MODEL INVESTIGATIONS OF CABLE STAYED BRIDGES

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DEDICATION

This thesis is dedicated to my wife
and to the memory of my parents.

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MODEL INVESTIGATION OF CABLE STAYED BRIDGES

ABSTRACT

The use of models as a direct aid in design of cable stayed bridges has been investigated.

It was taken into account that the cable stayed bridge displays a nonlinear behaviour due to large displacements and bending moment - axial force interaction. To evaluate the internal stresses and displacements under the action of dead and live loads, the model analysis was performed in two stages. In the first stage, the behaviour of the structure was assumed to be linear. In the second stage, the data obtained on the basis of linearity was adjusted by taking into account the actual nonlinear behaviour of the cable stayed bridge system.

To reduce stresses and displacements due to dead load by post-tensioning the cables, a procedure was developed to determine the magnitude of the post-tensioning forces required in cables to attain the reduction specified.

The experimental data obtained as a result of the model investigation, was compared with a theoretical analysis performed on the CDC 3300 digital computer.

ACKNOWLEDGEMENTS

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NOTATION

A	Area of cross section
A_D	Area of stiffening girder cross section
A_i	Area of cross section of cable "i"
AJ	Vector of displacements due to dead load
AM	Matrix of member end axial and shear forces and bending moments in the actual structure due to combined joint loads
AMA	Matrix of member end axial and shear forces and bending moments in the actual structure
AMAN	Vector of negative envelopes of axial and shear forces and bending moments
AMAP	Vector of positive envelopes of axial and shear forces and bending moments
AMF	Vector of member end axial and shear forces and bending moments due to post-tensioning of cables
AMJ BMJ	Matrix of member end axial and shear forces and bending moments in the released structure due to unit loads corresponding to the joint displacements
AMQ	Matrix of member end axial and shear forces and bending moments in the released system, due to unit values of the redundants
AML	Matrix of member end axial and shear forces and bending moments in the released structure due to combined joint loads
AMR	Matrix of member end axial and shear forces and bending moments in the restrained structure
AR	Matrix of reactions in the actual structure
ARF	Vector of reactions due to post-tensioning of cables
ARL	Matrix of reactions in the released structure due to combined joint loads
ARQ BRQ	Matrix of reactions in the released structure due to unit values of the redundants
A_{TB}	Area of tower cross section at bottom
A_{TT}	Area of tower cross section at top

C_0 Reduction factor

DL Dead load

DJ Matrix of joint displacements in the actual structure

DJF Vector of displacements due to post-tensioning of cables

DQL Matrix of displacements in the released structure corresponding to the redundants and due to combined joint loads

E Young's modulus of elasticity

F Flexibility matrix of the system

FM Square diagonal matrix of member flexibilities

G Modulus of elasticity in shear

h Height of cross section

I Moment of inertia of cross section

JD Matrix of displacements due to unit post-tensioning forces applied along the cables

J_D Moment of inertia of stiffening girder cross section

J_{D1} Moment of inertia of stiffening girder cross section for members 15, 16, 24 and 25

J_{D2} Moment of inertia of stiffening girder cross section for members 13,14, 17, 18, 22, 23, 26, 27 and 31

J_{TB} Moment of inertia of tower cross section at bottom

J_{TT} Moment of inertia of tower cross section at top

K Elastic support spring constant

k_A Scale reduction factor for area of cross section

k_E Scale reduction factor for modulus of elasticity

k_I Scale reduction factor for moment of inertia of cross section

k_L Scale reduction factor for length

k_M	Scale reduction factor for bending moment
k_N	Scale reduction factor for axial forces
k_p	Scale reduction factor for concentrated load
k_Q	Scale reduction factor for shear force
k_q	Scale reduction factor for distributed load acting on a surface
k_t	Temperature of prototype divided by temperature of model
k_w	Scale reduction factor for distributed load acting on a bar
k_y	Scale reduction factor for deflection
k_α	Scale reduction factor for linear expansion
k_ε	Scale reduction factor for strain
k_ν	Scale reduction factor for Poisson's ratio
k_ρ	Scale reduction factor for density of material
k_σ	Scale reduction factor for unit stress
L	Length of member
LL	Live load
M	Bending moment
M_r	Bending moment at location "r" due to dead load
M_r^i	Bending moment at location "r" due to the action of a unit load applied along cable "i"
M_r^j	Bending moment at location "r" due to post-tensioning of cables,as indicated in substructure "j"
N	Axial force
N_i	Axial force in cable "i" due to dead load
N_i^f	Axial force in cable "i" due to dead load and post-tensioning
P	Vertical, concentrated load, acting on the bridge
Q	Matrix of redundants

u_i	Horizontal displacement at location "i"
v_i	Vertical deflection at location "i"
V	Shear force
X	Vector of post-tensioning forces in cables
X_i	Post-tensioning force to be applied in cable "i" to reduce bending moments and displacements due to dead load

I

INTRODUCTION

1. Introductory note.

The cable stayed bridge is a modern and economical solution for medium span bridges, that is, for bridges having spans too large to be covered by a nonstiffened girder and too small to justify a suspension bridge. This type of bridge, may be described as a stiffened girder system supported elastically at intermediate points by inclined cables. The cables are suspended from towers located at the interior supports. Sketches of several typical cable stayed bridge systems are shown in Appendix No. 1.

The analysis and design of existing cable stayed bridges have been performed by analytical methods. In some cases, however, as for example, the George Street Bridge over the river Usk, at Newport, England [1], model investigations have been carried out, in addition to the analytical calculations, in order to verify the validity of the mathematical model on which the analysis was based.

Structural models may be used, however, not only to verify final analytical calculations, but also as direct tools for design of bridge systems. If the analysis is performed on a structural rather than on a mathematical model, the information obtained, more closely represents the actual behaviour of the structure.

The purpose of this research is to apply the above concept to the particular case of cable stayed bridges.

The reason that the cable stayed bridge has been selected for this application is that, it has been used extensively after the Second World War and it presents a great potential for the future development of bridges. Although the extensive application of modern cable stayed bridges started only in the last fifteen years, earlier types of similar bridge systems were known for almost two centuries, as shown in the brief historical review of the next section.

2. Historical Note

The idea of carrying a main bridge girder by inclined cables anchored to a tower has probably originated from its architectural counterpart, the tower (or mast), connected by cables to a rigid anchorage. Available information [2,3] takes us back as far as 1784 when C.J. Loscher brought out the idea of building a bridge similar to the cable stayed system (Freiburg, Germany.) Later in 1817, a footbridge, 110 feet long, stiffened with inclined suspension members was built in England.

Several other bridges having the main girder stiffened by inclined members were known from the nineteenth century. One of them, a 256 foot bridge over the river Saale (Neinburg, Germany, 1824), collapsed due to overloading by a crowd of people. Two others were the Poyet system (1821, France),

Fig. 1, and the Hatley system (1840, England), Fig. 2.

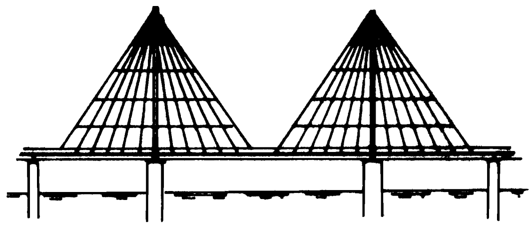


FIG. 1 BRIDGE SYSTEM POYET

The Poyet bridge system had the stiffening members converging to the top of the tower, whereas the Hatley system had parallel harp-shaped chains.

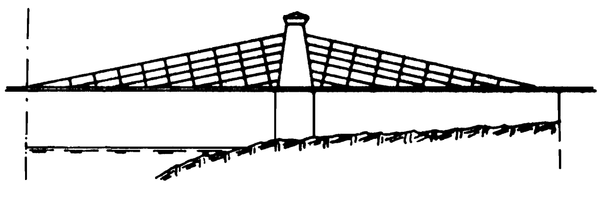


FIG. 2 BRIDGE SYSTEM HATLEY

The Albert Bridge [4] over the Thames River in London, England, (1873) had the stiffening members converging to the top of the tower as in the Poyet system, Fig. 3. The bridge had a main span of 400 feet.

FIG. 3 THE ALBERT BRIDGE OVER THE THAMES RIVER, LONDON

Another bridge, actually a combination of cable stayed and classical suspension bridge dates back to 1868. This is the Franz-Joseph Bridge [4] over the river Moldau in Prague, Checkoslovakia, Fig. 4.

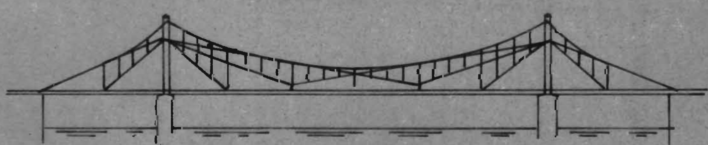


FIG. 4 THE FRANZ-JOSEPH BRIDGE OVER THE RIVER MOLDAU, PRAGUE

The Gisclard bridge system was developed in France, in 1908, Fig. 5, and had the main span girder suspended elastically by vertical ties. The ties were connected to a member which transferred the load to inclined stiffeners converging to the top of the tower.

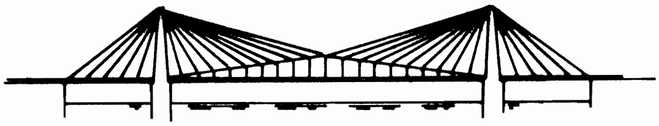


FIG. 5 THE GISCLARD BRIDGE SYSTEM

After the Second World War, the rapid advancement of bridge construction brought about the need to develop new concepts in bridge design. In order to achieve economy of both material and cost, designers have gone back to the cable stayed bridge concept. A leading role in the new development of this bridge system may be attributed to Dischinger [4]. Since 1950, a number of cable stayed bridge systems have been built in various parts of the world (Appendix No. 1). At present, this type of bridge system is being applied more and more by designers all around the world.

If we refer to Canadian or American achievements in the field of cable stayed bridges, the most representative is the "Papineau Bridge" (Montreal, 1969) having a main span of 790 feet. At the present time, however, the application of cable stayed bridges in Canadian design practice is only in its initial stage.

3. Basic concepts regarding the use of models as a direct aid in design of cable stayed bridges. Outline of research program.

At the present time, the analysis and design of bridges is being carried out on mathematical models. Structural models are employed only in the case of large bridge systems, to verify the analytical computations.

In designing a structural system, it should be taken in account, however, that a theoretical investigation is based on simplifying assumptions regarding the connections and the supports of that system, and its overall behaviour. This is the reason why the actual distribution of stresses in a structure is not identical to the one predicted by its mathematical model. By employing however, a structural model constructed in accordance with the theory of similitude, the information obtained represents more closely the behaviour of the prototype, than the results of a theoretical analysis.

To ascertain the possibility of carrying out the design process of a cable stayed bridge system on a small scale structural model, this process has been examined in detail.

For a cable stayed bridge with a given geometrical layout, the design may be divided into the following steps:

a) A preliminary set of sectional properties is assumed for each member of the system.

b) On the basis of the sectional properties assumed in step "a", stresses and displacements due to applied loads on the system are determined.

c) Maximum stresses and displacements obtained in step "b" are compared with stresses and displacements allowed by technical specifications. If the difference is larger than permitted by design codes, a new set of sectional properties is chosen and steps "b" and "c" are repeated until a specified relation between the sectional properties assumed and those determined as a result of the investigation performed, is obtained.

The cable stayed bridge displays a nonlinear elastic behaviour. This nonlinearity is due to large displacements and bending moment-axial force interaction. For a nonlinear structure, the principle of superposition does not apply, and therefore, it is not possible to determine critical stresses and displacements by superposition of influence line ordinates. In this case, the analysis should be carried out by loading the system with its full dead and live loads.

To analyse a cable stayed bridge by a "direct" method, that is by applying the full dead and live load on the model, difficulties are experienced by the fact that the range of standard structural sections available for design of the model is limited. To overcome this problem, all sectional properties of the model must be increased by a constant factor. This causes however, an equivalent increase of the total weight to be applied on the model in order to satisfy the similitude condition $k_y = k_L$. As a consequence, unless a special loading system which would enable the simulation of relatively large concentrated loads or of a distributed load, is employed, the procedure of increasing proportionally all sections becomes impractical and other methods have to be found.

If instead of a true model, a distorted model, which does not satisfy the similitude condition $k_A = k_L 2$ is employed, (a detailed discussion of similitude conditions is presented in Part II, Section 2), the stiffening girder may be designed as a square or rectangular bar. Such bars are available in relatively small sizes. Comparative theoretical calculations performed have indicated however, that the errors in evaluating bending moments and axial and shear forces by disregarding the condition $k_A = k_L 2$ may amount to as much as 8.6%. This represents a relatively large variation and therefore, a distorted model does not represent an exact answer to the above problem.

The solution developed in this research is to carry out the analysis on two models. The first model, which is designated as model "A", is employed to determine influence lines. On this basis, the linear stage of the analysis may be performed. The only load applied on model "A" is a relatively small concentrated load which may be handled with no difficulty. This allows the design of model "A" with relatively large structural sections. To predict the nonlinear behaviour of the prototype, a second model, which is designated as model "B", is employed. On this model, "B", the ratio between nonlinear and linear stresses and displacements is determined by loading the model in two stages. First a fraction of the dead and live loads is applied such that the displacements of the system remain small and its behaviour may be considered as linear. Then, the model is loaded with its full dead and live loads under which it displays its real, nonlinear behaviour. Strains and displacements are recorded for both cases. By comparing the two readings, that is by dividing the second reading by the first ones, the nonlinearity of the system is determined.

As model "B" is loaded with its full load, relatively small structural sections have to be employed in order to keep the total weight to be applied on the system at a minimum. Consequently, model "B" has to be designed as a distorted model. As both readings are determined on the same model, the same distortion factor is employed however, and this factor is cancelled when the second reading is divided by the first one.

By multiplying stresses and displacements recorded on model "A" by the nonlinearity factors determined on model "B", the real, nonlinear stresses and displacements are obtained.

An alternative solution to the above problem is to employ a special loading system as stated earlier in this section. In this case, standard structural sections may be used and one model only will be required for the analysis of a cable stayed bridge system.

A model investigation has been planned and carried out on the basis of the method presented above. The details regarding model "A" are described in Chapter II and the investigation of the nonlinear behaviour (model "B") is described in Chapter III.

An additional problem encountered in the analysis of a cable stayed bridge system is the calculation of post-tensioning forces required in cables to reduce bending moments and displacements due to dead load. To obtain these forces, bending moments and displacements are determined, at selected locations, first due to unit forces applied successively along each cable of the system and then due to dead load. Then, by expressing the condition that the maximum bending moments or the displacements should be reduced to a specified value, the post-tensioning forces are determined by solving a number of equations equal to the number of cables.

An investigation regarding the determination of post-tensioning forces is presented in Chapter V.

The procedure discussed above, affords a good method of analysis of a cable stayed bridge. If the analysis is to be repeated with different sectional properties in order to refine the design of the cross sections, the model should have interchangeable or adjustable sectional properties such that one may repeat a model investigation without having to construct a completely new model. During this investigation, the possibility of obtaining such a model was examined.

To compare the information determined experimentally with the theoretical data, a procedure for theoretical analysis of cable stayed bridges has been established and is presented in Chapter IV.

II

DETERMINATION OF INFLUENCE LINES OF A CABLE STAYED BRIDGE ON A SMALL SCALE STRUCTURAL MODEL

1. Planning of the model investigation

Planning of the model investigation for determining influence lines of a cable stayed bridge has been carried out in accordance with the object of this research as outlined in Chapter 1.

Before the design of the small scale model was started, it was necessary to reach decisions with regard to the following factors.

a) The size of the model

b) Materials to be employed for the model

c) Fabrication methods

d) Instrumentation, loading and recording of data

a. The size of the model

The main considerations taken into account in selecting the scale reduction factor for length, were the availability of standard structural sections as discussed in Chapter I, Section 3. In addition to these considerations, it was taken into account that on a larger size model, the errors in recording strains and displacements would be less than on a smaller size model. At the same time, as the scale reduction factor for length increases, the manufacturing tolerances become more difficult to satisfy. A smaller model, however, would require less laboratory space and would be easier to manipulate.

As a consequence of the above considerations, and taking into account as well the space designated for the model in the Civil Engineering Laboratory, it was decided to employ a length scale reduction factor of $k_L = 100$. A smaller scale reduction factor would have resulted in a model too large to be accomodated within the space available. At the same time, a larger scale reduction factor could not be justified.

b. Materials

In deciding on the particular material to be employed in the model investigation, the similitude conditions to be satisfied by the materials of the model and of the prototype were borne in mind. The prototype was assumed to be of structural steel, this being the material used generally for the construction of cable stayed bridges. If the influence of shear strain on the magnitude of displacements is negligible, any material exhibiting linear stress-strain properties would be adequate for the model if the loads are applied so that stresses remain below the elastic limit. If this influence is to be considered, however, a material having the same E and G as the prototype, should be employed, as will be discussed in more detail in Section 2.

Consequently, it was decided to employ steel, as this material satisfies most completely the requirements specified.

Its manufacturing and instrumentation is relatively simple, it has a high heat dissipation factor, its elastic properties do not change in time and it satisfies similitude requirements for shear strain, as opposed to, for example, plastics which do not satisfy this condition.

c. Fabrication methods

To decide what types of sections to select in the design of the model, fabrication methods for small scale structural models were reviewed [5,6]. Of special interest was the M.I.T. report R66-45 "Fabrication Techniques for Small Scale Structural Models" [7].

The decision adopted was to employ standard commercial sections for the girder and for the cables, and structural steel plate, cut and milled to the required size for the towers.

After examining several alternatives, it was decided to design the tower-girder and cable-girder connections as bolted, and the cable-tower connections as bolted-welded.

d. Instrumentation methods

The data to be determined during the model investigation are bending moments in the stiffening girder and axial forces in cables under a unit load applied vertically, at constant intervals on the girder.

The number of locations to be instrumented to acquire data should be equal to the degree of static indeterminancy of the system. All other data may then be obtained from conditions of static equilibrium.

To determine the ordinates of influence lines of axial forces in cables, the axial strains have to be measured. For the influence lines of bending moments in girder, one method is to measure maximum strains due to bending and to multiply them by $\frac{2EI}{h}$ such that the bending moments are obtained.

A second method is to determine u_i, v_i, θ_i, u_j, v_j, θ_j, the vertical, horizontal and rotational displacements at joints i and j, Figure 6, and to calculate the axial and shear forces and the bending moment at joints i and j from Equation (1)

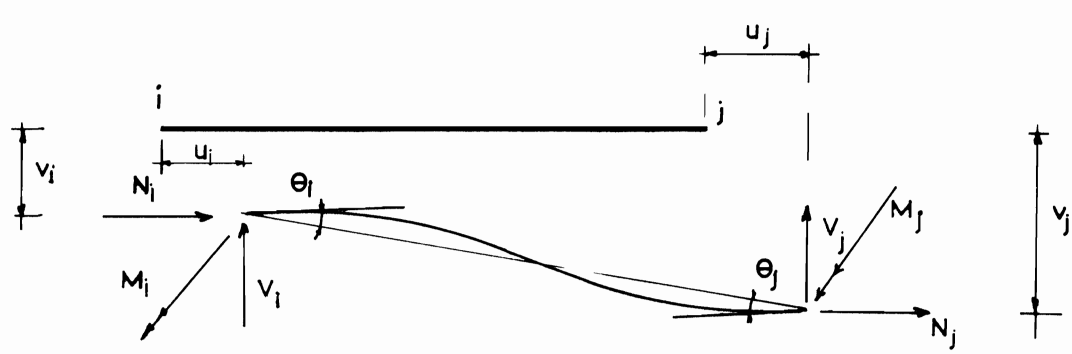


FIG. 6 TYPICAL BAR BEFORE AND AFTER DEFORMATION

$$\{A\}_{ij} = [S]_{ij} * \{D\}_{ij} \tag{1}$$

In Equation (1)

$$\{A\}_{ij} = \{N_i, V_i, M_i, N_j, V_j, M_j\}$$
$$\{D\}_{ij} = \{u_i, v_i, \theta_i, u_j, v_j, \theta_j\} \tag{2}$$

and

$$[S]_{ij} = \text{stiffness matrix of bar ij.} \tag{3}$$

By the second method influence lines are obtained by measuring displacements instead of strains.

For the acquisition of data in this investigation, it was decided to employ the first method. The second method was applied as an illustration only. The reason for this is, that by applying the first method, the information may be acquired with the aid of a small computer as will be shown later in more detail.

Influence lines of displacements may be obtained by employing microscopic scales, transducers, dial gages or by photographic methods [8,9,10,11,12]. For this investigation, dial gages were employed.

2. Design and description of the model

a. Similitude conditions

The cable stayed bridge system displays a nonlinear elastic structural behaviour. Consequently, the similitude conditions to be satisfied by the scale reduction factors of the governing variables of this system have to be valid in the nonlinear domain. Their derivation must be based on principles of dimensional analysis, as these principles are not restricted by the law of superposition.

Langhaar [13], Murphy [14], Beaujoint [15], and Preece [16] have applied the principles of dimensional analysis as laid down by Buckingham [17] to the general case of structural similitude.

The similitude requirements as established by Preece for the general case of a structure are:

$$k_\varepsilon = 1 \qquad (4)$$

$$k_\nu = 1 \qquad (5)$$

$$k_L = \text{constant in all directions} \qquad (6)$$

$$k_\alpha k_t = 1 \qquad (7)$$

$$k_L = k_y \qquad (8)$$

$$k_\sigma = k_E \qquad (9)$$

$$k_\sigma k_{L^2} = k_P \qquad (10)$$

$$k_q = k_\sigma \qquad (11)$$

$$k_\rho k_L = k_\sigma \qquad (12)$$

The scale reduction factors employed in equations (4) to (12) are defined in Notation.

For the case of a nonlinear elastic cable bridge system, the condition represented in Equation (4) may be disregarded as it applies only when the nonlinearity is due to the behaviour of the material itself and not to that of the system.

Equation (5) represents the condition that both the model and the prototype should be made of the same material. For the case of a simply supported steel girder with a ratio of length to height equal to 15, for example, it may be shown that if this condition is disregarded, the error is approximately 1%. To neglect the condition represented in Equation (5) is equivalent to neglecting the contribution of shear strain when the magnitude of the elastic displacement is computed.

It may be pointed out here that the contribution of shear strain to the magnitude of the elastic displacements is rather difficult to take into account in a mathematical model in the case of classical methods of computations, as the force, displacement or energy equations become very complex. Also, if a computer program is employed, the stiffness matrix of the members will contain more terms and hence, more computing time will be required to perform the calculations. If bending moments and axial and shear forces are determined however, on small scale structural models which satisfy the similitude condition expressed in Equation (5), the influence of shear strain in the value of elastic displacements is included without additional effort.

Equation (6) represents the condition that the longitudinal and cross sectional dimensions of the prototype and of the model have to be related by the same scale factor k_L. This condition is extremely difficult to satisfy in practice. If the assumption is made that sectional properties are represented by I and A only, Equation (6) may be replaced on the basis of Buckingham's Pi theorem by $k_I = k_L 4$ and $k_A = k_L 2$.

It may be shown that if the cross sectional properties are represented by I and A only, the error involved for a span to length ratio of 15 is less than 1%.

Equation (7) represents the conditions to be satisfied by the temperature and linear expansion scale factors. In this investigation, however, the data readings are recorded at constant temperatures and the bridge system has no constraints which produce temperature stresses and, therefore, this condition may be disregarded.

Equation (8) is valid for all cases of nonlinear elastic similitude and must be satisfied in this research.

Equation (9) indicates that the scale reduction factor for unit stress is equal to the scale reduction factor for modulus of elasticity. For this investigation $k_E = 1$ and hence $k_\sigma = 1$.

In the method applied here, only internal forces are considered however, and not unit stresses, hence Equation (9) does not represent a similitude condition which needs to to be satisfied in this investigation.

Equation (10) may be arranged more conveniently by employing Equation (9), as $k_E k_L 2 = k_p$. Equation (11) was not used in this investigation as the distributed load applied on the bridge system is given in lb. per inch and not in lb. per square inch. It may be shown that if the distributed load is given in lb. per inch, the equivalent condition is $k_W = k_E k_L$.

Equation (12) does not apply for the case of statical loads and may be disregarded.

Consequently, for a nonlinear elastic cable stay bridge system subjected to vertical statical loading, the similitude conditions are

$$k_Y = k_L \tag{13}$$

$$k_A = k_L 2 \tag{14}$$

$$k_E k_L 2 = k_p \tag{15}$$

$$k_\nu = 1 \tag{16}$$

$$k_I = k_L 4 \tag{17}$$

$$k_E k_L = k_W \tag{18}$$

Employing the same principles as in the deriviation of the above conditions, similitude requirements between bending moments, axial and shear forces acting on the prototype and bending moments, axial and shear forces acting on the model may be established. These are

$$k_M = k_p k_L \tag{19}$$

$$k_Q = k_p \tag{20}$$

$$k_N = k_p \tag{21}$$

If Equations (13) to (21) are satisfied, the model is a true model subject to the limitations stated earlier when discussing Equation (6). If any one of the Equations (13) to (21) is not satisfied, the model is distorted. In practice, it is very hard to satisfy simultaneously Equations (14) and (17) because of the limited range of small sections commercially available.

If Equation (13) is not satisfied, the model ceases to display a nonlinear behaviour. It may be shown that for linear similitude, the condition expressed by this equation is not essential, however. If only information for influence lines are to be determined, Equation (13) may be disregarded.

Equations (13) to (18) inclusive will be employed in part (b) for the design of the model. Equations (19) to (21) will be used to predict axial forces and bending moments of the prototype from axial forces and bending moments determined on the model.

b. Sectional properties and geometry of the model.

This stage of the model design consists of the reduction to scale of the dimensions of the prototype, according to the similitude conditions.

For the case when the geometry selected for the bridge system to be designed is similar to the geometry of an existing bridge with known sectional properties and the interior and exterior constraints of both systems are similar, a procedure to determine the sectional properties of the new bridge, based on principles of similitude, is suggested in Appendix No. 2.

The sectional properties and geometry of the prototype selected for this investigation are shown on Figure 7. The bars are marked by numbers from 1 to 31. The cables are designed symmetrically about the towers and about the centre line of the bridge system. Hence, $A_1 = A_6 = A_7 = A_{12}$, $A_2 = A_5 = A_8 = A_{11}$, $= A_3 = A_4 = A_9 = A_{10}$. The materials assumed for the prototype were CSA G40.12 steel for the girder and towers and wire rope bridge strand for the cables. The modulus of elasticity of the G40.12 steel has been taken equal to E = 29,000 ksi and of the cables as equal to E = 18,000 ksi.

It has been assumed that the bridge prototype has 6 lanes and its deck is supported by 2 box girders. The sectional properties represented in Figure 7 are for one girder only.

To determine the geometry and the cross sections of the model, first k_A and k_I have been calculated. As stated earlier in this section, k_L has been selected as equal to 100. Hence, from Equation (14), $k_A = k_{L2} = 10^4$ and according to relation (17) $k_I = k_{L4} = 10^8$.

Dividing the sectional properties of the prototype by

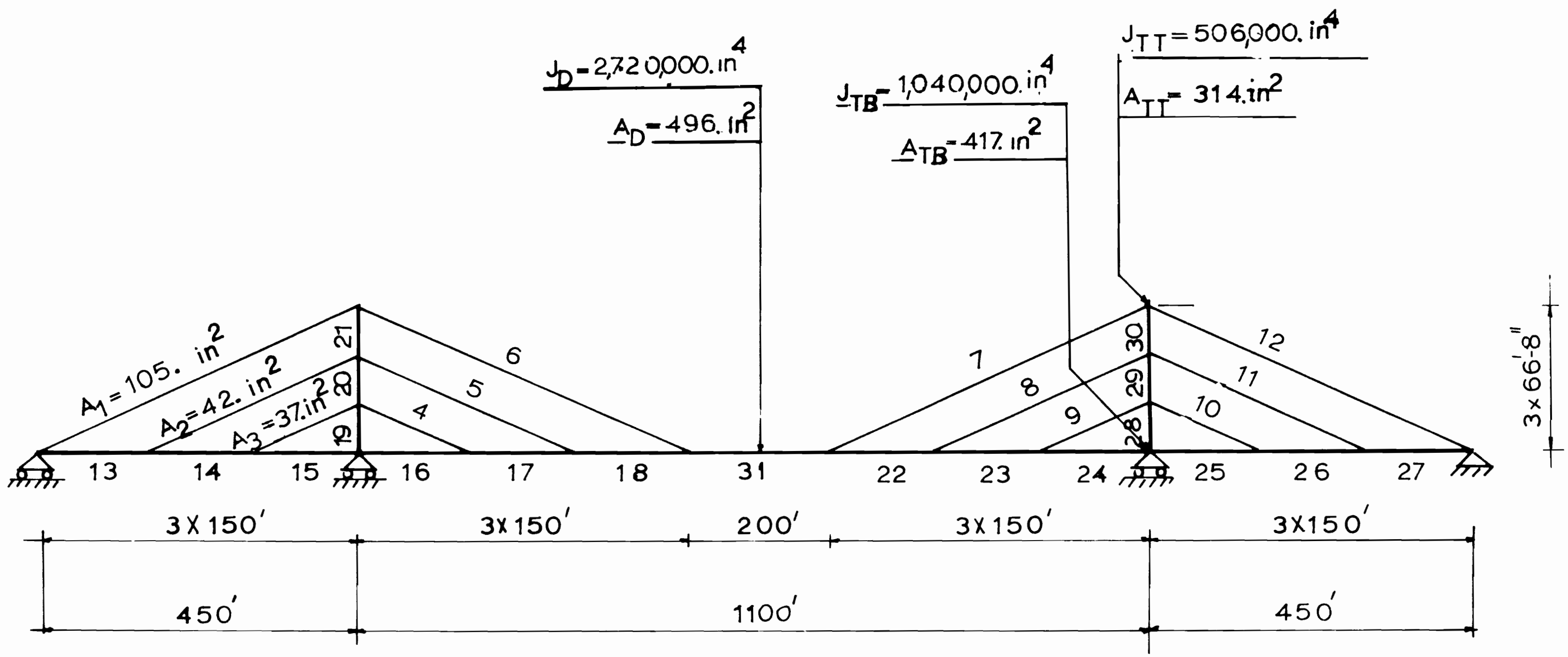


FIG. 7 GEOMETRY AND SECTIONAL PROPERTIES OF BRIDGE PROTOTYPE

the above scale factors

$$A_1^m = A_6^m = A_7^m = A_{12}^m = \frac{A_1^p}{k_A} = \frac{105}{10^4} = 0.01050 \text{ in}^2$$

$$A_2^m = A_5^m = A_8^m = A_{11}^m = \frac{A_2^p}{k_A} = \frac{42}{10^4} = 0.00420 \text{ in}^2$$

$$A_3^m = A_4^m = A_9^m = A_{10}^m = \frac{A_3^p}{k_A} = \frac{37}{10^4} = 0.00370 \text{ in}^2$$

$$J_D^m = \frac{J_D^p}{k_I} = \frac{2.72 \times 10^6}{10^8} = 0.02720 \text{ in}^4$$

$$A_D^m = \frac{A_D^p}{k_A} = \frac{496}{10^4} = 0.04960 \text{ in}^2 \qquad (22)$$

$$J_{TB}^m = \frac{J_{TB}^p}{k_I} = \frac{1.04 \times 10^6}{10^8} = 0.01040 \text{ in}^4$$

$$A_{TB}^m = \frac{A_{TB}^p}{k_A} = \frac{417}{10^4} = 0.04170 \text{ in}^2$$

$$J_{TT}^m = \frac{J_{TT}^p}{k_I} = \frac{.506 \times 10^6}{10^8} = 0.00506 \text{ in}^4$$

$$A_{TT}^m = \frac{A_{TT}^p}{k_A} = \frac{314}{10^4} = 0.03140 \text{ in}^2$$

For statical loads acting in a vertical plane, the above values may be increased or decreased proportionally. In order to employ commercially available sections, after multiple trials, the above sectional properties were increased by a factor of C = 31.6. Consequently,

$$A_1^m = A_6^m = A_7^m = A_{12}^m = 0.332 \text{ in}^2$$

$$A_2^m = A_5^m = A_8^m = A_{11}^m = 0.132 \text{ in}^2$$

$$A_3^m = A_4^m = A_9^m = A_{10}^m = 0.117 \text{ in}^2$$

$$J_D^m = 0.860 \text{ in}^4$$

$$A_D^m = 1.570 \text{ in}^2$$

$$J_{TB}^m = 0.331 \text{ in}^4 \qquad (23)$$

$$A_{TB}^m = 1.320 \text{ in}^2$$

$$J_{TT}^m = 0.160 \text{ in}^4$$

$$A_{TT}^m = 1.010 \text{ in}^2$$

To obtain the longitudinal dimensions of the model, the corresponding dimensions of the prototype were divided by $k_L = 100$.

The geometry of the model and the sectional properties requiered are shown on Figure 8.

c. Selection of sections for the model

The sections chosen for the girder were 2 - 2" x 1" x 3/16" channels (U.S. Steel Corporation, Catalog of Special Sections, Section C-597). The channels were made of ASTM-A7 steel and had a cross sectional moment of inertia in the vertical plane of bending of $I = 0.84 \text{ in}^4$. The area of the cross section was 1.53 in^2. The modulus of elasticity was assumed as equal to $E = 29{,}000$ ksi. The above area and moment of inertia were close to the requiered sizes, the

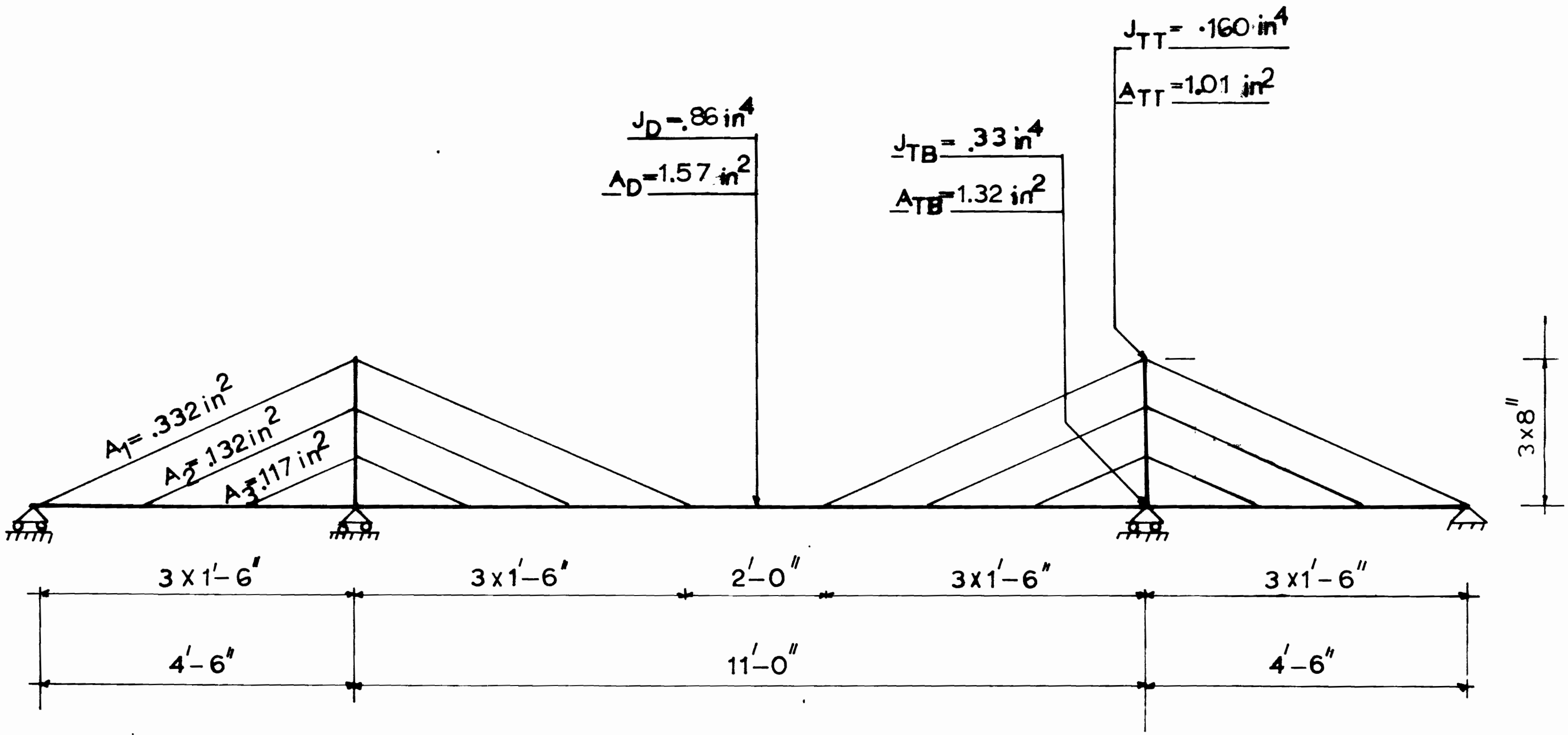


FIG. 8 GEOMETRY AND REQUIRED SECTIONAL PROPERTIES OF THE MODEL

differences being 2.32% for moment of inertia and 2.61% for area.

Before selecting channels, tubular sections were considered. This would have led, however, to a more complex type of cable-girder and tower-girder connection that would be more difficult to manufacture.

The towers were designed as variable rectangular shapes milled from C-1020 steel, having a nominal modulus of elasticity of E = 29,000 ksi.

For the cables, round bars were used. Preliminary investigations made with flexible cables did not give satisfactory results as their elastic properties were not constant. The bars were made of AISI C12L15 steel and had a nominal modulus of elasticity of E = 29,000 ksi. Consequently, $A_1 \ldots A_{12}$ had to be recalculated by decreasing the values given in (23) in the ratio of 18/29 = 0.62. Thus

$$\begin{aligned} A_1^m = A_6^m = A_7^m = A_{12}^m &= 0.332 \times 0.62 = 0.206 \text{ in}^2 \\ A_2^m = A_5^m = A_8^m = A_{11}^m &= 0.132 \times 0.62 = 0.081 \text{ in}^2 \\ A_3^m = A_4^m = A_9^m = A_{10}^m &= 0.117 \times 0.62 = 0.073 \text{ in}^2 \end{aligned} \tag{24}$$

The sizes selected were ½ inch round bars for members 1, 6, 7, 12 and 5/16 inch round bars for all other cables. For members, 3, 4, 8, 9, the deviation was about 6% from the requirements set forth in Figure 8 as closer sections were not commercially available.

A schematic view of the model with dimensions and sectional properties, is shown on Figure 9. Figure 10 represents a photograph of the model taken in the instrumentation stage.

d. Design of connections

The connections of the model were designed taking into account the following basic requirements:

- Eccentric application of forces should be avoided as much as possible.

- The connections should enable pretensioning of cables in order to eliminate any possible compression forces in cables during the loading process.

A. <u>Girder-cable connection</u>

The girder-cable connection is represented in Figure 11. The parts are made of C-1020 steel. The bolts are ASTM A325 high strength. The connection is movable, and it may be located at any place along the deck. Postensioning of rods is done by turning the nut "1".

B. <u>Tower-cable connection</u>

The tower-cable connection is shown in Figure 12. The parts are made of C-1020 steel. The particular design of this connection enables the cables to act centrally on the tower.

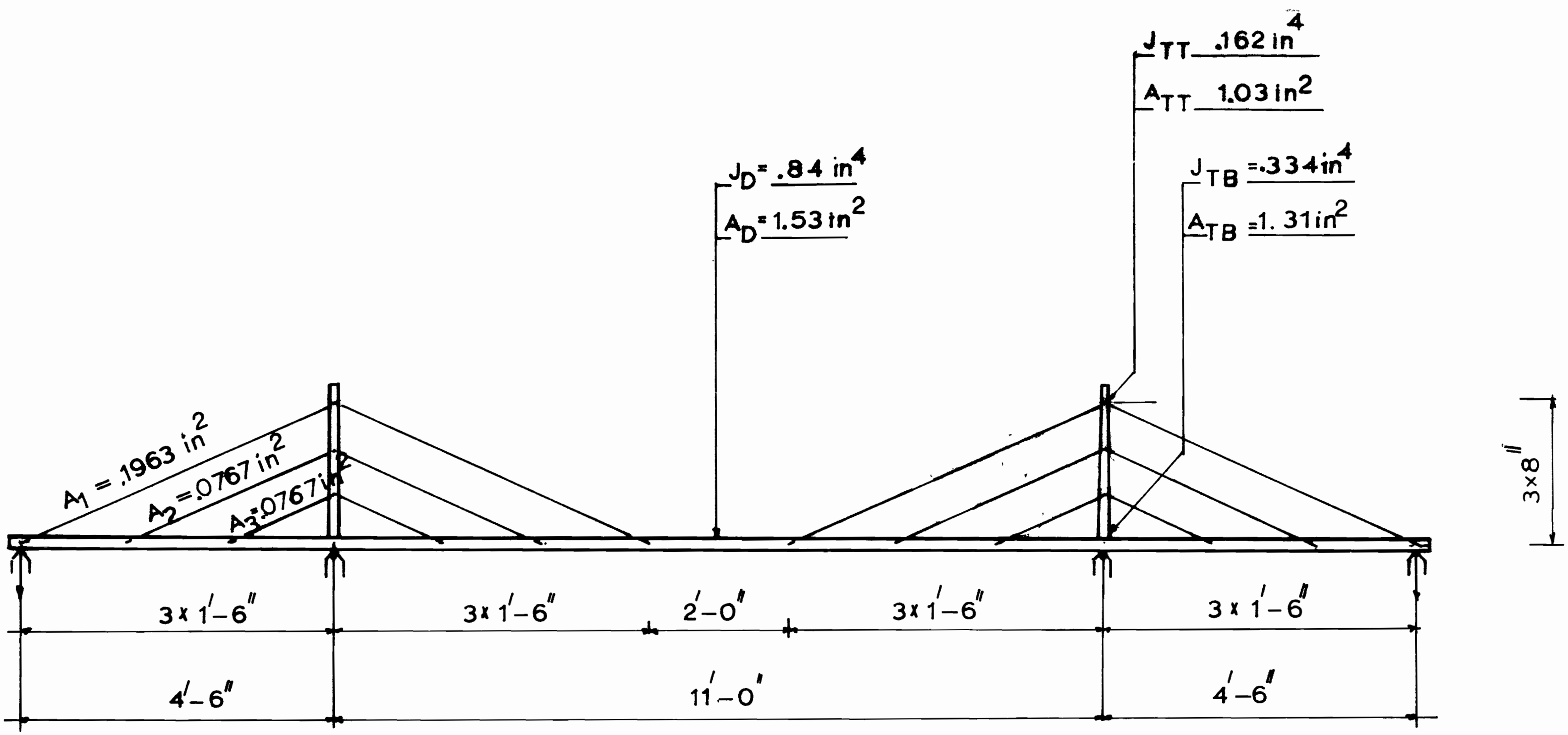
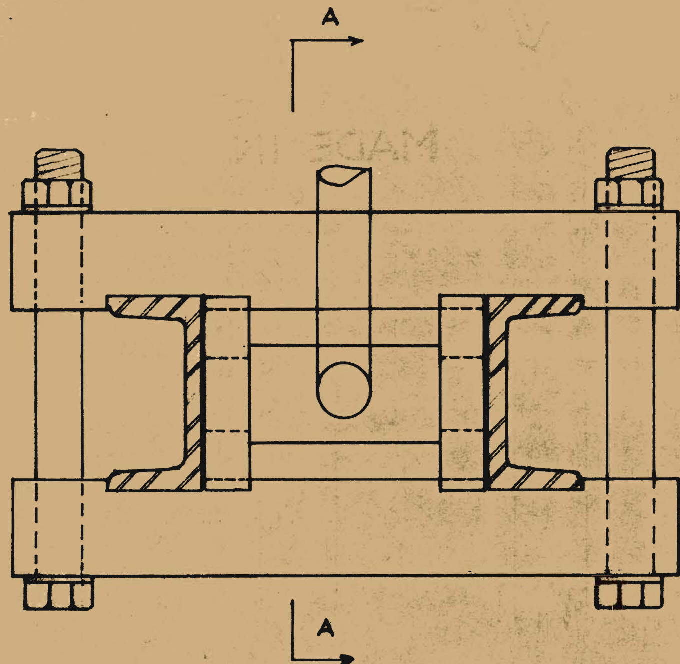
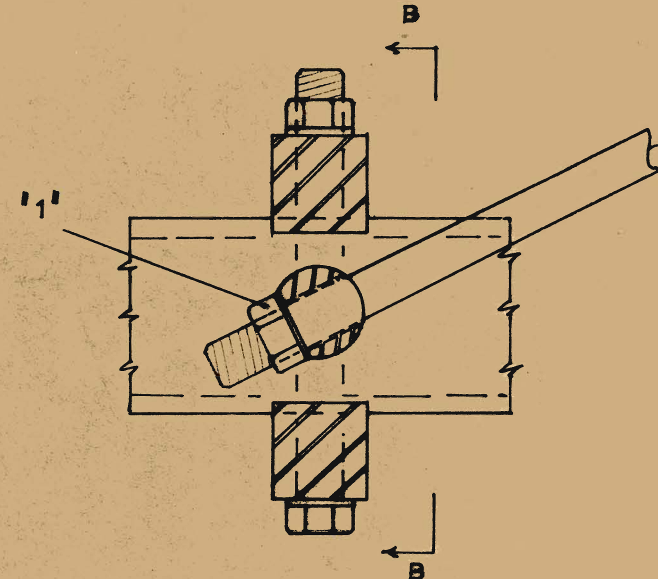


FIG. 9 DIMENSIONS AND SECTIONAL PROPERTIES OF THE MODEL

FIG. 1o MODEL FOR DETERMINATION OF INFLUENCE LINES. GENERAL VIEW.



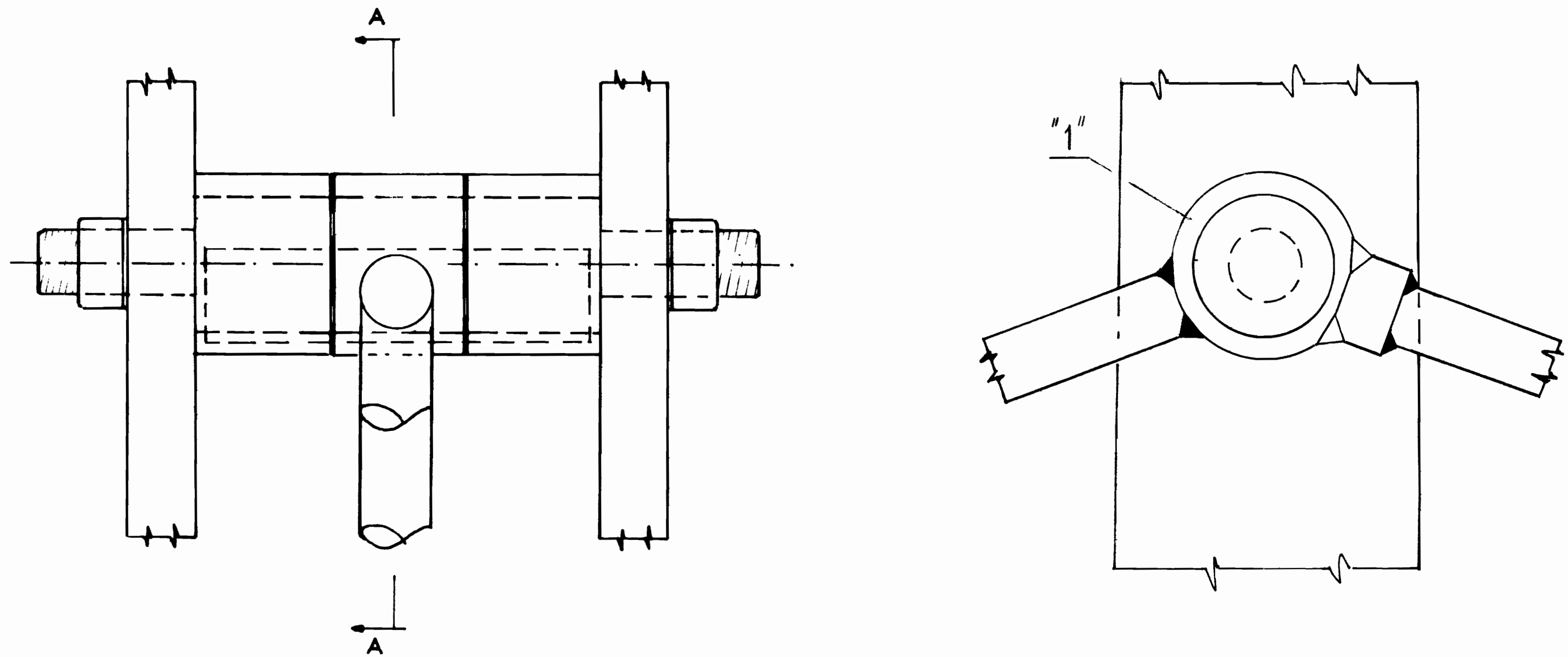
a. CROSS SECTION B-B



b. LONGITUDINAL SECTION A-A

FIG. 11 GIRDER-CABLE CONNECTION

SCALE $1/2'' = 1''$



a. FRONT VIEW

b. SECTION A-A

FIG. 12 TOWER-CABLE CONNECTION

SCALE 3/4" = 1"

C. Tower-girder connection

The tower-girder connection is shown on Figure 13. The connection is designed such that it may be employed both as hinged or rigid. With the square bars "1" positioned as shown, the connection is rigid. By moving the square bars laterally to the adjacent set of holes, the connection becomes hinged. In this investigation the rigid connection only is considered. A photograph of these connections is shown in Figure 14.

e. Design of the supports

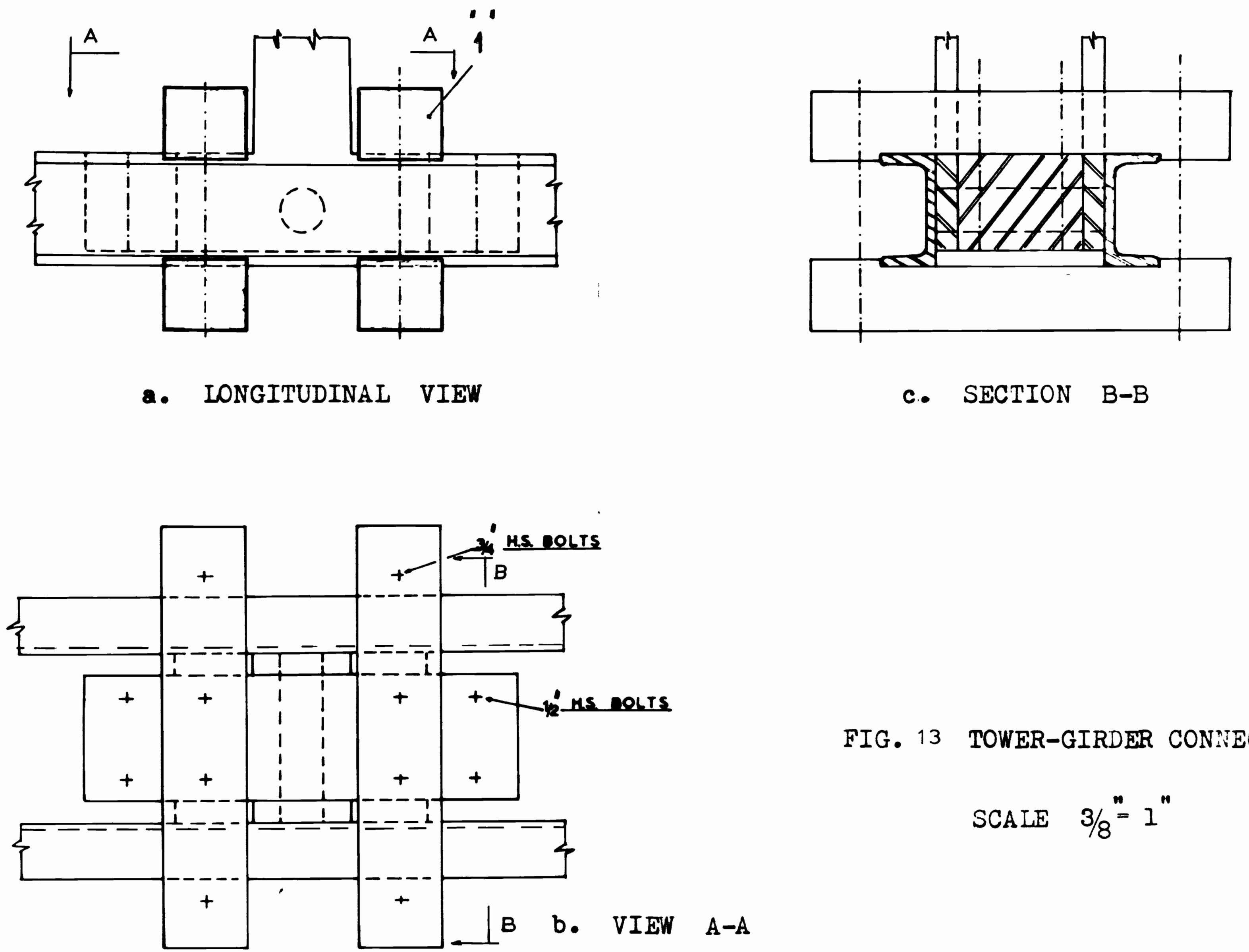
The supports of the bridge model have been designed to satisfy the following requirements:

A. The supporting area shall be as small as possible.

B. All supports except one should be free to move horizontally. Hence, no constraints inducing interior stresses will be applied on the model.

C. The end supports should be capable of withstanding uplift forces.

A detail of the interior support is shown on Figure 15. The end support is similar. The contact width between the supports and the girder channels is 1/16 in. To assure free movement of the supports, a layer of 1/32 in. of grease was inserted between the support block and the alignment plate. The alignment plates were provided with leveling screws.



a. LONGITUDINAL VIEW

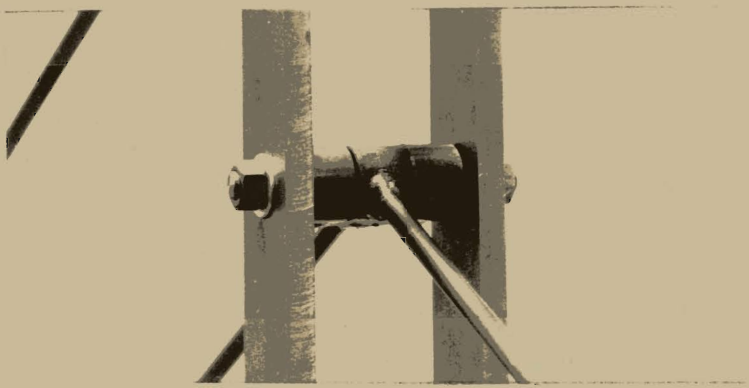
b. VIEW A-A

c. SECTION B-B

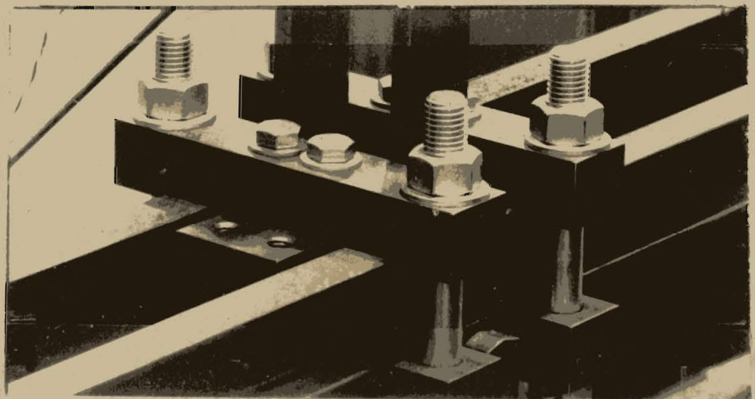
FIG. 13 TOWER-GIRDER CONNECTION

SCALE $3/8" = 1"$

a. GIRDER-CABLE CONNECTION



b. CABLE-TOWER CONNECTION



c. GIRDER-TOWER CONNECTION

FIG. 14 CONNECTIONS

To hold down the end supports, the device shown schematically on Figure 16 was provided. The roller bearings enable the channels to move freely in case of longitudinal shortening of the girder.

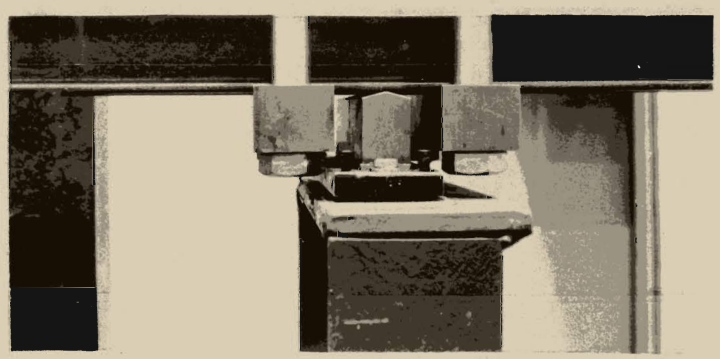
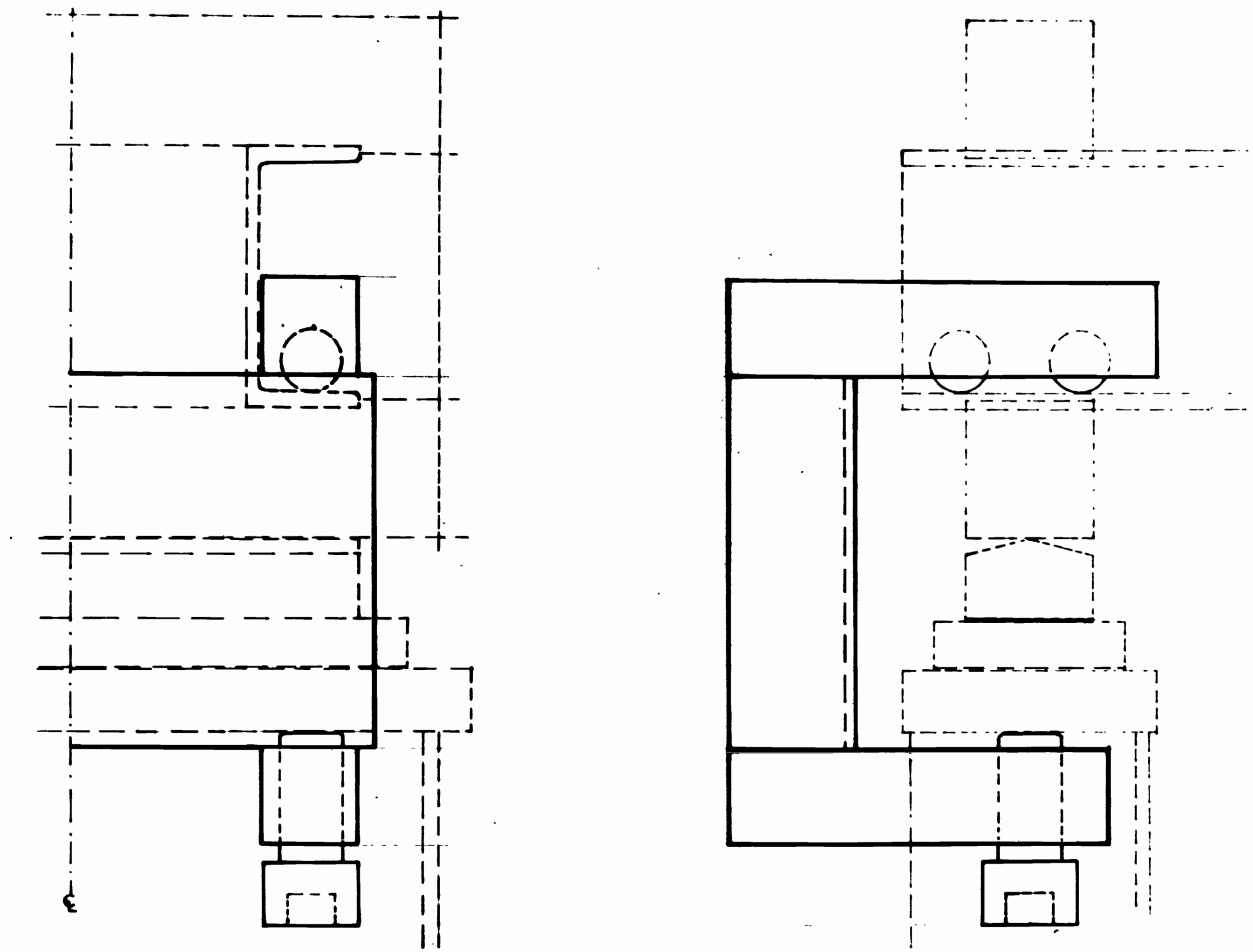


FIG. 15 SUPPORT DETAIL

f. Foundation

The foundation of the bridge model was constructed of reinforced concrete piers and steel channels.



A. FRONT VIEW

B. LATERAL VIEW

FIG. 16 UPLIFT FORCES DEVICE

Figure 17 represents the foundation during erection. For the case in which another model has to be tested on the same foundation, the vertical supporting channels may be moved horizontally or vertically. The vertical angles have been provided as supports for dial gages. The water level seen in the picture was employed to align the top bearing plates at the same level.

3. Manufacturing of the model

As a consequence of the principles adopted in design to employ commercial sections, wherever possible, to provide simple connections and to avoid welding of built-up small size sections, the model could be manufactured in the existing machine shop of the Faculty of Engineering without the need of special equipment. The machine shop equipment includes lathes, drilling, milling and welding machines, a band saw and grinders.

The fabrication was carried out within a tolerance of less than 1/32 in. for the cross sectional dimension and of less than 1/16 in. in the longitudinal direction.

4. Instrumentation

The instrumentation employed consisted of strain gages, dial gages and apparatus to acquire the strain gage information.

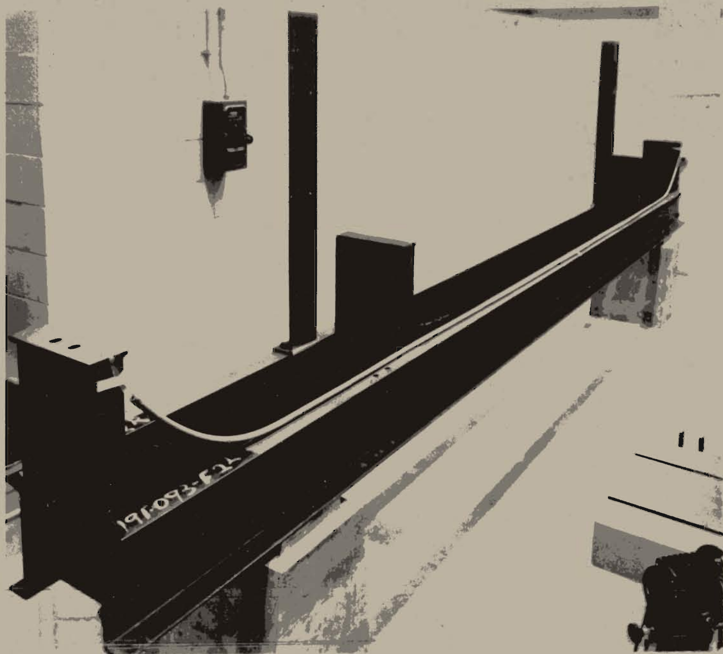
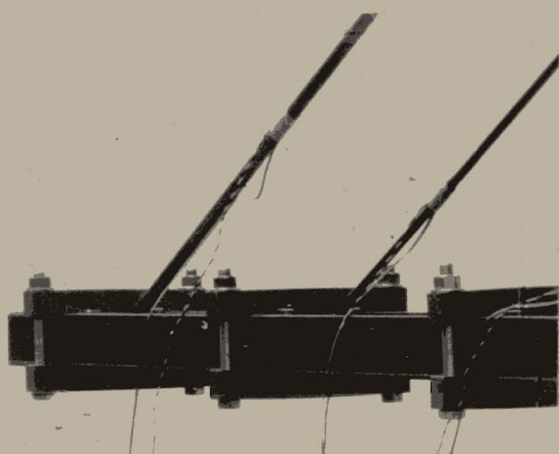


FIG.17 FOUNDATION



a. CABLE

b. CHANNEL

FIG. 18 STRAIN GAGE DETAILS

a. Strain gages

The strain gagelayout is represented on Figure 19 and details are shown on Figure 18.

Strain gages were applied on the stiffening girder as shown on Figure 19. These gages enable the reading of strains at extreme fibers and in the center of the web of the stiffening girder. From this information, the bending moment at interior support can be determined. To obtain strains in cables, two strain gages were applied, one opposite to the other, on each cable, Figure 19. The arrangement described, provides information from seven locations. The system investigated has a degree of statical indeterminancy equal to fourteen. Because of symmetry, however, only seven redundants need to be determined.

The strain gages employed were 1/4 in. Micro-measurement precision strain gages type EA-06-250BG-120. They were temperature compensated for steel and had a resistance of 120 ohms. The gage factor was 2.11 at 75^{o}F.

In the first stage of the investigation, the strains were acquired, one at a time, with a Budd Portable Digital Strain Indicator P-350 and its companion Portable 10 Channel Switch and Balance Unit SB-1.

In addition, a Baldwin Switching and Balancing unit SR-4, attached to the SB-1 unit, was also employed.

In the second stage of the investigation, the strain

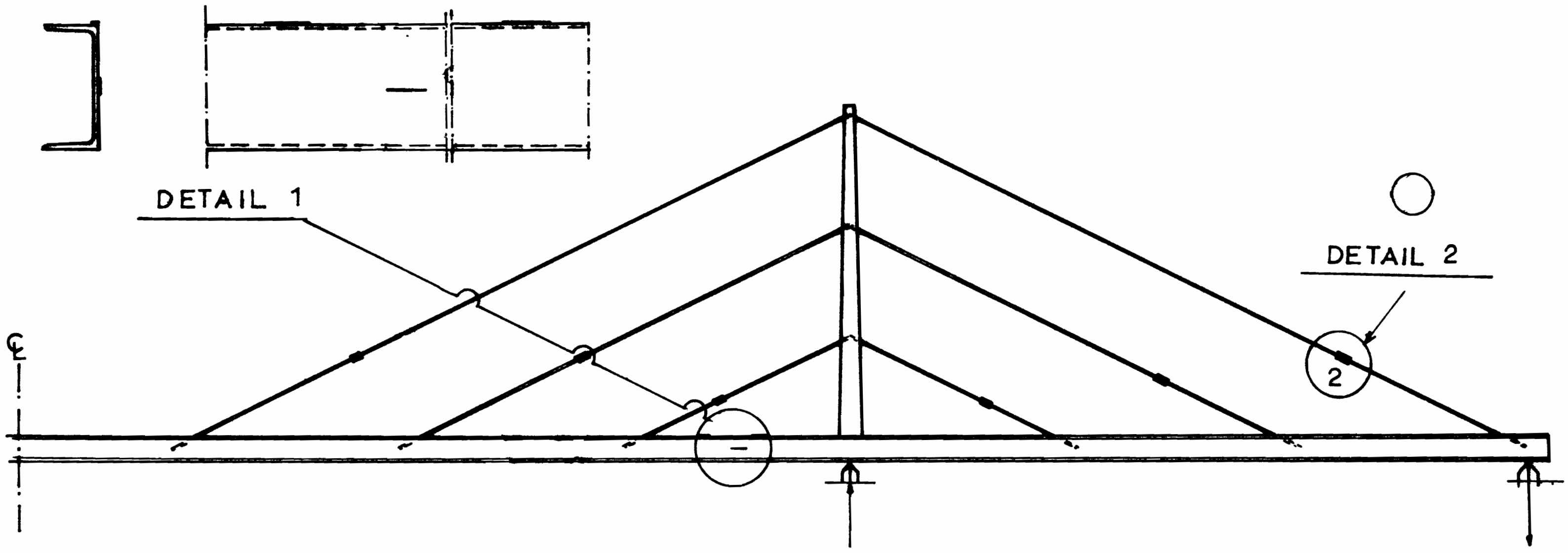


FIG. 19 STRAIN GAGE LAYOUT

data were acquired with a DS-366 Data Acquisition System. This system consists essentially of five parts.

A. A strain gage conditioner. This part acquires data from the various channels.

B. A signal amplifier

C. An analog to digital converter

D. A PDP-8/L 4k digital computer

E. A teletype console

The Data Acquisition System acquires data from the channels, performs a limited number of calculations and prints out the information on the typewriter.

b. Dial gages

The dial gage arrangement for determination of deflections is shown on Figure 20. Deflections were determined on half of the bridge only as the structure is symmetrical. Dial gages from 1 to 5 measure vertical deflections at joints 2, 3, 5, 6, 7 and dial gage 6 registers the horizontal displacement at joint 17. Joint numbers are indicated above or at the right of each joint on Figure 20 and dial gage numbers are indicated in a circle below or at the left of the joint.

The above arrangement allows the determination of vertical deflections of the stiffening girder and horizontal deflection at top of tower. It does not provide all infor-

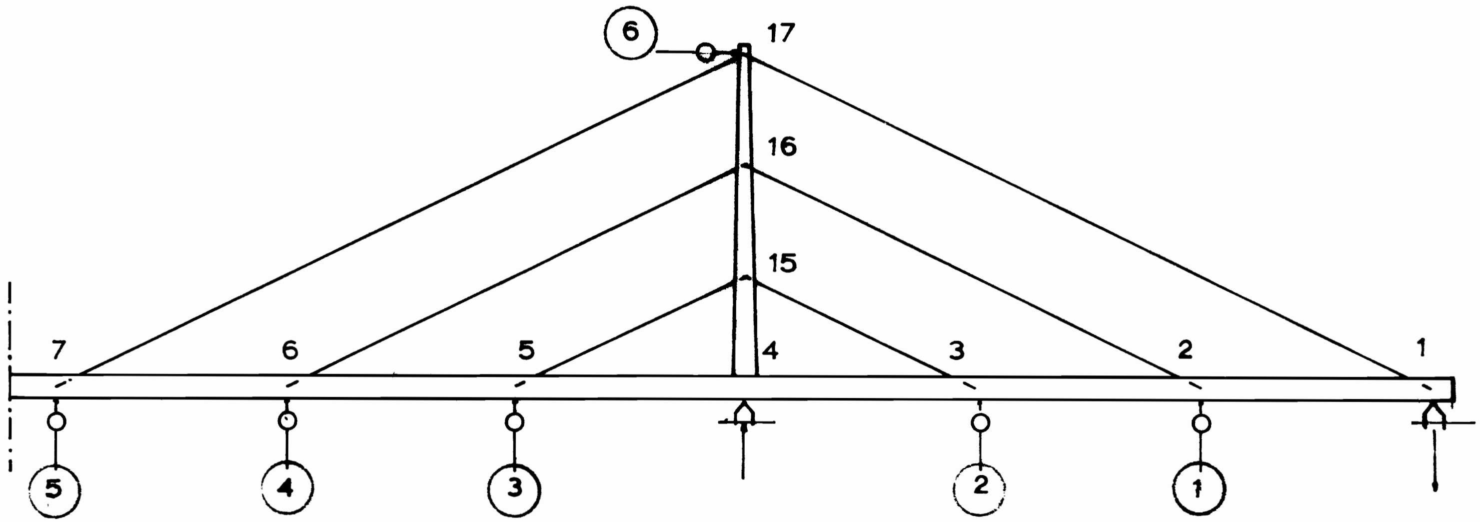


FIG. 20 DIAL GAGE ARRANGEMENT FOR DETERMINATION OF DEFLECTIONS

mation required to determine bending moments, axial and shear forces at joints, as described in Section 1 of this Part, Equations (1), (2), (3).

To obtain bending moments, and axial and shear forces from displacements, a dial gage arrangement as shown on Figure 21 was employed. Conversion of dial gage readings into joint displacements is shown in Appendix 3.

5. Loading, calibration and acquisition of data.

a. Loading

To determine influence lines for axial forces, bending moments and deflections, a concentrated load has been applied on the model at intervals of six inches. The load was simulated by two special loading devices "A" and "B". Device "A" was designed to apply a concentrated load at joints and at locations between joints where device "B" could not be employed due to interference with cables. Device "B" was designed for locations between joints. The schematic representation of both loading devices is shown on Figure 22 and details are represented on Figure 23. By turning the nut which connects the loading rod with the loading device a concentrated force is applied on the model.

Device "A" was designed such that there was no interference with the dial gages. Device "B" was provided with a torsional resistant support so as to counteract the torsional moment developed due to the application of loads out-

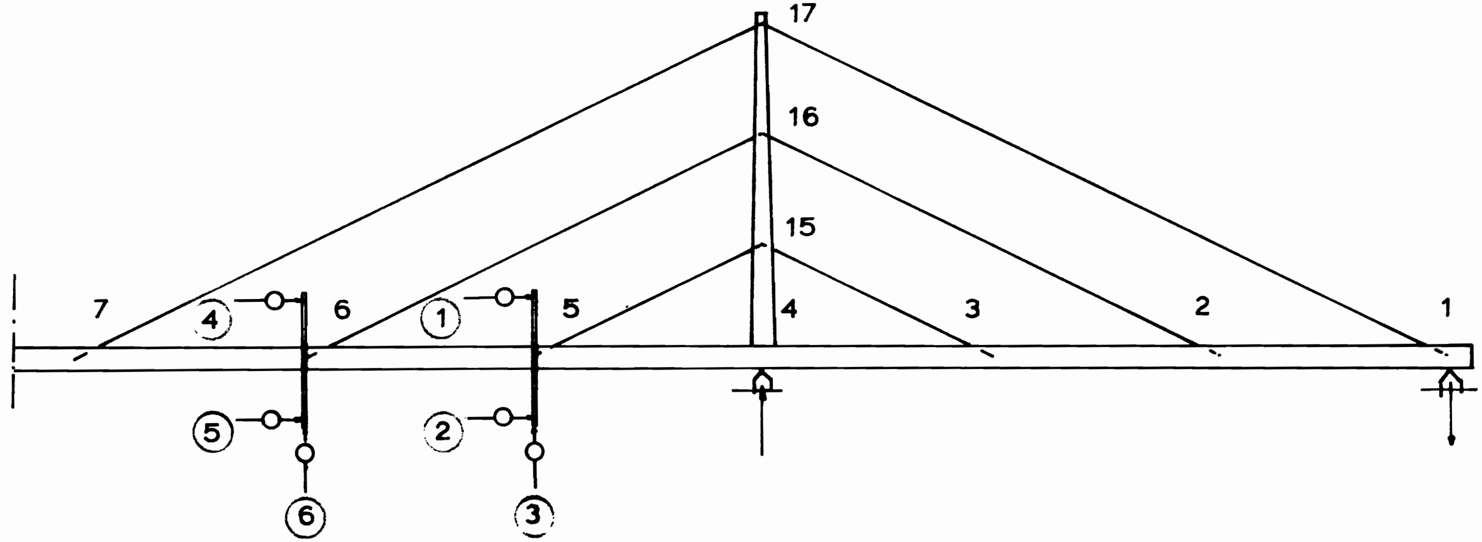


FIG. 21 DIAL GAGE ARRANGEMENT FOR DETERMINATION OF BENDING MOMENTS, AXIAL AND SHEAR FORCES

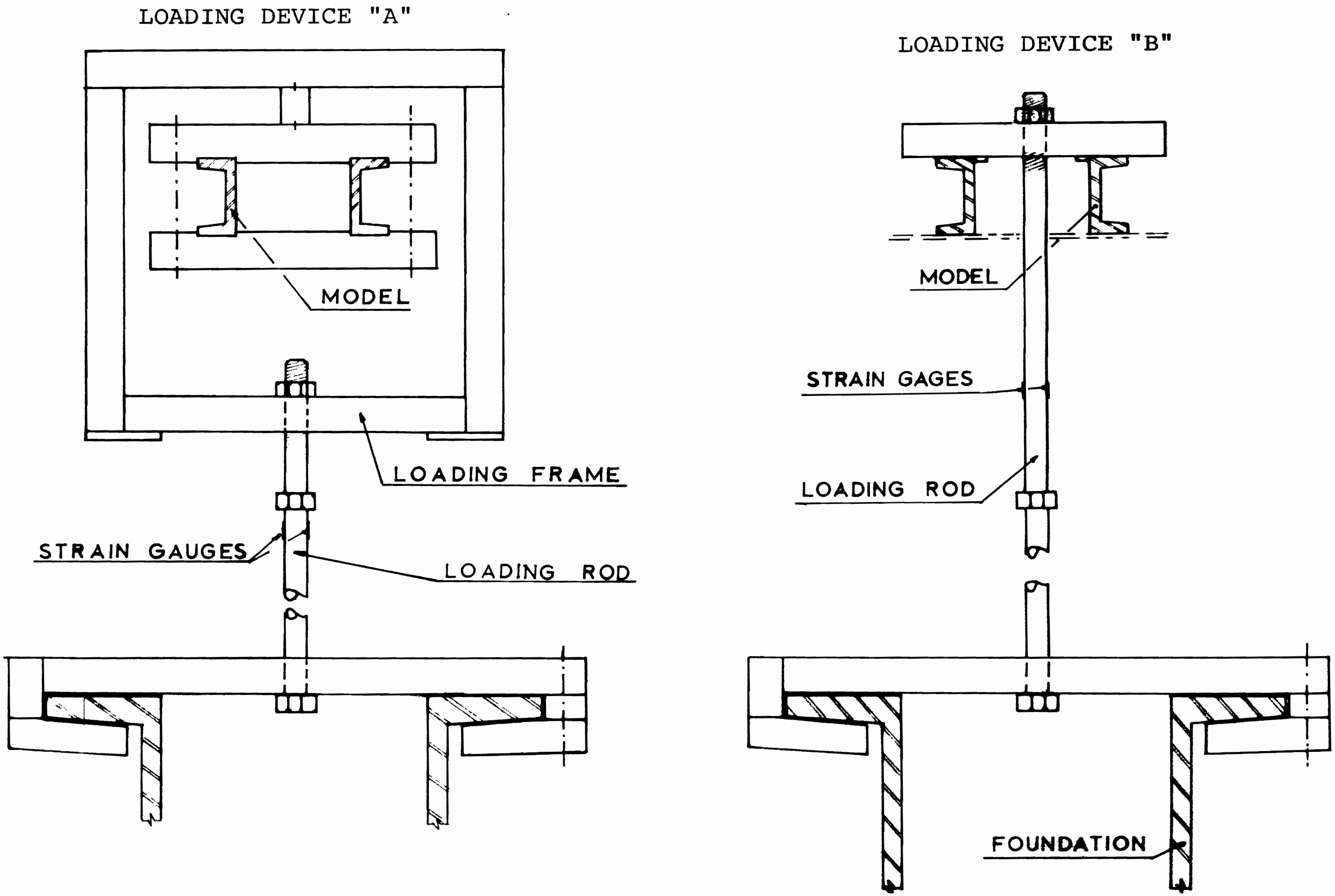
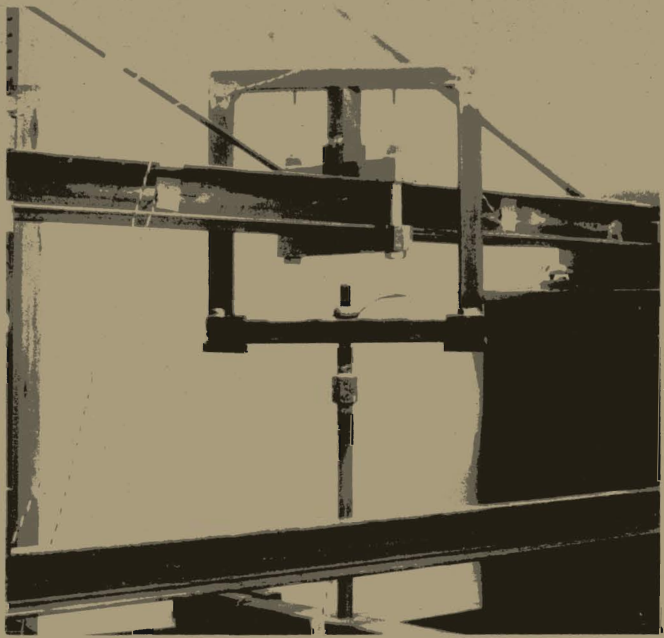
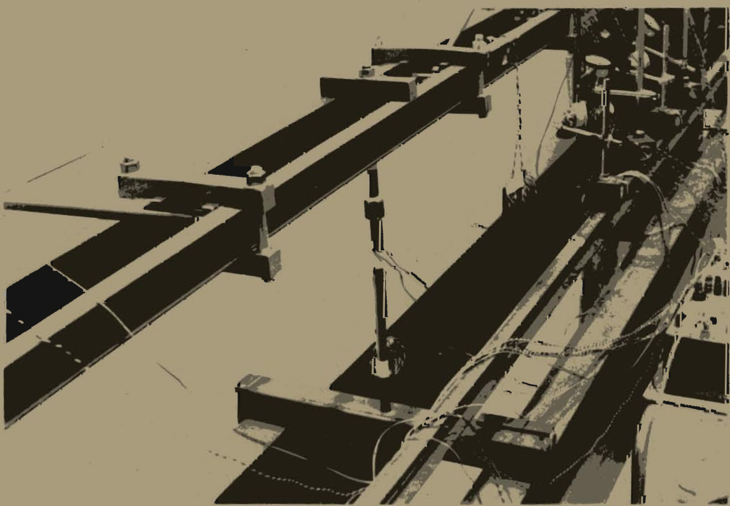


FIG. 22 LOADING DEVICES, SCHEMATIC REPRESENTATION



a. DEVICE "A"



b. DEVICE "B"

FIG. 23 LOADING DEVICES, DETAILS

side the shear center of the section.

During the loading process, the magnitude of the concentrated load was maintained at less than 2,500 lbs. The displacements induced in the system were proportional to the applied loads. To achieve this, two reading were taken for each measurement and their linearity verified.

b. Calibration

Before loading the model and recording the data,the girders, cables and loading rods were calibrated in order to verify whether the data obtained from the strain gage readings corresponded to the assumptions made regarding modulus of elasticity and linearity of stress strain distribution.

The following items were calibrated:

A. C-597 channel

B. ½ in and 5/16 in. diameter round bars

C. loading rods "A" and "B"

A. <u>Calibration of Channel</u>

The C-597 channel was made of ASTM-A7 steel with a nominal modulus of elasticity of E = 29,000 ksi. The loading scheme, instrumentation and the calibration data recorded, are shown on Figure 24.

For a modulus of elasticity assumed as indicated

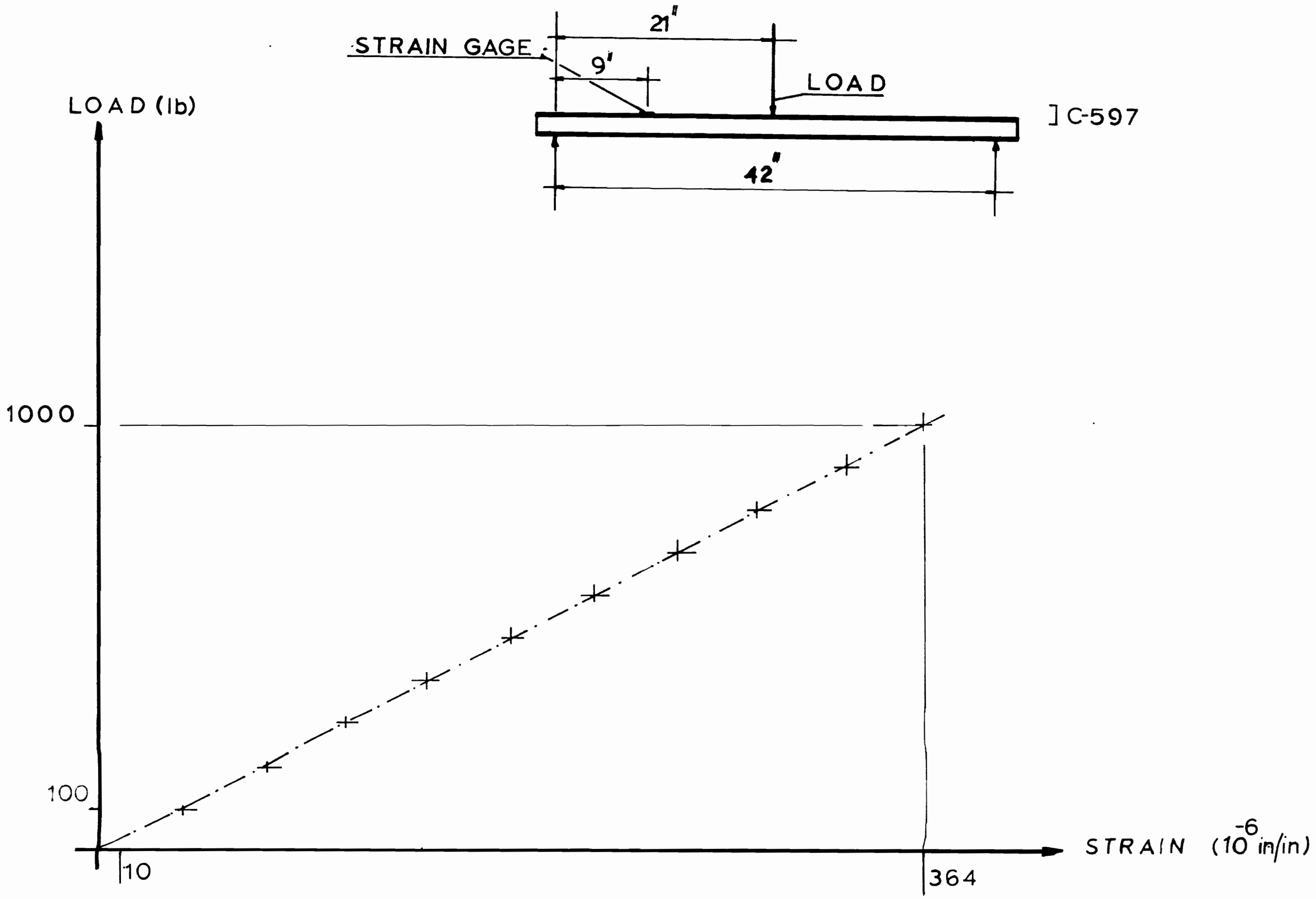


FIG. 24 CALIBRATION OF 2" CHANNEL

above, the theoretical strains were 1.6% less than the strains determined experimentally. Accordingly, from the calibration data, a bending strain of 40 μin/in was taken as equivalent to a bending moment of 1000 lb. in. acting on the girder cross section.

B. Calibration of ½ in and 5/16 in diameter round bars.

The ½ in and 5/16 diameter round bars had a nominal modulus of elasticity of E = 29,000 ksi and were made of AISI C12L15 steel. The loading scheme, instrumentation and the data recorded are shown on Figure 25.

For a modulus of elasticity assumed as indicated above, the actual strains were slightly larger than the theoretical strains. Thus from the calibration data, a normal force of 1000 lb was taken as equivalent to a normal strain of 169 μin/in for the 5/16 in diameter bar.

C. Calibration of loading rods "A" and "B"

The loading rods were made of C-1020 steel with a nominal modulus of elasticity of E= 29,000 ksi. The loading scheme, instrumentation and data recorded are shown on Figure 26.

From the calibration data, a normal force of 1000 lb was taken as equivalent to 109 μin/in for loading rod "A" and to 136 μin/in for loading rod "B".

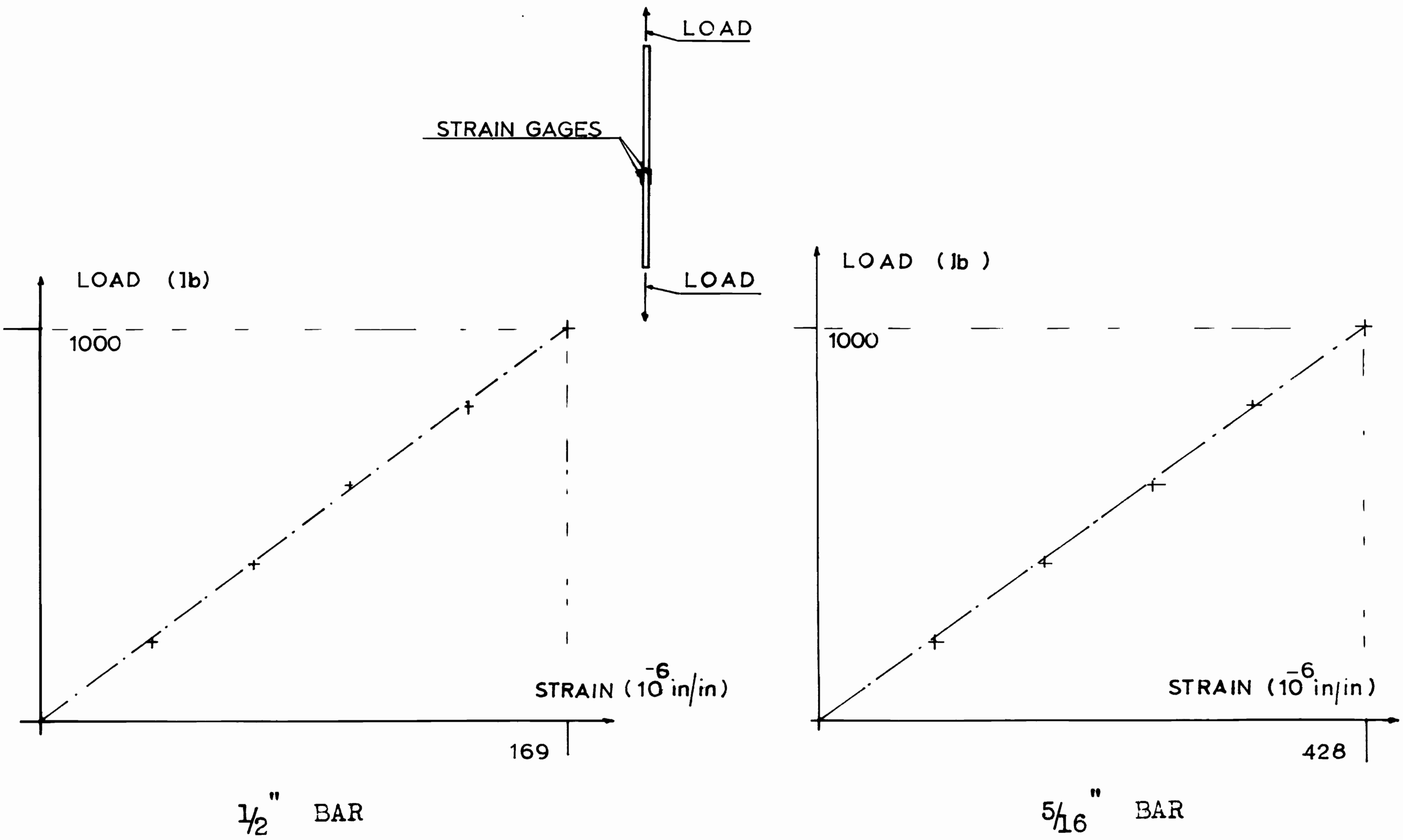
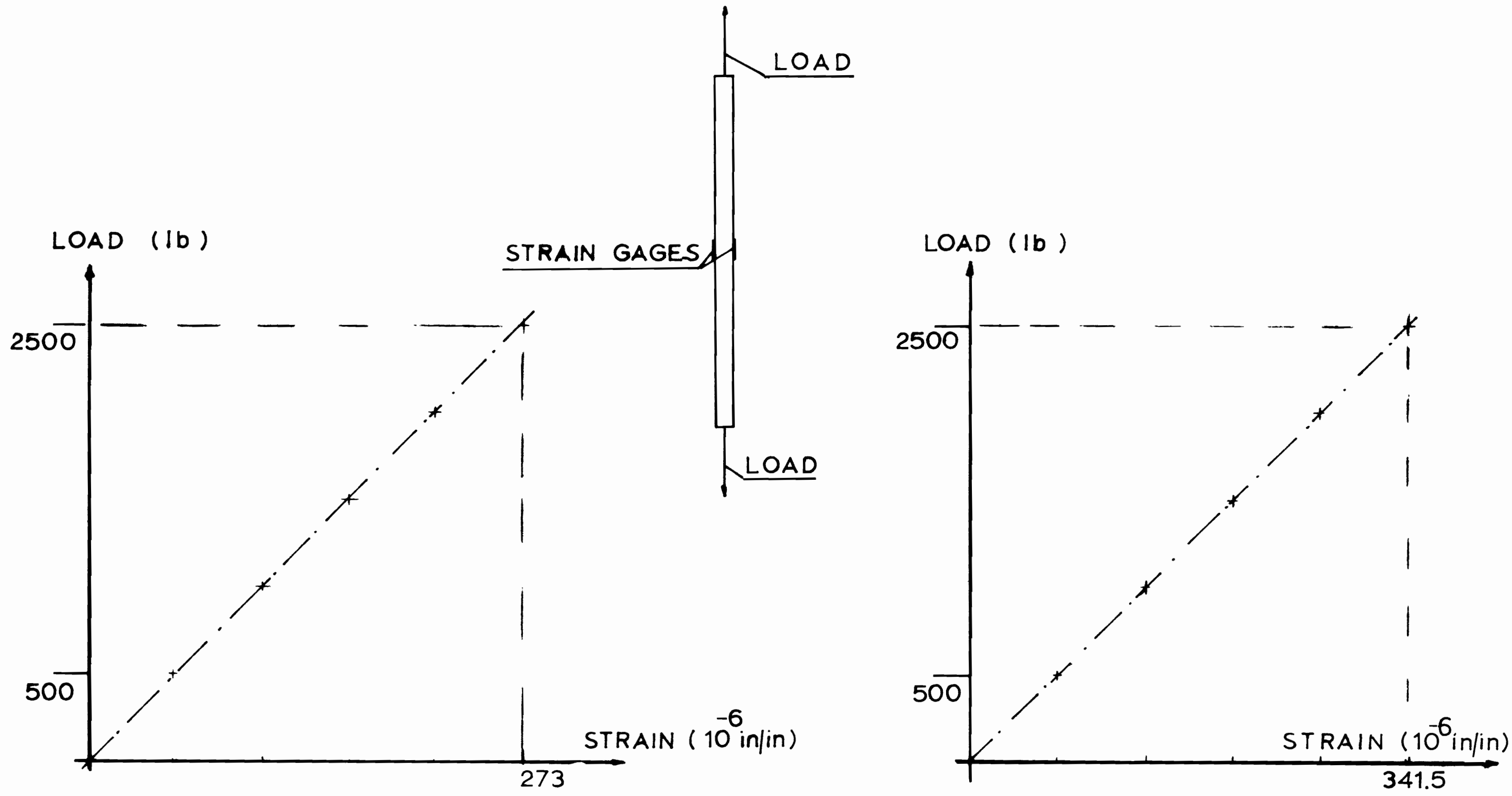


FIG. 25 CALIBRATION OF 1/2" AND 5/16" DIAMETER ROUND BARS.



LOADING ROD "A"

LOADING ROD "B"

FIG. 26 CALIBRATION OF LOADING RODS "A" AND "B"

c. Recording of data

The data recorded were strains and displacements due to a unit load applied at intervals of six inches along the bridge girder. The locations studied are shown on Figs. 19 and 20.

The strain readings were converted to bending moments and axial forces and compared with theoretical values, determined on a mathematical model. The results are represented in Figs. 27 to 39.

All influence lines correspond to the bridge prototype simulated by the model represented in Fig. 9.

The influence line ordinates of the bending moment, Fig. 33, are expressed in inches. Hence, the bending moments calculated employing these ordinates will be given in force-inch units.

To calculate influence line ordinates of displacements, Figs.34 to 39, E was expressed in lb/in^2, I in in^4, L in inches and p in lbs. Hence, the ordinates are expressed in in/lb and the resulting displacements will be in inches.

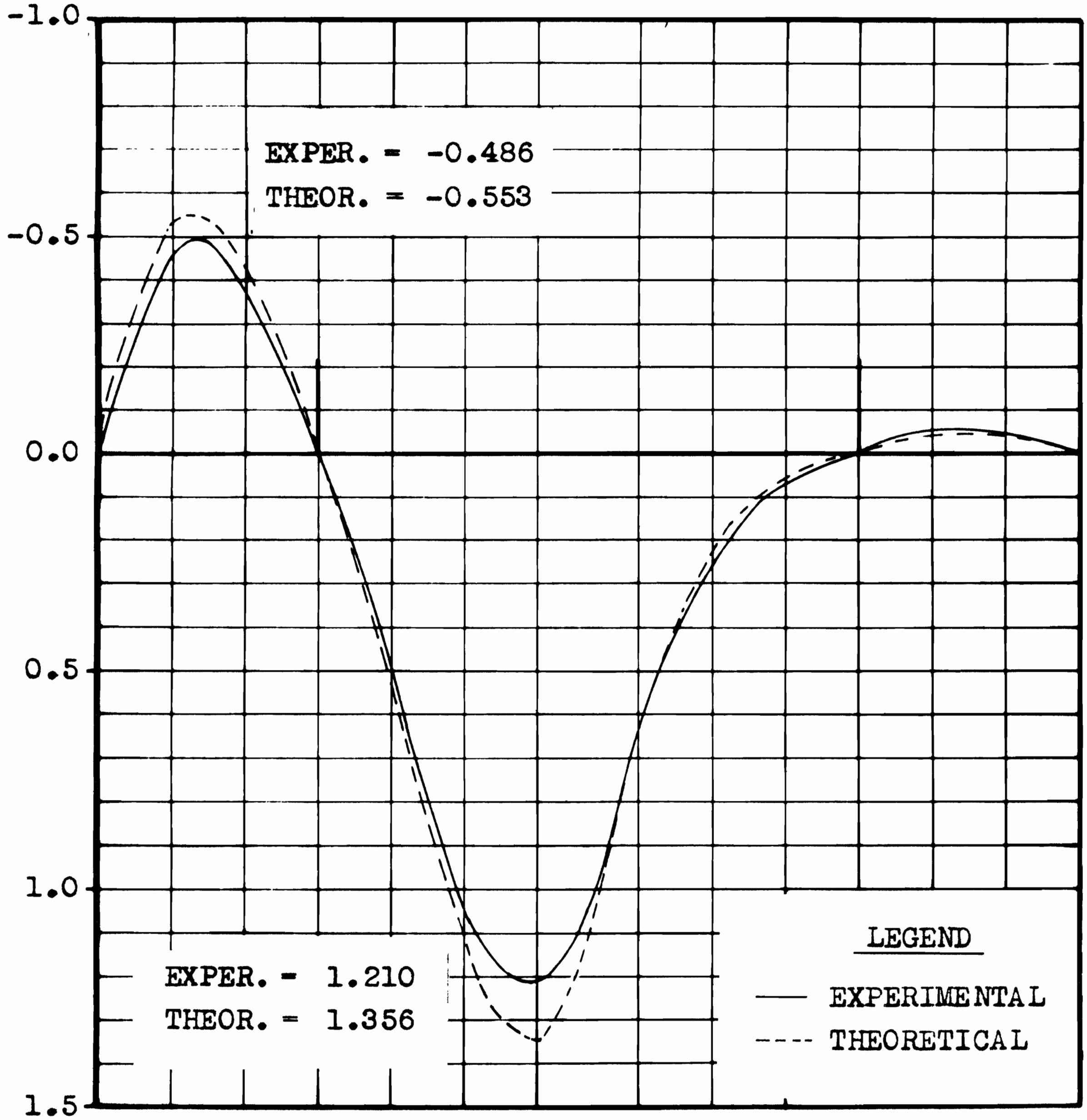


FIG. 27 INFLUENCE LINE OF AXIAL FORCE IN CABLE "1"

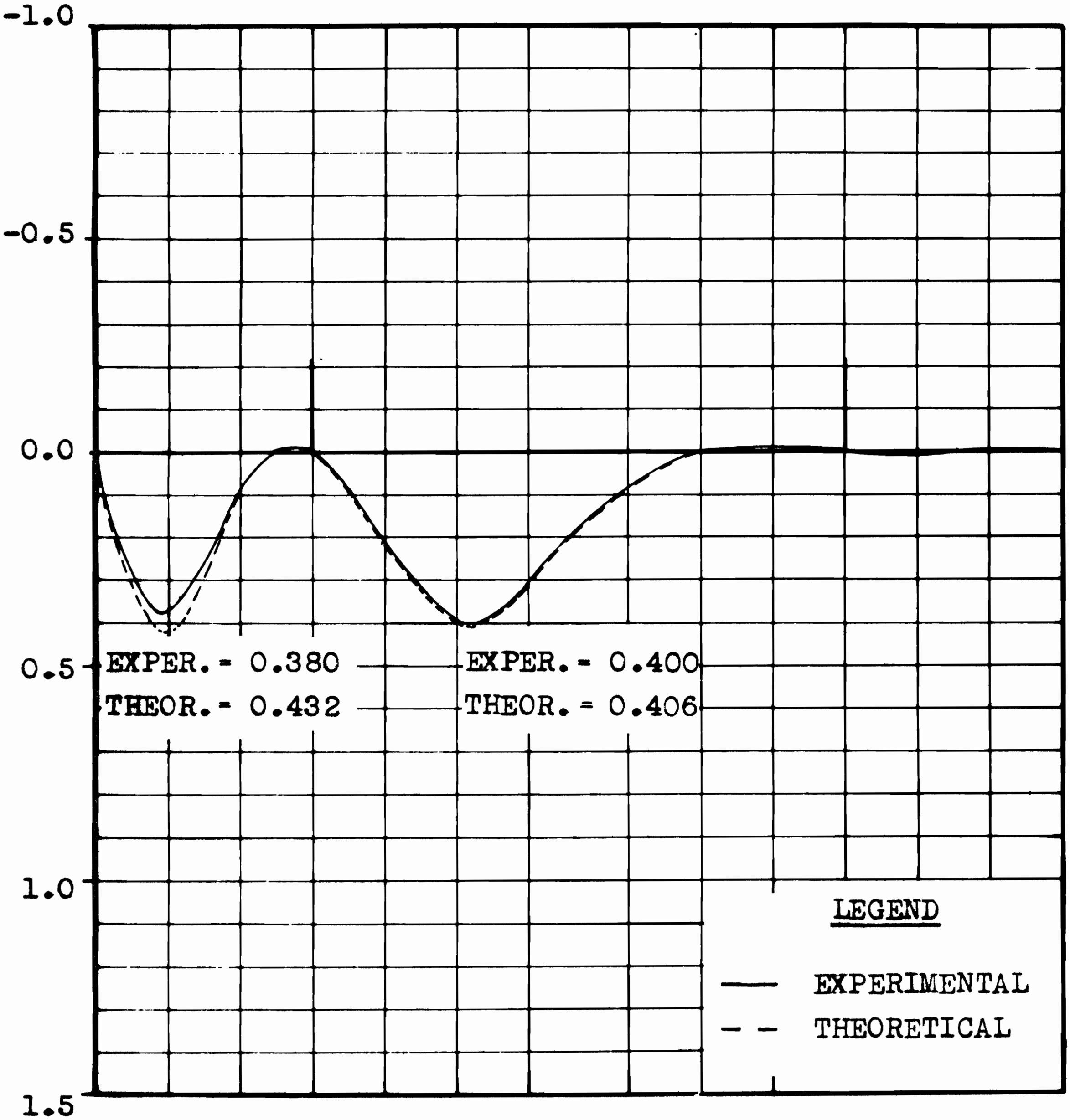


FIG. 28 INFLUENCE LINE OF AXIAL FORCE IN CABLE "2"

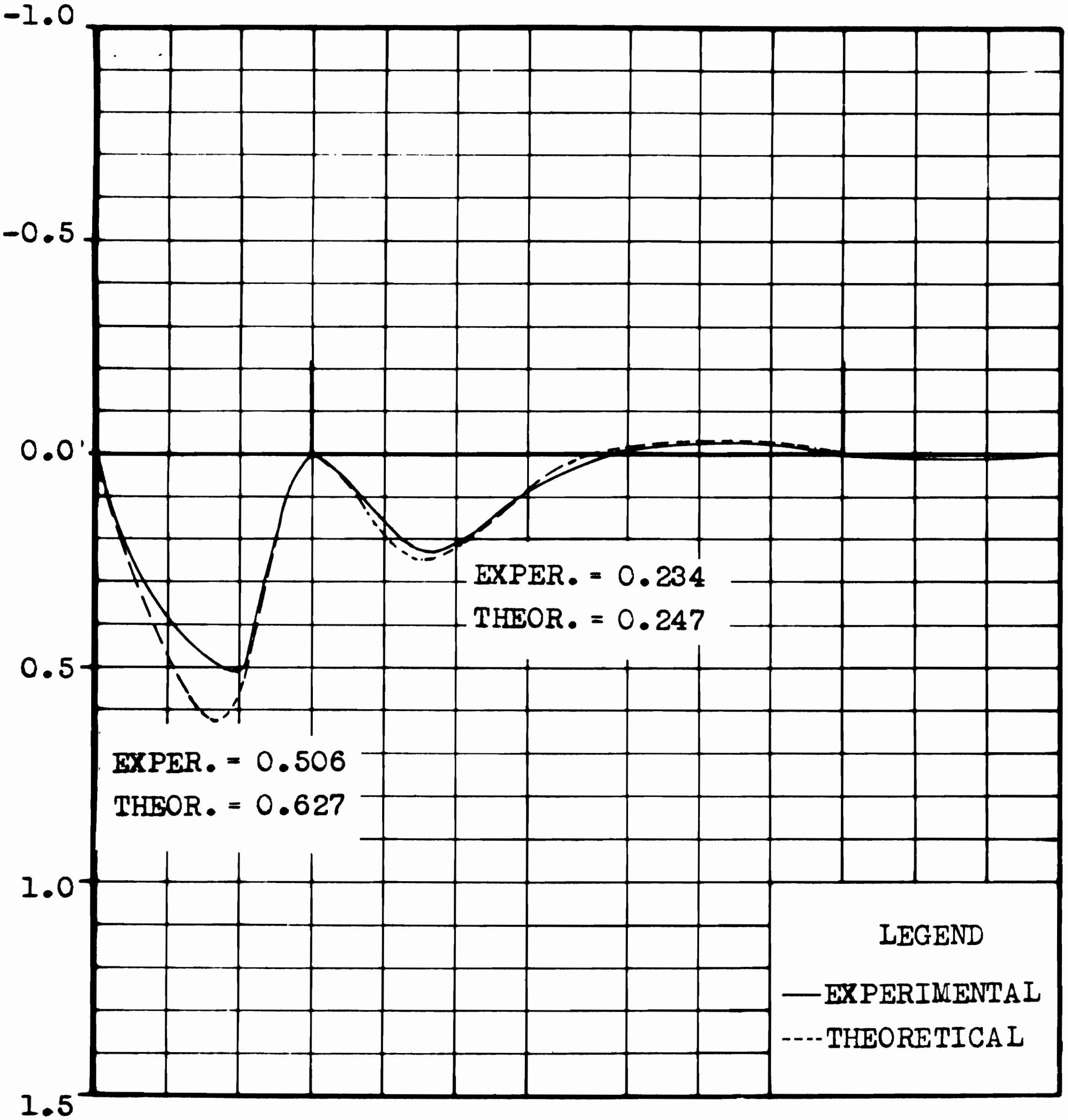


FIG. 29 INFLUENCE LINE OF AXIAL FORCE IN CABLE "3"

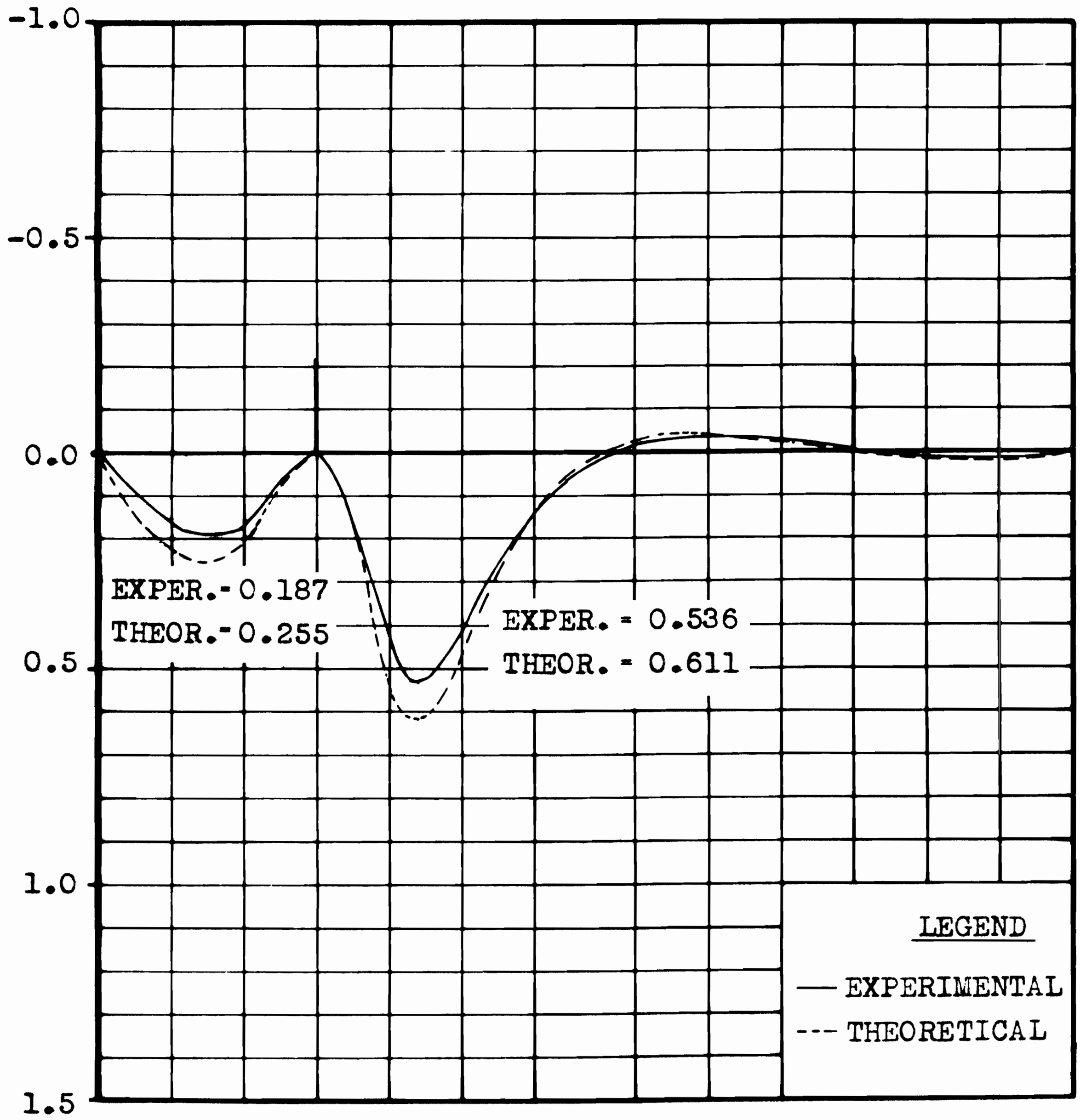


FIG. 30 INFLUENCE LINE OF AXIAL FORCE IN CABLE "4"

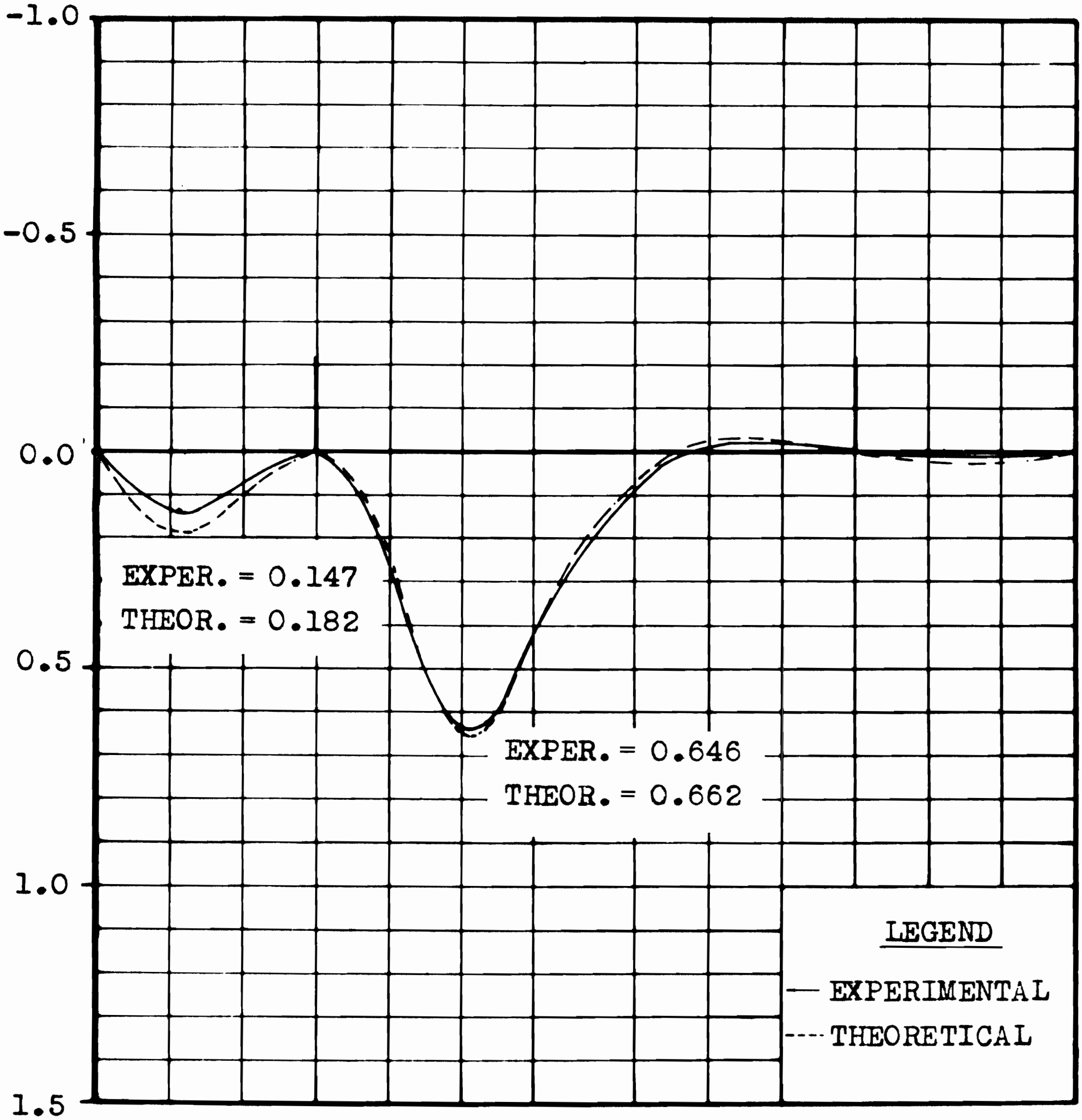


FIG. 31 INFLUENCE LINE OF AXIAL FORCE IN CABLE "5"

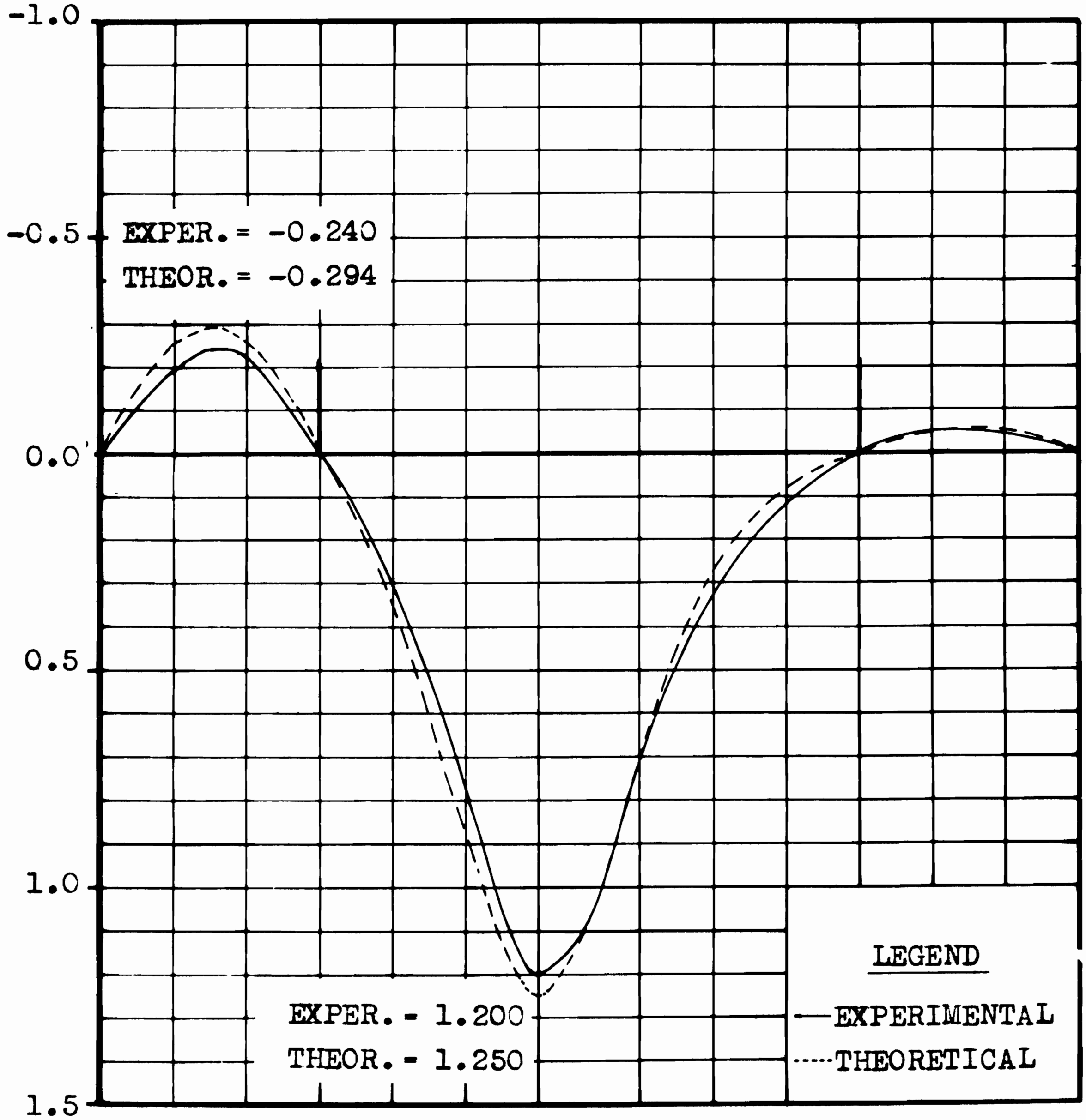


FIG. 32 INFLUENCE LINE OF AXIAL FORCE IN CABLE "6"

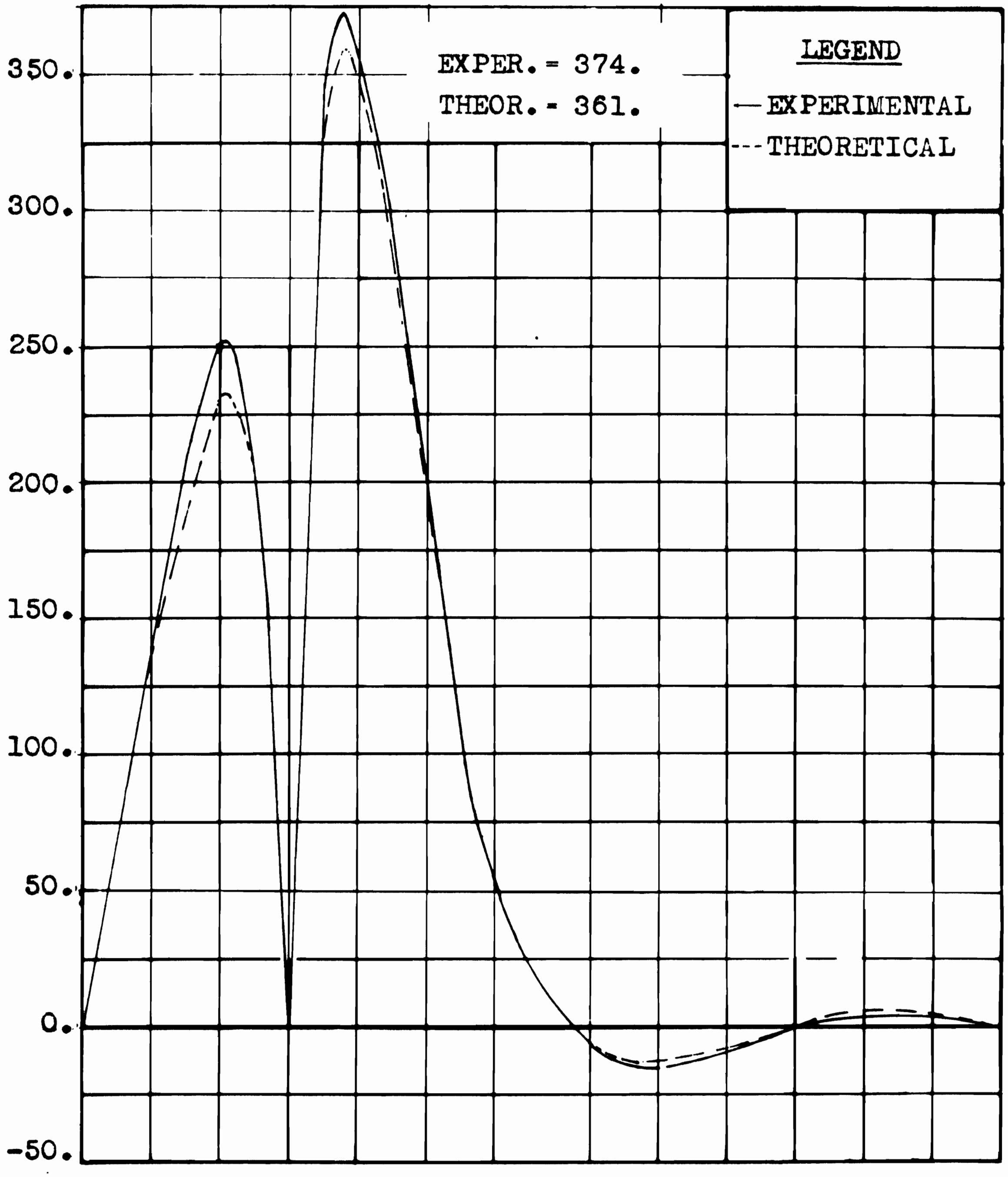


FIG. 33 INFLUENCE LINE OF BENDING MOMENT AT INTERMEDIATE SUPPORT

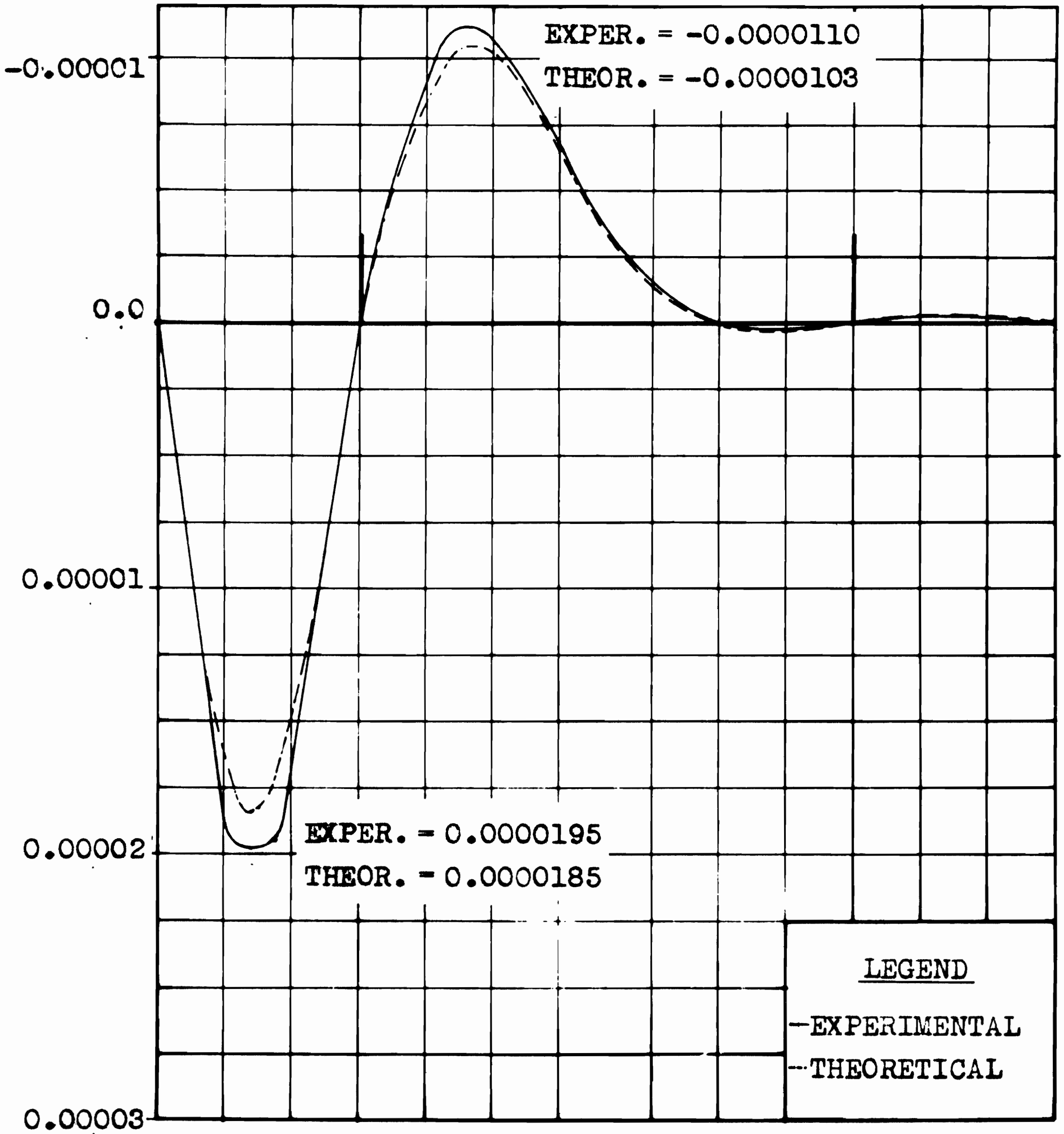


FIG. 34 INFLUENCE LINE OF VERTICAL DEFLECTION AT JOINT "2"

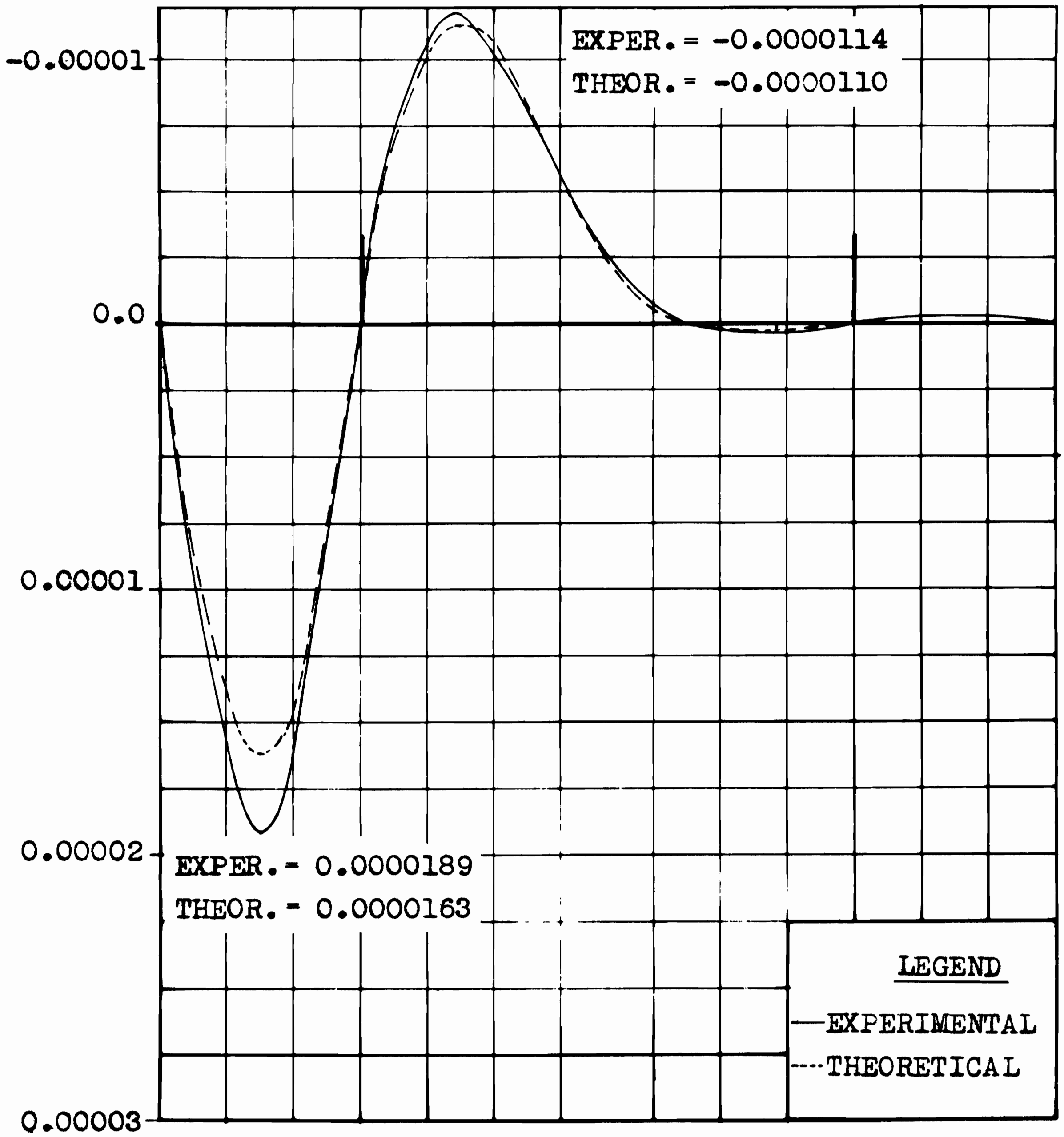


FIG. 35 INFLUENCE LINE OF VERTICAL DEFLECTION AT JOINT "3"

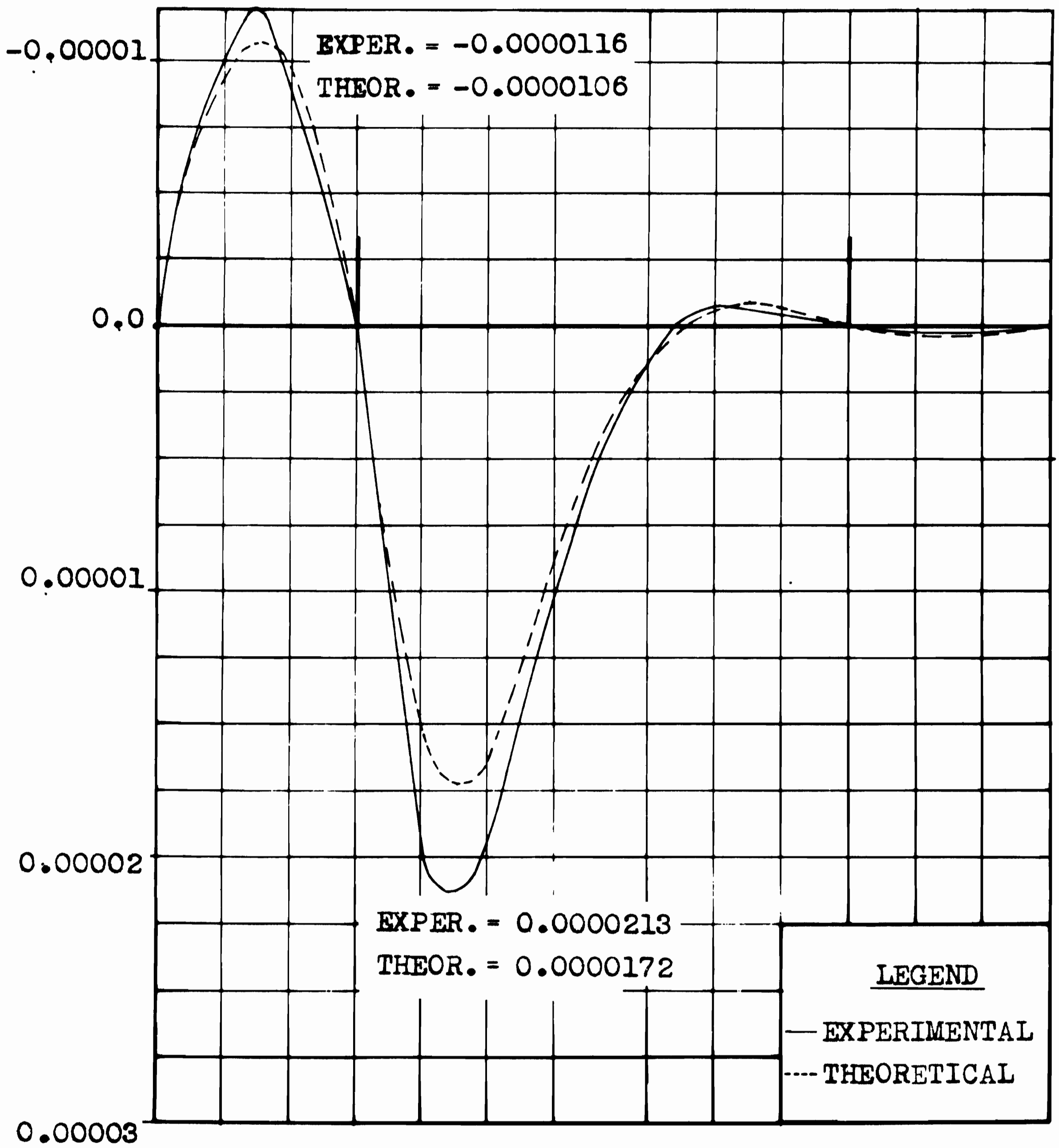


FIG.36 INFLUENCE LINE OF VERTICAL DEFLECTION AT JOINT "5"

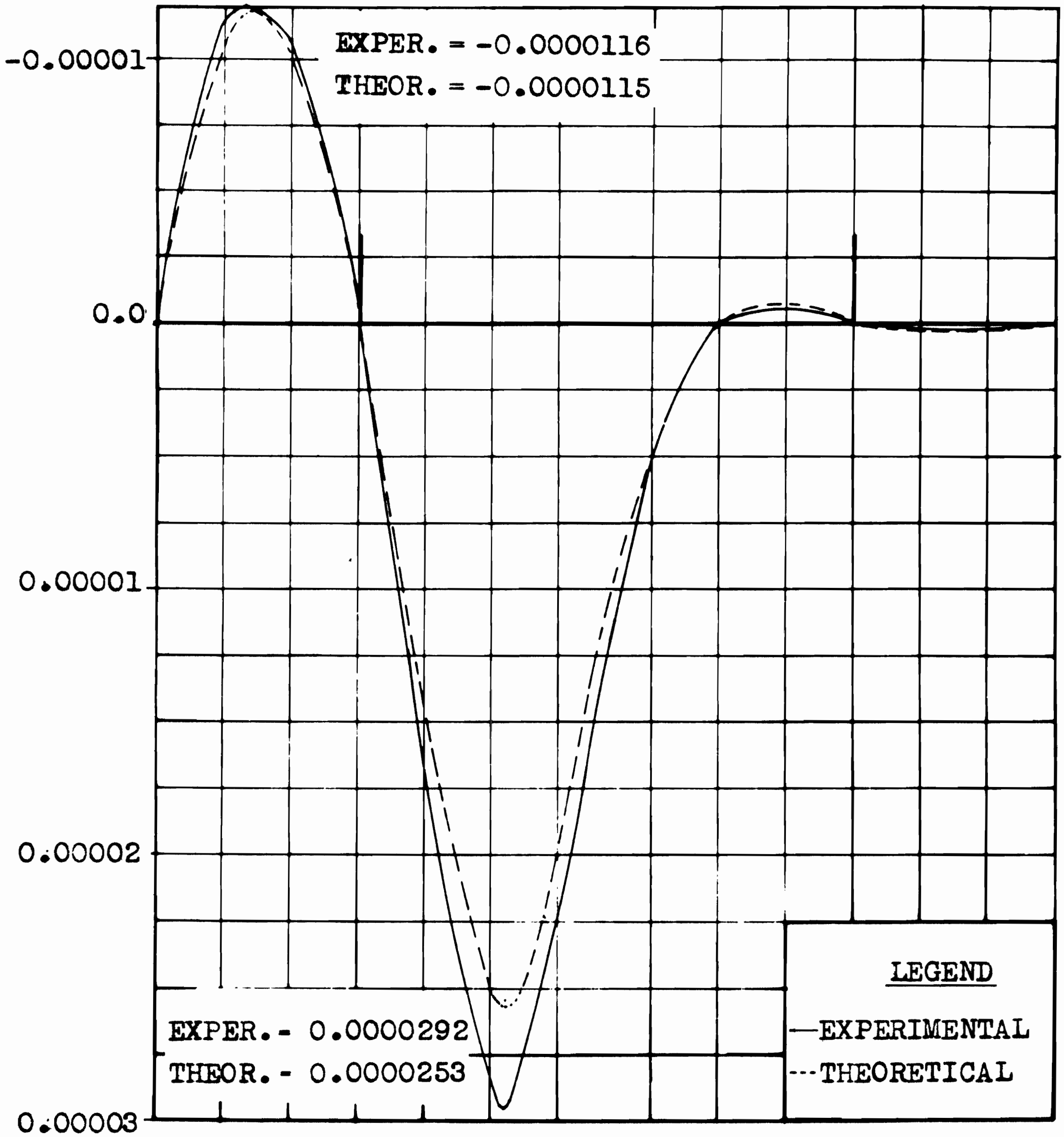


FIG. 37 INFLUENCE LINE OF VERTICAL DEFLECTION
AT JOINT "6"

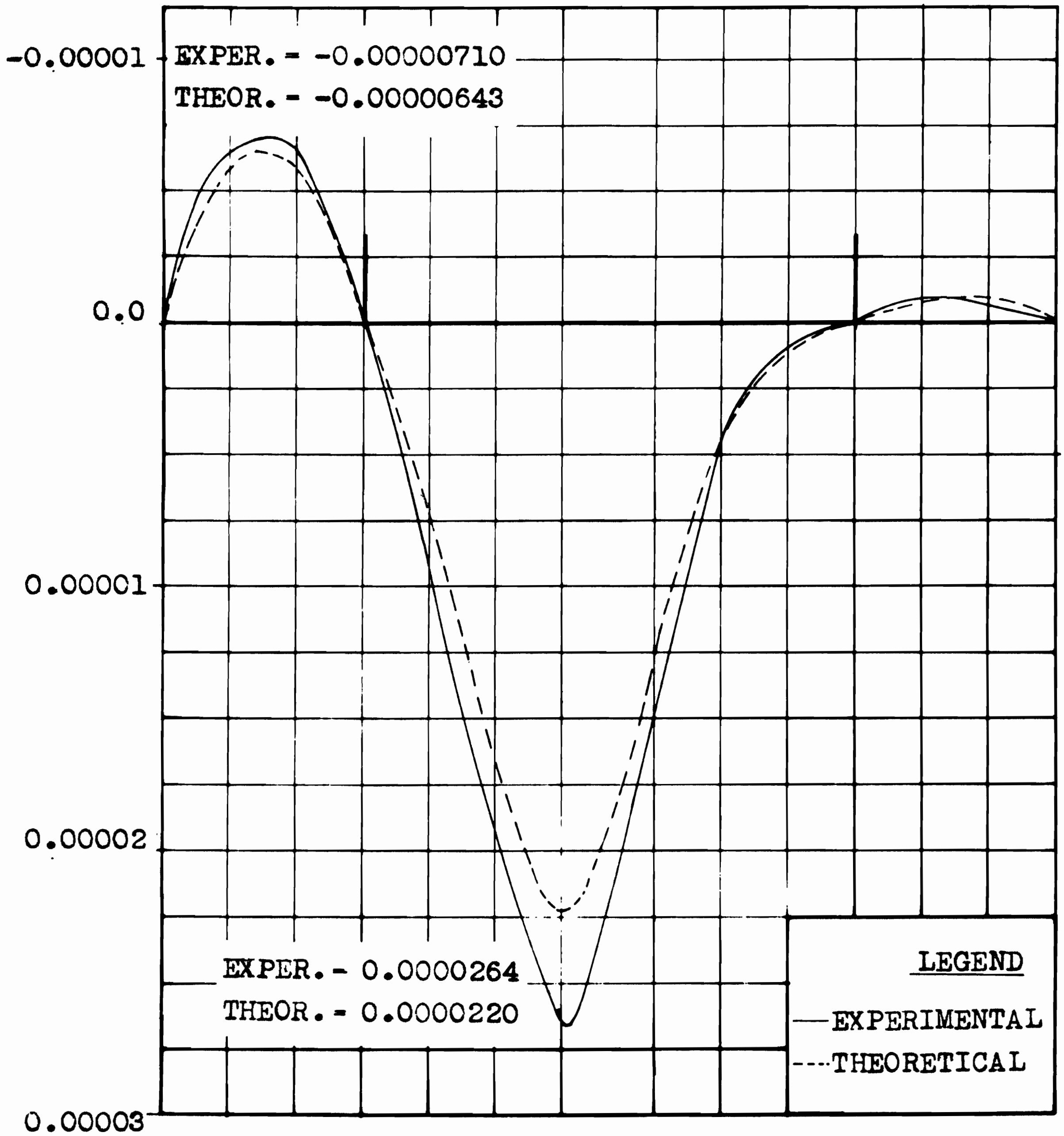


FIG. 38 INFLUENCE LINE OF VERTICAL DEFLECTION AT JOINT "7"

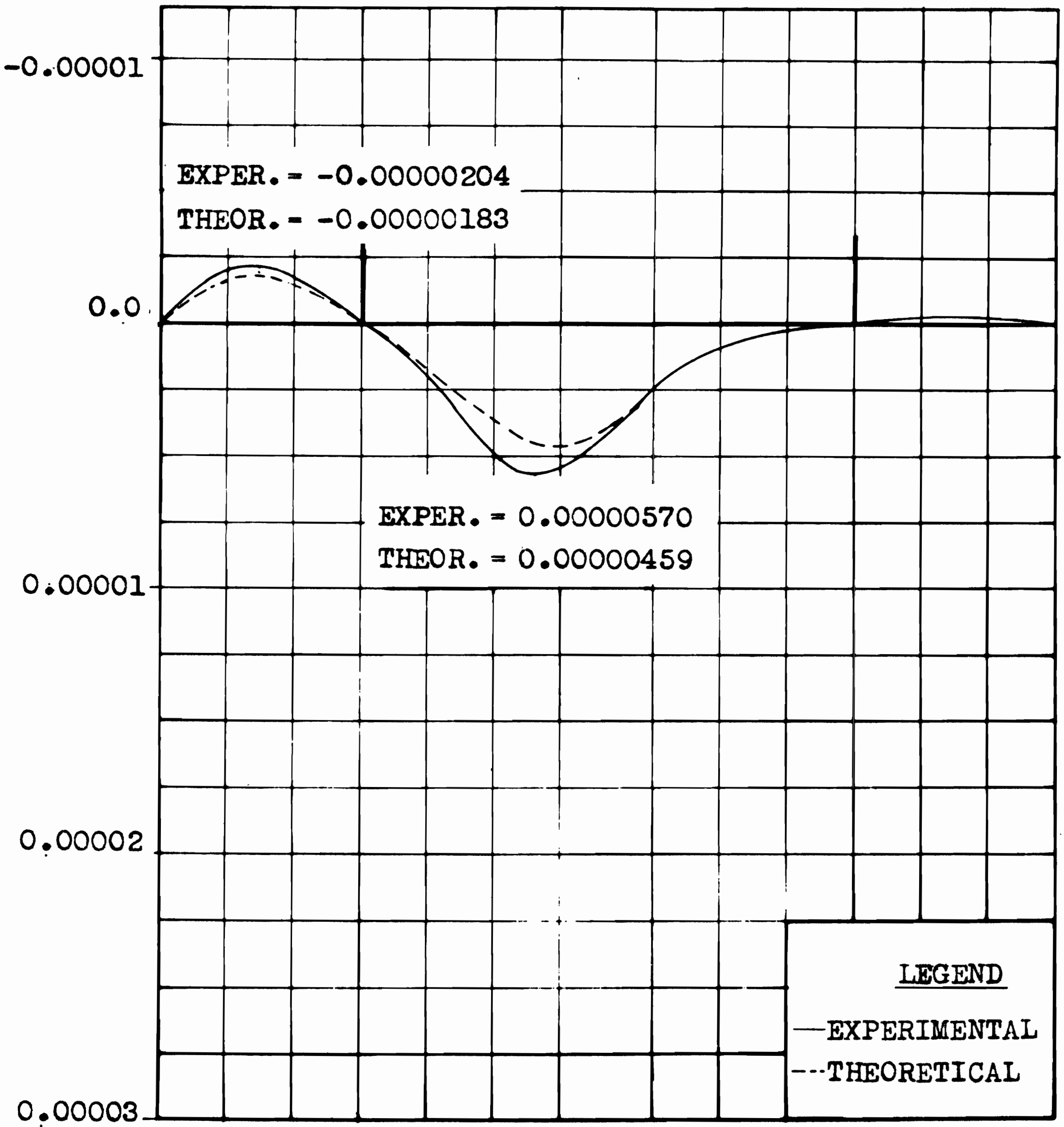


FIG. 39 INFLUENCE LINE OF HORIZONTAL DISPLACEMENT AT TOP OF TOWER

6. Conclusions

Figures 27 to 39 represent influence lines of axial forces in cables 1 to 6, of the bending moment at the interior support, of vertical deflections at joints 2, 3, and 5 to 7 and of horizontal displacements at joint 17. For the bridge system investigated, once the influence lines of bending moments and axial forces at the locations indicated are known, influence lines at any other locations may be obtained from conditions of static equilibrium.

As shown in Figures 27 to 39, the influence lines determined experimentally have essentially the same configuration as the influence lines determined analytically. The differences between maximum ordinates are given in Table No.1.

The deviations are due essentially to the simplifying assumptions with regard to connections, supports, influence of shear strain, etc., made in the mathematical model and also to some extent due to errors in recording strains and displacements.

The purpose of this investigation was to initiate, plan, and carry out an experimental method of analysis to be employed in design of cable stayed bridges. The main goal of such a method is to enable one to make more accurate assumptions regarding the connections of the structure, compared to those used in a mathematical model. A theoretical analysis relies essentially on three basic connection types whereas an experimental investigation may employ any

TABLE NO. 1 COMPARISON OF EXPERIMENTAL AND THEORETICAL MAXIMUM INFLUENCE LINE ORDINATES

	EXPERIMENTAL	THEORETICAL	DIFFERENCE	
			NUMERICAL VALUE	PERCENT
Axial force in cable 1	1.210	1.356	+.146	+12.00
2	.400	.406	+.006	+ 1.50
3	.506	.627	+.121	+23.90
4	.536	.611	+.075	+14.00
5	.646	.662	+.016	+ 2.48
6	1.200	1.250	+.050	+ 4.16
Bending moment at interior support	375×10^{3}	361×10^{3}	14×10^{3}	+ 3.72
Vertical deflection at joint 2	$.195 \times 10^{-4}$	$.185 \times 10^{-4}$	$-.010 \times 10^{-4}$	- 5.12
3	$.189 \times 10^{-4}$	$.163 \times 10^{-4}$	$-.026 \times 10^{-4}$	-13.70
5	$.213 \times 10^{-4}$	$.172 \times 10^{-4}$	$-.041 \times 10^{-4}$	-19.20
6	$.292 \times 10^{-4}$	$.253 \times 10^{-4}$	$-.039 \times 10^{-4}$	-13.30
7	$.264 \times 10^{-4}$	$.220 \times 10^{-4}$	$-.044 \times 10^{-4}$	-16.70
Horizontal displacement at joint 17	$.570 \times 10^{-4}$	$.459 \times 10^{-4}$	$-.111 \times 10^{-4}$	-19.50

possible type of connection. Also, in a theoretical analysis, many simplifying assumptions are made and numerous second order effects are neglected, whereas on a structural model such simplifications are not required and hence, the model represents the actual structure more closely.

It is true that at this stage of the development of model investigations, a theoretical analysis involves less time, cost and effort. The advantages of a model investigation are, however, valuable as they provide more accurate predictions for design.

For these reasons it is felt that model investigations will gain more and more recognition as a design tool in the future.

II

INVESTIGATION OF THE NONLINEAR BEHAVIOUR OF A CABLE STAYED BRIDGE, ON A STRUCTURAL MODEL

1. Planning of the model investigation

Planning of the model investigation of the nonlinear behaviour of a cable stayed bridge has been carried out in accordance with the basic concepts outlined in Chapter 1.

The general considerations regarding the size of the model, materials to be employed for the model, fabrication methods and instrumentation and recording of data, presented in connection with the model for determination of influence lines, (model "A"), were also taken into account in the planing of this investigation. On the basis of the same considerations, it was decided to employ steel as the material for model "B".

The size of the model was dictated mainly by the space available in the Civil Engineering Laboratory. A length scale reduction factor of $k_L = 200$ was selected.

Instrumentation methods similar to those used for the influence line model were applied.

2. Design and description of the model

a. Geometry and sectional properties of the model

The design of the model was carried out on the basis of the similitude conditions indicated in Chapter II, Sect. 2.

The prototype selected for this investigation was, with the exception of the stiffening girder, the same as the one employed for determination of influence lines. For the stiffening girder, the sectional properties were taken as variable, Figure 40.

The sectional properties of the prototype were designed symmetric with regard to the center line of the system and the cables were symmetric with respect to the towers.

The materials assumed for the bridge prototype were the same as considered previously for model "A".

The scale reduction factors for area and moment of inertia k_A and k_I were determined from the conditions $k_I = k_L{}^4$ and $k_A = k_L{}^2$. With $k_L = 200$ this gives $k_A = 4 \times 10^4$ and $k_I = 16 \times 10^8$.

Dividing the sectional properties of the prototype by the above scale factors:

$$
\begin{aligned}
A_1^m = A_6^m = A_7^m = A_{12}^m &= \frac{A_1^p}{k_A} = \frac{105}{4\text{x}10^4} = 0.002625 \text{ in}^2 \\
A_2^m = A_5^m = A_8^m = A_{11}^m &= \frac{A_2^p}{k_A} = \frac{42}{4\text{x}10^4} = 0.001050 \text{ in}^2 \\
A_3^m = A_4^m = A_9^m = A_{10}^m &= \frac{A_3^p}{k_A} = \frac{37}{4\text{x}10^4} = 0.000925 \text{ in}^2 \\
J_{D1}^m &= \frac{J_{D1}^p}{k_I} = \frac{2.72\text{x}10^6}{16\text{x}10^8} = 0.001700 \text{ in}^4
\end{aligned}
\tag{25}
$$

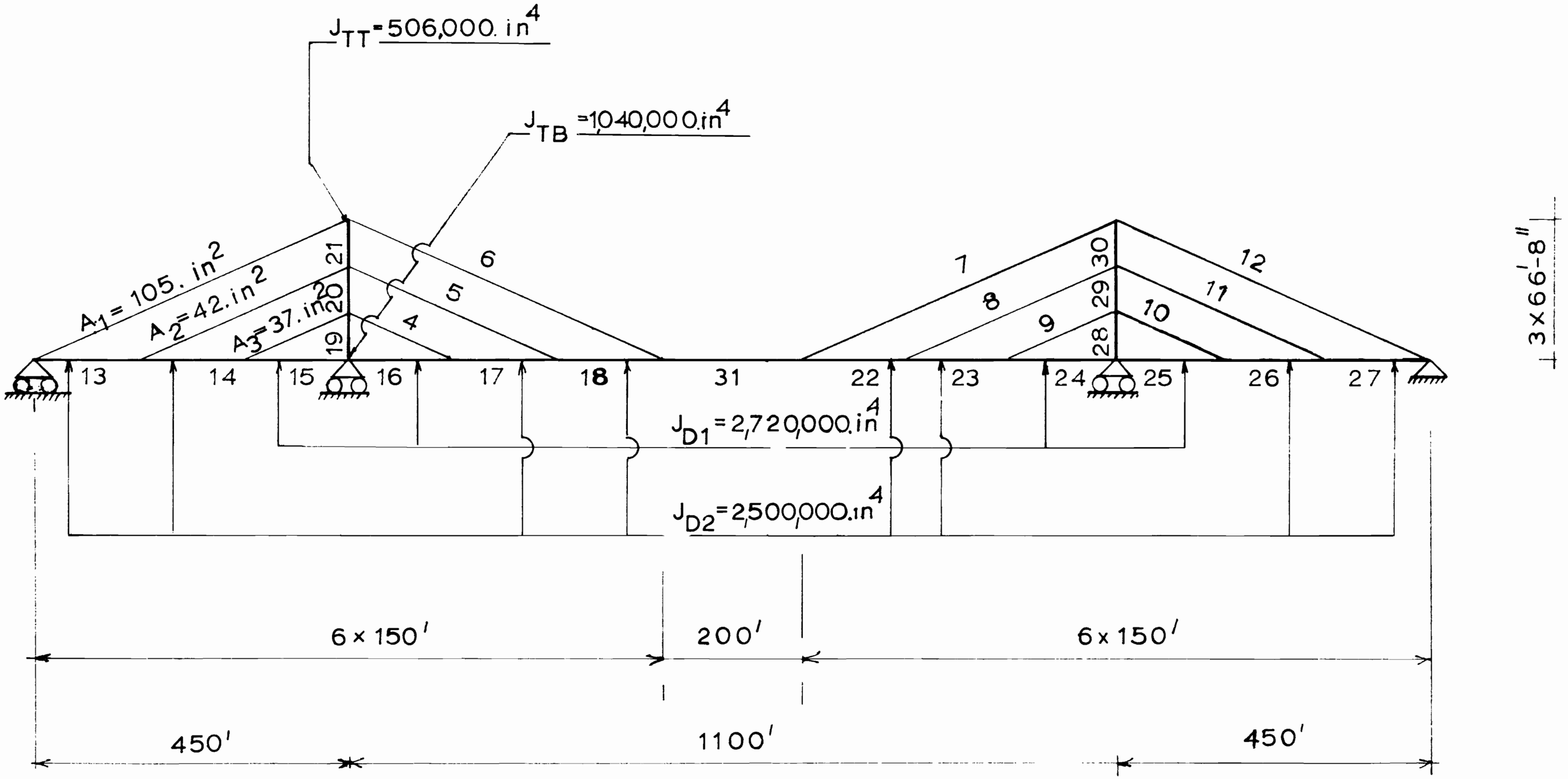


FIG. 4o DIMENSIONS AND SECTIONAL PROPERTIES OF THE PROTOTYPE

$$J^m_{D2} = \frac{J^p_{D2}}{k_I} = \frac{2.50 \times 10^6}{16 \times 10^8} = 0.00156000 \text{ in}^4$$

$$J^m_{TB} = \frac{J^p_{TB}}{k_I} = \frac{1.04 \times 10^6}{16 \times 10^8} = 0.0006500 \text{ in}^4 \qquad (25)$$

$$J^m_{TT} = \frac{J^p_{TT}}{k_I} = \frac{0.506 \times 10^6}{16 \times 10^8} = 0.00031625 \text{ in}^4$$

To satisfy the connection design requirements, it was necessary to increase the above sectional properties by a proportionality factor C= 4.37. This factor was obtained following several trials.

Consequently

$$A^m_1 = A^m_6 = A^m_7 = A^m_{12} = 4.37 \times .002625 = .01145 \text{ in.}^2$$

$$A^m_2 = A^m_5 = A^m_8 = A^m_{11} = 4.37 \times .00105 = .00458 \text{ in.}^2$$

$$A^m_3 = A^m_4 = A^m_9 = A^m_{10} = 4.37 \times .000925 = .00404 \text{ in.}^2$$

$$J^m_{D1} = 4.37 \times .0017 = .00742 \text{ in.}^4 \qquad (26)$$

$$J^m_{D2} = 4.37 \times .00156 = .00682 \text{ in.}^4$$

$$J^m_{TB} = 4.37 \times .00065 = .00284 \text{ in.}^4$$

$$J^m_{TT} = 4.37 \times .00032 = .00138 \text{ in.}^4$$

The geometry of the model and the sectional properties required are shown in Figure 41.

b. Design and selection of model sections

The stiffening girder was designed of two rectangular sections (3/8" x 1/2") made of AISI C-1020 steel and having a nominal modulus of elasticity of E = 29,000 ksi. The width of the girder was reduced from 3/8 in to 0.36 in respectively 0.335 in. to satisfy the stiffness requirements corresponding to J_{D1} respectively J_{D2}. The actual moments of inertia obtained were 0.0075 in^4 and 0.0069 in^4 which represents a deviation of 1.06%, respectively 1.16% from the requirements calculated in Equation (26).

The towers were designed as variable rectangular shapes milled from C-1020 steel plate having a thickness of 3/16 inches. The nominal modulus of elasticity of the steel plate was E = 29,000 ksi. The deviation from the nominal sectional properties at bottom of tower was 1.4%.

The cables were designed of C-1095 drill rods with a normal modulus of elasticity E = 29,000 ksi. For cables 1, 6, 7, and 12, drill rods were not available and QQW-471 music wire, reheated to 900^{o}F was employed. Reheating was necessary to reduce ductility in order to achieve a better grip at the cable-girder connection.

As the modulus of elasticity of the prototype cable

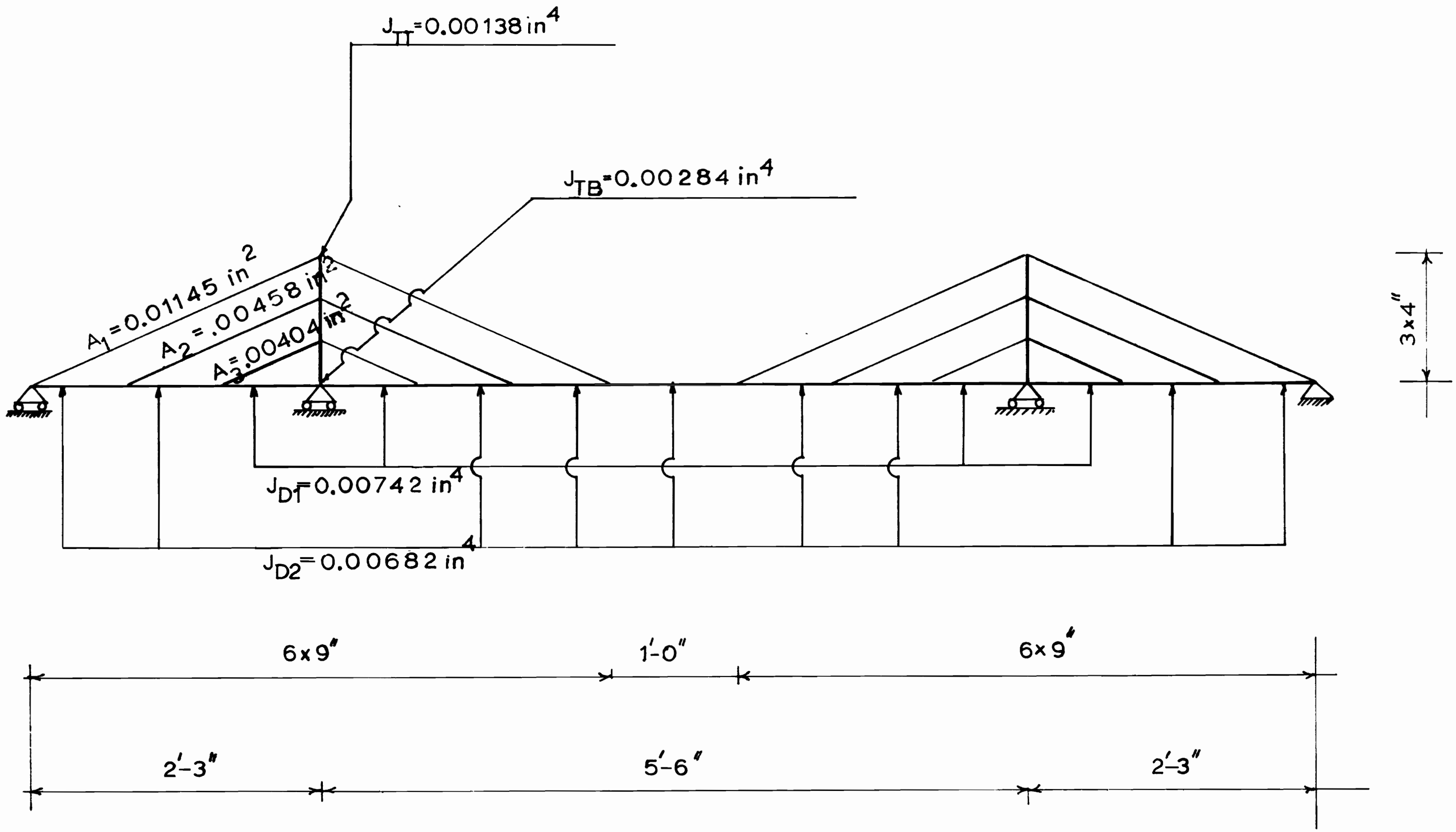


FIG. 41 DIMENSIONS AND REQUIRED SECTIONAL PROPERTIES OF THE MODEL

was E = 18,000 ksi it was necessary to recalculate the cable areas by decreasing the values given in Equation (26) by the ratio $\frac{18}{29}$ = 0.062.

Consequently

$$A_1^m = A_6^m = A_7^m = A_{12}^m = .62 \times .01145 = .0071 \ \text{in.}^2$$

$$A_2^m = A_5^m = A_8^m = A_{11}^m = .62 \times .00458 = .00284 \ \text{in.}^2 \qquad (27)$$

$$A_3^m = A_4^m = A_9^m = A_{10}^m = .62 \times .00404 = .00250 \ \text{in.}^2$$

The sizes selected were 2 - 67/1000 inches music wire for cable 1, 2 - 42/1000 inches drill rod for cable 2 and 2 - 40/1000 inches drill rod for cable 3. The deviations from Equation (27) were 1% for cable 1, 1.47% for cable 2 and 0.48% for cable 3.

The actual dimensions and sectional properties of the model are presented in Figure 42.

c. Design of connections

The connections were designed on the basis of the same principles as for the influence line model, that is, to avoid as much as possible, eccentric application of forces and to facilitate post-tensioning of cables.

A. <u>Girder-cable connection</u>

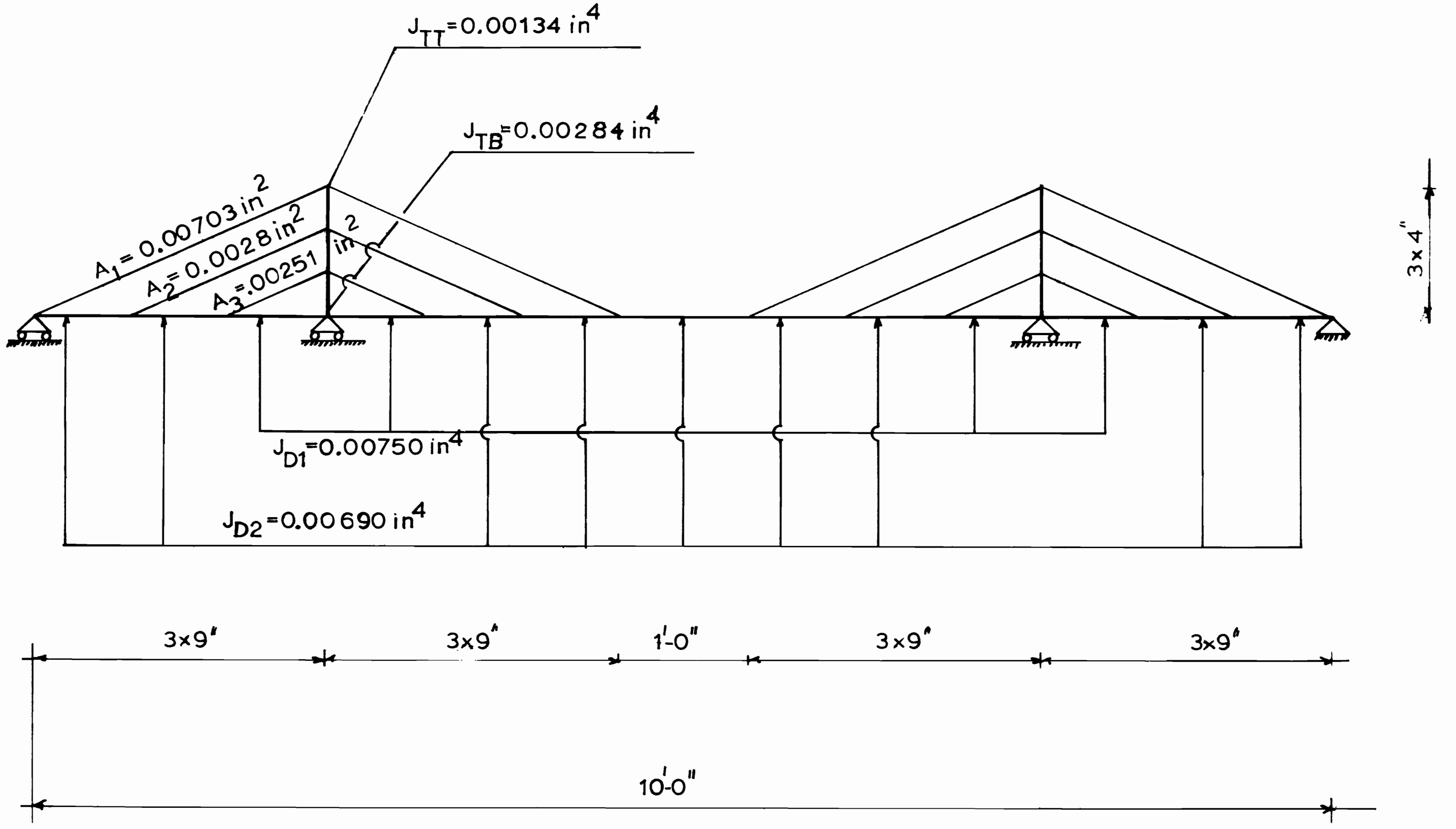


FIG. 42 DIMENSIONS AND ACTUAL SECTIONAL PROPERTIES OF THE MODEL

The girder-cable connection is represented in Figure 43. The parts are made of C-1020 steel. Post-tensioning of cables is achieved by turning the nut "1".

B. Tower-cable connection

The tower-cable connection is shown in Figure 44. The parts are made of C-1020 steel. Fixity of cables is achieved by turning the screws "1".

C. Tower-girder connection

The tower-girder rigid connection is represented in Figure 45. This connection may be modified to a hinge if required.

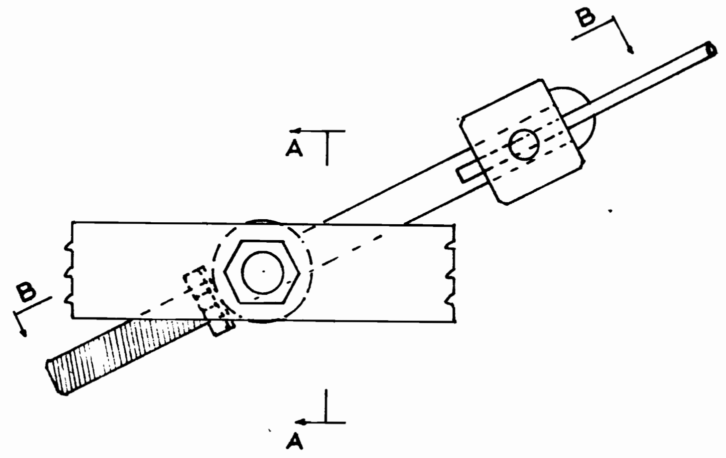
A photograph of the above connections is shown in Figure 46.

d. Foundation

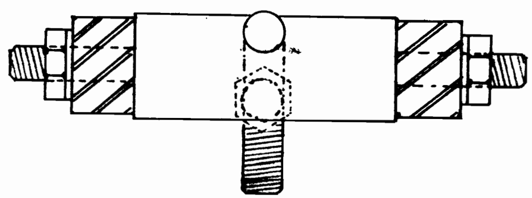
The foundation of the model was made of steel, Figure 47. Four vertical supporting channels, one for each support of the bridge system, were provided. The vertical channels were connected for stability by two horizontal channels, allowing sufficient free space for the loads applied on the model.

e. Supports

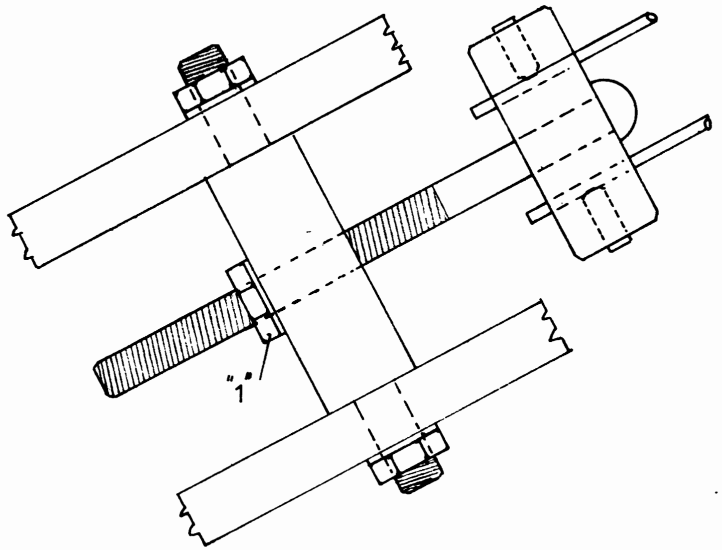
The supports are similar to the type employed for model "A", except that the sizes are smaller, Figure 48.



a. LONGITUDINAL VIEW



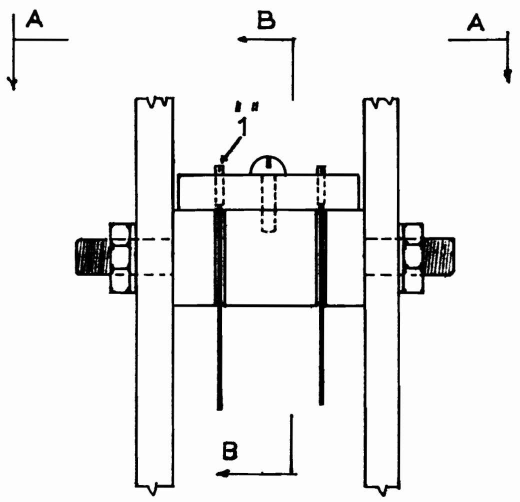
b. SECTION "A-A"



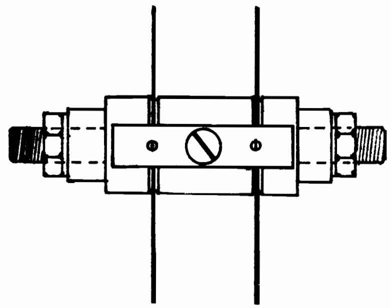
c. VIEW "B-B"

FIG. 43 GIRDER - CABLE CONNECTION

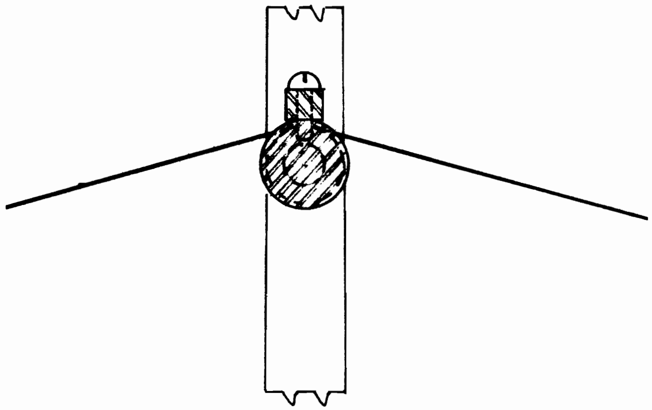
SCALE 1" = 1"



a. LATERAL VIEW



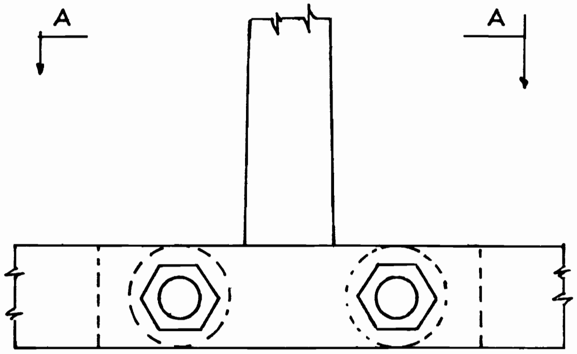
b. VIEW "A-A"



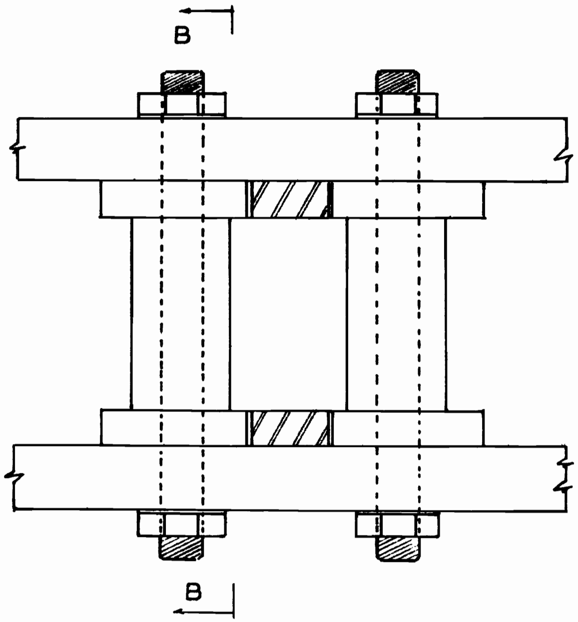
c. SECTION "B-B"

FIG. 44 TOWER - CABLE CONNECTION

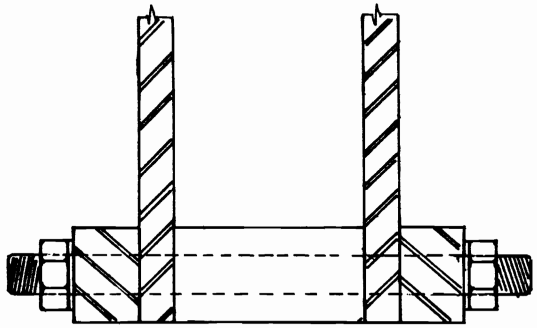
SCALE 1" = 1"



a. LATERAL VIEW



b. VIEW "A-A"

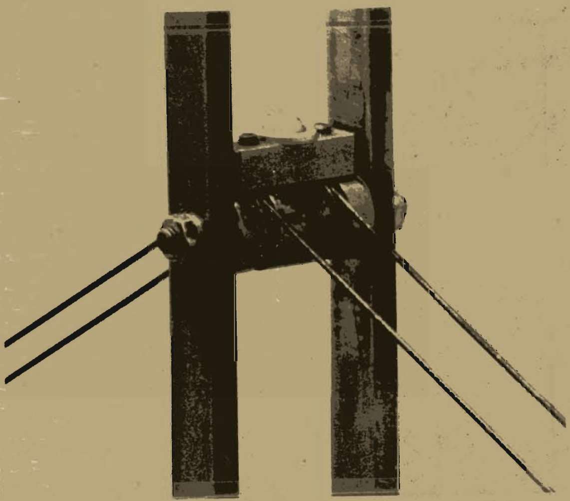


c. SECTION "B-B"

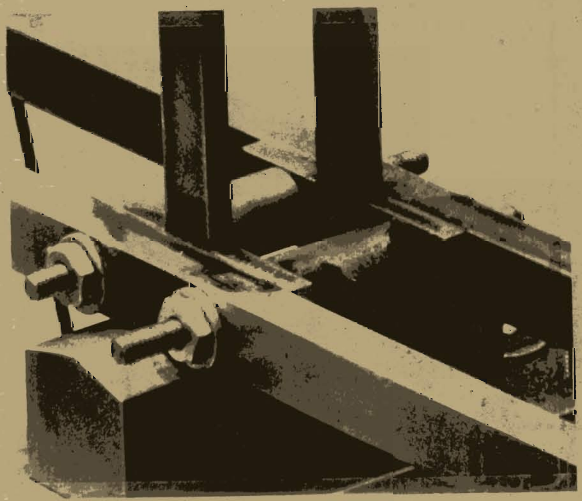
FIG. 45 TOWER - GIRDER CONNECTION

SCALE $1'' = 1''$

a. GIRDER-CABLE CONNECTION



b. TOWER-CABLE CONNECTION



c. TOWER-GIRDER CONNECTION

FIG. 46 CONNECTIONS

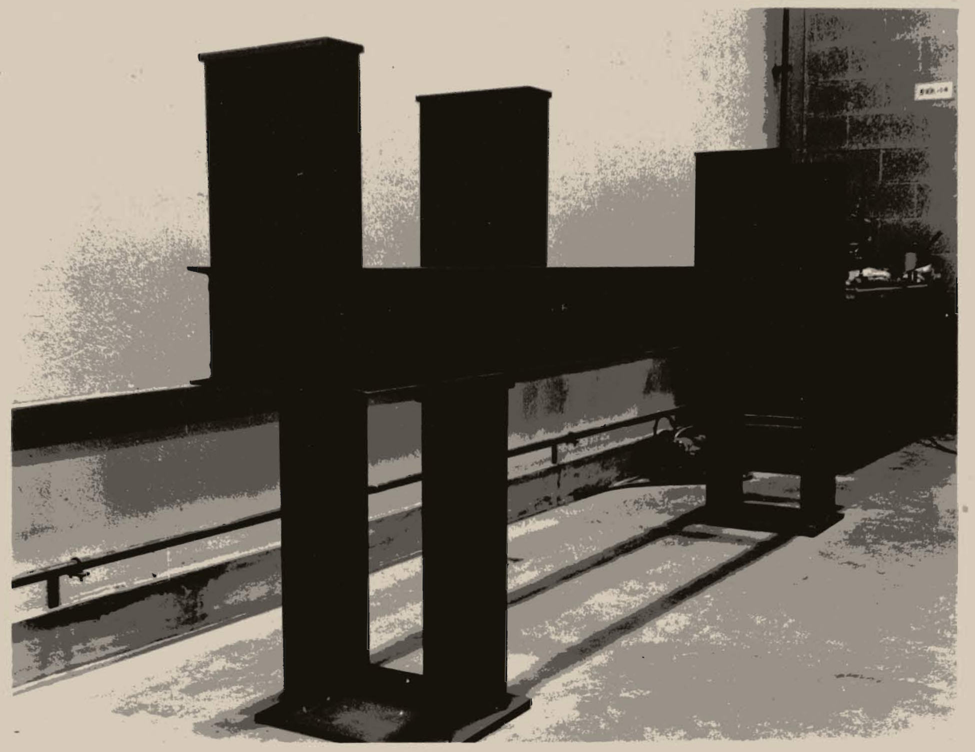


FIG. 47 FOUNDATION

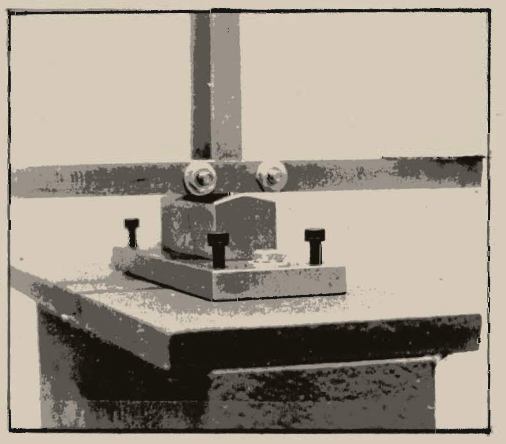


FIG. 48 SUPPORT

3. Manufacturing of the model

The model was manufactured in the machine shop of the Faculty of Engineering.

The simplicity of the stiffening girder and tower sections enables an easy adjustment of the model cross sections if sectional properties need to be changed. This may be achieved by milling off the girder or tower cross section or both. If necessary, the music wires and drill rods may be removed and new sizes may be installed.

Fabrication was carried out with a tolerance of less than $\frac{1}{100}$ in. for cross sectional sizes and less than $\frac{1}{16}$ in. for longitudinal dimensions.

4. Instrumentation

Axial forces in cables and bending moments in the stiffening girder have been determined with electrical strain gages. For displacements, dial gages have been employed. Acquisition of data from strain gages was performed with the apparatus described previously in Chapter II, Section 4.

a. Strain gages

The strain gage layout is shown in Figure 49. Two strain gages have been applied, one opposite to the other, on each of the 3/16 in. tensioning bolts. For the bending moments, three strain gages have been applied on the stiffening girder as shown in Figure 49. These gages enable to

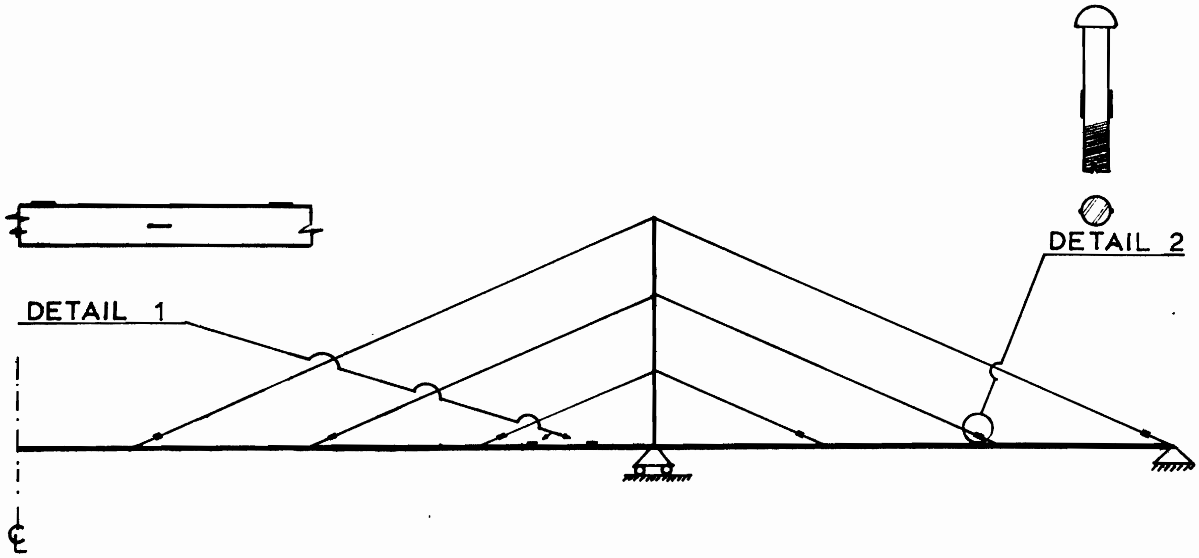


FIG. 49 STRAIN GAGE LAYOUT

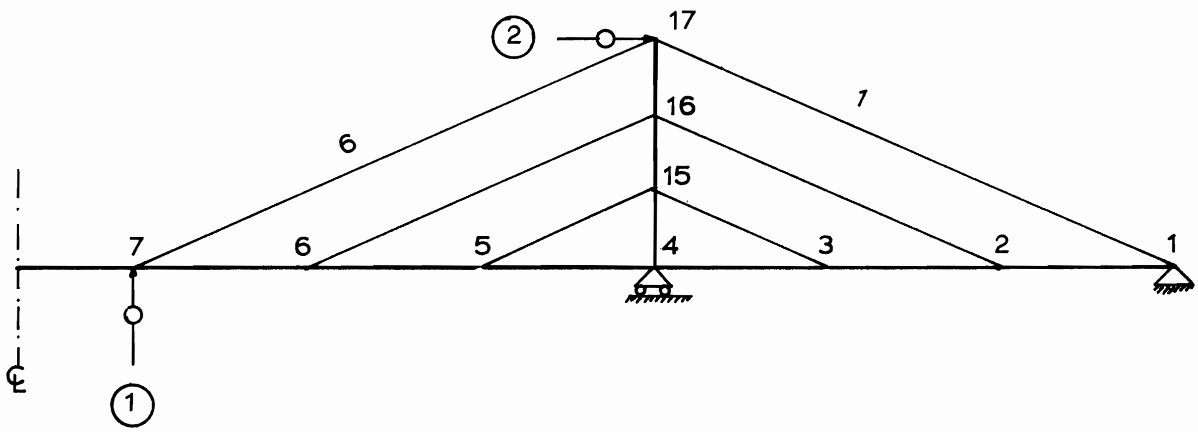


FIG. 50 DIAL GAGE LAYOUT

read strains at extreme fibres and at the center of the stiffening girder, from which the bending moment at interior support may be calculated.

The strain gages employed were 1/4 in. Micro measurements precision strain gages type EA-06-250BG-120. They were temperature compensated for steel and had a resistance of 120 ohms.

b. Dial gages

The dial gage arrangement is shown in Figure 50. Dial gage 1 measures vertical deflections at joint 7 and dial gage 2 determines the horizontal displacement at joint 17. Joint numbers are indicated above, or at the right of each joint in Figure 50 and dial gage numbers are indicated in a circle below or at the left of the joint.

5. Loading and Recording of Data

a. Loading

To determine the upper limit for which the applied loads are still proportional to internal forces, a concentrated load was applied at joint 7 and the axial force in cable 6 due to the action of this load was recorded, Figure 51. The concentrated force was applied with a tension rod in a manner similar to that described in Chapter II, Sect. 5. The application of the load was made in ten increments of 35.7 lbs. each, to a total of 357 lbs. A concentrated force of 357 lbs. applied at joint 7 will develop an axial force in cable 6 equivalent to the force corresponding to full dead and live load applied on the model.

The results of this test are plotted in Figure 51.

As represented in Figure 51, the cable displays linear behaviour up to a load equal to 0.3P, where P = 357 lbs. Only after the value of 0.3P is exceeded does the axial force in cable 6 increase at a higher rate than the applied load.

On this basis, a load equal to 20% of the total load was chosen for the first step of loading, as described in Section 1.

A photograph of the model during loading is presented in Figure 52. Loads were applied at joints and midway between joints. The weights employed were #11 reinforcing bars, ten inches long, of an average of 4.42 lbs. each. The weights were suspended from short square tubes, applied on the stiffening girder, as shown in Figure 52.

The dead load applied on the model was 118 lbs./ft. and the live load 29.5 lbs./ft. This is equivalent according to the similitude condition $k_W = k_L k_E$ to 5,400 lbs./ft. dead load and 1,350 lbs./ft. live load on the prototype.

b. Recording of Data

The data recorded were strains and displacements at

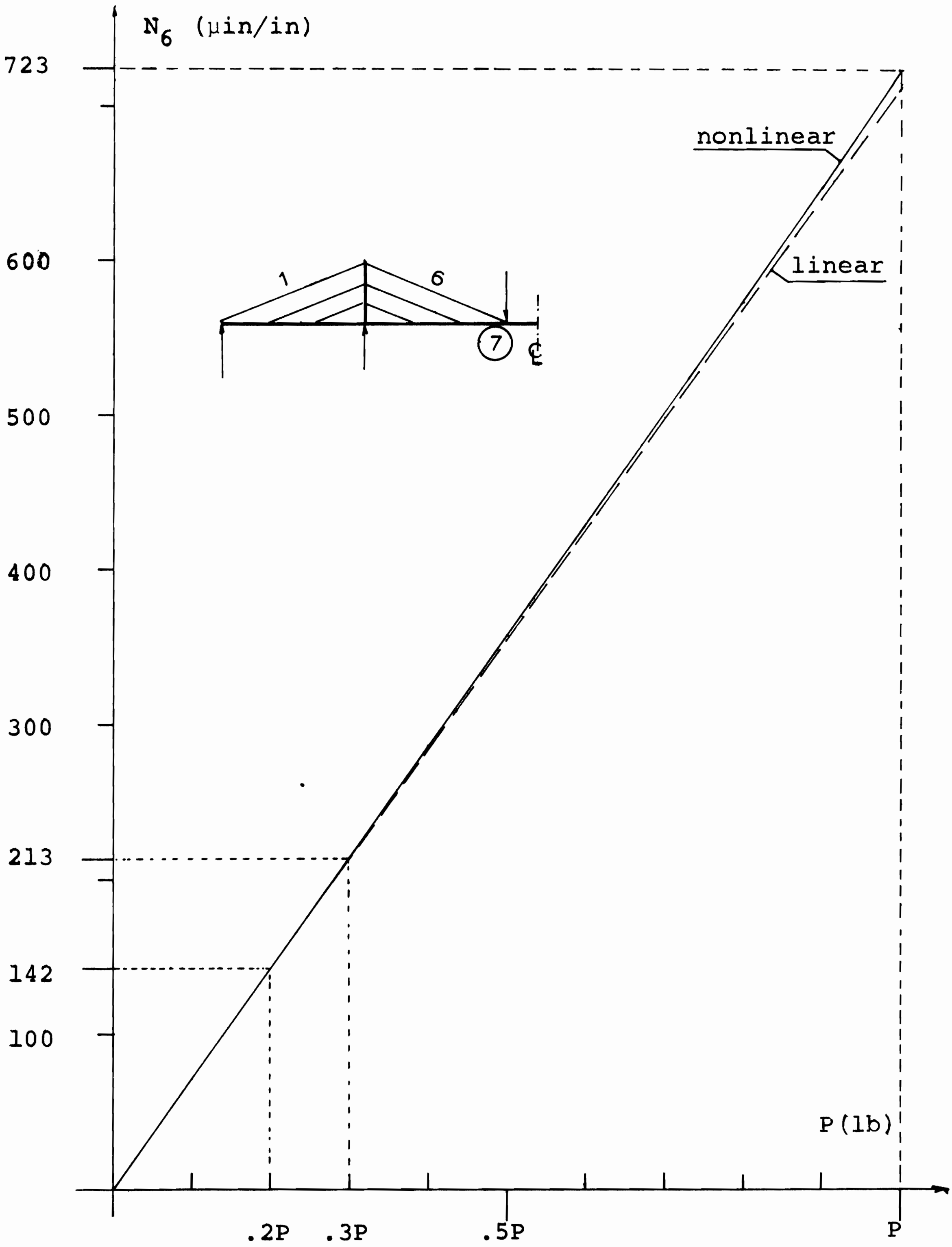


FIG. 51 NONLINEARITY OF CABLE 6 UNDER THE ACTION OF A VERTICAL CONCENTRATED LOAD AT JOINT 7

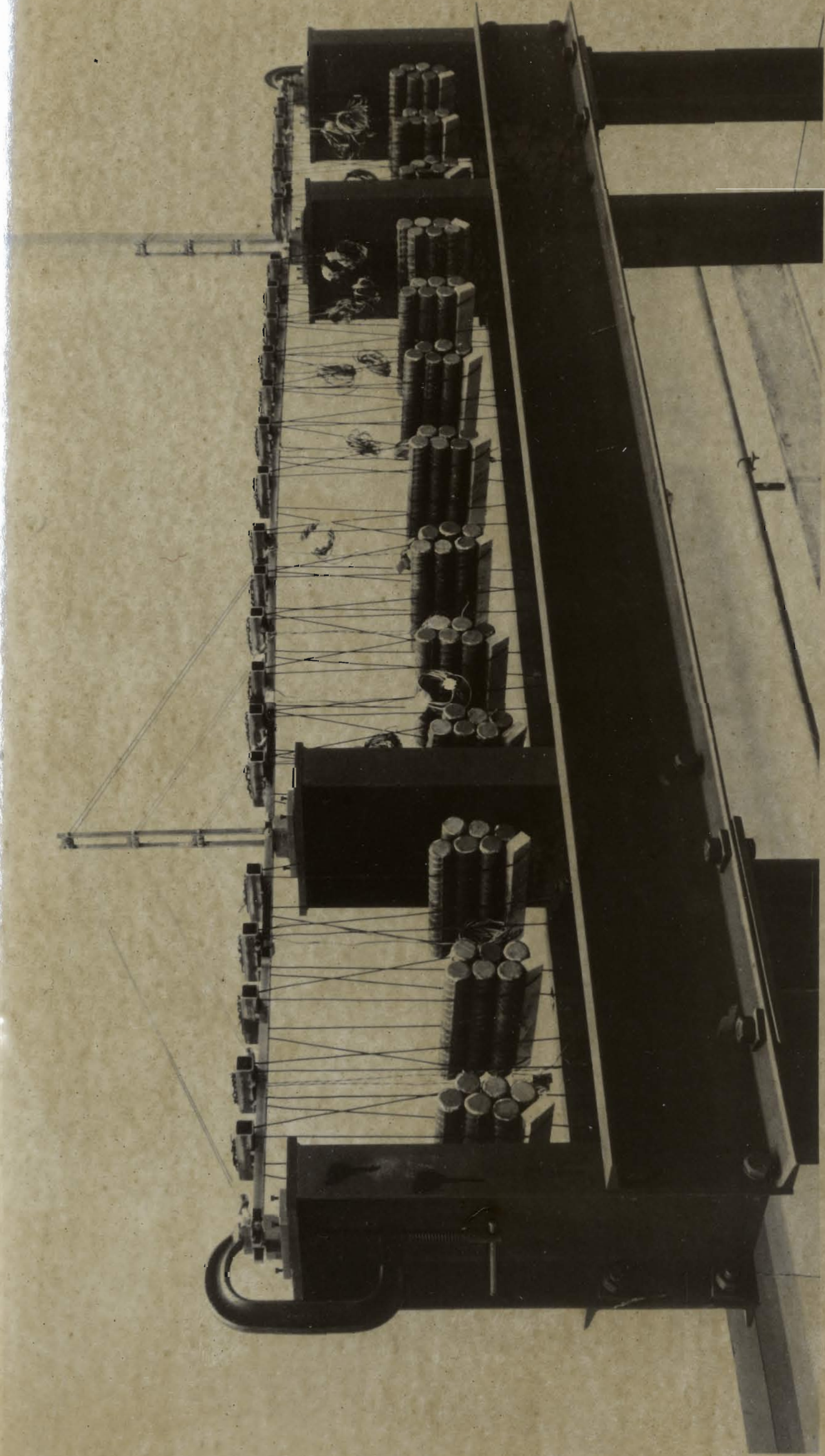


FIG. 52 GENERAL VIEW OF MODEL "B".

the locations indicated in Figure 49 and Figure 50 for the two cases of loading specified.

The strain readings of loading cases 1 and 2 were then compared in order to get the increase or decrease in stresses and strains due to the nonlinear behaviour of the structure.

The results obtained are shown in Table No.2. The strains indicated in this table are in micro inches per inch and the displacements are given in 10^{-2} in.

6. Conclusions

Table No. 2 contains a set of information which may be applied readily in design to increase or decrease the cross sections determined on the basis of a linear analysis.

The influence of nonlinearity was found to be smaller for the cables and larger for the bending moment in the stiffening girder at the location investigated. The result obtained for the bending moment should not be generalized however, as the influence of nonlinearity on the bending moment in the stiffening girder will vary with the location.

The influence of nonlinearity is larger for cables 1 and 6, than for the others, as these cables undergo the largest change of angle due to the displacement of the stiffening girder and tower.

The model employed had adjustable sectional properties.

Table No. 2 INFLUENCE OF NONLINEARITY ON STRESSES AND DISPLACEMENTS

Location and internal force or displacement investigated	Strains in μin/in and displacements in 10^{-2} in.			Difference due to nonlinear behaviour %
	20% of total load (linear)	Actual total load (nonlinear)	Comparative total load (linear)	
Axial Force in Cable 1	193.00	1018.00	965.00	+5.50
2	108.00	555.00	540.00	+2.78
3	98.00	495.00	490.00	+1.02
4	106.00	540.00	530.00	+1.85
5	113.00	584.00	565.00	+3.86
6	185.00	976.00	925.00	+5.52
Bending Moment at Interior Support	113.00	607.00	565.00	+7.42
Displacement at Joint 7	4.91	25.51	24.55	+3.90
Displacement at Joint 17	1.15	6.30	5.75	+9.55

The cross sections of the stiffening girder and tower may be decreased by milling, and cables may be interchanged without difficulty.

The investigation as a whole is relatively easy to perform and the cost of material and technical work was less than $2.000.00. This makes the procedure described a very practical tool for taking into account the nonlinear behaviour in the design of cable stayed bridges.

IV

STRUCTURAL ANALYSIS OF CABLE STAYED BRIDGES

1. Preliminary considerations

To compare the experimental data of the model investigation with the theoretical, a procedure for the structural analysis of cable stayed bridges has been developed.

This procedure contains both classical and computer methods of structural analysis.

Several general basic methods may be employed to carry out the structural analysis of a cable stayed bridge. For linear analysis the "slope-deflection" or any force or energy method could provide us with the conditions required to determine the unknown redundants and thus to solve the problem. The same applies for nonlinear analysis, except that the solution of the virtual work, continuity or energy equations becomes in this case, more cumbersome and numerical iteration methods have to be employed.

Compared with suspension bridges, cable stayed bridges display a different structural behaviour. The loads acting on the stiffening girder of a cable stayed bridge are transferred to the cables at connections, whereas, in the case of a suspension bridge, transverse forces due to the action of suspension rods, are applied along the cables.

The analysis of a cable stayed bridge may be divided in two parts. In the first part, bending moments, axial and shear forces and deflections due to dead and live loads are determined. In the second part, post-tensioning forces required in the cables in order to reduce the stresses and deflections calculated in stage one, to specified values, are calculated.

The analysis presented is limited to the case of a two dimensional cable stayed bridge system.

2. Analysis by Classical Methods

By classical methods, we define here the methods of analysis which do not require the use of a digital computer.

a. Analysis due to dead and live loads.

To analyse a cable stayed bridge by classical methods, first a system of equations equal in size to the number of redundants, is written. The procedure of assembling these equations is well-known. Energy or virtual work conditions or compatibility requirements for slopes and deflections at joints, may be employed.

The main difficulty of an analysis by classical methods lies in solving this system of equations. However, several procedures, listed below, will simplify the calculations required. These procedures are :

A. Selection of bending moments at fixed and flexible supports as redundants.

B. Application of the "Beam On Elastic Supports" analogy.

C. Use of symmetry and asymmetry to reduce the number of equations.

A. Selection of bending moments as redundants.

The selection of bending moments at fixed and flexible supports as redundants permits the writing of a five moment equation at each support. This yields a banded well-conditioned system of equations. If the internal forces in cables are selected as redundants, the resulting system of equations is not banded and hence, more difficult to solve than in the previous case. Also, the calculation of bending moments in the stiffening girder becomes a lengthy operation.

B. "Beam On Elastic Supports" analogy.

The "Beam On Elastic Supports" analogy, Fig.53, has been suggested by Smith[18]. Intended by its author as a

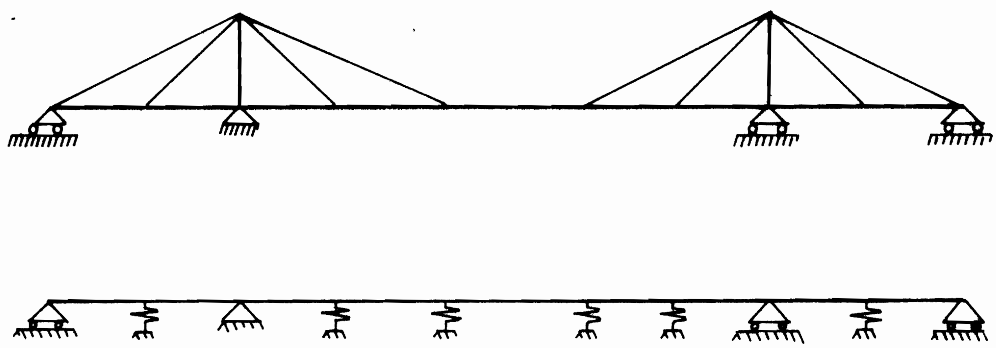


FIG. 53 BEAM ON ELASTIC SUPPORTS ANALOGY

computer method, it may also be used as a classical method. Smith shows how to determine the equivalent spring constant for any specific cable stayed bridge system. For the case of movable cable to tower connections, that is, when cables are free to slide along their supports, if the shortening of the tower is neglected, the elastic support spring constant K - the vertical force needed to develop a unit displacement - Fig. 54, may be obtained as follows :

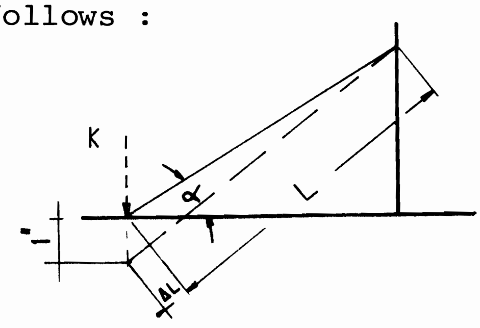


FIG. 54 ELASTIC SUPPORT SPRING CONSTANT

If N is the internal force in the cable, then from Fig. 54

$$\Delta L = \sin\alpha \tag{28}$$

also

$$\sin\alpha = \frac{NL}{EA} \tag{29}$$

but

$$K = N\sin\alpha \tag{30}$$

If Equation (29) is rearranged and substituted into Equation (30), the value of K, the elastic support spring constant, is obtained.

$$K = \frac{EA}{L}\sin^2\alpha \tag{31}$$

If shortening of the tower is also considered, the spring constant may be derived in a similar manner and its expression becomes

$$K = \frac{1}{\dfrac{H_t}{A_t E_t} + \dfrac{L_c}{A_c E_c \sin^2\alpha}} \tag{32}$$

Where H_t is the height of the tower. The suffix "t" applies to sectional and elastic properties of the tower and the suffix "c" to the corresponding properties of the cable.

Further details regarding how to determine K for a more general case, are outlined by Smith.

It should be pointed out that this procedure may be applied either in conjunction with available tables for continuous beams on elastic supports or by carrying out a complete analysis. In the later case, the solution of the five moment system of equations, written on the basis of continuity of slopes and displacements at joints, may be obtained with the five diagonal algorithm presented in Appendix 4. This algorithm represents an extension of the well-known three diagonal algorithm.

C. Symmetry and asymmetry.

The use of symmetry and asymmetry allows the reduction of the system of equations to one half its initial size. Structurally, this is equivalent to the substitution of the two subsystems shown in 55.b and 55.c with the system represented in Fig. 55.a.

The right end support on Fig. 55.b is capable of resisting bending moments only. It will prevent the right end from rotation but, it will not prevent it from moving upwards or downwards. In other words, the vertical reaction at this point will be zero.

For the cable stayed bridge represented in Fig. 55, by decomposing the vertical load into a symmetrical plus an asymmetrical load the 10 x 10 system of equations is reduced to two 5 x 5 systems which may be solved with less difficulty than the 10 x 10 system.

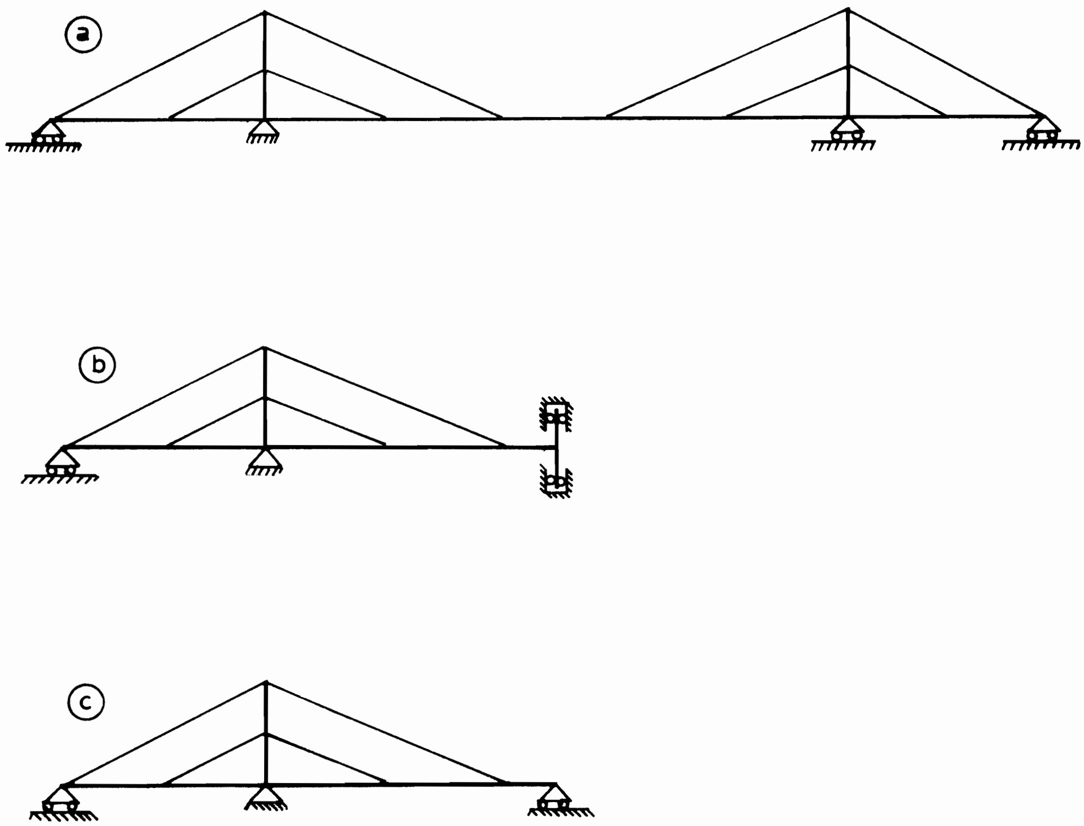


FIG. 55 USE OF SYMMETRY AND ASYMMETRY IN CABLE STAYED BRIDGE SYSTEMS ANALYSIS

The application of the above procedure will reduce the amount of work required to carry out a theoretical analysis. In the cases where the size of the system of equations is larger than 8 x 8 however, the calculations become cumbersome and the use of a digital computer is recommended.

b. Calculation of post-tensioning forces.

To determine the post-tensioning forces required in cables in order to reduce the maximum bending moment in the stiffening girder due to the action of dead load, to a specified value, a procedure, as described further, was developed.

Consider the bridge system represented in Fig. 56 and assume that the post-tensioning forces in the cables satisfy the following conditions :

$$X_1 = X_6 = X_7 = X_{12}$$
$$X_2 = X_5 = X_8 = X_{11}$$
$$X_3 = X_4 = X_9 = X_{10}$$

The post-tensioning forces may be computed following the steps described below :

A. Bending moments and deflection diagrams of the bridge system, due to the action of dead load are determined and the locations of the maximum bending moment and maximum deflection are established. Let the location of the maximum bending moment be indicated by "r" and the corresponding bending moment by M_r.

B. Consider the substructures represented in Fig. 57.a, 57.b,57.c, subjected to unit forces acting along the cables removed from the main bridge system. If reduction of bending moments is taken as the condition for determination of post-tensioning forces, bending moments at "r" and axial forces in the remaining cables, for

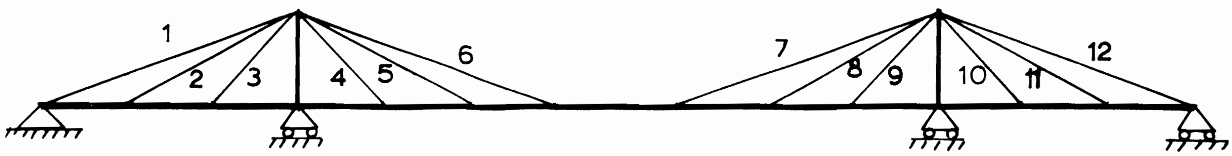


FIG. 56 POST-TENSIONING FORCES IN CABLES. BASIC SYSTEM.

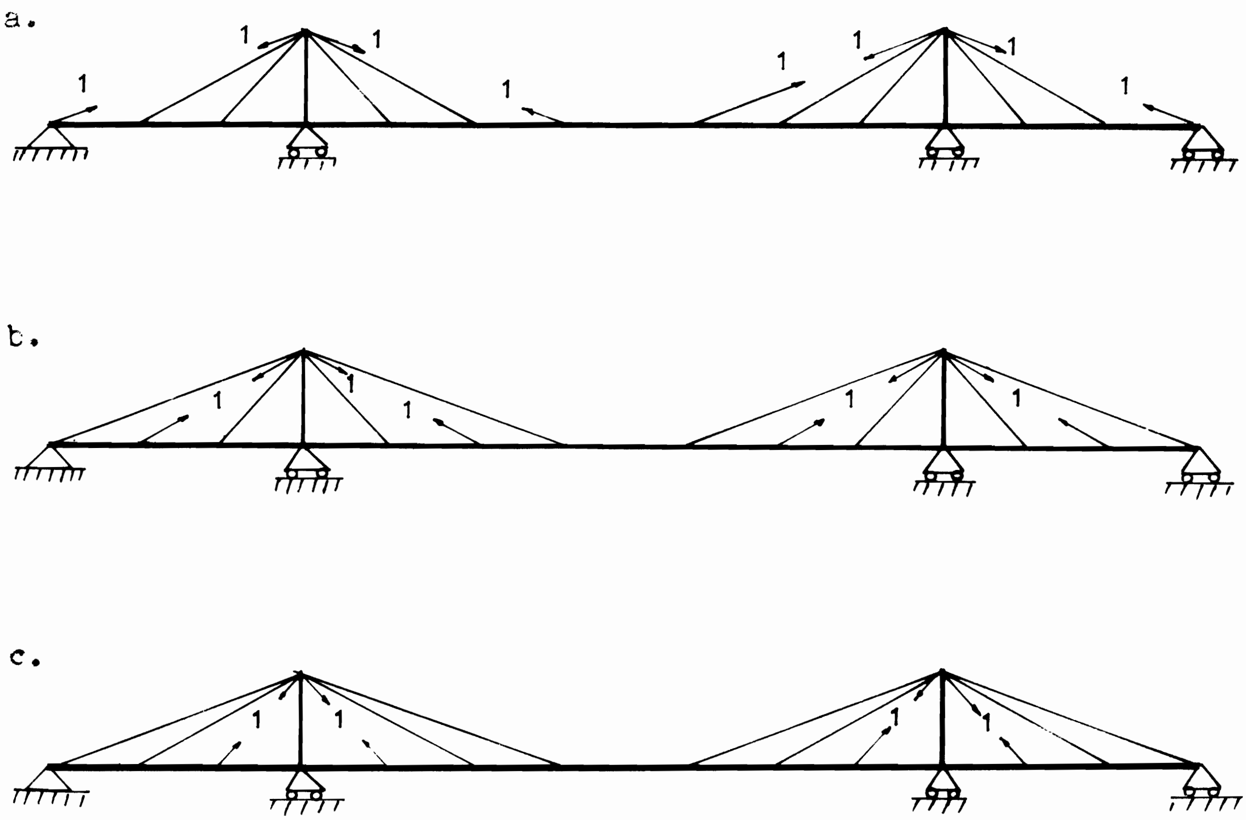


FIG. 57 POST-TENSIONING FORCES IN CABLES. SUBSTRUCTURES.

each of the three cases considered are calculated. Specifically, the following axial forces and bending moments are determined :

Case 1 Substructure a. N_2^a to N_5^a, N_8^a to N_{11}^a, M_r^a

Case 2 Substructure b. $N_1^b, N_3^b, N_4^b, N_6^b, N_7^b, N_9^b, N_{10}^b, N_{12}^b, M_r^b$ (33)

Case 3 Substructure c. N_1^c, N_2^c, N_5^c to $N_8^c, N_{11}^c, N_{12}^c, M_r^c$

In expression (33), N_i^a, N_i^b, N_i^c, where i varies from 1 to 12, are axial forces in cables due to unit loads applied as represented in Figures 57.a, 57.b, 57.c, and M_r^a, M_r^b, M_r^c are the corresponding bending moments at location "r".

The above values may be determined analytically or on a structural model.

C. X_1, the post-tensioning force in cables 1, 6, 7 and 12 and X_2, X_3, the post-tensioning forces in cables 2, 5, 8, 11, respectively 3, 4, 9, 10 are determined such that M_r is reduced to $C_0 M_r$ where $C_0 < 1$ is a reduction factor.

To reduce M_r to $C_0 M_r$ the post-tensioning forces must satisfy the condition:

$$M_r + X_1 M_r^a + X_2 M_r^b + X_3 M_r^c = C_0 M_r$$

The above expression may be written as

$$X_1M_r^a + X_2M_r^b + X_3M_r^c = M_r(C_0-1) \qquad (34)$$

Equation (34) indicates that there are many possible combinations of X_1, X_2, X_3 and two of the three unknown post-tensioning forces may be selected arbitrarily. If the sum of the unit stresses due to dead load and post-tensioning is specified, however, to be identical for cables 1, 2, 3, that is

$$\frac{N_1^f}{A_1} = \frac{N_2^f}{A_2} = \frac{N_3^f}{A_3} \qquad (35)$$

then, for a given C_0, the post-tensioning forces in cables X_1, X_2, X_3, are calculated as follows :

First, X_2 and X_3 are expressed as functions of X_1, employing Equation (35). Then X_2 and X_3 are substituted in Equation (34) and an expression for X_1 is obtained.

To express X_2 and X_3 as functions of X_1, Equation (35) is rearranged as a system of two equations with two unknowns.

$$\begin{aligned} a_{11}X_2 + a_{12}X_3 &= A_{11} + A_{12}X_1 \\ a_{21}X_2 + a_{22}X_3 &= A_{21} + A_{22}X_1 \end{aligned} \qquad (36)$$

where

$$
\begin{aligned}
a_{11} &= N_1^b - \frac{A_1}{A_3} \cdot N_3^b \\
a_{12} &= N_1^c - \frac{A_1}{A_3} \cdot N_3^c \\
a_{21} &= N_2^b - \frac{A_2}{A_3} \cdot N_3^b \\
a_{22} &= N_2^c - \frac{A_2}{A_3} \cdot N_3^c
\end{aligned}
\tag{37}
$$

and

$$
\begin{aligned}
A_{11} &= \frac{A_1}{A_3} \cdot N_3 - N_1 \\
A_{12} &= \frac{A_1}{A_3} \cdot N_3^a - N_1^a \\
A_{21} &= \frac{A_2}{A_3} \cdot N_3 - N_2 \\
A_{22} &= \frac{A_2}{A_3} \cdot N_3^a - N_2^a
\end{aligned}
\tag{38}
$$

Solving the system of equations (36), X_2 and X_3 may be expressed as

$$
\begin{aligned}
X_2 &= B_{11} + B_{12}X_1 \\
X_3 &= B_{21} + B_{22}X_1
\end{aligned}
\tag{39}
$$

where

$$
\begin{aligned}
B_{11} &= \frac{A_{11}a_{22} - A_{21}a_{12}}{B} \\
B_{12} &= \frac{A_{12}a_{22} - A_{22}a_{12}}{B} \\
B_{21} &= \frac{A_{21}a_{11} - A_{11}a_{21}}{B} \\
B_{22} &= \frac{A_{22}a_{11} - a_{21}A_{12}}{B}
\end{aligned}
\tag{40}
$$

In Equations (40), B is the determinant of the system of Equations (36), that is

$$B = a_{11}a_{22} - a_{21}a_{12} \tag{41}$$

Substituting X_2 and X_3 from Equation (39) into Equation (34) and rearranging

$$X_1 = \frac{M_r(C_o-1) - B_{11}M_r^b - B_{21}M_r^c}{M_r^a + B_{12}M_r^b + B_{22}M_r^c} \tag{42}$$

Once X_1 is known, X_2 and X_3 may be determined from Equation (39).

The final forces in cables due to post-tensioning and dead load are

$$N_i^f = N_i + X_iN_i^a + X_2N_i^b + X_3N_i^c \tag{43}$$

where i varies from 1 to 12.

The concept developed above is applicable to any cable stayed bridge system.

NUMERICAL EXAMPLE.

An example based on the method outlined above, has been worked out for the cable stayed bridge system represented in Fig. 58. The bending moment at an intermediate support due to a uniformly distributed dead load of 6000 lbs per linear foot is equal to 960,000 k-in. C_o was taken as equal to 0.5.

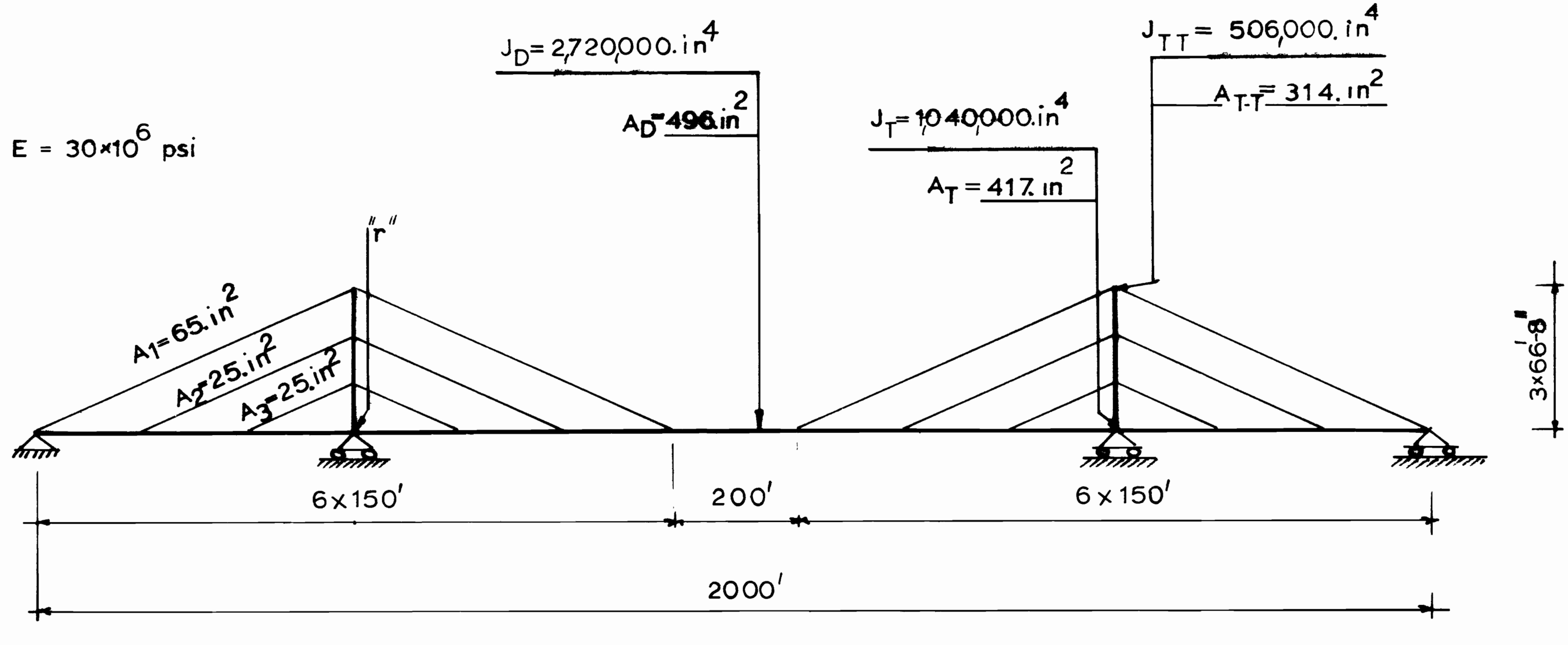


FIG. 58 BRIDGE SYSTEM SUBJECTED TO DEAD LOAD AND POST-TENSIONING FORCES IN CABLES.

Axial forces in the cables due to the dead load are given in Equation (44).

$$
\begin{aligned}
&\text{Cables 1 and 12, } N_1 = N_{12} = 2{,}840.00 \text{ kips}\\
&\text{Cables 2 and 11, } N_2 = N_{11} = 1{,}550.00 \text{ kips}\\
&\text{Cables 3 and 10, } N_3 = N_{10} = 1{,}370.00 \text{ kips}\\
&\text{Cables 4 and 9, } N_4 = N_9 = 1{,}372.00 \text{ kips}\\
&\text{Cables 5 and 8, } N_5 = N_8 = 1{,}550.00 \text{ kips}\\
&\text{Cables 6 and 7, } N_6 = N_7 = 2{,}842.00 \text{ kips}
\end{aligned}
\tag{44}
$$

The values of N_i^a, N_i^b, N_i^c, where i varies from 1 to 12, and of M_r^a, M_r^b, M_r^c, computed for the equivalent three substructures, are given in Table No. 3. The bending moments are given in lb-in and the axial forces in lb.

Table No. 3

AXIAL FORCES AND BENDING MOMENTS AT INTERMEDIATE SUPPORTS DUE TO UNIT LOADS. NUMERICAL EXAMPLE

Axial forces and bending moments	Substructure		
	a	b	c
N_1	1.00	- 0.47	- 0.06
N_2	- 0.97	1.00	- 0.21
N_3	- 0.19	- 0.49	1.00
N_4	- 0.30	- 0.43	1.00
N_5	- 0.85	1.00	- 0.20
N_6	1.00	- 0.47	- 0.06
N_7	1.00	- 0.47	- 0.06
N_8	- 0.85	1.00	- 0.20
N_9	- 0.30	- 0.43	1.00
N_{10}	- 0.19	- 0.49	1.00
N_{11}	- 0.97	1.00	- 0.21
N_{12}	1.00	- 0.47	- 0.06
M_r^j	-13.00	-199.00	-325.00

With the above data X_1, X_2 and X_3 may be determined from Equations (37) to (42), and N_i^f, where i varies from 1 to 12, may be determined from Equation (43).

From Equation (37)

$$a_{11} = -0.47 - \frac{65}{25} \times (-0.49) = +\ 0.80$$
$$a_{12} = -0.06 - \frac{65}{25} \times 1.00 = -\ 2.66$$
$$a_{21} = 1.00 - \frac{25}{25} \times (-0.49) = +\ 1.49$$
$$a_{22} = -0.21 - \frac{25}{25} \times 1.00 = -\ 1.21$$

With the above values B may be calculated from Equation (41).

$$B = 0.80 \times (-1.21) - (1.49) \times (-2.66) = 3.00$$

From Equation (38)

$$A_{11} = \frac{65}{25} \times 1370.00 - 2840.00 = 725.00$$
$$A_{12} = \frac{65}{25} \times (-0.19) - 1.00 = 1.495$$
$$A_{21} = \frac{25}{25} \times 1370.00 - 1550.00 = -180.00$$
$$A_{22} = \frac{25}{25} \times (-0.19) - (-0.97) = 0.78$$

B_{11}, B_{12}, B_{21} and B_{22} may be determined now from Equations (40).

$$B_{11} = \frac{725.00 \times (-1.21) - (-180.00) \times (-2.66)}{3}$$
$$= -452.00$$
$$B_{12} = \frac{-1.495 \times (-1.21) - 0.78 \times (-2.66)}{3}$$
$$= 1.29$$
$$B_{21} = \frac{-180.00 \times 0.80 - 725.00 \times 1.49}{3} = -408.00$$

$$B_{22} = \frac{0.78 \times 0.80 - 1.49 \times (-1.495)}{3} = 0.95$$

With the above information, X_1 may be determined from Equation (42)

$$X_1 = \frac{0.96\text{x}10^6 \text{x}(0.5-1.0)-(452.00)\text{x}(-199.00)-(-408.00)\text{x}(-325.00)}{-13.00+(1.29)\text{x}(-199.00)+(0.95)\text{x}(-325.00)}$$
$$= 1225.00 \text{ kips}$$

X_2 and X_3 may be calculated from Equation (39)

$$X_2 = -425.00 + 1.29 \times 1225.00 = 1130.00 \text{ kips}$$

$$X_3 = -408.00 + 0.95 \times 1225.00 = 754.00 \text{ kips}$$

Finally, N_i^f may be determined from Equation (43)

$$N_1^f = N_{12}^f = 2840.00+1225.00\text{x}1.00+1130.00\text{x}(-0.47)+754.00\text{x}(-0.06)$$
$$=3490.00 \text{ kips}$$

$$N_2^f = N_{11}^f = 1550.00+1225.00\text{x}(-0.97)+1130.00\text{x}1.00+754.00\text{x}(-0.21)$$
$$=1335.00 \text{ kips}$$

$$N_3^f = N_{10}^f = 1370.00+1225.00\text{x}(-0.19)+1130.00\text{x}(0.49)+754.00\text{x}1.00$$
$$=1335.00 \text{ kips}$$

$$N_4^f = N_9^f = 1372.00+1225.00\text{x}(-0.30)+1130.00\text{x}(-0.43)+754.00\text{x}1.00$$
$$=972.00 \text{ kips}$$

$$N_5^f = N_8^f = 1551.00+1225.00\text{x}(-0.85)+1130.00\text{x}1.00+754.00\text{x}(-0.20)$$
$$=1489.00 \text{ kips}$$

$$N_6^f = N_7^f = 2842.00+1225.00\text{x}1.00+1130.00\text{x}(-0.47)+754.00\text{x}0.06$$
$$=3490.00 \text{ kips}$$

Unit stresses in cables 1,2,3 and 6 are identical. To achieve the same condition for cables 4 and 5, A_4 should be decreased and A_5 increased, and the analysis repeated until the unit stresses are identical in all cables.

3. Analysis by digital computer

To develop a computer program for a cable stayed bridge, either the stiffness or the flexibility method, or both, may be applied.

If the flexibility method is employed, bending moments at fixed and flexible supports should be chosen as redundants, in order to obtain a well-conditioned, banded, flexibility matrix.

As in the previous case of the classical analysis, the computer methods will be presented, first for dead and live load and then for post-tensioning forces.

a. Analysis due to the action of dead and live loads.

A. Linear analysis

Based on the flexibility method, a computer program for analysis of a cable stayed bridge, has been developed. The program reads input data regarding the geometry and sectional properties of the system and calculates the values for plotting the following :

a) Influence lines for bending moments, axial and shear forces, displacements and reactions.

b) Envelopes of maximum bending moments, axial and shear forces for the most critical combination of dead and live loads.

The computer program developed, applies to a bridge system having an overall geometry and supports as represented in Fig. 59. The connections between towers and the stiffening girder are fixed and the cable-tower and cable-girder connections are hinged.

The basic steps of the flow chart of the computer program for influence lines and envelopes are given in the following pages. For the system considered, the redundants have been chosen as shown in Fig. 60. The redundants are indicated by Q_i, where i varies from 1 to 14.

In the flow chart developed, steps 2 and 3 represent the statements required to read and store the geometrical and sectional properties of the system to be analyzed. This data is employed in step 4 to determine

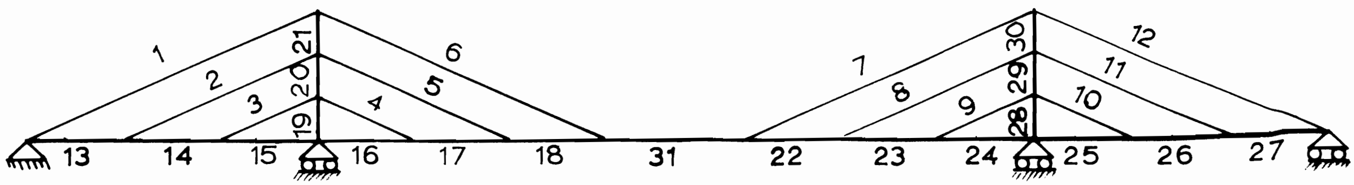


FIG. 59 CABLE STAYED BRIDGE SYSTEM ANALYSED BY THE FLEXIBILITY METHOD

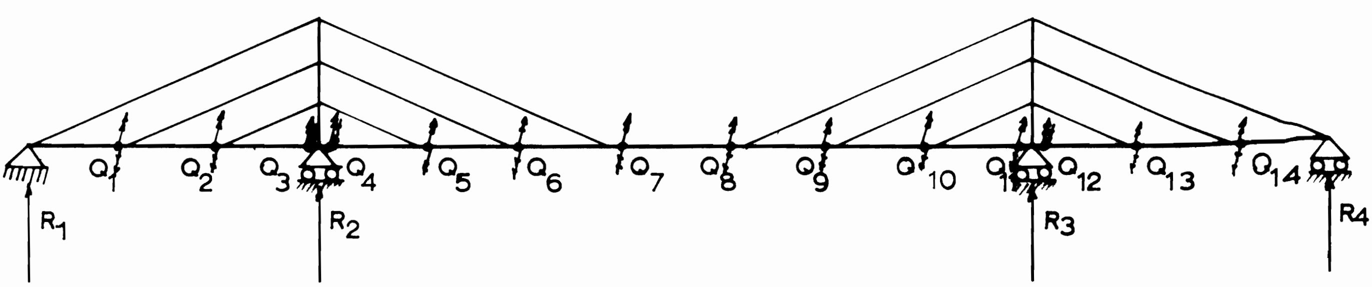
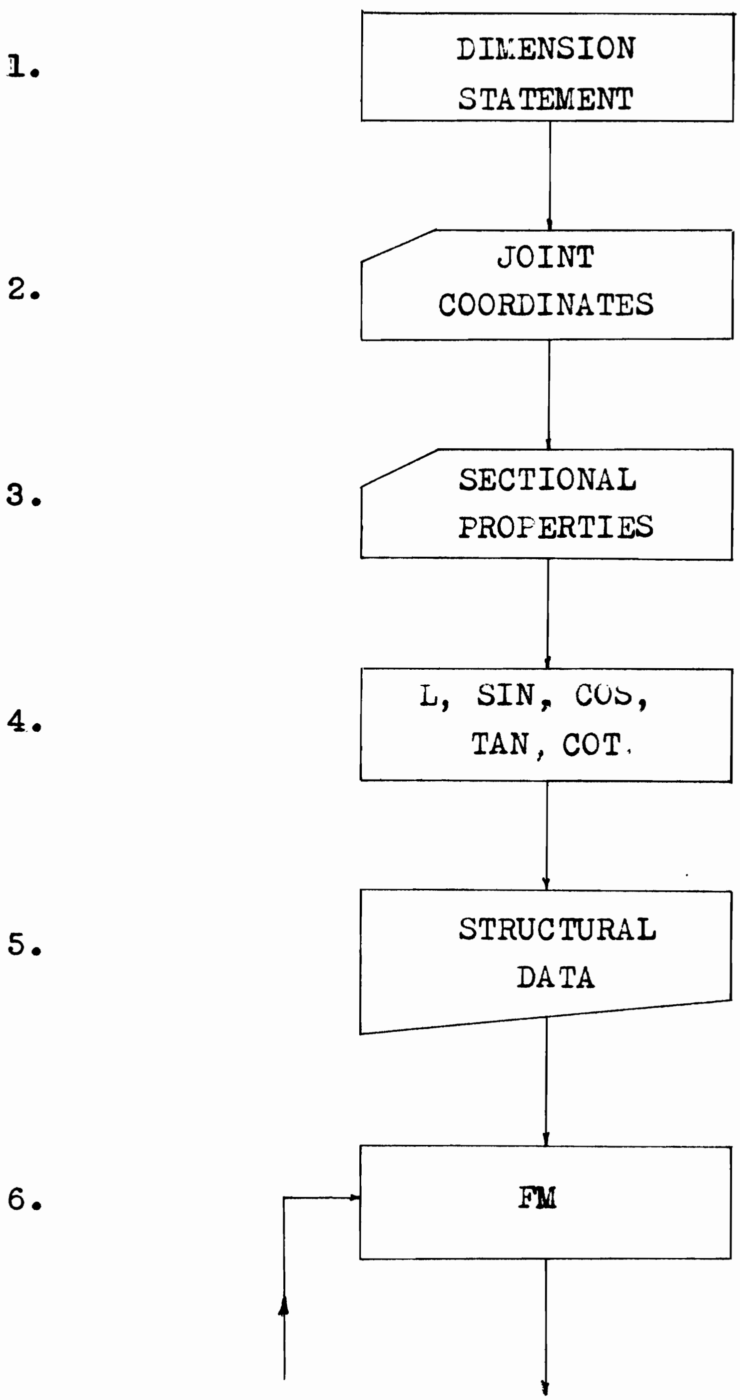
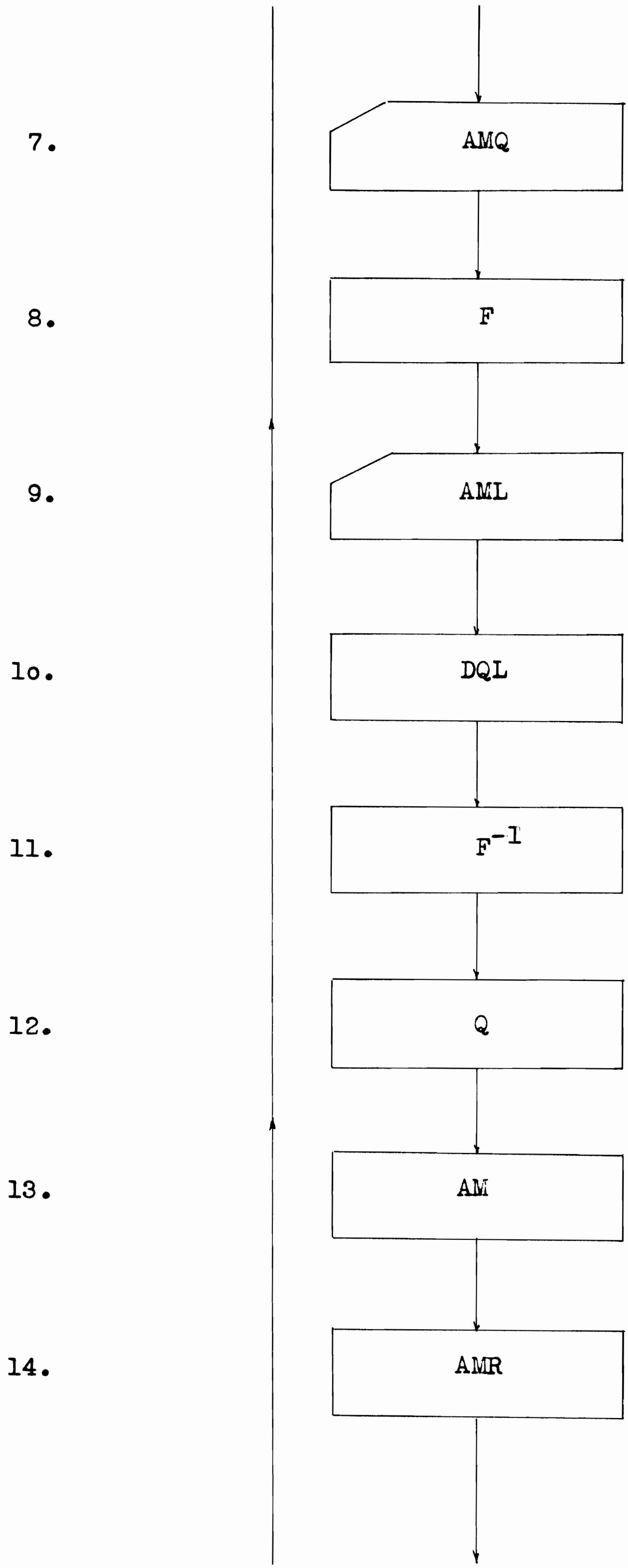


FIG. 60 SELECTION OF REDUNDANTS FOR ANALYSIS DUE TO DEAD AND LIVE LOADS

FLOW CHART

COMPUTER PROGRAM FOR DETERMINATION OF INFLUENCE LINES AND ENVELOPES DUE TO DEAD AND LIVE LOADS.




7.
AMQ
8.
F
9.
AML
10.
DQL
11.
F^{-1}
12.
Q
13.
AM
14.
AMR

15.

16.

17.

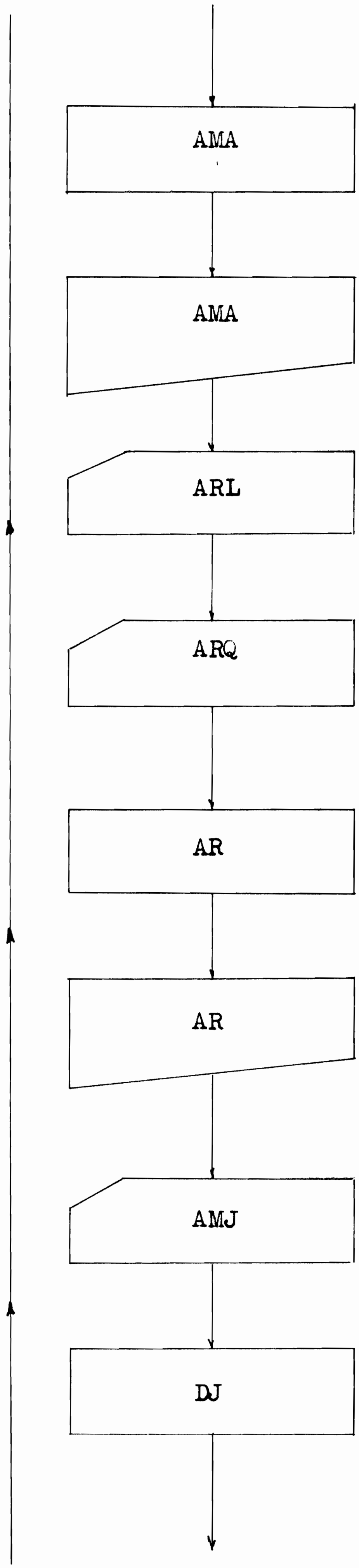
18.

19.

2o.

21.

22.

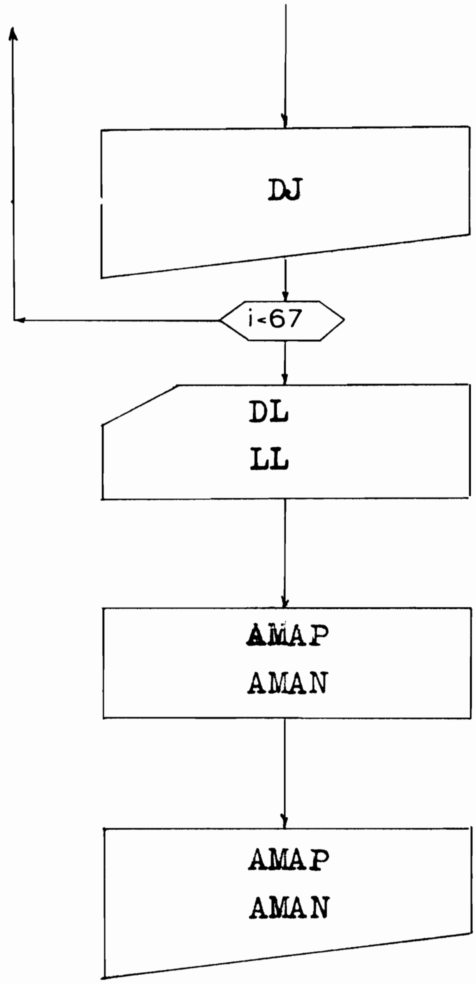


23.

24.

25.

26.



sin, cos tan and cot functions of the angles between the cables and the stiffening girder.

Steps 6 to 23 were developed on the basis of the computer methods described by Gere and Weaver[19,20]. To determine influence line ordinates for 67 locations of the unit load (the intervals taken along the girder were one fifth of the length of one member) steps 6 to 23 were repeated in a DO loop 67 times. The total computer time required to execute the program on the CDC 3300 computer is 3 minutes, 8 seconds.

The output, steps 16, 20 and 23, consists of influence coefficients for bending moments, axial and shear forces, reactions and displacements. The displacements calculated are shown on Fig. 61.

Steps 24 to 26 determine the values required to plot the envelopes of bending moments, axial and shear forces for the most critical combination of dead and live loads. Step 24 reads DL, the uniform distributed dead load and LL, the uniform distributed live load. Step 25 scans through the matrix AMA of axial and shear forces and bending moments at member ends. The general form of AMA is

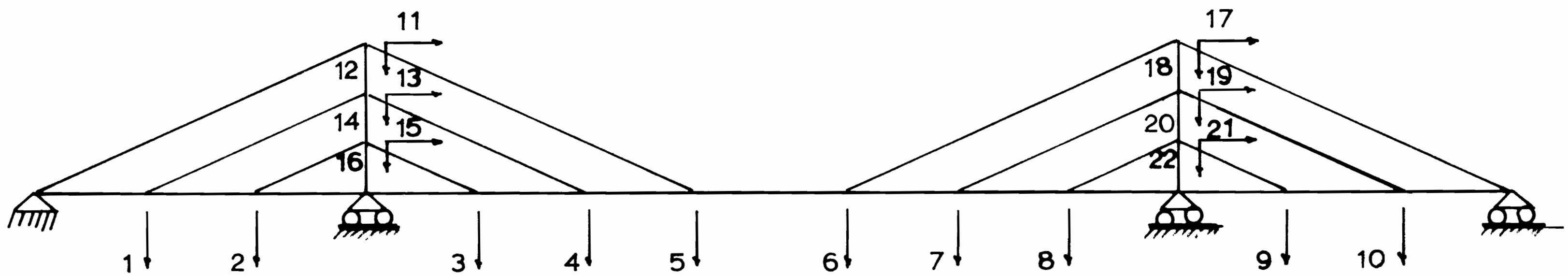


FIG. 61 DISPLACEMENTS

$$\text{AMA} = \begin{bmatrix} \text{AMA}_{1,1} & \text{AMA}_{1,2} & \cdots & \text{AMA}_{1,68} \\ \text{AMA}_{2,1} & \text{AMA}_{2,2} & \cdots & \text{AMA}_{2,68} \\ \ldots\ldots\ldots\ldots\ldots & & & \\ \text{AMA}_{93,1} & \text{AMA}_{93,2} & \cdots & \text{AMA}_{93,68} \end{bmatrix} \quad (45)$$

Columns 2 to 68 in Equation (45) contain bending moments (lines 3,6, ..93), axial forces (lines 1,4 .. 91) and shear forces (lines 2,5 .. 92) at member ends for 67 locations of the unit load along the stiffening girder. Column 1 contains member ends bending moments, axial and shear forces due to a uniformly distributed load of 1 kip per linear foot, along the stiffening girder.

To obtain moment envelopes, each third line, starting at column 2, is scanned and all positive terms are accumulated successively in a column vector AMAP. The same is done for the negative terms which are added and stored in AMAN. The next operation consists of multiplying AMAP by $\frac{L \times LL}{5.00}$ and the first column of AMA by $\frac{DL}{1000.00}$ and adding the results. This gives the final AMAP, that is the ordinates of the bending moments at bar ends due to the most critical combination of dead and live loads. The same procedure is employed for AMAN and also for axial and shear forces.

The total computer time required to calculate and print the envelopes is 15 seconds.

The computer program was written in USASI FORTRAN language for the Control Data Corporation (CDC) 3300 computer. This machine has 80k words of core storage (one word is equal to twenty-four bits) which represents a memory roughly equivalent to 320k bites on the IBM 360 series. The computer has full floating point and character hardware; eight disk drives with a total capacity of about 65 million characters; 5 tape units, 1 printer, 2 terminals, 1 card reader, one punch, one plotter and a multiplexer connected to the TWX network.

A listing of the computer program is given in Appendix 5.

B. Nonlinear Analysis

The nonlinearity of the cable stayed bridge system is caused by large displacements and bending moments-axial force interaction. Relations between stresses and strains at any cross-section are assumed to be linear.

Analysis of plane frames which display the above type of nonlinearity has been studied extensively in the past decade[21,22,23,24]. Saafan[21] developed a method which permits the performance of a nonlinear analysis by successive interations of linear subroutines.

The first step of the analysis determines a vector of displacements based on the initial geometry of the system and on the external loads. In the second step, an additional displacement vactor, due to the difference between the joint loads and the resultants of internal bending moments and axial and shear forces at each joint is determined. In performing the second step, the stiffness matrix of the system is assembled on the basis of the deformed geometry and of the axial loads determined in step one.

Each subsequent step "i" uses data determined in the previous step, "i-1".

The iteration stops when the last displacement vector obtained is a negligible fraction of the total displacement.

b. <u>Calculation of post-tensioning forces</u>.

After erection, the cable stayed bridge is under the action of dead load only. The bending moments and deflections of the stiffening girder may be reduced by post-tensioning the cables.

A procedure which facilitates the reduction of the maximum bending moment due to dead load by post-tensioning the cables, has been presented previously in

Section 2, in connection with classical computation methods. This procedure may be programmed on a digital computer by extending the concept developed initially.

The released structure is chosen as shown in Fig. 62. To determine unit displacements and bending moments due to unit loads applied along the cables, twelve substructures are considered. Each substructure consists of the original structure with one cable removed. Substructure No. 1 is represented in Fig. 63.

The basic equations for this case are

$$M_r^1 X_1 + \ldots M_r^{12} X_{12} = M_r (C_o - 1) \tag{46}$$

$$\frac{N_1^f}{A_1} = \frac{N_2^f}{A_2} = \ldots\ldots = \frac{N_{12}^f}{A_{12}} \tag{47}$$

and

$$\begin{aligned}
a_{2,2}X_2 + a_{2,3}X_3 + \cdots a_{2,12}X_{12} &= A_{2,1} + A_{2,2}X_1 \\
a_{3,2}X_2 + a_{3,3}X_3 + \cdots a_{3,12}X_{12} &= A_{3,1} + A_{3,2}X_1 \\
\ldots\ldots\ldots\ldots\ldots\ldots \\
a_{12,2}X_2 + a_{12,3}X_3 + a_{12,12}X_{12} &= A_{12,1} + A_{12,2}X_1
\end{aligned} \tag{48}$$

Equations (46), (47) and (48) correspond to equations (34), (35) and (36). A_{ij} and a_{ij} may be expressed as in Equations (38) and (37).

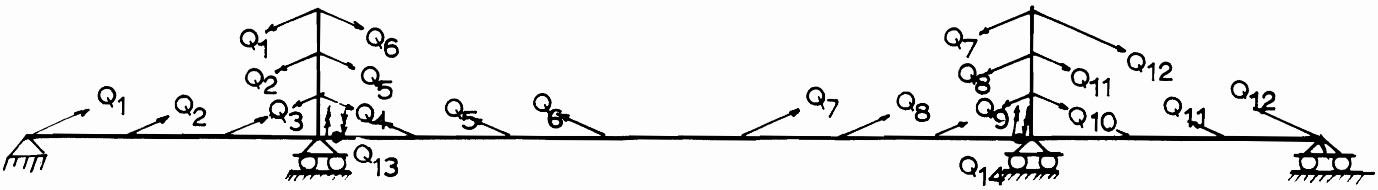


FIG. 62 SELECTION OF CABLES AS REDUNDANTS

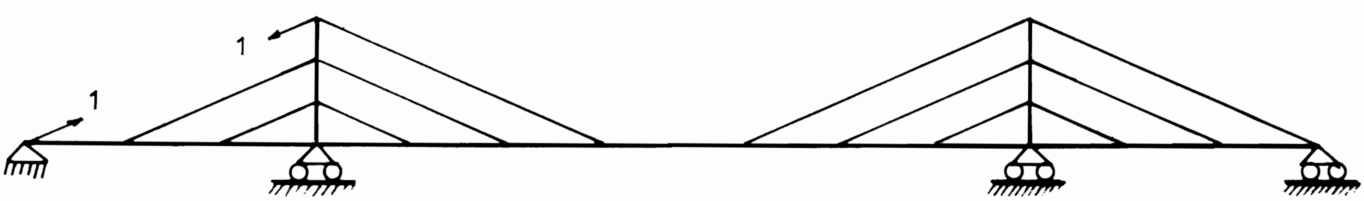


FIG. 63 SUBSTRUCTURE FOR CALCULATION OF POST-TENSIONING FORCES.

Equations (48) may be written in matrix form as

$$[a]*\{X\} = \{A\} \tag{49}$$

where

$$[a] = \begin{bmatrix} a_{2,2} & a_{2,3} & \cdots & a_{2,12} \\ a_{3,2} & a_{3,3} & \cdots & a_{3,12} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{12,2} & a_{12,3} & \cdots & a_{12,12} \end{bmatrix} \tag{50}$$

$$\{X\} = \{X_2, \quad X_3 \quad \ldots \; X_{12}\} \tag{51}$$

$$\{A\} = \{A^1\} + \{A^2\}*X_1 \tag{52}$$

In Equation (52)

$$\{A^1\} = \{A_{2,1}, \quad A_{3,1} \quad \ldots \; A_{12,1}\} \tag{53}$$

and

$$\{A^2\} = \{A_{2,2}, A_{3,2}, A_{4,2} \cdots A_{12,2}\} \tag{54}$$

From Equation (49)

$$\{X\} = [a]^{\overline{*}1}\{A\} \tag{55}$$

Relation (46) may be rewritten as

$$M_r(C_o-1) = M_r^1 X_1 + (M_r)*\{X\} \tag{56}$$

where

$$(M_r) = (M_r^2, M_r^3, M_r^4 \ldots M_r^{12}) \tag{57}$$

Substituting {X} from Equation (55) and taking Equation (52) into account, the relation shown in Equation (56) becomes

$$M_r(C_o-1) = M_r^1X_1 + (M_r)*[a]^{-1}*\{A^1\} + (M_r)*[a]^{-1}*\{A^2\}X_1 \qquad (58)$$

X_1 may be obtained now from

$$X_1 = \frac{M_r(C_o-1) - (M_r)*[a]^{-1}*\{A^1\}}{M_r^1 + (M_r)*[a]^{-1}*\{A^2\}} \qquad (59)$$

and X may be calculated from Equation (55).

With X_1 and X known, N_i^f may be determined from

$$N_i^f = N_i + X_1N_i^1 + \ldots X_{12}N_i^{12} \qquad (60)$$

where i varies from 1 to 12

The above procedure yields identical unit stresses in all cables.

Another criterion for determining the post-tensioning forces in cables is the reduction of displacements due to dead load by a specified reduction factor.

A procedure was developed to achieve this reduction. This procedure consists of determining first the displacements, bending moments, axial and shear forces, and reactions due to a unit force applied successively along each cable of the bridge. Next, a system of equations is written to express the condition that the sum of the displacements due to the unknown post-tensioning forces in the cables shall be opposite in sign to the displacements due to dead load and equal in absolute value to a fraction of these displacements. By solving this system of equations, the unknown post-tensioning forces are determined.

Finally, the bending moments, axial and shear forces, displacements, and reactions due to post-tensioning, are determined from the information obtained initially by applying unit forces along each cable.

A computer program was written in FORTRAN based on the above principles. The structure considered is the same as for the analysis for dead and live loads, Fig. 59. This time, however, the cables had to be chosen as redundants, Fig. 62.

The program consists of two parts. The first part contains the following steps:

1. The sectional properties and joint coordinates of the structure are read into the computer memory.

2. Sin, cos, tan and cot functions of the angles between the main girder and cables

are calculated.

3. The matrices AMQ, BRQ, BMJ are calculated and stored in the computer.

4. Matrix FM is calculated and stored.

The second part of the computer program contains the following basic steps:

1. Matrix F is computed.

2. Cable 1 is removed from the structure and the substructure shown in Fig. 63 is obtained. In this substructure, the displacements indicated in Fig. 61, the bending moments, axial and shear forces at all member ends, and the reactions due to a unit load applied along cable 1 are determined.

The procedure in detail is as follows:

2.1 The column corresponding to cable 1 in matrix AMQ is stored in AML.

2.2 The flexibility matrix F_1 of the substructure shown in Fig. 63 is generated by removing the row and column corresponding to cable 1 from matrix F.

2.3 The matrix F_1 is modified by increasing all terms on the main diagonal of the first 12 rows by $\frac{L}{EA}$. The term corresponding to cable 1, however, remains equal to zero.

2.4 The vector of displacements associated with the released substructure 1, DQL_1, is calculated from

$$DQL_1 = AMQ_1^T * FM_1 * AML$$

2.5 The vector Q1 of the unknown redundants of the released substructure 1, is calculated from

$$Q1 = -F_1^{-1} * DQL_1$$

2.6 The bending moments, axial and shear forces due to a unit load applied along cable 1 are calculated

$$AM_1 = AMQ_1 * Q1 + AML$$

Steps 2.1 to 2.6 are repeated for all 12 substructures. Vectors AM_1 to AM_{12} are stored in matrix AM.

2.7 The matrix of displacements indicated in Fig. 61 due to unit loads acting along cables 1 to 12 is computed.

$$DJ = BMJ^T * FM * AM$$

2.8 The reactions due to unit loads acting along cables 1 to 12 are calculated.

$$AR = BRQ^T * Q + ARL$$

3. The post-tensioning forces in cables are determined such that deflections 1 to 10 and displacements 11 and 17 will be reduced by a factor $C_o < 1$.

3.1 Displacements due to the action of the dead loads are read into the computer. Vertical deflections 1 to 10 and horizontal displacements 11 and 17 are then multiplied by C_o and stored in vector AJ.

3.2 Matrix JD is assembled from the first eleven rows and row 17 of matrix DJ.

3.3 The post-tensioning forces in cables are determined from

$$X = JD^{-1} * AJ$$

4. Final bending moments, axial and shear forces, reactions and displacements due to post-tensioning are calculated from

$$AMF = AM_1X_1 + \ \ldots\ AM_{12}X_{12}$$
$$DJF = DJ_1X_1 + \ \ldots\ DJ_{12}X_{12} \qquad (61)$$
$$ARF = AR_1X_1 + \ \ldots\ AR_{12}X_{12}$$

In Equations (61) AM_1 to AM_{12}, DJ_1 to DJ_{12} and AR_1 to AR_{12} are the corresponding column vectors in matrices AM, DJ and AR.

The procedure developed above allows the determination of the post-tensioning forces to be applied in cables to reduce displacements due to dead load.

The computer program, second part, is listed in Appendix 6.

4. Conclusions

The classical and matrix methods of structural analysis presented in this Chapter provide the means of determining the stresses and displacements due to dead and live loads and the post-tensioning forces required in cables to reduce these stresses and displacements by a specified reduction factor C_0. Although the methods described have been worked out primarily to compare the results of the experimental investigation performed with theoretical data, they may be employed as general methods of structural analysis of cable stayed bridges.

V

EXPERIMENTAL DETERMINATION OF POST-TENSIONING FORCES IN CABLES

1. Introduction

This Chapter describes the experimental procedure applied to determine the post-tensioning forces required in cables, to reduce the bending moments and displacements of the stiffening girders and towers of a cable stayed bridge.

The post-tensioning forces may be applied before or after the erection of the cable stayed bridge.

The technique employed consists of determining bending moments and displacements due to the action of dead load, and bending moments, axial forces and displacements due to unit forces applied successively along each cable of the system. From this data, post-tensioning forces required in cables to reduce bending moments and displacements due to dead load, are determined employing the procedures outlined in Chapter IV, Sections No. 2 and No. 3.

The investigation was carried out on model "A" which is described in Chapter II.

2. Post-tensioning forces in cables to reduce the maximum bending moment at intermediate supports

An experimental investigation was carried out to determine the post-tensioning forces required in the cables to reduce the maximum bending moment due to dead load at the intermediate supports.

The first step of the investigation was to determine the axial forces in the cables and the maximum bending moment at the intermediate supports, due to the action of the dead load. The prototype investigated is represented in Fig. 7. The dead load assumed for the prototype, was 5400 lb/ft which corresponds to 5400 x 31.6/100 = 1705 lb/ft for the model.

The axial forces and the bending moment were determined from the corresponding influence lines. The maximum bending moment at the intermediate support, M_r, was equal to 832×10^6 lb-in. The axial forces in cables are given in Equations (62).

$$
\begin{aligned}
N_1 &= N_{12} = 2.308 \times 10^6 \text{ lb} \\
N_2 &= N_{11} = 1.309 \times 10^6 \text{ lb} \\
N_3 &= N_{10} = 1.068 \times 10^6 \text{ lb} \\
N_4 &= N_9 \;\; = 1.081 \times 10^6 \text{ lb} \\
N_5 &= N_8 \;\; = 1.301 \times 10^6 \text{ lb} \\
N_6 &= N_7 \;\; = 2.286 \times 10^6 \text{ lb}
\end{aligned}
\qquad (62)
$$

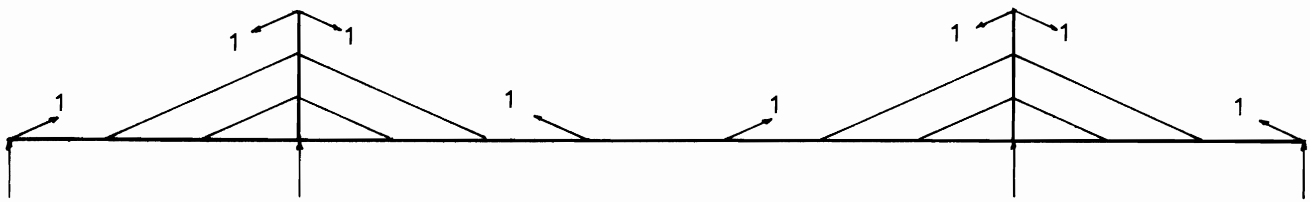
The next step of the investigation was to determine axial forces in the cables and the maximum bending moments at the intermediate supports, under the action of unit forces applied successively on the three substructures represented in Fig. 64. To achieve this, tension was applied along one cable at a time by tightening the nut at the cable-girder connection, Fig. 11. The data determined from the post-tensioning of the individual cables was then superimposed to obtain the conditions indicated in Fig. 64.

The axial forces in the cables and the bending moments at the intermediate supports due to unit forces applies along the cables, as indicated in Fig. 64, are given in Table No. 4.

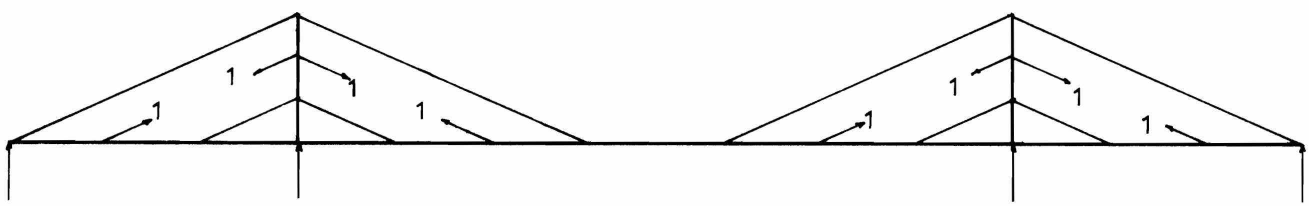
Table No. 4

POST-TENSIONING OF CABLES. AXIAL FORCES AND BENDING MOMENT AT INTERMEDIATE SUPPORTS. EXPERIMENTAL DATA.

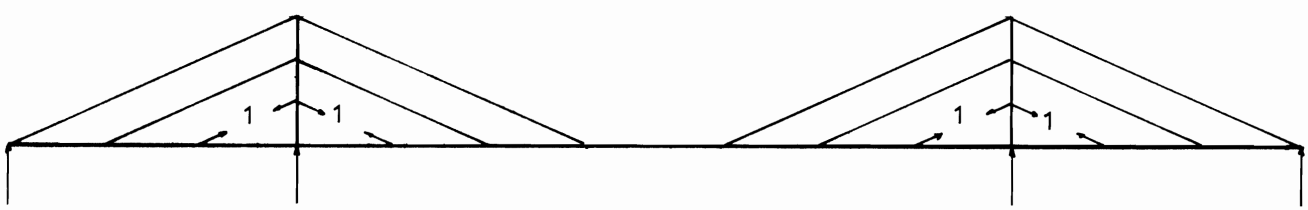
Axial forces and bending moment	Substructure		
	a	b	c
N_1	1.000	-0.445	-0.049
N_2	-0.974	1.000	-0.164
N_3	-0.210	-0.396	1.000
N_4	-0.335	-0.385	1.000
N_5	-1.000	1.000	-0.200
N_6	1.000	-0.431	-0.077
M_r^j	-20.4	-188.5	-376.0



a. Substructure "a"



b. Substructure "b"



c. Substructure "c"

FIG. 64 SUBSTRUCTURES

The units employed in Table No. 4 are lb-in for bending moment and lb for axial forces.

The reduction factor for M_r, has been selected equal to $C_o = 0.6$.

The post-tensioning forces determined from the data given in Table No. 4 and Equations (62), by employing the procedure outlined in Chapter IV, Section 2, were equal to

$$
\begin{aligned}
X_1 &= 995{,}000 \text{ lb} \\
X_2 &= 835{,}000 \text{ lb} \\
X_3 &= 457{,}000 \text{ lb}
\end{aligned}
\tag{63}
$$

To compare the experimental procedure described above with the theoretical method presented in Chapter IV, Section 2, the axial forces in cables and the maximum bending moment at an intermediate support, under the action of dead load and of unit forces applied successively along each cable, were calculated. The maximum bending moment at an intermediate support, due to dead load, was equal to 870×10^6 lb-in. The axial forces in cables due to dead load, are given in Equations (64).

$$
\begin{aligned}
N_1 = N_6 = N_7 = N_{12} &= 2.55 \times 10^6 \text{ lb} \\
N_2 = N_5 = N_8 = N_{11} &= 1.39 \times 10^6 \text{ lb} \\
N_3 = N_4 = N_9 = N_{10} &= 1.23 \times 10^6 \text{ lb}
\end{aligned}
\tag{64}
$$

The theoretical axial forces in cables and the bending moment at the intermediate supports, due to unit forces applied along cables, are given in Table No. 5.

Table No. 5

POST-TENSIONING OF CABLES. AXIAL FORCES AND BENDING MOMENT AT INTERMEDIATE SUPPORTS. THEORETICAL DATA.

Axial forces and bending moment	Substructure		
	a	b	c
N_1	1.000	-0.480	-0.060
N_2	-1.010	1.000	-0.226
N_3	-0.182	-0.416	1.000
N_4	-0.280	-0.395	1.000
N_5	-0.955	1.000	-2.400
N_6	1.000	-0.500	-0.064
M_r^j	-9.3	-173.0	-329.0

The units employed in Table No. 5 are lb-in for bending moment and lb for axial forces.

On the basis of the data indicated in Equations (64) and Table No. 5, the post-tensioning forces required in cables to reduce the maximum bending moments at the intermediate supports by $(1-C_o) = 0.4$, were computed following the procedure outlined in Chapter IV, Section 2. The values of these forces are given in Equations (65).

$$
\begin{aligned}
X_1 &= 1{,}035{,}000 \text{ lb} \\
X_2 &= 970{,}000 \text{ lb} \\
X_3 &= 496{,}000 \text{ lb}
\end{aligned}
\tag{65}
$$

The above forces compare favorably with those in Equations (63) the differences being 3.86% for X_1, 11.1% for X_2 and 7.85% for X_3. Explanation for these differences have been given in Chapter II, Section 6.

3. Post-tensioning forces in cables to reduce displacements due to dead load.

An investigation was performed to reduce the displacements due to dead load of the stiffening girders and towers of the cable stayed bridge represented in Fig. 7, by employing a model technique in connection with the digital computer.

The first step of the investigation was to determine the displacements under the action of dead load. Six displacements were considered. (Vertical displacements 1 to 5 and horizontal displacement 6, Fig. 20). The conditions required to reduce these six displacements by a constant factor determine the magnitude of the six unknown post-tensioning forces in the cables.

The dead load assumed was 5400 lb/ft on the prototype which corresponds to 1705 lb/ft on the model.

The displacements due to dead load were determined by employing the influence lines obtained previously in Chapter 2. The results are given in Equations (66).

$$
\begin{aligned}
v_1 &= 6.25 \text{ in} \\
v_2 &= 3.08 \text{ in} \\
v_3 &= 17.00 \text{ in} \\
v_4 &= 42.40 \text{ in} \\
v_5 &= 53.00 \text{ in} \\
u_6 &= 10.38 \text{ in}
\end{aligned}
\tag{66}
$$

The next step of the investigation was to determine the six displacements specified above, under the action of unit forces applied successively along each cable of the system.

The displacements recorded, were increased by the corresponding scale factor to obtain predicted values for the prototype. The results are represented in Table No. 6.

To determine the post-tensioning forces in the cables required to reduce the displacements given in Equations (66) by a factor of $(1-C_o)$, where $C_o = 0.6$, the principles developed in Chapter IV, Section 3, were followed. The displacement matrix JD is obtained from Table No. 6 and the vactor AJ from Equations (66). To calculate the vector X of post-tensioning forces, a computer subroutine was written for the inversion of the matrix JD and its multiplication by the vector AJ multiplied by $(1-C_o)$. The program reads the displacements from data cards and prints out the post-tensioning forces.

Table No. 6

DISPLACEMENTS DUE TO UNIT FORCES APPLIED ALONG CABLES. EXPERIMENTAL DATA

Cable	Displacements (microinches)					
	1	2	3	4	5	6
1	7.610	6.520	-7.900	-17.100	-21.200	-13.500
2	-4.840	-1.550	-2.300	-4.820	-3.750	-1.460
3	-2.180	-2.950	-1.180	-1.270	-0.442	-0.059
4	-1.240	-1.100	-2.850	-2.370	-0.706	-0.254
5	-2.170	-1.210	-2.640	-7.750	-5.010	-0.561
6	3.670	3.460	-4.620	-12.300	-18.100	2.990
7	0.876	0.635	-0.796	-3.400	-9.250	-1.840
8	-0.147	-0.177	0.266	0.326	-0.681	-0.118
9	-0.085	-0.085	0.113	0.198	0.142	0.000
10	0.000	0.000	0.054	0.108	0.081	0.000
11	0.000	0.000	0.052	-0.129	-1.060	-0.207
12	0.775	0.565	-0.680	-3.420	-9.860	-2.060

The post-tensioning forces obtained by solving the equation JD * X = $AJ(1-C_o)$ are given in Equations (67).

In order to compare the experimental procedure described above with the theoretical method presented in Chapter IV, Section 3, the displacements 1 to 6 under the action of dead load and unit forces applied successively along cables 1 to 12, were computed employing the computer program listed in Appendix No. 6. The results obtained are given in Table No. 7 and Equations (68).

$$
\begin{aligned}
X_1 &= 2.05 \times 10^5 \text{ lb} \\
X_2 &= 5.66 \times 10^5 \text{ lb} \\
X_3 &= 7.55 \times 10^5 \text{ lb} \\
X_4 &= 3.50 \times 10^5 \text{ lb} \\
X_5 &= 3.95 \times 10^5 \text{ lb} \\
X_6 &= 3.39 \times 10^5 \text{ lb}
\end{aligned}
\tag{67}
$$

Table No. 7

DISPLACEMENTS DUE TO UNIT FORCES APPLIED ALONG CABLES. THEORETICAL DATA.

Cable	Displacements (microinches)					
	1	2	3	4	5	6
1	7.170	6.320	-7.610	-15.600	-18.900	-11.800
2	-4.640	-1.460	-2.290	-4.270	-3.320	-1.290
3	-2.140	-2.850	-0.955	-1.100	-0.412	-0.044
4	-1.100	-1.030	-2.740	-2.140	-0.632	-0.225
5	-1.910	-1.140	-2.500	-6.930	-4.410	-0.479
6	3.180	3.330	-4.550	-10.500	-15.400	2.590
7	0.775	0.626	-0.784	-2.950	-8.160	-1.620
8	-0.148	-0.170	0.271	0.288	-0.609	-0.111
9	-0.069	-0.069	0.103	0.188	0.136	0.029
10	-0.040	-0.040	0.060	0.107	0.065	0.015
11	0.000	-0.025	0.058	-0.116	-0.967	-0.188
12	0.725	0.554	-0.660	-2.930	-8.940	-1.770

$$
\begin{aligned}
X_1 &= 2.06 \times 10^5 \text{ lb} \\
X_2 &= 5.81 \times 10^5 \text{ lb} \\
X_3 &= 6.93 \times 10^5 \text{ lb} \\
X_4 &= 3.17 \times 10^5 \text{ lb} \\
X_5 &= 3.62 \times 10^5 \text{ lb} \\
X_6 &= 3.46 \times 10^5 \text{ lb}
\end{aligned}
\qquad (68)
$$

A comparison between Equations (66) and (68) indicates that the difference between experimental and theoretical post-tensioning forces varies between -8.2% and +0.5%.

4. Conclusions

The investigations performed have indicated essentially that bending moments, displacements and axial forces in cables due to the action of unit forces applied along the cables, and post-tensioning forces in cables to reduce bending moments and displacements in the stiffening girder, may be determined by employing structural models.

The use of experimental data, rather than theoretical, enables the investigator to obtain a closer representation of the actual behaviour of the prototype - in this case a better prediction of the post-tensioning forces - as many of the assumptions made in design, employing the mathematical model, are not required in the case of a structural model.

VI

CONCLUSIONS

The object of this research was to develop an experimental technique for the application of small scale structural models as direct tools in the analysis and design of cable stayed bridges.

On the basis of the considerations presented in Chapter I, an investigation was performed to achieve the objective stated above.

To carry out the design of a cable stayed bridge by using a physical rather than a mathematical model, once the layout of the bridge is defined, the next operation is to select a set of sectional properties. This may be done by applying the method described in Appendix 3. After the initial set of sectional properties is obtained, a model may be designed by following the principles and techniques described in Chapter II. If a suitable loading system is available, this model is sufficient to carry out the analysis in both the linear and nonlinear domains. Otherwise, for the nonlinear part of the analysis, a second model may be designed by following the principles and techniques described in Chapter III.

For the design of an actual cable stayed bridge, a set of internal stresses and displacements is obtained and compared with the corresponding stresses and displacements allowed by the specifications governing the design of that bridge. If the differences are larger than the acceptable values specified, the sectional properties need to be modified and the investigation repeated.

To modify the sectional properties of model "A", it should be borne in mind that for determination of influence lines, only the relative ratios between the sectional properties of the stiffening girder, cables and towers are essential and not their absolute values. As a consequence, at least one of the above parts of the cable stayed bridge, may be left unchanged. Naturally, the part to be left unchanged should be the one most difficult to modify - in this case, the stiffening girder.

The problem of modifying the sectional properties of model "A" becomes more difficult if the stiffening girder itself needs to be adjusted. This may be done however, by decreasing the width or the thickness of the flange or the web thickness.

For model "A", the towers may be adjusted relatively easily on the milling machine. The same is true for the round bars. It should be pointed out, however, that for an easy adjustment of the cables, modification of the present cable-tower connection is recommended. This may be accomplished by welding a round tube, filleted on the inside to part "1", Fig. 12, and filleting the cable rod at its upper end so that it may be removed from the tube. In adjusting the cables it is not necessary to modify their cross-section along the full length. It is sufficient to do so on 1/3 to 1/4 of the cable length only. This implies the use of an equivalent rather than actual value for the area of the cable cross-section. It makes however, the adjustment of the cables relatively easy to perform.

The adjustment of model "B" may be performed by milling the towers and the stiffening girder. To adjust the cables, the wires need to be replaced with new wires. This operation may be carried out however, at an insignificant cost.

If the experimental data is recorded with an analog-digital computer, as the one employed in this investigation, the technique developed represents a convenient and practical design method.

The results of the above research may be applied, equally well, to virtually any type of cable stayed bridge.

To bring the model techniques into the design office, it is necessary however, to have the design by models recognized as an accepted method by including it in bridge codes and specifications. It is to be hoped that the future will bring about such a development, and the model as a design tool, will become a working aid in the bridge and structural engineering office.

SELECTED REFERENCES

1. Brown, Ch.D., "Design and Construction of the George Street Bridge over the River Usk at Newport, Monmouthshire", Proceedings of the Institution of Civil Engineers, London, 32, 1965.

2. Feige, A., "The Evolution of German Cable Stayed Bridges: an overall survey", Acier-Stahl-Steel, No. 12, December, 1966.

3. Thul, H., "Cable-stayed Bridges", Stahlbau-Tagung Munchen 1966, Heft 19, Stahlbau-Verlag, Koln, 1967 (in German).

4. Homberg, H., "Influence Lines for Cable-Stayed Bridges", Stahlbau, No. 24, February, 1955 (in German)

5. Breen, J.E., "Fabrication and Tests of Structural Models", ASCE Journal of the Structural Division, June, 1968.

6. Benito, C., "Building of Models", Bulletin Rilem, No. 8, September,1960.

7. Litle, W.A., and Foster, D.C., "Fabrication Techniques for Small Scale Structural Models", M.I.T. Report R66-45, October, 1966.

8. Hanson, N.W., Carpenter, J.E., "Structural Model Testing - A Profile Plotter", PCA Development Bulletin, D68, 1964.

9. M.I.T., Dept. of Civil Engineering, "Datalink - A Users Manual", M.I.T. Report T65-03.

10. Rocha, M.M., and Borges, J.F., "Photographic Method For Model Analysis of Structures", Laboratorio de Engenharia Civil, Publication No. 18, 1951, Lisbon.

11. Leonhardt, F., and Andra, W., "A Simple Method to Draw Influence Lines for Slabs", Bulletin Rilem, No. 11, June, 1961.

12. Landdeck, E.N., "A Direct Method for Model Analysis", ASCE Journal of the Structural Division, January, 1956.

13. Langhaar, H.L., "Dimensional Analysis and Theory of Models", John Wiley & Sons, Inc., New York, 1951.

14. Murphy, G., "Similitude in Engineering", The Ronald Press Co., New York, 1950.

15. Beaujoint, N., "Similitude and Theory of Models", Bulletin Rilem, No. 7, June, 1960.

16. Preece, B.W., and Davies, J.D., "Models for Structural Concrete" CR Books Ltd., London, 1964.

17. Buckingham, E., "On Physically Similar Systems: illustration of the use of dimensional equations",The Physical Review, Vol. 4, No. 4, 1914.

18. Smith, B.S., "The Single Plane Cable-Stayed Girder Bridge: a method suitable for computer use", Proceedings of the Institution of Civil Engineers, London, 37, 1967.

19. Gere, J. M., and Weaver, W.Jr., "Analysis of Framed Structures", D. Van Nostrand, 1965.

20. Weaver, W.Jr., "Computer Programs for Structural Analysis", D. Van Nostrand, 1966.

21. Saafan, S.A., "Nonlinear behaviour of Structural Plane Frames". Proc. Amer. Soc. Civ. Eng., Vol. 89, No. ST4, August, 1963.

22. W. Merchant and D.M. Brotton, "A generalized method of analysis of Elastic Plane Frames", I.A.B.S.E. Symposium, Rio de Janeiro, 1964.

23. Miller, M.A., Brotton, D.M., and Merchant W., "A Computer Method for the Analysis of Nonlinear Elastic Plane Framework", International Symposium on Use of Computers in Structural Engineering, Dept. of Civil Engineering, University of Newcastle, England, 1966.

24. Connor, J.J., et al., "Nonlinear Analysis of Elastic Framed Structures"., Proc. Amer. Soc. Civ. Eng., Vol. 94 ST6, June, 1968.

25. Troitsky, M.S. and Lazar, B. "Model Investigations of Cable Stayed Bridges", Report No. 1, Sir George Williams University, Faculty of Engineering, Sept., 1969.

26. Troitsky,M.S. and Lazar B., "Model Investigations of Cable Stayed Bridges - Influence Lines" Report No. 2, Sir George Williams University, Faculty of Engineering, February, 1970.

27. Troitsky, M.S. and Lazar, B. "Model Investigations of Cable Stayed Bridges - Structural Analysis of the Prototype", Report No. 3, Sir George Williams University, Faculty of Engineering, April , 1970.

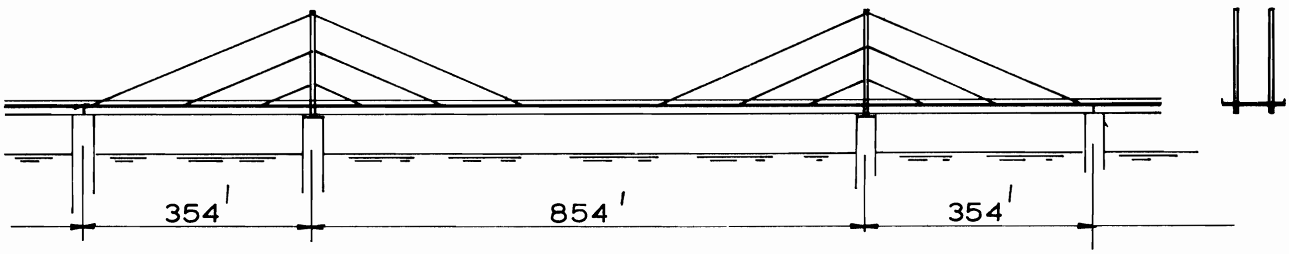
28. Troitsky, M.S. and Lazar, B., "Model Investigations of Cable Stayed Bridges - Nonlinear Behaviour", Report No. 4, Sir George Williams University, Faculty of Engineering, August 1970.

29. Troitsky, M.S. and Lazar, B., "Model Investigations of Cable Stayed Bridges - Post-tensioning of Cables", Report No. 5, Sir George Williams University,Faculty of Engineering, September 1970.

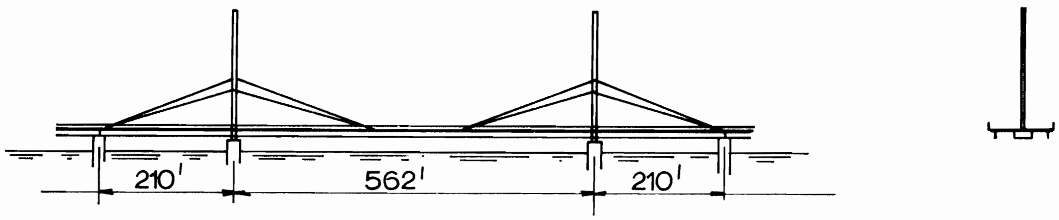
APPENDIX 1

CABLE STAYED BRIDGES

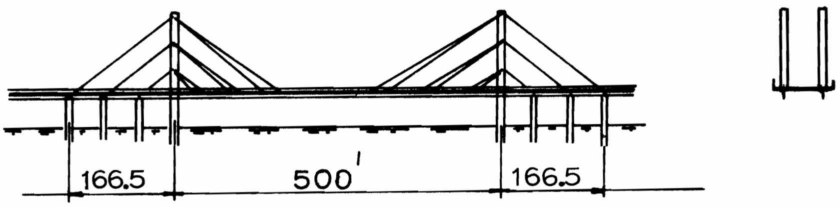
Structural Systems



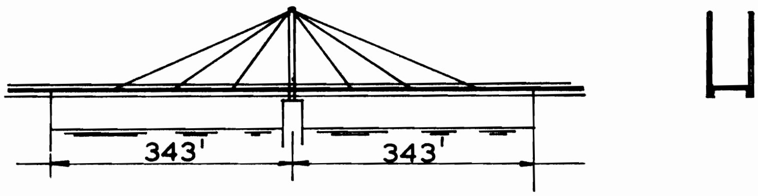
North Bridge at Dusseldorf, Germany, 1958



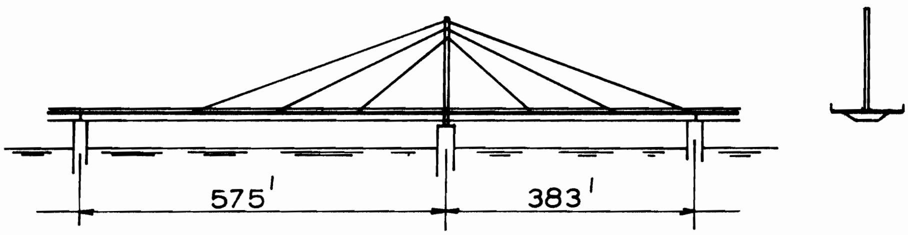
Elbe Bridge at Hamburg, Germany, 1962



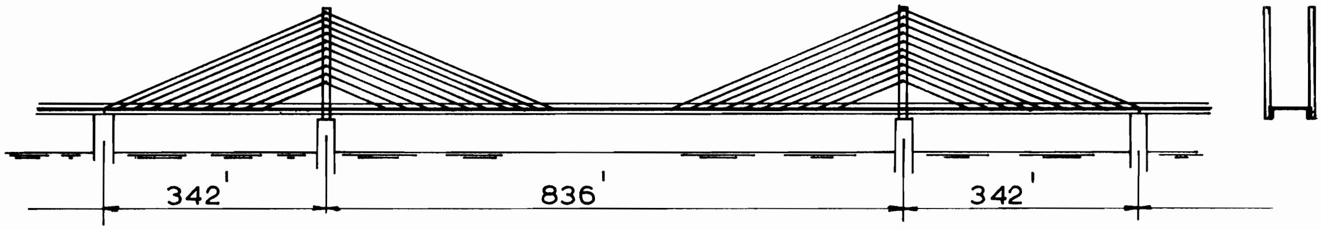
George Street Bridge, Newport, England, 1964



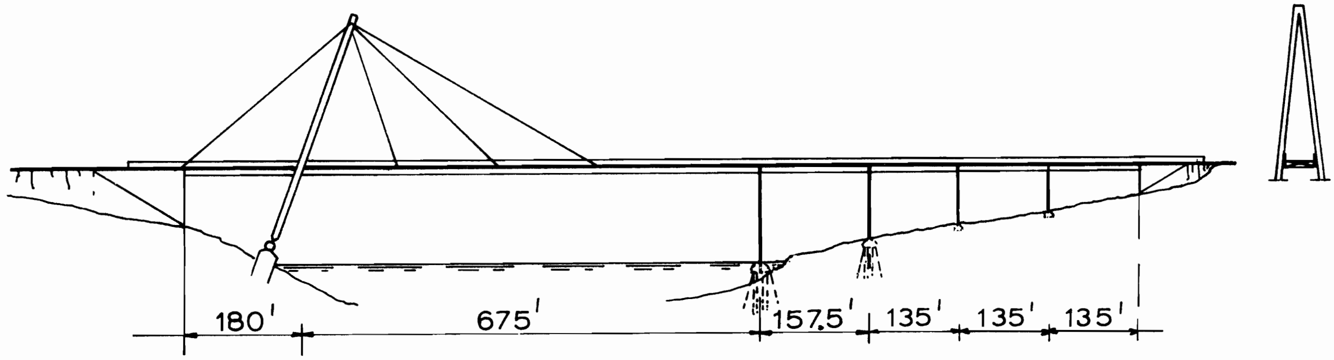
St-Florent-Le-Vieil Bridge, France, 1965



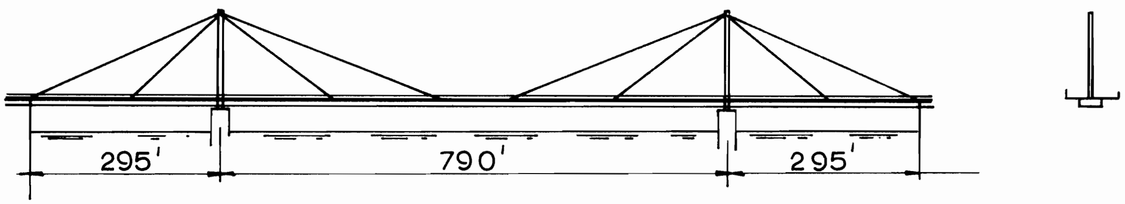
Rhine Bridge at Maxau, Germany, 1966



Rhine Bridge at Rees, Germany, 1967



Batman Bridge, Australia, 1968



Papineau Bridge, Montreal, Canada, 1969

APPENDIX 2

DETERMINATION OF SECTIONAL PROPERTIES OF A BRIDGE SYSTEM BY APPLYING THE THEORY OF SIMILITUDE

Notations

f	Unit bending stress at extreme fibers
M	Maximum bending moment in main girder
S	Section modulus of main girder
A	Area of main girder cross-section
L	Length of main span
k_1	A dimensionless constant depending on locations of loads and the type and locations of supports
p^d	Uniformly distributed dead load $p^d = p^s + p^t$
p^s	Uniformly distributed dead load given by the structural elements of the bridge system
p^t	Uniformly distributed dead load of surfacing, railing, etc. (non-structural elements of the bridge system)
p^l	Uniformly distributed live load
p	Uniformly distributed total load $(p = p^d + p^l)$
d	Density of main girder material
k_A	Scale reduction factor for area of cross-section
k_s	Scale reduction factor for section modulus
k_w	Scale reduction factor for load p
k_L	Scale reduction factor for length of span
k_h	Scale reduction factor for height of cross-section
k_I	Scale reduction factor for moment of inertia of cross-section
k_p^s	Scale reduction factor for load p^s

The procedure which follows represents a method of calculating the initial sectional properties for the design of a bridge system with a given geometry, if the sectional properties of a geometrically similar bridge system are known. The method is based on principles of similitude and is restricted to the case when both systems are geometrically similar and of the same material. It may be extended to systems made of different materials.

Basic Assumptions

$$
\begin{aligned}
k_s &= k_A k_L \\
k_I &= k_s k_L \\
p_n^t &= p_e^t \\
f_n &= f_e
\end{aligned}
\tag{1}
$$

The scale reduction factors "k" of Eq. 1 are defined at the beginning of this Appendix, p^t is the uniformly distributed dead load caused by the non-structural elements of the bridge system and f is the unit bending stress at extreme fibers. Subscript "e" applies to the parameters of the bridge system with known sectional properties and subscript "n" to the parameters of the bridge system with sectional properties to be determined.

From $f_n = f_e$

$$\frac{M_n}{S_n} = \frac{M_e}{S_e} \qquad (2)$$

In Eq. 2, M is the maximum bending moment in the stiffening girder and S is the corresponding section modulus.

Substituting $M = k_1 pL^2$ in Eq. (2)

$$k_1 \frac{P_n L_n^2}{S_n} = \frac{P_e L_e^2}{S_e} \qquad (3)$$

In Eq. 3, k_1 is a dimensionless constant depending on the location of loads and the type and location of supports and L is the length of the main span.

Equation (3) may be rearranged to

$$k_S = k_p k_L^2 = \frac{p_n^s + p_n^t + p_n^1}{p_e^s + p_e^t + p_e^1} k_L^2 \qquad (4)$$

In Eq. 4, p^s is the uniformly distributed dead load contributed by the structural elements of the bridge system and p^1 is the uniformly distributed live load.

From $p_n^s = dA_n$ and $p_e^s = dA_e$ where d is the density of the stiffening girder material and A is the area of the girder cross-section, corresponding to M,

$$\frac{p_n^s}{p_e^s} = k_w^s = \frac{A_n}{A_e} = k_A = \frac{k_s}{k_l} \qquad (5)$$

and Eq. (4) may be written as

$$k_s = \frac{\frac{k_s}{k_L} p_e^s + p_e^t + p_n^l}{p_e} k_L^2 \qquad (6)$$

Solving Eq. (6) with respect to k_s gives

$$k_s = \frac{p_e^t + p_n^l}{p_e - p_e^s k_L} k_L^2 \qquad (7)$$

The information on the right side of Equation (7) is known. Hence k_s may be calculated. The next step is to obtain k_I and k_A from Equation (1).

Example

Given

$$p_e = 8500 \text{ lb/ft}$$

$$p_e^s = 4200 \text{ lb/ft}$$

$$p_e^t = 1300 \text{ lb/ft}$$

$$p_n^l = 2000 \text{ lb/ft}$$

$$k_L = 1.25$$

k_s is first calculated from (7)

$$k_s = \frac{1300 + 2000}{8500 - 4200 \times 1.25} \times 1.25^2 = 1.585$$

Next k_I and k_A are computed.

$$k_I = 1.585 \times 1.25 = 1.98$$

$$k_A = \frac{1.585}{1.25} = 1.27$$

With k_I and k_A the I and A of the bridge to be designed are determined from the I and A of the geometrically similar bridge with known sectional properties.

APPENDIX 3

DETERMINATION OF JOINT DISPLACEMENTS WITH DIAL GAGES

A procedure was developed to determine u, v, θ, the vertical, horizontal and rotational displacement at joints by employing dial gages only. The joint considered in this application is a girder-cable joint. The procedure may be extended, however to any type of joint whose displacements are in a vertical plane and may be defined by three parameters.

In order to amplify the readings and thus reduce errors, vertical square bars were cemented to the top and bottom joint bars, Fig. A3.1.

On Fig. A3.1, a,b and c are the readings obtained. To determine the unknown displacements, first θ must be written as a function of a and b. Next, two equations are formed expressing the relashionship between u, v, a, b and c. With three conditions, the three unknowns u, v, θ may be determined.

If in Fig. A3.1 positive dial gage readings are taken to the left and negative readings to the right,

$$\tan \theta = \frac{a - b}{2D} \qquad (1)$$

To obtain the two relations between a, b, c and u, v, the procedure is the following, Fig. A3.1.

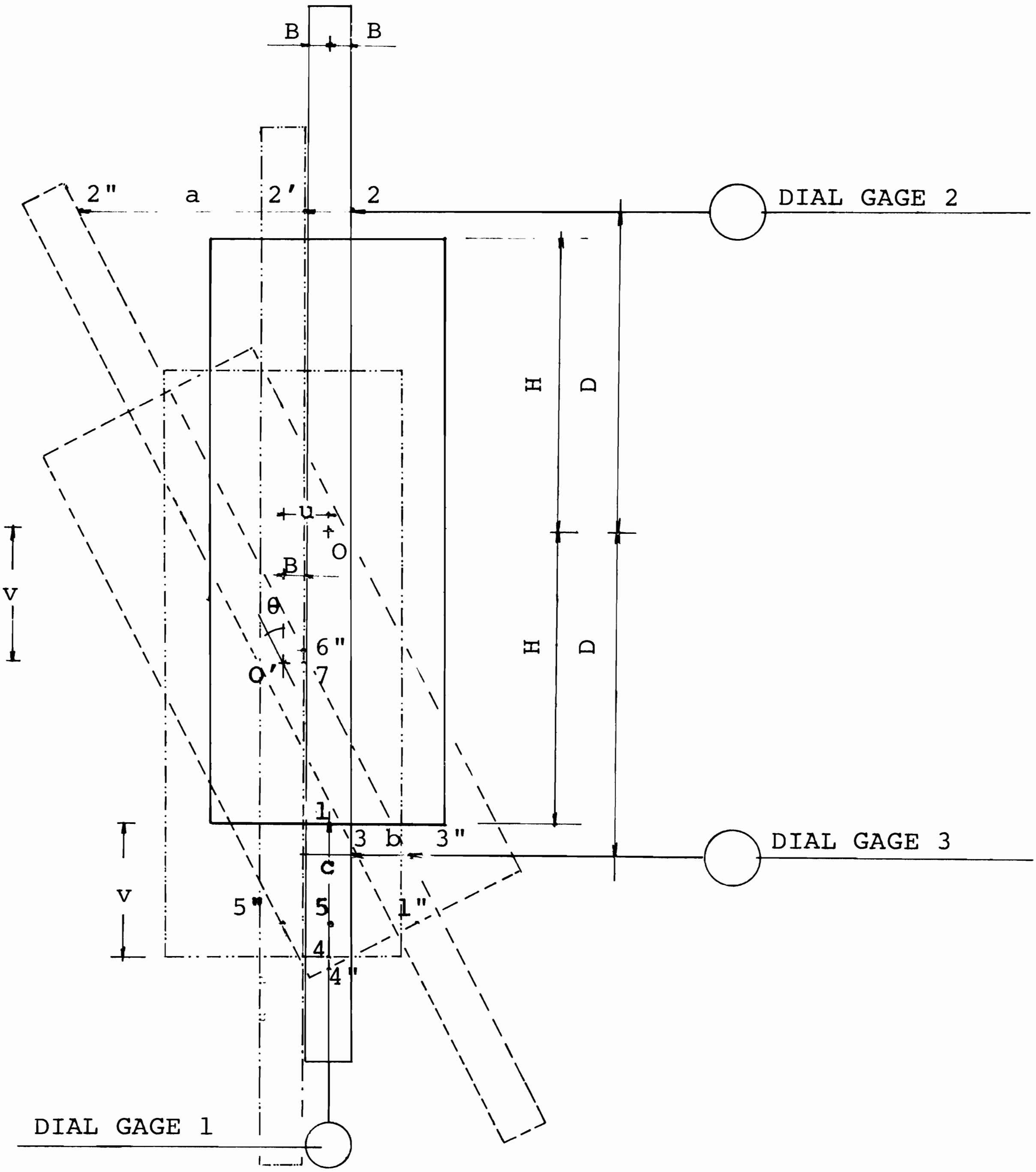


FIG. A3.1 DETERMINATION OF JOINT DISPLACEMENTS u, v, θ

WITH DIAL GAGES

For the first relation

$$L_{5,4} = H - H\cos\theta = H(1 - \cos\theta)$$

$$L_{5'',1''} = H\sin\theta$$

$$L_{5,5''} = u$$

$$L_{5,1''} = H\sin\theta - u$$

$$L_{5,4''} = \tan\theta\ (H\sin\theta - u)$$

but

$$v - L_{5,4} = c - L_{5,4''}$$

or

$$v - H(1 - \cos\theta) = c - (H\sin\theta - u)\tan\theta$$

This may be written as

$$u\tan\theta - v = H(\sin\theta\tan\theta + \cos\theta - 1) - c = P_1 \qquad (2)$$

In the above equations, $L_{i,j}$ indicates the distance between points i and j.

For the second relation

$$L_{0',7} = B\cos\theta$$

$$L_{6'',7} = B\sin\theta$$

$$L_{6'',2'} = D + v - L_{6'',7} = D + v - B\sin\theta$$

$$L_{2',2''} = (D + v - B\sin\theta)\tan\theta$$

but

$$u + B - L_{0',7} = a - L_{2',2''}$$

or

$$u + B(1 - \cos\theta) = a - (D + v - B\sin\theta)\tan\theta$$

This may be written as

$$u + v\tan\theta = B(\sin\theta\tan\theta + \cos\theta - 1) - D\tan\theta + {} + a = P_2 \qquad (3)$$

With the above relations the following calculations should be carried out:

1. Determine $\tan\theta$ from (1)
2. Determine P_1 and P_2 from (2) and (3)
3. Determine u and v from

$$\begin{aligned} u &= (P_1\sin\theta + P_2\cos\theta)\cos\theta \\ v &= (P_1\sin\theta + P_2\cos\theta)\sin\theta - P_1 \end{aligned} \qquad (4)$$

The above equations for u and v were obtained by solving (2) and (3).

Equations (4) have been verified graphically and the results obtained confirm the analytical procedure described.

APPENDIX 4

FIVE DIAGONAL MATRICES

The solution of banded systems of equations may be handled by matrix inversion (inversion by the adjoint matrix, pivotal elimination, decomposition, partitioning), by relaxation or other methods. If the band is made up of no more than three terms per row, a very rapid solution of the system of equations is the well known three diagonal algorithm

A similar algorithm has been developed for the more general case of banded matrices having five terms per row. This is for example the case of a system of equations of a continuous beam on elastic supports.

Consider the matrix equation

$$[A] * \{X\} = \{P\}$$

where [A] is a five diagonal matrix having the form

$$[A] = \begin{bmatrix} c_1 & d_1 & e_1 & 0 & 0 & 0 & 0 \\ b_2 & c_2 & d_2 & e_2 & 0 & 0 & 0 \\ a_3 & b_3 & c_3 & d_3 & e_3 & 0 & 0 \\ 0 & a_4 & b_4 & c_4 & d_4 & e_4 & 0 \\ \hline 0 & 0 & a_{n-2} & b_{n-2} & c_{n-2} & d_{n-2} & e_{n-2} \\ 0 & 0 & 0 & a_{n-1} & b_{n-1} & c_{n-1} & d_{n-1} \\ 0 & 0 & 0 & 0 & a_n & b_n & c_n \end{bmatrix} \qquad (1)$$

and $\{X\}$, $\{P\}$ are column vectors having each a number of n terms.

$$\{X\} = \{x_1, x_2, x_3 \ldots\ldots\ldots x_n\} \tag{2}$$

$$\{P\} = \{p_1, p_2, p_3, \ldots\ldots p_n\} \tag{3}$$

If p_k is the general term of $\{P\}$, then

$$p_k = a_k x_{k-2} + b_k x_{k-1} + c_k x_k + d_k x_{k+1} + e_k x_{k+2} \tag{4}$$

The algorithm developed for x is of the form

$$x_k = A_k x_{k+1} + B_k x_{k+2} + C_k \tag{5}$$

where A_k, B_k and C_k are determined by employing condition (4).

Equation (5) is valid for any value of k. Hence

$$x_{k-1} = A_{k-1} x_k + B_{k-1} x_{k+1} + C_{k-1} \tag{6}$$

and

$$x_{k-2} = A_{k-2} x_{k-1} + B_{k-2} x_k + C_{k-2} \tag{6a}$$

or employing Eq. (6).

$$x_{k-2} = A_{k-2} A_{k-1} x_k + A_{k-2} B_{k-1} x_{k+1} + A_{k-2} C_{k-1} + B_{k-2} x_k + C_{k-2} \tag{6b}$$

rearranging Eq. (6b).

$$x_{k-2} = (A_{k-2} A_{k-1} + B_{k-2}) x_k + A_{k-2} B_{k-1} x_{k+1} + A_{k-2} C_{k-1} + C_{k-2} \tag{7}$$

Substituting Eqs. (6) and (7) in Eq. (4) and rearranging the result in a form similar to Eq. (5), A_k, B_k and C_k are obtained.

$$A_k = - \frac{a_k B_{k-1} A_{k-2} + b_k B_{k-1} + d_k}{a_k (A_{k-2} A_{k-1} + B_{k-2}) + b_k A_{k-1} + c_k}$$

$$B_k = - \frac{e_k}{a_k (A_{k-2} A_{k-1} + B_{k-2}) + b_k A_{k-1} + c_k} \tag{8}$$

$$C_k = + \frac{p_k - a_k (A_{k-2} C_{k-1} + C_{k-2}) - b_k C_{k-1}}{a_k (A_{k-2} A_{k-1} + B_{k-2}) + b_k A_{k-1} + c_k}$$

for $k = 3.....n$

For $k = 1$ and $k = 2$

$$c_1 x_1 + d_1 x_2 + e_1 x_3 = p_1 \tag{9}$$

$$b_2 x_1 + c_2 x_2 + d_2 x_3 + e_2 x_4 = p_2$$

From Equations (9) and (5) A_k, B_k, C_k for $k = 1$ and $k = 2$ are obtained

$$A_1 = - \frac{d_1}{c_1} \qquad B_1 = - \frac{e_1}{c_1} \qquad C_1 = \frac{p_1}{c_1} \tag{10}$$

$$A_2 = \frac{b_2 e_1 - d_2 c_1}{c_1 c_2 - b_2 d_1}$$

$$B_2 = - \frac{c_1 e_2}{c_1 c_2 - b_2 d_1} \tag{11}$$

$$C_2 = \frac{c_1 p_2 - p_1 b_2}{c_1 c_2 - b_2 d_1}$$

To solve a system of equations, first A_{k-1}, B_{k-1}

C_k for $k = 1..n$ are determined. Then $\{X\}$ is obtained by backsubstitution. For $k = (n-1)$ and $k = n$, $e_k = e_{n-1} = e_n = 0$. Hence, $B_{n-1} = B_n = 0$. For $k = n$, $d_n = 0$, hence $A_n = 0$.

Consequently

$$x_n = C_n \tag{12}$$

Next from Equation (5), x may be obtained for all values of n

$$\begin{aligned} x_{n-1} &= A_{n-1}\, x_n + C_{n-1} \\ x_{n-2} &= A_{n-2}\, x_{n-1} + B_{n-2}\, x_n + C_{n-2} \end{aligned} \tag{13}$$

The above method is applicable for both classical and computer calculations. An example of an application of the above algorithm for solving a system of 6 equations in 6 unknowns by hand calculation is given at the end of this Appendix.

A program has also been written for the CDC 3300 digital computer. For convenience, the input data has been arranged vertically rather than on the diagonal. For an example chosen (n = 10), the above procedure needed substantially less computer time as compared with a solution of the system of equations by a Gauss Jordan elimination. One other advantage of this algorithm is the reduction of computer core required. The program is given at the end.

Example of solving a 6 x 6 system of equations by hand calculations

Solve the following system of equations by applying the five diagonal alogrithm:

$$
\begin{array}{rcrcrcrcrcrcr}
x_1 & + & 2x_2 & + & x_3 & & & & & & & = & -4 \\
x_1 & + & x_2 & + & 2x_3 & + & x_4 & & & & & = & -1 \\
x_1 & + & 2x_2 & + & 3x_3 & + & x_4 & + & 2x_5 & & & = & -2 \\
 & & x_2 & - & x_3 & + & 2x_4 & + & x_5 & - & x_6 & = & 2 \\
 & & & & x_3 & + & 2x_4 & + & x_5 & + & 2x_6 & = & 8 \\
 & & & & & & 2x_4 & + & 3x_5 & + & x_6 & = & 9
\end{array}
$$

The coefficients of $x_1 \ldots x_6$ represent the matrix [A] and the right hand side terms represent the vector {P}.

First, $A_1 \ldots A_6$, $B_1 \ldots B_6$, and $C_1 \ldots C_6$ are calculated from Eqs. (10), (11), and (8).

	1	2	3	4	5	6
A	-2	1	-1/2	-1/3	-5/3	0
B	-1	1	1	1/3	0	0
C	-4	-3	-1	5/3	13/3	2

Now applying Eqs. (12) and (13), {x} is obtained

{x} = {1, -2, -1, 2, 1, 2}

COMPUTER PROGRAM FOR THE SOLUTION OF A FIVE DIAGONAL SYSTEM OF EQUATIONS, APPLYING THE FIVE DIAGONAL ALGORITHM.

```
CXX    THE PROGRAM WILL READ A 5 DIAGONAL MATRIX WITH N ROWS,
CXX    WHERE ONLY THE 5 NON-ZERO VALUES OF EACH ROW ARE READ, AS WELL AS
CXX    P, WHERE P IS THE COLUMN VECTOR RESULTING FROM THE MULTIPLICATION
CXX    OF A BY THE COLUMN VECTOR X, THE LATTER BEING CALCULATED BY THE
CXX    PROGRAM USING A 5 DIAGONAL ALGORITHM.
C***INPUT***   NO. OF ROWS IN A IN COLS. 1-10 OF CARD 1, RIGHT JUSTIFIED.
C***INPUT***   ANY OPTIONAL TITLE IN COLS. 11-76.
C***INPUT***   READ A(K), B(K), C(K), D(K), E(K), WHERE K IS THE ROW,
C***INPUT***   AND THE 5 VALUES ARE THE RESPECTIVE ELEMENTS IN THE
C***INPUT***   5 DIAGONALS, IN FORMAT 5F10. WHERE NON-EXISTANT
C***INPUT***   LEAVE BLANK. A BLANK CARD FOLLOWS, THEN THE ELEMENTS OF P
C***INPUT***   IN FORMAT 7F10.
       COMMON ARRAY1(5,100),ARRAY2(5,105)
       DIMENSION ITITLE(16),A1(5,100),A2(5,105)
       EQUIVALENCE(ITITLE,ARRAY2),(A1,ARRAY1),(A2,ARRAY2)
CA.1   READ INTO N THE NO. OF ROWS, ALSO TITLE.
       READ 1010, N, ITITLE
1010   FORMAT(I10,4X16A4)
CA.2   PRINT TITLE. CHECK IF N WITHIN LIMITS.
       PRINT 1011, ITITLE,N
1011   FORMAT(20X16A4/20XI10,1X6HROWS, 12H5 DIAGONALS )
       IF(N .LT. 5 .OR. N .GT. 100) GO TO    10
       GO TO 11
10     PRINT 1012
1012   FORMAT(20X30HNO. OF ROWS GT 100 OR LT 5     )
       STOP
CB.1   READ A.
11     DO 20 I=1,N
20     READ 1020, (ARRAY1(J,I),J=1,5)
1020   FORMAT(5F10.0)
CC.1   READ BLANK CARD.
       READ 1030,(ITITLE(I),I=1,3)
1030   FORMAT(3A4)
       IF(ITITLE(1) .NE. 4H        .OR. ITITLE(2).NE. 4H        .OR.
      1ITITLE(3) .NE. 4H        )GO TO 30
       GO TO 31
30     PRINT 1031
1031   FORMAT(20X42HBLANK CARD MISSING OR NO OF ROWS INCORRECT)
       STOP
CD.1   READ THE N VALUES OF P IN FORMAT 7F10. N2 IS NO. OF CARDS.
31     DO 40 I=1,N ,7
       N3=I +6
40     READ 1040, (ARRAY2(1,J),J=I ,N3)
1040   FORMAT(7F10.0)
CE.1   GET ALPHA, BETA, GAMMA FOR K=1 AND 2.
       ARRAY2(2,1)=-ARRAY1(4,1)/ARRAY1(3,1)
       ARRAY2(3,1)=-ARRAY1(5,1)/ARRAY1(3,1)
       ARRAY2(4,1)=ARRAY2(1,1)/ARRAY1(3,1)
       A2(2,2)=(A1(2,2)*A1(5,1)-A1(4,2)*A1(3,1))/(A1(3,1)*A1(3,2)-
      1A1(2,2)*A1(4,1))
```

```
      A2(3,2)=A1(3,1)*A1(5,2)/(A1(2,2)*A1(4,1)-A1(3,1)*A1(3,2))
      A2(4,2)=(A1(3,1)*A2(1,2)-A2(1,1)*A1(2,2))/(A1(3,1)*A1(3,2)-
     1A1(2,2)*A1(4,1))
CF.1  GET ALPHA, BETA AND GAMMA FOR THE REST OF K.
      DO 60 I=3,N
      TEMP=
     1(A1(1,I)*(A2(2,I-2)*A2(2,I-1)+A2(3,I-2))+A1(2,I)*A2(2,I-1)+A1(3,I
     1))
      A2(2,I)=-(A1(1,I)*A2(3,I-1)*A2(2,I-2)+A1(2,I)*A2(3,I-1)+A1(4,I))/
     1TEMP
      A2(3,I)=-A1(5,I)/TEMP
60    A2(4,I)=(A2(1,I)-A1(1,I)*(A2(2,I-2)*A2(4,I-1)+A2(4,I-2))-A1(2,I)*
     1A2(4,I-1))/TEMP
CG.1  GET X(N), X(N-1)
      A2(5,N)=A2(4,N)
      A2(5,N-1)=A2(2,N-1)*A2(5,N)+A2(4,N-1)
CH.1  GET X(K)
      N3=N-1
      DO 80 I=2,N3
      N2=N-I
80    A2(5,N2)=A2(2,N2)*A2(5,N2+1)+A2(3,N2)*A2(5,N2+2)+A2(4,N2)
CI.1  PRINT REST OF HEADING AND RESULTS.
      PRINT 1090
1090  FORMAT(/34X36HMATRIX EQUATION AX=P, GIVEN A AND P //38X1HA48X1HX
     122X1HP//)
      PRINT 1092,(A1(J,1),J=3,5),A2(5,1),A2(1,1)
1092  FORMAT(27X3F13.2,10XF13.4,10XF13.2)
      PRINT 1093, (A1(J,2),J=2,5),A2(5,2),A2(1,2)
1093  FORMAT(14X4F13.2,10XF13.4,10XF13.2)
      N=N-2
      DO 91 I=3,N
91    PRINT 1091,(A1(J,I),J=1,5), A2(5,I), A2(1,I)
1091  FORMAT(1 X5F13.2,10XF13.4,10XF13.2)
      PRINT 1094, (A1(J,N+1),J=1,4),A2(5,N+1),A2(1,N+1)
1094  FORMAT(1X4F13.2,23XF13.4,10XF13.2)
      PRINT 1095,(A1(J,N+2),J=1,3),A2(5,N+2),A2(1,N+2)
1095  FORMAT(1X3F13.2,36XF13.4,10XF13.2)
      STOP
      END
```

APPENDIX 5

COMPUTER PROGRAM FOR DETERMINATION OF INFLUENCE LINES OF A CABLE STAYED BRIDGE

```
      PROGRAM CSBRIDGE
      DIMENSION X(20),Y(20),IZJ(31),IZK(31),AXJ(31),AXK(31),E(31),MJ(31)
     1,MK(31),L(31),SA(12),CA(12),TA(12),CTA(12),CT(8),V(4),R(31),I(31)
      DIMENSION FM(31,5),AMQ(93,14),AMQT(14,93),FAM(14,93),F(14,14),A(31
     1),LA(14),MA(14)
      DIMENSION VUL(26,4),AMUL(93,26),VUM(26,4),VUMT(4,26),AMUM(93,26)
      DIMENSION LEN(31),AMJ(93,23),AMJT(23,93)
      DIMENSION VULT(4,26),ARQ(4,14)
      DIMENSION DUMMY(218)
      EQUIVALENCE (DUMMY(1),MJ(1)),(DUMMY(32),MK(1)),(DUMMY(63),IZJ(1)),
     1(DUMMY(94),IZK(1)),(DUMMY(125),AXJ(1)),(DUMMY(156),AXK(1)),(DUMMY(
     2187),E(1))
      EQUIVALENCE (VUM,VULT)
      EQUIVALENCE (AMUM,AMQ),(AMUL,AMQT,AMJT),(AMUM,AMJ)
      COMMON   AMUM,AMUL
      REAL L,LEN,IZJ,IZK,I,LSC,LSE,LS,LT
C     NC IS NO OF CABLES
      NC=12
      NC1=NC+1
      LOAD=68
C     N=NO OF JOINTS, M=NO OF MEMBERS
      READ 1,N,M
    1 FORMAT(2I3)
      MT=3*M
      PU=1000./12.
      DO 2 J=1,N
    2 READ 3,X(J),Y(J)
    3 FORMAT(2F10.4)
      DO 9 K=1,M
      READ 5,IZJ(K),IZK(K),AXJ(K),AXK(K),E(K),MJ(K),MK(K)
    5 FORMAT(2(E9.4,1X),2F10.4,E9.4,1X,2I2)
    9 E(K)=E(K)/(10.**7)
C
C     FINDING LENGTH OF MEMBERS AND PRINT
      BUFFER OUT(7,1)(DUMMY(1),DUMMY(218))
      DO 8 N=1,M
      J=MJ(N)
      K=MK(N)
      XCO=X(J)-X(K)
      YCO=Y(J)-Y(K)
      L(N)=SQRT(XCO**2+YCO**2)
    8 L(N)=12.*L(N)
C     THIS PART OF THE PROGRAM WILL CALCULATE ANGULAR FUNCTIONS AND LSE
C
      I7=IFUNIT(7)+2
      GO TO(69,7,69,69,69),I7
    7 SA(1)=(L(19)+L(20)+L(21))/L(1)
      SA(2)=(L(19)+L(20))/L(2)
      SA(3)=L(19)/L(3)
      SA(4)=L(19)/L(4)
      SA(5)=(L(19)+L(20))/L(5)
      SA(6)=(L(19)+L(20)+L(21))/L(6)
      SA(7)=(L(28)+L(29)+L(30))/L(7)
      SA(8)=(L(28)+L(29))/L(8)
      SA(9)=L(28)/L(9)
      SA(10)=L(28)/L(10)
      SA(11)=(L(28)+L(29))/L(11)
      SA(12)=(L(28)+L(29)+L(30))/L(12)
      DO 100 K=1,12
```

```
      CA(K)=SQRT(1.-SA(K)**2)
      TA(K)=SA(K)/CA(K)
  100 CTA(K)=1./TA(K)
      CT21=CTA(2)-CTA(1)
      CT32=CTA(3)-CTA(2)
      CT45=CTA(4)-CTA(5)
      CT56=CTA(5)-CTA(6)
      CT87=CTA(8)-CTA(7)
      CT98=CTA(9)-CTA(8)
      CT1011=CTA(10)-CTA(11)
      CT1112=CTA(11)-CTA(12)
      DO 101 K=1,M
  101 R(K)=1./L(K)
C     PART 3
C
      LSE=L(13)+L(14)+L(15)
      LSC=L(16)+L(17)+L(18)
      LS=LSE+LSC
      LT=L(19)+L(20)+L(21)
      V(1)=(R(31)*LSC+1.)/LSE
      V(2)=R(31)+V(1)
      V(3)=R(31)*(LS/LSE)
      V(4)=R(31)*(LSC/LSE)
C
      BUFFER OUT(19,1)(L(1),L(31))
      I1=IFUNIT(19)+2
C     THE FOLLOWING STATEMENTS ASSUME A UNIFORM BEAM=3
      DO 499 J=1,31
      A(J)=AXJ(J)
  499 I(J)=IZJ(J)
C
C
C     CALCULATION OF FM MAT USING LEAST AMOUNT OF MEMORY POSSUBLE
C
      GO TO(69,1050,69,69,69),I1
 1050 DO 340 J=1,NC
      FM(J,1)=L(J)/(E(J)*A(J))
      DO 340 K=2,5
  340 FM(J,K)=0.
      DO 20 J=NC1,M
      FM(J,1)= L(J)/(E(J)*A(J))
      FM(J,2)= L(J)**3/(3.*E(J)*I(J))
      FM(J,3)= L(J)**2/(2.*E(J)*I(J))
      FM(J,4)=FM(J,3)
   20 FM(J,5)= L(J)/(E(J)* I(J))
      BUFFER OUT (5,1)(FM(1,1),FM(31,5))
      I5=IFUNIT(5)+2
      GO TO (69,327,69,69,69),I5
C      MATRICES AMQ,AML,ETC.
C     CABLE STAYED BRIDGE-UNIFORM AND POINTS LOADS
C     X AND Y DIMENSION IS NO OF JOINTS,REMAINING-DIMENSION IS NO OF MEMBERS
      DIMENSION MK(31)      ,IZJ(31),IZK(31),AXJ(31),AXK(31),E(31),MJ(31)
      DIMENSION X(70),L(31),LEN(31),LA(14),MA(14),AML2(6324)
      DIMENSION MJK(2,68),PJK(2,68),JK1(2,68),AML3(93)
      DIMENSION MAMA(93,68),AMAA(31,68),AMAS(31,68),AMAM(31,68)
      DIMENSION AMP(31),AMN(31),SMP(31),SMN(31),BMP(31),BMN(31)
      DIMENSION FM(31,5),AMQ(93,14),AMQT(14,93),FAM(14,93),F(14,14)
      DIMENSION VUL(26,4),AMUL(93,26),VUM(26,4),VUMT(4,26),AMUM(93,26)
      DIMENSION VULT(4,26),AML(93,70),DQL(14,68),Q(14,68),AMMQ(93,68)
```

```
      DIMENSION AM(93,68),AMR(93,68),AMA(93,68),ARL(4,68),ARQ(4,14)
      DIMENSION ARLL(4,68),AR(4,68),X1(3162),Y1(3162)
      DIMENSION AML1(1632),FAM2(14,93),DQL2(14,68),Q2(14,68)
      DIMENSION DUMMY(218)
      DIMENSION AMQ2(93,14),E1(70),P(70),MO(70),S(70),AMJT(23,93)
      DIMENSION DJ1(23,34),DJ(23,68),AMX(23,93),II1(68)
      EQUIVALENCE (AML(1),AMMQ(1),AM(1),AMR(1),AMA(1)),(AML(1),AMUL(1))
      EQUIVALENCE (AML(1),MAMA(1),AML2(1))
      EQUIVALENCE (AML(1),AMAM(1)),(AML(2109),BMP(1)),(AML(2140),BMN(1))
      EQUIVALENCE (AML(1),AMAA(1)),(AML(2109),AMP(1)),(AML(2140),AMN(1))
      EQUIVALENCE (AML(1),AMAS(1)),(AML(2109),SMP(1)),(AML(2140),SMN(1))
      EQUIVALENCE (AML(2419),AMUM(1)),(AML(4837),AML1(1)),(AML(1),FAM(1)
     1)
      EQUIVALENCE (VUM(1),VULT(1)),(AR(1),ARLL(1),AML3(1))
      EQUIVALENCE (AML(1303),AMQT(1)),(AML(1303),DQL(1))
      EQUIVALENCE (AML(4465),DQL2(1)),(AML(3163),FAM2(1)),(AML(3163),AMQ
     1(1)),(AML(1),AMJT(1),DJ(1)),(AML(3163),AMX(1)),(AML(5302),DJ1(1))
      EQUIVALENCE (AML(4465),Q(1)),(AML(1),AMQ2(1)),(AML(1303),Q2(1))
      EQUIVALENCE (VUM(1),VULT(1)),(AR(1),DUMMY(1))
      EQUIVALENCE (AML(1),X1(1)),(AML(3163),Y1(1)),(ARL(1),F(1))
      EQUIVALENCE (DUMMY(1),MJ(1)),(DUMMY(32),MK(1)),(DUMMY(63),IZJ(1)),
     1(DUMMY(94),IZK(1)),(DUMMY(125),AXJ(1)),(DUMMY(156),AXK(1)),(DUMMY(
     2187),E(1))
      COMMON/1/AML,FM,VUM,VUMT,ARL
      REAL L,LEN,MO,MAMA,IZJ,IZK
      REAL MJK
      REWIND 7
      BUFFER IN(7,1)(DUMMY(1),DUMMY(218))
      M=31
      MT=3*M
      LOAD=68
      NRE=4
      PU=1000./12.
      NR=14
      I7=IFUNIT(7)+2
      GO TO(69,5,69,69,69),I7
    5 PRINT 14
   14 FORMAT(1H1,55X,17HMEMBER PROPERTIES,///)
      PRINT 7
    7 FORMAT(9X,111HMEMBER          JOINT(J)        JOINT(K)   LENGTH(FT)
     1 IZJ            IZK            AXJ            AXK             E)
      BUFFER IN(19,1)(L(1),L(31))
      DO 621 I=1,67
  621 II1(I)=I
      I4=IFUNIT(19)+2
      GO TO (69,4,69,69,69),I4
    4 DO 9  K=1,M
    9 PRINT 6,K,MJ(K),MK(K),L(K),IZJ(K),IZK(K),AXJ(K),AXK(K),E(K)
    6 FORMAT(3(4X,I10),4X,F10.4,2(4X,E10.4),2(4X,F10.4),4X,E10.4)
      BUFFER IN(5,1)(FM(1,1),FM(31,5))
      I2=IFUNIT(5)+2
      GO TO (69,2,69,69,69),I2
    2 BUFFER IN(3,1)(F(1,1),F(14,14))
      I5=IFUNIT(3)+2
      GO TO (69,232,69,69,69),I5
  232 BUFFER IN(2,1)(VUMT(1,1) ,VUMT(4,26))
      I3=IFUNIT(2)+2
      GO TO (69,206,69,69,69),I3
  206 BUFFER IN(3,1)(AMUM(1,1),AMUM(93,26))
```

```
      I3=IFUNIT(3)+2
      GO TO (69,3,69,69,69),I3
    3 BUFFER IN(4,1)(VULT(1,1),VULT(4,26))
      I1=IFUNIT(4)+2
      GO TO (69,1,69,69,69),I1
    1 BUFFER IN(2,1)(AMUL(1,1),AMUL(93,26))
      I4=IFUNIT(2)+2
      GO TO (69,207,69,69,69),I4
  207 BUFFER IN(4,1)(ARQ(1,1),ARQ(4,14))
      I2=IFUNIT(4)+2
      GO TO (69,205,69,69,69),I2
  205 REWIND 2
      REWIND 3
      REWIND 4
      REWIND 5
C     FORMATION OF AML
C
      CALL EQUIA(L,E1,X,PJK,MJK,JK1)
C     FORMATION OF JK1 MATRIX IS DONE IN SUBROUTINE CAA
C     DEFINE JK1(1,1) AND JK1(2,1)
      JK1(1,1)=1
      JK1(2,1)=2
      JK1(1,68)=25
      JK1(2,68)=26
      DO 11 K1=1,52,17
      IBIG=0
      K2=K1+16
      DO 10 K3=K1,K2
      JJ=JK1(1,K3)
      KK=JK1(2,K3)
      DO 10 I1=1,MT
      IBIG=IBIG+1
   10 AML1(IBIG)=PJK(1,K3)*AMUL(I1,JJ)+PJK(2,K3)*AMUL(I1,KK)+MJK(1,K3)*A
     1MUM(I1,JJ)+MJK(2,K3)*AMUM(I1,KK)
      BUFFER OUT(3,1)(AML1(1),AML1(IBIG))
      I3=IFUNIT(3)+2
      GO TO (69,11,69,69,69),I3
   11 CONTINUE
      REWIND 3
      CALL UNIA(L,AMUL,AMUM,AML3,PU,S,P,MO)
      DO 12 I1=1,4744,1581
      I2=I1+1580
      BUFFER IN(3,1)(AML2(I1),AML2(I2))
      I3=IFUNIT(3)+2
      GO TO(69,12,69,69,69),I3
   12 CONTINUE
      DO 13 J=1,93
   13 AML2(J)=AML3(J)
      REWIND 3
      BUFFER OUT (5,1)(AML(1,1),AML(93,34))
      I4=IFUNIT(5)+2
      GO TO (69,718,69,69,69),I4
  718 BUFFER OUT(5,1)(AML(1,35),AML(93,68))
      I5=IFUNIT(5)+2
      GO TO (69,210,69,69,69),I5
  210 REWIND 5
C     MULTIPLICATION OF DQL=FAM*AML
      BUFFER IN(1,1)(FAM2(1,1),FAM2(14,93))
      I1=IFUNIT(1)+2
```

```
      GO TO (69,101,69,69,69),I1
  101 BUFFER IN (5,1)(AML(1,1),AML(93,34))
      I5=IFUNIT(5)+2
      GO TO (69,758,69,69,69),I5
  758 LOAD2=LOAD/2
      DO 759 IA=1,NR
      DO 759 KA=1,LOAD2
      DQL2(IA,KA)=0.
      DO 759 JA=1,MT
  759 DQL2(IA,KA)=DQL2(IA,KA)+FAM2(IA,JA)*AML(JA,KA)
      DO 760 IA=1,NR
      DO 760 KA=1,LOAD2
  760 DQL(IA,KA)=DQL2(IA,KA)
      DO 761 IA=1,NR
      DO 761 KA=1,MT
  761 FAM(IA,KA)=FAM2(IA,KA)
      BUFFER IN (5,1)(AML(1,35),AML(93,68))
      I4=IFUNIT(5)+2
      GO TO (69,766,69,69,69),I4
  766 REWIND 5
      LOAD3=LOAD2+1
      DO 762 IA=1,NR
      DO 762 KA=LOAD3,LOAD
      DQL(IA,KA)=0.
      DO 762 JA=1,MT
  762 DQL(IA,KA)=DQL(IA,KA)+FAM(IA,JA)*AML(JA,KA)
C
C     FORMATIIN OF Q
      DO 20 IA=1,NR
      DO 20 KA=1,LOAD
      Q(IA,KA)=0.
      DO 20JA=1,NR
   20 Q(IA,KA)=Q(IA,KA)+F(IA,JA)*DQL(JA,KA)
      BUFFER OUT(2,1)(Q(1,1),Q(14,68))
      I4=IFUNIT(2)+2
      GO TO(69,764,69,69,69),I4
  764 REWIND 2
C     AMMQ=AMQ*Q
      BUFFER IN(1,1)(AMQ(1,1),AMQ(93,14))
      I4=IFUNIT(1)+2
      GO TO(69,767,69,69,69),I4
  767 DO 765 IA=1,MT
      DO 765 KA=1,LOAD2
      AMMQ(IA,KA)=0.
      DO 765 JA=1,NR
  765 AMMQ(IA,KA)=AMMQ(IA,KA)+AMQ(IA,JA)*Q(JA,KA)
      BUFFER OUT(4,1)(AMMQ(1,1),AMMQ(93,34))
      I4=IFUNIT(4)+2
      GO TO(69,768,69,69,69),I4
  768 DO 791 KA=1,NR
      DO 791 IA=1,93
  791 AMQ2(IA,KA)=AMQ(IA,KA)
      DO 792 IA=1,NR
      DO 792 KA=1,LOAD
  792 Q2(IA,KA)=Q(IA,KA)
      DO 769 IA=1,MT
      DO 769 KA=LOAD3,LOAD
      AMMQ(IA,KA)=0.
      DO 769 JA=1,NR
```

```
  769 AMMQ(IA,KA)=AMMQ(IA,KA)+AMQ2(IA,JA)*Q2(JA,KA)
      BUFFER OUT(4,1)(AMMQ(1,LOAD3),AMMQ(MT,LOAD))
      I4=IFUNIT(4)+2
      GO TO(69,770,69,69,69),I4
  770 REWIND 4
C     AM=AML+AMMQ
C     AML IN 5  AMMQ IN 4  AM TO 4
      DO 774 J1=1,2
      BUFFER IN(4,1)(X1(1),X1(3162))
      BUFFER IN(5,1)(Y1(1),Y1(3162))
      I4=IFUNIT(4)+2
      GO TO (69,771,69,69,69),I4
  771 I5=IFUNIT(5)+2
      BACKSPACE 4
      GO TO (69,772,69,69,69),I5
  772 DO 773 J=1,3162
  773 X1(J)=X1(J)+Y1(J)
      BUFFER OUT(4,1)(X1(1),X1(3162))
      I4=IFUNIT(4)+2
      GO TO(69,774,69,69,69),I4
  774 CONTINUE
      REWIND 4
      REWIND 5
      BUFFER IN(4,1)(AM(1,1),AM(93,34))
      I4=IFUNIT(4)+2
      GO TO (69,216,69,69,69),I4
  216 BUFFER IN(4,1)(AM(1,35),AM(93,68))
      I5=IFUNIT(4)+2
      GO TO (69,217,69,69,69),I5
  217 REWIND 4
C
C     AMR IS CALCULATED BY WRITING A DO LOOP FOR EACH DISCONTINUITY
C     SUBROUTINES ARE CALLED TO 1.ASSIGN X AND LENGTH OF MEMBER TO
C                                 CORRESPONDING LOADS USING SUB.DISTANCE
C                               2.THE ELEMENTS OF THE MATRIX ARE THEN
C                                 CALCULATED USING SUBROUTINE ASSIGN
C     FOR X=0,450 LOADS 2,16
      DO 49 J=1,93
      DO 49 K=1,68
   49 AMR(J,K)=0.
      JX=12
      DO 200 LO=2,12,5
      JL=LO+3
      JX=JX+1
  200 CALL DISTANCE(JX,LO,JL,X,LEN,L,AMR,PU)
C
C     FOR X=1100,1500 LOADS 39,53
      JX=21
      DO 201 LO=39,49,5
      JL=LO+3
      JX=JX+1
  201 CALL DISTANCE(JX,LO,JL,X,LEN,L,AMR,PU)
C     FOR X=900,1100 LOADS 32,38
      JX=31
      LO=32
      JL=37
      CALL DISTANCE(JX,LO,JL,X,LEN,L,AMR,PU)
C
C     FOR X=450,900 LOADS 17,31
```

```
      JX=15
      DO 202 LO=17,27,5
      JL=LO+3
      JX=JX+1
  202 CALL DISTANC2(JX,LO,JL,X,LEN,L,AMR,PU)
C
C     FOR X=1550,2000
      JX=24
      DO 203 LO=54,64,5
      JL=LO+3
      JX=JX+1
  203 CALL DISTANC2(JX,LO,JL,X,LEN,L,AMR,PU)
      BUFFER OUT(5,1)(AMR(1,1),AMR(93,34))
      I5=IFUNIT(5)+2
      GO TO (69,220,69,69,69),I5
  220 BUFFER OUT(5,1)(AMR(1,35),AMR(1,68))
      I4=IFUNIT(5)+2
      GO TO(69,780,69,69,69),I4
C
C     AMA=AM+AMR
  780 REWIND 5
      DO 784 J1=1,2
      BUFFER IN(4,1)(X1(1),X1(3162))
      BUFFER IN(5,1)(Y1(1),Y1(3162))
      I4=IFUNIT(4)+2
      GO TO(69,781,69,69,69),I4
  781 I5=IFUNIT(5)+2
      GO TO(69,782,69,69,69),I5
  782 BACKSPACE 5
      DO 783 J=1,3162
  783 X1(J)=X1(J)+Y1(J)
      BUFFER OUT(5,1)(X1(1),X1(3162))
      I4=IFUNIT(5)+2
      GO TO(69,784,69,69,69),I4
  784 CONTINUE
      REWIND 4
      REWIND 5
      BUFFER IN(5,1)(AMA(1,1),AMA(93,34))
      I5=IFUNIT(5)+2
      GO TO (69,218,69,69,69),I5
  218 BUFFER IN(5,1)(AMA(1,35),AMA(93,68))
      I6=IFUNIT(5)+2
      GO TO (69,219,69,69,69),I6
  219 REWIND 5
      BUFFER OUT(16,1)(AMA(1,1),AMA(93,68))
      I1=IFUNIT(16)+2
      GO TO(69,1220,69,69,69),I1
 1220 PRINT 830
  830 FORMAT(72H1STRESS RESULTANTS FOR DEAD LOAD (CASE 1) AND MOVING LOA
     1DS(CASE 2 TO 67),//////)
      PRINT 813
  813 FORMAT(7H+MEMBER64X,10HLOAD CASES)
      DO 811 JJ=1,12
      J1=(JJ-1)*6+1
      J2=J1+5
      IF(JJ.NE.1)GO TO 693
      PRINT 694,(II1(I6),I6=J1,J2)
  694 FORMAT(6(/),24X,6(6X,8H    CASE ,I2,2X),///)
      GO TO 842
```

```
  693 IF(J1.NE.67)GO TO 815
      J2=67
      PRINT 840,(II1(I6),I6=J1,J2)
  840 FORMAT(6(/),1H1,24X, (6X,8H   CASE ,I2,2X),///)
      GO TO 842
  815 PRINT 812,(II1(I6),I6=J1,J2)
  812 FORMAT(6(/),1H1,24X,6(6X,8H   CASE ,I2,2X),///)
  842 DO 811 I=1,31
      I1=3*I-2
      I2=I1+1
      I3=I1+2
      PRINT 816,(AMA(I1,J),J=J1,J2)
  816 FORMAT(13X,12HNORMAL FORCE,1X,6(6X,E12.6))
      PRINT 817,I,(AMA(I2,J),J=J1,J2)
  817 FORMAT(3X,I2,8X,11HSHEAR FORCE,2X,6(6X,E12.6))
      PRINT 818,(AMA(I3,J),J=J1,J2)
  818 FORMAT(13X,11HBEND MOMENT,2X,6(6X,E12.6))
      PRINT 841
  841 FORMAT(/)
  811 CONTINUE
C     MJK AND PJK ARE GENERATED AGAIN
      CALL EQUIA(L,E1,X,PJK,MJK,JK1)
C     FORMATION OF JK1 MATRIX IS DONE IN SUBROUTINE CAA
C     DEFINE JK1(1,1) AND JK1(2,1)
      JK1(1,1)=1
      JK1(2,1)=2
      JK1(1,68)=25
      JK1(2,68)=26
      DO 441 J=1,68
      JJ=JK1(1,J)
      KK=JK1(2,J)
      DO 441 K=1,NRE
  441 ARL(K,J)=PJK(1,J)*VULT(K,JJ)+PJK(2,J)*VULT(K,KK)+MJK(1,J)*VUMT(K,J
     1J)+MJK(2,J)*VUMT(K,KK)
      K1=1
      CALL UNIV(L,VULT,VUMT,ARL,K1,NRE,PU,S,P,MO)
      BUFFER IN(2,1)(Q(1,1),Q(14,68))
      I4=IFUNIT(2)+2
      GO TO(69,785,69,69,69),I4
  785 CALL GMPRD(ARQ,Q,ARLL,NRE,NR,LOAD)
      BUFFER IN(1,1)(AMJT(1,1),AMJT(23,93))
      CALL GMADD(ARL,ARLL,AR,NRE,LOAD)
      BUFFER OUT(17,1)(AR(1,1),AR(4,67))
      I1=IFUNIT(17)+2
      GO TO (69,1221,69,69,69),I1
 1221 PRINT 631
  631 FORMAT(73H1VERTICAL REACTIONS FOR DEAD LOAD (CASE 1) AND MOVING LO
     1AD (CASE 2 TO 67),//////)
      PRINT 16,(II1(I),I=1,4)
   16 FORMAT(1X9HLOAD CASE,4(15X9HREACTION ,I2),///)
      DO 17 J=1,67
   17 PRINT 18,J,( AR(K,J),K=1,4)
   18 FORMAT(1X,5HLOAD ,I2,2X,4(15XE11.4),/)
      I6=IFUNIT(1)+2
      GO TO (69,211,69,69,69),I6
C     THE CALCULATION OF AMJT*FM=AMX FOLLOWS
  211 REWIND 1
      NJD=23
      JZ=MT-2
```

```
      DO 63 N=1,NJD
      DO 63 J=1,JZ,3
      JA=J+1
      JB=J+2
      K=JB/3
      AMX(N,J)=AMJT(N,J)*FM(K,1)
      AMX(N,JA)=AMJT(N,JA)*FM(K,2)+AMJT(N,JB)*FM(K,3)
   63 AMX(N,JB)=AMJT(N,JA)*FM(K,4)+AMJT(N,JB)*FM(K,5)
      BUFFER IN(4,1)(AM(1,1),AM(93,34))
      I4=IFUNIT(4)+2
      GO TO (69,223,69,69,69),I4
  223 CALL GMPRD(AMX,AM,DJ1,NJD,MT,LOAD2)
      BUFFER OUT(1,1)(DJ1(1,1),DJ1(23,34))
      BUFFER IN(4,1)(AM(1,1 ) ,AM(93,34))
      I1=IFUNIT(1)+2
      I4=IFUNIT(4)+2
      GO TO (69,224,69,69,69),I1
  224 GO TO (69,225,69,69,69),I4
  225 CALL GMPRD(AMX,AM,DJ1,NJD,MT,LOAD2)
      BUFFER OUT(1,1)(DJ1(1,1),DJ1(23,34))
      I2=IFUNIT(1)+2
      GO TO (69,226,69,69,69),I2
  226 REWIND 1
      BUFFER IN(1,1)(DJ(1,1),DJ(23,34))
      I1=IFUNIT(1)+2
      GO TO (69,227,69,69,69),I1
  227 BUFFER IN(1,1)(DJ(1,35),DJ(23,68))
      I1=IFUNIT(1)+2
      GO TO (69,228,69,69,69),I1
  228 BUFFER OUT(18,1)(DJ(1,1),DJ(23,68))
      I1=IFUNIT(18)+2
      GO TO (69,1222,69,69,69),I1
 1222 DO 212 J=1,23
      DO 212 K=1,68
  212   DJ(J,K)=  DJ(J,K)/10000000.
      PRINT 632
  632 FORMAT(74H1JOINT DISPLACEMENTS FOR DEAD LOAD (CASE 1) AND MOVING L
     2OADS(CASE 2 TO 67),//////)
      PRINT 229
  229 FORMAT(13H+DISPLACEMENT43X17HLOCATION OF LOADS)
      DO 230 JJ=1,12
      J1=(JJ-1)*6+1
      J2=J1+5
      IF(J1.NE.67)GO TO 231
      J2=67
      PRINT 644,(II1(I6),I6=J1,J2)
  644 FORMAT(6(/),11X, (6X,8H   CASE ,I2,1X),///)
      GO TO 643
  231 PRINT 234,(II1(I6),I6=J1,J2)
  234 FORMAT(6(/),11X,6(6X,8H   CASE ,I2,1X),///)
  643 DO 230 J=1,22
  230 PRINT 233,J,(DJ(J,K),K=J1,J2)
  233 FORMAT(/5XI2,4X,6(6X,E11.4),/)
      GO TO 645
   69 PRINT 6969
 6969 FORMAT(2X,18HITS ALL $*(=/+$ UP)
  645 STOP
      END
      SUBROUTINE GMADD(A,B,R,N,M)
```

```
C     TO ADD  2 MATRICES
      DIMENSION A(1),B(1),R(1)
      NM=N*M
      DO 10 I=1,NM
   10 R(I)=A(I)+B(I)
      RETURN
      END

      SUBROUTINE GMTRA(A,R,N,M)
      DIMENSION A(1),R(1)
      IR=0
      DO 10 I=1,N
      IJ=I-N
      DO 10 J=1,M
      IJ=IJ+N
      IR=IR+1
   10 R(IR)=A(IJ)
      RETURN
      END

      SUBROUTINE GMPRD(A,B,R,N,M,L)
      DIMENSION A(1),B(1),R(1)
      IR=0
      IK=-M
      DO 10 K=1,L
      IK=IK+M
      DO 10 J=1,N
      IR=IR+1
      JI=J-N
      IB=IK
      R(IR)=0.
      DO 10 I=1,M
      JI=JI+N
      IB=IB+1
   10 R(IR)=R(IR)+A(JI)*B(IB)
      RETURN
      END

      SUBROUTINE DISTANCE(JX,LO,JL,X,E,L,AMR,PU)
      DIMENSION X(70),E(31),L(31),AMR(93,68)
      REAL L
      JQ=LO-1
      X(JQ)=0.
      IA=3*JX-2
      JD=IA+1
      JE=IA+2
      DO 1 K=LO,JL
      E(K)=L(JX)
      JC=K-1
      IF(K.NE.35)GO TO 3
      X(K)=X(JC)+20.*12.
      GO TO 4
    3 X(K)=X(JC)+30.*12.
    4 AMR(JD,K)=(X(K)**2*(3.*E(K)-2.*X(K)))/E(K)**3
    1 AMR(JE,K)=-X(K)**2*(E(K)-X(K))/E(K)**2
      AMR(JD,1)=PU*L(JX)/2.
      AMR(JE,1)=-PU*L(JX)**2/12.
      RETURN
      END
```

```
      SUBROUTINE DISTANC2(JX,LO,JL,X,E,L,AMR,PU)
      DIMENSION X(70),E(31),L(31),AMR(93,68)
      REAL L
      JQ=LO-1
      X(JQ)=0.
      IA=3*JX-2
      JD=IA+1
      JE=IA+2
      DO 1 K=LO,JL
      JC=K-1
      E(K)=L(JX)
      X(K)=X(JC)+30.*12.
      AMR(JD,K)=-(E(K)-X(K))**2*(2*X(K)+E(K))/E(K)**3
    1 AMR(JE,K)=X(K)*(E(K)-X(K))**2/E(K)**2
      AMR(JD,1)=-PU*L(JX)/2.
      AMR(JE,1)= PU*L(JX)**2/12.
      RETURN
      END

      SUBROUTINE CAA(LO,JX,JL,JO,KO,L,E,X,P,MO,JK1)
      DIMENSION X(70),L(31),P(2,68),MO(2,68),E(70),JK1(2,68)
      REAL L,MO
C     CALCULATES AML FOR POINT LOAD CONDITIONS
      JJ=LO-1
      X(JJ)=0.
      DO 1 J=LO,JL
      JK1(1,J)=JO
      JK1(2,J)=KO
C
C     E DENOTES LENGTH OF MEMBERS WHEN CALCULATING PARMETERS IN THE PROGRAM
C     SINCE THE ORIGINAL VALUE OF LENGTH(L) MUST BE PRESERVED FOR LATER USE
C
      E(J)=L(JX)
      JJ=J-1
C     THE FOLLOWING 3 STATEMENTS ARE AS A RESULT OF THE DIS.AT MEMBER 31
      IF(J.NE.35)GO TO 7
      X(J)=X(JJ)+20.*12.
      GO TO 9
    7 X(J)=X(JJ)+30.*12.
    9 P(1,J)=(E(J)-X(J))**2*(2.*X(J)+E(J))/E(J)**3
      P(2,J)=X(J)**2*(3.*E(J)-2.*X(J))/E(J)**3
      MO(1,J)=X(J)*(E(J)-X(J))**2/E(J)**2
    1 MO(2,J)=X(J)**2*(E(J)-X(J))/E(J)**2
      RETURN
      END

      SUBROUTINE EQUIA(L,E1,X,P,MO,JK1)
      DIMENSION X(70),L(31),P(2,68),MO(2,68),E1(70),JK1(2,68)
      REAL L,MO
C     MTA IS NO OF ROWS IN MATRIX
C     THIS PART OF PROGRAM CALCULATES EQUIVALENT JOINT LOADS
C     FOR STRESS RESULTANTS,POINT LOADS (AML)
C     FOR LOADS 2 THRU 31
C
C     JO =JOINT AT LEFT  KO=JOINT AT RIGHT  LO=LOAD  JL=END OF DO LOOP
      JO=-1
      JX=12
      DO 10 LO=2,27,5
```

```
      JO=JO+2
      KO=JO+1
      JL=LO+4
      JX=JX+1
   10 CALL CAA(LO,JX,JL,JO,KO,L,E1,X,P,MO,JK1)
C     FOR MIDDLE PART OF STRUCTURE
      JX=31
      JO=13
      KO=14
      LO=32
      JL=38
      CALL CAA(LO,JX,JL,JO,KO,L,E1,X,P,MO,JK1)
C
C     FOR THE RIGHT HAND SIDE OF THE STRUCTURE
      JO=13
      JX=21
      DO 11 LO=39,64,5
      JO=JO+2
      KO=JO+1
      JL=LO+4
      JX=JX+1
      IF(JL.NE.68)GO TO 11
      JL=67
   11 CALL CAA(LO,JX,JL,JO,KO,L,E1,X,P,MO,JK1)
      RETURN
      END

      SUBROUTINE UNIV(L,AMUL,AMUM,AML,MT1,MT,PU,S,P,MO)
      DIMENSION AML(4,68),S(70),AMUL(4,26),AMUM(4,26),P(70),MO(70),L(31)
      REAL L,MO
C     CALCULATES ARL FOR UNIFORM LOAD CONDITIONS
C     THIS PART OF PROGRAM CALCULATES EQUIVALENT JOINT LOAD FOR A
C     UNIFORM LOAD
C
C     FIRST ASSIGN LENGTHS OF MEMBERS CORRESPONDING TO JOINT NUMBERS
      JJ=-1
      DO 210 J=13,18
      JJ=JJ+2
      JK=JJ+1
      S(JJ)=L(J)
  210 S(JK)=L(J)
      S(13)=L(31)
      S(14)=L(31)
      JJ=13
      DO 214 J=22,27
      JJ=JJ+2
      JK=JJ+1
      S(JJ)=L(J)
  214 S(JK)=L(J)
C
C     NOW EQUIVALENT JOINT LOADS ARE CALCULATED
C     PU=UNIFORM LOAD   NOTE-MO MUST BE REAL-CHECK WITH OTHER PROGRAM
      DO 212 J=1,26
      P(J)=PU*S(J)/2.
  212 MO(J)=PU*S(J)**2/12.
C
C     NOW THE AML MATRIX IS CALCULATED ACCORDING TO PAGE 36
C
      DO 213 J=MT1,MT
```

```
      AML(J,1)=0.
      DO 213 K=1,26
  213 AML(J,1)=AMUL(J,K)*P(K)+AMUM(J,K)*MO(K)+AML(J,1)
      RETURN
      END

      SUBROUTINE UNIA(L,AMUL,AMUM,AML,       PU,S,P,MO)
      DIMENSION AML(93  ),S(70),AMUL(93,26),AMUM(93,26),P(70),MO(70),L(3
     11)
      REAL L,MO
C     CALCULATES AML FOR UNIFORM LOAD CONDITIONS
C     THIS PART OF PROGRAM CALCULATES EQUIVALENT JOINT LOAD FOR A
C     UNIFORM LOAD
C
C     FIRST ASSIGN LENGTHS OF MEMBERS CORRESPONDING TO JOINT NUMBERS
      MT=93
      JJ=-1
      DO 210 J=13,18
      JJ=JJ+2
      JK=JJ+1
      S(JJ)=L(J)
  210 S(JK)=L(J)
      S(13)=L(31)
      S(14)=L(31)
      JJ=13
      DO 214 J=22,27
      JJ=JJ+2
      JK=JJ+1
      S(JJ)=L(J)
  214 S(JK)=L(J)
C     NOW EQUIVALENT JOINT LOADS ARE CALCULATED
C     PU=UNIFORM LOAD   NOTE-MO MUST BE REAL-CHECK WITH OTHER PROGRAM
      DO 212 J=1,26
      P(J)=PU*S(J)/2.
  212 MO(J)=PU*S(J)**2/12.
C
C     NOW THE AML MATRIX IS CALCULATED
C
      DO 213 J=1   ,MT
      AML(J   )=0.
      DO 213 K=1,26
  213 AML(J   )=AMUL(J,K)*P(K)+AMUM(J,K)*MO(K)+AML(J   ).
      RETURN
      END
         FINIS
```

APPENDIX 6

COMPUTER PROGRAM FOR REDUCTION OF STRESSES AND DISPLACEMENTS DUE TO DEAD LOAD BY POST-TENSIONING OF CABLES.

```
      PROGRAM PART2
      DIMENSION A(31),LA(14),MA(14),FAM(14,93),F(14,14),E(31),L(31)
      DIMENSION BMJ(93,22),BMQ(93,14),BRQ(4,14),FM(31,5),BRQ1(14,4)
      DIMENSION AML(93),FM1(31,5),AMQ(93,14),FAM1(14,93),FAM3(22,93)
      DIMENSION AMQ1(93,14)
      DIMENSION QSTORE(14,12)
      DIMENSION F1(14,14),DQL(14),Q(14),AM1(93,12),DJ1(22,12),AR1(4,12)
      DIMENSION FX(13,13),F4(14,14)
      DIMENSION DUMMY(93)
      DIMENSION DJ(22,1),ARL(4),AJ(12),X(12) ,AMA(93),DJA(22),ARA(4)
      DIMENSION JD(12,12)
      DIMENSION LA1(12),MA1(12)
      EQUIVALENCE (BMJ,BMQ,BRQ,AMQ),(BMJ(57),BRQ1)
      EQUIVALENCE (FM1,FM),(FAM3,FAM1,FAM),(F,F1)
      EQUIVALENCE (DUMMY,L),(DUMMY(32),E),(DUMMY(63),A)
      COMMON BMJ,FM1,FAM3
      REAL L,KAY,JD
      NR=14
      LOAD=68
      M=31
      MT=3*M
      PU=1000./12.
      PUU = 6.
      NC=12
      READ 4,KAY
    4 FORMAT(F10.4)
      BUFFER IN(5,1)(DUMMY(1),DUMMY(93))
      GO TO(69,1,69,69,69),UNITSTF(5)
    1 CONTINUE
      BUFFER IN(2,1)(BMQ(1,1),BMQ(93,14))
      GO TO (69,31,69,69,69),UNITSTF(2)
   31 REWIND 2
C
C     FM IS BUFFERED IN
C
      BUFFER IN(4,1)(FM1(1,1),FM1(31,5))
      GO TO (69,35,69,69,69),UNITSTF(4)
   35 REWIND 4
C
C     MULT OF AMQT* FM1=FAM1(93,14)
C     AMQ IS USED INSTEAD OF AMQT TO SAVE MEMORY
C
      JZ=MT-2
      DO 27 N=1,NR
      DO 27 J=1,JZ,3
      JA=J+1
      JB=J+2
      K=JB/3
      FAM(N,J)=AMQ (J,N)*FM(K,1)
      FAM(N,JA)=AMQ (JA,N)*FM(K,2)+AMQ (JB,N)*FM(K,3)
   27 FAM(N,JB)=AMQ (JA,N)*FM(K,4)+AMQ (JB,N)*FM(K,5)
C
C     F4 IS F1 WITH ALL ROWS AND COLUMNS INTACT
C     MULT OF AMQT*FAM1=F4(14,14)
C
      DO 47  I4=1,NR
      DO 47  I2=1,NR
      F4(I4,I2)=0.
```

```
      DO 47  J1=1,MT
   47 F4(I4,I2)=F4(I4,I2)+FAM(I4,J1)*AMQ(J1,I2)
C
C     DO LOOP STARTS HERE
C
      DO 30 I1=1,12
      DO 33 J=1,14
      DO 33 K=1,14
   33 F1(J,K)=F4(J,K)
C     REMOVE ROW AND COLUMN I1 FROM F1
      DO 34 J=1,14
      F1(I1,J)=0.
   34 F1(J,I1)=0.
C     FORMATION OF NEW F1
      DO 38 I2=1,12
      IF(I2.EQ.I1)GO TO 38
      F1(I2,I2)=F1(I2,I2)+L(I2)/(E(I2)*A(I2))
   38 CONTINUE
      DO 32 J=1,93
   32 AML(J)=BMQ(J,I1)
C      SET ROW I1 OF AML EQUAL TO ZERO
      I2 = 3*I1-2
      I3 = I2+1
      I4=I2+2
      AML(I2)=0.
      AML(I3)=0.
      AML(I4)=0.
C
C      CALCULATE DQL
C      GET FIRST FAM
C
C      STORE AMQ IN AMQ1
      DO 550 I=1,93
      DO 550 J=1,14
  550 AMQ1(I,J) = AMQ(I,J)
C      REMOVE COLUMN I1 FROM AMQ1
      DO 551 I=1,93
  551 AMQ1(I,I1) = 0.
C      MULTIPLY AMQ1T*FM1 TO GET FAM
      JZ=MT-2
      DO 555 N=1,NR
      DO 555 J=1,JZ,3
      JA=J+1
      JB=J+2
      K=JB/3
      FAM(N,J)=AMQ1(J,N)*FM(K,1)
      FAM(N,JA)=AMQ1(JA,N)*FM(K,2)+AMQ1(JB,N)*FM(K,3)
  555 FAM(N,JB)=AMQ1(JA,N)*FM(K,4)+AMQ1(JB,N)*FM(K,5)
C      NOW CALCULATE DQL
C     DQL(14)=FAM1*AML
      DO 39 IA=1,NR
      DQL(IA)=0.
      DO 39 JA=1,MT
   39 DQL(IA)=DQL(IA)+FAM(IA,JA)*AML(JA)
C
C     CALL MINV TO GET F=F**(-1)
C
      I4=I1-1
      I5=I1+1
```

```
      DO 83 J=1,I4
      DO 93 K=1,I4
   93 FX(J,K)=F(J,K)
      DO 83 K=I5,NR
      K9=K-1
   83 FX(J,K9)=F(J,K)
      DO 84 J=I5,NR
      J9=J-1
      DO 94 K=I5,NR
      K9=K-1
   94 FX(J9,K9)=F(J,K)
      DO 84 K=1,I4
   84 FX(J9,K)=F(J,K)
      NR1=NR-1
      CALL MINV(FX,NR1,D,LA,MA)
      DO 86 J=1,I4
      DO 96 K=1,I4
   96 F(J,K)=FX(J,K)
      DO 86 K=I5,NR
      K9=K-1
   86 F(J,K)=FX(J,K9)
      DO 87 J=I5,NR
      J9=J-1
      DO 97 K=I5,NR
      K9=K-1
   97 F(J,K)=FX(J9,K9)
      DO 87 K=1,I4
   87 F(J,K)=FX(J9,K)
C
C        MULTIPLY INV OF F BY -1
C
      DO 715 J=1,NR
      DO 715 K=1,NR
  715 F(J,K)=F(J,K)*(-1.)
C
C     Q(14)=-F(-1)*DQL
C
      DO 40 IA=1,NR
      Q(IA)=0.
      DO 40 JA=1,NR
   40 Q(IA)=Q(IA)+F1(IA,JA)*DQL(JA)
C
C        STORE Q IN QSTORE
C
      DO 560 I=1,NR
  560 QSTORE(I,I1) = Q(I)
C
C     CALCULATION OF AM1(93,I1)=AMQ*Q FOR 12 CASES
C
      DO 41 IA=1,MT
      AM1(IA,I1)=0.
      DO 41 JA=1,NR
   41 AM1(IA,I1)=AM1(IA,I1)+AMQ(IA,JA)*Q(JA)
      DO 53 I=1,NC
      J=3*I-2
   53 AM1(J,I1)=Q(I)
C
      DO 580 I=1,MT
  580 AM1(I,I1)=AM1(I,I1)+AML(I)
```

```
   30 CONTINUE
C
      PRINT 55
   55 FORMAT(63H1AXIAL FORCES IN CABLES 1 TO 11 DUE TO A UNIT FORCE IN C
     1ABLE 12,/////)
      DO 56 J=1,NC
   56 PRINT 57,(QSTORE(J,K),K=1,NC)
   57 FORMAT(12(1X,E10.2),/)
C
C       DISPLACEMENTS
C
      DO 561 I1=1,12
      TEMP = FM(I1,1)
      FM(I1,1)=0.
C     CALCULATION OF FAM3(22,93)=BMJT*FM1
      BUFFER IN (1,1)(BMJ(1,1),BMJ(93,22))
      GO TO(69,42,69,69,69),UNITSTF(1)
   42 REWIND 1
      DO 43 N=1,22
      DO 43 J=1,JZ,3
      JA=J+1
      JB=J+2
      K=JB/3
      FAM3(N,J)=BMJ(J,N)*FM(K,1)
      FAM3(N,JA)=BMJ (JA,N)*FM(K,2)+BMJ (JB,N)*FM(K,3)
   43 FAM3(N,JB)=BMJ (JA,N)*FM(K,4)+BMJ (JB,N)*FM(K,5)
      FM(I1,1)=TEMP
C
C     DJ1(22,I1)=FAM3*AM1(93,I1)
C
      DO 565 I=1,NR
  565 Q(I)=QSTORE(I,I1)
      DO 44 IA=1,22
      DJ1(IA,I1)=0.
      DO 44 JA=1,MT
   44 DJ1(IA,I1)=DJ1(IA,I1)+FAM3(IA,JA)*AM1(JA,I1)
      DO 575 I=1,22
  575 DJ1(I,I1)=DJ1(I,I1)/10.**7
  561 CONTINUE
C
C       REACTIONS
C
C     AR1(4,I1)=BRQT*Q          +  ARL
C
      DO 563 I1=1,12
      BUFFER IN (3,1)(BRQ1(1,1),BRQ1(14,4))
      GO TO (69,61,69,69,69),UNITSTF(3)
   61 REWIND 3
      DO 564 I=1,4
      ARL(I) = BRQ1(I1,I)
  564 BRQ1(I1,I)=0.
      DO 567 I=1,NR
  567 Q(I)=QSTORE(I,I1)
      DO 621 IA=1,4
      AR1(IA,I1)=0.
      DO 62 JA=1,14
   62 AR1(IA,I1)=AR1(IA,I1)+BRQ1(JA,IA)*Q(JA)
  621 AR1(IA,I1) = AR1(IA,I1) + ARL(IA)
  563 CONTINUE
```

```
      PRINT 65
   65 FORMAT(60H1BENDING MOMENTS,AXIAL AND SHEAR FORCES DUE TO POSTENSIO
     1NING,/////)
      DO 63 J=1,93
   63 PRINT 64,(AM1(J,K),K=1,12)
   64 FORMAT(12(1X,E10.2),/)
      PRINT 71
   71 FORMAT(45H1DISPLACEMENTS DUE TO POSTENSIONING OF CABLES,////)
      DO 72 J=1,22
   72 PRINT 73,(DJ1(J,K),K=1,12)
   73 FORMAT(12(1X,E10.2),/)
      PRINT 74
   74 FORMAT(31H1REACTIONS DUE TO POSTENSIONING,////)
      DO 75 J=1,4
   75 PRINT 76,(AR1(J,K),K=1,12)
   76 FORMAT(12(1X,E10.2),/)
      BUFFER IN(18,1)(DJ(1,1),DJ(22,1))
      GO TO(69,115,69,69,69),UNITSTF(18)
  115 DO 100 J=1,11
  100 AJ(J)=DJ(J,1)*(KAY-1.)*PUU/10.**7
      AJ(12) = - AJ(11)
      DO 101 J=1,12
      DO 102 K=1,11
  102 JD(K,J)=DJ1(K,J)
  101 JD(12,J)=DJ1(17,J)
      DO 610 I=1,6
      K=13-I
      JD(12,K)=-JD(11,I)
  610 JD(11,K)=-JD(12,I)
      CALL MINV(JD,NC,D1,LA1,MA1)
C     MULT OF JD **(-1)*AJ=X(12)
      DO 104 J=1,12
      X(J)=0.
      DO 104 K=1,12
  104 X(J)=X(J)+JD(J,K)*AJ(K)
      PRINT 531
  531 FORMAT(78H1FINAL POSTENSIONING FORCES TO APPLY IN CABLES TO REDUCE
     1 ACTION OF DEAD LOAD,////)
      PRINT 521,X
  521 FORMAT(//,46X,E10.3)
C     MULT COL I OF AM,AR, AND,DJ BY X(I)
      DO 611 I=1,12
      K=3*I-2
  611 AM1(K,I)=1.
      DO 106 K=1,93
      AMA(K)=0.
      DO 106 J=1,12
  106 AMA(K)=AM1(K,J)*X(J)+AMA(K)
      DO 107 K=1,22
      DJA(K)=0.
      DO 107 J=1,12
  107 DJA(K)=DJ1(K,J)*X(J)+DJA(K)
      DO 108 K=1,4
      ARA(K)=0.
      DO 108 J=1,12
  108 ARA(K)=AR1(K,J)*X(J)+ARA(K)
      PRINT 111
  111 FORMAT(///,1X69HFINAL AXIAL AND SHEAR FORCES AND BENDING MOMENTS D
     1UE TO POSTENSIONING,////)
```

```
      PRINT 109,AMA
  109 FORMAT(3(20X,E20.3),/)
      PRINT 112
  112 FORMAT(///,1X,40HFINAL DISPLACEMENTS DUE TO POSTENSIONING,/////)
      PRINT 110,DJA
  110 FORMAT(41X,E15.3)
      PRINT 113
  113 FORMAT(///,1X,36HFINAL REACTIONS DUE TO POSTENSIONING,/////)
      PRINT 114,ARA
  114 FORMAT(4(E30.3))
      GO TO 77
   69 PRINT 6969
 6969 FORMAT(8H1NO GOOD)
   77 STOP
      END
         FINIS
$OBJ,LGO
$AUX,ALIB,ADIR
```