Modification of Spiral Bevel Gears

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ABSTRACT

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A new method of spiral bevel gears' modification is proposed in this thesis. The method relys on the modification of the reference cones and the cone distance of the bevel pinion and gear. The modification greatly improves meshing conditions of the pair but more importantly it increases the load rating with regard to both bending and contact stress. The work was prompted by the limitations of the methods currently available. The principle of the method is explained, the relevant analysis is presented, the governing formulas are derived and supporting charts and recommendations are given. The only limitations are the occurrence of interference and pointed teeth as in the spur gears. The applicability of the proposed method is demonstrated step by step in a case study of a pair of spiral bevel gears of a drive gearbox for SGW - 250 coal mining conveyor in the fully mechanized underground coal working place. The design procedure and necessary formulas are discussed throughout the case study. The case study clearly reveals the benefits of such a modification; They are the bending stress and contact stress reduction of about 30%. These improvements in gear drive will however vary for other case but they will remain substantial.

The machining of such modified spiral bevel gears is feasible and can be carried out by standard machines and cutters already used for conventional spiral bevel gears. This aspect is also briefly discussed in the thesis.

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NOMENCLATURE

a	Center distance before modification
a_F	Tooth engagement time in each circle
a_v	Center distance of the virtual gears
a'	Modified Center distance
a^*	Virtual center distance factor before modification
a'^*	Virtual center distance factor after modification
a'_v	Center distance of the virtual gear after modification
a''_v	Process center distance after modification
Δa	Center distance increment due to cone distance change ΔR
Δa_v	Center distance of virtual gear increment due to addendum modification
Δa_r	Center distance increment of the virtual gear due to addendum modification
Δa_t	Center distance increment of virtual gear due to teeth thickness modification
A_1, A_2	Assembly distance
A_{a1}, A_{a2}	The distance of the pitch apex to the crown
b	Gear face width
C_1, C_2	The influence coefficient of the tooth thickness modification
C^*	Top clearance factor
d	Diameter of the cutter
d_1, d_2	Reference circle diameters of the pinion and gear
d_{a1}, d_{a2}	Addendum circle diameter
d_{v1}, d_{v2}	Virtual reference circle diameters of the pinion and gears
d'_1, d'_2	Pitch circle diameters of the pinion and gear
d'_{v1}, d'_{v2}	Virtual pitch circle diameters of the pinion and gears

f_F	Calculating factor of the dynamic factor
F_{tm}	Tangential force at the midpoint of the face width
g^*	Line of action length
g_m^*	Line of action length at the midpoint of tooth width
G	Converter of dimension
h_a	Addendum height
h_F	Maximum height of the force applying point
h_{F1}, h_{F2}	Standard dedenda
h'_{F1}, h'_{F2}	Modified dedenda
h_{nx}	Addendum height at point X of normal section
h_{Fa1}, h_{Fa2}	Dedendums from reference circles of the pinion and gear
h'_{F1}, h'_{F2}	Dedendums from pitch circles of the pinion and gear
h^*	Tooth height factor
H_{a1}, H_{a2}	The distances of crown to back
Δh_{t1}, Δh_{t2}	Addendum increment come from tooth thickness modification
ΔH	Tooth height variation coefficient
K_a	Application factor
K_F	Load combined factor for bending stress
K_h	Combined loading factor for contact stress
K_v	Dynamic factor
K_y	The ratio of the combined tooth form factor
K_{cc}	Center distance change ratio
K_{F1}, K_{F2}	Equal bending strength factors
$K_{F\alpha}$	Transverse load distribution factor of contact stress
$K_{F\beta}$	Longitudinal load distribution factor of bending stress
$K_{H\alpha}$	Transverse load distribution factor for pitting resistance
$K_{H\beta}$	Longitudinal load distribution factor

	for pitting resistance
m	Module of bevel gears
m_n	Module of normal section
m_0	Initial module
m_t	Transverse module of the large end
m_v	Module on the transverse section of the virtual helical gear at the large end of reference circle
m_x	Transverse module of the virtual gear at the cone distance R_x
m_{nm}	Normal module at the midpoint of the face width
m_{nx}	Module on the normal section of the virtual gear at the cone distance equals to R_x
m'	Module on pitch circle.
m_x'	Module on the transverse section of the virtual helical gear at the large end of the pitch circle
M	Conversion factor of midpoint of the face width
n	Speed (r.p.m)
N	Number of load cycles of the tooth
P	Power rating of the electric motor
r_1, r_2	Reference circle radii
r_{e1}, r_{e2}	Back-cone radii of the pinion and gears
r_{v1}, r_{v2}	Reference circle radii of the virtual gears
r_{av1}, r_{av2}	Virtual addendum circle radii
r_{Fv1}, r_{Fv2}	Virtual dedendum root circle radii
r_1', r_2'	Pitch circle radii
r'_{v1}, r'_{v2}	Pitch circle radii of virtual gears
r^*_{v1}, r^*_{v2}	Virtual reference radii factor at midpoint of the

face width of the pinion and gear

r^*_{av1}, r^*_{av2} Virtual tip circle radii factor of the pinion and gear

r^*_{bv1}, r^*_{bv2} Virtual base radii factors of at the midpoint of the pinion and gear

Δr_{v1}, Δr_{v2} Increments of the reference circle radii

R Standard cone distance before modification

R_m Cone distance at the midpoint after modification

R_0 Cone distance before modification

R_x Cone distance at point x

R_z Tooth surface roughness

R_{bvn} Base circle radius of the virtual gear

R_{Fnv} Normal section transition curve radius

R' Cone distance after modification

ΔR Cone distance increment

S_1, S_2 Tooth thicknesses of the pinion and gears

S_{a1}, S_{a2} Top land widths

S_{nF} The tooth root width of the normal section

S_{nx} Tooth thickness at point X of the normal section of the virtual gear

S_{p1}, S_{p2} Tooth thicknesses on the pitch circle of the pinion and gears

S_{Fmin} Safety factor in bending stress

S_{Hmin} Safety factor in pitting fatigue

S'_{nF} Modified tooth root width of the normal section

S^*_{a1}, S^*_{a2} Top land width factors

ΔS_1, ΔS_2 Tooth thickness increments of the pinion and gear for addendum modification

ΔS_i	Total tooth thickness variation
ΔS_n	Tooth thickness variation of the reference circle
ΔS_{nF}	The increment of the tooth root width of the normal section
ΔS_{ri}	Tooth thickness increment due to addendum modification
ΔS_{ti}	Tooth thickness increment due to tooth thickness modification
t	Circle pitch distance on the reference circle
t_h	Design gear life
t'	Circlar pitch on the pitch circle
t'_p	Circular pitch distance on the pitch circle
t^*_b	Virtual base circle pitch factor at the midpoint of face width
t^*_L	Pitch factor of the large end
T	Torque output of the electric motor
T_1, T_2	Torque transmitted by the pinion and gear
Δt	Increment of circular pitch
u	Speed ratio
u_0	Generating speed ratio
u_v	Virtual gear ratio
V_m	Velocity of the midpoint of the face width
X_1, X_2	Long and short addendum modification coefficient of the pinion and gear
X_{t1}, X_{t2}	Tooth thickness modification coefficient
X_Σ	Total addendum modification coefficient
$X_{t\Sigma}$	Total tooth thickness modification coefficient
y	Center distance increment of unit module
Y	Center distance departure coefficient

Y_1 Combined bending stress factor

Y_2 Cutter radius influence factor

Y_F Tooth form factor for bending stress

Y_F^* Tooth form factor of modified bevel gear

Y_x Size factor

Y_ε Contact ratio factor

Y_β Spiral angle factor

Y_{N1} Working life factor

Y_{p1} Combined bending stress limit factor

Y_{Fs1}, Y_{Fs2} Combined factor of the pinion and gear

Y_{sa1}, Y_{sa2} Stress correction factor

Y_{sat} Stress concentration factor

Y_{Rrelt} Tooth surface condition factor

$Y_{\rho prel}$ Relative influence factor of the cutter edge radius

$Y_{\delta relt}$ Relative sensitive factor of the material

Z_1, Z_2 Teeth number of the pinion and gears

Z_b Modification effect factor

Z_E Elasticity factor

Z_H Zone factor

Z_K Bevel gear factor

Z_L Lubricant factor

Z_n Life factor for pitting fatigue

Z_0 Number of teeth of the generating gear

Z_p Combined factor of contact stress limits

Z_R Surface roughness factor

Z_v Velocity factor

Z_W Surface hardness factor

Z_x	Size factor
Z_ε	Contact ratio factor for contact stress
Z_β	Spiral angle factor
Z_{mv}	Average virtual tooth number i.e $(Z_{v1}+Z_{v2})/2$
Z_{v1}, Z_{v2}	Virtual gear teeth number of the pinion and gear
Z_{vm}	Virtual tooth number at the midpoint of the face width
Z_{vn1}, Z_{vn2}	Normal virtual gear tooth number
Z_{vmin}	Minimum number of teeth of virtual gear with out undercut
α	Pressure angle of the reference circle on the transverse virtual gear
α_L	Pressure angle on the large end of the reference circle
α_m	Average tooth profile angle at the midpoint of the reference circle of the transverse section
α_n	Normal pressure angle
α_0	Transverse pressure angle
α_t	Tooth profile angle of the convex surface
α_{a1}, α_{a2}	Tip pressure angle of the pinion and gear
α_{nF}	The force application angle
α_{nm}	The pressure angle on the normal section of midpoint of the face width
α_{vL}	Transverse engagement angle of the pitch circle
α_{vm}	Transverse engagement angle at the midpoint of the face width
α'_L	Mesh angle at the large end
α'_m	Transverse engagement angle at midpoint of the face width
α_{nm1}, α_{nm2}	Average pressure angle of the normal section
α'	Pressure angle at the pitch circle
α'_{m1}, α'_{m2}	Pressure angles at the pitch circle at the midpoint of

the face width

β	Spiral angle of the large end of the virtual gear
β_L	Reference circle spiral angle of the large end
β_m	Spiral angle at the midpoint of the face width
β_0	Standard spiral angle
β_x	Spiral angle at the cone distance R_x
β_{bm}	Spiral angle at the base circle
β'	Spiral angle at the pitch circle
ψ	percent elongation in area
ϕ_R	Face width factor
ϕ_R^*	Face width calculation factor
δ	One half of the reference cone angle
δ_f	Root cone angle
δ_p	Reference cone angle of a generated bevel gear
δ_s	Percent elongation
δ_{a1}, δ_{a2}	Addendum angle of the pinion and gear
δ'_1, δ'_2	Pitch cone angle of the pinion and gear
δ''	Reference cone angle after modification
$\Delta\delta_1$, $\Delta\delta_2$	Increments of reference cone angle
ε_α	Transverse contact ratio
ε_β	Overlapping ratio
ρ_{F1}^*, ρ_{F1}^*	Root profile curvature radii factor
ρ_1^* ρ_2^*	Involute curvature radius factors on the pitch point of the pinion and gear
ρ_{a1}^*, ρ_{a2}^*	Involute curvature radius factor on tip point of the pinion and gear
ρ_{ao}^*	Fillet radius of the cutter

Σ	Shaft cross angle
σ_b	Breaking strength
σ_H	Surface contact stress of the pinion and gear
σ_s	Yield strength
σ_{F1}, σ_{F2}	Bending stress of the pinion and gear
σ_{Fp1}, σ_{Fp1}	Allowable bending stress
σ_{HP}	Allowable pitting fatigue stress of the pinion and gear
σ_{Flim}	Bending fatigue endurance
σ_{Hlim}	Endurance limit for contact stress
ϑ_F	Root cone angle of pinion and gear.
$\vartheta_{F\Sigma}$	Total dedendum angle of the equal tooth height tooth form
$\vartheta_{F\Sigma D}$	Total dedendum angle of the double reduced tooth form
$\vartheta_{F\Sigma S}$	Total dedendum angle of the pinion and gear (for standard tappered tooth form only)
$\vartheta_{F\Sigma T}$	Total dedendum angle of reduction root cone tooth form
ϑ'_{F1}, ϑ'_{F2}	Dedendum angles of the pitch circle of the pinion and gear (for standard reducing tooth form only)
$\Delta\vartheta_F$	Semiangle of the cone angle increment
$\Delta\vartheta'_{F1}$, $\Delta\vartheta'_{F1}$	Dedendum angle of the pitch circle

CHAPTER ONE
INTRODUCTION

The transmission of rotary motion from one shaft to another occurs in nearly every machine one can imagine. Gears constitute one of the best means available for transmitting this kind of motion. The gear mechanism is characterized by its constant transmission speed, high loading capacity, high working efficiency and long durability. The application of gear mechanism has a long history. As early as 152 BC, people began to use the mechanism in ancient instruments [1]. Since then, gear quality has continuously improved. However, the high speed and high power requirements of today's mechanical systems have challenged engineers to perfect the design of meshing gears.

The relative position of two shafts can be parallel, inter crossing or intersecting. Bevel gears are typically employed to transfer motion between two intersecting shafts. Although bevels are often employed for shaft angle of 90°, they may be applied to almost any shaft angle.

There are three types of bevel gears, straight, spiral and zerol. Each of these types can have various tooth forms as listed in Table 1 [2]. Straight bevels are the oldest, the simplest and yet the most widely used. Their teeth are straight and tapered and if extended inward, would intersect the gear axis. In recent years, cutting machines have been designed to crown the sides of the teeth in their longitudinal direction. They are known as Coniflex gears. These gears are capable of transmitting heavier loads than old style straight bevel gears under the same conditions. The tooth curvature of Coniflex gears

Table. 1. Various Tooth Forms of Straight Spiral and Zerol Bevels [2]

Tooth form	Bevel gear types	Production volume	Uses	Advantages
Involute	All	None	Difficult to manufacture	None from a practical point of view
Octoid	Straight (Coniflex*) Spiral Zerol*	Small to moderate	Most bevel gears of coarser than 10 DP (diametral pitch) which do not lend themselves to higher production methods	Requires simple tool and universal machine for producing both gears and pinions
Spherical	Spiral Zerol	Moderate to large	Principally used for gears of 10 DP and finer	Requires relatively simple tool and universal machine for producing both gears and pinions
Nongenerated (Helixform* and Formate*)	Spiral Zerol	Large	Low cost, high quality Limited to gears of 2.5:1 ratio and higher	Requires two basically different machines (but same simple tool) for producing gear and mating pinion efficiently. Process produces gears very rapidly. Universal machines available for producing both members where quantities insufficient to justify two separate machines
Revacycle*	Straight	Large	Principally automotive differential and farm implement gears	Requires generating broach-type cutter and universal machine for producing both gears and pinions in single operation from the solid blank. Process is very rapid

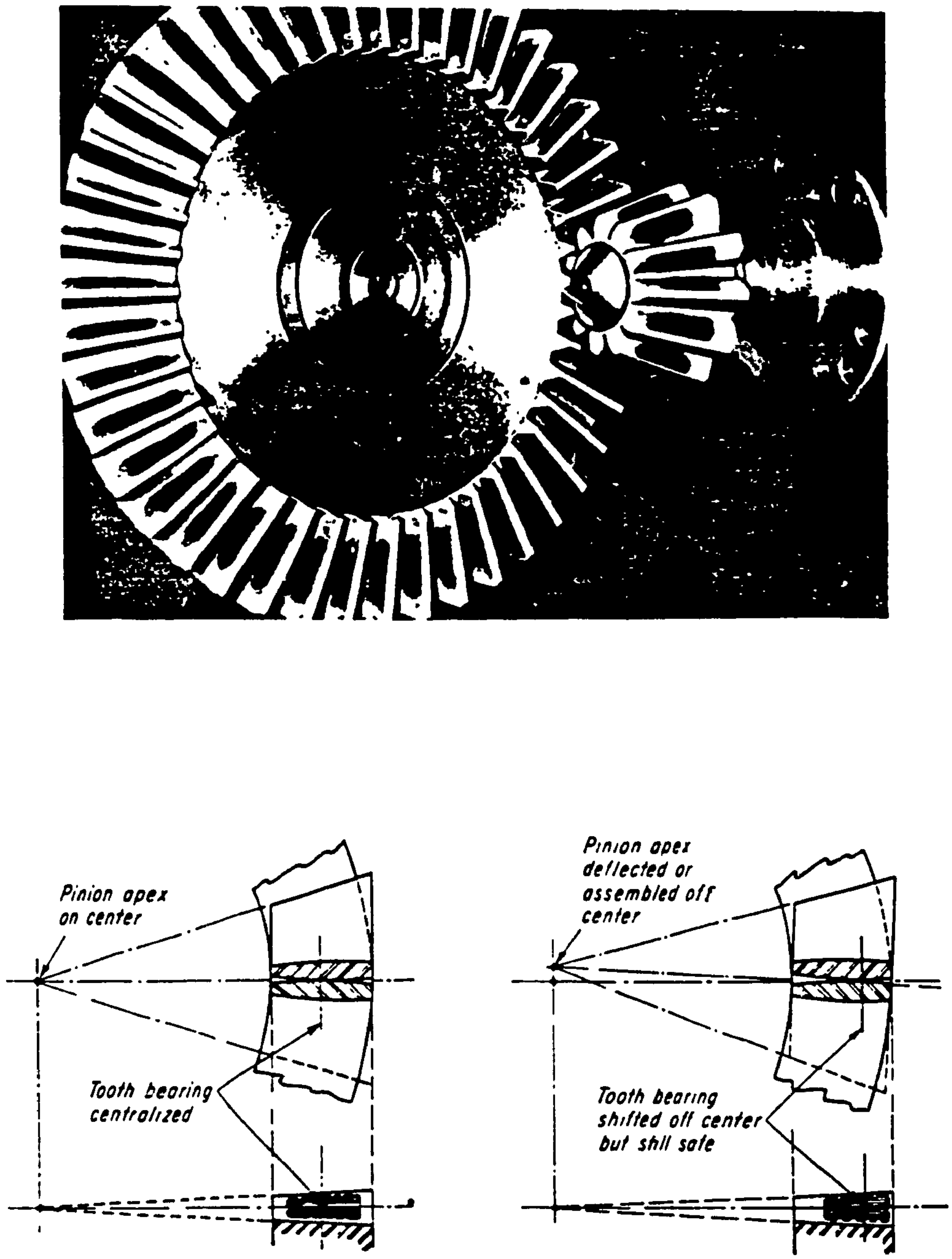


Fig. 1.1 Tooth Curvature of Coniflex Gears [3]

is shown in Figure 1.1.

Spiral bevels have curved oblique teeth which contact each other gradually and smoothly from one end to the other as illustrated in Figure 1.2. A spiral bevel gear can be seen as the assembly of an infinite number of short face-width straight bevels which are angularly displaced one another. Well-designed spiral bevel gears have more than two pairs of teeth in contact at all times. The overlapping tooth action transmits motion smoother and quieter than that of straight bevel gears. Therefore, spiral bevels have replaced straight bevels in many applications where high speeds, high loads, small gear size and quiet operation are demanded.

Zerol bevels, as illustrated in Figure 1.3, have curved teeth similar to those of the spiral bevels but with zero spiral angle at the middle of the face width and little end thrust. Straight and Zerol bevels are used where lower speeds and lighter load are required and where space, gear weight and mounting are primary concern.

1.1 The Major Types of Damages and Failures of Bevel Gears

Although the manufacturing and application of bevels began long time ago, the strength of the gear still remains a problem to be solved. In practice, the majority of gear mechanism breakdowns are usually due to the failure of bevel gears. The problem becomes intensified as higher speed and power are demanded in today's mechanical systems. The damage and failure of bevel gears can be classified in to the following four types:

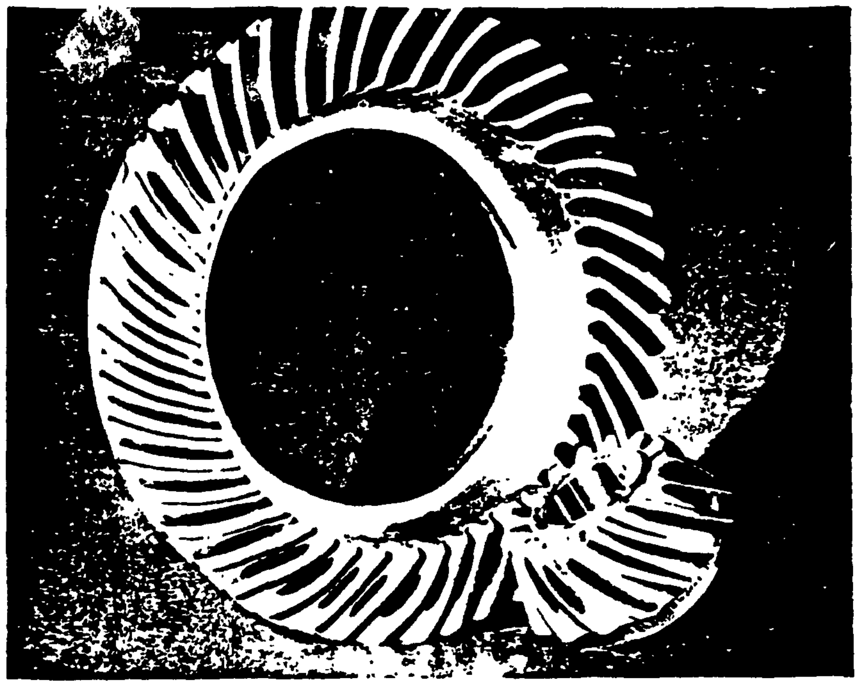


Fig. 1.2 Spiral Bevel Gear Design. [4]

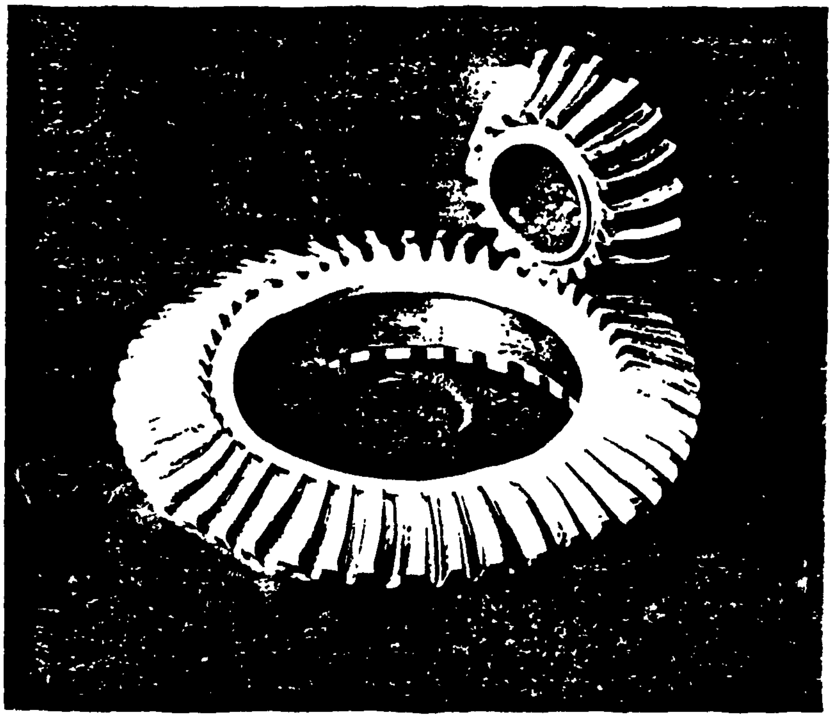


Fig. 1.3 Zerol Bevel Gears [5]

a) Tooth breakage.

b) Tooth surface failure:

1) Pitting.

2) Tooth surface wear.

3) Plastic flow

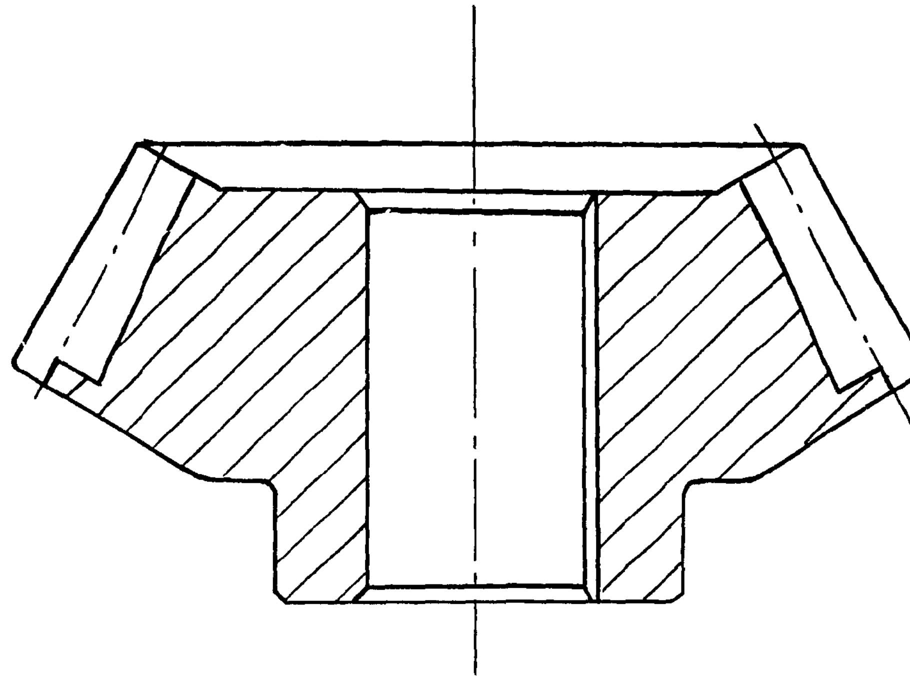
c) Damages caused by heat.

d) Other damage.

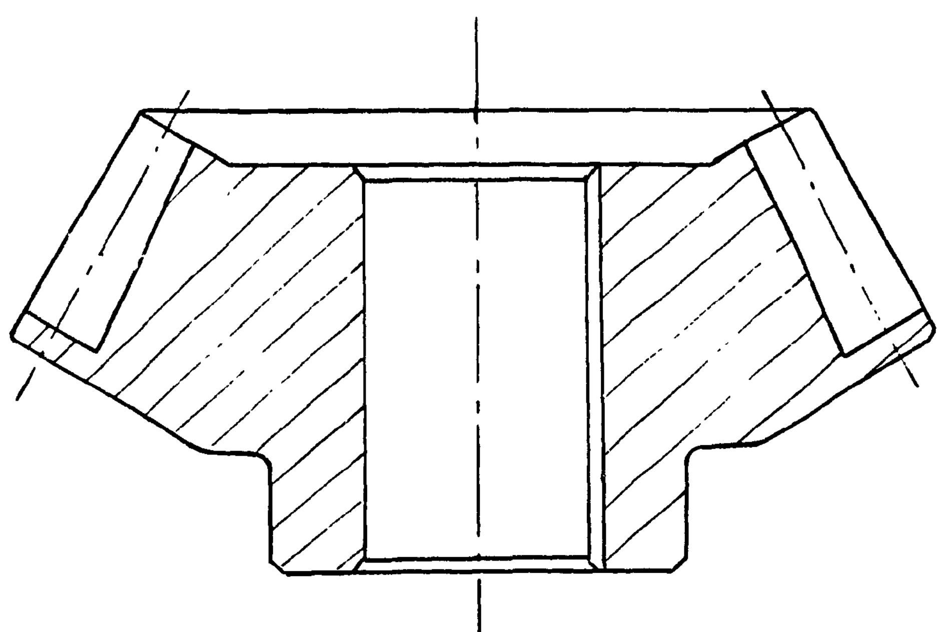
The major damage and failure are tooth breakage and flank damage Pitting and Wear [6].

1.2 Review of Previous Work

Abundant research has been conducted to improve the strength of bevels. The past work can be divided into three areas. In the first area, concentration has been focused on the development of new alloys for gears. Although the work in this area was successful, the cost of new materials is still very high. It should be pointed out that this particular topic is beyond the scope of this investigation. The second area is characterized by the employment of new manufacturing techniques and processes to improve the strength, durability, noise control, transmission quality and accuracy of gears. Al-Shareedah [7] suggested that the strength of bevel gears could be increased substantially if a web support is provided to the back of the teeth as shown in Figure 1.4. The web can be obtained by either cutting gears through a special gear manufacturing operation or through the technique of gear forging. By using plate analysis, it can be concluded that the bending strength of the teeth with web support is 2.5 times as much as that of teeth



b) With Partial Web Support



a) With Full Web Support

Fig. 1.4 Forging of Bevels With Web Support without web support. Nevertheless, no improvement has been achieved to the pitting resistance and transmission quality. Besides, machining is very difficult and the quality of forged gear is usually poor.

The third area deals with manufacturing techniques known a modification methods. These methods address the generation process, at the finishing instance of cutting. Figure 1.5 illustrates one of the methods - center distance modification. As the machining process proceeds, the reference line of the rack cutter gradually moves towards the reference circle of the workpiece. At the end of the machining, if the reference line is tangent to the reference circle (hence becoming the pitch line) a standard gear is formed. Otherwise, any non zero Xm is the distance between the reference line and the pitch line, will result a modified gear.

Another modification method is call the tooth thickness modification. Figure 1.6. shows the engagement of a blade of a cutter and a tooth of a workpiece during the cutting process. If the blade is so adjusted that the tooth thickness on the reference circle of a workpiece, S is made equal to $\frac{\pi m}{2}$, a standard gear will be formed. Otherwise, any other value of S will cause variation of tooth thickness and a tooth thickness modified gear is resulted.

Satoshi and Yasuji [8] presented a study of the effect of addendum modification on the bending fatigue strength of spur gears made of normalized steel. Theoretical analysis was performed with regard to the effect of addendum modification on the stresses at the tooth root fillet in the case of tip loading. They concluded that the value of root

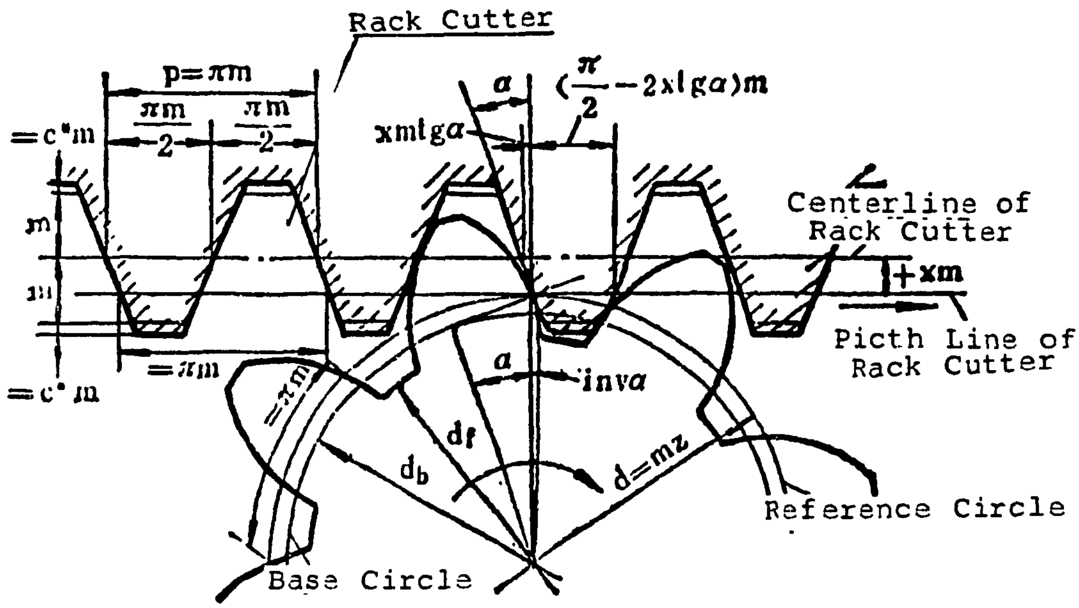


Fig. 1.5 Teeth Modification by basic Rack Offset [9]

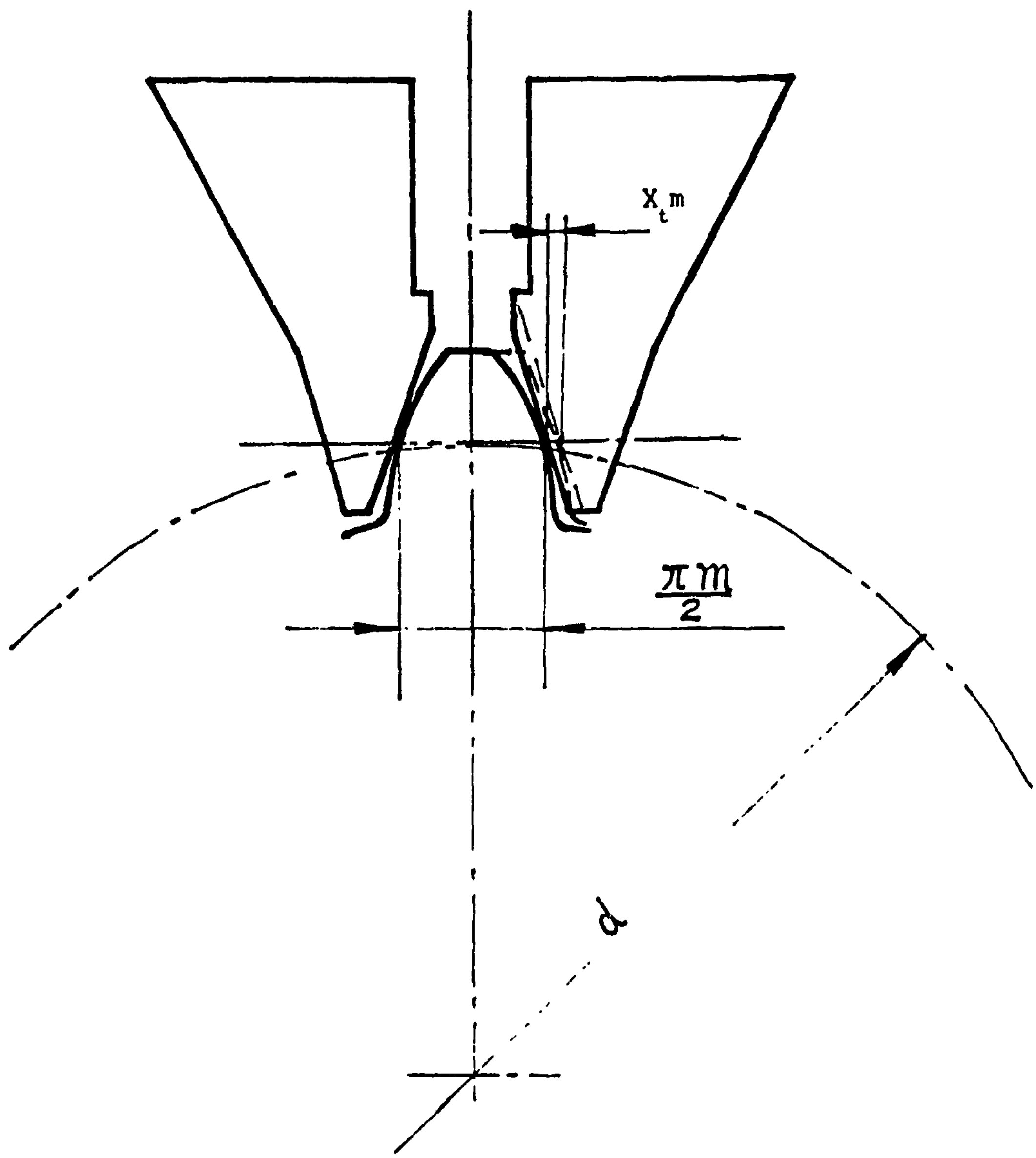


Fig. 1.6. Tooth Thickness Modification

stress factor for calculating true stress at the root fillet decreases with an increasing value of addendum modification coefficient. Thus, the bending fatigue strength of normalized steel gears could be improved significantly by selecting the proper amount of addendum modification. They extended the research to helical gears and almost the same conclusions were reached. [10] The above conclusions were verified by Teruaki, Hidak, Takeshi and Ishida [11] using photo elastic techniques to evaluate the stress level on internal gears. Merritt [12] suggested that the modification method used in spur gears could be applied to bevel gears. But since the pitch cone of generation coincides with the pitch cone of engagement, the modified bevels are always of the "long and short addendum" type, as explained in the next paragraph analogous to spur gears at standard centers.

Liang [13] summarized all kinds of modification methods available for bevels and found that only zero modifications are in application. They could be classified as:

a) Long and short addendum modification, i.e.

$$X_\Sigma = X_1 + X_2 = 0$$

where

X_Σ: Total addendum modification coefficient

X_1: addendum modification coefficient of the pinion

X_2: addendum modification coefficient of the gear

b) Combined Addendum and tooth thickness modification, i.e.

$$X_\Sigma = X_1 + X_2 = 0$$

$$X_{t\Sigma} = X_{t1} + X_{t2} = 0$$

where

$X_{t\Sigma}$: total tooth thickness modification coefficient

X_{t1}: tooth thickness modification coefficient of the pinion

X_{t2}: tooth thickness modification coefficient of the gear

Since the sum of modification coefficient is equal to zero, the above modifications were named as Zero Modification.

Liang concluded that zero modification can only balance the strength between pinion gear and gear. Moreover, since $X_{\Sigma} = X_{t\Sigma} = 0$, the central distance a_v, and pitch t on virtual gears are not changed. The reference circle and pitch circle overlap together. This means that the reference cone and pitch cone overlap together and the reference cone angle δ remains unchanged. See Figure 2.3.

1.3 Major Limitations of the Zero Modification Methods

The zero modification methods have the following limitations:

a) The total tooth number is limited by the minimum tooth number without undercut, i.e.

$$Z_{v1} + Z_{v2} \geq 2Z_{vmin}$$

Consequently, the reduction of gearbox volume is hampered.

b) The pitting resistance cannot be improved because the mesh angle remains unchanged.

c) When gear ratio is unity, it is impossible to increase the gear load capacity by this method.

d) For the spiral angle smaller than 25°, it is impossible to have more than two pairs of tooth in contact during the mesh action, in other words, the contact ratio can not be larger than two.

1.4 The Objective and Outline of This Investigation

The objective of this investigation is to outline a new tooth modifications, the addendum modification and tooth thickness modification, for improving the load rating of bevel gears. Traditionally, modification is performed on the standard pitch circle of virtual gears while the reference circle remains unchanged. The present modification method, however, modifies the reference circle and keeps the pitch circle constant.

Chapter 2 introduces the principle of the relative modification method. Two different approaches of the relative addendum modification, tooth thickness modification and the combined modification are discussed as well as their effects on the geometric variations of tooth form.

Chapter 3 discusses the tooth generation of the modified bevel gears and establishes the equations of pertinent process parameters. Chapter 4 presents a sample design of a pair of spiral bevel gears using the relative modification method in order to illustrate the application of the proposed method.

Finally, in chapter 5, general conclusions are drawn and recommendation for future work is made.

CHAPTER 2

THE PROPOSES MODIFICATION

2.1 The Basic Design of Bevel Gear

Bevel gears have pitch surfaces which are cones. These cones roll together without slipping, as shown in Figure 2.1. The true shape of a bevel gear tooth is obtained by taking a spherical section through the tooth, where the center of the sphere is at the common apex, as illustrated in Figure 2.2. Thus, as the radius of the sphere increases, the surface area becomes larger. With the number of teeth unchanged, the size of the tooth is increased as larger and larger spherical sections are taken. For bevel gear teeth, the action and contact conditions should be viewed on a spherical surface instead of a plane surface as in the case for spur gears. The projection of bevel gear teeth on the surface of a sphere is indeed a difficult and time-consuming problem. Fortunately, an approximation is available which simplifies it into a problem of ordinary spur gears. This method is called Tredgold's approximation. As long as the bevel gear has eight or more teeth, Tredgold's approximation is accurate enough for practical purposes [14]. This approximation has been universally used for bevel gear design.

In using Tredgold's approximation, a back cone is formed of elements which are perpendicular to the elements of the pitch cone at the large end of the teeth. This is shown in Figure 2.3. The length of

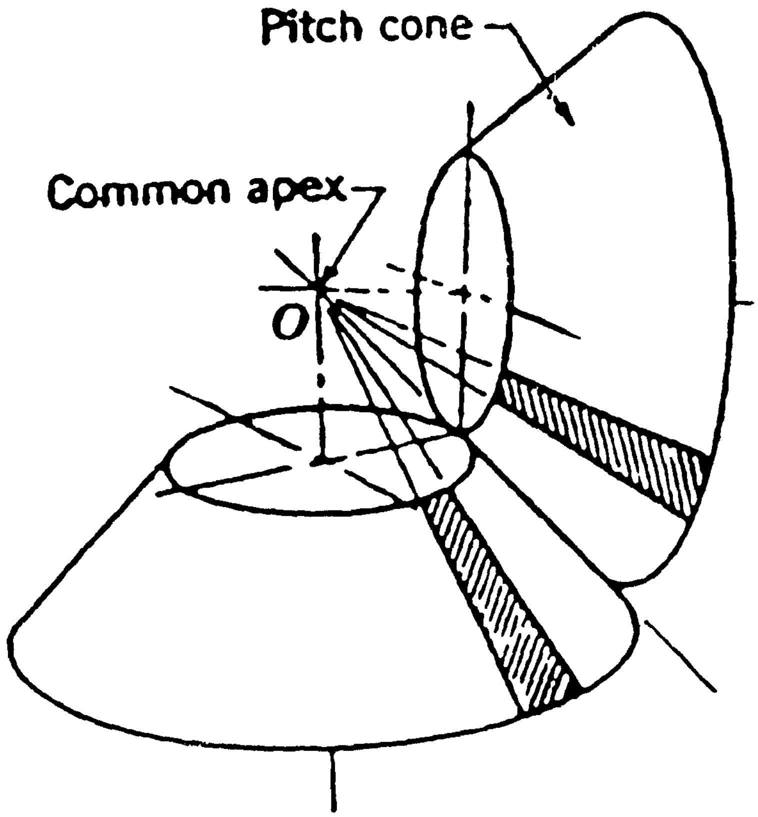


Fig. 2.1. The Pitch Cone of Bevel Gears is a pair of Cones Which Have Pure Rolling Contact [15]

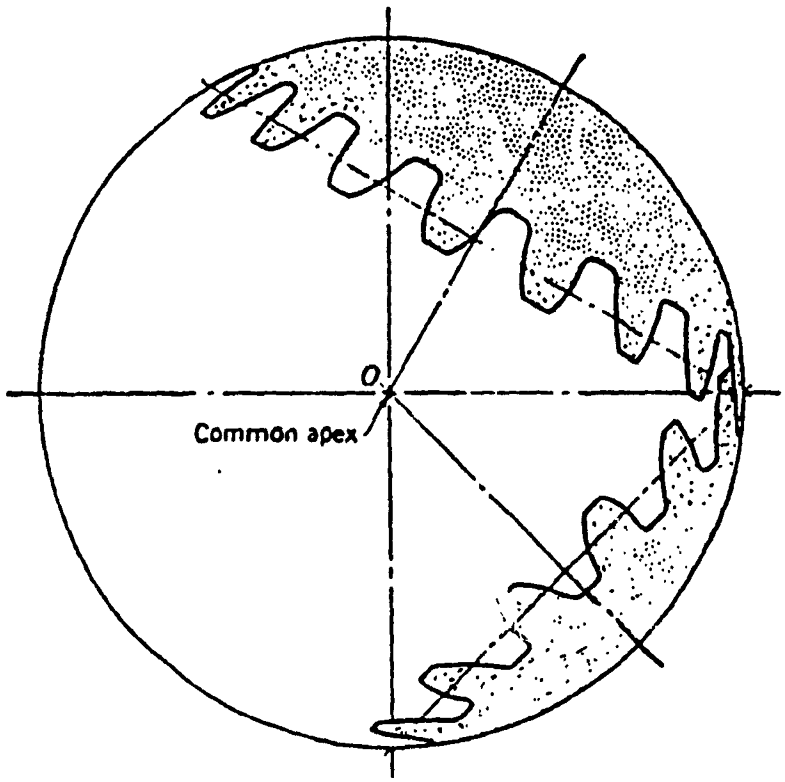


Fig. 2.2. A Spherical Section of Bevel-Gear Teeth [16]

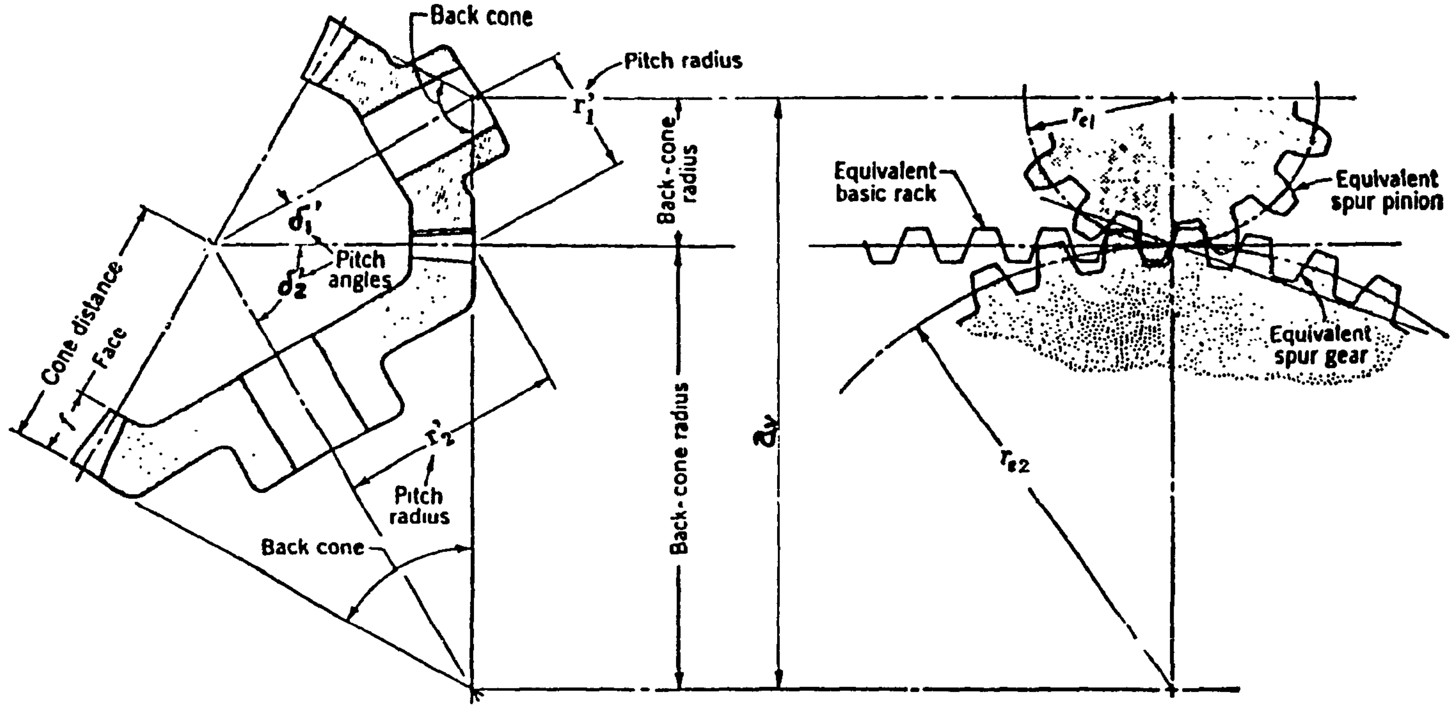


Fig. 2.3. Tredgold's Approximation [17]

a back-cone element is called the back-cone radius. An equivalent spur gear, so called equivalent gear or virtual gear, is constructed in such a way that its pitch radius r_e is equal to the back-cone radius. By using Tredgold's approximation, a pair of equivalent spur gears is thus obtained, which may be subsequently used to define the tooth profiles. They can also be used to determine the tooth action in exactly the same manner as for ordinary spur gears and the results correspond closely to those of bevel gears.

Two kinds of modification methods can be used to improve the tooth strength and working conditions of bevels gears: one is the conventional modification method and the other is this new modification method. Conventional modification applies an angle modification on the virtual gear by changing the pitch circle radius. Nevertheless, the radius of the reference circle and the module on the reference circle remain unchanged. The limitation of the conventional modification method is that the sum of modification coefficient X_{Σ} must be zero, otherwise, the center distance of the virtual gear is changed and the shafts angle is affected. Since bevels are designed for a fixed shaft angle, it is impossible to apply nonzero modification by the conventional method. It is desirable in design to realize the nonzero modification without changing the shaft cross angle. This is accomplished via the new modification which is outlined in the next section.

2.2 The Principle of Proposed Modification

The new modification approach applies an angle modification on the virtual gear by changing the radius of the equivalent spur gear reference circle. The radius of the pitch circle, however, remains the same as shown in Figure 2.4. The advantage of the new modification over the conventional one is that the modification has no effect on the cross angle of shafts and nonzero modification can be achieved.

Two ways of modification have been widely practiced. One is the modification along the radial direction. The other is the modification along the direction of tooth thickness which is called tooth thickness modification. There are many ways to realize addendum modification. In this investigation however, only two approaches of addendum modification will be discussed. The first approach is to modify the radius of the reference circle on a virtual gear, while the second approach, is to alter the cone distance. For more details of cone distance , refer to Figure 2.2.

It is common practice to perform both addendum and tooth thickness modification in a design in order to improve the strength and working conditions of bevel gears. For this reason, the combined modification rather than the individual modification will be studied. In this chapter, the combined effects of the addendum modification imposed on the reference circle coupled with tooth thickness modification will be fully discussed.

2.3 Reference Cone Modification

This method applies modification of the reference circle. The pitch circle, however remains unchanged. From Figure 2.4 it can be seen that the modification causes variations of the radius of the equivalent spur

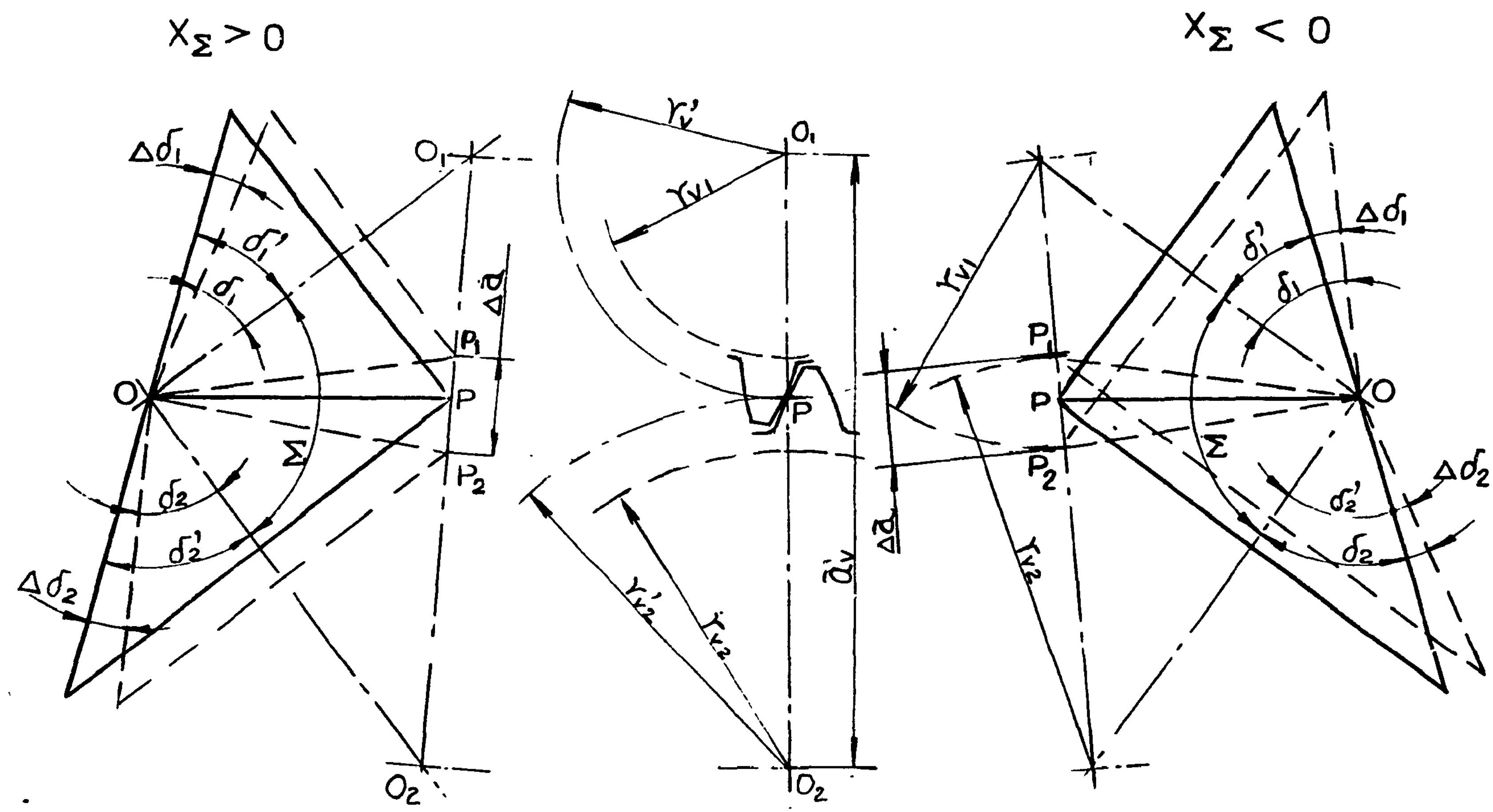


Fig. 2.4 Reference Cone Modification

gear and consequently, changes the center distance of the virtual gears.

The standard center distance of virtual gears is

$$a_v = O_1P_1 + O_2P_2$$

$$= r_{v1} + r_{v2} \qquad (2.1)$$

where r_{v1} and r_{v2} are the radii of the standard equivalent spur gear of the pinion and gear respectively.

The center distance of the modified virtual gear is

$$a'_v = O_1O_2$$

$$= r'_{v1} + r'_{v2} \qquad (2.2)$$

where r'_{v1} and r'_{v2} are the radii of pitch circles of the pinion and gear respectively.

The center distance increment due to modification is

$$\Delta a = P_1P + P\,P_2$$

$$= P_1P_2$$

$$= \Delta r_{v1} + \Delta r_{v2} \qquad (2.3)$$

Let the ratio of Δr_{v1} and Δr_{v2} be equal to the speed ratio u_v,

i.e. $$\frac{\Delta r_{v2}}{\Delta r_{v1}} = u_v \qquad (2.4)$$

and then Δr_{v1} as well as, Δr_{v2} can be written as

$$\Delta r_{v1} = \frac{\Delta a_v}{u_v + 1} \tag{2.5}$$

$$\Delta r_{v2} = \frac{\Delta a_v \cdot u_v}{1+u_v} \tag{2.6}$$

Define the center distance ratio K_{cc} as

$$K_{cc} = \frac{a'_v}{a_v} \tag{2.7}$$

The radii of the modified equivalent circles, r'_{v1} and r'_{v2}, have the form

$$r'_{v1} = K_{cc} \cdot r_{v1} \tag{2.8}$$

$$r'_{v2} = K_{cc} \cdot r_{v2} \tag{2.9}$$

For 90° cross shaft angle, since the virtual gear tooth numbers of the pinion and gear, Z_{v1} and Z_{v2} are

$$Z_{v1} = \frac{Z_1}{\cos\delta'_1} \tag{2.10}$$

$$Z_{v2} = \frac{Z_2}{\cos\delta'_2} \tag{2.11}$$

where Z_1 the tooth number of pinion

Z_2 the tooth number of gear

δ'_1 the pitch cone angle of pinion

δ'_2 the pitch cone angle of gear

the speed ratio u_v is given by

$$u_v = \frac{Z_{v2}}{Z_{v1}}$$

$$= \frac{Z_2 \cos\delta_1'}{Z_1 \cos\delta_1'}$$

$$= u \frac{\frac{\overline{OE}}{\overline{OP}}}{\frac{\overline{OF}}{\overline{OP}}}$$

$$= u \frac{r_2}{r_1}$$

$$= u^2 \qquad (2.12)$$

where u is the bevel gear ratio.

Substituting Eq'n (2.12) into Eq's (2.5) and (2.6) gives

$$\Delta r_{v1} = \frac{\Delta a}{1 + u^2} \qquad (2.13)$$

$$\Delta r_{v2} = \frac{\Delta a \cdot u^2}{1 + u^2} \qquad (2.14)$$

2.4 Tooth Thickness Modification

Tooth thickness modification can be used to overcome the defects arising from addendum modification. It is well known that if the addendum modification coefficients X_1 and X_2 are too small or too large, undercut and over-thin topland will be resulted. In these cases, the strength of the bevel tooth is greatly reduced and tooth modification becomes necessary. Even if the above phenomena do not occur, the modification can be performed to improve the strength. The improvement of tooth generation techniques has made the tooth thickness modification easy to be realized on conventional machining equipment.

2.5 Characteristics of Tooth Thickness Modification

The main difference between the conventional tooth thickness modification and the new tooth thickness modification is that the conventional modification is applied to the pitch circle. Consequently, the radius of the pitch circle is changed. The reference circle however, is not changed.

In the new modification method, modification is performed on the reference circle. As a result, the radius of the reference circle is changed rather than that of the pitch circle.

2.5.1. Tooth thickness increment

The tooth thickness variation of gear and pinion are given by

$$\Delta S_1 = m \cdot X_{t1} \tag{2.15}$$

$$\Delta S_2 = m \cdot X_{t2} \tag{2.16}$$

where

$m = \frac{t}{\pi}$ is the module of the bevel gear which is equal to the ratio of pitch distance on reference circle and π

X_{t1}, X_{t2} are tooth thickness modification coefficients for the pinion and gear respectively.

2.5.2. Pitch increment variation

The increment of pitch Δt, is the sum of ΔS_1 and ΔS_2

$$\Delta t = \Delta S_1 + \Delta S_2$$
$$= (X_{t1} + X_{t2})m$$
$$= X_{\Sigma t} \cdot m \qquad (2.17)$$

2.5.3. Center distance change Δa_t

The tooth thickness change will cause an increment Δa_t of the center distance , as shown in Figure 2.5. This can be found from

$$\Delta a_t = \Delta h_{t1} + \Delta h_{t2}$$
$$= \frac{\Delta S_1}{2\ \tan\alpha} + \frac{\Delta S_2}{2 \cdot \tan\alpha}$$
$$= \frac{X_{t\Sigma} \cdot m}{2\tan\alpha} \qquad (2.18)$$

2.6 Combined Modification

In the previous sections, the addendum modification and the tooth thickness modification were discussed. These two modification are rarely used separately. Combined modification is normally employed in bevel gear design. In this section, the geometric variations of the combined modification are discussed. The variation of the tooth thickness is illustrated in Figure 1.6.

2.6.1. The tooth thickness increment

The tooth thickness increment due to the thickness modification is

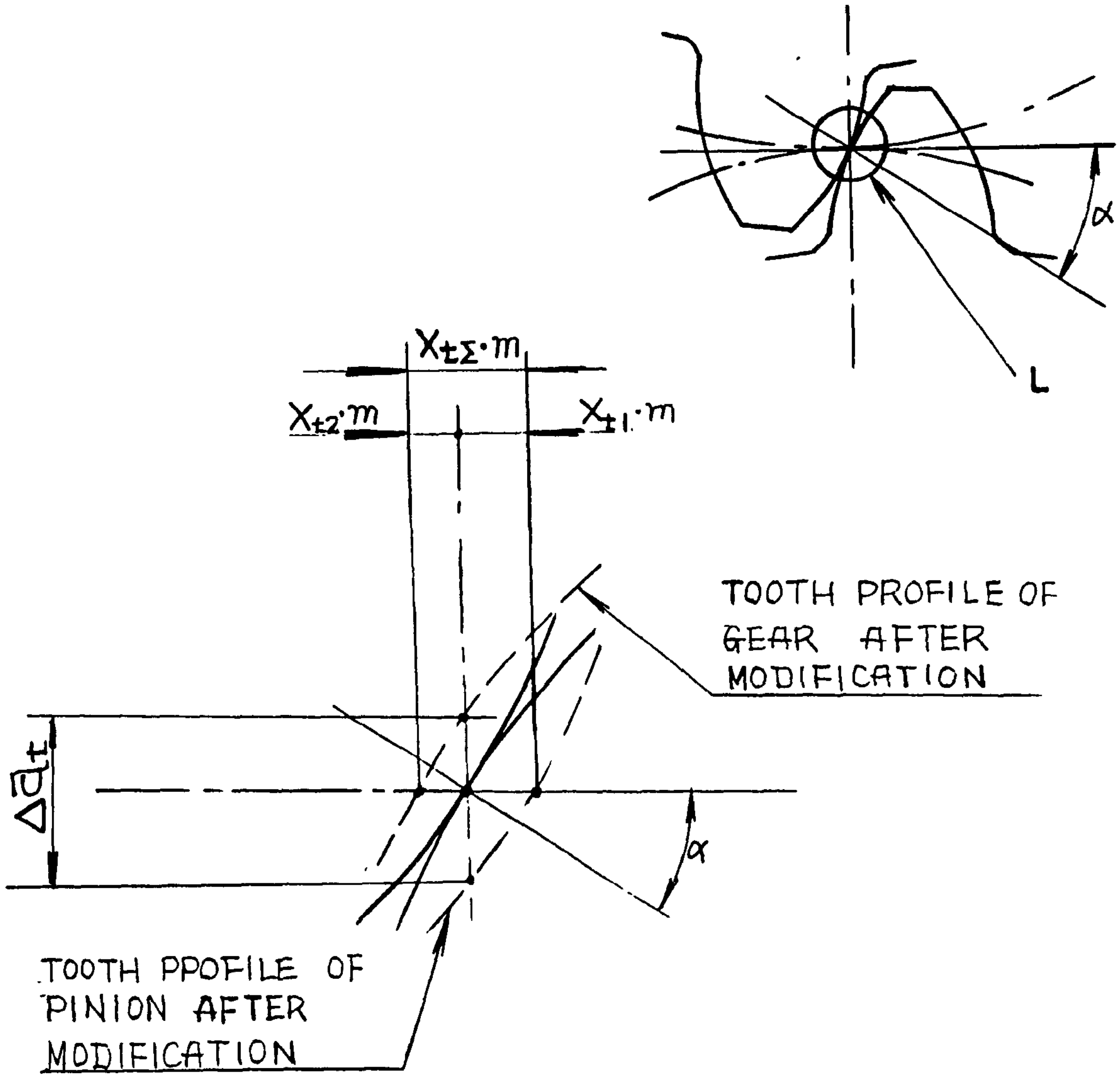


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Fig. 2.5. Central Distance Increment Caused by Tooth Thickness Modification

$$\Delta S_{ti} = m\, X_{ti} \qquad i = 1,2 \tag{2.19}$$

Tooth thickness variation due to reference circle modification is

$$\Delta S_{r} = 2\, X\cdot m\cdot \tan\alpha \tag{2.20}$$

So the total tooth thickness variation ΔS_i is

$$\Delta S_{i} = \Delta S_{ti} + \Delta S_{ri} = (X_{ti} + 2X_{i}\cdot\tan\alpha)\cdot m \qquad i = 1,2 \tag{2.21}$$

The tooth thickness of modified bevel gear is

$$S_{i} = (\frac{\pi}{2} + X_{ti} + 2X_{i}\cdot\tan\alpha)\cdot m \tag{2.22}$$

2.6.2. Circular pitch t on reference circle

When backlash is not considered, the pitch distance on the reference circle is the sum of the tooth thickness S_1 and S_2

$$\begin{aligned} t_{1} &= t_{2} \\ &= S_{1} + S_{2} \\ &= m\ (\ \pi + X_{t\Sigma} + 2\cdot X_{\Sigma}\cdot\tan\alpha\ ) \end{aligned} \tag{2.23}$$

2.6.3. Pressure angle at the pitch point

The pitch distance at the pitch circle t'_p is

$$\begin{aligned} t'_{p} &= S_{p1} + S_{p2} \\ &= \pi\cdot K_{cc}\cdot m \end{aligned} \tag{2.24}$$

in which

$$S_{p1} = \frac{d_{v1}'}{dv1} \cdot S_1 - d_{v1}'(\mathrm{inv}\ \alpha' - \mathrm{inv}\ \alpha)$$

$$= K_{cc} \ [\ S_1 - d_{v1} - (\mathrm{inv}\ \alpha' - \mathrm{inv}\ \alpha)] \qquad (2.25)$$

$$S_{p2} = \frac{d_{v2}'}{dv2} \cdot S_2 - d_{v2}'(\mathrm{inv}\ \alpha' - \mathrm{inv}\ \alpha)$$

$$= K_{cc} \ [\ S_2 - d_{v2} - (\mathrm{inv}\ \alpha' - \mathrm{inv}\ \alpha)] \qquad (2.26)$$

where the involute function of α' and α are

$$\mathrm{inv}\ \alpha' = \tan\alpha' - \alpha'$$

$$\mathrm{inv}\ \alpha = \tan\alpha - \alpha$$

Substituting Eq's (2.25) and (2.26) into Eq'n (2.24) gives

$$\pi m = (S_1 + S_2) - (d_{v1} + d_{v2})\cdot(\mathrm{inv}\alpha' - \mathrm{inv}\alpha) \qquad (2.27)$$

Replacing $(S_1 + S_2)$ by the Eq'n (2.23), and noticing that

$$d_{v1} + d_{v2} = m\cdot(Z_{v1} + Z_{v2}) \qquad (2.28)$$

the expression for the pressure angle on the pitch circle is finally found to be

$$\mathrm{inv}\ \alpha' = \frac{\tan\alpha}{Z_{vm}} \left(X_{\Sigma} + \frac{X_{t\Sigma}}{2\tan\alpha} \right) + \mathrm{inv}\ \alpha \qquad (2.29)$$

The geometric variations along the radial direction caused by the combined modification will be developed in the next section.

2.6.4. The center distance change Δa

The center distance change Δa has two parts, Δa_r and Δa_t. These are duc to the reference circle modification and the tooth thickness modification respectively.

$$\Delta a = \Delta a_r + \Delta a_t \tag{2.30}$$

Let the center distance departure coefficient y be defined by

$$y = \frac{\Delta a_v}{m} \tag{2.31}$$

or

$$y = \frac{a'_v - a}{m} \tag{2.32}$$

If the ratio of center distance $K_{cc} = a_v'/a_v$ is introduced, y can also be written as

$$y = (\frac{a_v'}{a_v} - 1)\cdot\frac{a_v}{m}$$

$$= (K_{cc} - 1)\cdot Z_{mv} \tag{2.33}$$

2.6.5. The coefficient of tooth height variation ΔH

In the process of tooth profile generation, the center distance of cutter and work piece has to be adjusted. From the previous sections, it is seen that the tooth thickness modification does not affect this distance. The center distance the of process, denoted by a_v'' is given by

$$a_v'' = a_v + \Delta a_v = a_v + X_\Sigma\cdot m \tag{2.34}$$

Comparing Eq's (2.34) and (2.32) gives

$$a_v'' - a_v' = X_\Sigma \cdot m - m \cdot y \tag{2.35}$$

The coefficient of tooth height variation ΔH is defined as the center distance variation per unit module and has the following expression

$$\Delta H = \frac{a_v'' - a_v'}{m}$$
$$= X_\Sigma - y \tag{2.36}$$

The tooth height of bevel gears after combined modification is given as

$$h = (2h_a^* + c^* - \Delta H) \cdot m \tag{2.37}$$

where

h_a^* is the addendum coefficient, and
c^* is the top clearance coefficient

The addendum height h_a becomes

$$h_a = (h_a^* + X - \Delta H) \cdot m \tag{2.38}$$

2.6.6. Module and cone distance

Spiral bevel gears have both normal modules m_{nx} and transverse modules m_x. These two parameters have the following relationship with respect to the spiral angle β_x and cone distance R_x

$$m_{nx} = m_x \cdot \cos \beta_x \tag{2.39}$$

$$m_x = \frac{R_x}{R} \cdot m \tag{2.40}$$

Therefore m_{nx} can be expressed as

$$m_{nx} = \frac{R_x}{R} \cdot m \cdot \cos \beta_x \tag{2.41}$$

where the basic module m is defined on the large end of the transverse virtual gear.

2.7. Cone Distance Modification Method

In the traditional modification methods, the cone distance R can not be changed. In the previous section, the combined reference circle modification and tooth thickness modification were discussed. It is obvious that the modification of the center distance of the virtual gear can be carried out by changing the cone distance of the bevel gear. In this section, the alternative addendum modification, cone distance modification and the combined effects of cone distance and tooth thickness modification will be discussed.

2.7.1 The principle of cone distance modification

The cone distance modification is realized by reducing or extending the cone distance R to obtain the reduced or increased center distance of the virtual gear as shown in Figure 2.6. The important characteristic of this method is that the shaft cross angle is not changed.

Assume R is the cone distance of the standard bevel gear and R' is the cone distance of the modified bevel shown in Figure 2.7. The

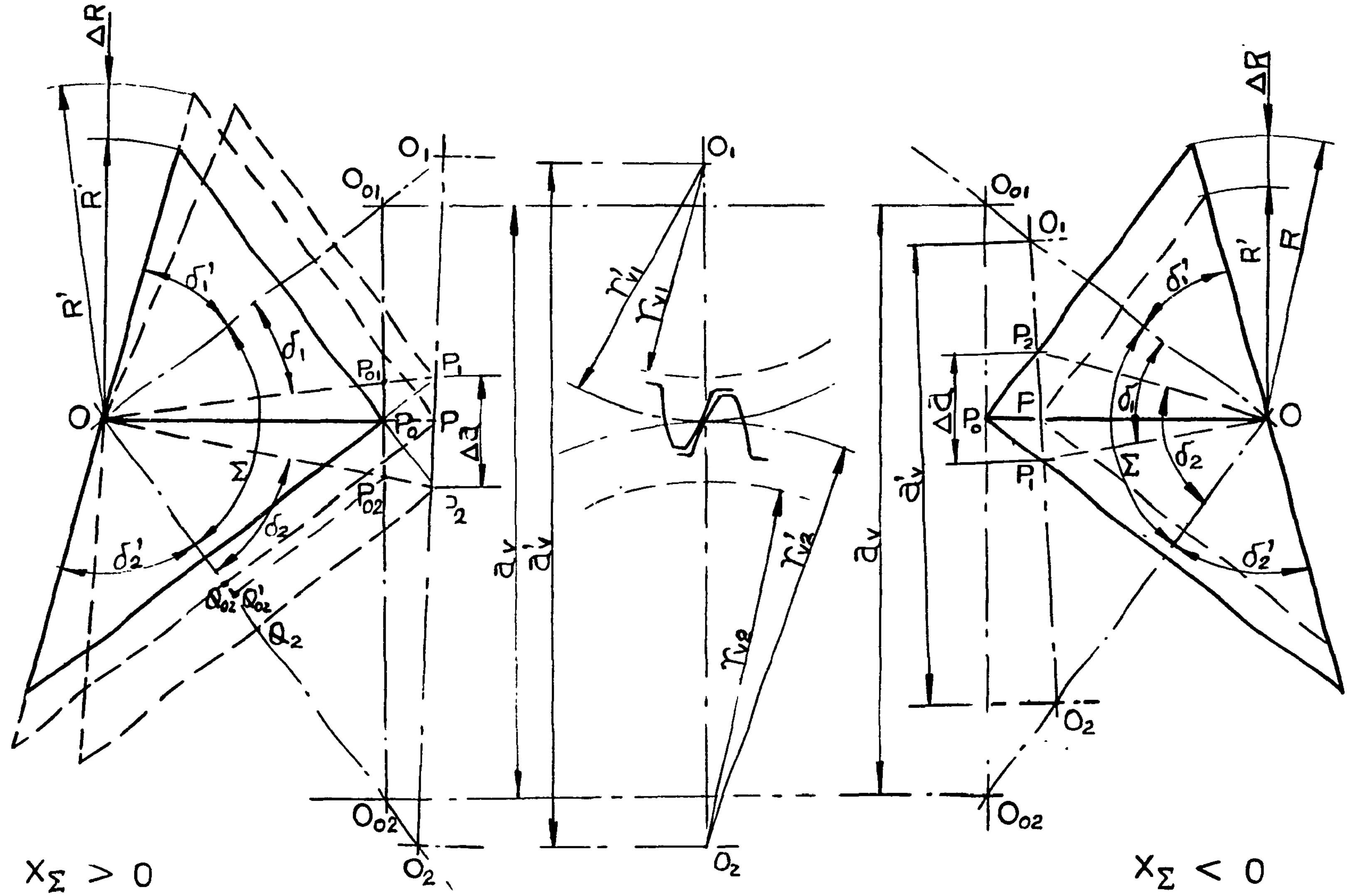


Fig. 2.6 Cone Distance Modification Method

difference in cone distance is:

$$\Delta R = R' - R$$

$$R' = OP$$

$$R = OP_o$$

$$\Delta R = OP - OP_o$$

$$= P_oP \qquad (2.42)$$

The center distance variation due to the cone distance change ΔR can be obtained readily from shown Figure 2.6. as

$$\Delta a = \Delta R \; (\tan\delta'_1 + \tan\delta'_2) \qquad (2.43)$$

or

$$\Delta R = \frac{\Delta a}{\tan\delta'_1 + \tan\delta'_2} \qquad (2.44)$$

where

δ'_1 pitch cone angle of pinion

δ'_2 the pitch cone angle of gear

when the cross angle of the shafts is 90^0

$$\tan\delta'_1 + \tan\delta'_2 = \frac{1 + u^2}{u} \qquad (2.45)$$

and

$$\Delta R = \Delta a \frac{u}{1 + u^2} \qquad (2.46)$$

2.8 General Discussion

After discussing the geometric variations of modification of bevel, the resulting improvement in the strength and other aspects of the bevel will be studied in the following sections:

2.8.1 Surface durability

The failure of the surface of gear teeth, generally called wear, consists of such common forms as pitting, scoring and abrasion. Wear is almost inevitable in gear transmission. It reduces the transmission accuracy, weakens the bending strength of bevels gears and possibly leads to tooth breakage. Two most common forms of wear are abrasive wear and adhesive wear. Abrasive wear occurs when hard particles are present during the sliding action. Adhesive wear occurs when two tooth bodies slide over each other at very high temperature. By increasing the pressure angle, the wear and scuffing can also be reduced.

Pitting is the most common form of gear tooth surface failure due to the high contact stress during the repetitive mesh action. The contact stress σ_H has the following from [18]:

$$\sigma_H = Z_H \cdot Z_E \cdot Z_\varepsilon \cdot Z_\beta \cdot Z_b \cdot \sqrt{\frac{k_H \cdot F_{tm}}{d_{m1} \cdot b} \cdot \frac{\sqrt{u^2+1}}{u}} \tag{2.47}$$

where

Z_H	the zone factor
Z_E	the elastic factor
Z_ε	the contact ratio factor
Z_β	the spiral angle factor

The influence of combined modification on σ_H is through the zone factor Z_H, which is given by [19]:

$$Z_H = 2 \cdot \sqrt{\frac{\cos\beta_m}{\sin 2\alpha'_{tm}}} \tag{2.48}$$

From the above relationship, it is evident that Z_H can be reduced by increasing the pressure angle α'_{tm}. From Eq'n (2.29) it can be easily deduced that this is possible if the sum of modification coefficients X_Σ is positive. It is also easy to show that if the pressure angle α'_{tm} is increased from 20° to 25°, Z_H can be reduced by about 10%. Since the contact stress σ_H is proportional to Z_H, the same percentage of reduction of contact stress is obtained. Larger pressure angles will further reduce the contact stress σ_H. However the contact ratio is reduced and poor transmission smoothness will be resulted. So the selection of a modification coefficient has to compromise between the low contact stress and the high contact ratio. As a criterion, the contact ratio ε is usually maintained at 1.2 or higher.

2.8.2. The minimum tooth number limit

Bevel gears are usually employed to reduce the rotation speed and change the rotation direction. The reduction of tooth number of bevel gears can significantly reduce the size of the gearbox. However, the selection of tooth number is limited by the undercut condition. In order to avoid undercut, the tooth number must be greater than the minimum tooth number limits Z_{vmin}. Unlike the conventional method, it is possible to lower the tooth number limit by applying the new modification method. This can be seen from the relationship:

$$Z_{vmin} = \frac{2h^*}{\sin^2\alpha'_{tm}} \tag{2.49}$$

For $h^*_a = 0.85$, the minimum tooth number limit Z_{vmin} is reduced from 15 to 9, if pressure angle α'_{tm} is increased from 20^0 to 25^0.

2.8.3. Contact ratio

Contact ratio is defined as the average number of pairs of teeth in contact. Generally, gears should not be designed to have contact ratio less than 1.2, because inaccurate mounting might reduce the contact ratio even more, thus increasing the possibility of impact between the teeth as well as the noise level. A contact ratio greater than two is difficult to be realized by conventional methods if the teeth width and spiral angle are both small. From the modification coefficient limit curves shown in Figure 2.8, a contact ratio greater than 2 is easily achieved if X_Σ is negative. Under this condition, the mesh action is smoother because more than two pair of tooth share the load.

2.8.4 Bending strength

The bending strength of gear teeth is the ability of the tooth root to resist crack and tooth breakage. It depends on the endurance limit of rotating-beam specimen. The bending fatigue strength depends on bending stress σ_F which is proportional to the tooth form factor Y_F.

The bending stress of tooth can be calculated by using the following relationship: [20]

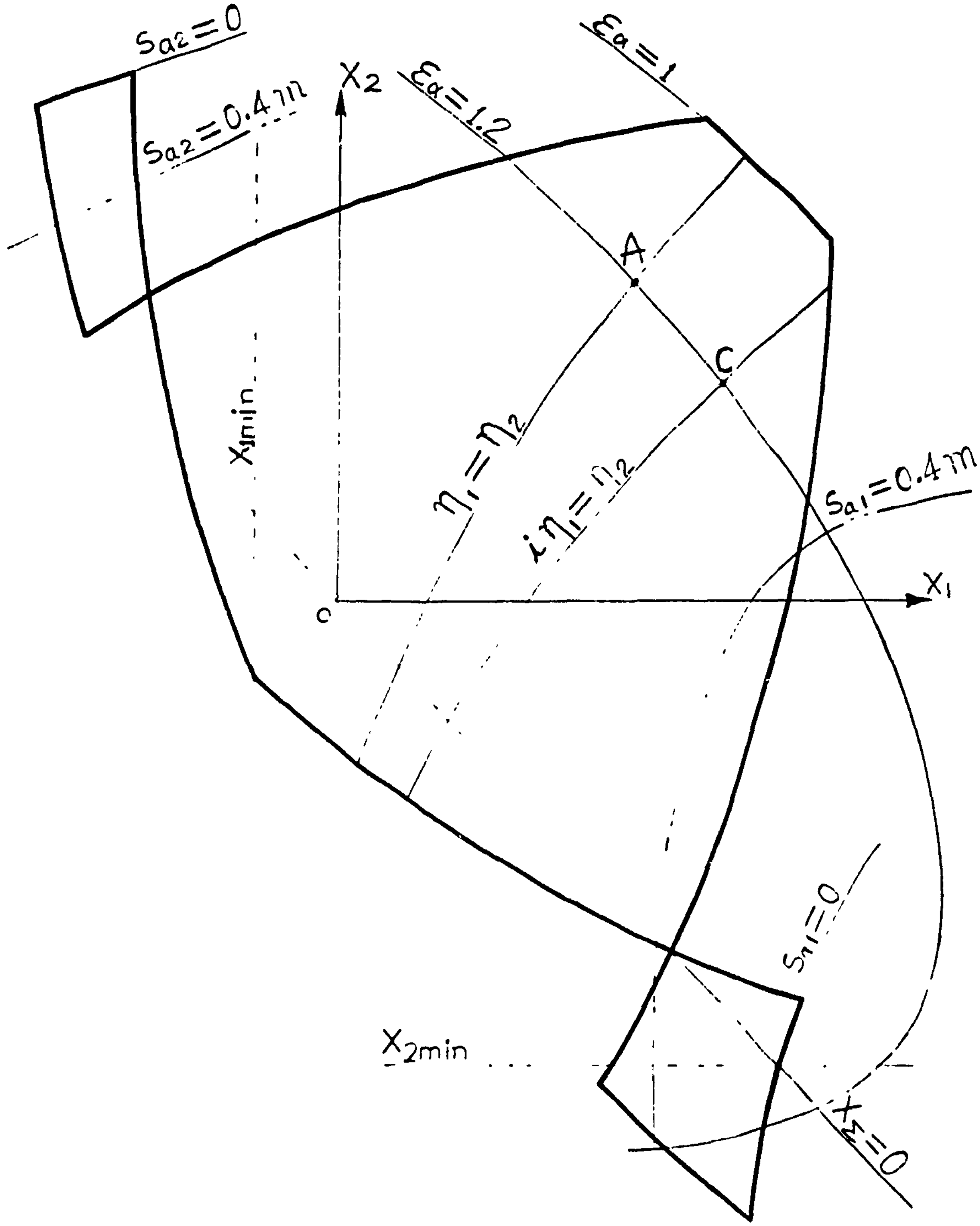


Fig. 2.7. Limit Curves For Modification

$$\sigma_F = \frac{F_{tm} \cdot K_F \cdot Y_{FS} \cdot Y_F \cdot Y_\beta \cdot Y_\varepsilon}{b \cdot m_{nm}} \tag{2.50}$$

Where

- b the face width
- Y_F tooth form factor
- Y_ε contact ratio factor
- Y_β spiral angle factor

The combined modification will cause the variation of coefficient Y_F which, in turn changes the bending stress. The tooth form coefficient Y_F has the following expression. [21]

$$Y_F = \frac{6\left(\frac{h_F}{m_n}\right) \cdot \cos\alpha_{nF}}{\left(\frac{S_{nF}}{m_n}\right)^2 \cos\alpha_n} \tag{2.51}$$

where

- h_F the maximum height of force apply point
- α_{nF} the force apply angle
- S_{nF} the tooth root width

From Figure 2.9 it can be observed that the tooth thickness variation of the normal virtual gear on the reference circle ΔS_n is

$$\Delta S_n = X_t \cdot m_n \tag{2.52}$$

The tooth variation of normal virtual gear on tooth root circle is

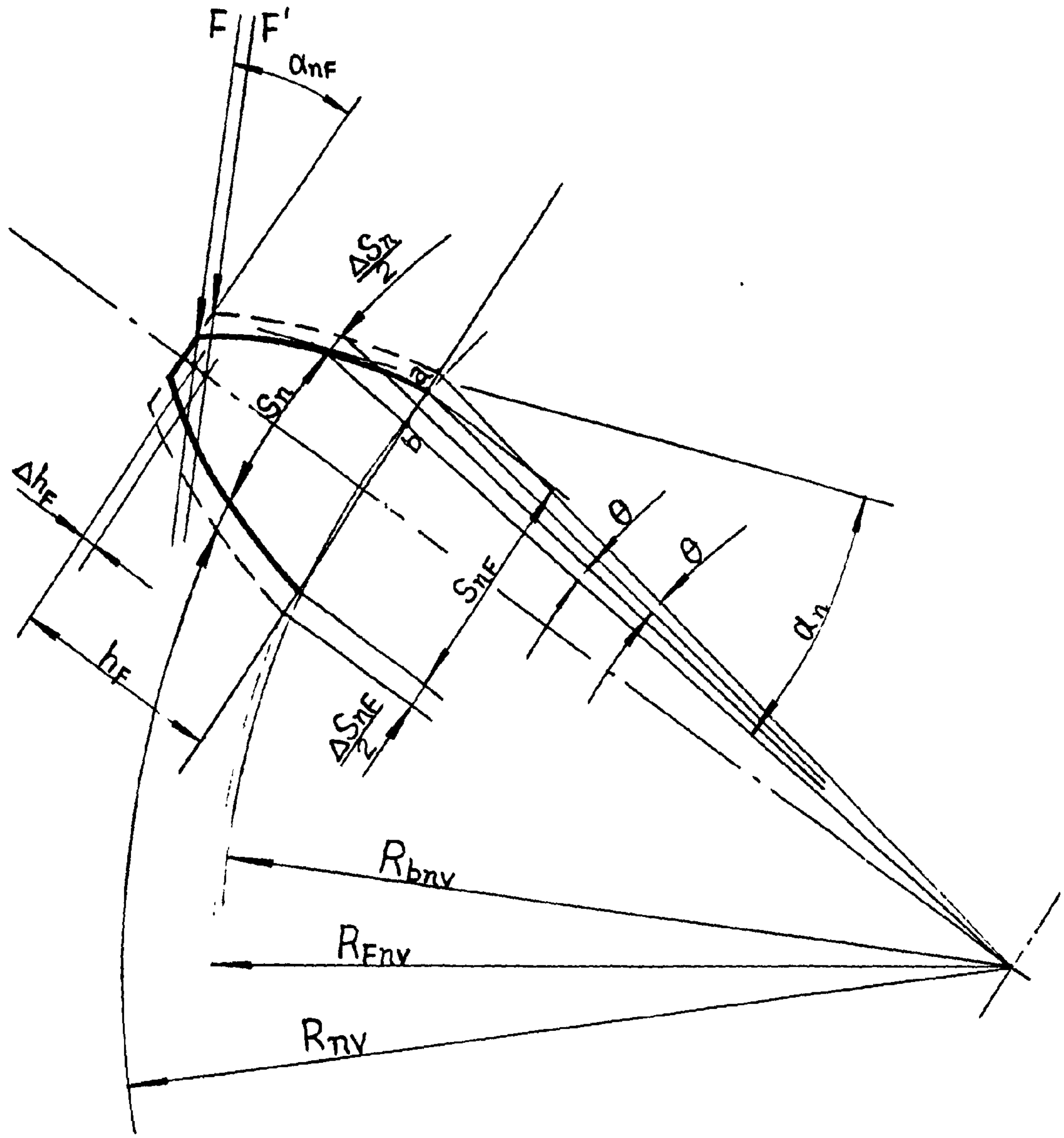


Fig. 2.8. Normal Section of Bevel Tooth

$$\Delta S_{nF} = X_t \cdot m_n \cos\alpha_n \tag{2.53}$$

Denoting the tooth form coefficient of modified bevel by Y'_F and the tooth form coefficient ratio by C gives

$$Y'_F = \frac{6(\frac{h_F}{m_n})\cdot\cos\alpha_{nF}}{(\frac{S'_{nF}}{m_n})^2\cos\alpha_n} \tag{2.54}$$

$$C = \frac{Y'_F}{Y_F}$$

$$= (\frac{S'_{nF}}{S_{nF}})^2$$

$$= (\frac{S_{nF}}{S_{nF} + \Delta S_{nF}})^2$$

$$\approx 1 - \frac{2\Delta S_{nF}}{S_{nF}} \tag{2.55}$$

When an estimation is made on the normal section, the root radius R_{bnv} is approximately taken as the base radius R_{FnV} and the tooth thickness on the root circle has the form:

$$S_{nF} = [\frac{\pi}{2} + 2\cdot X\cdot\tan\alpha + X_t + (\frac{2(h_a^* + X)\cdot\sin\alpha_n}{\cos^2\alpha_n})]\cdot m_n\cdot\cos\alpha_n \tag{2.56}$$

Applying Eq'n (2.56) and Eq'n (2.53) to Eq'n (2.55) gives

$$C = 1 - \frac{2X_t}{0.5\cdot\pi + 2\cdot X\cdot \tan\alpha + X_t + \dfrac{2\cdot(h_a^* + X)\cdot \sin\alpha_n}{\cos\alpha_n^2}} \tag{2.57}$$

By neglecting the influence of the addendum modification coefficient and letting $h_a^* = 0.85$ and $\alpha_n = 20^0$, the following expression is resulted:

$$C \approx 1 - 0.9\cdot X_t + 0.4\cdot X_t^2 \tag{2.58}$$

For a positive modification coefficient X_t, the tooth form coefficient ratio is always less then unity, thus the bending stress is reduced.

CHAPTER 3

DESIGN PARAMETERS AND MANUFACTURING PROCESS

Bevel gears are designed in pairs with each component produced separately. For a single gear, there is no pitch circle but the reference circle. Thus the machining of the modified bevel gear can only be realized by using the reference cone generating method. The reference cone modification discussed thus far, implies that the reference cone angle of a modified bevel gear has a relative pressure angle variation $\Delta\delta$. The corresponding effect of this angle difference on the virtual gear is the addendum modification. The generation of the radially modified bevel gear can be achieved by adjusting the relative position between the work piece and the cutter according to the shifting distance which is determined by the modification coefficient X_{Σ}. The ordinary machine tool and cutter are handy for this purpose. The conventional single blade method or adjustable duplex helical method [22], is also applicable to the generation of bevel gears with relative modification. The primary difficulty encountered is the adjustment of machine tools imposed by the variations of the basic parameters due to the modification. Those parameters such as pitch and module etc. must be accurately calculated .

3.1 The Characteristics of Reference Cone Generation.

The machining of modified bevel driven gears is realized by the generation motion between the work piece and cutter on the reference cone. In order to optimize production, the key elements of the generation method must be followed.

i) The tooth profile of the modified bevel gear changes with different modification coefficients. Thus a former cutter for generating the modified bevel gear does not possess interchangeability. Since the design and manufacturing cost of former cutters is high, it is more economical to utilize single blade or adjustable duplex helical cutters. Unless large production runs are involved.

ii) The final position of the cutter feeding must be at the point where the reference cone surface of generating gear tooth is tangent to the reference cone of the tooth flank.

iii) The gear ratio of the machine tool must change with different generating gears. For a flat-top taped generating gear, assuming δ_p and Z_p to be the reference cone angle and tooth number of a generated bevel gear respectively, and Z_o the tooth number of the cutter, then the required gear ratio has the form of

$$u_o = \frac{Z_o}{Z_p} = \frac{\cos\delta_f}{\sin\delta_p} \tag{3.1}$$

where δ_f is the root cone angle.

3.2 Derivation of the Reference Cone Angle Change

i) For Δr modification

Refer to Figure 2.4. In right-angle triangle ΔOPP_1 and ΔOPP_2, the pitch cone distance R' is equal to OP and

$$P_1P = OP \cdot \tan\angle P_1OP$$

$$= R' \cdot \tan\Delta\delta_1 \tag{3.2}$$

$$P_2P = OP \cdot \tan\angle P_2OP$$

$$= R' \cdot \tan\Delta\delta_2 \tag{3.3}$$

The central distance variation is

$$\Delta a = a'_v - a_v$$

$$= PP_2$$

$$= P_1P + P_2P$$

$$= R' \cdot (\tan\Delta\delta_1 + \tan\Delta\delta_2) \tag{3.4}$$

Since

$$u_v = \frac{P_1P}{P\,P}$$

then

$$u_v = \frac{R' \tan\Delta\delta_2}{R' \tan\Delta\delta_1}$$

$$= \frac{\tan\Delta\delta_2}{\tan\Delta\delta_1}$$

and

$$\tan\Delta\delta_1 = \frac{\tan\Delta\delta_2}{u_v} \tag{3.5}$$

Applying Eq'n (3.5) into Eq'n (3.4) gives:

$$\tan(\delta'_1 - \delta_1) = \tan\Delta\delta_1$$

$$= \frac{\Delta a}{R'} \left(\frac{1}{1 \div u_v} \right) \qquad (3.6)$$

$$= \frac{\Delta a}{R_o} \left(\frac{1}{1 + u_v} \right) \qquad (3.7)$$

$$\tan(\delta'_2 - \delta_2) = \tan\Delta\delta_2$$

$$= \frac{\Delta a}{R'} \left(\frac{u_v}{1 + u_v} \right)$$

$$= \frac{\Delta a}{R_o} \left(\frac{u_v}{1 + u_v} \right) \qquad (3.8)$$

When the cross angle is a right angle

$$u_v = u^2$$

Eq'n (3.7) and Eq'n (3.8) can be rewritten as follows

$$\tan(\delta'_1 - \delta_1) = \frac{\Delta a}{R_o} \left(\frac{1}{1 + u} \right) \qquad (3.10)$$

$$\tan(\delta'_2 - \delta_2) = \frac{\Delta a}{R_o} \left(\frac{u^2}{1 + u} \right) \qquad (3.11)$$

From the above equations, it is obvious that the change of the cone angle is proportional to the central distance variation Δa.

3.3 Difference of Design Parameters and Manufacturing Process

The selection of geometric parameters is important, since it influences the manufacturability and production cost. Improper selection can even make machining impossible. The parameters related to the cutter include the profile angle α_o, the nominal spiral angle β_o.

The gear ratio of machine tool and the variation of the reference angle are related to the adjustment of the machine tool.

3.3.1 Nominal pressure angle standardization

The transverse tooth form of a spiral bevel gear at the midpoint section is not symmetrical, but normal tooth form can be approximately designed into symmetrical. In this investigation, the pressure angle at the reference circle on the midpoint section is chosen as the standard pressure angle, i.e

$$\alpha_{nm1} = \alpha_{nm2}$$

$$= \alpha_{o} \tag{3.12}$$

3.3.2 Nominal spiral angle standardization

The average transverse profile angle α and its corresponding normal profile angle α_n have the following relationship:

$$\tan\alpha = \frac{\tan\alpha_n}{\cos\beta} \tag{3.13}$$

Assume the average transverse pressure angle on the midpoint section is α_m. From Eq'n (3.12) and Eq'n (3.13), it can be seen that

$$\tan\alpha_m = \frac{\tan\alpha_{nm}}{\cos\beta_m} \tag{3.14}$$

Again, assume the average pressure angle of the pitch circle on the midpoint of face width is α'_m. From Eq'n (2.29), it can be concluded that

$$\alpha'_{m1} = inv^{-1}[\cdot \frac{\tan\alpha_{m1}}{Z_{vm}} \cdot (X_{\Sigma} + \frac{X_{t\Sigma}}{2\tan\alpha_{m1}}) + inv\alpha_{m1}]$$

$$\alpha'_{m1} = inv^{-1}[\frac{\tan\alpha_{o}}{Z_{vm}\cos\beta_{m1}} \cdot (X_{\Sigma} + \frac{X_{t\Sigma}\cos\beta_{m1}}{2\tan\alpha_{o}}) + inv(\tan^{-1}\frac{\tan\alpha_{o}}{\cos\beta_{m1}})] \quad (3.15)$$

$$\alpha'_{m2} = inv^{-1}[\frac{\tan\alpha_{o}}{Z_{vm}\cos\beta_{m2}} \cdot (X_{\Sigma} + \frac{X_{t\Sigma}\cos\beta_{m2}}{2\tan\alpha_{o}}) + inv(\tan^{-1}\frac{\tan\alpha_{o}}{\cos\beta_{m2}})] \quad (3.16)$$

For a pair of meshing bevel gears, the pressure angle of the pitch point is equal, ie.

$$\alpha'_{m1} = \alpha'_{m2} \quad (3.17)$$

The condition can be met by setting

$$\beta_{m1} = \beta_{m2}$$
$$= \beta_{o} \quad (3.18)$$

in which β_{o} is the standard spiral angle.

3.4 Transformation of Datum Surface

The design datum surface is usually chosen on the transverse section of the larger end of the virtual gear. However, the process datum surface is at the midpoint of the normal section of the generating gear. The parameters based on the design datum surface must be transferred to the process datum before machining.

3.5 Measurement of Dimensions

In order to check the quality of bevel gears, checking dimensions must be given. They are usually selected based on the normal section of the large end. Since the cone distance of the check point depends on the measuring point, the cone distance of the checking point must be specified. The parameters of the reference circle on the normal virtual gear are usually selected as checking dimensions. The tooth thickness S_{nx} and addendum height h_{nx} change as different cutters and tooth generating methods are used. Refer to Figure 3.1. For depth-wise tapered spiral bevel tooth, the formulas for S_{nx} and h_{nx} are

$$S_{nx} \approx G \cdot S \cdot [1 - \frac{1}{6} \cdot (\frac{S}{d_v})^2] \cdot \cos\beta \tag{3.19}$$

$$h_{nx} \approx G \cdot (h_a + \frac{S^2}{4d_v} \cdot \cos^4\beta) \tag{3.20}$$

where G is the converter of measurement for the checking point which has a cone distance R_x and it can be expressed as

$$G = 1 - \frac{S}{4R} \cdot \sin2\beta \tag{3.21}$$

For nonzero relative modification , the cone distances of a driven gear and driving gear at their large ends are different. This can be verified from the following derivation.

For Δr modification, refer to Figure 2.4. Draw lines PO_{o2} and QO_{o2} perpendicular to the axis line OO_2, it can be observed that

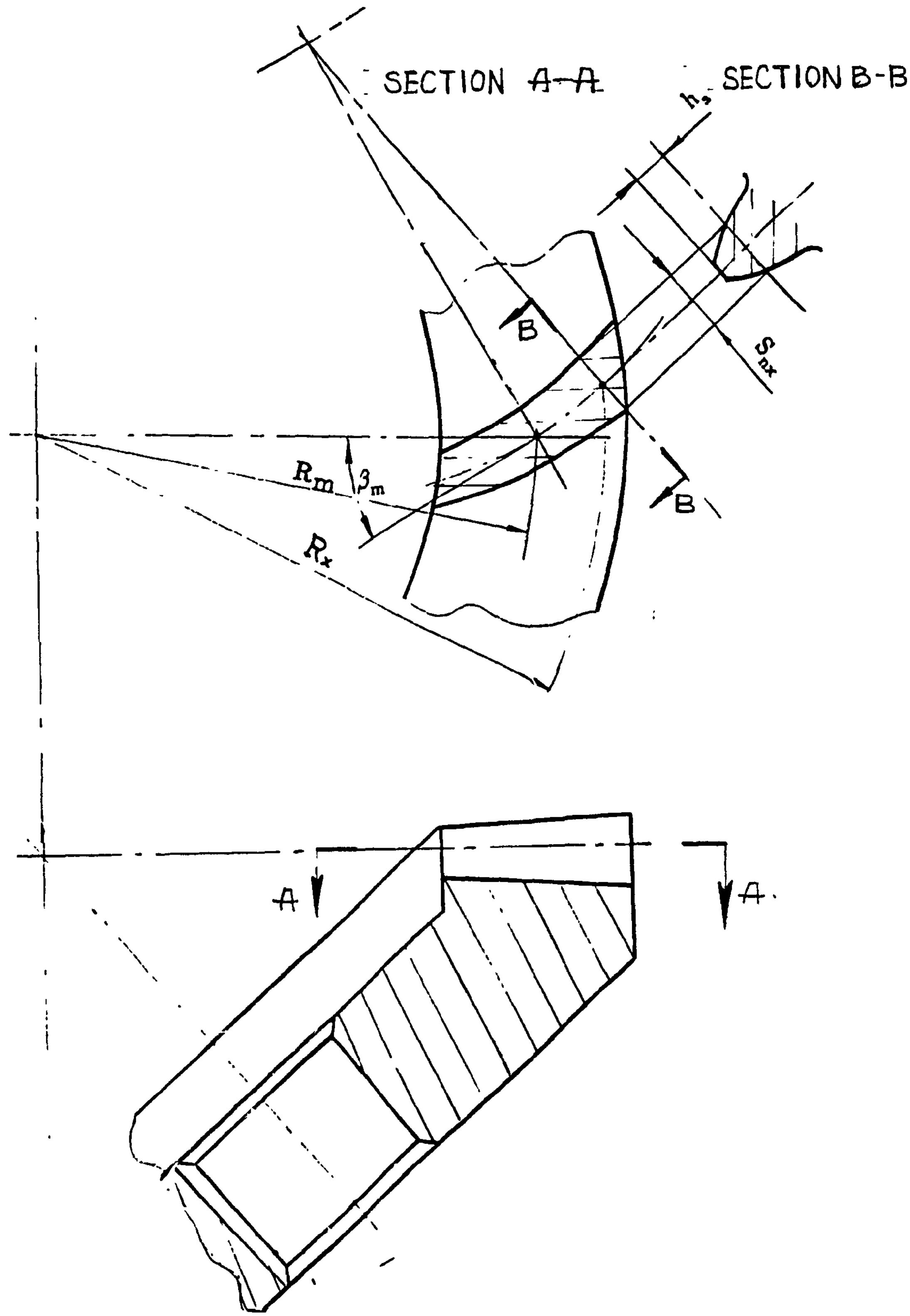


Fig. 3.1 The Measurement Dimensions of Spiral Bevel Gear

$$\frac{P_{o2}Q'_{o2}}{P_2Q_2} = \frac{OP_{o2}}{OP}$$

$$= \frac{OP_o}{OP}$$

$$= \frac{R_o}{R'}$$

$$= \frac{1}{K_{cc}} \qquad (3.22)$$

$$R_2 = OP_{o2}$$

$$= \frac{OP_{o2}Q'_{o2}}{\sin\delta_2}$$

$$= \frac{P_2Q_2}{K_{cc}\sin\delta_2}$$

$$= \frac{R_o\sin\delta'_2}{K_{cc}\sin\delta_2} \qquad (3.23)$$

$$R_1 = \frac{R_o\sin\delta'_1}{K_{cc}\sin\delta_1} \qquad (3.24)$$

For ΔR modification, refer to Figure 2.7. Draw lines P_2Q_2 and P_2Q_{o2} perpendicular to OO_2 and to intersect axis OO_2. The following relationship can be established:

$$R_2 = OP_2$$

$$= \frac{P_2Q_2}{\sin\angle P_2OQ_2}$$

$$= \frac{P_oQ_{o2}}{\sin\delta_2}$$

$$= \frac{0.5 \cdot m_o \cdot Z_2}{\sin\delta_2}$$

$$= R_o \frac{\sin\delta_2'}{\sin\delta_2} \qquad (3.25)$$

$$R_1 = R_o \frac{\sin\delta_1'}{\sin\delta_1} \qquad (3.26)$$

3.6 Machine Tool Selection

Since the tooth generating method for bevel gears with a relative modification is the same as that of the standard ones, machine tools used to produce standard ones can also be utilized to generate relative modified bevels. No special purpose machine tool and cutter are required. Machine tools such as Gleason 116 (U.S.A), Y 22160, YJ 2250 (P.R.C), Spiromatic No.2 (Swiss), 528C, 5284 (R.U.S.S) and ZFTKK (Germany) can all be adopted. For bevel gear cutters, since the standardization of bevel gear parameters is fully considered in design, the conventional cutters are applicable. Such as the single blade cutter with profile angle of 22.5°, 20°, 17.5°, 16° and 14.5°, and adjustable duplex cutter, can all be used. No modification is necessary. The standardization of designing parameters is very important. It greatly reduces the production cost and widen the application scope.

During the selection of the machine tool, the available equipment facilities of individual factory and the volume of production must also be taken into account.

CHAPTER 4
CASE STUDY

The design example discussed in this chapter is a practical problem. It deals with a pair of spiral bevel gears installed in the flexible fly bar conveyor Type S.G.W - 250 gearbox, which is used on the fully mechanized coal working face conveyor. In this gearbox, the bevel gears are subjected to extreme working conditions and frequently break down. In the original configuration of the gearbox, the bevel gears were designed as replaceable components.

It is known that increased bending strength and pitting resistance of helical and spur gears can be easily obtained through the conventional modification. But the application of conventional modification to bevel gears is limited by the fixed cross angle requirement and hence, only zero modification (i.e $X_1 + X_2 = 0$; $X_{t1} + X_{t2} = 0$) can be used to balance the bending strength between pinion and gear. This means that the increased strength of the pinion is obtained at the expense of weakening the strength of the gear. Although in the past, many attempts have been made to solve this problem, solution is still unknown to this practical application. The above-mentioned new modification method offers the potential of significant improvements in such a design.

4.1 Design Requirements

The design requirements are as follows

(1) Speed ratio u: $Z_1 = 14$; $Z_2 = 47$

$u = Z_1/Z_2$

$= 3.36$

(2) Addendum factor h_a^*: $h_a^* = 0.85$

(3) Top clearance factor C^*: $C^* = 0.188$

(4) Design gear life t_h:

(4.1) Gear life of five years

(4.2) 52 weeks a year

(4.3) 6 normal working days a week

(4.4) 3 shifts a day

(4.5) 5 hours normal running for each shift a day

$t_h = 23400$ (hours)

(5) Application factor K_a: $K_a = 1.25$ [23]

A conveyor is usually considered to withstand a medium shock.

(6) Longitudinal load distribution factor for the bending stress $K_{F\beta}$:

Refer to Table 2.

$K_{F\beta} = 1.8$

(7) Transverse load distribution factor $K_{H\beta}$ for the contact stress:

Refer to Table 2.

TABLE 2 LOAD DISTRIBUTION FACTOR

SUPPORTING / SUPPORTING STIFFNESS		SUPPORTING METHOD OF PINION AND GEAR: BOTH PINION AND GEAR BEAR SPAN		ONE SPAN ANOTHER "OVERHUNG"		BOTH PINION AND GEAR"OVERHUNG"	
		K_{FB}	K_{HB}	K_{FB}	K_{HB}	K_{FB}	K_{HB}
VERY GOOD		1.2	1.3	1.35	1.5	1.5	1.7
GENERAL	CROWN TEETH	1.4	1.5	1.6	1.8	1.8	2.0
	NONCROWN TEETH	1.6	1.8	1.8	2.1	2.0	2.3
WORSE	CROWN TEETH	1.55	1.7	1.75	2.0	1.9	2.2
	NONCROWN TEETH	2.2	2.4	2.5	2.8	2.8	3.1

Note: 1. Crown teeth inclouding all spiral bevel gear teeth.

2. Noncrown teeth means that spur bevel gear teeth which is not cut to crown teeth.

$K_{H\beta} = 2.1$

(8) Shaft angle Σ: $\Sigma = 90^{\circ}$

Item (6) and (7) can be calculated by many methods, most of them are complex. In this thesis, the recommended selections of $K_{F\beta}$ and $K_{H\beta}$ are listed in Table 2. Their magnitudes are about 90% of the Japan Gear Manufacture Association (JGMA)recommended values, and are closer to the American Gear Manufacture Association (AGMA) [24] recommended data.

(9) Material and heat treatment:

For comparison purpose, the same material as that in the original components of the S.G.W. - 250 conveyor is assumed for this sample design. A comparison of the result can also be conducted through experimental tests afterwards. The design specifications are listed bellow :

Material: 20 Cr.Mn.Ti.

(a) Chemical composition: C = 0.2%; Cr = 1%; Ti = 0.8%.

(b) Mechanical properties of the material [25]:

breaking strength	$\sigma_b = 1100\ N/mm^2$
yield strength	$\sigma_s = 850\ N/mm^2$
percent elongation	$\delta_s = 10\%$
percent reduction in area	$\psi = 45$

(c) Heat treatment [26]: carbonized.

RC = 56 ~ 62

where RC is Rockwell hardness.

(10) Bending fatigue endurance, $\sigma_{F\ell im1}$ and $\sigma_{F\ell im2}$:

$$\sigma_{F\ell im1} = \sigma_{F\ell im2}$$

$$= 400 \ (N/mm^2) \quad [27]$$

(11) Values of endurance limits $\sigma_{H1} = \sigma_{H2}$ [28]:

$$\sigma_{H\ell im1} = \sigma_{H\ell im2}$$

$$= 1500 \ (N/mm^2)$$

(12) Level of accuracy [29]:

ISO - 9-8-8 EF

(Equivalent to JB 180-60, Grade 9-8-8 Dc Chinese standard)

(13) Diameter of the cutter, d :

$$d = 304.8 \ (mm)$$

(14) Fillet radius of the cutter, ρ^*_{ao}:

$$0.3 < \rho^*_{ao} < 0.4. \ (mm)$$

(15) Spiral angle at the midpoint of face width, β_m:

$$\beta_m = 35^\circ.$$

(16) Cutter profile angle:

$$\alpha = 20^\circ.$$

(17) Failure Possibilities and Probability :

Major: tooth breaking and pitting

Minor: tooth surface wear

Probability of failure: 0.1%. [30]

(18) Power rating of the electric motor:

P = 125 (kw).

(19) Speed of the pinion:

1480 (r.p.m).

(20) Torque output of the electric motor, T :

$$T = 9550 \cdot \frac{\text{Rated Power (KW)}}{\text{Speed (R.P.M.)}}$$

$$\approx 806.58 \text{ (N·m)}$$

4.2 The Initial Design

(23) Torque transmitted by the pinion, T_1:

$$T_1 = T$$

$$= 806.58 \text{ (N.m)}$$

(24) Reference circle diameter of pinion d_1: (According to the pitting resistance design formula) [31]

$$d_1 = 770 \cdot \sqrt[3]{\frac{K_a \cdot K_{H\beta} \cdot T_1}{u \cdot (\sigma_{Hlim})^2}} \qquad (4.1)$$

$$= 109.268$$

(mm) (25) Selection of the

diameter of the pinion, d_1:

$$d_1 = 112 \text{ (mm)}$$

(26) Selection of the tooth number of the pinion, Z_1:

$$Z_1 = 14$$

(27) Selection of the tooth number of the gear, Z_2 :

$$Z_2 = 47$$

(28) Transverse module of the large end, m_t :

$$m_t = d_1/Z_1$$

$$= 8 \text{ (mm)}$$

(29) Speed ratio u :

$$u = Z_2/Z_1$$

$$\approx 3.357$$

(30) Pitch angle of the pinion, δ'_1:

$$\delta'_1 = \tan^{-1}(\frac{1}{u}) \qquad (4.2)$$

$$= 16.587^\circ$$

(31) Pitch cone angle of the gear, δ'_2 :

$$\delta'_2 = \Sigma - \delta'_1 \qquad (4.3)$$

$$= 73.413^\circ$$

(32) Virtual gear ratio u_v :

$$u_v = u^2 \tag{4.4}$$

$$= 11.27$$

(33) Cone distance before modification, R_o:

$$R_o = \frac{d_1}{2\sin\delta'_1} \tag{4.5}$$

$$= 196.168 \text{ (mm)}$$

(34) Gear face width b :

$$\frac{R_o}{3.5} < b < \frac{R_o}{3} \tag{4.6}$$

$$48.334 < b < 56.36 \text{ (mm)}$$

Choose b = 60 (mm)

(35) Gear face width factor ϕ_R :

$$\phi_R = b/R_o \tag{4.7}$$

$$\approx 0.3059$$

4.3. Feasible Range of the Modification Coefficient

Different applications of a spiral bevel gear have different design requirements. In order to increase the pitting resistance and prevent tooth breakage, most of the spiral bevel gears should be designed with spiral angle $\beta_m > 25^o$. In this design, the total contact ratio $\varepsilon > 2$ and $X_\Sigma > 0$ are required to reduce major failures of bevel gears in practice.

4.3.1 Preliminary choice of addendum modification coefficient

The following calculation is based on the assumption that the basic tooth form is the tooth form of transverse virtual gear at the midpoint of face width.

(37) Virtual tooth number of the pinion, Z_{v1} :

$$Z_{v1} = \frac{Z_1}{\cos\delta'_1} \tag{4.8}$$

$$= 14.608$$

(38) Virtual tooth number of the gear, Z_{v2}:

$$Z_{v2} = \frac{Z_2}{\cos\delta'_1} \tag{4.9}$$

$$= 164.638$$

(39) Normal virtual tooth number of the pinion, Z_{vn1} :

$$Z_{vn1} = \frac{Z_{v1}}{\cos^3\beta_m} \tag{4.10}$$

$$= 26.576$$

(40) Normal virtual tooth number of the gear, Z_{vn2} :

$$Z_{vn2} = \frac{Z_{v2}}{\cos^3\beta_m} \tag{4.11}$$

$$= 299.527$$

(41) Virtual reference radius factor of the pinion, r^*_{v1}:

$$r^*_{v1} = 0.5 \cdot Z_{v1} \qquad .4.12)$$

$= 7.304$

(42) Virtual reference radius factor of the gear, r^*_{v2} :

$$r^*_{v2} = 0.5 \cdot Z_{v2} \qquad (4.13)$$

$= 82.319$

(43) Virtual center distance factor before modification, a^* :

$$a^* = r^*_{v1} + r^*_{v2} \qquad (4.14)$$

$= 89.623$

(44) Tangent function of the pressure angle at the midpoint of tooth width, $\tan\alpha_m$:

$$\tan\alpha_m = \tan\alpha_2/\cos\beta_2 \qquad (4.15)$$

$= 0.4443$

(45) Transverse pressure angle at the midpoint of the reference circle, α_m :

$\alpha_m = 23.9568^\circ$

(46) Involute function of the pressure angle at the midpoint of face width, $\mathrm{inv}\alpha_m$:

$\mathrm{inv}\alpha_m = \mathrm{inv}23.9568^\circ$

$= 0.0262$

(47) Pitch distance factor of the virtual base circle at the midpoint of the face width, t_b^* :

$$t_b^* = \pi\cos\alpha_m \tag{4.16}$$

$$= 2.8710$$

(48) Virtual base radius factor at the midpoint of face width of the pinion, r_{bv1}^*:

$$r_{bv1}^* = r_{v1}^* \cdot \cos\alpha_m \tag{4.17}$$

$$= 6.6748$$

(49) Virtual base radius at the midpoint of face width of the gear, r_{bv2}^* :

$$r_{bv2}^* = r_{v2}^* \cos\alpha_m \tag{4.18}$$

$$= 75.2274$$

From [32], it is found that the sum of the addendum modification coefficients for virtual spur or heli l gears, is between -0.8 and 1.2. In this thesis, there will be a discussion on the feasible range of modification coefficient with which the above mentioned special design requirements in section 4.1 are met.

In addendum modification, four kinds of constraint conditions must be satisfied. They are:

(a) No undercut occurs.

(b) Tip of the tooth is not too thin.

(c) No interference occurs.

(d) Transverse contact ratio ε_α is greater than 1.2.

In order to balance the bending strength and increase the pitting resistance, positive modification on the pinion and negative modification on the gear are applied. The upper limit of the positive modification is limited by the tip thickness and the lower limit of the negative modification is limited by the condition that no undercut occurs.

For different cutters, the formula for checking interference condition is different. In this calculation, however, only the rack cutter is considered.

From Figure 15, [33], the feasible regions for the conventional modification are:

$$- 0.6 < X_\Sigma < + 1.2$$

$$- 0.3 < X_1 < + 0.8$$

$$- 0.5 < X_2 < + 0.7$$

With the tooth thickness modification, the feasible region is extended to :

$$-0.8 < X_\Sigma < +1.6$$

$$-0.4 < X_1 < +1.8$$

$$-0.9 < X_2 < +0.8$$

(a) The condition of undercut:

Undercut usually occurs when a large and negative modification coefficient is applied on the bevel gear with a small tooth number. In order to avoid undercut, the following condition should be satisfied

[34].

$$X \geq h_a^* - \frac{Z_v \sin^2\alpha_m}{2}$$

$$X_1 \geq -0.35$$

$$X_2 \geq -12.72$$

The following three constraint conditions (b),(c) and (d) are related to the transmission quality. The requirements are conflicting and compromise has to be made according to the application conditions. Modification coefficient can be optimized to obtain best transmission quality.

(b) Verification of the topland thickness S_{a1} :

The topland will become too thin when a large modification coefficient is applied on the pinion which has a small teeth number [35].

$$S_{a1} = d_{a1} \cdot \left(\frac{\pi}{2 \cdot Z_{v1}} + \frac{2X_1 \cdot \tan\alpha_m}{Z_{v1}} + \mathrm{inv}\alpha - \mathrm{inv}\alpha_{a1}\right) \qquad (4.20)$$

When S_a is too small, the bending strength on the tip is not sufficient. Usually, S_a should be greater than 0.2m. In the new method, since the large tooth thickness modification is introduced, S_a can be reduced to $S_a \geq 0.1m$.

(c) Transverse contact ratio ε_α [36]:

$$\varepsilon_\alpha = \frac{1}{2\pi} \left[Z_{v1} \cdot (\tan\alpha_{a1} - \tan\alpha') + Z_{v2} \cdot (\tan\alpha_{a2} - \tan\alpha')\right] \qquad (4.21)$$

The transverse contact ratio is a very important factor. A large contact ratio will help reducing noise and provide smooth torque transmission. The selection of ε_α depends on the working conditions. As assumed initially, this design is specifically for the production in the underground coal working face. And such an environment can absorb noise, so $\varepsilon_\alpha \geq 1.1$ can be chosen.

(d) Interference check:

Gear interference occurs when the total modification coefficient is too large. The kind of cutter being used is also influential. For a rack cutter, in order to avoid the interference, the following relation should be satisfied: [37]

$$\tan\alpha' - u_v \cdot (\tan\alpha_{a2} - \tan\alpha') \geq \tan\alpha_m - \frac{4 \cdot (h_a^* - X_1)}{Z_{v1} \cdot \sin 2\alpha_m} \quad (4.22)$$

$$\tan\alpha' - \frac{1}{u_v} (\tan\alpha_{a1} - \tan\alpha') \geq \tan\alpha_m - \frac{4 \cdot (h_a^* - X_2)}{Z_{v2} \cdot \sin 2\alpha_m} \quad (4.23)$$

With different modification coefficients, the results of those items are different. In order to find the best engagement condition of modified bevel gears, calculations are done with the coefficients in the region of $-0.8 < X_\Sigma < 1.6$ and $0.1 < X_1 < 0.8$ with a step increment of 0.02.

From the calculation results, X_Σ can be chosen from a larger feasible region because of the application of the new modification method. In this example, it is the goal to improve the bending strength and the pitting resistance of the pinion. By choosing $X_\Sigma = 0.36$, $X_1 = 0.68$ and $X_2 = -0.32$, the calculation result reveals that the noninterference condition is satisfied, and :

$$S^*_{a1} = 0.3833$$

$$\varepsilon_\alpha = 1.1281$$

$$\alpha' = 24.4626^\circ$$

4.3.2 Selection of the tooth thickness modification coefficient

Initial decision can be made by referring to Figure 16 [38]. From this figure , $X_{t1} = 0.10$ is chosen.

(50) Working cycles:

$$N_1 = 60 \cdot n_1 \cdot t_1 \quad (4.24)$$

$$= 2 \times 10^9$$

$$N_2 = N_1/u \quad (4.25)$$

$$= 5.95 \times 10^8$$

(51) Bending strength factors, K_{F1} and K_{F2} :

Both of them are chosen as unity since N_1 and N_2 are greater than 10^7.

(52) Combined factor of the pinion and gear, Y_{Fs1} and Y_{Fs2} :

$$Y_{Fa1} = 2.03$$

$$Y_{Fa2} = 2.1$$

$$Y_{sa1} = 2.075$$

$$Y_{sa2} = 2.00$$

$$Y_{Fs1} = Y_{sa1} \times Y_{Fa1}$$

$$= 2.03 \times 2.075$$

$= 4.21$

$Y_{Fs2} = Y_{sa2} \times Y_{Fa2}$

$= 2.00 \times 2.13$

$= 4.26$

(53) Ratio of the combined tooth form factors K_Y :

$$K_Y = \frac{Y_{FS1}}{Y_{FS2}} \qquad (4.26)$$

$= 0.9882$

(54) The influence coefficient of the tooth thickness modification of the pinion, C_1:

In ISO standard ISO / TC60 / WG6 271D - 274D, the effect of the tooth thickness modification is not considered. It is not reasonable. These are accounted through Eq'n (2.58) i.e:

$$C = 1 - 0.9X_t + 0.4X_t^2 \qquad (4.27)$$

$C_1 = 0.914$

(55) Influence coefficient of the tooth thickness modification of the gear, C_2:

$$C_2 = C_1 \cdot \frac{K_Y}{K_F} \qquad (4.28)$$

$= 0.9032$

And again, C_2 satisfies the following formula :

$$C_2 = 1 - 0.9X_{t2} + 0.4X_{t2}^2 \qquad (4.29)$$

(56) Modification coefficient of the gear, X_{t2} :

Substituting $C_2 = 0.9032$ into Eq'n (4.29), one can get:

$X_{t2} = 0.113$

(57) Total tooth thickness modification coefficient $X_{t\Sigma}$:

$$X_{t\Sigma} = X_{t1} + X_{t2}$$
$$= 0.213$$

(58) Transverse engage angle at the midpoint of the face width α_{vm}:

$$\alpha_{vm} = inv^{-1}\left[\frac{\tan\alpha_m}{a^{\bullet}}\cdot\left(X_1 + X_2 + \frac{X_{t\Sigma}}{2\tan\alpha_m}\right) + inv\alpha_m\right] \quad (4.30)$$

$$\alpha_{vm} = inv^{-1}0.0287$$
$$= 24.7867^{\circ}$$

(59) Center distance ratio K_{cc} :

$$K_{cc} = \cos\alpha_m \,/\, \cos\alpha_{vm} \quad (4.31)$$
$$= 1.0066$$

(60) Center distance departure factor Y :

$$Y = Z_{vm}(K_{cc} - 1) \quad (4.32)$$
$$= 0.5915$$

(61) Addendum variation coefficient ΔH :

$$\Delta H = X_{\Sigma} - Y \quad (4.33)$$
$$= -0.2315$$

ΔH could be positive, negative or zero, because ΔH is a function of α'. X_{Σ} and $X_{t\Sigma}$ have been included in the formula of α', this can be seen in Eq'n (2.29). Y could be larger than X_{Σ}. This is different from conventional concept, when $\Delta H < 0$, the tooth height is not reduced but increased. That is why it was named as "Tooth Height Change Coefficient" instead of "Tooth Height Reduce Coefficient".

From the limited curves for tooth thickness modification of Figure 4.1, one can see that the design with $\Delta H = -0.2315$ is close to the constrained boundary of the coefficient limits. This means that the first choice of $X_{t1} = 0.1$ is not very desirable. It is necessary to modify it and recalculate item (54), (55), (56), (58), (59),(60) and (61), until ΔH falls into the specified location as shown on Table 3. After such a procedure is carried out repeatedly, with $X_{t1} = 0.065$ and $X_{t2} = 0.077$, it can be obtained that $\Delta H = -0.172$, which is around the midpoint of the range. Normally, during the calculation it is necessary to modify the $X_{t\Sigma}$ from time to time, so as to make ΔH in the specific location. Up to now, all of the necessary basic parameters have been decided.

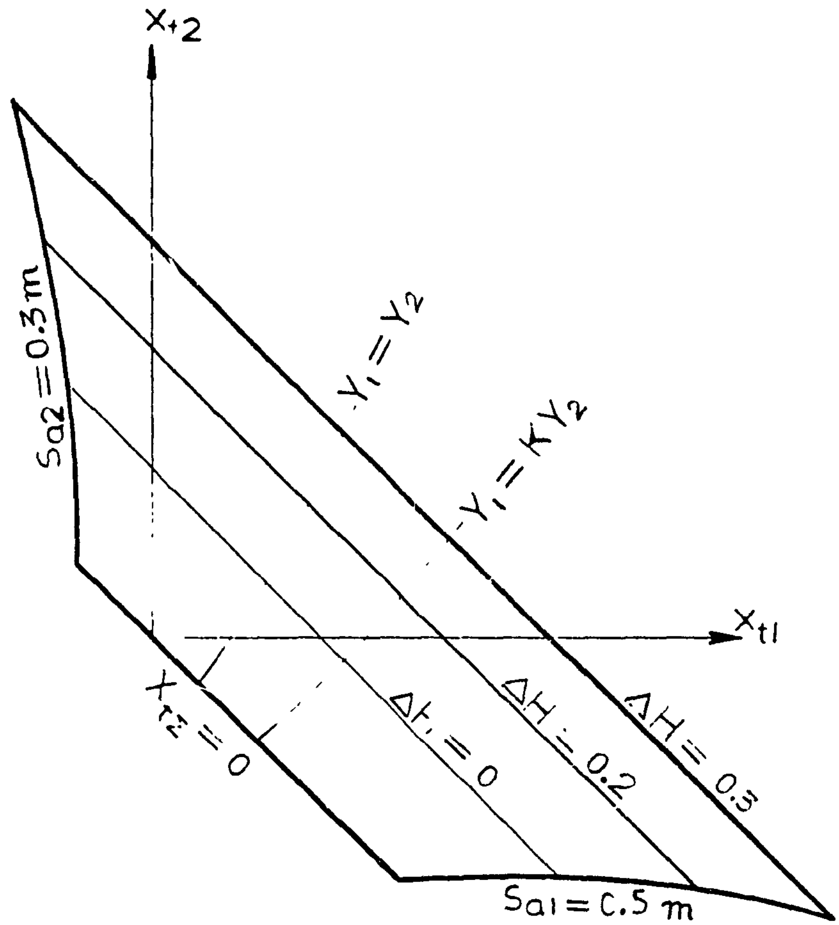


Fig. 4.1. Limit Curvers For Tooth Thickness Modification

X_{t1}	C_1 (54)	C_2 (55)	X_{t2} (56)	$X_{t\Sigma}$ (57)	$inva'_n$	a'_n (58)	K_{cc} (59)	Y (60)	ΔH (61)	
0.1	0.914	0.903	0.114	0.214	0.0291791	24.8816	1.007	0.627	-0.267	
0.09	0.922	0.911	0.104	0.194	0.0290675	24.7581	1.0063	0.565	-0.205	
0.08	0.931	0.920	0.093	0.163	0.0288946	24.7115	1.006	0.538	-0.178	
0.07	0.939	0.928	0.083	0.153	0.0288388	24.6936	1.0058	0.520	-0.16	
0.065	0.943	0.932	0.078	0.143	0.028783	24.6812	1.0057	0.511	-0.151	
0.06	0.947	0.936	0.074	0.134	0.0287328	24.6676	1.0056	0.502	-0.142	

Table. 3. Calculation Results of Items (54) (55) (56) (57) (58) (59) (60) and (61)

4.4 Check of the Bending Stress

Before calculating the geometric dimensions, strength of the driving and gears has to be checked. The following items from (62) to (89) are for this purpose.

(62) The cone distance increment ΔR:

$$\Delta R = \frac{u}{1 + u^2} \cdot Ym \tag{4.34}$$

$$= 1.128$$

(63) Cone distance of the large end after modification, R':

$$R' = R_o \frac{\sin\delta'}{\sin\delta} \tag{4.35}$$

$$R'_1 = 197.34$$

$$R'_2 = 197.601$$

(64) Face width factor ϕ'_R:

$$\phi'_R = b \,/\, R' \tag{4.37}$$

$$= 0.304$$

(65) Conversion factor of the midpoint, M:

$$M = 1 - 0.5 \cdot \phi'_R \tag{4.38}$$

$$= 0.848$$

(66) Circular velocity at the midpoint of the face width, V_m:

$$V_m = \frac{\pi \cdot d_1 \cdot n_1 \cdot M}{6 \times 10^4} \tag{4.39}$$

$= 7.\cdot6 (m/s)$

(67) Dynamic factor of the working condition, K_v [39]:

$$K_v = f_F \cdot (K_{350N}) + 1 \tag{4.40}$$

Since

$$\frac{Z_1 \cdot V_m}{100} = 1.03$$

and

$$K_{350N} = 0.175$$

and

$$K_a \cdot F_{tm} / b = 352 \ (N)$$

$f_F = 1$ calculating factor of dynamic factor

then

$$K_v = 1.173$$

(68) Combined load factor K_F:

$$K_F = K_a \cdot K_v \cdot K_{F\beta} \cdot K_{F\alpha} \tag{4.41}$$

$$= 2.6393$$

According to the ISO standard, $K_{F\alpha}$ should be less than ε_α. So $K_{F\alpha}$ = 1 can be chosen [40].

(69) The relative influence factor of the cutter edge radius $Y_{\rho Prel}$:

It depends on the fillet factor of the tooth root, ρ^*_{ao}. By selecting ρ_{ao} = 0.38 from [41], it can be obtained tnat

$$Y_{\rho Prel} = 1$$

(70) Involute curvature radius factor on the tip point of the pinion, ρ^*_{a1} [42]:

$$\rho^*_{a1} = [(r^*_{v1} + h^*_a + X_1 - \Delta H)^2 - (r^*_{bv1})^2]^{1/2} \quad (4.42)$$

$$= 6.2310$$

(71) Involute curvature radius factor on the tip point of the gear, ρ^*_{a2}:

$$\rho^*_{a2} = [(r^*_{v2} + h^*_a + X_2 - \Delta H)^2 - (r^*_{bv2})^2]^{1/2} \quad (4.43)$$

$$= 35.0714$$

(72) Line of action length at midpoint of tooth width

$$g^*_m = (r^*_{bv1} + r_{bv2}) \cdot \tan\alpha'_n \quad (4.44)$$

$$= 37.6357$$

(73) Contact ratio ε_α [43]:

$$\varepsilon_\alpha = (\rho^*_{a1} + \rho^*_{a2} - g^*_m) / t_b \quad (4.45)$$

$$= 1.2772$$

(74) Contact ratio factor Y_ε [44]:

$$Y_\varepsilon = 1/\varepsilon_\alpha \quad (4.46)$$

$$= 0.7830$$

(75) Spiral angle factor Y_β [45]:

$$Y_\beta = 1 - \beta_m / 120^\circ \quad (4.47)$$

$$= 0.7083$$

(76) The ratio of the cutter radius and the cone distance of the

midpoint of the face width, $d/2R_m$:

$$d/2R_m = \frac{d}{2 \cdot R' \cdot M} \tag{4.48}$$

$$= 0.9109$$

(77) Cutter radius influence factor Y_2:

$$Y_2 = 1$$

(78) Combined bending stress factor of the pinion, Y_1:

$$Y_1 = Y_{FS1} \cdot C_1 \cdot Y_\beta . Y_\varepsilon . Y_2 \tag{4.49}$$

$$= 2.2018$$

(79) Circular force at the midpoint of the face width, F_{tm}:

$$F_{tm} = \frac{2 \cdot T_1 \times 10^3}{d_1 \cdot M} \tag{4.50}$$

$$= 16985 \text{ (N)}$$

(80) Normal module at the midpoint of the face width, m_{nm}:

$$m_{nm} = M \cdot m \cdot \cos\beta_m \tag{4.51}$$

$$= 5.557$$

(81) Bending stress of the pinion and gear, σ_{F1} and σ_{F2}:

$$\sigma_{F1} = \frac{F_{tm} \cdot K_F \cdot Y_1}{b \cdot m_{nm}} \tag{4.52}$$

$$= 363.5 \quad (\text{N/mm}^2)$$

$$\sigma_{F2} = \sigma_{F1} / K_Y$$

$= 367.6 \quad (N/mm^2)$

Compare σ_{F1} and σ_{F2} with those of the conventional design method by knowing

$\sigma_{F1} = 507.6 \quad (N/mm^2)$ [46]

$\sigma_{F2} = 519.3 \quad (N/mm^2)$ [47]

The new method decreases the bending stresses of the gear and pinion by about 28% and 29% respectively.

(82) Dimension factor Y_x [48]:

$$Y_x = 1.21m^{(-1/4)} \qquad (4.53)$$

$$= 1$$

(83) Relative sensitivity factor of the material, $Y_{\delta relt}$ [49]:

Since $Y_{sa} = 2.076$

then $Y_{\delta relt} = 1.01$

(84) Surface condition factor Y_{Rrelt} [50]:

Tooth surface roughness grade is $\sqrt[3.2]{\ }$ $R_z = 4.17$ (μm)

$Y_{Rrelt} = 1.025$

(85) Stress concentration factor Y_{sat} [51]:

$Y_{sat} = 2$

(86) Working life factor Y_{N1}:

For $N_1 > 10^7$, $Y_{N1} = 1$

(87) Combined bending stress limit factor of the pinion, Y_{P1}:

$$Y_{P1} = Y_{n1} \cdot Y_x \cdot Y_{sat} \cdot Y_{\delta rel} \cdot Y_{Rrelt} \tag{4.54}$$
$$= 2.05$$

(88) Safety coefficient in the bending stress, S_{Fmin}:

For 99.9% reliability, $S_{Fmin} = 1.5$

(89) Allowable bending stress limits, σ_{FP1} and σ_{FP2}:

$$\sigma_{FP1} \approx \sigma_{FP2}$$
$$= Y_{P1} \cdot \sigma_{Flim} / S_{Flim} \tag{4.55}$$
$$= 535.6 \quad (N/mm^2)$$

(90) Confirmation:

Since $\sigma_{F1} < \sigma_{FP1}$ and $\sigma_{F2} < \sigma_{Fp}$, the bending strength is sufficient.

4.5 Check of the Pitting Endurance

(91) Combined load coefficient K_h:

$$K_h = K_a \cdot K_v \cdot K_{H\beta} \cdot K_{H\alpha} \tag{4.56}$$
$$= 3.084$$

Since $K_{H\alpha} < \varepsilon_\alpha$, $K_{H\alpha}$ is here chosen as unity [52].

(92) Zone factor for Hertzian pressure at pitch point, Z_H:

Zone factor Z_H can be calculated by Eq'n (2.48)

$$Z_H = 2\cdot\sqrt{\frac{\cos\beta_{bm}}{\sin 2\alpha'_{tm}}}$$

$$= 2.13$$

(93) Elasticity factor of the material, Z_E [53]:

$$Z_E = 189.8\cdot\sqrt{N/mm^2}$$

(94) Overlapping ratio ε_β :

$$\varepsilon_\beta = \frac{b\cdot\tan\beta_m}{\pi\cdot m}\cdot\frac{1}{(1 - 0.5\phi_R)} \quad (4.58)$$

$$= 1.9735$$

(95) Contact ratio factor Z_ε [54]:

For $\varepsilon_\beta \geq 1$

$$Z_\varepsilon = \sqrt{\frac{1}{\varepsilon_\alpha}} \quad (4.59)$$

$$= 0.8849$$

(96) Spiral angle factor Z_β [55]:

$$Z_\beta = 0.5\cdot(1 + \sqrt{\cos\beta_m}\ ) \quad (4.60)$$

$$= 0.9525$$

(97) Addendum modification effect factor Z_b [56]:

ISO / TC60 / WG6 /272D recommends the following formula for calculating Z_b

$$Z_b = \sqrt{\frac{\rho_{c1}\cdot\rho_{c2}}{\rho_{b1}\cdot\rho_{b2}}} \tag{4.61}$$

in which ρ_{c1} and ρ_{c2} are 'he involute curve radii of normal virtual gears of the pinion and gear respectively. By referring to Figure 4.2, their product can be found as following:

$$\rho_{c1}\cdot\rho_{c2} = \frac{(1 - 0.5\cdot\phi_R)^2\cdot m^2\cdot Z_1\cdot Z_2\cdot\sin^2\alpha_{nm}}{4\cdot\cos^4\beta_{bm}\cdot\cos\delta_1'\cdot\cos\delta_2'} \tag{4.62}$$

$$= 6416$$

Since the spiral angle on the base circle is:

$$\beta_{bm} = \tan^{-1}(\tan\beta_m\cdot\cos\alpha_{tm}) \tag{4.63}$$

$$= 32.6146^\circ$$

and the addendum circle radius r_{avnm} is

$$r_{avnm} = \frac{M\cdot m}{\cos^2\beta_{bm}}\left[\frac{Z}{2\cdot\cos\delta'} + (h^* + X - \Delta H)\right] \tag{4.64}$$

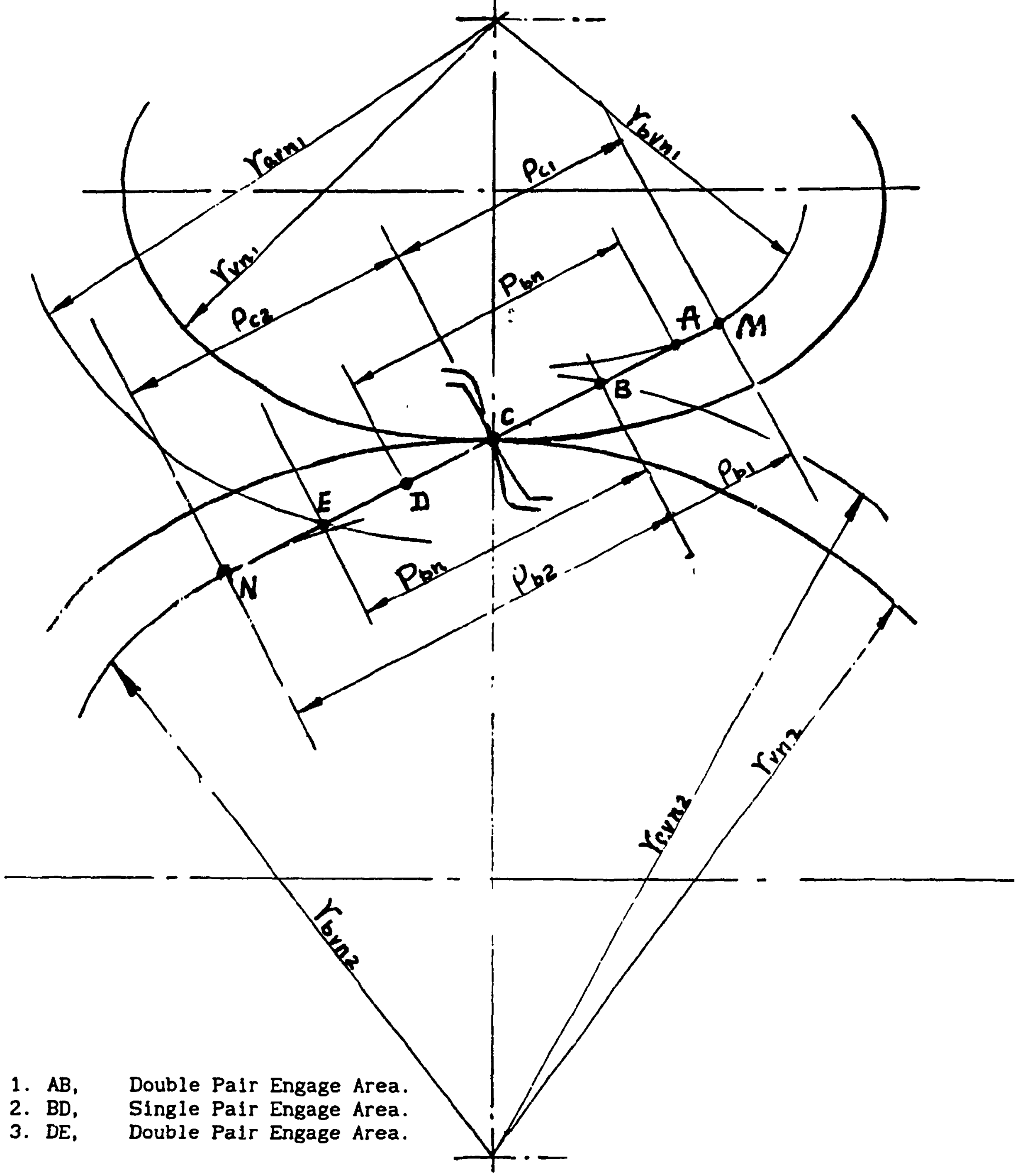
$$= 85.9211$$

and the base circle radius r_{bvnm1} is

$$r_{bvnm1} = \frac{M\cdot m\cdot Z\cdot\cos\alpha_{nm}}{2\cdot\cos^2\beta_{bm}\cdot\cos\delta'} \tag{4.65}$$

$$= 65.6265$$

then the pressure angle of the midpoint of the face width on the normal section of the tooth tip can be found as



1. AB, Double Pair Engage Area.
2. BD, Single Pair Engage Area.
3. DE, Double Pair Engage Area.

Fig. 4.2. Normal Virtual Gear of Midpoint of Face Width

$$\alpha_{anm1} = \cos^{-1}\left(\frac{r_{bvnm1}}{r_{avnm1}}\right) \tag{4.66}$$

$$= 40.1997^{\circ}$$

Eventually,

$$\rho_{b1} = M\cdot m\cdot \cos\alpha_{0}\cdot\frac{Z_{1}\cdot\tan\alpha_{anm1}}{2\cdot\cos^{2}\beta_{bm}\cdot\cos\delta'_{1}} - \pi\cos\beta_{m} \tag{4.67}$$

$$= 39.052$$

and

$$\rho_{b2} = \frac{M\cdot m\cdot\sin\alpha_{nm}}{2\cdot\cos^{2}\beta_{bm}}\cdot\left(\frac{Z_{1}}{\cos\delta_{w1}} + \frac{Z_{2}}{\cos\delta_{w2}}\right) - \rho_{b1} \tag{4.68}$$

$$= 254.0452$$

Substituting all of the parameters found in Eq'n (4.62), (4.67) and (4.68) to Eq'n (4.61), one can get $Z_b = 0.787$.

(98) Bevel gear factor Z_k:

According to the recommendation of ISO /TC60 / WG6 272D, the influences of the tooth profile difference between bevel gear and virtual gear, assembling error and rigidity variation etc. have to be considered. They all influence the pitting resistance. Usually Z_k = 0.85 is used.

(99) Combined coefficient of the contact stress, Z_h:

$$Z_{h} = Z_{H}\cdot Z_{E}\cdot Z_{\varepsilon}\cdot Z_{\beta}\cdot Z_{b}\cdot Z_{k}\cdot \tag{4.69}$$

$$= 77.35$$

(100) Contact stress σ_{h1}:

$$\sigma_{H1} = Z_h \cdot \sqrt{\frac{K_h \cdot F_{tm}}{d_{m1} \cdot b} \cdot \frac{(u^2 + 1)^{1/2}}{u}} \tag{4.70}$$

$$= 758 \ (N/mm^2)$$

$$\sigma_{H2} \approx \sigma_{H1}$$

$$\approx 758 \ (N/mm^2)$$

Compare σ_{H1} and σ_{H2} with those of the conventional design method by known $\sigma_{H1} = \sigma_{H2} = 1119$ (N/mm^2). The new method decreases the contact stress about 30%.

(101) Life factor of the pitting fatigue, Z_n:

$$Z_N \approx Y_n$$

$$= 1$$

(102) Surface hardness factor Z_w:

$$Z_w = 1$$

(103) Lubricant factor Z_L:

Assume No 40 oil is chosen as the lubricant. So $\mu = 65$ (mm^2/s) and $Z_L = 0.93$.

(104) Velocity factor Z_v [57]:

Velocity of the pitch point at the midpoint of the face width.

Since $V = V_m / M$

$= 8.679$ (m/s)

then $Z_v = 0.98$

(105) Average roughness of the tooth surface, R_z:

Surface roughness [3.2]

$$R_z = \frac{R_{z1} + R_{z2}}{2} \cdot \frac{100}{A_w} \tag{4.71}$$

$= 4.17$ (μ/m)

in which $A_w = \dfrac{1 + u_v}{u}$

(106) Coefficient of the surface roughness of the tooth surface, Z_R [58]:

$Z_R = 0.97$

(107) Size factor of the pinion and gear, Z_x:

$$Z_x = 1.05 - 0.005m_n \tag{4.72}$$

≈ 1.01

(108) Combined factor for the contact stress limits, Z_P:

$$Z_P = Z_n \cdot Z_w \cdot Z_L \cdot Z_v \cdot Z_R \cdot Z_x \tag{4.73}$$

$= 0.852$

(109) Safety factor for the contact stress S_{Hmin}:

$S_{Hmin} = 1.25$

(110) Allowable pitting fatigue stress limits of the pinion and

gear, σ_{HP}:

$$\sigma_{HP} = Z_P \cdot \sigma_{Hlin} / S_{Hmin} \tag{4.74}$$

$$= 1048 \quad (N/mm^2)$$

(111) Contact stress safety confirmation:

Since $\sigma_{H1} = \sigma_{H2}$

$< \sigma_{HP}$

the safety condition is satisfied.

4.6 Verification the engagement properties of the large end of the bevel gears

(114) Spiral angle of the large end on reference circle, β_L:

$$\beta_L = \sin^{-1}[\frac{R'}{d_2}(1 - M^2) + M \cdot \sin\beta_m] \tag{4.75}$$

$$= 41.9291^{\circ}$$

(115) Tangent Function of the pressure angle on the large end of the reference circle:

$$\tan\alpha_L = \tan\alpha_o / \cos\beta_L \tag{4.76}$$

$$= 0.4892$$

(116) Pressure angle on the large end of the reference circle α_L:

$$\alpha_L = \tan^{-1} 0.4892$$

$$= 26.0679^{\circ}$$

(117) Involute function of the mesh angle at the large end, $\text{inv}\alpha'_L$:

$$\text{inv}\alpha'_L = \frac{\tan\alpha}{Z_{vm}}\cdot\left(X_\Sigma + \frac{X_{t\Sigma}}{2\cdot\tan\alpha}\right) + \text{inv}\alpha_L \tag{4.77}$$

$$= 0.03178$$

(118) Pressure angle of the large end, α'_L:

$$\alpha'_L \approx 26.75^\circ$$

(119) Center distance factor after modification, α'^{*}

$$a'^{*} = a^{*}\cdot\cos\alpha \;/\; \cos\alpha' \tag{4.78}$$

$$= 90.1554$$

(120) Center distance departure coefficient Y:

$$Y = a'^{*} - a^{*}$$

$$= 0.5324$$

(121) Tooth height variation coefficient ΔH:

$$\Delta H = X_\Sigma - Y \tag{4.79}$$

$$= -\,0.172$$

(122) Involute curvature factor of the pinion at the pitch point, ρ_1^{*}:

$$\rho_1^{*} = r_{bv1}\cdot\tan\alpha'_L$$

$$= 3.3644$$

(123) Involute curvature factor of the gear at the pitch point, ρ_2^{*}:

$$\rho_2^{*} = r_{bv2}\cdot\tan\alpha'_L \tag{4.80}$$

$= 37.9177$

(124) Line of action length factor g^{*}:

g^{*} can be expressed in terms of the sum of (122) and (123). So

$g^{*} = 41.2821$.

(125) Virtual tip circle radius of the pinion, r^{*}_{av1}

$$r^{*}_{av1} = r^{*}_{v1} + h^{*}_{a} + X_{1} - \Delta H \tag{4.81}$$

$= 9.006$

(126) Virtual addendum radius of the gear, r^{*}_{av2}

$$r^{*}_{av2} = r^{*}_{v2} + h^{*}_{a} + X_{1} - \Delta H \tag{4.82}$$

$= 83.0291$

(127) Tip pressure angle of the pinion, α_{a1}:

$$\alpha_{a1} = \cos^{-1}\left(\frac{r_{bv1}}{r_{av1}}\right) \tag{4.83}$$

$= 42.2381^{\circ}$

(128) Tip pressure angle of the gear, α^{*}_{a2}:

$$\alpha^{*}_{a2} = \cos^{-1}\left(\frac{r_{bv2}}{r_{av2}}\right) \tag{4.84}$$

$= 27.0414^{\circ}$

(129) Tooth tip profile curvature radius of the pinion, ρ^{*}_{a1}:

$$\rho^{*}_{a1} = \sqrt{r_{av1}^{2} + r_{av1}^{2}} \tag{4.85}$$

$$= 6.1694$$

(130) Tip tooth profile curvature radius of the gear, ρ^{*}_{a2}:

$$\rho^{*}_{a2} = \sqrt{r_{av12}^{2} + r_{av2}^{2}} \tag{4.86}$$

$$= 37.7442$$

(131) Root profile curvature radius factor of the gear, ρ^{*}_{F2}:

$$\rho^{*}_{F2} = g^{*} - \rho^{*}_{a1} \tag{4.87}$$

$$= 35.1127$$

(132) Root profile curvature radius factor of the pinion, ρ^{*}_{F1}:

$$\rho^{*}_{F1} = g^{*} - \rho^{*}_{a2} \tag{4.88}$$

$$= 3.5379$$

(133) Pitch factor on the large end, t^{*}_{L}:

$$t^{*}_{L} = \pi \cdot \cos\alpha_{L} \tag{4.89}$$

$$= 2.822$$

(134) The ratio of the center distance change, K_{cc}:

$$K_{cc} = a'^{*} / a^{*} \tag{4.90}$$

$$= 1.0058$$

4.7 Geometrical Dimension Design and Analysis

(135) Virtual reference radius of the pinion, r_{v1}:

$$r_{v1} = m \cdot r^{*}_{v1} \tag{4.91}$$

$$= 58.432 \text{ (mm)}$$

(136) Virtual reference radius of the gear, r_{v2}:

$$r_{v2} = m \cdot r^{*}_{v2} \tag{4.92}$$

$$= 658.552 \text{(mm)}$$

(137) Reference circle diameter of the pinion, d:

$$d_1 = m \cdot Z_1 \tag{4.93}$$

$$= 112 \quad \text{(mm)}$$

(138) Reference circle diameter of the gear, d:

$$d_2 = m \cdot Z_2 \tag{4.94}$$

$$= 376 \quad \text{(mm)}$$

(139) Pitch diameter of the pinion, d':

$$d'_1 = k_{cc} \cdot d_1 \tag{4.95}$$

$$= 112.650 \text{ (mm)}$$

(140) Pitch diameter of the gear, d'_2:

$$d'_2 = k_{cc} \cdot d_2 \tag{4.96}$$

$$= 378.181 \text{ (mm)}$$

(141) Standard dedenda of the pinion, h_{F1}:

$$h_{F1} = m \cdot (h^{*}_{a} + C^{*} - X_1) \tag{4.97}$$

$$= 2.864 \text{ (mm)}$$

(142) Standard dedenda of the gear, h_{F2}:

$$h_{F2} = m\cdot(h_a^* + C^* - X_2) \quad (4.98)$$

$$= 10.864 \text{ (mm)}$$

(143) Virtual pitch circle radius of the pinion, r'_{v1}:

$$r'_{v1} = K_{cc}\cdot r_{v1} \quad (4.99)$$

$$= 58.771 \text{ (mm)}$$

(144) Virtual pitch radius of the pinion, r'_{v2}:

$$r'_{v2} = K_{cc}\cdot r_{v2} \quad (4.100)$$

$$= 662.372 \text{ (mm)}$$

(145) Virtual root circle radius of the pinion, r_{Fv1}:

$$r_{Fv1} = r_{v1} - h_{F1} \quad (4.102)$$

$$= 55.568 \text{ (mm)}$$

(146) Virtual dedendum radius of the pinion, r_{Fv2}:

$$r_{Fv2} = r_{v2} - h_{F2} \quad (4.104)$$

$$= 647.688 \text{ (mm)}$$

(147) Modified dedenda the pinion, h'_{F1}:

$$h'_{F1} = r'_{v1} - r_{Fv1} \quad (4.105)$$

$$= 3.203 \text{ (mm)}$$

(148) Modified dedenda of the gear, h'_{F2}:

$$h'_{F2} = r'_{v2} - r_{Fv2} \quad (4.106)$$

$$= 14.684 \text{ (mm)}$$

(149) Dedendum angle on the pitch circle of the pinion, θ'_{F1}:

$$\theta'_{F1} = \tan^{-1}\frac{h'_{F1}}{R'} \quad (4.108)$$

$$= 0.930^\circ$$

(150) Dedendum angle on the pitch circle of the gear, θ'_{F2}: θ'_{F2} =

$$\tan^{-1}\frac{h'_{F2}}{R'} \quad (4.109)$$

$$= 4.2565^\circ$$

Item (149) and (150) are usable only for standard reducing tooth form.

(151) Total dedendum angle of the pinion and gear, $\theta_{F\Sigma S}$: (For standard tapered tooth form)

$$\theta_{F\Sigma S} = \theta'_{F1} + \theta'_{F2} \quad (4.110)$$
$$= 5.1865^\circ$$

(152) Total dedendum angle of the double reduced tooth form, $\theta_{F\Sigma D}$:

$$\theta_{F\Sigma D} = (1 - 2\cdot R'\sin\beta' / d_2)\cdot(\frac{180\cdot\sin\delta'_2}{Z_2\cdot\tan\alpha_2\cdot\cos\beta_m}) \quad (4.111)$$

$$= 4.5508^\circ$$

(153) Total addendum angle of tapper ed tooth form, $\theta_{F\Sigma T}$:

$$\theta_{F\Sigma T} = 1.3\cdot\theta_{F\Sigma S} \quad (4.112)$$

$$= 6.7425^\circ$$

(154) Total dedendum angle of the equal addendum tooth form, $\theta_{F\Sigma}$:

Choose the smaller one of (152) and (153), namely (152),

$\theta_{F\Sigma} = 4.5588^{\circ}$

(155) Semiangle of the cone angle increment, $\Delta\theta_F$:

$$\Delta\theta_F = (\theta_{F\Sigma} - \theta_{F\Sigma S}) / 2 \qquad (4.113)$$

$$= -0.6297^{\circ}$$

(156) Root angle from the pitch circle of the pinion, θ'_{F1}:

$$\theta'_{F1} = \theta'_{F1} + \theta_F \qquad (4.114)$$

$$= 0.3003^{\circ}$$

(157) Root angle on the pitch circle of the gear, θ'_{F2}:

$$\theta'_{F2} = \theta'_{F2} + \theta_F \qquad (4.116)$$

$$= 3.6268^{\circ}$$

(158) Root cone angle of the pinion, δ_{F1}:

$$\delta_{F1} = \delta'_1 - \theta'_{F1} \qquad (4.117)$$

$$= 16.2178^{\circ}$$

(159) Root cone angle of the gear, δ_{F2}:

$$\delta_{F2} = \delta'_2 - \theta'_{F2} \qquad (4.118)$$

$$= 69.7862^{\circ}$$

(160) Addendum angle of the pinion, δ_{a1}:

$$\delta_{a1} = \delta'_1 + \theta'_{F2} \qquad (4.119)$$

$= 20.2138^\circ$

(161) Addendum angle of the gear, δ_{a2}:

$$\delta_{a2} = \delta'_2 + \theta'_{F1} \tag{4.120}$$

$= 73.5133^\circ$

(162) Reference cone angle of the pinion, δ_1:

$$\delta_1 = \delta'_1 - \tan^{-1}\frac{Y \cdot m}{(1 + u_v) \cdot R_2} \tag{4.121}$$

$= 16.4856^\circ$

(163) Reference cone angle of the pinion, δ_2:

$$\delta_2 = \delta'_2 - \tan^{-1}\frac{u_v}{(1 + u_v) \cdot R_2} \cdot \frac{Y \cdot m}{R_2} \delta_2 \tag{4.122}$$

$= 72.0705^\circ$

(164) Virtual addendum circle radius of the pinion, r_{av1}:

$$r_{av1} = r^*_{a1} \cdot m \tag{4.123}$$

$= 72.048$ (mm)

(165) Addendum circle radius of equivalent gear, r_{av2}:

$$r_{av2} = r^*_{a2} \cdot m \tag{4.124}$$

$= 664.168$ (mm)

(166) Addendum diameter of the pinion, d_{a1}:

$$d_{a1} = 2 \cdot r_{av1} \cdot \cos\delta'_1 \tag{4.125}$$

$= 138.100$ (mm)

(167) Addendum diameter of the gear, d_{a2}:

$$d_{a2} = 2 \cdot r_{av2} \cdot \cos\delta'_2$$

$$= 383.6428 \text{ (mm)}$$

(168) Circular tooth thickness factor on the reference circle of the pinion, S_1^* & tooth thickness S_1:

$$S_1^* = \frac{\pi}{2} + 2 \cdot X_1 \cdot \tan\alpha + X_{t1} \qquad (4.126)$$

$$= 2.3011$$

$$S_1 = S_1^* \cdot m$$

$$= 18.724 \text{ (mm)}$$

(169) Circular tooth thickness on the reference circle of the gear, S_2:

$$S_2 = m \cdot \left(\frac{\pi}{2} + 2 \cdot X_1 \cdot \tan\alpha + X_{t1}\right) \qquad (4.127)$$

$$= 10.6777 \text{ (mm)}$$

(170) Total tooth depth h:

$$h = m \cdot (2 \cdot h_a^* + C^* - \Delta H) \qquad (4.128)$$

$$= 16.48 \text{ (mm)}$$

(171) Addendum on the pitch circle of the pinion, h'_{a1}:

$$h'_{a1} = h - h'_{F1} \qquad (4.129)$$

$$= 13.277$$

(172) Addendum on the pitch circle of the gear, h'_{a2}:

$$h'_{a2} = h - h'_{F2} \tag{4.130}$$
$$= 1.796$$

(173) Distance between pitch apex to the crown of the pinion, A_{a1}:
See Figure 4.2.

$$A_{a1} = R'\cos\delta'_1 - h'_{a1}\sin\delta_1 \tag{4.131}$$
$$= 185.923 \text{ (mm)}$$

(174) The distance between pitch apex to the crown of the gear, A_{a2}:
See Figure 4.2.

$$A_{a2} = R'\cos\delta'_2 - h'_{a2}\cdot\sin\delta'_2 \tag{4.132}$$
$$= 54.315 \text{ (mm)}$$

(175) Assembly distance of the pinion and gear, A_1 and A_2:

Those two parameters are taken from the original design and are given as follows:

pinion: $A_1 = 205$ (mm)

gear: $A_2 = 188.4$ (mm)

(176) The distance between crown and the back of the pinion, H_{a1}:
See Figure 4.2.

$$H_{a1} = A_1 - A_{a2} \tag{4.133}$$
$$= 19.068 \text{ (mm)}$$

(177) The distance between crown and the back of the gear, H_{a2}:
See Figure 4.2.

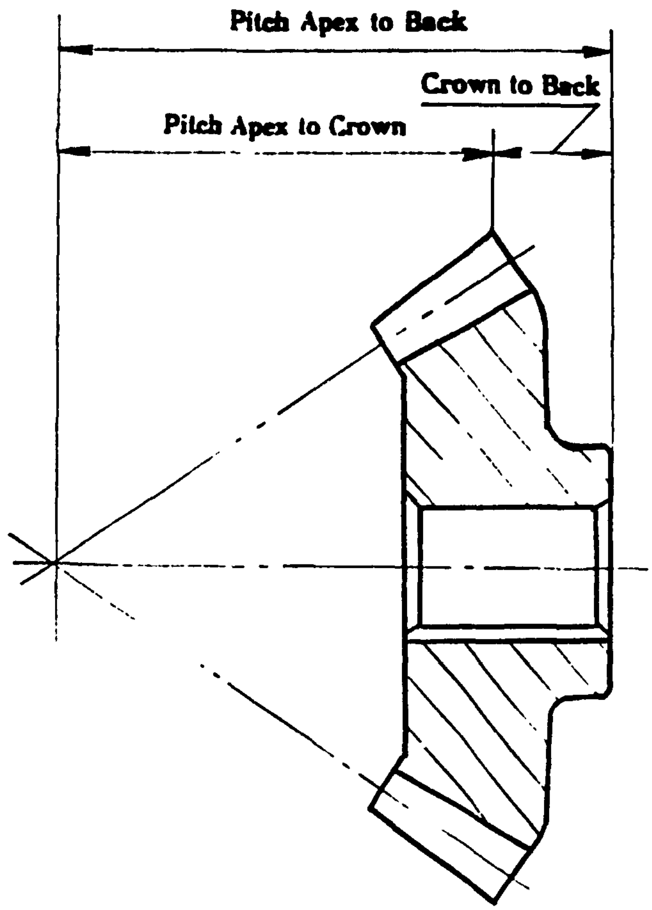


Fig. 4.3. The Distance of Pitch Apex to Crown and Crown to Back.

$$H_{a2} = A_2 - A_{a2} \tag{4.134}$$

$$= 134.085 \text{ (mm)}$$

4.8 Design Layout

Two detail drawings Figure 4.4 and 4.5 have been attached for illustrating this design.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

Computer literature survey shows that no work related to the new modification methods have been done. Let alone the practical application. Based on the analysis presented in this thesis, the new modification method has many advantages over the conventional modification method.

Theoretical analysis and sample design in the foregoing chapters have demonstrated that the principle of the new modification of spiral bevels is correct. The conventional modification can be considered as a special case of the new modification. All of the characteristics associated with the conventional modification method can be easily realized by the new modification method, and the action quality obtained with the new modification method is much better. But most of the characteristics realized by the new method are impossible for the conventional method.

5.1 Conclusions

i) Theoretical analysis has shown that the bending stress and contact stress imposed on bevel gears can be reduced by 28% and 30% respectively as compared with those with traditional modification method. Thus under the same application condition, the working life of bevel gears is lengthened, and the possibility of tooth breakage is reduced.

ii) Since pressure angle is increased and contact stress is reduced after modification, the scuffing problem, a troublesome one in large power transmission, could be improved. For the same reason, the wear can also be reduced.

iii) The contact ratio larger than two can be easily obtained with negative modification coefficient. This is very important, especially when transmission smoothness and noise reduction are required.

iv) The problem of undercut and overly thin topland could be easily cured by using positive tooth thickness modification coefficient . Thus the severity of the above mentioned problems caused by the addendum modification with large coefficient can be reduced. Better modification results can be obtained through the proper selection of addendum modification coefficients and tooth thickness modification coefficients.

v) The machining of the modified bevel gears can be easily realized by using the existing general cutting machine tools and cutters. This is of great importance in the applications of this new method.

5.2 Recommendations For Future Works

In order to apply this method, the following work are necessary and may be considered for further investigation.

i) Since the design process is tedious, complex and time consuming, to improve it, computerization of the design process is necessary.

ii) Laboratory tests under different working condition are necessary to verify the theoretical results. Comparison of the economical effects can also be made.

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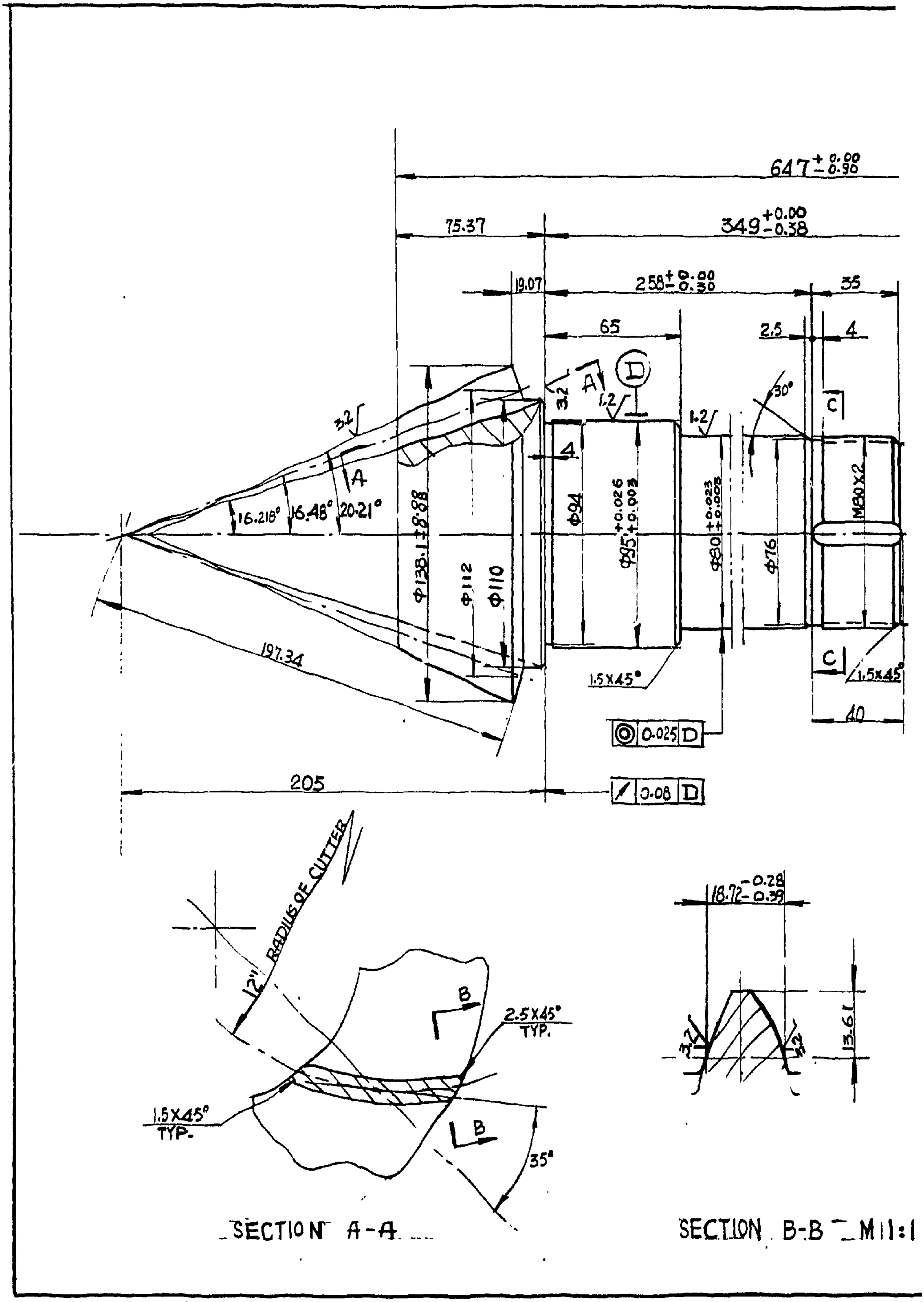
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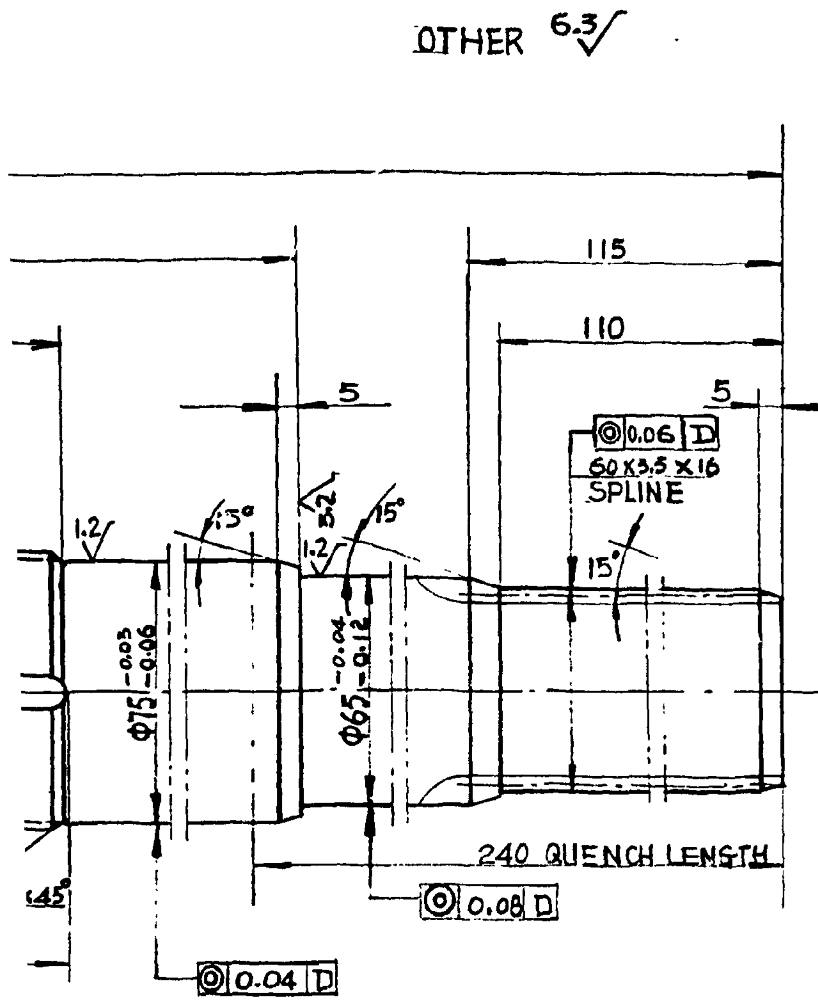
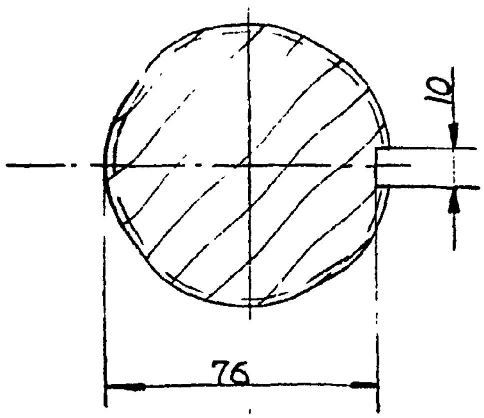
75.37
349 +0.00 −0.38
19.07
258 +0.00 −0.30
35
65
2.5
4
3.2
A
D
1.2
1.2
30°
C
4
A
16.218°
16.48°
20.21°
Φ138.1±8.88
Φ112
Φ110
Φ94
Φ95 +0.026 +0.003
Φ80 +0.023 +0.003
Φ76
M80X2
197.34
1.5X45°
0.025 D
C
1.5X45°
40
205
0.08 D
RADIUS OF CUTTER
12"
B
2.5X45° TYP.
1.5X45° TYP.
B
35°
18.72 −0.28 −0.39
3.2
3.2
13.61
SECTION A-A
SECTION B-B — M11:1



CHARACTERISTIC OF MESH		
1 Number of teeth	Z_1	14
2 Module	m	8
3 Tooth profile angle	α	20°
4 Addendum coefficient	h_a^*	0.85
5 Addendum Modification Coefficient	X_1	0.68
6 Tooth thickness Modification coef.	X_{t1}	0.065
7 Top clearence coef.	C^*	0.188
8 Tooth height	h	16.48
9 Spiral angle	β_m	35°
10 Spiral direction	right	
11 Accuracy grade	ISO 9-8-8 EF	
12 Pitch error accu.	$\delta_{t\Sigma}$	0.28
13 Max. circular runout of outer diameter	δ_{ej}	0.2
14 Error of pitch	δ_t	0.045
15 Drawing No. of engagement component	27S01-1	



SECTION C-C.

NOTES

1 The surface of teeth and spline is carbonized for 1.2~1.8mm deep approximately.

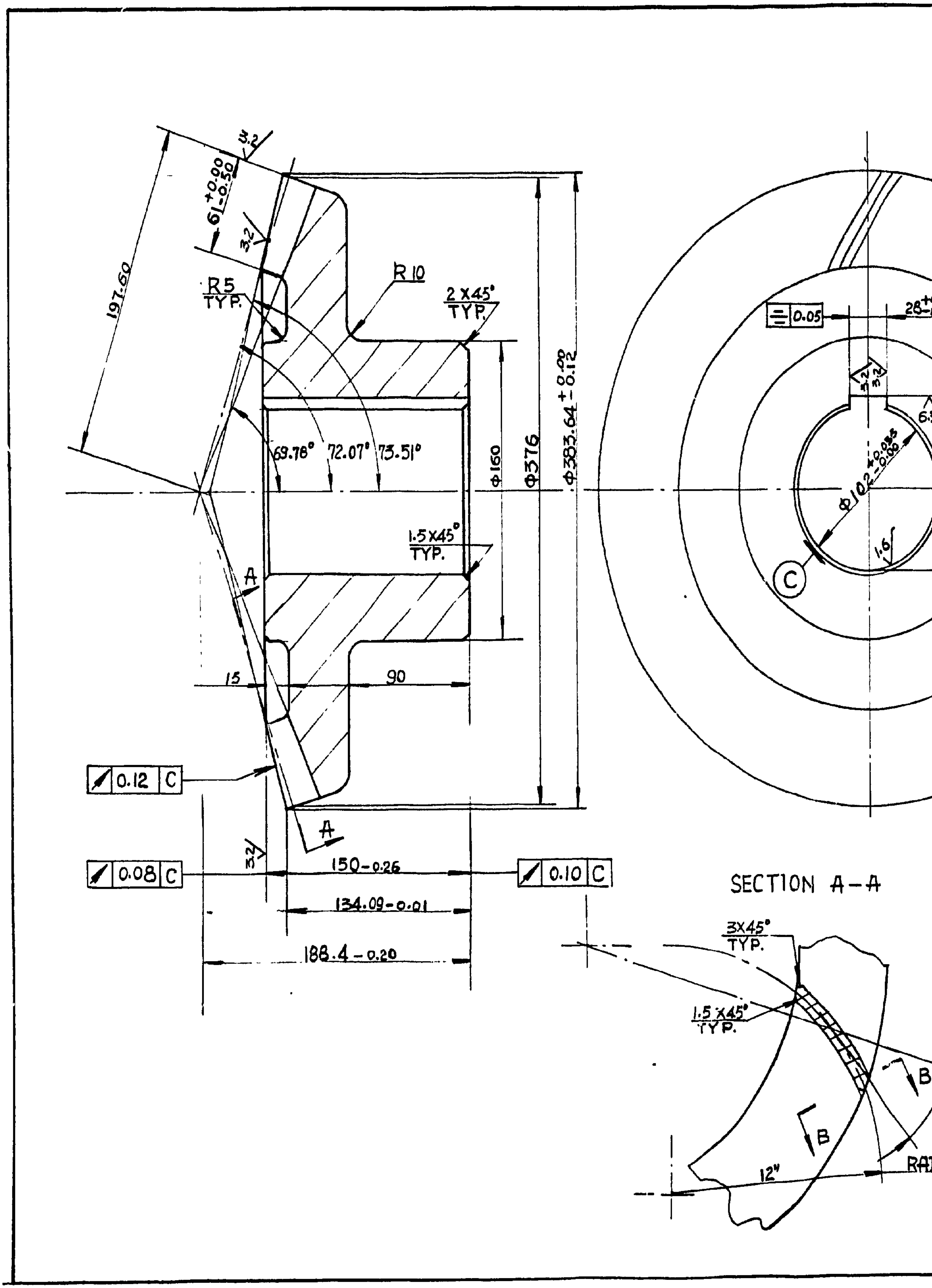
2 Quench hardness
Surface of teeth: RC 56~62
Surface of spline: RC 30~42

3 Cutting Method
Gleason simple duplex method

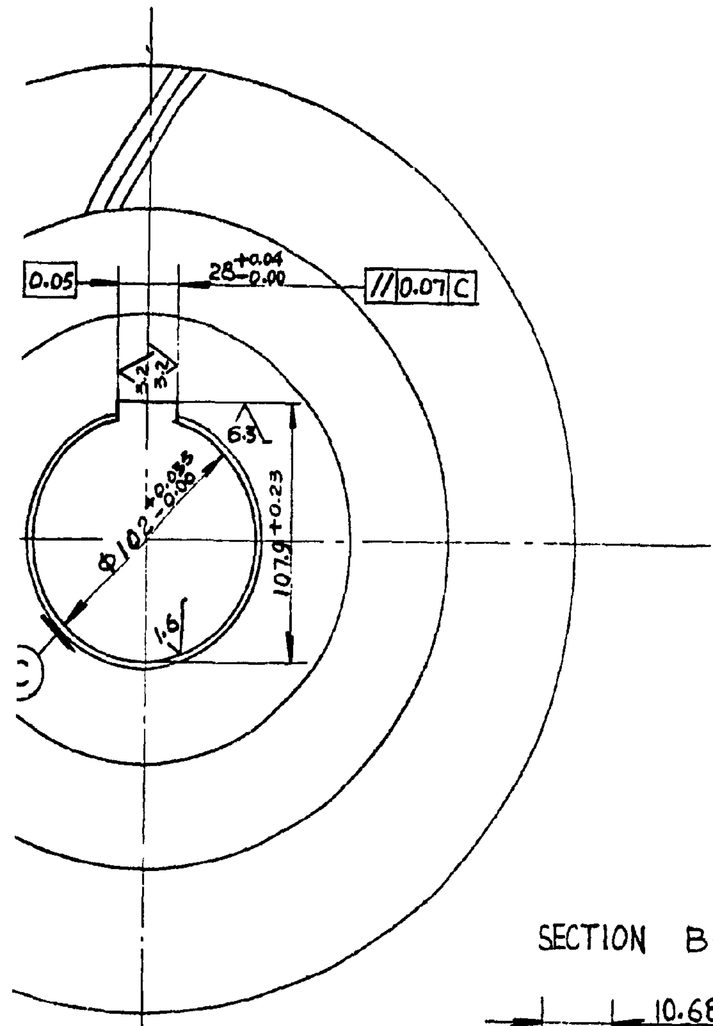
Fig. 4.4

CONCORDIA UNIVERSITY DEPARTMENT OF MECHANICAL ENGINEERING					
Drawing by Checked by	X.L	Gearbox of S.G.W. - 250 Conveyor Spiral Bevel Pinion			
Data	July 1989	Scale	1:2	Drawing No.	27S01-1

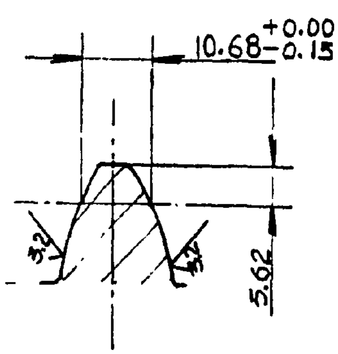
1:1



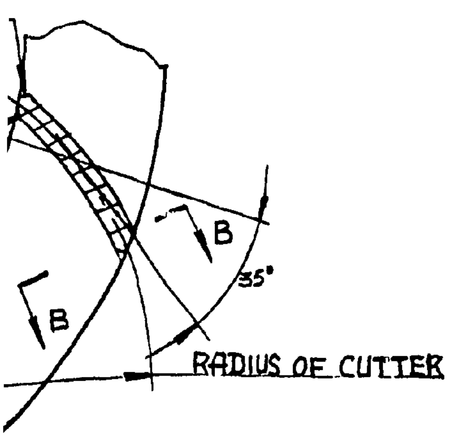
3.2
61 +0.00 -0.50
197.60
R5 TYP.
R 10
2 X45° TYP.
69.78°
72.07°
73.51°
ϕ160
ϕ376
ϕ383.64 +0.00 -0.12
1.5X45° TYP.
A
15
90
0.12 C
0.08 C
0.10 C
150-0.26
134.09-0.01
188.4-0.20
0.05
C
1.6
SECTION A-A
3X45° TYP.
1.5X45° TYP.
B
12"



SECTION B-B M 1:1



ON A-A



CHARACTERISTIC OF MESH		
1 Number of teeth	Z_2	47
2 Module	m	8
3 Tooth profile angle	α	20°
4 Addendum coefficient	h_a^*	0.85
5 Addendum Modification Coefficient	X_2	− 0.32
6 Tooth thickness Modification coef.	X_{t2}	0.077
7 Tooth height	h	16.48
8 Spiral angle	β_m	35°
9 Spiral direction	left	
10 Accuracy grade	ISO 9-8-8 EF	
11 Pitch error accu.	$\delta_{t\Sigma}$	0.28
12 Max. circular runout of outer diameter	δ_{ej}	0.2
13 Error of pitch	δ_t	0.045
14 Drawing No. of engagement component	27S01 - 2	

NOTES

1 The surface of teeth is carbonized for 1.2 ~ 1.8mm deep approximately.

2 Quench hardness
Surface of teeth: RC 54 ~ 60

3 Cutting Method
Gleason simple duplex method

Fig. 4.5

CONCORDIA UNIVERSITY DEPARTMENT OF MECHANICAL ENGINEERING					
Drawing by Checked by	X.L	Gearbox of S.G.W. - 250 Conveyor Spiral Bevel Gear			
Data	July 1989	Scale	1:2	Drawing No.	27S01-2