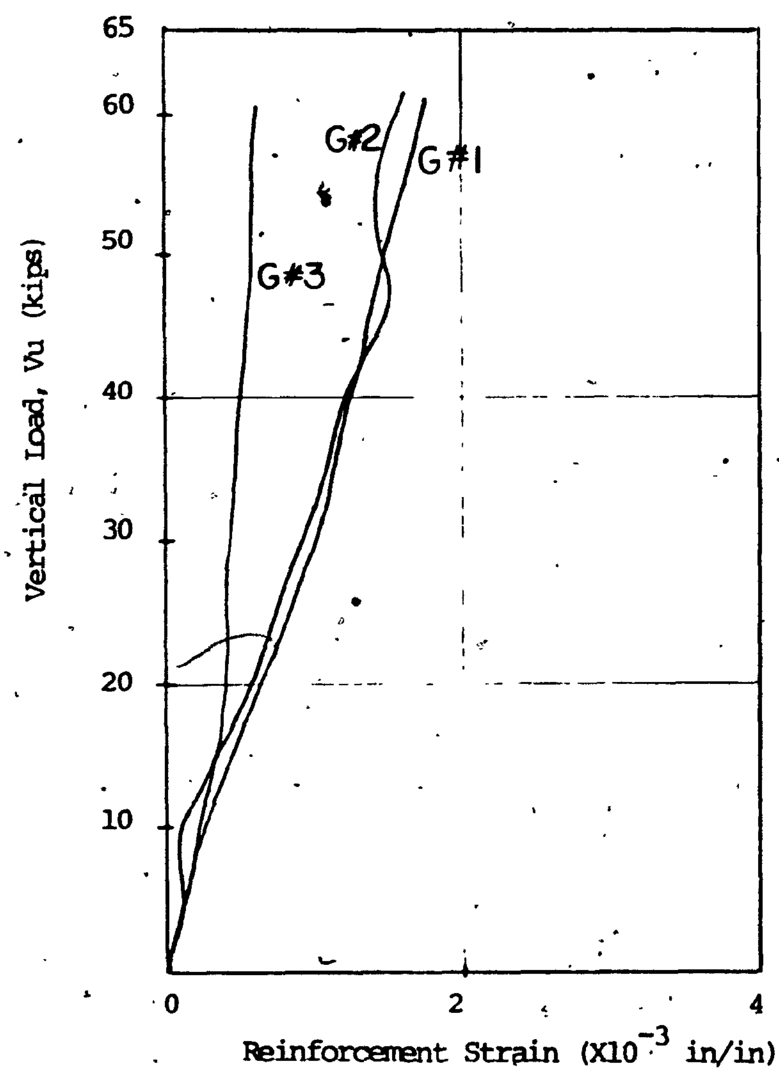


Fig. C12 Load-Strain Relationship for Speciment B3R

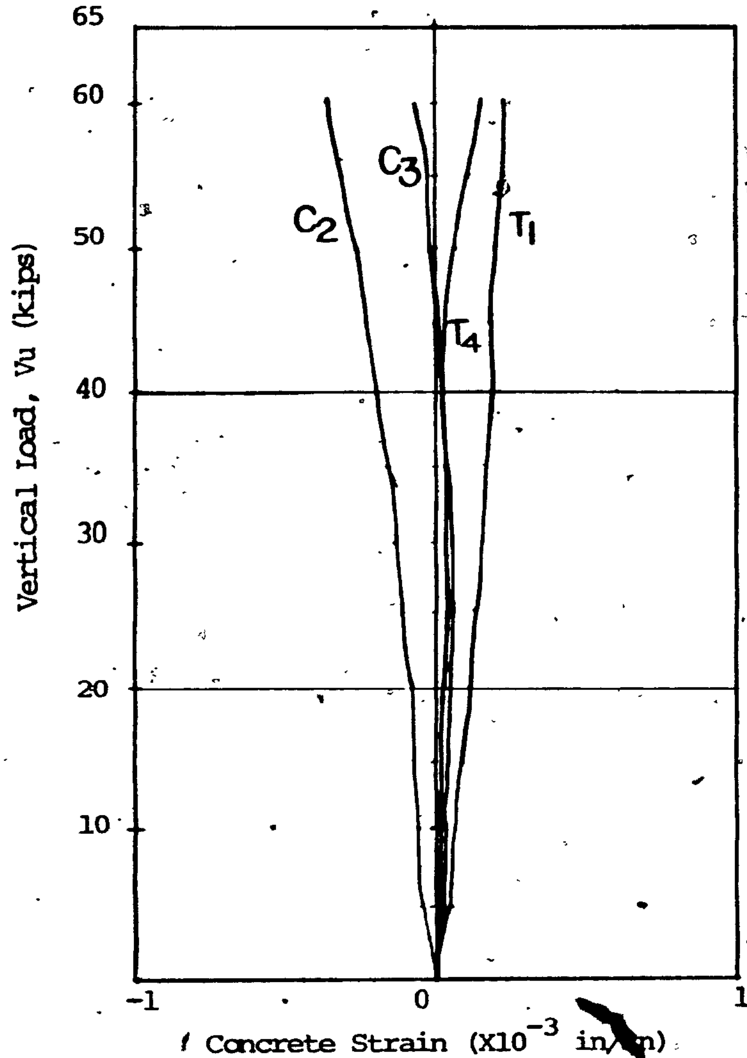


Fig. C13 Load Strain Relationship for Specimen B3R

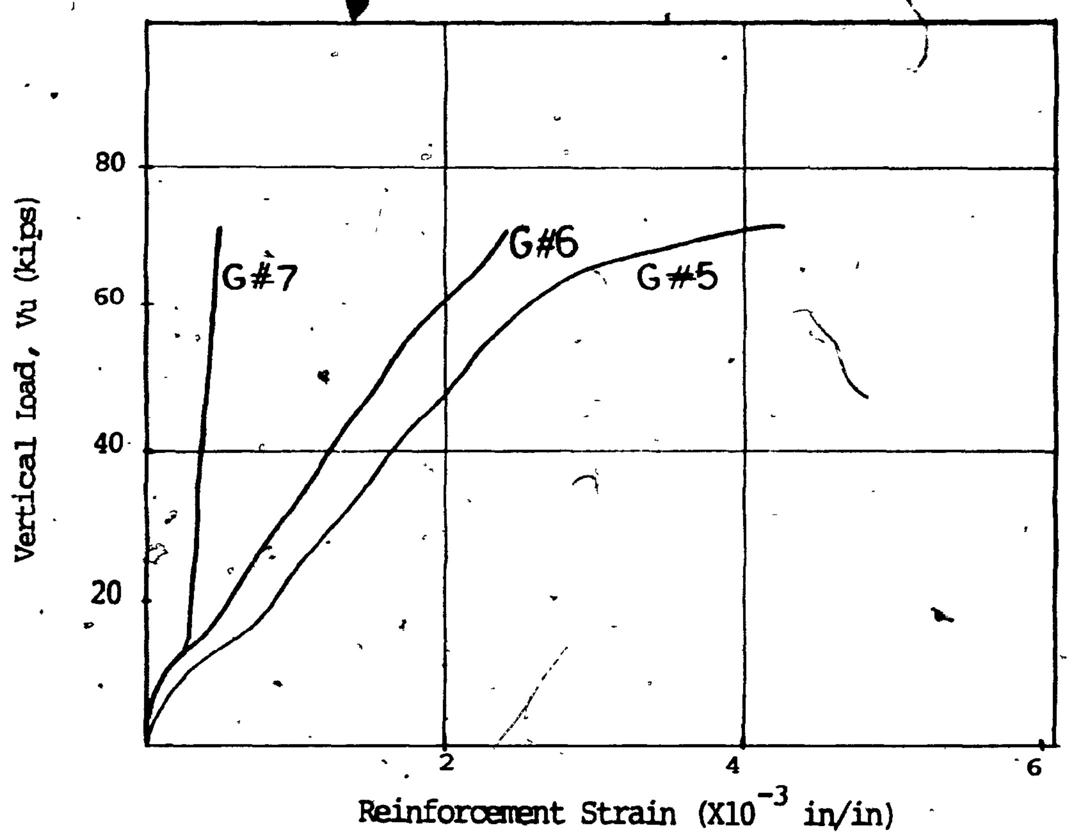


Fig. C14 Load Strain Relationships for Specimen B3L

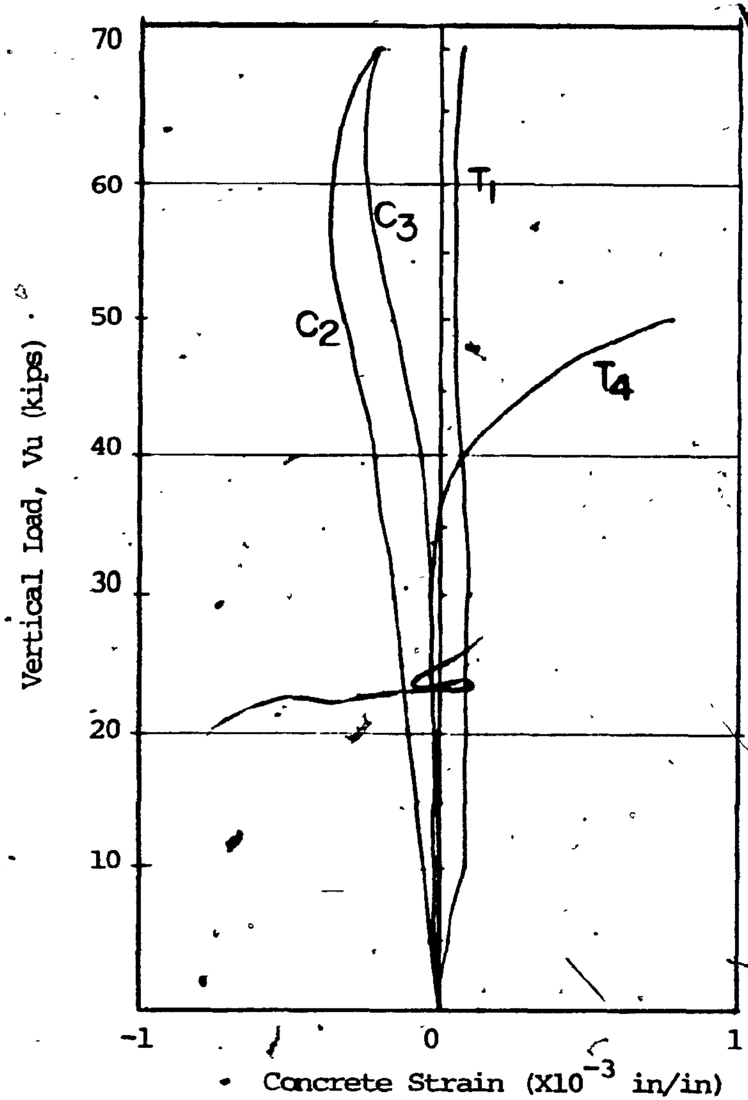


Fig. C15 Load-Strain Relationship for Specimen B3L

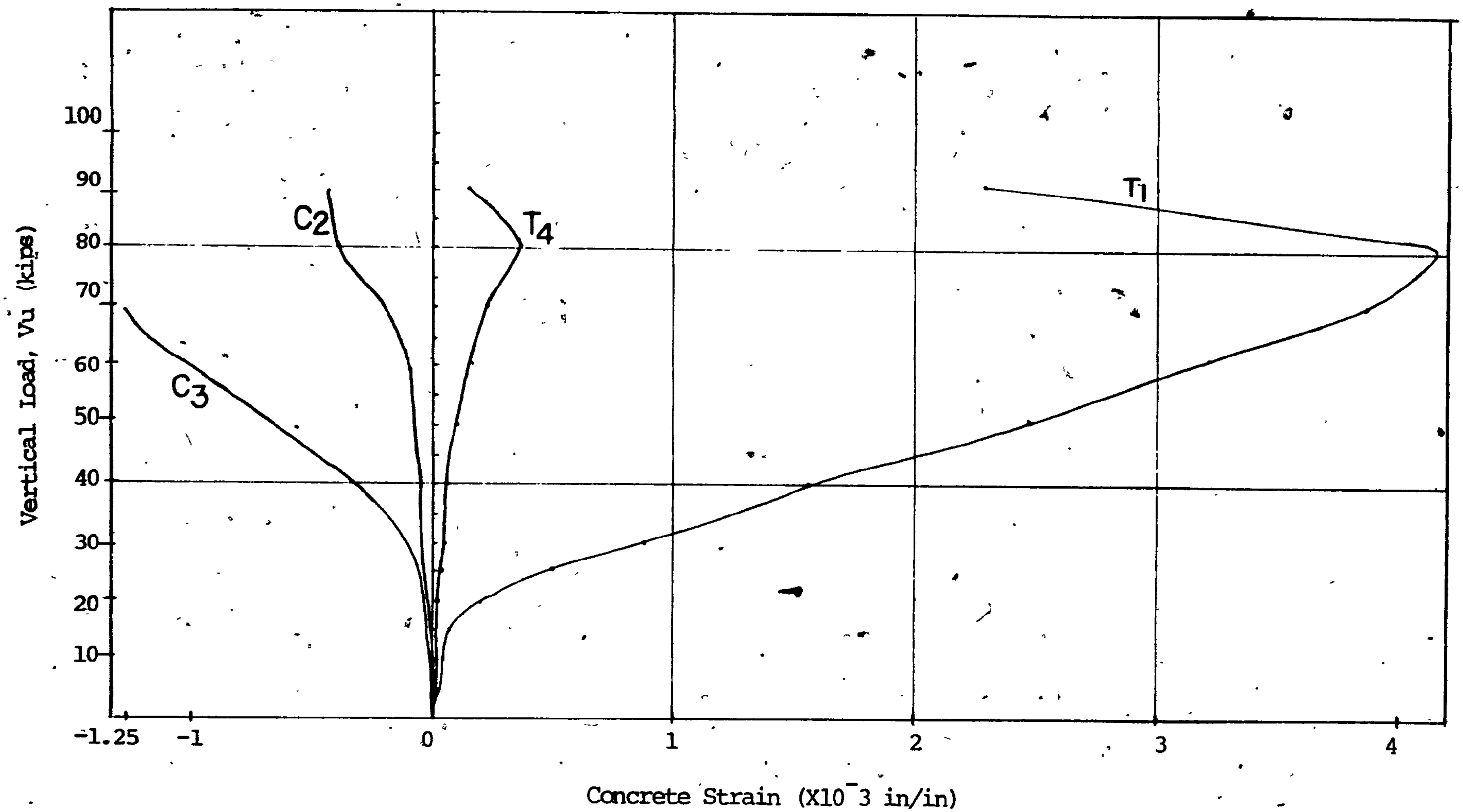


Fig. C16 Load-Strain Relationship for Specimen B4R

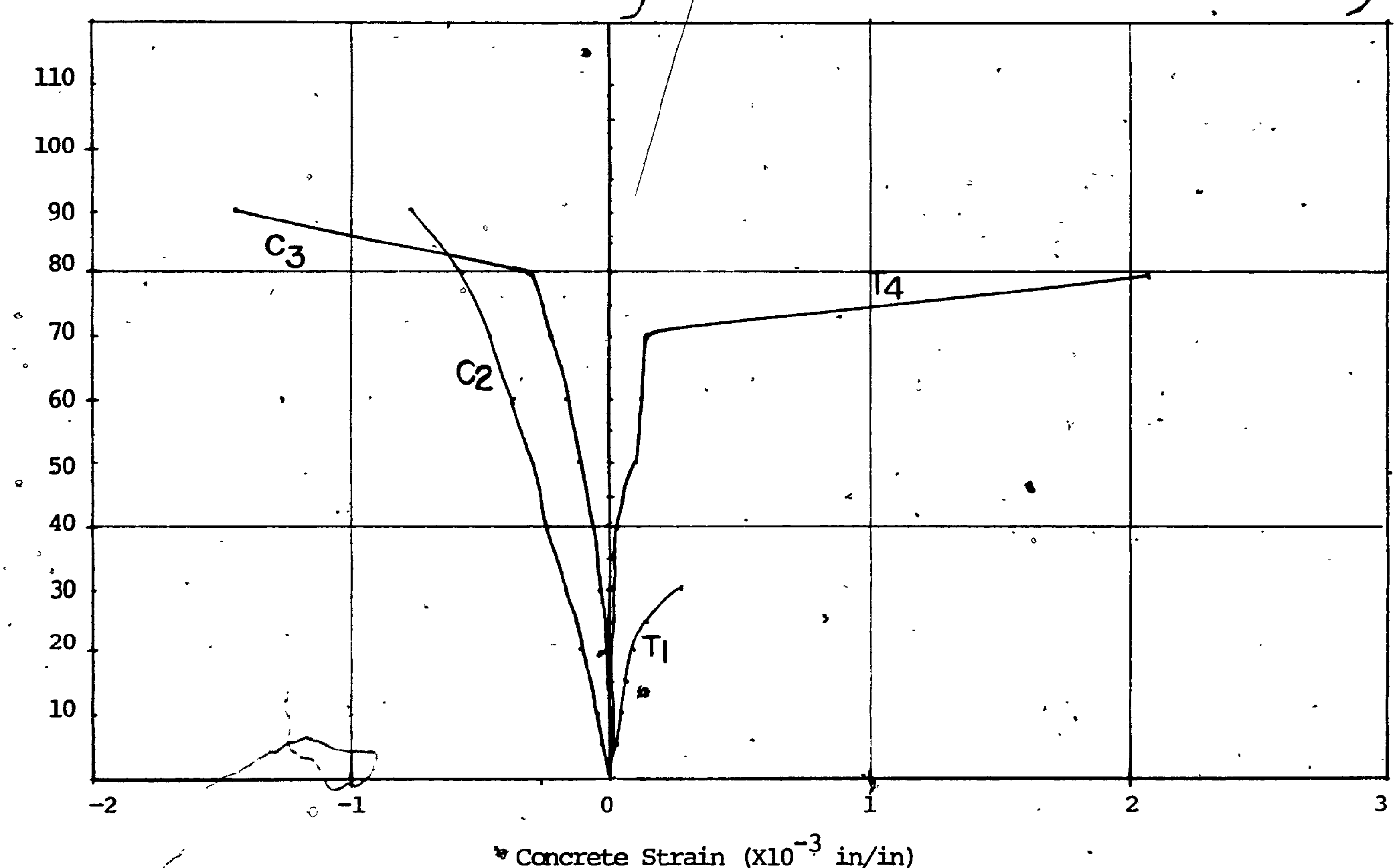


Fig. C17 Load-Strain Relationship for Specimen B4L

APPENDIX D

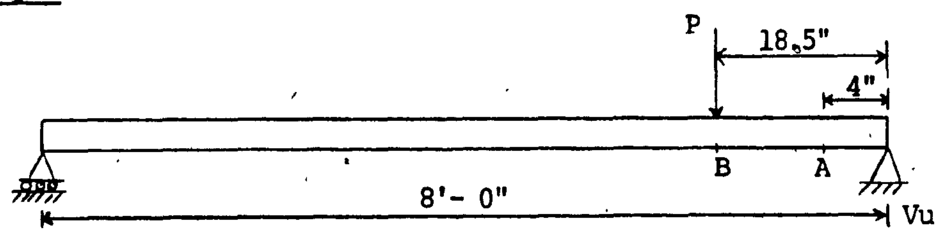
NUMERICAL EXAMPLE OF DAPPED-END

Numerical Example

Beam 1

The beam was designed to carry ultimate shear strength of Vu = 110k and assuming steel flexural strength fy = 60 ksi (420 mpa) concrete strength, f'c = 4000 psi (28 mpa).

Step 1



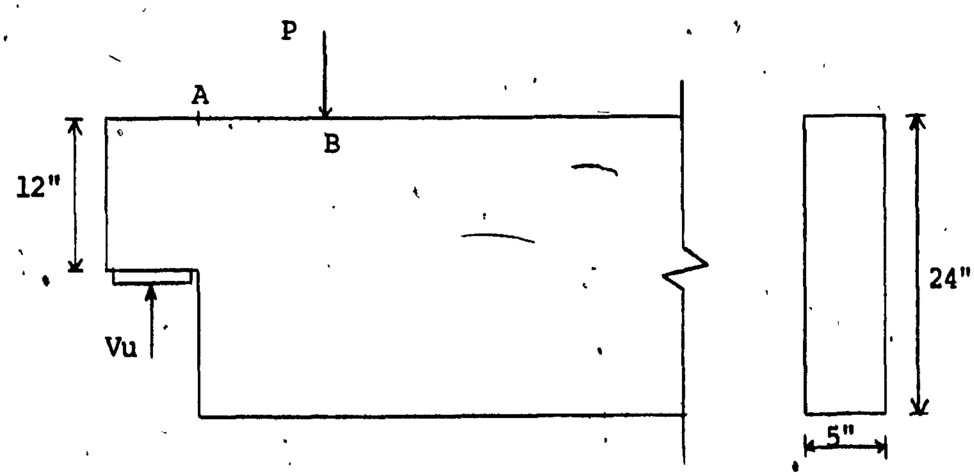
$$V_u = 110^k$$

$$M_B = V_u \times 18.5$$
$$= 1730 \text{ k in}$$

$$M_A = V_u \times 4$$
$$= 374 \text{ k in}$$

Step 2

The chosen cross sections of the dapped-end and cross section of full beam: 5" x 12" and 5" x 24" respectively (see sketch below).



Step 3

From equation (12) and (15)

$$Vu = (C + T)\ \phi\ /\sqrt{2}$$

or $$Vu = 1.41\ \phi\ Awf\ fy$$

or $$Vu = 0.7\ \phi\ f'c\ b\ h_D$$

Assume limited yield strength of steel for dapped-end, fy = 40 ksi.

Therefore,

$$Awf = \frac{Vu}{\phi 1.41\ fy}$$

$$= \frac{110}{0.85 x 1.41 x 60}$$

$$= 1.53\ in^2$$ Use 4 x 20M and 2 x 10M (see sketch below)

$Awf = 2.12\ in^2 > 1.53\ in^2$ ok.

Check capacity of Vu corresponding to the strength of concrete. From equation (15b):

$$Vu = 0.7\ \phi\ f'c\ b\ h_D$$

$$= 0.7 \times 0.85 \times 4 \times 5 \times 12$$

$$= 143^k > 100^k \quad ok.$$

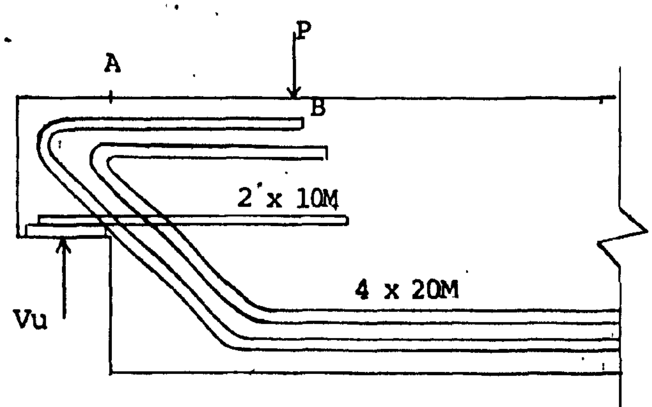
Therefore expected failure due to steel strength.

Step 4

It was assumed, f'c = 4000 psi.

Step 5

Dapped-end Reinforcement (flexure)



Flexural tension strength of steel

$$T = A_{wf}\ f_y$$

$$= 1.95 \times 60 = 117^k$$

Horizontal component of above;

$$T_{HI} = 117 \times \cos 45^\circ$$

$$= 82.7^k$$

Effective depth: $12" - 0.5" - \dfrac{0.78}{\cos 45^\circ} - \dfrac{1}{2\cos 45^\circ} = 9.69"$

Additional tension strength of horizontal confinement reinforcement

2 x 10M,

$$T_H = 0.12 \times 2 \times 60 = 14.4k$$

Depth of compression stress block:

$$C = 0.85\ f'c\ b\ a$$

For underreinforced, C = T, then:

$$A = \frac{T_{HI} + T_H}{0.85\ f'c\ b} = \frac{82.7 + 14.4}{0.85 \times 4 \times 5} = 5.7"$$

$$M = T\ (d_D - a/2)$$

$$= 97.1\ (9.69 - 5.7/2)$$

$$= 664k\ \text{in} > 374\ \text{k in} \quad \text{ok.}$$

Step 6

Moment at section B-B, M = 1730 k in

$$T = 117\ k$$

Effective depth: $d = 24" - 0.75" - 0.375" - 0.75" - 0.5"$

$$= 21.63"$$

Since steel yielded before concrete crushing, ie:

$$C = T$$

Stress block depth:

$$A = \frac{T}{0.85f'cb} = \frac{117}{0.85x4x5} = 6.88"$$

$$M = T\ (d - a/2)$$

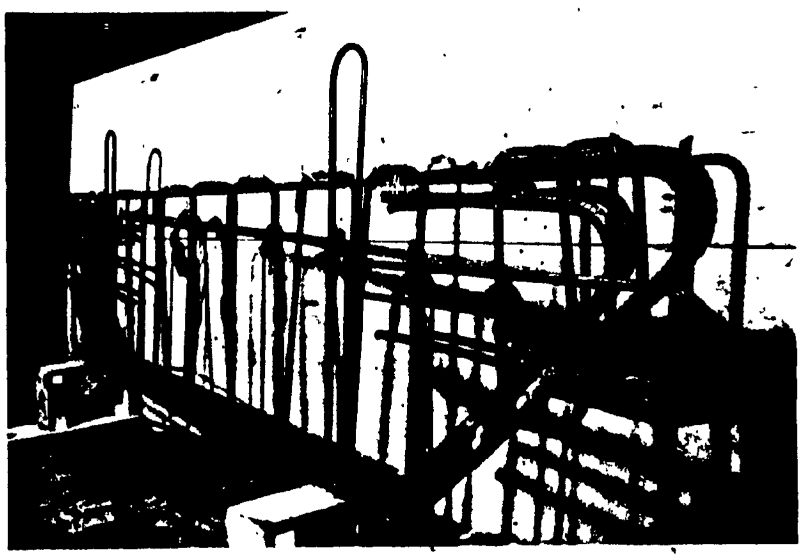
$$= 117\ (21.63 - 6.88/2)$$

$$= 2128\ k\ in > 1730\ k\ in.\ ok$$

Since all of the requirements are satisfied, therefore repetition of procedure is not needed.

Step 7

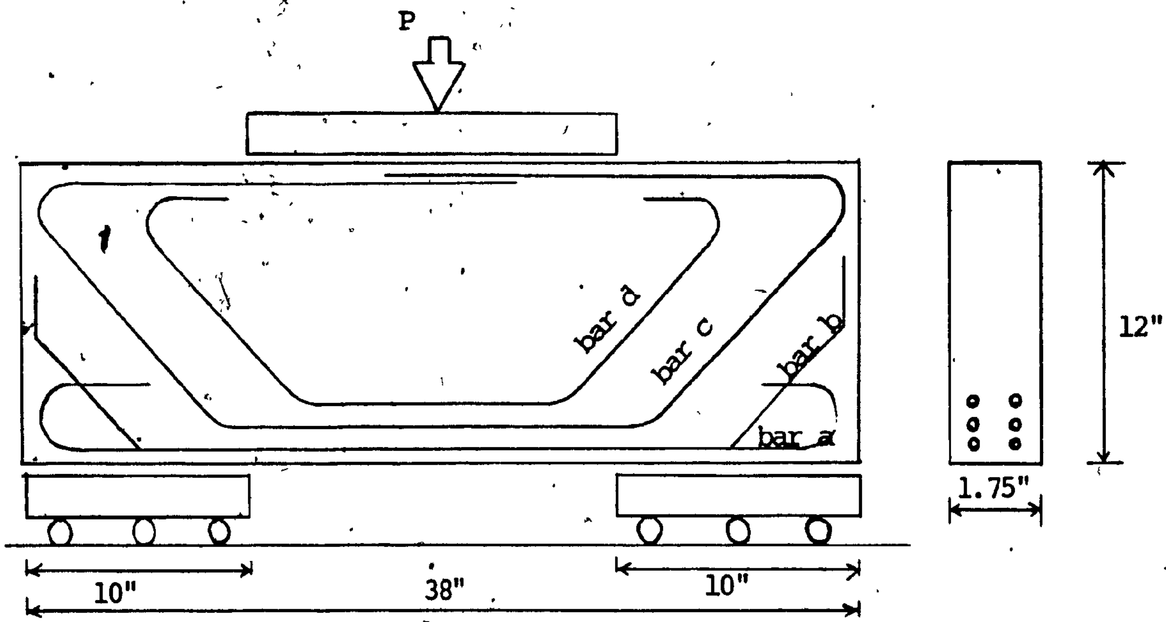
All stirrups are provided for structural purpose only. The final arrangement of the reinforcement shown below:



A4

APPENDIX E

ULTIMATE CAPACITY OF THE TEST SAMPLES



Series A

Sample No. 1

$f'c = 2150$ psi

$fy = 60$ ksi

Reinforcement

a: 1	c: 2
b: 1	d: 2

According to the F.B. diagram theory,

$$Vu = \sqrt{2}\,\phi\,C \quad \text{or} \quad \sqrt{2}\,\phi\,T$$

$$C = 0.7\, f'c\, b\, h/\sqrt{2}$$

$$Vu = 0.7\, f'c\, b\, h\, \phi$$

$$= 0.7 \times 2150 \times 1.75 \times 12 \times 0.85$$

$$= 26{,}864 \text{ lb}$$

$$\boxed{Vu = 26.86k}$$ Due to concrete

$$T = Awf\, fy \;;\quad Awf = (Avf + Ahf)/\sqrt{2}$$

$$Vu = \phi \sqrt{2}\, T$$
$$= \sqrt{2}\, \phi\, Awf\, fy$$
$$= \sqrt{2}\, \phi\, (Ahf + Avf)\, /\sqrt{2}\, fy$$
$$= \phi\, (Afh + Avf)\, fy$$ for horizontal and vertical reinforcement

$$Vu = \phi \sqrt{2}\, As\, fy$$ for inclined reinforcement

$$Vu = \phi\, (\sqrt{2}\, As + Ahf)\, fy$$
$$= 0.85\ (1.41 \times 4 \times 0.2 + \text{" X).")} \times 60$$
$$= \underline{77.9}\ k$$ (Steel yielded)

Series A

Sample No. 2 — Reinforcement

f'c = 2150 psi — a: 1 c: 2

fy = 60 ksi — b: 1 d: 2

All properties are same as sample 1, therefore it will have the same shear strength.

Vu = 26.86 k	(Concrete)
Vu = 77.9 k	(Steel)

Series A

Sample No. 3 — Reinforcement

f'c = 2150 psi — a: 1 c: 2

fy = 60 ksi — b: 1 d: 2

$$Vu = 0.7\, f'c\, b\, h\, \phi$$
$$= 0.7 \times 2150 \times 1.75 \times 12 \times 0.85/1000$$
$$= \underline{26.86}^{k}$$ (Concrete strength)

$$V_u = (1 \times 0.2 + 3 \times 0.2 \times 1.41) \times 60 \times 0.85$$
$$= \underline{\underline{53.3}}\,^k \quad \text{(Steel yielded)}$$

Series B

Sample No. 1

f'c = 4846 psi

fy = 60 ksi

Reinforcement			
a:	1	c:	2
b:	1	d:	2

$$V_u = 0.7\, f'c\, b\, h\, \phi$$
$$= 0.7 \times 4846 \times 1.75 \times 12 \times 0.85$$
$$= 60{,}550 \text{ lb}$$
$$= \underline{\underline{60.55}}\,k \quad \text{(Concrete)}$$

$$V_u = (2 \times 0.2 + 4 \times 0.2 \times 1.41) \times 60 \times 0.85$$
$$= \underline{\underline{77.9}}\,k \quad \text{(Steel)}$$

Series B

Sample No. 2

f'c = 4846 psi

fy = 60 ksi

Reinforcement			
a:	1	c:	2
b:	1	d:	2

$V_u = 60.55\,k$ (Concrete)

$V_u = 77.9\,k$ (Steel)

Series B

Sample No. 3

f'c = 4846 psi

fy = 60 ksi

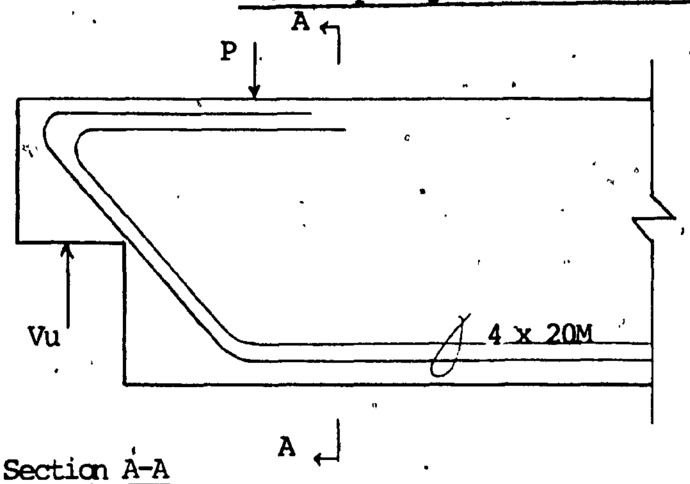
Reinforcement			
a:	1	c:	2
b:	1	d:	2

$V_u = \underline{\underline{60.55}}\,k$ (Concrete)

$$V_u = (1 \times 0.2 + 3 \times 0.2 \times 1.41) \times 60 \times 0.85$$
$$= \underline{\underline{53.3}}\,k \quad \text{(Steel)}$$

B1R

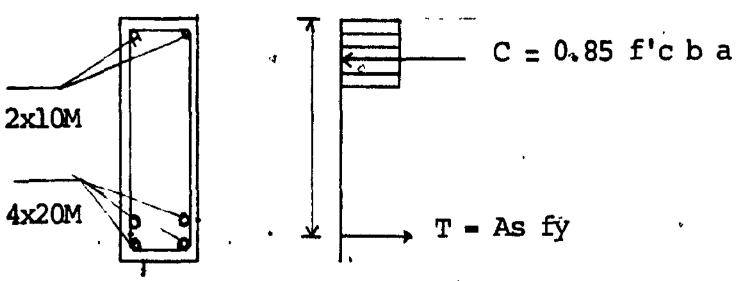
Check Capacity of Main Reinforcement



fy (exp) = 61402 psi

f'c (ave) = 4928 psi

Section A-A



$d = 24'' - 0.75'' - 0.375'' - 0.75'' - 0.5''$

$d = 21.63''$

$A_s = 4 \times 0.487 = 1.95 \text{ in}^2$

$T = A_s f_y$

$= 1.95 \times 61402$

$= 119{,}734 \text{ lb}$

$T = \underline{119.7k}$ $\qquad$ $C = 0.85 f'_c b a$

$T = C$

$119734 = 0.85 \times 4928 \times 5 \times a$

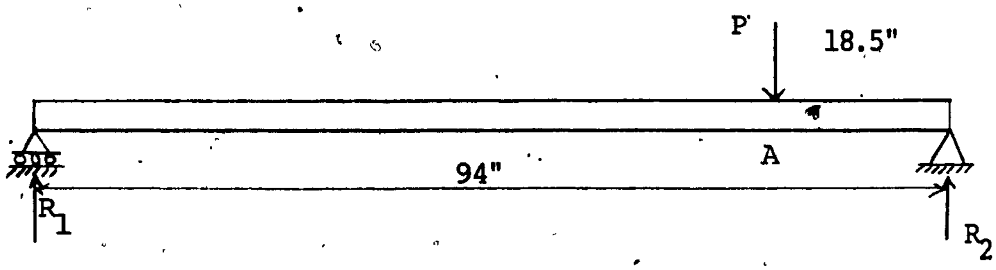
$a = 5.72''$

$M_{(A-A)} = T\,(d - a/2)$

$= 119734\,(21.63 - 5.72/2)$

$= 2{,}247{,}407$ lb in

$M_{(A-A)} = 187.28$ kft



$Ma = 0$

$P\,(77.5) = R_2\,(94)$

$R_2 = 0.824\,P$

$M_{(A)} = 0.824\,P\,(18.5)$

$= 15.24\,P$

Theoretical Ultimate P

$15.24\,P = 2247.4$

$P = 147.5$ k

$R_2 = Vn = 121.5k$

$Vu = 0.9 \times 121.5$

$= 109.4$ k

Check capacity of dapped-end reinforcement

$$T = As\ fy$$
$$\doteq 1.95 \times 61.4$$
$$= 119.7\ k$$

$$THI = 119.7 \cos 45^{\circ}$$
$$= 84.6\,k$$

Stress block depth

$$C = 0.85\ f'c\ b\ a_D$$

Since it is underreinforced, ie

$$C = T$$

$$a_D = \frac{THI + Th}{0.85\ f'c\ b} = \frac{84.6 + (0.12 \times 2 \times 61.4)}{0.85 \times 5 \times 5} = 4.67"$$

$$d_D = 12" - 0.5" - 0.78/\cos 45^{\circ} - 1/2\cos 45^{\circ}$$
$$= 9.69"$$

$$M = T\ (d_D - a/2)$$
$$= 99.3\ (9.69 - 4.67/2)$$
$$= 730\ k\ in > 374\ k\ in$$

According to free body diagram,

$$C = 0.7\ f'c\ b\ h/\sqrt{2}$$

$$= 0.7 \times 4928 \times 5 \times 12\ /\sqrt{2}$$

$$= 146{,}354\ \text{lb}$$

Therefore,

$$\boxed{C = 146.4\,k}$$

$$Twf = Awf\ fy$$

$$= (1.95 + 0.245 \sin 45^{o}) \times 61.4$$

$$= 2.12\ in^2 \times 61.4$$

$$= 130.1k$$

Ultimate shear strength

<u>Concrete:</u>

$$Vu = 1.41\ \phi\ C$$

$$= 1.41 \times 0.85 \times 146.4$$

$$= 175.4\ k$$

<u>Steel:</u>

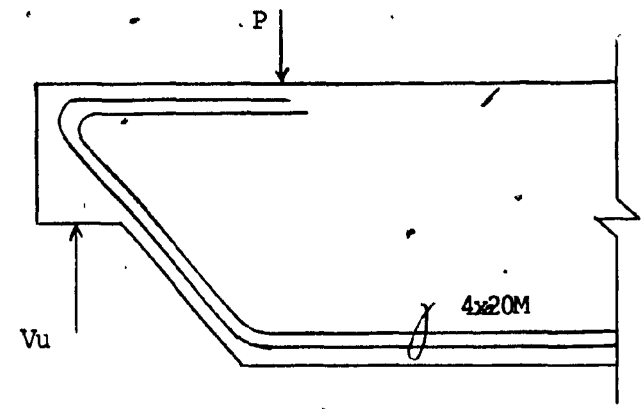
$$Vu = 1.41\ \phi\ Twf = \phi\ (Ah\ fy + As\ \sqrt{2}\ fy)$$

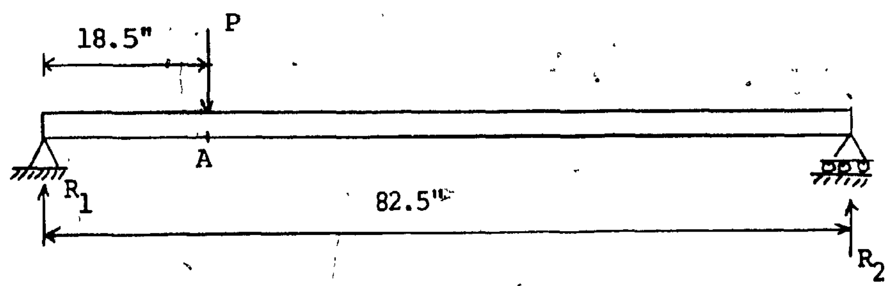
$$= 1.41 \times 0185 \times 130.1$$

$$= 155\ k$$

B1L

Check capacity of Main reinforcement





$$M@1 = 0$$

$$P(64) = R_2\ (82.5)$$

$$R_2 = 0.776\ P$$

$$M_A = 0.776P \times 18.5$$

$$= 14.35\ P$$

Maximum moment capacity of dapped-end beam, Mu

$$Mu = 2247.4\ k\ in$$ (calculated above).

$$14.35\ P = 2247.4$$

$$P = 156.6\ k$$

$$R_2 = 156.6 \times 0.776$$

$$= 121.5\ k$$

B1L (Cont'd.)

$$Vu = \phi R_2$$
$$= 0.9 \times 121.5$$
$$= 109.4 \text{ k}$$

Check capacity of dapped-end reinforcement

Moment capacity calculated has the same amount as it was calculated for speciment B1R.

$$M = 730 \text{ k in} > 374 \text{ k in}$$

Ultimate shear strength

The ultimate shear strength had the same strength as it was calculated for speciment B1R.

Concrete:

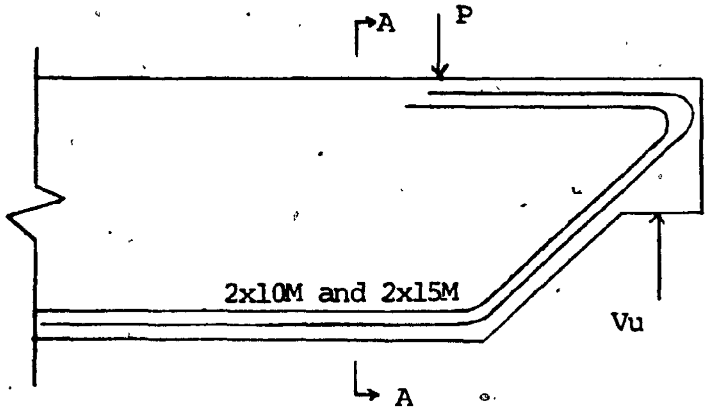
$$Vu = 175.4 \text{ k}$$

Steel:

$$Vu = 155 \text{ k}$$

B2R:

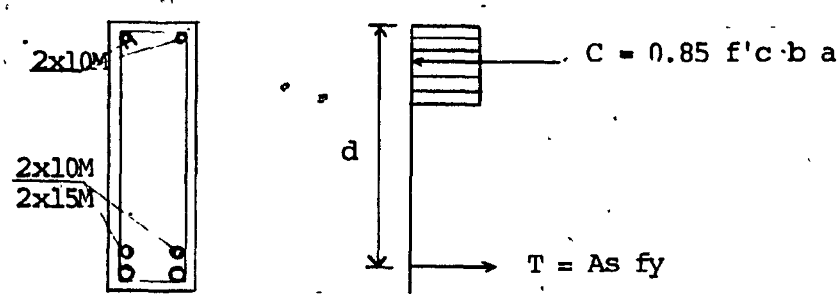
Check capacity of main reinforcement



$f_y = 73932$ psi

$f'_c = 4265$ psi

Section A-A



$$d = 24'' - 0.75'' - 0.375'' - \frac{2\,(0.2739)\,0.30 + 2(0.1217)\,1.4969}{2(0.2739) + 2(0.1217)}$$

$$= 22.2''$$

$$A_s = 0.7913 \text{ in}^2$$

$$T = A_{wf}\, f_y$$

$$= 0.7913 \times 73932$$

$$= 58{,}480 \text{ lb}$$

Therefore,

$$T = 58.5\ ^{k}$$

B2R (Cont'd)

$$T = C$$

$$58480 = 0.85 \times 4265 \times 5 \times a$$

$$a = 3.22 \text{ in}$$

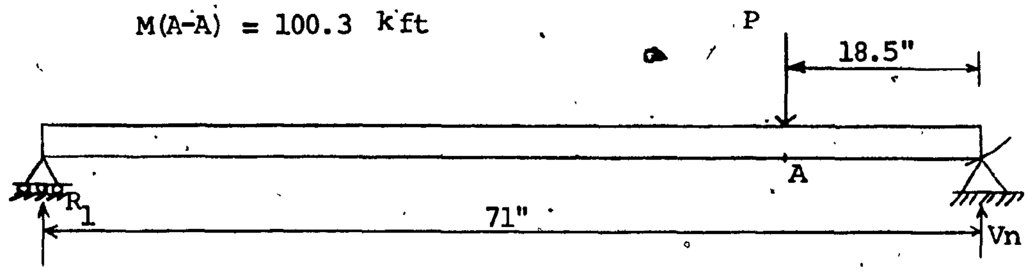
$$M(A\text{-}A) = T\ (d - a/2)$$

$$= 58480\ (22.2 - 3.22/2)$$

$$= 1204,103 \text{ lb in}$$

Therefore,

$$M(A\text{-}A) = 100.3 \text{ k ft}$$



$$\Sigma M_1 = 0$$

$$P(52.5) = Vn\ (71)$$

$$Vn = 0.739\ P$$

$$M_A = Vn\ (18.5)$$

$$= 0.739P\ (18.5)$$

$$= 13.67\ P$$

$$13.67P = 1204.1\text{k}$$

Therefore,

$$P = 88.10\text{k}$$

$$Vn = 88.10 \times 0.739$$

$$Vn_{max} = 65.10 \text{ k} \qquad Vu_{max} = 0.9 \times 65.10$$

$$= 58.6\text{k}$$

<u>Check capacity of dapped-end reinforcement</u>

$$T = As\ fy$$
$$= 0.791 \times 73932/1000$$
$$= 58.5\ k$$

$$THI = 58.50\ Cos\ 45^{o}$$
$$= 41.36\ k$$

Stress block depth:

$$C = 0.85\ f'c\ b\ a$$

$$a_D = \frac{THI + Th}{0.85\ f'c\ b}$$

$$= \frac{41.36 + (0.12 \times 2 \times 73.9)}{0.85 \times 4.26 \times 5} = 3.26"$$

$$d_D = 12" - 0.5" - 0.6/Cos\ 45^{o} - 1/2\ Cos\ 45^{o}$$
$$= 9.94"$$

$$M = T\ (d_D - a/2)$$
$$= 59.1\ (9.94 - 3.26/2)$$
$$= 491\ k\ in > 68 \times 4 = 272\ k\ in$$

Check ultimate shear strength

$$C = 0.7\ f'c\ b\ h\ /\sqrt{2}$$

$$= 0.7 \times 4265 \times 5 \times 12\ /\sqrt{2}$$

$$= 127{,}042\ \text{lb}$$

$$= 127.0\ \text{k}$$

$$Twf = Awf \times fy = (0.8 + 0.245 \sin 45^{\circ}) \times 74.3 = 72.3\ \text{k}$$

Steel:

$$Vu = 1.41\ \phi\ Twf$$

$$= 1.41 \times 72.3 \times 0.85$$

$$= 86.65\ \text{k}$$

Therefore,

$$\boxed{Vu_{max} = 86.65\ \text{k}} \qquad \text{(steel yielded)}$$

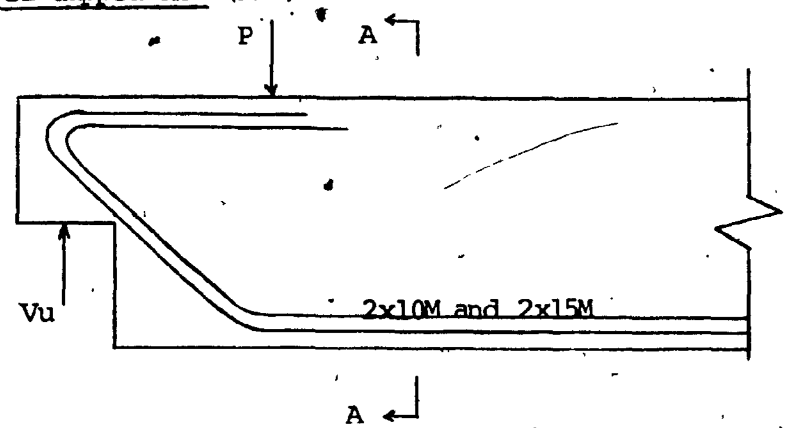
Concrete:

$$Vu = 0.7\ f'c\ \phi\ bh$$

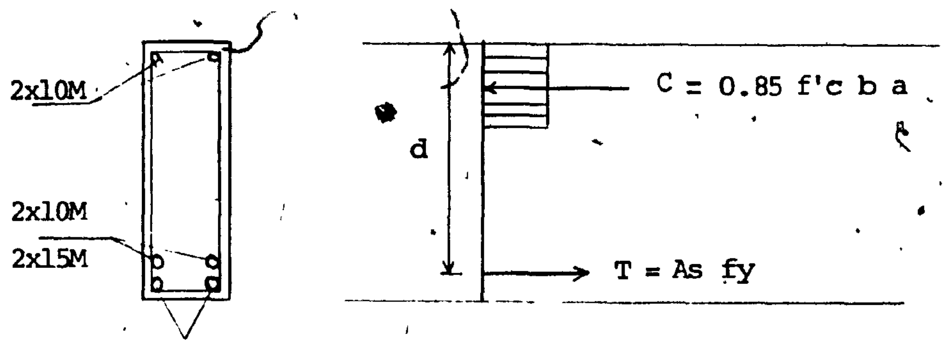
$$= 0.7 \times 4265 \times 5 \times 12 \times 0.85/1000$$

$$= 152.3\ \text{k} \qquad \text{(concrete failure)}$$

Check capacity of dapped-end (B2L)



Section A-A



$d = 24" - 0.75" - 0.375" - 0.66"$

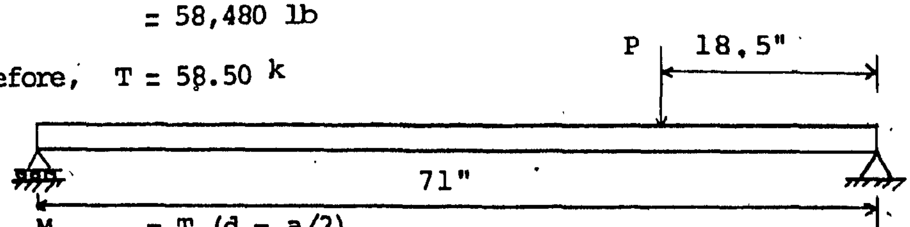
$= 22.2"$

$T = As\ fy$ $\qquad$ $T = C$

$= 0.7913 \times 73932$ $\qquad$ $a = 3.22$ in

$= 58,480$ lb

Therefore, $T = 58.50$ k



$M_{(A-A)} = T\ (d - a/2)$

$= 58480\ (22.2 - 3.22/2)$

$= 1204,103$ lb in $= 100.3$ kft

B2L (Cont'd.)

$$P(52.5) = R_2(71)$$

$$R_2 = 0.739\ P$$

$$M_A = 0.739P \times 18.5$$

$$= 13.67\ P$$

Maximum moment capacity, Mu

$$Mu = 1204.1\ k\ in$$

$$13.67P = 1204.1$$

$$P = 88.1\ k$$

$$R_2 = 88.1 \times 0.739$$

$$= 65.03\ k$$

$$Vu = \phi\ R_2$$

$$= 0.9 \times 65.03$$

$$= 58.53\ k$$

Check capacity of dapped-end reinforcement

Moment capacity calculated has the same amount as it was calculated for speciment B2R.

$$M = 491\ k\ in > 72.5 \times 4 = 290\ \ kin$$

B2L (Cont'd.)

Ultimate shear strength

The ultimate shear strength has the same strength as it was calculated for speciment B2R.

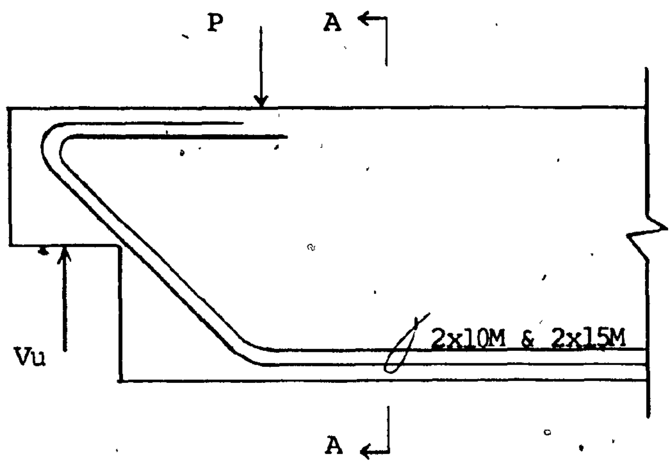
For concrete:

$V_u = 152.3\ ^k$

For steel:

$V_u = 86.65\ ^k$

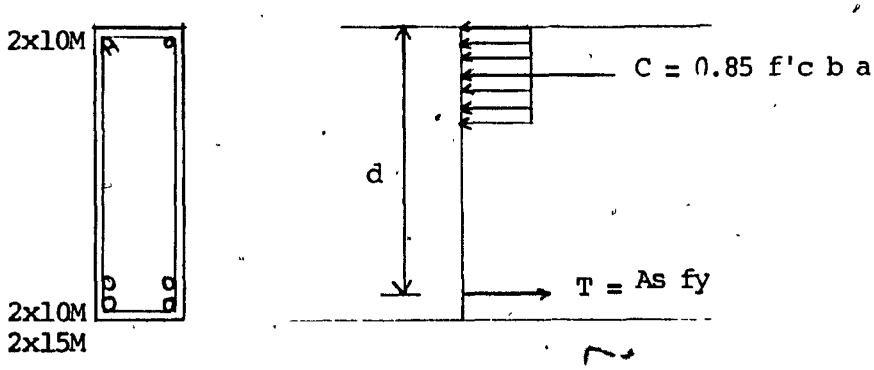
Check capacity of Main reinforcement (B3L)



f_y = 74260 psi

f'_c = 4141 psi

Section A-A



$$d = 24" - 0.75" - 0.375" - \frac{(2x0.2739)\ 0.3 + 2(0.1217)\ 1.4969}{(2 \times 0.2739) + 2(0.1217}$$

$d = 22.2"$

$A_s = (2 \times 0.2739) + (2 \times 0.1217)$

$= 0.7913\ in^2$

$T = A_s f_y$

$= 0.7913 \times 74260$

$= 58,762$ lb

Therefore,

$T = 58.76$ k

B3L. (Cont'd.)

$$T = C$$

$$58762 = 0.85 \times 4141 \times 5 \times a$$

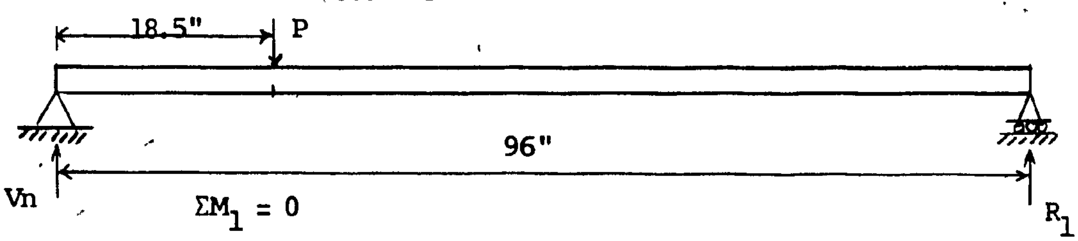
$$a = 3.34"$$

$$M_{(A-A)} = T\ (d - a/2)$$

$$= 58762\ (22.2 - 3.34/2)$$

$$= 1{,}206{,}383 \text{ lb in}$$

$$= 100.53^{k} \text{ ft}$$



$$\Sigma M_1 = 0$$

$$P(77.5) = Vn\ (96)$$

Therefore,

$$Vn = 0.807\ P$$

$$M_{(A)} = 0.807\ P \times 18.5$$

$$= 14.93\ P$$

$$14.93P = 1206.4$$

$$P = 80.8^{\ k}$$

$$Vu_{max} = 80.8 \times 0.807 \times 0.85$$

$$= 58.68^{k}$$

Therefore,

$$\boxed{Vu_{max} = 58.68\ k}$$

Check capacity of dapped-end reinforcement

$$T = As\ fy$$
$$= 0.791 \times 60$$
$$= 58.76\ k$$
$$THI = 58.76 \cos 45^{\circ}$$
$$\doteq 41.5\ k$$

Stress block depth:

$$C = 0.85\ f'c\ b\ a_D$$

Since it is underreinforced, ie.

$$C = T$$

$$a_D = \frac{THI + TH}{0.85\ f'c\ ba} = \frac{41.5 + (0.12 \times 2 \times 60)}{0.85 \times 4.14 \times 5} = 3.17''$$

$$d_D = 12'' - 0.5'' - 0.6/\cos 45^{\circ} - 1/2 \cos 45^{\circ} = 9.94''$$

$$M = T\ (d_D - a_D/2)$$
$$= 41.5\ (9.94 - 3.17/2)$$
$$= 346\ k\ in > 70 \times 4 = 280\ k\ in$$

Ultimate shear strength

$$C = 0.7\ f'c\ bh\ /\sqrt{2}$$

$$= 0.7 \times 4142 \times 5 \times 12\ /\sqrt{2}$$

$$= 123{,}278\ lb$$

$$= 123.4\ k$$

$$Twf = Awf\ fy$$

$$= (0.8 + 0.245 \sin 45^{o}) \times 74.3$$

$$= 72.3\ k$$

Steel:

$$Vu = 1.41\ \phi\ Twf$$

$$= 1.41 \times 0.85 \times 72.3$$

$$= 86.65^{k}$$

Therefore,

$$Vu_{max} = 86.65^{k}$$

Concrete:

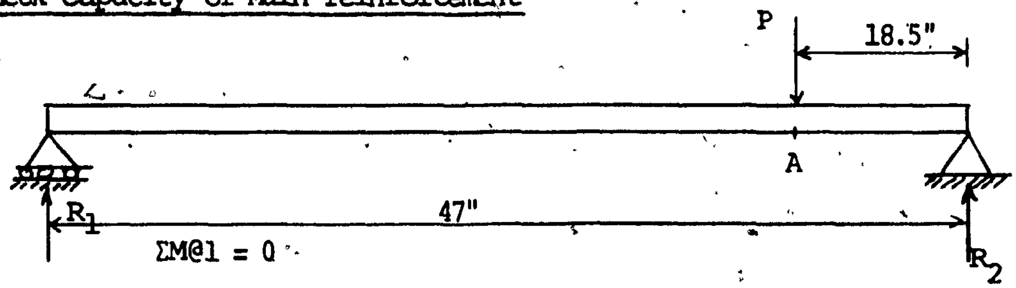
$$Vu = \phi\ C$$

$$= 1.41 \times 123.4 \times 0.85$$

$$= 147.8^{k}$$

B3R

Check capacity of Main reinforcement



$$\Sigma M@1 = 0$$

$$P(28.5) = R_2\ (47)$$

$$R_2 = 0.606\ P$$

$$M_A = R_2\ (18.5)$$

$$= 0.606\ (18.5)\ P$$

$$= 11.21\ P$$

Maximum moment capacity of dapped-end beam, Mu

$$Mu = 1206\ \text{kin} \qquad \text{(same as B3L)}$$

$$11.21P = 1206$$

$$P = 107.5\ k$$

$$R_2 = 107.5 \times 0.606$$

$$= 65.1\ k$$

$$Vu = \phi R_2$$

$$= 0.9 \times 65.1$$

$$= 58.6\ k$$

Check capacity of dapped-end reinforcement

It has the same moment resistance as it was calculated for specimen B3L. Therefore;

$$M = 344^{k}\text{ in} > 65 \times 4 = 260 \text{ k in, therefore, O.K.}$$

Ultimate shear strength

The shear strength exactly the same as it was calculated for specimen B3L.

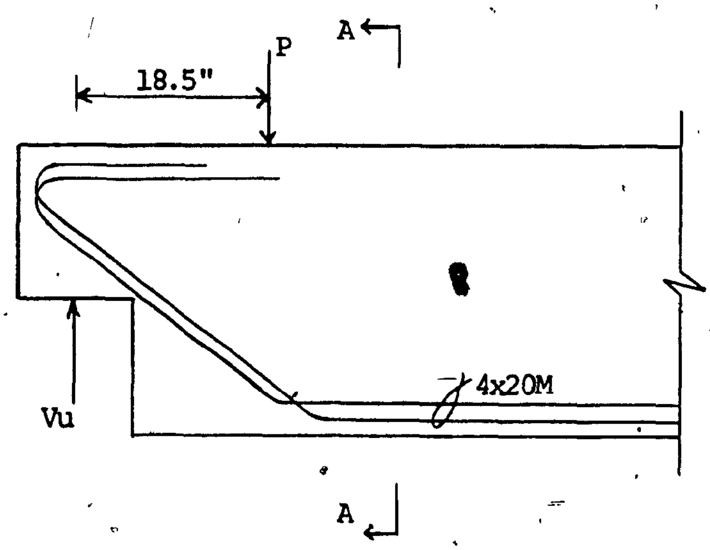
Concrete:

$$Vu = 147.8 \text{ k}$$

Steel:

$$Vu = 86.65 \text{ k}$$

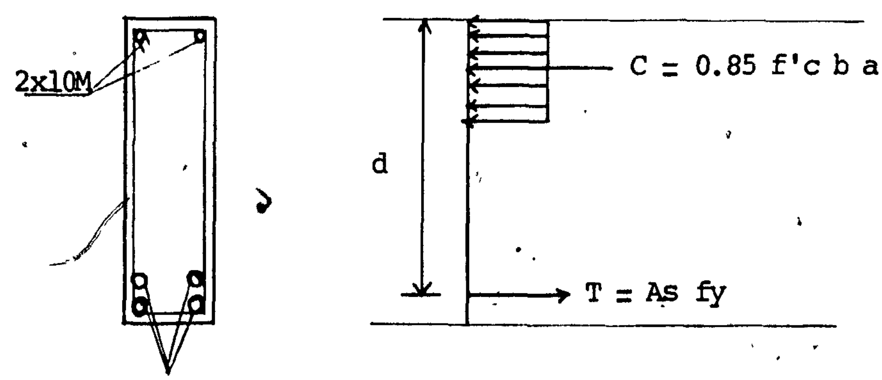
Check capacity of Main reinforcement (B4R)



fy(exp) = 61402 psi

f'c(ave) = 3991 psi

Section A-A



$d = 24" - 0.75" - 0.375" - 0.75" - 0.5"$

$= 21.63"$

$As = 4 \times 0.487 = 1.95 \text{ in}^2$

$T = As\ fy$

$= 1.95 \times 61402$

$= 119,734 \text{ lb}$

$= 119.7 \text{ k}$

$T = C$

$119734 = 0.85 \times 3991 \times 5 \times a$

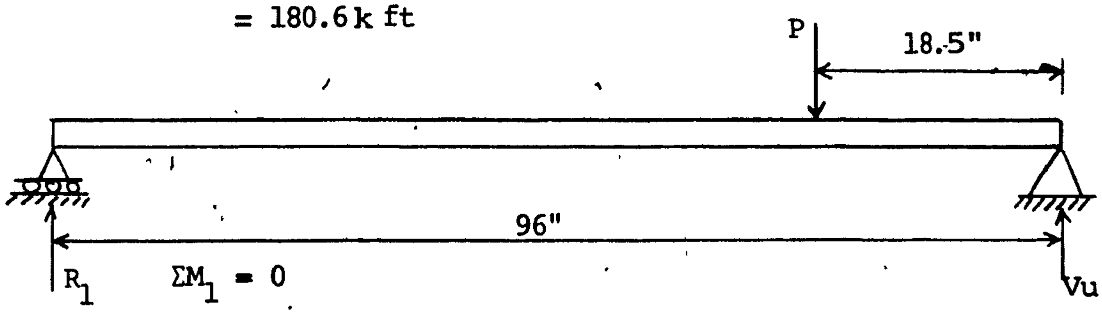
$a = 7.059$

$M_{(A-A)} = T\ (d - a/2)$

$= 119734\ (21.63 - 7.1/2)$

$= 2,167,784$ lb in

$= 180.6$ k ft



$\Sigma M_1 = 0$

$P(77.5) = R_2\ (94)$

$R_2 = 0.824\ P$

$M_{(A)} = 0.824P\ (18.5)$

$= 15.24\ P$

Theoretical Ultimate P

$15.24\ P = 21677$

$P = 142.2$ k

$Vn = 117.1$ k

$Vu = \phi\ Vn = 105.3$ k

Ultimate shear strength

$$Twf = Awf\ fy$$

$$= (1.95 + 0.24 \sin 45^{\circ}) \times 61.4$$

$$= 130.1\ k$$

$$C = 0.7\ f'c\ b\ h/\sqrt{2}$$

$$= 0.7 \times 3991 \times 5 \times 12\ /\sqrt{2}$$

$$= 118{,}880\ lb$$

$$C = 118.8\ k$$

Concrete:

$$Vu = 1.41\ \phi\ C$$

$$= 1.41 \times 0.85 \times 118.8$$

$$= 142.4\ k$$

Steel:

$$Vu = 1.41\ \phi\ Twf$$

$$= 1.41 \times 0.85 \times 130.1$$

$$= 155\ k$$

Check capacity of dapped-end reinforcement

$T = 119.7$

$THI = T \cos 45^{\circ}$

$= 119.7 \times \cos 45^{\circ}$

$= 84.6$ k

Stress block depth:

$C = 0.85 f'c\, b\, a_D$

Since it is underreinforced, ie

$C \doteq T$

$$a_D = \frac{THI + TH}{0.85 f'c\, b} = \frac{84.6 + (0.12 \times 2 \times 61.4)}{0.85 \times 3.99 \times 5} = 5.86''$$

$d_D = 12'' - 0.5'' - 0.78/\cos 45^{\circ} - 1/2 \cos 45^{\circ}$

$= 9.34''$

$M = T\,(d_D - a_D/2)$

$= 99.3\,(9.34 - 5.86/2)$

$= 636$ kin $> 90 \times 4 = 360$ kin, therefore, O.K.

Speciment B4L

All the calculations and the strengths of this specimen are the same as those calculated for specimen B4R, since it had same properties and same support length condition.